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I am submitting herewith a dissertation written by Michael K. Smith entitled "The Evolving Structure of Statistical Knowledge in High and Low Math Anxious Students." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Psychology.

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THE EVOLVING STRUCTURE OF STATISTICAL KNOWLEDGE
IN HIGH AND LOW MATH ANXIOUS STUDENTS

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The completion of a dissertation is in many ways a coming of age, a time when part of one's life is over and another is about to begin. Who we were and who we are about to become has been casually affected by many, but only significantly influenced by a select few.

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ABSTRACT

The problems students encounter in learning statistics at a college level was the focus of the present investigation. To ascertain these difficulties, three major concerns that dealt with mathematics, probability, and statistics were explored: 1) The mathematics background of incoming students, particularly any type of "math anxiety"; 2) The intuitive/everyday conceptions of probability, particularly as emphasized by Piaget; and 3) The formal/cognitive structuring of statistical material learned in the actual course itself.

Twenty introductory psychological statistics students were selected for the study. Ten of these were HIGH MATH ANXIOUS and ten were LOW MATH ANXIOUS based on scores on the Mathematics Anxiety Rating Scale. Each of these twenty were given several instruments: an initial open-ended interview concerning the experience of mathematics; an everyday probability explanation task; weekly self-report sheets during the course; and a final interview that tapped experiences in the course, together with problem solving and problem sorting tasks.

Results indicated that "math anxiety" tended to have developed from and be centered on formal mathematics classrooms, as opposed to everyday uses of mathematics. Intuitive concepts of probability tended to be split between those concepts involving a correct notion of chance and irreversibility and those showing incorrect assumptions of determinism (such as the reversibility of probabilistic phenomena and the so-called gambler's fallacy). The tasks which measured formal learning

in the classroom revealed that students very often make intuitive, concrete mistakes in learning concepts; for example, a bimodal distribution was "seen" as two separate distributions instead of one. The integration of material clustered on two major categories, those of PROBABILITY AND HYPOTHESIS TESTING. In general, anxiety level showed no effect on performance.

Implications for classroom teaching include being aware of background anxieties student display about mathematics combined with explaining the nature of probability which sets it apart from other types of mathematics courses.

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CHAPTER I

REVIEW OF LITERATURE

Introduction

In the mid fourth century B.C., Plato, then an elderly academic at the University of Athens, a school which he had founded, was hired as a consultant to the King of Asia Minor. The King, having read Plato's latest bestseller The Republic, was inspired by the author's concept of the philosopher-king, an idea which appealed to him as an avenue to earthly and perhaps even immortal success. Feeling that he already had many regal virtues, as a consequence of his position, he needed only to acquire the tools of logic, particularly mathematics, upon which Plato had designed his ideal government. Thus, Plato was dispatched to the shores of Asia Minor, with a suitable per diem, to teach the King and his sons the elements of geometry. After two days of drawing trapezoids, parallelograms, and squares in the sand, learning the ins and outs of interior and exterior angles, and attempting to prove theorem after theorem, the King became so frustrated that he sent Plato packing. Vowing never to waste time on math again, the King went on to conquer the rest of Asia and certain parts of Greece.

Although this first recorded example of "math anxiety" may have no bearing on Western civilization, it does illustrate nicely the importance that mathematics was to play in Western culture (Kline 1953). Above the entrance to Plato's school was the dictum "Let no one enter

here who does not know Geometry" (Durant 1966); until this day, entrance to any major or minor college or university would be denied without a suitable background in mathematics. Thus, when we consider the problems that people have in learning mathematics and statistics, we must keep in mind the knowledge that mathematics has been considered an extremely important element of education. From the Pythagoreans and Plato, to Descartes and Newton, to the statistical sciences which inspired most of modern psychology, mathematics has not only been a tool for modelling the universe but the methodology for conceptions of human nature as well.

In examining the problems in learning mathematics, we must also consider the learning of mathematics partially as a subset of the psychology of learning. Thus, the issues of types of instruction, what is learned, and transfer of training which, in different ways, have dominated the concerns of behaviorists, gestaltists, and cognitivists, will have to be discussed. Furthermore, the issue of who controls the learning process--either the interested individual or the reluctantly shaped and reinforced student--will have to be considered also. We must also consider two features that may be relevant only to the discipline of mathematics itself--the content of mathematical material and the importance placed by society on the mathematical curriculum. In the former, the work of Piaget on the learning of mathematics and probability will be discussed, particularly his claims as to the unique logical structure of those disciplines and their relevance to thought

processes in general. In the latter, I will consider the issues of "math anxiety" and the motivational problems that a student faces in learning material that she/he is confronted with from elementary school through college. Only by considering the world of the learner and the material to be learned can we approach an understanding of the difficulties involved in the learning and the teaching of mathematics.

Drill vs. Insight: Thorndike vs. Wertheimer

For nearly as long as mathematics has been considered an important discipline for people to learn, there has been a history of efforts to teach it. Edward L. Thorndike, one of this century's leading learning theorists and educational psychologists, criticized the techniques behind the teaching of the "old math":

The faith in indiscriminate reasoning and drill was one aspect of the faith in general mental discipline, the value of mathematical thought for thought's sake and computation for computation's sake being itself so great that what you thought about and what you computed with were relatively unimportant (1926, p. 96).

Thorndike was criticizing the teaching of arithmetic and algebra which existed in educational curriculum in this country from the early 1880's. This approach emphasized the value of mathematical reasoning in its own right, as a subject whose rudiments could easily transfer to the learning of other disciplines; thus, algebra textbooks, for instance, could deal with "bogus" and "fantastic problems" because the content was not nearly as important as the general principle behind the content.

Thorndike did not believe in the transfer of learning which was the basis of the early mathematical curricula; instead, he argued for

a "functional" approach to the teaching of mathematics. In considering the topic of algebra, for instance, Thorndike proclaimed that "algebra is a useful subject, but its utility varies enormously." To prove this, he examined all the uses of algebra in encyclopedic articles, textbooks, and scholarly articles in various disciplines. He concluded that only a small percentage of the varied techniques taught in high school classes were actually used in practical applications.

To teach algebra successfully, then, became a matter of emphasizing the practicality of the material to be taught.

It has been customary to select and arrange the problem materials almost wholly from the point of view of the algebraic techniques to be applied. . . . From the functional point of view, emphasizing ability to use algebra in solving problems which life will offer, it seems desirable to consider the lives of boys and girls and men and women as students, citizens, fathers and mothers, lawyers, doctors, business men or nurses, and select problems which they may usefully solve and which are properly solved by algebraic methods (p. 109).

This functionalistic philosophy produced a number of implications for the construction of algebraic problems and texts: 1) Real life problems should be used as much as possible. "Solving problems in school is for the sake of problem solving in life" (p. 154). 2) Problems should be solved as originals, i.e., not in reduced form or involving merely the use of a routine. 3) Pupils should "feel some need for a procedure and some purpose in learning it before they learn it" (pp. 152-3). Sections of the textbook should be divided according to disciplines, business, social science, science, etc., instead of by algebraic techniques; the principles and problems to be solved may

have to be relearned in each particular area, for the transfer of technique may not hold across different areas.

Not only should problems be important but they should be designed to teach what is fundamental and important to algebra. Thorndike devised lists of phenomena essential to algebra, through examinations of textbooks and talks with teachers. Instead of endless chapters of unrelated exercises, Thorndike felt that algebra could be grouped into a few basic principles, including the notion of symbolism, ability to read formulas, ability to evaluate and solve formulas, first in one unknown and then in two or more, and, finally, the ability to read graphs (statistical knowledge). Things to be avoided included elaborate computations, unreal or useless problems, unnecessary vocabulary, undesirable terms and definitions, superfluous connections, and shortcuts.

To teach each of these various techniques, and to insure their strength and use after the course ended, Thorndike emphasized the formation of "bonds," i.e., the learning of specific rules and places to apply these rules. Teachers should set up a particular formula, show its usefulness, establish any symbolism, variables, or abstractions involved in it, and then let the student work problems. Drill was very important for learning and should be allotted time depending on the importance of each topic. Important principles should have more time spent on them; earlier texts had allotted equal time to all techniques, a fact that Thorndike found appalling. Spaced drill should be included;

once a principle is introduced, practice on it should continue at various intervals of the course. In this, Thorndike is arguing from the psychological literature on massed vs. distributed practice, with the belief that the best learning occurs when students are retested at various intervals on the different techniques.

Thorndike strongly emphasized the formation of bonds or habits. He felt this to be the most essential part of the learning process:

Learning to compute algebraically is not only, or chiefly, learning rules and how to apply them; it is also building up a hierarchy of habits or connections or bonds which clarify, reinforce and, in part, create the understanding of what the rules mean and when to apply them (p. 246).

How about the learning of general principles which supercede habits?

"It so often happens that the really effective principle is the product of habits, not their producer. . . . Principles are not, as a rule, general but rather are limited to the fields where they have had habitual operation" (p. 245). The learning of fixed rules and bonds is so important that the teaching of shortcuts can often be detrimental:

"Every such rule which a pupil finds that he can break (as he can all such) without getting wrong answers means a risk that he will lose respect for and confidence in the really imperative rules" (p. 239).

To teach effectively the entire discipline of algebra, an instructor must build all the necessary bonds, step by step, with appropriate practice, to insure the student's efficient use of techniques. Thorndike presented evidence, that, at the end of a year of algebraic drill, easy bonds, such as $3x + 5x = 8x$ could be done 99 times out of 100 in $2\frac{1}{2}$

seconds or less. Achievement was emphasized, along with the fact that good bonds could produce a quick response from students. Thorndike was insistent that all bonds to be acquired should be thoroughly practiced, including changing the form of the bond. For example, he argued that "any disturbance whatsoever in the concrete particulars reasoned about will interfere somewhat with the reasoning, making it less correct or slower or both" (p. 458). To show this experimentally, he instructed students on forming the square of $(x + y)$ and the factors of $(x^2 - y^2)$. Then he presented them with the problem of squaring $(b_1 + b_2)$ and factoring $(1/x^2 - 1/y^2)$. Although, in principle, the latter two are exactly the same as the former two, Thorndike reported that, of 97 graduate students, error rates increased 6% to 41%. The "mind is ruled by habit throughout" (p. 458) and to insure transfer of learning one might have to teach each bond separately.

The theory behind Thorndike's techniques for learning mathematics was criticized and has continued to be criticized for its simplistic reliance on bond formation and not on the learning of general principles or even the meanings behind the bonds. The student is seen as someone needing step by step help in the acquisition of material and the transfer of learning is minimal at best. The success of Thorndike's own programs is difficult to determine, as are all subsequent programs (to be discussed later). Measuring success in the learning of algebra acquires a massive amount of time for the number of bonds which must be formed; thus, Thorndike generalized from simple laboratory results to the broader domain of the classroom.

The criticism that meaning was ignored in Thorndike's learning paradigm was raised early by Brownell (1928) and later by Wertheimer (in his 1945 book of collected articles). Wertheimer, representing the Gestalt school which was to present a formidable challenge to Thorndike's functionalism and other American psychologies of the time, lamented of the changed factors problem that "What kind of education had they received, that such slight changes, which in fact did not at all touch the character of the problem, should cause so much trouble?" (1959, pp. 153-4). He felt that students in Thorndike's classes were learning by rote, and not productively or by understanding the general principles behind procedures. In his classic work, he referred to two types of responses, A and B, which guided reasoning. A response of the B type would learn the factors of $(x^2 - y^2)$ by rote and then be troubled by the changed conditions of $(1/x^2 - 1/y^2)$; an A response would immediately see that the latter is only a slight variation of the former. As Wertheimer suggests, "we must differentiate, and very strongly so, between blind associations, blind habits, blind experiences, on the one hand, and the actuality of thinking, of grasping the inner relation between operations and their reasonable results" (p. 167). Someone who has learned the principle behind factoring should be able to tell immediately that an equation like $(a^2 + 2ab - b^2) = ?$ is impossible while $(a^2 - 2ab + b^2)$ is just a variation of $(a^2 + 2ab + b^2)$! In this way, a student will be able to transfer his training much more easily or use productive thinking as Wertheimer suggested instead of being bound by blind habit.

In Wertheimer's scheme, the most effective learning comes from an understanding of the inner structure of problems, or by "insight" into their fundamental nature. This can best be accomplished by letting the student "do the task, to let him find the necessary steps, but in a reasonable way, by proceeding from structurally easy tasks to structurally more difficult ones . . ." (p. 164). Wertheimer explored this in a number of investigations from studies of children solving simple parallelogram problems to famous mathematicians and scientists, such as Gauss and Einstein, in their discovery of new theories.

Wertheimer pointed out two traditional types of learning which he felt were more limited than his own productive thinking: traditional logic and classical associationism. "Some psychologists would hold that a person is able to think, is intelligent, when he can carry out the operations of traditional logic correctly and easily" (1945, p. 6). The operations Wertheimer had in mind included definition, analysis, abstraction, generalization, inferential and syllogistic reasoning--in short, the processes of the logician or the geometer (cf. Plato or Euclid). Contrasted with this is the associationistic approach, where thinking is the product of the contiguous succession of ideas ruled by habit, past experience, or chance. Key elements in this approach include bonds, frequency, recency, trial and error learning--in short, the classic Thorndikian position. In opposition to both theories, Wertheimer advanced a theory of productive thinking, a theory in which the learner is viewed not as a logic machine nor as susceptible to the

random winds of past associations, but one in which the learner is actively seeking the inner relation of material. To understand a subject matter means being able to handle it, to use it, in all its different transformations (cf. the factors problem). This recognition of structural features and transformations leads to an understanding that is not piecemeal but wholistic. For transfer of learning to occur, the learner cannot be taught effectively either deductively or habitually but only productively; that is, where he/she gains insight into the structural qualities of the domain. Wertheimer emphasized the structure of material, a fact which existed for him in algebra as well as geometry. We do have a reemphasis on the value of the domain of math itself, a fact which Thorndike had criticized; for Wertheimer math offers the possibility "to work in a marvellously pure way, by getting directly at the inner consistency of materials and operations."

Harry Harlow (1949) would later argue, however, that learning by insight could perhaps be explained by people or animals "learning how to learn." In a series of experiments using discrimination problems with monkeys, "learning sets" seemed to emerge; i.e., the rewarded object, despite being located in varying positions, could be recognized more quickly after dozens of trials rather than one or two. As such, Harlow argued that "insight" may be merely a "learning how to learn," with learning facilitated, as Thorndike believed, with appropriate trial and error procedures.

This clash between bond formation/rote learning and productive thinking, between habitual drill and discovery of rules, between math

as practical and math as pure, has continued throughout the twentieth century and troubles us today. On the one hand (in the followers of Thorndike, and influenced by Skinner and the behaviorist notion of reinforcement) there are many attempts to reform curriculum and to test classroom conditions to determine the most effective factors in the learning process. On the other hand, there are the repeated counter attempts to show that drill, behavioristic methods such as programmed learning, and often impractical curriculum were not the most effective manner of teaching mathematics. The former will be concerned with the best environment for learning or formation of bonds; the latter will be concerned with the most effective way of having the student learn the inner relations of mathematics. In the next sections, the recent literature surrounding both sides of the argument will be reviewed; in both cases, we will see an inability to discern even reasonably what is the most effective manner of teaching mathematics.

The "New Math": Curriculum Reform of the 50's and 60's

The dissatisfaction with the rote learning/drill method of instruction, first voiced by Brownell and Wertheimer, was to continue into the 1950's, when educators and psychologists began to revise theoretical notions associated with the learning process. At the same time, an historical phenomena occurred which, if it did not necessarily improve the teaching of mathematics, at least spurred millions of dollars of government research into it.

On October 4, 1957, the Russians launched Sputnik and all at once, Americans viewed themselves as behind in the space race, in the race for world superiority. After the disastrous launch of our own Vanguard I in December (an event which the press sarcastically labeled "Kaputnik"), government officials and citizens were even more concerned with the future of Americans in science and engineering and our ability to catch up with the Russians. How could this have happened and what could be done to prevent it?

Social and political movements and causes can spur interest and/or research into psychological problems; from the early Sputnik to the ghetto children of the sixties, the feminists of the seventies, or the back to basics we are witnessing today. In the late fifties and early sixties, this new concern with the teaching of science and particularly mathematics led to two international conferences of scientists and educators on the goals of mathematics: one at Woods Hole, Massachusetts, in 1959 and the famous Cambridge Report from Cambridge, Massachusetts, in 1963 (see Dessart and Frandsen 1973; Resnick and Ford 1981). Both groups called for a restructuring of the mathematics curriculum as the panacea for learning problems; children should be learning organized concepts and not drill in specific and arbitrary material. As Bruner remarked in 1960 (see Resnick and Ford, p. 104), in a statement that was nearly the opposite of the Thorndikian approach,

The curriculum of a subject should be determined by the most fundamental understanding that can be achieved of the underlying principles that give structure to that subject. Teaching

specific topics or skills without making clear their context in the broader fundamental structure of a field is uneconomical in several deep senses. In the first place, such teaching makes it exceedingly difficult for the student to generalize from what he has learned to what he will encounter later. In the second place, learning that has fallen short of a grasp of general principles has little reward in terms of intellectual excitement. . . . Third, knowledge one has acquired without sufficient structure to tie it together is knowledge that is likely to be forgotten.

From these conferences and other interests at the time, a revolution in the study of mathematics occurred including the introduction of concept-oriented mathematics (the New Math), discovery learning, and programmed learning.

Dessart and Frandsen reviewed the impact, techniques, and success of these different programs for the teaching of mathematics (1973). In a major five-year study, conducted by the Minnesota National Laboratory, on the effectiveness of the new curricula versus conventional methods for teaching mathematics, Rosenbloom and Ryan (1968) showed that, on all major dependent measures of mathematics ability, experimental programs did not significantly increase or decrease student performance. In a follow-up study by Ryan (1968), based on students' attitudes, interest, and perceptions of proficiency, there was once again no major difference between old and new methods.

The effects of another major curriculum innovation, that of programmed learning, a technique not originally devised by Skinner but enlarged and put to use by him, also showed no unequivocal differences on performance between PSI (as it came to be called) and conventional curriculum (see Dessart and Frandsen 1973). Other behavioral variables,

aside from curriculum, were considered to be important in the teaching and learning of mathematics. The design of the efficient classroom, in the Skinnerian mode, should allow for the control of all important reinforcement contingencies. As Dessart and Frandsen (1973), however, noted hardly any behavioral variables produced unequivocal results in the teaching of mathematics: from homogeneous (students of the same ability) to heterogeneous classes; from small to large classes; from team teaching to single teaching; from assigning homework to not assigning homework; from the use of teaching machines, programmed learning textbooks, to computers. In all cases, no single method or combination of methods produced clear statistical results as to their effectiveness in raising students achievement in mathematics.

Another curriculum change, paralleling and challenging the behavioral changes and promoting a different conception of student learning, was that of "discovery learning." Its essential aim was to restructure mathematics by devising curricula such that students could "discover" rules and concepts instead of being taught them explicitly or by rote. In a major review of the effectiveness of discovery learning, Shulman and Keislar (1966) remark that "there is no evidence that supports the proposition that having students encounter a series of examples of a generalization and then requiring them to induce the rule is superior to teaching the rule first and asking the students to apply it to a wide variety of examples" (p. 191). Furthermore, they comment on the general strategy of classroom teaching of mathematics:

In any event, two conclusions for classroom instruction are clear: (1) There is no useful way of posing a broad question regarding how much and what kind of guidance the teacher should apply. The question should always be formulated for a specific context including the type of subject, the maturity of the pupils, prior learning experiences, and so on. (2) Regardless of which way of talking about learning by discovery is adopted, no single teaching method is likely to accomplish the wide range of cognitive and affective objectives discussed at the conference (p. 186).

Despite these gloomy predictions for the learning and teaching of mathematics, we should remember that this pessimism is perhaps more shared by researchers and educators than by students. During the decade of the sixties and early seventies, scores on ETS sponsored nationwide tests continued to rise and to serve as a ruler against which future and current students would be judged. Thus, in spite of learning controversies, students of the early Space Age continued to learn mathematics.

The "New Mind": The Cognitive Revolution

Although Gestalt psychologists had emphasized mental qualities such as insight, American concerns with "mind" and "consciousness" came much later. The cognitive revolution of the sixties and seventies, spurred on not only by an interest in Piaget's work but also by the use of computers as a model for human thinking and reasoning, explored and reassessed many topics in the learning of mathematics. Mathematics provided a very appropriate model for use on the computer; the logic, discreteness, and well-formedness of mathematical problems provided the tools for simulation and testing of theories about the learning of mathematics. Furthermore, the reappraisal would be influenced by

critiques of past research; Thorndike had emphasized the formation of bonds but had done little research on which bonds were easy to acquire, which difficult, and why; Wertheimer had explored errors that people make in certain types of geometry problems but his investigations had been limited; and the decade of curriculum reform in the sixties had emphasized a restructuring of the content of mathematics but the assessment of effectiveness had always been in terms of global scores on "math" tests, with little concern for specific problems in specific areas for specific learners.

Cognitive researchers sought to remedy the lack of knowledge about mathematics per se and individual learners; in Resnick and Ford's (1981) words, "It is this dual knowledge--knowledge of the structure of mathematics and knowledge of how people think, reason, and use their intellectual capabilities--that furnishes the ingredients for a psychology of mathematics" (p. 4). In studying mathematics, researchers would now attempt to offer theoretical models that could account for three factors: 1) correspondence--how much one's mental picture of a mathematical phenomena mapped onto the correct mathematical concept; 2) integration--how much interrelatedness there was between different mathematical concepts; and 3) connectedness--how much knowledge of one domain transferred to knowledge of other domains. Most of the work, so far, in cognitive psychology or in cognitive science has been with the first two concerns.

For example, Suppes and Groen (1967; also Groen and Parkman 1972) studied elementary school children's ability to solve simple addition

problems of the form, $2 + 7 = ?$. Using a reaction time methodology, in the belief that the cognitive steps a student would use in solving this problem could be factored out by noticing difference in time to solution, the authors postulated three models by which this problem could be solved. Model A stated that the addition would be carried out by starting at 0; add 2; then add 7 and report result. Model B stated that you start at 2; add 7; and report result. Model C claimed that a person started with the larger unit, in this case 7; and then added the smaller, or 2, and report the result. In studies using all possible combinations of two digits, the authors found that Model C provided the more accurate match with data. Students when confronted with a simple addition problem tend to start with the larger number and then add the smaller to it.

Lankford (1972) studied children's solution processes in addition, subtraction, multiplication, and division. From the use of thinking aloud protocol analysis, he claimed that the errors of most pupils are highly individualized but often systematic, i.e., they follow a logic unique to the student. For example, in multiplying 304 by 6 the answer written down was 1804, where the student failed to carry the 2. From talking to this student, however, a different logic emerged; he had multiplied 6 by 4 equals 24; written 4; carried 2; multiplied 0 by 2; written 0; and then 6 by 3 equals 18; for an answer of 1804. Here the problem was not a technical one in carrying, but one in knowing what to do with what is carried.

The lack of understanding of interrelatedness of concepts is argued for in the case of addition and subtraction by Greeno (1978; 1980) and by Resnick (1980; 1981). Greeno argues that, in theory, addition and subtraction are parallel processes; the same mental process should see that $m + n = mn$ can be reversed to produce $mn - n = m$. Students, however, take awhile to learn this. In Resnick's example, a student is given the subtraction problems of $36 - 27 = ?$ and $27 - 36 = ?$. In both cases, one child's algorithm for solution was, in the first column, subtract the large number from the smaller one, indicating clearly a misunderstanding of the process of subtraction. Resnick, however, was able to train a mental picture of what is involved through the use of Dienes blocks; in this way the child was able to see and form a model which represented subtraction as the inverse of addition.

Resnick (1980) also discussed the issue of transfer of training through the use of a problem first explored by Wertheimer, finding the area of a parallelogram. Wertheimer had shown that few students really grasp the essence of the parallelogram problem, that of transforming the figure to a rectangle and then solving. Resnick had students first learn to transform non-rectangular figures to rectangular ones, of all shapes and sizes, by using scissors and paper. They were also taught to measure areas through the use of small building blocks, a technique which can be used easily with squares and rectangles but not with an angular shape like a parallelogram. After such practice, the students were given the problem of finding the area of a parallelogram. Only

50% of the students invented the solution by themselves; the other 50% could only get it after the experimenter had suggested the use of transformation. Thus, in one sense, we could argue for Wertheimer's A-B distinction and the use of insight by some and not by others; in Resnick beliefs, however, students who did not get it simply did not have a connection between the two procedures of measurement and transformation.

Although research of this type has been extended to other mathematical domains (see Greeno 1978 for geometry; Parkman and Groen 1971 for simple numerical equations), its primary emphasis has been on simple phenomena and the representation of solution strategies within these simple phenomena. Errors are seen as systematic, if idiosyncratic; the lack of interrelatedness of concepts is seen as a lack of logical connectiveness between concepts, a factor which may be susceptible to training. Currently, the cognitive revolution is seen as the primary mover for research in the learning of mathematics; with its emphasis on the subject matter of mathematics, applied concerns, and individual learning processes, its aim, the psychology of mathematics for instruction, is seen by Resnick and Ford (1981) to be in the vanguard of this revolution (p. 253).

The applications and evaluations of this cognitive approach have yet to be tested. The emphasis on the "new mind" has, to this point, been with small concerns--simple problems of addition, subtraction, geometry, etc. The large-scale factors of motivation in learning and the issue of transfer of training have only begun to be addressed.

The "Piagetian Mind" I: On the Learning of Math and Logic

In any discussion of the learning of mathematics, it is almost impossible not to mention the work of Piaget. Piaget, in a number of works (1952, 1956, 1960, 1964), has argued for what he calls the logico-mathematical development of thinking in the child, such that the child progresses from his view of reality (which is prelogical and prenumerical) to the adult conception of number and logic. The development takes place in a series of stages, with each stage represented by a certain set of operations which the child performs on the environment. Thus, a major assumption behind the growth from sensorimotor to formal operations is the child's gradual working on or restructuring of himself to the environment or the environment to himself (the processes of accommodation and assimilation). The child cannot be shown, in most cases, the path that must be taken; instead

In the area of logico-mathematical structures, children have real understanding only of that which they invent themselves, and each time that we try to teach them something too quickly, we keep them from reinventing it themselves. Thus, there is no good reason to try to accelerate this development too much; the time which seems to be wasted in personal investigation is really gained in the construction of methods (quoted in Copeland, 1970, p. 22).

Thinking is the development of logic which precedes mathematics; only in mathematics and formal scientific reasoning do we reach the culmination of adult logico-mathematical processes. The present review will outline some of the major findings as they apply to the learning of mathematics (see also Copeland 1970, Flavell 1977 in addition to Piaget's works); then discuss their implications for teaching as

contrasted with the behavioral/cognitive strategies already outlined; and proceed finally to a lengthy discussion of one of Piaget's works which is usually not discussed in summaries of his findings--that of his lately translated book on The Origin of the Idea of Chance in Children (1975, written in 1951) which outlines the growth of the child's conception of probability and statistics, in many ways a formulation counter to the mathematics of algebra or geometry which he discussed in his other works.

Before any understanding of arithmetic operations can occur (such as addition, subtraction, multiplication, and division), certain logical antecedents must be developed first. The child must understand the notions of classification (the beginnings of concepts, words, and the properties that relate to these), seriation or ordering (that concepts can be grouped by relations, such as smallest to largest), and logical connectiveness (the operations which allow the reformulation of concepts, such as conjunction, disjunction, implicative structures, negations, and the use of symbolism, other than words). The absence of these properties are typical of the preoperational stage; in the concrete operational development the child has learned these operations, learned the idea of conservation of quantity and number, which implicitly suggest the notion of reversibility of operations. For example, take the operation of larger than, which is a prerequisite for addition: a child is presented with pictures of a lot of poppies, a small number of bluebells, and asked which is more? He replies, "poppies." Asked if a poppy is flower, and a bluebell is flower, then

which is more--the bunch of flowers or the bunch of poppies? A pre-operational child will reply in the latter indicating a lack of the overall concept of flowers and the relation larger than. Until this is acquired, in its logical and symbolic form, the child will never learn the fundamentals behind arithmetics. Using notation, the previous example becomes $P(\text{oppies}) + B(\text{bluebells}) = F(\text{lowers})$, which means $P = F - B$, which implies $P < F$ and $B < F$.

These logical notions must develop first before a child will be able to understand that the concepts of addition, subtraction, multiplication, and division are all logically related to one another, build on each other, and are logically reversible upon each other: subtraction is the reverse of addition, multiplication is simplified addition, and division is reverse multiplication. Before a child develops these concepts, all attempts at teaching will be mere exercises in verbal learning not mathematical thinking.

The development of geometric thinking also presupposes the logical development of certain constructs, particularly those involving spatial relationships. Furthermore, Piaget remarked that the

study of the child's discovery of spatial relationships--what may be called the child's spontaneous geometry--is no less rewarding than the investigation of his number concepts. A child's order of development in geometry seems to reverse the order of historical discovery (in Copeland, p. 147, 1970).

By the latter statement, Piaget meant that the child progresses from a topological view of space to a Euclidean one of lines, shapes, and angles. At first, many Euclidean concepts are not developed: Continuity and infinity (infinite number of points on a straight line),

measurement in one dimension (the shortest path between two points is a straight line), conservation of distance and length, and finally the notion of measurement in two dimensions, which develops the concept of Cartesian coordinates from thinking about maps. Only when the latter developed are we finally able to move to a full understanding of geometry in one plane and its later extension into projective geometry (including volumes of spheres, etc. and the notions of projections by shadows).

The preceding paragraphs, which presented a brief survey of major operations learned by children, are underlaid by two major suppositions: 1) The child must invent these operations for himself before they are truly understood and 2) Most children develop these on their own and come to a logico-mathematical view of the world by the period of the formal operational thinking in adolescence. As such, these assumptions have several implications for teaching and research. First, a teacher must not try to teach concepts before a child is ready for them. This implies an understanding of development and a willingness to structure the classroom environment such that materials are available to help the child discover and invent operations (see Copeland 1970 for the construction of a Piagetian mathematics laboratory). Secondly, research into development can be served best by studying the development of operations and perhaps by the errors that are made. In this sense, many of the cognitive psychology experiments on the child's learning of simple math (such as Resnick 1980; Greeno

1980) are relevant to, and seek to extend, Piaget's work. The notion of mental maps, and connections between concepts, is one in which he strongly believed. Finally, since the child invents these operations, it becomes fruitless to try and train him in these concepts or try to speed up his development (a finding which even recently is suggested in Resnick and Ford's review of mathematics learning). Piaget himself is probably the best, last word on the subject:

This question (of accelerating learning) never fails to amuse students and faculty in Geneva, for they regard it as typically American. Tell an American that a child develops certain ways of thinking at seven, and he immediately sets about to try to develop those same ways of thinking at six or even five years of age. Investigators in countries other than America have tried to accelerate the development of logical thinking, and we have available today a considerable body of research on what works and what doesn't work. Most of the research has not worked. It hasn't worked because experimenters have not paid attention to equilibrium theory. The researchers have tried to teach an answer, a particular response, rather than to develop operations. They have tried to teach the child that of course the hot dog (shaped piece of clay) will weigh as much as the clay ball; just put both on a two-pan balance and you'll see. But the child is completely unconvinced unless he shuffles the data around in his mind, using one or more of the operations I've described. Learning a fact by reinforcement does not in and of itself result in mental adaptation (in Copeland, 1970, pp. 20-21).

The "Piagetian Mind" II: On the Probabilistic Nature of the World

"Almost every common action seems, in fact, to require the notion of chance as well as a sort of spontaneous estimate of the more or less probable character of a feared or expected event," Piaget and Inhelder state in the opening of their book The Origin of the Idea of

Chance in Children (1975), a work which represents the only detailed work done on the acquisition of concepts of chance by either children or adults. If such an intuition is so widespread, it is only natural to ask if this ability is inborn or develops later. The aim of their book is to show that the adult concepts of chance and probability develop in a series of stages, much like logical operations progress through four stages. The adult conception of chance, which differs from that held by both the primitive mind and by the mind of the small child, runs contrary to two major types of causality, determinism and miracles.

On the one hand, it differs from purely mechanical determinism, whose spatial and temporal links in the ideal state are reversible, because probability implies the intervention of irreversible phenomena. On the other, by interference within causal series, probability excludes categorically the concept of miracles (p. xvi).

It assumes that the idea of random irreversible mixture does have laws; in fact, probability itself derives secondarily from a primary deterministic search for order and its causes, the prime characteristics of the beginnings of logical thought in concrete operations. The examination of Why questions, particularly ones that adults have trouble answering, provide an illustration. Why doesn't Lake Geneva go all the way to Berne? Why isn't there a spring in our garden? Why are you so tall and yet have small ears? In his search for order, the child has yet to understand that there are some circumstances to which there are no reasons, but that things are due to chance interferences in biology, geology, etc., and can only be explained probabilistically if at all.

It is only by abandoning the concept of miracle and the strong logical belief that everything has a clear cause that the child can come to believe in a probabilistic ordering, the idea that chance governs certain phenomena.

Piaget and Inhelder's work centers around three topics which they feel are crucial to the development of the concept of chance: 1) the notion of chance in physical reality, including the ideas of random mixture and irreversibility, centered and uniform distributions, and fortuitous uniform distributions; 2) the idea of random drawing and intuitions of probability, including chance vs. miracle in a game of heads and tails, random drawing of pairs, and the quantification of probabilities; and, finally, 3) combinative operations (which supposedly lie at the heart of chance), including conceptions of combination, permutation, and arrangement. Each of these major concepts is explored through simple experiments with children at one of three stages of development: preoperational, concrete operational, or formal operational. As with most of Piaget's work, the authors find clear differences on each of the concepts by stage. Since the major purpose of the present work lies more in the concepts of probability, some of the experimental techniques and major findings that Piaget and Inhelder report will be discussed here. These will be followed by a discussion of their conclusions in light of the previous work on the learning of mathematics already reviewed.

To test the notions of chance in physical reality and the concepts of random mixture, the authors first designed a tilt board. This

is a rectangular box, resting on a pivot which allows it to seesaw. In the box, eight white marbles are placed on one side, eight black on the other, separated by a divider. The child is asked what will happen when the box is tilted forward and then tilted back again. Preoperational children predict that the balls will mix, but in a certain order: they will follow definite paths, will crisscross (white to black side, e.g.), and will, more than likely, finally return to their starting position. Mixing is an accident or a momentary disturbance; not until formal operations will the child realize that the mixture of balls is most likely irreversible and, therefore, very unlikely to return to the original order.

To test an intuitive idea of distributions, the authors constructed a wooden inclined plane, with a certain number of slots at the bottom, and fed by a funnel at the top. Balls can be rolled down the plane through the funnel and then disperse at random to the slots in the bottom of the box. If there are two slots, then we should expect an equal distribution between the two. As the number of slots increase, given that the funnel opening is in the center of the plane, we should expect the balls to form a normal distribution, particularly as more and more balls are rolled (law of large numbers). If the funnel is to one side, then a skewed distribution should follow. Preoperational children feel that the distributions in the two slots case will be by turns (each slot has a turn); with three or more slots, all slots will be equal. Balls will either follow a principle of repetition (that

where the last one went) or compensation (it is the other slots turn); factors, however, which are the basis of the gambler's fallacy. Only in formal operations does the child have a notion of centered and symmetric distributions, along with chance collisions as the basis for this.

Finally, in the last test of chance ideas before preceding to probability, Piaget and Inhelder demonstrated the reasoning behind a constant random relationship in conflict with a fortuitous uniform distribution. They designed an aluminum disc, with a spinner in the middle, with the disc separated into different colored sections. When the spinner is spun, the chances of landing on any colored section are approximately equal and random. The authors, however, tricked the child--the spinner was magnetized and a magnet placed under one color so that it would show up very often. In the preoperational stage, a child feels that, in the first case, he can predict where the spinner will stop, either in an isolated instance or as a function of preceding spins. In the latter case, he feels that the repeated landing on one color is not strange and tries to explain it by such factors as spinning the spinner in the same way, with the same speed, etc. Only later does the concept of chance in spins occur and the notion that, in the latter case, something is "fixed."

In the second section of their book, Piaget and Inhelder are more concerned with the intuitive beginnings of more formal definitions of probability. To test this, they play games of chance, alternately

with little counters with crosses on one side and an X on the other, or with different colored marbles. In playing the simulation heads and tails game with counters, preoperational children exhibit once again a gambler's fallacy, i.e., what occurs on the next toss of the counter can be predicted from the previous toss. Furthermore, when the authors "trick" the children by inserting ten counters that are "fixed" (crosses on both sides) and then drop all of them, preoperational children are not surprised at the results--it could happen, even if it happens two or three times in a row. Formal operational children, however, catch onto the trick immediately and look at the counters. In an experiment where different numbers of different colored marbles are put in a sack and the child asked to predict the most probable outcome of drawing, the authors show a lack of systematic probability for young children. In a sack with six red marbles and two blue which is most likely--Red; six blue and two red--Red/ why? because I like red a lot. Finally, the learning of the specific quantification of probabilities (how likely is red or blue) also takes time to develop.

In the last section of their book, the authors discuss the development of three types of operations which underlie probabilistic thinking; those of combinations (AB, BC, AC), permutations (AB, BA, BC, CB, AC, CA), and arrangements (THE PRECEDING SIX PLUS AA, BB, CC). Children given colored chips to work with and told to construct these phenomena are not able to find systematic patterns for construction until formal operational thinking. Early stages exhibit fortuitious

combinations which, in many cases, leave out certain possibilities; only later does the child understand a systematic ordering of possibilities that allows for an errorfree, rapid construction of all possible alternatives.

The development of ideas of chance and probability proceed through various stages (with a logical development of their own) much as arithmetic or geometric thinking also progresses from an egocentric, non-conserving relationship among objects to a system that allows formal operations. Chance and probability, however, offer an interesting counterpoint to the usual systems of hypothetico-deductive logic (such as occur in physics and in other types of mathematics) which Piaget normally uses by way of demonstration for this level of thought. Whereas hypothetico-deductive operations admit of a determinism and a reversibility (deriving Archimedes principles, e.g.), the notions of chance have to admit of an indeterminism, the possibility of the fortuitious. In the preoperational stage, the child's thought "oscillates between the predictable and the unforeseen, but nothing for him is either predictable for certain (by which we mean deriving from necessity) or absolutely unforeseen (that is, fortuitious)" (p. 214). Only later does the child understand that certain events are completely fortuitious, and that certain phenomena, such as heads and tails, e.g., although not perfectly predictable, do have a certain ordering, a certain distribution to them.

This ordering, however, is not like the deterministic relationships the child is discovering in other domains.

The intervention of chance in the cognitive processes presents . . . a decisive importance concerning the limits of operative activity . . . it defines a kind of reality in which the whole is no longer equal to the sum of its parts, because of the lack of additive composition, but obeys its own laws which are laws of equilibrium sui generis (p. 240).

This notion that physical reality has a chance, indeterminate element as an essential component lends to logic and mathematics (and life) a less definite flavor, one which will have to vacillate between a complete understanding of phenomena and one that admits of failure, of a lack of specific knowledge. Growing up means that certain questions can be answered only probabilistically, while others may not be answerable at all. This is the stake probability strikes at the heart of empirical rational science, a lesson a child must learn, but even few adults accept.

Piaget and the Computer

An extremely interesting and soon to be very prevalent extension of both Piaget and the cognitive concern with simulation and artificial intelligence comes in the work of Seymour Papert (1980). Papert cautions against the way current society and educational systems not only produce a fear of mathematics but also a fear of learning in general. Children normally love to learn and they have to learn to have trouble with mathematics and other subjects, finally producing the "intelligent adults (who) turn into passive observers of their own incompetence" (p. 39). He argues against an educational curriculum that is outdated and based on notions of "school math," an abstract entity which a child learns by rote, which few adults understand and is based on pre-twentieth

century measuring instruments. (For example, why learn and deal with the equation $y = x^2$ which describes a parabola? The reason--parabolas are easily drawn with a string and a pencil.) Education does not allow children to invent concepts as Piaget claims; nor even provides materials for their discovery if the willingness of teachers is there.

Our education culture gives mathematics learners scarce resources for making sense of what they are learning. As a result our children are forced to follow the very worst model for learning mathematics. This is the model of rote learning, where material is treated as meaningless; it is a dissociated model (p. 47).

To counter these trends, Papert and his colleagues developed the concept of Turtle Geometry, a forerunner of the powerful Logo language, both of which are easily accessible to any home computer owner. The Turtle Geometry concept is that the operator develops his own Micro-World, with words as commands, words which the child has defined. By creating different types of complex shapes, the child learns about planning and designing a figure, about subgoals and their relationship to a whole, and about spontaneous geometry as Piaget used the term. As such Turtle Geometry has three principles, which Papert claims makes it a powerful alternative to traditional education: 1) the continuity principle: "The mathematics must be continuous with well-established personal knowledge from which it can inherit a sense of warmth and value as well as 'cognitive' competence," 2) the power principle: "It must empower the learner to perform personally meaningful projects that could not be done without it," and 3) the principle of cultural resonance: "The topic must make sense in terms of a larger social context." And in a final word:

I have spoken of Turtle geometry making sense to children. But it will not truly make sense to children unless it is accepted by adults too. A dignified mathematics for children cannot be something we permit ourselves to inflict on children, like unpleasant medicine, although we see no reason to take it ourselves (all p. 54).

The Math Learner as Non-Rational: The Issue of Anxiety

The preceding sections, from Thorndike through Wertheimer, to the curriculum reform of the sixties, to the cognitive psychology of today, have all shared an underlying assumption about learning, particularly about mathematics learning: that the learner is a rational, or at least manipulable machine, we might say, whose failure in mathematics occurs either because of weak connections, insufficient insight, or a lack of the proper logical mental processes. In all of this work there is little discussion of the learner as "emotional" or "anxious" or "nervous" about the learning of mathematics. In fact, in Resnick and Ford's (1981) survey of work on the psychology of mathematics for instruction there is not a single mention of the concept of "math anxiety" and its possible effects on the learning process. This concern with logical mental processes--which is the concern of the cognitivists--or the lack of relevance of inner mental processes to learning--which followed from the behavioristic theories of Thorndike, Watson, and Skinner--needs to be offset by at least a glance at the relationship of anxiety to performance.

The issue of anxiety and behavior is not new (Freud, 1966), nor the issue of anxiety and mathematics. In this century, Brown (1933)

noticed the anecdotal correlation between attitude and performance in a statistics class. Students would make comments at the end of the course such as "I'd learn just as much statistics and have a lot more fun if I spent these two hours on the golf links"; "I never before sat in a class in which the English language was spoken and understood so little of what was being said"; "I'm blinded by the statistical fireworks"; and "Believe me, if I get a mark in this course, I'll never take another" (all p. 61).

The concern with math anxiety increased in the early 1970's until now, partly due to the concern over the drop in mathematics scores on a national scale as reflected by evaluations on ETS sponsored testing. The effect of attitude on mathematics performance was labeled as "math anxiety" by Richardson and Suinn (1973), as "mathophobia" by Lazarus (1974), or as the "I can't syndrome" by Tobias (1978); in all three cases, the authors claim that math anxiety is to be differentiated from a general anxiety, a test anxiety, or a general school anxiety. Tobias (1978) even claims an inherent sex bias in math anxiety, with females accounting for more of the phenomena than males. She feels that the explanation for this is due to socialization factors; females are more often stereotyped as dumb in mathematics, incapable of complex calculations, as better in the kitchen than in a spaceship, etc. The general finding of better mathematics performance for males than for females is well substantiated in psychology along with the recent evidence for increased math anxiety among females (Tobias 1978). The explanation

for the phenomena varies from Tobias (1978) "enculturation hypothesis" to an inherent genetic difference as suggested by Benbow and Stanley (1980).

The relationship between anxiety and performance is more questionable. Kincaid and Austin-Martin (1981) report that for high anxious people, anxiety is unrelated to mathematics achievement, whereas for low anxious people there is a positive relationship. Smith and McIlwaine (1982), in a study of undergraduate psychology students in statistics classes, noticed that, for males, anxiety decreased with good performance in the course but that initially high levels of anxiety for females did not change with good performance in the course. Furthermore, math anxiety scores were a significantly better predictor of final grade for females than for males. Handler (1983) argued that scores on math anxiety tests (such as developed by Richardson and Suinn 1973; see also Suinn, Edie, Nicoletti, and Spinelli 1973) decreased as competence in mathematics increased. This occurred in a treatment group that received only mathematics instruction and also in a treatment group that received mathematics plus mathematical anxiety reduction exercises. Morris, Kellway, and Smith (1978) reported high correlations between mathematics anxiety scores and final grade performance for psychology majors but not for mathematics majors. Thus, it becomes unclear if mathematics anxiety is function of competence, of a real anxiety, of academic discipline, or a function of sex.

The relationship between anxiety and mathematics--or perhaps we should say the questionable relationship--now goes hand in hand with

the questionable relationship of learning theories and curriculum reform in the learning of mathematics. Although the cognitive revolution is attempting to iron out difficulties in the logical, conceptual learning of mathematics, this approach will never fully contribute to a theory of mathematical understanding unless some notion of anxiety mathematics is taken into account. This anxiety may stem from the fact that mathematics often takes on great importance in the life of an individual. From elementary school through high school and college and even into some graduate schools, the stress on mathematics and related subjects is a prerequisite or basis for successful completion. Thus, mathematics cannot be escaped or avoided, if desired, unless a student drops out of school. Mathematics is used as a basis of comparison between people or sexes; mathematics tests are part and parcel of admission to colleges and professional schools; and, although English and verbal scores share these conditions, mathematics has a much larger symbolic history in the generation of culture (Kline 1953). At the least, mathematics is a tiny shadow that follows every student; at the most it is a hurdle that many seek to go around and many others fail to jump. That this ubiquity would have an effect on learning seems undeniable; its extent, along with the problems of logical approaches to the learning of mathematics, will be discussed in the final section, where a general model for the learning of mathematics will be proposed.

General Discussion

I think it is clear, from the review of literature to this point, that the problems people have in learning mathematics are not nearly

as difficult as the problems researchers have in studying the learning of mathematics. From the Old Math to the New Math, from the concern with pure mathematics to practical mathematics, from rote learning to productive thinking, from teaching the child to having him/her invent or discover procedures, from math interest to math anxiety, all seem to play a role in influencing the learning of mathematics and no one, to this point, seems sufficient in itself as the best method. In this final section, I will try to show the strong and weak points of all the relevant literature so far surveyed, and then offer suggestions for the study of the learning of mathematics, including a hypothetical model (based on Wertheimer) for the learning process.

One overriding weakness of almost all prior literature is in the definition of what is being studied, namely, "mathematics." Whether it be defined as arithmetic, algebra, geometry, etc., mathematics is such a broad term composed of so many different subjects and materials that it becomes exceedingly difficult to test if all the concepts are sufficiently learned. This especially complicates any programs designed to test experimental strategies. In the curriculum reform movement of the sixties, for example, even if the concept of restructuring was valid, there remains the difficult problem of knowing if teachers were actually using the materials or using them in the way they were designed to be used. As such it makes evaluation and precise control of conditions difficult; especially since the programs would extend over a period of several months to a year or more. Trying to control children

or adolescents' learning for this long a period would seem outside the limits of scientific experimental designs.

Another weakness that most programs and evaluation procedures of this type share is what I call the "democratic fallacy," i.e., a belief that all children start at scratch, are taught the material in an even fashion, all practice and are interested equally, and that all learn equally. Individual variations are merely random error; the only major differences would then be due to the method of instruction. Most research proves that this is not the case; if treated equally there are no unequivocal differences in any of the literature as to which factors promote good learning. This may be a function of weak designs or weak definitions of variables; however, we should also consider here the arguments of both Wertheimer and Piaget. Both claim that in learning mathematics, which has a logical structure to it and is not mere verbal habits, a child must come to "see" relationships; the discovery or invention of concepts is not necessarily something that can be predicted or even controlled. The individual difference factor is a most important variable, as Piaget would say, and cannot be speeded up by behavioral strategies.

The individual as a "rational" learner of mathematics must be offset by the learner as "emotional," i.e., that the learning of mathematics has different values for each child, something for which experimental assessment programs do not control. The past history of dealing with mathematics, whether successful or unsuccessful, will

certainly affect the way a student looks at a new year: the new teacher and the new material will either be an enjoyable extension of previous learning or a foreboding continuation of something that is already anxiety provoking. From the value placed on mathematics by parents, teachers, and society, each individual learner approaches its study from his/her unique dealings with the subject.

To address fully this individual difference factor means that we have to understand the individual's interest (or motivation we might say). Piaget would claim that certain ways of looking at the world are going to develop naturally; however, even he feels that access to certain materials or certain types of question-answer sessions will help a child understand concepts more readily. For someone who is self-motivated, i.e., wants to learn something, there is almost no stopping them--from abstract mathematics, to Rubik's cube, to video games, etc., early adolescent children show a remarkable ability to learn the most complicated things. If the child does not have an "interest," then, at the very worst, the teacher will be able to teach him nothing (examples of apathy, disinterest, etc.), or the teacher will have to develop such an interest in the child.

It is in the developing of interest that I think we can tie together many of the procedures and concepts talked about to this point. The argument over pure vs. practical mathematics is not very interesting to a mathematician or someone who thoroughly enjoys and understands mathematics. To a student in high school or college, however, forced

to take a certain subject, the issue of "Why is it this important?" becomes significant. In trying to make the material "important" (and thus something more than verbal habits), we can group together various strategies: Thorndike's functionalism, the discovery learning of the sixties, Piaget's invention hypotheses, and Papert and the computer concern with realistic simulated worlds of thought. Other factors can influence interest to be sure: a favorite teacher, a pleasant classroom, etc., and these may be worthy of study for the learning of mathematics.

For something like mathematics, which must be studied from elementary school through college, the maintaining of an element of interest must be more personally directed and this, I also think, can be better controlled if a student can recognize the "Why of what he/she is doing." In answering the "Why" part we must also consider returning part of the control of the learning process back to the student.

Since mathematics stays with the person for the length of schooling and since mathematics, in general, builds on previously learned material, it is essential that a student try to learn material fully at each step. In one sense this is not entirely possible, because the introduction of new concepts changes how one deals with old concepts. The concern with insight, productive thinking, discovery, invention, and the mental maps of the cognitivists are all arguing for an integrated knowledge of the material and the processes leading to such integration. The way mathematics is normally presented in classes is by fiat; this is what happens when you do such and such.

Mathematical invention, however, does not seem to happen in this way (Poincare 1948). Talks with professional mathematicians may alleviate myths people have about how math is done (Handler 1983) to make them realize that the mathematics they now study was in itself invented and that new mathematics that is worked on is not discovered immediately.

Papert and his idea of computer simulation of worlds, I believe, will help in the learning the rational, integrated part of mathematics. The computer is a powerful tool which allows a student to construct things by himself or herself, to show in a sophisticated way certain mathematical phenomena, and to keep track of individual performance, thus alleviating the democratic fallacy. This is only the first step and must be accompanied by what Papert further states: that the learning of mathematics has to have value for the student and not just be something forced upon him by adults. For people to learn mathematics well they must see value in learning mathematics; this can only come from an interest which is centered on growth rather than an "uninterest" focused on punishment or anxiety. Even adults have mythic conceptions about the underlying importance of mathematics, ideas which derive from the past and not the present.

We know today that mathematics does not possess the qualities that in the past earned for it universal respect and admiration. Mathematics was regarded as the acme of exact reasoning, a body of truths in itself, and the truth about the design of nature. How man came to the realization that these values are false and just what our present understanding is constitute the major themes [which now need to be addressed] (Kline, 1980, preface).

This concern with value and interest in mathematics is particularly relevant in adolescence. Past the peak of Piagetian formal operations, the adolescent is confronted with the types of mathematics that may impede or propel him in later career decisions: from algebra I and II, to geometry, to trigonometry and calculus the nature of mathematics changes into a more formalistic, logically rigorous and abstract exercise that builds on itself year by year. At a time in life when he/she is learning that everything can be questioned and experimented with, is a time when adults want them to question the least. The "Why" of learning algebra can become combined with the "Why" of adult rules, and the answer may produce resistance or acceptance of both.

Starting then, at the formal operational stage with the learning of algebra, I would like to propose a model of learning, based on Wertheimer, which combines several key elements already discussed. Presented in Figure I, the view assumes that learning precedes from initially blind rote or verbal learning processes, to one of limited logical inferences and of a more integrated knowledge of how concepts are connected. Finally, learning achieves the level of productive thinking, in which a student recognizes a concept no matter what the particular transformation. This process of learning is one that progresses from rote habit to creative insight. (A slightly similar version of this model has been presented by Gagne (1965), who proposed that learning progressed from simple stimulus-response learning to the more complicated processes of concept formation and problem solving. However, Gagne saw a continuity in processes in the different types of learning; the present

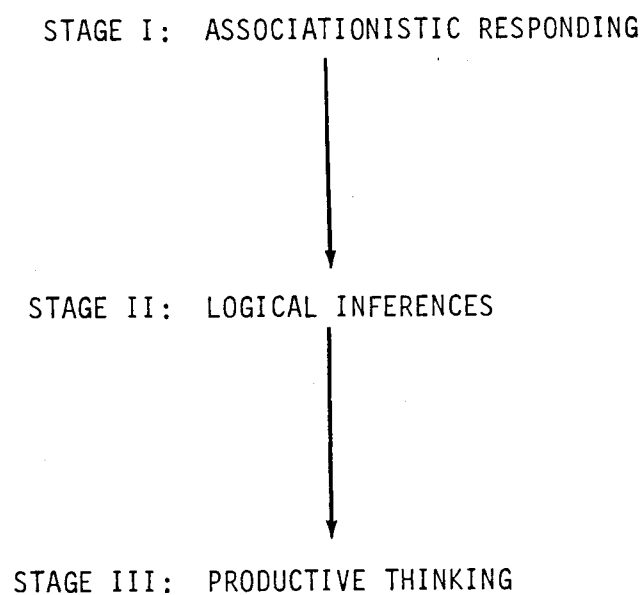


Figure I. Model of Learning Process.

model suggests more of a discontinuity, with productive thinking qualitatively different from associationistic responding.) In the initial phase, a confused or anxious student or a time-pressured competent one, may seek out whatever connections will make the grade; in the second stage, the rules and principles behind blocks of material will be recognized; in the third stage, the structures which give rise to the rules will be discerned. Initially the student may ask "Why?" and replies to his own question with, "Who cares. It doesn't make sense but I'll learn it for the grade." In latter stages the "Why?" is answered by "Yes, I see the relationship." Progress through all three stages can only occur if interest and value promote and guide learning. Different college age students start at different stages; different material is learned to various levels. There is little question, however, than few students advance through all three levels; in fact, we might argue, based on the failing math standards today and the way our educational environment is structured, that most students are at the first or, at most, second level.

At the heart of all these discussions, then, is in replacing value in the mathematics curriculum. Adolescents and other students must see value in what they are learning if they are to progress from associationistic responding to productive thinking. Any study of mathematics learning, therefore, must examine the value that students place on the material they are being required to master. Only then can the problems in the actual material be placed in the perspective of its perceived usefulness. This combination of value and content

must guide any investigation; any model of learning that is ultimately formulated will have to incorporate both considerations.

CHAPTER II

STATEMENT OF PROBLEM AND METHODS

Statement of Problem

How can we study the problems people have in learning statistics? How do these problems relate to the problems in learning mathematics, in general, and in teaching in both disciplines? In the preceding chapter, a number of general themes have been indicated that must be considered in studying the learning of mathematics and/or the acquisition of skills in general. First, we have to understand the world of the learner, i.e., the importance that "math" or "statistics" has in the life of the person. Secondly, as Piaget recommends, we need to understand the intuitive, almost unreflected, knowledge of statistical concepts a person develops while growing up. These can reflect either the Piagetian stages and their transitions or, more simply, everyday concepts of probability and/or statistics a person would be exposed to during development. Third, as both the Gestalt psychologists and the Cognitive Scientists argue, we must be concerned with the acquisition of skill itself, i.e., how the person comes to use and represent concepts which underlie the formal domain of statistics. In this component, we should strive to understand how a person "sees" a statistical concept and how this "seeing" changes (if at all) as the person progresses from a novice to a skilled user (or, at least, to a non-novice stage). This last aspect suggests a need to understand that learning

itself, especially of a multifaceted domain, will take place across time and may be learned in various stages. Only by combining these separate strands--the Phenomenal, the Piagetian, and the Gestalt/Cognitive--can we properly understand the phenomenon of the way in which statistics is learned.

To understand the world of mathematics for the person, or the meaning that mathematics plays in his/her life, means trying to explore personal interest and/or anxiety related to mathematics. Practically every person in our culture is exposed to learning mathematics as a basic skill, from first grade through college. For some, mathematics becomes focal as an interest and a career choice, i.e., those selecting mathematics as a vocation or a very strongly math-related discipline. For others, mathematics becomes focal as something that hinders the choice of career, something to be avoided in school, or, at the least, something that always produces the feeling that "I am just not good at it." The relationship between anxiety and performance is somewhat uncertain from the experimental literature (Handler 1983; Smith and McIlwaine 1981); the general distaste for mathematics itself which can manifest itself in a math-anxious person might lead to an avoidance of the topic, to the extent of taking courses or choosing careers which do not involve it. In considering the learning of statistics--a topic which is required of psychology majors, and of other disciplines as well--we must consider the problem of how learners confront the course initially. What kinds of expectations, beliefs, attitudes, anxieties, etc., do learners bring with them at the beginning of a course?

Part of this task can be accomplished through rating scales that attempt to capture the person's feelings about mathematics. The Mathematics Anxiety Rating Scale (MARS), developed by Suinn (1972), attempts to capture feelings of anxiety related to mathematics events from both everyday life and the classroom: examples range from "adding two numbers in your head" to "taking a math quiz." In psychometric evaluations of the MARS scale, Suinn et al. (1973) report that the scale does not intercorrelate with other measures of test anxiety, indicating that math anxiety may, in fact, be a separate phenomenon.

This scale is limited in that it does not fully capture what phenomenologists would call the "first person" experiences of the person and how they feel about their math background. To supplement these rating scales, a more phenomenological inquiry, developed along lines suggested by Colaizzi (1973) and Dapkus (1983), will be used. The purpose of this instrument will be to assess autobiographical information about mathematics and general circumstances and feelings the person has associated with mathematics and mathematics classes, factors a person will bring with him/her at the start of a new course. In more general terms, a phenomenological approach seeks to capture the first person experiences of a person or at least that person's own reflections on his/her experiences (Colaizzi 1973; Giorgi 1970; Pollio 1982). It attempts to view a topic such as mathematics, e.g., as a "figure" against the "ground" of a person's life; at certain times in a person's life mathematics has become important such that now a certain general "feeling" or "experience" of mathematics can be reported to an interviewer.

These self-reports contain cues not only to the experience of mathematics for that person but also, when tabulated across individuals, may reveal patterns for the more general experience of mathematics. As such, phenomenological methods seek to uncover unreflected aspects of a person's experience whereas rating scales can at best only validate what an experimenter already considers important.

Since the focus in this study is on statistics and probability, and not just on mathematics in general, an attempt must be made to describe the person's understanding of statistical concepts before he or she starts a formal course in statistics. For people who are psychology majors (in fact, for most college level juniors and seniors in non-natural science areas), previous exposure to formal treatments of probability and statistics are probably rare. As Piaget has reminded us, the notions of probability underlie some of the ways we look at everyday phenomena. Thus, it seems important to tap these notions independently of formal course work. To accomplish this, two different types of procedures will be used. First, using material from the Piaget and Inhelder (1975) book on the origin of chance, several of the Piagetian games will be administered to each subject. These games are designed to tap formal operational thinking of probabilistic concepts, from notions of random mixture in nature, to intuitive probabilities in the game of heads and tails, to patterns in combinations. As such, each person will be measured for intuitive notions of probability before he or she is exposed to formal considerations in a

classroom. Secondly, common sense notions of probability and sampling can be tapped by administering a test of explanation for everyday probability concepts. By having subjects try to explain statements such as "A 40 percent chance of rain tomorrow" to "Studies show that smoking may be related to cancer" we can tap understandings of probability that a person would be exposed to as part of growing up in this culture. Whereas a phenomenological approach assesses the experience of subjects as they see it, these games may be able to capture understandings which they might not be able to report directly.

Once the person starts learning the actual formal material of statistics, it is important to study the process of learning phenomenally. The learner must not be viewed as in a laboratory, dispassionately reacting to arbitrary stimuli and making logical, unemotional responses in the course of learning. Instead, learning must be considered as a part of the self-world of the person, i.e., where the learner experiences changes in what is learned and how he or she feels about what is learned. In this respect, first-person statements of what has been learned or experienced must be incorporated in the investigation. Throughout the quarter, each subject was asked to fill out sheets reporting on what they found easy to learn, difficult to learn, and their general attitude toward material. At the end of the quarter, another interview questionnaire, designed along the lines of the phenomenological questionnaire used initially, was administered to look for changes in the experience of the world of mathematics for the person.

We must also consider this learning as a part of the acquisition of skill, as both Gestalt psychologists and Cognitive Scientists have argued. In this case, each subject is learning material for which there already is a more skillful person present, i.e., the teacher. Secondly, the nature of the material itself is such that concepts build on each other in a supposedly logical fashion. For this reason, it was necessary to assess how well each person progressed toward mastery of material. To do this, more formal measures of evaluation were given at the end of the quarter, including a look at final grades, a problem solving experiment (in which a person was presented with easy to difficult problems designed to assess how much "insight" into material occurred) and a problem sorting experiment (to assess how each person structured the domain of statistics).

These measures, in toto, should seem to provide a more accurate picture of the experience of learning statistics for the person. From intuitive, everyday concepts and background math experience, to stages in learning throughout the quarter, to a final assessment of experience and skill at the end of the quarter, a person's progress in the acquisition of skill and knowledge has been assessed, a progression which not only was designed to tap formal knowledge but also the experience of learning the material as seen from the first-person perspective of the learner.

Methods

Subjects. Students enrolled in Psychology 3150, Introductory Statistics, lecture section, taught by the Experimenter, were given extra credit for participating in this study. At the beginning of the course, each student was given the Mathematics Anxiety Rating Scale (MARS), developed by Suinn (see Appendix A-1). On the basis of scores on this scale, 20 students were selected for further participation on the basis of the following criteria: 10 students scoring the highest on the scale were designated as the HIGH ANXIOUS MATH GROUP (hereafter HI ANXIOUS), 10 students scoring the lowest on the scale were designated the LOW ANXIOUS MATH GROUP (hereafter LO ANXIOUS). When a specific student declined to participate the next highest or lowest person was chosen until there were 10 in each group.

Initial interview materials. After selection of both groups all participants, during the first week and a half of the quarter, were given a tape-recorded 1½ hour interview conducted by the Experimenter. The Experimenter was blind as to which students were in which group as student assistants scored the MARS and selected the students for each group. Interview times, however, were arranged by the Experimenter.

The initial interview consisted of three components:

1. An open-ended interview designed to assess the "experience of mathematics" (see Appendix A-2).

2. Explanation of Everyday Probability Statements (see Appendix A-3). This was designed to assess a commonsense understanding of frequent statements using probability and/or statistical notions.
3. A test of Formal Operations as related to concepts of probability (see Appendix A-4). This task was designed to measure the degree of formal operational thinking about ideas of chance as outlined in Piaget and Inhelder's book The Origin of the Idea of Chance in Children (1975).

Week-by-week materials. Both groups of students were asked to fill out a self-report experience sheet at least two times a week (see Appendix A-5). Given 9 weeks of the quarter, after the initial week of selection, there were approximately 18 self-report sheets per student. Extra credit was made contingent upon completion of these sheets so as to insure a high return rate.

The purpose of the sheets was to ascertain the difficulties students were having in learning the specific content of statistics including concepts which they felt were difficult to master; concepts which were not difficult; and concepts which had to be looked up from previous lessons. In this way, two factors were assessed: 1) how students integrated and/or segmented the material of statistics in the learning process and 2) to determine if HI ANXIOUS differed from LO ANXIOUS people in the speed and nature of their acquisition of statistical concepts.

Final interview materials. During the last week of the quarter, a final interview was conducted with all 20 students. The interview consisted of 4 components, the final 2 of which were tape-recorded. The five components included:

1. A readministration of the MARS scale to assess attitudes toward mathematics at the end of the course.
2. A problem sorting experiment, to assess the degree of integration of the concepts covered in the entire course (see Appendix A-6). A total of 50 statistical problems was selected, 5 each from the 10 different domains of statistics taught in the quarter. Students were asked to sort problems in categories, such that each category represented problems having a similar statistical concept.

The following two measures were tape-recorded:

3. A Final Interview about attitudes and experience in the course (see Appendix A-7). Modeled on the initial interview, this instrument sought to investigate the experience of mathematics during the quarter, while the student had been involved in an actual mathematics related class.
4. A Verbal Problem session was conducted to test the depth of understanding of statistical concepts without the aid of notes or textbook. This task consisted of three separate components: 1) a test of knowledge about distributions and measures of central tendency; 2) a test of understanding

about formal ideas of binomial probability; and 3) a test of comprehension of formal notions of chi-square and hypothesis testing (see Appendixes A8-A11 for specific instructions and examples of test materials).

Scoring

This section will discuss the various scoring procedures for each of the major instruments used in the present study. The method of obtaining reliability, together with reliability estimates, will also be presented.

Initial Interview

MARS questionnaire. All 98 of the items composing the scale were rated on a 1 to 5 scale, with 1 representing "not at all anxious" and 5 standing for "very much anxious." All single items were analyzed, along with a total MARS score. This total score was a summation of all scores on all 98 items for each person.

The MARS was also subdivided into two scales, one representing items related to an EVERYDAY use of mathematics, another composed of items on a mathematics CLASS usage. To obtain reliability on this dimension, two independent judges rated each of 98 items on the EVERYDAY/CLASS distinction. Judges were in complete agreement on 82 of the 98 items, for a reliability percentage of 84%. The remaining 16 items were classified into one of the two categories after discussion among the two raters and the investigator.

Initial open-ended interview. All 20 interviews were transcribed and then read by three independent judges who were asked to score on two major areas: 1) to record specific answers to the questions that had been asked; and 2) to note general themes or patterns which they felt had emerged that could describe the experience of math anxiety or the lack of it. These general themes were then collapsed across the three judges to form four major dimensions: Reactions Specific to the Mathematics Class, Descriptions of Mathematics Itself, Past Experiences in Mathematics, and Other People's Attitudes Towards Mathematics. All of these themes had emerged from all independent judges; other minor themes that occurred were combined into one of these four categories.

After this inductive content analysis had been completed, all 20 interviews were rescored by two other raters. These raters were instructed to record specific statements by each of the subjects that belonged in each of the four major categories. These word-for-word responses were then analyzed for major patterns by the investigator.

Everyday Probability Statements

All 20 explanations for each statement were sorted into different categories independently by two judges: the experimenter and a senior level psychology/statistics undergraduate major. The procedure consisted of having the undergraduate, who was blind to which condition each subject was in, list all occurrences to each question. The statements were then randomly listed on a separate page for judgment by both

parties. In this way, the experimenter was also blind to which statement belonged to which subject, given this procedure and the time between the initial interviews and the present analysis. Agreement scores ranged from 80 to 95%. Disagreements as to category placements were resolved after independent classification. All classifications were made to determine the number of different categories of answers for each statement, with the judges bearing in mind the intuitive notions of probability behind each statement.

Piagetian Experiments

In most cases, subjects were rated as to whether or not they possessed the formal operational concept as presented by Piaget and Inhelder (1975). Scoring, then, consisted mainly of a correct/incorrect dichotomy. In all exercises, judgments were made by two independent judges (as in the previous sections), with agreement rates for coding of protocols varying between 80 to 100%.

Weekly Self-Reports

Each of the ten weeks of the course subjects were enrolled in concerned different statistical concepts. For each week, subjects were asked to note the number of homework problems/examples or in-class problems/examples that had proven to be difficult for them to understand. The number of these difficulties were summed for each student for each group for each of the ten weeks.

Final Interview

MARS and final open-ended interview. Both procedures were scored in the same way as in the initial interview. The final open-ended interview, however, produced different themes than those of the initial interview.

Problem solving exercises. There were three problem solving exercises; these involved measures of central tendency, binomial probability, and chi-square. For the test of knowledge about central tendency, subjects were asked to locate the mean, mode, and median for five different distributions. Scoring was as follows: 3--all three correct; 2--two correct; 1--one correct; 0--none correct. The binomial probability task and the chi-square task were both scored for two variables: Recognition of the table and Problem Solving using the table. Both variables were coded as either Correct or Incorrect.

Problem sorting exercise. In this exercise, each subject sorted the 50 problems into as many different categories as they felt appropriate. Analysis consisted of constructing a 50x50 grid and cross-tabulating, for each subject, the problems which they had sorted into similar piles. For example, if a pile contained the problems 5, 10, 15, and 20 the grid would reflect all possible pairs: 5 with 10, 15, 20; 10 additionally with 15, 20; and 15 with 20. In this way, it could be empirically determined how often subjects coded problems as belonging together.

CHAPTER III

RESULTS

Subjects

Of the 20 people selected to participate in this study on the basis of their MARS score, 15 were female and 5 were male, with the distribution for the High/Low groups being, respectively, 7 females, 3 males and 8 females, 2 males. This ratio of approximately two and a half or more, females to males, is slightly higher than the general sex distribution of psychology majors due perhaps to the influx of female students from other majors such as nursing, communications, speech pathology, who are required to take Psychological Statistics 3150. Only one female low subject dropped the course and thus did not complete all experimental material.

Initial MARS Results

Each MARS test item is rated on a 1 to 5 scale, with 1 representing "little anxiety" and 5 "very much anxiety." Given 98 items, the theoretically possible minimum and maximum scores are 98 and 490, respectively.

The high group had an initial mean of 267.7 on the MARS with a standard deviation of 42.0; the low group had an initial mean of 169.3, with an s of 17.9; as expected, these differences were significant ($t = 7.4$, $p < .05$). Furthermore, these averages correspond to

averages for social science and psychology majors as reported in prior studies (Brush 1978; Morris, Kellaway, and Smith 1978).

To evaluate differences among the 98 items, t-tests were conducted on all 98 items, based on the high-low dichotomy. Results of this analysis indicated that high and low groups differed on 53 of the 98 items, with the High Group having significantly higher scores on all 53 items. Appendix B, Table 10, presents the 53 items on which the two groups differ, along with the respective high-low means for each item. Appendix B, Table 11, presents the remaining 45 items on which the two groups did not differ significantly.

The items in both the significant and the non-significant sets were also scored on a dimension distinguishing between an everyday, concrete use of mathematics, numbers, or percentages (hereafter called the EVERYDAY dimension), such as are used in computing sales tax, bank accounts, bets, etc., and a class-related use of mathematics (hereafter called the CLASS dimension), including performance in class on tests, or studying or thinking about formal mathematics outside of class. Of the 98 items on the MARS scale, a total of 52 were coded as EVERYDAY and 46 were coded as CLASS.

To determine how the high and low groups differed on this EVERYDAY/CLASS dimension, the 53 items from Appendix B, Table 10, on which the two groups differed, were split into the EVERYDAY/CLASS distinction. Non-significant items were also divided in the same way. If the dimension of EVERYDAY/CLASS is not important as contributing to general math anxiety, then we should expect an equal division of items. Table 1

Table 1. Number of Items on Which High and Low Groups Differ and Don't Differ on the EVERYDAY/CLASS Dimension on the Pre-Test MARS

	CLASS	EVERYDAY
Differ	41	12
Don't Differ	5	40

presents the results of this analysis in which it can clearly be seen that an equal split has not occurred: the two groups did not differ on 40 of the 45 EVERYDAY items, while they did differ on 41 of the 53 CLASS items ($\chi^2 = 40.96$, $df = 1$, $p < .01$). These results clearly indicate that math anxiety for the high group, as measured by the MARS scale, stems from classroom math situations and not from the applications of mathematics to everyday life.

Phenomenological Interviews

This section will summarize the responses and the patterns that emerged from an analysis of initial interviews, first for the high anxious students and then for the low anxious group. Portions of a sample interview are included in Appendix C. As a context for these descriptions, it must be remembered that the statistics course in which all subjects were enrolled is a required course, one essential to graduation in their separate majors. In the sections below, responses in quotation marks should be considered as representing the words of

subjects, with slight paraphrases and condensations. In most cases, the statements in quotations are in response to direct questions of the experimenter. In some cases, which are noted below, responses are condensations of answers to two or more questions; in others, subjects talked freely without specific elicitation and responses were then grouped into the more specific categories to which they applied.

High Anxious Patterns

Reactions specific to the mathematics class. Responses to the initial question "How did you feel about taking statistics before the course started?" produced similar responses in this group: "petrified," "dread," "real frightened," "anxiety," "a block against math," "put off as long as possible," "worried," or "hesitant." In general, feelings of discomfort about approaching this particular class manifested themselves very quickly in the interviews.

Responses to the question "Describe the experience of being in a math class" in general displayed the same type of uncomfortableness: "utter confusion in class, freeze on tests"; "always uncomfortable in math classes"; "dislike math . . . really anxious on tests"; "uncomfortable in class . . . don't want to be there"; "apprehensive in class . . . not sure about tests"; "confusion . . . frustration"; "hate math . . . intimidates me"; "scary"; "confusion . . . panic"; or "too time consuming." Many students also commented on a "fear of failure" in a math class.

Descriptions of mathematics itself. In response to the question "How are math classes different from/similar to other subjects," many students reported that math classes were different from other subjects in that mathematics was more cut and dried than subjects like English or history, where more opinions could be expressed. Furthermore, many students described the content of mathematics itself as abstract or impractical. Typical comments included, "it's like puzzles but formulas are too abstract . . . it's cut and dried . . . get this right or you don't get it at all"; "puzzles but nothing practical"; "ambiguous . . . hard to plug in formulas . . . terrific if they work out . . . but awful if they don't"; "only like applied stuff"; "have to get it exactly right . . . make one mistake and it throws everything off . . . formulas bother me and not everyday math"; "orderly . . . anxieties about story problems and theorems"; "right or wrong . . . either understand it or you don't"; "logical . . . abstract thinking . . . if you can do it, kind of fun . . . if not real frustrated"; "intimidating . . . complex formulas"; and "math all numbers . . . not practical . . . math language."

In response to the question "In your daily life, when are you most aware of mathematics," subjects generally reported that they had no problem with practical applications of mathematics or mathematics they encountered at work. Whether the math centered on cashiering, bank accounts, accounting at work, or even dividing up dinner checks, all subjects reported that these kind of tasks presented no difficulty or that when a mistake was encountered it could be quickly corrected.

Past experiences in mathematics. In response to the question "What previous math courses have you taken?" and the follow-up probe "Tell me about times when you were comfortable/uncomfortable in math class," all 10 subjects reported at least one past event centered around a mathematics classroom in which feelings of embarrassment, humiliation, or failure became focal. Obviously, these may not be "exact" memories; however, they still are memories that now have become a part of the individual's perception of mathematics classes. The experiences included events occurring from second grade through college.

Typical responses were "In second grade, the teacher said 'find page 343' and I didn't know how to do it"; "had rheumatic fever in ninth grade . . . had to get a tutor who didn't teach me anything . . . got into class and didn't know what I was doing"; "in third grade, had rote drills on multiplication . . . finish a page and run up in front of class . . . slowest to finish are the last ones up front . . . teacher made fun of me . . . first time I got a bad grade"; "algebra II teacher came into class high . . . got on my nerves . . . tried to argue with her but she just stared me down . . . turned me off to math"; "ninth grade algebra . . . teacher would throw things up there . . . said 'What is your problem?' . . . I said, 'Why does this have to be so perfect?' . . . just barely got a C . . . traumatic"; "tenth grade anxieties about algebra word problems . . . hate to put things on board"; "fourth grade . . . had trouble with division tables . . . parents went crazy about it"; "fifth grade fractions . . . cheated

on homework . . . couldn't do it later . . . algebra II . . . got a D . . . hated teacher . . . embarrassed about board work"; "sixth grade fractions . . . had to get a tutor . . . real embarrassing"; "bad teacher in college . . . said to hell with it."

Other people. In response to the question "How does your experience in math compare to others?" 8 subjects responded that their feelings of fear or anxiety were similar to other people's (most people are like this), 2 felt that others were better than them, creating feelings of envy.

Curiously, in response to the question, "How would you feel if you were worse than anyone else in math?" all 10 basically reported that "I am," indicating, it seems, extreme feelings of inferiority about the relationship of self to others in this domain. When asked the opposite question, about feelings if you were "better than anyone else," responses were varied: "good but not willing to work that hard"; "fantastic"; "go to engineering school"; "wouldn't bother me . . . I like being on top"; "go into engineering or computer science"; "feel good, more intelligent"; "good feeling"; "smug"; "sympathetic to others"; "no different cause I'm not."

In response to the question about taking future mathematics courses, all 10 said no more mathematics was planned unless perhaps it was required of some graduate program they might enter.

Taking statistics is like. Responses to the question "Taking statistics is like . . ." included "cleaning the kitchen floor, it has

to be done"; "taking castor oil"; "pulling a tooth that hurts . . . once it's over it feels good"; "math but it's applied"; "paying your dues"; "walking off the edge of a cliff"; "taking chemistry, can't do it either"; "pain in the ass"; "taking statistics"; with one no answer.

In summary, then, it seems clear that the experience of mathematics for high anxious students is somewhat set apart from other subjects. Mathematics is a more fearful, dreaded experience, particularly within the confines of an actual class. Here, the student's self-world is potentially subjected to a number of threats, from embarrassment and humiliation to a time pressure that tends severely to reduce performance. Mathematics is seen as a perfect, logical exercise and within this frame the self is an always imperfect, almost illogical being forced to exist in a world not of its choosing.

Low Anxious Patterns

Reactions specific to the mathematics class. In response to the question "How did you feel about statistics before the course started?" patterns for the low anxious group were not quite as fearful or panicky as for the high group. However, with some students there still was no love lost for mathematics or statistics, and for two others there was an initial dread only because they had a bad experience in a previous course in statistics. Thus, initial reactions to statistics ranged from "thought I would get it over early"; "didn't think much about it"; "a little apprehensive"; "had to take it to graduate"; "had to be better than my previous class"; "panicky because I didn't know anything about

it"; to one "scared to death, because you hear people talk about this class" and another "pure unadulterated terror."

In response to the question "Describe the experience of being in a math class," descriptions were in general much more positive, in the sense that mathematics class was not seen as particularly uncomfortable. Some report that they liked mathematics, did well in it, or took it with friends, thus making it more interesting. One said that you had to concentrate more in mathematics, one didn't particularly like to go to class but was comfortable when she got there, and another was only bored. Thus, math classes are a more accepted or even enjoyed experience for high than for low math anxious students. Some comments included "I remember doing well . . . I liked people looking to be for help"; "I generally liked math . . . I thought it was interesting and fun"; "math wasn't terrifying . . . I always took it with friends so it was just another class"; "I like math but a lot of people don't"; "I was always reluctant to go to math class but felt comfortable once I got there."

Descriptions of mathematics itself. In response to the question "How are math classes different from/similar to other classes," most low anxious students reported mathematics as being no different from other subjects they had taken. Some felt that mathematics might be a little drier, but others reported on the fun of it. Some typical comments included "Math was fun . . . the abstract things and geometrical designs were like art"; "Math is a little drier than other classes

. . . it can be silly but I tolerate it"; "It really depends on the teacher . . . if they're interesting, it's fun"; "I loved algebra and geometry . . . people thought I was crazy." In general, mathematics was just not differentiated from other classes as it was for the high anxious students.

Past experiences in mathematics. In response to the question "Tell me about times when you were comfortable/uncomfortable in math class," only one person reported a specific memory, such as was evidenced for the high anxiety group. This even occurred in first grade and centered around a fear of failure in the classroom. Two other students reported bad experiences in a previous statistics course, but this centered on the apparent difficulty and ambiguity of the course. All others reported no disastrous events in the past and generally felt comfortable in mathematics class most of the time.

Other people. In response to the question "How does your experience in mathematics compare to other people," students often reported that others were afraid of mathematics, or apprehensive about it, that others were bored and anxious to get out of class, or that others don't pay attention in class. Some typical comments included "Other people seem to be afraid of math . . . I'm not, though I don't particularly look forward to it"; "Most people kind of panic"; "Others don't seem to pay attention"; "Others seemed to have trouble in class"; "Others always seemed anxious to go or about to fall asleep"; "Other people look more frustrated"; and "Others don't pay attention, they get lost."

The self, then, in general, is not anchored at a low point in relation to other students. Answers to the question, "worse in math," revealed a fear of actually sinking to this level. Typical responses included "I would feel like shit I don't want to be worse"; "'what's wrong,' how did I mess up"; "not so good, don't like to be bad at anything"; "terrible, just the pits"; "embarrassed"; "depressed, let down"; "wouldn't upset me." Only two answered like the high group in the sense of "I am." Responses to the opposite question, "better than others," were also "good" as for the high group, indicating that these people still do not feel as if they are better than anyone else, nor do they feel they are worse.

Taking statistics is like. Responses to the question "Taking statistics is like . . .," included "a challenge, a great challenge"; "any math class I've had"; "kind of learning a new language"; "any other math class except geometry"; "research"; "learning all over again . . . starts simple, gets hard, then is simple again"; "any other math class"; "Oh shit, here we go"; and "jumping into an ice cold pond." There was one no response.

In summary, then, for the low anxious people, mathematics is often seen as just another class, to be enjoyed or tolerated as the particular course or teacher dictates. Mathematics has not become the focus of any particularly bad experiences; in fact, mathematics class may even be a place to enjoy ones self or to enjoy others.

Explanation of Everyday Probability Statements

The purpose of this task is to assess an understanding of intuitive notions of probability encountered in statements, phrases, or slogans that an adult might have to deal with in life, via television, newspapers, magazines, or radios. These notions of probability and related topics preceded more formal considerations which the student encountered in a classroom.

Each student was presented orally with 5 statements, one at a time, and then asked, after each item, to explain in his or her own words what each statement meant and from where in the statement the numbers (if any) were derived. Each of the five statements produced 20 explanations; 10 for each member of the high anxious group and 10 for each member of the low group. The categories derived will be discussed for each statement individually, although collectively the data are presented in Table 2.

Statement 1

The first statement to explain was "I heard on the radio that there was a 40 percent change of rain tomorrow." This statement sought an explanation for the calculation of the 40 percent figure, a number which, in real life, is determined empirically by the Weather Service calculating how many days in the past with the same environment conditions produced rain. As such, some notion of empirical calculation of probability was being queried.

Table 2. Everyday Probability Explanations

	High	Low
Statement 1. 40% Chance of Rain		
A. Don't Know	1	3
B. Slightly Aware	5	6
C. Empirical	4	1
Statement 2. 3 out of 4 Doctors Surveyed		
A. Sampling Bias	9	10
B. No Sampling Bias	1	0
Statement 3. 7% Risk of Breast Cancer		
A. Know Something	3	5
B. Don't Know	7	5
Statement 4. Smoking May be Related to Cancer		
A. Know Something	9	5
B. Don't Know	1	5
Statement 5. Tennessee a 7 Point Favorite		
A. Specific Calculation/Don't Know	9	7
B. Relative Calculation	1	3

Three categories emerged from the inductive classification procedure. In the DON'T KNOW category, responses such as "pull it out of their hats"; "no idea how they got it"; or "60% chance it won't rain" would be given. A SLIGHTLY AWARE category contained responses that indicated that, in some way, calculations based on some type of observations would be used: "observe fronts"; "get it from surrounding fronts"; "from weather machines." A third category, EMPIRICAL, contained answers which expressed a certain amount of understanding of

the probabilistic nature of such calculations: "use data from past, under certain conditions rain will be produced"; "same conditions on another day"; or "what happened before that would predict rain."

From Table 2 we can see that the modal coding for both the High and Low groups was the Slightly Aware category. More high anxious students, however, responded Empirical than the low group. (The chi-square was non-significant; chi-square = 2.89, $df = 2$, $p > .05$.) In general, though, fewer than half of the students in each group had any notion of the probabilistic nature of the statement.

Statement 2

The second statement was "3 out of 4 doctors surveyed preferred Trident sugarless gums for their patients." This statement explored the concept of sampling and sampling legitimacy, i.e., when numbers such as "3 out of 4" are used to what kind of sample are they being referred. The idea of a random versus a non-random sample (perhaps only 4 doctors were used or doctors that were friends of the company) was also investigated.

As can be seen from Table 2, essentially everyone explained this proposition in the same way. For example: "Trident company probably interviewed doctors of their own employees"; "where'd they get the doctors?"; or "I want to know the population . . . how many surveyed." Only one person was unable to give any explanation of the statement. The chi-square value was obviously non-significant (chi-square = 1.05, $df = 1$, $p > .05$).

Statement 3

Statement 3 tested once again for the notion of sampling and sampling size but also employed a conditional probability statement. It went as follows: "The average risk level for breast cancer for women in this country is 7%, but rises to 16% if two generations before have had it." The first 7% is dependent upon some type of sample that can determine "average risk level," whether the sample be a small study or maybe even a large census survey. The 16% is a conditional statement: breast cancer average risk levels rise if two generations before have had it.

Subject explanations for this statement centered basically on a DON'T KNOW/KNOW SOMETHING dichotomy. Many subjects simply could not reply to the statement, except with negative comments or restatements: "can't grasp it or figure it out"; "totally confusing to me"; or "some type of research was done." Know Something explanations suggested different types of large scale research to determine risk level ("population figures"; "large hospital studies") and explained briefly the conditional statement.

Slightly more Don't Know explanations were given (60%) than Know Something explanations (40%). There was no difference, however, due to anxiety level ($\chi^2 = .83$, $df = 1$, $p > .05$).

Statement 4

Statement 4 was "Studies have shown that smoking may be related to cancer." The key word in this construction was "related," i.e.,

trying to tap some intuitive notions about the concepts of association and/or prediction. Furthermore, some classification of the notion of "studies" was also sought.

Once again, subject explanations tended to split between a DON'T KNOW/KNOW SOMETHING distinction. Don't Know explanations tended to center on the incomprehensibility of the statement: "I can't comprehend that"; "It's just a statement"; or "don't know." Know Something explanations centered on some way that this type of information could be obtained: "compare smokers versus non-smokers . . . who turns up with cancer"; "large number of case histories of who got cancer"; or "test large number of rats."

Seventy percent of all subjects knew something about the statement, with 90% of the high anxiety group explaining it versus only 50% of the low anxiety group. This result was marginally significant at the .05 level ($\chi^2 = 3.81$, $df = 1$, $p > .052$).

Statement 5

Statement 5 was "This past weekend Tennessee was a 7 point favorite to beat Washington State." This statement attempted to elicit responses that centered on the fact that some numerical estimates (like point spreads) can be simply calculated guesses. Strengths and weaknesses of the different teams may be considered but often the point spread is adjusted by the betting done on both teams and nothing else.

Subject explanations were grouped into two categories: RELATIVE CALCULATION vs. SPECIFIC CALCULATION/DON'T KNOW. Relative calculation responses indicated that the strengths and weaknesses of the two teams were somehow weighted and then the point spread determined. Specific calculation statements suggested that the past performances of the two specific teams were used. (In this case, the two teams had never met before.) Don't Know explanations were much as before.

Nearly all explanations were of the Specific Calculation/Don't Know category (80%), with most of these being of the Don't Know type. Once again, there was no difference due to anxiety factor (chi-square = 1.25, df = 1, $p > .05$).

In summary, it seems that many of these statements produced great difficulty for most students. Everyday occurrences of probabilities are probably often accepted without question, or with the stipulation that the numbers must come from a specific, set source, such as the "weather bureau," "sportcasters," or somebody else. In general, there was little or no difference due to anxiety level.

Formal Operational Probabilistic Thinking:

Piagetian Experiments

In this section, intuitive notions of the idea of probability, as explored in the Piaget and Inhelder book The Origin of the Idea of Chance in Children (1975) were investigated. As in other parts of Piagetian theory, the notion of probabilistic concepts centers around a developmental sequence, progressing from pre-operational through

concrete operational to formal operational. The difference between this book and other formal operational tasks attributed to Piaget is striking: more commonly cited experiments center on the development of causal, logical, deductive, mechanistic thinking whereas the notions of chance center on more probabilistic conceptions of the universe. The purpose of the following exercises, selected from the book, was to assess the level and kind of formal operational probabilistic thinking before students encounter formal descriptions of these topics in a class.

The experiments center on three major topics--the notion of random mixture in nature, the idea of the probabilistic nature of outcomes (as in heads and tails), and the concept of combinations and laws governing combinational activity.

1. Random mixture and irreversibility. The concept being tested in this set of exercises was the notion of random mixture in nature, i.e., that once certain orders are disturbed or upset the resulting interaction is not only random but also irreversible (unlikely to return to the original starting state). To test this a tilt board was constructed along the lines of Piaget and Inhelder (1975). This consisted of a rectangular box, with a divider in the middle at one end. Furthermore, the box rested on a center pivot, so that it could be tilted in either direction.

In this experiment, eight small white marbles were placed on one side of the divider, eight small black marbles on the other side

of the divider. Before the box was tilted, subjects were asked: What would happen when the box was tilted?; What would happen to the black and white marbles? Formal operational thinking should have predicted a random mixture of white and black, without any kind of predictability (while pre-operational thinking would have predicted a regular criss-crossing of balls, white to one side, black to the other). The results are presented in Table 3 (for this and the other Piagetian experiments).

Table 3. Results of Piagetian Exercises

	High	Low
1. Tilt Board		
A. Random Mixture--correct	10	10
B. Irreversibility		
No Chance	4	4
Small Chance	3	2
High Probability	3	4
2. Poker Chips		
A. A Priori 50/50		
Correct	3	6
Incorrect	7	4
B. Law of Large Numbers		
Aware	6	7
Don't Know	4	3
C. Number of Drops to Determine Fake Chips	1.8	1.9
D. Number Chips to Stop with "5"	3.2	3.1
E. Number Chips to Stop with "10"	4.2	4.3
3. Combinations		
A. Saw Pattern	1	1
B. Mechanical Correct	5	5
C. Mechanical Incorrect	4	3
D. Don't Know	0	1

For this experiment, all 10 subjects in both conditions correctly predicted a completely random mixture of the marbles.

Once the board had been tilted, subjects were then asked to predict that, if the tilting process would continue for approximately 1000 times, would the black and white marbles ever return to their starting positions? For Piaget, a formal operations answer would be that it is highly unlikely for the balls to go back the way they were once the order had been upset. In other words, this operation is irreversible. Subject responses tended to fall into three categories. Some said that there was NO CHANCE of the balls returning to the way they were; some said there was a SMALL CHANCE (1 in a 1000, for instance); the others assigned a HIGH PROBABILITY, e.g., it probably will happen or a 20% chance it will happen. Table 3 presents a breakdown by high and low anxiety conditions. Overall 40% of the people are completely in formal operations (NO CHANCE), 25% in the SMALL CHANCE category, and the remaining 35% in the HIGH PROBABILITY condition. The chi-square for math anxiety was non significant (chi-square = .34, df = 1, $p > .05$).

2. Estimated probabilities: a game of heads and tails. Essentially, the game of heads and tails was played to determine intuitive notions of such formal conceptions as binomial probability. In this exercise, 10 red chips were used, with each chip having an X and an O marked on either side.

In the first experiment, a subject was asked to predict what the outcome would be if all ten chips were dropped at once; they were also asked to explain their reasoning. For Piaget, evidence of formal operational thinking would be some indication that a 5/5 split is the most likely to occur (an a priori probability) on the first drop, on the second drop, etc. Subject explanations were scored for this notion of likelihood and for the notion of independence, i.e., that the first drop had no effect on the second. From Table 3, it can be seen that 50% did not get the notion of a priori probability or independence correct. In fact, in many cases, they demonstrated the so-called gambler's fallacy, i.e., that what happened on one drop would affect the next. If 6 heads, 4 tails came up, they often predicted that the next drop would be 4 heads, 6 tails. There was no difference due to anxiety ($\chi^2 = 1.82$, $df = 1$, $p > .05$).

Subjects were then asked to predict the most likely outcome if 50 chips were used, 100 chips, and then a 1000 chips. They were then asked if a 50/50 split was more likely with 50 chips or a 1000 chips. The concept being tested was the law of large numbers, i.e., that as the number of trials increase, large deviations away from the mean are more and more unlikely. From Table 3, 65% of the people were aware of this concept. Once again, there was no difference due to anxiety ($\chi^2 = .22$, $df = 1$, $p > .05$).

At this point, subjects were asked once again to predict the outcome of drop of another 10 chips. Before the actual drop, however,

the chips were "switched," i.e., 10 were substituted that had X's on both sides. In this exercise, Piaget was interested in testing the tolerance for highly unlikely outcomes. For his formal operational children, once they saw 10 X's on the table in front of them, they became highly suspicious and either picked up the chips or asked if they were fake. Pre-operational children, on the other hand, felt that this outcome was not unlikely, even if it happened two or three times in a row.

In this experiment, then, the "fake" chips were continually dropped until a subject thought something "funny" was going on, either by picking up the chips or saying something. (From questioning afterwards, apparently no one had seen the experimenter switch chips.) The average number of drops to determine fake chips was calculated and is presented in Table 3. Overall, it took two drops before subjects thought something fishy was going on (range was from 1 to 4). There was no difference due to anxiety ($t = 0.25$; $df = 18$; $p > .05$).

Following this experience, subjects knew of the existence of both real chips and fake chips. In the next experiment, subjects were told that the experimenter had 5 chips in his hand, but it was not revealed to subjects if the chips were "real" or "fake." The chips were shuffled and then one chip turned over at a time. The subjects' task was to say when they could determine if real or fake chips were being used. In actuality, the 5 chips were fake, so that each successive drop of a chip turned up an X. The average number of chips turned

over before a subject knew they were fake was calculated. Also the process was repeated with 10 chips and the average number before fake was spotted was calculated. The averages are presented in Table 3. The average number of chips to stop with on the 5 chips was 3, with 10 it was 4. This fits in with Piaget's notion that formal operational thinking will tolerate a certain amount of uncertainty before "fake" is assumed. Preoperational children, on the other hand, may require all instances before they are willing to assume that the "fake" chips are being used. There was no difference due to anxiety on either task ($t = 0.27$, $df = 18$, $p > .05$; $t = 0.01$, $df = 18$, $p > .05$).

3. Combinations. Finally, the concept of combinations and the patterns behind combinations were tested. Subjects were presented with three chips, one white, one blue, one red, and explained the concept of all possible combinations; in this case there were three (white-blue, white-red, blue-red). They were told that white-blue and blue-white were the same thing.

In the following experiment, subjects were presented with 4 different colors, then 5, then 6 and asked each time for all possible combinations. The scoring in this task was done on the last set of 6 chips. Subjects were observed and listened to to see if they obtained the correct number of combinations (in this case 15) by a pattern (i.e., number of combinations with 5 chips is 10, plus the new five with a sixth chip), by mechanical counting and rearrangement of chips, or if

they did not know how to do it at all. One additional category was added, that of mechanical counting which produced the wrong number of combinations.

Data are presented in Table 3. Only 10% of all subjects reported a pattern. Half of the students counted the correct number of combinations, while 35% counted them incorrectly. Only one person failed to do the task at all. By combining the don't know response with the mechanically incorrect, there was no difference due to anxiety level ($\chi^2 = 0$).

Weekly Self-Reports

Subjects in the present study also were asked to complete weekly self-report sheets on various attitudes and problems related to statistics. A cursory examination of these sheets (an average of approximately 10 per subject) revealed that very few of the many sections were thoroughly and consistently completed by subjects. Only one question seemed to produce reliable data for all subjects; that question was phrased "As you were reading the material or looking over lecture notes or studying for a test, which concepts and/or examples did you have trouble understanding?" This section will present an analysis of responses to this question.

Ten major concept areas of statistics were covered in the course, approximately one per week. The number of examples, concepts, or problems reported to be difficult were tabulated for each of the ten week/topics and broken down by anxiety level. The results are presented in Table 4.

Table 4. Frequencies of Difficult Concepts

	High	Low
Frequency Distribution	29	21
Correlation	5	6
Probability	14	15
Binomial Probability	5	1
Chi Square	6	6
Normal Distribution	12	11
Sampling	5	10
Hypothesis Test & Confidence Levels	3	4
One Population	3	2
Two Populations	1	0
Misc.	1	4

The largest number of concepts reported to be difficult for both groups occurred in week one material that covered introductory concepts such as distribution and measures of central tendency and variability. The second largest value for both groups was the third week dealing with probability; the third largest value was for week six dealing with the concept of the normal distribution. These three types accounted for 65% of all concepts reported difficult for the high group and 59% of all difficult concepts for the low group. Overall, then, there was

no difference due to anxiety ($\chi^2 = 7.05$, $df = 9$, $p > .05$) in the concepts found to be difficult by students in both groups.

It is interesting that difficulties arise at, what seems from a teaching point of view, to be crucial junctures in the course. In the first week, students seem to have to learn a new language and a new way of thinking about material. Thus, the reported difficulties with concepts such as variance, standard deviation, and the setting up of distributions do not seem surprising. The topic of correlation, which occurred during week two as taught in this course, was a bivariate application of variances and graphing and thus was related to the first week's material. The next major introduction of terminology and concepts occurred first with probability and then later with the normal distribution. Both binomial probability and χ^2 were developed from key ideas in probability theory. Sampling, hypothesis testing, one and two population statistics all were derived from notions of the normal distribution. It seems, in general, that students reported most difficulty at points in the course when totally new concepts were introduced; once the concept was mastered, however, related concepts present less difficulty.

Problem Solving Experiment

In this section, subjects were presented with three different sets of statistical stimuli and asked to report, verbally, their answers to the tasks. The three sets included 1) measures of central tendency using 5 graphs of different types of distributions; 2) a binomial

probability chart; and 3) a chi-square chart. In each case, more formal statistical material was tested at a recall level, with the additional element of transfer of training also being considered. Once again, these tasks were given at the end of the course.

1. Measures of central tendency. In this task, subjects were presented with graphs of 5 types of different distributions (without the following labels, of course): 1) Normal; 2) Positively Skewed; 3) Negatively Skewed; 4) Bimodal; and 5) Rectangular. In each case, the subject was asked to make a mark on the graph corresponding to the mean, mode, and median of each distribution. As a reminder, scoring was as follows: 3--all three correct; 2--two correct; 1--one correct; and 0--zero correct. The results for each of the five graphs broken down by math anxiety condition are presented in Table 5.

For the normal distribution, 70% of all subjects correctly identified the fact that the mean, mode, and median are in the center of the graph. This was not surprising, since the normal distribution was taught in the course. However, 30% of the group still mislabeled parts of the graph. Of these, two subjects could not do the task at all, one located three different values, and one reported no mode existed. For the purpose of evaluating differences due to anxiety level, the data was regrouped into Correct vs. Incorrect categories (the latter containing scores 0, 1, 2). Using this procedure, there was no difference due to anxiety level ($\chi^2 = .43$, $df = 1$, $p > .05$).

Table 5. Results of Problem Solving Exercises on Measures of Central Tendency

Distribution	Hi	Lo
A. Normal		
0	0	2
1	1	0
2	1	1
3	8	6
B. Positively Skewed		
0	0	1
1	5	5
2	3	2
3	2	1
C. Negatively Skewed		
0	0	1
1	6	4
2	2	2
3	2	2
D. Bimodal		
0	1	1
1	1	3
2	6	1
3	2	4
E. Rectangular		
0	0	2
1	0	2
2	6	0
3	4	5

The remaining four graphs were essentially exercises in transfer of training, since none had been explicitly labeled nor had measures of central tendency calculated in class. The negatively and positively skewed distributions showed errors of basically the same nature: 50%

of the subjects on both graphs made a joint two-error mistake by reversing the position of the median and the mean. In a skewed distribution, the extreme values cause the mean to be pulled toward the skewed end, while the median will be closer to the mode. Subjects basically reversed this distinction and located the two values more on a visual basis, i.e., the median was located about halfway across the X-axis while the mean was seen closer to the peak (mode). Reasoning consisted of the fact that the median was the halfway number, while the mean was more the average or representative value. There was no difference due to anxiety level for either case (chi-square = .28, $df = 1$, $p > .05$ for the positive skew; chi-square = .01, $df = 1$, $p > .05$ for the negative skew).

For the bimodal distribution, 30% of all subjects correctly labeled the mean and median as being at the center (or valley) of the X-axis and the graph as being bimodal, i.e., two modes or peaks in this case. For people who made only one mistake, the most common error was in the handling of the mode, i.e., since there are two high points this might change the mode, make it not exist, or make it an average value. The most common two mistake problem occurred in locating two means and two modes. In this case, subjects saw the graph as two separate distributions (i.e., two normal curves next to each other), each with a set of three center values. Here, once again, a visual problem was encountered, in which two distributions were seen rather than a single bimodal distribution. There was a difference due to

anxiety level ($\chi^2 = 8.3$, $df = 1$, $p < .05$) with the low group outperforming the high group.

Finally, on the rectangular distribution, 45% of the group identified the mean and median as the same, halfway across the X-axis, and that all values had the same frequency, meaning there was no mode. The most common single mistake was in the handling of the mode; several reported two modes (the top corners of the rectangles) or left it blank. For those who missed more than one concept, the most common mistake was in not knowing the location of either the mean or the median. A constancy of values (the rectangular appearance) produced difficulty in finding the "center" or "middle" values. There was no difference due to anxiety level ($\chi^2 = .46$, $df = 1$, $p > .05$).

2. Binomial probability. Subjects were presented with a single xeroxed page from a binomial probability table taken from their textbook. The table was without headings and contained only the symbols normally associated with this type of table, namely n , p , and x . Two questions were asked: 1) What table is this? The purpose was to test for recognition and possibly a brief explanation of the function of the table. 2) Using the table, solve the following problem: If you have a family of 10 children, with boys and girls being equally likely, what is the probability of getting exactly 5 boys out of 10 children? Since binomial probability was covered early in the quarter, both questions examined a retention of learning at a recognition level.

Data for problem recognition and problem solution are presented in Table 6. As can be seen 63% of the subjects failed even to recognize

Table 6. Results of Binomial Probability Problem Solving Exercises

	Recognition		Solution	
	Correct	Incorrect	Correct	Incorrect
<u>Anxiety</u>				
High	5	5	3	4
Low	2	7	4	5

the table as being one of binomial probability. The most commonly reported label for the table was chi-square or a z-table with explanations centering on the p-level at the top of the table. This symbol was recognized as a probability level but misidentified as the alpha level of a chi-square table or as the critical values of a z-table. Altogether recognition was very low, with anxiety not being a factor (chi-square = 1.57, df = 1, $p > .05$).

Problem solution was at the same level, with 63% incorrect. The correct solution to this task would be to identify that $n = 10$, $x = 5$, and $p = .05$ and then $p(x) = .246$, by using the table. Either people could not work the problem at all, got started but did not find all correct values (e.g., locate $n = 10$, $p = .05$, and then take the first probability from the table), or guessed that the correct probability was 50% (5 out of 10). Once again, there was no difference due to

anxiety level ($\chi^2 = .42$, $df = 1$, $p > .05$). In a few cases, subjects who did not recognize the table could work the problem.

3. Chi-square and hypothesis testing. A xerox of a chi-square table was presented to subjects, without any word labels (only symbols), in the last section of the problem solving exercises. Once again, subjects were asked to recognize the table, give a brief explanation, and then solve a problem using the table. Problem solution, in this case, was to explain the use of a particular chi-square value from the table. Why this particular value? What is it used for? The task attempted to tap verbal knowledge of hypothesis testing and the concept of a critical value.

Data for problem recognition and problem solution appear in Table 7. As can be seen 79% of all subjects recognized the table and briefly explained the symbols, df and α . The four other subjects labeled it as a z -score table. There was no difference due to anxiety level ($\chi^2 = 1.55$, $df = 1$, $p > .05$), although 9 out of 10 of the high anxiety group correctly recognized the table as opposed to only 6 out of 9 of the low anxious group.

Problem solution, overall, was closer to a 50/50 level (53% correct, 47% incorrect). Explanations in this case seemed to be all or none, i.e., complete recall and explanation of the concepts of null hypothesis, alternative hypothesis, and critical values or no explanation at all. Once again, there was no significant difference due to anxiety level ($\chi^2 = 2.55$, $df = 1$, $p > .05$); however, the high

Table 7. Results of Chi-Square Problem Solving Exercises

	Recognition		Solution	
	Correct	Incorrect	Correct	Incorrect
<u>Anxiety</u>				
High	9	1	7	3
Low	6	3	3	6

and low groups displayed different trends in performance, with the high group scoring 70% correct, the low 33%.

Problem Sorting Experiment

In this task, subjects were presented with stimuli representing the different statistical concepts covered in the course and asked to sort them into categories containing "similar" items. More specifically, there were ten major conceptual areas discussed in this introductory course on statistics: 1) frequency distributions; 2) correlation; 3) probability; 4) binomial probability; 5) chi-square; 6) normal distribution; 7) sampling; 8) hypothesis testing; 9) one population statistics; and 10) two population statistics. For each unit, 5 exemplars were chosen from a test file that accompanied the main text used in the course. Students did not have access to this test file. Each exemplar was placed on a separate slip of paper; there were 50 problems altogether, 5 from each of the 10 major divisions of the course.

Instructions to subjects were "On the following slips of paper you will find problems similar to those covered in the course. Please sort the problems into as many piles (categories) as you wish, such that each pile contains problems representing a similar statistical principle or concept. You can have as many or as few piles as you wish. I am interested in your judgments of similarity. Take as long as you wish." After all problems had been sorted, subjects were asked to provide a label for each category.

Two sample problems will be presented, one chosen from the probability chapter, one from One Population statistics. From the Probability unit: Two cards are drawn, without replacement, from an ordinary bridge deck. What is the probability that both are aces? From One Population statistics: The motor vehicle bureau claims at least 45% of all new cars registered are subcompacts. To check this claim, the police department randomly sampled new car dealers, and found sales of 200 new cars included 85 subcompacts. Is there enough evidence to reject the motor vehicle bureau's claim at the .10 level of significance?

This particular task, then, attempted to determine the level of integration of concepts by students in the course. Would the original structure as presented in the course be maintained or would certain areas coalesce?

The average number of categories, for the high and low groups, is presented in Table 8. As can be seen the high anxious group averaged 9.9 categories, very close to the original 10; the low group averaged

Table 8. Results of Problem Sorting Exercises

	High	Low
A. Number Categories	9.9	8.0
B. Number Problems in Largest Category	13.8	17.0
C. Number of Problems in Second Largest Category	9.6	10.4

slightly less at 8.0. The difference, however, was not statistically significant ($t = 1.25$, $df = 14$, $p > .05$).

Although the number of categories approximated the original construction of the task, the density (number of problems) in each pile varied quite a bit. To examine the distribution of concepts, the average number of problems in the first largest category and the average number of problems in the second largest category were calculated for both groups, with the data appearing in Table 8. For the low group, the first largest category included 17.0 problems, the second 10.4; for the high group, the respective figures were 13.8 and 9.6. Neither is statistically significant ($t = 2.05$, $df = 14$, $p > .05$ for the first category; $t = 1.23$, $df = 14$, $p > .05$ for the second category). It is interesting to note the percentage of problems accounted for by the first two largest categories. For the low group, the first category represented 34% of all problems, the second 21%; for the

high group, the respective figures are 28% first, 19% second. Thus, it seems that a certain amount of integration of statistical material did occur for both groups.

In order to evaluate the nature of these integrations, an examination of the labels used to represent categories was made. This evaluation indicated that the most frequent labels for either the first or second pile were first "Probability" or "Simple Probability" (covering problems from the probability, binomial probability, and chi-square sections) followed by "Hypothesis Testing," or "Samples and Populations" (covering problems from the hypothesis testing, one and two population statistics units). The normal distribution and sampling chapters were sometimes by themselves; at other times they were split between probability and hypothesis testing.

To examine these intuitive categories empirically, a 50 x 50 matrix was formed that tabulated how often each of the 50 problems was grouped by all of the subjects into similar categories. For example, let's assume that one subject created a unique pile of cards containing problem numbers 1, 2, 3, 4, and 5. In the matrix, entries would be made showing that problem 1 co-occurred with problems 2, 3, 4, and 5, that problem 2 co-occurred with 1, 3, 4, and 5, etc. In a similar manner, codings were listed for problems 3, 4, and 5. Each individual cell of the matrix, then, indicated how often a pair of problems was placed in a similar category for all subjects. Eight of the high anxious subjects and seven of the low anxious subjects completed this

task; making for a total number of possible entries equal to 15 for any pair of problems.

Clusters of problems were found by examining which sets of problems were paired most often with each other. A cut off criterion of more than 50% (or eight or more of the fifteen possible codings) was used to determine clusters. For example, if problems 1, 2, 3, 4, 5 formed a set, according to this criterion, it would mean that all possible pairs of these five (1-2, 1-3, 1-4, 1-5, 2-3, 2-4, 2-5, 3-4, 3-5, and 4-5) would have contained eight or more entries in the matrix. Upon inspection of the data, high and low groups produced similar results; for this analysis, the combined data will be considered.

Using this analysis, two major clusters emerged from the data. One cluster basically represented PROBABILITY problems, much as the intuitive labels had suggested. Of the 10 possible problems that came from the simple probability and binomial probability chapters, 9 of these (90%) formed a single group. Only two other problems, involving the normal distribution, were included in this set. A second major cluster centered on HYPOTHESIS TESTING and represented problems for the hypothesis testing chapters together with one and two population statistics. Of the 15 possible problems in these units, 11 (or 73%) formed a set. Two additional problems, coming from the chi-square unit, also clustered with this set. These two clusters account for 24 of the possible 50 that were sorted.

Two smaller clusters also were noted. One represented NORMAL DISTRIBUTION AND SAMPLING and contained 5 problems (out of 10 in the

two units). The final cluster centered on the concept of CORRELATION and had 4 problems as a set (out of 5 in this chapter). All other items either clustered alone or with less than 4 other problems.

The type of integration that has occurred is slightly different from that found in the weekly reports of difficult concepts. The initial difficulty with terminology and early concepts, such as variance and standard deviation, have now been absorbed into the uses of these concepts, i.e., more particularly in hypothesis testing, z-scores, or correlations. Although students reported no great difficulty with the concepts of hypothesis testing after they moved past the normal distribution, it seems from the problem sorting experiment that the two are somehow set apart from each other. Finally, probability is by itself; it is treated differently in both the weekly self-reports and in the problem sorting experiment.

Final Mars Results

Another MARS scale was administered to all subjects at the end of the course. Once again, there were 98 items, scored on a 1-5 scale (1--not at all anxious to 5--very much anxious). An examination of the differences between the high and low groups on individual questions, together with a comparison of pre- and post-class total MARS scores will be reported in this section.

High and low groups still differed on 32 scale items at the end of the course (see Appendix B, Table 12) as opposed to 53 items at the beginning of the quarter. A breakdown of the items on the

EVERYDAY/CLASS dimension previously discussed was still found to reveal a significant difference between highs and lows: high anxiety students still score higher on class-related mathematics items than on the everyday statements ($\chi^2 = 36.4$, $df = 1$, $p < .05$) (see Table 9).

Table 9. Posttest Differences on the Everyday/Class Distinction

	Class	Everyday
Differ	29	3
Don't Differ	17	49

Although this difference was now somewhat smaller than in the pre-test, it was still large and statistically significant. A survey of some of the questions which no longer differentiated the two groups revealed some perhaps predictable outcomes of the structure of the course: items such as "reading the word 'statistics,'" "taking an examination (quiz) in a math course," "sitting in a math class and waiting for the instructor to arrive," and "meeting your math teacher while walking on campus." Anxiety, then, seems to have been reduced only within a specific context and did not seem to generalize to mathematics in general.

A 2 x 2 ANOVA was also conducted on the total MARS scores for all students, using anxiety level (high-low) and time (pre-post) as factors. Predictably, the main effect of anxiety was significant ($F = 49.9$, $df = 1$, $p < .05$). Furthermore, the main effect of time was also significant ($F = 8.5$, $df = 1$, $p < .05$) indicating a general decrease in anxiety scores at the end of the quarter. There was no interaction effect ($F = 2.2$, $df = 1$, $p > .05$). From Figure 2 we can see the general picture: high and low anxiety students still maintain their initial difference on the MARS, although both groups tended to show lower scores.

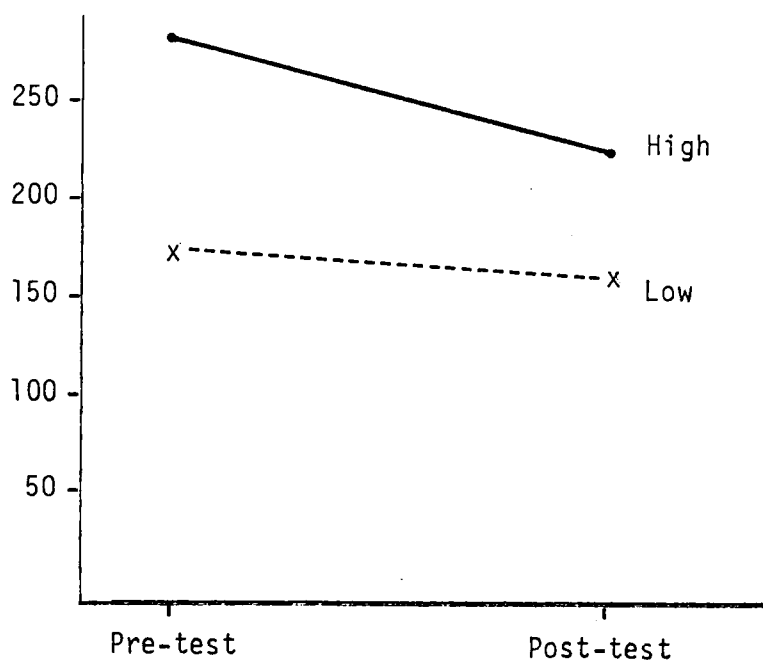


Figure 2. Pre- and Post-test Differences on MARS Scores as a Function of Anxiety.

Final Interviews

A final interview was also conducted with each student to try and ascertain his or her feelings about the statistics course, problems they encountered, and their present and future attitudes and expectations in regard to mathematics. A series of questions, much like the initial interviews, was asked of each subject.

For the final interviews, however, similar attitudes emerged from all students interviewed. These themes may be summarized as follows: "the course was not what I expected," "the course was easier than I thought," "after the first quiz or two, I started feeling comfortable in class, but now I'm glad it's over." Most students commented that statistics was not as bad as they thought, particularly in the distinction that it did not involve the high school algebra and geometry which they thought it might. However, all of the students still commented that no future mathematics related courses were in their schedules, unless they were absolutely required. Both high and low anxiety students reported similar statements.

CHAPTER IV

DISCUSSION

The present research has attempted to explore the problems students have in learning statistics at a college level. In order to explore these issues, different approaches and/or techniques were utilized: 1) an experiential point of view, informed by the phenomenological method which seeks to understand the self-world of the mathematics learner; 2) an intuitive/everyday perception of probability; and 3) a formal/cognitive approach, influenced by cognitive science, which attempted to represent the more formal knowledge structures that were acquired in the learning of a particular domain. The answer to the question, "What are the problems people have in learning statistics?" which is presented in this final section, must be informed by all three avenues of investigation.

The Self-World of the Mathematics Learner

From the experimental point of view a number of results have been found. The analysis of the Mathematics Anxiety Rating Scale (MARS) revealed that people who are highly anxious about mathematics seem to display this anxiety most strongly in the context of a mathematics classroom. On the other hand, everyday use of mathematics (totaling up bills, checkbooks, loans, etc.) seems to be a non-threatening experience for both high and low math anxiety subjects. It seems that formal

classroom approaches to mathematics are the focus of anxiety, at least as measured by the MARS scale.

The phenomenological interviews conducted with students in this study not only corroborate results produced by the MARS but also provided a number of themes defining the world of the math anxious student. These interviews have shown, in general, that the world of mathematics seems to be very different for those who are highly anxious in comparison to those who are not. The anxieties, once again, seem to focus on classroom mathematics and not on its everyday, practical applications. The latter set of tasks seems to provide no trouble for either group: cashiering, balancing checkbooks, dividing up dinner checks all are instances of applied mathematics (and, in some cases, difficult applied mathematics) that present no trouble even to highly anxious mathematics students. The everyday world of mathematics is not only not intimidating but seems to be reasonably well mastered.

The situation changes dramatically once we enter the formal world of the classroom. For the highly math anxious student, a number of themes emerged: 1) The initial entry into a statistics classroom is foreshadowed by feelings of an assault upon self, i.e., the class is dreaded, feared, apprehensive, and anxiety provoking; 2) These feelings, which stem from an overall dislike of mathematics, seem to have their origins in key early memories for the student. These early memories center around classroom experiences (from elementary school to high school) in which the student was forced to perform in a class

and could not do it. Feelings of humiliation, embarrassment, or fear of failure resulted which seems to shadow them through future mathematics courses; 3) The world of other people becomes one in which the self is anchored at the lowest point. Typical responses to the question, How would you feel if you were worse than anyone else in mathematics? become "I am," indicating feelings of inferiority felt in this particular subject; 4) These feelings, together it seems with the nature of the material, tend to make mathematics different from other subjects. Mathematics is seen as an abstract, logical, impractical, right or wrong exercise in which the self must be entirely perfect or an utter failure, a belief which tends to focus extreme amounts of anxiety on timed test situations.

This sense of the "differentness" of mathematics applies, to a certain extent, to the world of the low anxious math student. Here, mathematics may be enjoyed but the enjoyment centers around the puzzle solving, "I can get it right" nature of mathematics, particularly of high school algebra and geometry. For some students in this group, mathematics is tolerated: "I can get through it, even if I don't like it." For the low anxious student mathematics is not nearly as feared as for the high anxious student although its formal nature still sets it apart from other courses that must be taken.

This sense of anxiety drops, to a certain extent, by the end of the course, for both groups, as measured by the MARS scale. This would seem to be due to the structure of the course, which was designed

to counter some of the pressures felt by the high anxious people. (Specific implications for teaching strategies will be discussed later.)

It may also be attributable to two other factors: 1) receiving a good grade in the course (most students were graded on a contract basis and received A's); or 2) completing the last required mathematics related course. The anxiety, if reduced, however, seems merely to be dormant; a potential future formal mathematics situation may be expected to reawaken it once again.

For the learning of statistics, then, the student learner's self-world has already been conditioned in its attitude towards anything related to mathematics. In general, for the low anxious math student, the world of classroom mathematics is tolerated or even enjoyed for its puzzle solving, albeit impractical nature. For the high anxious math student, the abstractness of formal mathematics generates an entirely different world, a world in which one knows immediately when he or she has failed if an answer is not gotten completely right. As such, mathematics seems to provide an assault on the self which cannot be denied or even defended against--"I am the worst."

Intuitive/Everyday Concepts of Probability

Attitudes towards mathematics are not all the student brings to the formal learning of statistics; in addition, everyday, intuitive notions of probability are embedded within the conceptual world of the beginning student. These notions include concepts of number, probability, and sampling such as occur in advertisements, newspaper articles,

or even sportscasts. Furthermore, according to Piaget, formal operational thinking produces a certain way of looking at chance and the laws of chance; structures which, again, are part of the logical apparatus of any adult. This section will present the empirical findings that pertain to these concepts and structures.

Explanations of everyday statements that include probabilities (such as "40% chance of rain tomorrow," "3 out of 4 doctors surveyed preferred Trident sugarless gum for their patients") produced mixed results. Students tended to understand the use of samples (either large or small) in producing certain kinds of results; e.g., that a small biased sample could be the basis for some statements (the advertisement for "3 out of 4 doctors") versus the large scale samples needed to support a statement such as "Studies have shown that smoking may be related to cancer." These same students, however, fail in the notion of sampling and empirical evidence when they relate to a natural phenomenon such as the weather. Typical explanations for the 40% chance of rain centered on statements such as "they observe the sky" or "they use weather machines." The notion of probabilistic tendencies and large samples on which conclusions are based seems to hold only in the explanation of human phenomena and not in the explanation of natural events. Natural events are assumed to have more predictable causes.

This notion of a deterministic versus a chance universe is central to Piaget and Inhelder's views concerning the development of chance in children and adults. Most of Piaget's work, previous to his work on

chance, centered on the development of logical, scientific, deterministic thinking. The progression from preoperational thinking to formal operational thinking implied a deepening understanding on the child's part that the world was composed of causes and effects and laws embodying them. Concrete operations stressed the notion of reversibility, i.e., if the ball of clay is stretched lengthwise, then we still have as much clay as when it was a single ball. Changes that made the clay balls "look" different when, in fact, they contained the same amount of clay were mediated by an underlying principle of conservation. Furthermore, in formal operations, an event which, on first glance, seemed to be totally irregular, proved, upon inspection, to be regulated by a law; e.g., in demonstrations of the intuitive discovery of Archimedes principle by children. The universe was seen to be lawful, regular, and deterministic.

Into this schema comes the late work by Piaget and Inhelder on probability. The word "late" is used for two reasons: 1) the book was not translated until 1975, and 2) most textbook discussions as yet have not incorporated these more recent ideas of Piaget. In this 1975 book, Piaget and Inhelder argue for a more probabilistic and non-deterministic conception of reality. Certain events in nature are now irreversible including everything from the effects of hurricanes and volcanoes to the random dispersion of marbles. Furthermore, events are not totally predictable; the outcomes of coin tosses display patterns although not perfect regularity. Formal operational thinking, then,

must almost be Januslike: one side striving for determinism, the other mindful of the patterned, but still chance, nature of the universe.

This tension between determinism and chance, between reversibility and irreversibility, is clear in results produced by the Piagetian experiments conducted with the subjects of this study. On the one hand, they were aware of the probabilistic nature of certain events, as evidenced in the following: 1) when the tiltboard, composed of eight white marbles on one side and eight black on the other, was slanted, students accurately predicted a random, non-regular mixture of the marbles; 2) when asked the odds of the marbles returning (reversibility) to their old position, more than 50% said that this was highly unlikely; 3) in the tossing of coins, when asked to predict the likely distribution of 50 versus 1000 coins, most students were aware of the law of large numbers, i.e., that unusual distributions were less likely the more you increase the size of the sample; and 4) when "trick" coins were used (all heads) most students caught on quickly that something was wrong, i.e., the event of 10 heads was just too unlikely, especially if it happened twice in a row. All of these results provide evidence for formal operational understanding of probability as described by Piaget and Inhelder.

On the other hand, certain results of the present study lean toward a deterministic understanding of the nature of events, as indicated by the following: 1) a high percent of subjects felt that the odds were good for the white and black marbles to return to their original position on the board, i.e., that the event was reversible;

2) in the tossing of coins, students often displayed a gambler's fallacy, i.e., that the outcome of one toss would affect the next one. For example, if the toss were 7 heads, 3 tails, they might predict the reverse on the next toss, a sort of lawful equilibrium at work; 3) in the use of the "trick" chips, several students still felt that the outcome of 10 heads 3 or more times in a row was not all that unlikely; and 4) on the one activity, that of combinations, where there was a pattern to be discovered, students did not uncover the regularity but proceeded in a mechanical fashion.

That college age students are not entirely in formal operational thinking as defined by Piaget is not a new finding. For these types of probabilistic exercises, however, the results are relatively unknown and seem to suggest a problem for the learning of statistics. The tension between cause and effect, between determinism and chance, is at the heart of the formal learning of statistics and probability. The formal representations of this discipline emphasize the learning of certain lawful regularities, such as the binomial theorem, which posit that general distributions can be predicted but not specific outcomes. Furthermore, the notion of sampling is a probabilistic concept, which combines with the notion of likely and unlikely samples to produce a number of significant statistical concepts. Since statistics deals with the notions of sample spaces and sometimes arbitrary critical values affecting decisions, it is often unlike other types of mathematics; this discrepancy between right and wrong answers to problems

(as in algebra and geometry) and probabilistic answers (as in statistics) might prove difficult for a student learning statistical material. That students are not entirely formally operational in their thinking suggests a dichotomy that has long troubled disciplines as disparate as physics and psychology; that of the conflict between order and chaos in the world and between regularity and chance in the mind of the perceiver of the world.

Formal/Cognitive Acquisitions

The everyday world of mathematics and intuitive conceptions of probability provide the background against which to judge the formal acquisitions of statistical material. The introductory statistics course, which all subjects in this study completed, was designed to teach basic notions of distributions, descriptive statistics, probability sampling, as well as the elements of inferential statistics and hypothesis testing. The tasks used in examining this knowledge attempted to measure how many of these concepts had been acquired, how much transfer of training occurred on specific concepts, and to what extent the material had been integrated.

Starting with the acquisition of material, it is curious to note how much difficulty students had in even recognizing certain tables and formulas that had been used during the quarter. The presentation of the binomial probability chart produced less than 50% correct recognition and less than 50% correct problem solution. The recognition rate for the chi-square table was considerably higher, although the

rate for correct explanation of hypothesis testing was only slightly over 50%. Admittedly this type of recognition task is a difficult one, especially for students having had only one quarter of instruction. Still it points to a rather quick memory loss of knowledge which had only recently been shown to be at a reasonable level of performance.

The test of knowledge about measures of central tendency in five different distributions produced some of the same results. The first graph, that which concerned the normal distribution, was a test of recognition of a concept that had been covered extensively in the course; in this case, a high percentage of people correctly identified mean, mode, and median. The other four distributions required some transfer of principles since these distributions had not been particularly emphasized in the course. Results concerning the remaining four distributions were not so straightforward. For both skewed distributions, students often confused the position of the mean and the median; for the bimodal graph, students fairly often labeled it as two distinct distributions, producing incorrect means and medians. Finally, the rectangular distribution also seemed to produce a number of varied mistakes.

Although students may not often produce the "correct" solution from a formal statistical point of view, we must inquire into the nature of their "mistakes." A case can be made that students have not yet come to "see" statistics in the same way a more formally trained person does. Instead, their way of "seeing" problems is different and perhaps more intuitive. To a certain extent, there is a confusion of

"lookalike symbols": the chi-square distribution table is mislabeled a t-distribution or a z-distribution, a confusion which, at an elementary level, is perhaps understandable. In the solution of the binomial probability problem (what are the odds of getting 5 boys in 10 children) there was a problem in only a partial use of the table (implying some recall or reinsight into the way it was used) or students replied "commonsensically" that the odds were 50%. The latter intuitive judgment, which discounts the role of probability distributions, nevertheless displays an everyday approach founded on what seems sensible. In explaining measures of central tendency, students often made certain types of "visual" mistakes: on the skewed distribution, the "middle" point (for the median) was picked halfway across the graph; for the bimodal distribution, it was "seen" as two distributions; and on the rectangular distribution, it was often "seen" as having no middle to it. These "confusions" belie a knowledge of more formal reasoning although they do instruct us on how the student is seeing these problems.

It is in the student's view of statistics that we can combine some of the ideas of Wertheimer and Piaget mentioned in the introduction. For Wertheimer, the notion of productive thinking implies a deeper knowledge behind the surface appearance of problems. Insights are the entry into this more productive world. For students, at an introductory level, however, "insights" into statistics are closer to a common sense understanding of their world than to a strict, formal

statistical view. They see concepts, such as "median," "mean," "distribution," and "probability" in a visual sense that has yet to develop into the more formal reasoning necessary for a mature conceptualization of statistics.

Piaget's concept of formal operational thinking must then be recast within the concept of statistical learning. For Piaget, the concept of this stage of thinking implies the ability to handle symbols, to understand the intuitive sense behind symbols, and to discover laws describing their use. Clearly, students at an introductory level are not at a "formal operational" stage of statistical knowledge. Much of their thinking remains "concrete" (almost visual) and has yet to move to the abstract realm. This is not to imply a "fault" on their part; it is simply an indication that the acquisition of more formal disciplines, such as statistics, may produce stages of learning mirroring the stages suggested for other domains by Piaget. In learning a new domain we should expect some "Type B" or even "concrete operational" thinking. The use of a new symbolic language does not imply immediate familiarity with it; "formal operational" thinking may only come slowly and be quite dependent on the specific domain of knowledge to which it is applied.

Although students do not seem to display a deeper, more formal, understanding of statistical material, they do evidence a curious integration of material at the introductory level. The problem sorting experiment, which tapped student representations of statistics, showed that, in general, two categories, from both the intuitive and

empirical sorts, accounted for about 50% of all problems sorted. These problem categories were labeled "probability" or "hypothesis testing," producing a dichotomous division of the introductory domain of statistics.

Why this particular split? Probability is the basis of hypothesis testing and, in some way, should not be seen as different. One possible explanation draws on the distinction between determinism and chance elaborated in the previous section. The type of probability problems a student encounters usually present a great deal of difficulty, for such problems involve the representation of sample spaces that are hard to visualize (as in the binomial probability problem). Hypothesis testing, on the other hand, in all forms encountered in the usual undergraduate statistics course, involves a more rigid, rote procedure: that of the null hypothesis, alternative hypothesis, alpha levels, critical values, and decisions. In this sense, it is more "deterministic," i.e., it has a set procedure which, if followed, will product a "correct" result. The conceptualization of probability problems are not so straightforward. This dichotomy, then, may reflect a certain degree of integration and a certain type of non-integration; probability (chance) is seen as different from hypothesis testing (determinism).

The formal learning of statistics, then, involves many challenges for the student. A new world of symbols must be mastered; intuitive ideas must be recast into unfamiliar formats; and the search for rote procedures must be tempered by certain elements of more probabilistic

(chaotic) concepts. This transition from concrete to formal, from rote to productive, is accomplished in stages, in which the student slowly comes to "see" differently; that is, to "see" in a more formal statistical way. That the transition is not all of one piece is suggested by present results and is not surprising in the light of many similar results reported by Piaget for other knowledge domains.

The Issue of Anxiety and Performance

The relationship between anxiety and performance, as discussed in the introduction, is an unclear one. In the present set of studies, however, the most consistent finding, spread across both initial and final tasks, was that different levels of anxiety did not affect performance either positively or negatively. Students in both high and low anxious groups tended to display the same level of initial learning on intuitive/everyday probability tasks as well as the same level of acquisition, transfer, and integration of more formal concepts. How can this result be understood when the world of the highly anxious math student has been shown to be so clearly different from that of his/her low anxious counterpart?

The lack of difference on the explanation of everyday probability concepts and the Piagetian tasks is most easily explained. The MARS scale and the phenomenological interviews both corroborated the fact that the anxieties about mathematics that develop are centered on more formal, classroom mathematics learning. In other words, the anxiety tends to have its greatest psychological impact within more pressured,

classroom situations. Both everyday concepts and the Piagetian experiments test more commonsense, even public, uses of probabilistic thinking that occur outside the classroom. That there are no differences in performance on these tasks is not surprising.

That performance, however, was essentially the same within the more structured environment of a statistics course is more puzzling. Several factors and/or hypotheses might account for this lack of difference. First, the actual structure of this particular statistics course may have been important. It must be remembered that the course was taught in such a way as to minimize those factors which highly anxious students mention as most threatening (e.g., board work, totally abstract word problems, timed test pressures, etc.). Secondly, the content of statistics is different from that of algebra and geometry. Many students expect to find concepts from the latter two domains discussed in the course; only a very small number of algebraic concepts are used in teaching of statistics at an introductory level in psychology. Students often are surprised that the actual terminology and computational formulas of statistics are not carryovers or extensions of earlier courses. Thirdly, we should consider the fact that this is a required course and essential to graduation for all students in it. That they have to pass it may force many of them to perform in a situation that they fear, or even detest, but nevertheless must endure.

That they endure and perform as well in class as the non-anxious group, however, does not insure their future use or continual understanding of statistical concepts. For the math-anxious student,

classroom mathematics is avoided at all costs, unless they are forced to take courses to meet degree requirements. Future courses in mathematics are ruled out along with, in some cases, future options that may involve the taking of mathematics. Many psychology students fear the Graduate Record Examination, a test in which half of the items are based on high school algebra and geometry (Smith 1984). Performance is not strictly the issue; we must also consider the choice to perform. Some students may avoid any need to display future performance because of a fear of mathematics. In this case, we would have to say that math anxiety does have an effect on behavior related to mathematics.

Criticisms and Extensions

Any experimental investigation always seeks to answer two questions: 1) What was found from the present study? and 2) Where do we go from here? Previous sections have attempted to answer the first question, by seeking answers to the varied informal and formal problems that students encounter in learning statistics. The second question itself really has two parts: 1) how could the present study be improved? and, 2) what implications are there for the teaching of statistics?

As with all empirical studies, we must be concerned with the question of sample size, i.e., would the same pattern have emerged if more people were added to the study. Both high and low anxiety groups were composed of only ten people; a small number when considered psychometrically but still a reasonable sample size considering the

variety of quantitative and qualitative data collected. Each student was involved in approximately 6 hours of interviews and problem solving sessions, not to mention the records of weekly problems that were kept.

Certainly we have to be concerned with the question of sample size; at the same time, if the results of any study are robust we would expect them to carry over to much larger samples. In this sense, we are all influenced by the work of Skinner, Piaget, and even Freud who argued for more definite patterns emerging from small n's. As such, the concern with sample size becomes another empirical question; will other investigation replicate results found in the present study?

We must be concerned, however, with the generalizability of some of the results, particularly those obtained at the end of the course. The attitudes towards mathematics which were reflected in the final interviews and the final MARS scale could certainly have been influenced by the specific teacher of this course. With other teachers, other attitudes and changes in anxiety may have occurred. Future research will have to control for the factor of teacher and even teaching method in determining the relationship between anxiety and performance. This relationship may also have been influenced in the present study by the fact that students were working on a contract basis for a grade. This meant that the student knew that he or she could obtain an A if he or she completed a certain number of assignments. Perhaps, with this structure, the "pressure" that math anxious students normally report may have been lessened.

Similarly, the structure of statistics which emerged from the problem solving and problem sorting exercises may change if different teaching strategies and/or a different order of presentation of concepts had been used. Future research will also have to address this question.

There is one major area which can be extended, based on results of this study. It has been argued that the problems in the learning of statistics and probability are domain specific, i.e., mastery of this realm involves a certain kind of "seeing" and a certain kind of symbol usage the student had not encountered before. Thus, all kinds of "specific" problems arise: with probability, with distributions, with hypothesis testing, etc. As such, each of these subdomains must be investigated much more fully. Experimenters can design problems and tasks to test more specifically all of the processes and problems encountered at each stage of the learning of statistics. Only in this more detailed way, may we hope to understand fully the representation and integration students make of statistical concepts.

One possible path that this future research could take would involve exploring the problem solving processes and decision rules that differentiate experts and novices. These types of analyses have been offered by cognitive scientists in various domains; for example, see Chase and Simon 1973 for chess; Chi, Feltovich, and Glaser 1981 for physics; and Johnson 1980 for medical problem solving. In all cases, the structure of material in a specific domain is explored, via well-structured problems, to determine the differences in how highly trained

people and beginners represent concepts. This type of analysis has not been done for any realm of statistical knowledge and could be an appropriate extension of some of the results presented in this study.

The conflict between intuitive probabilities and formal concepts also could be documented more thoroughly. Only 5 statements of everyday probability were used in this study; these could be significantly expanded. More Piagetian experiments could be used, along with more detailed protocol analysis to try and specify the exact nature of the split between determinism and chance.

Finally, we must talk about the future, i.e., what will these students be like months from now, concerning the issue of math anxiety and the representation of statistical knowledge. Long-term studies might provide the most beneficial way to answer the question concerning problems people have in learning statistics.

Implications

Since the learning of statistics generally occurs in classroom contexts, the results of this study would seem to have their most immediate implications for the teaching of statistics. In this realm a number of suggestions may be offered, ranging from strategies to counter math anxieties in the self-world, to problems with the more formal nature of the teaching of statistics.

To overcome the fears and anxieties that certain students have developed about mathematics over the years is beyond the scope of any one particular teacher. The pattern displayed by the high anxiety

group, however, does present a number of temporary changes that can be instituted in the classroom. First, students must feel that they will not suffer humiliation or embarrassment. This can be achieved to some degree by not requiring threatening, isolating situations, i.e., board work, recital in front of the class, random calling upon people to answer questions, etc. Second, the asking of questions can be encouraged either directly, by making sure the teacher is patient and responds to all types of questions, or indirectly, by having students submit questions anonymously on sheets of paper. Third, "feelings of inferiority" can perhaps be countered by having students work in small groups (in class or out); in this manner, common problems will be shared and feelings that "I am the worst" can be offset by others who have the same opinion. Finally, time pressure can be reduced by having weekly quizzes instead of massive midterms and finals; in this way, the amount of "failure" at any particular test is small in the scope of the entire quarter. We might even consider removing time pressure altogether, by using various types of take home exercises and tests.

For the learning of the formal nature of statistics, it must be remembered that the student is encountering both a new language and a new kind of "seeing." As such, the use of new symbols should initially be kept to a minimum, be presented clearly, and reinforced with exercises that test acquisition and transfer. Since there is a carryover from everyday/intuitive probabilities that might interfere with learning, an initial part of the course should emphasize such commonsense

correspondences. Notions of chance, determinism, the gambler's fallacy, and other concepts should be discussed. Within the formal domain, all abstract concepts could be prefaced with concrete illustrations, allowing a student a more helpful path towards understanding the basis behind any abstractions. Finally, the split between probability and hypothesis testing should be emphasized and explained. In all cases, the abstract world can be made more intelligible through concrete examples well before students are expected to deal in the abstract logic of statistics.

The world of statistics and mathematics is a curious one indeed. By a college level, it has been preceded by years of formal schooling and by years of intuitive dealings with mathematics. In the classroom itself, a new world of terminology is presented, which sometimes may seem like Greek and at other times to approach the commonsense world. That statistics deals with the conflict between determinism and chance makes it more challenging than other branches of mathematics. That teaching at a college level involves students at once new to statistics but yet old to mathematics is equally frustrating and stimulating. It is this conflict between what a teacher thinks he or she knows and what students think they are learning that unites both teaching and learning and insures no definitive end to either process.

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APPENDIXES

APPENDIX A

INSTRUMENTS USED IN STUDY

APPENDIX A-1

NAME _____

Total Score _____

MATHEMATICS ANXIETY RATING SCALE (MARS)

The items in the questionnaire refer to things and experiences that may cause fear or apprehension. For each item, place a check (✓) in the box under the column that describes how much you are frightened by it nowadays. Work quickly but be sure to consider each item individually.

	Not at all	A little	A fair amount	Much	Very much
1. Determining the amount of change you should get back from a purchase involving several items.	☐	☐	☐	☐	☐
2. Having someone watch you as you total up a column of figures.	☐	☐	☐	☐	☐
3. Having someone watch you as you divide a five digit number by a two digit number.	☐	☐	☐	☐	☐
4. Being asked to add up $976 + 777$ in your head.	☐	☐	☐	☐	☐
5. Dividing a five digit number by a two digit number in private with pencil and paper.	☐	☐	☐	☐	☐
6. Calculating a simple percentage, e. g., the sales tax on a purchase.	☐	☐	☐	☐	☐
7. Listening to a salesman show you how you would save money by buying his higher priced product because it reduces long term expenses.	☐	☐	☐	☐	☐
8. Listening to a person explain how he figured out your share of expenses on a trip, including meals, transportation, housing, etc.	☐	☐	☐	☐	☐
9. Having to figure out how much it will cost to buy a product on credit (figuring in the interest rates).	☐	☐	☐	☐	☐
10. Totaling up a dinner bill that you think overcharged you.	☐	☐	☐	☐	☐
11. Telling the cashier that you think the dinner bill was incorrect and watching the cashier total up the bill.	☐	☐	☐	☐	☐

TOTAL _____

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	Not at all	A little	A fair amount	Much	Very much
12. Being treasurer for a club.	☐	☐	☐	☐	☐
13. Totaling up the dues received and the expenses of a club you belong to.	☐	☐	☐	☐	☐
14. Adding up $976 + 777$ on paper.	☐	☐	☐	☐	☐
15. Doing a word problem in algebra.	☐	☐	☐	☐	☐
16. Solving a problem such as: If $x = 11$, and $y = 3$, then the results of x/y is equal to _____?	☐	☐	☐	☐	☐
17. Solving the problem such as: If $x = 12$, and $y = 4$, then the ratio of x to y is equal to _____?	☐	☐	☐	☐	☐
18. Determining the grade point average for your last term.	☐	☐	☐	☐	☐
19. Reading an article on the basketball team, showing what percentage of free throws each player made, the percentage of field goals made, the total number attempted, etc.	☐	☐	☐	☐	☐
20. Reading an historical novel with many dates in it.	☐	☐	☐	☐	☐
21. Counting the number of pages left in a novel you are engrossed in.	☐	☐	☐	☐	☐
22. Guessing at the number of people attending a dance you're at.	☐	☐	☐	☐	☐
23. Buying a math textbook.	☐	☐	☐	☐	☐
24. Watching someone work with a slide rule.	☐	☐	☐	☐	☐
25. Watching a teacher work an algebraic equation on the blackboard.	☐	☐	☐	☐	☐
26. Signing up for a math course.	☐	☐	☐	☐	☐
27. Listening to another student explain a math formula.	☐	☐	☐	☐	☐
28. Walking into a math class.	☐	☐	☐	☐	☐
TOTAL	_____	_____	_____	_____	_____

	Not at all	A little	A fair amount	Much	Very much
29. Having to compute the miles/gallon on your car.	☐	☐	☐	☐	☐
30. Watching someone work with a calculator.	☐	☐	☐	☐	☐
31. Looking through the pages of a math text.	☐	☐	☐	☐	☐
32. Working on an income tax form.	☐	☐	☐	☐	☐
33. Reading your W-2 form (or other statement showing your annual earning and taxes).	☐	☐	☐	☐	☐
34. Studying for a math test.	☐	☐	☐	☐	☐
35. Starting a new chapter in a math book.	☐	☐	☐	☐	☐
36. Walking on campus and thinking about a math course.	☐	☐	☐	☐	☐
37. Meeting your math teacher while walking on campus.	☐	☐	☐	☐	☐
38. Reading the word "Statistics."	☐	☐	☐	☐	☐
39. Sitting in a math class and waiting for the instructor to arrive.	☐	☐	☐	☐	☐
40. Solving a square root problem.	☐	☐	☐	☐	☐
41. Signing up for a course in Statistics.	☐	☐	☐	☐	☐
42. Checking over your monthly bank statement.	☐	☐	☐	☐	☐
43. Taking the math section of a college entrance exam.	☐	☐	☐	☐	☐
44. Having someone explain bank interest rates as you decide on a savings account.	☐	☐	☐	☐	☐
45. Raising your hand in a math class to ask a question.	☐	☐	☐	☐	☐
46. Reading and interpreting graphs or charts.	☐	☐	☐	☐	☐
TOTAL	—	—	—	—	—

	Not at all	A little	A fair amount	Much	Very much
47. Reading a cash register receipt after your purchase.	☐	☐	☐	☐	☐
48. Figuring the sales tax on a purchase that costs more than \$1.00.	☐	☐	☐	☐	☐
49. Having a person illustrate to you the best way to divide your money into a savings and a checking account.	☐	☐	☐	☐	☐
50. Figuring out which of two summer job offers is the most lucrative: where one involves a lower salary, room and board, and travel, while the other one involves a higher salary but no other benefits.	☐	☐	☐	☐	☐
51. Reading a formula in chemistry.	☐	☐	☐	☐	☐
52. Hearing a lecture in a social science class where the instructor is commenting on some figures, e. g., the percentage of each socio-economic group who voted Republican.	☐	☐	☐	☐	☐
53. Taking an examination (quiz) in a math course.	☐	☐	☐	☐	☐
54. Taking an examination (final) in a math course.	☐	☐	☐	☐	☐
55. Hearing two of your friends exchanging opinions on the best way to calculate the cost of a product.	☐	☐	☐	☐	☐
56. Having someone ask you to recheck his figures in a simple calculation, such as division, or addition.	☐	☐	☐	☐	☐
57. Being asked by a friend to answer the question: how long will it take to get to Denver if I drive at 30 miles per hour?	☐	☐	☐	☐	☐
58. Studying for a driver's license test and memorizing the figures involved, such as the distances it takes to stop a car going at differing speeds.	☐	☐	☐	☐	☐
59. Hearing friends make bets on a game as they quote the odds.	☐	☐	☐	☐	☐
60. Playing cards where numbers are involved, e. g., bridge or poker.	☐	☐	☐	☐	☐
TOTAL	_____	_____	_____	_____	_____

	Not at all	A little	A fair amount	Much	Very much
61. Hearing a friend try to teach you a math procedure and finding that you cannot understand what he is telling you.	☐	☐	☐	☐	☐
62. Scheduling my daily routine to allocate set times for classes, for study time, for meals, for recreation, etc.	☐	☐	☐	☐	☐
63. Juggling class times around at registration to determine the best schedule.	☐	☐	☐	☐	☐
64. Deciding which courses to take in order to come out with the proper number of credit hours for full time enrollment.	☐	☐	☐	☐	☐
65. Working a <u>concrete, everyday</u> application of mathematics that has meaning to me, e. g., figuring out how much I can spend on recreational purposes after paying other bills.	☐	☐	☐	☐	☐
66. Working on an abstract mathematical problem, such as: "If x = outstanding bills, and y = total income, calculate how much you have left for recreational expenditures."	☐	☐	☐	☐	☐
67. Being given a set of numerical problems involving addition to solve on paper.	☐	☐	☐	☐	☐
68. Being given a set of subtraction problems to solve.	☐	☐	☐	☐	☐
69. Being given a set of multiplication problems to solve.	☐	☐	☐	☐	☐
70. Being given a set of division problems to solve.	☐	☐	☐	☐	☐
71. Picking up the math text book to begin working on a homework assignment.	☐	☐	☐	☐	☐
72. Being given a homework assignment of many difficult problems which is due the next class meeting.	☐	☐	☐	☐	☐
73. Thinking about an upcoming math test one week before.	☐	☐	☐	☐	☐
74. Thinking about an upcoming math test one day before.	☐	☐	☐	☐	☐

TOTAL

_____	_____	_____	_____	_____
-------	-------	-------	-------	-------

	Not at all	A little	A fair amount	Much	Very much
75. Thinking about an upcoming math test one hour before.	☐	☐	☐	☐	☐
76. Thinking about an upcoming math test five minutes before.	☐	☐	☐	☐	☐
77. Talking to someone in your class who does well about a problem and not being able to understand what he is explaining.	☐	☐	☐	☐	☐
78. Waiting to get a math test returned in which you expected to do well.	☐	☐	☐	☐	☐
79. Waiting to get a math test returned in which you expected to do poorly.	☐	☐	☐	☐	☐
80. Walking to math class.	☐	☐	☐	☐	☐
81. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.	☐	☐	☐	☐	☐
82. Picking up a math textbook to begin a difficult reading assignment.	☐	☐	☐	☐	☐
83. Being called upon to recite in a math class when you are prepared.	☐	☐	☐	☐	☐
84. Not knowing the formula needed to solve a particular problem.	☐	☐	☐	☐	☐
85. Receiving your final math grade in the mail.	☐	☐	☐	☐	☐
86. Opening a math or stat book and seeing a page full of problems.	☐	☐	☐	☐	☐
87. Being responsible for collecting dues for an organization and keeping track of the amount.	☐	☐	☐	☐	☐
88. Getting ready to study for a math test.	☐	☐	☐	☐	☐
89. Listening to a lecture in a math class.	☐	☐	☐	☐	☐
90. Figuring out your monthly budget.	☐	☐	☐	☐	☐
TOTAL	☐	☐	☐	☐	☐

	Not at all	A little	A fair amount	Much	Very much
91. Being given a "pop" quiz in a math class.	☐	☐	☐	☐	☐
92. Seeing a computer printout.	☐	☐	☐	☐	☐
93. Having to use the tables in the back of a math book.	☐	☐	☐	☐	☐
94. Being told how to interpret probability statements.	☐	☐	☐	☐	☐
95. Asking your math instructor to help you with a problem that you don't understand.	☐	☐	☐	☐	☐
96. Being asked to explain how you arrived at a particular solution for a problem.	☐	☐	☐	☐	☐
97. Tallying up the results of a survey or poll.	☐	☐	☐	☐	☐
98. Acting as secretary, keeping track of the number of people signing up for an event.	☐	☐	☐	☐	☐
TOTAL	—	—	—	—	—
Total Score	_____				

APPENDIX A-2

INITIAL INTERVIEW: OPEN-ENDED QUESTIONS

1. How did you feel about taking statistics before this course started?
2. How do you expect to do in this course?
3. How would you complete the following phrase: Taking statistics is like _____.
4. What do you think statistics is about? How does statistics relate to the study of psychology? to mathematics?
5. Describe the experience of being in a math class. How did you feel, what did you notice about time, your body, others around you?
6. How is your experience of mathematics different from/similar to that of other people? How can you tell?
7. In your daily life, when are you most aware of mathematics? least aware of math?
8. How are math classes different from/similar to other subjects you've taken?
9. What previous math courses have you taken? How have you done in these courses? Tell me about a time when you felt very comfortable in a math class. Tell me about a time when you felt uncomfortable in a math class.
10. What were your parents feelings about mathematics?
11. Have you noticed any changes in your attitudes towards mathematics as you've gotten older? How did these changes come about?
12. How would you feel if you were better than anyone else in math? worse than anyone else?
13. Do you plan on taking any more math-related courses?

APPENDIX A-3

INSTRUCTIONS: Please try to explain what each statement means as clearly as you can. In particular, what does each statement mean to you and where do you think the numbers involved in the statement come from?

STATEMENT:

1. I heard on the radio that there is a 40 percent chance of rain tomorrow.
2. 3 out of 4 doctors surveyed preferred Trident sugarless gum for their patients.
3. The average risk level for breast cancer for women in this country is 7%, but rises to 16¢ if two generations before have had it.
4. Studies have shown that smoking may be related to cancer.
5. This past weekend Tennessee was a 7 point favorite to beat Washington State.

APPENDIX A-4

PIAGETIAN PROBABILITY GAMES

I. Tilt Board

- A. Concept of RANDOM MIXTURE--will balls mix randomly or in a pattern?
- B. Concept of IRREVERSIBILITY--will balls return eventually to their starting position?

II. Heads and Tails

- A. Concept of A-Priori probability--will most likely drop be 50/50?
- B. Concept of LAW OF LARGE NUMBERS--as you increase the number of coins, will the probability of eccentric combinations decrease? (All heads or tails, e.g.)
- C. Concept of FIXEDNESS--how many attempts will be necessary to recognize a fix, i.e., that coins have been switched and they are all heads.
- D. Concept of RUNS GIVEN 5 CHIPS--given five chips, how likely is it to get 1, 2, 3, 4, or 5 heads in a row.
- E. Concept of RUNS GIVEN 10 CHIPS--given ten chips, how likely is it to get 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10 chips in a row.

III. Colored chips

- A. Concept of COMBINATIONS--will subject see a pattern to the number of possible combinations for 4, 5, 6, or more colored chips?

APPENDIX A-5

SELF-REPORT STUDY SHEET FOR STATISTICS 3150

NAME _____

PART I: TO BE COMPLETED AT THE BEGINNING OF YOUR STUDY SESSION

DATE:

TIME BEGUN:

TIME ENDED:

ASSIGNMENT ON WHICH YOU'RE WORKING:

1. How would you describe what you're thinking or feeling right now, at the beginning of this study session?

PART II: TO BE COMPLETED DURING OR AFTER STUDY SESSION

2. As you were reading the material or looking over lecture notes or studying for a test, which concepts and/or examples did you have trouble understanding? Briefly explain the trouble you had.
3. Which concepts and/or examples were clear to you?
4. Which problems at the end of the chapter or from homework assignments did you attempt to solve but were unable to work? Where did you have trouble?
5. Which problems did you solve?
6. In studying and working on this material, what, if anything, have you had to look up from previous lessons? (Formulas, definitions, examples, etc.)
7. Did you notice anything that you understand now that you didn't understand before?

8. Is there anything that you noticed about the material and/or yourself while you were studying?
9. Do you have any questions about anything related to the course?
10. How would you describe how you feel or what you're thinking, now, at the end of this study session?

APPENDIX A-6

PROBLEM SORTING EXPERIMENT

DIRECTIONS: I am going to give you 50 slips of paper, on each of which is printed a problem involving some type of statistical analysis which you have learned this quarter. Your task will be to sort these problems into piles. Each pile should contain problems which you feel are similar to each other, based on a common statistical concept or technique. You do not have to solve these problems but merely sort into as many piles as you feel necessary.

(AFTER THIS IS COMPLETED)

Now that you have these piles, please write out for me briefly why you sorted these problems the way you did. What does each pile represent (what kinds of problems)?

TASK CONSTRUCTION:

Five problems from each of the following domains (covering topics explained in this particular statistics course) will be selected from a workbook which accompanies R. Johnson's Elementary Statistics, the text used in this course. Students do not have access to this workbook.

DOMAINS:

- I. Frequency Distributions--Univariate Descriptive Statistics
- II. Simple Probability
- III. Binomial Probability
- IV. Hypothesis Testing and Statistical Inference
- V. Chi-Square
- VI. Normal Distribution
- VII. Sampling Distributions
- VIII. One Population Statistics
- IX. Two Population Statistics
- X. Linear Correlation and Regression

APPENDIX A-7

FINAL INTERVIEW: OPEN-ENDED QUESTIONS

1. How do you feel about statistics now that this course is almost finished?
2. Complete this phrase: Taking statistics is like _____.
3. Describe your experience of being in a statistics class this quarter. How did you feel, what did you notice about time, your body, others around you?
4. Tell me about a time when you felt very comfortable in statistics class. Tell me about a time when you felt uncomfortable.
5. Did you notice any changes in your attitudes or feelings towards statistics throughout this quarter?
6. How has your experience of statistics been different from/similar to other people in the course?
7. How has statistics been different from/similar to other courses you've taken this quarter?
8. What problems, if any, have you had with this course?
9. Do you plan to take any more math related courses?
10. What are the important concepts that have been covered in this course? Explain in your own words or by using paper and pencil what each concept means.

APPENDIX A-8

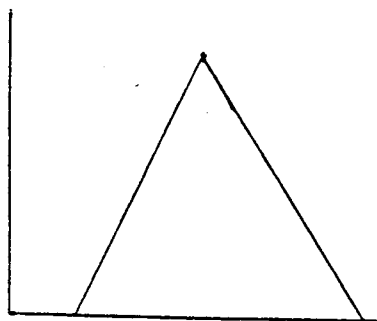
PROBLEM SOLVING EXPERIMENT

DIRECTIONS: During this part of your final interview, I am going to give you a few problems which I would like you to solve right now. Instead of working silently and writing out your answers, however, I would like you to talk out loud as you are trying to solve the problem. That is, tell me whatever comes to mind as you are thinking about the problem.

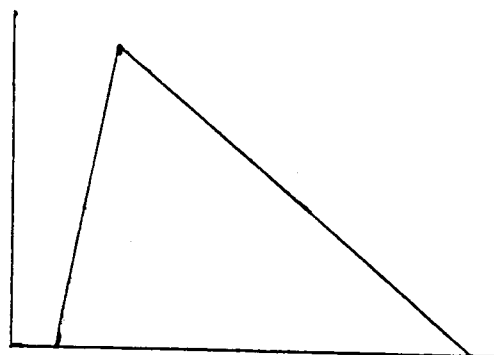
TASK CONSTRUCTION: Problems and questions taken from 3 major areas of material covered in this course will be the focus of this verbal problem part of the experiment. Each problem or problem set within the 3 domains has been selected to allow for flexibility of questioning on the part of the interviewer and to test the depth of knowledge attained by the student. The 3 domains and the specific problems include:

- I. Distributions and Measures of Central Tendency and Dispersion: The student will be presented with five different graphs, each of a different distribution, and asked to indicate on the graph the following quantities: mean, median, mode. The student will also be asked to explain the judgments which are made. Labels will not appear (see Appendix A-9).
- II. Binomial Probability: The student will be presented with part of a binomial probability table and asked to explain how the table is used, particularly what n , x , and p are and what happens when these values change (see Appendix A-10).
- III. Chi-Square and Hypothesis Testing: The student will be presented with a chi-square table and be asked to explain how it is used, particularly with concern to differing p values and differing df (see Appendix A-11).

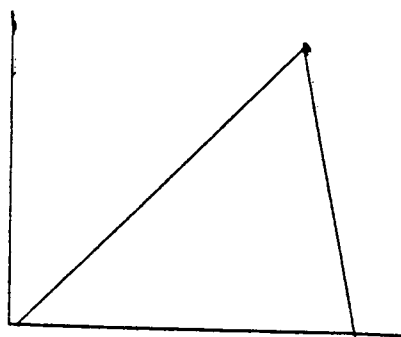
APPENDIX A-9



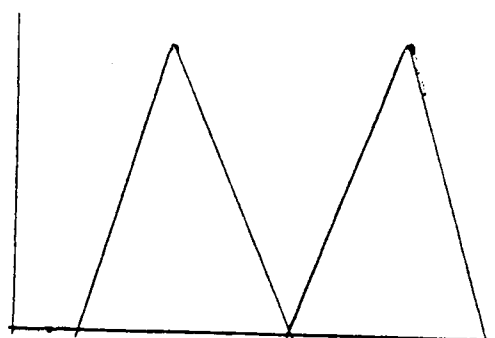
SYMMETRIC



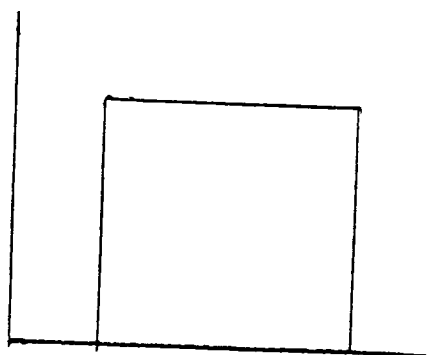
POSITIVELY SKEWED



NEGATIVELY SKEWED



BIMODAL

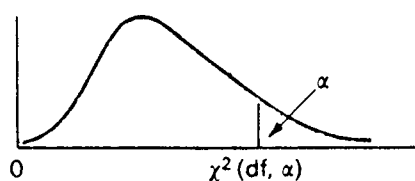


RECTANGULAR

APPENDIX A-10

n	z	p														z
		.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99		
9	0	914	630	387	134	040	010	002	0+	0+	0+	0+	0+	0+	0	
	1	083	299	387	302	156	060	018	004	0+	0+	0+	0+	0+	1	
	2	003	063	172	302	267	161	070	021	004	0+	0+	0+	0+	2	
	3	0+	008	045	176	267	251	164	074	021	003	0+	0+	0+	3	
	4	0+	001	007	066	172	251	246	167	074	017	001	0+	0+	4	
	5	0+	0+	001	017	074	167	246	251	172	066	007	001	0+	5	
	6	0+	0+	0+	003	021	074	164	251	267	176	045	008	0+	6	
	7	0+	0+	0+	0+	004	021	070	161	267	302	172	063	003	7	
	8	0+	0+	0+	0+	0+	004	018	060	156	302	387	299	083	8	
9	0+	0+	0+	0+	0+	0+	002	010	040	134	387	630	914	9		
10	0	904	599	349	107	028	006	001	0+	0+	0+	0+	0+	0+	0	
	1	091	315	387	268	121	040	010	002	0+	0+	0+	0+	0+	1	
	2	004	075	194	302	233	121	044	011	001	0+	0+	0+	0+	2	
	3	0+	010	057	201	267	215	117	042	009	001	0+	0+	0+	3	
	4	0+	001	011	088	200	251	205	111	037	006	0+	0+	0+	4	
	5	0+	0+	001	026	103	201	246	201	103	026	001	0+	0+	5	
	6	0+	0+	0+	006	037	111	205	251	200	088	011	001	0+	6	
	7	0+	0+	0+	001	009	042	117	215	267	201	057	010	0+	7	
	8	0+	0+	0+	0+	001	011	044	121	233	302	194	075	004	8	
	9	0+	0+	0+	0+	0+	002	010	040	121	268	387	315	091	9	
10	0+	0+	0+	0+	0+	0+	001	006	028	107	349	599	904	10		
11	0	895	569	314	086	020	004	0+	0+	0+	0+	0+	0+	0+	0	
	1	099	329	384	236	093	027	005	001	0+	0+	0+	0+	0+	1	
	2	005	087	213	295	200	089	027	005	001	0+	0+	0+	0+	2	
	3	0+	014	071	221	257	177	081	023	004	0+	0+	0+	0+	3	
	4	0+	001	016	111	220	236	161	070	017	002	0+	0+	0+	4	
	5	0+	0+	002	039	132	221	226	147	057	010	0+	0+	0+	5	
	6	0+	0+	0+	010	057	147	226	221	132	039	002	0+	0+	6	
	7	0+	0+	0+	002	017	070	161	236	220	111	016	001	0+	7	
	8	0+	0+	0+	0+	004	023	081	177	257	221	071	014	0+	8	
	9	0+	0+	0+	0+	001	005	027	089	200	295	213	087	005	9	
10	0+	0+	0+	0+	0+	001	005	027	093	236	384	329	099	10		
11	0+	0+	0+	0+	0+	0+	0+	004	020	086	314	569	895	11		
12	0	886	540	282	069	014	002	0+	0+	0+	0+	0+	0+	0+	0	
	1	107	341	377	206	071	017	003	0+	0+	0+	0+	0+	0+	1	
	2	006	099	230	283	168	064	016	002	0+	0+	0+	0+	0+	2	
	3	0+	017	085	236	240	142	054	012	001	0+	0+	0+	0+	3	
	4	0+	002	021	133	231	213	121	042	008	001	0+	0+	0+	4	
	5	0+	0+	004	053	158	227	193	101	029	003	0+	0+	0+	5	
	6	0+	0+	0+	016	079	177	226	177	079	016	0+	0+	0+	6	
	7	0+	0+	0+	003	029	101	193	227	158	053	004	0+	0+	7	
	8	0+	0+	0+	001	008	042	121	213	231	133	021	002	0+	8	
9	0+	0+	0+	0+	001	012	054	142	240	236	085	017	0+	9		

APPENDIX A-11



df	Amount of γ in Right-hand Tail									
	0.995	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010	0.005
1	0.0000393	0.000157	0.000982	0.00393	0.0158	2.71	3.84	5.02	6.64	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	6.00	7.38	9.21	10.6
3	0.0717	0.115	0.216	0.352	0.584	6.25	7.82	9.35	11.4	12.9
4	0.207	0.297	0.484	0.711	1.0636	7.78	9.50	11.1	13.3	14.9
5	0.412	0.554	0.831	1.15	1.61	9.24	11.1	12.8	15.1	16.8
6	0.676	0.872	1.24	1.64	2.20	10.6	12.6	14.5	16.8	18.6
7	0.990	1.24	1.69	2.17	2.83	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	14.7	17.0	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.58	5.58	17.2	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	18.6	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.90	7.04	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	23.5	26.3	28.9	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	29.6	32.7	35.5	39.0	41.4
22	8.64	9.54	11.0	12.3	14.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.0	13.1	14.9	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.9	15.7	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	34.4	37.7	40.7	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	36.7	40.1	43.2	47.0	49.7
28	12.5	13.6	15.3	16.9	18.9	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.1	17.7	19.8	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	40.3	43.8	47.0	50.9	53.7
40	20.7	22.2	24.4	26.5	29.1	51.8	55.8	59.3	63.7	66.8
50	28.0	29.7	32.4	34.8	37.7	63.2	67.5	71.4	76.2	79.5
60	35.5	37.5	40.5	43.2	46.5	74.4	79.1	83.3	88.4	92.0
70	43.3	45.4	48.8	51.8	55.3	85.5	90.5	95.0	100.0	104.0
80	51.2	53.5	57.2	60.4	64.3	96.6	102.0	107.0	112.0	116.0
90	59.2	61.8	65.7	69.1	73.3	108.0	113.0	118.0	124.0	128.0
100	67.3	70.1	74.2	77.9	82.4	114.0	124.0	130.0	136.0	140.0

Adapted from E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, vol. I (1962), pp. 130-131. Reprinted by permission of the Biometrika Trustees.

APPENDIX B

MARS SCALE

Table 10. MARS Scale Items on Which High and Low Groups Differ

Question	Means	
	Low	High
4. Being asked to add up 976+777 in your head.	1.9	2.9
8. Listening to a person explain how he figured out your share of expenses on a trip, including meals, transportation, housing, etc.	1.2	2.3
9. Having to figure out how much it will cost to buy a product on credit (figuring in the interest rates).	1.7	2.8
12. Being treasurer for a club.	1.2	1.9
13. Totaling up the dues received and the expenses of a club you belong to.	1.0	2.0
15. Doing a word problem in algebra.	2.2	3.7
16. Solving a problem such as: If $x=11$, and $y=3$, then the results of x/y is equal to ____.	1.3	3.1
17. Solving the problem such as: If $x=12$, and $y=4$, then the ratio of x to y is equal to ____.	1.4	3.0
23. Buying a math textbook.	1.4	3.0
25. Watching a teacher work an algebraic equation on the blackboard.	1.3	3.2
26. Signing up for a math course.	2.2	4.1

Table 10 (Continued)

Question	Means	
	Low	High
27. Listening to another student explain a math formula.	1.7	3.8
28. Walking to a math class.	1.7	3.6
31. Looking through the pages of a math text.	1.3	3.0
34. Studying for a math test.	2.3	4.4
35. Starting a new chapter in a math book.	2.1	4.1
36. Walking on campus and thinking about a math course.	1.4	3.8
37. Meeting your math teacher while walking on campus.	1.1	2.8
38. Reading the word "Statistics."	2.2	4.0
39. Sitting in a math class and waiting for the instructor to arrive.	1.2	2.6
40. Solving a square root problem.	1.5	3.4
41. Signing up for a course in Statistics.	2.3	4.0
45. Raising your hand in a math class to ask a question.	1.5	3.3
47. Reading a cash register receipt after your purchase.	1.0	1.4

Table 10 (Continued)

Question	Means	
	Low	High
53. Taking an examination (quiz) in a math course.	2.3	4.4
54. Taking an examination (final) in a math course.	3.1	4.8
67. Being given a set of numerical problems involving addition to solve on paper.	1.0	1.4
68. Being given a set of subtraction problems to solve.	1.0	1.5
69. Being given a set of multiplication problems to solve.	1.0	1.5
70. Being given a set of division problems to solve.	1.0	1.6
71. Picking up the math text book to begin working on a homework assignment.	1.7	3.3
72. Being given a homework assignment of many difficult problems which is due the next class meeting.	2.6	4.1
73. Thinking about an upcoming math test one week before.	2.1	4.1
74. Thinking about an upcoming math test one day before.	3.0	4.7
75. Thinking about an upcoming math test one hour before.	3.2	5.0

Table 10 (Continued)

Question	Means	
	Low	High
76. Thinking about an upcoming math test five minutes before.	3.3	4.7
77. Talking to someone in your class who does well about a problem and not being able to understand what he is explaining.	2.7	4.2
78. Waiting to get a math test returned in which you expected to do well.	2.5	4.3
80. Walking to math class.	1.1	2.9
81. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.	1.9	4.0
82. Picking up a math textbook to begin a difficult reading assignment.	2.1	4.0
83. Being called to recite in a math class when you are prepared.	1.8	3.3
84. Not knowing the formula needed to solve a particular problem.	2.9	4.8
86. Opening a math or stat book and seeing a page full of problems.	2.5	3.9
88. Getting to study for a math test.	1.8	3.5
89. Listening to a lecture in a math class.	1.2	2.8

Table 10 (Continued)

Question	Means	
	Low	High
91. Being given a "pop" quiz in a math class.	2.7	4.3
93. Having to use the tables in the back of a math book.	1.6	2.9
94. Being told how to interpret probability statements.	1.6	4.0
95. Asking your math instructor to help you with a problem that you don't understand.	1.5	3.1
96. Being asked to explain how you arrived at a particular solution for a problem.	1.6	3.6
97. Tallying up the results of a survey or poll.	1.4	2.3
98. Acting as secretary, keeping track of the number of people signing up for an event.	1.1	1.6

Table 11. MARS Scale Items on Which High and Low Groups Do Not Differ

Question	Means	
	Low	High
1. Determining the amount of change you should get back from a purchase involving several items.	1.3	1.4
2. Having someone watch you as you total up a column of figures.	1.9	2.6
3. Having someone watch you as you divide a five digit number by a two digit number.	1.9	2.4
5. Dividing a five digit number by a two digit number in private with pencil and paper.	1.1	1.1
6. Calculating a simple percentage, e.g., the sales tax on a purchase.	1.2	1.7
7. Listening to a salesman show you how you would save money by buying his higher priced product because it reduces long-term expenses.	1.3	1.9
10. Totalling up a dinner bill that you think overcharged you.	1.5	1.7
11. Telling the cashier that you think the dinner bill was incorrect and watching the cashier total up the bill.	1.3	1.8
14. Adding up 976+777 on paper.	1.5	1.2
18. Determining the grade point average for your last term.	1.4	1.7

Table 11 (Continued)

Question	Means	
	Low	High
19. Reading an article on the basketball team, showing what percentage of free throws each player made, the percentage of field goals made, the total number attempted, etc.	1.5	1.8
20. Reading an historical novel with many dates in it.	1.5	1.7
21. Counting the number of pages left in a novel you are engrossed in.	1.1	1.2
22. Guessing at the number of people attending a dance you're at.	1.0	1.1
24. Watching someone work with a slide rule.	1.8	2.2
29. Having to compute the miles/gallon on your car.	1.4	1.6
30. Watching someone work with a calculator.	1.4	1.6
32. Working on an income tax form.	2.4	3.2
33. Reading your W-2 form (or other statement showing your annual earning and taxes).	1.6	2.1
42. Checking over your monthly bank statement.	1.8	2.1
43. Taking the math section of a college entrance exam.	2.4	3.5

Table 11 (Continued)

Question	Means	
	Low	High
44. Having someone explain bank interest rates as you decide on a savings account.	1.6	2.0
46. Reading and interpreting graphs or charts.	2.0	2.7
48. Figuring the sales tax on a purchase that costs more than \$1.00.	1.6	1.3
49. Having a person illustrate to you the best way to divide your money into a savings and a checking account.	1.4	1.7
50. Figuring out which of two summer job offers is the most lucrative: where one involves a lower salary, room and board, and travel, while the other one involves a higher salary but no other benefits.	1.5	1.6
51. Reading a formula in chemistry.	2.7	3.4
52. Hearing a lecture in a social science class where the instructor is commenting on some figures, e.g., the percentage of each socio-economic group who voted Republican.	1.3	1.6
55. Hearing two of your friends exchanging opinions on the best way to calculate the cost of a product.	1.3	1.8

Table 11 (Continued)

Question	Means	
	Low	High
56. Having someone ask you to recheck his figures in a simple calculation, such as division, or addition.	1.1	1.8
57. Being asked by a friend to answer the question: how long will it take to get to Denver if I drive at 30 miles per hour?	1.4	2.0
58. Studying for a driver's license test and memorizing the figures involved, such as the distances it takes to stop a car going at differing speeds.	1.4	1.5
59. Hearing friends make bets on a game as they quote the odds.	1.5	1.6
60. Playing cards where numbers are involved, e.g., bridge or poker.	1.2	1.3
61. Hearing a friend try to teach you a math procedure and finding that you cannot understand what he is telling you.	2.8	3.7
62. Scheduling my daily routine to allocate set times for classes, for study time, for meals, for recreation, etc.	1.4	1.6
63. Juggling class times around at registration to determine the best schedule.	1.8	2.0
64. Deciding which courses to take in order to come out with the proper number of credit hours for full time enrollment.	1.6	1.4

Table 11 (Continued)

Question	Means	
	Low	High
65. Working a <u>concrete, everyday</u> application of mathematics that has meaning to me, e.g., figuring out how much I can spend on recreational purposes after paying other bills.	1.2	1.7
66. Working on an abstract mathematical problem, such as: "If $x =$ outstanding bills, and $y =$ total income, calculate how much you have left for recreational expenditures."	1.9	2.9
79. Waiting to get a math test returned in which you expected to do poorly.	3.4	3.9
85. Receiving your final math grade in the mail.	3.2	4.2
87. Being responsible for collecting dues for an organization and keeping track of the amount.	1.2	1.7
90. Figuring out your monthly budget.	1.5	1.8
92. Seeing a computer printout.	1.9	2.8

Table 12. MARS Scale Items on Which High and Low Anxiety Groups Differ on the Posttest

Question	Means	
	High	Low
10. Totaling up a dinner bill that you think overcharged you.	1.4	1.0
15. Doing a word problem in algebra.	3.2	2.2
16. Solving a problem such as: If $x=11$, and $y=3$, then the result of x/y is equal to ____.	2.3	1.2
18. Determining the grade point average for your last term.	1.8	1.1
25. Watching a teacher work an algebraic equation on the blackboard.	2.9	1.6
26. Signing up for a math course.	3.2	1.7
27. Listening to another student explain a math formula.	2.5	1.3
28. Walking into a math class.	2.6	1.4
34. Studying for a math test.	3.4	1.9
35. Starting a new chapter in a math book.	3.3	1.8
36. Walking on campus and thinking about a math course.	2.9	1.6
43. Taking the math section of a college entrance exam.	3.5	2.6
54. Taking an examination (final) in a math course.	3.7	2.3

Table 12 (Continued)

Question	Means	
	High	Low
61. Hearing a friend try to teach you a math procedure and finding that you cannot understand what he is telling you.	3.5	2.4
71. Picking up the math book to begin working on a homework assignment.	2.6	1.4
72. Being given a homework assignment of many difficult problems which is due the next class meeting.	3.3	2.3
73. Thinking about an upcoming math test one week before.	2.8	1.6
74. Thinking about an upcoming math test one day before.	3.6	2.2
75. Thinking about an upcoming math test one hour before.	4.0	2.7
76. Thinking about an upcoming math test five minutes before.	4.1	2.7
77. Talking to someone in your class who does well about a problem and not being able to understand what he is explaining.	3.7	2.6
78. Waiting to get a math test returned in which you expected to do well.	3.4	2.0
80. Walking to a math class.	2.2	1.1

Table 12 (Continued)

Question	Means	
	High	Low
81. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.	3.4	2.0
82. Picking up a math textbook to begin a difficult reading assignment.	3.4	2.2
85. Receiving your final math grade in the mail.	3.3	1.9
86. Opening a math or stat book and seeing a page full of problems.	3.2	1.9
88. Getting ready to study for a math test.	2.7	1.6
89. Listening to a lecture in math class.	2.2	1.3
93. Having to use the tables in the back of a math book.	2.3	1.2
96. Being asked to explain how you arrived at a particular solution for a problem.	2.8	1.8
97. Tallying up the results of a survey or poll.	2.0	1.2

APPENDIX C

PORTIONS OF AN INITIAL INTERVIEW

I = Interviewer

S = Subject

I: Tell me what you were feeling or thinking about taking Statistics before the course started.

S: Much anticipation. I only signed up for it because I needed to take it. I put off every other math course, but I felt I had to get this one out of the way. I haven't taken 1540 or 1550 yet.

I: You have to take those?

S: Yes, but I'm putting them off until, like, Summer School, because I'm very bad at it. I don't know, for some reason, I think I've talked myself into it, but I really am. From past high school experience, I've not shown to be really good at math.

I: O.K. What did you mean by anticipation?

S: Not anticipation . . . like, anxiety. I don't anticipate, or get excited about the class at all. Sorry. It's just the word Statistics. Psychology doesn't bother me; just Statistics. I didn't know what that meant, whether it had to do with word problems or what. Like I said, I've not been known to do too well with that. Anything with math always makes me nervous. Stay with simple problems . . .

I: Describe to me what it's like, the experience of being in a math class, or a class that deals with math.

S: Utter confusion--no, normally, I can understand--I can certainly grasp what's going on if it's going really slowly. Like being tested on it, I'll just freeze. I guess it just takes me a long time to get the right answer, to understand what's going on. I have to go back and have someone explain it to me. I don't grasp it right off the bat. I did good in geometry, where you can draw it out, but algebra just . . . well, I don't have much common sense. Maybe that's why I don't do well in algebra.

I: So a math class is a little bit of confusion . . .

S: Yeah.

I: Especially tests?

S: I think it's basically . . . yeah, especially tests. I can study long hours and get into a test and just not understand what we're

supposed to be doing. I think I've talked myself into thinking I won't understand it, so I just don't understand it. But usually, if I look at it a long time . . .

I: Why? Why do you think so?

S: I don't know. I think it's because I was bad in high school or something. I like English, history and that type of thing. I was just never too good at it. But actually, I do find that I enjoy working with numbers; it's like working out a little puzzle or something--that's interesting. When the pressure's off, I do a lot better, than like when I'm being required to do it. Like formulas. Formulas, as a whole, don't go over well with me.

I: Why?

S: I don't know. I guess they're too abstract. I want, I like things stated just so, so I can understand it, like charted down. When it's too abstract, you have to supply things, and figure them out. I like things where you can look at them, and memorize and you know. Like, History is good, cause you can relate to other things. And English is good. But math--it's too cut and dried. Get this right or you don't get it at all.

I: Do you not memorize the formula, or . . .?

S: It's just applying the numbers to the formula and getting the right answer that . . . I can usually remember what the formula is, it's just doing it that makes it hard. It's actually calculating the formula, doing the whole thing that makes it hard.

I: And that's what puts the pressure on?

S: Yes. I think I feel lost in my capabilities of doing math. It's just a feeling of utter helplessness, like I don't know what I'm doing. I mean, I've tried where I thought I've done it right, and it's been totally wrong. It's something really basic that I've messed up.

I: What happens when you get a wrong answer?

S: I usually give up. No . . . Actually, I thought I could do a lot better, it's just that I . . . but I've found that I can do it a lot better just working as a salesperson. I'm really not as slow as I thought I was, until like, on your questionnaire you had, when people were looking at you, I start feeling the pressure again. Maybe it's like I've got a phobia against teachers. Or math teachers. You know . . . next?

I: What about your experience in mathematics? Do you think of it as similar to or different from other people?

S: I think it's similar to a lot of people. I think a lot of people have trouble with math, as far as the people I've talked to.

VITA

Michael K. Smith was born in London, Kentucky, on December 31, 1952. He attended London Elementary School until the third grade. His family moved to Knoxville, Tennessee, when he was nine years old. He attended Pond Gap Elementary School and West High School and was graduated from the latter in 1970. He attended The University of Tennessee, Knoxville, and was graduated from there in 1976 with a Bachelor's Degree in History. He entered the graduate program in psychology in 1976, also at The University of Tennessee, Knoxville, and was graduated with a Ph.D. in 1984.

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