

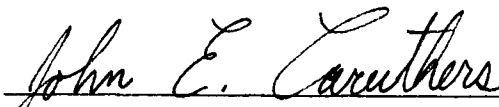
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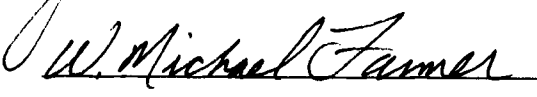
I am submitting herewith a dissertation written by Kao-Huah Huang entitled "Monte Carlo Particle Dispersion (MoCaPD) Model of Battlefield Obscuration." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Mechanical Engineering.



Walter Frost, Major Professor

We have read this dissertation
and recommend its acceptance:









Accepted for the Council:



The Graduate School

MONTE CARLO PARTICLE DISPERSION (MoCaPD)
MODEL OF BATTLEFIELD OBSCURATION

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Kao-Huah Huang

March 1984

ACKNOWLEDGMENT

The work reported herein was supported by FWG Associates, Inc. under contract from the U.S. Army Research Office and Battelle, Research Triangle Park, North Carolina (Contract No. DAAG29-81-D-0100).

The author is indebted to Mr. Gary Nelson, Mr. Bob Dekinder, and Mr. James F. O'Bryon, U.S. Army Aberdeen Proving Ground, Maryland. and Dr. Robert A. Sutherland, Atmospheric Sciences Laboratory, White Sands Missile Range, New Mexico, for their support of this research.

Special thanks should go to Dr. Walter Frost, the director of Atmospheric Science Division, University of Tennessee Space Institute, the President of FWG Associates, Inc., for his valuable guidance and assistance ensured the success of this program. Also, the suggestions and discussions with Dr. W. Michael Farmer has been most valuable.

The author is further indebted to Dr. John E. Caruthers, Dr. Frank Collins, and Dr. K. C. Reddy for their review of the present work. Thanks also due to Mrs. Mary Lo, Institute librarian for her great assistance of the author's research effort.

Sincerest appreciation is given to the author's family, especially to his mother, Mrs. Lien-Jin L. Huang, and his wife Ching-Li for their persistent encouragement and constant support.

At last but far from the least, the author would like to thank Mrs. Barbara Smith, Mrs. Debbie Batchelor, and Mr. Robert Suttles for their efforts to translate the manuscript into presentable form.

ABSTRACT

The essential input for an optical study of short-term dispersion of battelfield obscurants is the number density spatial distribution for each particle size at a particular line of sight. These number density distributions should account for ground-level source effect and must be spatially and temporally dependent. The basic Monte Carlo particle dispersion (MoCaPD) model approach consists of computing an ensemble or collection of time histories of random particle trajectories referenced to a fixed coordinate system. These random functions of position and time and of prevailing meteorological conditions are stored in a data file and then used to carry out statistical analyses or "real-time" simulations. The motion of particles is statistically consistent with the elementary Lagrangian turbulence properties of the lower atmospheric boundary layer. Moreover, the wind fields encountered by nearby particles are coherent (spatially correlated). The MoCaPD model includes all of these features and can predict temporal and spatial particle size distributions and number density probabilities of smoke or dust obscuring the view of a target detection system located relative to a smoke source.

The capabilities of the model are shown by comparison of the computed results with data measured from obscurant field trials. The obscurant material examined include red phosphorus (RP), hexachlorethane (HC), and Arizona road dust. Reasonable agreement between theory and experiment is demonstrated.

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NOMENCLATURE

A	Cross-sectional area of a particle
AP	Aerosol photometer
C	Mass concentration of the aerosol
C_D	Drag coefficient of particles
CL	Concentration path length (gm/m^2)
$D(w,z)$	Lag filter function
D_i	Largest particle diameter in the i th size interval
D_0	Initial particle diameter
DPG	Dugway Proving Ground
F	Fraction of acid concentration in water
$H(f,\vec{r})$	Filter function in frequency domain
HC	Hexachlorethane
L	Transmission path length
L'	Monin-Obukhov stability length scale
LOS	Line of sight
LPCI	Low pressure cascade impactor
N	Total number of particles
N_i	Total number of i th size particles that have been released
N_{oij}	Total number of i th size particles from source # j
P	Discrete size probability
Pr	Prandtl number
$P_j(D_i)$	Probability of mass distribution of j th source (i th size particles)
PMS	Particle measurement systems
PSI	Particle sizing interferometer

Q	Volumetric strength of source (m^3/sec)
Q_E	Extinction efficiency
R_1, R_2	Distance from particle to source in Equation 4.1
Re	Reynolds number
$R(r)$	Auto-correlation function
$R_L(\tau)$	Lagrangian time correlation function
RP	Red phosphorus
SWIV	Smoke Week IV
T	Transmittance
T_E	Eulerian time scale
T_L	Lagrangian time scale
U	Velocity induced by the jet (m/s)
U_0	Velocity at exit of nozzle (m/s)
V	Mean wind velocity (m/s)
V_a	Velocity of the particle relative to air
$\bar{W}_V$	Vertical mean wind speed
$\bar{W}_H$	Horizontal mean wind speed
$\bar{W}(\vec{r})$	Mean wind vector
a	Decay parameter of coherence function (Equation 3.42)
a_1	Constant $a_1 = V/\Lambda_x$
a_3	Constant $a_3 = V/\Lambda_z$
b	Constant $b = a_3/\sqrt{3}$
b	Distance from the centerline of the jet to the point where velocity $U = 1/2 U_m$ (Equation 4.4)
b_0	Radius of nozzle exit
f	Frequency

g	Acceleration of gravity
$h(t, \vec{r})$	Filter function in time domain
k	Wave number
k_1	Constant $k_1 = 2a_1\sigma_x/\pi$
k_3	Constant $k_3 = 3a_3\sigma_x^2/\pi$
ℓ	Integral length scale
m	Mass of particle
$\dot{m}$	Total mass release rate of the obscurant
$\dot{m}(\hat{t}_j)$	Mass emission rate from source #j
$\hat{n}_i(x_j, t)$	Number of particles that reached the line of sight
$p(D_i, x_j, t)$	Probability of the number density of size i at time t (those particles that came from source #j)
t_ℓ	Lifetime of particle
u_*	Friction velocity
u, v, w	Velocity near the source region (m/s)
$w(\vec{r}, t)$	Turbulent fluctuation component
z_0	Surface roughness parameter

Greek Symbols

α	Mass extinction coefficient
β	Effective heat transfer coefficient
$\gamma^2(\hat{\eta})$	Coherent function
ΔT	Dewpoint temperature depression
ϵ	Turbulence viscous dissipation per unit mass
$\hat{\eta}$	Reduced frequency, $\hat{\eta} = f z/\bar{V}$
η	Constant $\eta = z/b$ in Equation 4.2
κ	von Karman constant

λ	Wavelength (μm)
Λ	Turbulence length scale
μ	Constant $\mu = \exp(-\Delta t/T_L)$ in Equation 3.32
μ_2	Second moment of particle size distribution
μ_3	Third moment of particle size distribution
ν	Constant $\nu = (1 - \mu^2)^{1/2}$ in Equation 3.32
ξ_o	Constant $\xi_o = 1/[a(k\Delta z)_{\max}]$
ρ_o	Density of obscurant material
ρ_a	Air density
ρ_N	Particle number density
σ	Turbulent intensity
$\overline{\sigma}$	Mean extinction cross section
τ	Lag time
ϕ	Relative humidity in Equation 4.16
$\phi(f)$	Power density spectrum
$\phi(z/L')$	Stability parameter in Equation 2.5

1.0 INTRODUCTION

The objective of this research is to demonstrate the applicability and validity of the Monte Carlo Particle Dispersion Model (MoCaPD) methodology to war game scenarios and smoke generator design.

Battlefield obscurants, in particular, smoke munitions, can impact heavily on the effectiveness of existing and proposed electro-optical weapon systems. The use of obscurants can result in transmission losses and in beam spreading in active electro-optical systems. These effects can limit, for example, the performance of laser-designation, homing missiles. Although the results from field tests can provide a considerable amount of information on the effectiveness of the operational systems, it is not economical to test all systems under all types of atmospheric conditions. Since models are a viable tool in system evaluation and war-gaming, there is a need to develop simple but realistic battlefield smoke diffusion models to relate electromagnetic wave propagation conditions to the prevailing meteorology and to the obscurant source characteristics. A simple model should provide a systematic parameterization of short-term dispersion from ground-level sources.

Numerous working models of the atmospheric diffusion of smoke have been derived by analytical techniques (for summaries see Gifford (1975), Pasquill (1975), Drake et al. (1979), and Hosker (1980)). These models have been useful in a number of specialized applications. However, they are deficient in at least two important areas:

1. The ability to predict a given particle size distribution and/or number density as a function of time and spatial position along a prescribed line of sight, and
2. The ability to account for the characteristics of the smoke generator or munition in terms of the obscurant particulate and chemical composition.

Some of the dispersion models were developed especially for smoke obscurant studies, such as the HECSON model (Dumbauld and Bjorklund (1977)), the SOM II model (Baker and Pinkley (1978)), and the ACT I model (Gomez et al. (1979)). However, they were developed using the Gaussian formulation of cloud transport and diffusion and are not able to predict the number density distribution for each particle size at a line of sight, and the particle size distribution is an essential parameter for specifying the optical characteristics of an obscurant. The MoCaPD model is able to overcome these and other limitations with appreciable detail, including the effects of temporal mean wind speed variation with height over flat terrain and the qualitative effect of wind speed variation due to terrain features. Simulated turbulent fluctuations which reproduce the statistical properties of the prevailing atmospheric turbulence can also be superimposed on the mean wind fields. However, the wind fluctuation that a particle encounters during its motion must be described by Lagrangian turbulence. Moreover, particles that are released at separate source positions should experience a coherent wind field rather than independent wind fields. These last features were not included in the original MoCaPD model (Frost et al (1982)).

This dissertation first conceptually describes the MoCaPD model in Section 2. The computational procedure is outlined by illustrating the flow of the analysis from the introduction of the particle trajectory calculation to the final solution for the optical properties at a given position and time downwind of the obscurant source. Section 3 describes the methodology to generate coherent Lagrangian turbulence that is a physically realistic model of the atmospheric environment. Both Eulerian and Lagrangian numerical turbulence generators are reviewed, and an estimation of the Lagrangian time scale is discussed. Comparisons of the computed results with experimental data is given by Section 4. Three trials from a large field trial involving smoke and obscurants, Smoke Week IV (SWIV), are chosen to represent different obscurant features and different release methods. The results lead to the conclusion that the MoCaPD model is a viable tool for studying the sensitivity and trends in optical properties of battlefield environments associated with variation of the parameters governing the source characteristic and the meteorological conditions.

2.0 MoCaPD MODEL

2.1 Model Concept

The MoCaPD model computes the individual trajectories of many particles released into a simulated turbulent wind field. The equations governing the motion of the particle are the classical force balance equations with the aerodynamic drag, lift, and moment forces included as functions of Reynolds number and the Cunningham correction factor. A large number of particles for each size increment in the distribution of interest is released. The particle position and time are collected from which statistical analysis can be carried out. Figure 2.1 illustrates typical computed trajectories for three different sizes of particles; namely, 200, 100, and 20 μm . Only six trajectories are plotted for clarity along with the average trajectory. In general, 50 to 1000 particles of each size are released in a routine analysis. The wind field is composed of a logarithmic temporal mean wind with superimposed turbulence. In Figure 2.1, it is clearly seen that the smaller particles are more sensitive to the high frequency fluctuations and thus follow a more erratic path while the larger particles respond only to the lower frequency disturbances and are less erratic.

The method for application of the MoCaPD model to smoke obscuration studies is shown in Figure 2.2. In this figure, the particles are released from a smoke generator located 2 m above the terrain. The trajectory of each particle is computed and those that pass through a given volume element located at x_1 , y_1 , and z_1 are recorded with respect to time. Then the time history of the number of particles within the volume element is constructed. In other words, the probability that a

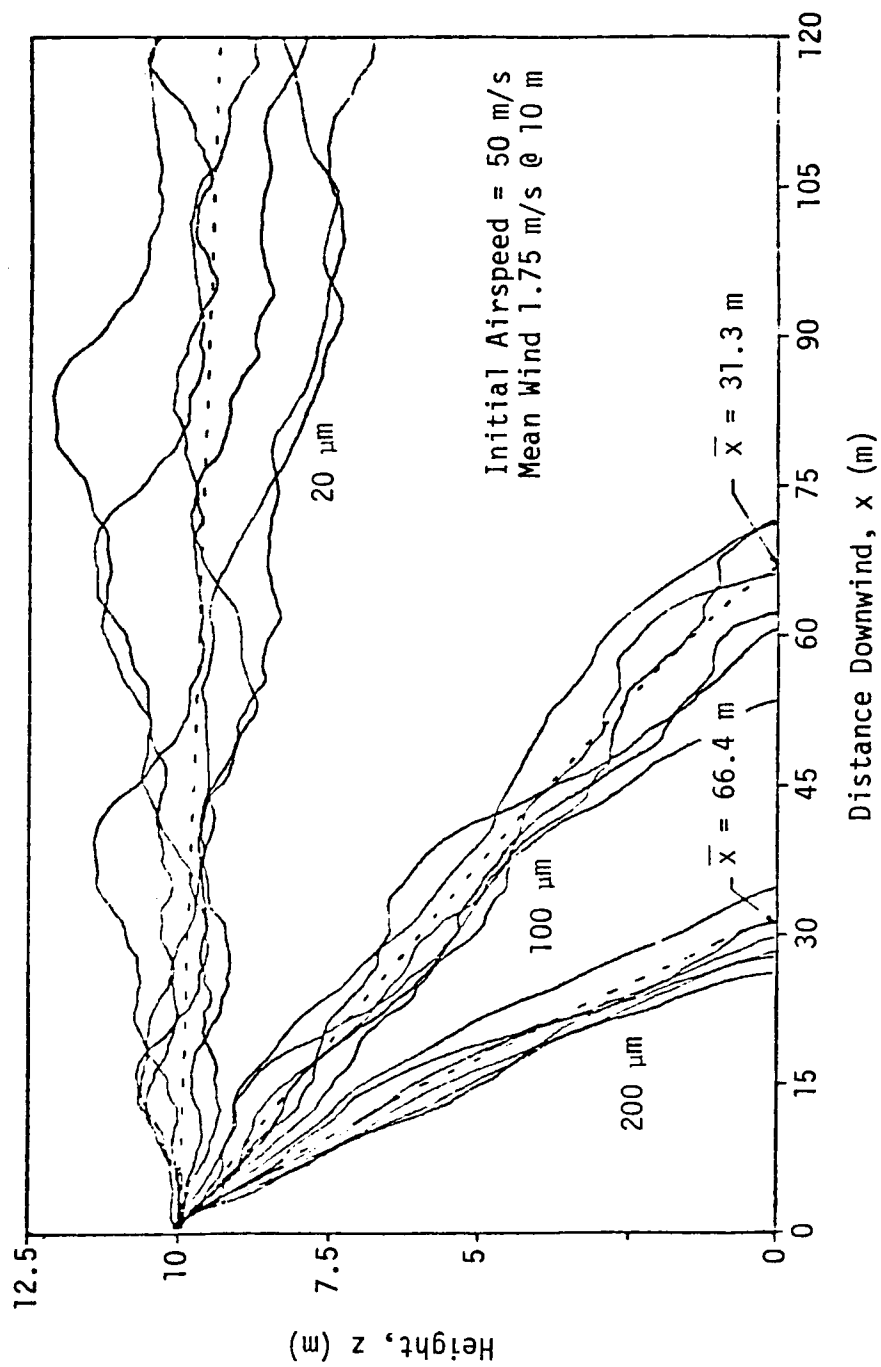


Figure 2.1 Trajectories of particles ejected at a height of 10 m into an atmospheric boundary layer.

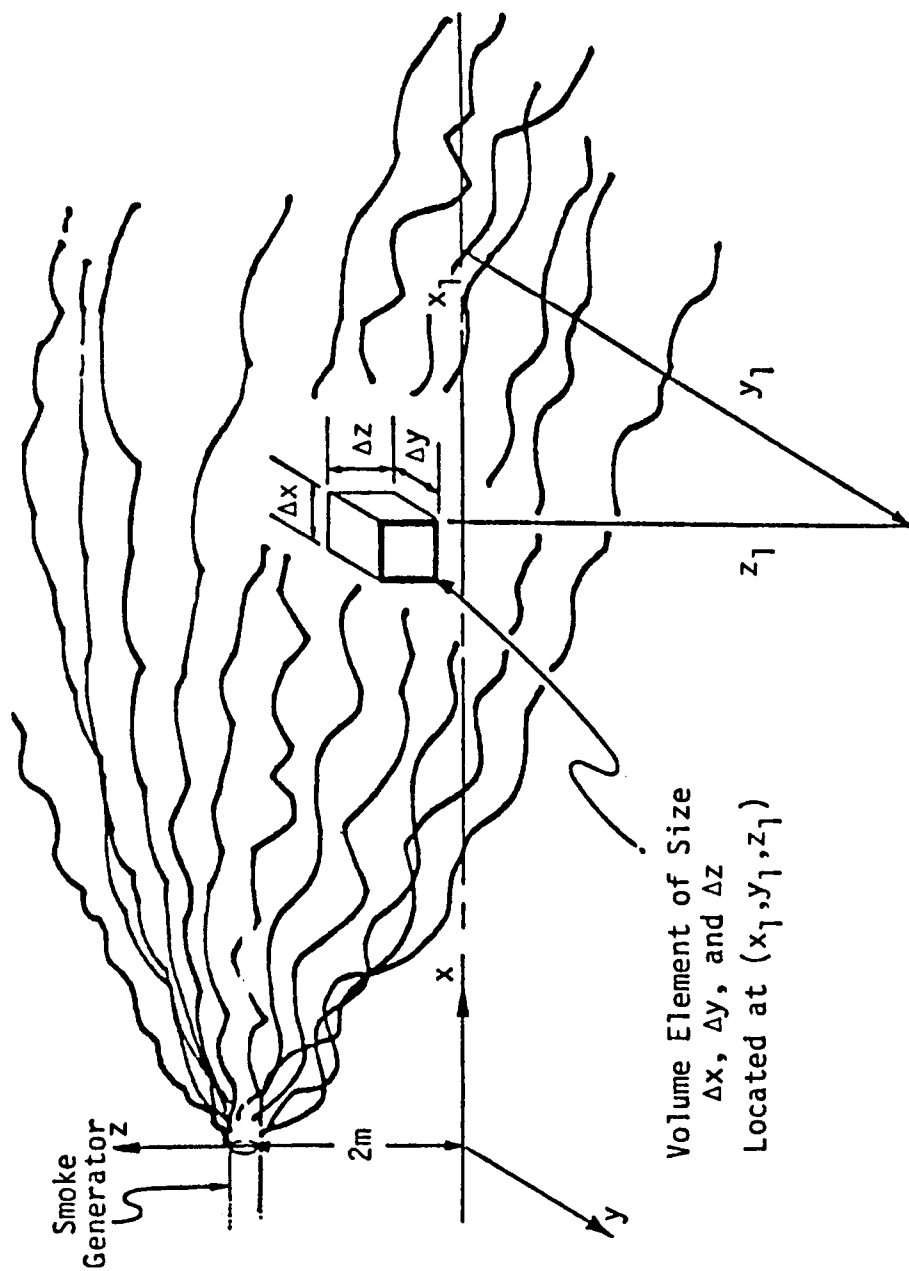


Figure 2.2 Illustration of a simple dispersion problem and the coordinate system.

given size particle will appear in a selected volume element in space at a given time can be determined.

2.2 Computational Procedure

The basic computational procedure of the MoCaPD model is schematically illustrated in Figure 2.3. The three-dimensional equations of motion based on a force balance on a particle are set up. The major forces affecting the particle are the aerodynamic forces, which are strongly influenced by the wind field and turbulence levels; the aerodynamic drag, lift, and moment coefficients, which yield the aerodynamic forces, are input as functions of the Reynolds number and the Cunningham correction factor. The physical and chemical properties of the particles enter the calculation through the density, particle size, and shape. Also, hygroscopic growth, evaporation, coagulation, and other chemical and physical effects can be modeled along the trajectory by inputting appropriate mathematical models.

The obscurant source characteristics must be known to give the initial mathematical conditions which are input to the Runge-Kutta or finite difference techniques that are used for solving the governing differential equations.

The output from the calculation, which is then a function of the inputs described above, is a collection or an ensemble of particle trajectories or position-time histories for each particle size of interest. This ensemble of particle trajectories is then used to carry out the statistical analysis required by the user for performance evaluation, field test planning, experimental data interpretation, or

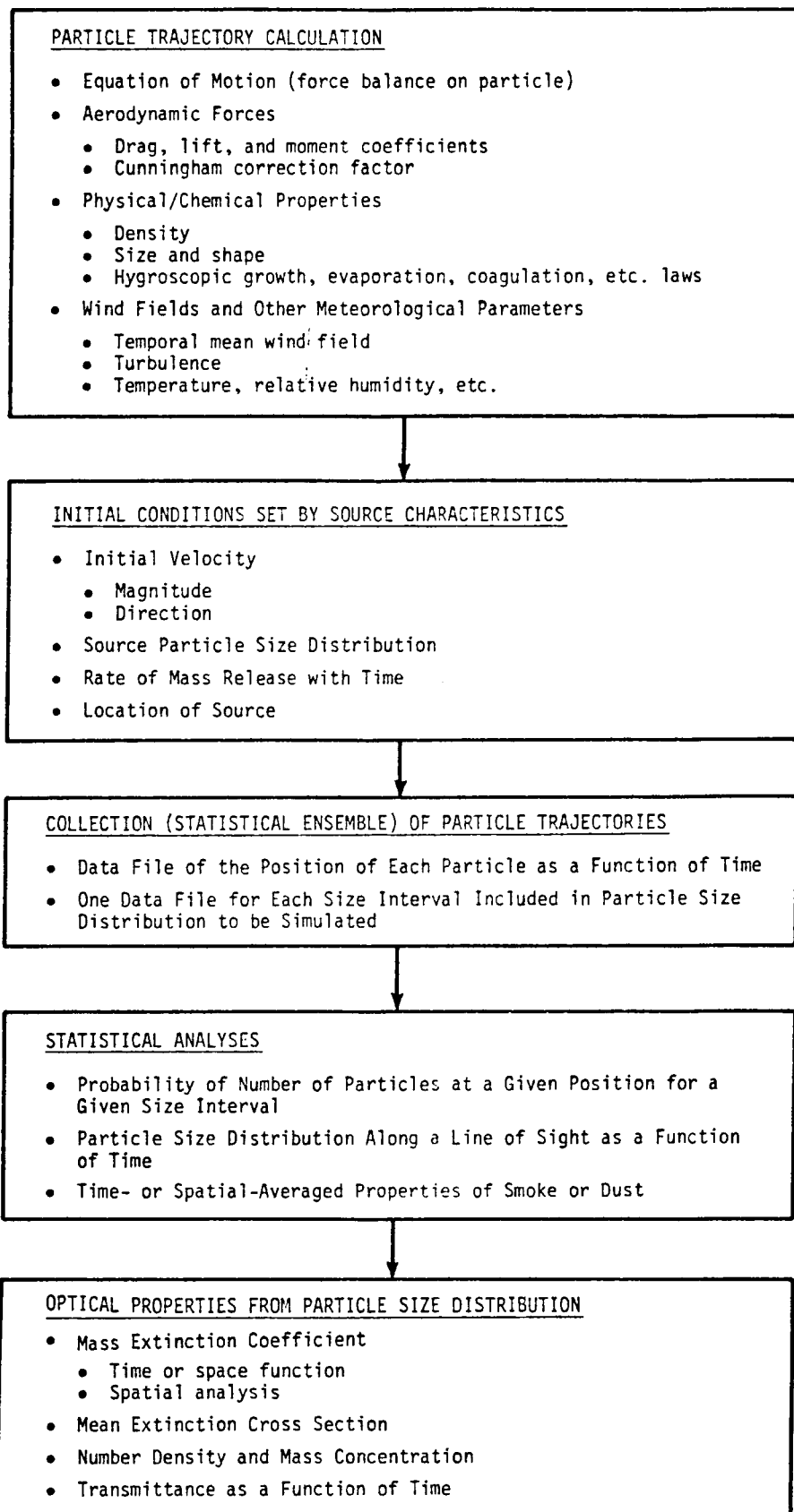


Figure 2.3 Basic computational procedure of the MoCaPD model.

parametric studies for general guidelines and sensitivity investigations of the parameters of an obscuration system.

By simple bookkeeping computations, which can be easily carried out, such parameters as the number or mass density of particles can be calculated at a given time interval. These, of course, are conditional probabilities which are based on the inputs, which are the source characteristics, the wind field, and the physical/chemical properties of the aerosol material.

Finally, the particle size distribution computed from the statistical analyses are used as inputs to determine the optical properties of the modeled battlefield environment. The computed optical properties are statistical in nature, which, of course, is also true of real battlefield environments. Typical parameters which can be computed as a function of time and space are the mass extinction coefficient, the mean extinction cross section, mass concentration, number density, and finally from these, the transmittance as a function of time and position in space. Thus, the computational procedure has the capability of closing the loop from the source of the obscurant to the space- and time-dependent optical properties of the environment. In the following subsections, a brief description is given of the five different stages in the computation.

2.2.1 Trajectory Calculations

2.2.1.1 Equations of Particle Motion. The equations governing the particle motion in an inertial Cartesian coordinate system fixed to the earth (with the positive z axis vertical to the earth) are:

$$\begin{aligned}
\ddot{m}\ddot{x} &= - \frac{1}{2} \rho_a A C_D (\dot{x} - W_x) |V_a| \\
\ddot{m}\ddot{y} &= - \frac{1}{2} \rho_a A C_D (\dot{y} - W_y) |V_a| \\
\ddot{m}\ddot{z} &= - \frac{1}{2} \rho_a A C_D (\dot{z} - W_z) |V_a| - mg
\end{aligned} \tag{2.1}$$

where m is the mass of a particle with cross-sectional area A ; ρ_a is the air density; and C_D is the drag coefficient defined as a function of Reynolds number; W_x , W_y , and W_z are the three instantaneous wind velocity components; V_a is the velocity of the particle relative to the air; and g is the acceleration of gravity. A more complete system of equations can be employed if the particles have lift and moment properties (see Frost and Koepf (1981), Huang and Frost (1982)). Assumptions currently made in the model, which can be readily relaxed as needed, are that gravity, temperature, and density are constant and the particles are spherical. The drag law for spheres as a function of Reynolds number and the Cunningham correction factor are currently used to compute C_D . The drag coefficient of a sphere is shown in Figure 2.4.

These conditions apply to a passive obscurant of spherical shape. The model can handle non-spherical particles by adding aerodynamic lift forces to the right-hand side of the equations. The effects of variable particle size due to hygroscopic growth, evaporation, coalescence, etc. can also be modeled by adding to the computer program appropriate equations governing the physical phenomena. The evaporation and growth laws must be functions of time to allow the new diameter of the particle to be computed at each time in the calculation. Evaporation is currently modeled with the relationship given by Trayford and Welch (1977):

$$d = d_0 (1 - t/t_\ell)^{1/2} \tag{2.2}$$

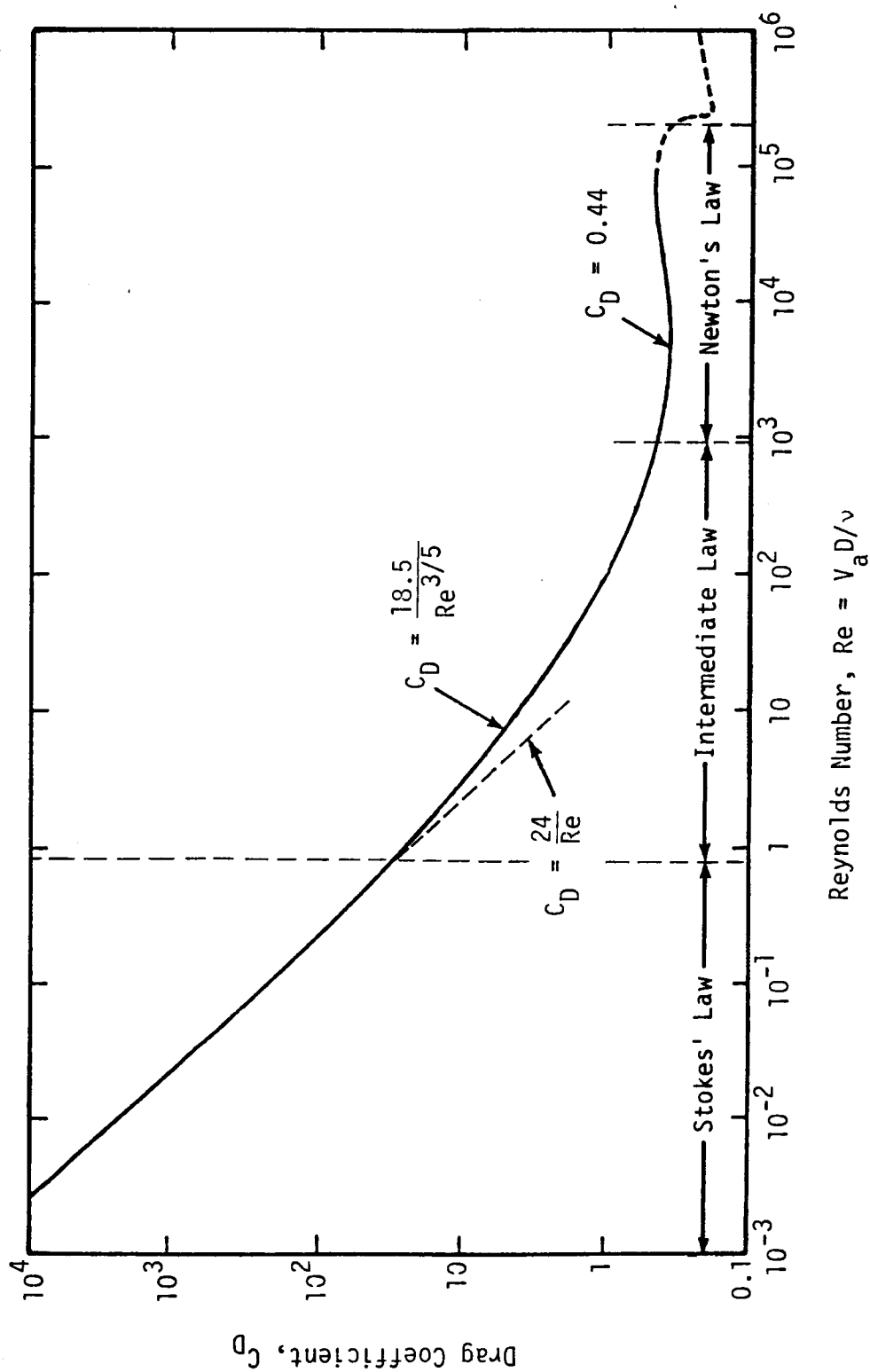


Figure 2.4 Friction (or drag coefficient) for spheres moving relative to a fluid with a velocity, V_a (Bird et al. (1960)).

The equation predicts the droplet diameter, d , as a function of time, t ; d_0 represents the initial diameter of the particle as it leaves the spray nozzle; and t_ℓ is given by:

$$t_\ell = d_0^2 / \beta \Delta T \quad (2.3)$$

where ΔT is the dewpoint temperature depression (a measure of relative humidity) and β is an effective heat transfer coefficient, expressed as a function of Prandtl number and Reynolds number by

$$\beta = 84.76(1 + 0.3 \text{Pr}^{1/3} \text{Re}^{1/2}) \times 10^{-12} \quad (2.4)$$

The initial conditions, which are dependent on the source characteristics, give the initial particle position, size, and velocity. The effect of the source will be discussed later.

2.2.1.2 Wind Field. The instantaneous wind velocity is composed of a time-averaged wind vector, $\vec{W}(\vec{r})$, and a fluctuating component, $\vec{w}(\vec{r}, t)$. A number of mean wind fields can be incorporated into MoCaPD, but currently the mean wind field simulated is that of the lower planetary boundary, which is reasonably and accurately described by a log-linear law (Frost et al. (1978), Haugen (1973)) in the form:

$$\begin{aligned} \overline{W}_H &= \frac{u_*}{\kappa} \left(\ln \left(\frac{z + z_0}{z_0} \right) + \psi(z/L') \right) \\ \overline{W}_z &= 0 \end{aligned} \quad (2.5)$$

where u_* is the friction velocity determined from the mean wind at a given reference height; κ is von Karman's constant; z_0 is the roughness parameter, which is a function of terrain and vegetation; and ψ is a stability parameter which is a function of $\zeta = z/L'$, where L' is the

Monin-Obukhov stability length scale. In this equation, $\bar{W}_H$ is the horizontal component of the mean wind and $\bar{W}_Z$ is the vertical component (see Figure 2.2 for the coordinate system).

The turbulent wind field is formed by superimposing a simulated turbulence fluctuation onto the mean wind field, i.e.,

$$\vec{W}(\vec{r}, t) = \vec{W}(\vec{r}) + w(\vec{r}, t) \quad (2.6)$$

Figure 2.5 illustrates the basic idea of obtaining the turbulent component of wind. A Gaussian white noise is computer-generated and passed through a digital filter which is designed to produce a random signal having the same statistical properties as the turbulence found in the atmosphere. More details will be given in Section 3. Figure 2.6 gives an example of both the measured and simulated data. Because the records are random, a one-to-one comparison cannot be made, but the two records are statistically similar, as will be demonstrated in Section 3.

However, turbulence which is generated in this way and employed by the previous version of the MoCaPD model (Frost et al. (1982)) is applicable to particles emitted at a single point. Thus, if it is assumed that the two particles illustrated in Figure 2.7 enter the turbulent field at the same time, then the model would predict identical trajectories, which is incorrect. On the other hand, to treat particles released at separate positions as experiencing independent turbulence time histories is also incorrect because the turbulent flow is strongly coherent over short distances.

Figure 2.8 shows measured turbulent time histories of the longitudinal wind velocity component measured at three stations separated laterally in a horizontal line perpendicular to the mean wind (see

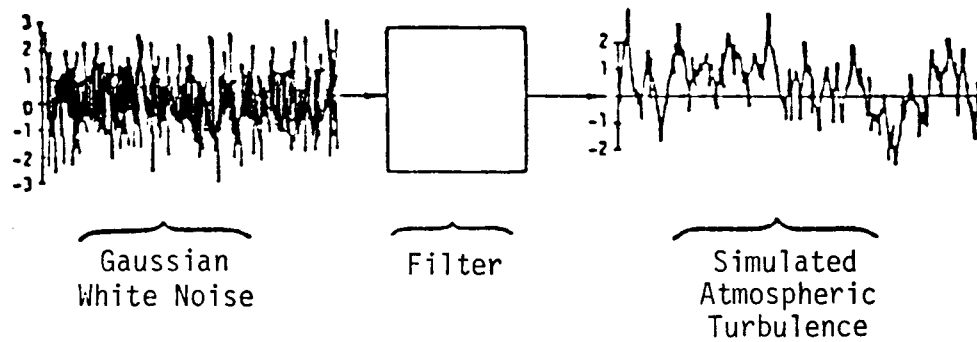


Figure 2.5 Turbulence simulation technique.

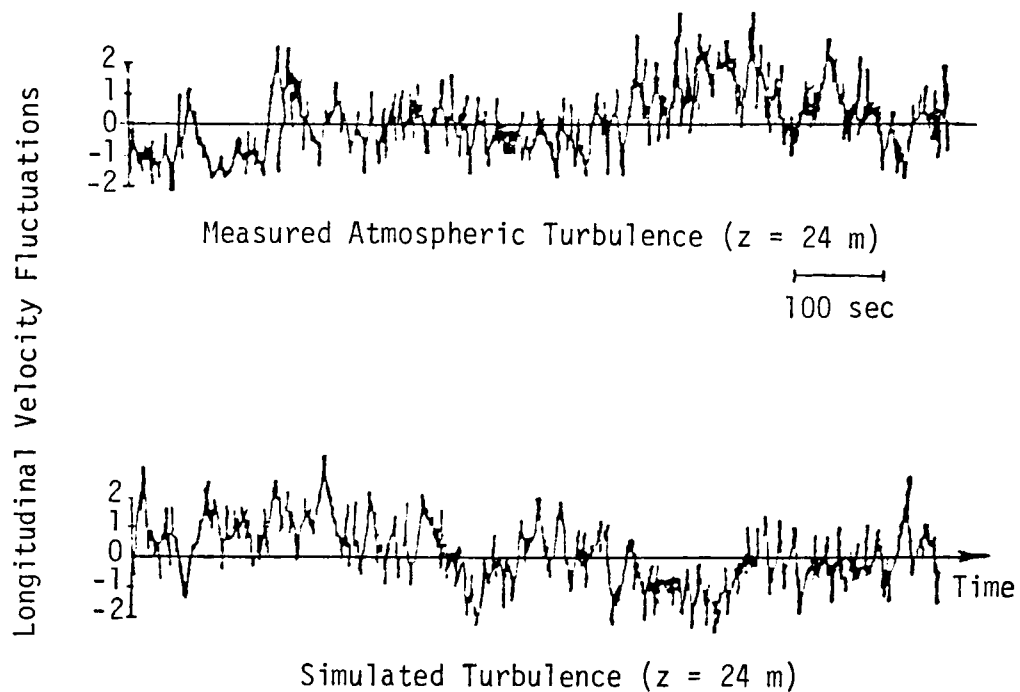


Figure 2.6 Time histories of tower-measured and simulated turbulence.

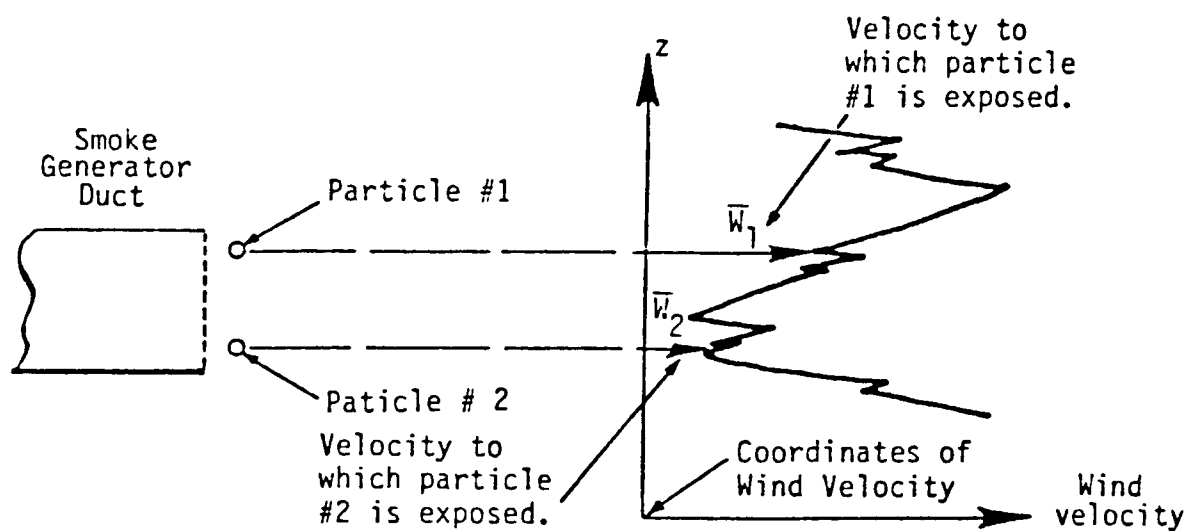


Figure 2.7 Wind velocity experienced by two different particles exiting the smoke generator.

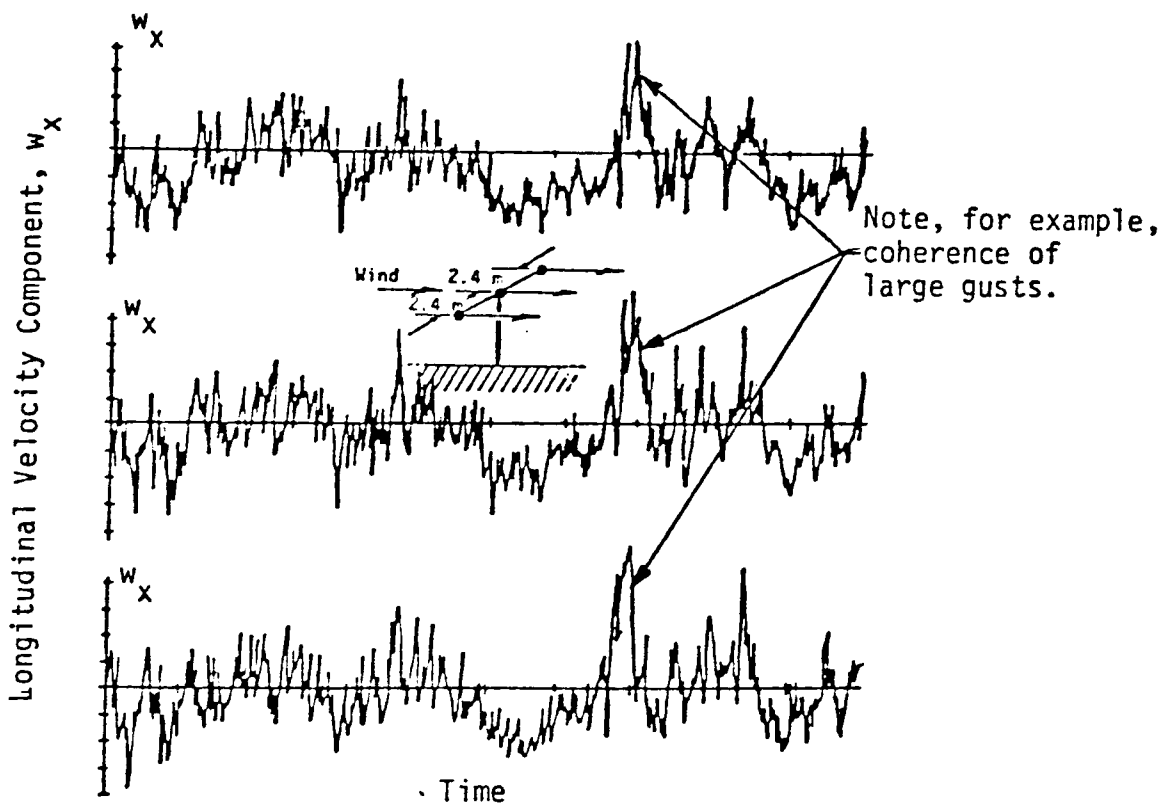


Figure 2.8 Illustration of the coherent structure of the longitudinal velocity component in the lateral direction ($\cdot$ indicates the position of measurements).

insert in this figure). One observes very strong coherence between long-period (or low-frequency) gusts with less or no coherence between short-period (or high-frequency) gusts. The coherence between eddies of large sizes causes the smoke to meander, and the lack of coherence between eddies of small sizes causes dispersion. No other current dispersion models appear to take the specific details of the coherence into effect. Details of the coherent turbulence generator which is employed in the current MoCaPD model will be given in Section 3.

Another problem with the previous formulation is that the simulated turbulence was described in an Eulerian system which would describe the turbulence measured at a fixed point. However, actual observations of smoke dispersion indicate that a smoke puff often appears to preserve its direction of travel with little change over times substantially longer than those over which the fixed-point measurements display appreciable variations of direction. Pasquill (1974) points out that this is the result of the difference of spectra between the Eulerian and Lagrangian description of the turbulence. He indicates the slope of the Eulerian description of turbulence in the inertial subrange fits a $-5/3$ frequency law while the Lagrangian description of turbulence fits a -2 frequency law. Details of this difference will be given in Section 3.

2.2.2 Source Characteristics

The source characteristics which are required as input to the solution of the governing equations are the initial velocity (magnitude and direction), particle size distribution of the particles issuing from the source, rate of the obscurant mass/particles released as a function

of time, and the position of the source relative to the location to the measurement position. Four typical source types which can be qualitatively modeled are munitions, smoke generators, explosive dust clouds, and aerial releases. Figure 2.9 illustrates schematically how these various sources can be modeled. Details of each source type are given by Frost et al. (1982).

An example will demonstrate the effect of the source characteristic on battlefield obscuration. Two typical types of sources used for battlefield obscuration are the smoke grenade and the nozzle-type smoke generator. These sources are elevated above the terrain at a 2-m height, as shown in Figure 2.10. Considering a simplified source modeling technique, the magnitude of initial velocity is Gaussian randomly distributed ($\dot{x}_i = 20 \pm 5$ m/s) for the smoke grenade, and the angle of the initial velocity is uniform randomly distributed. For the nozzle-type smoke generator, the magnitude of the initial velocity is chosen as constant ($\dot{x}_i = 10$ m/s) and the angle of the initial velocity is oriented parallel to the x axis. Before showing the results, the procedure for the collection of the particle trajectory and the statistical analysis of the number densities will be discussed.

2.2.3 Collection (Statistical Ensemble) of Particle Trajectories

A computer program which performs the computations described above allows rapid generation of many particle trajectories as functions of time and spatial position for realistic atmospheric conditions and specific source emission characteristics. Two major applications of the computer program are possible: one is to compute particle size distribution and number density on a "real-time" basis in conjunction with a

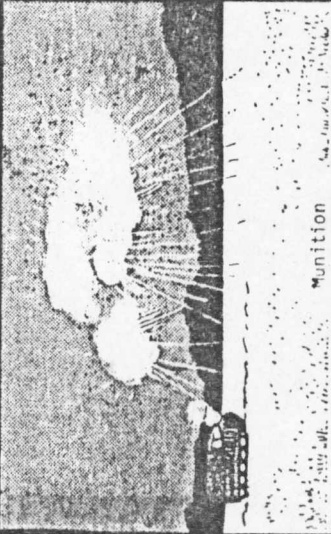
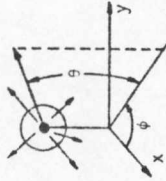
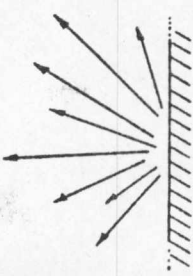
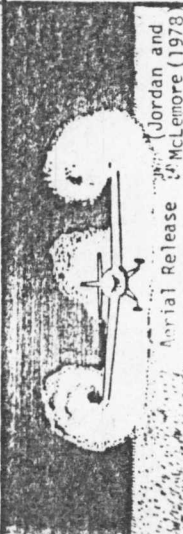
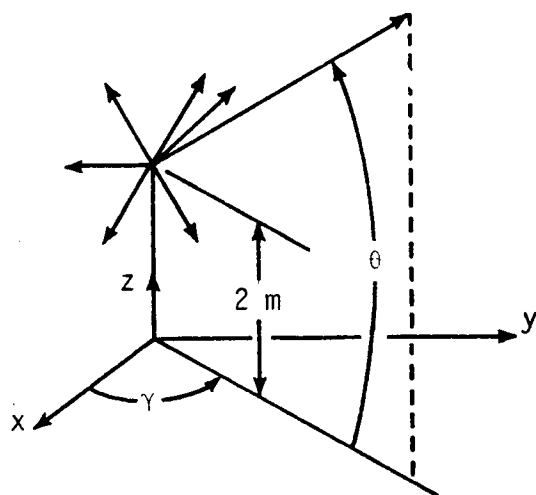
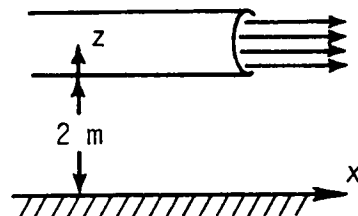
SOURCE TYPE	SIMULATION	INPUT REQUIREMENTS
		• Velocity direction and magnitude are selected with a random generator. • Release rate varies with time. • Particle size distribution.
		• Velocity and direction deterministic. • Release rate constant. • Particle size distribution.
		• Velocity magnitude and direction with a random number generator. • Rate of release. • Soil-type properties. • Particle size distribution.
		• Velocity magnitude deterministic, direction selected within limits randomly. • Rate of release function of system pressure and nozzle selection. • Particle size distribution specified by manufacturer.

Figure 2.9 Illustration of modeling techniques and required inputs for source characterization.



(a) Smoke grenade.



(b) Nozzle.

Figure 2.10 Simple source modeling technique.

weapons system performance simulation and the other is to store the trajectories in a data file for further statistical analysis.

Under the first approach, the particle size distribution and number density at a specific location or along a given line of sight can be computed for each time step of the weapon system performance computation. Consider a simple problem illustrated in Figure 2.2. Particles are released from a smoke generator located at a 2-m height. A collecting volume is located downstream at x_1 , y_1 , and z_1 .

Several factors influence the number of particles which pass through the volume element:

1. Size and initial velocity of the particles leaving the smoke generator.
2. The rate and size distribution of the particles released.
3. Mean wind speed and random turbulent eddies encountered along the trajectory.

4. Mean temperature and humidity and the random temperature fluctuations encountered along the trajectory for evaporating particles. Additional factors influencing the particle behavior are coalescence, nucleation, inter-particle forces, and chemical reactions.

5. Spatial location and size of the volume element.

Of course, the mean wind and turbulence will be strongly influenced by the terrain features and surface roughness, as well as by the release mechanism. Thus, the statistics are conditions depending on the terrain as well as the listed factors.

In this simple analysis the horizontal mean wind speed is described by a log-linear profile, which implies flat homogeneous terrain. Also, all effects described in Factor #4 above are at this time considered negligible. A random wind speed fluctuation which satisfies prescribed statistical properties of atmospheric turbulence is superimposed on the mean wind.

Instead of choosing the collecting "volume" (Δx , Δy , Δz) as shown in Figure 2.2, a line of sight slab is more interesting for optical studies. The collecting volumes are taken as infinite slabs in the lateral y direction and are segmented into vertical increments or bins of $\Delta z = 0.1$ m each in the vertical direction (see Figure 2.11). The size of each volume element is thus $\Delta x = 0.05$ m, $\Delta y = \pm\infty$, and $\Delta z = 0.1$ m.

2.2.4 Statistical Analysis

After the number of particles for each size has been counted at each line of sight, the probability distribution of the number density

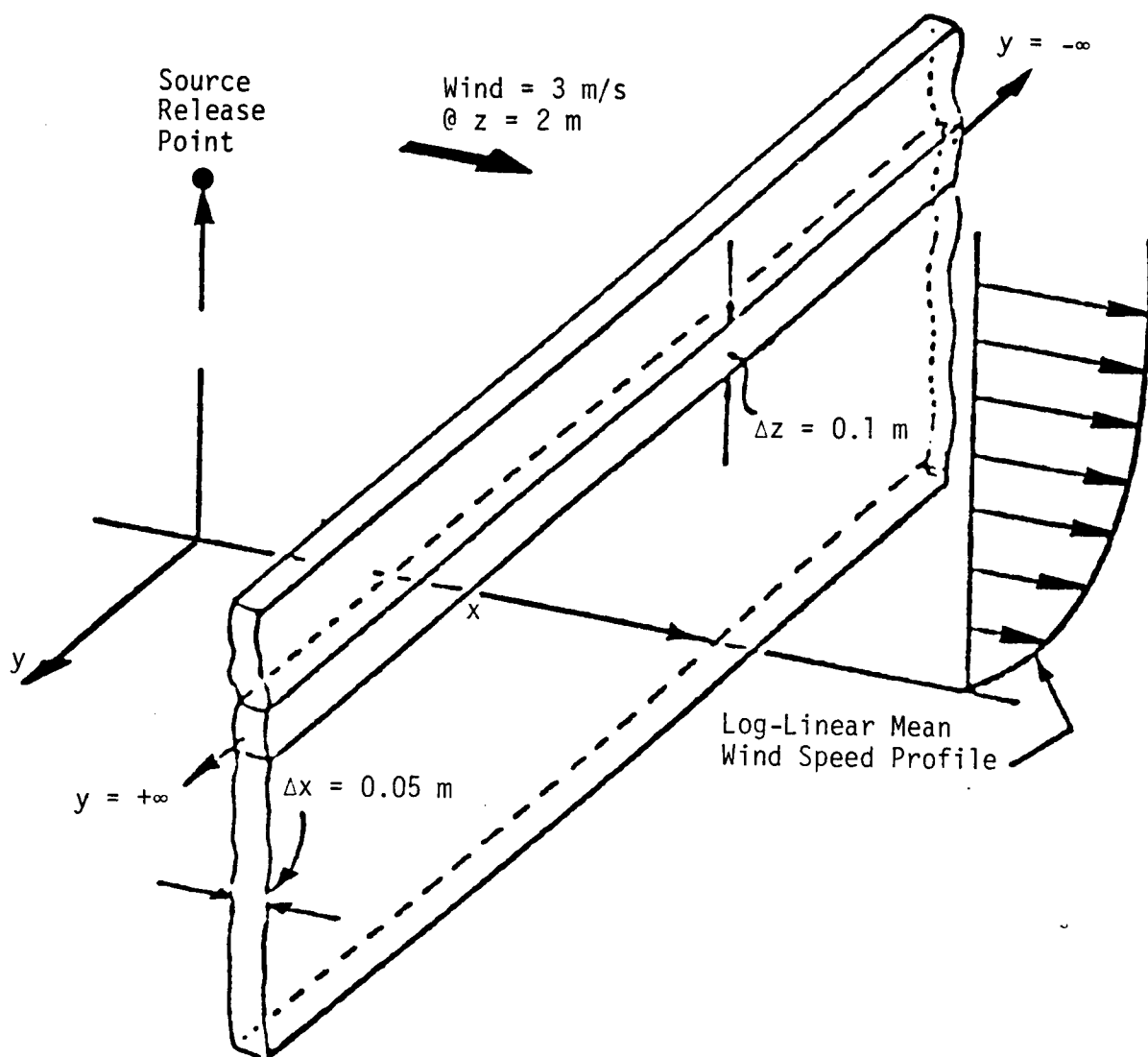


Figure 2.11 Illustration of a line of sight volume element and the coordinate system.

can be carried out for each volume element. Now reconsider the example which was mentioned in Section 2.2.2. The mean wind speed was taken to be 3 m/s at the 2-m height, and the mean wind direction was aligned to the x axis. Volume elements in which particles are to be sampled are located at various positions up to 500 m downstream of the source.

Five hundred particles for each size (1, 50, and 100 μm diameter) were released. The turbulence which was encountered by the particles was Eulerian turbulence. This is not quite right as indicated in Section 2.2.1.2; however, it will not affect the effect of the source which we are explaining.

The influence of the source on the particle dispersion is illustrated in Figure 2.12 for simple grenade and nozzle source models. This figure shows the probability that one or more particles will be in a volume element as a function of vertical and horizontal position from the release point. These probabilities are averaged over the time of the simulation, which was essentially 170 seconds. Inspection of the curves shows that the distribution of particles from the grenade is initially broader in the vertical direction and that from the nozzle, on the other hand, is relatively concentrated at the 2-m height, and this concentration remains clearly defined for 200 m downstream. One also notices a meandering of the plume where the higher concentrations initially drift downward and then upward, exiting at the downstream end of the figure at a height above the injection height.

Figure 2.13 shows the effect of the type of source and size of particles on the downstream probability distribution. The source effect is again the same as in Figure 2.12. The settling of the heavier particles is clearly seen. Figure 2.14 again shows the influence of the vertical position in the volume element on the computed probability of one or more particles being in the volume element for different particle sizes for the same source shown in Figure 2.11. The symbols represent the computed values with the curve faired through to computed points.

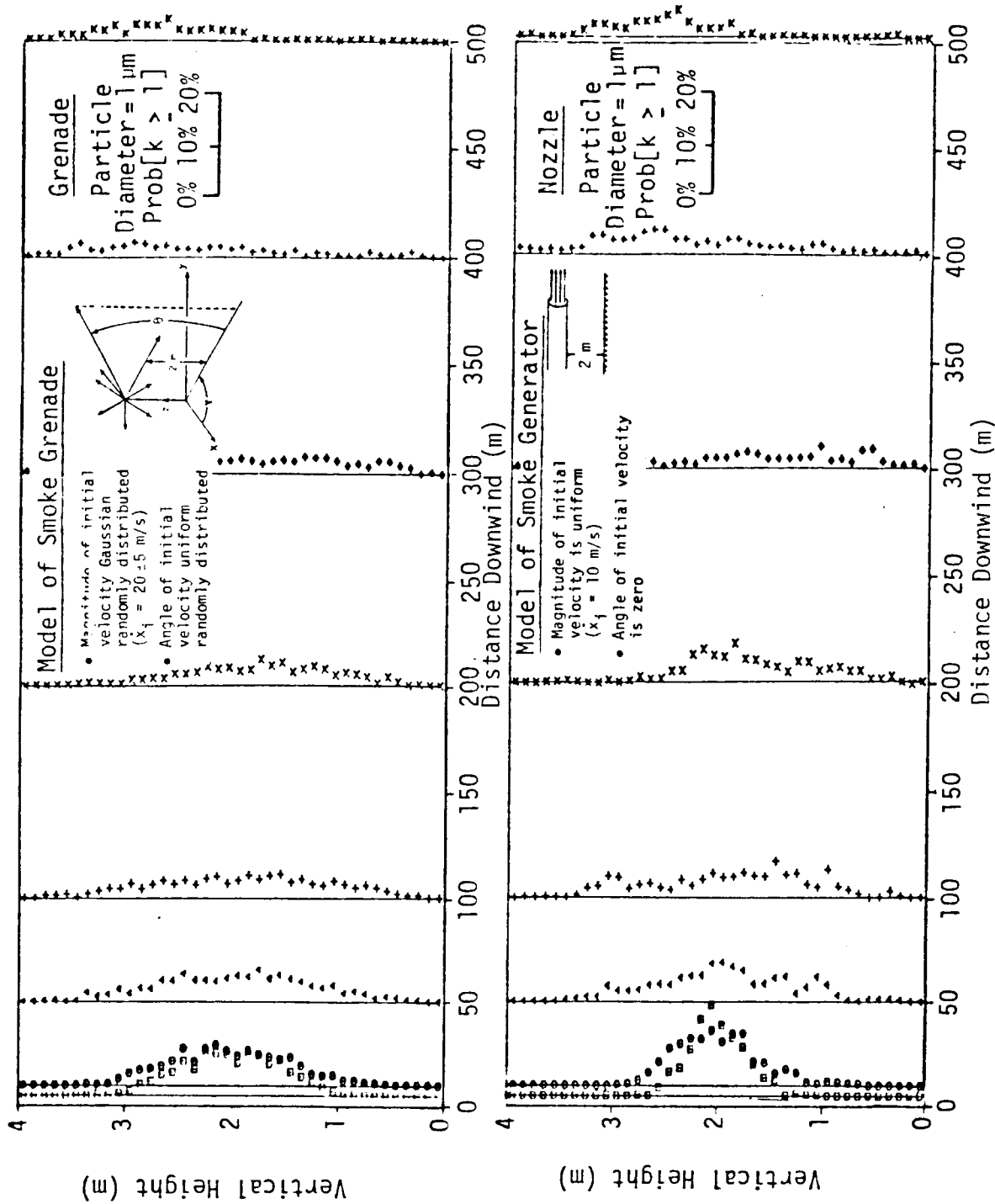


Figure 2.12 Percent of total particles released in the volume element as a function of vertical position.

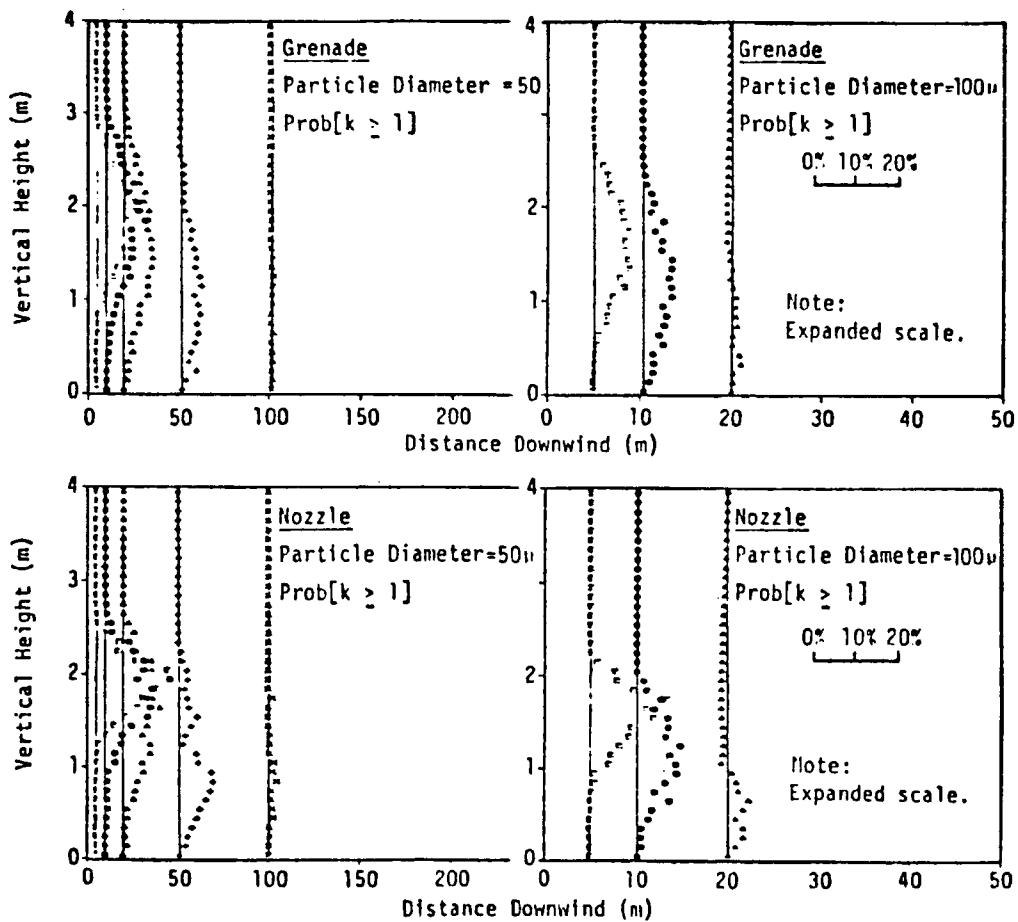


Figure 2.13 Effect of source type and particle size on the probability distribution of particle in a volume element as a function of vertical and horizontal position.

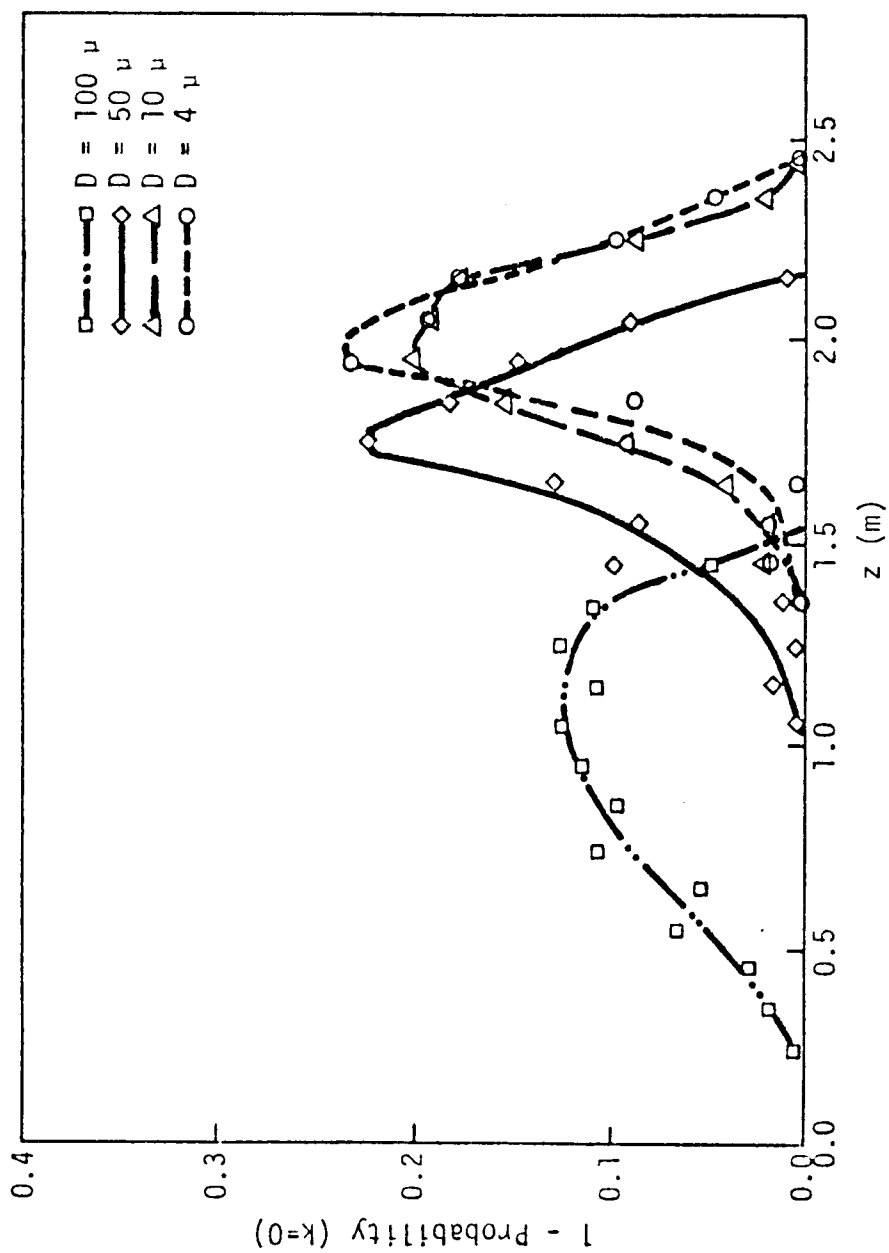


Figure 2.14 Probability of one or more particles in the volume element as a function of vertical position (source at 2-m height).

Because they are statistical estimates, the data do not fit the curves perfectly. The small particles are simply carried with the wind and disperse relatively uniformly about the nozzle height of $z = 2$ m. The heavier $100\text{ }\mu\text{m}$ particles precipitate out due to the gravity force and higher probabilities occur at lower values of z . It is also interesting to note that particles of size $10\text{ }\mu\text{m}$ or less tend toward Gaussian probability shapes, in keeping with the Gaussian plume model. Note also the effect of wind shear, which causes the 100 to $50\text{ }\mu\text{m}$ particle distributions to fair out at a lower vertical level.

2.2.5 Optical Properties and Particle Size Distributions

Farmer (1981) has reviewed basic quantities needed to optically characterize an obscurant cloud. One basic parameter is the mean extinction cross section. The mean extinction cross section is determined experimentally either from size distribution measurements when the extinction efficiency is known or from values of transmittance, path length, and number density estimates. Given the size distribution in M discrete size increments, the probability (P_i) of obtaining a particular particle size in the i th size interval is given by:

$$P_i = \frac{n_i}{N} \quad (2.7)$$

where n_i is the number of particles within the i th size interval and N is the total number of particles.

$$N = \sum_{i=1}^M n_i \quad (2.8)$$

the mean extinction cross section, $\bar{\sigma}$, is written:

$$\bar{\sigma} = \frac{\pi}{4} \sum_{i=1}^M Q_{Ei} D_i^2 P_i \quad (2.9)$$

where Q_{Ei} is the extinction efficiency for D_i , the largest particle diameter in the i th size interval. For spherical particles, Q_{Ei} can be computed from the Mie scattering theory when the index of refraction of the obscurant material is known. Thus for modeling the mean extinction cross section, the particle size distribution is required.

Farmer et al. (1981) computed mean extinction cross sections from particle size distributions measured with a particle sizing interferometer (PSI) during the Smoke Week III program. Figure 2.15 shows a plot of $\bar{\sigma}$ for Trial 18 as a function of time. Note that during the trial, $\bar{\sigma}$ changes by nearly three orders of magnitude. Farmer reports that the broad range of values of $\bar{\sigma}$ with time are to be expected when the particle size distribution measurements are examined. Data obtained from Smoke Week III shows that the size distributions have space- and time-dependences which presumably can be correlated through considerations involving atmospheric turbulence and wind velocity. In fact, Farmer et al. (1981) reports that phenomena observed for several trials during Smoke Week III clearly indicate spatial and temporal differences in the measured distributions. This suggests that the extinction cross section and thus the mass extinction coefficients are spatially and temporally dependent. This dependence is very likely associated with atmospheric disturbances as well as with chemical kinetics of the smoke cloud, and a realistic dispersion model must account for these effects.

Particle number density, ρ_N , can be found independently of the measured size distribution. Examination of the Beer-Bouguer law as expressed by

$$T = e^{-\bar{\sigma} \rho_N L} \quad (2.10)$$

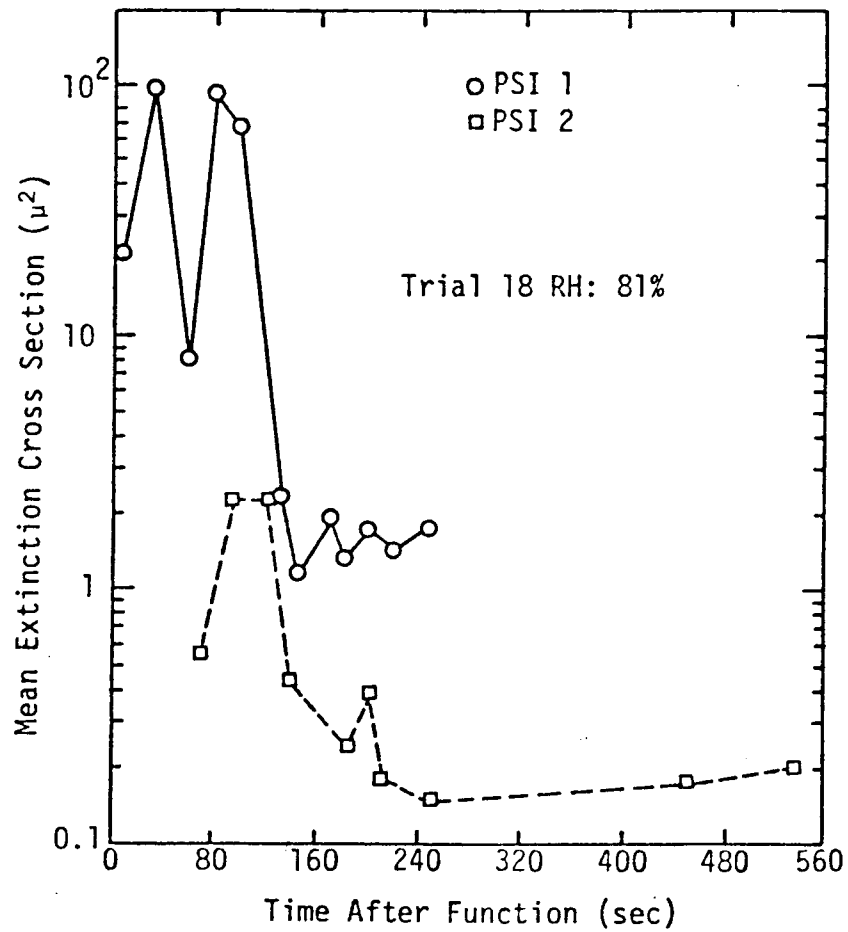


Figure 2.15 Mean extinction cross section as a function of time after munition detonation for Trial 18 (Farmer et al. (1981)).

where T , the transmittance and L , the transmission path, shows that a measured number density, path length, and transmittance measurement also yield $\bar{\sigma}$, i.e.,

$$\bar{\sigma} = \frac{-\ln(T)}{\rho_N L} \quad (2.11)$$

Hence, $\bar{\sigma}$ can be estimated directly using three (two when L can be assumed constant) relatively independent experimental measurements--transmittance, path length, and number density. The transmittance, however, cannot be analytically modeled without knowledge of the particle size distribution. Thus for modeling purposes, this approach is not very useful. Moreover, routinely used transport and diffusion models do not provide number density. The recent approach by Ohmstedde and Stenmark (1981) makes an initial attack on this problem.

The mass extinction coefficient, referred to simply as the "extinction coefficient," is often used to characterize the optical characteristics of an obscurant rather than $\bar{\sigma}$. The mass extinction coefficient, α , is defined as:

$$\alpha = \frac{\bar{\sigma}}{\frac{\pi}{6} \rho_o \mu_3} \quad (2.12)$$

where ρ_o is the material density of the obscurant and μ_3 is the third moment of the particle size distribution or, equivalently, $\pi\mu_3/6$ is the mean particle volume. α is thus the mean extinction cross section per mean particle mass. α can be computed using particle size data by computing $\bar{\sigma}$ as indicated above. Hence:

$$\alpha = \frac{3 \sum_{i=1}^M Q_{Ei} P_i D_i^2}{2\rho_o \sum_{i=1}^M P_i D_i^3} \quad (2.13)$$

where the third moment of the size distribution is given by:

$$\mu_3 = \sum_{i=1}^M P_i D_i^3 \quad (2.14)$$

Thus when ρ_o and Q_{Ei} are known, data from the particle size distribution can be used directly to estimate α . In cases involving some smokes, such as phosphorus, α is approximately independent of size distribution for mid-far infrared wavelengths; however, C is not and the size distribution requirement still applies. The need for particle size to characterize an obscurant optically is again evident.

Values of α can also be determined from transmittance values when the mass concentration, C , of the aerosol is known. In this case, the Beer-Bouguer law can be expressed as:

$$T = e^{-\alpha CL} \quad (2.15)$$

which can be solved to yield:

$$\alpha = \frac{-\ln T}{CL} \quad (2.16)$$

Dispersion models currently reported in the literature will predict time-averaged values of the mass concentration; however, the transmittance cannot be modeled analytically. Although, historically, time-averaged values of $\ln T$ and CL have been measured to determine α , this approach cannot be used in analytical simulations of weapon system performance.

3.0 COHERENT TURBULENCE SIMULATION

The idea of atmospheric turbulence simulation is based on filtering Gaussian random noise signals to produce an output signal having the same statistical properties as the turbulence found in the atmosphere. There are two categories of turbulence simulation: one is Eulerian turbulence and the other is Lagrangian turbulence. Most of the models concerned with Eulerian turbulence simulation are concerned with designing a filter function while Lagrangian simulation models seek an appropriate correlation function. In the case of the particle trajectories calculation, the wind components which the particles encounter are Lagrangian turbulence. Also, the inter-level coherence is particularly important in the smoke simulation, as indicated in Section 2. In the MoCaPD model, a Lagrangian coherence turbulence generator has been used. This section first reviews the Eulerian and Lagrangian turbulence simulation models, followed by a discussion of the relationship between Eulerian and Lagrangian turbulence. Then, the z-transformation technique to generate the Lagrangian coherent turbulence is introduced.

3.1 Review of Eulerian Turbulence Model

Eulerian turbulence simulation models have been developed by Neuman and Foster (1970), Reeves et al. (1976), and Wang and Frost (1980). The basic procedure is illustrated in Figure 2.5. A Gaussian white noise is computer generated and passed through a digital filter to provide the output which is the simulated time history of the turbulence. The form of the filter function is

$$|H(f)|^2 = \phi(f) \tag{3.1}$$

where $\phi(f)$ is the power density spectrum for the atmospheric turbulence being simulated. There are several kinds of spectrum functions which are commonly used in atmospheric turbulence simulation such as: (1) the von Karman spectrum, (2) the Dryden spectrum, and (3) the Kaimal spectrum.

The von Karman spectrum function, given by Hinze (1975), can be written as

$$\begin{aligned}\phi_x(\omega) &= \sigma_x^2 \frac{2\Lambda_x}{\pi V} \frac{1}{\left\{1 + \left[1.339 \frac{\Lambda_x}{V} \omega\right]^2\right\}^{5/6}} \\ \phi_y(\omega) &= \sigma_y^2 \frac{\Lambda_y}{\pi V} \frac{1 + \frac{8}{3} \left[1.339 \frac{\Lambda_y}{V} \omega\right]^2}{\left\{1 + \left[1.339 \frac{\Lambda_y}{V} \omega\right]^2\right\}^{11/6}} \\ \phi_z(\omega) &= \sigma_z^2 \frac{\Lambda_z}{\pi V} \frac{1 + \frac{8}{3} \left[1.339 \frac{\Lambda_z}{V} \omega\right]^2}{\left\{1 + \left[1.339 \frac{\Lambda_z}{V} \omega\right]^2\right\}^{11/6}}\end{aligned}\tag{3.2}$$

where the subscripts x, y, and z represent longitudinal, lateral, and vertical fluctuations, respectively; ω is angular frequency; Λ is the integrated length scale; σ is the turbulence intensity; and V is the longitudinal mean wind speed.

For the Dryden spectra, one chooses the longitudinal spatial autocorrelation function of the longitudinal fluctuation to be a simple exponential (Hughes and Lazebny (1965), Neuman and Foster (1970))

$$R_x(f) = \sigma_x^2 e^{-r/\Lambda_x}\tag{3.3}$$

where r is the correlation interval.

From the equation of continuity, Batchelor (1953) has shown the longitudinal and vertical auto-correlation functions are related by the equation

$$R_z(r) = R_x(r) + \frac{r}{2} \frac{\partial R_x(r)}{\partial r} \quad (3.4)$$

Substituting Equation 3.3 into Equation 3.4 for homogeneous turbulence, the vertical spatial auto-correlation function is found as:

$$R_z(r) = \sigma_z^2 \left(1 - \frac{r}{2\Lambda_z} \right) e^{-r/\Lambda_z} \quad (3.5)$$

The Dryden power spectrum can be obtained by the substitution of Equations 3.3 and 3.5 into the Fourier transform relation expressed as

$$\phi(\omega) = \frac{1}{\pi} \int_0^{\infty} R(r) e^{-j\omega r} dr \quad (3.6)$$

resulting in

$$\phi_x(\omega) = \frac{\Lambda_x}{V} \frac{2\sigma_x^2}{\pi} \frac{1}{1 + \left(\frac{\Lambda_x}{V} \omega \right)^2} \quad (3.7)$$

Similarly for the lateral and vertical directions,

$$\phi_y(\omega) = \frac{\Lambda_y}{V} \frac{\sigma_y^2}{\pi} \frac{1 + 3 \left(\frac{\Lambda_y}{V} \omega \right)^2}{\left\{ 1 + \left(\frac{\Lambda_y}{V} \omega \right)^2 \right\}^2} \quad (3.8)$$

$$\phi_z(\omega) = \frac{\Lambda_z}{V} \frac{\sigma_z^2}{\pi} \frac{1 + 3 \left(\frac{\Lambda_z}{V} \omega \right)^2}{\left\{ 1 + \left(\frac{\Lambda_z}{V} \omega \right)^2 \right\}^2} \quad (3.9)$$

The simulated atmospheric turbulence which employs the Dryden spectrum exhibits a ω^{-2} falloff at high frequencies while the von Karman spectrum falls off as $-5/3$. Busch and Larsen (1972) and Wang and Frost (1980) compared both spectra with experimental data and concluded that the von Karman spectrum gave a better description of measured data. However, the Dryden spectrum is much easier to handle mathematically, especially in dealing with the z-transformation.

Another spectrum function suggested by Kaimal (1973) has been found to correlate experimental data well (Frost 1980) and also allows for the effects of atmospheric stability on the turbulence to be simulated. The Kaimal spectra for the three turbulence components are given by:

$$\frac{f}{\sigma_{\alpha}^2} \phi_{\alpha}(f) = \frac{0.164(\hat{\eta}/\hat{\eta}_{o\alpha})}{1 + 0.164(\hat{\eta}/\hat{\eta}_{o\alpha})^{5/3}} \quad (3.10)$$

where f is the frequency and the subscript α denotes one of the coordinates x , y , and z ; σ_{α} is turbulence intensity; and $\hat{\eta}$ is a reduced frequency defined as $\hat{\eta} = fz/V(z)$. Equation 3.10 is a good approximation to the turbulence spectra for a stable atmosphere. The reduced frequency $\hat{\eta}_{o\alpha}$ is a function of the Richardson number (Frost et al. (1978)). These spectra were extended to neutral conditions by Frost et al. (1978) using values of σ_{α} and $\hat{\eta}_{o\alpha}$ given in Table 3.1.

TABLE 3.1. Recommended Values of σ_{α} and $\hat{\eta}_{\alpha}$ for Neutral Atmospheric Conditions.

$\sigma_x = 2.5 u_*$	$\hat{\eta}_{ox} = 0.0144$
$\sigma_y = 1.6 u_*$	$\hat{\eta}_{oy} = 0.0265$
$\sigma_z = 1.3 u_*$	$\hat{\eta}_{oz} = 0.0962$

Frost and Lin (1983) compared these three analytical spectra models (von Karman, Dryden, and Kaimal) with experimentally computed spectra. It was shown that some of the data are best fit to a von Karman or a Kaimal spectrum, as shown in Figure 3.1, whereas some of the data are best fit to the Dryden spectrum, as shown in Figure 3.2. The spectra which are plotted (Figures 3.1 and 3.2) are in a normalized form, i.e., $G_x(\hat{n}) = \hat{n} \phi_x(\hat{n}) / \sigma_x^2$.

3.2 Method of Discrete Transformation

For the filter system shown in Figure 3.3, the output $y(t)$ is given by the convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau \quad (3.11)$$

where $x(t)$ is input history and $h(\tau)$ is the response function. The Fourier transformation of Equation 3.11 is

$$Y(\omega) = H(\omega) X(\omega) \quad (3.12)$$

The corresponding relationship of the power spectral density function between the input and output can be written as

$$\phi_y(\omega) = |H(\omega)|^2 \phi_x(\omega) \quad (3.13)$$

Introducing the input as wide-band white noise for which the spectrum $\phi_x(\omega)$ is constant, adjustable to unity, gives

$$\phi_y(\omega) = |H(\omega)|^2 \quad (3.14)$$

Then a filter function $H(\omega)$ can be obtained which contains a desired output spectrum. For example, if the desired output is the longitudinal and vertical Dryden spectrum given by Equations 3.7 and 3.9, then

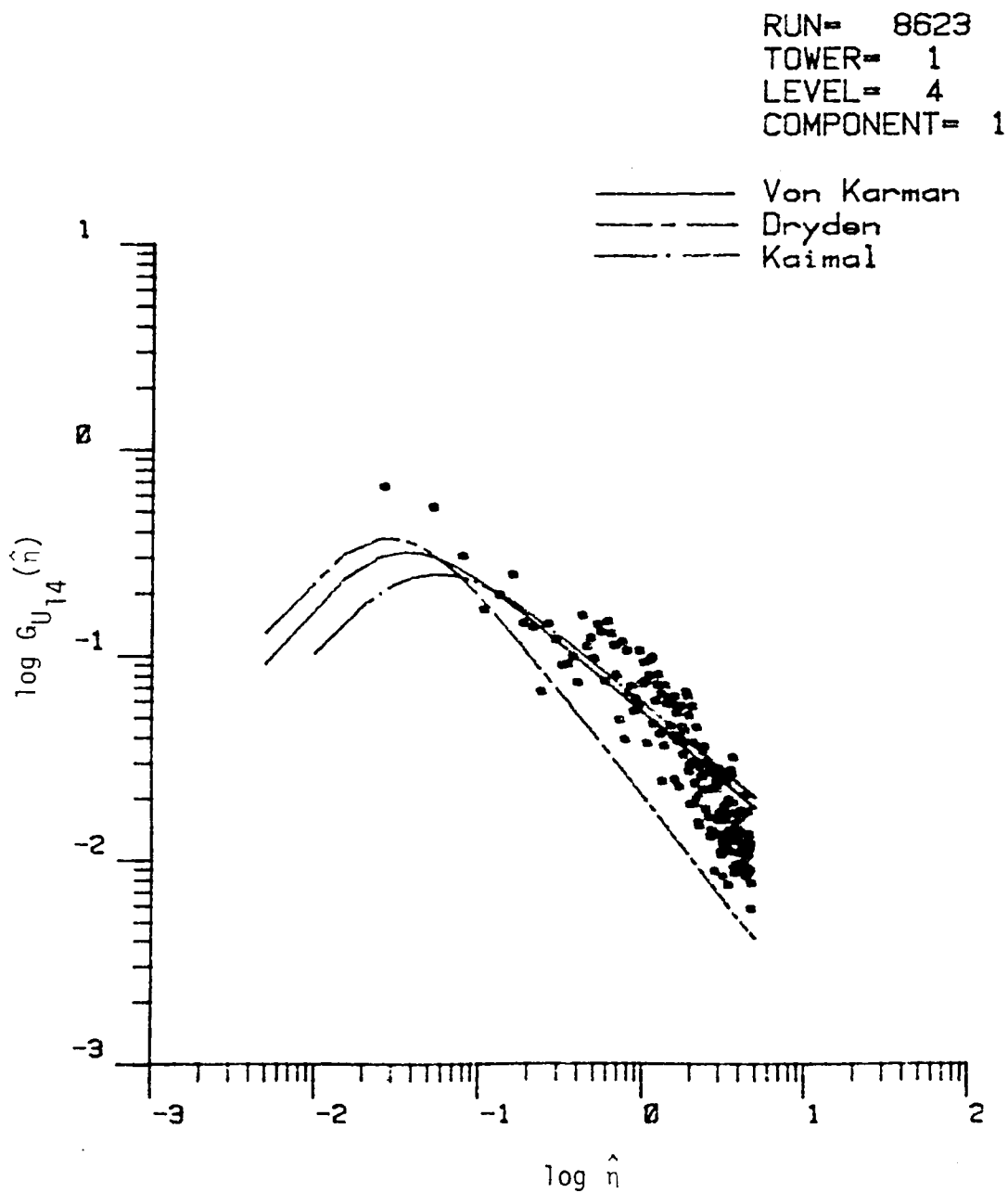


Figure 3.1 Comparison of longitudinal turbulence spectrum with three correlation models, Run 8623 (Frost and Lin (1983)).

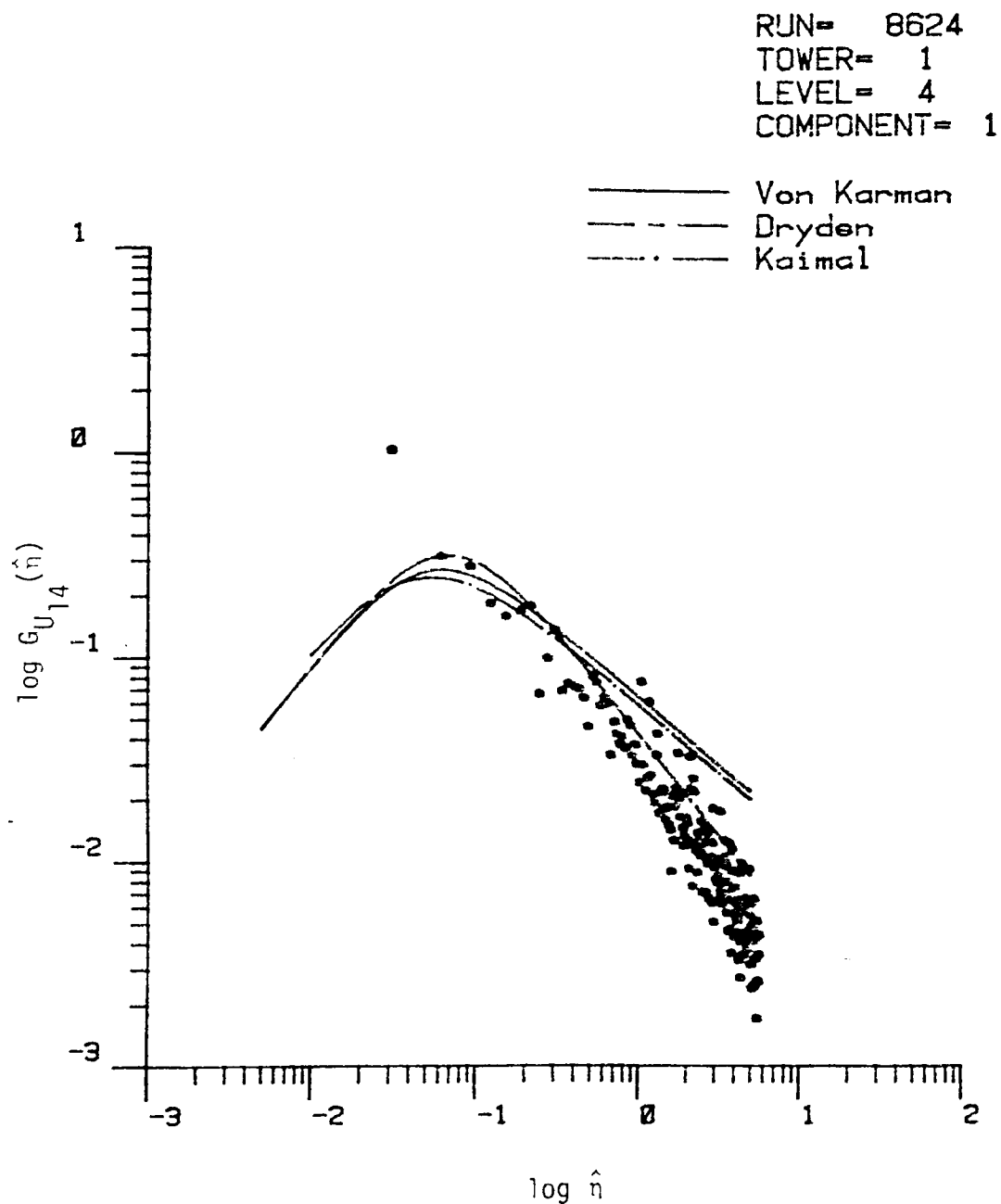


Figure 3.2 Comparison of longitudinal turbulence spectrum with three correlation models, Run 8624 (Frost and Lin (1983)).

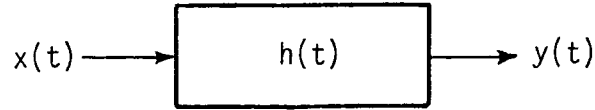


Figure 3.3 Simple filter system.

$$|H_1(\omega)|^2 = \frac{k_1}{a_1^2 + \omega^2} \quad (3.15)$$

$$|H_3(\omega)|^2 = \frac{k_3(b^2 + \omega^2)}{(a_3^2 + \omega^2)^2} \quad (3.16)$$

where the subscripts 1 and 3 represent the longitudinal and vertical components, respectively, and

$$a_1 = \frac{V}{\Lambda_x}, \quad k_1 = \frac{2a_1\sigma_x^2}{\pi}$$

$$a_3 = \frac{V}{\Lambda_z}, \quad b = \frac{a_3}{\sqrt{3}}, \quad k_3 = \frac{3a_3\sigma_z^2}{\pi}$$

One solution for $H(\omega)$ from these equations is

$$H_1(\omega) = \frac{\sqrt{C_1}}{a_3 + j\omega} \quad (3.17)$$

$$H_3(\omega) = \frac{\sqrt{C_3}(b + j\omega)}{(a_3 + j\omega)^2} \quad (3.18)$$

There are two discrete transformation techniques available to generate the digitized time series output y_n based on the digitized input x_n and the filter function given by Equations 3.17 and 3.18. They are the fast Fourier transformation (FFT) and the z-transformation. The procedure for using the FFT technique to generate time series turbulence is as follows. First use the FFT to transform a set of Gaussian white

noise x_n to the frequency domain X_m , as

$$X_m = \sum_{n=0}^{N-1} W_{mn} x_n \quad (3.19)$$

where

$$W_{mn} = \exp\left(-j \frac{2\pi mn}{N}\right) \quad (3.20)$$

Then the output Y_m is equal to

$$Y_m = H_m X_m \quad (3.21)$$

where inside the filter, H_m should specify the turbulence length scale, turbulence intensity, and mean wind velocity. Finally, use the inverse FFT to transform the output from the frequency domain Y_m to the time domain y_n , as

$$y_n = \frac{1}{N} W_{mn}^* Y_m \quad (3.22)$$

where

$$W_{mn}^* = \exp\left(j \frac{2\pi mn}{N}\right) \quad (3.23)$$

The second transformation technique uses the z-transformation, which is defined as

$$X(z) = \sum_{m=0}^{\infty} x(m\Delta t) z^{-m} \quad (3.24)$$

It is a powerful operational function which may be applied to discrete-time systems while the Laplace transformation is applied to continuous-time systems. Details of the z-transformation technique are given by Neuman and Foster (1970) and Wang and Frost (1980). When the z-transformation is applied to Equation 3.12 with the filter function given by Equations 3.17 and 3.18, the output of the longitudinal

turbulence component is

$$y_n = Cy_{n-1} + Dx_{n-1} \quad (3.25)$$

where the constants are given by

$$C = \exp\left(-\frac{V}{\Lambda_x} \Delta t\right)$$

$$D = \sigma_x \left(\frac{2\Lambda_x}{\pi V}\right)^{1/2} (1 - C) \quad (3.26)$$

Similarly, the vertical turbulence component can be written as

$$y_n = C_1 y_{n-1} + C_2 y_{n-2} + D_1 x_{n-1} + D_2 x_{n-2} \quad (3.27)$$

where the constants are

$$C_1 = 2\exp[-a_3 \Delta t]$$

$$C_2 = -\exp[-2a_3 \Delta t] \quad (3.28)$$

$$D_1 = \sqrt{k_3} \left[\frac{b}{a_3^2} + e^{-a_3 \Delta t} \left(\frac{a_3 - b}{a_3} \Delta t - \frac{b}{a_3} \right) \right]$$

$$D_2 = \sqrt{k_3} \left[\frac{b}{a_3^2} \left(e^{-a_3 \Delta t} - 1 \right) - \frac{a_3 - b}{a_3} \Delta t \right] e^{-a_3 \Delta t}$$

and a_3 , b , and k_3 are the same constants as given in Equation 3.16.

Compared to the FFT method (Equation 3.21), the difference equations in Equations 3.25 and 3.27 are very efficient for computation. It can be completely solved by basic arithmetic operation of multiplication, addition, and subtraction. Another advantage for these recurrent equations is that all the constants of Equations 3.25 and 3.27 are functions of the turbulence length scale, turbulence intensity, and mean wind speed, and they could be changed from step-by-step if a

quasi-time-invariant system is assumed. This characteristic is really helpful when Lagrangian turbulence along a particle trajectory is going to be simulated.

The length scale Λ and the turbulence intensity are functions of height, z , atmospheric stability conditions, $\phi(z/L')$, and the surface roughness, z_0 . The parameter L' is the Monin-Obukhov stability length. Equations for predicting the turbulence intensity values are given in Frost et al. (1978) as:

$$\sigma_z = \frac{0.52V}{\left\{ \ln \left(\frac{z}{z_0} + 1 \right) + \phi(z/L') \right\}}$$

$$\sigma_y = \sigma_z [0.583 + 1.39 \times 10^{-3} z]^{-0.8} \quad (3.29)$$

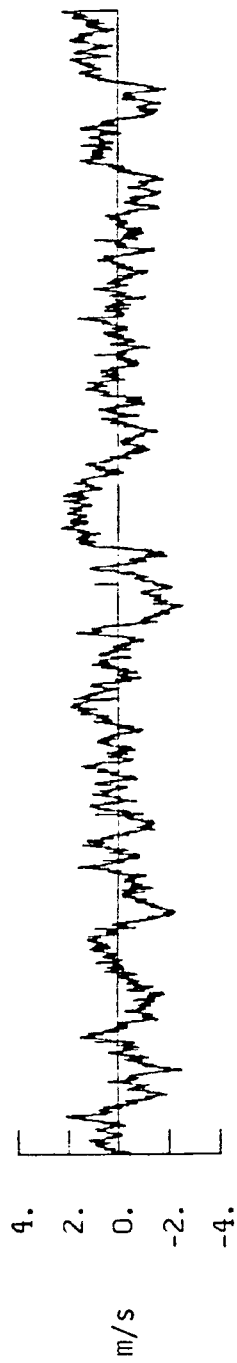
$$\sigma_x = \sigma_z [0.177 + 2.74 \times 10^{-3} z]^{-0.4}$$

The units of V are m/s and z is in meters. The turbulence length scale for the Dryden spectrum is given in Barr et al. (1974) as:

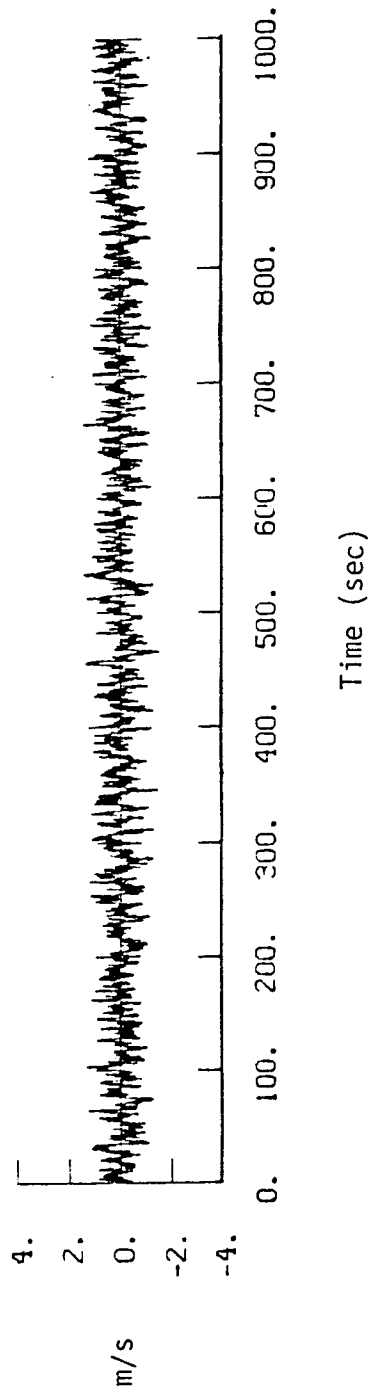
$$\Lambda_x = \Lambda_y = \begin{cases} 13.48(3.28z)^{1/2} & \text{for } z \leq 533 \text{ m} \\ 533 & \text{for } z > 533 \text{ m} \end{cases} \quad (3.30)$$

$$\Lambda_z = \begin{cases} z & \text{for } z \leq 533 \text{ m} \\ 533 & \text{for } z > 533 \text{ m} \end{cases}$$

Figure 3.4 shows the time series of both the longitudinal and vertical components and the simulated turbulence using the Dryden spectrum. The mean wind speed was taken to be 5 m/s at a 10-m height. Figure 3.5 shows the comparison of the theoretical Dryden spectrum curve and the simulated turbulent spectrum. As expected, both the longitudinal and vertical components fit the theoretical Dryden curve well. In the low-frequency part of the vertical component, the spectra becomes



(a) Longitudinal component of the Dryden model.



(b) Vertical component of the Dryden model.

Figure 3.4 Simulated Dryden spectrum turbulence ($V = 5 \text{ m/s}$, $z = 10 \text{ m}$, $\Delta t = 0.1 \text{ sec}$).

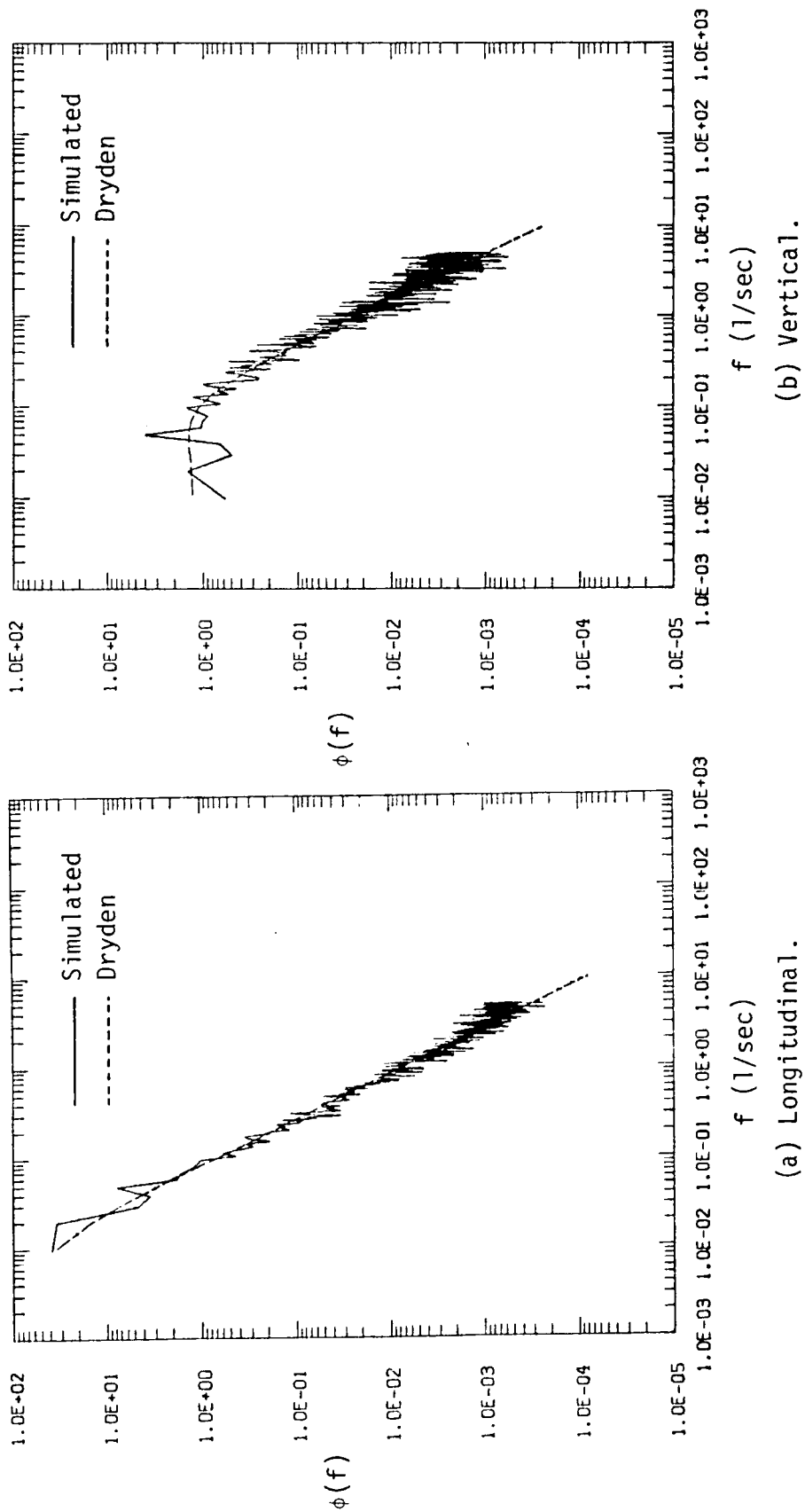
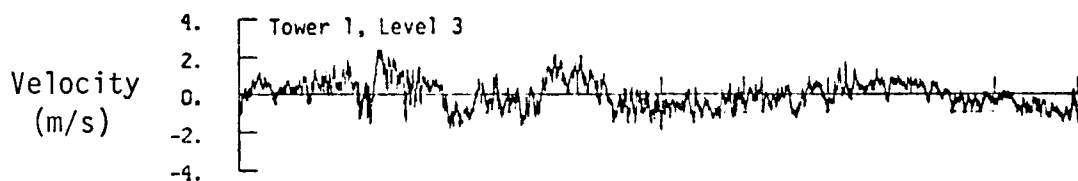


Figure 3.5 Comparison of the theoretical curve with simulated turbulence spectra ($V = 5$ m/s, $z = 10$ m, $\Delta t = 0.1$ sec).

flat because the vertical length scale is shorter than the longitudinal length scale. A set of tower measurement data was obtained from NASA/MSFC for Tower 1, Level 3. The anemometer was installed at a height of 12 m. Longitudinal mean wind speed was $V = 3.66$ m/s, vertical turbulence intensity was $\sigma_z = 0.6326$, and the integrated length scale was $\Lambda_z = 25$ m. In this case, the vertical length scale Λ_z was twice the height while Equation 3.30 indicates $\Lambda_z = 12$ m. Figure 3.6 shows the comparison of the time histories of the measured data and simulated turbulent data with the same vertical length scale and turbulent intensity. One cannot expect one-to-one comparison; however, in a statistical sense, the spectrum distribution is quite similar, as shown in Figure 3.7.

3.3 Review of Lagrangian Turbulence Simulation

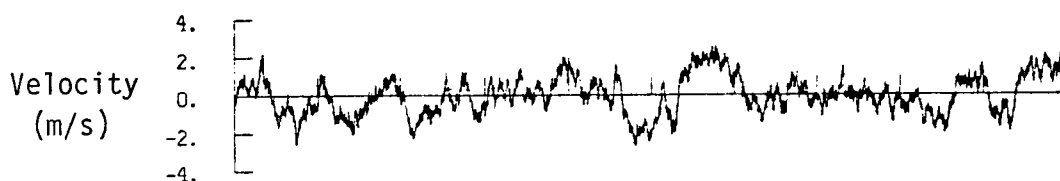
The preceding discussions have been concerned with the turbulence indicated by variations of velocity with time considered as measured by an instrument which is at a fixed location. This yields the Eulerian formulation of turbulence. In the Lagrangian turbulence simulation, the measurement is assumed to be made while the instrument is moving with the fluid. Generally, this is the only type of measurement which is meaningful, and it is in such terms that the effects of diffusion must be described in practice. It is obvious that the diffusion of a cloud of airborne material will depend on the development of the turbulence which affects the particular elements of the cloud containing the obscurant as it is carried along. In other words, interest centers on the variations of the velocity which would be observed by following the motion of fluid elements.



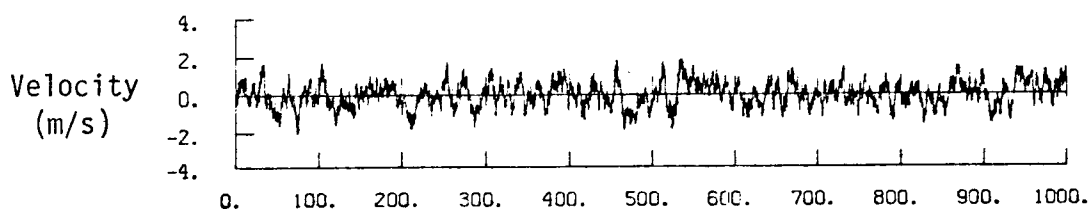
(a) Longitudinal measured data.



(b) Vertical measured data.

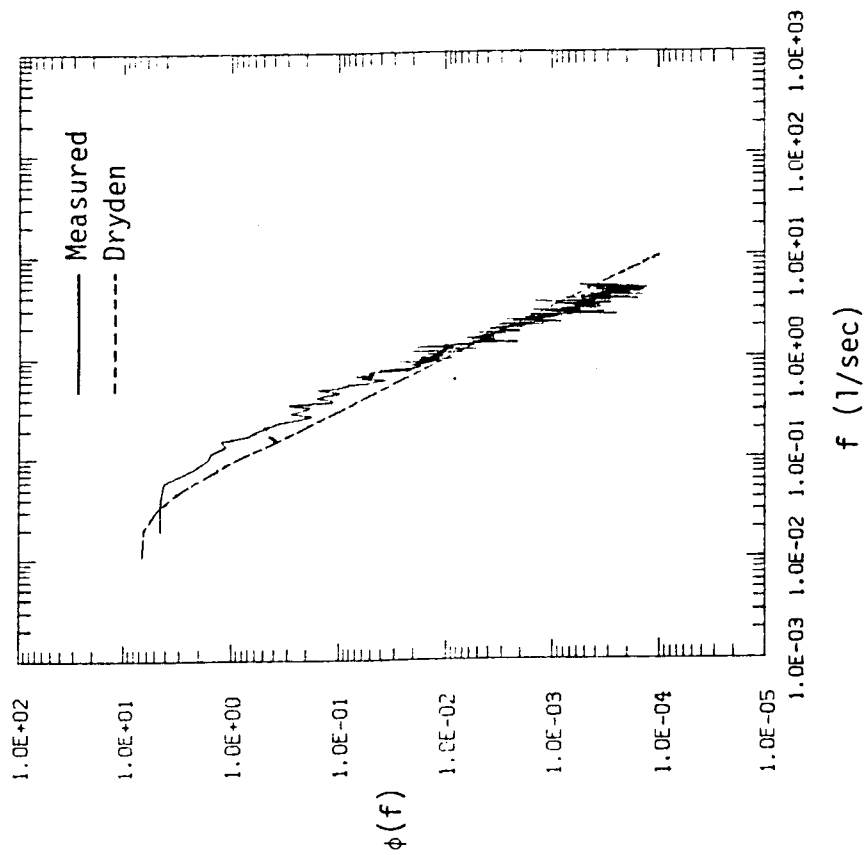


(c) Longitudinal simulated data.

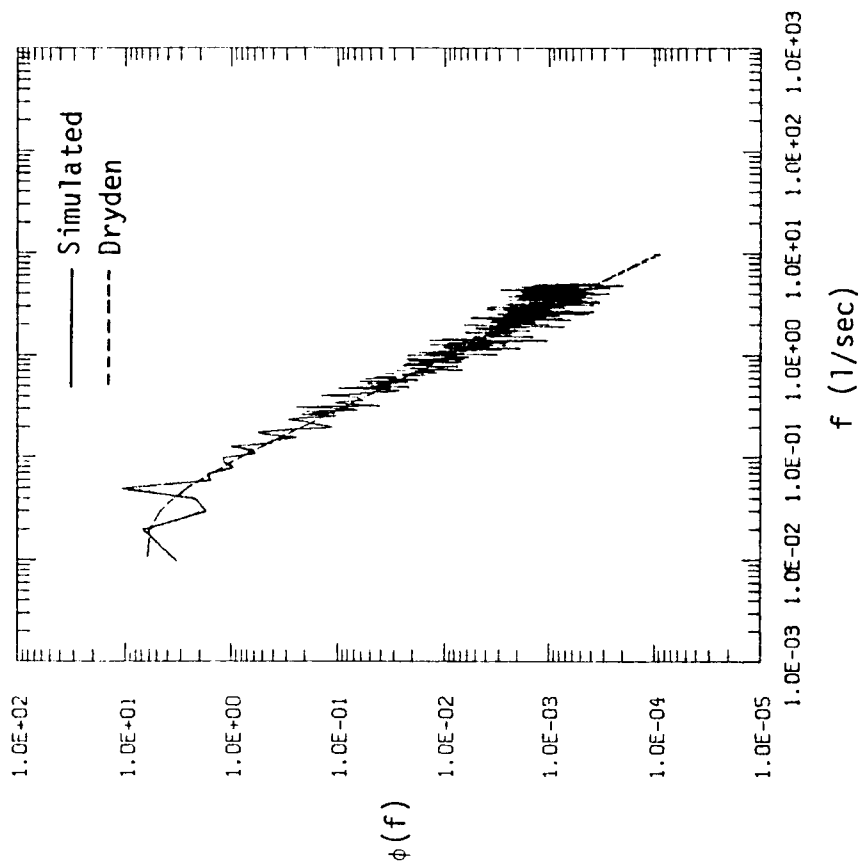


(d) Vertical simulated data.

Figure 3.6 Comparison of measured data and simulated data for both longitudinal and vertical components.



(a) Measured data.



(b) Simulated data.

Figure 3.7 Comparison of both the spectrum of measured and simulated data with theoretical curve.

The Lagrangian approach to turbulent diffusion was introduced by Taylor (1921), who examined the case of one-dimensional dispersion in homogeneous turbulence. Many difficulties have been encountered while trying to apply Taylor's method to realistic problems, largely because of inadequate knowledge of the turbulence statistics. In recent years, some statistical theories resulting from Taylor's idea have been widely applied to atmospheric diffusion problems. Gifford (1981) pointed out this breakthrough because Obukhov (1959) proposed that the evolution of a diffusing air particle's motion in the atmosphere forms a Markov process and can be described by the Fokker-Planck equation in physical space.

Thompson (1971), Hall (1975), Reid (1979), and Wilson (1981a, b, c) employed a Markov chain to simulate the fluid particle trajectories in which the particle velocities were represented by a Markov sequence. The equation of motion of a fluid particle which was used is the Langevin's equation as

$$\frac{dw}{dt} = -\alpha w + \lambda \xi(t) \quad (3.31)$$

where w is turbulent fluctuation, $\xi(t)$ is Gaussian white noise with zero mean and unit variance. Legg and Raupach (1982) showed the relation between the Langevin's equation to the existing Markov chain models of turbulent dispersion and found the coefficient $\alpha = 1/T_L$ and $\lambda = \sigma_w \sqrt{2/T_L}$; where T_L is the Lagrangian integral time scale. The Markov sequence (or Markov chain) is given a turbulent velocity at time t and the turbulent velocity at $t + \Delta t$ is given by

$$w(t + \Delta t) = \mu w(t) + \nu \sigma_{w_E} \xi(t) \quad (3.32)$$

where $\mu = \exp(-\Delta t/T_L)$; $\nu = (1 - \mu^2)^{1/2}$; T_L is Lagrangian time scale, the

time step $\Delta t \ll T_L$; and σ_{wE} is the Eulerian velocity variance. The turbulent velocity expressed by Equation 3.32 is called Lagrangian simulated turbulence. Multiplying $w(t)$ on both sides of Equation 3.32 and taking the average results in

$$\frac{\overline{w(t + \Delta t) w(t)}}{\overline{w^2(t)}} = e^{-\Delta t/T_L} \quad (3.33)$$

Equation 3.33 is a Lagrangian time correlation function of the turbulence and can be written as

$$R_L(\tau) = \exp(-\tau/T_L) \quad (3.34)$$

It is still not clear how to determine the Lagrangian time scale. Some of the useful results have been achieved only at the expense of rather intuitive hypotheses and broad assumptions. Reid (1979) found an expression for T_L in terms of the height above the surface z and the friction velocity u_* based on the vertical eddy diffusivity for heat transfer in the neutral surface layer due to an infinite area of uniform thermal stress at the surface. This expression is

$$T_L = 0.26 z/u_* \quad (3.35)$$

Joynt and Blackman (1976) used $T_L = z/\sigma_z$. According to Frost et al. (1978), the vertical turbulence intensity $\sigma_z = 1.3 u_*$ for the neutral atmosphere, which leads to $T_L = 0.77 z/u_*$. Hall (1975) employed $T_L = 2.47/\overline{W}_x(z)$ for neutral conditions. Substituting the neutral wind profile law, expressed at Equation 2.5, then leads to

$$T_L = 2.47 \frac{\kappa z}{u_* \ln \left(\frac{z}{z_0} + 1 \right)} \quad (3.36)$$

where κ is the von Karman constant, $\kappa = 0.4$. Equation 3.35 not only varies with height, but is a function of surface roughness.

Comparison between the Markov sequence (Equation 3.32) and the longitudinal component of the z-transformation (Equation 3.25) indicates that both are identical except the coefficient of the second right-hand side term. However, Equation 3.32 is a Markov sequence for the vertical component of turbulence while Equation 3.25 is for the longitudinal component. This is because the Markov sequence assumes that the vertical correlation function is simply an exponential function (Equation 3.34) while the Dryden spectrum uses Equation 3.5 for the vertical correlation.

3.4 The Relation Between Lagrangian and Eulerian Fluctuations

Pasquill (1974) indicates some of the sophisticated representations of the relation between Lagrangian and Eulerian fluctuations which have been developed. Basically, the analysis between the Lagrangian and Eulerian descriptions depend on one or another of three main principles (Pasquill (1974)):

1. The frozen eddy hypothesis.
2. The conjecture, first specifically stated by Corrisn (1959), that after sufficiently long migration times, particles may be considered to have velocities which are unbiased samples of the turbulent velocities at positions in a Eulerian reference frame.
3. The recognition of the difference in Eulerian and Lagrangian spectral characteristics which is implied by the inertial subrange concept.

First, in the frozen eddy hypothesis, particles may be considered to move through the fixed eddy pattern at a speed which is statistically represented by σ_x , and so the Lagrangian integral time scale may be

expected to be related closely to λ/σ_x , while the integral time scale for fluctuations at a fixed point (Eulerian) is $\lambda/\bar{W}_x(z)$ where λ is the integral length scale and $\bar{W}_x(z)$ is the longitudinal mean wind speed. The ratio of the two time scales, T_L/T_E , should be of the order $\bar{W}_x(z)/\sigma_x$ which in the atmosphere is typically of the order of ten.

Based on the second approach, Saffman (1963) and Philip (1967), assuming isotropic turbulence to give T_L/T_E as a function of $\bar{W}_x(z)/\sigma_z$ where σ_z is the vertical turbulence intensity, found

$$\frac{T_L}{T_E} = b \bar{W}_x(z)/\sigma_z \quad (3.37)$$

A value of $b = 0.35$ appears to be an adequate representation of Philip's curves.

The third approach was first applied by Corrisn (1963) with the assumption that the Eulerian and Lagrangian spectra may be completely represented by their inertia subrange forms, i.e., the region where turbulence decreases with frequency according to the $-5/3$ law. Both Eulerian and Lagrangian spectrum are written as

$$\begin{aligned} \phi_E(f) &= C u^{2/3} \epsilon^{2/3} f^{-5/3} & f \geq f_E \\ &= 0 & f < f_E \end{aligned} \quad (3.38)$$

$$\begin{aligned} \phi_L(f) &= B \epsilon f^{-2} & f \geq f_L \\ &= 0 & f < f_L \end{aligned} \quad (3.39)$$

where C and B are constants; u is velocity; and ϵ is the turbulence viscous dissipation per unit mass. Pasquill (1968) modified both Equations 3.38 and 3.39 to more realistic forms of spectra which have

finite density at low frequencies and reduce to the inertial subrange form at large f . The Eulerian spectrum corresponding to Equation 3.38 is

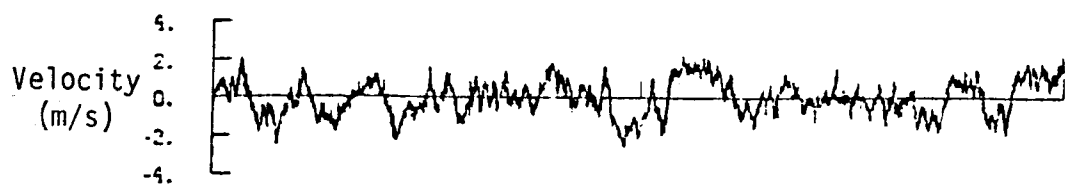
$$f\phi_E(f) = \frac{\sigma_E^2 f_E / f_{mE}}{\left[1 + \frac{3}{2} f_E / f_{mE}\right]^{5/3}}, \quad \frac{f_E}{f_{mE}} = 4T_E f_E \quad (3.40)$$

and the Lagrangian spectrum was suggested as

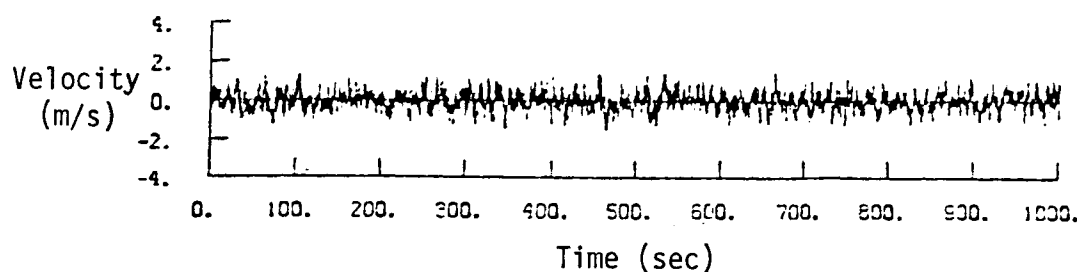
$$f\phi_L(f) = \frac{\sigma_L^2 f_L / f_{mL}}{(1 + f_L / f_{mL})^2}, \quad \frac{f_L}{f_{mL}} = 4T_L f_L \quad (3.41)$$

where all subscripts E represent Eulerian and L represents Lagrangian, f_{mE} or f_{mL} are the frequencies where the Eulerian or Lagrangian spectra are a maximum. It is interesting to point out that Equation 3.39 (Eulerian spectrum) has a very similar form to the von Karman spectrum given by Equation 3.2 while Equation 3.40 (Lagrangian spectrum) is similar to the Dryden spectrum given by Equation 3.7.

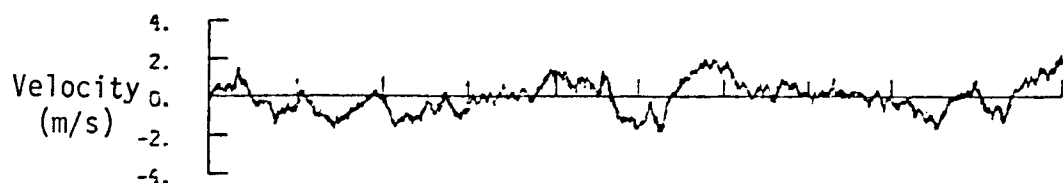
According to the above discussion, there are two main reasons for choosing the Dryden spectrum. First, it is easy to manipulate mathematically. Second, the spectra distribution is closed to that of Lagrangian turbulence. In a simplified case of assumed homogeneous turbulence for which the turbulence intensity is constant, then the only difference between the Lagrangian and Eulerian turbulence formulations is in the length scale. Figure 3.8 (a) and (b) show the Eulerian turbulence histories which were generated by the z-transformations given by Equation 3.25 and 3.27, respectively, while Figure 3.8 (c) and (d) show the Lagrangian turbulence generated by the same equations but three times the Eulerian length scale, i.e., $\Lambda_L / \Lambda_E = 3$. The differences of



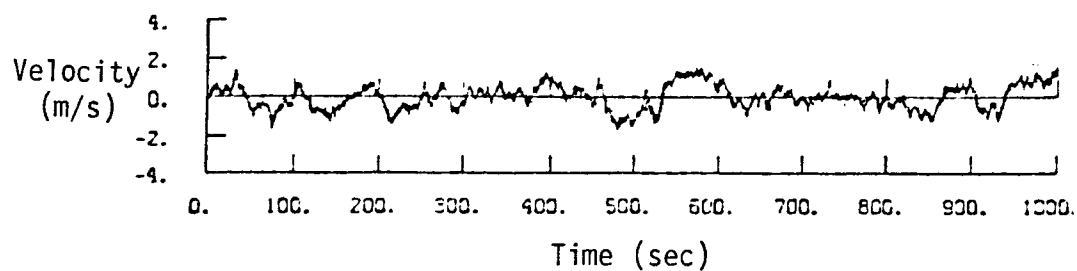
(a) Eulerian longitudinal component of the Dryden model.



(b) Eulerian vertical component of the Dryden model.



(c) Lagrangian longitudinal component of the Dryden model.



(d) Lagrangian vertical component of the Dryden model.

Figure 3.8 Comparison of Eulerian and Lagrangian turbulence time histories for both longitudinal and vertical components ($\Lambda_L/\Lambda_E = 3$).

the spectral distributions are shown in Figure 3.9 (a) and (b) for both longitudinal and vertical components, respectively. It is clear that the Lagrangian spectrum has been shifted with respect to the Eulerian spectrum. The intensity at high frequencies is decreased compared to the Eulerian spectrum while the energy at low frequencies remains the same or is increased. Another method for generating the Lagrangian turbulence is by tracing the history of a particle. The particle position for each time step can be calculated by using Equation 2.1, then the turbulence intensity and the Eulerian length scale can be found from Equations 3.29 and 3.30 for a given particle position. The desired Lagrangian length scale is assumed to be three times the Eulerian length scale. Then Lagrangian turbulence can then be generated by using Equations 3.25 and 3.27 step-by-step by changing the turbulence intensity and length scale corresponding the particle position. In other words, Equations 3.25 and 3.27 could be adapted to inhomogeneous motion. Figure 3.10 shows the Lagrangian turbulence so obtained while Figure 3.11 shows the particle trajectory. From an examination of the vertical fluctuations in Figure 3.10 and the vertical displacement in Figure 3.11, there are two times when the particle has been brought up to a higher altitude, namely, at around 300 m and 700 m downstream. Whenever the particle reaches high altitudes, the turbulence length scale increases, resulting in the domination of low-frequency fluctuations (between 75 and 300 seconds) while the high-frequency fluctuations dominates at low altitude (the first 50 seconds and the last 100 seconds).

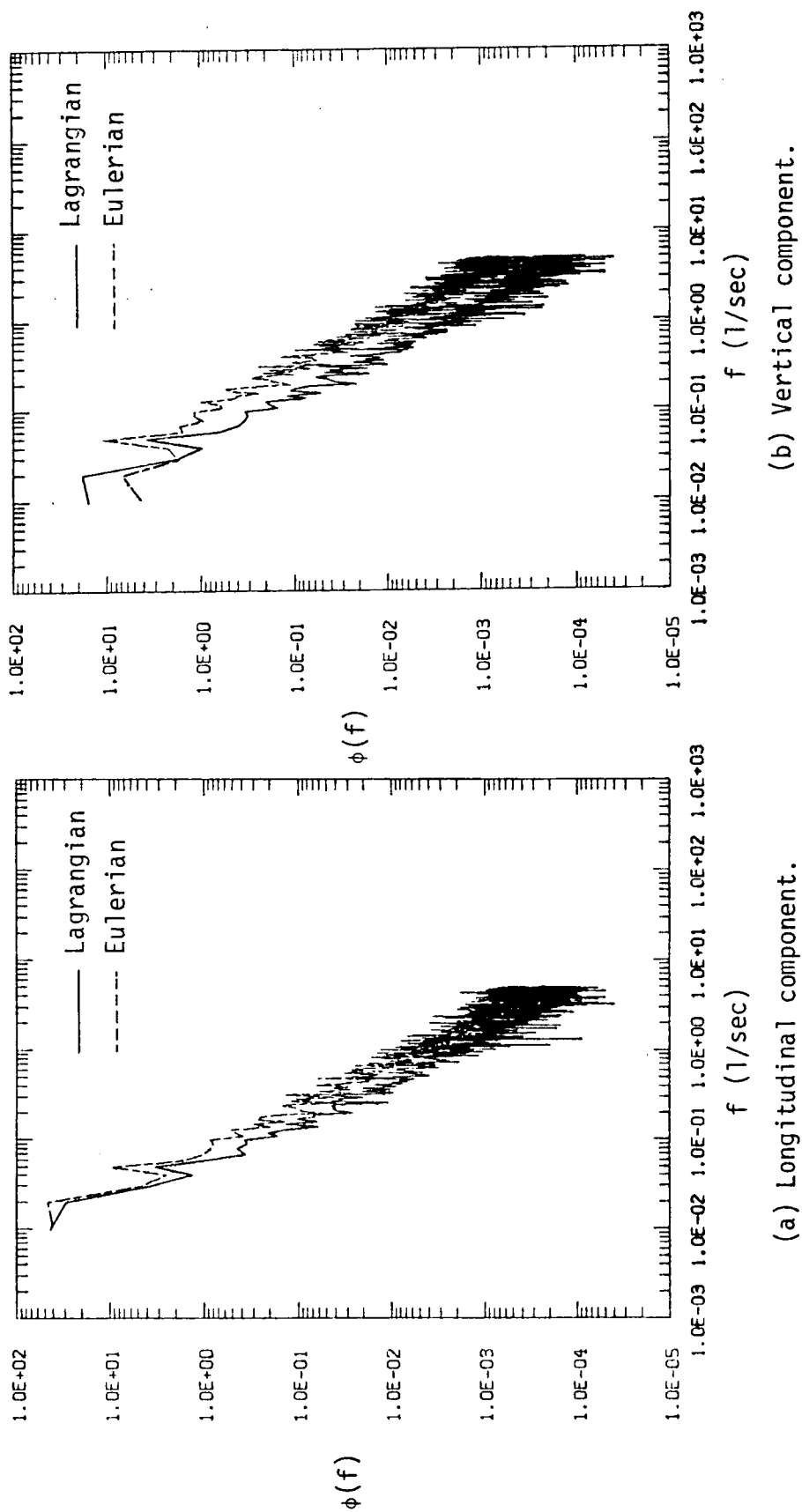


Figure 3.9 Comparison of the Lagrangian and Eulerian turbulence spectra in a homogeneous field.

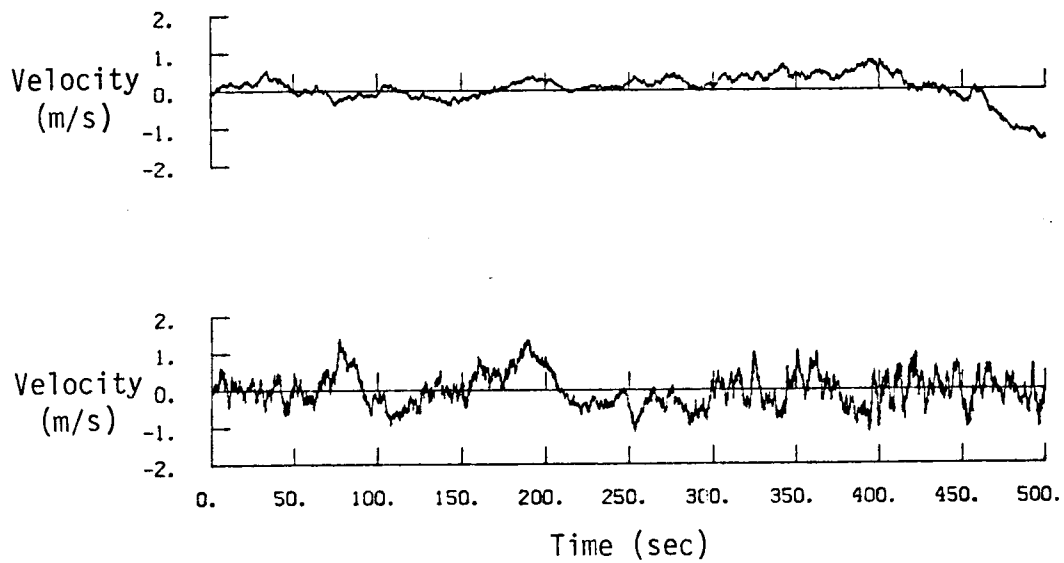


Figure 3.10 Lagrangian turbulence along a particle trajectory in a non-homogeneous field.

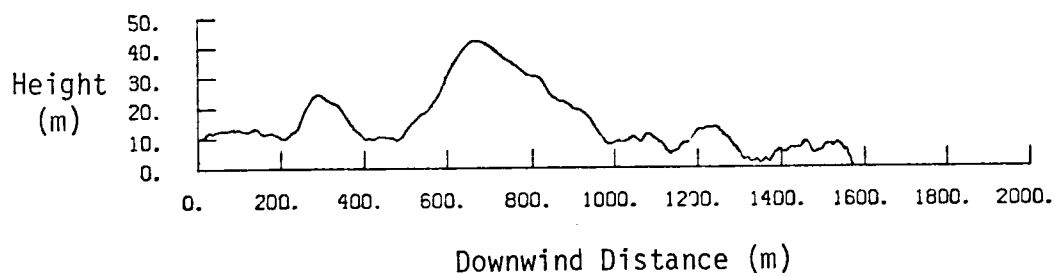


Figure 3.11 Example of particle trajectory.

3.5 Use of Z-Transformation Technique to Develop Coherent Lagrangian Turbulence Generator

As indicated in Section 2, it is necessary to simulate spatial correlations with neighboring particles when dealing with short-range smoke dispersion problems. The coherence function is a two-point spatial spectrum normalized by the product of the one-point spectra at the separation points. Thus, the coherence function is an expression of the correlation or coherence between the fluctuations of the individual frequencies.

The coherence between wind velocity components in the atmospheric surface layer follows the relationship (Davenport (1961), Haugen (1973), Pielke and Panofsky (1976))

$$\gamma^2(\hat{\eta}) = \exp(-a\hat{\eta}) \quad (3.42)$$

where a is a nondimensional parameter called the decay parameter, as listed in Table 3.2; $\hat{\eta}$ is defined as $f\Delta z/V$, V is the average mean wind speed between the two measuring positions; f is frequency; and Δz is the separation distance between the two positions. Figure 3.12 is a plot of the coherence function. The plot illustrates that at low frequencies ($f \rightarrow 0$) or small separation distance ($\Delta z \rightarrow 0$), the coherence approaches unity. This means that if $z_1 - z_2 = \Delta z$, the fluctuation "felt" at z is well correlated or equal to that felt at z_2 . For high frequencies ($f \rightarrow \infty$) or large separation distance ($\Delta z \rightarrow \infty$), there is no correlation between a fluctuation "felt" at z_1 and that "felt" at z_2 .

The coherence is a statistical property of the turbulence. The three-component inter-level coherent turbulence generator was developed by Perlmutter et al. (1977) for aircraft trajectory studies, followed by

TABLE 3.2 The Estimates of the Coefficient a of Different Stable Conditions (Brook (1975)).

Component	$\hat{n}$ Range	Stable	Neutral Stable	Unstable
Longitudinal	all - $\hat{n}$	16.8	17.0	16.8
	$\hat{n} < 0.12$	13.4	13.2	16.8
Lateral	all - $\hat{n}$	12.5	12.8	10.4
	$\hat{n} < 0.12$	14.4	13.4	11.6

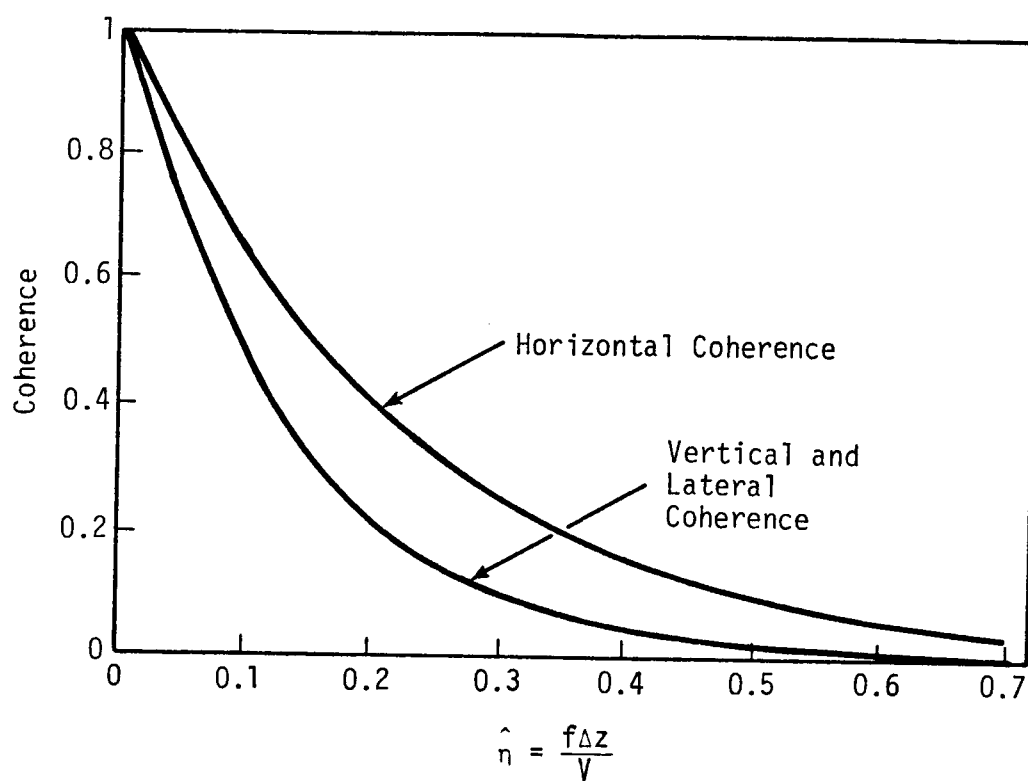


Figure 3.12 Coherence function.

Wang and Frost (1980), who used the Fourier transform technique to generate two-level fixed-point coherent turbulence. Perlmutter et al. (1977) employed several Gaussian noise sources, $\hat{x}_m(k)$, where k is the wave number. The m th noise passed through a lag filter $D_m(k,z)$ and the outputs were summed over all the noise to give a composite process $Y_c(k,z)$. Figure 3.13 illustrates how this process works. The composite process $Y_c(k,z)$ is then passed through a shaping filter $\hat{H}(k,z)$ to obtain the process $Y(k,z)$. Thus we can write

$$\hat{Y}(k,z) = A\hat{H}(k,z) \sum_{m=-M}^M D_m(k,z) \hat{x}_m(k) \quad (3.43)$$

where the lag filter $D_m(k,z)$ is defined as

$$\begin{aligned} D_{-m}(k,z) &= C_m \exp[-jkzd_m] \\ D_0(k,z) &= C_0 \\ D_{+m}(k,z) &= C_m \exp[jkzd_m] \end{aligned} \quad (3.44)$$

where

$$d_m = \frac{m\pi}{(k\Delta z)_{\max}}$$

The coefficients A and C_m in Equations 3.43 and 3.44 were found by Perlmutter et al. (1977) to be

$$\begin{aligned} C_0^2 &= 2\xi_0 \left\{ 1 - \exp\left[-\frac{1}{2\xi_0}\right] \right\} \\ C_m^2 &= \frac{2\xi_0 \left\{ 1 - (-1)^m \exp\left[-\frac{1}{2\xi_0}\right] \right\}}{[1 + (2m\pi\xi_0)^2]} \end{aligned} \quad (3.45)$$

where $\xi_0 = 1/[a(k\Delta z)_{\max}]$ and a is a constant of the coherence function, as shown in Table 3.2.

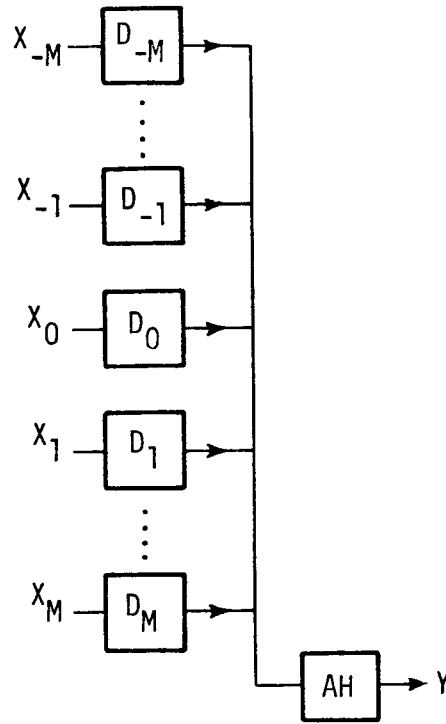


Figure 3.13 Lattice representation of coherent turbulence generator (Perlmutter et al. (1977)).

$$A^2 = \frac{1}{\left\{ c_0^2 + 2 \sum_{m=1}^M c_m^2 \right\}} \quad (3.46)$$

The shaping filter function $\hat{H}(k, z)$ was chosen as the Dryden spectrum because the Dryden spectrum has the best fit of the Lagrangian turbulence spectrum; moreover, there is an advantage to employing this spectrum when the z-transformation technique is used. The general form of the filter function $H(f)$, which represents the Dryden filter, was given by Wang and Frost (1980) as

$$H(f) = \frac{c + jdf}{[a + jf]^2} \quad (3.47)$$

where the constants a , c , and d are listed in Table 3.3. Incorporating Equation 3.47 into Equation 3.43, then utilizing the z-transformation

TABLE 3.3 Coefficients a, c, and d for the Dryden Spectrum.

	a	c	d
Longitudinal Component of the Dryden Spectrum (Equation 3.7)	$\frac{1}{2\pi} \left(\frac{V}{\Lambda} \right)$	$\frac{\sigma}{2\pi^2} \left(\frac{V}{\Lambda} \right)^{3/2}$	$\frac{\sigma}{\pi} \left(\frac{V}{\Lambda} \right)^{1/2}$
Lateral or Vertical Component of the Dryden Spectrum (Equations 3.8 or 3.9)	$\frac{1}{2\pi} \left(\frac{V}{\Lambda} \right)$	$\frac{\sigma}{\pi^2} \left(\frac{V}{2\Lambda} \right)^{3/2}$	$\frac{\sigma}{\pi} \left(\frac{3V}{2\Lambda} \right)^{1/2}$

results in a recursive-type difference equation. The output of this difference equation gives a sequence of turbulent velocities which contain the Dryden spectrum. The detailed derivation is given in the appendix. The discrete output y_{n+1} of Equation 3.43 through z-transformation is given as

$$y_{n+1}(z) = c_1 y_n(z) + c_2 y_{n-1}(z) + A \sum_{m=0}^M C_m [d_1 x_{m,n+i} + d_2 x_{m,n+i-1}] + [d_1 x_{m,n-i} + d_2 x_{m,n-(i+1)}] \quad (3.48)$$

where

$$i = \frac{m\pi z}{V \epsilon_0 \Delta t}, \quad \epsilon_0 = \left(\frac{\omega \Delta z}{V} \right)_{\max}, \quad \text{and}$$

$$c_1 = 2 \exp[-2a\pi\Delta t]$$

$$c_2 = -\exp[-4a\pi\Delta t]$$

$$d_1 = \frac{c}{a^2 b} + \frac{c_1}{2} [(d - c/a)b\Delta t - c/a^2 b]$$

$$d_2 = \frac{c_1}{2} \left[\frac{c}{a^2 b} \left(\frac{c_1}{2} - 1 \right) - \left(d - \frac{c}{a} \right) b\Delta t \right] \quad (3.49)$$

The constants a, c, and d are listed in Table 3.2 and $b = (2\pi)^{1/2}$.

In order to reproduce accurately the coherence function γ^2 given in Equation 3.42, the value of $\Delta z_{\max}$ in Equation 3.44 must be selected approximately. Wang and Frost (1980) indicates that $\Delta z_{\max}$ is approximately equal to 2 or 3. Employing the turbulence intensity (Equation 3.29) and length scale (Equation 3.30) into Equation 3.48 results in the Eulerian time sequence turbulence being generated for a given level. Coherent turbulence for another level can be generated by inputting a different height. Figure 3.14 shows two coherent turbulence time histories for 10-m and 9-m levels. Figure 3.15 compares the calculated coherence functions of the simulated and measured turbulence with the exponential curve given by Equation 3.42. The figure indicates that the calculated coherence functions are described appropriately by the exponential curve. Finally, the coherent Lagrangian turbulence for two neighboring particles can be produced by Equation 3.48 incorporating the relationship between the Lagrangian and Eulerian formulations as indicated in Section 3.4. Again, this Lagrangian turbulence generated by the recursive-type formula (Equation 3.48) can be adapted to inhomogeneous motion.

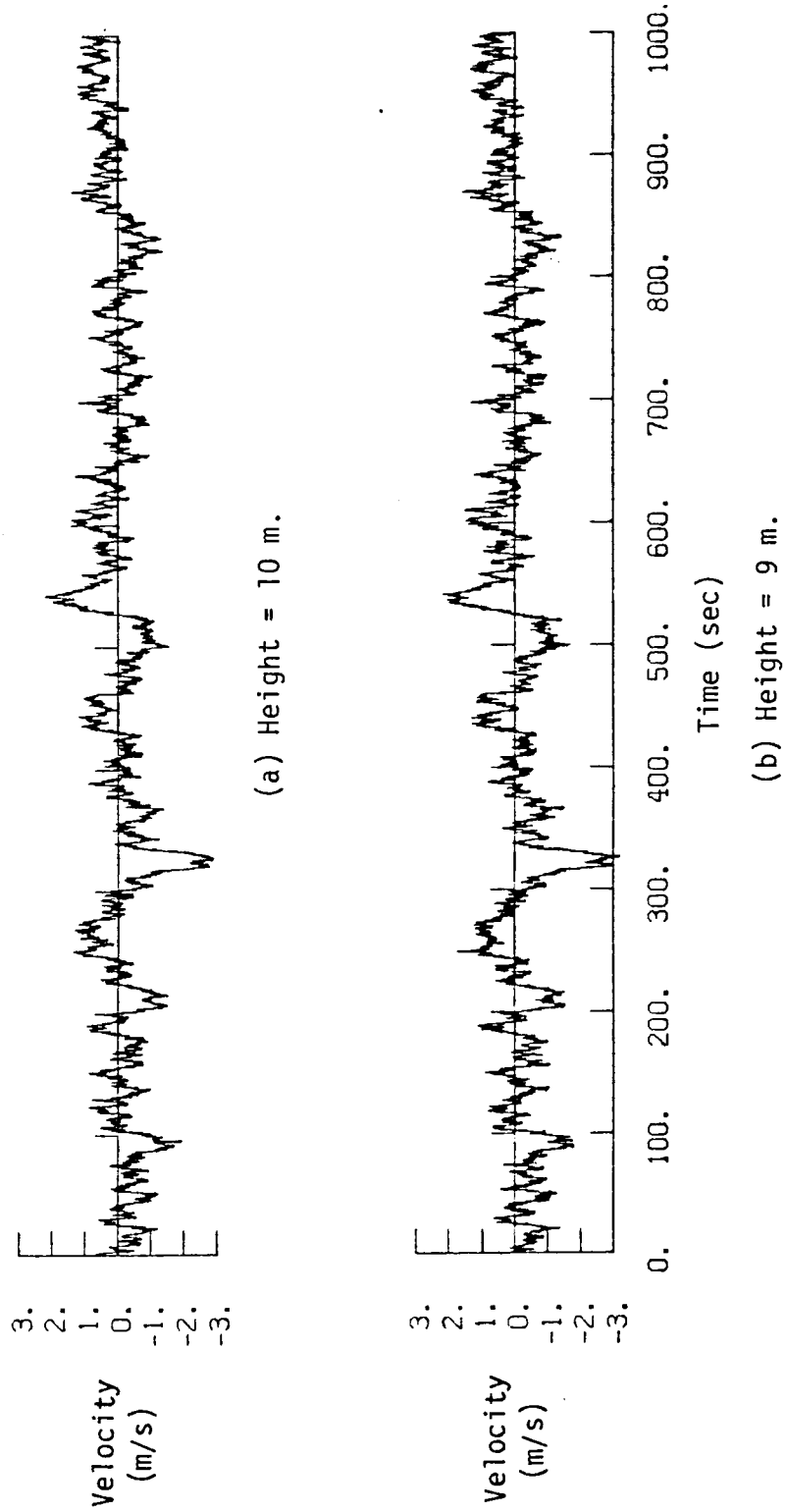


Figure 3.14 Simulated inter-level coherent turbulence.

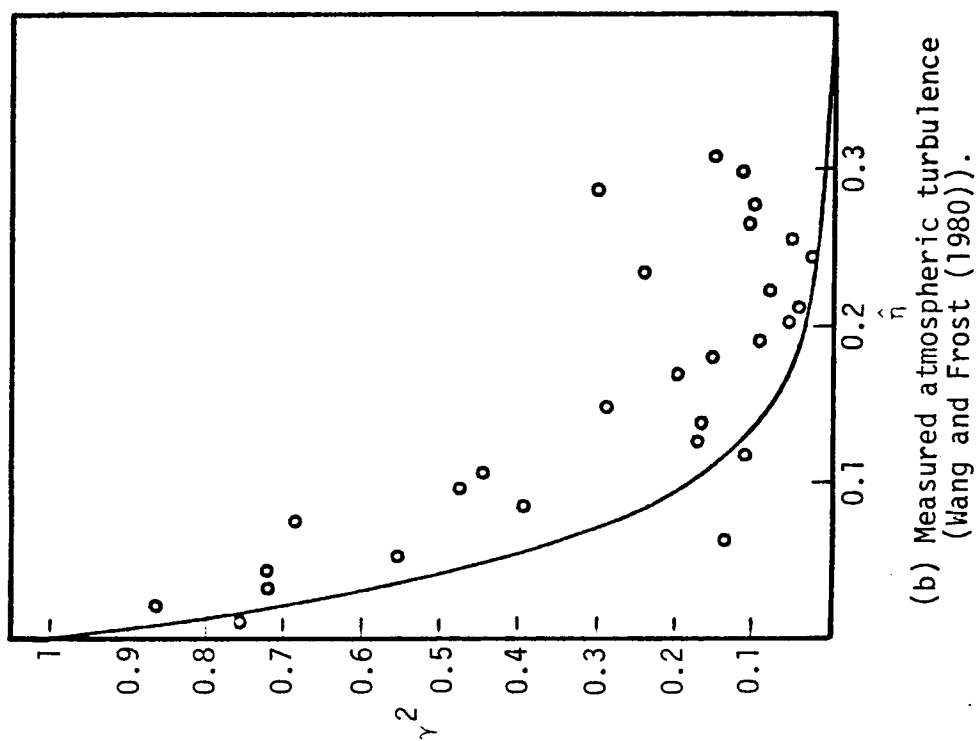
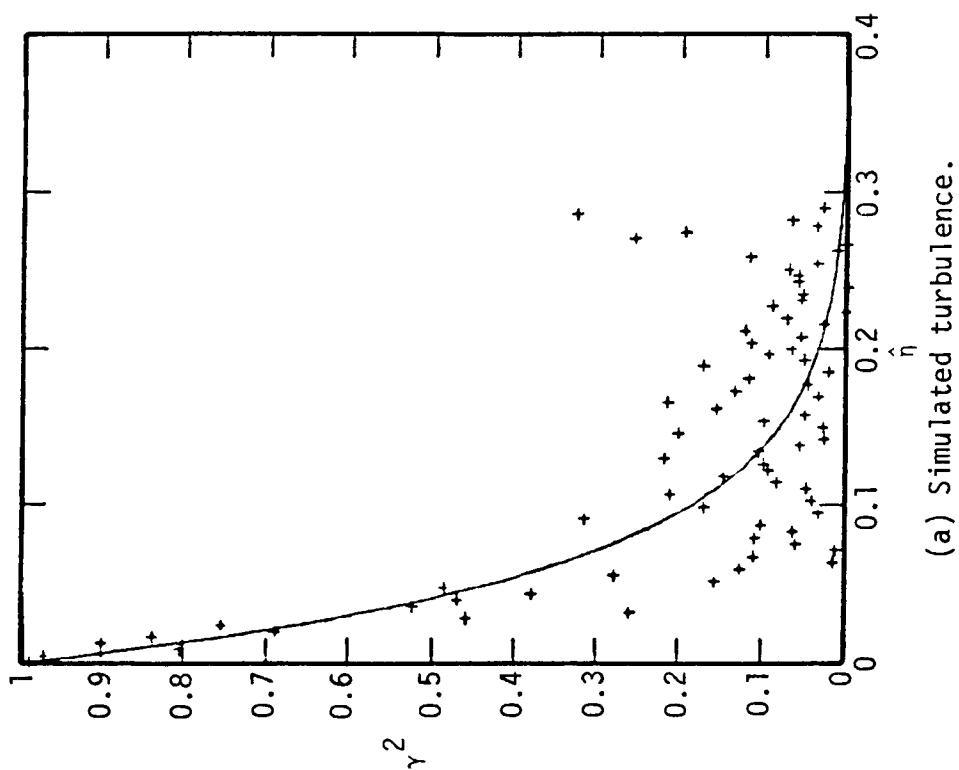


Figure 3.15 Statistical coherence estimates of simulated and measured atmospheric turbulence (compared with Equation 3.42, where $a = 17$, $\Delta z_{\max} = 3$).

4.0 MODEL EVALUATION WITH FIELD TEST DATA

In this section the MoCaPD model is evaluated by comparison with field test data. Three trials of Smoke Week IV (Nelson et al. (1981)) data, each having different obscurant materials, were used for the comparison. Those were: (1) Trial 8, which contained red phosphorus (RP), (2) Trial 16, which contained Hexachlorethane (HC), and (3) Trial 2, which contained Arizona road dust. The first two materials, RP and HC, are hygroscopic smokes while the road dust is non-hygroscopic. Since the different trials had different source release methods, the MoCaPD model had to take this into consideration. Examples of modeling the source characteristics are discussed first. The mechanism for the formation of phosphorus and HC smoke will be reviewed. Then the method for handling the non-linear multi-source effect will be illustrated, especially when the multi-sources are not fired at the same time (as Trial 16). Then comparison of the computed results and measured data will be given for each of these trials.

4.1 Modeling the Source

When the smoke particle leaves the source, it is driven by the fluid flow which emanates from the point source. The flow pattern near the source is totally different from the far field equilibrium flow pattern. In the current model, potential flow is assumed to exit near the source region. Certain types of sources are used frequently for the smoke tests. These include the grenade and the generator. Figure 4.1 illustrates the flow pattern from a three-dimensional source (the grenade) in a uniform flow. An image source of the same strength exists below

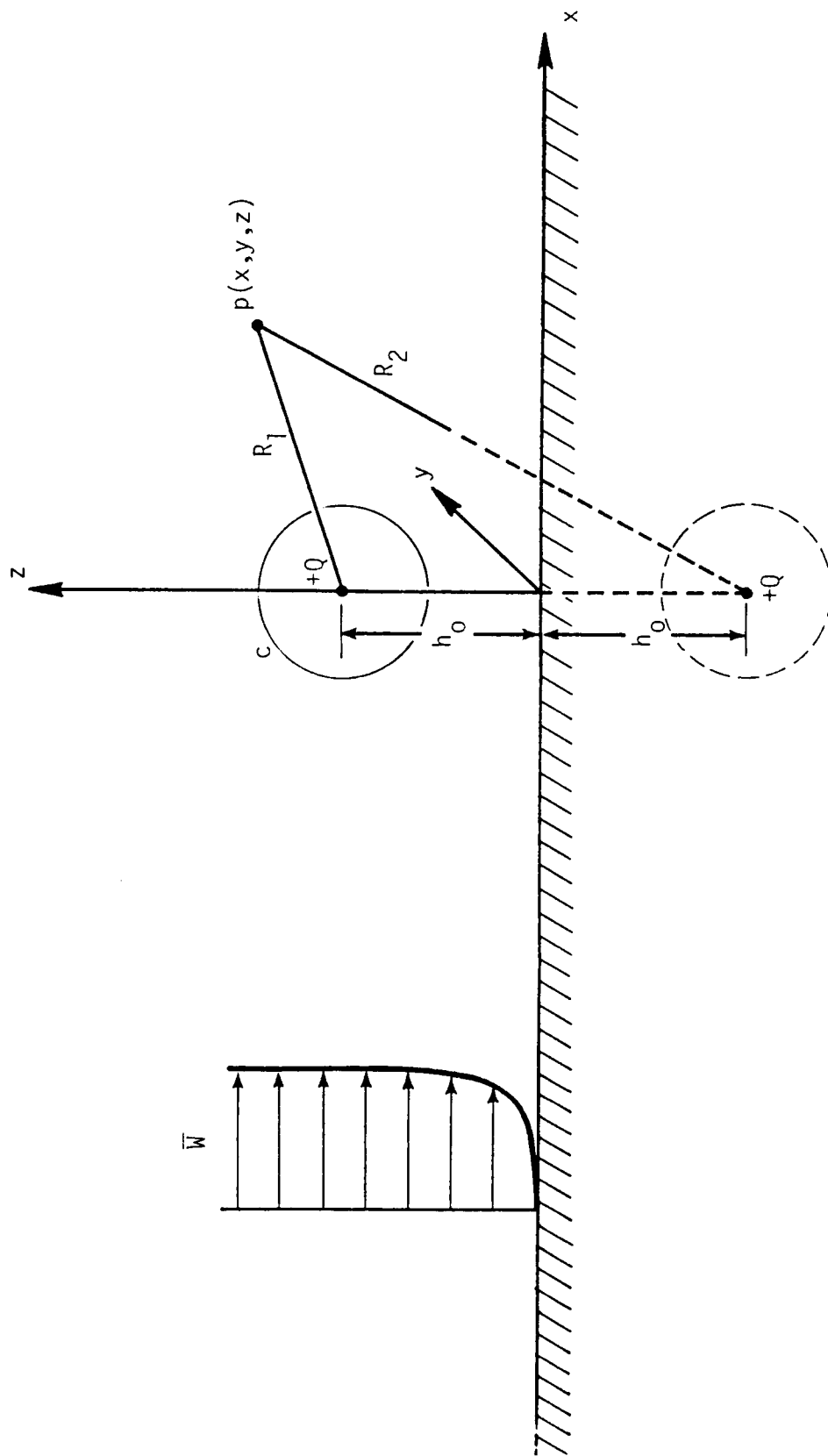


Figure 4.1 Superposition of source and surrounding flow.

the ground to create the ground boundary. The three components of velocities at a given point, $p(x,y,z)$, can be written as (Yuan (1967))

$$\begin{aligned}u(x,y,z) &= \frac{Q}{4\pi} \left(\frac{x}{R_1} + \frac{x}{R_2} \right) + \bar{W}_x \\v(x,y,z) &= \frac{Q}{4\pi} \left(\frac{y}{R_1} + \frac{y}{R_2} \right) + \bar{W}_y \\w(x,y,z) &= \frac{Q}{4\pi} \left(\frac{z - h_0}{R_1} + \frac{z + h_0}{R_2} \right)\end{aligned}\tag{4.1}$$

where

$$R_1 = [x^2 + y^2 + (z - h_0)^2]^{3/2}$$

$$R_2 = [x^2 + y^2 + (z + h_0)^2]^{3/2}$$

$\bar{W}_x$ and $\bar{W}_y$ are the two horizontal components of the free stream velocity, and Q is the strength of the source (the quantity of fluid per unit time flowing out through curve C).

The other type of obscurant source is a generator, as illustrated in Figure 4.2. A plane compound jet is assumed, which is a plane jet surrounded by fluid flowing past it in the same direction as the jet. Rajaratnam (1976) indicates that the velocity distribution in the fully developed region of the component jet could be written as:

$$\frac{U}{U_m} = \exp[-0.6749 \eta^2 (1 + 0.027 \eta^4)]\tag{4.2}$$

where $\eta = z/b$ and the length scale b is the distance from the jet centerline to the point where the velocity is $U = 1/2 U_m$, as illustrated in Figure 4.2. The velocity at the center line, $U_m(x)$, decreases when x increases in the following manner:

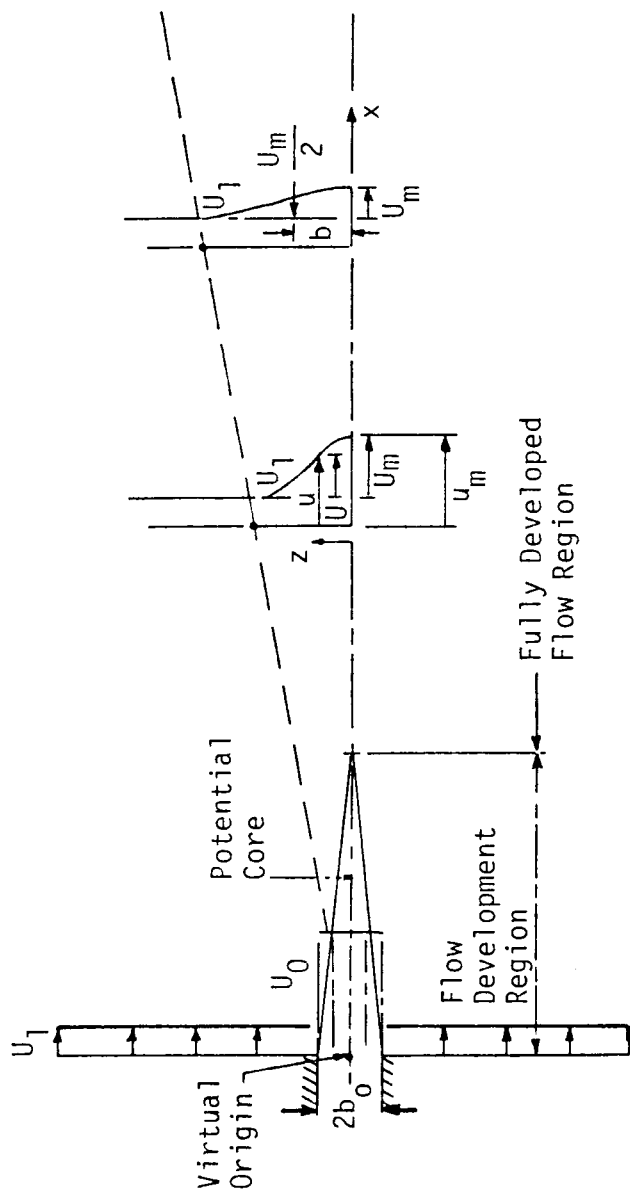


Figure 4.2 Definition sketch for compound jets (Rajaratnam (1976)).

$$\frac{U_m}{\sqrt{U_o(U_o - U_1)}} = \frac{3.41}{\sqrt{x/b_o}} \quad (4.3)$$

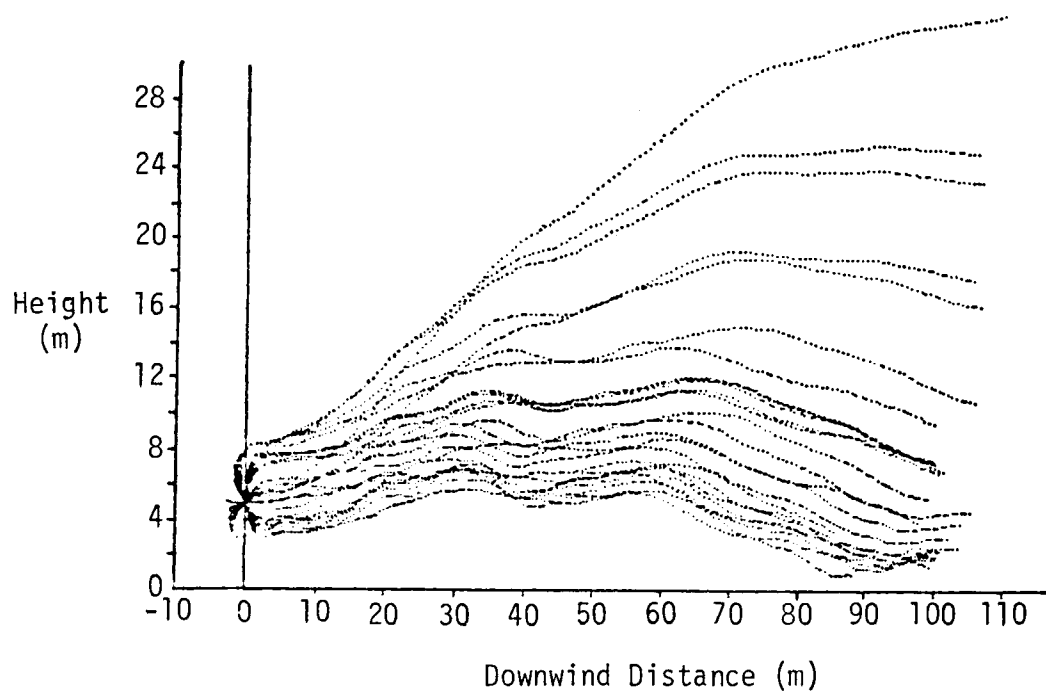
The relation for the length scale, b , can be written as

$$\frac{b}{b_o} = 0.118 \frac{x}{b_o} \frac{1}{1 + \frac{0.41}{\sqrt{\alpha(\alpha - 1)}} \sqrt{x/b_o}} \quad (4.4)$$

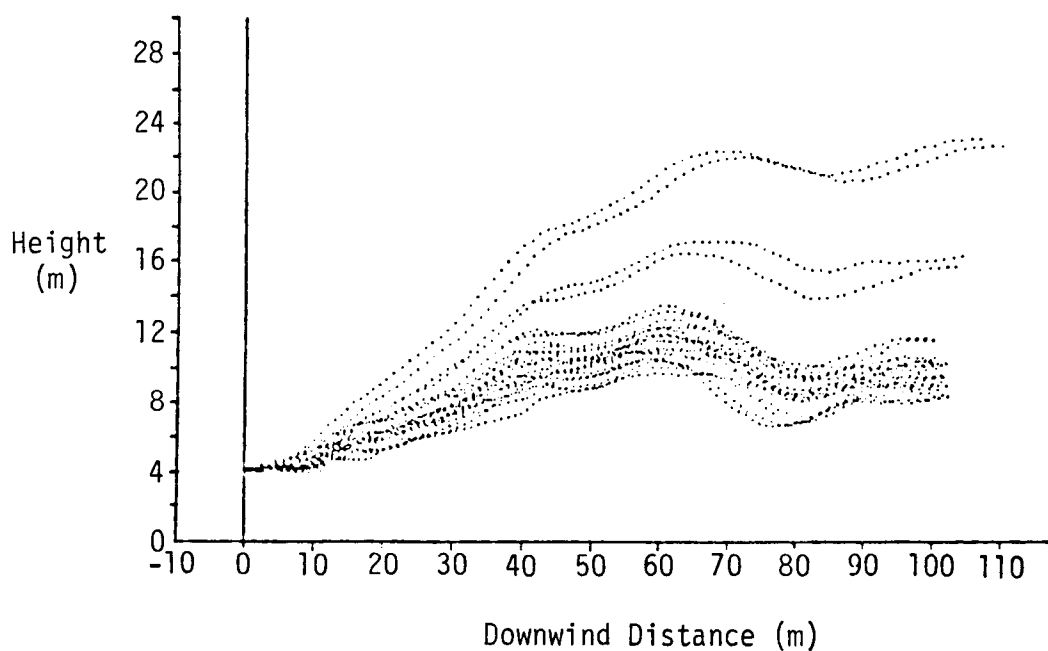
where $\alpha = U_o/U_1$ and U_o is exit velocity of jet, U_1 is velocity of the surrounding flow. Finally, the total flow velocity is

$$u(x,z) = U_1 + U(x,z) \quad (4.5)$$

Figure 4.3 shows the trajectories of the particles which leave the two sources. In this demonstration, a grenade and a Mitey Mite generator were used. The volumetric flow rate of the grenade, Q , was estimated to be $100 \text{ m}^3/\text{s}$. The mean wind speed of the free stream was taken to be 2 m/s . Five particles were grouped together to form a puff. Each puff was released in a random direction from the grenade and the five particles in the puff were spread in a 30° fan about the random direction. The total flow velocity that drives the particles is the mean wind speed plus the induced velocity from the source (no turbulence was included at this point in the computations) when the induced velocity from the source dropped to less than 50 percent of the mean wind speed, the the coherent Lagrangian turbulence $w(\vec{r},t)$ was introduced into the flow (Equation 2.6). For the generator simulation, the puffs are released from the nozzle into a 20° fan, and the particles were uniformly distributed across the nozzle exit plane. Only 25 particle (5 puffs) trajectories are plotted in Figure 4.3 for clarity. These five puffs were released sequentially in 0.1-second intervals. As expected,



(a) Grenade.



(b) Generator.

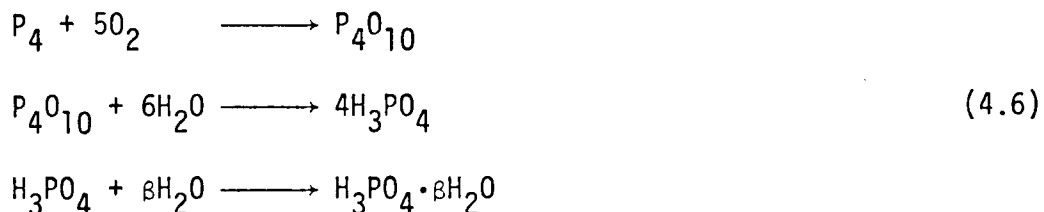
Figure 4.3 Particle trajectories from grenade and generator types of sources.

the particles from the grenade are sprayed in a broader pattern than the particles from the generator. The strongly coherent flow let the particles meander in the same pattern while the puffs are diffused in the downstream direction. This demonstration is only for five puffs and does not indicate the pattern to be expected from a continued operation.

4.2 Characteristics of Obscurant Smoke

Extensive knowledge of the physical constants of standard military obscurant smokes is required to produce a good smoke model. This information must be spatially and temporally dependent. Because a canister or grenade is exploded and then burned, it produces a smoke generation rate that is highly dependent on time. Smoke is produced for only several minutes and then the source is exhausted. Figure 4.4 shows the time histories of the emission rates from three different kinds of obscurant materials (Nelson (1981)). Two kinds of military smokes (phosphorus and HC) are reviewed below (Tarnove (1980), Milham and Anderson (1983), Rubel (1983)):

1. Phosphorus smoke: Phosphorus, on ignition in air, burns to form phosphorus pentoxide, P_4O_{10} . The pentoxide then combines chemically with atmospheric moisture to produce orthophosphoric acid, H_3PO_4 . The H_3PO_4 , being hygroscopic, absorbs more moisture from the air, thus producing the droplets of the phosphorus smoke product which are composed of an aqueous solution of orthophosphoric acid. The mechanisms for the chemical reaction given by:



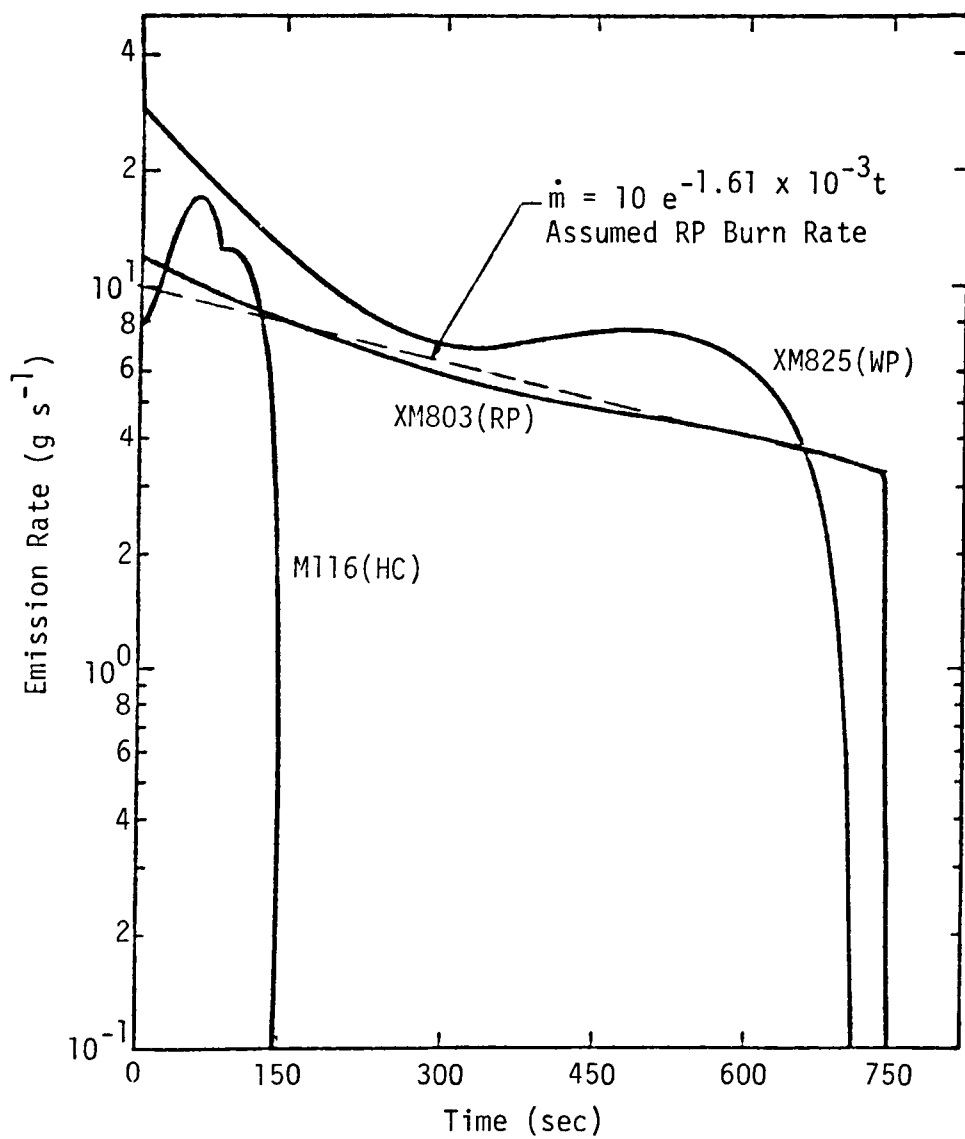


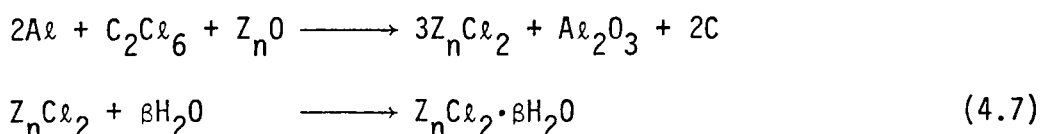
Figure 4.4 Emission rate as function of time.

where β is governed by the RH-equilibrium hydrated concentration. The product of this chemical reaction is a phosphorus-derived smoke. The amount of smoke generated is described by the yield factor which is defined as the increase in the mass of the acid solution with increasing relative humidity, i.e.,

$$\text{yield factor} = \frac{\text{mass of acid solution}}{\text{mass of acid}}$$

Details of the yield factor are given by Tarnove (1980) and Rubel (1983).

2. Hexachlorethane smoke (HC): HC is a pyrotechnic smoke mixture which is produced through the following process:



where β is a function of the RH. The water absorption characteristics of the HC are given by Rubel (1983) and Milham and Anderson (1983).

4.3 Optical Properties of Multi-Sources

As mentioned previously, the emission of a source is dependent on time. Moreover, on the battlefield, multi-sources are generally applied with sources located at different positions. This results in an increased non-linear effect. Figure 4.5 schematically illustrates the source contribution problem. Suppose that at time t , the number density of particles, $\hat{n}_i(x_j, t)$, which came from source #j on the line of sight is calculated. Those puffs which came from different sources take different times, t_1, t_2, t_3 , to travel to the line of sight. Let the rate of mass emission by the source #j, which is emitted at time $\hat{t}_j$, be expressed as $\dot{m}(\hat{t}_j)$ and let

$$\hat{t}_j = t - t_j \quad (4.8)$$

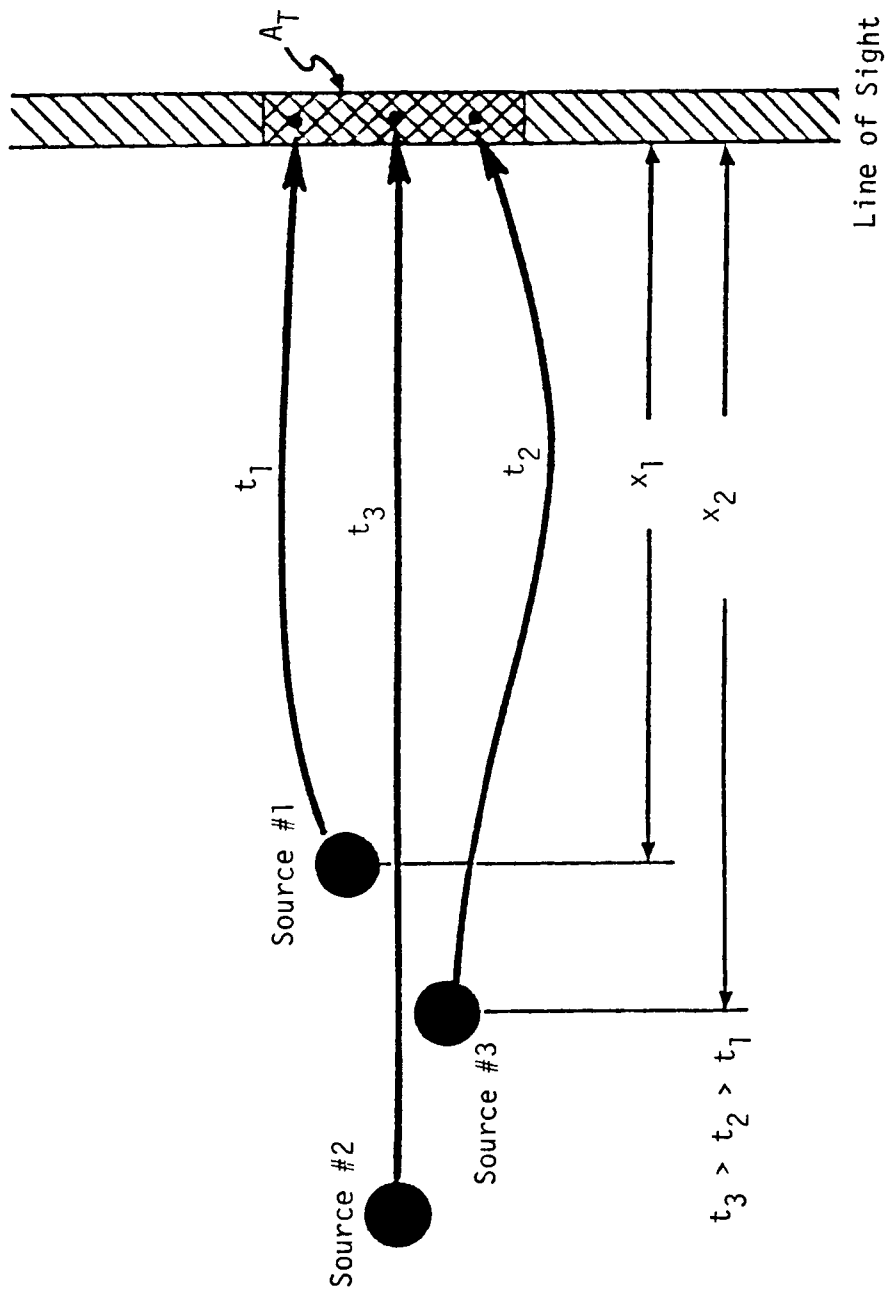


Figure 4.5 Schematic of source contribution.

where j is the source number (1, 2, 3 ...) and t_j is the particle travel time which can be expressed as $t_j = x_j/V$ where V is the mean wind speed and x_j is the distance from the source to the line of sight.

The concentration path length, CL , is calculated as:

$$CL(t) = \sum_{i=1}^{N_D} \sum_{j=1}^{N_S} \frac{\dot{m}(\hat{t}_j) \Delta t P_j(D_i) p(D_i, x_j, t)}{A_T} \quad (4.9)$$

where A_T is the cross sectional area of the line of sight; $\dot{m}(\hat{t}_j)$ is the j th source emission rate; Δt is time interval for the sampling; $P_j(D_i)$ is the probability of mass distribution of size D_i from source j ; and $p(D_i, x_j, t)$ is the probability of the number density of size D_i at time t (those particles came from source # j).

$$p(D_i, x_j, t) = \frac{\hat{n}_i(x_j, t)}{N_i} \quad (4.10)$$

where N_i is total number of particles of size D_i being released at each time interval and $\hat{n}_i(x_j, t)$ is the number of particles of size D_i that reach the line of sight.

The extinction coefficient, as discussed in Section 2.2.5, can be expressed as

$$\alpha(t) = \frac{3 \sum_{i=1}^{N_D} Q_{Ei} D_i^2 \hat{p}(D_i)}{2\rho_o \sum_{i=1}^{N_D} D_i^3 \hat{p}(D_i)} \quad (4.11)$$

where Q_{Ei} , the extinction efficiency for the largest particle diameter in the i th size interval, will be discussed later; ρ_o is the density of the particle; and $\hat{p}(D_i)$ is the number probability density of particles of size D_i at the line of sight. For the multi-source case, $\hat{p}(D_i)$ can be

written as:

$$\hat{p}(D_i) = \frac{\sum_{j=1}^{N_S} n_i(x_j, t)}{\sum_{j=1}^{N_S} \sum_{i=1}^{N_D} n_i(x_j, t)} \quad (4.12)$$

where N_S is the number of sources and $n_i(x_j, t)$ is the number of particles which are contributed by source #j, which can be expressed as

$$n_i(x_j, t) = \frac{\hat{n}_i(x_j, t)}{N_i} N_{o_{ij}} \quad (4.13)$$

where N_i and $\hat{n}_i(x_j, t)$ were defined previously and $N_{o_{ij}}$ is the total number of particles of the i th size from source #j. $N_{o_{ij}}$ can be determined from the mass rate by

$$N_{o_{ij}} = \frac{\dot{m}(\hat{t}_j) \Delta t P_j(D_i)}{\frac{\pi \rho_o}{6} D_i^3} \quad (4.14)$$

Substituting Equations 4.11, 4.12, and 4.13 into Equation 4.10 yields the expression for the extinction coefficient as

$$\alpha(t) = \frac{3 \sum_{i=1}^{N_D} D_i^{-1} Q_{Ei} P(D_i) \sum_{j=1}^{N_S} \hat{n}_i(x_j, t)}{2\rho_o \sum_{i=1}^{N_D} P(D_i) \sum_{j=1}^{N_S} \hat{n}_i(x_j, t)} \quad (4.15)$$

The extinction efficiency, Q_E , is a function of the particle index-of-refraction which, in turn, is a function of the particle material and wavelength of the light used for observation. It is generally believed that phosphorus munitions generate smoke that eventually (times the order of 30 to 60 minutes) becomes droplets containing orthophosphoric

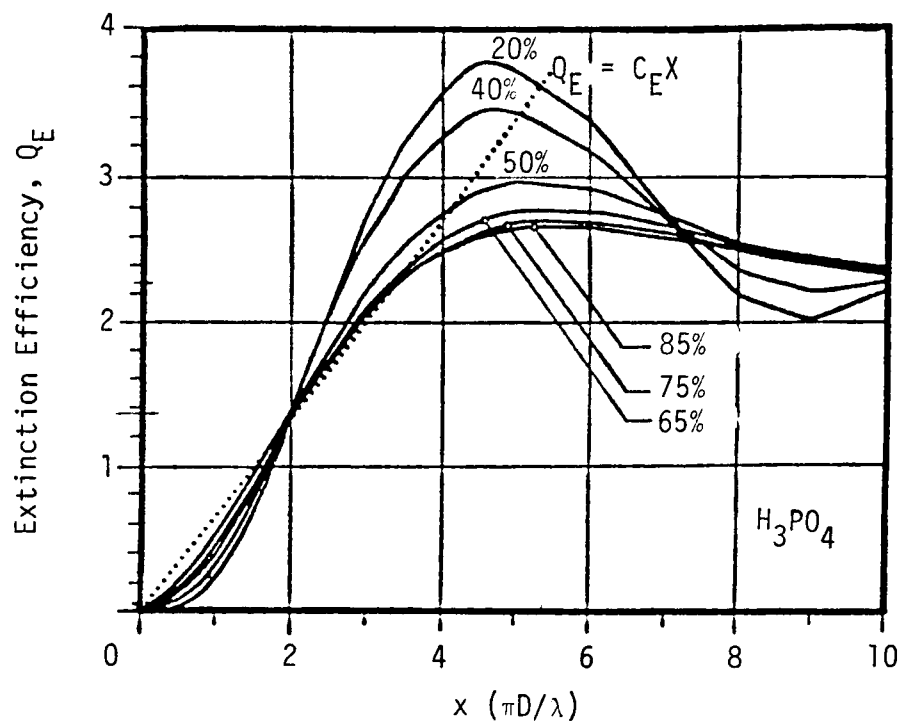
acid and that HC munitions generate smoke which becomes droplets containing zinc chloride. Using the Mie scattering theory, Q_E can be computed as a function of $\pi D/\lambda$ where D is the particle diameter and λ is the wavelength of interest. Figure 4.6 (a) and (b) give Q_E for orthophosphoric acid, which is thought to be a good approximation for new phosphorus smoke for a wavelength at $3.4 \mu\text{m}$ and a poor approximation at $9.75 \mu\text{m}$ (Farmer (1980)). The extinction efficiency Q_E is plotted in Figures 4.6 and 4.7 for varying fractions of orthophosphoric acid concentration in water, F . The relation between F and the relative humidity, ψ , is

$$F = \frac{329.8 - 323.4 \psi}{380.6 - 265.1 \psi} \quad (4.16)$$

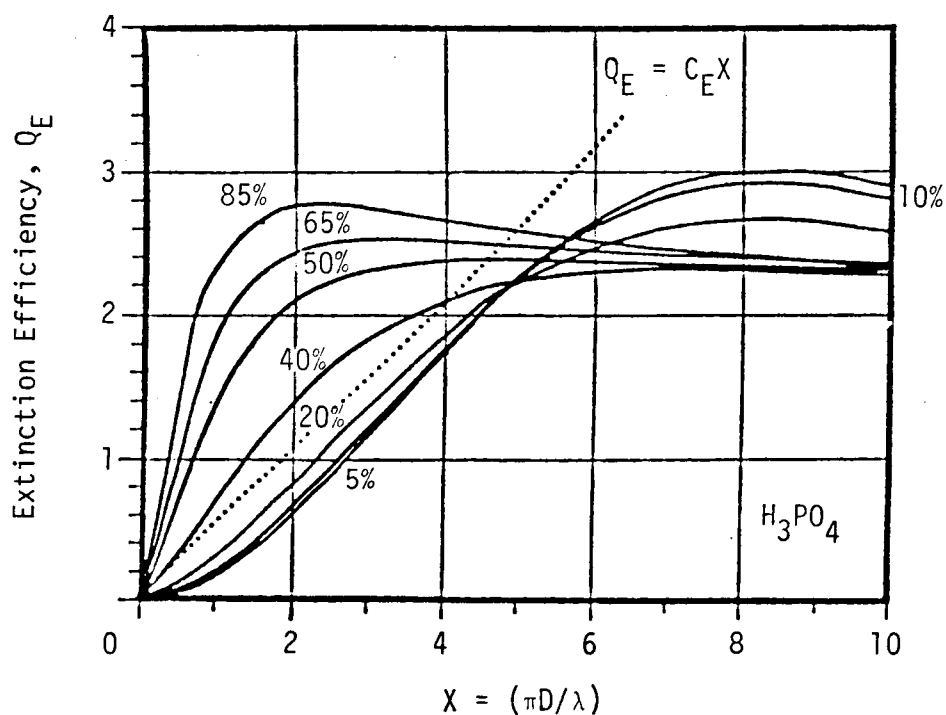
where the relative humidity is in the range $0.1 \leq \psi \leq 0.9$.

4.4 Comparison of the Computed Results and Smoke Week IV Data

Smoke Week IV was conducted by the Office of the Project Manager, Smoke/Obscurants, November 2-9, 1981, on Test Area 1, Redstone Arsenal, Huntsville, Alabama. The overall layout of the test area is shown in Figure 4.8 (Nelson et al. (1982)). The arrangement of the DPG (Dugway Proving Ground) transmissometer, and the location of the meteorology tower is shown in Figure 4.9. Along the line of sight (LOS), transmittance was measured for 0.4-0.7, 0.74-0.95, 1.06, 3.4, 8-12, and $10.6 \mu\text{m}$. Approximately 70 m to the north and south of LOS1 were two sets of transmissometers operating at $3.4 \mu\text{m}$. Each set consisted of a transmissometer LOS, one at 2-m vertical level while the other one was at a 7-m vertical level. All of the meteorological data were measured from the 50-m high MET tower, which was located near the center of the grid.

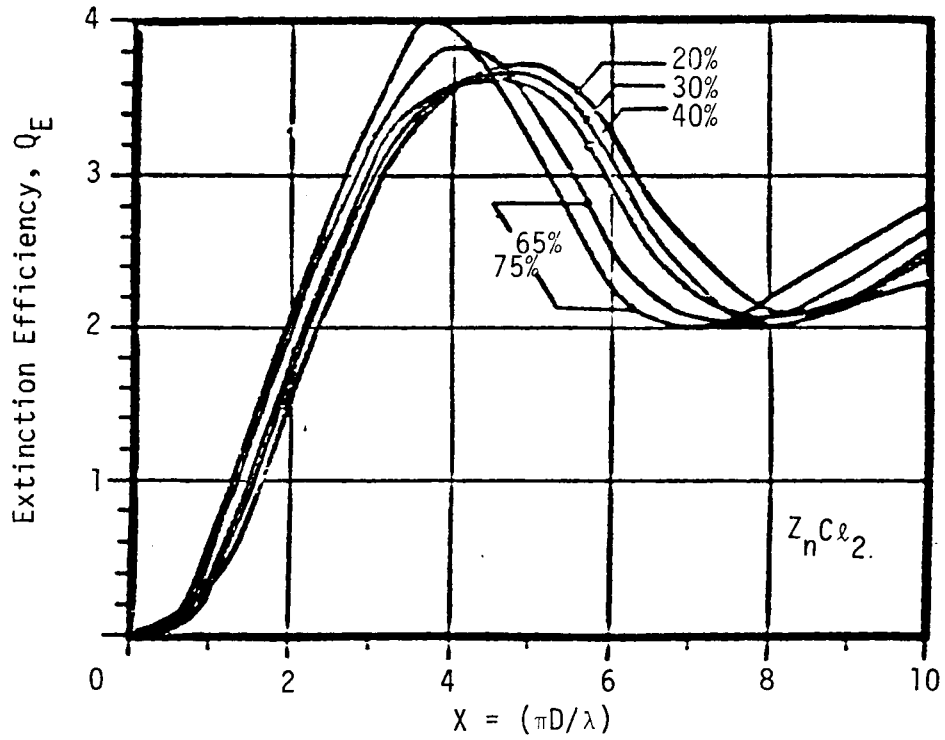


(a) 3.4 μm wavelength.

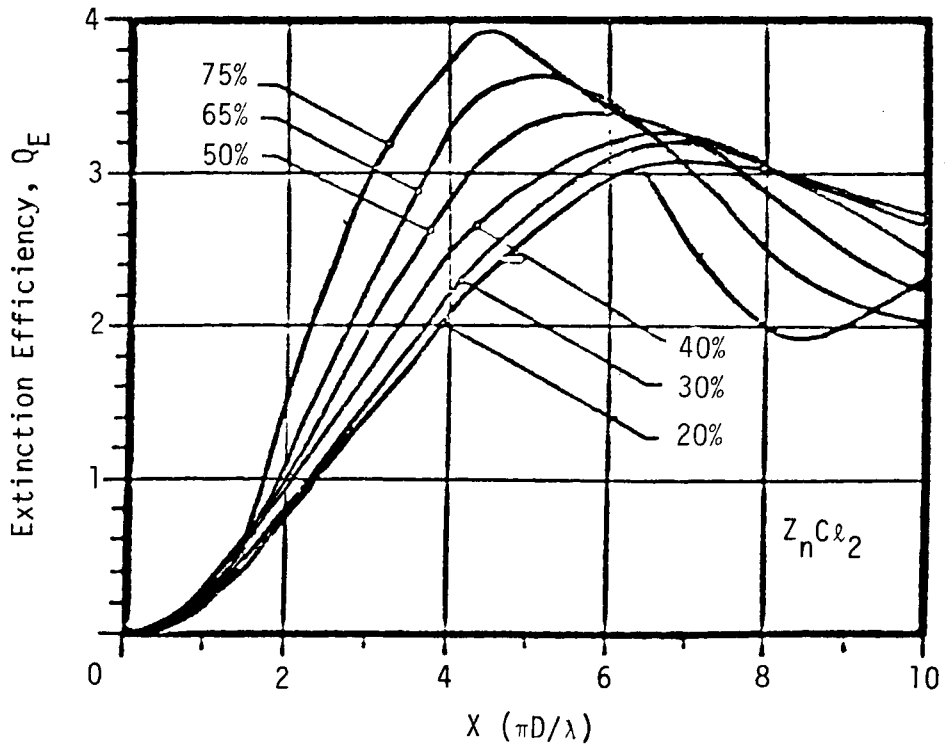


(b) 9.75 μm wavelength.

Figure 4.6 Extinction efficiency as a function of $\pi D/\lambda$ for ortho-phosphoric acid in varying concentrations in water, F (Farmer (1980)).



(a) 3.4 μm wavelength.



(b) 9.75 μm wavelength.

Figure 4.7 Extinction efficiency as a function of $\pi D/\lambda$ for zinc chloride in varying concentrations of water, F (Farmer (1980)).

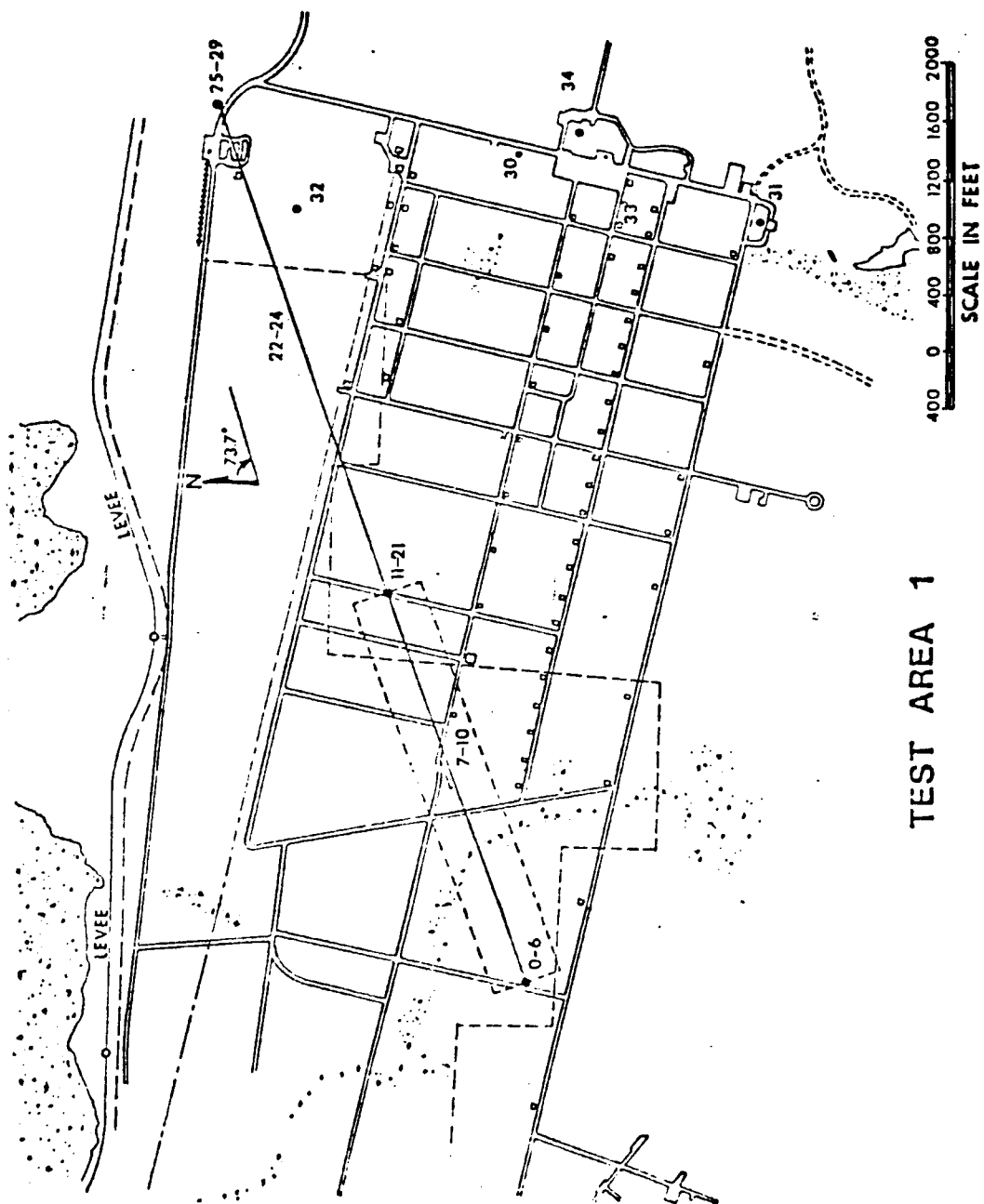


Figure 4.8 Overall test site layout (Nelson et al. (1982)).

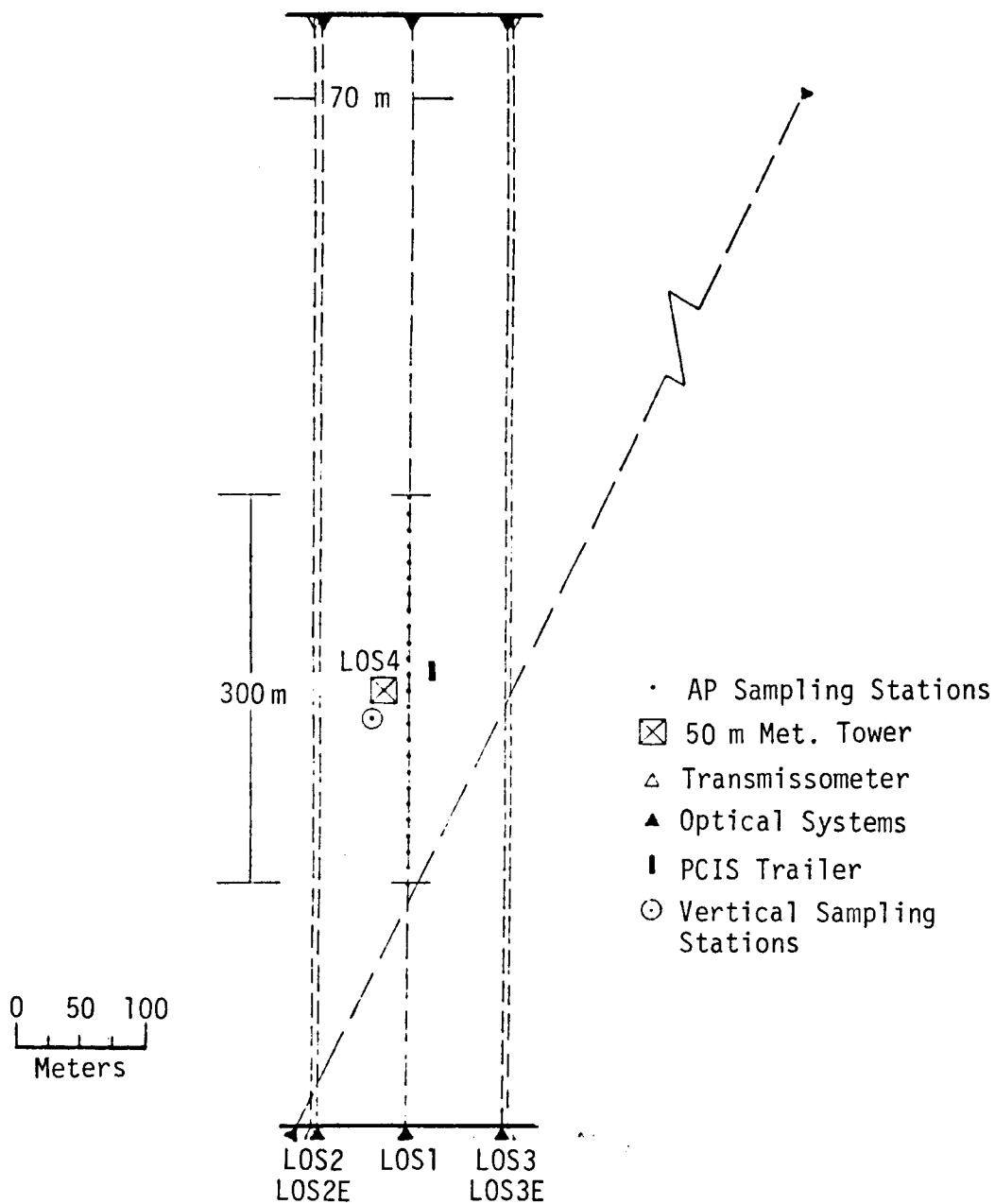


Figure 4.9 Location of DPG transmissometer lines of sight (Nelson et al. (1982)).

The following three subsections discuss the comparison between the computed results and the measured data from these three trials.

4.4.1 A Simulation of Trial 8 from Smoke Week IV

The obscuration source for Trial 8 in Smoke Week IV was a red phosphorus munition. Eight L8A1 grenades consisting of 2.88 kg (6.34 lbs) were detonated from 5.5-m (18-ft) high poles located in an area upwind of the grid center. The first grenade was 10 m south of LOS1 while the last (eighth) was about 75 m south of LOS1. The arrangement of grenades in the test field is illustrated in Figure 4.10. The meteorological data are listed in Table 4.1; the wind speed was 2.25 m/s at a 2-m height, the wind direction was 125 degrees, and the relative humidity was 59 percent. The particle mass distribution from the source was assumed to be the same as the measured data from the low pressure cascade impactor (LPCI), which was located near the LOS1. Figure 4.11 shows the mass distribution as a function of particle diameter for Trial 8, as measured by the particle measurement system (PMS) and the particle size interferometer (PSI) (Farmer (1983a)). This is the weakest assumption in the simulation since the particle distribution of the munition is expected to be different from that measured at a downstream location.

In this trial (Trial 8), two models have been applied. The first one is an incoherent simulation where it is assumed that the particles are independent of each other, as was the case with the previous MoCaPD model. The second one is the current MoCaPD model which assumed that the movements of particles were correlated with each other. The number of particles along LOS1 for each particle size from each source $\hat{n}_i(x_j, t)$ has been calculated substituting $\hat{n}_i(x_j, t)$ into Equation 4.15, the values

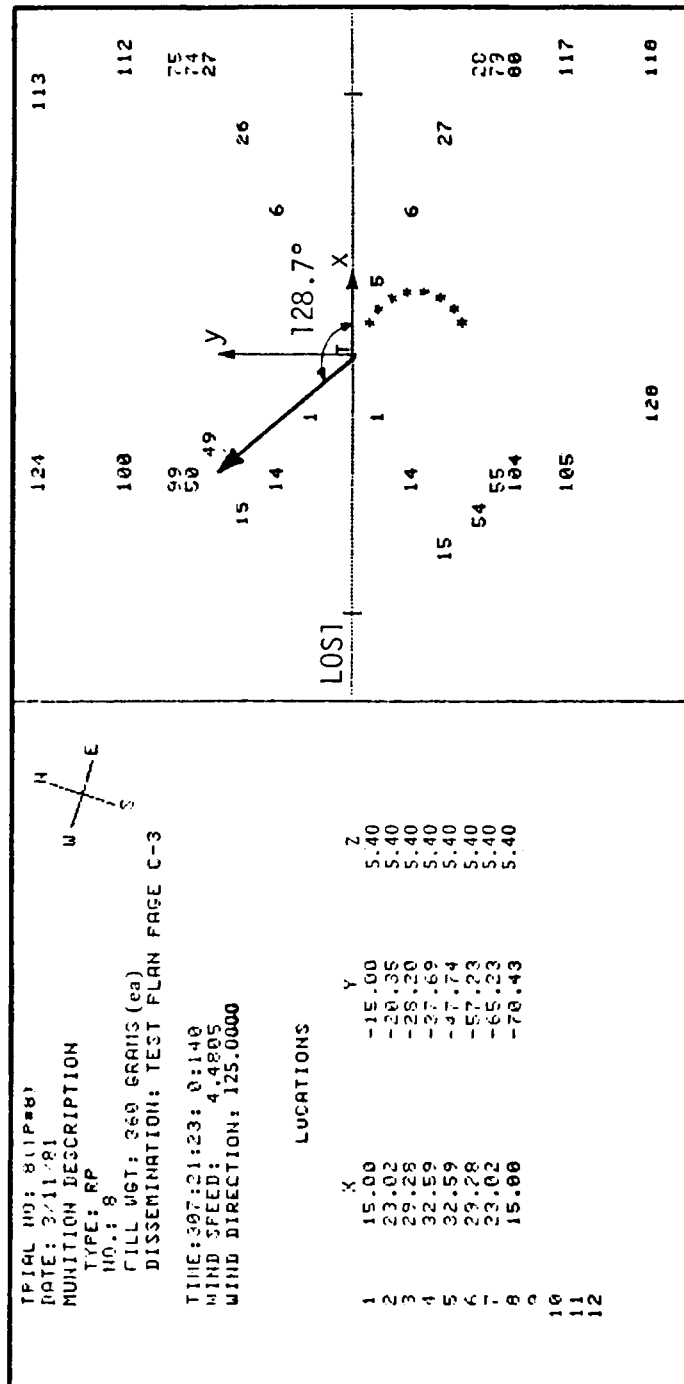


Figure 4.10 Munition arrangement in Smoke Week IV, Trial 8 (Nelson et al. (1982)).

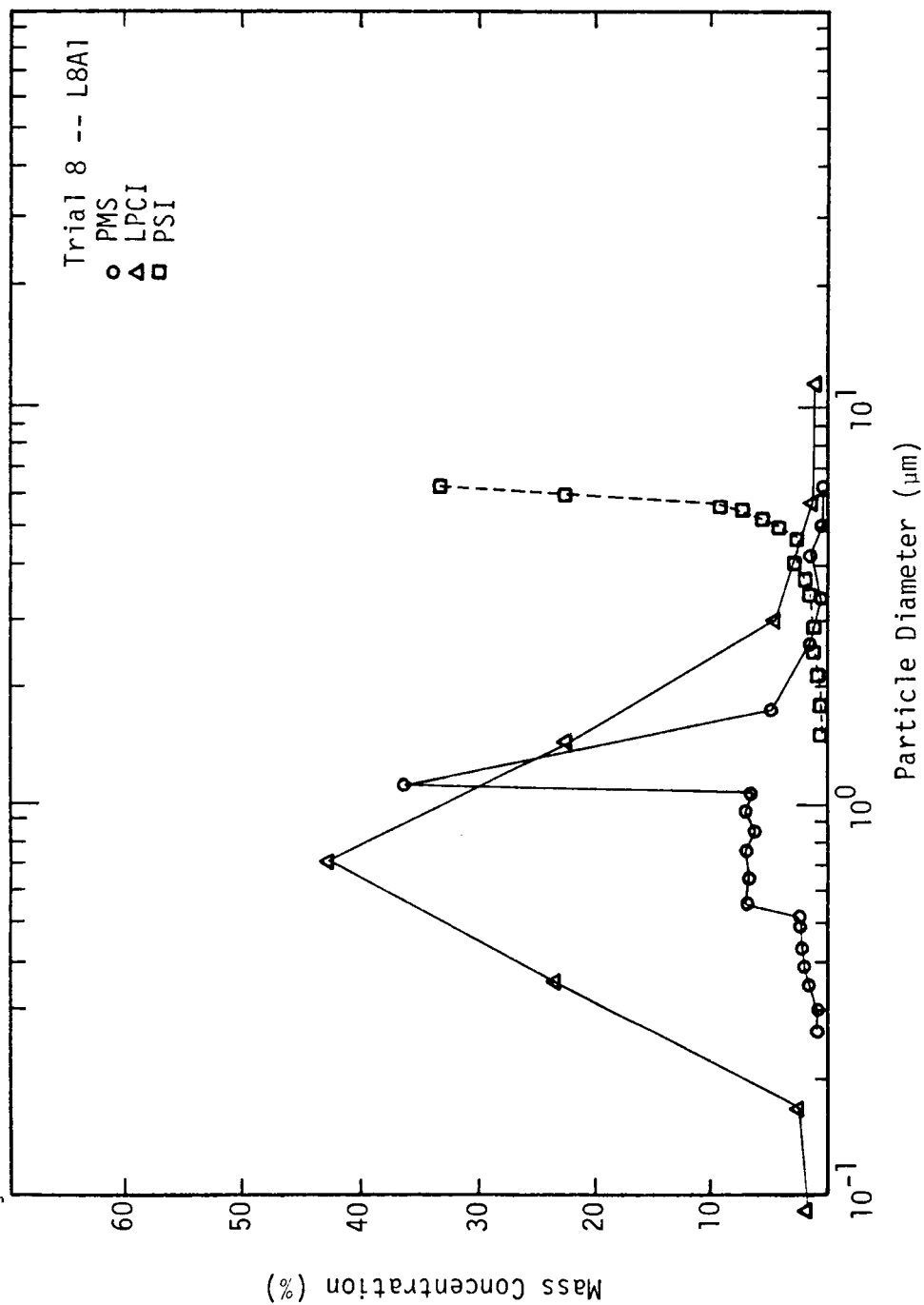


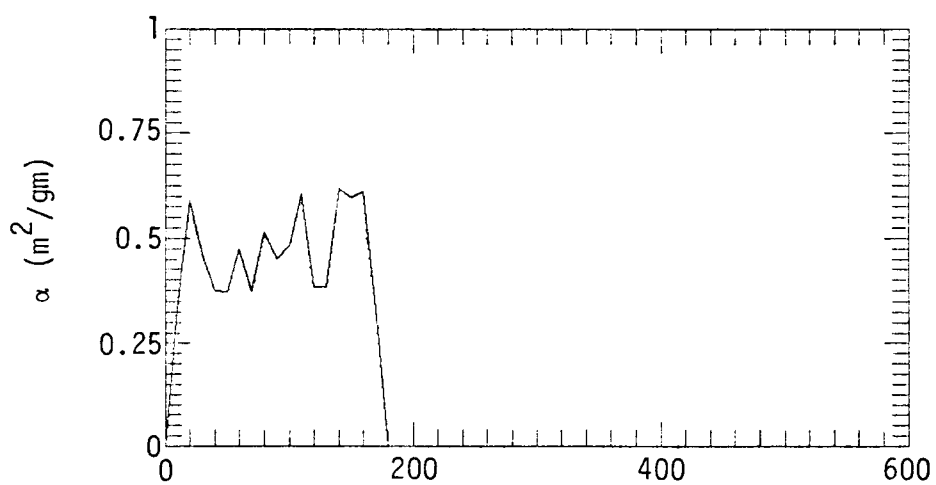
Figure 4.11 Mass concentration as a function of particle diameter for Trial 8, RP grenades, as measured by the PCI, PMS, and PSI (Farmer (1983a)).

TABLE 4.1 Meteorological Input Data.

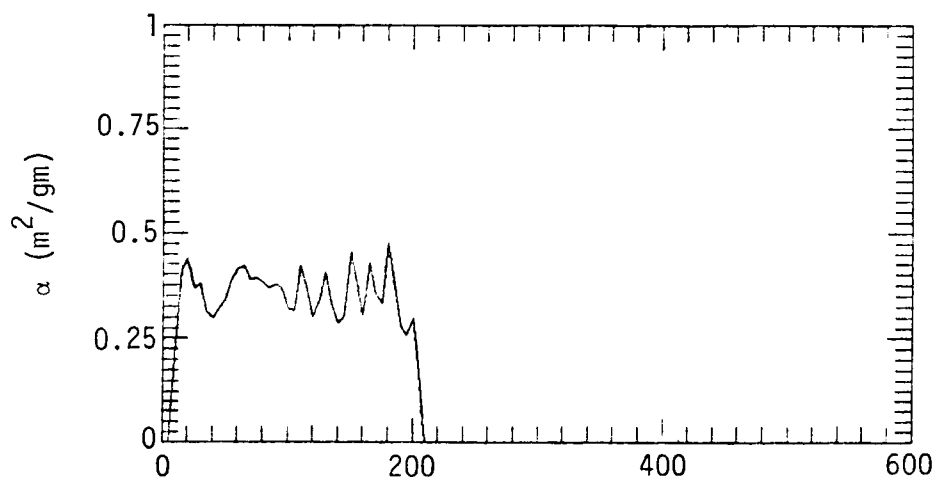
Smoke Week IV Trial No.	Trial 2	Trial 8	Trial 16
Wind Speed at 2-m Height	1.7 m/s	2.25 m/s	1.2 m/s
Averaged Wind Direction	131°	125°	158°
Relative Humidity	60°	59°	95°

of the density of red phosphorus at RH = 59 percent as 1.46 gm/cc (Farmer (1983a)).

Figure 4.12 shows the extinction coefficient α at 3.4 μm wavelength computed by both the incoherent and the coherent MoCaPD models. An integral time interval of 10 seconds was used for the incoherent MoCaPD model while 5 seconds was chosen for the coherent MoCaPD model. Both versions of the MoCaPD model predicted the extinction coefficient to be in the range 0.3 to 0.6 m^2/gm . Measured data are not available in this trial (Trial 8) to compare with the computed α ; however, the experimental laboratory data of Milham and Anderson (1983) indicate that the α value of red phosphorus at 3.4 μm wavelength is in the range of 0.3 to 0.7 m^2/gm when the RH is in the range of 60 percent to 70 percent. Values of α , which the MoCaPD model predicted, are well within this range. Figure 4.13 shows the extinction coefficient measurements for phosphorus smoke at 3.4 μm wavelengths from four other tests, H^3S , CSL, SWII, Phase IIA (Farmer (1980)). Values of α remain in the range of 0.2 to 0.5 m^2/gm up to a relative humidity equal to 60 percent.



(a) Incoherent MoCaPD model.



(b) Coherent MoCaPD model.

Figure 4.12 Comparison of extinction coefficient computed by both incoherent and coherent MoCaPD model.

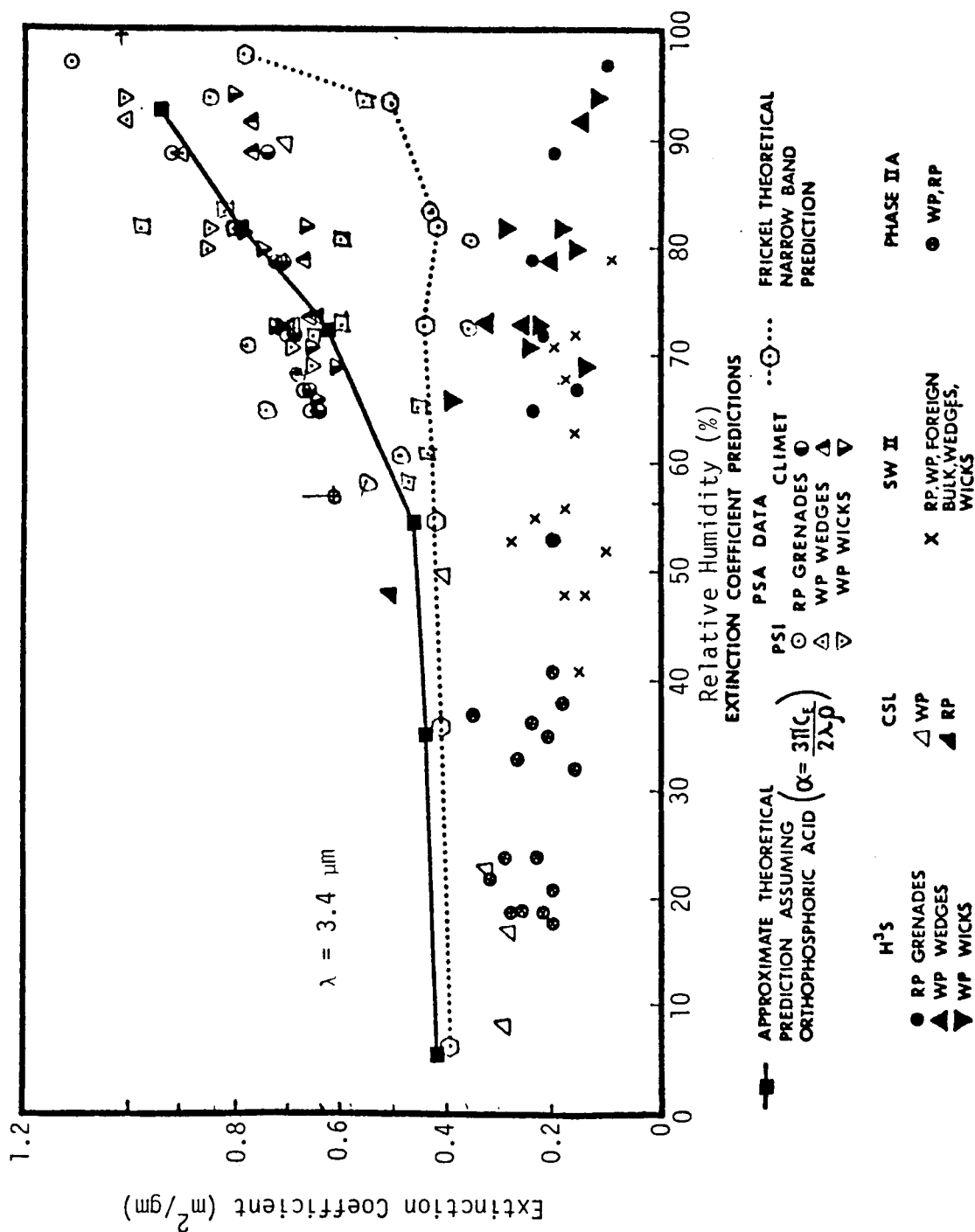


Figure 4.13 Comparison of extinction coefficient measurement for $3.4 \mu\text{m}$ wavelengths from four different tests with approximate theoretical values for phosphorus smokes (Farmer (1980)).

Farmer (1983b) points out that the measured α for phosphorus is highly unstable during the first 1 to 2 minutes from release and then becomes relatively constant. This might be due to the localized variations in relative humidity. In the MoCaPD model, constant RH is assumed so that any transient effect of this nature would not be predicted.

Substituting the number density $\hat{n}_i(x_j, t)$ into Equation 4.10, the value of the concentration path length CL can be calculated from Equation 4.9. The emission rate $\dot{m}(t)$ of RP, as shown in Figure 4.4, can be fit with the equation

$$\dot{m} = 10 e^{-at} \quad (4.17)$$

where $a = 1.61 \times 10^{-3}$. The yield factor was chosen equal to 4.5 as estimated from Milham and Anderson (1983) for phosphorus at a RH = 59 percent. Figure 4.14 shows the CL distribution as a function of time for both versions of the MoCaPD model and the experimental measurements of Dugway Proving Ground using the MoCaPD model and the aerosol photometer (AP). The figure indicates that both the previous MoCaPD (incoherent) and current MoCaPD (coherent) predict that the maximum of CL is around 5 gm/m^2 to 7 mg/m^2 , but the result from the AP is a factor of 2 to 3 higher than the predictions. However, Farmer (1983a) reports values of concentration C measured with a number of different instruments for Trial 8 in Smoke Week IV which range from 0.2 gm/m^3 to 0.7 gm/m^3 and the results from LPCI (operated by State University of New York) gave $C = 0.04 \text{ gm/m}^3$. For an L value of 5 m, the values of C predicted by the MoCaPD model are in the range of 0.6 to 1.0 gm/m^3 . Therefore, the computed results are definitely in the range of the measured values. As seen in Figure 4.14(a), the distribution of CL

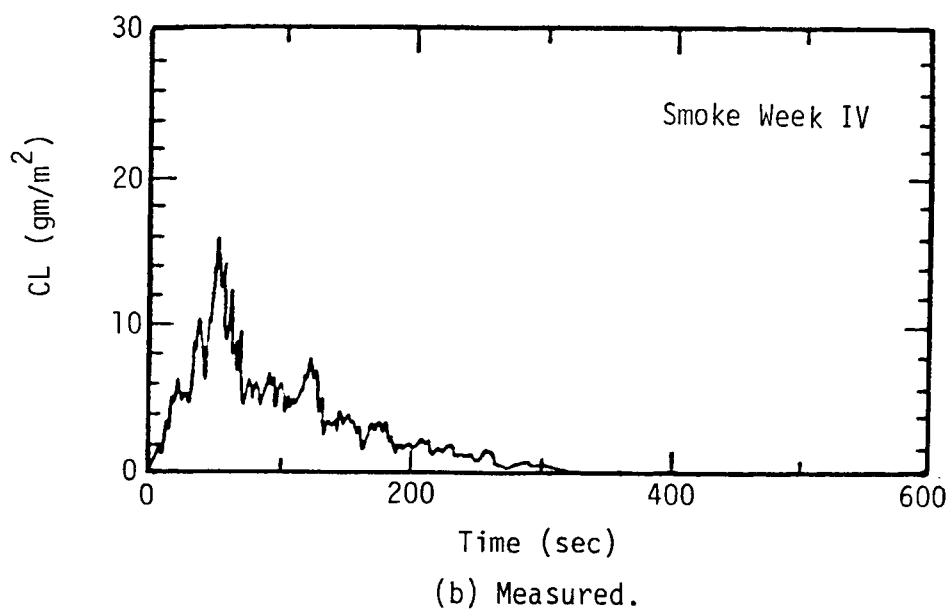
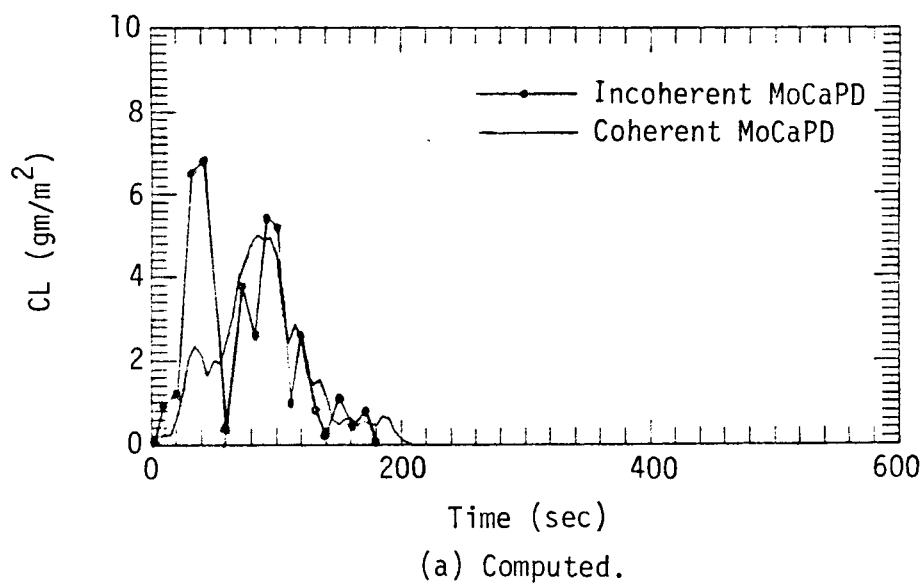
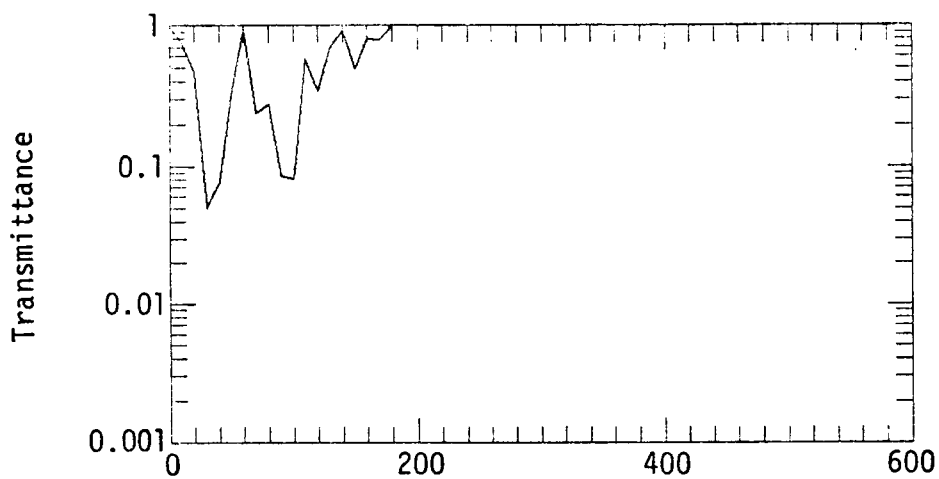


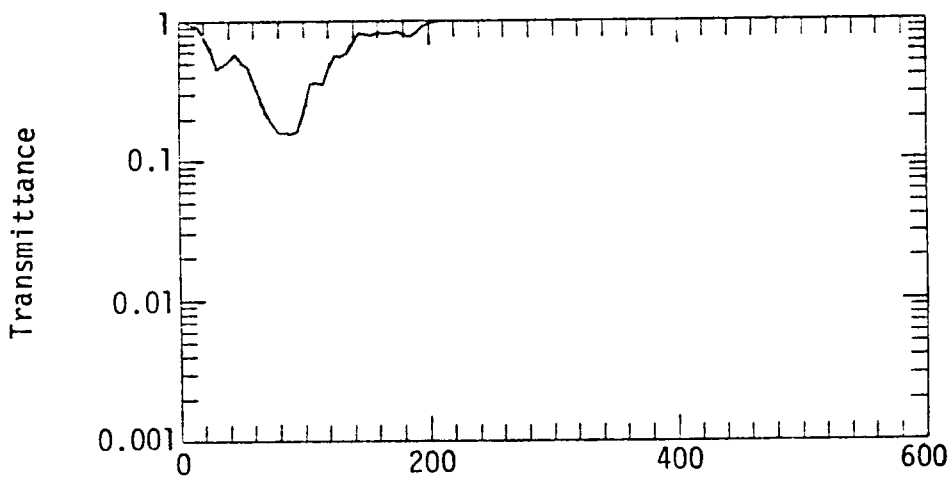
Figure 4.14 Comparison of concentration path length computed by the MoCaPD model and measured data from Smoke Week IV.

calculated by the incoherent MoCaPD model fluctuates greatly and is even very small at a particular time while the coherent MoCaPD predicts a smoother result, as would be expected. Figure 4.15 shows the comparison of the computed and measured transmittance as a function of time. The model used in Figure 4.15(a) is the incoherent MoCaPD model and again results are highly fluctuating. This is because the particles are released sequentially, one at a time, in the incoherent MoCaPD model. In the coherent MoCaPD model, however, groups of particles are released sequentially and the turbulence which the particles encounter is coherent. The particles in each group are diffused during the downstream motion, so the particle distribution is more uniform, i.e., there is less chance that a hole can occur. Figure 4.6(b) shows the comparison of the transmittance computed by the coherent MoCaPD model and the measured data. The agreement of the magnitudes is good, but the time of minimum transmittance at LOS1 has been shifted. Exact correspondence between the measurements and computations cannot be expected, the turbulent wind field cannot be simulated exactly. In addition, the grenades that were used in Smoke Week IV, Trial 8, were L8A1. The emission rate of L8A1 is different from that of XM803 grenades. Also, the smoke release methods are different for the L8A1 and XM803 grenades, and a fixed source height was assumed which was not correct for this particular trial.

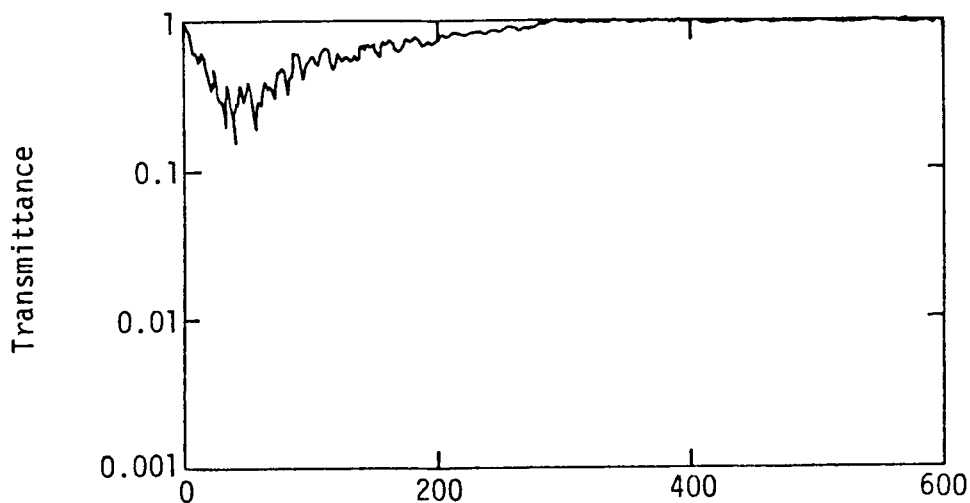
Figure 4.16 shows the distribution of the concentration path length, CL, transmittance, T, and extinction coefficient, α , at a wavelength of 3.4 μm and a 2.5-m height while Figure 4.17 shows the distribution of CL, T, and α for 3.4 μm wavelength at a 5-m height. It is noticed that the mean value of extinction coefficient does not change



(a) Incoherent MoCaPD model.



(b) Coherent MoCaPD model.



(c) Measured data.

Figure 4.15 Comparison of transmittance of $3.4\ \mu\text{m}$ wavelength computed by incoherent and coherent MoCaPD model and measured data from Trial 8, Smoke Week IV, RP.

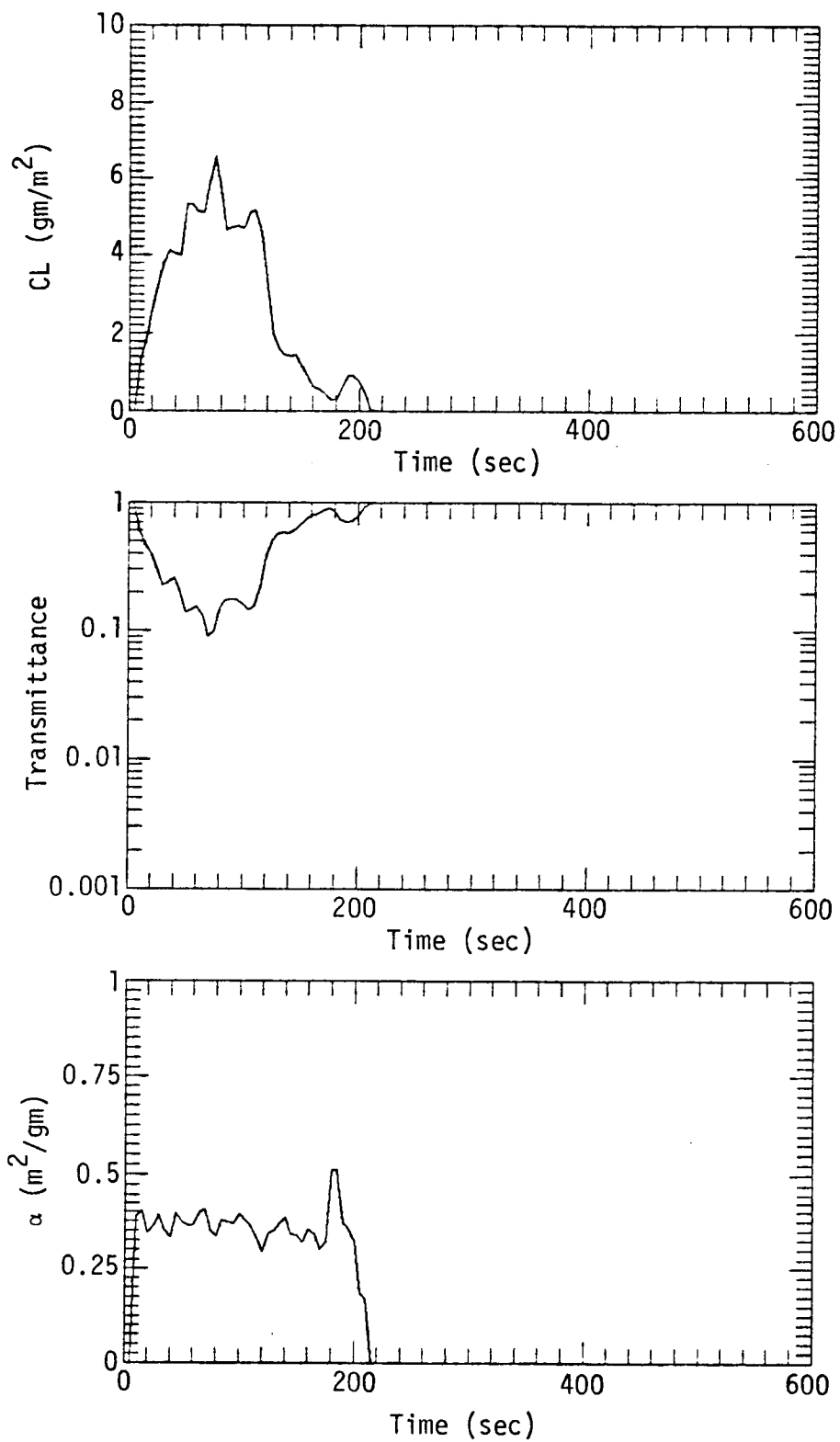


Figure 4.16 The distribution of concentration length (CL), transmittance, and extinction coefficient (α) of $3.4 \mu\text{m}$ wavelength at 2.5-m height for Trial 8, Smoke Week IV, RP.

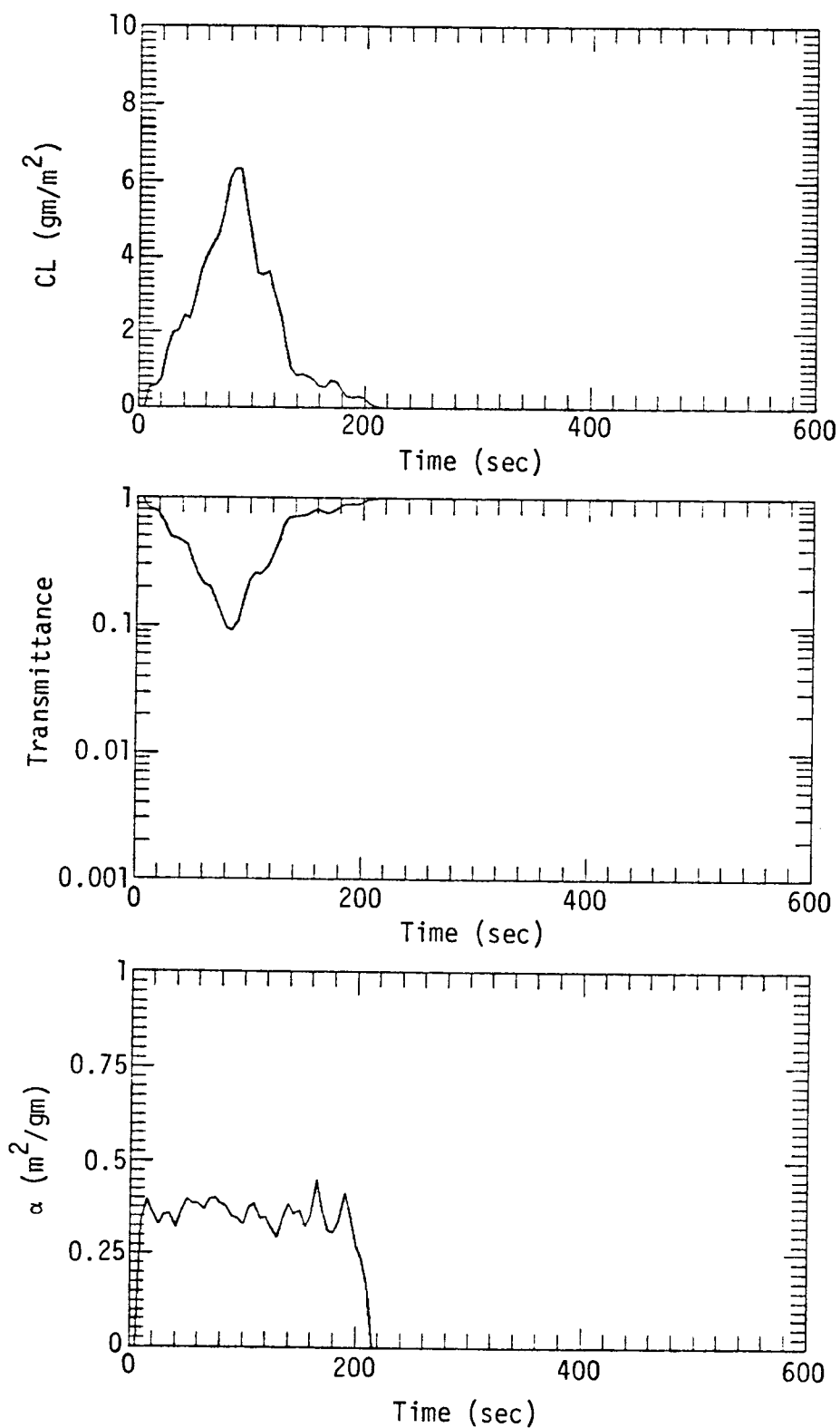
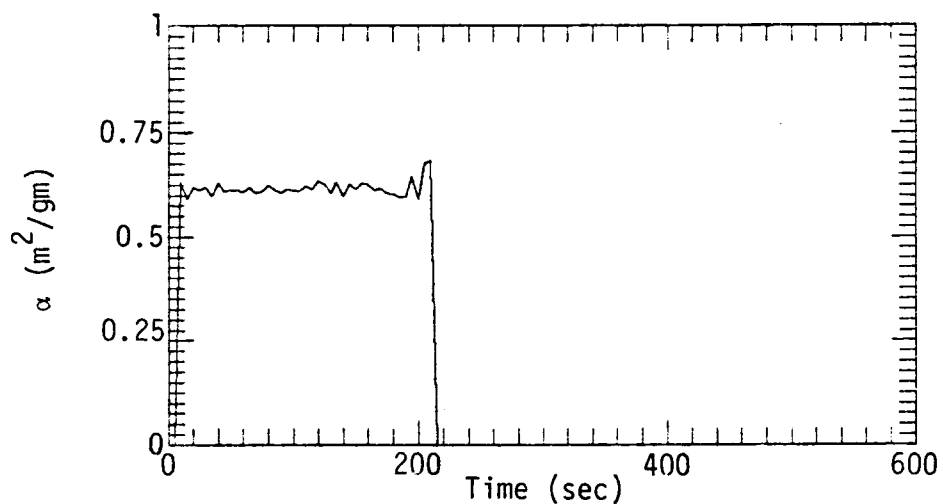


Figure 4.17 The distribution of concentration length (CL), transmittance, and extinction coefficient (α) of $3.4 \mu\text{m}$ wavelength at 5-m height for Trial 8, Smoke Week IV, RP.

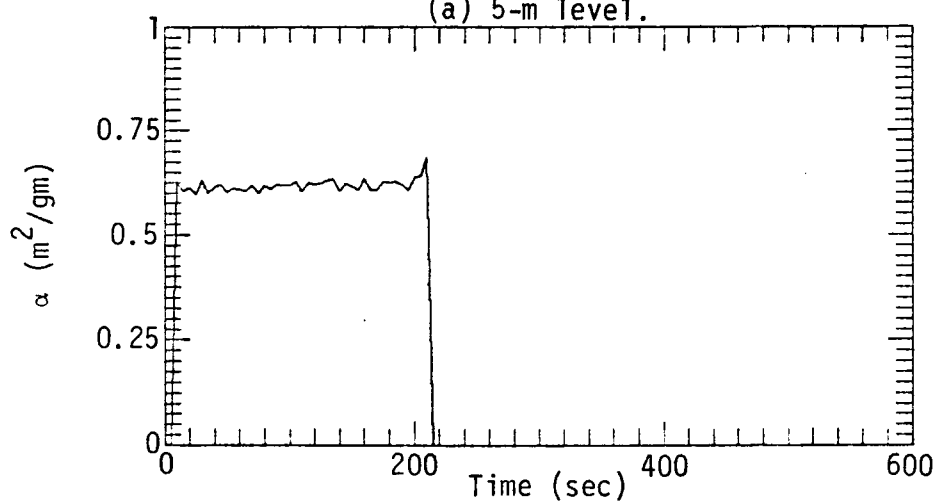
with increasing level. However, there are appreciable differences in the concentration distribution at the different levels. The extinction coefficient was also calculated for a 9.75 μm wavelength. Data of the extinction efficiency, Q_E , for 9.75 μm can be obtained from Figure 4.6(b), which is the best data presently available but known to be a relatively poor approximation to the Q_E values. Figure 4.18 shows the distribution of the extinction coefficient as a function of time for different source heights; it is quite constant with time and does not vary with source height. The mean value of α for 9.75 μm wavelength is $0.6 \text{ m}^2/\text{gm}$, which is twice the value of α at 3.4 μm . Figure 4.6(b) clearly indicated that Q_E is sensitive to $\pi D/\lambda$ for the 9.75 μm , wavelength when the concentration of orthophosphoric acid in water, F , is greater than 50 percent (i.e., when the relative humidity less than 73 percent). However, the extinction efficiency, Q_E , for 3.4 μm (see Figure 3.4(a)) is much smaller than Q_E at 9.75 μm for orthophosphoric acid. This is the reason why α at 9.75 μm is greater than α at 3.4 μm . The comparison of the computed transmittance at 9.75 μm wavelength and the measured transmittance at 10.6 μm wavelength is shown in Figure 4.19. The distribution of Q_E at 9.75 μm is believed to be closed to Q_E at 10.6 μm wavelength. The computed transmittance, however, is lower than the measured value.

4.4.2 A Simulation of Trial 16 from Smoke Week IV

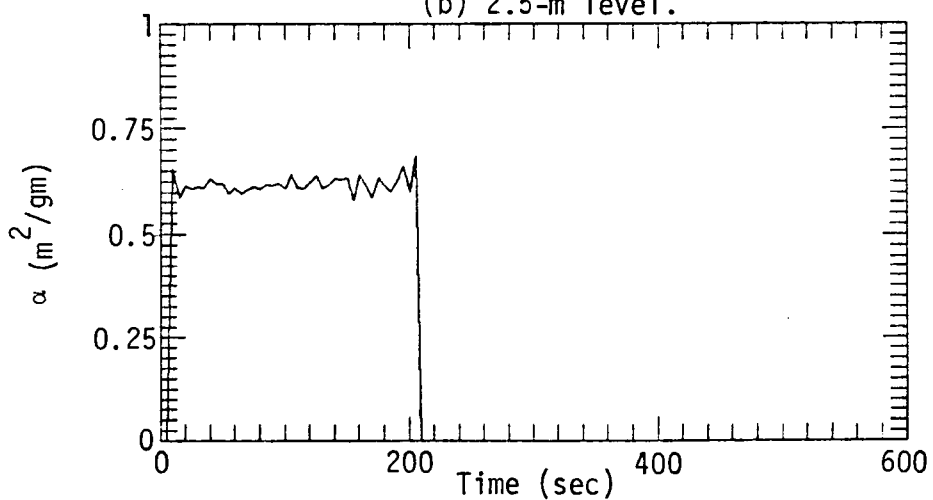
The source of the smoke for Trial 16 during Smoke Week IV was hexachloroethane (HC). Six M116 rounds (155 mm) were simulated by detonating six groups of canisters, each group containing three M1 and one M2 canisters. The total HC fill weight for the 24 canisters was



(a) 5-m level.

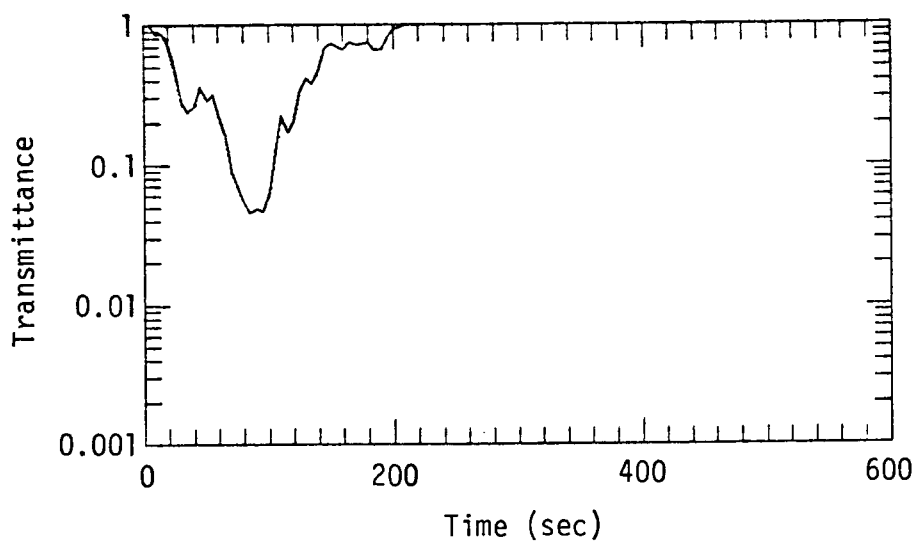


(b) 2.5-m level.

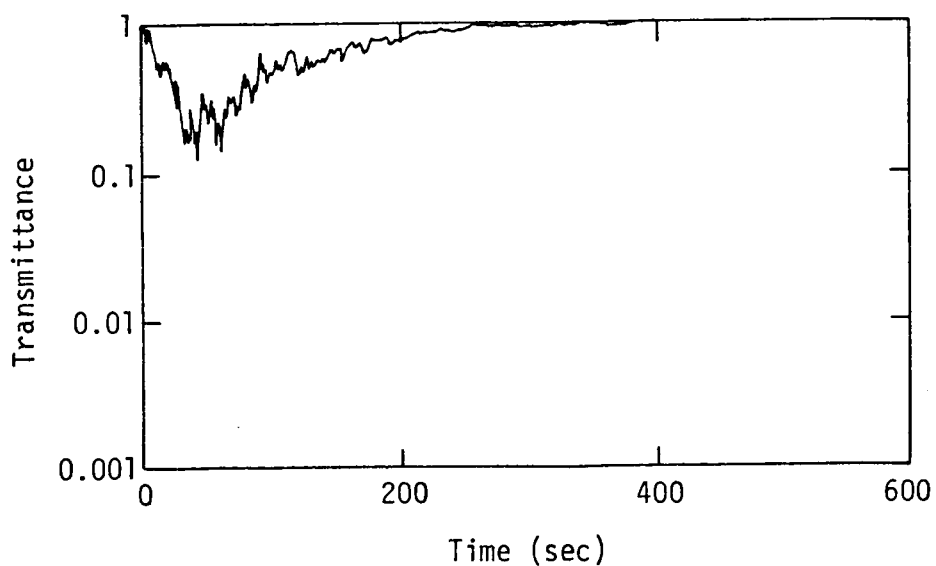


(c) 2-m level.

Figure 4.18 The extinction coefficient of $9.75 \mu\text{m}$ wavelength at different levels for Trial 8, Smoke Week IV, RP.



(a) MoCaPD model ($9.75 \mu\text{m}$).



(b) Measured data ($10.6 \mu\text{m}$).

Figure 4.19 Comparison of transmittance computed by the MoCaPD model and measured data at far-infrared range.

113.8 pounds. The six groups were positioned in a Lazy W pattern as shown in Figure 4.20. The groups were fired sequentially at 10-second intervals, the most westerly group being fired first. The meteorological conditions that existed during the tests are listed in Table 4.1; the wind speeds were marginally low (1.2 m/s at 2-m heights), and the relative humidity was 95 percent. The mass distribution for each particle size range is listed in Table 4.2.

In this field trial simulation, particles were collected at LOS2, which was 70-m upwind from LOS1, as shown in Figure 4.20. The density of HC in this trial was equal to 1.15 gm/cc. Substituting the number distribution, $\hat{n}_i(x_j, t)$, which came from each source at time t , into Equation 4.15, α can be calculated. Similarly, from Equation 4.9 the

TABLE 4.2 Mass Concentration Distribution for Trial 2 and Trial 16, Smoke Week IV.

Particle Size (μm)	Trial 2 (%)	Trial 16 (%)
0.3 - 0.4	---	2.1
0.4 - 0.5	---	9.0
0.5 - 0.6	---	8.9
0.6 - 0.8	---	30.0
0.8 - 1.0	1.0	16.0
1.0 - 1.5	1.5	34.0
1.5 - 2.0	5.5	---
2.0 - 3.0	5.0	---
3.0 - 5.0	77.0	---
5.0 - 6.0	10.0	---

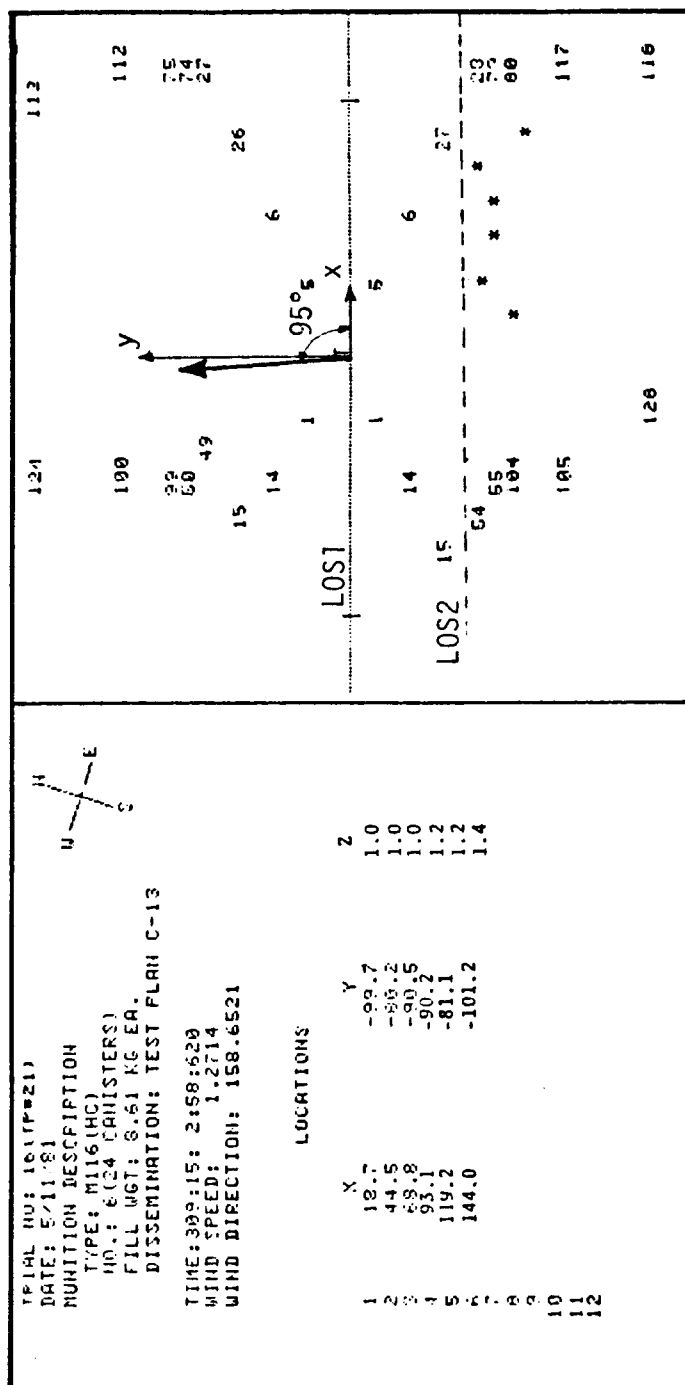


Figure 4.20 Munition arrangement in Smoke Week IV, Trial 16.

concentration path length, CL, can be obtained. Figure 4.21 shows the extinction coefficient at a wavelength of $3.4\ \mu\text{m}$ as a function of time at the 2-m level along LOS2, the average value of α was equal to $0.25\ \text{m}^2/\text{gm}$. The value of α reported by Farmer (1983a) was $0.063\ \text{m}^2/\text{gm}$. Thus, the computed α is four times higher than the result from Farmer (1983a). However, the value of α given by Milham and Anderson (1983) is around $0.64\ \text{m}^2/\text{gm}$ at 85 percent relative humidity, and the result from the H^3S test (Farmer (1980)) gave $\alpha = 0.1\ \text{m}^2/\text{gm}$ for HC at RH = 93 percent for $3.4\ \mu\text{m}$ wavelength. The distribution of the concentration path length, CL, as a function of time is shown in Figure 4.22. No measured datum of CL is available for LOS2. The comparison of the computed and measured transmittance at $3.4\ \mu\text{m}$ at LOS2 is shown in Figure 4.23. In this trial, the yield factor of source material HC is 1.6 (Milham and Anderson (1983)). The prediction of the transmittance from the coherent MoCaPD model was of the same order of magnitude as the measured data. In this particular trial (Trial 16) the ignition time of the sources was different from each other and, therefore, the contribution of each source to LOS was highly non-linear, as discussed in Section 4.3. Figures 4.24 and 4.25 show the distribution of the concentration path length, CL, the transmittance, T, and the extinction coefficient, α , at $3.4\ \mu\text{m}$ for Trial 16, Smoke Week IV, at heights of 2.5 m and 5 m, respectively. Again, the distribution of CL and T is significantly different for different heights while the extinction coefficient, α , is kept relatively constant.

The distribution of the extinction coefficient for $9.75\ \mu\text{m}$ at different heights is shown in Figure 4.26. Again, there is no

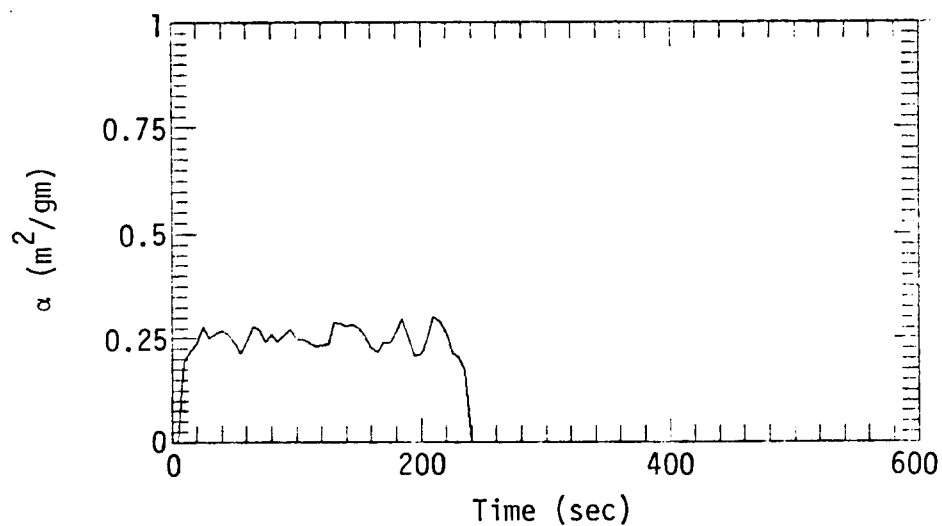


Figure 4.21 The extinction coefficient of $3.4 \mu\text{m}$ wavelength as a function of time for Trail 16, Smoke Week IV, HC.

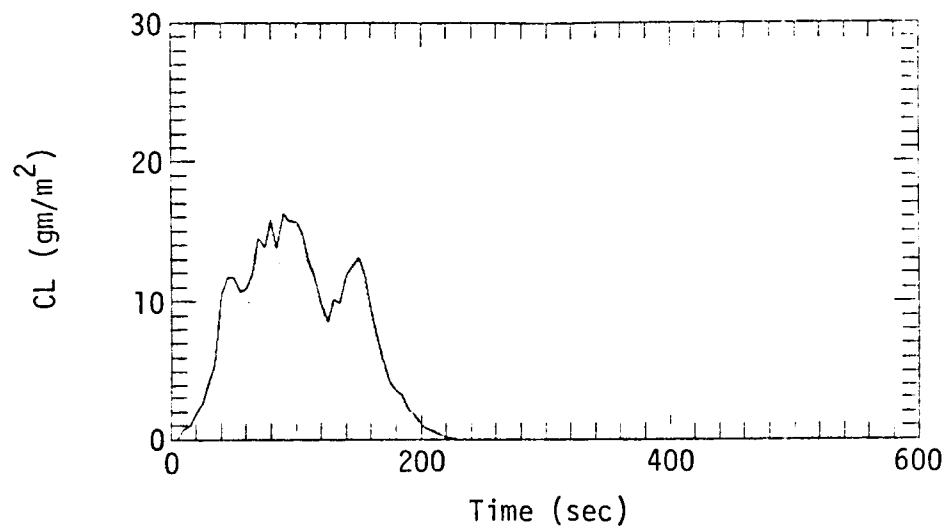
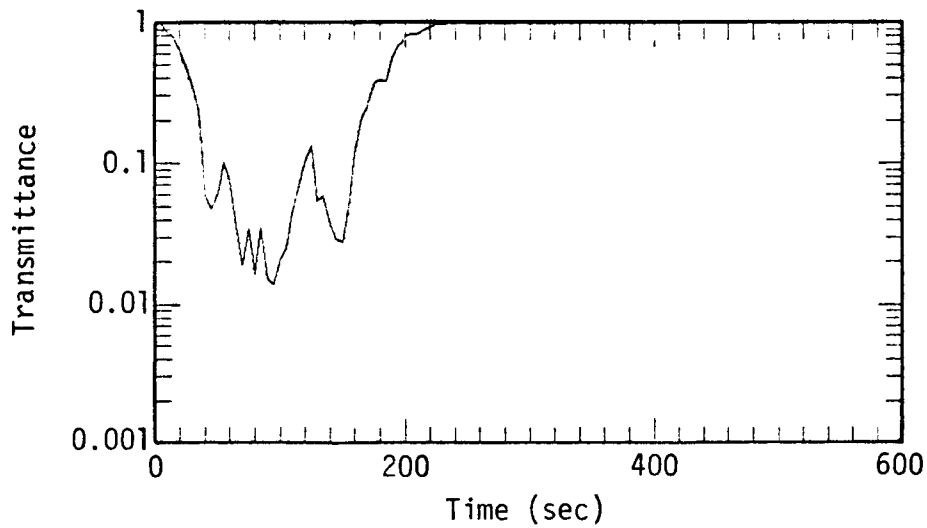
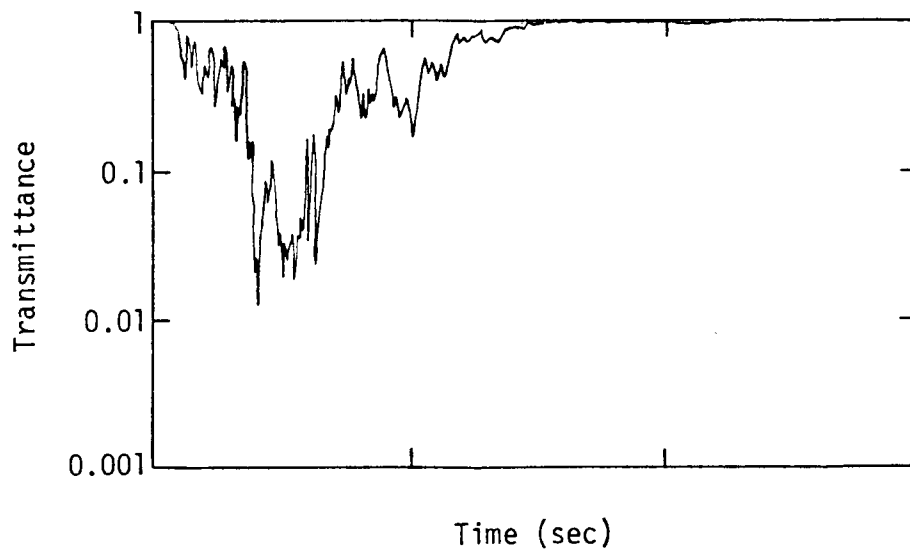


Figure 4.22 The concentration path length distribution as a function of time for Trial 16, Smoke Week IV, HC.



(a) MoCaPD model.



(b) Measured data.

Figure 4.23 The comparison of transmittance of computed data and measured data at $3.4 \mu\text{m}$ wavelength for Trial 16, Smoke Week IV, HC.

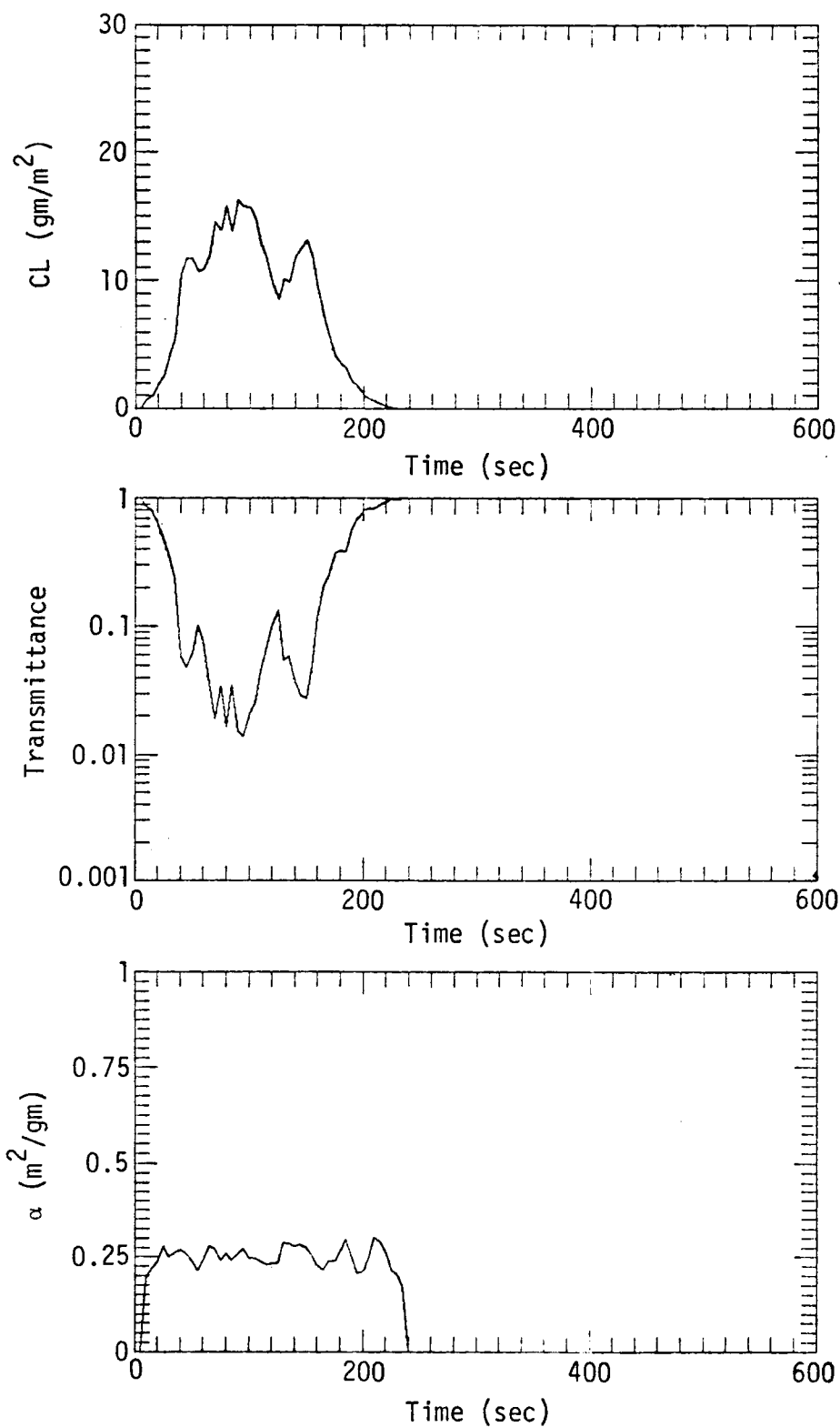


Figure 4.24 The distribution of concentration length (CL), transmittance, and extinction coefficient (α) of $3.4 \mu\text{m}$ wavelength at 2.5-m height for Trial 16, Smoke Week IV, HC.

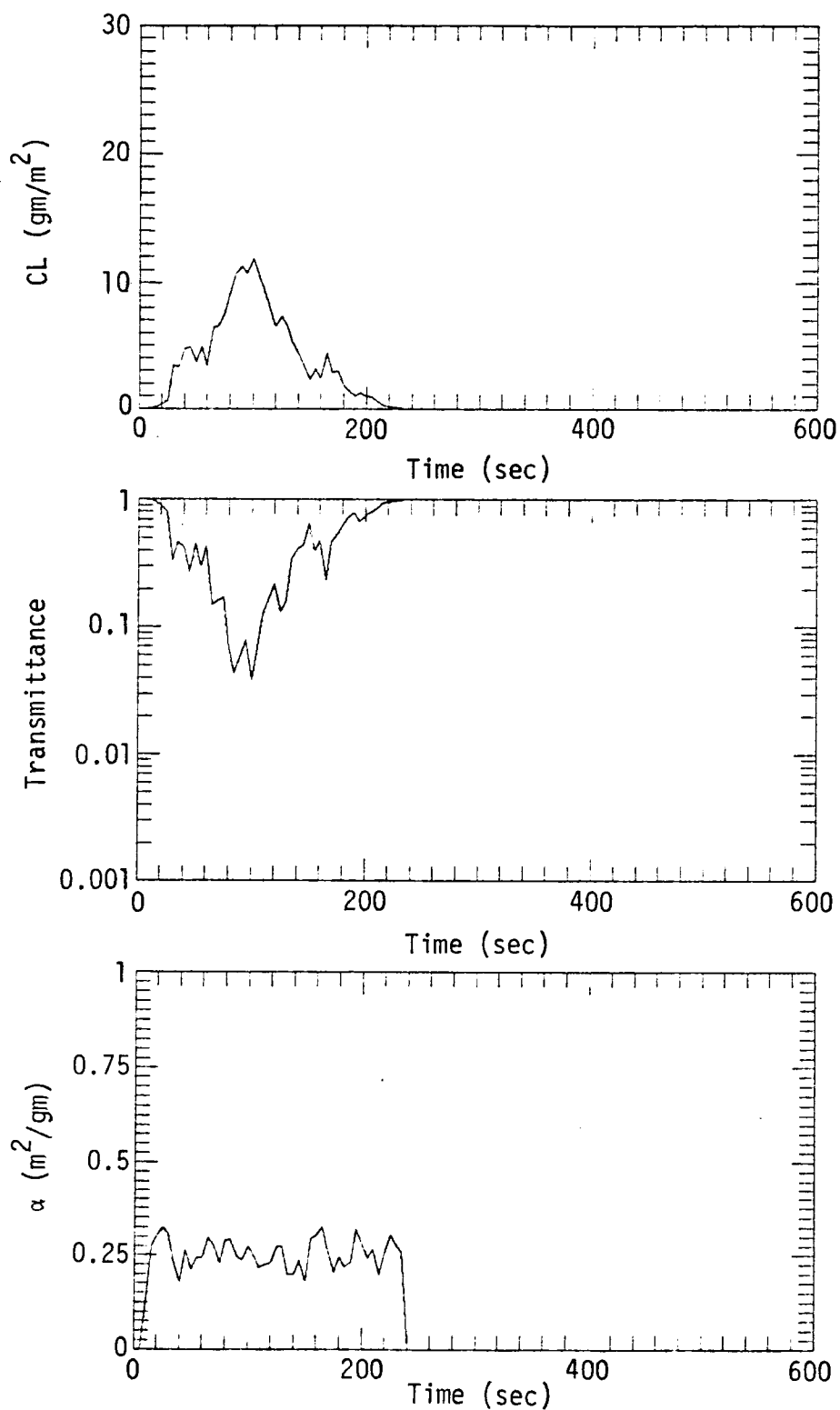
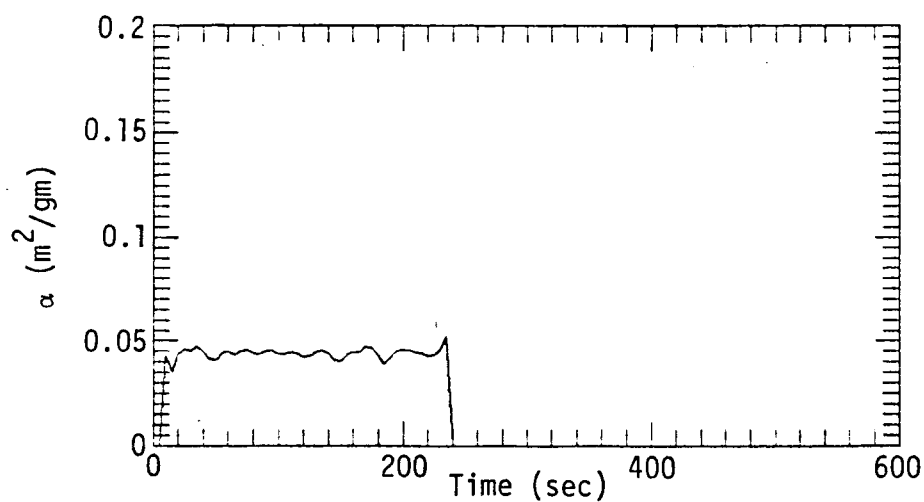
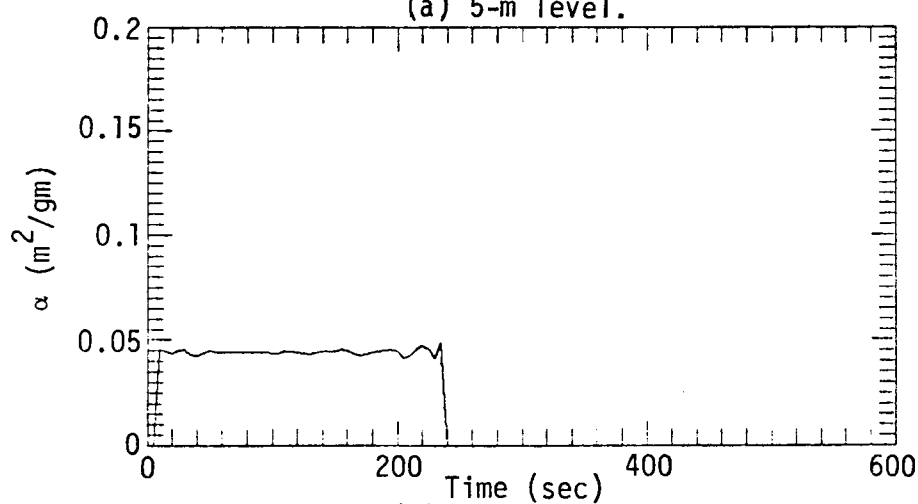


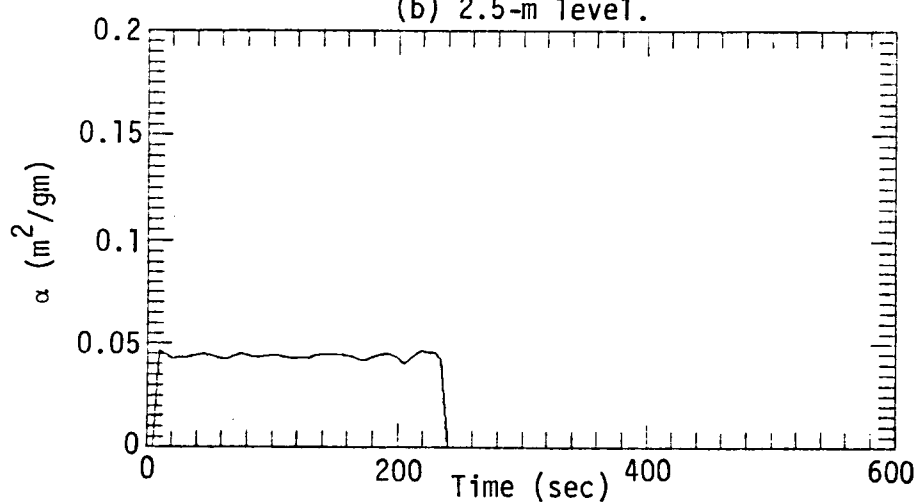
Figure 4.25 The distribution of concentration length (CL), transmittance, and extinction coefficient (α) of $3.4 \mu\text{m}$ wavelength at 5-m height for Trial 16, Smoke Week IV, HC.



(a) 5-m level.



(b) 2.5-m level.



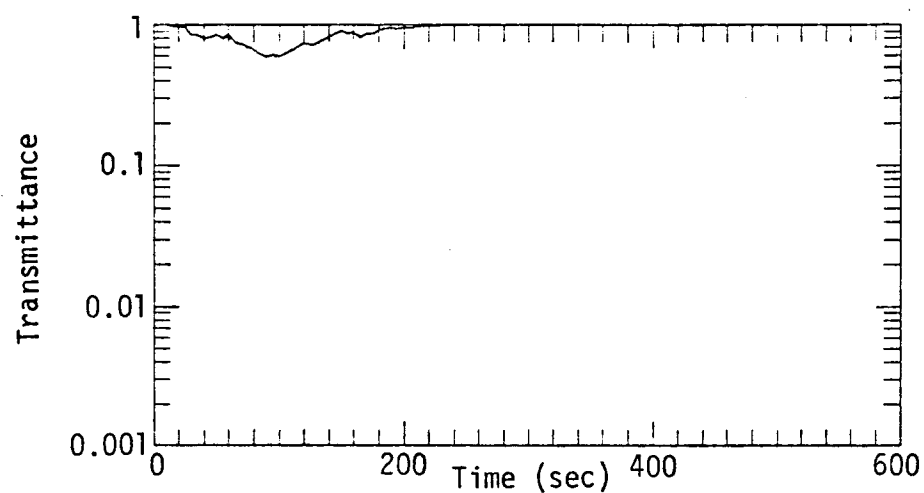
(c) 2-m level.

Figure 4.26 The extinction coefficient of $10.6 \mu\text{m}$ wavelength at different levels for Trial 16, Smoke Week IV, HC.

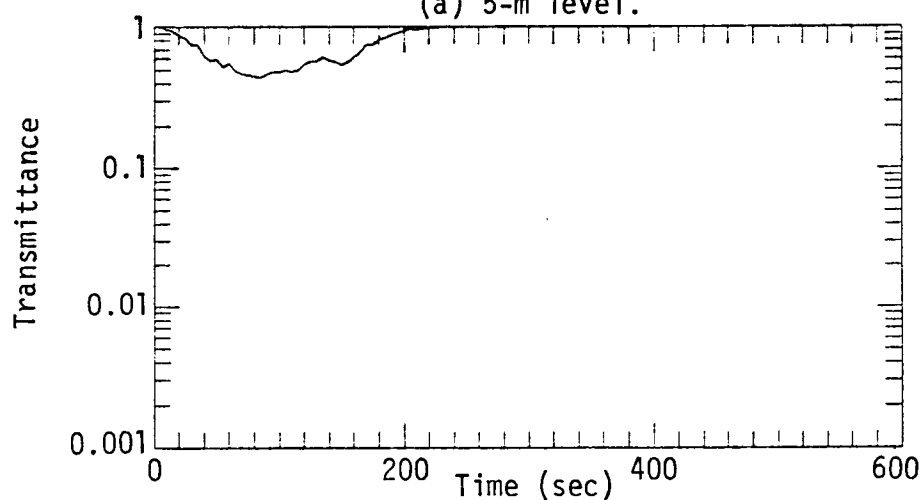
appreciable difference in the mean value of α for different heights; however, the mean value of α at $9.75\ \mu\text{m}$ is $0.045\ \text{m}^2/\text{gm}$, which is around one-fifth the value of α at $3.4\ \mu\text{m}$. In this trial, the relative humidity was 95 percent; consequently, the concentration fraction of zinc chloride in water, F , was only 17 percent. (It is assumed that the relation between the concentration fraction of zinc chloride in water and the relative humidity is the same as that for phosphorus (Equation 4.16).) In Figure 4.6 it was shown that for such a low F and such a long wavelength ($9.75\ \mu\text{m}$), the extinction efficiency, Q_E , is lower compared to that at $3.4\ \mu\text{m}$. Generally speaking, the extinction coefficient of a HC smoke is not as high as a phosphorus smoke because the size of the particle is smaller and the extinction efficiency of zinc chloride is smaller, especially in the case of low relative humidity (high concentration acid, F). The distribution of the transmittance for $9.75\ \mu\text{m}$ wavelength is shown in Figure 4.27 for different source levels. The minimum value of transmittance for $9.75\ \mu\text{m}$ is 0.5, while it was equal to 0.01 at $3.4\ \mu\text{m}$. This large change simply occurs because the extinction of HC is one order of magnitude lower at $3.4\ \mu\text{m}$ than at $9.75\ \mu\text{m}$.

4.4.3 A Simulation of Trial 2 from Smoke Week IV

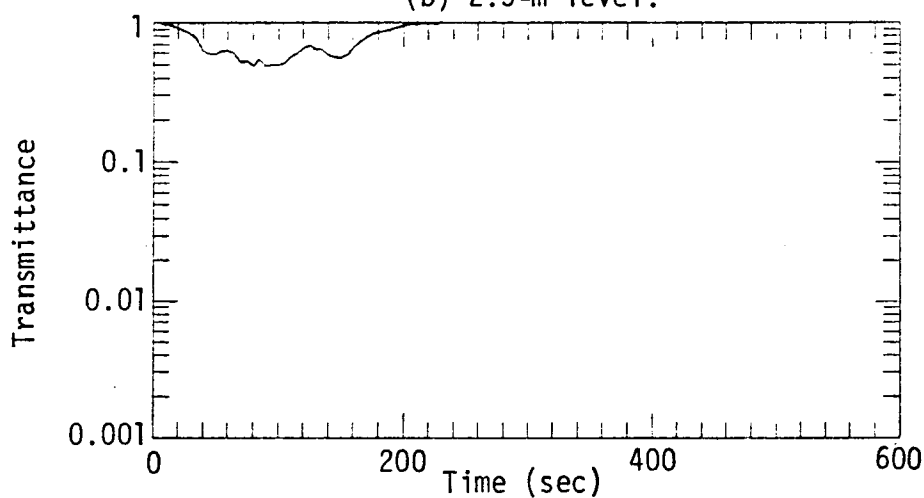
In Trial 2 of the Smoke Week IV field test, three Mitey Mite generators dispensing 22.7 kg (50 lbs) of Arizona road dust were operated. The main element of the Arizona road dust is silicon, which has a density of $2.2\ \text{gm/cc}$. Three generators were positioned to the southeast, around 90 m away from the grid center. The three generators were placed 10 m apart, as shown in Figure 4.28. The meteorological



(a) 5-m level.



(b) 2.5-m level.



(c) 2-m level.

Figure 4.27 The transmittance of $10.6\ \mu\text{m}$ wavelength at different levels for Trial 16, Smoke Week IV, HC.

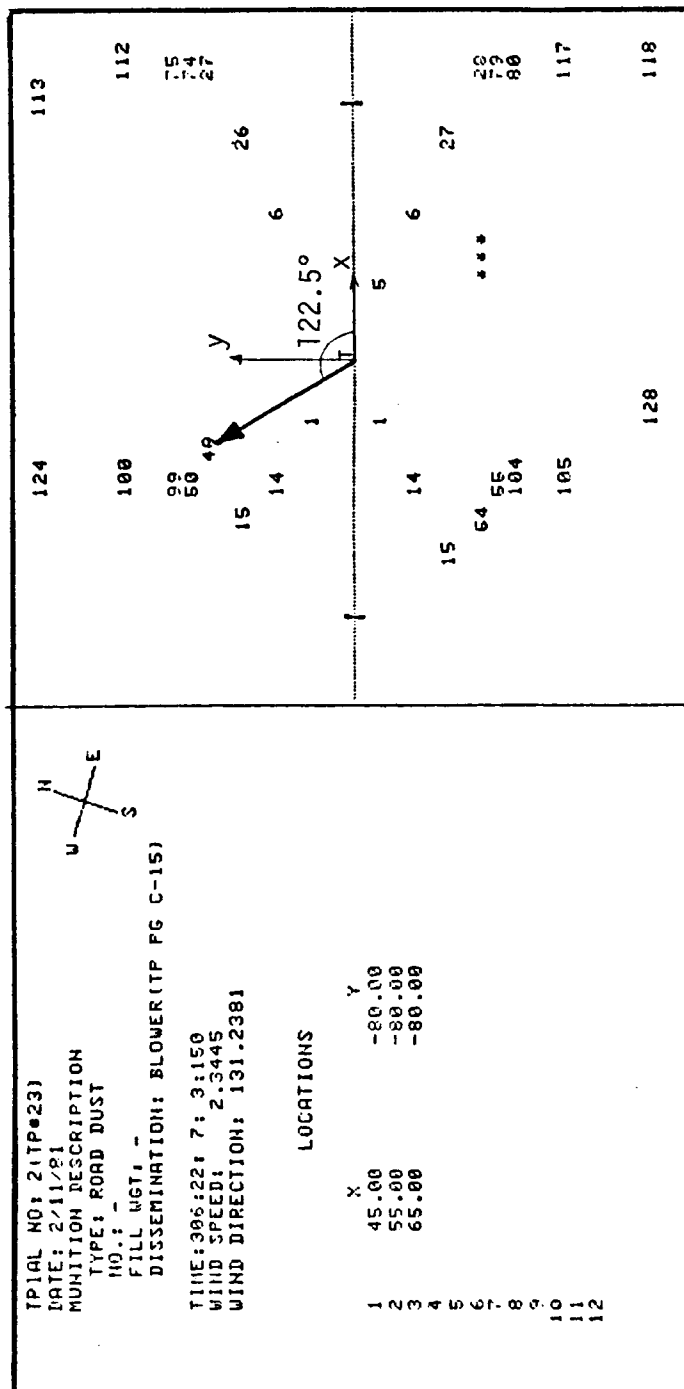


Figure 4.28 The arrangement of Mitey Mite generator in Trial 2, Smoke Week IV test.

data during the test are listed in Table 4.1. The speed was 1.7 m/s at a 2-m height and the wind direction was 131 degrees from true north, which is equivalent to 122.5 degrees in the grid system shown in Figure 4.28. The relative humidity was 60 percent. However, it should not affect the results of the computation because the road dust can be assumed to be non-hygroscopic. The mass distribution for each particle size range is listed in Table 4.2.

The exit diameter of the Mitey Mite generator is 10 cm (4 inches), and the exit velocity was estimated to be 8 m/s. The source exit was assumed to be 1.5 m above the ground. The technique for simulating this type of generator source was described in Section 4.1. In this simulation trial, particles were released for 300 seconds in 5-second intervals. During each 5-second interval, 100 puffs (each puff containing 5 particles) were released for each size of particle. Particles were collected at the main sampling line, LOS1, to obtain the number density of each size $\hat{n}_i(x_j, t)$. In order to calculate α from Equation 4.20, the extinction efficiency, Q_E , of the Arizona road dust must be known. Since silicon is the major element of Arizona road dust, the refractive index of silicon was assumed to represent the refractive index of this dust. However, the absorption property of silicon is so great and particles are of a sufficient size that the extinction efficiency can be assumed equal to 2 (Van De Hulst (1967)). An interesting point is that the contribution to α from small particles is greater than the contribution from larger particles (α is inversely proportional to particle size, Equation 4.20). Figure 4.29 shows the distribution of α as a function of time. There are two spikes in the α distribution which never occurred in the RP and HC trials.

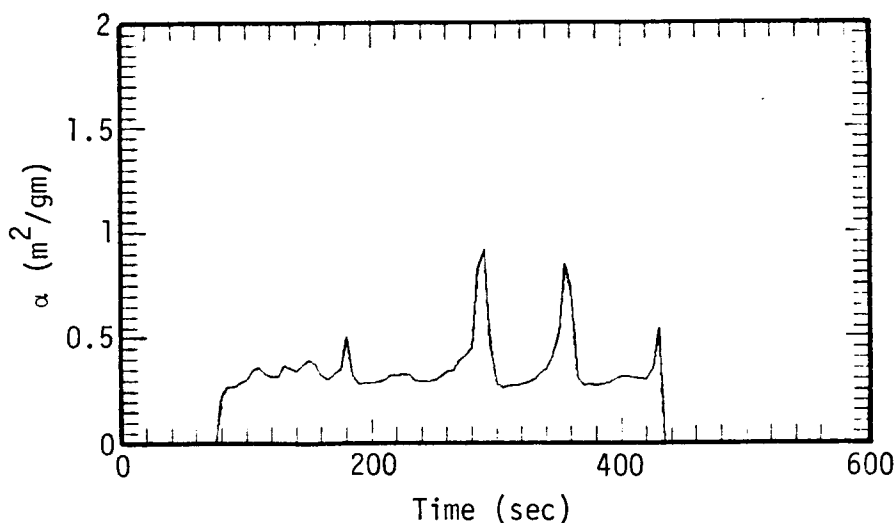


Figure 4.29 The distribution of extinction coefficient as a function of time for 3.4 μm wavelength for Trial 2, Smoke Week IV, Arizona road dust.

This occurs in the model because sometimes small particles are pre-dominant at the LOS. This might be a phenomenon which actually occurs. Figure 4.29 indicates that α is in the range between 0.3 m^2/gm to 0.5 m^2/gm . The reported value from the PMS measurement is 2.478 m^2/gm for Arizona road dust at 3.4 μm wavelength. However, since the extinction efficiency, Q_E , is assumed to be equal to the constant ($Q_E = 2$), the mass extinction cross section can be approximated as:

$$\bar{\sigma} = \frac{\pi}{4} D^2 Q_E \quad (4.18)$$

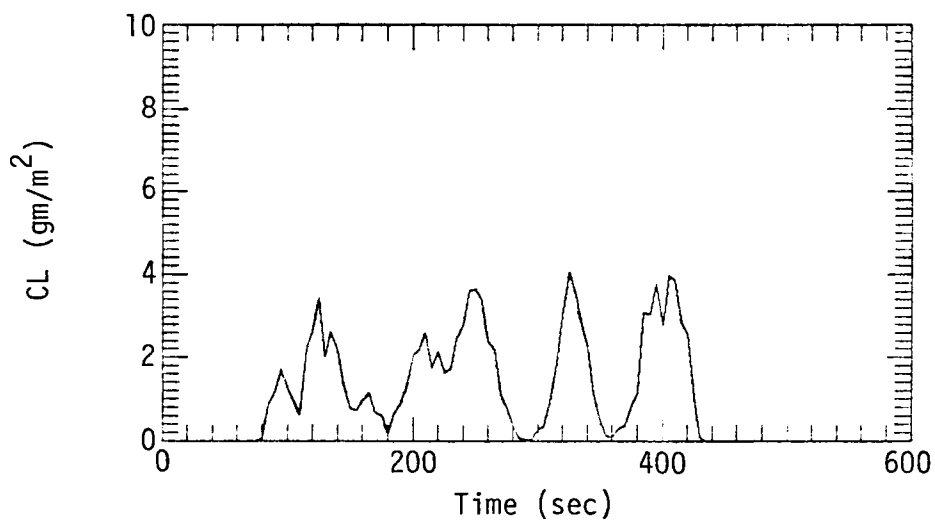
Substituting Equation 4.18 into Equation 2.12, the approximated extinction coefficient, α , can be written as

$$\alpha = \frac{3}{\rho_o D_{32}} \quad (4.19)$$

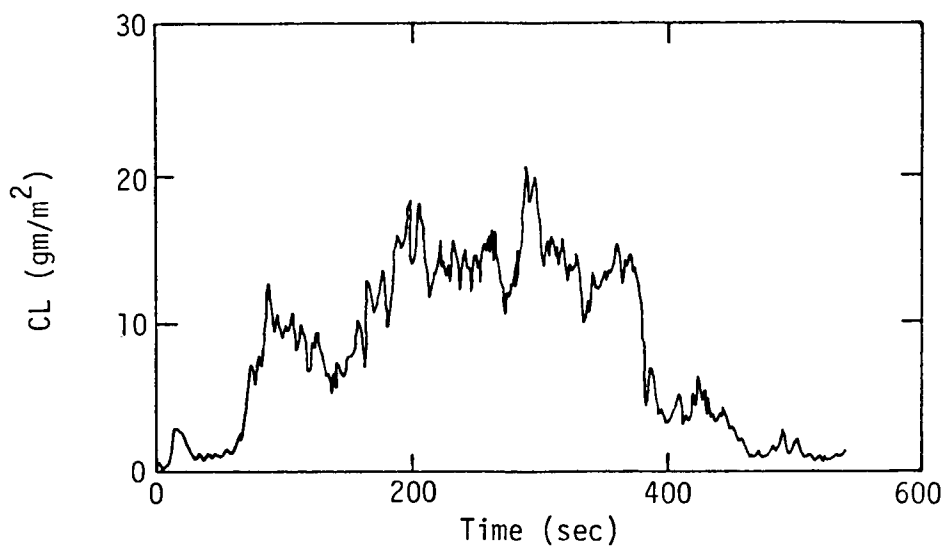
where $D_{32} = \mu_3/\mu_2$. The volume/area mean, D_{32} , is only 0.5 for PMS instrumentation, while it is 5.28 for PSI and 8.55 for PSA (no data is available for LPCI). This presents quite a broad range of D_{32} values

from different instruments. If $D_{32} = 5.28$ is used for the analysis, then the calculated mean is $0.26 \text{ m}^2/\text{gm}$ (from Equation 4.19, $\rho_o = 2.2 \text{ gm/cc}$) and α is $0.16 \text{ m}^2/\text{gm}$ for $D_{32} = 8.55$. The prediction of α from the MoCaPD model is close to the PSI measurement. It should be mentioned since α is very sensitive to the particle size for small particles (as indicated before) that a larger α ($\approx 2.8 \text{ m}^2/\text{gm}$) could exist for $D_{32} = 0.5$. A proper particle size distribution is an extremely important input for the MoCaPD model to predict a reasonable value of α .

Figure 4.30 shows the comparison of the computed and measured values of CL. The measured CL, which is from the DPG, is three times higher than the computed result from MoCaPD. Both indicate clearly that the smoke reaches to LOS1 only after 80 seconds. Figure 4.31 compares the computed and measured transmittance time histories at $3.4 \text{ }\mu\text{m}$ wavelength. Although the phase is shifted a little, the MoCaPD model prediction has the right trend and order of magnitude. It is interesting that both results show the histories of the transmittance are wavy, indicating that there is some chance for holes to exist in the smoke. This is probably due to the effect of a meandering wind, and it frequently happens for nozzle-type smoke generators. Figures 4.32 and 4.33 show the concentration path length, transmittance, and extinction coefficient at 2.5-m and 5-m heights, respectively. Again, the results are more variable than those for the grenade and canister.

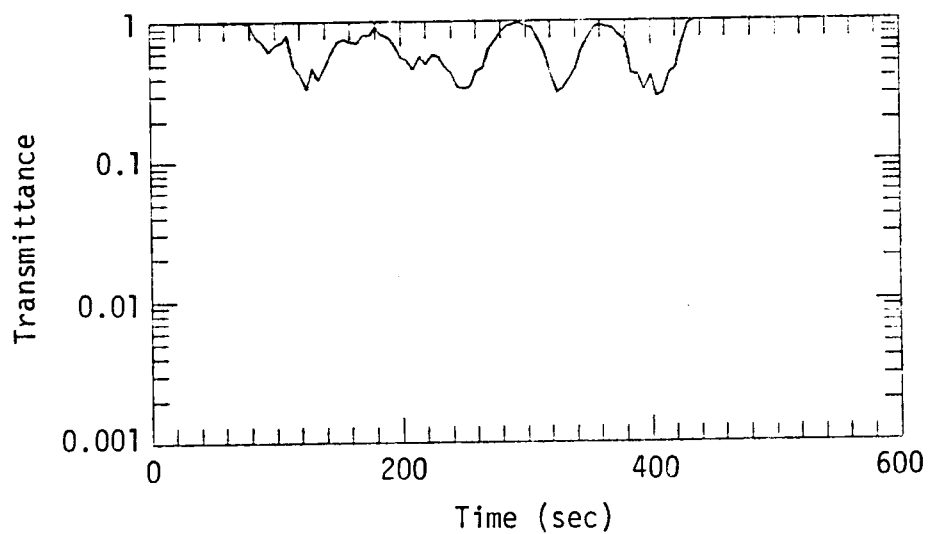


(a) MoCaPD model.

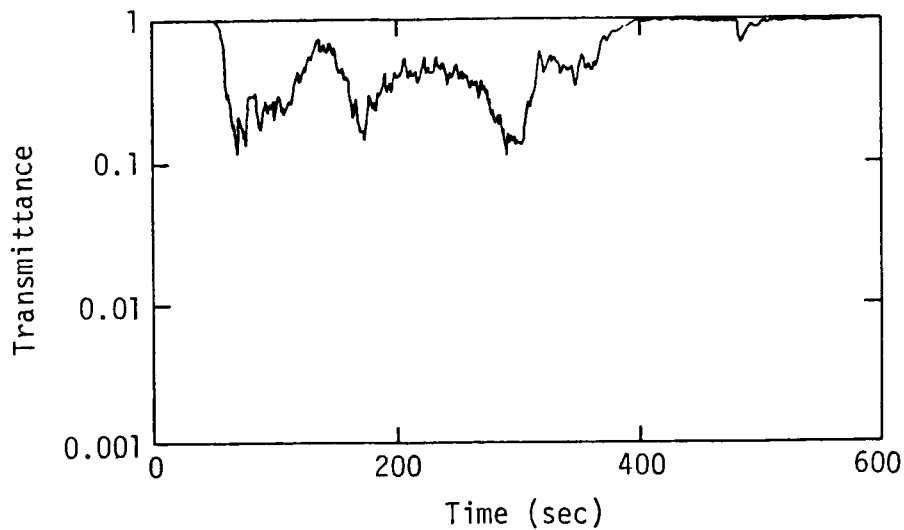


(b) Measured data.

Figure 4.30 Comparison of concentration path length computed by the MoCaPD model and measured data at LOS1 for Trial 2, Smoke Week IV, Arizona road dust.



(a) MoCaPD model.



(b) Measured data.

Figure 4.31 Comparison of transmittance computed by the MoCaPD model and measured data at LOS1 for Trial 2, Smoke Week IV, Arizona road dust.

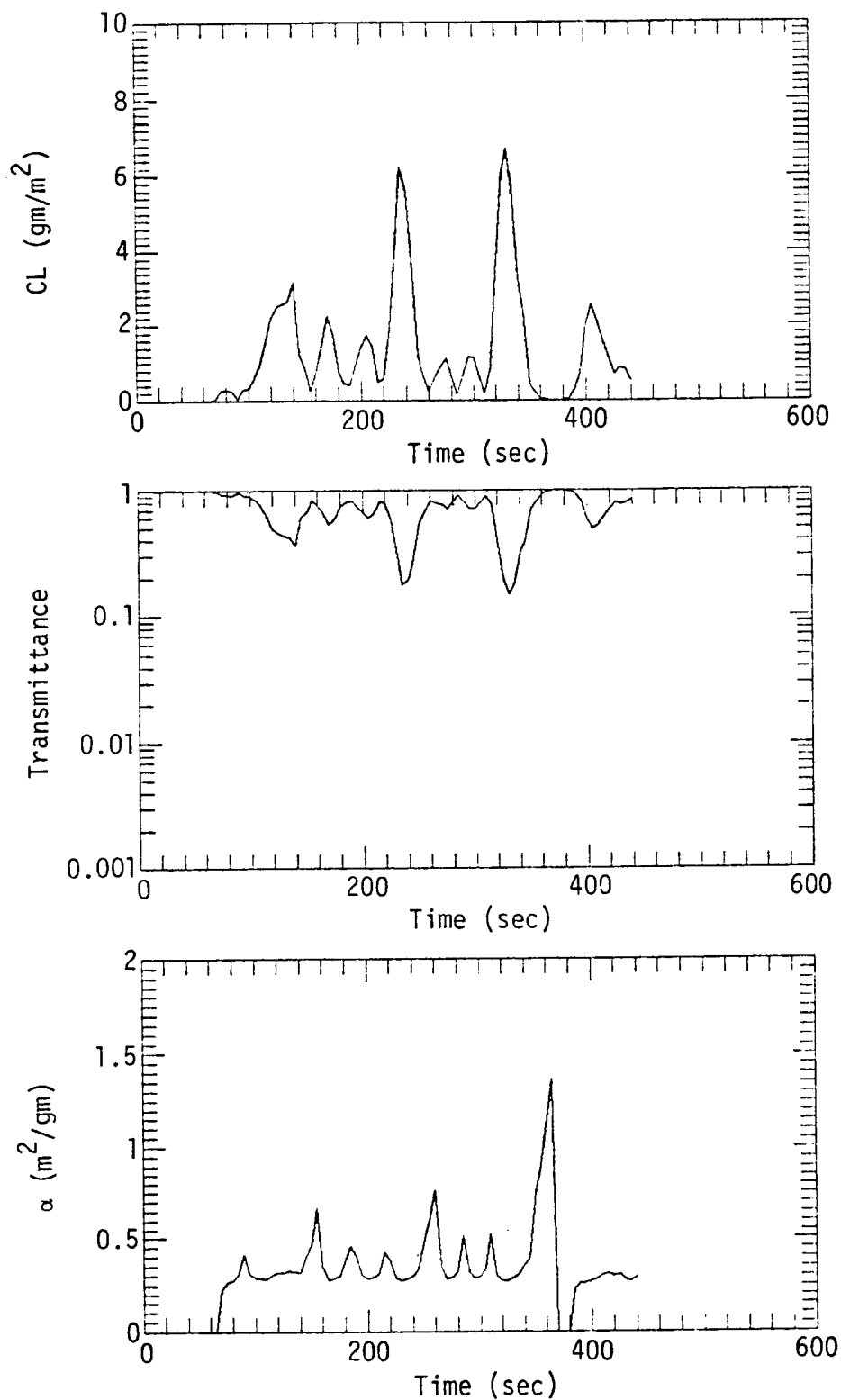


Figure 4.32 The distribution of concentration path length (CL), transmittance, and extinction coefficient (α) of $3.4 \mu\text{m}$ wavelength at 2.5-m height for Trial 2, Smoke Week IV.

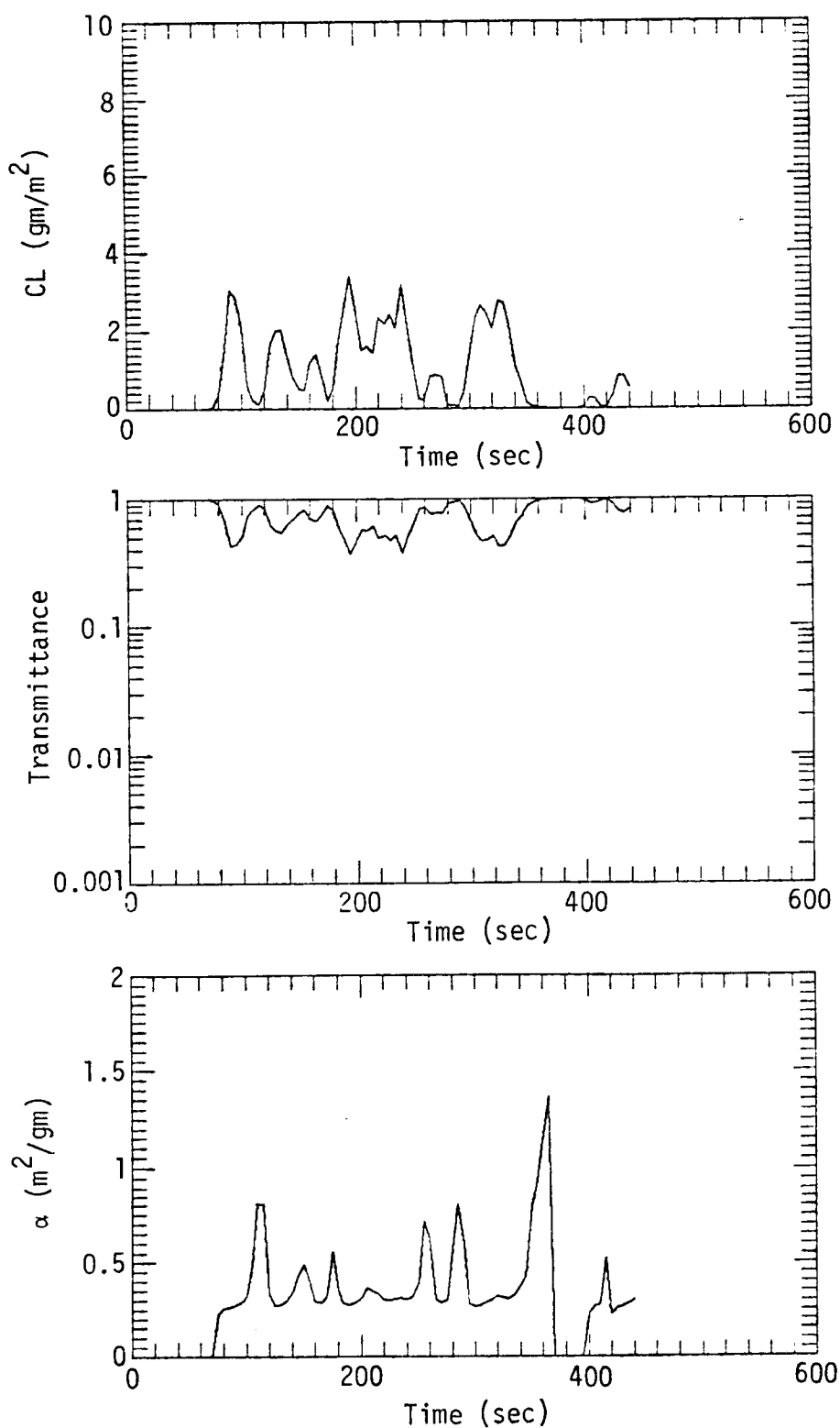


Figure 4.33 The distribution of concentration path length (CL), transmittance, and extinction coefficient (α) of $3.4 \mu\text{m}$ wavelength at 5-m height for Trial 2, Smoke Week IV.

5.0 CONCLUSIONS

None of the analytical diffusion models can predict instantaneous electro-optical properties in a battlefield environment, firstly, because the actual process is not deterministic but random in nature, and secondly, because many of the physical processes, particularly those that govern the chemical change of the aerosols, are not at this time readily modeled. However, the major shortcoming of the stochastic model is that the motion of each pseudo-particle is determined independently of any consideration of any other pseudo-particles. The MoCaPD model is a numeric experimental method that has been applied to overcome such defects. It can be concluded from the results of the reported study that:

1. The MoCaPD model has the capability of taking account of the source characteristics, such as the particle number density, the method of obscurant dissemination, and the non-linear effects caused by multiple sources.
2. The MoCaPD model incorporates the atmospheric characteristics. It has a unique algorithm which generates inter-level coherent Lagrangian turbulence, which has the same statistical properties (both intensity and length scale) as found in the atmosphere.
3. The MoCaPD model can calculate the optical properties of the environment as a function of time and position.

Some additional information would enhance the usefulness of the model, such as the development of the particle size distribution for

generic types of sources, through laboratory or small field tests. The particle size distribution for hygroscopic smokes is a strong function of the relative humidity; however, the relative humidity may change strongly in the near field of obscurant dissemination so the particle size distribution may be a function of time. Additionally, the estimation of realistic Lagrangian time scale and the inclusion of the stability condition into the turbulence generator should be accomplished.

The comparison of the computer results with experimental measurements was reasonable. The MoCaPD model was shown to provide unique capabilities for battlefield obscuration studies which heretofore have not been available using existing dispersion models. The application of the MoCaPD model can be applied to other fields. For example, the model was successfully applied to study the drift of agricultural chemical released from an aircraft (Frost and Huang (1981)). Also, the MoCaPD model may be able to simulate the particle sprays in combustion processes. In summary, the following tasks are recommended:

1. Carry out parametric studies for passive obscurants,* and investigate the influence of the source and micrometeorological variables and provide general criteria and rules-of-thumb about the trends in the obscurant diffusion which are relative to changes in these parameters. For example, studies as to the sensitivity of computer results due to

*Passive obscurants: Obscurant's properties cannot be affected by environment.

variations in input parameters would provide insight as to measurement resolution required for field studies.

2. Extend the computer program to include effects of active obscurants.** Incorporate models of evaporation/condensation and hygroscopic particle growth. Develop the particle interactive forces, which then can be used to incorporate coalescence and other inter-particle force effects into the MoCaPD model.
3. Compare computed results for active obscurants with existing experimental data and with other analytical models to establish the validity of the new model.

**Active obscurants: Obscurant's properties can be affected by environment.

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APPENDIX

APPENDIX

Z-TRANSFORMATION TECHNIQUE FOR TURBULENCE SIMULATION WITH COHERENCE

Perlmutter et al. (1977) developed a multi-filter system to provide turbulence simulation including the coherence function. They defined the two-point spectral function $\phi(\omega, z_1, z_2)$ at different heights z_1, z_2 to be

$$\phi(k, z_1, z_2) = A^2 H(k, z_1) H^*(k, z_2) \sum_{m=-M}^M D_m(k, z_1) D_m^*(k, z_2) \quad (A.1)$$

where $H(k, z)$ is the frequency response function; $D(k, z)$ is a lag filter; and k is the wave number. The schematic system for this inter-level coherence model is illustrated in Figure A.1.

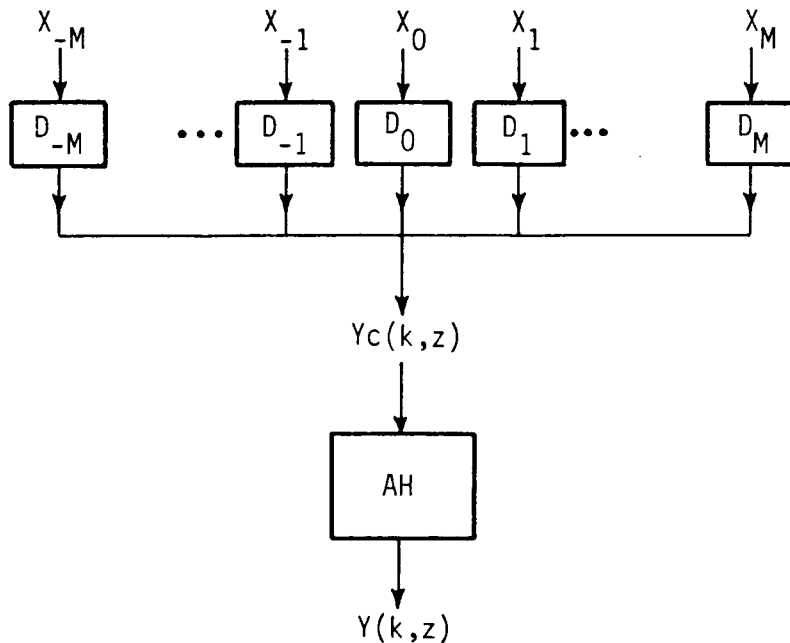


Figure A-1. The interlevel coherence model.

Perlmutter et al. (1977) used Equation A.7 to make a best fit to the true wind coherence exponential function given by Equation 3.42. Then the constant C_m can be written as

$$C_0^2 = 2\xi_0 \left\{ 1 - \exp\left(-\frac{1}{2\xi_0}\right) \right\}$$

$$C_m^2 = \frac{2\xi_0 \left\{ 1 - (-1)^m \exp\left(-\frac{1}{2\xi_0}\right) \right\}}{[1 + (2m\pi\xi_0)^2]} \quad (A.8)$$

where

$$\xi_0 = \frac{1}{a(k\Delta z)_{\max}}$$

Substituting Equations A.3 and A.4 into Equation A.2 gives

$$\hat{Y}(k, z) = \hat{H}(k, z) \sum_{m=0}^M \left(e^{\frac{j m \pi z k}{(k \Delta z)_{\max}}} \hat{\chi}_m(k) + e^{\frac{-j m \pi z k}{(k \Delta z)_{\max}}} \hat{\chi}_{-m}(k) \right) B_m \quad (A.9)$$

where

$$B_0 = \frac{C_0 A}{2}, \quad B_m = C_m A \text{ for } m \neq 0$$

Changing the parameter of Equation A.9 from wave number k to angular frequency ω gives

$$\hat{Y}(\omega, z) = \hat{H}(\omega, z) \sum_{m=0}^M \left(e^{j \omega g_m(z)} \hat{\chi}_m(\omega) + e^{-j \omega g_m(z)} \hat{\chi}_{-m}(\omega) \right) B_m \quad (A.10)$$

where

$$g_m(z) = \frac{m \pi z}{V \xi_0}, \quad \xi_0 = \left(\frac{\omega \Delta z}{V} \right)_{\max}$$

and V is reference wind speed. The expression for $\hat{Y}(\omega, z)$ in Equation A.10 is the sum of $Y_m(\omega, z)$, where $Y_m(\omega, z)$ is given by

$$\hat{Y}_m(\omega, z) = B_m \hat{H}(\omega, z) \left(e^{j \omega g_m(z)} \hat{\chi}_m(\omega) + e^{-j \omega g_m(z)} \hat{\chi}_{-m}(\omega) \right) \quad (A.11)$$

Several independent horizontally homogeneous Gaussian white noise processes, $X_m(k)$, are input to the lag filter function $D_m(k,z)$. The m th noise passes through lag filter D_m and the output is summed over the noises to give a composite process $Y_c(k,z)$. The composite process $Y_c(k,z)$ then proceeds through a shaping filter $A\hat{H}(k,z)$ to obtain the process $Y(k,z)$. Thus, it can be written as

$$\hat{Y}(k,z) = A\hat{H}(k,z) \sum_{m=-M}^M D_m(k,z) \hat{X}_m(k) \quad (A.2)$$

where A is a constant multiplier and $D_m(k,z)$ is assumed to be a pure lag (Perlmutter et al. (1977)).

$$D_{-m}(k,z) = C_m e^{-jkzd_m}$$

$$D_0(k,z) = C_0 \quad m = 1, 2, \dots, M \quad (A.3)$$

$$D_m(k,z) = C_m e^{jkzd_m}$$

Notice from Equation A.3 that C_m and d_m are equal for D_ℓ and $D_{-\ell}$ where C_m is a constant which needs to be determined and d_m is written as

$$d_m = \frac{m\pi}{(k\Delta z)_{\max}} \quad (A.4)$$

Substituting Equations A.3 and A.4 into Equation A.1, then

$$\phi(k, z_1, z_2) = A^2 H(k, z_1) H^*(k, z_2) \sum_{m=-p}^p C_m^2 \exp[(z_1 - z_2)d_m] \quad (A.5)$$

The coherence function is defined as

$$\gamma^2(k, z_1, z_2) = \frac{\phi(k, z_1, z_2) \phi^*(k, z_1, z_2)}{\phi(k, z_1, z_2) \phi(k, z_2, z_2)} \quad (A.6)$$

Substituting Equation A.5 into Equation A.6, then

$$\gamma^2(k, z_1, z_2) = \frac{\left\{ C_0^2 + 2 \sum_{m=1}^p C_m^2 \cos(k\Delta z d_m) \right\}^2}{\left\{ C_0^2 + 2 \sum_{m=1}^M C_m^2 \right\}^2} \quad (A.7)$$

From Wang and Frost (1980), a general form of the filter function $\hat{H}(f, z)$, which can be used to represent the Dryden filter for both the longitudinal and lateral turbulence components, is given by:

$$\hat{H}(f, z) = \frac{c + jdf}{[a + jf]^2} \quad (\text{A.12})$$

where the coefficients a , c , and d are listed in Table 3.3, they are functions of velocity, turbulence intensity, and length scale. Since

$$\hat{H}(\omega, z) = \left(\frac{1}{2\pi} \right)^{1/2} \hat{H}(f, n)$$

then Equation A.12 can be rewritten as

$$\hat{H}(\omega, z) = \frac{b^3 c + jdb\omega}{[ab^2 + j\omega]^2} \quad (\text{A.13})$$

where $b = (2\pi)^{1/2}$. Taking the Laplace transformation of $\hat{H}(\omega, z)$ gives

$$\hat{H}(s, z) = \frac{b^3 c + dbs}{(ab^2 + s)^2} \quad (\text{A.14})$$

Now consider a unit step function $x(t)$ as the input to the system. The Laplace transform of $x(t)$ is

$$X(s) = \mathcal{L} [x(t)] = \frac{1}{s} \quad (\text{A.15})$$

For a linear system, the Laplace transformation of Equation A.11 is then given by

$$Y_m(s) = B_m H(s, z) [e^{sg_m(z)} x_m(s) + e^{-sg_m(z)} x_{-m}(s)] \quad (\text{A.16})$$

Substituting Equations A.14 and A.15 into Equation A.16 then gives

$$Y_m(s) = B_m \frac{b^3 c + dbs}{(ab^2 + s)^2} \frac{1}{s} [e^{sg_m(z)} + e^{-sg_m(z)}] \quad (\text{A.17})$$

Expanding in partial fractions results in

$$Y_m(s) = B_m \left\{ \frac{c/a^2 b}{s} - \frac{c/a^2 b}{ab^2 + s} + \frac{(d - c/a)b}{(ab^2 + s)^2} \right\} [e^{sg_m(z)} + e^{-sg_m(z)}] \quad (\text{A.18})$$

and the inverse Laplace transformation provides the output $y_m(t)$

$$y_m(t) = B_m \left\{ \frac{c}{a^2 b} [S_{-g_m}(t) - e^{-ab^2(t+g_m)}] + (d - c/a) b(t + g_m) e^{-ab^2(t+g_m)} \right. \\ \left. + \frac{c}{a^2 b} [S_{g_m}(t) - e^{-ab^2(t-g_m)}] + (d - c/a) b(t - g_m) e^{-ab^2(t-g_m)} \right\} \quad (A.19)$$

where

$$S_{-g_m}(t) = \begin{cases} 0 & \text{when } -g_m(t) < t < 0 \\ 1 & \text{when } t < -g_m(t) \end{cases}$$

and

$$S_{g_m}(t) = \begin{cases} 0 & \text{when } 0 < t < g_m(t) \\ 1 & \text{when } t > g_m(t) \end{cases}$$

Now taking the z-transformation of Equation A.19 gives

$$Z[y_m(t)] = B_m \left\{ z^i \left(\frac{c}{a^2 b} \left(\frac{z}{z-1} - \frac{z}{z - e^{-ab^2 \Delta t}} \right) - (d - c/a) b \Delta t z \frac{d}{dz} \left(\frac{z}{z - e^{-ab^2 \Delta t}} \right) \right. \right. \\ \left. \left. + z^{-i} \left(\frac{c}{a^2 b} \left(\frac{z}{z-1} - \frac{z}{z - e^{-ab^2 \Delta t}} \right) - (d - c/a) b \Delta t z \frac{d}{dz} \left(\frac{z}{z - e^{-ab^2 \Delta t}} \right) \right) \right\} \\ = B_m \left\{ z^i \left(\frac{c}{a^2 b} \left(\frac{1 - e^{-ab^2 \Delta t}}{z - e^{-ab^2 \Delta t}} \right) + (d - c/a) b \Delta t (z - 1) \frac{e^{-ab^2 \Delta t}}{(z - e^{-ab^2 \Delta t})^2} \right) \right. \\ \left. + z^{-i} \left(\frac{c}{a^2 b} \left(\frac{1 - e^{-ab^2 \Delta t}}{z - e^{-ab^2 \Delta t}} \right) + (d - c/a) b \Delta t (z - 1) \frac{e^{-ab^2 \Delta t}}{(z - e^{-ab^2 \Delta t})^2} \right) \right\} \\ \left(\frac{z}{z-1} \right) \quad (A.20)$$

where $i = g_m(z)/\Delta t = \frac{m\pi z}{v\epsilon_0 \Delta t}$. Letting $C_1 = 2 \exp[-ab^2 \Delta t] = 2 \exp[-2a\pi \Delta t]$, then Equation A.20 becomes

$$y_m(z) = \frac{B_m}{\left(z - \frac{c_1}{2}\right)^2} \left\{ z \frac{c}{a^2 b} + \frac{c_1}{2} \left((d - c/a) b \Delta t - \frac{c}{a^2 b} \right) \right. \\ \left. + \frac{1}{2} \left(\frac{c}{a^2 b} \left(\frac{c_1}{2} - 1 \right) - (d - c/a) b \Delta t \right) \right\} (z^i + z^{-i}) x(z) \quad (A.21)$$

Taking the inverse z-transformation and since $y(t)$ is the summation of y_m , we get

$$y_{n+1} = c_1 y_n + c_2 y_{n-1} + \sum_{m=0}^M B_m \left\{ [d_1 x_{m,n+i} + d_2 x_{m,n+i-1}] \right. \\ \left. + [d_1 x_{m,n-i} + d_2 x_{m,n-(i+1)}] \right\} \quad (A.22)$$

where

$$c_1 = 2 \exp[-2a\pi\Delta t]$$

$$c_2 = -\exp[-4a\pi\Delta t]$$

$$d_1 = \frac{c}{a^2 b} + \frac{c_1}{2} \left((d - c/a) b \Delta t - \frac{c}{a^2 b} \right)$$

$$d_2 = \left(\frac{c}{a^2 b} \left(\frac{c_1}{2} - 1 \right) - (d - c/a) b \Delta t \right) \frac{c_1}{2}$$

VITA

Kao-Huah Huang was born in Kaohsiung, Taiwan, The Republic of China, on March 1, 1953. He received his elementary and high school education in Kaohsiung. In June 1976 he graduated from The National Cheng Kung University, Tainan, Taiwan, where he received his Bachelor of Science degree with a major in Aeronautical Engineering. In the following two years, from October 1976 to August 1978, he served in the China Air Force as a second lieutenant. From September 1978 to July 1979 he was employed by The National Cheng Kung University as a teaching assistant.

He began his graduate studies at The University of Tennessee Space Institute in September 1979 as a graduate research assistant for the Atmospheric Science Division. In December 1980, he received the Master of Science degree in Aerospace Engineering and continued at the Space Institute in the Doctor of Philosophy program.

In April 1982, he accepted a position of Research Engineer with his present employer, FWG Associates, Inc., in Tullahoma, Tennessee. He received the Doctor of Philosophy degree in Mechanical Engineering in March 1984.

He was married to Ching-Li in December 1982. They have a daughter Grace, four months old.