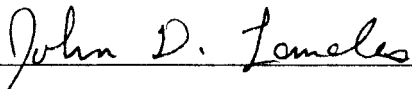


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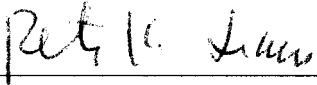
I am submitting herewith a dissertation written by Christopher D. Wilson entitled "Fracture Toughness Testing with Notched Round Bars." I have examined the final copy of this dissertation for form and content and recommend it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Engineering Science.

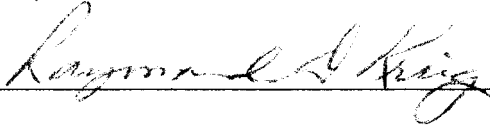


John D. Landes, Major Professor

We have read this dissertation and recommend its acceptance:







Accepted for the Council:



Associate Vice Chancellor
and Dean of the Graduate School

Fracture Toughness Testing
with
Notched Round Bars

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Christopher D. Wilson
August 1997

Dedication

This dissertation is dedicated to my wife Tammy. Without her love and support this work could not have been accomplished. She has been a constant encouragement to me throughout my graduate studies.

In addition to Tammy, this work is dedicated to our parents Wesley and Wilda Wilson and Allan and Margaret Atkins. They have been a tremendous influence on our lives and have provided many of the opportunities which have made this dissertation possible.

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Abstract

Current fracture toughness testing is conducted with planar specimens, such as the compact tension C(T) specimen and the single edge bend SE(B) specimen. The crack front in a planar specimen has a constraint condition which varies through the thickness from plane stress on the lateral surfaces to plane strain in the middle of the specimen. This change in constraint makes the geometry of planar specimens three-dimensional; however, two-dimensional analyses based on the assumption of plane stress or plane strain are often performed.

An alternative specimen geometry is the notched round bar (NRB). This axisymmetric geometry has the advantage of constant constraint in the circumferential direction. Additionally, the NRB geometry can be modeled using an axisymmetric formulation—essentially a two-dimensional analysis.

The suitability of the notched round bar geometry for fracture toughness testing was studied. Notched round bars with finite notch root radii were loaded to failure in tension to generate load-displacement curves. In an effort to simplify test methodology, no fatigue precracking was done. Four different materials with a range of strength and hardening characteristics were tested: 2024-T351 aluminum alloy, overaged 2024, a modified A302B pressure vessel steel, and nylon 6/6. Three different notch root radii ρ were used. The apparent fracture toughness values using finite ρ were linearly extrapolated to a zero notch root radius to infer sharp-crack fracture toughness.

For the 2024-T351 and the modified A302B NRB specimens, apparent fracture toughness K_{Jc} versus $\sqrt{\rho}$ was extrapolated to $\sqrt{\rho} = 0$. The extrapolated fracture toughness values closely agreed with fracture toughness values obtained from precracked C(T) specimens. The NRB results for modified A302B were used to determine T_0 , the reference temperature in a three-parameter Weibull model for characterizing the ductile to brittle transition. The resulting NRB master curve agreed closely to the master curve developed from precracked C(T) specimens.

For the overaged 2024 and the nylon 6/6 NRB specimens, apparent fracture toughness $J = J(\rho)$ versus crack extension Δa were plotted. A J_Q construction, similar to the construction in the ASTM E813 J_{Ic} test method, was used to determine apparent initiation fracture toughness $J_Q(\rho)$. A plot of $J_Q(\rho)$ versus ρ was linearly extrapolated to $\rho = 0$. The NRB results compared favorably to the results obtained from precracked C(T) and SE(B) specimens. However, the amount of stable crack growth obtained with the NRB geometry was very small—an order of magnitude smaller than that obtained with the planar specimens.

In performing this research, three key technological advances in fracture mechanics and plasticity were used. The first advance was the previously mentioned extrapolation procedure to relate apparent fracture toughness for notched specimens with sharp-crack fracture toughness. The second advance was the use of the load separation method to determine J and Δa without direct crack length measurement. The third advance was the inclusion of hydrostatic stress effects on yielding in the nonlinear finite element analyses of the NRB geometry. The application of these three technological advances was essential to the successful outcome of this research.

The conclusions reached in this study are threefold. First, the notched round bar geometry is a suitable alternative to current planar geometries for K -based initiation fracture toughness testing. Second, the NRB geometry is an attractive alternative for K_{Ic} fracture toughness testing in the transition range of ferritic steels. Third, although a J - R curve can be developed using the NRB geometry, the small amount of stable crack growth obtained severely limits the use of notched round bars. Based on these conclusions, it is recommended that further studies involving the notched round bar be made.

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List of Symbols and Acronyms

a	(a) Crack Length (b) Constant in Drucker-Prager Yield Function
b	Remaining Ligament ($W - a$)
c	Weibull Exponent
k	Yield Strength in Simple Shear
k_t	Stress Concentration Factor
m	(a) Exponent in Power Law for $G()$ (b) Proportionality Constant Between $CTOD$ and J
n	Strain Hardening Exponent in Ramberg-Osgood Stress-Strain Equation
p_f	Cumulative Probability for Failure in Weibull Analysis
r	Neck Radius of a Notched Round Bar
r_c	Distance from Crack Tip to Cleavage Initiation Site
r_p	Second-Order Plastic Zone Size
r_y	First-Order Plastic Zone Size
v	Displacement Due to a Crack
v_{el}	Elastic Displacement Due to a Crack
v_{pl}	Plastic Displacement Due to a Crack
A	Area
A_{pl}	Area Under the Load-Plastic Displacement Curve
B	Thickness
C	Compliance (Reciprocal of Stiffness)
E	Young's Modulus
E'	Effective Young's Modulus $E/(1 - \nu^2)$ for Plane Strain
K_ε	Critical Stress Intensity Factor
K_I	Stress Intensity Factor
K_{Ic}	Plane Strain Fracture Toughness Based on LEFM

K_{Jc}	Elastic-Plastic Fracture Toughness Equal to $\sqrt{J_c E}$
K_Q	Conditional Fracture Toughness Based on K
K_0	Weibull Scale Parameter
J	Elastic-Plastic Strain Energy Release Rate
J_c	Elastic-Plastic Fracture Toughness for Cleavage
J_{Ic}	Plane Strain Fracture Toughness Based on EPFM
J_Q	Conditional Fracture Toughness Based on J
$J-R$	Resistance Curve Fracture Toughness Based on EPFM
P	Load
P_N	Normalized Load
P_Q	Load for Calculating K_Q
P_5	5% Secant Offset Load
R	Nominal Radius of a Notched Round Bar
$S_{i,j}$	Separation Constant
T_0	Reference Temperature for Transition Fracture Toughness Master Curve
U	Strain Energy
W	Width
$\mathcal{G}$	Elastic Strain Energy Release Rate
$\mathcal{G}_c$	Critical Elastic Strain Energy Release Rate
δ_t	Crack-Tip Opening Displacement
η_{pl}	Dimensionless Parameter That Relates Plastic Work to J_{pl}
ν	Poisson's Ratio
ρ	Notch Root Radius
σ_c	Theoretical Cohesive Strength
σ_{eff}	von Mises or Effective Stress
σ_{ult}	Ultimate Tensile Strength
σ_{ys}	0.2% Offset Yield Strength
σ_Y	Flow Stress
σ_0	Yield Strength in Uniaxial Tension
$\theta^{1/4}$	Size Adjustment Ratio

ASTM	American Society for Testing and Materials
CRB	Cracked Round Bar
COD	Crack Opening Displacement
CTOD	Crack-Tip Opening Displacement
C(T)	Compact Tension
EPFM	Elastic-Plastic Fracture Mechanics
E399	ASTM E399 Fracture Toughness K_{Ic} Test Method
E602	ASTM E602 Sharp-Notch Strength Test Method
E813	ASTM E813 Fracture Toughness J_{Ic} Test Method
E1152	ASTM E1152 Fracture Toughness $J-R$ Test Method
E1290	ASTM E1290 Fracture Toughness $CTOD$ Test Method
E1820	ASTM E1820 Common Test Method for Fracture Toughness
FAD	Failure Assessment Diagram
LEFM	Linear Elastic Fracture Mechanics
MVC	Microvoid Coalescence
NRB	Notched Round Bar
ORNL	Oak Ridge National Laboratory
PBS	Percent Bending Stress
SE(B)	Single Edge Notch Bend
1T	One-Inch Thickness
1/2T	One-half Inch Thickness

Chapter 1

Introduction

Many structures contain crack-like defects. Structures made from materials that behave in a brittle fashion can be seriously weakened by the stress concentration near a crack tip. In such structures, fracture can occur with little advanced warning—even though the working stresses are well below the yield strength. The traditional strength of materials approach is not sufficient to guarantee structural integrity in these cases. Clearly, a different approach is needed.

The fracture mechanics approach, combining information about applied stress and crack size, has proved to be a successful alternative to the strength of materials approach. A parameter combining stress and crack size in a structure is compared to a material property quantifying the material's resistance to crack growth. This property, known as fracture toughness, is used in an analogous manner as the yield strength in the strength of materials approach.

Several different parameters are currently used to characterize the crack-tip stress field. When the deformation is in the elastic range, the crack-tip stress field can be described using K , the stress intensity factor. Thus, K is the governing parameter for Linear Elastic Fracture Mechanics (LEFM). LEFM was first applied to ship structures and aerospace structures in the 1950s. When the deformation is in the plastic range, the crack-tip stress field can be described equivalently using the J -Integral (often referred to as simply J) or the crack-tip opening displacement (CTOD). Thus, J and CTOD can be used as governing parameters for Elastic-Plastic Fracture Mechanics (EPFM). EPFM was first applied in power generation industry during the late 1960s.

All these parameters can be determined for a given crack geometry and boundary conditions. In most cases, handbooks containing compilations of solutions are used to determine K or J . Examples of these handbooks are “The Stress Analysis

of Cracks” [1] for K solutions and “An Engineering Approach for Elastic-Plastic Fracture Analysis,” [2] for J solutions. For CTOD calculations, the use of the COD Design Curve approach [3] is prevalent. If none of these parameters are valid for a particular geometry and boundary condition, plastic collapse analysis is used to determine if failure will occur. The ranges of applicability of LEFM, EPFM, and plastic collapse are shown in Fig. 1.1. In many cases, a useful design approach is to develop a failure assessment diagram or FAD [4], where fracture toughness and collapse are both considered.

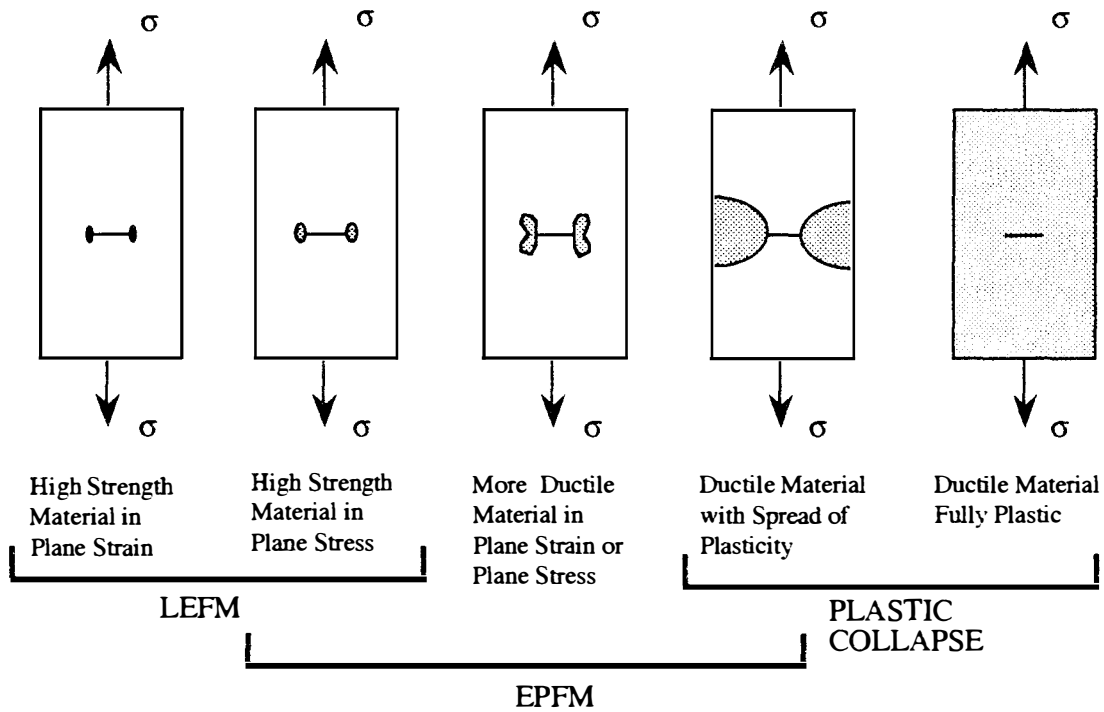


Figure 1.1: Range of Fracture Behavior

Once a parameter has been calculated to quantify the crack-tip stress field, it must be compared to a suitable measure of fracture toughness. The American Society for Testing and Materials (ASTM) has developed fracture toughness testing standards based on K , J , and CTOD over the past twenty-five years. The first ASTM fracture toughness testing standard, E399 “Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials,” [5] was published in 1970. In 1981, E813 “Standard Test Method for J_{Ic} , A Measure of Fracture Toughness” [6] was published. In 1987, E1152 “Standard Test Method for Determining J - R Curves” [7] was

published. Finally, in 1989, E1290 “Standard Test Method for Crack-Tip Opening Displacement (CTOD) Fracture Toughness Measurement” [8] was published.

In the chapters that lie ahead, the stress analysis of cracks and fracture toughness testing are discussed. Problems with current fracture toughness testing are described. A possible solution to these problems is given in the form of a new test specimen geometry—the *notched round bar* shown with a smooth tensile bar in Fig. 1.2.

Previous work with notched round bars and cracked round bars is noted and extended. Three key technological advances in fracture mechanics and plasticity theory are discussed. These technologies are used in a research program which focuses on determining the suitability of notched round bars for fracture toughness testing. Results and conclusions are presented for the work reported here.

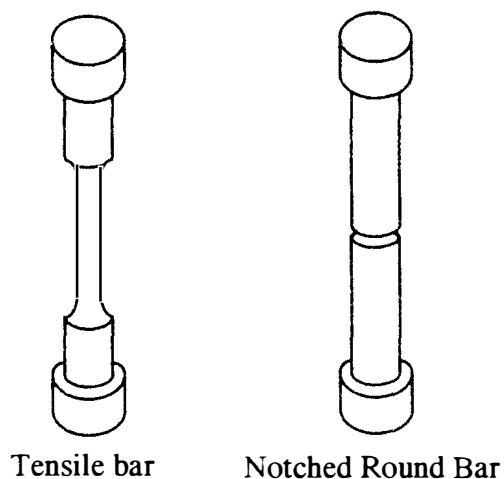


Figure 1.2: The Notched Round Bar

Chapter 2

Stress Analysis of Cracks

In this chapter, the stress analysis of cracks is reviewed. First, the elastic stress field is described in terms of K . The elastic stress field approach is limited by crack-tip plasticity. After plasticity is discussed, the energy release rate approach is given and the connection between it and the elastic stress field is presented. Finally, an elastic-plastic stress approach based on J which is valid when K -based analysis breaks down is described.

2.1 Elastic Stress Field Approach

The stress intensity factor approach was developed by Irwin [9] and Williams [10] in the 1950s. The approach is based on the elastic stress field very near the crack tip. The mode I (the tensile or opening mode) stress components near the crack tip (Fig. 2.1) have the form

$$\begin{aligned}\sigma_{xx} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right), \\ \sigma_{yy} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right), \\ \tau_{xy} &= \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}, \\ \tau_{xz} &= \tau_{yz} = 0, \\ \sigma_{zz} &= \begin{cases} 0 & \text{plane stress} \\ \nu(\sigma_{xx} + \sigma_{yy}) & \text{plane strain,} \end{cases}\end{aligned}\tag{2.1}$$

where r is the distance from the crack tip, θ is the angle of inclination, and K_I is called the mode I stress intensity factor because its value governs the intensity of the stress field. A complete discussion of the elastic stress field is given in textbooks by Broek [11], Ewalds and Wanhill [12], and Anderson [13].

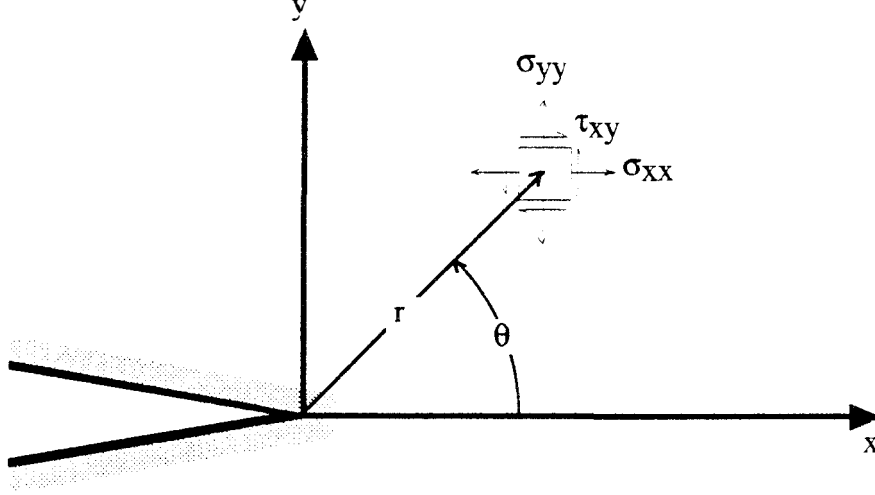


Figure 2.1: Crack-Tip Geometry and Stresses

Along the plane ahead of the crack (shown as the line ($\theta = 0$) in Fig. 2.1), the stresses are

$$\begin{aligned}\sigma_{xx} &= \sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}, \\ \tau_{xy} &= 0.\end{aligned}\tag{2.2}$$

The crack-tip stresses of Eq. 2.2 are proportional to $r^{-1/2}$, approaching infinity as r approaches zero. The stresses for all mode I problems have the form given by Eq. 2.2 regardless of boundary conditions and crack length. K_I is a function of nominal stress or load, crack size, and component geometry.

The stress intensity factor K_I is the basic parameter in LEFM because it controls the magnitude of the stress singularity of Eq. 2.2. K_I has the dimensions of stress $\times$ length^{1/2}. For a given specimen shape, crack length, and the load, the stress intensity factor can usually be determined from a handbook. For example, the stress intensity factor for a crack in a infinitely wide plate ($W \gg 2a$) shown in Fig. 2.2 is

$$K_I = \sigma\sqrt{\pi a}.\tag{2.3}$$

where σ is the nominal applied stress. If the width W is not large compared to the crack length $2a$, then the stress intensity factor given by Eq. 2.3 must be modified to account for finite width. For planar geometries (constant thickness), K_I is usually written as

$$K_I = \sigma \sqrt{\pi a} f\left(\frac{a}{W}\right), \quad (2.4)$$

where $f(a/W)$ is a function which modifies the K_I solution for a crack in a infinitely wide plate to account for finite width. For the plate in Fig. 2.2 (often referred as a center cracked panel), the solution for K_I is

$$K_I = \sigma \sqrt{\pi a} \sqrt{\sec\left(\frac{\pi a}{W}\right)}. \quad (2.5)$$

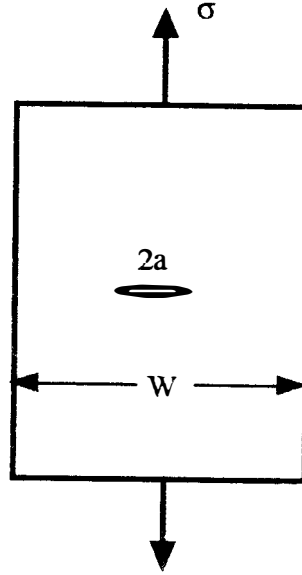


Figure 2.2: Griffith Crack Problem

Unstable crack growth occurs when the applied K_I approaches its critical value K_c , a material property. Some material/structure combinations undergo some stable crack growth before failure. In the next section, crack-tip plasticity is discussed. The amount of plasticity at a crack tip affects the amount of stable crack growth that can occur.

2.2 Crack-Tip Plasticity

In the previous section, it was shown that the mathematical stress field has a $r^{-1/2}$ singularity at the crack tip. Certainly, a real material cannot withstand infinite stresses. When the yield strength is exceeded near the crack tip, a limited amount of plastic flow occurs. Therefore, the theoretically infinite stresses are prevented by the presence of the crack-tip plastic zone shown in Fig. 2.3. Plasticity makes the crack behave as if it were longer than its actual length because the crack-tip plastic zone results in larger crack-tip displacements and reduced local stiffness. Irwin estimated the first-order (elastic analysis) size of the plastic zone by letting σ_{yy} in Eq. 2.2 equal the yield stress σ_{ys} and solving for the distance r_y . For thin plates (whose constraint is referred to as plane stress)

$$r_y = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2. \quad (2.6)$$

For thick plates (whose constraint is referred to as plane strain), an additional component of stress, σ_{zz} , inhibits plastic flow, making the plastic zone smaller by a factor of three

$$r_y = \frac{1}{6\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2. \quad (2.7)$$

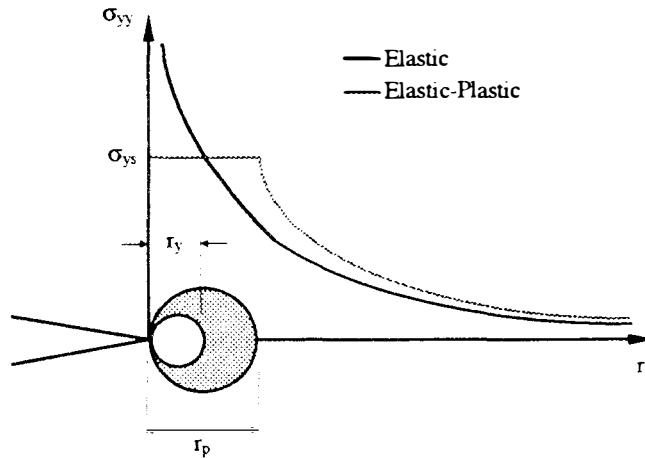


Figure 2.3: Plastic Zone Approximations

Irwin estimated the second-order (elastic-plastic analysis) size of the plastic zone r_p by redistributing the stresses from the crack tip to the first-order plastic zone

$$\sigma_{ys} r_p = \int_0^{r_y} \sigma_{yy} dr, \quad (2.8)$$

which leads to the result

$$r_p = \frac{1}{\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2 = 2 r_y. \quad (2.9)$$

Irwin showed that the actual stress intensity factor was approximated by treating the crack as having effective length $a + r_y$. This approximation amounts to moving the crack tip to the center of the second-order plastic zone in Fig. 2.3.

For LEFM to be valid, the crack-tip plastic zone must be contained inside a region where the stress intensity factor controls the magnitude of the stress field. To ensure this, the crack length should be about one order of magnitude larger than the plastic zone size.

2.3 Energy Release Rate Approach

An alternative to the elastic stress field approach (K_I approach) of LEFM is the energy release rate approach. In the early 1920s, Griffith [14] proposed that if the rate of change in the elastic energy dU stored in a cracked body equaled or exceeded the work required to produce a new fracture surface area dA , then the crack would grow. Unstable crack growth occurred when the applied $\mathcal{G}$ approached its critical value $\mathcal{G}_c$, a material constant

$$\mathcal{G}_{\text{applied}} = -\frac{dU}{dA} \rightarrow \mathcal{G}_c. \quad (2.10)$$

Irwin later showed that the stress intensity factor approach was equivalent to the energy release rate approach. The relationship between $\mathcal{G}$ and the mode I stress intensity factor K_I is

$$\mathcal{G} = \frac{K_I^2}{E'}, \quad (2.11)$$

where $E' = E$ for plane stress and $E' = E/(1 - \nu^2)$ for plane strain. The dimensions of $\mathcal{G}$ are work $\div$ area or FL/L^2 where F is the force dimension and L is the length

dimension. $\mathcal{G}$ is also called the crack driving force because its dimension can be expressed as F/L (force per unit length).

For the infinitely wide plate ($W \gg 2a$) in Fig. 2.2, the applied strain energy release rate is

$$\mathcal{G} = \frac{\pi \sigma^2 a}{E'}. \quad (2.12)$$

Since the stress intensity factor and energy release rate approach are equivalent, unstable crack growth occurs when $\mathcal{G} \rightarrow \mathcal{G}_c$ or $K_I \rightarrow K_c$.

2.4 Elastic-Plastic Stress Field Approach

When the plastic zone size is no longer small in comparison to the cracked body's geometry, LEFM is no longer valid and EPFM must be used. The J -Integral, introduced by Rice [15] in 1968, is a nonlinear extension of the energy release rate concept described in the previous section. Rice used nonlinear elasticity and assumed that no unloading occurs in the cracked body. However, this simplifying approach, referred to as deformation plasticity, ignores the fact that the actual plastic deformation in metals is irreversible. For a linear elastic material, $J = \mathcal{G}$. For a nonlinear elastic material,

$$J = -\frac{dU}{dA}, \quad (2.13)$$

where dU is interpreted not as the energy released from the body when a crack extends, but rather as the difference in energy of a cracked body with fracture surface area A and a cracked body with a incrementally larger fracture surface $A + dA$ (Fig. 2.4). This distinction must be made because of the irreversible work done by crack extension in an elastic-plastic material and will be important for a growing crack.

Rice found a path independent line integral that was equal to $-dU/dA$ using nonlinear elasticity

$$J = \int_{\Gamma} \left(W dy - T_i \frac{\partial u_i}{\partial x} ds \right), \quad (2.14)$$

where W is the strain energy density $W = \int \sigma_{ij} d\epsilon_{ij}$, T_i is the traction vector normal to the contour, u_i is the displacement vector normal to the contour, and ds is a length element along a closed contour Γ around the crack tip (Fig. 2.5).¹

¹Subscripts refer to indicial notation with a range convention of 1 to 3.

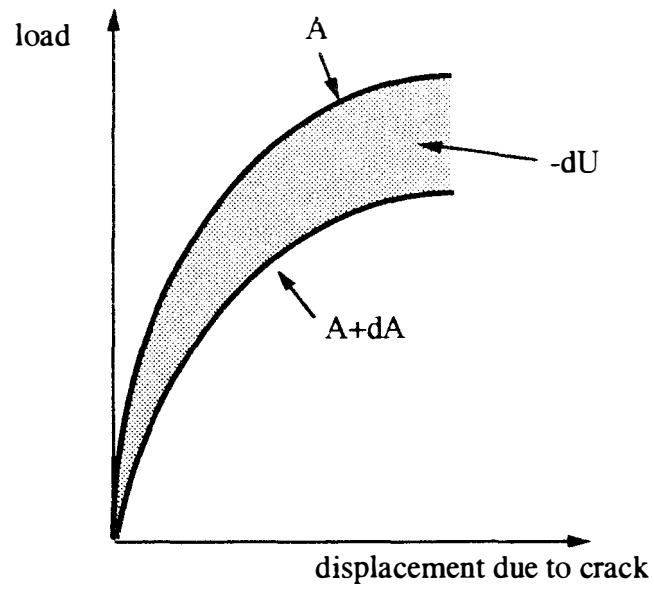


Figure 2.4: Graphical Definition of J

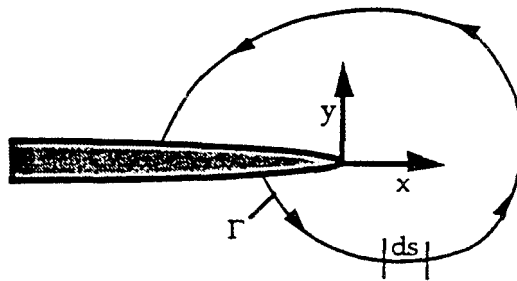


Figure 2.5: Arbitrary Contour Around a Crack Tip [13]

Hutchinson [16] and Rice and Rosengren [17] independently showed that J is a stress intensity parameter in a nonlinear elastic material represented by a power law hardening relationship

$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0} \right)^n, \quad (2.15)$$

σ_0 is the yield stress, ϵ_0 is the yield strain, α is a dimensionless constant and n is the strain hardening exponent. For such a material, the stresses and strains are given by

$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n, \theta), \quad (2.16)$$

and

$$\epsilon_{ij} = \alpha \epsilon_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{n}{n+1}} \tilde{\epsilon}_{ij}(n, \theta), \quad (2.17)$$

where I_n is an integration function depending on n and constraint (either plane stress or plane strain), and $\tilde{\sigma}_{ij}$ and $\tilde{\epsilon}_{ij}$ are dimensionless functions of n and θ . For an elastic material ($n = 1$), the singularity ($r^{-1/(n+1)}$) of Eq. 2.16 reduces to the $1/\sqrt{r}$ singularity of LEFM.

The J -Integral defines the magnitude of the so-called HRR singularity, much like K defines the magnitude of the linear elastic singularity. The HRR singularity predicts infinite stresses as $r \rightarrow 0$, just as the K singularity. Physically, this situation cannot occur since the large strains at the crack tip cause the crack to blunt, creating a stress-free surface at $r = 0$. Although the small strain analysis that leads to the HRR singularity does not consider the blunted crack tip, J uniquely characterizes the elastic-plastic crack-tip field in the region surrounding the crack-tip, just as K uniquely characterizes the elastic crack-tip field surrounding the crack-tip. This uniqueness implies that a critical value of J (or K) is a size-independent measure of fracture toughness.

Chapter 3

Fracture Toughness Testing

In the previous chapter, it was mentioned that when a crack-tip parameter (K , $\mathcal{G}$, or J) reaches a critical value, then unstable crack growth would occur. This critical value is a material property called fracture toughness. Fracture toughness is a quantity which characterizes the resistance to crack growth in the presence of cracks or crack-like defects. Fracture toughness should not be confused with toughness, the area under the stress-strain diagram determined from a tensile test of a smooth (unnotched) specimen. Fracture toughness testing requires a notched or cracked test specimen.

In this chapter, the two basic mechanisms of quasi-static fracture are defined. Single value fracture toughness and fracture toughness in terms of a resistance curve are reviewed. Three current ASTM fracture toughness test methods and one draft fracture toughness test method are described.

3.1 Fracture Mechanisms

A fundamental understanding of fracture mechanisms is necessary before defining fracture toughness. There are two basic mechanisms of monotonic, time-independent fracture: cleavage and microvoid coalescence. Cleavage is a fracture mechanism characteristic of specific crystal planes that have low resistance to cracking. Cleavage often occurs with little or no plastic deformation. Fracture surfaces created by cleavage usually have a smooth, faceted, shiny appearance. Ductile microvoid coalescence (MVC) is the plasticity-induced formation, growth, and joining of tiny voids in the crack-tip region. Fracture surfaces created by MVC have a rough, dimpled, dull appearance. Regardless of the fracture mechanism, the material property that quantifies the material's resistance to cracking is fracture toughness.

Fracture toughness can be described as a material property which characterizes the resistance to crack growth and failure from monotonic loading due to the presence of crack-like defects in a material undergoing deformation. The dimensions of fracture toughness depend on the crack-tip parameter used in the test method. For energy release rates $\mathcal{G}$ or J , fracture toughness has the dimensions of FL/L^2 . For stress intensity factor K_I , fracture toughness has the dimensions of stress $\times$ length^{1/2}. Fracture toughness can be converted from energy release rates to stress intensity factors using Eq. 2.11.

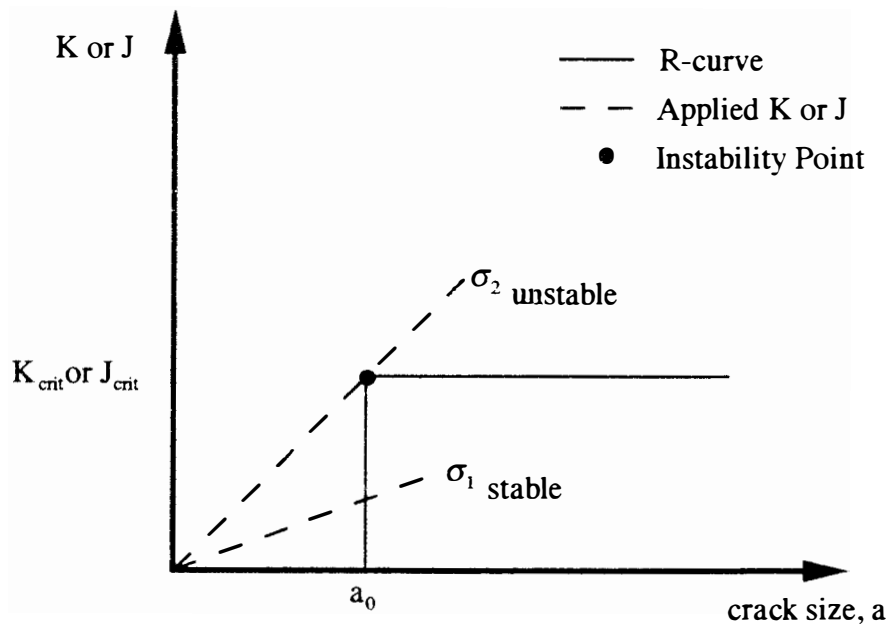
Material behavior has a considerable influence on fracture toughness. Fracture toughness is much less when crack extension occurs by cleavage instead of ductile fracture because cleavage is an intrinsically brittle mode of fracture involving the separation of atomic bonds along well-defined crystallographic planes.

A single value of fracture toughness is usually sufficient when a test specimen fails by cleavage because the cleavage fracture mechanism is typically unstable. For the MVC fracture mechanism, some stable crack growth occurs, implying that the material's fracture toughness changes with crack extension. A single value of fracture toughness is no longer a complete description; a plot of material *resistance* to crack growth, R , versus crack extension, Δa , is needed for a complete description.

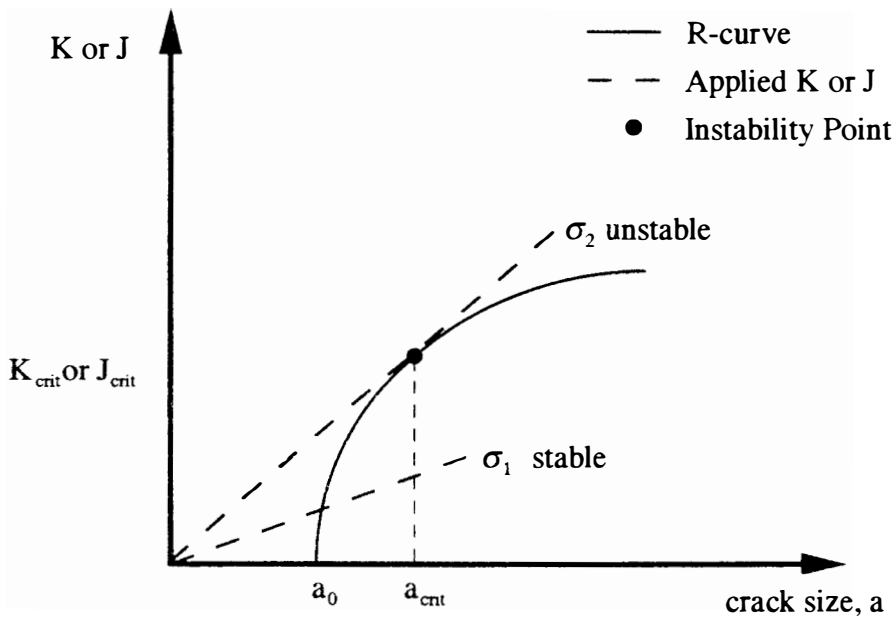
In Fig. 3.1a, the R -curve for an ideally brittle material is plotted versus crack length a . For a given applied stress σ_1 , the applied K or J can be plotted for a range of crack sizes. If an initial crack size is a_0 and the material's resistance to crack growth is given by the step function R -curve, then the situation is stable and the crack does not grow. If the stress is increased slightly, the situation is still stable. However, if the stress level increases from σ_1 to σ_2 , a point of instability is reached and unstable crack growth occurs. The single value of K (or J) at this instability K_{crit} (or J_{crit}) uniquely describes the resistance to unstable crack growth.

When the initial part of the R -curve has a finite slope as in Fig. 3.1b, some stable crack growth can occur before instability is reached. In this case, as the stress increases from σ_1 to σ_2 , the crack growth from initial length a_0 to a critical length a_{crit} . Unstable crack growth occurs at a_{crit} because the applied K or J -curve is tangent to the material's R -curve.

The R -curve is usually plotted with change in crack size Δa as the abscissa instead of a . In Fig. 3.2, the initial crack size a_0 is subtracted from all subsequent crack sizes a , so that $\Delta a = a - a_0$. The initial slope of the R -curve is usually very steep. For that reason, the line given by the initial slope of the R -curve is called the blunting line since very little crack growth occurs there. The term "blunting" refers to the fact that the sharp crack is stretching without extending further into



(a) R-Curve for Brittle Material



(b) R-Curve for Ductile Material

Figure 3.1: R-curve Behavior in Terms of R versus a

the material. Instead, the crack tip tends to round out or blunt in this part of the R -curve.

Cleavage fracture has a flat or falling R -curve (unstable crack growth leading to failure) as in Fig. 3.1a, whereas, microvoid coalescence (MVC) usually has a rising R -curve (some stable crack growth before failure) as in Fig. 3.1b. For MVC, the initiation of stable crack growth can be used as a conservative single value of fracture toughness. ASTM has developed fracture toughness tests for initiation fracture toughness and fracture toughness in terms of a resistance curve.

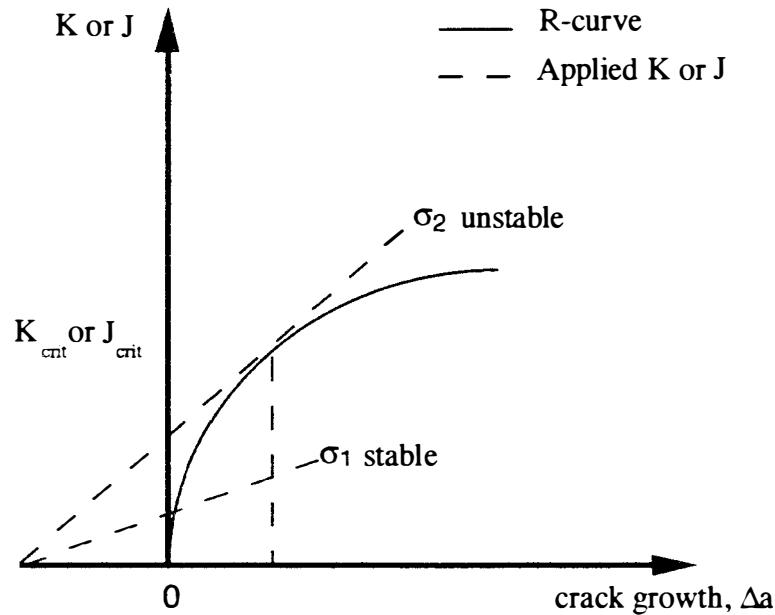


Figure 3.2: R-curve Behavior in Terms of R versus Δa

3.2 ASTM Fracture Toughness Tests

Historically, the use of a particular fracture toughness test can be related to the location of interest on a plot of fracture toughness versus temperature. Fig. 3.3 is a qualitative plot of the ductile to brittle transition of ferritic steels. The K_{Ic} test was applied to the lower shelf of the curve. The J_{Ic} and J - R curve tests were applied to the upper shelf of the curve. The CTOD test was the only standard method

for determining fracture toughness in the ductile to brittle transition. A draft test method using K_{Jc} (which may soon become an ASTM standard test method [18]) is also applicable in the ductile to brittle transition. In Fig. 3.3, the extreme portion of the curve was not characterized by fracture toughness at all because the stress field associated with using J breaks down. In this region, failure can be predicted using a plastic collapse analysis. Note that the failure (fracture) mechanism is printed below the fracture toughness curve, and the parameter used to assess failure (fracture) is printed above the curve. In the following subsections, three ASTM test methods based on K and J are described. The draft test method based on K_{Jc} is also described.

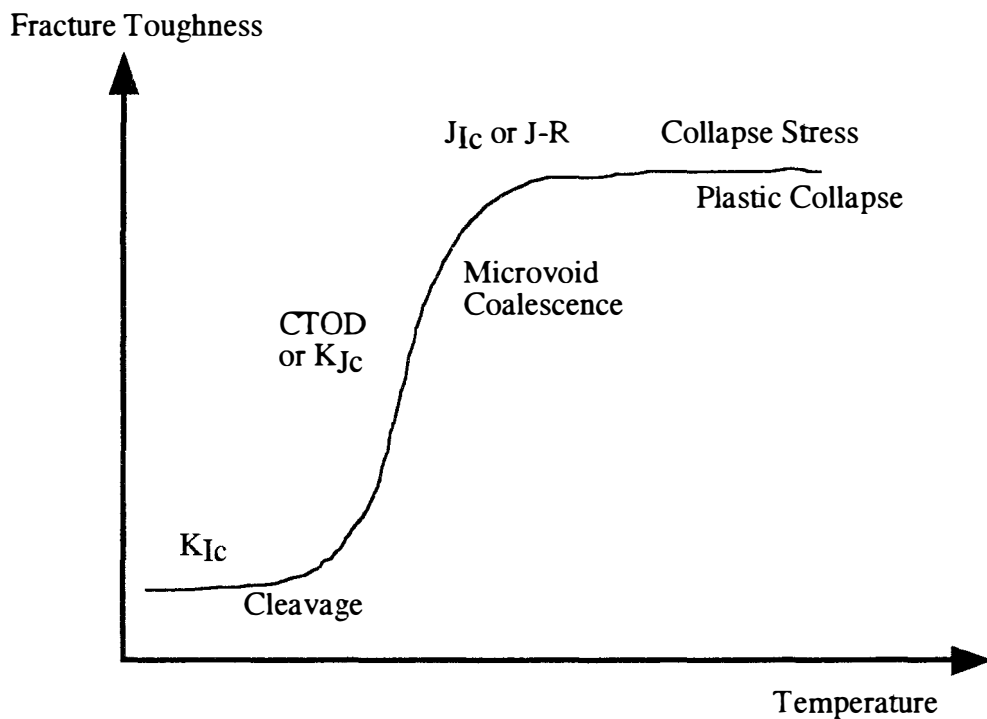


Figure 3.3: Ductile to Brittle Transition

3.2.1 E399 K_{Ic} Test

When the plastic zone size is small compared to the crack size and contained inside a region where K controls the crack-tip stress field, critical stress intensity factors K_c can be determined experimentally. However, K_c depends on specimen thickness and constraint. A schematic plot of K_c versus thickness is given in Fig. 3.4. For

small thicknesses, the stress state is plane stress and K_c varies considerably. As the thickness increases, the stress state transitions from plane stress to plane strain. Beyond a certain thickness, the value of K_c tends to approach a constant lower limit, designated as K_{Ic} , the plane strain fracture toughness. K_{Ic} is considered a material property. ASTM E399 is a widely accepted test method for determining K_{Ic} .

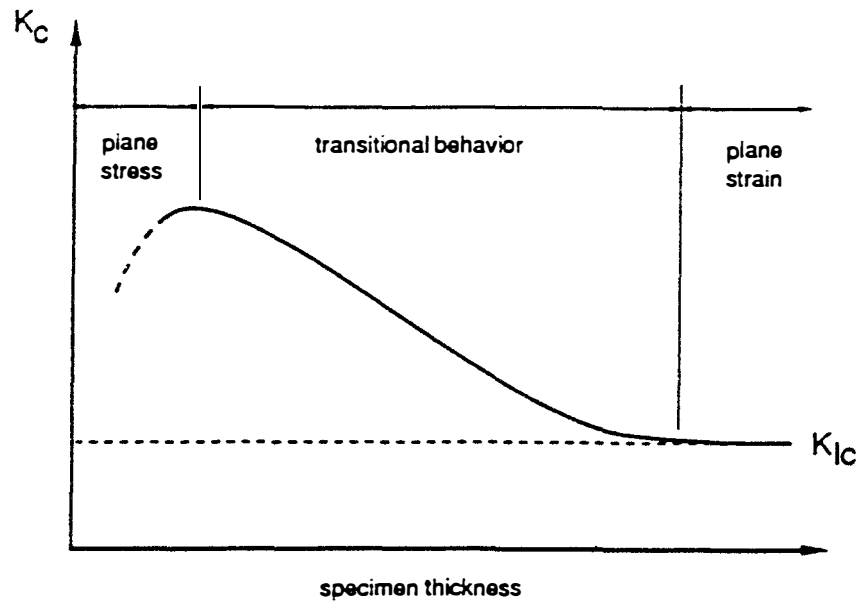
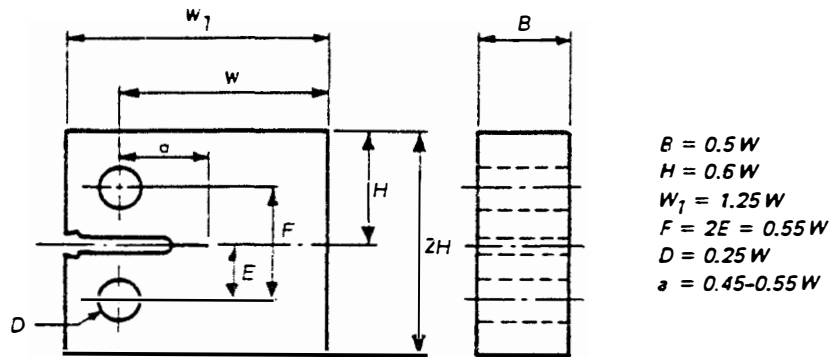
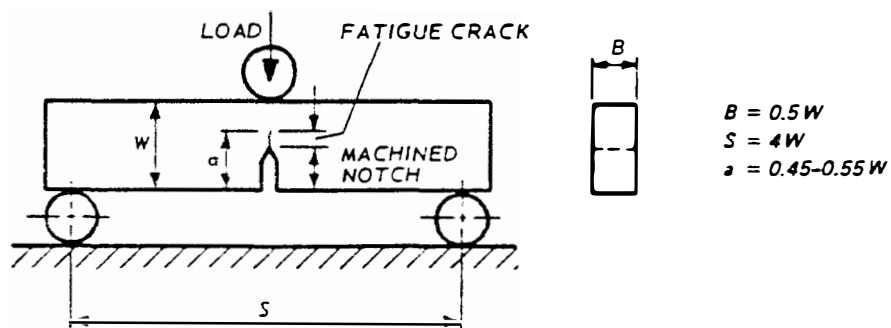


Figure 3.4: Fracture Toughness versus Specimen Thickness

The two most commonly used test specimens for K_{Ic} testing are the compact tension C(T) specimen and single edge bend SE(B) specimen shown in Fig. 3.5. For the C(T) specimen, load is applied by pins placed in the two holes and pulled in opposite directions. Displacement is measured on the front-face (the face the notch is cut into) of the C(T) specimen. For the SE(B) specimen, load is applied in three-point bending. Displacement is measured on the front-face of the SE(B) specimen as well. Very stringent requirements are placed on the machining of the notches in both specimen geometries. The specimens must be fatigue precracked, and the reproducibility of symmetric fatigue crack fronts depends, in part, on the machining of the notch.



Compact Tension C(T) Specimen



Single Edge Bend SE(B) Specimen

Figure 3.5: C(T) and SE(B) Specimen Geometries [12]

The steps in performing a K_{Ic} test are summarized below.

1. Determine the specimen dimensions crack length a , thickness B , and width W . To ensure plane strain conditions for the test, some estimate of the expected K_{Ic} should be made. The largest practical thickness is often used.
2. Fatigue precrack the specimen until $0.45 \leq a/W \leq 0.55$. The purpose of fatigue precracking is to create an ideal planar crack with a crack-tip radius of practically zero. Care must be taken not to use too high of a precracking load, thus avoiding having to account for any plasticity effects from the fatigue cycles.
3. Load the specimen to failure, recording the load-displacement (P - v) curve.
4. Examine the fracture surface of the specimen (Fig. 3.6). Measure the length of the fatigue precrack along the lateral surfaces and three points equally spaced in the interior. For K calculations, an average crack length $\bar{a}$ is used

$$\bar{a} = \frac{a_1 + a_2 + a_3}{3}. \quad (3.1)$$

For a valid K_{Ic} test, the values of a_1 , a_2 , and a_3 must be within 5% of $\bar{a}$ and the surface crack lengths a_s must be within 10% of $\bar{a}$.

5. Determine the critical load P_Q from the P - v curve. Generally, there are three ways to determine P_Q , depending on the characteristics of the P - v curves shown in Fig. 3.7. The initial slope of the P - v curve is determined, and a secant line is drawn to the curve using a slope equal to 95% of the initial slope. For a Type I P - v curve, the critical load $P_Q = P_5$, where P_5 is the load at the intersection of the P - v curve and the secant line. In some cases, a sudden crack extension and arrest takes place. This phenomenon is called pop-in and is shown as Type II. In this case, P_Q is taken as the maximum load preceeding the pop-in. Finally, Type III P - v curves exhibit a nearly perfect elastic behavior. In this case, $P_Q = P_{max}$. To further ensure that excessive plasticity does not exist, a valid test requires that

$$\frac{P_{max}}{P_Q} \leq 1.1. \quad (3.2)$$

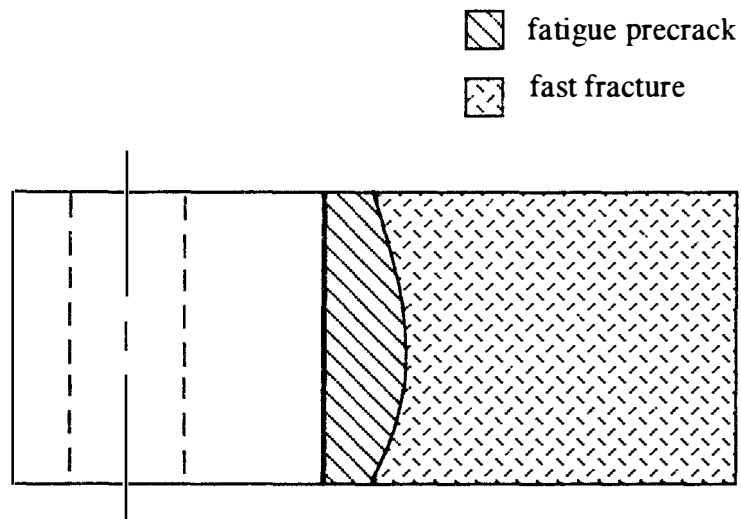


Figure 3.6: E399 Crack Front Measurement

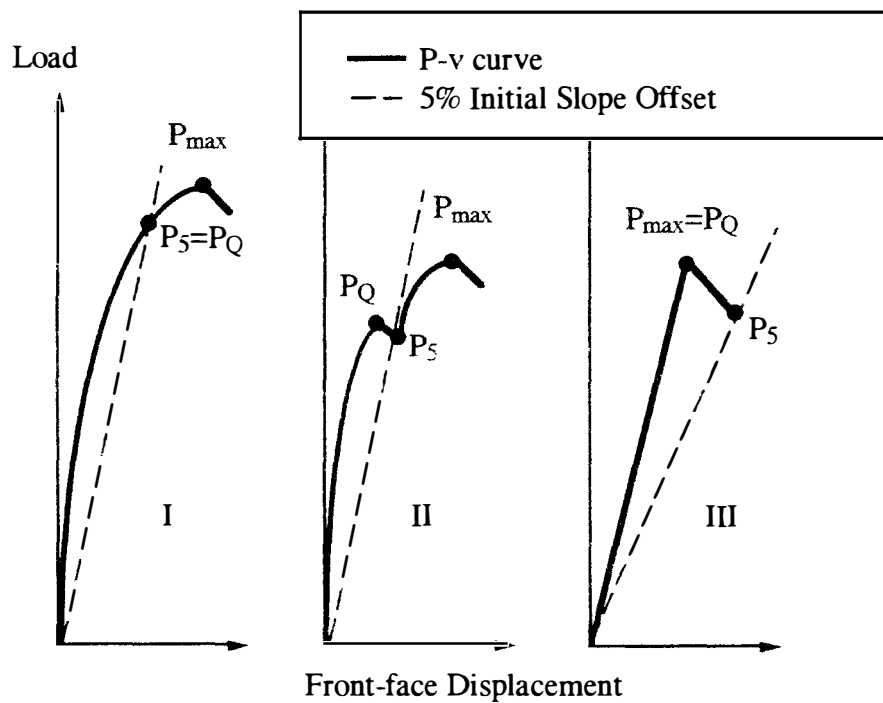


Figure 3.7: E399 $P-v$ Curves

6. Calculate the conditional fracture toughness K_Q using P_Q and $\bar{a}$. For the C(T), K_Q is given by

$$K_Q = \frac{P_Q}{BW^{1/2}} f\left(\frac{a}{W}\right), \quad (3.3)$$

with

$$f\left(\frac{a}{W}\right) = \frac{(2 + a/W)}{(1 - a/W)^{3/2}} \left[0.886 + 4.64 \left(\frac{a}{W}\right) - 13.32 \left(\frac{a}{W}\right)^2 + 14.72 \left(\frac{a}{W}\right)^3 - 5.6 \left(\frac{a}{W}\right)^4 \right]. \quad (3.4)$$

7. Check the size requirements to ensure plane strain conditions prevailed during the test. The crack length a , uncracked remaining ligament $W - a$, and thickness B must be much larger than the plane strain plastic zone size given by Eq. 2.7. The size requirements are

$$a, B \geq 2.5 \left(\frac{K_Q}{\sigma_{ys}} \right)^2 \approx 50r_y. \quad (3.5)$$

If K_Q meets these size requirements and the other requirements on crack front straightness and avoiding excessive plasticity, then $K_{Ic} = K_Q$.

Historically, Brown and Srawley [19, 20] developed the size requirements for the K_{Ic} test in the 1960s. They experimented with a high-strength maraging steel and 7075-T6 aluminum alloy. They determined that the 95% secant offset line allowed for approximately 2% stable crack growth during the test. This plasticity allowance is similar to using a 0.2% offset for determining σ_{ys} in a tensile test.

3.2.2 E813 J_{Ic} Test

The E813 J_{Ic} test method has two approaches for determining J_{Ic} : the multiple specimen approach (the original approach) and a single specimen approach. This discussion will focus on the single specimen approach.

Like the E399 K_{Ic} test, compact tension C(T) specimens and single edge bend SE(B) specimens are used. The configuration of the specimens is slightly different from their E399 counterparts. For example, the C(T) specimen used for J_{Ic} testing

has a deeper ($a/W > 0.5$) starter notch than the C(T) used for K_{Ic} testing. Additionally, the starter notch is straight—not a chevron notch as for K_{Ic} testing. For J_{Ic} testing, displacement is measured on the line of action of the load (load-line). In the K_{Ic} test, displacement is measured on the front-face of the specimen. A schematic of a C(T) for J_{Ic} testing is shown in Fig. 3.8.

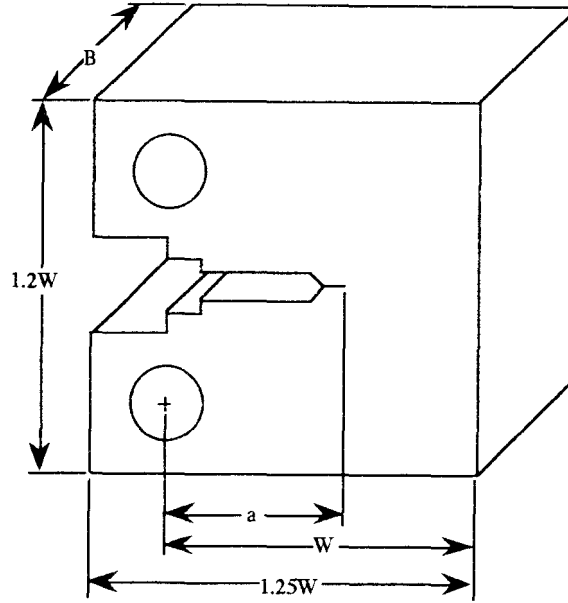


Figure 3.8: C(T) Specimen Geometry for J_{Ic} Testing

The steps in performing a single specimen J_{Ic} test are summarized below.

1. Determine the specimen dimensions crack length a , thickness B , and width W . As with K_{Ic} testing, the largest practical thickness is often used.
2. Fatigue precrack the specimen until $0.5 \leq a/W \leq 0.75$.
3. Load the specimen to a predetermined displacement. Then unload the specimen partially. Reload the specimen and then unload again, building a load-displacement (P - v) curve shown in Fig. 3.9. The partial unloadings, which are always elastic, are used to determine the current crack length using compliance. Compliance $C = v/P$ can be thought of as the reciprocal of stiffness.

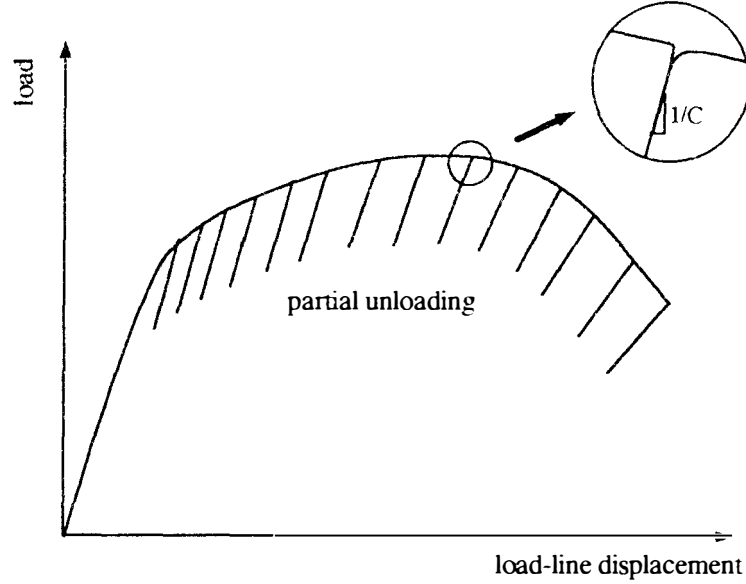


Figure 3.9: Unloading Compliance for Crack Length Determination

Compliance is a function of crack size. Once the compliance is measured, the current crack length can be determined. This process should continue until the specimen is visually near complete failure.

4. Completely unload the specimen, heat tint it to mark the final crack front, chill it, and then reload the specimen to completely fracture it.
5. Examine the fracture surface of the specimen and measure the stable crack growth as shown in Fig. 3.10. For J calculations, an average crack length $\bar{a}$ is used

$$\bar{a} = \frac{1}{8} \left[\frac{1}{2} (a_{s1} + a_{s2}) + a_1 + \cdots + a_7 \right], \quad (3.6)$$

where a_{s1} , a_{s2} are surface crack measurements from the load-line to the fatigue precrack and $a_1, \dots, a_7$ are equally spaced interior measurements of crack length. The total stable crack growth Δa is measured in the same manner.

6. Calculate J for the crack lengths determined by unloading compliance.

7. Plot J versus Δa as in Fig. 3.11, and construct the blunting line given by

$$J = 2\sigma_Y \Delta a, \quad (3.7)$$

where $\sigma_Y = 1/2 (\sigma_{ys} + \sigma_{uts})$ is the flow stress or effective yield strength that accounts for hardening. Construct the two exclusion lines parallel to the blunting line at offsets of $\Delta a = 0.15$ mm and $\Delta a = 1.5$ mm. Fit the J - Δa data points between the exclusion lines using a power law

$$J = C_1(\Delta a)^{C_2}. \quad (3.8)$$

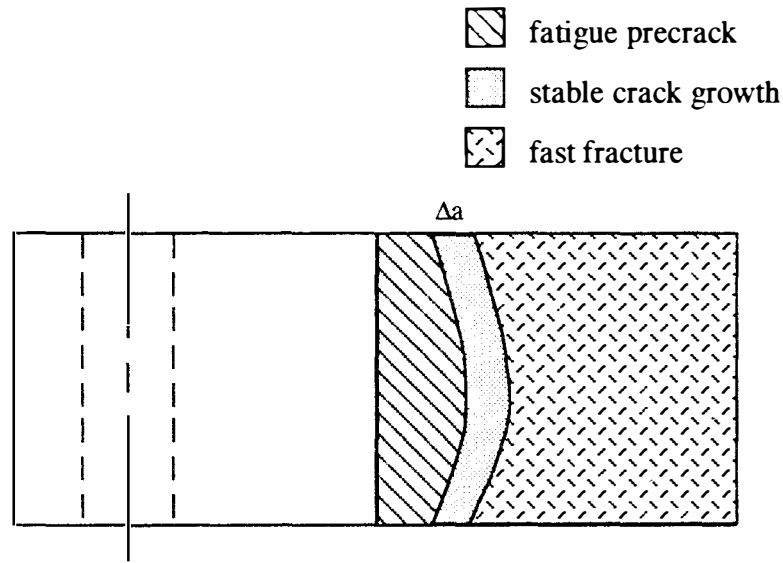


Figure 3.10: E813 Crack Front Measurement

8. Determine the conditional fracture toughness J_Q from the intersection of a 0.2 mm offset line parallel to the blunting line (Eq. 3.7) and the power law curve fit (Eq. 3.8).
9. Check the size requirements to ensure that a valid test has taken place

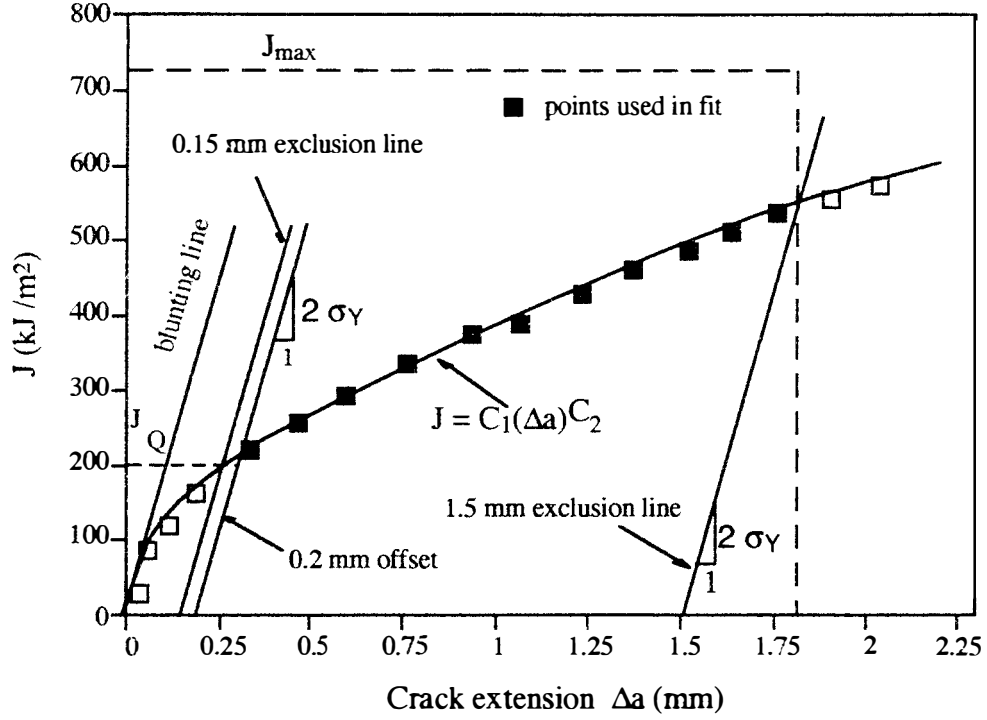


Figure 3.11: E813 J - R Curve Construction

$$b, B \geq 25 \frac{J_Q}{\sigma_Y}, \quad (3.9)$$

where $b = W - a$ is the uncracked ligament, and B is the specimen thickness. If J_Q meets the size requirements and the other requirements on crack-front straightness and avoids excessive plasticity by having

$$J < J_{max} = \frac{b\sigma_Y}{15}, \quad (3.10)$$

then the test is valid and $J_{Ic} = J_Q$.

For completeness, the equations necessary to calculate J in the J_{Ic} test are given for the C(T) specimen. The crack length a/W in terms of compliance C for the C(T) specimen is

$$\begin{aligned} \frac{a}{W} = & 1.000196 - 4.06319u_{LL} + 11.242u_{LL}^2 - 106.043u_{LL}^3 \\ & + 464.335u_{LL}^4 - 650.677u_{LL}^5, \end{aligned} \quad (3.11)$$

where

$$u_{LL} = \frac{1}{1 + \sqrt{BE'C}}. \quad (3.12)$$

Once the crack length a/W is determined, J can be calculated.

The calculation of J is divided into elastic and plastic parts

$$J = J_{ei} + J_{pl}, \quad (3.13)$$

where

$$J_{ei} = \frac{K^2(1 - \nu^2)}{E'}, \quad (3.14)$$

and

$$J_{pi} = \frac{\eta_{pl} A_{pl}}{bB}. \quad (3.15)$$

The plastic work A_{pl} is the plastic area under the load versus load-line displacement curve in Fig. 3.12. The term bB is the remaining uncracked area of the specimen. The term η_{pl} is a dimensionless parameter that relates the plastic work done on a specimen to the crack growth resistance defined in terms of J_{pl} .

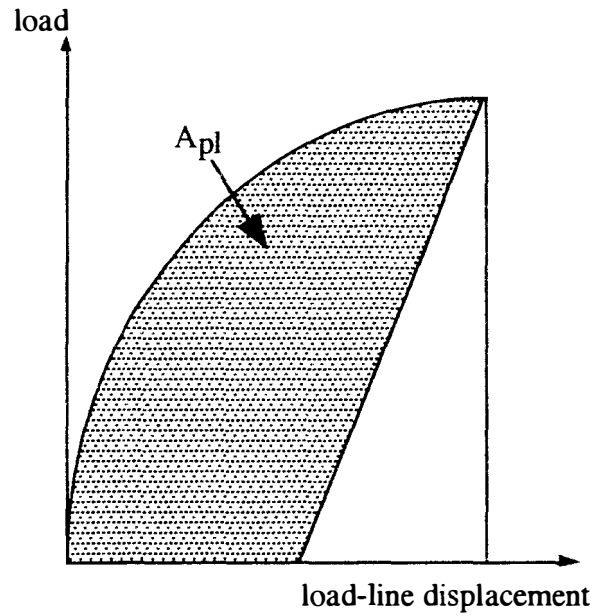


Figure 3.12: Area Used in J Calculations

For the C(T) specimen, η_{pl} is defined as

$$\eta_{pl} = 2 + 0.522 \frac{b}{W}. \quad (3.16)$$

Historically, the size requirements in the J_{Ic} test were developed by Begley and Landes [21, 22] in the early 1970s. Landes and Begley tested A533B pressure vessel steel (yield strength of 70 ksi) and a Ni-Cr-Mo-V rotor steel (135 ksi).

The blunting line in Fig. 3.11 was used to allow for the fact that some of the apparent crack extension was due to the blunting of the crack tip. If the crack is assumed to blunt in the shape of a semicircle, then the apparent crack extension due to blunting is

$$\Delta a_{blunt} = \frac{\delta_t}{2}, \quad (3.17)$$

where δ_t is the CTOD value at the crack-tip. CTOD and J are related through the following equation

$$\delta_t = m \frac{J}{\sigma_Y}. \quad (3.18)$$

Taking m as unity, Eqs. 3.17 and 3.18 give the blunting line defined previously by Eq. 3.7.

The 0.15 mm exclusion line was used because small crack extensions usually had a large scatter when J values were determined. The 1.5 mm exclusion line ensured that Δa was generally less than 6% of the remaining ligament. Between the two exclusion lines, the value of J calculated using the original ligament b_0 was approximately the same as using the actual ligament b . The horizontal exclusion line $J = J_{max}$ is given by

$$J_{max} = \frac{b\sigma_Y}{15}. \quad (3.19)$$

This exclusion line is used in the same spirit as Eq. 3.2 for the K_{Ic} test, separating ductile crack growth from the regime of plastic collapse.

3.2.3 E1152 J - R Test

In cases where ductile instability analyses of large structures are needed, the complete J - R curve is developed using the E1152 test method. E1152 addresses a very important issue that was not addressed in the E813 test—how to calculate J for a growing crack.

The original definition of J given by Rice modeled elastic-plastic deformation as nonlinear elastic. The crack was treated as stationary. This model is valid as long as no unloading occurs. The use of unloading compliance to determine crack lengths in E813 and E1152 would seem to invalidate the model. However, Rice showed that J could be treated as a nonlinear energy release rate using deformation plasticity, making this possible for a growing crack.

For a growing crack, the P - v record is shown in Fig. 3.13. J is calculated using Eq. 3.13 with both elastic and plastic components of J being calculated incrementally because the deformation plasticity P - v record depends on crack size. This incremental calculation of J was not performed in the J_{Ic} test because the interest there was on initiation fracture toughness, where the crack extension should be small. The J values are updated at each unloading point because the crack length can be updated using compliance.

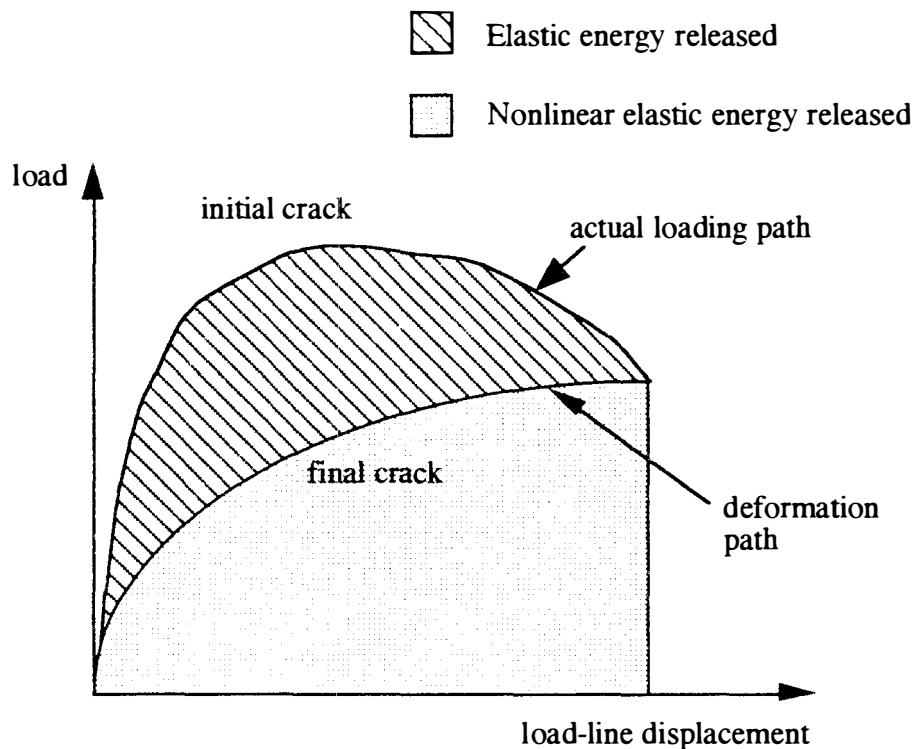


Figure 3.13: Deformation Plasticity for a Growing Crack

Limits are placed on the maximum value of J and the maximum value of Δa . For a valid J - R test,

$$B, b_0 \geq \frac{20J_{max}}{\sigma_Y}, \quad (3.20)$$

where b_0 is the initial ligament size. The maximum crack extension allowed is

$$\Delta a_{max} \leq 0.1b_0. \quad (3.21)$$

Note that the J_{max} in E1152 is smaller than the J_{max} allowed in E813 (Eq. 3.19, but is larger than the J_{Ic} size requirement (Eq. 3.9). The size restrictions are imposed in an attempt to obtain geometry independent R -curves that can be applied to structures.

3.2.4 Draft Transition Fracture Toughness Test Method

The fourteenth draft test method (1997) quantifies fracture toughness of ferritic steels in the 275 to 825 MPa (40 to 120 ksi) range of yield strength that experience cleavage cracking in the ductile to brittle transition. In the draft test method, fracture toughness is expressed in terms of an elastic-plastic stress intensity factor, K_{Jc} , that is determined from J_c at the onset of cleavage fracture

$$K_{Jc} = \sqrt{J_c E}. \quad (3.22)$$

where J_c is calculated using a procedure similar to the one described in the E813 test method.

The draft test method uses specimen geometries, fatigue precracking, and crack length measurement procedures that are similar to the E399, E813, and E1152 test methods. The major difference between the draft test method and the previous methods is the use of Weibull statistics to predict the transition fracture toughness curve. The statistical effect of specimen size on K_{Jc} in the transition range is modeled using a weakest-link theory embodied in a three parameter Weibull distribution of fracture toughness values.

The J -Integral at crack instability is denoted as J_c and is converted to a K -based fracture toughness value using Eq. 3.22. A valid K_{Jc} is obtained if $K_{Jc} < K_{Jc(limit)}$ where $K_{Jc(limit)}$ is given by

$$K_{Jc(limit)} = \sqrt{\frac{Eb_0\sigma_{ys}}{30}}. \quad (3.23)$$

where b_0 is the remaining ligament of the specimen.

A median K_{Jc} value can be determined if at least six valid K_{Jc} results are determined at a single temperature. A three-parameter Weibull model relates K_{Jc} and the cumulative probability for failure at or before K_{Jc} , p_f

$$p_f = 1 - \exp \left\{ - \left[\frac{K_{Jc} - K_{min}}{K_0 - K_{min}} \right]^c \right\}, \quad (3.24)$$

where K_0 is the Weibull scale parameter to be determined, c is the Weibull exponent, assumed to be four, and K_{min} is a lowerbound fracture toughness for ferritic steels, taken as 20 MPa $\sqrt{m}$ (18.2 ksi $\sqrt{in}$).

The actual Weibull exponent c can be determined by calculating the slope of graph of $\ln[-\ln(1-p_f)]$ versus $\ln(K_{Jc} - K_{min})$. The probability values p_f are assigned to K_{Jc} values after the K_{Jc} values are ranked in order of increasing magnitude. Cumulative probability values for failure are assigned to each ranked $K_{Jc(i)}$ using

$$p_{f(i)} = \frac{i - 0.3}{N + 0.4}, \quad (3.25)$$

where $i = 1$ is the lowest ranked K_{Jc} value, $i = 2$ is the next ranked K_{Jc} value, and $i = N$ is the highest ranked K_{Jc} value.

The Weibull parameter K_0 can be calculated for $c = 4$ from

$$K_0 = \left[\sum_{i=1}^N \frac{K_{Jc(i)} - K_{min}}{N - 0.3068} \right]^{1/4} + K_{min}, \quad (3.26)$$

where N is the number of specimens. The estimated median K_{Jc} of the test population is

$$K_{Jc(med)} = (K_0 - K_{min}) (\ln 2)^{1/4} + K_{min}. \quad (3.27)$$

The draft test method allows the development of a fracture toughness versus temperature curve (sometimes referred to as a master curve). The curve for 1T (one-inch thick) compact specimens is given by

$$K_{Jc(med)} = 30 + 70 \exp[0.019(T - T_0)] \text{ MPa}\sqrt{m}, \quad (3.28)$$

where T is the test temperature ($^{\circ}C$) and T_0 is the reference temperature ($^{\circ}C$). A typical plot is shown in Fig. 3.14. T_0 is determined using

$$T_0 = T - \frac{1}{0.019} \ln \left[\frac{K_{Jc(med)} - 30}{70} \right], \quad (3.29)$$

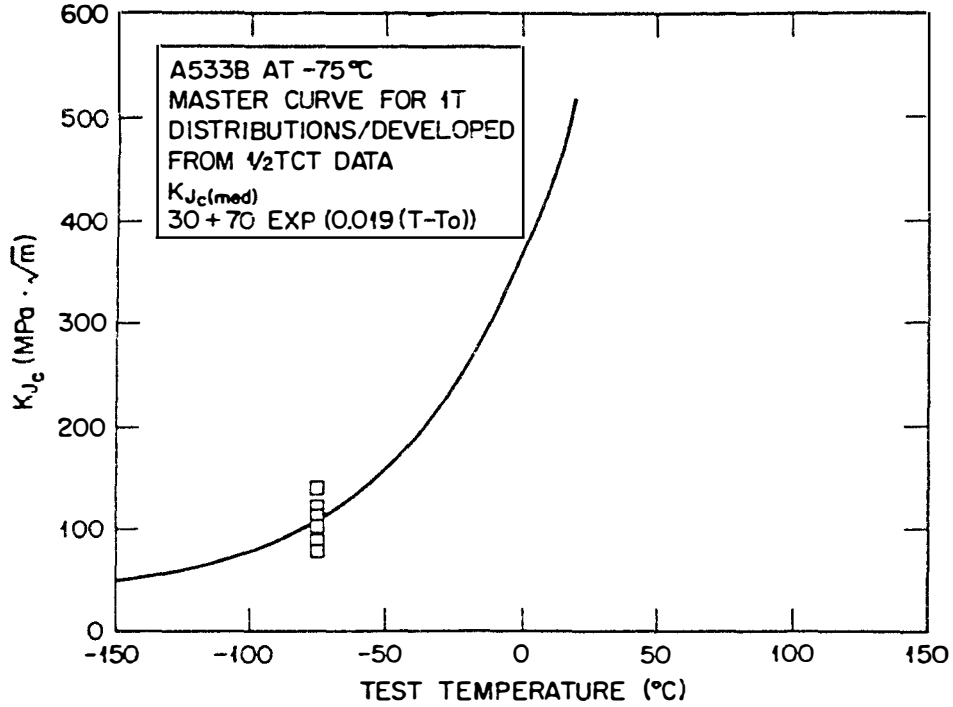


Figure 3.14: Typical Plot of K_{Jc} versus Temperature [18]

where $K_{Jc(med)}$ is given in units of $\text{MPa}\sqrt{\text{m}}$ and temperature T and T_0 are given in $^{\circ}\text{C}$.

The preceding discussion focused on the development of a master curve using 1T (one-inch thick) specimens. When a specimen size other than a 1T is used, a size adjustment must be made to adjust the K_{Jc} data to its 1T equivalent using

$$K_{Jc(1T)} = 20 + [K_{Jc(x)} - 20] \left(\frac{B_x}{B_{1T}} \right)^{1/4}, \quad (3.30)$$

where $K_{Jc(x)}$ and $K_{Jc(1T)}$ have the units of $\text{MPa}\sqrt{\text{m}}$. The thickness B_x is the nominal thickness of the test specimen and B_{1T} is the thickness of a 1T (1 in. or 50 mm). After the size adjustment is made, Eq. 3.26 is used to determine K_0 , Eq. 3.27 is used to determine $K_{Jc(med)}$, and Eq. 3.29 is used to determine T_0 .

The reference temperature T_0 is the temperature where the median K_{Jc} for 1T size compact specimens is $100 \text{ MPa}\sqrt{\text{m}}$ ($90.9 \text{ ksi}\sqrt{\text{in}}$). The value of T_0 should be relatively independent of the test temperature chosen. However, the selection of a test temperature that would lead to a median K_{Jc} close to $100 \text{ MPa}\sqrt{\text{m}}$ is recommended, and a means of estimating this temperature is provided in the draft test method.

Chapter 4

The Need for Change

In this chapter, the problems in current fracture toughness tests are reviewed. In addition, the factors leading to high costs in testing are discussed. Possible solutions are postulated. One possible solution—**starting over**—is presented as the main thrust of this research.

4.1 Problems in Current Fracture Toughness Tests

As previously mentioned in the chapter on fracture toughness testing, there are three major problem areas in using the current test standards. First, knowing the material response in advance or at least having an educated guess is necessary with current methods. Second, there are many inconsistencies between the practices described in the current tests. Third, the current tests are complicated and tend to get more complicated as new revisions are implemented.

1. *Knowing Material Response in Advance*—Using the three ASTM test methods discussed requires the user to decide a priori which test to apply. If the material does not respond to loading as expected, then the test is invalid, and the specimen and valuable testing time are wasted. This scenario is not uncommon in K_{Ic} testing.
2. *Varying Constraint Specimen Geometries*—The standard specimens have planar geometries. For thin planar specimens, a plane stress condition exists and the constraint is nearly uniform through the thickness. For thick planar specimens, a plane stress condition exists on the lateral surfaces of the specimen and transitions toward a plane strain condition in the middle of the specimen. If the planar specimen is thick enough, a state of plane strain will exist in the

middle. Thus, the constraint will always vary from plane stress to plane strain (or nearly plane strain) in planar specimens.

3. *Inconsistencies Between the Current Test Methods*—There are several inconsistencies which can be addressed.

- Crack front measurements in the K_{Ic} test involve taking two surface measurements and three interior measurements along the crack front; crack front measurements in the J_{Ic} test involve seven interior measurements. Both choices were rather arbitrary. Regardless of the test method, the uneven crack growth that occurs in planar specimens is a problem that cannot be removed from the test.
- Two arbitrary yield strengths are used in the fracture toughness tests: 0.2% offset yield strength (σ_{ys}) and effective yield strength (σ_Y) taken to be the average of the 0.2% offset yield strength and the tensile ultimate strength to account for strain hardening.
- The m -factor used in relating J and K to CTOD (Eq. 3.18) is inconsistently used. The value of m depends on strain hardening and is not unity for elastically loaded metals. Shih [23] has determined that m varies from 1.25 to 2.5 for common structural steels.

4. *Complicated Procedures*—All the current tests are much more complicated than standard tensile testing. These complications may even discourage some industries from performing qualitative fracture toughness testing.

4.2 The High Cost of Current Fracture Toughness Tests

In comparison to tensile testing, fracture toughness testing using the current ASTM fracture toughness test standards can be very expensive. During the past 25 years, the standard fracture toughness testing procedures have relied on test specimens which are expensive to fabricate. These specimens must be fatigue precracked—taking both operator and machine time. The standard specimens require expensive instrumentation to record the load-line displacement. These factors make fracture toughness tests much more expensive to perform than tensile tests.

4.3 Possible Solutions

4.3.1 Common Test Method

One possible solution to simplify fracture toughness testing and reduce testing costs is use the common test method [24], ASTM E1820 “Standard Test Method for Measurement of Fracture Toughness” . In 1996, ASTM approved the common test method which provides, in a single test method, the procedures and guidelines found in the E399, E813, E1152, and E1290 test methods. These four current fracture toughness tests were developed over a thirty year span by engineers and scientists with different backgrounds and different problems. One of the great strengths of ASTM is its diversity; however, thirty years of diversity has led to some unnecessary duplication and inconsistency in the fracture toughness tests. Many of the differences in test procedures do not have a sound technical basis. The purpose of the common test method is to allow the material’s response to loading dictate the results of the test. The common test method will allow the material’s response to dictate the results of the test. The common test method should alleviate many of the inconsistencies between the current test methods. Unfortunately, the methodology in the common test method will be more complicated. This is the price of versatility.

4.3.2 Starting Over

Another possible solution to simplify fracture toughness testing is to start anew. The notched round bar was rejected as a specimen geometry by The Committee on Rapid Inexpensive Tests for Determining Fracture Toughness [25] in the 1970s because of high load capacity requirements and difficulty in fatigue precracking. The difficulty with high load requirements was imaginary—conventional wisdom at the time dictated that a round bar with a very large diameter, 5.0 in., would be necessary to maintain the constraint level of a 1.0 in. thick planar specimen. The difficulty with fatigue precracking was real—asymmetric crack growth occurred when there was a small load misalignment. However, if smaller diameter (much less than 1.0 in.) specimens are used and if the precracking requirement can be removed, then the notched round bar seems likely to be a reasonable alternative to current test geometries.

The notched round bar has two advantages over planar fracture toughness specimens. First, the notched round bar loaded in tension is essentially a two-dimensional (axisymmetric) geometry which develops equal constraint around the crack front. Second, notched round bars will reduce manufacturing and testing costs.

Chapter 5

Stress, Strength, and Fracture of Round Bars

In this chapter, previous work with notched round bars is discussed and extended. The stress analysis of notched round bars as well as strength approach to failure of notched round bars is reviewed. Stress analysis and tests involving fatigue precracked round bars are also discussed.

5.1 Notched Round Bars

In this section, the work of Neuber [26] and Creager [28] are presented. Neuber's work was an elastic solution to the notched round bar loaded in tension. Creager's work relates the stress intensity factor to the notched (not cracked) round bar. In addition, early work on notched round bar tensile testing is discussed. Sachs and coworkers [29, 30, 31] pioneering work in notched bar tensile testing is reviewed.

5.1.1 Stress Analysis

In 1937, Neuber [26] published a monumental work on the stress analysis of notches in linear elastic materials. A deeply notched round bar loaded in tension is shown in Fig. 5.1. Neuber used a hyperbolic notch profile with the transformations

$$x = \sinh u \cos v, \quad (5.1)$$

$$y = \cosh u \sin v, \quad (5.2)$$

which gives an orthogonal set of coordinates using

$$\left(\frac{x}{\sinh u}\right)^2 + \left(\frac{y}{\cosh u}\right)^2 = 1, \quad (5.3)$$

$$\left(\frac{y}{\sin v}\right)^2 - \left(\frac{x}{\cos v}\right)^2 = 1. \quad (5.4)$$

In this set of coordinates, the lines $u = \text{constant}$ are ellipses and the lines $v = \text{constant}$ are hyperbolas. For $u = 0$, the surface of the ellipsoid u collapses to the cross-section of the neck of the notched bar.

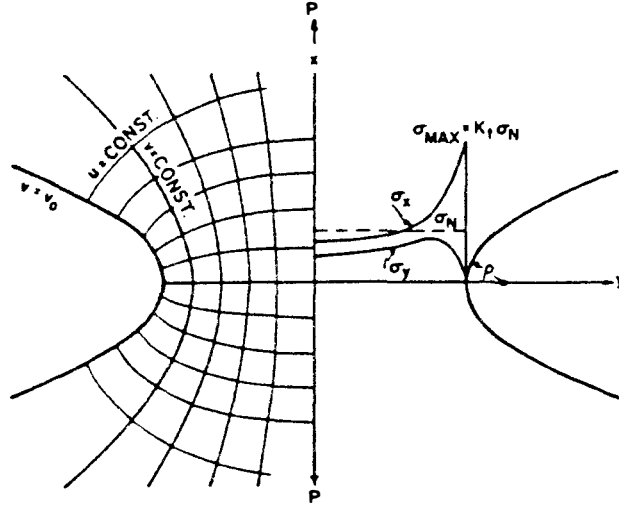


Figure 5.1: Notched Round Bar Geometry—Notch Neck Region [26]

The stresses for $u = 0$ are given by

$$\sigma_{xx} = \frac{1}{\cos v} [B - C(1 + D)] + \frac{1}{\cos^2 v} (-A + B + C \cos^2 v), \quad (5.5)$$

$$\sigma_{\theta\theta} = \frac{1}{\cos v} \left[\frac{A}{1 + \cos v} - B - (2 - D)C \right] \quad (5.6)$$

$$\sigma_{yy} = \frac{1}{\cos v} \left[\frac{-A}{1 + \cos v} - (2 - D)C \right] + \frac{A - B}{\cos^3 v} \quad (5.7)$$

where

$$D = 2(1 - \nu), \quad (5.8)$$

$$C = -\frac{\tilde{\sigma}}{2} \frac{1 + \cos v_0}{1 + (2 - D) \cos v_0 + \cos^2 v_0}. \quad (5.9)$$

$$A = (D - 1)(1 + \cos v_0)C, \quad (5.10)$$

$$B = A - C \cos^2 v_0. \quad (5.11)$$

In the above equations, $\tilde{\sigma}$ is the net section stress

$$\tilde{\sigma} = \frac{P}{\pi r^2}, \quad (5.12)$$

and v_0 is the nondimensional coordinate at the surface of the notched round bar chosen so that

$$\tan^2 v_0 = \frac{r}{\rho}. \quad (5.13)$$

where r is the smallest radius of the notch (along $u = 0$) and ρ is the smallest radius of curvature of the hyperbolic notch.

Along $u = 0$, a typical stress distribution is shown in Fig. 5.1. The stress concentration factor k_t relating $\sigma_{xx(max)}$ to net section stress $\tilde{\sigma}$ ($\sigma_{xx(max)} = k_t \tilde{\sigma}$) is

$$k_t = \frac{r/\rho(r/\rho + 1)^{1/2} + 0.8r/\rho + 1.3[(r/\rho + 1)^{1/2} + 1]}{r/\rho + 0.6(r/\rho + 1)^{1/2} + 2}, \quad (5.14)$$

where a Poisson's ratio of 0.3 is assumed. For $r/\rho = 25$, the stress concentration factor is $k_t = 5.17$. For $r/\rho = 12.5$, the stress concentration factor is $k_t = 3.71$. For $r/\rho = 6.25$, the stress concentration factor is $k_t = 2.70$.

It must be remembered that Neuber's solution for a notched round bar assumes that the notch is deep. Thus, Neuber's results do not depend on the nominal radius of the bar, R , shown in Fig. 5.2. Finite geometry effects for the notched round bar can be found in Peterson's Handbook [27].

In 1966, Creager [28] examined the stress distribution of a notched plate in terms of K_I . For the local notch geometry shown in Fig. 5.3, Creager derived the stress field in terms of polar coordinates r and θ . It should be noted that r in Fig. 5.3 and Eq. 5.15 is a radial coordinate—not the notch neck radius shown in Fig. 5.2. The stresses near a notch with a finite notch root radius are

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) - \frac{K_I}{\sqrt{2\pi r}} \frac{\rho}{2r} \cos \frac{3\theta}{2}, \quad (5.15)$$

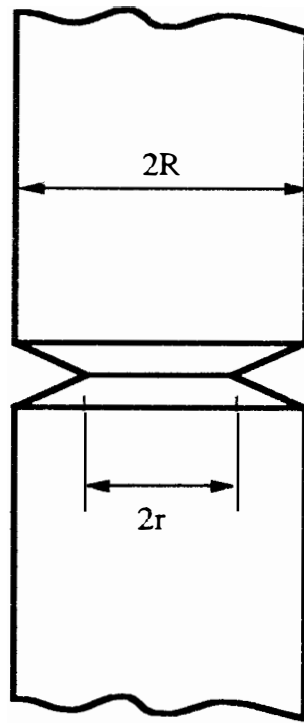


Figure 5.2: Notched Round Bar Geometry—Finite Geometry

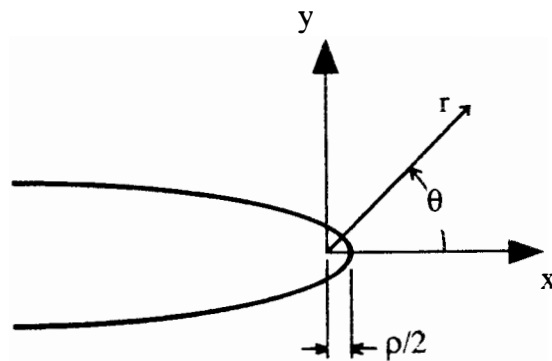


Figure 5.3: Blunt Notch Geometry

$$\begin{aligned}
\sigma_{yy} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + \frac{K_I}{\sqrt{2\pi r}} \frac{\rho}{2r} \cos \frac{3\theta}{2}, \\
\tau_{xy} &= \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + \frac{K_I}{\sqrt{2\pi r}} \frac{\rho}{2r} \sin \frac{3\theta}{2}, \\
\tau_{xz} &= \tau_{yz} = 0, \\
\sigma_{zz} &= \begin{cases} 0 & \text{plane stress} \\ \nu(\sigma_{xx} + \sigma_{yy}) & \text{plane strain,} \end{cases}
\end{aligned}$$

These field equations are similar to the equations for the mathematically sharp crack. The singularity is at the focal point of the notch and not at the notch tip. As $\rho/r \rightarrow 0$, the field equations reduce to those for the sharp crack.

5.1.2 Strength Approach to Failure

In the 1940s, Sachs and coworkers [29, 30, 31] studied the stress concentration effects of notched round bars made of heat treated low alloy steels. They found that the effect of notch root radius was related to the tensile strength of the alloy. Notch strength dropped abruptly with decreasing root radius for higher strength levels. The notched round bar was shown to be useful in distinguishing differences in notch strength among metals having similar yield strengths.

In 1955, Lubahn [32] discussed the applicability of notched bar data to strength criteria. He postulated that fracture begins at the notch root in sharply notched bars because the very high local strains have more effect on the fracture process than the “ductility-killing triaxiality” in the interior section ahead of the notch. For mild notches, he postulated that fracture starts in the interior section because the loss of ductility caused by the high level of stress triaxiality was more important than the small strain concentration at the root of a mild notch.

In 1960, Geil and Carwile [33] studied the effect of notch geometry on tensile properties of annealed commercially pure titanium at 100, 25, -78 , and -196 C. They varied both notch depth and notch root radius in their tests. Later in 1963, Kaufman and Johnson [34] studied notch sensitivity in aluminum alloy sheets. They proposed that the ratio of notch tensile strength to tensile yield strength as a criterion for rating the notch sensitivities of metals.

In 1972, Kaufman [35] proposed using tensile tests on notched round bars for screening thick aluminum alloy plate, extrusions, and forgings for fracture toughness. Kaufman noted several advantages in using notched round bars. First, the notched

round bar tests were relatively inexpensive compared to E399 fracture toughness testing, requiring only one lathe setup. Second, the tests were conducted with the same equipment used for tensile testing. Third, the data could be readily recorded and analyzed by experienced tensile testing technicians. Kaufman stated that such tests would not replace the need for fracture toughness tests, but would provide the means for screening large sample populations in alloy development and quality control programs.

Kaufman examined the the effect of notch root radius on the notched tensile strength of 1 3/8-in. 2024-T351 aluminum plate. Note the size effect in Fig. 5.4—the larger diameter specimen has lower notched tensile strength. The size effect is due to the greater theoretical stress concentration factor ($k_t \approx \sqrt{r/\rho}$, where r is the radius of the notch neck and ρ is the notch root radius) for the larger specimen. Also note that for notch root radii $0.0002 \leq \rho \leq 0.0007$ in., there is a plateau where the notched tensile strength reaches a minimum. In the summary of his work, Kaufman describes the effect of misalignment on the measured notched tensile strength. He noted that small amounts (0.010 in.) of eccentricity significantly reduced the notched tensile strength.

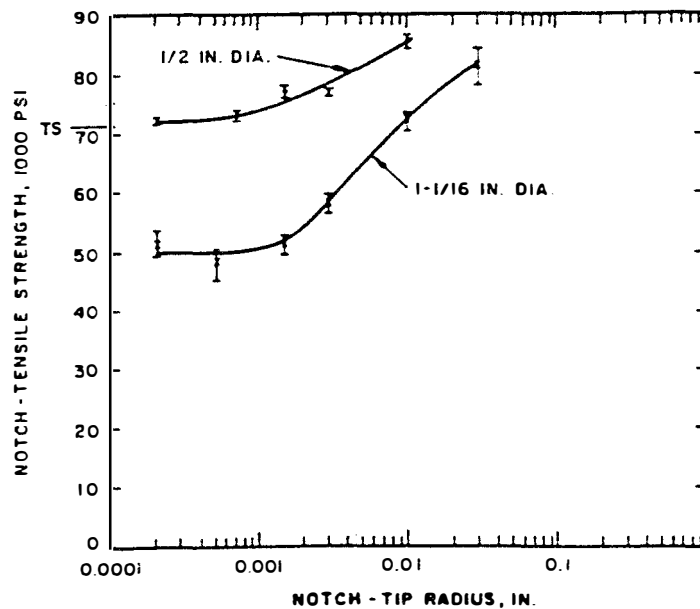


Figure 5.4: Effect of Notch Root Radius on Notch Tensile Strength [35]

In 1974, ASTM published E602 “Standard Method for Sharp-Notch Tension Testing with Cylindrical Specimens” [36]. The test method requires notch root radii less than or equal to 0.0007 in. and applies only to materials that can be machined consistently to such small radii. Aluminum and magnesium alloys are materials that have been tested with this method.

In the E602 test method, the sharp-notch strength of a notched round bar is calculated using the maximum load during the test and the original net cross-sectional area. The ratio of sharp-notch strength to yield strength has been correlated to plane strain fracture toughness when the ratio does not exceed 1.3. The ratio is not a quantitative measure of resistance to crack growth, but it may be useful as a screening test to compare materials or for quality control.

5.1.3 Fracture Approach to Failure

In 1965, during the early development of fracture toughness testing methodology, many different test specimen geometries, including the notched round bar, were critically examined by Brown and Srawley [19]. Tensile and fatigue properties had long been determined using notched round bars, and some of the earliest fracture toughness tests were naturally attempted with this geometry. The major difficulties with using notched round bars were the issue of how sharp a notch must be to simulate fatigue precrack and the lack of an accurate K_I calibration. These problems led Brown and Srawley to favor other specimen geometries.

5.2 Cracked Round Bars

In this section, the stress analysis of cracked round bars is presented. Solutions for K_I and J are given. The results are extended to include compliance C and η_{pl} for round bars. In addition, the fracture mechanics approach to failure of notched round bars will be discussed.

5.2.1 Stress Analysis

The elastic stress field near the crack tip in a cracked round bar is given by Eqs. 2.1 with the stress intensity factor reported in the handbook of Tada, Paris, and Irwin [1]

$$K_I = \tilde{\sigma} \sqrt{\pi(R-r)} f\left(\frac{r}{R}\right), \quad (5.16)$$

where $\tilde{\sigma}$ is the net section stress ($P/\pi r^2$), and $f(r/R)$ is given by

$$f\left(\frac{r}{R}\right) = \frac{1}{2}\sqrt{\frac{r}{R}} \left[1 + \frac{1}{2}\frac{r}{R} + \frac{3}{8}\left(\frac{r}{R}\right)^2 - 0.363\left(\frac{r}{R}\right)^3 + 0.731\left(\frac{r}{R}\right)^4 \right]. \quad (5.17)$$

The compliance (due to the introduction of a crack) $C = v/P$ for a cracked round bar can be found by equating Eqs. 2.10, and 2.11

$$\mathcal{G} = -\frac{dU}{dA} = \frac{K_I^2}{E'}. \quad (5.18)$$

This equation can be rewritten in terms of load and compliance since $U = 1/2 P^2 C$ as

$$\mathcal{G} = -\frac{1}{2}P^2 \frac{dC}{dA}, \quad (5.19)$$

where dA is the area of new crack surface created

$$dA = -2\pi r dr. \quad (5.20)$$

Combining the previous equations yields a nondimensional expression for the compliance derivative

$$\frac{d(RE'C)}{d(r/R)} = 4 \frac{1 - r/R}{(r/R)^3} \left[f\left(\frac{r}{R}\right) \right]^2. \quad (5.21)$$

There is no closed-form integral formula to determine the nondimensional compliance due to the crack; however, a symbolic mathematical analysis program, Maple V [37], gives a series solution as

$$\begin{aligned} RE'C_{crack} = & 0.066795 \left(\frac{r}{R}\right)^8 - 0.15215 \left(\frac{r}{R}\right)^7 + 0.20179 \left(\frac{r}{R}\right)^6 - 0.044254 \left(\frac{r}{R}\right)^5 \\ & + 0.19522 \left(\frac{r}{R}\right)^4 - 0.53021 \left(\frac{r}{R}\right)^3 + 0.6755 \left(\frac{r}{R}\right)^2 + \frac{R}{r} + C_{int} \end{aligned} \quad (5.22)$$

For $r/R = 1$, there is no crack and hence, the compliance due to the crack is zero. This condition leads to $C_{int} = -1.4127$.

The total compliance is equal to the sum of the compliance without the crack, $L/(\pi R(1 - \nu^2))$ and the compliance due to the crack

$$RE'C = \frac{L}{\pi R(1 - \nu^2)} + 0.066795 \left(\frac{r}{R}\right)^8 - 0.15215 \left(\frac{r}{R}\right)^7 + 0.20179 \left(\frac{r}{R}\right)^6$$

$$-0.044254 \left(\frac{r}{R}\right)^5 + 0.19522 \left(\frac{r}{R}\right)^4 - 0.53021 \left(\frac{r}{R}\right)^3 + 0.6755 \left(\frac{r}{R}\right)^2 + \frac{R}{r} - 1.4127. \quad (5.23)$$

where L is the gage length over which the total compliance is measured.

Rice, Paris and Merkle [38] estimated J_{pl} for a deeply cracked round bar in terms of the load P and the displacement at the loading point due to the presence of the crack v_{pl}

$$J_{pl} = \frac{1}{2\pi r^2} \left(3 \int_0^{v_{pl}} P dv_{pl} - P v_{pl} \right). \quad (5.24)$$

Beyond the analysis of Rice, Paris and Merkle, a power-law hardening material can be assumed. In this case, the load P as a function of plastic load-line displacement can be written as

$$P = A_n \left(\frac{v_{pl}}{r} \right)^{1/n}, \quad (5.25)$$

where A_n is a constant having the dimension of load. In this case, J_{pl} becomes

$$J_{pl} = \frac{1}{2\pi r^2} \frac{2n-1}{n+1} P v_{pl}. \quad (5.26)$$

For large n (perfect plasticity), J_{pl} is

$$J_{pl} = \frac{1}{\pi r^2} P v_{pl}. \quad (5.27)$$

Another way to express J_{pl} is in terms of η_{pl} and A_{pl} (the plastic area beneath the load versus load-line displacement curve). In this case, J_{pl} becomes

$$J_{pl} = \frac{\eta_{pl} A_{pl}}{\pi r^2}, \quad (5.28)$$

where it should be recalled that η_{pl} is a dimensionless parameter that relates the plastic work done on a specimen to crack growth resistance defined in terms of J_{pl} . For the cracked round bar, η_{pl} is

$$\eta_{pl} = 1 - \frac{1}{2n}. \quad (5.29)$$

The calculation of plastic displacement can be made by separating the total displacement measured during a test into elastic and plastic components

$$v_{pl} = v_{total} - v_{el}, \quad (5.30)$$

where v_{el} can be calculated using the previously derived compliance formula.

5.2.2 Fracture Approach to Failure

Brown [39] noted that only a small amount of K_{Ic} data had been generated by 1975 from fatigue precracked round bars because of the difficulty with maintaining concentric crack growth during fatigue cycling. Precracking was often done in reverse bending on a lathe. A second difficulty came in trying to determine when the crack had grown to the desired length for the toughness test.

In the late 1980s, Stark and Ibrahim [40, 41, 42] performed fracture toughness tests on small fatigue precracked round bars with bar radius R of 4.2 mm and a notch radius r of approximately 2 mm so that $r/R \approx 0.5$. Using the maximum load in the test to calculate K_{Ic} , their results were within 3% of 1T C(T) data for a high-strength aluminum alloy. They used finite element analyses to determine two size requirements for a valid K_{Ic} test. First, the fatigue crack must be at least twice r_y in depth. Second, the average axial stress across the final ligament at fracture must not exceed 2.5 times the yield strength so that general yielding does not precede fracture. Stark and Ibrahim did report that a significant number of their specimens had eccentric final ligaments. They analyzed the specimens in question to determine a bending correction to the stress intensity factor.

In 1995, Giovanola, et al. [43] used small precracked round bars to measure the fracture toughness of Ti-10V-2Fe-3Al, HY-130 steel, and A508 class 2A steel. These materials were selected because they have a wide range of fracture toughness to yield strength ratios and were previously well-characterized. The specimens had diameter of 16 mm and initial ligament (notch diameter before precracking) of 8 mm. Precracking was done on a lathe until the ratio of crack depth a to nominal specimen radius R varied from 0.56 to 0.78. Eccentricity increased with increasing crack size, varying from 0.10 to 0.75 mm. In an arrangement similar to a standard tensile test, crack opening was measured using a 25.4-mm gage length. The maximum load was used for calculating fracture toughness based on fractographic evidence and analysis of the load-displacement records.

For the Ti-10V-2Fe-3Al, K_{Ic} values obtained using cracked round bars were about 15% lower than determined using a conventional test specimen. For the embrittled HY-130, fractographic analysis revealed that initiation and propagation of the crack were cleavage driven. For the ductile HY-130, fractographic evidence revealed that crack initiation occurred at maximum load, followed by a rapid “quasi-stable” growth for a certain distance before complete instability. For the A508 material, the cracked round bars gave similar toughness values as conventional specimens. Crack eccentricity occurred in specimens of all three materials.

In contrast to using global stress field approaches based on K or J to describe the mechanics of fracture, the French research group F.M. Beremin [44]-[47] developed a local approach to the fracture mechanics of cracked round bars in the 1980s. Their local approach required precise elastic-plastic finite element analyses combined with a damage function to model void growth leading to ductile fracture. They measured ductile crack initiation, stable crack growth, and instability on fatigue precracked round bars. They simulated void growth using a model proposed by Rice and Tracey [48] in 1969. The model requires the determination of parameters which locally describe the change in size of a cavity as a function of hydrostatic stress, yield strength, and plastic strain. The local approach assumes that failure occurs when a critical void density is reached in a material. F.M. Beremin concluded that a local criterion can be used in more complex situations than global criteria based on K or J .

Chapter 6

Key Technological Advances

In this section, three key technological advances in fracture mechanics and plasticity theory are discussed. First, the relationship between fracture toughness obtained from notched specimens and cracked specimens was experimentally determined at Westinghouse Research Laboratories [49, 50] in the 1970s. Adapting this relationship to notched round bars will eliminate the need for expensive fatigue precracking. Second, the load separation method for calculating η -factors was developed by Paris, Ernst, and Turner [51] and Ernst, Paris, and Landes [52] in the early 1980s. This method was extended to relate load, gage displacement, and crack length in a convenient form by Landes and Herrera [53, 54] in the 1980s. Using this method will eliminate the need for crack length measurements as required by current J_{Ic} and J - R test methods. Third, the inclusion of hydrostatic stress in the yield function for metal plasticity theory was studied by Richmond [55, 56] and colleagues in the 1970s. Including the effects of hydrostatic stress on yielding will be important in the accurate computational modeling of the mechanical behavior of the notched round geometry.

These three advances are not widely known or used in the fracture mechanics and plasticity technical communities. However, they are integral parts of the research program discussed in this dissertation.

6.1 Relating Notches to Cracks

The relationship between the theoretical stress concentration factor k_t and stress intensity factor K was derived by Irwin [57] as

$$K_I = \lim_{\rho \rightarrow 0} \frac{\sqrt{\pi}}{2} \sigma_{max} \sqrt{\rho}, \quad (6.1)$$

where $\sigma_{max} = k_t \sigma_{nominal}$ and ρ is the notch root radius. This result provides a bridge between the mechanics of a notch and the mechanics of a crack.

The bridge between the material responses of a notch and a crack is more difficult to quantify. In 1973, Clark and Logsdon [49] tackled the problem of using notches in place of a fatigue precrack for the fracture toughness evaluation of a porcelain ceramic. They performed fracture toughness tests on notched single edge bend specimens using various notch radii. In addition, they performed fracture toughness tests using fatigue precracked single edge bend and compact tension specimens—achieving the same value for fracture toughness.

In 1977, Begley, Logsdon, and Landes [50] applied a similar procedure to determine J_{Ic} for ASTM A471 NiCrMoV rotor steel and 6061-T651 aluminum alloy using center notched panels (essentially the same geometry shown in Fig. 2.2) and compact tension specimens. In addition, they performed fracture toughness tests using fatigue precracked specimens and found excellent agreement with the extrapolated value of fracture toughness for a zero notch radius (Fig. 6.1) from the notched (no fatigue precrack) specimens. For the J - R curve slopes (Fig. 6.2), the notched specimen results were identical to the fatigue precracked specimen results.

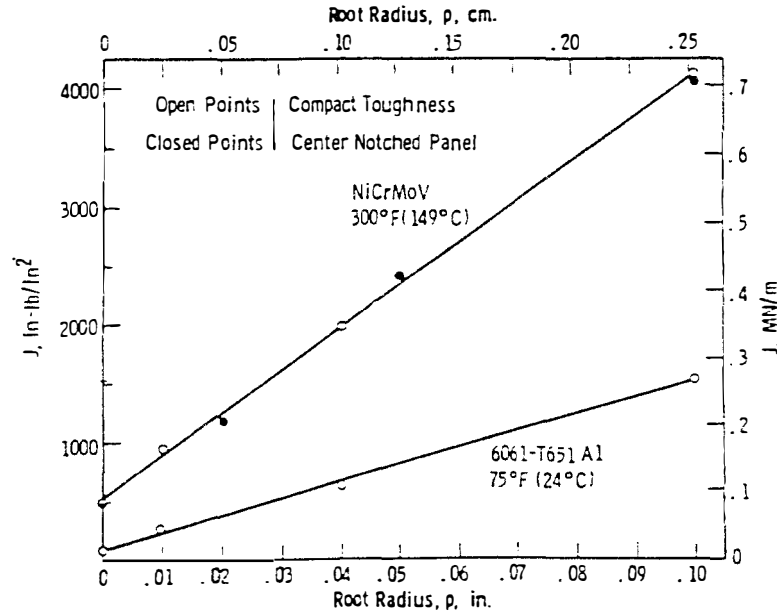


Figure 6.1: Critical J versus Notch Root Radius [50]

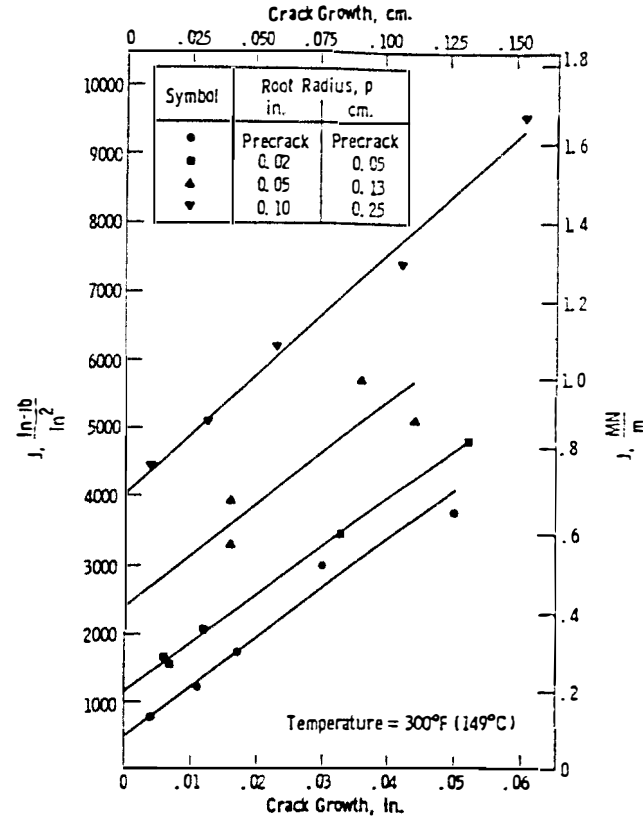


Figure 6.2: J - R Curves for A471 NiCrMoV Center Notched Panels [50]

More recently in 1996, Reiff and Ernst [58] determined that initiation values of CTOD could be calculated by extrapolating apparent CTOD values from notched Charpy specimens to a zero notch root radius. They used four different sizes of notch root radii and observed a linear increase in CTOD. By extrapolating to a zero notch root radius, they obtained CTOD values which equaled those obtained from fatigue precracked Charpy specimens.

6.2 Load Separation and Normalization Methods

Conventional J - R tests require the measurement of crack growth. To simplify J - R analysis and testing, the load separation and normalization methods were derived. The load separation method is based on the relationships developed by Ernst, et al. [51, 52], where load P , plastic displacement v_{pl} due to the crack, and crack length a are related using a separable product of two functions. Landes and Herrera [53, 54] developed the normalization method for determining a J - R curve from the load-

displacement curve without having to measure crack growth. The normalization method is based on load separation and was proved successful for many planar notch geometries, including the compact tension C(T) and the single edge bend SE(B) specimens used in fracture toughness testing, by Sharobeam and Landes [59].

In this research program, the analysis of Sharobeam and Landes will be extended from a planar specimen to the notched round bar geometry. If the load in a notched round bar can be separated into two multiplicative functions

$$P = G\left(\frac{r}{R}\right) H\left(\frac{v_{pl}}{r}\right), \quad (6.2)$$

where $v_{pl} = v - v_{el}$ is the plastic displacement due to the presence of a notch (or crack), and r is the notch radius, and R is the bar radius, then it has been shown that η_{pl} can be determined for use in Eq. 3.15. It should be noted that the function G has the dimensions of area, and the function H has the dimensions of stress.

Consider the load-displacement test records of two notched round bars with the same nominal radius R , but with different notch radii r_i and r_j . A parameter $S_{i,j}$ can be defined for a fixed plastic displacement v_{pl} as

$$S_{i,j} = \frac{P(r_i)}{P(r_j)} \Big|_{v_{pl}}, \quad (6.3)$$

and substituting Eq. 6.2 into Eq. 6.3 yields

$$S_{i,j} = \frac{G\left(\frac{r_i}{R}\right) H\left(\frac{v_{pl}}{r_i}\right)}{G\left(\frac{r_j}{R}\right) H\left(\frac{v_{pl}}{r_j}\right)}. \quad (6.4)$$

Power law expressions for G and H have been successfully used for several different notched specimen geometries by Landes and Herrera [53, 54] and Sharobeam and Landes [59]. In this work, assume that

$$G\left(\frac{r}{R}\right) = \pi R^2 \left(\frac{r}{R}\right)^m, \quad (6.5)$$

and that

$$H\left(\frac{v_{pl}}{r}\right) = A_n \left(\frac{v_{pl}}{r}\right)^{1/n}, \quad (6.6)$$

where m is a constant that depends on the specimen geometry, and n is the Ramberg-Osgood strain hardening exponent (Eq. 2.15). H can be expressed as a product of two functions

$$H\left(\frac{v_{pl}}{r}\right) = A_n \left(\frac{v_{pl}}{R}\right)^{1/n} \left(\frac{r}{R}\right)^{-1/n}. \quad (6.7)$$

Using Eqs. 6.5 and 6.7, Eq. 6.4 can be written as

$$S_{i,j} = \frac{\left(\frac{r_i}{R}\right)^{m-1/n}}{\left(\frac{r_j}{R}\right)^{m-1/n}}. \quad (6.8)$$

Eq. 6.8 implies that $S_{i,j}$ is a constant for fixed values of r_i and r_j and is not a function of v_{pl} . The constant $S_{i,j}$ is called a separation constant since it represents the condition where load can be represented by a separable product of functions.

If r_j is taken as a reference value of notch radius, then the separation constants $S_{i,j}$ for different r_i are given by

$$S_{i,j} = \alpha \left(\frac{r_i}{R}\right)^{m-1/n}, \quad (6.9)$$

where α is a constant. On a log-log graph of $S_{i,j}$ versus r_i/R , a straight line would indicate a power-law fit with slope (exponent) $m - 1/n$. If n is already determined for the material being used, then a value of m for the specimen geometry can be determined.

The load separation can be used to determine η_{pl} as follows. First, recalling that the energy rate interpretation of J_{pl} is

$$J_{pl} = -\frac{dU_{pl}}{dA}, \quad (6.10)$$

where U_{pl} is the plastic potential energy represented by the area A_{pl} under the load versus displacement curve. For the notched round bar, $dA = -2\pi r dr$ and J_{pl} becomes

$$J_{pl} = \frac{1}{2\pi r} \frac{dU_{pl}}{dr}. \quad (6.11)$$

in terms of η_{pl} , J_{pl} can be expressed as

$$J_{pl} = \frac{\eta_{pl} A_{pl}}{\pi r^2}, \quad (6.12)$$

where πr^2 is the area of the uncracked ligament.¹ Equating Eqs. 6.11 and 6.12 and solving for η_{pl} yields

¹Recall that for planar specimens the area of the uncracked ligament is bB , where b is the width of the ligament $W - a$ and B is the thickness of the specimen.

$$\eta_{pl} = \frac{r}{2} \frac{1}{A_{pl}} \frac{dU_{pl}}{dr}. \quad (6.13)$$

The plastic potential energy U_{pl} is the integral of the P - v_{pl} curve

$$U_{pl} = \int_0^{v_{pl}} P dv_{pl}. \quad (6.14)$$

For separable behavior, U_{pl} (or equivalently A_{pl}) can be written as

$$U_{pl} = \int_0^{v_{pl}} G\left(\frac{r}{R}\right) H\left(\frac{v_{pl}}{r}\right) dv_{pl}, \quad (6.15)$$

or

$$U_{pl} = G\left(\frac{r}{R}\right) \int_0^{v_{pl}} H\left(\frac{v_{pl}}{r}\right) dv_{pl}. \quad (6.16)$$

Substituting Eq. 6.7 into Eq. 6.16 yields

$$U_{pl} = \pi R^2 \left(\frac{r}{R}\right)^m \left(\frac{r}{R}\right)^{-1/n} \int_0^{v_{pl}} A_n \left(\frac{v_{pl}}{R}\right)^{1/n} dv_{pl}. \quad (6.17)$$

For convenience, define a new function $\tilde{G}$ as

$$\tilde{G}\left(\frac{r}{R}\right) = \pi R^2 \left(\frac{r}{R}\right)^{m-1/n}, \quad (6.18)$$

Substituting $\tilde{G}$ into Eq. 6.17 yields

$$U_{pl} = \tilde{G}\left(\frac{r}{R}\right) \int_0^{v_{pl}} A_n \left(\frac{v_{pl}}{R}\right)^{1/n} dv_{pl}. \quad (6.19)$$

and taking the derivative of U_{pl} in Eq. 6.19 with respect to r yields

$$\frac{dU_{pl}}{dr} = \frac{1}{R} \frac{d\tilde{G}\left(\frac{r}{R}\right)}{d\left(\frac{r}{R}\right)} \int_0^{v_{pl}} A_n \left(\frac{v_{pl}}{R}\right)^{1/n} dv_{pl}. \quad (6.20)$$

Substituting Eq. 6.20 into Eq. 6.13 yields

$$\eta_{pl} = \frac{1}{2} \frac{r}{R} \frac{1}{\tilde{G}\left(\frac{r}{R}\right)} \frac{d\tilde{G}\left(\frac{r}{R}\right)}{d\left(\frac{r}{R}\right)}. \quad (6.21)$$

The derivative of $\tilde{G}$ with respect to r/R is

$$\frac{d\tilde{G}\left(\frac{r}{R}\right)}{d\left(\frac{r}{R}\right)} = \left(m - \frac{1}{n}\right) \left(\frac{r}{R}\right)^{-1} \tilde{G}\left(\frac{r}{R}\right). \quad (6.22)$$

Substituting Eqs. 6.18 and 6.22 into Eq. 6.21 yields

$$\eta_{pl} = \frac{1}{2} \left(m - \frac{1}{n} \right). \quad (6.23)$$

Referring back to Eq. 6.9, it is seen that η_{pl} is one-half of the slope of the line given by the log-log plot of $S_{i,j}$ versus r_i/R . In Eq. 6.23, η_{pl} has its dependence on specimen geometry through m and its dependence on material hardening through n .

To this point in the discussion, the focus has been on applying load separation to characterize J_{pl} for a stationary crack. Recall that the load-displacement P - v curve for a growing crack (shown in Fig. 3.13 in the discussion of the E1152 J - R test method) actually has a combination of plasticity and crack growth contained in it. A series of P - v curves for constant crack lengths is shown in Fig. 6.3. The intersections of the constant crack length P - v curves with the actual P - v curve of a growing crack denote the actual crack lengths during the loading process.

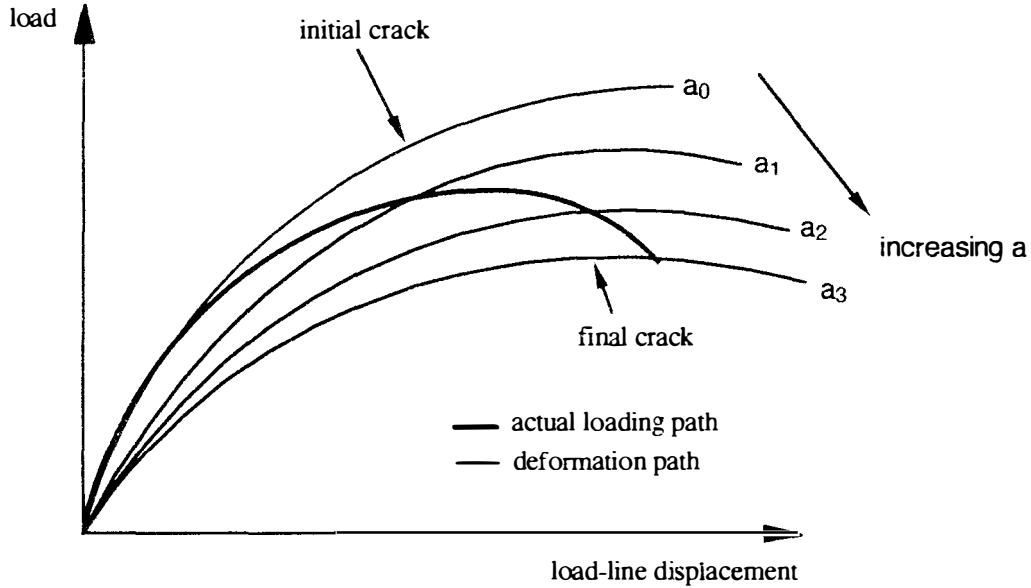


Figure 6.3: Load-Displacement Curves for Growing and Stationary Cracks

The normalization method is an extension of the load separation method which allows the crack lengths during the loading process to be determined indirectly. The relationship between load P , displacement v , and crack length a can be determined

using the load separation functions G , H , and the compliance function C . If any two of the three parameters (P , v , and a) are known, then the third parameter can be determined. For the notched round bar, the crack length a is $R - r$. In the derivation that follows, the normalized notch neck radius r/R will be used instead of using a because G and C are conveniently written as functions of r/R .

The first step in the normalization method is to write the displacement v as the sum of its elastic and plastic parts

$$v = v_{el} + v_{pl}. \quad (6.24)$$

The elastic displacement can be written in terms of normalized compliance $RE'C$ (given by Eq. 5.23) as

$$v_{el} = RE'C \left(\frac{r}{R} \right) \frac{P}{RE'}. \quad (6.25)$$

The plastic displacement can be found by rewriting Eq. 6.2 in terms of a normalized load P_N

$$P_N = \frac{P}{\tilde{G} \left(\frac{r}{R} \right)} = A_n \left(\frac{v_{pl}}{R} \right)^{1/n}, \quad (6.26)$$

where P_N has the dimensions of stress since P has the dimension of load and $\tilde{G}$ has the dimension of area. Recall that the constant A_n has the dimension of stress.

Solving Eq. 6.26 for v_{pl} yields

$$v_{pl} = R \left(\frac{P_N}{A_n} \right)^n = R \left(\frac{P}{A_n \tilde{G} \left(\frac{r}{R} \right)} \right)^n \quad (6.27)$$

Substituting Eq. 6.18 into the above equation gives

$$v_{pl} = R \left(\frac{P}{A_n \pi R^2 \left(\frac{r}{R} \right)^{m-1/n}} \right)^n. \quad (6.28)$$

Finally, Eqs. 6.25 and 6.28 into Eq. 6.24 gives

$$v = RE'C \left(\frac{r}{R} \right) \frac{P}{RE'} + R \left(\frac{P}{A_n \pi R^2 \left(\frac{r}{R} \right)^{m-1/n}} \right)^n. \quad (6.29)$$

or in terms of η_{pl} ,

$$v = RE'C \left(\frac{r}{R} \right) \frac{P}{RE'} + R \left(\frac{P}{A_n \pi R^2 \left(\frac{r}{R} \right)^{2\eta_{pl}}} \right)^n. \quad (6.30)$$

Thus, a single equation relating P , v , and r/R is obtained. Once the constants A_n , n , and η_{pl} are determined, Eq. 6.30 can be used to determine the missing parameter when any two of three parameters (P , v , and r/R) are known. From the load-displacement curve, P and v are known and r/R is known initially. After crack growth occurs, r/R can be determined using Eq. 6.30.

6.3 Hydrostatic Stress Effects in Metal Plasticity

Classical metal plasticity assumes that hydrostatic pressure has a negligible effect on the yield strength and postyield behavior of metals. In the 1940s, Bridgman [60, 61] conducted a series of experiments on smooth (unnotched) tensile bars for a variety of materials, including many common metals. Under the conditions of hydrostatic pressures up to 3100 MPa (450 ksi), he found no significant effect on the yield point until the higher pressures were reached. He found that the major effect of hydrostatic pressure was increased ductility. In other words, much larger strains before fracture were obtained when hydrostatic pressure was applied as a boundary condition to a tensile test. Additionally, Bridgman found that the material volume in the gage section did not change for very large plastic strain changes. Therefore, a metal was assumed to have incompressible plastic strains. These two experimental observations about metals—no influence of hydrostatic pressure on yielding and incompressibility for plastic strain changes—are two of the basic tenets of classical metal plasticity.

Generations of material scientists and engineers have studied classical metal plasticity based on Bridgman's two observations. Plasticity textbooks by Hill [62] in 1950 and Mendelson [63] in 1968 are examples of classical plasticity. Even modern treatments on plasticity, such as Lubliner [64] in 1990 and Stouffer and Dame [65] in 1996 assume that there is no hydrostatic pressure dependence for yield and incompressibility as fundamental assumptions for metal plasticity. These assumptions are also the basis for metal plasticity in commercial finite element programs. For example, the theory manual of ABAQUS [66] directs potential users to use a yield criterion for metals that has no dependence on hydrostatic stress.

It is important to examine hydrostatic stress and incompressibility in mathematical terms. To simplify the discussion, assume that the principal stresses are given by σ_1 , σ_2 , and σ_3 . The principal stresses are physical quantities that do not de-

pend on the coordinate system chosen by the observer. In determining the principal stresses, a cubic equation must be solved

$$\sigma^3 - I_1\sigma^2 - I_2\sigma - I_3 = 0, \quad (6.31)$$

where σ is a principal stress and I_1 , I_2 , and I_3 are functions of the principal stresses that are invariant—they do not change with coordinate system.

In terms of a cartesian coordinate system x - y - z , the stress invariants are

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}, \quad (6.32)$$

$$I_2 = \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 - (\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx}), \quad (6.33)$$

$$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\tau_{xy}\tau_{yz}\tau_{zx} - (\sigma_{xx}\tau_{yz}^2 + \sigma_{yy}\tau_{zx}^2 + \sigma_{zz}\tau_{xy}^2). \quad (6.34)$$

The hydrostatic or mean stress σ_m is defined as

$$\sigma_m = \frac{1}{3}I_1. \quad (6.35)$$

and the hydrostatic pressure is $p = -1/3 I_1$.

Bridgman's first observation that the hydrostatic pressure had no effect on the yield behavior of metals until very large hydrostatic pressures led engineers to develop a plasticity theory that subtracts the mean stress from the principal stresses. The resulting stresses are called the deviatoric stresses S_1 , S_2 , and S_3 , and are written as

$$S_1 = \sigma_1 - \frac{1}{3}I_1, \quad (6.36)$$

$$S_2 = \sigma_2 - \frac{1}{3}I_1, \quad (6.37)$$

$$S_3 = \sigma_3 - \frac{1}{3}I_1 \quad (6.38)$$

The deviatoric stress invariants are the coefficients J_1 , J_2 , and J_3 of the equation

$$S^3 - J_1S^2 - J_2S - J_3 = 0, \quad (6.39)$$

where the invariants are functions of the principal stresses and the mean stress

$$J_1 = 0, \quad (6.40)$$

$$J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2], \quad (6.41)$$

$$J_3 = (\sigma_1 - \sigma_m)(\sigma_2 - \sigma_m)(\sigma_3 - \sigma_m). \quad (6.42)$$

The deviatoric stress invariants can also be written in terms of the deviatoric stresses

$$J_2 = \frac{1}{2} (S_1^2 + S_2^2 + S_3^2), \quad (6.43)$$

$$J_3 = S_1 S_2 S_3. \quad (6.44)$$

For classical metal plasticity, a yield function is a function $f = f(\sigma_1, \sigma_2, \sigma_3)$ such that when $f < 0$ the material behavior is elastic. For $f = 0$, the material behavior is plastic and yielding occurs. Assuming that yield is independent of the hydrostatic stress leads to a yield function $f = f(J_2, J_3)$. The von Mises yield function is commonly used for metal plasticity. It states that yield depends only on J_2

$$\sqrt{J_2} = k, \quad (6.45)$$

where k is the yield strength in pure shear. For hardening materials, k is a function of plastic strain. The square root of $3J_2$ is commonly referred to as the von Mises or effective stress and can be written in terms of the principal stresses

$$\sigma_{eff} = \sqrt{3J_2} = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}. \quad (6.46)$$

If the yield strength in pure tension $\sigma_0 = \sqrt{3} k$ is used, Eq. 6.45 can be written as

$$\sigma_{eff} = \sigma_0. \quad (6.47)$$

Squaring both sides of Eq. 6.45 leads to

$$J_2 = k^2. \quad (6.48)$$

This equation can be interpreted as a surface in the principal stress space (a coordinate system with axes given by principal stresses σ_1 , σ_2 , and σ_3). Such a surface is known as a yield surface. For the von Mises yield function, the yield surface is a circular cylinder of radius k whose axis is defined in the direction of the hydrostatic stress. Any cross-section taken perpendicular to the cylinder axis will be a circle of radius k . A von Mises yield surface is shown in Fig. 6.4.

The intersection of the yield cylinder with any plane perpendicular to the cylinder axis in principal stress space is a curve known as a yield locus. For the von Mises yield surface, every yield locus is a circle.

The discussion thus far has focused on the initial yield surface, where a material first starts to yield. Many materials exhibit strain hardening, a process where the

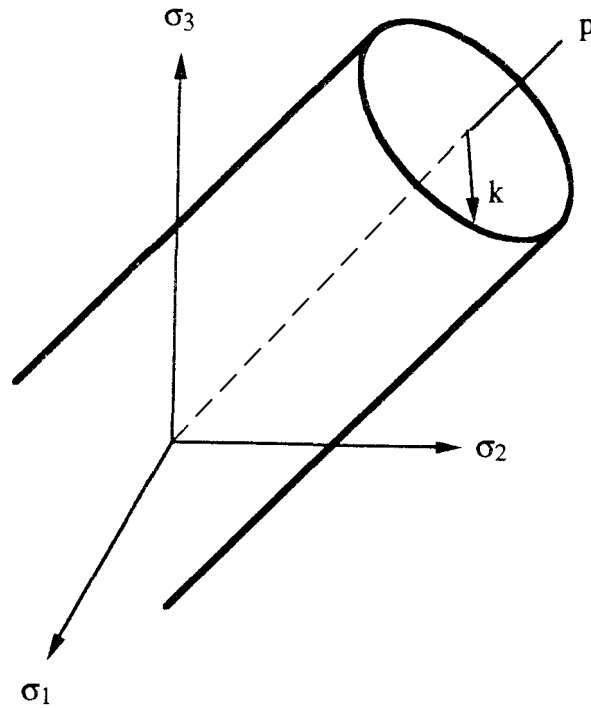


Figure 6.4: von Mises Yield Surface

yield surface changes shape or location or both as the material is plastically deformed. For many metals, isotropic hardening is an appropriate approximation. Essentially, isotropic hardening means that the yield surface expands equally in all directions. For the von Mises yield function, the radius k of the yield surface grows larger.

Isotropic hardening implies that the tensile and compressive yield behaviors are the same. While this is approximately true for some materials, it is not true in general. Some materials exhibit different magnitudes of yield strength for tension than for compression. This effect is called the Bauschinger effect and becomes important whenever stresses are to be predicted after a reversal of loading. Kinematic hardening is often used to describe materials with a pronounced Bauschinger effect. For kinematic hardening, the new yield surface is assumed to have the same radius k as the initial yield surface. However, the axis of the yield surface shifts in principal stress space.

Different metals have differing amounts of Bauschinger effect. Purely isotropic hardening represents an extreme of no Bauschinger effect. Kinematic hardening represents the other extreme. However, a linear combination of the two models

is useful in describing real materials. In practice, isotropic hardening is easier to implement and is more often used.

The second basic tenet of classical metal plasticity is incompressibility. It was observed that volume change during plastic deformation is nearly elastic. In terms of principal strain increments $d\epsilon_1$, $d\epsilon_2$, and $d\epsilon_3$, the sum of the plastic strain increments $d\epsilon_{ii}^{pl}$ (or plastic dilation rate) must be zero. Mathematically, this condition is

$$d\epsilon_{ii}^{pl} = d\epsilon_1^{pl} + d\epsilon_2^{pl} + d\epsilon_3^{pl} = 0. \quad (6.49)$$

The relationship between plastic strain increments and the yield condition is given by an associated flow rule. Thus, a flow rule governs the postyield behavior of a material. The general form of an associated flow rule is

$$d\epsilon_i^{pl} = d\lambda \frac{\partial f}{\partial \sigma_i}, \quad (6.50)$$

where f is the yield function and $d\lambda$ is a positive scalar. This type of flow rule is also called a normality flow rule because it assumes that the strain increment is normal to the yield surface. Drucker and Prager [67] showed that the plastic dilation rate $d\epsilon_{ii}^{pl}$ can be summed from Eq. 6.50 to obtain

$$d\epsilon_{ii}^{pl} = 3d\lambda \frac{\partial f}{\partial I_1}, \quad (6.51)$$

where I_1 is the first stress invariant. Recalling that the hydrostatic stress is $1/3 I_1$, it can be seen that if the yield function f does not depend on hydrostatic stress, then the plastic dilation rate $d\epsilon_{ii}^{pl}$ must be zero. This is how the two observations of Bridgman and the two tenets of classical plasticity based on them are related.

Although classical metal plasticity has a great deal of “inertia” in engineering practice, the basic tenets were challenged in the 1970s by the experiments of Richmond, Spitzig, and Sober [55, 56]. They studied the effects of hydrostatic pressure up to 1100 MPa (160 ksi) on the yield strength of four steels (4330, 4310, maraging steel, and HY80). These pressure levels were significantly less than those used by Bridgman years earlier.

Richmond found that the yield strength was a linear function of hydrostatic pressure. For high-strength steels, Richmond found that a yield function identical to one proposed by Drucker and Prager [67] for soils described the yielding process. The Drucker-Prager yield function is

$$\sigma_{eff} = \sigma_0(\epsilon_{pl}) + a I_1, \quad (6.52)$$

where a is a material constant related to the theoretical cohesive strength of the material σ_c . The theoretical cohesive strength is the stress required to overcome cohesive forces between neighboring atoms. The cohesive strength is usually expressed as a fraction of Young's modulus E . Dieter [68] gives the range of theoretical cohesive strength for metals as $E/15$ to $E/4$ with a typical value of $E/5.5$. As with the von Mises yield function, σ_0 the yield strength for pure tension and is a function of plastic strain ϵ_{pl} .

Graphically, a can be interpreted as the slope of the graph of σ_{eff} versus I_1 as shown in Fig. 6.5. The value of σ_{eff} for $I_1 = \sigma_{uniaxial}/3$ corresponds to the typically reported yield strength for a tensile test. The value of I_1 for $\sigma_{eff} = 0$ corresponds to the theoretical cohesive strength of the material. For the 4330 steel shown in Fig. 6.6, the yield strength was 1475 MPa and the cohesive strength was 59000 MPa. For $E = 200000$ MPa, the theoretical cohesive strength was approximately $0.3E$.

The ratio a/σ_0 was found to be nearly constant for five steels and single crystal iron. This suggests that a/σ_0 is a property of the bulk iron lattice, just as the elastic constants E and ν are properties of the lattice structure.

In terms of a yield surface, the Drucker-Prager yield function is a cone whose apex is at a hydrostatic stress equal to the cohesive strength. This is shown schematically in Fig. 6.7. For small to moderate amounts of hydrostatic stress, this cone would locally be approximated by a cylinder. For this situation, the von Mises yield function would give comparable results to the Drucker-Prager yield function. However, for high levels of hydrostatic stress, the Drucker-Prager yield function is preferable to von Mises.

For sharply notched or cracked geometries, the development of large tensile hydrostatic stresses is well-known. The use of the Drucker-Prager yield function should result in more accurate modeling and simulation of the mechanics of notches and cracks. However, the Drucker-Prager yield function is rarely used for metal plasticity.

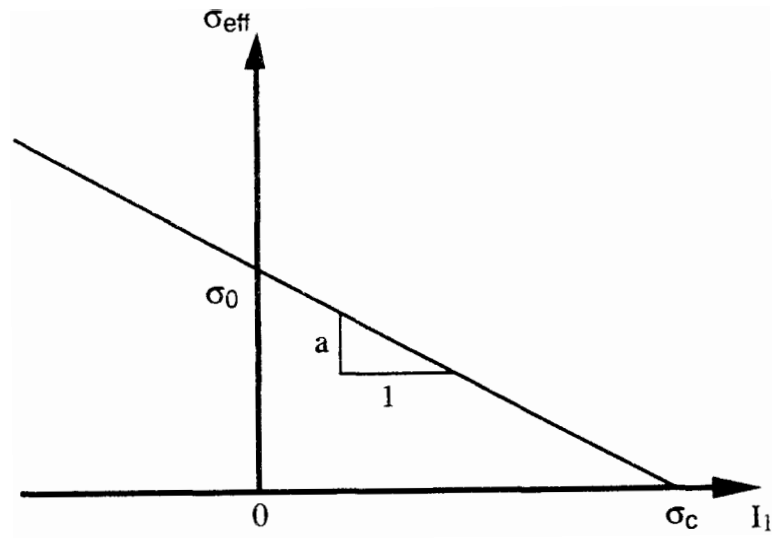


Figure 6.5: Schematic of σ_{eff} versus I_1

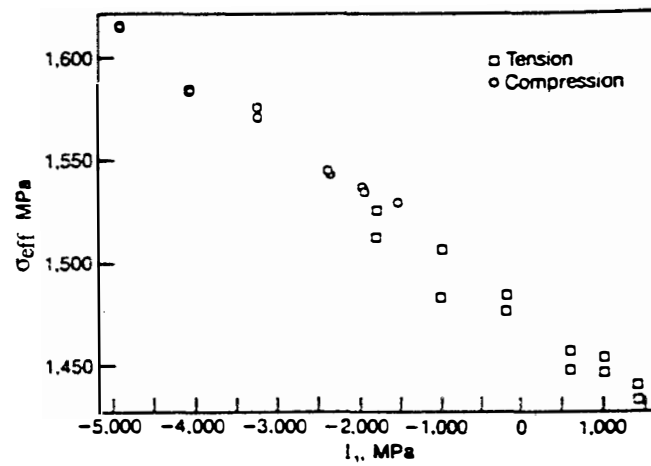


Figure 6.6: σ_{eff} versus I_1 for 4330 Steel [55]

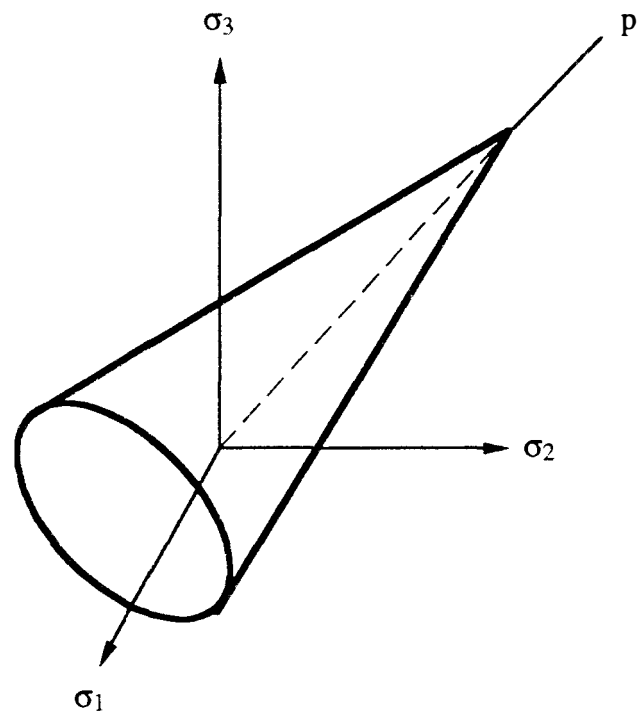


Figure 6.7: Drucker-Prager Yield Surface

Chapter 7

Research Plan

A combined experimental and analytical research program was designed to study the suitability of the notched round bar (NRB) for fracture toughness testing. Four materials with a wide range of mechanical behavior were tested: 2024-T351 aluminum alloy, overaged 2024 aluminum alloy, a modified ferritic A302 grade B pressure vessel steel, and nylon 6/6. The aluminum alloy and the nylon were tested at room temperature, while the A302B steel was tested at -67°C (-88.6°F). Tensile tests were conducted for each material with the resulting engineering stress-engineering strain curves shown in Fig. 7.1. A summary of tensile properties is given in Table 7.1.

Table 7.1: Tensile Properties for Four Materials

	2024-T351	Overaged 2024	Mod. A302B	Nylon 6/6
E psi	10.4×10^6	10.4×10^6	29.5×10^6	0.29×10^6
ν	0.33	0.33	0.3	0.4
σ_{ys} ksi	56	26	91	5.1
σ_{uts} ksi	69	45	110	6.5
n	15	5	8	3

7.1 Experimental Program

The NRB specimen details are shown in Fig. 7.2. The aluminum alloys and the pressure vessel steel specimens had a nominal radius R of 0.25 in. and a neck radius r of 0.125 in., thus $r/R = 0.5$. The nylon specimens had a slightly different design

with a nominal radius R of 0.1875 in. and a neck radius r of 0.09375 in. All specimens had a notch flank angle of 45° . All notch root radii ρ had a tolerance of ± 0.001 in.

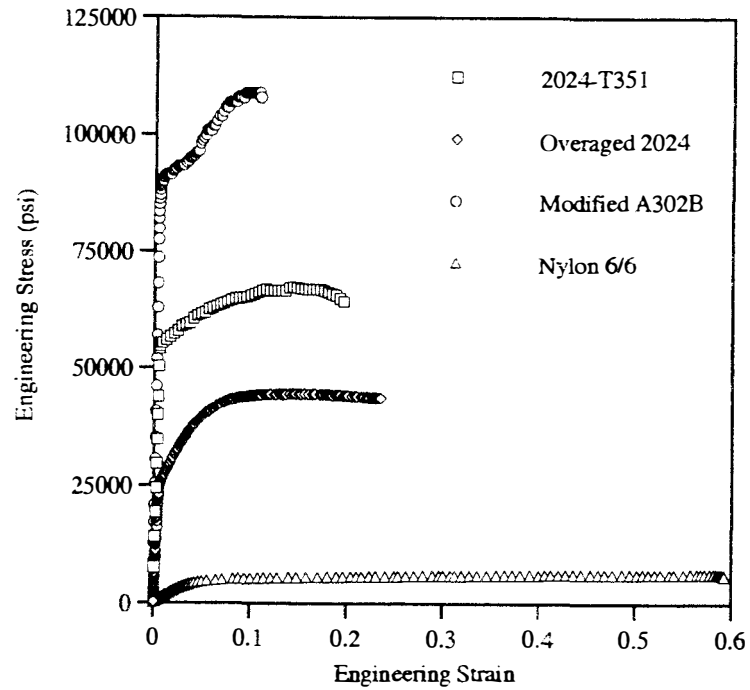


Figure 7.1: Engineering Stress-Engineering Strain Curves for Four Materials

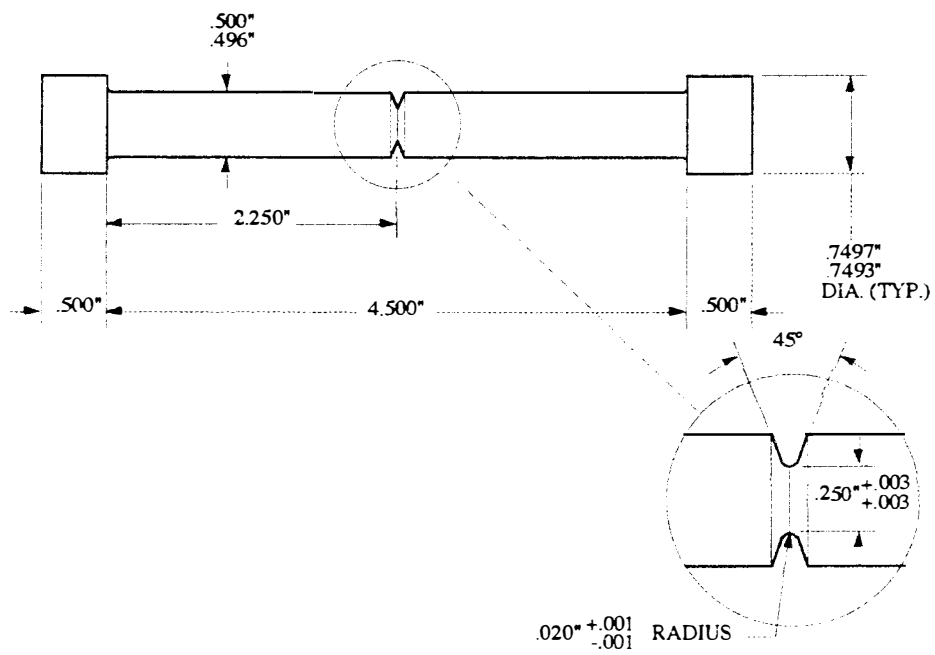


Figure 7.2: Engineering Drawing of the Notched Round Bar Specimen

To maintain proper alignment, buttonhead specimen ends were used in place of threaded ends. A mechanical gripping apparatus used in low-cycle fatigue testing was used with the buttonhead ends with a percent bending stress PBS of less than 5% up to 30 ksi, where PBS is defined as

$$PBS = \frac{\Delta\sigma_{max}}{\sigma_{nominal}} \times 100, \quad (7.1)$$

where $\Delta\sigma_{max}$ is the difference between the maximum outer fiber stress and the nominal stress $\sigma_{nominal}$ in the specimen. An alignment straight (unnotched) specimen with a set of four strain gages was used to determine the PBS in the method prescribed by the E602 test method for sharp-notch tensile testing [36]. E602 allows no more than 10% bending stress for a nominal tensile stress of 30 ksi.

All tests were conducted on screw-driven universal testing machines with constant crosshead motion. An extensometer with a gage length of 0.4 in. and a 10% extension range was used in all tests. Load and gage displacement data were recorded with a X-Y plotter and digitized afterward.

A total of four test series were conducted. K_{Ic} testing was performed on the 2024-T351 aluminum alloy with 1T-C(T) and NRB specimens. J_{Ic} and J - R curve testing was performed on the overaged 2024 aluminum alloy using both 1/2T-C(T) and NRB specimens. K_{Jc} testing was performed on the A302B pressure vessel steel using NRB specimens. McCabe and Swain [69] at the Oak Ridge National Laboratory (ORNL) performed J_{Ic} and J - R curve testing on the A302B steel using 1T-C(T) specimens. J_{Ic} and J - R curve testing was performed on the nylon 6/6. Zhou, Landes, and Huang [70] previously performed J_{Ic} and J - R curve testing on the nylon 6/6 using SE(B) specimens.

The 2024-T351 aluminum specimens were machined from a one-inch thick plate. Standard K_{Ic} fracture toughness tests were conducted using fatigue precracked 1T-C(T) specimens. In addition, nonstandard blunt-notch 1T-C(T) specimens with varying notch root radii ρ were tested. NRB specimens with nominal radius R of 0.25 in. and neck radius r of 0.125 in. ($r/R = 0.5$) with varying notch root radii were tested. The notch root radii ρ chosen were 0.005, 0.010, and 0.020 in. The test matrix is given in Table 7.2.

Standard J_{Ic} and J - R curve fracture toughness tests were conducted using fatigue precracked 1/2T-C(T) specimens. In addition, nonstandard blunt-notch 1/2T-C(T) specimens with varying notch root radii ρ were tested. NRB specimens with nominal radius R of 0.25 in. and notch neck radius r of 0.125 in. ($r/R = 0.5$) with

varying notch root radii were tested. The notch root radii ρ chosen were 0.005, 0.010, and 0.020 in. The test matrix is given in Table 7.3.

Transition fracture toughness testing of a ferritic steel, modified A302B, was performed using the notched round bar geometry. NRB specimens with a nominal radius R of 0.25 in. and notch neck radius r of 0.125 in. ($r/R = 0.5$) with varying notch root radii were tested at -67°C (-88.6°F). The notch root radii ρ chosen were 0.005, 0.010, and 0.020 in. The test matrix is given in Table 7.4. The results of this test series were compared to the unpublished results of McCabe and Swain of ORNL [69].

Fracture toughness testing of a polymeric material, nylon 6/6, was performed using notched round bars. NRB specimens with nominal radius R of 0.1775 in. and small neck radius r of 0.089 in. ($r/R = 0.5$) with varying notch root radii were tested. The notch root radii ρ chosen were 0.005 and 0.010 in. All tests were conducted using the same constant stroke rate (0.01 in/min) as used in the tensile tests. The test matrix is given in Table 7.5. The results of this test series were compared to the results of Zhou, Landes, and Huang [70].

Table 7.2: K_{Ic} Test Matrix for 2024-T351 Aluminum Alloy

Geometry	ρ in.	Number of Specimens
C(T)	Precrack	2
C(T)	0.005	3
C(T)	0.010	2
C(T)	0.020	2
NRB	0.005	2
NRB	0.010	2
NRB	0.020	2

Table 7.3: J_{Ic} and J - R Test Matrix for Overaged 2024 Aluminum Alloy

Geometry	ρ in.	Number of Specimens
C(T)	Precrack	3
C(T)	0.005	2
C(T)	0.010	2
C(T)	0.020	2
NRB	0.005	2
NRB	0.010	3
NRB	0.020	3

Table 7.4: K_{Jc} Test Matrix for A302B Pressure Vessel Steel

Geometry	ρ in.	Number of Specimens
NRB	0.005	3
NRB	0.010	3
NRB	0.020	3

Table 7.5: J_{Ic} and J - R Test Matrix for Nylon 6/6

Geometry	ρ in.	Number of Specimens
NRB	0.005	4
NRB	0.010	4

7.2 Analytical Program

In conjunction with the experimental program, an analytical program was conducted. Nonlinear finite element analyses were used to model the load-displacement response of both aluminum alloys tested. Finite element analyses were not conducted for the modified A302B or the nylon 6/6; however, the η_{pl} resulting from the finite element analyses of the two aluminum alloys were generalized as a function of hardening exponent n . In addition to modeling the $r/R = 0.5$ specimen geometry for $\rho = 0.005, 0.010$, and 0.020 in., finite element models were constructed for $r/R = 0.4, 0.45, 0.55$, and 0.6 . The load separation method was applied to the resulting $P-v$ curves to determine η_{pl} for the the NRB geometry. Once η_{pl} was determined using Eq. 6.23, the experimental $P-v$ data was reduced and fracture toughness was determined by the extrapolation of apparent fracture toughness versus ρ for J -based analysis or $\sqrt{\rho}$ for K -based analysis.

The Sandia National Laboratory computer program FASTQ [71] was used for preprocessing of meshes and boundary conditions. A commercial finite element program, ABAQUS [66], was used for the finite element analyses and postprocessing of results.

Axisymmetric models of notched round bars were developed with FASTQ in three levels of mesh refinement in the notch region. A schematic showing the region of the notched round bar actually modeled is given in Fig. 7.3.

The Q4 element type was used for all meshes. A coarse mesh typically had 250 elements in the notch region. Medium and fine meshes typically had 500 and 1000 elements, respectively, in the notch region. An example of a coarse mesh is given in Fig. 7.4.

The FASTQ results were translated into an ABAQUS input file and the boundary conditions and job control commands were added using a text editor. Large strain analysis and reduced integration options were used. The nonlinear material response was modeled using isotropic hardening with a von Mises yield criterion, and again with a Drucker-Prager yield criterion. The actual true stress versus plastic strain data was used as input for the hardening curve.

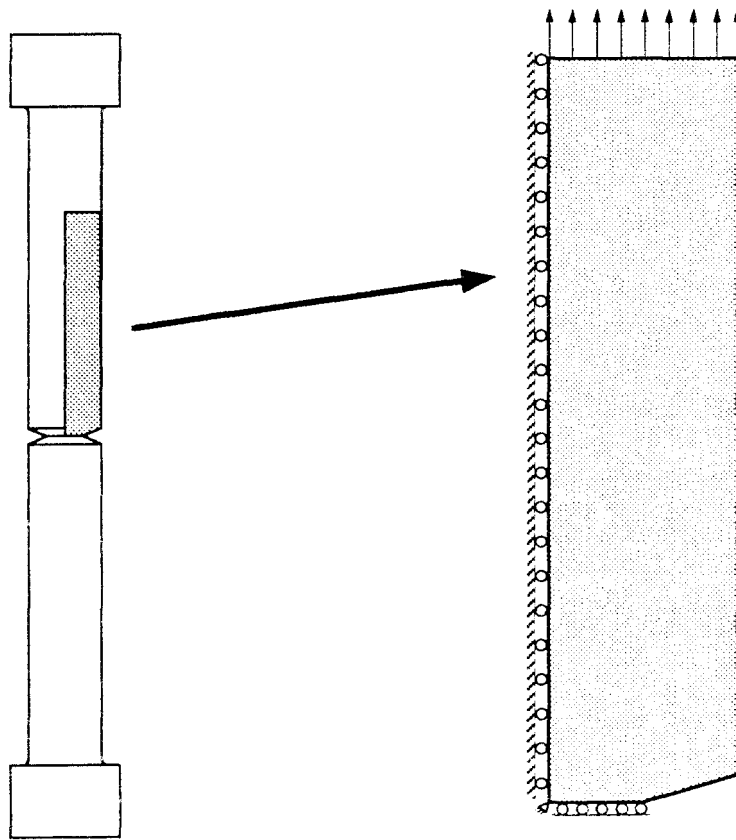


Figure 7.3: Schematic of Axisymmetric Model of a Notched Round Bar

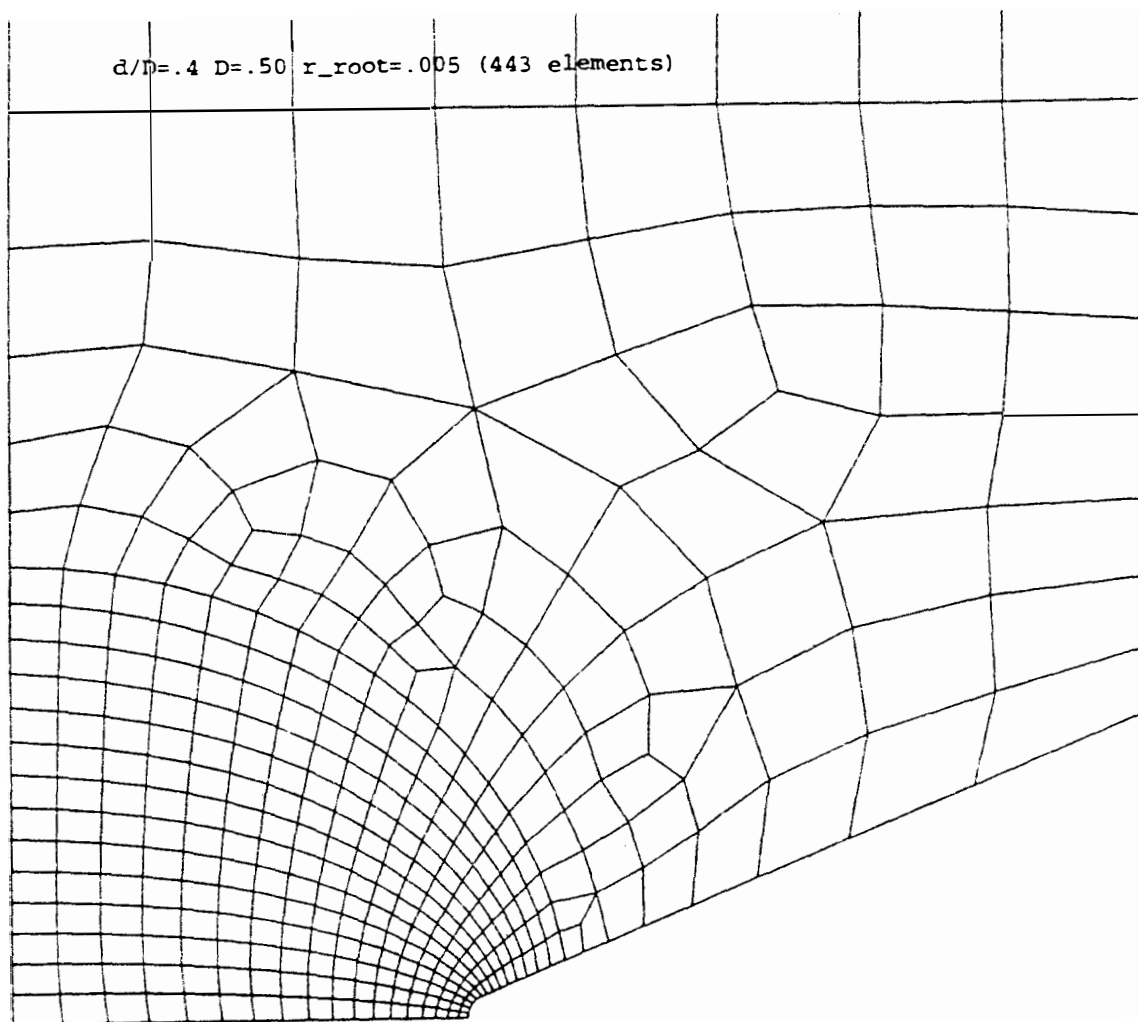


Figure 7.4: Coarse Mesh in the Notch Region

Chapter 8

Results

8.1 K_{Ic} Testing of 2024-T351 Aluminum Alloy

The experimental NRB load-displacement P - v curves for 2024-T351 are shown in Figs. 8.1–8.3. For comparison, a representative P - v curve for each notch root radius ρ is shown in Fig. 8.4. The P - v records for all three values of ρ seem to follow a common curve with the smallest ρ failing at a lower load. It was observed that all specimens failed before the load reached an instability point predicted by $dP = 0$.

Also shown in Figs. 8.1–8.3 are finite element results using a von Mises yield function and a Drucker-Prager yield function. The finite element results show that the von Mises yield function consistently overpredicts the actual load-displacement response of the NRB specimens. Failure loads were overpredicted by approximately 10% for all notch root radii.

Finite element results with the Drucker-Prager yield function were iteratively determined. Recall that the Drucker-Prager yield function is given by

$$\sigma_{eff} = \sigma_0 - aI_1, \quad (8.1)$$

where a is a constant to be determined for each material considered. The constant a is the slope of the graph of σ_{eff} versus I_1 . Using an initial yield strength σ_0 of 55 ksi (from tensile test data) and guessing a theoretical cohesive strength $\sigma_c = 0.18E$ lead to a reasonable matching of both 2024-T351 tensile test data and NRB test data. After a couple of iterations, the final value of σ_c used in the analyses was 0.176 E. This is equivalent to using $a = -.03$ in the Drucker-Prager yield function. The resulting P - v curves are shown in Figs. 8.1–8.3 for notched round bars and in Fig. 8.5 for a smooth tensile bar.

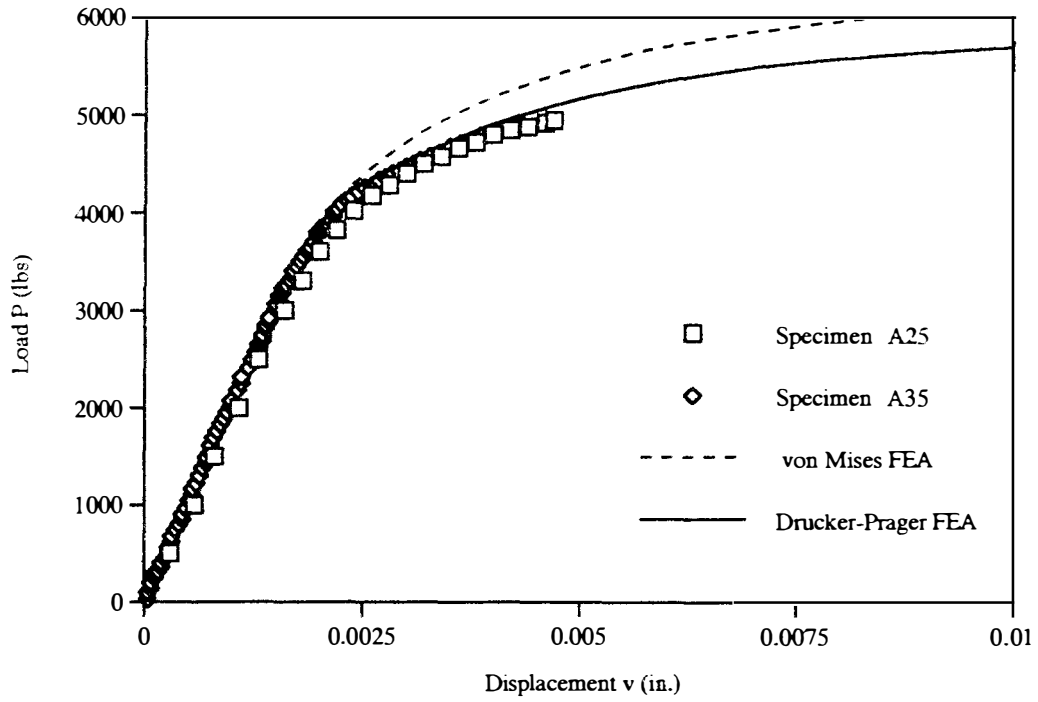


Figure 8.1: NRB Load-Displacement Results ($\rho = 0.005$ in.) for 2024-T351

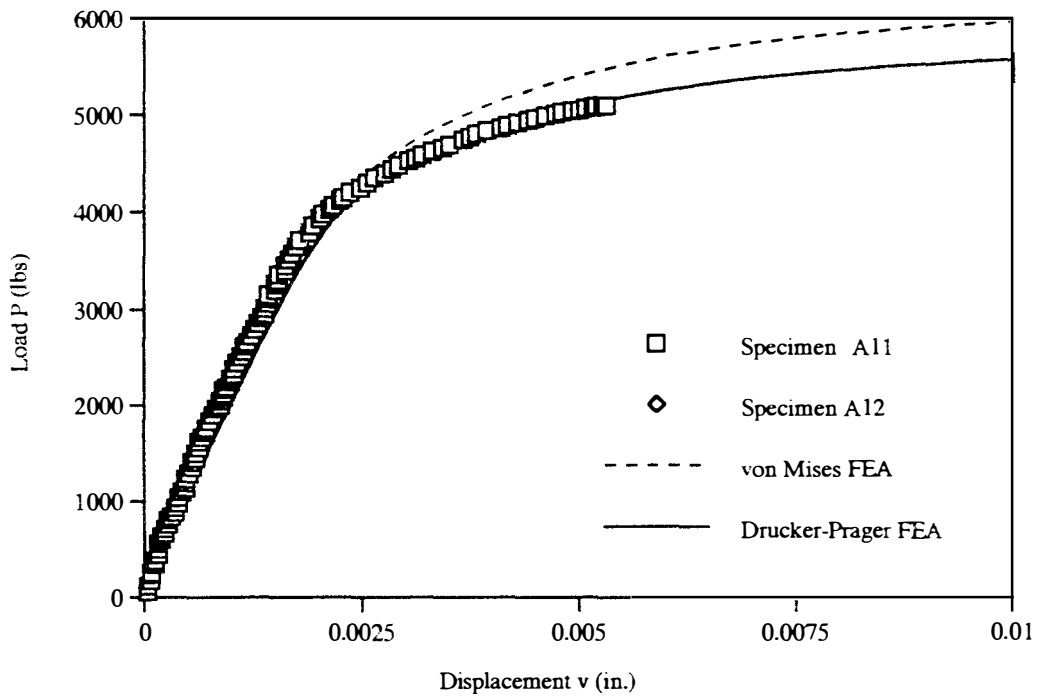


Figure 8.2: NRB Load-Displacement Results ($\rho = 0.010$ in.) for 2024-T351

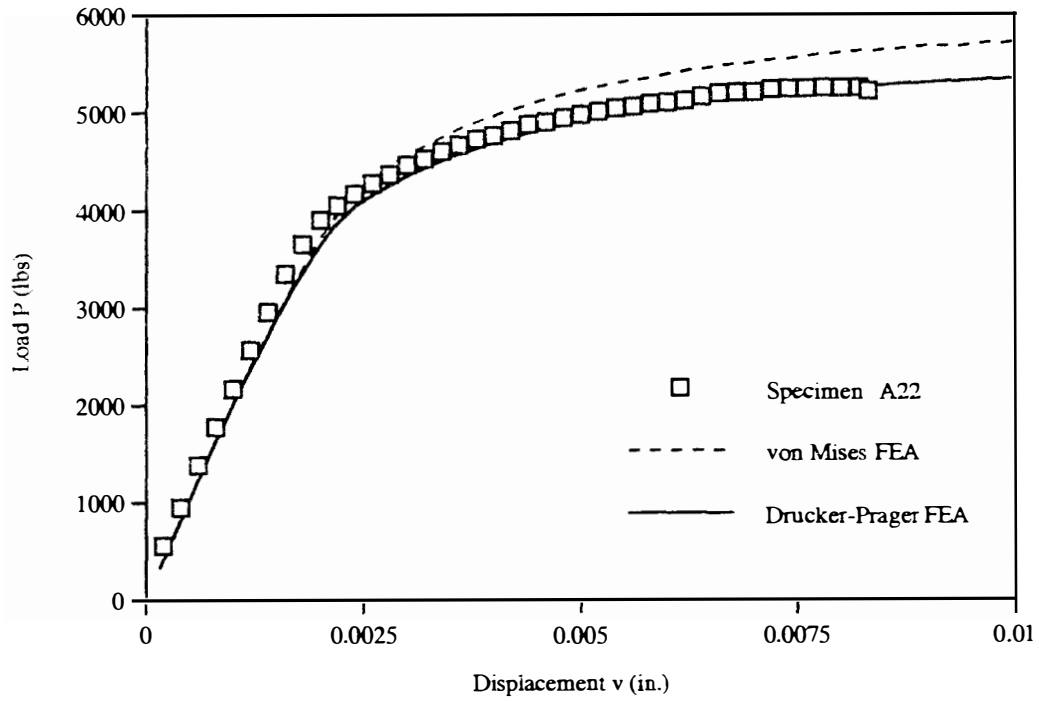


Figure 8.3: NRB Load-Displacement Results ($\rho = 0.020$ in.) for 2024-T351

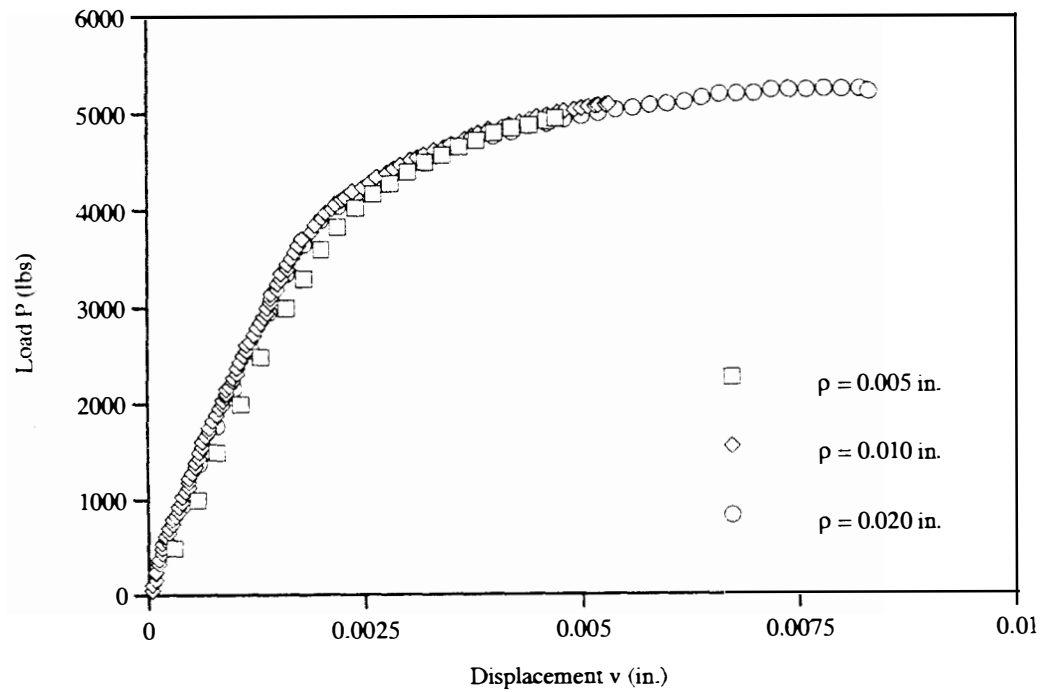


Figure 8.4: NRB Load-Displacement Results for All ρ for 2024-T351

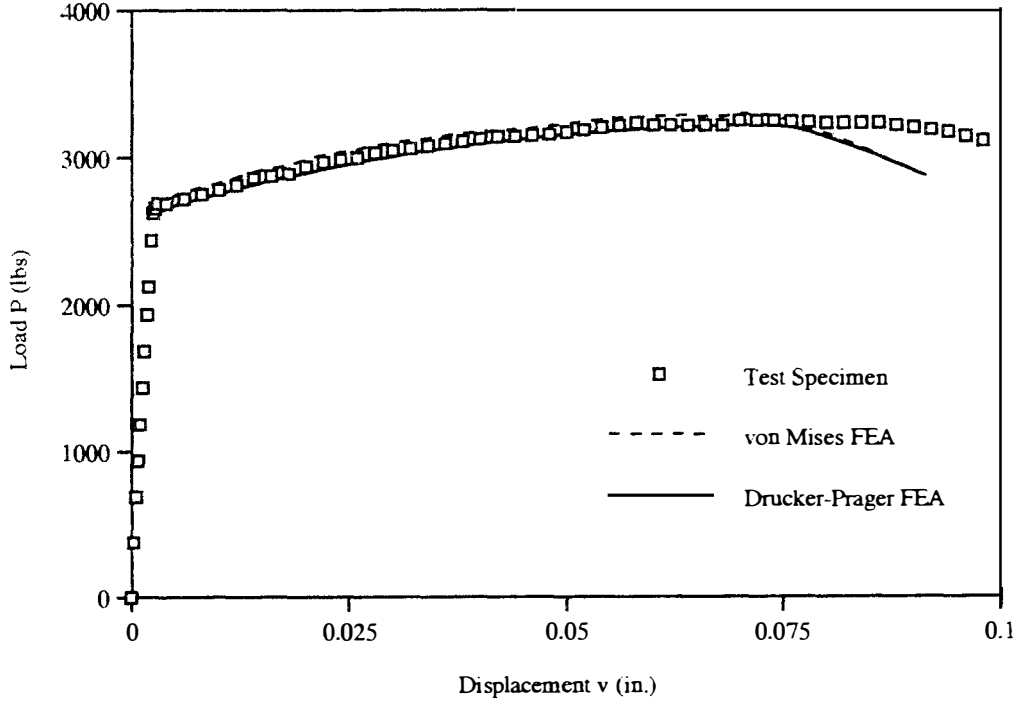


Figure 8.5: NRB Load-Displacement Results for a Tensile Test of 2024-T351

Once the Drucker-Prager yield function was calibrated by iteratively determining a , it was used in finite element analyses to generate load-displacement curves to be used in the load separation method. The load separation method requires either experimental or analytical $P-v_{pl}$ curves for several different values of r/R . Using coarse meshes similar to the one shown in Fig. 7.4, $P-v_{pl}$ curves were generated for $r/R = 0.4, 0.45, 0.5, 0.55, 0.6$. These analytical $P-v_{pl}$ curves are plotted in Fig. 8.6.

The separation constants $S_{i,j}$ defined in Eq. 6.3 were calculated using the $r/R = 0.5$ $P-v_{pl}$ as the reference “ j ” curve in the load separation method. $S_{i,j}$ versus v_{pl} is plotted in Fig. 8.7. The separation constants $S_{i,j}$ are essentially constant for a given r/R , except at very small plastic displacements. At these v_{pl} , the load does not separate into a product of functions of r/R and v_{pl}/r , and the method breaks down. A log-log plot of $S_{i,j}$ versus r/R is shown in Fig. 8.8. Recalling the discussion following Eq. 6.23, the slope of the $S_{i,j}-r/R$ curve is $2\eta_{pl}$. A value of $\eta_{pl} = 0.869$ for a 2024-T351 notched round bar was calculated.

Once η_{pl} was determined, apparent fracture toughness values from the experimental $P-v$ curves were determined using J calculations as outlined in the E813 test method. The values of J_c were converted to K_{Jc} using Eq. 3.22. These results are shown in a graph of apparent fracture toughness K_{Jc} versus $\sqrt{\rho}$ in Fig. 8.9. Extrapolating to a zero notch root radius gave $K_{Jc} = 22.3 \text{ ksi}\sqrt{\text{in}}$ for the NRB specimens.

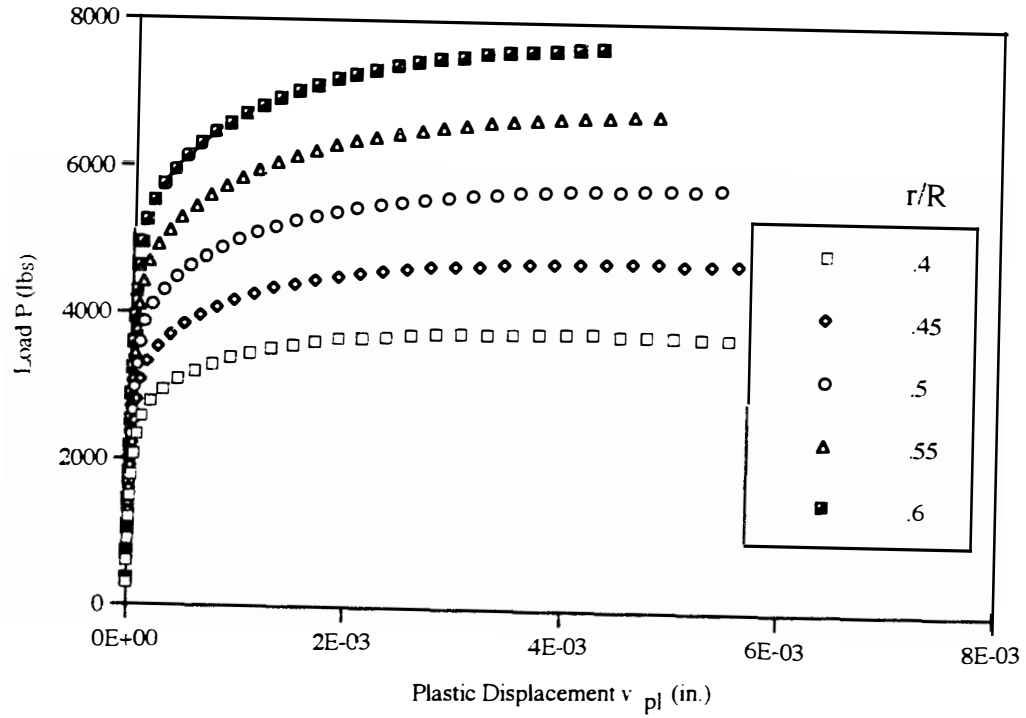


Figure 8.6: P - v_{pl} Curves Generated by Finite Element Analyses of 2024-T351

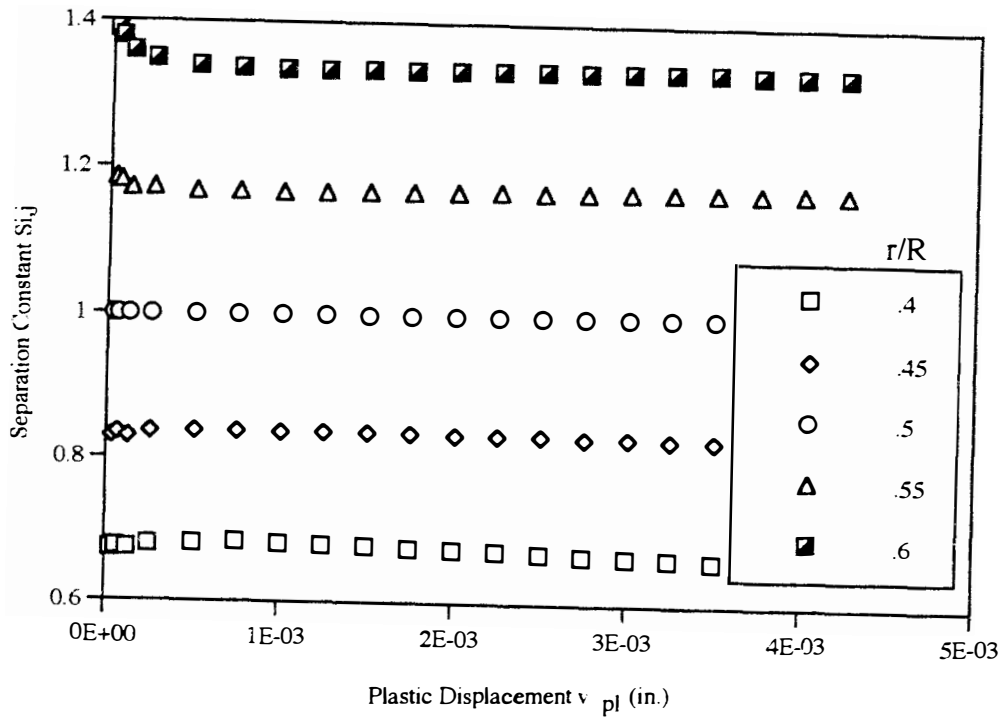


Figure 8.7: $S_{i,j}$ - v_{pl} Curves Generated by Finite Element Analyses of 2024-T351

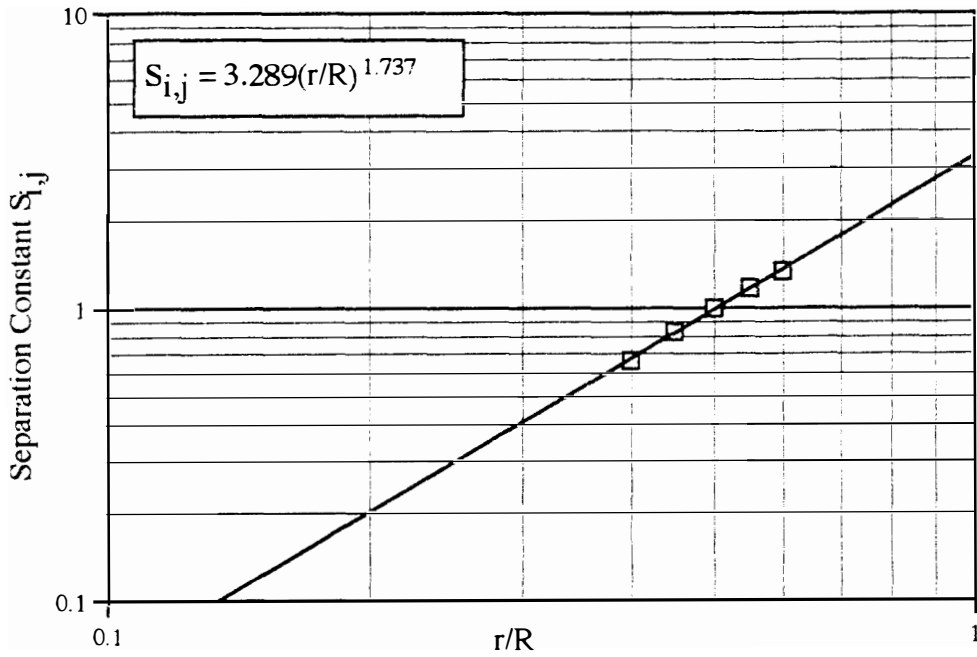


Figure 8.8: log – log Plot of $S_{i,j}$ versus $\frac{r}{R}$ for 2024-T351

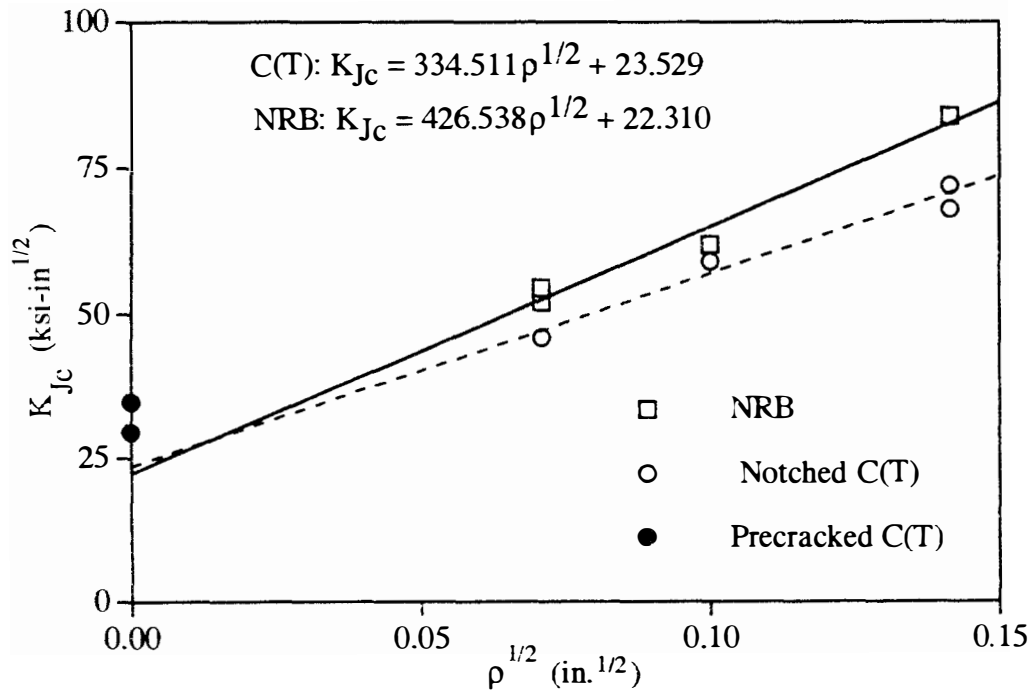


Figure 8.9: Extrapolated K_{Jc} Test Results for 2024-T351

In addition to NRB specimens, compact tension C(T) specimens were also tested. The fatigue precracked C(T) specimens were analyzed in the same manner as the NRB specimens. The average value of K_{Ic} for the precracked C(T) specimens was $33 \text{ ksi}\sqrt{\text{in.}}$. Apparent K_{Ic} values were determined from the blunt-notch C(T) specimens. Extrapolating to a zero notch root radius gave $K_{Ic} = 23.5 \text{ ksi}\sqrt{\text{in.}}$. Thus, the value of fracture toughness extrapolated from both NRB and blunt notched C(T) specimens were lower than the fracture toughness calculated using precracked C(T) specimens. The results are typical of the statistical scatter in fracture toughness data.

8.2 J_{Ic} and J - R Curve Testing of Overaged 2024 Aluminum Alloy

The experimental notched round bar load-displacement P - v records for the overaged 2024 are shown in Figs. 8.10–8.12. For comparison, a representative P - v curve for each notch root radius ρ is shown in Fig. 8.13. The P - v curves for all three values of ρ follow a common curve to a point where unloading occurs. Unlike the previously described 2024-T351 results, all the overaged 2024 test records fail beyond a point where the differential load dP is zero. This observation indicates that some ductile crack growth is occurring before fracture.

Also shown in Figs. 8.10–8.12 are finite element results using the Drucker-Prager yield function. The finite element results closely approximate the experimental results through the linear region of the P - v curve and into the nonlinear region to within approximately 10% of the maximum load. Beyond this point, the finite element results overpredict the load-displacement behavior of the notched round bars. It is plausible to assume that the deviation occurs at a point where the ductile crack growth begins and the finite element formulation is no longer representative of the physical situation.

As with the 2024-T351 material, the finite element results using the Drucker-Prager model were iteratively determined. Using an initial yield strength of 26 ksi and guessing a theoretical cohesive strength $\sigma_c = 0.1E$ lead to a reasonable match between the finite element results and the test records. The final value of a in the Drucker-Prager yield function was -0.03 —the same value that was iteratively found in the finite element analyses using 2024-T351. The fact that the value of a was the same for both the 2024-T351 and the overaged 2024 may indicate that a is a material property.

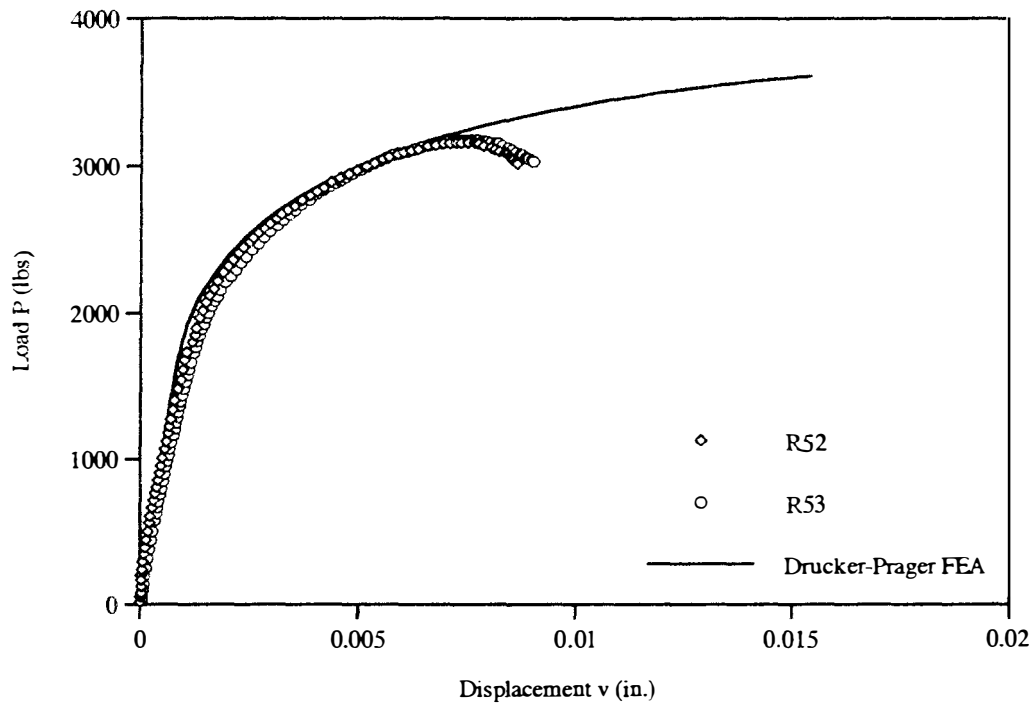


Figure 8.10: NRB Load-Displacement Results ($\rho = 0.005$ in.) for Overaged 2024

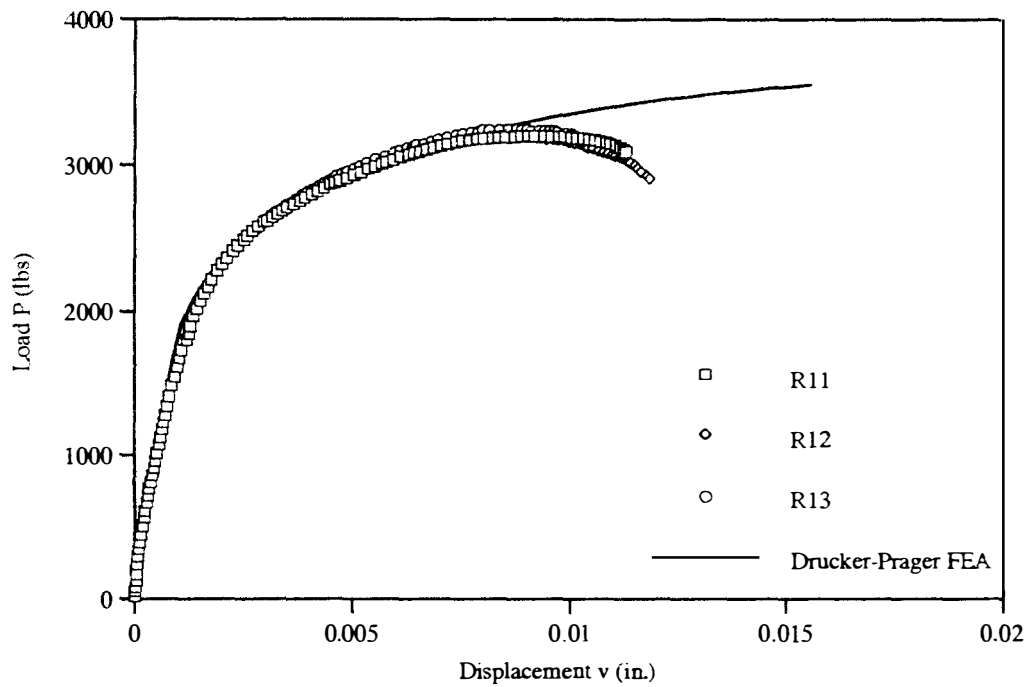


Figure 8.11: NRB Load-Displacement Results ($\rho = 0.010$ in.) for Overaged 2024

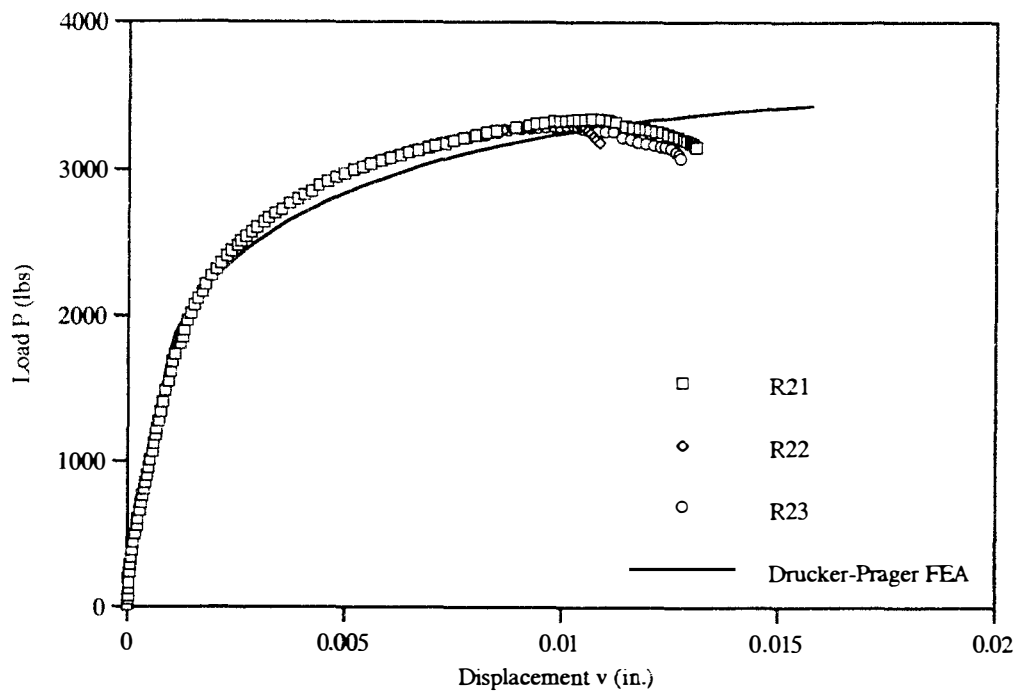


Figure 8.12: NRB Load-Displacement Results ($\rho = 0.020$ in.) for Overaged 2024

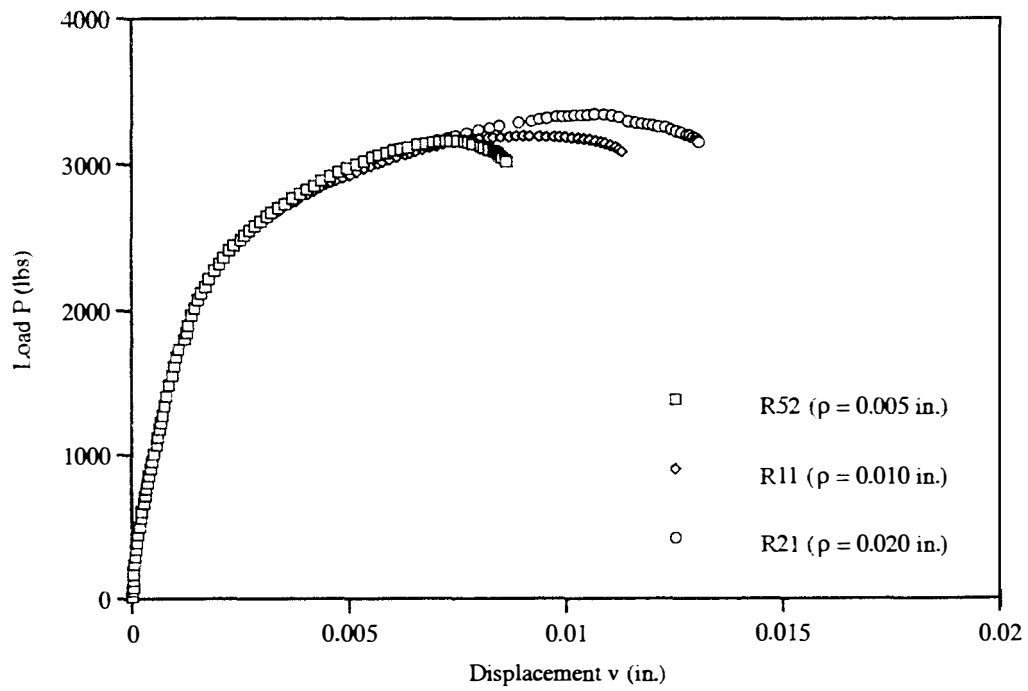


Figure 8.13: NRB Load-Displacement Results for All ρ for Overaged 2024

The same procedure used to perform the load separation analysis of the 2024-T351 notched round bars was performed using the overaged 2024 notched round bars. The value of η_{pl} for the overaged 2024 NRBs was determined to be 0.7345. This value of η_{pl} was used to calculate J for the notched round bar using Eq. 6.12.

After η_{pl} was determined, the normalization method (embodied in Eq. 6.30) was used to iteratively determine the apparent change in r/R as ductile crack growth occurred in a test specimen. The crack length a was defined as

$$a = R - r = R \left(1 - \frac{r}{R} \right). \quad (8.2)$$

For each P - v data pair, a value of a was calculated. J was calculated in accordance with the E813 test method. The resulting J - R curves are plotted in Fig. 8.14.

For comparison, the J - R curves from precracked 1/2T-C(T) and notched (without precracking) 1/2T-C(T) specimens are plotted in Fig. 8.15. The maximum crack extension Δa found in any of the NRB test records was 0.015 in., while the maximum crack extension found in the notched C(T) test records was 0.090 in. Clearly, the notched round bar geometry did not develop a significant amount of crack extension.

The blunting line and the 0.2 mm (0.008 in.) offset lines are also shown in Fig. 8.14. According to the E813 test method, the intersection of the fitted J - R curve with the 0.2 mm offset line is the conditional fracture toughness J_Q . However, none of the NRB J - R curves extended to this intersection. Thus, the current methodology was not used to determine J_Q for the notched round bars.

In this research program, J_Q was taken as the J value at the intersection of the blunting line and the power law curve fit of the J - R curve for data points clearly off the blunting line. For the NRB test records shown in Fig. 8.14, an exclusion line parallel to the blunting, but offset by a distance of 0.001 in. was used. It is recognized that such a construction for precracked specimens would result in very low values of J_Q compared to the E813 test method. However, the E813 test method assumes that sharp cracks are used and that blunting occurs only with subsequent loading. This is not the case for the notched round bar which has a finite notch root radius before monotonic loading. A comparison of J_Q values developed in this manner for the NRB specimens and J_Q values developed using the E813 test method for 1/2T-C(T) specimens is shown in Fig. 8.16.

Extrapolating the apparent J_Q values for the NRB specimens to $\rho = 0$ in. in Fig. 8.16 gave $J_Q = 112$ in-lb/in². The value of J_Q from the precracked 1/2T-C(T) specimens was 110 in-lb/in². It was surprising to note that the apparent J_Q values for the notched 1/2T-C(T) specimens did not follow a linear relationship with

notch root radius ρ for $\rho = 0.020$ in. If this notch root radius is excluded from an extrapolation to zero notch root radius, then the predicted J_Q value is essentially the same as the precracked results. Reexamining Fig. 8.15 did not reveal any reason for the anomalous behavior of the larger notch root radius 1/2T-C(T) specimens.

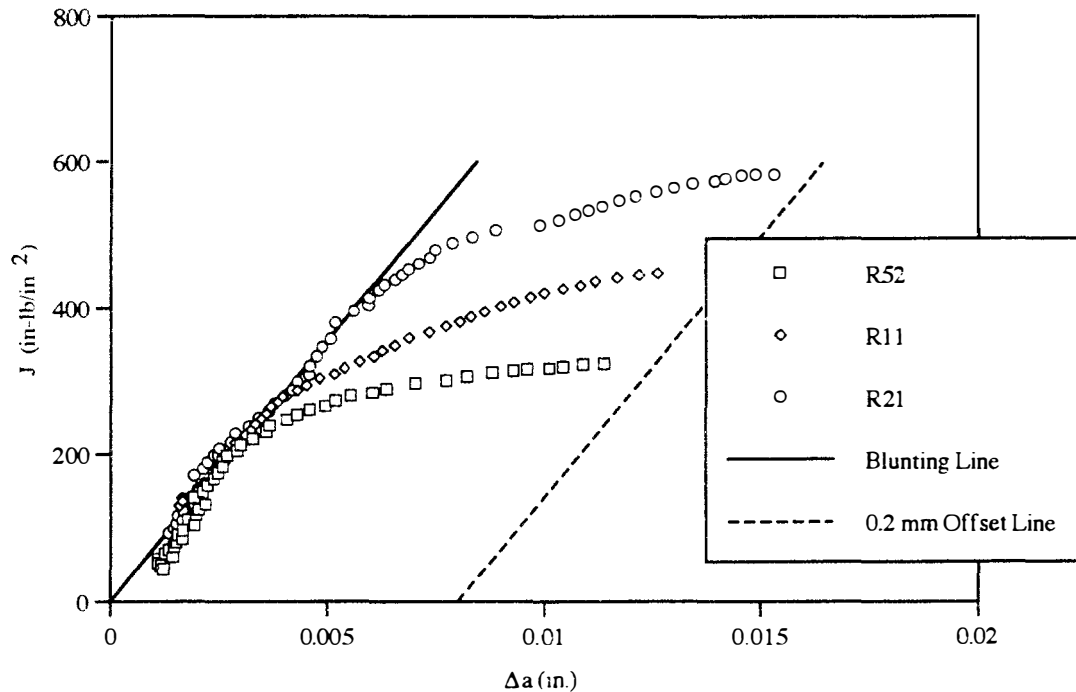


Figure 8.14: NRB J - R Curve Results for All ρ for Overaged 2024

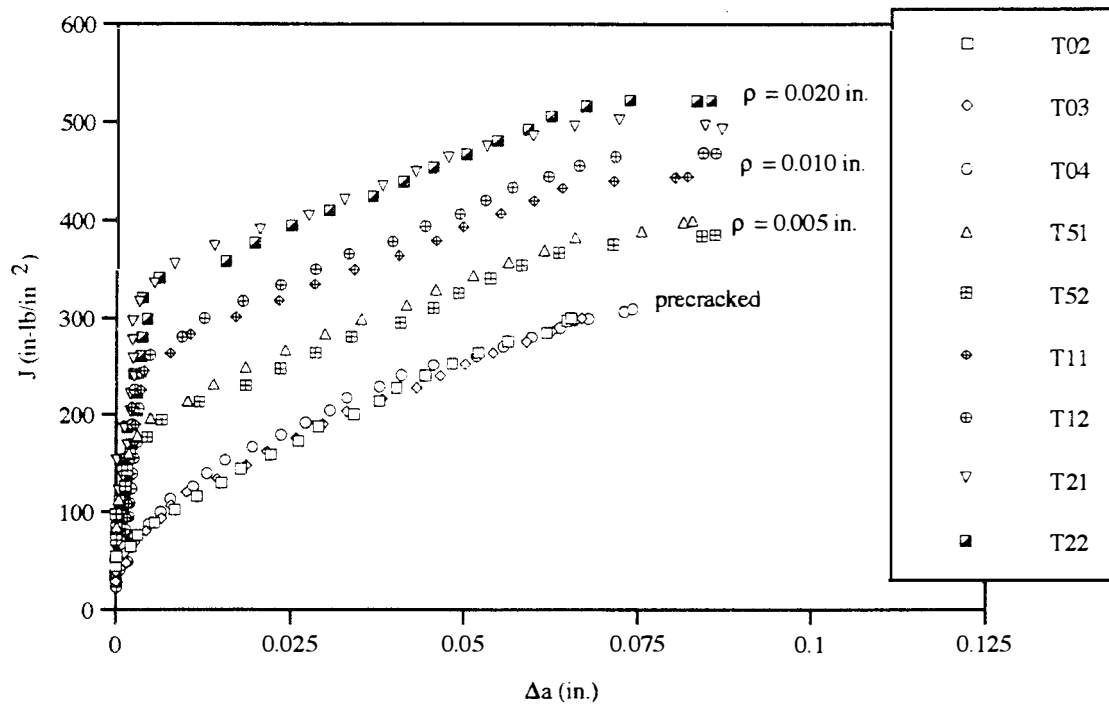


Figure 8.15: 1/2T-C(T) J - R Curve Results for All ρ for Overaged 2024

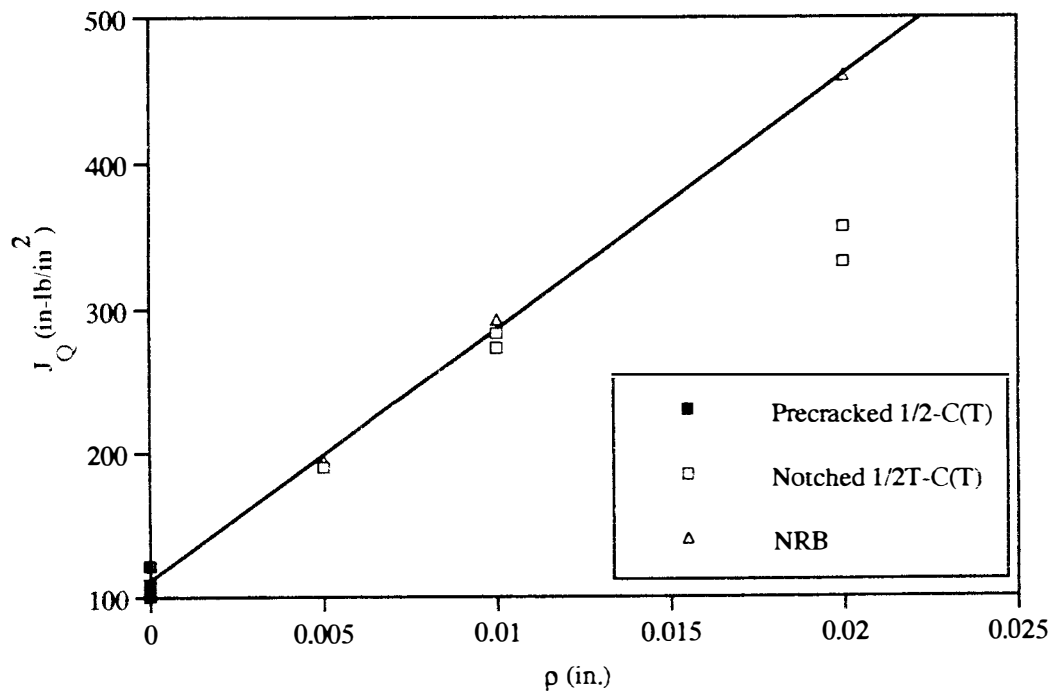


Figure 8.16: Extrapolated J_Q Test Results for Overaged 2024

8.3 K_{Jc} Testing of A302B Pressure Vessel Steel

The load-displacement P - v curves for the K_{Jc} test matrix listed in Table 7.4 are shown in Figs. 8.17-8.19. An important observation to be made is the scatter in the elastic slopes (stiffness) of the P - v curves. Three specimens were tested for each notch root radius $\rho = 0.005, 0.010, 0.020$. One test record in the $\rho = 0.005$, X12, had a significantly different elastic slope—30% less than the other two specimens. Similarly, one test record in the $\rho = 0.030$, X31, had a stiffness that was 70% lower than the other two specimens in the group. All the specimens, regardless of notch root radius, should have approximately the same elastic slope. In both cases of significant slope deviation, the extensometer was not well seated and slipped during loading.

Calculations for K_{Jc} were performed following the draft test method. Since the K_{Jc} values were not generated using precracked specimens, the values are referred to as apparent K_{Jc} values. A plot of apparent K_{Jc} versus $\sqrt{\rho}$ is shown in Fig. 8.20. A value of $K_{Jc} = 161 \text{ ksi}\sqrt{\text{in}}$ was found by extrapolating a linear curve fit to $\rho = 0$. Clearly, the apparent K_{Jc} values are biased by their respective notch root radii.

To use the draft test method, the data must be treated as a single—not three separate groups. It was necessary to develop a scheme to remove the notch root radius bias from the data and essentially create a single data set from the three notch root radii groups. A simple scheme to adjust for bias is shown in Fig. 8.21. The mean apparent K_{Jc} values were determined from the linear curve fit for each notch root radius. Then the difference between each actual data point and the mean value at a given ρ was calculated. The actual data point was then moved down to $\rho = 0$ and placed at the same relative position with respect to the mean value from the linear fit at $\rho = 0$. The transformed values of apparent K_{Jc} for $\rho = 0.005 \text{ in.}$ are shown in Fig. 8.21. All data points were adjusted in this manner.

After the adjustment for notch root radius bias was made, the procedure in the draft test method for the Weibull analysis of the data was performed. The results are shown in Fig. 8.22. The Weibull slope of 4 and a $K_{min} = 20 \text{ MPa}\sqrt{\text{m}}$ were used to determine a reference temperature $T_0 = -100 \text{ C}$. For comparison, McCabe and Swain [69] determined T_0 to be -87.3 C using 1T-C(T) specimens. The resulting master curves for both the NRB tests and the test using 1T-C(T) specimens are shown in Fig. 8.23.

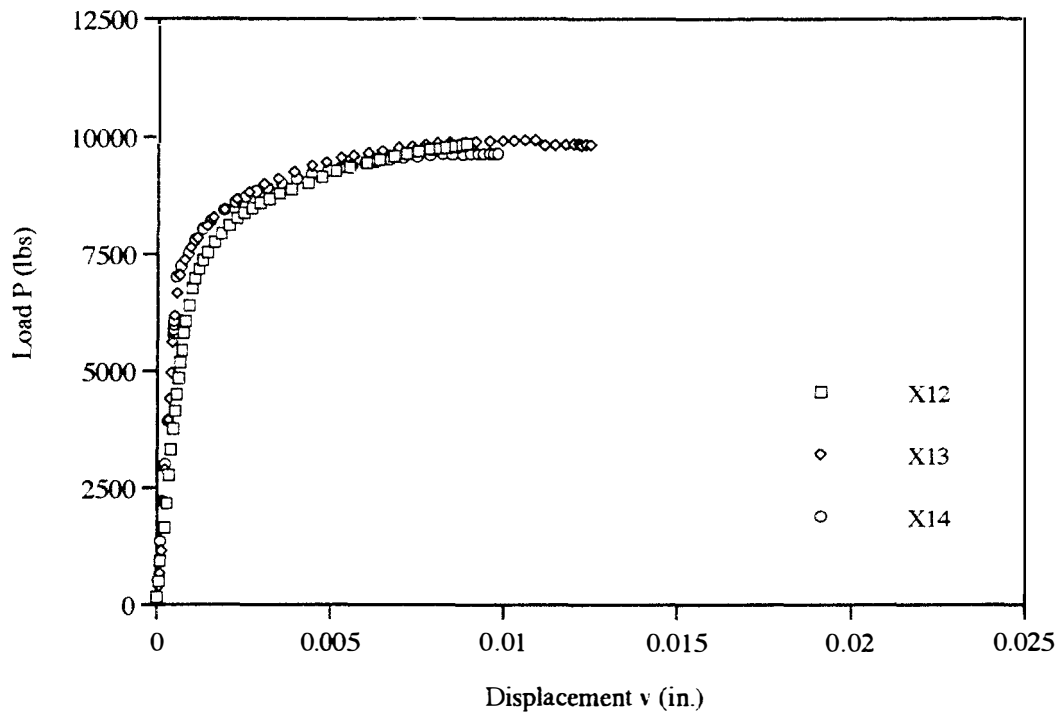


Figure 8.17: Load-Displacement $P-v$ Curves for A302B, $\rho = 0.005$ in.

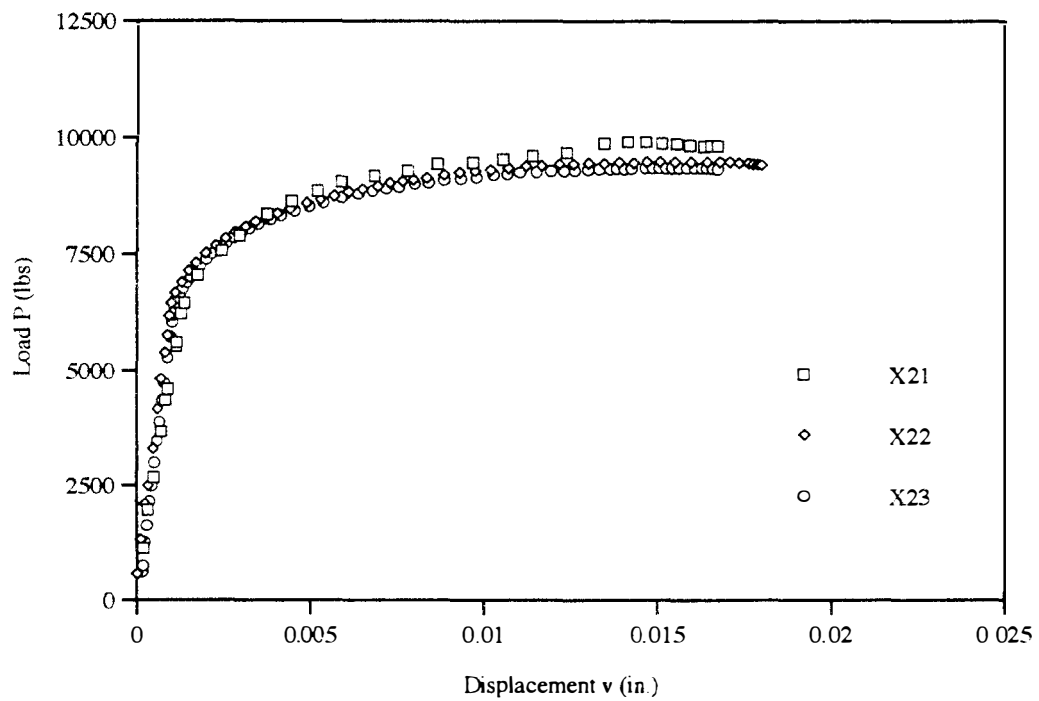


Figure 8.18: Load-Displacement $P-v$ Curves for A302B, $\rho = 0.010$ in.

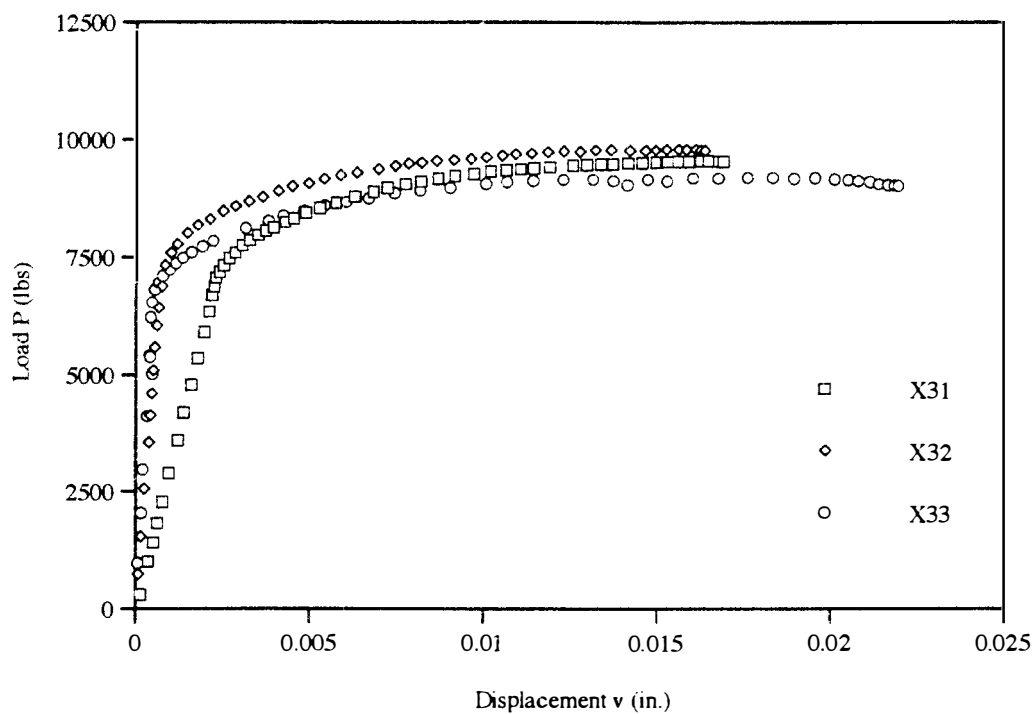


Figure 8.19: Load-Displacement P - v Curves for A302B, $\rho = 0.020$ in.

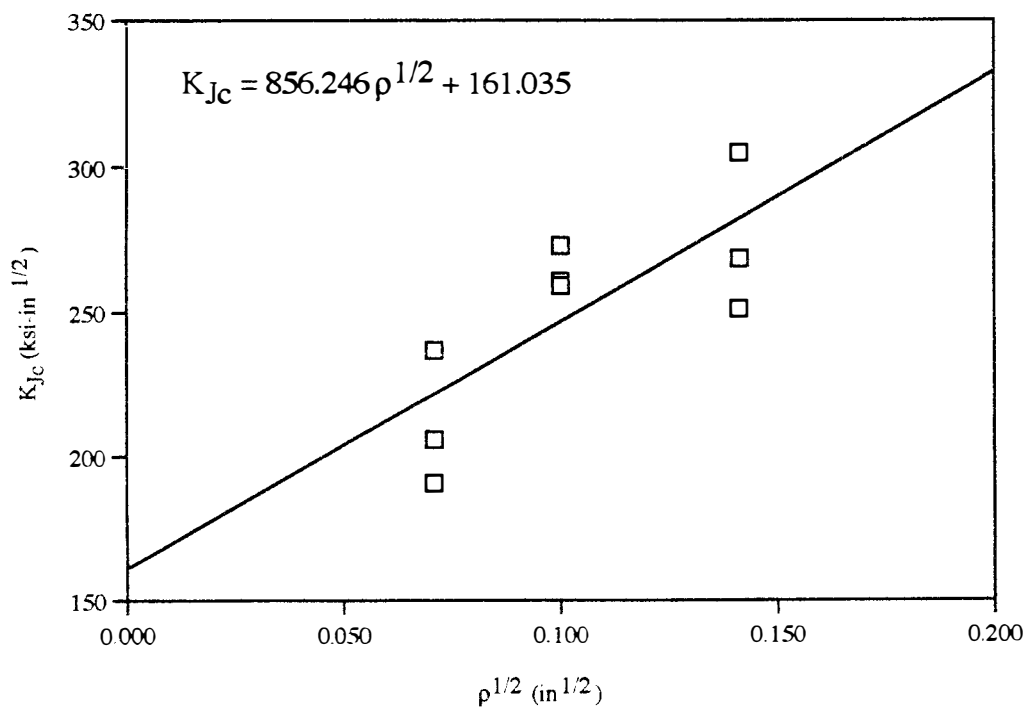


Figure 8.20: K_{Jc} versus $\sqrt{\rho}$ for A302B NR.B Specimens

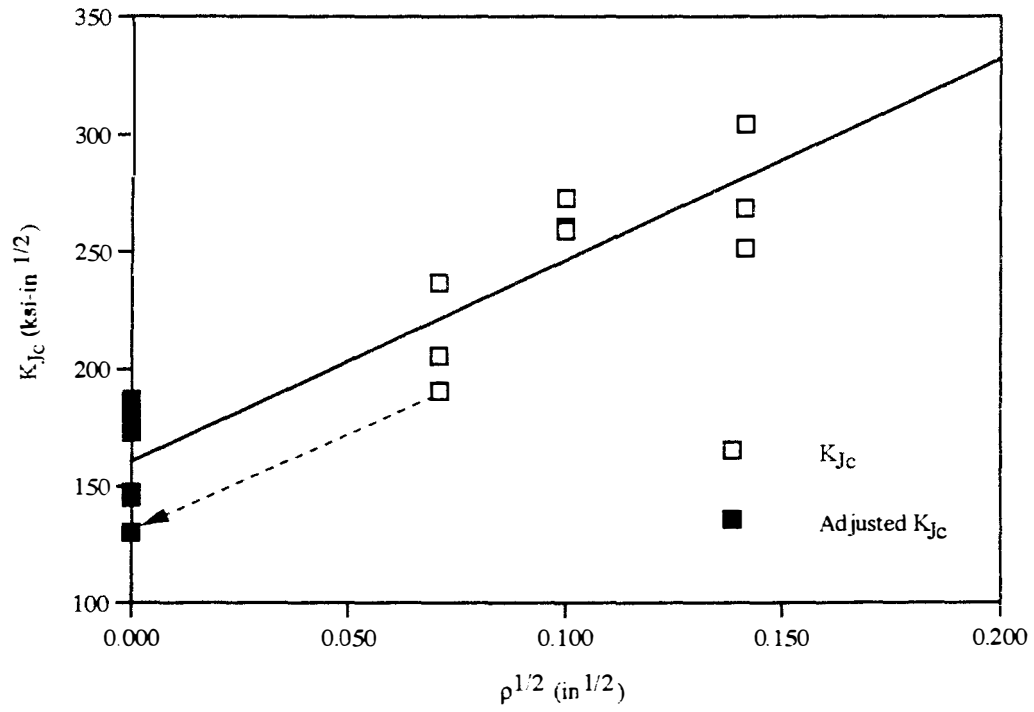


Figure 8.21: Bias Adjustment for K_{Jc} versus $\sqrt{\rho}$ for A302B

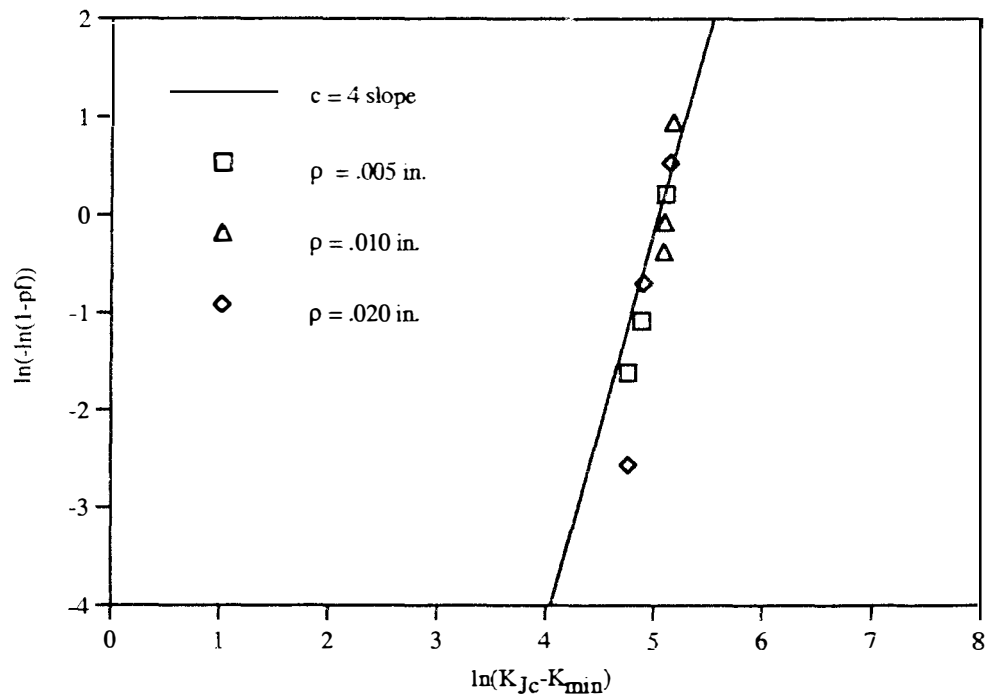


Figure 8.22: Weibull Plot for A302B NRB Specimens

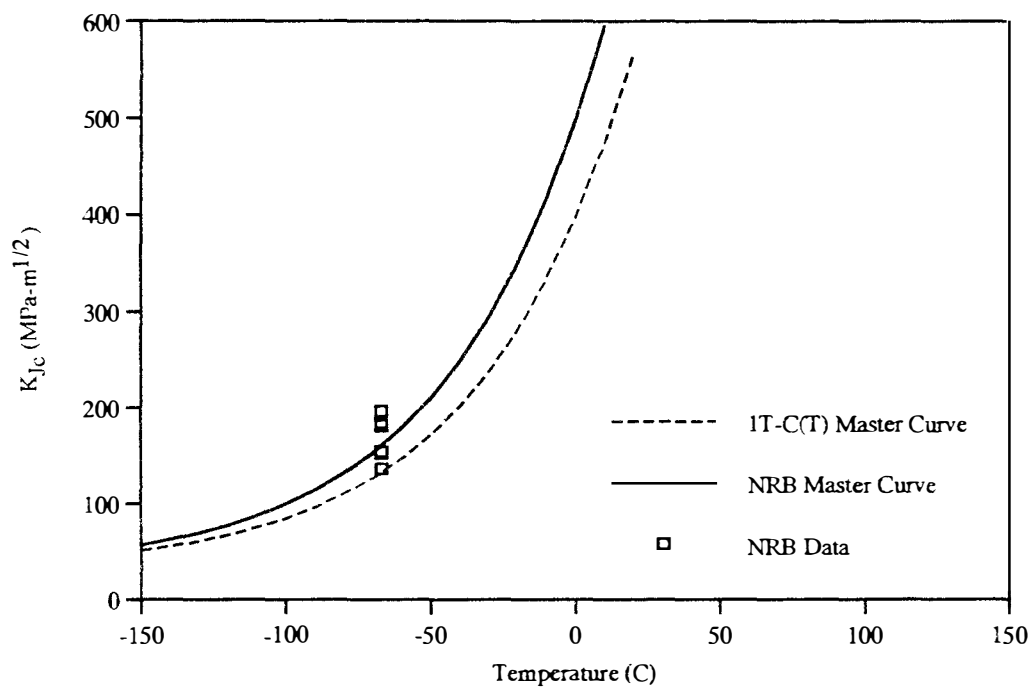


Figure 8.23: Master Curves for K_{Jc} from NRB and C(T) Specimens

The size adjustment in the draft test method is based on the length of crack front in a planar specimen. For the NRB specimens used in this test series, the length of crack front (notch front) is the circumference $2\pi r \approx 0.8$ in. The size adjustment ratio $\theta^{1/4}$ is

$$\theta^{1/4} = \left(\frac{B_x}{B_{1T}} \right)^{1/4}. \quad (8.3)$$

where the circumference of the NRB is treated as an equivalent planar specimen thickness. For $B_x = 0.8$ in., the size adjustment ratio $\theta^{1/4} = 0.95$. This size adjustment may not be applicable for a NRB.

If the size adjustment is generalized to an area in the crack (notch) front region, then the result is equivalent to a length of crack front adjustment for a planar specimen. However, the result is very different for the notched round bar. In Fig. 8.24 the area $dA = Br_c$ for a planar specimen is shown. The distance r_c can be interpreted as the distance from the crack (notch) tip to the cleavage initiation site by Heerens, Read, Cornec, and Schwalbe [72]. They studied the scatter in fracture toughness data in the ductile to brittle transition using C(T) specimens and found $r_c \approx 1.5 - 2.0$ mm (0.06 to 0.08 in.).

A similar area approach (Fig. 8.24) for a notched round bar would give

$$dA = \pi r^2 - \pi(r - r_c)^2. \quad (8.4)$$

Using an area approach for a size adjustment for planar specimens yields

$$\theta^{1/4} = \left(\frac{B_x r_c}{B_{1T} r_c} \right)^{1/4}, \quad (8.5)$$

which is of course nothing more than the current size adjustment. However, the size adjustment between a NRB specimen and a 1T planar specimen is

$$\theta^{1/4} = \left(\frac{\pi r^2 - \pi(r - r_c)^2}{B_{1T} r_c} \right)^{1/4}. \quad (8.6)$$

For example, if $r_c = 1.5$ mm is used to adjust the NRB K_{Jc} data, the size adjustment is $\theta^{1/4} = 0.82$. The resulting expression for $K_{Jc(1T)}$ is

$$K_{Jc(1T)} = 20 + [K_{Jc(x)} - 20] \theta^{1/4}. \quad (8.7)$$

If $K_{Jc(med)} = 161.6 \text{ MPa}\sqrt{\text{m}}$ is used, then $K_{Jc(1T-med)} = 136.1 \text{ MPa}\sqrt{\text{m}}$. The resulting T_0 is -89 C. This result compares very favorably to T_0 of -87.3 C found by McCabe and Swain [69].

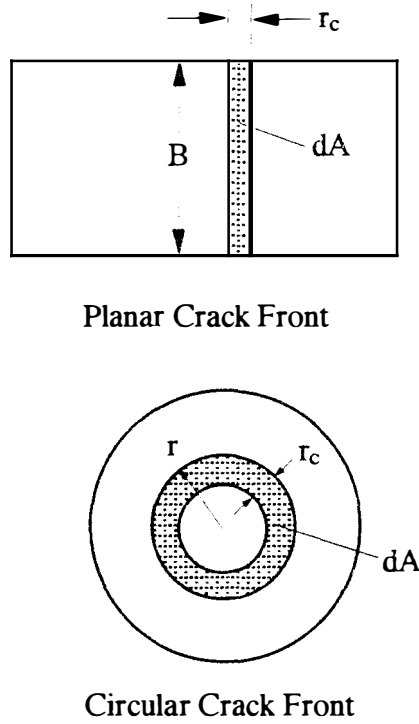


Figure 8.24: Areas Used for K_{Jc} Size Adjustments

8.4 J_{Ic} and J - R Curve Testing of Nylon 6/6

The experimental NRB P - v test records for nylon 6/6 are shown in Figs. 8.25 and 8.26. For both notch root radii, there was a significant amount of unloading in the notched round bar load-displacement records. The $\rho = 0.005$ in. test records showed a slightly higher maximum load than the test records for $\rho = 0.010$ in. It was observed that the $\rho = 0.010$ in. test records tended to fail at slightly larger displacements than the NRB specimens with $\rho = 0.005$ in. Crack growth was visually observed from the notch surface, unlike the previously discussed 2024 aluminum alloys and the modified A302 steel.

In Fig. 8.27, The J - R curves of two representative specimens are shown. The maximum J values in the two test records are almost identical. However, the $\rho = 0.010$ in. specimen had a slightly larger amount of crack extension. Since crack growth was visually observed, the nylon 6/6 NRB J - R curves were analyzed using the E813 test method. J_Q was determined using the intersection of the 0.2 mm offset line and a power law fit to the J - R data between the 0.15 and 1.5 mm exclusion lines in Fig. 3.11. The resulting apparent J_Q values for $\rho = 0.005$ and 0.010 in. were

essentially the same, 185 and 187 in-lb/in², respectively. In Fig. 8.28, the resulting extrapolation to $\rho = 0$ yielded a sharp-crack fracture toughness of 183 in-lb/in². For comparison, the single edge bend SE(B) results of Zhou, Landes, and Huang [70] are also shown in Fig. 8.28. They found an average J_Q of 173 in-lb/in². The NRB results were higher, but the difference is within the typical scatter in fracture toughness data.

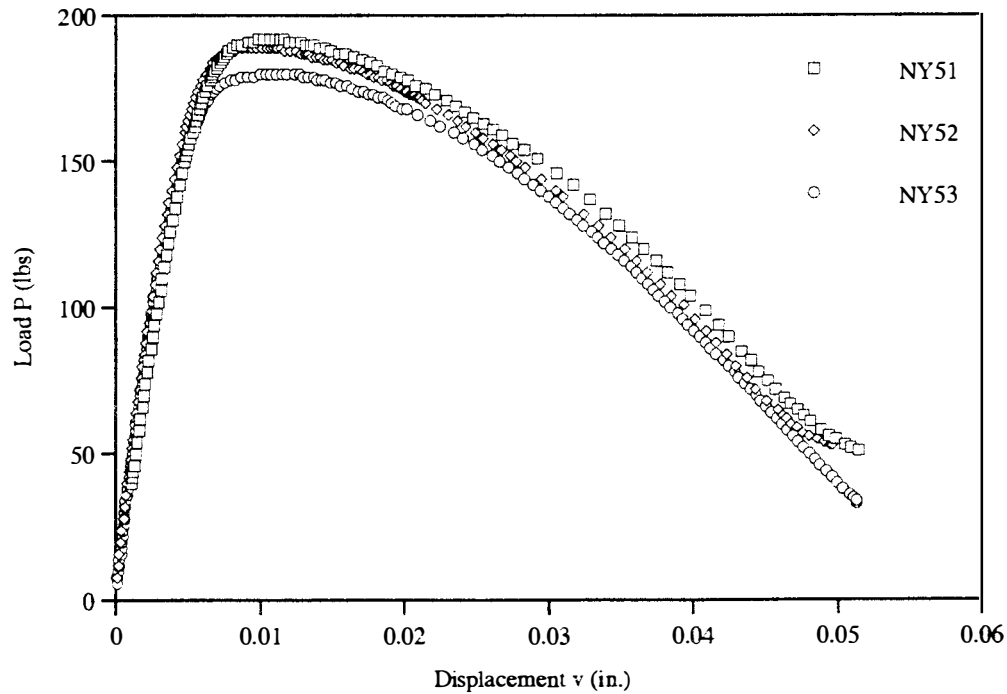


Figure 8.25: Load-Displacement P - v Curves for Nylon 6/6, $\rho = 0.005$ in.

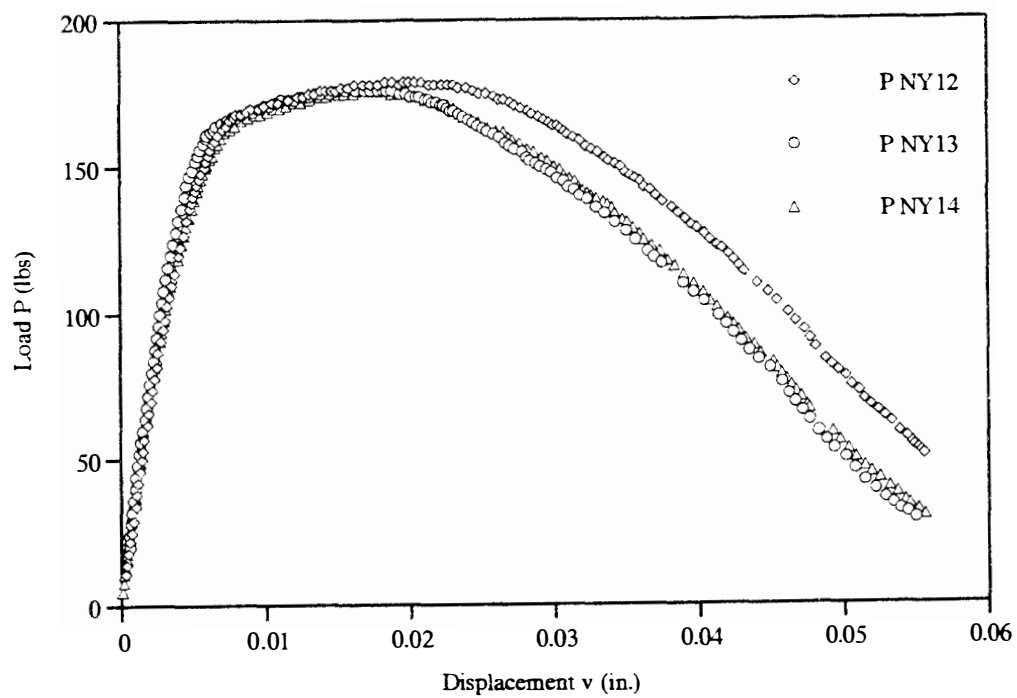


Figure 8.26: Load-Displacement $P-v$ Curves for Nylon 6/6, $\rho = 0.010$ in.

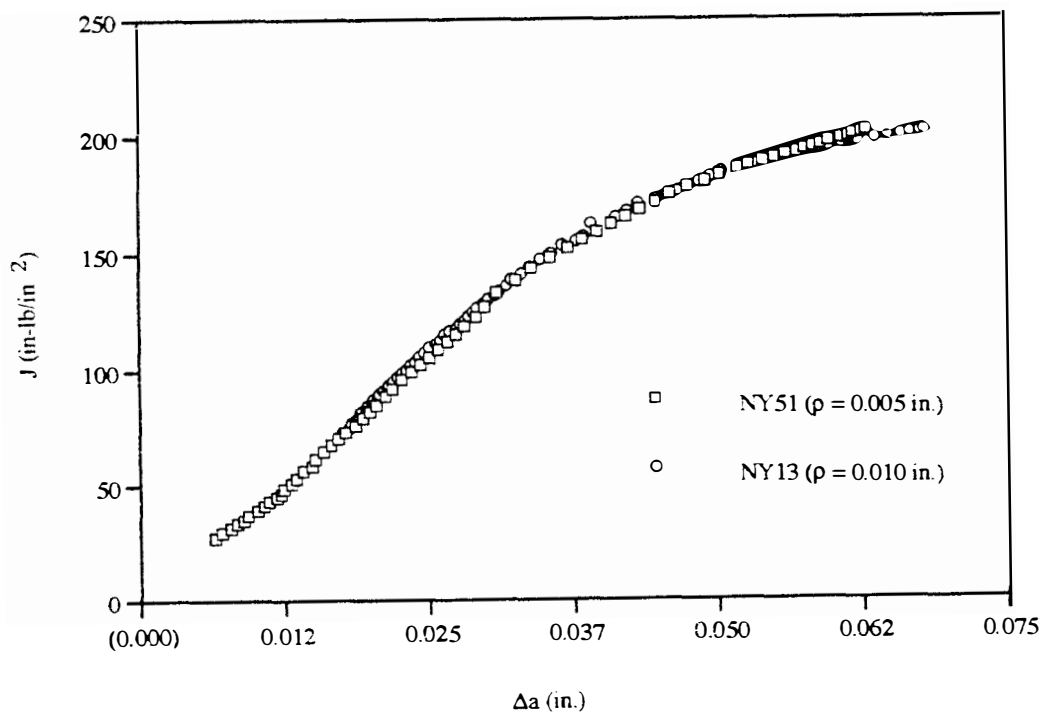


Figure 8.27: $J-R$ Curves for Nylon 6/6 NRB Specimens

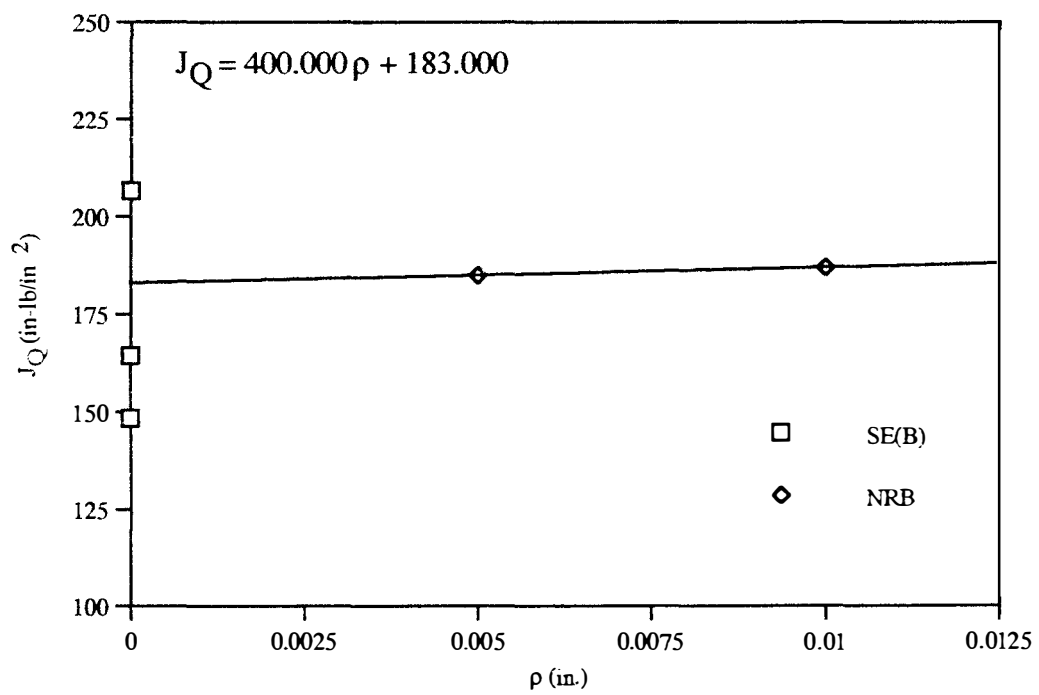


Figure 8.28: J_Q versus ρ for Nylon 6/6 NRB Specimens

8.5 Discussion

The notched round bar geometry gave comparable results to traditional fracture toughness test specimen geometries, such as the compact tension and single edge bend geometries, for a variety of materials. For initiation fracture toughness of 2024-T351, the fracture toughness determined using notched round bars slightly underestimated the result obtained with precracked C(T) specimens. The E399 test methodology was not suitable for the NRB geometry because of the large amount of plasticity found in the load-displacement record. The increased plasticity level was expected because of bluntness of the notch root radius versus a sharp precrack. Therefore, a valid K_{Ic} using the E399 test method was not obtained.

However, a critical value of J , J_c was determined for the 2024-T351 using the E813 test methodology and converted to K_{Ic} . The extrapolation of apparent fracture toughness K_{Ic} to zero notch root radius ρ using a graph of apparent K_{Ic} versus $\sqrt{\rho}$ worked well for both the NRB geometry and the notched, but not precracked, C(T) geometry. In fact, the apparent fracture toughness values for a given notch root radius for both the C(T) and NRB geometries lie very close together (Fig. 8.9). Reexamining the results of Begley, Logsdon, and Landes [50] in Fig. 6.2 reveals a similar result for a NiCRMoV steel. Both C(T) and center notched panels gave the same extrapolation line to J_c .

For the overaged 2024 aluminum, the NRB specimens produced a much smaller amount of stable crack growth than notched C(T) specimens. However, the two geometries gave the same extrapolation line in a plot of apparent J_Q versus ρ . In terms of calculating initiation fracture toughness, the NRB geometry is a suitable alternative to the C(T) geometry. However, if a substantial J - R curve is sought, the NRB does not provide sufficient stable crack growth. In this case, a planar specimen must be used.

For the modified A302B steel, the NRB geometry produced a master curve that was very similar to the master curve generated using precracked C(T) specimens. If the proposed size adjustment is made to the NRB results, then the master curve determined using the NRB specimens is essentially the same as the master curve using C(T) specimens. Since, no notched, nonprecracked C(T) specimens were used, no observation concerning the invariance of the apparent fracture toughness versus notch root radius slope was made.

For the nylon 6/6, the NRB specimens produced observable surface crack growth. The resulting apparent J - R curves were very similar to those produced using precracked specimens. The slope of the extrapolation line of apparent frac-

ture toughness versus notch root radius was essentially zero. This implies that the smallest notch root radius possible could be used without extrapolation to obtain a sharp-crack fracture toughness of nylon 6/6. The J - R curves presented in this work were developed using the E813 construction. This construction does not correct for crack growth in the J calculation. A more accurate J - R curve could be developed using the construction method outlined in the E1152 J - R test method.

Based on the test results previously discussed, the key technological advance of relating apparent fracture toughness to notch root radius ρ and extrapolating to zero ρ to infer sharp-crack fracture toughness was validated. The other key technological advances used in this research—the load separation and normalization methods and the inclusion of hydrostatic stress effects on yield using the Drucker-Prager yield function—were also validated.

For the first time, the load separation method was used for the NRB geometry to determine a functional form for η_{pl} , the proportionality term relating plastic work and J_{pl} . The resulting η_{pl} included both a geometry dependence and an explicit dependence on material hardening. The normalization method again proved useful in determining crack length without direct crack measurement. Although the amount of stable crack growth calculated in the NRB test results was smaller than the amount seen in planar specimens, the normalization method produced J - R curves which were successfully used to determine initiation fracture toughness of overaged 2024 aluminum and nylon 6/6.

A large part of the success of the load separation and normalization methods was due to the accuracy of the finite element analyses. The load-displacement curves developed using the Drucker-Prager yield function were used in the separation and normalization process. The effect of hydrostatic tensile stress was found to be significant for notched round bars. Clearly, the Drucker-Prager yield function was more accurate than the traditionally used von Mises yield function.

Chapter 9

Conclusions and Recommendations

The results presented in the previous chapter clearly demonstrated that the notched round bar geometry is a suitable alternative to the currently used planar specimen geometries. The NRB geometry is particularly well-suited for initiation fracture toughness testing where the construction of a J - R curve is not required. The NRB geometry is a reasonable alternative for fracture toughness testing using the E399 test method. Instead of determining K_{Ic} , a J_c value should be determined and then converted to K_{Jc} . This change in methodology is needed because of the larger amount of plasticity that develops in a notched versus a precracked specimen.

For fracture toughness testing in the ductile to brittle transition, the NRB geometry performed extremely well. The K_{Jc} results obtained using a modified version of the draft test method were very close to the results using precracked C(T) specimens. The modification made to the methodology to remove the bias of notch root radius worked well. A modified size adjustment to account for the difference between round bars and planar specimens was proposed.

For J - R curve testing, the NRB is probably not as useful as the planar specimens because the NRB does not develop a large amount of stable crack growth. However, if initiation fracture toughness J_{Ic} is required, the methodology presented here can be used. In other words, the NRB geometry should provide enough stable crack growth to support a J_Q construction similar to the E813 test method. For materials like the overaged aluminum, which do not have observable surface crack growth, the J_Q construction in the E813 test method must be modified. The proposed change is to forego the use of the 0.2 mm offset line for determining J_Q . Instead, J_Q should be taken from the actual blunting line. For materials such as the nylon 6/6, which had observable surface crack growth, the existing J_Q construction in the E813 test method is sufficient.

Overall, the NRB geometry should be attractive to industries that do not currently use precracked, planar specimens. The notched round bar does not require a servohydraulic testing machine because precracking is not necessary. If the normalization method is used for determining crack length, then unloading compliance is not required. In fact, the NRB geometry is no more difficult to test than a straight tensile bar. Two key issues must be remembered when using the NRB geometry. First, proper alignment must be maintained. The alignment requirements in the E602 test method was found to be sufficient in this research program. Second, the notch root radius of each specimen should be carefully measured before testing. The extrapolation procedure may be sensitive to the measurement of notch root radius.

In terms of a cost savings over using precracked planar specimens, the NRB geometry may provide a marginal advantage. When compared on a per test basis, the NRB is significantly cheaper to test. However, the methodology presented here requires the use of several notch root radii and at least, duplicate tests at a given notch root radius. For example, the E399 test method recommends that at least three precracked specimens be used. If notched round bars are used with duplicate tests at three notch root radii, then six test specimens would be needed. It is estimated that a single NRB test costs about one-half the amount of testing a precracked, planar specimen. Thus, testing six notched round bars would cost the same as three precracked, planar specimens.

Some cost advantage in using the NRB geometry might be gained if the observations that for a given material, the slope of the apparent fracture toughness versus ρ (or $\sqrt{\rho}$) is invariant. If this invariance can be further proved, then fewer notched round bars could be used to determine the same fracture toughness result. It might be possible to test NRB specimens with a single notch root radius and use the slope of the apparent fracture toughness versus ρ line to extrapolate to a sharp-crack fracture toughness.

Specific recommendations focus on the further development of a test methodology for using notched round bars. Size requirements for valid test results must be established for the NRB geometry. A combination of more experimental results and nonlinear finite element analysis of the notch neck region will be needed in determining size requirements. For the case of transition fracture toughness testing, a larger volume of test results are needed to confirm or reject the scheme for removing notch root radius bias. Additionally, the size adjustment proposed for the NRB geometry in developing a master curve should be further explored.

As a final recommendation, the notched round bar should be studied as a possible test specimen geometry for mixed-mode I-III fracture toughness testing. All

the work presented in this dissertation has focused on mode I—the opening or tensile mode of crack-tip deformation. The other two modes of crack-tip deformation are mode II, the inplane shear or sliding mode and mode III, the out of plane shear or tearing shear mode. While planar specimens are more suitable for mode I-II fracture toughness testing, they are not well-suited for mode I-III fracture toughness testing. On the otherhand, the notched round bar is well-suited for mode I-III fracture toughness testing. In such a test, a combination of tension and torsion would be used to develop fracture toughness in terms of K_I and K_{III} . There are no existing standardized test methods for mixed-mode fracture toughness testing.

These recommendations should be brought to the attention of interested members of ASTM E08 Committee on Fatigue and Fracture. In particular, those members using the E602 test method [36] for sharp-notch strength of round bars should have an interest in the work presented here. Many of the results of this dissertation could be incorporated in a modified version of the E602 test method.

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