

An Analysis of Post-Secondary Mathematics Instructors

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Degree

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DEDICATION

This dissertation is dedicated to my family, my wife Jillian, our son Stephen, my parents, and our dog Tysen.

To Jillian and Stephen, thank you for putting up with me as I have been working on this paper. I know it has taken a lot of time out of all our schedules and I am thankful that you have afforded me the time to complete my work. Jillian, thank you listening to me talk so much about this study and for giving me advice and encouragement. Stephen keep growing and learning and becoming the smart and creative young man that I know you are.

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ABSTRACT

The purpose of this study was to determine significant similarities and differences among post-secondary mathematics instructors with a focus on how they teach and engage students in learning mathematics. Prior research confirms that some of the similarities and differences among post-secondary mathematics education practices fit within categories that include teaching preparation, academic background, and cultural issues. Therefore, the study sought to explore these characteristics and to possibly identify others, with the intention of improving the overall effectiveness of mathematics teaching at the post-secondary level. Data were collected from eight post-secondary mathematics instructors through a screening survey and informal interviews. For data analysis procedures, a coding process was used that was motivated by guidelines from Bhattacharya (2017) and Hatch (2002) to analyze the interview data. This method of analysis enables the development of themes from the interview data which was used to understand the participants teaching philosophies and practices with regards to teaching post-secondary mathematics. Three themes were revealed by the data concerning the participants' beliefs regarding teaching post-secondary mathematics. These were 1) student-centered instructor beliefs, 2) content-centered beliefs, and 3) beliefs on student practices. Three themes were revealed by the data concerning practices the participants deem relevant to the teaching and learning post-secondary mathematics. These were 1) student-centered instructor practices, 2) content-centered practices, and 3) student practices. The analysis of the data revealed potential influences of the participants' beliefs on the practices they find relevant to teaching and learning, as well as potential implications of these influences on mathematics and mathematics education.

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CHAPTER ONE

INTRODUCTION

The study of college mathematics has changed significantly over recent years. The demand for students with a working knowledge of subjects like calculus has only increased. Today, professions requiring calculus make up 5% of the workforce, which is a proportion that is growing at a rate 50% higher than overall job growth (Bressoud, 2018). It is no secret that science, technology, engineering, and mathematics (STEM) related careers are in demand; and those in demand jobs are requiring more post-secondary education than in the past. According to the *Georgetown University Center on Education and the Workforce*, in 2010, 93% of technical and STEM professions required post-secondary education and training credentials. In addition, in 2010 it was predicted that STEM occupations would grow by 24% between 2010 and 2020, and by 2020 94% of STEM professions will require post-secondary education. Nationally, 80% of the fastest growing occupations depend on STEM skills, and the growth in STEM jobs is six times greater than non-STEM jobs (Noonan, 2017). STEM jobs with bachelor's degree or higher result in a wage premium of over 14% compared to non-STEM jobs with similar educational requirements (Rothwell, 2014).

During August 2011, the Obama administration's Council on Jobs and Competitiveness announced a new plan to increase the number of earned degrees in engineering by 10,000 each year. Furthermore, in 2012 the President's Council of Advisers on Science and Technology, (PCAST), published a report calling for one million additional STEM degrees over the next decade, and projected a 17% increase in the need for STEM graduates (Bressoud, Camp, & Teague, 2012; Holm, 2016). Most recently, the Trump administration announced its vision for STEM education for the next five years (Klein, 2018). This vision includes developing

partnerships between schools and local businesses to bolster work-based learning, helping students learn science, technology, engineering, and math, or STEM concepts through project-based learning, or by solving real-world problems, and boosting digital literacy. Across administrations and for almost ten years, STEM education has been an area of emphasis nationally.

Prompted by these reports and the reality of STEM jobs, the Mathematical Association of America, and the National Council of Teachers of Mathematics have advocated that students not move too quickly through preliminary courses just to ensure that necessary courses like calculus are on their high school transcripts and college applications. They claim that this rush to earn high school credit for these courses is ineffective and counter-productive (Bressoud, 2015).

Bressoud, Camp, & Teague (2012) outlines the progression of the teaching of calculus from the 1950's through 2012. In 1950, the Advanced Placement Calculus program began, which provides an opportunity for secondary students to take challenging curricula through college-level courses. Originally, calculus was a small part of the early AP exams. The emphasis was on unfamiliar problems that drew of all areas of mathematics. This emphasis changed through the 1950's to the knowledge of calculus. The 1970's and 1980's saw the AP program accelerate through school districts in effort to raise secondary school quality by challenging the best students to engage in college-level work while still in high school. By the early 1980's, over 200,000 students were studying calculus in high school, but only 15% of these students actually took the AP Calculus exam.

With this illustration about calculus, in 1987 the MAA started considering the effectiveness of teaching calculus in secondary schools. Through its Committee on the Undergraduate Program in Mathematics, the MAA published a report which made

recommendations on the teaching of calculus in secondary schools. This report identified five problem areas:

1. Secondary school teacher qualifications and expectations.
2. Student qualifications and expectations.
3. The effect of repeating a course in college after having experienced success in a similar secondary school course.
4. College placement.
5. Lack of communication between secondary schools and colleges

The committee also found that many students were short-changing their mathematical preparation so they could make it to calculus by grade twelve. The calculus course they took in high school was nothing like the calculus courses which were offered at the college-level. In response to the five identified problem areas, the committee made seventeen recommendations. The two most important recommendations initiated a joint policy statement by the MAA and NCTM. These recommendations were

1. The calculus course offered in the 12th grade should be treated as a college-level course.
2. Students who enroll in a calculus course in secondary school should have demonstrated mastery in algebra, geometry, trigonometry, and coordinate geometry.

Around twenty years later, the AP situation had improved. By 2009, close to one-third of the 1.8 million high school students who went to college took calculus while in high school, and over 50% of those who studied calculus in high school took the AP exam, where only 15% took it in the early 1980's. However, with the large numbers of students completing an AP calculus

course and taking the AP calculus exam, what was a program designed to engage and challenge the most talented students has turned into an expectation for all students who are going to college, and are able to keep up with the fast past curriculum. By the 2011-12 academic year, over 650,000 students were studying calculus in high school, 50,000 more than in 2009. It is also expected that every school should offer AP calculus or some calculus course offering college credit. Bressoud et al. (2012) claims that too many students are being accelerated through their math curriculum which is sacrificing their preparation and knowledge of algebra, geometry, and trigonometry.

It now seems reasonable to determine what effective practices can be instituted by math departments to produce successful math majors, well prepared to enter their chosen profession, whether that be teaching, industry, or research in academia. Although there is no single formula for successful teaching of college mathematics, there is considerable evidence that separates certain practices that have proven to be successful from those that are generally ineffective. Teachers who study this evidence can learn much from the experiences of others (MAA & AAC, 1990). Some of these known practices are discussed in the literature review of this study.

Issues and Problem Statement

Even with this growing need for in-depth study of post-secondary mathematics education, there is the lack of research in teacher education, professional development, and effective teaching practices (Speer et al. 2010). Speer et al. (2010) goes on to describe how there is a diverse and growing body of literature where researchers have considered aspects of collegiate mathematics teaching, but without direct focus on what teachers say, do, and think about in collegiate classrooms in an extensive or detailed way. And while there is a considerable amount of research in K-12 mathematics education, the implications of this body of research does not

transfer directly to post-secondary mathematics education because of unique issues and characteristics that define post-secondary mathematics teaching and learning. Some of these issues and characteristics include the following:

1. The difference in teaching preparation of novice post-secondary teachers as compared to novice K-12 teachers (Lewis & Tucker, 2009)
2. The academic background of novice post-secondary teachers (Nardi, 2015).
3. The overall culture of post-secondary mathematics and teaching (Dawkins, 2013; Jaworski, 2017).

It is typical for novice post-secondary teachers, usually new graduate teaching assistants, to arrive with no teaching experience, not even the standard in-service teaching experience one would expect from a new K-12 teacher (Lewis & Tucker, 2009). Furthermore, the academic backgrounds of these novice instructors usually consist of an undergraduate degree in mathematics or a related science, rather than in mathematics education. While these prior qualifications and experiences are important, the newcomers need a broadened understanding of how to read and converse in the unfamiliar area of mathematics teaching and learning (Nardi, 2015). Furthermore, the prior experience of undergraduate and graduate mathematics can bring with it a certain culture of teaching and learning (Dawkins, 2013; Jaworski, 2017). This culture can sometimes bring certain impressions and assumptions about what constitutes a correct solution, a valid argument, or a rigorous proof. When transitioning to the scholarly community of mathematics education from pure mathematics, novice instructors can encounter a rethinking of epistemological beliefs. As Nardi (2015) writes, this is even more true for those who arrive in mathematics education study from a purely mathematical background.

Given the importance of preparing post-secondary mathematics instructors, it is reasonable to identify effective practices. The focus of this dissertation study was to conduct an in-depth study to examine post-secondary instructors of mathematics in order to determine instructional practices that support substantial engagement with mathematics. This identification of practices contributes to efforts that inform the improvement of instruction in post-secondary mathematics.

Purpose of the Study

The purpose of this study is to determine significant similarities and differences among post-secondary mathematics instructors with a focus on how they teach and engage students in learning mathematics. Research confirms some of the unique characteristics of post-secondary mathematics instruction (Dawkins, 2013; Jaworski, 2017; Lewis & Tucker, 2009; Nardi, 2015). Some of the similarities and differences among post-secondary mathematics education practices fit within categories that include teaching preparation, academic background, and cultural issues. Therefore, the study sought to explore these characteristics and to possibly identify others, with the intention of improving the overall effectiveness of mathematics teaching at the post-secondary level. The full details of the implementation of the study are included in chapter three entitled methodology. However, I will state that the data collection method involved individually conducted interviews of post-secondary mathematics instructors at a large university located in the Southeastern United States. Participants were selected based on the courses they taught over the past three years. These courses were divided into three categories, which include freshmen and sophomore level courses, junior and senior level courses, and graduate level courses.

Research Questions

Based on the statement of purpose of this study, I strived to answer the following questions:

1. What are the similarities and differences among the teaching philosophies of post-secondary mathematics instructors?
2. What are the specific practices identified by post-secondary mathematics instructors as critical to the teaching and support of students learning post-secondary mathematics?
3. How do the teaching philosophies of post-secondary mathematics instructors influence their practices which are used to teach and support students learning post-secondary mathematics?

I conducted in-depth qualitative interviews as the method of data collection. The details of his process are included as chapter three of this dissertation.

The Motivation and Significance for the Proposed Study

Motivation for the proposed study. My motivation for pursuing this study initiated with my personal experience as an undergraduate and later graduate student in mathematics, where I personally experienced an overwhelmingly drastic transition from freshman and sophomore level mathematics courses to junior and senior mathematics level courses. This transition involved changing from coursework where procedural based computations comprise a large percentage of the content, to coursework where generalization and abstraction are the primary focus and computations are practically non-existent (Alcock & Simpson, 2002). It is important to note that abstraction is not a characteristic only of upper-level mathematics. Subjects such as calculus and linear algebra, which are usually studied at the freshman and

sophomore level, can be heavily abstracted. For this reason, we should regard abstraction as being relative to the subject being studied. For example, an eighth grader studying elementary algebra could reasonably find an equation in the real variable x to be abstract. While at the same time, a student of graduate mathematics may find the same equation to only be a generalization. It should be noted that I use the term *generalization* to mean a representation of a concrete object. In this case x represents an unknown real number, which is reasonably concrete. In contrast, *abstract* could mean that x represents an unknown element of an ordered field, which is not necessarily concrete. Therefore, abstraction involves how a topic or idea is taught or presented to students. Furthermore, this example shows that the proposed study has significance for K-12 education as well as post-secondary.

The shock of transition to the abstract is not an uncommon event for undergraduates. The literature provides examples of how past learning from secondary, university freshman, and sophomore level courses affect learning at the upper undergraduate level. Many students have difficulties with post-secondary mathematics because of inadequate preparation from their school experience for the degree of abstraction and formalization experienced at the university level (Jaworski, 2016). In upper level mathematics classes like differential equations, fundamentals of mathematics, and abstract algebra, students see abstract concepts for the first time (Celik & Shaqlaih, 2017).

Another motivation for initiating this study is the consideration of two agendas for teaching mathematics that impact how students engage with the content. These two agendas are *knowledge of mathematics* and *knowledge about mathematics* (Lampert, 1990). I believe that both agendas are critical, and the proposed study allows for better understanding of how effective instructors address these two agendas in their own teaching.

Zooming in on these two agendas, Lampert (1990) states that at every level of schooling, and for all students, reform in math education should include students making conjectures, abstracting mathematical properties, explaining their reasoning, validating and discussing their assertions, and questioning their own and others thinking. Lampert's study focused on a 5th grade classroom doing a lesson on exponents. Students were given the freedom to suggest their own hypotheses to answer questions given by the teacher. In this way, students were allowed the freedom to exert their own mathematical authority. After explaining how the lesson proceeded Lampert describes the teacher as a representation of what it means to know mathematics. The teacher serves as a role model for the students that demonstrates what it looks like for an expert to know mathematics in the way that the students show know it. She even refers to the need to prove to her students any mathematical assertions, and make explicit the knowledge being used to construct a mathematical argument. The teacher must also have the ability to follow students' arguments as they "zig zag" around the math that supports their assertions. Lampert's study highlighted two agendas. One was related to students learning the skills and knowledge necessary to use mathematics as a tool. Lampert refers to this as the *knowledge of mathematics* or mathematical content. The other agenda involved the students learning how to participate in mathematical discourse. Lampert refers to this as *knowledge about mathematics* or mathematical practice. (Polya, 1954) suggests that *knowledge about mathematics* starts with guessing before proving, then combining observations, trying and trying again until your reasoning works.

My personal experience in mathematics includes both of Lampert's agendas. However, I have always included Lampert's *knowledge about mathematics* as a component of mathematical content. More specifically, I refer to this as *mathematical wisdom*. My definition of *mathematical wisdom* includes Lampert's concept of *knowledge about mathematics*, and is a

learned skill that is acquired with experience. However, in addition to Lampert's concept, mathematical wisdom includes the ownership of mathematical knowledge for oneself independent from what may have been learned in the classroom. For example, I remember being in my algebra courses in high school and calculus courses in college, but I do not remember the specific moments I learned most of the individual ideas or concepts from algebra or calculus that I now teach. They have been replaced by my own interpretations and derivations. Again, this only something I achieved through experience.

How mathematics is taught at the post-secondary level impacts the opportunities that students experience that are truly engaging with abstraction and substantial mathematical discourse. Jaworski (2016) explains how students gain powerful skills in abstraction and discourse that impact their work as engineers and scientists in society. Furthermore, learning at the university level is also seen as acculturation in advanced mathematics (Borko 2004; Dawkins, 2013; Jaworski 2016). This acculturation includes learning methods of abstraction and formalization necessary for advanced mathematics.

The importance of teaching mathematics in this way extends to the preparation of preservice teachers. Hodge, Gerberry, Moss, & Staples (2010) also compiled opinions of university professors concerning the important characteristics of successful pre-service mathematics teachers. One relevant opinion is that teachers should have the ability to think *mathematically*, which is equated to understanding notions of abstraction and proof, which is also referred to as the “culture” of mathematics which is passed on to students also mentioned by Borko (2004); Dawkins, (2013); Jaworski (2016). Hodge, Gerberry, Moss, & Staples (2010) cites upper-level mathematics courses which relate content to future teaching strategies as being effective in serving the needs of future secondary mathematics teachers.

In summary, this study is motivated by the need for students of post-secondary mathematics to transition from the procedural based mathematics of secondary education to the abstract and rigor of post-secondary mathematics found in calculus and proof-based concepts (Alcock & Simpson, 2002). Some Important aspects of this transition include the acculturation into advanced mathematics (Borko 2004; Dawkins, 2013; Jaworski 2016), and the acquisition of knowledge of and about mathematics (Lampert, 1990; Polya, 1954). I will now mention the significance of this study and why it is worthy our attention.

Significance for the proposed study. This study is significant in several ways. I will begin this section with a statement about the significance of post-secondary mathematics education and why it is worthy of study. Second, I discuss how the study contributes to efforts that bring together the fields of mathematics and mathematics education. Third, I discuss how the study is positioned to contribute to the knowledge base about effective teaching practices at the post-secondary level. This is important for the preparation of a number of professions, including future mathematics teachers. Fourth, I explain how the study can potentially inform the preparation of graduate students and faculty in mathematics. Fifth, the methodological approach of focusing on positive cases highlights the voices and perspectives of the instructors themselves.

First, as stated in the introduction, calculus is required by 5% of professions in the current workforce, and the proportion of professions requiring calculus is growing at a rate 50% higher than overall job growth (Bressoud, 2018). However, beyond the external importance of learning mathematics, there is a need for research in post-secondary mathematics teaching purely within the fields of mathematics education and mathematics.

Second, this research would serve to strengthen the connection and cooperation between these two fields. Dörfler (2003) raises the points a) mathematics and mathematics education operate as two cultures, and b) there is the potential to bridge the gap between the two fields through research. Regarding this, Dörfler (2003) lists several points that illustrate a disconnect between the fields of mathematics and mathematics education. These disconnects include the following:

1. Mathematics and mathematics education are rarely in the same department and without structural and organizational separation
2. Mathematics and mathematics education teaching are separate where there is very little crossover between a mathematics professor teaching a mathematics education course or a mathematics education professor teaching a math course.
3. The education of mathematicians is highly homogenized and standardized all over the world, and the demands of content covered are widely accepted. Math educators come from diverse fields with few having a mathematical education beyond a certain level due to less homogeneity in background and academic careers.
4. Mathematics and mathematics education are generally organized into separate professional organizations. For example, professional mathematicians are usually members of the Mathematical Association of American and the American Mathematical Society. Mathematics educations are usually members of the National Council of Teachers of Mathematics and the Association of Mathematics Teacher Educators.
5. There are wide spread assumptions and misconceptions on both sides about what the people in the other scientific community do and what their work is about.

6. Negative opinions and prejudices persist about the respective other field and its representatives exist.

Regarding his second point, Dörfler (2003) puts forth a strand of research that can bridge the gap between the two fields. He defines this research as “mathematicology”. He posits that mathematics is a product of the activity of mathematicians and has been standardized, conventionalized, and corroborated by extended experience and usage. Consequently, mathematics education must study and investigate mathematics as it is practiced, produced, and used in all its forms. In this sense mathematics education must study mathematics in a broad sense that includes mathematics as a human phenomenon, an activity, and a product as well. This is the type of study of mathematics that Dörfler refers to as “mathematicology”. Dörfler states that mathematicology should not lose sight of the human being, which means that it should not detach itself from the people who do the mathematics, in other words the mathematicians and the students. Dörfler provides many examples of research topics for mathematicology that include the study of mathematics and of human activity in mathematics. Some include:

1. The quality and structure of mathematical concepts and theories.
2. Symbols and symbolization in mathematics
3. Abstraction and generalization
4. Metaphors, mathematical language, and discourse
5. Relations between mathematics and other sciences

These examples of research topics can enable mathematicology to develop theoretical descriptions and models of mathematical activities and processes, with the goal of reflecting on what mathematicians and learners of mathematics do or do not do. Dörfler also provides potential usages and insights from mathematicology. These include:

1. Informing and motivating research in mathematics education
2. Making curricular decisions that are informed and legitimized by a deeper understanding of mathematics
3. Organizing, planning, designing, and monitoring applications of mathematics
4. Talking about mathematics as a body of knowledge and as a human activity
5. Epistemological, social, and psychological analyses of mathematics might lead to substantiated critique of certain practices in mathematics, for example the exactification of calculus in the early 19th century
6. Benefits of historical studies for mathematics education
7. Conveying the intentions and characteristics of mathematics to a broader audience
8. Recognizing mathematics as a language, and treating it as such in school, like learning a foreign language where one not only studies grammar and structure, but also the culture in which the language is embedded.

To summarize Dörfler's mathematicology, it can be concisely defined as research in mathematics education that studies and investigates mathematics as it is learned, practiced, produced, and used by people. It seeks to develop theoretical descriptions and models of mathematical activities and processes as human activity. Dörfler views mathematicology as a way of building functional and mutual enriching relationships between mathematics and mathematics education by increasing collaboration between mathematicians and educators. This could be accomplished by looking for issues in mathematics that have features analogous to aspects of mathematics education. It is the responsibility of both mathematicians and educators to increase communication and connections between both groups (Fujita et al., 2000). I am

including Dörfler's mathematicology as an example of bridging the academic gap between mathematics and mathematics education.

Limitations and Delimitations

This research had factors that limited the study in specific ways. For this reason, the methodology of this study took steps to ensure the credibility of the results. In this section I will address the limitations that relate to the participants and being the researcher.

The primary limitation of this study involved myself as a data collection tool. I therefore will declare any potential bias or assumptions that might otherwise affect the outcomes of the study. Creswell (2014) states that a researcher should clarify the bias they bring to the study, and that self-reflection creates an open and honest narrative that will resonate well with readers.

Some of the study participants were colleagues or former instructors of mine. This is almost unavoidable because the university where the study participants teach is the same university where I studied mathematics. This was an advantage to this study. The rapport I have with some of the participants did aid in the depth and richness of their responses in the interviews.

I am limiting the scope of this study to the effective teaching of post-secondary mathematics. This is to provide focus and clarity. I gathered a total of eight participants for the study. This delimitation was chosen to attempt to achieve depth into the understanding of the participants' individual teaching practices to learn what specifically makes them effective at teaching post-secondary mathematics. Choosing a larger sample would have decreased the likelihood of achieving this depth of understanding. Given the timeline of this study, I have chose to delimit the collection of data to interviews that will be conducted individually. I transcribed all the interview data myself, thereby completely immersing myself in the data.

Theoretical Framework

The theoretical foundation of this study is based on the philosophical ideas of Dörfler (2003). One of the end goals of this study is to help bridge the gap between mathematics and mathematics education. I see Dörfler's mathematicology as an excellent viewpoint through which to view this research. With that being said, I included interview questions that served to investigate mathematics as it is practiced, produced, taught, and learned by people. I not only questioned the participants about mathematical content, but also about mathematics as a product of human beings and as a human activity.

Dörfler provided several examples of research that includes the study of mathematics as a human activity. Some of these examples include recognizing mathematics as a language with its own grammar and structure in the same way that a foreign language is treated in school, making curricular decisions that are informed and legitimized by a deeper understanding of mathematics. These two examples accurately represent the way that I view mathematics, as a language, as well as how educational decisions about mathematics should be made, by an informed and legitimized deep understanding of mathematics.

In additions I also draw on the philosophical ideas of Imre Lakatos, Paul Ernest, Reuben Hersh, and George Polya (MAA, 2017; Polya, 1954). Their ideas share the common root of social constructivism where mathematical objects are socially constructed abstract objects, constructed by the mathematical community (MAA Notes, 2017). A goal of Lampert (1990) was to have students prove their mathematical assertions. In terms of rigor, proof interpreted by social constructivism is what the mathematical community says it is. What counts as legitimate mathematical justification depends on the audience and time. Therefore, the validity of the proofs written by the students in the Lampert study was something to being negotiated by the

classroom community. I applied this point of view when writing my interview questions and applied this idea to interpreting the interview data as I conducted the analysis for the study.

I will draw on Lakatos's concept of proof described in Currie (1976) as being motivated from a concern of the growth of informal mathematical knowledge rather than mathematical ontology. Specifically, when trying to prove a conjecture that is believed to be true, we derive the proof from accepted premises. If the conjecture is believed to be false then we attempt to produce a counterexample that would refute the claim made by the conjecture. In the case of proving the conjecture, the premises usually take the form of axioms which are fundamental truths that form the basis for the discipline. Since this is a growing branch of mathematics, a complete set of reasonable axioms or a list of counterexamples may not exist. Lakatos involves decomposing the conjecture into a collection of "smaller" sub-conjectures that are applicable to some other body of mathematical knowledge, where those sub-conjectures imply the result in the original conjecture.

The key part of this concept is the negotiating of the relevant sub-conjectures. This is how I envision interpreting what counts as a legitimate practice of post-secondary instructors. The similarities and differences detailed in the analysis and results section will be negotiated by the interview data. While this list of similarities and differences may not create a definitive characterization of an post-secondary instructor, it would develop a list of sufficient characteristics which would give a sufficient basis for an instructor to be considered effective under the lens of social constructivism, as grounded in the contexts in which the analyzed occurs.

Reflexivity Statement

My personal philosophy concerning teaching post-secondary mathematics can be summarized in two major points. The first is the development of *mathematical wisdom*. The second is the successful assimilation into the culture of post-secondary mathematics.

First, I define *mathematical wisdom* as consisting of components of *knowledge about mathematics* defined by Lampert (1990) and Polya (1954) which includes the ability to participate in mathematical discourse, and the ability to combine mathematical observations to provide the reasoning necessary to build a mathematical argument. More specifically, *mathematical wisdom* is the well-practiced ability to understand and use mathematics as a language to communicate ideas to others via a mathematical argument. It is with mathematical wisdom that I believe one can begin to see the “big picture” with respect to mathematics and teaching.

As a student of both undergraduate and graduate level mathematics, I know from experience the requirements needed to succeed in a field that demands abstractness and rigor. In my opinion, to be effective at teaching this level of mathematics it is necessary for instructors to see the “big picture” of the content they teach. This requires being proficient in mathematics that is at a higher level than the mathematics they would normally teach. It also requires a broad vision of mathematics.

For example, to be effective at teaching freshman level calculus would require being familiar with senior level advanced calculus. With the goal of having students see the big picture at their level of mathematics, I have found being proficient in upper-level undergraduate and graduate level mathematics to be an invaluable teaching tool. Take for example the content studied in multivariable calculus. The primary goal in that course should be to explore the

similarities and differences among the fundamental concepts of integration, differentiation and continuity as compared to the same concepts in single variable calculus. In my experience, having a point of view that includes knowledge of matrix algebra, linear transformations, Taylor's theorem in several variables, and ideas of existence and uniqueness, all of which are not usually studied in multivariable calculus, is necessary to plan a lesson effective in facilitating students' understanding of these similarities and differences.

As a young student of secondary level mathematics and eventually early post-secondary undergraduate mathematics, I had no concept of the "big picture". This had to do with the culture of mathematics in K-12 schools which emphasizes speed, procedures, and memorization (Stigler & Hiebert, 1999). As I entered undergraduate level study I took the typical freshman and sophomore level math classes which included the three-semester calculus sequence, linear algebra, and ordinary differential equations. Thinking back on that experience I realize that I had no clue as to what the "big picture" was concerning these courses, and how all the content fit together. I remember my linear algebra professor telling me that the individual concepts as they only related to that course were not important, it was how they would be used later in other courses that was important. At the time, I could not understand what he meant because of my lack of experience. Now I realize exactly what he meant since I have had the experience to use those concepts in a deeper and much more meaningful way.

Finally, my concept of mathematical wisdom includes facilitating students' deeper and broadened understanding of important concepts early on during the foundational study, rather than waiting on the topics to reappear in a later course. I make it a goal to point out connections between the different STEM courses that students are taking. Some examples include linear

algebra and differential equations, or differentiability in multivariable calculus and the concept existence and uniqueness that a student may see in an abstract proof-based class.

The culture of college and university level mathematics is unique and significantly different from K-12 mathematics since it is characterized by proving theorems using precise definitions (Alcock & Simpson, 2002; Dawkins, 2014; Weber, 2004). This characterization not only distinguishes K-12 mathematics from post-secondary mathematics, it also tends to distinguish mathematics from mathematics education. Perhaps this is due to the lack of literature on post-secondary teaching practices (Speer et al. 2010). I believe the culture of post-secondary mathematics promotes a lack of knowledge of effective teaching practices among some teaching mathematicians. This is true even when the duties of a university mathematician include teaching. It should be noted that I am not criticizing the teaching that university professors do. More than likely, their method of teaching is influenced by their beliefs about mathematics, students, and education, as well as their knowledge of the material being covered (Weber, 2004).

Personally, I choose not to separate the mathematics from the practices I find to be effective at teaching post-secondary math. I see mathematics education as an area of mathematics in the same way that analysis, algebra, topology are areas of mathematics. In this way, mathematics education is an area of mathematics that intersects all other areas of mathematics, in a way similar to how linear algebra permeates through so many different areas of mathematics. Furthermore, I do not separate the job of teaching mathematics from the job of doing mathematics. At some point every student of mathematics realizes that teaching someone else a concept helps reinforce and strengthen their own understanding of that concept. It helps you explore ways of thinking of the concept that you may not have considered before. Furthermore, it should go without saying that doing and speaking the language of mathematics

facilitates the ability to teach mathematics. In this way, teaching mathematics and doing mathematics are one and the same for me. For me this is a necessary characteristic of an effective instructor. I should note that it is not my intention to find flaws or problems with current methods of instruction, but to add to the already developing body of knowledge and literature which speaks to the issue of effective post-secondary teaching. This study will only examine cases of instructors who are known to be effective at teaching post-secondary mathematics. As a graduate student of both mathematics and mathematics education, I believe I am in a unique position to strengthen the connection between these two fields which too often seem deliberately segregated.

Definition of Terms

The following is a list of definitions and ideas that are important to the study:

Post-Secondary Mathematics: Mathematical concepts and topics which are traditionally studied in courses at the elementary calculus level and above. Specifically, this includes undergraduate calculus, differential equations, linear algebra, abstract algebra, analysis, advanced calculus, numerical analysis, and probability.

Post-Secondary Mathematics Instructors: Mathematics instructors who routinely teach courses at the elementary pre-calculus level and above.

Mathematical Wisdom: Mathematical wisdom consists of components of *knowledge about mathematics* defined by Lampert (1990) and Polya (1954) which includes the ability to participate in mathematical discourse, and the ability to combine mathematical observations to provide the reasoning necessary to build a mathematical argument. Specifically, *mathematical wisdom* is the ability to understand and use mathematics as a language to communicate ideas to others via a mathematical argument.

Mathematically Abstract: A mathematical concept or topic is abstract if it has been generalized to the point where any dependence on a real-world context has been removed.

Direct Association: An association between a belief and a practice is *direct* if a participant's belief is associated to a practice which the participant says is actually performed by someone as opposed to being hypothetical practice

Indirect Association: An association between a belief and a practice is *indirect* if a participant's belief is associated to a practice which the participant does not say is actually performed by someone or is clearly stated as a hypothetical practice.

Internal Practice: A practice is *internal* if it is a practice of a participant.

External Practice: A practice is external if it is not a practice of a participant and is a practice of another individual.

Organization of the Study

This dissertation is organized into five chapters. Chapter One contains the introduction to the study, the statement of the problem, the purpose of the study, the motivation and significance, limitations, delimitations, a definition of terms list, the theoretical framework of the study, and a reflexivity statement. Chapter Two discusses a review of the literature that concerns teaching practices at the post-secondary level as well as the secondary level. Chapter Three explains the research methodology and data analysis procedures of the study. Chapter Four presents the findings from the data. Chapter Five discusses the implications from the data analysis.

CHAPTER TWO

LITERATURE REVIEW

The purpose of the review of the literature is to provide context for the proposed study. I will address three areas of scholarship relevant to the study. The first area provides context to post-secondary education by defining the mathematics and challenges that take place within post-secondary mathematics instruction. The second area of research focuses on the challenge of teaching mathematics at the post-secondary level. The third area focuses on scholarship addressing K-12 mathematics education, given that the research in this area is extensive and applicable in some respects to post-secondary education.

Mathematics at the Post-Secondary Level

As mentioned previously, post-secondary mathematics has an intended goal of producing theory that is rooted in abstraction. The strength of abstraction is that it makes possible the ability to build theory, which consequently yields results that are applicable to many different mathematical situations. It would not be worthwhile to have a mathematical result that is only useful for one specific application.

While building theory, it is very easy to consider ideas that are rooted purely in mathematics and not consider questions that concern the “real world”. This is not to say that real world applications are not important. In fact, they are. This leads us to some of the specifics about post-secondary mathematics. Higher-level post-secondary math is traditionally divided into two branches, pure mathematics and applied mathematics. Applied mathematics is mathematics developed for some sort of practical real world use, and frequently used in other disciplines, (e.g. the natural sciences, etc). Pure mathematics on the other hand is mathematics for its own sake. It exists with the intended goal of gaining a deeper understanding of

mathematical ideas themselves. However, applied and pure mathematics have deep connections. There are multiples examples throughout the history of mathematics that important real world applications often develop from and during the study of pure mathematics. A few examples include the Riemann Hypothesis and its connection to prime numbers and numerical encryption, or topological equivalences called homeomorphisms and applications in computer graphics and biology (Ingram, 2017). One may even say that pure mathematics was the catalyst for Alan Turing's first computer, which was a machine developed during World War II that help the allies break the German Enigma machine which was used to transmit coded messages.

Another distinction within post-secondary mathematics can be captured with the term "higher-level". Most of the freshmen and sophomore courses in undergraduate mathematics are not considered to be pure or applied, but simply foundational. Later junior and senior level classes tend to have a pure or applied focus, but even then, a traditional undergraduate program in mathematics is not necessarily labeled as pure or applied. Students are required to take a breadth of courses that span both branches. Courses that are considered pure are abstract algebra and topology. Applied courses include differential equations, and numerical analysis. Some courses like real analysis and probability can fit both branches. Real analysis involves the rigorous study of real-valued functions which includes the subject of calculus. It is here where the literature review of this study will begin.

The Post-Secondary Challenges

This section of the review examines various challenges associated with teaching post-secondary mathematics based on existing literature. The focus includes a discussion of the challenge itself and possible solutions as documented through literature.

The student preparation challenge. One of the post-secondary challenges is how entering students are developing in areas of understanding abstraction and proof. They need further experiences with abstraction and proof to truly be prepared to tackle post-secondary mathematics coursework. The recognition of this problem is not an indictment of secondary instructors, but a point related to curriculum standards and the culture of teaching mathematics. Typically, abstraction and proof are not a focus of secondary mathematics. In many cases, a student may first encounter abstraction or proof in an introduction to abstract mathematics course, or even a course in calculus (Celik & Shaqlaih, 2017; Jaworski, 2016).

The calculus example. Because of the predominance of calculus in current literature on post-secondary mathematics education, and the ability of calculus to illustrate the importance of and transition to abstraction and proof, I will begin with an overview from calculus and then tease out challenges that students typically face with a focus on abstraction.

A typical differential calculus course begins with a significantly different method of studying the behavior of real functions than any method used in an algebra course. This method is an infinite process called a limit. Limits allow a method of studying the slope of a function whose graph is not a line. We should all remember studying linear functions in an algebra course which are functions defined algebraically by an equation of the form $y = mx + b$. The slope of the line is the number m , and the b corresponds to the point $(0, b)$ on the graph of the line. Introductory calculus provides a way of finding the slope m for any non-linear function for which a special limit, called the *derivative*, exists. The derivative concept is a possible example of a student's first encounter with a potential abstraction or higher-level generalization. This concept is derived from a fundamental idea with which students should already be familiar, the slope of a line. I am including this motivation and the following derivation to ground readers

with and without a mathematical background in a fundamental calculus concept to identify calculus as a potential first mathematical abstraction for novice students of post-secondary mathematics.

For students and former students of mathematics, recall from elementary algebra that the slope of a line which passes through the points (x_1, y_1) and (x_2, y_2) graphed on the x, y plane is the real number $m = \frac{y_2 - y_1}{x_2 - x_1}$. In addition, recall function notation that relates the coordinates of the points (x_1, y_1) and (x_2, y_2) with a function called f whose graph passes through these points. This function notation takes the form $f(x_1) = y_1$ and $f(x_2) = y_2$. This notation allows us to rewrite the formula for slope as $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$. The advantage of this form of the slope formula is that it represents the slope of the line which passes through the two points using the notation of a function f which also passes through the same two points. Students then use the limit process on the expression $\frac{f(x) - f(c)}{x - c}$, with $(x, f(x))$ and $(c, f(c))$ being the two points in question. The limit process uses the notation $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c} = f'(c)$ and yields the *derivative* of the function f at the point $(c, f(c))$. This limit process can be done algebraically or numerically. A numerical process involves calculating the slope of several different lines, essentially using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ repeatedly, a process with which students are usually very familiar. Physically, the derivative $f'(c)$ represents a rate of change of motion like speed. Geometrically, the derivative represents the “linearization” of the graph of function $f(x)$ for values of x which are near c . Visually this linearization makes the graph of $f(x)$ look like the graph of a line for values of x which are near c . Finally, algebraically the derivative allows the student to treat the

algebraic expression which defines the non-linear function $f(x)$ as a linear function defined by the familiar $y = mx + b$.

For a person who is unfamiliar with formal mathematics, the derivative can be illustrated among other things as the velocity or speed of a moving object, for example a car traveling on a highway. The speedometer of your car is providing you with the speed of your car at any moment in time. This is precisely the derivative of a function which represents the position of your car as it moves along the road.

The abstraction in the derivative concept is the broadening of the concept of slope that allows the inclusion of non-linear functions, as well including an interpretation of slope at a specific point on the graph of a function, instead of the elementary rate of change between two points which students learn in early algebra courses. This interpretation of rate of change at a point allows the inclusion of a wider list of applications.

Students usually spend a lot of class time learning different ways to calculate the derivative of a function in different situations, requiring the use of different rules which are usually derived in class. After learning the basic rules, students can then use the derivative in applications. The derivative has many applications in the real world. A fundamental use of the derivative is its representation as the rate of change of a quantity, for example the rate of change of the position of an object, also known as velocity. This makes the derivative a very versatile piece of information. A typical student in first semester calculus will apply the derivative to various real world examples like:

1. Calculating the velocity or acceleration of a moving object
2. Calculating the rate of change of one quantity that is influenced by a related rate of change of another quantity

3. Optimizing real world scenarios like maximizing the profit from the sale of a product or minimizing the cost of producing a product
4. Purely mathematical examples like minimizing the distance between two points on the graph of a function
5. The extremely important concepts of linear approximation and differentials which bring the geometric and algebraic representations of the derivative together into one “equation”. These concepts also serve as the bridge between differential and integral calculus.

This summarizes some of the typical fundamentals of differential calculus.

Integral calculus takes what students have learned about the derivative as a rate of change and answers the “reverse” question. For example, if you know the velocity of an object at any moment, what then is the change of position of the object, or the distance the object has traveled? This requires being able to derive the function whose derivative you already know, a process called integration. Here is where students are typically introduced to the most important result in calculus called the Fundamental Theorem of Calculus. This theorem characterizes the relationship between the derivative and the integral, making use of the linear approximation and differential concepts mentioned earlier. Next students learn various methods of integration that are based on some of the rules for finding derivatives, but in general integration is a much more difficult process that at its core is a “guess and check” idea. The course usually ends with an in-depth study of infinite series, which are infinite sums of numbers that may be finite, called a *converging* series, or may be something other than finite, called a *diverging* series. Students derive and apply various tests to determine the convergence or divergence of an infinite series. This leads to the concept of an infinite series of functions of a variable x , where the series may

converge or diverge for different values of x . Finally, with these tests for convergence and divergence, and the idea of a series of functions, students encounter Taylor's theorem which provides an approximation of a k -times differentiable function by a polynomial of degree k . This theorem provides results that are applicable to real world problems and pure mathematics.

Lastly multivariable calculus, typically a third semester course, examines the same ideas as single variable differential and integral calculus, but with functions of several variables as opposed to one. The primary goal in this course is to characterize the similarities and differences among fundamental concepts of limits, slope, derivatives, and integration in several variables compared to the same concepts in one variable. Some of the noticeable differences are the representation of the derivative and slope as a vector instead of a number, the infinite number of possible ways of considering a limit as opposed to the finite number of ways in one variable, and the connection between integration and vector fields. As mentioned earlier, it is common for students to encounter difficulties while considering abstract ideas, and calculus can serve as a student's first encounter with abstraction within their post-secondary experience. I will now discuss what literature has to say about possible solutions to this issue.

Literature on effectiveness in calculus instruction. Bressoud and Zorn (2018) discuss current issues and efforts to support students in calculus. Large numbers of students with the goal of entering a STEM related field enroll in calculus courses each year. However, a large percentage will fail to pass this course. Champion and Mesa (2017) provide evidence that by the time a student enters the ninth grade, their current mathematical knowledge, course placement, and socio-economic status will predict whether they will complete a calculus course before leaving high school. Furthermore, only 19% of high school students are predicted to complete a calculus course in high school. In the past, most students in calculus courses were the best and

most prepared among their peers. Now there is an influx of students who are not prepared to take the course. This situation is due partially to the overall increase of undergraduate enrollment over the last two decades. According to the *National Center for Education Statistics*, between 2000 and 2017 total undergraduate enrollment in degree granting post-secondary institutions increased from 13.8 million to 16.8 million students. Furthermore, by 2028 total undergraduate enrollment is expected to increase to 17.8 million students. In addition, the *National Science Foundation* cites a survey administered by the Higher Education Research Institute at UCLA which reports that in the year 2000 approximately one-third of all freshmen planned to study a STEM field. By 2016 this proportion rose to 45%. Since calculus serves as a “gatekeeper” course for many STEM related fields, it is now a goal of many public universities to implement changes with respect to their calculus and pre-calculus courses to combat high failure rates.

An example of this change can be found at the University of Evansville with their *Resequencing Calculus* project. Dwyer and Gruenwald (2017) describes how the traditional three semester calculus sequence is being re-sequenced to more closely align with the other STEM courses in engineering, chemistry, biology, economics, or physics. Specifically, some topics from the traditional calculus II, (integral calculus), and calculus III, (multi-variable calculus), courses are being re-ordered to allow for an easier transition from calculus in one variable to calculus in several variables. Sequences and series which are traditionally in calculus II have been replaced by vectors, functions of several variables, and double integrals, which are usually first studied in calculus III. Sequences and series have been moved to calculus III. This not only eases the transition from functions of one variable to functions of several variables, but also moves the most difficult topics in calculus II to calculus III which evens out the difficulty

among all the courses. Also, Taylor polynomial approximations are now presented in calculus I where future engineers can learn it from a mathematician before they use it in an engineering context. The University of St. Thomas, along with other numerous institutions, has piloted this program. Success was measured as the percentage of calculus I enrollees who eventually passed calculus III. The re-sequenced calculus success rate was 58.3% compared to the traditional three-semester calculus sequence whose success rate was approximately 30%. I mention this example specifically to highlight the modification of texts and the reshaping of curriculum as an example of supporting positive change in post-secondary mathematics teaching.

One of the unique challenges that calculus provides many students is being the first course where the path to a solution is not something that can be provided by an algorithm. Calculus solutions tend to be more analytical and “proof-like”. Some may consider this a property of abstraction. It should be noted that this is not an inherent property of calculus. Traditional methods of teaching that involve “passive learning” enable algebra courses to be taught in a manner that promotes memorization and repetition. Algebra can be taught in a manner that supports inquiry and abstraction (Roschelle et al, 2008). It therefore seems reasonable that any example of effective teaching in calculus would require a method of transition from algorithmic based solution methods to analytical based methods. This could be described as a shift to a more “active learning” approach.

Active learning. Currently, U. S. mathematics classrooms have been described using passive learning where teachers demonstrate procedures by working through several examples in a lecture form. Some teachers may present long demonstrations during a lesson and give students the rest of the time to work on assignments (Stigler & Hiebert, 1999). This could describe the typical classroom in the U. S.

To further explore this idea of active learning, consider the work of Petress (2008). *Active learning* can be described as exhibiting the following traits of student behavior: not overtly dependent on a teacher, asking questions of clarification, challenging ideas and procedures, following up learning sessions with personal inquiries, connecting the most recently learned concepts with previously learned concepts, and keeping an enthusiastic attitude. Petress (2008) goes on to list many traits of passive and active learning in terms of student behavior.

Adams and Dove (2018) gives an example of active learning in the calculus classroom using the “flipped classroom”. The flipped classroom can be described as a reversal of the traditional roles of class time and time spent outside of class. Prior to class students will gather information for the forthcoming class meeting, perhaps viewing lecture videos. Class time is used for more active learning experiences (Petress, 2008). The study done by Adams and Dove (2018) focuses specifically on the needs of Millennials who believe that effective instruction should involve variety, collaboration, and the interactive use of technology. It is worth noting that Bressoud and Zorn (2018) highlights how the current make up of calculus courses includes a large percentage of underprepared students. Even with the diversity of the average classroom changing, due to an increasing population of non-traditional students, the results of Adams and Dove (2018) are applicable to the current population of underprepared students due to the number of current calculus students being in the Millennial generation. The results of a study performed by Adams and Dove (2018) on a flipped calculus course of 23 students reported 19 students completing the course and quantitative data suggesting that the flipped class had strong significant impacts on the students’ learning and retention of content. In every measure of achievement defined by the study, the flipped classroom outperformed the traditional classroom.

I mention this study not as an endorsement of flipped classrooms, nor as an indictment of traditional classrooms, but to highlight non-traditional class structure as a potential method of enhancing post-secondary mathematics instruction.

A final example of active learning in calculus is Crawford et. al (2018), where active learning takes the form of discovery based learning. Crawford explains in the introduction by quoting *Beyond Crossroads* from AMATYC, that “effective mathematics instruction should require students to be active participants”. The mathematics department at Jacksonville University describes their success at effective instruction through discover based learning as a process where students take the lead making conceptual sense of a topic, for example deriving methods to evaluate integrals using integration by parts in calculus II. This might involve having students work in groups, presenting their problems on white boards, and solving open-ended questions or real-world problems with the skills they have previously acquired. While this article did not report results on an intervention or study, it did report conclusions regarding properties of effective teaching in calculus. Some of these include minimizing the length of lectures, introducing technology to enable students to explore visualizations more easily, and requiring students to write since writing requires students to bring all pieces of a solution together. Discovery based learning, as an example of active learning places Crawford in line with Chickering & Gamson (1989) which identified seven principles of good practice in undergraduate education. They include student-faculty contact, cooperation among students, active learning, prompt feedback, time on task, high expectations, and communicating high expectations.

The instructional practices of *resequencing* and *active learning* are both possible solutions that were made by instructors. This is important since this study aims at discovering

more choices of instructional practices made by post-secondary mathematics instructors. Ideally, this study's goal was to obtain effective instructional practices for learning mathematics at and above the calculus level, extending into elementary proof writing, abstract algebra, analysis, etc.

Literature outside of an explicit focus on calculus provides recommendations for enhancing post-secondary mathematics instruction. This literature includes Graves et al. (2009) which speaks of assessing effectiveness in teaching mathematics. It lists creating a student growth atmosphere, recognizing different learning styles, and adapting teaching styles, highlighting available resources and aids for learning, and considering feedback from multiple sources as qualities that must be developed to effectively teach in a problem-solving mathematics curriculum. These are in addition to Collier (1999) who identifies classroom performance, curriculum planning, materials development, and assessment as primary aspects of effective teaching. The intended goal of this study, to discover new instructional practices for learning mathematics at and above the pre-calculus level, should confirm some of the effective practices as well as adding others.

Post-Secondary Mathematics Teaching Challenge

Though related to the previous section about student challenges, I will now discuss literature that speaks to the challenges related to teaching post-secondary mathematics. Professional mathematical associations like the *American Mathematical Association* and the *Mathematical Association of America* have goals to enact meaningful change in current post-secondary teaching. Holm (2016) describes the efforts of many of the professional mathematical associations to transform post-secondary education in mathematics. The group *Transforming Post-Secondary Education in Mathematics*, which was founded by Phillip Griffiths through cooperation with the Carnegie Corporation of New York, works to support the mathematics

community to negotiate the changes in education due to the increased financial pressure on colleges, universities, and students, as well as the demands on mathematics departments who frequently teach the largest percentage of their institutions' student bodies.

An AMS Task Force on Excellence, pre-TPSE, first took an historical accounting of post-secondary education in mathematics to help explain the current state of the field. The task force proclaimed, "To ensure their institution's commitment to excellence in mathematics research, doctoral departments must pursue excellence in their instructional programs" (Ewing, 1999, p. 3). As part of this recommendation, departments must maintain a relevant and broad curriculum. In addition to what they must teach, departments must also address questions of how to teach mathematics (Ewing, 1999). The task force advanced three areas that merited a department's initial attention (Lewis & Tucker, 2009). These areas were:

1. Harness the power of technology to improve teaching and learning.
2. Leadership matters. Success in this area depends upon the value assigned to it by a department's leadership.
3. Invest in teaching graduate students to be good teachers.

Holm highlights the teacher education of graduate students extensively. Preparing graduate students to teach, a particular role for research universities, is intertwined with any discussion of undergraduate education. Graduate students represent the future of the professoriate (Holm, 2016). TPSE has brought together many of the professional associations that include post-secondary mathematics education as part of their primary mission, like the MAA, AMS, and the Society for Industrial and Applied Mathematics, SIAM. This cooperation between associations has highlighted some impressive programs at research universities. The AMS Committee on Education has noted the programs at the University of Michigan, and the

University of Illinois. Michigan's math department was previously noted by Ewing (1999) as having a culture that encourages and rewards innovation, one that is well rounded, that strikes a balance between teaching and research, and that supports the work of students and colleagues at all levels. The University of Illinois was noted for its collaboration with engineering faculty to develop workshops, where students of calculus are able to use their knowledge to solve real world applications. This is a connection to Dwyer and Gruenwald (2017) and the *Resequencing Calculus* program, where traditional calculus topics were rearranged to align with other science courses for the purpose of making connections.

Upon reflection, Holm posits that the mathematics community must maintain the bridges between research in mathematics education and practitioners of mathematics education, particularly at the post-secondary level. I use Holm (2016) to highlight the importance of teacher education in post-secondary mathematics education, specifically the teacher education of graduate students. The remainder of this section will include literature on post-secondary mathematics teacher education. This will highlight the importance of considering university support that strengthens graduate assistant teaching. This can be accomplished through department initiatives, campus-wide centers on teaching and learning, and stronger connections between colleges, departments of education and mathematics departments.

Graduate teaching assistantships. As a former graduate teaching associate of a Ph.D. granting department of mathematics, I can speak to some of the common issues encountered by math graduate teaching assistants, GTAs. Unlike degree programs in mathematics education where applicants usually have a master's degree and several years of teaching experience, applicants of a doctoral program in mathematics typically only hold a bachelor's degree in mathematics or a related science, and have little to no teaching experience (DeChenne et al.,

2012; Speer et al., 2005). Therefore, a GTA's aptitude for quality teaching is usually not able to be considered because of the lack of teaching experience. Given the fact that their assistantship involves teaching mathematics and not simply doing mathematics, it also seems reasonable to value an applicant's ability to perform quality teaching.

Lewis and Tucker (2009) provides two important reasons for a mathematics department to make it a priority to prepare graduate students to be successful teachers. The first reason is for the department's own self-interest. Most mathematics departments employ graduate students as teaching assistants. These teaching assistants usually lead recitation sections of a course or are fully responsible for their own classes. Furthermore, these classes are primarily at the freshman level (Miller et al., 2018; Speer et al., 2005). If a math department cannot show quality teaching in the courses taught by graduate students, the institution may decide to use the graduate student funds on full-time lecturers, which are typically more expensive to employ. Secondly, the future jobs for which graduate students will apply will undoubtedly require a teaching responsibility, with an expectation of some teaching experience (Speer et al., 2005). Lewis and Tucker, (2009) suggests some ways a department might invest in helping graduate students learn to teach. These include a **teaching assistant orientation** where the department, during a week-long program, would introduce graduate students to each other, the department, and their teaching duties. The orientation could focus on fundamental skills like recognizing the importance of good performance in the classroom, and balancing duties as a graduate student and duties as a teaching assistant. Beyond establishing these fundamental skills for all GTA's, another suggestion is to have faculty lecturers serve as **teaching mentors** for GTA's, as well as having senior GTA's meet and work with new GTA's and offer their guidance and experience as part of their teaching assistant duties. As GTAs complete their assigned duties, it is suggested that they be **carefully**

supervised while deciding how much authority and independence they should be granted over their own classes. Having a course coordinator write a common exam and doing grading in groups to offer advice to less experienced GTAs is suggested. During the first year of teaching, GTAs could participate in a **teaching seminar** as part of their regular coursework. This seminar could include topics such as classroom management, creating syllabi, and writing exams. The department's **value system** could also place teaching duties as a priority for GTAs, where there is a very public expectation of quality teaching in order to maintain the teaching assistant appointment.

Faculty support and development in teaching STEM. The development and support of faculty is another area of importance. This section focuses on the development and support of faculty in STEM areas that would serve to address the President's Council of Advisors on Science and Technology's, published report calling for one million additional STEM degrees over the next decade, and a projected increase of 17% in the need for STEM graduates (Bressoud, Camp, & Teague, 2012; Holm, 2016), as well as development and support of faculty to become more effective in classroom teaching.

Faculty support and development in teaching. Motivated by the support and development of graduate teaching assistants, I will now discuss topics associated with the support and development of faculty in teaching.

Teaching and technology. In the graduate teaching assistantship section, it was mentioned that there is a need for faculty support and development purely within the area of teaching. I will briefly mention a few areas where this development can take place. First, Jesso & Kondratieva (2016) suggest the structured use of technology in the classroom to facilitate the introduction of advanced topics like the derivative in calculus. In this case technologies such as

computer algebra systems and dynamic geometry software could be used to model the algebra and geometric interpretations of the derivative. Beyond this one simple example there are many applications for the use of technology in the classroom that could extend an instructor's ability to teach more effectively and efficiently. Jesso & Kondratieva (2016) conducted a study to determine what would motivate an instructor to employ technology in teaching mathematics. Based on quantitative and qualitative data from the surveys and interviews of 13 instructors of undergraduate mathematics in post-secondary institutions in Newfoundland, Jesso & Kondratieva made the following conclusions.

The use of technology for exploratory and creative activities with students takes place mostly on an individual basis, only occasionally, and is very topic specific. This is due to the varying levels of familiarity and experience with technology outside of teaching among the instructors. However, if a topic is known to be challenging for students, and the instructor is familiar with a technological tool, they may choose to employ that tool if it is beneficial for the students. At the same time, the instructors are aware of the possible side effects and unwanted outcomes like developing dependency on the technology, or futile learning of higher-level mathematics using computer algebra systems without prior fluency of arithmetic and algebra skills, and proper understanding of fundamentals.

Classroom discourse. Significant empirical research has shown that student learning can be improved when instructors move from traditional lecture style instruction to more student-centered, interactive instruction (Handelsman et al., 2004; Henderson & Dancy, 2011). One important and well researched student-centered method of instruction is classroom discourse. Herbel-Eisenmann et al. (2013) explore the efforts to design material for supporting secondary

mathematics teacher in using Teacher Discourse Moves in order to develop classroom discourse that is productive for students' learning.

The Culture Challenge

This section considers the challenges that students may face during a period of acculturation into the mathematical community (Cobb, 1989; Dawkins, 2013; Yackel & Cobb, 1996). The nature of learning mathematics at any level is distinguished by the teacher's initiate and guidance in the development of social norms that sustain classroom microcultures characterized by explanation, justification, and argumentation (Dawkins 2013; Wood, Cobb, & Yackel, 1991). Learning mathematics involves several components that relate to the overall culture that exists within and outside the classroom. Some of these components include curriculum and technology. At the post-secondary level, components that contribute to the overall culture include teaching and research responsibilities. These components are briefly discussed in this section.

Curriculum. Recall that in the significance section of this paper, I cited observations made by Döfler (2003) concerning the culture differences between mathematics and mathematics education. I will now develop this issue more as part of the literature review.

Bishop (1991) gives a broad overview of the fundamental issues of mathematical culture and its implications on teaching mathematics. He gives a viewpoint that teaching children to do mathematics emphasizes knowledge as a way of *doing*, while mathematical education should be concerned with finding a way of *knowing* mathematics, which requires *thinking* about mathematics. To quote John Dewey, "*It's not the doing that matters; it's the thinking about doing.*" Teaching mathematics as a way of *doing* mathematics has greatly influenced the development of curriculum in most countries around the world.

The connection between *doing* and curriculum is blatant. Starting with basic arithmetic and its operations of addition, subtraction, multiplication, and division, learning mathematics amounts to *doing* computational exercises starting with positive integers, followed by rational numbers including fractions and decimals. But what do they know? What have they learned? Later work in algebra develops basic skills in *doing* problems like solving a standard list of equations that increase in difficulty and simplifying a standard list of algebraic expressions that increase in difficulty. But what do they know? What have they learned? It is important to note that it is not the problems themselves that are the issue, but the mindset that places so much importance on this particular skill of *doing*. This continues through geometry and trigonometry where these subjects are developed as an area where students apply their computational and procedural skills. But what do they know? What have they learned? Finally, those who survive will make the transition into post-secondary math starting with calculus and progress through differential equations where they will encounter a long list of textbook problems involving derivatives, integrals, and equations involving derivatives and integrals waiting to be solved. But what do they know? What have they learned?

Technology in Curriculum. Bishop continues to explain that this mindset turns mathematics into a “users” curriculum instead of “knowers” curriculum where the mastered skills are a wide ranging “took-kit” for the user. The obvious conclusion to this type of learning is the “user” becoming a human calculator, which of course is a mechanical device that was designed to perform the same procedures that the curriculum demands, but with more speed and accuracy. At the time of this book’s writing, 1991, handheld calculators were very common, with the Texas Instrument TI-81 begin introduced in 1990. At present, we have even more advanced computing technologies, which include Wolfram Mathematica, Wolfram Alpha,

Maple, and Matlab which harness the power of laptop and desktop computers to do far more complicated and faster computations than any handheld calculator ever could. So, the usefulness of having students master procedural calculations as the primary objective of mathematics education is even more ridiculous now than it was in 1991. So how do we, in 2020, design curriculum that facilitates students understanding and ability to think analytically, and not reduce learning down to simple mastery of computational procedures?

A possible first step is identifying a proper use of technology that allows students to take advantage of modern 21st century tools (NCTM, 2015). It should be noted that there are advantages to students mastering certain computational procedures. For example, elementary school arithmetic typically has students master arithmetic of rational numbers (e.g. fractions with integer numerator and denominator) (NCTM, 2015). Post-secondary instructors would likely disagree with saying students are proficient in the arithmetic of fractions, because most of the time they are not. The primary reason can be an over use of calculators too early in the learning process. One may ask why it is important for students to master this skill when technology can do it effortlessly. The answer is embedded in the broader view of mathematics that is required of anyone wanting to facilitate students' deeper understanding of the ideas implied by this basic arithmetic.

A broader view of mathematics is the connection between unique factorization of whole numbers into primes, and the fundamental theorem of algebra which implies the unique factorization of polynomials into primes. Recall that a factorization is a representation in the form of a product of terms (e.g. $72 = 2^3 \cdot 3^2$, or $x^4 + 6x^3 - 3x^2 - 52x - 60 = (x - 3)(x + 2)^2(x + 5)$). The requirement of finding the least common denominator as part of adding or subtracting rational numbers extends to the arithmetic of adding or subtracting rational

expressions in algebra, which then extends to the reverse procedure of decomposing rational expressions into partial fractions, a technique used in integration in second semester calculus. So 4th grade arithmetic has implications on freshman level calculus. This cannot be ignored. So how can technology fit into this effectively?

We must first identify the main problem associated with the use of technology. Danesi (2016) provides some context to the use of technology. Citing Marinovic (2015).

Let us start by defining some boundaries. Most often educators and the media describe digital technologies and their use in very general terms, considering them as a universal panacea. The truth is that while technologies develop and change with rapid speed, “core processes in educational institutions evolved with glacial speed over many years and are not susceptible to rapid change” (Duin et al. 2001). Also, examples of what worked or did not work at “place A,” do not automatically extend to “place B,” and the diversity of technologies, contexts, and ways in which they are used make extrapolations difficult. The world is still faced with a “digital divide,” which exists even in the highly developed countries, and while the divide becomes less an issue of access to technology and more an issue of how it is used, it continues to exist in different forms. For the present purposes, it is also relevant that there are many more studies that address social phenomena related to the adoption of technology, rather than the cognitive consequences of its use.

According to Martinovic, the main problem with the use of technology is that it is subject to variation. The solution lies in viewing technology, and any activity involving technology, as an extension of the classroom and not a replacement. Martinovic suggests a back and forth process where students engage with the technology, but then return to pure reasoning. Essentially the technology serves its purpose and is then put aside. It does not serve as the foundation for the lesson. Furthermore, to make the pure reasoning portion seem relevant in a modern society, it is suggested that topics fit in with a culture that blends and meshes popular themes with serious ones.

In a modern society, technology has an influence on curriculum, which in turn has an influence on culture (Bishop, 1991; Döfler, 2003). Culture influences the effectiveness of teaching. Therefore, the use of technology is important to the effectiveness of teaching.

Returning to the overall question of culture, mathematics versus mathematics education, even though Bishop's writing is 28 years old it is very comprehensive in describing a culture of mathematics education that even now in 2019 persists in many forms. I will now provide more recent examples of this type of educational mindset.

Culture of the college or university math departments. Research institutions employ professors who are traditionally charged with two important responsibilities, generating new knowledge and educating students (Anderson et al., 2011). While these two responsibilities are supposed to be balanced, in practice that are usually imbalanced in favor of research. The teaching component is usually labeled as a "teaching load". That "load" can sometimes be reduced if a professor goes above and beyond in the area of research and raising outside funding (Anderson et al., 2011). Departmental and university cultures often do not adequately support or reward effective teaching, but contributions to research with publication and grant support externally and internally can be rewarded with promotions and salary increases. Teaching is rarely rewarded from the outside and only minimally from within the department (Anderson et al., 2011; Taylor 2010; Wilson, 2010). What are the possible effects, positive and negative, on the quality of education from a department where research is typically valued more over teaching? Examples from Wilson (2010) include:

1. Faculty members enjoying too much autonomy in the classroom to the point where there is too much variation in what is taught in different sections, making it impossible to tell exactly what students have learned.
2. A study from UC Riverside showed that college students spend ten fewer hours studying that in 1961, but college grades on average have gone up, meaning that

either students are learning more, much faster than in the past, or professors are demanding less while giving better grades.

3. There is little external incentive to make courses more rigorous and grading stricter.
4. Professors may be given grants to assess and rework basic courses. This is especially true in community colleges where there are mandates to build co-requisite courses where students earn college credit while satisfying a remediating requirement.
5. Universities have added new tracks to graduate programs in education that teach doctoral students how to evaluate the effectiveness of teaching.

Anderson et al. suggests seven initiatives that would aid research institutions in STEM education. These include:

1. Educating faculty about research on learning
2. Creating awards and named professorships that provide research support for outstanding teachers
3. Requiring excellence in teaching for promotion
4. Creating teaching discussion groups
5. Creating cross-disciplinary programs in college-level learning
6. Providing ongoing support for effective STEM teaching
7. Engaging chairs, deans, and presidents

The Student Mindset Challenge

Typically an issue addressed at the K-12 level, the *student mindset* is an issue that persists at the post-secondary level as well. There is an impression among many students that mathematical ability is something you are born with. Those who were not gifted the ability to understand mathematics can only expect marginal performance in the mathematics classroom.

Jo Boaler and Carol Dweck have done extensive research into *mathematical mindsets*. The foundation for this mindset can start as early as birth. One reason that students in the United States have a fixed mindset is the praise they receive from parents and teachers (Boaler, 2016). Researchers have found that the praise parents gave their babies between birth and age three predicted their mindsets five years later (Gunderson et al., 2013). Therefore, the mathematical mindset a child has early in life can affect their mindset in secondary mathematics, and eventually in post-secondary mathematics. Consequently, it seems reasonable to attempt to instill a positive mathematical mindset early on in a student's career, as well as combating negative mathematical mindsets, which some refer to as math phobia.

Henrich et al. (2016) addresses negative attitudes in post-secondary mathematics within a quantitative literacy course. While this is not higher-level mathematics within the focus of this study, Henrich did cite specific examples of negative math mindsets from which we can gain insight. The following quotes are provided by Henrich:

Math has been, and probably always will be, my sworn enemy. Just thinking about the subject makes my skin crawl a bit. My mathematical history is a series of disasters and disappointments.

I currently see my weakness in math as a real roadblock in my life both academically and personally. I tend to turn my back on activities and classes that involve mathematical thought. I rely on the people around me to help if there is a quantitative problem to solve. This is something I find extremely embarrassing.

Instructional Practices

As previously stated, the Mathematical Association of America, MAA, has been at the forefront of post-secondary mathematics education in terms of research. The MAA maintains an online publication platform called Problems, Resources, and Issues in Mathematics Undergraduate Students, also known as PRIMUS. PRIMUS is a leading journal for the publication of articles concerning the teaching of collegiate mathematics. Its focus is on

pedagogical initiatives in college-level mathematics and mathematics education (MAA, 2018).

In addition, the MAA also publishes an open access journal called the *Instructional Practices Guide* which is a guide to evidence-based instructional practices in undergraduate mathematics.

I will continue by mentioning some of the highlights of this guide.

The *Instructional Practices Guide* is designed as a “how to” guide focused on mathematics instruction at the undergraduate level. It is based on the concept that effective teaching is supported by three foundational types of practices: **classroom practices, assessment practices, and course design practices**. I will briefly discuss these different practices.

Classroom Practices. Classroom practices mentioned by the instructional practices guide include building a classroom community, responding to student contributions in the classroom, and collaborative learning strategies, among others.

Building a classroom community. The guide states that community and sense of belonging are more likely to flourish in classrooms where the instructor incorporates student-centered learning approaches (Slavin, 1996; Rendon, 1994). Establishing norms for active engagement or taking steps to increase a student’s sense of belonging to the classroom community impacts the quality of student engagement in the classroom (MAA, 2018). The guide lists several tips that can help facilitate the establishment of the norms required to have more interaction in the classroom. These include the following:

1. Promote a friendly atmosphere while attending to students who are members of marginalized communities.
2. Establish the importance of arriving prepared for class by completing assigned readings, videos, or homework before coming to class.
3. Model reaching consensus to arrive at resolutions to questions posed in class.

4. Remind students that when working in groups it is important to listen carefully and with respect.
5. Focusing on the mathematics and critique ideas, but do not criticize people.
6. Help students take responsibility for their own learning by asking them to share strategies and questions with the goal of communicating their reasoning.

Responding to student contributions in the classroom. Student contributions in the classroom are discussed to address how responding to incorrect answers, incomplete explanations, faulty arguments, and students' struggles affect student learning. Research shows that productive struggle can help students develop persistence and confidence (Edwards and Beattie, 2016). Furthermore, one of the most powerful ways an instructor can build community and student confidence is to reframe errors (MAA 2018). The practices guide lists the following tips for responding to student contributions.

1. Create a safe space for incorrect answers.
2. Keep a poker face by asking students to justify the reasoning behind their answers no matter what.
3. Focus on reasoning. Encourage students to share their reasoning without fear of discouragement from negative reactions.
4. Distinguish between types of errors. There is a difference between a typo and a common misconception or confusion.
5. Identify correct aspects of a solution. Even though a solution may be incorrect, the student may have done some good work to get there.
6. Keep in mind that speaking in front of peers and the instructor is risky.

Collaborative learning strategies. This refers to learning that takes place as small groups of students focus on open-ended complex tasks, as opposed to cooperative learning which is a more structured small group learning that focuses on foundational or traditional knowledge with group roles (MAA, 2018). Examples of collaborative learning include paired board work and small group work. Paired board work first necessitates that all students demonstrate their knowledge at the board in a classroom. Students are paired at the board. One is the scribe and the other is the quality controller. The scribe writes the mathematics on the board. The quality controller monitors the quality of the mathematics displayed, attending to precision of notation, correctness, and accuracy. After each problem, students rotate roles so that each student has multiple opportunities to serve in both roles. This method allows students to share their reasoning publicly and allows instructors to formatively assess students' knowledge and skills (MAA, 2018).

Small group work provides opportunities for students to work together and grow in their learning. The guide discourages placing low-performing students with high-performing students and suggests the following strategies to use for grouping students.

1. Balance student personalities so that more vocal students are grouped with less vocal students.
2. Regroup students often so that they work with a variety of students from class.
3. Use different grouping strategies such as using random generators or drawing from a deck of cards.
4. Use more strategic approaches such as grouping based on declared majors or interests.

5. Avoid allowing students remain in groups when the dynamic of the group impedes student learning.

Assessment practices. Regarding assessment, the instruction guide cites Hanson and Mohn (2011) who state that assessment is regarded as an essential element for learning in terms of finding and recording increased knowledge or skills. Assessment is divided into parts, formative and summative. Formative assessments are primarily used to inform the direction in which instructors might modify their lessons based on student performance, while summative assessment are conducted with the purpose of evaluating student proficiency with regard to one or more learning outcomes (MAA, 2018).

For implementing formative assessment, the guide cites Black and William (1998, 2009) by identifying five strategies for implementing formative assessments.

1. Clarify and share learning intentions and criteria for success.
2. Engineer effective classroom discussions and other learning tasks that elicit evidence of student understanding.
3. Provide feedback that moves learners forward.
4. Activate students as instructional resources for one another.
5. Active students as the owners of their own learning.

Characteristics of exemplary summative assessments from the instructional practices guide include the following:

1. Authenticity: reflects real life experiences
2. Challenging: stimulates the learner to apply knowledge
3. Coherent: serves as a guide for the student to achieve the learning goal
4. Engaging: attracts the learner's interest

5. Respectful: sensitive to the individual learner's beliefs and values
6. Responsive: includes a feedback mechanism to assist the student in the learning process
7. Rigorous: requires applied understanding of learning to achieve a successful outcome
8. Valid: provides information that is useful to meet the intended learning outcomes.

Examples of different assessment options include writing assessments, oral presentations, group projects, online homework systems, classroom polling systems, assessing via technology, and written reflections and portfolios.

Finally, I will discuss course design practices as described by the instructional practices guide.

Course design practices. The guide describes design practices as thoughtful planning which includes more than just writing notes. The instruction should be designed for all students, not just some. Key aspects of course and lesson design are identifying goals for student learning, selecting instructional strategies to achieve those goals, and choosing methods to assess student learning. Further properties include 1) designing instruction is more than just planning content, 2) instructional design should aim to effect meaningful change, 3) design for student-centered learning must be in sync with evidence-based practices.

When instructors are designing lessons they should ask themselves the following questions.

1. Who are the students in this course and what knowledge and skills will they bring?
2. What are the course learning goals?
3. What does learning look like in the context of this course?
4. What promotes student participation in the course?
5. How is this course inclusive?

6. How will I provide my students with feedback?
7. How will I gather information to improve the course?

Examples of course design considerations include designing for equity, designing the learning environment, designing mathematical activities and interactive discussions, designing homework, and designing a flipped classroom

Effectiveness in K-12 Mathematics

This section of the literature review includes literature concerning effective teaching at the K-12 level. I will begin with Deborah Ball's *High-Leverage Practices*. In Ball and Forzani (2010) titled *Teaching Skillful Teaching*, Ball defines high-leverage practices as skills that underlie effective teaching and are most likely to affect student learning. In addition, high-leverage practices are teaching practices by which the proficient enactment by a teacher is likely to lead to comparatively large advances in student learning (Ball, Sleep, Boerst, & Bass, 2009). Ball and her colleagues developed a preliminary set of criteria to help with their identification of high-leverage practices for beginning mathematics teachers. These included the following:

Criteria based on examination of the work of mathematics teaching

Supports work that is central to mathematics

1. Helps to improve the learning and achievement of all students
2. Is done frequently when teaching mathematics
3. Applies across different approaches to teaching mathematics.

Criteria necessitated by teacher education context

4. Can be articulated and taught
5. Is accessible to learners of teaching

6. Can be revisited in increasingly sophisticated and integrated acts of teaching
7. Is able to be practiced by beginners in their field-based setting

Two goals for the practices shaped the considerations of the researchers. The first was the transferability of the practices to be used in any teaching setting, regardless of variations such as curricula or teaching style. The second was the “teachability” of the practices. They sought to choose practices that lend themselves to careful instruction within their teacher education courses. As stated in Ball and Forzani (2010), teaching demands special kinds of knowledge and skills that most individuals do not naturally possess. Therefore, training teachers for actual practice to masters these unnatural skills is crucial. The practice of teaching effectively is learnable.

A list of 19 practices is provided by Teacher Works, an organization associated with the department of education at the University of Michigan. These practices are not written to be only associated with mathematics education, but with education in general. Regardless, all of them could be used for mathematics education. Ball’s background as an elementary mathematics teacher certainly makes this reasonable. These 19 practices can be interpreted as 19 methods of effective teaching. In this review of literature these practices can be considered *axioms* of effective teaching. This section of the review will attempt to highlight instances of Ball’s 19 methods used as effective math instruction. I will first highlight the practice of *leading a group discussion*.

In a group discussion, the teacher and all of the students work on specific content together, using one another’s ideas as resources. The purposes of a discussion are to build collective knowledge and capability in relation to specific instructional goals and to allow students to practice listening, speaking, and interpreting. The teacher and a wide range of students contribute orally, listen actively, and respond to and learn from others’ contributions.

Ball et al. (2011) discuss the preparation necessary for teachers to lead mathematics discussions. Like most practices in teaching, facilitating a discussion is complex and takes time to learn. Ball describes the importance and difficulties of effectively facilitating mathematical discussions, especially with beginning teachers. The study itself involves decomposition of the high-leverage practice of leading a classroom discussion into “nested” practices that include the nature of the work being discussed, the mathematics being discussed, and the interaction that happens during the discussion. Ball describes how this decomposition of discussion is difficult because of the unpredictability of what a teacher will need to say or do in a discussion. This is especially important when deciding whether to pursue a student’s solution method or question. With respect to the mathematics in the discussion, Ball suggests that this affects beginning teachers due to the complexity of the mathematics and the need to manage the discussion.

Further connections to Ball’s 19 practices are the NCTM effective mathematics teaching practices. These include establishing mathematics goals to focus learning, implementing tasks that promote reasoning and problem solving, using and connecting mathematical representations, facilitating meaningful mathematics discourse, posing purposeful questions, building procedural fluency from conceptual understanding, support productive struggle in learning mathematics, and eliciting and using evidence of student thinking (NCTM, 2014).

I will now take a slight detour from literature that focuses on what specifically happens in the classroom that defines effectiveness, and consider an article by Boaler (2002), that explores the diversity of theoretical perspectives that could aid in attaining other high-leverage practices, including those that focus on social and equity issues. Boaler (2002) states that

Ball, Chevallard, Lave, McDermott, and Zevenbergen all concern themselves with an extremely issue, that of breadth of thought and of open-ness. They remind us that theoretical perspectives plan an important role, prevailing against ideology and dogma. They also remind us that theories must be

employed with care and reflexivity, in order to preclude a form of narrowness that comes from the unquestionable acceptance of dominant paradigms. Chevallard and Zevenbergen urge mathematics educators to look beyond single frameworks, remaining open to the different ways that theories from within and outside mathematics education may illuminate some pressing and enduring questions in our field, such as those of social inequity. The different authors speak to the importance of a research knowledge that is broadly conceived, enhanced by theoretical reflexivity.

In this passage Boaler puts emphasis on social equity. This in line with Ball's practice of *learning about students' cultural, religious, family, intellectual, and personal experiences and resources for use in instruction*. Ball defines this practice as follows:

Teachers must actively learn about their particular students in order to design instruction that will meet their needs. This includes being deliberate about trying to understand the cultural norms for communicating and collaborating that prevail in particular communities, how certain cultural and religious view affect what is considered appropriate in school, and the topics and issues that interest individual students and groups of students. It also means keeping track of what is happening in students' personal lives so as to be able to respond appropriately when an out-of-school experience affects what is happening in school.

Boaler warns against a narrow mindedness in theoretical frameworks that may not give adequate perspective to question traditional practices which are familiar. Furthermore, she makes a similar claim about mathematics by recognizing how connections can be made in ways that you might least expect. Citing a study of Burton (1999), she refers to an interview of a mathematician who could not find anything about his ideas in standard research. Eventually a connection was made with a very ancient problem and some work from the 1960's. He goes on to state that this is common in mathematics. In this case, we can see how looking beyond the usual places can sometimes result in a breakthrough. Though not directly connected to one of Ball's practices, this could certainly be a practice of effective teaching at either the K-12 or post-secondary level. Another quote of Boaler makes this quite clear.

Burton and Singh both write about the work of mathematicians, focusing upon the act of "making connections". But what is this aspect of mathematical work? And has its character, as an action or mathematical practice, rather than a form of knowledge or knowing, contributed to its relative lack of attention in curriculum materials and teaching? For the act of making connections is not something students need to *know*, it is something they need to *do*. One could imagine a student with a broad knowledge of mathematical procedures and even a conceptual understanding of the relations between procedures, who still would not draw connections between different mathematical ideas, relations, or

representations as they work. They may do so, as knowledge and practice are intricately connected, but the act of doing so is not defined by the knowledge they possess. This raises the question of whether mathematics education researchers have focused too predominately upon knowledge categories, neglecting various mathematical actions that are so critical to mathematics work.

Finally, Boaler recognizes the work of the National Research Council who reviewed relevant research in K-8 math education (Kilpatrick, Swafford & Findell 2001). They created a framework for successful math learning which they called *mathematical proficiency*. The components of mathematical proficiency include:

1. Conceptual understanding: the ability to comprehend math concepts.
2. Procedural fluency: skill in carrying out procedures accurately, efficiently, and appropriately
3. Strategic competence: ability to formulate and solve mathematical problems
4. Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification
5. Productive disposition: inclination to see mathematics as sensible, useful, and worthwhile

It is this last component, productive disposition, that is closely related to the idea of mathematics acculturation that was mentioned by Dawkins (2013) at the beginning of this paper. Ben-Zvi and Arcavi (2001) also references acculturation while studying 7th grade students studying statistics with technology. Both Boaler (2002) and Ben-Zvi and Arcavi (2001) refer to acculturation as the practice of knowing mathematics that is the product of teaching.

Implications of K-12 Effectiveness on Post-Secondary Mathematics Education

Finally, we arrive at literature that gives focus to the implications on post-secondary mathematics education. The first study we consider is Crowley and Dunn (1993), which proposed

implications from the 1989 NCTM K-12 mathematics reform and standards on undergraduate mathematics. A quick review of the NCTM standards notes that the standards were organized into four grade sections, grades K-4, grades 5-8, and grades 9-12. Among each of these grade levels there were four general themes. They include (1) mathematics as problem solving, (2) mathematics as communication, (3) mathematics as reasoning, and (4) mathematical connections. These general themes easily correspond to the following high-leverage practice respectively: (1) explaining and modeling content, practices and strategies, (2) leading a group discussion, (3) eliciting and interpreting individual students' thinking, and (4) diagnosing particular common patterns of student thinking and developing in a subject-matter domain (Ball & Forzani, 2010). The general themes are defined below.

1. Mathematics as problem solving: Students should be challenged with both applied and theoretical problems from within and outside mathematics.
2. Mathematics as communication: Students should be able to “listen to”, read about, write about, speak about, reflect on, and demonstrate mathematical ideas.
3. Mathematics as reasoning: Students must make and test conjectures, formulate counterexamples, comprehend logical argument, assess the validity of arguments, and develop simple valid arguments.
4. Mathematical Connections: Students should be able to make connections between mathematical topics and between mathematics and other disciplines. They should have the ability to generate mathematical equivalences and mathematical models, as well as to understand.

In summary, this literature review consists of areas of scholarship which include post-secondary mathematics education and challenges that take place within post-secondary

mathematics instruction, challenges of teaching mathematics at the post-secondary level, and K-12 mathematics education.

Literature on effectiveness in calculus instruction includes:

- Bressoud and Zorn, (2018) and Champion and Mesa (2017) which give justification for the discussion by noting the large numbers of students entering STEM fields and the circumstances from outside their educational experience they have an effect on their mathematical performance.
- Dwyer and Gruenwald (2017) describes an attempt at resequencing the traditional calculus sequence to more closely align with other STEM courses to make STEM students' experience more effective and efficient.
- Noting the passive learning which is typical of U.S. classrooms (Stigler and Hiebert, 1999), we arrived at the instructional practices of the flipped classroom and active learning (Adams & Dove, 2018; Chickering & Gamson, 1989; Crawford et. al 2018; Petress 2008).

Literature on effectiveness in post-secondary mathematics education which does not focus specifically on calculus include Collier (1999), Graves et al., (2009), MAA (2018). Post-secondary mathematics teaching challenges include graduate teaching assistantships, (DeChenne et al., 2012; Deshler et al., 2015; Ewing, 1999; Holm, 2016; Jesso & Kondratieva, 2016; Handelsman et al., 2004; Lewis & Tucker, 2009; Miller et al., 2018; Speer et al., 2005). This challenge is important because of graduate teaching assistants being the future instructors of post-secondary education.

The culture challenge highlights differences between the cultures of mathematics and mathematics education (Anderson et al., 2011; Bishop, 1991; Danesi 2016; Döfler, 2003;

Marinovic, 2015; Taylor, 2010; Wilson, 2010). This section's importance is rooted in the attempt of this study's results providing insight into strengthening of the connection between mathematics and mathematics education.

The student mindset challenge explores the issue of students' impressions of their own ability in mathematics (Boaler, 2016; Gunderson et al., 2013; Henrich et al., 2016). This challenge, which is more familiar to K-12 literature, persists into post-secondary mathematics education.

Effectiveness in K-12 mathematics includes specific practices that are chosen by K-12 instructors that promote skillful teaching (Ball et al., 2011; Ball & Forzani, 2010; Ben-Zvi & Arcavi, 2001; Boaler, 2002; Burton, 1999; Crowley & Dunn, 1993; Kilpatrick, Swafford & Findell, 2001).

CHAPTER THREE

METHODOLOGY

In this section I will describe and justify the methodological choices for this study. As I mentioned, I sought to discover and explore the philosophies and practices that define the teaching of post-secondary mathematics from the perspective of instructors. In the chapter on findings I constructed a detailed description of these practices to learn how they may be used to develop meaningful professional development for future post-secondary mathematics teachers. Based on this goal, in-depth open-ended qualitative interviews were this study's research method.

The methodology I used was motivated by Wagener's study (2009) which examined the affective socialization process in mathematics doctoral study. Wagener used qualitative, semi-structured, in-depth interviews as the primary tool for data collection. This method of qualitative research motivated the methodology used in my study which will be explained in detail in the following subsections.

Qualitative research aims to work within the context of human experiences and to find the ways in which meaning is made from those experiences. This is not meant to create generalizations to a population like that of quantitative research, but to establish the meaning of a phenomenon from the views of participants (Bhattacharya, 2017 p.6; Creswell, 2014). The data collected is rich in description of people, places, and conversations, and not easily handled by statistical procedures (Bogdan & Biklen, 1992, p.2). Using interviews as the source of data allowed the study to reveal the intentions, personal perspectives, and insights that the participants have that concern the teaching of post-secondary mathematics.

Inductive analysis in qualitative research refers to “working up” from the data and assumes that the researcher is not starting the data analysis with any kind of preestablished testable hypothesis about the data. The process involves the researcher looking at the raw data, separating them into small analytical units of meaning called codes for further analysis, clustering similar analytical units of meaning and labeling them as categories, and identifying salient patterns after looking within and across categories, usually called themes (Bhattacharya, 2017). This describes the overall method used to analyze the data for this study.

Context and Participants

Context

This study took place at a large, public research university in the Southeastern United States. The university’s mathematics department offers several options for bachelors, masters and doctoral degrees. The faculty consists of 39 full time tenured or tenure-track faculty who do research in all areas major of pure and applied mathematics, and 31 non-tenure track lecturers who are full time instructors. The department student body consists of approximately 89 traditional graduate students, and an unknown number of students in a teaching masters degree program. The department also serves many undergraduate mathematics majors, as well as thousands of undergraduate non-math STEM and non-STEM majors in service courses. The overall undergraduate population is 23,290 students, and the overall graduate student population is 6,170 students as of Fall 2019.

Participant Selection

The selection process of the study participants was rooted in the intent and style of qualitative research. Creston (2007) defines the concept of purposeful sampling where the researcher selects individuals and sites for study because they can purposefully inform an

understanding of the research problem and central phenomenon in the study. In this study, I was interested in discovering and exploring the teaching philosophies and practices that define the teaching of post-secondary mathematics from the perspective of those instructors. In order to obtain the most useful data, sampling was based on participants being instructors who routinely or periodically teach post-secondary mathematics courses. For this study, this means courses at the pre-calculus level or above.

The reason for selecting only instructors of post-secondary mathematics is clear given the study's goal of studying the beliefs and practices of instructors of post-secondary mathematics. The reason for defining post-secondary mathematics courses to be courses at the pre-calculus level and above will help to ensure that the study has enough participants, while at the same time reasonably limiting the study to mathematics which is primarily taught at the post-secondary level and reasonably above the level of mathematics taught at the secondary level. This is done with the intentional goal of extending results already known about philosophies and practices at the K-12 level and not repeating the work of any past study. However, this study has applicability to K-12 mathematics since calculus and pre-calculus were included in the study, and both courses are offered at the high school level as well.

The decision for the number of participants in the study was made based on *access* which reflects a practical concern, and *relevance* which reflects a validity concern (Potter, 1996). Handling the practical concern of access, Creston (2007) suggests a guideline of qualitative research which states that one should collect extensive details on few sites or individuals. This would be as opposed to collecting minimal details on several sites or individuals. To handle the concern of validity, the study must make clear the accuracy of its results. A preliminary concern that should be dispelled is the potential for researcher bias. I have already handled this concern

in the limitations section in chapter one. To further support the validity of this study's results, the extensive details compiled from the data will allow transference from the context of this study to the context of the reader. Creswell (2007) describes *transference* as a process where readers can infer information to other settings and determine whether the findings are legitimate because of shared characteristics. A sample size of eight participants allowed for both access and relevance for the study's results.

In summation, this study, including qualitative one-hour interviews for data collection, took place at a large research university in the Southeastern United States. A sample of eight participants was selected. These participants were instructors of post-secondary mathematics, and post-secondary mathematics was defined as courses including pre-calculus and above.

The Participants

Six of the eight participants are full-time lecturers who are instructors whose primary duties are teaching. Out of these six, three teach primarily lower-level courses which included mathematical reasoning, statistics, college algebra, and pre-calculus. Each of these three have either undergraduate degrees in mathematics education or mathematics, or have completed extensive coursework in education. They also have teaching masters degrees in mathematics, a traditional masters degree in mathematics, or a Ph.D. in mathematics education. Two of these three lecturers identified as female. The other identified as male. The other three lecturers teach primarily higher-level lower division courses which included calculus I, II, III, and calculus for life sciences. One holds an undergraduate degree in mathematics education and a traditional masters degree in mathematics. The other two hold both undergraduate degrees and Ph.Ds in mathematics. One of these three lecturers identified as female. The other two identified as male.

The other two participants are full-time tenured or tenure-track professors whose primary duties including teaching primarily upper-division and graduate level mathematics courses, conducting research in mathematics, and advising doctoral candidates. The courses routinely taught by these participants include lower division linear algebra and abstract algebra, as well as graduate courses in number theory and abstract algebra. Both of these professors identified as male. A summary of the participants and their attributes is included below in Table 3.1.

Table 3.1: Participant Attributes

Pseudonym	Gender	Position	Courses	Highest Degree
Charlie	Male	Tenure-Track Professor	Upper-Division Undergraduate and Graduate	Ph.D. Mathematics
Jane	Female	Lecturer	Lower-Level Lower-Division Undergraduate	Teaching Masters Degree in Mathematics
Keith	Male	Lecturer	Lower-Level Lower-Division Undergraduate	Traditional Masters Degree in Mathematics
Lois	Female	Lecturer	Lower-Level Lower-Division Undergraduate	Ph.D. Mathematics Education
Marty	Male	Tenured Professor	Graduate	Ph.D. Mathematics
Sanborn	Male	Lecturer	Upper-Level Lower-Division Undergraduate	Traditional Masters Degree in Mathematics
Sarah	Female	Lecturer	Upper-Level Lower-Division Undergraduate	Ph.D. Mathematics
Tim	Male	Lecturer	Upper-Level Lower-Division Undergraduate	Ph.D. Mathematics

Data Collection Procedures

Initial Procedures

The chronological order of the data collection procedures of this study was as follows: First, I gained approval from the Institutional Review Board (IRB) of the university for the research. Once the study was approved, I informed the mathematics department associate head of the university, and he graciously forwarded my invitation email to the appropriate faculty. The nomination process for selecting participants for this study first involved determining which instructors at the university qualify as being instructors of post-secondary mathematics as previously defined. This involved the use of a survey that determined the mathematics courses that the instructors routinely teach to verify that they teach post-secondary mathematics courses as defined by the study. It also served to diversify the participating candidates according to gender and courses taught. I am a former member of the university's mathematics department as a graduate teaching associate, and an adjunct lecturer therefore I did not expect any significant difficulties with finding willing participants. I used the survey to determine a sample of instructors to ask to participate in the study. The survey is included in Appendix D. The procedure to determine participants was as follows:

1. The survey was only sent to full-time tenured, tenure-track, and non-tenure track lecturers, not post-doctoral faculty or graduate students.
2. To diversify the participant sample, I strived to obtain five male participants and five female participants.
3. To be selected, instructors must have taught classes at the pre-calculus level and above over the last three years of their full-time positions.

4. The courses relevant to this study were categorized into groups that include graduate level courses, junior and senior level undergraduate courses, and sophomore and freshman level undergraduate courses. It was a priority to diversify the participants to include instructors that teach courses that represent all three of these categories.

The survey was sent out a total of three times. Initially, eleven instructors expressed interest in the study by completing the survey. A formal invitation was extended to each of these eleven individuals. This invitation included the procedure for scheduling the interviews along with the written consent form which are included in Appendices A and B. The IRB was amended to allow the participation of all eleven instructors just in case all eleven responded to the invitation. In the end, eight of the eleven responded to the invitation to schedule an interview.

Data Collection Methods

Interviews were the primary data source for this study. The interviews were semi-structured and approximately one hour in length. With semi-structured interviews, one is confident about getting comparable data across participants, as opposed to rigidly structured interviews where the participant cannot tell his or her story personally in their own words, and the interview falls out of the qualitative range (Bogdan & Biklen, 1992). Qualitative in-depth interviews are very similar to ordinary conversations, just with pre-written questions that guide the conversation. This was more effective than formal structured interviews where participants' responses do not affect the direction of the interview (Potter, 1996). The in-depth interview purposefully served as the best method of data collection for this study. In-depth interviews bring out the affective and value-laden implication of the subjects' responses, to determine whether the experience had central or peripheral significance. They elicit the relevant personal

context, the idiosyncratic associations, beliefs, and ideas (Merton & Kendall, p. 545, 1946). In qualitative research, a single case or a small, nonrandom, purposeful sample is selected precisely because the researcher wishes to understand the particular, *in depth* (Merriam & Tisdell, 2016, p.254).

Interviews

Interviews were conducted individually on the university's campus in the participant's private office. I began by stating the purpose of the interview and asking for permission to begin. The first questions asked for the participants education background. Next, questions on the participants' beliefs on teaching and students were asked, and finally questions on classroom practices. I allowed the participants to answer each question for as long as reasonably possible. I only stopped to ask clarifying questions, and to follow up with questions to obtain as deep of a response as possible. All of the interviews were audio recorded, transcribed, and stored on my private personal computer. All participant identifiers were replaced with pseudonyms, and the pseudonym key was deleted after replacing the identifiers.

Data Analysis Procedures

Informal Data Analysis

For each participant, each interview was transcribed and informally analyzed during the time between consecutive interviews. This informal analysis was a simple read through of each transcription to determine the quality of the interview, and to determine ways to make later interviews more effective. After completing all interviews, the formal data analysis began. The formal analysis involved finding the big ideas that were discussed.

Formal Data Analysis

Inductive analysis in qualitative research refers to “working up” from the data and assumes that the researcher is not starting the data analysis with any kind of preestablished testable hypothesis about the data. The process involved looking at the raw data, separating them into small analytical units of meaning called codes for further analysis, clustering similar analytical units of meaning and labeling them as categories, and identifying salient patterns after looking within and across categories, usually called themes (Bhattacharya, 2017). I used this description of the process of qualitative data analysis, as well as a model by Hatch (2002) which was meant to provide a framework to guide researchers through inductive analysis. This describes the overall method used to analyze the data for this study.

The analysis started with first analyzing the data in an attempt to answer the second research question. What are the specific teaching practices identified by post-secondary mathematics instructors as critical to the teaching and support of students learning post-secondary mathematics?

1. Looking at the data and separating the data into small analytical units of meaning involved reading through all the interviews and identifying any and all practices stated by the participants that were practices of the participants, practices of other instructors, or practices of students. Each interview was read one at a time and every practice found was highlighted and labeled with a preliminary code that described the nature of the practice. If possible the code was an In Vivo code that described the practice in the participants’ own words. At this point in the analysis the significance of this practices was not considered. The only goal was to find and label all the practices mentioned by

the participants. Each interview was read several times to ensure all the practices were found. A list of practices was then compiled.

2. After identifying all practices mentioned in each individual interview, I started to look for common practices. These were practices mentioned in the interviews which were practices of the participants, their students, or other instructors and were common among the different interviews. This was done by selecting two practices off the compiled list at a time and rereading all the interview transcripts searching for quotes mentioning those two practices from among the data. This process continued until all the individual practices on the compiled list were exhausted. It was clear that several of the practices on the list were common practices among the participants, however the frequency with which these practices occurred among the participants was smaller than desired. I did not consider this to be unexpected since the background and teaching assignments of the instructors differed significantly. However, it was also clear that many of these practices fit within certain categories with common characteristics. Examples of these categories include practices concerning the facilitation of a specific classroom environment, teaching practices, and practices of students that yield products of mathematical content like mathematical solutions or proofs. I went through the list of practices and put each practice in an appropriate category if applicable. I also searched for more categories. In total I found six categories.
3. After exhausting all the practices by placing them in the appropriate categories, I began to look for common characteristics of the categories. The larger groups in which categories with common characteristics fit became themes, and the categories within

them were then referred to as subthemes. There were three themes, student-centered instructor practices, content-centered practices, and student practices.

The subthemes of the theme on student-centered instructor practices were *classroom environment practices* which included practices that facilitated a desired classroom environment, and *teaching practices* which were practices the participants used that focus more on people than mathematical content.

The subthemes of the theme on content centered practices were *philosophical practices* and *products of content*. Philosophical practices were practices that facilitate a type of behavior or understanding as opposed to facilitating the creation of a student product like solving a problem. Products of content included practices that facilitate students' ability to produce mathematical content.

The subthemes of the theme on student practices were characteristics of successful graduate students, characteristics of successful undergraduate students, and characteristics of unsuccessful students.

Figure 3.1, which describes this analysis, is included below.

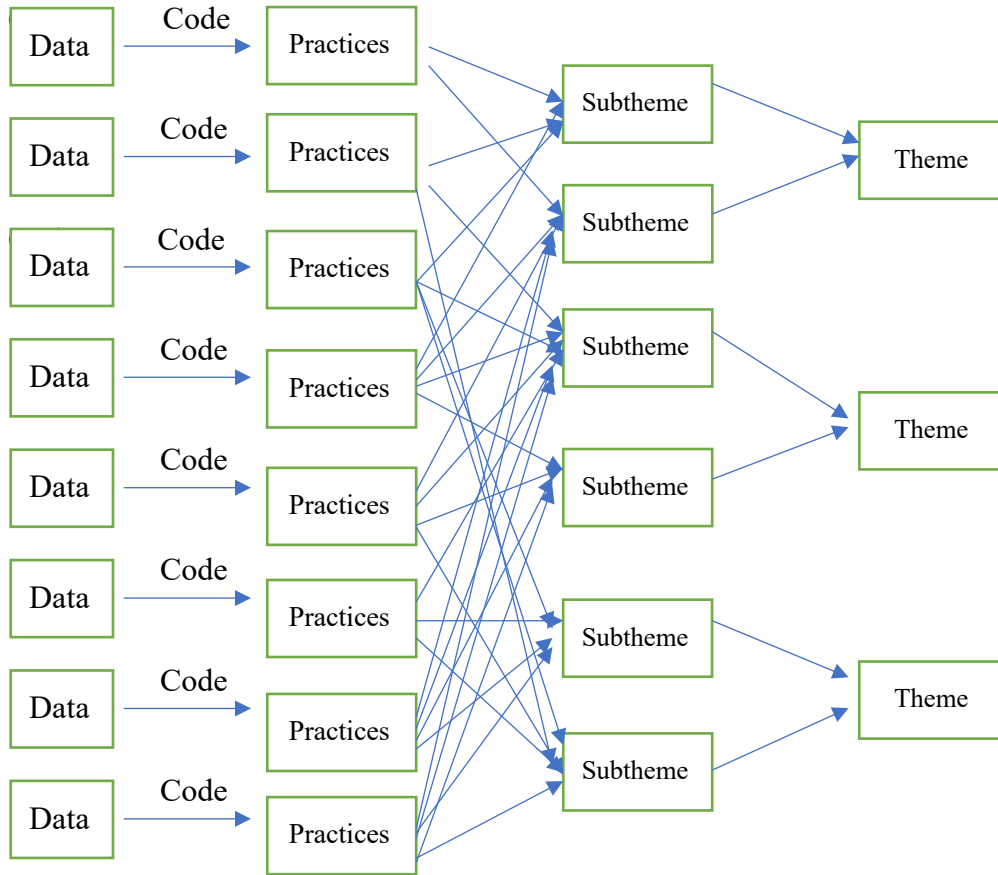


Figure 3.1: Coding for Practices

Next an analysis of the data was done in an attempt to answer the first research question. What are the similarities and differences among the teaching philosophies of post-secondary mathematics instructors? The philosophies of the participants were more difficult to identify than their practices because their responses to questions regarding their teaching philosophies did not necessarily begin with “my teaching philosophy is”, nor did statements regarding their philosophies fall within a single portion of the interview. Because of the nature of the participants’ responses when questioned about their teaching philosophies, the philosophies were identified by searching for statements that spoke about the beliefs of the instructors. The

decision to focus on beliefs as a way to determine the teaching philosophies is supported by literature. A teaching philosophy is a statement about personal beliefs and ideas about students' learning, mathematics, and teaching practices (Yoem et al., 2018). This includes beliefs about how students learn best along with associated effective teaching strategies (Medina & Draugalis, 2013). This will also be discussed in the chapter four.

Inspired by the third question which asks what influences the participants' teaching philosophies have on their teaching practices, I searched for the participants beliefs on teaching and learning by looking for beliefs that were associated to the practices which were already found. This proved very effective since I did not have to look for phrases that used the word philosophy. The method of analysis of the participants' beliefs that define their teaching philosophies went as follows.

1. I began by going through every practice within all subthemes of each major theme and found quotes which stated beliefs that inspired the associated practice. This gave natural rise to the themes and subthemes of the participants' beliefs. By finding associated beliefs to all the practices, the three major themes on beliefs were student-centered instructor beliefs, content-centered beliefs, and beliefs on student practices.

Similar to the subthemes of the major themes on student-centered instructor practices, the subthemes of the theme on instructor beliefs were beliefs on classroom environment and beliefs on teaching. The subthemes of the theme on content-centered beliefs were philosophical beliefs and beliefs on content. The subthemes of the theme concerning beliefs on student were beliefs on successful graduate students, undergraduate students, and unsuccessful students.

2. I also read through the data to find beliefs that may not have been associated to a practice.

Those beliefs were also included.

Figure 3.2, which describes this analysis, is included below.

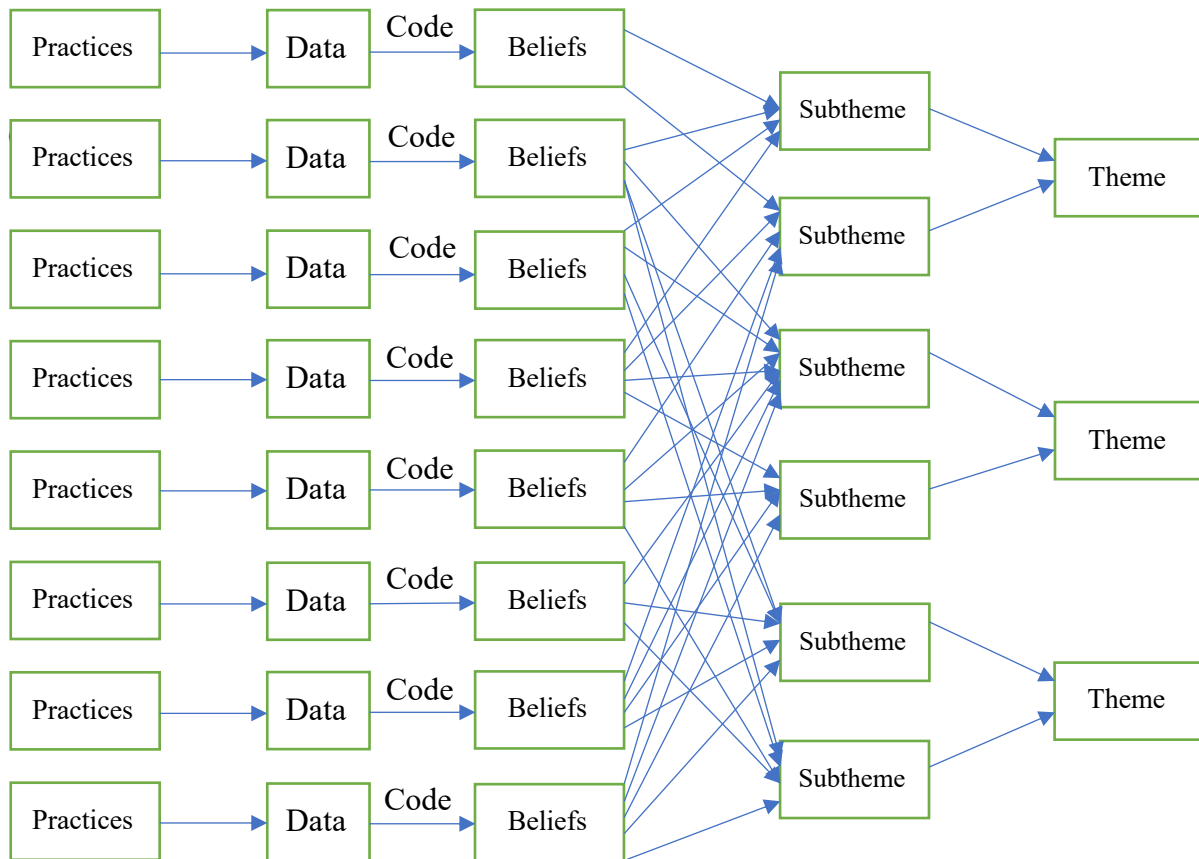


Figure 3.2: Coding for Beliefs

To answer the third research question concerning how the teaching philosophies of the instructors influence their practices, I analyzed the associations between the beliefs and practices found during the analysis for research questions one and two.

1. Because the associations between the instructors' beliefs and practices were so strong, there was a characteristic of the belief-practice association that became evident almost immediately. It was clear that some beliefs inspired practices that were actually used by instructors or students, and other beliefs inspired practices that were hypothetical in nature. By hypothetical I mean that it was not clear via a quote if the practice was actually used by an instructor or a student. For example, the quote may have included a phrase like, "an instructor should do this", or "a student should do that". The beliefs that were associated to practices which were actually used were said to be **directly** associated to the practice. The beliefs which were associated to the hypothetical practices were said to be **indirectly** associated to the practice. To answer the research question concerning how the instructors' beliefs influence their practices, every belief-practice association was labeled as either direct or indirect.
2. It was also evident that the practices revealed by the data were either practices of the participants or they were practices of students or other instructors. Practices which are actions or suggested actions of the participants themselves were referred to as **internal** practices. Practices which are actions or suggested actions of other individuals were referred to as **external** practices.

The characterizations of belief-practice associations and the practices were used to answer the third research question about how the participants beliefs influenced their practices. To increase the credibility of my data analysis I cross checked the codes for thirteen percent of the interview data with another qualitative researcher for intercoder reliability. We both met to discuss and compare our analysis. Our individual codings were approximately 90% identical. We discussed further and were able to arrive at 100% agreement. The qualitative researcher is a

doctoral student who also conducted a qualitative study for their dissertation and has completed the dissertation defense portion of the doctoral program.

In summation, during September and October of 2019 I conducted the semi-structured interviews, transcribed the interview data, and begin the analysis procedures. During November and December 2019 I began the write up of my findings and conclusions.

Using a theoretical framework of mathematicology and social constructivism, my qualitative study explored the characteristics of effective teaching of post-secondary mathematics. Eight participants at a large public research university in the Southeastern United States were chosen for the study. The study used the criteria that (1) the participants must be instructors of post-secondary mathematics, and (2) post-secondary mathematics will include pre-calculus courses and above. Each interview was audio recorded and transcribed, and an informal analysis was used to inform the interviews to come. After all the interviews were completed the data was inductively analyzed.

CHAPTER FOUR

FINDINGS AND INSIGHTS GAINED

This chapter presents the results of the study. Recall that the purpose of this study is to determine the teaching philosophies and teaching practices of post-secondary mathematics instructors, and to determine how their teaching philosophies influence their teaching practices. Specifically, the study seeks to answer the following research questions:

1. What are the similarities and differences among the teaching philosophies of post-secondary mathematics instructors?
2. What are the specific teaching practices identified by post-secondary mathematics instructors as crucial to the teaching and support of students learning mathematics?
3. How do the teaching philosophies of post-secondary mathematics instructors influence their teaching practices which are used to support students learning post-secondary mathematics?

I will proceed by detailing the themes which are relevant for each research question.

The Teaching Philosophies of Post-Secondary Mathematics Instructors

The first section of this chapter will address the first research question: What are the similarities and differences among the teaching philosophies of post-secondary mathematics instructors? During the interview process it was clear that many of the faculty members express their philosophies through their beliefs about teaching, mathematics, and students as evidenced by their comments and areas of emphasis in responses during the interviews. In this way their comments provided insights into their beliefs regarding mathematics teaching (Thompson, 1984). Scholars have long argued that beliefs impact how individuals teach (Ball, Thames, and Phelps, 2008; Dewey, 1933; Fennema & Franke, 1992). Participants of this study did not

necessarily make general summative statements that characterize their teaching philosophies. Therefore, this section will present the key ideas that define the participants' philosophies of teaching mathematics at the post-secondary level, based on an analysis of their interview comments.

In order to operationalize the concept of philosophy for the purposes of this study, I draw on relevant literature. A teaching philosophy is defined as a statement about personal beliefs and ideas about students' learning, mathematics, and teaching practices (Yoem et al., 2018). This includes beliefs about how students learn best along with associated effective teaching strategies (Medina & Draugalis, 2013). Personal beliefs and ideas imply that a personal teaching statement is uniquely determined by a person's background and experiences. Given the details of this definition, data concerning the participants' beliefs on student learning and mathematics will be used to provide context and help make sense of their teaching philosophies. I will then discuss the similarities and differences of the beliefs of the participants.

Three major themes were identified for the first research question. They include student-centered instructor beliefs, content-centered beliefs, and beliefs on student practices. Each of these themes included several examples and warranted the delineation of separate subthemes.

Student-centered instructor beliefs. Based on analysis of the data, the theme on student-centered instructor beliefs included two subthemes called beliefs on classroom environment and beliefs on teaching. The beliefs on classroom environment subtheme consists of positivity and math anxiety. The beliefs on teaching subtheme consists of active learning, mathematics PhD advising, and support for graduate teaching assistants.

Content-centered beliefs. Based on analysis of the data, the theme content-centered beliefs included two subthemes called philosophical beliefs and beliefs on content. The

philosophical beliefs subtheme consists of rigor at the lower division level and conceptual understanding of content. The beliefs on content subtheme consists of interpreting and writing proofs and the whole answer.

Beliefs on student practices. Based on analysis of the data, the theme beliefs on student practices theme included two subthemes called philosophical beliefs and beliefs on students. The philosophical beliefs subtheme consists of ideas including student responsibility and motivation of math PhD students. The beliefs on students subgroup consists of beliefs on successful undergraduate students, successful graduate students, and unsuccessful students. This complete listing is summarized as follows.

1. Student-Centered Instructor Beliefs
 - a. Beliefs on Classroom Environment
 - i. Positivity
 - ii. Math Anxiety
 - b. Beliefs on Teaching
 - i. Active Learning
 - ii. Math Ph.D. Advising
 - iii. Supporting Graduate Teaching Assistants
2. Content-Centered Beliefs
 - a. Philosophical Beliefs
 - i. Rigor at the Lower Division Level
 - ii. Conceptual Understanding
 - b. Beliefs on Content
 - i. Interpreting and Writing Proofs

- ii. The Whole Answer
- 3. Beliefs on Student Practices
 - a. Philosophical Beliefs
 - i. Student Responsibility
 - ii. Motivation of Math Ph.D. Students
 - b. Beliefs on Students
 - i. Characteristics of Successful Graduate Students
 - ii. Characteristics of Successful Undergraduate Students
 - iii. Characteristics of Unsuccessful Students

I will now discuss the theme consisting of student-centered instructor beliefs.

Student Centered Instructor Beliefs

Out of the eight faculty members participating in the study, four discussed beliefs about classroom environment.

Classroom environment. The beliefs related to classroom environment include *positivity in the classroom* and *math anxiety*. Three participants discussed beliefs about positivity in the classroom and two discussed beliefs about math anxiety.

Positivity. The classroom environment belief mentioned by the most faculty members was fostering a positive environment. Three out of the eight participants mentioned fostering positivity in some way as an important practice. These three participants included Sanborn with respect to higher level lower division math students, and Keith and Lois each with respect to freshman lower division math students. The following quotes illustrate positivity within the context of the students and mathematics being taught in each interviewee's classroom:

Sanborn: I want them to have a positive experience in a math class. I think [about] being positive all the time, being aware of inclusion and diversity and really trying to be welcoming, even overly so in some kind of capacity. So I think a positive experience in a math class is one thing I definitely want.

Lois commented, “I try to communicate effectively, respectfully, and with positivity.” Keith commented, “Give them confidence and let them know you're there to help. I feel that that's one of the big things you can do.”

Sanborn’s belief of fostering positivity in his classroom specifically involves creating an inclusive and diverse environment while Lois describes positivity in relation to how she communicates to her students. Keith describes wanting to have his students feel comfortable and confident in class. We see that these comments suggest that maintaining a positive classroom environment is an important belief that is crucial to the facilitation of students learning in a mathematics classroom.

Math anxiety. The second belief that the data revealed which helps to create a classroom environment that facilitates the learning of mathematics is combating math anxiety. Tobias (1993) defines *math anxiety* as the feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations. Furthermore, math anxiety can cause one to forget and lose one’s self confidence (Tobias, 1993). This math anxiety theme is mentioned in the literature as *math phobia* by Gunderson et al. (2013) and Henrich et al. (2016). Keith and Jane both addressed instances of math anxiety and its effects on freshman lower division students. Keith only mentions math anxiety by name when discussing his practice that helps students combat it, which is mentioned during the discussion of the research question on practices.

Consider Keith's comment below concerning his belief:

Keith: Sometimes you feel in a flipped class that you're not really teaching. You're more of a cheerleader. You can do this! You can do this! Yes, I know you can! Yes, it's tough, but we can [do it]. But I guess we're kind of a cheerleader in all of our classes too. We're on their side. We've got to keep their morale up, keep them going. Especially this time of year, right now when we've been going and going and going and it's near the end and you're tired. You've not had a break and they're not used to working this hard, this much, covering this much material this quickly. For them it is different. And it's tiring having to manage all that without mom and dad to help.

Jane's comments reflect a math anxiety theme, however, she discusses how it negatively impacts her students, noting that it is a goal of her teaching to help students feel comfortable with math.

The following comments are illustrative.

Jane: I want them to feel more comfortable to approach a problem. So part of my goal is to try to get them so they're not afraid to approach a math problem and use their critical thinking skills to approach the problem. The math anxiety for the level of students that I teach is pretty debilitating. It's so bad for some of them that they need to pick a different major so they don't have to deal with those classes.

Both Jane and Keith's comments address their roles as supporters and confidence builders in their classes. The next student-centered beliefs I discuss relate to teaching.

Beliefs on Teaching. The next two beliefs are student-centered instructor beliefs that focus specifically on teaching students or teaching how to teach students. They include *active learning*, *advising math Ph.D. students*, and *supporting graduate teaching assistants*.

Active learning. As mentioned in chapter three, Petress (2008) describes active learning as exhibiting the following traits: not being overtly dependent on a teacher, asking questions of clarification, challenging ideas and procedures, following up learning sessions with personal inquiries, connecting the most recently learned concepts with previously learned concepts, and keeping an enthusiastic attitude. Three of the eight faculty members' interview comments reflect traits of active learning as part of their teaching beliefs. This group of three includes Sanborn with respect to higher level lower division math students and mentoring graduate teaching assistants as they begin teaching; Susan with respect to mentoring graduate teaching assistants; and Lois with respect to freshman level lower division mathematics. The following quotes illustrate the active learning component of this group's beliefs on teaching

Lois: I encourage students' thinking and students' active participation. So active learning is a big deal. They are accustomed to active learning from high school. They are accustomed to something different. More teachers have taken up active learning, so I think the students' expectations have changed.

Lois indicates that students seem to expect active learning when they arrive in her classes. It is unclear whether she encourages active learning because the students expect it or if she emphasizes it because she believes it to be an important aspect of effective teaching. Perhaps it has to do with both of these reasons. Sanborn on the other hand links active learning to the content. Consider his comments:

Sanborn: Last year I really tried to incorporate more active learning. It really depends on how many homework questions they have. It works well with some sections but not all. Worksheets have really standardized *active learning* components. [The graduate students] are supposed to do all the worksheets in class.

Sarah: Get active learning going on in your classroom so you don't think that just because you said something, that they understood what you said. I don't think that graduate students or young teachers starting out often think that if you say something, that students may not understand what you said. It's not the case that they'll always remember what you said and understand what you said.

Sarah's comments reflect the importance of active learning. She conveys that active learning is important because it helps students to remember and understand what is said in class.

In summation, these quotes show that Lois uses active learning to elicit student participation, and to meet student expectations from their previous classes. Sarah uses it to confirm whether or not her students actually understand what she is teaching. She believes it is a poor assumption to believe students understand what is being taught in class without verification. She also sees it as a necessary skill to be taught to inexperienced teachers, graduate teaching assistants in particular. Sanborn incorporates active learning into materials he provides for the graduate teaching assistants he mentors. It should be noted that Sanborn and Sarah's inclusion of graduate teaching assistant mentoring connects to DeChenne et al. (2012), Deshler et al. (2015), Holm (2016), Lewis and Tucker (2009), Miller et al. (2018), Speer et al. (2005) from the literature. This gives weight to the significance of this belief. Graduate teaching assistants will also be discussed later in the student practices theme.

Math Ph.D. advising. In this section, the following beliefs apply to advisors of mathematics Ph.D. students, specifically what should be involved to properly advise these students. This theme arose from a question posed to both Marty and Charlie. The question considered what expectations a student should have of their advisor. Specifically, the question asked if it was reasonable to require an advisee to complete the research on what eventually

becomes an overly ambitious topic, and risk the student not completing their dissertation on time to graduate. The following quotes illustrate this theme:

Marty: No. [The student] has to say I'm learning it. I'm learning it and can somebody help me learn it better? So that's the first requirement, to try. And then the second important realization that a person ought to have is that I may or may not succeed in the goal that I've set myself up. And it's a question of how long I will try. You know, the Ph.D. has a time limit. You can't keep on doing it for 12 years. But if you have understood mathematics by then, to an extent that you have the courage and have a faith in yourself that you can study, you can carry this forward even when you haven't really succeeded in it right now. That is the only way getting through graduate school or earning a degree of Ph.D. has any value. Otherwise it has no value, whatsoever. Really speaking, the Ph.D. is just a degree. Degrees have no value. Education is a mechanism within a human being. It has nothing to do with degrees and certificates issued by other people.

Charlie: I think a good topic for a student can expand up or down. For students to spend multiple years actually working on a project and not have anything that's worth being a thesis is probably a bad topic. Sometimes it's hard to know. You know, doing research is a different thing than taking classes. It's kind of a transition. I don't think they should sort of be on their own with their problem. Their research should just be baseline, it doesn't have to be a major contribution. I don't think the significance of the contribution is necessarily a threshold.

Marty's response focused on beliefs and actions of the student as well as an overall philosophy on the purpose for obtaining a Ph.D. in mathematics. Charlie's response focused

more on the what the advisor should be doing in this instance rather than the student, as well as details on the transition from coursework to research. The clear difference in their opinions concerns the reasonableness of the requirements for a student to complete the research portion of their Ph.D. completely independently. Marty believes that there is only value in the Ph.D. if it represents primarily independent work requiring a long time of study, with the possibility that the student not actually finishing. The integrity of the Ph.D. process must be maintained, or the degree has no value. Charlie on the other hand, believes that a student's topic is adjustable, and that a good topic can expand up or down. Furthermore, for a student to actually work on a topic for a long period of time without any meaningful results is probably a sign of a bad topic, not necessarily an incapable student. Both Marty and Charlie mention that the student can receive help.

Neither Marty nor Charlie gave any direct indication as to how an advisor should help their advisee complete their dissertation. However, Marty's response included the statement:

Marty: If you have understood mathematics by then, to an extent that you have the courage and have a faith in yourself that you can study, you can carry this forward even when you haven't really succeeded in it right now.

This would seem to imply that success in a math Ph.D. program depends on a student's internal conviction, generated by their understanding of mathematics, which the participant believes is a by-product of success from completing the Ph.D. program up to the research portion.

Both Marty's and Charlie's comments reflect differing ideas about their roles in supporting graduate students' research in mathematics. Continuing with the topic of graduate students, the next set of beliefs related to supporting graduate teaching assistants (GTA's).

Supporting GTA's. The preparation for teaching graduate teaching assistants was also a significant idea found in the interview data. This is another connection to the literature noted by DeChenne et al., (2012), Ewing (1999), Holm, (2016), Lewis & Tucker, (2009), Miller et al., (2018), Speer et al., (2005). Sanborn, Lois, and Sarah made references to this idea. Some of the beliefs concerning expectations of GTA's include the connection between content covered in class and writing exams, conveying an attitude associated with communicating effectively, respectfully, and with positivity, and assessing what students understand and what they do not understand. Sanborn referenced the writing of exams in his interview.

Sanborn: That's one thing that very much bothers me with the other mentoring programs. The grad students are never once having to write an exam. And I definitely think that is part of teaching. Everything is pre-written. You as a lecturer have zero control over changing it, quizzes everything. For my class I definitely want [graduate students to write exams] because that is part of teaching. I think it's imperative to being an effective teacher. In my eyes the creation of an exam does not happen when you are actually writing it. The creation of the exam is when you're thinking about your lesson plans. That's where it starts.

This quote is of particular significance, providing direct connection to the literature provided by Lewis and Tucker, (2009), which states that teaching seminars for GTA's should cover writing exams. Lois spoke in reference to necessary communication skills that she not only attributed to herself when speaking about positivity, but also to her colleagues and GTA's.

Lois: I expect my colleagues or my GTAs to convey an attitude [that facilitates] communicating effectively, respectfully, and with positivity, which is sometimes hard to

articulate to them. [For example I might say], that thing you just said in the classroom, that might not convey the right attitude.

For Sarah, as noted previously in the active learning belief, she recognizes that beginning teachers should be aware of the need to determine what students understand and what they do not understand in class, since it is too easy for an inexperienced teacher to assume that just because they said something that the students actually made the connection.

Sarah: I think graduate students or young teachers starting out often think that if you say something that students understand what you said. They're going to remember what you said and they're going to understand what you said. And that's just not the case.

Lois echoed Sarah's statement in a slightly different way. She noted that displaying an enthusiastic attitude towards mathematics does not necessarily inspire students to be enthusiastic towards mathematics, nor does it necessarily justify the importance of mathematics to students.

Lois: [What are we doing and why are we doing it?] [Students] don't just get that. I used to think in my naivety that if I was just enthusiastic about math, they would just get it. That it's cool. Some of them do. You're enthusiastic about it, you know, look, it's cool and then that's enough and they think it's cool. But then I finally learned, they don't always just get it. I mean, you could be doing something and be like, obviously this is useful. Obviously this is cool. They didn't just automatically pick up on the relevance for what we were learning in class.

In summation, this subtheme on beliefs in teaching included beliefs relating to active learning, advising Ph.D. students in mathematics, and supporting graduate teaching assistants in mathematics. The beliefs concerning active learning involved students' active participation as a way of verifying what is actually being learned in class, as well as meeting students'

expectations from their high school experiences where active learning is a standard classroom practice. Advising Ph.D. students in mathematics included a belief that students should realize that it is their job to learn independently and view their advisor as someone that can help them learn in a better way. Another belief involved dissertation topics being allowed to expand up or down and students not being left alone on the problem they are solving. Supporting GTA's included a belief that writing exams is part of teaching and that GTAs should be involved in that process, as well as communicating with students in a positive and respectful way. Finally, there were beliefs concerning GTAs not assuming that students understand everything being taught to them or assuming that students are enthusiastic about mathematics. In addition to student-centered beliefs, the data also revealed beliefs on mathematical content which will be discussed next.

Content-Centered Beliefs

The next major theme revealed by the data focused on the particular mathematical content being studied in the classroom. Six of the eight participants commented on practices related to this theme. This theme is divided into two subthemes called philosophical beliefs and beliefs of content. Philosophical beliefs includes *rigor at the lower division level* and *conceptual understanding*. Beliefs on content includes *interpreting and writing proofs* and *the whole answer*. The whole answer is an In Vivo term which describes students' willingness and ability to consider the entire argument which led to the solution to a mathematical problem as the answer rather than the stereotypical numerical value at the very end. For example, in an elementary algebra course, students routinely solve different types of equations. Very often students think of a numerical solution to an equation as being the answer and all the work which led to the solution as unimportant. The participants who made comments concerning this idea

have beliefs which inspire them to facilitate students' understanding that the entirety of the work that led to a solution is the answer, not just the numerical value at the end.

An outline of the structure of this theme is included below.

Content-Centered Beliefs

- a. Philosophical Beliefs
 - i. Rigor at the Lower Division Level
 - ii. Conceptual Understanding
- b. Beliefs on Content
 - i. Interpreting and Writing Proofs
 - ii. The Whole Answer

Philosophical beliefs. Philosophical beliefs includes *rigor at the lower division level* and *conceptual understanding*. I will first discuss rigor at the lower division level.

Rigor at the lower division level. Interview questions concerning mathematical rigor at the lower division level specifically addressed how much rigor, in the form of mathematical proof, should an instructor use when explaining a concept, and how much rigor should be expected from students. Using the language of the participants, rigor will be defined as the degree to which an instructor poses the questions *why* with respect to a particular concept, gives examples of answering *why*, and the degree to which students are expected to answer *why* concerning a particular concept. This language is used in the following quote:

Charlie: I think students sometimes get intimidated by proof. We shouldn't think of it as there's computation and then there's proof. Proof is just answering the question *why*. Students should be able to answer the question *why* without being intimidated by it, and without getting hung up on quantifiers. They should be able to understand *why*.

Six out of the eight faculty members agreed that any type of rigor involving proof is unnecessary at the lower division level. The remaining two did not mention this type of rigor during the interview. When describing rigor, faculty used the phrases, big picture, basics, proofs, and deriving.

Marty's comments reflect the idea that it is not the students' job to prove everything, or in his words, see the big picture. He believes that to see the big picture requires a long time of practice. The following excerpt explains his thoughts.

Marty: At freshman, sophomore level they don't have to see the big picture at all. You can only see the big picture when you have total command over the small picture. The big picture is not something that should be taught at the beginning. So my philosophy of teaching is you have to build muscles then lift heavy weights. So the reason for that is mathematics is not something that can be understood at once. It is best understood by long practice.

Charlie states that in a non-honors lower division course, he wouldn't emphasize rigor. He starts to teach *why* things are true, or to introduce the concept of proof in an honors linear algebra course, which is a higher-level lower division course.

Charlie: In non-honors lower division courses I wouldn't put much emphasis on rigor. It's at the higher end of the lower division honors courses where I try to teach them more about why things are true.

Tim was very straightforward about the role of rigor, saying that these lower division students should simply know the basics. His comment, "They should know the basics, depending on the class", was in the context of pre-calculus and calculus II students.

Sarah made a similar comment to Tim, but with more detail as to what is actually taught in class, and what she expects her students to know and do. Her comments shared below will also tie into the next theme concerning students giving *the whole answer*.

Sarah: Just getting the basics of being a little more mature and formal about their mathematical arguments. Towards the latter half of the course I'm having them use the comparison theorem for integrals. And then all the tests and stuff for sequences and series. And there is where I start being really picky about what they write.

Sanborn disagrees with doing any type of rigorous proofs, but firmly believes in deriving some formulas in class. He explains his thoughts in the following excerpt.

Sanborn: Nothing. They should not have to do any proofs in my eyes, as far as rigorous proofs. Not at all. But that doesn't mean that you, [the teacher], shouldn't derive some formulas. I think that as you go along in the mathematical curriculum, the more derivation should be a part of the course. Because when you say the word derive do your students actually know what that means?

Thus, Sanborn's belief is somewhere in between no rigor and some rigor by deriving a formula or rule for his students, but not requiring that they understand everything about the derivation in the end. They are only required to know the rule and how to use it. He does address potential disadvantages of using class time to do derivations. This is described by the following quote:

Sanborn: However, there is a risk versus reward. The risk is the amount of time you spend on it. The reward is whether the people who are actually paying attention will get something out of it. For example, coming up with the derivative of sine using the formal limit definition. I think at this level it becomes a time where it's just too much. You should be aware of how much [time] it would take. I do like massaging that need to

derive things, but I don't think that I need to prove everything. Things that need more than 10 or 15 minutes, I don't know, it's not a good use of time.

Speaking within the context of mathematical reasoning, college algebra and statistics courses, Jane disagreed about this use of proofs. But instead of elaborating on where students should be in terms of proficiency with rigor after taking her class, she commented on how her students were not academically at the level they should be when they enter her course. This is illustrated by the following:

Jane: No. No proofs. And the rigor, I mean, obviously I want them to have the foundation. I want them to be able to [do the work].

All six of the eight participants who commented on rigor believe that rigor in the form of proofs should not be expected at the lower division level.

Conceptual understanding. There was also mentioning of conceptual understanding. In this case, seven of the faculty members made a reference to some kind of conceptual understanding of mathematics. By conceptual understanding the participants were referring to students having a deep understanding mathematics. The data includes references to the use of exam questions which test conceptual understanding, practicing the use of definitions, the need to relate mathematical concepts to other subjects, and the emphasis of working a problem with understanding rather than memorizing steps.

Charlie referenced topics and students in his linear algebra courses. It is interesting to note that at this level, Charlie is comfortable doing an exercise in class that he knows the majority of the students may not understand, but he believes it worth doing for the few students who will gain something from it.

Charlie: You sort of have the core material of the course, but sometimes you can throw in things that maybe a few students will get, not proof but like why things are related or why you do certain things. If I can say something in two minutes that two people in class will get, then that can be a worthwhile thing.

Charlie's point differs slightly from Sanborn's philosophy of maximizing time spent in class for the majority of students. This may be due to differences in teaching philosophies or styles, but it also may be due to the level of students being taught. A course in linear algebra is more likely to serve only mathematics majors than a calculus course.

Continuing with this subtheme, Sarah described her emphasis on conceptual understanding by referencing conceptual questions she asks on exams in calculus II.

Sarah: I try to ask some conceptual questions on an exam. [I am] basically not handing it to them in the names that they're used to but telling them the relationships that they need to be able to figure stuff out.

Sarah believes in having students demonstrate their conceptual understanding by asking questions that do not use the familiar names and examples of functions, but only give the relationships among functions where the students must answer questions knowing the relationships.

Marty's comments about conceptual understanding were likely motivated by his experiences from teaching mathematics majors, specifically referencing concepts taught in an advanced calculus course.

Marty: It is not possible without practicing the [epsilon-delta] definition [by] writing epsilon-delta proofs for limits to understand the definition. You don't have to do it in a day and night, but you have to do it until the **concept** actually just completely remains in

your subconscious so that you know that you don't have to make extra efforts to just artificially remember the definition.

Jane's statement concerning her freshman lower division students also suggests the need for conceptual understanding. She commented that students have the ability to apply their knowledge for purposes other than simply getting the right answer. She believes it is important to understand why things happen so her students can handle something new that they have never seen.

Jane: I definitely want them to know what they're doing things. Because I think they're better able to deal with something new that's thrown at them. If they just learned the mechanics, they can't apply it in other places. Unfortunately, that is what happens. So a lot of them are just [able] use to just using the calculator to do a lot of stuff. They just grab it right away. But as far as understanding why they're doing this and that, I think it's really important. And it's really hard because they just want the answer. I guess that's pretty typical. They want the answer. They just want to get there, and they don't want to know why things are happening. I think it's important to understand why things are happening.

Similar to Jane, Keith makes the point that having a conceptual understanding will give students the ability to apply mathematical concepts to other subjects.

Keith: They do need to learn that work ethic. Learn the vocabulary and understand the concepts. [Do not] just memorize a bunch of formulas, but understand why things work the way they do, and how they work, and how to put them together in other subjects.

Sanborn does not focus on step-by-step procedures. He believes in using conceptual understanding to figure things out.

Sanborn: I'm not a big fan of steps on problems. I am a big fan of going over how to solve something and really emphasize on how to figure it out.

Similar phrases among all the quotes are *why things work*, *how they work*, *why things are happening*, *why you do certain things*, and *why things are related*. These phrases were used by Keith, Sanborn, Jane, and Charlie which imply the need for students to answer questions about the specifics of the content they are learning. It seems as though the idea of conceptual understanding is similar to the idea of rigor, but it replaces the need for formal rigor which the majority of the participants stated was unnecessary at the lower division level.

In summary, within the philosophical beliefs subtheme there were beliefs concerning rigor in lower division undergraduate mathematics courses, as well as conceptual understanding in mathematics. For rigor in lower division mathematics, the general consensus was that rigor in the form of mathematical proofs is unnecessary. However, one participant commented that he does include derivations of rules and formulas in his class, but there is risk and reward for doing so. For conceptual understanding, six of the participants commented that conceptual understanding is important. These comments included thoughts on classroom activity, exam questions, intense focus on definitions, the ability to apply mathematics in other subjects, and putting an emphasis on figuring out a problem instead of following a predetermined sequence of steps. The next subtheme concerning content involves beliefs on content.

Beliefs of content. The next subtheme consists of beliefs concerning written mathematical content being taught in the participants' classrooms. This subtheme is divided into *interpreting and writing proofs* and *the whole answer*. Both of these are examples of student written work.

Interpreting and writing proofs. The first significant belief concerns a very important aspect of the traditional introduction to abstract mathematics or introduction to proofs course that undergraduate mathematics majors, typically at the junior level, take as a transition from lower division mathematics to upper division mathematics. This important aspect is the interpretation and construction of mathematical proofs, which are necessary at the upper division undergraduate and graduate levels (Cupillari, 2013). This is an important theme because many undergraduate mathematics majors have no experience with this flavor of mathematics before they enter the course. It is in this course where these students learn an important aspect of the mathematics they are studying. (Celik & Shaqlaih, 2017; Jaworski, 2016). Furthermore, the interpretation and writing of proofs tends to be a major challenge for many students at this level (Cupillari, 2013).

Charlie and Marty commented on their beliefs and were in the best position to provide an opinion since they teach upper level undergraduate and graduate students as part of their normal teaching assignment. They both agree that the primary reason for the challenge experienced by students at this level is that proofs in mathematics are only now, between sophomore and junior level classes, being made part of the everyday mathematics curriculum and should have been part of the curriculum from the beginning. To dispel the notion that this implies that junior level undergraduate rigor should be included at the lower division level, which would contradict the first theme of the chapter.

Charlie specifically mentions that proofs essentially answer the question *why* in mathematics. As you will see from his quote, there are many ways to phrase the *why* without directly calling it a proof or writing a traditional proof.

Charlie: I think students sometimes get intimidated by proof. We shouldn't think of it as there's computation and then there's proof. Proof is just answering the question *why*.

Students should be able to answer the question *why* without being intimidated by it, and without getting hung up on quantifiers. They should be able to understand *why*.

The following quote of Marty is confirmed by Celik & Shaqlaih (2017) and Jaworski (2016) which state that proofs are not a typical part of secondary curriculum. Even though this belief specifically addresses curriculum of upper level undergraduate mathematics majors, the quotes from Charlie and Marty imply a need for a change in lower level curriculum down to the secondary level. The change being needed to normalize the expectation of answering the question *why* in mathematics. The implications on secondary mathematics curriculum will be addressed further in the next final chapter. I mention it now to give significance to the belief.

Marty: Well [proof] type-oriented courses should be there right from the beginning. I won't call [the beginning] elementary for the grades they are in. They're equally tough as [higher level material]. And in calculus also, I do not really like that calculus courses do not actually allow you to practice small degrees of proof. Because there is no such thing as the mathematics of proof being different from mathematics. It's really one thing. It is artificially separated by people, in my opinion, who are completely misguided about such separations. And that creates unnecessary pain later on, especially for those who want to study mathematics. It is a belief among some high schools and so on that they are not really concerned with the proofs. But really speaking, proofs allow you to develop better methods. So to some extent they have to [do proofs]. They don't have ask for proficiency in proof writing. But I think understanding of proofs is necessary.

Another significant comment made by Marty concerns what he describes as an artificial separation of mathematics, specifically with respect to proofs. He believes that these separations actually hinder students, especially students who want to study mathematics.

The whole answer. Two of the participants mentioned a belief about the work of a problem being part of the answer. The title of this idea is a combination of a terms used by Sanborn, *the work is the answer* which Sanborn mentions concerning a practice he uses on pg. 140 and Sarah, *your whole argument*. This explains a student's inability or unwillingness to consider the work for a problem as part of the answer.

Sanborn: People think of math as getting the right answer, not [what does it mean].

Because to me, half the time I could care less with the [the "answer" is], you know. [One day a student] came in with a related rates problem I [assigned]. A 12-point problem. I gave checkmarks on all the places that he got points. He thought it was funny. He's like, for the final answer there's no check marks. I was like, no. You communicated all the right things. So that's relevant.

Sarah echoed Sanborn belief with the following by stating "It's your *whole argument* that I am grading. I really try and get them away from what's the answer. General mathematical maturity." These two quotes give examples of what Sarah and Sanborn expect their students to be able to do and understand in class. Sanborn conveys that he is trying to show his students what a mathematical solution actually is, specifically that it is more than just the right answer. Sarah describes the whole answer as the whole argument. In other words, the justification for the answer is part of the answer. It is very clear that Sanborn values the argument over the answer when he states that he could care less what the answer is. This idea of the whole argument or the whole answer is a similarity in the teaching philosophies of Sanborn and Sarah.

In summation, this subtheme on beliefs on content included beliefs on interpreting and writing proofs as well as students writing what the participants referred to as the whole answer. The consensus on proof writing in the upper division level of mathematics where it is required was that proofs should be thought of as nothing more than explaining why something is true. Furthermore, the necessity for explaining why something is true should be expected even in elementary mathematics. The whole answer concept included a belief that questioned what getting the right answer on a problem even means, as well as a belief that the entire mathematical argument is the answer. The next major theme revealed by the data concerns beliefs on the practices of students.

Beliefs on Student Practices

I will now discuss the final major theme revealed by the data concerning the beliefs of the participants, which includes two subthemes each consisting of beliefs on the practices of students that the participants deem responsible for the success or lack of success in learning post-secondary mathematics. The two subgroups, *philosophical ideas* and *habits of students* include and describe, based on the data, ideas and actions taken by students that help or hurt students' abilities to facilitate their own learning of post-secondary mathematics. An outline of the structure of this theme is included below.

a. Philosophical Beliefs

- i. Student Responsibility
- ii. Motivation of Math Ph.D. Students

b. Characteristics of Students

- i. Characteristics of Successful Graduate Students
- ii. Characteristics of Successful Undergraduate Students

iii. Characteristics of Unsuccessful Students

Philosophical beliefs. The philosophical beliefs subtheme consists of the beliefs on *student responsibility* and the *motivation of math Ph.D. students*. These beliefs concern a type of internal conviction that Marty, Tim, Charlie and Keith describe that students need to be successful.

Student responsibility. Three of the eight faculty members made specific statements regarding this belief. This includes Tim with respect to higher-level lower division math students, and Keith and Jane with respect to freshman level lower division math students. It is illustrated by participant quotes that an expectation of student responsibility is a common belief among instructors who teach different levels of mathematics. Tim commented, “The derivative is something you crank out in practice. I assume they really know how to do those.”

Jane: I would expect [students] to be at a certain level when they walk in. I'm not finding that they are. And that's really disturbing. They keep telling us the act scores keep going up and up and up. But I'm not really seeing it. I'm seeing a big divide. There are students that are comfortable with [doing math]. It's easy. They just have to do it. Then there are others that cannot. They can't work with fractions. They don't know what a log is. They just don't understand concepts it seems. So how do you move forward? I don't know. I still struggle with that.

Keith: So, you know, you want to teach them the math, but I feel a bit of an obligation to teach them, I don't know if responsibility is the right word. We'll say math responsibility and math accountability for their work or lack thereof, sometimes.

Based on their comments, Tim and Jane have a slightly different interpretation of student responsibility from Keith. Tim and Jane's comments on responsibility focus more on students'

proficiency with basics. Tim's comments on the derivative are specifically about students' ability to do something procedural. He describes it as something you crank out. Jane is describing proficiency with prerequisites that she believes students are responsible for knowing. Keith describes responsibility in more general in terms instead of mathematical terms, specifically referencing potential life challenges. It will be shown in the discussion of the second research question that he is the only person to mention how he facilitates this responsibility.

Motivation of math Ph.D. students. The next practice of students concerns the motivation of doctoral level students in mathematics. This was mentioned by both Marty and Charlie, the two faculty members who routinely teach this level of mathematics. Both of them mention a type of internal motivation needed to complete a doctoral program, but their opinions differ in terms of what that motivation looks like. Consider their comments:

Marty: For a Ph.D. student a burning passion for mathematics and learning and the ability to self-learn is required. You must hold within you some attraction towards it.

[Mathematics] is not some kind of a mechanical task like any other subject. I cannot tell the person go conduct the following 50 experiments and tabulate the results and plot the graphs by known methods. Use the software that I tell you to use, that's not pure mathematics. You have to be fired up about learning the subject even without actually having tremendous knowledge about it.

Charlie: To get through a graduate program you have to have some internal motivation to want to finish. At some point you just want to see it to the end, and you don't necessarily have the internal motivation to study the math for its own sake, but just to complete the process.

Marty sees the required motivation as being more intrinsic in nature, while Charlie recognizes that at some point a student may just want to finish and seeing the end may be motivation enough. However, Charlie does use the phrase *at some point* when talking about completing the Ph.D. process. So it is possible that the extrinsic motivation to finish is only significant towards the end of the process. Neither Charlie nor Marty discussed how students develop this motivation.

In summation, this subtheme on philosophical beliefs on student practices includes beliefs on student responsibility and motivation of math Ph.D. students. Beliefs concerning student responsibility included an expectation that students come prepared to do work at the level expected of them by having mastered material studied during previous courses. Another related belief was that not only should students be responsible enough to master the required material, but also that instructors have a duty to teach them this type of responsibility. Motivation of math Ph.D. students includes beliefs concerning what is required for students to complete a doctoral program in mathematics, specifically a burning passion for mathematics and the ability to self-learn. Another belief concerning doctoral students in mathematics completing their program differed slightly by stating that the motivation to finish does not always have to be intrinsic.

Beliefs on students. The final subtheme, beliefs on students, concerns beliefs on the characteristics and practices of students and lists some specific beliefs mentioned by the participants which describe characteristics of successful graduate and undergraduate students, as well as characteristics that describe unsuccessful students. Six of the participants, Marty, Charlie, Tim, Sarah, Jane, and Keith commented on at least one of these characteristics.

Characteristics of successful graduate students. We will start with comments related to characteristics of successful graduate students. Marty and Susan commented on practices they

find associated with successful doctoral students. Marty's comments were from the point of view of a seasoned doctoral advisor, and Sarah's were from the point of view of a former doctoral student in mathematics.

Marty commented, "So at that point in time in graduate school, what should happen is that now you know enough ways of learning by yourself so that now you don't need a classroom."

Sarah: I really think that once people get to the graduate level, they should have the chops to go off and figure it out themselves. I mean by third year or something like that [you] should be able to learn from no one. So I think it is kind of appropriate.

Marty has an expectation that students at this level have the ability to learn on their own, in other words they are completely independent. Note that this is consistent with his philosophy concerning the role of a Ph.D. advisor. Sarah describes advice she received from her doctoral faculty which would make her successful. She also echoes Marty expectation of independence when she describes her and her peers learning on their own, and her own belief that at the graduate level people should have the chops to go off and figure it out themselves, to use her words. "I really think that once people get to the graduate level, they should have the chops to go off and figure it out themselves. By your third year [you] should be able to learn from no one."

The next set of beliefs discussed which were revealed by the data concern practices of successful undergraduate students.

Practices of successful undergraduate students. Marty, Sarah, and Tim shared comments on beliefs that are associated with successful undergraduate students. For mathematics majors, both Marty and Sarah believe a liking or curiosity and self-effort is needed to be successful in mathematics.

Marty: In a hierarchical subject like mathematics, information does not equal knowledge. You need a lot of *self-effort*. You have to have some kind of *liking* for mathematical thoughts. If you are absolutely dejected by those kinds of thoughts, you are not going to be able to learn mathematics at all.

Sarah: For an actual math major you need that **driving curiosity** and just wanting to have the formality, getting into the more formal side of math. Beating your head against a wall a little bit on your homework problems and then having it [the wall] tumble down and thinking that's really exciting, only to see that there's another wall. It's kind of fun. You have to kind of get to the point where it takes perseverance. For undergraduate and graduate students, for so much of it, it wasn't how smart people were, it's how hard they were willing to work.

Sarah also mentions perseverance over raw intelligence as a trait of successful math majors.

In addition, Tim's response was in the context of students of calculus that may not be actual math majors. He believes that students who take their own time to do the work usually do well.

Tim: Try and learn this stuff by doing it. It's something you've got to do. That's probably my main theory about this. The main responsibility of learning is on the student. If you don't know ask me because that's what I prefer. I'll try and get them to come in and say let's talk after school. I'll be glad to talk to you.

This idea of being independent and mathematics being somewhat an individual endeavor is echoed in students' comments in the secondary level and graduate work as well (Boaler & Greeno, 1998; Wagener, 2010). The final set of beliefs discussed which was revealed by the data concern the practices of unsuccessful students.

Practices of unsuccessful students. Three faculty members, Sarah, Jane, and Keith all commented on characteristics that they notice about students who are unsuccessful in their mathematics classes. Sarah's comments are with respect to higher level lower division students, mainly calculus. Keith and Jane's comments are with respect to students in mathematical reasoning, college algebra, or pre-calculus.

Some of these practices include an unwillingness to seek help from the instructor which may originate from math anxiety, poor attendance, laziness, making excuses, and being distracted. Jane cites an unwillingness to seek help from the instructor as an issue with her students. Or if they did seek help it was from an online source. She also made a comparison to some of her own family members who were not willing to seek help or do a seemingly easy task for their own good. Her quote is as follows.

Jane: I have three daughters and they, it's so funny, would much rather text somebody than to pick up the phone and call them. I think students are a little bit fearful to come in for whatever reasons. In fact, I had several students miss class. I'll say do you want the notes. [They say], oh no I'll look it up on YouTube. So it's like, okay. Well what am I here for?

Keith cites laziness and distraction as issues with students in his class. This includes sporadic attendance, being distracted in class by using their phones for social media purposes, and not participating in group work while letting others in the group do all the work. His quote is as follows:

Keith: They're just daydreaming in class, or even sometimes just sporadically coming once a week, maybe coming to class. Occasionally they've missed for like two weeks, doing bad on quizzes, and not participating in group work. When we do have some group

work, they're just sitting there and letting everybody else do the work. Or you constantly see them having people in their group having to explain everything to them. They're not really engaged. They're just kind of sitting there letting everybody else do the work or heaven forbid pulling out their phone and trying to play on their phone. And I'm having to walk by and [tell them to] put [their] phone up. This is math class, not social media hour.

Finally, Sarah cites students making excuses for their mathematical mistakes as an issue. Some of her students try to justify their incorrect solutions. They do not want to admit that they have things to learn, and essentially want to be correct from the start. Her quote is as follows.

Sarah: I have students that I see not being very successful. Every time they make a mistake, they have an excuse for the mistake that they made rather than [admitting] they did something wrong and try to figure it out. People want to justify their incorrect actions. Basically, they're not seeing it as a process of I want to learn what the right thing to do is, they almost want to be right from the get-go.

In summation, the subtheme on beliefs of students included beliefs concerning successful graduate and undergraduate students, as well as beliefs concerning unsuccessful students. The beliefs concerning successful graduate students include students having the ability to learn without the classroom. This was echoed in the discussion of beliefs on student responsibility. Beliefs concerning successful undergraduate students include having a driving curiosity and a liking of mathematics that endures throughout the inevitable struggles and setbacks that occur while learning mathematics, as well as students taking responsibility for their own learning. Beliefs concerning unsuccessful students includes characteristics like laziness or an

unwillingness to work, as well as not admitting when a mistake is made by attempting to justify an incorrect solution.

This section describes in detail the beliefs of the participants concerning the teaching and learning of mathematics revealed by the data. Based on the analysis, these beliefs were divided into three major themes which include student-centered instructor beliefs, content-centered beliefs, and beliefs on student practices. Each major theme had two subthemes and each subtheme included two or more beliefs. The next section will take the results compiled concerning beliefs and visually determine the similarities and differences of the beliefs using figures which will represent the different beliefs.

Similarities and Differences Among the Teaching Philosophies

While discussing the beliefs of the participants it was clear that certain instructors' beliefs fit within certain themes and not in others. The beliefs that fit within subthemes will clearly have more similarities than those in different subthemes. Likewise, the beliefs within major themes will have more similarities than those in different major themes. The analysis used to compare and contrast beliefs starts within subthemes, expands outward within major themes, and finally concludes looking across major themes. Recall how the participants' beliefs are being used as a way of describing their philosophies

Similarities and Differences Among Student-Centered Instructor Beliefs

The beliefs in this theme concern facilitating positivity in the classroom, combating math anxiety, active learning, advising math Ph.D. students, and supporting graduate teaching assistants. Facilitating positivity in the classroom and combating math anxiety are both in the subtheme characterized by beliefs on classroom environment, while active learning, advising math Ph.D. students, and supporting graduate teaching assistants are in the subtheme

characterized by beliefs on teaching. I will start by discussing similarities and differences among the beliefs in the classroom environment subtheme.

Similarities and differences among beliefs on classroom environment. This discussion will begin with beliefs on positivity in classroom and follow with beliefs on math anxiety.

Positivity in the classroom. Beliefs on positivity in classroom were very similar. There stated beliefs were made by Lois, Keith, and Sanborn. The statements made by Sanborn and Keith were very general, mentioning examples of positivity that include inclusion diversity, being welcoming, and giving confidence. Lois' comment on her belief was not significantly different, but she did specifically focus her comment on positive communication.

Math Anxiety. Jane and Keith made similar comments regarding their beliefs on math anxiety. They used examples that include having students feel more comfortable so they are not afraid to approach a problem and keeping morale up. They also made reference to the level of anxiety that some students have where some students end up choosing different majors, or noting how students are unaccustomed to the demands of a college level math course and do not have the ability to manage all their responsibilities without help from others.

In summation, it is seen that Keith, Lois, and Sanborn each share beliefs on facilitating a positive classroom environment, while Jane and Keith share beliefs on this issue of math anxiety. This is represented visually with the Figure 4.1 below. Participants are joined by a line segment if they stated a belief on the same topic.

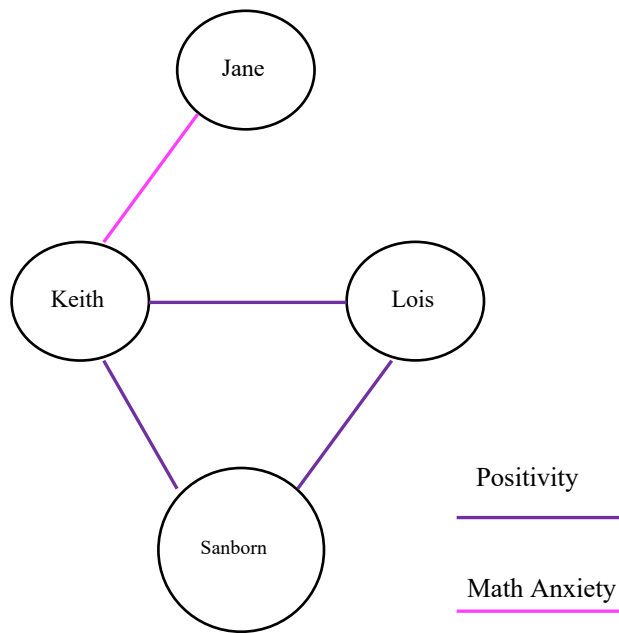


Figure 4.1: Beliefs on Classroom Environment

Next I will compare and contrast the beliefs on teaching.

Similarities and differences among beliefs on teaching. I will begin with beliefs on active learning, followed by math Ph.D. advising, and finally supporting graduate teaching assistants.

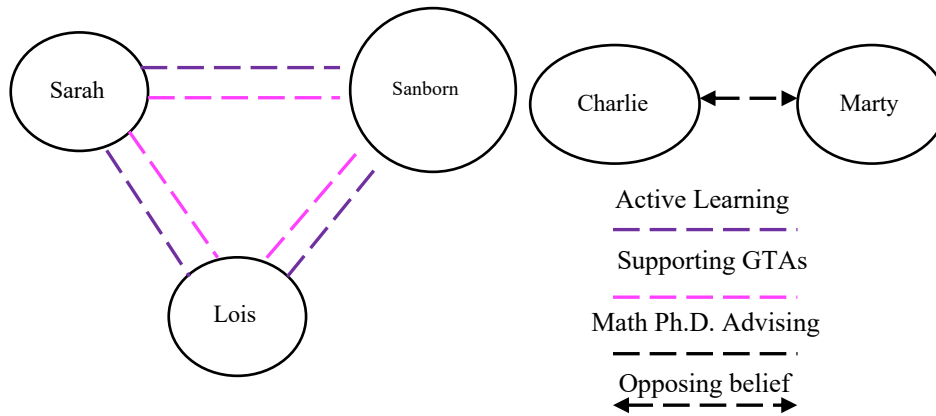
Active learning. Lois, Sarah, and Sanborn stated beliefs on active learning. Each referenced active learning for a different reason. Lois mentioned it out of a need to meet students' expectations because of the type of learning they are accustomed to from high school. Sarah's belief on active learning was motivated by the need for students to show what they understand and do not understand in class, as opposed to lecturing only and assuming that students understand everything that was said. Sanborn only mentioned active learning as it

relates to materials used by graduate teaching assistants and how it works well with some sections of his class but not all.

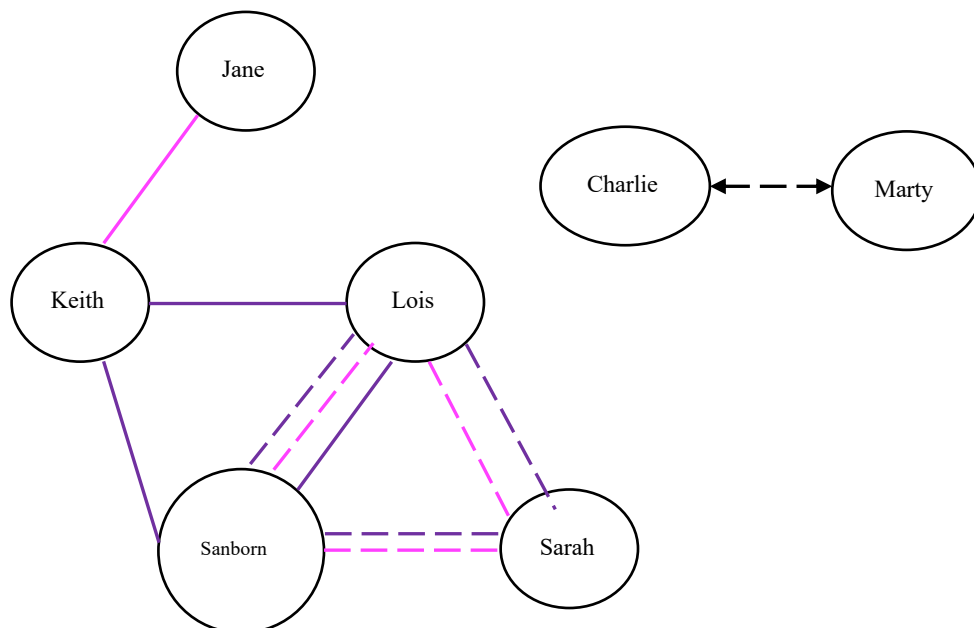
Math Ph.D. advising. Charlie and Marty were the only two participants in the position to comment on advising math Ph.D. students. Their beliefs differ with respect to expectations of the level of research being done by students. Marty's statement emphasized the importance of Ph.D. students being independent and learning on their own while thinking of their advisor or instructor as someone that can help them learn better. Furthermore, the Ph.D. degree has no value without the students completing a rigorous process. Charlie's belief was not as strict. He stated that students should not be on their own with their research problem, and that their problem could expand up or down to better suit the student.

Supporting graduate teaching assistants. Lois, Sarah, and Sanborn commented on beliefs concerning graduate teaching assistants. Sarah and Sanborn both commented on beliefs concerning expectations of GTAs' teaching. Sanborn spoke about the need for GTAs to take part in writing exams for their courses. He believes writing exams is directly connected to lesson planning. Sarah expectation concerned GTAs' abilities to determine whether their students understand what is being taught in class. Lois' comment different slightly in context. While speaking of facilitating a positive classroom environment, she mentioned an expectation of GTAs conveying a positive attitude while communicating to students.

In summation, it is observed that Lois, Sarah, and Sanborn all mutually share beliefs concerning active learning and supporting GTAs. Charlie and Marty have somewhat different beliefs concerning advising math Ph.D. students. This is represented visually by Figure 4.2 below. Participants are joined by a line segment if they stated a belief on the same topic.



By combining the Figure 4.1 and Figure 4.2 from these two subthemes we get visual of the similarities and differences of the participants beliefs within student-centered instructor beliefs theme represented by Figure 4.3.



With the exception of Marty and Charlie's somewhat opposing beliefs concerning advising math Ph.D. students, the remaining beliefs among the other participants are very similar. Next, I will compare and contrast the beliefs found within the theme on content-centered beliefs.

Similarities and Differences Among Content-Centered Beliefs

The beliefs in this theme concern rigor at the lower division level and conceptual understanding which are included in the subtheme on philosophical beliefs, and interpreting and writing proofs and the whole answer idea which were included in the beliefs on content subtheme. I will begin by discussing the similarities and differences among the beliefs in the philosophical beliefs subtheme.

Similarities and differences among philosophical beliefs. I will begin with beliefs on rigor at the lower division level followed by beliefs on conceptual understanding.

Rigor at the lower division level. Charlie, Jane, Marty, Sanborn, Sarah, and Tim commented on beliefs concerning rigor. There was complete agreement among these participants that rigor in the form of formal proofs should not be required at this level. They used phrases which included the following: students knowing the basics, not doing any proofs, knowing the basics and being a little more formal about their mathematical arguments, wanting students to have a foundation for learning, not having to see the big picture until they have total command over the small picture, expecting students to do traditional problems without particularly tricky ways. Only Sanborn stated something different than the other five by mentioning that deriving formulas should not be out of the question.

Conceptual understanding. Charlie, Jane, Keith, Marty, Sanborn, and Sarah all mentioned beliefs concerning conceptual understanding. Keith and Jane's beliefs were similar. They both mentioned conceptual understanding as a requirement for students in lower division courses using mathematics in different subjects or in something that they have never encountered. Marty commented on math majors needing to understand a definition which is crucial to a specific concept without making extra efforts to artificially remember the definition. Like Marty, Sarah illustrated her belief in the necessity for conceptual understanding by referring to math majors and students in higher-level math courses writing formal arguments. Similar to Marty, Sanborn described his belief in conceptual understanding by emphasizing how students should be able to figure out a method of solving a problem rather than artificially remembering a sequence of steps to solve the problem. Finally, Charlie described students developing conceptual understanding when he throws out ideas that maybe only a few students will understand, but then has his students work in groups where students will help each other.

In summation, it is observed that five participants Charlie, Jane, Marty, Sanborn, and Sarah all mutually share common beliefs on both rigor at the lower division level and conceptual understanding, while Tim shares a belief on rigor with these five and Keith shares a belief with these five on conceptual understanding. This is visually represented by the Figure 4.4 below. Participants are joined by a line segment if they stated a belief on the same topic. Next I will discuss the similarities and differences among beliefs on content subtheme.

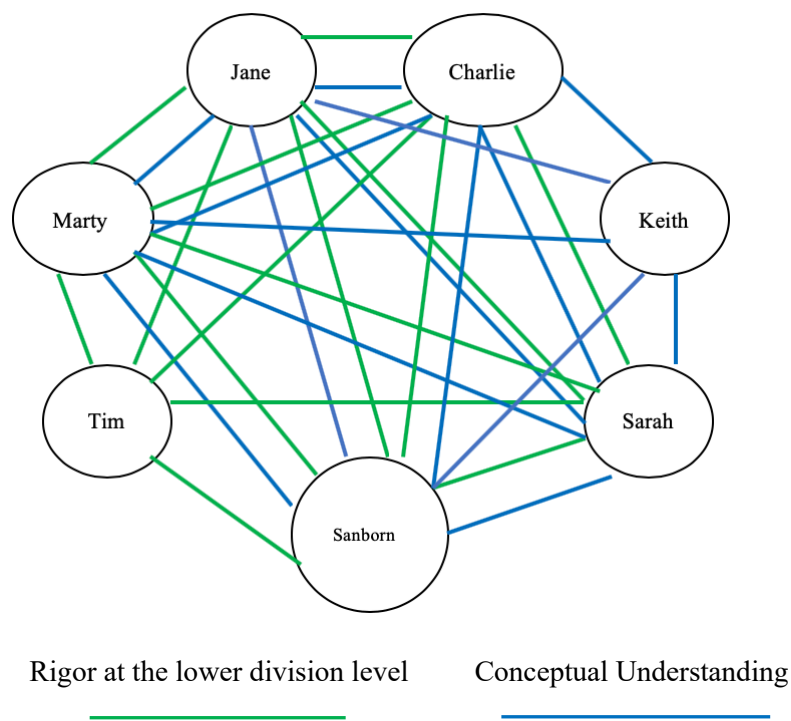


Figure 4.4: Philosophical Beliefs

Similarities and differences among beliefs on content. I will begin by discussing beliefs on interpreting and writing proofs followed by beliefs concerning the whole answer idea.

Interpreting and writing proofs. Marty and Charlie each shared their beliefs on undergraduate mathematics majors writing proofs. Recall that this was phrased as answering the question *why* concerning the truth of a mathematical statement. They were very much in agreement that answering *why* is a fundamental part of mathematics at any level, and that there should not be a specific moment when answering *why* should introduced into the curriculum. Answering *why* should be taught from the beginning. They also both agree that the level of expectation for answering *why* can be adjusted depending on the level of mathematics being taught.

The whole answer. Sanborn and Sarah each shared their beliefs on the whole answer idea. The code for this belief, the whole answer, was inspired by phrases used by Sanborn and Sarah. Sanborn commented that, “The work is the answer.” In his response in practices in the next section, and Sarah commented that “It’s your whole argument that I am grading”. They each believe in changing students’ perception of what a mathematical argument is by getting them away from thinking that the answer is a number at the end of a problem.

In summation, the beliefs on content have similarities within interpreting and writing proofs, with Marty and Charlie having similar beliefs, and the whole answer idea with Sanborn and Sarah having similar beliefs. This is visually represented by the Figure 4.5 below. Participants are joined by a line segment if they stated a belief on the same topic.

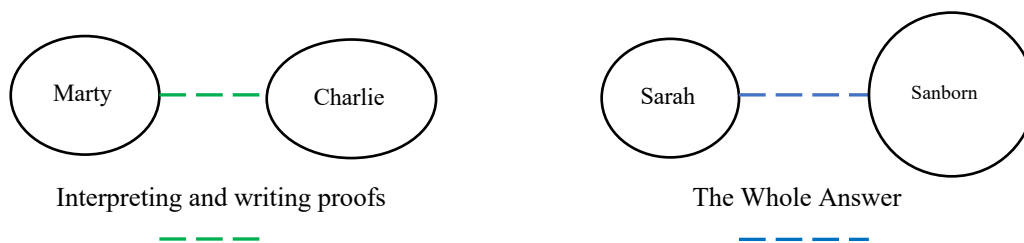


Figure 4.5: Beliefs on Content

By combining the Figure 4.4 and Figure 4.5 from these two subthemes we get a visual of the similarities and differences of the participants beliefs within student-centered instructor beliefs theme represented by Figure 4.6 below.

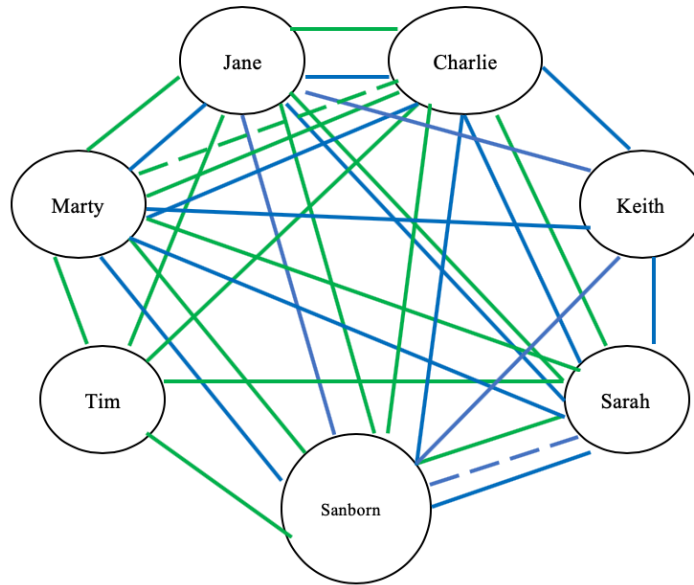


Figure 4.6: Content-Centered Beliefs

Similarities and Differences Among Beliefs on Student Practices

Finally, I will discuss the similarities and differences among the beliefs within the theme on student practices. The beliefs in this theme which were revealed by the data concern student responsibility and motivation of math Ph.D. students in the subtheme on philosophical beliefs, and characteristics of successful graduate and undergraduate students, and characteristics of unsuccessful students in the subtheme on characteristics of students. I will first discuss the philosophical beliefs subtheme.

Similarities and differences among the philosophical beliefs. I will begin with the beliefs on student responsibilities and then discuss the beliefs on the motivation of math Ph.D. students.

Student responsibility. Jane, Keith, and Tim all made comments concerning their beliefs on student responsibility. Jane and Tim were speaking about their students in lower-level courses, while Tim spoke about his students in higher-level lower division courses. Jane and Tim's beliefs were similar. They both commented that students have a responsibility to come into their classes being proficient on pre-requisite material they were supposed to learn in earlier classes. Jane commented, "I would expect [students] to be at a certain level when they walk in." Tim made comments on his students find derivatives in his calculus courses. He commented, "The derivative is something you crank out in practice. I assume they really know how to do those." Keith on the other hand not only mentioned his students' responsibility but also his obligation to facilitate their responsibility. He commented, "You want to teach them the math, but I feel a bit of an obligation to teach them math responsibility and math accountability for their work, or lack thereof." So there is a significant difference among the comments made by Jane and Tim compared to Keith. They are not opposing but certainly different.

Motivation of math Ph.D. students. Marty and Charlie commented on their beliefs concerning the motivation of math Ph.D. students. These beliefs were somewhat opposing. Marty's belief was illustrated by the following comment.

For a Ph.D. student a burning passion for mathematics and learning and the ability to self-learn is required. You must hold within you some attraction towards it. You have to be fired up about learning the subject even without actually having tremendous knowledge about it.

Charlie disagreed with Marty about the type of motivation that it takes to complete a Ph.D. in mathematics by stating the following:

To get through a graduate program you have to have some internal motivation to want to finish. At some point you just want to see it to the end, and you don't necessarily have the internal motivation to study the math for its own sake, but just to complete the process.

This would seem to oppose Marty's belief to a certain degree.

In summary, there is similarity between the beliefs of Jane and Tim and a difference in beliefs of Keith concerning student responsibility. There is also a difference in beliefs between Charlie and Marty concerning motivation of Math Ph.D. students. This is visualized by Figure 4.7 below. Participants are joined by a line segment if they stated a belief on the same topic.

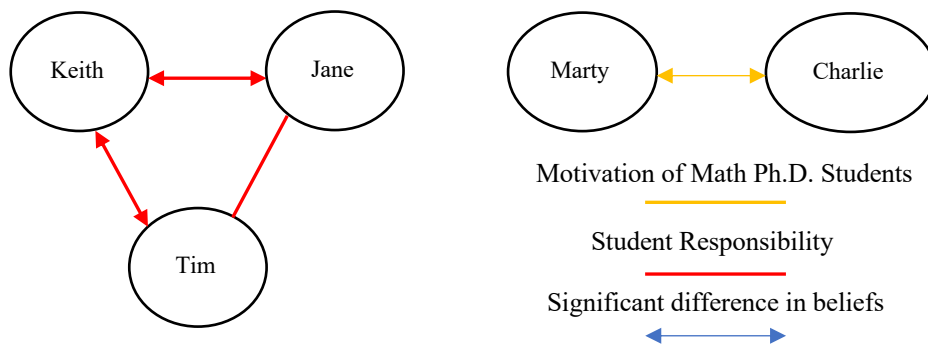


Figure 4.7: Philosophical Beliefs

Similarities and differences among beliefs on students. This theme consists of beliefs on the characteristics of successful graduate and undergraduate students, and characteristics of unsuccessful students. I will first discuss characteristics of successful graduate students followed by characteristics of successful undergraduate students, and finally characteristics of unsuccessful students.

Characteristics of successful graduate students. Marty and Sarah commented on characteristics of successful graduate students. Marty believes that successful graduate students should be able to learn independently without the need for the classroom. Sarah made similar comments except that she specifically said that independent should begin by their year of graduate school.

Characteristics of successful undergraduate students. Marty, Sarah, and Tim made comments on successful undergraduate students. Marty commented that in undergraduate mathematics students need a lot of self-effort and to have a liking for mathematical thoughts. Sarah made comments that were in agreement with Marty. She believes that math majors need a driving curiosity for the formal side of mathematics, and to enjoy the perseverance necessary to knock down walls to solve problems. Tim believes mathematics is something you have to try and do in order to be successful, so students need to take responsibility for their own learning which involves asking for help when they need it.

Characteristics of unsuccessful students. Jane, Keith, and Sarah each mentioned beliefs concerning students which are unsuccessful in their classes. Jane mentioned students' inability or unwillingness to come see her for help. Some of her students have a fear of talking to instructors in person. Keith commented on how unsuccessful students in his classes are likely to have poor attendance or daydream in class during the days that they do attend. In addition, they are not engaged during group work letting their other students do the work for them. Sarah describes students making excuses for mistakes rather than admitting they were wrong as a characteristic of a unsuccessful student.

In summation, there are similarities between the beliefs of Marty and Sarah concerning successful graduate students, similarities among the beliefs of Marty, Sarah, and Tim concerning

successful undergraduate students, and similarities among the beliefs of Jane, Keith, and Sarah concerning unsuccessful students. While there were differences among of the various statements the participants made there were no opposing beliefs. A visual of the similarities is provided below in Figure 4.8. Participants are joined by a line segment if they stated a belief on the same topic.

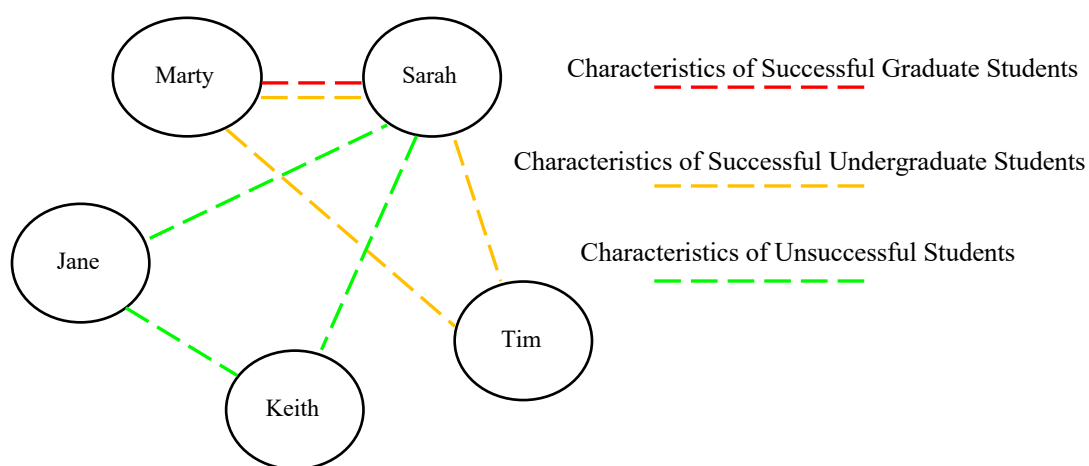


Figure 4.8: Beliefs on Students

Overall, beyond the clear differences in beliefs across the different major themes, differences among beliefs within individual themes and subthemes are present. A visual picture of these similarities and differences provides a much clearer representation than a written narrative could. An important fact to note is that participants with different backgrounds and different teaching assignments did have similarities in beliefs and not only differences.

The Teaching Practices of Post-Secondary Mathematics Instructors

The next section serves to answer the second research question. What are the specific teaching practices identified by post-secondary mathematics instructors as crucial to the teaching

and support of students learning mathematics? The practices identified in the interview data are actions taken by individuals that are associated to the beliefs of the participants identified in the previous section. These practices could be practices of the participants, their students, or other instructors. Because of the methods used to identify these practices as outlined in chapter three, the themes and subthemes consisting of these practices look quite similar to the themes and subthemes of the participants' beliefs. The reader should make note of this so this section does not appear to be repetition of the previous section. I will first outline practices, if any, mentioned by the participants which were motivated by the beliefs previously stated. This will begin by listing the themes and subthemes of practices.

1. Student-Centered Instructor Practices

- a. Classroom Environment

- i. Positivity

- ii. Math Anxiety

- b. Teaching Practices

- i. Active Learning

- ii. Math Ph.D. Advising

- iii. Supporting Graduate Teaching Assistants

2. Content-Centered Practices

- a. Philosophical Practices

- i. Rigor at the Lower Division Level

- ii. Conceptual Understanding

- b. Products of Content

- i. Interpreting and Writing Proofs

- ii. The Whole Answer
- 3. Student Practices
 - a. Philosophical Practices
 - i. Student Responsibility
 - ii. Motivation of Math Ph.D. Students
 - b. Characteristics of Students
 - i. Characteristics of Successful Graduate Students
 - ii. Characteristics of Successful Undergraduate Students
 - iii. Characteristics of Unsuccessful Students

Preliminary Definitions

During the analysis of the data it was clear that for the majority of the beliefs which described the instructors' teaching philosophies, there were associated learning or teaching practices the participating instructors found significant. Some of the associations between beliefs and practices were **direct** and some were **indirect**. By direct and indirect I mean the following: A **direct association** between a belief of the participant and a practice meant that the participant stated a practice that is actually performed. This could be a practice of the participant, a student, or another instructor. It is not only a practice that the instructor intends or recommends should be performed

For example, in the positivity section of the beliefs on classroom environment subtheme of the student-centered instructor beliefs theme on pg. 80, Sanborn said the following concerning his belief on a positive classroom environment.

Sanborn: I want them to have a positive experience in a math class. I think [about] being positive all the time, being aware of inclusion and diversity and really trying to be

welcoming, even overly so in some kind of capacity. So I think a positive experience in a math class is one thing I definitely want.

Later Sanborn said the following concerning a practice that he uses to facilitate a positive classroom environment.

Sanborn: I think that a positive experience and inclusion is important class. I always get there 15 minutes early. And the beauty of this is that in a 9:05am large lecture, no one is in there at 8am. So I can get there as early as I want, and I just start walking around. They're trained now where they leave every other row open so I can walk. And in a smaller class a lot of times [I walk around while] collecting homework problems. So I [ask] how are you doing? What's going on? So it's basically forcing a personal conversation with every one of my students before class starts. It's worked very nicely, even a smaller class [with 35 students]. At that point with 35 students, I'm also recording what homework problems they want to go over, because I do go over homework problems in the smaller class. So that's how I start before class.

This practice of Sanborn where he gets to class early is by definition **directly associated** to his belief because the practice is actually used. It is not only a good idea of a practice that has yet to be implemented or was not specifically. Sanborn even references the original belief in the first line of the statement of the practice. He refers to the specific details of how that practice is employed. Specifically, he talks about getting to class early and walking around to have a personal conversation with every one of his students before class.

Now take for example the following stated belief of Marty concerning rigor at the lower division level, which was part of the philosophical beliefs subtheme of the content centered beliefs theme.

Marty: At freshman-sophomore level they don't have to see the big picture at all. You can only see the big picture when you have total command over the small picture. The big picture is not something that should be taught at the beginning. So my philosophy of teaching is you have to build muscles then lift heavy weights. So the reason for that is mathematics is not something that can be understood at once. It is best understood by long practice.

Within this stated belief that freshman and sophomore level students do not have to see the big picture, there is a sentence at the end which says that the big picture is best understood by long practice. The implication being that students should practice for a long time in order to see the big picture. So we see that this implied practice of students is **indirectly associated** to the Marty's belief on rigor because he did not mention how he actually facilitates the practice for students, nor did he give any indication on how students would go about this practice for themselves. I will therefore define an **indirect association** between a belief and a practice as a relationship where the belief does not inspire a practice which the participant definitively states is actually performed, by the participant or another individual.

To summarize these two definitions, a **direct association** between a belief and a practice is a relationship where the belief connects to a practice which the participant *does* definitively state is actually performed, by the participant or another individual. An **indirect association** between a belief and a practice is a relationship where the belief connects to a practice which the participant *does not* definitively state is actually performed, by the participant or another individual.

Another characterization of the relationship between beliefs and practices gleaned from data analysis is whether the associated practice was a practice of a participant or a practice of

another individual, like a student or another instructor. A practice of a participant would be an action taken by the participant for the purposes of teaching or student learning. I refer to this practice as an **internal practice**. A practice of another individual would be an action taken by an individual other than a participant for the purposes of teaching or student learning. I refer to this practice as an **external practice**. Clearly the practices from the student practices theme would contain external practices. However, it also contained internal practices, and within the other two themes, student centered instructor practices and content centered practices, there were mixtures of internal and external practices.

For an example of an internal practice consider the statement Keith made about his beliefs on student responsibility, which was in the philosophical beliefs subtheme of beliefs on student practices theme on pg. 100. He made the following statement.

Keith: So, you know, you want to teach them the math, but I feel a bit of an obligation to teach them, I don't know if responsibility is the right word. We'll say math responsibility and math accountability for their work or lack thereof, sometimes.

In response to this belief on his obligation to teach the students in his classes responsibility, he mentioned the following practice on pg. 142.

Keith: I try to get them to come to office hours. I've learned to do that instead of calling them out in front of class. I'm just trying to figure out what's going on in their life at that point in time because you never know. There are so many different things, and then go from there, try to address some of those and give them ideas of things they can do. But then also talk about how they are preparing for this math class because things aren't going so well, and let's get a plan of attack to do better and take baby steps one step at a time.

It is clear that Keith is describing a practice that he takes upon himself to do. He goes out of his way to invite students come to office hours, and with individual students he develops a purposeful plan to help his students perform better in class. As defined in this study, this practice clearly qualifies as an internal practice.

For an example of an external practice, consider Sarah's beliefs on characteristics of successful graduate students, which was in the student characteristics subtheme of the student practices theme. She made the following statement on her beliefs.

Sarah: I really think that once people get to the graduate level, they should have the chops to go off and figure it out themselves. By your third year [you] should be able to learn from no one.

She then mentioned the following practice on pg. 125 with regards to students figuring out things for themselves: "Learn from a textbook. [In graduate school], we learned so much on our own working on homework."

Sarah describing a practice of learning from a textbook is not something she is facilitating for the students, but something the students have to do on their own. Therefore this practice is considered to be external.

Significance of belief-practice association. The distinctions made between direct and indirect associations between beliefs and practices, as well as the distinctions made between internal and external practices will be used to describe how the instructors' beliefs influence their practices. The number of direct associations, indirect associations, internal practices, and external practices which correspond to each of the three themes on beliefs, student-centered instructor beliefs (SCIB), content-centered beliefs (CCB), and beliefs on student practices (BSP) were counted. An analysis of direct associations, indirect associations, internal practices, and

external practices within each theme yielded percentages of associations. These direct or indirect associations to internal or external practices will be referred to as **belief-practice associations**. This will be discussed following a description of the practices, after which the results from the analysis will serve to answer the third research question which concerns the influence of the participants' beliefs on the associated practices.

I will now provide details for all the practices that were revealed through data analysis. This will involve describing the belief-practice association as either being direct or indirect, as well as the associated practice as being either internal or external.

Student Centered Instructor Practices

Classroom environment. Recall that four faculty members, Sanborn, Lois, Jane, and Keith, discussed beliefs they had which were student centered. I will discuss the practices mentioned by the faculty which are used to create a positive classroom environment and alleviate students' math anxiety.

Positivity. Sanborn's practice involves greeting and having a conversation with his students before class starts.

Sanborn: I think that a positive experience and inclusion is important class. I always get there 15 minutes early. And the beauty of this is that in a 9:05am large lecture, no one is in there at 8am. So I can get there as early as I want, and I just start walking around.

They're trained now where they leave every other row open so I can walk. And in a smaller class a lot of times [I walk around while] collecting homework problems. So I [ask] how are you doing? What's going on? So it's basically forcing a personal conversation with every one of my students before class starts. It's worked very nicely, even a smaller class [with 35 students]. At that point with 35 students, I'm also recording

what homework problems they want to go over, because I do go over homework problems in the smaller class. So that's how I start before class.

This practice is internal since it is an action taken by Sanborn himself, and is directly related to his belief on positivity since it is an action he actual takes. Relatedly, Lois gives examples of ways she communicates with her students with positivity.

“I provide prompt and substantive feedback to students. That could be grading papers well, or it could be just replying to emails. That could mean a lot of different things. But that's important in my environment.”

This practice is internal and is directly associated to its corresponding belief.

Keith’s comments relay a specific point about positivity. He believes in giving his students confidence and letting them know that he is there to help. He attempts to foster this confidence by having students attend office hours.

Keith: I try to get them to come to office hours. I've learned to do that instead of calling them out in front of class. I'm just trying to figure out what's going on in their life at that point in time because you never know. There are so many different things, and then go from there, try to address some of those and give them ideas of things they can do. But then also talk about how they are preparing for this math class because things aren't going so well, and let's get a plan of attack to do better and take baby steps one step at a time.

This practice is internal and directly associated to its corresponding belief, as was the case with the other examples. Next the practices revealed by the data concerning math anxiety will be discussed.

Math anxiety. Math anxiety was an issue recognized by Keith and Jane. Jane was involved in teaching a class with was specifically designed to address the math anxiety issue.

Jane: They started a math study skills class which I [and other instructor] taught. We had them do a math biography, like an autobiography, and to have them talk about their math experience. I was amazed at how many of them could list the elementary school teacher that told them, and they could name them, they could give their name, that “This teacher told me I wasn’t any good at math”. It just stayed with them. So they were convinced they weren’t good. It just stayed with them. So we were having them do some meditation to try to calm them down. They had student success come and give them career talks.

Jane starting the math study skills class is an internal practice and is directly associated to its corresponding belief.

Keith: I am currently piloting something called Math 100 which is math study designed for students whose background suggests they’re going to struggle in Math 119 [college algebra]. One of the sections we’re on right now is note taking skills. I finished up with some listening and anxiety skills, [specifically] dealing with math anxiety, test anxiety, how to calm yourself down, positive self-talk instead of negative self-talk, giving themselves confidence.

This practice is internal and is directly associated to its corresponding belief.

In summation, for the subtheme on classroom environment the data revealed practices concerning facilitating a positive classroom environment and helping students cope with math anxiety. The four practices on positivity were all internal practices and were all directly associated to the corresponding beliefs. The two practices concerning math anxiety were also internal and directly associated the corresponding beliefs. Next the practices which are associated to beliefs on teaching will be discussed.

Teaching practices. Teaching practices summarizes the practices mentioned by Lois, Sanborn, Sarah, Marty, and Charlie concerning their beliefs on active learning, advising PhD students in mathematics, or supporting graduate teaching assistants.

Active learning. Lois facilitates active learning in her mathematical reasoning classroom through group work and class projects that are hands-on activities.

Lois: They do something active learning in class every day. So it's not just me and slides. [I do] a 10-15 minute lecture at most of me showing them something. Then they do in class questions where they work in groups or they do a hands-on activity. I can do active learning by doing worksheets and clickers and online canvas questions. Most of the time if I ask really good questions, then I'm satisfied. But some of the time I'm going need it to be off the books and something where they're shooting something down the hallway, they've designed a hearing aid and they're testing it to see how well it's amplifying their sound, or they're bouncing balls in the racquetball court.

This practice of the students doing an active learning activity is external since it is something the students are doing and is directly associated to its corresponding belief. Furthermore, in a hypothetical discussion about teaching a higher-level pre-calculus course, Lois mentions continuing active learning in that course as well.

Lois: I would try to still continue active learning. I would want my class to look and feel the same where they get in groups and do hard problems together with my support during most of the class. And I would still do a 10 to 15 minute Ted talk at the beginning with of what are we doing and why are we doing it.

This practice is indirect since it is hypothetical, and it is internal since it is implemented by Lois.

Sanborn cited a belief in active learning as something he uses in class, but he did not mention any specific practices that he routinely uses that would be considered active learning. He did mention his graduate teaching assistants using worksheets in class that have an active learning component.

Sarah also referred to active learning as a practice which should be used by graduate students or inexperienced teachers as a tool to determine whether students understand what was taught in class. However, unlike Sanborn she did reference a few examples of active learning that she would use or that she would recommend others use. This is illustrated as follows:

Sarah: [The teachers] are going to have to be checking in, having the students do things so they can figure out whether or not students are actually understanding. So all the active learning stuff, teaching think-pair-share and other strategies [should be used] to get students to see what they know and don't know so the instructor can know what they don't know.

This practice is external since Sarah is speaking about the actions of other instructors and not herself. Furthermore, it is indirectly associated to Sarah's belief on active learning since Sarah does not mention that the practice is actually being used.

Math Ph.D. advising. Concerning the advising of Ph.D. students, Marty mainly spoke of expectations that graduate students should have regarding the Ph.D. process. As previously noted, his beliefs concerning the process centered around students being committed to learning. The only practice he noted of the advisor is helping the student learn in a better way.

Marty: If you [have] spent a lot of effort and time on it and you have meaningful questions because you get stuck somewhere, you don't see why something is the way it is, then your thesis advisor can shed some light on it. But if the thesis advisor sheds the

light on it right to begin with you will not have gained any appreciation about it, and in fact you will not at all understand. It will be of no use. So it's only at a proper time that a piece of advice from the thesis advisor has meaning [or] has value, and that person who is in graduate school has to realize that. [The student] has to say I'm learning it. I'm learning it and can somebody help me learn it better?

Given the fact that Marty believes a student should ask if someone can help them learn better, it is reasonable to believe that there is someone there to help them learn better. Furthermore, it is possible that it is a student's advisor. Therefore, it is also reasonable to think that a practice which Marty believes in is helping students learn in a better way. This practice of the advisor is internal and indirectly associated to the belief of advising since Marty does not give specifics on how he advises his students. Charlie on the other hand believes that an advisor should not be as hands off.

Charlie: I think a student who's passed their exams, they should be given a problem that they can solve to some extent with some help. I think that if a student is struggling on their topic, the advisor should try to shift it. I don't think they should sort of be on their own with their problem.

From this we can conclude that an advisor should help their student choose a problem that is suitable for the dissertation phase of their PhD. The practice of assisting the student with their problem is critical to the student's success. This practice is indirect since Charlie does not describe it as something that is actually done, and internal since it is the practice of an instructor.

Practices for supporting graduate teaching assistants (GTAs). To support his philosophy of GTAs writing exams Sanborn has his GTAs consider the writing of exams long before they are actually written.

Sanborn: You should be thinking about what is important and that should decide for you what to cover in class. You should at least be aware of it. You should be thinking about it as you go along.

This practice is indirectly associated to Sanborn's belief since it is not described as actually taking place and the practice is external since it would be a practice of GTAs.

Lois' beliefs concerning communication skills, which were previously discussed concerning her belief of a maintaining positive classroom environment, were also attributed to the GTAs that she mentors. The practices of communicating effectively, respectfully, and with positivity also apply here. Her quote will also be referenced here as follows.

Lois: [Examples include] providing prompt and substantive feedback to students. That could be grading papers well, or it could be just replying to emails, also professionalism and efficient use of class time in the class.

In this context the practice is indirectly associated to her belief since Lois did not mention observing the practice or giving an indication that it actually is used, and the practice is external since it is described as a practice of GTAs.

Sarah's practices concerning GTAs verifying their students' knowledge and understanding include the students actually doing something to confirm they have the understanding.

Sarah: [The instructors] need to be checking, having the students do things so that they can figure out whether or not students are actually understanding. [Use] strategies to be able to see [what the] students know and don't know, and then let the instructor also know what they don't know.

Similar to how this quote arose in the discussion about active learning, this practice is indirectly associated to her belief and the practice is external.

Lois' concern about GTAs assuming too much about their students' enthusiasm towards mathematics, similar to Sarah's concern about assuming students understand everything, was followed by her stating that GTAs need to justify the importance of the mathematics they are teaching. They have to literally say it out loud. Lois' comments are illustrative of this. She commented, "[What are we doing and why are we doing it?] You have to say it out loud. Unless I say, what are we doing, why are we doing this, they didn't make that connection."

This practice would be indirectly associated to her belief and the practice would be external since it is not explicitly witnessed by Lois, and is an implied practice of a GTA.

In summation, for the subtheme on teaching practices the data revealed practices concerning active learning, advising math Ph.D. students, and supporting graduate teaching assistants. There were three stated practices associated to beliefs on active learning. Two of the practices were indirect associated to their corresponding beliefs, one internal and one external. The other practice was external and directly associated to its corresponding belief. There were two stated practices associated to beliefs on advising math Ph.D. students. Both practices were internal and were both indirectly associated to their corresponding beliefs. There were four practices associated to beliefs on supporting graduate teaching assistants. All four were external practices and all were indirectly associated to their beliefs. The next practices to be discussed which were revealed by the data were those associated to beliefs on mathematical content.

Content Centered Practices

Next I move to practices which were inspired by beliefs on mathematical content. This section will list and describe the practices which were motivated by the individual beliefs

contained in each subtheme. An outline of the theme consisting of the subtheme and topics of the individual beliefs is provided below.

- a. Philosophical Practices
 - i. Rigor at the Lower Division Level
 - ii. Conceptual Understanding
- b. Products of Content
 - i. Interpreting and Writing Proofs
 - ii. The Whole Answer

Philosophical practices. The first subtheme was philosophical practices which included beliefs on rigor at the lower division level and conceptual understanding.

Rigor at the lower division level. For rigor at the lower division level, the overwhelming belief was that rigor, as previously defined, was not necessary at this level. Marty described this as seeing the big picture, and that this can only take place after a student first understands the small picture. He further described understanding the small picture as building muscles, and understanding the big picture as lifting heavy weights. Lastly, he said that building the muscles to understand the big picture only comes after a long practice with mathematics. Therefore, we can conclude that Marty suggests a practice that involves a long study with mathematics in order to answer questions concerning the *why* questions which define rigor.

Marty: Mathematics is not something that can be understood at once. It is best understood by long practice.

This implied practice would be indirectly associated to his belief and external. Marty did not witness it, nor is it his practice, but the implied practice of a student.

Charlie mentioned a practice that he does expect at the lower division level in lieu of rigor. This involves skills he expects his students to master in a calculus course.

Charlie: In calculus they have to differentiate, find maxima, all the various things on some list of problems, and I would expect them to be able to do those types of problems, without particularly tricky ways or anything.

The association to Charlie's belief is direct and the practice is external. Charlie does witness students doing these calculus problems, and it is a student practice.

Sarah's belief also concerned students knowing the basics. However, at the calculus II level, Sarah does have expectations of her students concerning the justification of the use of certain theorems. A practice she uses involves having her students show that certain conditions of the theorems are satisfied in order to receive full credit.

Sarah: As I'm doing the lectures, I make it clear this is what you have to show me to get the full credit. You have to show me that you satisfied these conditions. I don't make them show me in any great detail. I don't make them show me the derivative and then prove that it's negative, but just write down that you know it's supposed to be negative.

This association to the practice is direct since it is observed by Sarah, and the practice is internal since she shows students how to justify theorems during her lectures.

Sanborn mentioned deriving formulas as an alternative to rigor involving formal proofs. His practice of doing derivations is described as follows:

Sanborn: I definitely have to talk about that whenever we're deriving the derivative for inverse tangent for example. I'm [say] we're coming up with a rule so that afterwards we can just use the rule. That's what deriving means. Then I use the terminology of okay this is going to be an exercise in understanding. Follow me. If you got it, awesome. If you

didn't at the end of it that's fine. We're going to have a punchline. You've just got to know how to use the punchline afterwards.

The association between this practice and Sanborn's belief on rigor is direct since the practice is realized in his class. The practice itself is internal because it is the result of Sanborn's action.

Jane's only practice which was mentioned concerning this belief was wanting students to be able to do the prerequisite work necessary to be successful in her class.

Jane: I want them to be able to [do the work].

The implied practice of students doing the work is external, and it is indirectly associated with Jane's belief on rigor.

Conceptual understanding. To practice conceptual understanding in his upper division courses, Charlie uses group work where students with different abilities help each other. This practice is a by-product of his belief on going over things that only a few of his students may grasp.

Charlie: I have them work on group work problems, and I go around helping them and answer questions. I think working together through a problem, especially sort of more conceptual one where your mistakes aren't just going to be computational [is important]. It's hard to check someone else's work. But on a logic problem, I think it's easier to talk through the problem. And so I think it's good for them to talk through the problems. I found that students have different [abilities]. Students who have a better understanding of what's a correct proof, and how to approach the proof, can help the others. They learn from each other and they learn by explaining. It's also a good way to see how they understand quickly.

The association between Charlie's belief on conceptual understanding and this practice of having students do group work is direct since he actually sees his students do group work. It is an internal practice because he facilitates the group work as a teaching practice.

For practices that motivate conceptual understanding, Sarah described questions that she asks on exams.

Sarah: I give them two or three different series and ask which of these could apply to the integral test. I like to put things for the first and second fundamental theorems of calculus, giving them information about that function at a particular point and the values of the derivative at a particular point, having them put together the pieces of the fundamental theorem.

This practice illustrates Sarah's intent to have her students demonstrate their conceptual knowledge with a problem that requires them to actually use their knowledge. It has a direct association to her belief on conceptual understanding given that the action takes place in her class, and it is internal being one of Sarah's teaching practices.

Tim, speaking with respect to calculus II students mentioned that he mixes computational and conceptual examples.

Tim: Yea sometimes I just do [something purely computational], know how to integrate.

Other times I pull out some ideas and the concepts with it, so we mix them, I think.

Tim's practice is directly associated to his belief and is internal being his teaching practice.

Marty's belief in students needing a conceptual understanding was illustrated by the use of the epsilon-delta definition of limits typically seen in an advanced calculus class. The practice associated with this belief on the definition involves actually making calculations of delta with a given epsilon. This is illustrated as follows.

Marty: To understand the [epsilon-delta] definition is actually to write formal arguments or to make the calculations with epsilon. Given epsilon find delta. Actually, do it in several examples so that you understand what is involved.

This practice is indirectly associated to Marty's belief since he does not say that he witnessed his students practicing the epsilon-delta definition, and it is an external practice since it would be an action taken by his students.

Sanborn practices a little deception in order to motivate the need for his students to understand mathematical conceptually. His deception involves pretending that he does not remember or know how to complete a problem, forcing him to go back and rediscover something that he forgot.

Sanborn: There are times where I do lie in class where I'm like you know at first glance I didn't know how to do this problem. [to motivate a reason to derive something or discover something, no steps]. I want them to actually understand why things work out. This practice of Sanborn is directly associated to his belief on conceptual understanding since his method of deception is actually implemented, and it is internal since it is his practice.

Keith's belief on conceptual understanding concerns students using their knowledge in other subjects. The practice he uses to motivate his students' interest in using mathematics in other subjects is as follows:

Keith: Try to throw things out there, in real life situations or just talk about it. How would this apply in this situation? I don't always know specific examples in [different] areas, but just to get them to think about it. When I'm asked, when am I ever going to need to know this? I [say] pull your book out. Find the homework section and read the word problems.

Look at the word problems. That's where you are going to get to see some real-life scenarios where this can apply.

Keith's practice is directly associated to his belief since he is giving an example of what is actually done to motivated students' interest in conceptual understanding. It is an internal practice because he does refer to this practice as something he does.

In summation, for the subtheme on philosophical practices the data revealed practices concerning rigor at the lower division level as well as practices concerning conceptual understanding. For rigor at the lower division level there were three external practices two of which were indirectly associated to their corresponding beliefs and one which was directly associated to its corresponding belief. There were also two internal practices both of which were directly associated to their beliefs. For conceptual understanding there were five internal practices all of which were directly associated to their corresponding beliefs. There was also one external practice which was indirectly associated to its correspond beliefs. The next subtheme of practices revealed by the data are those associated to beliefs on products of mathematical content.

Products of content. This subtheme includes practices on interpreting and writing proofs and writing the whole answer.

Interpreting and writing proofs. With respect to interpreting and writing proofs, Charlie believes that a proof is nothing more than answering the question *why*, and that this should not be thought of something separate from something computational in mathematics. A practice is implied by the following quote.

Charlie: I don't think proofs should be one new thing that is completely different from what came before.

This implies that there should not be a single moment where answering the question *why* becomes relevant. From this we can infer that answering *why* should have been part of the curriculum from the beginning. The practice of answering *why* from the beginning would be indirectly associated to his beliefs on writing proofs since it is not something actually implemented, but only considered by Charlie. Furthermore, it is an external practice since it would have to be done by an earlier instructor.

Marty's suggested practice concerning this issue also focused on teaching previously learned material.

Marty: So that means instead of high schools focusing on calculus, they should focus more on elementary [material], let's say Euclidean geometry or algebra. Once you are well prepared and you understand by small geometric examples or algebraic examples what it means to write a proof, there doesn't have to be any kind of critical jump because it slowly develops.

Similar to Charlie, Marty's practice is indirectly associated to his belief, and an external practice.

The whole answer. This belief involves changing the perception of what an answer is in mathematics from consisting of just a number at the end of a problem. Sanborn's practice involves saying *the work is the answer* over and over again in class in order to change his students' perception.

Sanborn: I tried for the first the few weeks to [say] *the work is the answer. The work is the answer* every day in math class because I'm trying to change that conversation.

Sanborn's practice is directly associated to his belief since it is being implemented, and being an action taken by Sanborn it is internal.

Sarah's practice was not as well defined, but she described it in terms of goals she has for her students which include clear communication.

Sarah: I start being really picky about what they write and them not putting circles around anything. How do you explain the process that you use to get there? Getting them to just be a little clear about their communication. Get them not to try to write as little as possible.

Sarah's practice of being picky is directly associated to her belief in students writing a complete solution because it something that is actually implemented, even though how it is implemented is not very clear. However, when she states, "Get them not to try to write as little as possible." there is an underlying teaching practice. This would be an internal practice since it is implemented by Sarah.

In summation, for the subtheme on products of content the data revealed practices associated beliefs concerning interpreting and writing proofs as well as the whole answer idea. For interpreting and writing proofs there were two external practices which were indirectly associated to their corresponding beliefs. For the whole answer idea there were two internal practices that were directly associated to their corresponding beliefs. The final theme concerning student practices will not be discussed.

Student Practices

Student practices concerns the practices or actions of students that the participants believe either help or hurt student success. The theme beliefs about student practices was defined through two subthemes, philosophical practices which consisted of practices concerning student responsibility, and the motivation of math Ph.D. students, and characteristics of students which consists of practices of successful graduate and undergraduate students, and practices of

unsuccessful students. An outline of this theme is provided below, after which I will discuss the practices that arose from each subtheme.

Student Practices

a. Philosophical Practices

- i. Student Responsibility
- ii. Motivation of Math Ph.D. Students

b. Characteristics of Students

- i. Characteristics of Successful Graduate Students
- ii. Characteristics of Successful Undergraduate Students
- iii. Characteristics of Unsuccessful Students

Philosophical practices. The practices associated with student responsibility and motivation of math PhD students were described by Marty, Tim, and Keith.

Student responsibility practices. Tim's suggested practice that characterizes a responsible student in calculus is doing homework. "Do the homework. Study. Really work on it." This practice is indirectly associated to his belief since it is not witnessed by Tim, and it is external being something done by students and not Tim.

For Keith's college algebra, statistics, and pre-calculus students he describes a practice he uses to help students become successful in his class. From among Keith, Jane, and Tim, Keith is the only instructor who mentions a practice employed to facilitate student responsibility. This was also how Keith facilitated a positive classroom environment.

Keith: I try to get them to come to office hours. I've learned to do that instead of calling them out in front of class. I'm just trying to figure out what's going on in their life at that point in time because you never know. There are so many different things, and then go

from there, try to address some of those and give them ideas of things they can do. But then also talk about how they are preparing for this math class because things aren't going so well, and let's get a plan of attack to do better and take baby steps one step at a time. This practice is directly associated to Keith's belief on facilitating student responsibility since it is implemented. It is also internal being Keith's practice.

Motivation of math Ph.D. students. Marty mentioned a belief of motivated Ph.D. students of mathematics which is based on an attraction to mathematics. He also referred to a Ph.D. student having an attraction to towards mathematics that must be translated into practice as quote below.

“You must hold within you some attraction towards it. And that attraction has to be translated into practice.” He however did not specify what this practice is. It would appear to be an overall statement about a student's willingness to do the required work. This is clearly the practice of a student making it external, and since Marty didn't specify an actual instance of a student translating their attraction into practice it is indirectly associated to Marty's belief.

Charlie also stated a belief about the need for an internal motivation to complete a graduate program, and that the motivation need not be to study mathematics for its own sake. However, he did not associate this belief with a specific practice.

In summation, for the theme on philosophical practices the data revealed practices associated to beliefs concerning student responsibility and motivation of math Ph.D. students. For student responsibility there were two practices associated beliefs of the participants. One was external and indirectly associated to its corresponding belief. The other was internal and directly associated to its corresponding belief. For the motivation of math Ph.D. students there was one external practice that was indirectly associated to its corresponding belief. Next the

subtheme concerning the practices associated to characteristics of successful and unsuccessful students will be discussed.

Characteristics of students. The characteristics of students theme concerns practices that Marty, Charlie, Tim, Sarah, Jane, and Keith mentioned within their discussion of beliefs on successful undergraduate and graduate students, and unsuccessful students.

Characteristics of successful graduate students. For successful graduate students, in addition to his belief that students should be able to learn independently without the need of a classroom, Marty suggested the practice of being able to read books and articles that go beyond coursework. He commented, “So you have to be able to read book or articles which extend from where you have left [with coursework].” Marty’s practice is indirectly associated to beliefs on successful graduate students since his statement did not reference a specifically used practice. It is also external being a practice of another individual, namely a student.

Sarah also believes in students needing to learn independently, and also suggested when writing a mathematical argument, only including statements of which you are absolutely confident.

Sarah: You have to be clear on your argument. Don’t say anything you aren’t completely confident of. Some of the best advice I ever got is that you can’t fudge it. You have to be super confident and really clear on how one thing builds upon another. You have to make a really clear argument and there can’t be anything fuzzy.

This practice is indirectly associated to her beliefs in graduate student success since her statement does not reference her facilitating the practice nor observing it as an instructor. It is an external practice of a student. She also echoed Marty’s suggested practice through the following. “Learn from a textbook. [In graduate school], we learned so much on our own working on

homework.” The practice of successful students working on their own on homework is significant since it is associated to her belief that students’ driving curiosity should motivate them to persevere through difficulties they will encounter studying mathematics. This practice is also an indirect association to her belief and an external practice.

Characteristics of successful undergraduate students. Sarah shared practices they she saw in successful undergraduate students. Sarah believes hard work rather than intelligence makes for a successful student. She commented, “For so much of it, it wasn’t how smart people were, it’s how hard they were willing to work.” So it seems reasonable to conclude that putting in hard work is a necessary practice. She also mentions being willing to be wrong and working in groups as necessary practices.

Sarah: So I think begin willing to make mistakes and recognize that you are wrong is a huge asset. Being willing to throw out ideas that could be wrong. Being able to work in groups. It’s not something they come in knowing how to do but hold them to it.

The practice of working hard in her statement is indirectly associated to her belief as she does not mention facilitating this or observing this in her own students, and it is an external practice of students.

Tim simply mentions that being willing to work usually means you will do well. He comments, “People who really work at it usually do well. I do see kids that will just kind of show up and take the test and go away.”

Tim mentions actually seeing students who practice the opposite of what he believes is a practice of successful students, so this practice is directly associated to his belief. It is also an external practice of students.

Characteristics of unsuccessful students. Of the three faculty members who state beliefs and examples of practices of unsuccessful students, Jane, Keith, and Sarah, only Keith mention a practice that he employs to help these students. This was to send an email and ask if the student is ok, to see what is going on. He commented, “I’ll send an email just to try and see, Hey, what’s going on? Are you okay?” Keith’s action of sending an email is a directly associated practice of his beliefs and is internal being his own practice.

Jane mentioned a practice of students where they avoid asking for her notes when they miss class and use the internet instead. She commented, “In fact, I had several students miss class. I’ll say do you want the notes. [They say], oh no I’ll look it up on YouTube. So it’s like, okay. Well what am I here for?” This practice of Jane is a direct association to her belief and an external practice of students.

Sarah mentioned some of her students not admitting their mistakes and trying to justify incorrect answers. She commented, “Every time they make a mistake, they have an excuse for the mistake that they made rather than [admitting] they did something wrong and try to figure it out.” This practice is directly associated to her beliefs, being a practice that she observes, and is an external practice of students.

In summation, for the final subtheme consisting of practices associated to the participants’ beliefs on the characteristics of students, the data revealed practices associated to beliefs concerning successful graduate students, successful undergraduate students, and unsuccessful students. For successful graduate students there were three external practices each being indirectly associated to their corresponding beliefs. For successful undergraduate students there were two external practices, one indirectly associated to its corresponding belief and the other directly associated. Finally, for unsuccessful students there were two external practices

which were directly associated to their corresponding beliefs, and one internal practice which was directly associated to its corresponding belief.

To summarize this section on instructor and student practices, note that every practice associated to a corresponding belief was characterized as being external or internal. Recall that an **internal practice** is an action taken by a participant for the purposes of teaching or student learning. An **external practice** is a practice taken by another individual for the purposes of teaching or student learning. Every practice revealed by the data was considered and labeled as either being internal or external. Furthermore, the association between a belief and its associated practice was characterized by whether the associated practice was an actual observed practice or only a suggested or hypothetical practice. If the associated practice was a practice that was actually used then the association is said to be **direct**. If the associated practice is only suggested or hypothetical then the association is said to be **indirect**. It seems reasonable to consider direct associations to be stronger than indirect associations because the associated practices are guaranteed to be used or observed by the participants, as revealed by the data. In this section I have justified and labeled every belief-practice association, BPA, as being either direct or indirect. I have also justified and labeled every associated practice as being either internal or external. Below I have provided Table 4.1 below that gives a complete summary of all the belief practice associations and associated practices for all the participants. To make the chart concise but also readable I have used specific abbreviations to denote whether a BPA is direct or indirect and whether the associated practice is internal or external. DI means the BPA is direct and the associated practice is internal. INI means the BPA is indirect and the associated practice is internal. DE means the BPA is direct and the associated practice is external. INE means the BPA is indirect and the associated practice is external.

Table 4.1 Belief-Practice Associations and Associated Practices

	Lois	Jane	Keith	Sanborn	Sarah	Tim	Charlie	Marty
Positivity	DI	None	DI	DI	None	None	None	None
Math Anxiety	None	DI	DI	None	None	None	None	None
Active Learning	DE, INI	None	None	None	INE	None	None	None
Math Ph.D. Advising	None	None	None	None	None	None	INI	INI
Supporting GTAs	INE, INE	None	None	INE	INE	None	None	None
Rigor LD Level	None	INE	None	DI	DI	None	DE	INE
Conceptual Understanding	None	None	DI	DI	DI	DI	DI	INE
Interpreting and Writing Proofs	None	None	None	None	None	None	INE	INE
The Whole Answer	None	None	None	DI	DI	None	None	None
Student Responsibility	None	None	DI	None	None	INE	None	INE
Motivation of Math Ph.D. Students	None	None	None	None	None	None	None	INE
Successful Graduate Students	None	None	None	None	INE, INE	None	None	INE
Successful Undergraduate Students	None	None	None	None	INE	DE	None	None
Unsuccessful Students	None	DE	DI	None	DE	None	None	None

From the chart every BPA for every participant can be seen. Topics which concern the beliefs and practices are also conveniently group from top to bottom according to the themes and subthemes to which they belong. It is therefore easy to see how the BPAs for each participant are spread among the different themes and subthemes. This information will be used to answer the third research question which asks how the teaching philosophies or beliefs influences

practices. This is where the characterizations of the associations between belief and practices as well as the characterizations of practices will become useful.

Influences of Teaching Philosophies on Teaching Practices

The final research question asks about the influences of the instructors' teaching philosophies on their teaching practices. Specifically, the question asks the following. How do the teaching philosophies of post-secondary mathematics instructors influence their teaching practices which are used to support students learning mathematics?

Analysis on Belief-Practice Association (BPA)

First, each statement on beliefs made by a participant was categorized into the three themes on beliefs, student-centered instructor beliefs, SCIB, content-centered beliefs, CCB, and beliefs on student practices, BSP. To analyze the data for the purpose of answering the third research question, groups for each of SCIB, CCB, and BSP were made, and a participant was included in that group if the majority of their beliefs fit the corresponding theme. For example, a participant was included in the SCIB group if the majority of their beliefs revealed by the data fit the SCIB theme. There was an instance of a participant satisfying all three groups because of a tie in the number of beliefs that satisfied each theme. In this case, the participant was included all groups. Next, to determine how the practices of the participants in each of the three groups were influenced by their beliefs, the number of direct and indirect associations and internal and external practices were tallied. The percentages of each of type of association and each type of practice were also calculated. A chart showing the tallying of beliefs was included on pg. 152. These numerical data were used to make reasonable claims about the influence of a participant's beliefs on their practices or the practices of their students.

SCIB, CCB, and SPB majority. When focusing on the participants with the majority of their belief-practice associations fitting within each theme we arrive at data which can be used to make claims or conjecture as to how the beliefs or philosophies of the participants influence their practices.

Student-centered instructor beliefs. Lois and Jane had the majority of their belief-practice associations fit within the SCIB theme. It should be noted that Jane only had one belief-practice association in each theme, meaning that she technically has a majority of her belief-practice associations in all three themes. So her inclusion in theme may be considered somewhat trivial. Regardless, within this theme 62% of the BPAs were direct associations, 38% were indirect associations, 62% of the associated practices were internal practices, and 38% of the associated practices were external practices.

Content-centered practices. Marty, Charlie, Sanborn, and Jane all had the majority of their BPAs fit within the theme on content centered practices. Again, Jane was included with only one BPA. For this theme 37% of the BPA's were direct associations, and the remaining 63% were indirect associations. Furthermore, 44% of the associated practices were internal and the remaining 56% were external practices.

Student practices. Sarah, Tim, and Jane all had the majority of their BPAs fit within the theme on student practices. Again, Jane was included with only one BPA. For this theme 59% of the BPAs were direct associations, and the remaining 41% were indirect. 41% of the associated practices were internal and the remaining 59% were external practices.

These percentages from each of the three major themes is summarized by Table 4.2, 4.3, and 4.4 below.

Table 4.2 SCIB Majority

SCIB Majority	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Lois	2	3	2	3
Jane	1	0	1	0
Keith	2	0	2	0
Total	5	3	5	3
%	62	38	62	38

Table 4.3 CCB Majority

CCB Majority	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Marty	0	6	1	5
Charlie	2	2	2	2
Sanborn	4	1	4	1
Jane	0	1	0	1
Total	6	10	7	9
%	37	63	44	56

Table 4.4 BSP Majority

BSP Majority	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Sarah	4	5	3	6
Tim	2	1	1	2
Jane	2	1	1	2
Keith	2	0	2	0
Total	10	7	7	10
%	59	41	41	59

Results. Given the results from this analysis we may make some reasonable claims about the influences of the beliefs or philosophies of these participants on their associated practices. For associations between beliefs and practices we can see that for the participants who had a majority of their beliefs fall in the SCIB theme, their percentage of direct associations was slightly higher than those in the BSP theme, and even higher than the CCB theme. As a necessary consequence

the percentage of indirect associations between practices in the SCIB theme was less than both the BSP and CCB themes. From this we may conjecture that the beliefs of participants with a majority of their BPAs falling within the SCIB theme influence their practices to be more direct. By definition this means that their beliefs influence their practices to be developed and able to be implemented rather than being hypothetical like an indirect association.

We can also see that participants whose BPAs fall within the SCIB theme had a higher percentage of associated internal practices. So we may conjecture that these participants' beliefs influence their practices to be actions taken by them rather than actions taken by students or other instructors. We also see that the instructors with a majority of their practices falling within BSP have the smallest percentage of associated internal practices, and consequently the highest percentage of associated external practices. So we may conjecture that the beliefs of participants with a majority of their BPAs falling within the BSP theme influence their associated practices to be external. By definition this means that the associated practices are practices of another individual, for example a student or another instructor.

Significance of the results. A potential significance of these results is the ability to possibly predict how an instructor of post-secondary mathematics will perform in the classroom based on the results of some type of data collection and analysis. I use the term predict very loosely in this context since these results are not based on any type of rigorous statistical analysis. Furthermore, since the number of participants was quite small for an analysis using numerical data, I do not believe that the major significance of these results concerns which type of beliefs motivate which type of practices. The significance lies in the conjecture that certain beliefs may motivate certain practices. With that being said, this data collection could take the form of a survey of beliefs. The National Council of Teachers of Mathematics has created

survey of teacher beliefs published in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). This survey, entitled *Teaching and Learning Beliefs Survey*, gives an example of a method of data collection where the data consists of beliefs of K-12 mathematics instructors. This hypothetical survey could collect data consisting of the beliefs of potential instructors of post-secondary mathematics instructors and be used to develop professional development for teacher education programs. This could be professional development for future GTAs in a graduate program of mathematics, or possibly professional development for experienced instructors. This will be explored further in the next chapter which discusses the implications of the findings of this study.

CHAPTER 5

DISCUSSION AND IMPLICATIONS

The purpose of this study is to determine significant similarities and differences among post-secondary mathematics instructors with a focus on how they teach and engage students in learning mathematics. The findings in chapter four describe the beliefs of eight post-secondary mathematics instructors concerning the teaching and learning of post-secondary mathematics. In addition, the beliefs were compared and contrasted to describe the similarities and differences among the beliefs in an effort to describe the teaching philosophies of the eight participants. Finally, an analysis was done to determine how the teaching philosophies or beliefs of the participants influence their practices and the practices of their students. This chapter will address the study's relevance to mathematics and mathematics education, and to my own learning.

Relevance to Mathematics

Recall that in chapter four it was observed that there are associations between the beliefs of the participants and the practices they believe are important to the teaching and learning of mathematics. Furthermore, it was conjectured that these associations can help describe the influences that participants' beliefs have on their practices or the practices of their students. There were examples of beliefs which were indirectly associated to practices of the participants or their students that have potential implications to mathematics content studied at the secondary level. There were beliefs in the *beliefs on content* subtheme of the theme on *content-centered beliefs* concerning the transition from mathematics without proofs to mathematics requiring the interpretation and writing of proofs. These beliefs included the idea that there is no such thing as mathematics involving proofs being different from any other type of mathematics. The associated practice involved including proofs, or in the language of the participants answering

the question *why*, in foundational mathematics studied at the secondary level. For example, high schools could focus on Euclidean geometry or algebra where proofs could be a normal part of the content, rather than emphasizing calculus. This would mean changes not only at the secondary level, but in our broader culture of mathematics education that deems calculus as important to high school students who are university bound. Furthermore, at the post-secondary level there would be no need for a critical jump later to the mathematics of proof. The implication on secondary education would be a change to allow proofs to be a standard part of the curriculum. Of course, there would be the need to build up to this change in elementary and middle grades for this shift to happen successfully. To achieve the goal of students mastering formal reasoning, curriculum materials should include simple proofs starting in the intermediate grades (Ball & Bass, 2003; Fitzgerald, 1996; NCTM, 2000). Furthermore, NCTM (2000) in its standards for Pre-K through 12th grade states that reasoning and proof allow students to explore phenomena, justify results, and use mathematical conjectures. Proofs and mathematical arguments are formal ways of expressing particular kinds of reasoning and justification. With different expectations of sophistication at all grade levels, students should see and expect that mathematics makes sense (NCTM, 2000). Proof and justification allow sensemaking to be front and center within the curriculum.

Also, within the beliefs on content sub-theme, beliefs concerning students writing what participants referred to as *the whole answer* imply similar implications on secondary mathematics. Recall how Sanborn mentioned his practice of changing the conversation concerning what constitutes a complete solution to a problem in mathematics. This change of conversation could certainly happen within mathematics studied at the elementary or secondary levels.

Relevance to Mathematics Education

Research in mathematics education concerning post-secondary classroom practices has the potential to affect change in the access to the discipline of mathematics and change in attitudes and beliefs concerning mathematics. (Boaler, 2016; Gunderson et al. 2013; Henrick et al., 2016; Gutierrez & Dixon-Roman, 2011; MAA Notes, 2018). One of the significant results of the data were the percentages of direct and indirect belief-practice associations, as well as the percentages of internal and external practices within each of the three major themes. The results saw that the participants with the majority of their beliefs falling within the student centered instructor beliefs theme had a higher percentage of direct belief-practice associations, and as a consequence a lower percentage of indirect belief-practice associations as compared to the participants with the majority of their beliefs falling within the beliefs on student practices or content-centered beliefs themes. By definition this means that the group of participants having the majority of their reported beliefs falling within the student-centered instructor beliefs theme reported more practices which were actually used in the classroom as opposed to being hypothetical practices. This is significant since it might give an indication of how to judge how likely an instructor is to be prepared to implement a lesson using fully realized teaching practices.

Furthermore, the participants with the majority of their beliefs falling within the theme on student-centered instructor beliefs had a higher percentage of internal practices associated to their beliefs, and as a consequence a lower percentage of external practices associated to their beliefs as compared to the participants with the majority of the beliefs falling within the themes on content-centered beliefs and beliefs on student practices. By definition this means that the participants having the majority of their reported beliefs falling within the student-centered

instructor beliefs theme reported more practices implemented by the participants themselves as opposed to practices used by students or other instructors. Therefore, it seems reasonable to conjecture the possibility that instructors whose beliefs are more student centered tend to have more developed and implementable personal practices. This is significant because it would seem to give a potential indicator of how prepared an instructor might be when they walk into the classroom. It also might indicate how creative an instructor is to come up with their own practices instead of needing to find practices from someone else.

It should be noted though that these results are not meant to imply a likelihood of an instructor's practices being directly associated to their beliefs if their beliefs are centered on student practices. Nor is it meant to imply that an instructor is more likely to exhibit more internal practices if their beliefs are more student centered. The implication of these results is that further research should be conducted to determine whether these results could be generalized. This could take the form of more qualitative research or quantitative research to explore whether or not these relationships which emerged in this study are evident in other analyses, and what these findings potentially mean for student-centered practices and mathematical learning.

Practical Implications

The practical implications of this study are found within teacher education and mathematics faculty professional development. Considering the designing of professional development for experienced and inexperienced post-secondary mathematics instructors is a meaningful byproduct of the results of this study.

Professional Development

Professional development for graduate teaching assistants. Recall that graduate teaching assistants account for a significant percentage of inexperienced faculty in college mathematics departments. The professional development of graduate teaching assistants is an important issue to college mathematics departments (DeChenne et al., 2012; Lewis and Tucker 2009; Miller et al., 2018; Speer et al., 2005). Literature suggests some teaching orientations, teaching mentors, supervision, and teaching seminars as possible ways to help GTAs improve their teaching (Lewis and Tucker, 2009).

This study saw beliefs in the practice of supporting GTAs as being significant in the theme on student-centered instructor beliefs. The beliefs included thoughts on GTAs being responsible for the development of their own exams, communicating effectively, respectfully, and with positivity, and also not taking for granted that students are enthusiastic about mathematics or that they always understand what is being taught in class without confirmation. Some practices that were associated with these beliefs were having GTAs deciding what was important and what should be covered in class, providing substantive feedback to students, using strategies like active learning to determine whether students are actually understanding what is being taught in class, and letting students know exactly what they are learning and why they are learning it. These beliefs and associated practices can be incorporated into professional development for inexperienced incoming graduate teaching assistants to help facilitate their transition from being not only students but teachers. This can be achieved through a combination of experiences. First, professional development hosted the University Teaching and Learning center, for example, that offers sessions about best practices, including ways to involve students more actively in class sessions, can be productive for potentially shifting to a practice-

based view of teaching mathematics. In addition, professional learning sessions co-facilitated by mathematics and mathematics education faculty hold promise in this regard. Further, co-teaching a mathematics course with an experienced instructor, and including a reflective seminar for GTAs, can increase GTA learning about effective student-centered teaching practices.

Professional development for inexperienced and experienced instructors. There were also data from the study which can be used to aid in the creation of professional development for all post-secondary instructors, those with experience and those with little experience. There were several practices from each of the three different themes that are relevant to the design of potential professional development and in equipping instructors with the tools necessary to teach their students more effectively.

The theme on student-centered instructor practices revealed practices that could be used to create a positive classroom environment. These practices included the following:

1. Have one-on-one conversations with students before class is the opportunity presents itself
2. Providing prompt and substantive feedback to students like well graded papers, or reply to emails
3. Designing curriculum that could help students cope with their math anxiety
4. Designing class projects that have a strong active learning component.
5. Having students do group work involving hard problems with the teacher's support
6. Using active learning and think-pair-share strategies to verify whether students are actually learning what is being taught in class

7. For advising Ph.D. students in mathematics, determining the proper time to offer advice to aid is students understanding without undermining the student's appreciation for the material

Practices revealed by the content-centered practices theme which could help students with mathematics and also improve their point of view on mathematics included the following.

1. Seeing mathematics as something that should not be understood all at once, but after a significant period of practice
2. Having students master a standard collection of problems without the need for tricks before expecting advanced rigor
3. Having students justify conditions needed to use theorems.
4. Deriving formulas to understand where rules used in class come from.
5. Using group work to have students learn from each other
6. Having students solve problems which are somewhat indirect requiring them to put pieces of information together
7. Making calculations with advanced definitions or theorems in order to understand them, for example the epsilon-delta definition of limits
8. Using a little deception in order to create a scenario where deriving a formula is necessary, for example the derivative of inverse tangent
9. Using real life situations to motivate students to care about why mathematics works the way it does, and how to see its use in other subjects
10. Seeing proofs or the idea of proofs earlier in a student's program of study so there is no critical jump when proofs become common place

11. Changing the conversation in mathematics so justification for an answer is thought of as part of the answer. Have students provide the whole answer
12. Have students be clear about their communication and stop trying to write as little as possible.

Practices from the student practices theme which could help students facilitate their own understanding of mathematics and become more responsible included the following.

1. Impress upon students the need to really work on their homework.
2. Impress upon students the need to come to office hours to get help. Try to address more than mathematics when you talk to students. Try to determine what is going on in their lives. If they are not doing well talk about how to prepare for class so they have a plan of attack.
3. Help students translate the attraction they have for mathematics into practices.
4. Facilitate the ability of graduate students of mathematics to be able to read articles and books that extend from where they left in class.
5. Stress to graduate students that in a proof they should only write things down of which they are completely confident.
6. Facilitate students' abilities to learn from a textbook.
7. Impress upon students the need to be willing to make mistakes and that recognizing when you are wrong is a huge asset.
8. Impress upon students the need to see their instructor when they are absent in order to determine they missed in class and to get help.

My Learning

When I began this doctoral program I was still a relatively new graduate from a Masters program in pure mathematics. I had been a graduate teaching associate in that department teaching mathematics courses for two years. I was also a faculty member at a high school for a year teaching several honors and dual enrollment mathematics courses. Later I took a faculty position in the math department at a small community college. Therefore it goes without saying that my background had been primarily in mathematics, either studying mathematics or teaching mathematics, rather than studying educational theory.

After my first year in the new position I gained a deeper appreciation for the ways in which my students learned. I became interested in using the skills I learned in my Masters program to break a mathematical concept down to its most basic components in order to structure a lesson that would create a classroom environment where mathematics would seem as natural and fluid as possible, and not seem unreasonably abstract or foreign. I knew from my own experience as a high school student and as a freshman in college that mathematics can easily come across as boring and meaningless if concept are not given a well prepared motivation, and the significance and implications of the concept are unknown. This led me to consider building my knowledge of practices that would help me further improve and develop my ideas on teaching mathematics.

The former dean the mathematics and sciences division at my institution encouraged me to apply to a doctoral program focusing in mathematics education. When I entered the program I was very nervous about whether I would fit in with the other graduate students. I was previously in a five-year post-baccalaureate program in mathematics and mathematics education with a cohort of students also earning their secondary education license. In that group I felt accepted

but also felt somewhat out of place given that my goals were very different than those of my peers. Now being in a doctoral level graduate program with others students whose backgrounds were not necessarily in mathematics made me feel even more unsure of myself. I did not know how well I would do in the coursework. I lacked confidence in my ability to write well in the style required for the program. Most importantly I did not know if I would fit in.

I began the program very conservatively only taking one class each term or the first year. This seemed appropriate as I was a non-traditional student working a full-time job. Since the program requires a minimum number of years teaching, I assumed that there would be many more students teaching full time while in the program. To my surprise there were few students in any of my courses who were also teaching or working another job. Almost all my peers were traditional graduate students with graduate teaching assistantships and had much more free time to focus on coursework than I did. Some were even involved in research projects with faculty or other students. At this point I not only felt inadequate in my coursework but also in research.

Things turned around during my second year when I took a couple of courses in the mathematics department which focused on teaching mathematics with a heavy emphasis on proper conceptual motivation. It was in these classes where I felt most at home in a world that mixed mathematics and teaching. Many of the ideas I saw in that class I use today in my calculus courses. The course also rejuvenated my interest in discovering new teaching practices that not only helped my students learn, but also merged my love for mathematics with my love for teaching.

I have frequently felt that mathematics and mathematics education are treated by many as two completely separate worlds where mathematicians who teach never talk about educational issues, and educators who teach methods and practices never talk about mathematics. This is the

area where I feel I can have the most influence as someone that can bridge the worlds of mathematics and mathematics education, specifically in areas of post-secondary mathematics that focus on higher-level undergraduate mathematics. As mentioned in the literature review of my study, there is a lack of research in teaching practices of post-secondary mathematics, as compared to research in secondary mathematics. So this is where my ideas involving research began. The comprehensive exam portion of the program gave me a great opportunity to explore some of my ideas of motivation for concepts, specifically derivatives in calculus and basic set theory in an introduction to proofs course.

This study gave me the opportunity to discover how other instructors with a background similar to my own think about mathematics and teaching. I was pleasantly surprised by how many of the participants had strong opinions concerning teaching. These opinions on the teaching and learning of mathematics could only have come from individuals who teach post-secondary mathematics on a regular basis and have several years if not decades of experience. I hope that the findings of my study inspire or form the basis for further research in teaching post-secondary mathematics.

After five years in the program I feel more confident in my abilities, but I know that learning in any field is a process that never stops. I see learning mathematics or learning educational theory as languages. While I am certainly more fluent in the language of mathematics I have not forgotten where I started in mathematics and how long it took me to get to where I am now. I therefore expect the same to be true of education. I expect to continue to expand my understanding until I have a fluent understanding of concepts at the highest level.

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APPENDICES

Appendix A

Invitation

Good day,

My name is Jimmy Miller. I am a Mathematics Education PhD candidate in the UTK department of Theory and Practice in Teacher Education, currently in the dissertation phase. My study involves analyzing the teaching practices of post-secondary mathematics instructors. I am contacting full time mathematics instructors in your department to find a group of 8-10 willing participants to take part in an individual one-hour interview where we will discuss teaching college level mathematics.

My experience in both pure mathematics and mathematics education has motivated me to study the teaching practices of instructors whose primary teaching duties involve teaching post-secondary mathematics to undergraduate math majors and graduate students. I have earned a BS and MS in mathematics, and an MS in math education from UTK. I am currently in my sixth year as an assistant professor in the math department at Roane State Community College.

The goal of my study is to determine practices that are effective at teaching undergraduate math majors and graduate students. Specifically, I want to answer the following questions:

1. What are the similarities and differences among the teaching philosophies of post-secondary mathematics instructors?
2. What are specific teaching practices identified by post-secondary mathematics instructors as critical to the teaching and support of students learning abstract mathematics?
3. How do the teaching philosophies of post-secondary mathematics instructors influence the teaching practices which are used to teach and support students learning abstract mathematics?

I would appreciate you completing a very brief survey to determine the courses you have taught over the last three years. I will use the results of the survey to select a sample of UT mathematics faculty to invite to participate in an individual one-hour interview. The link to the survey is posted below.

I appreciate your response and thank you for your time. If you have any questions please feel free to contact me at jmille54@utk.edu

Jimmy Miller
UTK PhD Candidate

Appendix B

Interview Consent Form

INSTRUCTIONS

Good day. My name is Jimmy Miller. Thank you for considering to participate in my study. I am looking for the views of post-secondary mathematics instructors concerning teaching. My research project focuses on teaching post-secondary mathematics, specifically the teaching practices used by post-secondary mathematics instructors that make instruction effective. I am not performing an evaluation of you. I am trying to learn more about post-secondary mathematics teaching practices which can hopefully be used to design professional development to support and inform new teachers.

You are being invited to participate because of the results of an initial survey to which you responded. This form contains information that will help you decide if you would like to participate in this research study. Please take the time to read this carefully, and if you have any questions please do not hesitate to contact me.

With your permission, I would like to conduct an interview with you. Our conversation will be recorded so I can get all the details while maintaining an attentive conversation with you. The audio from the interview will be transcribed and analyzed using qualitative research methods.

The interview should last approximately one hour, and your participation with this study will only involve our meeting this fall semester. I have specific questions that I would like to ask you, so if time begins to run short I may need to interrupt you to complete the interview.

CONSENT FORM INSTRUCTIONS

Please take a moment to read and sign the release form. Note that only the researchers on this project will have access to the interview recordings which will be destroyed after they are transcribed. I also ask that you sign a form which meets all our human subject requirements. This document states that: (1) your information will remain confidential, (2) your participation is voluntary and you may stop at any time, and (3) we do not intend to inflict any harm.

You will not receive any direct benefit from your participation in this research project other than the potential benefit from the things we learn from the analysis of the data.

This research is considered to be no more than minimal risk, meaning there is no more expected risk to you than what you would experience on a typical day. There is the potential risk of loss of confidentiality, as someone could find out you were in the study. This risk is highly unlikely because of the procedures that will be used to protect your information.

IRB NUMBER: UTK IRB-19-05288-XP

If you agree to participate in this study I will assign you a pseudonym which will be used instead of your name on all research materials before I analyze them for the study. The code key that relates pseudonyms to participant names and the consent forms will be stored separately from the research materials to prevent your name from being linked to them. The materials will be stored on my personal password protected computer and UT's password protected Google Drive. No information which could identify you will be shared in publications or presentations about this study. If in the future I wish to include your name, recordings, or any information that could identify you I will ask for your written permission.

Declining to participate in this study will involve no penalty or loss of benefits to which you would otherwise be entitled. Furthermore, you may discontinue participation at any time without penalty or loss of benefits to which you would otherwise be entitled.

It is possible that your research information may be used for future research studies, or shared with other researchers in those future research studies without obtaining additional consent from you. If so, all identifiable information will be removed before any future use or sharing with other researchers.

CONTACT INFORMATION

If you have any questions about this research you may contact me, Jimmy Miller, at jmille54@utk.edu or (865) 696-8214 or my advisor, Dr. Lynn Hodge, at lhodge4@utk.edu or (865) 974-8778.

If you have questions or concerns about your treatment in this research or your rights as a research participant, please contact the University of Tennessee IRB Compliance officer at 865-974-7697 or utkirb@utk.edu.

CONSENT

I have read the above information and have received a copy of this form. I agree that participating in this research study includes allowing Jimmy Miller to use the interview audio/video recording and written transcription for research purposes. I agree to participate in this study.

Participant's Name (printed) _____

Participant's Signature _____ *Date* _____

IRB NUMBER: UTK IRB-19-05288-XP

IRB APPROVAL DATE: 07/18/2019

Appendix C

Interview Guide

QUESTIONS

A. Academic Background

1. What is your educational background?
2. Did complete a teacher education program?
 - a) Or, have you taken any education classes?
 - b) Or, have you participated in professional development for teaching?

B. Beliefs on Teaching and Students

3. What is your philosophy for teaching undergraduate students; specifically, can you describe your teaching philosophy as it relates to the transition from freshmen and sophomore level math courses to junior and senior math courses?
4. What is your philosophy for teaching graduate students?
5. Do you have a written teaching philosophy?
6. How would you define a successful undergraduate mathematics student?
7. How would you define a successful graduate mathematics student?

C. Classroom Practices

8. Can you describe a typical class session?
9. Can you describe in what ways you have flexibility and in what ways you have limitations in what and how you are allowed to teach.
10. What things do you do in the classroom that you believe help students understand mathematics more conceptually?

11. How do you facilitate students' transition from high-school mathematics to rigorous, abstract, proof-based mathematics?

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Appendix D

Teaching Methods of Post-Secondary Mathematics Instructors

Thank you for your time completing this survey. You are receiving this survey because you have been identified as full-time mathematics instructor within your department. Please read the following statements before continuing with the survey.

Hello. My name is Jimmy Miller. I am a PhD candidate in Mathematics Education at the University of Tennessee, Knoxville. The intent of this study is to gain understanding into the teaching practices of post-secondary mathematics instructors at the undergraduate and graduate level. Your participation in this survey is voluntary and should only take a few minutes to complete.

Following this survey you may be contacted for an interview, if you indicate interest. Please direct any questions to me at jmille54@utk.edu.

Please provide responses to the following questions to the best of your ability.

* Required

1. What is your last name? *

Your answer

2. What is your first name? *

Your answer

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3. What is your gender?

Your answer

4. What is your position within the department of mathematics, (i.e lecturer, assistant professor, etc). *

Your answer

5. How long have you been teaching post- secondary (college-level) mathematics? *

Your answer

6. Please indicate which of the following courses you have taught over the last three years. (Check all that apply.) *

- ☐ Calculus I (Math 141/147)
- ☐ Calculus II (Math 142/148)
- ☐ Calculus III (Math 241/248)
- ☐ Math for Life Sciences I (Math 151)
- ☐ Math for Life Sciences II (Math 152)
- ☐ Differential Equations I (Math 231/237)
- ☐ Matrix Algebra I (Math 251)
- ☐ Introduction to Abstract Mathematics (Math 300/307)
- ☐ Introduction to Analysis (Math 341)

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- ☐ Introduction to Abstract Algebra (Math 351)
- ☐ Probability and Statistics (Math 323)
- ☐ Numerical Algorithms (Math 371)
- ☐ Combinatorics (Math 421)
- ☐ Probability (Math 423)
- ☐ Statistics (Math 424)
- ☐ Differential Equations II (Math 431)
- ☐ Partial Differential Equations (Math 435)
- ☐ Advanced Calculus (Math 441)
- ☐ Complex Variables (Math 443)
- ☐ Matrix Algebra II (Math 453)
- ☐ Analysis I (Math 447)
- ☐ Analysis II (Math 448)
- ☐ Number Theory (Math 450)
- ☐ Abstract Algebra (Math 451)
- ☐ Abstract Algebra I (Math 457)
- ☐ Abstract Algebra II (Math 458)
- ☐ Differential Geometry (Math 462)
- ☐ Numerical Analysis (Math 471)
- ☐ Numerical Algebra (Math 472)

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- ☐ Statistics II (Math 526)
- ☐ Analytical Applied Math I (Math 515)
- ☐ Analytical Applied Math II (Math 516)
- ☐ Enumerative Combinatorics I (Math 521)
- ☐ Enumerative Combinatorics II (Math 522)
- ☐ Probability I (Math 523)
- ☐ Probability II (Math 524)
- ☐ Statistics I (Math 525)
- ☐ Ordinary Differential Equations I (Math 531)
- ☐ Ordinary Differential Equations II (Math 532)
- ☐ Partial Differential Equations I (Math 535)
- ☐ Partial Differential Equations II (Math 536)
- ☐ Real Analysis (Math 545)
- ☐ Complex Analysis (Math 546)
- ☐ Modern Algebra I (Math 551)
- ☐ Modern Algebra II (Math 552)
- ☐ Number Theory I (Math 555)
- ☐ Number Theory II (Math 556)
- ☐ Topology I (Math 561)
- ☐ Topology II (Math 562)

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- ☐ Riemannian Geometry I (Math 567)
- ☐ Riemannian Geometry II (Math 568)
- ☐ Numerical Mathematics I (Math 571)
- ☐ Numerical Mathematics II (Math 572)
- ☐ Optimization (Math 577)
- ☐ Mathematical Ecology I (Math 581)
- ☐ Mathematical Ecology II (Math 582)
- ☐ Optimal Control Theory (Math 585)

Interest in Follow-up Interview

Please complete the following information if you are interested in potentially participating in a follow-up interview to discuss teaching post-secondary mathematics. The interview will last approximately one hour and will be conducted either in-person or via Zoom video conferencing. Not all those that indicate interest in participating will be selected.

7. Email Address *

Your answer

8. Work Phone Number

Your answer

SUBMIT

Never submit passwords through Google Forms.

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Appendix E

Participant Narratives

Charlie

I will now discuss Charlie in detail. Charlie studied mathematics as an undergraduate and earned his bachelors degree from a university in the northeastern United States. He also completed a PhD in mathematics from a large university in the western United States, and completed a post-doctoral program at a different university in the northern United States. He held a graduate teaching assistantship while earning his PhD where he completed a one semester course in teaching and a one semester course on mentoring newer GTAs. He is a tenured/tenure-track faculty member and reported teaching courses in linear algebra and undergraduate and graduate abstract algebra at the university where the study took place.

Charlie's beliefs and practices were found within the student-centered, content-centered, and student-practices themes.

Student-Centered Instructor Beliefs

For the student-centered instructor theme, Charlie mentioned beliefs regarding math PhD advising in the beliefs on teaching subtheme.

Beliefs on Teaching

Math PhD advising. I think a good topic for a student can expand up or down. For students to spend multiple years actually working on a project and not have anything that's worth being a thesis is probably a bad topic. Sometimes it's hard to know. You know, doing research is a different thing than taking classes. It's kind of a transition. I don't think they should sort of be on their own with their problem. Their research should

just be baseline, it doesn't have to be a major contribution. I don't think the significance of the contribution is necessarily a threshold.

Student-Centered Instructor Practices

Charlie also mentioned a practice regarding math PhD advising.

Math PhD advising. I think a student who's passed their exams, they should be given a problem that they can solve to some extent with some help. I think that if a student is struggling on their topic, the advisor should try to shift it. I don't think they should sort of be on their own with their problem.

From this we can conclude that an advisor should help their student choose a problem that is suitable for the dissertation phase of their PhD. The practice of assisting the student with their problem is critical to the student's success. This practice is indirect since Charlie does not describe it as something he has actually done, and internal since it is the practice of an instructor.

Content-Centered Beliefs

For the content-centered beliefs theme, Charlie mentioned beliefs regarding rigor at the lower division level and conceptual understanding within the philosophical ideas subtheme, as well as interpreting and writing proofs within the products of content subtheme.

Philosophical Beliefs

Rigor at the lower division level. In calculus they have to differentiate, find maxima, all the various things on some list of problems, and I would expect them to be able to do those types of problems, without particularly tricky ways or anything. It's at the higher end of the lower division honors course. There is where I try to, um, sort of teach them more about **why, why things are true.**

Conceptual understanding. You sort of have the core material of the course, but sometimes you can throw in things that maybe a few students will get, not proof but like **why things are related or why you do certain things.** If I can say something in two minutes that two people in class will get, then that can be a worthwhile thing.

Beliefs on Content

Interpreting and writing proofs. I think students sometimes get intimidated by proof. We shouldn't think of it as there's computation and then there's proof. Proof is just answering the question *why*. Students should be able to answer the question *why* without being intimidated by it, and without getting hung up on quantifiers. They should be able to understand *why*

Content-Centered Practices

For the content-centered practices, Charlie mentioned practices regarding rigor at the lower division level and conceptual understanding within the philosophical ideas subtheme, as well as interpreting and writing proofs within the products of content subtheme.

Philosophical Practices

Rigor at the lower division level. In calculus they have to differentiate, find maxima, all the various things on some list of problems, and I would expect them to be able to do those types of problems, without particularly tricky ways or anything.

The association to Charlie's belief is direct and the practice is external. Charlie does witness students doing these calculus problems, and it is a student practice.

Conceptual understanding. I have them work on group work on problems, and I go around help them and answer questions. I think working together through a problem, especially sort of more conceptual one where your mistakes aren't just going to be

computational [is important]. It's hard to check someone else's work. But on a logic problem, I think it's easier to talk through the problem. And so I think it's good for them to talk through the problems. I found that students have different [abilities]. Students who have a better understanding of what's a correct proof, and how to approach the proof, can help the others. They learn from each other and they learn by explaining. It's also a good way to see how they understand quickly.

The association between Charlie's belief on conceptual understanding and this practice is direct since he actually sees his students do group work. It is an internal practice because he facilitates the group work as a teaching practice.

Products of Content

Interpreting and writing proofs. I don't think proofs should be one new thing that is completely different from what came before.

This implies that there should not be a single moment when answering the question *why* becomes relevant. From this we can infer that answering *why* should have been part of the curriculum from the beginning. The practice of answering *why* from the beginning would be indirectly associated to his beliefs on writing proofs since it is not something actually implemented, but only considered by Charlie. Furthermore, it is an external practice since it would have to be done by an earlier instructor.

Beliefs on Student Practices

For the beliefs on student practices theme, Charlie mentions beliefs on the motivation of math PhD students, which was within the philosophical ideas subtheme.

Philosophical Beliefs

Motivation of math PhD students. To get through a graduate program you have to have some internal motivation to want to finish. At some point you just want to see it to the end, and you don't necessarily have the internal motivation to study the math for its own sake, but just to complete the process.

Charlie did not mention any practices associated with this belief. The dissemination of Charlie's belief-practices associations is summarized below.

Charlie	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Student Centered	0	1	1	0
Content Centered	2	1	1	2
Student Practices	0	0	0	0
Totals	2	2	2	2
%	50	50	50	50

Jane

I will now discuss Jane in detail. Jane earned an undergraduate degree in mathematics education from a large university in the southeastern United States. She also earned an undergraduate degree in engineering. She did not mention from what institution. She also earned a master degree in mathematics with an emphasis on teaching from a different large university in the southeastern United States. During the time between earning her two undergraduate degrees she taught high school mathematics. The length of time was not revealed. After earning her degree in mathematics she worked as an engineer for seven years. She has been a lecturer at the university where the study took place for seven years. She routinely

teaches courses in mathematical reasoning, statistics, and finite math. She has taught business calculus in the past.

Jane's beliefs fit within the theme on student-centered instructor beliefs, content-centered beliefs, and beliefs on student practices. For the student-centered instructor beliefs theme Jane stated beliefs on classroom environment including math anxiety.

Student-Centered Instructor Beliefs

Beliefs on Classroom Environment

Math anxiety. I want them to feel more comfortable to approach a problem. So part of my goal is to try to get them so they're not afraid to approach a math problem and use their critical thinking skills to approach the problem. The math anxiety for the level of students that I teach is pretty debilitating. It's so bad for some of them that they need to pick a different major, so they don't have to deal with those classes.

Jane also had a practice associated with her belief on math anxiety.

Student-Centered Instructor Practices

Classroom Environment

Math anxiety. They started a math study skills class which I [and other instructor] taught. We had them do a math biography, like an autobiography, and to have them talk about their math experience. I was amazed at how many of them could list the elementary school teacher that told them, and they could name them, they could give their name, that "This teacher told me I wasn't any good at math". It just stayed with them. So they were convinced they weren't good. It just stayed with them. So we were having them do some meditation to try to calm them down. They had student success come and give them career talks.

This practice is internal and is directly associated to its corresponding belief.

Content-Centered Beliefs

Jane had beliefs within the theme of content-centered beliefs. This included the philosophical ideas subtheme for both rigor at the lower division level and conceptual understanding.

Philosophical Beliefs

Rigor at the lower division level. No. No proofs. And the rigor, I mean, obviously I want them to have the foundation. I want them to be able to [do the work].

Conceptual understanding. I definitely want them to know what they're doing things. Because I think they're more able to deal with something new that's thrown at them. If they just learned the mechanics, they can't apply it in other places. Unfortunately, that is what happens. So a lot of them are just so use to just using the calculator to do a lot of stuff. They just grab it right away. But as far as understanding why they're doing this and that, I think it's really important. And it's really hard because they just want the answer. I guess that's pretty typical. They want the answer. They just want to get there, and they don't want to know why things are happening. I think it's important to understand why things are happening.

Content-Centered Practices

Philosophical Practices

Rigor at the lower division level. I want them to be able to [do the work].

The implied practice of students doing the work is external, and it is indirectly associated with Jane's belief on rigor.

Beliefs on Student Practices

Jane's beliefs on student practices concerned student responsibility in the philosophical beliefs subtheme, and characteristics of unsuccessful students in the characteristics of students subtheme.

Philosophical Beliefs

Student responsibility. I would expect [students] to be at a certain level when they walk in. I'm not finding that they are. And that's really disturbing. They keep telling us the act scores keep going up and up and up. But I'm not really seeing it. I'm seeing a big divide. There are students that are comfortable with [doing math]. It's easy. They just have to do it. Then there are others that cannot. They can't work with fractions. They don't know what a log is. They just don't understand concepts it seems. So how do you move forward? I don't know. I still struggle with that.

Characteristics of unsuccessful students. I have three daughters, and they, it's so funny would much rather text somebody than to pick up the phone and call them. I think students are a little bit fearful to come in for whatever reasons. In fact, I had several students miss class. I'll say do you want the notes. [They say], oh no I'll look it up on YouTube. So it's like, okay. Well what am I here for?

Practices of Students

Jane's only mentioned one practice in this theme which concerned characteristics of unsuccessful students.

Characteristics of Students

Characteristics of unsuccessful students. In fact, I had several students miss class. I'll say do you want the notes. [They say], oh no I'll look it up on YouTube. So it's like, okay. Well what am I here for?

This practice of Jane is a direct association to her belief and an external practice of students.

The dissemination of Sanborn's belief-practice associations is summarized below.

Jane	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Student Centered	1	0	1	0
Content Centered	0	1	0	1
Student Practices	1	0	0	1
Totals	2	1	1	2
%	67	33	33	67

Keith

I will now discuss Keith in detail. Keith studied mathematics at the undergraduate level at a small private school in the southeastern United States. He completed a bachelor of arts in mathematics. He also completed a master of science in mathematics from a large public university in the southeastern United States. During the period when he was completing his undergraduate degree he completed mathematics education coursework. He was not very specific about which courses, but he mentioned that he took all the coursework necessary until the student teaching portion of the education program. He decided to focus his time solely on completing his masters degree in mathematics when he realized he could teach at the community college or university level. During his masters program he took a graduate mathematics for teachers. Keith currently works as a lecturer in the mathematics department of the university where the study took place. He has participated in a few seminars on professional development.

He reported teaching pre-calculus on the preliminary survey but also teaches finite math, college algebra.

Keith's beliefs and practices were found within the student-centered, content-centered, and student-practices themes.

Student-Centered Instructor Beliefs

For the student-centered instructor beliefs theme, Keith mentioned beliefs regarding positivity and math anxiety within the classroom environment subtheme.

Classroom Environment

Positivity. Give them confidence and let them know you're there to help. I feel that that's one of the big things you can do.

Math anxiety. Sometimes you feel in a flipped class that you're not really teaching.

You're more of a cheerleader. You can do this! You can do this! Yes, I know you can!

Yes, it's tough, but we can [do it]. But I guess we're kind of a cheerleader in all of our classes too. We're on their side. We've got to keep their morale up, keep them going.

Especially this time of year, right now when we've been going and going and going and it's near the end and you're tired. You've not had a break and they're not used to working this hard, this much, covering this much material this quickly. For them is different. And it's tiring having to manage all that without mom and dad to help.

Student-Centered Instructor Practices

Keith mentioned practice concerning positivity and math anxiety within the classroom environment subtheme.

Positivity. I try to get them to come to office hours. I've learned to do that instead of calling them out in front of class. I'm just trying to figure out what's going on in their life

at that point in time because you never know. There are so many different things, and then go from there, try to address some of those and give them ideas of things they can do. But then also talk about how they are preparing for this math class because things aren't going so well, and let's get a plan of attack to do better and take baby steps one step at a time.

This practice is internal and directly associated to its corresponding belief.

Math anxiety. I am currently piloting something called Math 100 which is math study designed for students whose background suggests they're going to struggle in Math 119 [college algebra]. One of the sections we're on right now is note taking skills. I finished up with some listening and anxiety skills, [specifically] dealing with math anxiety, test anxiety, how to calm yourself down, positive self-talk instead of negative self-talk, giving themselves confidence.

This practice is internal and is directly associated to its corresponding belief.

Content-Centered Beliefs

Keith mentioned a belief regarding conceptual understanding which is within the philosophical ideas subtheme.

Philosophical Beliefs

Conceptual understanding. They do need to learn that work ethic. Learn the vocabulary and understand the concepts. [Do not] just memorize a bunch of formulas, but understand why things work the way they do, and how they work, and how to put them together in other subjects.

Content-Centered Practices

Keith also mentioned a practice regarding conceptual understanding.

Conceptual understanding. Try to throw things out there, in real life situations or just talk about it. How would this apply in this situation? I don't always know specific examples in [different] areas, but just to get them to think about it. When I'm asked, when am I ever going to need to know this? I [say] pull your book out. Find the homework section and read the word problems. Look at the word problems. That's where you are going to get to see some real-life scenarios where this can apply.

Keith's practice is indirectly associated to his belief since he is only giving examples of what can be done to motivated students' interest in conceptual understanding. It is an external practice because he does not refer to this practice as something he does, but only refers to it as something that can be done.

Beliefs on Student Practices

For the beliefs on student practices theme, Keith mentioned beliefs concerning student responsibility within the philosophical ideas subtheme, as well as characteristics of unsuccessful students within the characteristics of students subtheme.

Philosophical Beliefs

Student responsibility. So, you know, you want to teach them the math, but I feel a bit of an obligation to teach them, I don't know if responsibility is the right word. We'll say math responsibility and math accountability for their work or lack thereof, sometimes.

Characteristics of Students

Characteristics of unsuccessful students. They're just daydreaming in class, or even sometimes just sporadically coming once a week, maybe coming to class. Occasionally, if they've missed for like two weeks, But doing bad on quizzes, not participating in group work. When we do have some group work, they're just sitting there and letting everybody

else do the work. Or you constantly see them having people in their group having to explain everything to them. They're not really engaged. They're just kind of sitting there letting everybody else do the work or heaven forbid pulling out their phone and trying to play on their phone. And I'm having to walk by and [tell them to] put [their] phone up. This is math class, not social media hour.

Student Practices

Keith also mentioned practices concerning both student responsibility and characteristics of unsuccessful students.

Philosophical Practices

Student responsibility. I try to get them to come to office hours. I've learned to do that instead of calling them out in front of class. I'm just trying to figure out what's going on in their life at that point in time because you never know. There are so many different things, and then go from there, try to address some of those and give them ideas of things they can do. But then also talk about how they are preparing for this math class because things aren't going so well, and let's get a plan of attack to do better and take baby steps one step at a time.

This practice is directly associated to Keith's belief on facilitating student responsibility since it is implemented by Keith. It is also internal being Keith's practice.

Characteristics of unsuccessful students. I'll send an email just to try and see, Hey, what's going on? Are you okay?

Keith's action of sending an email is a directly associated practice of his beliefs and is internal being his own practice.

The dissemination of Keith's belief-practices associations is summarized below

Keith	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Student Centered	2	0	2	0
Content Centered	1	0	1	0
Student Practices	2	0	2	0
Totals	5	0	5	0
%	100	0	100	0

Lois

I will now discuss Lois in detail. Lois earned an undergraduate degree at a small university in the southeastern United States where she studied mathematics, mathematics education, and English education. She also earned a masters degree in mathematics with an emphasis in teaching and a Ph.D. in mathematics education from a large university in the southern eastern United States. She previously taught mathematics at the high school level and is currently a lecturer at the university where the study took place teaching courses in mathematical reasoning and basic calculus. She routinely implements and facilitates professional development for college and high school faculty.

All of Lois' beliefs and practices were found in the theme which was student-centered. For this theme mentioned beliefs on establishing a positive classroom environment, teaching using active learning and supporting graduate teaching assistants.

Student-Centered Instructor Beliefs

Beliefs on Classroom Environment

Beliefs on positivity. I try to communicate effectively, respectfully, and with positivity.

Beliefs on Teaching

Beliefs on active learning. I encourage students' thinking and students' active participation. So active learning is a big deal. They are accustomed to active learning

from high school. They are accustomed to something different. More teachers have taken up active learning, so I think the students' expectations have changed.

Beliefs on graduate teaching assistants. I expect my colleagues or my GTA's to convey an attitude [that facilitates] communicating effectively, respectfully, and with positivity, which is sometimes hard to articulate to them. [For example I might say], that thing you just said in the classroom, that might not convey the right attitude.

Lois noted that displaying an enthusiastic attitude towards mathematics does not necessarily inspire students to be enthusiastic towards mathematics, nor does it necessarily justify the importance of mathematics to students.

Lois: [What are we doing and why are we doing it?] [Students] don't just get that. I used to think in my naivety that if I was just enthusiastic about math, they would just get it. That it's cool. Some of them do. You're enthusiastic about it, you know, look, it's cool and then that's enough and they think it's cool. But then I finally learned, they don't always just get it. I mean, you could be doing something and be like, obviously this is useful. Obviously this is cool. They didn't just automatically pick up on maybe the relevance for learning what we were learning in class.

Student-Centered Instructor Practices

Classroom Environment

Practices which facilitate positivity.

Lois also had practices associated to fostering a positive classroom environment.

Teaching Practices

Active learning. They do something active learning in class every day. So it's not just me and slides. [I do] a 10-15 minute lecture at most of me showing them something. Then

they do in class questions where they work in groups or they do a hands-on activity. I can do active learning by doing worksheets and clickers and online canvas questions. Most of the time if I ask really good questions, then I'm satisfied. But some of the time I'm going need it to be off the books and something where they're shooting something down the hallway, they've designed a hearing aid and they're testing it to see how well it's amplifying their sound, or they're bouncing balls in the racquetball court.

This practice is external since it is something the students are doing and is directly associated to its corresponding belief.

Furthermore, in a hypothetical discussion about teaching a higher level pre-calculus course, Lois mentions continue active learning in that course as well.

Lois: I would try to still continue active learning. I would want my class to look and feel the same where they get in groups and do hard problems together with my support during most of the class. And I would still do a 10 to 15 minute Ted talk at the beginning with of what are we doing and why are we doing it.

This practice is indirect since it is hypothetical, and it is internal since it is a practice of Lois.

Practices for supporting GTAs. [Examples include] providing prompt and substantive feedback to students. That could be grading papers well, or it could be just replying to emails, also professionalism and efficient use of class time in the class.

In this context the practice is indirect since Lois did not mention observing the practice and external since it is described as a practice of GTAs.

Lois: [What are we doing and why are we doing it?] You have to say it out loud. Unless I say, what are we doing, why are we doing this, they didn't make that connection.

This practice would be indirect and external since it is not explicitly witnessed by Lois, and is an implied practice of a GTA. The dissemination of Lois' belief-practices associations is summarized below.

Lois	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Student Centered	3	2	3	2
Content Centered	0	0	0	0
Student Practices	0	0	0	0
Totals	3	2	3	2
%	60	40	60	40

Marty

I will now discuss Marty in detail. Marty earned an undergraduate degree in mathematics from an institution outside of the United States. He earned a PhD in mathematics from a large university in the midwestern United States. He did not complete any courses in education or any formal training in teaching. He is a tenured/tenure-track professor at the university where the study took place and teaches courses in graduate combinatorics, number theory, and abstract algebra.

Student-Centered Instructor Beliefs

Marty mentioned a belief concerning PhD advising for students of mathematics. This was in the teaching subtheme.

Beliefs on Teaching

Math PhD advising. No. [The student] has to say I'm learning it. I'm learning it and can somebody help me learn it better? So that's the first requirement, to try. And then the second important realization that a person ought to have is that I may or may not succeed in the goal that I've set myself up. And it's a question of how long I will try. You know,

the PhD has a time limit. You can't keep on doing it for 12 years. But if you have understood mathematics by then, to an extent that you have the courage and have a faith in yourself that you can study, you can carry this forward even when you haven't really succeeded in it right now. That is the only way getting through graduate school or earning a degree of PhD has any value. Otherwise it has no value, whatsoever. Really speaking, the PhD is just a degree. Degrees have no value. Education is a mechanism within a human being. It has nothing to do with degrees and certificates issued by other people.

Student-Centered Instructor Practices

For the theme on student-centered instructor practices, Marty mentioned practices for math PhD advising.

Teaching Practices

Math PhD advising. If you [have] spent a lot of effort and time on it and you have meaningful questions because you get stuck somewhere, you don't see why something is the way it is, then your thesis advisor can shed some light on it. But if the thesis advisor sheds the light on it right to begin with you will not have gained any appreciation about it, and in fact you will not at all understand. It will be of no use. So it's only at a proper time know that a piece of advice from the thesis advisor has meaning [or] has value, and that person who is in graduate school has to realize that. [The student] has to say I'm learning it. I'm learning it and can somebody help me learn it better?

Given the fact that Marty believes a student should ask if someone can help them learn better, it is reasonable to believe that there is someone there to help them learn better. Furthermore, that person should be the student's advisor. Therefore, it is also reasonable to think that a practice

which Marty believes in is helping students learn in a better way. This practice of the advisor is internal and indirectly associated to the belief of advising since Marty does not give specifics on how he advises his students.

Content-Centered Beliefs

For the theme on content centered beliefs, Marty mentioned beliefs concerning rigor at the lower division level and conceptual understanding from the philosophical ideas subtheme, as well as interpreting and writing proofs from the products of content subtheme.

Philosophical Beliefs

Rigor at the lower division level. At freshman, sophomore level they don't have to see the big picture at all. You can only see the big picture when you have total command over the small picture. The big picture is not something that should be taught at the beginning. So my philosophy of teaching is you have to build muscles then lift heavy weights. So the reason for that is mathematics is not something that can be understood at once. It is best understood by long practice.

Conceptual understanding. It is not possible without practicing the [epsilon-delta] definition [by] writing epsilon-delta proofs for limits to understand the definition. You don't have to do it in a day and night, but you have to do it until the **concept** actually just completely, uh, remains in your subconscious so that you know that you don't have to make extra efforts to just artificially remember the definition.

Beliefs on Content

Interpreting and writing proofs. Well [proof] type-oriented courses should be there right from the beginning. I won't call [the beginning] elementary for the grades they are in. They're equally tough as [higher level material]. And in calculus also, I do not really

like that calculus courses do not actually allow you to practice small degrees of proof. Because there is no such thing as the mathematics of proof being different from mathematics. It's really one thing. It is artificially separated by people, in my opinion, who are completely misguided about such separations. And that creates unnecessary pain later on, especially for those who want to study mathematics. It is a belief among some high schools and so on that they are not really concerned with the proofs. But really speaking, proofs allow you to develop better methods. So to some extent they have to [do proofs]. They don't have ask for proficiency in proof writing. But I think understanding of proofs is necessary.

Content-Centered Practices

For the theme on content-centered practices, Marty mentioned practices concerning rigor at the lower division level and conceptual understanding from the philosophical ideas subtheme, as well as interpreting and writing proofs from the products of content subtheme.

Philosophical Practices

Rigor at the lower division level. Mathematics is not something that can be understood at once. It is best understood by long practice.

This implied practice would be indirect and external. Marty did not witness it, nor is it his practice, but the implied practice of a student.

Conceptual understanding. To understand the [epsilon-delta] definition is actually to write formal arguments or to make the calculations with epsilon. Given epsilon find delta. Actually, do it in several examples so that you understand what is involved.

This practice is indirectly associated to Marty's belief since he does not witness his students practicing the epsilon-delta definition, and it is an external practice since it would be an action taken by his students.

Products of Content

Interpreting and writing proofs. So that means instead of high schools focusing on calculus, they should focus more on elementary [material], let's say Euclidean geometry or algebra. Once you are well prepared and you understand by small geometric examples or algebraic examples what it means to write a proof, there doesn't have to be any kind of critical jump because it slowly develops.

Similar to Charlie, Marty's practice is indirectly associated to his belief, and an external practice.

Beliefs on Student Practices

For the beliefs on student practices theme Marty mention beliefs concerning motivation of math PhD students from the philosophical ideas subtheme and characteristics of undergraduate and graduate students from the characteristics of students subtheme.

Philosophical Beliefs

Motivation of math PhD students. For a PhD student a burning passion for mathematics and learning and the ability to self-learn is required. You must hold within you some attraction towards it. [Mathematics] is not some kind of a mechanical task like any other subject. I cannot tell the person go conduct the following 50 experiments and tabulate the results and plot the graphs by known methods. Use the software that I tell you to use, that's not pure mathematics. You have to be fired up about learning the subject even without actually having tremendous knowledge about it.

Beliefs on Students

Characteristics of successful graduate students. So at that point in time in graduate school, what should happen is that now you know enough ways of learning by yourself so that now you don't need a classroom.

Characteristics of successful undergraduate students. In a hierarchical subject like mathematics, information does not equal knowledge. You need a lot of **self-effort**. You have to have some kind of **liking** for mathematical thoughts. If you are absolutely dejected by those kinds of thoughts, you are not going to be able to learn mathematics at all.

Student Practices

For the student practices theme, Marty mentioned practices concerning motivation of math PhD students from the philosophical ideas subtheme, as well as characteristics of successful graduate students from the characteristics of students subtheme.

Philosophical Practices

Motivation of math PhD students. You must hold within you some attraction towards it. And that attraction has to be translated into practice.

He however did not specify what this practice is. It would appear to be an overall statement about a student's willingness to do the required work. This is clearly the practice of a student making it external, and since Marty didn't specify an actual instance of a student translating their attraction into practice it is indirectly associated to Marty's belief.

Characteristics of successful graduate students. So you have to be able to read book or articles which extend from where you have left [with coursework].

Marty's practice is indirectly associated to beliefs on successful graduate students since his statement did not reference a specific example. It is also external being a practice of another individual, namely a student.

The dissemination of Sanborn's belief-practices associations is summarized below.

Marty	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Student Centered	0	1	1	0
Content Centered	1	2	0	3
Student Practices	1	1	0	2
Totals	2	4	1	5
%	33	67	17	83

Sanborn

I will now discuss Sanborn in detail. Sanborn studied mathematics education as an undergraduate, earning a degree in secondary mathematics education from a large university in the southeastern United States. During his tenure as an undergraduate he earned 60 hours of education coursework. He then earned a masters degree in mathematics from a different large university in the southeastern United States. He is a lecturer at the university where the study took place, and reported teaching pre-calculus, calculus I, calculus II, and calculus III on the initial survey. He occasionally attends conferences in mathematics and mathematics education.

Sanborn's beliefs and practices were found within the student-centered and content-centered themes. For the student-centered theme Sanborn reported beliefs concerning establishing a positive classroom environment and teaching using active learning, supporting graduate teaching assistants. He reported practices on rigor at the lower division level, conceptual understanding, and the whole answer.

Student-Centered Instructor Beliefs

Beliefs on Classroom Environment

Beliefs on positivity. I want them to have a positive experience in a math class. I think [about] being positive all the time, being aware of inclusion and diversity and really trying to be welcoming, even overly so in some kind of capacity. So I think a positive experience in a math class is one thing I definitely want.

Beliefs on Teaching

Beliefs on graduate teaching assistants. That's one thing that very much bothers me with the other mentoring programs. The grad students are never once having to write an exam. And I definitely think that is part of teaching. Everything is pre-written. You as a lecturer have zero control over changing it, quizzes everything. For my class I definitely want [graduate students to write exams] because that is part of teaching. I think it's imperative to being an effective teacher. In my eyes the creation of an exam does not happen when you are actually writing it. The creation of the exam is when you're thinking about your lesson plans, that's where it starts.

Beliefs on active learning. Last year I really tried to incorporate more active learning. It really depends on how many homework questions they have. It works well with some sections but not all.

Worksheets have really standardized *active learning* components. [The graduate students] are supposed to do all the worksheets in class.

Student-Centered Instructor Practices

Classroom Environment

Sanborn had a practice associated to fostering a positive classroom environment.

Practices which facilitate positivity. I think that a positive experience and inclusion is important class. I always get there 15 minutes early. And the beauty of this is that in a 9:05am large lecture, no one is in there at 8am. So I can get there as early as I want, and I just start walking around. They're trained now where they leave every other row open so I can walk. And in a smaller class a lot of times [I walk around while] collecting homework problems. So I [ask] how are you doing? What's going on? So it's basically forcing a personal conversation with every one of my students before class starts. It's worked very nicely, even a smaller class [with 35 students]. At that point with 35 students, I'm also recording what homework problems they want to go over, because I do go over homework problems in the smaller class. So that's how I start before class.

This practice is internal since it is an action taken by Sanborn himself, and is directly related to his belief on positivity since it is an action he actual takes.

Teaching Practices

Sanborn also had a practice for supporting graduate teaching assistants.

Practices for supporting GTAs. You should be thinking about what is important and that should decide for you what to cover in class. You should at least be aware of it. You should be thinking about it as you go along.

This belief is indirect and external since it is not actually observed and is the practice of an individual other than Sanborn.

Content-Centered Beliefs

Sanborn also had beliefs within the theme on content-centered beliefs. This includes the philosophical ideas subtheme for both rigor at the lower division level and conceptual understanding, as well as the products of content subtheme for the whole answer.

Philosophical Ideas

Rigor at the lower division level. Nothing. They should not have to do any proofs in my eyes, as far as rigorous proofs. Not at all. But that doesn't mean that you, [the teacher], shouldn't derive some formulas. I think that as you go along in the mathematical curriculum, the more derivation should be a part of the course. Because when you say the word derive do your students actually know what that means?

However, there is a risk versus reward. The risk is the amount of time you spend on it. The reward is whether the people who are actually paying attention will get something out of it. For example, coming up with the derivative of sine using the formal limit definition. I think at this level it becomes a time where it's just too much. You should be aware of how much [time] it would take. I do like massaging that need to derive things, but I don't think that I need to prove everything. Things that need more than 10 or 15 minutes, I don't know, it's not a good use of time.

Conceptual understanding. I'm not a big fan of steps on problems. I am a big fan of going over how to solve something and really emphasize on how to figure it out.

Beliefs on Content

The whole answer. People think of math as getting the right answer, not [what does it mean]. Because to me, half the time I could care less with the [the "answer" is], you know. [One day a student] came in with a related rates problem I [assigned]. A 12-point

problem. I gave checkmarks on all the places that he got points. He thought it was funny. He's like, for the final answer there's no check marks. I was like, no. You communicated all the right things. So that's relevant.

Content-Centered Practices

Sanborn had practices associated to his beliefs on rigor at the lower division level, conceptual understanding, and the whole answer.

Philosophical Ideas

Rigor at the lower division level. I definitely have to talk about that whenever we're deriving the derivative for inverse tangent for example. I'm like we're coming up with a rule so that afterwards we can just use the rule. That's what deriving means. Then I use the terminology of okay this is going to be an exercise in understanding. Follow me. If you got it, awesome. If you didn't at the end of it that's fine. We're going to have a punchline. You've just got to know how to use the punchline afterwards.

The association between this practice and Sanborn's belief on rigor is direct since the practice is realized in his class. The practice itself is internal because it is the result of Sanborn's action.

Conceptual understanding practices. Sanborn: There are times where I do lie in class where I'm like you know at first glance, I didn't know how to do this problem. [to motivate a reason to derive something or discover something, no steps]. I want them to actually understand why things work out.

This practice of Sanborn is directly associated to his belief on conceptual understanding since his method of deception actually implemented, and it is internal since it is his practice.

Products of Content

The whole answer. Sanborn: I tried for the first the few weeks to [say] **the work is the answer. The work is the answer** every day in math class because I'm trying to change that conversation.

Sanborn's practice is directly associated to his belief since it is being implemented, and being an action taken by Sanborn it is internal.

The dissemination of Sanborn's belief-practice associations is summarized below.

Sanborn	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Student Centered	1	1	1	1
Content Centered	3	0	3	0
Student Practices	0	0	0	0
Totals	4	1	4	1
%	80	20	80	20

Sarah

I will now discuss Sarah in detail. Sarah studied mathematics as an undergraduate at a large public university in the midwestern United States earning a bachelors degree in mathematics. She also studied mathematics at a large public university in the southern United States earning a PhD in mathematics. She taught as a teaching assistant in a supplemental instructor program in graduate school where she incorporated active learning into her lessons. She also taught as an independent instructor during her third year of school where used methods of inquiry-based learning. She is a lecturer at the university where the study took place and reported teaching courses in college algebra, basic calculus, calculus II, and calculus for life sciences.

Sarah's beliefs and practices were found within the student-centered, content-centered, and student practices themes.

Student-Centered Instructor Beliefs

For the student-centered theme Sarah reported beliefs concerning active learning and supporting graduate teaching assistants in the beliefs on teaching subtheme.

Beliefs on Teaching

Active learning. Get active learning going on in your classroom, so you don't think that just because you said something that they understood what you said. I don't think that graduate students or young teachers starting out often think that if you say something that students may not understand what you said. It's not the case that they'll always remember what you said and understand what you said.

Supporting graduate teaching assistants. I think graduate students or young, you know, teachers starting out often think that if you say something that students understand what you said. They're going to remember what you said and they're going to understand what you said. And that's just not the case.

Student-Centered Instructor Practices

Sarah's student-centered instructor practices were within both active learning and supporting graduate teaching assistants of the teaching practices subtheme.

Teaching Practices

Active learning. [The teachers] are going to have to be checking in, having the students do things so they can figure out whether or not students are actually understanding. So all the active learning stuff, teaching think-pair-share and other strategies [should be

used] to get students to see what they know and don't know so the instructor can know what they don't know.

This practice is external since Sarah is speaking about the actions of other instructors and not herself. Furthermore, it is indirectly associated to Sarah's belief on active learning since Sarah is not witnessing the practice for herself.

Supporting graduate teaching assistants. [The instructors] need to be checking, having the students do things so that they can figure out whether or not students are actually understanding. [Use] strategies to be able to see [what the] students know and don't know, and then let the instructor also know what they don't know.

Similar to the use of part of this quote in the discussion about active learning, this will be an indirect and external practice.

Content-Centered Beliefs

For the theme on content-centered beliefs, Sarah mentioned beliefs concerning rigor at the lower division level, and conceptual understanding within the philosophical ideas subtheme, as well as the whole answer within the products of content subtheme.

Philosophical Beliefs

Rigor at the lower division level. Just getting the basics of being a little more mature and formal about their mathematical arguments. Towards the latter half of the course I'm having them use the comparison theorem for integrals. And then all the, you know, tests and stuff for sequences and series. And there is where I start being really picky about what they write.

Conceptual understanding. I try to ask some conceptual questions on an exam. [I am] basically not handing it to them in the names that they're used to but telling them the relationships that they need to be able to figure stuff out.

Beliefs on Content

The whole answer. It's your **whole argument** that I am grading. I really try and get them away from what's the answer. General mathematical maturity.

Content-Centered Practices

Sarah's practices were within the philosophical ideas subtheme concerning rigor at the lower division level and conceptual understanding, as well as the products of content subtheme concerning the whole answer.

Philosophical Practices

Rigor at the lower division level. As I'm doing the lectures, I make it clear this is what you have to show me to get the full credit. You have to show me that you satisfied these conditions. I don't make them show me in any great detail. I don't make them show me the derivative and then prove that it's negative, but just write down that you know it's supposed to be negative.

This association to the belief is direct since it is observed by Sarah, and the practice internal since she shows students how to justify theorems during her lectures.

Conceptual understanding. I give them two or three different series and ask which of these could apply to the integral test. I like to put things for the first and second fundamental theorems of calculus, giving them information about that function at a particular point and the values of the derivative at a particular point, having them put together the pieces of the fundamental theorem.

This practice illustrates Sarah's intent to have her students demonstrate their conceptual knowledge with a problem that requires them to actually use their knowledge. It has a direct association to her belief on conceptual understanding given that the action takes place in her class, and it is internal being one of Sarah's teaching practices.

Products of Content

The whole answer. I start being really picky about what they write and them not putting circles around anything. How do you explain the process that you use to get there? Getting them to just be a little clear about their communication. Get them not to try to write as little as possible.

Beliefs on Student Practices

The beliefs that Sarah mentions on this theme are found within the subtheme on the characteristics of students. This includes characteristics of successful undergraduate and graduate students, as well as unsuccessful students.

Characteristics of Students

Characteristics of successful graduate students. I really think that once people get to the graduate level, they should have the chops to go off and figure it out themselves. By your third year [you] should be able to learn from no one.

Characteristics of successful undergraduate students. For an actual math major you need that **driving curiosity** and just wanting to have the formality, getting into the more formal side of math. Beating your head against a wall a little bit on your homework problems and then having it [the wall] tumble down and thinking that's really exciting, only to see that there's another wall. It's kind of fun. You have to kind of get to the point where it takes perseverance.

Characteristics of unsuccessful graduate students. I have students that I see not being very successful. Every time they make a mistake, they have an excuse for the mistake that they made rather than [admitting] they did something wrong and try to figure it out. People want to justify their incorrect actions. Basically, they're not seeing it as a process of I want to learn what the right thing to do is, they almost want to be right from the get-go.

Student Practices

The practices that Sarah mentions were also found in the same three areas.

Practices of Students

Characteristics of successful graduate students. You have to be clear on your argument. Don't say anything you aren't completely confident of. Some of the best advice I ever got is that you can't fudge it. You have to be super confident and really clear on how one thing builds upon another. You have to make a really clear argument and there can't be anything fuzzy.

Learn from a textbook. [In graduate school], we learned so much on our own working on homework.

Characteristics of successful undergraduate students. So I think begin willing to make mistakes and recognize that you are wrong is a huge asset. Being willing to throw out ideas that could be wrong. Being able to work in groups. It's not something they come in knowing how to do but hold them to it.

Characteristics of unsuccessful students. Every time they make a mistake, they have an excuse for the mistake that they made rather than [admitting] they did something wrong and try to figure it out.

The dissemination of Sarah's belief-practices associations is summarized below.

Sarah	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Student Centered	0	2	0	2
Content Centered	3	0	3	0
Student Practices	1	3	0	4
Totals	4	5	3	6
%	44	56	33	67

Tim

I will now discuss Tim in detail. Tim studied mathematics as an undergraduate earning a bachelors degree in mathematics from a large university in the midwestern United States. He also earned a PhD in mathematics from the same university. During his time as a graduate student he worked as a graduate teaching assistant but received no formal training in teaching, nor did he take any courses in education. Tim is a lecturer at the university where the study took place. He reported teaching pre-calculus, calculus 1, calculus 2, and geometry on the initial survey.

Tim's beliefs and practices were found within the themes on content-centered beliefs and beliefs on student practices.

Content-Centered Beliefs

Tim's belief in this theme concerned rigor at lower division in the philosophical ideas

Philosophical Beliefs

Rigor at the lower division level. They should know basics, depending on the class.

Content-Centered Practices

Tim's student-centered instructor practice was with regards to conceptual understanding.

Philosophical Practices

Conceptual understanding. Yea sometimes I just do [something purely computational], know how to integrate. Other times I pull out some ideas and the concepts with it, so we mix them, I think.

Tim's practice is directly associated to his belief and is internal being his teaching practice.

Beliefs on Student Practices

Tim's beliefs on student practices concerns student responsibility within the philosophical ideas subtheme and characteristics of successful undergraduate students.

Philosophical Beliefs

Student responsibility. The derivative is something you crank out in practice. I assume they really know how to do those.

Characteristics of Students

Characteristics of successful undergraduate students. Try and learn this stuff by doing it. It's something you've got to do. That's probably my main theory about this. The main responsibility of learning is on the student. If you don't know ask me because that's what I prefer. I'll try and get them to come in and say let's talk after school. I'll be glad to talk to you.

Student Practices

Tim mentioned practices associated to both of his beliefs on student responsibility and characteristics of successful undergraduate students.

Philosophical Practices

Student responsibility. Do the homework. Study. Really work on it.

This practice is indirect since it is not witnessed by Tim, and it is external being something done by students and not Tim.

Characteristics of Students

Characteristics of successful undergraduate students. People who really work at it usually do well. I do see kids that will just kind of show up and take the test and go away.

Tim mentions actually seeing students who practice the opposite of what he believes is a practice of successful students, so this practice is directly associated to his belief. It is also an external practice of students.

The dissemination of Tim's belief-practice associations is summarized below.

Tim	# of Direct Associations	# of Indirect Associations	# of Internal Practices	# of External Practices
Student Centered	0	0	0	0
Content Centered	0	0	1	0
Student Practices	1	1	0	2
Totals	1	1	1	2
%	50	50	33	67

Appendix F

Deborah Ball's High-Leverage Practices:

- 1. Leading a group discussion:** In a group discussion, the teacher and all of the students work on specific content together, using one another's ideas as resources. The purposes of a discussion are to build collective knowledge and capability in relation to specific instructional goals and to allow students to practice listening, speaking, and interpreting. The teacher and a wide range of students contribute orally, listen actively, and respond to and learn from others' contributions.
- 2. Explaining and modeling content, practices, and strategies:** Explaining and modeling are practices for making a wide variety of content, academic practices, and strategies explicit to students. Depending on the topic and the instructional purpose, teachers might rely on simple verbal explanation, sometimes with accompanying examples or representation. In teaching more complex academic practices and strategies, such as an algorithm for carrying out a mathematical operation or use the metacognition to improve comprehension, teachers might choose a more elaborate kind of explanation that we are calling "modeling". Modeling includes verbal explanation, but also thinking aloud and demonstrating.
- 3. Eliciting and interpreting individual students' thinking:** Teachers pose questions or tasks that provoke or allow students to share their thinking about specific academic content in order to evaluate student understanding, guide instructional decisions, and surface ideas that will benefit other students. To do this effectively, a teacher draws out a student's thinking through carefully-chosen questions and tasks and considers and checks alternative interpretations of the student's ideas and methods.
- 4. Diagnosing particular common patterns of student thinking and development in a subject-matter domain:** Although there are important individual and cultural differences among students, there are also common patterns in the ways in which students think about and develop understanding and skill in relation to particular topics and problems. Teachers who are familiar with common patterns of student thinking and development and who are fluent in anticipating or identifying them are able to work more effectively and efficiently as they plan and implement instruction and evaluate student learning.
- 5. Implementing norms and routines for classroom discourse and work:** Each discipline has norms and routines that reflect the ways in which people in the field construct and share knowledge. These norms and routines vary across subjects but often include establishing hypotheses, providing evidence for claims, and showing one's thinking in detail. Teaching students what they are, why they are important, and how to use them is crucial to building understanding and capability in a given subject. Teachers may use explicit explanation, modeling, and repeated practice to do this.

- 6. Coordinating and adjusting instruction during a lesson:** Teachers must take care to coordinate and adjust instruction during a lesson in order to maintain coherence, ensure that the lesson is responsive to students' needs, and use time efficiently. This includes explicitly connecting parts of the lesson, managing transitions carefully, and making changes to the plan in response to student progress.
- 7. Specifying and reinforcing productive student behavior:** Clear expectations for student behavior and careful work on the teacher's part to teach productive behavior to students, reward it, and strategically redirect off-task behavior help create classrooms that are productive learning environments for all. This practice includes not only skills for laying out classroom rules and managing truly disruptive behavior, but for recognizing the many ways that children might act when they actually are engaged and for teaching students how to interact with each other and the teacher while in class.
- 8. Implementing organizational routines:** Teachers implement routine ways of carrying out classroom tasks in order to maximize the time available for learning and minimize disruptions and distractions. They organize time, space, materials, and students strategically and deliberately teach students how to complete tasks such as lining up at the door, passing out papers, and asking to participate in class discussion. This can include demonstrating and rehearsing routines and maintaining them consistently.
- 9. Setting up and managing small group work:** Teachers use small group work when instructional goals call for in-depth interaction among students and in order to teach students to work collaboratively. To use groups effectively, teachers choose tasks that require and foster collaborative work, issue clear directions that permit groups to work semi-independently, and implement mechanisms for holding students accountable for both collective and individual learning. They use their own time strategically deliberately choosing which groups to work with, when, and on what.
- 10. Building respectful relationships with students:** Teachers increase the likelihood that students will engage positive, individual relationships with them. Techniques for doing then include greeting students positively every day, having frequent, brief, "check in" conversations with students to demonstrate care and interest, and following up with students who are experiencing difficult or special personal situations.
- 11. Talking about a student with parents or other caregivers:** Regular communication between teachers and parents/guardians supports student learning. Teachers communicate with parents to provide information about students' academic progress, behavior, or development; to seek information and help; and to request parental involvement in school. These communications may take place in person, in writing, or over the phone. Productive communications are attentive to considerations of language and culture and designed to support parents and guardians in fostering their child's success in and out of school.

- 12. Learning about students' cultural, religious, family, intellectual, and personal experiences and resources for use in instruction:** Teachers must actively learn about their particular students in order to design instruction that will meet their needs. This includes being deliberate about trying to understand the cultural norms for communicating and collaborating that prevail in particular communities, how certain cultural and religious views affect what is considered appropriate in school, and the topics and issues that interest individual students and groups of students. It also means keeping track of what is happening in students' personal lives so as to be able to respond appropriately when an out-of-school experience affects what is happening in school.
- 13. Setting long and short term learning goals for students:** Clear goals referenced to external standards help teachers ensure that all students learn expected content. Explicit goals help teachers to maintain coherent, purposeful, and equitable instruction over time. Setting effective goals involves analysis of student knowledge and skills in relation to established standards and careful efforts to establish and sequence interim benchmarks that will help ensure steady progress toward larger goals.
- 14. Designing single lessons and sequences of lessons:** Carefully-sequenced lessons help students develop deep understanding of content and sophisticated skills and practices. Teachers design and sequence lessons with an eye toward providing opportunities for student inquiry and discovery and include opportunities for students to practice and master foundational concepts and skills before moving on to more advanced ones. Effectively-sequenced lessons maintain a coherent focus while keeping students engaged; they also help students achieve appreciation of what they have learned.
- 15. Checking student understanding during and at the conclusion of lessons:** Teachers use a variety of informal but deliberate methods to assess what students are learning during and between lessons. These frequent checks provide information about students' current level of competence and help the teacher adjust instruction during a single lesson or from one lesson to the next. They may include, for example, simple questioning, short performance tasks, or journal or notebook entries.
- 16. Selecting and designing formal assessments of student learning:** Effective summative assessments provide teachers with rich information about what students have learned and where they are struggling in relation to specific learning goals. In composing and selecting assessments, teachers consider validity, fairness, and efficiency. Effective summative assessments provide both students and teachers with useful information and help teachers evaluate and design further instruction.
- 17. Interpreting the results of student work, including routine assignments, quizzes, tests, projects, and standardized assessments:** Students work is the most important source of information about the effectiveness of instruction. Teachers must analyze student productions, including assessments of all kinds, looking for patterns that will guide their efforts to assist specific students and the class as a whole and inform future instruction.

18. Providing oral and written feedback to students: Effective feedback helps focus students' attention on specific qualities of their work; it highlights areas needing improvement; and delineates ways to improve. Good feedback is specific, not overwhelming in scope, and focused on the academic task, and supports' perceptions of their own capability. Giving skillful feedback requires the teacher to make strategic choices about the frequency, method, and content of feedback and to communicate in ways that are understandable by students.

19. Analyzing instruction for the purpose of improving it: Learning to teach is an ongoing process that requires regular analysis of instruction and its effectiveness. Teachers study their own teaching and that of their colleagues in order to improve their understanding of the complex interactions between teachers, students, and content and of the impact of particular instructional approaches. Analyzing instruction may take place individually or collectively and involves identifying salient features of the instruction and making reasoned hypotheses for how to improve.

VITA

Jimmy Edward Miller was born in Athens, TN and raised in Madisonville, Tennessee. After graduating from Sequoyah High School he attended the University of Tennessee in Knoxville, Tennessee where he earned a Bachelor of Science in Mathematics in 2009, and a Master of Science in Mathematics Education in 2010 as part of a five year program. He immediately returned to graduate school in 2010 to pursue a graduate degree in mathematics, earning a Master of Science in Mathematics in 2013 and working as a graduate teaching associate teaching courses in mathematical reasoning, college algebra, and calculus. Jimmy then taught high school and dual enrollment mathematics classes for Monroe County Schools in Tennessee from 2013-2014. the fall of 2014 he accepted a position at Roane State Community College as an Instructor of Mathematics, and during spring 2015 accepted a position of Assistant Professor of Mathematics. During fall 2015 Jimmy began pursuing a Doctor of Philosophy in Mathematics Education at the University of Tennessee in Knoxville, Tennessee. As of spring 2020, Jimmy is in his sixth year as an Assistant Professor of Mathematics at Roane State Community College and was recommend for tenure and promotion to Associate Professor of Mathematics. He will serve as the chair of the mathematics department starting in fall 2020.