To the Graduate Council:

I am submitting herewith a thesis written by Mukta Lal Kar entitled "Analysis of Capacitorless Bandpass Filters." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

Roy D Joseph

Roy D. Joseph, Major Professor

We have read this thesis
and recommend its acceptance:

Eugene C Hutchmann

Lloyd W Crawford

H C Reddy

Accepted for the Council:

Hilton A Smith

Vice Chancellor
Graduate Studies and Research

6

Cop. 2

ANALYSIS OF CAPACITORLESS BANDPASS FILTERS

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Mukta Lal Kar

December 1976

1346215

ACKNOWLEDGEMENTS

The author wishes to express his deep appreciation to his work supervisor, Dr. E. C. Huebschman and to his academic advisor, Dr. Roy D. Joseph for their continuous encouragement and guidance. Thanks are due to Dr. K. C. Reddy and Dr. L. W. Crawford for their comments and suggestions from time to time. The author extends his thanks to Mrs. Nanette Lyons for typing this manuscript. Finally, the author acknowledges his wife's co-operation and encouragement during the period this work was performed.

ABSTRACT

In an effort to build bandpass filters using the latest LSI technology a number of capacitorless active circuits have been proposed. In each of these the actual design differs from the ideal bandpass in the sense that a numerator zero appears. This work studies the significance of this characteristic in such circuits and gives the conditions under which its effects can be neglected; specifically treated are the 3 dB bandwidth, as well as the positioning of the passband and the peak frequency.

TABLE OF CONTENTS

CHAPTER		PAGE
I.	INTRODUCTION	1
	State of the Art	1
	Modeling of Operational Amplifier	2
	Capacitorless Bandpass Filter	6
II.	DESCRIPTION AND ANALYSIS OF BANDPASS FUNCTIONS	12
	Ideal Bandpass Function	12
	Bandpass Function With a Finite Zero in the Numerator	15
III.	SUMMARY OF RESULTS	28
IV.	CONCLUSION	31
BIBLIOGRAPHY		33
APPENDIX		35
VITA		37

LIST OF FIGURES

FIGURE		PAGE
1.	Open Loop Response of a Typical Operational Amplifier	3
2.	Open Loop Response of An Internally Compensated 741 Amplifier	5
3.	Circuit for Voltage Transfer Function in Equation 4	8
4.	Circuits for Transfer Function in Equation 6	10
5.	Typical Magnitude Responses of Bandpass Functions	13
6.	Root-Locus Plot of Equation 22	17
7.	Magnitude Responses of Non-Ideal Bandpass Functions	25

NOMENCLATURE

a	Numerator zero for non-ideal bandpass function
b	Bandwidth of ideal bandpass function
f	Frequency
s	Complex frequency
A	Gain of amplifier
B	Bandwidth
G	Passband gain of bandpass function
G_m	Maximum passband gain
Q	Quality factor
T	Time constant
α	Gain adjustment
K, β, γ	Resistor ratios
δ, σ	Resistor ratios
ϕ	Square of the maximum passband gain
Ψ	An intermediate variable, $\Psi = (1 - a^2)/(1 + a^2)$
ω	Angular frequency
ω_m	The frequency at which G_m occurs
$\bar{\omega}$	Center of the passband
ω_1	Lower 3 dB frequency
ω_2	Upper 3 dB frequency

CHAPTER I

INTRODUCTION

I. STATE OF THE ART

During the last two decades solid state technology has progressed to the point where inductorless active filters have become a reality. Inductorless active filters ordinarily require monolithic operational amplifiers (integrated out of resistors, diodes and transistors) as well as external capacitors and resistors.

Using present LSI (Large Scale Integration) technology it is impossible to fabricate a capacitance of more than a few picofarads within a micro-space area. If any required capacitance can be simulated indirectly by associated passive and active devices, it will be possible to integrate an active filter section within a "TO" (typical round package style) or a "DIP" (Dual-in-line package style) pack. This effort will result in an appreciable reduction in cost per filter section through the use of large scale production techniques. In addition, less space will be required for a filter section and the total component count will go down, increasing system reliability.

A number of approaches have been reported in recent years for the elimination of capacitance. In most cases the limited bandwidth characteristics of active devices have been used. It is well known that the gain of a transistor decreases

with increasing signal frequency. Beyond a certain frequency its gain falls off. The utilization of this low-pass characteristic of a transistor in constructing a filter section was first discussed by Capparelli and Liberatore (1).[1] Similarly, the differential voltage gain of an operation amplifier is frequency dependent and its approximate mathematical characteristics have been worked out by a number of authors (2, 3). A single pole model (6 dB/octave fall-off outside passband) of the commerically available operational amplifier is used in most cases to avoid complexity in the design equations.

II. MODELING OF OPERATIONAL AMPLIFIER

In reference (2) Moschytz discusses the different two-port characteristics, the gain stability, and the sensitivity of the non-ideal operational amplifier both in inverting and non-inverting modes. The frequency dependent open-loop differential voltage gain can be approximated by a rational function with finite real poles as

$$A(s) = A_o \prod_{i=1}^{n} \frac{1}{s + \omega_i} \tag{1}$$

where A_o is the d.c. gain and $\omega_1 < \omega_2 < \ldots < \omega_n$. Also ω_n is less than or equal to unity gain bandwidth of the amplifier. The unity gain bandwidth is defined as the frequency at which the voltage gain crosses the 0 dB line (see in Figure 1). For

---

[1] Numbers in parentheses refer to similarly numbered references in the Bibliography.

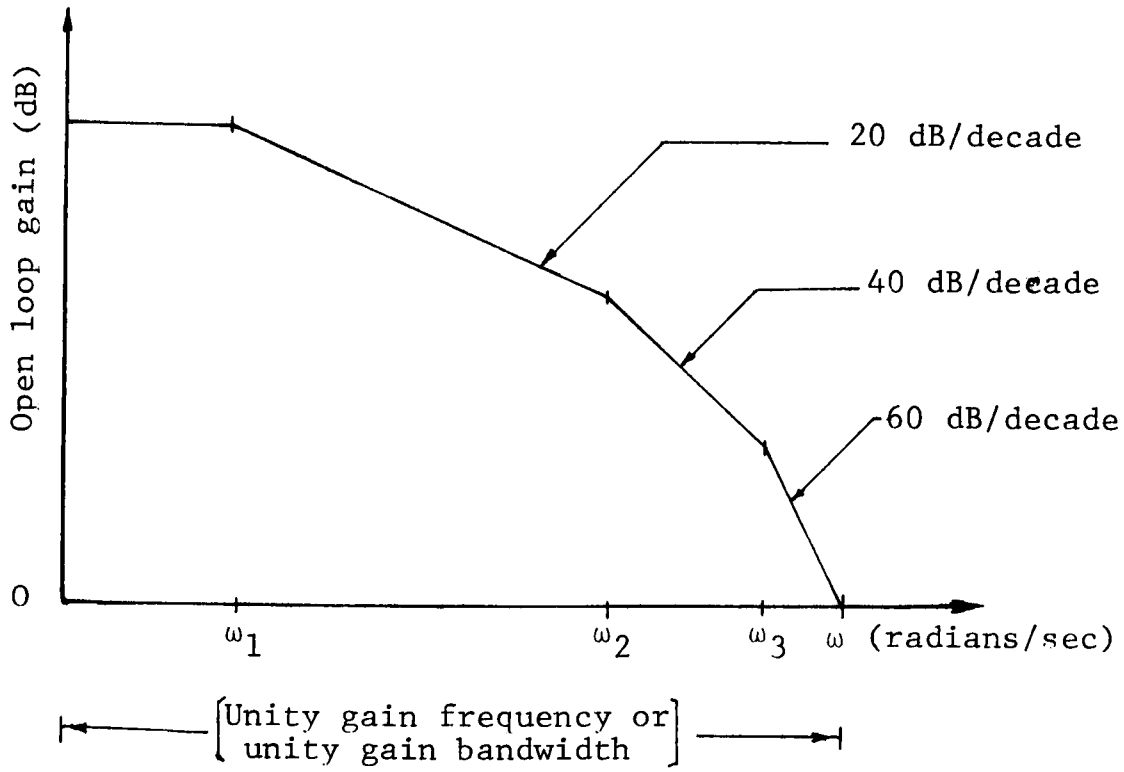


Figure 1. Open loop response of a typical operational amplifier.

a typical case A(s) can be written as

$$A(s) = \frac{A_o}{(s + \omega_1)(s + \omega_2)(s + \omega_3)} \qquad (2)$$

This representation gives a Bode plot of the form shown in Figure 1. Here ω_1, ω_2, and ω_3 are the corner frequencies in the band of interest.

In constructing active filters, the experimental design will approach the desired one more closely if more operational amplifier poles are taken into consideration in the design procedure. However, if one does not require the response in the higher frequency range, the poles at ω_2, ω_3, etc., can be neglected (3). In any operational amplifier the singularities away from the dominant one (nearest to origin) can be made negligible by employing a proper compensation scheme (2). Such compensation ensures a 6 dB/octave (20 dB/decade) roll-off rate for the frequency less than the unity gain cross-over frequency. By this we achieve greater stability as a closed-loop system component (4). There are two such compensation techniques for operational amplifiers - internal and external. Reference (5) studied the effect of internal compensation on the 741 operational amplifier. The use of a compensating capacitor of 30pF in an intermediate stage produces a large separation between the first and second most dominant poles.

The internally compensated 741 monolithic amplifier has the open-loop response shown in Figure 2. The single pole

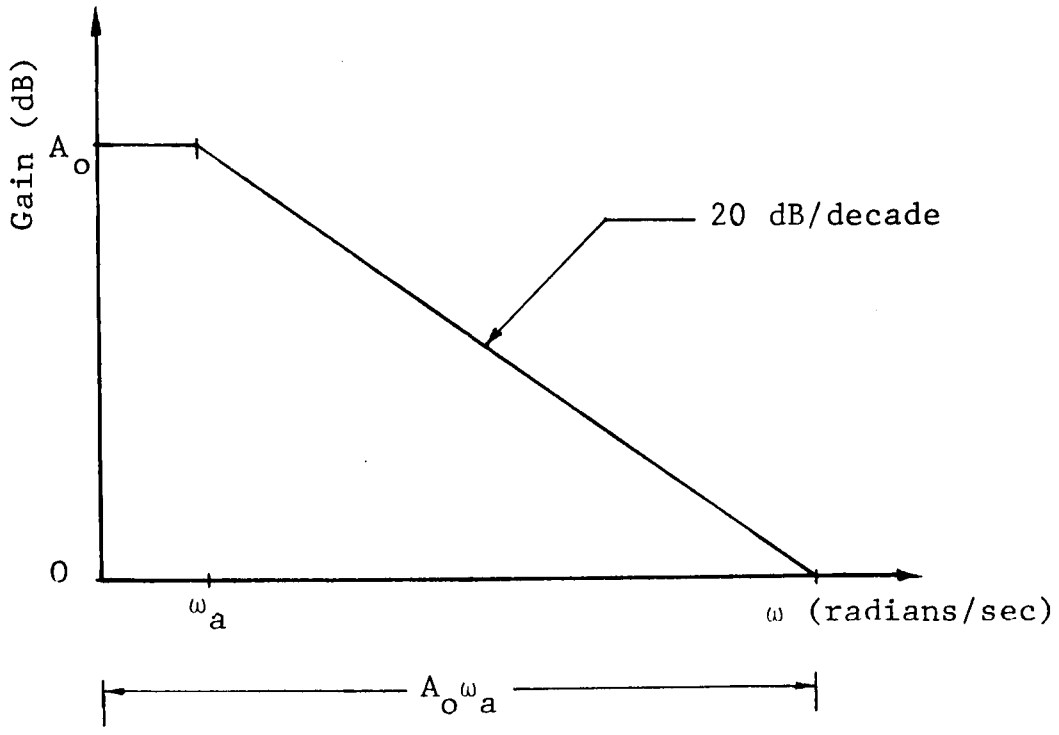


Figure 2. Open loop response of an internally compensated 741 amplifier.

model of such a 741 amplifier is approximated by

$$A(s) = - \frac{A_o \omega_a}{s + \omega_a} \qquad (3)$$

Here A_o is the open-loop d.c. gain of the amplifier and ω_a is the open-loop 3 dB bandwidth (frequency) in radians per second. For a compensated 741 $A_o \geq 50{,}000$ and $\omega_a \simeq 20$ rad/sec.

The product $A_o \omega_a$ is usually known as the gain-bandwidth product of an amplifier. This one pole model of an operational amplifier is used extensively, particularly in the area of active filters. By using this single pole model, complexity in design equations can be avoided without introducing any appreciable error at the lower frequencies.

III. CAPACITORLESS BANDPASS FILTER

The utilization of the operational amplifier pole to eliminate an external capacitor in the design of a second order bandpass function was first discussed by Rao and Srinivasan (7). Here the pole of an operational amplifier along with an external capacitor and a few resistors has been used to obtain a high Q (quality factor) bandpass function. For a particular center frequency, independent control of Q and G (midband gain) is achieved by variable resistors. Compensation schemes to protect against variations in center frequency due to changes in bias voltage and temperature have also been suggested. With this circuit it is difficult to tune the center frequency

because it is necessary to either vary four resistors simultaneously or keep their ratios constant, in pairs. Although detailed experimental information is lacking, this work first demonstrated that an active filter design might be possible without using any external capacitors.

Proceeding in the same way, Rao and Srinivasan (8) later designed the capacitorless bandpass section shown in Figure 3. The voltage transfer function for this circuit is

$$\frac{V_2(s)}{V_1(s)} = \frac{\left(1 + \frac{K}{A_o}\right) + s\left(\frac{TK}{A_o}\right)}{s^2\left(\frac{T^2K}{A_o^2}\right) + s\left(\frac{T}{A_o}\left(1 + \frac{2K}{A_o}\right)\right) + 1} \tag{4}$$

where amplifier gain $A = A_o/(1 + sT)$ and $K = 1 + (R_b/R_a)$. For $(K/A_o) << 1$ Equation 4 reduces to

$$\frac{V_2(s)}{V_1(s)} = \frac{1 + s\left(\frac{TK}{A_o}\right)}{s^2\left(\frac{T^2K}{A_o^2}\right) + s\left(\frac{T}{A_o}\right) + 1}. \tag{5}$$

Compared to their previous circuit using an amplifier and one external capacitor, this design provides lower sensitivity figures with respect to bias voltage and temperature. The quality factor Q and the midband gain are also made insensitive to variations in temperature and bias voltage. This particular design does not allow an independent tuning capability for center frequency and quality factor Q. The

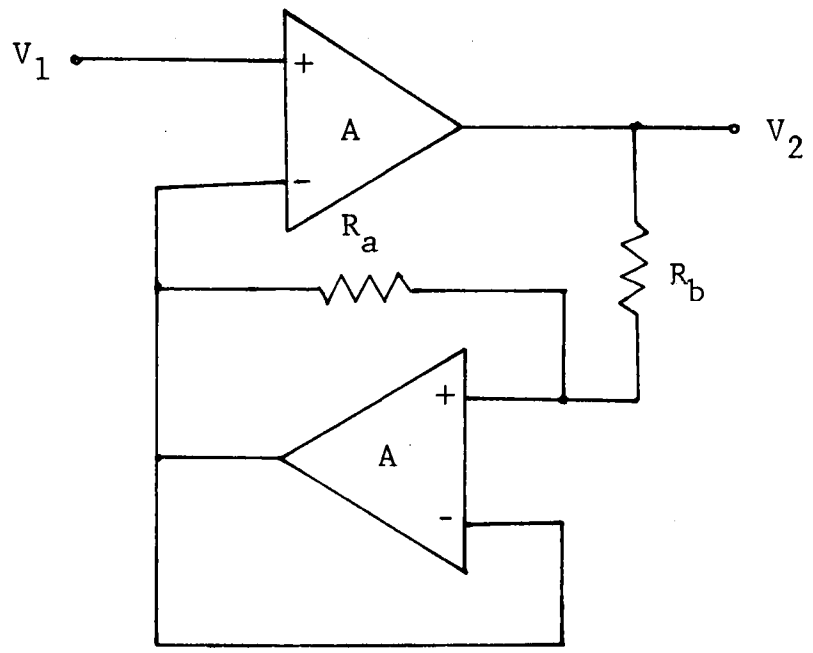


Figure 3. Circuit for voltage transfer function in Equation 4.

possibility of realizing a capacitorless lowpass section was also discussed.

In 1975, R. Schaumann (9) presented a tunable active bandpass section without any external capacitor. He used two amplifier blocks - one in the forward and one in the feedback paths of the second order system; they are shown in Figure 4. The overall system transfer function is

$$T_B(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{K_1}{\alpha T_1}\left(s + \frac{1}{T_2}\right)}{s^2 + s\left(\frac{1}{T_1} + \frac{1}{T_2}\right) + \frac{1 + K_1K_2}{T_1T_2}} \tag{6}$$

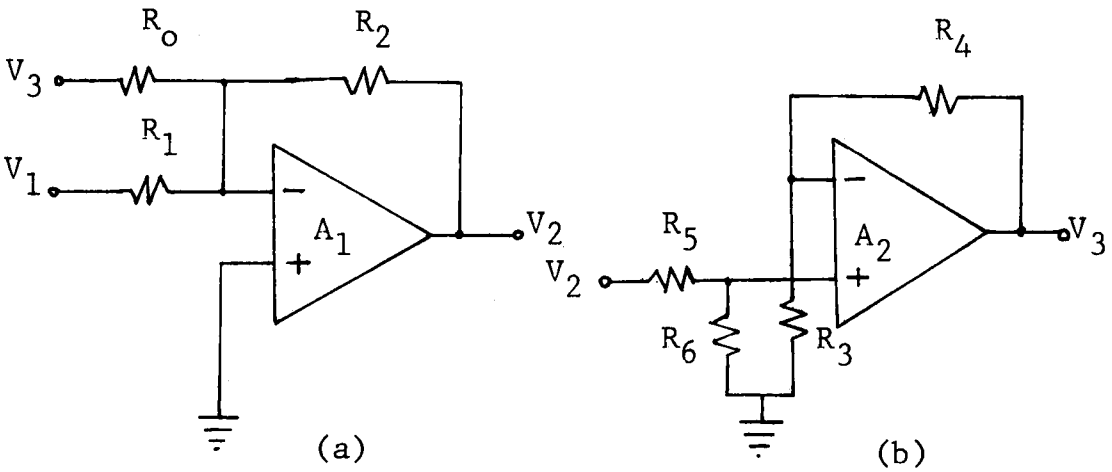
$$\text{with } A_i(s) = \frac{A_{oi}}{1 + sT_{oi}}, \quad i = 1, 2, \tag{7a}$$

$$K_1 = \frac{\gamma}{1 + \frac{\delta}{A_{o1}}}; \quad T_1 = \frac{T_{o1}}{1 + \frac{A_{o1}}{\delta}} \tag{7b, c}$$

$$\alpha = \frac{R_o}{R_1}; \quad \gamma = \frac{R_2}{R_o}; \quad \delta = 1 + \gamma + \frac{R_2}{R_1} \tag{8a, b, c}$$

$$K_2 = \frac{\beta\sigma}{1 + \sigma/A_{o2}}; \quad T_2 = \frac{T_{o2}}{1 + \frac{A_{o2}}{\sigma}}; \quad \sigma = 1 + \frac{R_4}{R_3}. \tag{9a, b, c}$$

The sensitivities of center frequency f_o and quality factor Q with respect to external passive resistors are very small. For medium to high Q operation, stability of center frequency with respect to bias voltage and temperature variations can be achieved through proper compensation techniques (using devices



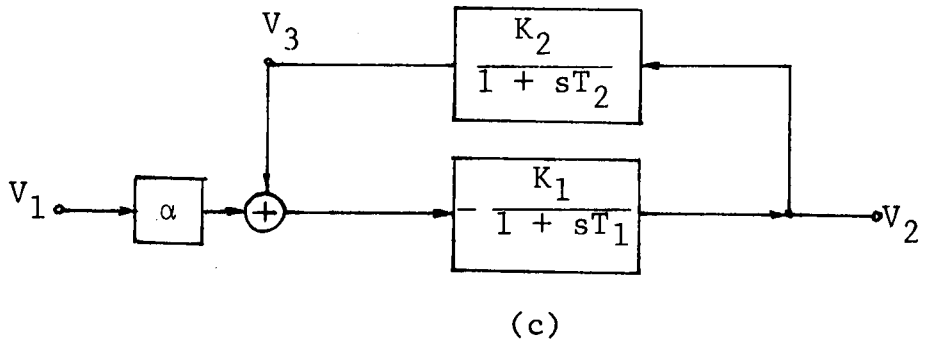


Figure 4. Circuits for transfer function in Equation 6.

like JFET or thermistor). One of the remarkable advantages of this circuit is that the center frequency can be set to any value over a wide frequency range with a desired Q value. The center frequency is set by using a variable resistor in place of R_6 and desired Q value can be obtained by adjusting R_o. One of the shortcomings of this circuit is that at high Q values, the center frequency deviates from the expected one by an appreciable percentage. As a result, the pass-band, obtained from the practically implemented circuit deviates from the theoretically designed one.

If we observe the voltage transfer functions in Equations 5 and 6 carefully we notice that a finite zero appears in the numerator of both equations. The zero should be at the origin for an ideal bandpass function. The effect of a finite zero may be negligible in some cases depending on its closeness to origin. However its effect on the bandwidth, position of passband, midband gain and ω_m (the frequency at which peak occurs in a bandpass function) can not be neglected. One must study these effects critically before designing a capacitorless active bandpass filter section. The work reported here is the detailed analysis of such a "bandpass" function with a finite zero in the numerator.

CHAPTER II

DESCRIPTION AND ANALYSIS OF BANDPASS FUNCTIONS

I. IDEAL BANDPASS FUNCTION

An ideal bandpass function is written as

$$\frac{V_2(s)}{V_1(s)} = G(s) = \frac{\omega_o s}{s^2 + \frac{\omega_o s}{Q} + \omega_o^2} \tag{10}$$

where ω_o is the frequency at which the gain of Equation 10 is maximum and Q is the quality factor related to the selectivity or fall off rate near ω_o.

If ω_o is normalized to unity ($s \rightarrow s/\omega_o$) Equation 10 can be written as

$$G(s) = \frac{s}{s^2 + bs + 1} \tag{11}$$

where for convenience we let $b = \frac{1}{Q}$. A typical plot of Equation 11 showing the variation in magnitude with increasing frequency is given in Figure 5. Since G(s) must be stable and does not have negative distinct poles (and so not realizable by an RC network alone) we restrict b to the values $0 < b < 2$. To find the 3 dB bandwidth of the bandpass function in Equation 11 consider the square of the magnitude of G(s) on j-axis

$$|G(j\omega)|^2 = \frac{\omega^2}{(1 - \omega^2)^2 + b^2\omega^2} \tag{12}$$

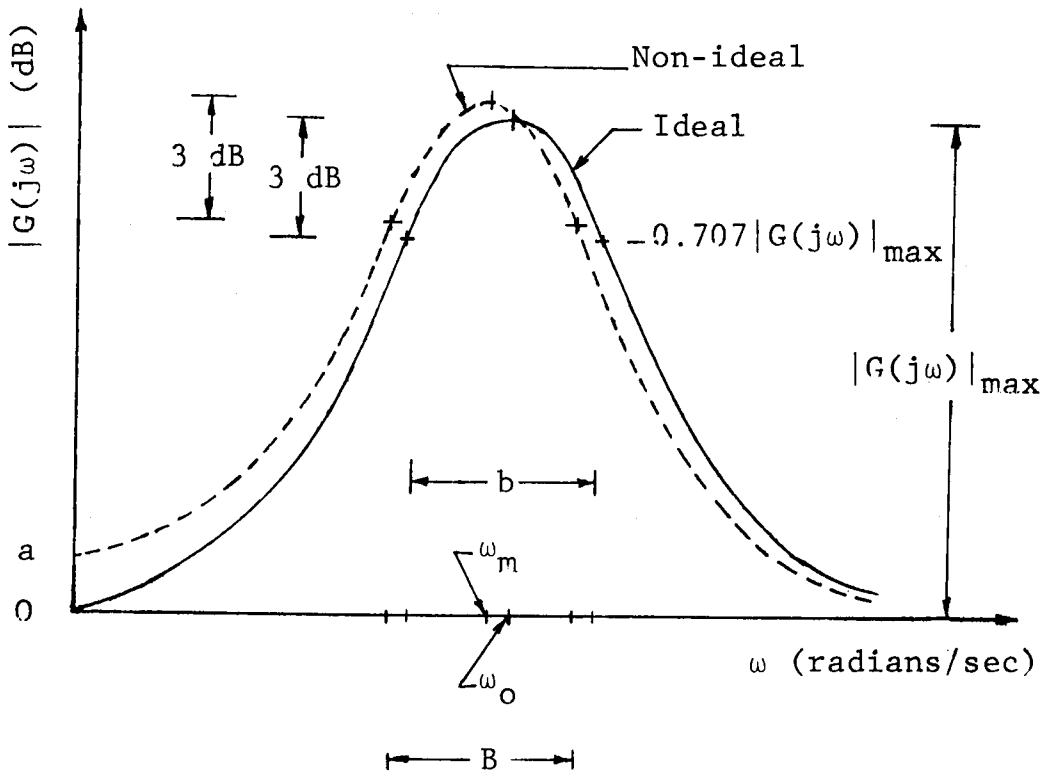


Figure 5. Typical magnitude responses of bandpass functions.

It is easy to show that $|G(j\omega)|^2$ is maximum at $\omega = 1$ and that $|G(j\omega)|^2_{max} = |G(j)|^2 \simeq 1/b^2$. To calculate the 3 dB (half power) frequencies we use

$$\frac{\omega^2}{(1 - \omega^2)^2 + b^2\omega^2} = \frac{|G(j)|^2}{2} = \frac{1}{2b^2} \tag{13}$$

Rewriting we get

$$\omega^4 - \omega^2(2 + b^2) + 1 = 0. \tag{14}$$

Equation 14 has the solution

$$\omega^2 = \frac{b^2 \pm b(b^2 + 4)^{\frac{1}{2}}}{2} + 1. \tag{15}$$

The desired positive roots of Equation 15 are

$$\omega_{1,2} = \frac{\mp b + (b^2 + 4)^{\frac{1}{2}}}{2} \tag{16a, b}$$

The 3 dB bandwidth, B, is calculated as

$$B = \omega_2 - \omega_1 = b = \frac{1}{Q}. \tag{17}$$

Note that $\omega_2\omega_1 = 1 = \omega_o^2$, so ω_o is the geometric mean of the 3 dB frequencies. ω_o is not the arithemtic mean, $\bar{\omega}$, of ω_1 and ω_2 unless $b^2 << 4$. Then $\bar{\omega} = (b^2 + 4)^{\frac{1}{2}}/2 \simeq 1$. In this case the passband is centered at $\omega_m = 1$ and is not skewed toward lower 3 dB frequency (the frequency at which $|G(j\omega)|_{max}$ occurs is taken as ω_m in this work). It is found that the passband is effectively centered at $\omega_m = 1$ (within 1% when $b < 0.2$ or equivalently $Q > 5$). It is clear that we should not consider $b > 0.2$ if we desire an ideal bandpass characteristic which is centered at $\omega_m = 1$.

II. BANDPASS FUNCTION WITH A FINITE ZERO IN THE NUMERATOR

A normalized "bandpass" transfer function with a finite zero in the numerator may be expressed as

$$G(s) = \frac{s + a}{s^2 + bs + 1}; \quad a \geq 0. \tag{18}$$

The magnitude of this function is also shown in Figure 5 and can be expressed as

$$|G(j\omega)|^2 = \frac{\omega^2 + a^2}{(1 - \omega^2)^2 + b^2\omega^2}. \tag{19}$$

We have omitted any gain constant in Equation 18 since it only effects magnitude of peak, but not position of peak or passband. For the sake of simplicity we let $\omega^2 = x$ so for real ω, $x \geq 0$. Equation 19 then becomes

$$|G(j\omega)|^2 = \frac{x + a^2}{(1 - x)^2 + b^2x}. \tag{20}$$

To find the maximum value of $|G(j\omega)|^2$ we set

$$\frac{d|G(j\omega)|^2}{dx} = 0$$

which after algebraic manipulation gives

$$x^2 + 2a^2x - a^2(2 - b^2) - 1 = 0 \tag{21}$$

or equivalently

$$1 + \frac{2a^2\left(x + \frac{b^2 - 2}{2}\right)}{x^2 - 1} = 0 \qquad (22)$$

or

$$1 + \frac{2a^2(x - x_o)}{x^2 - 1} = 0.$$

In order to find the roots x_m of Equation 21 look at the root-locus of Equation 22 in terms of the parameter $2a^2$ as shown in Figure 6. As long as $b^2 \leq 2$ one root is greater than or equal to zero, so Equation 20 has a peak for all values of "a". For the case $b^2 > 2$, making x_o negative, the root locus requires that $2a^2 < 2/(b^2 - 2)$ if Equation 21 is to have a positive root and $|G(j\omega)|^2$ a peak. Since this value of b corresponds to $Q < 0.707$ it is of little practical importance and will not be considered further.

To calculate the frequency at which the maximum of $|G(j\omega)|^2$ (i.e. peak) occurs, we solve Equation 21 for its positive root and get

$$x_m = \omega_m^2 = -a^2 + ((a^2 + 1)^2 - a^2b^2)^{\frac{1}{2}}. \qquad (23)$$

Inspection of root-locus shows that $\omega_m^2 \leq 1$. Insertion of this value of x_m in Equation 20 gives

$$|G(j\omega)|^2_{max} = \frac{1}{2(\omega_m^2 - 1) + b^2}. \qquad (24)$$

The 3 dB (or half power) frequencies can be obtained by writing

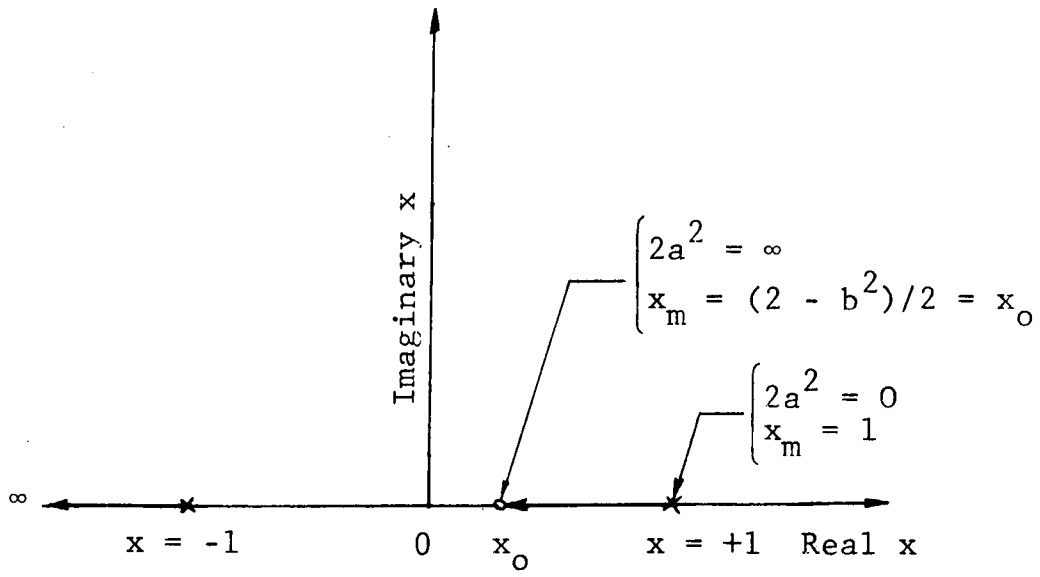


Figure 6. Root-locus plot of Equation 22.

$$|G(j\omega)|^2 = \frac{|G(j\omega)|^2_{max}}{2}$$

or

$$\frac{x + a^2}{(1 - x)^2 + b^2 x} = \frac{\phi}{2} \tag{25}$$

where $|G(j\omega)|^2_{max} = G_m^{\ 2} = \phi$.

The solution of Equation 25 is given by

$$x_{1,2} = \frac{-(b^2\phi - 2\phi - 2) \mp ((b^2\phi - 2\phi - 2)^2 - 4\phi(\phi - 2a^2))^{\frac{1}{2}}}{2\phi}. \tag{26}$$

The 3 dB frequency points ω_1 and ω_2 are

$$\omega_1 = + (x_1)^{\frac{1}{2}} \tag{27a}$$

$$\omega_2 = + (x_2)^{\frac{1}{2}}. \tag{27b}$$

For illustrative purposes and to verify later results, the exact values of ω_1, ω_2, and ω_m have been calculated using Equations, 23, 26, 27a, and 27b for a set of representative values of a and b. These are shown in Table I in the appendix.

From Figure 5 if is clear that certain values of a will yield only one 3 dB point. To ensure two 3 dB (half power) points we require that

$$\frac{G_m^{\ 2}}{2} \geq |G(0)|^2 = a^2 \tag{28}$$

Using Equation 24 for G_m^2 this yields

$$2a^2(2(\omega_m^2 - 1) + b^2) \leq 1. \tag{29}$$

Since $\omega_m^2 \leq 1$ Equation 29, and thus Equation 28, are satisfied if

$$a^2b^2 \leq 1/2 \tag{30}$$

Thus, we shall assume that $ab < (1/2)^{\frac{1}{2}}$ so two 3 dB points are certain to exist.

Consider Equation 23 for ω_m^2 rewritten as

$$\omega_m^2 = - a^2 + (a^2 + 1)\left(1 - \frac{b^2}{\left(a + \frac{1}{a}\right)^2}\right)^{\frac{1}{2}} \tag{31}$$

Note that since $a + \frac{1}{a} > 2$, then

$$\frac{b^2}{\left(a + \frac{1}{a}\right)^2} < \frac{b^2}{4}.$$

Thus if $\frac{b^2}{4} << 1$ so an ideal passband would be centered then

$$\omega_m^2 = - a^2 + (a^2 + 1) = 1. \tag{32}$$

This result can be verified by inspecting ω_m in Table I in the appendix. Therefore if $b < 0.2$ we may use $\omega_m^2 = 1$ when estimating G_m^2. After estimating G_m^2 the 3 dB points can be obtained from

$$|G(j\omega)|^2 = \frac{G_m^2}{2}. \tag{33}$$

Inserting $\omega_m^2 = 1$ in Equation 19 gives

$$G_m^2 = \frac{1 + a^2}{b^2}. \tag{34}$$

On the other hand, Equation 24 gives

$$G_m^2 = \frac{1}{b^2} \tag{35}$$

Since we have two approximations for G_m^2 (Equations 34 and 35) it first must be determined which provides a better estimate of G_m^2. To determine this, we calculate the square of the error resulting from each approximation

$$A_1 = \left(\frac{1}{b^2} - G_m^2\right)^2 \tag{36}$$

$$B_1 = \left(\frac{1 + a^2}{b^2} - G_m^2\right)^2. \tag{37}$$

To ascertain which of the above squared error terms in greater we calculate the difference between A_1 and B_1 using the exact value of G_m^2 from Equation 24

$$B_1 - A_1 = \frac{a^2}{b^4}\left(\frac{2(2 + a^2)(x_m - 1) + a^2b^2}{2(x_m - 1) + b^2}\right) \tag{38}$$

The denominator of Equation 38 is always positive since it is

the inverse of a squared magnitude (see Equation 24). B_1 will be less than A_1 if

$$x_m - 1 + \frac{a^2b^2}{2(2 + a^2)} < 0. \tag{39}$$

Inserting the value of x_m from Equation 23 and realizing that $b^2 \leq 2$ if a peak is to exist for all values of a Equation 39 becomes

$$a^2(b^2 + 4) > -8 \tag{40}$$

which is clearly satisfied for all values of a. Thus, for reasonable values of Q($Q > 5$) $B_1 < A_1$ easily, independent of a.

Since $B_1 < A_1$, $(1 + a^2)/b^2$ is used for G_m^2 to give a minimum error. Thus Equation 33 is written as

$$\frac{\omega^2 + a^2}{(1 - \omega^2)^2 + b^2\omega^2} = \frac{1 + a^2}{2b^2} \tag{41}$$

which simplifies to

$$(1 - a^2)\omega^2 + 2a^2 = \frac{1 + a^2}{b^2}(\omega^2 - 1)^2 \tag{42}$$

Completing the square in $(\omega^2 - 1)$ gives

$$(\omega^2 - 1)^2 - b^2\left[\frac{1 - a^2}{1 + a^2}\right](\omega^2 - 1) - b^2 = 0 \tag{43}$$

Solving for ω^2 yields

$$\omega^2 = \frac{b^2\Psi \pm b(b^2\Psi + 4)^{\frac{1}{2}}}{2} + 1 \tag{44}$$

where

$$\Psi = \frac{1 - a^2}{1 + a^2} \tag{45}$$

Note that for $0 < a < \infty$ the value of Ψ lies in the interval $-1 < \Psi < 1$. From Equation 44 we see that the 3 dB frequencies satisfy

$$\omega_2^2 = \frac{b^2\Psi + b(b^2\Psi^2 + 4)^{\frac{1}{2}}}{2} \tag{46a}$$

$$\omega_1^2 = \frac{b^2\Psi - b(b^2\Psi^2 + 4)^{\frac{1}{2}}}{2} \tag{46b}$$

The 3 dB bandwidth (B) in the square form can be deduced from Equation 46a and 46b as

$$B^2 = (\omega_2 - \omega_1)^2 = \omega_2^2 + \omega_1^2 - 2\omega_1\omega_2$$

$$= b^2\Psi + 2 - 2(1 + (\Psi - 1)b^2)^{\frac{1}{2}} \tag{47}$$

We see that for $a = 0$, that is, $\Psi = 1$ (implying the case of ideal bandpass function) Equation 47 yields

$$B^2 = b^2 \tag{48}$$

as we would expect from Equation 17. In Equation 47 if we let

$$(1 + (\Psi - 1)b^2)^{\frac{1}{2}} \simeq 1 + \frac{1}{2}(\Psi - 1)b^2 - \frac{1}{8}(\Psi - 1)^2 b^4$$

by using the first 3 terms of the Taylor Series, Equation 47 becomes

$$B^2 = b^2 + \frac{1}{4}(\Psi - 1)^2 b^4$$

or

$$B = b\left[1 + \frac{(\Psi - 1)^2 b^2}{4}\right]^{\frac{1}{2}} \tag{49}$$

The Taylor series is valid since $b < 0.2$ ($Q > 5$). The effect of a non zero "a" on bandwidth can be easily determined from Equation 49. The additional term $(\Psi - 1)^2 b^2/4$ appears due to this finite zero. Also notice that for $\Psi \neq 1$ the bandwidth is larger than the ideal situation ($B > b$).

The largest deviation from the ideal bandwidth occurs when $b = 0.2$ (smallest Q) and $\Psi = -1$ ($a \to \infty$). Inserting these values of b and A in Equation 49 we get $B = 0.20396$. Error $= (B - b)/b = 1.98\%$. To limit the bandwidth error to 1% we require

$$\frac{B - b}{b} = (1 + \frac{1}{4}(\Psi - 1)^2 b^2)^{\frac{1}{2}} - 1 < 0.005$$

This inequality reduces to

$$(\Psi - 1)^2 < 1 \tag{50}$$

Inequality 50 is true for $0 < \Psi < 2$, but in our particular case Ψ takes on values only between -1 and +1. Thus error in B will be less than 1% for $0 < \Psi < 1$ or equivalently when $0 < a^2 < 1$.

The center of the passband ($\bar{\omega}$) in square form can be written as

$$\bar{\omega}^2 = \left(\frac{\omega_2 + \omega_1}{2}\right)^2 = \frac{\omega_2^2}{4} + \frac{\omega_1^2}{4} + \frac{\omega_1\omega_2}{2} \tag{51}$$

Insertion of ω_1 and ω_2, from Equations 46a and 46b respectively, yields

$$\bar{\omega}^2 = \frac{1}{4}(b^2\Psi + 2 + 2(1 + b^2(\Psi - 1))^{\frac{1}{2}}) \tag{52}$$

If we approximate $(1 + b^2(\Psi - 1))^{\frac{1}{2}}$ by $1 + b^2(\Psi - 1)/2$ Equation 52 becomes

$$\bar{\omega}^2 = 1 + (b^2/4)(2\Psi - 1) \tag{53}$$

Equation 53 shows us an important effect of "a" on the passband. The non ideal center of passband is greater than 1 for $\frac{1}{2} < \Psi < 1$. This implies that passband shifts to the right of the ideal one for $a < (1/3)^{\frac{1}{2}} \simeq 0.58$; for $-1 < \Psi < \frac{1}{2}$, that is, $a > (1/3)^{\frac{1}{2}}$, the passband shifts to the left. These are shown pictorially in Figure 7. It is interesting to note that to center the passband exactly at 1 we should select $\Psi = \frac{1}{2}$ or $a = (1/3)^{\frac{1}{2}}$ and not $a = 0$ (as might be expected). This is so because even an ideal bandpass is centered at 1 only if $b^2 \to 0$.

If the center of the passband $\bar{\omega}$ is to be within 1% of the peak frequency $\omega_m = 1$ we must have

$$(\bar{\omega} - 1)^2 < (0.005)^2 \tag{54}$$

Using a three term Taylor series expansion for $\bar{\omega}$ (from Equation

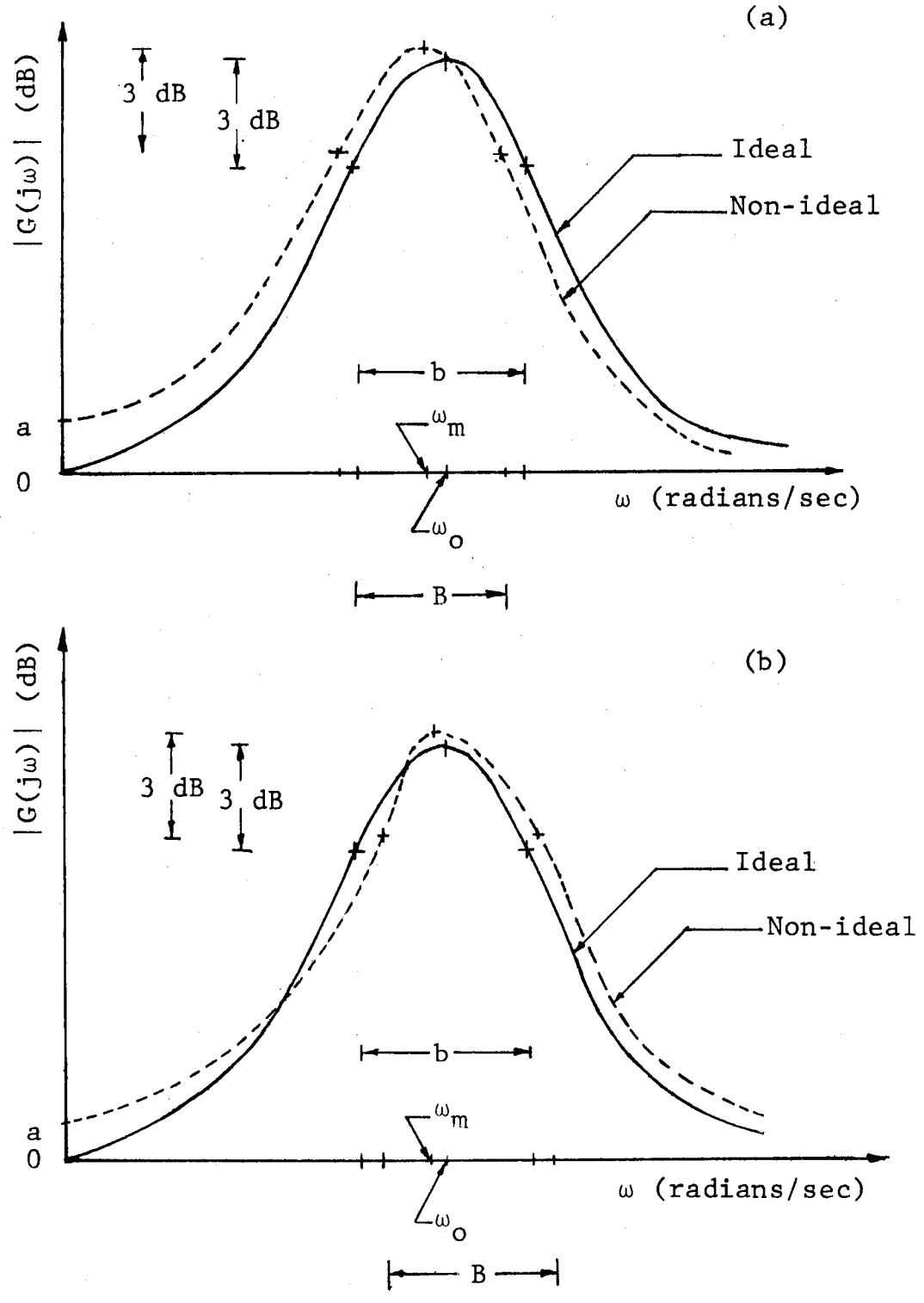


Figure 7. Magnitude responses of non-ideal bandpass functions. (a) $a > 0.577$ where $\bar{\omega} < 1$; (b) $a < 0.577$ where $\bar{\omega} > 1$.

53) this results in

$$(2\Psi - 1)^2 < \frac{16(10)^{-4}}{b^4} \tag{55}$$

Since $b \leq 0.2$ this inequality is satisfied when

$$(2\Psi - 1)^2 < 1 \tag{56}$$

Thus, the center of the passband is within 1% of ω_m when $0 < \Psi < 1$ or equivalently when $0 < a^2 < 1$. Note that the largest error in $\bar{\omega}$ occurs when $b = 0.2$ and $\Psi = -1$. In that case error is approximately 1.51%

To check the validity of the approximations used representative calculations are made using the simplified formulas (Equations 49 and 53). The 3 dB points are found from B and ω using,

$$\omega_1 = \bar{\omega} - \frac{B}{2} \tag{57a}$$

$$\omega_2 = \bar{\omega} + \frac{B}{2} \tag{57b}$$

These results are given in Table I in the appendix. Comparisons of the respective values in the tables indicates that there is no significant difference between the two results (one obtained without using any approximations and other using approximations.)

In our study of the nonideal bandpass function we have determined that its magnitude characteristic will have two 3 dB points if $a^2b^2 < \frac{1}{2}$. Assuming $Q > 5$ both the bandwidth

B and the center frequency $\bar{\omega}$ will be within 1% of the desired b and 1, respectively, as long as $0 < a < 1$; for $0 < a < 3^{-\frac{1}{2}}$ $\bar{\omega}$ is right of 1 while for $a > 3^{-\frac{1}{2}}$, $\bar{\omega}$ in left of 1. In the larger interval $0 < a < \infty$ the bandwidth B will be within 2% of b while the center frequency $\bar{\omega}$ will differ from 1 by not more than 1.52%.

CHAPTER III

SUMMARY OF RESULTS

The preceding work analyzed both ideal and non-ideal (with a finite zero in the numerator) bandpass functions. This work has been carried out to increase our understanding of the important parameters involved in the design of complex filters. This knowledge will be most useful when constructing a filter section which takes advantage of the latest technological developments. These designs may take the form of a capacitor-less active filter as introduced by either Rao and Srinivasan (8) or Schaumann (9). In both bandpass realizations a finite zero appears in the numerator.

From the analysis of the ideal bandpass function we know that the peak frequency ω_m is always the geometric mean of the two 3 dB frequencies. ω_m approaches the arithmetic mean for the same two frequencies if $b^2 << 4$. Calculation shows that if $b < 0.2$, that is $Q > 5$, the ideal bandpass characteristic will be centered within 1% of ω_m. Because of this it is assumed that the ideal bandpass characteristics desired are centered at the peak frequency ($Q > 5$).

For the non-ideal case the effects of the finite zero on 3 dB bandwidth, shift of ω_m from ideal peak frequency ($\omega_o = 1$), and deviation or shift of passband from the ideal have been studied. The effect on the passband due to a finite zero could be a serious one if the value of Q is very high when the passband center frequency is high.

Let us investigate the effects of the finite zero "a" in particular when the designs of Rao and Srinivasan (8), and Schaumann (9) are used. In the first case the voltage transfer function of Equation 5 can be rewritten as

$$\frac{K^{\frac{1}{2}}(1/K^{\frac{1}{2}} + s_1)}{s_1^2 + s_1/K^{\frac{1}{2}} + 1} = \frac{G_o(s_1 + a)}{s_1^2 + bs_1 + 1} \tag{58}$$

where the normalized frequency variable

$$s_1 = \frac{s}{(A_o/TK^{\frac{1}{2}})} \quad \text{and } G_o = \text{Gain constant}$$

Comparing Equation 58 with Equation 18 we have

$$a = \frac{1}{K^{\frac{1}{2}}}$$

$$b = \frac{1}{K^{\frac{1}{2}}}$$

where $K = 1 + R_a/R_b$. The desired $Q = 1/b = K^{\frac{1}{2}}$, so in the design $a = 1/Q$ also. To ensure two 3 dB points Equation 30 requires that $Q > 1.19 (1/Q^4 < 1/2)$ in this case. To get B and $\bar{\omega}$ within 1% of desired values we need $0 < a < 1$; in this design this means $0 < 1/Q < 1$ or $Q > 1$. Thus for any reasonable Q value (say $Q > 5$) design gives two 3 dB points and results in a center frequency and bandwidth within 1% of those desired.

For the Schaumann (9) circuit we normalize frequency to $s_1 \rightarrow s/((1 + K_1K_2)/T_1T_2)^{\frac{1}{2}}$ in Equation 6 in a similar way and obtain an equivalent "a" as

$$a = \frac{1}{T_2} \cdot \frac{1}{\omega_o^{\frac{1}{2}}} = \frac{1}{(1 + K_1K_2)^{\frac{1}{2}}} = \frac{1}{2Q}$$

assuming the amplifiers are alike. Equation 30 then results in $Q > 1/2^{\frac{1}{4}} = 0.84$ to insure two 3 dB points. For all practical purposes Q is greater than 5 for bandpass filters, making a very small. The effects are negligible in this case also.

CHAPTER IV

CONCLUSION

From our investigation of the bandpass designs of Rao and Srinivasan, and Schaumann it is clear that their designs give accpetable bandpass filters for high Q applications. Only slight changes in peak frequency, bandwidth and passband position are experienced if $Q > 5$.

Lower Q designs may be necessary when making a cascade realization of higher order functions. In these cases it is necessary to verify that $a^2b^2 < \frac{1}{2}$ and $a^2 < 1$ so a true bandpass function is achieved and the passband and 3 dB bandwidth are not changed more than 1%. Since a depends on the Q and f_o of the desired bandpass function its value can only be checked after the particular relization technique is selected.

It should also be mentioned that there is a limitation in selecting the center frequency of the bandpass in the low and high frequency range. This can be seen by considering the following equations from reference (9)

$$f_o = \left(\beta f_{c_1} f_{c_2}\right)^{\frac{1}{2}} \tag{59}$$

$$Q = (\beta\gamma\delta)^{\frac{1}{2}}\left(\left(\frac{f_{c_1}}{f_{c_2}}\right)^{\frac{1}{2}} + \frac{\delta}{\sigma}\left(\frac{f_{c_2}}{f_{c_1}}\right)^{\frac{1}{2}}\right)^{-1} \tag{60}$$

where f_o is the center frequency, $0 < \beta < 1 (\beta = R_6/(R_5 + R_6))$, f_{c_1} and f_{c_2} are the gain-bandwidth products of the amplifiers,

and γ, δ and σ are given in Equations 8b, c and 9c. If the amplifiers are the same, Equations 59 and 60 reduce to

$$f_o = \beta^{\frac{1}{2}} f_c \tag{61}$$

$$Q = (\beta\gamma\delta)^{\frac{1}{2}}(1 + \delta/\sigma)^{-1} \tag{62}$$

From Equations 61 and 62 we observe that if f_o is selected very low by fixing β, then Q of the circuit will be low also. Again from Equation 61, $f_{o\ max} \leq f_c$, that is, f_o can be selected only as large as the gain-bandwidth product of the amplifier.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Capparelli, F., and Liberatore, A., "Active Bandpass Network with only Resistors and Passive Elements", Electronics Letters, Vol. 8, No. 2, pp. 43-44, 27th January, 1972.

2. Moschytz, G. S., "The Operational Amplifier in Linear Active Networks", IEEE Spectrum, Vol. 7, pp. 42-50, January, 1970.

3. Mack, R. J., and Fiddler, J. D., "Simple 2nd - Order Amplifier Model for Differential and Common-mode Gain", Electronics Letters, Vol. 12, No. 6, pp. 136-137, March 18, 1976.

4. Mitra, S. K., Analysis and Synthesis of Linear Active Networks, pp. 456-462, John Wiley, New York, 1969.

5. Wooley, Bruce A., Wong, Sing-Yul J., and Pederson, Donald O., "A Computer Aided Evaluation of the 741 Amplifier", IEEE Journal of Solid State Circuits, Vol. SC-6, No. 6, pp. 357-361, December , 1971.

6. Budak, Aram, and Petrela, Dashamir M., "Frequency Limitations of Active Filters Using Operational Amplifier", IEEE Transactions on Circuit Theory, Vol. CT-19, No. 4, pp. 322-328, July 1972.

7. Rao, Radhakrishna K., and Srinivasan, S., "A Bandpass Filter Using the Operational Pole", IEEE Journal of Solid-State Curcuits, pp. 245-246, June 1973.

8. Rao, Radhakrishna K., and Srinivasan, S., "Low-Sensitivity Active Filters Using the Operational Amplifier Pole", IEEE Transactions on Circuits and Systems, Vol. CAS-21, No. 2, pp. 260-262, March, 1974.

9. Schaumann, Rolf, "Low-Sensitivity High-Frequency Tunable Active Filter Without External Capacitors" IEEE Transactions on Circuits and Systems, Vol. CAS-22, No. 1, pp. 39-44, January, 1975.

APPENDIX

APPENDIX

REPRESENTATIVE VALUES OF PEAK AND 3 dB FREQUENCIES

TABLE I

PEAK(ω_m) AND 3 dB FREQUENCIES(ω_1,ω_2) FOR A NON-IDEAL BANDPASS FUNCTION

a	b	ω_m	Lower 3 dB frequency		Upper 3 dB frequency	
			ω_1*	ω_1**	ω_2*	ω_2**
0.2	0.2	0.999615	0.904220	0.904220	1.104223	1.104225
0.2	0.1	0.999915	0.951057	0.951057	1.051057	1.051057
0.2	0.03	0.999991	0.985095	0.985095	1.015095	1.015095
0.2	0.02	0.999996	0.990042	0.990042	1.010042	1.010042
0.2	0.01	0.999999	0.995011	0.995011	1.005010	1.005010
1	0.2	0.994975	0.894680	0.894488	1.095192	1.095486
1	0.1	0.998748	0.948715	0.948687	1.048778	1.048812
1	0.03	0.999888	0.984887	0.984886	1.014888	1.014889
1	0.02	0.999950	0.989950	0.989949	1.009950	1.009950
1	0.01	0.999988	0.994987	0.994987	1.004988	1.004988

a - Magnitude of finite zero

b - Bandwidth of ideal bandpass function

ω_m - Frequency at which $|G(j\omega)|_{max}$ occurs

* - Exact (i.e. using Equations 27a and 27b)

** - Approximated (i.e. using Equations 57a and 57b)

VITA

Mukta Lal Kar was born in Howrah, West Bengal, India, on January 20, 1949. He obtained his Pre-University (science) degree from Presidency College, Calcutta University in 1964. The same year he entered the Jadavpur University, Calcutta - 32 and in August, 1969 he received his Bachelor of Engineering degree in Electronics and Telecommunications. After a short employment of 3 months in the Electronics and Radar Development Establishments, Bangalore, India, he joined the Electronics (Rocket System) Group of Vikram Sarabhai Space Center, Trivandrum, India, in December 1969. He was there until he obtained a Research Assistantship to study for the M. S. in Electrical Engineering at The University of Tennessee. He received the M. S. degree in December 1976.

He is married to the former Manjula Biswas, West Bengal, India and they have a son, Partha.