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Prof C. A. Perkins.

T H E S I S

A DEMONSTRATION OF THE APPLICATION OF ALTERNATING CURRENT THEORY TO TELEPHONE LINES.

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B. O. Adkerson

Under the direction of

Prof. C. A. Perkins.

University of Tennessee Knoxville, Tenn.

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In this thesis we shall deal with alternating current fundamentals and fundamental measurements of very small quantities of energy, varying in frequency between 200 and 2,000 cycles per second. This means that all values can best be shown by the use of curves.

For the purpose of calculations, all energy must be in the sine wave form, but this will be so if the original wave shape is analyzed into its component parts by the means of Fourier's Theorem as will be explained later; thus enabling measurements to be made of, and exact values computed for each of the numerous frequencies, which is necessary for the transmission of speech.

In order that the subject may be discussed in a logical sequence, let us first see how spoken words may, by means of an electric current, be conveyed and approximately reproduced at a point many miles distant from the place where they originate; then discuss the various factors governing their transmission and influencing their clearness.

The motion of the air molecules when transmitting a sound wave is to and fro in the direction of transmission. If this motion be recorded upon a tape which moves uniformly and at right angles to the path of vibration of the molecules, a record called the graph or wave form of the sound is obtained. In the case of articulate sounds, this motion of the air particles is a highly irregular one, but in the case of musical sounds or prolonged vowel sounds, the motion is a regularly repeated or cyclical one. (See Drawing 1).

The air particles upon coming into contact with the transmitter diaphragm cause it to vibrate; which, by varying the pressure upon and therefore the resistance of the carbon granules, causes the current to be varied in a manner somewhat similar to the variation of the air particles causing it. The slight distortion being due to the natural frequency of vibration of the diaphragm.

This varying current of very small energy has its potential incresed by passing through an induction coil (sometimes passing through a repeating coil in addition) before entering the line. Being a periodic current, it attenuates as it flows along the line.

Upon reaching the other end of the line and passing through other coils, it passes around the wire wound upon the poles of the receiver magnets; thus varying its magnetism and causing it to set the diaphragm, placed in front of it, to vibrating. (Additional distortion resulting from the natural frequency of the diaphragm). The air in proximity to the receiving diaphragm is therefore set in vibration in a manner which is not very dissimilar to that of the air which is before the transmitter. This repetition is, however, far from being perfect.

In order to reduce this dissimilarity as much as possible, it is necessary to make tests upon all objects which have a tendency to distort the wave shape from the form possessed by the air particles which are acting upon the transmitter.

Fourier's Theorem states that any single valued periodic curve, no matter how irregular it may be, provided it does not exhibit discontinuities, may be resolved into a series of sine waves consisting of the fundamental and some or all of the harmonics. (See Appendix Page 1). The variation of the transmitter resistance impresses upon the induction coil a voltage wave similar to the wave form of the air particles vibrating before the diaphragm. By Fourier's Theorem this wave may be analyzed into its component sine waves; each frequency of voltage producing its own current, as if this frequency were acting alone. (See Appendix page 3).

Consonant sounds couse transient waves which are very complex in shape, but which in the electrical transmission of the voice take care of themselves in as far as intelligibility is concerned. Vowels produce a sufficient number of periodic cycles to be considered a steady state. By the above theorem these vowel wave shapes may be analyzed into their component sine waves. Therefore, in order to determine the voice transmission characteristics of a circuit, it is only necessary to ascertain its effect upon transmitted sine waves of frequencies between 200 and 2,000.

The question which naturally arises now is; if a sine wave voltage be impressed upon one end of a line, will it maintain its shape while being transmitted, or will its final form vary from that of the sine wave? This question is fully answered by the following properties of the sine wave which show that the original form is maintained. (See Appendix, page 4)

1. Sine waves of the same frequency can be added or subtracted, in or out of phase, and they will still obey the sine law.
2. Sine waves may be passed through a condenser and will still obey the sine law.
3. If a sine wave be applied to the primary of a transformer, a sine wave will be induced in the secondary.
4. Sine waves may be passed through an inductance and will still obey the sine law.
5. Sine waves may be passed through a resistance and will still obey the sine law.
6. There can always be selected a combination of sine waves which will so add as to form any periodic current wave shape,

regardless of how irregular, provided it does not exhibit discontinuities.

In order to maintain intelligent conversation by telephone between two points, there are four essential conditions which must be satisfied.

1. Energy sufficient to operate them must reach the receivers placed at the ends of the line.
2. The quality of the sound must be maintained.
3. The absence of interference, such as is caused by crosstalk and power induction, must be secured.
4. Disconcerting effects must be eliminated from the line.

The percentage of energy required for intelligent commercial conversation is one-tenth of one percent against a constant impedance. This is from the average of a large number of tests made by the Bell Telephone System engineers. When the line is very long, the energy, on account of being used up in the line, may fall below this value and must be increased. This is done by impressing the incoming current, after its potential has been increased by a transformer, upon the grid of a vacuum tube and using the resulting plate current to traverse the remainder of the line.

To preserve the required quality, it is necessary to transmit the component frequencies between 200 and 2,000 per second with the least possible variation in percentage loss of energy in the different frequencies. Where circuit must be designed for one predominating frequency, it has been found that best results are obtained by using 800. The time relation between frequencies is not of very great importance. This means that the same spoken word may have different irregular wave shapes and still be almost identical in sound.

Crosstalk in central offices and cables is prevented by twisting together the wires forming an individual circuit, and then twisting together all pairs that run parallel to one another for any appreciable distance; by enclosing in soft iron covers all relays and induction coils whichaare placed close together. On open lines crosstalk, as well as power induction, is prevented by transposing at definite intervals the wires forming the individual circuits, and the various circuits with respect to one another; the method of transposition used depending upon the circuits on the line, as well as the kind and the transpositions of any power line from which induction is to be avoided. The effect upon conversation caused by induced currents of various frequencies is shown by drawing 2.

Reflection loss is that caused by the reflection of the waves from some point in the line back to the sending end. When waves pass from a medium of one density into a medium of a different density, the energy is partly transmitted, partly absorbed, and partly reflected. This occurs at the junction of dissimilar impedances, which is the density of the electric medium, such as the junction of an open wire line and a cable, or more particularly at the junction of a loaded with a non-loaded circuit. Reflection loss is reduced as much as possible by using cable only when absolutely necessary, and joining dissimilar lines by means of inequality ratio repeating coils, which will be described later.

To obtain the best transmission possible,,it is necessary to make numerous and various tests; to determine if a line is transmitting at its maximum efficiency; to locate any conditions in the circuit which are detrimental to its efficiency; to obtain data which will enable better circuits and instruments to be designed.

The instruments with which the investigations for this thesis were made consisted of a Wheatstone bridge, impedance bridge, capacity bridge, and 35 miles of artificial standard cable. As the tests were made to determine the behavior of the line and instruments while transmitting sine wave currents, it was essential that such currents be used to make the tests. These were obtained by means of a vacuum tube oscillator which will later be described in detail.

The Wheatstone bridge is shown in fig. 1, drawing 3, and consist of four arms: r_1, r_2, and r, each containing variable, non-inductive resistance, and x, the unknown resistance whose value is to be determined. Between the junction of arms $r_2 - x$ and $r_1 - r$, a galvanometer is connected, while a battery is connected between the junction of $r_1 - r_2$ and $x - r$. When a balance of the bridge is obtained, indicated by no current flowing through the galvanometer, the value of the unknown resistance is determined by means of the formula derived on page 6 of appendix.

The impedance bridge is shown by drawing 4. It contains four arms, two of which, r_1 and r_2, are formed of 1,000 ohm coils wound non-inductively; another by the variable non-inductive resistance r; and the last contains the object whose impedance is to be determined. Between the junctions of the arms $r_2 - z$ and $r_1 - r$, is connected a telephone receiver to detect the alternating current, while between the junction of arms $r_1 - r_2$ and $r - z$, is connected the source of alternating current in such a way that it must pass through four D.P. D.T. switches before reaching the bridge arms. It is impossible to make an inductance which does not contain resistance; therefore, each inductometer has wired with it a compensating resistance. The small fixed inductance is provided so that it can be placed in opposition to the main inductance, thus obtaining readings down to zero. The shielded transformer is provided to protect the impedance bridge from any unbalances which may be in the oscillator circuit.

In the bridge the inductance may be switched from one arm to the other, being put in series with the unknown line when the latter has a capacity or minus (-) reactance, and in series with the resistance r when it has an inductive or plus (+) reactance. The ratio arms being equal, when a balance has been obtained the resistance r is equal to the resistance component of the line impedance, and the inductance L is equal to the effective inductance component.

The capacity bridge is shown by fig. 2 of drawing 3. The only way that it differs from the impedance bridge is that the variable capacity is permanently connected into the variable resistance arm of the bridge. As in the impedance bridge, when a balance has been obtained, the resistance r is equal to the resistance component of the line impedance, and the capacity C is equal to the effective capacity component.

The standard telephone cable is one that contains 19 gauge B & S copper conductors, having a resistance of 88 ohms and a capacity of .054 mf. per mile. Artificial cable is made by arranging condensers and resistances of the proper value in the manner as shown in drawing 5; keys being provided so as to enable any desired length of the cable to be used.

The sine waves of different frequencies were obtained by the use of a vacuum tube oscillator, the wiring diagram of which could not be given on account of its confidential nature.

The oscillator used was a vacuum tube instrument working on the principle of a part of the alternating current energy from the output returning to the input through a tuned circuit. Filters were used to eliminate the harmonics, thereby making a pure sine wave for each frequency used.

The output of the oscillator is regulated by means of a variable series-shunt resistance combination. Between the output control resistance and the output terminals of the oscillator is inserted a shielded repeating coil, which protects the apparatus to be used with the oscillator from any unbalances which are in the oscillator circuit. The windings of this repeating coil have a capacity effect and the grounded shield grounds the midpoint of this so formed condenser.

By means of the above named instruments, used in connection with the vacuum tube oscillator as a source of sine wave current of variable frequency, the physical properties of telephone instruments and circuits were investigated from the standpoint of transmitting and handling alternating currents varying from 5 mil to 1 micro-ampere in strength, and from 200 to 2,000 cycles per second.

Transmitter

The transmitter consists of carbon granules packed loosely in a cup which is closed in front by the vibrating diaphragm; therefore, it has no inductance. Pressure upon the diaphragm causes the resistance of the transmitter to change. In ordinary usage, this variation is about ± 25% of normal. The average current flowing is indicated by the circle and the maximum and minimum variations are shown by drawing 6.

Receiver

The receiver consists of a permanent horse shoe magnet having coils of fine wire wrapped around its pole pieces. The passage of a varying current through these coils causes the flux to vary, thus setting the diaphragm in vibration. The direct current resistance is constant and has a value of 72 ohms, while the behavior of the receiver when handling alternating current is shown by drawings 7, 8, and 9, and the influence of the receiver upon a subscribers sub-set is shown by drawing 10. These drawings clearly illustrate the increase in inductance and resistance caused by the presence of the stationary diaphragm decreasing the magnetic reluctance; the still further reduction caused by its vibration, and the pronounced effect produced at its natural frequency of vibration. While this has the effect to overvaluate the energy in the waves near the natural frequency of the diaphragm, it can occur with the least detriment at the frequency which is most important in voice transmission.

Repeating Coils

A repeating coil is simply a transformer wound upon a core of high permeability, and which is constructed so as to reduce the hysteresis and eddy current to a minimum. The four windings, two for the primary and two for the secondary, are alternately spaced around the circular core in order to obtain the best balance and distribution of flux.

Repeating coils, like other instruments, are designed to give their most efficient services under certain conditions; thus, we have repeating coils for voice transmission, for the transmission of both voice and ringing currents, and inequality-ratio coils for the junction of dissimilar lines.

The inductance of the repeating coils with their secondaries open was found to be greater than could be measured by the impedance bridge, therefore, it was found necessary to give them some kind of termination; 2,000 ohms of non-inductive resistance and 35 miles of artificial standard cable were used for this purpose.

Drawings 11 and 12 give the impedance curves of coil 25-A, drawings 13 and 14 the curves of coil 55-A; with terminations of 2,000 ohms non-inductive resistance and 35 miles of artificial cable respectively; and drawings 15 and 16 the curves of coil 55-E. The last is a coil having an inequality ration of 2.66 to 1. Drawing 15 shows the curves when sending in the low side with the secondary terminated by 2,000 ohms of non-inductive resistance, while drawing 16 shows the curves when sending in the high side with the secondary terminated by 35 miles of artificial standard cable.

By comparison of the curves for coils 25-A and 55-A when they have the same termination, it will be seen that at all frequencies coil 55-A has less inductance and resistance, which shows it is the coil best suited for transmission of voice alone, while 25-A is adapted for both voice and ringing currents.

Lines

A line is a circuit composed of two parallel conductors, two wires or the ground and one wire. At the present time, however, the grounded lines have been abounded in all except rural districts; hence a line may be considered as composed of two parallel wires.

Every line has certain constants (r, L, C, S) which will affect the transmission of alternating current over it. The effect of which constants upon the transmission of alternating energy over a line containing them has been considered on page 9 of appendix. If the numerical value of the above quantities be determined and substituted in the derived expressions, their quantitative effect upon voice transmission will be obtained. As a practical illustration, a line composed of No. 12 N. B. S. gauge copper wires speced one foot between centers, supported with other wires on a pole line at an average distance of 20 feet above ground, was chosen; its S being measured and r, L, and C being computed mathematically. The substitution of these quantities gave values as shown on page 9 of appendix. From an inspection of the numerical values of the attenuation and wave length constants given on page 11 of the appendix, it will be seen that both increase with an increase of frequency. As explained under quality, the time relation of the higher frequencies does not greatly affect the sound of the word, although their presence is essential. The increased attenuation constant for the high frequencies has a tendency to entirely eliminate them. While for intelligent conversation only the frequencies between 200 and 2,000 are required, there are some sounds which are almost indistinguishable on account of their containing energy in the region of the higher frequencies which is eliminated by the increased attenuation constant. Drawing 17 shows some of these sounds and the frequency region in which their energy lies.

By inspection of the form of the attenuation constant, as shown on page 8 of the appendix, it will be seen that the constant may be reduced and transmission improved by any of the following four methods:

1. Increasing the inductance per mile of line.
2. Decreasing the insulation resistance of the line.

3. Decreasing the capacity of the line.

4. Decreasing the resistance by increasing the size of conductor. A combination of some or all of them may be used. The easiest of the above methods to place into practical application is the first; from experimental investigation it being found that for a given amount of iron and copper in the form of impedance coils it results in a smaller attenuation constant to employ them in the Pupin fashion as coils in series than in any other way. By comparison of the constants for a loaded and a non-loaded line, it will be found that for the former the attenuation constant is less for all frequencies, and has a smaller variation between the frequencies for which computations are made; also, its velocity of wave propagation varies only slightly with different frequencies. Thus a loaded line is better adapted to the transmission of speech as the different frequencies attenuate more nearly alike, and the form of the original sound wave is better maintained by the almost constant velocity of wave propagation. It is impossible, however, to obtain a distortionless line by means of load coils properly spaced, as the conductor resistance varies with the temperature, and the insulation resistance varies with the time of day and the condition of the weather. Fig. 18 illustrates an extreme case of variation in the leakage of a line expressed in megohms per mile. This is an 8 gauge copper circuit between Jacksonville and West Palm Beach, Fla., a distance of about 300 miles, and is typical of any circuit located along a seashore in a warm climate.

As has been stated on page 5, when waves pass from a medium of one density into a medium of a different density, the energy is partly transmitted, partly absorbed, and partly reflected. The insertion of impedance coils in a line forms points whose density varies from that of the line; therefore, we are confronted by the problem of energy absorption and reflection at the coils. Pupin mathematically solved this problem*, and from his solution formed the following law.

"If there be a non-uniform line loaded with inductance coils at equal intervals, and if we consided the total inductance and resistance to be smoothly distributed along the line, then these two lines, the non-uniform and uniform, having the same total resistance and inductance, will be electrically equivalent for transmission purposes as long as one half of the distance between two adjacent coils expressed as a fraction of 2π taken as a wave length, is an angle so small that its sine has practically the same numerical value as that angle in circular measure".

By applying this law to an instance when there are ten coils per wave length, it will be found that one half the angle exceeds its sine by only 1.9%. (Appendix, page 12).

When an electric current travels along a wire it is surrounded by a magnetic field (drawing 19, fig. 2) and a dielectric field (fig. 3), the combination field surrounding the circuit being as shown. (fig. 4). When the electromagnetic wave reaches the end of the line it is reflected; with a reversed magnetic component if the end be open (fig. 1), and a reversed electric component if it be closed (fig. 5). If there be in any line a part in which there is greater inductance or capacity per unit of length than at other parts, these lumps of inductance or capacity will cause a partial reflection of the wave in a manner which will be a combination of the above. This reflected component travels back to the sending end and there unites with the outgoing wave; increasing or diminishing its value according as it is in or out of phase, there-

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*The mathematical solution being somewhat long, the reader desiring it is referred to "The Propagation of Electric Currents in Telephone and Telegraph Conductors" by J. A. Fleming.

by causing the impedance of the line to vary as the frequency of the sending current is changed.

The above fact makes possible a method of locating any irregularity which may be in the line. The line is tested by means of the impedance bridge, and the equivalent inductance and resistance curves plotted - equivalent inductance meaning the inductance which remains af after the capacity has been neutralized, or in case the capacity predominates, the inductance which is necessary to neutralize the remaining capacity. In the latter event the inductance is said to be negative or minus (-) in order to distinguish it from the former which is said to be positive or plus (+) - by means of the frequency interval between two adjacent maximum or minimum points on these curves, the distance from the sending end to the point where the irregularity occurs may be determined in the manner described on page 12 of the appendix. If the line contains irregularities the curves will have humps superimposed on them at periodic intervals if the irregularities be at some distance from the sending office, or will have their general shape distorted if they be nearby.

Drawings 20 and 21 are impedance curves of the Atlanta-Birmingham non-loaded circuit 13-14, and loaded circuits 3-4 and 5-6 strapped together at Birmingham, the load coils containing 245 milhenrys and being spaced 7.88 miles apart. By the application of the derived rules the irregularity of both lines will be found to locate at Birmingham, which is at a distance of 146 miles, and is caused by the cable in that city. The distortion of the loaded lines at the higher frequency being caused by the cable in Atlanta, at which place the tests were made.

In order to prevent reflection from the open or short-circuited receiving end of the line in the above test, it was necessary to make the line equal to one extending to infinity. This may be accomplished by continually adding to the end additional lines of the same impedance, as is actually done in the handling of long distance calls between points very widely separated; or by determining the impedance of the given open-circuited line and them terminating the line with this impedance, thus obtaining a line of the same unit impedance but twice its original length; again determining its impedance with the termination and adding this impedance to the first termination; a line of the same unit impedance but of four times its original length is obtained. By continuing this process indefinitely a line of infinite length, possessing the same impedance per unit of length as the original line, is obtained. The impedance of an infinite line may also be theoretically calculated as shown on page 14 of appendix.

The results of a large number of experiments have shown that non-loaded and loaded lines terminated by non-inductive resistance of 700 and 2,000 ohms respectively, display the characteristics of an infinite line. When a line is infinite, nothing that may be done with the receiving end will affect its impedance. A test of the 35 miles of artificial standard cable was made with both the impedance and the capacity bridge, and it was found that short circuiting the receiving end in no way affected its impedance; therefore, it is equivalent to an infinite line. Drawings 22 and 23 show the results of these tests.

From the preceding discussion, it is seen that the same alternating current theory which applies to the transmission of power, can also be applied to the transmission of the human voice, as follows:

1. Analysis of the human voice into its component frequencies.
2. Considering each of these frequencies necessary for intelligibility.

3. Dealing with quantities of energy miniature in comparison, of frequencies much higher, and of such nature as to cause capacity and the slightest amount of inductance to be given great importance in the physical construction of all circuits.

The only difference between the two is that in power lines the receiving end impedance has more effect upon sending end impedance than the characteristic impedance of the transmission line, while in telephone lines, if the circuit is very long, the receiving end impedance has negligible effect upon the sending end impedance.

This application of alternating current theory to telephone lines can be used not only to improve transmission and locate irregularities, but also to predetermine the action of a circuit and thus enable it to be designed to meet the requirements of any of the different types of service.

A P P E N D I X

FOURIER'S THEOREM

Fourier's Theorem states that any single valued periodic curve, regardless of how irregular it may be, provided it does not exhibit discontinuities, may be resolved into a series of sine waves consisting of the fundamental and some or all of the series of harmonics.

Let Y be the ordinate of any such curve; then Y may be expressed by the series

$$Y = A_0 + A_1 \sin pt + B_1 \cos pt + A_2 \sin 2pt + B_2 \cos 2pt + A_3 \sin 3pt + B_3 \cos 3pt + - - - - - .$$

where $p = \frac{2\pi}{T}$, and T is the time of the fundamental sine curve. If the zero point of the fundamental sine curve be taken as the origin, then pt is the abscissa corresponding to Y.

The value of the constants A_0, A_1, B_1, A_2, B_2, A_3, B_3, etc., may be found by the aid of the following theorems:

1. The average value of $\sin \theta$ or $\cos \theta$, taken over a complete cycle, is zero.

$$\sin \theta_{av} = \frac{1}{2\pi}\int_0^{2\pi} \sin \theta \, d\theta = \frac{1}{2\pi}\Big[- \cos \theta\Big]_0^{2\pi} = 0$$

$$\text{Cos } \theta_{av} = \frac{1}{2\pi}\int_0^{2\pi} \cos \theta \, d\theta = \frac{1}{2\pi}\Big[\sin \theta\Big]_0^{2\pi} = 0.$$

2. The average value of $\sin^2 \theta$ and of $\cos^2 \theta$, taken over a complete period, is 1/2.

$$\cos 2\theta = \text{Cos}^2 \theta - \sin^2 \theta = 1 - 2\sin^2 \theta = 2 \cos^2 \theta - 1$$

$$\sin^2 \theta_{av} = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} \, d\theta - \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} \cos 2\theta = \frac{1}{2}.$$

$$\cos^2 \theta_{av} = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} \, d\theta + \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} \cos 2\theta = \frac{1}{2}.$$

since by theorem 1 the average values of $\sin \theta$ and $\cos \theta$ are zero.

3. The average value of $\sin n\theta \cos m\theta$, $\sin n\theta \sin m\theta$, and $\cos n\theta \cos m\theta$, taken over a complete period, is as shown below:

$$\sin (n + m)\theta = \sin n\theta \cos m\theta + \cos n\theta \sin m\theta$$

$$\underline{\sin (n - m)\theta = \sin n\theta \cos m\theta - \cos n\theta \sin m\theta}$$

(a) $\sin n\theta \cos m\theta = \frac{1}{2} \sin (n + m)\theta + \frac{1}{2} \sin (n - m)\theta$

$$\cos (n + m)\theta = \cos n\theta \cos m\theta - \sin n\theta \sin m\theta$$

$$\underline{\cos (n - m)\theta = \cos n\theta \cos m\theta + \sin n\theta \sin m\theta}$$

(b) $\sin n\theta \sin m\theta = \frac{1}{2} \cos (n - m)\theta - \frac{1}{2} \cos (n + m)\theta$

(c) $\cos n\theta \cos m\theta = \frac{1}{2} \cos (n - m)\theta + \frac{1}{2} \cos (n + m)\theta$.

From theorem 1 it is clear that the average value of (a) is zero, and of both (b) and (c) is zero except when $n = m$, in which case it is 1/2.

If a number of equal spaced ordinates be drawn through a complete cycle of the curve to be analysed, then the average value of these ordinates will be the value of A_0, because the average value of all the sine and cosine terms is zero. Let both sides of the series be multiplied by sin pt, we have

$$Y \sin pt = A_0 \sin pt + A_1 \sin^2 pt + B_1 \cos pt \sin pt +$$
$$A_2 \sin 2pt \sin pt + B_2 \cos 2pt \sin pt + - - - .$$

Upon taking the average of the right hand side, all the terms disappear except the second which equals 1/2. Hence $Y \sin pt_{av} = \frac{A_1}{2}$, or A_1 equals twice the average value of $Y \sin pt$. Successive multiplications of the series by cos pt, sin 2pt, cos 2pt, etc., and taking the average value will give

B_1 = twice the average value of $Y \cos pt$.

A_2 = twice the average value of $Y \sin 2pt$.

B_2 = twice the average value of $Y \cos 2pt$. etc.

By letting $A_1 = c \cos \theta$, and $B_1 = c \sin \theta$

$$A_1 \sin pt + B_1 \cos pt = c \sin pt \cos \theta + c \cos pt \sin \theta$$

$$= c \sin (pt + \theta) = \sqrt{A_1^2 + B_1^2} \sin (pt + \theta).$$

where $c = A_1/\cos\theta = B_1/\sin\theta$, $\qquad \tan\theta = B_1/A_1$

$\cos\theta = A_1/\sqrt{A_1^2 + B_1^2}$ $\qquad c = \sqrt{A_1^2 + B_1^2}$.

Substituting these values the series may be written in the form

$$Y_o = A_o + \sqrt{A_1^2 + B_1^2} \sin (pt + \theta_1) + \sqrt{A_2^2 + B_2^2} \sin (2pt + \theta_2) + \sqrt{A_3^2 + B_3^2} \sin (3pt + \theta_3) + - - - - - .$$

in which $\sqrt{A_1^2 + B_1^2}$, $\sqrt{A_2^2 + B_2^2}$, $\sqrt{A_3^2 + B_3^2}$, etc., are the amplitudes of the different harmonics and θ_1, θ_2, θ_3, etc., are the phase angles.

Voltage Application to Inductance.

Let the voltage across an inductance be given in the form of a Fourier's series,

$$e = E_1 \sin (2\pi ft - \theta_1) + E_2 \sin 2(2\pi ft - \theta_2) + E_3 \sin 3(2\pi ft - \theta_3).$$

Substituting this value in $e = ri + L\, di/dt$, if r may be neglected,

$$L \frac{di}{dt} = E_1 \sin (2\pi ft - \theta_1) + E_2 \sin 2(2\pi ft - \theta_2) + E_3 \sin 3(2\pi ft - \theta_3) + - - - - .$$

Multiplying both sides of the equation by dt and then integrating,

$$Li + C = -\frac{E_1}{2\pi f} \cos (2\pi ft - \theta_1) - \frac{E_2}{4\pi f} \cos 2(2\pi ft - \theta_2) - \frac{E_3}{6\pi f} \cos 3(2\pi ft - \theta_3) - - - - .$$

The constant of integration is equal to zero, because the current cannot have a unidirectional component without a commutating device or electric valve of some sort.

$$i = -\frac{E_1}{2\pi fL} \cos (2\pi ft - \theta_1) - \frac{E_2}{4\pi fL} \cos 2(2\pi ft - \theta_2) - \frac{E_3}{6\pi fL} \cos 3(2\pi ft - \theta_3) - - - - - .$$

Which means that each harmonic in the voltage produces its own current, <u>as if this harmonic were acting alone</u>.

Properties of Sine Waves

1. Sine waves of the same frequency can be added or subtracted, in or out of phase, and will still obey the sine law.

Assume two sine waves which may be represented by the equations

$i_1 = I_1 \sin \Phi$, and $i_2 = I_2 \sin (\Phi + c)$

Let $I_1 = (A + B)$, $I_2 = (A - B)$, $\Phi = (\theta - D)$, $(\Phi + c) = (\theta + D)$

$$i_r = I_1 \sin \Phi + I_2 \sin (\Phi + c)$$

$$= (A + B)(\sin \theta \cos D - \cos \theta \sin D) + (A - B)(\sin \theta \cos D + \cos \theta \sin D)$$

$$= \sin \theta \cos D(A + B + A - B) + \cos \theta \sin D(A - B - A - B)$$

$$= 2 A \sin \theta \cos D - 2 B \cos \theta \sin D$$

As $(A \cos D)$ and $(B \sin D)$ are constants, we may let

$A \cos D = K \cos h$, and $B \sin D = K \sin h$.

$$i_r = 2K \sin \theta \cos h - 2K \sin h \cos \theta$$

$$= 2K \sin (\theta - h).$$

2. Sine waves may be passed through a condenser and will still obey the sine law.

The quantity of electricity stored in a condenser, called the charge, equals the average current flowing into the condenser multiplied by the time during which it flows. $Q = \int i\, dt$, where i is the charging current at any instant. The charge is found to be directly proportional to the applied voltage. $Q = Ce = \int i\, dt$. $i = C\, de/dt$.

If the voltage applied to the capacity circuit be represented by $e = E_m \sin 2\ ft$, the current will be

$$i = C \frac{d(E_m \sin 2\pi ft)}{dt}$$

$$= C \frac{E_M\ 2\pi f \cos 2\pi ft\ dt}{dt}$$

As $I_{av} = 4fCE_m$, and $I_m = I_{av} \times \pi/2$.

$$i = I_m \cos 2\pi ft = I_m \sin (2\pi ft - 90^\circ).$$

This is a sine wave of current which leads the sine wave of voltage by 90°.

3. If a sine wave be applied to the primary of a transformer, a sine wave will be induced in the secondary.

The electromotive force is proportional to the time rate of change of flux, $e \neq n \frac{d\phi}{dt}$. If the core is not too highly saturated, so that the flux varies as the current, this may without appreciable error, be written $e = M \frac{di}{dt}$. By the assumption of a sine wave current this becomes

$$e = M \frac{d(I_m \sin 2\pi ft)}{dt}$$

$$= M \frac{I_m 2\pi f \cos 2\pi ft \; dt}{dt} = M I_m 2 f \sin (2 ft - 90^\circ).$$

As the induced voltage $E_{im} = 2\pi fMI_m$

$$e = E_m \cos 2\pi ft = E_m \sin (2\pi ft + 90^\circ).$$

The sine wave of **voltage** therefore lags 180° behind the sine wave of voltage which produces it.

4. Sine waves may be passed through an inductance and still obey the sine law.

$$e = n \frac{d\phi}{dt}$$

$$= L \frac{di}{dt}$$

$$= L \frac{d(I_m \sin 2\pi ft)}{dt}$$

$$= L \frac{I_m 2\pi f \cos 2\pi ft \; dt}{dt}$$

As the induced voltage $E_{im} = 2\pi fLI_m$

$$e = E_m \cos 2\pi ft = E_m \sin (2\pi ft + 90^\circ).$$

The induced voltage being 180° out of phase with the applied voltage, the resultant will be

$$e_r = E_{1m} \sin 2\pi ft - E_{2m} \sin (2\pi ft + 90^\circ).$$

Which the proof of the first property shows to be a sine wave.

5. Sine waves may be passed through resistance and will still obey the sine law.

$$e = ri + L\frac{di}{dt}$$

As circuit contains only resistance $L\frac{di}{dt} = 0$

$$e = ri = r(I_m \sin 2\pi ft).$$

Which shows that there is a resistance voltage loss $r(I_m \sin 2\pi ft)$; this loss being a sine wave, the first property proves that the resultant voltage is of the same form.

6. There can always be selected a combination of sine waves which will so add as to make any alternating current wave shape, regardless of how irregular.

This synthesis follows as a result of Fourier's Theorem which is discussed in detail on page 1 of this appendix.

Formula for Wheatstone Bridge

In order for the galvanometer not to deflect there must be no current flowing through it, to satisfy this condition

$$r_2 : x :: r_1 : R$$

$$x = R\frac{r_2}{r_1}.$$

Attenuation and Wave Length Constants

Let a transmission line consisting of two very long parallel wires be considered, having a resistance r ohms per mile of line, a capacity C farads per mile, an inductance L henrys per mile, and a dielectric conductance of S mhos per mile.

If v and i be the potential difference between and the current in the wires at point x, then at point x + fx it will be $v + \frac{dv}{dx} fx$ and $i + \frac{di}{dx} fx$. The potential difference being partly expended in driving the current against the ohmic resistance, and partly in overcoming the back electromotive force due to inductance. Similarly, the change of current in the same length is due partly to charging the wires and partly to conduction across the dielectric. This may be expressed by the differential equations as follows:

$$\frac{dv}{dx} = ri + L\frac{di}{dt}$$

$$\frac{di}{dx} = Sv + C\frac{dv}{dt}.$$

If both v and i are taken as simple sine functions of time which can be represented by $v = V \sin(pt + \theta)$ and $i = I \sin pt$, differing by a phase angle θ but having equal frequency n such that $p = 2\pi n$; then

$$\frac{di}{dt} = p\, I \cos pt$$
$$\frac{dv}{dt} = p\, V \cos(pt + \theta).$$

Hence if we denote the periodic current by a simple vector representing its maximum value, then a vector p times as long and at right angles to it will represent the maximum value of $\frac{di}{dt}$.

If therefore, any line be taken to represent rI, or the maximum value of ri, then for the maximum value of $L\frac{di}{dt}$ we must draw a line to the same scale representing LpI at right angles to the vector rI. The vector sum of these lines, rI + jpLI, will be the line representing $\frac{dv}{dx}$. Hence when the time variation of v and i are simple harmonic, in place of the above differential equations can be written the vector equations

$$\frac{dV}{dx} = (r + jpL)I$$
$$\frac{dI}{dx} = (S + jpC)I$$

where V and I are the maximum values of v and i during the period.

Differentiating each of the above equations with regard to x and separating the variables, we obtain

$$\frac{d^2V}{dx^2} = (r + jpL)(S + jpC)V = P^2V$$

$$\frac{d^2I}{dx^2} = (r + jpL)(S + jpC)I = P^2I$$

Where $P = \sqrt{r + jpL}\,\sqrt{S + jpC} = \alpha + j\beta.$

Squaring $\alpha^2 - \beta^2 + 2j\alpha\beta = (rS - p^2LC) + j(pLS + prC)$

Equating steps $\alpha^2 - \beta^2 = rS - p^2LC$

$2\alpha\beta = p(LS + rC)$

Equating sizes $\alpha^2 + \beta^2 = \sqrt{(r^2 + p^2L^2)(S^2 + p^2C^2)}$

Solving $\alpha = \sqrt{1/2\left[\sqrt{(r^2 + p^2L^2)(S^2 + p^2C^2)} + (rs - p^2LC)\right]}.$

$$\beta = \sqrt{1/2\left[\sqrt{(r^2 + p^2L^2)(S^2 + p^2C^2)} - (rS - p^2LC)\right]}.$$

α being the attenuation constant, β the wave length constant, and $P = \alpha + j\beta$ the propagation constant of the line.

By adding and subtracting $2p^2CLrS$ to and from the product $(r^2 + p^2L^2)(S^2 + p^2C^2)$ the above expressions for α and β can be thrown into the forms

$$\alpha = \sqrt{1/2\left[\sqrt{(Sr + p^2LC)^2 + p^2(LS - Cr)^2} + (Sr - p^2LC)\right]}$$

$$\beta = \sqrt{1/2\left[\sqrt{(Sr + p^2LC)^2 + p^2(LS - Cr)^2} - (Sr - p^2LC)\right]}.$$

If the value of the primary constants are such that $LS - Cr = 0$, the above expressions reduce to the forms

$$\alpha = \sqrt{Sr}$$

$$\beta = p\sqrt{CL}.$$

The last is known as a distortionless line, because waves of all frequencies attenuate equally and travel with the same velocity.

Thus, given the values of r, L, C, and S per mile of line, the value of the attenuation constant α, and hence of $e^{-\alpha}$ may be calculated, giving the attenuation per mile of line. Also the value of the wave length constant β, which the above formula gives in radians, may be calculated. As the angle of the vector denoting the current or potential is shifted backward by $\beta\frac{180}{\pi}$ degrees per mile, after traveling $\frac{2\pi}{\beta}$ miles the phase has shifted backward 360° and the cycle as regards phase begins to be repeated. The length $\frac{2\pi}{\beta} = \lambda$ is therefore called the wave length.

The wave velocity W equals the frequency times the wave length,

$$W = n\lambda$$

But $\quad p = 2\pi n$

And $\quad \lambda = \frac{2\pi}{\beta}$

Therefore $W = \frac{p}{\beta}$

Thus we find that the velocity of the wave is a function of the frequency n, except in the special case when β varies as p.

Line Computations

Inductance

$$L = \left[2 \log_e\left(\frac{d - r}{r}\right) + 0.5\right]10^{-9}$$ Equation for inductance.

Transferring from Naperian to common system of logarithms,

$$L = \left[4.61 \log\left(\frac{d - r}{r}\right) + 0.5\right]10^{-9}$$ henrys per cm. of conductor.

$$= \left[74.1 \log\left(\frac{d - r}{r}\right) + 8.05\right]10^{-5}$$ " " mile " " .

Substituting the valuew from the line; No. 12 N. B. S. gauge copper, spacing 1 ft., average distance from ground 20 ft., in vicinity of other wires.

$$L = \left[74.1 \log\left(\frac{12 - .0404}{.0404}\right) + 8.05\right]10^{-5}$$

$$= (74.1 \times 2.472 + 8.05)\ 10^{-5}$$

$$= 191.3 \times 10^{-5} = .00191 \text{ henrys per mile of conductor}$$

$$= 2 \times .00191 = .00382 \text{ henrys per mile of circuit.}$$

Capacity

$$C = \frac{1}{2\,V^2 \log_e \left(\frac{d-r}{r}\right)}$$ Equation for capacity, per cm. of wire in electromagnetic units.

Reducing to common system of logarithms and microfarads per mile of wire to neutral.

$$C = \frac{3.883}{100 \log \left(\frac{d-r}{r}\right)}$$

$$= \frac{3.883}{100 \times 2.472}$$

$$= .0156 \text{ mf.}$$

$= C/2 = .0078$ mf. capacity between wires.

Resistance

$$r = K\frac{L}{A}$$

$K = 1.6 \times 10^{-6}$ ohms per cm. cube or 9.7 ohms per circular mil foot at 0° C, for the grade of copper used

$r = r_0(1 + \alpha t)$ $\alpha = .004$. Variation with temperature.

$r = 9.7(1 + .004 \times 21) = 10.5$ ohms for 70° F $= 21^\circ$ C.

$r = \frac{10.5 \times 5{,}280 \times 2}{80.8 \times 80.8} = 17$ ohms per mile of circuit.

Insulation Resistance

Let l be the length of line, r the resistance of the voltmeter, E the testing battery voltage, D the voltmeter reading.

$$R = \left(\frac{E}{D} - 1\right) \times l \times r$$

$= \left(\frac{120}{23} - 1\right) \times 146 \times 100{,}000$, substituting values of line.

$$= 61{,}600{,}000 \text{ ohms}$$

$$S = 1/R = .0000000162$$

Line Constants

Referring to the equations derived on page 8 for the attenuation and wave length constants, and substituting the values of R, L, C, and

S, as obtained above for an unloaded line; variation of frequency will give results as follow:

Non-loaded Lines

$r = 17$ ohms, $C = .0078$ mf., $L = .00382$ henrys, $S = 0$ nearly.

$$n = 80$$

$Lp = 1.92$, $Cp = 3.92/10^6$, $\sqrt{r^2 + p^2L^2} = 17.12$

$2\alpha^2 = \left(\frac{3.92}{10^6} \times 17.12 - \frac{7.53}{10^6}\right)$ $\alpha = .00546.$

$2\beta^2 = \left(\frac{3.92}{10^6} \times 17.12 + \frac{7.53}{10^6}\right)$ $\beta = .00611$

$\lambda = 2\pi/\beta = 1040$ miles

$W = p/\beta = 82,200$ miles per second.

$$n = 800$$

$\alpha = .01127$, $\beta = .02965$, $\lambda = 212$ mi., $W = 169,500$ mi. per sec.

$$n = 8000$$

$\alpha = .01415$, $\beta = .27450$, $\lambda = 22.9$ mi. $W = 183,200$ mi. per sec.

Loaded Lines

On loaded lines there are placed loading coils of 245 milhenrys and 16 ohms, spaced 7.88 miles apart. Considering the coils alone, $L = .245/7.88 = .03105$ henrys per mile, $r = 16/7.88 = 2$ ohms (nearly). The constants of the line per mile will now be,

$L = .00382 + .03105 = .03487$ henrys, $r = 17 + 2 = 19$ ohms

$C = .0078$, $S = 0$ (nearly)

Solving for the constants as above, we obtain:

$$n = 80$$

$\alpha = .00391$, $\beta = .00922$, $\lambda = 682$ mi., $W = 54,500$ mi. per sec.

$$n = 800$$

$\alpha = .00774$, $\beta = .083$, $\lambda = 75.7$ " $W = 60,500$ " " "

$$n = 8000$$

$\alpha = .01222$, $\beta = .828$, $\lambda = 7.59$ " $W = 60,700$ " " "

Standard Cable

$r = 88$ ohms, $C = .054$ mf., $L = .001$ henrys, $S = 0$ (nearly)

$n = 800$

$\alpha = .106$, $\beta = .1108$, $\lambda = 56.7$ mi., $W = 45,400$ mi. per sec.

Line Equivalency

"If there be a non-uniform cable line loaded with inductance coils at equal intervals, and if we consider the total inductance and resistance to be smoothly distributed along the line, then these two lines, the non-uniform and the uniform, having the same total resistance and inductance, will be electrically equivalent for transmission purposes as long as one half of the distance between two adjacent coils expressed as a fraction of 2π taken as the wave length, is an angle so small that its sine has practically the same numerical value as that angle in circular measurement."

Applying this rule to the loaded line having a frequency of 800;

$\lambda = 75.7$ mi. $\quad \frac{75.7}{7.88} = 9.62$ coils per wave length.

$\frac{1}{2}\lambda = \frac{360^\circ}{2 \times 9.62} = 18.7^\circ = .3267$ radians.

Sine $18^\circ\ 42' = .3206$

we fine that $\frac{1}{2}\lambda$ exceeds $\sin \frac{1}{2}\lambda$ by 1.9%.

Accordingly it is clear that if there are at least nine coild per wave, the non-uniform line is for that frequency practically equivalent to a line in which the same inductance and resistance is uniformly distributed.

Location of Irregularity by Impedance Measurements

The equation for impedance of a circuit is $Z = E/I = \sqrt{r^2 + (2\pi fL - 1/2\pi fC)^2}$. As the instrument used (impedance bridge) measuered effective inductance equivalent to the last two quantities, or $(2\pi f L_1)^2 = (2\pi f L - 1/2\pi f C)^2$, we may write $Z = E/I = \sqrt{r^2 + (2\pi f L_1)^2}$.

If a potential be applied to this line it will cause a current to flow; both of which upon reaching the receiving end will be reflected back to the sending end, there affecting the sending potential and current as if additional impedance $Z_r = E_r/I_r$ had been added to the line. The resistance of the line being a constant quantity, the effect will be that L_1 will vary according as L_r adds or subtracts from it; the maximum value being when they add in phase, and the minimum when they add 180° out of phase.

In order to reach the sending end in phase with the outgoing wave, the point of reflection must be at a distance of one half of a wave length; $D_1 = \frac{1}{2}\lambda_1$. In order to reach the sending end 180° out of phase, the point of reflection must be at a distance of one quarter of a wave length; $D_2 = \frac{1}{4}\lambda_2$. Calling the velocity of the wave W, we have

$W = n\lambda, \qquad n = W/\lambda,$

$n_1 = W/2D_1$, where n_1 is the frequency at which L is maximum

$n_2 = W/4D_2$, where n_2 is the frequency at which L is minimum.

The place where reflection occurs being a fixed point, $D_1 = D_2$, and by subtraction is obtained $n_1 - n_2 = W/4D$, W being nearly constant when taken for a given line and frequencies between 200 and 1,500. It is easier, however, to count the frequency interval between two adjacent maximum or minimum points on the equivalent inductance curve, therefore, $2(n_1 - n_2) = 2W/4D$. Solving for D, $D = W/2(n_1 - n_2)$. Hence to obtain the distance in miles of the irregularity from the sending end, divide half of the velocity of wave propagation for the circuit by the frequency interval between two adjacent maximum points on the equivalent inductance curve.

Applying this rule to the non-loaded and loaded lines on which tests were made, using the W computed for a frequency of 800 we obtain:

Non-Loaded

$$D = W/2(n_1 - n_2) = 169,500/2(n_1 - n_2) = 84,750/(n_1 - n_2) \text{ miles.}$$

Loaded

$$D = W/2(n_1 - n_2) = 60,500/2(n_1 - n_2) = 30,250/(n_1 - n_2) \text{ miles.}$$

or, since the loading coils are spaced 7.88 miles apart

$$D = 30,250/7.88(n_1 - n_2) = 3,800/(n_1 - n_2) \text{ loading sections.}$$

These results check approximately with the rule used by the American Telephone and Telegraph Company, which gives 90,000 and 3,200 as the numbers to be divided by the frequency interval between two adjacent maximum or minimum points on the equivalent inductance curve, in order to obtain the distance in miles and loading sections from the sending end to the location of the irregularity.

The difference between the computed values and those used by the above named company is due to the variation of the computed values of L and C from their actual values for wires strung on poles; they here being affected by the varying height above the ground; number, spacing, and condition of the other circuits of the line in a manner which could not be easily determined by any mathematical law, and have, therefore, been totally ignored in computing the values of the inductance and capacity. The greater error of the loaded line being caused by its variation from its equivalent uniform line, for which the velocity of wave propagation was figured.

Characteristic Impedance of an Infinite Line

Let a line of infinite length be assumed, which has the following values for its constants:

C = Capacity per unit of length.

r = Resistance (loop) per unit of length.

L = Series inductance per unit of length.

g = Conductance, or leakage from wire to wire, per unit of length.

$Y = 1/Z$ = Admittance.

Z = Impedance.

Such a circuit will absorb energy from another circuit and will transmit energy with an efficiency which depends upon the above quantities. Its impedance, if line is infinitely long and of homogeneous construction, must be constant looking toward infinity from any point.

Let two points, 1 and 2, be taken and the distance between them be called Dl; the entire length of line being l.

Then $E_1 - E_2 = DE, \quad I_1 - I_2 = DI$

$E_1/I_1 = E_2/I_2$ As impedance is constant.

$\frac{I_1 - I_2}{I_1} = \frac{E_1 - E_2}{E_1}$, or $\frac{DI}{I} = \frac{DE}{E}$. Laws of proportion.

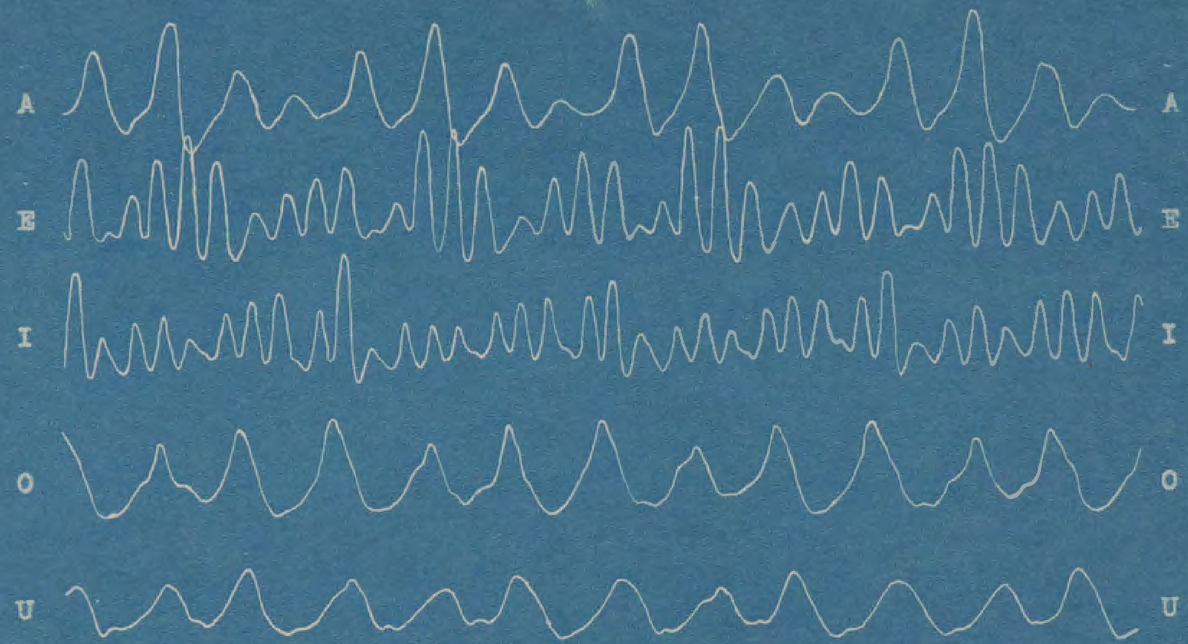
$$DE \doteq Z\ DI = I\sqrt{r^2 + (2\pi fL)^2}.$$

$$DI = Y\ DE = E\sqrt{g^2 + (2\pi fC)^2}.$$

As $E\ DI = I\ DE$, $E^2\sqrt{g^2 + (2\pi fC)^2} = I^2\sqrt{r^2 + (2\pi fL)^2}$.

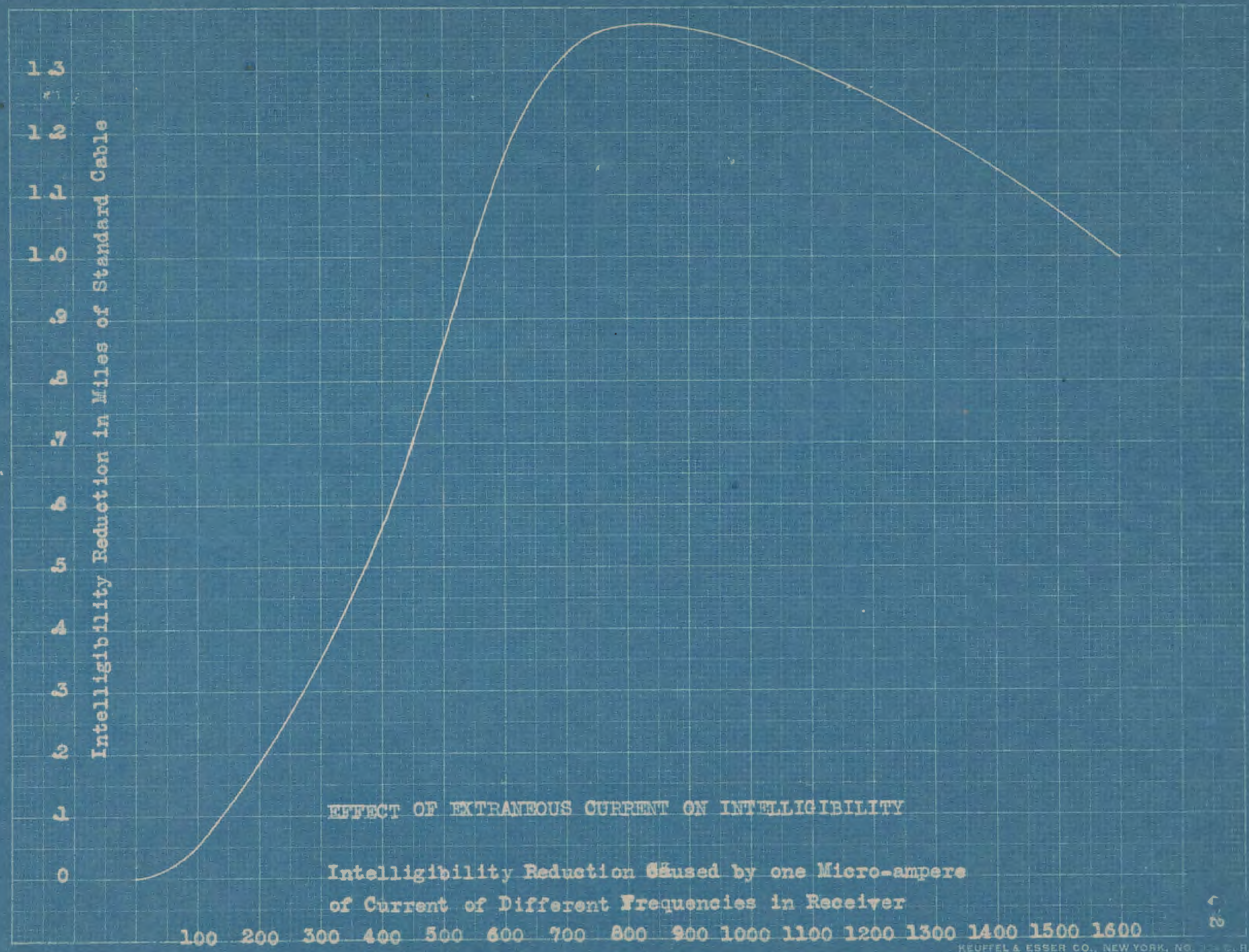
$$\frac{E^2}{I^2} = \frac{\sqrt{r^2 + (2\pi fL)^2}}{\sqrt{g^2 + (2\pi fC)^2}}$$

$$\frac{E}{I} = \sqrt[4]{\frac{r^2 + (2\pi fL)^2}{g^2 + (2\pi fC)^2}}.$$



WAVE FORMS OF THE VOWELS A, E, I, O, U, TAKEN WITH THE OSCILLIOGRAPH.

Miller.


Intelligibility Reduction in Miles of Standard Cable
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600
EFFECT OF EXTRANEOUS CURRENT ON INTELLIGIBILITY
Intelligibility Reduction Caused by one Micro-ampere
of Current of Different Frequencies in Receiver
KEUFFEL & ESSER CO., NEW YORK

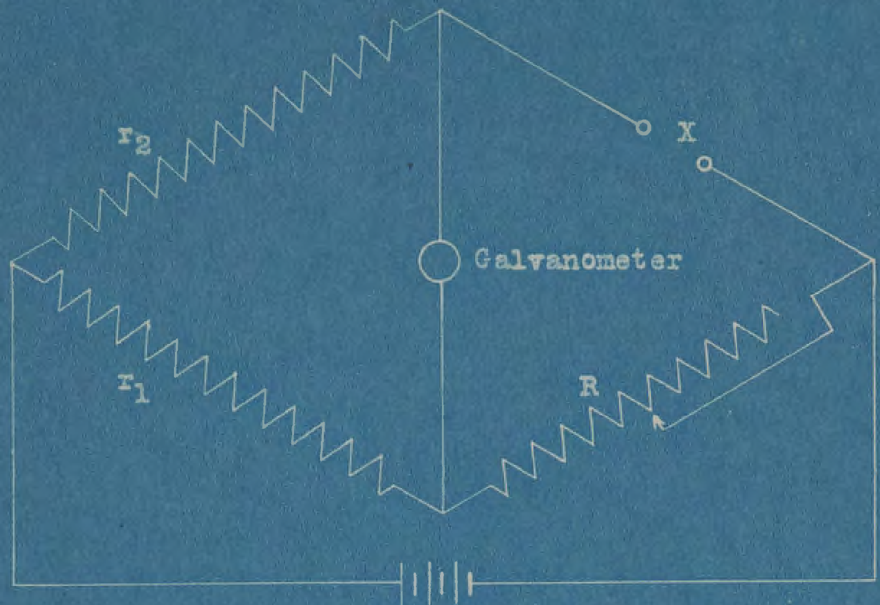


FIG. 1.

WHEATSTONE BRIDGE.

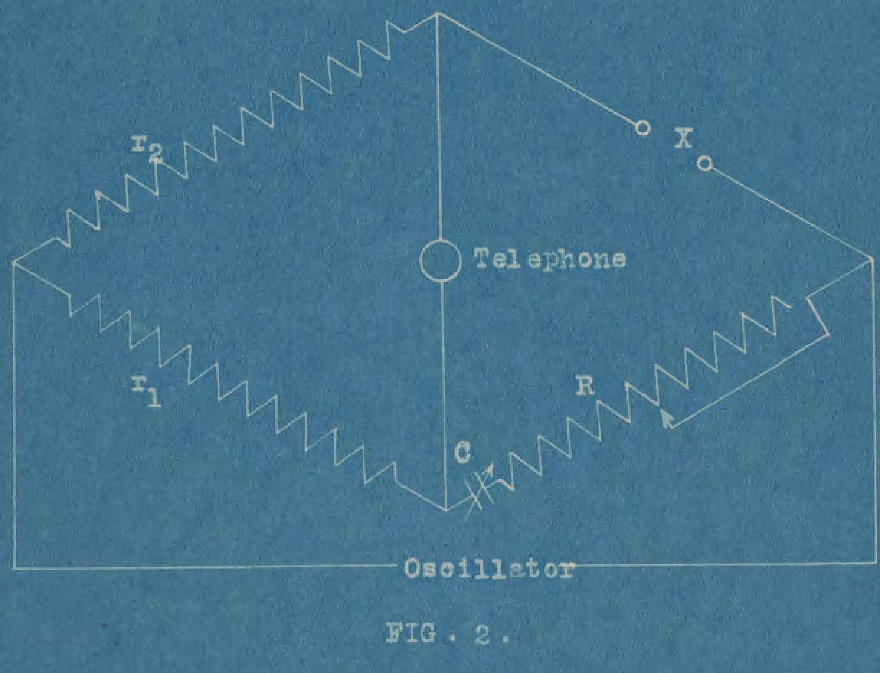


FIG. 2.

CAPACITY BRIDGE.

IMPEDANCE BRIDGE

FIG. 1.
THEORY.

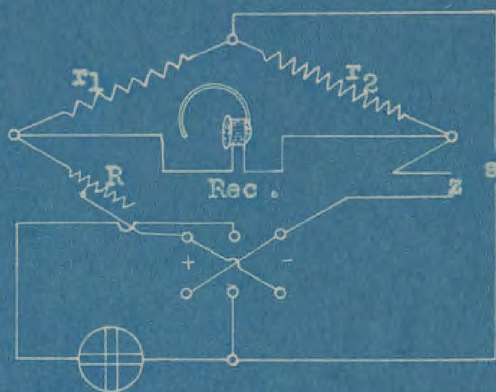
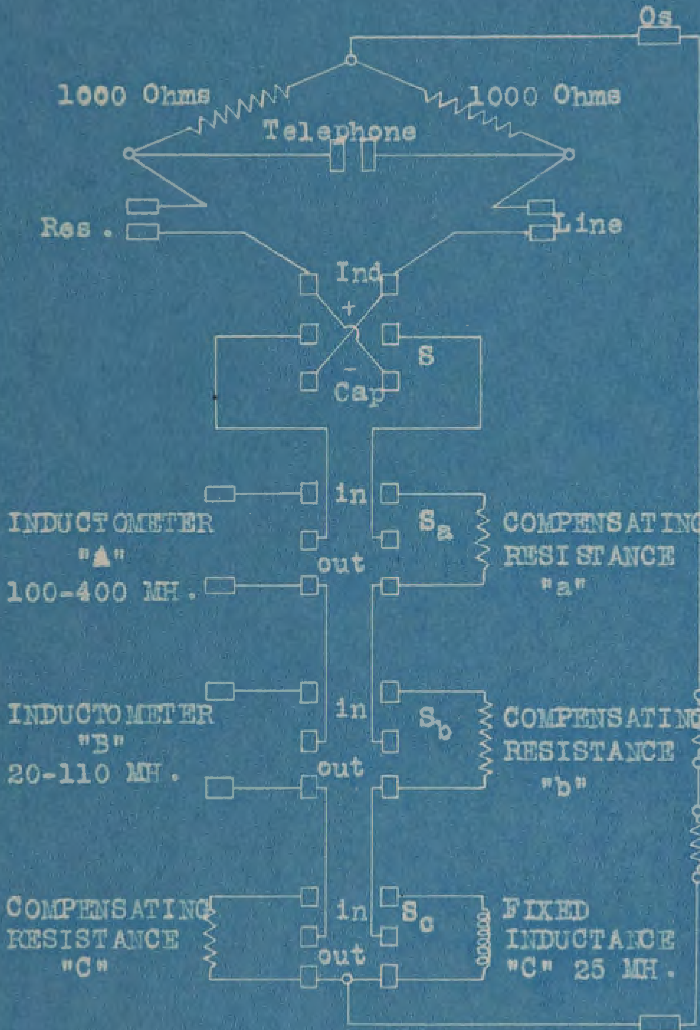
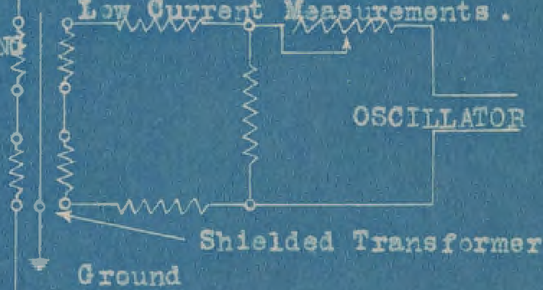


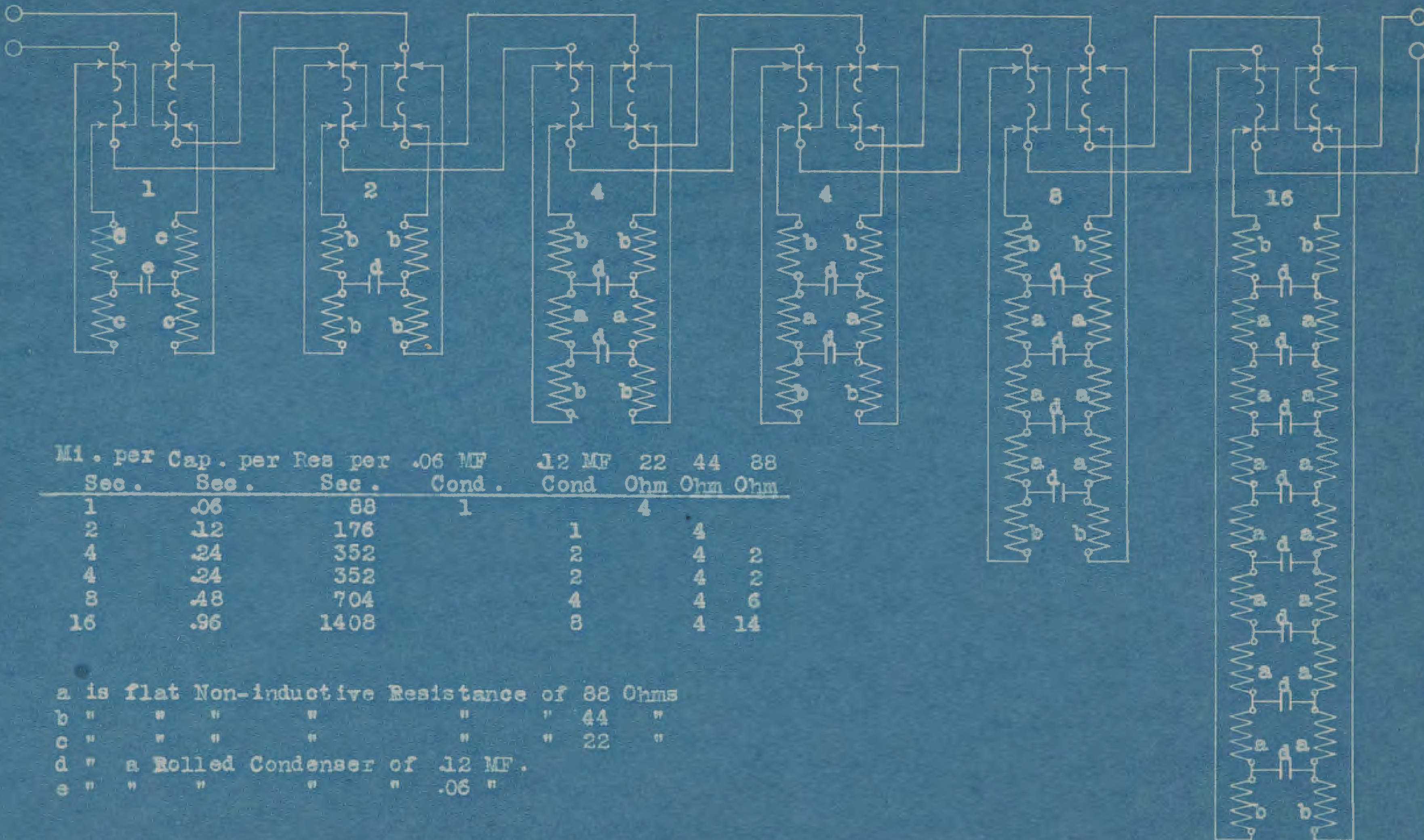
FIG. 2.
ACTUAL ARRANGEMENT IN DIAGRAMMATIC FORM.



Apparatus for
Low Current Measurements.

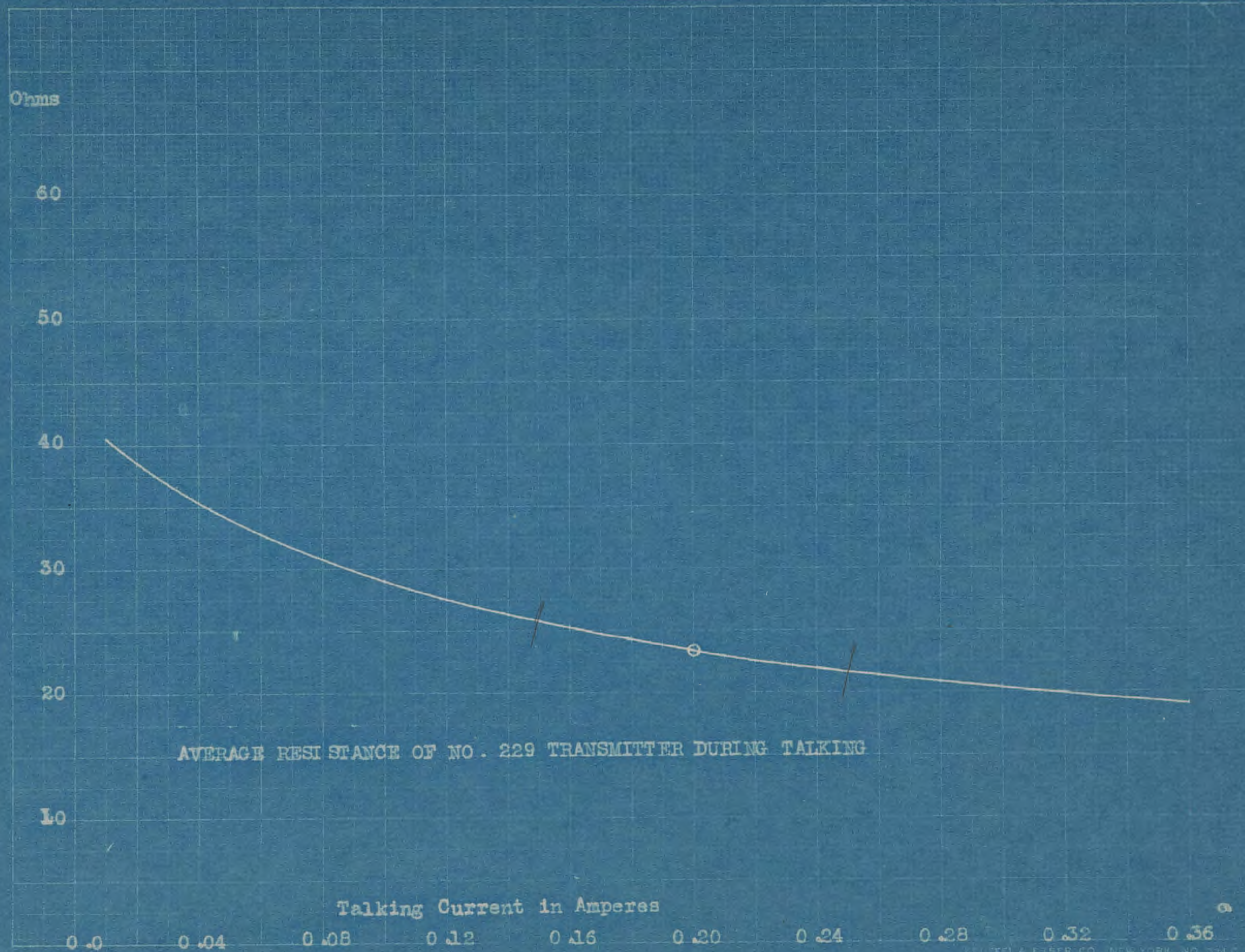


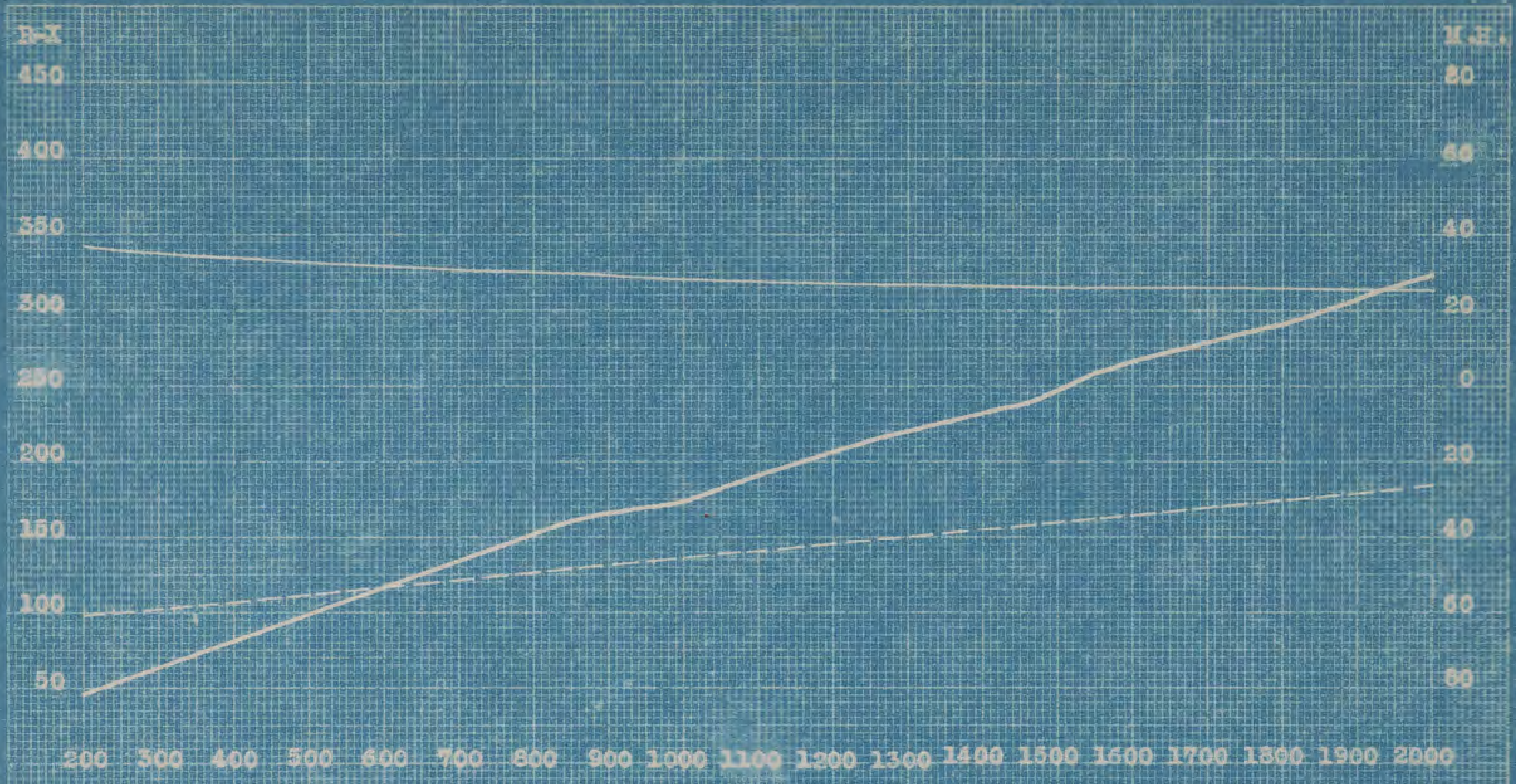
35 MILE ARTIFICIAL CABLE



Mi. per Sec.	Cap. per Sec.	Res per Sec.	.06 MF Cond.	.12 MF Cond	22 Ohm	44 Ohm	88 Ohm
1	.06	88	1		4		
2	.12	176		1		4	
4	.24	352		2		4	2
4	.24	352		2		4	2
8	.48	704		4		4	6
16	.96	1408		8		4	14

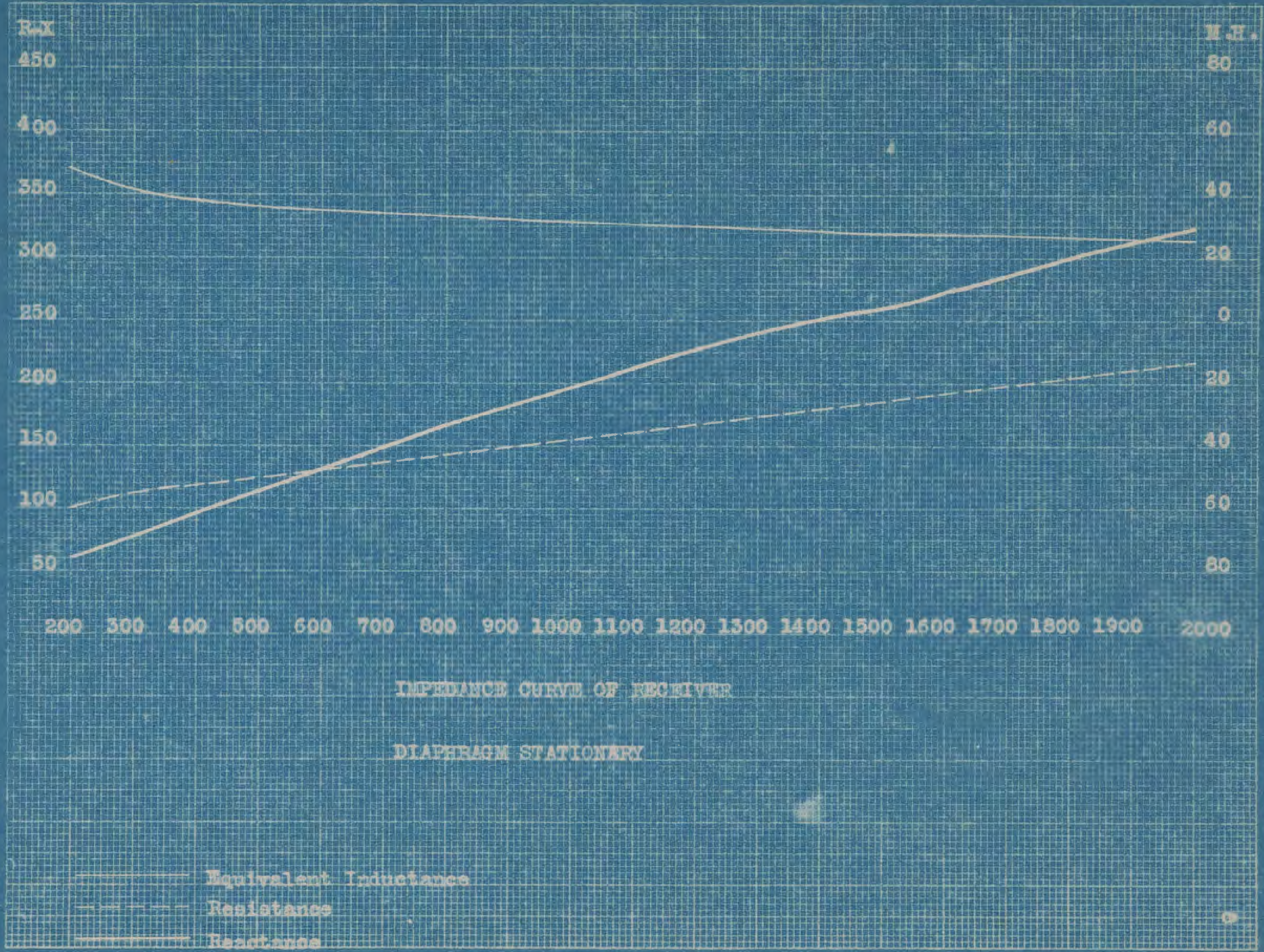
a is flat Non-inductive Resistance of 88 Ohms
b " " " " " " 44 "
c " " " " " " 22 "
d " a Rolled Condenser of .12 MF.
e " " " " " .06 "


Ohms
60
50
40
30
20
10
AVERAGE RESISTANCE OF NO. 229 TRANSMITTER DURING TALKING
Talking Current in Amperes
0.0
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
0.36
KEUFFEL & ESSER CO., NEW YORK, NO. 354 C



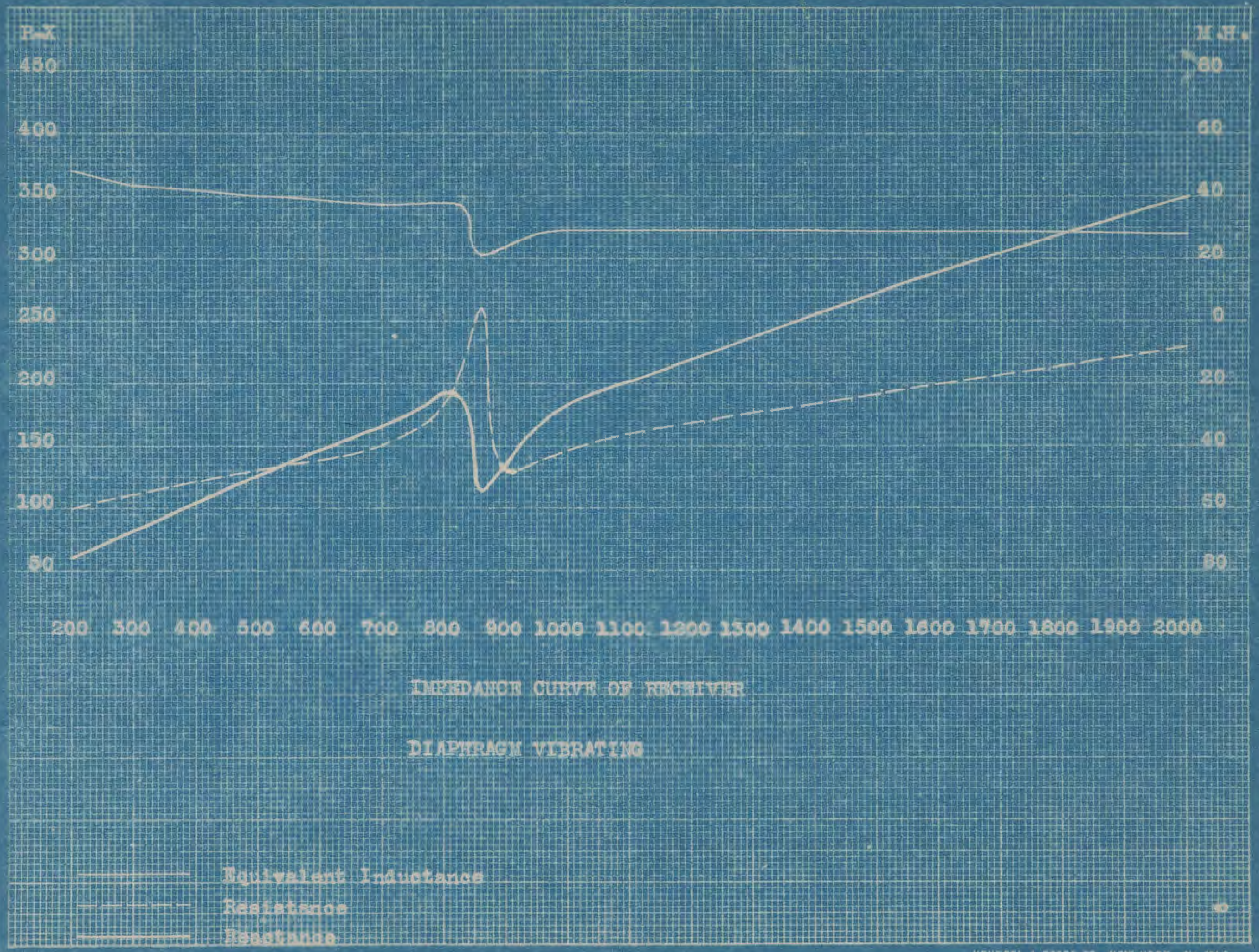
IMPEDANCE CURVE OF RECEIVER WITHOUT DIAPHREM

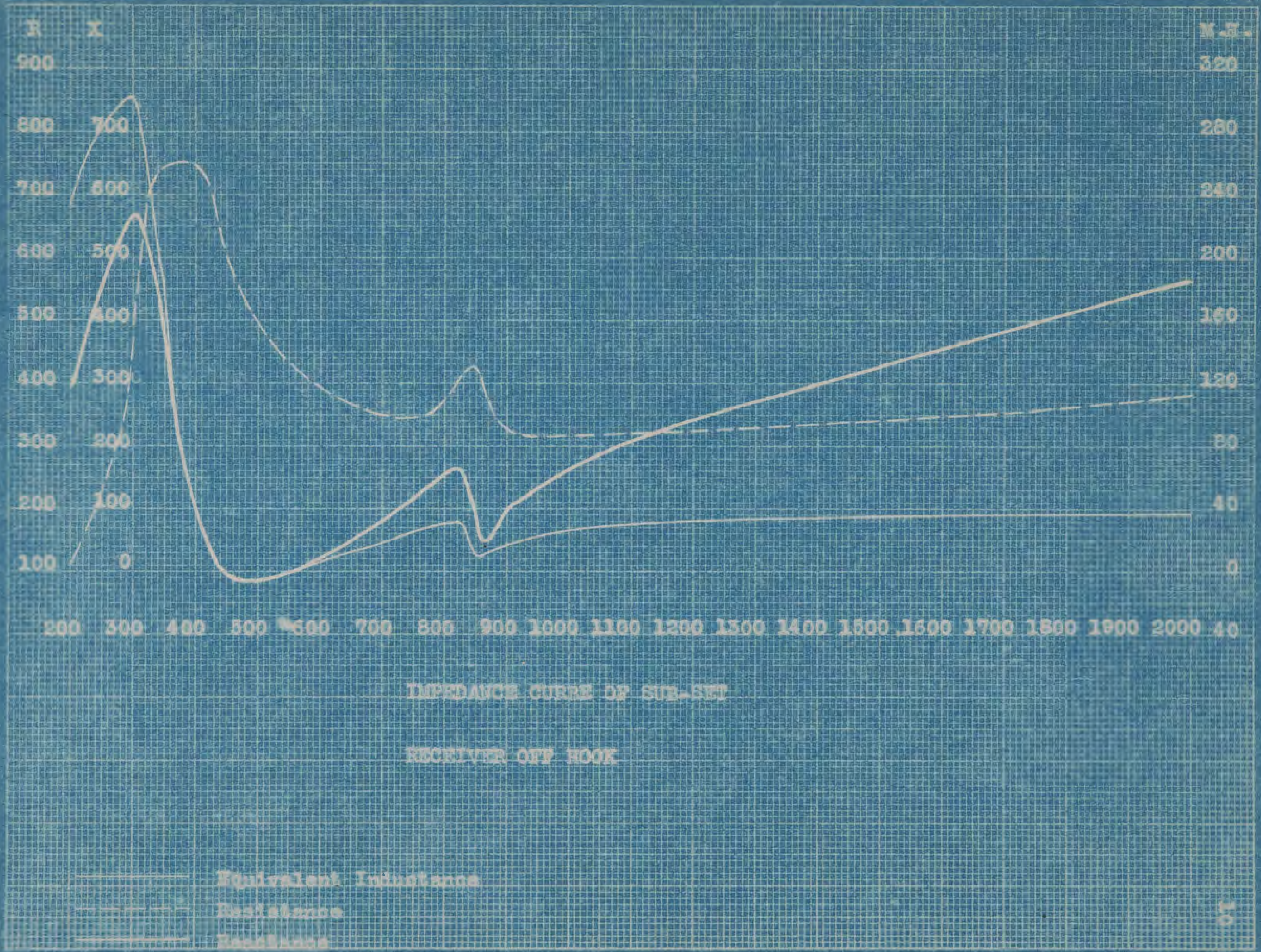
Reactance
Equivalent Inductance
Resistance



IMPEDANCE CURVE OF RECEIVER

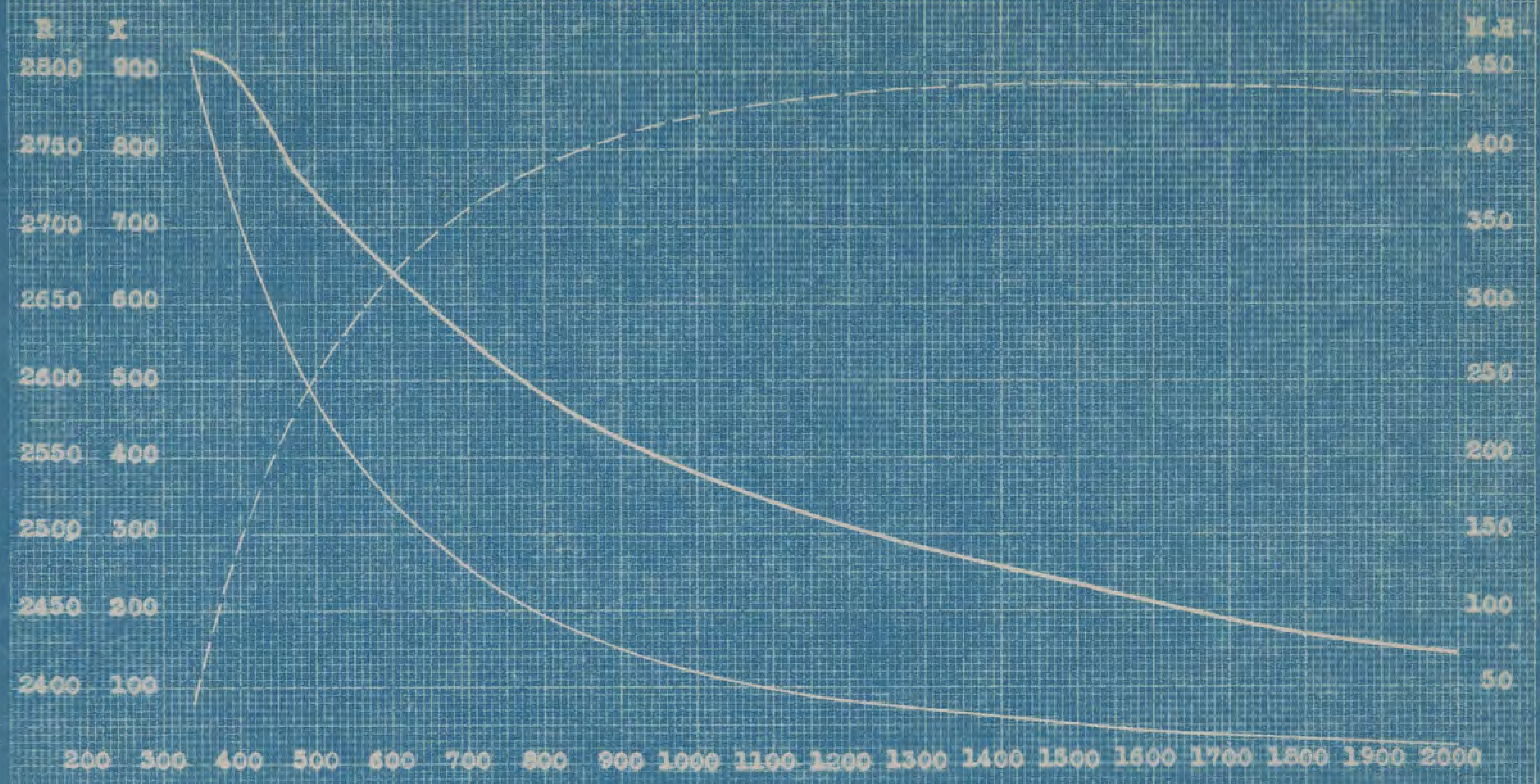
DIAPHRAGM STATIONARY


R-X
450
400
350
300
250
200
150
100
50
M.H.
80
60
40
20
0
20
40
60
80
200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
IMPEDANCE CURVE OF RECEIVER
DIAPHRAGM VIBRATING
Equivalent Inductance
Resistance
Reactance



IMPEDANCE CURVE OF SUB-SET

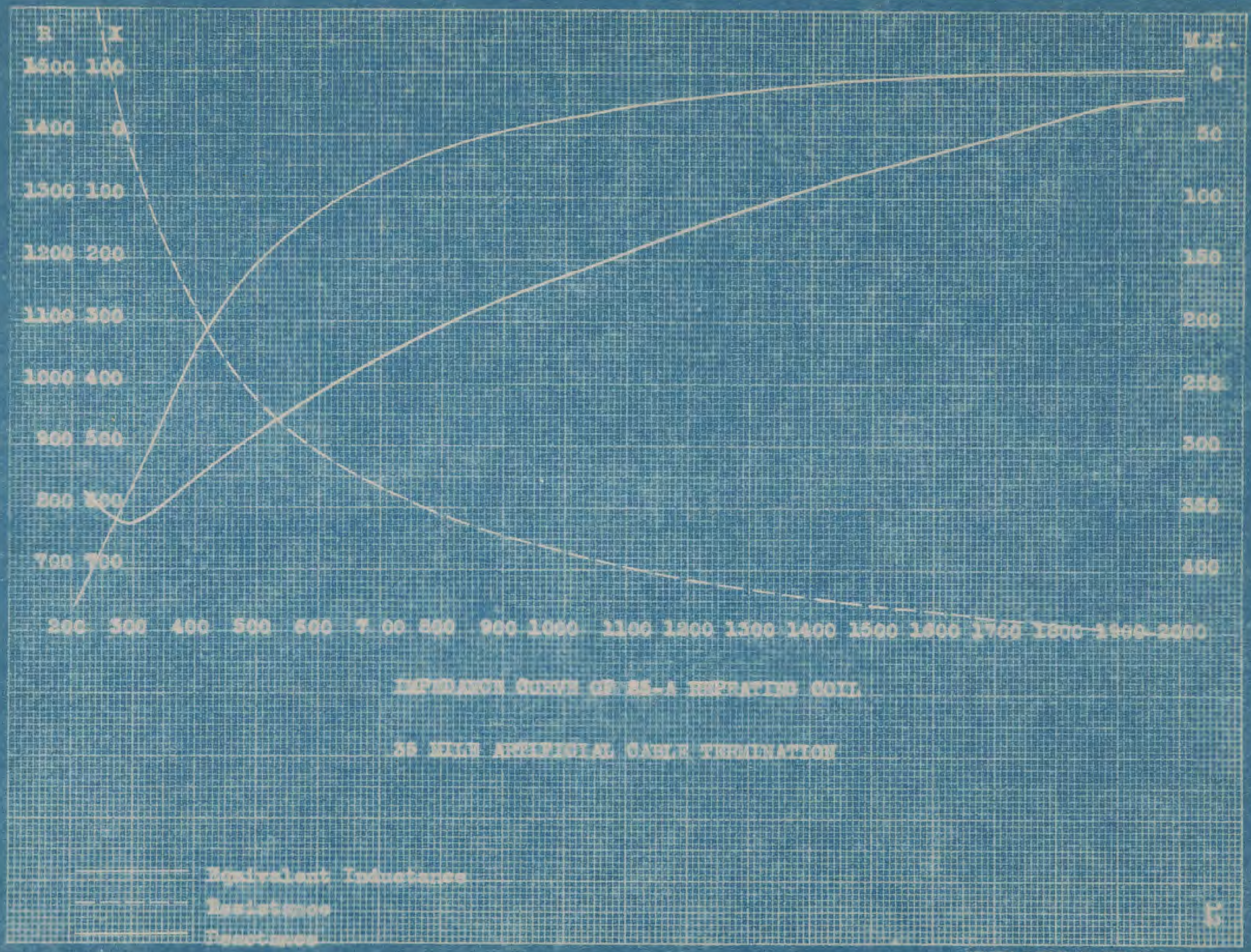
RECEIVER OFF HOOK

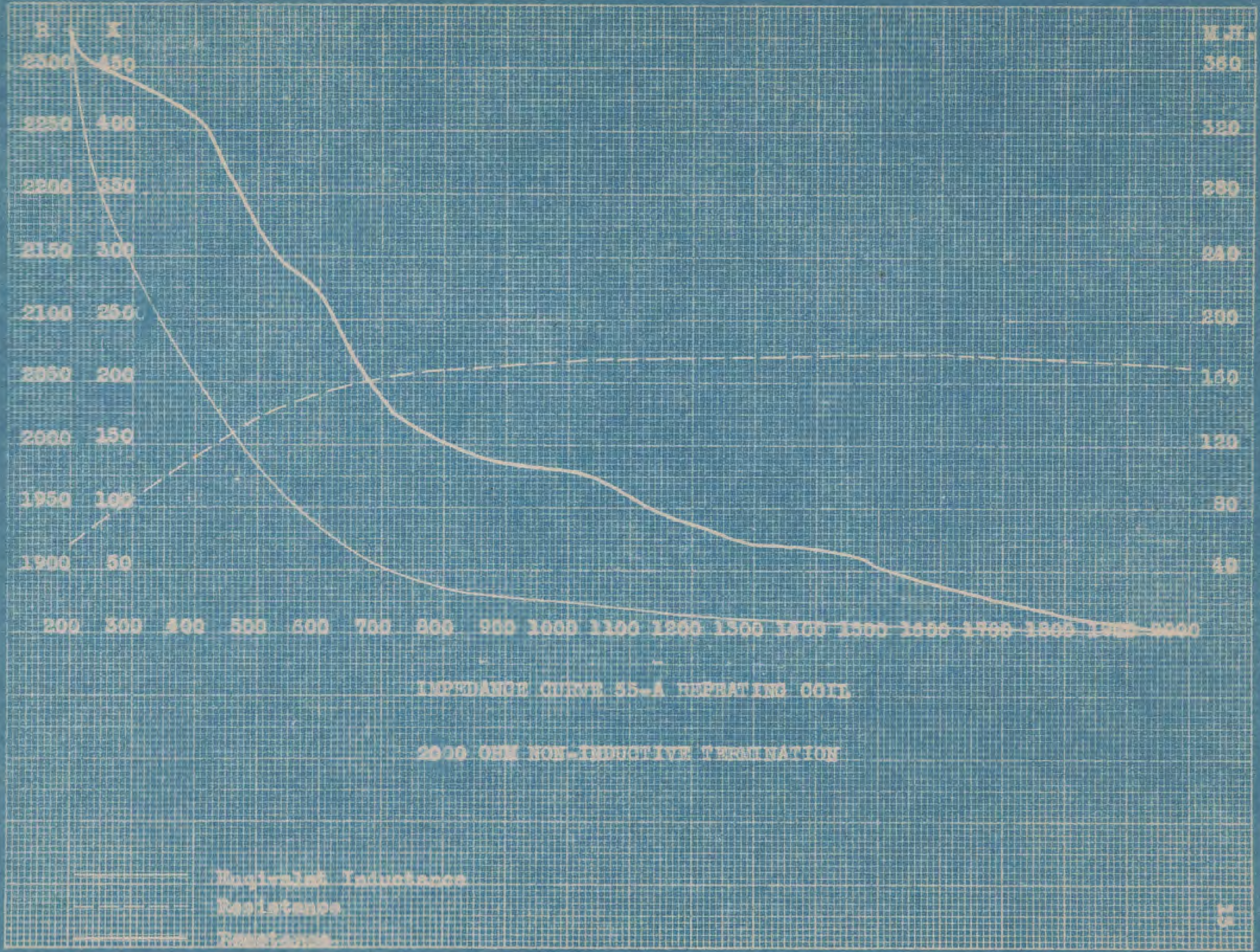


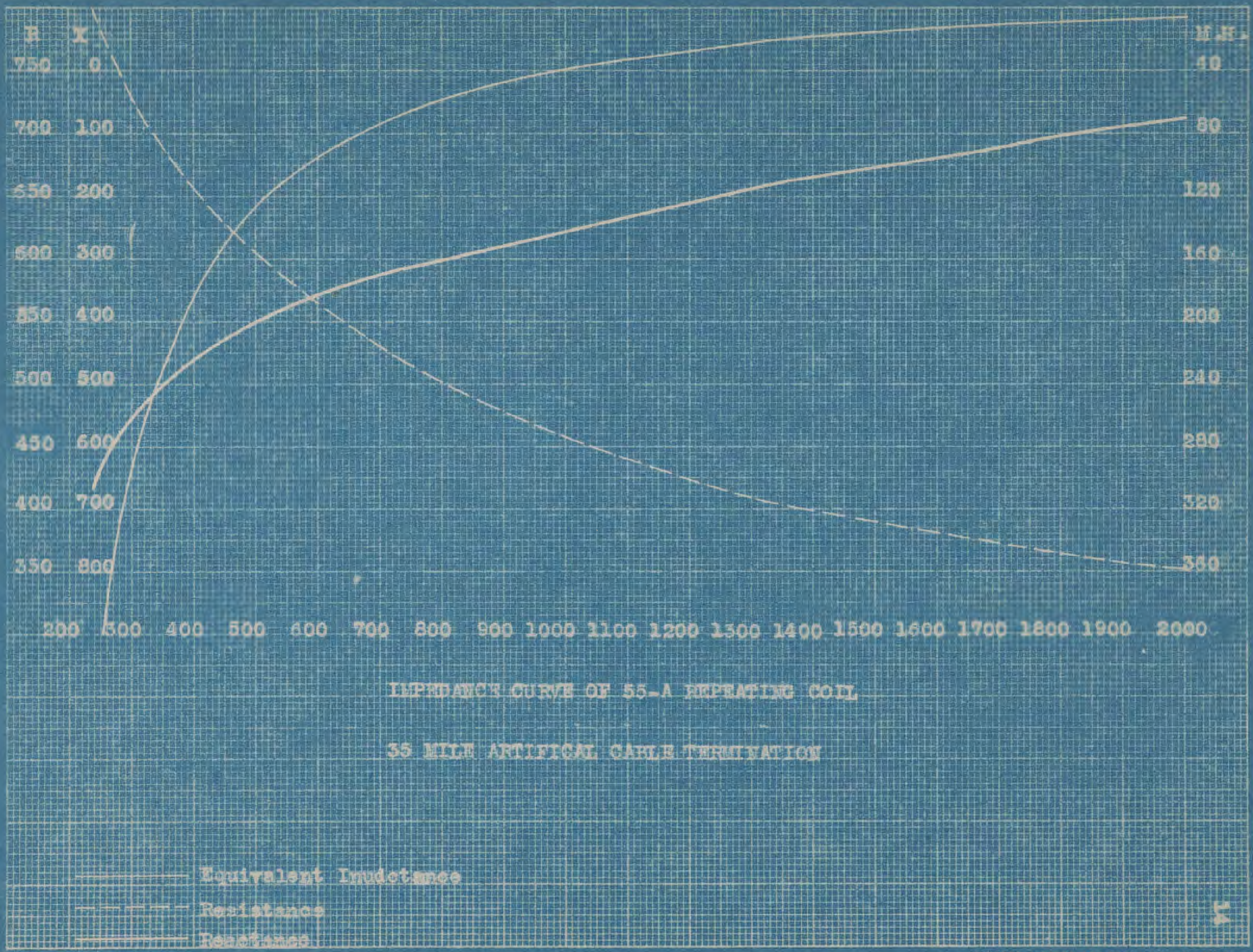
IMPEDANCE CURVE OF 25-A REPEATING COIL

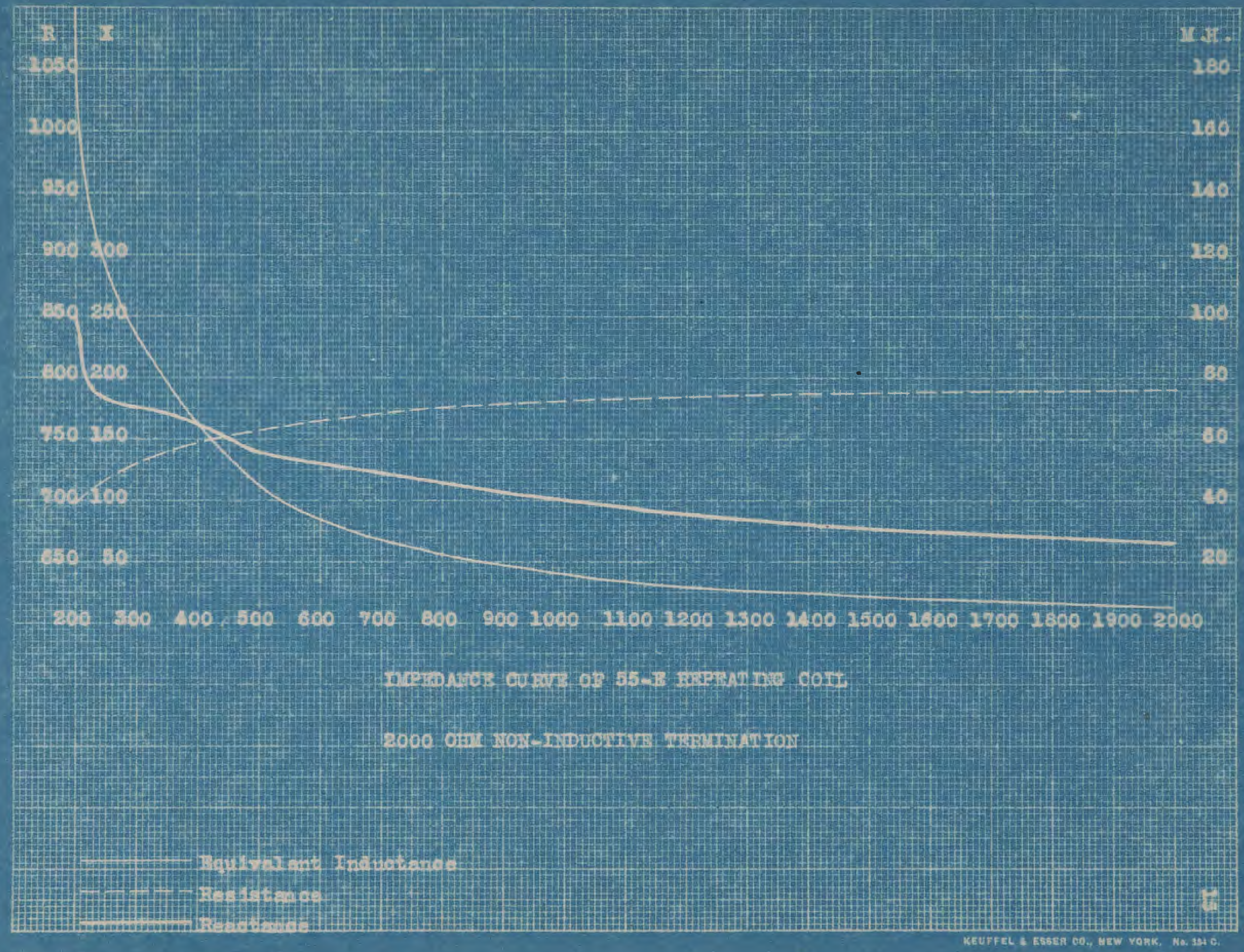
2000 OHM NON-INDUCTIVE TERMINATION

Equivalent Inductance
Resistance
Reactance

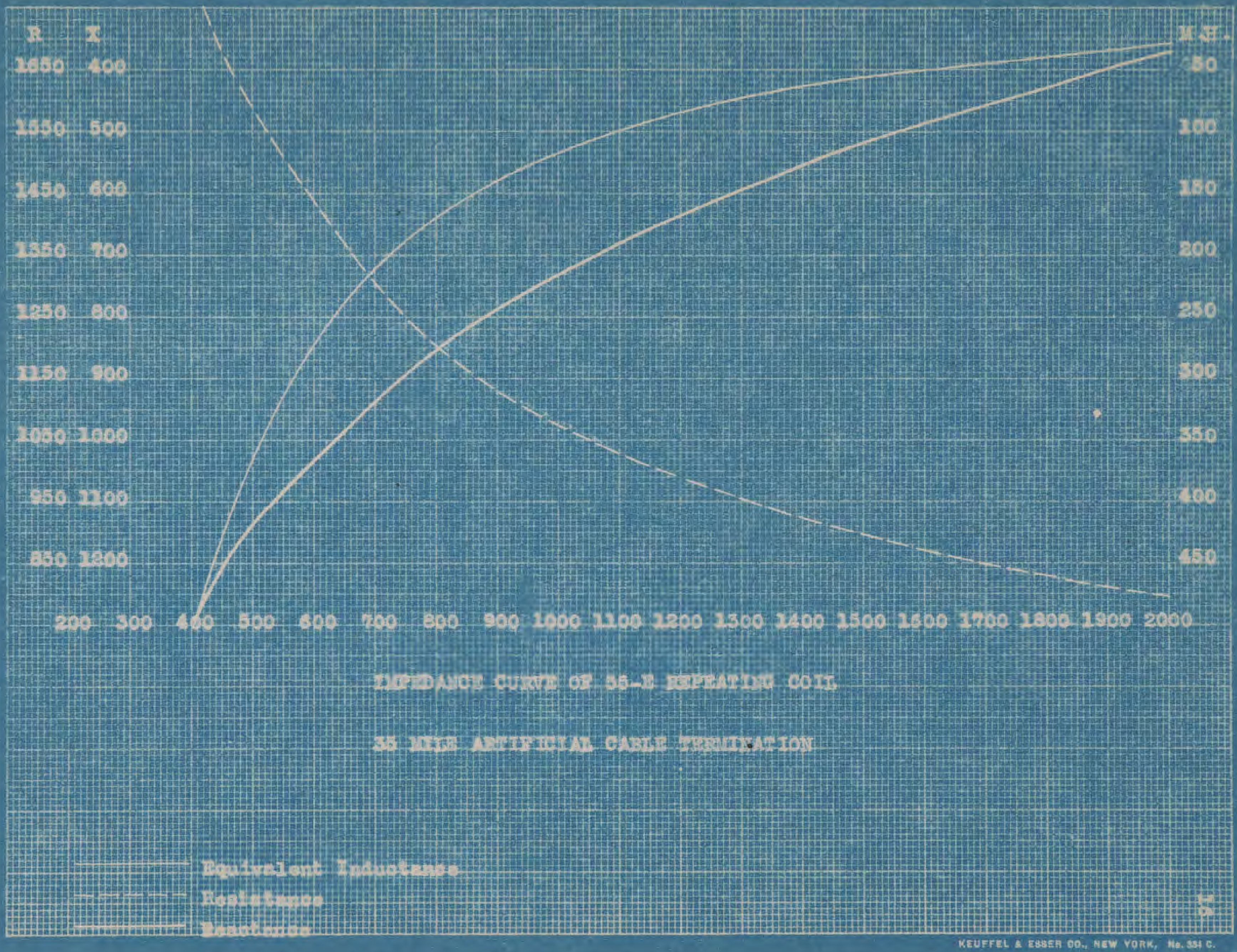

R
X
M.H.
1500 100
1400 0
1300 100
1200 200
1100 300
1000 400
900 500
800 600
700 700
0
50
100
150
200
250
300
350
400
200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
IMPEDANCE CURVE OF 25-A REPEATING COIL
35 MILE ARTIFICIAL CABLE TERMINATION
Equivalent Inductance
Resistance
Reactance
C


R
X
M.H.
2300
2250
2200
2150
2100
2050
2000
1950
1900
450
400
350
300
250
200
150
100
50
360
320
280
240
200
160
120
80
40
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
1600
1700
1800
2000
IMPEDANCE CURVE 55-A REPEATING COIL
2000 OHM NON-INDUCTIVE TERMINATION
Equivalent Inductance
Resistance

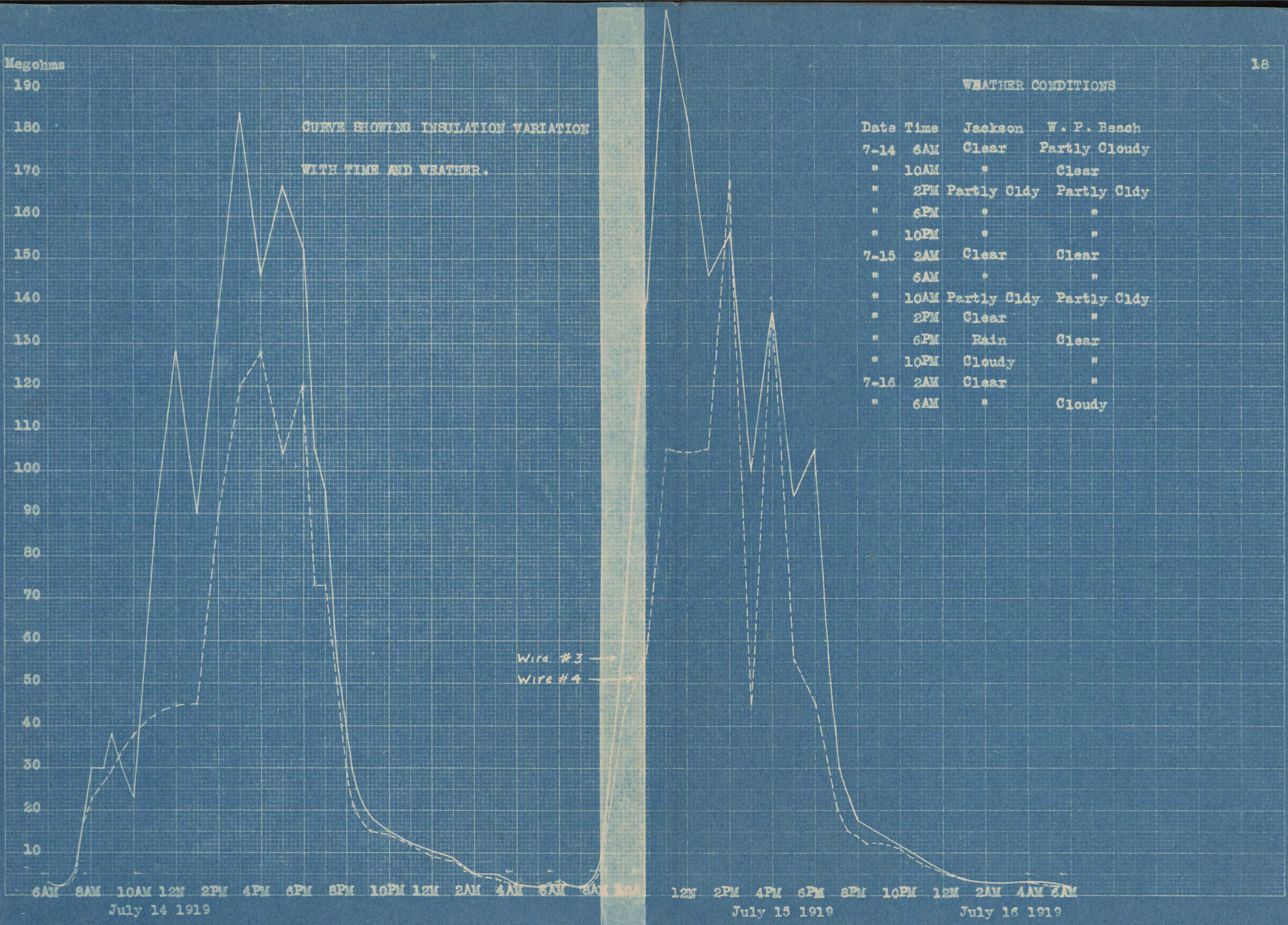

R X
750 0
700 100
650 200
600 300
550 400
500 500
450 600
400 700
350 800
M.H.
40
80
120
160
200
240
280
320
360
200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
IMPEDANCE CURVE OF 55-A REPEATING COIL
35 MILE ARTIFICAL CABLE TERMINATION
Equivalent Inudctance
Resistance
Reactance
14



R
X
M.H.
1050
1000
950
900 300
850 250
800 200
750 150
700 100
650 50
180
160
140
120
100
80
60
40
20
200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
IMPEDANCE CURVE OF 55-E REPEATING COIL
2000 OHM NON-INDUCTIVE TERMINATION
Equivalent Inductance
Resistance
Reactance
KEUFFEL & ESSER CO., NEW YORK. No. 354 C.

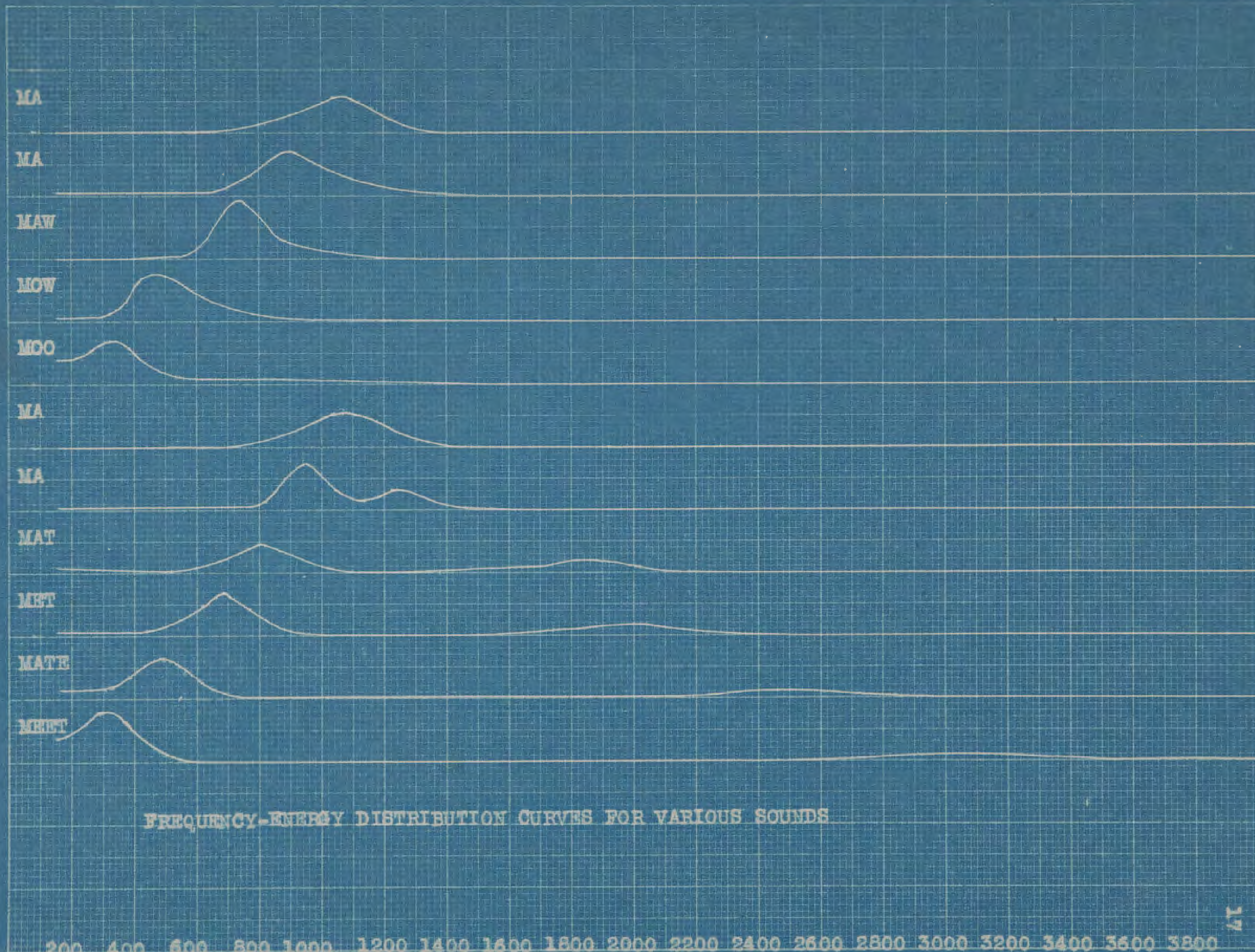

R X
1650 400
1550 500
1450 600
1350 700
1250 800
1150 900
1050 1000
950 1100
850 1200
M.H.
50
100
150
200
250
300
350
400
450
200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
IMPEDANCE CURVE OF 35-E REPEATING COIL
35 MILE ARTIFICIAL CABLE TERMINATION
Equivalent Inductance
Resistance
Reactance
KEUFFEL & ESSER CO., NEW YORK, No. 354 C.

CURVE SHOWING INSULATION VARIATION WITH TIME AND WEATHER.



WEATHER CONDITIONS

Date	Time	Jackson	W. P. Beach
7-14	6AM	Clear	Partly Cloudy
"	10AM	"	Clear
"	2PM	Partly Cldy	Partly Cldy
"	6PM	"	"
"	10PM	"	"
7-15	2AM	Clear	Clear
"	6AM	"	"
"	10AM	Partly Cldy	Partly Cldy
"	2PM	Clear	"
"	6PM	Rain	Clear
"	10PM	Cloudy	"
7-16	2AM	Clear	"
"	6AM	"	Cloudy



FREQUENCY-ENERGY DISTRIBUTION CURVES FOR VARIOUS SOUNDS

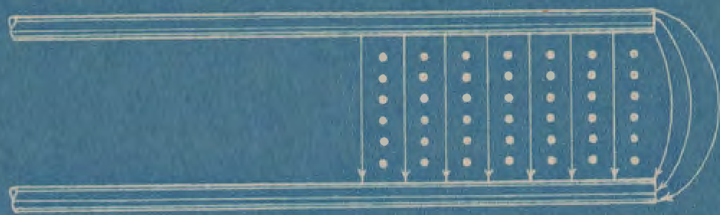


FIG. 1.

WAVE REFLECTION FROM OPEN CIRCUIT LINE

FIG. 2.

MAGNETIC FIELD OF SINGLE WIRE AND OF CIRCUIT.

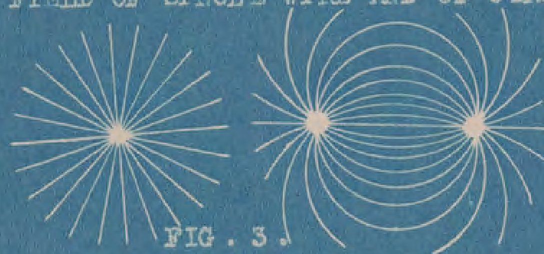


FIG. 3.

DIELECTRIC FIELD OF SINGLE WIRE AND OF CIRCUIT.

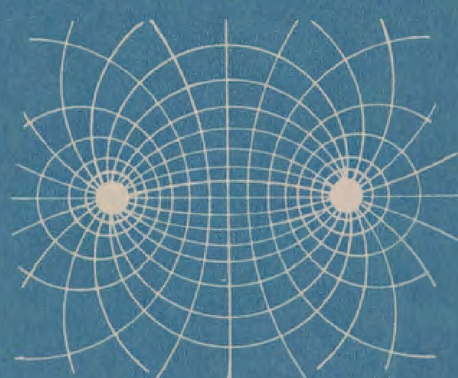


FIG. 4.

COMBINED MAGNETIC AND DIELECTRIC FIELD OF CIRCUIT.

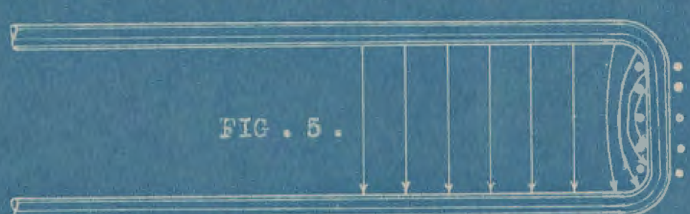
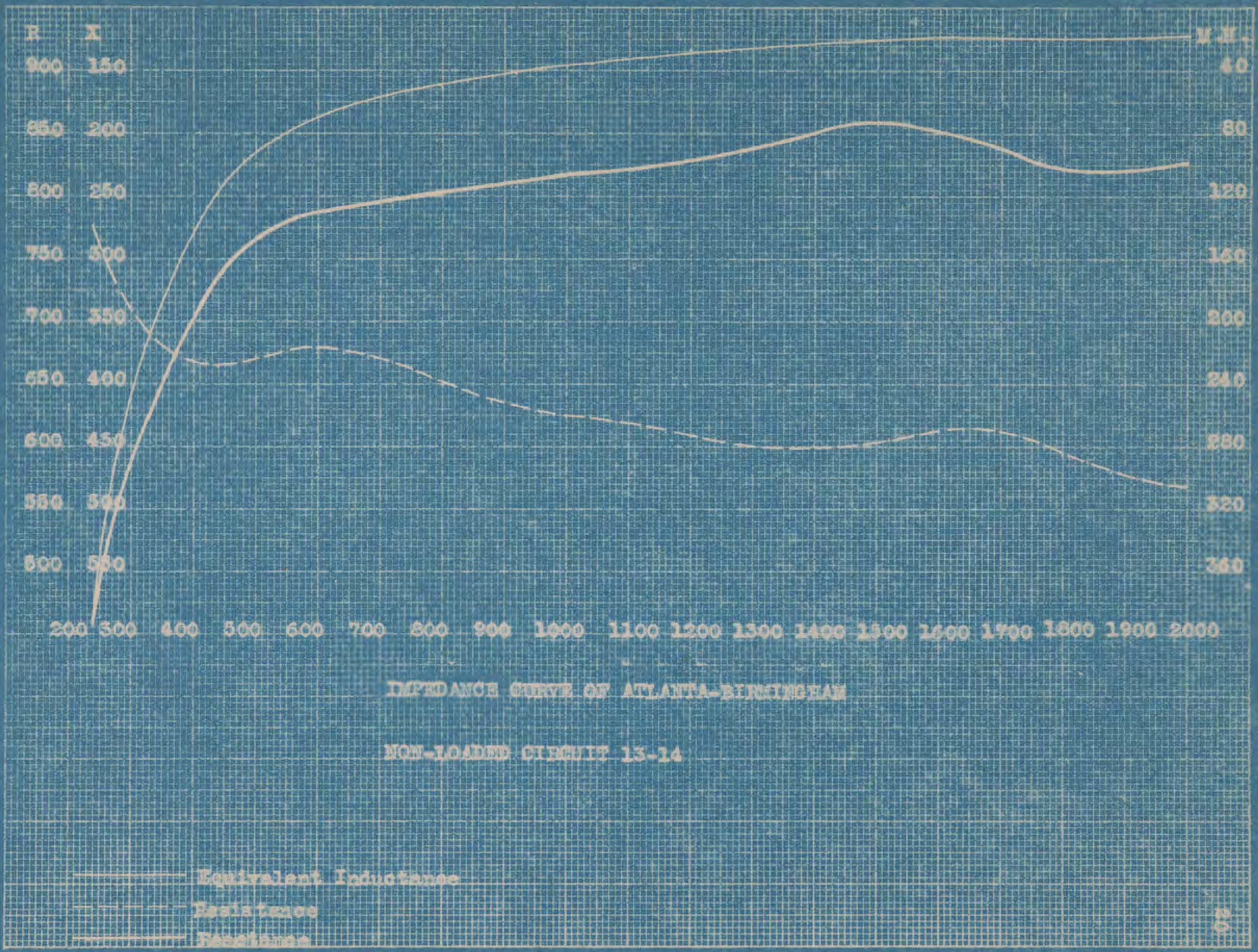
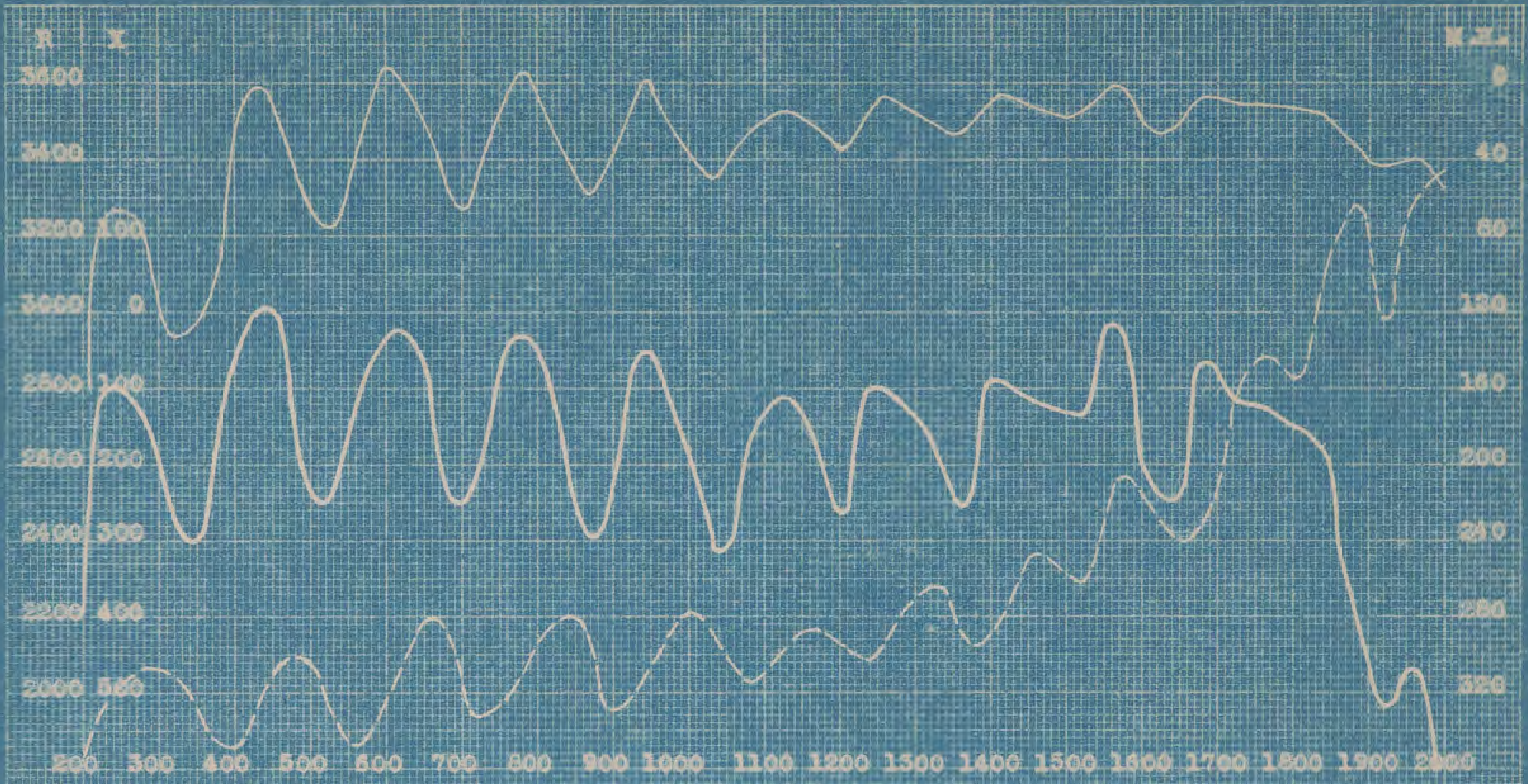


FIG. 5.

WAVE REFLECTION FROM SHORT CIRCUIT LINE.



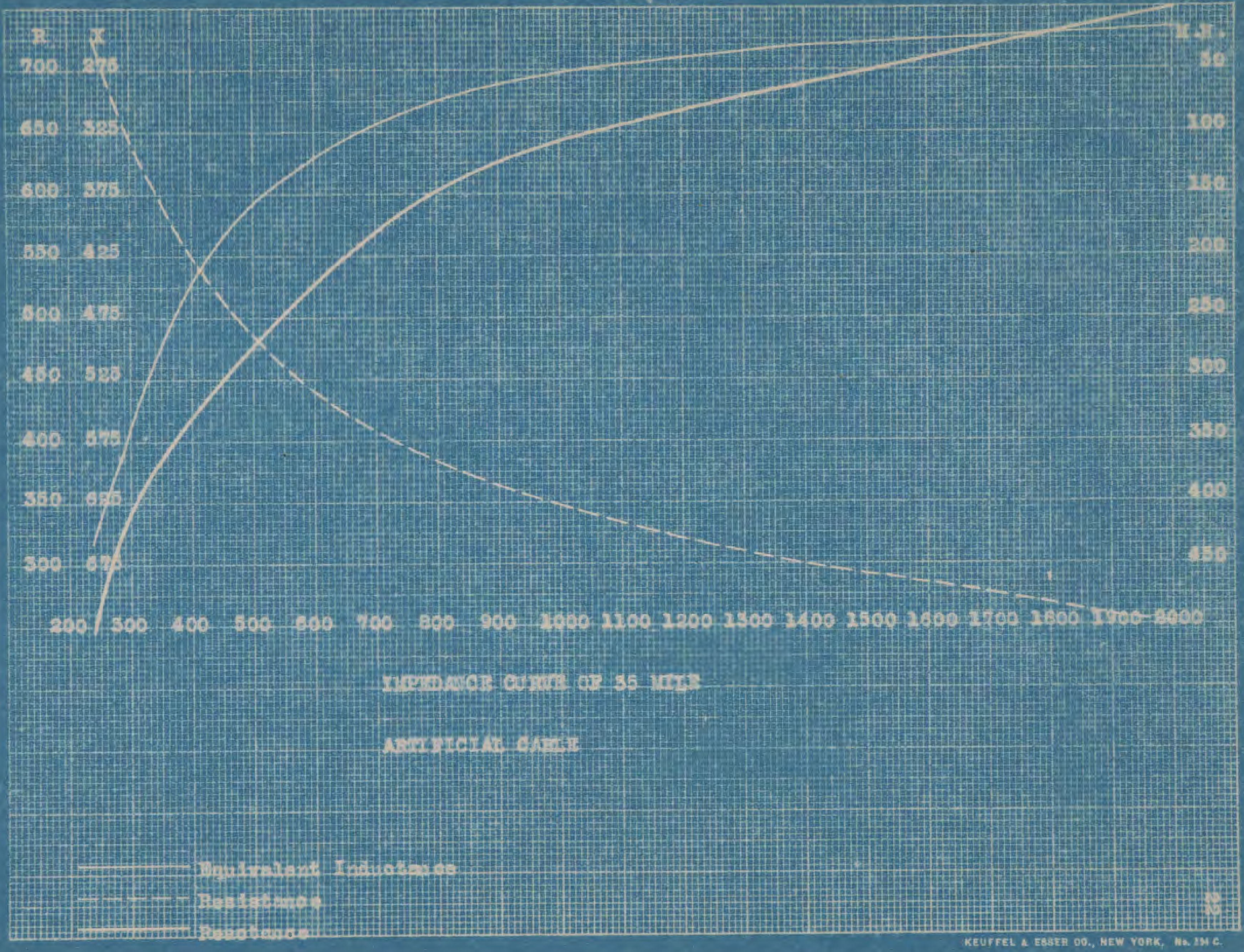
R
X
900 150
850 200
800 250
750 300
700 350
650 400
600 450
550 500
500 550
M.H.
40
80
120
160
200
240
280
320
360
200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
IMPEDANCE CURVE OF ATLANTA-BIRMINGHAM
NON-LOADED CIRCUIT 13-14
Equivalent Inductance
Resistance

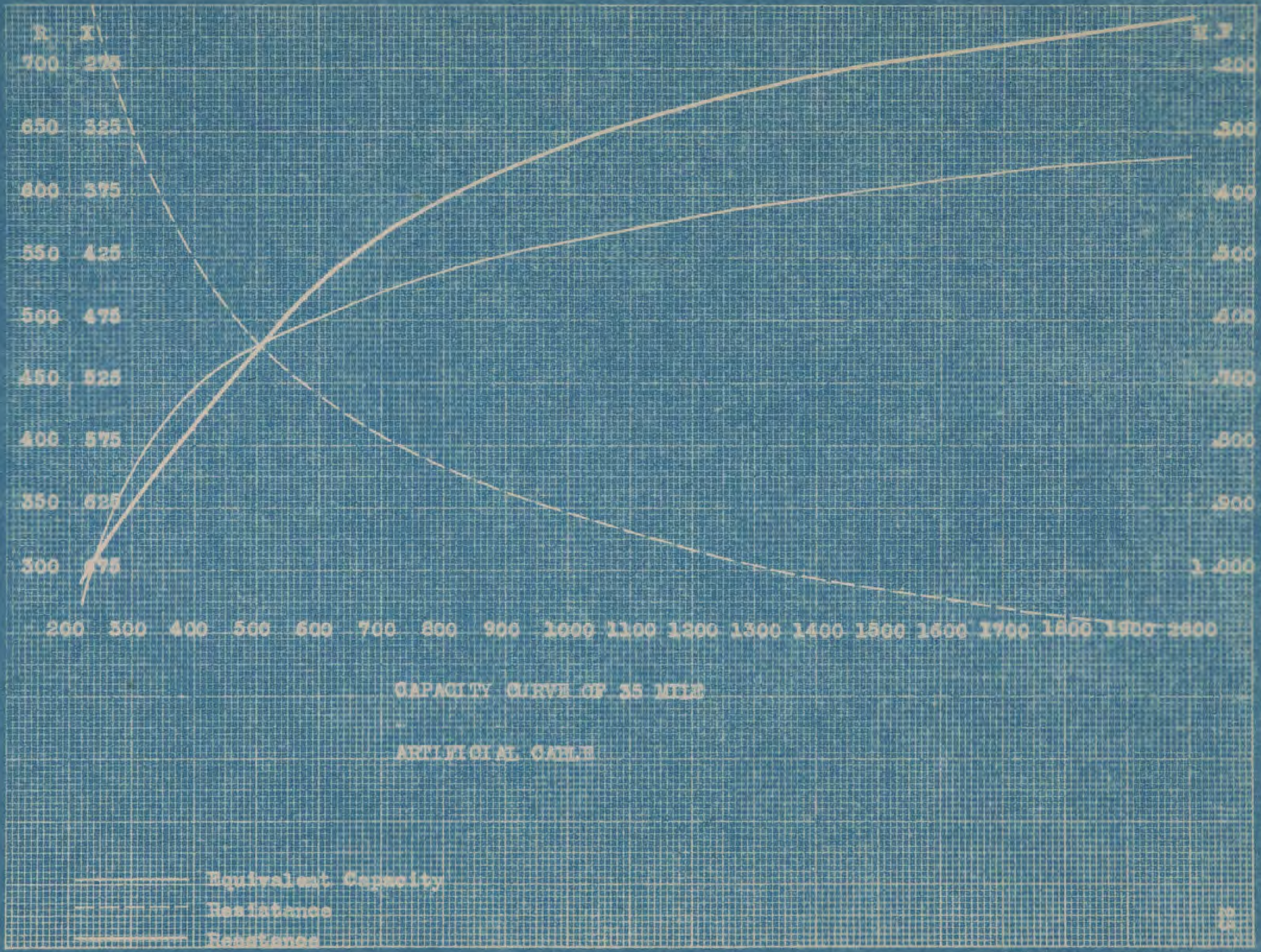


IMPEDANCE CURVE OF ATLANTA-BIRMINGHAM

LOADED CIRCUITS 3-4 AND 5-6 STRAPPED

Equivalent Inductance
Resistance


R
X
M.H.
700 275
650 325
600 375
550 425
500 475
450 525
400 575
350 625
300 675
50
100
150
200
250
300
350
400
450
200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
IMPEDANCE CURVE OF 35 MILE
ARTIFICIAL CABLE
Equivalent Inductance
Resistance
Reactance
KEUFFEL & ESSER CO., NEW YORK, No. 214 C.


R X
700 275
650 325
600 375
550 425
500 475
450 525
400 575
350 625
300 675
M.F.
.200
.300
.400
.500
.600
.700
.800
.900
1.000
200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
CAPACITY CURVE OF 35 MILE
ARTIFICIAL CABLE
Equivalent Capacity
Resistance
Reactance