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DISTRICT-NEIGHBORHOOD COVERING PROBLEMS: MODEL DEVELOPMENT AND MULTIOBJECTIVE SOLUTION TECHNIQUES

A Thesis
Presented for the
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Dedicated to all unknown scientists who treasure
the idea of a spaceship earth.

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ABSTRACT

This thesis developes several neighborhood impact location models. The obnoxious characteristics associated with public facilities are discussed, and three basic elements concerned with the minimization of neighborhood impact are introduced and a number of formulations are developed within a general multiobjective frame work.

The set of noninferior solutions to multiobjective location models are subdivided into two sets: 1) the set of strong noninferior solutions and 2) the set of weak noninferior solutions. Three routines are suggested to identify noninferior solutions. The first one is a target routine which will identify a predetermined number of solutions as close to the trade-off curve as possible, using points on the trade-off curve as targets. The second one is a modified constraint routine which will generate the set of weak noninferior solutions automatically using the set of strong solution as starting solutions. The third one is a unified noninferior set estimation routine which can be used to generate the complete set of noninferior solutions.

Two data sets which are frequently referred to in literature are used in this thesis. It is found that the routines developed in this thesis generate compatilble results efficiently. Furthermore, the problem formulations dealing with neighborhood impact are shown to identify

interesting solutions as compared to those locations models not containing neighborhood impact criteria.

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INTRODUCTION

1.0 PUBLIC AND PRIVATE FACILITY LOCATION MODELS

In 1964, Hakimi developed a procedure to locate a switching center on a communication network which could also be used to locate a police station on a highway system (1). His work stimulated research in this field. Within ten years, about 500 papers on location modeling were published (16). These models have been developed for a wide variety of purposes.

Revelle, Marks, and Liebman (2) have classified facility location models into two broad categories: 1) Private Sector and 2) Public Sector.

The major difference between public and private facility models rests on the identification of a realistic objective function. Any optimal location model solution is based on the assumption that objectives are usually well defined and quantifiable. For private enterprises, the main objective is to maximize the profit or the profit-to-cost ratio. The cost of the material used to manufacture marketable items, the profit per item, costs of labor, transportation and capital costs are well defined. Constraints such as the minimum number of items required, the maximum quantity of raw material available, and maximum production capacity are usually known. Even the detrimental aspects, such as the emission of pollutants to the atmosphere, the deterioration of water quality, the excessive noise levels, which are frequently limited to various degrees by Federal or local laws, can be incorporated as constraints to the private facility location models.

Unlike private facility location models, the objectives in public facility location models are varied and not easily quantified. These objectives involve many of the concerns of the private sector and includes other social and political concerns as well. The decision maker, charged with the responsibility of locating public facilities, is responsible to the interests of different groups. The main concerns of public facility location models may be social costs, social benefit, health hazards, environmental degradation, aesthetic, ecology, equity, to name but a few. Even if these objectives can be quantified, they are not usually measured in commensurate units. In fact, for this very reason Liebman defined public facility location problems as wicked problems (3). "In wicked problems," says Liebman: "the set of alternatives is too large, too diverse, and too little agreed upon. The decision-maker has difficulty even considering their ranges, or defining their boundaries. The selection of a solution from among alternatives involves, to a very large degree, resolution of conflicting goals, a function which should not done by models."

The main function of an analyst is to obtain the problem description from the decision maker, formulate the description in mathematical form, solve and generate different alternative solutions, and let the decision maker make his choice after weighing the interests of different groups.

This thesis deals with special properties of public facility location models. A wide variety of public sector location models have been considered by researchers. ReVelle et al. (2) suggested two main groups: 1) ordinary services and 2) extraordinary or emergency services. Ordinary services location models include location of post offices, schools, highways, public buildings, parks, public housings, environmental services such as water treatment, liquid and solid waste disposal facilities, offshore oil platforms, nuclear and fossil fuel power plants, rain gages, and air monitors. In emergency or extraordinary service models, the location of fire stations, police stations, emergency ambulance stations, hospitals, and civil defense units are included.

2.0 TWO UNDERLYING MODELS

Of all of the above location problems, two general models appear to form the basis for many applications: The p-Median problem initiated by Hakimi (1) and the Maximal Covering Location Problem (MCLP) by Church and ReVelle (5).

Hakimi defined the p-Median problem as an approach to find the optimum location of a switching center in a communication network such that the total length of wire is minimized. The p-Median problem (PMP) can be defined in the following way:

> Minimize the total weighted travel distance associated with a network of demand nodes by locating p-facilities on the network (on arcs or at nodes) where each demand point is served by its closest facility.

In the Maximal Covering Location Problem (MCLP) , Church and ReVelle (5) tried to maximize the population which can be served within a stated service distance or time, given a limited number of facilities. The Maximal Covering Location Problem (MCLP) is defined as (5):

> Maximize coverage (population covered) within a desired service distance S by locating a fixed number of facilities.

The PMP has been applied to locate power plants (7), health clinics (10), regional prisons (8), schools (61), solid waste wastesheds (34), and waste water treatment plants (20) whereas the MCLP has been applied to locate oil platforms (61), air monitors (59), rain gages (63), fire stations (30), ambulance depots (2), and health clinics (60).

3.0 NEGATIVE-BENEFIT ASPECTS OF A FACILITY

The PMP and MCLP consider only the beneficial aspects of a location. Many facilities, e.g. power plants, sewage treatment plants, solid waste disposal sites, incinerators, regional prisons, fire stations, ambulance stations, defense units are obnoxious or semi-obnoxious due to environmental pollution, aesthetic value, or safety reasons. Even though these facilities are indispensible for the convenience and the benefit of people living in a community, no one wants to suffer the nearby direct impact or negative benefit associated with these facilities.

Many authors have implicitly or explicitly expressed concerns on the negative impacts of a facility. Orloff has written a paper exclusively on this subject (9). He suggested that the area immediately surrounding a population center should be regarded as an infeasible area for facility location because the neighboring impact would be too high if a facility was placed there. His objective was to minimize the number of facilities needed to serve all the population centers.

In considering the possible impact of a power plant on its neighboring population along with other objectives, Church and Cohon (7) employed two criteria. The first one was to minimize the population or the exposure within the impact area when locating a set of power plants. The second one was to maximize the weighted distance which was calculated as the product of the distance of a population center to its closest power plant and the population of the center.

Church and Garfinkel (8) investigated a one-facility maximum median (maxian) problem. They dealt with the problem of locating a point on a network so as to maximize the sum of its weighted distance to the nodes.

Austin(6) and Church and Bell (12) investigated the general benefit function which varies with distance. This benefit function might represent an algebraic sum of all related parameters. Their problem was to maximize the total benefits associated with serving a network of demand nodes by locating a number of facilities on the network.

4.0 MULTIOBJECTIVE MODELS

The papers mentioned previously are mostly aiming at the optimization of one objective -- namely weighted distance, coverage, or impact to the neighborhood. In the real world (especially in locating public facilities) where different purposes should be fulfilled, many objectives are to be considered simultaneously. These objectives may concern aggregate travel distance, coverage of demands, neighborhood impact, balance of load, air quality, water quality, noise level, odor, ecological maintenance, worse case performance, equity, and valuable resource preservation, among others. If it is economically and technologically feasible, many of these objectives should be formulated and solved as a multiple objective problem. The multiobjective problem is important because in considering different objectives simultaneously, a solution which is good or satisfactory in all objectives may be obtained. If not, a set or a subset of noninferior trade-off solutions which provides the decision maker (DM) with different choices may be found. Church and Cohon (7) defined a noninferior solution in the following way.

> A solution is noninferior if there is no other feasible solution which yields a higher value of one objective without yielding lower values of at least one other objective.

Hwang et al.(15) did a relatively complete survey on the methods and applications of Multiple Objective Decision Making (MODM). All methods were classified, elucidated, and then illustrated by a simple example. Application of these methods to 12 classes of real world problems were briefly explained. The methods are explained in Chapter One of this thesis.

A review of most of the existing methods of MODM was also given in a survey by Cohon and Marks (20). Moreover, the advantages and disadvantages of the methods were discussed by Church and Cohon (7).

5.0 OBJECTIVES OF THIS THESIS

The objectives of this thesis will be twofold. The first objective is to further investigate neighborhood impact on location modeling. In this investigation, the obnoxious characteristics associated with a facility will also be discussed. Moreover, a number of formulations are given for different neighborhood impact assumptions within the context of the problem used in this thesis.

The second objective is to formulate a multiobjective problem using weighted distance, coverage, and neighborhood impact as objectives. The set of strong noninferior solutions (defined in Chapter III) will be generated by solving this formulation using a weighting method. The set of weak noninferior solutions (also defined in Chapter Three) will be investigated by three routines. The first one is a target routine which will identify a predetermined number of solutions as close to the trade-off curves as possible, using points on the trade-off curves as targets. The second one is a modified constraint routine which will generate the set of weak noninferior solutions automatically using the set of strong solution as starting solutions. The third one is a unified noninferior set estimation routine which will generate the complete set of noninferior solutions directly. The MPSX Linear programming system (51) is employed to solve the problems optimally, and a modified Teitz And Bart algorithm is implemented to solve the problems heuristically.

The Reverse Teitz And Bart (RTAB) routine by Church and Huber (19) will be used to generate a band of close-to-optimal solutions for comparison. The results of computer runs using routines developed will be presented in the last chapter.

CHAPTER I

LITERATURE REVIEW

1.0 INTRODUCTION

This chapter presents a general review of network location models. For models which are historically important or which underlie this thesis, the objectives, formulation, constraints, definition of variables, and the techniques to find the solution will be discussed.

2.0 LOCATION ANALYSIS IN A CONTINUOUS DECISION SPACE

In 1926, Weber (21) considered the location of a factory between two resources and a single market. This problem was defined on a continuous plane. Since that time, Kuhn and Kuenne (22), Cooper (23), and Weisfield (53) have independently developed iterative solution processes for solving a generalized Weber problem. The generalized Weber problem is one where several factories are located between the same resource base and many market centers. Each factory would serve the markets that are closest to it. An extended Weber problem which identifies one facility to serve two or more demand points can be defined as:

$$\text{Minimize} \quad Z = \sum_{i=1}^{n} Wi * \left[(Xi-Xp)^2 + (Yi-Yp)^2 \right]^{1/2} \quad . \ . \ . \ . \ (1\text{-}1)$$

where:

Wi = the weight attached to the ith point (resource, population, or goods demanded)

Xi,Yi= the location of the ith point relative to some fixed Cartesian Co-ordinate system

Xp,Yp= the unknown co-ordinate of the central point p

n = the number of markets which are to be served

The objective is to minimize the weighted distance for all users receiving commodities from their closest factories. The only unknowns are Xp and Yp. Since the objective function is continuous and differentiable, partial differentiation can be used to find the condition of a global maximum.

$$\frac{\partial Z}{\partial X_p} = \sum_i W_i \left(X_i - X_p \right) \Big/ d_{ip} = 0 \qquad \text{(1-2)}$$

$$\frac{\partial Z}{\partial Y_p} = \sum_i W_i \left(Y_i - Y_p \right) \Big/ d_{ip} = 0 \qquad \text{(1-3)}$$

Expressing Xp and Yp in (1-2),(1-3) as dependant variables,

$$X_p = \sum_i \frac{W_i X_i}{d_{ip}} \Big/ \sum_i \frac{W_i}{d_{ip}} \qquad \text{(1-4)}$$

$$Y_p = \sum_i \frac{W_i Y_i}{d_{ip}} \Big/ \sum_i \frac{W_i}{d_{ip}} \qquad \text{(1-5)}$$

The iterative method for solving the extended Weber problem is:

1. Choose an arbitary starting point, for example, the centroid of the demand points, as Xp and Yp.
2. Calculate Xp, Yp from equation (1-4) and (1-5).
3. If the differences between the current Xp, Yp and the previous Xp, Yp are negligible, Stop. Otherwise, go to step 2 with new values of Xp and Yp.

Due to geographical, political, resource limitation, or other constraints, the continuous decision space assumption is not always valid. In 1964, Hakimi (1) attacked the problem of locating a central point on a network of discrete decision space. His objective was to find the optimum location of a switching center on one of the nodes of a communication network such that the total length of connecting wire was minimized. His work initiated much interest in the field of facility location on a network. The generalized form of this problem is:

> Minimize the total weighted travel distance associated with a network of demand nodes by locating p-Facilities on the network (on arcs or at nodes) where each demand point is served by its closest facility.

This problem is called the p-Median problem. Hakimi was able to show that there will always be an optimal solution to this type of problem which will consist entirely of nodes. This important proof forms the basis for most p-Median solution techniques that search for the best p-Nodal solution.

3.0 p-MEDIAN PROBLEM

In 1970, ReVelle and Swain formulated the p-Median problem for the location of p-Facilities on a network of N demand points and solved their formulation optimally by linear programming. The formulation is defined as follows:

$$\text{Minimize } Z = \sum_{i=1}^{N} \sum_{j=1}^{N} Ai*Dij*Xij \qquad \ldots\ldots\ldots (1\text{-}6)$$

subject to:

$$\sum_{j=1}^{N} Xij = 1 \qquad i = 1,2,\ldots,N \qquad (1\text{-}7)$$

$$Xjj - Xij \geqslant 0 \quad \text{for all } i,j \text{ pair, } i \neq j \qquad (1\text{-}8)$$

$$\sum_{j=1}^{N} Xjj = P \qquad \ldots\ldots\ldots\ldots \qquad (1\text{-}9)$$

$$Xij = (0,1) \qquad \ldots\ldots\ldots\ldots \qquad (1\text{-}10)$$

where:

N = the number of demand points

i = the index of a demand node

j = the index of a potential site

$Xij = \begin{cases} 1 & \text{if } i \text{ is served by } j \\ 0 & \text{otherwise} \end{cases}$

Ai = the demand at node i

Dij = the shortest distance between node i and potential site j

P = the number of central facilities allowed

The objective is to minimize the total weighted distance in locating P facilities. This objective is one of the most important measures of the effectiveness of a given locational configuration. The smaller the average distance, the less costly is the transportation, and the more accessible the system is to its users. In fact, the minimization of transportation cost is one of the main concerns of both private and public location analysis.

Constraints of type (1-7) insure that each node i is fully assigned. Constraints of type (1-8) limit assignment to only those nodes which are self assigned. Constraint

(1-9) restricts the number of facilities (self assignments) to equal P. Contraints of type (1-10) require that the variables Xij may only be 1 or 0.

In this formulation the distance matrix can be asymmetric, i.e. Dij may not be equal to Dji, but all demand nodes are assumed to be potential facility sites. This assumption is not always true. Many demand points may not satisfy the geographic, resource, or human factor requirements of a facility location. We can generalize the above formulation to take this fact into account by summing j over J which is a set of feasible sites, instead of summing j over 1 to N.

3.1 PMP solution techniques - optimal

The above PMP formulation can be solved by linear programming by relaxing the integer requirements (contraints of type (1-10)) to non-negativity requirements. The solution of this relaxed problem (the Xij values) may or may not be all integer. If Xij are all integers, the optimal solution to the integer formulation is found. This happens with great regularity (80% of the time (5)). If they are not all integers, the branch and bound method can be employed to obtain an optimal integer solution.

Other methods that can be used to solve the PMP optimally were developed by Garfinkel et al.(52), Khumawala (26), El-Shaieb (13), Narula et al.(56), Cornuejols et al.(57), and Geoffinion (55). The relaxation methods with subgradient optimization (e.g. Narula et al.(56), Cornuejols et al. (57)) appear to be the best procedures (62) for solving the p-Median problem.

3.2 PMP solution Techniques - Heuristic

For large problems, which frequently occur in practical applications, many of the optimal approaches may not be usable. Heuristic methods are developed with the goal of faster computational time or finding a solution in the absence of an optimal technique. Of course, the weakness of the heuristic methods is that optimality cannot be guaranteed.

3.2.1 PMP Solution Technique: Maranzana Heuristic

Maranzana (24) developed a heuristic method to solve the PMP by partitioning. Starting with a random (or any logical) solution of p-Facility locations, demand points are assigned to their closest facility thereby forming P partitions of demand points. A new facility is chosen in each partition to give best service to all demand points within that partition. Then new partitions are developed using current facility locations. The heuristic cycle is carried on until no improvements are possible.

This method looks attractive at first sight, because it agrees with the intuition of dividing the decision spaces into P partitions within which P separate medians can be found. Nevertheless, the results of this approach rely heavily on the starting solution, and the chance of getting an optimal solution is small (25,27). In spite of its weakness, this approach is still very appealing because of its low computational cost.

3.2.2 PMP Solution Technique: Teitz and Bart Heuristic

A frequently used heuristic method for solving the PMP was developed by Teitz and Bart (25). To find p-Facility locations to serve a set of demand nodes, this method starts with a random initial solution of p-Facility locations, then each potential facility is tried in turn to substitute for one of the current facilities. If there is improvement in the objective, the potential facility will be encoded as a current facility, and the facility to be removed will be the one that gives the largest improvement in the objective function upon removal. The heuristic terminates when no further substitution of current facilities by any potential facility is possible.

Mathematically, the Teitz and Bart Method can be expressed in the following way:

Let:

Dij = the shortest distance between nodes i and j

Wij = the weighted distance of serving demand i
= Ai*Dij

Ai = the demand at i

iZbj = the change in weighted distance associated with demand i, when the potential facility at site b replaces the facility at site j

Zbj = the total net change in weighted distance when the potential facility at site b replaces the facility at site j

$= \sum_{i} iZbj$

F = the set of current facility sites

k = the index of a site in F such that facility site k is closest to i

$$Dik = \min_{j \in F} Dij$$

F' = the set of current facility sites excluding site k
$= F - \{ k \}$

s = the index of a site in F such that facility site s is second closest to i

$$Dis = \min_{j \in F'} Dij$$

The term iZbj is defined as the change of weighted distance when potential site b replaces site j when only demand node i is considered. For example, let site k be the closest facility to node i before replacement of site j by potential site b, and let site b be the closest facility to node i after substitution, then the weighted distance was Wik; the new weighted distance is Wib. Therefore the change in weighted distance, iZbj, is equal to the difference of Wib and Wik.

Suppose that one of the current facilities is to be replaced by a potential site b, the following steps are necessary to determine the net effect of such a replacement:

1. Find iZbj for each $j \in F$

2. Find Zbj by summing iZbj (over i) for $j \in F$

3. Find the minimum of Zbj for all $j \in F$

4. For this minimization problem, if the minimum Zbj is less than zero, there is decrease in total weighted distance, then potential site b can replace j to gain a better value of the objective.

To find iZbj, the net change in weighted distance associated with demand i upon replacement of site j by site b, three cases need to be considered.

1) Dib $\leqslant$ Dik $\leqslant$ Dis

2) Dik $<$ Dib $\leqslant$ Dis

3) Dik $\leqslant$ Dis $<$ Dib

CASE 1). Dib is shortest:
Upon substitution of b for any of the current locations $j \in F$, demand i will be assigned to b, the change in weighted distance is:

iZbj = Wib - Wik for all $j \in F$

CASE 2). Dib greater than Dik but less than Dis:
The weighted distance associated with demand i will not be changed if j is not the closest facility to i:

iZbj = 0 for all $j \in F'$

But if b replaces k (the closest facility to i),

iZbk= Wib - Wik

CASE 3). Dib greater than Dis:
In this case the second closest facility becomes the closest when b replaces k. Then:

iZbk = Wis - Wik
iZbj = 0 for $j \in F'$

After all iZbj are found for each i and j, the total change in weighted distance by replacement of any j F by a site b can be computed by summing iZbj over i. This method usually out performs the Maranzana method in solving the PMP (2).

Church and Bell (12) generalized this method to maximize benefit functions which vary with distance. The term iZbj was defined as change of benefits instead of change of weighted distances. Zbj was summed up in the same manner. To maximize, the largest Zbj was chosen. If it was greater than zero, then replacement is made.

The modified Teitz and Bart (MTAB) heuristic is applicable to all location problems with the following conditions: 1) the benefit function varies with distance and 2) the demands are serviced by the closest facility. These assumptions are implied in all models in this thesis, thus the MTAB will be used extensively.

4.0 SET COVERING AND MAXIMAL COVERING LOCATION PROBLEM

The original p-Median Problem minimizes the average weighted distance of the system, but there is no restriction on the distance a user might be required to travel to its closest available facility. The maximal service distance, i.e. the maximum distance that any user would have to travel to reach a facility, is an important factor for many applications. A model that identifies the minimal number and the location of facilities that insures no demand point will be farther than the maximal service distance from a facility was developed by Toregas in 1971 (10). This model was called the Location Set Covering Problem (LSCP). It is not always practical because the budget available for construction of facilities is normally limited, and the number of facilities required to achieve total coverage within a given maximal covering distance could be larger than the budget allows. Should this occur, the goal of total coverage can be relinquished. A model which maximizes coverage with the construction of a limited number of facilities is more realistic. This problem was defined by Church and ReVelle in 1974 and called the Maximal Covering Location Problem (MCLP).

Mathematically, the MCLP can be written as follows (5):

MCLP1:

$$\text{Maximize } Z = \sum_{i \in I} Ai * Yi$$

subject to:

$$\sum_{j \in Ni} Xj \geq Yi \quad \text{for all } i \in I \quad \ldots\ldots\ldots \quad (1\text{-}11)$$

$$\sum_{j \in J} Xj = P \quad \ldots\ldots\ldots\ldots\ldots \quad (1\text{-}12)$$

$$Xj = (0,1) \quad \text{for all } j \in J \quad \ldots\ldots \quad (1\text{-}13)$$

$$Yi = (0,1) \quad \text{for all } i \in I \quad \ldots\ldots \quad (1\text{-}14)$$

where:

I = the set of demand nodes

J = the set of potential facility sites

i = the index of a demand node

j = the index of a potential facility site

$$X_j = \begin{cases} 1 & \text{if a facility is allocated to site } j \\ 0 & \text{otherwise} \end{cases}$$

$$Y_i = \begin{cases} 1 & \text{if node } i \text{ is within the maximal covering distance of a facility} \\ 0 & \text{otherwise} \end{cases}$$

A_i = the population at node i

P = the number of facilities to be allocated

$N_i = \{ j \mid D_{ij} \leq S_i \}$, service subset of node i

S_i = the maximal covering distance or time

D_{ij} = the shortest distance (or time) from node i to site j

N_i is the subset of potential facilities that is eligible to cover demand node i. The objective is to maximize the number of people covered within the maximal covering distance. Constraints of type (1-11) allow the coverage index Y_i to equal to one only when one or more facilities are established within S_i units of demand node i. Constraint (1-12) limits the total number of facilities to be allocated. Constraints of type (1-13) and (1-14) are the integer requirements of X_i and Y_j respectively.

An equivalent formulation was also presented by Church and ReVelle (5). It was derived by substituting $(1-Y'_i)$ for Y_i where:

$$Y'_i = \begin{cases} 1 & \text{if demand node } i \text{ is not covered} \\ 0 & \text{otherwise} \end{cases}$$

The equation (1-11) is transformed into:

$$\sum_{j \in N_i} X_j \geq 1 - Y'_i \qquad \text{for all } i \in I \quad . \; . \; . \; . \; (1\text{-}11a)$$

which is equivalent to:

$$\sum_{j \in Ni} Xj + Y'i \geqslant 1 \quad \text{for all } i \neq I \quad . \; . \; . \; . \; (1\text{-}11b)$$

This inequality implies that either

$$\sum_{j \in Ni} Xj \geqslant 1 \quad \text{or} \quad Y'i = 1$$

Upon variable substitution, the objective can be written as:

$$\text{Maximize} \quad Z = \sum_{i \in I} Ai * (1 - Y'i)$$

After multiplying Ai into (1-Y'i), the first term is a known constant, and the second term is the maximization of a negative quantity which is equivalent to the minimization of the positive value of the quantity. The transformed formulation can be expressed as:

MCLP2:

$$\text{Minimize} \quad Z = \sum_{i \in I} Ai * Y'i$$

subject to:

$$\sum_{j \in Ni} Xj + Y'i \geqslant 1 \quad \text{for all } i \in I \quad . \; . \; . \; (1\text{-}15)$$

$$\sum_{j \in J} Xj = P \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1\text{-}16)$$

$$Xj = (0,1) \quad \text{for all } j \in J \quad . \; . \; . \; (1\text{-}17)$$

$$Y'i = (0,1) \quad \text{for all } i \in I \quad . \; . \; . \; (1\text{-}18)$$

This model minimizes the population that is not covered by any of the P facilities located on a network. It is equivalent to the former one since only variable subsititution is used. This formulation is preferred because

X_j and Y'_i need not be greater than one to satisfy constraints of type (1-15) and the objective function keeps each Y'_i as small as possible, thus it is not necessary to limit the upper bound of X_j and Y'_i to 1 when solving this formulation by linear programming. To solve the first MCLP fomulation (MCLP1), it is necessary to add the upper bounds to the Y_i variables.

The MCLP formulation maximizes coverage when complete coverage is not possible due to the limit on the number of facilities. There is no guarantee of how far the uncovered demand nodes may be. Since it is unfair for those who are being ignored altogether, a constraint may be added to the MCLP to bring every demand node within some mandatory distance of a facility (5).

$$\sum_{J \in M_i} X_j \geq 1 \text{ for all } i \in I \quad \ldots\ldots\ldots \quad (1\text{-}15a)$$

where:

M_i = a set of potential sites which is within a mandatory distance of node i

$= \{ J \mid D_{ij} \leq T_i \}$

T_i = Mandatory distance of node i

The mandatory distance T_i is always greater than S_i.

The MCLP is a more realistic formulation than the LSCP because normally the decision maker has a limited budget with which to build facilities. It is the special feature of maximizing coverage that has made this problem a favorite for a number of applications.

4.1 MCLP Solution Techniques

The formulation of the MCLP was originally intended to be solved by linear programming. The branch and bound method is employed to resolve the fractional solutions should they occur. In addition, Church (54) developed a Greedy Adding (GA) algorithm and Greedy Adding with Substitution (GAS) algorithm to solve this problem. These two approaches are simple and easy to understand.

Starting with an empty solution set, GA adds, one at a time, the best facility locations to the solution set. The first location that GA selects is the one which covers the largest population. The second, third and so on are selected to cover the largest population that is not covered in the preceding step. The process stops when the number of facilities in the solution is p, the maximum number of facilities allowed by the budget. The GA algorithm never removes any facility location from the solution set. It is possible (and frequently the case) that initially chosen facilities become superfluous as a result of the addition of more facilities which partially or totally cover those populations which were covered by the sites chosen initially.

The Greedy Adding with Substitution (GAS) algorithm improves the GA by including a subroutine that eliminates unneeded facilities. The GAS algorithm determines new facility locations at each iteration just as the GA did, but in addition, seeks to improve the solution at each iteration by trying to replace each facility one at a time by other potential sites. If replacement is possible, the potential site chosen to replace a particular facility is the one which gives the greatest improvement in the objective.

The GAS algorithm is very similar to the Teitz And Bart (TAB) algorithm. With minor modifications, both can be applied to solve the PMP or MCLP. The TAB algorithm tries to find the best location of a fixed number of facilities by trying to use a potential site to replace one of the current facilities. The GAS algorithm tries to find the best location of P facilities by trying to replace a current facility with one of the potential sites.

Both TAB and GAS cannot guarantee optimality. A solution may exist which cannot be improved by replacing any single facility, but it is possible that it can be improved by replacing two, three or more facilities simultaneously. Shen Lin (30) investigated this property in his attempt to solve the Travelling Salesman Problem (TSP). The TSP attempts to find the shortest path that a salesman can take to visit all destinations once and finish at the starting point. He introduced the concept of an λ-optimum algorithm where λ is a positive integer. In the context of facility location problem, a solution is defined as λ-optimal if it is impossible to obtain a better objective by replacing any λ of the current facilities by a set of λ potential sites.

When TAB and GAS algorithms terminate, it is impossible to obtain a better objective by replacing any one of the current facilities by any one of the potential sites, therefore they can be classified as a 1-optimum algorithms. Better objectives can be obtained by a 2-optimum algorithm, 3-optimum algorithm, etc. To identify the locations of P facilities on a network, a p-optimum algorithm would give an optimal solution because a p-optimum algorithm would exhaust all the possible combinations. Of course, this approach is too time consumming to be practical. We will use TAB and GAS, the 1-optimum algorithms, for their convenience and economy. It has been reported that these algorithms (5) identify optimal solutions frequently.

An algorithm similar to GA and GAS was formerly developed by Ignizio and Harnett (28) to solve the LSCP. The Ignizio algorithm replaces only the facility sites whose contribution to total coverage is less than that by the last added facility, whereas GAS replaces any facility in the solution set by another facility if the resultant coverage increases. Another algorithm developed by Kuehn and Hamburger (29) for warehouse location can also be modified to solve the MCLP.

5.0 THEORETICAL LINK BETWEEN PMP AND MCLP

In 1976, Church and ReVelle reported that the MCLP can be formulated and solved as a PMP. Consider a PMP on a network where the distance Dij is replaced by D'ij, where:

$$D'ij = \begin{cases} 0 & \text{if } Dij \leqslant Si \\ 1 & \text{otherwise} \end{cases}$$

The modified distance D'ij is eaual to 0 if i is within cover distance of j, or equal to 1 if i is not within cover distance of j. The objective becomes:

$$\text{Minimize} \quad Z = \sum_{i=1}^{N} \sum_{j=1}^{N} Ai*D'ij*Xij$$

subject to:

$$\sum_{j=1}^{N} Xij = 1 \qquad i = 1,2,\ldots,N \qquad (1\text{-}19)$$

$$Xjj - Xij \geq 0 \quad \text{for all } i,j \text{ pairs, } i \neq j \quad (1\text{-}20)$$

$$\sum_{j=1}^{N} Xjj = P \qquad \ldots\ldots\ldots\ldots \quad (1\text{-}21)$$

$$Xij = (0,1) \qquad \ldots\ldots\ldots\ldots \quad (1\text{-}22)$$

Other variables are as defined previously.

For a given configuration, each node i assigns to its closest facility. If node i is serviced by a facility within the distance Si, then the objective Ai*D'ij*Xij=0 since D'ij =0. However, if node i is serviced by its closest facility which is farther than Si distance units away, then the objective Ai*D'ij*Xij = Ai since both D'ij and Xij are equal to one. This becomes a problem that minimizes population that is not covered by any facility, which is the same as the MCLP2 formulated previously. Thus, it is possible to solve the MCLP as a PMP. Any method applicable to solve the PMP can be used to solved the MCLP as well.

6.0 IMPACT MODELS

Many papers have dealt with some of the negative impacts of facility locations (6,7,8,9,10,11,12,13). The following is a review of the general strategy used by different researchers.

6.1 Donut Shaded feasible area by Orloff

Orloff was one of the earlier investigators who expressed concerns for the obnoxious characteristics of a public facility in a location model. He defined a donut shaped area about a discrete population center to be served. The area of the donut was regarded as the region of feasibility for locating a facility to serve the population center. The area between the donut and its geometric center (donut hole) was regarded as infeasible for facility location. He suggested that the area in the donut hole was too close to the population center and would create too high an impact if a facility was placed within the donut hole area. His objective was to minimize the number of facilities needed to completely serve a number of discrete population centers where facilities were placed in the intersections of the donut shaped regions. This problem is a variant of the Location Set Covering Problem (LSCP) which was first solved in 1971 (10).

Orloff's contributions were the definition of the donut shaped feasible area using a political power index and the implementation of a random generation scheme to generate the most probable location area. The former contribution should be regarded as an extension of Toregas' approach to define a coverage distance (10). The second contribution, the generation of the most probable location area of semi-obnoxious facilities, was made by varying two variables. But these two variables were difficult to relate to the political preferences of the people living within the population centers.

6.2 Maxian Problem by Church and Garfinkel

Church and Garfinkel (8) tried to mitigate the neighborhood impact of locating an obnoxious facility on a network by maximizing the sum of weighted distances of the facility to the nodes. The formulation was:

$$\text{Maximize} \quad T(x) = \sum_{i \in N} Ai * Dix$$

Where x is a point on the undirected graph G = (N,A), with node set N and arc set A. Ai is the demand at i and Dix is the shortest distance from node i to point x. They proved that there existed a finite set which must contain such a facility location. The method to identify the facility location on the nodes or along the arcs was also derived.

6.3 Benefit Functions by Austin, Church and Bell

To relate the costs, the positive benefits, and the negative benefits of providing a facility, Austin introduced the concept of optimization of benefit objective functions (6). He described four classes of public facilities and suggested a benefit function for each class. Church and Bell suggested two more benefit functions (12). The shapes of the benefit functions are shown in Fig. 1-1. (All Fig. are included in the Appendix).

Form A might represent a facility of high nuisance value. The overall impact becomes more beneficial to people residing further away up to some point where benefits drop to zero. Form B might represent a fire or a police station where accessibility is a prime factor, and negative benefit a second consideration. The maximum benefit accrues at some reasonable distance from the facility where service provided by the facility is still good. Form C might represent a school whose students are required to travel through a dangerous area. Form D might represent a public facility in which the effect is fairly constant over the area it serves. Jurisdictional boundaries could truncate the source area and create a discontinuous response curve. Form E might represent a facility whose net benefit function decreases with increasing distancce. Form F might represent a very obnoxious facility which is to be located as far away as possible.

Church and Bell suggested that these functions should be solved in the following way:

$$\text{Maximize } Z = \sum_i \sum_j NBij * Xij$$

subject to:

$$\sum_j Xij = 1 \quad \text{for } i=1,2,\ldots,n \qquad \ldots . \ (1\text{-}23)$$

$$\sum_j Xjj = P \quad \text{for } i=1,2,\ldots,n \qquad \ldots . \ (1\text{-}24)$$

$$Xij - Xjj + \sum_{k \in Bij} Xkk \geqslant 0 \quad \text{for } \underset{j \neq i}{j=1,2,\ldots n} \qquad (1\text{-}25)$$

$$Xij = (0,1) \ \ldots\ldots\ldots\ldots . \ (1\text{-}26)$$

where:

NBij = the benefit associated with serving area i given that the facility at j is the closest established facility.

Bij = the set of sites k that are closer to area i than site j.

Other notation is as defined previously.

The objective is to maximize the total benefit associated with serving a network of demand nodes by locating p-Facilities on the network. Constraints of type (1-23) maintain that each node must either assign to itself or assign to another node. Constraint (1-24) restricts the number of self assignments to be equal to P. Constraints of type (1-25) maintain that an assignment is made to the closest established facility. Constraints of type (1-26) maintain that Xij be either 0 or 1.

The solution of the above formulation by linear programming tended to be fractional and not easily resolved by branch and bound, but the solution by heuristic procedure yielded many interesting results as presented by Church and Bell (12).

6.4 Impact Objectives by Church and Cohon

In 1976, Church and Cohon considered the impact of a power plant to its neighboring area uitilizing two different criteria: 1) minimize the population or exposure within a given distance of facility sites; 2) maximize the distance of the facilities from population centers.

For case 1), Church and Cohon (7) first assumed that the risk to the population was associated with the mere presence of one or more facilities within a given impact distance. The objective was expressed as follows:

$$\text{Minimize } Z = \sum_{i} Ai * Zi$$

Subject to the additional constraint:

$$-Xj + Zi \geqslant 0 \quad \text{for all } i \text{ and all } j \varepsilon IAi$$

where:

$$Zi = \begin{cases} 1 & \text{if one or more facilities are placed within a given impact distance of node } i \\ 0 & \text{otherwise} \end{cases}$$

IAi = a set of facility j which is within impact area of node i.

Other notation is as defined previously.

Church and Cohon also considered the case where a larger risk was assumed to associate with more facilities established within an impact area, the objective was expressed as:

$$\text{Minimize } Z = \sum_{i} \sum_{j \varepsilon IAi} Ai * Xj$$

The objective was to minimize the total number of people impacted by each and every facility.

Church and Cohon also captured the potential dependence of risk to populations on the capacity of plants located within the impact area. The objective was expressed as follows:

$$\text{Minimize } Z = \sum_{i} \sum_{k} \sum_{j \varepsilon IAi} Ai * Sjk$$

where:

Sjk = the size in units of capacity established at site j using technology k.

k = the type of technology implemented at a particular site

For case 2), the objective was to maximize the distance of facilities from population centers. It was given mathematically as follows:

$$\text{Minimize } Z = \sum_{i} \sum_{j} (M-Dij) * Yij$$

Subject to the additional contraint:

$$-Fr + \sum_{j \varepsilon Tri} Yij \geqslant 0 \quad \text{for } i \varepsilon I \text{ and } r=1,2,\ldots,ri$$

where:

M = the largest interpoint distance

$$Yij = \begin{cases} 1 & \text{if the closest established faculty to i is j} \\ 0 & \text{otherwise} \end{cases}$$

Tri = the set of r closest sites to i

Dij = shortest distance from point i to point j

ri = total number of sites j within impact distance of node i

$$Fr = \begin{cases} 1 & \text{if a facility is built at the rth potential site from i} \\ 0 & \text{otherwise} \end{cases}$$

The additional contraints are included to define the values of the variables Yij for a particular facility configuration. This is another version of the Maxian Problem in which the distance of the facilities from populations is maximized.

7.0 MULTIOBJECTIVE SOLUTION

The objectives of the PMP, the MCLP, and neighborhood impact models are important objectives in public facility location analysis. These objectives, together with any other required objectives, should be considered simultaneously for a complete representation of a real world problem. This becomes a multiobjective problem which can be solved by vector optimzation techniques initiated by Kuhn and Tucker (4). The vector optimization techniques yield a large set of alternative solutions unless some preference information is given. Based on the stage at which the preference information is provided, vector optimization techniques (or multiobjective solution methods) fall into four categories (15). They are:

1. Methods for which no articulation of preference information is given
2. Methods where a priori articulation of preference information is given
3. Methods for progressive articulation of preference information is given
4. Methods where a posteriori articulation of preference information is given

These categories will be explained in the following sections.

7.1 No articulation of preference

Methods in this category need no information on the preference of different objectives in solving a multiple objective optimization problem. The assumption is either all objectives reach their maximum value simultaneously, or the objectives are in commensurate units so that an optimal solution can be found by minimizing the absolute values or the squares of the differences between the objectives and an ideal solution. The advantage of this method is that in the process of obtaining the solution, the decision maker is not required to give his/her preferences.

7.2 A Priori Articulation of preference

Methods in this category require that preference information be given to the analyst before the problem is actually solved. The information may be weights on the objectives, constraints on the objectives, a multiobjective utility function, or a goal and the priorities for the objectives. When the weights are given, the objectives in the multiobjective problem are combined to form one objective, thus it is equivalent to solving a single objective problem. When constraints on all except one objectives are given, it becomes a single objective problem with constraints on the remaining objectives. If a multiobjective utility function with all necessary parameters is given, the problem degenerates into a single objective problem again. When a goal (a target solution) and a priority for each objective are given, a new single objective problem can be formulated to minimize the weighted deviations of each objective from the target. When only the priorities of the objectives are given, the objectives can be ranked in decreasing order of importance. The preferred solution obtained by this method is one which optimizes the objectives starting with the most important and proceeding according to the order of importance of the objectives. Some of the methods in the a priori preference category include the utility theory by Raiffa (37,38), best compromise solution by Geoffrin (39), and goal programming by Charnes and Cooper (40).

7.3 Progressive Articulation of Preference

Methods in this category require constant feedback in between the decision maker and the analyst. Each time, the decision maker is asked about some preference information based upon the current solution. The analyst then generates another solution for the decision maker. This process is repeated until the decision maker is satisfied or some other terminal criteria come into consideration.

The methods in this category are also referred to as iterative methods due to the constant decision maker - analyst interactions. Well known examples of the step method are: the iterative procedure of Benayoun, et al (41); surrogate worth tradoff method developed by Haimes and Hall (42) and Haimes et al (43); the Geoffrion method and interactive goal programming by Geoffrion (44,45,46) and by Dyer (47).

7.4 A Posteriori Articulation of preference

Methods in this category determine a subset of the complete set of nondominated solutions to the multiobjective problem. The decision maker chooses the final solution from this subset based on unindicated or non-quantifiable criteria. Methods in this category do not require any preference from the decision maker before execution takes place. One serious problem is that a large number of solutions may be needed, and it becomes impossible for the decision maker to choose a good solution. This is especially true when there are more than three objectives. The methods in this category are also called generating methods since the generation of a number of alternative solutions is made before a decision maker needs to be invlolved.

Generating methods include the weighting and constraint methods, the noninferior set estimation (NISE) method and the multiobjective simplex method. In weighting methods, the objectives are converted into one super objective by multiplying each objective by its associated weight. Many problems are solved for a varied set of weights. In constraint methods, constraints are given to all objectives except one. The unconstrainted objective becomes the only objective. The problem is then solved for a number of different constraint values for each of the objectives under constraint.

The Noninferior Set Estimation Method is an efficient version of the weighting method. Since this method will be used in this thesis, the NISE method is explained in detail in the following section.

7.4.1 NISE Method

The Noninferior Set Estimation (NISE) method originally developed by Cohon et al (17) seeks a subset of the noninferior solutions between two objectives of a vector optimization problem. A noninferior set is a set in which no solutions are dominated; i.e. no solutions are better than any of the elements in the set in all objectives. The objective function is expressed as an additive separable function of two objectives with a weight associated with each objective to show the relative importance of the respective objective. It is assumed that the decision space is convex. A set is said to be convex if any point on the line segment joining any two points of the set is still an element of the set. Mathematically the core of the problem can be expressed as follows:

$$\text{Maximize } Z = W1 * Z1 + W2 * Z2 \qquad \text{(1-27)}$$

Subject to $X \in R$

where:

Z1 = the first objective (as a function of X)

Z2 = the second objective (as a function of X)

W1 = the relative importance of objective one

W2 = the relative importance of objective two

X = the vector of decision variables

R = the region of feasibility

The weights W1 and W2 are defined so that they sum up to 1. Both Z1 and Z2 are functions of X. The constraint on X is that the vector X be feasible. This method first identifies two solutions, one of them is found by optimizing objective one and the other by optimizing objective two. Then the weights W1 and W2 are calculated from these two solutions. A third solution is identified by using the current weights, and added to the set of solutions. If the solution is located above the line joining the first two solutions in the objective space, then an adjacent pair is chosen among the current existing solutions. The weights of this pair are calculated, and a new solution identified. The procedure stops when no new solutions can be identified between any pair of the adjacent solutions.

To explain this process in detail, let us define the following terms (refer to Fig. 1-2):

Let:

PI	=	the string set of the objectives
Pi	=	(Z1(i), Z2(i)), the ith point in the string set PI
SEGi	=	segment i, the segment between Pi and Pi+1
AIi	=	area of infeasible solution of SEGi
AUi	=	undominated and feasible area of SEGi
ADi	=	dominated area of SEGi
Ui	=	upper bound of SEGi
Ki	=	maximum improvement of the ith segemment
	=	Perpendicular distance from Ui to SEGi

The string set, PI, is an array of the first and second objectives used to store all solutions that are generated. PI is arranged in descending order of either the first or the second objective.

Segment i delineates two areas: 1) ADi, the area closer to the origin than Segment i, is dominated by segment i because there exists at least one point on Segment i that is better than any point in ADi in both objectives and 2) AUi is the undominated and feasible area of Segment i. All possible noninferior solutions of Segment i are located within this area.

The first solution, P1, can be identified by letting W1 = 0, W2 = 1 in the overall objective. The second solution, P2, is identified by putting W1 = 1, W2 = 0 in the overall objective. As shown by Cohon and Marks (20), to identify any solution in the undominated area between these 2 solutions, or between any adjacent pair in the string set, W1 and W2 must satisfy the property:

$$W1/W2 = -m \quad \ldots\ldots\ldots\ldots \quad (1\text{-}28)$$

where m, the gradient of Pi and Pi+1, is:

$$m = [Z2(i)-Z2(i+1)]/[Z1(i)-Z1(i+1)] \quad \ldots \quad (1\text{-}29)$$

To set the sum of W1 and W2 to one, we have

$$W1 + W2 = 1 \quad \ldots\ldots\ldots\ldots \quad (1\text{-}30)$$

Solving (1-28) and (1-30), we have

$$W1 = \frac{(Z1(i)-Z1(i+1))}{(Z1(i)-Z1(i+1)-Z2(i)+Z2(i+1))} \quad \ldots \quad (1\text{-}31)$$

$$W2 = \frac{(Z2(i+1)-Z2(i))}{(Z2(i)-Z2(i+1)-Z1(i)+Z1(i+1))} \quad \ldots \quad (1\text{-}32)$$

With these weights, we can solve formulation (1-27). The solution is one which is within the undominated area that has the longest perpendicular distance from Segment i. The worst case is when the solution is some point on Segment i. In this case, the perpendicular distance from the solution to Segment i is zero, which means no solution exists in region AUi. To test whether the solution, Pt, is within the undominated area, one can use the equation of Segment i. Let it be:

$$W1 * Z1 + W2 * Z2 = C1$$

W1 and W2 are found in equations (1-31) and (1-32).

$$C1 = W1 * Z1(i) + W2 * Z2(i) \quad \ldots\ldots (1\text{-}32)$$

Then find the equation of the line passing through the point, Pt, parallel to Segment i. Since the lines are parallel, the slope is just the same. The equation is:

$$W1 * Z1 + W2 * Z2 = C2$$

where:

$$C2 = W1 * (T1) + W2 * (T2) \quad \ldots\ldots (1\text{-}33)$$

T1 = the value of objective one of Pt

T2 = the value of objective two of Pt

If C2 = C1, then Pt is a point on Segment i. We can set the maximum possible improvement Ki to zero and try another segment. If C2 > C1, then Pt is undominated. This point is to be merged into the string set. The maximum possible improvements in the new segments can be identified by the following steps (refer to Fig. 1-3):

1. Find the equation of the line passing through the new solution point, Pt, parallel to Segment i. Name it Lt. Lt is the outer limit of any feasible solution in Segment i.
2. Find the point of intersection (Up1) of Lt and line PiUi. It is the Upper limit of the new segment PiPt. Find the intersection (Up2) of Lt and line Pi+1Ui. It is the upper limit of the new segment Pi+1Pt.

3. Find the possible improvements, Ka and Kb, of these two new segments. They are the perpendicular distances from the respective upper limits to the segments.

Now merge Pt in the string set. Record Ka, Kb, Up1, Up2. Then search for the segment with the largest possible improvement. The process of defining the weights, identifying the new solutions, computing the possible improvements and merging are repeated until no new points can be identified, or until the maximum possible improvement is less than an acceptable bound, or until the number of cycles exceeds a fixed amount. The algorithm is summarized as follows:

1. Find the first point (P1) of the string set by maximizing Z2 subject to the feasibility of X. Find the second point (P2) of the set by maximizing Z1 subject to the feasibility of X. Then calculate the largest possible improvement Ki, let n = 2.
2. Find i such that Ki is maximum for i = 1, 2, . . ., n-1. If the maximum Ki equals to 0, or the number of iterations is exceeded, stop. Continue if otherwise.
3. Find W1 and W2 of the ith segment by equations (1-31) and (1-32).

Then solve formulation (1-27). Calculate C1 by (1-32) and C2 by (1-33). If C2 < C1, set Ki = 0, and return to 2. Continue if otherwise.

4. Find Ka, Kb, the maximum possible improvements in the segment between Pi and Pt and in the segment between Pt and Pi+1 as explained in Fig. 1-3.

5. Merge this solution into the string set in the following way:

P'n	=	Pn	for n=1, 2, . . . , i
P'n+2	=	Pn+1	for n=i, i+1, . . . , N
P'n+1	=	Z	for n=i+1
K'n	=	Kn	for n=1, 2, . . . , i-1
K'n+2	=	Kn+1	for n=i, i+1, . . . , N
K'i	=	Ka	
K'i+1	=	Kb	

where P' is the array of new P values, K' is the array of new K values.

Go to 2

CHAPTER II

MODEL DEVELOPMENT

1.0 INTRODUCTION

The purpose of this chapter is to introduce several definitional forms for neighborhood impact. These definitional forms will be integrated into four types of model formulations that deal with minimizing neighborhood impact. A multiobjective problem with neighborhood impact will be given and solution approaches will also be suggested.

2.0 METHODS OF MINIMIZING NEIGHBORHOOD IMPACT

Many facilities, even though very beneficial in their function, are obnoxious in some way. These include power plants, solid waste transfer facilities, sanitary landfills, airports, roads, rail lines, fire stations, and incinerators. The impact on a neighborhood around a sanitary landfill and a solid waste transfer station, for example, includes increased truck traffic, heavy equipment noise, odors, increased rat populations and other disease-carrying vectors as well as declining aesthetic value. The negative impacts represent externalities which result in decreased property value and environmental quality.

Another good example of neighborhood impact is that of a power plant. A power plant discharges gaseous pollutants like sulphur dioxide, oxides of nitrogen, and carbon monoxide to the atmosphere. These gases are not only damaging to the health of the neighboring population, but also to the neighboring property and vegetation (49). Particulates from stack gases result in increased dust fall and suspended solids on the neighboring area. This further depreciates the value of the property and increases the incidence of respiratory diseases in the neighboring area (49). If wet cooling is used, a power plant discharges water vapor into the atmosphere which turns into fog on cold days. This decreases visibility and increases the driving hazards in the neighboring area. Other damages include the discharge of waste heat into streams and air, increases in the risk of nuclear radiation, and depreciation of the neighboring aesthetic values.

Thus, associated with many public facilities, there are some obnoxious characteristics. The neighborhood impact associated with these obnoxious characterics can be defined as any damage, hazard, devaluation, and inconvenience that is caused by the presence of a semi-obnoxious facility to the neighboring demand nodes. We will also call neighborhood impact as negative benefit or simply impact. One approach which may be used to minimize neighborhood impact is to incorporate better engineering in system design. A second approach is to improve operation and maintenance. When both methods can not mitigate all impacts, then one should consider siting such facilities in ways that will minimize nneighborhood impact.

The impact of an area by a facility is a function of three basic elements. First, impact is a function of the distance from the facility. For example, those living within three blocks of a fire station are disturbed each time the station responds to a call. Whereas, those living two miles away from the station are disturbed only when there is a fire in their area.

Second, impact is a function of the capacity of the facility. A 1000 MW power plant will need two times as much coal, discharge more water vapor and waste heat into the atmosphere than a 500 MW power plant, thus increasing the impact.

Third, impact is a function of the population around the facility. For example, in a nuclear power plant accident, the total damage to a town with 1000 people will be about two times as much as the damage to a town with 500 people, if both populations are of equal distance from the plant. The population here can be generalized to include human population or valuable assets which include properties, natural resources, or ecological order that may be devaluated by the mere presence of the facility.

In general, the neighborhood impact should increase under the following conditions:

1. increases in population
2. increases in the capacity of facility
3. decreases in the distance between population center and facility

In order to mitigate the impact in locating facilities on a network, one can consider several types of approaches:

1. Define an infeasible area of facility location around the demand nodes such that no facilities can be placed there:

   This limits the closest distance that a facility is allowed to be placed relative to a population center. Population centers cannot be too close to each other, or the infeasible district cannot be too large, otherwise, the infeasible area defined for each population center may overlap to form a continuous area such that no feasible area exist.

2. Maximize the total weighted distance from demand nodes to their closest facilities:

   This is the reverse of the p-Median Problem. The facility locations which give a maximum total weighted distance is to be identified. The total weighted distance can be defined in two ways: 1) It can be the sum of weighted distances within each partition; or 2) it can be the sum of weighted distances between all pairs of demand points and facility points.

3. Assign a negative benefit quantity to each site and locate facilities by minimizing the sum of these negative benefit quantities:

   Define a negative benefit quantity associated with each potential site, then locate the facilities such that the total negative benefit is minimized. This negative benefit is a collective constant term which represents the impact on population, environment, aesthetic value, and ecological order. The variation of impact with distance is not explicitly considered using this criterion.

4. Maximize a benefit quantity which is a function of distance from the closest facility while locating a set of facilities:

   Define a benefit function which varies with distance, then maximize the total benefit. The function is an algebraic sum of all positive and negative benefits.

5. Minimize the population within a defined impact area of any facility by locating a set of facilities:

   An impact area surrounding a site is defined. Within this area a population is considered as impacted. Outside of this area the population is considered as not impacted. The population is supposed to be impacted only by the closest facility. When two facilities are placed within the impact distance of the population, the impact to the population is only counted once.

6. Minimize the sum of population impacted by each facility while locating a set of facilities:

   This criterion is the same as the preceding criterion except that the impact is counted twice when two facilities are impacting a population.

7. Minimize the sum of the products of the population within an impact area of a facility times the capacity of the facility while locating a set of facilities:

   The impact to a population is assumed to depend on: 1) the population; and 2) the total capacity of the facilities within a defined impact area of the population. The impact of a given facility would be defined as the population impacted by the facility times the capacity of the facility.

8. Maximize the product of weighted distance times the capacity of the facility where each demand is assigned to the closest facility:

   In this criterion, the basic elements in impact, including population, distance, and capacity are considered simultaneously.

Some of these criteria have been investigated by researchers. Orloff (9) investigated Criterion 1) as explained in Chapter One. Church and Garfinkel (8) investigated Criterion 2) in their effort to locate obnoxious facilities on a network. Church and Cohon (7) briefly explored Criteria 5) and 6) as two of the many objectives in locating regional energy facilities. Formulations were proposed, but specific problems were not solved. Church and Bell (12) and Austin (6) utilized criterion 4) with different forms of benefit functions. Solutions of their problem obtained by linear programming tended to be fractional and somewhat difficult to resolve by branch and bound method. Heuristic solutions by a modified Teitz and Bart method proved to be an acceptable approach. Nevertheless, more research should be done to identify the applicable benefit functions for practical applications. In the following sections, formulations of criteria 5, 6, 7, and 8 will be given. Later, they will be modified to take into account the variations of neighborhood impacts with distances from facilities.

3.0 POPULATION WITHIN IMPACT AREA

The total population within an impact distance of any facility is a simple method of estimating neighborhood impact. Utilizing this impact measure, Church and Cohon defined the following location problem (7):

Minimize the total population within a given distance of the facilities while locating p-Facilities on a network

Church and Cohon designated this problem as the Minimal Impact Location problem (MILP). It seeks the location of p obnoxious or semi-obnoxious facilities on a network of N demand nodes such that the total population within an impact distance of any facility is minimized. An impact area is an area within the defined impact distance of a facility. The impact to a population within an impact area of two or more facilities will be counted only once. Defined on a network of nodes and arcs, a linear programming formulation of this problem is given as follows:

MILP:

Minimize $Z = \sum_{i \in I} Ai*Zi$ . . . . (2-1)

subject to:

$Zi - Xj \geq 0$ for all i j pairs, where $Dij \leq Si$ . . . (2-2)

$\sum_{j \in J} Xj = p$ . . . . (2-3)

$Xj = (0,1)$ for all $j \in J$ . . . . (2-4)

$Zi = (0,1)$ for all $i \in I$ . . . . (2-5)

where:

I = the set of demands

J = the set of potential sites

i = the index of a demand node

j = the index of a potential site

A_i = the demand at i

p = the number of facilities to be allocated

D_{ij} = the shortest distance from demand i to site j

S_i = the impact distance of demand i, i.e. the distance that defines the impact neighborhood of demand i.

$$X_j = \begin{cases} 1 & \text{if a facility is allocated to } j \\ 0 & \text{otherwise} \end{cases}$$

$$Z_i = \begin{cases} 1 & \text{if demand } i \text{ is within the impact distance of any site} \\ 0 & \text{otherwise} \end{cases}$$

This is an alternate form of the impact formulation by Church and Cohon (7).

Constraints of type (2-2) require that $Z_i = 1$ when a potential site within the impact distance S_i of demand i is utilized. Constraint (2-3) limits the number of facilities to be allocated. Constraints of type (2-4) and (2-5) give the integer requirements of X_j and Z_i.

The input data to this model are: I, J, A_i, p, D_{ij}, and S_i. The decision variables are X_j and Z_i. The necessary data is easily defined except S_i. Orloff suggested S_i be related to the political index of demand i. The higher the political index, the larger is the impact distance. He assumed that the political index was related to the average income. Church and Cohon (7) suggested that the impact distance depend on the degree of obnoxiousness of facilities. In the location of nuclear power plants, the long term dosage of radiation and the probability of risks to the neighboring population should be used to define the impact distance.

The MILP can be solved by linear programming by relaxing the integer requirements (2-4) and (2-5) to non-negativity requirements. Instead of minimizing the total population within impact areas of facilities, the impact areas may be minimized by allocating two or more facilities on one potential site. To avoid this, all Xj's should be limited to less than or equal to 1. The branch and bound method can be used to resolve any fractional solutions which may happen to occur. The modified Teitz and Bart algorithm as introduced by Church and Bell (12) can be adopted to solve this problem heuristically.

The MILP is insensitive to the number of times that an individual demand area is impacted. Since the population within the intersections of impact areas is to be counted only once, the facilities will quite frequently be located in a cluster in an area of minimum population where individuals are impacted a multiple number of times. Another more reasonable formulation can be developed to improve the sensitivity of the MILP.

4.0 SUM OF POPULATION WITHIN IMPACT AREAS OF EACH FACILITY

A population may be impacted by each of several nearby facilities, so the actual effect to that population could be a function of each of the nearby facilities. One estimate of impact is to calculate the total population living within the impact area of a facility and then sum over all located facilities. If a population is within the impact areas of two different facilities, they get counted twice; once for each facility. The unit of this measure would be person-facility, which is one person living within the impact area of one facility. Here, obnoxiousness is assumed to be directly proportional to the number of facilities and the number of people. One person within impact areas of four facilities is assumed to be as bad as four persons within the impact area of one facility. The problem can be stated as follows:

> Minimize the total number of person-facilities when locating p obnoxious facilities on a network of N demand nodes. A person-facility is one person living within the pre-specified impact distance of one facility.

The mathematical formulation in a network context can be given as follows:

MPFLIP:

$$\text{Minimize } Z = \sum_{i=1}^{N} Ai * \left(\sum_{j \in Ni} Xj \right) \quad \ldots\ldots\ldots \text{(2-6)}$$

subject to:

$$\sum_{j \in J} Xj = p \quad \ldots\ldots\ldots \text{(2-7)}$$

$$Xj = (0,1) \quad \text{for all } j \ldots. \text{(2-8)}$$

where:

N = total number of demand points

Ai = the population at demand node i

p = the number of facilities to be built

$$X_j = \begin{cases} 1 & \text{if a facilities to be built at site } j \\ 0 & \text{otherwise} \end{cases}$$

$N_i = \{ j \mid D_{ij} \leq S_j \}$, the facility sites that have an impact area that include demand i

S_j = impact distance of site j

Other notation is as previously defined.

This formulation will be called the Minimal Person-Facility Impact Location Problem (MPFILP).

The content inside the parentheses in the objective equation (2-6) is the total number of facilities that impacts node i. The overall objective is to minimize the total number of people impacted by each facility. Constraint (2-7) limits the total number of facilities to be built. Constraints of type (2-8) give the integer requirements of the assignment index Xj. The input data to the model are N, J, Ai, P, Ni, and Sj. Sj, the impact distance of potential site j, can be a constant or a variable input array, depending on the degree of complexity required. Equivalently, Sj can be replaced by Si which is the distance within which node i is considered to be impacted. If a demand is more populous, more important, or more powerful, its impact distance could be longer than the impact distance of a less populous, less important, and/or less powerful demand node.

Linear programming can be used to solve this problem. The integer requirements of Xj are relaxed to non-negativity requirements. The branch and bound method can be used to resolve the fractional solutions should they occur.

Another equivalent formulation of minimizing the total person-facilities is listed as follows:

$$\text{Minimize} \quad Z = \sum_{j \in J} A'j * Xj \qquad \ldots\ldots\ldots\ldots \quad (2\text{-}9)$$

subject to:

$$\sum_{j \in J} Xj = p \qquad \ldots\ldots\ldots\ldots \quad (2\text{-}10)$$

$$Xj = (0,1) \text{ for all } j \quad \ldots\ldots \quad (2\text{-}11)$$

where:

$A'j$ = total population impacted by site j

$= \sum_{i \in Mj} Ai$

$Mj = \{ i \mid Dij \leq Sj \}$, set of demands that are impacted by site j

Other terms are as defined previously.

The objective here is to minimize the total population under impact of each facility. Constraints of type (2-10) require that p facilities are to be chosen.

This formulation is a special case of the well known Knapsack problem (58) except that the objective is minimizing the neighborhood impact instead of maximizing the total priority of items to be included in a knapsack. A Knapsack problem is one that maximizes the priorities of items to be put into a knapsack with a volume or a weight constraint. The A'j is analogous to the priority and the constraint (2-10) is analogous to the volume or weight constraint.

The Knapsack problem with zero-one coefficients and an integer right hand side in the constraint can be easily solved optimally by ordering the priorities of the items and picking one by one the most important items until limited by the constraint. The current problem can be solved by the same method, but instead of picking the largest A'j values, the smallest A'j values are selected. This solution method is superior to linear programming and branch and bound in terms of execution time.

5.0 THE PRODUCT OF IMPACTED POPULATION TIMES FACILITY CAPACITY

The Minimal Impact Location Problem (MILP) and the Minimal Person-Facility Impact Location Problem (MPFILP) are concerned with the population within the impact area and the total number of facilities that are impacting a population. The capacity of a facility could also be an important factor. The impact to a person living within the impact area of a facility may be larger if the capacity of the facility is doubled. Let us define a new quantity - a person-capacity unit. A person-capacity unit is one person living within the impact area of a facility operating at a load of one unit of capacity. Obnoxiousness is assumed to be directly proportional to the number of people and the number of capacity units. For example, four persons living within the impact area of one capacity unit will be assumed to be the same as one person living within the impact areas of four capacity units. The following formulations will not be concerned with how many facilities are operating, but the distribution of capacity among p facilities. It is assumed that each demand i is serviced by its closest available facility. The problem can be stated as follows:

> Minimize the total number of person-capacity units when locating P obnoxious facilities and allocating a given amount of capacity among the facilities on a network of N demand nodes.

A capacity unit is a measurement of the load of a facility. For an electric power plant, one million watts(MW) can be regarded as one capacity unit. Thus a 1000 MW electric power plant can be considered as operating at 1000 capacity units. By the same token, one million gallons per day (MGD) would be one capacity unit for a sewage treatment plant; a 100 MGD plant will operate at 100 capacity units. The mathematical formulation can be given as follows:

MPCILP:

$$\text{Minimize:} \quad Z = \sum_{j \in J} \sum_{i \in M_j} A_i * \sum_{i \in I} (A_i * X_{ij}) \quad \ldots\ldots(2\text{-}30)$$

subject to:

$$\sum_{j \in J} X_{ij} = 1 \quad \ldots\ldots\ldots\ldots(2\text{-}31)$$

$$\sum_{j \varepsilon J} Xjj = p \quad \ldots\ldots\ldots\ldots (2\text{-}32)$$

$$Xjj - Xij \geqslant 1 \quad \text{for all } i,j,i \neq j \quad \ldots\ldots (2\text{-}33)$$

$$Xij - Xjj + \sum_{k \varepsilon Bij} Xkk \geqslant 0 \quad \text{for all } i,j \text{ such that } i \neq j \ldots (2\text{-}34)$$

$$Xij = (0,1) \quad \ldots\ldots\ldots\ldots (2\text{-}35)$$

where:

$Xij = \begin{cases} 1 & \text{if demand } i \text{ is serviced by facility } j \\ 0 & \text{otherwise} \end{cases}$

$Bij = \{ k \mid Dik < Dij \}$, a set of potential site k such that k is closer to i than j

$Mj = \{ i \mid Dij \leqslant Sj \}$, a set of demand that are impacted by site j

Sj = impact distance of site j

The notation and constraints are as defined previously.

The formulation will be called the Minimal Person-Capacity Impact Location Problem (MPCILP). This is basically the problem defined by Church and Cohon (7).

The objective of this formulation is to minimize the sum of the product between population under impact and the capacities of all the closest facilities impacting the population. The content inside the parenthesis in the objective function (2-30) is the capacity of the facility j. The input data are: I, J, P, Ai, Sj, Dij. The variables to be determined are the assignment indices Xij. The most important variables are Xjj which indicate where facilities are located.

Constraints of type (2-31) insure that each node i is fully assigned. Constraint (2-32) restricts the number of facilities (self assignments) to be allocated. Constraints of type (2-33) limit assignments to only those nodes which are self assigned. These three constraints are just the same as those in the PMP. Constraints of type (2-34) require that all demand nodes are serviced by their closest facility. If there are no facilities built in between demand i and facility j, then the sum of Xkk is zero in constraints of type (2-34), leaving Xij - Xjj equal to or greater than 0. If Xjj is equal to 1, then Xij would be 1. Constraints of type (2-35) require that the assignment variables be 1 or 0.

When the Xjj values are fractional, constraints of type (2-34) are not restrictive enough to guarantee closeness. For example,

Let:

$D11 < D12 < D13 < D14 < D15$

$X11=.2, \quad X22=.3, \quad X33=.7, \quad X44=1.0$

If both closeness constraints and constraints of type (2-31) are effective, we should have:

$X11=.2, \quad X12=.3, \quad X13=.5$

But applying constraints of type (2-34) with i=1, j=2:

$X12 - X22 - X11 > 0$

$X12 - .3 - .2 > 0$

$X12 > .1$ . . . . (a)

When $i=1$, $j=3$:

$$X13 - X33 + X22 + X11 \geq 0$$

$$X13 - .7 + .3 + .2 \geq 0$$

$$X13 \geq .7 - .3 - .2$$

$$X13 \geq .2 \quad \dots\dots\dots\dots\dots \text{(b)}$$

Constraints (a) and (b) do not give exact values of X12 and X13. A more restrictive form can be structured as:

$$Xij - Xjj + \sum_{k \varepsilon Bij} Xik \geq 0 \text{ for all } i,j,i \neq j \quad (2\text{-}36)$$

Constraints of type (2-36) require the total service provided to a demand i by all facilities closer to i than j plus the service by facility j, would be equal to or greater than that can be provided by facility j. These constraints are just as functional as Constraints of type (2-34) when Xij are integers.

To show how this set of constraints performs when Xij are not integers, let us write out the constraints of type (2-36) in actual numbers as defined in the preceding example:

When $i=1$, $j=2$

$$X12 - X22 + X11 \geq 0$$

$$X12 - .3 + .2 \geq 0$$

$$X12 \geq .1 \quad \dots\dots\dots\dots\dots \text{(c)}$$

When $i=1$, $j=3$

$$X13 - X33 + X11 + X12 \geq 0$$

$$X13 - .7 + .2 + X12 \geq 0$$

$X12 + X13 \geqslant .5$ . . . . (d)

If constraints (2-35) are used, by adding (a) and (b), we have:

$X12 + X13 \geqslant .1 + .2 = .3$

If constraints (2-36) are used, by adding (c) and (d), we have:

$X12 + X13 \geqslant .5$

If closeness is obeyed,

$X12 + X13 = .8$

We can conclude that constraints of type (2-37) are tighter constraints than constraints of type (2-35), but no contraint will guarantee proximity when the solution is fractional.

The Minimal person-capacity Impact Location Problem (MPCILP) can be solved by linear programming with the integer requirement relaxed. If solutions are fractional, the proximity requirements may be violated. No intepretations of the solutions are valid before the branch and bound method is used to resolve the fractions. The modified Teitz and Bart algorithm as introduced by Church and Bell can be used to solve this problem heuristically.

6.0 PRODUCT OF WEIGHTED DISTANCE AND CAPACITY

Another approach to handle obnoxiousness is to maximize the sum of the product between weighted distance and capacity of facilities. This would give a great incentive to place facilites such that large population centers are as far away from large facilities as possible. Mathematically, this approach can be structured as:

MPWCLP:

$$\text{Maximize } Z = \sum_{j \varepsilon J} \sum_{i \varepsilon I} Ai * Dij * \sum_{i \varepsilon I} (Ai * Xij)$$

subject to:

$$\sum_{j \varepsilon J} Xij = 1 \quad \ldots\ldots\ldots\ldots (2\text{-}38)$$

$$\sum_{j \varepsilon J} Xjj = p \quad \ldots\ldots\ldots\ldots (2\text{-}39)$$

$$Xjj - Xij \geqslant 1 \quad \text{for all } i,j,i \neq j \quad \ldots\ldots (2\text{-}40)$$

$$Xij - Xjj + \sum_{k \varepsilon Bij} Xkk \geqslant 0 \quad \text{for all } i,j \text{ such that } i \neq j \quad \ldots (2\text{-}41)$$

$$Xij = (0,1) \quad \ldots\ldots\ldots\ldots (2\text{-}42)$$

The notation is as previously defined.

This formulation will be called the Minimal Product of Weighted Distance and Capacity Location Model (MPWCLP). The solution to this formulation will consist of facilities located at the periphery of the clusters of populations. Most of the demand would be serviced by one facility so that facilities will generally have a large capacity. The closest facility assignment constraints would tend to force all facilities into one remote region. In spite of the attempt to minimize neighborhood impact, this model would identify facilities too concentrated in a remote area to provide good service.

7.0 MODIFICATION OF IMPACT FORMULATIONS BY DISTANCE

Up to this point, four neighborhood impact models have been presented. The first three models involve an impact distance Sj (or Si). Within this distance, a population is regarded as impacted. Outside of the distance, the population is regarded as uninfluenced. Since the effect of neighborhood impact usually decreases with the distance from facilities, we can define an impact factor which decreases with distance. An impact factor is a number ranging from 1 to 0 which gives the degree of impact a facility j exerts on the population at node i. For example, the impact factor, IPij, can be:

$$IPij = \begin{cases} 1. & \text{if } 0 \leq Dij \leq S1 \\ .75 & \text{if } S1 < Dij \leq S2 \\ .5 & \text{if } S2 < Dij \leq S3 \\ .25 & \text{if } S3 < Dij \leq S4 \\ 0 & \text{if } S4 < Dij \leq S5 \end{cases}$$

where: $S1 < S2 < S3 < S4 < S5$ are fixed distances

or:

$$IPij = \{1.-a\}^{*} Dij \quad \text{for } Dij \leq 1/a$$

where: a is a constant

in general:

$$IPij = f(Dij)$$

where: $0 \leq f(Dij) \leq 1$

The population Ai multipled by the impact factor IPij will give the degree that the population at node i is impacted by the facility at j. Equivalently, the product can be regarded as the population equivalent at node i that is impacted by the facility at j. The equivalent population can be defined as follows:

$$EPOPij = IPij * Ai$$

where:

EPOPij = Equivalent population of node i that is impacted by facility j

IPij = Impact factor of i by j

Ai = population at node i

The MPFILP and MPCILP can easily be modified by substituting the equivalent population for the population at each demand node. For the ease of reference, the modified forms are presented in the following sections.

7.1 Distance Modification on MPFILP

The MPFILP after modification with impact factor becomes:

MPFILP2:

$$\text{Minimize } Z = \sum_{i \varepsilon I} \sum_{j \varepsilon J} IPij * Ai * Xj$$

subject to:

$$\sum_{j \varepsilon J} Xj = p \qquad \ldots\ldots\ldots(2\text{-}44)$$

$$Xj = (0,1) \quad \text{for all } j \ldots.(2\text{-}45)$$

The notation is as previously defined.

The constraints and the solution methods are similar to the MPFILP. The extra input data are impact factors. In the objective, j is summed over all potential sites in this formulation, whereas j was summed over all elements within the impact subset of i in the MPFILP (distance unmodified) formulation.

7.2 Distance Modification on MPCILP

The MPCILP after modification with impact factor becomes:

MPCILP2:

$$\text{Minimize } Z = \sum_{i \varepsilon I} \sum_{j \varepsilon J} IPij * Ai * \sum_{i \varepsilon I} Ai * Xij$$

subject to:

$$\sum_{j \varepsilon J} Xij = 1 \quad \ldots\ldots\ldots\ldots (2\text{-}47)$$

$$\sum_{j \varepsilon J} Xjj = p \quad \ldots\ldots\ldots\ldots (2\text{-}48)$$

$$Xjj - Xij \geqslant 1 \quad \text{for all } i,j,i \neq j \quad \ldots . (2\text{-}49)$$

$$Xij - Xjj + \sum_{k \varepsilon Bij} Xkk \geqslant 0 \quad \text{for all } i,j \text{ such that } i \neq j \quad .. (2\text{-}50)$$

$$Xij = (0,1) \quad \ldots\ldots\ldots\ldots (2\text{-}51)$$

The notation is as previously defined.

The constraints and the solution methods are the same as the MPCILP. Moreover, the extra input data the variation of j are the same as that of the MPFILP2.

7.3 Distance Modification on MILP

To incorporate the impact factor into the MILP, considerably more work is required. Since no facilities except the closest one are assumed to be impacting a population center, the concept of equivalent population under impact can not be applied directly. Suppose that impact factor function can be represented by N different values. Let Mk be the impact factor value at the Kth segment, Sk-1 be the distance from a node at which the Kth segment begins, Sk be the distance from a node at which the Kth segment ends. If the closest facility from node i lies within the Kth segment, the impact factor of facility j on node i is Mk. The Mk values are illustrated in Fig. 2-1.

To formulate the MILP with distance modifications, three sets of constraints will be used.

1. Total facility allocated is P.
2. The impact comes from only one facility.
3. The impact must come from the closest facility.

The mathematical formulation can be listed as follows:

$$\text{Minimize } Z = \sum_{i} \sum_{k} Mk * Yik * Ai$$

subject to:

$$\sum_{j \in J} Xj = p \quad \ldots\ldots\ldots\ldots \quad (2\text{-}54)$$

$$Xj - \sum_{l=1}^{k} Yil \leqslant 0 \quad \text{for all } i, j \quad \text{where } S(k-1) \leqslant Dij \leqslant S(k) \quad \ldots \quad (2\text{-}56)$$

$$Yik = (0,1) \quad \ldots\ldots\ldots\ldots \quad (2\text{-}57)$$

$$Xj = (0,1) \quad \text{for all } j \quad \ldots\ldots \quad (2\text{-}58)$$

where:

N = number of different impact factor values

M_k = the value of the kth segment impact factor

$$Y_{ik} = \begin{cases} 1 & \text{if the closest facility is allocated at the kth impact segment of demand i} \\ 0 & \text{otherwise} \end{cases}$$

S_{k-1} = the distance from a node at which the kth segment begins

S_k = the distance from a node at which the kth segment ends

Other notation is as previously defined.

The objective of this formulation is to minimize the sum of impact of all nodes by their closest facility. Y_{ik} is the impact index of node i by facilities at the kth segment. Constraint (2-54) limits the number of facilities to be allocated. Constraints of type (2-56) require that only the closest facility will impact node i. The last two contraints give the integer requirements of the X_j and Y_{ik}.

The mathematics of this formulation were originated by Church in his investigations of different forms of benefit functions in public facility location (50).

This model requires impact factors as extra data. The solution methods are the same as the MILP.

8.0 APPLICATIONS OF NEIGHBORHOOD IMPACT MODELS

The solutions to pure neighborhood impact problems consist mostly of nodes at the periphery of the demand cluster. This model is practical when travel distance is not a major factor. One application may be that of locating a house or business away from high crime area (8). The demand in node i in this case is the relative danger of node i. The location of regional prison, highly contagious disease convalescence center, or defense unit can also make use of this model.

9.0 MULTIOBJECTIVE FACILITY LOCATION FORMULATION

It has been shown that the important aspects in facility location analysis are weighted distance, coverage, and neighborhood impact. In many cases it appears that all three objectives are important and should be considered simultaneously. This can be expressed as a multiobjective problem (vector optimization) using three variable weights to show the relative importance of those three objectives. The multiobjective problem is expressed in the following formulation.

NCWDLP:

$$\text{Minimize } Z = \sum_{j \varepsilon J} \sum_{i \varepsilon I} W1 * Dij * Ai * Xij + \sum_{j \varepsilon J} \sum_{i \varepsilon I} W2 * D'ij * Ai * Xij + \sum_{i \varepsilon I} W3 * Ai * Zi \quad \ldots\ldots (2\text{-}60)$$

subject to:

$$\sum_{j \varepsilon J} Xij = 1 \quad \text{for all } i \varepsilon I \quad \ldots\ldots (2\text{-}61)$$

$$\sum_{j \in J} Xjj = p \quad \ldots\ldots\ldots\ldots\ldots \text{(2-62)}$$

$$Xjj - Xij \geq 0 \text{ for all } i,j \text{ pairs } \underset{i \neq j}{} \quad \ldots\ldots \text{(2-63)}$$

$$Zi - Xjj \geq 0 \text{ for all } i,j \underset{Dij \leq Si}{} \quad \ldots \text{(2-64)}$$

$$Xij = (0,1) \quad \ldots\ldots\ldots\ldots\ldots \text{(2-65)}$$

$$Zi = (0,1) \quad \ldots\ldots\ldots\ldots\ldots \text{(2-66)}$$

where:

Dij = the shortest distance (or time) from i to j

W1 = the relative importance of the PMP

W2 = the relative importance of the MCLP

W3 = the relative importance of the MILP

Si = the impact distance of node i

Ci = the maximal covering distance of node i

$$Xij = \begin{cases} 1 & \text{if node } i \text{ is served by site } j \\ 0 & \text{otherwise} \end{cases}$$

$$Zi = \begin{cases} 1 & \text{if node } i \text{ is within impact distance of any facility} \\ 0 & \text{otherwise} \end{cases}$$

D'ij = the modified distance to convert MCLP TO PMP

$$= \begin{cases} 1 & \text{if } Dij \geq Ci \\ 0 & \text{otherwise} \end{cases}$$

This formulation will be called the Neighborhood Cover Weighted Distance Location Problem (NCWDLP). The Maximal Covering Location Problem (MCLP) is expressed as a special case of the p-Median Problem (PMP) to decrease the number of unknowns. This will decrease the time and the cost required to solve this problem by computer. For neighborhood impact, the Minimal Impact Location Problem (MILP) formulation is used for the sake of simplicity, however, the Minimal Person-facility Impact Location Problem (MPFILP) or other formulations can equivalently be used, if compatible.

The objective of this formulation is to minimize the sum of three terms, i.e. weighted distance, coverage, and neighborhood impact. They are weighted by W1, W2, W3 respectively. Normally the sum of W1, W2, W3 adds up to 1. When either W1, W2 or W3 is one, and the corresponding two weights are zero, it becomes a single objective problem of minimizing the non-zero weighted objective. When the weight of any objective is set to zero, that objective is dropped out, it becomes a two dimensional problem of the remaining objectives. When none of the weights are set to zero, it is a three dimensional problem.

Constraints of type (2-61) insure that each node i is fully assigned. Constraints of type (2-62) restricts the number of facilities (self assignments) to be allocated. Constraints of type (2-63) limit assignments to only those nodes which are self assigned. Constraints of type (2-64) require that Zi = 1 when a potential site within impact distance of demand i is self assigned. Constraints of type (2-65) and (2-66) give the integer requirements of Xij and Zi.

The impact distance Si and the maximal covering distance Ci are assumed to vary with each demand node i. For a more powerful demand node i, Ci will be relatively small, but Si will be relatively large. Instead of being a function of demand node, the impact distance and the maximal covering distance can be a function of the characteristics of a facility. Thus Si and Ci would be replaced by Sj and Cj. For a very obnoxious facility with little covering ability, the Sj will be relatively large and the Cj will be relatively small. Normally, the covering distance is larger than the impact distance.

The formulation listed above can be modified in a number of ways. The MILP can be formulated in the same manner as it was originally developed by Church. If desired, the mandatory closeness constraint can also be added.

$$\sum_{j \in Mi} Xjj \geq 1 \quad \text{for } i \in I$$

where:

Mi = Mandatory service subset of i

= subset of potential facility j such that j is within mandatory distance of i

$= \{ j \ J \mid Dij \leq Ti \}$

Ti = the mandatory closeness distance of node i

The neighborhood impact can be measured by person-facility instead of the population under impact. The third term in the objective could be replaced by:

$$\sum_{i \in I} \sum_{j \in Ni} W3 * Ai * Yj$$

The weights W1, W2, and W3 show the relative importance of the weighted distance, coverage, and neighborhood impact objectives. When an objective is regarded as important, the weight associated with it should be large, or vice versa. If accessibility is the main factor, the weighted distance is to be minimized, thus W1 should be large. If coverage is important, W2 should be large. And if neighborhood impact is important, W3 should be large. The importance of each term depends on the preference of the general public - the true owner of public facilities. But the importance of each objective is difficult to define and quantify. These preferences are seldom known at the planning stage of facility construction. The role of the analyst thus is to generate enough choices by varying the weights. It is up to the decision maker to make the final choice after weighing the preferences of the public.

9.1 Coverage and Neighborhood formulation

A fire station is required within a reasonable cover distance for life and property protection, but no one wants it next door. The location of a fire station thus has two objectives: 1) maximize coverage; 2) minimize the neighborhood impact. This problem can be solved by the previous multiobjective formulation by putting the relative importance of PMP to zero. Another more efficient formulation can be derived by combining the MCLP with any one of the impact problems. The MILP will be used for the sake of simplicity. The problem can be stated as follows:

> Maximize coverage (population covered) within a desired service distance S by locating a fixed number of facilities, keeping the neighborhood impact as low as possible.

Mathematically, this can be written as follows:

MCMILP:

$$\text{Minimize } Z = \sum_{i \in I} (Wc * Ai * Y'i + Wn * Ai * Zi)$$

subject to:

$$\sum_{j \in J} Xj = p \quad \ldots\ldots\ldots\ldots\ldots \quad (2\text{-}67)$$

$$\sum_{j \in Ni} Xj + Y'i \geq 1 \quad \text{for all } i \quad \ldots\ldots \quad (2\text{-}68)$$

$$Zi - Xj \geq 0 \quad \text{for all } i,j \text{ such that } D(i,j) \leq Si \quad \ldots\ldots \quad (2\text{-}69)$$

$$Xj = (0,1) \quad \ldots\ldots\ldots\ldots \quad (2\text{-}70)$$

$$Y'i = (0,1) \quad \ldots\ldots\ldots\ldots \quad (2\text{-}71)$$

$$Zi = (0,1) \quad \ldots\ldots\ldots\ldots \quad (2\text{-}72)$$

where:

$$Xj = \begin{cases} 1 & \text{if a facility is allocated to site } j \\ 0 & \text{otherwise} \end{cases}$$

$$Y'i = \begin{cases} 1 & \text{if demand node } i \text{ is not covered} \\ 0 & \text{otherwise} \end{cases}$$

$$Zi = \begin{cases} 1 & \text{if demand node } i \text{ is impacted} \\ 0 & \text{otherwise} \end{cases}$$

$Ni = \{ j \mid Dij \leq Ci \}$, set of all potential sites j which can cover i

Ci = the covering distance of node i

S_i = the impact distance of node i

W_c = the relative importance of coverage

W_n = the relative importance of impact

Other notation is as defined previously.

This problem will be called the Maximal Coverage Minimal Impact Location Problem (MCMILP). The objective of this formulation is to minimize the population that is not covered within the defined cover distance and the population that is within impact distance. Constraint (2-67) restricts the number of facilities to be allocated. Constraints of type (2-68) require that the noncoverage index of node i, Y'_i, to be equal to one when no facilities are allocated within cover distance from demand node i. Constraints of type (2-69) require that $Z_i = 1$ when a site within the impact distance S_i of demand i is utilized. Constraints of type (2-70), (2-71), and (2-72) give the integer requirements of X_i, Y'_i, and Z_i.

The input data to this model are: I, J, A_i, P, D_{ij}, C_i, S_i. The decision variables are X_j, Y'_i, and Z_i. C_i and S_i are subject to variations as previously discussed. Solution techniques are the same as those of the Neighborhood Cover Weight Distance Location Problem (NCWDLP).

The formulation will be solved more efficiently by linear Programming than the NCWDLP. Because X_j values instead of the X_{ij} values are being solved, the number of constraints as well as the number of variables to be solved are greatly reduced. The proximity requirement is not explicitly expressed in this formulation, thus the solution will come out to be integers more often than the NCWDLP.

10.0 APPLICATION OF MULTIOBJECTIVE FACILITY LOCATION MODELS

Many real world problems have been formulated and solved as multiple objective problems. Hwang summarized the applications into twelve broad areas. They are: academic planning, econometrics and development planning, financial planning, health care planning, manpower planning, media planning, production planning, public administration, system reliability and maintenance policy, transportation planning, and water resources management. There are, however, just a few applications of multiple objective problems to facility locations.

One of these few formulations was the location of fire station in Baltimore City by Schilling et al. (30). In one formulation the objectives were population protection and property protection. In another formulation, the objectives were primary protection and secondary protection. Both formulations were constrained by the availability of resources, i. e. the number of ambulance depots allowed. Rider did more or less the same study for New York City (31). In 1977, Borreani - Ostaneil and Capellaro (32) presented a multiobjective, linear, mixed-integer model formulation dealing with problems of allocating and dimensioning school services in an industrial town in Italy. Estimates of need in the near future and the qualitative and structural characteristic of the population were considered. The objectives were investment costs, displacement time (i.e., the time spent travelling between classes), and double turns of students (i.e., the use of the same structure by two different groups of students in a day). Flavell and Salkin (33) located training and retraining centers for a government agency in Great Britain such that a satisfactory allocation of funds be made to each region. The objective was to maximize the training and retraining opportunities for individuals.

Church and Cohon (7) presented a multiobjective locational analysis of the regional energy facility siting problem. The objectives were to minimize facility costs, minimize water transfers, maximize equity of plant distribution, and minimize population safety impact. The following constraints were used: minimum capacity and concentration constraints, fuel mix constraints, water quality requirements, water requirements for heat dissipation, air pollution constraints and constraints required for formulation of the objectives. Church (34) presented a model to allocate solid waste treatment center in the Tennessee Valley area. His main objectives were to minimize the weighted distance, maximize the coverage, and maximize the balance of load. The main constraint was the number of facilities to be constructed.

The multiobjective models developed in this thesis are conceptual models which consider simultaneously the importance of impact, coverage, and weighted distance in the location of obnoxious or semiobnoxious facilities. These models can be applied with minor modifications to location of solid waste processing plants, solid waste transfer stations, ambulance stations, fire stations, power plants, health depots, or locations of any facilities with obnoxious characteristics. Constraints or objectives specific to the facilities under considerations can be added to the model as required.

The complexity of the modern world demands understanding, quantification, and integration of most aspects of life. The attempt to quantify the travel distance and coverage in locating facilities has been made by many researchers. An attempt have been made to quantify the neighborhood effect and to integrate those quantified objectives for more exact representation of the real world by the multiobjective formulation. If it is possible, more objectives should be considered. Nevertheless, the state of art has not been developed enough to quantify, interrelate, and optimize on all these objectives. More research is needed.

CHAPTER III

SOLUTION TECHNIQUES

1.0 INTRODUCTION

This chapter will summarize the techniques used in solving the formulations developed in Chapter Two. Since most problems are cast as multiobjective location problems, specific multiobjective approaches will be discussed. The trade-off curve between objectives for problems having a convex region of feasibility is usually easy to estimate when the objective and constraints are linear. However, the location problems introduced in Chapter Two do not have convex regions of feasibility. Although the boundaries define regions that are convex, only integer solutions within the regions are feasible. This means that the resulting trade-off curves are not continuous and could appear with scallops (dents) along the discrete point surface. Fig. 3-1 gives an example of such a surface of discrete points. Points on the surface are classified as either strong or weak based on their relationship to the surrounding points. A noninferior point is weak if it lies below a line segment connecting two noninferior points. A noninferior point is strong if it lies on or above a line segment connecting any two noninferior points.

An approach which makes use of linear programming will be applied to solve the discrete noninferior trade-off set for the neighborhood impact and coverage formulation. A target approach, a Weak Noninferior Set Estimation (WENISE) approach, and a Unified Noninferior Set Estimation (UNISE) approach will be developed to find the noninferior trade-off solutions between any pair of objectives heuristically. Last of all, the Reverse Teitz and Bart algorithm will be used to generate solutions so that a comparison test of the heuristics can be made.

2.0 SOLUTION METHODS INTRODUCED PREVIOUSLY

Two obvious approaches used to solve the formulations of Chapter II are 1) utilizing linear programming with branch and bound method and 2) utilizing the modified Teitz and Bart (MTAB) algorithm given by Church and Bell (7). The linear programming approach can be easily structured on a large scale linear programming system like MPSX.

To solve the multiobjective problems, the Noninferior Set Estimation (NISE) method developed by Cohon et al. (17) will be used. The NISE method was originally developed to find the trade-off curve between two objectives in a continuous space problem. The noninferior areas, upper bounds, lower bounds, and maximum possible improvements can be defined in the plane of the two objectives.

The trade-off surface for discrete problems is composed of both weak and strong noninferior solutions. The NISE method (which is a specialized weighting method) can be used to generate only the strong noninferior solutions for this type of problem. This method will be referred to as the Strong Noninferior Set Estimation (SONISE) method in this thesis. Weak noninferior solutions can also be of interest to a decision maker, and should be identified as well. Since the NISE method cannot be used to generate these solutions, several different approaches will be introduced which can identify such solutions.

3.0 GENERATING BOTH WEAK AND STRONG NONINFERIOR SOLUTIONS

The constraint method can be used to generate all noninferior solutions, however, this is not an efficient approach for many integer problems. This is due to the fact that many problems may have to be solved before even one weak noninferior solution is found. In addition, a guarantee that all discrete weak solutions can be found within a reasonable amount of computer time cannot be made. Because of this, the use of the traditional constraint method will not be utilized. Thus an effort will be directed toward a modified weighting approach.

To identify the set of all noninferior solutions between two objectives, we can identify all strong noninferior solutions by the SONISE method. Each pair of adjacent strong noninferior solutions can be investigated for the existence of nearby weak noninferior solutions. Let A, B be a pair of adjacent strong noninferior solutions, where:

$$A = (Z11, Z12)$$

$$B = (Z21, Z22)$$

In Fig. 3-2, the area above segment AB is infeasible, for any feasible solution in this area would have been identified by the SONISE method. Segment AB is actually the upper bound for weak noninferior solutions in triangle ABC. The area to the left of or below point C (Z11,Z22) is inferior because any solution in that area is inferior to solution A, or B, or both.

The existence of any weak noninferior solution in this area can be identified by two different approaches: 1) maximize the deviation of the noninferior solution to be identified from point C in the direction of line segment AB; 2) Minimize the deviation of noninferior solutions from specified targets on line segment AB. These criteria are shown graphically in Fig. 3-3.

4.0 GENERATING COMPLETE SET OF NONINFERIOR SOLUTIONS: THE COVERAGE - NEIGHBORHOOD PROBLEM

Making use of the first criterion to generate weak noninferior points, we can develop a formulation to find the best weak noninferior solution (if one exists) between an adjacent pair of strong solutions of a coverage - impact formulation (Refer to Fig. 3-4). Consider the following formulation:

WENISP:

Maximize $Z = M1$

subject to:

$$\sum_{i \epsilon I} Ai * Yi - M1 \geqslant Z11 \quad \ldots\ldots\ (3\text{-}1)$$

$$\sum_{i \epsilon I} Ai * Z1 - M1 \geqslant Z22 \quad \ldots\ldots\ (3\text{-}2)$$

$$\sum_{j \epsilon J} Xj = p \quad \ldots\ldots\ (3\text{-}3)$$

$$\sum_{j \epsilon Ni} Xj - Yi \geqslant 0 \text{ for all } i \quad \ldots\ldots\ (3\text{-}4)$$

$$Zi - Xj \geqslant 0 \text{ for all } i,j \text{ such that } Dij \leqslant Si \quad \ldots\ldots\ (3\text{-}5)$$

$$Xj = (0,1) \quad \ldots\ldots\ (3\text{-}6)$$

$$Yi = (0,1) \quad \ldots\ldots\ (3\text{-}7)$$

$$Zi = (0,1) \quad \ldots\ldots\ (3\text{-}8)$$

where:

$Xj = \begin{cases} 1 & \text{if a facility is allocated to site } j \\ 0 & \text{otherwise} \end{cases}$

$Yi = \begin{cases} 1 & \text{if demand node } i \text{ is covered} \\ 0 & \text{otherwise} \end{cases}$

$Zi = \begin{cases} 1 & \text{if demand node } i \text{ is impacted} \\ 0 & \text{otherwise} \end{cases}$

$Ni = \{ j \mid Dij \leqslant Ci \}$, set of all potential site j which can cover i

Ci = the covering distance of node i

Si = the impact distance of node i

M1 = a positive contant

Z11 = the coverage value of point C

Z22 = the impact value of point C

Other notation is as defined previously.

The formulation will be called the Weak Noninferior Search Problem (WENISP) for the Maximal Coverage Minimal Impact Location Problem (MCMILP). It is exactly the same as the Maximal Coverage Minimal Impact Location Problem defined previously except that the objective is to maximize the improvement from point C where constraints of type (3-1) and (3-2) are used to define M1.

This formulation can be used to identify a solution within the feasible noninferior area of Triangle ABC by maximizing M1, the largest possible improvement from point C. M1 is equal to min(D1,D2) where D1, D2 are as shown in Fig. 3-4. If M1 comes out to be zero after solving this formulation, no improvement is possible within segment AB and either point A or point B will be identified as a solution to this problem. That means there are no weak noninferior solutions within this area. If M1 is not zero and an integer solution exists, the new solution can be merged into the string set. A string set is a set of points which are arranged in ascending order of one of the elements. The whole set of noninferior solutions between two selected objectives can be generated by exhaustively investigating for weak solutions between every adjacent pair of solutions in the string set.

5.0 TARGET ROUTINE

A second approach can be developed to identify weak solutions by minimizing the deviation from the solutions to a target on line segment AB. This is similar to the Goal Programming technique originally proposed by Charnes and Cooper (40). This method requires the decision maker to set a target that he/she wishes to attain. Then the preferred solution is the one which minimizes the deviation from the target. A set of strong noninferior solutions of any two objectives can be found by the SONISE method. For each pair of adjacent strong noninferior solutions, a target can be defined on the line segment joining them. This target is defined on a line segment that bounds the region where the weak noninferior solutions exists. The following problem can now be written where the second criterion is utilized:

Minimize $Z = \max [M1, M2]$ . . . . (3-9)

Subject to $X \in R$

where:

$M1 = S1(Z1' - Z1(X))$

$M2 = S2(Z2' - Z2(X))$

Z1' = the target value of the first objective

Z2' = the target value of the second objective

Z1(X) = value of the first attainable objective

Z2(X) = value of the second attainable objective

S1 = relative importance of the first objective

S2 = relative importance of the second objective

X = the vector of decision variables

R = the region of feasibility

The objective is to find the facility configuration such that the difference between the objective and the target is minimized. S1 and S2 are factors to convert the measurements of two objectives into compatible units. The ratio of S1/S2 is the gradient or the direction of optimization.

One approach to solve the above formulation is to use a mixed integer programming package. Another approach would be to structure a heuristic based upon vertex substitution, as follows:

1. Provide a starting solution and a target on the line segment connecting strong noninferior solutions.
2. Calculate the objectives of the current solution. Find the maximum difference between the objectives of the current solution and the respective values of the target.
3. Check to see if a potential site can replace one of the current facilities. Use the MTAB procedure to calculate the change in each of the objectives with such an exchange. Then find the maximum difference of the new objective values with the respective values in the target.

4. If the maximum difference of step 3 is smaller than the maximum difference of step 2, the potential site is encoded as a facility, and the facility that is removed is the one that results in the greatest move towards the target. If every potential site has been tried, and no replacements are possible, stop. Otherwise, go to step 3.

Instead of one target on a chosen segment, we can choose a fixed number of targets equally spaced throughout the length of the segments. The number of targets depend on the complexity of the problem desired or on the availability of computer facilities. By optimizing toward the targets, we will generate the approximate shape of the noninferior trade-off curve.

The decision maker can provide a fixed number of targets in this approach, but the targets can also be determined interactively such that only targets which identify solutions within the region of greatest interest on the trade-off curves are used. The process is explained in the following section.

6.0 SONISE FOLLOWED BY WENISE ROUTINE

Starting with the string set of solutions developed by the SONISE method, the Weak Noninferior Set Estimation (WENISE) routine calculates the bounded region (BORE) containing any possible weak noninferior solutions under each segment. The segment with the largest BORE area will be selected. The mid-point of this segment will be used as a target. The target routine (which can be developed as a subprogram) will be called. If a weak noninferior solution is not found within the BORE area of this segment, the BORE area of this segment is eliminated from further analysis; and another segment is investigated. If a weak noninferior solution is found within this segment, this point is merged into the string set and the BORE area of this segment is broken down into two BORE areas. This routine will continue on investigating new segments until no new noninferior points can be identified (All BORE areas have been tested without success). The following section will explain how the noninferior areas are defined.

6.1 Noninferior areas for the WENISE routine

The set of strong noninferior solutions are drawn in Fig. 3-5. The initial BORE areas are the right angled triangles below the segments joining two adjacent strong solutions. The area below the right angled triangles are inferior because they are dominated by at least one strong solution.

We assume that the greatest resolution of the trade-off will exist by examining the region of the trade-off curve with the largest BORE area. Let us find that segment and put a target D on the mid-point of this segment (refer to Fig. 3-4). The direction of optimization, G, is the line joining the target and point C. Since G = S1/S2, we can take S2 as unity in (3-9), then S1 = G. G is actually a factor which makes D1 as important as D2. Suppose we identify a point E after solving (3-9) by using D as the target (refer to Fig. 3-6). The area EKCJ is inferior to point E. The area IHF is infeasible, because any solution in this area should have a smaller Z and should have been identified instead of E. Thus, the noninferior areas in this segment is broken into 2 areas: trapezoid AKIH and trapezoid EJBF. Point E and the BORE areas can be added to the string set, and another pair of adjacent solutions with largest BORE area is chosen from the string set for identification of the next weak solution.

Suppose EJBF is chosen next. Let the mid-point of EB be L. Suppose JL intersect EF or FB in M, where M is a point on EJBF (refer to Fig. 3-7). We will choose M as the target, and the direction of optimization is JM. If there are no distinct solutions in this segment, this area is infeasible. If a point N is identified, area NSJK is inferior to N, and area OPFQ is infeasible. Thus the noninferior area in EJBF is broken into ESNQ and OKBP.

Suppose the next largest BORE area is the rectangle ESNQ, we can go through the same process as in EJBF, and will find Q as the target and SQ as the direction (refer to Fig. 3-8). We can treat this area in the same way as we treated EJBF.

Suppose the solution N is identified in a different location when EJBF in Fig. 3-7 is investigated. The BORE areas are sectioned differently. See Fig. 3-9. Area SJKN is domininated by N, and area QOP is infeasible. BORE area NKBP is still a trapezoid, but BORE area ESOQF is a pentagon. If this segment is to be investigated next, we can let U be the mid-point of EO (refer to Fig. 3-10). Let SU intersect FQ at V. We will choose V as the target and SV as the direction of optimization. We may identify a solution within any of the three subareas ESOQXW,WXQZYF, or FYZ as shown in Fig. 3-10, but no matter where the solution appears, the resulting BORE areas will be broken into rectangles, trapezoids, or pentagons. They are illustrated in Fig. 3-11, Fig. 3-12, Fig. 3-13. These BORE areas can be handled as described previously.

Thus the initial BORE areas are triangles, and after successive identification of noninferior solutions, the areas will break down into rectangles, trapezoids, or pentagons.

7.0 UNISE ROUTINE

The methods that we used to identify the complete set of noninferior solution were divided into two steps: 1) use the SONISE routine to identify the set of strong solutions; and 2) use the WENISE method to find the set of weak noninferior solutions between all pairs of strong solutions. These steps can be combined into one. We will call it the Unified Noninferior Set Estimation (UNISE) method.

This method can be best explained by an example. Suppose there are only seven noninferior solutions on the objective space of a 2-objective problem as shown in Fig. 3-14. Note that solution A, D, and G are strong solutions, and solution B, C, E, and F are weak solutions. We can identify solution A and solution G easily by solving two single-objective problems. The solutions and the BORE areas are shown in Fig. 3-15. The target and the direction of optimization are also shown. We can proceed to find the third solution by calling the target subproblem. The target to be supplied to the subprogram is:

$$Z1' = Z71$$

$$Z2' = Z12$$

and the direction of optimization:

$$S1/S2 = [Z12-Z72] \ / \ [Z71-Z11]$$

putting S2=1, S1 is:

$$S1 = [Z12-Z72] \ / \ [Z71-Z11]$$

We will identify solution E because the deviation from target T to solution E is the smallest. The resulting BORE areas and their targets are shown in Fig. 3-16. The direction of optimizations are along the diagonals passing through the targets. Assume that the BORE areas in AIEJ is the largest, we will investigate this segment for the next solution. Using J as target and IJ as direction of optimization, we will identify solution D. Using L as the target and KL as the direction, we will identify solution F. The results are shown in Fig. 3-17.

By the same approach, we will identify solution C in segment AMDN, nothing in QFRE, nothing in OEPD, and nothing in FSGU. The results are shown in Fig. 3-18. The only BORE areas left behind are AVCW and CYDE. We will identify solution B in AVCW and nothing in CYDE. The results are shown in Fig. 3-19. AVCW is broken down into two more noninferior segments, but we will not be able to identify any solution in these segments. The set of BORE areas becomes null, and no more executions are required.

Thus the set of noninferior solutions, including the strong and the weak solutions, can be found by the UNISE method. If, due to limited funds for analyses, we want to terminate the execution after identifying solution A and G and calling the target subprogram four times, we will identify solutions A, C, D, E, F, and G, leaving only solution B unidentified. That means that the execution can terminate at any time with a good approximation of the noninferior trade-off curve.

8.0 RESOLVING DIFFICULTIES WITH HEURISTICALLY DERIVED SOLUTIONS

The above discussions of BORE areas and solutions are based on one assumption: that of the ability to identify the optimal solution for any given problem. Using linear programming will always give the optimal solution, but the answer may be fractional, which is not acceptable to the integer location problem. The branch and bound method can be used to resolve the fractions but this can be tedious and time consumming. The heuristic approach which we have developed appears to be the only available alternative. The heuristic approach does not, however, gaurantee that optimal solution will be identified. One problem is that a solution within a BORE area may not be identified when that area is being investigated when one actually exists. To conclude that the area contains no weak solutions is obviously an erroneous conclusion.

Several possible options may be selected to mitigate this situation. First, we can call the target subprogram for each target and direction of optimzation with different starting solutions, and choose the best solution found. If this solution is within the appropriate BORE area, new BORE areas are calculated, and then the solution and the BORE areas are listed in the string set. But if the best solution is not in the appropriate BORE area, then the solution is neglected, and the BORE area is removed from further analysis. This method was used to identify the complete noninferior set by SONISE and WENISE methods.

Second, for each solution identified, one can test whether it is noninferior to any solution in the string set. If it is, it can be inserted and the BORE areas redefined for this region in the string set. If not, one can generate another solution with a different starting solution until a good solution is found or the number of trials exceeds a predetermined number. The detail is shown in the following iteration:

Let:

N = the maximum number of times the target subprogram is to be called.

K1 = the maximum number of different starting solutions we want to provide to the target subprogram for each target and direction of optimization.

1. Identify the best solution in objective one and then in objective two. Calculate the BORE area of these two points.
2. Identify the segment with largest BORE area. If all BORE areas are zero or the total number of iterations is greater than N, stop. Otherwise, set K = 0.
3. Provide a starting solution.
4. Call the target subprogram. Set N = N + 1.
5. If the current solution is noninferior to any solutions in the string set, go to 6. Otherwise, set K = K + 1. If K is less than K1, go to 3. Otherwise, set the current BORE area to zero, go to 2.

6. Test whether the current solution lies within the current BORE area or not. If not, decrease the BORE area by a factor (e.g. 10). Then insert the current solution and the resulting BORE areas in an appropriate place in the string set.
7. Check the string set for inferior solution which may arise as a result of noninferior solutions generated in later steps. If a point is found inferior, it is deleted from the string set. Go to 2.

The main difference between this and the preceding approaches is that this approach stores all noninferior solutions generated. Thus it is more efficient in finding the set of noninferior solutions. It is implemented by the UNISE method.

No matter which approach is used, due to the heuristic nature of the problem, the best set of noninferior solution cannot be guaranteed. Nevertheless, nearly all solutions are within 5% of optimality. In practice, these solutions are regarded as acceptable.

9.0 REVERSE TEITZ AND BART ROUTINE

The Reverse Teitz and Bart Routine (RTAB) developed by Church and Huber (19) generates many close-to-optimal solutions by reversing the process developed by Teitz and Bart. Starting with the facility configuration of strong noninferior solutions generated by SONISE, RTAB tries to replace the facilities by one, two, or more facilities. If the objective values of the resulting facility configurations are within, say 5% , of the solutions in SONISE, they are recorded as close-to-optimal solutions. We will use this approach to identify some of these solution so that a comparision of the heuristics can be made.

CHAPTER IV

RESULTS AND CONCLUSIONS

1.0 INTRODUCTION

This chapter presents the solutions to some of the problems discussed and developed in Chapter II and Chapter III. Table 4-1 (all the tables are included in the Appendix) presents a list of the different models introduced in Chapter II. Each of those models was solved optimally with the exception of the Neighborhood Impact, Coverage, and Weighted Distance Location Problem. The table also gives a brief description of each model. The data sets used and the difficulties encountered in both optimal and heuristic approaches are dicussed. An analysis of the results is also attempted.

2.0 DATA AND SOLUTIONS

Two data sets which are frequently referred to in literature (12,19) are used in this thesis. One of them is Toregas and Schilling's 30 node network of major cities in New York (10); the other one is Swain's 55 node network on a Cartesian plane (49). The population at each node, the property value at each node, and the shortest distance matrix are presented for the Toregas/Schilling data in Table 4-2. The population and the Cartesian coordinates for each node is presented for the Swain data in Table 4-3. The spatial distribution of the nodes of these two data sets are also shown in Fig. 4-1 and Fig. 4-2.

All covering models which are discussed in Chapter Two were solved by The Mathematical Programming System Extended (MPSX) (51). This system is provided by the IBM Corporation and was used to solve the formulations optimally. A FORTRAN program was written for use in the DEC system 10 for each formulation in order to transform the input parameters into a data file compatible with the MPSX system. For each problem a data file and the MPSX code was submitted as a job to the IBM 3031 system. The heuristic solutions were generated by FORTRAN programs written and executed on the DEC system 10 at The University of Tennessee. Input to all impact models includes:

1. Number of nodes considered
2. Number of facilities to be allocated
3. Number of different impact segments
4. Distances of impact segments
5. Coverage distance (c)
6. Impact factors (Mk) for impact segments
7. Population (Ai) at each node
8. Cartesian coordinates of each node or the shortest distance Matrix (Dij)

The input parameters used to generate tables and figures in the following pages are arbitrarily chosen, however, other parameters can be used.

With the exception of the MPFILP which is solved by the Knapsack algorithm introduced previously, all heuristics incorporated the Modified Teitz and Bart (MTAB) algorithm. The output from the computer programs included the configuration of facilities, the objective values, and the set of noninferior solutions.

3.0 MILP AND MPFILP SOLUTIONS

There was no difficulty encountered when solving the Minimal Impact Location Problem (MILP) as formulated in Chapter Two. In each case, the heuristic solutions to this formulation were the same as the optimal solution identified by linear programming. For the MILP with distance modification, only one problem was attempted. This solution was fractional. It was, however, easily resolved by branch and bound. The solutions and input parameters to these two models are depicted in Table 4-4 and the configurations of facility locations are shown in Fig. 4-3 and Fig. 4-4. As shown in Table 4-4, when the MILP is solved, facilities are allocated to nodes 52, 53, 54, 55; when the MILP with distance modification is solved, facilities are allocated to nodes 50, 52, 53, 55.

The solutions to the Minimal Person-Facility Impact Location Problem (MPFILP) using the Knapsack Problem Method was similar to that of the MILP. This is understandable because both the MPFILP and MILP solutions are very likely to be at nodes which have the least population. With distance modification, the solution to the MPFILP is identical to the solution of the MILP with distance modification. The results are also shown in Table 4-4.

4.0 MPCILP SOLUTIONS

The optimal solution to the Minimal Person-Capacity Impact Location Problem using the first nine points of the Swain data set was fractional. This solution was difficult to resolve by the branch and bound method. As mentioned before, the constraints that require assignment to the closest established facility are not functional when solutions are not integers. It is not possible to obtain an optimal solution to the MPCILP on the Swain data unless some constraints and variables are selectively added or deleted (e.g. ReVelle and Rosing (18)), because there are more constraints and variables than the MPSX system allows. The heuristic solution is the same as the solution to the MILP. The objectives and the input parameters are also presented in Table 4-4.

5.0 DISCUSSIONS OF THE IMPACT MODELS

The solutions to different impact models are found to be generally the same as shown in Table 4-4. The nodes which are far away from the central cluster of demand points are frequently chosen as sites for facilities. These models may be applicable to some real world location problems but are limited on the basis of the definition of the objective.

Minimizing neighborhood impact should be considered as one of the objectives in multiobjective location models. The following sections deal with multiobjective location problems when minimum neighborhood impact is included.

6.0 MCMILP SOLUTIONS

The set of strong noninferior solutions to the Maximal Coverage Minimal Impact Location Problem (MCMILP) were generated by linear programming and consists of eight distinct points. Seven of the eight points are noninferior (The seventh and the eighth solutions are alternative solutions to the Maximal Covering Location problem, however, the seventh solution is superior to the eighth solution in terms of the impact objective). The input paramters and resulting solutions are shown in Table 4-5. All problems except one terminated in all integer solutions. The fractional solution was resolved by branching on one variable (refer to Fig. 4-5).

Using the heuristic SONISE routine, the same set of strong noninferior solutions was generated. The set of strong noninferior solutions is plotted in Fig. 4-6. Even though the tradeoff curve is composed of discrete points (either weak or strong), a line has been drawn connecting all strong solutions. This line is not the tradeoff curve, but can be used to identify the weak noninferior solutions when they exist. In subsequent figures, all strong noninferior solutions will be connected by discrete line segments in order to easily identify weak solutions.

The configuration of each solution is plotted in Fig. 4-7, 4-8, 4-9, 4-10, 4-11, 4-12, 4-13, and 4-14. As shown in the figures, there is a gradual trend of locating facilities from the periphery to the central area of the cluster of demand.

A lower bound line which is 5% less than the optimal values is included in Fig. 4-6. The points on the line are defined as:

$$X_i = X'_i - (X'_{max} - X'_{min}) * 5\%$$

$$Y_i = Y'_i - (Y'_{max} - Y'_{min}) * 5\%$$

where:

X_i = X axis of the ith element of the 95% curve

Y_i = Y axis of the ith element of the 95% curve

X'_i = X axis of the ith optimal solution

Y'_i = Y axis of the ith optimal solution

X'_{max} = the maximum X value of the optimal solution

X'_{min} = the minimun X value of the optimal solution

Y'_{max} = the maximum Y value of the optimal solution

Y'_{min} = the minimum Y value of the optimal solution

To find a set of weak noninferior solutions by the Target routine, a fixed number of equally spaced targets were identified on the line segments joining the set of strong solutions generated by the heuristic method. The solutions resulting from the Target routine are shown in Table 4-6 and Fig. 4-15. All solutions except one are points on, or very close to, the curve of strong noninferior solutions. Although this one solution is not within the 5% band of the strong noninferior solutions, it may be the best one within that segment (see Fig. 4-16).

A more efficient method to find the complete set of noninferior solutions is the WENISE routine which is a subprogram called after the completion of the SONISE routine. The results are shown in Table 4-7 and Fig. 4-17. From Fig. 4-17, we can see that only one weak noninferior solution is identified. An inspection of the RTAB results in Fig. 4-18 shows that there are not many close-to-optimal solutions. With the exception of a few noninferior solutions within the segment between solution (6) and solution (7), all noninferior solutions are identified.

The UNISE routine, which is another approach to identify noninferior solutions, identified four out of seven strong noninferior solutions (refer to Table 4-8 and Fig. 4-19). However, all solutions are within the 5% band of the strong noninferior solutions.

7.0 NCWDLP SOLUTIONS

The Neighborhood Cover Weighted Distance Location Problem (NCWDLP) is a large problem to be solved optimally. The solutions are fractional and somewhat difficult to be resolved by the branch and bound method. Computer programs using heuristic methods were developed to generate a trade-off curve of any two objectives of the NCWDLP. The solutions of the trade-off between neighborhood impact and weighted distance are shown in Fig. 4-20 to Fig. 4-23 and Table 4-9 to Table 4-11. Fig. 4-23 shows that no RTAB solutions are above the strong trade-off solutions. This implies that the SONISE solutions are probably optimal. The densities of both the WENISE solutions and the UNISE solutions vary directly with the density of the RTAB solutions; i.e., in the area where RTAB identifies a lot of solutions, WENISE and UNISE will also identify many solutions. All solutions generated by the WENISE Routine, the Target Routine, and the UNISE Routine are very close to the SONISE solutions. Furthermore, they give a good approximation of the noninferior trade-off curve.

Solutions to the trade-off between population coverage and weighted distance are shown in Fig. 4-24 to Fig. 4-27 and Table 4-12 to Table 4-14. They show the same property as the trade-off solutions between the neighborhood impact and the weighted distance. From Fig. 4-24 and Fig. 4-27, we can see that the WENISE routine identified all the weak noninferior solutions that RTAB has identified. From Fig. 4-26, we know that the UNISE routine identified all but the fourth strong noninferior solutions. The Target Routine does not perform as well as expected. Two of the solutions are not within the 5% band of the strong noninferior solutions.

8.0 SOLUTIONS TO PROPERTY COVERAGE VERSUS POPULATION COVERAGE MODEL

In 1974, Schilling (30) generated a set of strong noninferior trade-off solutions between property coverage and population coverage. His solutions will be reproduced here for two purposes: 1) to find all weak noninferior solutions that he had not identified; 2) to test whether the routines developed in this thesis give compatible results using his data set. Schilling's Property Coverage versus Population Coverage Model is solved optimally by the same basic idea employed by the WENISP. To begin with, a strong trade-off curve between the objectives is found both by optimal method and heuristic methods. The results of these two methods are identical. They are also identical to Schilling's results.

To show how well the weak noninferior search problem (WENISP) performs when implemented to identify weak noninferior solutions, the segment between solution 4, (342,418), and solution 5, (398,284), was chosen. The lower left corner of this segment, (ZZ1,ZZ2), was equal to (342,284). A few weak solutions were found within this area, although a lot of branching was required. The branching trees are shown in Fig. 4-29, Fig. 4-30, and Fig. 4-31. We can see that a total of 39 nodes are required to identify 4 integer noninferior solutions. The WENISE Routine was also used to solve the same problem. It identified a good noninferior trade-off set as well as 3 out of 4 solutions from WENISP. All results are presented in Fig. 4-28 and Table 4-15.

Two trade-off sets of this problem were also generated heuristically, one by the Target Routine and the other by the UNISE Routine. They are presented in Fig. 4-32 and Fig. 4-33. The RTAB solutions were also determined and included with Fig. 4-28, Fig. 4-32, and Fig. 4-33. From these figures, it can be seen that the SONISE Routine, the WENISE Routine, the Target Routine, as well as the RTAB Routine generate compatible results. It is important to note that there is no guarantee that all the weak noninferior solutions will be detected by any of the approaches except the WENISP approach.

9.0 COMPUTER TIME REQUIREMENTS AND ACCURACY OF DIFFERENT METHODS

Heuristic methods are less time consuming but cannot guarantee optimality. The chance of getting an optimal solution can be increased by increasing the number of trials using different starting solutions. Different starting solutions can be used, for example, to identify different results among which the best solution is selected. This chosen solution may be optimal if it is based on a large number of starting solutions.

In Table 4-16, a list of the total computer time used by different routines is given.

The total time used is the time used to compile and execute the programs. For many of the problems, more computer time than normally required has been used. One reason for this is that for each solution shown in the preceding tables, as many as eight different starting solutions have been used. Another reason is that an attempt has been made to exhaust all the noninferior solutions. As a result, each solution shown in the table represents the best of a set of one or more solutions which were generated. When applying the preceding solution methods to actual problems, it may not be cost effective to exhaust the complete set of noninferior solutions, thus the total time requirement will be much reduced.

10.0 CONCLUSIONS

We have introduced different surrogates to measure neighborhood impact based on three basic elements. They are the distance of a facility to a population center, the population at the population center, and the capacity of the facility. With these basic elements in mind, models were developed to mitigate the effect of neighborhood impact.

Although the single objective neighborhood impact model is probably not directly applicable, it is too important to be totally neglected. It can, in fact, be easily incorporated with other objectives to form a multiobjective problem. In this thesis, a multiobjective model of neighborhood impact, coverage, and weighted distance is formulated.

Three routines have been developed to solve the multiobjective models. With different starting solutions, these three methods frequently identify the optimal noninferior solutions. Nevertheless, as shown in Fig. 4-25, these heuristic routines do fail to generate satisfactory results some of the time in spite of the fact that different starting solutions have been used. This is a manifestation of the nature of heuristic routines.

The frequency of of not finding optimal solutions seems to vary with the data set. For data sets whose numeric values vary dramatically, there is a greater chance of identifying a solution which depart drastically from the optimal solution. The reason is that there may not exist a series of solutions each different by one site from the previous for which the objective value does have a gradual change towards the optimal value. In this case, a stable facility configuration may be established and it is not possible to replace any one of the current facilities by any one of the potential sites.

It is quite likely that a 2-Optimum Algorithm (30) can be applied to improve the probability of attaining an optimal solution. This is a good topic for future research.

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APPENDIX

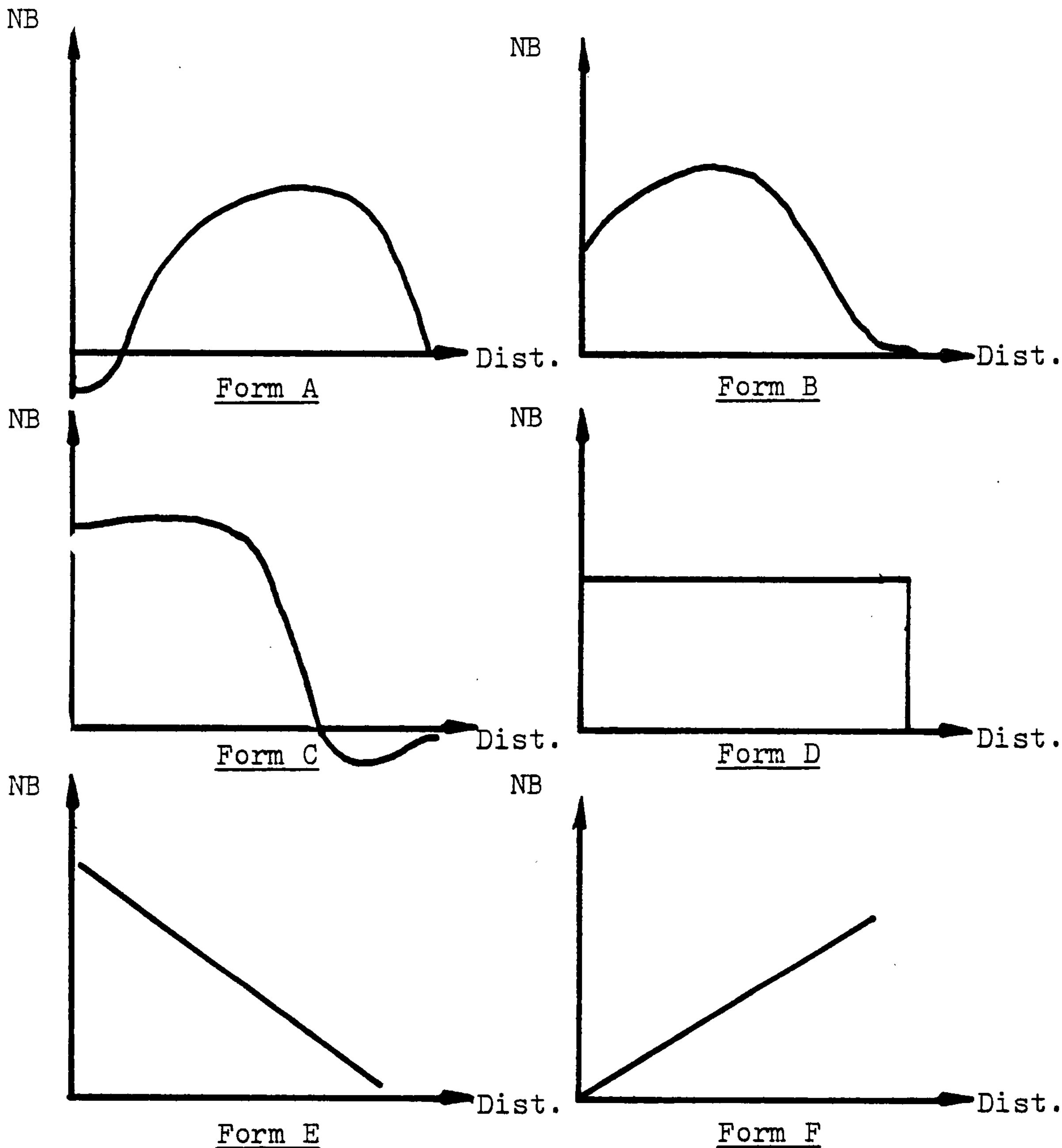


Fig. 1-1. Benefit Functions by Austin, Church and Bell.

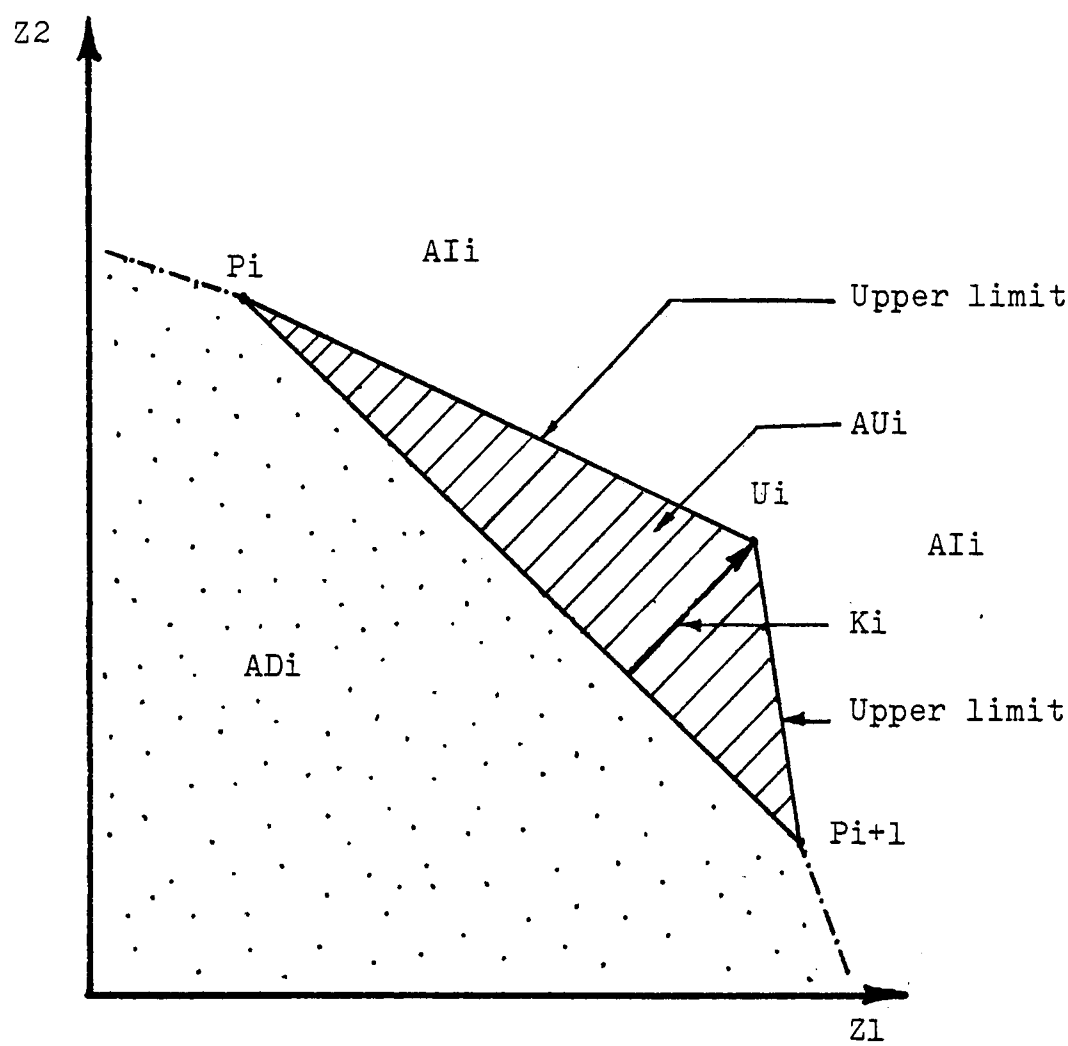


Fig. 1-2. The ith Element and the Associated Quantities of the String Set.

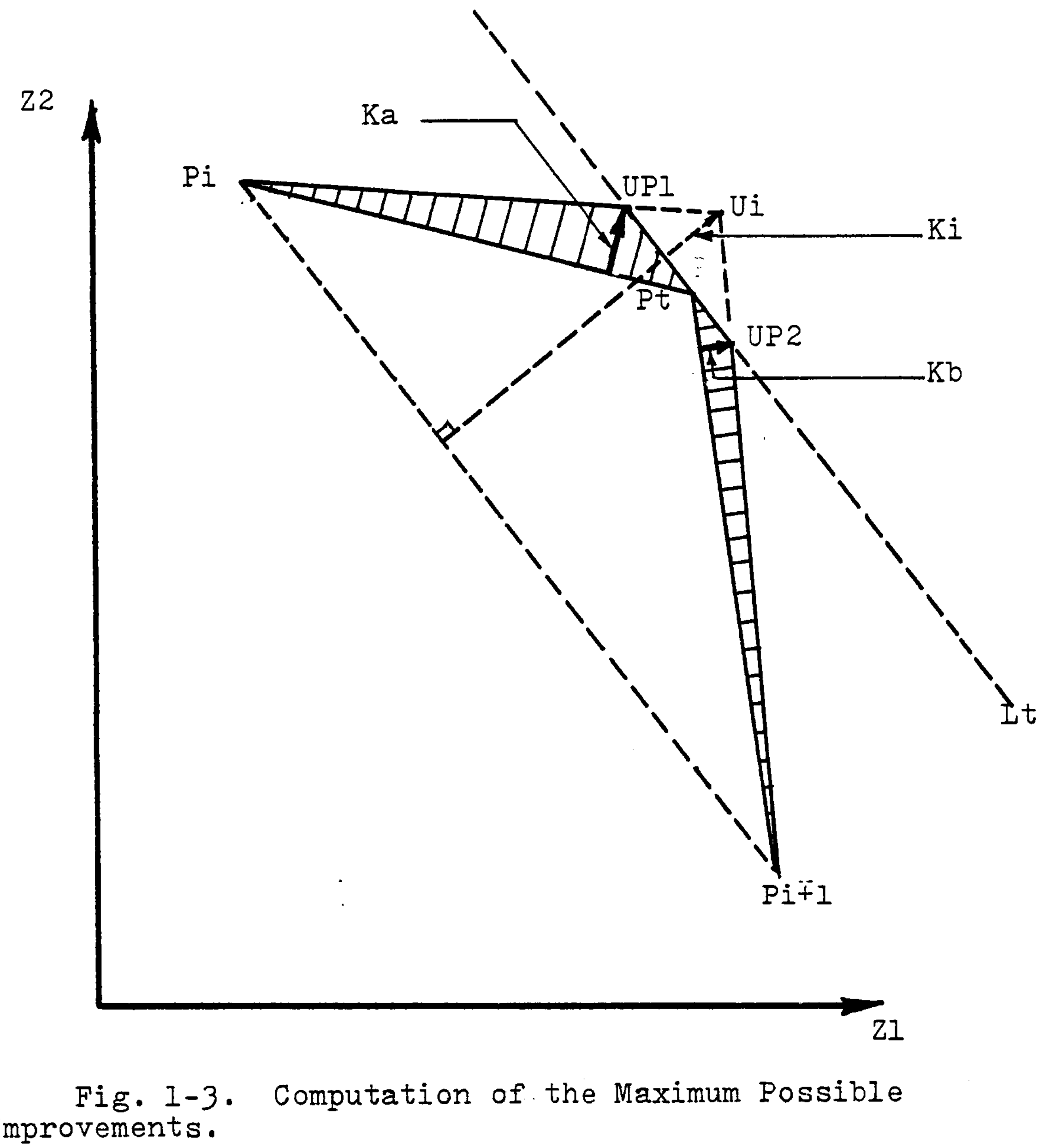


Fig. 1-3. Computation of the Maximum Possible Improvements.

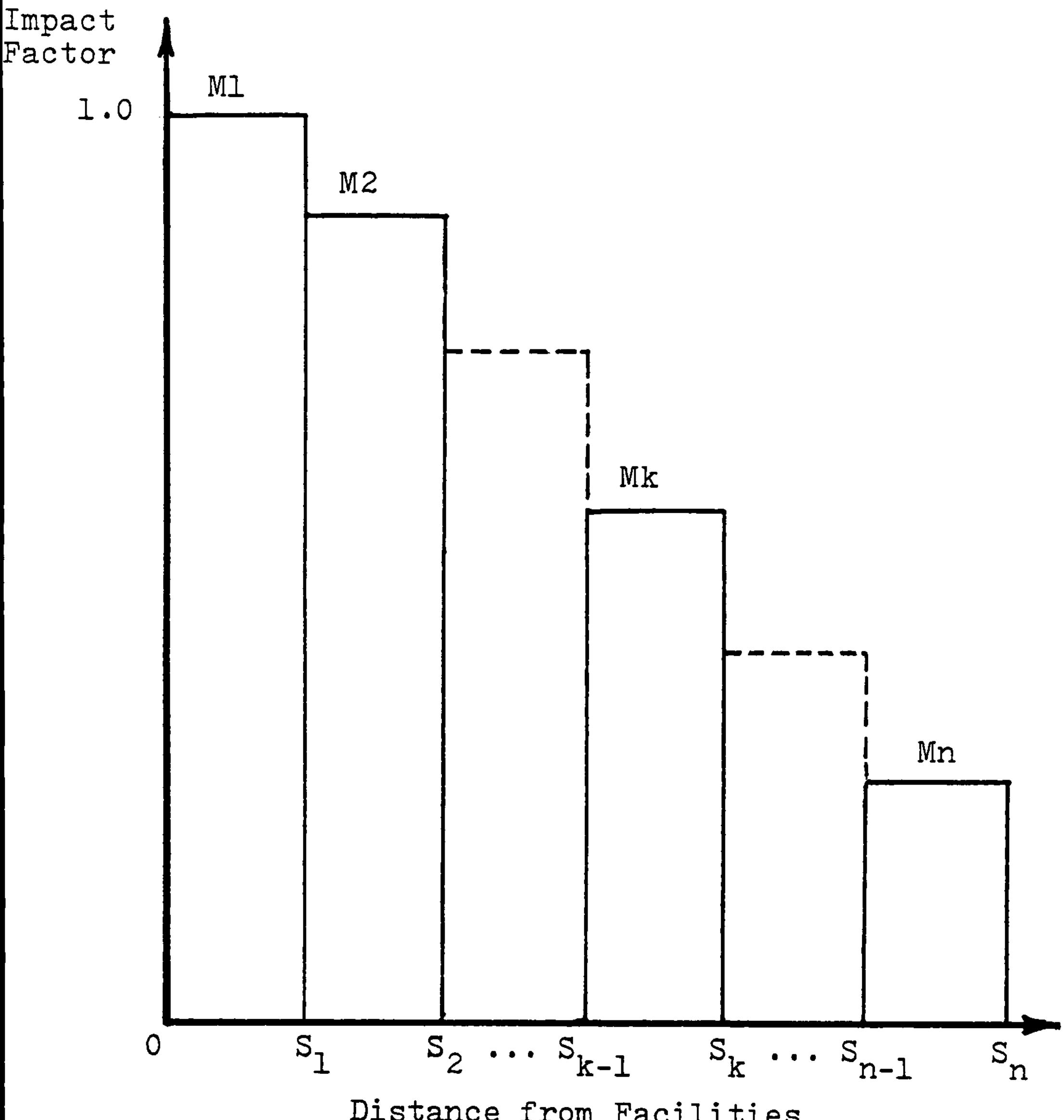
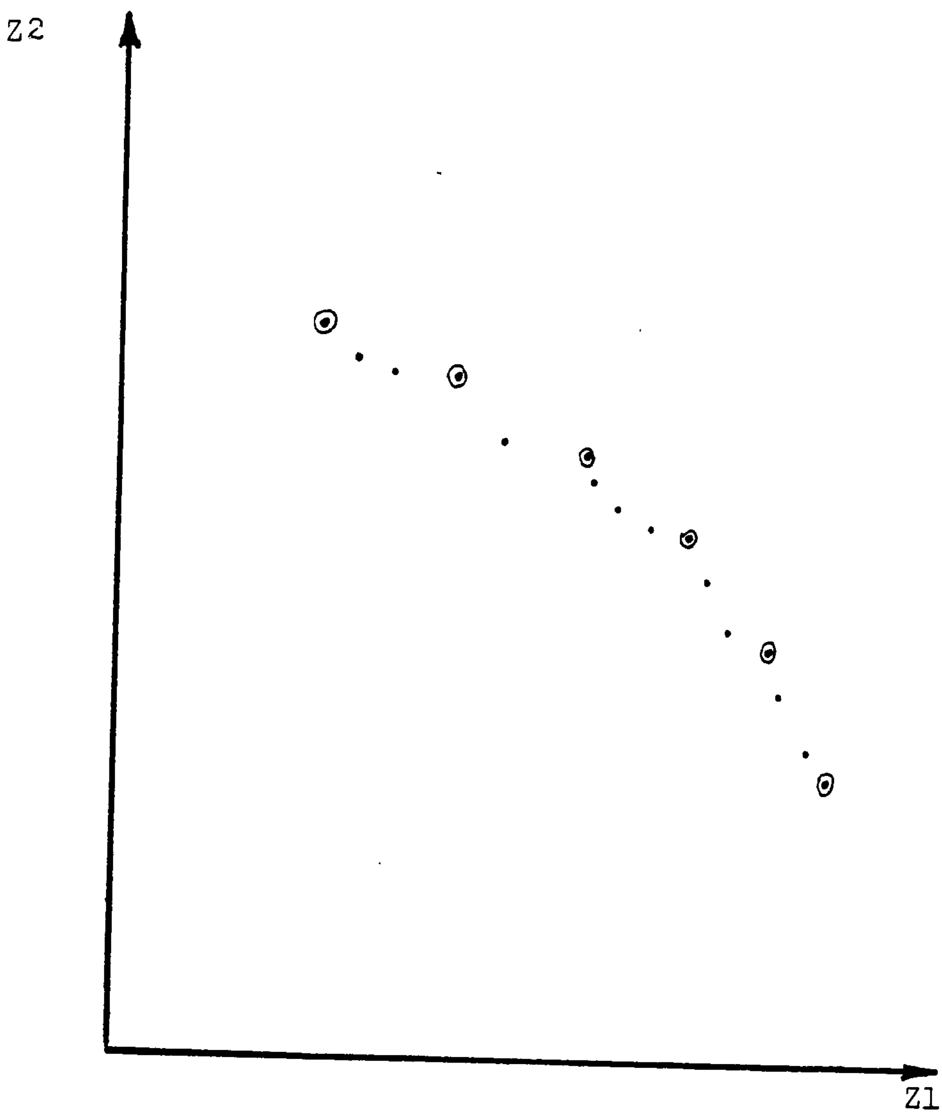


Fig. 2-1. Impact Factor Versus Distance from Facilities.



Strong Noninferior Points

Weak Noninferior Points

Fig. 3-1. Surface of Discrete Objective Space.

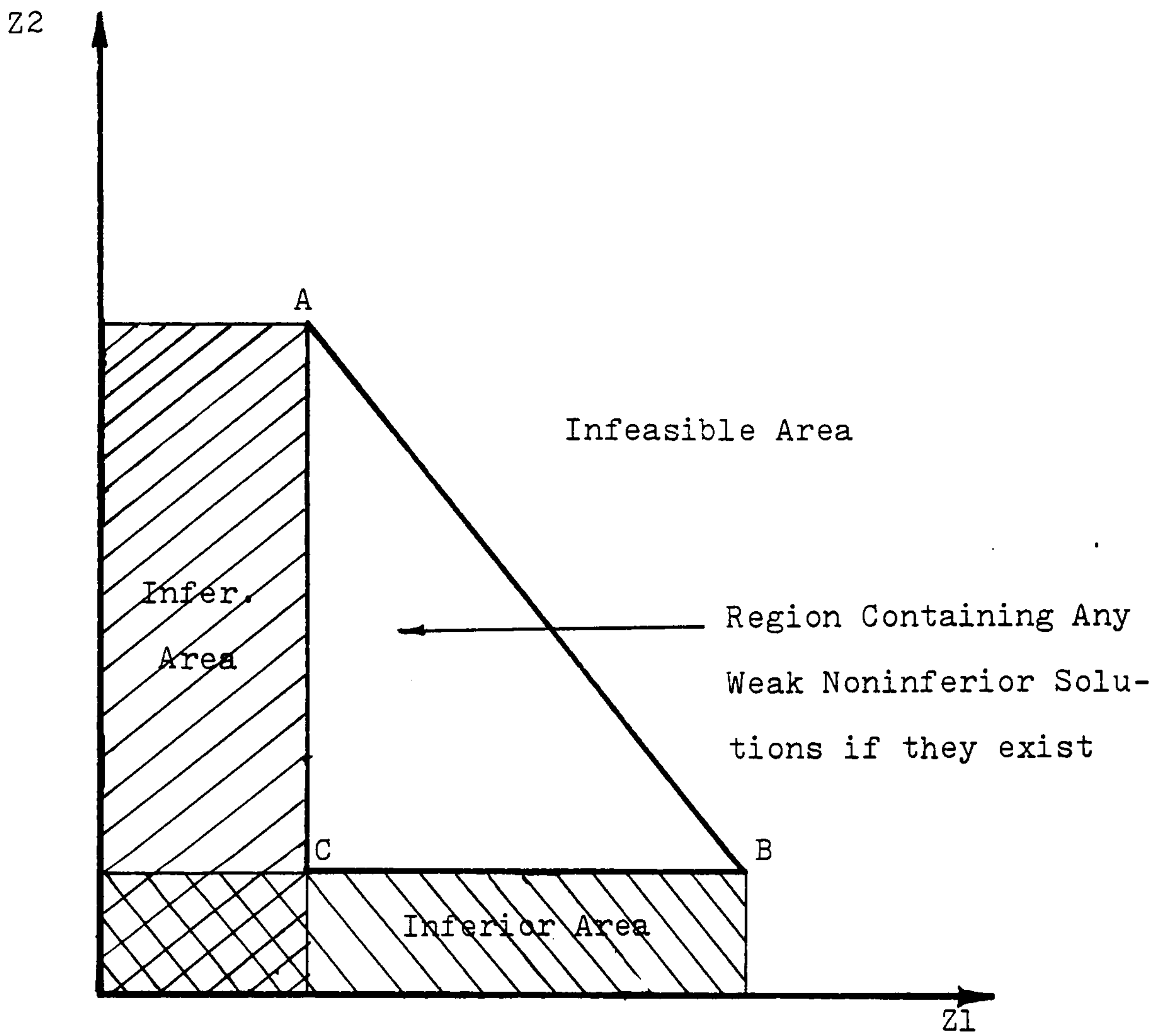


Fig. 3-2. Region for which Weak Noninferior Solutions may exist in Objective Space.

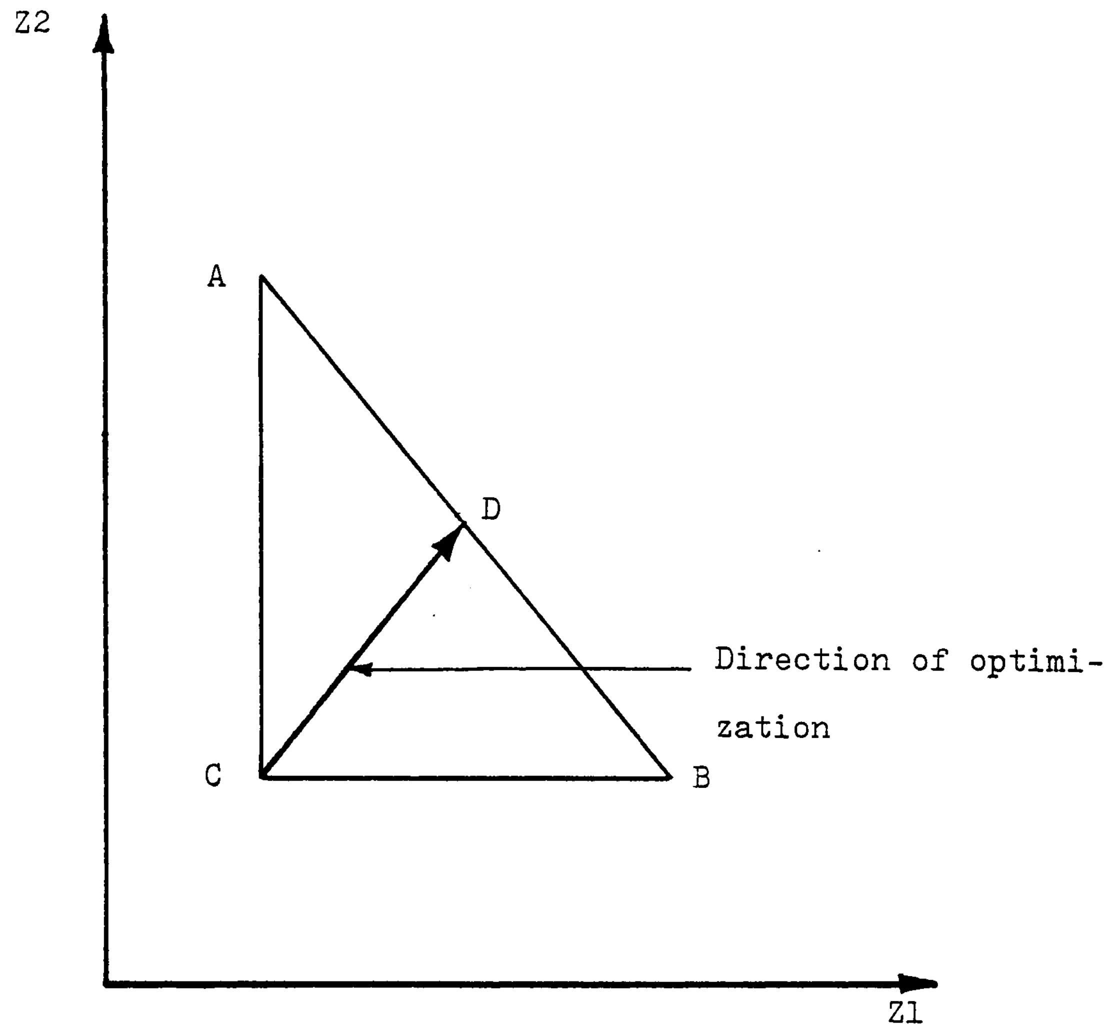


Fig. 3-3. Method for Maximize Deviation from Point C or Minimize Deviation from Target D.

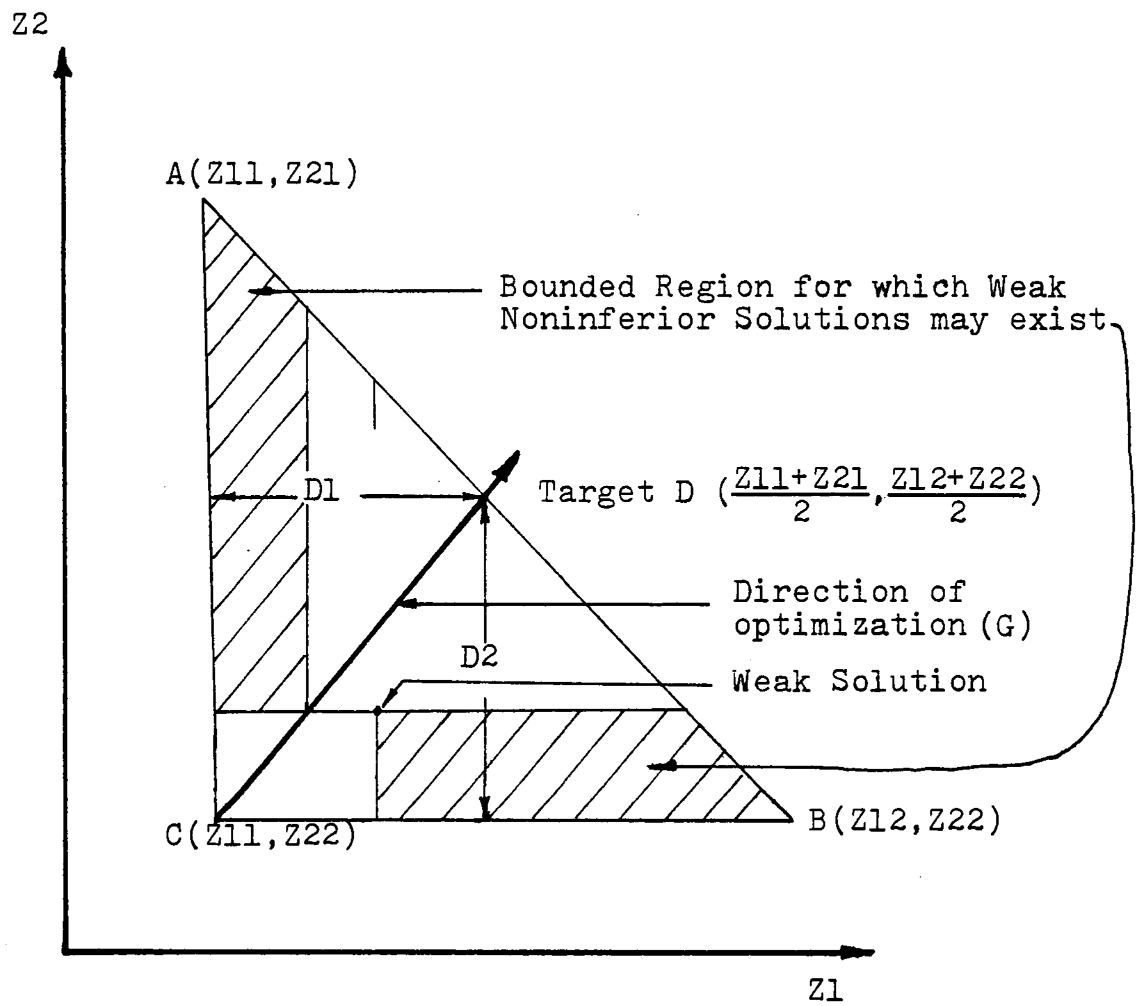
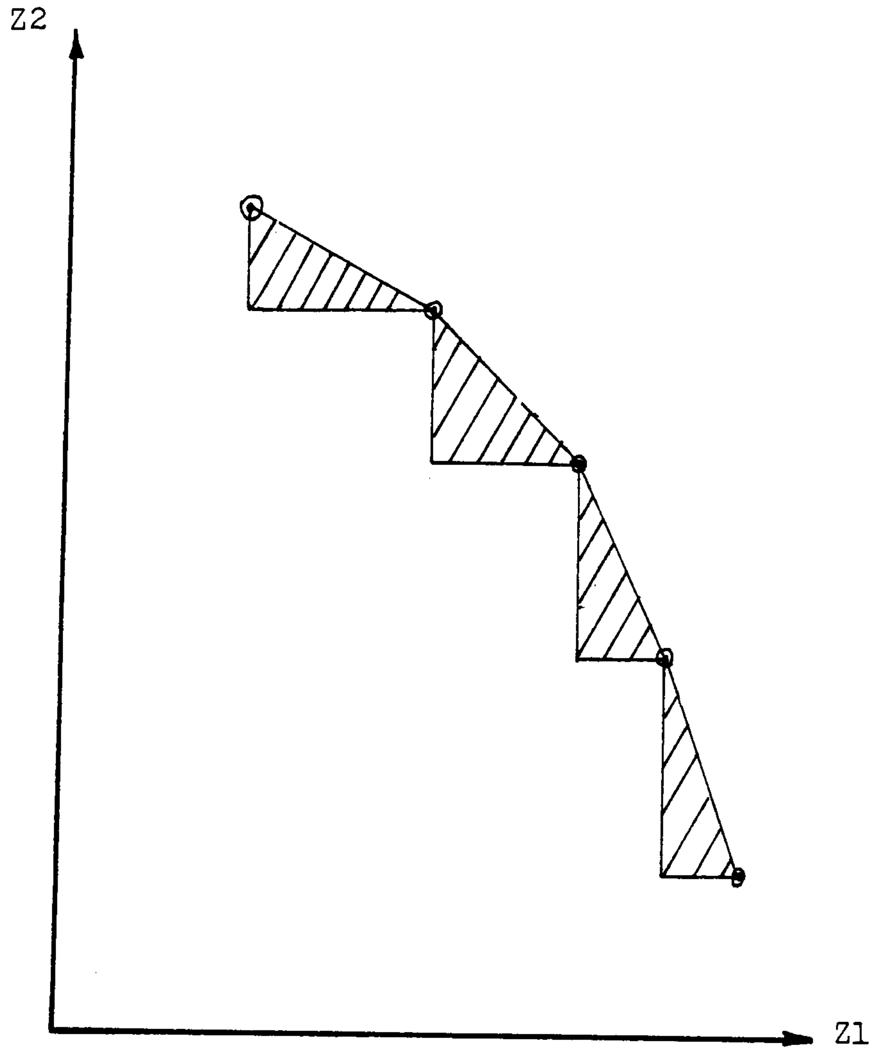


Fig. 3-4. Weak Noninferior Solution between a Pair of Strong Noninferior Solutions.



⊙ Noninferior Solution from SONISE

▨ Bounded Regions for which Weak Noninferior Solutions may exist

Fig. 3-5. Bounded Regions for which Weak Noninferior Solutions May Exist after the Application of SONISE.

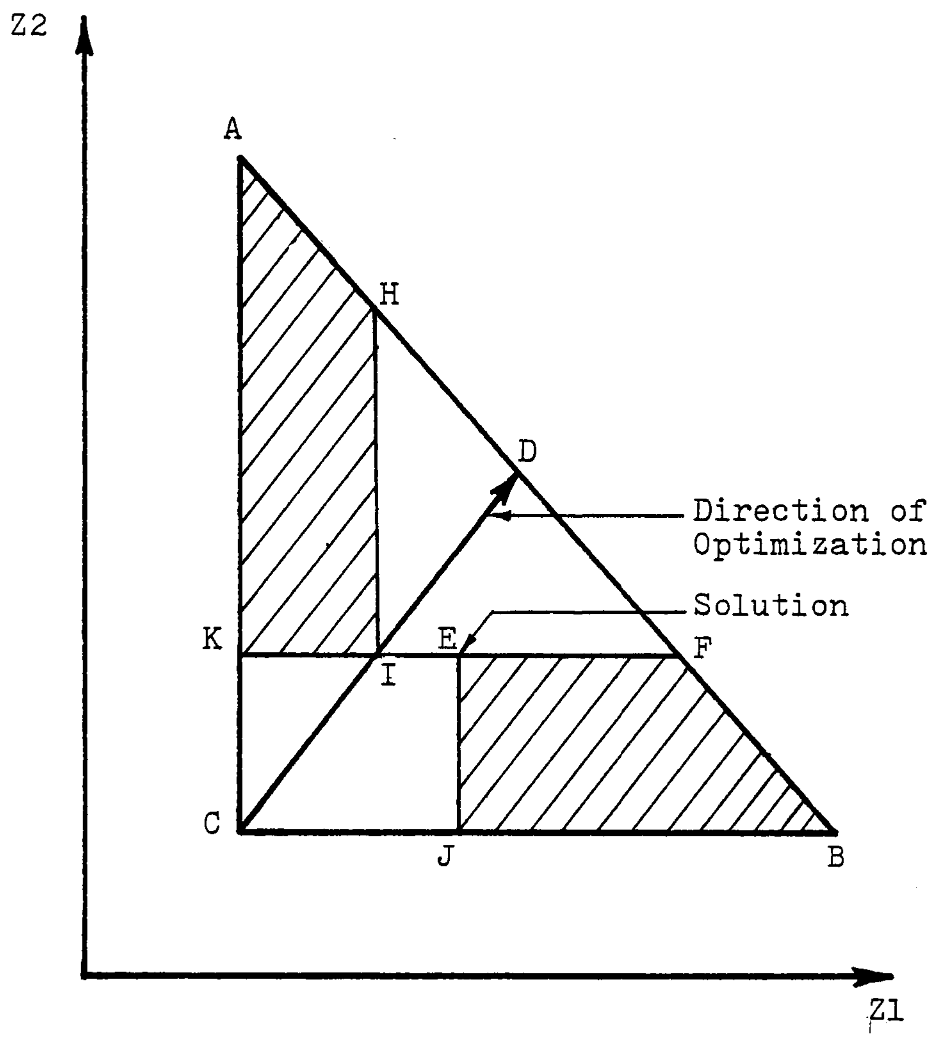


Fig. 3-6. Bounded Regions for which Weak Noninferior Solutions may Exist after identifying the First Weak Solution.

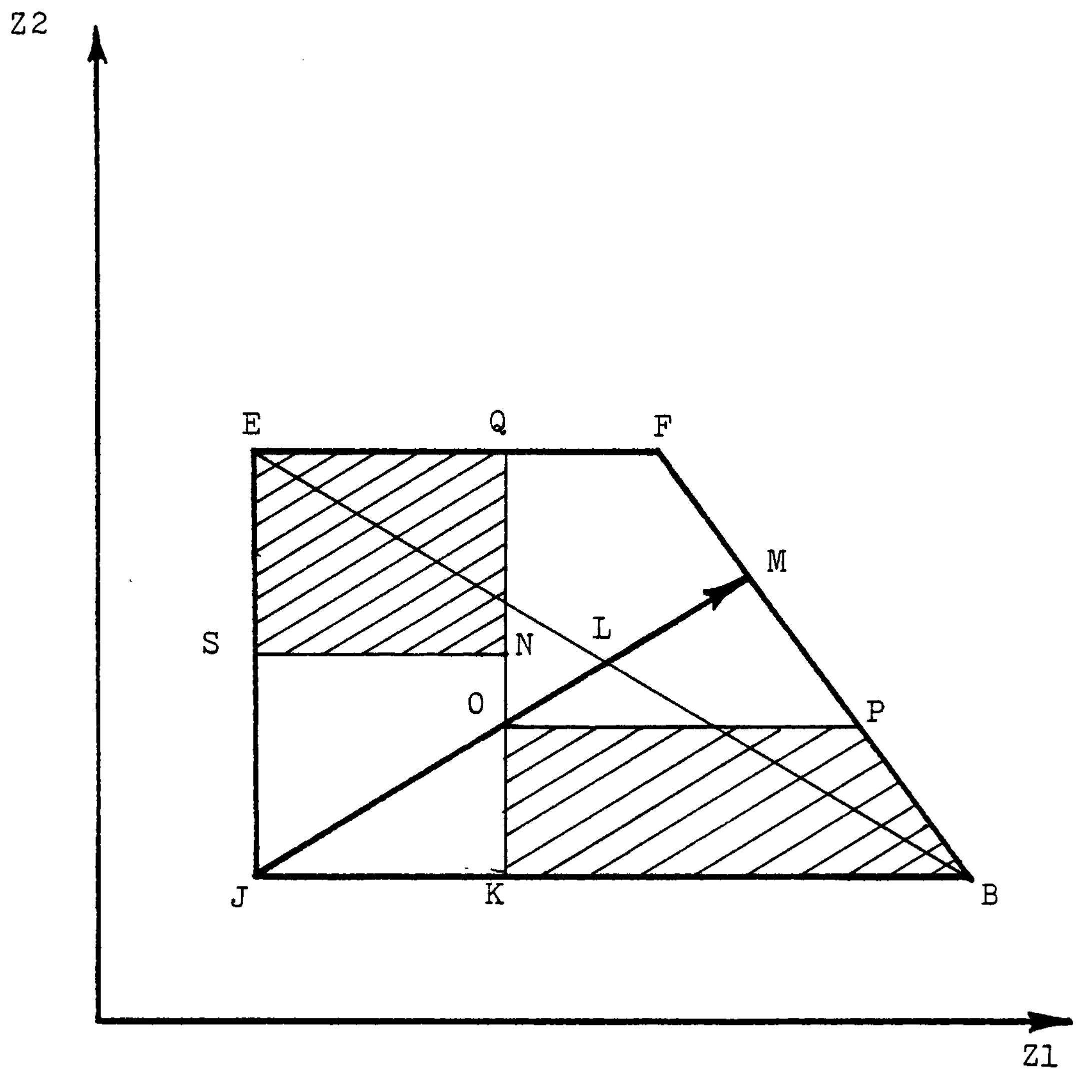


Fig. 3-7. Bounded Regions for which Weak Noninferior Solutions May Exist after Identifying the Second Weak Solution.

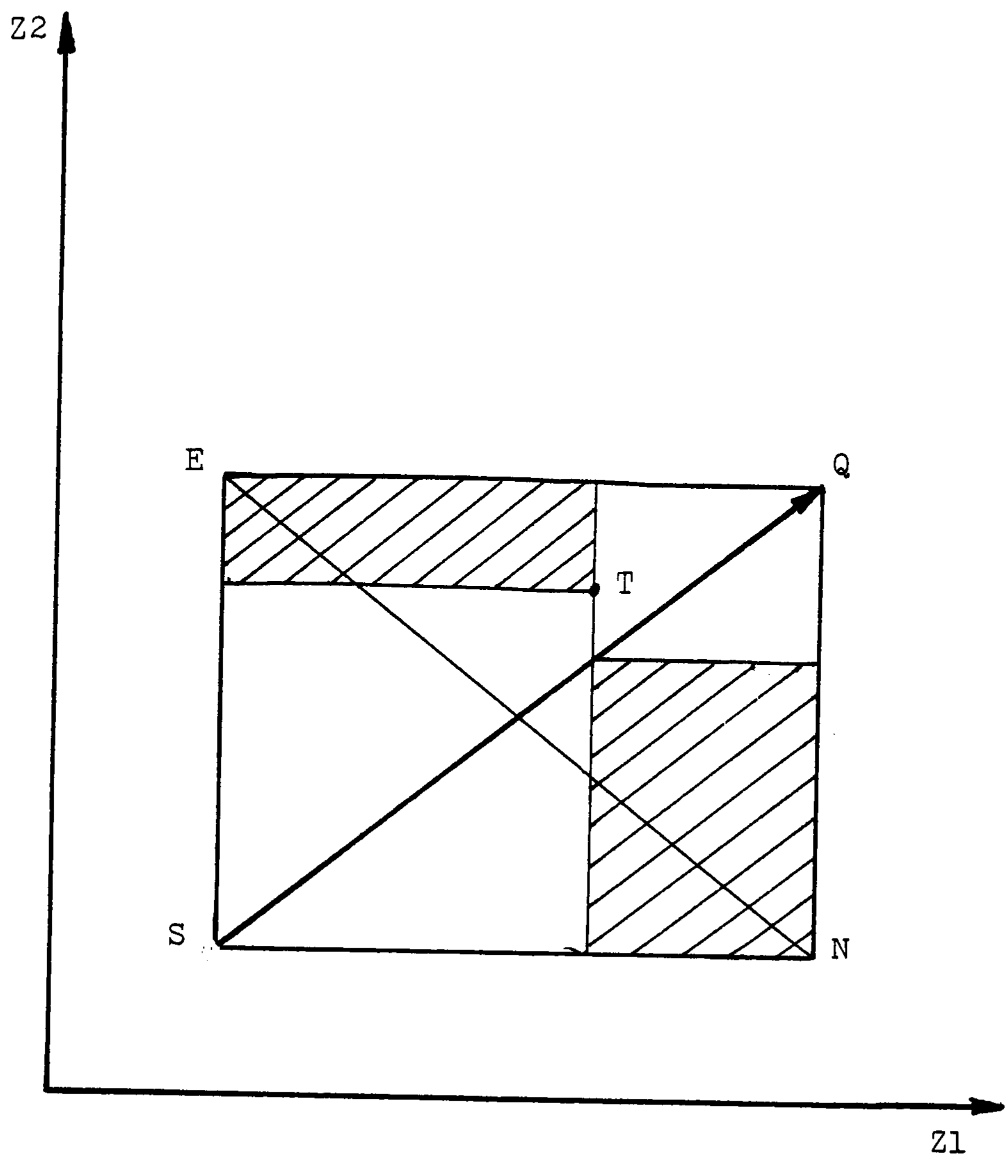


Fig. 3-8. Bounded Regions for which Weak Noninferior Solutions May Exist after Identifying the Third Weak Solution.

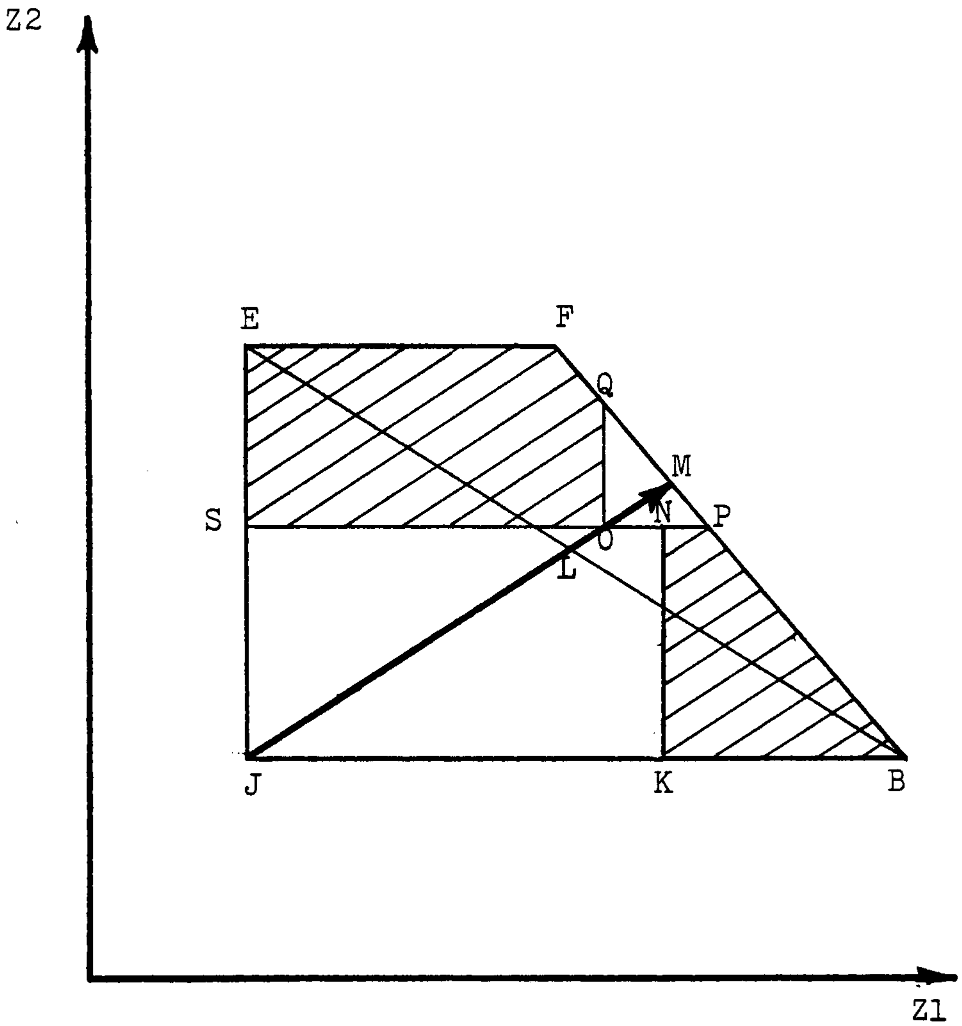


Fig. 3-9. Bounded Regions for which Weak Noninferior Solutions May Exist if Solution N is Identified.

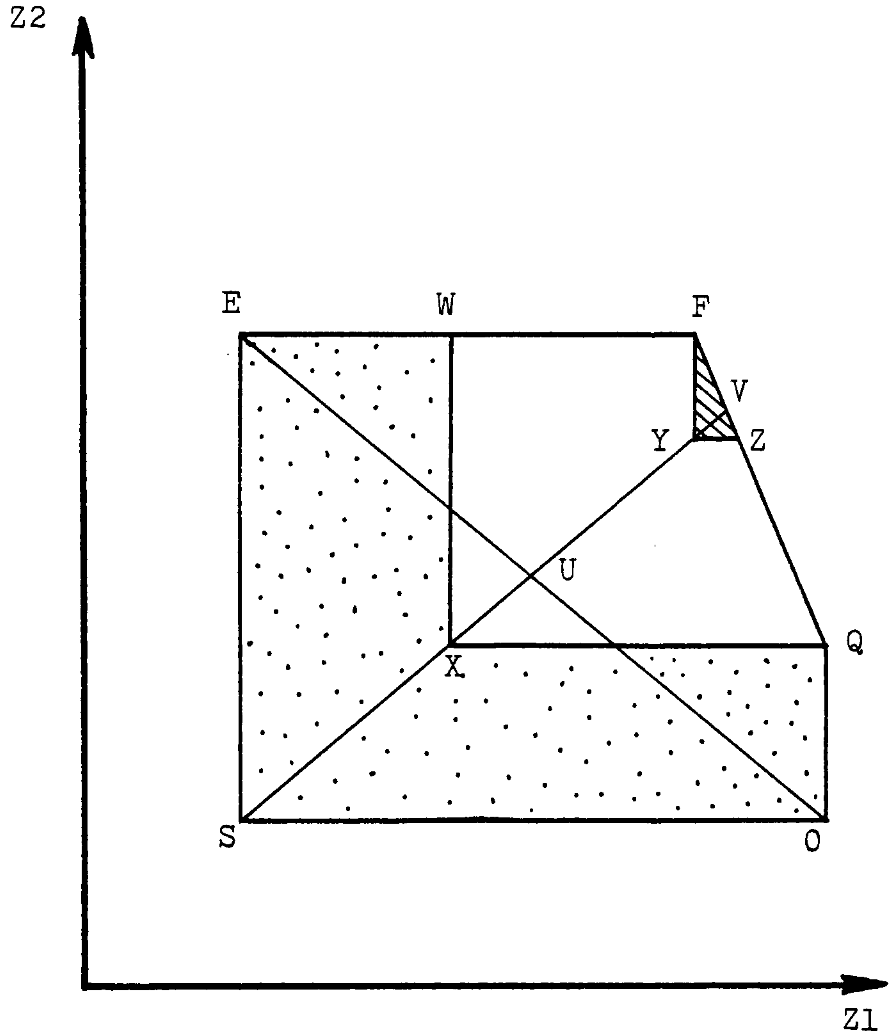


Fig. 3-10. Subregions of Future Solutions: ESOQXW, WXQZYF, FYZ.

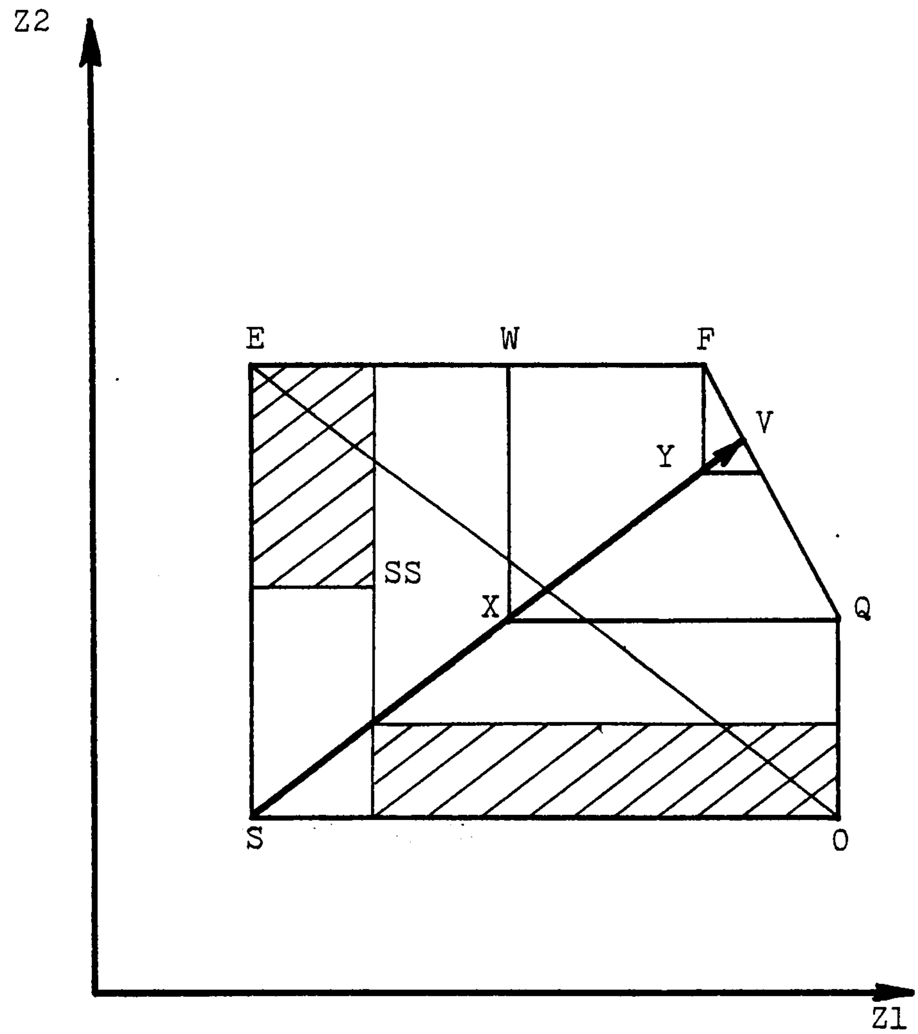


Fig. 3-11. The Resulting Bounded Regions generated by the Solution SS within ESOQXW.

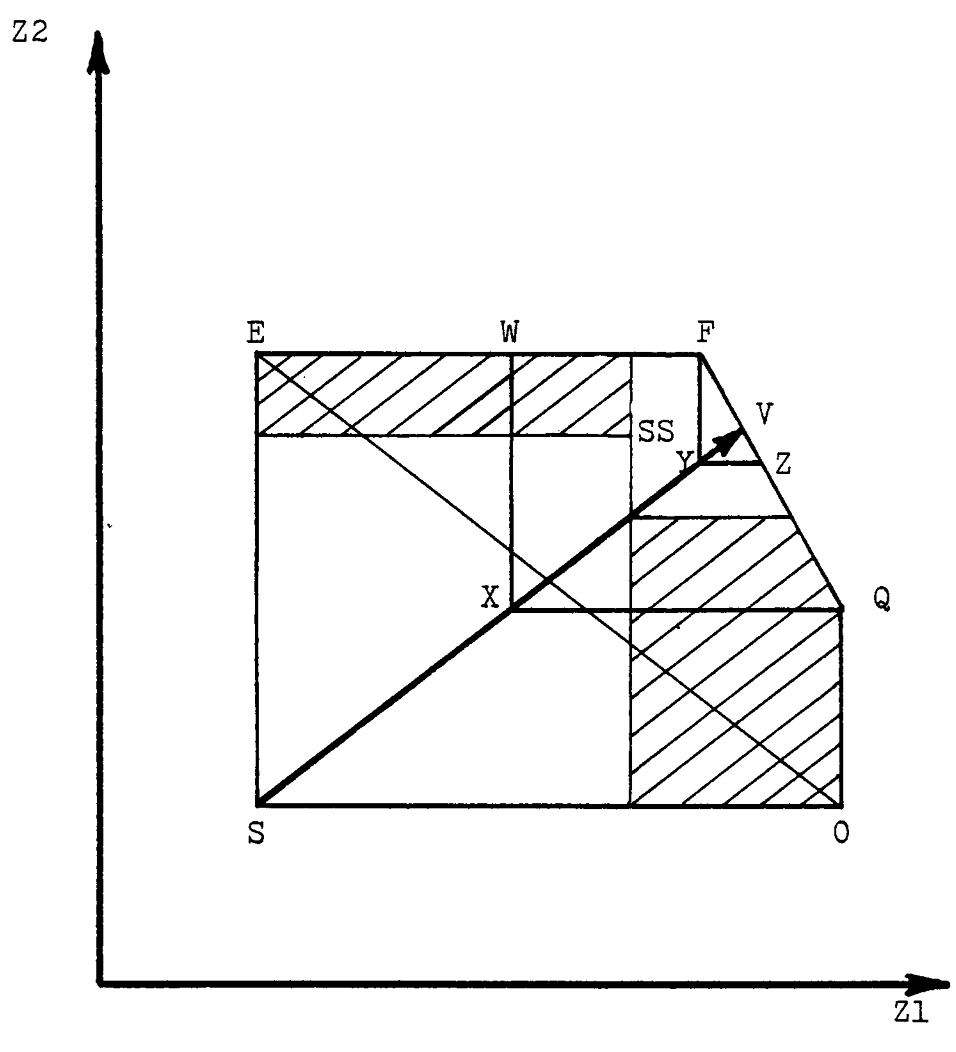


Fig. 3-12. The Resulting Bounded Regions generated by the Solution SS within WXQZFY.

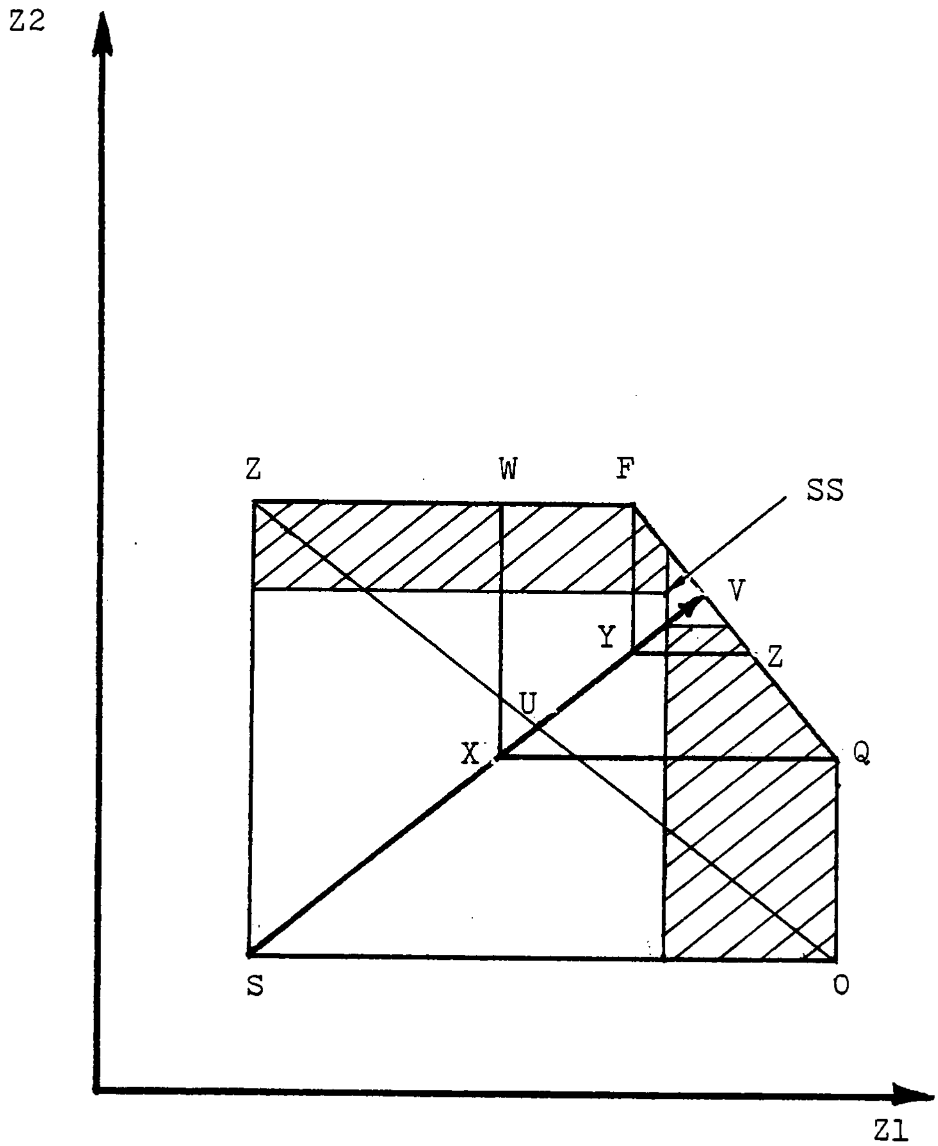


Fig. 3-13. The Resulting Bounded Regions generated by the Solution SS within FYZ.

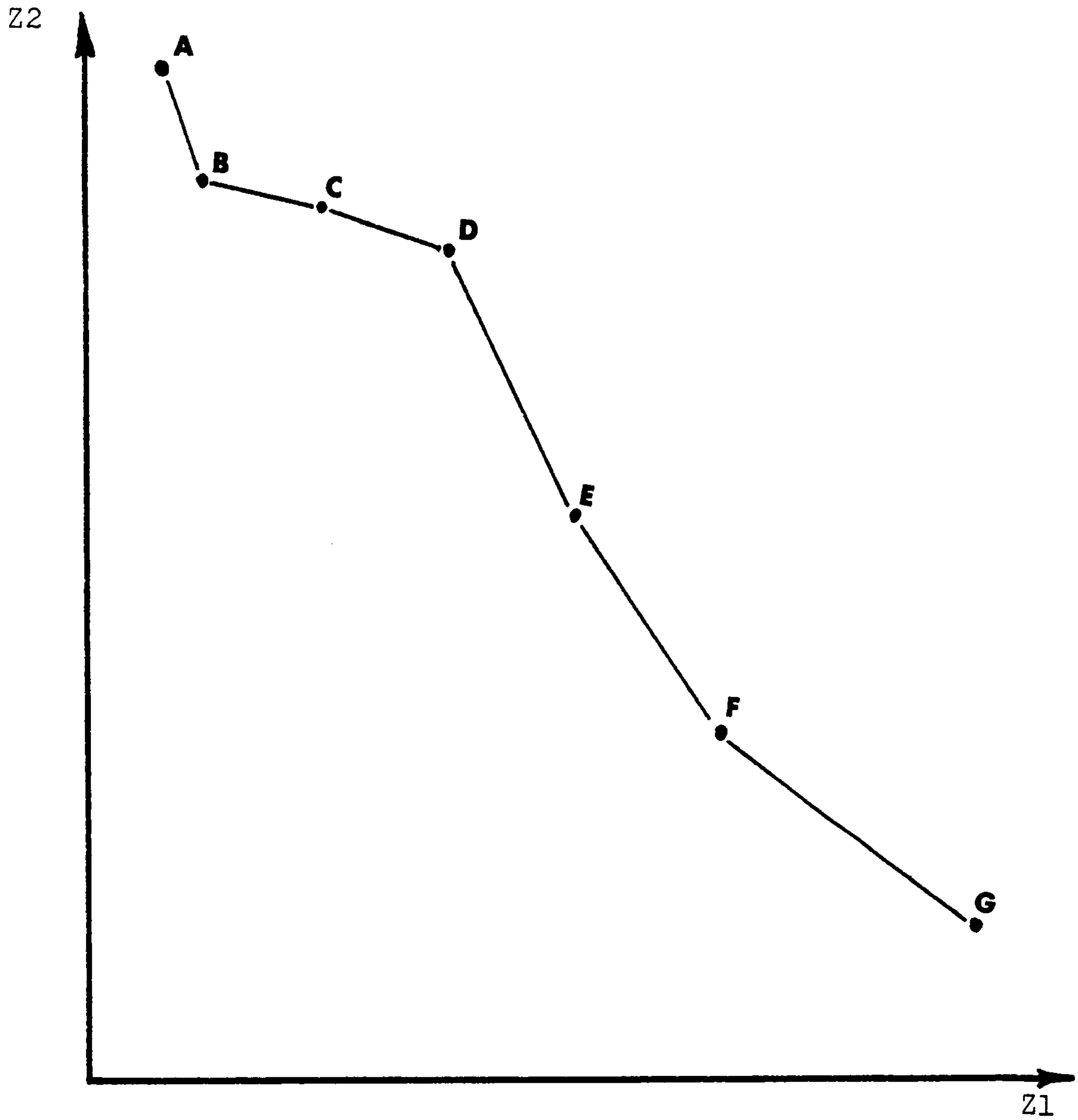


Fig. 3-14. A Complete Set of Noninferior Solutions.

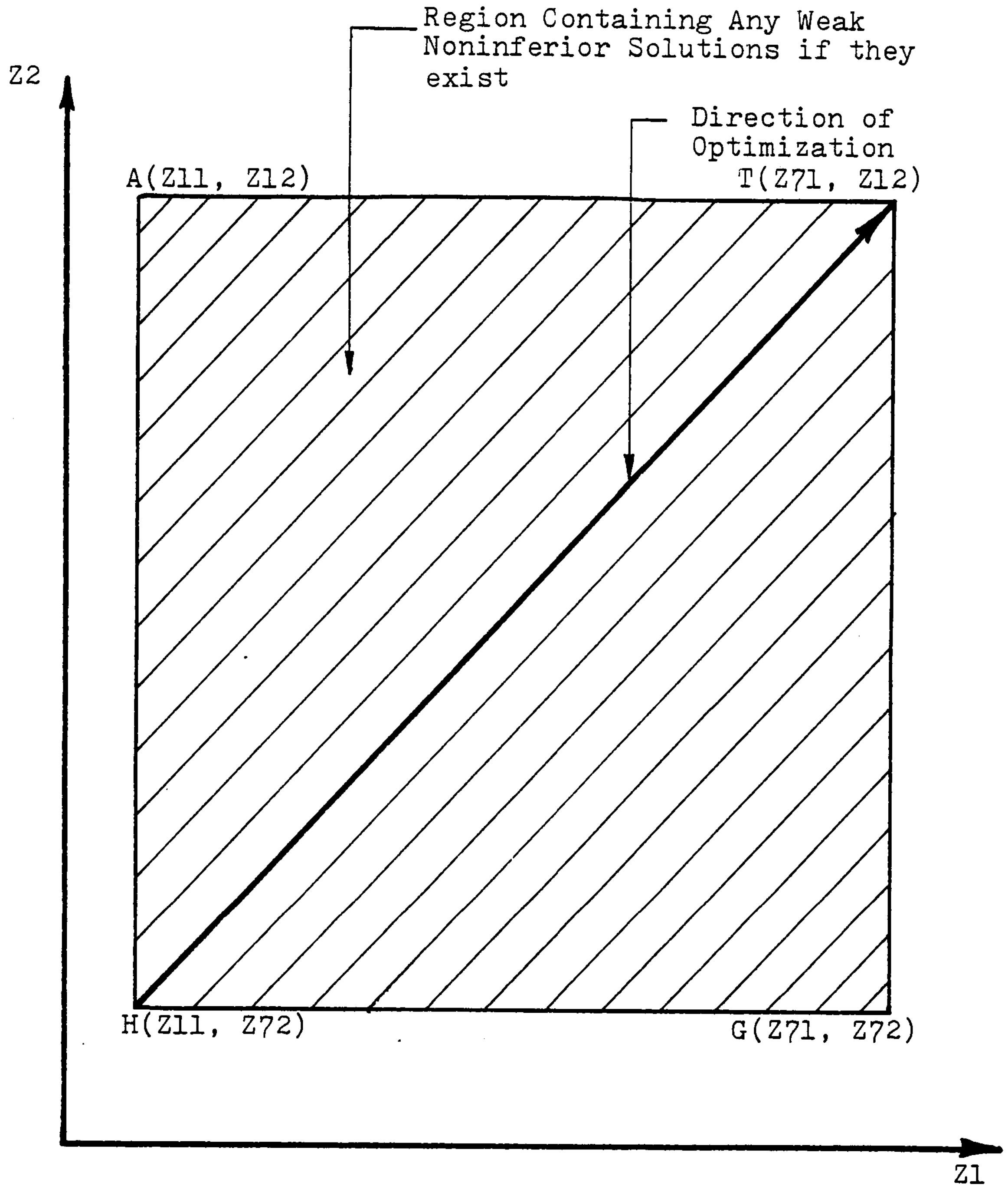


Fig. 3-15. Identification of a Complete set of Noninferior Solutions by UNISE (Run 1).

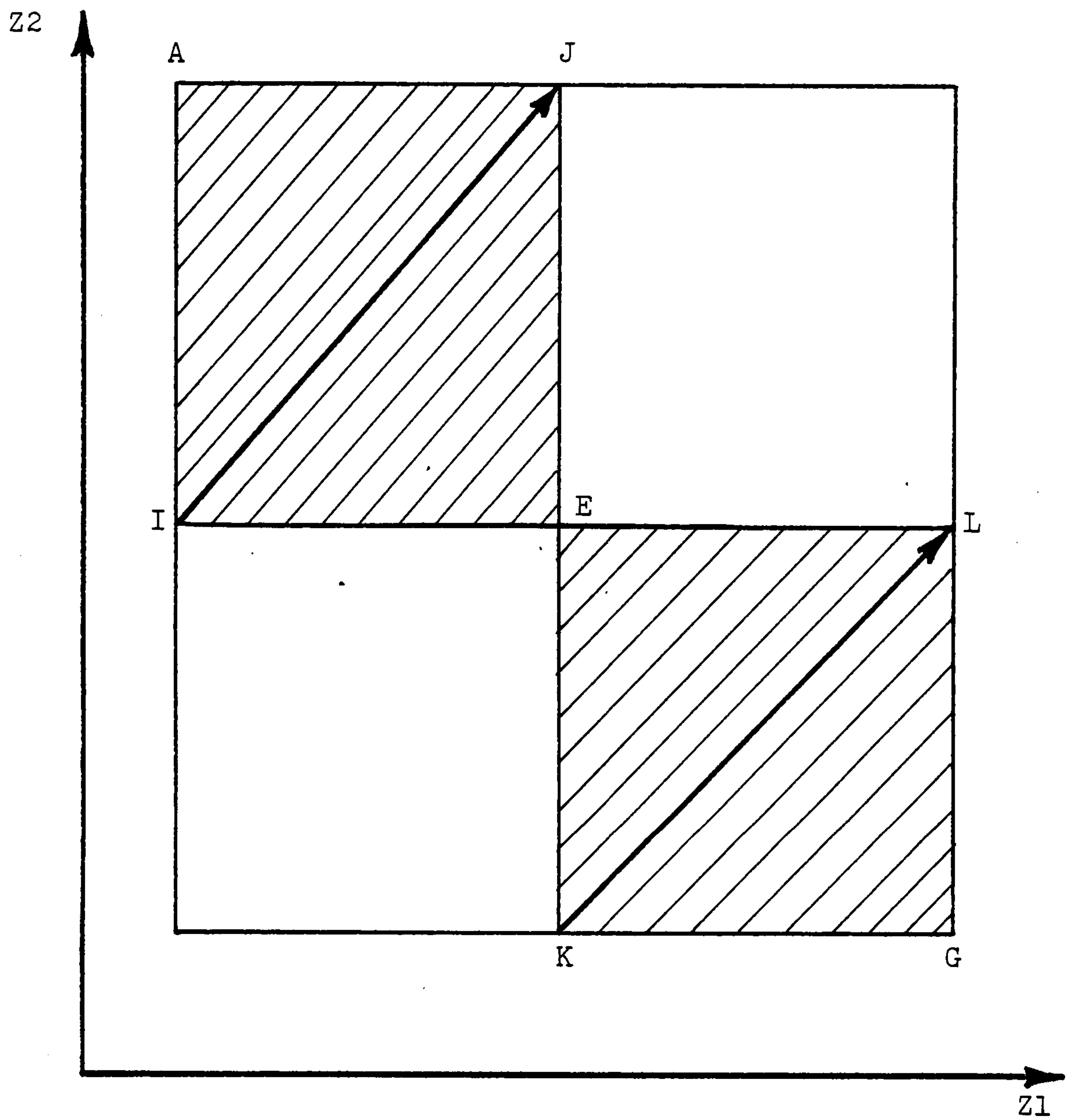


Fig. 3-16. Identification of a Complete set of Noninferior Solutions by UNISE (Run 2).

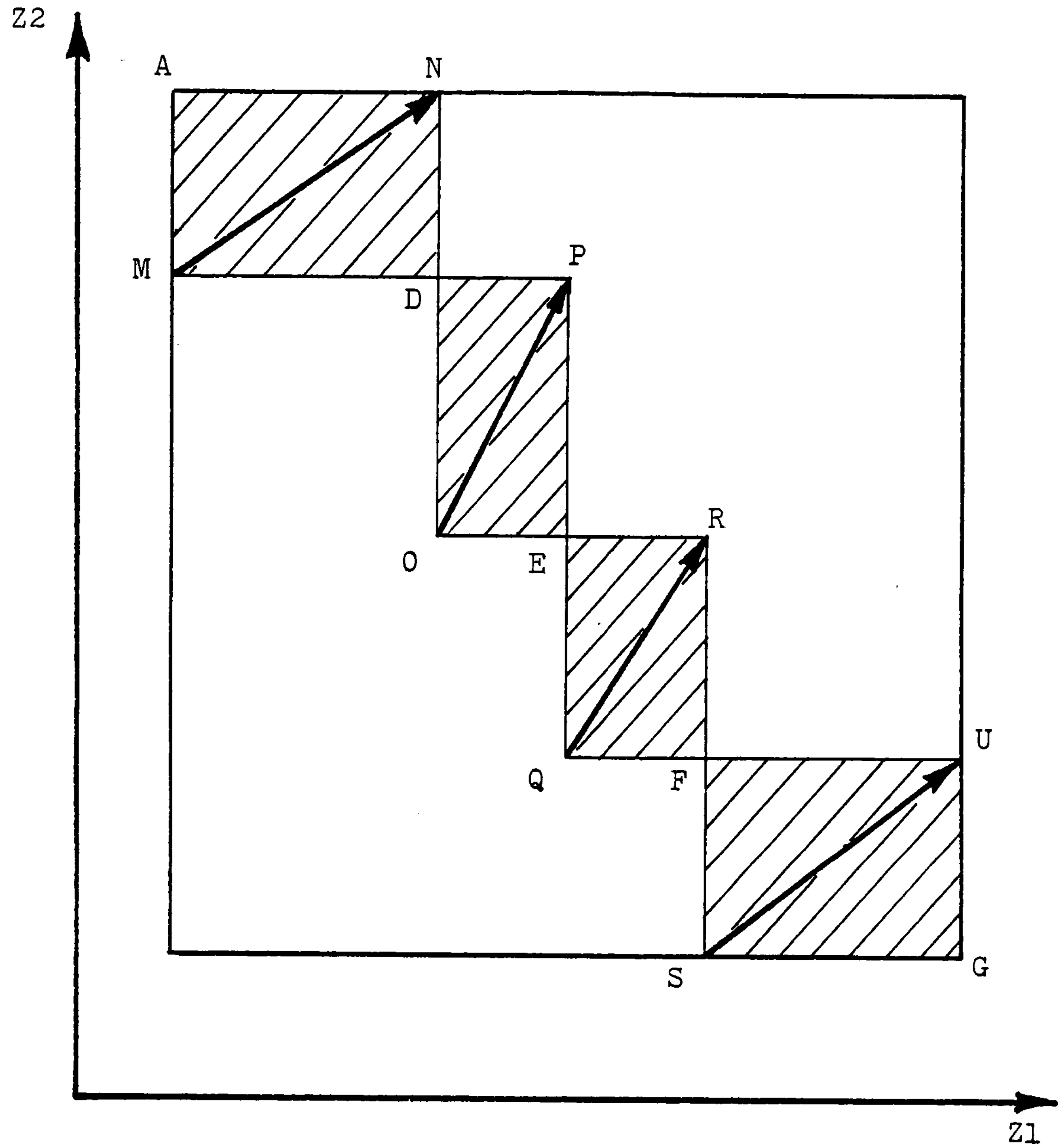


Fig. 3-17. Identification of a Complete set of Noninferior Solutions by UNISE (Run 3).

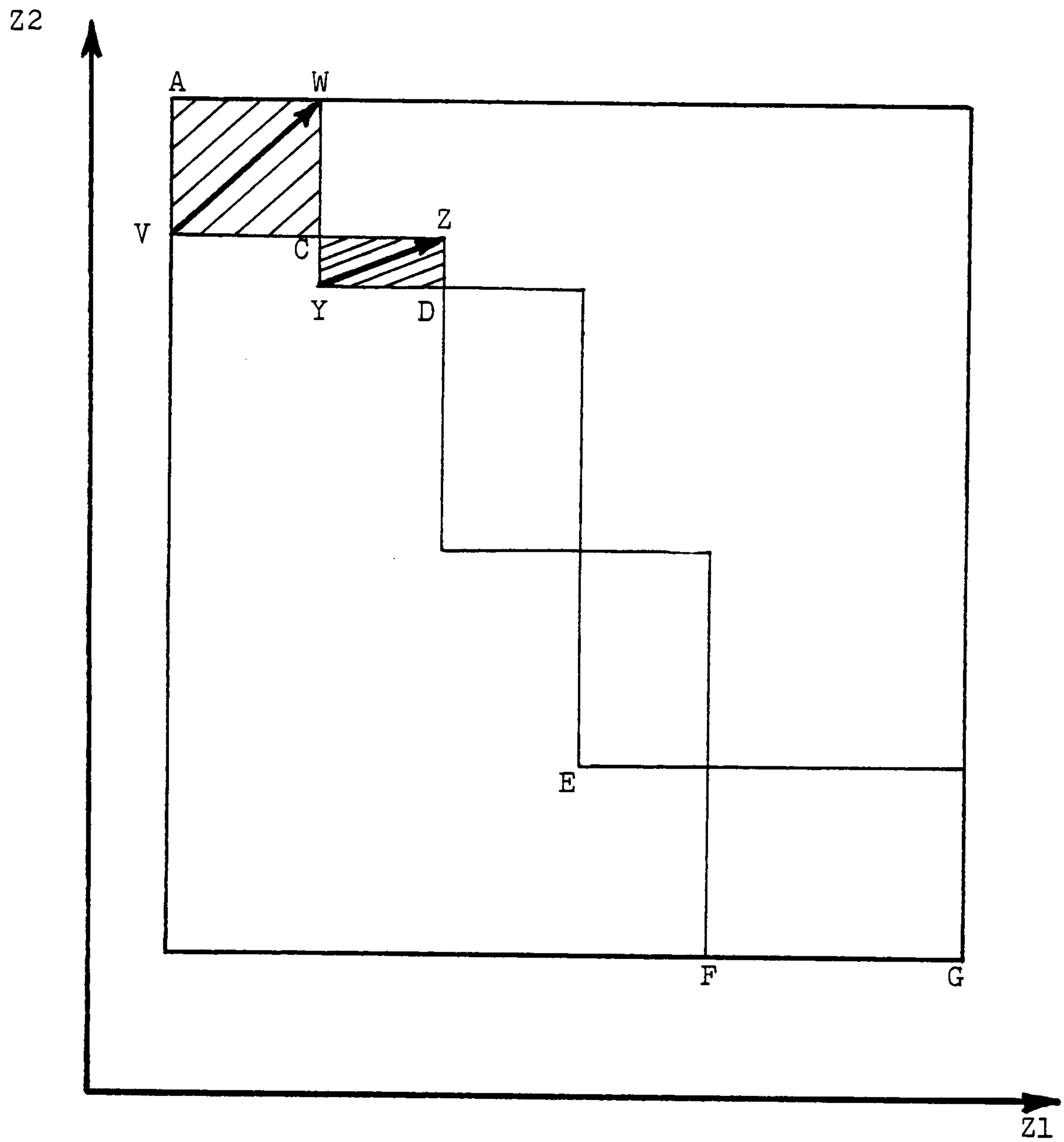


Fig. 3-18. Identification of a Complete set of Noninferior Solutions by UNISE (Run 4).

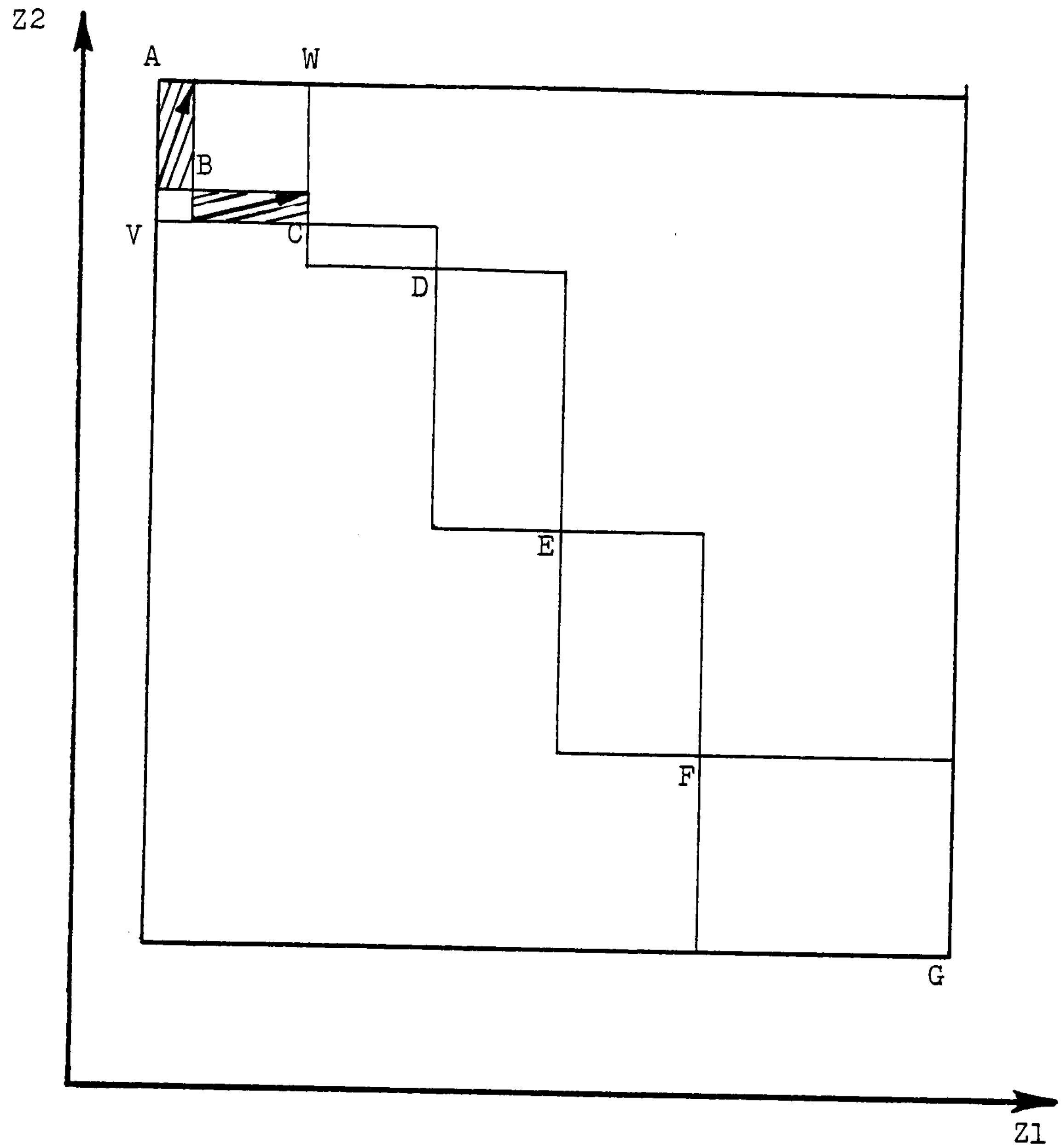


Fig. 3-19. Identification of a Complete set of Noninferior Solutions by UNISE (Run 5).

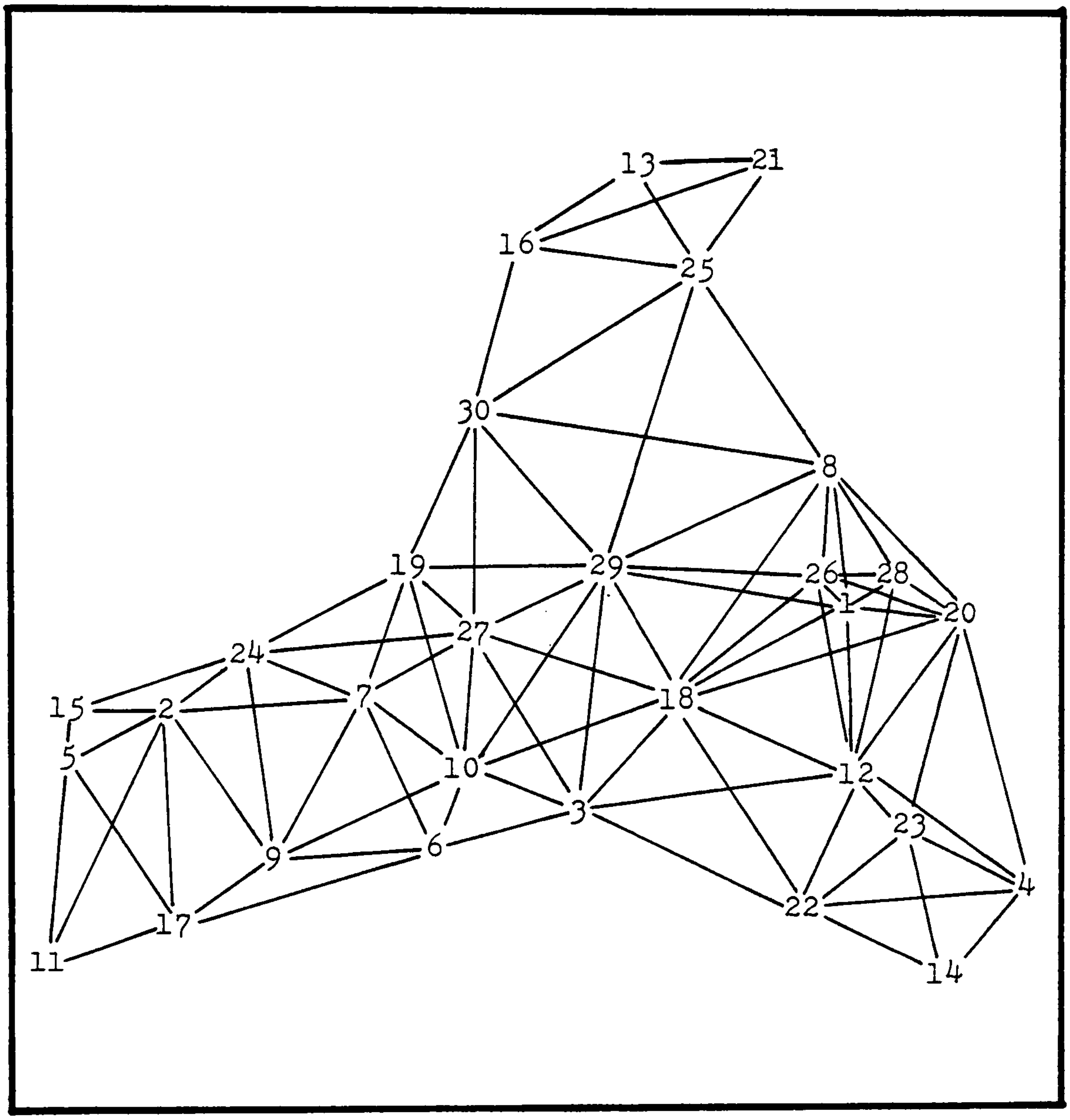


Fig. 4-1. Layout of Demand Nodes on a Plane: Toregas' Data.

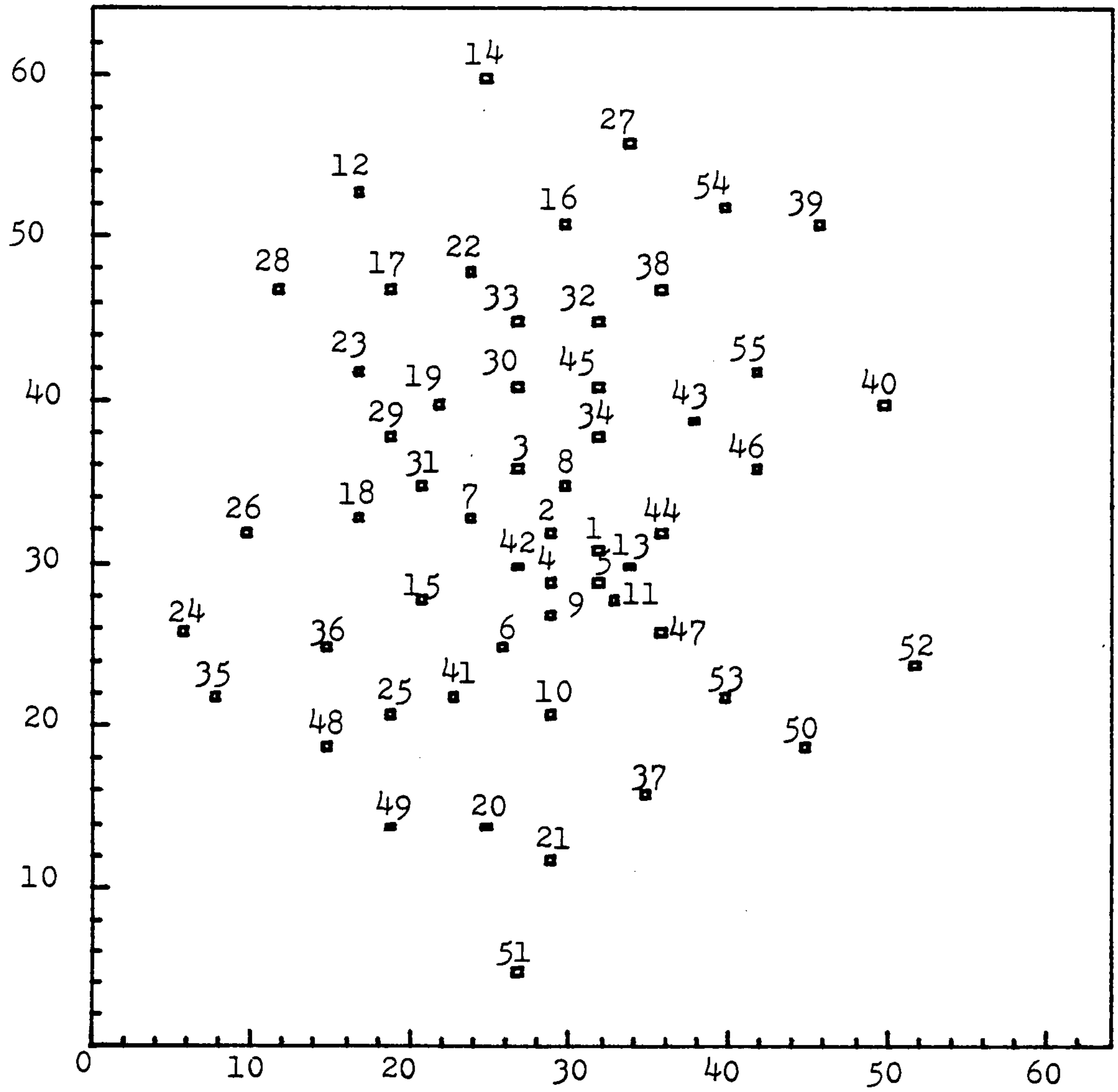
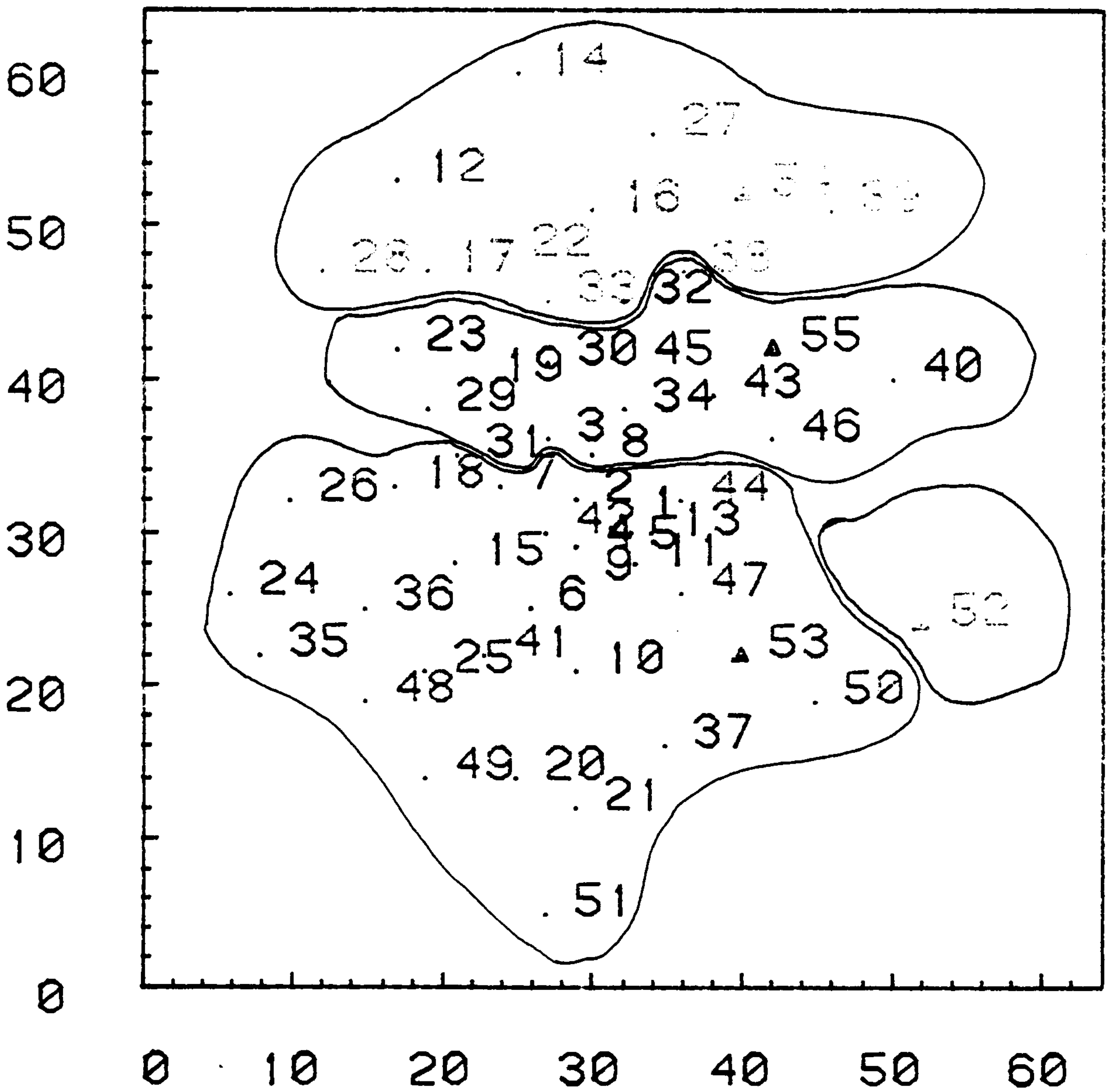


Fig. 4-2. Layout of Demand Nodes on a Plane: Swain's Data.



▲ Facility Locations of the Solutions to MILP and MPFILP

Fig. 4-3. Configuration of Solutions to MILP and MPFILP.

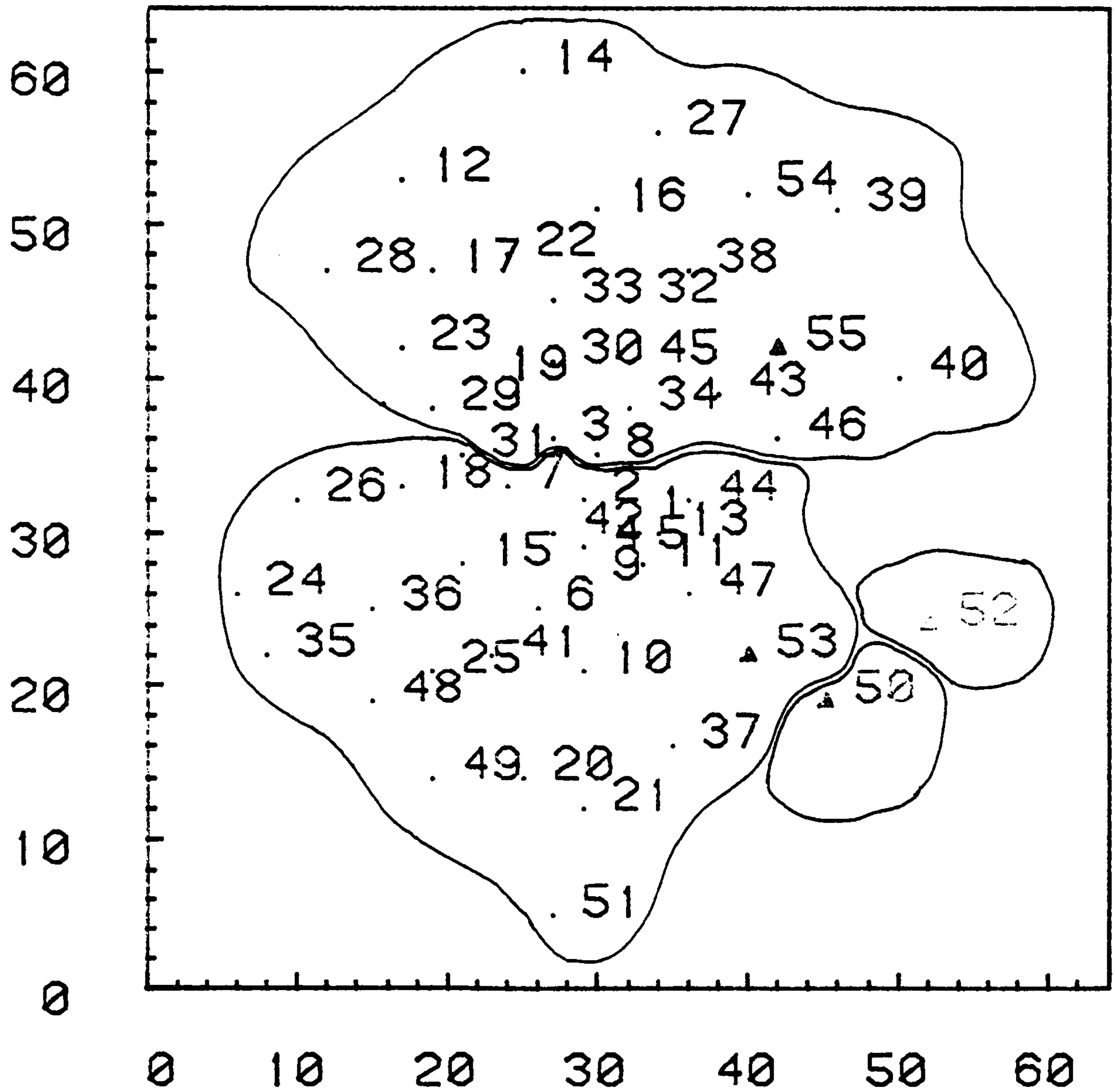


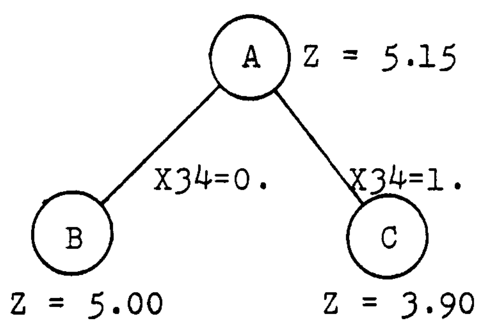
Fig. 4-4. Configuration of Solutions to MILP and MPFILP with Distance Modification.

Weights used: Wc = .027

Wn = .973

Optimal Integer Solution found:

Node B



Configurations:

A	X34 = 0.5	X45 = 0.5
	X48 = 1.0	X53 = 1.0
	X54 = 1.0	
B	X47 = 1.0	X48 = 1.0
	X53 = 1.0	X54 = 1.0
C	X34 = 1.0	X45 = 1.0
	X48 = 1.0	X53 = 1.0

Fig. 4-5. Resolving a Fractional Solution to MCMILP by Branch and Bound Algorithm.

Data: SWAIN. No. of facilities: 4.

Coverage distance: 15. Impact distance: 3.

Impact factor: 1.

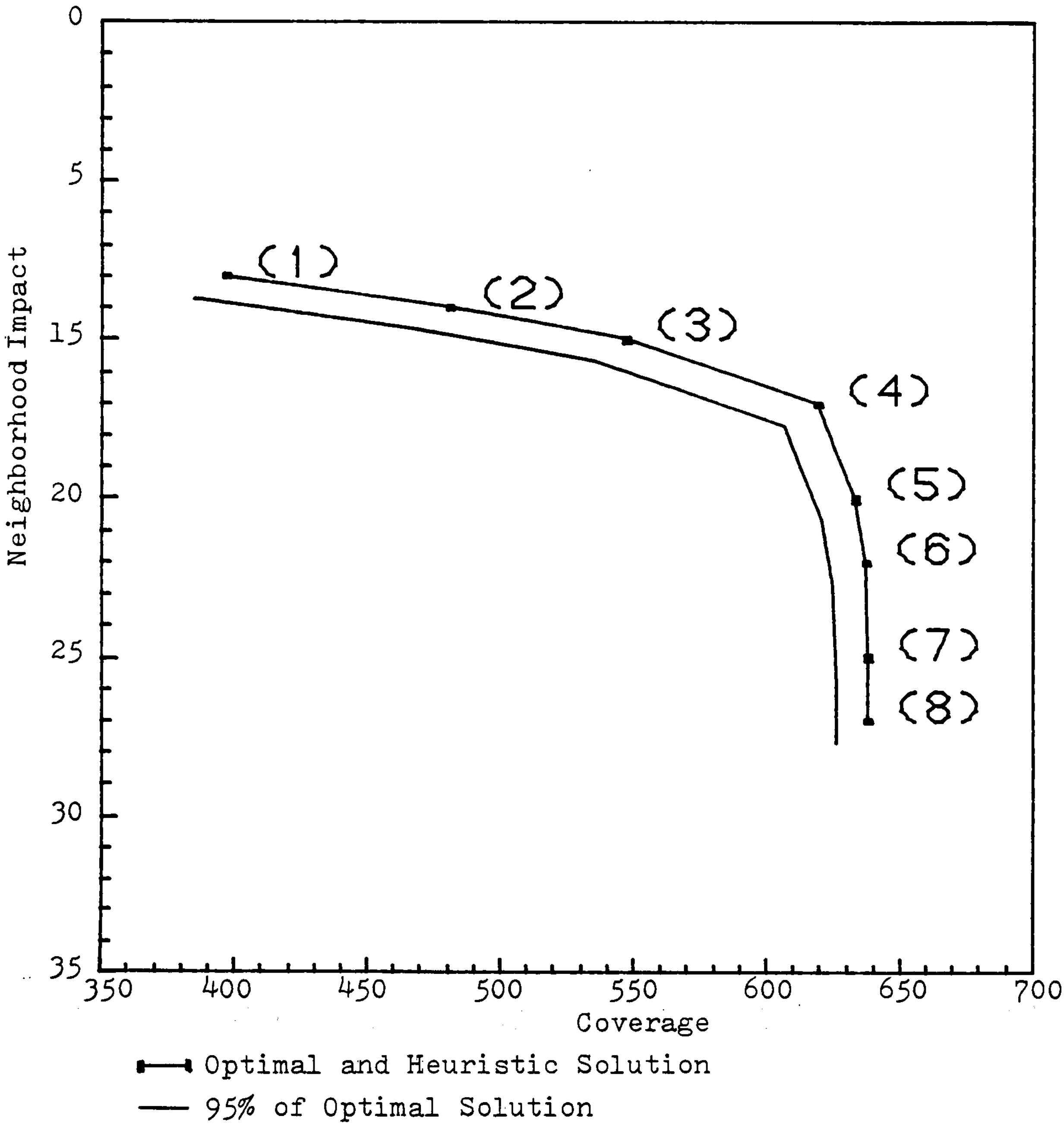
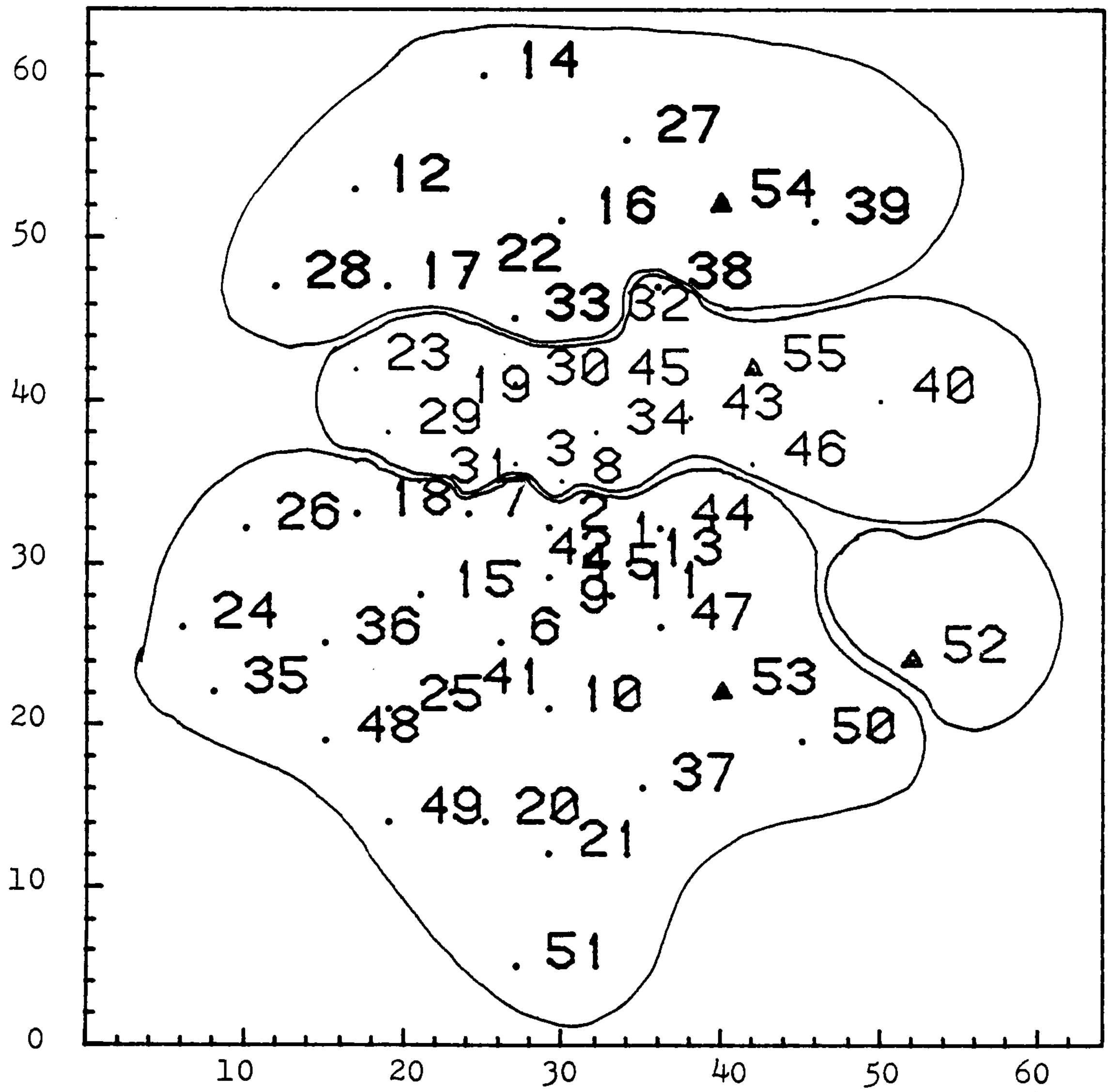
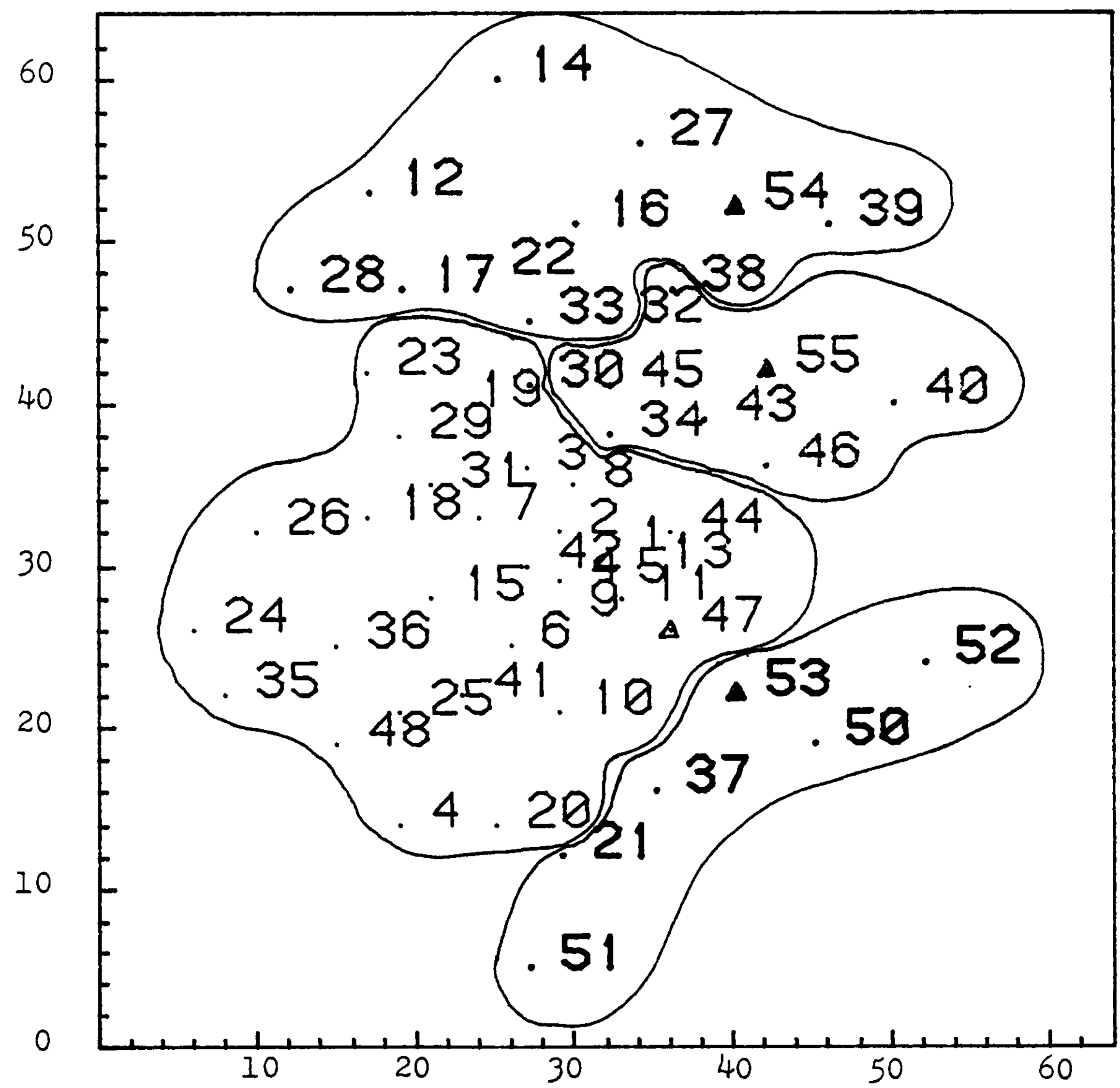


Fig. 4-6. Optimal and Heuristic Solutions of MCMILP.



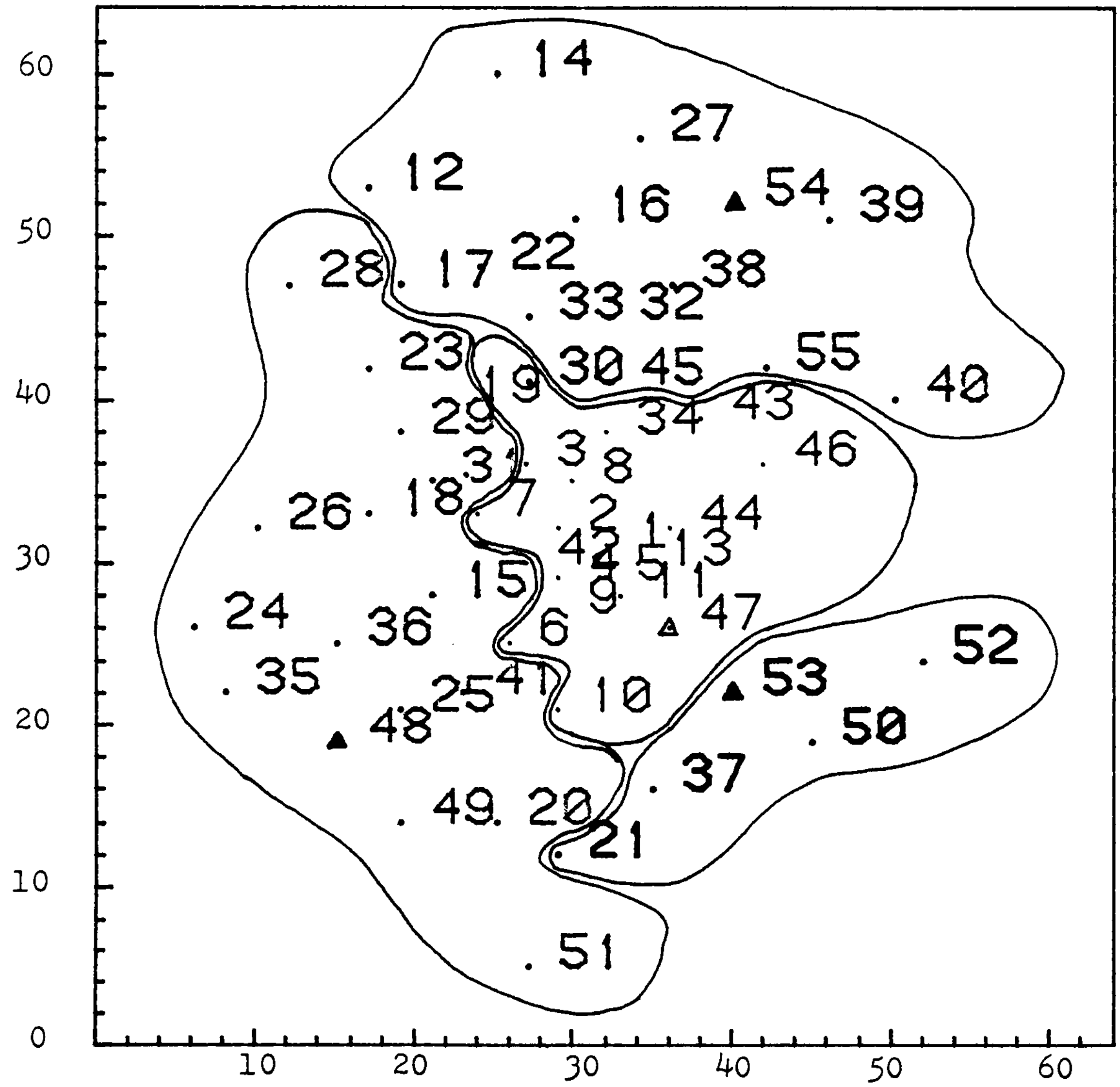
Δ Facility and its partition.

Fig. 4-7. The Facility Configuration of MCMILP Solution Point (1) of Table 4-5.



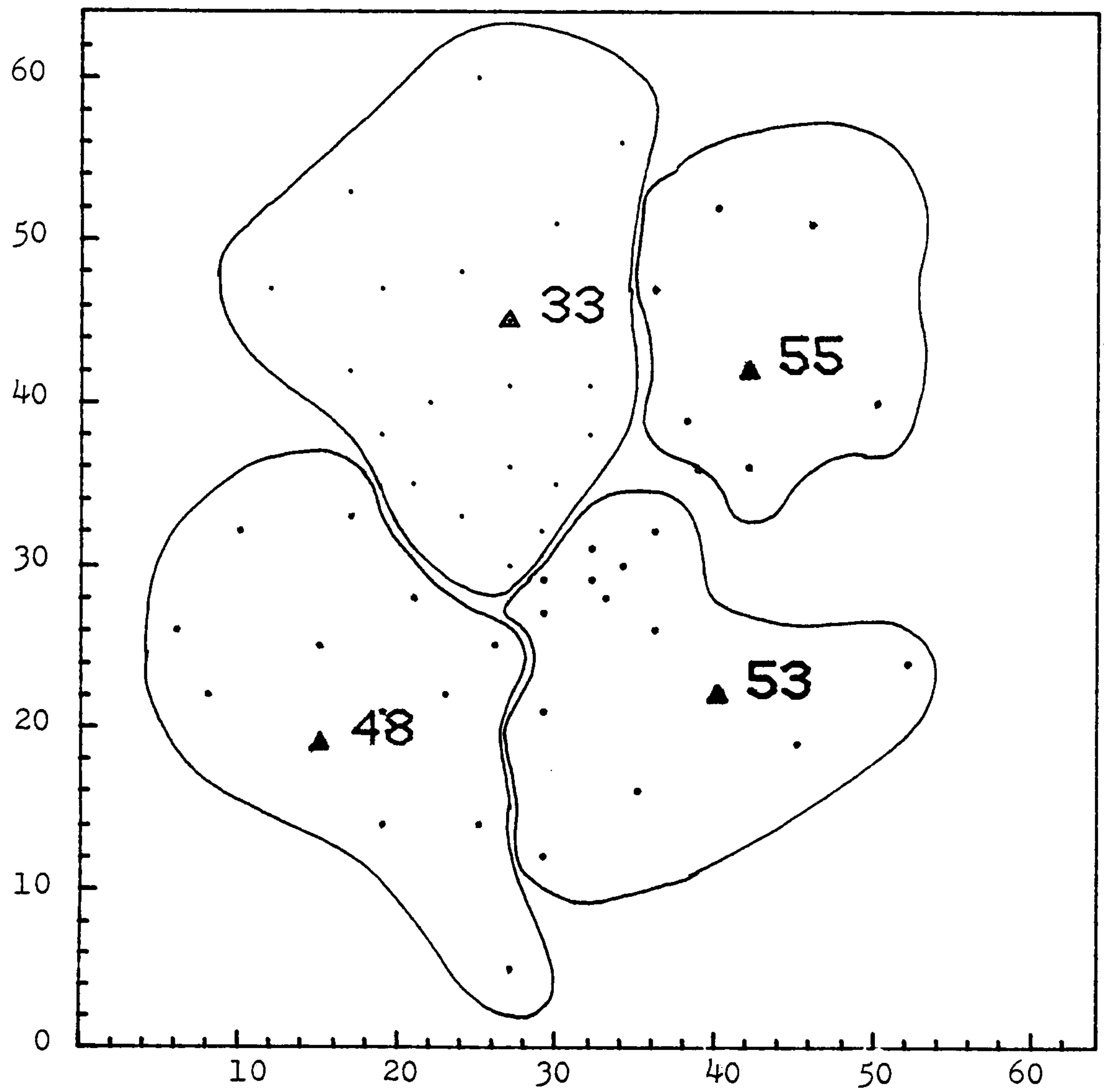
△ Facility and its partition.

Fig. 4-8. The Facility Configuration of MCMILP Solution Point (2) of Table 4-5.



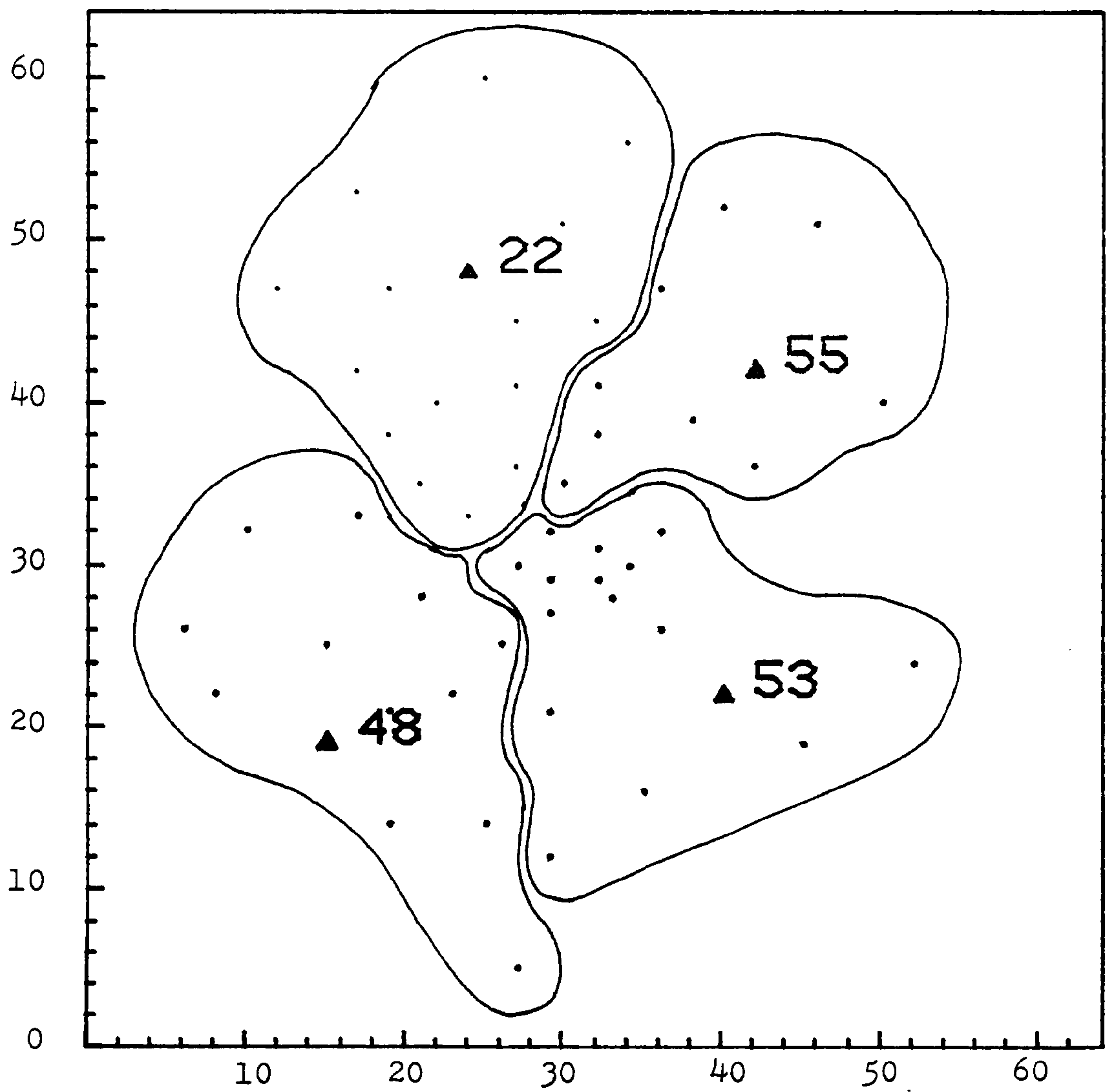
△ Facility and its partition.

Fig. 4-9. The Facility Configuration of MCMILP Solution Point (3) of Table 4-5.



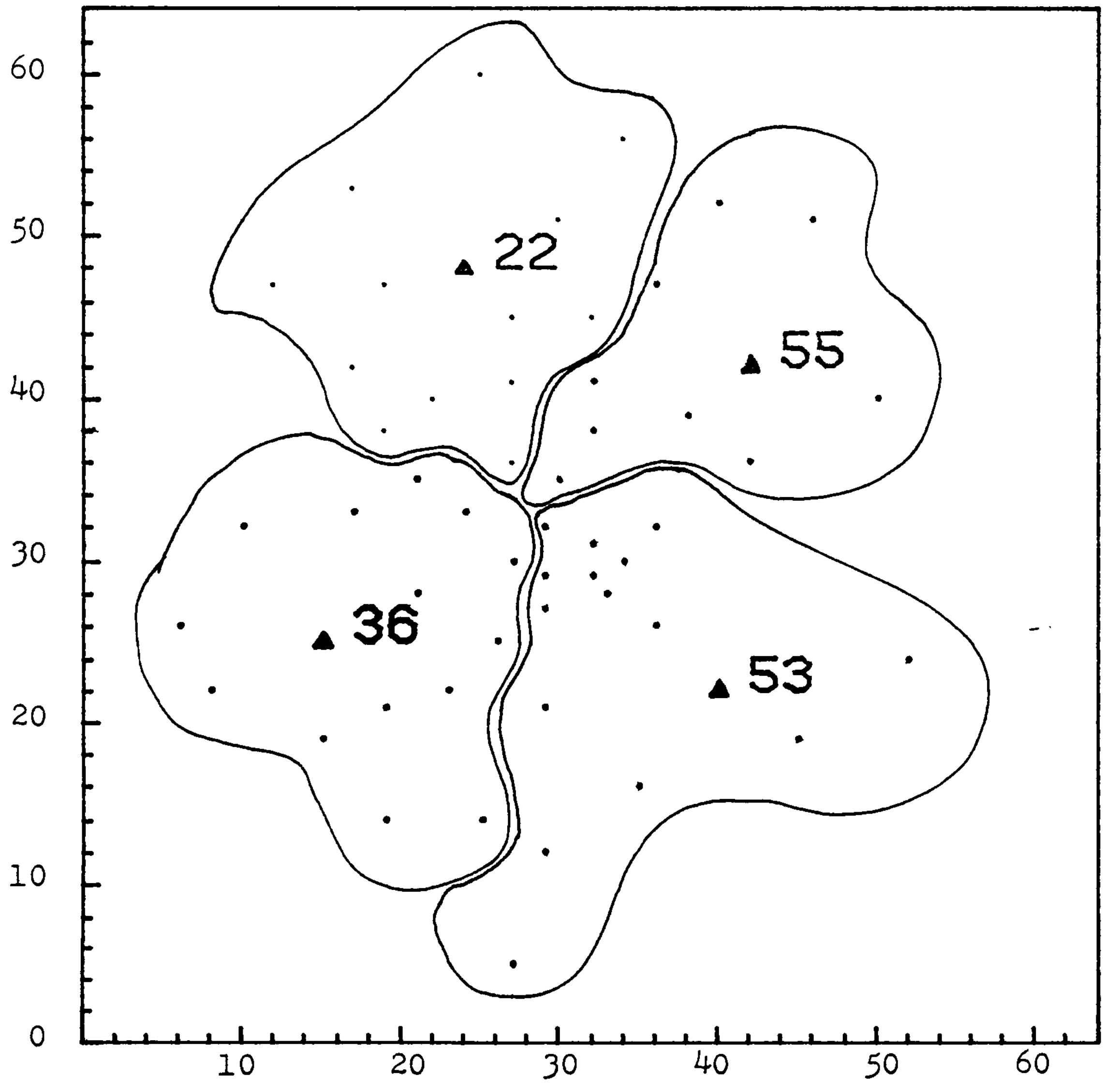
△ Facility and its partition.

Fig. 4-10. The Facility Configuration of MCMILP Solution Point (4) of Table 4-5.



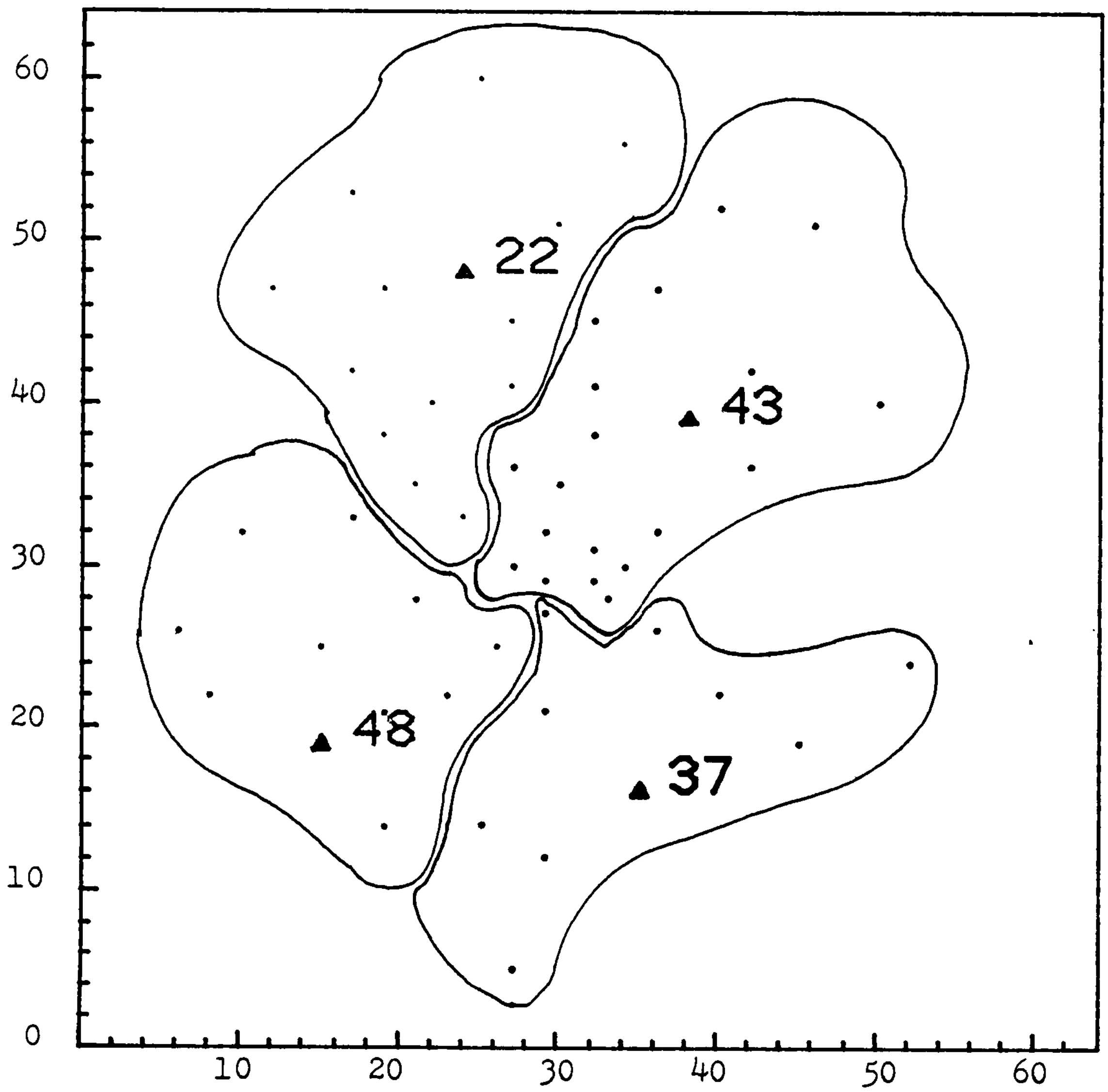
△ Facility and its partition.

Fig. 4-11. The Facility Configuration of MCMILP Solution Point (5) of Table 4-5.



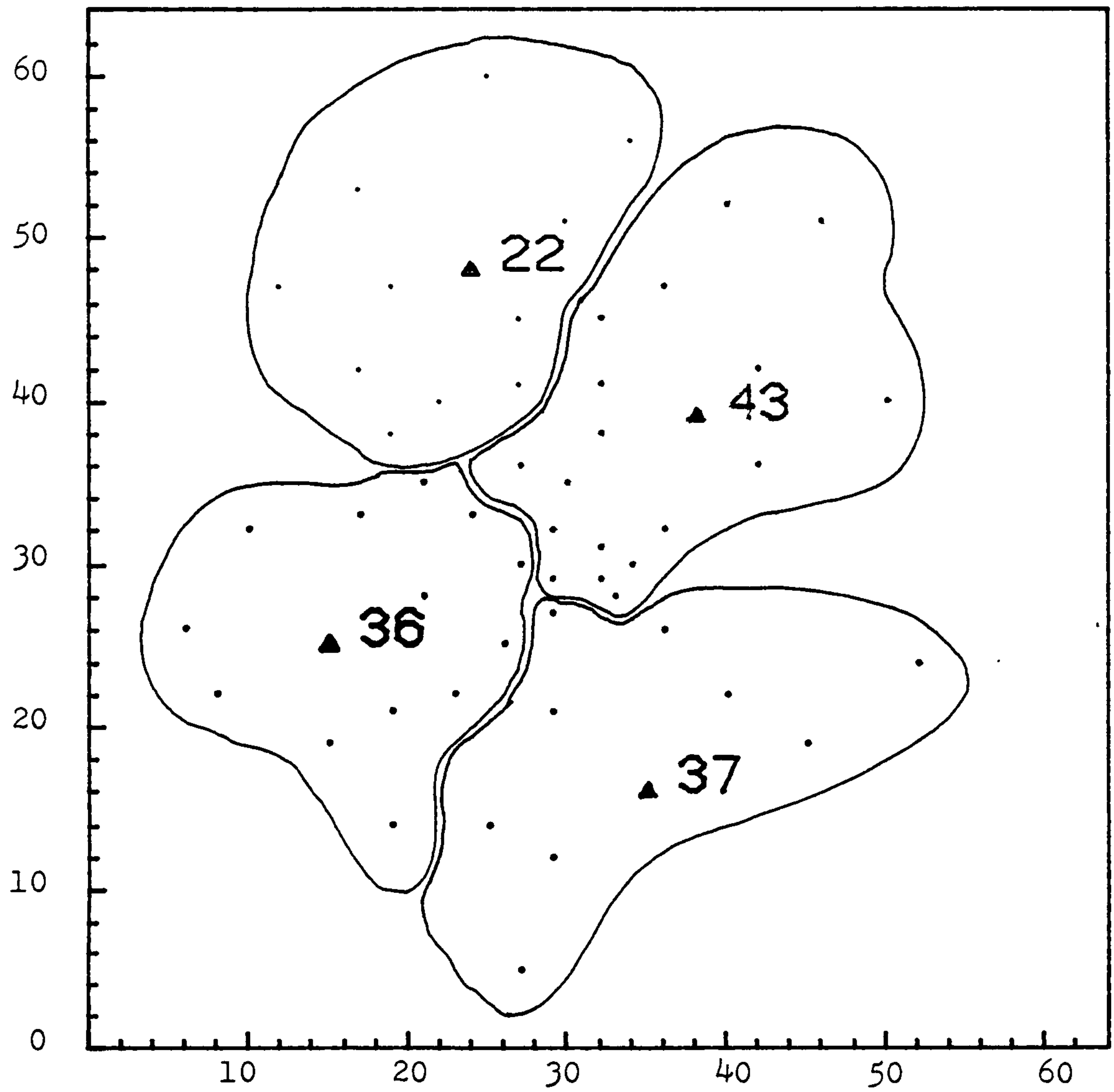
△ Facility and its partition.

Fig. 4-12. The Facility Configuration of MCMILP Solution Point (6) of Table 4-5.



△ Facility and its partition.

Fig. 4-13. The Facility Configuration of MCMILP Solution Point (7) of Table 4-5.



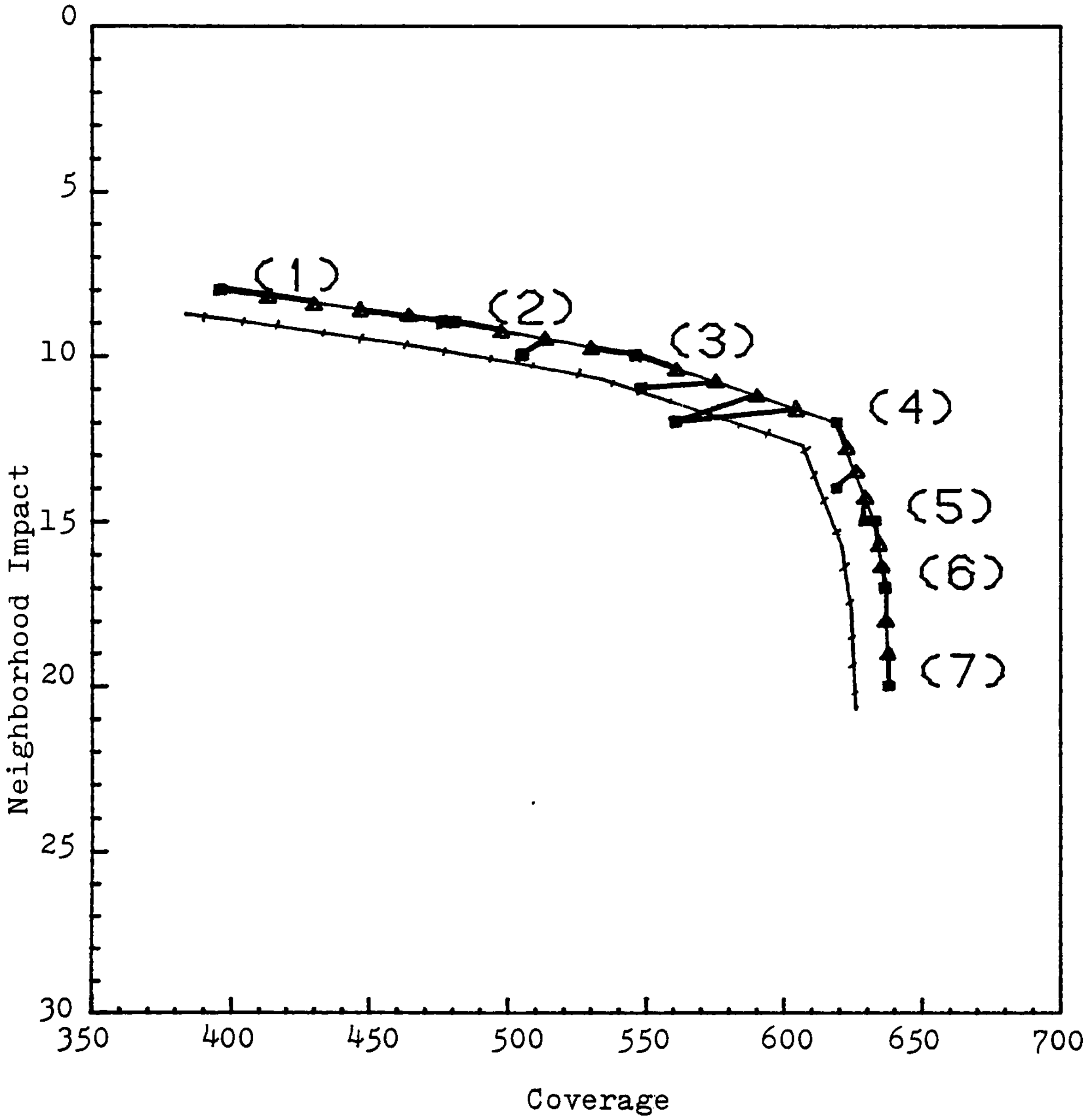
△ Facility and its partition.

Fig. 4-14. The Facility Configuration of MCMILP Solution Point (8) of Table 4-5.

Data: SWAIN No. of Facilities: 4

Coverage distance: 15 Impact distance: 3

Impact factor: 1.



—— Optimal Solutions ▫ Weak Solutions by Target Routine

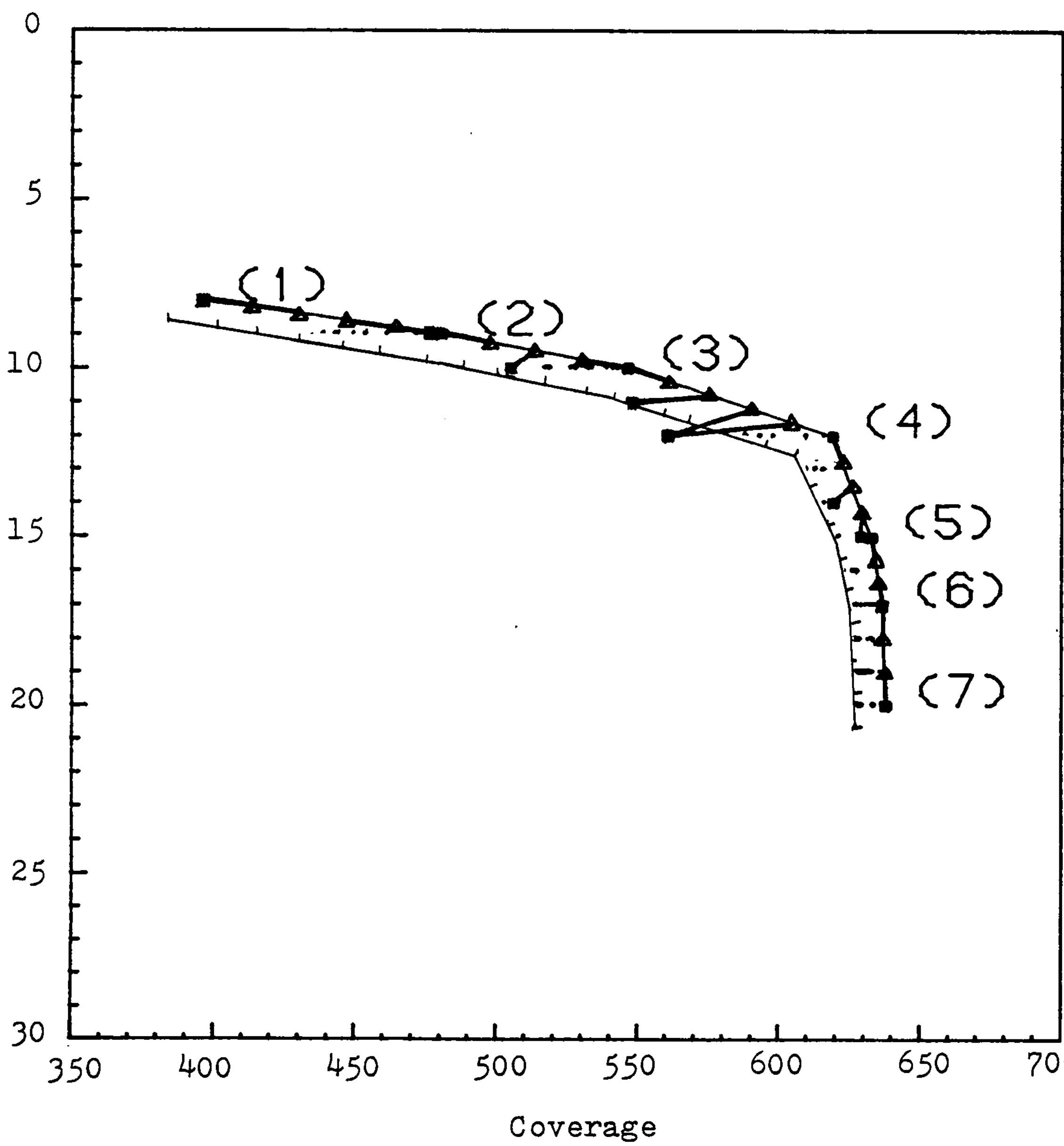
Δ Targets -+- 95% of Optimal Solutions

Fig. 4-15. Weak Solutions of MCMILP by Target Routine.

Data: SWAIN No. of Facilities: 4

Coverage Distance: 15. Impact Distance: 3.

Impact factor: 1.



Optimal Solutions Weak Solutions by Target Routine

Targets 95% of Optimal Solutions

• RTAB Solutions

Fig. 4-16. Weak Solutions of MCMILP by Target Routine with Solutions by RTAB.

Data: SWAIN No. of Facilities: 4

Coverage Distance: 15. Impact Distance: 3.

Impact Factor: 1.

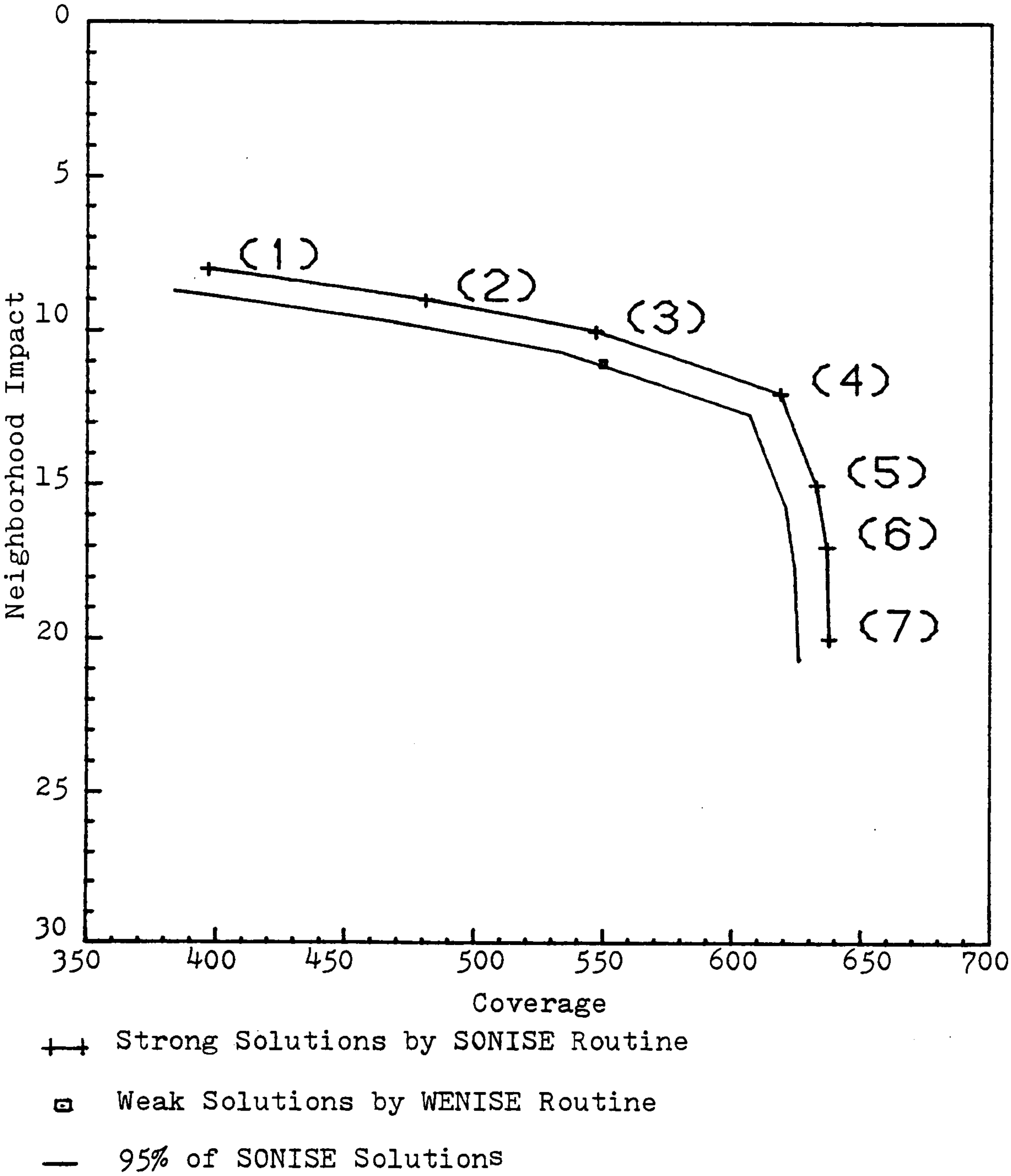
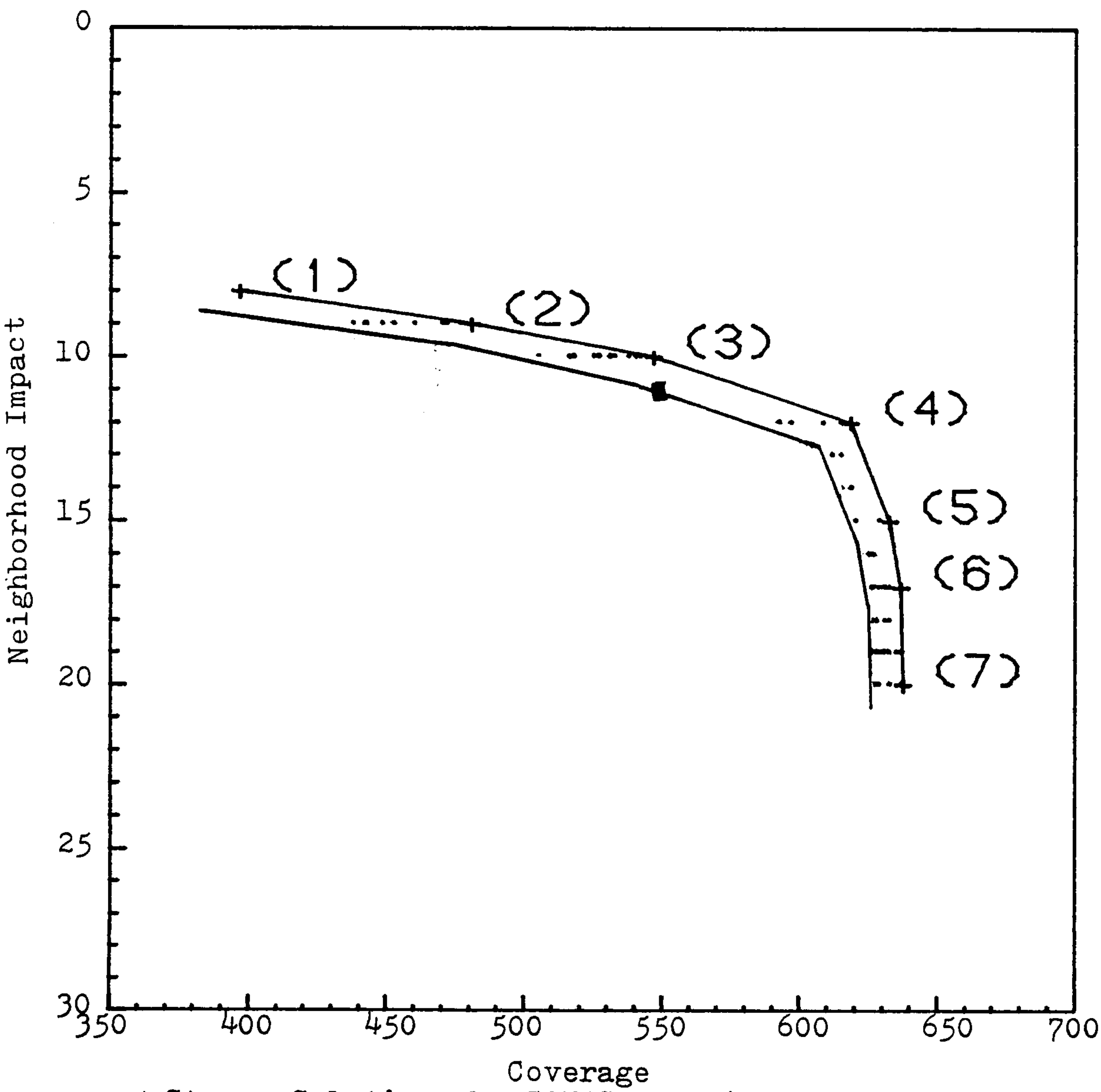


Fig. 4-17. Noninferior Solutions of MCMILP by SONISE Routine and then by WENISE Routine.

Data: SWAIN No. of Facilities: 4

Coverage Distance: 15 Impact Distance: 3

Impact Factor: 1



++ Strong Solutions by SONISE Routine

▫ Weak Solutions by WENISE Routine

— 95% of SONISE Solutions · Solutions by RTAB

Fig. 4-18. Noninferior Solutions to MCMILP by SONISE Routine and then by WENISE Routine, with Solutions by RTAB.

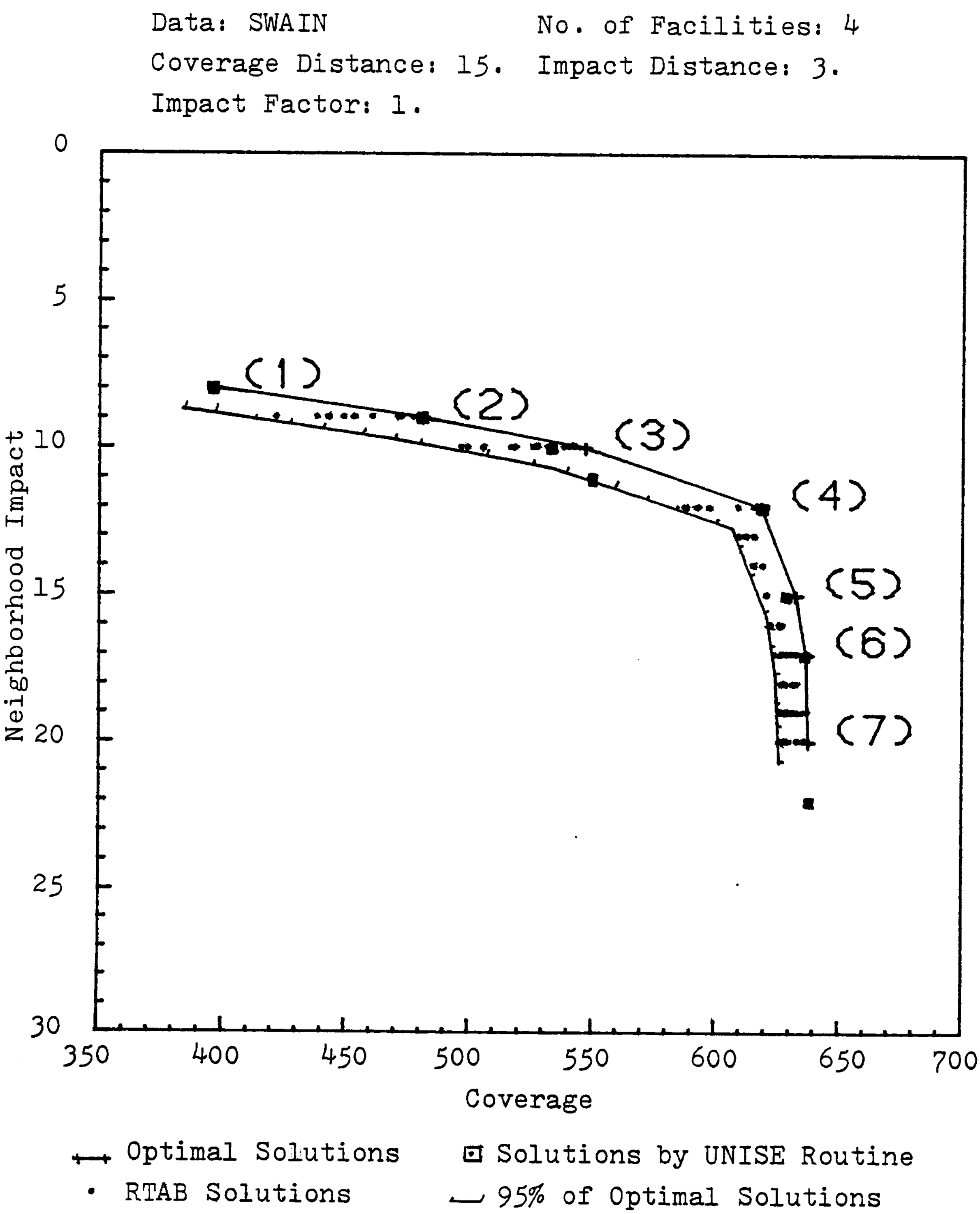
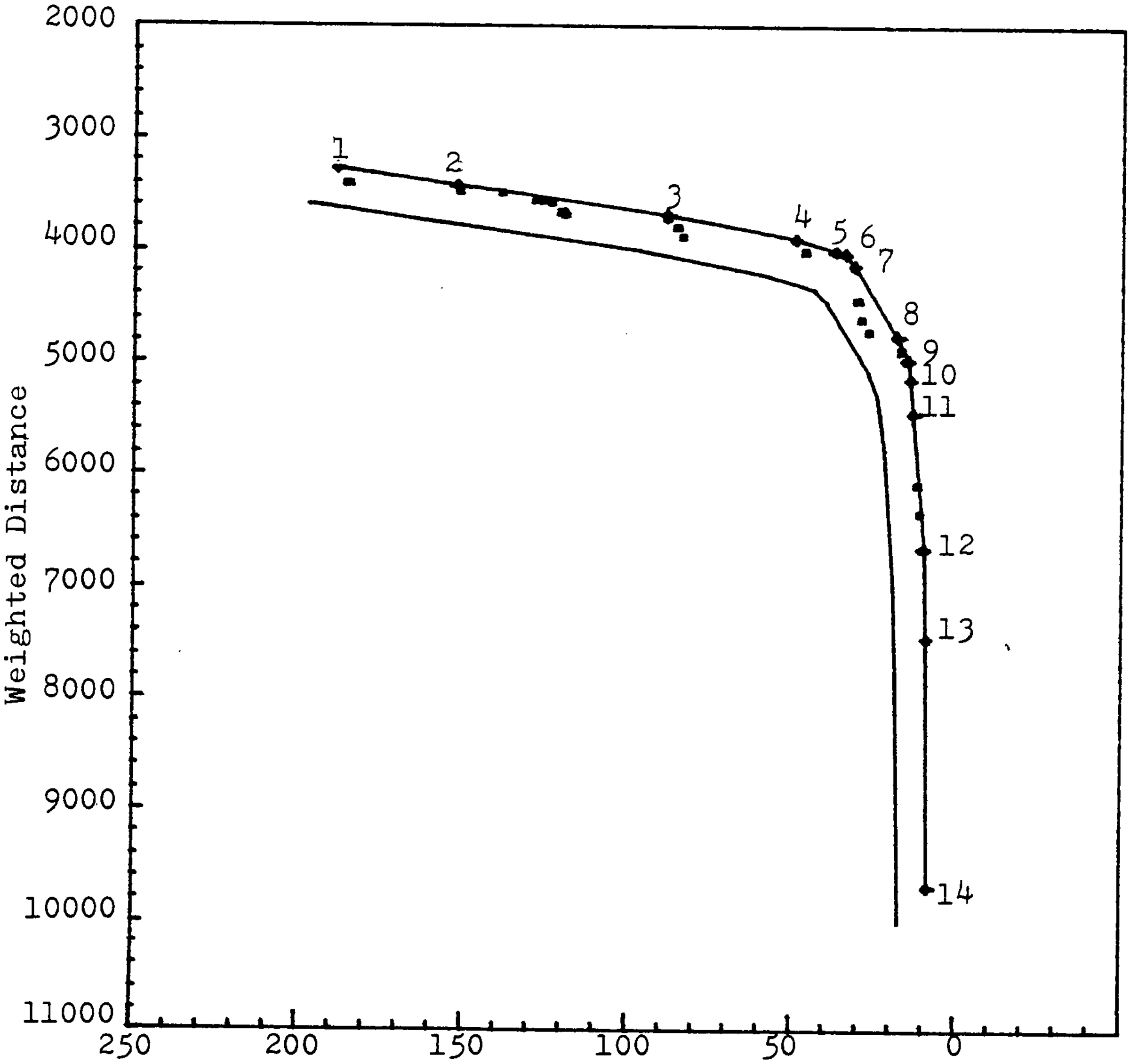


Fig. 4-19. Noninferior Solutions of MCMILP by UNISE Routine with Solutions by RTAB.

Objectives: Nei. Imp. and Wt. Dist. Data: SWAIN

Coverage Distance: 15 No. of Facilities: 4

Impact Distance: 3 Impact Factor: 1



++ Strong Solutions by SONISE Routine

▫ Weak Solutions by WENISE Routine

— 95% of SONISE Solutions

Fig. 4-20. Noninferior Solutions to NCWDLP by SONISE Routine and then by WENISE Routine (Nei. vs. Wt. D.).

Objectives: Nei. Imp. and Wt. Dist.

Data: SWAIN | No. of Facilities: 4

Coverage Distance: 15 | Impact Distance: 3

Impact Factor: 1

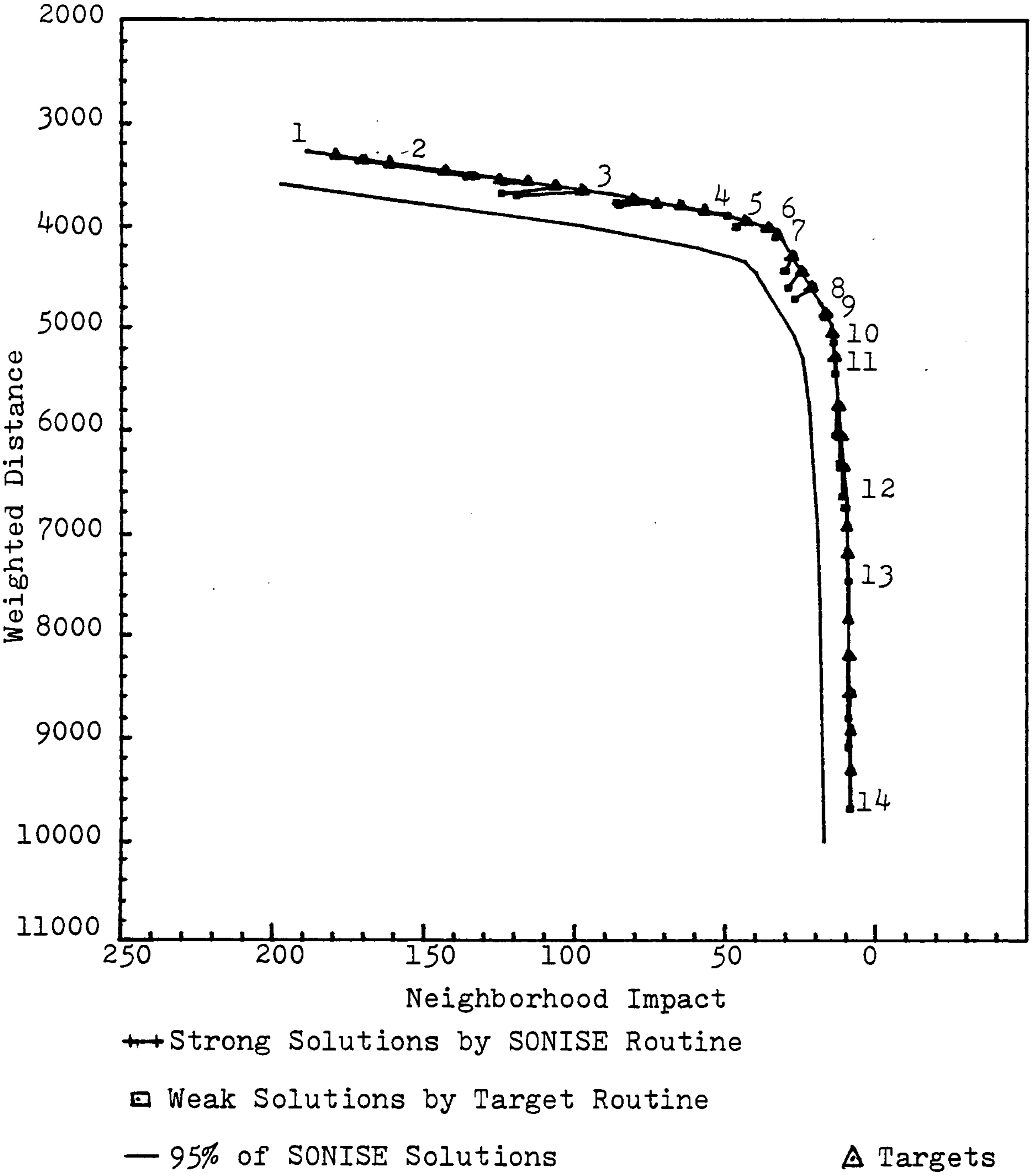


Fig. 4-21. Weak Solutions to NCWDLP by Target Routine (Nei. vs. Wt. D.).

Objectives: Nei. Imp. and Wt. Dist.

Data: SWAIN | No. of Facilities: 4

Coverage Distance: 15 | Impact Distance: 3

Impact Factor: 1

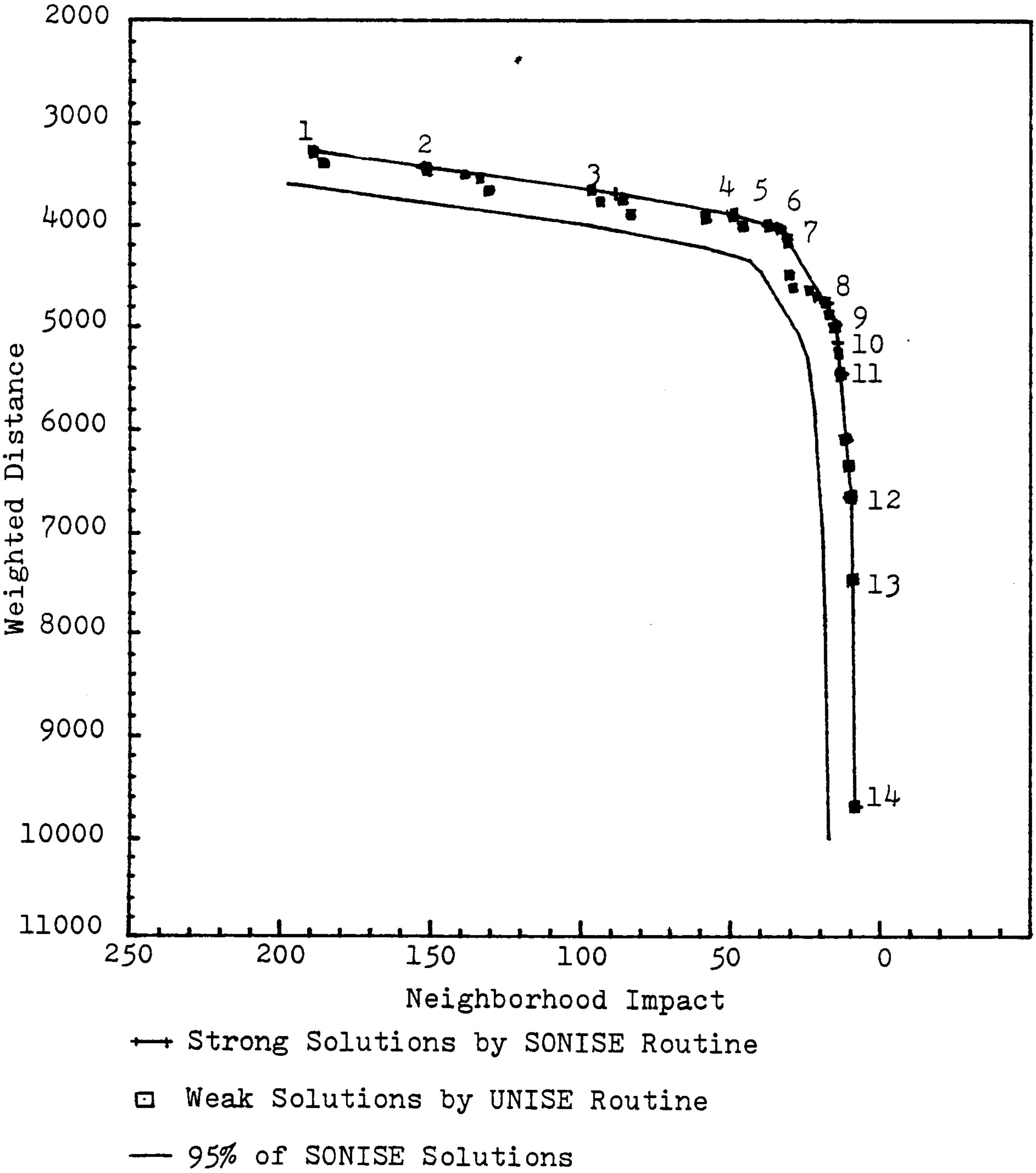


Fig. 4-22. Noninferior Solutions to NCWDLP by UNISE Routine (Nei. vs. Wt. D.).

Objectives: Nei. Imp. and Wt. Dist.

Data: SWAIN No. of Facilities: 4

Coverage Distance: 15 Impact Distance: 3

Impact Factor: 1

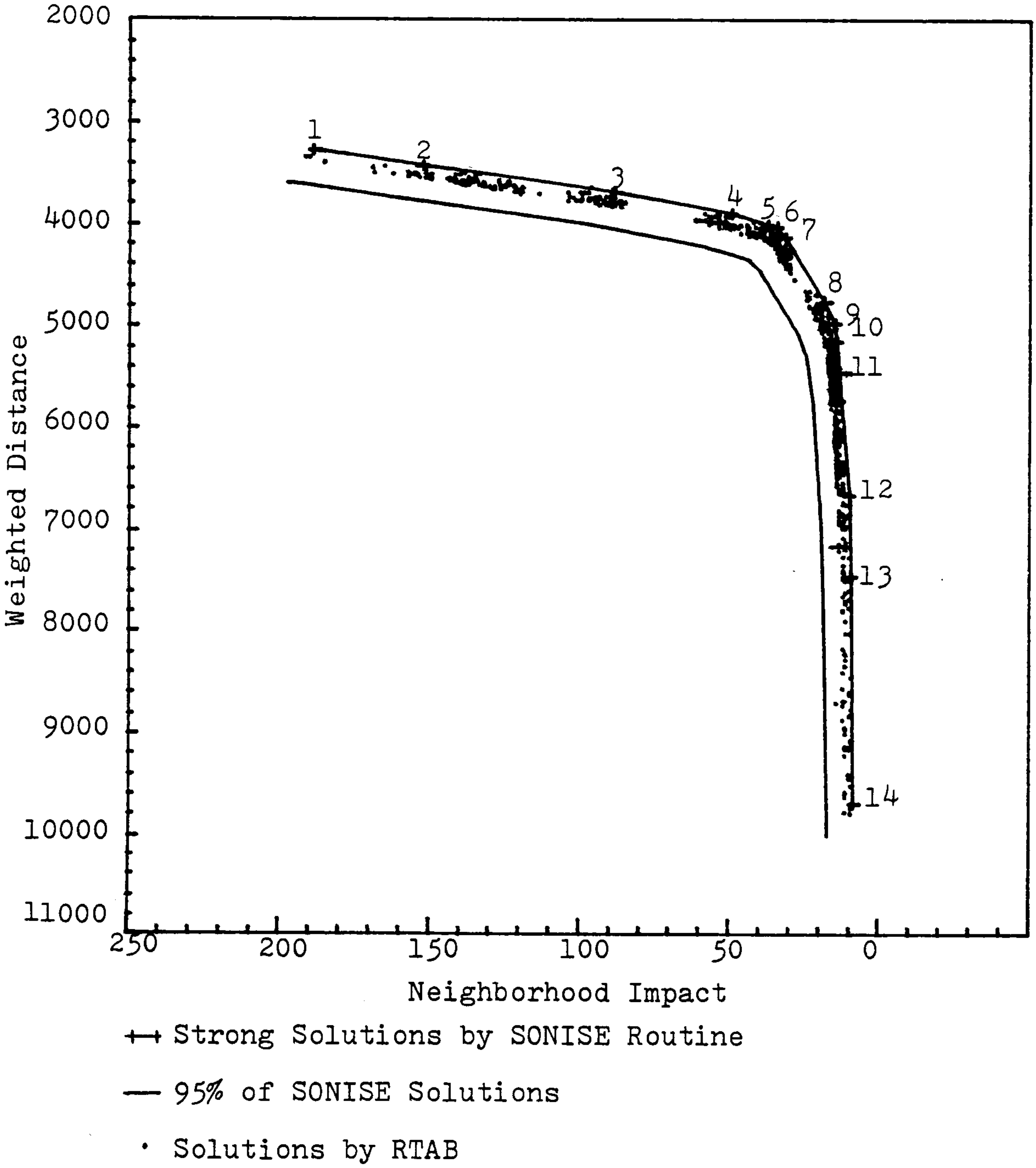
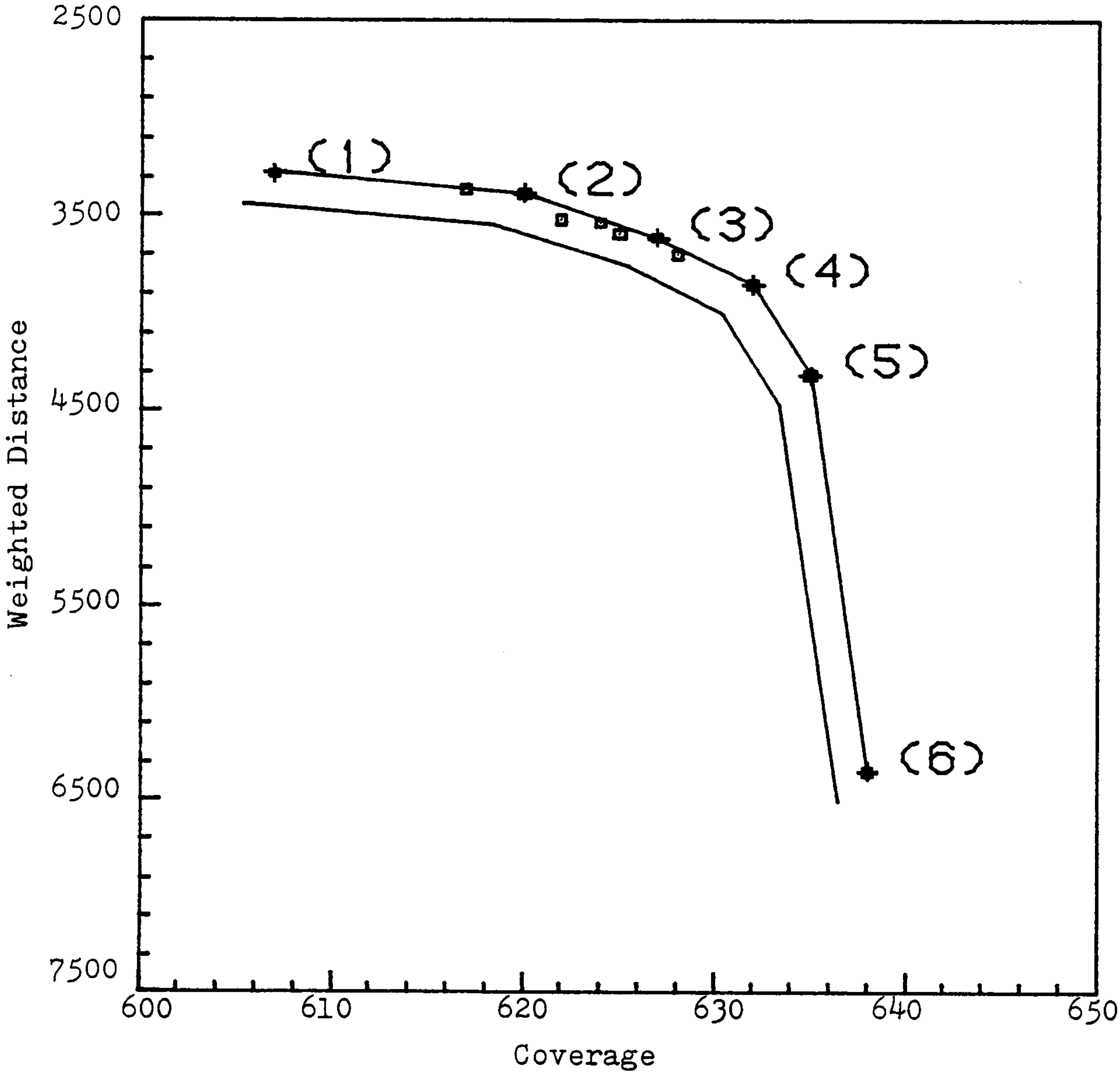


Fig. 4-23. Solutions to NCWDLP by RTAB (Nei. vs. Wt. D.).

Objectives: Coverage and Wt. Dist. Data: SWAIN

Coverage Distance: 15 No. of Facilities: 4

Impact Distance: 3 Impact Factor: 1



Strong Solutions by SONISE Routine

Weak Solutions by WENISE Routine

— 95% of SONISE Solutions

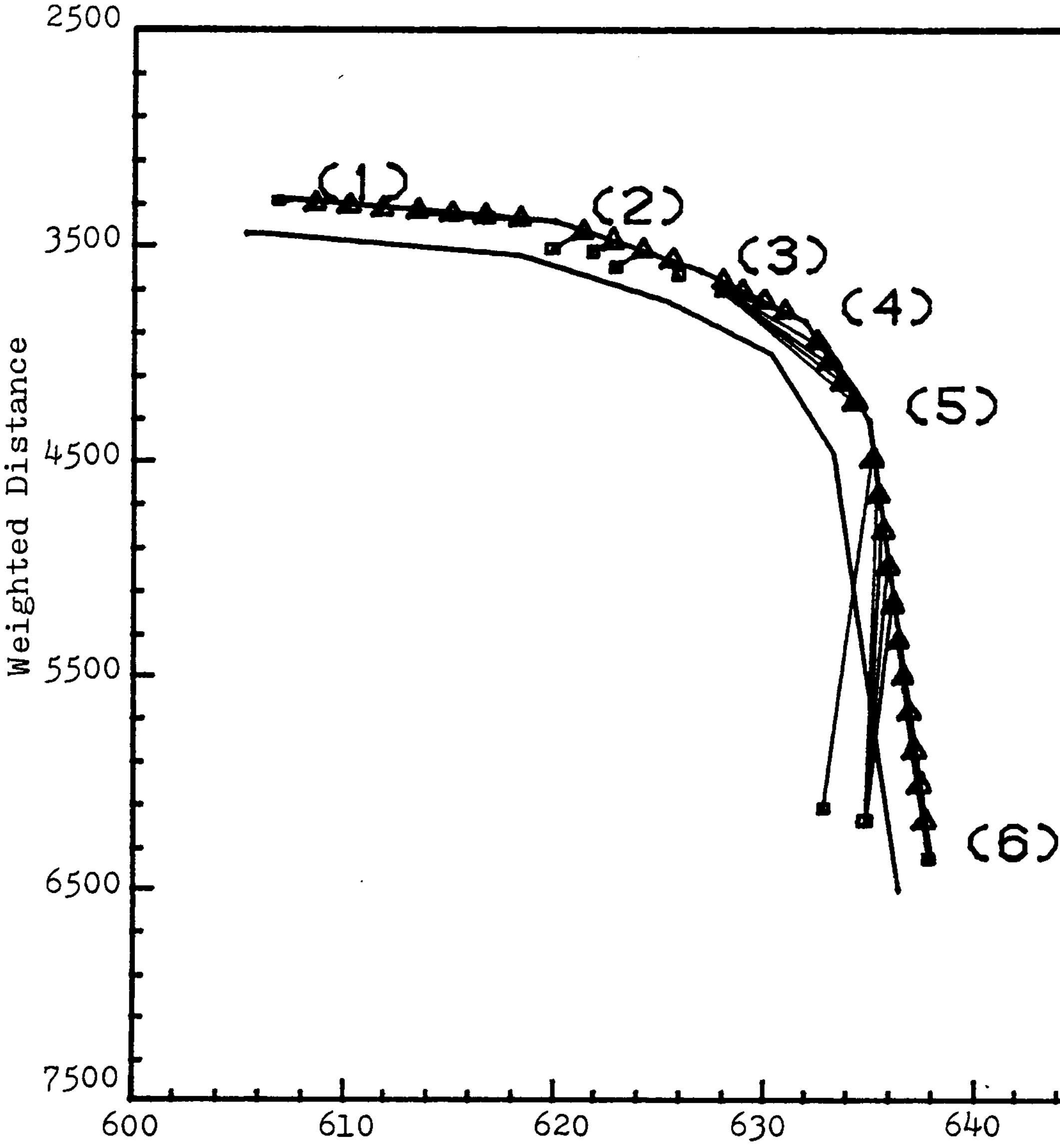
Fig. 4-24. Noninferior Solutions to NCWDLP by SONISE Routine and then by WENISE Routine (Cov. vs. Wt. D.).

Objectives: Coverage and Wt. Dist.

Data: SWAIN — No. of Facilities: 4

Coverage Distance: 15 — Impact Distance: 3

Impact Factor: 1



Strong Solutions by SONISE Routine

Weak Solutions by Target Routine

95% of SONISE Solutions

Δ Targets

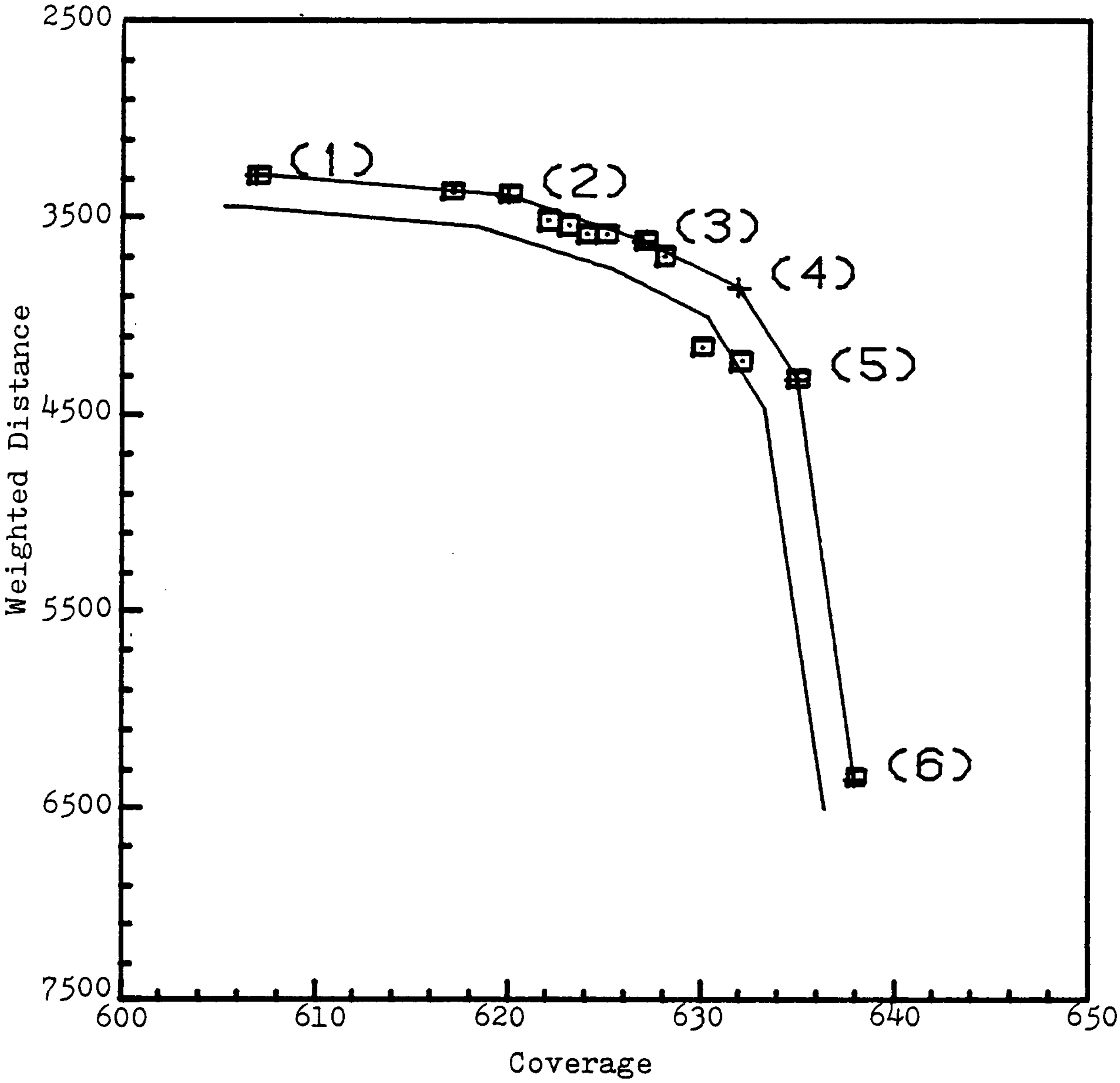
Fig. 4-25. Weak Solutions to NCWDLP by Target Routine (Cov. vs. Wt. D.).

Objectives: Coverage and Wt. Dist.

Data: SWAIN No. of Facilities: 4

Coverage Distance: 15 Impact Distance: 3

Impact Factor: 1



++ Strong Solutions by SONISE Routine

⊡ Weak Solutions by UNISE Routine

— 95% of SONISE Solutions

Fig. 4-26. Noninferior Solutions to NCWDLP by UNISE Routine (Cov. vs. Wt. D.).

Objectives: Coverage and Wt. Dist.

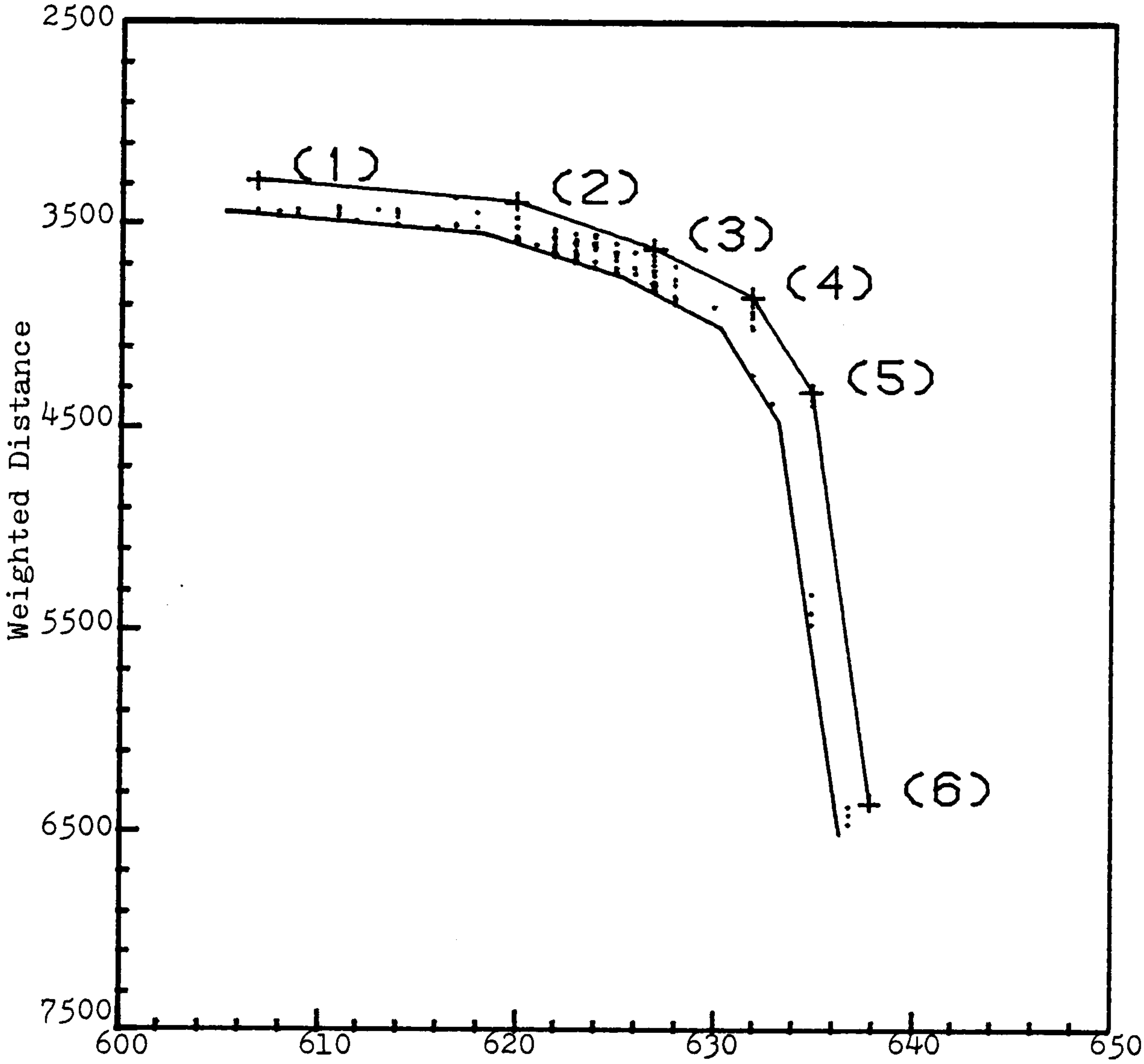
Data: SWAIN

No. of Facilities: 4

Coverage Distance: 15

Impact Distance: 3

Impact Factor: 1



Strong Solutions by SONISE Routine

95% of SONISE Solutions

• Solutions by RTAB

Fig. 4-27. Solutions to NCWDLP by RTAB Routine (Cov. vs. Wt. D.).

Data: Toregas No. of Facilities: 3

Population Cov. Dist.: 75 Property Cov. Dist. : 75

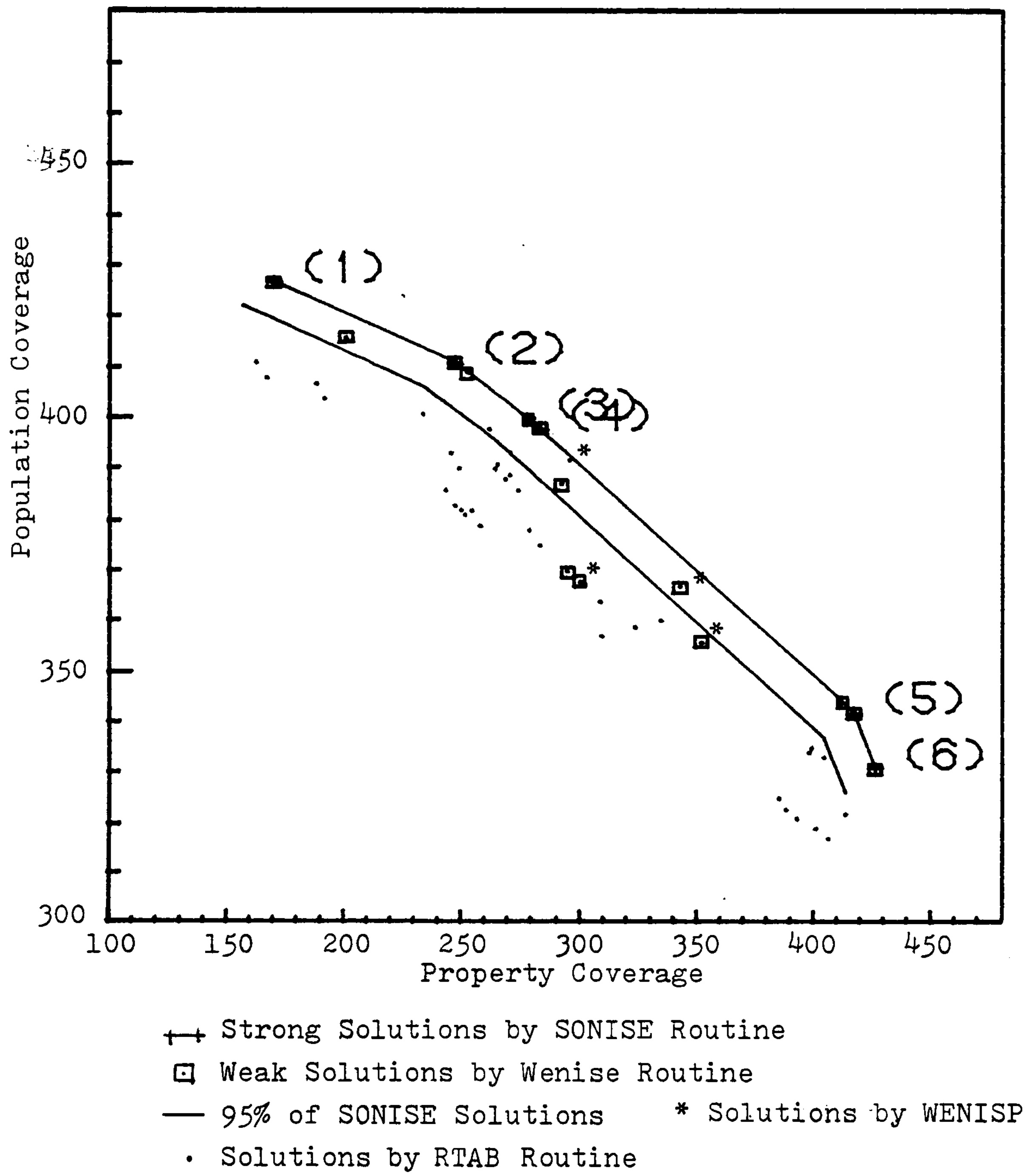
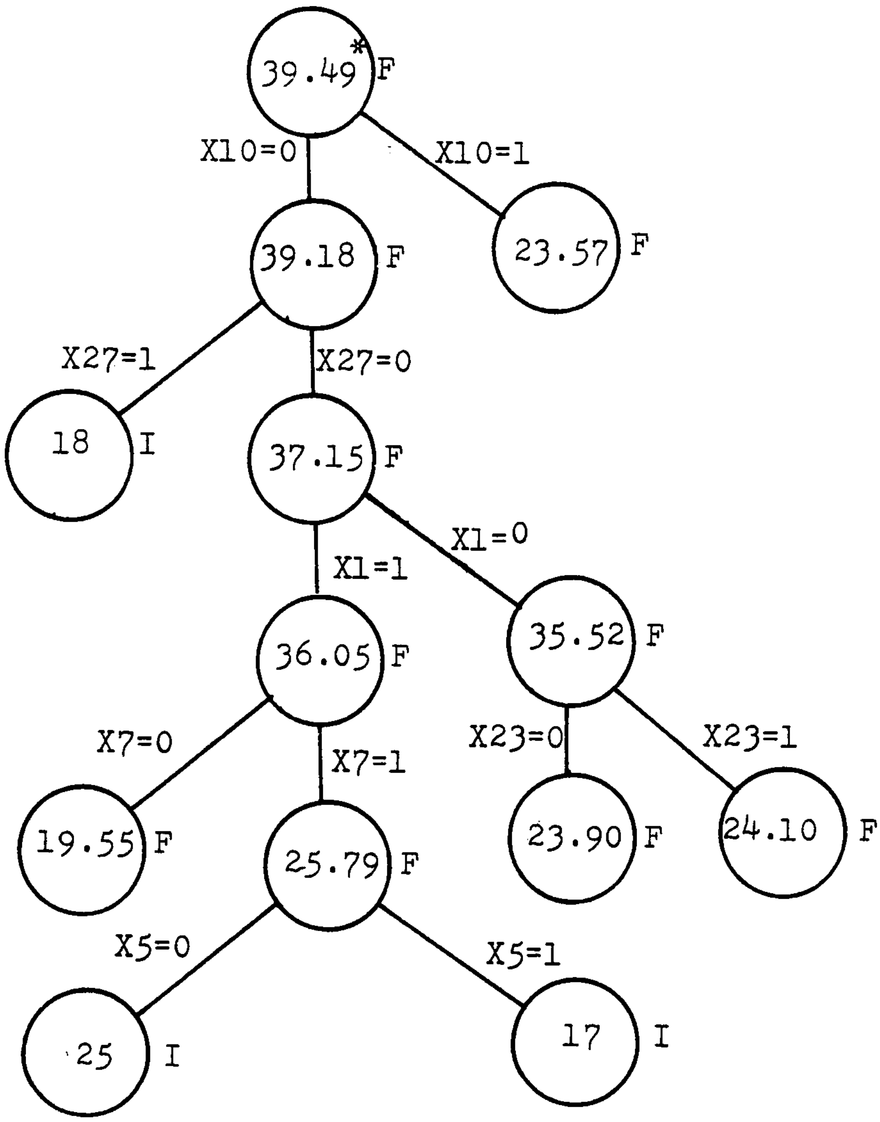


Fig. 4-28. Noninferior Solutions to Cover Versus Cover Model by SONISE Routine and then by WENISE Routine.

ZZ1=284

ZZ2=342



Note:

F Fractional Solution

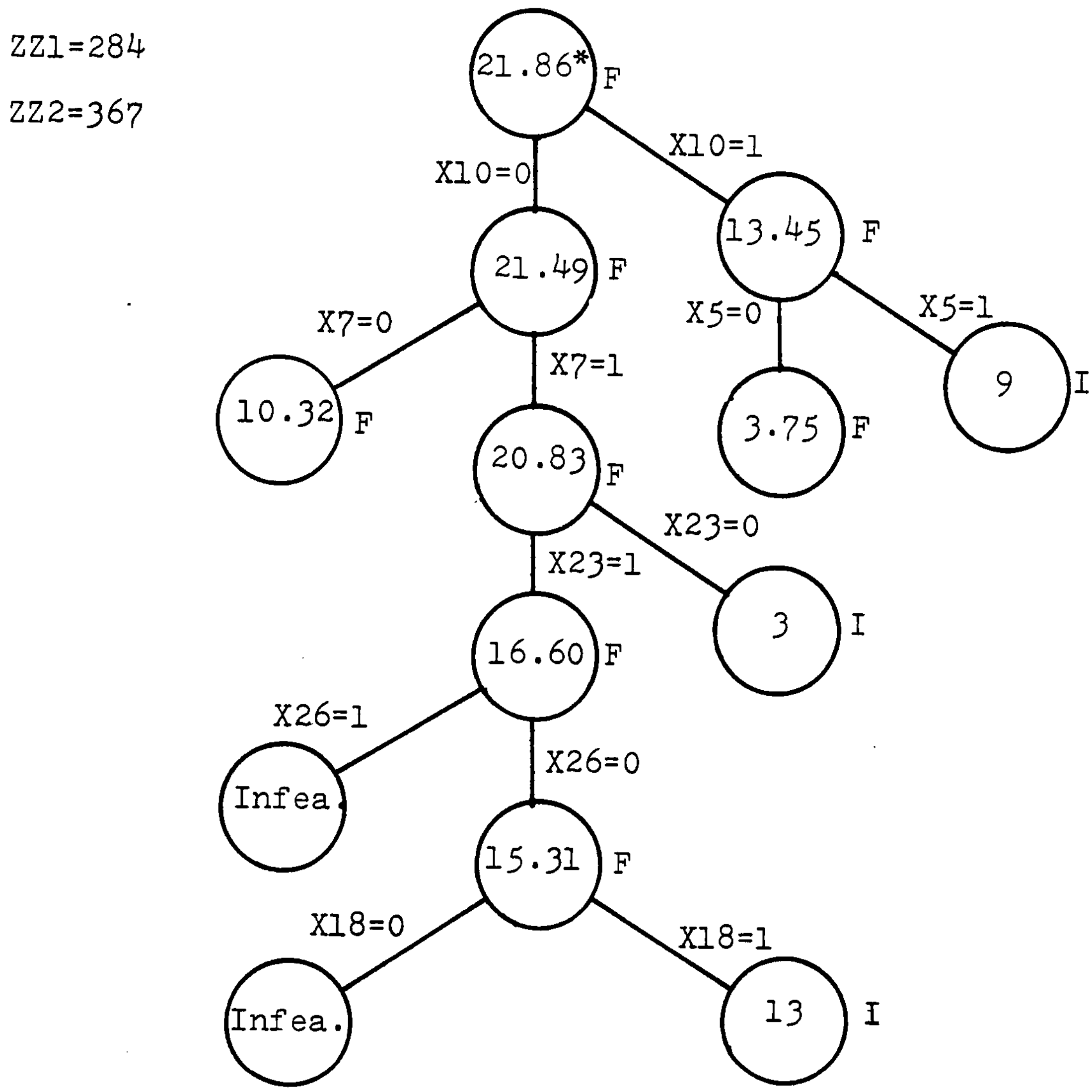
I Integer Solution

* Objective Value

Optimal Integer Solutions:

Obj.	Configuration
25	1 7 18
17	1 5 7

Fig. 4-29. Identifying an Weak Integer Solution to Cover Versus Cover Model by Branch and Bound (ZZ1=284,ZZ2=342).



Note:

F Fractional Solution

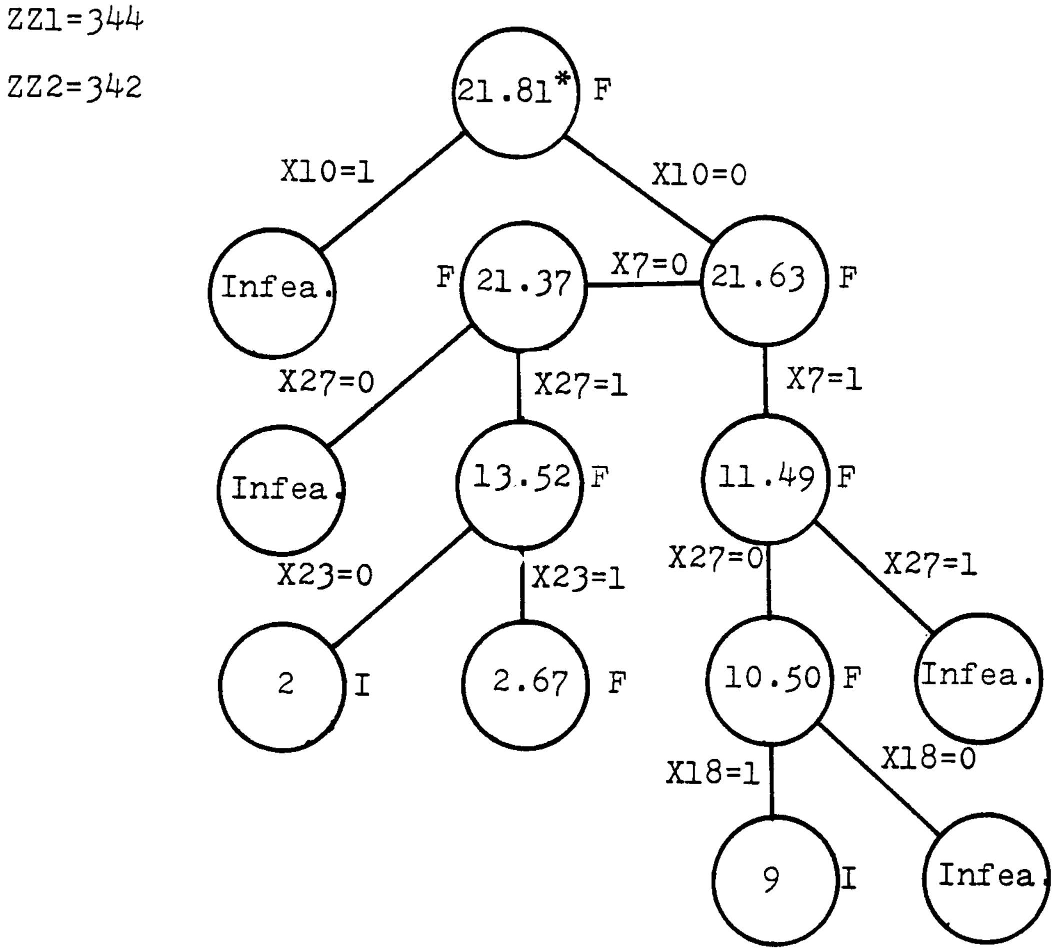
I Integer Solution

* Objective value

Optimal Integer Solution:

Obj.	Configuration
13	7 18 23

Fig. 4-30. Identifying an Weak Integer Solution to Cover Versus Cover Model by Branch and Bound (ZZ1=284, ZZ2=367).



Note:

F Fractional Solution

I Integer Solution

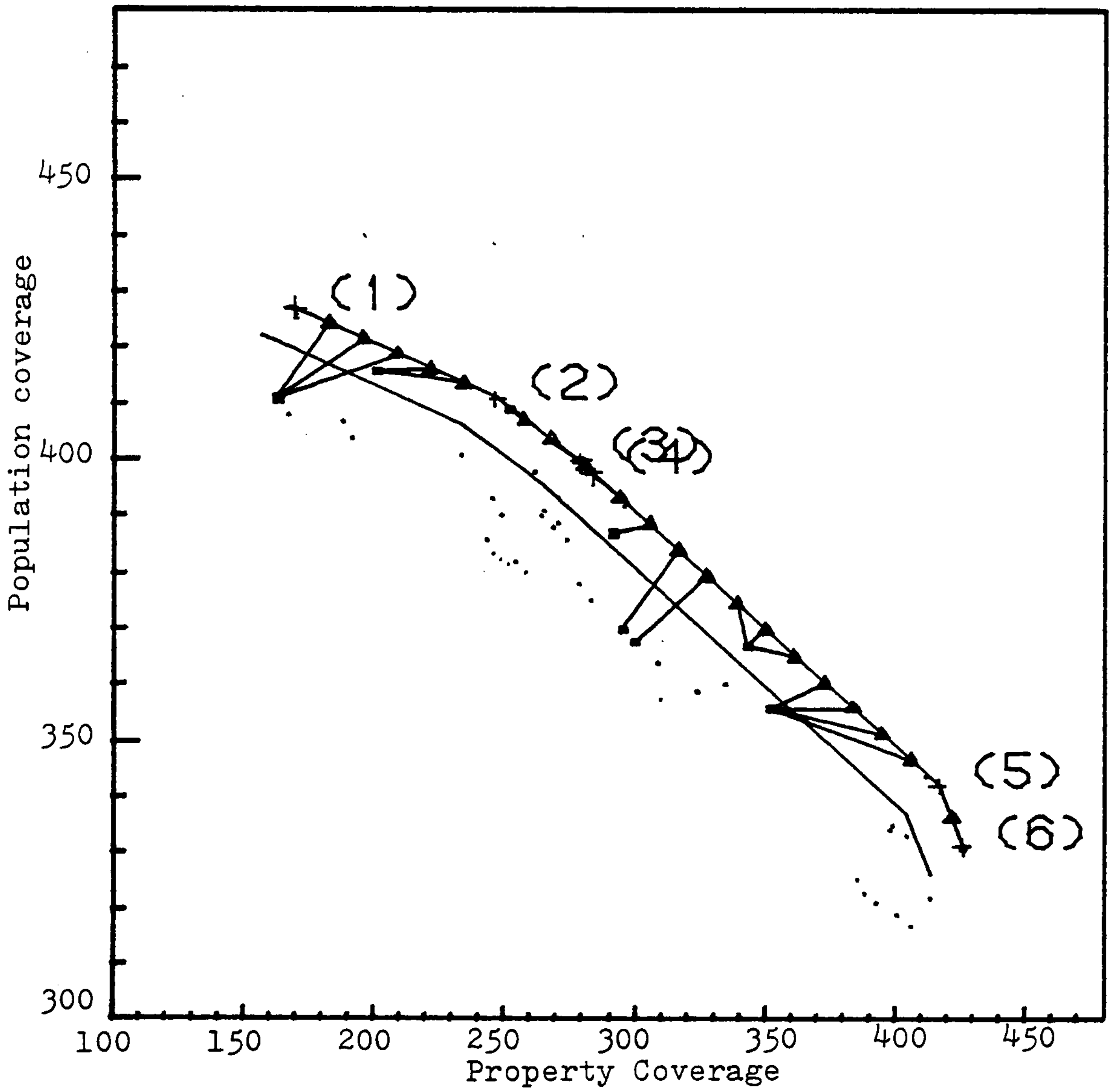
* Objective Value

Optimal Integer Solution	
Obj.	Configuration
9	7 12 18

Fig. 4-31. Identifying an Weak Integer Solution to Cover Versus Cover Model by Branch and Bound (ZZ1=344, ZZ2=342).

Data : Toregas No. of Facilities: 3

Population Cov. Dist.: 75 Property Cov. Dist.: 75



Strong Solutions by SONISE Routine

Solutions by Target Routine

Targets

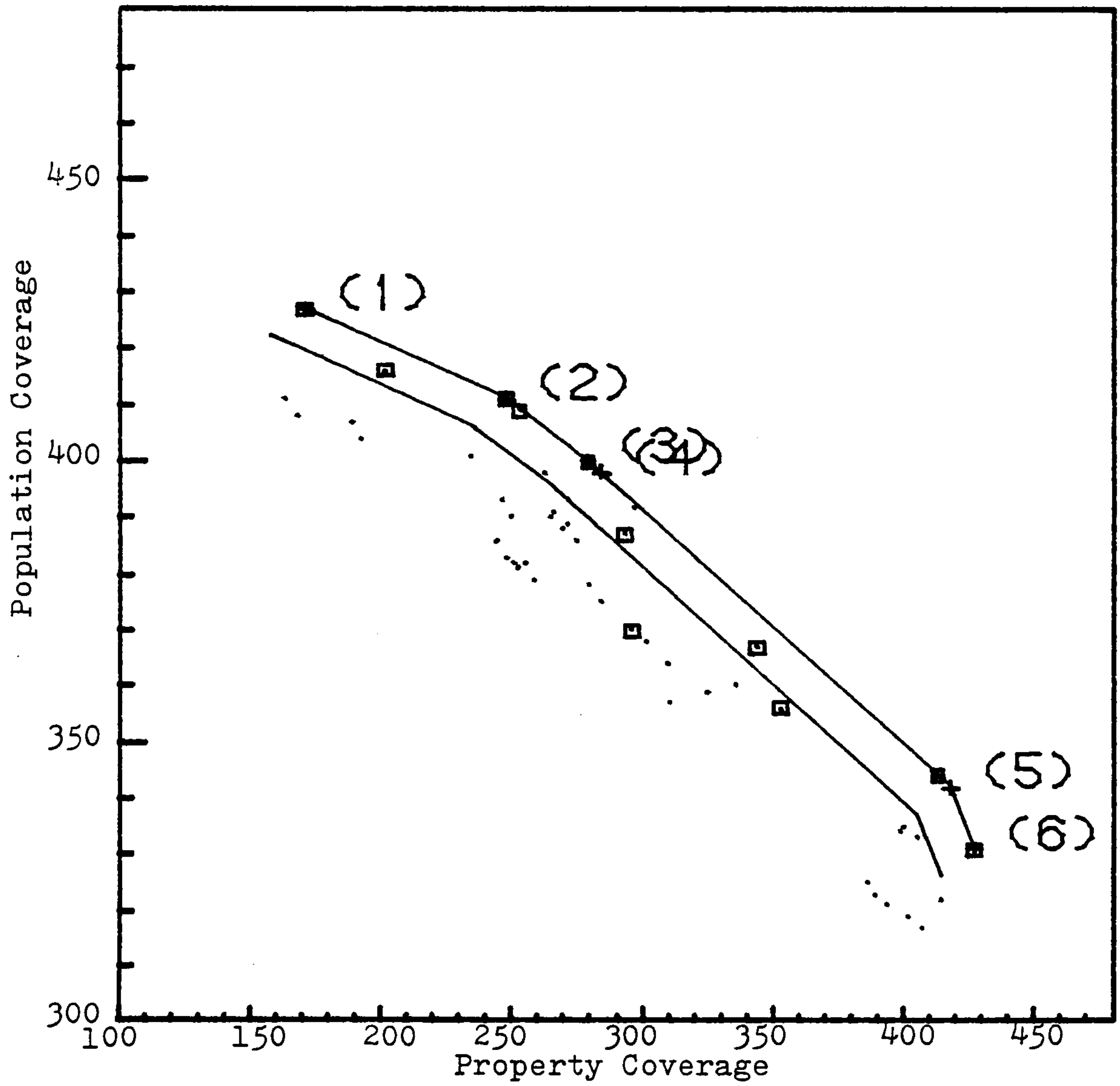
95% of SONISE Solutions

Solutions by RTAB Routine

Fig. 4-32. Weak Solutions to Cover Versus Cover Model by Target Routine.

Data: Toregas No. of Facilities: 3

Population Cov. Dist.: 75 Properties Cov. Dist.: 75



Strong Solutions by SONISE Routine

Noninferior Solutions by UNISE Routine

95% of SONISE Solutions

Solutions by RTAB Routine

Fig. 4-33. Noninferior Solutions to Cover Versus Cover Model by UNISE Routine

TABLE 4-1

A LIST OF THE DIFFERENT MODELS INTRODUCED IN CHAPTER II

Model Name	Symbol	Description	Heuristic used
Minimal Impact Location Problem	MILP	Minimize the total population within a given distance of any facility	MTAB
Minimal Impact Location Problem with distance modification	MILP with dist. mod.	Same as the MILP but the neighborhood impact varies with distance	MTAB
Minimal Person-Facility Impact Location Problem	MPFILP	Minimize the total population within a given distance of each facility	MTAB
Minimal Person-Facility Impact Location Problem with distance mod.	MPFILP with Dist. Mod.	Same as the MPFILP but the neighborhood impact varies with distance	MTAB
Minimal Person-Capacity unit Location Problem	MPCILP	Minimize the total number of person-capacity units	MTAB
Minial Person-Capacity Unit Location Problem with distance mod.	MPCILP with dist. Mod.	Same as the MPCILP but the neighborhood impact varies with distance	MTAB
Maximal Coverage Minimal Impact Location Problem	MCMILP	Two dimensional model, using neighborhood impact and coverage as objectives	SONISE WENISE Target UNISE
Neighborhood Cover Weighted Distance Location Problem	NCWDLP	Three dimensional model, using neighborhood impact, coverage, and weighted distance as objectives	SONISE WENISE Target UNISE

TABLE 4-2

TOREGAS' 30 NODE NETWORK

Population: (Pop(i),i=1,30)

71	62	56	39	35	21	20	19	17	17
16	15	14	12	12	11	10	10	9	9
9	8	8	8	8	7	6	6	6	6

Property Value: (Pr(i),i=1,30)

6	6	6	6	7	8	8	8	8	9
9	9	10	10	11	12	12	14	15	16
17	17	19	20	21	35	39	56	62	71

Distance Matrix: ((D(i,j),j=1,30),i=1,30)

0	260	140	138	297	196	194	51	257	171
355	54	203	149	311	234	300	78	172	38
209	112	74	230	157	16	145	7	93	176
260	0	159	370	37	121	66	274	60	113
101	292	290	333	51	224	77	206	106	296
338	264	312	33	286	244	115	259	167	166
140	159	0	211	196	56	93	170	117	46
215	137	272	174	210	206	160	62	114	178
291	105	157	139	239	129	78	144	92	148
138	370	211	0	407	267	304	181	328	257
426	90	333	61	421	370	371	176	306	100
339	106	70	350	287	152	270	137	229	312

TABLE 4-2 (Continued)

Distance Matrix: ((D(i,j),j=1,30),i=1,30)

297	37	196	407	0	153	103	311	92	150
69	329	327	370	19	261	74	243	143	333
375	301	349	70	323	281	152	296	204	203
196	121	56	267	153	0	60	226	61	34
159	193	285	230	172	219	104	118	113	234
327	161	213	106	275	185	91	200	128	161
194	66	93	304	103	60	0	208	67	47
167	226	243	267	117	177	123	140	59	230
291	198	246	46	239	178	49	193	101	119
51	274	170	181	311	226	208	0	275	201
375	105	152	200	325	198	330	108	186	81
158	163	125	244	106	41	159	48	107	190
257	60	117	328	92	61	67	275	0	87
111	254	310	291	111	244	56	179	126	295
358	222	274	60	306	245	116	260	168	186
171	113	46	257	150	34	47	201	87	0
193	179	251	220	164	185	138	93	79	209
293	151	199	93	241	160	57	175	94	127
355	101	215	426	69	159	167	375	111	193
0	352	391	389	88	325	55	277	207	393
439	320	372	134	387	344	216	359	268	267
54	292	137	90	329	193	226	105	254	179
352	0	257	95	343	287	297	86	216	65
263	58	20	265	211	69	180	61	146	229
203	290	272	333	327	285	243	152	310	251
391	257	0	352	341	66	366	257	184	233
60	315	277	257	46	193	194	200	193	124
149	333	174	61	370	230	267	200	291	220
389	95	352	0	384	380	334	181	288	151
358	69	75	313	306	164	252	156	241	322
311	51	210	421	19	172	117	325	111	164
88	343	341	384	0	275	93	257	157	347
389	315	363	84	337	295	166	310	218	217
234	224	206	370	261	219	177	198	244	185
325	287	66	380	275	0	300	205	118	270
126	311	307	191	92	218	128	233	141	58
300	77	160	371	74	104	123	330	56	138
55	297	366	334	93	300	0	222	182	338
414	265	317	110	362	289	172	304	224	242
78	206	62	176	243	118	140	108	179	93
277	86	257	181	257	205	222	0	130	116
263	124	106	179	211	67	94	82	64	147
172	106	114	306	143	113	59	186	126	79

TABLE 4-2 (Continued)

Distance Matrix: ((D(i,j),j=1,30),i=1,30)

207	216	184	288	157	118	182	130	0	208
232	219	236	73	180	156	36	171	79	60
38	296	178	100	333	234	230	81	295	209
393	65	233	151	347	270	338	116	208	0
239	123	76	266	187	52	181	37	129	212
209	338	291	339	375	327	291	158	358	293
439	263	60	358	389	126	414	263	232	239
0	321	283	305	52	199	242	206	199	172
112	264	105	106	301	161	198	163	222	151
320	58	315	69	315	311	265	124	219	123
321	0	60	244	269	127	183	119	188	253
74	312	157	70	349	213	246	125	274	199
372	20	277	75	363	307	317	106	236	76
283	60	0	285	231	89	200	81	166	249
230	33	139	350	70	106	46	244	60	93
134	265	257	313	84	191	110	179	73	266
305	244	285	0	253	214	85	229	137	133
157	286	239	287	323	275	239	106	306	241
387	211	46	306	337	92	362	211	180	187
52	269	231	253	0	147	190	154	147	120
16	244	129	152	281	185	178	41	245	160
344	69	193	164	295	218	289	67	156	52
199	127	89	214	147	0	129	15	77	160
145	115	78	270	152	91	49	159	116	57
216	180	194	252	166	128	172	94	36	181
242	183	200	85	190	129	0	144	52	70
7	259	144	137	296	200	193	48	260	175
359	61	200	156	310	233	304	82	171	37
206	119	81	229	154	15	144	0	92	175
93	167	92	229	204	128	101	107	168	94
268	146	193	241	218	141	224	64	79	129
199	188	166	137	147	77	52	92	0	83
176	166	148	312	203	161	119	190	186	127
267	229	124	322	217	58	242	147	60	212
172	253	249	133	120	160	70	175	83	0

TABLE 4-3

SWAIN'S 55 NODE NETWORK

Population: (Pop(i),i=1,55)

71	62	56	39	35	21	20	19	17	17
16	15	14	12	12	11	10	10	9	9
9	8	8	8	8	7	6	6	6	6
6	5	5	5	5	5	5	4	4	4
4	4	4	4	3	3	3	3	3	3
3	2	2	2	2					

Cartesian Co-ordinates of Population Centers:
((X(i),Y(i)),i=1,55)

32	31	29	32	27	36	29	29	32	29
26	25	24	33	30	35	29	27	29	21
33	28	17	53	34	30	25	60	21	28
30	51	19	47	17	33	22	40	25	14
29	12	24	48	17	42	6	26	19	21
10	32	34	56	12	47	19	38	27	41
21	35	32	45	27	45	32	38	8	22
15	25	35	16	36	47	46	51	50	40
23	22	27	30	38	39	36	32	32	41
42	36	36	26	15	19	19	14	45	19
27	5	52	24	40	22	40	52	42	42

TABLE 4-4

SOLUTIONS TO MILP AND RELATED PROBLEMS

Data: SWAIN

Formulation	Neighborhood Distance(s)	Impact Factor(s)	Optimal Solution Obj.	Optimal Solution Time (sec)	Optimal Solution Facility Location	Heuristic Solution Obj.	Heuristic Solution Time (sec)	Heuristic Solution Facility Location
MILP	3	1	8.0	12.45	52,53,54,55	8.0	.60	52,53,54,55
MILP#	1,2,4,8	1,.5,.25,.1	10.9	48.0*	50,52,53,55	10.9	.66	50,52,53,55
MPFILP	3	1	8.0	9.20	52,53,54,55	8.0	.29	52,53,54,55
MPFILP#	1,2,4,8	1,.5,.25,.1	11.4	11.03	50,52,53,55	11.4	.30	50,52,53,55
MPCILP	3	1			**	1280.0	.86	52,53,54,55
MPCILP#	1,2,4,8	1,.5,.25,.1			**	1982.1	1.01	52,53,54,55

Note: * Branch and bound method is required

** Problems too large to be solved

\# Problems solved with distance modification

TABLE 4-5

STRONG SOLUTIONS TO MCMILP USING OPTIMAL METHOD

Data: SWAIN — No. of Facilities: 4

Coverage Distance: 15 — Impact Distance: 3

Impact Factor: 1

No.	Wc	Wn	Configuration of Solution	time (sec)	Coverage	Neighborhood Impact
1	.000	1.000	52 53 54 55	12.45	397	8
2	.013	.987	47 53 54 55	9.24	481	9
3	.018	.982	47 48 53 54	8.12	547	10
4	.055	.945	33 48 53 55	8.03	619	12
5	.217	.783	22 48 53 55	9.49	633	15
6	.345	.655	22 36 53 55	12.13	637	17
7	.200	.800	22 37 43 48	12.48	638	20
8*	1.000	.000	22 36 37 43	11.20	638	22

Note: * This solution should be regarded as inferior to solution 7.

TABLE 4-6

WEAK SOLUTIONS TO MCMILP USING TARGET ROUTINE

Data: SWAIN — No. of Facilities: 4

Coverage Distance: 15 — Impact Distance: 3

Impact Factor: 1

No.	Target Cov.	Target Nei.	Gradient	Solutions Starting				Solutions Final				Coverage	Neighbor-hood
1	413.80	8.20	0.01	49	48	43	44	52	53	54	55	397.00	8.00
2	430.60	8.40	0.01	49	48	43	44	52	53	54	55	397.00	8.00
3	447.40	8.60	0.01	49	48	43	44	52	53	54	47	477.00	9.00
4	464.20	8.80	0.01	49	48	43	44	52	53	54	47	477.00	9.00
5	497.50	9.25	0.02	49	48	43	44	55	53	54	47	481.00	9.00
6	514.00	9.50	0.02	49	48	43	44	52	48	53	47	505.00	10.00
7	530.50	9.75	0.02	49	48	43	44	54	48	53	47	547.00	10.00
8	561.40	10.40	0.03	49	48	43	44	53	48	54	47	547.00	10.00
9	575.80	10.80	0.03	49	48	43	44	49	48	54	47	548.00	11.00
10	590.20	11.20	0.03	49	48	43	44	47	48	38	53	561.00	12.00
11	604.60	11.60	0.03	49	48	43	44	53	48	38	47	561.00	12.00
12	622.50	12.75	0.21	49	48	43	44	53	48	55	33	619.00	12.00
13	626.00	13.50	0.21	49	48	43	44	53	48	43	33	619.00	14.00
14	629.50	14.25	0.21	49	48	43	44	53	48	54	22	629.00	15.00
15	634.33	15.67	0.50	49	48	43	44	22	48	55	53	633.00	15.00
16	635.67	16.33	0.50	49	48	43	44	22	48	43	53	637.00	17.00
17	637.33	18.00	3.00	49	48	43	44	53	48	43	22	637.00	17.00
18	637.67	19.00	3.00	49	48	43	44	37	48	43	22	638.00	20.00

TABLE 4-7

NONINFERIOR SOLUTIONS TO MCMILP, USING SONISE ROUTINE AND THEN WENISE ROUTINE

Data: SWAIN — No. of Facilities: 4

Coverage Distance: 15 — Impact Distance: 3

Impact Factor: 1

No.	Weights Wc	Weights Wn	Solutions Starting				Solutions Final				Coverage	Neighborhood Impact
			(Strong Solutions)									
1	.000	1.000	22	36	37	43	52	53	54	55	397	8
2	.013	.987	52	53	54	55	47	53	54	55	481	9
3	.018	.982	52	53	54	55	47	48	53	54	547	10
4	.055	.945	52	53	54	55	33	48	53	55	619	12
5	.217	.783	33	48	53	55	22	48	53	55	633	15
6	.345	.655	33	48	53	55	22	36	53	55	637	17
7	.167	.833	22	36	37	43	22	37	43	48	638	20
8@	1.000	.000	22	36	37	43	22	36	37	43	638	22

TABLE 4-7 (Continued)

No.	Target Cov.	Target Nei.		Solutions Starting	Solutions Final	Coverage	Neighbor-hood
					(Weak Solutions)		
9*	547	120	0.03	47 48 53 54	41 48 54 55	550	11

Note: @ This solution to the maximal coverage problem should be regarded as inferior to solution 7.

* This target is the lower left corner instead of the normally defined upper right corner of the BORE area.

TABLE 4-8

NONINFERIOR SOLUTIONS TO MCMILP, USING UNISE ROUTINE

Data: SWAIN — No. of Facilities: 4

Coverage Distance: 15 — Impact Distance: 3

Impact Factor: 1

No.	Target Cov.	Target Nei.	Gradient	Solutions Starting				Solutions Final				Coverage	Neighbor-hood
1	0	1*	--	37	36	22	43	55	54	53	52	397	8
2	477	10	0.02	47	54	53	52	47	54	53	55	481	9
3	397	12	0.02	55	54	53	52	41	54	53	52	533	10
4	545	12	0.01	33	55	53	48	41	55	54	48	550	11
5	578	15	0.06	31	54	53	52	33	55	53	48	619	12
6	578	22	0.17	31	54	53	52	48	22	53	54	629	15
7	629	22	0.78	48	22	53	54	36	22	53	55	637	17
8	1	0*	--	37	36	22	43	37	36	22	43	638	22

Note: * This is the weight used to find the optimal solution of coverage or neighborhood impact.

TABLE 4-9

NONINFERIOR SOLUTIONS TO NCWDLP, USING SONISE ROUTINE AND THEN WENISE ROUTINE (NEI. VS. WT. D.)

Objectives: Neighborhood Impact and Weighted Distance

Data: SWAIN No. of Facilities: 4

Coverage Distance: 15 Impact Distance: 3

Impact Factor: 1

No.	Weights Wn	Weights Wd	Solutions Starting				Solutions Final				Neighborhood Impact	Weighted Distance
			(Strong Solutions)									
1	0.000	1.000	5	10	20	35	1	3	22	41	188	3282.86
2	0.815	0.185	41	1	22	3	41	1	22	7	152	3419.71
3	0.791	0.209	16	37	52	2	22	9	36	8	88	3692.76
4	0.844	0.156	41	1	22	3	10	8	22	36	49	3894.20
5	0.903	0.096	41	8	22	47	41	8	22	31	37	4003.22
6	0.931	0.069	10	8	22	36	41	8	22	47	34	4033.54
7	0.973	0.027	41	1	22	3	41	33	47	8	31	4135.89
8	0.983	0.017	41	33	47	8	41	33	47	31	18	4747.61
9	0.990	0.010	41	33	47	31	41	54	47	30	15	4974.99
10	0.996	0.004	41	33	47	8	48	54	47	31	14	5137.17
11	0.997	0.003	48	54	47	31	48	55	47	33	13	5438.26
12	0.999	0.001	48	54	47	31	48	54	47	55	10	6661.19
13	0.999	0.001	48	54	47	55	53	54	47	55	9	7453.47
14	1.000	0.000	37	20	25	23	55	54	53	52	8	9675.09

TABLE 4-9 (Continued)

No.	Target Nei.	Target Wt. Dist.	Gradient	Solutions Starting				Solutions Final				Neighborhood Impact	Weighted Distance
				(Weak Solutions)									
15	188	3419.71	3.77	41	1	22	3	41	1	33	3	185	3408.34
16	152	3489.18	4.96	41	1	22	31	10	1	22	31	151	3476.10
17	152	3559.04	5.36	41	1	22	7	41	1	22	31	138	3489.18
18	138	3559.04	5.82	41	2	16	29	41	2	16	23	128	3548.36
19	152	3692.76	4.27	41	1	22	7	41	2	16	29	126	3559.04
20	126	3658.02	16.50	41	2	16	29	41	2	22	29	123	3584.29
21	126	3692.76	3.52	41	2	16	29	41	2	32	29	120	3658.02
22	120	3692.76	1.09	41	2	32	29	41	2	38	29	119	3683.46
23	88	3870.36	35.52	22	9	36	8	33	9	36	8	85	3795.12
24	88	3894.20	5.17	22	9	36	8	33	9	48	8	83	3870.36
25	49	4003.22	9.09	10	8	22	36	10	8	33	36	46	3996.55
26	31	4606.64	235.37	41	33	47	8	41	38	47	8	30	4428.21
27	31	4747.61	47.06	41	33	47	8	48	38	47	8	29	4606.64
28	29	4747.61	12.82	48	38	47	8	48	54	47	8	27	4714.08
29	18	4974.99	75.79	41	33	47	31	41	38	47	31	17	4879.18
30	13	6661.19	407.65	48	55	47	33	48	55	47	38	12	6089.84
31	12	6661.19	285.68	54	55	47	33	54	55	47	41	11	6332.48

TABLE 4-10

WEAK SOLUTIONS TO NCWDLP, USING TARGET ROUTINE (NEI. VS. WT. D.)

Objectives: Neighborhood Impact and Weighted Distance

Data: SWAIN   No. of Facilities: 4

Coverage Distance: 15   Impact Distance: 3

Impact Factor: 1

No.	Target Nei.	Target Wt. Dist.	Gradient	Solutions Starting				Final				Nei.	Wt. Dist.
1	179.00	3317.82	3.77	10	8	22	36	10	2	22	36	135.00	3502.20
2	170.00	3351.78	3.77	10	8	22	36	10	2	22	36	135.00	3502.20
3	161.00	3385.74	3.77	10	8	22	36	10	2	22	36	135.00	3502.20
4	142.86	3458.71	4.27	10	8	22	36	10	2	22	36	135.00	3502.20
5	133.71	3497.72	4.27	10	8	22	36	10	2	22	36	135.00	3502.20
6	124.57	3536.73	4.27	10	8	22	36	10	2	22	36	135.00	3502.20
7	115.43	3575.74	4.27	10	8	22	36	41	2	22	29	123.00	3584.29
8	106.29	3614.75	4.27	10	8	22	36	20	2	33	36	124.00	3674.37
9	97.14	3653.76	4.27	10	8	22	36	33	2	41	36	119.00	3708.60
10	80.20	3733.05	5.17	10	8	22	36	9	8	22	48	86.00	3768.01
11	72.40	3773.34	5.17	10	8	22	36	9	8	33	36	85.00	3795.12
12	64.60	3813.63	5.17	10	8	22	36	10	8	22	36	49.00	3894.20
13	56.80	3853.91	5.17	10	8	22	36	10	8	22	36	49.00	3894.20
14	43.00	3948.71	9.09	10	8	22	36	10	8	33	36	46.00	3996.55

TABLE 4-10 (Continued)

	Target			Solutions									
No.	Nei.	Wt. Dist.	Gradient	Starting				Final				Nei.	Wt. Dist.
15	35.50	4018.38	10.11	10	8	22	36	41	8	22	29	37.00	4016.63
16	32.50	4084.72	34.12	10	8	22	36	41	8	22	55	33.00	4112.10
17	27.75	4288.82	47.06	10	8	22	36	41	8	38	47	30.00	4428.21
18	24.50	4441.75	47.06	10	8	22	36	38	8	41	52	29.00	4603.96
19	21.25	4594.68	47.06	10	8	22	36	52	8	54	41	27.00	4711.40
20	16.50	4861.30	75.79	10	8	22	36	47	38	31	41	17.00	4879.18
21	14.50	5056.08	162.18	10	8	22	36	31	48	47	54	14.00	5137.17
22	13.50	5287.71	301.09	10	8	22	36	55	33	48	47	13.00	5438.26
23	12.25	5743.99	407.65	10	8	22	36	47	41	55	38	13.00	6040.87
24	11.50	6049.72	407.65	10	8	22	36	54	48	47	41	12.00	6316.24
25	10.75	6355.46	407.65	10	8	22	36	54	47	48	51	11.00	6645.34
26	9.67	6925.28	792.28	10	8	22	36	54	47	48	53	10.00	6739.44
27	9.33	7189.38	792.28	10	8	22	36	55	47	54	53	9.00	7453.47
28	8.83	7823.74	2221.62	10	8	22	36	55	53	47	54	9.00	7453.47
29	8.67	8194.01	2221.62	10	8	22	36	52	53	48	54	9.00	8796.73
30	8.50	8564.28	2221.62	10	8	22	36	52	53	54	49	9.00	9087.73
31	8.33	8934.55	2221.62	10	8	22	36	52	53	54	55	8.00	9675.09
32	8.17	9304.82	2221.62	10	8	22	36	52	53	54	55	8.00	9675.09

TABLE 4-11

NONINFERIOR SOLUTIONS TO NCWDLP USING UNISE ROUTINE (NEI. VS. WT. D.)

Objectives: Neighborhood Impact and Weighted Distance

Data: SWAIN No. of Facilities: 4

Coverage Distance: 15 Impact Distance: 3

Impact Factor: 1

	Target			Solutions									
No.	Wt. Dist.	Nei.	Gradient	Starting				Final				Wt. Dist.	Nei.
1	0.00	1*	--	5	10	25	35	41	1	22	3	3283.86	188
2	3419.71	188	3.77	41	1	22	3	41	1	33	3	3408.34	185
3	3507.66	188	14.92	41	1	22	3	41	1	22	7	3419.71	152
4	3489.18	152	4.96	41	1	22	31	10	1	22	31	3476.10	151
5	3512.48	152	4.88	41	1	22	7	41	1	22	31	3489.18	138
6	3645.81	188	3.93	41	1	22	3	41	11	22	3	3512.48	133
7	3645.81	133	3.60	41	11	22	3	41	11	33	3	3635.24	130
8	4205.45	188	5.98	41	1	22	3	41	11	22	8	3645.81	96
9	3768.01	96	12.22	41	11	22	8	41	11	33	8	3748.16	93
10	3889.80	96	6.42	41	11	22	8	48	9	22	8	3768.01	86
11	3889.80	86	4.35	48	9	22	8	48	9	33	8	3870.36	83
12	4205.45	96	9.03	41	11	22	8	18	6	22	8	3889.80	58
13	3996.55	58	8.90	18	6	22	8	36	10	22	8	3894.20	49

TABLE 4-11 (Continued)

No.	Target Wt. Dist.	Target Nei.	Gradient	Solutions Starting				Solutions Final				Wt. Dist.	Nei.
14	4205.45	58	13.15	18	6	22	8	36	10	33	8	3996.55	46
15	4033.54	46	3.08	47	41	22	8	31	41	22	8	4003.22	37
16	4205.45	46	17.41	36	10	33	8	47	41	22	8	4033.54	34
17	4487.97	34	113.61	47	41	22	8	47	41	33	8	4135.89	31
18	4680.14	34	36.52	41	28	32	8	41	52	32	8	4487.97	30
19	4680.14	30	21.35	41	52	32	8	41	52	38	8	4603.96	29
20	4680.14	29	9.52	41	47	22	31	41	47	22	34	4640.59	23
21	5874.77	34	87.86	41	28	32	8	41	47	22	31	4680.14	21
22	5146.85	21	116.68	41	47	22	31	41	47	33	31	4747.61	18
23	5007.51	18	86.63	41	47	33	31	41	47	38	31	4879.18	17
24	5874.77	17	363.96	41	47	36	33	41	47	55	30	5007.51	15
25	5438.26	15	215.37	41	47	55	30	41	47	55	33	5219.84	14
26	6316.24	15	436.24	41	47	55	30	48	47	55	33	5438.26	13
27	6316.24	13	877.99	48	47	55	33	54	47	55	33	6089.84	12
28	6645.34	12	555.50	54	47	55	33	54	47	55	41	6332.48	11
29	6928.11	11	595.63	54	47	55	41	54	47	55	48	6661.19	10
30	7561.60	10	633.50	54	47	52	49	54	47	55	53	7453.47	9
31	1.00	0*	--	5	10	20	37	52	53	54	55	9675.10	8

Note: * This is the weight used to identify the optimal solution of neighborhood impact or weighted distance.

TABLE 4-12

NONINFERIOR SOLUTIONS TO NCWDLP, USING SONISE ROUTINE AND THEN WENISE ROUTINE (COV. VS. WT. D.)

Objectives: Coverage and Weighted Distance

Data: SWAIN — No. of Facilities: 4

Coverage Distance: 15 — Impact Distance: 3

Impact Factor: 1

No.	Wc	Wd	Solutions Starting				Solutions Final				Coverage	Weighted Distance
			(Strong Solutions)									
1	0.00	1.000	37	36	22	43	3	41	22	1	607	3283.86
2	0.942	0.058	3	41	22	1	3	25	22	5	620	3383.23
3	0.990	0.010	3	41	22	1	18	20	16	2	627	3607.24
4	0.996	0.004	37	36	22	43	9	36	22	43	632	3850.29
5	0.998	0.002	25	4	43	22	11	22	25	43	635	4317.21
6	1.000	0.000	37	36	22	43	37	36	22	43	638	6347.85

TABLE 4-12 (Continued)

No.	Target Cov.	Target Wt. Dist.	Gradient	Solutions Starting				Solutions Final				Cov.	Wt. Dist.
								(Weak Solutions)					
7	607	3383.23	7.64	3	41	22	1	3	25	22	1	617	3365.34
8	620	3526.18	71.48	37	42	14	15	10	1	22	18	622	3516.90
9	622	3607.24	16.21	18	20	16	2	18	10	16	2	624	3537.15
10	624	3607.24	23.36	37	42	14	15	20	1	22	18	625	3586.66
11	632	4317.21	155.64	22	36	43	37	22	36	1	37	628	3695.56

TABLE 4-13

Weak Solutions to NCWDLP, Using Target Routine (Cov. vs. Wt. D.)

Objectives: Coverage and Weighted Distance

Data: SWAIN　　No. of Facilities: 4

Coverage Distance: 15　　Impact Distance: 3

Impact Factor: 1

	Target			Solutions									
No.	Cov.	Wt. Dist.	Gradient	Starting				Final				Cov.	Wt. Dist.
1	608.63	3296.28	7.64	22	36	1	37	22	41	1	3	607.00	3283.86
2	610.25	3308.70	7.64	22	36	1	37	22	41	1	3	607.00	3283.86
3	611.88	3321.12	7.64	22	36	1	37	22	41	1	3	607.00	3283.86
4	613.50	3333.54	7.64	22	36	1	37	22	25	1	3	617.00	3365.34
5	615.13	3345.96	7.64	22	36	1	37	22	25	1	3	617.00	3365.34
6	616.75	3358.39	7.64	22	36	1	37	22	25	1	3	617.00	3365.34
7	618.38	3370.81	7.64	22	36	1	37	22	25	1	3	617.00	3365.34
8	621.40	3428.03	32.00	22	36	1	37	22	36	2	10	620.00	3502.20
9	622.80	3472.83	32.00	22	36	1	37	22	18	1	10	622.00	3516.90
10	624.20	3517.64	32.00	22	36	1	37	22	36	2	21	623.00	3588.44
11	625.60	3562.44	32.00	22	36	1	37	22	36	2	37	626.00	3625.76
12	628.00	3655.85	48.61	22	36	1	37	22	36	1	37	628.00	3695.56
13	629.00	3704.46	48.61	22	36	1	37	22	36	1	37	628.00	3695.56

TABLE 4-13 (Continued)

	Target			Solutions									
No.	Cov.	Wt. Dist.	Gradient	Starting				Final				Cov.	Wt. Dist.
14	630.00	3753.07	48.61	22	36	1	37	22	36	1	37	628.00	3695.56
15	631.00	3801.68	48.61	22	36	1	37	22	36	1	37	628.00	3695.56
16	632.60	3943.67	155.64	22	36	1	37	22	36	1	37	628.00	3695.56
17	633.20	4037.06	155.64	22	36	1	37	22	36	1	37	628.00	3695.56
18	633.80	4130.44	155.64	22	36	1	37	22	36	1	37	628.00	3695.56
19	634.40	4223.83	155.64	22	36	1	37	22	36	1	37	628.00	3695.56
20	635.25	4486.43	676.88	22	36	1	37	22	18	43	20	633.00	6110.26
21	635.50	4655.65	676.88	22	36	1	37	22	18	43	21	635.00	6171.36
22	635.75	4824.87	676.88	22	36	1	37	22	18	43	21	635.00	6171.36
23	636.00	4994.09	676.88	22	36	1	37	22	18	43	21	635.00	6171.36
24	636.25	5163.31	676.88	22	36	1	37	22	18	43	21	635.00	6171.36
25	636.50	5332.53	676.88	22	36	1	37	22	36	43	37	638.00	6347.85
26	636.75	5501.75	676.88	22	36	1	37	22	36	43	37	638.00	6347.85
27	637.00	5670.97	676.88	22	36	1	37	22	36	43	37	638.00	6347.85
28	637.25	5840.19	676.88	22	36	1	37	22	36	43	37	638.00	6347.85
29	637.50	6009.41	676.88	22	36	1	37	22	36	43	37	638.00	6347.85
30	637.75	6178.63	676.88	22	36	1	37	22	36	43	37	638.00	6347.85

TABLE 4-14

NONINFERIOR SOLUTIONS TO NCWDLP, USING UNISE ROUTINE (COV. VS .WT. D.)

Objectives: Coverage and Weighted Distance

Data: SWAIN   No. of Facilities: 4

Coverage Distance: 15   Impact Distance: 3

Impact Factor: 1

No.	Cov.	Target Wt. Dist.	Gradient	Solutions Starting				Solutions Final				Cov.	Wt. Dist.
1	0	1.00*	--	37	36	22	43	3	41	22	1	607	3283.86
2	607	3419.71	19.41	3	41	22	1	3	25	22	1	617	3365.34
3	617	3455.10	29.92	3	25	22	1	3	25	22	5	620	3383.23
4	620	3526.18	71.48	43	23	47	8	22	10	18	1	622	3516.90
5	607	4154.16	37.84	3	41	22	1	3	25	22	11	623	3542.23
6	623	3671.92	64.84	11	6	54	16	18	6	1	16	624	3586.16
7	624	3671.92	85.76	18	6	1	16	18	20	1	22	625	3586.66
8	625	3695.56	36.30	18	20	1	22	18	20	1	16	627	3622.79
9	623	4154.16	87.42	3	25	22	11	1	36	22	37	628	3695.56
10	607	6347.85	98.84	3	41	22	1	18	20	16	11	630	4154.16
11	628	4154.16	229.30	52	47	54	53	9	36	43	22	632	4232.67
12	632	4383.26	50.20	11	25	17	39	11	25	22	43	635	4317.21
13	1	0.00*	--	37	36	22	43	37	36	22	43	638	6347.85

Note: * This is the weight used to identify the optimal solution of coverage or weighted distance.

TABLE 4-15

NONINFERIOR SOLUTIONS TO PROPERTY COVERAGE VERSUS POPULATION COVERAGE MODEL USING SONISE, WENISE, AND WENISP ROUTINE

Data: Toregas — Population Coverage Distance: 75

No. of Facilities: 3 — Property Coverage Distance: 75

No.	Wprop.	Wpop.	Solutions: Starting			Solutions: Final (Strong Solutions)			Property Coverage	Population Coverage
1	0.000	1.000	5	6	23	5	6	23	171	427
2	0.200	0.800	5	6	23	5	6	26	248	411
3	0.273	0.727	5	6	23	5	10	26	279	400
4	0.294	0.706	5	10	26	5	10	1	284	398
5	0.318	0.682	5	10	26	5	27	1	418	342
6	1.000	0.000	5	6	23	27	5	12	427	331

TABLE 4-15 (Continued)

No.	Target Prop.	Target Pop.	Gradient	Solutions Starting			Solutions Final			Property Coverage	Population Coverage
				(Weak Solutions)							
7	171	411	0.22	5	6	23	5	10	23	202	416
8	248	400	0.35	5	6	26	5	6	1	253	409
9	284	370	2.33	5	10	1	5	10	12	293	387
10	284	367	0.52	5	10	1	5	7	26	296	370
11**	284	367	1.00		--			--		297	392
12*	296	367	0.06	5	7	26	5	7	1	301	368
13*	284	342	0.42	5	10	1	18	7	1	344	367
14*	344	342	0.34	18	7	1	18	7	12	353	356
15*	353	342	0.22	5	27	1	5	27	26	413	344

Note: * Solution identified both by WENISE Routine and WENISP Routine.

** Solution identified just by WENISP Routine.

TABLE 4-16

COMPUTER TIME USED BY DIFFERENT ROUTINES

Data: SWAIN No. of Facilities: 4

Coverage Distance: 15 Impact Distance: 3

Impact Factor: 1

No.	Objectives	Method	No. of times that Target Routine is called	Total time used (sec)	No. of Distinct Noninferior solutions	Time per each Noninferior solution	No. of Optimal solutions
1	Cov. vs Nei.	Optimal (MPSX)	--	82.14	7	10.2 7	7
2	Cov. vs Nei.	SONISE	32	16.95	7	2.42	7
3	Cov. vs Nei.	SONISE & Target	52	27.54	8	3.93	7
4	Cov. vs Nei.	SONISE & WENISE	68	33.90	8	4.84	7
5	Cov. vs Nei.	UNISE	46	25.6	5	5.12	4

VITA

Wing-Sing Tsui was born in Kwang Tung, China on January 9, 1953. Four years later, the Tsui family moved to Hong Kong. Upon completion of his primary and secondary education in Hong Kong, he attended one year in The Cheng Kung University in Taiwan and then joined The University of Tennessee, Knoxville in 1975.

He received a Bachelor of Science degree in Engineering Science in 1978. In the same year, he was admitted to the graduate school of The University of Tennessee, Knoxville and was awarded the degree of Master of Science, with a major in Environmental Engineering in 1980.