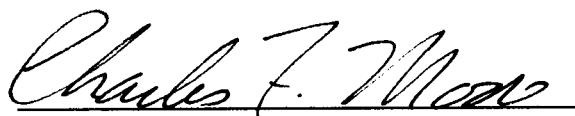


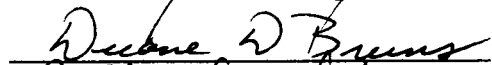
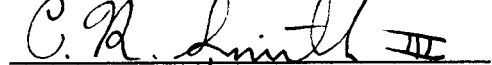
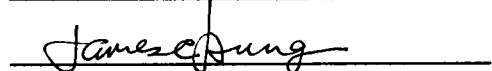
To the Graduate Council:

I am submitting herewith a dissertation written by Joe Mark Keeton entitled "A General-Purpose Multivariable Controller Using a Singular Value Decomposition Structure." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Chemical Engineering.



Charles F. Moore, Major Professor

We have read this dissertation
and recommend its acceptance:

Accepted for the Council:



The Graduate School

A GENERAL-PURPOSE MULTIVARIABLE CONTROLLER USING A
SINGULAR VALUE DECOMPOSITION STRUCTURE

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Joe Mark Keeton

June 1984

ABSTRACT

A relatively new control technique, long used in the numerical analysis field, is singular value decomposition (SVD). Given the process gain matrix, the decomposition can be performed, resulting in the matrices U , Σ , and V . The U and V matrices have special properties which lend to implementation in the SVD controller. Development of this controller is briefly outlined, and studies into the practical problems of application are presented. In order to test the controller on a realistic system, a detailed nonlinear first-principles simulation of a coal-fired power plant is developed. The performance of the SVD controller is compared with that of a conventional multivariable single input-single output controller, and with another new technique, the multivariable tuning regulator. The SVD controller is found to perform quite well and, for the example of the coal-fired power plant, to be a robust algorithm.

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I. INTRODUCTION

Multivariable systems have always presented a unique problem to the control engineer, that of interaction. Much effort has been expended over the years in an attempt to deal with process interaction. Many control techniques are available, none of which is without major difficulties.

This work focuses on interaction and the practical problems it causes in the control of a process. A number of different techniques and strategies are compared and evaluated using a detailed simulation of a fossil fired power plant.

The techniques included in this study cover a broad range of control sophistication, from a simple single input-single output control to the multivariable PI tuning regulator. Also developed and presented in this work are several structured controllers using singular value decomposition orientations. The study is organized and presented as follows:

- Section II is a background of the work in multivariable control. Several well-established multivariable controller design techniques are outlined and discussed. These include multivariable single input-single output control, several types of decoupling schemes, frequency domain design strategies such as inverse Nyquist array and characteristic loci, and time domain techniques such as optimal and modal control.

- Development of the singular value decomposition controller is outlined in Section III. It also describes the special case of a two input, two output system. For this case, the Given's rotation interpretation is used, and the development of equations for rotation angles is outlined.
- Section IV describes an investigation into the controller structure. The possibility of rotating the proportional and integral modes through different angles is examined, and conclusions drawn as to the best controller orientation.
- Various possibilities for tuning the controller are discussed in Section V. Studies are performed on several different process models in order to achieve a consistent set of tuning rules. The results are summarized in a statement of suggested tuning procedures.
- Section VI describes the development of a detailed nonlinear coal-fired power plant model. The application, tuning, and results of three control strategies are reported: conventional multivariable single input-single output, singular value decomposition, and another new technique, the multivariable tuning regulator.
- Conclusions are made regarding the performance of the SVD controller and its practicality. Recommendations are presented for the direction of future work.

II. BACKGROUND

To understand the multivariable control problem, it is helpful to examine single-loop systems and consider the complexities added by multiple loops.

Single-loop feedback control systems are designed to control one "dependent" variable (pressure, temperature, composition, etc.) by manipulating one "independent" variable (typically valve position). The dependent variable is measured and compared to a set point. Any deviation from the setpoint is detected by the controller, which adjusts the independent manipulated variable in the required direction. For most systems a satisfactory degree of control may be attained utilizing the single-loop method. A distillation column can be considered as an example. Very effective control of the distillate composition is often achieved by manipulating either the distillate flow rate or the reflux flowrate. Similarly, the bottoms composition is often satisfactorily controlled by manipulating either the bottoms flow rate or the steam rate to the reboiler. If control of only one of these compositions is desired, good control can be usually obtained using the single-loop method.

If both distillate and bottoms compositions are to be controlled, however, problems are often encountered in terms of interaction between the control loops. A change in the reflux flowrate affects the bottoms composition along with the distillate composition. Similarly, a change in steam rate to the reboiler would affect both bottoms and distillate compositions. Not only must the

individual control loops be considered, but also the interaction between the loops. This interaction can cause the two controllers to "fight" each other when tightly tuned, so that in many systems tight control of two interacting variables is not possible with conventional techniques.

Many industrial examples of multivariable problems are available. In the distillation example, it has been shown that a minimum of energy is consumed by a column when both distillate and bottoms compositions are controlled to their setpoints (1). The classical control of only one product has been preferred to the more complicated dual composition control strategies because energy has been relatively inexpensive. With the increasing cost of energy, and the estimate that distillation consumes 40% of the energy used in a chemical plant, it becomes advantageous to develop a more sophisticated control scheme (2). Other unit operations in a plant exhibit similar characteristics, and the economics of investing in advanced control strategies has become more attractive.

Control Methods

Most of the existing literature on multivariable control can be grouped into several areas. Among them are multivariable single input-single output control (MVSISO), decoupling schemes, inverse Nyquist array, characteristic loci, optimal control, and modal control.

Multivariable Single Input-Single Output

The simplest in terms of implementation (and the conventional approach) is to use a single input-single output strategy which basically ignores the problem of interaction. Measarovic and others (3,4) have shown that in many cases this simple approach is superior to techniques which attempt to decouple the system. Niederlinski (5) considers two systems: an interacting (non-decoupled) and a non-interacting (decoupled) scheme. He derives the disturbance transfer function for each and calculates the deviation ratio, defined as the deviation with control divided by the deviation without control. He concludes that for the kind of interaction present in many unit operations, the non-decoupled scheme should result in superior disturbance attenuation. He uses an example from Rosenbrock (6) to illustrate his point by tuning proportional-integral controllers on a distillation column model using the Ziegler-Nichols method, but does not go further in terms of tuning parameter optimization.

For a non-decoupled scheme the only design question is controller pairing--that is, matching sensor and actuator through a controller in a manner which gives the least interaction. Consider for example Figures 1 and 2. For a 2x2 system either m_1 or m_2 can be manipulated to control c_1 , the other being left to control c_2 . To aid in the pairing decision, Bristol (7) developed the relative gain array. Not only can the relative gains help determine which input variables to manipulate to control particular output variables, but

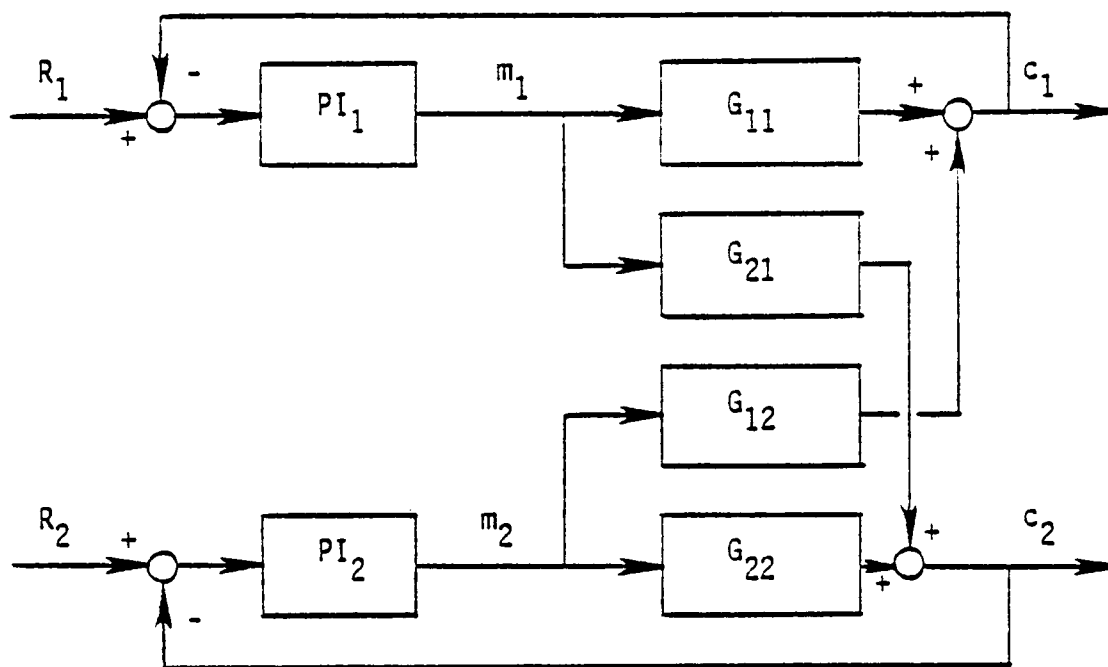


FIGURE 1. "FORWARD" CONTROLLER LOOP PAIRING.

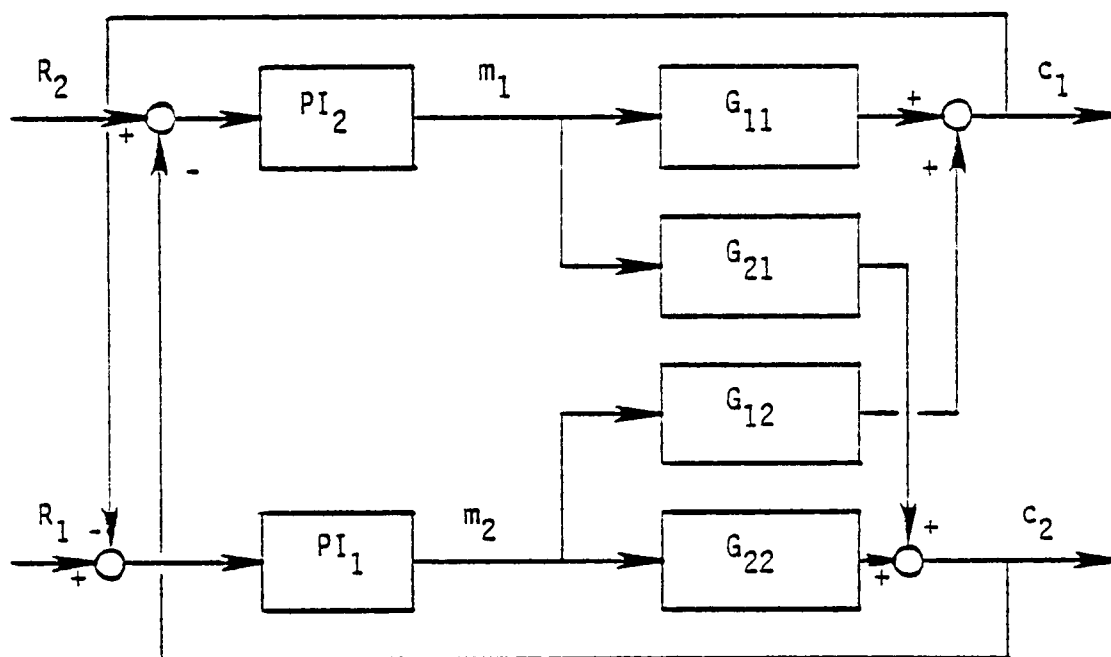


FIGURE 2. "REVERSE" CONTROLLER LOOP PAIRING.

they also give some indication of the degree of success the pairing is likely to exhibit.

A general 2x2 block diagram is shown in Figure 3. Shinskey (8) demonstrated the relative gain array derivation for a 2x2 system. The array consists of the relative gains, λ_{ij} defined by

$$\lambda_{ij} = \frac{(\partial c_i / \partial m_j)_m}{(\partial c_i / \partial m_j)_c} , \quad (\text{Eq. 1})$$

where c_i is the i th controlled variable and m_j is the j th manipulated variable. The subscripts m and c indicate holding all other manipulated and controlled variables constant, respectively. It is easy to determine the numerator of these terms experimentally by simple step tests holding the other manipulated variables constant. The partial derivative in the denominator, however, is relatively difficult to experimentally evaluate. To circumvent this difficulty, Shinskey suggests determining the p -canonical representation of the system, which is done by performing step tests and fitting the constants in Equations 2 and 3:

$$c_1 = K_{11}m_1 + K_{12}m_2 \quad (\text{Eq. 2})$$

$$c_2 = K_{21}m_1 + K_{22}m_2 . \quad (\text{Eq. 3})$$

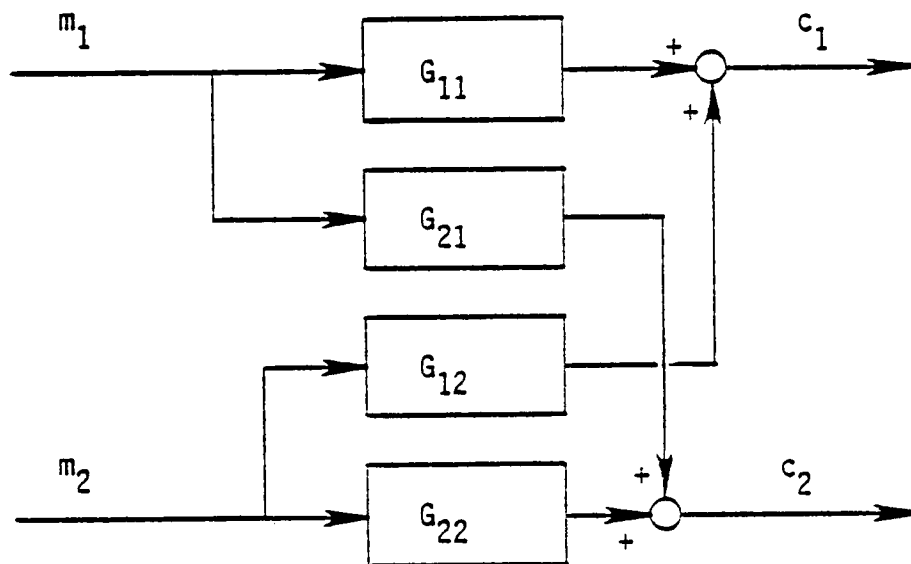


FIGURE 3. GENERAL 2x2 MULTIVARIABLE PROCESS.

The numerator of the relative gain is then given by:

$$\left. \frac{\partial c_1}{\partial m_1} \right|_{m_2} = K_{11} \quad . \quad (\text{Eq. 4})$$

The denominator is found by substituting for m_2 in Equation 3 to yield:

$$c_1 = K_{11}m_1 + K_{12} \left(\frac{c_2 - K_{21}m_1}{K_{22}} \right), \quad (\text{Eq. 5})$$

from which

$$\left. \frac{\partial c_1}{\partial m_1} \right|_{c_2} = K_{11} - \frac{K_{12}K_{21}}{K_{22}} \quad . \quad (\text{Eq. 6})$$

The relative gain is then given by:

$$\lambda_{11} = \frac{1}{1 - K_{12}K_{21}/K_{11}K_{22}} \quad . \quad (\text{Eq. 7})$$

One of the interesting and useful properties of the relative gain array is that the elements of the rows and columns sum to 1.0. In a 2x2 system, once λ_{11} has been found the rest of the array is determined by:

$$\lambda_{22} = \lambda_{11} \quad (\text{Eq. 8})$$

and

$$\lambda_{12} = \lambda_{21} = 1 - \lambda_{11} \quad . \quad (\text{Eq. 9})$$

A matrix manipulation technique is available for systems with more than two inputs and outputs.

Once the array has been calculated, the variables should be paired to give the largest absolute values on the diagonal. The nearness of the off-diagonal elements (in absolute value) to the diagonal elements is a measure of the degree of interaction and thus a measure of the success a pairing is likely to have.

Another method of system analysis is the singular value decomposition presented by Klement and Laub (9) and applied to an azeotropic distillation column by Downs and Moore (10). The singular value decomposition involves decomposing the gain matrix, A , into three matrices:

$$A = U\Sigma V^T, \quad (\text{Eq. 10})$$

where U and V are both unitary matrices and Σ is a diagonal matrix containing the singular values, σ_i . The ratio of $\sigma_{\max}/\sigma_{\min}$ is the condition number, a measure of the difficulty of controlling the multivariable process. U and V are both orthonormal matrices which measure the degree of interaction and can be interpreted using a coordinate rotation.

Decoupling Schemes

The most prevalent non-interacting multivariable control method is p-canonical decoupling (11-18).

The first work done in this area was by Freeman (12,13) and Kavanagh (14,19) who, working independently, published papers outlining synthesis procedures using matrix notation and p-canonical models. Waller (17) gives a fairly general discussion of the types of matrix decouplers discussed here. In the p-canonical scheme the process matrix, G , is multiplied by a decoupling matrix, H , to achieve a system which looks to the individual control loops like a diagonal (non-interacting or decoupled) process. Figure 4 is a general block diagram representation of this system. The resulting diagonal matrix, D , is given by the equation:

$$GH = D . \quad (\text{Eq. 11})$$

The only restriction on D is that the off-diagonal elements be zero. At least three different methods have been proposed to choose diagonal elements. Zalkind (20) suggests forcing the diagonal elements to 1 so that the effective matrix is:

$$GH = D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \quad (\text{Eq. 12})$$

This approach is sometimes called "perfect decoupling." If the modeled process contains lag elements then some of the elements of H will contain pure leads which are physically impossible to implement. Therefore this scheme can be practically used only if the process

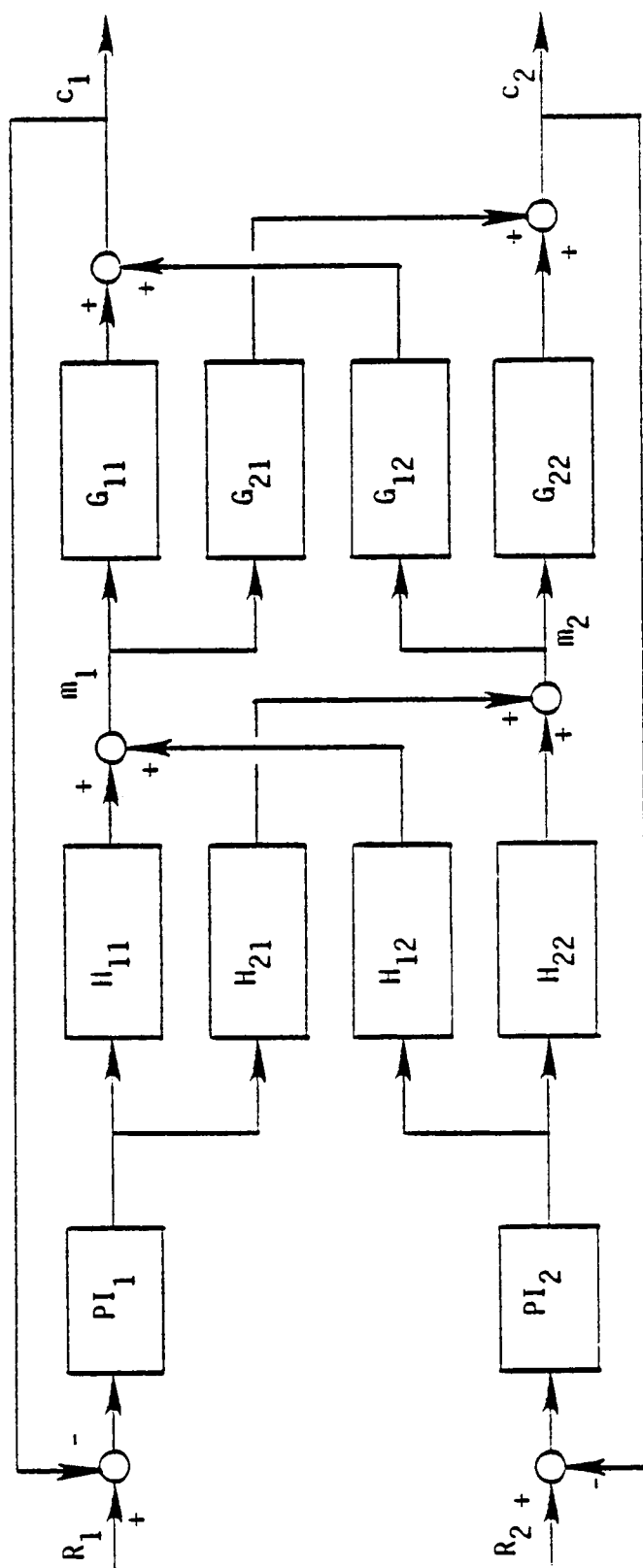


FIGURE 4. P-CANONICAL DECOUPLER WITH CONTROLLERS.

model is steady state. Zalkind makes the comment, "the non-interacting controller network usually carries too much load, leaving the conventional controllers with nothing to do."

Luyben (16) introduces the term "ideal decoupling" to describe the result if the diagonal elements of D are set to their corresponding elements in G, so that the effective matrix is:

$$GH = D = \begin{bmatrix} G_{11} & 0 \\ 0 & G_{22} \end{bmatrix} \quad . \quad (\text{Eq. 13})$$

Waller comments that this leads to rather complicated expressions for the elements of H which may at times be impossible to implement. In general, H is given by:

$$H = \frac{1}{G_{11}G_{22} - G_{12}G_{21}} \begin{bmatrix} G_{22}D_{11} & -G_{12}D_{22} \\ -G_{21}D_{11} & G_{11}D_{22} \end{bmatrix} \quad . \quad (\text{Eq. 14})$$

For the ideal decoupling case, D_{11} would be set to G_{11} and D_{22} would be set to G_{22} .

A more logical choice would be to set some of the elements in H (Waller, Zalkind, Luyben). Luyben introduces the term "simplified decoupling" to describe the resulting system if the diagonal elements of H are set to 1. The system is then completely specified as:

$$GH = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} 1 & -\frac{G_{12}}{G_{11}} \\ -\frac{G_{21}}{G_{22}} & 1 \end{bmatrix}$$

(Eq. 15)

$$= \begin{bmatrix} G_{11} - \frac{G_{12}G_{21}}{G_{22}} & 0 \\ 0 & G_{22} - \frac{G_{12}G_{21}}{G_{11}} \end{bmatrix} .$$

Luyben (16) and Waller (17) point out that the decoupling elements are easier to implement than in the first two schemes.

Waller points out that any two elements of H can be set to unity, so long as they are not in the same column. Thus, if dead time considerations make one of the elements in Equation 14 impossible to implement, any of Equations 16, 17, or 18 could be used:

$$H = \begin{bmatrix} 1 & 1 \\ -\frac{G_{21}}{G_{22}} & -\frac{G_{11}}{G_{12}} \end{bmatrix} \quad \text{(Eq. 16)}$$

$$H = \begin{bmatrix} -\frac{G_{22}}{G_{21}} & -\frac{G_{12}}{G_{11}} \\ 1 & 1 \end{bmatrix} \quad \text{(Eq. 17)}$$

$$H = \begin{bmatrix} -\frac{G_{22}}{G_{21}} & 1 \\ 1 & -\frac{G_{11}}{G_{12}} \end{bmatrix} . \quad \text{(Eq. 18)}$$

Shinskey (8) points out that there are two problems with the p-canonical decoupling scheme: initialization and constrained operation. Ideally, initialization of the two decoupler outputs to begin automatic control should occur without "bumping" the system. In reality, however, the initial values are not known. Automatic control must begin and an error must be detected. The error must then be compensated for through controller action. The second problem, constrained operation, is more serious. If one of the manipulated variables is constrained at some value, both output variables cannot be controlled. Using the p-canonical decoupling strategy, the controller will attempt to maintain both variables at their set points by manipulating the unconstrained variable. Since this is impossible, the variable will eventually become saturated. Shinskey suggests that these problems can be avoided by utilizing the v-canonical decoupler shown in Figure 5. In this system the two decoupling terms have the effect of canceling the interaction terms of the process matrix. The main problem with the v-canonical decoupling scheme is the presence of the feedback loop formed by the two decouplers which can be unstable. The stability question arises from the total loop gain and total loop dynamics. A strong possibility exists that the resulting loop will be positive, causing almost certain instability. The condition occurs when there is an even number of sign changes in the process gain elements.

If instabilities exist due to feedback problems, a solution is to eliminate one of the decoupling elements, leaving a partially decoupled system (Figure 6). The problem then would be deciding which

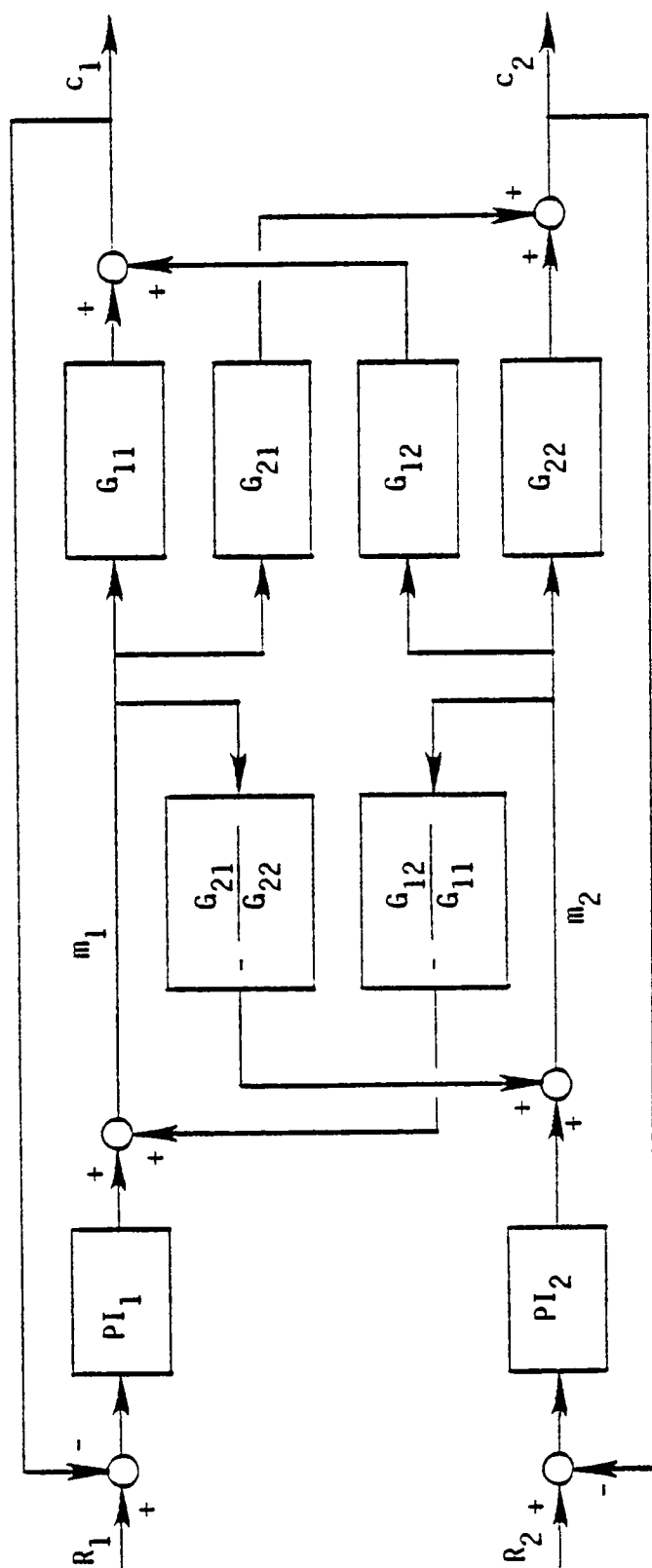


FIGURE 5. V-CANONICAL DECOUPLER WITH CONTROLLERS.

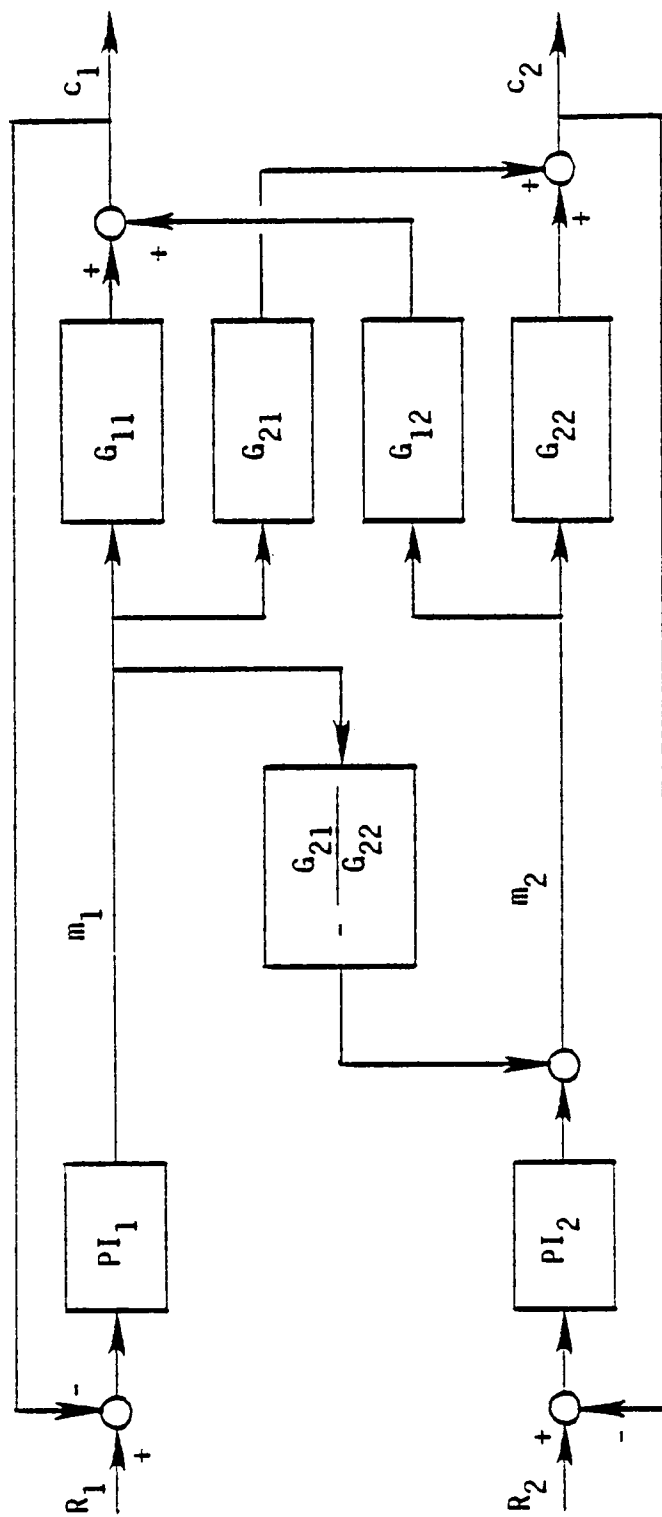


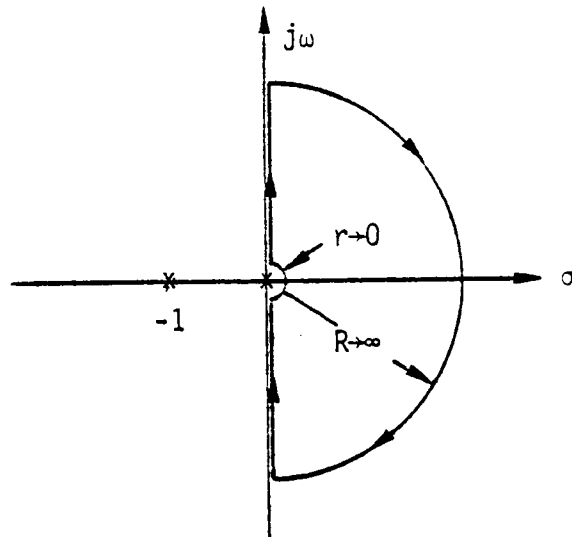
FIGURE 6. PARTIAL DECOUPLER WITH CONTROLLERS.

loop to decouple. The choice can be made on the basis of several considerations, such as choosing the more important controlled variable, or the relative speed of response of the system. If one loop is fast-responding it will not likely be affected by changes in the slow-responding loop, whereas the slow loop might be disturbed by changes in the fast loop. This is similar to classical feedforward control.

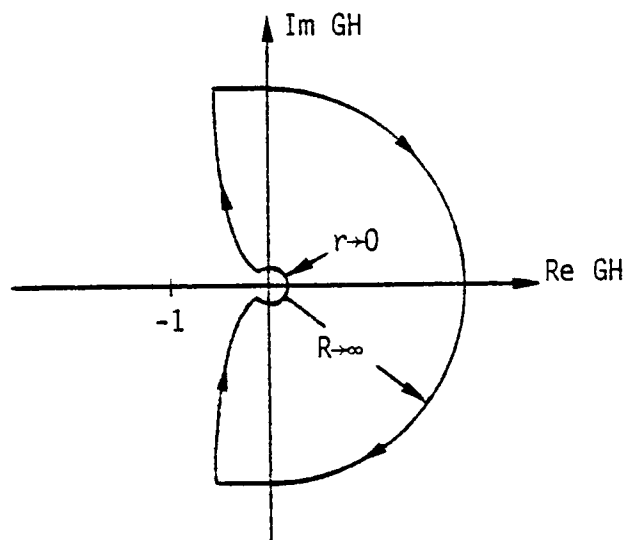
Inverse Nyquist Array

Nyquist analysis (21-25) is a graphical frequency response method for determining absolute and relative stability of closed-loop control systems. It is superior to such techniques as Routh-Hurwitz in several ways. Routh-Hurwitz and others generally provide a method for determining only absolute stability. Without an indicator of the relative stability the technique is difficult or impossible to use for controller design. These other methods can only be applied to systems whose characteristic equation is a finite polynomial in s . This means that they cannot treat dead time ($e^{-\theta s}$) exactly.

Nyquist analysis for a single-variable system is well developed. A "Nyquist Path" is drawn in the s -plane which completely encloses the right half of the plane, avoiding any poles and zeros of $G(s)H(s)$. (In this context $G(s)$ is the process transfer function and $H(s)$ the feedback transfer function.) This insures that every pole and zero of $GH(s)$ in the right half plane is enclosed by the Nyquist Path. An example is shown in Figure 7a for $GH(s) = \frac{1}{s(s+1)}$.



(a) Nyquist Path (s-plane).



(b) Nyquist Plot (GH-plane).

FIGURE 7. NYQUIST ANALYSIS FOR $GH(s) = \frac{1}{s(s+1)}$.

Each leg of the Nyquist Path is then mapped into the GH plane, shown in Figure 7b for this example. The Nyquist Stability Theorem in effect says that the closed-loop system, $\frac{G}{1+GH}$, is stable if the Nyquist plot encircles the $(-1, 0j)$ point the same number of times (in the same direction) as it encircles the origin. If the system is unstable, or only marginally stable, many techniques and rules of thumb are reported for modification of $H(s)$ to produce a desirable Nyquist Plot.

Application of the Nyquist Stability Theorem to multivariable systems is less developed. In the standard feedback configuration of Figure 8, $G(s)$ is the matrix of process transfer functions, $H(s)$ the feedback matrix (usually the identity matrix of appropriate size), and $K(s)$ the controller matrix to be designed. The vectors $\underline{r}(s)$, $\underline{e}(s)$, $\underline{u}(s)$, and $\underline{y}(s)$ are setpoints, errors, manipulated variables, and controlled variables, respectively. The inverse Nyquist array technique (INA) was introduced and developed by Rosenbrock (26, 27, 28) and discussed by others (29-36). Rosenbrock defines a matrix $Q(s)=G(s)K(s)$, then takes the inverse $\hat{Q}(s)=Q(s)^{-1}$. The inverse Nyquist array is the set of Nyquist diagrams of $\hat{q}_{ij}$, the elements of $\hat{Q}(s)$. Rosenbrock's technique involves selecting $K(s)$ so that $\hat{Q}(s)$ is diagonally dominant. A set of approximate single-loop control problems, corresponding to the diagonal elements of the INA, is solved. Rosenbrock suggests three criteria for stability. The first is that the diagonal diagrams of the INA should each encircle the $(-1, 0j)$ point and the origin the same number of times (the classical

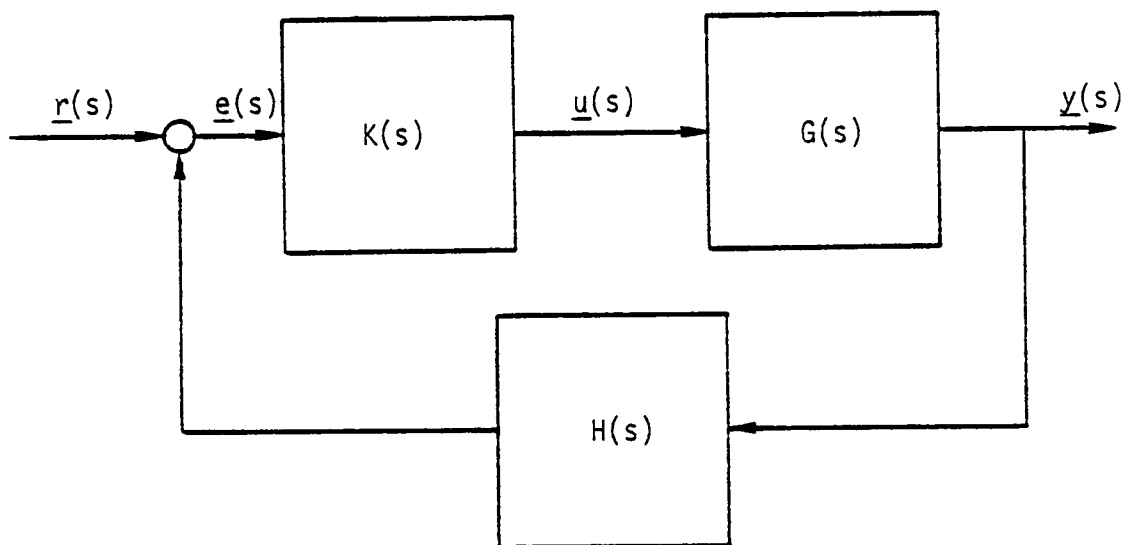


FIGURE 8. STANDARD MULTIVARIABLE FEEDBACK CONFIGURATION.

Nyquist criteria). The second and third requirements refer to the principle of diagonal dominance. They require that, for all s , each diagonal element of $\hat{Q}(s)$ must have a larger magnitude than the sum of the off-diagonal elements in the same row. This insures that the interactive effects are small.

To make $\hat{Q}(s)$ diagonally dominant, Rosenbrock outlines a procedure for selecting $K(s)$. He proves that any $K(s)$ can be generated by the multiplication of three matrices, $K_a K_b(s) K_c(s)$. K_a , which is a constant matrix, is usually taken to be $G(0)^{-1}$ so the system $G(s)K_a$ is diagonal at steady state. $K_b(s)$ is chosen to meet the second and third stability criteria. Finally the elements of $K_c(s)$, which is diagonal, are chosen using conventional single-variable Nyquist design techniques.

Practical experiments have been conducted that show the INA design method is tolerant of large model inaccuracies and system nonlinearities, and that on-line tuning is not difficult (32).

Characteristic Locus

The Characteristic Locus (CL) is a technique which has received significant attention (36-45), though it is less developed than the inverse Nyquist array. It has been suggested that the Characteristic Locus is more powerful and general than the Inverse Nyquist Array for control system analysis, but that it does not provide as much insight and guidance for controller design (36).

Much of the development work leading to the CL method was done by MacFarlane and Belletrutti (37,41). It is similar to the INA

technique and, as with Rosenbrock's method, the controller is formed by the cascading of several transfer function matrices. The design procedure typically involves four phases: stability, integrity, interaction, and performance. In each phase basic controllers, $K_i(s)$, are designed iteratively to achieve part of the design objective for the closed-loop system. The final controller is given by $K(s) = \prod K_i(s)$, $i=1,2,\dots$. Specific forms for $K_i(s)$ have been suggested to achieve certain desired effects, such as elementary transformation and eigenvalue adjustment (41).

MacFarlane (40) comments that the iterative nature of the controller calculations is well suited to an interactive graphics design method.

Optimal Control

Substantial work has been reported on optimal control, which is a natural outgrowth of the state space representation of linear dynamic systems (39,40,46-62). The state space, or time domain, description differs from the classical frequency domain techniques in that the former is closer to physical reality and is useful for providing statistical descriptions of system behavior. System dynamics are represented by the first-order vector/matrix differential equation

$$\begin{aligned}\dot{\underline{x}} &= \underline{A}\underline{x} + \underline{B}\underline{u} \\ \underline{y} &= \underline{C}\underline{x} \quad ,\end{aligned}\tag{Eq. 19}$$

where $\underline{x}$ represents the state vector of dimension $n \times 1$, $\underline{u}$ the vector of manipulated variables (a scalar in the single input-single output case), and $\underline{y}$ the vector of controlled variables (also a scalar in the single input-single output case). In the single-variable case, an n th order differential equation is described by n states and n first order differential equations. In the multivariable case, several differential equations are described as the combination of first order equations. The state variables may correspond to actual physical measurements and/or time derivatives of these measurements.

Analogous to the continuous-time representation of Equation 19, a discrete-time formulation can also be developed:

$$\begin{aligned}\underline{x}_{i+1} &= A\underline{x}_i + B\underline{u}_i \\ \underline{y}_{i+1} &= C\underline{x}_{i+1}\end{aligned}\quad (\text{Eq. 20})$$

Much of the work done in the continuous domain has a direct analog in the discrete domain (57).

Optimal control seeks to optimize a scalar objective function, which may be any combination of state and control variables. In general an analytical solution cannot be found. However, a specific case which is sufficiently flexible for many applications does lend itself to analytical solution. This is a system whose state equation is of the form of Equation 19, with a quadratic objective function given by:

$$J = \int_0^{t_f} [\underline{x}^T Q \underline{x} + \underline{u}^T R \underline{u}] dt, \quad (\text{Eq. 21})$$

where t_f is the final time. By changing the matrix elements in the objective function (Q and R) the nature of the control can be changed. A minimum energy controller, minimum time controller, tracking controller, or a controller having a combination of these and other characteristics can be created.

The optimal control law described by Equation 21 is known as the "linear-quadratic problem" (LQP) (56, 58, 62). The LQP is the only general optimal control problem which has a closed-form analytical solution (39). Much work is going on in other areas of optimal control, and the LQP is only a special example which provides a flavor of the "optimum" concept.

Athans and Falb (48) demonstrate the derivation for a servo problem (no disturbance). The controller is of the form:

$$\underline{u} = -K \underline{x}. \quad (\text{Eq. 22})$$

Upon substitution into Equation 21, the cost function, J, is minimized when:

$$K = R^{-1} B^T P, \quad (\text{Eq. 23})$$

where P is the symmetric matrix solution to the Riccati equation:

$$P + PA + A^T P - PBR^{-1}B^T P + Q = 0. \quad (\text{Eq. 24})$$

In Equations 23 and 24, R and Q are as defined in the objective function, and A and B are the matrices comprising the state space representation of Equation 19.

Typical of the considerations when choosing R and Q is the discussion by Edgar (39). Depending on the circumstances, the cost of control can be viewed in different ways. If maintaining setpoints is more important than all other considerations there is essentially no cost of control. In this case, R in the objective function would be set to zero, and the feedback matrix of equation 22 would blow up. (This is intuitively apparent, because if control action is free, why not call for an infinite amount?) The solution to this problem is typically to set R at smaller and smaller values until a constraint is violated. (This is an example of the frequently trial-and-error nature of choosing R and Q .)

On the other hand, when there is a cost of control, the problem becomes that positive and negative action generally do not cost the same. For instance, increasing steam flowrate is a positive cost and should be penalized, whereas decreasing steam flow is a benefit (negative cost) and should be encouraged. One solution is to formulate the objective function so the control effort is normalized about a true zero-cost point, rather than the steady state. In the case of Equation 25:

$$J = \int_0^{t_f} [\underline{x}^T Q \underline{x} + (\underline{u} - \underline{u}_0)^T R (\underline{u} - \underline{u}_0)] dt, \quad (\text{Eq. 25})$$

where $\underline{u}_0$ is zero-cost control, the optimal feedback is given by Equation 26, where $\underline{v}$ is the open-loop part:

$$\underline{u} = -K\underline{x} + \underline{v}. \quad (\text{Eq. 26})$$

All feedback systems designed in the LQP formation are asymptotically stable. This, together with its "optimal" nature, is one of the main advantages of the technique. One of the disadvantages is that all the state variables must either be measured or estimated. The estimation step can consume much computing time, depending on the sophistication of the estimator. Another disadvantage is that no satisfactory method of on-line tuning is available. This can be a substantial problem due to the inadequacies of computer models in control system design.

Modal Control

Modal control (sometimes referred to as eigenvalue placement or pole placement) was introduced by Rosenbrock (63) in 1962. It has since been discussed at length by many authors (i.e. 63-75). The goal of this technique is to use state feedback to shift the eigenvalues of the system to the desired locations. If the open-loop response is unstable or only marginally stable it would be useful to shift the eigenvalues further into the left half complex plane. Conversely, if

the open-loop system is too sluggish it would be necessary to shift the engenvalues.

For the open-loop state Equation 19 and the feedback control of Equation 22, the closed-loop state equation becomes:

$$\dot{\underline{x}} = A\underline{x} - BK\underline{x} \quad (\text{Eq. 27})$$

or

$$\dot{\underline{x}} = (A - BK)\underline{x} \quad (\text{Eq. 28})$$

Closing the loop shifts the eigenvalues from those of A to those of (A-BK). It has been shown that for a completely observable and controllable system each of the n eigenvalues may be arbitrarily placed (73).

An analytical technique has been developed for calculation of the feedback matrix, K, which involves the assumption that K can be represented as the dyadic product of two vectors, $K=pq^T$. Davison and Chatterjee describe the generation of p and q to shift each pole to its desired location (67). The result is that the loop gains are directly proportional to the shifts in the eigenvalues. Edgar mentions some unpublished work implementing modal control on several chemical system models, which was successful and reliable (39).

Advantages of the pole placement controller design technique include a simple, fast calculation and considerable freedom in allocating loop gains. This is due to the wasted degrees of freedom (also considered by some to be a major drawback). For an

n-dimensional system, K contains n^2 elements; yet only n elements are required to specify n poles. Another drawback is that the process leads to a simple proportional controller, with no guidance as to design of dynamic compensating elements. In addition, no correlation exists between the pole locations and the transient response because of the significant effect of transfer function zeros. This also makes the controller difficult to tune on-line.

Summary

Several available multivariable control strategies have been described. They range from simple to complex, from the point of view of both conceptualization and implementation.

Perhaps the greatest advantage of multivariable single input-single output control is its simplicity. This also contributes to ease of on-line tuning. The disadvantage is that this strategy does nothing about the interaction and gives the poorest performance. The decoupling schemes are also conceptually simple, but can be complex in implementation. Inverse Nyquist array design is tolerant of model inaccuracies and nonlinearities, and can be tuned on-line. It is, however, an iterative and interactive design technique which requires some experience for effective use. Optimal control is a true synthesis method and, in the LQP case, an explicit solution is available. The cost function can be related to an economic specification. All the state variables must be either measured or estimated, a great deal of computing time per timestep is

not uncommon, and no method is currently available for on-line tuning. Modal control enables the designer to exactly place the system poles, and considerable freedom is available in doing so. No control of the zeros is available, however, and adding dynamic compensation is difficult.

There are several other multivariable control system design techniques, each with their advantages and disadvantages. These include incomplete state feedback, Lyapunov functions, and adaptive control. The purpose of the next section is to develop the mathematical background for a relatively new multivariable design technique, the singular value decomposition (SVD).

III. SINGULAR VALUE DECOMPOSITION CONTROLLER

The singular value decomposition (SVD) is a very powerful numerical analysis tool which has recently found application in the process control area. Many standard linear algebra textbooks (i.e. 76, 77, 78) contain good discussions of SVD from the mathematician's viewpoint. The work of Moore (79), however, gives the concepts a physical interpretation which is necessary for control application. Smith (80) incorporates these ideas and proposes a controller structure called the singular value decomposition controller. Smith and Keeton (81) have both done studies which show the concept is promising, performing substantially better than multivariable single input-single output control.

SVD Controller Structure

The general multivariable control problem is to find the vector of manipulated variables, $\underline{x}$, which gives the desired vector of outputs, $\underline{b}$ in the steady state input/output relation:

$$M\underline{x} = \underline{b} , \quad (\text{Eq. 29})$$

where M is the steady-state gain matrix. In some instances the inverse of the gain matrix M may be multiplied by the vector of desired outputs to give the required inputs. Not all systems, however, have steady state gain matrices which are square and nonsingular. For many nonsquare process gain matrices a vector of

inputs may be calculated using the pseudo-inverse:

$$\underline{x} = (M^T M)^{-1} M^T \underline{b} . \quad (\text{Eq. 30})$$

In general the pseudo-inverse is said to find the minimum norm least squares solution to Equation 29. That is, the solution will give the "smallest" vector $\underline{x}$ which yields an output vector "closest" to the desired $\underline{b}$. One requirement for use of the pseudo-inverse is that, for an $n \times m$ process gain matrix M , the rank of M must equal the minimum of n and m . For example, if there are less inputs than outputs the matrix must have full column rank, a condition not always satisfied. Another problem with the pseudo-inverse is that it is not always a robust algorithm (82). A more robust technique for solving Equation 29 exists in the form of the singular value decomposition (SVD). Numerically reliable methods exist for calculating the SVD of a matrix; the system does not require a square matrix or full rank.

Any matrix (square, rectangular, full rank, or less than full rank) may be decomposed into three matrices:

$$M = U \Sigma V^T . \quad (\text{Eq. 31})$$

For M an $n \times m$ process gain matrix, Σ will also be $n \times m$, and will be diagonal. There will be r diagonal elements, where r is the rank of M , and these elements will be the singular values. U will be a $n \times n$ orthogonal matrix, the column vectors of which describe the principle

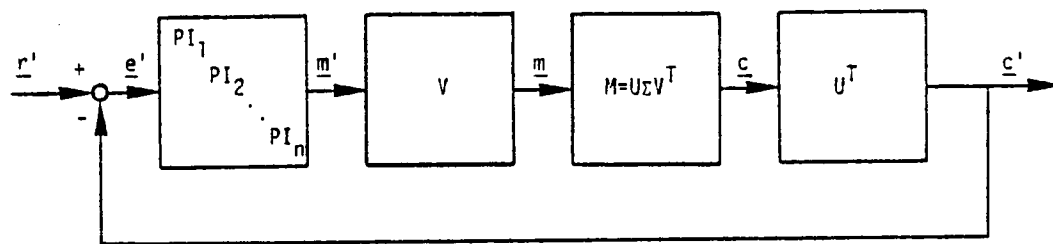
and degenerate axes of the Image of M . V is a $m \times m$ orthogonal matrix whose column vectors describe the principle and degenerate axes of the complement to the Kernel of M . Because Σ is diagonal its "inverse" is found by transposing it and inverting each of the non-zero diagonal elements. Similarly the orthogonal nature of the U and V matrices permits inverting by simply transposing the rows and columns, suggesting another method of solution for Equation 29:

$$\underline{x} = V\Sigma^{-1}U^T \underline{b} \quad . \quad (\text{Eq. 32})$$

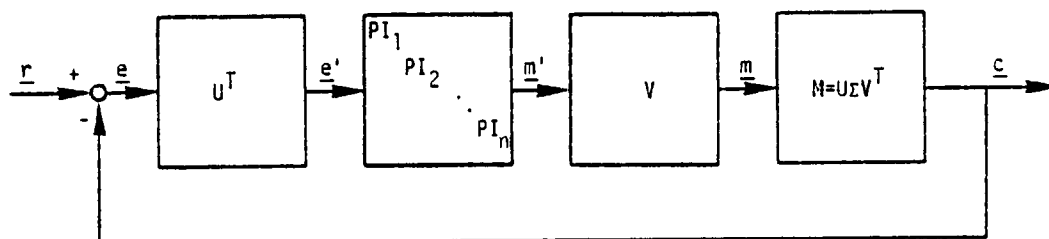
As previously discussed this technique is robust, and may be used for any matrix, regardless of its complexities. This technique also gives the minimum norm least squares solution.

In the form of Equation 32, the SVD is a feedforward configuration which does not take advantage of the disturbance rejection/model inaccuracy tolerance of feedback control. The ease of inverting the U and V matrices, however, suggests the configuration shown in Figure 9a. The $\underline{m}'$ and $\underline{e}'$ were named "structured variables" by Smith (80). Unfortunately the setpoints and output variables in this case are in terms of structured variables, which have no intuitive physical interpretation (other than a weighted average error of each controller). This problem may be solved using block diagram manipulation, resulting in the form of Figure 9b.

The columns of U are an orthogonal basis for the Image of M . Also the columns of V are an orthogonal basis for the complement of the Kernel of M . The effect of the SVD controller in Figure 9b is as



(a) Before Block Diagram Manipulation.



(b) After Block Diagram Manipulation.

FIGURE 9. SINGULAR VALUE DECOMPOSITION CONTROLLER.

follows. First the error signals are projected onto the orthogonal basis for the Image of M (multiplied by $U^{-1} = U^T$). In this domain the signals are independent and a standard single-loop control algorithm is applied, which generates the structured manipulated variables. These signals are projected from their basis in the complement of the Kernel of M back to the normal basis (multiplied by $(V^T)^{-1} = V$). In the steady state this results in the minimum norm least squares solution.

This controller structure has application for any $n \times m$ process gain matrix. The special case of a square 2×2 system, however, permits a simplification which has significant potential for on-line tuning.

2 x 2 Process and the Given's Rotation

The conventional singular value decomposition, as well as the existing computer codes for its calculation, results in a Σ with all positive singular values, and some order the σ_i 's in decreasing magnitude along the diagonal. This is useful for system analysis, but less so for controller design. If the sign restriction is relaxed, for a 2×2 system the SVD can be represented using a Given's rotation. In this form the singular values are not guaranteed to be positive, but the U and V can be represented as in Equation 33:

$$U = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$V = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}. \quad (\text{Eq. 33})$$

Visualization shows the matrices in Equation 33 have the effect of rotating a vector through an angle of β or θ , respectively. The rotation is effected without changing its length, which is a characteristic of all orthogonal matrices. U may be completely specified by only one parameter, β . The same is true of V , specified by one parameter, θ . This enables the U and V matrices to be "tuned" using only one parameter each.

The concept of vector rotation can be applied in discussion of the controller action in the 2×2 case. In Figure 10, the controlled variables are first subtracted from their setpoints to produce a vector of error signals. The vector is multiplied by U^T (rotated through an angle of $-\theta$) to produce the projection onto the Image of M . These structured error signals are independent in the steady-state. The independent nature enables standard single-loop control to be effectively employed, producing structured manipulated variables. This vector is finally multiplied by V (rotated through an angle of β) which has the effect of changing the basis from the complement of the Kernel of M to the normal basis. The resulting vector of manipulated variables will, in the steady state, be the minimum norm least squares solution. It is also desired to minimize interaction during the transient response.

Computer algorithms available for SVD computation can handle matrices with dimensions in the hundreds. This capacity is not

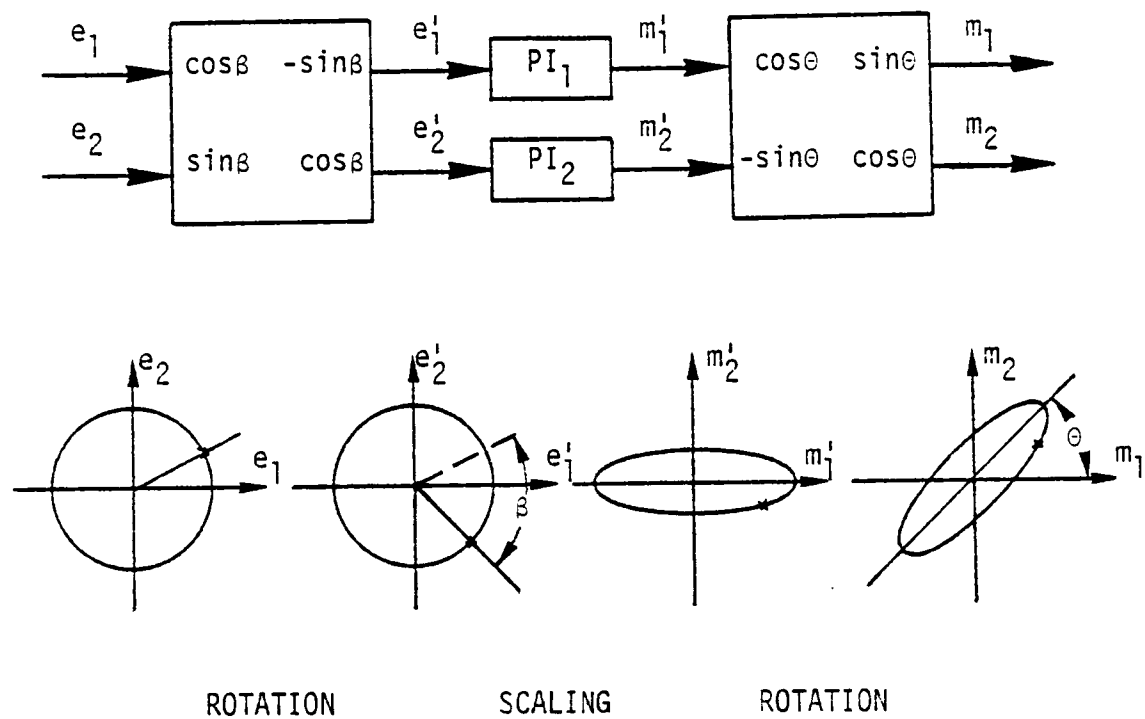


FIGURE 10. GEOMETRICAL INTERPRETATION OF 2x2 SVD CONTROLLER.

required for 2×2 systems, and an analytical expression for the computation would be desirable.

The columns of V form an orthogonal basis for the Image of M . A unit ball of inputs is mapped by M into an ellipse, as see in Figure 11. The columns of V correspond to the major and minor axes of this ellipse. The minor axis is 90° off from the major axis, which is reflected in the Given's rotation form of Equation 33. If the first column,

$$\underline{v}_1 = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \quad (\text{Eq. 34})$$

is assigned to the major axis, the second column,

$$\underline{v}_2 = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \quad (\text{Eq. 35})$$

will correspond to the minor axis. Next the θ which maximizes the output $\underline{b}$ in Equation 36 must be found:.

$$M\underline{v}_1 = \underline{b} . \quad (\text{Eq. 36})$$

The "maximum" will be defined in terms of the 2-norm, $\zeta = \sqrt{b_1^2 + b_2^2}$.

Substituting parameters into Equation 36 for M and $\underline{v}_1$:

$$\begin{aligned} \zeta^2 = & (m_{11}^2 + m_{21}^2)\cos^2\theta - 2(m_{11} m_{12} + m_{21} m_{22})\cos\theta \sin\theta \\ & + (m_{12}^2 + m_{22}^2)\sin^2\theta . \end{aligned} \quad (\text{Eq. 37})$$

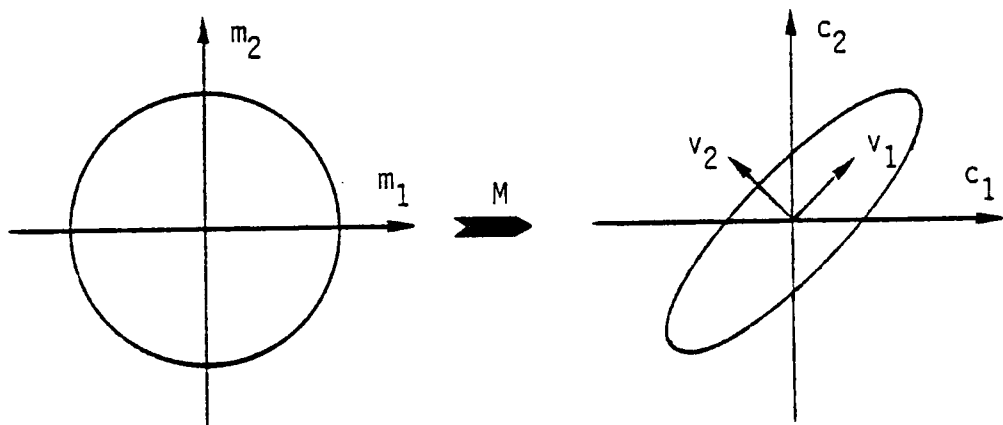
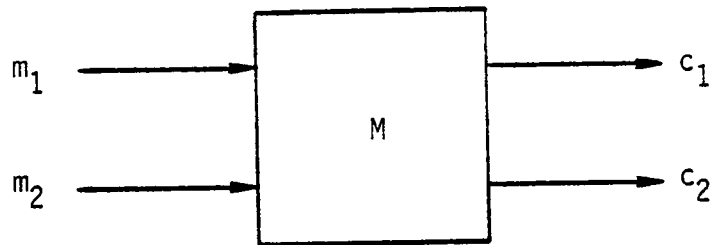


FIGURE 11. MAPPING OF UNIT BALL BY PROCESS GAIN MATRIX, M .

Taking the derivative:

$$\begin{aligned} \frac{d(\xi^2)}{d\theta} &= 2(m_{12}^2 + m_{22}^2 - m_{11}^2 - m_{21}^2)\cos\theta \sin\theta \\ &\quad - 2(m_{11} m_{12} + m_{21} m_{22})(\cos^2\theta - \sin^2\theta) . \end{aligned} \quad (\text{Eq. 38})$$

Setting the derivative to zero and solving for θ , using trigonometric identities:

$$\theta = \frac{1}{2} \tan^{-1} \left[\frac{2(m_{11} m_{12} + m_{21} m_{22})}{m_{12}^2 + m_{22}^2 - m_{11}^2 - m_{21}^2} \right] . \quad (\text{Eq. 39})$$

But this only guarantees that $d(\xi^2)/d\theta = 0$. To insure it is a maximum the second derivative must be taken:

$$\begin{aligned} \frac{d(\xi^2)}{d\theta^2} &= 2(m_{12}^2 + m_{22}^2 - m_{11}^2 - m_{21}^2)(\cos^2\theta - \sin^2\theta) \\ &\quad + 8(m_{11} m_{12} + m_{21} m_{22})\cos\theta \sin\theta . \end{aligned} \quad (\text{Eq. 40})$$

For the θ to produce a maximum ξ (and thus ξ^2) the second derivative must be negative. Again using trigonometric identities, the resulting inequality is:

$$(m_{12}^2 + m_{22}^2 - m_{11}^2 - m_{21}^2)\cos 2\theta + 2(m_{11} m_{12} + m_{21} m_{22})\sin 2\theta < 0 . \quad (\text{Eq. 41})$$

If this inequality does not hold (the left-hand side is positive), the calculated θ corresponds to a minimum (the minor axis of the Image). The angle corresponding to the maximum is found by simply adding 90° .

The relation that the orthogonal complement to the Kernal of M is equal to the Image of M^T is employed in development of equations for β (U). The same development is applied as for θ (V), except the transpose of M is used. Thus the only difference is m_{12} and m_{21} are reversed in Equations 39 and 41, resulting in:

$$\beta = \frac{1}{2} \tan^{-1} \left[\frac{2(m_{11} m_{21} + m_{12} m_{22})}{m_{21}^2 + m_{22}^2 - m_{11}^2 - m_{12}^2} \right] \quad (\text{Eq. 42})$$

(Eq. 43)

$$(m_{21}^2 + m_{22}^2 - m_{11}^2 - m_{12}^2) \cos 2\beta + 2(m_{11} m_{21} + m_{12} m_{22}) \sin 2\beta < 0 .$$

To find the singular values σ_1 and σ_2 which make up the diagonal of Σ , Equation 31 is manipulated to give:

$$\Sigma = U^T M V. \quad (\text{Eq. 44})$$

When the elements of M and the expressions for U and V are substituted into Equation 44, the resulting expressions for the singular values are:

$$\sigma_1 = \frac{m_{11}\cos\theta - m_{12}\sin\theta}{\cos\beta} \quad (\text{Eq. 45})$$

$$\sigma_2 = \frac{m_{21}\sin\theta + m_{22}\cos\theta}{\cos\beta} \quad (\text{Eq. 46})$$

Thus, an algorithm simple enough to be stored on a programmable calculator is available for decomposing a 2 x 2 system. Given the gain matrix, θ is calculated using Equation 39. This value is checked by testing the inequality of Equation 41. If this fails, 90° is added to the value of θ . These steps are repeated for β using Equations 42 and 43. Finally Equations 45 and 46 are used to calculate the singular values.

These analytical expressions provide a simplification which is beneficial in the application of a 2 x 2 SVD controller. Without them an extensive computer algorithm would be required, making calculation in the field very cumbersome.

Previous Applications

Smith (80) performed studies applying the SVD controller to several process models, including the column A and column C models of McAvoy (18) and the transfer function model of Wood and Berry (83). He decomposed the steady-state transfer function matrices to give U and V, then performed extensive studies into tuning the proportional and integral modes. During most of his work the proportional gains and integral reset times were optimized using a computer search routine (84). Smith showed that using a SVD controller with optimized gains

and reset times can result in performance which is improved over standard multivariable single input-single output (MVSISO) control.

Keeton (81) also compared the SVD controller with standard MVSISO control, and with several other multivariable control techniques. His studies were done on a detailed nonlinear simulation of a 20 tray n-octane/n-pentane binary distillation column. He compared MVSISO control, p-canonical decoupling, v-canonical decoupling, partial decoupling, SVD control, and other techniques. He concluded the SVD controller performed better than most other strategies, and there was potential for application once further research and development had been conducted.

During the work of both investigators only the steady state gain matrix was decomposed. For the most part, all tuning studies were performed using a PATTERN trial and error search for the optimum.

Summary

A relatively new structure, the singular value decomposition controller, has been introduced. The concept is based on a powerful numerical analysis tool, which has only recently found application in the process control field. At present the decomposition cannot be performed on a process transfer function matrix. Rather a process gain matrix is decomposed, resulting in the U and V matrices necessary for controller implementation. If the steady state gain matrix is the one decomposed, the steady state control action is the minimum norm least squares solution.

The controller was introduced, and the special case of a 2×2 system explored. In the 2×2 case a Given's rotation interpretation can be used for the singular value decomposition. A set of equations was developed for the β and θ angles corresponding to the U and V matrices, respectively.

The previous studies into application by Smith (80) and Keeton (81) were mentioned. They found the controller performed well, and emphasized that much work remained before implementation could be attempted.

The motivation for the work reported here was a desire to bring together all the ideas on SVD controller design, refine them, and put them into a workable form. It was also desired to make a comparison between the SVD controller and other advanced multivariable controller design techniques. The following sections describe this development.

IV. SVD ANGLE TUNING OPTIMIZATION

Section III described the SVD controller, and discussed the special case of a 2×2 system. The Given's rotation provides a useful simplification in that the U and V matrices may be "tuned" by adjusting the β and θ angles, respectively. This provides a total of six well focused tuning parameters. The angles β and θ deal with the interaction and the four PI parameters (gain and integral reset time for each loop) deal with the process dynamics. This definition is not available for higher order systems. For example a multivariable single input-single output PI controller for a 3×3 system has six elements which require tuning. To specify each of the U and V matrices for the SVD controller requires four parameters. The number of parameters which must be tuned increases from 6 to 14, effectively prohibiting on-line optimization. The 2×2 case is small enough to be tuned on-line, and the procedure is discussed in Chapter V. This chapter explores the different possibilities for starting values. The work described was done using the UTCC DEC-10 computer for process simulation, control, and optimization.

Simulation

Process models were simulated using z-transform-derived difference equations. Five processes were examined in this phase of the work: the three distillation column models of Weischedel and McAvoy (18), the column model of Wood and Berry (83), and a process which includes elements which sag (exhibit an initial response in the opposite direction from its final value). The transfer function

matrix of McAvoy's column A is given in Equation 47:

$$G(s) = \begin{bmatrix} \frac{0.58e^{-1s}}{(1+6.3s)(1+1.96s)} & \frac{-0.45e^{-1s}}{(1+5.68s)(1+3.0s)} \\ \frac{0.35e^{-1.3s}}{(1+5.0s)(1+0.67s)} & \frac{-0.48e^{-1s}}{(1+4.7s)(1+0.36s)} \end{bmatrix}. \quad (\text{Eq. 47})$$

The SVD angles corresponding to the steady state gain matrix for this process are $\beta=-38.7^\circ$ and $\theta=44.1^\circ$. The transfer function matrix for column B is given by Equation 48:

$$G(s) = \begin{bmatrix} \frac{0.562e^{-1s}}{(1+7.74s)(1+7.74s)} & \frac{-0.516e^{-1.5s}}{(1+7.1s)(1+7.1s)} \\ \frac{0.344e^{-1.5s}}{(1+15.8s)(1+0.5s)} & \frac{-0.393e^{-1s}}{(1+13.8s)(1+0.4s)} \end{bmatrix}. \quad (\text{Eq. 48})$$

The steady state gain matrix is similar, and similar decomposition angles are produced: $\beta=-34.3^\circ$, $\theta=44.5^\circ$. The time constants for column C were divided by 10 to put them on the same order of magnitude as the other processes:

$$G(s) = \begin{bmatrix} \frac{0.471e^{-1s}}{(1+3.07s)(1+3.07s)} & \frac{-0.495e^{-2s}}{(1+2.85s)(1+2.85s)} \\ \frac{0.749e^{-1.7s}}{(1+5.7s)(1+5.7s)} & \frac{-0.832e^{-1s}}{(1+5.05s)(1+5.05s)} \end{bmatrix}. \quad (\text{Eq. 49})$$

The SVD angles corresponding to the column C steady state gain matrix are $\beta=121.4^\circ$ and $\theta=47.6^\circ$. The distillation column of Wood and Berry had gains too large for the general process subroutine; they were all scaled by a factor of 0.1 which produced SVD angles of $\beta=-41.8^\circ$ and $\theta=62.7^\circ$:

$$G(s) = \begin{bmatrix} \frac{1.28e^{-1s}}{(1+16.7s)} & \frac{-1.89e^{-3s}}{(1+21.0s)} \\ \frac{0.66e^{-7s}}{(1+10.9s)} & \frac{-1.94e^{-3s}}{(1+14.4s)} \end{bmatrix}. \quad (\text{Eq. 50})$$

Finally a sagging process was generated of the form in Equation 51:

$$G(s) = \begin{bmatrix} \frac{2.0(1-8.0s)}{(1+2.0s)(1+12.0s)} & \frac{-0.8(1-14.125s)}{(1+1.6s)(1+9s)} \\ \frac{0.7}{(1+9.0s)} & \frac{1.6}{(1+12.0s)} \end{bmatrix}. \quad (\text{Eq. 51})$$

In this system the first controlled variable sags in response to either manipulated variable. The steady state SVD angles are $\beta = -36.6^\circ$ and $\theta = 5.0^\circ$.

Smith discussed the considerations for choosing a time-step size when conducting tuning optimization studies. Many iterations are necessary, which indicate the desirability of a large Δt . Smith showed, however, that large time-steps result in a rough contour of cost function versus controller gains. He demonstrated a relatively smooth response can be obtained by using a small Δt . No specific guidelines are available for specifying timestep size. Trial-and-error experiments showed that for the transfer function models of Equations 47-51 a Δt of approximately 0.1 was needed to eliminate the local minima in the search process.

Control

More than one controller mode is available for implementation on structured variables in the SVD controller. The simplest is a

proportional (constant gain) controller, but allows steady state offset between the process outputs and the setpoints. Integral control is used to eliminate the offset. Integral control alone, however, is characterized by slow response and high oscillation. A combination of proportional and integral is common in the process industries. The addition of derivative control is possible, but the additional tuning required proved to be prohibitive in this work. Thus, two-mode PI control was used on the structured variables in the SVD controller.

Since the simulations were digital in nature (as any industrial application of the controller) a digital algorithm for implementation was needed. The velocity algorithm (Equation 52) is one commonly used

$$m_{i+1} = m_i + K \left[(e_{i+1} - e_i) + \frac{\Delta t}{\tau_I} e_{i+1} \right] \quad (\text{Eq. 52})$$

because of desirable characteristics of bumpless transfer to automatic mode and no reset windup. These characteristics are not afforded in the SVD controller. In the single-variable case reset windup is avoided because the manipulated variable (the output from the controller algorithm) is limited at its constraint. In the case of the SVD controller, however, the output from the PI algorithm is the structured manipulated variable. The structured variable has no constraint, and is combined with each of the other structured manipulated variables (according to the matrix V) to produce the actual physical manipulated variables. It would be difficult to

determine that a given structured variable has reached a constraint. An alternative is to monitor each of the physical manipulated variables, and to stop integrating in each loop once any variable saturates.

The problem of bump-on-transfer for the SVD controller has a solution which is more obvious. When the switch from manual to automatic is made, the current value of the manipulated variables is read. This vector is converted to a vector of structured variables by simply multiplying by $V^{-1}=V^T$, which produces the initial values of the structured manipulated variables.

Optimization

For each controller orientation and process, the control parameters were tuned by hand to achieve a good response. These parameters were used as starting values in a PATTERN optimization routine (84). PATTERN permits the user to specify any desired cost function. The integral absolute error (IAE), integral squared error (ISE), and integral of time times absolute error (ITAE) are commonly used criteria. In general minimum ISE gives the fastest response, but with much oscillation. Minimizing ITAE gives slower response, but oscillations die out more quickly. Minimum IAE gives a response between the other two. Tight controller tuning using IAE can still result in "ringing" however, as shown in Figure 12 for McAvoy's column C. This figure shows the system response to sequential setpoint changes with control parameters of $K_1=10.7$, $\tau_{11}=9.1$, $K_2=-3.4$, and

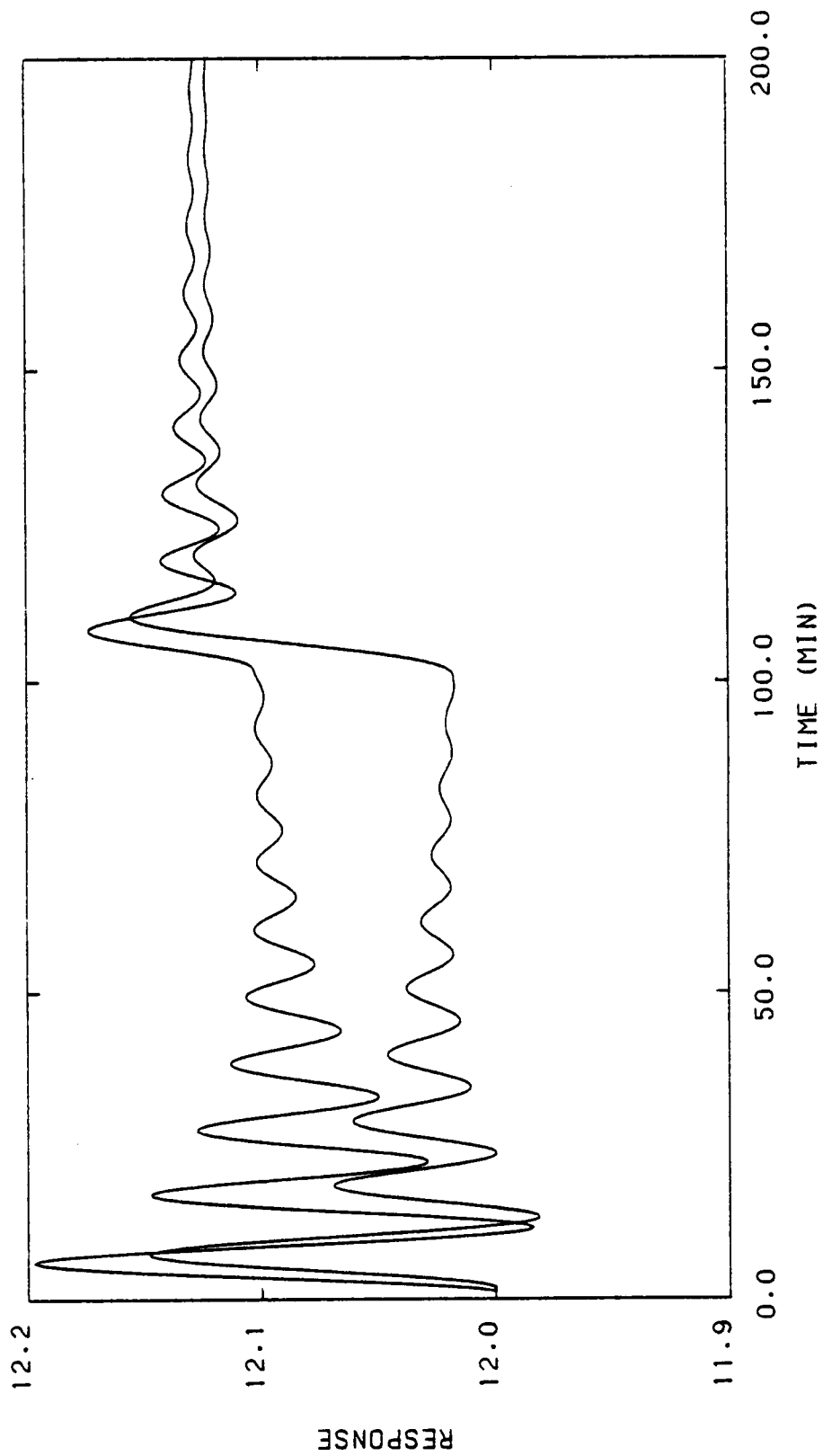


FIGURE 12. COLUMN C RESPONSE TO IAE TUNING.

$\tau_{I2}=0.6$. The response is oscillatory, and for any real system this tuning would be considered far from optimum.

Another available cost criteria modifies the integral absolute error by adding a term which penalizes controller movement, as in Equation 53:

$$\text{COST} = \int (|s-c| + a \left| \frac{dm}{dt} \right|) dt \quad . \quad (\text{Eq. 53})$$

In this equation s represents the setpoint (or vector of setpoints), c the controlled variable (or vector of controlled variables), and m the manipulated variable (or vector of manipulated variables). The parameter " a " is a weighting factor which is adjusted so that the resulting optimum tuning parameters give the desired response. No specific guidelines exist for setting " a ". For the column C process several values were tried until a response which approximated quarter-amplitude damping was achieved. The response to sequential setpoint changes is shown in Figure 13, and corresponds to optimum tuning parameters of $K_1=9.0$, $\tau_{I1}=11.9$, $K_2=-1.0$, and $\tau_{I2}=1.2$. The value of $a=0.005$ was typical for most of the processes examined, and the value used for all the optimization studies reported here.

Tuning Optimization Scenario

For a given process and controller orientation different test scenarios will result in different optimum tuning parameters. For instance tuning for disturbance inputs will give different parameters than tuning for setpoint changes. Also, the optimum tuning parameters

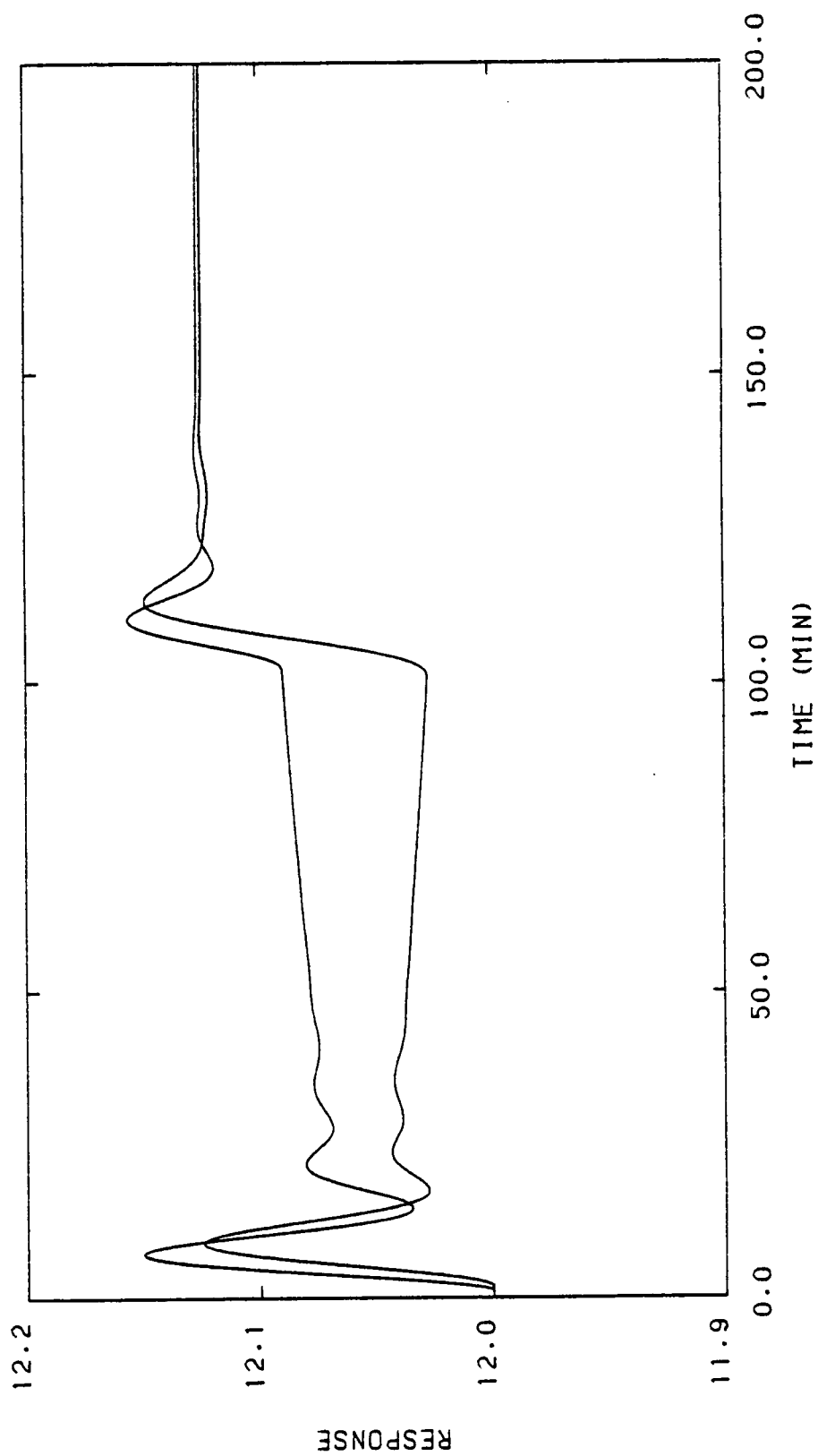


FIGURE 13. COLUMN C RESPONSE, $a = 0.005$.

vary for each setpoint change. Some rationale for choosing the scenario is necessary.

Much work on this question was done using all five process models described at the beginning of this chapter. Results using the column A model of Weischedel and McAvoy were typical, and will be used to illustrate the conclusions.

Varying setpoint changes were examined first. Because the systems considered were linear in nature, the size of the inputs had no effect on the optimum tuning parameters, except the cost. All possible combinations are represented by a unit ball locus of setpoints, as shown in Figure 14. Because the system is linear and scaling has no effect, the lower half of the unit ball gives the same results as the upper half. Tuning was optimized at 15° intervals along the upper half of the unit ball.

Initial values to begin the optimizations for this study were selected as follows. Setpoint inputs at 45° were used, corresponding to the conventional sequential unit setpoint changes. The rotation angles were fixed at the SVD values, calculated using Equations 39 and 42 and the steady-state gain matrix. The proportional and integral modes were tuned by hand to give a good response. These values (the SVD angles and the K 's and τ_I 's) were used to begin the optimization for all cases.

The optimization results are shown in Table 1. The setpoint changes corresponding to each angle are shown, together with the optimum tuning and its associated cost. (Note that in these simulation studies the process input was modeled as a milliamp signal

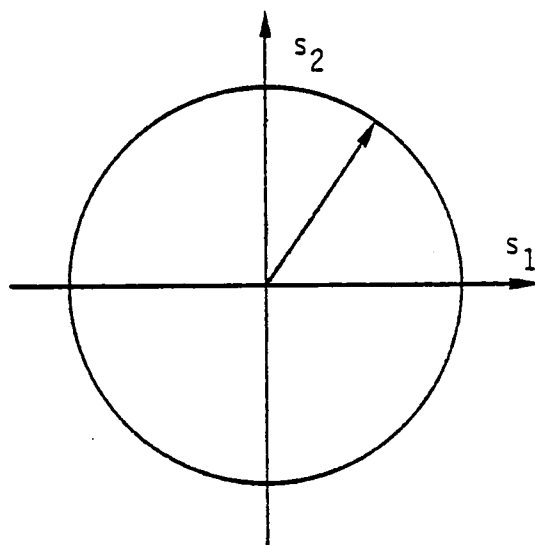


FIGURE 14. UNIT BALL LOCUS OF SETPOINT CHANGES.

TABLE 1
OPTIMIZED TUNING PARAMETERS FOR McAVOY'S
COLUMN A FOR DIFFERENT SETPOINT CHANGES

Angle	Setpoint		Optimum Tuning						COST
	1	2	K_1	τ_{I_1}	K_2	τ_{I_2}	β	θ	
15°	12.9659	12.2588	2.3	7.5	-11.9	4.2	-28.0	39.0	6.5
30°	12.866	12.5	2.3	7.7	-11.0	3.7	-30.0	40.0	6.7
45°	12.7071	12.7071	2.3	7.7	-10.0	3.3	-33.0	41.0	6.5
60°	12.5	12.866	2.3	6.7	-11.3	3.1	-25.0	38.0	5.5
75°	12.2588	12.9659	2.3	7.2	-10.9	3.2	-45.0	43.0	4.6
90°	12.0	13.0	2.0	3.8	-12.2	1.7	-38.0	37.0	2.9
105°	11.7412	12.9659	2.0	7.9	-12.1	4.1	-57.0	43.0	5.7
120°	11.5	12.866	2.7	6.4	-9.8	1.9	-17.0	31.0	5.6
135°	11.2929	12.7071	2.5	8.6	-9.4	3.2	-35.0	41.0	6.7
150°	11.134	12.5	2.7	8.6	-11.6	3.5	-30.0	39.0	6.7
165°	11.0341	12.2588	2.3	7.5	-11.7	4.1	-28.0	39.0	6.5
180°	11.0	12.0	1.8	8.3	-13.0	6.0	-34.0	44.0	6.0

constrained between 4 and 20 and with a steady state value of 12. Therefore all inputs are deviations from 12ma.) The importance of these results is the consistency of the optimum tuning. Regardless of the setpoint change combination, the optimum tuning parameters seemed to all be approximately the same. This is especially significant in the case of the rotation angles, because they seemed to all be near the SVD angles corresponding to the steady state gain matrix. This seems to indicate that if the majority of system upsets are setpoint changes, the SVD angles will be near optimum.

The other possibility is the presence of significant disturbance inputs, which implies the need to use a disturbance input test scenario for tuning optimization. Disturbances are more general than setpoint changes in that setpoint changes affect the system only through the controller and the process transfer function. Disturbances, on the other hand, can enter the system at different points and by many transfer functions, depending on the nature of the disturbance. One common condition is for the disturbance to share the same process dynamics as the manipulated variables. For generality, step disturbances of this type were assumed and applied to the system as a series of vectors rotated in 15° intervals about the unit ball. The resulting optimum tuning was close to that achieved using setpoint changes (Table 1 for McAvoy's column A). The cost was generally lower, but the optimum rotation angles were again close to the SVD angles for the steady state gain matrix. This implies that if the disturbances act primarily through the process transfer function matrix, the SVD angles will be near optimum.

The disturbance does not have to act through the process transfer function, however. In the example of a distillation column a feed upset (either flowrate or composition) will affect the product split in a manner completely different than changing either reflux flowrate or steam to the reboiler. In any real process it is impossible to identify all the different disturbances, much less model them. If modeling were possible, tuning could be optimized using these disturbances in the test scenario. Several different disturbance transfer functions were applied to the systems examined. When tuning was optimized, the consistency experienced using setpoint changes was no longer present. Rather than values of β ranging from -57° to -17° centered around the SVD β of -38.7° , values were obtained ranging from -63° to 51° with no obvious pattern. Similar results were observed with θ .

The exercise was carried out on several different processes, with consistently the same results. Generally disturbance inputs, which are impossible to predict, gave optimum rotation angles which were random. This is because disturbance inputs which act through different transfer functions tend to excite different frequencies of the process, thus seeing a different effective process gain matrix. Different gain matrices decompose into different rotation angles, as evidenced in the optimizations.

The conclusion drawn from this phase of the work is that the most general optimization test scenario is a sequence of setpoint changes. From Table 1, the setpoint changes implemented have little effect on the optimum tuning, and the values at 45° are typical. Thus

in the remaining work of this dissertation the test scenario used will be sequential unit setpoint changes.

SVD Controller Tuning

To illustrate the power of the singular value decomposition controller, each of the five transfer functions described at the beginning of Section IV (Equations 47-51) were tuned. PATTERN was used for tuning parameter optimization. The COST criteria was in the form of Equation 53, which includes a term to penalize controller movement. The optimization scenario was sequential unit setpoint changes. (In the case of McAvoy's column C this represented an unreachable output. For this process the scenario was changed to sequential setpoint steps of 0.1.) The simulation inputs and outputs were in the form of milliamps, scaled between 4.0 and 20.0 and with initial values of 12.0.

Rotation Angles Fixed at SVD

The first step in this analysis was to tune each process using conventional multivariable single input-single output control. Each system was tuned by hand to minimize the COST function of Equation 53. These proportional gains and integral reset times were used as starting values in the PATTERN search.

The tuning was performed using the SVD controller with rotation angles (β and θ) fixed at the calculated values (from Equations 39 and 42). This controller was tuned by hand and these values used as starting parameters in a PATTERN search. Results are shown in Table 2

TABLE 2
TUNING COMPARISON BETWEEN MVSISO CONTROL AND
SVD CONTROL WITH FIXED ROTATION ANGLES

Process	Optimum Tuning						COST
	K_1	τ_{I_1}	K_2	τ_{I_2}	β	θ	
McAvoy's Column A							
MVSISO	3.9	3.3	-6.7	4.9	0.0	0.0	14.88
SVD	2.9	7.8	-13.1	4.0	-38.7	44.1	9.70
McAvoy's Column B							
MVSISO	15.0	2.0	-34.0	7.0	0.0	0.0	30.16
SVD	13.0	8.0	-29.0	9.0	-34.3	44.5	30.31
McAvoy's Column C							
MVSISO	9.0	11.9	-1.0	1.2	0.0	0.0	7.15
SVD	-1.1	11.5	23.5	2.6	121.4	47.6	2.85
Wood and Berry Model							
MVSISO	13.1	4.4	-0.8	4.5	0.0	0.0	17.91
SVD	1.2	17.7	-2.8	5.9	-41.8	62.7	16.80
Sagging Process							
MVSISO	0.33	14.5	0.89	14.1	0.0	0.0	49.32
SVD	0.31	12.3	0.43	9.6	-36.6	5.0	44.07

for each of the five processes used for illustration. It is evident that the SVD controller generally gave improved performance. McAvoy's column B is the only process which showed no improvement. Column A, however, showed a 35% improvement in performance as measured by COST. It is of interest that the steady state gain matrices and the rotation angles for both processes are very similar. This is an illustration of the fact that the dynamics of a process play an important role in controller performance. The dynamics are virtually the only difference between columns A and B.

Column C shows a dramatic (60%) improvement in performance. The response to MVSISO control is shown in Figure 13. Figure 15 shows the response to SVD control, where the angles (β and θ) are fixed at their calculated values. This response also demonstrates the dramatic improvement in performance for SVD control.

Optimum Rotation Angles

If steady state (or slow) control is the only requirement, the SVD rotation angles (β and θ) calculated from the steady state gain matrix are the best. In a plant, however, disturbances can occur which need to be compensated for quickly. In most processes, setpoint changes need to be made occasionally, and a fast response to these changes is needed. Thus the optimum frequency at which to decompose the process transfer function matrix may be other than zero. To investigate this, each of the five processes were tuned again. The angles β and θ were included to add two more degrees of freedom in the tuning optimization.

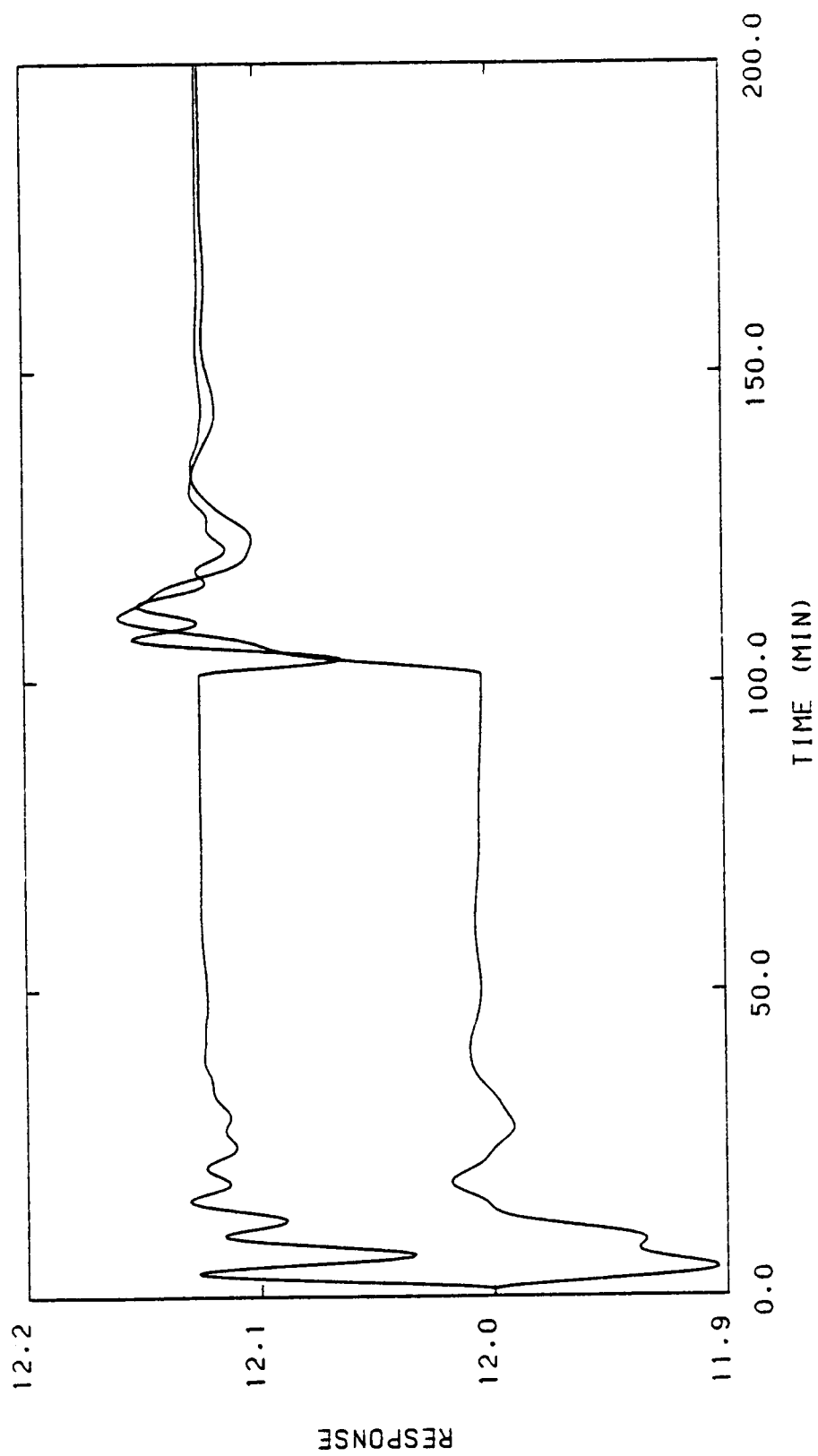


FIGURE 15. COLUMN C RESPONSE TO SVD CONTROL: ANGLES FIXED AT CALCULATED VALUES.

The results of the tuning are shown in Table 3 with the MVSISO results and the results holding the rotation angles constant. In every case at least some improvement in performance was realized. This included McAvoy's column B, which had shown no improvement using the calculated rotation angles. Column C also showed some improvement, though much less than its improvement upon applying the calculated rotation angles. (That is, the optimized angles were only a slight improvement over the calculated angles.) The column C response to sequential setpoint changes, using the optimum tuning from Table 3, is shown in Figure 16. Comparing with Figure 15, the improvements are barely noticable, which was true of all five processes tested. The improvement in COST was slight (less than 10%) and the response plot showed an improvement which at times was barely discernable.

Independent Mode Rotation

Decomposing the steady state gain matrix insures the structured variables are independent in the steady state. This gives no information about the transient, however. The optimum rotation angles for a setpoint change transient are different from the values calculated at steady state. Going one step further, the two controller modes, proportional and integral, affect the system at different frequencies. The proportional action is instantaneous, responding to high frequency (or fast) system upsets. Integral action, however, is slow, and is sometimes thought of as having a steady state calibrating effect. Thus the two modes see a "different"

TABLE 3
TUNING COMPARISON WITH OPTIMUM SVD ROTATION ANGLES

Process	Optimum Tuning						COST
	K_1	τ_{I_1}	K_2	τ_{I_2}	β	θ	
McAvoy's Column A							
MVSISO	3.9	3.3	-6.7	4.9	0.0	0.0	14.88
SVD	2.9	7.8	-13.1	4.0	-38.7	44.1	9.70
Optimum	2.3	7.7	-10.0	3.3	-33.0	41.0	9.18
McAvoy's Column B							
MVSISO	15.0	2.0	-34.0	7.0	0.0	0.0	30.16
SVD	13.0	8.0	-29.0	9.0	-34.3	44.5	30.31
Optimum	13.0	4.8	-30.9	7.7	-19.3	30.5	28.68
McAvoy's Column C							
MVSISO	9.0	11.9	-1.0	1.2	0.0	0.0	7.15
SVD	-1.1	11.5	23.5	2.6	121.4	47.6	2.85
Optimum	-1.2	7.3	27.6	3.5	148.4	47.6	2.47
Wood and Berry Model							
MVSISO	13.1	4.4	-0.8	4.5	0.0	0.0	17.91
SVD	1.2	17.7	-2.8	5.9	-41.8	62.7	16.80
Optimum	1.3	17.3	-3.3	7.1	-46.8	65.7	16.53
Sagging Process							
MVSISO	0.33	14.5	0.89	14.1	0.0	0.0	49.32
SVD	0.31	12.3	0.43	9.6	-36.6	5.0	44.07
Optimum	0.33	12.3	1.00	11.1	-37.9	3.6	40.19

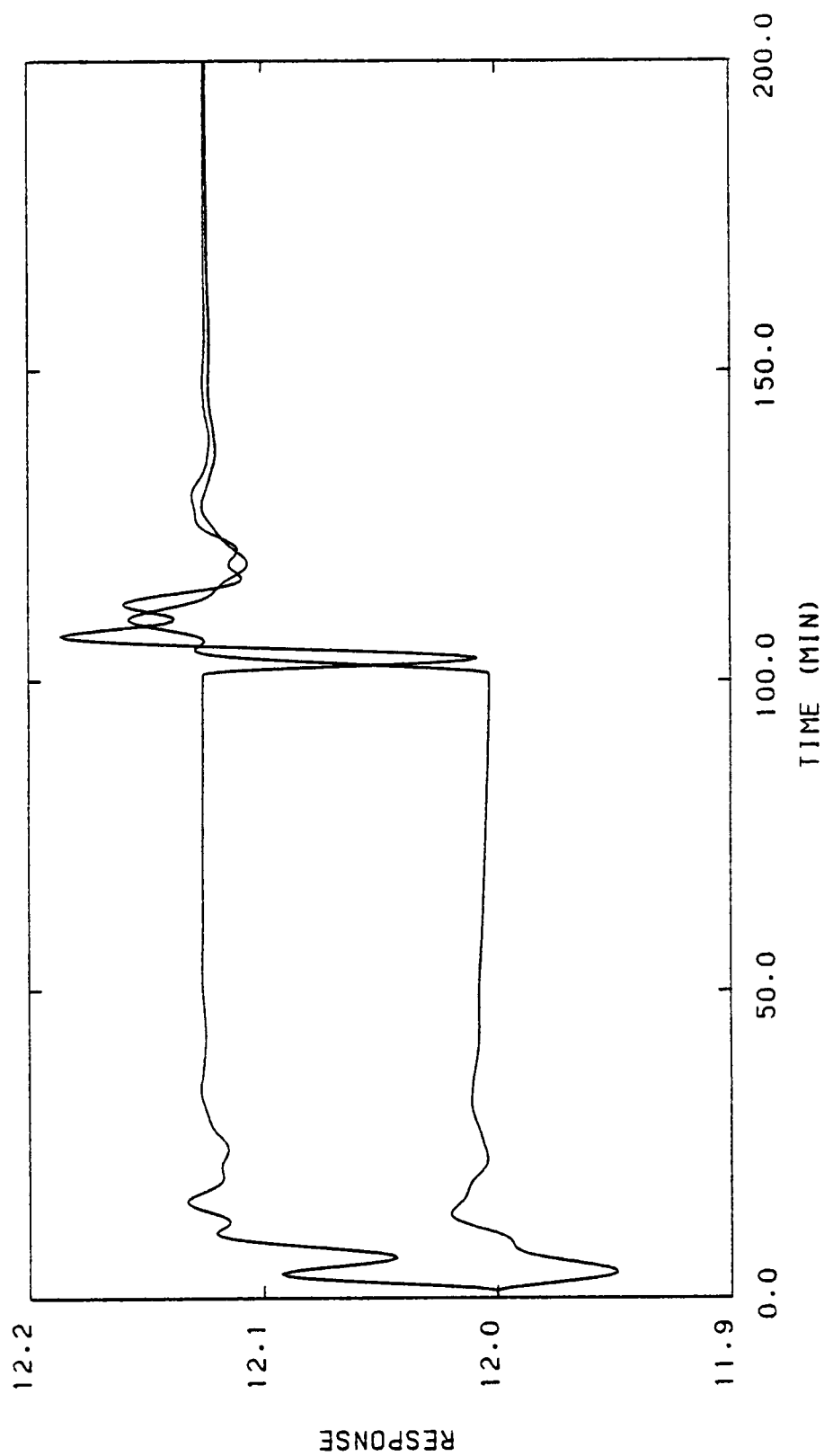


FIGURE 16. COLUMN C RESPONSE TO SVD CONTROL: OPTIMUM ANGLES, MODES ROTATED TOGETHER.

gain matrix corresponding to their respective frequencies. It would be natural that rotating the separate modes through different rotation angles might significantly improve performance.

To investigate the possibility, each process was tuned using the configuration shown in Figure 17. The U_p and V_p are specified by β_p and θ_p , respectively, and the U_I and V_I by β_I and θ_I . For this tuning there are eight parameters to be optimized. Initial values to begin the optimization were taken to be the optimum tuning found when rotating both modes together.

The results are shown in Table 4. In most cases a slight improvement in COST was realized. Also in most cases the optimum angles were not far from the values calculated using the steady state gain matrix. The significant exception to this was the sagging process. A 20% improvement in COST was realized and the optimum angles were significantly different than the calculated values. The plotted response also showed a significant improvement. This is due to the sagging nature of the response. The short-term, or high frequency, component of the process response is in a direction which is opposite the long-term, or low frequency, final value. Thus, at the frequency range of each of the two controller modes the dynamic gain matrix is radically different. (Some components will even have different signs.) These different gain matrices will decompose into significantly different rotation angles, as evidenced in Table 4.

McAvoy's column C showed a 23% improvement in COST. The optimum angles were not far from the calculated values, however. The separate mode rotation reduces the interaction and permits tighter

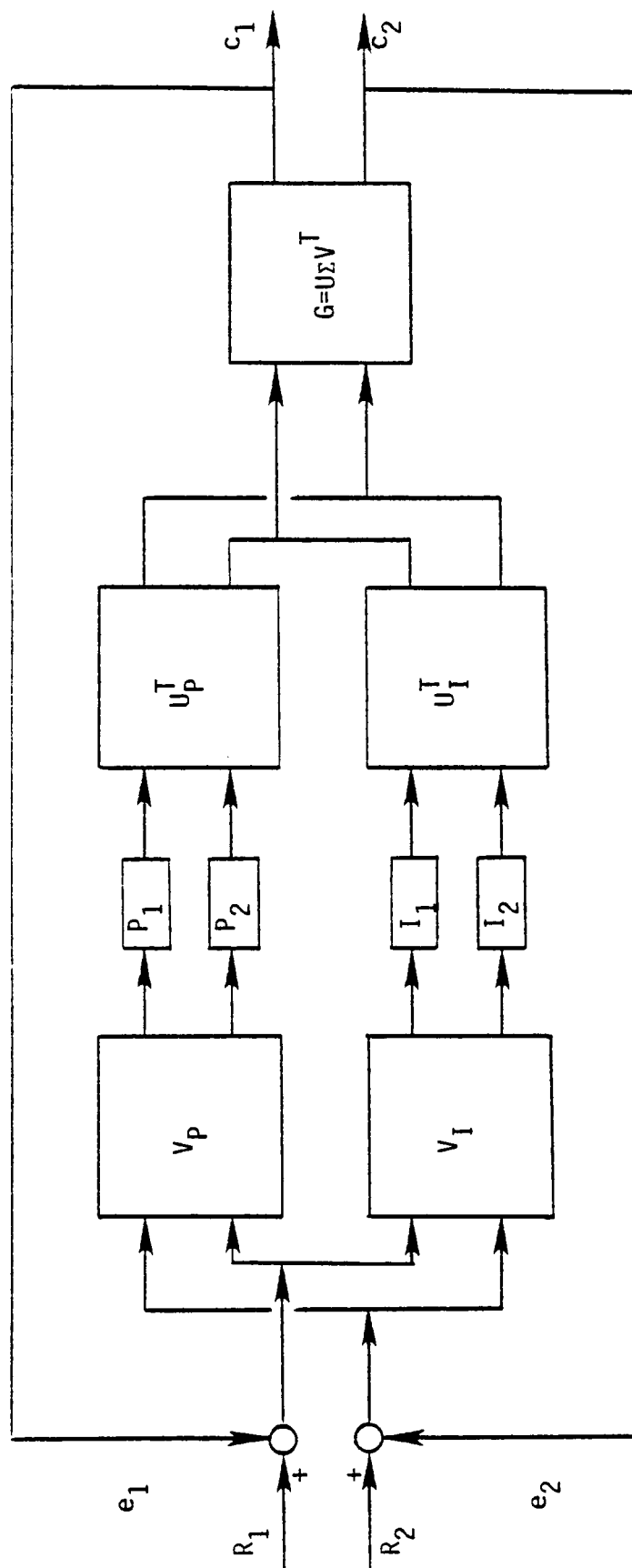


FIGURE 17. SVD CONTROLLER STRUCTURE WITH INDEPENDENT MODE ROTATION.

TABLE 4

TUNING COMPARISON WITH PROPORTIONAL AND INTEGRAL
MODES ROTATED THROUGH DIFFERENT ANGLES

Process	Optimum Tuning								COST
	K_1	τ_{I_1}	K_2	τ_{I_2}	β_I	β_P	θ_I	θ_P	
McAvoy's Column A									
MVSISO	3.9	3.3	-6.7	4.9	0.0	0.0	0.0	0.0	14.88
SVD	2.9	7.8	-13.1	4.0	-38.7	-38.7	44.1	44.1	9.70
Together	2.3	7.7	-10.0	3.3	-33.0	-33.0	41.0	41.0	9.18
Separate	2.7	7.7	-12.2	3.4	-44.7	-30.7	43.1	42.1	8.96
McAvoy's Column B									
MVSISO	15.0	2.0	-34.0	7.0	0.0	0.0	0.0	0.0	30.16
SVD	13.0	8.0	-29.0	9.0	-34.3	-34.3	44.5	44.5	30.31
Together	13.0	4.8	-30.9	7.7	-19.3	-19.3	30.5	30.5	28.68
Separate	11.4	5.3	-28.9	7.6	-26.3	-16.3	37.5	40.5	28.09
McAvoy's Column C									
MVSISO	9.0	11.9	-1.0	1.2	0.0	0.0	0.0	0.0	7.15
SVD	-1.1	11.5	23.5	2.6	121.4	121.4	47.6	47.6	2.85
Together	-1.2	7.3	27.6	3.5	148.4	148.4	47.6	47.6	2.47
Separate	-3.5	15.1	19.4	2.8	122.4	91.4	54.6	47.6	2.00
Wood and Berry Model									
MVSISO	13.1	4.4	-0.8	4.5	0.0	0.0	0.0	0.0	17.91
SVD	1.2	17.7	-2.8	5.9	-41.8	-41.8	62.7	62.7	16.80
Together	1.3	17.3	-3.3	7.1	-46.8	-46.8	65.7	65.7	16.53
Separate	1.3	17.3	-3.3	7.1	-46.8	-46.8	65.7	65.7	16.53

TABLE 4 (Continued)

Process	Optimum Tuning							COST
	K_1	τ_{I_1}	K_2	τ_{I_2}	β_I	β_P	θ_I	θ_P
Sagging Process								
MVSISO	0.33	14.5	0.89	14.1	0.0	0.0	0.0	0.0
SVD	0.31	12.3	0.43	9.6	-36.6	-36.6	5.0	5.0
Together	0.33	12.3	1.00	11.1	-37.9	-37.9	3.6	3.6
Separate	0.37	13.8	0.33	3.6	-47.6	2.4	-15.0	31.0

tuning. This leads to a faster (although more oscillatory) response as shown in Figure 18.

Optimum Proportional Mode Rotation

To optimize the tuning rotating each of the two modes independently requires adjusting eight parameters. This is prohibitive in any real application. The number of parameters can be reduced by two by fixing the integral mode rotation at the calculated values. The rationale for this simplification is that the integral mode can be thought of as a steady state controller, thus the steady state gain matrix is appropriate for angle determination.

This strategy was investigated by tuning each of the five processes. Two sets of initial values were tried. First the proportional angles were started at the values calculated from the steady state gain matrix. (These were the angles at which the integral mode was fixed.) Then the proportional angles were started at zero (as in the MVSISO controller). The optimum angles converged to the same point each time.

The results are shown in Table 5. There is generally little improvement over control where the angles are fixed at their calculated values. For two of the processes the COST is larger than for optimized angles rotated together. The conclusions for these processes is that the strategy offers nothing over simply fixing the values because of the large part the integral mode plays in the dynamic response. With integral reset times short, as they are for these processes, the frequency at which they excite the system does

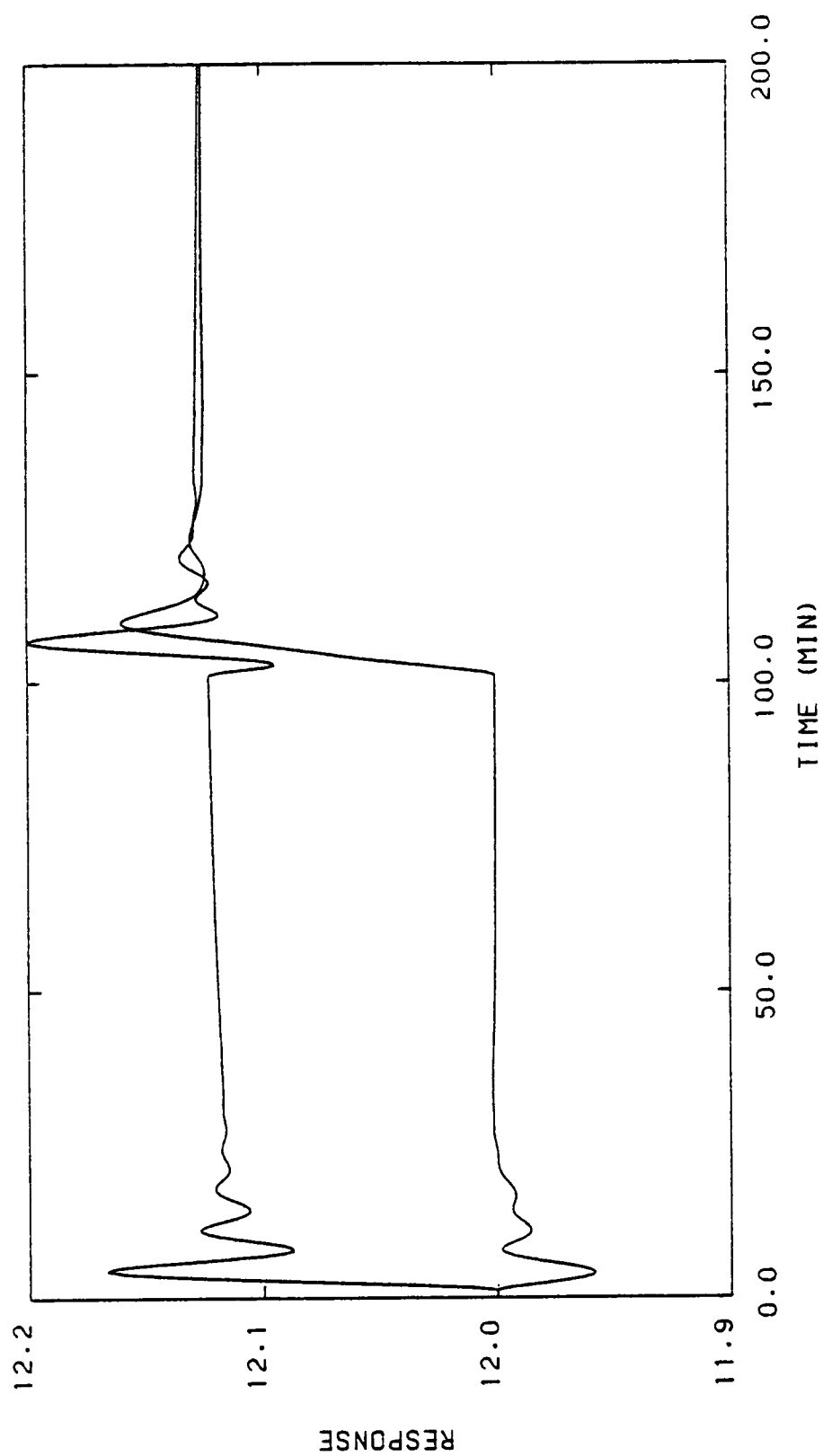


FIGURE 18. COLUMN C RESPONSE TO SVD CONTROL: OPTIMUM ANGLES, MODES ROTATED INDEPENDENTLY.

TABLE 5

TUNING COMPARISON WITH FIXED INTEGRAL MODE ROTATION
AND OPTIMUM PROPORTIONAL MODE ROTATION

Process	Optimum Tuning						COST
	K_1	τ_{I_1}	K_2	τ_{I_2}	$\beta(p)$	$\theta(p)$	
McAvoy's Column A							
MVSISO	3.9	3.3	-6.7	4.9	0.0	0.0	14.88
SVD	2.9	7.8	-13.1	4.0	-38.7	44.1	9.70
Together	2.3	7.7	-10.0	3.3	-33.0	41.0	9.18
P Separate	2.6	6.7	-12.3	3.5	-51.7	46.1	9.17
McAvoy's Column B							
MVSISO	15.0	2.0	-34.0	7.0	0.0	0.0	30.16
SVD	13.0	8.0	-29.0	9.0	-34.3	44.5	30.31
Together	13.0	4.8	-30.9	7.7	-19.3	30.5	28.68
P Separate	12.0	4.0	-30.8	7.7	-19.0	28.0	28.61
McAvoy's Column C							
MVSISO	9.0	11.9	-1.0	1.2	0.0	0.0	7.15
SVD	-1.1	11.5	23.5	2.6	121.4	47.6	2.85
Together	-1.2	7.3	27.6	3.5	148.4	47.6	2.47
P Separate	-2.0	12.5	20.5	2.6	151.4	47.6	2.21
Wood and Berry Model							
MVSISO	13.1	4.4	-0.8	4.5	0.0	0.0	17.91
SVD	1.2	17.7	-2.8	5.9	-41.8	62.7	16.80
Together	1.3	17.3	-3.3	7.1	-46.8	65.7	16.53
P Separate	1.2	17.7	-2.8	5.9	-41.8	62.7	16.80
Sagging Process							
MVSISO	0.33	14.5	0.89	14.1	0.0	0.0	49.32
SVD	0.31	12.3	0.43	9.6	-36.6	5.0	44.07
Together	0.33	12.3	1.00	11.1	-37.9	3.6	40.19
P Separate	0.29	11.1	0.73	15.9	-34.0	18.0	43.44

not approximate zero (steady state). For processes where integral control is used in a calibrating mode (reset times which are several times the largest time constant) the interpretation of a steady state controller could be more appropriate. In this case a strategy of the type discussed here might prove beneficial.

Summary

Several process models have been used to illustrate the effectiveness of the singular value decomposition controller. It has been found to be effective, and a powerful process control tool.

A COST criteria was developed which includes integral absolute error plus a weighted derivative of the manipulated variables to penalize controller action. A weighting of 0.005 was found to be effective for prevention of ringing in the processes studied.

A sensitivity study was conducted which concluded that the best general test scenario for tuning optimizations is a set of sequential setpoint changes of the same magnitude. Disturbance inputs are too general and unpredictable in terms of their transfer function matrix.

Optimizing the rotation angles, both together and independently, produced interesting results. The usefulness may be academic, however, because of the complication of on-line tuning. The possibility of on-line tuning is investigated in the next section.

V. ON-LINE TUNING STUDIES

Several detailed tuning optimizations were performed, and for each process model examined thousands of iterations were run to optimize the tuning in various configurations. These studies minimized very fine differences in COST, but the resulting differences in system responses were many times indistinguishable. An example of this can be seen by comparing Figures 16 and 18. The differences in system response are minor, but the COST was reduced by 19%. In other cases and configurations the COST difference was even less, as was the difference in response.

Generally, the strategies which produce slight improvements in COST are more involved in terms of tuning requirements. A trade-off must be made between improved performance and effort expended on tuning. It is impractical to do such optimizations on a real process. The hundreds of iterations required would not be possible on a distillation column, for example. The continual system upsets would produce a great deal of product not meeting specifications, and interfere with the smooth operation of the rest of the plant. The fine tuning which minimizes COST for a given test scenario does not generally give the minimum COST for another input. Thus, effort expended in tight tuning for one set of inputs is wasted unless all inputs are the same. (This is illustrated in Table 1 for McAvoy's column A.)

If an SVD controller is to be practical for industrial application, some general tuning rules are required, analogous to

Ziegler-Nichols tuning rules for PID controllers. These tuning rules should result in controller settings which are sufficiently general to give good performance under a wide range of conditions. They should require a minimum of trial and error, and be computationally simple.

This section explores the development of a set of tuning rules for an industrial SVD controller, and examines several different schemes for fixing the SVD rotation angles. A procedure for setting the proportional gains and integral reset times is explored. Finally, a complete set of procedures for controller tuning is outlined.

Options for Setting Rotation Angles

Several techniques are available for finding an appropriate set of rotation angles. An obvious possibility is a trial and error search for the angles. Another is to develop a gain matrix and calculate the SVD angles. Several techniques for rotating the proportional and integral modes through different angles also exist. Each is examined, and the merits evaluated.

Trial and Error Search

Most techniques discussed in Section IV involve performing step tests and developing a gain matrix to calculate the SVD rotation angles. It is desirable to minimize the number of calculations. If appropriate rotation angles could be reliably determined using trial and error techniques, all calculation would be eliminated.

This approach was tried on many processes. Initially the proportional and integral modes were tuned with rotation angles of

zero (effectively a multivariable single input-single output system). This had the effect of forcing the angles of zero to be a local optimum.

In certain cases, the angles could be effectively searched by not pre-biasing the system with any tuning. Equal tuning was applied to each loop (for instance proportional gains of 2.0 with no integral control). Under these conditions a special relationship exists between the two rotation angles, β and θ . The absolute values of the angles have no effect, only the difference between the angles. For instance, with identical tuning parameters in each loop the system responds the same with rotation angles of $\beta=90^\circ$, $\theta=45^\circ$ as with angles of $\beta=45^\circ$, $\theta=0^\circ$. This becomes obvious when the controller is interpreted from a geometric viewpoint. Referring to Figure 10, the U^T matrix rotates the error signals through an angle of β . The structured (rotated) error signals are acted on by the proportional and integral modes to scale the rotated error signals. If the proportional and integral tuning is the same in each loop, the unit ball of inputs is not converted into an ellipse, but remains a circle. This circle is rotated back through an angle of θ . The overall effect of the controller will be to multiply the error signals, and rotate them through an angle of $(\beta-\theta)$.

Thus, when the loops are similar in tuning requirements, an effective simplification is available for finding the difference between the rotation angles. An example is shown in Tables 6 and 7. Table 6 shows the successful step-by-step trial and error angle tuning for McAvoy's column A. Table 7 shows the step-by-step angle tuning for

TABLE 6
TRIAL AND ERROR TUNING OF ROTATION ANGLES
FOR McAVOY'S COLUMN A

Trial	K_1	K_2	β	θ	COST
1	2.0	2.0	0	0	399.84
2			10	0	578.55
3			0	10	319.03
4			0	20	272.17
5			0	30	242.66
6			0	50	210.38
7			0	70	197.40
8			0	90	196.84
9			0	110	208.44
10			0	80	195.70←
11			0	85	195.92
12	2.0	1.0	0	80	196.55
13			10	90	205.06
14			-10	70	189.45
15			-20	60	184.05
16			-30	50	180.42
17			-40	40	178.62←
18			-50	30	178.71
19			-45	35	178.70
20			-35	45	179.29

TABLE 7
TRIAL AND ERROR TUNING OF ROTATION ANGLES
FOR McAVOY'S COLUMN C

Trial	K_1	K_2	β	θ	COST
1	2.0	2.0	0	0	77.24
2			10	0	694.65
3			0	10	38.12
4			0	20	24.62
5			0	30	21.07
6			0	40	19.53
7			0	50	18.69
8			0	60	18.24
9			0	70	17.99
10			0	80	17.90←
11			0	90	17.94
12			0	85	17.91
13	2.0	1.0	0	80	18.36
14			10	90	19.90
15			-10	70	17.97
16			-20	60	17.71
17			-30	50	17.60
18			-40	40	17.61
19			-35	45	17.59←

McAvoy's column C, which was not successful. The initial section of Table 6 illustrates the search for the angle difference. Proportional gains were set at 2.0 and no integral control was used. The COST was monitored for each trial, and the angle difference optimized to the nearest 5° . The optimum angle difference was 80° . The second section of the table shows the search for the optimum absolute angles. An arbitrarily different PI tuning was chosen (in this case $K_1 = 2.0$, $K_2=1.0$, integral mode turned off). This insures the absolute angles will be significant, not just the difference. Keeping the difference between the angles constant, the pair was adjusted to minimize COST, again to within 5° . The resulting angles were $\beta=-40^\circ$, $\theta=40^\circ$. This compares well with the values calculated from the steady state gain matrix of $\beta=-38.7^\circ$, $\theta=44.1^\circ$.

A typical example of systems for which the trial and error technique does not work is McAvoy's column C. Table 7 shows the step-by-step trial and error search for appropriate rotation angles. The optimum angle difference was 80° , and the absolute angles were tuned to $\beta=-35^\circ$, $\theta=45^\circ$. This compares to values calculated from the steady state gain matrix of $\beta=121.4^\circ$, $\theta=47.6^\circ$. Even the angle difference did not agree with the calculated value.

The trial and error method for finding rotation angles was tested on several process models. Approximately half the cases were successful. "Successful" is defined in this context as finding the same angles which would be calculated from the steady state gain matrix. PI tuning was attempted using the angles which resulted from the trial and error search. In every case where different angles were

found the same result occurred: the angles calculated from the gain matrix gave better performance than the angles which resulted from the trial and error search.

The conclusion is the method is not useful for tuning SVD controllers. There are some interesting points brought up by this study, but the technique is too unreliable to be included as part of the tuning procedure. Another problem is, again, too many iterations are involved for implementation on a chemical process.

Fixing Angles at Calculated Values

A second technique for finding an appropriate set of rotation angles is to develop a steady state gain matrix and calculate the angles. One of the principle advantages is that no trial and error is involved. Two open-loop step tests are performed, one in each loop. From these tests a steady state gain matrix is evaluated. Using this matrix and Equations 39 and 42, the angles θ and β are calculated. Step tests are easy to perform on a unit. More importantly, the disruption is required only once. The unit can return to normal steady state operation in a matter of hours, rather than the days or weeks required for iterative techniques.

Another advantage is the calculations required are simple. They can be automated in the plant computer or a hand-held programmable calculator.

The technique works consistently well. As in Tables 2 through 5, a majority of improvement using the various more involved forms was realized upon implementing an SVD controller of this form. Slight

reductions in COST were seen when the rotation angles were optimized from the calculated values. These reductions were generally small, however, compared with the first improvement. (McAvoy's column B is an exception to this general trend.)

One disadvantage of fixing the angles at their calculated values is these may not be optimum. As mentioned, the true optimum is generally not far from the calculated values.

Another drawback was witnessed in some rare cases. In these cases (e.g., sagging process), the SVD controller with the calculated angles performed little better than multivariable single input-single output (MVSISO) control. In no case, however, did the SVD controller with the calculated angles perform worse than MVSISO control.

The conclusion is that this is a good technique. It is simple to perform; it is effective in that it almost always shows significant improvement over MVSISO control; and the drawbacks are limited, none of which cause it to perform worse than MVSISO control.

Integral Mode at Calculated Values

A third method for fixing rotation angles involves rotating the proportional and integral modes separately. This approximately doubles the number of calculations required to perform the control algorithm, which generally is no problem. There is a difficulty, however, in dealing with the increased degrees of freedom. The number of parameters to be tuned are increased from four for MVSISO, to six for SVD where proportional and integral modes are rotated together, and to eight for SVD where modes are rotated separately. It is clear

that some simplification must be made in the tuning. One possibility is to fix the integral mode rotation at the calculated values and the proportional mode rotation at zero. This approach has intuitive appeal, as the calculated values of the rotation angles are based on the steady state gain matrix. This matrix is essentially the dynamic transfer function matrix at a frequency of zero. When the integral mode is interpreted as steady state or calibrating control, the angles produced from the steady state gain matrix are optimum.

The use of integral mode, however, is not steady state control. With reset times on the same order as the system time constants, there is definitely a major impact on the transient response.

A further problem with this approach is there is no basis for starting the proportional mode rotation angles at zero. Proportional control excites much higher system frequencies than integral control, so the matrix to use in the decomposition is not the steady state gain matrix. One approach is to decompose the matrix at some arbitrarily high frequency, which would probably give better starting values. Unfortunately high frequency matrix decomposition has another problem. Calculating the gain matrix at a frequency other than zero requires a full process transfer function matrix. To develop such a matrix is very involved, and would probably prove to be beyond the abilities of a plant technician tuning the controller.

This approach was tried for several cases. In every case better results were achieved by setting the rotation angles for both modes to the calculated values than for setting just the integral mode angles. This is further illustrated by the results shown in Tables 4

and 5. Both tables show the results for optimizing proportional mode rotation. (In Table 4 the integral mode rotation is being optimized simultaneously. Table 5 shows fixing the integral mode rotation and optimizing the proportional.) In both cases the optimum proportional mode rotation was near the angles calculated from the gain matrix. This implies the calculated values are better than angles of zero as starting points for any optimization.

This technique does not offer improvement over simply fixing the rotation angles at the values calculated from the steady state gain matrix. The added calculational requirements are significant, and no improvement in performance is realized.

Modified Gain Matrix

A fourth method for finding appropriate rotation angles also involves the idea of rotating the modes separately. This technique has the same intuitive appeal as that discussed above, but with a more sound theoretical basis. It involves decomposing the matrix at different frequencies for the different modes. The proportional mode rotation angles are calculated using the gain matrix at a high frequency. The integral mode rotation angles are calculated using the gain matrix at a lower frequency. This reduces the degrees of tuning freedom from eight back to six: the proportional gains and integral reset times in each loop, the "proportional frequency," and the "integral frequency."

The problem is then one of finding the correct frequency at which to decompose for each mode. Unfortunately there is no "correct"

frequency which can be determined from the transfer function matrix. If the integral reset times were tuned the same in each loop, the "integral frequency" would then be $1/\tau_I$. Two problems occur with this approach: first, the reset times are rarely tuned the same, and second the proportional and integral modes are not generally tuned until after the rotation angles are fixed.

For proportional angle determination a limit will sometimes be approached. As frequency increases, the gain matrix will change, with each element decreasing in magnitude to zero at a frequency of infinity. For some systems, the decomposition angles will approach a limit as the frequency approaches infinity. For these systems, the proportional angles can be taken as that limit.

So in two special cases there is some guidance for choosing the proportional frequency and integral frequency. In general these will not be available. The idea of a proportional and integral frequency can be used in a trial and error tuning scheme, however. Rather than search for the optimum $\beta_p, \theta_p, \beta_I, \theta_I$, a search is made for ω_p and ω_I . Two options are available for starting values. ω_I should be set at zero. ω_p could be started at zero or at an arbitrary high value. Both were tried for several cases. Setting ω_p to a high value rarely produced good results. The best frequencies from which to start optimizations tended to be zero for both modes. The optimum frequencies still tended to be very near zero even after extensive searching. This confirms the results in Table 4, p. 67, where the four angles were optimized independently. The optimum angles (both

proportional and integral) were near the angles calculated from the zero-frequency gain matrix.

This method is similar to the last one discussed because little improvement in performance is realized for the significant effort expended in doing the added calculations. Again, calculating the gain matrix at some frequency other than zero requires a full process transfer function matrix. From a practical viewpoint, evaluating such a matrix is too involved for the technician tuning the controller.

Proportional and Integral Mode Tuning

Given a good technique for specifying the SVD rotation angles, tuning the proportional and integral parts of the controller is still necessary. Standard Ziegler-Nichols tuning rules are often inadequate when applied to multivariable systems. This is true of the SVD controller as well. If the controller is put into manual and step tests performed on the process, the resulting tuning parameters are not usually appropriate. In the MVSISO case, interaction is often such a problem that the tuning must be backed off considerably from the Ziegler-Nichols parameters. In the case of the SVD controller, the parameters which are picked off the response curve do not represent the system seen by the PI control.

The proportional/integral part of the SVD controller sees a structured process. If the rotation angles are those calculated from the steady state gain matrix, the structured process will be decoupled in the steady state. If the PI part of the controller is put into manual and step tests performed on the structured manipulated

variables, the response of the structured error signals will represent the system seen by the PI controller. If this response is the one used to tune based on Ziegler-Nichols, the parameters will probably give good results. There is some interaction during the transient, but as the Ziegler-Nichols parameters are somewhat loose, the system typically works well.

As an example of this technique, Figure 19 shows the response of the structured error signals to sequential unit step changes in the structured manipulated variables. For the first variable, the response suggests Ziegler-Nichols tuning of $K=-1.62$, $\tau_I=10.93$. For the second, tuning parameters of $K=29.90$ and $\tau_I=4.07$ are suggested. These numbers are close to the PATTERN-optimized values of $K_1=-1.1$, $\tau_{I1}=11.5$, $K_2=23.5$, $\tau_{I2}=2.6$. The results for column C were typical of the processes which were tested.

This technique cannot be used directly for the case where proportional and integral modes are rotated independently. In this case there are two sets of structured manipulated and structured controlled variables. Given that all four angles (β_p , θ_p , β_I , and θ_I) have been fixed and are different, four step tests are required to develop four response curves. These response curves can be used with rules for P-only and I-only tuning. One drawback is that if any set of β and θ are not the values calculated from the steady state gain matrix, the structured variables for that set will not be independent. Thus considerable modification (probably backing off) of the parameters is necessary.

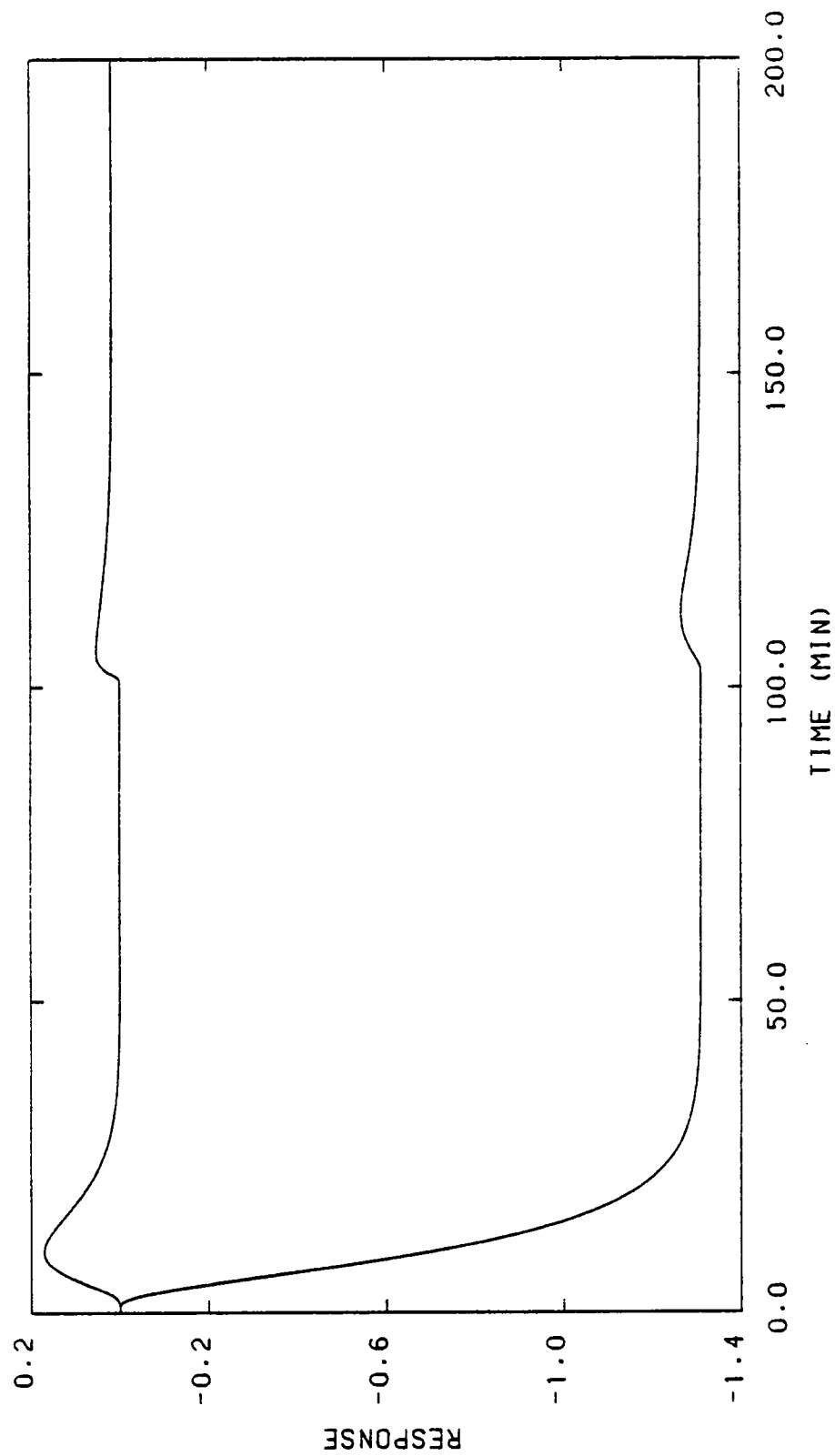


FIGURE 19. COLUMN C OPEN LOOP STEP RESPONSE OF STRUCTURED VARIABLES.

The conclusion is this is a good technique for finding an initial set of tuning parameters, in the same sense that Ziegler-Nichols is a good technique for finding initial tuning in the single variable case. The parameters are preliminary, however, and should be fine tuned after they are applied.

Summary

Various techniques for tuning a SVD controller have been investigated. Since it is not practical to do extensive optimizations in an industrial setting, the goal was to develop a set of simple, straightforward tuning rules.

A trial and error search for rotation angles, with no prior knowledge, was impractical. The method was successful only about half the time, and too many trials (with their system upsets) were required.

Decomposing the transfer function matrix at different frequencies produced interesting results. The major drawback preventing use in an industrial setting is the necessity of developing a transfer function matrix which contains all the system dynamics. Interesting work has been done in this area, but its use in a controller tuning procedure is not practical.

The best consistent, straightforward tuning procedure is as follows:

1. Perform step tests on the open loop system to develop a steady state gain matrix.

2. Calculate the SVD rotation angles (Equations 39 and 42).
3. Use these rotation angles in a structure where the proportional and integral modes are rotated together.
4. Perform step tests on the structured manipulated variables to develop response curves.
5. Use these curves to develop initial PI tuning parameters using Ziegler-Nichols tuning rules.
6. Fine tune the system on-line.

This procedure gave consistently good response, with significant improvement over multivariable single input-single output control in almost all cases.

VI. FOSSIL PLANT MODEL AND CONTROL

The concept of a singular valve decomposition controller has been developed and discussed in detail. The development of tuning rules has been explored at length using transfer function models from the literature. In order to test the SVD controller under realistic conditions, a system was simulated and used for control studies. These studies included three major phases: conventional MVSISO control, multivariable tuning regulator control, and SVD control. The purpose was to compare SVD control with existing techniques and with another example of advanced control.

The model was a detailed first-principles simulation of a coal-fired power plant. Historically, large coal-fired power plants have been designed for base-load operation. In this mode, the plant operates in the upper 15% of rated power, its most efficient range, and generation demand does not vary significantly for long periods. Under these ideal conditions little control action is needed, and plant designers have provided for limited maneuvering capability. Utilities must maintain a specific system-wide maneuverability, however, for two reasons: they must follow the daily cycle of load demand, and they must maintain a "spinning reserve" in case a generating unit is lost. The addition of large nuclear plants, which have no effective maneuvering capability, has led utilities to cycle the large fossil plants over a wider power range than assumed during design (85, 96, 97).

It is possible to improve the cycling capability of fossil plants through improved control strategies (88-92). Many studies have been made regarding improved tuning of existing control systems. There are significant structural problems in some older systems, however, which limits the potential improvements. Some experiments with feedforward configurations have netted better results. There have also been several investigations into application of modern control theory, which appear very successful in the modeling phase (93, 94). Still, there have been relatively few industrial applications of the available theory.

The work reported in this section consists of modeling a generic drum-type reheat boiler and studying several multivariable controller design strategies. The model is described in detail. Results of conventional multivariable single input-single output control are reported. Multivariable tuning regulator (MTR) control is applied to the plant model, and the results compared with MVSISO control. Finally the SVD controller is applied and results compared to use of both MVSISO and MTR control.

Model Description

The mathematical model developed for this work includes the boiler/drum and superheat/reheat sections of a subcritical drum-type reheat unit. This non-linear first-principles model was designed specifically for performing studies of steam temperature/flow dynamics under normal cycling conditions. Several authors have discussed boiler-turbine-generator models of this nature (e.g. 95-98) and the

transients reported by Hillery (85) were used to tune the dynamics of the model. The plant model was broken into eleven sections, each treated in a modular fashion:

- Coal mill/Furnace
- Steam generator waterwalls
- Drum
- Primary superheater
- Superheat spray
- Secondary superheater
- Throttle valves
- High pressure turbine
- Reheat spray
- Reheater
- Low pressure turbine

Figure 20 is a schematic diagram of the plant as modeled.

The available manipulated variables were feeder mill stroke rate, superheat and reheat furnace burner tilt angles, turbine throttle valve position, and superheat and reheat spray valve positions. The controlled variables were total generated power, steam temperature into the high pressure turbine, and steam temperature into the low pressure turbine. Pressures, temperatures, and flow rates throughout the plant were available for monitoring.

Coal Mill/Furnace

The coal crusher was a ring roller mill in which the pulverized coal feed rate approximates a first order lag of the mill stroke rate. The time constant of the lag is approximately the ratio of the mass of coal in the crusher zone of the mill to the coal feed rate.

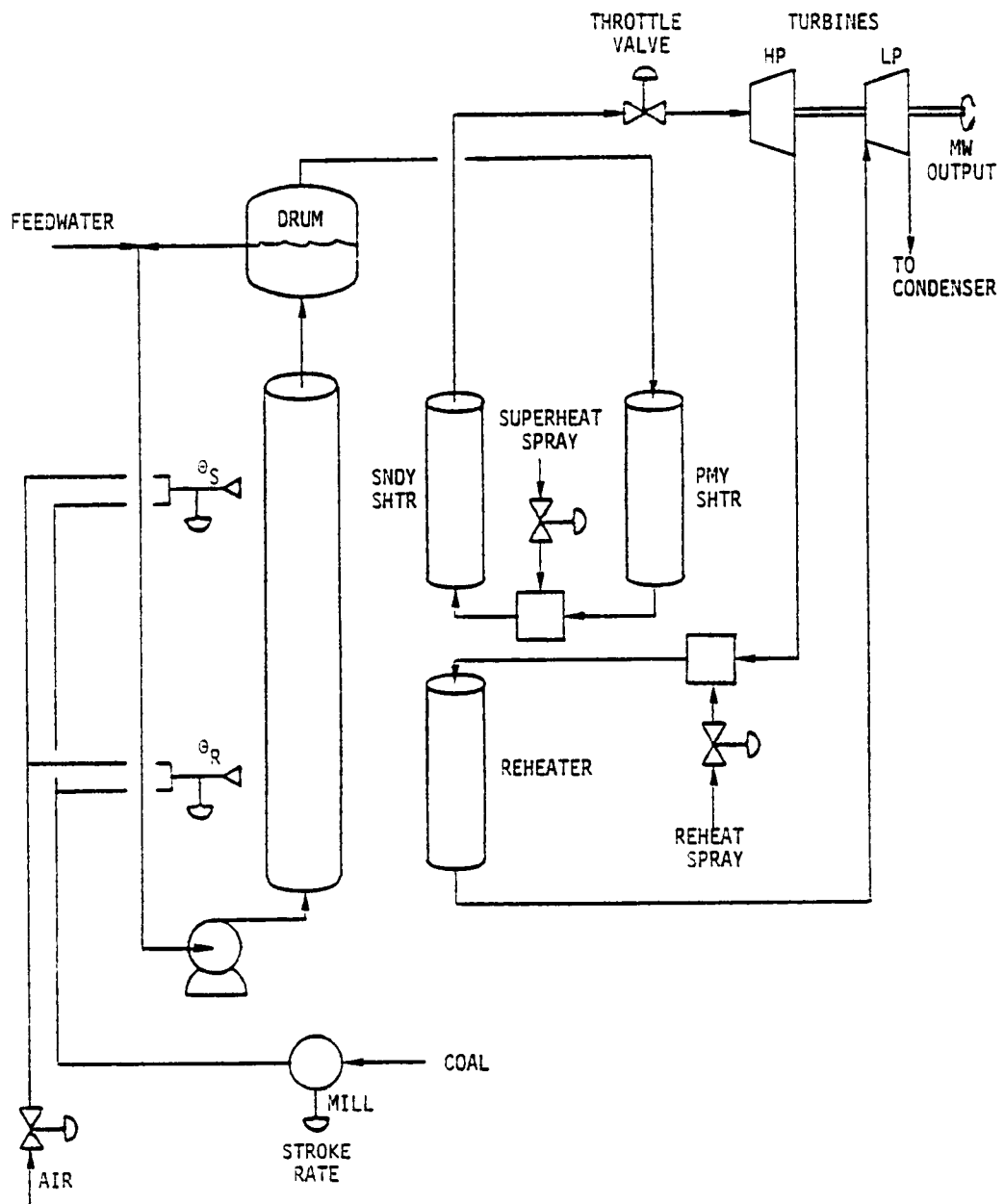


FIGURE 20. COAL-FIRED POWER PLANT MODEL SCHEMATIC.

The coal had a heating value of 12,000 BTU/lb as-received coal. A carbon/hydrogen/sulphur/nitrogen/oxygen/ash content typical of bituminous coal was used, producing a heating value of 14,241 BTU/lb dry ash-free (DAF) coal and a stoichiometric air requirement of 10.746 lb air/lb DAF coal. 15% excess air was used. An equation was derived which yielded adiabatic flame temperature, actual flame temperature, and flue gas temperature in terms of excess air and heat removal.

The furnace was split into two sections: the superheat furnace and the reheat furnace (see Figure 20). It was assumed that half the coal/air mixture was burned in each furnace. Radiant heat transfer occurs between the flames of each furnace and the waterwalls (where the steam is generated). The flue gases then pass on to the superheaters and reheater in the superheat furnace and reheat furnace, respectively. The flame temperature in each furnace will depend on, among other things, the amount of heat radiated to the waterwalls. This in turn depends on the radiant heat transfer coefficient, which can be adjusted by changing the form factor. This is the effect of the burner tilt. Each of the burner nozzles can be tilted $\pm 30^\circ$, effectively distributing the heat between the waterwalls and the flue gas. For instance, if the superheat burner nozzle is tilted down the radiant heat transfer coefficient will increase. More heat is transferred to the waterwalls, and more steam is generated. At the same time, the flue gas is cooler and less heat is delivered to the superheaters. More steam is generated, but it does not contain as much superheat. Burner tilt angles can be an effective manipulated variable.

Steam Generator Waterwalls

A differential equation was developed describing the heat balance on the waterwall metal mass. The heat into the wall is radiant heat from the combustion flame and is proportional to the difference in temperature raised to the fourth power. The heat out is that conducted into the saturated water on the water side of the wall. This is boiling heat transfer, and the rate of steam generation can be calculated directly from the heat transferred and the latent heat at the drum pressure.

Drum

The pressure in the drum is related to the rate of steam generation and the flow of steam out of the drum by Equation 54:

$$C_{pSG} \frac{dP_{Drum}}{dt} = W_{Gen} - W_{Drum} \quad , \quad (Eq. 54)$$

where W_{Gen} is the rate of steam generation, W_{Drum} is the rate of steam flow out of the drum, and C_{pSG} is the circulation loop capacitance, given by Equation 55

$$C_{pSG} = \frac{1}{h_{fg}} \left[m_{SG} \frac{\partial h_f}{\partial P_{Drum}} + m_{Drum} C_{pmtl} \frac{\partial T_{Drum}}{\partial P_{Drum}} \right] \quad , \quad (Eq. 55)$$

where h_{fg} = enthalpy of vaporization at drum pressure
 m_{SG} = mass of water in the drum and waterwalls
 h_f = saturated liquid enthalpy
 P_{Drum} = drum pressure

m_{Drum} = mass of metal in drum
 $C_{p_{\text{mtl}}}$ = specific heat of drum metal
 T_{Drum} = saturation temperature at drum pressure.

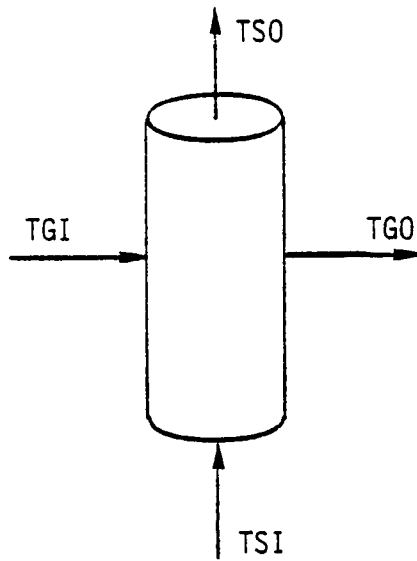
Throttle Valves

The position of the throttle valves controls the flow of steam around both loops (the high pressure turbine loop and the low pressure turbine loop). Choked flow is assumed to occur through the valves, so the flow is a function of the upstream pressure and valve opening only (i.e., independent of downstream pressure). The flow loss coefficients are adjusted so that a 50 psi drop occurs between the drum and the throttle valves in the steady state. A 50 psi drop is also maintained across the throttle valves in the steady state.

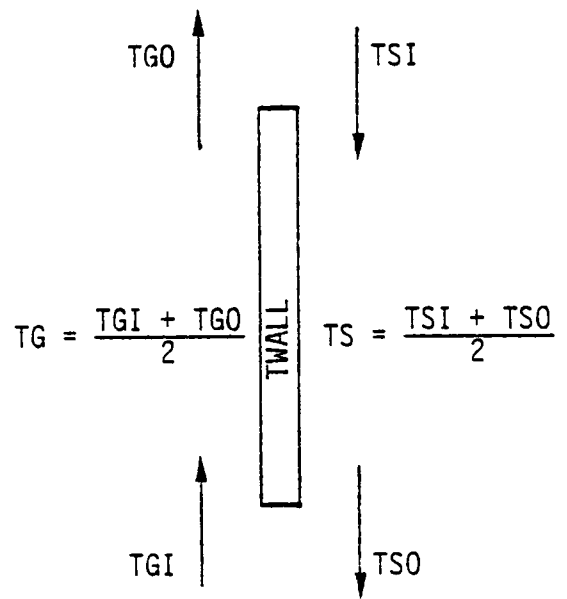
Flow through the high pressure turbine is assumed to lag the throttle valve flow by a 0.5 second time constant. Flow around the low pressure turbine loop and through the low pressure turbine lags the high pressure turbine flow by a 5.0 second time constant (96). Dead time is added to the properties transport based on assumed pipe sizes, lengths, and steam flowrate.

Superheater/Reheater Models

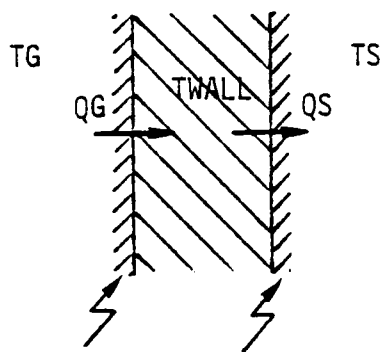
Figure 21 illustrates the development of models for the superheaters and the reheater. The overall objective is to calculate both the gas and steam outlet temperatures given the gas and steam inlet temperatures. It is first necessary to define an average gas side temperature and an average steam side temperature. Given these



(a) Overall Approach.



(b) Development of Average Temperatures.



Heat transfer coefficients,
proportional to flowrate.

(c) Heat Transferred and Wall Temperature.

FIGURE 21. DESIGN OF SUPERHEATER AND REHEATER MODELS.

temperatures and the wall metal temperature, a differential equation can be written describing the heat balance on the wall. From this information the total heat removal/addition rate for the gas and the steam can be determined. The new steam outlet temperature can be calculated from the heat addition rate and the water properties subroutine. The new gas outlet temperature can be calculated from the heat removal rate and the equation describing the flue gas temperature.

Superheat Spray/Reheat Spray

The superheat and reheat sprays add cold water to the steam in an effort to control steam temperature. They are modeled as an adiabatic mixing process. The cold water (70°F) is added with a steady state mass and energy balance (i.e. no dynamics are modeled), giving the new steam enthalpy. The steam temperature is determined using the water properties routine.

Turbine Models

In order to model the turbines it was necessary to fix the downstream pressures. In the steady state the pressure upstream of the high pressure turbine was 1900 psia. It is assumed that the HP turbine is a constant entropy process between the superheated upstream conditions and saturated vapor at 155 psia. Thus downstream conditions during the transients were calculated using the entropy of the inlet steam and a pressure of 155 psia.

Similarly, for the low pressure turbine the downstream pressure (that in the condenser) is assumed constant at 3.6 psia. A constant

entropy calculation across the LP turbine yields the downstream conditions.

In both cases the MW generation is taken as the difference between inlet and outlet enthalpies multiplied by the steam flowrate. All calculations assume an ideal turbine (100% efficiency).

Modeling Results/Identification

Table 8 is a summary of the important assumptions and steady state conditions used in the model. Open loop tests were performed on the system, and results are shown in Figures 22 through 28. Each figure shows the response of four controlled variables (total MW generation, drum pressure, HP turbine inlet temperature, and LP turbine inlet temperature) to one of seven manipulated variables. These responses compared favorably with those obtained for an operating plant by Hillery (85).

A nonlinear least squares routine from IMSL, using a Levenberg-Marquardt algorithm, was used to fit transfer function models to the open loop responses. The form of each transfer function was determined from the shape of the step response. The result was a transfer function matrix representing the dynamics of the plant around the 100% power range (Table 9).

Conventional Control Strategy

Multivariable single input-single output control was applied to the system first. Since there were only four controlled variables, only four of the seven manipulated variables could be used. Relative gain analysis was performed on the steady state gain matrix to

TABLE 8

ASSUMPTIONS AND INITIAL CONDITIONS FOR BOILER MODEL

Coal Heating Value	12,000 BTU/lb as received 14,241 BTU/lb dry ash-free
Stoichiometric Air Required	10.746 lb/lb DAF coal
Air Used	15% excess
Generation Capacity	200 MW
High Pressure Turbine	82.08 MW
Low Pressure Turbine	117.92 MW
Coal Feed Rate	44.68 lb/s
Flame Temperature	2522°F
Pressures	
Drum/Waterwall	2000 psia
Throttle Valve	1950 psia
HP Turbine Inlet	1900 psia
HPT Outlet/Reheater/LPT Inlet	155 psia
LP Turbine Outlet/Condenser	3.6 psia
Temperatures	
Drum/Waterwall	636°F
HP Turbine Inlet	1000°F
LP Turbine Inlet	1000°F
Superheat/Reheat Spary Water	70°F

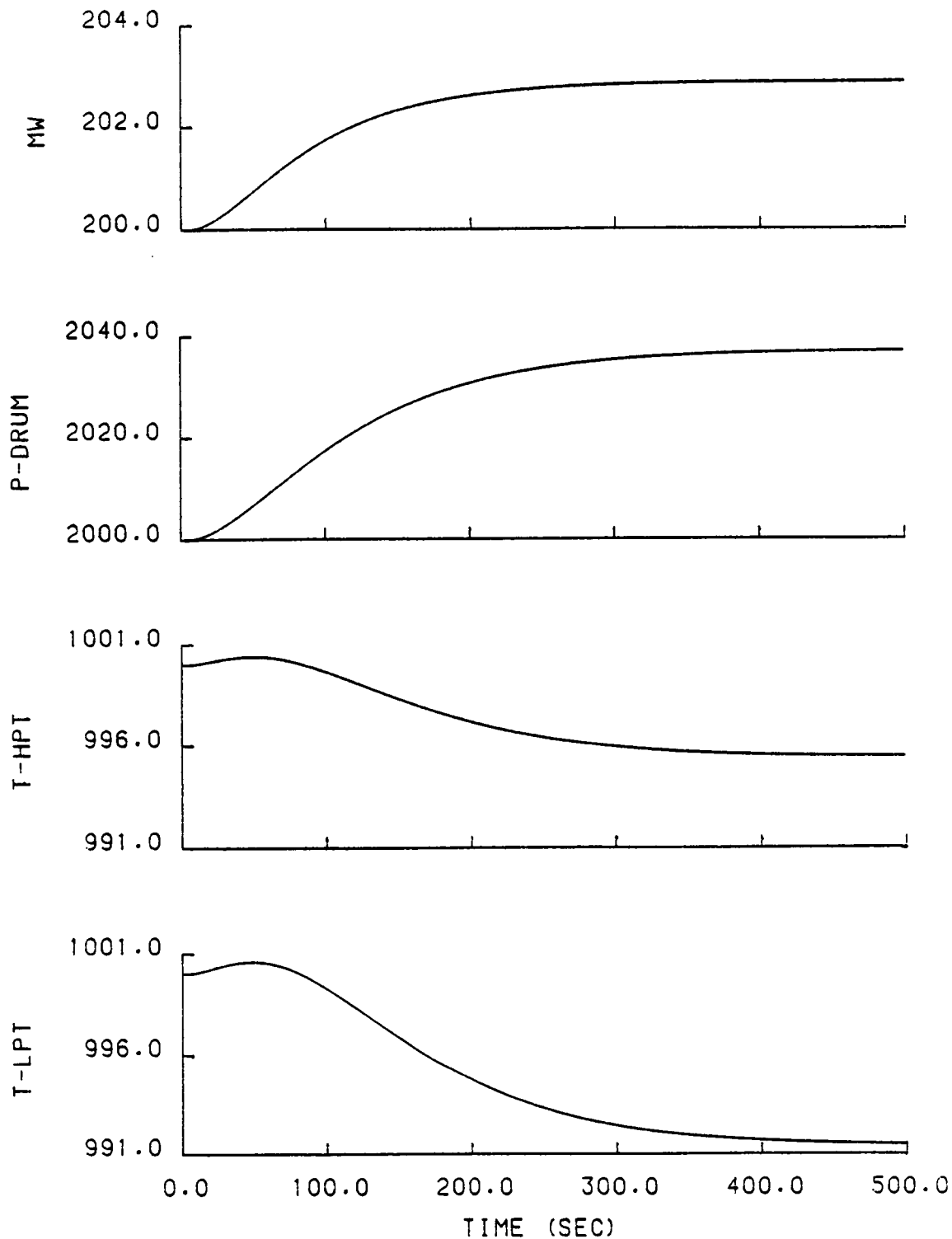


FIGURE 22. POWER PLANT RESPONSE TO STEP CHANGE IN COAL MILL STROKE RATE.

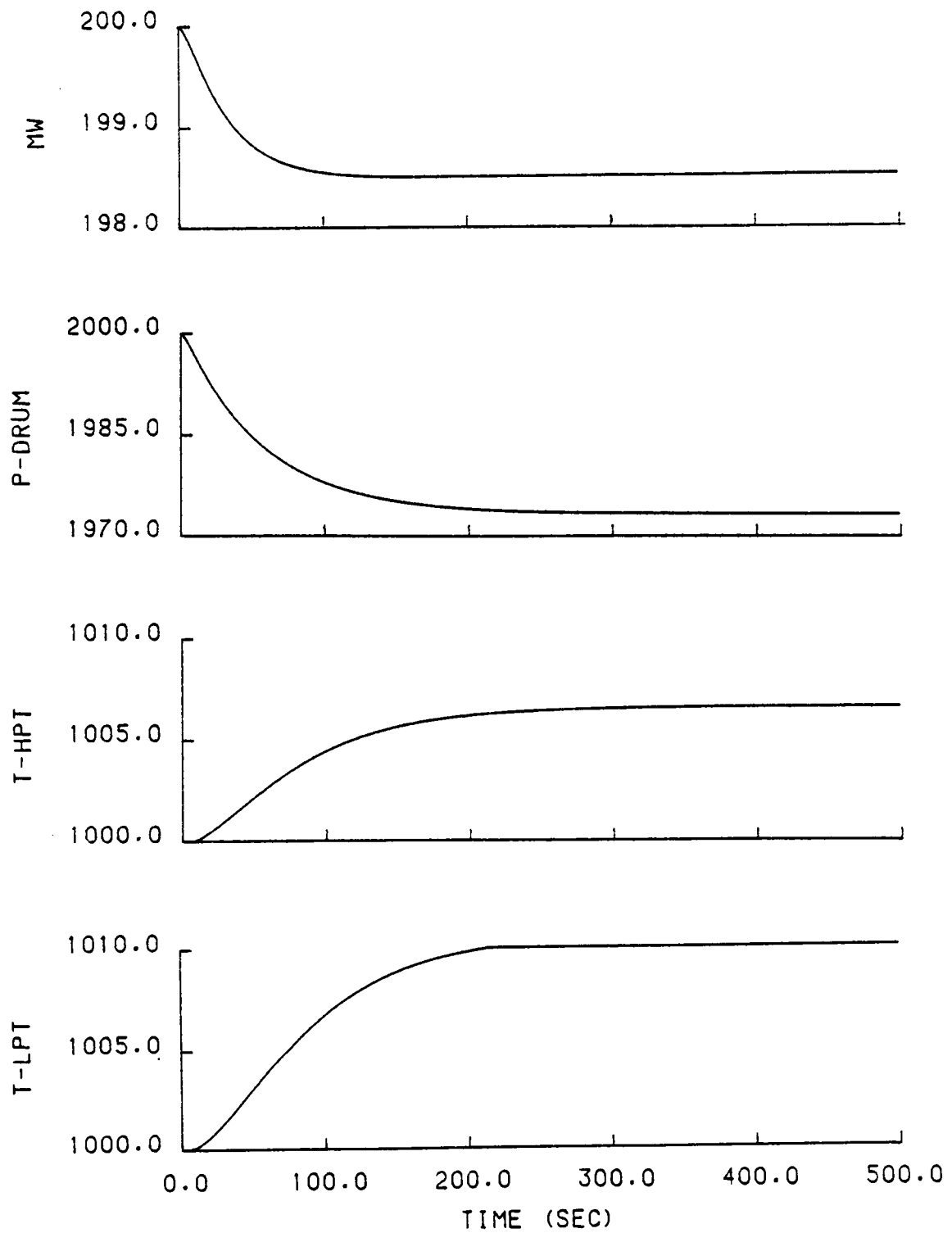


FIGURE 23. POWER PLANT RESPONSE TO STEP CHANGE IN PRIMARY AIR.

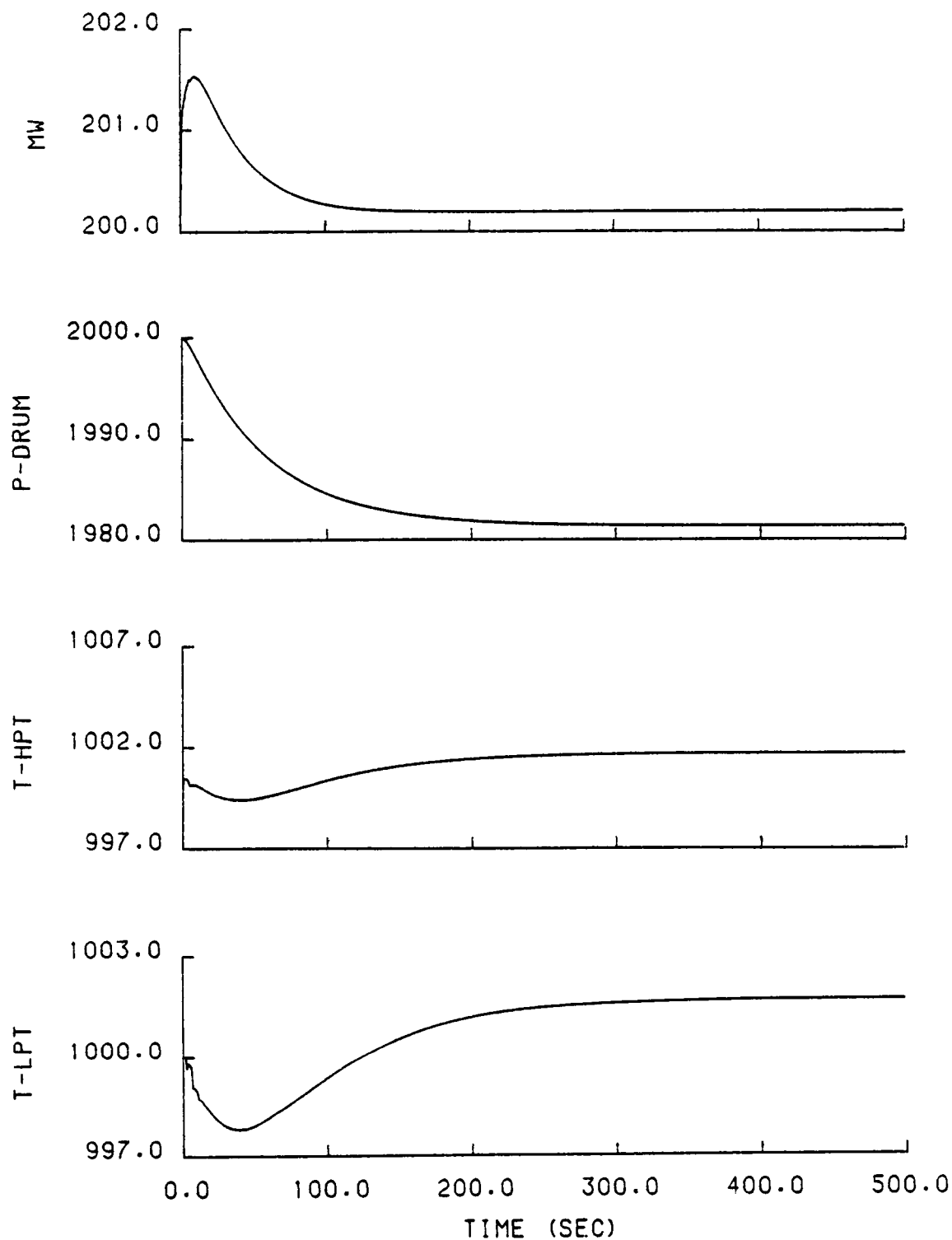


FIGURE 24. POWER PLANT RESPONSE TO STEP CHANGE IN THROTTLE VALVE POSITION.

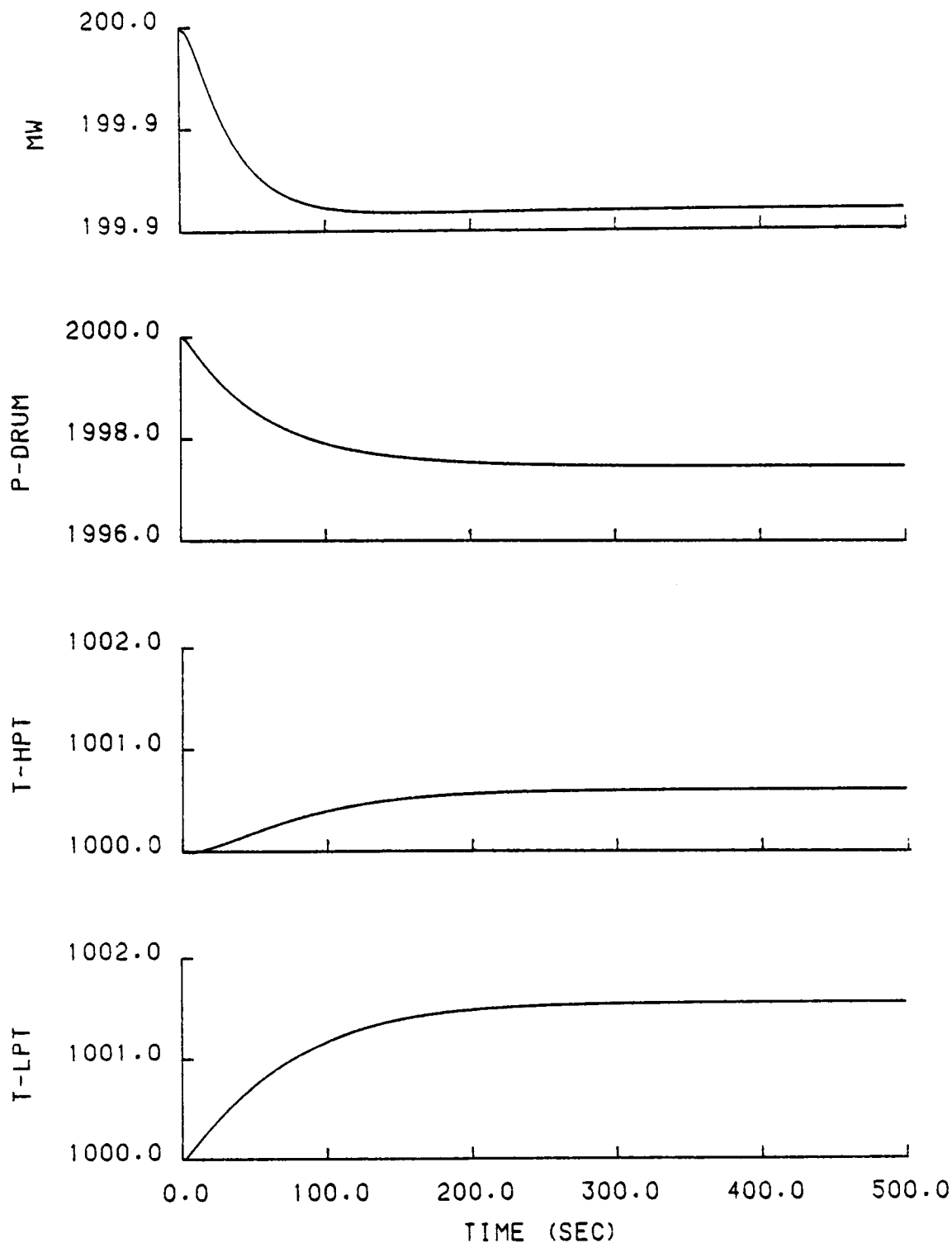


FIGURE 25. POWER PLANT RESPONSE TO STEP CHANGE IN REHEAT FURNACE BURNER TILT ANGLE.

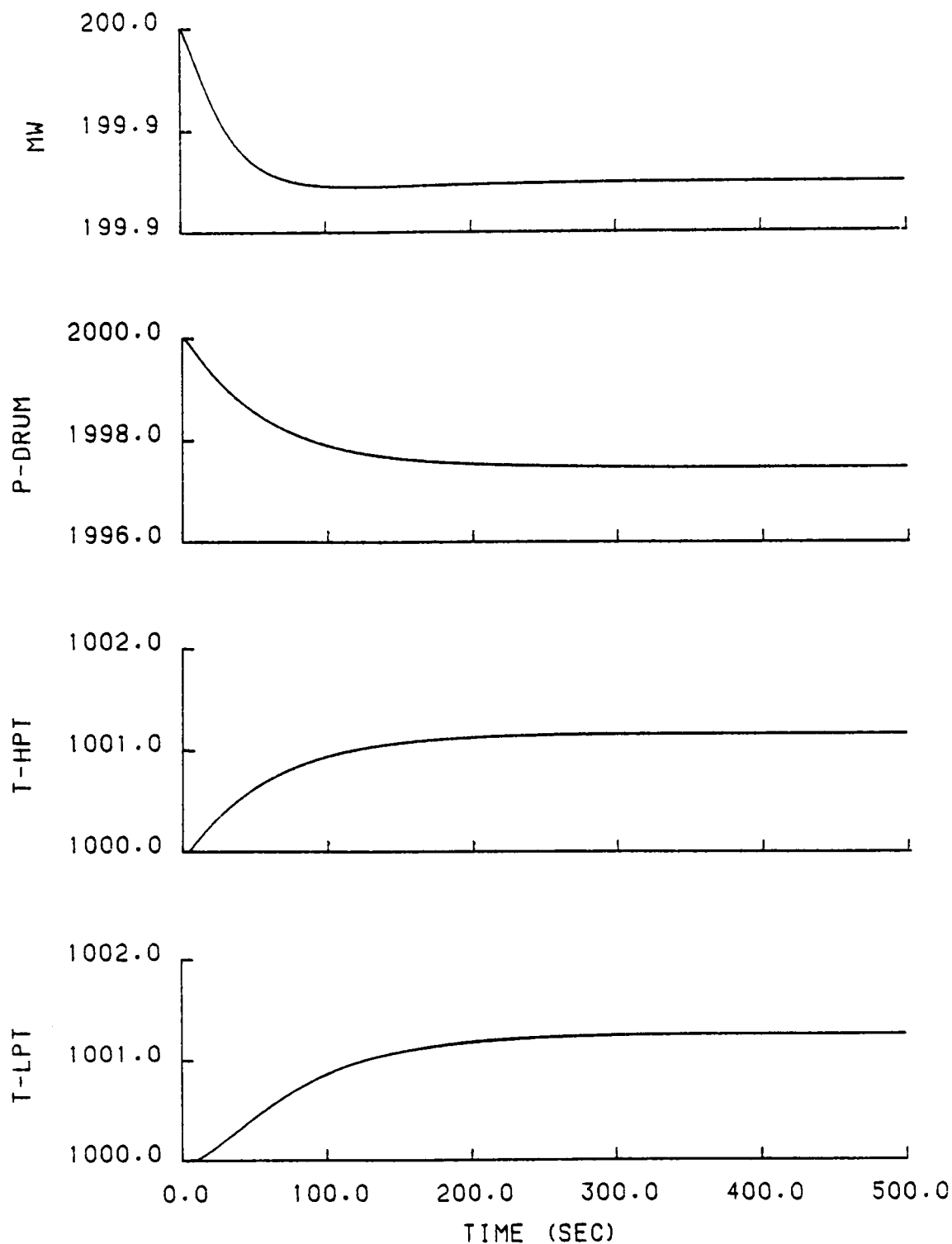


FIGURE 26. POWER PLANT RESPONSE TO STEP CHANGE IN SUPERHEAT FURNACE BURNER TILT ANGLE.

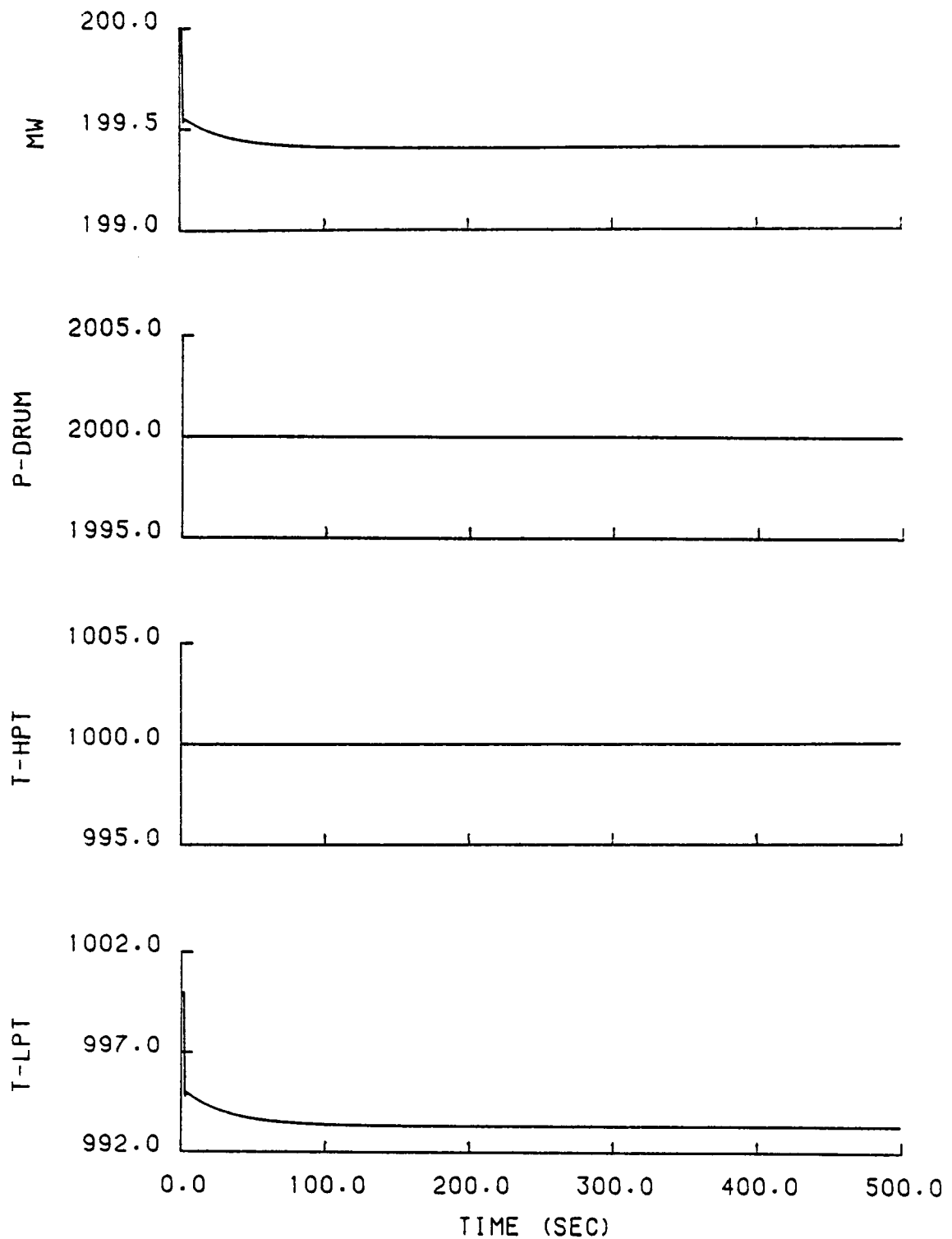


FIGURE 27. POWER PLANT RESPONSE TO STEP CHANGE IN REHEAT SPRAY.

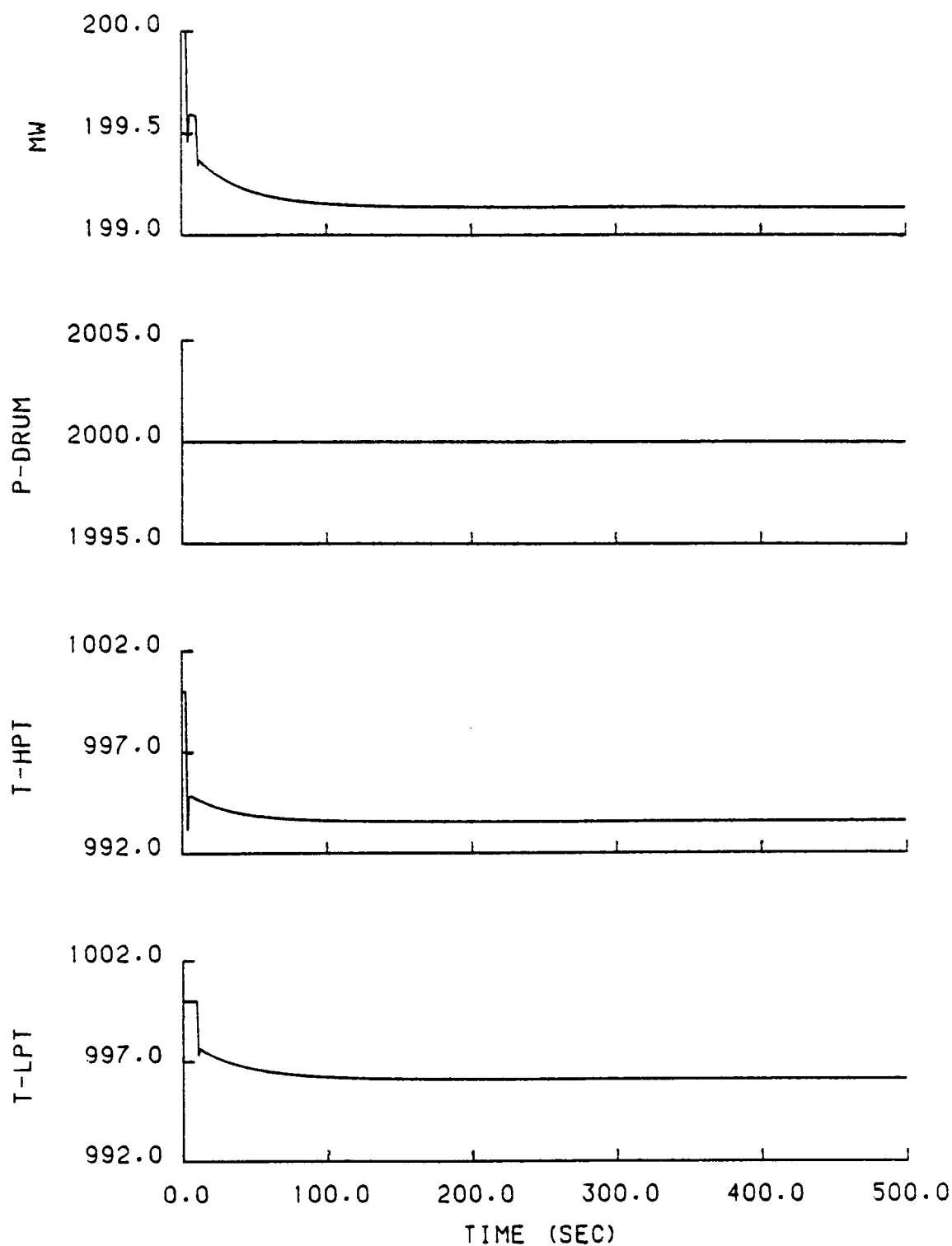


FIGURE 28. POWER PLANT RESPONSE TO STEP CHANGE IN SUPERHEAT SPRAY.

TABLE 9
TRANSFER FUNCTION MATRIX REPRESENTATION OF BOILER MODEL

STROKE										
AIRVLV	$\frac{6.18}{(1+37.7s)(1+57.4s)}$	$\frac{-294}{(1+33.9s)}$	$\frac{20.8(1+335s)}{(1+3.68s)(1+31.75s)}$	$\frac{-8.59}{(1+33.0s)}$	$\frac{-7.64}{(1+25.6s)}$	$\frac{-0.74}{(1+1.63s)}$	$\frac{-0.95}{(1+11.7s)}$	$\begin{bmatrix} MW \\ P_{DRUM} \\ T_{HPT} \\ T_{LPT} \end{bmatrix}$		
AVALVE	$\frac{80.3}{(1+63.2s)(1+62.6s)}$	$\frac{-5580}{(1+57.0s)}$	$\frac{-1970}{(1+57.4s)}$	$\frac{-239}{(1+55.1s)}$	$\frac{-239}{(1+55.1s)}$	0	0			
THETAR	$\frac{-10.3(1-61.3s)}{(1+73.1s)(1+75.3s)}$	$\frac{1336}{(1+35.9s)(1+50.7s)}$	$\frac{178(1-76.7s)}{(1+45.8s)(1+44.6s)}$	$\frac{55.4}{(1+41.2s)(1+50.0s)}$	$\frac{110}{(1+58.5s)}$	0	$\frac{-6.44}{(1+7.52s)}$			
THETAS	$\frac{-22.6(1-48.9s)}{(1+76.9s)(1+70.3s)}$	$\frac{2160}{(1+46.5s)(1+46.5s)}$	$\frac{172(1-198s)}{(1+35.9s)(1+53.4s)}$	$\frac{145}{(1+10.0s)(1+58.2s)}$	$\frac{117}{(1+40.8s)(1+40.8s)}$	$\frac{-8.25}{(1+1.60s)}$	$\frac{-4.85}{(1+13.6s)}$			
WRSRV								$=$		
WSSPRV										

determine the best four, and the pairing to use. It was concluded that power generation should be controlled by manipulating the throttle valve, drum pressure by manipulating the coal feeder stroke rate, and the high and low pressure turbine inlet steam temperatures by manipulating the superheat and reheat spray valves, respectively. The arrangement corresponds to the classical boiler-following-turbine strategy employed in many plants.

Tuning for each of the four loops was done by hand, then optimized using a PATTERN search routine. At first the COST function was integral absolute error (IAE), total for all loops. Since the four controlled variables had different units (MW, psia, and °F), the contribution of each to the IAE was weighted to reflect that variable's importance. The tuning optimized with this procedure exhibited ringing however, especially in the throttle valve. To compensate for this, a term was added to the COST function to penalize throttle valve movement. When the tuning was optimized in this manner the resulting responses closely resembled those reported by Hillery (85).

Tuning was optimized for a step change of -10% in power demand (MW). The system response to the change is shown in Figure 29. The MW response is good, and the steam temperature excursions are not extreme. (Steam temperature swings of more than 20°F cause thermal stresses on turbine blades, which can lead to cracking.) The drum pressure swing is large, however. (Large drum pressure excursions cause excessive boiler tube stress, which eventually leads to stress corrosion cracking.)

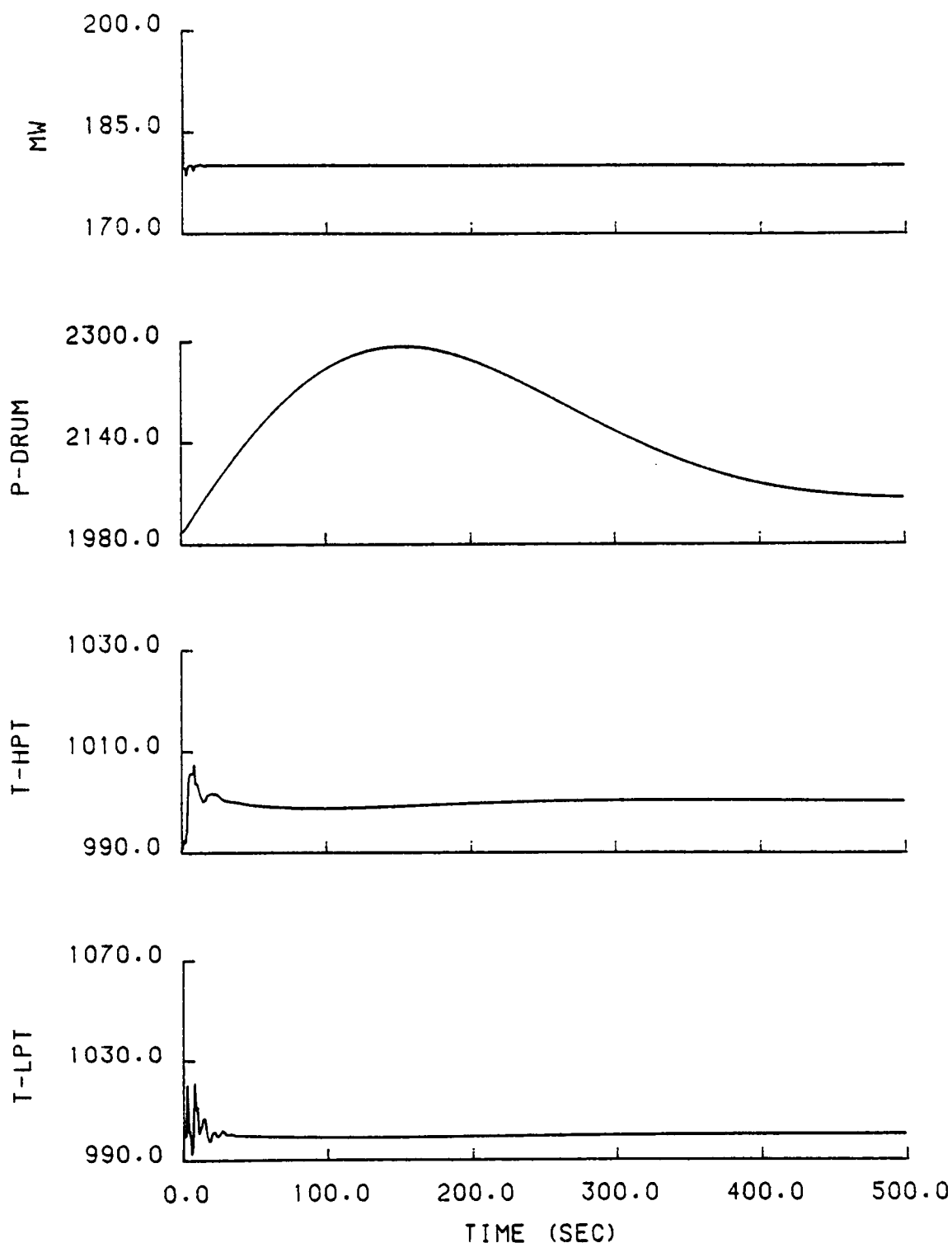


FIGURE 29. RESPONSE OF PLANT UNDER MVSISO CONTROL TO -10% STEP CHANGE IN POWER DEMAND.

Step changes are rare in practice, occurring only in emergency situations. The most common maneuver is a slow (less than 10% per minute) ramp. Figure 30 shows the system response to a -10% per minute ramp to 75% power. Generated power follows the ramp well, but the other variables exceed allowable limits. Clearly, using this control strategy a 10% per minute ramp to 75% power is too large a change too fast.

To achieve the increased maneuverability that utilities desire, much improvement would be realized if ramp rates could be increased to 30% per minute. Figure 31 shows the system response to a ramp set point decrease of 30% per minute to 75% power. The power generation tracks the set point nearly perfectly, but the three other controlled variables exceed limits.

The MVSISO control strategy is adequate for small (less than 10%) and slow (less than 10% per minute) set point changes. This conclusion is corroborated by experience of the power industry. A different control technique is clearly required if increased maneuverability is to be achieved.

The Multivariable Tuning Regulator

A recent development in multivariable controller design is the introduction of the multivariable tuning regulator (100,101,102). The multivariable tuning regulator (MTR) splits the proportional and integral modes apart, as shown in Figure 32.

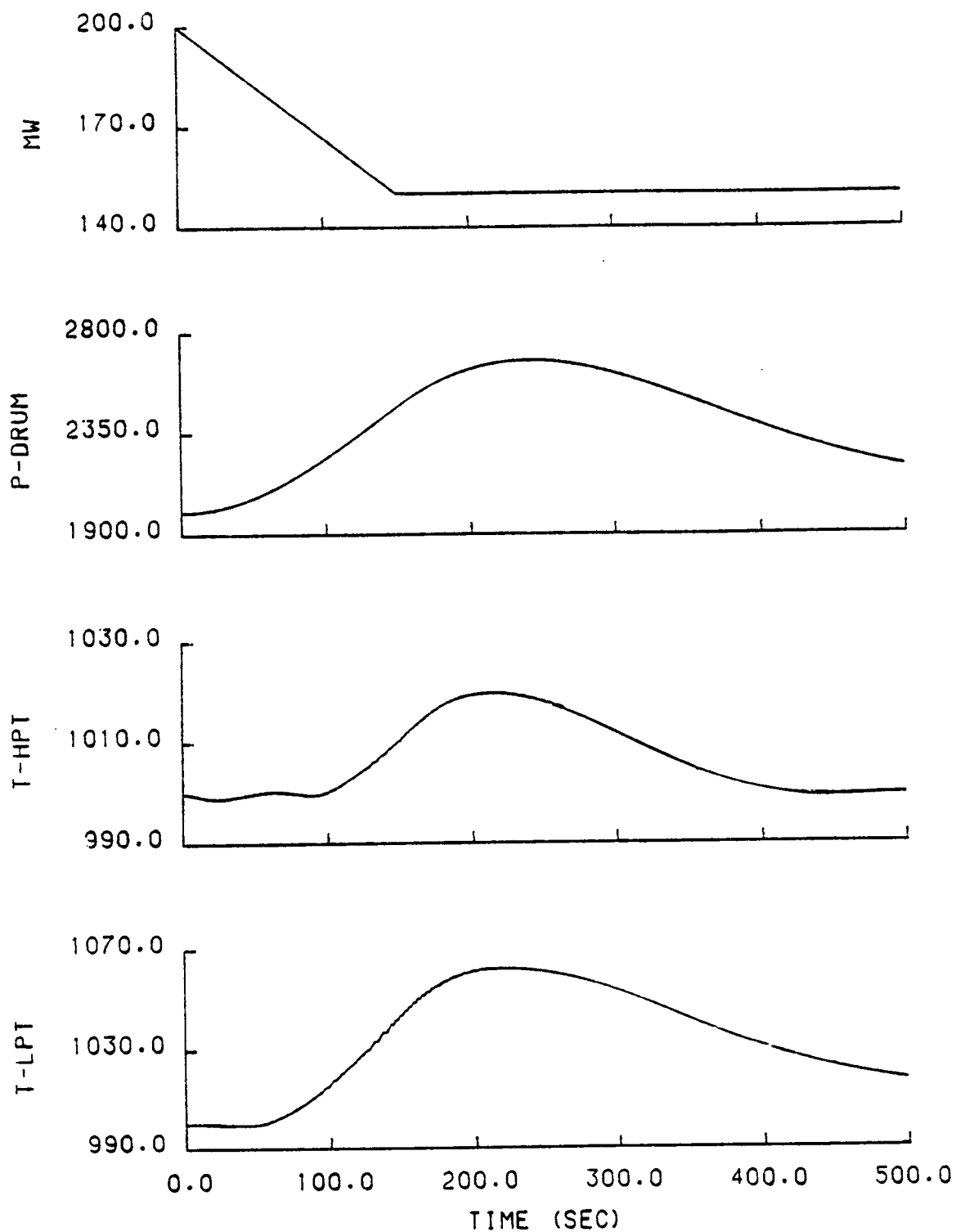


FIGURE 30. RESPONSE OF PLANT UNDER MVSISO CONTROL TO -10%/MIN RAMP TO 75% POWER.

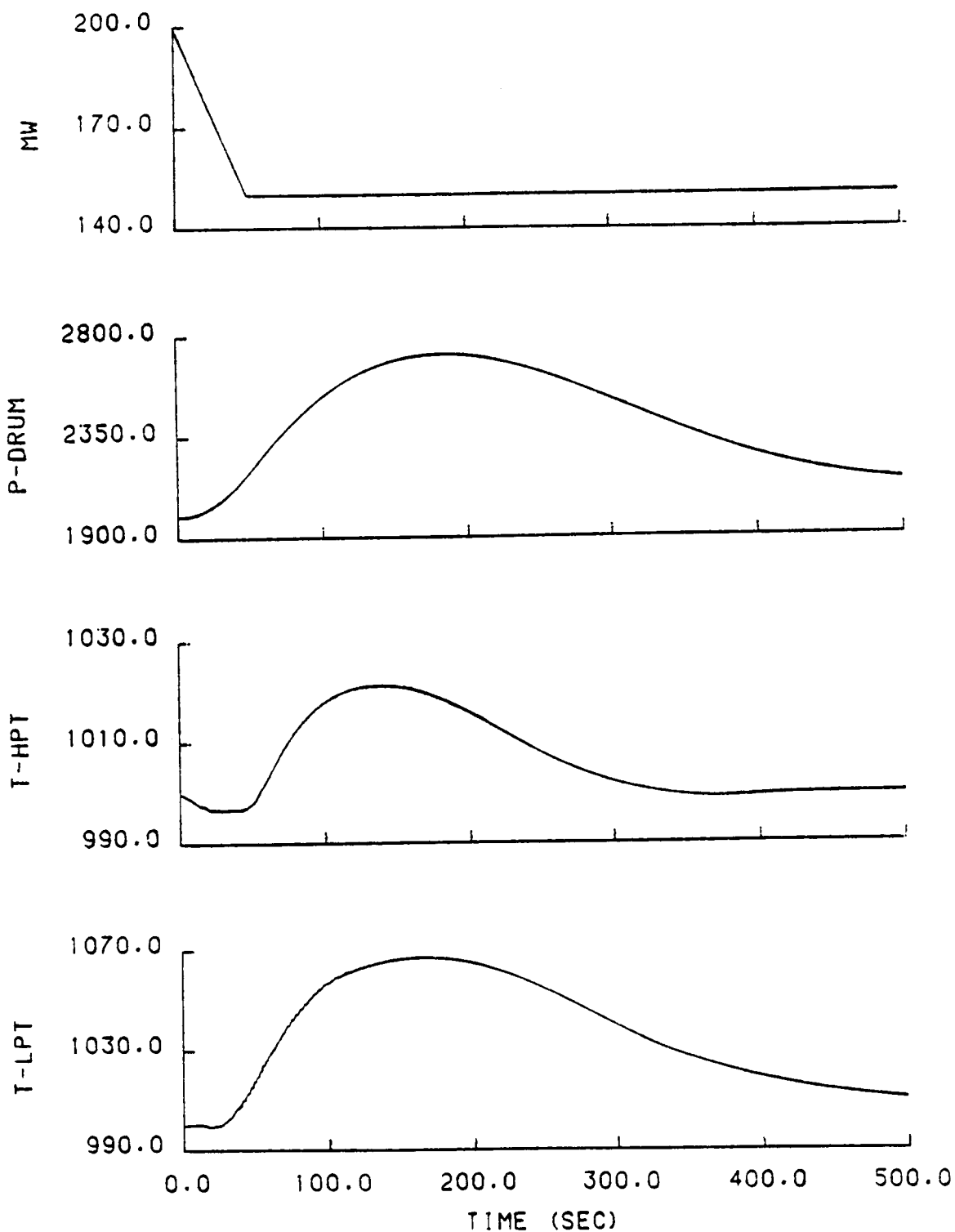


FIGURE 31. RESPONSE OF PLANT UNDER MVSISO CONTROL TO -30%/MIN RAMP TO 75% POWER.

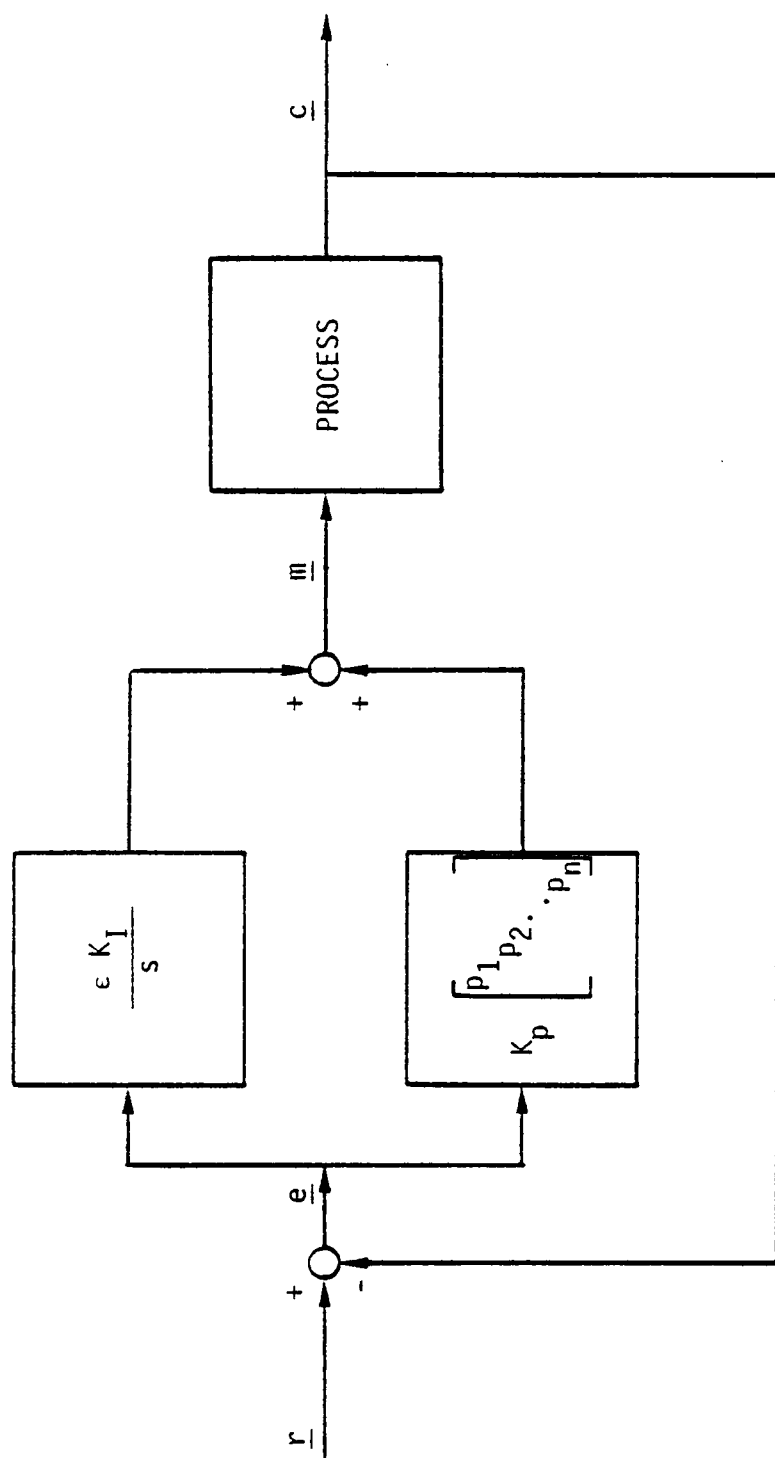


FIGURE 32. STRUCTURE OF THE MULTIVARIABLE TUNING REGULATOR.

The integral mode is a matrix which acts on the errors, $\underline{e}$, as $K_I \epsilon/s$. In this expression $1/s$ represents the integrator. K_I is the inverse (or pseudo-inverse in the case of a nonsquare system) of the steady state gain matrix. ϵ is an adjustable (scaler) parameter used to tune the integral mode. In this configuration the integral action is thought of as a steady state controller.

The proportional mode is a different matrix, formulated as the combination $K_p P$ where P is the diagonal matrix shown in Figure 32. K_p is the inverse (or pseudo-inverse) of the matrix of initial slopes of open loop step responses. Given the open loop responses (e.g. Figures 22 through 28), a matrix is formed of the slopes at time zero. K_p is the inverse of the matrix. The diagonal matrix P contains the proportional gains, p_i , for each loop. Because the proportional mode acts instantaneously, it is interpreted as "seeing" a process gain matrix which looks like the derivative matrix.

As for the MVSISO controller, the tuning was optimized for a step change of -10% in the MW setpoint. Figure 33 shows the system response to this input. The power generation does not respond as fast as in the MVSISO case, but there is a significant improvement in drum pressure control. None of the pressure or temperature constraints is approached, and a larger step change could be made with no problem.

As before, the desired power was decreased from 100% to 75% at rates of 10% per minute and 30% per minute. The responses to these changes are shown in Figures 34 and 35, respectively. In both cases, power generation tracks the setpoint well and the drum pressure has

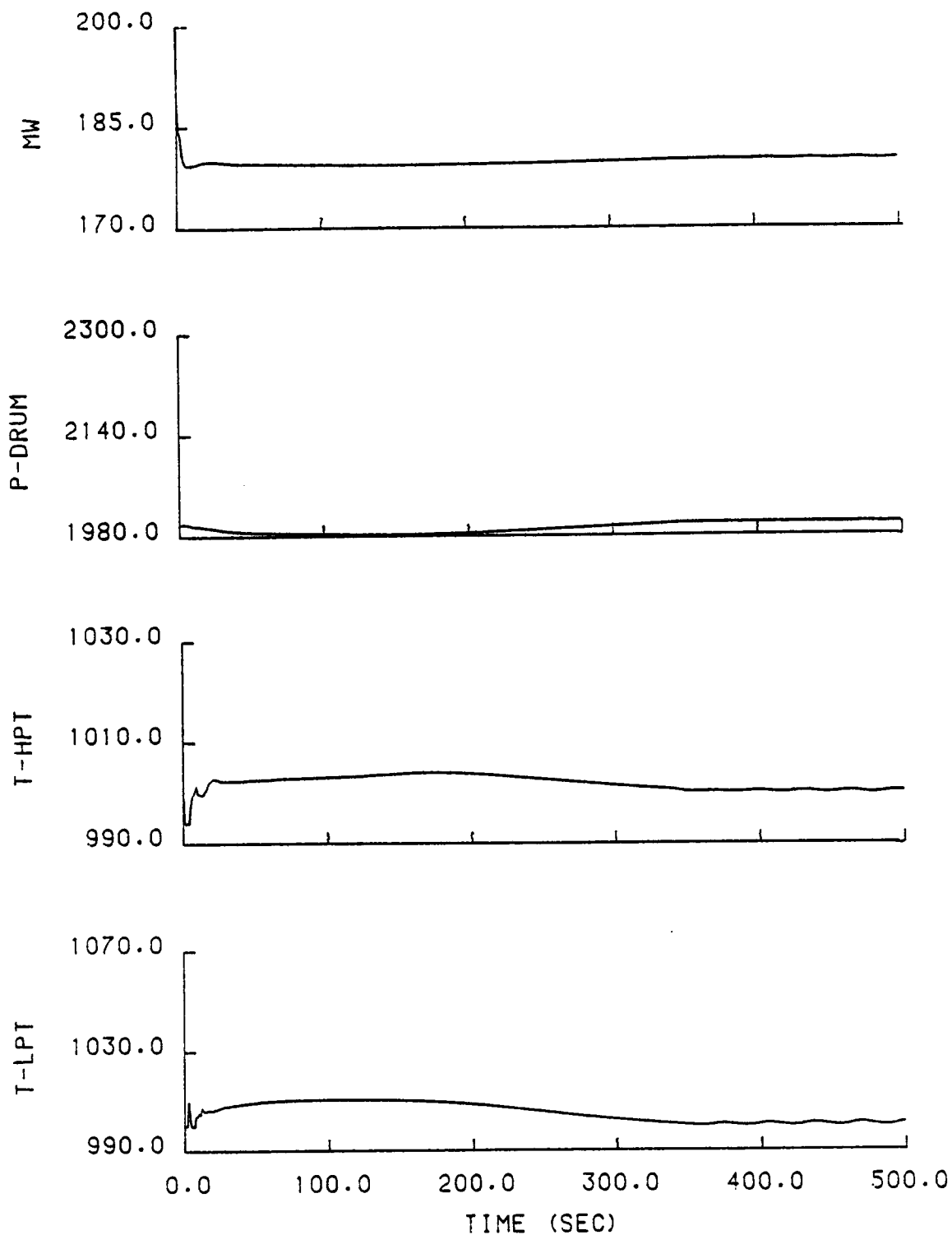


FIGURE 33. RESPONSE OF PLANT UNDER MTR CONTROL TO -10% STEP CHANGE IN POWER DEMAND.

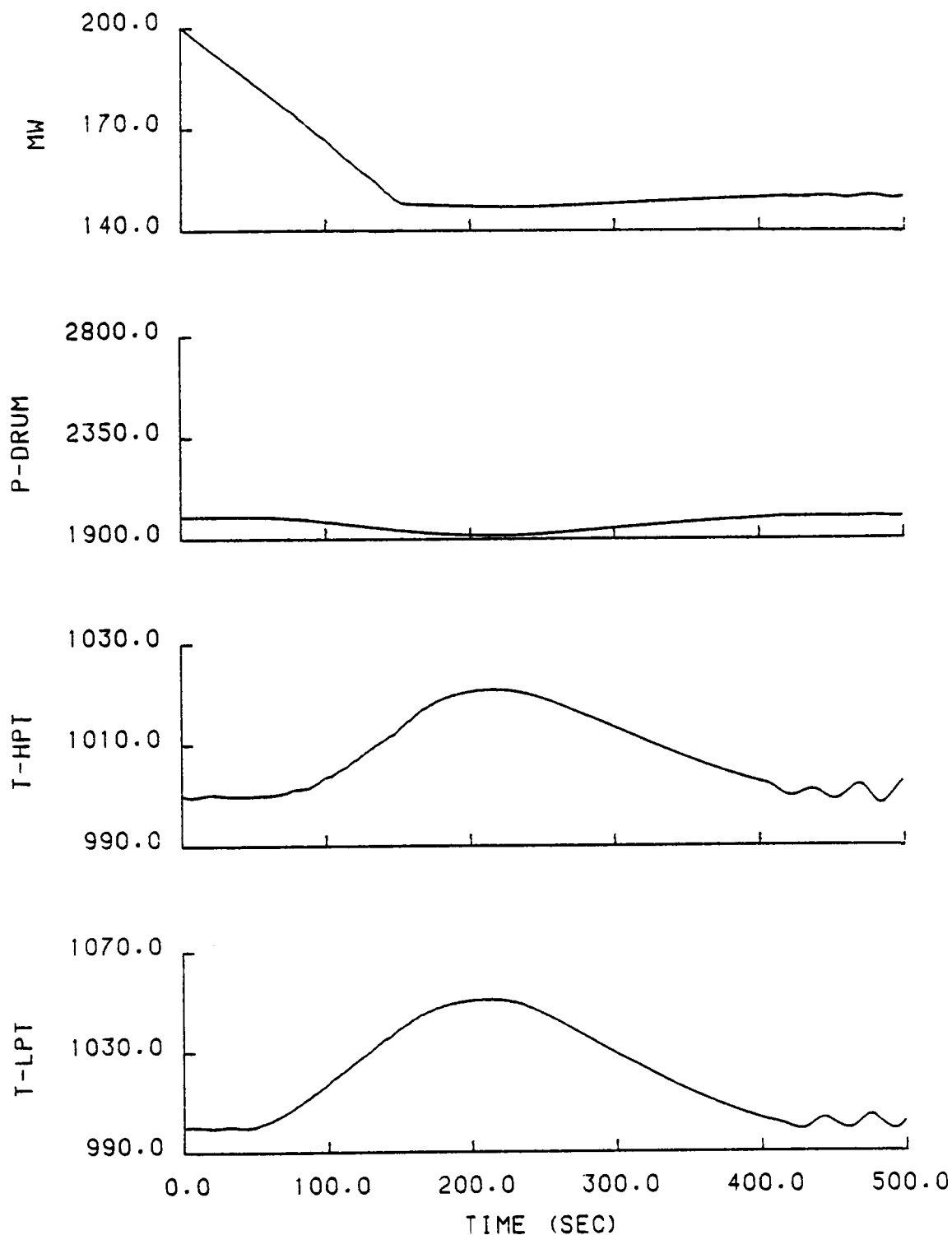


FIGURE 34. RESPONSE OF PLANT UNDER MTR CONTROL TO -10%/MIN RAMP TO 75% POWER.

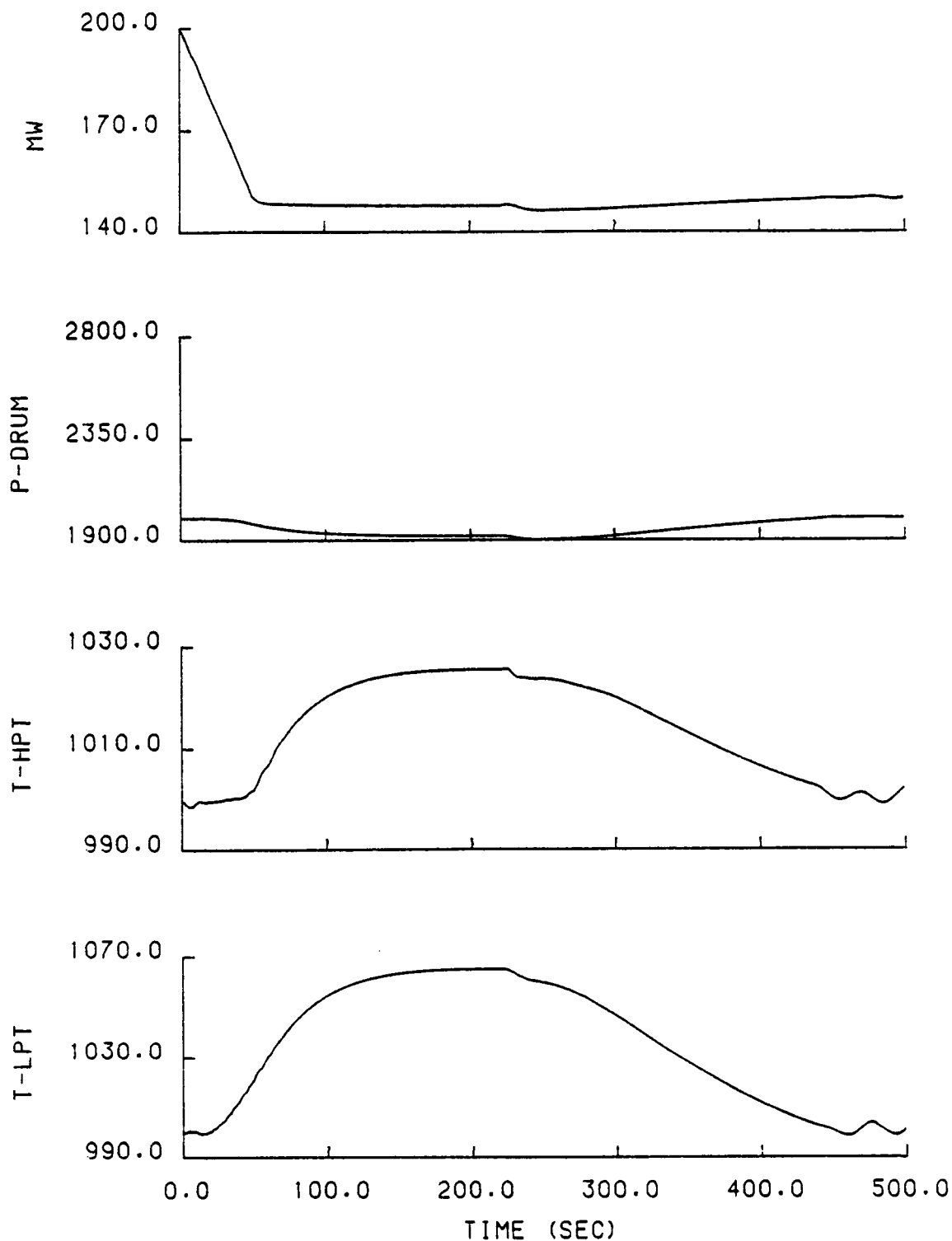


FIGURE 35. RESPONSE OF PLANT UNDER MTR CONTROL TO -30%/MIN RAMP TO 75% POWER.

little deviation. In both cases the temperature excursions are significant, however. One reason the ramps to 75% power are less well behaved than the step to 90% is the nonlinearity of the system, an explanation which also applies to the MVSISO case. Tuning which was optimized in the upper 10% of the operating range is no longer optimum at 75% power. Nonlinearities cause the system dynamics to differ significantly in different operating ranges.

In an effort to study this, the multivariable tuning regulator was optimized (the ϵ and p_i 's) using each of the ramps as the test scenario. The tuning was optimized for a 25% decrease in setpoint at the same two rates (10% per minute and 30% per minute, Figures 36 and 37). The performance is improved over that using the step-optimized parameters, but there are still significant excursions. Also, as with the step-optimized tuning parameters, the steam temperatures exhibit a small amount of ringing toward the end of their response. This might be corrected by the addition of a penalty for spray valve movement in the COST function.

In addition to the nonlinearities, there is another explanation for the poorer MTR performance during ramp changes. The proportional matrix K_p is the (pseudo-) inverse of the derivative matrix (the matrix of slopes of the step response curves). The response of the system to a ramp input in each of the manipulated variables is different than to a step. For physical systems in which the order of the denominator is greater than the order of the numerator, the slope of the response to a ramp input will be zero at time equal zero. The

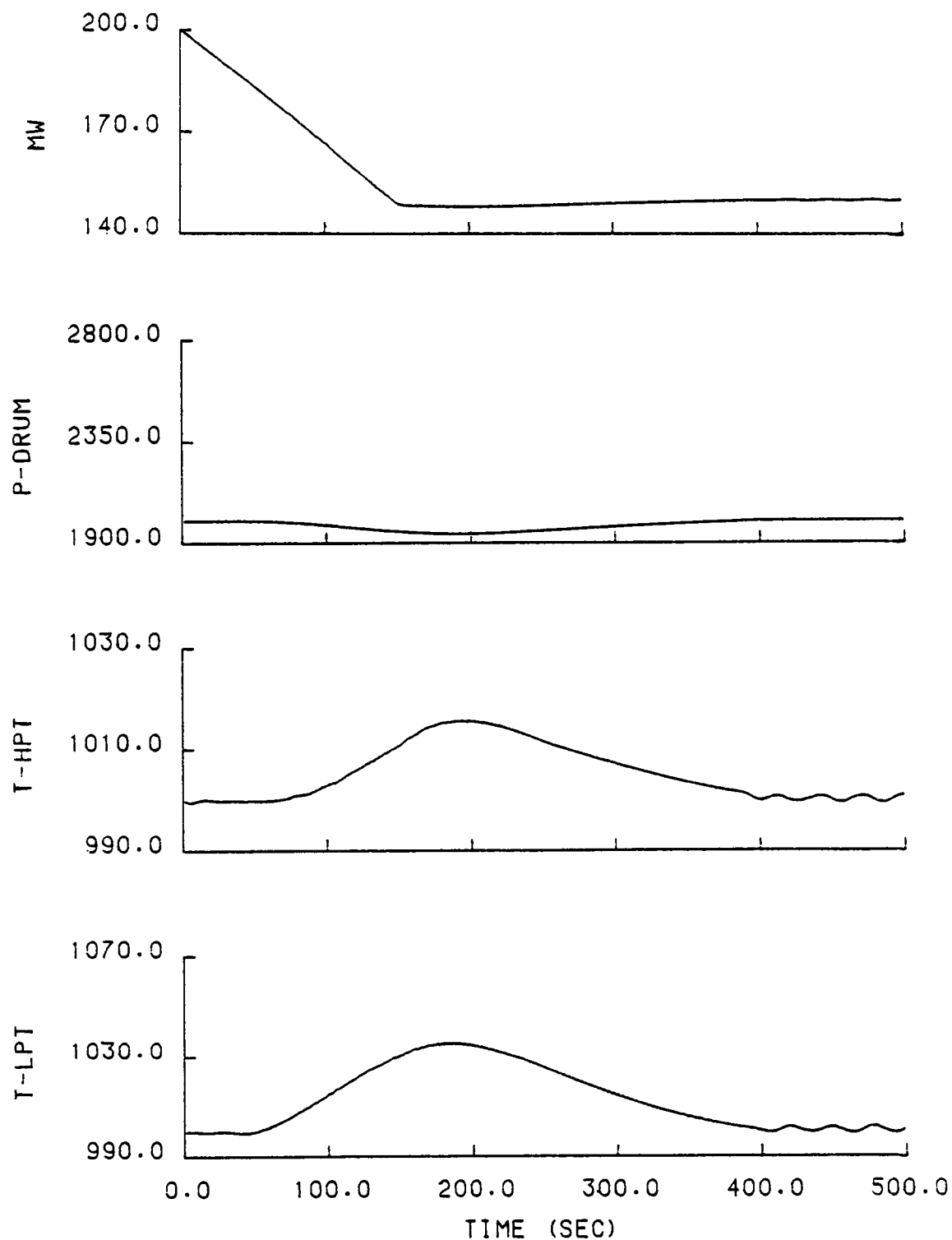


FIGURE 36. RESPONSE OF PLANT UNDER MTR CONTROL TO -10%/MIN RAMP TO 75% POWER, OPTIMUM TUNING.

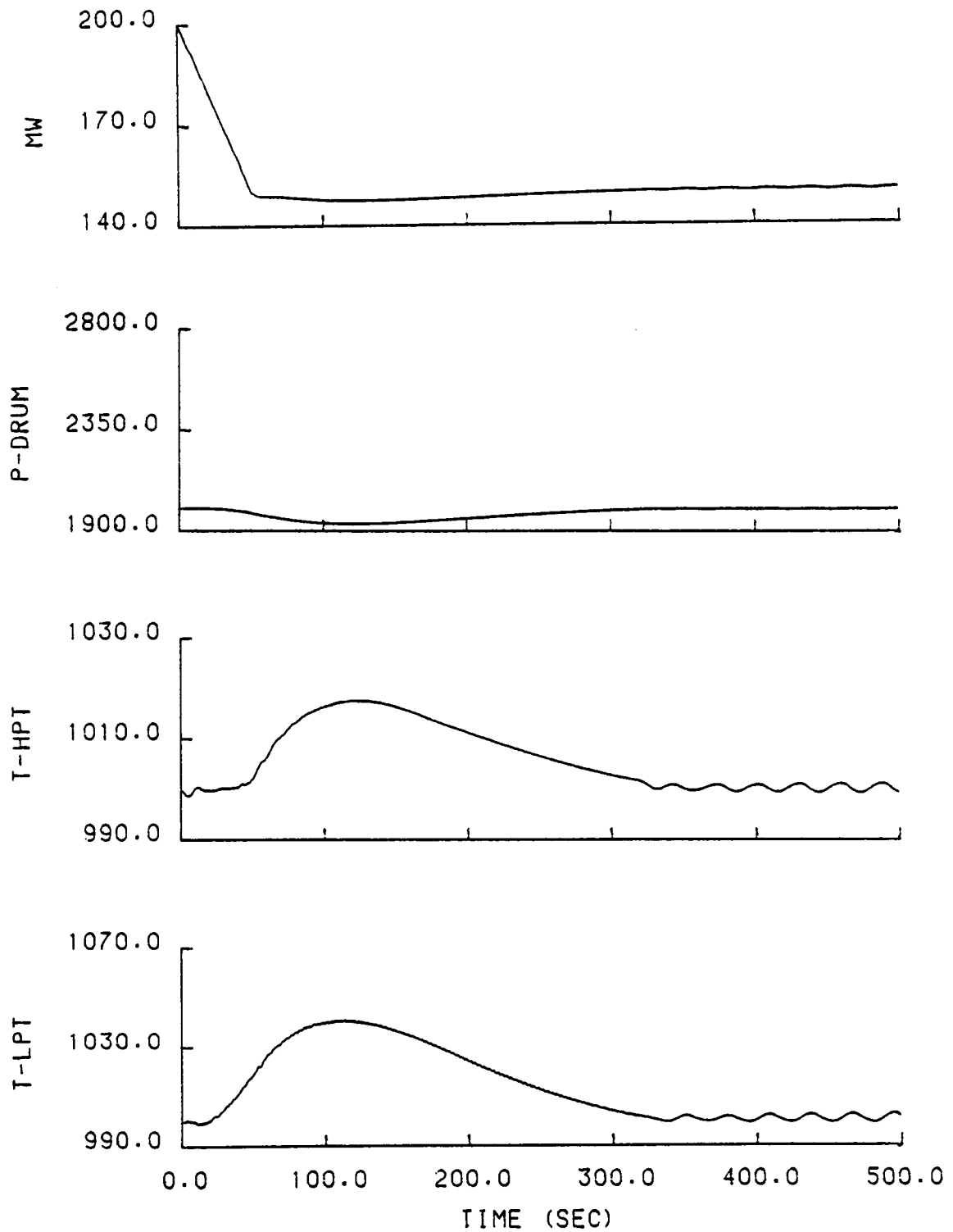


FIGURE 37. RESPONSE OF PLANT UNDER MTR CONTROL TO -30%/MIN RAMP TO 75% POWER, OPTIMUM TUNING.

matrix K_p , determined from step responses, is inappropriate for any other input.

The multivariable tuning regulator, in this example, performed well for the step setpoint change. It gave significant improvement over MVSISO control in maintaining drum pressure. For cases of ramp inputs, the temperature control was only slightly better than MVSISO.

Singular Value Decomposition Control

The 4×7 steady state process gain matrix was decomposed using an IMSL singular value decomposition routine. The resulting U and V matrices were implemented in an SVD control structure. As before, the proportional and integral tuning was optimized for a -10% step change in generation demand. The system response to this step change is shown in Figure 38. The MW response is not as fast as in the MVSISO structure, but the drum pressure control is nearly perfect. The steam temperature control is also good, with no variable approaching its constraint. As in the MTR case, a larger step change can be made before problems are encountered.

As before, 25% decreases in power demand were made at 10% per minute and 30% per minute (Figures 39 and 40). This is where significant improvement over the other techniques is exhibited. In both cases, good control of all variables is maintained. The different process characteristics at 75% power have not adversely affected the control. In this example the SVD controller has proven to be very robust. The reason for this robustness is the

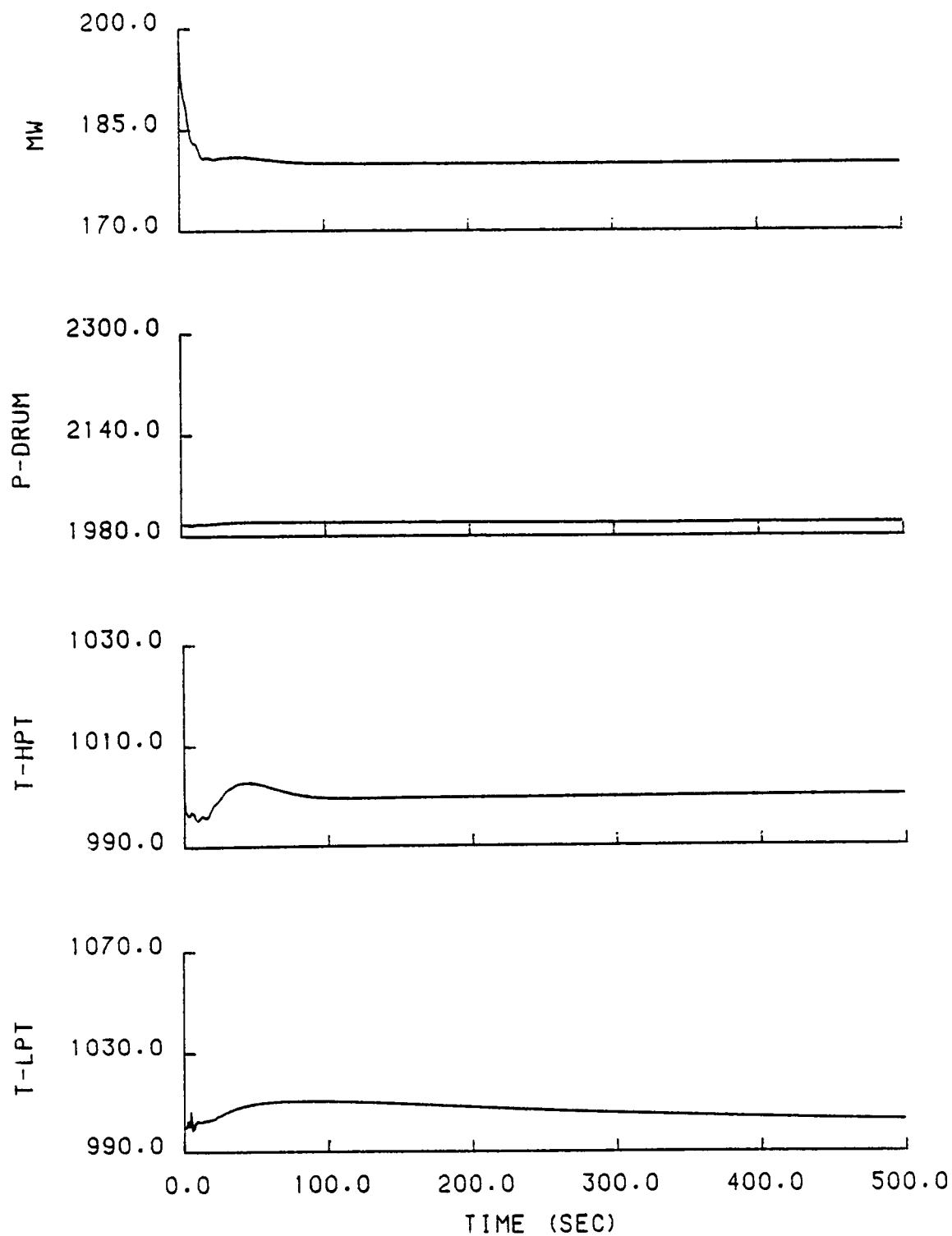


FIGURE 38. RESPONSE OF PLANT UNDER SVD CONTROL TO -10% STEP CHANGE IN POWER DEMAND.

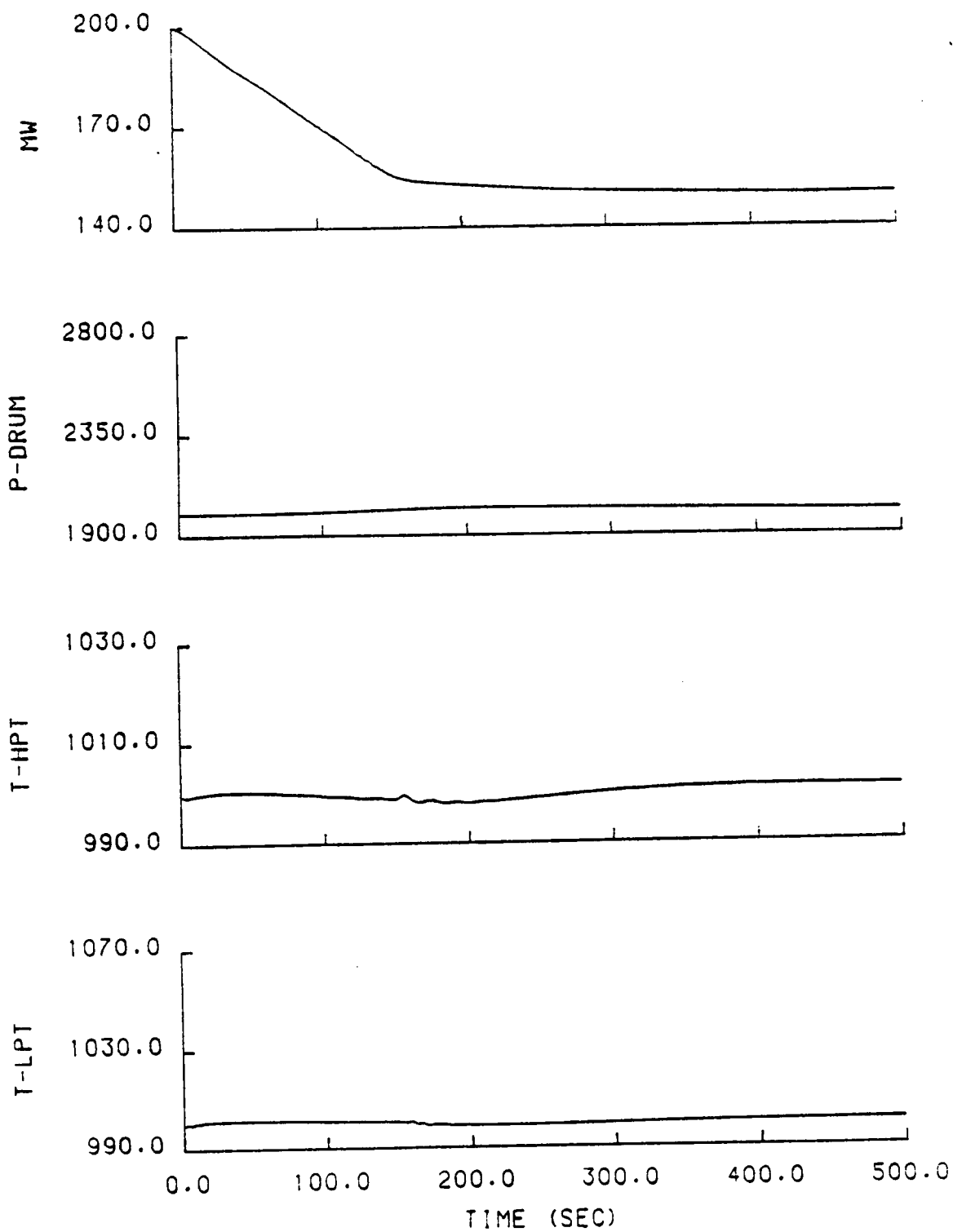


FIGURE 39. RESPONSE OF PLANT UNDER SVD CONTROL TO -10%/MIN RAMP TO 75% POWER.

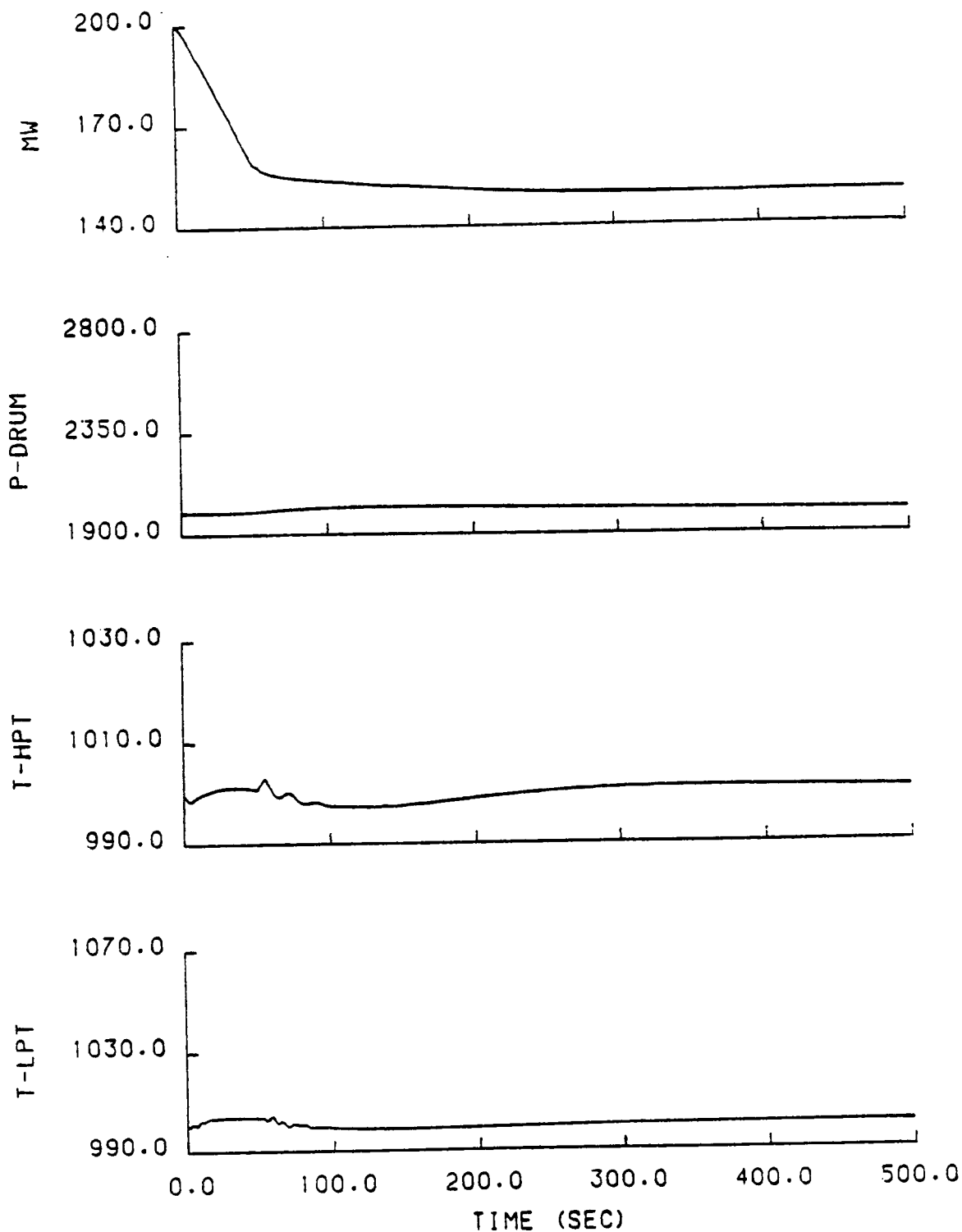


FIGURE 40. RESPONSE OF PLANT UNDER SVD CONTROL TO -30%/MIN RAMP TO 75% POWER.

orthogonality of the structured variable space. If the system were almost singular, small deviations in the plant parameters would require large changes in the optimum tuning parameters. In an orthogonal space, however, small changes in plant parameters require equally small changes in control.

Summary

In order to test the singular valve decomposition controller under a realistic set of conditions, a detailed nonlinear first principles simulation of a coal-fired power plant has been developed. The model was designed specifically to study steam temperature/flow dynamics for the purpose of control system analysis.

Standard multivariable single input-single output control was applied to the system. The responses with this conventional control compared favorably with actual plant transients reported in the literature, helping validate the model. This control showed significant interaction, which in practice limits the size and rate of power changes.

A new multivariable control technique, the multivariable tuning regulator, was implemented on the process. Significant improvement was realized for the step input on which it was designed. Problems were encountered however, when ramp inputs to different power levels were attempted. Drum pressure control was still significantly improved over MVSISO, however.

The singular valve decomposition controller was implemented and tuned for the step power change. These tuning parameters were also applied to -10% and -30% per minute ramp changes. In all cases the SVD controller worked well, with few interaction problems. One of the most significant implications of this result involves the robustness of the controller. The MVSISO and the MTR controllers had problems when maneuvering the plant to power levels other than that for which they were designed. In contrast, the SVD controller performed well in regions with different plant dynamic characteristics.

The conclusion is that significant potential exists for improving fossil-fired power plant maneuverability. Singular valve decomposition control appears to be a good candidate technique for achieving this improvement.

VII. CONCLUSIONS AND RECOMMENDATIONS

The interaction between manipulated and controlled variables causes many problems for the control engineer in both the design and implementation of integrated systems. Often these problems cannot be simply solved using conventional single loop concepts. This work investigated the practical effectiveness of singular value decomposition (SVD) in the design of multivariable strategies.

First, several SVD controllers were designed and evaluated in an attempt to develop practical guidelines which minimize application difficulties. The form chosen is shown in Figure 10, where the U and V matrices are defined in terms of two angles of rotation and apply to both modes (proportional and integral).

It was generally concluded that in some cases a subtle advantage can be realized by separating the proportional and integral modes and optimizing the angles of rotation for each mode. By far the simplest and most generally reliable approach was to set the angles of rotation according to a steady state analysis. These angles can be calculated as functions of the steady state gain matrix using Equations 39, 41, 42, and 43.

Various rules for tuning the proportional and integral modes were explored and again the simplest seemed to be the most reliable. The standard Ziegler-Nichols method, applied to the structured variables, seemed to be adequate.

The SVD controller was also studied and evaluated in a simulated total plant situation. The plant chosen was a coal-fired

power plant with seven manipulated variables and four controlled variables. The SVD performance was evaluated and compared with conventional control as well as with a relatively new strategy, the multivariable tuning regulator. The results were quite promising. The SVD controller substantially improved the overall plant maneuverability, and significantly decreased the drum pressure and steam temperature excursions. The SVD controller was found to be a robust technique. The control designed at 100% power worked well at 75% power where, due to the nonlinearities, the plant dynamics were different. The MVSISO control and the multivariable tuning regulator did not work as well when moved to 75% power. One reason for the robustness exhibited by SVD is the orthonormality of the structured system. If the physical system is close to singular, small perturbations in the plant characteristics will have large effects on the performance. Since the structured system is orthonormal, small perturbations in the plant characteristics will have equally small effects on performance. This robustness is a very desirable characteristic.

In conclusion, the concept is a practical one from the standpoint of physical implementation. In terms of controller design, simple step tests are all that is required to develop a steady state gain matrix, from which the U and V matrices can be calculated. In terms of hardware, the SVD strategy can be implemented on any relatively sophisticated programmable microprocessor-based control system.

Recommendations

Further work on the SVD controller needs to be conducted in several areas. First, the problem of scaling should be addressed. If the manipulated and controlled variables are scaled differently (for example if the units are changed) the steady state gain matrix will be affected, resulting in a different decomposition. In a practical sense this will not be a problem in that all variables seen by the computer will be the same (milliamps, volts, etc.). This needs to be understood from a theoretical viewpoint, however.

Dynamic effects are a problem not dealt with in the current SVD structure. Some effort in this direction has been made by decomposing the process transfer function matrix at a frequency other than zero. An even more effective technique might be available in terms of dynamic U and V matrices.

The next logical step for development of the SVD concept would involve constructing an SVD controller and implementing it on a 2 x 2 multivariable process such as a distillation column. The results from this experimentation would be of great interest, and should prove that the SVD structure is useful for improved multivariable control.

The power plant model developed for this work was useful for the control studies conducted here. One weakness of the plant model was lack of detail. For instance, the steam piping between the drum and the high pressure turbine was modeled as one node for flow calculations. Another weakness was the lack of plant-specific information. The model was based on a generic drum-type reheat unit, with no exact design details (e.g., pipe lengths, drum volume). Both

of these weaknesses would affect the numbers which result from the control studies, but not the results. For example, the optimum tuning parameters would be different for a more detailed model, but the general results (in terms of potential performance improvements) should be the same. Before conclusive results can be stated for the applicability of any control strategy to a given power plant, a much more detailed, plant-specific model would be required. For the purposes of comparative control studies, however, the model in its present form is very useful.

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APPENDIX

```

C*****
C*****
C
C                               BOILER.FOR
C
C   A ROUTINE WRITTEN TO SIMULATE A COAL-FIRED POWER PLANT FOR
C   THE PURPOSE OF CONTROL SYSTEM STUDIES.
C
C                               WRITTEN BY:      J. M. KEETON
C                               DATE:            8/24/83
C                               LAST REVISION:    2/16/84
C
C*****
C*****

REAL HF,HFG,HHPTO,HRSPTY,HSSPTY,HS1I,HS1O,HS2I,HS2O,HS3I,HS3O
REAL HWW,KLOSS,KVALVE,MDRUM,MSG,MW,MWHPT,MWLPT,MWW
REAL DHPT(100),DRHT(100),DLPT(100),HLPTI,HLPTO,C(4),M(7)

C*****      CONTROL PARAMETERS

COMMON /CONTR/ GAIN(4),TAU(4),SP(4),ICONT,DELT

ICONT=0
SP(1)=200.0
SP(2)=2000.0
SP(3)=1000.0
SP(4)=1000.0
GAIN(1)=0.4
TAU(1)=2.0
GAIN(2)=0.4
TAU(2)=0.0
GAIN(3)=0.4
TAU(3)=0.0
GAIN(4)=0.4
TAU(4)=0.0

C*****      END CONTROL PARAMETERS

C   INPUT INITIAL VALUES AND CONSTANTS

AIRVLV=0.5
AVALVE=0.9494
COST=0.0
CPMTL=0.2
DELT=0.1
FIRE=44.6847
GHR=0.2905806
GHS1=0.381888
GHS2=0.225036
HRSPTY=38.0
HSSPTY=38.0
HWW=780.871585

```

```

ICOUNT=0
KLOSS=0.6205084792
KVALVE=290.4571
MDRUM=10000.0
MSG=30000.0
MWW=10000.0
PDRUM=2000.0
PNOM=1950.0
PTHROT=1950.0
QG1=47030.87
QG2=47030.87
QG3=92379.6
QRADR=64031.47
QRADS=64031.47
QS1=47030.87
QS2=47030.87
QS3=92379.6
RAD=1.67400365E-9
SHR=0.5105785
SHS1=0.671015
SHS2=0.3954105
STROKE=44.6847
TAUF=67.0
TAUFLO=4.0
TAUHPT=0.5
TAULPT=5.9
TDRUM=636.0
TG10=1383.57
TG20=1963.01
TG30=1404.69
THETAR=0.0
THETAS=0.0
TS10=771.33
TS20=1005.23
TS30=1000.0
TWW=800.0
WDRUM=275.76
WHPT=275.76
WLPT=275.76
WRSPRY=0.0
WSSPRY=0.0
WTHROT=275.76

TAUWW=MWW*CPMTL/HWW
ISTEPS=INT(5.0/DELT)
ITER=INT(500.0/DELT)
DO 100 I=1,100
  DHPT(I)=1477.4
  DRHT(I)=1195.5
  DLPT(I)=1000.0
  CONTINUE

```

100

C***** BEGIN THE ITTERATIONS

```

DO 5000 ICOUNT=1,ITER
TIME=DELT*ICOUNT

C   GIVEN MILL STROKE RATE, FIND COAL FIRING RATE
      CALL LAG(STROKE,FIRE,1.0,DELT,TAUF)

C   FIND THE AIR FLOW RATE
      AIRFLO=1104.4181*AIRVLV

C   GIVEN AIR FLOW, FIRING RATE, AND QRAD, FIND FLAME TEMPERATURE
      F=FIRE/2.0
      A=AIRFLO/2.0
      CALL TEMP(TFLAMR,F,A,QRADR)
      CALL TEMP(TFLAMS,F,A,QRADS)

C   CALCULATE RADIANT HEAT TRANSFER COEFFICIENTS
      RADR=RAD*(1.0-0.5*TAN(THETAR))/2.0
      RADS=RAD*(1.0-0.5*TAN(THETAS))/2.0

C   GIVEN FLAME AND WATERWALL TEMPERATURES, FIND QRAD
      QR=RADR*((TFLAMR+460.0)**4-(TWW+460.0)**4)
      QS=RADS*((TFLAMS+460.0)**4-(TWW+460.0)**4)
      QRADR=0.8*QRADR+0.2*QR
      QRADS=0.8*QRADS+0.2*QS
      QRAD=QRADR+QRADS

C   CALCULATE QRAD/HWW FOR WATERWALL TEMPERATURE SUBROUTINE, AND ADD
C   TO TDRUM
      X=TDRUM+QRAD/HWW
      CALL LAG(X,TWW,1.0,DELT,TAUWW)

C   EVALUATE THE CIRCULATION LOOP CAPACITANCE
      CALL VJPROP(1,PDRUM,TDRUM,HF,X)
      HFG=X-HF
      X=PDRUM+1.0
      CALL VJPROP(1,X,XT,XF,XG)
      CPSG=(MSG*(XF-HF)+MDRUM*CPMTL*(XT-TDRUM))/HFG

C   FIND STEAM GENERATION RATE
      WGEN=HWW*(TWW-TDRUM)/HFG

C   FIND DRUM PRESSURE
      PDRUM=PDRUM+(WGEN-WDRUM)*DELT/CPSG

C   STEAM FLOW AROUND THE LOOPS

```

```

WAVG=(WDRUM+WTHROT)/2.0
PTHROT=SQRT(PDRUM*PDRUM-WAVG*WAVG/((KLOSS*KLOSS))
WTHROT=KVALVE*AVALVE*PTHROT/PNOM
CALL LAG(WTHROT,WDRUM,1.0,DELT,TAUFLO)
WHPT=WTHROT
CALL LAG(WHPT,WLPT,1.0,DELT,TAULPT)
WRHT=(WHPT+WLPT)/2.0

```

C EVALUATE STEAM CONDITIONS AROUND THE LOOP

```
WGAS=F+A
```

C PRIMARY SUPERHEATER

```

CALL VJPROP(1,PDRUM,TDRUM,X,HS1I)
CALL SHTR1(TDRUM,TS10,QS1,WDRUM,TG20,TG10,QG1,WGAS,DELT)
HS10=HS1I+QS1/WDRUM
CALL VJPROP(3,PDRUM,TS10,X,HS10)
Q=QRADS+QG2+QG1
CALL TEMP(TG10,F,A,Q)

```

C SUPERHEAT SPRAY

```
HS2I=(WSSPRY*HSSPRY+WDRUM*HS10)/(WSSPRY+WDRUM)
```

C SECONDARY SUPERHEATER

```

CALL VJPROP(3,PDRUM,TS2I,X,HS2I)
CALL SHTR2(TS2I,TS20,QS2,WDRUM,TFLAMS,TG20,QG2,WGAS,DELT)
HS20=HS2I+QS2/WDRUM
CALL VJPROP(3,PDRUM,TS20,X,HS20)
Q=QRADS+QG2
CALL TEMP(TG20,F,A,Q)

```

C THROTTLE VALVE

```

PHPTI=PTHROT*SQRT(AVALVE)
IHPT=INT(1089.9/(DELT*WAVG))
IF(IHPT.GT.100) IHPT=100
CALL VJPROP(3,PHPTI,THPTI,X,DHPT(IHPT))

```

C HIGH PRESSURE TURBINE

```

S=-5.56869835E-5*PHPTI+7.035123967E-4*DHPT(IHPT)+0.63430817
IF(S.LE.1.5668) HHPTO=820.7376*S-91.3317
IF(S.GE.1.5668) HHPTO=(S-0.142372093)/0.001192389006
MWHPT=(DHPT(IHPT)-HHPTO)*WHPT/947.088

```

C REHEAT SPRAY

```

IRHT=INT(1195.46/(DELT*WRHT))
IF(IRHT.GT.100) IRHT=100
HS3I=(WRSPPRY*HRSPPRY+WRHT*DRHT(IRHT))/(WRSPPRY+WRHT)

```

C

REHEATER

```

CALL VJPROP(3,155.0,TS3I,X,HS3I)
CALL RHTR(TS3I,TS30,QS3,WRHT,TFLAMR,TG30,QG3,WGAS,DELT)
HS30=HS3I+QS3/WRHT
CALL VJPROP(3,155.0,TS30,X,HS30)
Q=QRADR+QG3
CALL TEMP(TG30,F,A,Q)

```

C

LOW PRESSURE TURBINE

```

ILPT=INT(623.25/(DELT*WRHT))
IF(ILPT.GT.100) ILPT=100
TLPTI=DLPT(ILPT)
S=0.000348*TLPTI+1.554453
CALL VJPROP(2,155.0,TLPTI,X,HLPTI)
IF(S.LE.1.8713) HLPTO=678.7265*S-27.8509
IF(S.GT.1.8713) HLPTO=(S-0.062679962)/0.00163461539
MWLPT=(HLPTI-HLPTO)*WLPT/947.088
MW=MWHPT+MWLPT

```

C

UPDATE THE DEAD TIME TABLES

```

DO 200 I=100,2,-1
DHPT(I)=DHPT(I-1)
DRHT(I)=DRHT(I-1)
DLPT(I)=DLPT(I-1)
200 CONTINUE

DHPT(1)=HS20
DRHT(1)=HHPTO
DLPT(1)=TS30

```

C

CALL THE CONTROL SUBROUTINE

```

C(1)=MW
C(2)=PDRUM
C(3)=THPTI
C(4)=TLPTI
M(1)=STROKE
M(2)=AIRVLV
M(3)=AVALVE
M(4)=THETAR
M(5)=THETAS
M(6)=WRSPRY
M(7)=WSSPRY
MM3=M(3)
CALL CNTRL(C,M)
COST=COST+5.0*ABS(M(3)-MM3)
COST=COST+20.0*ABS(SP(1)-C(1))*DELT
COST=COST+1.0*ABS(SP(2)-C(2))*DELT
COST=COST+4.0*ABS(SP(3)-C(3))*DELT
COST=COST+4.0*ABS(SP(4)-C(4))*DELT

```

```

STROKE=M(1)
AIRVLV=M(2)
AVALVE=M(3)
THETAR=M(4)
THETAS=M(5)
WRSPRY=M(6)
WSSPRY=M(7)

```

C WRITE OUT THE RESULTS

```

IADD=IADD+1
IF(IADD.EQ.ISTEPS) WRITE(22,1000) TIME,MW,PDRUM,THPTI,TLPTI
IF(IADD.EQ.ISTEPS) IADD=0

```

5000 CONTINUE

```

CALL PLOT

```

1000 FORMAT(1X,7F12.4)

```

STOP
END

```

C*****

```

SUBROUTINE LAG(XIN,XOUT,GAIN,DELT,TAU)

```

C ROUTINE FOR A FIRST ORDER LAG

```

A=EXP(-DELT/TAU)
B=GAIN*(1.0-A)

XOUT=A*XOUT+B*XIN

RETURN
END

```

C*****

```

SUBROUTINE SHTR1(TSI,TSO,QS,WS,TGI,TGO,QG,WG,DELT)

```

C ROUTINE TO CALCULATE HEAT TRANSFERED TO/FROM TUBE WALLS IN
C THE PRIMARY SUPERHEATER

```

REAL M
IF(I.EQ.1) GO TO 100
M=22610.0

```

```

CP=0.15
UGCON=0.3250426
USCON=0.3517842
T=1188.4775
I=1
100 CONTINUE

TS=(TSI+TSO)/2.0
TG=(TGI+TGO)/2.0
UG=UGCON*WG
US=USCON*WS
X=M*CP/(UG+US)
A=EXP(-DELT/X)
BG=UG*(1.0-A)/(UG+US)
BS=US*(1.0-A)/(UG+US)

T=A*T+BG*TG+BS*TS

QQS=US*(T-TS)
QQG=UG*(TG-T)
QS=0.8*QS+0.2*QQS
QG=0.8*QG+0.2*QQG

RETURN
END

```

C*****

SUBROUTINE SHTR2(TSI,TSO,QS,WS,TGI,TGO,QG,WG,DELT)

C ROUTINE TO CALCULATE HEAT TRANSFERED TO/FROM TUBE WALLS IN
C THE SECONDARY SUPERHEATER

```

REAL M
IF(I.EQ.1) GO TO 100
M=16192.0
CP=0.15
UGCON=0.2327735
USCON=0.2519240
T=1565.27
I=1
100 CONTINUE

TS=(TSI+TSO)/2.0
TG=(TGI+TGO)/2.0
UG=UGCON*WG
US=USCON*WS
X=M*CP/(UG+US)
A=EXP(-DELT/X)
BG=UG*(1.0-A)/(UG+US)

```

BS=US*(1.0-A)/(UG+US)

T=A*T+BG*TG+BS*TS

QQS=US*(T-TS)

QQG=UG*(TG-T)

QS=0.8*QS+0.2*QQS

QG=0.8*QG+0.2*QQG

RETURN

END

C*****

SUBROUTINE RHTR(TSI,TSO,WS,TGI,TGO,QG,WG,DELT)

C ROUTINE TO CALCULATE HEAT TRANSFERED TO/FROM TUBE WALLS IN
C THE REHEATER

REAL M

IF(I.EQ.1) GO TO 100

M=33576.0

CP=0.15

UGCON=0.4826815

USCON=0.5223921

T=1321.82

I=1

100 CONTINUE

TS=(TSI+TSO)/2.0

TG=(TGI+TGO)/2.0

UG=UGCON*WG

US=USCON*WS

X=M*CP/(UG+US)

A=EXP(-DELT/X)

BG=UG*(1.0-A)/(UG+US)

BS=US*(1.0-A)/(UG+US)

T=A*T+BG*TG+BS*TS

QQS=US*(T-TS)

QQG=UG*(TG-T)

QS=0.8*QS+0.2*QQS

QG=0.8*QG+0.2*QQG

RETURN

END

C*****

SUBROUTINE TEMP(T,FIRE,AIRFLO,Q)

C ROUTINE TO CALCULATE FLUE GAS TEMPERATURE GIVEN COAL FIRING
C RATE, AIR FLOW RATE, AND HEAT REMOVAL HISTORY

A=0.3027*FIRE+0.2371*AIRFLO
B=6.8935E-5*FIRE+1.3773E-5*AIRFLO
T=-A/B+SQRT(A*A/(B*B)-2./B*(Q-11527.95*FIRE-77.*A-2964.5*B))

RETURN
END

C*****

SUBROUTINE CNTRL(C,M)

C SUBROUTINE FOR CONVENTIONAL MVSISO CONTROL

REAL C(4),M(4),E(4),EM1(4)
COMMON /CONTR/ GAIN(4),TAU(4),SP(4),ICOUNT,DELT

IF(ICOUNT.GT.0) GO TO 200
DO 100 I=1,4
100 EM1(I)=0.0
200 ICOUNT=1

DO 300 I=1,4
E(I)=SP(I)-C(I)
M(I)=M(I)+GAIN(I)*(E(I)-EM1(I)+E(I)*DELT/TAU(I))
300 EM1(I)=E(I)

C LIMIT THE OUTPUTS

IF(M(1).GE.75.0) M(1)=75.0
IF(M(1).LE.0.0) M(1)=0.0
IF(M(2).GE.1.0) M(2)=1.0
IF(M(2).LE.0.0) M(2)=0.0
IF(M(3).GE.1.0) M(3)=1.0
IF(M(3).LE.0.0) M(3)=0.0
IF(ABS(M(4)).GE.0.5236) M(4)=0.5236*M(4)/ABS(M(4))
IF(ABS(M(5)).GE.0.5236) M(5)=0.5236*M(5)/ABS(M(5))
IF(ABS(M(6)).GE.50.0) M(6)=50.0*M(6)/ABS(M(6))
IF(ABS(M(7)).GE.50.0) M(7)=50.0*M(7)/ABS(M(7))

RETURN
END

C*****

```

      SUBROUTINE CNTRL(C,M)

C     SUBROUTINE FOR SVD CONTROL

      REAL UT(4,4),V(7,7),C(4),M(7),E(4),EM1(4),EP(4),MP(4)
      COMMON /CONTR/ GAIN(4),TAU(4),SP(4),ICOUNT,DELT

      IF(ICHK.GE.1) GO TO 300
      DO 100 I=1,7
100    READ(24,*) (V(I,J),J=1,7)
      DO 200 I=1,4
200    READ(24,*) (UT(I,J),J=1,4)
      ICHK=1
300    ICOUNT=ICOUNT+1
      IF(ICOUNT.GT.1) GO TO 500
      DO 400 I=1,4
400    EM1(I)=0.0
500    CONTINUE

C     IF ANY VALUE IS CONSTRAINED, STOP INTEGRATING

      DELTAT=DELT
      IF(M(1).GE.75.0.OR.M(1).LE.0.0) DELTAT=0.0
      IF(M(2).GE.1.0.OR.M(2).LE.0.0) DELTAT=0.0
      IF(M(3).GE.1.0.OR.M(3).LE.0.0) DELTAT=0.0
      IF(ABS(M(4)).GE.0.5236) DELTAT=0.0
      IF(ABS(M(5)).GE.0.5236) DELTAT=0.0
      IF(ABS(M(6)).GE.50.0) DELTAT=0.0
      IF(ABS(M(7)).GE.50.0) DELTAT=0.0

C     CALCULATE THE ERROR SIGNAL

      DO 600 I=1,4
600    E(I)=SP(I)-C(I)

C     MULTIPLY BY U-TRANSPOSE

      DO 700 I=1,4
      EP(I)=0.0
      DO 700 J=1,4
700    EP(I)=EP(I)+UT(I,J)*E(J)

C     DEVELOP THE STRUCTURED CONTROLLER ACTION

      DO 900 I=1,4
900    MP(I)=GAIN(I)*(EP(I)-EM1(I)+DELTAT*EP(I)/TAU(I))

C     SHIFT THE ERROR SIGNALS

      DO 1000 I=1,4
1000   EM1(I)=EP(I)

C     MULTIPLY BY V

```

```

      DO 1100 I=1,7
      DO 1100 J=1,4
1100   M(I)=M(I)+V(I,J)*MP(J)

```

C LIMIT THE OUTPUTS

```

      IF(M(1).GE.75.0) M(1)=75.0
      IF(M(1).LE.0.0) M(1)=0.0
      IF(M(2).GE.1.0) M(2)=1.0
      IF(M(2).LE.0.0) M(2)=0.0
      IF(M(3).GE.1.0) M(3)=1.0
      IF(M(3).LE.0.0) M(3)=0.0
      IF(ABS(M(4)).GE.0.5236) M(4)=0.5236*M(4)/ABS(M(4))
      IF(ABS(M(5)).GE.0.5236) M(5)=0.5236*M(5)/ABS(M(5))
      IF(ABS(M(6)).GE.50.0) M(6)=50.0*M(6)/ABS(M(6))
      IF(ABS(M(7)).GE.50.0) M(7)=50.0*M(7)/ABS(M(7))

      RETURN
      END

```

C*****

 SUBROUTINE CNTRL(C,M)

C SUBROUTINE FOR MULTIVARIABLE TUNING REGULATOR CONTROL

```

      REAL IMAT(7,4),PMAT(7,4),C(4),M(7),E(4),EM1(4),MP(7),MI(7)
      COMMON /CONTR/ GAIN(4),TAU(4),SP(4),ICOUNT,DELT

      IF(ICHK.GE.1) GO TO 300
      DO 100 I=1,7
100    READ(24,*) (IMAT(I,J),J=1,4)
      DO 200 I=1,7
200    READ(24,*) (PMAT(I,J),J=1,4)
      ICHK=1
300    ICOUNT=ICOUNT+1
      IF(ICOUNT.GT.1) GO TO 500
      DO 400 I=1,4
400    EM1(I)=0.0
500    CONTINUE

```

C IF ANY VALUE IS CONSTRAINED, STOP INTEGRATING

```

      DELTAT=DELT
      IF(M(1).GE.75.0.OR.M(1).LE.0.0) DELTAT=0.0
      IF(M(2).GE.1.0.OR.M(2).LE.0.0) DELTAT=0.0
      IF(M(3).GE.1.0.OR.M(3).LE.0.0) DELTAT=0.0
      IF(ABS(M(4)).GE.0.5236) DELTAT=0.0
      IF(ABS(M(5)).GE.0.5236) DELTAT=0.0
      IF(ABS(M(6)).GE.50.0) DELTAT=0.0

```

```

        IF(ABS(M(7)).GE.50.0) DELTAT=0.0
C    CALCULATE THE ERROR SIGNAL
        DO 600 I=1,4
600    E(I)=SP(I)-C(I)
C    FORMULATE THE P-PART
        DO 700 I=1,7
        MP(I)=0.0
        DO 700 J=1,4
700    MP(I)=MP(I)+PMAT(I,J)*(E(J)-EM1(J))*GAIN(J)
C    FORMULATE THE I-PART
        DO 800 I=1,7
        MI(I)=0.0
        DO 800 J=1,4
800    MI(I)=MI(I)+IMAT(I,J)*E(J)*DELTAT*TAU(1)
C    ADD TO THE PREVIOUS OUTPUT
        DO 900 I=1,7
900    M(I)=M(I)+MI(I)+MP(I)
C    SHIFT THE ERROR SIGNALS
        DO 1000 I=1,4
1000    EM1(I)=E(I)
C    LIMIT THE OUTPUTS
        IF(M(1).GE.75.0) M(1)=75.0
        IF(M(1).LE.0.0) M(1)=0.0
        IF(M(2).GE.1.0) M(2)=1.0
        IF(M(2).LE.0.0) M(2)=0.0
        IF(M(3).GE.1.0) M(3)=1.0
        IF(M(3).LE.0.0) M(3)=0.0
        IF(ABS(M(4)).GE.0.5236) M(4)=0.5236*M(4)/ABS(M(4))
        IF(ABS(M(5)).GE.0.5236) M(5)=0.5236*M(5)/ABS(M(5))
        IF(ABS(M(6)).GE.50.0) M(6)=50.0*M(6)/ABS(M(6))
        IF(ABS(M(7)).GE.50.0) M(7)=50.0*M(7)/ABS(M(7))

        RETURN
        END

C*****

```

SUBROUTINE VJPROP(MODE,PP,TT,HHF,HHG)

```

      REAL HHF,HHG,PSAT(28),TSAT(28),HF(28),HG(28),P(28),T(13)
      REAL H(28,13),HHH(13)
C
C   THIS IS A ROUTINE TO RETURN WATER (SATURATED LIQUID, SATURATED
C   VAPOR, AND SUPERHEATED VAPOR) PROPERTIES, DEPENDING ON THE MODE.
C
C           MODE      GIVEN      THE ROUTINE RETURNS
C           1         P         T-SAT,HF-SAT,HG-SAT
C           2         T,P        H
C           3         P,H        T
C
C   THE ROUTINE USES A CRUDE LINEAR INTERPOLATION BETWEEN POINTS TAKEN
C   FROM THE STEAM TABLES OF KEENAN, KEYES, HILL, AND MOORE. THE
C   PRESSURE DATA VARY BY 100 PSI INCREMENTS BETWEEN 400 PSIA AND 3000
C   PSIA, THE TEMPERATURE IN 100 F INCREMENTS BETWEEN 400 F AND 1500 F.
C   THE DATA ARE READ FROM UNIT 21.
C
C   IF THIS IS THE FIRST TIME CALLED, READ IN THE TABLES
      IF(ICOUNT.GE.1) GO TO 300
      ICOUNT=ICOUNT+1
C
      DO 100 I=1,28
      READ(21,*) PSAT(I),TSAT(I),HF(I),HG(I)
100    CONTINUE
C
      READ(21,*) (T(J),J=1,13)
C
      DO 200 I=1,28
      READ(21,*) P(I),(H(I,J),J=1,13)
200    CONTINUE
C
300    CONTINUE
C
C   GIVEN THE PRESSURE, FIND THE SATURATION T, HG, AND HF
      DO 400 I=1,28
      IF(PP.LE.PSAT(I)) GO TO 500
      IF(I.EQ.28) WRITE(5,6000)
400    CONTINUE
C
500    CONTINUE
C
      XP=(PP-PSAT(I-1))/(PSAT(I)-PSAT(I-1))
      TTT=XP*TSAT(I)+(1.0-XP)*TSAT(I-1)
      THHF=XP*HF(I)+(1.0-XP)*HF(I-1)
      THHG=XP*HG(I)+(1.0-XP)*HG(I-1)
C
      IF(MODE.EQ.2) GO TO 2000
      IF(MODE.EQ.3) GO TO 3000
      IF(MODE.NE.1) WRITE(5,5900)
C
C   IF THIS IS MODE 1, WE HAVE THE ANSWER

```

```

      TT=TTT
      HHF=THHF
      HHG=THHG

      RETURN

2000  CONTINUE

C    FOR MODE 2, CHECK TO SEE THAT THE TEMPERATURE IS ABOVE SATURATION
      IF(TT.LE.TTT) WRITE(5,6100)

C    FIND THE INTERVAL IN WHICH THE TEMPERATURE LIES, AND FIND XT
      DO 2200 J=1,13
      IF(TT.LE.T(J)) GO TO 2300
      IF(J.EQ.13) WRITE(5,6200)
2200  CONTINUE

2300  CONTINUE

C    INTERPOLATE TO FIND HG
      XT=(TT-T(J-1))/(T(J)-T(J-1))
      UPH=XT*H(I-1,J)+(1.0-XT)*H(I-1,J-1)
      DOH=XT*H(I,J)+(1.0-XT)*H(I,J-1)
      HHG=XP*DOH+(1.0-XP)*UPH
      HHF=0.0

      RETURN

3000  CONTINUE

C    FOR MODE 3, CHECK THAT THE ENTHALPY IS ABOVE SATURATION
      IF(HHG,LT.THHG) WRITE(5,6300)

C    FOR EACH INTERVAL, EVALUATE THE ENTHALPY AND CHECK FOR TEMP INTERVAL
      DO 3100 J=1,13
      HHH(J)=XP*H(I,J)+(1.0-XP)*H(I-1,J)
      IF(HHG.LE.HHH(J)) GO TO 3200
      IF(J.EQ.13) WRITE(5,6400)
3100  CONTINUE

3200  CONTINUE

      XH=(HHG-HHH(J-1))/(HHH(J)-HHH(J-1))
      TT=XH*T(J)+(1.0-XH)*T(J-1)
      HHF=0.0

      RETURN

5900  FORMAT(' ILLEGAL MODE NUMBER')

```

```

6000  FORMAT(' PRESSURE OUT OF BOUNDS, HIGH')
6100  FORMAT(' TEMPERATURE TOO LOW FOR SUPERHEAT')
6200  FORMAT(' TEMPERATURE OUT OF BOUNDS, HIGH')
6300  FORMAT(' ENTHALPY TOO LOW FOR SUPERHEAT')
6400  FORMAT(' ENTHALPY OUT OF BOUNDS, HIGH')

```

END

C*****

SUBROUTINE PLOT

C THIS IS A ROUTINE TO PLOT OUT THE SIMULATION RESULTS

```

      DIMENSION CHAR(7),Y(200,6),ALINE(100),YMAX(6),YMIN(6)
      DIMENSION ICOUNT(100),X(200),YLBL(6,11)
      DOUBLE PRECISION HEAD(7)
      DATA BLANK/' ','/ ',CHAR/'1','2','3','4','5','6','*'/
      DATA HEAD/'MW','PDRUM','THPTI','TLPTI','THPTI','TLPTI','TIME'/

```

CLOSE(UNIT=22)

C READ IN THE DATA

```

      DO 100 I=1,200
100    READ(22,*,END=200) X(I),(Y(I,J),J=1,4)
200    NDATA=I-1

```

C FIND THE MAXIMUM AND MINIMUM VALUES

```

      DO 400 J=1,4
      YMAX(J)=Y(1,J)
      YMIN(J)=Y(1,J)
      DO 300 I=1,NDATA
      IF(Y(I,J).LT.YMIN(J)) YMIN(J)=Y(I,J)
300    IF(Y(I,J).GT.YMAX(J)) YMAX(J)=Y(I,J)
400    CONTINUE

```

C SCALE ALL THE DATA

```

      DO 600 J=1,4
      IF(YMAX(J).NE.YMIN(J)) GO TO 525
      DO 500 I=1,NDATA
500    Y(I,J)=0.5
      GO TO 600
525    CONTINUE
      DO 550 I=1,NDATA
550    Y(I,J)=(Y(I,J)-YMIN(J))/(YMAX(J)-YMIN(J))
600    CONTINUE

```

C FILL THE LABEL TABLE

```

        DO 650 J=1,4
        DO 640 K=1,11
        YLBL(J,K)=YMIN(J)+0.1*(K-1)*(YMAX(J)-YMIN(J))
640     CONTINUE
650     CONTINUE

C      WRITE OUT THE HEADING

        DO 700 J=1,4
        WRITE(23,6010) CHAR(J),HEAD(J),(YLBL(J,K),K=1,11)
700     WRITE(23,*)
        WRITE(23,6040) HEAD(7)

C      DRAW THE PLOT

        DO 1100 I=1,NDATA

C      FILL THE VECTOR ALINE

        DO 800 K=1,100
        ALINE(K)=BLANK
800     ICOUNT(K)=0

        DO 900 L=1,4
        LL=INT(100.0*Y(I,L))
        ALINE(LL)=CHAR(L)
900     ICOUNT(LL)=ICOUNT(LL)+1

        DO 1000 M=1,100
1000    IF(ICOUNT(M).GT.1) ALINE(M)=CHAR(7)

C      WRITE OUT ALINE

        ICOU=ICOU+1
        IF(ICOU.EQ.20) WRITE(23,6020) X(I)
        IF(ICOU.NE.20) WRITE(23,6030) X(I),(ALINE(J),J=1,100)
        IF(ICOU.EQ.20) ICOU=0

1100    CONTINUE

        WRITE(23,6040)

6010    FORMAT(1H ,A1,1X,A10,2X,11(F7.2,3X))
6020    FORMAT(1H ,9X,F7.2,1X,10('+****/****'),'+')
6030    FORMAT(1H ,9X,F7.2,1X,'*',100A1,'*')
6040    FORMAT(1H ,6X,A10,1X,10('+****/****'),'+')

        RETURN
        END

```

VITA

Joe Mark Keeton was born in Paducah, Kentucky. He attended elementary schools in Oak Ridge, Tennessee, San Jose, California, and Chattanooga, Tennessee. He was graduated from Naperville Central High School in Naperville, Illinois, near Chicago. He received a Bachelor of Science degree in Chemical Engineering from The University of Tennessee, Knoxville, in 1979, a Master's degree in 1981, and a Doctor of Philosophy degree in 1984.