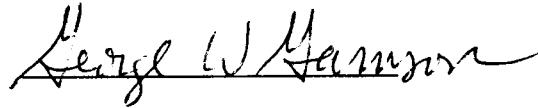


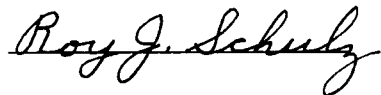
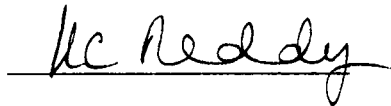
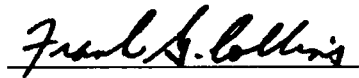
To the Graduate Council:

I am submitting herewith a dissertation written by Herbert Thomas entitled "A Numerical Simulation of Axisymmetric, Steady-State Plasma Flow with Self-Induced and Applied Magnetic Fields". I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy with a major in Aerospace Engineering.



George W. Garrison, Major Professor

We have read this dissertation
and recommend its acceptance:



Accepted for the Council:



Vice Provost
and Dean of the Graduate School

**A Numerical Simulation of Axisymmetric,
Steady-State Plasma Flow with Self-Induced
and Applied Magnetic Fields**

A Dissertation

Presented for the

Doctor of Philosophy Degree

The University of Tennessee, Knoxville

Herbert Thomas

August 1991

Für meine Eltern

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ABSTRACT

An axisymmetric, implicit, finite-difference algorithm was formulated for the steady-state performance prediction of Arcjets and MPD-Thrusters subject to both self-induced and applied magnetic fields. Thermal non-equilibrium was taken into account. The azimuthal momentum equation was added to the algorithm to accommodate swirl flow due to applied magnetic fields. The electrical conductivity was calculated by Saha's equation and Hall effects were included.

The code was validated by comparisons to simple test cases. A parameter study was performed based on a geometry that allows comparison to other numerical data and experimental results. Good agreement was found between the predictions of the new code and the previously published numerical and experimental results.

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NOMENCLATURE

a	speed of sound	E	electric field w.r.t.laboratory
B	magnetic induction	F	body force per unit volume
c	speed of light	h	Planck's constant
c	particle random thermal velocity	I_{sp}	specific impulse
		I_t	total current
c_v	specific heat at constant volume	j	conduction current
		J	Jacobian
c_p	specific heat at constant pressure	J	electric current vector
		k	Boltzmann's constant
dt	incremental time step	K_n	Knudsen number λ/l
d	Debye shielding distance	l	length
e	electronic charge	m	particle mass
e	total energy per unit mass	$\dot{m}$	total mass flow

M	Mach number	U_e	electron diffusion velocity
n	number density	x,y	spatial coordinates
p	hydrostatic pressure	Z	number of charges
P_r	Prandtl number	α	degree of ionization
		α	wall inclination
q	heat flux vector	γ	ratio of specific heats
Q	collision cross section	∂	partial
r	radius	δ_e	energy loss factor
R	gas constant	ε	ionization energy
Re	Reynolds number	η	viscosity
R_m	magnetic Reynolds number	λ	mean free path
$\mathfrak{R}$	universal gas constant	μ_0	magnetic permeability
S	magnetic force number	ν	kinematic viscosity, collision frequency
t	time	ρ	mass density
T	temperature	ρ_e	electron mass density
u	velocity vector	σ	electrical conductivity
u_e	electron velocity vector	τ	shear stress tensor

$\omega\tau$ Hall parameter

ϕ electric field potential

Subscripts

ave average

ref reference value

e electron

rs between species r and s

h heavy particles

tr translational

H Hall

r,x,y,z coordinate directions

I ion

1 referring to first

n neutral

2 referring to second

Superscript

n time step

$\rightarrow$ vector quantity

CHAPTER I

INTRODUCTION

Building an infrastructure in space requires new space-vehicle propulsion systems designed for specific tasks. Increasingly, those tasks require high total mission velocities, a measure of the energy necessary to perform a certain mission. Hence, a high propellant exhaust velocity is necessary in order to minimize propulsion system mass [1]. Due to the inherent limit of chemical propulsion systems to deliver high exhaust velocities, a wide array of new propulsion systems offering higher exhaust velocities is being investigated [2],[3]. One of the advanced propulsion systems under consideration is a class of electromagnetic accelerators which uses the interaction of applied electric and magnetic fields to accelerate a plasma.

Due to the complex physical processes within electromagnetic accelerators, a complete predictive analytical model is not available to provide a complete understanding of their performance and improvements depend on a better understanding of physical processes within the device. Experimental investigations provide information about the effects taking place in electromagnetic thrusters, while numerical modeling allows parameter studies and, once numerical codes include the major physical processes, allows for performance predictions. The major thrust of this dissertation is a contribution towards the inclusion of the important physical effects into a numerical model.

The major assumptions and physical limitations underlying this model are as follows:

- *axisymmetry*
- *monatomic gas with negligible electron excitation energy*
- *infinite reaction rates for ionization*

- *thermal ionization dominant*
- *thermal non-equilibrium, i.e. different electron and heavy particle temperature*
- *non-zero Hall parameter*
- *finite magnetic Reynolds number*
- *cross sections and transport properties are functions of temperature and density*
- *single and double ionization*
- *applied and induced magnetic fields*
- *radiation effects negligible*
- *Sutherland viscosity law applicable*
- *neglected effects of the electrode sheath region*

This dissertation starts with a discussion of the technical background in which this work is embedded. After describing some of the previous attempts to perform numerical simulation of electromagnetic thrusters the modeling equations are outlined. Comparisons with analytical and numerical results are provided. Finally, the code is applied to a particular thruster configuration, where viscosity and an applied magnetic field are included, and the quantitative and qualitative accuracy of the code is discussed.

CHAPTER II

BACKGROUND

2.1 MOTIVATION FOR ELECTRIC PROPULSION RESEARCH

Present and future space missions (for example the creation of a permanent space station, satellites with long on-orbit life-times, orbital maneuvering and transfer vehicles as well as interplanetary missions) require higher amounts of propulsion energy than most current applications. The increased demand for propulsive energy reflects itself in increasing propellant mass and associated with that, increased size and costs.

Propellants with high specific energies minimize the propellant mass required for a mission. Research is now being performed on exotic chemical propellants, like metastable helium He^* and He_2^* [4],[5], to raise the specific energy content of the propellant above that achieved by LH_2/LO_2 engines, which currently provide the highest level of thrust per pound of propellant mass flow, or specific impulse. The exotic chemical propellants raise the maximum obtainable specific impulse to 3150 sec for He^* [6] and 2000 sec for He_2^* [7] as compared to a maximum of approximately 500 sec for LH_2/LO_x .

Efficient use of the propellant is another means to reduce propellant mass. In chemical propulsion systems, this requires high pressure combustion chambers and high area ratio nozzles, both increasing the exhaust velocity of the propulsion system.

A simple relationship between exhaust velocity and the ratio of final to initial mass is given by [1]:

$$\frac{m_f}{m_0} = \exp\left(\frac{-\Delta v}{u_e}\right) \quad (4.1)$$

where Δv is the total change in mission velocity (a measure of the energy necessary to perform a certain mission), u_e is the exhaust velocity, m_f is the propellant mass and m_0 is the initial total mass of the vehicle. To maintain this ratio, higher total mission velocities require higher exhaust velocities and hence, propellant energy. It is obvious from equation 4.1, that the exhaust velocity of the propulsion system should be as high as possible to minimize the propellant mass fraction of the vehicle. A parameter utilized to compare different propellants and propulsion systems is the "specific impulse" mentioned before, and defined by [8]:

$$I_{sp} = \frac{\int F dt}{g_0 \int \dot{m} dt} \quad (4.2)$$

or, if thrust and propellant flow are constant:

$$I_{sp} = \frac{F}{\dot{m} g_0} = \frac{F}{\dot{w}} \quad (4.3)$$

Another useful parameter to compare thrusters is the overall efficiency, defined as the ratio of the kinetic energy of the jet to the input power.

$$\eta = \frac{1}{2} \frac{u^2}{P_{inp}} \quad (5.1)$$

Modern liquid hydrogen/liquid oxygen chemical propulsion systems are limited to specific impulses of about 500 sec, while future more energetic propellants (like metastable Helium) are expected to achieve up to 3150 sec. [6].

2.2 COMPARISON OF ELECTRIC PROPULSION SYSTEMS

The specific impulse available from chemical systems is inherently limited by the chemically-bound energy of the propellants. Other systems which use applied electric and magnetic fields do not suffer from such limitations, specifically, electric propulsion can provide extremely high specific impulse levels.

There are three essentially different categories of electric propulsion systems, distinguished by the major acceleration mechanism employed [9]:

- a.) *electrothermal thrusters*
- b.) *electromagnetic thrusters*
- c.) *electrostatic (ion) thrusters*

2.2.1 Electrothermal Thruster

The common feature of electrothermal propulsion systems (Figure 7.1) is the addition of energy to the propellant by electrical heating. The random thermal energy in the propellant is then transformed into axially directed kinetic energy by an expansion of the hot propellant gas through a nozzle. Since the energy addition is not limited to the chemically bound energy of the propellant the specific impulse can be increased. The limiting factor for the electrothermal device is set by the maximum allowable surface temperatures of the heating elements of containment structure. Electrothermal devices attempt to limit the contact of the hot fluid with the thruster walls by adding energy at the core of the flow and establishing a layer of cooler fluid between the hot core and the wall. One differentiates between different classes of electrothermal accelerators depending on the way the core-fluid is heated.

The simplest electrothermal thruster is the resisto-jet, whereby a resistive heating element delivers energy to the fluid by conduction and convective heat transfer. The heated fluid is then expanded through a nozzle [9]. Since the propellant is heated by contact with a hot physical object, the heating element, the maximum temperature of the fluid is determined by the maximum allowable temperature of the heating element and the efficiency of the energy transfer between the heating element and the propellant. Obtainable exhaust velocities are therefore limited. However, even though the Isp level is usually below 300 second, the resisto-jet offers several advantages over chemical propulsion in the small-scale thrust range. Several Newtons of thrust can be produced, and even more importantly, the possible use of waste fluids and gases as propellant make the resisto-jet a prime candidate for space-station applications [9].

The Arcjet (Figure 7.2) is another type of electrothermal device. Arcjets heat the working fluid by using an electric discharge between a cathode and an anode. The anode, in general, also

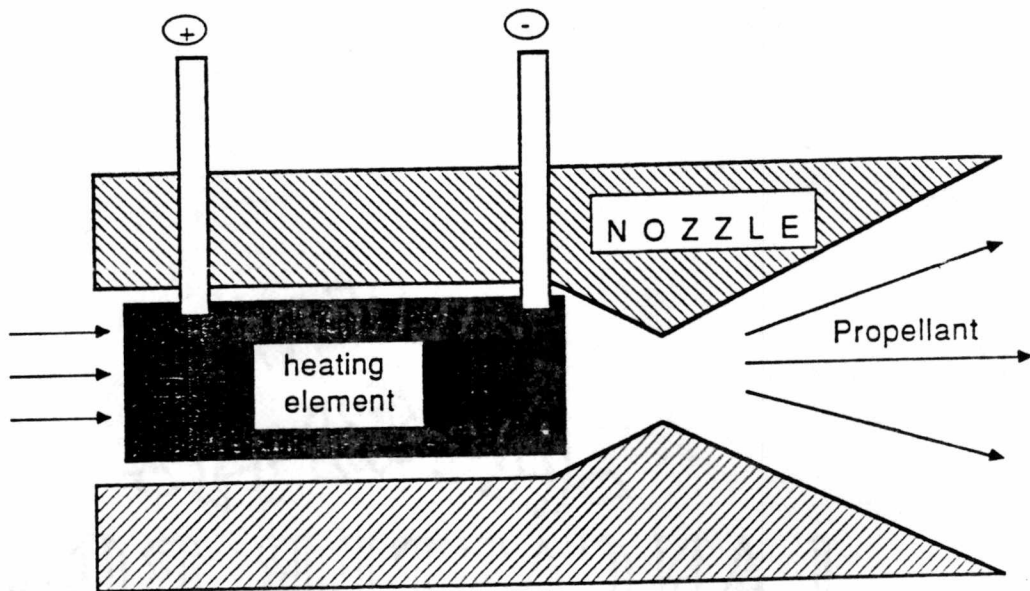


Figure 7.1: Electrothermal thruster (resisto jet)

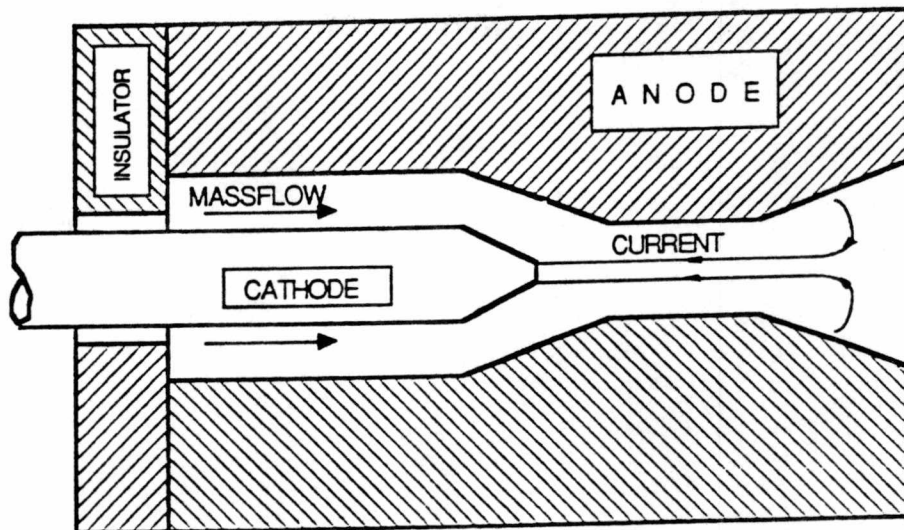


Figure 7.2: Electrothermal thruster (Arcjet thruster)

serves as the nozzle. Injecting the fluid between the cathode and anode with an azimuthal velocity component (swirl flow) keeps the wall attachment point of the arc moving and also serves as a "cold" layer between the hot arc and the wall. The arc is therefore confined to a small volume at the core of the device and higher temperatures can be sustained in the arcjet, as compared to the resisto-jet. Attachment to the walls occurs only at the cathode tip and the anode, which does not limit the temperature within the arc itself. The attachment of the arc to the cathode does, however, decrease the lifetime of a thruster due to cathode erosion. Nonetheless, higher values of the specific impulse, as compared to the resistojet, can be achieved.

Using hydrazine as propellant, arcjets have produced an Isp of 700 seconds [11] and have shown overall efficiencies of 30 % [5]. Hydrogen arcjets have achieved specific impulse values that approach 1500 sec.

The prime objectives of arcjet investigations are stable operation of the arcjet, energy loss mechanisms and a reduction in cathode erosion rates to increase lifetime. Stable operation of the arcjet can be realized by using a vortex flow to push the arc down through the constrictor or nozzle throat.

Heat loss to the anode and frozen flow losses are major energy loss mechanisms in arcjet operation. Frozen flow losses occur because of incomplete recombination of ions and neutral particles and incomplete recombination of dissociated molecules. Electrode erosion can be influenced by various procedures, such as cooling of the anode, starting methods, reversing polarity and imposing an external magnetic field [5].

In order to circumvent the material temperature limits caused by a continuous operation of an arcjet, pulsed electrothermal (PET) thrusters have been developed. These thrusters provide multi-megawatt input power levels in microsecond bursts, with milliseconds between pulses [10]. The PET thruster offers the additional benefit of pressure levels beyond 100 atm [12]. At these pressure levels

the propellant fluid is close to equilibrium, and most dissociation and ionization energy can be recovered in the nozzle of these systems [13]. Due to the higher (burst) input power, Isp levels of up to 3000 seconds can be realized, and efficiencies of up to 56 % were demonstrated [5].

Because cathode erosion, energy loss to the anode and frozen flow losses severely limit the performance of arcjets, and to a lesser degree the performance of PET thrusters, other configurations have been studied that eliminate the electrodes altogether. Electrodeless discharge thruster concepts offer the possibility to circumvent the limitations of arcjets caused by electrode erosion and heat loss.

One possible technique to heat the propellant is to apply microwave power in a plasma discharge. These devices are called microwave electrothermal thrusters (MET). A high efficiency for power absorption, high power density, and external control of the discharge location, shape, and volume are cited as major benefits [4]. Power coupling efficiencies, i.e. the percentage of input energy being absorbed by the plasma, in excess of 95 % were demonstrated, and specific impulse levels of up to 600 seconds with helium propellant have been reported [14]. A drawback is that the energy is absorbed primarily near the surface of the volume, i.e. near the thruster walls.

Plasmas can also be generated and sustained by focussed laser beams where absorption of power from an optical beam occurs by the process of inverse bremsstrahlung [15],[16] of free electrons in the plasma absorbing the beams. Because the optical frequency of the sustaining laser beam is greater than the plasma frequency, the beam can propagate well into the interior of the plasma, offering a significant advantage over microwave discharges [17]. Steady-state plasmas with temperatures of 10000° K or more can be generated by lasers far away from walls (see figure 10.1). Consequently, a reduction in the heat loss to the walls can be achieved and limitations due to material

temperature limits can be avoided. Both the MET and the laser sustained plasma thruster are in early stages of development, but offer great potential.

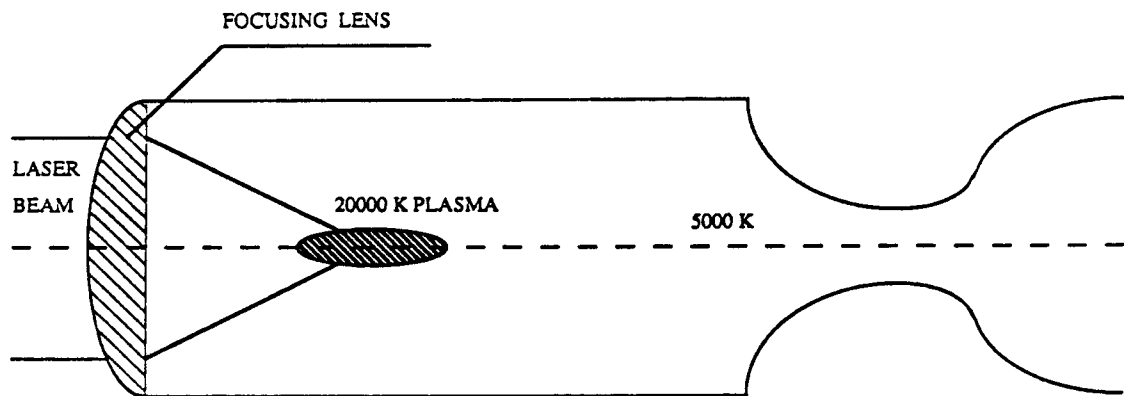


Figure 10.1: Schematic of a laser sustained plasma thruster

2.2.2 Electromagnetic Thruster

The electric propulsion concepts discussed so far are based on converting electrical energy into the thermal energy of the working fluid, and transforming the random thermal energy into directed kinetic energy, differing from chemical propulsion systems only in the way the energy is transferred to the working fluid.

A different acceleration mechanism is employed in electromagnetic thrusters. Here, the major acceleration is due to electromagnetic forces that act directly on the fluid, while heating of the propellant does not significantly increase the thrust of the device [18]. The electromagnetic forces, called Lorentz forces, occur when a current and a magnetic field are not aligned with each other.

Magnetoplasmadynamic (MPD) thrusters consist of a coaxial arrangement of a center-body cathode and an anode, separated by an insulator at the upstream portion through which the propellant is injected into the thruster (see Figure 11.1). Given a strong enough electric field, the propellant ionizes and becomes electrically conducting. The electric current raises the temperature of the electrically conducting propellant which is then defined as a plasma. Either a constricted arc or a diffuse discharge will form between anode and cathode and a significant current can be carried between the two electrodes.

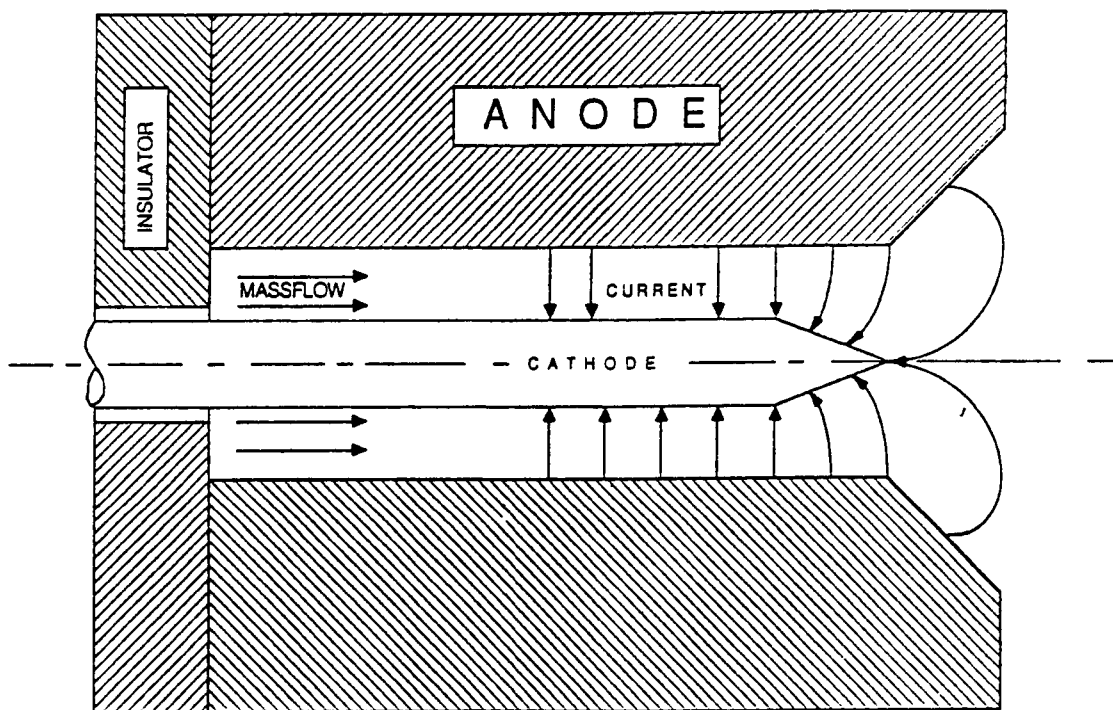


Figure 11.1: Schematic of an MPD thruster

A strong current in the radial direction creates a substantial magnetic field in the azimuthal or circumferential direction. The radial component of the current interacts with the induced magnetic field in the azimuthal direction to create an axially directed body force, which accelerates the plasma through the nozzle. The axial component of the current directed toward the upstream portion of the thruster interacts with the induced magnetic field to force the plasma towards the center line, tending to increase the pressure at the center line. High exhaust velocities can be realized, because the Lorentz force accelerates the plasma directly. This is reflected in a specific impulse of up to 4500 seconds [19]. Depending on the input power level, up to 100 Newtons of thrust can be provided by MPD thrusters with Argon propellant flow rates of 6 gms/sec at input currents of approximately 23 kA [20]. At this current level, voltage fluctuations were observed, indicating a discharge instability, but severe cathode erosion did not result. The onset of cathode erosion limits the lifetimes of MPD thrusters [21]. Overall efficiencies of 30 to 50 % have been demonstrated using hydrogen propellant [22],[23], but efficiencies in general have not exceeded 30% for most propellants [4].

In several proposed electromagnetic devices use is made of the Hall current which is induced in a direction perpendicular to the applied electric and magnetic fields. An electron entering a magnetic field is deflected perpendicular to the surface defined by the direction of the magnetic field and the electron velocity vector. If the electron does not collide with other particles, it will continue to gyrate with a radius depending on the strength of the magnetic field and the velocity of the electron. This particular radius is called the gyro radius or the Larmor radius. If, however, the electron collides with other particles, the electron will be deflected away from this helical path. The ratio of the mean free path between collisions, λ , to the Larmor radius, r_L , determines the nature of the transport properties, a large ratio indicating significant directional dependencies of the transport

properties [24]. The ratio of the mean free path between collisions and the average Larmor radius is known as the Hall parameter ($\omega\tau$) and is defined as [25]:

$$\omega\tau = \frac{\lambda_e}{r_L} = \frac{eB}{m_e n Q c_e} \quad (13.1)$$

with Q being the electron momentum collision cross section for the atoms and molecules involved and c_e being the mean random thermal velocity of the electrons [25].

$$c_e = \sqrt{\frac{8kT_e}{\pi m_e}} \quad (13.2)$$

The more distance the electrons can travel on the average before they collide with other particles, the greater their deflection from the direction of the electric field. This condition corresponds to a large Hall parameter and occurs predominantly in low density devices. The result of all of the deflections that the electrons experience traveling through a magnetic field gives rise to a current, referred to as the Hall current. Since the current carried by the electrons is proportional to the number of charge carriers and their velocity

$$\vec{j}_e = -en_e \vec{u}_e \quad (13.3)$$

an increased deflection represents an increase of the Hall current. Electromagnetic accelerators operating with large Hall parameters ($\omega\tau > 1$) possess large Hall currents, and are therefore referred to as Hall current thrusters.

A Hall current thruster has a an electrode arrangement like the MPD thruster. As with the MPD thruster, a current is established between cathode and anode. However, if a strong magnetic field is applied in the axial direction and the mass density of the device is low, a Hall current in the azimuthal direction is formed. Interaction of this Hall current with the applied magnetic field causes a radial compression of the plasma, while the interaction of the Hall current with its induced magnetic field in radial direction causes an acceleration of the plasma.

Any MPD thruster operated with an applied magnetic field essentially displays some characteristics of a Hall thruster, and the difference between the two devices is somewhat arbitrarily given by the dominant current, direct or Hall current. Due to the existence of additional self-induced currents and magnetic fields, the acceleration mechanisms become very complex. Analysis usually is based on the fields and currents of first and second order strength.

2.2.3 Electrostatic Thruster

In electrostatic propulsion systems, electrically charged particles (ions and electrons) are accelerated by an applied stationary electric field. The force acting on the charged particles is proportional to the charge and the strength of the electric field. The ions are created in a hollow cathode discharge and accelerated towards an "accelerator" grid that is at a lower electrical potential than the ion source. After passing through the accelerator grid, the ion beam is neutralized by an injection of free electrons. Due to the low mass of the ions, and the strong accelerating force created by the electric field, the ions achieve very high exhaust velocities. Specific impulse levels of 5000 seconds and above can be achieved, but even 30 KW thrusters produce only milli-Newtons of thrust.

The overall efficiency of ion thrusters is about 70% [26],[27]. Figure 15.1 illustrates the concept of an ion thruster.

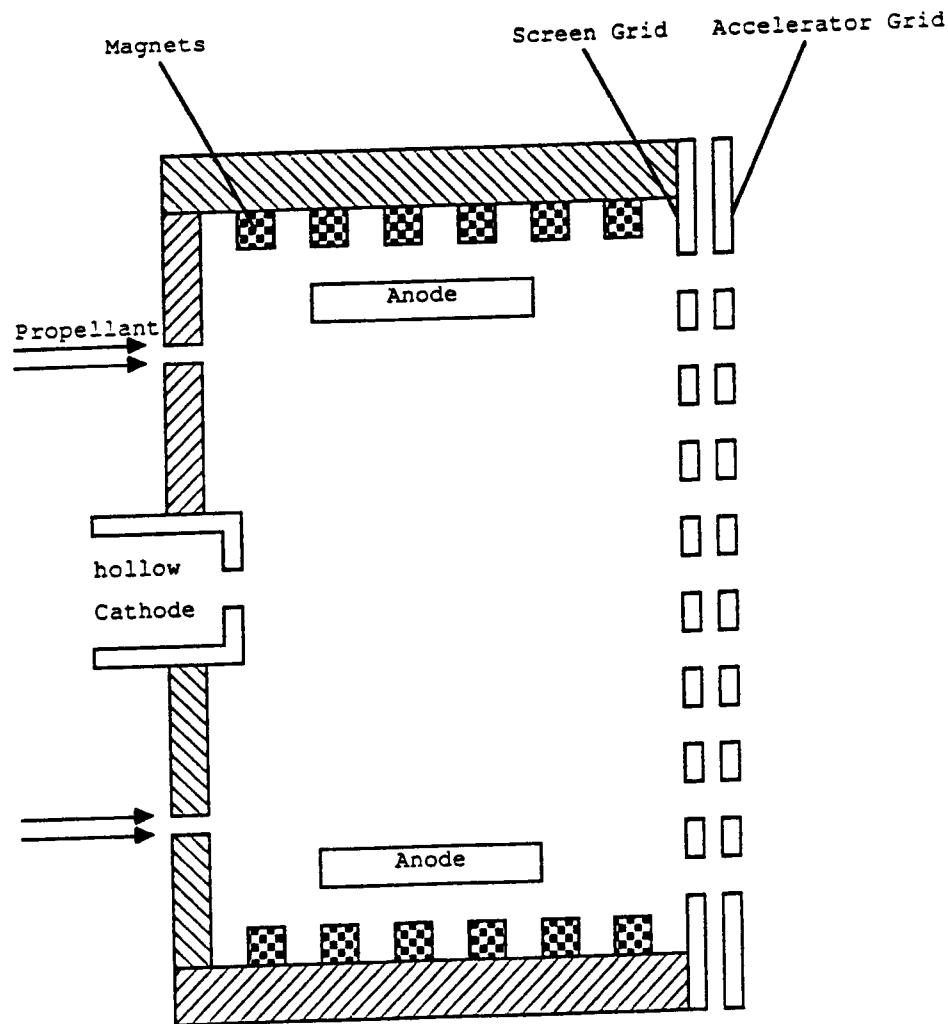


Figure 15.1: Schematic of the most popular design of an ion thruster

2.3 STATE OF THE ART

Resisto-jets are the most developed electrothermal-type thrusters and are already being used on satellites for station keeping. Ion thrusters and low power (1-2 kW) arcjets are also fairly well

developed and are planned to be used on satellites within a few years. High power arcjets (of interest for orbit transfer) require additional development to increase their efficiency and longevity. Heat losses to the anode are thought to be a major contributor to the energy loss in high power arcjets [28], and excessive cathode erosion at high current density limit the life-time.

Erosion of the accelerator grid is a major life-time limiting factor for ion thrusters and is being investigated theoretically and experimentally [29],[30].

MPD thrusters are several years away from actual flight, but due to the promise of high specific impulse in combination with fairly high thrust warrant continued research. An improved understanding of the physical processes is imperative for the development of MPD and Hall current thrusters to produce a flight-ready propulsion unit.

MET and laser sustained plasma thrusters are the least developed of the thruster types, and will require several years of additional research effort to achieve flight-readiness.

A comparison between the different thrusters is given in Figure 16.1.

TYPE	achievable THRUST LEVEL (Newtons)	I_{sp} (sec)	OVERALL EFFICIENCY
CHEMICAL	$>10^6$	<500	$>90\%$
ARCJET/RESISTOJET	>1	1500/300	30%
MPD-THRUSTER/PET	>100	3000	50%
ELECTROSTATIC	$< 10^4$	5000	70%

Figure 16.1: Comparison of different thruster concepts

2.4 PROPOSED ELECTRIC PROPULSION APPLICATIONS

Three areas of activity in space could benefit from electric propulsion:

- a.) *station-keeping*
- b.) *orbit maneuvering and transfer*
- c.) *interplanetary exploration*

2.4.1 Station Keeping

High and low thrust propulsion system for the space station were investigated by Jones [31]. The advantage of having a "dual-mode" propulsion system for the space station lies in the optimal use of different propulsion systems for the different applications. While sufficient thrust is available for large scale motions of the space station, subtle maneuvers are frequently required that need thrust levels that will not interfere with the low gravity scientific research to be performed on board the space station. Studies clearly indicate that utilizing waste water and other fluids as propellant will alleviate the waste disposal problem for the space station. Due to its inherent ability to function with different propellants, resisto-jets will perform the low thrust requirements.

Application of electric propulsion devices for station keeping of satellites is also under consideration. Because station keeping of satellites requires minute amounts of thrust, electric propulsion devices are actually preferable to chemical thrusters. Resistojets are presently used for station-keeping and 1.8 kW arcjets will be used on the Telstar IV communications satellite to be flown in 1994. In the near future, Ion thrusters are supposed to perform the station keeping task of a european satellite [32],[33].

2.4.2 Orbit Maneuvering and Transfer

Orbit transfer missions are required to transport a payload from one orbit to another, usually requiring a change in altitude and inclination. Especially important are the transfer of payloads from the U.S. space transportation systems Space Shuttle's low earth orbit (LEO) to a geosynchronous orbit (GEO), resupply, spacecraft retrieval, and relocation of old or inoperative spacecraft for space debris management [34].

If fast round-trip times are imperative, high thrust is required and chemical propulsion is therefore well suited for these missions. However, if the trip-time is not of major concern (as is very often the case), electric propulsion can provide significant advantages over chemical propulsion due to the increased payload capabilities, a result of the higher specific impulse levels of electric propulsion systems. Manvi and Fujita [34] considered an ammonia and hydrogen arcjet for orbit transfer missions, and compared the results with a Xenon ion thruster. They found that the Xenon ion-thruster is an excellent device for a nuclear powered OTV, if long trip-times are acceptable. Arcjets, due to their higher thrust levels, are attractive if shorter trip-times are required.

Rudolph and Ogg [35] compared an advanced liquid oxygen/liquid hydrogen chemically propelled orbit transfer vehicle (OTV), including aerobraking, with an argon fueled MPD thruster OTV. Extrapolations of performance data for the MPD thruster to the early 21st century were used and the availability of megawatt power levels was assumed. They reported expendable MPD thruster OTVs not to be competitive with the advanced liquid oxygen/liquid hydrogen thruster OTVs, unless payloads exceeded 10^5 Kg. However, a reusable MPD thruster OTV was found to be significantly less expensive than the chemically propelled OTV even for payloads as small as 10000 Kg.

As a specific example, they concluded that the MPD thruster OTV can deliver a 10000 Kg payload from LEO to GEO in 30 days at 85% of the cost of the chemically propelled OTV. The

same MPD thruster OTV could deliver a 20000 Kg payload in roughly 80 days at half the cost of an advanced chemical OTV.

The most cost-effective OTV propulsion system depends on the payload mass and even more significantly on the allowable trip-time. Shortest trip-times will favor conventional chemical propulsion, less stringent trip-time requirements favor MPD thrusters and arcjets, and for high trip-time flexibility the ion thruster offers the most cost-effective solution.

2.4.3 Interplanetary Missions

Sauer [36] investigated the use of solar electric propulsion (SEP) for several future planetary missions. In SEP's, the power comes from solar cells, i.e. photo-electric power. The missions investigated included a comet rendezvous - asteroid flyby mission (CRAF), an orbiter mission to Saturn (CASSINI) and a comet nucleus sample return (CNSR) mission. All of these missions are high Δv missions, demanding high launch vehicle lift capabilities when performed as all ballistic missions. The CNSR mission, particularly, would likely require several Shuttle launches. According to Sauer, all the missions mentioned above could be enabled by or benefit from the use of a moderately sized solar electric propulsion system.

CHAPTER III

DEVELOPMENT OF CFD MODEL FOR MPD THRUSTERS

3.1 INTRODUCTION

Obviously, electric propulsion will play an important role in future space endeavors. Each electric thruster concepts has peculiar strengths and weaknesses and, therefore several different propulsion systems will be needed to optimally perform different missions. However, significant thruster improvements will require a clear understanding of the physical processes involved and the ability to predict the performance of thrusters. Performance predictions will enable the design of reliable, inexpensive and optimized thrusters. Only if the design of thrusters can be taken from an Edisonan approach toward a directed development effort, will electric propulsion achieve operational status.

Numerical modeling of arcjets and MPD-Thrusters can significantly enhance the design of improved-performance electromagnetic accelerators because simulations permit inexpensive design variation and trade-off studies. For this purpose, the significant physical processes within electromagnetic accelerators need to be acceptably well simulated. Thus, accurate physical models are necessary to predict the performance well enough for this purpose.

A model of plasma flow through electromagnetic thrusters will be outlined in this chapter. Important physical processes will be described for different thruster types and previous work will be reviewed. The equations that are used in this work to model the plasma flow through electromagnetic accelerators and the implementation of a numerical code will be discussed. Finally, test cases for the validation of the code will be presented.

3.2 SIGNIFICANT PROCESSES

Electrostatic thrusters (ion engines) are very low density devices and therefore lend themselves to numerical simulations via Monte Carlo methods [37],[38].

PET thrusters are high density devices and can therefore accurately be described by a continuum description of the problem. The high density in the arc-chamber also implies near-equilibrium conditions, which simplifies the calculation of conductivity and plasma composition. However, since PET thrusters present highly unsteady conditions, a numerical scheme should be time-accurate. Furthermore, after passing through the throat of the nozzle, the plasma density decreases rapidly and the chemistry probably becomes frozen. An appropriate criterion for the transition from equilibrium to frozen flow should therefore be implemented in a code for plasma-based thruster modeling.

Electrothermal and electromagnetic accelerators operate at medium to low density levels ($\approx 10^{-3}$ kg/m³). The frequency of molecular collisions occurring at these density levels are insufficient to ensure equilibrium conditions within the device. Therefore, the chemical composition, particularly the degree of ionization can deviate significantly from equilibrium conditions.

Furthermore, thermal equilibrium cannot be assumed. Atoms, ions and electrons will not have the same energy levels, and separate species energy equations are required. The deviation of the electron temperature from the heavy particle temperature is particularly important, because the degree of ionization is calculated from Saha's equation (see equation 42.2 and 42.3) using the electron temperature.

Since performance improvements were reported for devices operating in an applied magnetic field [39],[40], it is desirable to include the effect of applied magnetic fields. This, in particular, necessitates the inclusion of the azimuthal momentum equation, since a swirl flow in the azimuthal direction is the consequence of an applied axial magnetic field and radial current density.

Another important factor for electrothermal accelerators is energy loss through radiation. At temperature levels above 10000° Kelvin and pressures close to 1 atm, which exist in electrothermal and PET-type thrusters, the temperature distribution of the fluid is not only determined by conduction and convective heat-transfer, but also by radiation. Since the temperature of the gas has a profound impact on the composition of the propellant and the degree of ionization, a correct description of the flow field should include radiation effects.

Another consequence of the high temperatures in electrothermal and electromagnetic accelerators is a low Reynolds number. Argon gas in MPD thrusters, for example, exhibits a Reynolds number of approximately 20000. The velocity profile throughout the device is therefore significantly influenced by viscous effects and those effects should therefore be taken into account.

Multiple ionization occurs at higher temperatures and, due to the energy consumed to accomplish multiple ionization, the temperature of the plasma is limited. In addition, the electrical transport properties change when multiple ionization exists. These effects should be taken into account when modeling certain thrusters.

3.3 LITERATURE REVIEW

Attempts to numerically model the acceleration mechanisms within electromagnetic accelerators started in the 80's with the onset of powerful super computers and renewed interest in advanced propulsion concepts. Theoretical research on electromagnetic accelerators is presently being performed in Japan, Germany and the United States.

In their earlier work, Kimura et. al. [41] used a finite element method to investigate the current distribution in electromagnetic accelerators with applied magnetic fields. However, the

gasdynamic model was quasi-one-dimensional, certainly inadequate to describe the problem accurately, since a major effect of an applied magnetic field is the creation of an azimuthal swirl flow.

More recently Nishida, Kaita and Tanaka [42] developed another code, using a total variation diminishing (TVD) MacCormack scheme for the axisymmetric Euler equations and an implicit forward time - centered space (FTCS) for the electromagnetic field. They applied this code to an arcjet, operating with Argon as propellant. They specified subsonic inlet conditions and took non-equilibrium ionization and recombination into account (finite reaction rate chemistry). Thermal equilibrium was assumed and viscous effects were neglected. The same base code was used by Tanaka et. al. [43] to investigate the performance of a bell-shaped nozzle. They found that the Lorentz force and the Joule heating (equation 33.1 and 33.2) are only significant in the constrictor region and may be neglected in the nozzle. Nonequilibrium ionization-recombination reactions, however, are important in the expansion region and should therefore be included. The highest degree of ionization was found to be just upstream of the constrictor, which agrees well with the findings of Sleziona et. al. [44], who found the highest electron temperature at the constrictor.

Extensive work in the numerical simulation of self-field thrusters has also been done in Germany over the past several years. Two codes were developed by 1987, one using a finite-difference method to solve a one-dimensional fluid-flow, two-dimensional electromagnetic problem [45],[46], based on earlier work [47],[48], and another one using a modified MacCormack forward differencing scheme to solve the time-dependent, two-dimensional gasdynamic problem for a simple cylindrical geometry [45],[49] to evaluate the starting transient of MPD thrusters as well as pulsed thrusters.

Computed results for the current contour lines and thrust values of the first code were compared to existing experimental data of a nozzle-type thruster (DT2-IRS) [50]. The theoretically determined thrust agreed very well with experimental data for currents in excess of 2.5 kAmps, but

was under-predicted for lower current levels. Fairly good agreement between the calculated and measured current contour lines was also demonstrated, even though an isothermal electron temperature was assumed.

The second, two-dimensional unsteady code, was applied to a coaxial, straight duct geometry with two centered electrodes, with insulators upstream and downstream. The argon mass flow was 0.02 kg/sec, an entrance Mach number of 1.2, an initial temperature 8000°K, and the final input current, after a transient time of 200 μ s, was 12000 Amps. After 200 μ s, the axial velocity component had decreased from about 2500 m/sec to about 2000 m/s with a velocity peak of approximately 3000 m/sec at the downstream portion of the cathode. The Mach number displayed two steep decreases at the two interfaces between the electrodes and their neighboring insulators, followed by a rebounding beyond the pre-interface value immediately after the interface. An overall decrease to subsonic flow occurred in the early upstream portion of the electrodes. From its lowest value of approximately 0.7, the Mach number increased steadily to a choked exit condition (Mach number equal unity). The temperature displayed a fairly monotonic increase to approximately 18000°K, with only small peaks at the electrode-insulator interfaces after 200 μ s. Again, the thrust was predicted very well (within 3 %), but computed mass flow was not conserved. The deviation from the mass conservation law was attributed to the McCormack scheme [46].

Improvements to these codes were reported in 1988 [51],[52]. The one-dimensional flow code was extended by addition of the electron energy equation and therefore eliminated the assumption of an isothermal electron temperature. A fully ionized gas was, however, still assumed. Comparisons with measurements [53] showed a fairly accurate calculation of the electron temperature in the downstream region, but an increasing under-prediction of the electron temperature towards the device. Experimental values for the electron temperatures were not given for the discharge region. The highest electron temperature for an input current of 3000 Amps and a mass flow rate of 0.8 g/sec was calculated in the constrictor region to be almost 40000°K.

The time-dependent code was extended to more complex geometries. Since the original code displayed difficulties solving problems with high ratios of current to mass flow, an existing finite volume gasdynamic code was extended to include electromagnetic effects. However, since time-accurate calculations are very time consuming, only steady-state calculations were performed [52]. The code was applied to the same geometry as in the previous year, but the mass flow rate was reduced to 6 g/sec and the current increased to 10000 Amps. For this case, the temperature stayed almost constant, except for the downstream interface of the electrodes and the insulators, where a steep increase in temperature was calculated, the temperature going from approximately 10000°K to about 40000°K. It is interesting to note that the temperature increase was the same at the cathode and anode, which, due to a predicted current concentration (energy flux density increase) at the cathode, seems unreasonable. Noteworthy also is the fact that a fairly homogeneous current distribution was calculated, making the steep rise in temperature at the downstream portion of the electrodes inexplicable. The axial velocity distribution, however, exhibited a stronger and more monotonic increase close to the cathode. The code was then applied to a similar geometry, but included the expansion region. In addition, there were no insulators on the cathode sides, and the cathode did not extend throughout the computational region, therefore more closely resembling an actual thruster. The geometry used for the computations and its physical counterpart, the ZT1-IRS MPD Thruster, are depicted in figures 26.1 and 26.2, respectively.

For the described case, a peak temperature occurred immediately downstream of the cathode tip on the axis of symmetry with a temperature of more than 45000°K. The mass flow rate for this case was also 6 g/sec, but the input current was 6000 Amps.

During the 21st International Electric Propulsion Conference (IEPC), a significantly improved code was presented [54] as well as a new code including chemical reactions [55] to determine the flow field in vacuum chambers.

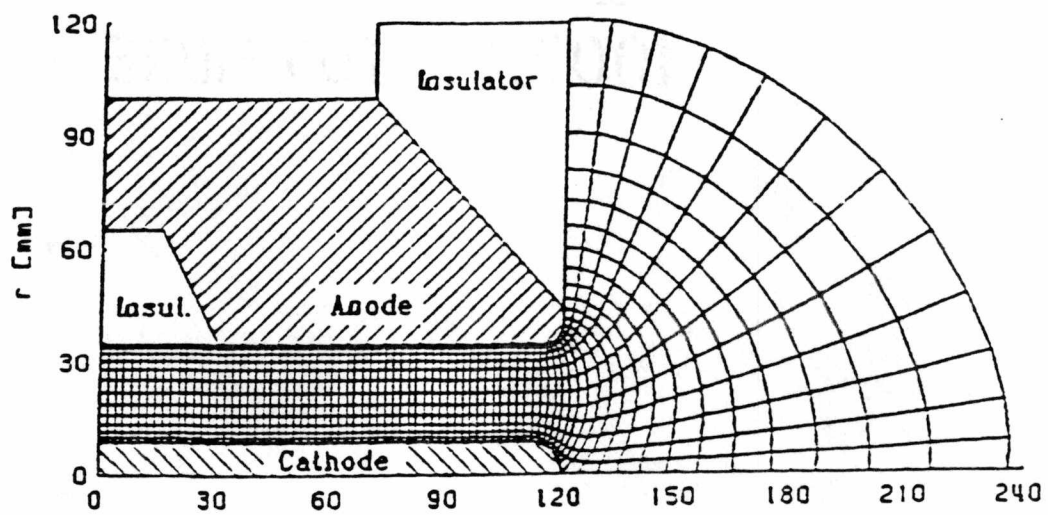


Figure 26.1: Simplified ZT1-IRS geometry used for computation (taken from [52])

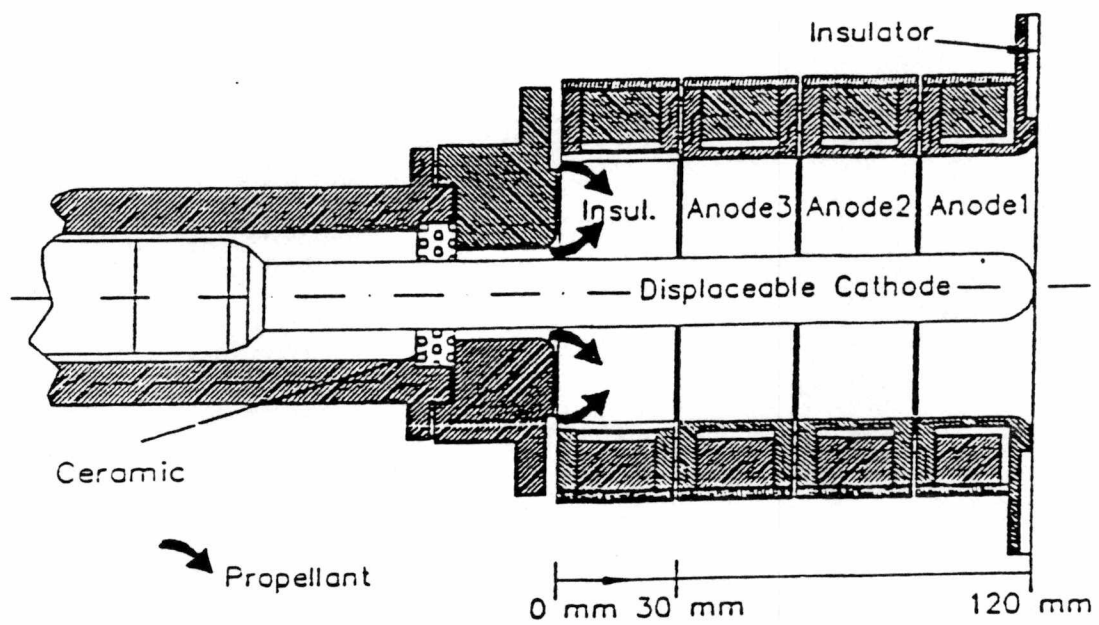


Figure 26.2: ZT1-IRS cylindrical continuous MPD thruster (taken from [52])

Not only did the improved code include some viscous effects, but it also introduced a separate electron energy equation to determine the electron temperature. Transport properties, such as the thermal conductivity and the viscosity were calculated using gas-kinetic models. A argon mass flow rate of 2 g/sec and a current of 6000 Amps were specified for the above described geometry of a "real" thruster geometry (Figure 26.1). The heavy particle temperature at the inlet was assumed to be 300°K, while the electron temperature at the inlet and exit plane was set to 5000°K. The electron temperature increased quickly with axial distance to about 10000°K, with a maximum value of 13000°K on the axis of symmetry just downstream of the cathode tip. The fairly steep rise of the electron temperature after the inlet suggests the existence of fairly strong current densities in that region. The heavy particle temperature, however, increased only modestly in that region, and displayed high temperatures up to 10000°K only in a small region on the axis of symmetry just downstream of the cathode tip. Considering the lower mass flow rate specified in this case, as compared to the similar test case of [52] for the same current, one would expect higher temperatures than reported.

The velocity profile reported for this case showed a significant increase in velocity at the upstream portion of the cathode and anode, with significantly higher speeds there, than in the expansion area. Interestingly, the streamlines closely follow the 90° turn at the anode, suggesting a significant affect of the back pressure on the development of the jet in the expansion region (see Figure 28.1).

Even though not applied to MPD thrusters, the code described in [55] is noteworthy, since it combines the solution of the Navier-Stokes equations with the magnetic induction equation, the electron energy equation and a set of chemical reaction equations to compute the flow-field and the temperature distribution in a vacuum chamber.

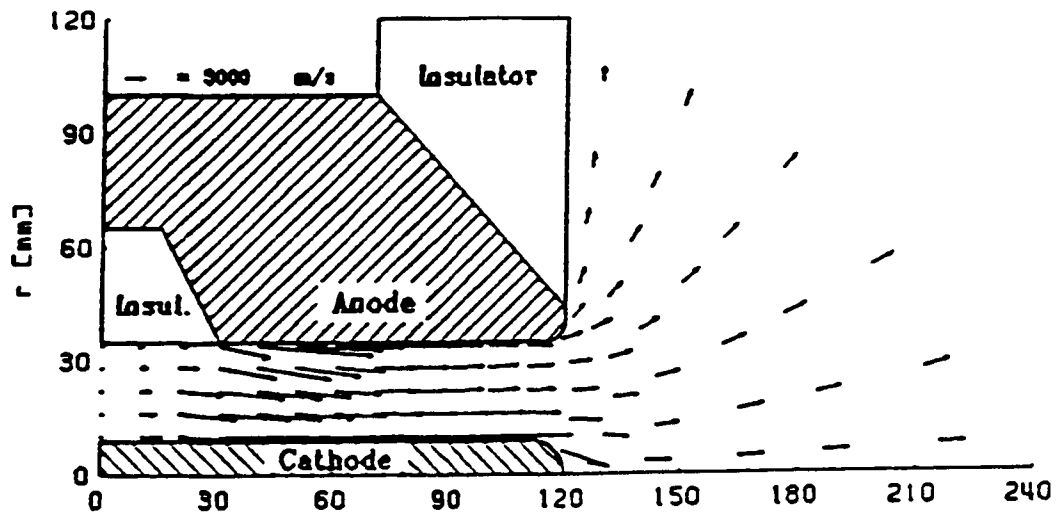


Figure 28.1: Velocity vector distribution (taken from [53])

Martinez-Sanchez and Heimerdinger [56] investigated a two-dimensional, fully ionized and isothermal plasma at finite magnetic Reynolds numbers by a finite volume Newton-Raphson scheme. They showed that tailoring of the channel shape in an MPD thruster can be used to influence the current streamlines, thereby perhaps providing a means of building higher performance thrusters with longer lifetimes.

Martinez-Sanchez et. al. of the Massachusetts Institute of Technology (MIT) provided several new numerical solutions at the 21st IEPC, each tackling a separate problem area.

Niewood and Martinez-Sanchez prepared a quasi one-dimensional solution to MPD thrusters [57]. The model included separate electron and heavy species temperatures, varying conductivity and ionization fractions, collisional energy transfer between heavy particles and electrons, averaged viscosity and ambipolar diffusion, electron heat conduction and velocity slip. They found that thermal

equilibrium is not a viable assumption for MPD thrusters and that viscosity and area variation play a significant role in determining thruster performance.

Special attention to nonequilibrium effects is presented in [58]. A classical kinetic approach to nonequilibrium ionization was offered, using a multi-level description (electronically excited states) of atoms or ions. Multi-level electronically excited states turned out to be irrelevant at temperatures higher than the energy gap between the first excited state and the continuum.

Niewood, Preble, Hastings and Martinez-Sanchez [59] investigated plasma instabilities, which may turn out to be a major performance limiting factor. In particular, they examined two instabilities, which could be excited in an MPD thruster. These instabilities are the modified two-stream instability and the electrothermal instability. They showed that the plasma can evolve to an equilibrium state in the presence of the modified two-stream instability and that the electrothermal instability correlates well with the experimentally observed onset of oscillations near the anode at a critical current ("onset"). Operation of thrusters with higher current levels than those resulting in onset, lead to rapid erosion and therefore reduce the lifetime of thrusters .

Electromagnetic accelerators possess inherently complex and not fully understood physical processes. A complete numerical simulation of all processes, even a complete simulation of all significant processes, is out of reach of today's computers, and insufficient experimental data on transport properties and cross sections provide for a high error margin in any simulation. Any attempt to simulate electromagnetic thrusters via numerical calculations will therefore be a compromise between the desirable effects to be included in a model and computation limitations. Any code, therefore, investigates certain important aspects, but should not be expected to include all possibly important effects.

The goals of the present research is to simulate the magnetic annular arc (MAARC) thruster [60],[61]. This thruster operates with an applied magnetic field, and the effects of an applied magnetic field are therefore significant in any modeling attempt. A profound effect of an applied

magnetic field is the creation of a swirl flow and the azimuthal momentum equation is therefore included in the Navier-Stokes equations. In addition, low magnetic Reynolds numbers (of the order of 10000) indicate highly viscous flow, and one would expect viscous effects to play a significant role in the development of the flow field. Solving the viscous Navier-Stokes equations is therefore preferred over the solution of the Euler equations. The transport properties are extremely important for accurate simulations, and they can vary over a wide range in electromagnetic thrusters due to the significant changes in temperature and density. The electrical conductivity and the Hall parameter (equation 13.1) depend on the electron temperature, which can significantly differ from the heavy particle temperature, depending on the chemical composition of the plasma and the degree of electromagnetic interaction. The electron temperature is therefore calculated in this code. Additionally, double ionization and variations in the cross sections with temperature and density are taken into account to provide fairly accurate transport properties.

3.4 MODELING EQUATIONS

3.4.1 Flow Field Conservation Equations and Boundary Conditions

In free molecular flow it is assumed that few collisions occur between the particles so that the behavior of single particles or collections of particles can be simulated. In continuum flow, there are sufficient particles to have a significant number of collisions, so that the properties at any point in the flow field can be described by an average value, determined by the molecular collisions. This model is used in collision-dominated problems and leads to the commonly used Navier-Stokes equations. Between these two extremes, the free molecular flow and the collision dominated flow, a region exist that cannot accurately be described by either procedure.

MPD-type thrusters have flow rates in the gram/second range, flow velocities of up to 10000 meters/second and temperatures in excess of 10000 degree Kelvin. These conditions lead to mass density levels of 10^{-4} to 10^{-3} kg/m³ and particle density levels of 10^{20} /m³. The mean free path, λ , for these conditions is approximately 10^{-4} meters, and with an inter-electrode spacing of about 10^{-2} meters a Knudsen number

$$K_n = \frac{\lambda}{l} \quad (31.1)$$

of about 10^{-2} exists, where l represents a reference length. This Knudsen number represents the limit of viscous (collision dominated) flow [62] (see Figure 31.1). Therefore, numerical simulation of MPD

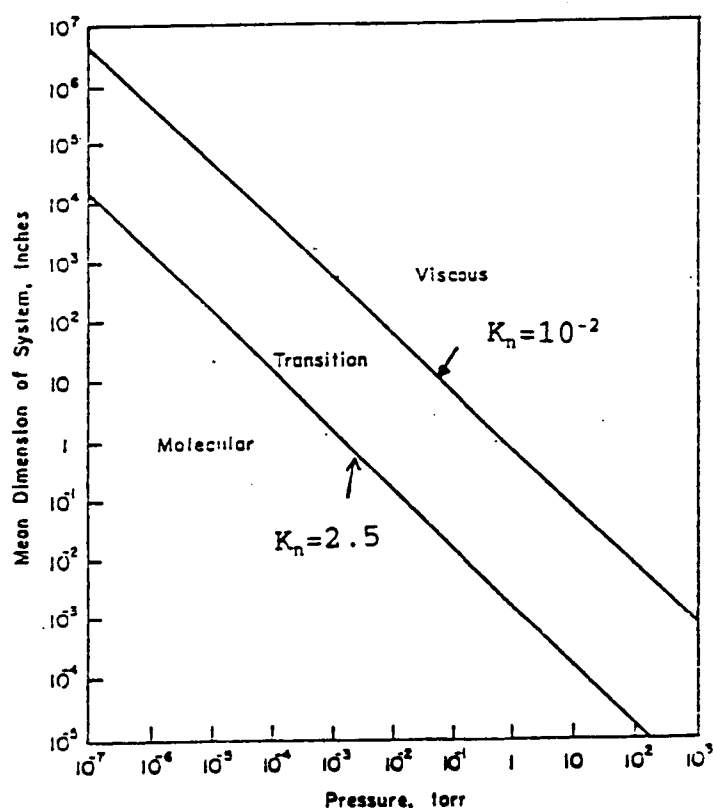


Figure 31.1: Knudsen number and flow regimes as function of pressure and characteristic dimensions

(taken from [62])

thrusters using the collision dominated approach should be valid throughout most of the device and the Navier-Stokes equations can be used.

Values of the Knudsen number above 10^2 represent the transition region for which no numerical modeling is possible at this time. Hence, a Navier Stokes code, the PARC code, was chosen.

The PARC code (see Appendix B.1) was selected for several reasons. First, the code is a fairly robust and efficient Navier Stokes solver. Additionally, the code had been used extensively for internal and external flows and had thereby proven its reliability. Furthermore, the PARC code was extended concurrently to include equilibrium chemistry [79] and [80], and it seemed therefore desirable to use the same code as a basis for this work, in order to ease the combination of the two extended codes.

The version of the PARC code utilized is capable of solving the steady-state, ideal gas Navier Stokes equations for two-dimensional and axisymmetric flows and can be applied to a variety of geometries and boundary conditions [65],[66]. A time-accurate solver is not required for performance predictions. Relinquishing the ability to perform time-accurate calculations simplifies the code and speeds up the computations. Even though the flow through an electromagnetic accelerator is not axisymmetric, the changes occurring in the azimuthal direction fluctuate rapidly and average out. Effects due to the three-dimensional nature of the flow should therefore not significantly influence the performance predictions made with an axisymmetric code. Concurrently at UTSI, work was being performed to improve the basic PARC code to allow for equilibrium and non-equilibrium chemical reactions [79], and a bonding of the real gas code with electromagnetic effects promised to deliver a versatile code. The PARC code appeared to be a reasonable compromise between simplicity, speed and expandability in order to address this problem.

The interaction of an applied magnetic field in the axial direction with a radial current creates forces in the azimuthal direction and cause a swirling flow. The momentum conservation

equation in the azimuthal direction, therefore, has to be included in the set of equations, even though gradients in the azimuthal direction are neglected.

The Navier Stokes equations are modified by the addition of the Lorentz force

$$\vec{F}_L = \vec{J} \times \vec{B} \quad (33.1)$$

and the overall energy equation is modified by inclusion of the Joule heating term

$$\text{Joule heating} = \vec{J} \cdot \vec{E} \quad (33.2)$$

The five conservation equations for mass, momentum and overall energy are then written in strong conservative form as follows for the axisymmetric case. The axial, radial and azimuthal direction are denoted by x , r and z , respectively.

mass:

$$\frac{\partial \rho r}{\partial t} + \frac{\partial (\rho r u)}{\partial x} + \frac{\partial (\rho r v)}{\partial r} = 0 \quad (33.3)$$

x-momentum:

$$\frac{\partial (\rho r u)}{\partial t} + \frac{\partial (\rho r u^2 + p r)}{\partial x} + \frac{\partial (\rho r u v)}{\partial r} = \frac{1}{Re} \left(\frac{\partial r \tau_{x,x}}{\partial x} + \frac{\partial r \tau_{x,r}}{\partial r} \right) \quad (33.4)$$

$$+ S R m r (J_r B_z - J_z B_r)$$

y-momentum (radial):

$$\begin{aligned} \frac{\partial(\rho r v)}{\partial t} + \frac{\partial(\rho r u v)}{\partial x} + \frac{\partial(\rho r v^2 + p r)}{\partial r} = \frac{1}{Re} \left(\frac{\partial r \tau_{x,r}}{\partial x} + \frac{\partial r \tau_{r,r}}{\partial r} - \tau_{z,z} \right) \quad (34.1) \\ + p + \rho w^2 + S Rm r (J_z B_x - J_x B_z) \end{aligned}$$

z-momentum (azimuthal):

$$\begin{aligned} \frac{\partial(\rho r w)}{\partial t} + \frac{\partial(\rho r u w)}{\partial x} + \frac{\partial(\rho r v w)}{\partial r} = \frac{1}{Re} \left(\frac{\partial r \tau_{x,z}}{\partial x} + \frac{1}{r} \frac{\partial r^2 \tau_{r,z}}{\partial r} \right) \quad (34.2) \\ - \rho v w + S Rm r (J_x B_r - J_r B_x) \end{aligned}$$

energy:

$$\begin{aligned} \frac{\partial r e}{\partial t} + \frac{\partial[r u(e+p)]}{\partial x} + \frac{\partial[r v(e+p)]}{\partial r} = S Rm r (J_x E_x + J_r E_r) \quad (34.3) \\ + \frac{1}{Re} \frac{\partial}{\partial x} \left(r u \tau_{xx} + r v \tau_{xr} + r w \tau_{xz} + \frac{r \mu}{Pr(\gamma-1)} \frac{\partial a^2}{\partial x} \right) \\ + \frac{1}{Re} \frac{\partial}{\partial r} \left(r u \tau_{xr} + r v \tau_{rr} + r w \tau_{rz} + \frac{r \mu}{Pr(\gamma-1)} \frac{\partial a^2}{\partial r} \right) \end{aligned}$$

The variables used to nondimensionalize the above equations and the nondimensional parameters are given in Appendix A.1.

The total energy e in the overall energy equation consists of:

- 1.) *the internal energy of the heavy particles, atoms and ions,*
- 2.) *the internal energy of the electrons*
- 3.) *the kinetic energy of directed motion and*
- 4.) *the ionization energy*

$$e = \rho \left(c_v T_{\text{ave}} + \frac{1}{2} \bar{u}^2 + \alpha_1 C_1 + \alpha_2 C_2 \right) \quad (35.1)$$

where ρ is the density, c_v is the specific heat at constant volume, u is the velocity, α_1 the degree of single ionization, α_2 the degree of double ionization and c_1 and c_2 are given by equation 37.4.

There is no energy associated with the rotation or vibration of molecules, since only monatomic gases are being considered and rotation and vibration are absent from monatomic gases [67]. Furthermore, the energy in electronic excitation is neglected and therefore, the only contribution towards the internal energy is due to the translational energy associated with the temperature of the species.

The equation of state is based on the assumption of an ideal gas with constant specific heats, consistent with the above assumptions of monatomic gases and negligible electronic excitation.

The total pressure is equal to the sum of the partial pressures p_i according to Dalton's rule [68].

$$P = \sum_s n_s k T_s = k (n_n T_n + n_i T_i + n_{ii} T_{ii} + n_e T_e) \quad (35.2)$$

Defining the heavy particle number density as [69].

$$n_H = n_n + n_i + n_{ii} \quad (36.1)$$

and the first and second degree of ionization as

$$\alpha_1 = \frac{n_i}{n_H} \quad \alpha_2 = \frac{n_{ii}}{n_H} \quad (36.2)$$

the pressure can be written as

$$p = n_H k T_H + n_e k T_e \quad (36.3)$$

since the heavy particles are assumed to be in thermal equilibrium.

The electron number density can be expressed by the first and second degree of ionization.

$$n_e = n_i + 2n_{ii} = n_H (\alpha_1 + 2\alpha_2) \quad (36.4)$$

and therefore

$$p = n_H k T_{ave} \quad (36.5)$$

The temperature used in this equation is an averaged temperature, consisting of the temperature of the electrons and the temperature of the heavy particles.

$$T_{ave} = T_h + (\alpha_1 + 2\alpha_2) T_e \quad (37.1)$$

with α_1 and α_2 being the first and second degree of ionization, respectively. Using this average temperature keeps the Navier-Stokes equations unchanged.

The pressure is related to the conservative flow variables through the equation of state, where the temperature is being replaced by the internal energy and the specific heat

$$T_{ave} = C_v u \quad (37.2)$$

where u is the internal energy. The internal energy is expressed by the total energy e .

$$p = (\gamma - 1) \left[e - \frac{1}{2} \rho (u^2 + v^2 + w^2) - \alpha_1 C_1 \rho - \alpha_2 C_2 \rho \right] \quad (37.3)$$

with

$$C_1 = \frac{\epsilon_1 e}{\gamma (\gamma - 1) k} \quad C_2 = \frac{\epsilon_2 e}{\gamma (\gamma - 1) k} \quad (37.4)$$

ϵ_1 and ϵ_2 are the first and second ionization energies, respectively. The above relations hold for the non-dimensionalizations as described in Appendix A.1.

Algorithm for the flow conservation equations

There are two distinctly different approaches to the numerical solution of differential equations. The first approach expresses an unknown variable in terms of known variables, and is therefore called an explicit method. This method is easily coded, but requires small time steps to keep the algorithm stable.

The second approach requires the solution of a matrix equation to find the unknown variables and is called an implicit algorithm. The coding is generally more cumbersome than with the explicit methods and the solution of the matrix equation requires more computing time. This disadvantage, however, is offset by being able to use larger time-step sizes, which are possible due to the inherently greater stability of those algorithms. Additional information about the algorithms and the implicit scheme used in this work can be found in [65],[66].

An implicit algorithm is used to solve the flow conservation equations, based on an implicit factorization scheme, as presented by Beam and Warming [70],[71]. Local linearizations are applied to the nonlinear terms and an approximate factorization of the two-dimensional implicit operator into two one-dimensional operators is used. This results in block-tridiagonal matrices, or block-pentadiagonal matrices, if implicit fourth order smoothing is added to the basic conservation equations. Both matrices can easily be solved, but are time-consuming. The solution of the block matrices can be circumvented by a diagonalization of the blocks in the implicit operators. This procedure leads to easily solvable scalar tridiagonal or pentadiagonal matrix equations. Time-accurate solutions, however, can not be produced.

The first and second order spatial derivatives are approximated by central differencing with second order accuracy. A detailed description of the algorithm can be found in Appendix B.1.

Boundary conditions

The option list of boundary conditions for the PARC2D flow field solver is quite extensive and is described in [65] and [66].

The boundary conditions used for the calculations performed in this work are as follows:

- 1.) *subsonic entrance condition*
- 2.) *subsonic or supersonic exit conditions, depending on the downstream conditions*
- 3.) *Isothermal, no-slip wall*
- 4.) *Adiabatic, slip wall*

For the subsonic entrance condition, the total pressure and total temperature are prescribed and the flow is assumed to be normal to the entrance plane.

The exit condition can be subsonic or supersonic. In case of subsonic exit conditions, the static pressure is prescribed, and the density and velocity are then extrapolated from upstream values. For supersonic exit boundary conditions, all flow variables are extrapolated from upstream values.

Physical walls are treated with the no-slip, isothermal boundary condition. Non-physical walls, such as the center line are treated as velocity-slip, adiabatic walls. Figure 40.1 shows the application of the boundary conditions to the MPD thruster geometry.

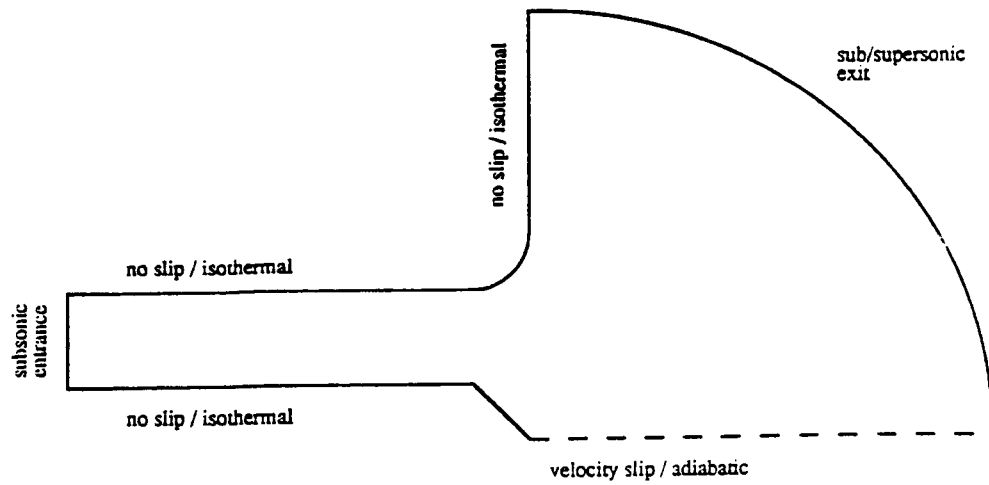


Figure 40.1: Application of boundary conditions

3.4.2 Magnetic Induction Equation and Boundary Conditions

The electromagnetic problem is fully described by Maxwell's equations in combination with the generalized Ohm's law, which relates the current density to the electric and magnetic fields. Maxwell's equations are given by [72]:

$$\frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} - \nabla \times \vec{B} = -\mu_0 \vec{J} \quad (40.1)$$

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \vec{E} = 0 \quad (40.2)$$

$$\nabla \cdot \vec{B} = 0 \quad \nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_0} \quad (41.1)$$

where ρ_e is the charge density and Ohm's law including Hall effects by:

$$\vec{J} = \sigma (\vec{E} + \vec{u} \times \vec{B}) - \frac{\omega_e \tau_e}{|\vec{B}|} (\vec{J} \times \vec{B}) \quad (41.2)$$

Taking the curl of Ohm's law, substituting the curl of the electric field in Maxwell's equation, and replacing the current density by the curl of the magnetic field, one obtains the magnetic induction equation, which expresses the electromagnetic problem only in terms of the magnetic field.

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \left[\frac{1}{\sigma \mu_0} (\nabla \times \vec{B}) \right] - \nabla \times (\vec{u} \times \vec{B}) + \nabla \times \left[\frac{\omega_e \tau_e}{\sigma |\vec{B}| \mu_0} ((\nabla \times \vec{B}) \times \vec{B}) \right] = 0 \quad (41.3)$$

For axisymmetric flow, the electric field in the azimuthal direction is zero, since it is equal to the change of the voltage in azimuthal direction. Implementing this result and the assumption of axisymmetry in Maxwell's equations (see equations 40.1 and 40.2) shows that the magnetic field in the axial and the radial direction does not change with time. Therefore, the system of three partial differential equations for the magnetic field reduces to one equation for the induced magnetic field in the azimuthal direction. As before, the axial, radial and azimuthal direction are denoted by x , r , and z , respectively.

$$\begin{aligned}
0 = & \frac{\partial B_z}{\partial t} + \frac{\partial v}{\partial r} B_z - \frac{\partial w}{\partial r} B_r + v \frac{\partial B_z}{\partial r} - \frac{\partial w}{\partial x} B_x + \frac{\partial u}{\partial x} B_x + u \frac{\partial B_z}{\partial x} + w \frac{B_r}{r} \\
& + \frac{1}{Rm} \left[\frac{1}{\sigma^2} \frac{\partial \sigma}{\partial r} \left(\frac{B_z}{r} + \frac{\partial B_z}{\partial r} \right) + \frac{1}{\sigma^2} \frac{\partial \sigma}{\partial x} \frac{\partial B_z}{\partial x} - \frac{1}{\sigma} \left(-\frac{B_z}{r^2} + \frac{1}{r} \frac{\partial B_z}{\partial r} + \frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial r^2} \right) \right. \\
& + \frac{\partial \alpha}{\partial r} \left(B_z \frac{\partial B_z}{\partial x} + B_r \frac{\partial B_r}{\partial x} - B_r \frac{\partial B_x}{\partial r} \right) + \frac{\partial \alpha}{\partial x} \left(-\frac{B_z^2}{r} - B_z \frac{\partial B_z}{\partial r} + B_x \frac{\partial B_r}{\partial x} - B_x \frac{\partial B_x}{\partial r} \right) \\
& \left. + \alpha \frac{B_r}{r} \left(\frac{\partial B_x}{\partial r} - \frac{\partial B_r}{\partial x} \right) + \alpha B_r \left(\frac{\partial^2 B_r}{\partial x \partial r} - \frac{\partial^2 B_x}{\partial r^2} \right) - 2\alpha \frac{B_z}{r} \frac{\partial B_z}{\partial x} + \alpha B_x \left(\frac{\partial^2 B_r}{\partial x^2} - \frac{\partial^2 B_x}{\partial x \partial r} \right) \right]
\end{aligned} \tag{42.1}$$

The electrical conductivity and the Hall parameter can be determined after calculating the electron number density from Saha's equation for the first and second degree of ionization, based on local thermodynamic equilibrium. Saha's equation is:

$$K_1 = \frac{n_i n_e}{n_0} = \frac{\alpha_1 (\alpha_1 + 2\alpha_2)}{(1 - \alpha_1 - \alpha_2)} n_h = \left(\frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} \frac{2Z_1^{elec}}{Z_0^{elec}} \exp\left(-\frac{I_1}{kT}\right) \tag{42.2}$$

$$K_2 = \frac{n_{ii} n_e}{n_i} = \frac{\alpha_2}{\alpha_1} (\alpha_1 + 2\alpha_2) n_h = \left(\frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} \frac{2Z_2^{elec}}{Z_1^{elec}} \exp\left(-\frac{I_2}{kT}\right) \tag{42.3}$$

with α_1 and α_2 given by equation (36.2) as described in [25] and [69]. The transformation of the magnetic induction equation into computational coordinates is described in Appendix A.2.

The induction equation in the azimuthal direction is a time-dependent, second order, non-linear elliptic partial differential equation. The time-dependence can be used as an iteration mechanism to obtain a steady-state solution. Contrary to an exact solution procedure, the degree of accuracy can be chosen with an iterative solution algorithm. Choosing a "rough" convergence criteria for the magnetic field strength can significantly speed up the overall iteration procedure, because very accurate solutions are not necessary in the early stages of the overall iteration (for a description of the overall iteration scheme see page 51).

A Thomas algorithm [73] was employed to solve the system of equations resulting from a Gauss-Seidel scheme. The Gauss - Seidel method avoids the inversion of one sparse, big matrix for every time step. Instead, the matrix inversion is replaced by the iterative solution of a system of tri-diagonal equations. This system can be vectorized easily and therefore be computed simultaneously on a vector processor.

All boundaries for the electromagnetic problem can be classified as being either conductors or insulators. Here, insulators are not necessarily physical insulators, but can be "free" boundaries acting as insulators, as for example, the exit plane or the center line, while conductors are physical boundaries.

Insulators

Currents can not pass through insulators. Using this condition in Maxwell's equation (40.1 and 40.2) leads to the boundary condition for the magnetic field along an insulator.

$$\nabla \times \vec{B} = 0 \quad \Rightarrow \quad \frac{\partial r B_z}{\partial r} = 0 \quad (43.1)$$

For axisymmetric flow the magnetic field times the radius is a constant along an insulator.

$$B_z \cdot r = \text{constant} \quad (44.1)$$

Conductors

The electric field is perpendicular to a perfectly conducting surface and the direction of the electric field is therefore known by the slope of the conductor. Additionally, the curl of the electric field is zero for steady-state conditions, according to Maxwell's equations (40.1 and 40.2). Knowing the geometry of the electrodes and the total current going through the electrodes, the distribution of the induced magnetic field is only a function of the electrical conductivity and the Hall parameter. Expressing the current densities in Ohm's law by the curl of the induced magnetic field by Maxwell's equations (40.1 and 40.2), Ohm's law (equation 41.2) can be written only in terms of the electric field and the magnetic field.

$$\begin{pmatrix} J_x \\ J_r \end{pmatrix} = \frac{1}{Rm\tau} \begin{pmatrix} \frac{\partial r B_z}{\partial r} \\ -\frac{\partial r B_z}{\partial x} \end{pmatrix} = \frac{\sigma E_r}{1 + (\omega\tau)^2} \begin{pmatrix} 1 + \Omega^2 B_x^2 & \Omega^2 B_x B_r - \Omega B_z \\ \Omega^2 B_x B_r + \Omega B_z & 1 + \Omega^2 B_r^2 \end{pmatrix} \begin{pmatrix} \alpha \\ 1 \end{pmatrix} \quad (44.2)$$

with α given by the inclination of the electrode and Ω given by

$$\Omega = \frac{\omega\tau}{|B|} \quad (44.3)$$

The derivative perpendicular to the wall can be eliminated by the two equations 44.2. Only one differential equation along the boundary remains with the induced magnetic field and the electric

field as variables. The value of the induced magnetic field at the downstream side of the cathode is known to be zero, because at the cathode tip (lying at the center line) the induced magnetic field must be zero due to the assumed symmetry at this point. Since all other boundaries downstream are regarded as insulators, the induced magnetic field at the downstream side of the anode must be zero, too. The induced magnetic field strength is also known at the upstream sides of the conductors, because the total current going through the electrodes determines the change in the magnetic field from the downstream to the upstream sides of the electrodes by [54].

$$B_z = -\frac{\mu_0}{2\pi r} I \quad (45.1)$$

The distribution of the induced magnetic field along the conductor can now be solved. An initial guess for the electric field enables the solution of the differential equation along the conductor to provide a first guess for the induced magnetic field. The upstream value of the induced magnetic field is compared to the value provided by equation 45.1 and the electric field value is then adjusted along the electrode to improve the conformity between the two values for the induced magnetic field at the upstream portion of the electrode. This process is repeated until convergence of the two values for the induced magnetic field is achieved.

Only two conducting surfaces, i.e. one anode and one cathode boundary, can be specified in the code. Even though the free stream exit boundaries are not physical insulators, they are generally treated as such. To minimize the effect of those boundaries on the general solution of the magnetic induction equation, those boundaries are placed at a significant distance away from the main current. After a numerical solution has been obtained, the placement of the exit boundary must be checked. If the current density and the induced magnetic field are negligible small (as compared to

the inter-electrode values) close to the exit boundary, the distance of the boundary from the inter-electrode region is sufficient to ensure an accurate numerical modeling (negligible effect of exit boundary on solution).

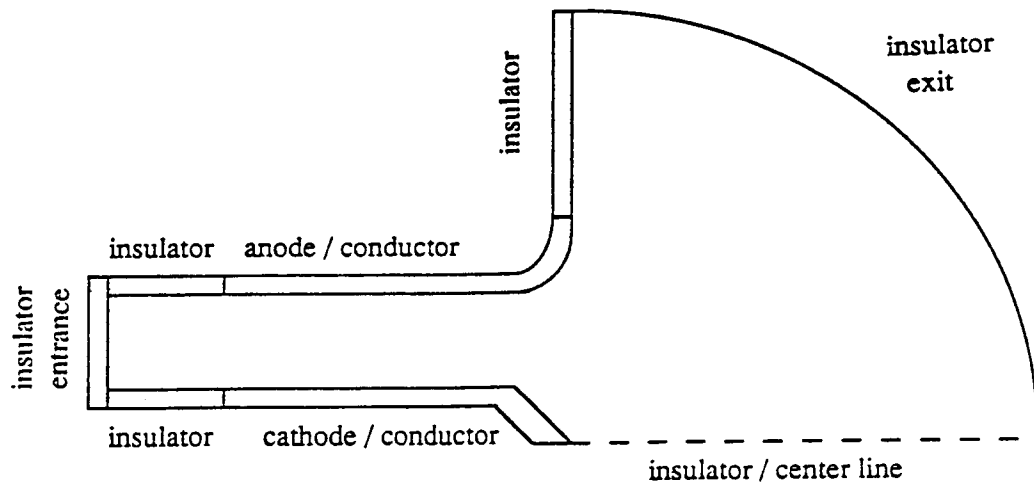


Figure 46.1: Boundary conditions for the induction equation

3.4.3 Electron Energy Conservation Equation

Insufficient collisions may occur in an MPD-type thruster to equilibrate the electrons to the same temperature as the heavy particles. However, it is the temperature of the electrons that determines the ionization and hence the electrical conductivity and the Hall parameter of a plasma. It is therefore necessary to calculate the temperature of the electrons in a plasma thruster and in order to do this, the species energy equation for the electrons must be solved in addition to the overall energy equation.

The two-dimensional electron energy equation, as it applies to plasmas with applied currents and electromagnetic fields, is derived in Sutton and Sherman [24] and can also be found in [74],[75] and [76] in similar form.

$$\begin{aligned}
& \frac{\partial}{\partial t} \left[\rho_e \left(\frac{1}{2} |u_e|^2 + \frac{3}{2} \frac{kT_e}{m_e} \right) \right] + \frac{\partial}{\partial x} \left[\rho_e u_{ex} \left(\frac{1}{2} |u_e|^2 + \frac{3}{2} \frac{kT_e}{m_e} \right) \right] \\
& + \frac{\partial}{\partial y} \left[\rho_e u_{ey} \left(\frac{1}{2} |u_e|^2 + \frac{3}{2} \frac{kT_e}{m_e} \right) \right] + \frac{\partial q_{ex}}{\partial x} + \frac{\partial q_{ey}}{\partial y} + \frac{\partial}{\partial x} (u_{ex} p_e) + \frac{\partial}{\partial y} (u_{ey} p_e) \\
& - \frac{\partial}{\partial x} (u_{ex} \tau_{exx} + u_{ey} \tau_{eyx}) - \frac{\partial}{\partial y} (u_{ex} \tau_{eyx} + u_{ey} \tau_{eyy}) = \\
& = -n_e e (u_{ex} E_x + u_{ey} E_y) + 2m_e n_e \sum_r \frac{\delta_{er} v_{er}^*}{m_r} \left(\frac{3}{2} kT_r - \frac{3}{2} kT_e \right) \\
& + m_e n_e \sum_r v_{er}^{**} [(u_{rx} - u_{ex}) u_{rx} + (u_{ry} - u_{ey}) u_{ry}]
\end{aligned} \tag{47.1}$$

where the summation index r is over all the species present.

Neglecting the effect of viscosity on the electron temperature and assuming (see Appendix A.3)

$$u_e^2 \ll u^2 \quad v_{es}^{**} \ll v_{es}^* \quad (48.1)$$

the following electron energy equation can be derived (see Appendix A.3).

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n_e k T_e \right) - \frac{\partial}{\partial x} \left[5 k T_e \left(\frac{1}{2} n_e u + \frac{j_x}{e} \right) \right] - \frac{\partial}{\partial y} \left[5 k T_e \left(\frac{1}{2} n_e v + \frac{j_y}{e} \right) \right] \quad (48.2)$$

$$- \frac{\partial}{\partial x} \left(\eta_e \frac{\partial T_e}{\partial x} \right) - \frac{\partial}{\partial y} \left(\eta_e \frac{\partial T_e}{\partial y} \right) =$$

$$= j_x E_x + j_y E_y + 3 n_e m_e k \sum_r \frac{v_{e,s}^*}{m_r} (T_r - T_e)$$

This equation is solved to determine the electron temperature in the plasma.

Three possibilities exist for the specification of the boundary condition for the electron energy equation. The temperature of the wall can be specified, i.e. the wall is isothermal, the temperature gradient normal to the wall is set to zero, i.e. the wall is adiabatic, or the electron temperature is determined by a balance of the right hand side of the electron energy equation (equation 48.2).

$$\vec{J} \cdot \vec{E} = 3 n_e m_e k \sum_r \frac{v_{e,s}^*}{m_r} (T_e - T_r) \quad (48.3)$$

In case of an isothermal or adiabatic boundary condition, the electron temperature is independent of the heavy particle temperature. Physically the most reasonable assumption for the boundary value of the electron temperature seemed to be the third option, whereby the electron temperature is determined as a function of the heavy particle temperature. Contrary to [54], in which the electron temperature is arbitrarily prescribed at the entrance and exit of the thruster, the electron temperature in this work is a function of the heavy particle temperature and the transport properties.

The algorithm used to solve the electron energy equation is the same as described in section 3.4.2 for the magnetic induction equation.

3.4.4 Interaction of the Individual Codes

The collision cross sections depend strongly on the electron temperature, and they can vary over several orders of magnitude within the plasma of a thruster. The large discrepancy between the cross sections at different portions of the thruster during operation causes significant differences in the transport properties. Those transport properties are therefore only poorly estimated during early stages of the iteration process. A consequence of this is the need to "pre-iterate" between the induction equation, the transport properties (cross sections) and a form of the electron energy equation, since the full electron energy equation is unstable, if the transport properties are not estimated correctly. Since the major terms of the electron energy equation are the terms on the right-hand-side, namely the Joule heating term and the energy transfer term between heavy particles and electrons, the full electron energy equation can be estimated fairly well by regarding only the right-hand-side. It is, therefore, only those two terms that are used for the pre-iteration (depicted on page 51).

Specifically, the pre-iteration consists of first calculating the cross sections and the transport properties based on the initial starting values. With the transport properties given, the induction equation can be solved to arrive at a first approximation of the current density and electromagnetic field distribution. The knowledge of the electromagnetic variables provides the information for the Joule heating term, being used on the right-hand-side of the electron energy equation. The Joule heating term is balanced by the energy exchange between the heavy particles and the electrons, thereby estimating the temperature of the electrons. With the change in the electron temperature, the cross sections and the transport properties change and need to be recalculated. If the change in electron temperature has become negligible, a new solution for the electromagnetic problem is calculated, and the iteration process for the electron temperature and the cross sections is repeated. This iteration process is continued until the induction equation, the cross sections and the right-hand-side of the electron energy equation have converged.

After the pre-iteration is concluded, the main iteration cycle starts, in which the Navier Stokes equations are solved with the electromagnetic forcing terms (Lorentz force and Joule heating) added to the equations. The iteration proceeds as before, with the only difference being that the full electron energy equation is now solved.

After full convergence is achieved, the code prints the output and plot files (formatted for TECPLOTTM software), as well as the restart files.

SOLUTION PROCEDURE

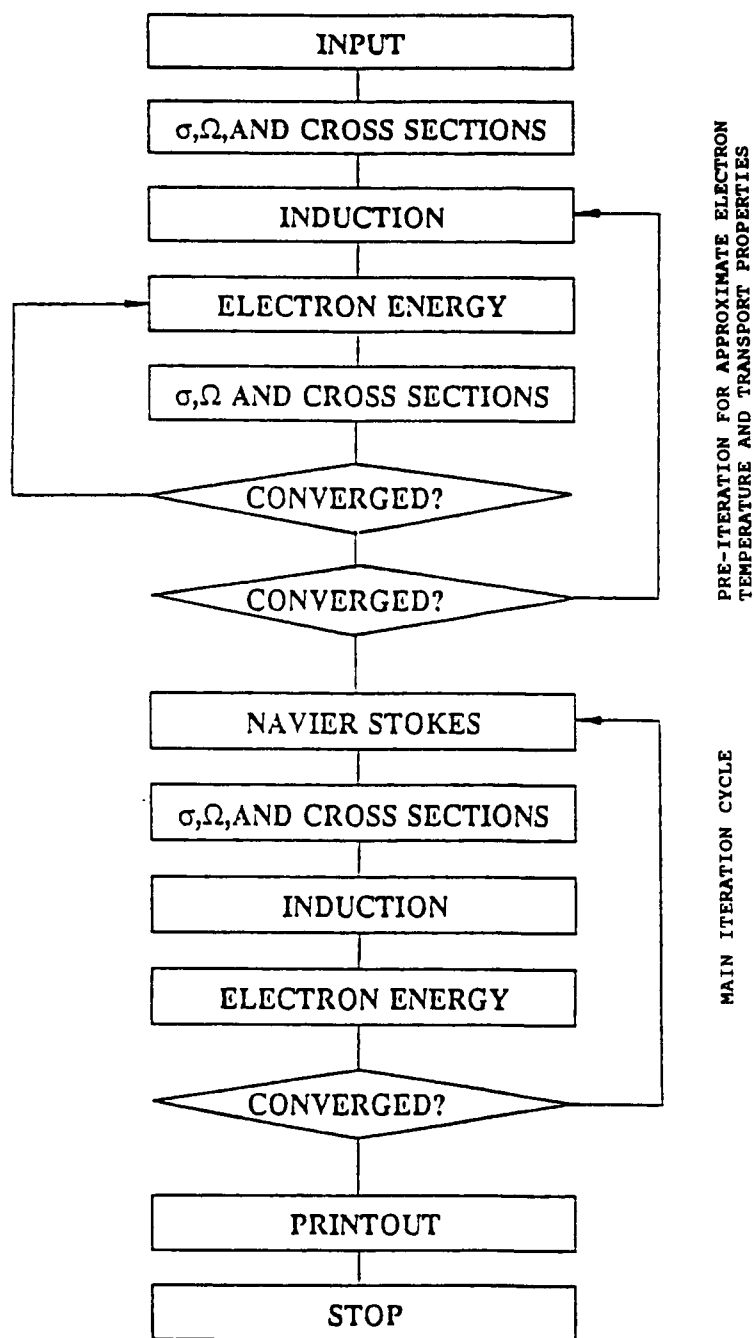


Figure 51.1: Iteration procedure

3.5 VERIFICATION OF THE CODE

Several aspects of the code were checked separately by running different test cases. The plots in all the described cases display the non-dimensionalized variables as they appear in the output from the PARC code (see Appendix A.1) or as they are used in the codes solving the magnetic induction equation (see Appendix A.2).

Test geometry 1: Two coaxial cylindrical electrodes, separated by insulators at the two sides, were used to check the general convergence of the code. The tests were conducted without an applied magnetic field, fluid motion or Hall parameter, unless otherwise specified.

The length of the duct was 0.21 meters, the inner radius (cathode) was 0.021 meters and the outer radius (anode) was 0.105 meters. The reference length used for this case was 0.21 meters. The grid consisted of an evenly distributed 100 x 50 grid.

The basic geometry is displayed in figure 53.1 where the computational region is marked by dashed lines.

The total current going from anode to cathode has to be constant (conservation of total current). Because the total current, I_{total} , penetrates a linearly decreasing surface area with decreasing radius, r , for a coaxial electrode arrangement, the current density in radial direction, j_r , varies inversely proportional to the radius.

$$j_r = \frac{I_{total}}{2\pi rL} \quad (52.1)$$

where L is the length of the device and is constant for this particular geometry. This relationship between the current density and the radius is displayed on page 56.

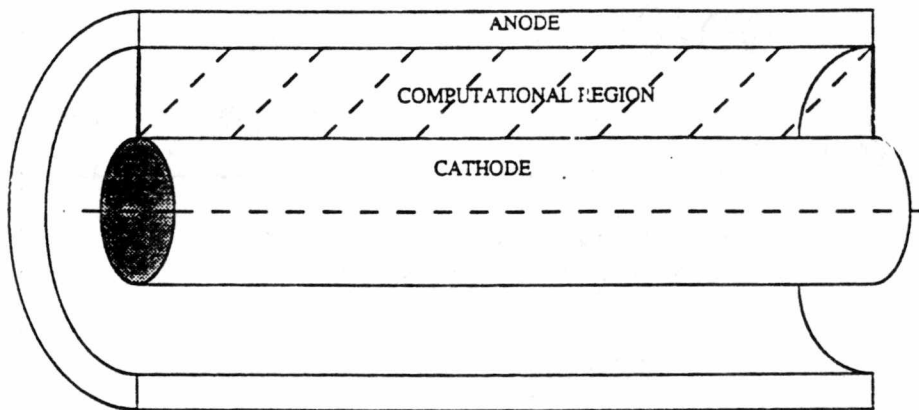


Figure 53.1: Basic geometry for test case 1 (dashed lines mark the region of computation, displayed in the following figures)

For the above described case the electric field has no components in the axial direction, because there is no voltage gradient in this direction (electrodes extend all the way to the sides)

$$\frac{\partial V}{\partial x} \equiv 0 \quad \Rightarrow \quad E_x \equiv 0 \quad (53.1)$$

with V being the voltage. In addition, since there is no fluid motion and the Hall parameter is set to zero, Ohm's law (equation 41.2) relates the current density to the electric field by

$$J_x = \sigma E_x \quad (54.1)$$

Therefore, the current density in axial direction, j_x , must be zero, as well. Ohm's law (equation 41.2) in radial direction shows that the electric field in radial direction, E_r , is proportional to the current density in radial direction, j_r ,

$$E_r = \frac{j_r}{\sigma} \quad (54.2)$$

for this case and, therefore, increases towards the cathode. This result is reflected on page 56, which shows the electric field lines on the right side of the figure to run straight from the anode (top) to the cathode (bottom). The electric field vectors at the bottom of page 56 show the increase of the electric field in radial direction, E_r , as explained above.

The increase of the current density from anode to cathode reveals itself in an increase of the induced magnetic field strength from anode to cathode, since the current density and the induced magnetic field are related by Maxwell's equations (40.1 and 40.2). Without a Hall effect and fluid motion, there is no current concentration caused by the existence of axial currents, because axial currents do not exist for the test geometry 1 with the aforementioned assumptions, as can be verified easily by Ohm's law (63.3). Instead, the current density is distributed evenly over the electrodes. This even distribution of the current density leads to a linear increase of the magnetic field from left to right along an r -equal-constant line.

$$\frac{\partial B_z}{\partial x} = j_r = \text{const.} \quad \Rightarrow \quad B_z(x) = a_1 j_r + a_2 \quad (54.3)$$

with a_1 and a_2 being constants, to be determined by the boundary conditions. Because there is no preferred direction of the induced magnetic field, the boundary condition was specified, such that the induced magnetic field has the same absolute value at the entrance and exit of the device, but with opposite signs. The linear increase of the induced magnetic field is clearly shown on page 57.

Obviously, because the current densities increase toward the cathode, the *change* of the induced magnetic field in axial direction increases toward the cathode, as well, but remains constant along all r -equal-constant lines. This behavior of the induced magnetic field is depicted on page 57.

Using the same geometry as before, the influence of an applied magnetic field in combination with rotating fluid motion was checked. Again, the Hall parameter was set to zero, but a constant azimuthal velocity component and a constant applied magnetic field in the axial direction were imposed on the flow. The solution for this case does not differ from the one above, except that the electric field will be increased or decreased by a constant amount depending on the sign of the velocity in the azimuthal direction and the applied magnetic field. This can be seen from Ohm's law (equation 41.2). With the above assumptions Ohm's law reduces to

$$j_y = \sigma (E_y + wB_x) \quad (55.1)$$

The current density is determined by the total current going through the device and is therefore constant. It follows that the magnitude of the electric field balances with the magnitude of the applied magnetic field times the circumferential velocity component.

The code showed the same graphical results as in the above case, but the numerical value of the electric field was changed according to the value of the azimuthal velocity times the applied magnetic field.

The influence of fluid motion on the distribution of the induced magnetic field was compared to an analytical solution. For a constant axial velocity, negligible Hall parameter and no applied magnetic field the following equation for the distribution of the induced magnetic field for the above geometry was derived for comparison purposes:

$$B_z(x) = \frac{E_y}{u} + \left(B_0 - \frac{E_y}{u} \right) \exp(\mu_0 \sigma u x) \quad (55.2)$$

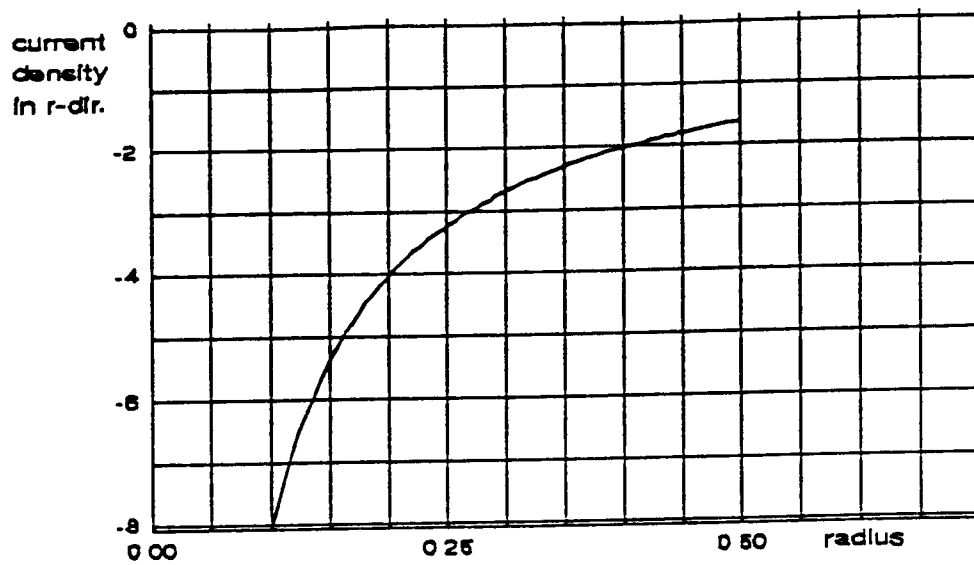


Figure 56.1: Radial current density component for test case 1

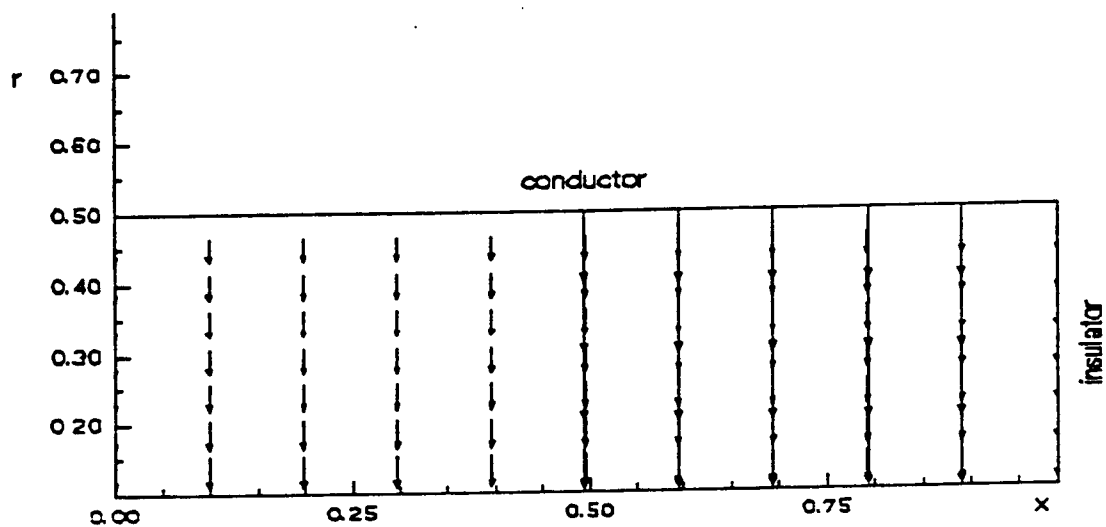


Figure 56.2: Electric field vector and field lines for case 1

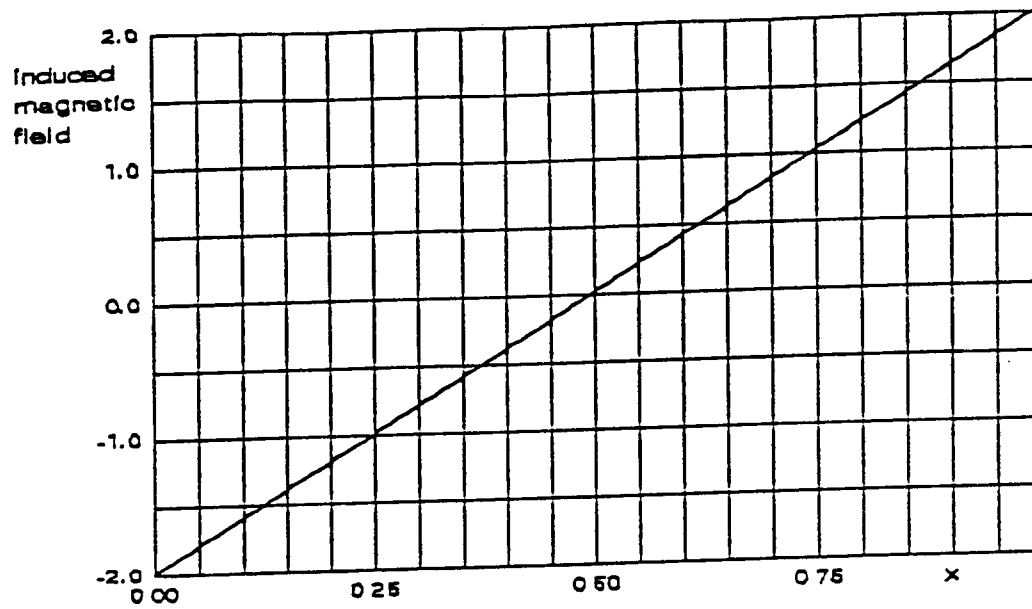


Figure 57.1: Induced magnetic field in the axial direction

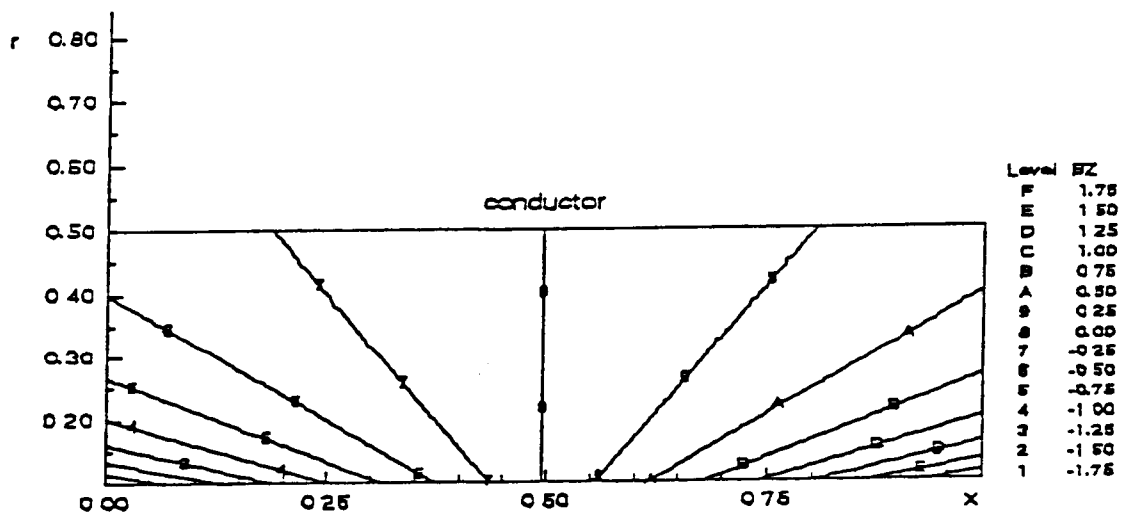


Figure 57.2: Two dimensional display of the induced magnetic field

This equation shows an exponential increase of the induced magnetic field, where the exponent depends on the electrical conductivity, the axial velocity, and the distance x , with μ_0 being the permeability of free space. This nondimensional combination is the magnetic Reynolds number, signifying the interaction of the fluid motion with the electromagnetic problem. Larger magnetic Reynolds numbers lead to stronger current concentrations at the downstream portion of the electrodes, showing the increasing effect of the fluid motion on the electromagnetic problem. This increase of the induced magnetic field at the downstream portion of the electrodes causes a current concentration at these points, and exhibits, therefore, an effect on the current distribution in electromagnetic accelerators similar to that of a non-zero Hall parameter.

Again, excellent agreement was established between the numerical solution and the analytical solution as can be seen from figure 59.1, which shows the numerical solution (indicated by solid dots) closely fitting the analytical solution, represented by the continuous line.

To verify the interaction of the electromagnetic effects in the Navier-Stokes solver, a simple one-dimensional numerical solution using a fourth order Runge Kutta method was produced for a constant Lorentz force in the axial direction and a constant Joule heating term for the same geometry as before. For a one-dimensional problem, the effects can be described by

$$\rho u \frac{\partial}{\partial x} \left(\frac{u^2}{2} \right) + \frac{p}{R} \frac{\partial u}{\partial x} + \frac{u}{r} (j_y B_z - j_z B_y - \rho u \frac{\partial u}{\partial x}) = j_x E_x + j_y E_y \quad (58.1)$$

The same case was reproduced with the Navier-Stokes solver. Both numerical results agreed very well, the maximum difference in all flow field variables being less than 1%. The small remaining difference is explainable by the higher order accuracy (4th order Runge Kutta) and finer grid

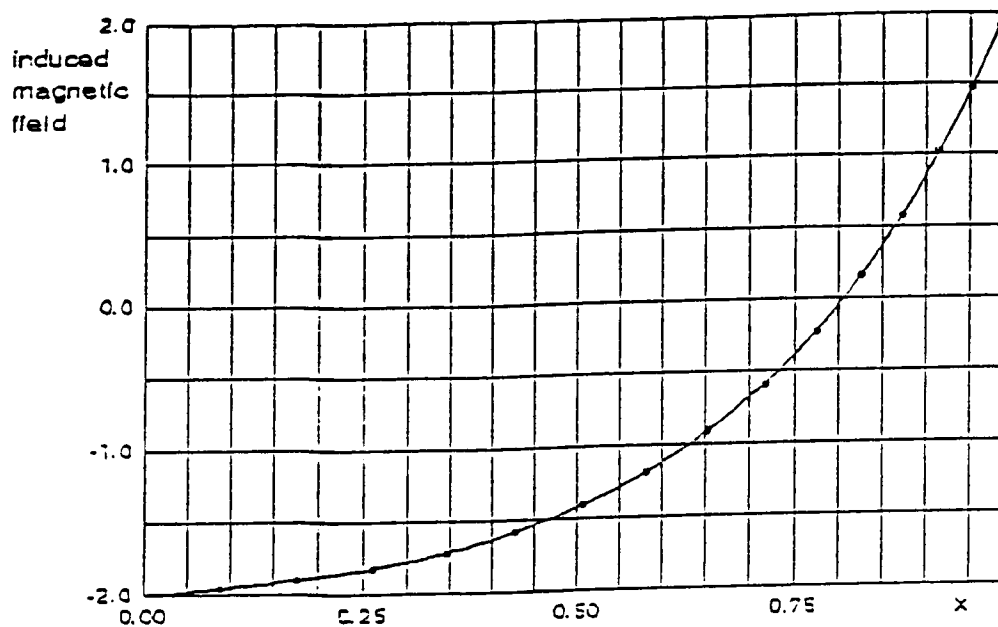


Figure 59.1: Comparison between analytical solution (solid line) and numerical solution (solid dots) of the induced magnetic field for a constant axial velocity

resolution (1000 points) of the simple one-dimensional code. The comparison for several nondimensionalized variables (see definitions in Appendix A.1) is shown in figure 60.1.

In this particular test case, the Lorentz force ($\mathbf{j} \times \mathbf{B}$) led to an almost 50% increase in the axial velocity. This increase in velocity lead to a decrease in temperature, even though a small Joule heating term was present. Because the Runge Kutta code is valid only for supersonic flow, a case with constant or decreasing temperature was desired in order to prevent choking.

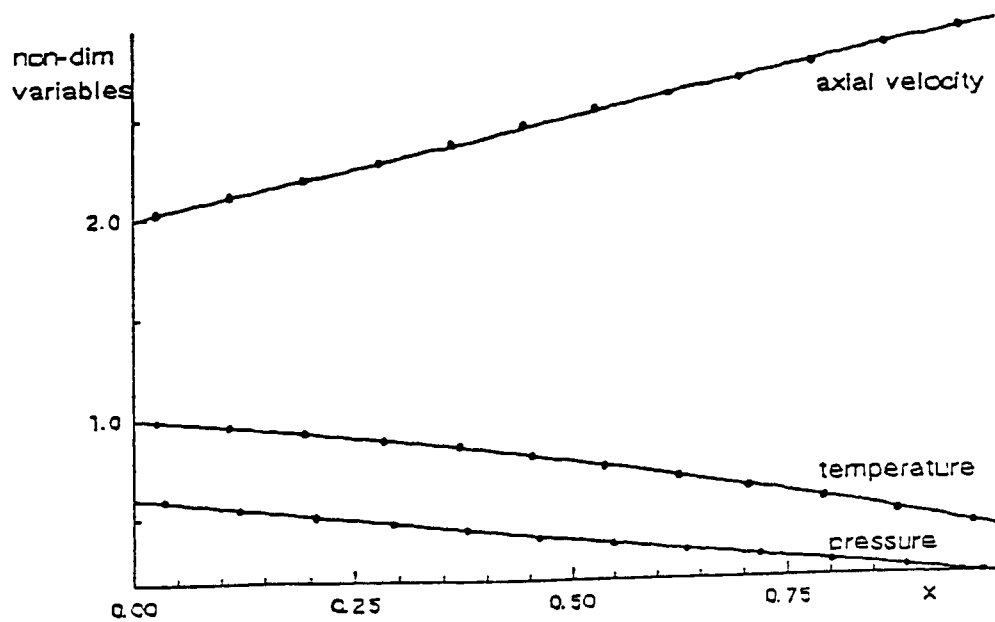


Figure 60.1: Comparison of Runge Kutta solution (solid lines) with PARC code (solid circles) for constant Joule heating and Lorentz force (grid 40 x 10)

Test geometry 2: The test geometry was changed in order to perform validation calculations with a non-zero Hall parameter. Whereas, in the previous cases the electrodes extended completely across the axial length, the electrodes in the following cases were confined to the mid-sections. Therefore, insulator sections were added to the sides of the electrodes. The test calculation was carried out without an applied magnetic field or fluid motion.

The length of the duct was 0.21 meters, the inner radius (cathode mid-section) was 0.021 meters and the outer radius (anode mid-section) was 0.105 meters. The reference length used for this case was 0.21 meters. The grid consisted of an evenly distributed 100 x 50 grid.

The basic geometry is displayed in figure 62.1 where the computational region is marked by dashed lines.

Neither an analytical solution, nor a simple numerical solution could be found for a non-zero Hall parameter and axisymmetric conditions. Therefore, only a quantitative analysis of the computation was performed, including the calculation of test variables. The divergence of the current density and the curl of the electric field were used as those variables. Both test variables should be zero for a fully converged solution.

A problem similar to the configuration above has been solved by several authors [24] and [76], i.e. a two-dimensional, segmented-electrode channel (verses an axisymmetric channel investigated in this work). In both aforementioned cases, however, a constant applied magnetic field was prescribed, contrary to the varying induced magnetic field used for the calculations in this work. Additionally, those cases were developed for segmented MHD generator flows (Faraday type generators), creating an axially directed electric field between the electrodes.

From these investigations, it is known that a non-zero Hall parameter and fluid motion cause a shift between the electric field and the current density. In addition, current concentrations occur at

the connecting points between electrodes and insulators, where the amount of concentration and the preferred direction of this concentration depend on the magnitude of the Hall parameter, the axial velocity component and the direction of the magnetic field.

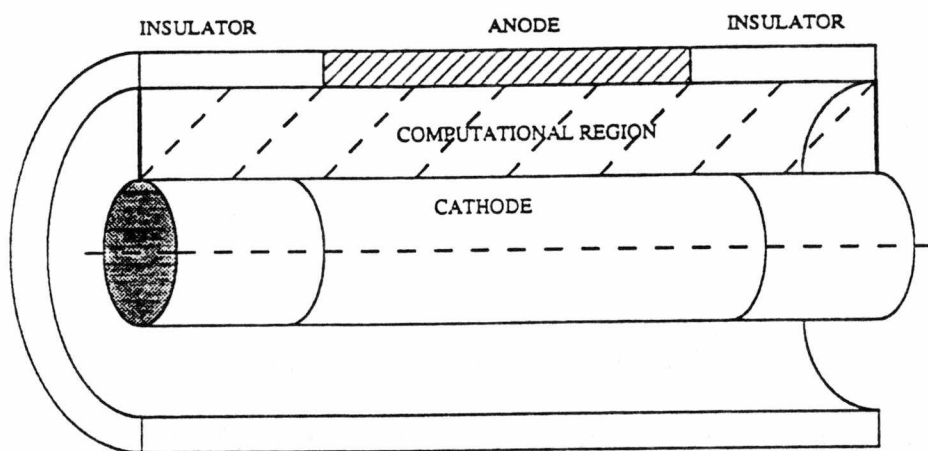


Figure 62.1: Basic geometry for a run with a non-zero Hall parameter

As can be seen from the following plots, the general behavior of the solution for the electric and magnetic fields, as well as for the current density correspond well to the previously discussed results.

Since there is no current passing through the insulators, the magnetic field must be constant along the insulators at the sides of the electrodes, as can be seen by Maxwell's equation (40.1 and 40.2). A stronger variation of the induced magnetic field at the current concentration points (at the intersection of insulators and electrodes) is also visible in figure 65.1, indicated by a concentration of lines of constant induced magnetic field. Also, since the current density has to increase toward the

center line (as explained above), higher values for the magnetic field result at a lower radius (for the same axial position).

A comparison between the streamlines of the electric field and the current density is given at the top and bottom of page 66. Both streamlines bend out towards the sides of the channel.

The "asymmetric" bulging of the streamlines is due to the axisymmetric geometry, which causes the radially directed electric field to increase toward the inner electrode. The axial component of the electric field, however, must be zero at both electrode surfaces. The ratio of E_y over E_x increases therefore toward the inner electrode, causing the "asymmetric" bulging of the streamlines.

It is interesting to note the angle of attachment of the streamlines at the electrodes. While the streamlines of the electric field are perpendicular to the electrode surface, the streamlines of the current density are tilted at an angle. This is caused by the Hall effect.

The differences between the case discussed here with the cases solved by [24] and [76] leads to markedly different results. Whereas in [24] and [76] only one current concentration point at the anode and one at the cathode was found, the case discussed here predicts two such points at the cathode, and a fairly evenly distributed current density at the anode. The fairly even distribution of the current density at the anode can be explained by the fact that the magnetic field strength and the current density at the anode are less than their respective values at the cathode (as discussed in the previous paragraphs). The axial current density at the electrode is given by

$$j_x = -\Omega j_y B_z \quad (63.1)$$

from Ohm's law (equation 41.2), with Ω given by equation 44.3. For these calculations Ω was constant. Hence, the ratio of j_x over j_y is smaller at the anode than at the cathode, leading to a more evenly distributed current density.

As mentioned before, the solution obtained by [24] and [76] were based on a constant applied magnetic field. For the case presented here, the only existing magnetic field was self-induced,

varying from -1.75 to +1.75 from the left side to the right side of the cathode. Since the radial component of the current density does not change it's sign, it is obvious from equation 64.1 that the axially directed current changes sign with the induced magnetic field. This is clearly visible on page 66.

The divergence of the current density was zero for all test runs. However, as can be seen from the contour plot (Figure 65.2), the curl of the electric field was not zero. Even large numbers for the curl of the electric field can be observed, especially at the interface of electrodes and insulators. There are two reasons for this: First, the induced magnetic field in the axial direction jumps at those interfaces, hence providing a discontinuity. In fact, at those interfaces, the electric field would have to assume two values to satisfy the boundary conditions for the electrodes and the insulators. Second, the curl of the electric field is calculated by taking two derivatives of the induced magnetic field; the first derivatives of the induced magnetic field with respect to the axial and radial directions give the current density and by Ohm's law the electric field, and the second gives the curl of this field. The derivatives are found in the computational coordinates, where, for example

$$\frac{\partial}{\partial x} = \xi_x \frac{\partial}{\partial \xi} + \eta_x \frac{\partial}{\partial \eta} \quad (64.1)$$

An error in the value of the magnetic field is therefore, multiplied by the metric for each differentiation. The term "metrics" is used to refer to the set of all first partial derivatives of the curvilinear coordinates with respect to the Cartesian coordinates [63]. In the displayed cases, the metrics were of the order of 200. Taking the second derivative, therefore, multiplies an error in the magnetic field by approximately 40000! Hence, even though large variations in the curl of the electric field exist at the interfaces of electrodes and insulators, the value for the magnetic field is accurate within one percent at the most troublesome calculation point.

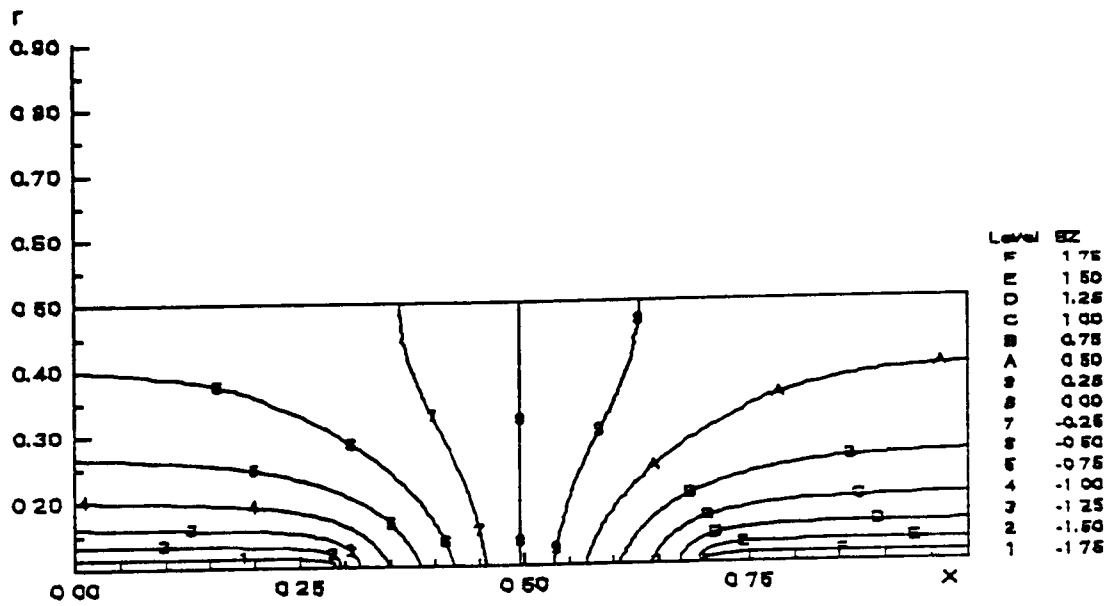


Figure 65.1: Induced magnetic field for case geometry 2

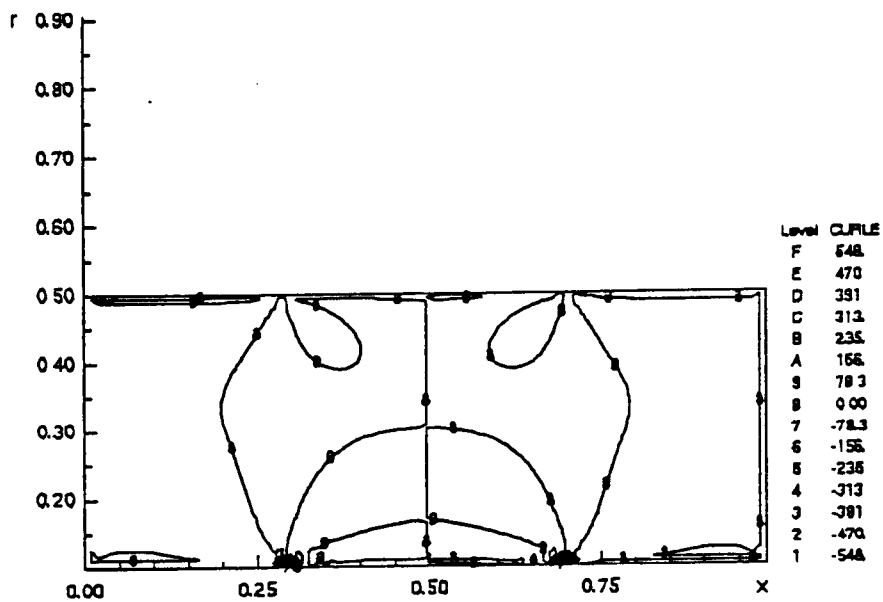


Figure 65.2: Contour plot of the curl of the electric field

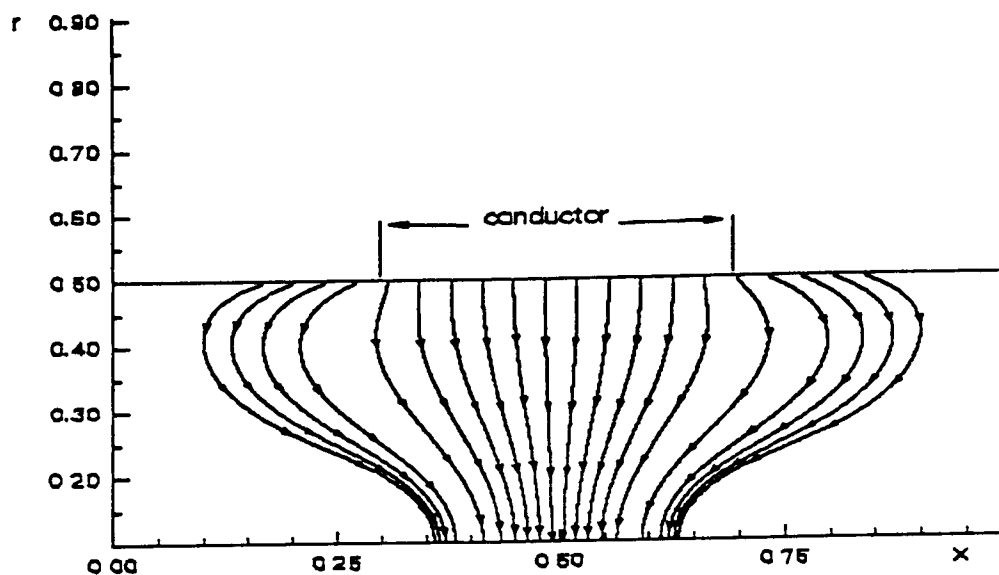


Figure 66.1: Streamlines of the electric field for case geometry 2

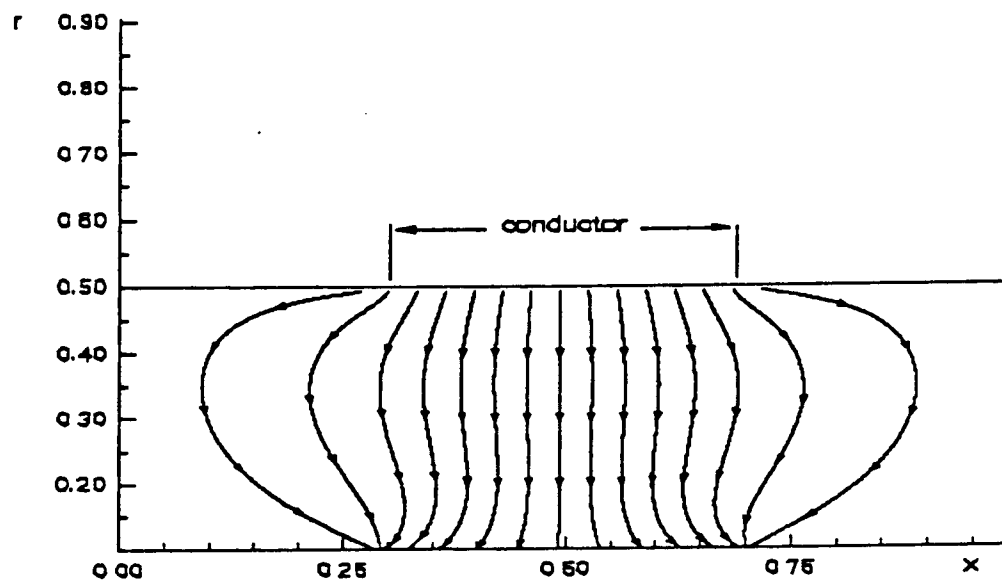


Figure 66.2: Streamlines of the current density for case geometry two

CHAPTER IV

APPLICATION OF THE CODE TO MPD FLOWS

4.1 GEOMETRICAL CONFIGURATIONS

Two different configurations were used to demonstrate the capability of the code. The first configuration was a simple, coaxial duct geometry, with a center cathode and outside anode (top of page 72) and was selected with the intent of demonstrating the basic physical processes. Calculations with and without an applied magnetic field were performed on this geometry. The other configuration was chosen similar to the geometry used in reference [53], since experimental and numerical data were available for this thruster. Several calculations were performed for this geometry with different input currents and applied magnetic field strengths. Argon was used as the propellant for all calculations. The applied magnetic field is constant and in the axial direction only.

The results of the calculations, presented graphically as non-dimensionalized variables, are discussed in this chapter.

4.2 STRAIGHT DUCT, NO APPLIED MAGNETIC FIELD

The geometry used for the first case is shown on page 72 and consists of two coaxial cylinders with an electrode mid-section, separated by insulators at the entrance and exit (left and right side) of the device. The total pressure and temperature were selected such that the initial entrance Mach number was 0.5, the heavy particle temperature 3500°K , and the density was 0.01 kg/m^3 . The

downstream boundary conditions then determine the converged value of the entrance conditions (entrance temperature, Mach number, pressure etc.), since the flow is subsonic. The total current input was 3000 Amps. The electron temperature at the entrance was calculated from the electron energy equation in the form

$$\vec{J} \cdot \vec{E} + 3n_e m_e k \sum_s \frac{\delta_s}{m_s} v_{e,s}^* (T_s - T_e) = 0 \quad (68.1)$$

where δ_s is the energy loss factor which was taken as unity for argon. The equation is a balance of the energy input due to ohmic heating with energy loss of the electrons to the heavy particles due to collisions. The far-field exit pressure was set to 1 Newton per square meter (10^{-5} bar) to allow full expansion of the gas.

A contour plot of the current densities in the axial and radial direction is shown at pages 72 and 73, respectively, and the current streamlines are depicted at the bottom of page 73. It can be seen, that the axial current density concentrates at the intersection of the electrodes and insulators, with larger values at the cathode. The radial current density increases towards the cathode (current conservation), and a small increase of the radial current from the upstream to the downstream side of the cathode is visible. The latter effect is due to the non-zero Hall parameter, which tends to increase the current concentration at the downstream side of the cathode. In general, the current streamlines run from the anode (top) to the cathode (bottom) in fairly straight lines and bulge outward at the edge of the conductors. This straight pattern of the current density streamlines between the electrodes and the small current density concentration at the downstream side of the cathode indicate a small Hall parameter.

A contour plot of the axial and radial electric field is shown at the top and bottom of page 74, respectively. The electric field is determined by the geometrical arrangement of the device and the distribution of electrical conductivity throughout the plasma. The simple geometrical arrangement leads to a fairly homogeneous electric field between the two electrodes. The axially directed electric field is almost zero throughout most of the device and exhibits significant values only close to the intersections of electrodes and insulators. These points represent discontinuities in the gradient of the induced magnetic field, which leads to non-zero values for the axially directed electric field along some portion of the electrodes. The radial component of the electric field exhibits essentially the same behavior as the radial component of the current density.

The induced magnetic field, depicted on page 75, exhibits largely the same behavior as in figure (65.1) for a test case with non-zero Hall parameter. At the entrance of the device, the magnetic field decreases inversely with increasing radius and is constant in the axial direction. Close to the electrodes, the magnetic field starts to increase due to the current flowing between the two electrodes, and eventually goes to zero at the cathode and anode "tip". The change in the magnetic field is slightly more rapid at the downstream portion of the electrodes, indicating a stronger current flow through the downstream portion of the electrodes as compared to the up-stream portion of the electrodes. As mentioned before, this is due to the Hall effect.

Page 75 shows the value of the curl of the electric field. As mentioned before (see chapter 3.5), the curl of the electric field should be zero for steady magnetic fields, and the deviation from zero is therefore indicative of the error associated with the computation. The only significant deviations of this variable from zero occurs at the intersections of the electrodes with the insulators. The discontinuities associated with these intersections are responsible for this behavior. It should be noted, however, that the numerical value of the curl of the electric field contains a magnification of the error associated with the induced magnetic field, since the curl of the electric field is derived by double differentiation of the magnetic field. In doing so, the numerical error of the induced magnetic

field is multiplied twice by the value of the metric terms (see equations 64.1). Because the metric terms are fairly large (>100), even large numbers of the curl of the electric field indicate fairly accurate calculation of the induced magnetic field. The numerical error for the induced magnetic field turns out to be less than 10^{-2} % at the electrode-insulator intersections.

The Mach number development throughout the device is depicted on page 76. The Mach number at the entrance is almost 0.5 and increases continuously toward a choked condition at the exit. A major increase in Mach number can be seen at the downstream portion of the device, indicating electromagnetic interaction. Also visible is the effect of the boundary layer on the Mach number distribution. About 20% of the channel flow is strongly influenced by the boundary layer, indicating the necessity to include viscous effects in thruster calculations.

The influence of the boundary layer is also clearly visible on page 76, depicting the velocity streamlines. Viscous effects narrow the passage for the plasma through the channel, manifested as a pinching of the velocity streamlines toward the exit. The velocity vectors throughout the device are delineated at the top of page 77. Again, viscous effects are visible, as well as a continuously increasing velocity.

The pressure and density distributions are portrayed on pages 77 and 78, respectively. Both decrease toward the exit, but while the pressure is insensitive to the boundary layer, the density is affected, again signifying the importance of viscous effects in thruster computations.

Pages 78 and 79 depict the heavy particle temperature and the electron temperature, respectively. Significant differences between the two temperatures are visible. While the heavy particles remain at a fairly constant temperature, with only slight increases at the cathode, the temperature of the electrons varies over a wide range. The temperature of the heavy particles (atoms and ions) and the electrons is approximately the same (3500°K) at the entrance. The heavy particle temperature increases about 20%, while the temperature of the electrons more than doubles. Clearly, this difference in heavy particle and electron temperature is significant and warrants the separate

calculation of the electron temperature, especially when the transport properties are calculated and not assumed to be constant.

Assuming constant transport properties or a fully ionized plasma is not justified as can be seen at the bottom of page 79, displaying the degree of ionization. A variation of the degree of ionization of more than one order of magnitude is visible. The strong dependence of the degree of ionization on the electron temperature becomes obvious by a comparison of the electron temperature distribution and the distribution of the degree of ionization, both exhibiting fairly similar patterns. Collision cross sections also depend very strongly on the species temperature and they can vary over several orders of magnitude throughout a thruster. Transport properties, therefore, depend strongly on the species temperature and can not be treated as constant. The significance of an accurate evaluation of the electron temperature is evident.

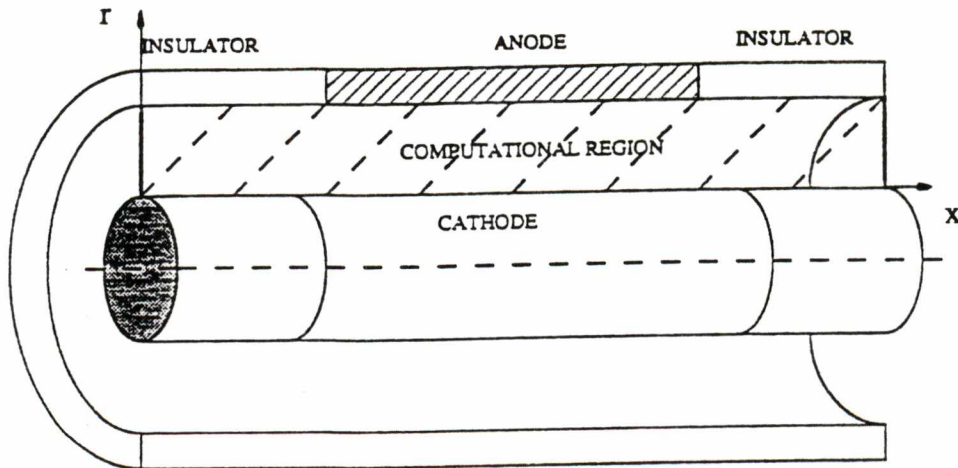


Figure 72.1: Straight duct geometry

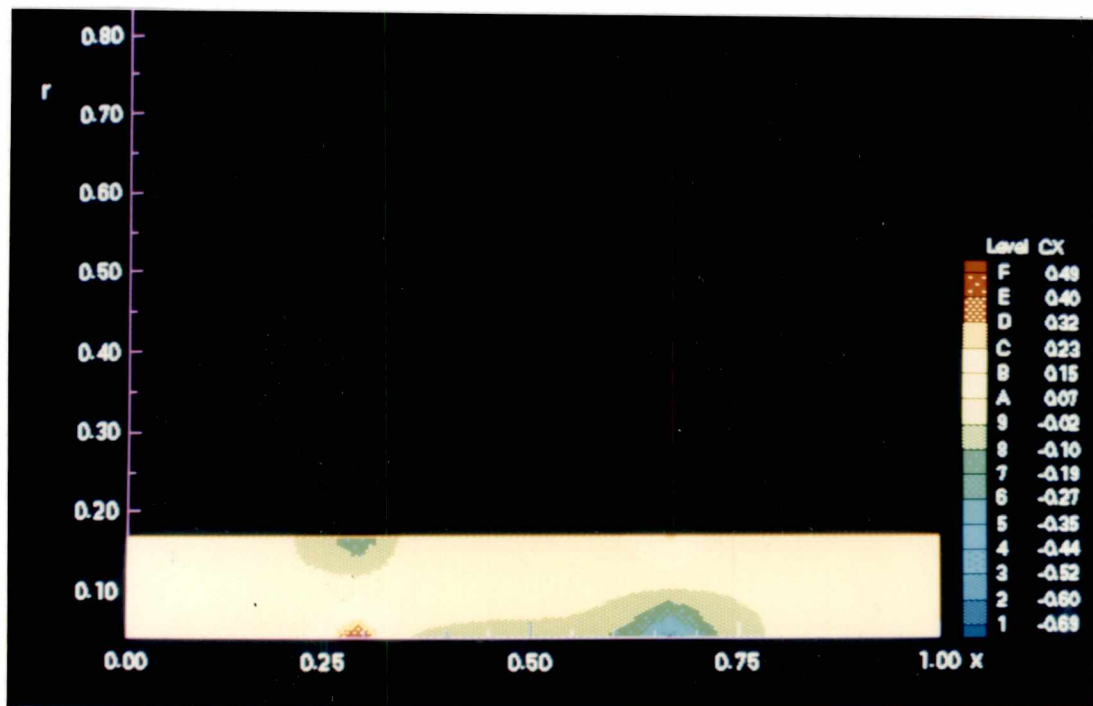


Figure 72.2: Axial component of the current density

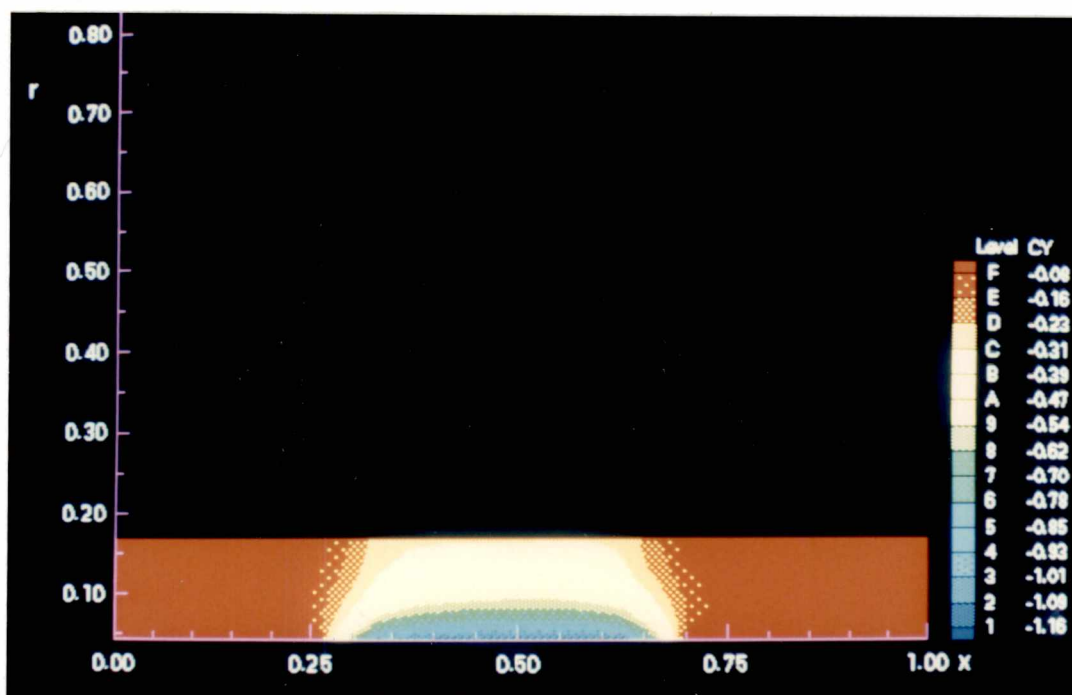


Figure 73.1: Radial component of the current density

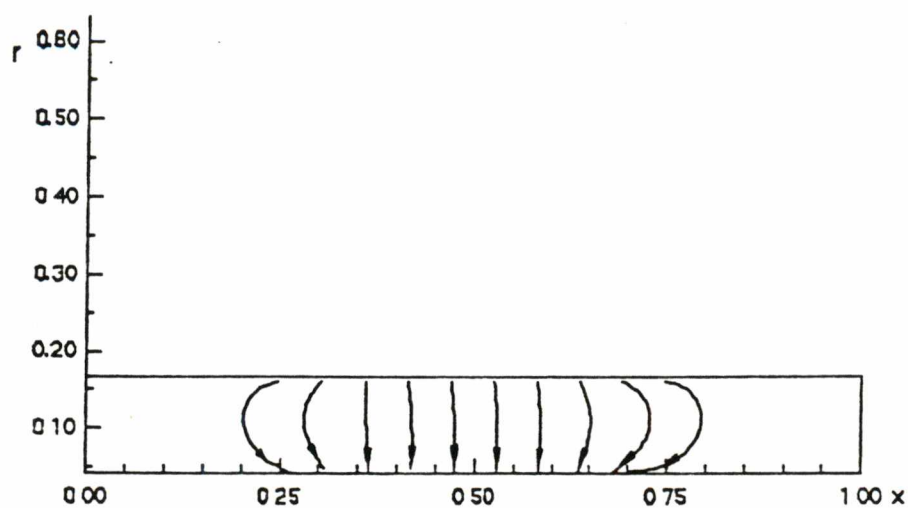


Figure 73.2: Current streamlines

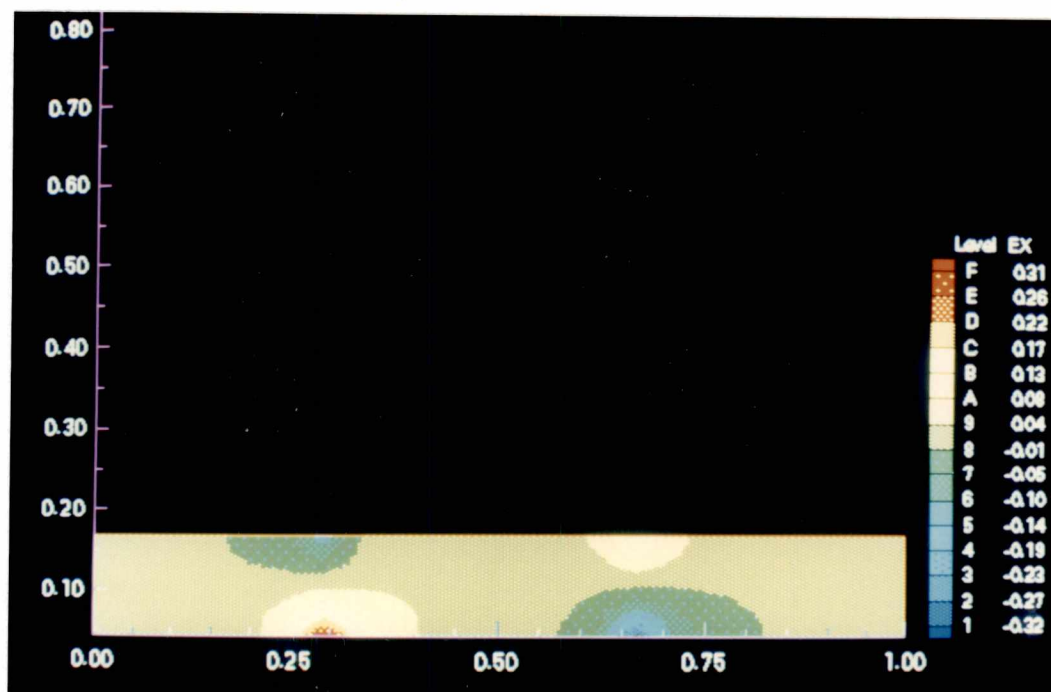


Figure 74.1: Axial component of the electric field

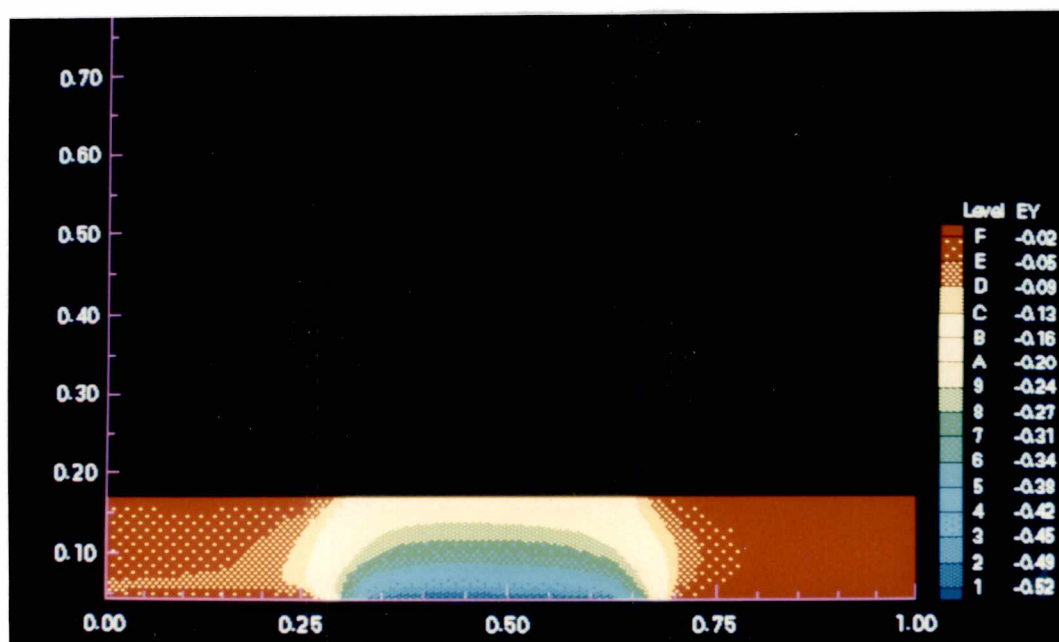


Figure 74.2: Radial component of the electric field

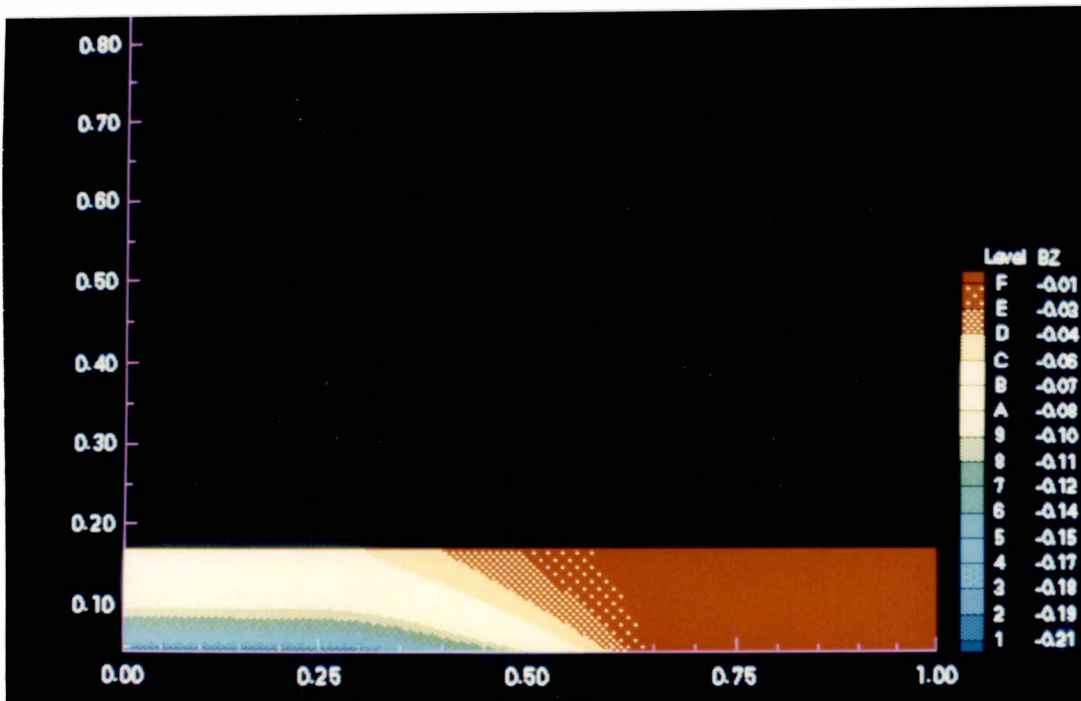


Figure 75.1: Induced magnetic field

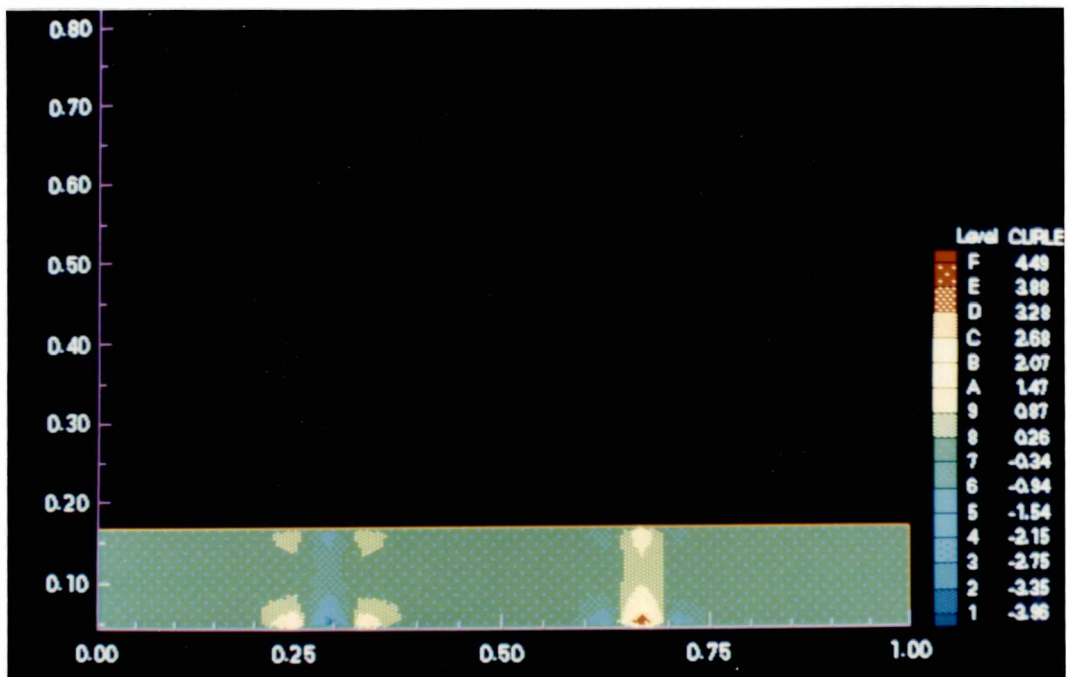


Figure 75.2: Curl of the electric field (test-variable should be zero)

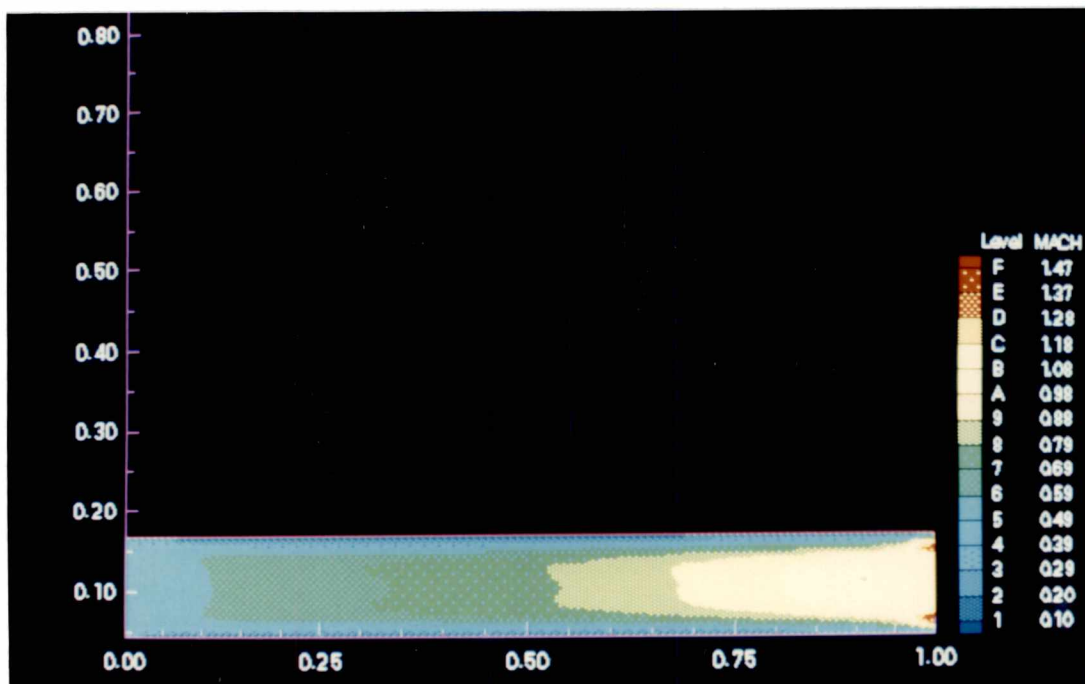


Figure 76.1: Mach number

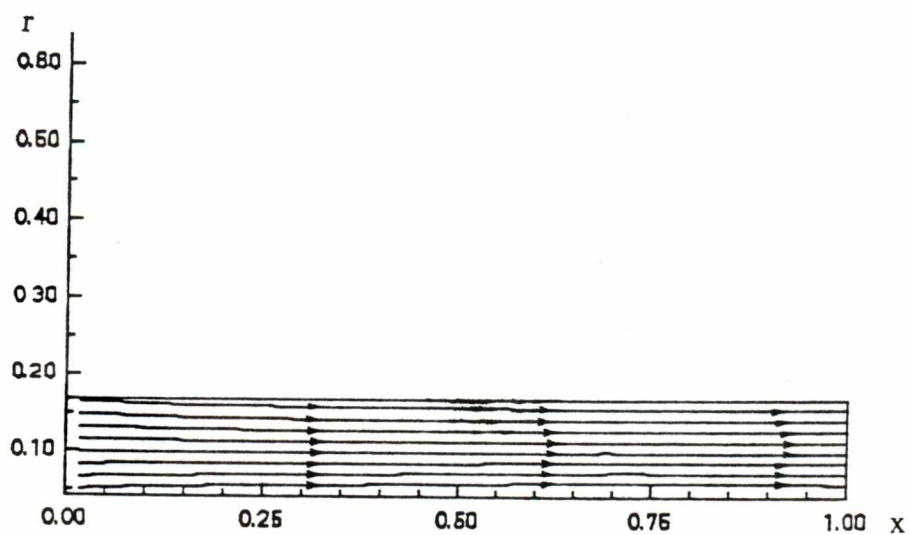


Figure 76.2: Velocity streamlines

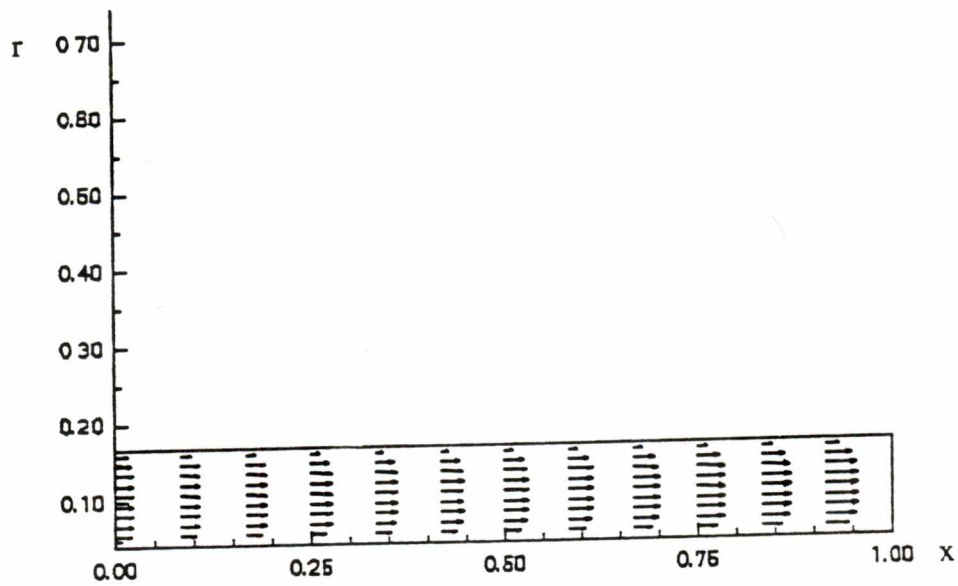


Figure 77.1: Velocity vector plot

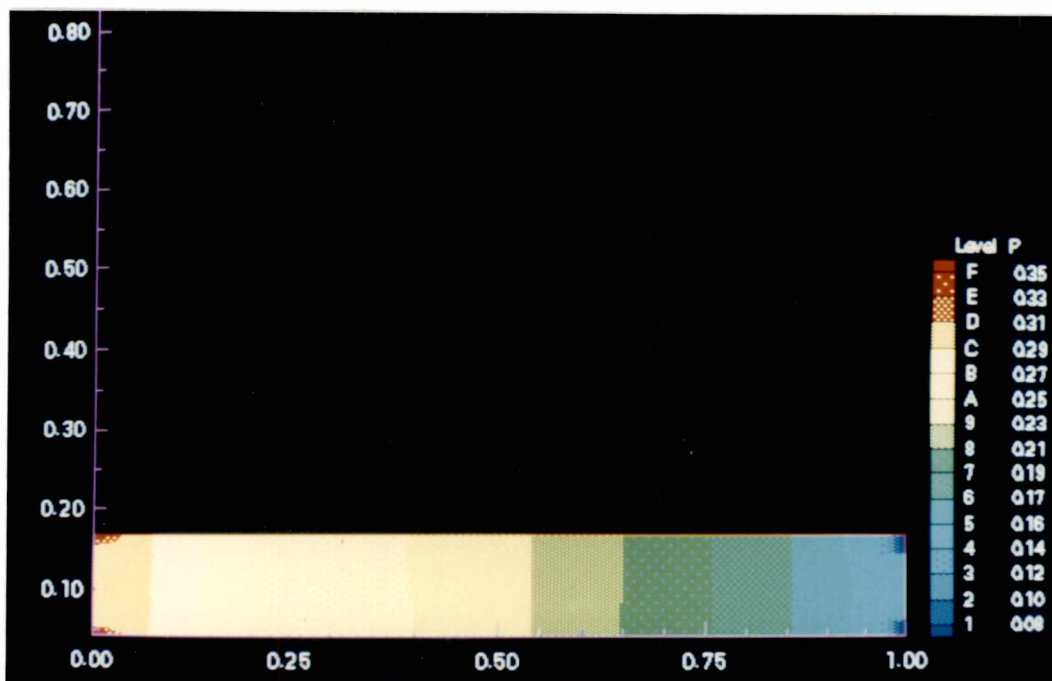


Figure 77.2: Pressure distribution

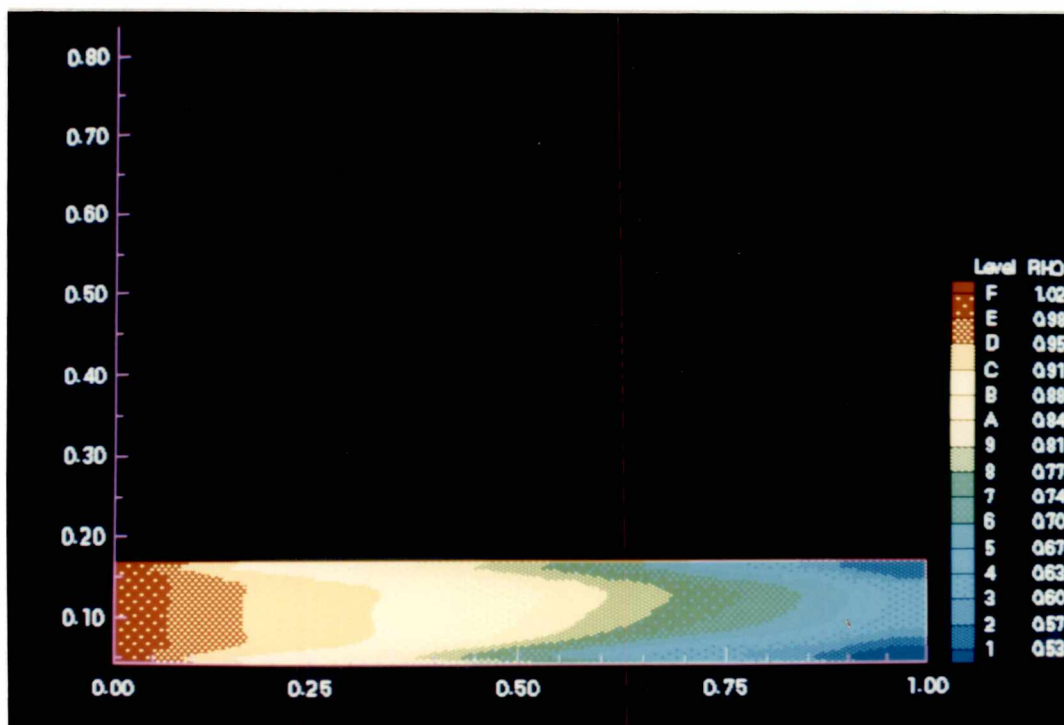


Figure 78.1: Density distribution

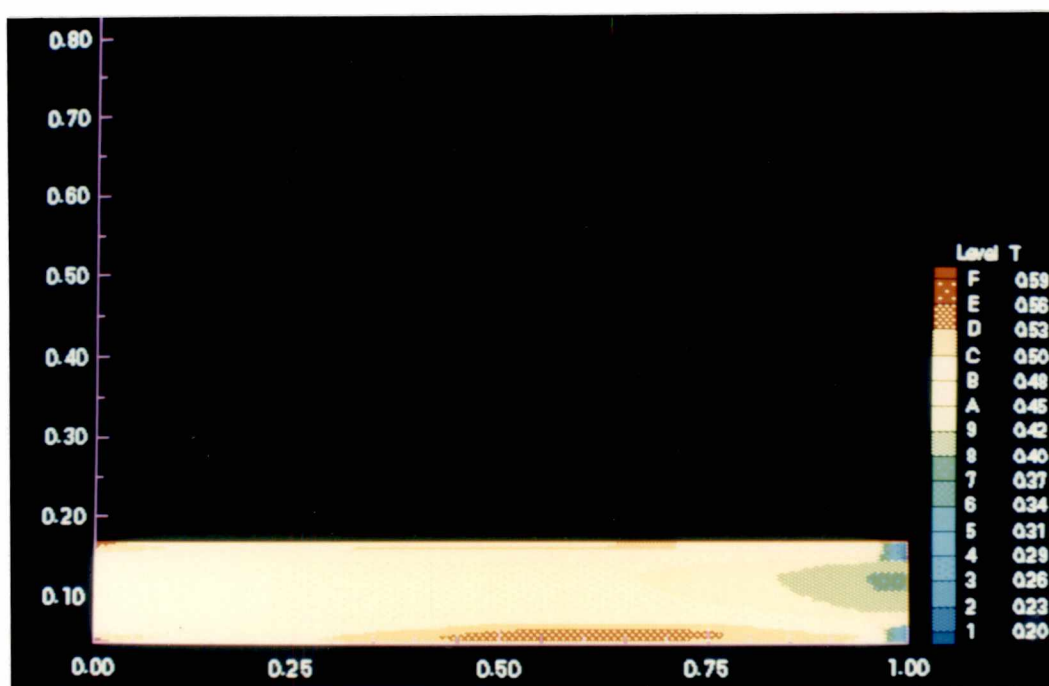


Figure 78.2: Heavy particle temperature

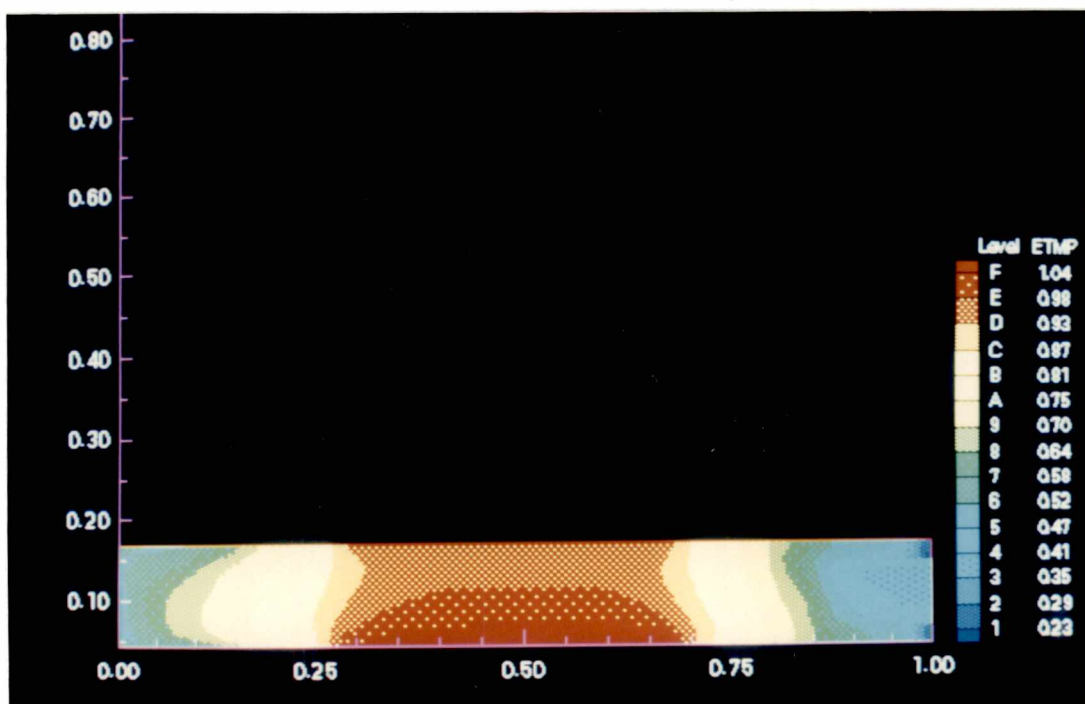


Figure 79.1: Electron temperature

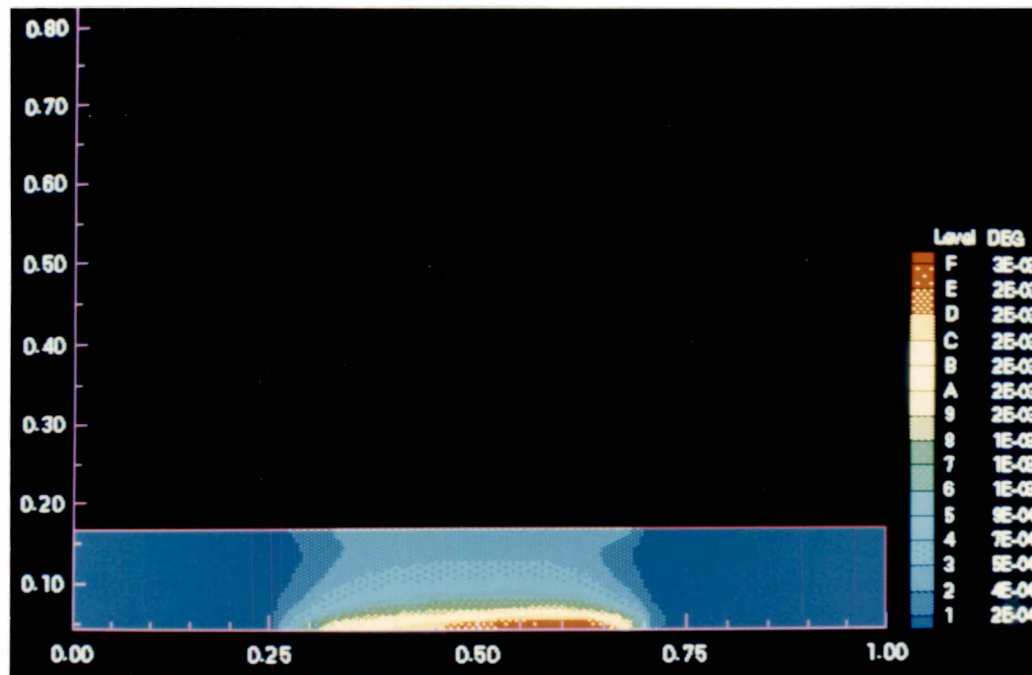


Figure 79.2: Degree of ionization

4.3 STRAIGHT DUCT WITH APPLIED MAGNETIC FIELD

The geometry and the entrance conditions are the same as in the previous case. For this case, however, an applied magnetic field of the same order as the azimuthal induced magnetic field is added. The axially directed applied magnetic field has a value of 0.015 Tesla or 150 Gauss, while the maximum induced magnetic field has a value of 0.0315 Tesla or 315 Gauss. With this value, the applied magnetic field has about half the strength of the induced magnetic field at the upstream portion of the duct at the inner radius, and is about twice as strong as the induced magnetic field at the upstream portion of the duct at the outer radius.

The effect of this applied magnetic field is not visible in any of the thermodynamic or electromagnetic variables. The only noticeable effect of the applied magnetic field is the induction of a swirl flow (see figure 81.1). The swirl flow increases toward the downstream portion of the electrodes, with higher values at the cathode, and then diminishes towards the exit. However, the induced swirl velocity is only about one twentieth of the maximum axial velocity, indicating that the electromagnetic acceleration due to the Lorentz force is small. The observed electromagnetic acceleration is fairly small, since the mass flow rate turned out to be fairly high for the specified entrance conditions (about 21 grams/second) and the input power is moderate.

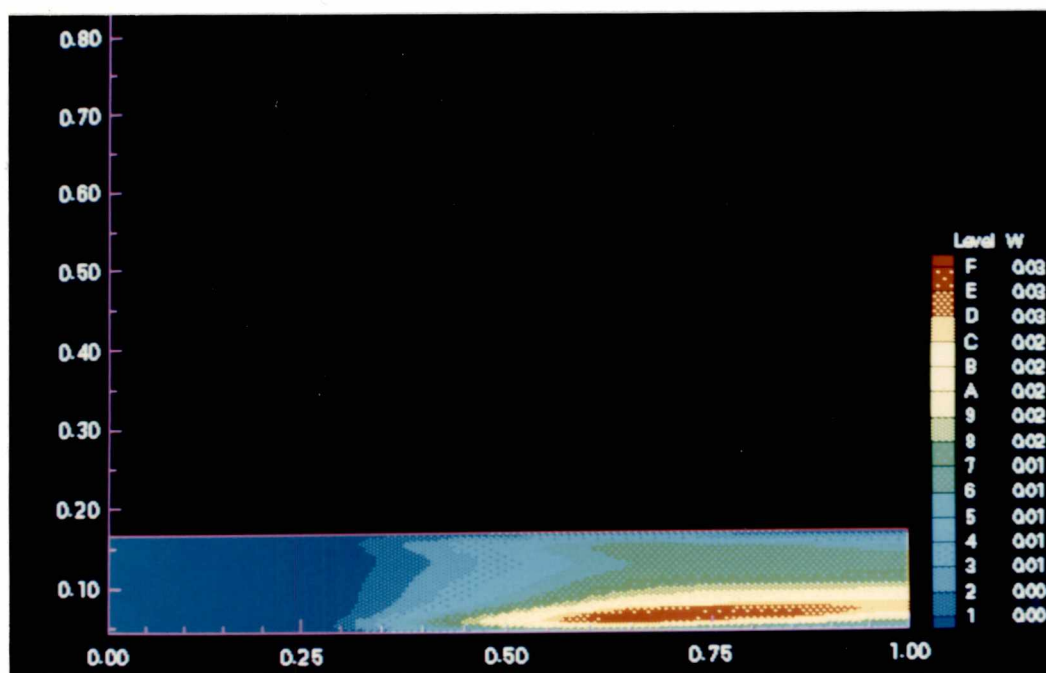


Figure 81.1: Swirl flow component of the velocity (azimuthal velocity)

4.4 MPD-TYPE THRUSTER WITH APPLIED MAGNETIC FIELD

In order to enable comparisons to existing data and other codes, a similar geometry as used by Sleziona et. al. [52] (see figure 26.1) was used to perform MPD-type thruster calculations. The only difference between the two geometries is an added short insulator section upstream of the cathode. As mentioned previously, the discontinuity in boundary conditions between electrodes and insulators causes problems at these interfaces. This effect is more pronounced if the insulator is perpendicular to the electrode. To circumvent possible problems, the short insulator section was added to the basic geometry. The distribution of electromagnetic fields should not be significantly influenced by this difference. The grid used to discretize the geometry is displayed on page 88. Due to the finite number of grid points, not enough grid lines end at the cathode tip to provide an adequate resolution of the rounded tip. The cathode tip, therefore, looks more like a spike.

Using input values for total pressure and temperature that approximated the entrance conditions in [53] (six grams per second mass flow rate) caused problems. When specifying an initial heavy particle temperature of 300°K and an electron temperature of 5000°K as in [53], the electron temperature decreased rapidly to a very small value, causing numerical problems when calculating the degree of ionization. The decline in electron temperature indicates, that the entrance value of 5000°K is not appropriate for the slightly modified configuration, used for the computations in this work. In [53], the electron temperature increases from the initial value of 5000°K to about 10000°K , indicating a strong current at the entrance of the thruster. This difference between the present calculation and the work reported in [53] is most likely caused by the insulator section added upstream of the cathode, preventing significant current densities at the entrance which would allow the electron temperature to stay above 5000°K .

In order to circumvent the decline of the electron temperature, a heavy particle entrance temperature of 3500°K was used and the electron temperature was determined according to the

electron energy balance, equation 68.1. The total pressure and temperature were specified to deliver a mass flow rate of six grams per second.

For these entrance conditions, however, the high thermal input due to Joule heating caused choking of the flow during the computations, with the choked condition occurring shortly after the entrance and back-flow occurring in parts of the entrance region. The computations were then aborted.

The code was then applied to the above geometry, but with an increased total pressure and temperature to increase the mass flow and thereby reduce the ratio of heat input to mass flow to prevent early choking of the flow. The entrance temperature for the heavy particles and the electron temperature were set and calculated as above. Compared to [53], these entrance conditions correspond to a higher energy flow entering the device. The "cold" gas thrust for these entrance conditions turns out to be approximately 16 Newton, while the cold gas thrust calculated in [53] is one Newton. The gas temperature is set to 3500°K at the wall, providing some heat loss, in addition to friction losses due to a finite viscosity. At the computational exit, a pressure of one milli bar was specified (also to prevent starting problems), and the electron temperature was again calculated by equation 68.1.

As before, the figures show non-dimensionalized variables and the flow enters the thruster on the left side. The grid size was 35 by 25 (in the axial and radial direction, respectively) and the propellant is argon.

Pages 88 and 89 show the axial and radial current density components. The largest axial current densities are visible at the cathode tip and the downstream corner of the anode. Since the current density is proportional to the electric field (neglecting the Hall effect) and the electric field should be perpendicular to the electrode surface, this is an expected result. But a significant current density in the axial direction is also visible throughout the area between the anode and the cathode, especially toward the upstream portion of the cathode. This is directly related to the Hall effect. Considering Ohm's Law (equation 41.2) for the axial direction and neglecting the electric field and the velocity

components (both should be zero at the surface), the axially directed current density is proportional to the radially directed current density times the induced magnetic field, where the proportionality factor is the Hall parameter. A significant axial Hall current toward the inlet is therefore to be expected, noting that:

$$j_x = -\frac{\omega\tau}{|\vec{B}|} j_r B_\theta \quad (84.1)$$

The effect of the Hall parameter is also observable on page 89, depicting the radial current density component. The radial component of the current density increases towards the centerline (cathode side), but also increases along the cathode toward the cathode tip, which is a typical behavior for flows with finite Hall parameters.

The effects can also be exhibited by a plot of the current streamlines, delineated at the bottom of page 89. The strong axial current directed toward the upstream portion of the thruster is visible, as well as the fairly perpendicular inclination of the current density at the cathode tip and the anode corner. The bulging of the current outward to the side of the electrodes is also noticeable.

Top and bottom of page 90 show the distribution of the electric field in the axial and radial directions, and page 91 shows the streamlines of the electric field. The actual pattern of the contour plots for the electric field in the axial and radial directions is very similar to the corresponding contour plots of the current density. One notable difference is apparent by comparing the streamline plots of the current density and the electric field at the electrode surface. While the current density shows a significant inclination at the electrode surfaces, the inclination of the electric field is much smaller at the conductors. The boundary condition imposed at the electrodes requires a vanishing axial component of the electric field parallel to the surface, while the current along the electrode surface is not restricted. The small inclination of the electric field at the electrode surfaces is due to

the one-sided difference approximation of the derivative of the induced magnetic field at the boundaries.

The current concentration at the cathode tip due to the Hall parameter and the fluid motion is also evident on page 91, showing the induced magnetic field. The induced magnetic field changes more slowly at the upstream portion of the thruster, than towards the cathode tip. Essentially, the induced magnetic field follows the same pattern as can be seen in the cases before, namely an inverse dependency on the radius, causing larger values at the cathode.

The velocity streamlines are depicted at the top of page 92 and the velocity vectors are given at the bottom of page 92. Again, as in the previous cases, the influence of the boundary layer within the duct can be seen by the pinching of the streamlines. Contrary to the results obtained by Sleziona et. al. [54], the streamlines clearly show a separation of the flow at the downstream anode corner, and do not follow the 90° turn of the anode. Also visible is entrainment, the fact that outside mass gets sucked towards the exhaust stream and becomes mixed with it. Entrainment is cited as a major problem with early experimental investigations of electromagnetic thrusters, since the "high" back pressure in the vacuum chamber almost ensured entrainment of gas in the vacuum chamber into the exhaust jet, leading to higher than expected specific impulse values. The acceleration within the duct and the formation of the exhaust jet can also be observed on page 92. The axial velocity achieves a maximum value of about 2000 m/sec, a relatively small value for an MPD thruster. As mentioned before, this is due to the higher mass flow rate used for the computation as compared to actual devices. Additionally, the device is operating primarily as an electrothermal thruster, rather than an electromagnetic thruster. Exit velocities as seen in electromagnetic thrusters can, therefore, not be expected.

A contour plot of the Mach number distribution is presented at the top of page 93. The strong acceleration in the expansion area is even more evident than shown on page 92. While the Mach number increases from 0.5 at the entrance to about one at the exit of the straight duct portion

of the thruster, the acceleration in the small, divergent portion of the cathode tip brings the Mach number to almost two.

An interesting result is given by the contour plot of the azimuthal velocity on page 93. As in the cases before, the azimuthal velocity (swirl flow) remains essentially zero, until the inter-electrode region is reached. From then on downstream, the azimuthal velocity increases, particularly near the cathode. The highest values for the azimuthal velocity are visible in a small "pocket" downstream of the cathode tip. From then on downstream the azimuthal velocity decreases. Since the radial component of the current density diminishes in this region (see page 89), this effect has to be attributed to the strong radial velocity component occurring in the region of the cathode tip. To explain the effects, it is necessary to look at the azimuthal momentum equation (see equation 34.2). Because of the steady, axisymmetric flow, derivatives with respect to time and azimuthal direction are zero. Neglecting viscous effects (the region of highest swirl velocity is at a distance far enough from the cathode tip to allow this simplification), and neglecting the change of the swirl flow with respect to the radial coordinate, only two terms remain [79] in the azimuthal momentum equation:

$$\frac{\partial \ln(w)}{\partial x} = - \frac{v}{r u} \quad (86.1)$$

An increase of the radial velocity, v , toward the center line must therefore lead to a significant increase in azimuthal velocity, w .

The pressure distribution presented on page 94 exhibits a similar behavior as in the previous cases. The pressure continuously diminishes throughout the duct and is fairly constant over the height of the duct. The density also displays the familiar behavior of diminishing values, but affected stronger by viscous effects as shown on page 94.

An interesting comparison can be made between the heavy particle temperature distribution (top of page 95) and the electron temperature distribution (bottom of page 95). The heavy particle

temperature distribution exhibits a similar pattern as in the previous cases, with a continuously increasing temperature towards the cathode tip, with the highest values obtained before the expansion area. The expansion area converts the random thermal energy into kinetic energy, thereby reducing the temperature. The region of the exhaust jet shows, therefore, a significantly reduced temperature as compared to the maximum. Physically inexplicable is the temperature increase above the exhaust jet, close to the exit boundary of the grid. This region, however, is not clearly an inflow or outflow boundary, as can be seen on page 92. The streamlines show a more tangential flow to the boundary in this region, a condition that leads to poor results at the boundary [63].

The electron temperature, however, displays a smoother transition from the entrance value to the hottest temperature at the cathode tip and anode edge with a subsequent cool-down to a fairly homogenous temperature at the exit of the computational region. The electron temperature stays above the heavy particle temperature throughout the expansion region and is not significantly affected by the decrease of the heavy particle temperature in the exhaust jet. These effects are probably due to the changed transport properties as compared to the inlet, reducing the coupling between the electron temperature and the heavy particle temperature. Additionally, the global velocity does not affect the electron temperature as much as the heavy particle temperature. The electron temperature is dominated by the energy input due to Joule heating and the currents (see equation 68.1).

The degree of ionization (shown on page 96) tracks the electron temperature very closely. Obviously, the density does not affect the degree of ionization as much as the electron temperature.

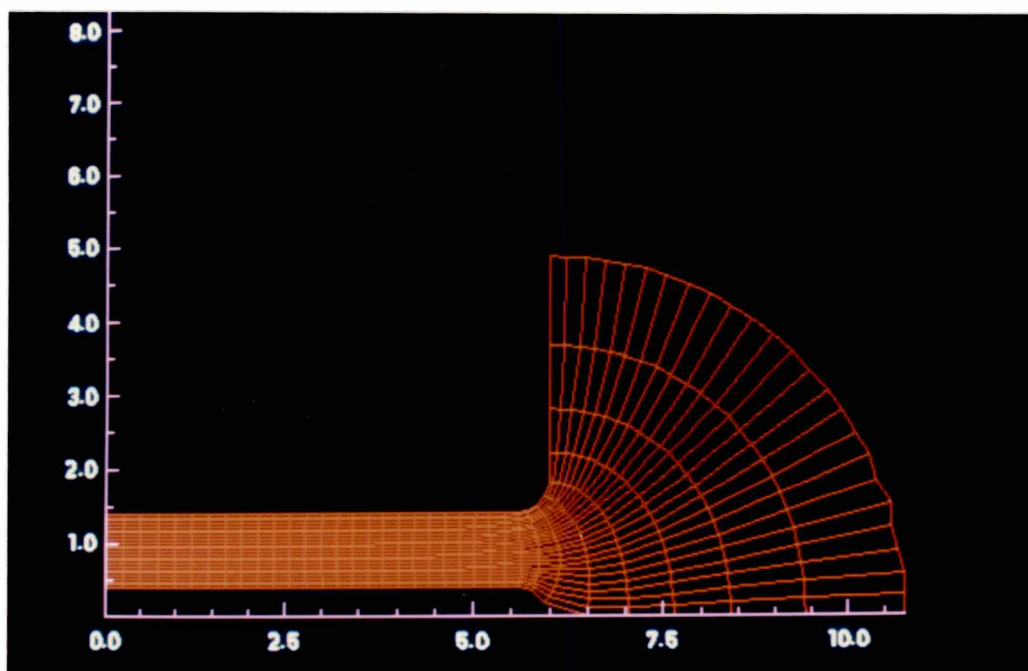


Figure 88.1: Grid used to discretize the MPD-thruster geometry

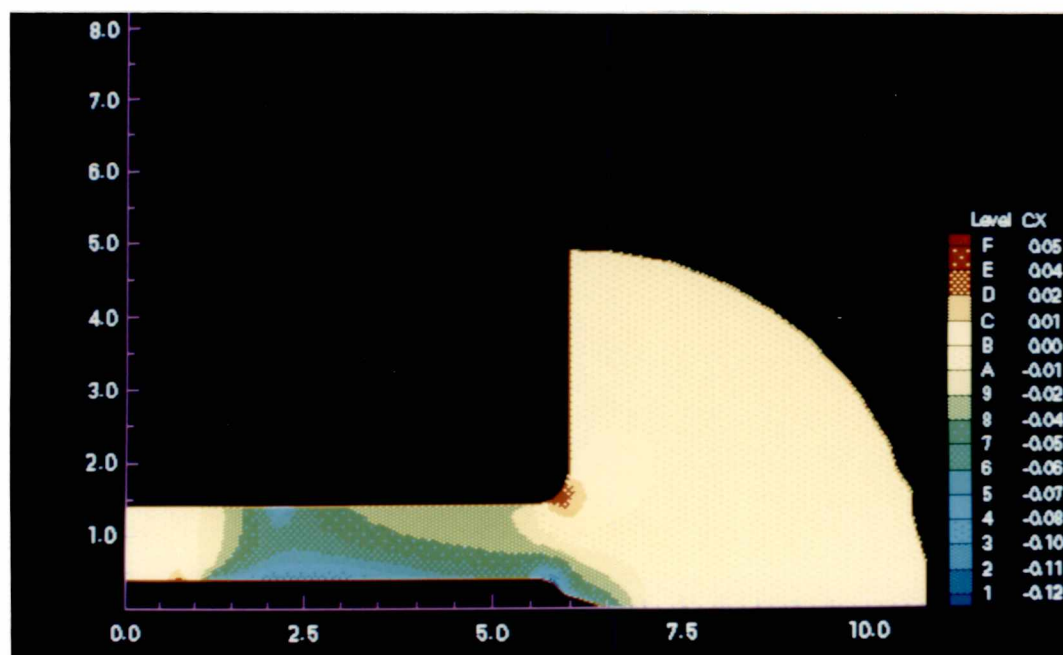


Figure 88.2: Axial component of the current density

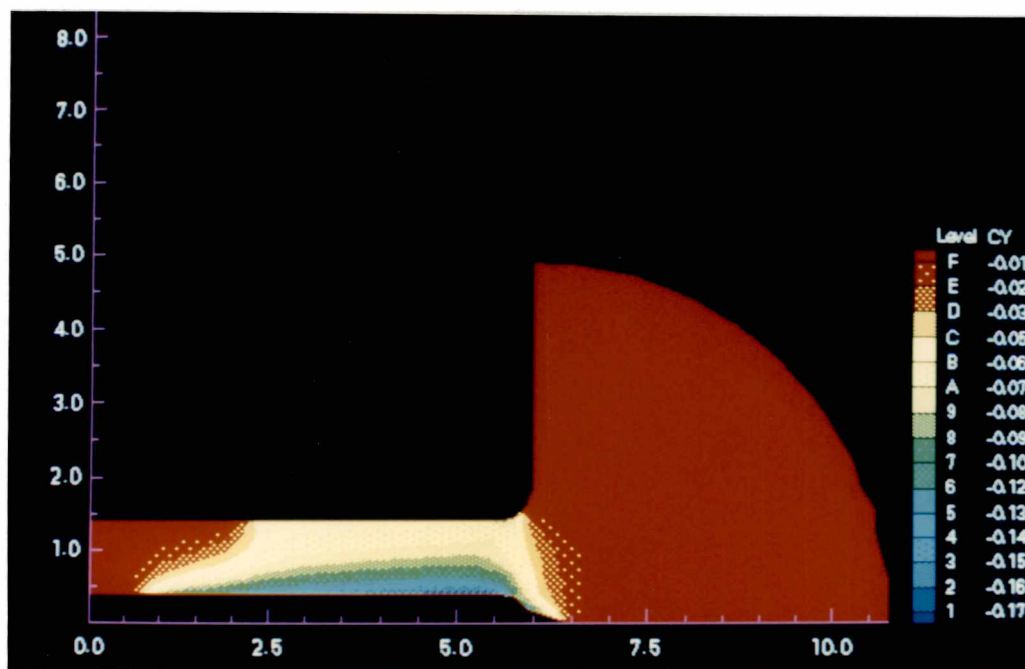


Figure 89.1: Radial component of the current density

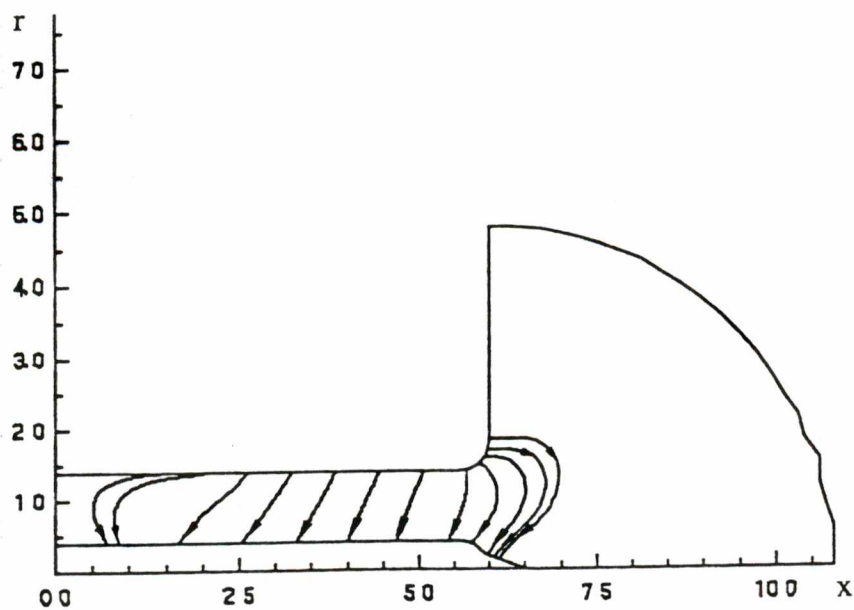


Figure 89.2: Current density streamlines

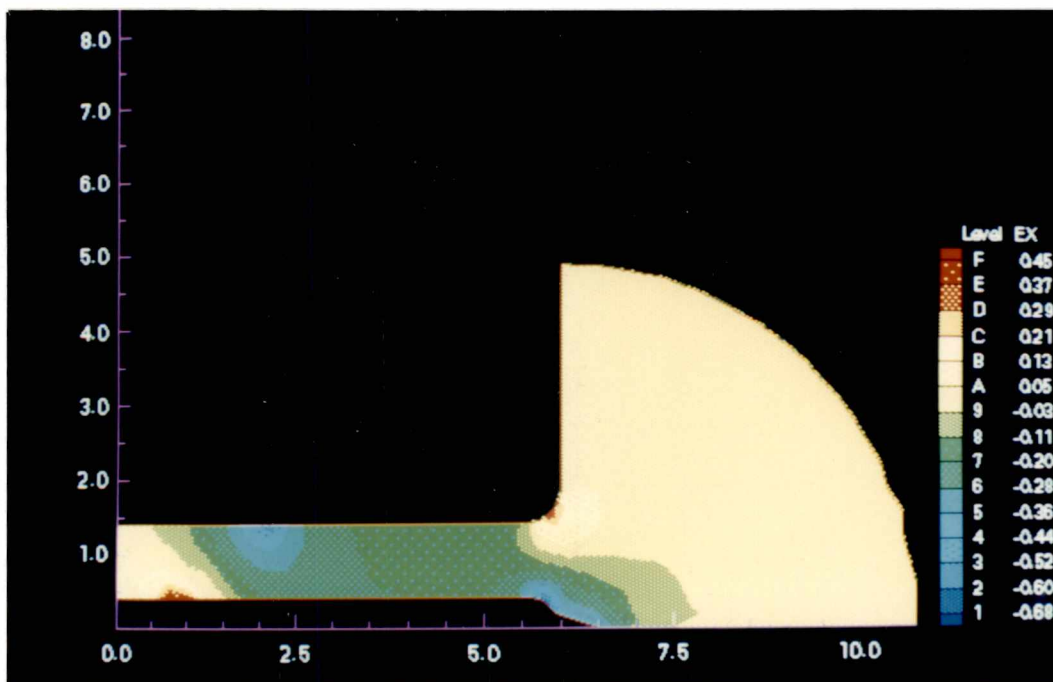


Figure 90.1: Axial component of the electric field

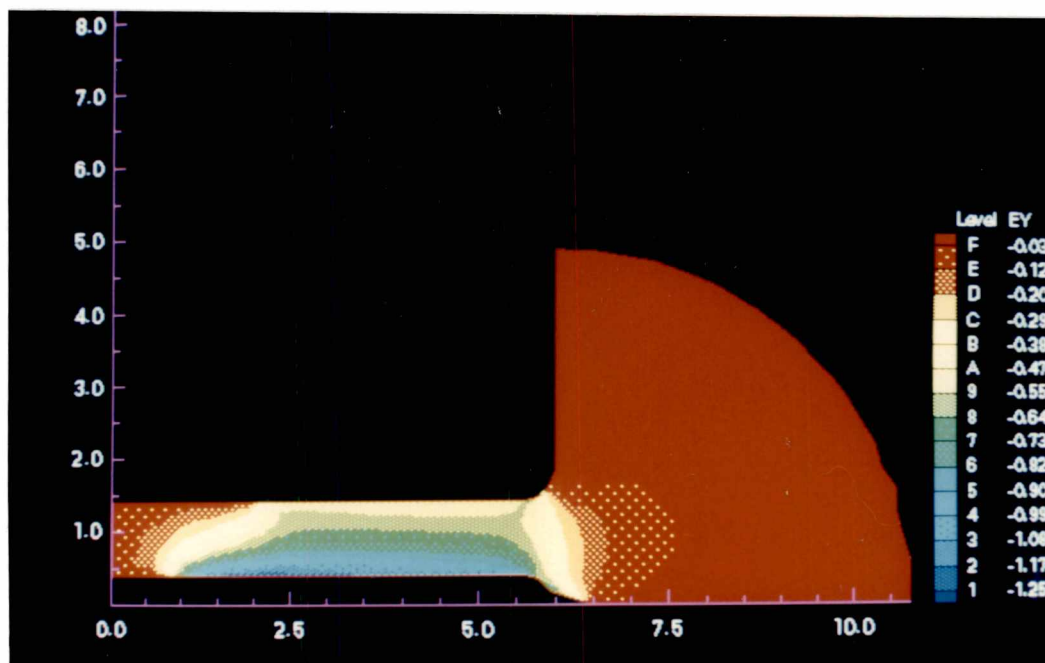


Figure 90.2: Radial component of the electric field

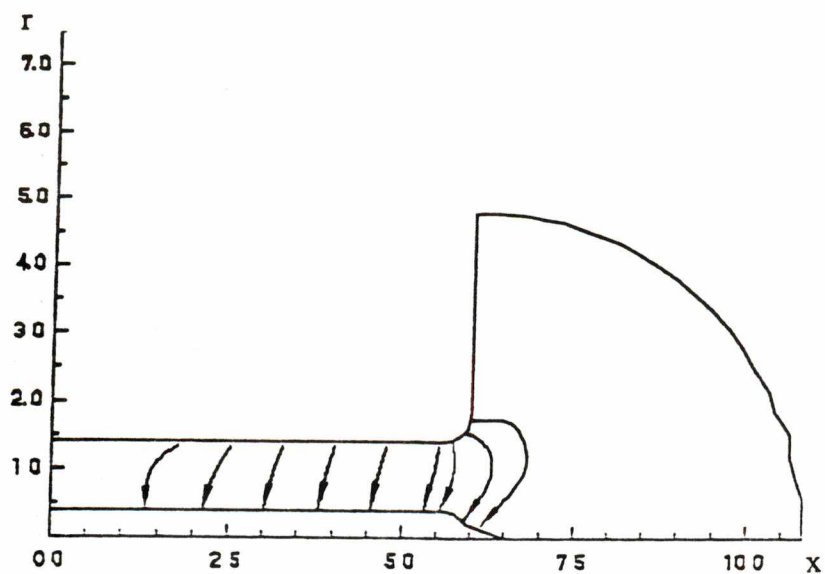


Figure 91.1: Electric field streamlines

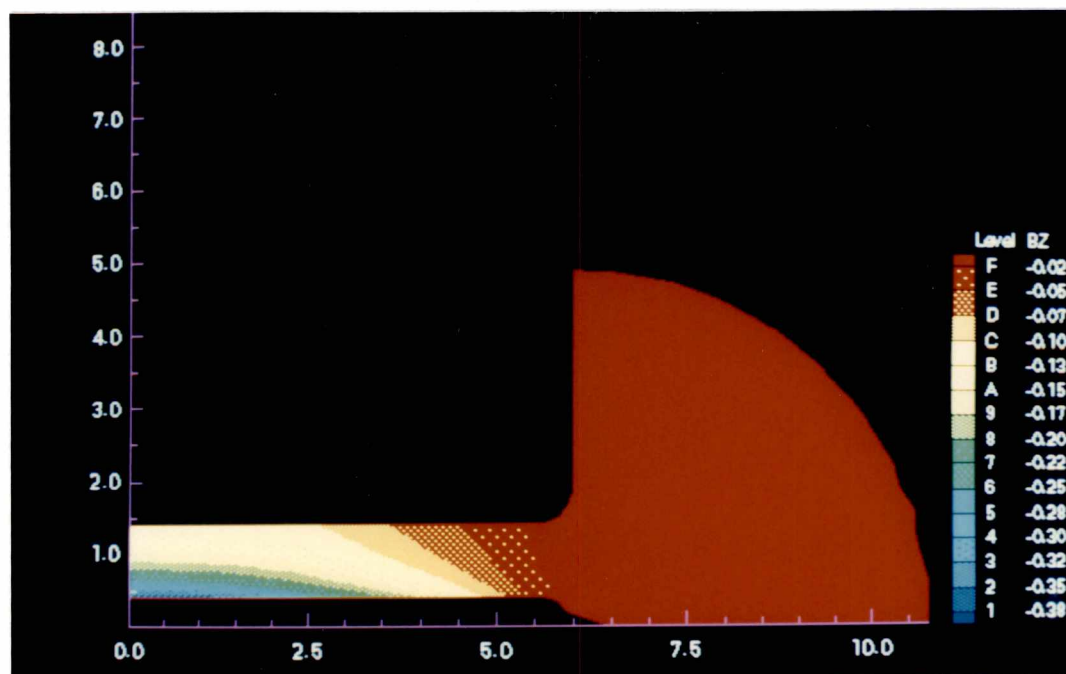


Figure 91.2: Induced magnetic field

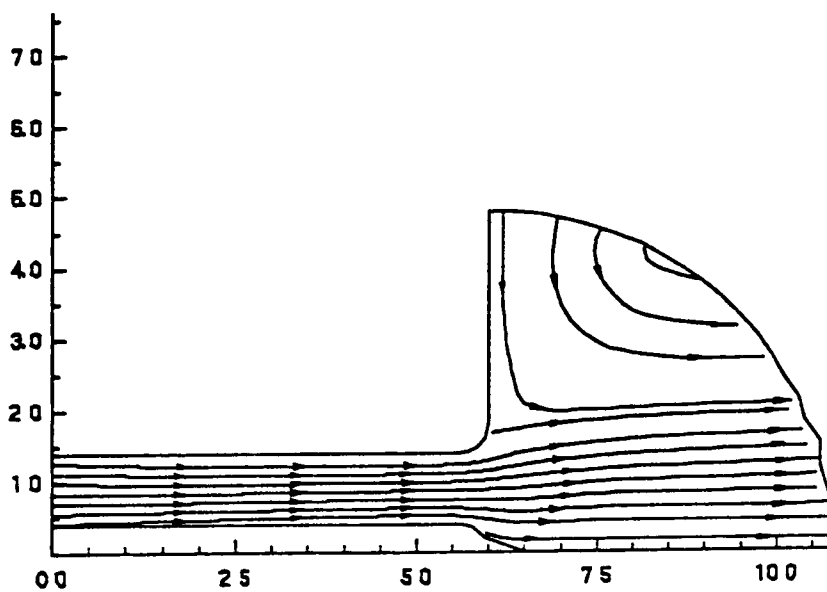


Figure 92.1: Velocity streamlines

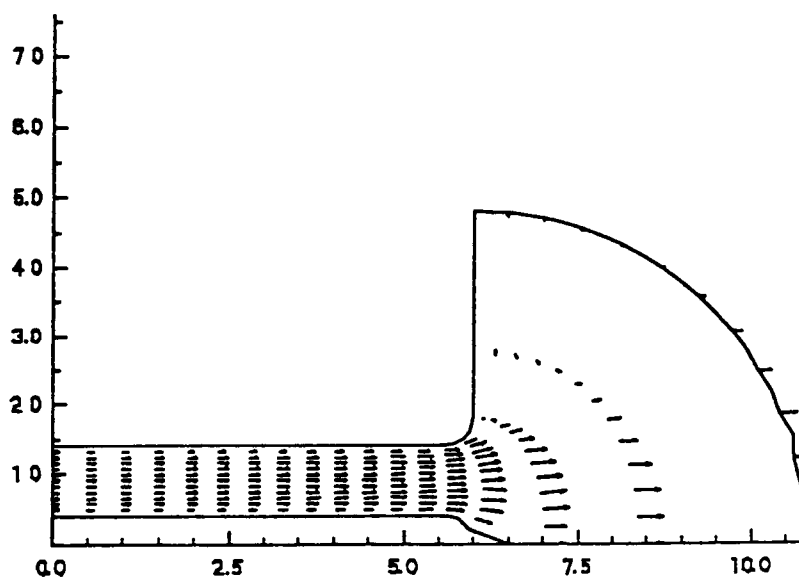


Figure 92.2: Velocity vector plot

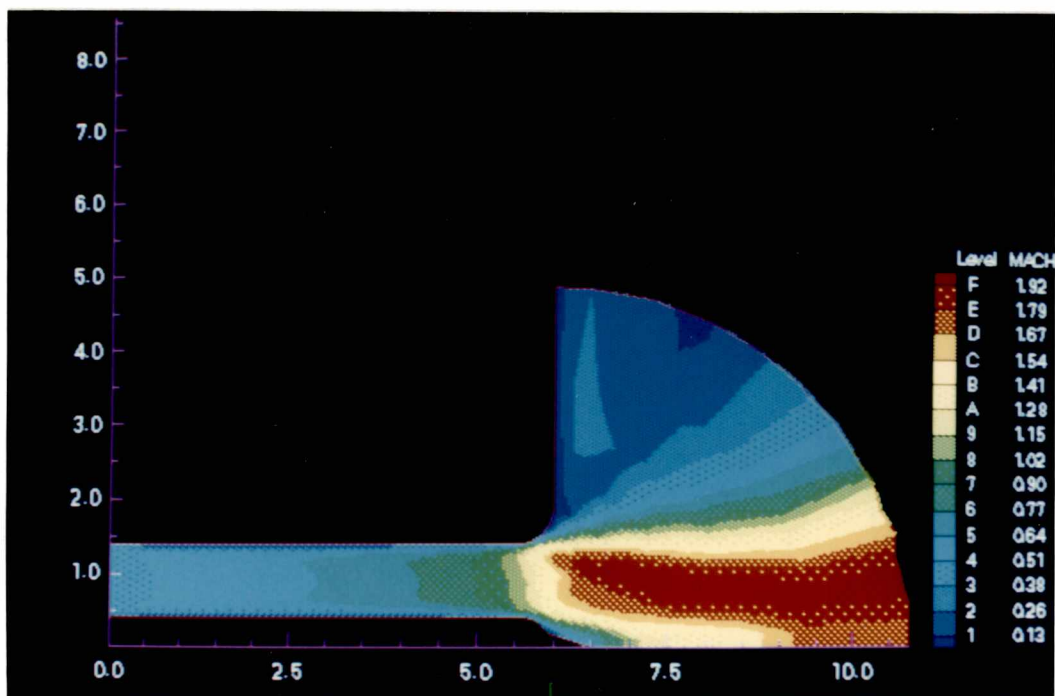


Figure 93.: Mach number distribution

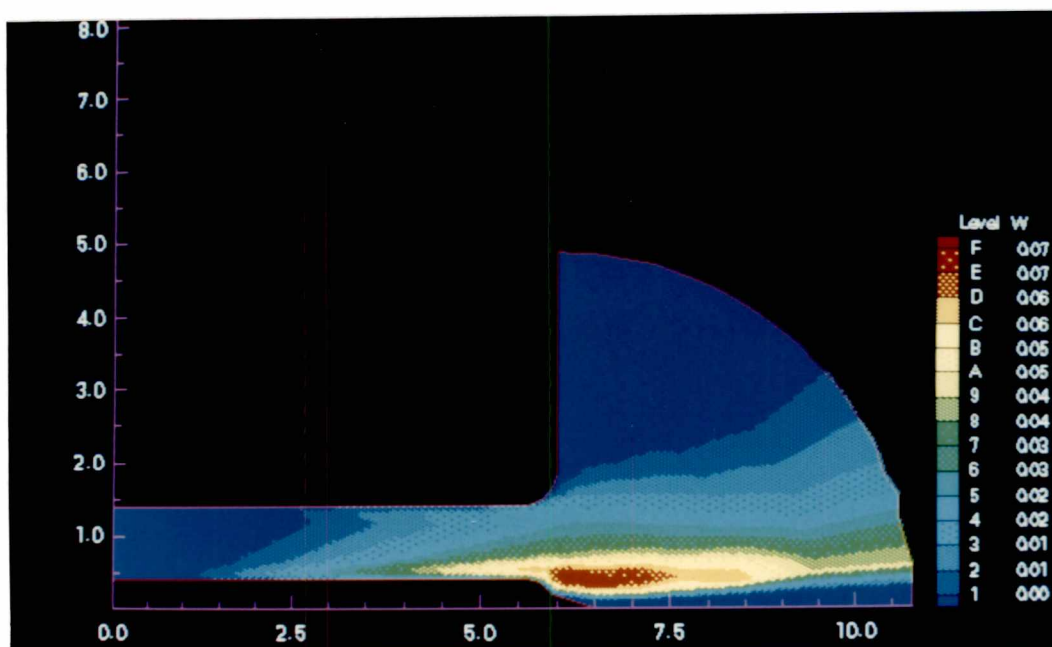


Figure 93.2: Azimuthal velocity component

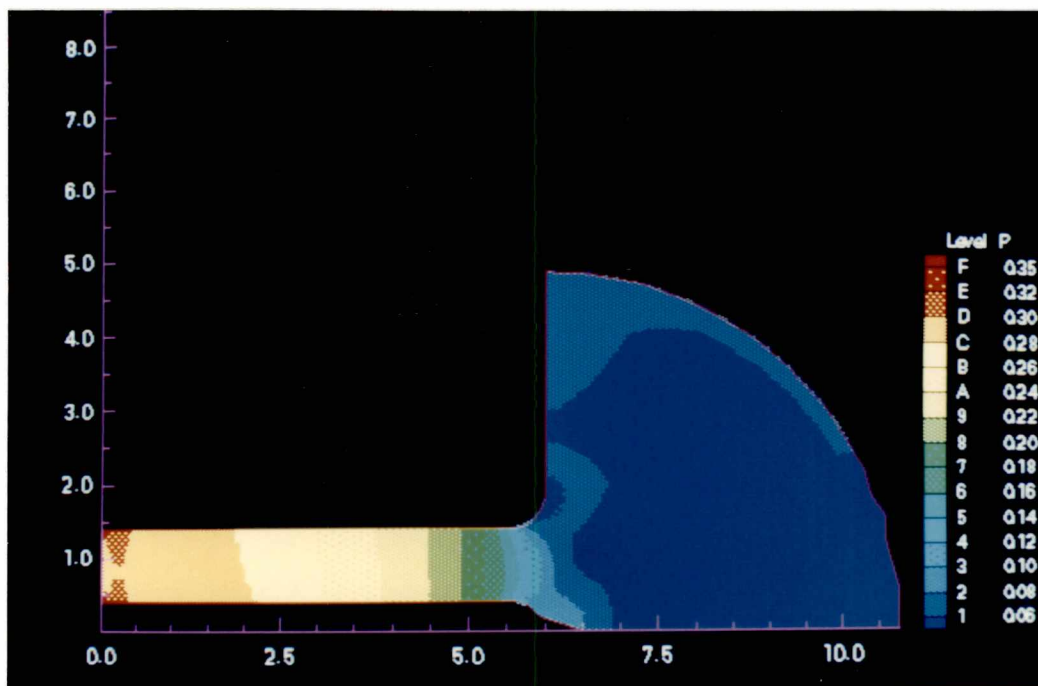


Figure 94.1: Pressure distribution

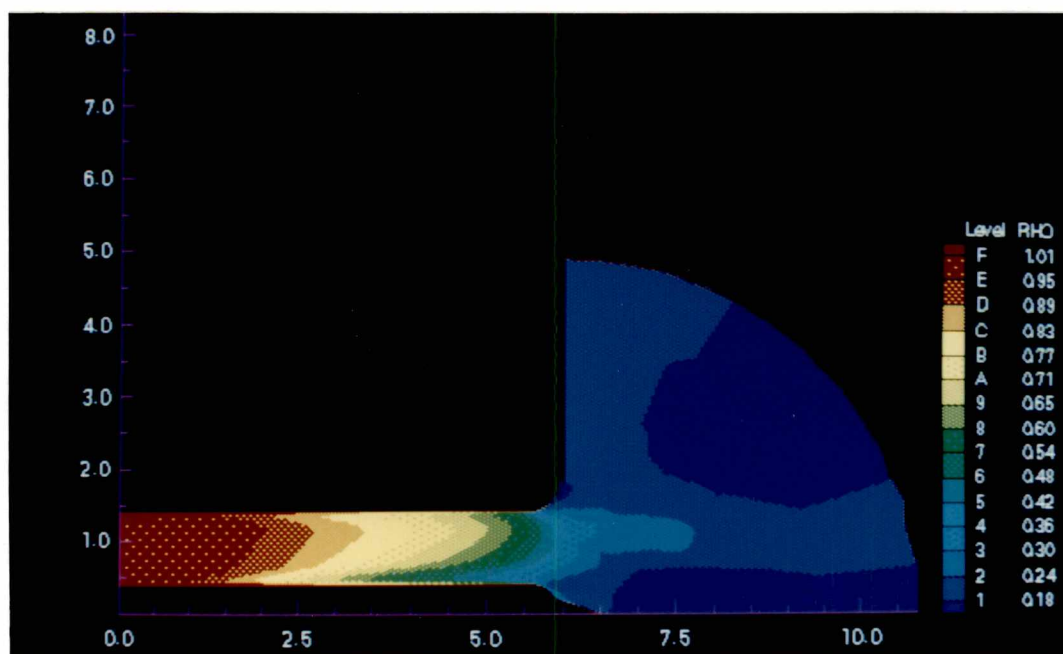


Figure 94.2: Density distribution

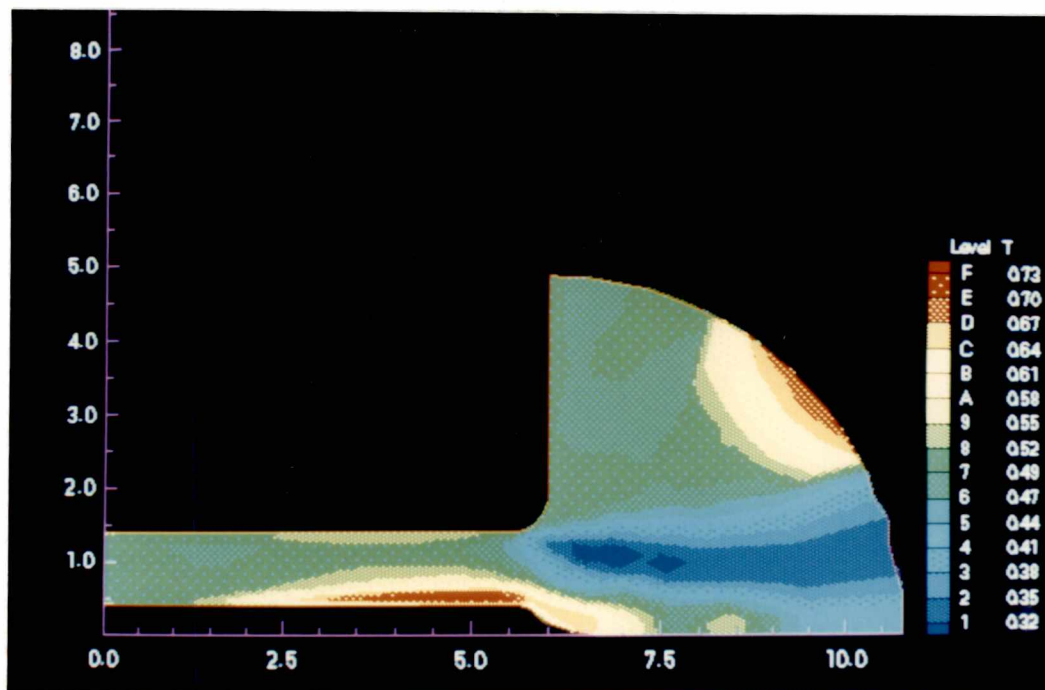


Figure 95.1: Heavy particle temperature distribution

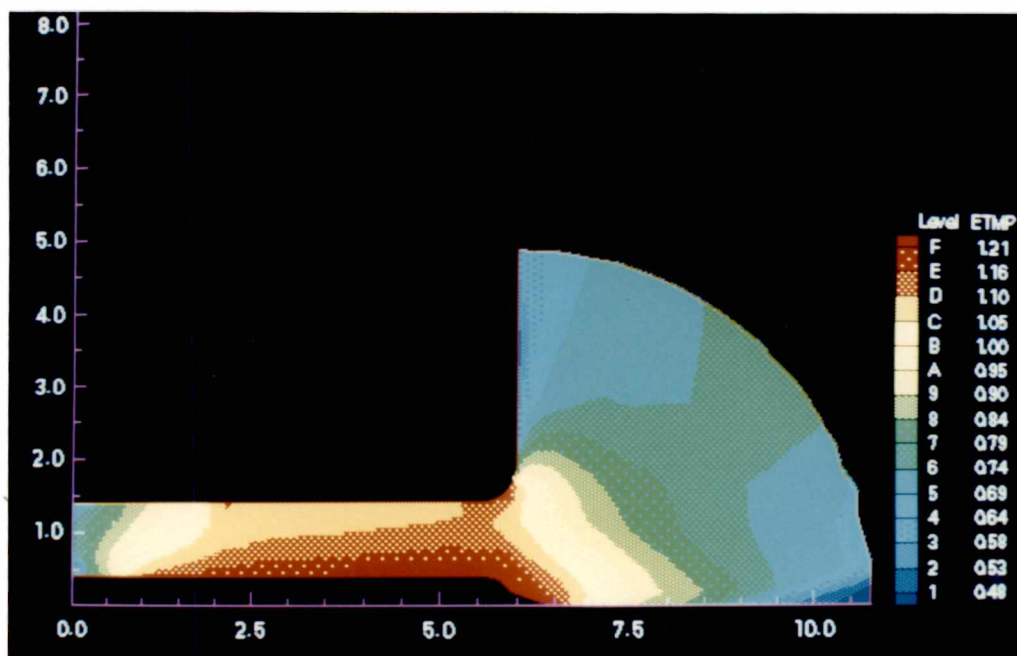


Figure 95.2: Electron temperature distribution

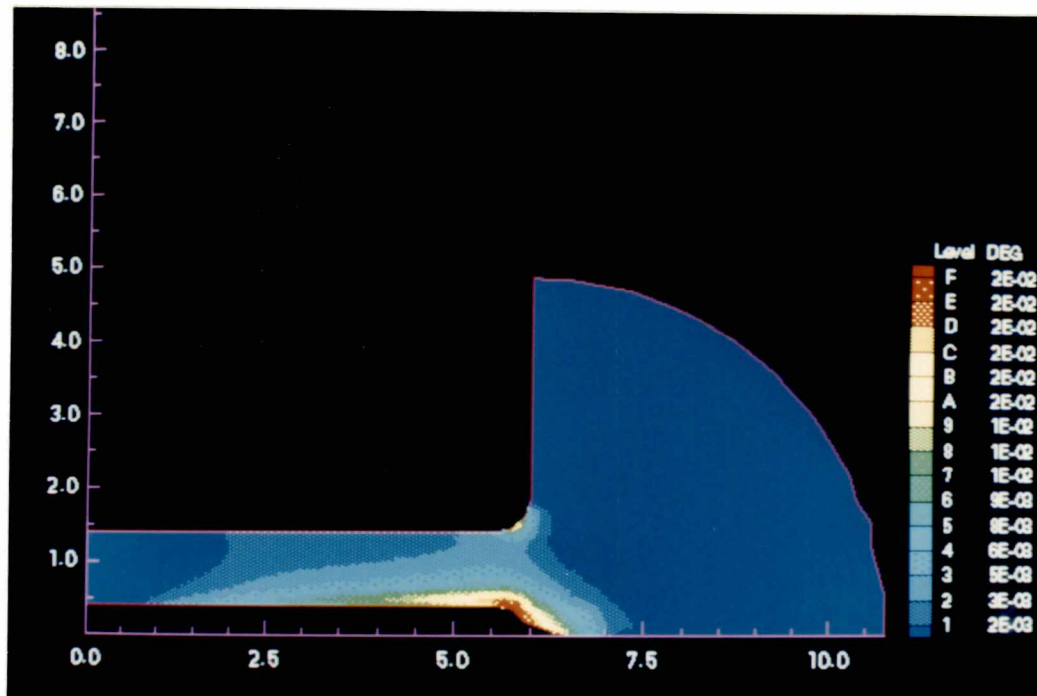


Figure 96.1: Degree of ionization

4.5 PARAMETER STUDY

Several cases were run with the geometry described in chapter 4.3 to perform a parameter study and visualize trends. Performance parameters, as well as thermodynamic and transport properties were investigated as a function of input current and applied magnetic field strength. A total of twelve runs was performed, with input currents of 3 kA, 7.5 kA, 12.5 kA and 17.5 kA and applied magnetic field strength of zero Tesla, 0.1 Tesla and 0.2 Tesla. The highest input power case (17.5 kA input current and 0.2 Tesla applied magnetic field strength) did not converge and is therefore missing on the plots presented in this chapter.

The voltage distribution is depicted on page 102 and compared to the experimentally obtained voltage distributions [82]. Five experimentally obtained voltage values are represented by the dashed line, where the measured voltage corresponds to different mass flow rates, ranging from 0.8 grams per second to 6 grams per second. The calculated voltage is about four Volts lower than measured, but the trend is clearly predicted by the calculations. Since the voltage drop at the electrodes is not taken into account in these calculations, a lower calculated voltage is reasonable. The calculated voltage compares favorably with the voltage found in [53]. The functional relationship between voltage and current does not significantly change with an applied magnetic field, except for the absolute value of the voltage, which increases with the applied magnetic field. The voltage increase becomes larger for increasing applied magnetic fields. Neglecting the Hall effect and assuming that the axial velocity and the induced magnetic field are not affected by the applied magnetic field, it can be deduced from Ohm's law (equation 41.2) in the radial direction that the electric field balances with the product of the applied magnetic field and the azimuthal velocity.

$$E_r + wB_\theta = \text{constant} \quad (97.1)$$

Since the azimuthal velocity itself increases with the applied magnetic field (due to the Lorentz force), a non-linear increase in the electric field, and thereby in the voltage, with an applied magnetic field appears reasonable.

The power verses input current for different magnetic fields is depicted on page 102. Since the power is proportional to the square of the input current, with the proportionality given by the electrical conductivity for a simplified Ohm's law, a non-linear increase in power with input current is expected. The relationship between power and input current is, however, not strictly quadratic, because higher input power leads to higher temperatures, and higher temperatures cause an increase in electrical conductivity. The voltage increases with increasing applied magnetic field strength, and the power exhibits, therefore, the same behavior as the voltage. Two experimentally measured power points have been added on page 102, showing an under-predicted power level, but a very accurate trend. This may be due to the electrode voltage drop which is neglected in these calculations.

The top of page 103 shows the thrust as a function of the input current for three applied magnetic fields. Overall, the thrust appears to vary with the square of the input current [81], as is also visible on page 103. Again, an increase in applied magnetic field strength causes an increase in thrust. Interesting, however, is that the increase in thrust is more pronounced for higher input currents. This indicates that the thrust is dominated by the Joule heating of the gas for lower input currents and by the Lorentz force acting on the gas for higher input currents.

The bottom of page 103 depicts the specific impulse as a function of input current for the three applied magnetic fields. As expected, the specific impulse increases nearly linear with the input current and also shows an increase with higher applied magnetic fields. The increase in specific impulse due to an applied magnetic field of 0.1 Tesla is about 10%. A similar increase in specific impulse by going from 0.1 Tesla to 0.2 Tesla can also be seen, indicating that the specific impulse scales proportionately to the applied magnetic field strength.

The mass flow rate, determined by

$$\dot{m} = \int_A \rho v dA \quad (99.1)$$

with A being the cross-sectional area, through which the flow passes, is shown on page 104. An increase in the applied magnetic field causes a reduction in the mass flow. As can be seen on page 106, the pressure at the entrance increases with increased power (increased current and applied magnetic field), which causes the ratio of total pressure (specified as input variable) to static pressure to drop. The entrance Mach number is therefore reduced, and since the temperature remains approximately equal, the entrance velocity is reduced. The density remains almost constant as well, and, therefore, the mass flow rate decreases with increasing power levels.

This effect diminishes as the input current is increased, indicating that the Lorentz force causes more acceleration of the flow, counteracting the Joule heating to a larger degree. This causes the choking condition to occur at higher mass flow rates.

The distribution of thermodynamic properties is plotted along the x-axis for four different cases. The four cases are:

- a.) zero applied magnetic field and 3 kA current*
- b.) 0.2 Tesla applied magnetic field and 3 kA current*
- c.) zero applied magnetic field and 12.5 kA current, and*
- d.) 0.2 Tesla applied magnetic field and 12.5 kA current*

Since changes are more pronounced for smaller radii, the values are taken from the fifth gridpoint above the cathode surface. Viscous effects, however, should be small at these grid points.

The bottom of page 104 shows the heavy particle and electron temperature distribution along the x-axis for the four cases mentioned above. It shows that the electron temperature is higher than the heavy particle temperature, and that the difference is more pronounced for smaller power. The energy transfer between heavy particles and electrons is responsible for this behavior. If the

temperature increases, as it does with increasing power, the electron-heavy particle collision cross sections and the degree of ionization increases, thereby increasing the collision frequency between electrons and heavy particles. The electron energy is therefore transferred to the heavy particles at a higher rate, causing the difference in those two temperatures to diminish.

Also visible is that the exit temperature depends more on the power than the current. In general, the temperature increases throughout the inter-electrode region, and then decreases in the expansion region ($x > 0.15$). An exception to this rule is case b, for which the temperature increases almost to the exit plane. This is due to the fact that the physics for this case changed essentially from being a device with only a self-induced magnetic field to a device with a predominantly applied magnetic field. The temperature drop in the expansion area is not as pronounced with the other high applied magnetic field case either. This indicates that the Joule heating contributes significantly to the thermodynamic properties in the expansion region, if the applied magnetic field is high. The current, however, is only minimally affected by the applied magnetic field. The additional Joule heating in the expansion area results, rather, from an increase in the electric field. This, again, can be explained by Ohm's law. The induced magnetic field is small enough in the expansion region to be neglected. If no applied magnetic field is present, Ohm's law is given by:

$$j_y = \sigma E_y \quad (100.1)$$

With an applied magnetic field, Ohm's law becomes:

$$j_y = \sigma (E_y + wB_x) - \Omega j_z B_x \quad (100.2)$$

Since the current remains essentially unchanged, the electric field has to increase due to the applied magnetic field. This increase in the electric field value with increased applied magnetic field in the

expansion area results in higher Joule heating, leading to an increase in temperature in the expansion area, if an applied magnetic field is present.

The transport properties are a strong function of the electron temperature. The same trend that appeared for the electron temperature distribution occurs for the viscosity (page 105), the electrical conductivity (page 105), and the Hall parameter (page 106). The non-dimensionalized value of the Hall parameter divided by the magnetic field strength, Ω , was cut off at one in the numerical calculations, since large values of Ω increase the non-linear portion of the induction equation (equation 41.3) and cause instabilities. Since the cut-off affects essentially the expansion region only, and the induced magnetic field is small in that region, the overall effect should be small.

The bottom of page 106 shows the pressure distribution for the above cases. In general, the pressure depends on the power. Small power levels affect the flow only minimally since the flow is mainly driven by the pressure drop from entrance to exit and the pressure decreases, therefore, throughout the duct and the expansion area. With increasing power, the inter-electrode temperature increases due to Joule heating. The increase in kinetic energy (velocity) is not high enough to cause the mass density to drop enough to offset the higher temperatures and the pressure must therefore increase.

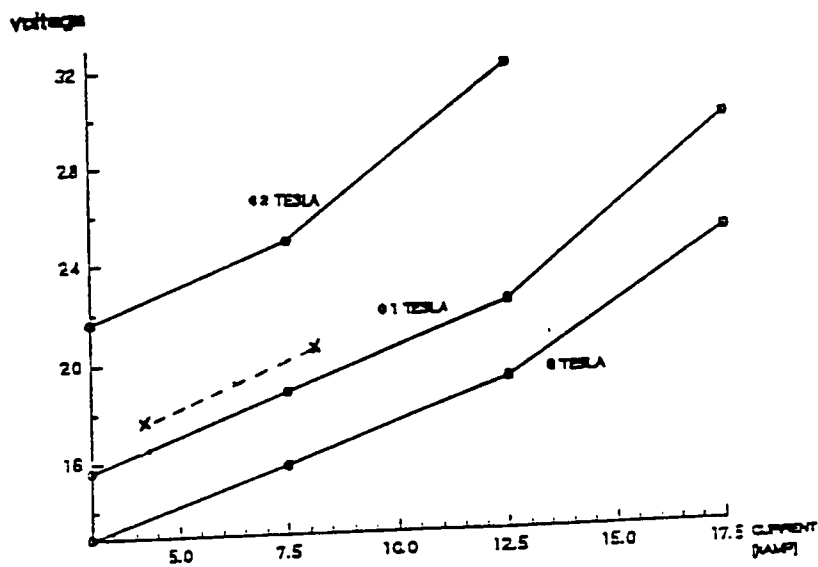


Figure 102.1: Voltage difference as a function of input current at specified applied magnetic fields

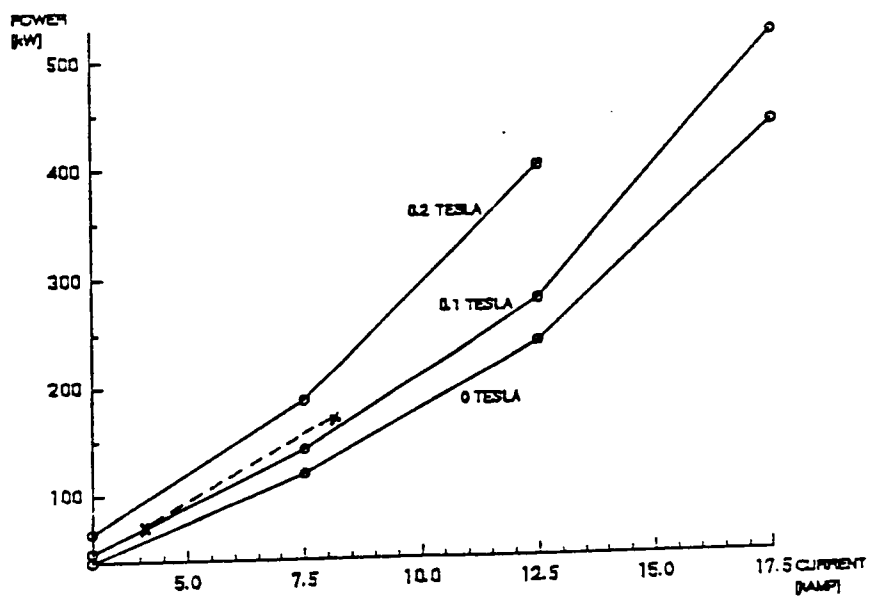


Figure 102.2: Power verses input current at specified applied magnetic fields

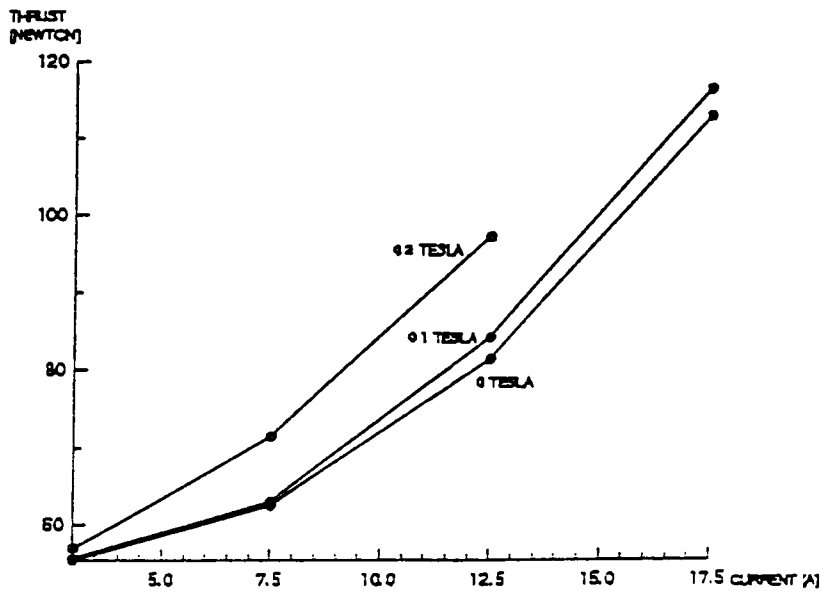


Figure 103.1: Thrust verses input current at specified applied magnetic fields

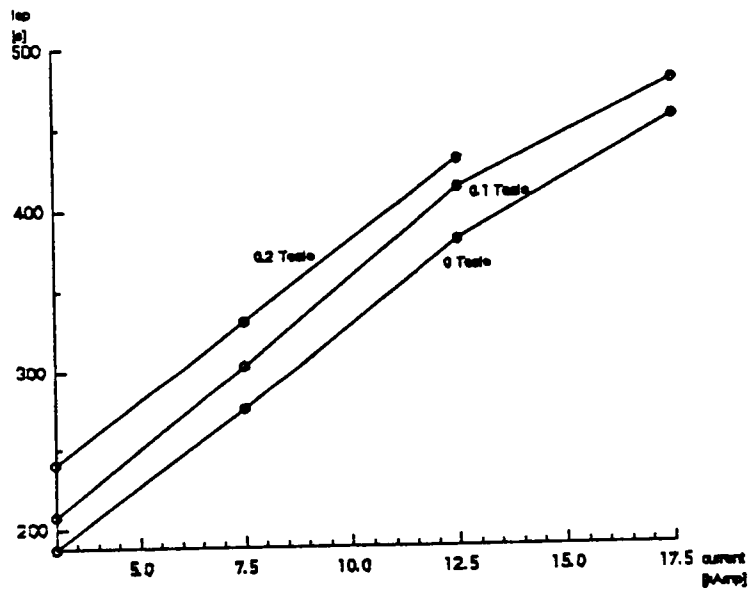


Figure 103.2: Specific impulse verses input current at specified applied magnetic fields

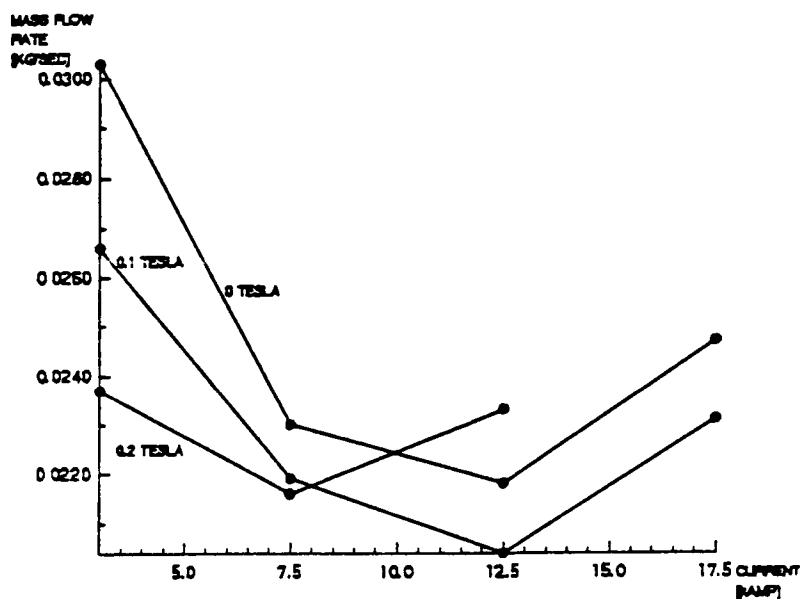


Figure 104.1: Mass flow rate verses input current at specified applied magnetic fields

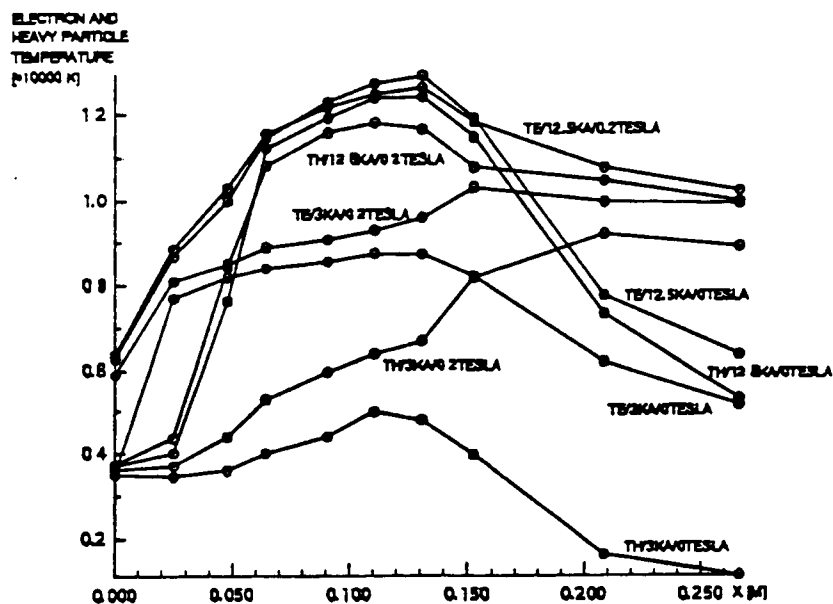


Figure 104.2: Heavy particle and electron temperature distribution

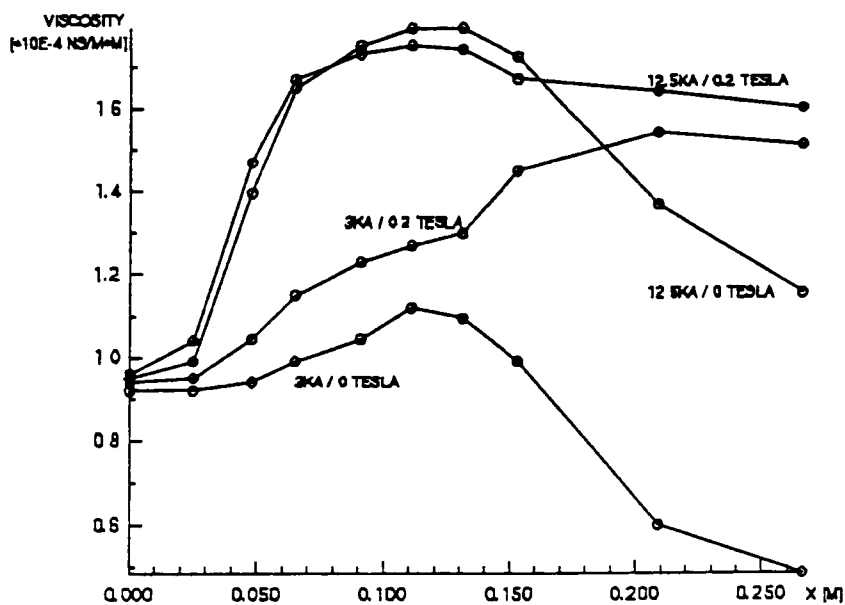


Figure 105.1: Viscosity distribution

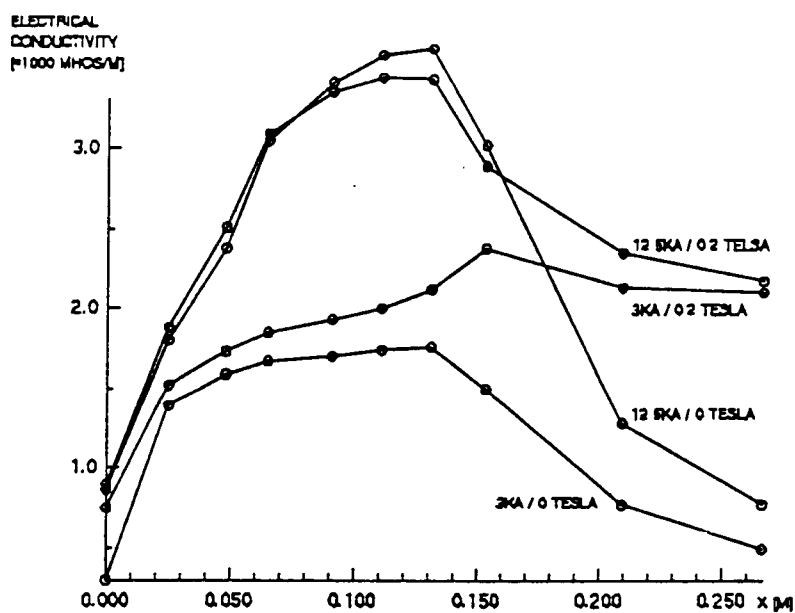


Figure 105.2: Electrical conductivity distribution

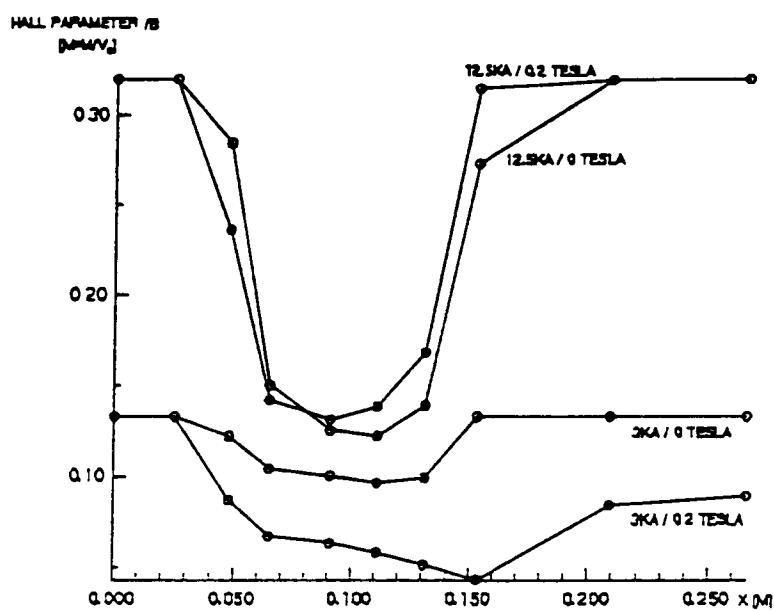


Figure 106.1: Hall parameter distribution

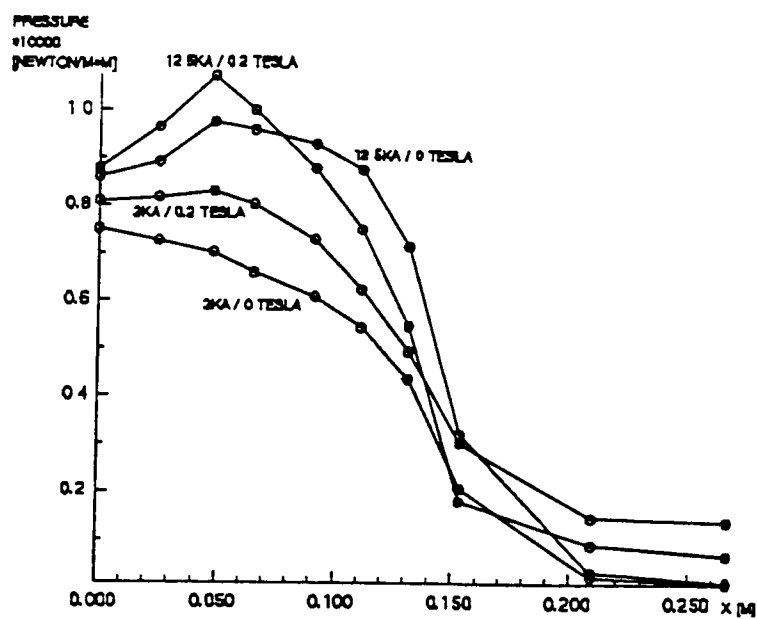


Figure 106.2: Pressure distribution

CHAPTER V

SUMMARY AND RECOMMENDATIONS

5.1 SUMMARY

An existing two dimensional/axisymmetric Navier-Stokes solver for ideal gases (PARC code) was extended to include electromagnetic effects and a two-temperature fluid.

The Lorentz forces and Joule heating were added to the Navier Stokes equations and the energy equation, respectively. Applied magnetic field effects were incorporated by adding the azimuthal momentum equation to the momentum conservation equations in the axial and the radial directions. The temperature in the Navier Stokes solver was split into a contribution due to the heavy particles and a contribution due to the electrons. Ionization and recombination effects were taken into account. The assumptions of an ideal gas with constant specific heats remained unchanged.

The boundary conditions applied in this study consisted of subsonic entrance flow, supersonic exit, no-slip isothermal and no-slip adiabatic walls, as well as an axis of symmetry boundary condition.

The Lorentz forces and the Joule heating term were evaluated separately from the Navier Stokes solver. Under the assumption of axisymmetric, steady-state conditions, the three magnetic induction equations for the magnetic field strength were reduced to one equation in the azimuthal direction. The Hall effect (non-zero Hall parameters) was included in the formulation for the magnetic induction equation.

Perfect conductors and perfect insulators were assumed to provide the appropriate boundary conditions for the induced magnetic field. Applied magnetic fields were assumed constant in time and

were prescribed. Once the magnetic field strength was found, the current density and the electric field were evaluated. Knowing the electromagnetic fields, the right-hand-side of the Navier Stokes equation was then specified.

The electron energy equation was solved to determine the temperature of the electrons. The thermal conductivity, the heat transfer coefficient, and the collision cross sections were iterated with the electron temperature and the mass density (from the Navier Stokes solver).

Adiabatic wall boundary conditions were used for the electron energy equation. In addition, equation 68.1 served to provide the electron temperature at the inflow and outflow boundaries by the energy balance between Joule heating and the energy transfer between the heavy particles and the electrons. The electron temperature and the mass density were used to evaluate the varying degree of ionization, the electrical conductivity, and the Hall parameter.

The interaction between all conservation equations were established by solving the Navier Stokes equations, the magnetic induction equation and the electron energy equation, iteratively. About 10000 iterations were required to obtain convergence, representing approximately ten hours of CPU time.

Several aspects of the code were verified by a multitude of tests and the code was then applied to three cases. A coaxial, straight duct geometry with electrodes in the midsection were calculated with and without an applied magnetic field. The major effect of an applied magnetic field was the occurrence of a swirl flow. Other parameters of the flow did not change appreciably due to the applied magnetic field. Finally, the code was applied to an MPD geometry closely resembling the geometry used by Sleziona et. al. [54]. It was found that choking and an instability occurred at the same mass flow rate and current value used by Sleziona and higher mass flow rates were required to obtain stable solutions. A parametric study was performed on the Sleziona geometry and the results appear reasonable.

The positive effect of an applied magnetic field on the performance of electrothermal and electromagnetic thrusters, as demonstrated by the parameter study, clearly indicates the significance of applied magnetic field thrusters. The significantly increased thrust due to an applied magnetic field for high current densities suggests that especially electromagnetic thrusters will benefit substantially from applied magnetic fields.

5.2 RECOMMENDATIONS

Even though the code described in this work contains several major effects occurring in arcjets and MPD thrusters, enhancements are possible and desirable.

Non-equilibrium ionization:

Currently, the code utilizes an equilibrium ionization model, the Saha equation, evaluated at the local electron temperature and gas pressure. In reality, a finite amount of time is required for the particles to adjust to the local conditions, and therefore, non-equilibrium may exist. This effect is especially important in the expansion region. The degree of ionization in the expansion region will be higher for finite-rate ionization as compared to equilibrium ionization, because the decline in temperature in that region does not lead to an immediate change in the degree of ionization. This can significantly alter the accuracy of energy-loss calculations, the direction of electric fields, and the distribution of current. Due to the possible significance of this effect, it is desirable to extend the current model to allow for non-equilibrium ionization.

Improved stability of the code is likely, if the finite-rate ionization is included in the implicit portion of the Navier-Stokes solver. This, however, necessitates a major change to the algorithm.

Chemical reactions:

The Navier-Stokes solver used for this work is based on the assumption of constant chemical composition of the gas. The ratio of specific heats is constant throughout. While this is a justifiable approximation for a monatomic gas, this assumption limits severely the selection of propellants that can be modeled. Interesting propellants, such as Hydrogen or Hydrazine, cannot be modeled accurately.

A version of the Navier-Stokes solver, developed concurrently at UTSI [80], [81], is capable of calculating the flow field including certain chemical reactions and updated transport properties based on the chemical composition of the gas.

A combination of this solver with the solver for the magnetic induction equation and the electron energy conservation equation is desirable, if the application of the code to non-monatomic propellants is desired. However, one must expect a significant increase in computation time, since the Navier-Stokes solver including chemical reactions is not as stable as the base code employed here. The addition of new source terms will most likely diminish the stability even further and achieving solutions to the full problem might take weeks.

Again, as discussed in the section on non-equilibrium ionization, significant deviations in the chemical composition can be expected in the expansion region, and therefore non-equilibrium chemical reactions would be required to describe this region, correctly. Extending the code to include finite rate chemistry with electromagnetic effects would be a significant undertaking.

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APPENDICES

APPENDIX A.1

TRANSFORMATION OF THE NAVIER STOKES EQUATIONS INTO COMPUTATIONAL SPACE

The transformation of the Navier-Stokes equations from Cartesian coordinates to general curvilinear coordinates can be found in [64] and follows the development of Viviand [75] and Vinokur [76]. The curvilinear coordinates and the Cylindrical coordinates (Cartesian: $y = r$) are related by

$$\tau = t \quad (119.1)$$

$$\xi = \xi(x, r, t)$$

$$\eta = \eta(x, r, t)$$

and

$$t = \tau \quad (119.2)$$

$$x = x(\xi, \eta, \tau)$$

$$r = r(\xi, \eta, \tau)$$

in the general case of a time-dependent grid.

The transformations are chosen, such that the grid spacing in the curvilinear space is uniform and of unit length. Application of the chain rule leads to

$$\begin{bmatrix} \partial_t \\ \partial_x \\ \partial_r \end{bmatrix} = \begin{bmatrix} 1 & \xi_t & \eta_t \\ 0 & \xi_x & \eta_x \\ 0 & \xi_y & \eta_y \end{bmatrix} \begin{bmatrix} \partial_\tau \\ \partial_\xi \\ \partial_\eta \end{bmatrix} \quad (120.1)$$

and

$$\begin{bmatrix} \partial_\tau \\ \partial_\xi \\ \partial_\eta \end{bmatrix} = \begin{bmatrix} 1 & x_\tau & r_\tau \\ 0 & x_\xi & r_\xi \\ 0 & x_\eta & r_\eta \end{bmatrix} \begin{bmatrix} \partial_t \\ \partial_x \\ \partial_r \end{bmatrix} \quad (120.2)$$

Taking the inverse of one of the matrix equations and comparing the terms, one can find the metric terms and the Jacobian to be

$$J^{-1} = r (x_\xi r_\eta - x_\eta r_\xi) \quad (120.3)$$

$$\xi_x = r J r_\eta \quad \xi_y = -r J x_\eta$$

$$\eta_x = -r J r_\xi \quad \eta_y = r J x_\xi$$

with r equal one for the two-dimensional case.

Defining

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ e \end{bmatrix} \quad E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ u(e+p) \end{bmatrix} \quad F = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ v(e+p) \end{bmatrix} \quad (120.4)$$

$$\begin{aligned}
E_v &= \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xr} \\ \tau_{xz} \\ f_5 \end{bmatrix} & F_v &= \begin{bmatrix} 0 \\ \tau_{xr} \\ \tau_{rr} \\ \tau_{rz} \\ g_5 \end{bmatrix} & G_v &= \begin{bmatrix} 0 \\ J_r B_z - J_z B_r \\ J_z B_x - J_x B_z \\ J_x B_r - J_r B_x \\ J_x E_x + J_r E_r \end{bmatrix} & H_v &= \delta \begin{bmatrix} 0 \\ 0 \\ p + \rho w^2 \\ -\rho vw \\ 0 \end{bmatrix}
\end{aligned} \quad (121.1)$$

where δ is one for axisymmetry and zero for two-dimensional flow, with

$$\begin{aligned}
\tau_{xx} &= \mu \left[2 \frac{\partial u}{\partial x} - \frac{2}{3} (\nabla \cdot \vec{u}) \right] \\
\tau_{xr} &= \mu \left[\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right] \\
\tau_{rr} &= \mu \left[2 \frac{\partial v}{\partial r} - \frac{2}{3} (\nabla \cdot \vec{u}) \right] \\
\tau_{xz} &= \mu \left[\frac{\partial w}{\partial x} \right] \\
\tau_{rz} &= \mu \left[r \frac{\partial}{\partial r} \left(\frac{w}{r} \right) \right] \\
f_5 &= u \tau_{xx} + v \tau_{xr} + w \tau_{xz} + \mu P r^{-1} (\gamma - 1)^{-1} \frac{\partial a^2}{\partial x} \\
g_5 &= u \tau_{xr} + v \tau_{rr} + w \tau_{rz} + \mu P r^{-1} (\gamma - 1)^{-1} \frac{\partial a^2}{\partial r}
\end{aligned} \quad (121.2)$$

for the axisymmetric case (x = axial, r = radial, z = azimuthal direction), where

$$\nabla \cdot \vec{u} = \frac{1}{r} \frac{\partial}{\partial r} (r v) + \frac{\partial u}{\partial x} \quad (121.3)$$

or with

$$\tau_{xx} = \mu \left[2 \frac{\partial u}{\partial x} - \frac{2}{3} (\nabla \cdot \vec{u}) \right] \quad (122.1)$$

$$\tau_{yy} = \mu \left[2 \frac{\partial v}{\partial y} - \frac{2}{3} (\nabla \cdot \vec{u}) \right]$$

$$\tau_{xy} = \mu \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]$$

$$f_4 = u \tau_{xx} + v \tau_{xy} + \mu Pr^{-1} (\gamma - 1)^{-1} \frac{\partial a^2}{\partial x}$$

$$g_4 = u \tau_{xy} + v \tau_{yy} + \mu Pr^{-1} (\gamma - 1)^{-1} \frac{\partial a^2}{\partial y}$$

for the two-dimensional case (r substituted by y), where

$$\nabla \cdot \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad (122.2)$$

the Navier-Stokes equations can conveniently be written as

$$\partial_t r Q + \partial_x r E + \partial_r r F = \frac{1}{Re} (\partial_x r E_v + \partial_r r F_v) + G_v r SRm + H_v \quad (122.3)$$

where r equals one for the two-dimensional case. The following non-dimensionalizations were used:

$$\rho = \frac{\rho_d}{\rho_{ref}} \quad \vec{u} = \frac{\vec{u}_d}{u_{ref}} \quad e = \frac{e_d}{\rho_{ref} u_{ref}^2} \quad (122.4)$$

$$p = \frac{p_d}{\rho_{ref} a_{ref}^2} \quad t = \frac{t_d a_d}{L_{ref}} \quad \mu = \frac{\mu_d}{\mu_{ref}}$$

The nondimensional parameters are defined as

$$Re = \frac{\rho_{ref} u_{ref} L_{ref}}{\mu_{ref}} \quad (123.1)$$

$$Pr = \frac{\mu_{ref} C_{p_{ref}}}{\kappa_{ref}}$$

$$Rm = \mu_0 \sigma_{ref} u_{ref} L_{ref}$$

$$S = \frac{B_{ref}^2}{\mu_0 \rho_{ref} u_{ref}^2}$$

Note the distinction between μ_0 , the magnetic permeability of vacuum, and μ_{ref} , the reference viscosity.

Application of the chain rule to the Navier-Stokes equations leads to

$$\begin{aligned} \partial_\tau \tau Q + \xi_t \partial_\xi \tau Q + \eta_t \partial_\eta \tau Q + \xi_x \partial_\xi \tau E + \eta_x \partial_\eta \tau E + \xi_y \partial_\xi \tau F + \eta_y \partial_\eta \tau F = \\ \frac{1}{Re} (\xi_x \partial_\xi \tau E_v + \eta_x \partial_\eta \tau E_v + \xi_y \partial_\xi \tau F_v + \eta_y \partial_\eta \tau F_v) + SR_m \tau G_v + H_v \end{aligned} \quad (123.2)$$

The PARC code uses the Navier-Stokes equations in a strong conservative form. To obtain this form, the Navier-Stokes equations are multiplied by $J^{(-1)}_{2d}$ and use is made of

$$\begin{aligned} \partial_\xi \left(\frac{\xi_x}{J} \right) + \partial_\eta \left(\frac{\eta_x}{J} \right) &= 0 \\ \partial_\xi \left(\frac{\xi_y}{J} \right) + \partial_\eta \left(\frac{\eta_y}{J} \right) &= 0 \end{aligned} \quad (123.3)$$

New vectors for Q, E, F etc. can now be defined, such that the convenient form of the Navier-Stokes equation can be kept

$$\begin{aligned}
 Q = J^{-1} \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ e \end{bmatrix} \quad E = J^{-1} \begin{bmatrix} \rho U \\ \rho u U + \xi_x p \\ \rho v U + \xi_y p \\ \rho w U \\ U(e+p) \end{bmatrix} \quad F = J^{-1} \begin{bmatrix} \rho V \\ \rho u V + \eta_x p \\ \rho v V + \eta_y p \\ \rho w V \\ V(e+p) \end{bmatrix}
 \end{aligned} \tag{124.1}$$

where the Jacobian is now for two dimensional or axisymmetric flow, with

$$U = \xi_x u + \xi_y v \tag{124.2}$$

$$V = \eta_x u + \eta_y v$$

The viscous flux terms are

$$E_v = J^{-1} (\xi_x E_{v_{old}} + \xi_y F_{v_{old}}) \tag{124.3}$$

$$F_v = J^{-1} (\eta_x E_{v_{old}} + \eta_y F_{v_{old}})$$

The viscous stress terms are also transformed

$$\tau_{xx} = \frac{\mu [4 (\xi_x u_\xi + \eta_x u_\eta) - 2 (\xi_y v_\xi + \eta_y v_\eta)]}{3} \quad (125.1)$$

$$\tau_{xr} = \mu (\xi_y u_\xi + \eta_y u_\eta + \xi_x v_\xi + \eta_x v_\eta)$$

$$\tau_{xz} = \mu (\xi_x w_\xi + \eta_x w_\eta)$$

$$\tau_{rr} = \frac{\mu [-2 (\xi_x u_\xi + \eta_x u_\eta) + 4 (\xi_y v_\xi + \eta_y v_\eta)]}{3}$$

$$\tau_{rz} = \mu (\xi_y w_\xi + \eta_y w_\eta)$$

$$\tau_{zz} = -\frac{2}{3} \mu (\xi_x u_\xi + \eta_x u_\eta + \xi_y v_\xi + \eta_y v_\eta)$$

$$f_5 = u\tau_{xx} + v\tau_{xr} + w\tau_{xz} + \mu Pr^{-1} (\gamma - 1)^{-1} (\xi_x \partial_\xi a^2 + \eta_x \partial_\eta a^2)$$

$$g_5 = u\tau_{xr} + v\tau_{rr} + w\tau_{rz} + \mu Pr^{-1} (\gamma - 1)^{-1} (\xi_y \partial_\xi a^2 + \eta_y \partial_\eta a^2)$$

The actual implementation of the resulting system of equations is described in appendix B.1.

APPENDIX A.2

TRANSFORMATION OF THE INDUCTION EQUATION INTO COMPUTATIONAL SPACE

The following non-dimensionalizations were used for the electromagnetic variables.

$$B = \frac{B_{dim}}{B_{ref}} \quad E = \frac{E_{dim}}{B_{ref} a_{ref}} \quad j = \frac{j_{dim}}{\sigma_{ref} E_{ref}} \quad (126.1)$$

The steady state magnetic induction equation can be written as (equation 41.3):

$$-\nabla \times (\vec{u} \times \vec{B}) + \frac{1}{Rm} \nabla \times \left[\frac{\nabla \times \vec{B}}{\sigma} + \alpha (\nabla \times \vec{B}) \times \vec{B} \right] = 0 \quad (126.2)$$

with α being

$$\alpha = \frac{\omega_e \tau_e}{|B|} \sigma \quad (126.3)$$

For the azimuthal direction this equation becomes:

$$\begin{aligned} 0 = & \frac{\partial v}{\partial r} B_z - \frac{\partial w}{\partial r} B_r + v \frac{\partial B_z}{\partial r} - \frac{\partial w}{\partial x} B_x + \frac{\partial u}{\partial x} B_x + u \frac{\partial B_z}{\partial x} + w \frac{B_r}{r} \\ & + \frac{1}{Rm} \left[\frac{1}{\sigma^2} \frac{\partial \sigma}{\partial r} \left(\frac{B_z}{r} + \frac{\partial B_z}{\partial r} \right) + \frac{1}{\sigma^2} \frac{\partial \sigma}{\partial x} \frac{\partial B_z}{\partial x} - \frac{1}{\sigma} \left(-\frac{B_z}{r^2} + \frac{1}{r} \frac{\partial B_z}{\partial r} + \frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial r^2} \right) \right. \\ & + \frac{\partial \alpha}{\partial r} \left(B_z \frac{\partial B_z}{\partial x} + B_r \frac{\partial B_r}{\partial x} - B_r \frac{\partial B_x}{\partial r} \right) + \frac{\partial \alpha}{\partial x} \left(-\frac{B_z^2}{r} - B_z \frac{\partial B_z}{\partial r} + B_x \frac{\partial B_r}{\partial x} - B_x \frac{\partial B_x}{\partial r} \right) \\ & \left. + \alpha \frac{B_r}{r} \left(\frac{\partial B_x}{\partial r} - \frac{\partial B_r}{\partial x} \right) + \alpha B_r \left(\frac{\partial^2 B_r}{\partial x \partial r} - \frac{\partial^2 B_x}{\partial r^2} \right) - 2\alpha \frac{B_z}{r} \frac{\partial B_z}{\partial x} + \alpha B_x \left(\frac{\partial^2 B_r}{\partial x^2} - \frac{\partial^2 B_x}{\partial x \partial r} \right) \right] \end{aligned} \quad (126.4)$$

with x , r and z denoting the axial, radial and azimuthal direction, respectively.

Using the following second order transformations

(127.1)

$$\begin{aligned}\frac{\partial^2 B_z}{\partial x^2} = & \xi_x \frac{\partial \xi_x}{\partial \xi} \frac{\partial B_z}{\partial \xi} + \xi_x^2 \frac{\partial^2 B_z}{\partial \xi^2} + \xi_x \frac{\partial \eta_x}{\partial \xi} \frac{\partial B_z}{\partial \eta} + \xi_x \eta_x \frac{\partial^2 B_z}{\partial \xi \partial \eta} + \eta_x \frac{\partial \xi_x}{\partial \eta} \frac{\partial B_z}{\partial \xi} + \\ & + \xi_x \eta_y \frac{\partial^2 B_z}{\partial \xi \partial \eta} + \eta_x \frac{\partial \eta_x}{\partial \eta} \frac{\partial B_z}{\partial \eta} + \eta_x^2 \frac{\partial^2 B_z}{\partial \eta^2} +\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 B_z}{\partial r^2} = & \xi_y \frac{\partial \xi_y}{\partial \xi} \frac{\partial B_z}{\partial \xi} + \xi_y^2 \frac{\partial^2 B_z}{\partial \xi^2} + \xi_y \frac{\partial \eta_y}{\partial \xi} \frac{\partial B_z}{\partial \eta} + \xi_y \eta_y \frac{\partial^2 B_z}{\partial \xi \partial \eta} + \eta_y \frac{\partial \xi_y}{\partial \eta} \frac{\partial B_z}{\partial \xi} + \\ & + \xi_y \eta_y \frac{\partial^2 B_z}{\partial \xi \partial \eta} + \eta_y \frac{\partial \eta_y}{\partial \eta} \frac{\partial B_z}{\partial \eta} + \eta_y^2 \frac{\partial^2 B_z}{\partial \eta^2} +\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 B_z}{\partial x \partial r} = & \left(\xi_x \frac{\partial \xi_y}{\partial \xi} + \eta_x \frac{\partial \xi_y}{\partial \eta} \right) \frac{\partial B_z}{\partial \xi} + \left(\xi_x \frac{\partial \eta_y}{\partial \xi} + \eta_y \frac{\partial \eta_y}{\partial \eta} \right) \frac{\partial B_z}{\partial \eta} + \xi_x \xi_y \frac{\partial^2 B_z}{\partial \xi^2} + \\ & \eta_x \eta_y \frac{\partial^2 B_z}{\partial \eta^2} + (\xi_x \eta_y + \eta_x \xi_y) \frac{\partial^2 B_z}{\partial \xi \partial \eta} +\end{aligned}$$

in addition to the transformation described in appendix A.1 leads to the induction equation in the computational space. The equation is split up into the left-hand-side, that includes only terms depending on the magnetic field in azimuthal direction, and the right-hand-side, that includes only the fixed applied magnetic field components in radial and axial direction. The left-hand-side is

the fixed applied magnetic field components in radial and axial direction. Multiplication of the equation by r , and including the r in B (i.e. $B_z = r B_z$ from now on), the left-hand-side is

$$\begin{aligned}
 & B_z \left(\xi_x \frac{\partial u}{\partial \xi} + \eta_x \frac{\partial u}{\partial \eta} + \xi_y \frac{\partial v}{\partial \xi} + \eta_y \frac{\partial v}{\partial \eta} \right) + \tag{128.1} \\
 & + \frac{\partial B_z}{\partial \xi} \left[u \xi_x + v \xi_y + \frac{1}{Rm\sigma^2} \left(\xi_x^2 \frac{\partial \sigma}{\partial \xi} + \eta_x \xi_y \frac{\partial \sigma}{\partial \eta} + \xi_y^2 \frac{\partial \sigma}{\partial \xi} + \eta_y \xi_y \frac{\partial \sigma}{\partial \eta} \right) \right. \\
 & \quad \left. - \frac{1}{Rm\sigma} \left(\xi_x \frac{\partial \xi_x}{\partial \xi} + \eta_x \frac{\partial \xi_x}{\partial \eta} + \xi_y \frac{\partial \xi_y}{\partial \xi} + \eta_y \frac{\partial \xi_y}{\partial \eta} \right) - \frac{1}{Rm} \left(\frac{\partial \alpha}{\partial x} \xi_y B_z - \frac{\partial \alpha}{\partial y} \xi_x B_z \right) \right] \\
 & + \frac{\partial B_z}{\partial \eta} \left[u \eta_x + v \eta_y + \frac{1}{Rm\sigma^2} \left(\xi_x \eta_x \frac{\partial \sigma}{\partial \xi} + \eta_x^2 \frac{\partial \sigma}{\partial \eta} + \xi_y \eta_y \frac{\partial \sigma}{\partial \xi} + \eta_y^2 \frac{\partial \sigma}{\partial \eta} \right) \right. \\
 & \quad \left. - \frac{1}{Rm\sigma} \left(\xi_x \frac{\partial \eta_x}{\partial \xi} + \eta_x \frac{\partial \eta_x}{\partial \eta} + \xi_y \frac{\partial \eta_y}{\partial \xi} + \eta_y \frac{\partial \eta_y}{\partial \eta} \right) - \frac{1}{Rm} \left(\frac{\partial \alpha}{\partial x} \eta_y B_z - \frac{\partial \alpha}{\partial y} \eta_x B_z \right) \right] \\
 & \frac{\partial^2 B_z}{\partial \xi \partial \eta} \left[\frac{-2}{Rm\sigma} (\xi_x \eta_x + \xi_y \eta_y) \right] + \frac{\partial^2 B_z}{\partial \xi^2} \left[\frac{-1}{Rm\sigma} (\xi_x^2 + \xi_y^2) \right] + \frac{\partial^2 B_z}{\partial \eta^2} \left[\frac{-1}{Rm\sigma} (\eta_x^2 + \eta_y^2) \right]
 \end{aligned}$$

The right-hand-side is:

$$\begin{aligned}
 & \frac{1}{Rm} \left[B_x \left[\left(\xi_x \frac{\partial \alpha}{\partial \xi} + \eta_x \frac{\partial \alpha}{\partial \eta} \right) \left(\xi_x \frac{\partial B_r}{\partial \xi} + \eta_x \frac{\partial B_r}{\partial \eta} - \xi_y \frac{\partial B_x}{\partial \xi} - \eta_y \frac{\partial B_x}{\partial \eta} \right) - \left(\xi_x \frac{\partial w}{\partial \xi} + \eta_x \frac{\partial w}{\partial \eta} \right) \right] \right. \\
 & B_r \left[\left(\xi_y \frac{\partial \alpha}{\partial \xi} + \eta_y \frac{\partial \alpha}{\partial \eta} \right) \left(\xi_x \frac{\partial B_r}{\partial \xi} + \eta_x \frac{\partial B_r}{\partial \eta} - \xi_y \frac{\partial B_x}{\partial \xi} - \eta_y \frac{\partial B_x}{\partial \eta} \right) - \left(\xi_y \frac{\partial w}{\partial \xi} + \eta_y \frac{\partial w}{\partial \eta} \right) \right] \\
 & \alpha B_x \left[\xi_x \frac{\partial \xi_x}{\partial \xi} \frac{\partial B_r}{\partial \xi} + \xi_x^2 \frac{\partial^2 B_r}{\partial \xi^2} + \xi_x \frac{\partial \eta_x}{\partial \xi} \frac{\partial B_r}{\partial \eta} + \xi_x \eta_x \frac{\partial^2 B_r}{\partial \xi \partial \eta} + \eta_x \frac{\partial \xi_x}{\partial \eta} \frac{\partial B_r}{\partial \xi} + \right. \\
 & \quad + \xi_x \eta_x \frac{\partial^2 B_r}{\partial \xi \partial \eta} + \eta_x \frac{\partial \eta_x}{\partial \eta} \frac{\partial B_r}{\partial \eta} + \eta_x^2 \frac{\partial^2 B_r}{\partial \eta^2} - \xi_x \xi_y \frac{\partial^2 B_x}{\partial \xi^2} - \xi_x \frac{\partial \xi_y}{\partial \xi} \frac{\partial B_x}{\partial \xi} \\
 & \quad - \xi_x \eta_y \frac{\partial^2 B_x}{\partial \xi \partial \eta} - \xi_x \frac{\partial \eta_y}{\partial \xi} \frac{\partial B_x}{\partial \eta} - \eta_x \xi_y \frac{\partial^2 B_x}{\partial \xi \partial \eta} + \eta_x \frac{\partial \xi_y}{\partial \eta} \frac{\partial B_x}{\partial \xi} \\
 & \quad \left. + \eta_x \eta_y \frac{\partial^2 B_x}{\partial \eta^2} + \eta_x \frac{\partial \eta_y}{\partial \eta} \frac{\partial B_x}{\partial \eta} \right. \\
 & \quad + \alpha B_r \left[\xi_x \xi_y \frac{\partial^2 B_r}{\partial \xi^2} + \xi_x \frac{\partial \xi_y}{\partial \xi} \frac{\partial B_r}{\partial \xi} + \xi_x \eta_y \frac{\partial^2 B_r}{\partial \xi \partial \eta} + \xi_x \frac{\partial \eta_y}{\partial \xi} \frac{\partial B_r}{\partial \eta} + \eta_x \xi_y \frac{\partial^2 B_r}{\partial \xi \partial \eta} \right. \\
 & \quad + \eta_x \frac{\partial \xi_y}{\partial \eta} \frac{\partial B_r}{\partial \xi} + \eta_x \eta_y \frac{\partial^2 B_r}{\partial \eta^2} + \eta_x \frac{\partial \eta_y}{\partial \eta} \frac{\partial B_r}{\partial \eta} - \xi_y \frac{\partial \xi_y}{\partial \xi} \frac{\partial B_x}{\partial \xi} - \xi_y^2 \frac{\partial^2 B_x}{\partial \xi^2} - \xi_y \frac{\partial \eta_y}{\partial \xi} \frac{\partial B_x}{\partial \eta} \\
 & \quad \left. \left. \xi_y \eta_y \frac{\partial^2 B_x}{\partial \xi \partial \eta} - \eta_y \frac{\partial \xi_y}{\partial \eta} \frac{\partial B_x}{\partial \xi} - \xi_y \eta_y \frac{\partial^2 B_x}{\partial \xi \partial \eta} - \eta_y \frac{\partial \eta_y}{\partial \eta} \frac{\partial B_x}{\partial \eta} - \eta_y^2 \frac{\partial^2 B_x}{\partial \eta^2} \right] \right]
 \end{aligned}
 \tag{129.1}$$

APPENDIX A.3:

DERIVATION OF THE ELECTRON ENERGY CONSERVATION EQUATION

The starting point for the derivation of the electron energy conservation equation as used in these calculations can be found in [24] and [73]. The basic electron energy conservation equation without viscous effects is given as

$$\begin{aligned}
 & \frac{\partial}{\partial t} \left[\rho_e \left(\frac{1}{2} \bar{u}_e^2 + \frac{3}{2} \frac{kT_e}{m_e} \right) \right] + \nabla \cdot \left[\rho_e \bar{u}_e \left(\frac{1}{2} \bar{u}_e^2 + \frac{3}{2} \frac{kT_e}{m_e} \right) \right] \\
 & + \nabla \cdot \bar{q}_e' + \nabla \cdot (\bar{u}_e p_e') = \\
 & = \bar{E} \cdot \bar{J}_e + n_e m_e \sum_s \nu_{es} (\bar{u}_s - \bar{u}_e) \cdot \bar{u}_s + \\
 & + 2n_e m_e \sum_s \frac{\delta_s}{m_s} \nu_{es} \left(\frac{3}{2} kT_s - \frac{3}{2} kT_e \right)
 \end{aligned} \tag{130.1}$$

with

$$\vec{J}_e = -\vec{u}_e n_e e \quad (131.1)$$

$$v_{ee}^* = \frac{n_i}{3n_e k T_e} \int Q_{ee} c'_e f_e dc'_e \quad (131.2)$$

$$v_{ee}^{**} = \frac{n_i}{n_e} \int Q_{ee} c'_e f_e dc'_e \quad (131.3)$$

where δ_e is the energy loss factor, which is set to one for the calculations performed in this work.

Neglecting smaller order terms such as

$$u_e'^2 \ll u^2 \quad v_{ee}^{**} \ll v_{ee}^* \quad (131.4)$$

one obtains

$$\begin{aligned} & \frac{\partial}{\partial x} \left[\frac{3}{2} n_e k T_e \right] - \nabla \cdot \left[\frac{5}{2} \vec{J}_e \frac{k}{e} T_e \right] + \nabla \cdot q_e - \nabla \cdot \left[J_e \frac{k}{e} T_e \right] = \\ & = \vec{E} \cdot \vec{J}_e + 3 n_e m_e k \sum \frac{\delta_e}{m_i} v_{ei}^* (T_i - T_e) \end{aligned} \quad (131.5)$$

where

$$q_e^j = -\eta_e \frac{\partial T_e}{\partial x^j} + \frac{5}{2} n_e k T_e V_e^j = -\eta_e \frac{\partial T_e}{\partial x^j} - \frac{5}{2} \vec{J}_e \frac{k}{e} T_e - \frac{5}{2} \vec{u}_e n_e k T_e \quad (131.6)$$

The thermal conductivity can be calculated by [73].

$$\eta_e' = \beta_e \kappa_e \quad (132.1)$$

$$\kappa_e = \frac{15}{4} k \frac{n_e}{\sum_j \alpha_j n_j \Delta_j^{(2)}(T_e)} \quad (132.2)$$

$$\beta_e = \frac{n_e}{\Delta_e^{(2)}(T_e)} \left[\frac{1}{\sum_i \frac{n_i}{\alpha_i \Delta_i^{(2)}(T_e)}} \right] \quad (132.3)$$

with

$$\alpha_j = 1 + \frac{\left(1 - \frac{m_i}{m_j}\right) \left(0.45 - 2.54 \frac{m_i}{m_j}\right)}{\left(1 + \frac{m_i}{m_j}\right)^2} \quad (132.4)$$

$$\Delta_j^{(2)} = \frac{16}{5} \sqrt{\frac{2m_i m_j}{\pi k T (m_i + m_j)}} \pi \Omega_j^{(2,2)} \quad (132.5)$$

The $\pi \Omega_{ij}^{(2,2)}$ are taken from [78].

Finally, one obtains the following electron energy equation, as it applies to electromagnetic accelerators.

$$\begin{aligned}
& \frac{\partial}{\partial \alpha} \left(\frac{3}{2} n_e k T_e \right) - \nabla \cdot \left(5 \bar{J}_e \frac{k}{e} T_e \right) - \nabla \cdot \left(\eta_e \nabla T_e + \frac{5}{2} \bar{u} n_e k T_e \right) = \\
& = \bar{E} \cdot \bar{J}_e + 3 n_e m_e k \sum_s \frac{v_{\infty s}}{m_s} (T_s - T_e)
\end{aligned} \tag{133.1}$$

Using the same transformation as described in Appendix A.1, one finds

$$\frac{\partial}{\partial \xi} \left[\xi_y \left(5 \frac{k}{e} J_r + \frac{5}{2} n_e k v \right) + \xi_x \left(5 \frac{k}{e} J_x + \frac{5}{2} n_e k u \right) \right] * J^{-1} r T_e + \tag{133.2}$$

$$\frac{\partial}{\partial \eta} \left[\eta_y \left(5 \frac{k}{e} J_r + \frac{5}{2} n_e k v \right) + \eta_x \left(5 \frac{k}{e} J_x + \frac{5}{2} n_e k u \right) \right] * J^{-1} r T_e -$$

$$\frac{\partial}{\partial \xi} \left(\frac{\xi_y \lambda_e}{r} \right) * J^{-1} r T_e - \frac{\partial}{\partial \eta} \left(\frac{\eta_y \lambda_e}{r} \right) * J^{-1} r T_e$$

$$+ \frac{\partial}{\partial \xi} \left[\xi_y \lambda_e \frac{\partial \xi_y J^{-1} T_e}{\partial \xi} + \xi_x \lambda_e \frac{\partial \xi_x J^{-1} T_e}{\partial \xi} \right]$$

$$+ \xi_y \lambda_e \frac{\partial \eta_y J^{-1} T_e}{\partial \eta} + \xi_x \lambda_e \frac{\partial \eta_x J^{-1} T_e}{\partial \eta} \Big]$$

$$+ \frac{\partial}{\partial \eta} \left[\eta_y \lambda_e \frac{\partial \eta_y J^{-1} T_e}{\partial \eta} + \eta_x \lambda_e \frac{\partial \eta_x J^{-1} T_e}{\partial \eta} \right]$$

$$+ \eta_y \lambda_e \frac{\partial \xi_y J^{-1} T_e}{\partial \xi} + \eta_x \lambda_e \frac{\partial \xi_x J^{-1} T_e}{\partial \xi} \Big]$$

This equation is discretized and yields the following equation:

$$\begin{aligned}
 & J^{-1} r T_{\phi j,k} \left[-\frac{1}{2} (\alpha_{1j+1,k} + 2\alpha_{1j,k} + \alpha_{1j-1,k}) \xi_{xj,k} - \frac{1}{2} (\alpha_{2j+1,k} + 2\alpha_{2j,k} + \alpha_{2j-1,k}) \xi_{yj,k} \right. \\
 & \quad \left. - \frac{1}{2} (\alpha_{3j,k+1} + 2\alpha_{3j,k} + \alpha_{3j,k-1}) \eta_{xj,k} - \frac{1}{2} (\alpha_{4j,k+1} + 2\alpha_{4j,k} + \alpha_{4j,k-1}) \eta_{yj,k} \right] \\
 & + J^{-1} r T_{\phi j+1,k} \left[-\frac{1}{2} \frac{\lambda_e \xi_j}{r_{j+1,k}} - \frac{5k}{2e} (j_{xj,k} \xi_{xj+1,k} + j_{rj,k} \xi_{rj+1,k}) \right. \\
 & \quad + \frac{5k}{4} \kappa_e (u_{xj,k} \xi_{xj+1,k} + u_{rj,k} \xi_{rj+1,k}) + \frac{1}{2} (\alpha_{1j+1,k} + \alpha_{1j,k}) \xi_{xj+1,k} + \frac{1}{2} (\alpha_{2j+1,k} + \alpha_{2j,k}) \xi_{yj+1,k} \\
 & \quad \left. + \frac{1}{8} (\alpha_{3j,k+1} - \alpha_{3j,k-1}) \xi_{xj+1,k} + \frac{1}{8} (\alpha_{4j,k+1} - \alpha_{4j,k-1}) \xi_{yj+1,k} \right] \\
 & + J^{-1} r T_{\eta j-1,k} \left[\frac{1}{2} \frac{\lambda_e \xi_j}{r_{j-1,k}} + \frac{5k}{2e} (j_{xj,k} \xi_{xj-1,k} + j_{rj,k} \xi_{rj-1,k}) - \frac{5k}{4} \kappa_e (u_{xj,k} \xi_{xj-1,k} + j_{rj,k} \xi_{rj-1,k}) \right. \\
 & \quad + \frac{1}{2} (\alpha_{1j,k} + \alpha_{1j-1,k}) \xi_{xj-1,k} + \frac{1}{2} (\alpha_{2j,k} + \alpha_{2j-1,k}) \xi_{yj-1,k} \\
 & \quad \left. - \frac{1}{8} (\alpha_{3j,k+1} - \alpha_{3j,k-1}) \xi_{xj-1,k} - \frac{1}{8} (\alpha_{4j,k+1} - \alpha_{4j,k-1}) \xi_{yj-1,k} \right] \\
 & J^{-1} r T_{\phi j,k+1} \left[-\frac{1}{2} \frac{\lambda_e \eta_j}{r_{j,k+1}} - \frac{5k}{2e} (j_{xj,k} \eta_{xj,k+1} + j_{rj,k} \eta_{rj,k+1}) + \frac{5k}{4} \kappa_e (u_{xj,k} \eta_{xj,k+1} + j_{rj,k} \eta_{rj,k+1}) \right. \\
 & \quad \left. + \frac{1}{2} (\alpha_{3j,k+1} + \alpha_{3j,k}) \eta_{xj,k+1} + \frac{1}{2} (\alpha_{4j,k+1} + \alpha_{4j,k}) \eta_{yj,k+1} \right]
 \end{aligned} \tag{134.1}$$

$$+\frac{1}{8}(\alpha_{1j+1,k}-\alpha_{1j-1,k})\eta_{y_{j,k+1}}+\frac{1}{8}(\alpha_{2j+1,k}-\alpha_{2j-1,k})\eta_{y_{j,k+1}}\Big]$$

$$J^{-1}rT_e|_{j,k-1}*\left[\frac{1}{2}\frac{\lambda_e\eta_y}{r}|_{j,k-1}+\frac{5}{2}\frac{k}{e}(j_{y,k}\eta_{y_{j,k-1}}+j_{r,j,k}\eta_{y_{j,k-1}})\right.$$

$$\left.-\frac{5}{4}kn_e(u_{y,k}\eta_{y_{j,k-1}}+j_{r,j,k}\eta_{y_{j,k-1}})\right]$$

$$\frac{1}{2}(\alpha_{3j,k}+\alpha_{3j,k-1})\eta_{y_{j,k-1}}+\frac{1}{2}(\alpha_{4j,k}+\alpha_{4j,k-1})\eta_{y_{j,k-1}}$$

$$\left.-\frac{1}{8}(\alpha_{1j+1,k}-\alpha_{1j-1,k})\eta_{y_{j,k-1}}-\frac{1}{8}(\alpha_{2j+1,k}-\alpha_{2j-1,k})\eta_{y_{j,k-1}}\right]$$

$$\frac{1}{8}J^{-1}rT_e|_{j+1,k+1}*\left[(\alpha_{1j+1,k}+\alpha_{1j,k})\eta_{y_{j+1,k+1}}+(\alpha_{2j+1,k}+\alpha_{2j,k})\eta_{y_{j+1,k+1}}\right.$$

$$\left.(\alpha_{3j,k+1}+\alpha_{3j,k})\xi_{y_{j+1,k+1}}+(\alpha_{4j,k+1}+\alpha_{4j,k})\xi_{y_{j+1,k+1}}\right]$$

$$-\frac{1}{8}J^{-1}rT_e|_{j-1,k+1}*\left[(\alpha_{1j,k}+\alpha_{1j-1,k})\eta_{y_{j-1,k+1}}+(\alpha_{2j,k}+\alpha_{2j-1,k})\eta_{y_{j-1,k+1}}\right.$$

$$\left.(\alpha_{3j,k+1}+\alpha_{3j,k})\xi_{y_{j-1,k+1}}+(\alpha_{4j,k+1}+\alpha_{4j,k})\xi_{y_{j-1,k+1}}\right]$$

$$-\frac{1}{8}J^{-1}rT_e|_{j+1,k-1}*\left[(\alpha_{1j+1,k}+\alpha_{1j,k})\eta_{y_{j+1,k-1}}+(\alpha_{2j+1,k}+\alpha_{2j,k})\eta_{y_{j+1,k-1}}\right.$$

$$\left.(\alpha_{3j,k}+\alpha_{3j,k-1})\xi_{y_{j+1,k-1}}+(\alpha_{4j,k}+\alpha_{4j,k-1})\xi_{y_{j+1,k-1}}\right]$$

$$\begin{aligned}
& + \frac{1}{8} J^{-1} r T_e^j \big|_{j-1, k-1} * \left[(\alpha_{1, j, k} + \alpha_{1, j-1, k}) \eta_{x, j-1, k-1} + (\alpha_{2, j, k} + \alpha_{2, j-1, k}) \eta_{y, j-1, k-1} \right. \\
& \quad \left. (\alpha_{3, j, k} + \alpha_{3, j, k-1}) \xi_{x, j-1, k-1} + (\alpha_{4, j, k} + \alpha_{4, j, k-1}) \xi_{y, j-1, k-1} \right]
\end{aligned}$$

The α 's are given by

$$\alpha_1 = \xi_x \lambda_e \quad (136.1)$$

$$\alpha_2 = \xi_y \lambda_e$$

$$\alpha_3 = \eta_x \lambda_e$$

$$\alpha_4 = \eta_y \lambda_e$$

APPENDIX B.1

NAVIER-STOKES SOLUTION ALGORITHM

Introduction

This appendix is separated into two parts. Part 1 describes the algorithm for the solution of the Navier-Stokes solver; part two describes the changes to the original Navier-Stokes solver, that are implemented to accommodate the two-temperature model and the electromagnetic effects.

B.1.1: Description of the Navier-Stokes solver

The equation to be solved by the Navier-Stokes solver can be put in the following form

$$\partial_t \tilde{Q} + \partial_x \tilde{E} + \partial_\eta \tilde{F} = \frac{1}{Re} [\partial_x \tilde{E}_v + \partial_\eta \tilde{F}_v] \quad (137.1)$$

The vectors are given in Appendix A.1

A differencing scheme from Beam and Warming is utilized to discretize the system of equations. Only the left-hand-side is regarded in the following, since the right-hand-side of the equations is time-lagged and not part of the following developments.

$$Q^{n+1} - Q^n + h \cdot (E_t^{n+1} + F_\eta^{n+1}) = h \cdot \frac{1}{Re} (RHS^n) \quad (137.2)$$

Here, the $n+1$ denotes the new, as of yet unknown value of the vectors, n to the already determined values, and h corresponds to the time step size.

Since nothing is known about the values at the time step $n+1$, these values need to be approximated.

The flux vectors are linearized in time by a Taylor series

$$\vec{E}^{n+1} = \vec{E}^n + A^n \Delta \vec{Q}^n + O(h^2) \quad (138.1)$$

$$\vec{F}^{n+1} = \vec{F}^n + B^n \Delta q + O(h^2)$$

with

$$A = \frac{\partial \vec{E}}{\partial \vec{Q}} \quad B = \frac{\partial \vec{F}}{\partial Q} \quad (138.2)$$

For an axisymmetric flow, including the azimuthal momentum equation, the matrices A and B are as follows.

$$A = \begin{bmatrix} 0 & \xi_x & \xi_y & 0 & 0 \\ -u\theta + \xi_x \phi^2 & \theta - (\gamma - 2)\xi_x \mu & \xi_y \mu - (\gamma - 1)\xi_x \nu & -\xi_x (\gamma - 1)w & \xi_x (\gamma - 1) \\ -v\theta + \xi_y \phi^2 & \xi_x \nu - (\gamma - 1)\xi_y \mu & \theta - (\gamma - 2)\xi_y \nu & -\xi_y (\gamma - 1)w & \xi_y (\gamma - 1) \\ -w\theta & \xi_x w & \xi_y w & \theta & 0 \\ \theta(\phi^2 - a_1) & \xi_x a_1 - (\gamma - 1)u\theta & \xi_y a_1 - (\gamma - 1)v\theta & -\theta(\gamma - 1)w & \gamma\theta \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & \eta_x & \eta_y & 0 & 0 \\ \eta_x \phi^2 - \mu \theta & \theta - (\gamma - 2) \eta_x \mu & \eta_y \mu - (\gamma - 1) \eta_x \nu & \eta_x (1 - \gamma) w & \eta_x (\gamma - 1) \\ \eta_y \phi^2 - \nu \theta & \eta_x \nu - (\gamma - 1) \eta_y \mu & \theta - (\gamma - 2) \eta_y \nu & \eta_y (1 - \gamma) w & \eta_y (\gamma - 1) \\ -\mu \theta & \eta_x w & \eta_y w & \theta & 0 \\ \theta (\phi^2 - a_1) & \eta_x a_1 - (\gamma - 1) \mu \theta & \eta_y a_1 - (\gamma - 1) \nu \theta & -\theta (\gamma - 1) w & \gamma \theta \end{bmatrix}$$

where the following abbreviations have been used

$$\phi^2 = \frac{\gamma - 1}{2} (\mu^2 + \nu^2 + w^2) \quad (139.1)$$

$$\theta_{for A} = \xi_x \mu + \xi_y \nu \quad \theta_{for B} = \eta_x \mu + \eta_y \nu$$

$$a_1 = \frac{e}{\rho} \gamma - \frac{(\gamma - 1)}{2} (\mu^2 + \nu^2 + w^2)$$

Implementing this linearization, however, does not yet provide an effective means of solving the system of equations, since a full matrix inversion is required. A major simplification can be achieved, by an approximate factorization of the two-dimensional operator into two one-dimensional operators as follows

$$\Delta \bar{Q}^n + h(\bar{E}^n + A^n \Delta \bar{Q}^n + \bar{F}^n + B^n \Delta \bar{Q}^n) = h(RHS) + O(h^2)$$

$$[I + h\partial_\xi A^n + h\partial_\eta B^n] \Delta \bar{Q}^n = \quad (140.1)$$

$$[I + h\partial_\xi A^n][I + h\partial_\eta B^n] \Delta \bar{Q}^n = h(RHS) + O(h^2)$$

Now, only the solution of two block tridiagonal matrices needs to be found. A further simplification of the solution procedure can be implemented by diagonalization of the matrices A and B.

$$\Lambda_\xi = T_\xi^{-1} A T_\xi \quad \Lambda_\eta = T_\eta^{-1} B T_\eta \quad (140.2)$$

The matrices T are the eigenvector matrices, while the Λ matrices are the eigenvalue matrices. With this the above matrix equation can be put into the following form.

$$T_\xi [I + h\partial_\xi \Lambda_\xi] N [I + h\partial_\eta \Lambda_\eta] T_\eta^{-1} \Delta Q^n = h(RHS^n) \quad (140.3)$$

$$N = T_\xi^{-1} T_\eta$$

Taking the eigenvector matrices outside the spatial derivatives decreases the time accuracy of the code; the speed of the code, however, is enhanced, since now only a tridiagonal system of equations needs to be solved.

B.1.2: Modifications to the Navier-Stokes solver to accommodate electromagnetic effects
 and two-temperature model

To ease the implementation of electromagnetic effects into a fluid dynamic code a modular approach is used in the development of the code for the electromagnetic problem. Therefore, the electromagnetic effects are not included in the implicit portion of the PARC code. Instead, the electromagnetic effects are introduced into the Navier- Stokes code as a time-lagged forcing term, similar to the viscous terms. This allows a separate calculation of the electromagnetic effects outside the Navier-Stokes solver. As the viscous terms, the electromagnetic effects are calculated with the previously calculated gasdynamic properties at every time-step of the Navier-Stokes code by a call to a subroutine. The calculated electromagnetic terms are then added to the time-lagged portion of the algorithm for the Navier-Stokes solver (see Appendix A). Therefore, the major changes in the Navier-Stokes solver consist only of a call to the subroutines, that determine the degree of ionization, the cross sections, and that solve the induction equation. In addition, the velocity components are stored before the call to the subroutines, solving the induction equation, because they are needed in the electromagnetic portion of the code. Furthermore, new input parameters are read in, that control the execution of the different codes.

Those parameters are NREST1, which is used to go through the initial pre-iteration cycle, and IEM, which allows the code to be run for a solution of the Navier Stokes equations, only, the Navier Stokes equations in combination with the magnetic induction equation and the solution of the induction equation, only. The iteration scheme can be seen in figure 51.1.

Because the Navier-Stokes code calculates only the total energy and therefore includes the translational energy of the electrons, changes in the Navier-Stokes solver to accommodate the two

temperature model are minimal. Only when the heavy particle temperature of the pressure is calculated, is it necessary to subtract the energy contents of the electrons from the total energy.

APPENDIX C.1

USER MANUAL

References [63] and [64] give very detailed instructions for the use of the PARC code, which represents the base code for this work. Therefore, only additions due to the added codes will be described here.

The restart file for the PARC code was extended to initialize the electron temperature (TE) and the azimuthal velocity (W). In addition, another restart file was created to initialize the electromagnetic variables used in the magnetic induction equation solver. There, the applied magnetic field is prescribed, and an initial guess for the induced magnetic field is calculated. In addition, the current densities in the axial, radial and azimuthal direction need to be specified.

The input file, specifying the program execution, boundary conditions and reference values, was also extended. In addition to the variable NMAX, giving the number of iterations of the Navier-Stokes solver, NMAXEM must be specified for the number of iterations for the induced magnetic field solver. Since the total number of iterations for the induced magnetic field solver is NMAX times NMAXEM (the induced magnetic field solver is called for each iteration cycle of the Navier-Stokes solver), NMAXEM should be carefully chosen.

In order to allow dimensionalized output variables, the gas constant (GASC) and the reference magnetic field strength were included (both in the SI system of units).

Several input variables were added for the induced magnetic field solver.

AION1	First ionization energy
AION2	Second ionization energy

ATW Atomic weight of the propellant
CURRAMP Total input current
BETAH Hall parameter (if $< 0 \Rightarrow$ calculated)
SIGG Electrical conductivity (used only for BETAH > 0)

The execution of the code is controlled by two variables. IEM determines, if the code is run as

- a) Navier Stokes solver, only (IEM=0)
- b) Navier Stokes solver with induced magnetic field (IEM=1)
- c) Induced magnetic field solver, only (IEM=2)

In addition, NREST2 specifies, if the code is run for the first time (NREST2 = 1), or if a pre-iteration has been performed (NREST2 = 0). When using the code for the first pre-iteration, it is suggested to keep NMAX at 1 and NMAXEM at 500 to 1000, even though other values will work. This combination, however, allows an iteration check for the induced magnetic field and the electron temperature after the first iteration of the Navier Stokes solver, and it allows a fairly good convergence speed between the electron temperature, the transport properties and the induced magnetic field.

The boundary conditions for the induction equation are specified by JTYPEEM and KTYPEEM for J=constant and K=constant lines, respectively. A value of 50 represents an insulator boundary, whereas a value of 30 specifies a conductor boundary. VJ and VK are zero for insulators and anode and one for the cathode. This distinction is necessary, since the code searches for the two path' from cathode to anode along the insulator walls.

JTET and KTET specify the boundary type for J=constant and K=constant walls, respectively. Setting this value to 10 fixes the electron temperature to the input value, given by ETJ and ETK. A value of five determines the electron temperature according to the energy balance

between Joule heating and energy transfer between heavy particles and electrons (equation 68.1). Any other value of ETJ and ETK leads to an adiabatic wall (for the electron temperature).

VITA

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