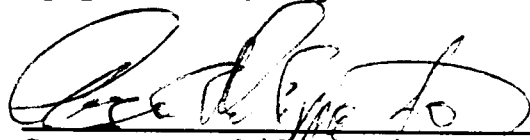


To the Graduate Council:

I am submitting herewith a dissertation written by Aris N. Athanassiou entitled "Heterogeneous Expectations and Common Knowledge Beliefs in Finance." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Finance.



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We have read this dissertation
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Associate Vice Chancellor
and Dean of The Graduate School

H E T E R O G E N E O U S E X P E C T A T I O N S
A N D C O M M O N K N O W L E D G E B E L I E F S
I N F I N A N C E

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

ARIS N. ATHANASSIOU

May 1992

To my father, Nicolas

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iv

that no formal proof can be produced, if not preceded by a deep intuitive understanding of the subject matter.

Aris

ABSTRACT

The assumptions of homogeneous expectations and common knowledge beliefs in Finance are examined. Relaxation of the assumption of common knowledge with respect to all relevant events has a profound effect on financial equilibrium pricing models. Outcomes can no longer be considered as being exogenously determined. They depend on the evolution of beliefs which necessarily follows any exogenous shock associated with non common knowledge beliefs. The recognition of the dependence of outcomes on beliefs-which are more likely to be heterogeneous and non common knowledge immediately after the initial shock-leads to the following view of financial markets. Financial markets can be best described by a dynamic (i.e. disequilibrium) model of the belief adjustment process. If beliefs converge to a common limit, the price of the asset converges to an equilibrium price and a pricing equilibrium is reached.

Pricing equilibria are topologically characterized and are found to be equivalent to common knowledge beliefs with respect to all relevant events. In addition, common knowledge beliefs are shown to imply homogeneous expectations. As a result, pricing equilibria are rational

expectations equilibria, since beliefs at equilibrium are homogeneous and are confirmed by the relevant observables.

A specific model of the belief adjustment process is derived in the form of a system of difference equations. Its attractors (i.e. equilibria) comply to the topological characterization of equilibrium, derived previously. Convergence of the process to equilibrium is not guaranteed but is favored by relatively uninhibited flow of information among investors. Further it is shown that the final equilibrium depends on both the initial disturbance and the belief adjustment process and as a result, financial assets do not have an intrinsic value determined exogenously. The value of a financial asset can only be specified within the particular market in which the asset is traded. Finally, the dynamic model developed is used in order to offer some possible explanations for "crashes" and "bubbles" the Weekend Effect and the Small Firm effect.

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OVERVIEW

The assumption of homogeneous expectations (also referred to as homogeneous priors or homogeneous beliefs) in its different forms constitutes a cornerstone in equilibrium models of modern Financial Theory. By assuming homogeneous expectations financial theorists attempted to solve the problem of aggregating individual rational behavior into a market level equivalent. Thus, the individual behavior model is considered to become the market model when it is summed across individuals.

In financial markets, however, it is collective behavior that determines prices. Each player who is concerned with profit should therefore consider the beliefs of all other players. As a result it has been suggested that a stronger assumption than homogeneous expectations at a specific point in time is needed to obtain the equilibrium pricing formulas associated with the aforementioned models-the assumption of common knowledge beliefs.¹

¹ A formal definition of the term "common knowledge" will be given in essay 2, but for now it can be thought of as the state in which each and every investor knows of an event, knows that all others know of the event, knows that all others know that the investor knows, and so on to infinity.

Homogeneous expectations, and common knowledge beliefs have been regarded as somewhat unrealistic, but necessary for aggregation assumptions, and the related objective was to (at least) partly relax them in an equilibrium framework. The position taken in this dissertation is somewhat different. Rather than attempting to relax these assumptions in an equilibrium framework, it will be shown that common knowledge is a necessary and sufficient condition for a pricing equilibrium. Furthermore, it will be shown that common knowledge implies homogeneous expectations and that the converse is not true. In addition, relaxation of the assumption of common knowledge with respect to all relevant events leads to dynamic representations of the belief adjustment process that need to be studied out of equilibrium. Further, "homogeneous priors/common knowledge" states, are locally stable equilibria of a dynamic process of opinion adjustment which starts from a disequilibrium point of "non common knowledge".

Specifically, the objectives of this dissertation are as follows:

- i) to substantiate the need and importance of the view in terms of the theoretical and empirical limitations of the current "state of the art" theoretical framework;
- ii) to show that common knowledge beliefs imply homogeneous expectations and constitute a necessary and sufficient

condition for any pricing equilibrium;

iii) to present a dynamic (disequilibrium) model of opinion adjustment in order to explore the relation of heterogeneous initial opinions with the final equilibrium pricing; derive the conditions for the existence of such equilibrium; and present the economic interpretation and policy implications of these conditions; and

iv) to offer explanations for some of the phenomena viewed as anomalies under the current "state of the art" theoretical framework.

The above objectives will be accomplished in three consecutive essays as follows: the first objective in Essay 1, the second objective in Essay 2, and the third and fourth objectives in Essay 3.

Essay 1, entitled "Static and Dynamic Views on Heterogeneous Expectations and Common Knowledge", is the motivation behind the two subsequent essays. First a literature review is presented in which the limitations of each particular view are underlined. These theoretical limitations naturally lead toward the view proposed in this work. In the last portion of Essay 1 some well documented empirical anomalies-such as crashes and speculative bubbles, the "weekend effect" and the "small firm" effect-are presented. Some explanation is given as to why and how the model presented in this work helps to resolve these anomalies.

Essay 2 is entitled "Theoretical Characterization of Equilibrium". Here theorems are developed which not only support the conjectures made in the first essay, but are also necessary to characterize equilibrium in the third essay. Specifically, common knowledge is proved to be a necessary and sufficient condition for equilibrium. Further, it is shown that common knowledge implies homogeneous expectations.

In the 3rd and final essay, "A Dynamic Model of the Opinion Adjustment Process", a simple but specific model of the suggested opinion adjustment mechanism is developed and analyzed in order to add resolution to the view proposed in this work. Conditions for convergence are found and it is shown that the equilibrium price depends on both the exogenous disturbance and the opinion adjustment process. In other words, in the final essay we examine the market between the elusive equilibria described by the currently used pricing models². The reason is that if the market often operates out of equilibrium, the study of disequilibrium provides insight and helps explain many of the phenomena considered anomalies (for example the "weekend effect", the "small firm effect", etc.) under the current state of theoretical knowledge.

² Such equilibria are indeed elusive, because as the market moves toward a specific equilibrium, exogenous disturbances such as arrival of new macroeconomic information deflect it towards a new one.

ESSAY 1

Static and Dynamic Views
on Heterogeneous Expectations
and Common Knowledge

1.1.0 Introduction

There have been several investigations concerning the effect of heterogeneity of information (and thus of expectations) on asset prices. For expositional purposes this body of literature is divided in three general classes. The basic assumptions supporting each approach along with a general outline of the associated method are presented. The theoretical limitations of each approach are underlined and the results are compared to the ones obtained later in this work. Further, it is shown how overcoming the theoretical flaws of each approach naturally leads to the view proposed in this work.

Next, the basic characteristics of the model are presented in the form of conjectures to be proven later. Finally, some well documented empirical anomalies are presented and suggestions are made as to how the model presented here might resolve them.

1.2.0 Heterogeneous Expectations Viewed Statically

In the first class of literature reviewed here belong works that view heterogeneous expectations statically and, consequently, overlook the problem of belief interaction^{3,4}. Most of these works have taken place either in the context of CAPM-type mean variance models (Lintner [1969], Miller [1977], Williams [1977], and Jarrow [1980]), or in an Arrow-Debreu contingent claims context (Varian [1985].)

The basic approach is as follows:

Before trading takes place each investor receives a noisy private signal $\tilde{Y}_i$ concerning the distribution of the future value of a given risky asset:

$$\tilde{Y}_i = \tilde{X} + \tilde{\epsilon}_i$$

where $\tilde{X}$ is normally distributed and represents the true distribution of the asset's future value. $\tilde{\epsilon}_i$ has all

³ A precise explanation of what is meant here will be given later.

⁴ This body of literature will be referred to as "static model literature".

"desirable" statistical properties^{5,6}. Then, individual investors solve their private-maximization-of expected-utility-problems given this signal. This exercise yields an expression which relates each individual's demand to the market price of the asset, the expected future value of the asset given his/her private signal, the variance of the asset value given this signal, and his/her degree of risk aversion.

At this point a market clearing condition is imposed and the market clearing equation is solved for market price. This market clearing price is a weighted average of individuals' expectations of the future value given the signals they received minus a risk adjustment factor. Given the specific distributional assumptions made about the signals-specifically that they are normally and independently distributed around "reality"-the market price becomes equal to the certainty equivalent present value of the future uncertain value of the asset.

The weight with which each individual's conditional expectation enters the market clearing price is larger the smaller the degree of risk aversion, and the variance of the

⁵ $\tilde{\epsilon}_i$'s are normally distributed with mean 0 and variance $\sigma_{\tilde{\epsilon}}^2$, and are mutually independent and independent of $\tilde{X}$.

⁶ Note that a true distribution of the asset's future value is assumed to exist independently of the signals received, or the behavior of investors (i.e. learning mechanism.)

future asset value given his signal, ceteris paribus. That is, the less risk averse individual i is, and the more precise his signal, the more heavily his opinion is weighted in the market clearing price⁷. Additionally the risk adjustment factor is smaller the less risk averse individuals are and the more precise their signals (smaller degree of heterogeneity.)^{8,9,10}

⁷ See Varian [1986], Miller [1977], and Lintner [1969].

⁸ This observation points towards a possible explanation for the "small firm effect". If small firms are associated with more uncertainty which suggests that investor expectations about them will be more diverse, then the risk adjustment factor will be higher for small firms. The latter will result in lower current prices and higher realized rates of return if heterogeneity of expectations is lessened with the passage of time, ceteris paribus.

⁹ If non observation of prices for a certain period leads to more heterogeneous opinion structures, then the above results point toward an explanation for the "weekend effect". The reason is that over the weekend there exists a time interval during which all major international exchanges are closed. However, in order to take advantage of the full explanatory potential of the results provided in the first class of literature, one should make multiperiod extensions of the model thus taking into account belief interaction. Development of such extensions along with a detailed discussion of all of the above points is one of the objectives of the Essay 3.

¹⁰ This result is consistent with Varian [1985] and Lintner [1969] but diverges from Miller's [1977] results. Assuming risk neutrality, and limited borrowing and short selling Miller concludes graphically that the asset's market clearing price is determined solely by investors who hold the "highest expectations". As a result, an increase in the spread of expectations which increases the number of investors holding relatively high expectations will increase the price of the asset.

Results pertaining to each individual step of the dynamic model developed in the 3rd essay are in agreement with the above conclusions. However, because of its static nature the basic model derived in the first class of literature is unable to follow the process of opinion adjustment through its steady state and thus some of the most important features of the dynamic pricing equilibrium¹¹ are overlooked.

Although, the properties of the equilibrium described in the first class of literature make intuitive sense, if individuals are rational such equilibrium is not really an equilibrium. The reason is that because the market clearing price takes into account all individual signals, its informational content with respect to the future value of the asset is superior to any particular individual signal. Thus, rational investors after observing the market clearing price are forced to review their expectations and a new trading cycle is produced endogenously.¹²

Equilibrium will be reached after the first market clearing only if investors ignored the informational content of prices. In other words, equilibrium will be reached after

¹¹ Dynamic pricing equilibrium or simply pricing equilibrium is the equilibrium of the dynamic system whose law of motion represents the opinion adjustment process. A formal definition of the term is given in Essay 2.

¹² For a detailed discussion see for example Grossman ['76].

the first trading cycle only if investors failed to recognize that the information reflected in prices with respect to the asset's future value is superior to any particular individual signal. The latter would characterize investors as being irrational. Arbitrage opportunities would exist for a rational investor within that environment. One would simply have to refrain from trading until the market clears and then, before realization of the future value of the asset, form a demand function given the first market clearing price. Since other investors fail to recognize the informational superiority of prices relative to their individual signals, they will ignore the market clearing price, thus retaining their original demand functions unmodified. Subsequently, the rational investor would engage in trading with other investors who are driven by inferior information. As a result, the new trading cycle would place this investor in an advantageous position relative to all others who did not revise their expectations given the first market clearing price. To avoid confusion the generally accepted convention is followed in referring to the market clearing condition mentioned above as intermediate equilibrium, or competitive equilibrium, or simply market clearing^{13,14}.

¹³ Such condition is different than "pricing equilibrium." As will be discussed in detail later, a pricing equilibrium may be reached after a series of market clearings.

¹⁴ Apart from the problem identified above, the approach adopted by the first class of literature contains an even deeper problem. Namely, the fact that a true distribution of the future asset value is assumed to exist independently of investors' actions. This problem is addressed in detail later, since it persists in the next class of literature. Additionally, its resolution is one of the major philosophical foundations of the view of the world proposed in this dissertation

1.3.0 Heterogeneous Expectations Viewed Dynamically with Equilibrium Asset Prices Independent of the Trading Process

In the second class of literature¹⁵ reviewed here the problem of market clearing not implying equilibrium has been clearly recognized (see Grossman [1976], Grossman and Stiglitz [1980], Feldman [1987], and Handa and Linn [1991].) The foundations are put in place by Grossman in his pioneering work "On the Efficiency of Competitive Stock Markets Where Traders have Diverse Information." [1976]. Grossman's solution goes as follows. Suppose investors realize that market clearing prices contain information (as they would if they're rational.) They will now condition their expectations about the future value of the asset not only upon their private signals, but also upon the market clearing price (which is a function $P^*(Y)$ of the vector of signals $Y = (Y_1, Y_2, \dots, Y_n)$.) In the market clearing condition, expectations are now conditioned not only upon private signals, but also upon the

¹⁵ This body of literature will be referred to as "literature which assumes independence of beliefs and outcomes".

market clearing price. In Grossman's [1976, p.576] terms:

Each period traders come to the market with another realization of $\tilde{Y}$, and another $\bar{P}_0(Y)$ is found where the auction stops. After many repetitions traders can tabulate the empirical distribution of $(\bar{P}_0, P_1)$ pairs (P_1 is the future value of the asset.) From this they get a good estimate of the joint distribution of $\bar{P}_0$ and P_1 . After this joint distribution is learned, traders will have an incentive to change their bids just as the market is about to clear. This follows from the fact that if everyone observes that the market is about to clear at $\bar{P}_0(Y)$, they can condition their beliefs on $\bar{P}_0(Y)$ and learn something more about P_1 . This changes their demands and thus the market will not clear at $\bar{P}_0(Y)$. Suppose instead that the market has been clearing for a long time with prices generated by $P_0^*(Y)$ a solution to 12 (i.e., the market clearing condition with expectations conditioned upon clearing price.) Then at any particular time, given that traders come to the market with some Y , if the market is about to clear at $P_0^*(Y)$, and traders then realize that $P_0^*(Y)$ is the equilibrium, they will not change their bids due to the new information they get about P_1 from $P_0^*(Y)$. $P_0^*(Y)$ is a self fulfilling

expectations equilibrium: when all traders think that prices are generated by $P_0^*(Y)$, they will act in such a way that the market clears at $P_0^*(Y)$ ¹⁶.

Subsequently, Grossman proceeds by construction. He assumes that there exist an equilibrium $P^*(Y)$ which is of the form

$$P^*(Y) = \alpha + \beta \bar{Y}$$

where $\bar{Y}$ is the average of all signals received. α is a function of the sum of the inverse of individual risk aversions and the true distributional characteristics of the future value of the asset. β is a function of the true distributional characteristics of the asset's future value. Grossman then proceeds to prove that $P^*(Y)$ is an equilibrium price function, which satisfies the market clearing condition. Additionally, it is unique within the class of all price functions linear to $\bar{Y}$ ¹⁷.

To prove that $P^*(Y)$ as constructed above is an

¹⁶ Note that contrary to the general description given here Grossman assumes that traders can change their bids right before the market clears, which is unrealistic given the mechanics of major world stock markets. Under such regime not even market clearing can be achieved before the final pricing equilibrium is reached. The difference however, is irrelevant to properties of the pricing equilibrium.

¹⁷ The question of whether there exist non linear price functions of $\bar{Y}$ which are also rational expectation equilibria to our knowledge remains unanswered.

equilibrium $P(Y)$ one needs to prove that $\bar{Y}$ is a sufficient statistic¹⁸ for $h_i(Y_i, \bar{Y} \mid \text{future value of the asset})$, where $Y_i, \bar{Y}$ denote realizations of $\tilde{Y}_i$ and $\bar{Y}$ respectively, and h_i is the joint distribution of $\tilde{Y}_i$ and $\bar{Y}$ conditional on the density of the future value of the asset. As Grossman demonstrates, this is true under the distributional assumptions made. Since $\bar{Y}$ is a sufficient statistic for h_i , the density of the future value of the asset, conditional on $\tilde{Y}_i = Y_i$ and $\bar{Y} = \bar{y}$, is independent of Y_i . Since Y_i is the only source of heterogeneity in Grossman's framework and expectations concerning the future value of the asset are independent of Y_i , expectations have become homogeneous. In other words, prices have symmetrized information (i.e., converted heterogeneous expectations to homogeneous expectations) and a rational expectations equilibrium¹⁹ has been reached²⁰. The

¹⁸ Let $\tilde{x}, \tilde{y}, \tilde{z}$ be random variables and let x, y, z denote their realizations respectively. Let $h(y, x \mid z)$ be the joint density function of $\tilde{y}$ and $\tilde{x}$ conditional on $\tilde{z}$. The random variable $\tilde{x}$ is a sufficient statistic for $h(y, x \mid z)$ if there exist functions $g_1(\cdot)$ and $g_2(\cdot)$ such that for all y and z :

$$h(y, x \mid z) = g_1(y, x) g_2(x, z)$$

By applying Bayes' rule to $f(z \mid y, x)$ (the density function of $\tilde{z}$ conditional upon $\tilde{y}$ and $\tilde{x}$) one can directly show that if $\tilde{x}$ is a sufficient statistic for $h(y, x \mid z)$ then $f(z \mid y, x)$ is independent of y .

¹⁹ In this particular context "rational expectations" imply that in equilibrium individuals understand the functional relation between the equilibrium price and the joint signal.

intuition is as follows: Individuals can infer $\bar{y}$ from $P^*(Y)$ because P^* is linear in $\bar{Y}$, and thus, invertible. In addition, since $\bar{Y}$ is a sufficient statistic for h_i , individual investors can infer the distributional characteristics of the future asset value from observed prices without paying attention to their private signal. Price is informationally superior to their private signal. In other words, heterogeneous expectations were transformed to homogeneous expectations by letting each investor observe prices and follow a logical process of opinion adjustment.²¹ Such price systems which symmetrize information by producing prices which are sufficient statistics are called fully revealing price systems. The corresponding rational expectations equilibria are called fully revealing rational expectations (FRRE) equilibria. FRRE equilibria imply not only that prices aggregate all relevant information, but also that investors are able to infer all relevant information from prices. Thus, private information cannot be used to generate excess profits. In addition, no trading strategy can beat the buy-and-hold strategy. FRRE equilibria are consistent with strongly

²⁰ It will be shown later in a more general framework that symmetrization or homogeneous expectations is a necessary condition for any pricing equilibrium.

²¹ In fact in the situation described here, common knowledge rather than simply homogeneous expectations has prevailed. This point will be discussed in detail in Essay 2.

efficient markets²². As stated above, a fully revealing price system makes private information redundant given equilibrium prices. Fully revealing rational expectations equilibria are identical to competitive equilibria in an artificial economy in which investors share their private information.²³

Fully revealing rational expectations equilibria are economically attractive. First, the price system aggregates information efficiently by revealing a price which is a sufficient statistic for individuals' private signals. Thus, private information cannot be used to generate excess profits. Second, equilibrium allocation cannot be improved upon by a central planner who has access to all the diverse information. Thus, FRRE equilibria result in efficient

²² In fact some interpretations of the strong form of the Efficient Market Hypothesis (EMH) which state that current prices fully reflect **all** private and public information, are identical to FRRE equilibria. Unfortunately, however, the strong form of the EMH is not defined very precisely and is subject to interpretation. The key in interpreting the strong form of the EMH is in the interpretation of the term "fully reflect". If the term is taken to imply that prices aggregate all relevant information (including private information) **and** that investors can infer all relevant information from prices, then strongly efficient markets are identical to FRRE equilibria. If however, the term "fully reflect" is interpreted simply as "aggregate" the two conditions are different. This is because the premise that prices aggregate or take into account all relevant information does not necessarily imply that investors can subsequently infer such information from prices. As a result, private information might remain private and consequently investors might use it to generate excess profits.

²³ See Huang and Litzenberger [1988, p.274].

allocations²⁴. Furthermore, fully revealing rational expectations equilibria add robustness to financial models which assume homogeneous beliefs by essentially claiming that heterogeneous beliefs cannot be sustained after investors have tabulated the true relation between the future price of the asset and the joint signal.

However, as Grossman and Stiglitz have pointed out (Grossman [1976] and Grossman and Stiglitz [1980]), FRRE equilibria involve the following paradox²⁵: As the equilibrium conveys information through price which is superior to each individual's private signal, optimal demands are independent of private signals. Optimal demands depend only upon equilibrium prices. But if each individual's demand is independent of his/her private signal how can the equilibrium price aggregate the information contained within all private signals. Furthermore, if information is costly it is irrational for any individual to collect information. And, if individuals do not collect information, there are no diverse beliefs to be aggregated by equilibrium prices in the first place.

The resolution proposed by Grossman and Stiglitz [1980] is the following: Markets can only be partially efficient -

²⁴ See Huang and Litzenberger [1988, p.273-5].

²⁵ This paradox is commonly referred to as the "classical conundrum" of efficient markets.

price system reflects information only partially-in the sense that price is not a sufficient statistic for an individual's private signal. In other words, even if all information is taken into consideration by the price formation mechanism, investors cannot infer all relevant information from prices only. In this case, an individual's demand for the asset in question will depend not only on price information but also on his/her private signal. Further, if the information of all individuals is only partially reflected in prices then there exists an incentive to collect information even at a cost. But under what conditions will price fail to be a sufficient statistic? Note that in the FRRE equilibrium considered there exists only one source of uncertainty, namely the future value of the risky asset. Individuals receive signals and the equilibrium price system reveals a sufficient statistic about individuals' private signals. The price system is fully revealing because it is a linear function of the sufficient statistic, and thus, invertible. When the equilibrium price of the risky asset increases (decreases) each individual can infer that the demand for the risky asset increased (decreased) because individuals have received bullish (bearish) signals on the average. Suppose however, that there exists an additional source of uncertainty, namely the supply of the risky asset. In such a case each individual will not be able to understand whether a price change occurred because all

other individuals received new signals, or because there was a change in the supply of the risky asset. If for example the price of the risky asset increased, investors could attribute the increase to either good news concerning the asset, or to a decrease in the supply of the risky asset. Consequently, each investor would be willing to acquire information, even at a cost, in order to identify the source of the price increase. In this case the information revealed by the price system is not superior to individual signals and thus the price system is not fully revealing. Now, individuals will be willing to buy information (i.e., private signals) at a cost, in order to benefit in the future. However, if all investors choose to become informed the price system would be again fully revealing and the paradox would reemerge. Instead, the proportion of investors who will choose to become informed will be such that the price system is informative to the point of offering an excess return to informed investors equal to the costs of information. As a result the proportion of informed investors will be endogenously determined^{26,27}.

Fully revealing and partially revealing rational

²⁶ For a concise presentation see Huang and Litzenberger [1988, p.278].

²⁷ One should note that the assumption of uncertain supply is not the most natural in a financial context. After all, the supply of stocks as used above is the number of shares outstanding, and thus is common knowledge.

expectations equilibria as described above imply that after investors reach rational expectations, that is, when they learn about the true functional relation between equilibrium prices and signals, markets remain in equilibrium continuously. This premise adds robustness to financial models which assume homogeneous expectations, because all of the above classes of models end up being observationally equivalent^{28,29}.

The main conclusions obtained in the second class of literature reviewed in this essay are in strong disagreement with the results derived in this dissertation. In the third essay it will be shown that equilibrium depends on the trading process, and thus, there does not exist a unique equilibrium price, given the initial signals. Uniqueness is guaranteed only after both the initial signals and the trading process are specified. As a result, the assumption that initial signals are unbiased estimates of the ultimate equilibrium is unfounded because the ultimate equilibrium cannot be specified ex-ante. In addition, it is shown that convergence to equilibrium is not guaranteed.

The above differences are due to the relaxation of one

²⁸ In the case of PRRE equilibria the equivalence holds up to a constant, determined among other things by the cost of information.

²⁹ See Huang and Litzenberger [1988, p.276].

essential assumption shared in the second class of literature—namely, that the true distribution of asset prices exists independently of the trading process. Thus it can be specified ex-ante (i.e., independence of outcomes and beliefs.) Indeed, if such distribution exists, then the results obtained in the second class of literature hold³⁰. However, the above assumption is invalid, as will be shown in Essays 2 and 3.

³⁰ If the true distribution exists and does not vary with time, it should be expected for individual beliefs to eventually zero-in on it.

**1.4.0 Heterogeneous Expectations Viewed Dynamically
with Equilibrium Asset Prices Depending
on the Trading Process**

Before examining the third class of literature in this section the following question is addressed, which arises naturally given the results obtained in the second class of literature, reviewed previously. If heterogeneous expectations and rationality always result in PRRE equilibria which are observationally equivalent, up to a constant, to markets of homogeneous expectations, why depart from homogeneous expectations assumptions in the first place if the additional modelling complications do not improve our understanding of financial markets? Rephrasing the question, why is an alternative view such as the one which will be proposed in this work needed? What are its advantages relatively to the view shared in the second class of literature? The answer to these questions constitutes the core of the motivation behind the view of the world which will be exposed within this essay and substantiated in the following two essays.

1.4.1 Dependence of Outcomes on Beliefs.

First, consider the paradox associated with FRRE equilibria. The resolution proposed offers a non paradoxical alternative, but does not really resolve the paradox which runs at a much deeper philosophical level. After all, if one starts with non-contradicting assumptions and proceeds through an internally consistent mathematically correct path, how can one end up with a paradox? If that happened the final paradox would indeed constitute proof that the initial assumptions were paradoxical. Simply put, the paradox associated with FRRE equilibria proves by contradiction that the initial assumptions on which FRRE equilibria are based are internally inconsistent.

Specifically, it will be argued that the assumption that there exists a real ex-ante specified distribution of future returns (specified before individuals receive their signals), and the assumption that each investor realizes that prices depend upon all other investor's beliefs (and thus, has an incentive to attempt to predict them) are contradictory. The reason is that the objective distribution of asset value depends upon investors' subjective beliefs about the distribution³¹.

³¹ As a simple example, suppose that an insignificant portion of investors receives erroneous negative information concerning a risky asset. Suppose further that the rest of the investors (the vast majority) knows of the erroneous signal but does not know what proportion of investors received the erroneous signal. The second portion of investors (investors

As will be shown in the two subsequent essays, relaxing the assumption that there exists a real, ex-ante specifiable distribution of future asset return, or alternatively recognizing that the outcome of an investor's trading activity is not determined solely by factors exogenous to the trading process (i.e., exogenous shocks), but by endogenous factors as

who did not receive the negative signal) should recognize that mistaken investors might have an effect on the market, and thus should try to estimate their opinion structure. Given their estimation they will adjust their expectations concerning the future value of the asset; and so on; The system might or might not reach an equilibrium eventually, but the potential equilibrium price will in general differ from the equilibrium price that would have emerged if the proportion of investors who received the erroneous signal were common knowledge in the beginning of the process. Additionally, what was considered to be an erroneous signal initially might end up being correct given the belief interaction that took place during the opinion adjustment process. This example is in accordance with Soros' [1987] description of his own investment strategy. Soros has apparently been successful over the last twenty years by implementing, he claims, a strategy not based on fundamentals but on future crowd behavior. For example, he believes that in the 60's a number of poorly informed investors became excited about rises in the reported annual earnings of conglomerates. The truly informed and rational investor in this case, says Soros, would not sell short in anticipation of an eventual collapse of conglomerate shares, but should instead buy in anticipation of further buying by uninformed investors. The initial rise in conglomerate stocks, due in part to rational investors like Soros, stimulated further uninformed investors by creating an increasing trend in prices, and allowed conglomerates to report further earnings increases through acquisitions. Eventually prices collapsed, but the collapse did not come until 1970.

Indeed, would it not be irrational for the informed investor possessing such information to give up the opportunity of making a profit? And further, would not such rational behavior give a further boost in prices, thus proving poorly informed investors not so poorly informed?

well (i.e., the trading activity of other investors) leads to the following view: **Financial markets can be best described by a dynamic system whose laws of motion are given by specifying the traders' learning behavior. If beliefs converge to any common limit, the above process converges towards an equilibrium which is a rational expectations equilibrium. Rational expectations equilibria are locally stable equilibria of the above process. At any time t before reaching equilibrium there exist incentives to collect information (not only prices, but also other relevant information such as trading volumes). Such information will help them form more accurate estimates about the near future. At any off-equilibrium region private information is taken into account even if it is costly and this is the way it gets incorporated into the final equilibrium.**³²

In the third class of literature³³, reviewed here, the dependence of outcomes on beliefs is partially, or fully

³² The resolution proposed here for the "classical conundrum" is different from the resolution proposed by Grossman. Grossman retained the initial assumptions, and by assuming in addition that there exists a source of uncertainty other than the uncertainty associated with the future value of the asset, he concluded that markets are continuously in a PRRE equilibrium as opposed to being continuously in a FRRE equilibrium. On the other hand, we proposed that the only way to truly resolve the paradox is to dispose of the internally inconsistent assumptions which led to it.

³³ This body of literature will be referred to as "literature which recognizes the dependence of outcomes on beliefs."

recognized (see Blume and Easley [1982], Figlewski [1978], Harrison and Kreps [1978], Anderson and Sonnenschein [1982], and Bray [1982].) Blume and Easley [1982] touch upon the problem when they argue (p. 341) that:

...fully rational learning would require each trader to take into account the effect of his learning, and the learning of others on equilibrium prices. However, the necessary structure for such an analysis requires a degree of sophistication on the part of traders that seems highly implausible.

But if one recognizes the effect of learning on the relation between signals and prices one will be forced to recognize that not even the fully rational model³⁴ need be correct out of equilibrium. Thus the question is not simply one of "degree of sophistication on the part of traders"³⁵. **Even fully rational investors adept at Bayesian learning will not be able to infer the correct relation between signals and prices because such relation is not correct out of equilibrium. Instead, they will infer alternative relations**

³⁴ What Grossman and Stiglitz refer to as the correct relation between signals and prices.

³⁵ Blume and Easley [1982] recognize the depth of the problem since they make similar arguments later.

which, if they converge, will ultimately reveal an alternative equilibrium relation. This relation will depend on the initial signals and the learning process which in its turn depends upon the mechanics of the market (for example, trading restrictions, form of the auction process, or the learning process in general.) The equilibrium relationship between signals and prices will be a self-fulfilling prophecy. It will be correct simply because everyone believes it to be correct. Thus, a predetermined distribution for the future value of the asset independent of the initial signals and the learning process does not exist³⁶. Bray [1982] has clearly recognized this point:

...the difficulty is that in many models of rational expectations equilibria, including that of Grossman and Stiglitz [1976], the objective distribution of variables depends upon agents' subjective beliefs about the distribution. Outside a rational expectations equilibrium the objective distribution of variables differs both from agents' subjective beliefs and from the objective distribution which would prevail in a rational expectations equilibrium. Agents may learn about the

³⁶ This premise is in accordance with the implications of propositions 1 and 2 of the following essay and will be formally proved in Essay 3.

relationship between variables, given their current beliefs. However, when they modify their beliefs in light of what they have learned, the relationship changes. Agents may attempt to learn about the relationship using Bayesian or classical statistical techniques which are based on a correct specification of the rational expectations equilibrium. In doing so they fail to take into account the dependency of outcomes on beliefs. Their estimation technique is based on misspecification of the situation³⁷. (pp. 318-9)

Although results derived in the two subsequent essays are generally in agreement with results in the third class of literature³⁸, it should be noted that works belonging to the third class of literature do not include a theoretical characterization of equilibrium, analogous to the 2nd essay in this work. One of the implications is that the corresponding models allow for non-rational expectations equilibria. Such equilibria have no economic meaning³⁹.

³⁷ A detailed elaboration on these points will be presented in the third essay.

³⁸ Specific results and comparison of works belonging to the third class of literature will be discussed in the 3rd essay.

³⁹ It will be shown in the following essay that all pricing equilibria are rational expectations equilibria. The reason is that if a rational investor's beliefs are not

1.5.0 Empirical Implications of the Proposed View

Apart from its theoretical basis the view proposed in this work would be little more than an intellectual exercise if it were devoid of potential to explain observed reality better. As noted above, the view proposed by Grossman [1976], and shared in the second class of literature, generates results which are observationally identical to the results of models where investors share homogeneous expectations at all times (which is equivalent to common knowledge⁴⁰). But the high tractability of the results is not costless.

confirmed in equilibrium then the investor in question will be driven to modify his beliefs, thus, disturbing equilibrium (see Feldman [1987].)

⁴⁰ It will be formally demonstrated in the next essay that homogeneous posterior expectations at all times is an outcome of common knowledge beliefs. If an event is not common knowledge, expectations, even if homogeneous initially, will be revised endogenously as more information about the opinion structure is revealed by consecutive trading cycles. The revised expectations will in general be heterogeneous. However, if expectations are forced by assumption to be homogeneous at all times then the effect of non-common knowledge on the future is ignored. In essence, assuming homogeneous expectations at all times leads to models which are equivalent to common knowledge models for all practical purposes.

1.5.1 Trading Volumes.

First consider trading volumes. In a world blessed with common knowledge by assumption, as in the case of homogeneous expectations static models, and in a world crowned with common knowledge by construction, as in the case of FRRE equilibria,^{41,42} there exists no theoretical basis for trading⁴³ unrelated to consumption. In FRRE equilibria an event (represented by the signals received) is not common knowledge but prices are. After investors have tabulated the true equilibrium relationship between prices and signals, events are logical consequences of prices. Thus, they are common knowledge (see Milgrom [1981].) As a result, events provide no basis for trading. Trading in such models can only have a consumption basis. However in the real world, daily trading volumes are too high to be the sole result of emerging consumption needs. For example, consider the case of a mutual fund. The current consumption needs of its owners are not directly linked to the composition of the fund. If consumption were the sole reason for trading, then the manager of the fund

⁴¹ PRRE equilibria offer a basis for trading between informed and uninformed traders only immediately after an exogenous disturbance.

⁴² For a theory of trading volume based on heterogeneous beliefs see Karpoff [1986].)

⁴³ It will be shown in the next essay that if an event is common knowledge then no trading will take place as a result of the exogenous disturbance represented by the event.

would trade in the securities composing the fund only as a result of trading in the shares of the fund. If an investor sold a share of the fund for $\$X$ and another investor bought two shares for $\$2X$, and no other transactions in the shares of the fund took place, then the manager would buy into the securities composing the fund for a total of $\$X$ (retaining the proportional composition of the fund.) Thus the volume of trading induced by the fund manager in the securities composing the fund should certainly be less than the volume of trading in the shares of the fund⁴⁴. This is not the case especially for funds addressed to investors with long term investing horizons.

Apart from the trading volume problem, several stock market anomalies have been documented in terms of the existing theoretical framework. The model presented here provides a resolution for some of these empirical anomalies.

1.5.2 The "Small Firm" Effect.

Reinganum [1981], Banz [1981], and others have identified a negative correlation between common stock returns and the aggregate market value of corresponding firms. Banz adjusts

⁴⁴ In the presence of transactions costs the manager of the fund would have an incentive to not continuously balance trading in the shares of a fund by trading in his components. The result however, would certainly hold over longer periods of time.

for risk, and finds that small firms produce larger risk adjusted returns than large firms. In addition, the effect appears robust even after adjusting for earnings yields of the securities (see Reinganum [1981]), or risk measurement effects associated with market values (see Reinganum [1982]). Keim [1983], Reinganum [1983], Constantinides [1984], Carleton and Lakonishok [1985-86], and others consider other explanations of the firm size effect. However, the phenomenon remains an anomaly.

The view proposed in this work, as well as the static view shared in the first class of literature reviewed in Essay 1, suggest an explanation for the "small firm effect" based on informational arguments. If small firms are associated with greater risk and less frequent trading, then it is likely that they are associated with more heterogeneous opinion structures, as well as with greater intrinsic uncertainty⁴⁵ that each investor faces with respect to the beliefs of other investors. Results provided by the first class of literature suggest that the risk adjustment factor is greater for firms associated with relatively higher heterogeneity in

⁴⁵ A formal definition of the term "intrinsic uncertainty" or "intrinsic risk" will be given in the second essay, but for now, it can be thought of as uncertainty concerning the opinion of others. On the other hand, "extrinsic uncertainty" or "extrinsic risk" is the risk associated with the direction and amplitude of exogenous disturbances.

expectations. Thus, the price of the risky asset is lower. If additional trading cycles result in reduction of the intrinsic uncertainty concerning the opinion of others and reduce heterogeneity, then "small firms" should be associated with higher average returns. Looking at the issue from a different but equivalent point of view, "small firms" produce higher returns on the average, because they are associated with higher intrinsic risk as opposed to higher extrinsic risk. Beta and other similar measures of risk cannot by construction reflect intrinsic risk and thus, returns for small firms appear abnormally high. In other words securities for which there is relatively little information available may be perceived as riskier than securities for which more information is available. This occurs because each investor is faced with greater uncertainty regarding the opinion of others (and thus the future value of the security) with respect to low information securities. Commensurate with that risk, investors require a premium to hold low information securities. If however, risk is measured in a CAPM framework it will appear that low information securities are associated with abnormal returns. To the extent that low market value securities have less information available it follows that in a CAPM framework "small firms" will appear to earn abnormal returns.

Empirical evidence that supports arguments of this type

is presented by Barry and Brown [1984]. They find that period of listing partly explains the "small firm" effect. Their purpose is to investigate the effect of sample length (i.e., period of listing) associated with different firms and not the effect of different degrees of heterogeneity in expectations and associated intrinsic uncertainty. Never-the-less, their results support the view presented here because longer periods of listing are likely to result in more homogeneous opinion structures if the dynamic system of opinion adjustment converges. However, short periods of listing are certainly not the only source of high heterogeneity and higher associated intrinsic risk. Cragg and Malkiel [1982] study the relation between ex-post returns and various measures of risk for common stocks and find that the measure of risk which performs best in their analysis is a measure of divergence of opinion about the asset returns. This empirical evidence is in strong support of the above arguments.

Theoretical results presented in the second class of literature reviewed in this essay, diminish considerably the potential of the first class of literature to explain the small firm effect for reasons presented earlier. On the other hand, the view of the world presented here, reinstates the potential of the first class of literature to explain the "small firm" effect. The reason is that heterogeneous expectations are sustained out of equilibrium. Thus, all of

the arguments made concerning the small firm effect are consistent with the view of the world in this dissertation. In addition, because of its dynamic nature, the model presented here has the ability to not only explain the effects of intrinsic uncertainty but also identify the effects of its reduction and the conditions under which such reduction will take place. As an example consider two identical firms A and B and a market-wide event relevant to both. Opinions however, concerning the exact effect of the event on firms A and B are heterogeneous. Suppose now that a particular investor knows with certainty the opinion of others at each and every point in time concerning the effect of the event on firm A but not on firm B. Then the investor knowing that the opinion of other is relevant in determining each firm's future price will prefer to invest in firm A ceteris paribus. Since all investors think similarly, firm A will have a higher current price than firm B. If eventually the effect of the event on firms A and B becomes public information⁴⁶-opinion structures

⁴⁶ Note that in general there are no exogenous events that turn the effect of previous events into public information. The process is rather endogenous. As will be proved in the third essay, opinions concerning firm B in the example might converge to a different equilibrium than for firm A, since investors do not know that the two firms were identical as hypothesized in the example. Perceived differences become real differences. In fact, the distinction between "real" and "perceived" in our world is artificial. In deed, the two firms of the example were really different at least in one respect: the way they were perceived.

have converged to a state of common knowledge-then the two firms will end up with the same price. Owners of firm B however, would have realized a higher return.

1.5.3 The "Weekend Effect"

French [1980] observed that returns on a portfolio composed of the 500 largest firms on the New York Stock Exchange were significantly negative on Mondays over the period 1953-1977⁴⁷. They were also significantly negative in each of the five year subperiods studied. Although the phenomenon was too weak to be directly exploitable by the general public through a trading rule, (transactions costs of .25% of the transaction value would eliminate all profits) it may still be considered an anomaly given the current theoretical framework.

The reason is that there exist market participants such as stock specialists who face transaction costs of less than .25% of the transaction value. In addition, investors who were going to trade anyway, have an incentive to delay purchases planned for Friday until Monday and execute sales planned for Monday on the preceding Friday. At present there exists no theoretical justification for the phenomenon which came to be

⁴⁷ For additional evidence on the "weekend effect" see for example Lakonishok and Levi [1982] and McInish and Wood [1985].

known as the "weekend effect".

The model presented here provides an explanation for the weekend effect because it suggests that prices behave differently after periods of non-observation. Over weekends there exists a time interval during which all major stock exchanges of the world are closed. As a result, each individual, as markets open on Monday, is faced with greater intrinsic uncertainty concerning the effect of events that took place over the weekend. Such increase in uncertainty would result in lower prices *ceteris paribus*. In addition, the proposed explanation suggests that the longer the period of non-observation the more pronounced the weekend effect will be. With the passage of time as new major exchanges in different time zones begin to operate the time gap between Friday closing and Monday openings is likely to diminish. Consequently, the explanation offered here for the "weekend effect" suggests that the effect is less pronounced in more recent periods. Empirical evidence which supports the above is provided by Lakonishok and Levi [1982]. They find that the negative return on Mondays is more substantial in the period 1968-73 (-0.208%) than in 1974-79 (-0.020%).

1.5.4 "Crashes" and Speculative "Bubbles"

As in the case of the crash of 1987 there have been instances where no event can be identified as the cause of

drastic price movements, usually referred to as crashes or speculative bubbles⁴⁸. Up to now no satisfactory explanation has been given to the most notable of these instances, the crash of Monday October 19, 1987.

The view proposed in this work provides an explanation for "crashes" and "bubbles", because it accounts for price movements which are not due to exogenous disturbances. As each player strives to learn the opinion of others, the dynamic system representing the opinion adjustment mechanism on the aggregate might exhibit cyclical or chaotic⁴⁹ behavior, or simply converge to a distant equilibrium for some parameter ranges. Thus, according to this view of the world, price movements-sometimes even drastic ones-are not always associated with specific exogenous disturbances (see DeLong, Shleifer, Summers, and Waldmann [1990].) In support of the above arguments is the empirical evidence presented by Shiller [1981]. He finds that different measures of stock price

⁴⁸ For theoretical results and empirical evidence on "crashes" and "bubbles" see De Bondt and Thaler [1985], Kindleberger [1978], Leland and Rubinstein [1988], Shiller [1988], and Smith, Suchanek and Williams [1988].

⁴⁹ A chaotic system can be defined informally as a non linear deterministic dynamic system which displays sensitivity to initial conditions. In other words, small changes in the initial conditions are greatly magnified as time evolves. A chaotic system yields time-series which are indistinguishable from random ones, at least through standard statistical tests. For a formal definition of chaos see for example Devaney [1990].

volatility over the past century appear to be far too high (5 to 13 times too high) to be attributed to new information that has a bearing on future real dividends. He notes (p. 434) that "The failure of the efficient market model is thus, so dramatic that it would seem impossible to attribute the failure to such things as data errors, price index problems, changes in the tax laws." In addition, controlling for changes in interest rates does not alleviate the phenomenon.

1.5.5 Some Additional Empirical Support

Further empirical support for the model presented in this work is provided by Martell and Philippatos [1974]. The authors test the Martingale hypothesis for the commodity futures market using two adaptive control filter type security trading models. Contrary to other authors, who used trading rules (i.e., filter sizes) that remain invariant across the trading horizon, Martell and Philippatos allow the trading rule to change in response to new information, based on the elasticity of price expectations. The adaptive mechanism governing the filter size changes rely on information theory and control theory concepts, thus, allowing for feedback control and continuous monitoring of the effects of various decision rules on the objective function (i.e., profit in the first model, and information gains in the second) independent of the determinants of change in the process. The authors find

that both models produce higher net profits with less risk than the "buy-and-hold" strategy and thus the Martingale hypothesis is not supported empirically.

The view of the world presented here provides for a model that explains such empirical evidence⁵⁰. Note that if one selected equilibrium prices only, excluding all intermediate equilibrium prices (i.e., market clearing prices prevailing before a pricing equilibrium is reached) then such prices would conform to the Martingale hypothesis. Thus, in our framework the question of "market efficiency" is one of "speed of adjustment of a market stimulated by information." Martell and Philippatos recognize this fact and designed their tests accordingly. This is the reason that the empirical methodology introduced in Finance by the above authors constitutes an appropriate methodology for testing the theoretical model presented in this work.

⁵⁰ Several statements made by Martell and Philippatos show that the theoretical model that they may have had in mind when designing their empirical methodology was very similar to the one presented here. They note (p. 497) for example that "this equilibrium state (pricing equilibrium) will prevail when we have extracted all possible information in the system."

1.6.0 Concluding Remarks

In this section the view proposed in this work is summarized and some of its implications are as follows.

Financial markets can be best described by a dynamic system whose laws of motion are given by specifying the traders' learning behavior. If beliefs converge to any common limit the above process converges towards an equilibrium which is a rational expectations equilibrium. Rational expectations are locally stable equilibria of the above process.

The basic differentiating elements of the above view with respect to the mainstream financial literature reviewed in this essay are:

1. The outcome of an investor's trading activity is not determined solely by factors exogenous to the trading process, but by endogenous factors as well, namely, the trading activity of other investors.⁵¹

2. Rational individual behavior may lead to non convergent aggregate behavior.⁵²

⁵¹ This premise is formally demonstrated in Essay 3.

⁵² This premise is formally demonstrated in essay 3.

3. Any state with identical beliefs is an equilibrium for the above process. Further any such equilibrium is a rational expectations equilibrium because it is a self fulfilling prophecy. As a result there does not exist a unique equilibrium for the dynamic process of opinion adjustment.⁵³

To this point, a dynamic approach is postulated-rather than postulating equilibrium and deriving relations between risk and return-as appropriate for describing a world with imperfect information⁵⁴. In addition, the questions of "why is such work important, and how does it add to our current knowledge" are addressed. The answers have basically two aspects, a theoretical aspect and an empirical one.

First, rational behavior does not guarantee convergence towards a unique equilibrium and common knowledge. In addition, the current state of the art theoretical paradigms involve some internal inconsistencies.

Second, the assumptions on which current equilibrium models are based have some serious implications-other than offering modeling convenience-which are inconsistent with observed reality. Several financial market anomalies have been

⁵³ These premises are formally demonstrated in the second and third essays.

⁵⁴ A world with imperfect information is in this case a world where heterogeneous signals are received and not shared, or more concisely a world not blessed with common knowledge at all times.

documented, but not necessarily explained fully in terms of the currently accepted theoretical framework such as CAPM and APT. Given the theoretical framework proposed here, some of these anomalies-for example the "crashes" and "bubbles", the Weekend Effect, the Small Firm Effect, and rejection of Martingales for describing return time series-may become "normal phenomena".

ESSAY 2

Theoretical Characterization of Equilibrium

2.1.0 Introduction

In this Essay the necessary mathematical formalism is developed. Theorems are proved in order to substantiate the conjectures made in the previous section concerning some of the properties and characteristics of the proposed view of the world.

Pricing equilibrium is shown within the introduced formalism to be essentially the state in which the beliefs of all individuals are supported by the observed time path of the relevant variables. Pricing equilibria differ from the Marshallian type of equilibrium of supply equals demand in that they encompass it but are not limited to it⁵⁵. Financial markets may remain in disequilibrium even after they clear. The reason is that in financial markets the source of disequilibrium is not related to the inability of markets to

⁵⁵ The Marshallian type of equilibrium is a necessary, but not a sufficient condition for a pricing equilibrium as defined here. As explained later in this essay, the reason is that the market clearing conditions might reveal information not held by all investors at the time they formed the demands which led to such market clearing conditions. Such new information will lead investors to modify their demand and enter a new trading cycle and so on to the point where no information is left to be revealed by a new trading cycle (e.g. pricing equilibrium.)

clear. After all, in financial markets, prices are assumed to be fully flexible and thus, a Keynesian-type disequilibrium such as downward sticky wages causing excess supply of labor, cannot be substantiated.

The source of disequilibrium in financial markets is the intrinsic uncertainty about the opinion structure which is reduced as consecutive market clearing conditions are revealed. **Equilibrium is reached only when all relevant information (concerning the belief structure) has been extracted from the system through consecutive trades and observation of the corresponding market clearing conditions.** Such equilibrium states where all activity ceases, until further exogenous disturbances are imposed on the system, are shown states of common knowledge. Further, it is demonstrated that the only potential equilibria are rational expectations equilibria. This general characterization of equilibrium is necessary in the third essay where a specific dynamic model representing the opinion adjustment mechanism is developed.

2.2.0 Definitions and their Interpretation

Let the unrevealed state of the world be $\omega \in \Omega$, where "cardinality of Ω " $< \infty$ ⁵⁶. Let the i th player's private information be described by a partition P_i on Ω ⁵⁷, $\forall i \in \{1, 2, \dots, n\}$. Let $P_i(\omega)$ be the element of P_i which contains ω and let each player receive a signal which reveals to him $P_i(\omega)$. In other words each player receives an ex-ante signal $P_i(\omega)$ which tells the player initially which element of P_i contains ω , but nothing more⁵⁸.

2.2.1 An Event.

Any event can be represented by a group of states. In other words, the nature of an event specifies the states which might be revealed.

⁵⁶ In other words, the number of states is assumed to be finite.

⁵⁷ For some theoretical and intuitive background on informational set partitioning see Appendix 2.1,

⁵⁸ An implicit assumption made in the common knowledge literature and maintained in the subsequent derivations is that individual signals cannot contradict each other. In other words, no pair of signals $P_i(\omega)$ and $P_j(\omega)$ can point to elements of the partitions P_i and P_j , respectively, which have null intersection.

Definition 1: An event is a closed set A such that $A \subseteq \Omega$.

The above definition is consistent with an Arrow-Debreu framework but is more general than its usual form, in the sense that several events may be associated with each state of nature and an event can transcend several states. For example, if payoffs are relevant events, in an Arrow-Debreu framework each possible payoff is usually associated with one state of nature. Here the states of nature are finer, in a lattice theory sense⁵⁹. Thus, a specific payoff may spread across several states.⁶⁰

2.2.2 Homogeneous Expectations.

Let Π be a probability measure on Ω . Then each player can associate with any event A a probability $p_i^1(A)$ which he will condition on his private signal after the signal is received. For example, if $A \cap P_i(\omega) = \{\emptyset\}$ then for player i , $p_i^1(A | P_i(\omega)) = 0$. The probability $p_i^1(A)$ conditioned upon whatever information player i possesses represents the i th player's

⁵⁹ The elements of lattice theory necessary in the remainder of this essay are presented in Appendix 2.1.

⁶⁰ The difference is of no importance if relevant events are mutually exclusive, for example possible payoffs. On the other hand, if events are not mutually exclusive, for example possible payoffs along with macroeconomic information, the difference becomes important. Since in the real world relevant events are not mutually exclusive, the approach adopted here is more appropriate for describing real events.

expectations or 1st order beliefs or prior beliefs with respect to A. Players are said to have homogeneous expectations if they attribute the same probability to all events.

Definition 2: Players 1, 2, ..., n are said to have homogeneous expectations or common priors or common first order beliefs if and only if for any event A such that $A \subseteq \Omega$, $p_i^1(A) = p_j^1(A) \forall i, j \in \{1, 2, \dots, n\}$.

2.2.3 Common Knowledge

Before introducing the working definition for common knowledge a number of definitions pertaining to a set of two players is introduced in order to provide some intuition to the concept of common knowledge.

Even if players i, j have common priors, the first order beliefs of player i are not generally known by player j and vice versa. Let the second order beliefs of player i, $p_i^2(A)$ be the ith player's opinion about the first order beliefs of player j. Equivalently, we can define 3rd, and higher order beliefs. If player j knows the probability that player i attributes to an event then player j is said to know the 1st order beliefs of player i. In this case (definition 3), player j faces no uncertainty concerning the 1st order beliefs of player i.

Definition 3: Player j is said to know $p_i^1(A) = a_i$ if and only if $p_j^2(A) = a_i$.

If player i knows that player j knows the probability that player i attributes to an event, then player i is said to know the 2nd order beliefs of player j , and player j knows the 1st order beliefs of player i . In this case (definition 4), player i faces no uncertainty concerning the correct beliefs of player j about the i th player's first order beliefs.

Definition 4: Player i is said to know player j knows $p_i^1(A) = a_i$ if and only if $p_i^3(A) = p_j^2(A) = p_i^1(A) = a_i$.

If player j (i) knows the probability player i (j) attributes to an event, and player i (j) knows that player j (i) knows the probability player i (j) attributes to an event, and ...⁶¹, then first order beliefs concerning an event are common knowledge for players i, j . In this case (definition 5), all uncertainty player i (j) faces with respect to the beliefs of player j (i) has been removed.

Definition 5: First order beliefs concerning an event A are common knowledge for players i, j if and only if $p_{j(i)}^{2n}(A)$

⁶¹ "... " signifies an infinite number of "knowledge layers".

$$= p_{i(j)}^{2n-1}(A) = a_{i(j)}, \quad \forall n \in I_+.$$

In definition 5 above all intrinsic uncertainty has been removed. As Guth and Philippatos [1988] note, in financial markets investors' collective behavior determines prices. Investors who are concerned with gains should therefore, consider beliefs of other investors, thus forming 2nd order beliefs. But if second order beliefs are important, investors should consider the second order beliefs of other investors, thus forming 3rd order beliefs; and so on to infinity. Thus, "intrinsic uncertainty" can be defined as the uncertainty associated directly with 2nd and higher order beliefs, and which interacts indirectly with and increases uncertainty associated with 1st order beliefs. Investors face uncertainty not only about "extrinsic" factors governing asset returns, but also face intrinsic uncertainty over the belief structures of other investors. The reason again, is that prices are determined endogenously in financial markets and thus, investors should take into account the market's collective behavior. The state at which all intrinsic uncertainty has been removed from the system is called a state of common knowledge.

Let R be the joint (finest common coarsening)⁶² of

⁶² For a definition of these terms, intuitive explanations, and examples see Appendix 2.1.

partitions $P_1, P_2, \dots, P_n$, and $R(\omega)$ the element of R which contains ω ⁶³. R represents the information held by a hypothetical player who is as informed as the least informed player, but not less. If an event contains the signaled element of R (i.e., $R(\omega)$) then this hypothetical player can assess with certainty that A will occur and A is common knowledge.

Definition 6⁶⁴ (Aumann [1976]): An event A is common knowledge among players 1, 2, ..., n if $R(\omega) \subseteq A$.^{65, 66}

If common knowledge does not hold, as the trading process reveals information and players attempt to determine the

⁶³ $R(\omega)$ is the element of the joint which contains all individual signals. If individual signals are not contradicting each other then the definition of the joint guarantees that all individual signals are contained within one and only one of its elements.

⁶⁴ Definitions 5 and 6 are equivalent if in definition 5 $a=1$, or if the 1st order beliefs about the event in definition 5 are taken to be the event in definition 6. For a demonstration see Aumann [1976, pp. 1236-7]. Definition 6 will be used as the working definition for the remainder of this work.

⁶⁵ Note that Aumann in his original definition used the word "meet" to describe "joint" in lattice theory terms.

⁶⁶ Common knowledge as defined here is the ultimate equilibrium emerging from a DeGroot [1974] type convergence of player's beliefs. A DeGroot process is a process in which each member of a hypothetical jury trusts the opinion of all other members of the jury and takes them into account when deciding on his/her own vote.

belief structure, players affect their own beliefs and consequently their opinions concerning the asset's value. As a result prices change endogenously until the ultimate pricing equilibrium of common knowledge has been reached (if the particular process converges to equilibrium.)

Guth and Philippatos [1988], using specific financial paradigms demonstrated that the assumption of homogeneous expectations is not sufficient for the derivation of the above equilibrium models. In their own words

It turns out that "homogeneous beliefs" assumptions still allow investors to face intrinsic uncertainty because while the members of the market may (objectively) agree in their beliefs, they would not typically know a priori that they share the same beliefs. In short, the agents may agree without knowing they agree. (p. 730)

As noted above, intrinsic uncertainty will lead to individual opinion adjustments which will generally result eventually in heterogeneous expectations. In the following pages the above ideas will be generalized by demonstrating that common knowledge is a necessary and sufficient condition for any pricing equilibrium.

2.2.4 Pricing Equilibria.

Let C be an one-to-one and onto correspondence from Ω to $\Theta \times X$, thus $\omega \mapsto (\theta, \chi)$ where θ describes payoffs, for example the value of securities in ω . χ describes occurrences that might be causally or statistically related to θ , for example macroeconomic events.

There are k securities in each state of the world and the wealth of each player in each state is characterized by the number of shares held, a vector $\bar{e}_i \in R_+^k$. One of the k securities is money. There are no restrictions on borrowing and lending.

In addition, to fully characterize a player we need:

i) A utility function, an operator $U_i(\cdot)$ from $\Theta \times R_+^k$ to R_+ such that $U_i(\theta, \bar{e}_i) = \text{"utility of the } i\text{th player in } \theta\text{"}$, $\forall \chi$ ⁶⁷. In essence that means that players are concerned with wealth but not with the conditions under which wealth is generated. U_i , $\forall i$ is smooth and if the resulting mapping is

⁶⁷ Note that time is not an argument of $U_i(\theta, \bar{e}_i)$. The reason is that in the subsequent derivations the utility problem of participating players needs to be examined at one specific point in time only. This in itself does not imply that the derived results pertain solely to one-period models. In order to confirm the validity of the derived results in multiperiod frameworks, it has to be confirmed that the assumptions made here allow each player to be concerned with a future beyond what is defined in this section as "pricing equilibrium." θ reflects all potential future contingencies including any informational contingencies since it is causally or statistically related to χ . Since player i , $\forall i$, when examining $U_i(\theta, \bar{e}_i)$ is concerned with θ , players are concerned with the future beyond the pricing equilibrium. Thus, the results derived in this essay are applicable to one-period as well as multiperiod models.

concave (strictly concave) in R_+ , player i is said to be weakly (strictly) risk averse. Additionally, the mapping is monotonically increasing in $e_i^1, e_i^2, \dots, e_i^k, \forall i$;

ii) prior beliefs about ω , $p_i(\omega)$; and

iii) prior informational partition P_i .

If A represents any event, a trade $t(\cdot)$ is an operator from Ω to R^{nk} such that $t(A) = (\bar{t}_1, \bar{t}_2, \dots, \bar{t}_n)$ where $\bar{t}_i$ represents i th player's net trade of securities (i.e., $\Delta \bar{e}_i = (\Delta e_i^1, \Delta e_i^2, \dots, \Delta e_i^k)$) given all information player i possesses concerning A . Call "non-cash" trade a trade which involves at least one security which is not money.

A trade is feasible if:

$$\sum_{i=1}^n \bar{t}_i = \vec{0} \quad (1).$$

Further, a trade is mutually acceptable if

$$E_i(U_i(\theta, \bar{e}_i + \bar{t}_i)) \geq E_i(U_i(\theta, \bar{e}_i)), \forall i.$$

At this point it is possible to define a pricing equilibrium within the introduced formalism as the state of the market where no trading-other than consumption related trading-takes place.

Definition 7: A **pricing equilibrium** is the state of the market where there exists no "non-cash" trade $t(A) > [0]$ such

that $t(A)$ is feasible and mutually acceptable⁶⁸.

What is implied by the above definition is that not only current $\vec{e}_i$'s are such that allocational Pareto optimality has been achieved (see Milgrom and Stokey [1982], pp. 21-2), but also that all relevant information has been extracted from the system by all players. Thus, there exists no further incentive for non-consumption trading. In other words, the observed (perceived) conditions support each and every player's identical beliefs. Therefore, no two players can enter a trade agreement in which the two participants would take positions supported by different beliefs as in every trade agreement made on a non-consumption basis⁶⁹.

⁶⁸ Any exchange solely in money balances is not considered a trade violating the definition of a pricing equilibrium. Further, given the assumption of unlimited borrowing and lending no pair of players will ever enter a trading agreement aiming solely at the liquidation of a security for consumptions purposes, unless both players are indifferent toward the trade. If indifference trades are assumed to never take place then any trade aiming solely at the satisfaction of consumption needs is an exchange solely in cash balances. In addition, any trade aiming at the satisfaction of consumption and speculative needs would take place even in absence of the associated consumption needs. Consequently, consumption has no effect on pricing equilibria.

⁶⁹ "Non consumption basis" implies here the set of incentives associated with buying or selling an asset which is not demanded for its own sake but for resale or potential resale in the future. In this case, that is if both trading partners are desiring the asset simply because it represents claim on some future cash flow (e.g. its future price), for the transaction to occur the two investors have to disagree on the characteristics of the future cash flow, and consequently,

As defined here, equilibrium, is more general than the Marshallian equilibrium. In fact the two definitions are equivalent under the very strict conditions of informationally perfect, complete and competitive markets. Under the above conditions investors have all relevant information before trading even starts. Thus, they are able in the first round of trading to achieve positions with which they will be forever content in the absence of any exogenous disturbance. Trading cycles are informationally redundant in perfect markets and thus, cannot change the belief structure of the market. Further, if the market is complete and competitive, the first market clearing price correctly represents the homogeneous aggregate belief structure. One round of trading is sufficient to produce a fully revealing equilibrium price. Under these conditions a pricing equilibrium is achieved when the price is

on the current price of the asset in question. An investor's estimate of the value of the asset is the price which will make the investor in question indifferent toward the transaction. If the two traders agree on the asset's value, a price equal to their common estimate will make them indifferent toward the transaction. At any higher price they will both desire to sell and at any lower price they will both desire to buy. In either case the trade would be infeasible.

On the other hand, if at least one of the traders desires the asset in question for its own sake, the two individuals need not disagree on the characteristics of the cash flow associated with the potential future resale of the asset in order for the transaction to take place. If for example, the asset in question is a used automobile, and both the buyer and seller know with certainty that the price of the car is determined now and in the future by the "blue book", the trade might still be desirable to both parties simply because they attribute different marginal utilities to owning the car.

such that supply (S) equals demand (D). After the " $S=D$ " condition is met in such markets all relevant information has been extracted from the system. No further incentives for trading will be produced endogenously.

However, under more realistic assumptions the " $S=D$ " condition does not produce a fully revealing equilibrium price. Information concerning the belief structure remains concealed in the system. Trade incentives outlive the fulfillment of the " $S=D$ " condition as the corresponding prices and volumes⁷⁰ are only revealing small pieces of the opinion structure puzzle in each trading cycle. As a result, the belief structure is consecutively modified.

⁷⁰ Note that when price is no more fully revealing, other variables, such as trading volumes may have informational value.

2.3.0 Pricing Equilibria and Common Knowledge

It is shown below that there exists a very concise way to describe the situation where all information has been extracted from the system, and consequently, there exist no further incentives for trading. It is the state of things defined as common knowledge. Common knowledge will be shown to be a necessary and sufficient condition for equilibrium.

Proposition 1: Common knowledge is a necessary and sufficient condition for any pricing equilibrium.

Proof: Let the market be initially at equilibrium and let P_i denote the informational partition of player i , including any information revealed by the trading process. Let R be the joint of P_i , $\forall i \in \{1, 2, \dots, n\}$. Since the market is at equilibrium note that e_i 's, $\forall i \in \{1, 2, \dots, n\}$, are Pareto optimal and there exists no trade which is mutually acceptable.

Now, suppose that an external disturbance is applied on the market. That is, each investor receives a private (i.e., differentiated) signal $P_i(\omega^*)$. If any event A affecting

and potentially causing trading is such that $R(\omega^*) \subseteq A$, event A (by definition 6) is common knowledge.

We will prove by contradiction that there exists no trade $t(A)$ such that $t(A)$ is mutually acceptable. In other words, given any disturbance the market is in equilibrium if after the disturbance all events affecting utility are common knowledge.

Suppose that there exists a trade $t(A)$ which is mutually acceptable and feasible.

Then $\forall i : i \in \{1, 2, \dots, n\}$ and $\forall \omega : \omega \in R(\omega^*)$ we have

$$E_i(U_i(\theta, \bar{e}_i + \bar{t}_i) | P_i(\omega)) \geq E_i(U_i(\theta, \bar{e}_i) | P_i(\omega)) \quad (2),$$

and assume that the inequality holds strictly at for at least one investor j .

Consider the operator $O_{R(\omega^*)}(\cdot)$ from Ω to $(0,1)$ which maps ω to 1 if and only if $\omega \in R(\omega^*)$ and to 0 otherwise. In other words, this operator yields 1 if the state it operates on is within the signaled element of the joint and 0 otherwise. Consider trade $\bar{t}_j' = \bar{t}_j O_{R(\omega^*)}(\omega)$. Clearly if t is feasible so is t' . Operator $O_{R(\omega^*)}(\cdot)$ along with the assumption that all events affecting utility are common knowledge allows for the subsequent decomposition of the expression $E_j(U_j(\theta, \bar{e}_j + \bar{t}_j'))$.

For investor j looking at $\bar{t}_j'$ before he receives his private signal we have

$$\begin{aligned} E_j(U_j(\theta, \bar{e}_j + \bar{t}_j')) &= E_j(E_j(U_j(\theta, \bar{e}_j + \bar{t}_j O_{R(\omega^*)}(\omega)) | P_j(\omega))) = \\ &E_j(E_j(U_j(\theta, \bar{e}_j) O_{R(\omega^*)}(\omega) | P_j(\omega))) + \end{aligned}$$

$$E_j(E_j(U_j(\theta, \vec{e}_j + \vec{t}_j) O_{R(\omega^*)}(\omega) \mid P_j(\omega)))$$

where $\bar{R}(\omega^*)$ is the complement of $R(\omega^*)$.

By the definition of joint R is coarser than P_j . In addition, given common knowledge with respect to A , $U_j(\theta, \vec{e}_j + \vec{t}_j)$ does not depend on $O_{R(\omega^*)}$ given $P_j(\omega)$. Consequently, $O_{R(\omega^*)}$ can be taken out of the conditional expectation expression. Thus, the above is equal to

$$\begin{aligned} & E_j(O_{\bar{R}(\omega^*)}(\omega) E_j(U_j(\theta, \vec{e}_j) \mid P_j(\omega))) + \\ & E_j(O_{R(\omega^*)}(\omega) E_j(U_j(\theta, \vec{e}_j + \vec{t}_j) \mid P_j(\omega))) \end{aligned}$$

which by relation (2) is greater than

$$\begin{aligned} & E_j(O_{\bar{R}(\omega^*)}(\omega) E_j(U_j(\theta, \vec{e}_j) \mid P_j(\omega))) + \\ & E_j(O_{R(\omega^*)}(\omega) E_j(U_j(\theta, \vec{e}_j) \mid P_j(\omega))) . \\ & = E_j(U_j(\theta, \vec{e}_j)) \end{aligned}$$

Thus for trader j , before his private signal were observed

$$E_j(U_j(\theta, \vec{e}_j + \vec{t}_j')) > E_j(U_j(\theta, \vec{e}_j))$$

which is a contradiction to the hypothesis of initial equilibrium. The sufficiency direction of the proposition has been proved.

Put simply it was shown that if an incentive to trade exists after a signal was received and if common knowledge holds, then the incentive existed before the signal was received. The latter contradicts the hypothesis of initial equilibrium. Thus, given any vector of initial private signals

if common knowledge holds, equilibrium has been reached. In other words common knowledge is a sufficient condition for equilibrium.⁷¹

To show the necessity direction of the "common knowledge $\Leftrightarrow$ pricing equilibrium" equivalence assume that the market is not initially at equilibrium. Thus, there exists a trade t_j which is feasible and mutually acceptable.

$$E_j(U_j(\theta, \vec{e}_j + \vec{t}_j)) \geq E_j(U_j(\theta, \vec{e}_j)).$$

In addition, consider signals $P_i(\omega)$ such that ω not in $R(\omega^*)$ ⁷². Following the same line of proof

$$E_i(U_i(\theta, \vec{e}_i + \vec{t}_i) | P_i(\omega)) > E_i(U_i(\theta, \vec{e}_i) | P_i(\omega)).$$

Thus, given any signal, if common knowledge does not hold equilibrium is not reached.⁷³//

Intuitively speaking, if common knowledge does not hold there exist m such that for at least two players i, j , $p_i^m(\theta)$ is different than $p_j^m(\theta)$. Such difference will result in a different opinion with uncertainty concerning the current price of the asset. No matter how small the difference, there

⁷¹ As will be explained in the following pages, under common knowledge private signals become equivalent to public announcements and thus, cannot be sources of heterogeneity in expectations.

⁷² The signals in this case may be interpreted as information revealed by the trading process as well as exogenous information.

⁷³ Proposition 1 allows to define equilibrium in the next section as a state of common knowledge.

will be a price, between prices implied by $p_i^m(\theta)$ and $p_j^m(\theta)$, and a volume which is low for small belief differences, and high degrees of strict risk aversion⁷⁴ that will support trade.

In order to shed some intuitive light on the first portion of the above proof, note that the real reason trading does not take place is that under common knowledge the fact that a trade is acceptable is also common knowledge. This leads each participant to the undeniable conclusion that the trade is not beneficial.

Put differently, common knowledge allows each participant to be fully informed with respect to the future value of the asset. As a result, there exists no basis for speculation⁷⁵. Each investor under common knowledge will ignore the private signal in favor of the "truth" which is revealed to the investor in question and which is in that sense independent of the private signal⁷⁶. Although, the prior signals might have been different making trade attractive to the "unsophisticated

⁷⁴ The reason that volume under such conditions will be small is that the speculative incentive is not large enough to cause large deviations from initial allocational Pareto optimality.

⁷⁵ The "truth" to which all investors agree is fully defined by ω and by the fact that ω is a state of common knowledge.

⁷⁶ A private signal can be viewed as an unsophisticated interpretation of the observed facts.

player"⁷⁷, the inferences made by the rational investor about the global belief structure will force the investor to neglect the direct (unsophisticated) inferences of his private signal⁷⁸. Private signals become equivalent to public announcements, or alternatively the economy becomes equivalent to an artificial economy in which investors publicly announce their private signals. This is indeed the way that common knowledge leads to homogeneous expectations with respect to the payoff variable. Thus investors reach homogeneous expectations with respect to prices, price distributions, and equilibrium relationships of the CAPM type, depending on the initial assumptions concerning the form of the exogenous disturbances. In the absence of any exogenous disturbance prices are constant and fully predictable in a common knowledge world. In other words, rationality does not allow the existence of "disagreed" common knowledge⁷⁹.

⁷⁷ "Unsophisticated player" implies here a player who does not consider higher order beliefs. Such players do not "play" in perfect markets.

⁷⁸ The paradox of assuming that common knowledge holds at all times is now clear. If individuals always ignore their private signals, which is to say they ignore the only means by which reality manifests itself, how can the aggregate opinion structure reflect this reality. In other words, how can an observation mechanism take measurements of the observed, while being isolated from it.

⁷⁹ A formal proof of this premise is given in the following section.

As will be shown in Essay 3, if one parts from the ideal and paradoxical conditions described above, the opinion adjustment mechanism through which information is sought and absorbed will ultimately reveal an alternative truth which in general will differ from the unique "truth" described above, and which will be closely related to the initial signals and learning mechanism⁸⁰ (market structure in general.)

⁸⁰ The dependence of outcomes (i.e. ultimately revealed truth) on the observation mechanism will be formally demonstrated in the last essay. However, it is not difficult to examine the truth of this premise within the topological framework introduced in the present essay. Two observations are important in understanding why the ultimately revealed equilibrium depends on the learning process. First, the partition of each investor on the state space represents whatever information the investor possesses, including any information revealed by the trading process (Milgrom and Stokey [1982].) Second, the identity of the joint of all partitions depends on each individual partition, and in addition the joint of all partitions fully defines all events that are common knowledge. If the trading process reveals any relevant information to investors, as in the case where common knowledge concerning the relevant events does not hold, it is natural to expect that investors will adjust their partitions in light of the information revealed by the trading process. Thus, the joint of all partitions will change as a result of trading. Consequently, the events defined by the joint as being common knowledge will change as well. In other words, the ultimately revealed equilibrium depends on the information that is released sequentially by the trading process and the way this information is processed by the learning mechanism.

2.4.0 Homogeneous Expectations and Common Knowledge

The next proposition clarifies the relation between common knowledge and homogeneous expectations. If a set of expectations is common knowledge the expectations cannot be heterogeneous.

Proposition 2: Common knowledge with respect to any set of expectations implies that the expectations are homogeneous.

Proof: Let A be an event and let p_i , $i = 1, 2, \dots, n$ denote the probability trader i attributes to A given the signal $P_i(\omega^*)$. Then

$$p_i = p(A \cap P_i(\omega^*)) / p(P_i(\omega^*)),$$

where $p(\cdot)$ denotes probability.

Let $R(\omega^*)$ be the element of the joint of P_i ($i = 1, 2, \dots, n$) which contains ω^* . Let also P_i^l , $l = 1, 2, \dots, L$ be all disjoint members of P_i that belong to $R(\omega^*)$. Then

$$\bigcup_{l=1}^L P_i^l = R(\omega^*)$$

Assume that expectations p_i are common knowledge for all i ($i = 1, 2, \dots, n$). Then p_i is constant given P_i^l , for any

$l^{81} \ (l=1,2,\dots,L).$

$$p_i = p(A \cap P_i^l(\omega^*)) / p(P_i^l(\omega^*)) \Leftrightarrow$$

$$p(A \cap P_i^l(\omega^*)) = p_i p(P_i^l(\omega^*)) \Leftrightarrow$$

$$\sum_{l=1}^L p(A \cap P_i^l(\omega^*)) = \sum_{l=1}^L p_i p(P_i^l(\omega^*)) \Leftrightarrow$$

$$p(A \cap \bigcup_{l=1}^L P_i^l(\omega^*)) = p_i p(\bigcup_{l=1}^L P_i^l(\omega^*)) \Leftrightarrow$$

$$p_i = p(A \cup R(\omega^*)) / p(R(\omega^*)),$$

thus, p_i is independent of i .⁸²//

Lemma 1: Any pricing equilibrium implies homogeneous expectations.

Proof: Directly follows from propositions 1 and 2.//

Proposition 2, and Lemma 1 formalize the notion presented in the common knowledge literature that the relevant assumption for substantiating equilibrium pricing models is common knowledge and not common priors (homogeneous beliefs.)

⁸¹ Otherwise, the expectations p_i of investor i would not be common knowledge among all other investors.

⁸² It should be noted at this point that the converse of the proposition 2 does not hold. If investors have homogeneous expectations at one point in time but do not realize it, they will be led to adjust their beliefs. The adjustment will generally result in heterogeneous expectations. This is the reason as Guth and Philippatos [1988] argued that the assumption of common knowledge rather than homogeneous expectations is needed in order to derive the equilibrium asset pricing models in Finance.

In fact Lemma 1, contrary to common intuition, but consistently with implicit notions in the common knowledge literature, reveals that homogeneous expectations are in a sense a "by-product" of common knowledge and equilibrium, rather than a sufficient condition for them.

Further the result presented as Lemma 1 implies that if equilibrium exists it is a rational expectations equilibrium since all expectations are confirmed in it⁸³. Non-rational expectations equilibria do not make economic sense. The intuitive reason is that if a rational investor's beliefs are not confirmed in equilibrium then the investor in question will be driven to modify his beliefs, thus, disturbing equilibrium⁸⁴. Thus, a dynamic model which exhibits convergence to non-rational expectations equilibria is not acceptable as a satisfactory representation of the opinion adjustment process. This is the basic problem of the literature which recognizes the dependence of outcomes on beliefs (i.e., third class of literature.) The dynamic models

⁸³ As Grossman has suggested, rational expectations equilibria are self-fulfilling prophecies. If all investors believe that prices will increase, in the absence of any new exogenous disturbances prices will indeed increase, and vice versa. The reason is simply that the outcome of an investor's trading activity is not determined solely by exogenous shocks, but also by the trading activity of his fellow investors.

⁸⁴ See Feldman [1987].

presented in it converge to non-rational expectations equilibria for some allowable parameter ranges.

2.5.0 Concluding Remarks

Results developed in this essay allow the complete shift of attention from the homogeneous expectations condition to the common knowledge condition as the state of opinion structure that characterizes equilibrium.

From Proposition 1 it should be noted that in a world with initial Pareto optimality and common knowledge at all times, with respect to any possible event, prices will be implicit in the sense that no trading will ever take place. Thus, although such implicit prices will be common knowledge they will not be observable.

The above notion brings to focus an implication of the common knowledge assumption which is not consistent with observable reality. At equilibrium there is no "speculative" trading; there is only "consumption" trading. Even accepting that some trading is due to emerging consumption needs does not support observed total daily transaction values which by far exceed any acceptable dollar value of emerging consumption needs.

In a world where all events are common knowledge prices contain no information whatsoever in the entropic sense (0

degree of surprise). Indeed, in this case the need for observing prices is absent since they are redundant with respect to events and states of nature⁸⁵. What is implied by the common knowledge assumption (common knowledge holding at all times) is that events contain some intrinsic truth not dependent upon the nature of the observer. Observer and observation exist without interconnections and thus can be studied independently. However, once the common knowledge assumption is relaxed such simplifications are no longer possible. As Harrison and Kreps [1978] note, investors with imperfect information (i.e., non-common knowledge beliefs) "...may be unable initially to achieve positions with which they will be forever content, and thus the current price may be affected by whether or not markets will reopen in the future." (p. 323)

Finally, by Lemma 1 it was formally demonstrated that expectations are necessarily homogeneous in equilibrium. Thus, the only potential equilibria are rational expectations equilibria.

⁸⁵ One does not need a thermometer (price) to feel cold or warm (form an opinion about the asset's true value), nor does the temperature depends on any way upon revelation of the thermometer reading.

Appendix 2.1

In this section some elements of Lattice theory necessary for the work presented in Essay 2 are given.

A partition P of a set Ω is a decomposition of Ω into mutually disjoint subsets. Mutually disjoint subsets are subsets which have a null intersection. The subsets in turn are called classes of the partition considered.

Example: Consider set $\Omega = \{1,2,3,4,5\}$. Two possible partitions on Ω would be $P_1 = \{1,2,3/4/5\}$ and $P_2 = \{1,2/3,4/5\}$.

A partition on the state space may represent an investor's information concerning future outcomes. Suppose for example that the elements of Ω above represent potential prices of a pharmaceutical research firm. Suppose further that the firm's only investment is a research project to find a cure for cancer. There exist three possibilities. Either a cure will have been found by the end of the period, or the line of research pursued will have been proved wrong and dropped, or the outcome of the research will still be

knows that in the first case above the price of the firm's stock will be \$5, in the second case \$1, \$2, or \$3 and in the third case \$4. Then this investor's information can be represented by partition P_1 above. If now the investor receives a signal that the price which will be revealed belongs in the third class of the partition—a cure for cancer has been found—the investor will know given the signal and the partition that the price of the asset will be \$5.

Definition 1: A coarsening of a partition P_i on a set Ω is a partition P_j such that for any class of P_i , P_i^l there exists a class of P_j , P_j^k such that $P_i^l \subseteq P_j^k$.

Example: Consider set Ω and partition P_1 on Ω as in the previous example. Two possible coarsenings of P_1 would be $P_3 = \{1,2,3,4/5\}$ and $P_4 = \{1,2,3,4,5\}$

If a partition is taken to represent an investor's entire set of information prior to receiving a signal then any coarsening of the partition will represent the information of a less informed investor. If for example an investor's information is represented by P_4 above and given the signal in the previous example the investor will not know if the firm's price will be \$1, \$2, \$3, \$4, or \$5.

Definition 2: A refinement of a partition P_i on a set Ω is a partition P_j such that for any class of P_i , P_i^1 there exists a class of P_j , P_j^k such that $P_j^k \subseteq P_i^1$.

Example: Consider P_1 as above. Two possible refinements of P_1 would be $P_5 = \{1, 2/3/4/5\}$ and $P_6 = \{1/2/3/4/5\}$.

One can notice directly from the above definitions that a partition is a coarsening of all its refinements and a refinement of all of its coarsenings. If a partition is taken to represent an investor's entire set of information prior to receiving a signal then any refinement of the partition will represent the information of a more informed investor.

Definition 3: The meet of partitions P_i on a set Ω is the coarsest common refinement of the partitions.

Example: Consider partitions P_1 and P_2 on Ω as in the previous example. The meet of P_1 and P_2 is $M = \{1, 2/3/4/5\}$. Although, P_6 is also a refinement of P_1 and P_2 , M is coarser and thus, P_6 is not the meet of P_1 and P_2 .

If a set of partitions represents the entire set of information of some investors prior to receiving a signal then the meet of the partitions in question represents the

information of a hypothetical investor who is as informed as the most informed investor in every region of the state space⁸⁶, but not more.

Definition 4: The joint of partitions P_i on a set Ω is the finest common coarsening of the partitions.

Example: Consider partitions P_1 and P_2 on Ω as in the previous example. The joint of P_1 and P_2 is $R=\{1,2,3,4/5\}$. Although, $P_4=\{1,2,3,4,5\}$ is also a coarsening of P_1 and P_2 , R is finer than P_4 and thus, P_4 is not the joint of P_1 and P_2 .

If a set of partitions represents the entire set of information of some investors prior to receiving a signal then the joint of the partitions in question represents the information of a hypothetical investor who is as informed as the least informed investor in every region of the state space, but not less.

⁸⁶ An investor might be more informed than another investor in one region of the state space but less informed than the same investor in another region of the state space.

ESSAY 3

**A Dynamic Model
of the Opinion Adjustment Process**

3.1.0 Introduction

In this essay some additional mathematical symbolism is introduced in order to allow the construction of a specific model of opinion adjustment which will add resolution to the picture of a world not blessed with common knowledge that is described in the previous essays. The model is formalized as a system of difference equations.

A discrete time framework is adopted, not necessarily for modelling convenience, but because it represents the opinion formation mechanism better than continuous time frameworks. A maintained assumption is that macroeconomic information is exogenous, and thus the initial signal vector is exogenous. All subsequent opinion structures, trading volumes and prices will be endogenous.

After a general description of the model is presented there is a derivation of individual demands which depend on wealth, risk aversion, and individual estimates of asset value and risk. Subsequently a market clearing condition is imposed and solved for market clearing price. Each individual estimate of the asset value depends on all other individual estimates which are revised each time the market clearing price proves

them erroneous⁸⁷. The process is formalized as a system of difference equations.

Conditions for convergence are derived and interpreted in terms of economic intuition. Subsequently, the equilibrium is shown to be a rational expectations equilibrium which is different than both the initial individual estimates and the equilibrium that would have emerged if common knowledge held initially.

The derived results are compared to results in the third class of literature where the dependence of outcomes on beliefs is recognized. Finally, explanations based on the proposed model are given for "crashes", "speculative bubbles", the "weekend effect", and the "small firm effect".

⁸⁷ Only at equilibrium are all individual estimates correct and need no further revisions.

3.2.0 The Model

In this section a dynamic model of the opinion adjustment process is presented. The model is acceptable, given previous results, since its attractors (i.e., equilibria) comply with the characterizations derived in Essay 2.

3.2.1 A General Description of the Process

Suppose that a financial market is composed of m investors, one risky asset, and a riskless asset. The market is initially at a pricing equilibrium and right before $t=0$ an exogenous disturbance (i.e., signal) E_i ($i=1,2,\dots,m$) is imposed on the system. Each investor then forms an initial opinion O_{i0} depending on all individual signals E_i ⁸⁸. At any subsequent point in time t , such that $t > 0$, each investor has an estimate O_{it} about the worth of the asset which takes into account the opinion of all others. Each investor is assumed to correctly estimate the initial signal vector and the opinion

⁸⁸ Signals E_{it} can be interpreted as expected values of the price of the asset, whereas opinions O_{it} can be interpreted as individual estimates of the market value of the asset taking into account expected values and estimated risk among other things. In other words an opinion O_{it} can be interpreted as the price that would make individual i indifferent between buying selling or not transacting a share of the risky asset.

of other investors at each point in time t given the observed market clearing conditions at t , but not the individual demands D_{it}' of other investors⁸⁹. An alternative and more realistic interpretation is to view the individual as knowing all possible prices of the asset but not the demand associated with each possible price. For example each individual might know that the potential range of prices is $\{0, 1/8, 2/8, \dots, 100\}$ but not the number of individuals who belong to each price class according to their beliefs, nor their individual risk aversion levels⁹⁰.

At each point in time t , such that $t \geq 0$, each participant comes into the market with a demand schedule D_{it}' which depends on his opinion about the value of the asset O_{it} among other things. An auctioneer calls out prices until P_t , the market clearing price, is found. Since no opinion adjustments take place during each round of the auction, the model approximates the real situation in which representatives of the investors (i.e., brokers) arrive at the auctioneer with priced buy and sell orders. The auctioneer allows no

⁸⁹ Suppose for example, that each investor ignores the risk aversion level of other participants or any other determinant of individual demands.

⁹⁰ The choice of interpretation does not affect the derived results. However, it affects the required formulation of the model. In the subsequent derivations the first interpretation is chosen, for expositional purposes, because it results in a slightly simpler formulation of the problem.

transactions at any price other than the market clearing price P_t . After the market clears each investor observes the market clearing price and trading volume (which contain information about the opinion structure) and forms a new demand schedule D_{it+1}' depending on his new value estimate O_{it+1} . In $t+1$ a new trading cycle begins and the market clears at P_{t+1} . Note that P_t is not an equilibrium price as defined in the second essay. Equilibrium is reached (if ever reached) after a number of market clearing trading cycles reveal all relevant information concerning the opinion structure.

To recapitulate, each period t consists of a trading cycle which clears the market and produces P_t , and an opinion adjustment which takes into consideration the market clearing conditions and produces all individual demands D_{it+1}' . Initially (i.e., before $t=0$) there are no previous trading cycles to be taken into account and investors form demands given all initial signals and their initial estimates concerning the demand schedules of others.

All individuals are price takers, and there are no restrictions for borrowing at the risk free rate which for arithmetic convenience will be assumed to be 0. Short selling (i.e., negative holdings) is allowed. Thus, each player can either invest in the risk free asset (i.e., cash) earning 0 return, or go long or short on the risky asset. Shares of the risky asset are assumed to be infinitely divisible. Finally,

it is assumed that consumption does not take place until equilibrium is reached⁹¹.

3.2.2 Individual Demand Determination.

Each investor i realizes in each period t that the estimate O_{it} might be erroneous in the sense that the asset value in his opinion might change after observing the next market clearing conditions at t . Suppose that he also estimates $O_{it+1} - O_t$ (i.e., the deviation of i th investor's current value estimate from the investor's next value estimate) to be distributed as $N(0, \sigma_{it}^2)$ ⁹². The distribution of each player's estimation error should be given a Bayesian interpretation. The error is not truly stochastic in nature. However, each investor attributes a subjective probability to

⁹¹ This assumption is made for arithmetic convenience and does not affect the derived qualitative results. As explained in the previous essay if indifference trades are assumed to not take place, and given unlimited borrowing and lending, consumption affects cash balances only. Thus, consumption affects wealth, and indirectly the numerical value of market clearing prices as any other change in the parameters determining demand would. However it does not affect the general market clearing price expressions nor the convergence conditions derived.

⁹² This myopic behavior is recursively optimal when investors are price takers. In this case, market clearing prices are out of any particular investor's control and in addition, each investor does not make an estimate unless it is necessary to make a decision which depends on this estimate. As a result, each investor's decisions incorporate all information available to the investor at the time the decisions are made.

each possible error range because he has limited information⁹³. At the beginning of each period t (after trading at $t-1$), investor i is endowed with wealth W_{it-1} and after trading at t with W_{it} .

After trading is completed in period t , each player i in order to determine the optimal demand D_{it+1}' (i.e., number of shares demanded) maximizes a two parameter utility function. This utility function is defined on the expected value conditional on P_{t+1} and estimated variance conditional on P_{t+1} of the increase in wealth associated directly with the investor's trading activity in the next trading cycle⁹⁴.

$$\max_{D_{it+1}'} U_{it} [E(W_{it} + D_{it+1}' (O_{it+2} - P_{t+1})),$$

$$\text{var}(W_{it} + D_{it+1}' (O_{it+2} - P_{t+1}))],$$

$$E(W_{it} + D_{it+1}' (O_{it+2} - P_{t+1})) =$$

$$W_{it} + D_{it+1}' (O_{it+1} - P_{t+1});$$

and

$$\text{var}(W_{it} + D_{it+1}' (O_{it+2} - P_{t+1})) =$$

$$D_{it+1}'^2 \text{var}(O_{it+2} - P_{t+1}) = D_{it+1}'^2 \sigma_{it+1}^2.$$

Thus, the objective function to be maximized with respect to D_{it+1}' becomes

$$U_{it} [W_{it} + D_{it+1}' (O_{it+1} - P_{t+1}), D_{it+1}'^2 \sigma_{it+1}^2]$$

⁹³ Normality can be replaced with a wide range of distributional assumptions without effect on the qualitative results.

⁹⁴ Maximizing the specified utility function is consistent with the myopic behavior of each investor which in turn is recursively optimal as explained previously.

Solving the first order conditions for D_{it+1}' yields

$$D_{it+1}' = ((-U_{it}^1/2U_{it}^2)/\sigma_{it+1}^2)(O_{it+1}-P_{t+1})$$

where the superscripts to U_{it} denote partial derivatives. The last expression is the demand schedule for investor i since it determines demanded shares for any price of the risky asset. Assume, for arithmetic convenience, that given a specific risky opportunity each investor desires to invest in it a fixed portion of his/her wealth, independently of the investor's wealth level⁹⁴.

$$d(-U_{it}^1/2U_{it}^2)/dW_{it} = 1/R_i$$

where R_i is a positive constant.

Thus,
$$-U_{it}^1/2U_{it}^2 = W_{it}/R_i,$$

and
$$D_{it+1}' = (W_{it}/(R_i\sigma_{it+1}^2))(O_{it+1}-P_{t+1}).$$

Defining
$$d_{it+1}' = W_{it}/(R_i\sigma_{it+1}^2),$$

we get
$$D_{it+1}' = d_{it+1}'(O_{it+1}-P_{t+1}).$$

In other words, an individual's demand for the risky asset is increasing in the investor's estimate of asset value and decreasing in risk aversion and perceived risk. Further, since $(O_{it}-P_t)$ represents the i th investor's estimate of how much the risky asset is overpriced or underpriced, the i th

⁹⁴ This assumption plays no role in the subsequent results, other than expositional convenience. As will become clear in section 3.2.4, the relevant result in this section (3.2.2) is the linearity of demand functions with respect to the quantity $(O_{it+1}-P_{t+1})$. Such linearity between demand and estimated overpricing or underpricing is not affected in any way by assumptions on the form of risk aversion.

individual's demand is positive when in his/her view the asset is underpriced (i.e., $O_{it} > P_t$) and negative when the asset is underpriced (i.e., $O_{it} < P_t$.)

3.2.3 Market Clearing Price Determination

At this point a market clearing condition is imposed which when solved for price will yield the market clearing price P_{t+1} .

$$\sum_{i=1}^m D_{it+1}' = 0 \Leftrightarrow$$

$$\sum_{i=1}^m (d_{it+1}' (O_{it+1} - P_{t+1})) = 0 \Leftrightarrow$$

$$\sum_{i=1}^m (d_{it+1}' O_{it+1}) - \sum_{i=1}^m (d_{it+1}' P_{t+1}) = 0 \Leftrightarrow$$

$$P_{t+1} = \sum_{i=1}^m ((d_{it+1}' / \sum_{i=1}^m d_{it+1}') O_{it+1}).$$

Defining $f_{it+1}' = d_{it+1}' / \sum_{i=1}^m d_{it+1}'$,

then $P_{t+1} = \sum_{i=1}^m (f_{it+1}' O_{it+1}).$

Thus, the market clearing price P_{t+1} is a weighted average of all individual estimates. By Walras' law if the market for the risky asset clears, the market for the riskless asset clears also. The lower an investor's risk aversion, the larger the weight that the investor's opinion carries in the determination of market clearing price. In addition, the lower the perceived estimation error, the more heavily the corresponding individual's opinion will be weighted in the

market clearing price. Finally, the higher the individual estimates for the value of the asset, the higher the market clearing price.

The above results are intuitively appealing. With two exceptions the results are consistent with results in the first class of literature discussed in detail in Essay 1. The first difference is that in this dissertation these results hold for any one-step transition, but not for equilibrium. In other words, the properties of market clearing prices contribute to the properties of equilibrium prices, but differ from them. To identify the properties of equilibrium, the dynamic behavior of the opinion adjustment process needs to be examined. In contrast, in the first class of literature these results are viewed as pertaining to equilibrium.

The second difference is that the market clearing price derived above does not include a risk adjustment factor analogous to the quantity subtracted from the otherwise similar market clearing price derived in the first class of literature. The intuitive reason is that the market clearing price in the first class of literature aggregates expected values of the asset price. Expected values do not reflect risk in themselves, and thus, a risk adjustment factor appears as directly reducing the market clearing price. Here, on the other hand, the market clearing price aggregates opinions for the value of the asset. Opinions are already adjusted for risk

and thus, if a risk adjustment factor appeared in the market clearing price expression above, risk would have been counted twice. In other words, the market clearing price in our model is implicitly adjusted for risk, since it aggregates opinions-not expected values-which are already adjusted for individual risk estimates.

Note, however, that if the initial opinion O_{i0} is expressed in terms of the initial individual signals E_j , which represent expected values rather than opinions, a risk adjustment factor will appear as directly reducing each individual opinion O_{i0} . Specifically,

$$O_{i0} = \sum_{j=1}^m ((d_{ij0}/\sum_{j=1}^m d_{ij0}) E_j) - (S/\sum_{j=1}^m d_{ij0}).^{95,96}$$

where S is the fixed stock supply (i.e., quantity of outstanding shares of the risky asset) and d_{ij0} is the i th investor's estimate for d_{j0}' .

Thus, each individual signal will enter an investors opinion with a greater weight, the less risk averse (i.e., lower R_i) the individual corresponding to the signal is, and the more precise the signal. Similarly the risk adjustment factor will be smaller when individuals in general are less

⁹⁵ See Appendix 3.1.

⁹⁶ The above expression for O_{i0} is identical to the expression derived for the equilibrium price in the first class of literature (static model literature) discussed in detail in the first essay. In our framework, however, as explained earlier equilibrium is far from being reached yet.

risk averse and their signals are more precise. In addition, the initial individual estimates O_{i0} of the asset value are inversely related to the fixed supply⁹⁷ of the risky asset.

3.2.4 Dynamic Behavior of the Process.

As shown in the previous section the market clearing price at each point in time t ($t \geq 0$) will be a weighted average of all individual estimates where the weights will be functions of individual demands. Specifically,

$$P_t = \sum_{i=1}^m f_{it}' O_{it}$$

where $f_{it}' = d_{it}' / (\sum_{i=1}^m d_{it}')$

Since weights depend on individual demands they are unknown to each participant, but assume that each participant i has generally an estimate $f_{i1}, f_{i2}, \dots, f_{im}$ of the real weights $f_{1t}', f_{2t}', \dots, f_{mt}'$. The estimates in general differ from the real weights. Right before $t=0$ an exogenous disturbance is applied on the system in the form of some heterogeneous informational signals which result in heterogeneous priors $\bar{O}_0$ concerning the value of the risky

⁹⁷ Note that here as in the first class of literature the fixed supply of the stock represents the capital invested in the risky opportunity and not merely the number of shares outstanding. If for example a firm issued stock in order to cover cash flow shortages related to current investments and not in order to undertake new investment opportunities, the corresponding increase in the number of shares outstanding should not be considered an increase in the fixed supply of the risky asset.

asset⁹⁸. At $t=0$ markets will open and will clear and a market clearing price P_0 will be revealed.

$$P_0 = \sum_{i=1}^m f_{i0} O_{i0}$$

P_0 in general will differ from player's i estimate O_{i0} . In addition a corresponding trading volume V_0 will be generated. The trading volume V_0 , if higher than the individual demand of player i will reveal to player i that others are also in disagreement. Player i , before forming an individual demand and trading again, will attempt to estimate the value of the asset in his opinion after observing the market clearing conditions at $t=1$ (O_{i1}), taking into account the opinion of others. The i th investor's estimate then will be

$$O_{i1} = \sum_{j=1}^m f_{ij} O_{j0}.$$
⁹⁹

In general the opinion structure transition is described as

⁹⁸ For a derivation of $\vec{O}_0$ given the initial disturbance see Appendix 3.1.

⁹⁹ The expression for O_{i1} is a direct implication of the expression derived for P_t ($t = 1$) since each investor is assumed to correctly estimate the opinions of other investors given the observed market clearing conditions. In addition, the opinion structure estimated after the previous market clearing can be viewed as each investors best estimate of the opinion structure after the next market clearing. In general, the estimates for O_{it} 's can be deterministic estimates or expected value operators and thus, should not be interpreted as random variables. However, even if the estimate for O_{it} is interpreted as a random variable it would be independent of f_{ij} since each investor is a price taker. Thus, the expression derived for O_{i1} would still hold.

$$\vec{O}_1 = F \times \vec{O}_0$$

It is assumed that individual i forms his estimate for the market clearing price P_1 using the same weights as previously. The reason is that each participant has no obvious reason to change his weight estimates, because although, the investor knows that they were erroneous in the first trading cycle, he/she also recognizes the fact that other individuals will change their demands. Thus, the investor's weight estimates might be proven correct in the next trading cycle.

In addition, it is assumed that the set of investors remains unchanged throughout the process. The last two assumptions allow for a stationary transition matrix¹⁰⁰. Although, it would be clearly more realistic to part with the last two assumptions¹⁰¹ the resulting mathematical complications would defeat the expositional purpose of this model. More importantly, the model in its present form has sufficient depth and width to demonstrate the properties of equilibrium and further to explain "crashes", "speculative bubbles", the "weekend effect" and the "small firm effect".

Markets will reopen again at $t=1$ and a new market

¹⁰⁰ A transition matrix the elements of which are independent of time.

¹⁰¹ For example, Laidler [1976] notes "...with the passage of time, any change in the behavior of the economy may be met not just by changing expectations, but also by changing the manner in which expectations are formed..." (p. 47)

clearing price P_1 and corresponding volume V_1 will be revealed, and so on. The process can be described by the m -dimensional system of linear difference equations

$$\vec{O}_t = Fx\vec{O}_{t-1} = F^{(t)}x\vec{O}_0, \quad t = 1, 2, \dots \quad (1)$$

For a pricing equilibrium to be reached all elements of $\vec{O}_t$ have to converge to a common limit O^* (see Essay 2.)

$$\lim_{t \rightarrow \infty} O_{it} = O^*, \quad \forall i \in \{1, 2, \dots, m\}$$

Such pricing equilibrium will be a self-fulfilling prophecy since for the equilibrium to be reached each individual must know that his/her estimate of the equilibrium price is correct, and additionally, the individual must know that all others have the same opinion. Thus, no further adjustments will take place in the absence of a new exogenous disturbance. Further such equilibrium will be a rational expectations equilibrium (expectations confirmed by observed reality) of common knowledge (by proposition 1 in Essay 2.) If such equilibrium is reached information has been symmetrized.

Now, let $f_{ij}^{(t)}$ denote the element in row i and column j of the matrix $F^{(t)}$. It follows from relation (1) above that a pricing equilibrium is reached if and only if there exists a vector $\vec{\phi} = (\phi_1, \phi_2, \dots, \phi_m)$ such that for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, m$

$$\lim_{t \rightarrow \infty} f_{ij}^{(t)} = \phi_j.$$

If the above is satisfied then $\phi_1, \phi_2, \dots, \phi_m$ are necessarily non-negative and $\sum_{i=1}^m \phi_i = 1$. Thus, when and if a

pricing equilibrium is reached the common estimate of the asset's worth will be $\sum_{i=1}^m \phi_i O_{i0}$.

3.2.5 Convergence Conditions and their Implications

In this section conditions for convergence of the opinion adjustment mechanism to a pricing equilibrium are identified. Matrix F is a stochastic matrix¹⁰². Thus, it can be viewed as the one step transition matrix of a Markov chain with m states and stationary transition probabilities. Thus, the standard limit theorems can be applied. In simple terms, a pricing equilibrium is reached if each investor takes into account the opinions of other investors.

Proposition 1: If all the recurrent states of the Markov chain representing the opinion adjustment mechanism, communicate with each other and are aperiodic, a pricing equilibrium is reached.

If on the other hand, the states of the Markov chain form at least two disjoint sets¹⁰³ or if the communicating states

¹⁰² A matrix is called stochastic when the elements of each and every one of its rows add to 1.

¹⁰³ In a Markov chain, two sets of states, i and j respectively, are called disjoint if and only if for every state i in the first set $f_{ij}=0$, and for every state j in the second set $f_{ji}=0$. In other words, states i and j do not communicate with each other.

in a single closed set are periodic¹⁰⁵, a pricing equilibrium is not reached. In economic terms the convergence conditions can be interpreted as follows: The condition of the states communicating with each other means that an investor takes into account the opinion of all other investors. If not a pricing equilibrium will never be reached. Suppose, that there exist four market participants but the first two take into account only the opinion of each other. Suppose, further, that the same holds for the other two participants. Then the transition matrix would be

$$\begin{bmatrix} f_{11} & f_{12} & 0 & 0 \\ f_{21} & f_{22} & 0 & 0 \\ 0 & 0 & f_{33} & f_{34} \\ 0 & 0 & f_{43} & f_{44} \end{bmatrix}$$

States 1 and 2 communicate with each other and thus, players 1 and 2 will reach homogeneous beliefs eventually. The same holds for players 3 and 4. However, the opinions of the entire group of all four individuals will not converge to a common limit and thus a pricing equilibrium will not be reached. Financially, an equilibrium is not reached in this

¹⁰⁵ A state i of a Markov chain is called periodic if and only if its period is greater than 1. If the period of a state is 1, the state in question is called aperiodic. Periodicity is a class property since all communicating states have the same period. Intuitively speaking, a state being aperiodic means that the system can remain in that state for any length of time before returning to any other state.

case simply because although all individuals have positive demands they do not recognize it and some individuals ignore the opinion of some others.

On the other hand, if there exist a group of investors who take into consideration the opinion of all other investors, but whose opinion is not taken into account by players outside this group, then a pricing equilibrium will be reached in which the opinions of investors in the group in question will bear no weight in the determination of the equilibrium price. Suppose that investors 1 and 2 represent institutional investors and investors 3 and 4 represent individual investors. If individual investors take into account the opinion of all other individual investors as well as the opinion of institutional investors, but institutional investors do not consider the opinion of individual investors, then individual investors will play no role in the determination of the final equilibrium price. For four investors of which the first two (institutional investors) do not consider the opinion of the last two, but the last two (individual investors) consider the opinion of all other investors the transition matrix will be

$$\begin{bmatrix} f_{11} & f_{12} & 0 & 0 \\ f_{21} & f_{22} & 0 & 0 \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix}$$

The opinions of individual investors (players 3 and 4) will have no bearing on the final pricing equilibrium (the equilibrium weights will be $(\phi_1, \phi_2, 0, 0)$). Individual investors recognizing this fact will choose to not participate in the final trading cycle thus incurring null demands.

Note that the derived results diverge from Harrison and Kreps's [1978] conclusion of global convergence to a unique equilibrium¹⁰⁵. The reason for such difference is due to the authors' assumption of perfect contingent foresight. That is in negotiating current price investors are assumed to form a clear and identical conception as to the price that will prevail in each future contingency. As the authors note (p. 335), however, "If one drops this utopian assumption, ... investors must turn to public information such as prices and trading volumes in order to discover what their fellow investors know and how they will react to incoming information." Such learning processes might exhibit instability as was shown.

¹⁰⁵ This is the only case in which proposition 1 is not consistent with works in which it is recognized that equilibrium is not independent of the learning process. Specifically, Proposition 1 is in agreement with Blume and Easley [1982] in that convergence to equilibrium is not guaranteed. By assuming a Bayesian learning process the above authors prove that beliefs are locally stable but globally unstable (i.e. convergence is not guaranteed.) Our result is also consistent with Bray [1982] who on the assumption that investors use different forms of least squares estimation concludes that potential instability is a real property.

Proposition 1 has some very important economic and policy implications. It indicates that the free flow of information favors price stability, whereas any impediments on the flow of information favor instability. At one extreme there is common knowledge, a state in which all information is available to all players. Under common knowledge equilibrium is imminent as shown in Essay 2. At the other extreme there is a process characterized by complete isolation of each and every investor. In this case no information is exchanged and equilibrium will never be reached. Prices are likely to exhibit violent instability. In a case more realistic than both extremes, information flows from one participant to all others but not freely (i.e., common knowledge is not immediately achievable.) In this case each consequent trading cycle reveals some additional pieces of the opinion structure puzzle until all pieces are revealed and equilibrium is reached¹⁰⁶. In the process, trading volumes are generated endogenously and prices exhibit some endogenous instability due to autonomous opinion adjustments. Since opinions are subject to autonomous changes, even if exogenous shocks are not violent, financial markets are potentially very unstable.

¹⁰⁶ If each investor had initially all information revealed to him during the process then investors would have shared the same initial estimates concerning the weights with which individual opinions enter the market clearing prices. A pricing equilibrium would have been reached in the first trading cycle.

As Davidson [1978, pp. 385-86] notes,

Recognizing the mercurial possibility of the economic system, man has, over time, devised certain institutions and rules of the game, which as long as they are operational, avoid such catastrophes by providing a foundation for a conventionality of belief in the stability of the system and hence in the quasi stability of expectations. It is the existence of spot and forward markets, money, and concurrent serial time-length money (forward) contracts and their enforceability, as well as the expectations that these institutions will continue to operate with continuity or 'orderliness' for the foreseeable future which limits the magnitude of E_e (elasticity of expectations) and keeps real world economic fluctuations in bounds¹⁰⁷.

¹⁰⁷ Stability was perhaps the reason that over the last years we observed a surge of new financial instruments which supposedly contribute to market completeness, but are priced as if markets were complete. Under mainstream financial theory, derivative instruments play no role other than reducing transactions costs and providing the general public with access to preferential rates of borrowing and divisibility. Under the proposed framework, however, derivative securities play a far more important role. They convey information about the opinion structure not contained in the price of the underlying asset, thus contributing to price stability (see for example Davidson [1978].) Skinner [1989] provides empirical evidence which strongly supports this view. He found that the listing of options on common stock is associated with a decline in the variance of the returns of these stocks (10-20% depending on the sampling

By the same token, organized financial markets also contribute to price stability by facilitating the propagation of information. This remark brings to focus some important policy implications of Proposition 1. Authorities should guarantee that the institutional structures which support market operations, function properly and will continue to do so in the future. Furthermore, disclosure requirements should be enforced since such requirements favor the informational flow, thus contributing to stability. Other than the above, authorities should not impose artificial restrictions on the operations of markets. Closing the markets in periods of high instability, imposing restrictions on the width of price fluctuations, and other such artificial impediments imposed upon the ability of markets to reflect the aggregate opinion, hinder the flow of information and are likely to contribute to further instability. Instead, authorities should guarantee the existence of market mechanisms which will continue to operate, and thus to act as informational media even in periods of high

period.) Controlling for market volatility did not alleviate the phenomenon. In addition, Skinner after testing several hypotheses, he concludes that none provides satisfactory explanation for the result. According to the theoretical view presented in this work the introduction of options on a stock should reduce autonomous variation (by revealing additional information concerning the opinion structure) but leave systematic risk, as measured by CAPM's beta, unaffected. Skinner, finds no significant change in the firms' beta around the time of options listing.

instability.¹⁰⁸

3.2.6 Equilibrium Determination and its Implications

If a pricing equilibrium is reached the equilibrium price depends on the initial disturbance and the opinion adjustment mechanism.

Proposition 2: If a pricing equilibrium is reached then beliefs are homogeneous and identical to the equilibrium price which is $\sum_{i=1}^m \phi_i O_{i0}$ where $\vec{\phi}$ is a solution to $\vec{\phi}F = \vec{\phi}$ along with $\sum_{i=1}^m \phi_i = 1$.

Proof: Given standard results of Markov Chain theory it is easy to calculate the equilibrium opinion of player i $\forall t: t \geq t_{\text{equilibrium}}$,

$$O_{it} = \sum_{i=1}^m \phi_i O_{i0},$$

where $\vec{\phi}$ is a solution to $\vec{\phi}F = \vec{\phi}$ along with $\sum_{i=1}^m \phi_i = 1$. Thus, O_{it} is independent of i . By applying the market clearing condition, it is easy to verify that the equilibrium price is identical to each individual belief. The equilibrium price is simply a weighted average of identical components.

From Proposition 2 it is easy to see that the final equilibrium price depends not only on the initial disturbance

¹⁰⁸ Perhaps the crash of 1987 and the subsequent period of instability would not be as severe as they were if the market mechanism were able to accommodate the enormous trading volumes without interruption.

(represented by the initial opinion structure $\vec{O}_0$), but also on the opinion adjustment mechanism (represented by the transition matrix F .) This result is consistent with results in section 2. The truth contained in the observed can only be defined in conjunction with the observer. For a given initial disturbance (i.e., event) the final price will depend on the learning process of investors.

Note that in the world described here (a world of non common knowledge), prices have a function in addition to market clearing. They reveal information about the structure of opinions. Such information affects each player's opinion and thus the opinion structure which in its turn affects prices. The mere fact that prices are observed affects them¹⁰⁹. Additionally, note the implicit analogue of this view to quantum physics. There's an inevitable interconnection between observer and observed¹¹⁰. Observations contain no

¹⁰⁹ As will be explained in the following section, this is the reason that the model could resolve anomalies of the weekend effect type: Prices react differently when observed continuously, and when they remain unobserved for a certain time period. Such systematic price behavior after of periods of non-observation could never be arbitrated-away.

¹¹⁰ Physicists are usually considered to be in an advantageous position with respect to economists. The reason is that physicists can construct artificially isolated (or almost isolated) environments in which experiments are conducted, whereas economists cannot. Quantum physics, however, have revealed at least one very important advantage economists have over physicists. Although, in both sciences observer and observed are inseparable, "investor behavior" is within the scope of economics, whereas "experimenter behavior"

intrinsic truth except in conjunction with the observation mechanism. Events relevant to financial markets are subject to interpretation and thus it is only the knowledge concerning the participants and the mechanism that enables us to evaluate the effect of an event (see the relative "truth" contained in it.)^{111,112}

Proposition 2 is partially consistent with results obtained in the third class of literature. We are in agreement with Bray [1982] and Blume and Easley [1982] in that

is outside the scope of physics.

¹¹¹ In fact if one had specific knowledge about the characteristics of each market participant one could construct a limit for uncertainty analogous to the Plank constant (i.e. an upper limit of the uncertainty characterizing the simultaneous determination of position and quantity of movement of an electron.)

¹¹² Compare the source, and essence of uncertainty to Von Newman's catastrophe of the infinite regress, and his classic demonstration that quantum mechanics entail an infinite regress of measurements before the quantum uncertainty can be removed: Any measuring device is itself a quantum system containing uncertainty. A second measuring device could be used to monitor the first one, but it would contain its own quantum uncertainty; and so on to infinity. Wigner and others have pointed out that this uncertainty is only terminated by decision of the experimenter. In the same fashion 1st order beliefs contain some intrinsic uncertainty. Second order beliefs could be used to remove that uncertainty, but they in turn entail some uncertainty; and so on to infinity. Again since knowledge of infinite order (common knowledge) is not achievable, the process ends and the intrinsic uncertainty is contained (specified) only through decision of the market participant. This observation suggests that common knowledge is nothing but a paradigm serving to concisely describe an "easily analyzable" attractor of the dynamic system of opinion adjustment.

equilibrium if ever reached will be different from both the investors' initial subjective beliefs and from the objective equilibrium that would prevail if all information concerning the opinion structure were common knowledge in the beginning of the process. However, proposition 2 diverges from the results obtained by the forementioned authors when it comes to the nature of equilibrium. The results derived in this dissertation suggest that at equilibrium opinions are homogeneous, and identical to the price of the asset (from proposition 2.) As Grossman argues, homogeneous opinions are self-fulfilling prophecies. The opinion of each and every participant is supported by the equilibrium, and thus the equilibrium is a rational expectations equilibrium.

However, Bray, and Blume and Easley conclude that opinion structures might converge to a non rational expectations equilibrium. Such equilibria as Feldman [1978] argues do not make economic sense. The intuitive reason is that if an investor is mistaken, at equilibrium it is trivial for the investor to realize that his/her beliefs are incorrect. As a result, the investor in question will modify his/her beliefs and the opinion structure will shift, "dragging equilibrium out of equilibrium". The difference in the results stems from differences in the assumptions concerning the opinion adjustment process. The dynamic process in this work is specifically designed to incorporate the properties of

equilibrium derived in the second essay. Bray as well as Blume and Easley do not characterize the equilibrium independently of any specific adjustment processes. As a result the seemingly rational processes adopted prove themselves to be irrational at equilibrium.

Finally, the derived results diverge from Harrison and Kreps' [1978] conclusions concerning the uniqueness of equilibrium. Contrary to Proposition 2 the above authors conclude that equilibrium is unique. The difference as explained previously is due to Harrison and Kreps' assumption of "perfect contingent foresight". In fact the authors draw caution to the direct relation between the uniqueness of equilibrium and the above assumption.

3.3.0 Some Empirical Anomalies Reexamined Theoretically

As explained in the first essay we can use the model derived in the previous section in order to explain some well documented empirical anomalies.

3.3.1 Crashes, Bubbles, and Cyclical Instability.

As shown in section 2.6 the equilibrium solution to the opinion adjustment process is $\sum_{i=1}^m \phi_i O_{i0}$ where $\vec{\phi}_{(1 \times m)}$ is a solution to $\vec{\phi} F = \vec{\phi}$ along with $\sum_{i=1}^m \phi_i = 1$. Such equilibrium price can be sharply higher (a bubble) or lower (a crash) than the initial average opinion (i.e., $(1/m) \sum_{i=1}^m O_{i0}$) or the initial average signal (i.e., $(1/m) \sum_{i=1}^m E_i$) which is directly related to the initial average opinion. Note that the average initial signal has no more significance than any other weighted average of the initial signals. However, the average signal is used as a point of reference in this section. To understand why, suppose that an observer of market activity adhered to the theory of fully or partially revealing

prices¹¹³. For such observer any increase or decrease in prices not directly associated with the initial signal would be unexplainable. Not only would such observer be surprised by price movements after the adjustment immediately following the disturbance, but also to find out that equilibrium price, more often than not, is quite different than the average initial signal¹¹⁴. Speculative "bubbles" and "crashes" would be considered anomalous occurrences. In the proposed framework, however, autonomous changes in prices will follow any exogenous disturbance¹¹⁵. Suppose, for example, that the market is composed of 2 investors. An exogenous disturbance is applied on the market which leads the two investors to form initial opinions for the price of the risky asset $O_{10} = 2$ and $O_{20} = 30$. In addition suppose that the true initial weights with which the estimates of the investors enter the market clearing price are $f_{10} = 3/4$ and $f_{20} = 1/4$. Assume that the two investors have the same variance estimates at any point in time and the same levels of risk aversion. Assume also that

¹¹³ Fully revealing and partially revealing prices are discussed in Essay 1, section 1.3.0.

¹¹⁴ The believer of fully and partially revealing prices requires the initial signals to be unbiased estimates of the equilibrium price.

¹¹⁵ The reason is that several applications of the F matrix may be needed in order for equilibrium to be reached. In financial terms, several trading cycles may be needed in order for all relevant information to become public (i.e. common knowledge)

the aggregate market value of the shares of the risky asset is very low relatively to the wealth of the two investors¹¹⁶. The last two assumptions allow the true demand weights to remain stationary throughout the process in this example. Now suppose that the estimates of the two investors concerning the true weights are $f_{11t}=1/2$, $f_{12t}=1/2$ and $f_{21t}=1/4$, $f_{22t}=3/4$, $\forall t$. It follows from Proposition 1 that an equilibrium will be reached and by Proposition 2 the equilibrium price will be $P^*=20.67$. However, it will take 7 trading cycles with corresponding market clearing prices \$9, \$17.75, \$19.94, \$20.48, \$20.62, \$20.66, \$20.67, in order for equilibrium to be reached¹¹⁷. The above price sequence will appear to the eyes of the observer who adhered to the theory of FRRE or PRRE equilibria as a speculative bubble.

Note, in addition, that relatively subtle exogenous disturbances¹¹⁸ can potentially cause violent price movements. The circumstances favoring such occurrences, although unprobable, are quite possible. The conditions under

¹¹⁶ Note that the last two assumptions, which allow the real demand weights to remain stationary throughout the process, are not made in any of the theoretical derivations.

¹¹⁷ True convergence will take several more cycles, but convergence to the nearest cent of equilibrium will be reached in 7 cycles.

¹¹⁸ Subtle exogenous disturbance implies a disturbance associated with a signal vector which has a mean close to the previous equilibrium and a small variance.

words, for a given initial disturbance, the F matrix that will yield equilibrium solutions which are sharply different than the average initial signal will be identified.

As shown, $\bar{\phi}$, the vector of equilibrium weights solves $\bar{\phi}F = \bar{\phi}$ along with $\sum_{i=1}^m \phi_i = 1$. Thus, it suffices to find F such that the equilibrium weights corresponding to an extreme region of the initial disturbance $\bar{O}_0$ are unusually high. Replacing one of the columns of the $\bar{\phi}F = \bar{\phi}$ linear system with $\sum_{i=1}^m \phi_i = 1$ and solving for ϕ_i ($i = 1$ for expositional convenience)

$$\phi_i = \begin{bmatrix} 0 & f_{21} & \dots & f_{m1} \\ 0 & f_{22}-1 & \dots & f_{m2} \\ \vdots & \vdots & & \vdots \\ 0 & f_{2m-1} & \dots & f_{mm-1} \\ 1 & 1 & \dots & 1 \end{bmatrix} / |F'| = |F_1'| / |F'|$$

where F' is $(F-[I])^T$ ¹²⁰ with the last of its rows replaced by $\bar{1}$. For a ϕ_i corresponding to an extreme region of the opinion structure to be high, $|F_i'|$ should be large relative to $|F'|$, that is F_i' should be heterogeneous relative to F' . Since F_i' is F' with its i th column replaced by the vector $[0, 0, \dots, 1]^T$, for $|F_i'| / |F'|$ to be high, the remaining columns of F' should be heterogeneous relative to F' . In economic terms the i th column of F' represents the i th player's

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T superscript signifies transpose.

of F' should be heterogeneous relatively to F' . In economic terms the i th column of F' represents the i th player's estimate about the weights individual estimates carry in the determination of the market clearing price. Thus, the i th individual's weight will be higher the more the individual agrees with others and the more others disagree among themselves. In other words, crashes and bubbles are likely to happen when among general disagreement there exist regions corresponding to extreme values of the opinion structure in which reigns relative agreement. The above result makes intuitive sense, since when investors generally disagree in their estimates of the form of the adjustment process, relatively homogeneous investors can attract others in sharing their beliefs. Player's who are "wrong" are likely to be proven "right" when other player's are "very wrong".

Still another case in which crashes or bubbles are likely is when a set of investors corresponding to an extreme and relatively homogeneous region of the opinion structure ignore the opinion of others but their opinion is not ignored. In other words, some vectors of the F matrix, corresponding to extreme and relatively homogeneous opinions, are formed by long series of 0's and short series of positive constants where the constants correspond to weights attributed to investors within the group. Results in section 2.5 suggest that the equilibrium price in this case will be close to the

extreme relatively homogeneous opinion of the group in question which ignores others, but is not ignored.

As an example consider the following scenario. Suppose that stock prices followed a sharply increasing trend over the last few months¹²⁰. Further, suppose that the majority of investors believe that the trend will continue, but disagree on the degree of further price increases. Some believe that prices will increase greatly, others believe that the increase will be imperceptible and others still are placed between the two extremes. Finally, suppose that a relatively small group of "fundamentalists" believe that prices are too high and will eventually return to sharply lower levels. Members of the latter group using fundamental analysis which is more or less standardized, and which utilizes publicly available data, such as Price-Earnings ratios and Dividend Payout ratios, arrive at similar conclusions concerning the level to which prices should drop. Thus, on one hand, there is a heterogeneous majority which takes into account all opinions, and on the other hand, a relatively homogeneous minority of fundamentalists which ignores the opinion of non-fundamentalists. The derived results suggest that in this case, although, the vast majority believes that prices will increase, prices will eventually decline reaching an

¹²⁰ As for example, during the months preceding August of 1987.

equilibrium level close to the homogeneous opinion of the fundamentalists. The only reason, however, prices will decline and the fundamentalists will be proven correct is that the fundamentalist group constitutes a relatively homogeneous niche which ignores an otherwise heterogeneous market and not because fundamental analysis is ex-ante "more correct" than the analysis followed by the majority of investors.

Finally, as shown in section 3.2.4, hindrances on the flow of information are likely to cause cyclical instability in prices. If there exist classes of investors in, or near, informational isolation from each other they will tend to continue to disagree. Price by itself is not a fully appropriate channel for bridging the informational gaps. Thus, instability prevails.

As an example consider again the situation described previously and assume further that as opinions decline, driving prices toward their new equilibrium level, very high volumes of trading are generated. Finally, suppose that the market structure not being able to handle the increased volume closes or allows the creation of long back-order queues¹²¹. In either case, the currently observed market clearing conditions do not properly take into account the current

¹²¹ The conditions in this hypothetical scenario were intentionally chosen to resemble those that prevailed on October 17, 1987 and the subsequent days.

aggregate opinion structure. In other words the informational link between investors has been cut or has deteriorated. As explained previously, the derived results suggest that in this case further instability will prevail.¹²²

3.3.2 The Weekend Effect.

As shown in the previous section the opinion of each investor concerning the price of the asset in period 0 includes a risk adjustment factor which is inversely related to the estimated Bayesian variance of the asset.

$$O_{j0} = \sum_{i=1}^m (d_{ji0} / \sum_{i=1}^m d_{ji0}) E_i - S / \sum_{i=1}^m d_{ji0}.$$

where d_{ji0} is j th investor's estimate of $d_{i0}' = W_i / (R_i \sigma_{i0}^2)$.

In periods during which the market is closed the channel of communication between investors is cut. The market fails to perform as the informational medium through which information about the belief structure is propagated. As shown previously such condition is likely to cause persistent heterogeneity of opinions and instability, thus increasing perceived risk (i.e., the j th investor's estimate of σ_{i0}^2 , $V_{i,j}$.) Commensurate with such risk, investors will perceive a relative advantage in investing at the risk free rate, and thus, the price of the

¹²² Several additional real examples of speculative bubbles along with empirical evidence on destabilizing rational speculation is presented in DeLong, Shleifer, Summers, and Waldman [1980].

risky asset will drop. In other words, the risk adjustment factor will increase, thus decreasing opinions O_{j0} and subsequently the market clearing price of the risky asset in period 0, P_0 ¹²³. In addition note that during weekends there exists a time interval¹²⁴ during which all major international stock markets are closed. If global events are as likely to happen in the weekend as in any other weekday, and in addition they are as likely to be positive as to be negative, then on the average prices should drop on Mondays.

Consider the following two hypothetical scenarios. In the first case, an important global event takes place on a Wednesday. In the second, the same hypothetical event takes place on a Saturday. In the former scenario U.S. investors for example will evaluate the effect of the disturbance on stock prices taking into account observed market clearing conditions. Even if the American organized markets are closed at the time of the event, investors will take into account market clearing conditions in other world markets as for example in the Japanese stock exchanges. In the second case, however, investors will be forced to evaluate the effect of the event without the aid of any market clearing conditions in

¹²³ See Appendix 3.1 and section 3.2.3.

¹²⁴ The approximate length of this time interval is 64 hours minus the time lag between the two exchanges which are furthest apart in time.

organized markets, since all international markets will be closed at the time of the disturbance. As a result, investors in the second case, operating in relative isolation from each other, are likely to reach more heterogeneous opinions concerning the disturbance. Recognizing the above will lead the investors in question to raise their estimates of future price variance. The reason is that the range of potential market clearing prices is limited by the range of the initial opinion structure¹²⁵. As a result, the risk adjustment factor when markets will reopen will be higher, and thus prices will be lower.

In order to evaluate the magnitude of the effect consider the following numerical application. Suppose that the average signal $(1 / m) \sum_{i=1}^m E_i$, is equal to \$100 in both scenarios above. All investors have the same wealth and risk aversion levels and in addition each investor estimates the wealth and risk aversion levels of all other investors to be equal to his own. Further, suppose that in the first scenario above investors have a common estimate of the variance of the asset

¹²⁵ Any opinion concerning price is a weighted average of all previous opinions. As a result, it cannot be higher than the highest previous opinion, or lower than the lowest. Further, all market clearing prices, including the first one, are a weighted average of all concurrent opinions. Thus, market clearing prices are also limited by the concurrent opinion range. It is easy to see by induction that all market clearing prices, including the equilibrium price, are limited by the range of the initial opinion structure.

price which is $\sigma_0^2 = 25$ and in the second scenario $\sigma'_0{}^2 = 40$. Each investor's estimates other investor variance estimates correctly in both scenarios. By directly applying the expression for O_{j0} derived in the Appendix 3.1, it is easy to calculate that if the risk adjustment factor reduces the average signal E_i by 10% in the first scenario, it will reduce the average signal by 16% in the second scenario. In general under the above conditions if a variance estimate of $\sigma_0^2 = 25$ reduces the initial average signal by 10% in the first scenario, a variance estimate of $\sigma'_0{}^2$ will reduce the average signal by $.4\sigma'_0{}^2$ in the second scenario. In addition by applying the expression derived for first market clearing price P_0 it is easy to see that P_0 will be proportionately lower in the second scenario.

The explanation provided here for the Weekend Effect suggests that the longer the period of non-observation the more pronounced the weekend effect will be. With the passage of time as new major exchanges in different time zones begin to operate the time gap between Friday closing and Monday openings is likely to shorten. Consequently, the explanation offered here for the "weekend effect" suggests that the effect is less pronounced in more recent periods. Empirical evidence which supports the above is provided by Lakonishok and Levi [1982]. They find that the negative return on Mondays is more substantial in the period 1968-73 (-0.208%) than in 1974-79

(-0.020%). Similar evidence is presented by French [1980].

Since the described effect is based on an intermediate property of the opinion adjustment process, and not on an equilibrium property, the explanation offered is practically identical with explanations consistent with the first class of literature¹²⁶ which views financial markets in a static manner. However, one important feature of the "Weekend Effect" escapes the static nature of the first class of literature. Although, the increased intrinsic uncertainty related to weekends will attenuate eventually as the market reaches equilibrium (if equilibrium is reached), the equilibrium price as demonstrated in section 3.2.6 is likely to be different than in the case where organized markets operated with no interruption.

3.3.3 The Small Firm Effect.

To understand the "small firm" effect within the proposed framework, the opinion adjustment process should be first examined in reverse. Suppose that firms A and B end up with the same equilibrium price per share after an exogenous shock. Further, suppose that firm B is a small, infrequently trading firm with a "low" public profile. Firm A is a frequently

¹²⁶ The first class of literature along with consistent explanations for the "Weekend Effect" is presented in Essay 1. For an alternative presentation see Huang and Litzenberger [1988, p.270].

trading, "high" profile firm¹²⁷ but the two firms are similar in all other respects. Consider the two firms immediately following the exogenous shock. Since the two firms are hypothesized to be similar, the initial signal structures with respect to the value of the two firms are likely to have the same mean. However, firm B, the low profile firm, is likely to be associated with more diverse signals. If investors recognize the above, the Bayesian estimates (i.e. subjective conditional estimates) for the variance of the estimation errors will be higher, ceteris paribus. As a result, the risk adjustment factor reducing each initial individual opinion will be higher for firm B¹²⁸. Therefore, the initial market clearing price of firm B, as a weighted average of all individual initial opinions, will be lower¹²⁹. Since it was hypothesized that the two firms ended up with the same price per share, firm B has realized a higher return.

Next, consider the process from the beginning. Suppose that firm D is a "low" profile infrequently trading firm, whereas C is a frequently trading "high" profile firm. As

¹²⁷ By characterizing a firm as "low" ("high") profile we imply that a relatively small (large) portion of the firm specific characteristics has become public information.

¹²⁸ See Appendix 3.1.

¹²⁹ See section 3.2.3.

explained above firm D will have a lower price per share immediately following the mean preserving exogenous shock. However, there exists no reason whatsoever for firm D to end up with a lower equilibrium price than firm C. Putting the two pieces together it can be concluded that infrequently trading, "low" profile firms will exhibit higher returns on the average. If small firms have in general a lower public profile and trade more infrequently than larger firms they should earn higher returns on the average. In other words, the initial belief structure associated with small firms involves greater intrinsic uncertainty than the belief structure associated with larger firms but in the end, common knowledge prevails in both cases. Since equilibria come after the elimination of greater intrinsic uncertainty, small firms should exhibit higher returns on the average.

In addition note that if returns were adjusted for risk in a CAPM framework the effect would persist. The reason is that the risk for which investors compensate by requiring a higher return for infrequently trading, low profile firms, is associated with endogenously produced variation and not with the firm's sensitivity to exogenous variation¹³⁰ which is

¹³⁰ If a low level of security research from analysts is associated, as would be expected, with more diverse opinion structures, Arbel and Strebel's [1982] findings support this idea. The above authors found that a low level of security research is not reflected in a firm's ex-post systematic risk, but is associated with higher average returns.

measured by a firm's Beta in a CAPM framework. However, if other measures of risk as for example divergence of opinion were used and the returns were adjusted for them, the effect could disappear. Empirical evidence presented in the first essay strongly supports this view¹³¹.

To add some intuition to the explanation offered for the "small firm effect" consider the following scenario. A small number of investors possesses inside information concerning two similar firms A and B. Further, assume that the only difference between Firms A and B is that firm A is a large "high profile" firm, associated with relatively homogeneous opinion structures, whereas firm B is a more obscure, small firm, associated with more heterogeneous opinion structures. In this case, it is natural for the investors possessing the inside information to expect that the public opinion concerning firm A is more likely to reflect their private information within a predictable and short time frame. The reason is that analysts of firm A, and the public in general are more likely to infer the inside information from highly visible market clearing conditions concerning firm A. By the same token it might take the public a great deal of time of

¹³¹ Cragg and Malkiel [1982] have studied the relation between ex-post returns and various measures of risk for common stocks and found that the measure of risk which performed best in their analysis was a measure of divergence of opinion about the asset returns.

unpredictable length to infer the inside information concerning firm B. In the process new exogenous disturbances or autonomous price changes might drive the price of B to very unpredictable levels. In this case the informed investors will prefer to invest in firm A, unless firm B promises a higher expected return. On the other hand, there exists no reason for firm A to end up with a higher price on the average. Thus, firm A will realize a higher return than firm A on the average.

In order to form a rough estimate concerning the magnitude of the "small firm effect" consider the following numerical application. Suppose that there exist two similar firms, firm A and firm B as in the example above. An exogenous shock is applied to both. It is natural to expect that in this case investors will have more homogeneous opinion structures and lower price variance estimates for firm A than for firm B. The reason, as explained, is that in the case of firm A tighter signal structures and highly visible and frequent market clearing conditions make convergence to a more predictable range of prices, within a shorter time period, more likely. Assume that all values of the relevant parameters are the same as in the numerical example in section 3.3.2, where firms A and B are analogous to the first and second scenario respectively. The risk adjustment factor reduces the initial average signal associated with firm A by 10%, and with

firm B by 16%. Although, at the final equilibrium price will depend on the opinion transition matrix associated with each firm, it is certain that for all potential transition matrices that conform to the convergence conditions derived previously, all uncertainty will be eliminated at equilibrium. Thus, at equilibrium variance estimates are 0. As a result, the risk adjustment factors will be 0 for both firms. Consequently, on the average the small firm B will realize a 16% return relatively to the initial average signal, whereas a firm A will realize a 10% return only.

As explained in Essay 1 the arguments used here to explain the small firm effect are essentially informational. In other words, it is suggested that the reason for the apparent negative correlation between common stock returns and the aggregate market value of corresponding firms is that firm size serves as a proxy for the divergence of opinions and the resulting intrinsic uncertainty. Empirical evidence presented by Arbel and Strebel [1982] supports the informational nature of these arguments. They found that the most neglected securities performed best in each of the years studied. In addition, they found that this neglected firm effect exists beyond the small firm effect, and moreover, their results suggested that the small firm effect is subsumed by the neglected firm effect. Barry and Brown [1984] showed empirically that a crude measure of differential information

(e.g., period of listing) is associated with the firm size anomaly. Although, relatively short periods of listing are not necessarily associated with greater intrinsic uncertainty, with the passage of time, the public profile of a firm should expand. As a result, investors should value a firm more homogeneously after exogenous shocks as the firm's period of listing increases. Thus, intrinsic uncertainty following exogenous shocks should decrease as period of listing increases, ceteris paribus.

Finally, note that although Arbel and Strebel, Barry and Brown, and others have used informational arguments to explain the small firm effect, they did not offer a complete theoretical demonstration of how specific informational characteristics of small firms might explain the firm size anomaly.

3.4.0 Concluding Remarks

In this essay it was shown that equilibrium prices depend on individual signals and the learning process followed by investors. Further, equilibrium prices are in general different than both the initial opinions and the equilibrium prices that would have emerged if common knowledge held immediately following the initial shock. Finally, if information is not propagated among investors during consecutive trading cycles, equilibrium will never be reached. How do these conclusions compare to the fundamentalist view? The essence of fundamentalism which dates back at least to Williams [1938] is that a stock has an intrinsic value stemming from its potential to pay dividends in the future. This is because a stock represents ownership of a firm and dividends represent income the firm earns for its owners. On the other hand, beginning with the idea that stock prices are created by investors and recognizing that investors may form different first or higher order beliefs, it is contended that there can be no objective intrinsic value for the stock without specifying many characteristics of the market within which the stock is traded. Instead, it was argued that the

equilibrium price revealed eventually (if ever) represents the market aggregation of individual attitudes and investor beliefs. The reason a particular investor adjusts his beliefs throughout the process is that the investor does not initially have all relevant information, and thus consecutive trading cycles reveal additional information which leads the investor in question to change his beliefs.

As beliefs change, the set of all relevant information, whether revealed or not, changes as well. As a result the complete set of information eventually revealed will differ in general from the initial complete set of information. Thus, the equilibrium price which reflects the complete set of information will in general differ from both the initial beliefs and the equilibrium price that would have emerged if the initial set of all relevant information was available to all investors. In other words, investors at any particular point in time may be unable to achieve positions with which they will be forever content, even in the absence of further exogenous shocks. Thus, the stock price will be affected by whether or not the markets will reopen in the future.

If markets do reopen a speculative phenomenon¹³² will appear. Keynes (1936, Ch. 12) attributes primary importance to

¹³² "Speculative phenomenon" implies here trading an asset which is not entirely demanded for its own sake, but also for resale (or potential resale) in the future.

this phenomenon and goes on to suggest that it might be better if markets never reopen¹³³. The properties of asset prices derived in this work may appear curious to a fundamentalist, but it is precisely these properties that allowed us to explain some well documented anomalies in the last portion of this essay.

¹³³ As shown, opinions and prices can change autonomously under real world intrinsic uncertainty and heterogeneity of opinions. Thus, reopening of markets can be associated with autonomous instability. If markets closed immediately after the first trading cycle following an exogenous disturbance, then intrinsic uncertainty would be of little concern to investors, and thus, prices would perhaps be more stable. The problem, however, with Keynes' statement is that autonomous trading cannot be separated from trading directly linked to exogenous disturbances. Thus, if markets were closed for autonomous trading they would be closed for non-autonomous trading as well. As a result, investors would find themselves further and further away from optimality with respect to ever changing real conditions to which they would not be able to adjust.

Appendix 3.1

Going through the same maximization-of-utility-with-respect to-demand problem as in section 3.2.2, but substituting opinion E_i for the initial signal O_{it} (there is no other information to be taken into account by each individual at this time)

$$D_{i0}' = d_{i0}' (E_i - P_0),$$

where

$$d_{i0}' = W_i / (R_i \sigma_{i0}^2),$$

where W_i is the i th individual's wealth before the initial disturbance, R_i is the i th individual's level of risk aversion, and σ_{i0}^2 the individual's estimate for the variance of the error in his signal.

The initial demand D_{i0} is notional in the sense that investors will never trade directly based on it. The reason is that individual investors being rational they realize that such demand is based only on their individual signals and as such it does not take into account the signals of other investors. Instead, each investor j will attempt to form an estimate P_{j0} of the corresponding notional market clearing price P_0 which will take into account all individual signals E_i in the following way. First each individual j forms an

estimate d_{ji0} for all notional d_{i0} 's since, as hypothesized initially, each individual knows the signals other investors received but not their individual demand functions. Subsequently, each individual will apply the market clearing condition which should take into account the fixed initial supply of the asset S (i.e number of shares outstanding) which is nothing more than a constant negative demand function¹³⁴.

$$\sum_{i=1}^m D_{ji0} - S = 0 \Leftrightarrow$$

$$\sum_{i=1}^m (d_{ji0} (E_i - P_{j0})) - S = 0 \Leftrightarrow$$

$$\sum_{i=1}^m (d_{ji0} E_i) - \sum_{i=1}^m (d_{ji0} P_{j0}) - S = 0 \Leftrightarrow$$

$$P_{j0} = \sum_{i=1}^m (d_{ji0} / \sum_{i=1}^m d_{ji0}) E_i - S / \sum_{i=1}^m d_{ji0}.^{135}$$

Since the estimated P_{j0} takes into account all information available at this point to investor j , it

¹³⁴ The fixed supply of the risky asset should appear in the initial notional market clearing condition and there only, unless a change occurs subsequently in the fixed supply of the asset (as for example when a firm issues new shares.) If the fixed stock supply is not taken into account initially, a problem of indeterminacy will arise. Several prices might clear the market depending on the fixed supply of the risky asset. On the other hand, in subsequent real (not notional) trading cycles, equating the total demand to 0 guarantees the constancy of the fixed supply S of the risky asset. In the case where the outstanding stock of the risky asset changes during the process, the reduction or increase of shares should be included as a constant in the market clearing condition corresponding to the time of the change.

¹³⁵ The above expression for P_{j0} is identical to the expression derived for the equilibrium price in the first class of literature discussed in essay 1. In our framework, however P_{j0} is only an initial estimate and a pricing equilibrium is far from being reached as yet.

constitutes his initial opinion O_{j0} concerning the value of the asset.

$$O_{j0} = \sum_{i=1}^m (d_{ji0} / \sum_{i=1}^m d_{ji0}) E_i - S / \sum_{i=1}^m d_{ji0}.$$

Thus, each individual signal E_i will enter the initial opinion of player j with a greater weight the less risk averse and the more wealthy individual j is estimated to be, and the more precise corresponding signal. Similarly, risk adjustment factor will be smaller when individuals are estimated to be less risk averse, and the signals are more precise. In addition, the initial opinion of each individual is inversely related to the fixed supply¹³⁶ of the risky asset, ceteris paribus.

¹³⁶ Note that here, as in the first class of literature, the fixed supply of the risky asset represents the capital invested in the risky opportunity and not merely the number of shares outstanding. If for example, a firm issued stock in order to cover cash flow shortages related to current investments, and not in order to undertake new investment opportunities, the increase in the number of shares outstanding should not be considered as an increase in the fixed supply of the risky asset.

S U M M A R Y A N D C O N C L U S I O N S

The major contribution of this research endeavor is to provide an alternative to the fundamentalist view in Finance, It shows that if the paradoxical assumption of common knowledge at all times with respect to all relevant events is relaxed, then a risky asset does not have an intrinsic value. Instead its value can only be determined within the particular market in which the asset is traded.

The essence of fundamentalism in finance, which dates back at least to Williams [1938] and which transcends almost all current state of the art theoretical models, is that a risky asset has an intrinsic value stemming from its potential to produce positive cash flows in the future. Financial models based on this view may be logically impeccable within their own assumptions and may even withstand some econometric tests. However, such models fail to recognize the dependence of outcomes on beliefs which are more likely to be heterogeneous rather than homogeneous at any given point in time. In models based on the fundamentalist view, realized events, for example asset prices, are assumed to be determined by the stable over-time path of the economic system. Consequently, any errors in

expectations are assumed to be reduced over time as investor beliefs "zero-in" on the equilibrium path of the financial markets. As a result, models based on the fundamentalist view, essentially permit the use of a certainty equivalent system with known parameters as the generator of the path towards which the real financial market has a propensity to approach. These models ignore the fact that "erroneous" expectations¹³⁷ are associated with trading which releases information in the period where such errors occur. Thus, whatever equilibrium the market converges to, if it converges at all, must be different than the equilibrium which would have been reached if the markets were in a state of common knowledge at all times.

Another major contribution of this work is to show that pricing equilibria are synonymous with common knowledge with respect to all relevant information. In addition, any pricing equilibrium is a rational expectations equilibrium since expectations at equilibrium are shown to be homogeneous and confirmed by the relevant observables (i.e., prices, volumes.) These properties of pricing equilibria are demonstrated in a general topological framework. Then they are used in order to

¹³⁷ Heterogeneity of expectations guarantees that tomorrow's market clearing conditions will prove most of today's expectations wrong, so that further endogenous trading and ever changing parameters of any stochastic approximation of market returns are inevitable in the real world.

construct a simple model of the opinion adjustment process¹³⁸. Subsequently the stability of the above model is examined. It is found that convergence of the opinion adjustment process to equilibrium is not guaranteed but is favored by relatively uninhibited flow of information among investors.

The last contribution of this research endeavor is to provide some possible explanations for some phenomena viewed as "anomalies" under the current state of the art financial framework. The explanations offered are direct consequences of the model of the opinion adjustment process developed in the third essay and rely on the crucial concepts of autonomous variation¹³⁹ and intrinsic uncertainty.

Finally, consider two distinct lines of further theoretical research which are worth pursuing. As explained in Essay 3 a restricting set of assumptions¹⁴⁰ has been imposed on the opinion adjustment process in order to allow for a

¹³⁸ Through this model it is formally demonstrated the dependence of outcomes on beliefs and the consequent dependence of the asset's value upon many of the characteristics of the market within which the stock is traded (i.e. risky assets have no intrinsic value)

¹³⁹ Autonomous variation implies price movements not directly associated with exogenous events, but due, as shown, to the existence of intrinsic uncertainty.

¹⁴⁰ These assumptions include fixing the set of investors, and in addition, fixing the estimates of each investor with respect to the weights with which the opinions of all other investors enter the market clearing equation.

stationary transition matrix. Since in reality not only expectations but also the way expectations are formed are prone to change, it would be theoretically desirable to relax these assumptions, thus, allowing the transition matrix to vary over time¹⁴¹. Except being a more realistic model of the opinion adjustment process, such model might also lead to a rigorous theory of trading volume.

The second line of theoretical research worth pursuing is related to the application of some of the concepts developed here in Macroeconomics. In addition, since no satisfactory theory can claim that the relation between stock prices and macroeconomic variables is one-directional¹⁴², it would be desirable to integrate a model representing the economy at large with a model representing the stock market mechanism.

In order to examine the application of some of the concepts developed here to macroeconomics first consider the Keynesian view of the economy. Keynes viewed an economic

¹⁴¹ Designing the structure of a realistic model with a non-stationary transition matrix is quite easy given results in the second essay, but the mathematical complications associated with its analysis are quite formidable. The reason is that there exists no standardized method for solving systems of difference equations with time-variant transition matrices.

¹⁴² Perhaps the most important drawback of all pricing models in finance, including the one developed here, is their inability to endogenize exogenous disturbances. By considering macroeconomic variables as exogenous, these models essentially assume that the level of stock prices has no effect whatsoever on the macroeconomy.

system as a system which "lurches" from one historical position to another without even necessarily being at equilibrium. In addition, he demonstrated within his assumptions that booms and slumps need not be the result of occasionally faulty entrepreneurial expectations, but that a monetary system could settle as a theoretical matter to any level of prices and employment¹⁴³.

The vague similarity between the views proposed here and the Keynesian results¹⁴⁴ is not accidental¹⁴⁵. Keynes' theory relied on the fundamental distinction between risk and uncertainty in the Knightian sense. For Keynes, as Knight¹⁴⁶ had maintained at about the same time risk is measurable and hence is distinctly different from uncertainty which is incapable of measurement in a probabilistic sense. Results in

¹⁴³ For an in depth presentation of Keynesian views see Davidson [1978]

¹⁴⁴ Contrary to the fundamentalist view in Finance which follows the spirit of Classical or Neoclassical Economics.

¹⁴⁵ The vague parallel between the views expressed in this dissertation and the Keynesian views is limited to similarities with Keynes' general idea that the distinction between risk and uncertainty is essential in economic modeling and that uncertainty is the root of economic disequilibrium. The parallel does not extend to the specific applications that Keynes pursued in the General Theory of Employment Interest and Money [1936]. For example, in this dissertation there is no assumption similar to Keynes' hypothesis that downward sticky wages cause excess supply of labor. According to the views expressed here, disequilibrium is never related to the market's inability to clear.

¹⁴⁶ see Knight [1937, p. xxiv.]

this dissertation, on the other hand, are based on the crucial assumption that each investor realizes that prices are created by investors and are subject to change along with investor beliefs. Thus, each investor has an incentive to estimate the beliefs of others. In addition, if common knowledge does not hold initially, investors in their attempt to estimate other investors' beliefs will affect the complete set of information which if eventually revealed will determine investor common knowledge beliefs and thus, equilibrium prices. In a framework of non-common knowledge, purely probabilistic views of the market become futile because investor's attempts to predict the future result in errors which in turn lead to actions (i.e., trading) which will affect the future outcomes that investors attempt to predict. In statistical terms, intrinsic uncertainty will severely bias any probabilistic characterization of extrinsic uncertainty¹⁴⁷. Thus, it is maintained that purely probabilistic views which eventually lead to equilibrium models are not adequate for describing the real world financial markets.

¹⁴⁷ Each round of trading in our framework resembles what Shackle [1955] calls a "crucial experiment". In a crucial experiment the decision maker believes in the possibility that the very choice of a course of action may alter the existing circumstances in such a manner which makes quick restoration of the situation, if the results turn-out to be undesirable, exceedingly costly if not impossible. In systems where crucial experiments are concerned, Shackle maintained, probability statements are not applicable to evaluating the possible outcomes of any action.

The above view is to an extent consistent with the general Keynesian view of the economy. The problem, however, with the Keynesian view is that after it refutes the capability of the calculus of probability to characterize uncertainty, it does little to characterize uncertainty otherwise. As a result, the Keynesian models lack the mathematical elegance, determinacy and tractability of classical economics¹⁴⁸.

At the risk of gross overstatement, it is suggested that the well developed and precise "intrinsic uncertainty" may replace the undeterminate but far more general Keynesian (i.e., Knightian) uncertainty in its role in the economy. The reason is that Keynesian uncertainty and intrinsic uncertainty lead to systems which need to be studied out-of-equilibrium, simply because "a world in which expectations are liable to be falsified cannot be described by the simple equations of the

¹⁴⁸ For example, Kenneth J. Arrow in response to a letter by Paul Davidson, in which the latter indicated that Keynes' theory was at variance with general equilibrium theory, the former responded: "I am not yet convinced that the distinction between uncertainty and risk is really vital... The problem in all this is to create something which is genuine theory, that is, a well articulated framework with implications. It is interesting that the parts of Keynesian theory that became popular were, as you suggest, a retrogression from Keynes' deeper insights; but this is no accident. One part of the theory was in fact capable of development even though it was an impoverished version of the whole; the other part, deeper though it was, could not in fact be developed." (see Davidson [1978, p.373].)

equilibrium path."^{149,150} In addition, intrinsic uncertainty is a well developed feature of any system the future of which is never known with certainty to those whose interacting behavior ultimately determines it. As a result the concepts related to intrinsic uncertainty may lead to a model of the economy which proceeds by first specifying the particular set of values at a moment in time -where these values need not be equilibrium ones-and then shows how the interaction of these non equilibrium values leads to change and potentially to an equilibrium the features of which have been prespecified.

Although, there is no doubt that the above research endeavor would constitute a formidable task, if successful, it will perhaps prove that the 50-year old perceived tradeoff

¹⁴⁹ Robinson [1963, p.25]

¹⁵⁰ Hicks for example indicated that he had mixed feelings in accepting the Nobel Prize in 1972 (which he shared with Arrow) for his work on general equilibrium and welfare economics. Hicks [1977, pp. v-vi) states: "This is work which has become part of the standard literature of what is called in modern controversy 'neo-classical economics. But it was done a long time ago, and it was with mixed feeling that I found myself honored for that work, which I myself felt to have outgrown..."

Where I now feel that I went wrong was in my attempt to represent the markets of that week [any particular period] as being in equilibrium, even in general equilibrium in the sense of my static theory."

between realism and precision¹⁵¹ is non existent after all.

¹⁵¹ Keynes [1973, vol.XIV, p. 2] for example responded as follows to a letter by Shoves in which the latter expressed the same need for precision as many modern Neoclassical scholars: "...you ought to not feel inhibited by a difficulty in making the solution precise. It may be that part of the error in classical analysis is due to that attempt. As soon as one is dealing with the influence of expectations and of transitory experience, one is, in the nature of things, outside the realm of the formally exact."

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