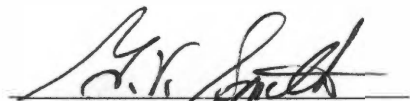


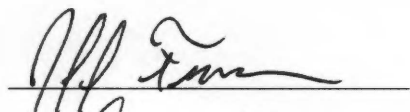
To the Graduate Council:

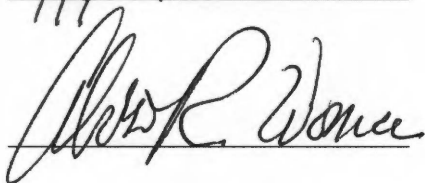
I am submitting herewith a dissertation written by Xinyan Wang entitled "Modification and Evaluation of Continuous Roll Prediction Model for Front Drive Mowers." I have examined the final paper copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Biosystems Engineering.


Paul D. Ayers, Major Professor

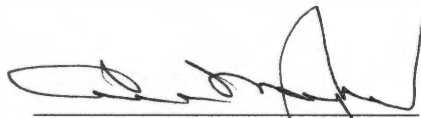
We have read this dissertation
and recommend its acceptance:







Accepted for the Council:



Vice Chancellor and Dean of
Graduate Studies

AG-VET-MED.

Thesis

2005b

.w24

**MODIFICATION AND EVALUATION OF CONTINUOUS
ROLL PREDICTION MODEL FOR FRONT DRIVE
MOWERS**

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Xinyan Wang

December 2005

DEDICATION

This dissertation is dedicated to my husband, Jiuqiu Gui, and to my daughter, Tian Gui for their constant love and support.

ACKNOWLEDGEMENTS

I wish to thank all those who helped me complete my Ph.D. in Biosystems Engineering & Environmental Sciences. Great thanks are given to the members of my graduate committee: Dr. Paul D. Ayers, Dr. G.V. Smith, Dr. Jeffrey Freeman, and Dr. Alvin Womac. Thank you for taking time from your extremely busy schedule to help me. I would like to thank my major professor Dr. Ayers for his guidance, suggestions and varied test methods. I am grateful to Dr. G.V. Smith for his suggestions in my ROPS height adjustable fitting design and the system control knowledge I learned from his class. Thank you Dr. Jeffrey Freeman for the mechanical dynamics knowledge you taught me. It helped me a lot. Thank you Dr. Alvin Womac for your focus, and attention to my project. I clearly remember you reminding me after my presentation for the Seminar 505. "There is yaw, the motions more complex. Remove the deck, do the test again".

To Craig A. Wagoner and Dr. William E. Hart who made the ROPS height adjustable fitting on my project, thank you for the support. Thanks are given to graduate students, Robert Comer, Adam Fiscor, Christopher Butler, Ken Swinson, Matthew Rice, Bryan Biss and Clifton Andrew Robertson for their help in making the roll test for this project. Finally, I wish to acknowledge John Deere, Inc for the donation of an F925 mower and CDC/NIOSH and Colorado State University for financially supporting this work.

ABSTRACT

The recently approved ASAE Standard S547 “Tip-Over Protective Structure (TOPS) for Front Wheel Drive Turf and Landscape Equipment” addressed a continuous roll prediction model for Roll-Over Protective Structure (ROPS) design. The existing model described in this Standard did not take into account the influence of the mower deck on the rollover behavior. In order to evaluate the accuracy of the original model, according to the ASAE S547 requirement, a 4.05 meters long and 3.42 meters wide slope of 35 degrees was constructed at the University of Tennessee. Lateral upset tests for Deere F925 front drive mower with regular and inverted ROPS of 1.9 m and 2.22 m were conducted. These tests indicated the mower deck influenced the rollover behavior. In addition, the mower yaw (rotation) and slide downhill were also observed. The existing model described in ASAE S547 did not accurately predict the roll behavior of the mower. Therefore, this project involved evaluating the accuracy of the existing continuous roll model and modifying it to include the mower deck size, yaw and slide downhill.

Due to the deck size, the mower underwent a more complex combination of roll, yaw and slide down hill. By adding the deck size, the tipping axes changed, therefore, the potential energy, the moment of inertia and kinetic energy changed. These changes affected the roll behavior. Due to the yaw, the direction of the tipping axis changed, the

equivalent slope angle increased, i.e. the equivalent slope became steeper, and the continuous roll tendency increased.

The existing model was revised to include 1) the deck size, 2) yaw and 3) slide down the slope. This revision included the tipping axes rotation and mass moment of inertia transformation. By increasing the ROPS height, the roll behavior could be changed from continuous roll to non-continuous roll. Using the revised model, the minimum ROPS height required to stop the roll (critical ROPS height (CRH)) could be determined. The CRH predicted by the revised model was 2.50 m, and 2.63 m, with the deck transportation and working position, respectively, and ROPS in the regular position (the top of ROPS tilting backward). The CRH predicted by the revised model for the ROPS in the inverted position (the top of ROPS tilting forward) was 2.00 m.

In order to evaluate the accuracy of the revised model, field tests with a Deere F925 mower at ROPS height of 1.90 m, 2.22 m, 2.42 m, 2.55 m, and 2.67 m were conducted. For Deere F925 mower with the regular ROPS, the measured CRH were 2.55 m with the deck in transportation position (up) and 2.67 m with the deck in working position (down). For Deere F925 mower with the inverted ROPS, field tests showed a CRH of 2.03 m with deck in mowing position.

The reason of the greater roll tendency with deck down is that the deck height influences two parameters, the initial vertical height to center of gravity, and the tipping axis (i.e. the distance between CG and the tipping axis). As the deck height

(D3) is decreased, although the initial vertical height to the CG decreases, the change of the tipping axis creates a larger potential energy. For F925 mower, the initial potential energy change (ΔPE) from first positions to second position increases from 4158 J to 4360 J as the deck height decreases from 0.2 m to 0.1 m. Therefore, there is greater tendency for the mower to roll with lower deck position than higher deck position.

In order to quantitatively evaluate the revised model prediction accuracy, an accuracy factor (AF) (the critical ROPS height (CRH) predicted by the revised model divided by the measured critical ROPS height (CRH))(expressed as a percentage) was defined. The accuracy factor (AF) of the revised model for deck up, deck down, and inverted ROPS (98, 98, and 99%, respectively) indicates a better prediction than the original model (85, 83, and 82%, respectively). Field lateral upset tests showed that the revised model result was very close to the field test result.

Model sensitivity analysis was conducted. As the deformation (T) of the slope and ROPS increased by 20%, the CRH only increased 0.1%. As the moment of inertia (MI) increased by 20%, the CRH only increased 1.1%. From 1.82 m to 1.52 m deck, as the deck extension width (Bm) decreased by 16.7%, the CRH only decreased by 0.9%. From 1.82 m deck to 2.28 m deck, as the deck extension width (Bm) increased by 25%, the CRH only decreased by 0.6%. By adding a 95 kg operator mass (m_1), the CRH

changed from 2.63 m to 2.67 m, therefore, the revised model was relatively insensitive to these parameters.

The model showed strong sensitivity to other factors. As the deck height (D3), the angle (LL2) between the tipping axis 3 and the longitudinal axis, the friction coefficient (Miu) and the horizontal distance (L3) between the center of gravity (CG) and deck/slope contact point increased by 20%, the CRH decreased by 1.6%, 1.6%, 6.8% and 9.5% respectively. As the slide distance (Ls) of CG, the height (H1) of center of gravity and the horizontal distance (L6) between CG and the ROPS impact point increased by 20%, the CRH increased by 1.9%, 4.1%, and 6.0% respectively. Therefore, the slight changes in these parameters caused a greater change in the roll behavior.

The deck size, yaw, and slide downhill had a greater influence on the mower roll behavior. Due to the deck size, the front tires of the mower are suspended, therefore, the weight shifts to the rear tire, and the rear tire receives greater friction force than front deck. This causes the front deck move faster than the rear tire. This situation results in the yaw of the mower. Due to the yaw, the angle between the longitudinal direction and the tipping axis connecting the ROPS impact point to deck/slope contact point decreased, therefore, the equivalent slope became steeper; finally, the roll tendency increased. From the accuracy factor (AF), the revised model is a better predictor of the front drive mower continuous roll than the original model.

Further studies should address the effect of operator weight. Due to the operator's weight, the weight of the mower, the center of gravity and the mass moment of inertia change, therefore, the roll behavior changes. It is essential to find empirical values of the yaw and the slide distance on various grasses. Due to the additional front structure, yaw and slide, the longer and wider test slope is needed for the adequate continuous roll testing.

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION.....	1
II. LITERATURE REVIEW.	4
2.1 Stability and Non-Continuous Roll Prediction Model for Tractor Design	4
2.2 Continuous Roll Prediction Model for Tractor ROPS Design.....	7
2.3 ROPS Standards Review.....	10
2.3.1 Previous ROPS (TOPS) Standards.....	10
2.3.2 Description of New ASAE S547 Standard.....	11
2.4 Summary.....	17
III. OBJECTIVES.....	19
IV. EVALUATION OF THE EXISTING MODEL FOR MOWER (OECD CODE 6).....	21
4.1 Test Vehicle.....	21
4.2 Model Prediction Result (OECD CODE 6).....	22

4.3 Field Tests	22
4.3.1 Test Slope and Procedures.....	25
4.3.2 Initial Field Test Results.....	27
 V. MODIFICATION OF THE EXISTING MODEL FOR	
MOWER (OECD CODE 6).....	33
5.1 Addition of the Size of the Mower Deck.....	37
5.1.1 Revised Tipping Axes.....	39
5.1.2 Revised Potential and Kinetic Energy due to Deck Size.....	42
5.1.3 Revised Moment of Inertia.....	63
5.2 Addition of Yaw Angle and Slide to the Mower Motion.....	67
 VI. EVALUATION OF THE REVISED MODEL.....	71
6.1 Measured Slide and Yaw.....	71
6.2 Revised Model Estimates of CRH.....	78
6.2.1 Reference Mower Dimensions.....	78
6.2.2 Revised Model Results.....	80
6.3 Critical ROPS Height by Field Test.....	81
6.3.1 Field Tests for the F925 Mower with Regular ROPS.....	83
6.3.2 Field Test for the F925 Mower with Inverted ROPS.....	86
6.4 Comparison of the CRH for the Original Model, the Revised Model and	
Field Test Results.....	88

6.4.1 Regular ROPS.....	88
6.4.2 Inverted ROPS.....	88
6.5 Accuracy Factor of the Revised Continuous Roll Model.....	90
6.6 Sensitivity Analysis of Revised Model.....	92
6.6.1 Sensitivity Analysis of Mower Dimensions.....	92
6.6.2 Sensitivity Analysis of Yaw and Slide.....	95
6.6.3 Sensitivity Analysis of Slope Factors.....	97
6.6.4 Influence of Operator Weight on Roll Behavior.....	99
6.6.5 Summary of Sensitivity Analysis.....	102
VII. CONCLUSIONS AND SUGGESTIONS.....	103
7.1 Conclusions.....	103
7.2 Suggestions.....	105
LIST OF REFERENCES.....	107
APPENDIXES	111
APPENDIX A A REVISED CONTINUOUS ROLL MODEL..	112
A.1 Flow Chart.....	112
A.2 Definition of Variables.....	116
A.3 Matlab Code.....	122

APPENDIX B MEASUREMENT OF PARAMETERS.....168

B.1 List of Parameters.....168

B.2 Measurement.....168

B.2.1 Determination of Angular Velocity from the Vehicle.....168

B.2.2 Center of Gravity.....169

B.2.3 Moment of Inertia Determination.....171

B.2.4. Determination of ROPS Deformation and Cone Penetration

Index.....174

VITA.....177

LIST OF TABLES

Table	Page
1. Input values of the F925 mower with regular ROPS.....	24
2. Initial field test results.....	29
3. Coordinates of CG and contact points in the system (X1, Y1, Z1).....	38
4. Field tests on polyethylene pad with regular ROPS.....	72
5. Field tests on polyethylene pad with inverted ROPS.....	75
6. The value of the yaw angle and other angles.....	80
7. Revised and original model results with regular ROPS.....	81
8. Revised and original model results with inverted ROPS.....	81
9. Field test results for F925 mower with regular ROPS.....	85
10. Field test results for F925 mower with inverted ROPS.....	87
11. The accuracy factor (AF) of the revised model.....	91
12. The accuracy factor (AF) of the original model.....	91

LIST OF FIGURES

Figure	Page
2-1. Tipping axes of sideways overturning.....	6
2-2. Definition of the continuous roll for B2 version.....	8
2-3. Definition of the continuous roll for B1 version.....	8
2-4. Definition of the continuous roll for B3 version.....	9
2- 5. Ramp for testing the non-continuous roll (from ASAE 2002).....	12
2-6. Inputs of OECD CODE 6.....	13
2-7. Outputs of OECD CODE 6 (B2 version).....	15
4-1. Test vehicle (Deere F925 mower).....	21
4-2. Sensitivity analysis on slope angle from the existing model.....	23
4-3. Test slope of 35 degrees for mower continuous roll.....	26
4-4. Rotation disks mounted on the mower.....	26
4-5. Field tests for the F925 mower with regular ROPS of 1.90 m.....	28
4-6. Field tests for the F925 mower with regular ROPS of 2.22 m.....	28
4-7. Field tests for the F925 mower with inverted ROPS of 1.90 m.....	29
4-8. The axes change of rotation due to the deck size.....	30
4-9. ROPS hits the slope.....	31
4-10. Yaw and slide were observed.....	31
4-11. Angular velocities for F925 mower with inverted ROPS of 1.90 m.....	32

5-1. Definition of the mower motion at positions 1 and 2.....	34
5-2. Definition of the mower motion at positions 3 and 4.....	34
5-3. Definition of the mower motion at positions 5 and 6.....	35
5-4. Definition of the mower motion at position 7.....	35
5-5. Definition of the equivalent force acting point S.....	36
5-6. Definition of (X1, Y1, Z1) coordinates and seven contact points.....	38
5-7. The tipping axes of B1 version (mower rolls back).....	40
5-8. The tipping axes of B2 version (mower rolls over the top of ROPS).....	40
5-9. The tipping axes of B3 version (mower rolls front).....	41
5-10. Definition of the longitudinal unstable equilibrium point (EQ).....	41
5-11. Revised origin of X''', Y''', Z''' system.....	43
5-12. Revised potential energy from position 1 to 2.....	43
5-13. Transformation of the coordinates from the original system to the system (X''' , Y''' , Z''').....	48
5-14. Relationship between the original slope angle and revised equivalent slope angle.....	50
5-15. The relationship between θ_1 and deck size.....	51
5-16. Motion of the mower from position 2 to position 3.....	53
5-17. Motion of the mower from position 3 to position 4.....	54
5-18. Mower motion from positions 4 to 5.....	55
5-19. Determine the angular velocity of CG (O_4) after the maximum sinkage of ROPS into the slope.....	57

5-20. Determine the angular velocity (O_1) of CG after the maximum sinkage of the rear tire into the slope.....	61
5-21. Definitions of K1, K2, K3 and K4 in position 3.....	61
5-22. Transfer the moment of inertia from old coordinates to new coordinates.....	63
5-23. Adding the roll, yaw and slide into the model.....	68
5-24. Potential energy change due to slide and yaw.....	69
6-1. ROPS just hits the slope.....	72
6-2. The angle when the mower continues to roll about the tipping axis 3.....	73
6-3. Inverted ROPS and Regular ROPS.....	73
6-4. Field test for F925 mower with inverted ROPS.....	74
6-5. The additional slide and yaw downhill for inverted ROPS.....	75
6-6. Measurement of yaw angle and slide distance on actual grass slope.....	76
6-7. Put the marks on contact points of deck and ROPS/grass slope.....	76
6-8. Measurement of the friction coefficient.....	77
6-9. Load cell sensor is used to measure the friction force.....	78
6-10. Definition of angles between the axes.....	79
6-11. Adjustable fitting for ROPS height extension.....	82
6-12. Mount the adjustable fitting for ROPS height extension to the mower frame...	82
6-13. Field test with the extended ROPS height (regular ROPS).....	84
6-14. A field test with regular ROPS height of 2.55 m (deck down).....	85
6-15. The relationship between ROPS height and continuous roll possibility.....	86

6-16. A field test with inverted ROPS height of 2.03 m.....	87
6-17. Comparison between model results and field test results for regular ROPS.....	89
6-18. Comparison between model results and field test results inverted ROPS.....	89
6-19. Error bar of the measured CRH due to stepwise adjustment of ROPS.....	92
6-20. Sensitivity analysis in deck height (D3).....	93
6-21. Sensitivity analyses in distance between CG and deck contact point.....	94
6-22. Sensitivity analyses in deck extend width (Bm).....	94
6-23. Sensitivity analyses in height of CG.....	95
6-24. Sensitivity analyses in distance between CG and ROPS impact point.....	96
6-25. Sensitivity analyses in moment of inertia.....	96
6-26. Sensitivity analyses in the angle (LL2) between the tipping axis 3 and the longitudinal axis.....	97
6-27. Sensitivity analyses in slide distance (Ls).....	98
6-28. Sensitivity analyses in friction coefficient of the slope.....	98
6-29. Sensitivity analyses in ROPS and slope deformation.....	99
6-30. Parameters changes due to the operator weight.....	100
6-31. The CRH increases due to the operator weight.....	101
A-1. Main flow chart of Matlab code.....	113
A-1-1. Flow chart of Matlab code (B3 Version).....	114
A-1-2. Flow chart of Matlab code (B1 Version).....	115
A-2. Definitions of W1, R2, H9 and angles C1 and L9.....	117
A-3. Definitions of W3, W5, W6, W7 and angles W4 and F1.....	118

A-4. Definitions of W9, W10, W11 and angle W8.....	119
A-5. Angle definitions.....	119
A-6. Determination of the new CG position (CG (W9, W10, W11)).....	120
A-7. Definitions of E2, VO and C2 for B1 Version.....	120
A-8. Definitions of E2, VO and C2 for B2 Version.....	121
A-9. Definitions of E2, VO and C2 for B3 Version.....	121
B-1. Angle disks are mounted on the mower.....	169
B-2. Determination of the roll moment of inertia using the pendulum method.....	173
B-3. The measurement of roll moment of inertia of mower.....	174
B-4. Connection of LVDT to 23X datalogger.....	175
B-5. Measurement of ROPS deflection.....	175
B-6. Measurement of Cone Penetration Index.....	176

CHAPTER I

INTRODUCTION

Each year lawnmower operators must cut the grass on sloped land increasing the risk of mower rollover. SAE standard J2194 does not insure protection of the operator clearance zone in a rollover of 180 degrees (ASAE, 2001C). It is likely that the continuous roll of the mower increases the opportunity for operator fatalities. According to the ASAE S547 criteria for a non-continuous roll, it is defined as after striking the surface of the test slope, the machine may lift itself from the surface by pivoting about the upper corner of the protective structure, but it shall not roll on to its top. The machine shall fall back on the side that struck first. Therefore, the continuous roll is defined as a condition in which the vehicle center of gravity passes over the top of the tipping axis after the ROPS contacts the slope.

Recently, the increased sales of commercial riding lawnmowers indicate a rise in the number of mowers in operation. The Outdoor Power Equipment Institute estimated that nearly 300,000 commercial turf outdoor power equipment units were shipped in 2004 model year (September 2003 to August 2004) (OPEI, 2004). Detailed surveys regarding lawnmower rollovers had not been conducted. However, general fatality statistics in the lawn and garden industry are available. The United States Department of Labor (USDOL), Bureau of Labor Statistics lists Standard Industrial Classification (SIC) code 078 (Landscape and Horticulture Services (including lawn mowing

services)) as producing 19.3% and 18.0% of the total agricultural fatalities in 2000 and 2001, respectively. The average fatality percentage for the previous 4-year period was 13.7 %. While the number of fatalities in the agricultural sector as a whole is decreasing, the fatality numbers for SIC 078 is increasing (USDL, 2003). In 2000, SIC code 078 accounted for 9.2% of the total non-highway vehicle overturn fatalities in the agricultural industry.

Many ROPS standards do not provide operator protection for a lateral roll greater than 90 degrees. The terms roll over and tip-over have the same meaning, but roll over is a more common term than tip-over, therefore, the Roll Over Protective Structure (ROPS) will be used instead of the Tip-Over Protective Structure (TOPS). A new standard developed for TOPS (or ROPS) design for front drive mowers had been approved by American Society of Agricultural Engineers (ASAE) in December 2002. The recently approved ASAE Standard S547 "Tip-Over Protective Structure (TOPS) for Front Wheel Drive Turf and Landscape Equipment" addressed a continuous roll prediction model for ROPS design (ASAE, 2002).

The model addressed in ASAE S547 originates from the Organization for Economic Co-operation and Development (OECD) Code 6, "OECD Standard code for the official testing of front mounted rollover protective structures on narrow-track agricultural and forestry tractors ", designed for narrow track tractors. This OECD model described in this standard does not account for the influence of the mower deck on the rollover behavior. Field tests for the F925 mower were conducted to evaluate the

accuracy of the existing OECD model. It predicted a non-continuous roll with ROPS height of 2.17 m while experimentally, a continuous roll was observed with extended ROPS height of both 2.22 m and 2.42 m. Previous lateral upset tests indicated the mower deck influenced the rollover behavior. Field tests also revealed mower yaw and slide downhill, which were not included in the original model. Therefore, this project involved evaluating the accuracy of the existing continuous roll model and modifying it to include the mower deck, and the yaw and slide downhill.

CHAPTER II

LITERATURE REVIEW

2.1 Stability and Non-Continuous Roll Prediction Model for Tractor Design

Numerous research studies had been conducted on agricultural vehicles roll stability on slopes. Much of the work focused on the initial loss of stability and non-continuous roll of tractors. Larson et al. (1976) was among the first who investigated the dynamic behavior of sideways overturning. Using a mathematical model they simulated the actual sideways overturning behavior on slopes and predicted when it would occur. Overturning tests were conducted with a John Deere 4020 tractor. The tests were conducted on a 17 deg slope for a 3, 6 and 9 in. high bumps. These tests showed the tractor to be more stable than simulations had predicted. The model predicted sideward tractor overturns for less severe conditions (the maximum roll angle and angular velocity) than those that would actually overturn the tractor. Larson and Liljedahl (1971) developed a mathematical model for simulating sideways tractor overturns and predicted conditions when they would occur. This model could be used to warn the driver of an impending overturning condition, or it might be used to signal an automatic control system that would prevent the tractor from overturning. Davis and Rehkugler (1974a, 1974b) developed a three-dimensional model of the tractor, comprising the main body, front axle, rear wheels, clutch and engine, which

has 10 total degrees of freedom of movement. The tractor body rotational motion, defined by the principal axes angular velocities and Euler parameters, is not restricted to small magnitudes. Validation results were obtained with a scale-model that was rolled onto a ramp to initiate overturning.

In lateral stability analysis, it is important to identify the tipping axis. The tipping axis depends on the track width of front and rear axles. A conventional tractor may tip sideways about two axes. The overturning motion first takes place at an axis connecting the roll center of the front axle (3) to the ground contact point of the rear wheel (2) as shown in figure 2-1. When the front axle assembly reaches a rotation stop (limited by swing angle), the second tipping motion occurs as the entire tractor rotates about an axis connecting the ground contact point of the front (1) and rear wheels (2) on the side of the tractor at which the initial overturning motion began (Smith et al., 1974).

Chisholm (1976a) developed a mathematical model to describe the dynamic behavior of tractor based on the force and displacement equations of equilibrium at each point where the tractor made contact with the ground during overturning. His model was used to simulate overturns down steep slopes and assess the amount of energy absorbed by the ROPS during a wide range of overturning accidents (Chisholm, 1979b). The model was validated by experiments in which a tractor was overturned down a simulated bank about 2 m high (Chisholm, 1979c). In order to more accurately simulate multiple rollovers, a more general relationship between the

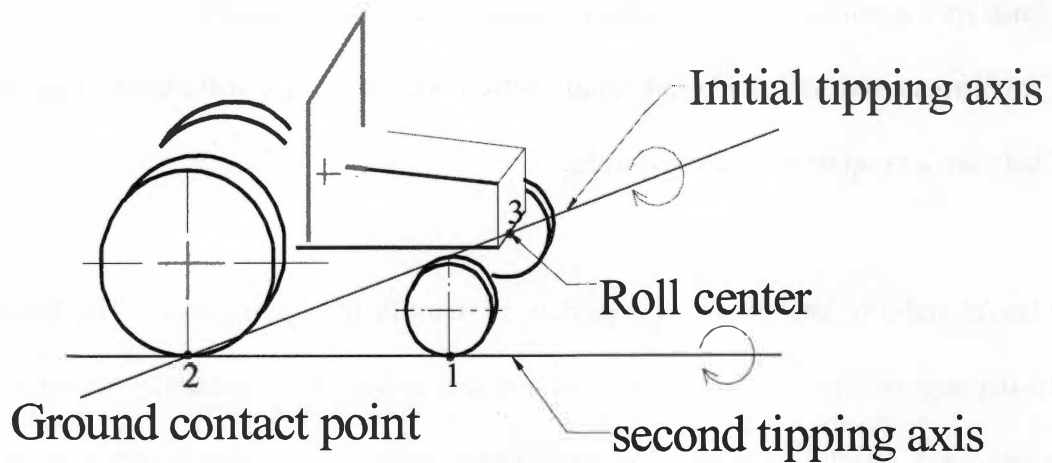


Figure 2-1. Tipping axes of sideways overturning.

transverse ground frictional force and the normal force was developed that included the slide distance and sinkage (Chisholm 1982).

Recently, Eger (2003) presented different modeling approaches for vehicle rollover. A simple two-degree of freedom model was used to develop a stability boundary and provide insight for rollover sequences. This model was only valid for quarter turns because the model assumed road-surface contact of the pivot point. This model has many limitations due to its simple character. A more sophisticated mechanical model employed a multibody approach and provided realistic simulation results. He suggested that the increased model complexity was not always necessary.

Many studies focus on the development of overturn stability monitoring. A quantitative value referred to as the stability index and a relevant measuring system have been developed from physical tractor operating conditions and tractor dynamics (Liu et al., 1995). Nichol et al (2004) developed a system that has the ability to provide critical information to the operator of a farm tractor about the tractor's stability on a side hill. The models reviewed indicate the initial loss of stability and not the potential for continuous rollover.

2.2 Continuous Roll Prediction Model for Tractor ROPS Design

After the initial loss of stability, the vehicle may continuously roll until the center of gravity is directly over the ROPS contact point for B2 version (see figure 2-2) or the top of the tipping axis for versions B1 or B3 (see figures 2-3 and 2-4). If the vehicle continues to roll down the slope, it is defined as a continuous roll (see figure 2-2). Little work had been done on evaluating continuous roll behavior of agricultural vehicles. Schwanghart (1973) developed a model for prevention of continuous rollover of agricultural tractors on a slope. He utilized conservation of energy to predict continuous roll behavior. This model was used by in the Organization for Economic Co-operation and Department (OECD) Standard Code 6 for official testing of front mounted rollover protective structures on narrow-track agricultural and forestry tractors (OECD, 2002). This model will be described in detail later in this chapter. The existing OECD model was also addressed by a new Standard, ASAE

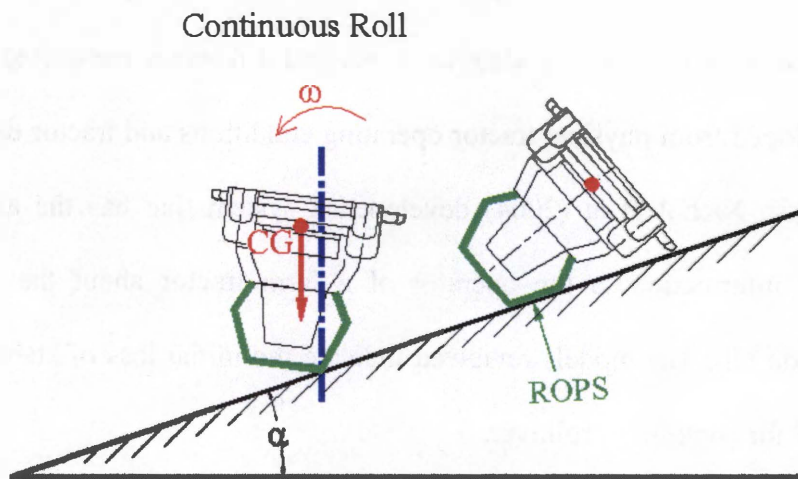


Figure 2-2. Definition of the continuous roll for B2 version.

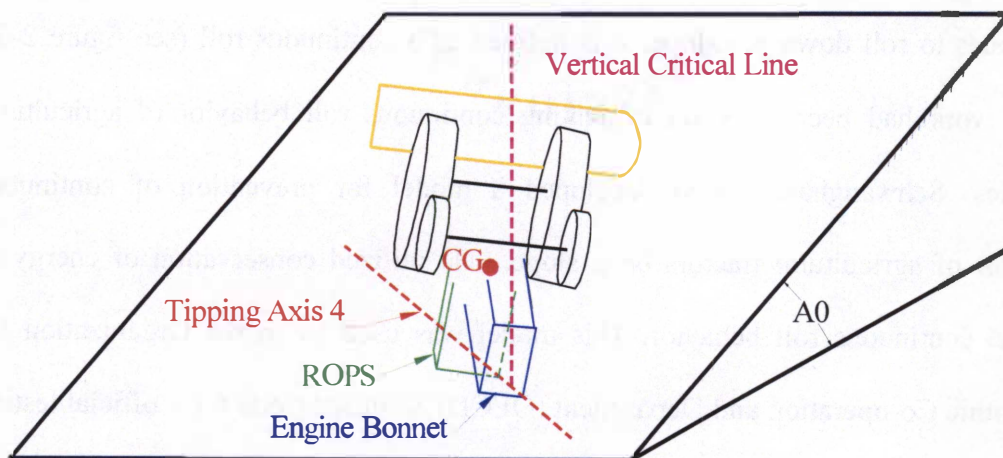


Figure 2-3. Definition of the continuous roll for B1 version.

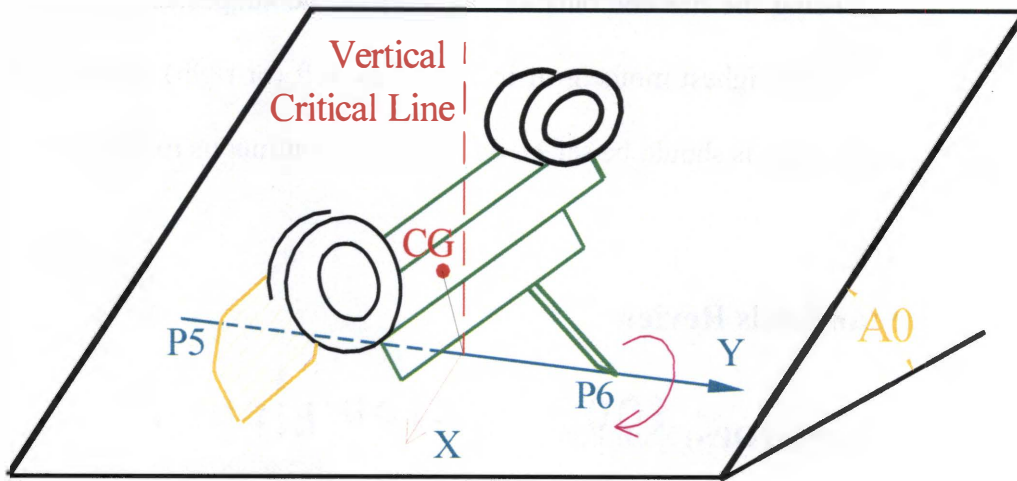


Figure 2-4. Definition of the continuous roll for B3 version.

S547, "Tip-Over Protective Structure (TOPS) for Front Wheel Drive Turf and Landscape Equipment".

Febo and Pessina (2001) tested Schwanghart's model with 17 tractors. In the real life, the same tractor can be alternatively equipped with four, or more different tire combinations. According to the OECD Standard Code and the EC Directive for the official testing of front-mounted rollover protective structures (ROPS) on narrow track width, wheeled agricultural and forestry tractors (OECD, 1995 and EC, 1987), the non-continuous roll test must be conducted with the tractor having the greatest tire diameter and the smallest cross-section for tires of that diameter. The test results showed that the tire combination giving the highest moment of inertia of the left (or

right) wheels about the tractor longitudinal axis is a more reliable parameter to be considered for choosing the tire combination. Therefore, he suggested that the tire combination giving the highest moment of inertia of the left (or right) wheels about the tractor longitudinal axis should be selected for the non-continuous rolling test.

2.3 ROPS Standards Review

2.3.1 Previous ROPS (TOPS) Standards

The terms rollover protective structures (ROPS) and tip-over protective structures (TOPS) had the same meaning, but the ROPS was more common term than TOPS, therefore, it was often used. Rollover protective structures (ROPS) are used to minimize the possibility of operator injuries during the vehicle roll. Standards had been developed for ROPS design and testing for agricultural vehicles including mowers. American National Standards Institute (ANSI) B71.4-1999 provided safety specifications for Commercial Turf Care Equipment (ANSI, 1999). ANSI (B71.4-1999) recommended that any loaded Commercial Turf Care Equipment weighing over 1430 lb (including attachments and 200 lb operator) be equipped with an “operator’s protective device”.

American Society of Agricultural Engineers (ASAE) Standards (S383.1 and S478) and Society of Automotive Engineers (SAE) Standards (J2194 and J1040) all appear to address ROPS design for wheeled agricultural and industrial tractors (SAE, 2000; ASAE, 2001a; ASAE, 2001b; ASAE, 2001c). None of these standards include the

continuous roll lateral upset test specified in ASAE S547 (ASAE, 2002). These standards only provide protection for 90-degree rolls on slope or ground.

ASAE Standard S547 “Tip-Over Protective Structure (TOPS) for Front Wheel Drive Turf and Landscape Equipment” was approved in December 2002. The S547 standard lateral upset test is designed to limit the roll angle during the side upset of a self-propelled, ride-on machine (ASAE, 2002). Within this standard there are two methods in which a machine may be tested. The first of which is the lateral upset where the vehicle is tipped down a slope. The second is a demonstration of prevention of rollover by calculation. A model taken from the Organization for Economic Co-operation and Development (OECD) Standard Code 6 is used for this calculation. Only one of these testing methods needs to be done to determine if the vehicle continuously rolls down a slope. The ASAE standard S547 required a non-continuous roll performance. Also section 1.2 of the ASAE S547 states that any vehicle that meets the requirements of ASAE standards S383 and S478 meets the performance requirements of this standard (ASAE, 2002).

2.3.2 Description of New ASAE S547 Standard

As stated in the S547 Standard, the lateral upset test shall be carried out on a natural earth slope or a test ramp at least four meters long (see figure 2-5). The slope of the surface shall be 33 degrees +5/-0 degrees. The surface shall be covered with a

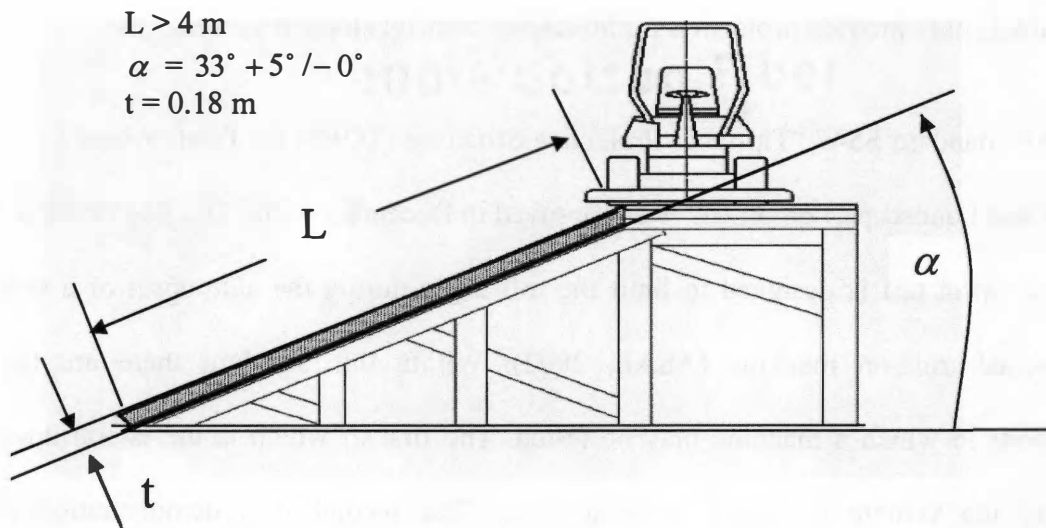


Figure 2-5. Ramp for testing the non-continuous roll (from ASAE 2002).

minimum of a 180 mm layer of material, which when measured in accordance with ASAE Recommendation R313; has a cone penetration index of A (235 ± 20) or B (335 ± 20) *psi* ((1619 ± 138) or (2308 ± 138) *kPa*). The cone penetration index is defined as the force per unit base area required push a 30° circular stainless steel cone through the soil, expressed in kilopascals. It is an index of soil strength and used to characterize the soil properties.

The ASAE S547 Standard also includes a new modeling component that would allow ROPS design without actual field upset testing. The model described in the Standard is from the OECD Standard Code 6 (OECD, 2002), "OECD Standard Code for the official testing of front mounted rollover protective structures on narrow-track

wheeled agricultural and forestry tractors". The existing model only evaluated the rotation of the tractor around the tipping axis (Schwanghart, 1973).

The model described in OECD Code 6 allows assumptions of the ROPS/slope deformation (0.2 meters) and the slope coefficient of elasticity (0.2) (embedded in the model). However, as it was originally developed for narrow track tractors, the mower deck sizes are ignored by the OECD CODE 6 model (see figure 2-6), i.e. the influence of the mower deck to determine the vehicle/slope contact points is not considered by the existing model. The slide and yaw downhill also are ignored by the existing model.

The inputs of the existing model include (from ASAE 2002):

Front tire width (B0) (m),

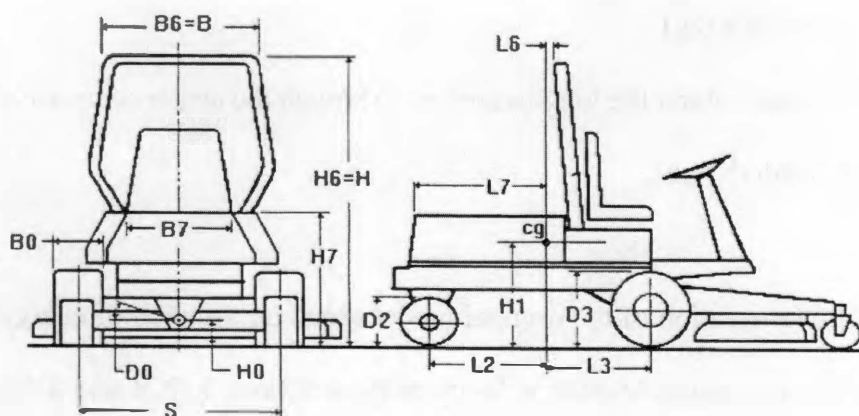


Figure 2-6. Inputs of OECD CODE 6.

Width of ROPS (B6) (m),

Width of the engine bonnet (B7) (m),

Rear axle swing angle (D0) (m),

Height of the rear tires (D2) (m),

Height of front tires (D3),

Height of the rear-axle pivot point (H0) (m),

Height of the center of gravity (H1) (m),

Height of ROPS at the point of impact (H6) (m),

Height of the engine bonnet (H7) (m),

Horizontal distance between the center of gravity and front axle (L3) (m),

Horizontal distance between the center of gravity and rear axle (L2) (m),

Horizontal distance between the center of gravity and the rear corner of the engine bonnet (L7) (m),

Horizontal distance between the center of gravity and the leading point of intersection of ROPS (L6) (m),

Tractor mass (MC) (kg)

Moment of inertia about the longitudinal axis through the center of gravity (Q) and

Rear track width (S) (m).

The governing equation of the original model used conservation of energy principles to determine the vehicle angular velocity at the positions 1, 2, 4 and 6 (see figure 2-7).

$$\Delta KE = \Delta PE \quad (2.1)$$

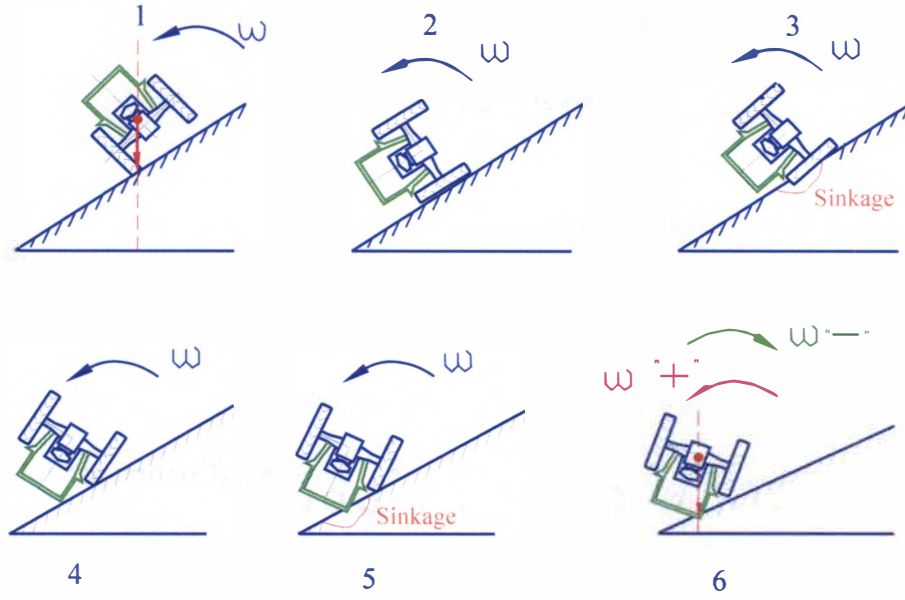


Figure 2-7. Outputs of OECD CODE 6 (B2 version).

Where,

$$\Delta KE = \frac{1}{2} J (\omega_2^2 - \omega_1^2) \text{ and} \quad (2.2)$$

$$\Delta PE = mg(h_2 - h_1). \quad (2.3)$$

thus

$$\frac{1}{2} J (\omega_2^2 - \omega_1^2) = mg(h_2 - h_1) \quad (2.4)$$

Where,

J = Moment of inertia about the longitudinal axis through the center of gravity ($kg \cdot m^2$),

ω = Angular velocity (rad / s),

m = Mass of the mower (kg),

g = Gravity acceleration and (m/s^2) and

h = Height of center of gravity (m) .

The outputs of the existing model are described below:

The outputs of the existing model are the roll behavior (Continuous roll or Non-continuous roll) and the angular velocities of center of gravity at the critical vehicle positions. Figure 2-7 shows the 6 critical vehicle positions as defined in the model (B2 version).

Position 1: it shows the center of gravity directly over the tipping axis.

Position 2: it is defined as when the top of the tire contacts the slope surface.

Position 3: it is after the sinkage of the top of the tire.

Position 4: it occurs when the top of the ROPS contacts the slope.

Position 5: it is at the maximum ROPS deflection and sinkage of the ROPS into the slope.

Position 6: it is defined as the point where the vehicle's center of gravity is directly over the ROPS contact point for the B2 Version.

For the B1 version, the position 6 is defined as the point where the vehicle's center of gravity is directly over the top of the tipping axis 4 connecting the ROPS/slope contact point to the engine bonnet/slope contact point (see figure 2-3). For the B3 version, the position 6 is defined as the point where the vehicle's center of gravity is

directly over the top of the tipping axis 3 connecting the deck/slope contact point to the ROPS/slope contact point (see figure 2-4). If the angular velocity is greater than 0 at position 6, then the vehicle rolls over itself, and the roll behavior is considered a “continuous roll” (Ayers and Wang, 2003). In this case, the ROPS does not pass the standard requirement. In addition, the existing model (OECD model) uses conservation of momentum principles and Newton’s Second Law to determine the angular velocities at positions 3 and 5.

2.4 Summary

Although much research had investigated the initial loss of stability of agricultural vehicles, little research had been done on evaluating their continuous roll behavior. Schwanghart (1973) developed a model for prevention of continuous overturning of agricultural tractors on a slope. His model utilized conservation of energy to predict continuous roll behavior. This model described in OECD code 6 was adopted by the ASAE S547 standard and is used for mower ROPS design.

The existing model (OECD model) was not developed for vehicles with front decks; therefore, it ignored the influence of the mower deck on roll behavior. The existing model also ignored the slide and yaw downhill. By adding the mower deck size, and degrees of freedom for yaw and slide downhill, many factors change, including: tipping axis, potential energy, and moment of inertia; therefore, the angular velocities

change. Finally, the roll behavior changes, therefore, the existing model needs to be evaluated prior to implementation for ROPS design for front drive mowers.

CHAPTER III

OBJECTIVES

The purpose of this project was to modify the existing continuous roll model (OECD code 6) described in ASAE S547 for front drive mower to include the mower deck, yaw and slide downhill, to accurately predict the roll behavior of a mower on a slope. Although the Standard S547 uses the term TOPS to describe the operator protective structure, rollover protective structure (ROPS) is the more common term, and will be used throughout this study.

The specific objectives reported are to:

1. Evaluate the accuracy of the existing continuous roll prediction model. The existing continuous roll prediction model, Schwanghart's model and OECD code 6 model was the same model.

The evaluation includes:

- Field upset test verification (roll behavior and angular velocities) using a Deere F925 commercial front drive mower with mower deck.
- Determination of measured and calculated critical ROPS height (CRH) (Critical ROPS height (CRH) is the ROPS height that the vehicle transitions from the continuous roll to non-continuous roll). Due to the bar corner shape (round shape or right angle), the height of ROPS impact point (H6) is changed. For the mower with the round corner shape of bar, it is shorter than the whole ROPS height.

2. Modify the existing continuous roll prediction model to include the size of the mower deck, mower slide and yaw down the slope.
3. Evaluate the accuracy of the revised model to predict the CRH using a Deere F925 mower. An accuracy factor is defined to quantitatively evaluate the model prediction accuracy.
4. Conduct model sensitivity analysis on model parameters including the mower deck size, the horizontal distance between the center of gravity and the leading point of intersection of ROPS, and the height of the center of gravity. Sensitivity analysis provides the degrees of influence of different factors on mower roll behavior.

CHAPTER IV

EVALUATION OF THE EXISTING MODEL FOR MOWER (OECD CODE 6)

4.1 Test Vehicle

In October 2002, John Deere Turf Care Inc provided a F925 front drive mower (Serial Number MOF925X180228) for model evaluation (see figure 4-1). The F925 mower has 16 kW Yanmar Diesel Engine, a 72-inch wide front deck and has a mass of approximately 899.5 kg. The mower was equipped with a ROPS designed to satisfy OSHA 1928, which does not have a continuous roll criterion. The Deere mower F925 has an original ROPS height at the impact point of 1.90 m.



Figure 4-1. Test vehicle (Deere F925 mower).

4.2 Model Prediction Result (OECD CODE 6)

The existing simulation model (OECD CODE 6) was programmed in Matlab. The input values for the F925 mower are shown in table 1. The center of gravity and the moment of inertia measurements are given in Appendix B. The existing model predicted angular velocities for a given ROPS height and final continuous/non-continuous roll decision. By adjusting the ROPS height and running the model, the critical ROPS height (CRH) was determined. The critical ROPS height (CRH) is the ROPS height where the vehicle rolls transitions from the continuous to non-continuous. Using a slope angle of 35 degrees, the existing model predicted the critical ROPS height of 2.17 m. To increase safety, the mower should be tested on a slope angle of 38 degrees. As the slope angle increases, the CRH predicted by the OECD model increases (see figure 4-2). The ASAE standard allowed a slope angle variation from 33 to 38 degrees, producing a CRH range from 2.14 m to 2.21 m respectively. The original OECD model used a slope angle of 33 degrees, which determined a low CRH.

4.3 Field Tests

In order to evaluate the accuracy of the existing model, five field tests were conducted on a slope of 35 degrees. The ROPS height and orientation were varied. The slope construction is discussed below.

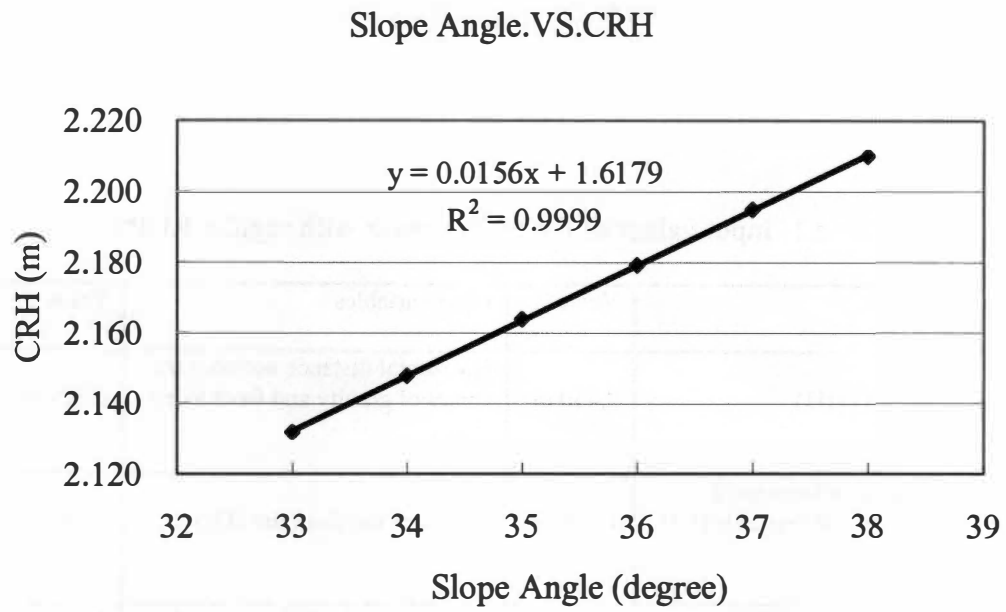


Figure 4-2. Sensitivity analysis on slope angle from the existing model.

Table 1: Input values of the F925 mower with regular ROPS

Input variables	Value	Input variables	Value
Height of the CG (H1)	0.450 m	Horizontal distance between the center of gravity and front axle (L31)	0.229 m
Horizontal distance between the center of gravity and rear axle (L2)	1.206 m	Height of the front tire (D3)	0.565 m
Height of the rear tires (D2)	0.360 m	Original height at the point of impact (H6)	1.90 m
Horizontal distance between CG and ROPS impact point (L6) (minus, if ROPS lies behind the plane of CG)	-0.471 m	Width of ROPS(B6)	0.775 m
Height of engine bonnet (H7)	0.845 m	Width of the engine cover (B7)	0.650 m
Horizontal distance between the center of gravity and the rear corner of the engine bonnet (L7)	1.050 m	Height of the rear-axle pivot point (H_0)	0.310 m
Rear track width (S)	0.880 m	Front tire width (B01)	0.190 m
Rear axle swing angle (D0)	0.102 rad	Tractor Mass (m)	899.5 kg
Moment of inertia About the longitudinal axis through the center of gravity (Q)	206.3 kgm ²		

4.3.1 Test Slope and Procedures

According to the requirement of ASAE S547 (ASAE, 2002), a test slope of 35 degrees was conducted at University of Tennessee (see figure 4-3). It is 4.05 meters long and 3.42 meters wide. The polyethylene pads were used because of their ability to be more consistent in the cone index values (the force per unit base area required to push a 30° circular stainless steel cone through the soil). If a natural earth slope is used, the restrictive cone index requirements of the standard may not have been met, as the cone index values would change as the soil dries. The average cone penetration resistance of the pad from 0 to 180 mm for the ten measurements was 320 *psi* (2205 *kPa*) for a size B cone, which is a small cone with a base area of 129 mm².

A hydraulic lift device was mounted in the top of the slope. The mower was placed above the plate driven by this lift device; the mower longitudinal centerline was parallel with the contour lines of the slope surface. The rotation disks were mounted on the mower (see figure 4-4). The wheels on the downhill side of the mower were placed on top of the test slope. The mower was then gradually lifted until the center of gravity was directly over the tipping axis. After the mower was just allowed to roll, a continuous or non-continuous roll was observed.



Figure 4-3. Test slope of 35 degrees for mower continuous roll.



Figure 4-4. Rotation disks mounted on the mower.

4.3.2 Initial Field Test Results

Initial field tests were conducted to evaluate the accuracy of the existing model for the F925 mower with a regular ROPS height of 1.90 m (3 tests), a raised ROPS height to 2.22 m (1 test) and an inverted ROPS height of 1.90 m (1 test) (see figures 4-5, 4-6 and 4-7). ROPS extensions, described in section 6.3, were used to raise the ROPS height from 1.90 m to 2.22 m. In addition, by inverting the ROPS, L6 could be changed from -0.471 m to -0.045 m. The negative value indicates that the ROPS lay behind the mower center of gravity.

The initial field test results are given in table 2. For the F925 mower, the continuous roll behavior was observed with original ROPS height at the impact point of 1.90 m, as was predicted by the existing model. The existing OECD model predicted a non-continuous roll, while a continuous roll was observed with ROPS height of 2.22 m. Field test results showed that the mower deck influences the roll behavior due to the axes change of rotation (figure 4-8). Field tests showed that there is discrepancy in the roll behavior predicted by the existing model and the field test. In addition to this discrepancy, the mower yaw (rotation) and slide downhill were also observed. This is because by adding the deck size, the front tires of the mower are suspended, therefore, the weight shifts to the rear tire, and the rear tire receives greater friction force than front deck. This causes the front deck move faster than the rear tire. This situation results in the yaw of the mower and a change in the direction of tipping axis. The equivalent slope angle changed, therefore, the roll behavior of the mower changed.



Figure 4-5. Field tests for the F925 mower with regular ROPS of 1.90 m.



Figure 4-6. Field tests for the F925 mower with regular ROPS of 2.22 m.



Figure 4-7. Field tests for the F925 mower with inverted ROPS of 1.90 m.

Table 2: Initial field test results

ROPS Position	ROPS Height at Impact Point (m)	Model Prediction Result	Field Test Result	CRH (Model) (m)
Regular (L6=-0.471m)	1.90	Continuous Roll	Continuous Roll	2.17
	2.22	Non- Continuous Roll	Continuous Roll	
Inverted (L6=-0.045m)	1.90	Non- Continuous Roll	Continuous Roll	1.67



Figure 4-8. The axes change of rotation due to the deck size.

Yaw and slide downhill were also observed (see figures 4-9 and 4-10). For the F925 mower with inverted ROPS, the existing model predicted a CRH of 1.67 m. Field test result produced a continuous roll with the inverted ROPS height of 1.90 m. The OECD does not calculate the value of angular velocity of CG. Thus, an angular velocity value of -1 was used to represent the non-continuous roll in order to distinguish the roll behavior.

According to ASAE S547, the field test should be conducted on both directions. The deck extension width on the left side of driver is greater than that on the right side of driver. Also, the deck shape is different on both sides. These two configurations will cause a different roll behavior. To maintain consistency in the field test results, the mower was always rolled to the left side of the driver.



Figure 4-9. ROPS hits the slope.



Figure 4-10. Yaw and slide were observed.

Figure 4-11 showed that the measured angular velocity and the predicted angular velocity at position 6 had a different sign. At position 6, the existing OECD model predicted a non-continuous roll while a continuous roll was observed. At the same time, additional downhill sliding was observed. Therefore, the existing model for the mower ROPS design needed to be modified to include the mower deck dimensions and the yaw and slide downhill.

Value of -1 was used to represent the non-continuous roll in order to distinguish the roll behavior due to the calculated value of angular velocity of CG was not available (see figure 4-11).

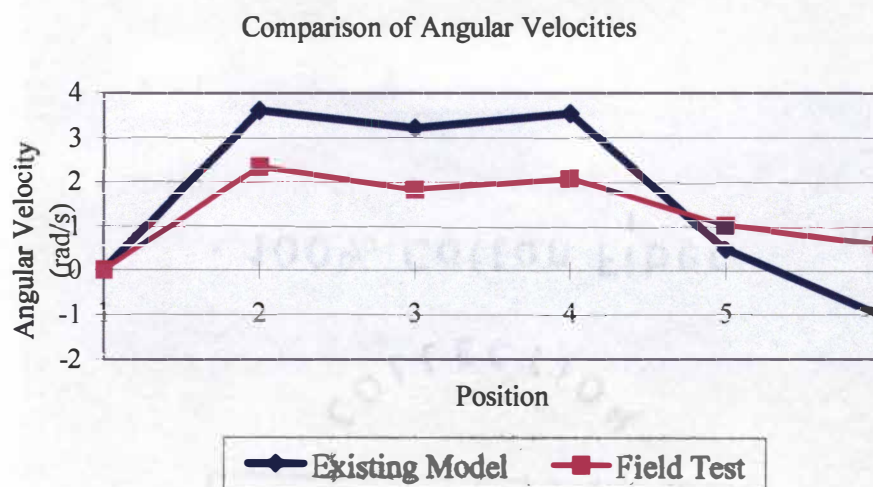


Figure 4-11. Angular velocities for F925 mower with inverted ROPS of 1.90 m.

CHAPTER V

MODIFICATION OF EXISTING MODEL FOR MOWER

(OECD CODE6)

The modification of the existing model included adding the deck size, yaw, and slide downhill into the model. The motion of the mower included seven critical positions. Figures 5-1, 5-2, 5-3 and 5-4 shows the 7 critical vehicle positions as defined in the revised model.

Position 1: it shows the center of gravity directly over the tipping axis. This position is the starting point of the model; the angular velocity of center of gravity at this position is zero. The existing OECD model starts at position 1 with the tipping axis at wheels (see figure 5-1).

Position 2: it is defined as when the deck and the top of the rear tire contacts the slope surface.

Position 3: it is after the sinkage of the top of the tire.

Position 4: it occurs when the top of the ROPS contacts the slope.

Position 5: it is at the maximum ROPS deflection and sinkage of the ROPS into the slope.

Position 6: After position 5, the mower rotates and sides about equivalent force acting point S. The equivalent force acting point S is defined as the intersection of the rotation plane 2 and the tipping axis 3 (see figure 5-5). This process is called the

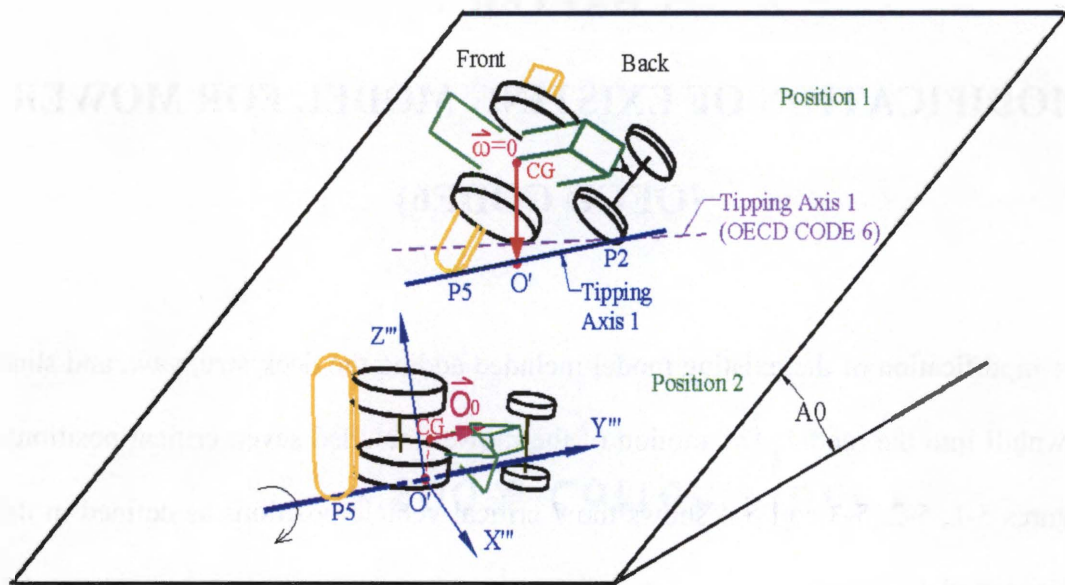


Figure 5-1. Definition of the mower motion at positions 1 and 2.

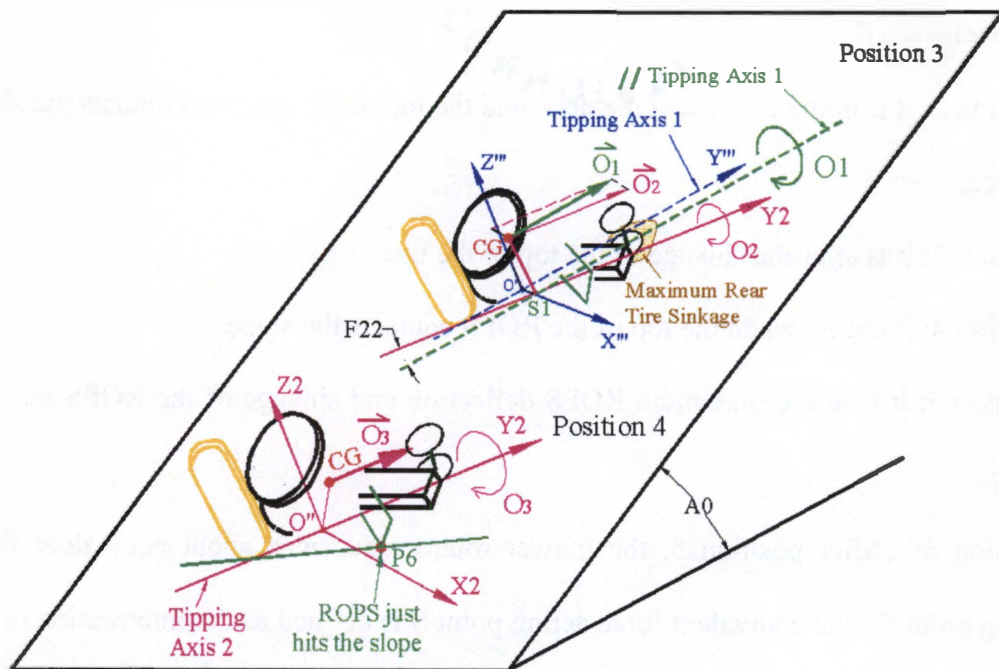


Figure 5-2. Definition of the mower motion at positions 3 and 4.

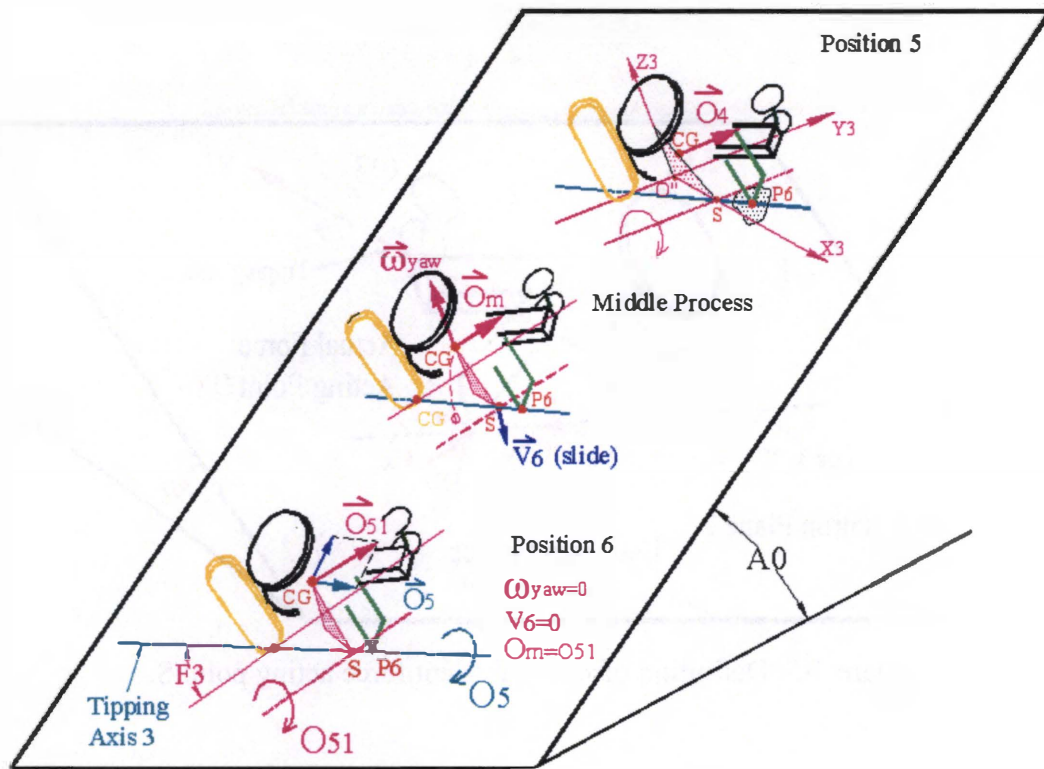


Figure 5-3. Definition of the mower motion at positions 5 and 6.

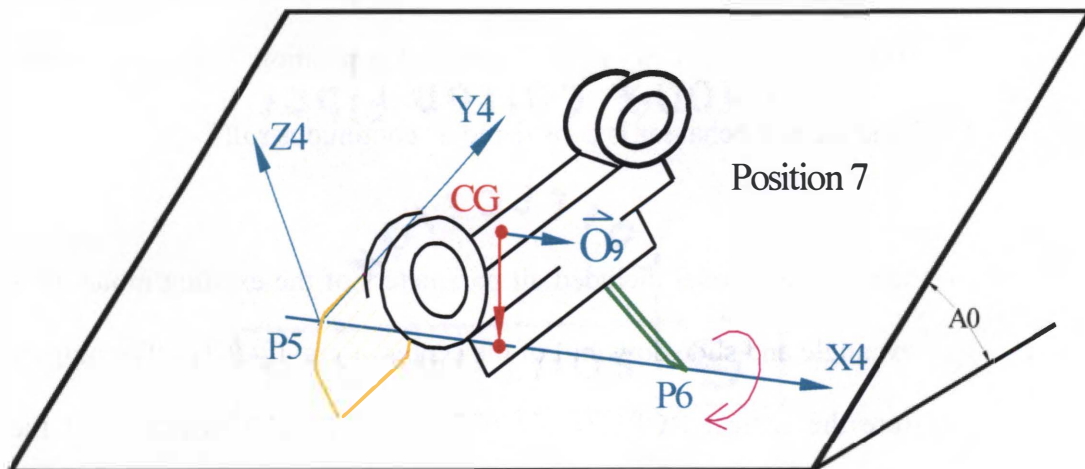


Figure 5-4. Definition of the mower motion at position 7.

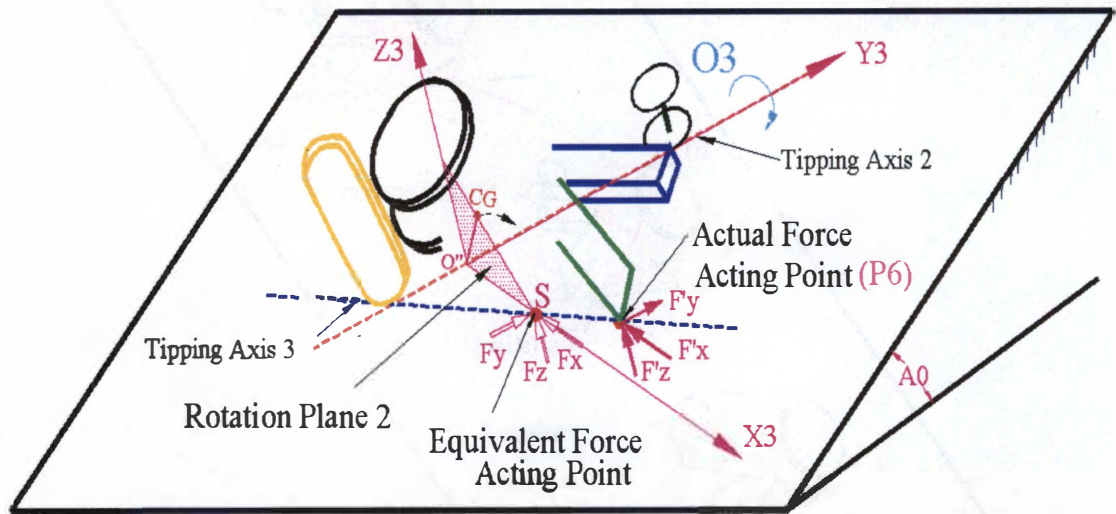


Figure 5-5. Definition of the equivalent force acting point S.

middle process. Then, after yaw and slide stop, the mower begins to roll about the tipping axis 3. This position is defined as position 6.

Position 7: it is at the point where the vehicle center of gravity is directly over the tipping axis 3. If the angular velocity is greater than 0 at position 7, then the mower rolls over itself, and the roll behavior is considered a “continuous roll”.

The inputs to the modified model included all parameters of the existing model plus the deck size, yaw angle and slide downhill of the center of gravity (CG). The outputs of the model were the critical ROPS height (CRH) and angular velocities of the mower at each of the 7 critical positions. The CRH is obtained by varying the ROPS height and running the program until the angular velocity at position 7 is zero. This

describes a roll to the mower front (B3 version). Situations where the mower rolls to the rear (B1 version) or over the top of the ROPS (B2 version) are discussed later.

The roll and movement of the mower required the transformation of coordinates from one position to another. There were also the transformations between the potential and kinetic energy. The quantitative calculations regarding the influence of deck size on the tipping axes, potential energy, kinetic energy, moment of inertia, and equivalent slope angle are discussed in detail.

5.1 Addition of the Size of the Mower Deck

Three parameters were added into the model to include the size of the mower deck. These are: 1) the horizontal distance between the contact point of the deck and the center of gravity (L_3), 2) the height of the contact point of the deck (D_3), and 3) the extended width of the deck measured from the center line of the front tire to the left end of the mower deck (B_m) (see figure 5-6). In order to study the quantitative influence of mower deck size on mower roll behavior, a right hand coordinate system (X_1, Y_1, Z_1) was defined. The XZ plane contained the CG. The XY plane was oriented on the longitudinal axis passing through points P2 and P3. The positive X_1, Y_1, Z_1 were vertical upward, horizontal rearward, and horizontal to the driver's right, respectively. In (X_1, Y_1, Z_1) coordinates system, the coordinates of the seven mower/slope contact points and the center of gravity are shown in table 3.

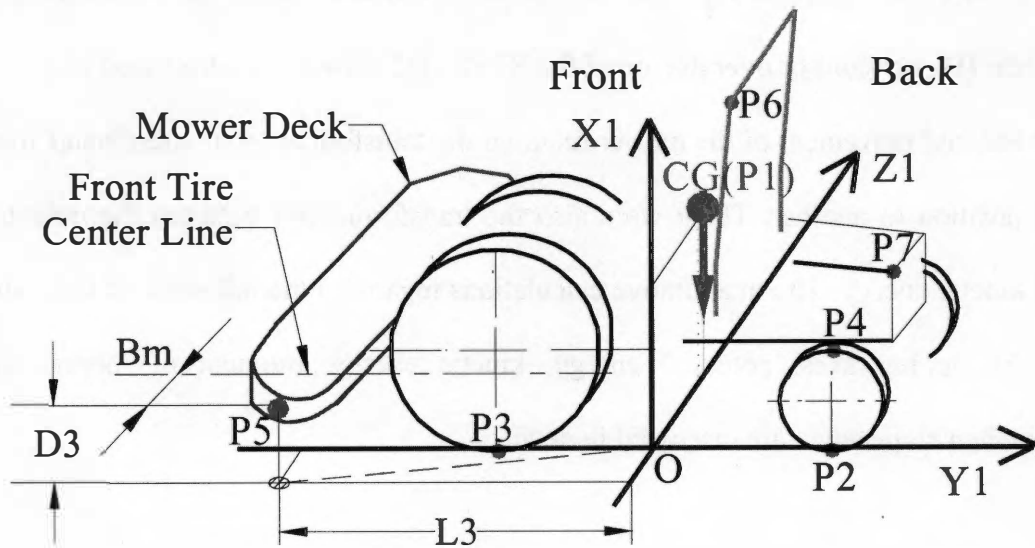


Figure 5-6. Definition of (X1, Y1, Z1) coordinates and seven contact points.

Table 3: Coordinates of CG and contact points in the system (X1, Y1, Z1)

Critical Points	Coordinates
CG	(X (1,1), Y (1,1), Z (1,1))
P2	(X (1,2), Y (1,2), Z (1,2))
P3	(X (1,3), Y (1,3), Z (1,3))
P4	X (1,4), Y (1,4), Z (1,4)
P5	X (1,5), Y (1,5), Z (1,5)
P6	X (1,6), Y (1,6), Z (1,6)
P7	X (1,7), Y (1,7), Z (1,7)

5.1.1 Revised Tipping Axes

Because adding the deck size to the model, the tipping axes changed; therefore, the moment of inertia, the equivalent slope angle, and the potential energy changed. These factors affected the mower roll behavior. The modified tipping axes are described below (Three versions are defined in order to distinguish the roll behaviors):

1. If the leading point of intersection of ROPS is in front of the longitudinal unstable equilibrium point, the mower rolls towards the back. This is defined as B1 version (see figure 5-7).
2. If the leading point of intersection of ROPS is near the longitudinal unstable equilibrium point, the mower directly rolls over the top of ROPS. This is defined as B2 version (see figure 5-8).
3. If the leading point of intersection of ROPS is behind the longitudinal unstable equilibrium point, the mower will roll towards the front. This is defined as B3 version (see figure 5-9). Tipping axes 1, 2 and 3 change from dashed lines to solid lines by adding the mower deck. The longitudinally unstable equilibrium point (EQ) is the intersecting point between the rotation plane 2 and the line passing through the ROPS/slope contact point and parallel to the tipping axis 2 (see figure 5-10)

From the field test, only the B3 version (the mower rolling towards the front) was observed. The other two situations did not occur. However, in order to cover all possible situations, the versions B1 and B2 were also included in the Matlab code.

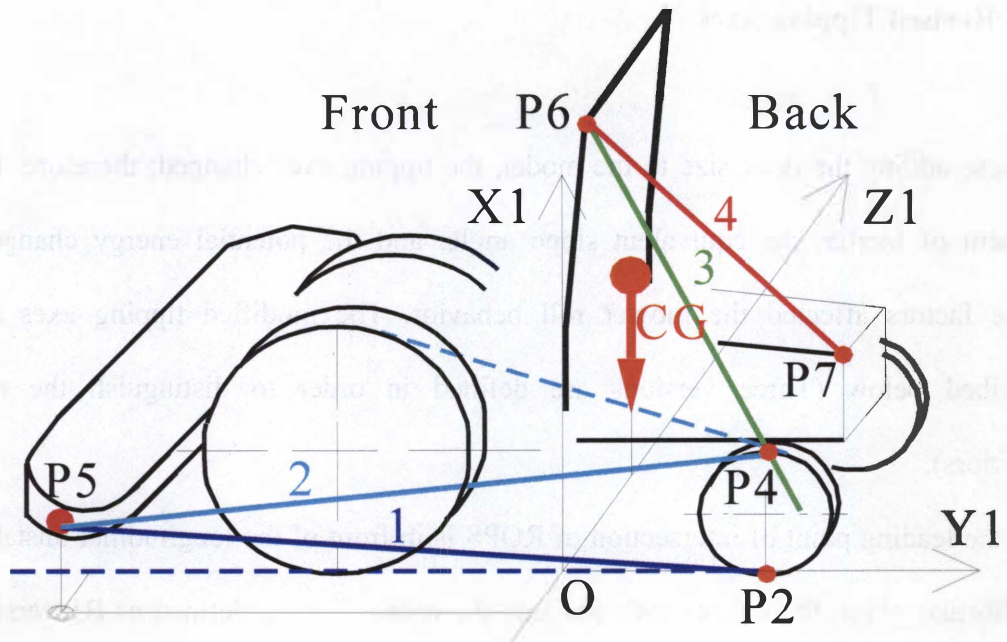


Figure 5-7. The tipping axes of B1 version (mower rolls back).

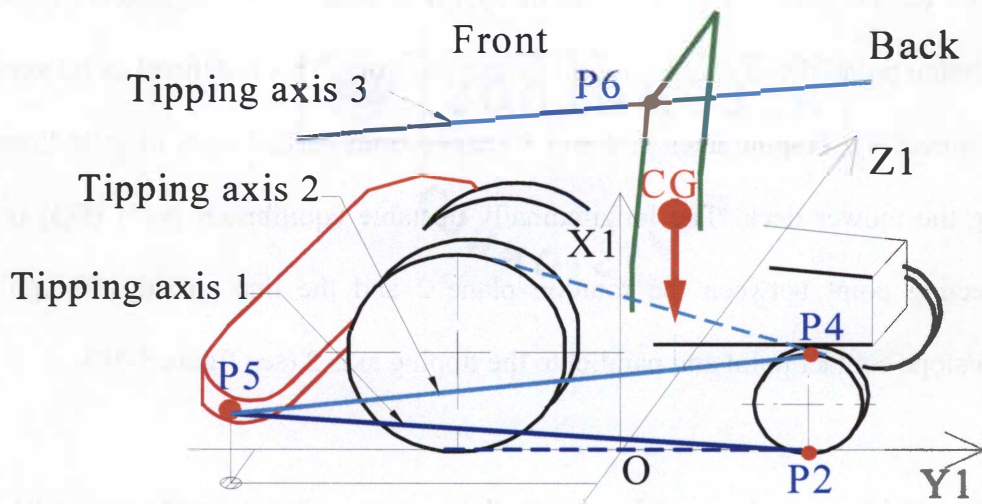


Figure 5-8. The tipping axes of B2 version (mower rolls over the top of ROPS).

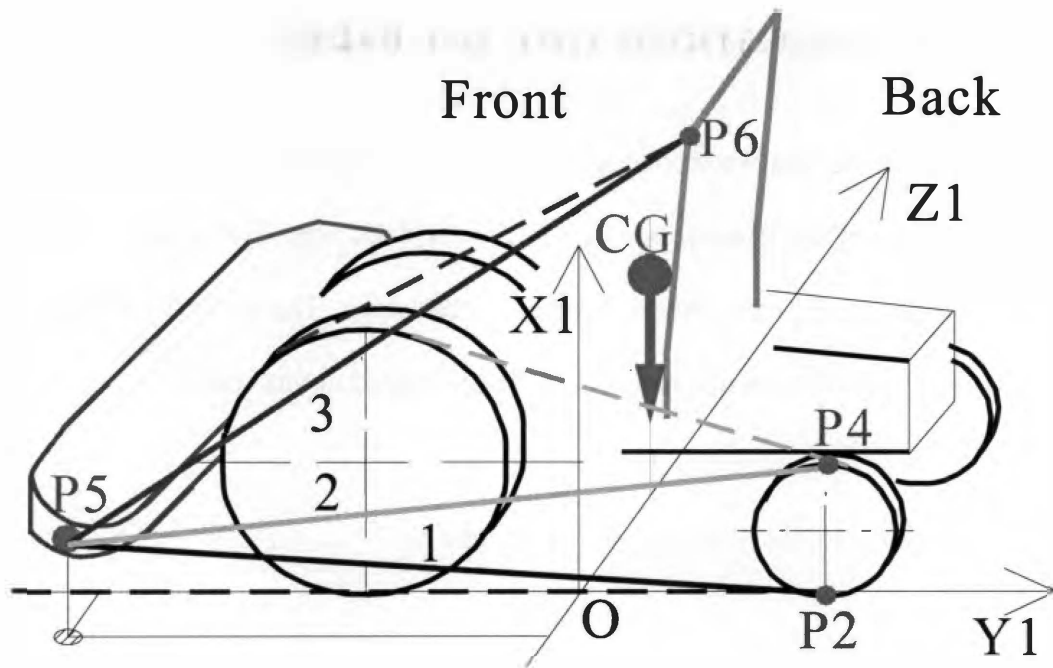


Figure 5-9. The tipping axes of B3 version (mower rolls front).

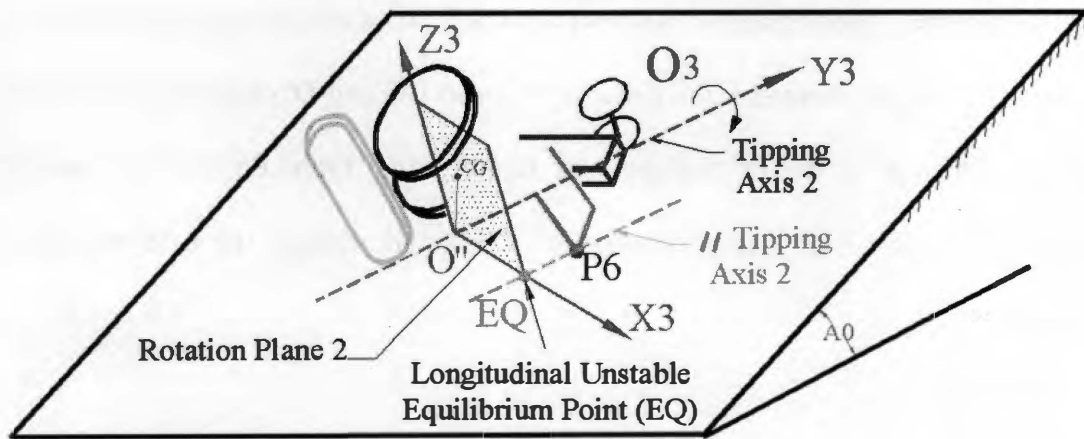


Figure 5-10. Definition of the longitudinal unstable equilibrium point (EQ).

5.1.2 Revised Potential and Kinetic Energy due to Deck Size

The three situations that were addressed in the revised potential energy calculations were: 1) change of the tipping axis due to mower deck size, 2) the slide of mower down the slope, and 3) the yaw of mower on the slope. The revised potential and kinetic energy calculations due to deck size are discussed in this section.

A. Revised potential energy from position 1 to position 2.

During the roll, the center of gravity rotates around the tipping axis in a plane that is perpendicular to the tipping axis. This plane is defined as a rotation plane (see figure 5-11). The following calculations were conducted in the rotation plane perpendicular to the corresponding tipping axis. The point O' is the intersection of tipping axis 1 and rotation plane 1 (see figure 5-11). From figures 5-11, as the tipping axis changes, the point O' changes; therefore, the distance between CG and O' changes. The potential energy changes as the O' changes (see figure 5-12). From Eq. 5.1, the potential energy change (ΔPE) is a function of CG height change (h) in two different positions.

$$\Delta PE = mgh \quad (5.1)$$

Where,

g = Gravity acceleration (m/s^2),

m = Mass of the mower (kg) and

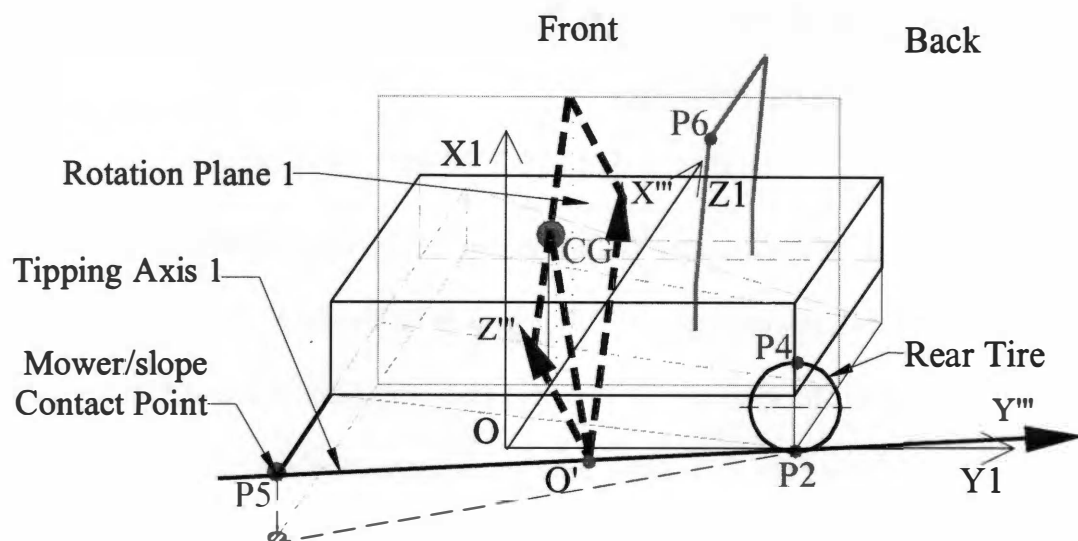


Figure 5-11. Revised origin of X''' , Y''' , Z''' system.

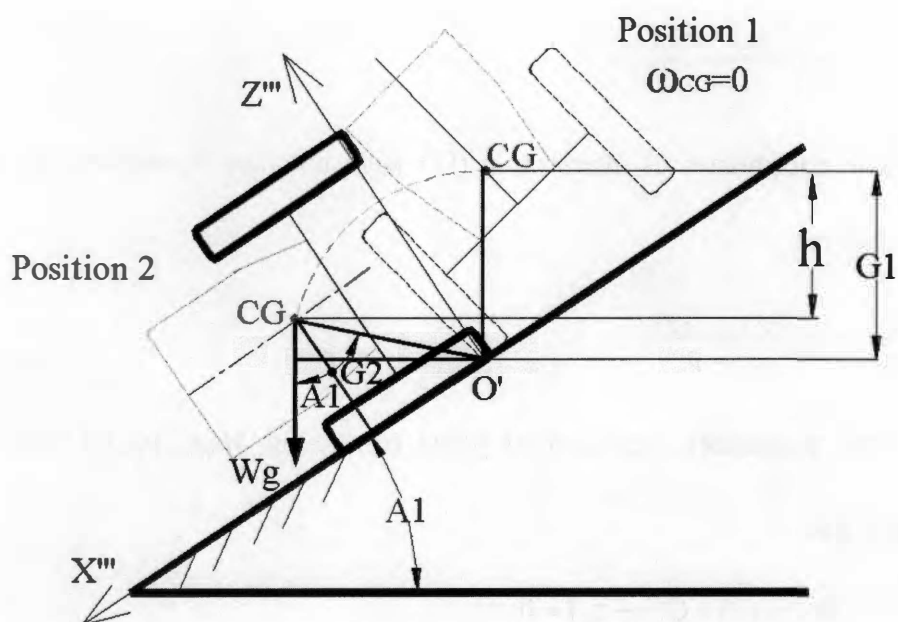


Figure 5-12. Revised potential energy from position 1 to 2.

$$\Delta PE = mgG1(1 - \cos(A1 + G2)). \quad (5.2)$$

The deck size affected G1, G2 and A1.

First, the influence of deck size on G1, the distance from the CG (W9, W10, W11) to O' (D52, D53, D54), is discussed. W9, W10 and W11 are the X, Y and Z location of new CG relative to the system (X1, Y1, Z1) (see Appendix A.2 and Matlab code). D52, D53 and D54 are the X, Y and Z location of point O' relative to the system (X1, Y1, Z1).

$$G1 = \sqrt{(D52 - W9)^2 + (D53 - W10)^2 + (D54 - W11)^2} \quad (5.3)$$

The following equation is for the line passing through point (x_1, y_1, z_1) , (x_2, y_2, z_2) .

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \quad (5.4)$$

$$x = \frac{x_2 - x_1}{y_2 - y_1} y + \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} \quad (5.5)$$

The specific coordinates of intersection (O') were obtained by solving the line and plane equations.

$$z = \frac{z_2 - z_1}{y_2 - y_1} y + \frac{z_1 y_2 - z_2 y_1}{y_2 - y_1} \quad (5.6)$$

The general mathematic equation of plane containing $M(x_0, y_0, z_0)$ with normal

$\vec{n} = \{A, B, C\}$ is:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 \quad (5.7)$$

Where,

$$A = (x_2 - x_1),$$

$$B = (y_2 - y_1) \text{ and}$$

$$C = (z_2 - z_1).$$

$$(x_2 - x_1)(x - x_0) + (y_2 - y_1)(y - y_0) + (z_2 - z_1)(z - z_0) = 0 \quad (5.8)$$

Introducing equation (5.5) and (5.6) into equation (5.8), the general equation for intersection coordinates is obtained.

$$y = \frac{A(x_0 - x_1) + B(y_0 - y_1) + C(z_0 - z_1)}{\frac{A^2}{B} + B + \frac{C^2}{B}} + y_1 \quad (5.9)$$

$$x = \frac{A}{B}y + \frac{x_1y_2 - x_2y_1}{B} \quad (5.10)$$

$$z = \frac{C}{B}y + \frac{z_1y_2 - z_2y_1}{B} \quad (5.11)$$

From the above equations, the intersection of tipping axis 1 and rotation plane 1 is obtained ($O' (D52, D53, D54)$). The rotation plane passes through CG (W9, W10, W11).

The tipping axis 1 passes through P5 $\left(D3, -L3, -\left(\frac{B0 - B01}{2} \right) \right)$ and P2 (0, L2, 0);

therefore, the normal is:

$$\vec{P5P2} = (0 - D3)\vec{i} + (L2 - (-L3))\vec{j} + \left(0 - \left(-\frac{(B0 - B01)}{2} \right) \right)\vec{k} \quad (5.12)$$

$$\vec{P5P2} = -D3\vec{i} + (L2 + L3)\vec{j} + \left(\frac{B0 - B01}{2} \right)\vec{k} \quad (5.13)$$

Where,

$$x_2 = 0,$$

$$y_2 = L2,$$

$$z_2 = 0,$$

$$x_1 = D3,$$

$$y_1 = -L3,$$

$$z_1 = -\frac{(B0 - B01)}{2},$$

$$A = -D3,$$

$$B = L2 + L3 \text{ and}$$

$$C = \frac{B0 - B01}{2}.$$

$$D53 = \frac{-D_3(W9 - D3) + (L2 + L3)(W10 + L3) + \left(\frac{B0 - B01}{2}\right)\left(W11 + \left(\frac{B0 - B01}{2}\right)\right)}{\frac{D_3^2}{(L2 + L3)} + (L2 + L3) + \frac{\left(\frac{B0 - B01}{2}\right)^2}{(L2 + L3)}} - L3 \quad (5.14)$$

$$D52 = \frac{-D3}{(L2 + L3)} D53 + \frac{D3L2}{(L2 + L3)} \quad (5.15)$$

$$D54 = \frac{\frac{(B0 - B01)}{2}}{(L2 + L3)} D53 + \frac{-\frac{(B0 - B01)}{2} L2}{(L2 + L3)} \quad (5.16)$$

From the above equations, the intersection (O') of tipping axis 1 and rotation plane 1 is a function of deck size; therefore, G1 is also a function of mower deck size (see equation 5.3).

Second, the influence of deck size on G2, which is the angle between the line CGO' and the Z''' -axis, is determined (see figure 5-12).

$$G2 = \arctan\left(\frac{X_{CG}'''}{Z_{CG}'''}\right) \quad (5.17)$$

Where,

$X_{CG}''' =$ X-coordinate of center of gravity (CG) relative to system (X''', Y''', Z''') and

$Z_{CG}''' =$ Z-coordinate of center of gravity (CG) relative to system (X''', Y''', Z''').

A rotation is considered positive if it is counterclockwise when looking down the axis toward the origin. The system changes from the original system (X1, Y1, Z1) to the new system (X''', Y''', Z''') with the coordinates of O' relative to the original system (D52, D53, D54) because of the deck size. The center of gravity (CG) is represented by (W9, W10, W11) in the original system and by (X_{CG}''' , Y_{CG}''' , Z_{CG}''') in the new system. The coordinate system (X1, Y1, Z1) is translated to point O' (D52, D53, D54), first. Then, the system (X', Y', Z') is rotated by an angle θ about X'-axis. Finally, the (X'', Y'', Z'') coordinate system is rotated by an angle φ about Z''-axis. Therefore, the system (X''', Y''', Z''') is built (see figure 5-13).

The relationship between the original system (X1, Y1, Z1) and new system (X''', Y''', Z''') is

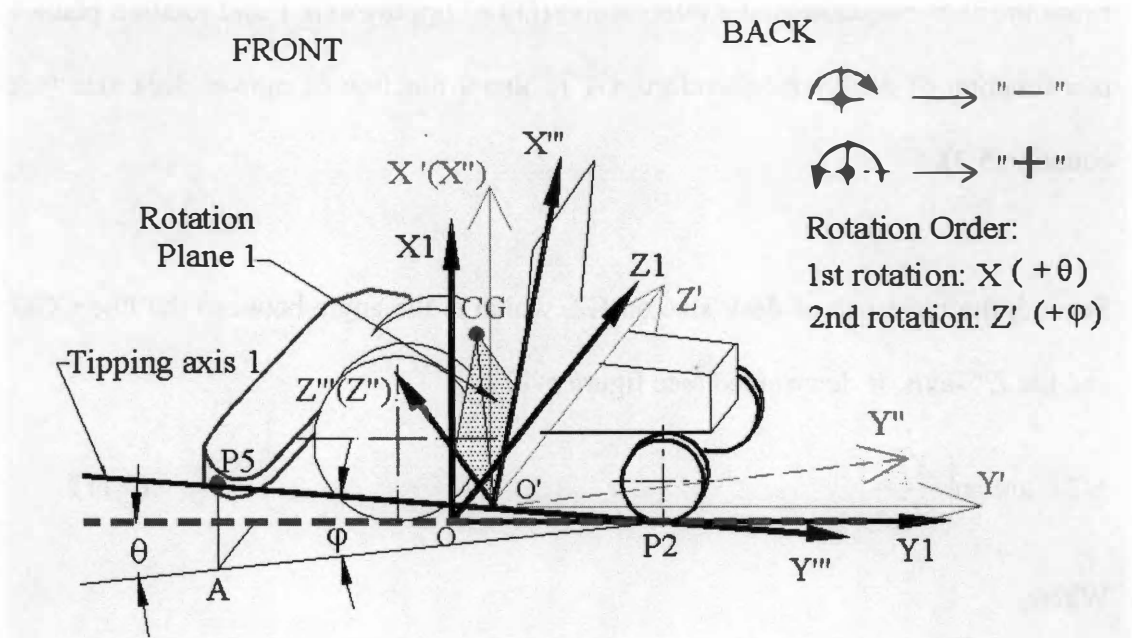


Figure 5-13. Transformation of the coordinates from the original system to system (X''', Y''', Z''') .

$$\begin{bmatrix} X''' \\ Y''' \\ Z''' \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} X1 - D52 \\ Y1 - D53 \\ Z1 - D54 \end{bmatrix} \quad (5.18)$$

$$\begin{bmatrix} X''' \\ Y''' \\ Z''' \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi \cos \theta & \sin \varphi \sin \theta \\ -\sin \varphi & \cos \varphi \cos \theta & \cos \varphi \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} X1 - D52 \\ Y1 - D53 \\ Z1 - D54 \end{bmatrix} \quad (5.19)$$

The following equation is the relationship between the new system (X''', Y''', Z''') and original system $(X1, Y1, Z1)$.

$$\begin{bmatrix} X''' \\ Y''' \\ Z''' \end{bmatrix} = \begin{bmatrix} (X1 - D52)\cos\varphi + (Y1 - D53)\sin\varphi\cos\theta + (Z1 - D54)\sin\varphi\sin\theta \\ -(X1 - D52)\sin\varphi + (Y1 - D53)\cos\varphi\cos\theta + (Z1 - D54)\cos\varphi\sin\theta \\ -(Y1 - D53)\sin\theta + (Z1 - D54)\cos\theta \end{bmatrix} \quad (5.20)$$

Where,

$$\varphi = \arctan\left(\frac{B0 - B01}{2L0}\right), \quad (5.21)$$

$$\text{and } \theta = \arctan\left(\frac{D3}{\sqrt{L0^2 + \left(\frac{B0 - B01}{2}\right)^2}}\right). \quad (5.22)$$

Using the above Eq (5.20), the following equations are obtained

$$X'''_{CG} = (W9 - D52)\cos\varphi + (W10 - D53)\sin\varphi\cos\theta + (Z1 - D54)\sin\varphi\sin\theta \quad (5.23)$$

$$Z'''_{CG} = -(W10 - D53)\sin\theta + (W11 - D54)\cos\theta \quad (5.24)$$

Introducing Eqs. (5.23) and (5.24) into Eq (5.17) yields the equation of the angle G2.

$$G2 = \arctan\left(\frac{(W9 - D52)\cos\varphi + (W10 - D53)\sin\varphi\cos\theta + (W11 - D54)\sin\varphi\sin\theta}{-(W10 - D53)\sin\theta + (W11 - D54)\cos\theta}\right) \quad (5.25)$$

From the above equation, the angle G2 is a function of angles θ and φ , as well as of the intersection coordinates. Therefore, it is a function of the deck size.

Finally, the influence of deck size on the equivalent slope angle ($A1$) is discussed. The equivalent slope angle is different than the original slope angle. The equivalent slope angle ($A1$) is defined as the angle of the line (OT), an intersection of rotation plane and the slope, with respect to the horizontal plane (see figure 5-14). It affects the mower roll behavior. By adding the mower deck size, the tipping axis 1 changes the angle by θ_1 degrees from the longitudinal direction; therefore, the direction of the rotation plane 1 also changes an angle of θ_1 degrees. The relationship between $A1, \theta_1$ and the original slope angle ($A0$) is shown below:

$$\tan(A1) = \frac{c}{e} = \frac{\frac{c}{f}}{\frac{e}{f}} = \frac{\frac{c}{f}}{\sqrt{\frac{e^2}{f^2}}} = \frac{\frac{c}{f}}{\sqrt{\frac{f^2 + d^2}{f^2}}} = \frac{\frac{c}{f}}{\sqrt{1 + \left(\frac{d}{f}\right)^2}} = \frac{\frac{c}{f}}{\sqrt{1 + \left(\frac{\frac{d}{a}}{\frac{f}{a}}\right)^2}} \quad (5.26)$$

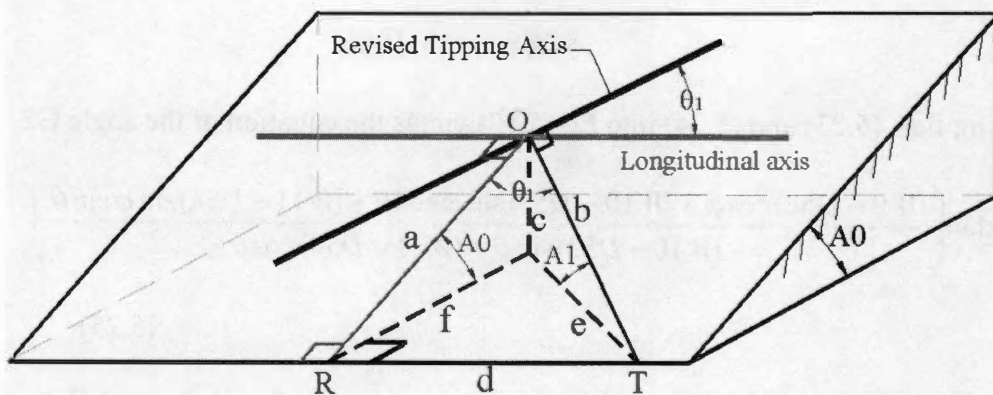


Figure 5-14. Relationship between the original slope angle and revised equivalent slope angle.

$$\tan(A1) = \frac{\tan(A0)}{\sqrt{1 + \left(\frac{\tan(\theta_1)}{\cos(A0)}\right)^2}} \quad (5.27)$$

$$A1 = \arctan \left(\frac{\tan(A0)}{\sqrt{1 + \left(\frac{\tan(\theta_1)}{\cos(A0)}\right)^2}} \right) \quad (5.28)$$

From figure 5-15, the following equations are obtained:

$$\theta_1 = \arctan \left(\frac{V1}{L2 + L3} \right) \text{ and} \quad (5.29)$$

$$\theta_1 = \arctan \left(\frac{\sqrt{(D3)^2 + \left(\frac{B0 - B01}{2}\right)^2}}{L2 + L3} \right). \quad (5.30)$$

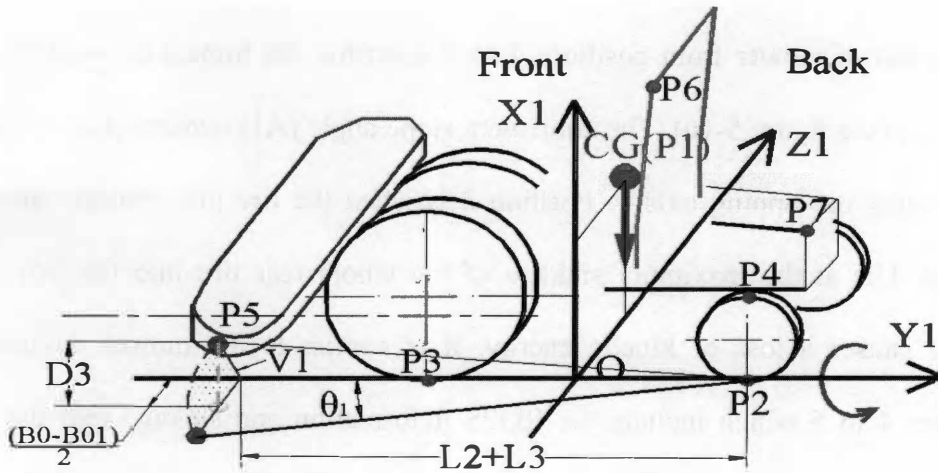


Figure 5-15. The relationship between θ_1 and deck size.

Where,

$\frac{B0}{2}$ = Extended width of the deck measured from the center line of the front tire to

the left end of the mower deck (m),

$B01$ = Width of the front tire (m),

$L3$ = Horizontal distance between the contact point of the deck and the center of gravity (m) and

$L2$ = Horizontal distance between the center of gravity and the rear axle (m).

From the above equation, the equivalent slope angle is a function θ_1 . The angle θ_1 is a function of deck size; therefore, the equivalent slope angle also is a function of deck size.

B. Revised kinetic energy from position 2 to position 3.

The motion of mower from positions 2 to 3 describes the impact of whole rear tire into slope (see figure 5-16). The equivalent slope angle ($A1$) occurs when the mower rolls around the tipping axis 1. Position 2 is when the tire just contacts the slope. Position 3 is at the maximum sinkage of the whole rear tire into the slope. This process causes a loss of kinetic energy. It is similar to the mower motion from positions 4 to 5 which include the ROPS deformation and sinkage into the slope. Since the mower motions from position 2 to 3 and position 4 to 5 involve sinkage, they are discussed together in section D.

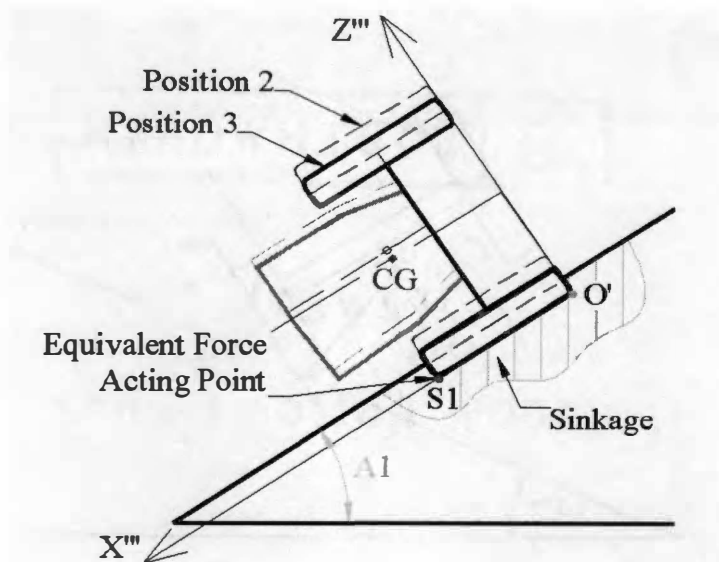


Figure 5-16. Motion of the mower from position 2 to position 3.

C. Revised potential energy from position 3 to position 4.

The motion of the mower from positions 3 to 4 describes the movement as the mower rolls around the tipping axis 2 until the ROPS hits the slope. Figure 5-17 shows the motions of the mower in the rotation plane 2. Point S is the equivalent force acting point (see figure 5-5). The actual force points are the deck/slope, rear tire/slope contact points and ROPS/slope contact point. The potential energy transforms into kinetic energy during this process. The angle A2 is the equivalent slope angle when the mower rolls around the tipping axis 2. As discussed before, it is also the function of deck size (see Appendix A). The calculations of the revised potential energy are similar to that from position 1 to 2.

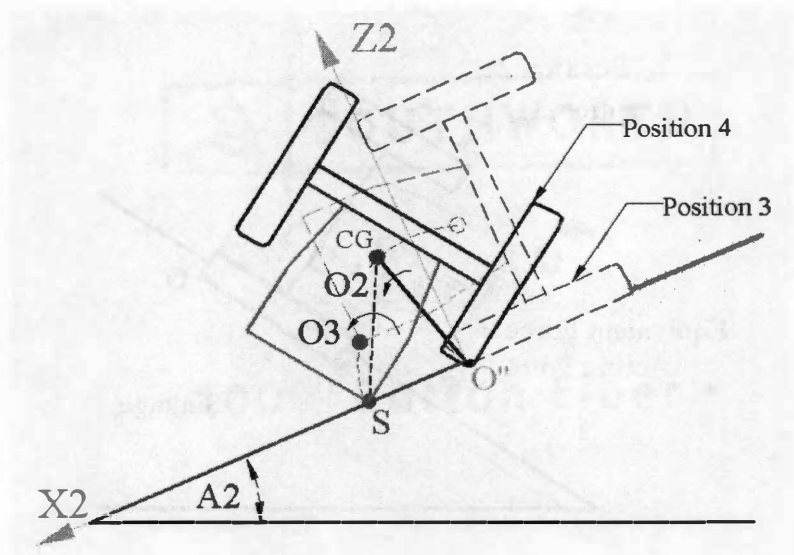


Figure 5-17. Motion of the mower from position 3 to position 4.

D. Revised kinetic energy from positions 4 to 5

The motion of the mower from position 4 to 5 describes the ROPS impact on the slope. Position 4 is defined as when ROPS hits the slope. Position 5 is at the maximum sinkage of ROPS into the slope (see figure 5-18). The following equations are used to determine the relationship between the angular velocity of the CG after the maximum sinkage of the ROPS into the slope, and the deck size. Because the intersection (O'') of the tipping axis 2 and rotation plane 2 and the equivalent force acting point (S) change by adding the deck size, they result in the change of the angular velocity of CG after the maximum sinkage of ROPS into the slope.

From Newton's second law and figure 5-18, the following equations are obtained.

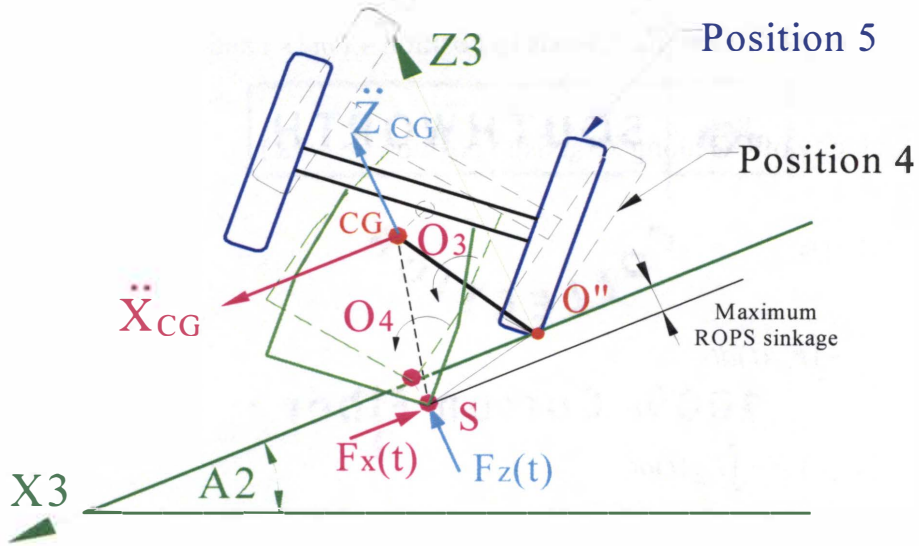


Figure 5-18. Mower motion from positions 4 to 5.

$$m \ddot{Z}_{CG} = F_z(t) \quad (5.31)$$

$$m \int \ddot{Z}_{CG} dt = \int F_z(t) dt \quad (5.32)$$

From figure 5-19, obtain

$$m (\dot{Z}_{CG2} - \dot{Z}_{CG1}) = \int F_z(t) dt \quad (5.33)$$

Where,

m = Mass of the mower (kg),

$\ddot{Z}_{CG}$ = Acceleration of CG about Z3-axis (m / s^2),

$F_z(t)$ = Impact Force of the mower along Z3-axis (N),

$\dot{Z}_{CG}$ = Velocity of CG along Z3-axis (m / s),

$\dot{Z}_{CG1}$ = Velocity of CG along Z3-axis in position 4 (m / s) and

$\dot{Z}_{CG2}$ = Velocity of CG along Z3-axis in position 5 (m / s).

$$m \ddot{X}_{CG} = -F_X(t) \quad (5.34)$$

$$m \int \ddot{X}_{CG} dt = - \int F_X(t) dt \quad (5.35)$$

$$m (\dot{X}_{CG2} - \dot{X}_{CG1}) = - \int F_X(t) dt \quad (5.36)$$

Where,

$\ddot{X}_{CG}$ = Acceleration of CG along X3-axis (m / s²),

$F_X(t)$ = Impact force of the mower along X3-axis (N) ,

$\dot{X}_{CG}$ = Velocity of CG along X3-axis (m / s),

$\dot{X}_{CG1}$ = Velocity of CG along X3-axis in position 4 (m / s) and

$\dot{X}_{CG2}$ = Velocity of CG along X3-axis in position 5 (m / s).

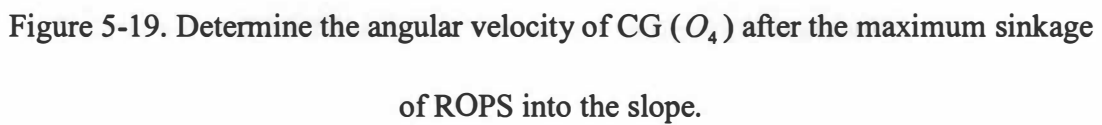
From Newton's second law for rotational motion and the figure 5-19, the following equations are obtained:

$$J_{CG} \dot{\omega}(t) = M_{CG}(t) \quad (5.37)$$

$$J_{CG} \int \dot{\omega}(t) dt = \int M_{CG}(t) dt \quad (5.38)$$

$$J_{CG} (O_4 - O_3) = \int (F_X(t) K_6 - F_Z(t) K_7) dt \quad (5.39)$$

$$J_{CG} (O_4 - O_3) = K_6 \int F_X(t) dt - K_7 \int F_Z(t) dt \quad (5.40)$$


$$\dot{X}_{\text{CG1}} = O_3 K_5, \quad (5.41)$$

$$\dot{Z}_{CG1} = -O_3 K_9, \quad (5.42)$$

From figure 5-19, as the ROPS just hits the slope, the velocity of point S is

$$V_s = -(K_7 + K_9)O_3, \quad (5.43)$$

Using the law of conservation of momentum, after impact with the slope, the velocity of the mower can be obtained as follows:

$$mV_1 + m_2V_2 = mV_1' + m_2V_2' \quad (5.44)$$

Where,

m_2 = Mass of the slope (kg),

V_1 = Velocity of the mower before impact (m / s),

V_1' = Velocity of the mower after the maximum sinkage of ROPS into the slope (m / s),

V_2 = Velocity of the slope before impact (m / s) and

V_2' = Velocity of the slope after the maximum sinkage of ROPS into the slope (m / s).

The following equation determines the relationship between the elasticity coefficient (U) of the slope and the velocities of two bodies.

$$U = \frac{V_2' - V_1'}{V_1 - V_2} \quad (5.45)$$

Solving the Eqs (5.44) and (5.45), the Eq (5.46) is obtained

$$V_1' = V_1 - \frac{1}{\left(1 + \frac{m}{m_2}\right)} (1 + U)(V_1 - V_2) \quad (5.46)$$

Because the mass of the slope is infinity ($m_2 = \infty$ and $V_2 = 0$), the velocity of the mower after impact into the slope is:

$$V_1' = -UV_1 \quad (5.47)$$

Therefore, the velocity (V_s') of the point S after the maximum sinkage of ROPS into the slope is

$$V_s' = -UV_s \quad (5.48)$$

$$V_s' = U((K_7 + K_9)O_3) \quad (5.49)$$

Where:

V_s' = Velocity of the equivalent force acting point (S) after maximum sinkage of ROPS into the slope (m/s) and

V_s = Velocity of the equivalent force acting point (S) when ROPS just hits the slope (m/s).

The mower starts to rotate around the point S (see figure 5-19). The angular velocity of CG about the point S is O_4 (see figure 5-19).

$$\dot{Z}_{CG2} = V_s' + (V_{CG/S})_z \quad (5.50)$$

$$\dot{Z}_{CG2} = V_s' + (r_2 O_4) \sin(\beta) \quad (5.51)$$

$$\dot{Z}_{CG2} = U((K_9 + K_7)O_3) + K_7 O_4 \quad (5.52)$$

$$\dot{X}_{CG2} = O_4 K_6 \quad (5.53)$$

Substituting Eqs (5.41) and (5.53) into Eq (5.36), Eq (5.54) is obtained.

$$m(O_3K_5 - O_4K_6) = \int F_X(t)dt \quad (5.54)$$

Substituting Eqs (5.41) and (5.53) into Eq (5.36), Eq (5.55) is obtained.

$$m(U(K_9 + K_7)O_3 + K_7O_4 + O_3K_9) = \int F_Z(t)dt \quad (5.55)$$

Substituting Eqs (5.42) and (5.52) into Eq (5.33), the angular velocity of the center of gravity (O_4) after ROPS impact into slope is obtained:

$$O_4 = \left(\frac{J_{CGY} + mK_5K_6 - mUK_7^2 - (1+U)mK_9K_7}{J_{CGY} + mK_6^2 + mK_7^2} \right) \Theta_3 \quad (5.56)$$

From the equation (5.56), the angular velocity of CG (O_4) is a function of K_5, K_6, K_7, K_9 and J_{CGY} , these factors are a function of deck size; therefore, the angular velocity of CG (O_4) is a function of the deck size.

Similarly, the angular velocity (O_1) of CG after the maximum sinkage of the rear tire into the slope from positions 2 to 3 can be determined by the following equations (see figures 5-20 and 5-21). The plane CGO'S1 is the rotation plane1 (see figure 5-20), therefore, the triangle P5O'S1 is a right-angled triangle ($\angle P5O'S1 = 90^\circ$).

$$K_1 = |Y'''(1,5)| \tan(F22) \quad (5.57)$$

$$O_1 = \left(\frac{J_{Y''} + mK_3^2 - UmK_4^2 - (1+U)mK_2K_4}{J_{Y''} + mK_3^2 + mK_4^2} \right) * \Theta_0 \quad (5.58)$$

Where,

$|Y'''(1,5)|$ = Absolute value of the y-coordinate of the point P5, relative to the system

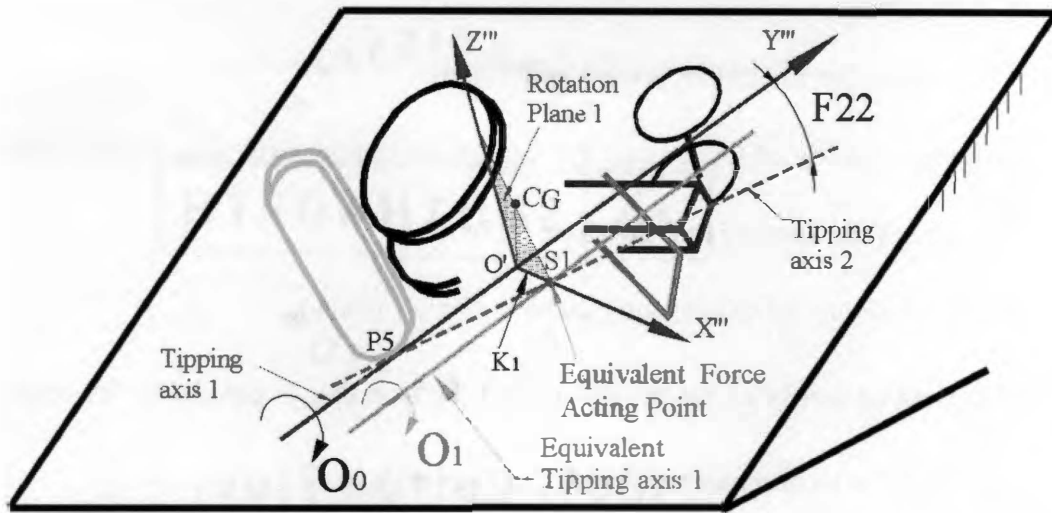


Figure 5-20. Determine the angular velocity (O_1) of CG after the maximum sinkage of the rear tire into the slope.

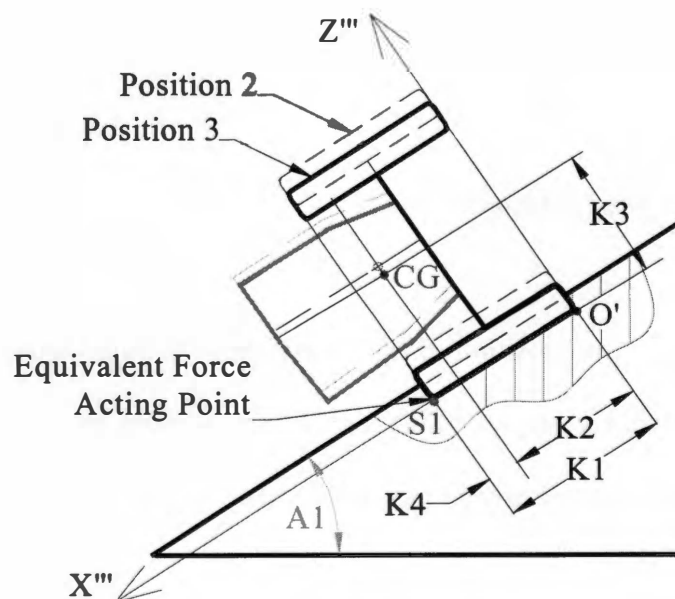


Figure 5-21. Definitions of K1, K2, K3 and K4 in position 3.

$(X''', Y''', Z''') \text{ (m)},$

$F_{22} = \text{Angle between the tipping axes 1 and 2 (rad)},$

$O_1 = \text{Angular velocity of CG around the equivalent tipping axis after the maximum sinkage of the rear tire into the slope (rad/s)},$

$O_0 = \text{Angular velocity of CG around the tipping axis 1 (rad/s)},$

$J_{Y'''} = \text{Moment of inertia of the mower around the axis that is parallel to the tipping axis 1 (Y''' axis) and passes through CG (kg} \cdot \text{m}^2 \text{)},$

$m = \text{Mass of mower (kg)},$

$K_1 = \text{X-coordinate of the point S, an equivalent force acting point, relative to (X''', Y''', Z''') system (m)},$

$K_2 = \text{X-coordinate of CG, relative to (X''', Y''', Z''') system (m)},$

$K_3 = \text{Z-coordinate of CG, relative to (X''', Y''', Z''') system (m), and}$

$K_4 = K_1 - K_2 \text{ (m)}.$

E. Revised energy from positions 5 to 6.

The revised energy from positions 5 to 6 is discussed in detail later (see section 5.2).

The deck size causes the change of the mower motion from positions 5 to 6. The yaw and slide downhill were observed in the field tests.

F. Revised energy from positions 6 to 7.

The motion of mower from positions 6 to 7 is the process by which the kinetic energy transforms to potential energy. The principle is the same as that from positions 1 to 2.

5.1.3 Revised Moment of Inertia

Because the tipping axes change, the moment of inertia around the tipping axis changes. The modified moment of inertia is described below (see figure 5-22). A right hand coordinate system (X, Y, Z) is fixed in CG, with the X, Y, Z directions being vertical upward, horizontal rearward, and horizontal to the driver right. The counterclockwise rotation direction is considered positive if look down from the top. Assuming that the axes X, Y and Z are principle axes, therefore, the inertia matrix

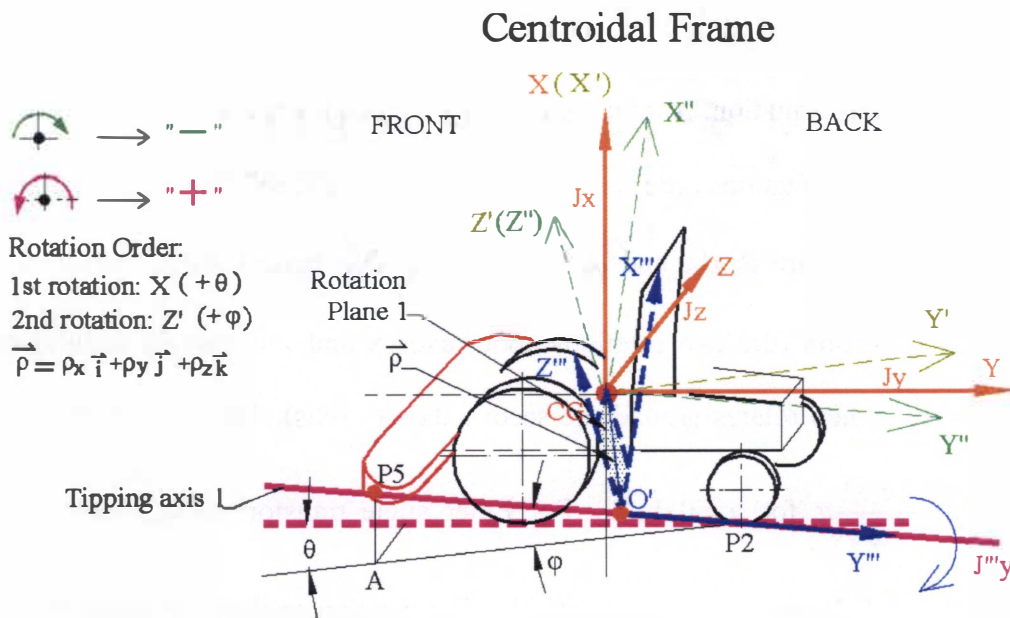


Figure 5-22. Transfer the moment of inertia from old coordinates to new coordinates.

about (X, Y, Z) is:

$$[J] = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \quad (5.59)$$

The moment of inertia around the tipping axis 1 can be obtained by the transformation and translation of the moment of inertia around the (X, Y, and Z). In sequence, these rotations are θ with respect to the X-axis, ϕ with respect to the Z'-axis, therefore, the moment of inertia about the system (X'', Y'', Z'') is determined. Finally, the moment of inertia around the system (X''', Y''', Z''') is obtained by using the parallel-axis theorem. The inertia matrix about the system (X''', Y''', Z''') is:

$$[J'''] = \begin{bmatrix} J'''_x & -J_{xy} & -J_{xz} \\ -J_{yx} & J'''_y & -J_{yz} \\ -J_{zx} & -J_{zy} & J'''_z \end{bmatrix} \quad (5.60)$$

There are two methods for the transformation of the moment of inertia. One is the Bryant angle transformation. Another is Euler angle transformation. For Bryant angle transformation, the rotations order is X, Y, and Z axes (yaw, roll, pitch), respectively. The rotation angle are θ_1, θ_2 , and θ_3 respectively. The Bryant angle matrix has a singularity condition (the two rotations about axes x and z cannot be individually distinguished) when rotates around the second axis (y axis). The singularity occurs where $\theta_2 = \frac{2n+1}{2}\pi$ for $n = 0, 1, 2, \dots$. For Euler angle transformation, the rotations order is about Z, X, and Z axes respectively. The rotation angle are ϕ, θ and Φ . The

Euler angle transformation has problems if the second rotational angle, θ is a multiple of π ($\theta = n\pi$) for $n = 0, 1, 2, \dots$

In this project situation, only two rotations were needed. The rotation order was X, Z axes, respectively, therefore, it can belong to either Bryant or Euler angle transformation. If it is considered Bryant angle transformation ($X(\theta_1), Y(\theta_2), Z(\theta_3)$), the rotation angle (θ_2) around Y-axis is zero. It meets the angle limitation. If it is considered Euler angle transformation ($Z(\phi), X(\theta), Z(\Phi)$), the rotation angle (θ) is smaller than $\frac{\pi}{2}$; it also meets the angle limitation, because only the deck width equals infinite, the rotation angle (θ) can equal $\frac{\pi}{2}$. In summary, in this project situation, the matrix transformation meets all angle limitations.

The transformation of the moment of inertia from the system (X, Y, Z) to the system (X''', Y''', Z''') is shown below:

$$J''' = (A_1 A_2)^T J (A_1 A_2) - m \tilde{\rho} \tilde{\rho} \quad (5.61)$$

$$J''' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{pmatrix} \begin{pmatrix} c\varphi & -s\varphi & 0 \\ s\varphi & c\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}^T \begin{pmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{pmatrix} \begin{pmatrix} c\varphi & -s\varphi & 0 \\ s\varphi & c\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} - m \tilde{\rho} \tilde{\rho} \quad (5.62)$$

$$J''' = \begin{bmatrix} c\varphi & s\varphi c\theta & s\theta s\varphi \\ -s\varphi & c\theta c\varphi & s\theta c\varphi \\ 0 & -s\theta & c\theta \end{bmatrix} \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \begin{bmatrix} c\varphi & -s\varphi & 0 \\ s\varphi c\theta & c\theta c\varphi & -s\theta \\ s\theta s\varphi & s\theta c\varphi & c\theta \end{bmatrix} - m \begin{bmatrix} 0 & -\rho_z & \rho_y \\ \rho_z & 0 & -\rho_x \\ -\rho_y & \rho_x & 0 \end{bmatrix} \begin{bmatrix} 0 & -\rho_z & \rho_y \\ \rho_z & 0 & -\rho_x \\ -\rho_y & \rho_x & 0 \end{bmatrix} \quad (5.63)$$

$$J''' = \begin{bmatrix} \Delta 1 & \Delta 2 & \Delta 3 \\ \Delta 4 & J_x \sin^2(\varphi) + J_y \cos^2(\theta) \cos^2(\varphi) + J_z \sin^2(\theta) \cos^2(\varphi) & \Delta 5 \\ \Delta 6 & \Delta 7 & \Delta 8 \end{bmatrix} - m \begin{bmatrix} -\rho_z^2 - \rho_y^2 & \rho_x \rho_y & \rho_x \rho_z \\ \rho_x \rho_y & -\rho_z^2 - \rho_x^2 & \rho_y \rho_z \\ \rho_x \rho_z & \rho_y \rho_z & -\rho_y^2 - \rho_x^2 \end{bmatrix} \quad (5.64)$$

Therefore, the moment of inertia around tipping axis 1 is obtained:

$$J'''_y = J_x \sin^2(\varphi) + J_y \cos^2(\theta) \cos^2(\varphi) + J_z \sin^2(\theta) \cos^2(\varphi) + m(\rho_x^2 + \rho_z^2) \quad (5.65)$$

Similarly, the moments of inertia around tipping axes 2 and 3 can also be obtained.

Where,

A1= Matrix of the direction cosines and sines between the longitudinal axis and line

AP2,

A2= Matrix of the direction cosines and sines between the line AP2 and tipping axis1,

A^T = Transpose matrix of A,

θ = Angle between the Y-axis and the tipping axis 1 (rad),

φ = Angle between line AP2 and the tipping axis 1(rad),

ρ = Vector where the beginning point is center of gravity and the end point is the intersecting point of rotation plane and tipping axis (m),

J_X = Moment of inertia of the mower around the X-axis ($kg \cdot m^2$),

J_Y = Moment of inertia of the mower around the Y-axis ($kg \cdot m^2$),

J_Z = Moment of inertia of the mower around the Z-axis ($kg \cdot m^2$),

J''_Y = Moment of inertia of the mower around the Y'' -axis ($kg \cdot m^2$),

$c\varphi = \cos(\varphi)$,

$s\varphi = \sin(\varphi)$,

$c\theta = \cos(\theta)$ and

$s\theta = \sin(\theta)$.

From Eq (5.64), the moment of inertia around the new tipping axis is a function of θ and φ (i.e. it is a function of deck size).

5.2 Addition of Yaw Angle and Slide to the Mower Motion

From the field tests, mower rotation (yaw) and slide down the slope were observed after the maximum sinkage of ROPS into the slope. The slide direction is almost along a direction perpendicular to the Y_2 axis. Therefore, the mower undergoes a combination of three motions including slide, roll and yaw down the slope around the equivalent force acting point S (see figure 5-23). The mower moves from position 5 to position 6 after yaw and slide (see figure 5-23). At position 6, assuming that the

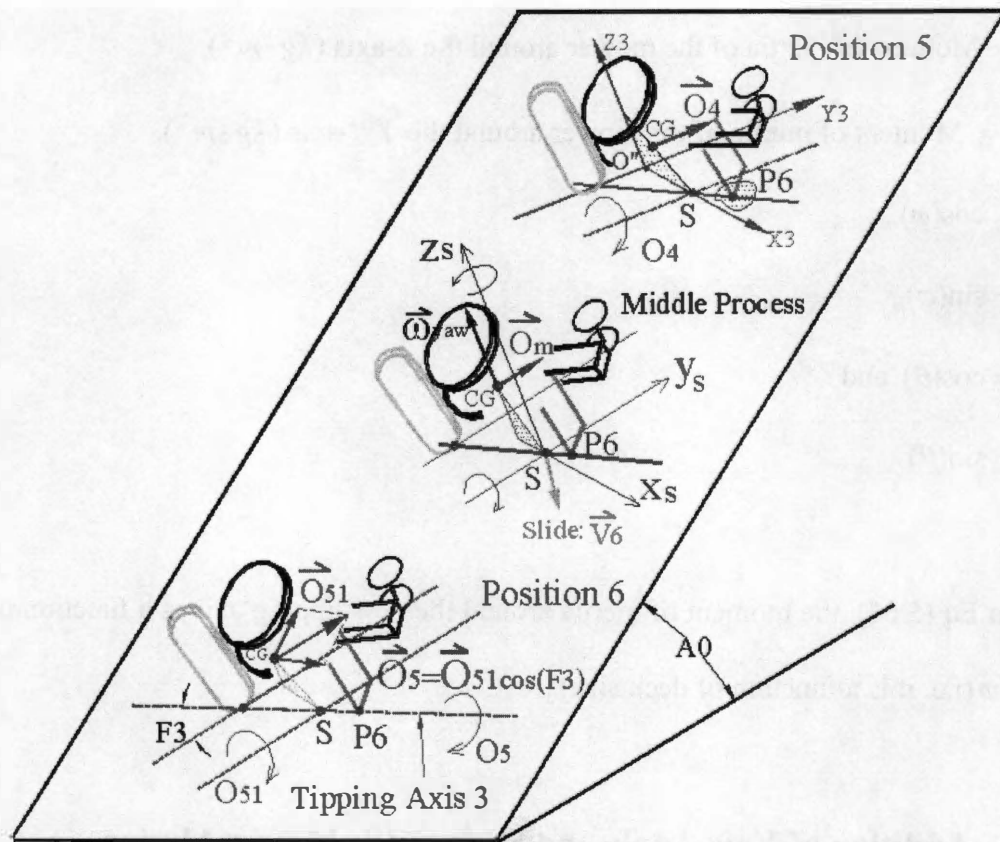


Figure 5-23. Adding the roll, yaw and slide into the model.

yaw angular velocity and slide velocity are zero, only the roll around the equivalent tipping axis exists with angular velocity O_{s1} . The mower will roll around the tipping axis 3 with the angular velocity O_s (see figure 5-23).

Figure 5-24 shows the potential energy changes due to the slide and yaw of the mower downhill in two dimensions. Because of the work due to the friction force, the conservation of energy equation is described as follows:

$$\frac{1}{2} J_s O_{s1}^2 + mgh_2 = \left(\frac{1}{2} J_s O_4^2 + mgh_1 \right) - F_s L_s \quad (5.66)$$

$$O_{s1} = \sqrt{O_4^2 + \left(\frac{2mgh - 2F_s * L_s}{J_s} \right)} \quad (5.67)$$

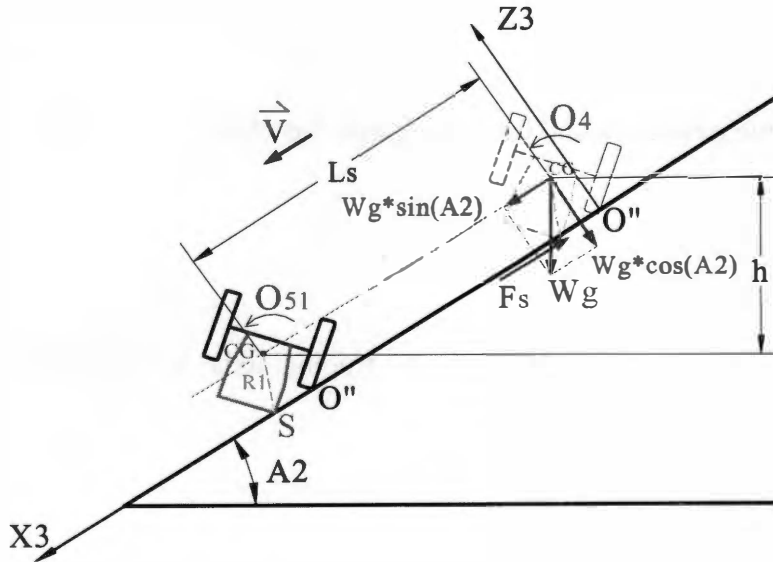


Figure 5-24. Potential energy change due to slide and yaw.

Where,

J_s = Moment of inertia about the tipping axis that passes through the point S and is parallel to the tipping axis 2 ($kg \cdot m^2$) (see figure 5-23),

O_{s1} = Angular velocity of the mower around the tipping axis that passes through the point S and is parallel to the tipping axis 2 at position 6 (rad / s) (see figure 5-23),

O_4 = Angular velocity of the mower around the tipping axis that passes through the point S and is parallel to the tipping axis 2 at position 5 (rad / s) (see figure 5-23),

F_s = Friction force (N),

L_s = Slide distance of CG (m),

m = Mass of mower (kg) and

g = Gravity acceleration (m / s^2).

The change in height of CG is determined as following:

$$h = h_1 - h_2 \quad (5.68)$$

The angular velocity of the mower around the tipping axis 3 is obtained as follows:

$$O_5 = O_{s1} \cos(F3) \quad (5.69)$$

Where,

O_5 = Angular velocity of the mower around the tipping axis 3 at position 6 (rad / s) and

$F3$ = Angle between the tipping axes 2 and 3 (rad).

CHAPTER VI

EVALUATION OF THE REVISED MODEL

6.1 Measured Slide and Yaw

The deck size, the yaw angle, the slide distance, and the friction coefficient are considered as inputs of the revised model. In order to measure the yaw angle and the slide distance during the mower continuous roll, one-meter grid marks are put on the slope (see figure 6-1). Nine field tests were conducted on the slope for the F925 mower with the regular ROPS (see table 4). The regular ROPS is defined as the top of ROPS tilting backward. The ROPS height was adjusted using the extension described in section 6.3. For the regular ROPS, the average slide distance on the polyethylene pad is approximately one-meter. The angle between the tipping axis 3 and the longitudinal direction after the mower undertakes a yaw and when the mower continues to roll was estimated from video footage. The angle between the tipping axis 3 and the longitudinal axis change because the mower yaws. The average value of this angle is about 15 degrees (see figure 6-2). This angle can be dependent on mower and slope conditions. It directly affects the equivalent slope angle.

Then, the ROPS was rotated 180-degrees so the top of ROPS tilts forward (see figures 6-3 and 6-4). This mounting position of ROPS is called an inverted ROPS. By doing this, the horizontal distance between the ROPS impact point and the center of gravity (L6) was changed. Three field tests on the slope for the F925 mower with

Table 4: Field tests on polyethylene pad with regular ROPS

ROPS Position	ROPS Height (m)	No of Tests
Regular	1.90	3
Regular	2.22	1
Regular	2.42	1
Regular	2.55	2
Regular	2.67	2



Figure 6-1. ROPS just hits the slope.

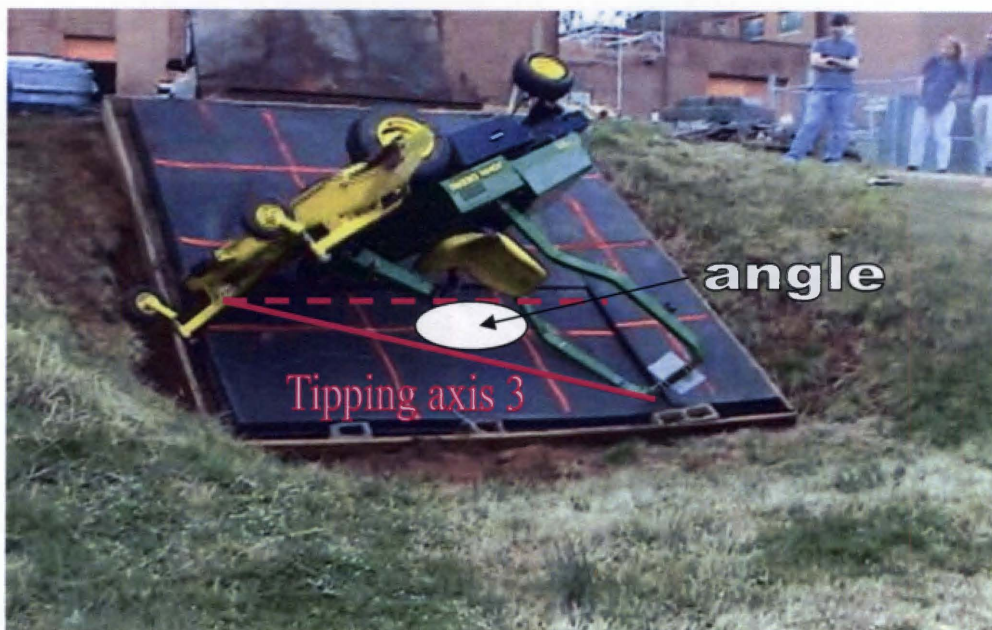


Figure 6-2. The angle when the mower continues to roll about the tipping axis 3.

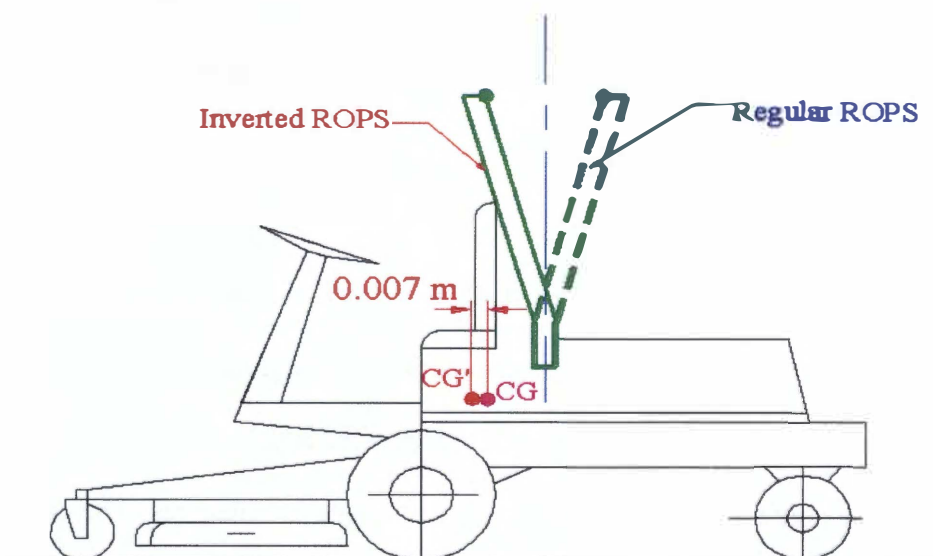


Figure 6-3. Inverted ROPS and Regular ROPS.



Figure 6-4. Field test for F925 mower with inverted ROPS.

the inverted ROPS were conducted (see table 5). For the inverted ROPS, more slides were observed. The average slide distance is about 1.2 meter. The average value of the angle between the tipping axis 3 and the longitudinal direction is about 5 degrees (see figure 6-5).

In order to obtain this angle and slide distance on actual grass slopes of 30 degrees, the measurement was conducted on a grass slope of 30 degrees (see figures 6-6 and 6-7). The slide distance of the ROPS/slope contact point is 0.51 m; the slide distance of the deck/slope contact point is 0.68 m. The yaw angle is 5 degrees. The direction of friction force always varied according to yaw. The influence of the slide on energy conservation needs to be explored. Friction coefficient measurements are needed.

Table 5: Field tests on polyethylene pad with inverted ROPS

ROPS Position	ROPS Height (m)	No of Tests
Inverted	1.90	1
Inverted	2.03	1
Inverted	2.22	1



Figure 6-5. The additional slide and yaw downhill for inverted ROPS.



Figure 6-6. Measurement of yaw angle and slide distance on actual grass slope.



Figure 6-7. Put the marks on contact points of deck and ROPS/grass slope.

With a low friction force, the slide increases the kinetic energy and the roll potential. The slide can either increase or decrease the roll potential depending on the friction coefficient.

An approximate measurement of the friction force was conducted by holding the mower with chains on the slope (see figure 6-8 and 6-9). The value of the pull force was collected by a force transducer. Based on the force transducer measurements, the friction coefficient can be obtained by the calculation. The average friction coefficient of polyethylene pads was 0.505. By using the same method, the friction coefficient of real grass can be measured.



Figure 6-8. Measurement of the friction coefficient.

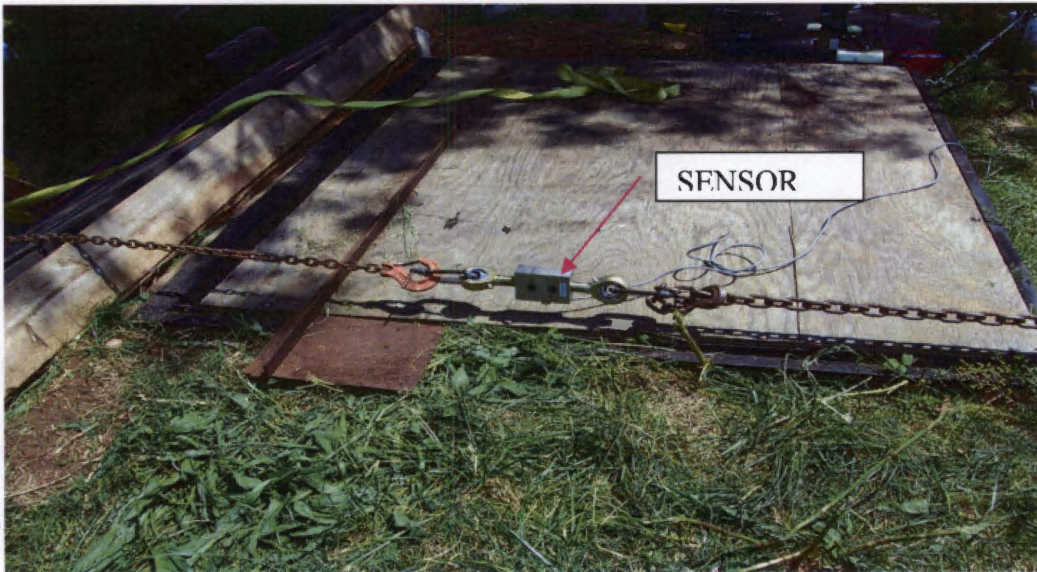


Figure 6-9. Load cell sensor is used to measure the friction force.

6.2 Revised Model Estimates of CRH

6.2.1 Reference Mower Dimensions

In order to run the program, the following dimensions are some important inputs.

A. Regular ROPS

The slide distance due to slide and yaw was 1.0 m. The angle (LL2) between the tipping axis 3 and the longitudinal axis was 15 degree after the mower rotates (see figure 6-10 and Appendix A.3). The yaw angle is the difference between the angle

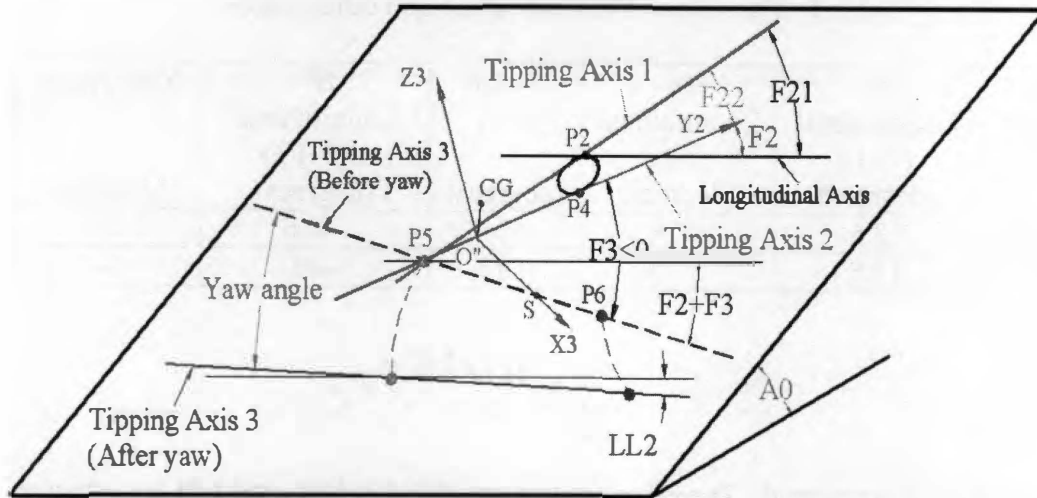


Figure 6-10. Definition of angles between the axes.

LL2 and the angle of F_2+F_3 (see figure 6-10 and table 6). The mowing deck height and transporting deck height are 0.1 m and 0.2 m respectively. The heights of the center of gravity are 0.45 m and 0.48 m respectively for mowing position and transportation position. The determination of center of gravity, the ROPS deformation (T) and the Cone Penetration Index (CI) are shown in Appendix B. The horizontal distance between the CG and the ROPS impact point is -0.471 m.

B. Inverted ROPS

The horizontal distance between the CG and the ROPS impact point is -0.045 m. The weight of ROPS is 70 (lb), therefore, center of gravity moves forward of 0.007 m

Table 6: The value of the yaw angle and other angles

ROPS height	I- Longitudinal (F21) (degrees)	II- Longitudinal (F2) (degrees)	II-III (F3) (degrees)	III- Longitudinal (F2+F3) (degrees)	Yaw Angle (degrees)
1.90	12.5	3.14	-43.7	-40.6	25
2.63	12.5	3.14	-52.6	-49	34

when the ROPS is inverted. The other parameters (L2, L3, L31, and L7) also change. The slide distance and yaw angle will change due to the change of the equivalent force acting point (S). The slide distance due to slide and yaw was 1.2 m. The angle (LL2) between the tipping axis 3 and the longitudinal axis was 5 degrees after the mower rotated (see figure 6-5).

6.2.2 Revised Model Results

A. Regular ROPS

Using the inputs described in section 6.2.1 and Appendix A.3, the revised model predicted CRH was of 2.50 m and 2.63 m with the deck in transportation and working position respectively (0.2 m and 0.1 m) (see table 7).

B. Inverted ROPS

Table 7: Revised and original model results with regular ROPS

	CRH (m) Transportation	CRH (m) Mowing
Revised Model	2.50	2.63
Original Model	2.17	2.17

Table 8: Revised and original model results with inverted ROPS

	CRH (m) Transportation	CRH (m) Mowing
Revised Model	1.92	2.00
Original Model	1.67	1.67

For inverted ROPS, the revised model predicted a CRH of 1.92 m and 2.00 m with the deck transportation and working position respectively (0.2m and 0.1 m) (table 8).

From the revised model results, the deck slide and yaw have a greater contribution to the mower continuous roll.

6.3 Critical ROPS Height by Field Test

In order to evaluate the accuracy of the revised model, the field tests were conducted for the F925 with a regular ROPS and an inverted ROPS when the deck was down and up. An adjustable fitting was made for the ROPS height extension (see figure 6-11). The space between two holes was 2.6 in (6.5 cm). The top end was connected to the ROPS; the bottom end was connected to the mower frame (see figure 6-12).

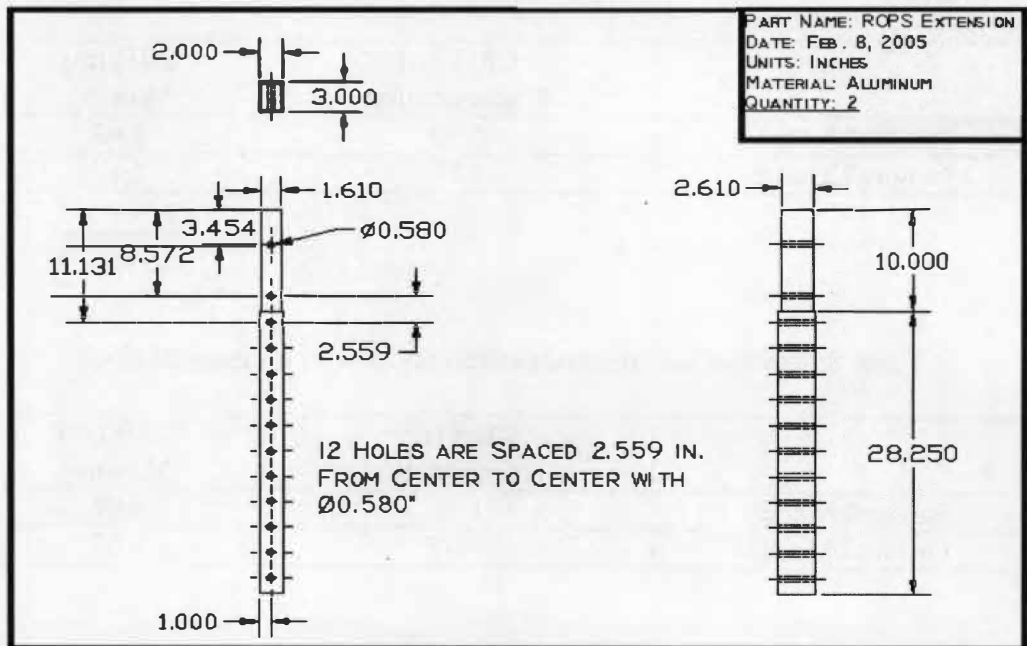


Figure 6-11. Adjustable fitting for ROPS height extension.



Figure 6-12. Mount the adjustable fitting for ROPS height extension to the mower frame.

This allowed the ROPS height at the impact point to be raised from 1.90 m to 2.67 m to evaluate the critical ROPS height.

6.3.1 Field Tests for the F925 Mower with Regular ROPS

Figures 6-13 and 6-14 show the field tests for the F925 mower with the regular ROPS. The distance between the ROPS impact point and the center of gravity was -0.471 m (negative, since the ROPS impact point lies behind of the center of gravity). Table 9 shows the field test results. In the field test, the F925 mower ends upside down at the bottom of the slope (see figure 6-14). Technically, it is a continuous roll; however, the angular velocity at position 7 is near zero. This position is defined as the critical roll. Therefore, the ROPS height in the critical roll is close to the estimated CRH. The field test also showed that the mower with the deck up was less likely to roll.

Due to variations of the continuous roll dynamics in the field tests, an expected theoretical field-test performance is shown in figure 6-15. To evaluate the model accuracy, the predicted critical ROPS height should be compared to the ROPS height producing a 50% possibility of continuous roll. Based on the table 9, the critical ROPS height (CRH) for the field tests is estimated as 2.55 m with deck up and 2.67 m with the deck down. These values are an estimate of the CRH from field tests. Many more tests would be needed to assure the statistical validity of these estimates.



Figure 6-13. Field test with the extended ROPS height (regular ROPS).



Figure 6-14. A field test with regular ROPS height of 2.55 m (deck down).

Table 9: Field test results for F925 mower with regular ROPS

ROPS Height (m)	Number of Test	Field Test Results		
		Continuous Roll	Continuous Roll	Continuous Roll
1.90	3	Continuous Roll	Continuous Roll	Continuous Roll
1.90	1	Continuous Roll (grass slope of 30 degrees)		
2.22	1	Continuous Roll		
2.42	1	Continuous Roll		
2.55	2	Critical Roll (deck down)	Non-Continuous Roll (deck up)	
2.67	2	Critical Roll (deck down)	Non-Continuous Roll (deck down)	

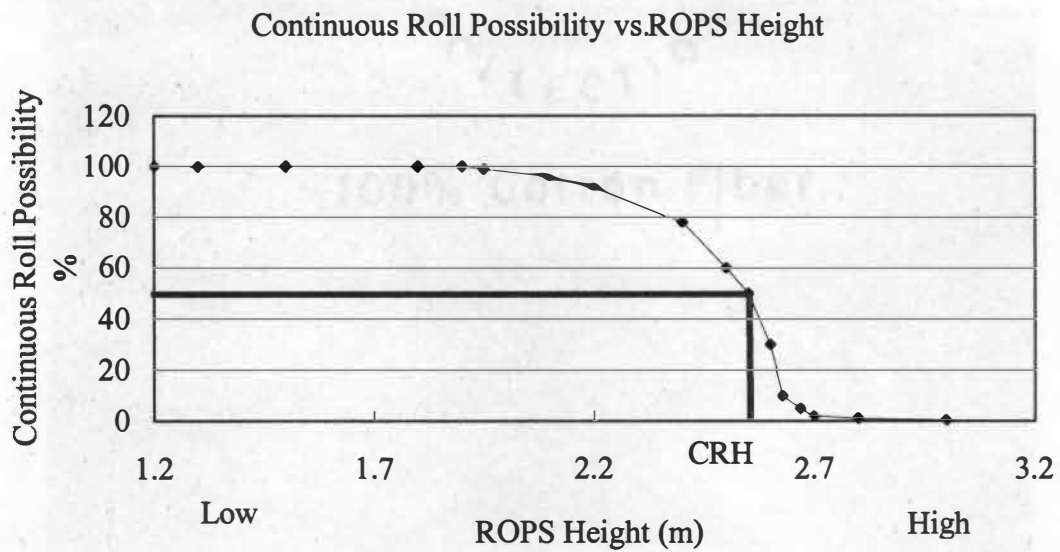


Figure 6-15. The relationship between ROPS height and continuous roll possibility.

6.3.2 Field Test for the F925 Mower with Inverted ROPS

A field test with an inverted ROPS height of 2.03 m was conducted. The critical roll was also observed (see figure 6-16 and table 10). When the ROPS height was increased to 2.22 m, non-continuous roll was observed.



Figure 6-16. A field test with inverted ROPS height of 2.03 m.

Table 10: Field test results for F925 mower with inverted ROPS

ROPS Height (m)	Number of Test	Field Test Results
1.90	1	Continuous Roll
2.03	1	Critical Roll
2.22	1	Non-Continuous Roll

6.4 Comparison of the CRH for the Original Model, the Revised Model and Field Test Results.

6.4.1 Regular ROPS

From sections 6.2 and 6.3, the accuracy of the revised model can be evaluated. For the regular ROPS, the measured CRH are 2.55 m and 2.67 m with deck in the transportation (up) and the working position (down) respectively. The CRH predicted by the original model is 2.17 m. The CRH predicted by the revised model is 2.50 m, and 2.63 m with the deck in the transportation (up) and the working position (down) respectively. Field tests show that the revised model result is much closer to the field test result (see figures 6-17).

6.4.2 Inverted ROPS

For the inverted ROPS, the horizontal distance between the CG and the ROPS impact point changes from 0.471 m to 0.045 m. The measured CRH is 2.03 m with deck in working position (down). The CRH predicted by the original model is 1.67 m. The CRH predicted by the revised model is 2.00 m with the deck in the working position (down). Field tests show that the revised model result is much closer to the field test result (see figures 6-18).

Model Results VS. Field Test Results

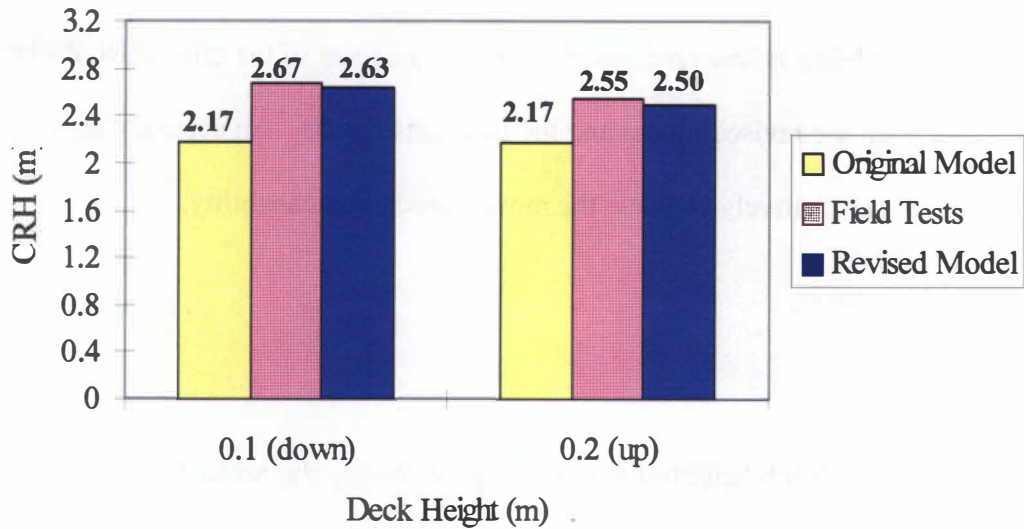


Figure 6-17. Comparison between model results and field test results for regular ROPS.

Model Results V.S. Field Test Results

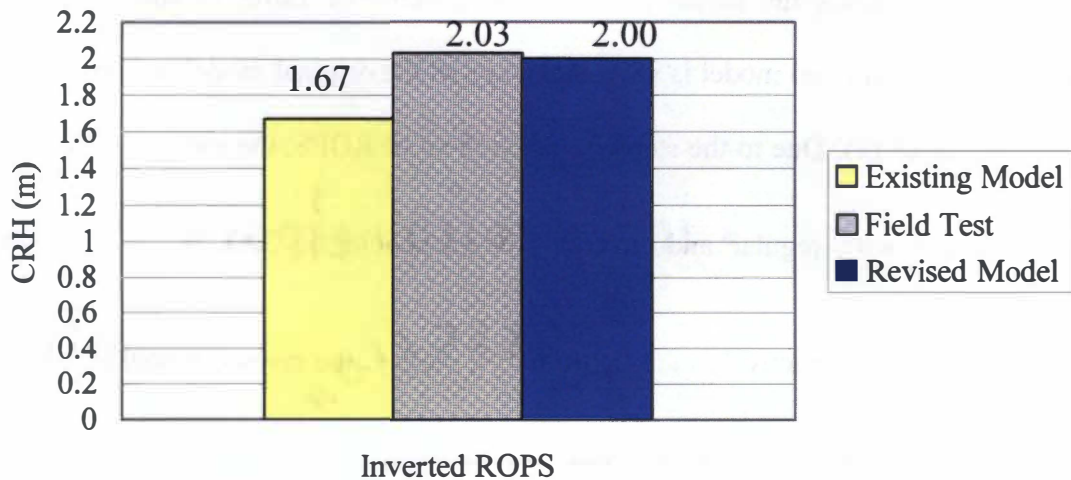


Figure 6-18. Comparison between model results and field test results for inverted ROPS.

6.5 Accuracy Factor of the Revised Continuous Roll Model

Prediction capability is analyzed based on the comparison of the critical ROPS height (CRH) between the revised model and the field data results. An accuracy factor (AF) is defined to quantitatively evaluate the mower prediction capability.

$$AF = \frac{CRH1}{CRH2} * 100\%$$

Where,

CRH1 = Critical ROPS height of the mower predicted by the revised model (m),

CRH2 = Critical ROPS height of the mower determined from the field test (m) and

AF= accuracy factor.

If the accuracy factor equals 100%, it indicates the model predicts accurately. If the factor is smaller than 100%, the model predicts unsafe conditions. A factor greater than 100% indicates the model predicts safe conditions. Table 11 shows that the accuracy of the revised model is 98%. Accuracy of the original model is from 81% to 85% (see table 12). Due to the stepwise adjustment of ROPS, the measured CRH for

F925 mower with regular and inverted ROPS should be $2.67 \pm \frac{0.065}{2}$ (m) and

$2.03 \pm \frac{0.065}{2}$ (m) respectively (see figure 6-19), clearly, the revised model is a better

predictor of the continuous roll behavior.

Table 11: The accuracy factor (AF) of the revised model

ROPS position	Deck Height (m)	CRH Revised Model Result (m)	CRH Field Test Result (m)	AF (%)
Regular (L6=-0.471m)	D3=0.1	2.63	2.67	98
Regular (L6=-0.471m)	D3=0.2	2.50	2.55	98
Inverted (L6=-0.045m)	D3=0.1	2.00	2.03	99

Table 12: The accuracy factor (AF) of the original model

ROPS position	Deck Height (m)	CRH Original Model Result (m)	CRH Field Test Result (m)	AF (%)
Regular (L6=-0.471m)	D3=0.1	2.17	2.67	81
Regular (L6=-0.471m)	D3=0.2	2.17	2.55	85
Inverted (L6=-0.045m)	D3=0.1	1.67	2.03	82

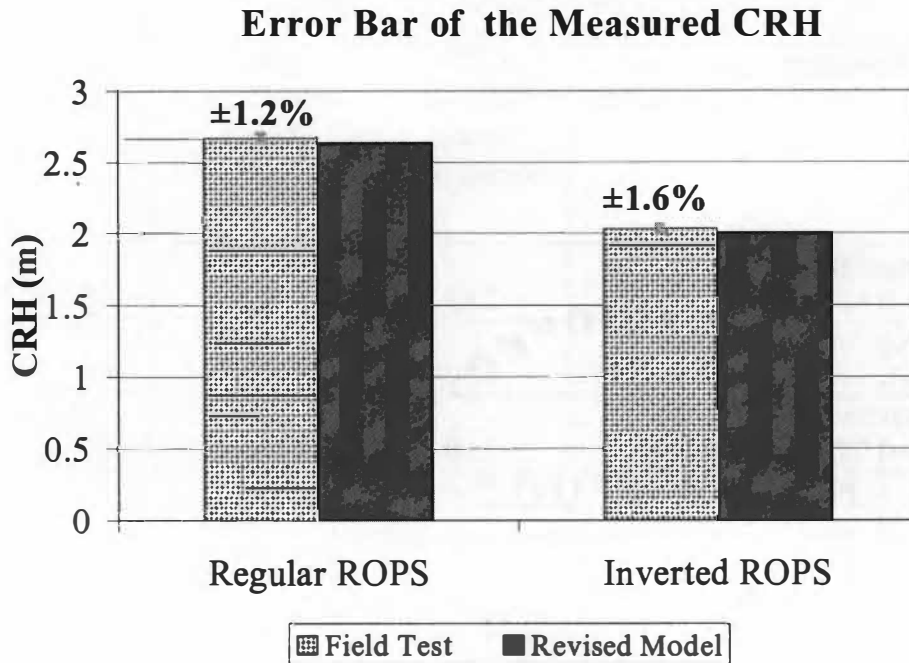


Figure 6-19. Error bar of the measured CRH due to stepwise adjustment of ROPS.

6.6 Sensitivity Analysis of Revised Model

In order to know how various parameters affect the roll behavior and the influence degree, a sensitivity analysis was conducted for the revised model.

6.6.1 Sensitivity Analysis of Mower Dimensions

Sensitivity analysis of roll parameters included: 1) Mower deck size (D3, Bm, L3), 2) Moment of inertia, 3) Height of the center of gravity, and 4) Horizontal distance between the center of gravity and the ROPS contact point. These factors will be adjusted within the expected range or +/- 20 %, in 10 % increments. In order to

conduct the sensitivity analysis for the proposed parameter, assuming the other parameters are constant (see table 1 and Appendix A.3), only the proposed parameter is changed. The yaw and slide are constant (see section 6.1). From figures 6-20 and 6-21, as D3 and L3 increases, the CRH decreases. Therefore, the continuous roll tendency decreases, and the mower is less likely to continuously roll. From figure 6-22, there is a peak point at the deck extension width of 0.504 m. If the deck extension width is greater than 0.504 m, as the Bm parameter increases the CRH decreases. If the deck extension width is smaller than 0.504 m, the CRH increases as the Bm parameter increases. If the mower mounts a 60 in deck, it is less likely to continuous roll than if it mounts a 72 in deck. If the deck width is same as the track width of the mower, the mower roll tendency obviously decreases. Although an increase in deck

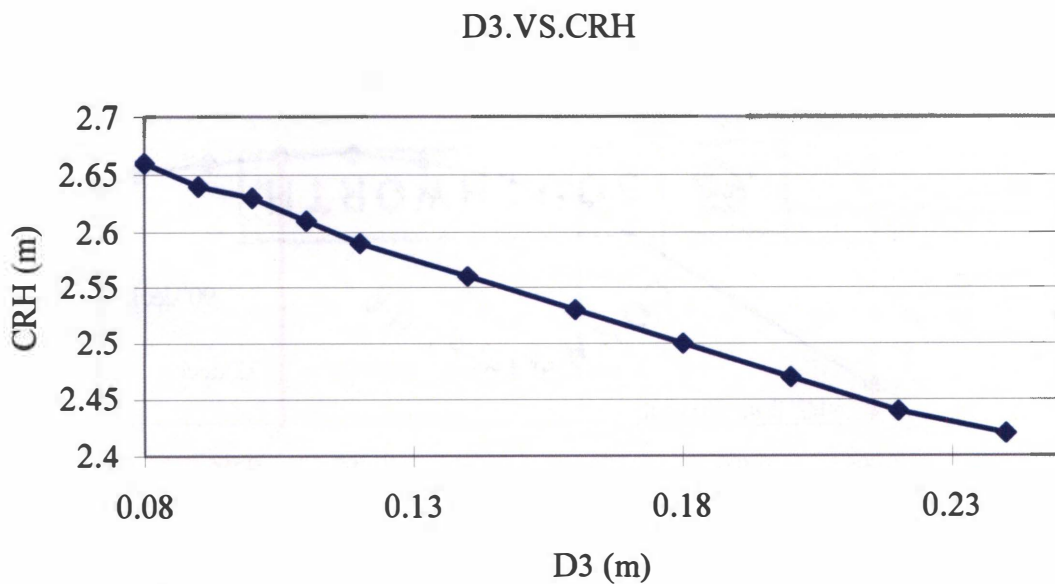


Figure 6-20. Sensitivity analysis in deck height (D3).

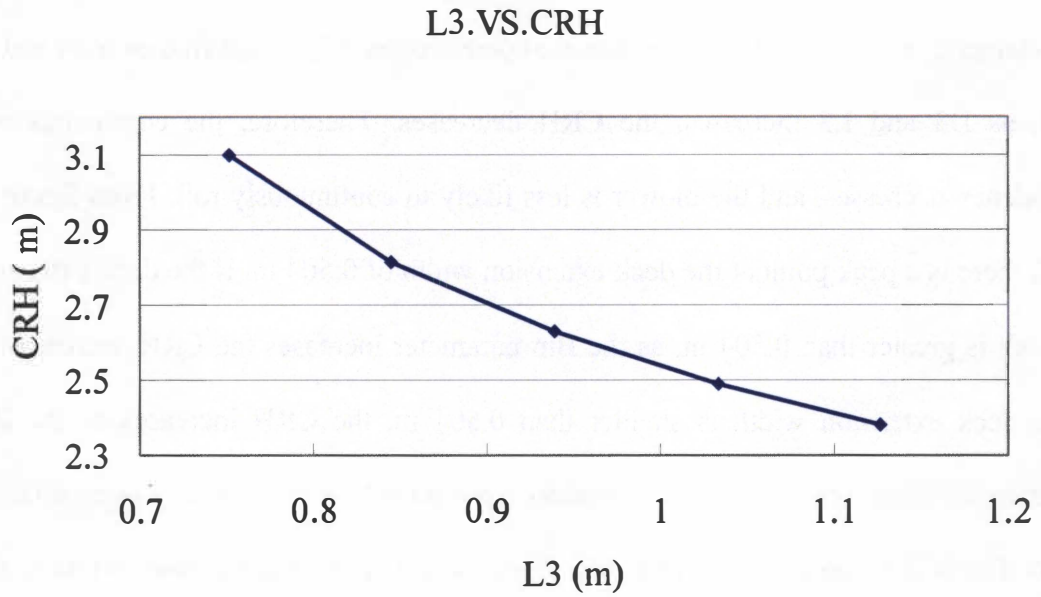


Figure 6-21. Sensitivity analyses in distance between CG and deck contact point.

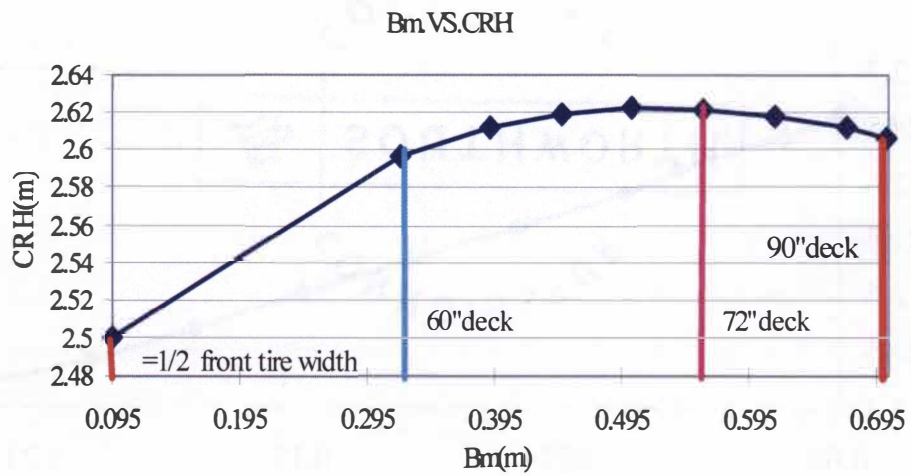


Figure 6-22. Sensitivity analyses in deck extension width (Bm).

size increases the CRH, it also increases the width of the vehicle base, making it more stable and less likely to start a roll. From figures 6-23, 6-24 and 6-25, as H1, L6 and MI increases, the CRH increases. Therefore, the roll tendency increases. The revised model is relatively insensitive to MI with CRH ranging from 2.60 m to 2.66 m with a $\pm 20\%$ variation to MI.

6.6.2. Sensitivity Analysis of Yaw and Slide

The angle (LL2) between the tipping axis 3 and the longitudinal axis changes as the mower undertakes a yaw. It will change the equivalent slope angle; therefore, it

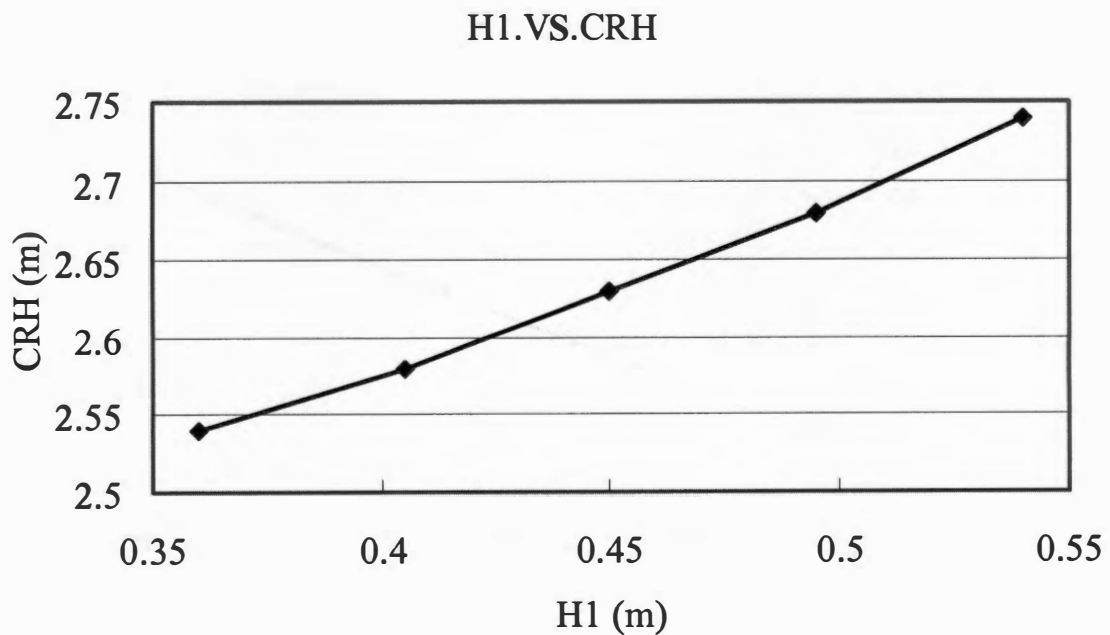


Figure 6-23. Sensitivity analyses in height of CG.

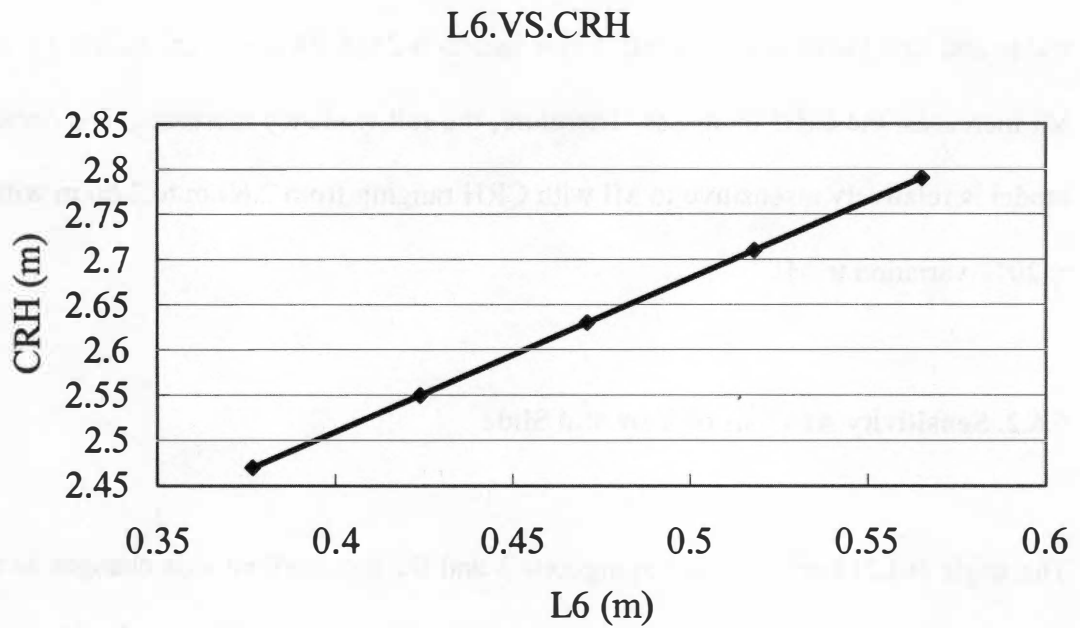


Figure 6-24. Sensitivity analyses in distance between CG and ROPS impact point.

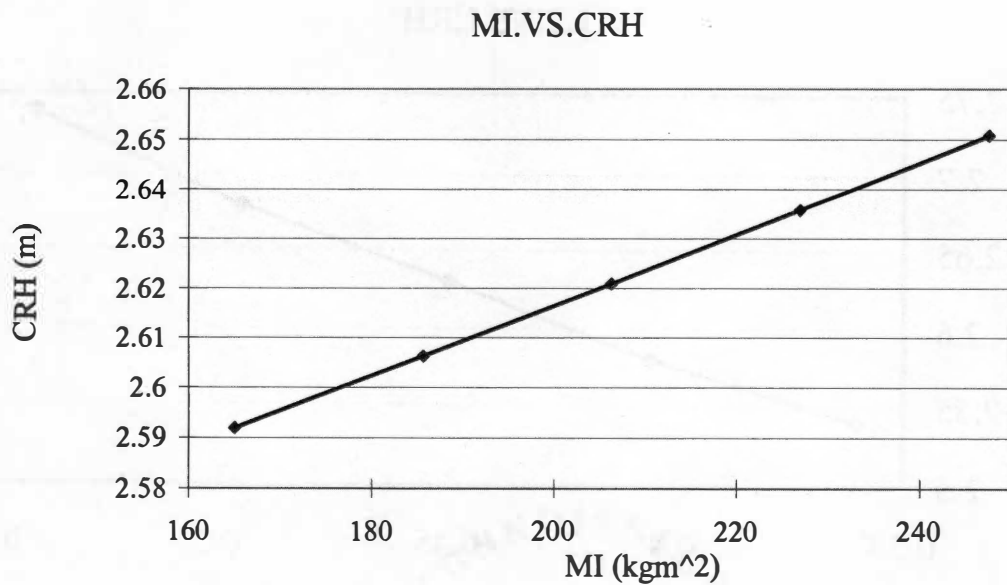


Figure 6-25. Sensitivity analyses in moment of inertia.

the roll behavior. Figure 6-26 shows the mower is less likely to continuously roll as the angle (LL2) increases. i.e., the mower is more likely to continuously roll as the yaw angle increases. Figure 6-27 shows that the mower continuous roll tendency increases as the slide distance (Ls) increases.

6.6.3. Sensitivity Analysis of Slope Factors

Figure 6-28 shows the revised model is more sensitive to the friction coefficient (μ). From figure 6-29, the model is insensitive to the ROPS and slope deformation (T).

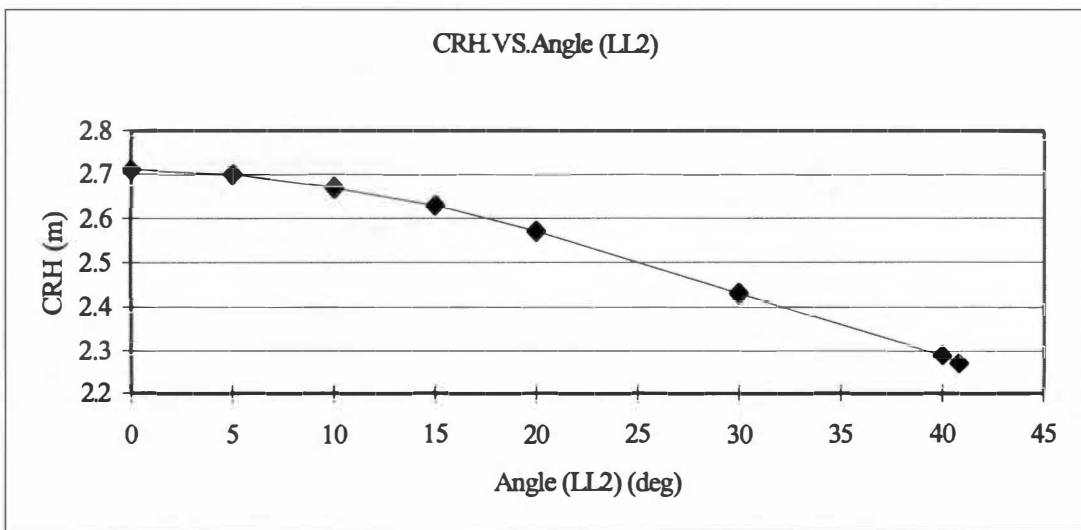


Figure 6-26. Sensitivity analyses in the angle (LL2) between the tipping axis 3 and the longitudinal axis.

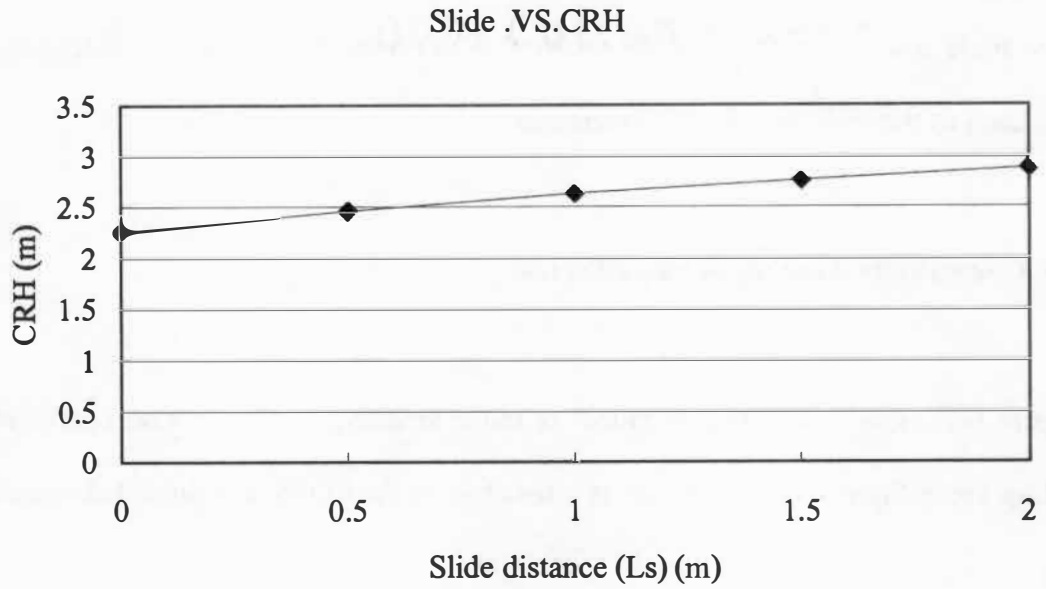


Figure 6-27. Sensitivity analyses in slide distance (Ls).

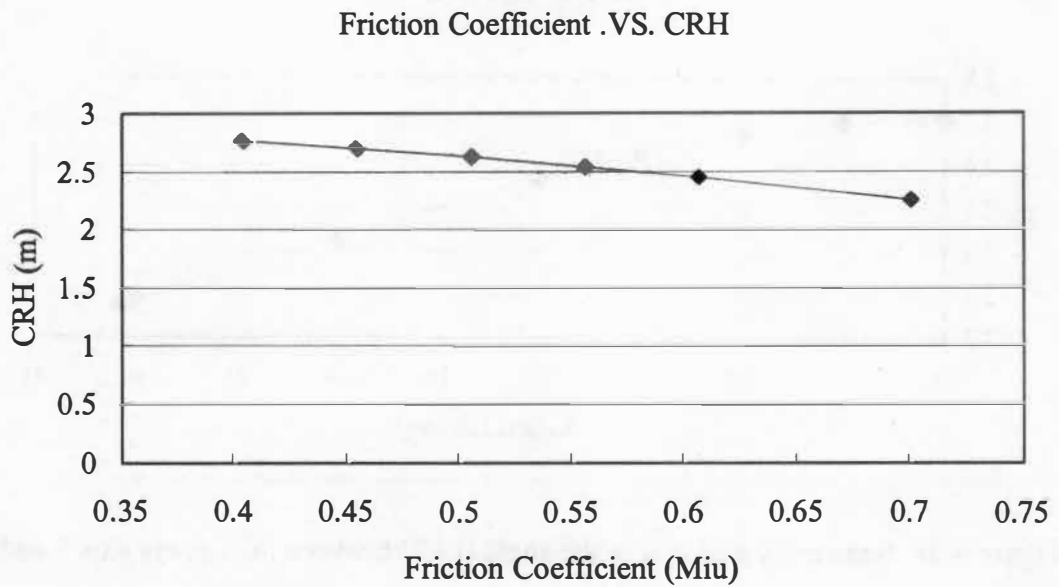


Figure 6-28. Sensitivity analyses in friction coefficient of the slope.

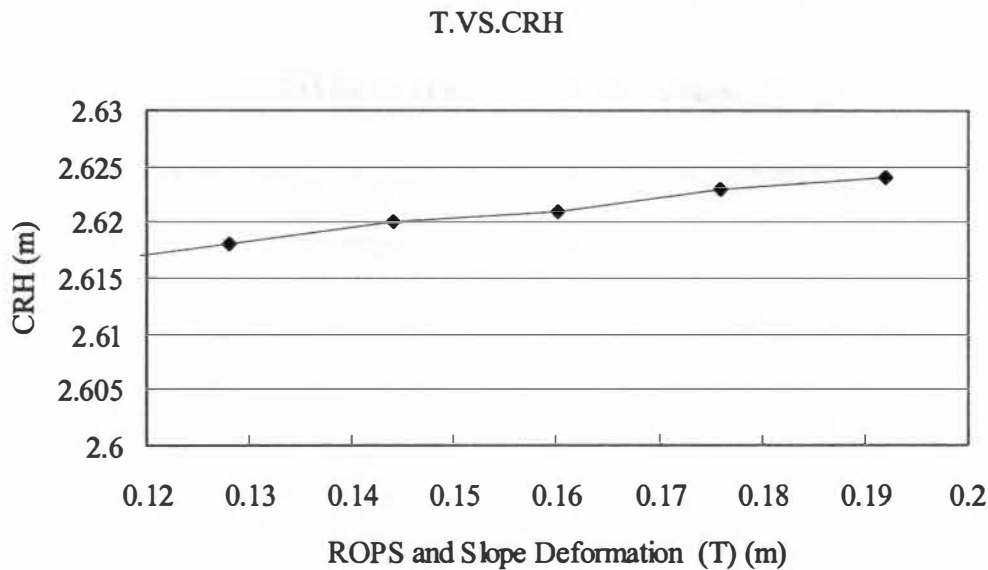


Figure 6-29. Sensitivity analyses in ROPS and slope deformation.

6.6.4 Influence of Operator Weight on Roll Behavior

To evaluate the influence of operator weight on roll behavior, A 95 kg operator weight located 150 mm above the lowest point of the operator-supporting surface of the seat and 250 mm forward of seat back is added to the model (ANSI, 1999). By adding the operator weight, the weight of the mower (W_t), the center of gravity (CG) and the mass moment of inertia changes, therefore, ten model parameters changed including the horizontal distance between the CG and the rear axle (L2), the horizontal distance between the CG and the deck/slope contact point (L3), the horizontal distance between CG and the front axle (L31), the horizontal distance

between CG and ROPS impact point (L6), the horizontal distance between CG and the engine bonnet (L7), height of CG (H1), roll moment of inertia (I_y), yaw moment of inertia (I_x), pitch moment of inertia (I_z), and mass of mower (m). Figure 6-30 showed the CG is changed due to the operator weight. From Eqs 6-1 and 6-2, the moment of inertia of mower around the CG can be obtained.

$$I = (J_{m1} + m_1 r_1^2) + (J_{m2} + m_2 r_2^2) \quad (6-1)$$

$$J_{m1} = \frac{m_1}{12} (a^2 + b^2) \quad (6-2)$$

Where,

I = Roll moment of inertia of mower around the CG ($kg \cdot m^2$),

J_{m1} = Moment of inertia of operator around the his/her center of gravity ($kg \cdot m^2$),

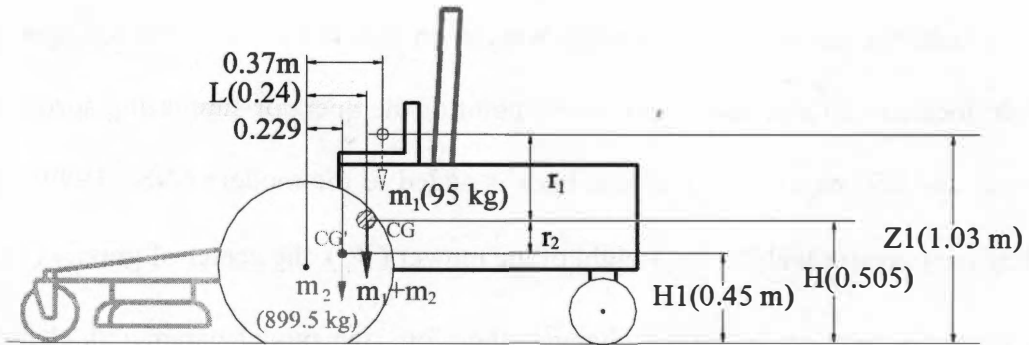


Figure 6-30. Parameters changes due to the operator weight.

r_1 = Horizontal distance between the operator's center of gravity and the mower's CG (m),

r_2 = Horizontal distance between the mower's CG with operator and the mower's CG without operator (m),

m_1 = Mass of the operator (kg),

m_2 = Mass of the mower without the operator (kg),

a = Operator's height (m), and

b = Operator's width (m).

In summary, by adding the operator's weight, the critical ROPS height (CRH) increases from 2.63 m to 2.67 m (see figure 6-31), therefore, the roll tendency increases.

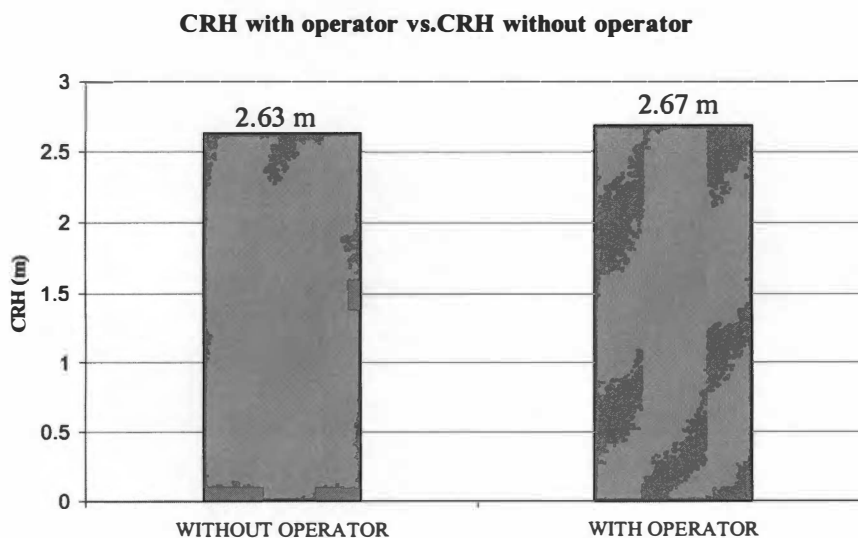


Figure 6-31. The CRH increases due to the operator weight.

6.6.5 Summary of Sensitivity Analysis

In summary, the revised model is most sensitive to parameters including the deck height (D3), the horizontal distance between the CG and the deck/slope contact point (L3), the height of CG (H1), the horizontal distance between the CG and the ROPS impact point (L6), the angle between the tipping axis 3 and the horizontal axis after the mower rotates (LL2), the slide distance of CG (Ls) and the friction coefficient (Miu). A slight change in these parameters will cause significant change in roll behavior. The model is less sensitive to parameters including the deformation of the slope and the ROPS (T), the moment of inertia (MI) and the deck extension width (Bm).

CHAPTER VII

CONCLUSIONS AND SUGGESTIONS

7.1 Conclusions

A new standard ASAE S547, “Tip-Over Protective Structure (TOPS) for Front Wheel Drive Turf and Landscape Equipment” addresses a continuous roll prediction model for the mower ROPS design. This model does not include the mower deck size; therefore, the accuracy of the model needed exploration. A field upset test slope was conducted, based on the standard requirements. The 35 degrees test slope is 4.05 meters long and 3.42 meter wide. The polyethylene pad was put on the slope. The average cone penetration resistance of the pad from 0 to 180 mm was 320 *kPa* for a size B cone, meeting the standard requirement. An F925 front drive mower was used to evaluate the continuous roll model. Five field tests were conducted to evaluate the accuracy of the original model with the F925 mower.

Continuous roll behavior was observed with the regular ROPS height of 1.90 m. However, the original model predicts a non-continuous roll with the ROPS height of 2.22 m, while the field test result shows a continuous roll. Yaw and slide downhill were also observed. Then, the ROPS was mounted in a way that the top of the ROPS tilts forward. It is defined as the inverted ROPS. For the F925 inverted ROPS, the original model predicts a critical ROPS height (CRH) of 1.67 m while field test shows a CRH of 2.03 m. There are larger discrepancies between the original model

results and field test results. Yaw and slide were also observed. It indicates that the mower deck has influence on the roll behavior. Therefore, the original model for the front drive mower ROPS design was modified to include the deck size and the yaw and slide downhill. For the F925 mower with the regular ROPS, the revised model predicts the critical ROPS heights (CRH) of 2.50 m and 2.63 m with deck in the transportation position and mowing position, respectively. The critical ROPS heights measured by field tests results are 2.55 m and 2.67 m respectively. For the F925 mower with the inverted ROPS, the critical ROPS heights predicted by the revised model and measured by the field test are 2.00 m and 2.03 m respectively. These results indicate the revised model can more accurately predict the continuous roll behavior.

From comparison of the measured CRH and the predicted CRH by the revised model and the existing model, the yaw and slide downhill has greater contribution to the mower roll behavior. The following specific conclusions are made:

1. The deck height has greater influence on the mower roll behavior. The yaw and slide downhill were also observed due to the deck size. It indicates the mower deck size affects the roll behavior.
2. The yaw affects the mower roll behavior. The equivalent slope angle will change due to the yaw angle after the ROPS hits the slope, therefore, the tendency of continuous roll changes.

3. The slide changes the potential energy difference in two different positions; therefore, it affects the mower roll behavior.
4. Three roll situations were observed, continuous roll, non-continuous roll, and a critical roll where the mower ends upside down. The ROPS height at the critical roll is a close estimate of the measured CRH.
5. From the accuracy factor that ranges from 98% to 99%, the revised model is a better predication than the original model. The original model produced accuracy factors of 81%, 82% and 85%.

7.2 Suggestions

The deck size, yaw angle, and slide downhill cannot be ignored in the ROPS design. These three parameters have significant influence on roll behavior. After reviewing the study's findings, two suggestions for future research and ROPS design industries can be offered.

Further studies should concern accurately measuring the yaw angle and the slide distance the mower undertakes at the moment it begins to continuous roll about tipping axis 3. Although the foam meets the standard requirement, it is not grass. It is essential to find empirical values of the yaw angle and the slide distance on the various grasses with more field tests. Due to the additional front structure influence, yaw and slide, the longer and wider test slope is needed.

For the matrix transformation of moment of inertia, if Bryant angle transformation ($X(\theta_1), Y(\theta_2), Z(\theta_3)$) is selected, the rotation angle around the second axis (Y axis) should be prevented from a multiple of 90° ($\theta_2 = \frac{2n+1}{2}\pi$). If Euler angle transformation ($Z(\phi), X(\theta), Z(\Phi)$) is selected, the rotation angle around the second axis (X axis) should be prevented from a multiple of 180° ($\theta = n\pi$).

The bracket for caster affects the roll behavior, for B3 version, the new tipping axis 4 should be added to the model. The weight of operator influences the position of center of gravity (CG)); it makes the CG upward and forward, therefore, it should be included to the model.

- 107

LIST OF REFERENCES

- American National Standards Institute (ANSI). 1999. ANSI /OPEI B71.4-1999. American national standard for commercial turf care equipment – safety specification. American National Standards Institute, Inc., New York, NY 10036.
- ASAE. 2001a. ASAE Standard S383.1. Rollover protective structures (ROPS) for wheeled agricultural tractors. 2001 ASAE Standards. ASAE, St. Joseph, MI 49085.
- ASAE. 2001b. ASAE Standard S478. Rollover protective structures (ROPS) for compact utility tractors. 2001 ASAE Standards. ASAE, St. Joseph, MI 49085.
- ASAE. 2001c. SAE Standard J2194. Roll-over protective structures (ROPS) for wheeled agricultural tractors. 2001 ASAE Standards. ASAE, St. Joseph, MI 49085.
- ASAE. 2002. ASAE Standard S547. Tip-over protective structures (TOPS) for front wheel drive turf and landscape equipment. December 2002. ASAE, St. Joseph, MI 49085.
- Ayers, P. D. and Wang, X. 2003. Model evaluation of new ASAE S547 lateral upset test. Distributed at the ASAE Section Meetings Paper. Chattanooga, TN, USA.
- Chisholm, C.J. 1979a. A mathematical model of tractor overturning and impact behavior. *J. Agric. Eng Res* 24: 375-394.
- Chisholm, C.J. 1979b. The effect of parameter variation on tractor overturning and impact behavior. *J. Agric. Eng Res* 24: 417-440.
- Chisholm, C.J. 1979c. Experimental validation of a tractor overturning simulation *J. Agric. Eng Res* 24: 395-415.
- Chisholm, C.J. 1982. Agricultural tractor overturning and impact behavior. In: A.H. Wickens (Ed.), *The dynamics of vehicles on roads and on tracks*. In: *Proc. 7th IAVSD Symposium*. Swets & Zeitlinger (1982), pp. 238–251
- Davis, D. C. and G. E. Rehkugler. 1974a. Agricultural wheel-tractor overturns, Part I: Mathematical model. *TRANS. ASAE* 17(3): 477-483.
- Davis, D. C. and G. E. Rehkugler. 1974b. Agricultural wheel-tractor overturns, Part II: Mathematical model verification by scale-model study. *TRANS. ASAE* 17(3): 484-488, 492.

Donham, K., D. Osterberg., M.L/Meyers, and C. Lehtola. 1999. Final report tractor risk abatement and control: the policy conference, September 10-12,1997. The University of Iowa, Iowa City, Iowa.

EC Directive (1987). Tests of front mounted roll-over protective structures on narrow track wheeled agricultural and forestry tractors. G.U. Serie Speciale n.86, June.

Eger, R. 2003. Modeling of rollover sequences. *Control Engineering Practice* 11: 209-216.

Febo, P. and Pessina, D. 2001. The effect of tire size on the non-continuous rolling test for narrow tractors. *J. Agric. Eng Res* 78(2): 147-152.

Larson, D. and J. B. Liljedahl. 1971. Simulation of sideways overturning of wheel tractors on side slopes. SAE Paper No. 710709, SAE, New York, N.Y.10001.

Larson, D.L., D.W. Smith and J.B. Liljedahl. 1976. The dynamics of three-dimensional tractor motion. *TRANS.ASAE* 19(1): 195-200.

Liljedahl, J. B., P. K. Turnquist, D. W. Smith and M. Hoke. 1998. Tractor and their power units. 4th Ed. Van Nostrand Reinhold, NY.

Liu, J., P. D. Ayers, and B. Nafziger. 1995. Development of stability index for tractors. In *Proceedings of the Second National Conference for NIOSH-sponsored Agricultural Health and Safety Center*, 27-28 March, Fort Collins, Colo.: Colorado State University.

Nichol, C.I, H.J. Sommer III and D.J.Murphy. 2004. Simplified overturn stability monitoring of agricultural tractors. *Journal of Agricultural Safety and Health*.Vol.11 (1): 99-108.

OECD. 1995. OECD Standard Code for the official testing of front mounted rollover protective structures on narrow-track wheeled agricultural and forestry tractors. Code 6. Paris, France.

OECD. 2002. OECD Standard Code for the official testing of front mounted rollover protective structures on narrow-track wheeled agricultural and forestry tractors. Code 6. Organisation for Economic Co-operation and Development, Paris, France.

Outdoor Power Equipment Institute (OPEI). 2004.
Website: <http://www.opei.org/newsroom>.

Schwanghart, V. H. 1973. Berechnungsmethode für das Umsturzverhalten eines Ackerschleppers am Hang (Method for calculating the overturning behavior of tractors on slopes). *Grundl. Landtech*, 23(6): 170-176.

Smith, D.W., J.V. Perumpral and J.B.Liljedahl. 1974. The kinematics of tractor sideways overturning. *TRANS. ASAE* 17(1): 1-3.

Society of Automotive Engineers (SAE). 1995. Developments in vehicle center of gravity and inertial parameter estimation and measurement. Paper No 950356. SAE, Warrendale, PA 15096.

SAE. 2000. SAE J1040. Performance criteria for rollover protective structures (ROPS) for construction, earthmoving, forestry, and mining machines. SAE, Warrendale, PA 15096.

United States Department of Labor (USDOL). 2003. Bureau of Labor Statistics. Injuries, Illnesses and Fatalities, Detailed Statistics for SIC Code 078. Webpage - www.bls.gov.

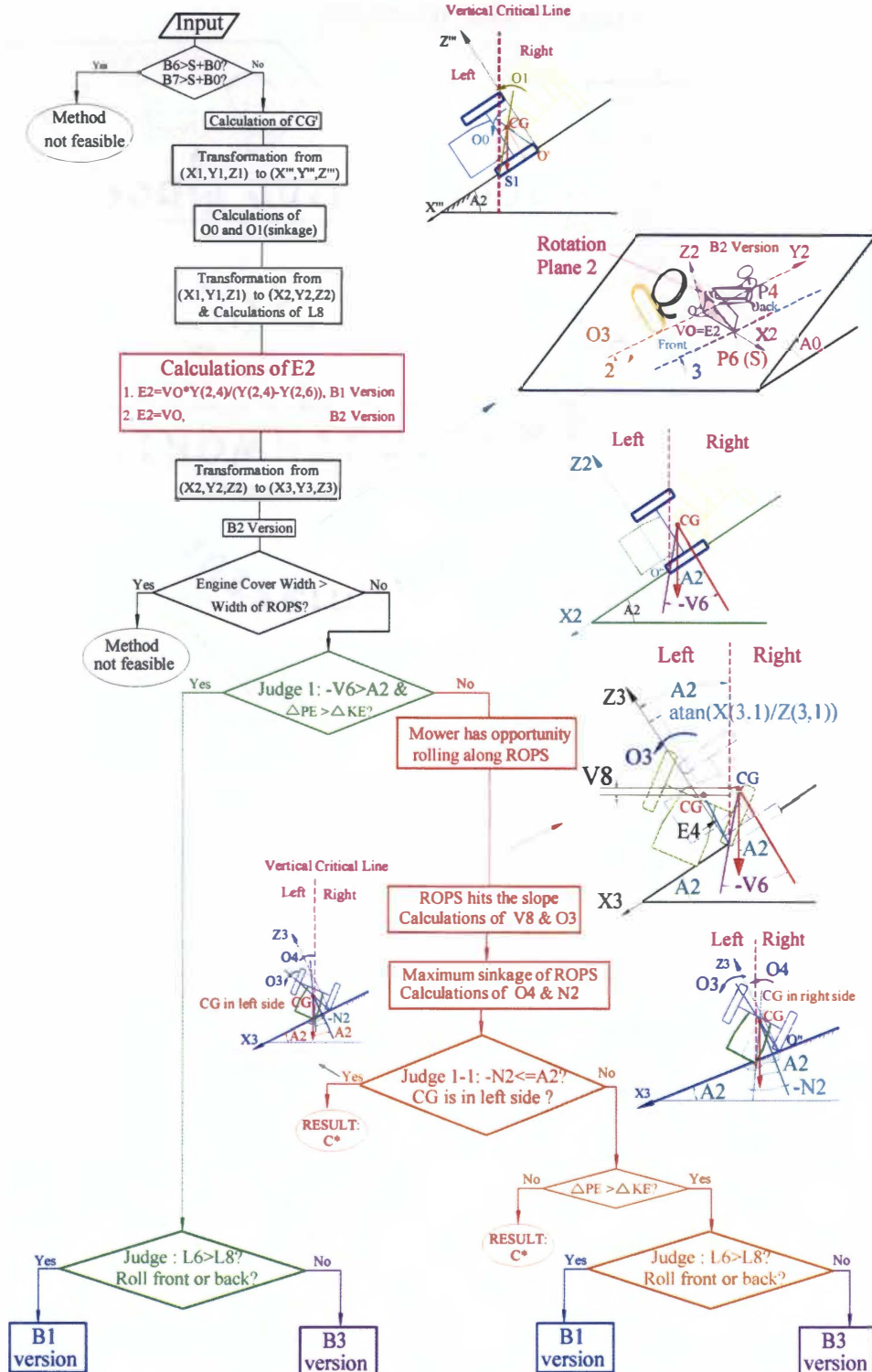
APPENDIXES

APPENDIX A

A REVISED CONTINUOUS ROLL MODEL

A.1 Flow Chart

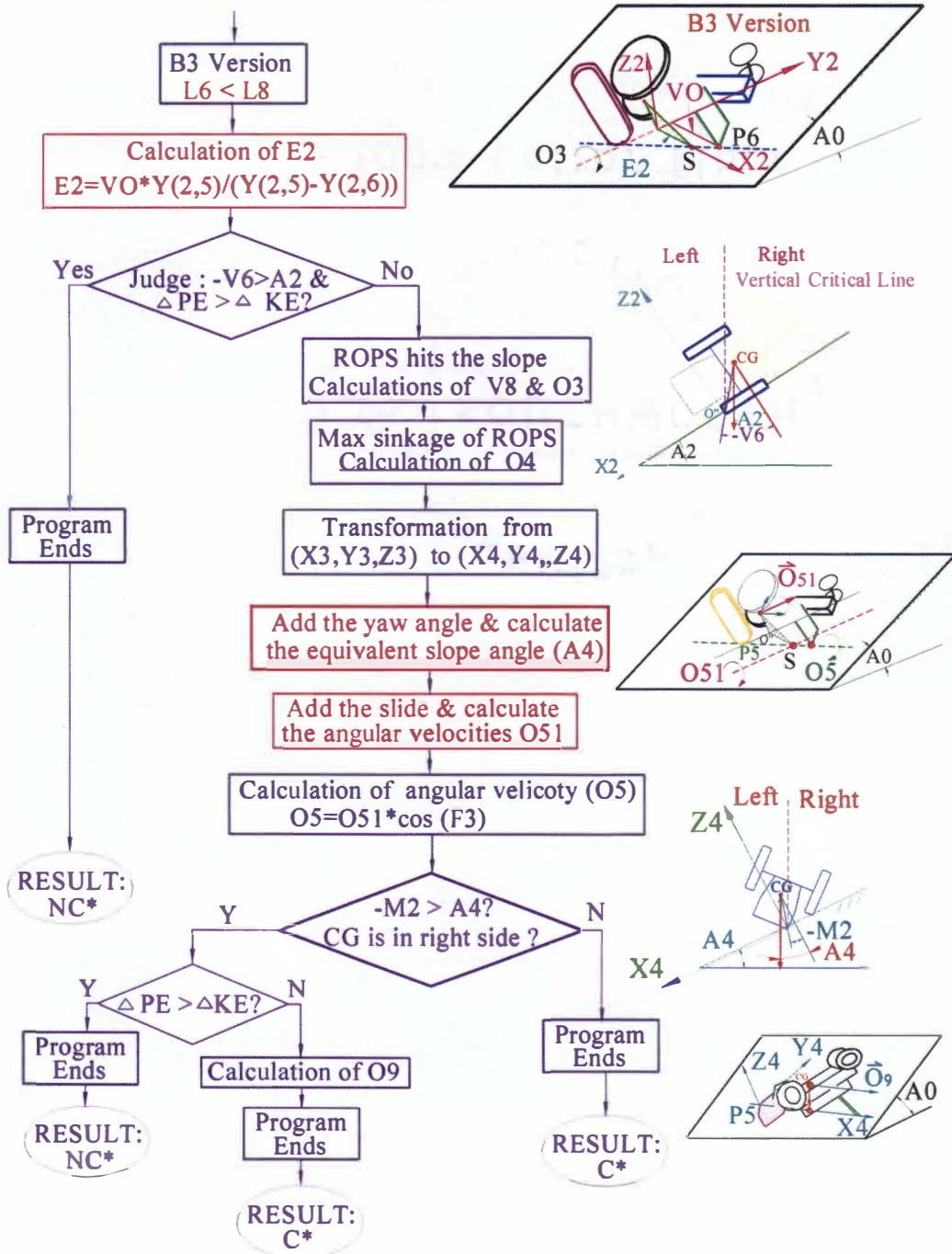
Figure A-1 shows the main flow chart of the Matlab code including B2 version. The flow chart for B3 version and B1 version are shown in figures A-1-1 and A-1-2, respectively. The data files for F925 mower with regular and inverted ROPS are put in the end of the Matlab code.



* C is "Continuous Roll", NC means "Non-Continuous Roll"

Figure A-1. Main flow chart of Matlab code.

Matlab Program Flow Chart (Continued)



* C is " Continuous Roll ", NC is " Non-Continuous Roll "

Figure A-1-1. Flow chart of Matlab code (B3 Version).

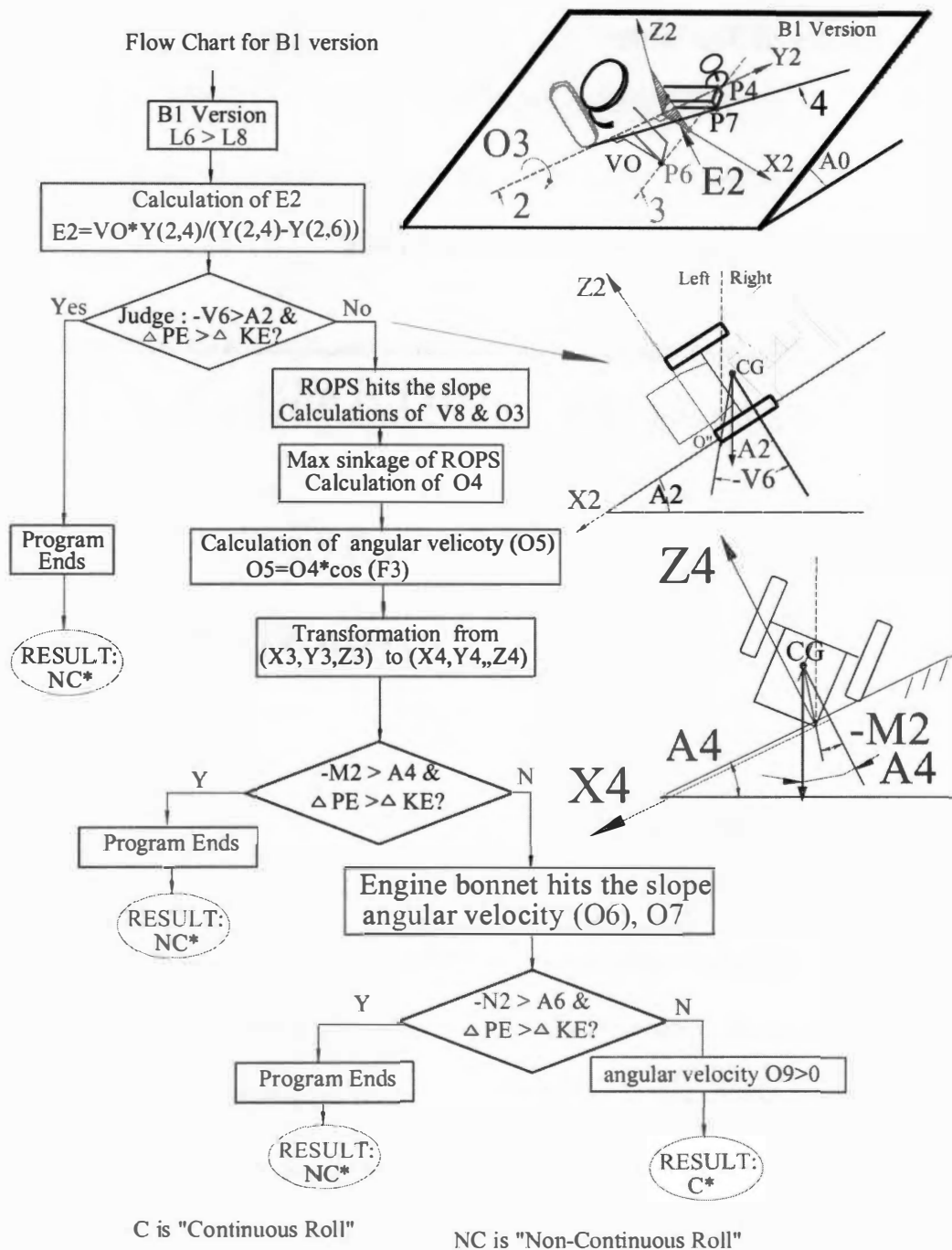


Figure A-1-2. Flow chart of Matlab code (B1Version).

A.2 Definition of Variables

The following are used for definition of variables.

The following equations are used to determine the new position of CG after the mower rolls about the initial tipping axis of D_0 degrees.

The blue color plane is a symmetric plane of mower (see figure A-2). The original position of CG is in this plane. $\triangle CGDE$ is a right-angled triangle (line $CGD \perp$ line ED),

$$\text{Length } CGD = H1$$

$$\text{Length } DE = L31$$

$$\text{Length } ECG = R2$$

$$\angle CGED = \angle C1$$

$\triangle BCE$ is a right-angled triangle (line $BC \perp$ line CE),

$$\angle BEC = L9$$

Let's build the line CGF perpendicular to line EB (see figure A-2), an intersection of the plane ABM and the symmetric plane. Also, let's build another line

FO , perpendicular to the initial tipping axis. Then, the rotation plane $CGFO$, was built. Therefore, the triangle $CGFE$ is a right-angled triangle (line $CGD \perp$ line EF).

$$\angle CGEF = \angle C1 - \angle L9$$

$$\text{Length } CGF = H9$$

$$\text{Length } EF = W1$$

Because the plane ABM is perpendicular to the symmetric plane, therefore, the triangle BEA is a right-angled triangle (line $BE \perp$ line AE).

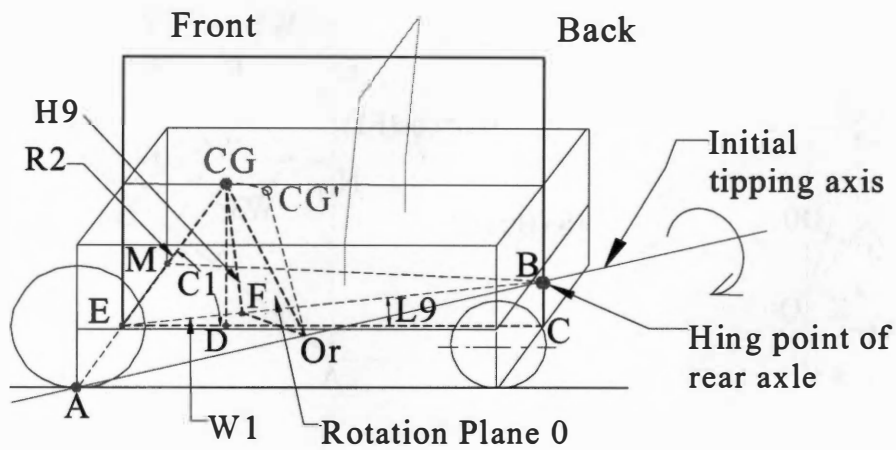


Figure A-2. Definitions of W1, R2, H9 and angles C1 and L9.

$$\text{Length } EB = W2$$

$$\text{Length } AE = \left(\frac{S1 + B01}{2} \right)$$

$$F1 = \angle ABE$$

The triangle F O, B is a right-angled triangle (see figure A-3).

$$\text{Line } FO, \perp \text{Line } O, B$$

$$\text{Length } FO, = W3$$

$$\text{Length } FB = W2 - W1$$

$$W4 = \angle CGO, F$$

$$W5 = \text{Length } CG'K$$

$$W6 = \text{Length } FK$$



W11 = Length AN (see figure A-4)

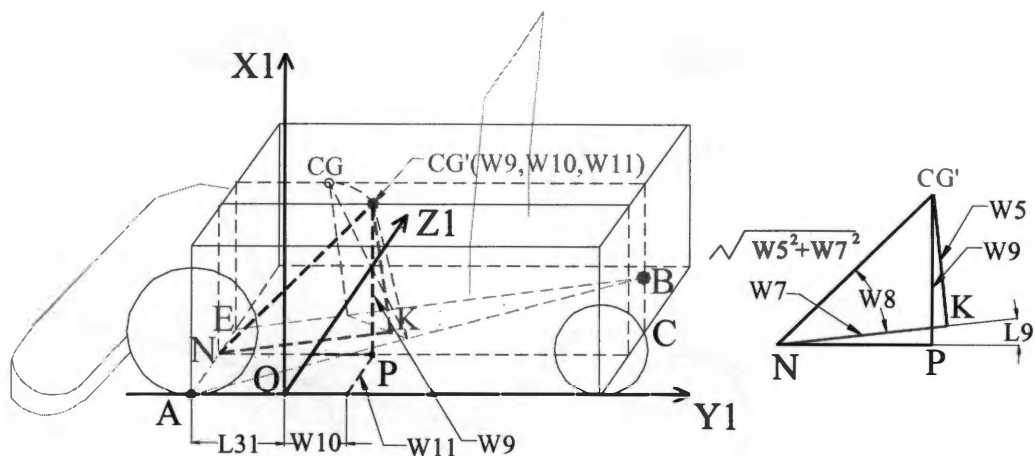


Figure A-4. Definitions of W9, W10, W11 and angle W8.

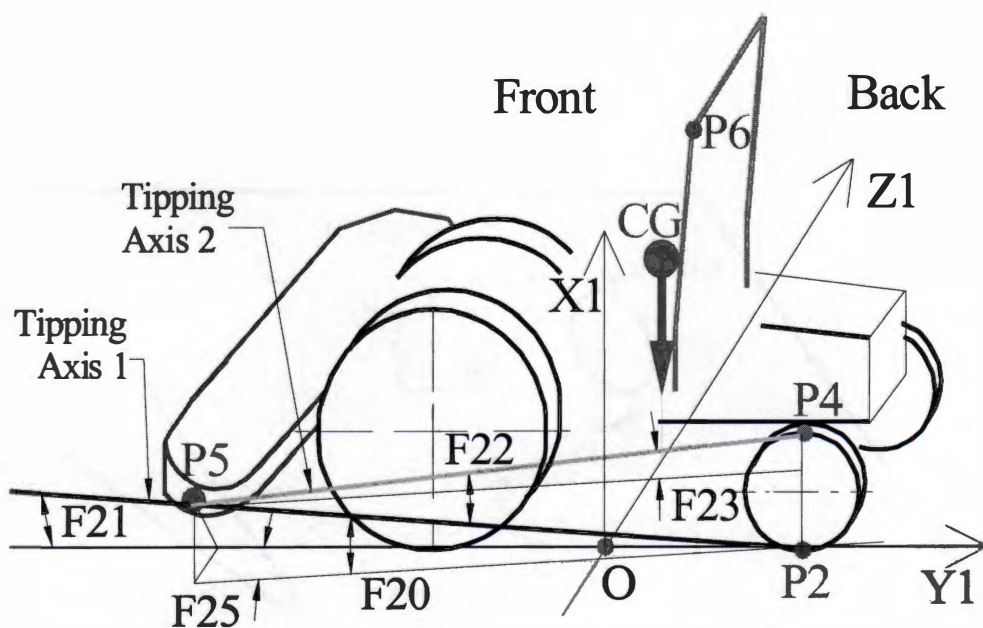
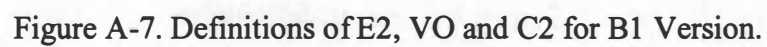


Figure A-5. Angle definitions.



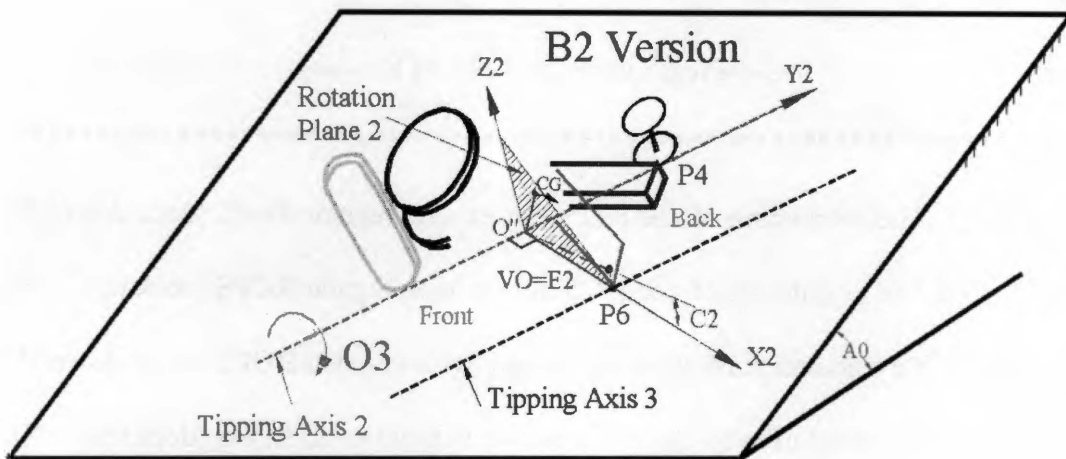


Figure A-8. Definitions of E2, VO and C2 for B2 Version.

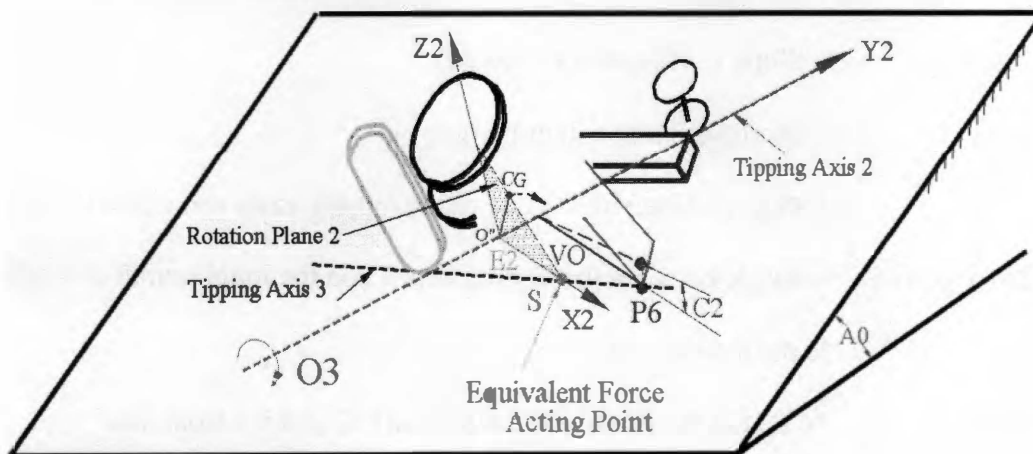


Figure A-9. Definitions of E2, VO and C2 for B3 Version.

A. 3 Matlab Code

```
clc

clear

disp('-----Please input the value of K-----')

disp('*****');

disp('* K=1 The calculation of the F925 mower with regular ROPS (deck down) *');

disp('* K=2 The calculation of the F925 mower with regular ROPS (deck up) *');

disp('*K=3 The calculation of the F925 mower with inverted ROPS (deck down) *');

disp('*K=4 The calculation of the F925 mower with inverted ROPS (deck up) *');

disp('*****');

K=input('K'); % SELECTION SWITCH

H6=input('H6'); % Height of ROPS impact point

MIU=0.505 % Friction coefficient of mower on slope

if K==1 % The calculation of F925 mower with regular ROPS

U=0.2; % Slope coefficient of elasticity

T=0.16; % The ROPS/slope deformation

LS=1.0 % Slide distance of CG due to the mower yaws and slides downhill.

LL2=15*pi/180 % Angle between the tipping axis 3 and the longitudinal axis after

% the mower yaws

L31=0.229; % L31 is the distance between the CG and the front axle

F925RE; % Open data file1 of F925 mower with regular ROPS (deck down:

%working position, the data file was put in the end of the code)

elseif K==2
```

U=0.2 % Slope coefficient of elasticity
 T=0.16 % The ROPS/slope deformation
 LS=1.0 % Slide distance of CG due to the mower yaws and slides
 %downhill.
 LL2=15*pi/180 % Angle between the tipping axis 3 and the longitudinal axis after
 %the mower yaws
 L31=0.229; % L31 is the distance between the CG and the front axle
 F925REUP % Data file 2 (F925REUP.m), regular ROPS, deck up
 elseif K==3 % The calculation of F925 with inverted ROPS (deck down)
 U=0.2; % Slope coefficient of elasticity
 T=0.16; % The ROPS/slope deformation
 LS=1.2 % Slide distance of CG due to the mower yaws and slides
 %downhill.
 LL2=5*pi/180 % The angle between the tipping axis 3 and the longitudinal axis
 %after the mower undertakes a yaw
 % L31 is distance between the CG and the center of the front axle
 % Because the ROPS is mounted in inversed direction, L31
 %changes.
 L31=0.222 % L31 is the distance between the CG and the front axle
 F925INMO1; % Open data file 3 of F925 mower, inverted ROPS (deck down)
 elseif K==4 % The calculation of F925 with inverted ROPS (deck up)
 U=0.2; % Slope coefficient of elasticity
 T=0.16; % The ROPS/slope deformation

```

LS=1.2          % Slide distance of CG due to the mower yaws and slides
                %downhill.

LL2=5*pi/180    % The angle between the tipping axis 3 and the longitudinal axis
                %after the mower undertakes a yaw

                % L31 is distance between the CG and the center of the front axle
                % Because the ROPS is mounted in inversed direction, L31
                %changes.

L31=0.222       % L31 is the distance between the CG and the front axle

F925INMO1UP;    % Open data file 4 of F925 mower with inverted ROPS, deck up
end

O01=0;          % Initial angular velocity of CG

A0=0.61;        % Slope angle

H1=HH(1);       % Height of the COG

L3=HH(2);       % Horizontal distance between the center of gravity and front axle

L2=HH(3);       % Horizontal distance between the center of gravity and rear axle

D3=HH(4);       % Height of front tire

D2=HH(5);       % Height of the rear tires

L6=HH(7);       % Horizontal distance between the center of gravity and the
                %leading point of intersection of ROPS

B6=HH(8);       % Width of ROPS

H7=HH(9);       % Height of the engine bonnet

B7=HH(10);      % Width of the engine cover

```

```

L7=HH(11);      % Horizontal distance between the center of gravity and the rear
                 %corner of the engine bonnet

H0=HH(12);      % Height of the rear-axle pivot point

S=HH(13);       % Rear track width

B0=HH(14);      % Two times deck extension width measured from the center of
                 %front tire

D0=HH(15);      % Rear axle swing angle

MC=HH(16);      % Mower Mass

Q=HH(17);       % Moment of inertia About the longitudinal axis through the
                 center of %gravity

SB=S+B0         % Total width of mower front track

if B6>S+B0       % Judge if Width of ROPS is greater than that of mower front
                 %track

disp('----The method is not available, because B6>SB-----')

break

elseif B7>S+B0   % Judge if the width of the engine cover is greater than that of the
                 %mower front track

disp('The method is also not available, because B7>SB')

break

end

G=9.8;          % Gravity acceleration

B01=0.19        % Width of front tire

%B2 Version begins (the point of impact of ROPS near the center of gravity)

```

B=B6; % Width of ROPS

H=H6; % Hight of ROPS impact point

% The following equations are the empirical equations from SAE standard. They are
%used to determine the moment of inertia. .

IYY=(H+H1)*(S+B01)*MC/9.4212 % Roll moment of inertia

L01=L31+L2; % Wheel base

IZZ=(H+H1)*L01*MC/4.2193 % Pitch moment of inertia

IXX=L01*(S+B01)*MC/2.2048 % Yaw moment of inertia

% From the above equations, the following empirical equation can be obtained.

% $IXX/IYY = (L01 * 9.4212) / ((H+H1) * 2.2048)$, therefore, $IXX \approx 3 * IYY$, the roll

%moment of inertia was measured, then, this empirical equation was used to

%calculate the yaw moment of inertia ($IXX \approx 3 * IYY$). The pitch moment of inertia

%(IZZ) was determined by the empirical equation from SAE standard. Finally, the

%roll moment of inertia around the tipping axes 1, 2, 3 and 4 are obtained.

R2=sqrt(H1.*H1+L31*L31) % Please see figure A-2, R2=Length ECG

C1=atan(H1./L31); % Please see figure A-2, C1=angle CGED

L0=L2+L3; % Horizontal distance between the ROPS/slope contact
%point and the rear axle.

V1=sqrt(D3.^2+(B0/2-B01/2).^2); %see figure

F21=atan(V1/L0) % Angle between the tipping axis 1 and longitudinal axis,
%Please see figure A-5.

F20=atan(D3/sqrt(L0*L0+(B0/2-B01/2).^2)); % Angle between the tipping axis 1
%and the Y1OZ1 plane. Please see figure A-5.

$F23 = \text{atan}((D2-D3)/\text{sqrt}(L0*L0+(B0/2-B01/2).^2));$ % Angle between the tipping
 %axis 2 and the Y1OZ1 plane. Please see figure A-5.

$F22 = F20 + F23;$ % Angle between tipping axes 1 and 2

$F2 = F22 + F21;$ % Angle between the tipping axis 2 and longitudinal
 %axis

% F25 is the angle between the longitudinal axis and the projection of tipping axes 1
 %and 2 in YOZ plane

$F25 = \text{atan}((B0/2-B01/2)/L0);$ % Please see figure A-1

% The CG changes from the original position to a new position due to the rear axle
 %swing angle. the CG means the original position of the center of gravity.

% the CG' means the new position of the center of gravity.

% The following equations are used to calculate the position of the center of gravity.

% Please see figures A-2, A-3 and A-4.

$L9 = \text{atan}(H0/L01);$

$H9 = R2.*\sin(C1-L9);$

$W1 = H9./\tan(C1-L9);$

$W2 = \text{sqrt}(H0.^2+L01.^2);$

$S1 = S./2;$

$F1 = \text{atan}((S1+0.5*B01)/W2);$

$W3 = (W2-W1).*\sin(F1);$

$W4 = \text{atan}(H9/W3);$

$W5 = \text{sqrt}(H9*H9 + W3*W3) * \sin(W4 + D0);$

$W6 = W3 - \text{sqrt}(W3*W3 + H9*H9) * \cos(W4 + D0);$

$$W7 = W1 + W6 * \sin(F1);$$

$$W8 = \text{atan}(W5 / W7);$$

$$NP = \sqrt{W5^2 + W7^2} * \cos(W8 + L9)$$

% W9, W10, W11 are x, y and z-coordinates of the CG' relative to the system (X1, Y1, Z1) respectively.

$$W9 = \sin(W8 + L9) * \sqrt{W5 * W5 + W7 * W7}; \quad \% \text{ x-coordinate of CG' relative to system (X1, Y1, Z1)}$$

$$W10 = NP - L31; \quad \% \text{ y-coordinate of CG' relative to system (X1, Y1, Z1)}$$

$$W11 = 0.5 * (S + B01) - W6 * \cos(F1); \quad \% \text{ z-coordinate of CG' relative to system (X1, Y1, Z1)}$$

% 2. Find the origin of system (X'', Y'', Z''), O' (D52, D53, D54),

% The Y''-axis passes through the tipping axis 1.

$$TT52 = (-D3)^2 / (L2 + L3) + L2 + L3 + (B0 - B01)^2 / (4 * (L2 + L3));$$

$$TT61 = -D3 * (W9 - D3) + (L2 + L3) * (W10 + L3) + 0.5 * (B0 - B01) * (W11 + 0.5 * (B0 - B01));$$

$$D53 = TT61 / TT52 - L3 \quad \% Y$$

$$D52 = ((-D3) / (L2 + L3)) * D53 + (D3 * L2) / (L2 + L3) \quad \% X:$$

$$D54 = ((B0 - B01) / (2 * (L2 + L3))) * D53 - ((B0 - B01) * L2 / (2 * (L2 + L3))) \quad \% Z:$$

% G1 is the distance from CG' (W9, W10, W11) to O' (D52, D53, D54)

$$G1 = \sqrt{(D52 - W9)^2 + (D53 - W10)^2 + (D54 - W11)^2}$$

$$A1 = \text{atan}(\tan(A0) / \sqrt{1 + (\tan(F21))^2 / (\cos(A0))^2}); \quad \% \text{ Equivalent slope angle for tipping axis 1}$$

% Determine the origin O''(F52, F53, F54) of the system (X2, Y2, Z2). Y2-axis

% passes through the tipping axis 2

$T44=(D2-D3)*(H1-D3)+(L2+L3)*L3+(B0-B01)*(S+B0)/4;$
 $TT5=(D2-D3)^2/(L2+L3)+L2+L3+(B0-B01)^2/(4*(L2+L3));$
 $F53=(T44/TT5)-L3$ % y-coordinate of O" relative to system (X1,Y1,Z1)
 $F52=((D2-D3)/(L2+L3))*F53+(D3*L2+D2*L3)/(L2+L3);$ %The x-coordinate of O"
 %relative to system (X1,Y1,Z1)

$F54=((B0-B01)/(2*(L2+L3)))*F53- ((B0-B01)*L2/(2*(L2+L3)));$
 % F54 is the z-coordinate of %O" relative to system (X1, Y1, Z1).
 %-----7 IMPORTANT POINTS COORDINATES relative to system (X1, Y1, Z1)
 %in initial position (position 0) -----

$X(1, 1)=H1;$ % x-coordinate of CG relative to system (X1,Y1,Z1) before the mower
 %rolls about the initial axis.

$Y(1, 1)=0;$ %y-coordinate of CG relative to system (X1,Y1,Z1) before the mower
 %rolls about the initial axis.

$Z(1, 1)=(S+B01)/2;$ % z-coordinate of CG relative to system (X1,Y1,Z1) before the
 %mower rolls around the initial axis.

$X(1, 2) = 0;$ % x-coordinate of P2(left rear tire/slope contact point) relative
 to %system (X1,Y1,Z1)

$Y(1, 2) = L2;$ % y-coordinate of P2 relative to system (X1,Y1,Z1)

$Z(1,2)=0;$ % z-coordinate of P2 relative to system (X1,Y1,Z1)

$X(1, 3) = 0;$ % x-coordinate of P3 (left front tire/slope contact point) relative
 %to system (X1,Y1,Z1)

$Y(1, 3)=-L3;$ % y-coordinate of P3 relative to system (X1,Y1,Z1)

```

Z(1,3)=0;          % z-coordinate of P3 relative to system (X1,Y1,Z1)

X(1, 4)=D2;        % x-coordinate of P4(top of the left rear tire)relative to system
                    %(X1,Y1,Z1)

Y(1, 4) =L2;       % y-coordinate of P4 relative to system (X1,Y1,Z1)

Z(1,4)=0;          % z-coordinate of P4 relative to system (X1,Y1,Z1)

X(1,5)=D3;         % x-coordinate of P5 (Deck/slope contact point)relative to system
                    %(X1,Y1,Z1)

Y(1,5)=-L3;        % y-coordinate of P5 relative to system (X1,Y1,Z1)

Z(1,5)=- (B0/2-B01/2); % z-coordinate of P5 relative to system (X1,Y1,Z1)

X(1, 6)=H;         % x-coordinate of P6 (ROPS/slope contact point)relative to
                    %system (X1,Y1,Z1)

Y(1, 6) = -L6;     % y-coordinate of P6 relative to system (X1,Y1,Z1)


Z(1,6)=(S+B01)/2-B/2; % z-coordinate of P6 relative to system (X1,Y1,Z1)

X(1 ,7)=H7;        % x-coordinate of P7(engine cover/slope contact point)relative to
                    %system (X1,Y1,Z1)

Y(1, 7) = L7;      % y-coordinate Y of P7 relative to system (X1,Y1,Z1)

Z(1,7)=(S+B01)/2-B7/2; % z-coordinate of P7 relative to system (X1,Y1,Z1)


O1=0;O2=0; O3=0;O4=0;O5=0;O6=0;O7=0; O8=0;O9=0; % initial values of angular
%velocities at critical positions

% In the code, the name of (XL,YL,ZL) is used insted of (X''',Y''',Z''').

```

```

% The new origin of the system (XL,YL,ZL) is the point O'(O'((D52,D53,D54)
%relative to system (X1,Y1,Z1))

% The (XL,YL,ZL) system is obtained, First rotate (F25) about X, then it rotates
%angle

% F20 around Z'-axis, assuming if look down to origin, counter-clockwise is positive.

% The following are to determine the x, y, and z coordinates of seven contact points
%relative to the system (XL, YL, ZL)

for (K = 1:7)

XL(1, K) =cos(F20)*(X(1,K)-D52)+sin(F20)*cos(F25)*(Y(1,K)-
D53)+sin(F20)*sin(F25)*(Z(1,K)-D54);

YL(1, K) =-sin(F20)*(X(1,K)-D52)+cos(F20)*cos(F25)*(Y(1,K)-
D53)+cos(F20)*sin(F25)*(Z(1,K)-D54);

ZL(1, K) =-sin(F25)*(Y(1,K)-D53)+cos(F25)*(Z(1,K)-D54);

end

% Because of the rear axle swing angle, the mower rolls about the initial axis. the
%center of gravity changes from CG to CG'.

% The following is to determine the x, y, and z coordinates of CG' relative to system
%(XL, YL, ZL).

XL(1, 9) =cos(F20)*(W9-D52)+sin(F20)*cos(F25)*(W10-
D53)+sin(F20)*sin(F25)*(W11-D54); % x-coordinates relative to system
(XL,YL,ZL)

YL(1, 9) =-sin(F20)*(W9-D52)+cos(F20)*cos(F25)*(W10-
D53)+cos(F20)*sin(F25)*(W11-D54);%y-coordinates relative to system (XL,YL,ZL)

```

$ZL(1, 9) = -\sin(F25) * (W10 - D53) + \cos(F25) * (W11 - D54);$
 $G22 = \sqrt{XL(1,1) * XL(1,1) + ZL(1,1) * ZL(1,1)}$
 $ZL(1, 9) = -\sin(F25) * (W10 - D53) + \cos(F25) * (W11 - D54);$ % z-coordinates relative to system (XL,YL,ZL)
 $G2 = \text{atan}(XL(1,9)/ZL(1,9))$ % The angle between the ZL-axis and the line CGO'
 %connecting CG' to O'.
 $G31 = G1 - G1 * \cos(A1 + G2)$ % The height difference of CG' from positions 1 to 2.
 % In the following equation, the empirical equation ($IXX \sim 3 * IYY$) was used
 $QQ1 = 3 * Q * (\sin(F20)).^2 + Q * (\cos(F20)).^2$
 $* (\cos(F25)).^2 + IZZ * (\sin(F25)).^2 * (\cos(F20)).^2 ;$ % MMI around the tipping axis
 %that passes the CG and is parallel to the tipping axis 1.
 $O0 = \sqrt{(2 * MC * G * G31 / (QQ1 + MC * G1 * G1)) + O01^2}$ % The angular velocity of
 %CG' at the position 2 (the top of left rear tire hits the slope).
 %The following equations describe the impact of the left rear tire on the slope.
 % After the top of left rear tire hits the slope, the center of gravity changes from CG'
 %to the original position CG. Because the center of gravity rotates in the plane that is
 %perpendicular to this tipping axis 1, The line O'S1 is perpendicular to
 % line P5O' (tipping axis 1).
 $K1 = \text{abs}(YL(1,5)) * \tan(F22)$ % the distance between the origin O' and equivalent
 %force acting point(S1).
 $K2 = XL(1,1)$ % x-coordinate of CG relative to system (XL,YL,ZL).
 $K3 = ZL(1,1)$ % z-coordinate of CG relative to system (XL,YL,ZL).
 $K4 = K1 - XL(1,1)$

```

DD1=QQ1+MC*K3*K3+MC*K4*K4;
O1=(QQ1+MC*K3*K3-U*MC*K4*K4-(1+U)*MC*K2*K4)*O0/DD1 % angular
%velocity of CG at position 3, the maximum sinkage of left rear tire into slope.
%The mower will roll about the tipping axis 2.The angular velocity is O2, a
%component of the angular velocity O1.
O2 = O1*cos(F22); % angular velocity of CG about tipping axis 2
%--- TRANSFORMATION OF THE COORDINATES FROM THE POSITION 2
%TO 3
% The new origin of system (X2, Y2, Z2) is O"(F52, F53, F54) relative to system
%(X1, Y1, Z1). First, the system (X1, Y1, Z1) translates to O"(F52, F53, F54).
% Second, the system (X1, Y1, Z1) rotates F25 about X, then rotates -F23(=F20-F22)
% about Z, counter-clockwise is positive if look down from the axis to the origin. The
%new system (X2,Y2,Z2) is obtained.
% The following are to determine the x, y, and z coordinates of seven contact points
%relative to system (X2, Y2, Z2)
for (K = 1:7)
X(2, K) =cos(-F23)*(X(1,K)-F52)+sin(-F23)*cos(F25)*(Y(1,K)-F53)+sin(-
F23)*sin(F25)*(Z(1,K)-F54); % x-coordinates relative to system (X2,Y2,Z2)
Y(2, K) =-sin(-F23)*(X(1,K)-F52)+cos(-F23)*cos(F25)*(Y(1,K)-F53)+cos(-
F23)*sin(F25)*(Z(1,K)-F54);% y-coordinates relative to system (X2,Y2,Z2)
Z(2, K) =-sin(F25)*(Y(1,K)-F53)+cos(F25)*(Z(1,K)-F54); % z-coordinates relative
%to system (X2,Y2,Z2)
end

```

% If the distance between the ROPS/slope contact point (P6) and the rotation plane is
 %zero, the mower can roll along ROPS. i.e. if the point P6 is in the rotation plane, the
 %mower has the opportunity to roll along the ROPS ($Y(2,6)=0$); By solving the
 %equation ($Y(2,6)=0$), a logic switch (L8) can be obtained as follows:

$$L8 = -F53 + ((\sin(F23) * (X(1,6) - F52) + \cos(F23) * \sin(F25) * (Z(1,6) -$$

$$F54)) / (\cos(F23) * \cos(F25)); \quad \% \text{ it is used to determine which ways the mower}$$

% rolls (B1 Version, B2 Version or B3 Version).

% If $L6=L8$, the mower rolls along ROPS; if $L6<L8$, the mower rolls to front; if
 % $L6>L8$, the mower rolls to the back.

$$A2 = \text{atan}(\tan(A0) / \sqrt{1 + (\tan(F2))^2 / (\cos(A0))^2}); \quad \% \text{ Equivalent slope angle}$$

%for tipping axis 2

$$C2 = \text{atan}(Z(2, 6) / X(2, 6));$$

% C2 is the angle that the mower needs to roll around the tipping axis 2 until the
 %ROPS hits the slope (see figure A-7).

$$T2 = T; \quad \% \text{ The deformation of the ROPS and slope.}$$

$$VO = \sqrt{X(2, 6)^2 + Z(2, 6)^2}; \quad \% \text{ The rotation radius of P6 about the tipping}$$

%axis 2 (see figure A-7)

$$E1 = T2 / VO; \quad \% \text{ The angle that the mower needs to roll about the tipping axis 2 due}$$

%to the deformation of the ROPS and slope.

$$E2 = (VO * Y(2,4)) / (Y(2,4) - Y(2, 6)); \quad \% \text{ This calculation is for B1 and B2 version.}$$

% E2 is the distance between the O" and the equivalent force acting point S

%(see figure A-8).

$$T3 = E1 * E2; \quad \% \text{ The radian that the point P6 underwent.}$$

```

E4=sqrt(X(2, 1) * X(2, 1) + Z(2, 1) * Z(2, 1)); % The rotation radius of CG about the
                                         %tipping axis 2

V6=atan(X(2, 1) / Z(2, 1)); %The angle between the slope and the line CGO"
                                         %connecting the CG to origin O".

% The following is to determine the coordintes of seven points
% after the mower rolls about the tipping axis 2 up to the maximum sinkages of
%ROPS into the slope.

%----- ROTATION OF THE TRACTOR FROM THE POSITION 3 TO 4 --
for( K =1:7)
if Z(2, K) == 0
E3 = -3.14159./ 2;
else
E3=atan(X(2, K)/ Z(2, K));
end

% The following is to determine the coordinates of seven points relative to system
%(X3,Y3,Z3).

% The system (X3, Y3, Z3) is same as the system (X2, Y2, Z2), but the mower rolls
%the angle of (E3+C2+E1) around the tipping axis 2,therefore, the coordinate values
%of seven points change.

X(3,K)=sqrt(X(2, K) * X(2, K) + Z(2, K) * Z(2, K)) *sin(E3 + C2 + E1)
Y(3,K) = Y(2, K)
Z(3,K)=sqrt(X(2, K) ^ 2 + Z(2, K) ^ 2) *cos(E3 + C2 + E1)
end

```

```

% The following if-else control flow statement is to determine who touches the
%ground first, either ROPS or engine bonnet. The maximum deformation of ROPS
%and slope is 0.16 m, therefore, it is the condition of the judgment.
if Z(3, 7) < -0.16

disp('Because Z(3, 7) < -0.16 ');

disp('%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%')

disp('The Engine Bonnet Touches The Ground Before the ROPS')

disp('Method is Not available')

disp('%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%')

else

Z(3, 6) = 0; % ROPS hits the slope before the engine bonnet.

% In the following equation, the empirical equation ( $I_{XX} \approx 3 \cdot I_{YY}$ ) was used

$$Q_3 = 3 \cdot Q \cdot (\sin(F_{23}))^2 + Q \cdot (\cos(F_{25}))^2 \cdot (\cos(F_{23}))^2 + I_{ZZ} \cdot (\sin(F_{25}))^2 \cdot (\cos(F_{23}))^2$$

% Q3 is the MMI around the axis that is parallel to tipping axis 2


$$V_5 = (Q_3 + MC \cdot E_4 \cdot E_4) \cdot O_2 \cdot O_2 / 2;$$



$$V_7 = E_4 \cdot (1 - \cos(-A_2 - V_6));$$

% Height change of CG from position 3 to the top of
%ROPS/slope contact point.

% The following is to judge if the mower can roll up to the top of the tipping axis 2
if -V6 > A2 & V7*MC*G > V5

% Judge 1: -V6 > A2 and Difference in PE > Difference in KE.

% The mower meets the above conditions; therefore, it cannot roll over the top of the
%tipping axis2.

V77=V7*MC*G; % Potential energy difference

```

```

disp('Because -V6>A2& V77>V5, The mower lost the opportunity to roll along
ROPS')

disp('The program will check if it has other opportunities, such as roll front or back')

disp('O0='); disp(O0); % angular velocity in position 2

disp('O1='); disp(O1); % angular velocity in position 3

disp('O2='); disp(O2); % compant of the angular velocity O1 in the direction of
tipping %axis 2

disp('O3='); disp(O3); % angular velocity in position 4

disp('O4=');disp(O4); % angular velocity in position 5

disp('O5=');disp(O5); % compant of the angular velocity O4 in the direction of
tipping %axis 3

% For B1 Version, the mower underwent eight positions. For B2 and B3 VERSION,
only %six positions.

disp('O6=');disp(O6); %angular velocity in position 6 (only for B1 version)

disp('O7=');disp(O7); %angular velocity in position 7 (only for B1 version)

disp('O8=');disp(O8); % compant of the angular velocity O7 in the direction of
tipping %axis 4

disp('O9=');disp(O9); %angular velocity in position 6(for B2 and B3 version).

%it is the angular velocity in position 8(for B1 version)

% The following is the second judgment, which way the program will go, B1 version
or %B3 VERSION

% Judge2: if L6<L8?

if L6<L8

```

```
% 111111---B3---B3---B3---111111
```

```
disp('The first running of B3 VERSION begins')
```

```
O3=0;O4=0;O5=0;O6=0;O7=0;O8=0;O9=0;
```

```
E2 = (VO* Y(2, 5)) / (Y(2, 5) - Y(2, 6)) % the distance between the O" and the
```

```
%equivalent force acting point S(B3
```

```
Version).
```

```
T3=E2*E1;
```

```
Z(3, 6) = 0;
```

```
% In the following equation, the empirical equation (  $IXX \sim 3*IYY$ ) was used
```

```
Q3=3*Q*(sin(F23))^2+Q*(cos(F25))^2*(cos(F23))^2+IZZ*(sin(F25))^2*(cos(F23))
```

```
^2 % MMI around the tipping axis that is parallel to the tipping axis 2 and
```

```
%passes through the CG
```

```
V5=(Q3+MC*E4*E4)*O2*O2/2 % The kinetic energy of the mower in the
```

```
%position 3
```

```
V7 = E4 * (1 - cos(-A2 - V6))
```

```
if -V6 > A2 & V7*MC*G>V5
```

```
% First judgment of the roll direction of the mower after the top of the rear tire
```

```
%contacts the slope.
```

```
% This situation means the CG is in the right side of the critical vertical line and the
```

```
%kinetic energy is not enough to lift the CG up to the top of the tipping axis 2
```

```
disp('***** The Roll Stops, "NC"*****')
```

```
disp('O0='); disp(O0); % angular velocity in position 2
```

```
disp('O1='); disp(O1); % angular velocity in position 3
```

```

disp('O2='); disp(O2); % component of the angular velocity O1 in the direction of
%tipping axis 2

disp('O3='); disp(O3); % angular velocity in position 4

disp('O4=');disp(O4); % angular velocity in position 5

disp('O5=');disp(O5); % component of the angular velocity O4 in the direction of
                    %tipping %axis 3

% For B1 Version, the mower underwent eight positions. For B2 and B3 VERSION,
%only six positions.

disp('O6=');disp(O6); %angular velocity in position 6 (only for B1 version)

disp('O7=');disp(O7); %angular velocity in position 7 (only for B1 version)

disp('O8=');disp(O8); % compant of the angular velocity O7 in the direction of
                    %tipping axis 4

disp('O9=');disp(O9); %angular velocity in position 6(for B2 and B3 version).

                    %it is the angular velocity in position 8(for B1 version)

disp('----- After First B3 VERSION(Roll Front) calculation, Roll Stops,"NC"-----
--')

else

    % This situation means the CG is in the left side of

    % critical vertical line. the program will judge if the kinetic energy is

    % enough to lift the CG upto the top of the tipping axis 2

    %V8 is the height change of CG between the position 3 and position 4

V8 = E4 * cos(-A2 - V6) - E4 * cos(-A2 - atan(X(3, 1) / Z(3, 1)));

```

```

% The following is to determine the angular velocity of CG in Position 4 (the ROPS
%just hits the slope)

O3= sqrt((2 * MC * G * V8) / (Q3 + MC* E4 * E4) + O2 * O2);

% O3 is the angular velocity of CG at position 4

% The following is to determine the angular velocity of CG at position 5

% (The maximum deformation of ROPS/slope)

K9 = X(3, 1);      % x-coordinate of CG relative to system (X3,Y3,Z3)

K5 = Z(3, 1);      % z-coordinate of CG relative to system (X3,Y3,Z3)

K6 = Z(3, 1) + T3; % z-coordinate of the point S, a equivalent force acting point.

K7 = E2 - X(3, 1); % x-coordinate between the point S and CG.

K8 = U;            % elasticity coefficient of the slope

DD2 = Q3 + MC* K6 * K6 + MC * K7 * K7;

O4 = (Q3 + MC * K5 * K6 - K8 * MC * K7 * K7 - (1 + K8) * MC* K9 * K7) * O3 /
DD2;

% O4 is the angular velocity at position 5 (The maximum deformation of
%ROPS/slope)

F3 =atan(VO/ (Y(3, 5) - Y(3, 6))) % the angle between the tipping axes 2 and 3

%TRANSFORMATION OF THE COORDINATES FROM THE POSITION 3 TO 4

% The origin of the system (X4,Y4,Z4) is the point P5, the deck/slope contact point
for(K = 1:7)

X(4, K) = X(3, K) *cos(F3) + (Y(3, K) - Y(3, 5)) * sin(F3); % x-coordinate of seven
%points relative to system (X4,Y4,Z4)

```

```

Y(4, K)=(Y(3,K)-Y(3,5))*cos(F3)-X(3,K)*sin(F3); % y-coordinate of seven points
                                                    %relative to system (X4,Y4,Z4)

Z(4, K) = Z(3, K); % z-coordinate of seven points relative to system (X4,Y4,Z4)

end

A4 = atan(tan(A0) / sqrt(1 + (tan(LL2)) ^ 2 / (cos(A0)) ^ 2)); % the equivalent slope
                                                    %angle after the yaw.

M1= sqrt(X(4,1).^2 + Z(4,1).^ 2); % Rotation radius of the CG around the tipping
axis 3

M2 =atan(X(4, 1) / Z(4, 1)); % the angle between the Z4-axis and the line CGP5.

% The following is to determine MMI around the axis that is parallel to the tipping
%axis 3 and passes through CG

F252=atan((B0+S-B6)/(2*(L3-L6))); % the angle between the projection of the
%tipping axis 3 in Y1OZ1 plane and the longitudinal axis.

F232=atan((H6-D3)/sqrt((L3-L6)^2+(S+B0-B6)^2/4)); % the angle between the
%tipping axis 3 and its projection.

% In the following equation, the empirical equation (  $I_{XX} \sim 3 * I_{YY}$  ) was used

Q5=3*Q*(sin(F232))^2+Q*(cos(F252))^2*(cos(F232))^2+IZZ*(sin(F252))^2*(cos(F
232))^2 % MMI around the axis that is parallel to the tipping axis 3 and passes
%through CG

R1=(E2-X(2,1))^2+Z(2,1)*Z(2,1) % the rolling radius around the equivalent force
%acting point S.

% The following equations are to determine the angular velocity of CG due to the
%slide downhill of the mower.

```

```

FS=MIU*MC*G*cos(A2)          % Friction force

O51=sqrt( O4^2+(2*MC*G*LS*sin(A2)-2*FS*LS1)/(Q3+MC*R1^2))

% O51 is the angular velocity of CG around the equivalent tipping axis.

%This axis passes through the equivalent force acting point S and is perpendicular to
%the tipping axis 2.

O5= O51* cos(F3);             % angular velocity of CG around the tipping axis 3

if -M2 > A4    % Judge if the position of CG is in the right side of the critical vertical
               %line when the maximum sinkage of ROPS into the slope occurs
               %In this case, the CG is in the right side of the critical vertical line.

%The program will judge if the kinetic energy is greater enough to lift the CG over
%the top of the tipping axis 3

M3 = M1 * (1 -cos(-A4 - M2));    % the height difference of CG we need to lift.

M4 = (Q5 + MC * M1 * M1) * O5 * O5 / 2 ; % the kinetic energy of the mower

if M3 * MC * G > M4    % Judge if the potential energy difference is greater than
%the kinetic energy difference.

% In this case, it indicates the kinetic energy is NOT enough to lift the CG over the
%top of the tipping axis 3;Therefore, the roll stops.

disp('***** The Roll Stops
*****');

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

```

```

disp('O4=');disp(O4);

disp('O5=');disp(O5);

disp('O6=');disp(O6);

disp('O7=');disp(O7);

disp('O8=');disp(O8);

disp('O9=');disp(O9);

disp('***** B3 VERSION(Roll Front) END, The Roll Stops."NC"
*****')

else

    % In this case, it indicates the kinetic energy is greater enough to lift the CG

    %over the top of the tipping axis 3; Therefore, the roll continues.

O9=sqrt(O5*O5 - 2*MC*G*M3./(Q5 + MC*M1.^2));

%O9 is the angular velocity in position 6 (B3 Version)

disp('=====Roll Continues =====');

% The outputs of angular velocities in critical positions.

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

disp('O4=');disp(O4);

disp('O5=');disp(O5);

disp('O6=');disp(O6);

disp('O7=');disp(O7);

```

```

disp('O8=');disp(O8);

disp('O9=');disp(O9);

disp('==== Roll continues for the mower B3 VERSION(Roll Front), "C"====');

end

else

    %In this case, the CG is in the left side of the critical vertial line, it means the

    %mower passes over the top of the tipping axis 3 when the maximum sinkage of

    %ROPS into the slope occurs; therefore, the roll continues.

disp('&&&&& After B3 VERSION(Roll Front) calculation, Roll Continues, no

need to calculate O6,O7,O8,O9 &&&&&&&');

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

disp('O4=');disp(O4);

disp('O5=');disp(O5);

disp('O6=');disp(O6);

disp('O7=');disp(O7);

disp('O8=');disp(O8);

disp('O9=');disp(O9);

disp('&&& The Roll Continues, "C" &&&&');

end

end

```

```

disp('++++++The first running of B3 VERSION(Roll Front) ends
+++++')

else    % The mower rolls to the back. The calculation in B1 version begins.

% L6>L8, B1 version

% 1111111-----B1----B1----B1-----111111

% First running of B1 version begins

disp('The program will check whether mower can roll to back (B1 VERSION)');

disp('Because L6>0,the program will go B1 version (Roll back, the impact point of
ROPS behind of the COG)');

disp('First running of B1 version begins')

% The following are the initial values of angular velocities in critical positions

O3 = 0;

O4 = 0;

O5 = 0;

O6 = 0;

O7 = 0;

O8 = 0;

O9 = 0;

Z(3,6)=0;

% In the following equation, the empirical equation ( $I_{XX} \sim 3 \cdot I_{YY}$ ) was used

 $Q3 = 3 \cdot Q \cdot (\sin(F23))^2 + Q \cdot (\cos(F25))^2 \cdot (\cos(F23))^2 + I_{ZZ} \cdot (\sin(F25))^2 \cdot (\cos(F23))^2$ 

% The moment of inertia of the mower around the axis that is parallel to the
%tipping axis 2 and passes through CG

```

$V5 = (Q3 + MC * E4 * E4) * O2 * O2 / 2;$

$V7 = E4 * (1 - \cos(-A2 - V6));$

% The following repeat the calculation process in B2 version

if -V6 > A2 & V7 * MC * G > V5

disp('##### The Roll Stops, "NC" #####')

% The outputs of the angular velocities in eight critical positions

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

disp('O4='); disp(O4);

disp('O5='); disp(O5);

disp('O6='); disp(O6);

disp('O7='); disp(O7);

disp('O8='); disp(O8);

disp('O9='); disp(O9);

disp(' Because -V6 > A2 & V7 * MC * G > V5, The Roll Stops, "NC" #####')

else

% In this case, CG passes over the top of tipping axis 2.

$V8 = E4 * \cos(-A2 - V6) - E4 * \cos(-A2 - \text{atan}(X(3, 1) / Z(3, 1)));$

% V8 is the height change of CG from position 3 to position 4

$O3 = \sqrt{2 * MC * G * V8 / (Q3 + MC * E4 * E4) + O2 * O2};$ % Angular velocity at

%position 4

% The following equations are to determine the angular velocity change due to the
 %impact of the ROPS into the slope

$$K9 = X(3, 1);$$

$$K5 = Z(3, 1);$$

$$K6 = Z(3, 1) + T3;$$

$$K7 = E2 - X(3, 1);$$

$$K8 = U;$$

$$DD2 = Q3 + MC * K6 * K6 + MC * K7 * K7;$$

$$O4 = (Q3 + MC * K5 * K6 - K8 * MC * K7 * K7 - (1 + K8) * MC * K9 * K7) * O3 /$$

DD2; % Angular velocity at position 5 (max sinkage of ROPS into slope)

$$F3 = \text{atan}(VO / (Y(3, 4) - Y(3, 6)));$$

$$O5 = O4 * \cos(F3); \quad \% \text{ angular velocity of CG around the tipping axis 3}$$

%---- TRANSFORMATION OF THE COORDINATES FROM 3 TO 4 -

for(K = 1:7)

$$X(4, K) = X(3, K) * \cos(F3) + (Y(3, K) - Y(3, 4)) * \sin(F3);$$

$$Y(4, K) = (Y(3, K) - Y(3, 4)) * \cos(F3) - X(3, K) * \sin(F3);$$

$$Z(4, K) = Z(3, K);$$

end

$$A4 = \text{atan}(\tan(A0) ./ \sqrt{1 + (\tan(F2 + F3)).^2} / (\cos(A0)).^2));$$

% A4 is the equivalent slope %angle due to the tipping axis 3

$$C3 = \text{atan}(Z(4, 7) ./ X(4, 7));$$

C4 = 0; % The deformation of the engine bonnet

C5 =sqrt(X(4, 7) * X(4, 7) + Z(4, 7) * Z(4, 7)); % The rotating radius of the energy

%bonnet around the tipping axis 3

C6 =C4./ C5; % the rotating angle due to the deformation of the engine bonnet

C7 = C5 * (Y(4, 6) - Y(4, 1)) / (Y(4, 6) - Y(4, 7)); % The equivalent rotation radius.

C8 = C6 * C7;

M1 =sqrt(X(4, 1) ^ 2 + Z(4, 1) ^ 2); % The rotating radius of CG

M2 =atan(X(4, 1)./ Z(4, 1));

%----ROTATION OF THE TRACTOR FROM THE POSITION 4 TO 5 -

% In order to let the engine bonnet contact the slope, the mower must roll angle by

%(C9+C3+C6) around the tipping axis 3

for(K =1:7)

if Z(4, K)~=0

C9 =atan(X(4, K) / Z(4, K));

else

C9 = -3.14159 / 2;

end

X(5, K) =sqrt(X(4, K) ^ 2 + Z(4, K) ^ 2) *sin(C9 + C3 + C6);

Y(5, K) = Y(4, K);

Z(5, K) =sqrt(X(4, K) ^ 2 + Z(4, K) ^ 2) *cos(C9 + C3 + C6);

end

Z(5, 7) = 0;

% In the following equation, the empirical equation ($I_{XX} \approx 3 \cdot I_{YY}$) was used

```

Q5 = Q*(cos(F2 + F3)).^2 + 3*Q*(sin(F2 + F3)).^ 2; % The moment of inertia
around %the axis that is parallel to the tipping axis 3 and passes through CG
M3 = M1*(1-cos(-A4 - M2));
M4 = (Q5 + MC* M1 * M1) * O5 * O5./ 2; % The kinetic energy of the mower in
                                     %position 5
if-M2 > A4 & M3* MC* G > M4 % Judge the CG position relative to the vertical
    %critical line and if the potential energy is greater than the kinetic energy
    disp('Roll stop')
    disp('O0='); disp(O0);
    disp('O1='); disp(O1);
    disp('O2='); disp(O2);
    disp('O3='); disp(O3);
    disp('O4=');disp(O4);
    disp('O5=');disp(O5);
    disp('O6=');disp(O6);
    disp('O7=');disp(O7);
    disp('O8=');disp(O8);
    disp('O9=');disp(O9);
else
    MM1 =M1*cos(-A4 -atan(X(5, 1) / Z(5, 1)));
    M5 = M1 *cos(-A4 -atan(X(4, 1) / Z(4, 1))) - MM1;
    O6 =sqrt(2 * MC * G * M5 / (Q5 + MC * M1 * M1) + O5 * O5);
    M6 = X(5, 1);

```

```

M7 = Z(5, 1);
M8 = Z(5, 1) + C8;
M9 = C7 - X(5, 1);
N1 = U;
DD3 = (Q5 + MC*M8*M8 + MC* M9*M9);
O7 = (Q5 + MC*M7*M8 - N1*MC*M9*M9 -(1+ N1)*MC*M6*M9)*O6./DD3;
F5 =atan(C5./(Y(5,6)-Y(5,7)));
A6 =atan(tan(A0) /sqrt(1+(tan(F2+F3+F5))^2./(cos(A0)).^2));
%---- ---TRANSFORMATION OF THE COORDINATES FROM THE POSITION
5 %TO 6 -----
for(K =1:7)
X(6, K) = X(5, K) *cos(F5) + (Y(5, K) - Y(5, 6)) *sin(F5);
Y(6, K) = (Y(5, K) - Y(5, 6)) *cos(F5) - X(5, K) *sin(F5);
Z(6,K)=Z(5,K);
end
O8 = O7 *cos(-F5);
N2 =atan(X(6, 1)./ Z(6, 1));
N3 =sqrt(X(6, 1).^ 2 + Z(6, 1).^ 2);
Q6 = Q*(cos(F2 + F3 + F5)).^2 + 3*Q*(sin(F2 + F3 + F5)).^ 2;
N4 = N3*(1-cos(-A6 - N2));
N5 = (Q6 + MC*N3*N3)*O8*O8./ 2;
P9 = (N4*MC*G - N5)./(N4*MC*G);
if -N2 > A6 & N4 * MC * G > N5

```

```

disp('The RoLL STOPS')

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

disp('O4=');disp(O4);

disp('O5=');disp(O5);

disp('O6=');disp(O6);

disp('O7=');disp(O7);

disp('O8=');disp(O8);

disp('O9=');disp(O9);

else

disp('The Roll Continues,"C"')

N6=-N4;

O9=sqrt(2 * MC* G * N6./(Q6+MC*N3*N3) +O8*O8);

disp('O9=');

disp(O9);

end

end

% The first running of B1 VERSION ends

disp('The first running B1 VERSION ends')

end

%The first running of B1 Version ends

```

```

end

else

% The program goes B2 VERSION

% Judge 1: If  $-V6 > A2$  & Difference in potential energy  $>$  Difference in kinetic
energy? %Result: No

% In this case, the CG is in the left side of the vertical critical line when the
maximum %sinkage of the rear tire into the slope, but the kinetic energy is great
enough to lift the %CG passing over the top of the vertical critical line.
% It means that the mower has the opportunity to roll along the ROPS, then, the
%program will judge if the kinetic energy is great enough to lift the CG passing over
%the top of the ROPS/slope contact point.

 $V8 = E4 * \cos(-A2 - V6) - E4 * \cos(-A2 - \text{atan}(X(3, 1) / Z(3, 1)))$ ;

 $O3 = \sqrt{2 * MC * G * V8 / (Q3 + MC * E4 * E4) + O2 * O2}$ ; % Angular velocity
when %ROPS hits the slope

% The following equations are to determine the angular velocity (O4) of CG around
the %equivalent tipping axis, This equivalent tipping axis passes through the
ROPS/slope %contact point and is parallel to the tipping axis 2

 $K9 = X(3, 1)$ ; % x-coordinate of CG relative to the system (X3,Y3,Z3)

 $K5 = Z(3, 1)$ ; % z-coordinate of CG relative to the system (X3,Y3,Z3)

 $K6 = Z(3, 1) + VO * E1$ ;

 $K7 = VO - X(3, 1)$ ;

 $K8 = U$ ;

 $DD2 = Q3 + MC * K6 * K6 + MC * K7 * K7$ ;

```

CC1=K8*MC*K7*K7

CC2=(1+ K8)*MC*K9*K7

CC3=Q3+MC*K5*K6

CC=-CC1-CC2+CC3

O4=CC*O3/DD2 % The angular velocity around the equivalent tipping axis.

N3=sqrt((X(3, 6) - X(3, 1)) ^ 2 + (Z(3, 6) - Z(3, 1)) ^ 2);

N2=atan(-(X(3,6)-X(3,1))/Z(3,1));

Q6=Q3+MC*N3*N3; % The moment of inertia around the axis that is parallel to the
%tipping axis 2 and passes through the ROPS/slope contact point (B2 version).

% The following if-else control flow statement is to judge if CG is in the left side of
%critical vertical line when the maximum deformation of ROPS and slope occurs.

% Please see Judge1-1 in flow chart

if -N2<=A2

% This situation means that the CG has been in the left side of the critical

%vertical line when the maximum deformation of ROPS and slope occurs.

%The mower rolls over ROPS; therefore, a continuous roll occurs.

disp('===== The Roll Continues =====')

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

disp('O4=');disp(O4);

disp('O5=');disp(O5);

```
disp('O6=');disp(O6);
```

```
disp('O7=');disp(O7);
```

```
disp('O8=');disp(O8);
```

```
disp('O9=');disp(O9);
```

```
disp('=====B2 VERSION, The Roll Continues,"C" =====')
```

```
else
```

```
% In this case, CG has been in the right side of the critical vertical line when the
```

```
%maximum deformation of the ROPS and slope occurs. Please see judge 1-1 in flow
```

```
%chart.
```

```
% The program will judge if the kinetic energy is great enough to lift the CG directly
```

```
%over the ROPS/slope contact point, therefore, the program goes from judge 1-1 to
```

```
%the judge1-2 in flow chart.
```

```
N4 = N3 * (1-cos(-A2 - N2));
```

```
N5 = (Q6) * O4 * O4 / 2
```

```
if N4*MC*G>N5
```

```
    % In this case, the kinetic energy is NOT enough to lift the CG directly over the
```

```
    %ROPS/slope contact point. Please see judge 1-2 in flow chart.
```

```
    % The mower lost the opportunity to roll along the ROPS, but it has other two
```

```
    %opportunities to roll, either rolling front (B3 version) or rolling back (B1
```

```
    %version)
```

```
disp('##### Judge if the mower can roll back or front
```

```
#####');
```

```
    % Outputs of the angular velocities in critical positions
```

```
disp('O0='); disp(O0);
```

```
disp('O1='); disp(O1);
```

```
disp('O2='); disp(O2);
```

```
disp('O3='); disp(O3);
```

```
disp('O4=');disp(O4);
```

```
disp('O5=');disp(O5);
```

```
disp('O6=');disp(O6);
```

```
disp('O7=');disp(O7);
```

```
disp('O8=');disp(O8);
```

```
disp('O9=');disp(O9);
```

```
% The second running B3 Version will begin
```

```
if L6<L8
```

```
%222222----B3----B3-----B3-----2222222
```

```
% The explain sentences are ignored because it is the second running of B3 version
```

```
disp(' Second Running of B3 Version Begins')
```

```
O3=0;O4=0;O5=0;O6=0;O7=0;O8=0;O9=0;
```

```
E2 = (VO* Y(2, 5)) / (Y(2, 5) - Y(2, 6))
```

```
T3=E2*E1;
```

```
Z(3, 6) = 0;
```

```
Q3=3*Q*(sin(F23))^2+Q*(cos(F25))^2*(cos(F23))^2+IZZ*(sin(F25))^2*(cos(F  
23))^2;
```

```
V5=(Q3+MC*E4*E4)*O2*O2/2;
```

```
V7 = E4 * (1 - cos(-A2 - V6));
```

```

if -V6 > A2 & V7*MC*G> V5

disp('***** The Roll Stops,"NC" *****')

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

disp('O4=');disp(O4);

disp('O5=');disp(O5);

disp('O6=');disp(O6);

disp('O7=');disp(O7);

disp('O8=');disp(O8);

disp('O9=');disp(O9);

disp('--- After first B3 VERSION(Roll Front) calculation, The Roll
Stops,"NC"')

else

V8 = E4 * cos(-A2 - V6) - E4 * cos(-A2 - atan(X(3, 1) / Z(3, 1)));

O3= sqrt((2 * MC * G * V8) / (Q3 + MC* E4 * E4) + O2 * O2)

K9 = X(3, 1);

K5 = Z(3, 1);

K6 = Z(3, 1) + T3;

K7 = E2 - X(3, 1);

K8 = U;

```

```

DD2 = Q3 + MC* K6 * K6 + MC * K7 * K7;

O4 = (Q3 + MC * K5 * K6 - K8 * MC * K7 * K7 - (1 + K8) * MC* K9 * K7) *

O3 / DD2;

F3 =atan(VO/ (Y(3, 5) - Y(3, 6)))

F23=F2+F3

% TRANSFORMATION OF THE COORDINATES FROM THE POSITION

%3 TO 4

for(K = 1:7)

X(4, K) = X(3, K) *cos(F3) + (Y(3, K) - Y(3, 5)) * sin(F3);

Y(4, K)=(Y(3,K)-Y(3,5))*cos(F3)-X(3,K)*sin(F3);

Z(4, K) = Z(3, K);

end

A4 = atan(tan(A0) / sqrt(1 + (tan(LL2)) ^ 2 / (cos(A0)) ^ 2));

M1= sqrt(X(4,1).^2 + Z(4,1).^ 2);

M2 =atan(X(4, 1) / Z(4, 1));

F252=atan((B0+S-B6)/(2*(L3-L6)));

F232=atan((H6-D3)/sqrt((L3-L6)^2+(S+B0-B6)^2/4));

Q5=3*Q*(sin(F232))^2+Q*(cos(F252))^2*(cos(F232))^2+IZZ*(sin(F252

))^2*(cos(F232))^2;

R1=(E2-X(2,1))^2+Z(2,1)*Z(2,1);

FS=MIU*MC*G*cos(A2);

O51=sqrt( O4^2+(2*MC*G*LS*sin(A2)-2*FS*LS)/(Q3+MC*R1*R1));

O5 = O51* cos(F3);

```

if -M2 > A4

M3 = M1 * (1 -cos(-A4 - M2));

M4 = (Q5 + MC * M1 * M1) * O5 * O5 / 2 ;

if M3 * MC * G > M4

disp('***** The Roll Stops *****');

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

disp('O4='); disp(O4);

disp('O5='); disp(O5);

disp('O6='); disp(O6);

disp('O7='); disp(O7);

disp('O8='); disp(O8);

disp('O9='); disp(O9);

disp('***** The Roll Stops (B3 VERSION),"NC"*****')

else

O9=sqrt(O5*O5 - 2*MC*G*M3./(Q5 + MC*M1.^2));

disp('=====The Roll Continues, "C"=====');

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

```

disp('O4=');disp(O4);

disp('O5=');disp(O5);

disp('O6=');disp(O6);

disp('O7=');disp(O7);

disp('O8=');disp(O8);

disp('O9=');disp(O9);

disp('==== The roll continues for the mower( B3 VERSION),"C")

end

else

disp('&&&& After B3 VERSION calculation, the roll continues, no need not
to calculate O6-O9 &&&&');

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

disp('O4=');disp(O4);

disp('O5=');disp(O5);

disp('O6=');disp(O6);

disp('O7=');disp(O7);

disp('O8=');disp(O8);

disp('O9=');disp(O9);

disp('&&&& The Mower Rolls Front. "C" &&&&&&')

end

```

end

```
%222222----B3----B3-----222222---END----END----
```

else

%The second running of B1 version begins

% 22222222----B1----B1----B1-----22222222

```
disp('The program will check whether the mower has the opportunity to roll back(
B1 VERSION)');
```

```
disp('Because L6>L8, the program will go B1 version(Roll Back)( point of impact  
of ROPS behind of the COG');
```

O3 = 0; O4 = 0; O5 = 0; O6 = 0; O7 = 0; O8 = 0; O9 = 0;

$$Z(3,6)=0;$$

% In the following equation, the empirical equation ($IXX \sim 3 \cdot IYY$) was used

$$Q3=3*Q*(\sin(F23))^2+Q*(\cos(F25))^2*(\cos(F23))^2+IZZ*(\sin(F25))^2*(\cos(F23))^2$$
$$V5 = (Q3 + MC * E4 * E4) * O2 * O2 / 2;$$
$$V7 = E4 * (1 - \cos(-A2 - V6));$$

% Repeat the process of B2 version

if -V6 > A2 & V7 * MC* G > V5

disp('##### The Roll Stops, "NC" #####')

```
disp('O0='); disp(O0);
```

```
disp('O1='); disp(O1);
```

```
disp('O2='); disp(O2);
```

```

disp('O3='); disp(O3);

disp('O4=');disp(O4);

disp('O5=');disp(O5);

disp('O6='); disp(O6);

disp('O7=');disp(O7);

disp('O8=');disp(O8);

disp('O9=');disp(O9);

disp(' Because -V6 > A2 & V7 * MC* G > V5, The Roll Stops, "NC" &&&&&')

else

% CG passes over the top of tipping axis 2

V8 = E4 *cos(-A2 - V6) - E4 *cos(-A2 -atan(X(3, 1)./ Z(3, 1))); % Height change of
CG %from position 3 to position 4

O3 =sqrt(2 * MC * G * V8./ (Q3 + MC*E4*E4) + O2 * O2); % Angular velocity at
%position 4

K9 = X(3, 1);

K5 = Z(3, 1);

K6 = Z(3, 1) + T3;

K7 = E2 - X(3, 1);

K8 = U;

DD2=Q3 + MC*K6*K6 + MC*K7* K7;

O4 = (Q3 + MC* K5 * K6 - K8 * MC * K7 * K7 - (1 + K8) * MC* K9 * K7) * O3 /
DD2;

F3 =atan(VO/ (Y(3, 4) - Y(3, 6)));

```

```

O5 = O4 *cos(F3);

%REM---- TRANSFORMATION OF THE COORDINATES FROM 3 TO 4 -

for( K = 1:7)

X(4, K) = X(3, K) *cos(F3) + (Y(3, K) - Y(3, 4)) *sin(F3);
Y(4, K) = (Y(3, K) - Y(3, 4)) *cos(F3) - X(3, K) *sin(F3);
Z(4,K)=Z(3,K);

end

A4 =atan(tan(A0)./sqrt(1 + (tan(F2 + F3)).^ 2 / (cos(A0)).^ 2));

C3 =atan(Z(4, 7)./ X(4, 7));

C4 = 0;

C5 =sqrt(X(4, 7) * X(4, 7) + Z(4, 7) * Z(4, 7));

C6 =C4./ C5;

C7 = C5 * (Y(4, 6) - Y(4, 1)) / (Y(4, 6) - Y(4, 7));

C8 = C6 * C7;

M1 =sqrt(X(4, 1) ^ 2 + Z(4, 1) ^ 2);

M2 =atan(X(4, 1)./ Z(4, 1));

%REM ----ROTATION OF THE TRACTOR FROM THE POSITION 4 TO 5 -

for(K=1:7)

if Z(4, K)~=0

C9 =atan(X(4, K) / Z(4, K));

else

C9 = -3.14159 / 2;

end

```

```

X(5, K) = sqrt(X(4, K) ^ 2 + Z(4, K) ^ 2) * sin(C9 + C3 + C6);
Y(5, K) = Y(4, K);
Z(5, K) = sqrt(X(4, K) ^ 2 + Z(4, K) ^ 2) * cos(C9 + C3 + C6);
end

Z(5, 7) = 0;

% In the following equation, the empirical equation ( $I_{XX} \sim 3 * I_{YY}$ ) was used
Q5 = Q*(cos(F2 + F3)).^2 + 3*Q*(sin(F2 + F3)).^ 2;
M3 = M1*(1-cos(-A4 - M2));
M4 = (Q5 + MC* M1 * M1) * O5 * O5./ 2;
if-M2 > A4 & M3* MC* G > M4
disp('The Roll Stops, "NC"')
disp('O0='); disp(O0);
disp('O1='); disp(O1);
disp('O2='); disp(O2);
disp('O3='); disp(O3);
disp('O4=');disp(O4);
disp('O5=');disp(O5);
disp('O6=');disp(O6);
disp('O7=');disp(O7);
disp('O8=');disp(O8);
disp('O9=');disp(O9);
disp('The Roll Stops, "NC"')
else

```

```

MM1 =M1*cos(-A4 -atan(X(5, 1) / Z(5, 1)));
M5 = M1 *cos(-A4 -atan(X(4, 1) / Z(4, 1))) - MM1;
O6 =sqrt(2 * MC * G * M5 / (Q5 + MC * M1 * M1) + O5 * O5);
M6 = X(5, 1);
M7 = Z(5, 1);
M8 = Z(5, 1) + C8;
M9 = C7 - X(5, 1);
N1 = U;
DD3 = (Q5 + MC*M8*M8 + MC* M9*M9);
O7 = (Q5 + MC*M7*M8 - N1*MC*M9*M9 -(1+ N1)*MC*M6*M9)*O6./DD3;
F5 =atan(C5./(Y(5,6)-Y(5,7)));
A6 =atan(tan(A0) /sqrt(1+(tan(F2+F3+F5))^2./(cos(A0)).^2));
%TRANSFORMATION OF THE COORDINATES FROM THE POSITION 5 TO 6
for(K =1:7)
X(6, K) = X(5, K) *cos(F5) + (Y(5, K) - Y(5, 6)) *sin(F5);
Y(6, K) = (Y(5, K) - Y(5, 6)) *cos(F5) - X(5, K) *sin(F5);
Z(6,K)=Z(5,K);
end
O8 = O7 *cos(-F5);
N2 =atan(X(6, 1)./ Z(6, 1));
N3 =sqrt(X(6, 1).^ 2 + Z(6, 1).^ 2);
Q6 = Q*(cos(F2 + F3 + F5)).^2 + 3*Q*(sin(F2 + F3 + F5)).^ 2;
N4 = N3*(1-cos(-A6 - N2));

```

```

N5 = (Q6 + MC*N3*N3)*O8*O8./ 2;
P9 = (N4*MC*G - N5)./(N4*MC*G);
if -N2 > A6 & N4 * MC * G > N5
disp('The Roll Stops, "NC"')
disp('O0='); disp(O0);
disp('O1='); disp(O1);
disp('O2='); disp(O2);
disp('O3='); disp(O3);
disp('O4=');disp(O4);
disp('O5=');disp(O5);
disp('O6=');disp(O6);
disp('O7=');disp(O7);
disp('O8=');disp(O8);
disp('O9=');disp(O9);
disp('The RoLL Stops, "NC"')
else
disp('The Roll Continues, "C"')
N6=-N4;
O9=sqrt(2 * MC* G * N6./(Q6+MC*N3*N3) +O8*O8);
disp('O9=');
disp(O9);
end
end

```

```

% The second running of B1 VERSION ends,

disp('SECOND run B1 VERSION Ends')

end

end

else

O9 =sqrt(-2* MC * G * N4 / (Q6) + O4 * O4);

disp('Because  $N4 * MC * G < N5$ , so, the roll continues. "C"');

disp('O0='); disp(O0);

disp('O1='); disp(O1);

disp('O2='); disp(O2);

disp('O3='); disp(O3);

disp('O4=');disp(O4);

disp('O5=');disp(O5);

disp('O6=');disp(O6);disp('O7=');disp(O7);

disp('O8=');disp(O8);disp('O9=');disp(O9);

disp('**** After B2 VERSION Calculation, The Roll Continues. "C" "*****')

end

end

end

%Till now all of opportunities for rollover have been checked, therefore, the program

%ends.

end

```

The following are data files for F925 mower with regular ROPS and inverted ROPS
(deck up, deck down)

Data file1 (F925RE.m) saves the data for F925 mower with regular ROPS (deck
down).

```
% Data file 1 (F925RE.m), regular ROPS (deck down)
```

```
% H1 L3 L2 D3 D2 H6 L6 B6 H7 B7 L7 H0 S
```

```
B0(2Bm) D0 Mc Q
```

```
HH=[0.45 0.939 1.206 0.1 0.36 1.90 -0.471 0.775 0.845 0.650 1.051 0.31 0.88  
1.12 0.102 899.5 206.3];
```

```
% Data file 2 (F925REUP.m), regular ROPS (deck up)
```

```
HH=[0.48 0.939 1.206 0.2 0.36 1.90 -0.471 0.775 0.845 0.650 1.051 0.31 0.88  
1.12 0.102 899.5 206.3];
```

```
%Data file 3 (F925INMO1.m), inverted ROPS (deck down)
```

```
HH=[0.45 0.932 1.213 0.1 0.36 1.90 -0.045 0.775 0.845 0.650 1.058 0.31 0.88  
1.12 0.102 899.5 206.3];
```

```
%Data file 4 (F925INMO1UP.m), inverted ROPS (deck up)
```

```
HH=[0.48 0.932 1.213 0.2 0.36 1.90 -0.045 0.775 0.845 0.650 1.058 0.31 0.88  
1.12 0.102 899.5 206.3];
```

APPENDIX B

MEASUREMENT OF PARAMETERS

B.1 List of Parameters

The parameters measured are shown below:

- 1) Angular velocity (ω)
- 2) Center of gravity (CG)
- 3) Moment of inertia
- 4) ROPS deformation (T) and Cone Penetration Index (CI)

B.2 Measurement

B.2.1 Determination of Angular Velocity from the Vehicle

To record the rotation angle of the mower, the angle disks are mounted on the mower (see figure B-1). The angular velocities are determined from the mover rotation between video frames (1/30 s). It is the value of the angle difference divided by the time difference between two video frames.

$$\omega = \frac{(\alpha_2 - \alpha_1)}{(Time2 - Time1)} \quad (B.1)$$

Where,



Figure B-1. Angle disks are mounted on the mower.

$\alpha_2 - \alpha_1 =$ Angle difference (degree) and

$$Time2 - Time1 = \frac{1}{30} (second) .$$

B.2.2 Center of Gravity

The position of center of gravity for the vehicle can be determined by the weighting method. The mathematical procedure utilized by Liljedahl et al. (1998) was used to calculate the unloaded center of gravity for the vehicle in the horizontal direction in front of the rear axle (X_{cg}) and in the vertical direction above the ground (Z_{cg}).

$$X_{cg} = \frac{R_f L}{W_t} \quad (\text{B.2.1})$$

Where,

R_f = force under the front axle,

L = wheelbase and

W_t = total vehicle weight.

$$\Delta r = r_r - r_f \quad (\text{B.2.2})$$

Where,

r_r = rear tire radius and

r_f = front tire radius.

$$L' = \sqrt{L^2 + (\Delta r)^2 - (n - r_r)^2} \quad (\text{B.2.3})$$

$$\tan \lambda_1 = \frac{(n - r_r)}{L'} \quad (\text{B.2.4})$$

$$\tan \lambda_2 = \frac{\Delta r}{L} \quad (\text{B.2.5})$$

$$\lambda = \lambda_1 + \lambda_2 \quad (\text{B.2.6})$$

$$h = \frac{W_t l - R_f' L}{W_t \tan \lambda} + \frac{R_f' \Delta r}{W_t} \quad (\text{B.2.7})$$

$$Z_{cg} = r_r + h \quad (\text{B.2.8})$$

Where,

L' = length of vehicle when lifted (m),

n = height of lifted vehicle from front axle to ground (m) and

R_f' = the force acting upward on the front axle when the vehicle is in the lifted position (N).

B.2.3 Moment of Inertia Determination

(a) The moment of inertia determination about the longitudinal (roll) axes passing through the center of gravity of the mower can be measured using the pendulum method (see figure B-2 and B-3).

The mower and the supporting sling form a compound pendulum that will oscillate with a period T given by

$$T = 2\pi \sqrt{\frac{I_0}{WR_0}} \quad (\text{B.2.9})$$

Using the parallel axis theorem, roll moment of inertia of mower about the center of gravity of the mower can be obtained.

$$I_t = I_0 - m_t R_0^2 \quad (\text{B.1.10})$$

Where,

T = Period of oscillation (s),

I_0 = Moment of inertia of mower about pivot O (kgm^2),

W = Weight of mower (N),

R_0 = Distance between pivot O and the center of gravity of the mower (m),

I_t = Moment of inertia of mower about the center of gravity of the mower (kgm^2)

and

m_t = Mower mass (kg).

(b) Using the SAE standard, the roll moment of inertia of mower about the center of gravity of the mower can be obtained (SAE, 1995).

$$I_t = \frac{(RH + CG) * TW * m_t}{K} \quad (\text{B.2.11})$$

Where,

TW = Track width (m),

CG = Center of gravity height (m),

RH = ROPS height (m),

m_i = Mass of vehicle (kg) and

K = Approximation constant (dependent on inertia property and vehicle class).

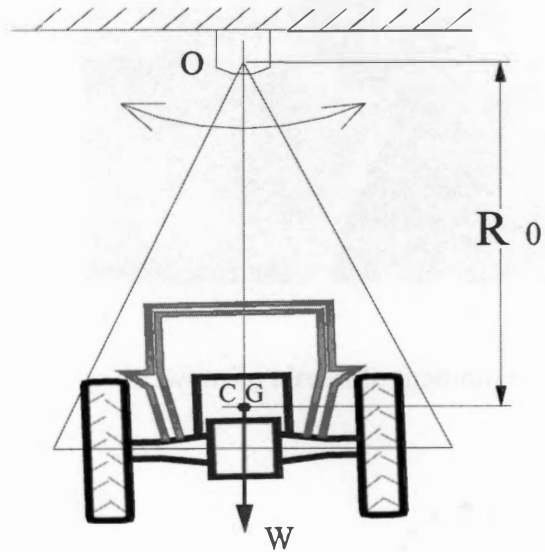


Figure B-2. Determination of the roll moment of inertia using the pendulum method.



Figure B-3. The measurement of roll moment of inertia of mower.

B.2.4.Determination of ROPS Deformation and Cone Penetration Index

Linear Variable Differential Transformer (LVDT) sensor is used to measure the deformation of ROPS. The field test data is recorded with CR23X datalogger by connecting the LVDT to the datalogger (see figures B-4 and B-5). The data is downloaded from the datalogger to a laptop with datalogger support software (PC208W3.1). The laptop processes the data with Excel. Cone Penetration readings were taken at the deck and ROPS impact points (see figure B-6).

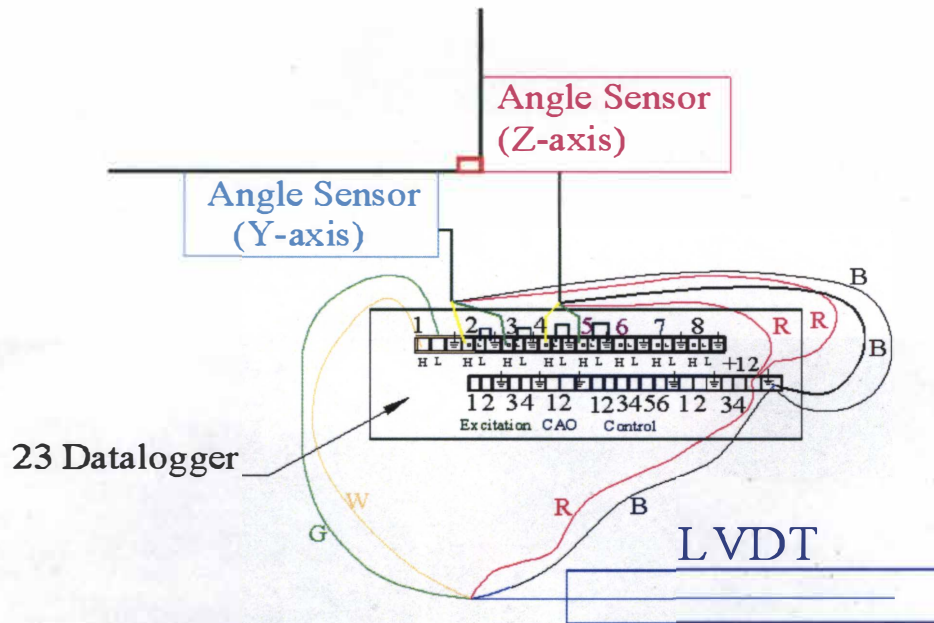


Figure B-4. Connection of LVDT to 23X datalogger.

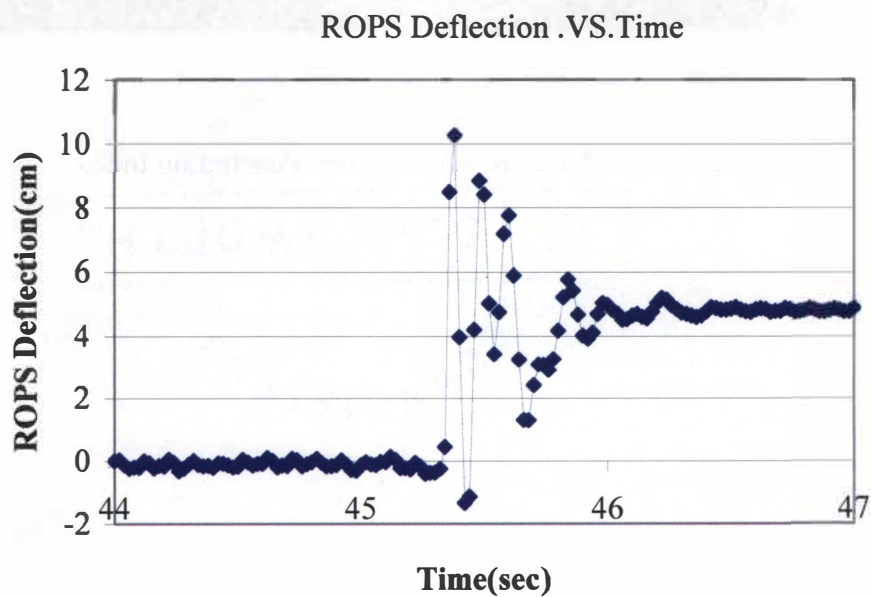


Figure B-5. Measurement of ROPS deflection.



Figure B-6. Measurement of Cone Penetration Index.

VITA

Xinyan Wang was born in Baoding, China on October 31, 1963. She received her bachelors' degree in Agricultural Machinery and Implement Design from Hebei Agricultural University, China in June 1984 and her master degree from China Agricultural University in June 1993. She had nine years of teaching and researching experience in Automobile and Tractor Engineering area at the China Agricultural University as an Assistant and Associate Professor.

...the ... of ...
...the ... of ...
...the ... of ...
...the ... of ...
...the ... of ...

25. SOUTHWORTH

