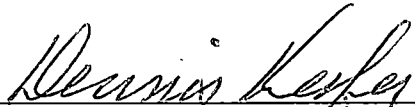
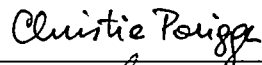
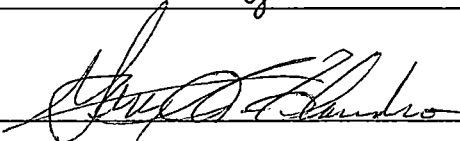
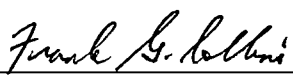


To the Graduate Council:

I am submitting herewith a dissertation written by Herbert Daniel Thomas entitled "Numerical Simulation of Pulsed Plasma Thrusters". I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Mechanical Engineering.


Dr. Dennis Keefer, Major Professor

We have read this dissertation
and recommend its acceptance:

Accepted for the Council:


Associate Vice Chancellor
and Dean of the Graduate School

NUMERICAL SIMULATION OF PULSED PLASMA THRUSTERS

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Herbert Daniel Thomas

May 2000

DEDICATION

This dissertation is dedicated to my wife

Jessica L. Thomas.

ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. Dennis Keefer, for his guidance and extreme patience during the course of my Ph.D. education. I also appreciate the assistance of the other professors on my dissertation committee: Dr. Frank Collins, Dr. Gary Flandro, and Dr. Christian Parigger. Dr. Montgomery Smith at UTSI was very helpful with the digital filtering and spectroscopic data reduction portions of this dissertation. Bob Rhodes and his knowledge of the MACH2 code was also vital to the completion of this study. I am also extremely grateful to the staff of the University of Tennessee Space Institute library (headed by Mary Lo), for their help with the vast literary searches required for this work. I would like to express my sincere appreciation to my family and friends in Knoxville, Tullahoma, and Cowan for their support and encouragement, without which none of this work would have been possible. A very special thanks goes to my wife Jessica for her extreme patience, understanding, and love.

ABSTRACT

MACH2 was used to simulate the plasma flow in cablegun (developed at UTSI) and pulsed plasma thruster (PPT-4, developed at the University of Illinois at Urbana-Champaign) geometries, and the corresponding results were compared to the experimental operation of the devices. The capabilities of MACH2 were extended by incorporating a new radiation model as well as new thermodynamic equation of state tables for plasmas containing either Teflon or a mixture of 39% Teflon and 61% copper (by mass). In developing the tables, the thermodynamic variables at low temperatures were calculated using the NASA Equilibrium Code (CEA). The high temperature calculations were performed by iterating the multi-component Saha equation. Cubic polynomials were then used to blend the high and low temperature calculations together over the intermediate temperature regions. The Saha-calculated specific heats contained large numerical fluctuations and were filtered using a (1,1)-fold trimmed mean filter. The data was also smoothed over suitable temperature intervals using cubic polynomials. Using spectroscopic data gathered in the cablegun experiments at UTSI, a preliminary analysis was performed to obtain a rough estimate of plasma temperature.

In the MACH2 simulations, the plasma absorption coefficient ("fox") and the propellant absorptivity were adjusted to give the correct values for plasma velocity and ablated mass. The resulting impulse bits for the cableguns were $6.89\text{e-}04 \text{ N}\cdot\text{s}$ for a plasma consisting of only Teflon and $1.84\text{e-}03 \text{ N}\cdot\text{s}$ for a plasma containing 39% Teflon and 61% copper (by mass). The corresponding specific impulses are 1254 s and 695 s, respectively. The PPT-4 thruster produced a total impulse of $2.03\text{e-}04 \text{ N}\cdot\text{s}$ and an

average specific impulse of 521 s. Although providing less impulse per shot, the PPT-4 thruster output more impulse per energy stored in the capacitor.

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1. INTRODUCTION

1.1 Statement of Problem

Due to the increasing desire to minimize the cost of spacecraft and to extend their useful lifetimes, new propulsion technologies are constantly investigated to reduce the required propellant mass that a spacecraft must carry or to maximize the payload for a given amount of fuel. The dynamics of this problem was first solved by K.E. Tsiolkolyskii, who developed the famous "rocket equation" to estimate vehicle performance [1]:

$$\frac{M_f}{M_0} = \exp\left(-\frac{\Delta V}{U_{ex}}\right) \quad (1.1.1)$$

where M_f is the spacecraft mass after the propulsive burn, M_0 is the initial mass, ΔV is the velocity increment imparted to the spacecraft, and U_{ex} is the rocket exhaust velocity. This expression shows that for a given initial mass and ΔV requirement, the corresponding propellant mass is reduced by maximizing U_{ex} . The exhaust velocities produced by conventional chemical rockets are limited by the available energies released from the chemical reactions. However, the propellant exhausts of electric propulsion systems can reach much higher speeds than chemical rockets but, typically, at much lower thrust levels. Therefore, for missions requiring only small amounts of thrust, such as spacecraft attitude control or for interplanetary missions where electric thrusters can operate over long periods of time, electric propulsion devices can offer substantial mass savings compared to spacecraft with chemical propulsion systems [2].

One area that has received renewed interest is the use of pulsed plasma thrusters

(PPT's) for spacecraft attitude control, precision pointing, and orbit maintenance [3].

Also, similar devices known as cableguns are currently used in plasma opening switches [4]. However, these devices suffer severe energy losses in the conversion of the electrical energy to exhaust jet kinetic energy. Therefore, studies have been undertaken to find ways to increase the overall efficiency of PPT's [5,6]. The objective of this dissertation is to aid in this effort by implementing a numerical model of the plasma flow in cablegun and PPT devices using MACH2 [7]. MACH2 is a time-dependent, two-dimensional, magnetohydrodynamics (MHD) simulation code. MACH2 employs an implicit, finite-volume solver of the Arbitrary Lagrangian Eulerian (ALE) variety. The capabilities of MACH2 have been extended by developing new radiation and ablation models and also by incorporating new thermodynamic equation of state tables that were created for the desired propellant plasma constituents.

1.2 Electric Propulsion Background

The first person credited to have considered converting electric energy to rocket exhaust kinetic energy is Robert Goddard who experimented with electric gas discharges in 1906 [8]. In 1929, Herman Oberth followed by contributing to the theoretical foundation of rocketry and devoted a chapter on electric propulsion in his book "Wege zur Raumschiffahrt" ("Possibilities of Space Flight") [9]. The first comprehensive study of the use of electric propulsion in space flight systems began in 1954 and was published by Ernst Stuhlinger [10].

Electric propulsion devices can be grouped into three general classes: electrostatic, electromagnetic, and electrothermal [2]. Most early development

concentrated on the electrostatic (or ion) thruster concept, which accelerates the positive ions in a plasma through an applied potential difference. The first operational ion engine model was developed at Rocketdyne Corporation in August of 1958 [8]. The first space test of an ion engine was in 1964 with the U.S. SERT I (Space Electric Rocket Test) engine, and the first long-time space flight test occurred in 1970 with SERT II. More recently, the NSTAR ion engine propelled the Deep Space 1 probe to a flyby of asteroid 9969 Braille on July 28, 1999 [11].

Electromagnetic (MPD or MHD) propulsion denotes concepts that accelerate ionized plasmas by utilizing Lorentz forces resulting from the interaction of external and/or internal magnetic fields with currents driven through the plasma. Steady devices have been extensively studied in the U.S. [12] and in Russia [13]. This dissertation focuses on the unsteady or pulsed plasma systems. These thrusters are driven by an RLC (resistance-inductance-capacitance) pulse circuit and can have traditional, rectangular geometries or coaxial forms. Basically, these devices operate by discharging a current across a solid propellant, which is ablated, ionized, and then accelerated out the thruster exit. The first operational PPT was employed on the ill-fated, Soviet Zond 2 spacecraft which was launched toward Mars in November of 1964 [6]. The United States followed in 1968 with the successful operation of a PPT on the LES-6 spacecraft. Other spacecraft that employed PPT's include the Synchronous Meteorological Satellite (SMS) in 1974 and the Navy TRANSIT navigation satellite system (TIP/NOVA) in 1981.

Electrothermal devices were first proposed in the 1950's [8]. In this concept, the propellant is heated electrically and then expanded through a nozzle. Resistojets and arcjets both fall into this category [2]. In a resistojet, the propellant is passed over an

electrically heated resistor, which convects heat into the propellant gas. An arcjet passes the propellant gas through an arc column, which transfers heat to the gas by ohmic heating, convection and radiation.

An alternative to the conventional, rectangular, breech-fed PPT is the cablegun [4]. This coaxial, pulsed device operates in a similar manner to the PPT mentioned above, except that the propellant gas is accelerated primarily by electrothermal rather than electromagnetic processes. The discharge from an RLC circuit radiates to a solid, polymer propellant, which ablates, ionizes, and then expands out the nozzle exit.

1.3 Renewed Interest in PPT's and Cableguns

Lately, there has been an increased interest in ablation-fed, pulsed plasma microdevices for use in satellite control and plasma opening switches. In December of 1999, a PPT will fly on the Earth Observer 1 (EO-1) to demonstrate the concept's attitude control capability [14]. The U.S. Air Force is considering the use of PPT's in future flights of their Mighty Sat II program. PPT's are also scheduled to fly on the New Millenium Interferometer spacecraft (Deep Space 3 mission) to be launched in 2001. This ambitious project will demonstrate the use of pulsed plasma thrusters for spacecraft translation, precision positioning, and formation flying. A schematic of the traditional, rectangular, breech-fed PPT is shown in Figure 1.3.1.

Pulsed plasma thrusters and cableguns offer several advantages over resistojets and other concepts considered for use in plasma opening switches and spacecraft attitude and control. The solid, polymer propellant is noncorrosive and requires no tankage and feedlines [6]. The devices can operate with zero warmup time. The thrusters can impart

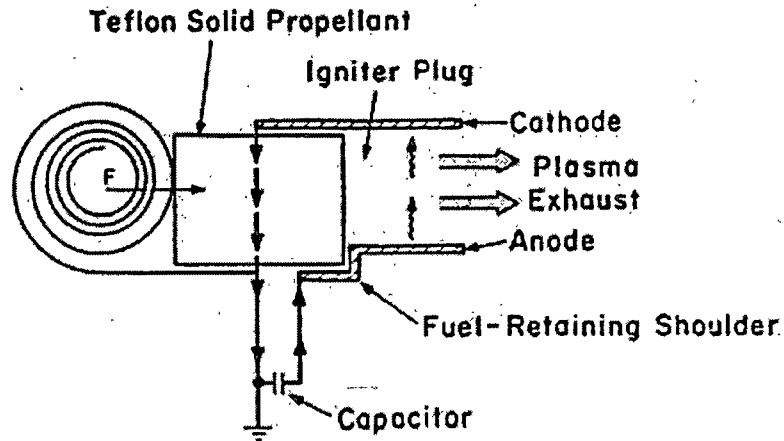


Figure 1.3.1: Traditional PPT Concept [15]

short, discrete increments of velocity change to the spacecraft. The PPT's and especially cableguns are scaleable to very small sizes and are based on simple concepts which can be easily implemented.

1.4 Scope of Dissertation

The scope of this dissertation is to simulate the plasma flow in coaxial PPT and cablegun geometries using MACH2 and then to compare the results with the experimental operation of the devices. The capabilities of MACH2 are extended by developing and implementing new radiation and ablation models in the plasma control volume and on the propellant boundary. Also, tables of thermodynamic, equation of state data are generated over desired density and temperature ranges for plasmas with desired propellant constituents.

The coaxial PPT modeled in this dissertation is designated PPT-4 and is under

development at the University of Illinois at Urbana-Champaign by Dr. Rodney Burton [16]. Burton proposes that PPT-4 (shown in Figure 1.4.1) operates mainly as an electrothermal device. This conclusion is investigated further in this study. Equations of state based on Teflon are used in the simulation of these devices.

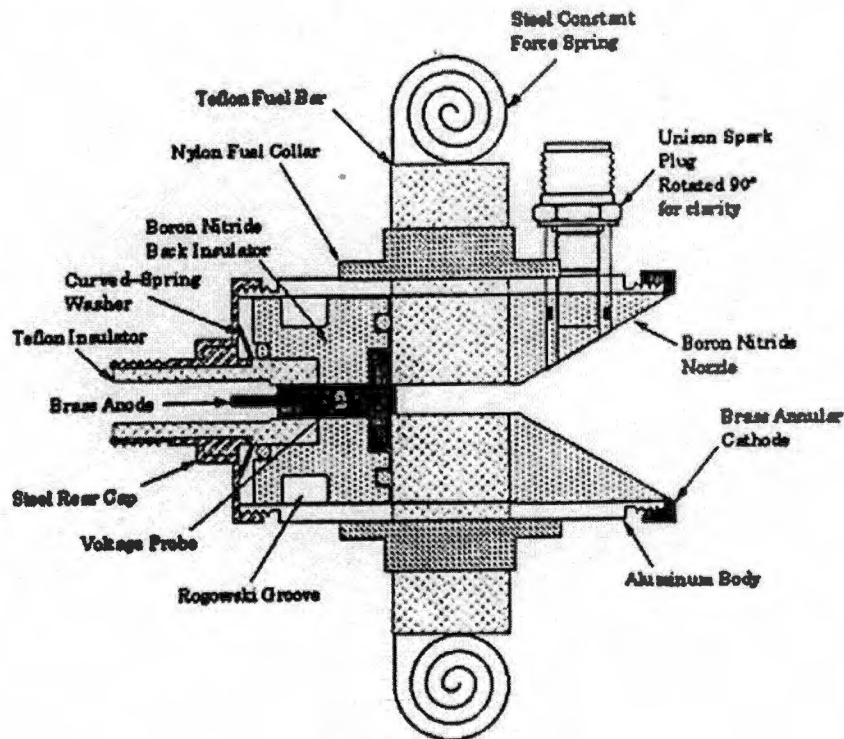


Figure 1.4.1: PPT-4 Geometry [16]

A typical representation of the cablegun modeled in this study is shown in Figure 1.4.2. During recent experiments at UTSI, the plasma exhaust contained mass ratios of 39% Teflon from the propellant and 61% copper from the cathode. Experiments using non-ablating, tungsten center electrodes were also performed, resulting in plasma exhausts containing only Teflon. Therefore, equation of state tables are generated for the

Teflon-only and Teflon-copper plasmas and used in the MACH2 simulations. Also, experimental spectroscopic data is used in a preliminary analysis to estimate the cablegun plasma temperature, which is compared to the MACH2 results.

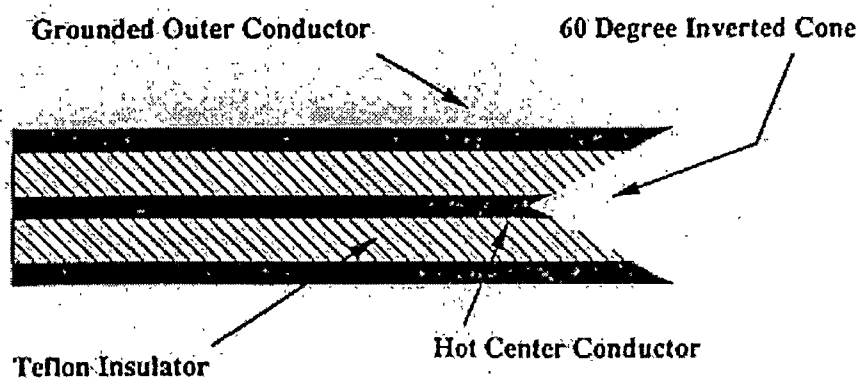


Figure 1.4.2: Typical Cablegun Geometry [2]

2. METHODOLOGY

2.1 Governing Equations

The MACH2 code is used to simulate the cablegun and PPT experiments. This program numerically integrates the time-dependent governing equations presented in this section, which includes the continuity, momentum, energy, and state equations along with the appropriate Maxwell electromagnetic relations [17]. In this study, the plasma is assumed to be a state of Local Thermodynamic Equilibrium (LTE), and the magnetohydrodynamic (MHD) approximation is applied (where appropriate) to the electromagnetic expressions.

2.1.1 MHD Relations

The well-known continuity equation is given by:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{v} = 0 \quad (2.1.1)$$

where ρ and $\vec{v}$ are the plasma density and velocity vector, respectively. The time derivative above uses the convective differential operator:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \quad (2.1.2)$$

Basically, equation (2.1.1) states that the rate of increase of mass in the control volume is equal to the net rate of mass flowing across the control surface.

The next governing expression is Newton's 2nd Law (or momentum equation), which relates surface and body forces to the time rate of change of momentum within the

control volume:

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \vec{J} \times \vec{B} + \nabla \cdot \vec{\sigma} \quad (2.1.3)$$

where p is the plasma pressure, $\vec{J}$ is the current density, $\vec{B}$ is the magnetic induction, and $\vec{\sigma}$ is the stress tensor, which can include elastic and viscous surface forces depending on the problem at hand. Note that the above momentum equation differs from that given in fluid mechanics texts by the inclusion of the electromagnetic, Lorentz body force $\vec{J} \times \vec{B}$.

In the plasma simulations, energy must be conserved at all times. Similar to the continuity equation, the conservation of energy states that the energy increase in the control volume must be equal to energy input minus output (sources-sinks) plus the net energy transported across the volume boundary. Letting u designate the internal energy per unit mass, the energy equation in MACH2 is:

$$\rho \frac{Du}{Dt} = -p \nabla \cdot \vec{v} + \nabla \times \vec{B} \cdot (\vec{\eta} \cdot \nabla \times \vec{B}) + (\vec{\sigma} \cdot \nabla) \cdot \vec{v} - \nabla \cdot \vec{F}_{\text{diff}} - \dot{q}_{\text{rad}} \quad (2.1.4)$$

where $\vec{\eta}$ is the electrical resistivity tensor, $\vec{F}_{\text{diff}}$ represents the sum of a limited thermal conduction and equilibrium radiation flux terms [17], and $\dot{q}_{\text{rad}}$ corresponds to radiation cooling rate due to emission. The first term on the right-hand side represents the flow work, while the second term corresponds to Joule heating. The third and fourth terms designate the diffusive energy transport.

2.1.2 Electromagnetic Relations

The conservation equations given above (along with the equations of state discussed in section 2.2.3) must be combined with relations for $\vec{J}$ and $\vec{B}$ to complete the

system of equations. The relation for the current density is given by Ampere's law:

$$\vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B} \quad (2.1.5)$$

where μ_0 is the permeability of free space, and the displacement current has been neglected due to the MHD approximation. The equation for the magnetic induction is derived from Faraday's law:

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \quad (2.1.6)$$

where $\vec{E}$ is the electric field intensity, which is given by the generalized form of Ohm's law:

$$\vec{E} = \bar{\eta} \cdot \vec{J} - \vec{v} \times \vec{B} + \frac{1}{en_e} (\vec{J} \times \vec{B}) \quad (2.1.7)$$

which includes the contribution due to the moving plasma (second term on right) and the Hall current (last term on right). The variable e represents the magnitude of the charge of an electron, and n_e is the electron number density. Substituting (2.1.7) into (2.1.6) and using (2.1.5) results in the desired equation for magnetic induction:

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \left(\frac{1}{\mu_0} \bar{\eta} \cdot \nabla \times \vec{B} \right) + \nabla \times (\vec{v} \times \vec{B}) - \nabla \times \left[\frac{1}{en_e \mu_0} (\nabla \times \vec{B} \times \vec{B}) \right] \quad (2.1.8)$$

This expression describes the local, time rate of change of the $\vec{B}$ field due to field diffusion, motion of the plasma, and the Hall effect.

2.1.3 Equations of State

The 11 equations given above describe 14 plasma properties (counting the electron number density and plasma temperature, T). Therefore, three more relations are required to close the system:

$$\begin{aligned}p &= p(\rho, T) \\u &= u(\rho, T) \\n_e &= n_e(\rho, T)\end{aligned}\tag{2.1.9}$$

Given a plasma density and temperature, the pressure, specific internal energy, and electron number density can be determined from generated thermodynamic tables. The method used to construct these equation of state tables is discussed later in section 2.4.

2.2 Boundary Conditions

Before integrating the above system, the values of the desired variables along the grid boundaries must be specified. These boundary conditions depend on the problem at hand and include hydrodynamic, thermal, and electromagnetic requirements [17]. The hydrodynamic conditions are specified at impermeable walls, inlets and outlets, and at the axis of cylindrical symmetry. The thermal boundary conditions are applied to either heat-conducting walls or insulators. The electromagnetic variables are specified at the surfaces of idealized electrical conductors and insulators and at the symmetry boundaries.

2.2.1 Hydrodynamic Boundary Conditions

Along a wall surface, the flow can be specified as free slip or no slip. For the free slip condition, the flow is restricted to the plane of the surface. Therefore, velocity components normal to the wall are set to zero:

$$\hat{n} \cdot \vec{v} = 0 \quad (2.2.1)$$

where $\hat{n}$ is the unit normal vector to the wall and is directed toward the grid exterior.

The no-slip condition sets all velocity components to zero at the wall:

$$\vec{v} = 0 \quad (2.2.2)$$

At an inlet, the flow properties can either be set to specified values:

$$\begin{aligned} \rho &= \rho_{\text{inlet}} \\ \vec{v} &= \vec{v}_{\text{inlet}} \end{aligned} \quad (2.2.3)$$

or assigned the values just inside the boundaries:

$$\begin{aligned} \hat{n} \cdot \nabla \rho &= 0 \\ \hat{n} \cdot \nabla \vec{v} &= 0 \end{aligned} \quad (2.2.4)$$

Similar conditions are also applied to the flow outlets. At the axis of cylindrical symmetry, the components of velocity normal to the axis are set to zero.

2.2.2 Thermal Boundary Conditions

The thermal boundary condition for a heat-conducting wall at a constant temperature is:

$$T = T_{\text{wall}} \quad (2.2.5)$$

In the case of an insulator, the conductive heat flux is zero at the wall:

$$\hat{n} \cdot \nabla T = 0 \quad (2.2.6)$$

For inlets and outlets, the boundary conditions are similar to the hydrodynamic expressions; the temperature can be specified, or the internal energy can be set to the value just inside the boundary:

$$\begin{aligned} T &= T_{\text{inlet}}, T_{\text{outlet}} \\ \hat{n} \cdot \nabla u &= 0 \end{aligned} \quad (2.2.7)$$

2.2.3 Electromagnetic Boundary Conditions

Due to the cylindrical symmetry of the problems analyzed in this dissertation, the electromagnetic boundary conditions are listed in cylindrical form. For a perfect conductor, three expressions model the conditions at the surface [17]:

$$\begin{aligned} J_{\theta} &= 0 \\ n_r J_z - n_z J_r &= 0 \\ \hat{n} \cdot \nabla(rB_{\theta}) &= 0 \end{aligned} \quad (2.2.8)$$

These equations result from the condition that the tangential component of the electric field and the normal component of the magnetic induction must be continuous through the surface of a perfect conductor. For the problems covered in this study, the radial and axial components of the magnetic induction are zero.

For a perfect electrical insulator, no current flows across the wall surface. Therefore, the normal current along the insulator is zero. This restriction leads to the following boundary relation:

$$rB_{\theta} = \frac{\mu_0 I}{2\pi} \quad (2.2.9)$$

where I is the total current flowing between the insulating boundary and the axis of

symmetry. At this cylindrical axis of symmetry, the electromagnetic boundary conditions are similar to the hydrodynamic conditions. The components of current density and magnetic induction normal to the axis are set to zero.

2.3 Radiation/Ablation Model

As stated above in section 1.4, an important objective of this dissertation is to develop a model to predict the amount of energy radiated to the propellant surface from the plasma flowfield. In this model, radiation energy is emitted from a differential volume element (dV_e), and a fraction of this energy intercepts a wall element (dA_w) located on the propellant surface (see Figure 2.3.1). To simplify the analysis, the plasma is assumed to be optically thin and dominated by diffuse, gray emission. Also, the propellant surface is assumed to be non-emitting and does not reflect the incoming radiation.

Kirchhoff's Law states that in order to maintain an energy balance within a black enclosure, the energy emitted from a body must equal that absorbed [18]. Therefore, the radiation density (power/volume) emitted from an isothermal, diffuse, gray volume without self-absorption is given by [19]:

$$Q_e = 4\kappa\sigma T_e^4 \quad (2.3.1)$$

where κ is the total absorption coefficient (which is equal to the total emission coefficient through Kirchhoff's Law), σ is the Stefan-Boltzmann Constant, and T_e is the temperature of the emitting volume element. The amount of this radiation that reaches dA_w is:

$$d^3q_w = Q_e dV_e f_r = 4\kappa\sigma T_e^4 dV_e f_r \quad (2.3.2)$$

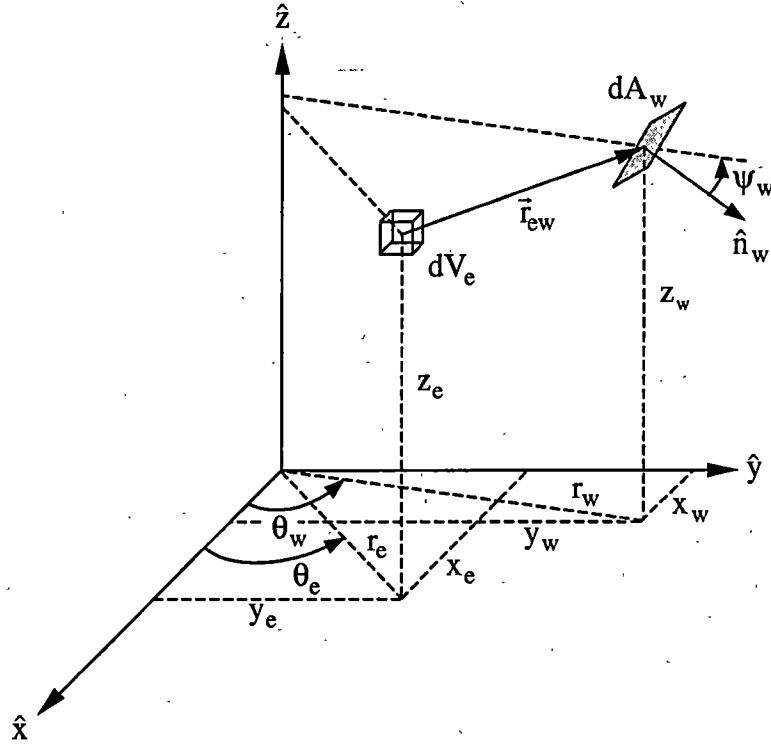


Figure 2.3.1: Radiation Model Geometry

The view factor, f_r , is the fraction of the total emitted energy that intercepts dA_w . In earlier attempts, the view factor was calculated by dividing 4π into the solid angle, $d\Omega$, subtended by viewing dA_w from the center of the emitting volume. The solid angle was approximated by projecting the finite surface of dA_w perpendicular to the incoming radiation:

$$f_r = \frac{d\Omega}{4\pi} = \frac{\hat{r}_{ew} \cdot \hat{n}_w dA_w}{4\pi r_{ew}^2} \quad (2.3.3)$$

This approximation is valid when dA_w is small compared to the distance, r_{ew} , from dV_e to dA_w . However, for emitting volumes close to the wall element ($r_{ew} \ll 1$), the view factors calculated from (2.3.3) can reach values greater than 0.5, which is not physically

viable. To obtain a better approximation to the view factor, the wall element is projected onto the surface of a sphere which has a radius equal to the distance between dV_e and dA_w (r_{ew}) and is centered about dV_e (see Figure 2.3.2). The projected area dA'_w is then divided by the total area of the sphere to give a more accurate relation for f_r :

$$f_r = \frac{dA'_w}{4\pi r_{ew}^2} \quad (2.3.4)$$

For convenience, the $\hat{z}'$ axis is parallel to the vector formed by $\vec{r}_1 - \vec{r}_2$, where $\vec{r}_1$ is the vector from dV_e to the center of the top surface of dA_w . $\vec{r}_2$ is the vector from dV_e to the bottom dA_w surface:

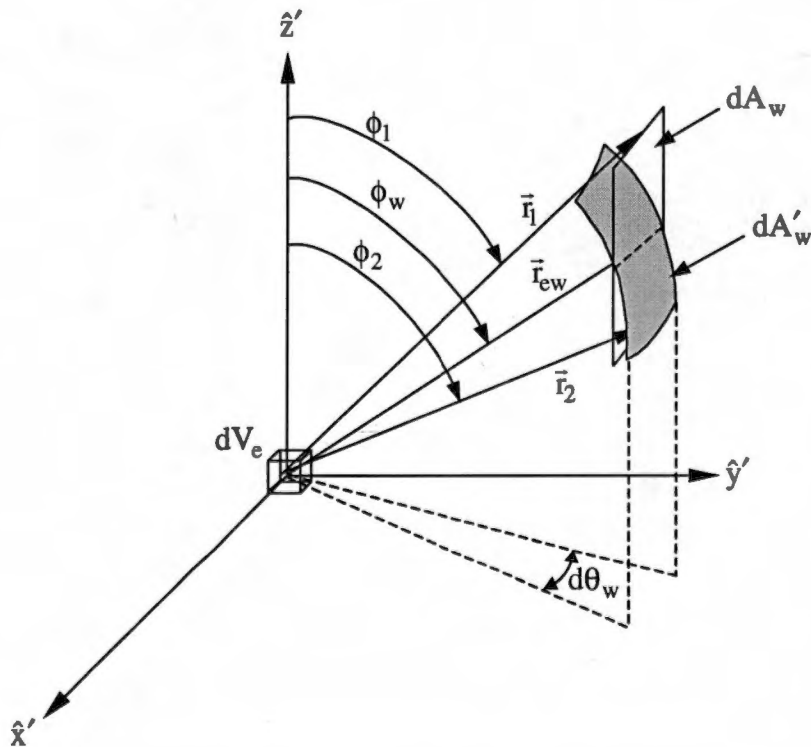


Figure 2.3.2: Radiation View Factor Geometry

$$\begin{aligned}\vec{r}_1 = & [r_{wup} \cos(\theta_w) - r_e \cos(\theta_e)]\hat{i} + [r_{wup} \sin(\theta_w) - r_e \sin(\theta_e)]\hat{j} \\ & + (z_{wup} - z_e)\hat{k}\end{aligned}$$

and

$$(2.3.5)$$

$$\begin{aligned}\vec{r}_2 = & [r_{wlow} \cos(\theta_w) - r_e \cos(\theta_e)]\hat{i} + [r_{wlow} \sin(\theta_w) - r_e \sin(\theta_e)]\hat{j} \\ & + (z_{wlow} - z_e)\hat{k}\end{aligned}$$

where the positions of the upper and lower wall surfaces are defined by r_{wup} , θ_w , z_{wup} and r_{wlow} , θ_w , z_{wlow} , respectively (in the global coordinate system shown in Figure 2.3.1). The corresponding magnitudes of $\vec{r}_1$ and $\vec{r}_2$ are:

$$r_1 = \sqrt{r_{wup}^2 - 2r_{wup}r_e \cos(\theta_e - \theta_w) + r_e^2 + (z_{wup} - z_e)^2}$$

and

$$(2.3.6)$$

$$r_2 = \sqrt{r_{wlow}^2 - 2r_{wlow}r_e \cos(\theta_e - \theta_w) + r_e^2 + (z_{wlow} - z_e)^2}$$

Therefore, the resulting unit vector for $\hat{z}'$ is given by:

$$\hat{z}' = \frac{1}{\sqrt{(r_{wup} - r_{wlow})^2 + (z_{wup} - z_{wlow})^2}} \begin{bmatrix} (r_{wup} - r_{wlow})\cos(\theta_w)\hat{i} \\ + (r_{wup} - r_{wlow})\sin(\theta_w)\hat{j} \\ + (z_{wup} - z_{wlow})\hat{k} \end{bmatrix} \quad (2.3.7)$$

Using this geometry, the expression for the projected, differential spherical surface element is:

$$d^2A'_w = r_{ew}^2 \sin(\phi_w) d\phi_w d\theta_w \quad (2.3.8)$$

The projected wall element is then obtained by integrating (2.3.8) from ϕ_1 to ϕ_2 :

$$dA'_w = r_{ew}^2 [\cos(\phi_1) - \cos(\phi_2)] d\theta_w \quad (2.3.9)$$

Substituting (2.3.9) into (2.3.4) results in the following relation for the view factor:

$$f_r = \frac{[\cos(\phi_1) - \cos(\phi_2)]d\theta_w}{4\pi} \quad (2.3.10)$$

where the cosines in the numerator are determined by taking the dot products of $\vec{r}_1$ and $\vec{r}_2$ with the unit vector, $\hat{z}'$:

$$\cos(\phi_1) = \frac{\vec{r}_1 \cdot \hat{z}'}{r_1} \quad \text{and} \quad \cos(\phi_2) = \frac{\vec{r}_2 \cdot \hat{z}'}{r_2} \quad (2.3.11)$$

Now that the view factor is known, the power radiated to dA_w from dV_e can be obtained from (2.3.2):

$$d^3q_w = \frac{\kappa\sigma T_e^4 dV_e}{\pi} \cdot [\cos(\phi_1) - \cos(\phi_2)]d\theta_w \quad (2.3.12)$$

The amount radiated to dA_w from a volume "toroid" located at $r = r_e$ and $z = z_e$ is calculated by substituting $dV_e = r_e dr_e dz_e d\theta_e$ into (2.3.12) and integrating θ_e from 0 to 2π :

$$d^2q_w(i, j) = \frac{\kappa\sigma T_e^4 r_e dr_e dz_e}{\pi} \cdot \left\{ \int_0^{2\pi} [\cos(\phi_1) - \cos(\phi_2)] d\theta_e \right\} d\theta_w \quad (2.3.13)$$

where the i, j indices correspond to the r_e, z_e location of the toroid. The radiation from the i, j toroid to the differential wall strip circumscribing 2π is found by integrating (2.3.13) from $\theta_w = 0$ to 2π . However, due to axisymmetry, the contribution of the toroid to the wall strip is constant for all θ_w . Therefore, θ_w can be fixed to an arbitrary value ($\pi/2$ for convenience), and the θ_w integration gives:

$$dq_w(i, j) = 2\kappa\sigma T_e^4 r_e dr_e dz_e \cdot \int_0^{2\pi} [\cos(\phi_1) - \cos(\phi_2)] d\theta_e \quad (2.3.14)$$

Finally, the total radiation to the wall strip from the emitting control volume is calculated

by summing (2.3.14) over all i, j toroids:

$$q_w = \sum_{j=1}^{j_{\text{cells}}} \sum_{i=1}^{i_{\text{cells}}} dq_w(i, j) \quad (2.3.15)$$

Once the radiation intercepted by dA_w is known, the mass flow rate ablated from the propellant surface can be determined:

$$\dot{m}_{\text{abl}} = \frac{\alpha q_w}{h_{fg}} \quad (2.3.16)$$

where α is the absorptivity of the solid teflon surface, and h_{fg} is the specific enthalpy change required to ablate the propellant material and convert the products to their gaseous form. Given the mass flow ablated, the velocity at which the propellant leaves the solid surface can be calculated from the following expression:

$$v_n = \frac{\dot{m}_{\text{abl}}}{\rho_{\text{abl}} dA_w} \quad (2.3.17)$$

where ρ_{abl} is the density of the propellant material in the ablated state, and dA_w is equal to $2\pi r_w dl_w$ (dl_w is the differential length of the wall strip). The velocity, v_n , is normal to the surface and is directed towards the centerline of the thruster (opposite the $\hat{n}_w$ vector in Figure 2.3.1). The corresponding velocity components are:

$$\begin{aligned} v_y &= -v_n \cos(\psi_w) \\ v_z &= v_n \sin(\psi_w) \end{aligned} \quad (2.3.18)$$

2.4 Plasma Thermodynamic Theory

The experiments that are simulated in this study contained plasma flows consisting of Teflon or a mixture of 39% Teflon from the propellant and 61% copper

from the cathode. To simulate these experiments using MACH2, tables of thermodynamic data must first be generated over desired density and temperature ranges. These tables consist of calculated pressures, molecular weights, internal energies, constant volume specific heats, and electron mole fractions at temperatures ranging from 500 to 100,000 K and densities of $1.0\text{e-}07$ to 10 kg/m^3 . Throughout this study, the plasma is assumed to be thermally perfect and in a state of local thermodynamic equilibrium.

At low temperatures, molecular as well as atomic species may be present. Therefore, the NASA Lewis Research Center's computer program CEA (Chemical Equilibrium with Applications) [20] is used to evaluate the plasma thermodynamic properties from 500 K to 6000 K. This code has been extensively used over this temperature range in a variety of problems. However, running CEA with only copper as an input and comparing the results to a previous NASA study [21] show that CEA severely overpredicts the ionization of copper. Therefore, the effects of ionization are not included in the results obtained from CEA. Also, when solid and liquid species are allowed in the computations, severe "noise" is observed in the molecular weights at low temperatures, and the values of molecular weight and internal energy are inconsistent with those expected at the high temperature limit where atomic species are dominant (due to dissociation). Hence, solid and liquid species are also removed from the CEA calculations.

For the higher temperatures ranging from about 3000 K to 100,000 K, only atomic species of C, F, and Cu and their corresponding ions are considered. The equilibrium mole fractions for the plasma constituents are obtained by iterating the Saha

equation (see section 2.4.2). These mole fractions, along with the calculated partition functions, are then used to determine the thermodynamic quantities of interest.

The energy level and degeneracy data used in the partition function computations originated from the NIST Database for Atomic Spectroscopy [22]. The carbon and fluorine levels originated from version 1.0 of Database 61 while the copper data was extracted from version 1.2 of Database 78. After these data files were compiled, the author noticed that version 2.0 of Database 78 existed at http://physics.nist.gov/cgi-bin/AtData/main_asd and contained levels for C, F, and Cu.

For each density, the CEA and Saha results are blended together using cubic polynomials over suitable temperature ranges. A different blending region is used for each density and is determined by trial and error to achieve the smoothest transition from the low temperature results to the high temperature values. From 500 K up to where the blending begins, the properties are set equal to the CEA output. Above the blending region, the Saha results are assigned to the properties.

2.4.1 NASA Equilibrium Code

As stated above, the NASA Equilibrium Code (CEA) is used in the thermodynamic computations over the range of lower temperatures due to the code's ability to calculate thermodynamic properties for molecular as well as atomic species. CEA is run up to 6000 K, which is the cut-off temperature for most of the code's tabulated data.

2.4.2 Saha Theory

The first step in predicting the thermodynamic properties of a plasma is to calculate the number densities of all the atomic species and the corresponding ions in the mixture. To facilitate the analysis, the plasma is assumed to be thermally perfect and in a state of local thermodynamic equilibrium (LTE). In an LTE plasma, energy level populations and particle velocities conform to their equilibrium (Maxwell-Boltzmann) distributions. However, radiation energy densities are not required to reach blackbody values [23]. Also, as mentioned earlier, only atomic/ionic species are considered.

Assuming LTE, Maxwell-Boltzmann statistics can be used to relate the number density of a z -times ionized atom to that of a $(z-1)$ -times ionized atom at a particular temperature and density. This results in the following expression known as the Saha equation [24]:

$$\frac{n_{a,z}}{n_{a,z-1}} = \frac{2Q_{a,z}}{n_e Q_{a,z-1}} \cdot \frac{(2\pi m_e kT)^{3/2}}{h^3} \cdot \exp\left(-\frac{E_{\infty,a,z-1} - \Delta E_{\infty,z}}{kT}\right) \quad (2.4.1)$$

where n_e is the electron number density, and $n_{a,z}$ represents the number density of the z -times ionized atomic species, denoted by “a”. $E_{\infty,a,z-1}$ is the ionization energy of the corresponding $(z-1)$ ion relative to its ground state. $Q_{a,z}$ is the internal, electronic partition function (only atomic species are considered) and is given by:

$$Q_{a,z} = \sum_j g_j \exp\left(-\frac{E_j}{kT}\right) \quad (2.4.2)$$

E_j is the energy level of an excited state j relative to the ground state for the ion, and g_j is the corresponding degeneracy of that level. The summation is carried out only up to the

lowered ionization potential (see below). Electric microfields in the plasma broaden the excited energy levels resulting in overlapping of the upper states. Therefore, the effective ionization energy is lowered to remove the upper level degeneracy. The Debye-Huckel [25] theory gives a correction of :

$$\Delta E_{\infty z} = \frac{z'e^2}{4\pi\epsilon_0\rho_D} \quad (2.4.3)$$

where z' is equal to 0 for nonionized atoms, 1 for single-charged ions, 2 for double-charged ions, etc., and ρ_D represents the Debye length [26]:

$$\rho_D = \sqrt{\frac{\epsilon_0 kT}{e^2 \left(n_e + \sum_a \sum_z z'^2 n_{a,z} \right)}} \quad (2.4.4)$$

Along with (2.4.1), the number density calculations must satisfy the conservation of mass:

$$n_a = \sum_z n_{a,z} \quad (2.4.5)$$

which states that the total number density for chemical species a must be equal to the sum of the ion number densities (including the nonionized state) for that species.

Equations (2.4.1) and (2.4.5) represent a linear system that can be solved for the unknown number densities, $n_{a,z}$. However, the system is largely dependent on the electron number density, not only in the Saha equation but in the ionization potential lowering as well (see equations. (2.4.3) and (2.4.4)). Therefore, one must guess a value for n_e , calculate the number densities for all species, and then check if n_e satisfies the conservation of charge according to:

$$n_e = \sum_a \sum_z z' \cdot n_{a,z} \quad (2.4.6)$$

The electron number density must be iterated until the guessed and calculated values coincide. The secant method [27] is an excellent iterative technique that converges for a wide range of guessed values of n_e . For this study, the criterion of convergence (fractional difference between guessed and calculated values) is set at $1.0e-10$.

Note that to solve the above problem, the total number densities for each species, n_a , in equation (2.4.5) must be known and are required to be consistent with the guessed electron number density and provided plasma mass density. This can be accomplished by manipulating the following expression for density:

$$\rho = \frac{n \cdot MW}{N_{av}} \quad (2.4.7)$$

where n is the total number density and N_{av} represents Avogadro's number in particles/kmol. The average molecular weight (kg/kmol) is given by:

$$MW = y_e MW_e + \sum_a y_a MW_a \quad (2.4.8)$$

where:

$$MW_e = N_{av} \cdot m_e \quad (2.4.9)$$

and

$$y_e = \frac{n_e}{n}, \quad y_a = \frac{n_a}{n} \quad (2.4.10)$$

Given initial mole fractions for the species of interest, $y_a^0 = \frac{n_a^0}{n^0}$, the relative amount of a particular species, a , to any of the other species (e.g. species 1) is known:

$$\frac{n_a}{n_1} = \frac{n_a^0}{n_1^0} = \frac{y_a^0}{y_1^0} \quad (2.4.11)$$

Here, the assumption is made that the relative amounts of the atomic species remain constant. Substituting (2.4.8) – (2.4.11) into (2.4.7) results in the following relation for chemical species 1:

$$n_1 = \frac{N_{av}(\rho - n_e m_e)}{MW_1 + \sum_{a=2}^{\text{\# of species}} \left(\frac{y_a^0}{y_1^0} \right) MW_a} \quad (2.4.12)$$

Note that the denominator in (2.4.12) is constant. Therefore, for a fixed density, n_1 depends only on n_e during the iteration. Once n_1 is determined, the other total species number densities are calculated from (2.4.11).

With the Saha iteration complete, the thermodynamic properties can now be determined from the calculated number densities of the plasma constituents. Since the plasma is assumed to be thermally perfect, the pressure is given by:

$$p = nkT \quad (2.4.13)$$

where the total number density is equal to:

$$n = n_e + \sum_a \sum_z n_{a,z} \quad (2.4.14)$$

The average molecular weight is calculated from (2.4.7).

The partition function, $Q_{a,z}$, is a critical parameter in the calculation of the thermodynamic properties of an LTE plasma. Maxwell-Boltzmann statistics predict that the molar internal energy of a plasma component is related to its internal partition function by the following equation [28]:

$$\hat{u}_{a,z} = \frac{3}{2} \hat{R}T + \frac{\hat{R}T^2}{Q_{a,z}} \left(\frac{\partial Q_{a,z}}{\partial T} \right)_p + \sum_{k=1}^{z-1} E_{\infty a,k} \quad (2.4.15)$$

where $\hat{R}$ is the universal gas constant. The first term in (2.4.15) represents the translational energy while the second corresponds to the energy of the internal electronic modes. The last term in (2.4.15) must be included because E_j is the energy relative to the ground state of ion z , not to the ground state of the un-ionized atom. Since the Saha iteration scheme is performed at constant density over the temperature range, an analytical expression for the derivative of the partition function with respect to temperature may be obtained easily (Sontagg, pp. 174-175):

$$\hat{u}_{a,z} = \hat{R}T \left(\frac{3}{2} + \frac{Q'_e}{Q_{a,z}} \right) + \sum_{k=1}^{z-1} E_{\infty a,k} \quad (2.4.16)$$

where, for convenience, Q'_e is defined as:

$$Q'_e = T \cdot \left(\frac{\partial Q_{a,z}}{\partial T} \right)_p = \sum_j g_j \left(\frac{E_j}{kT} \right) \exp \left(-\frac{E_j}{kT} \right) \quad (2.4.17)$$

Once the $\hat{u}_{a,z}$'s are known for all the plasma constituents, the internal energy of the mixture is simply the sum of the products of the component energies and the respective mole fractions:

$$\hat{u} = \sum_a \sum_z y_{a,z} \cdot \hat{u}_{a,z} \quad (2.4.18)$$

Dividing by the average plasma molecular weight, calculated from (2.4.7), gives the specific internal energy, u , in units of J/kg.

The constant volume specific heat is another useful thermodynamic property and is calculated by taking the partial derivative of u with respect to temperature at constant

density:

$$C_v = \left(\frac{\partial u}{\partial T} \right)_\rho \quad (2.4.19)$$

Again, since the Saha iteration scheme is performed at constant density over the temperature range, the corresponding u 's can be numerically differentiated without the use of any Maxwell relations. The derivatives are evaluated in the following manner:

$$\begin{aligned} \text{For } T = T_{\min} & \Rightarrow C_v(T) = \frac{u(T + dT) - u(T)}{dT} \\ \text{For } T = T_{\max} & \Rightarrow -C_v(T) = \frac{u(T) - u(T - dT)}{dT} \\ \text{For } T_{\min} < T < T_{\max} & \Rightarrow C_v(T) = \frac{u(T + dT) - u(T - dT)}{2dT} \end{aligned} \quad (2.4.20)$$

Note, a numerical derivative is required here since the derivative of (2.4.18) contains terms involving the mole fractions as well as the partition functions. Therefore, unlike the procedure used to obtain (2.4.16), this cannot be done in analytic form.

2.4.3 Blending Method

The CEA and Saha generated results are blended together using cubic polynomials over a suitable temperature blending range. At each density, a blending region is chosen by trial and error to produce a smooth transition from the low temperature data to the high temperature values. The polynomial coefficients are calculated to match the endpoint values and slopes for each thermodynamic property. A different blending region is used for the electron mole fraction blending. In this study, y_e is blended from zero (and zero slope) at a chosen low temperature to the initial value in

the Saha data. The lower temperature is selected such that no overshooting occurs in the blending near the Saha value.

2.5 Numerical Scheme: MACH2

The governing equations described above are integrated using MACH2. MACH2 is a multidimensional, Cartesian or axisymmetric, finite-volume, time-dependent, general purpose MHD solver. All important aspects of this code, including theory and operation, are described in detail in references [7] and [17].

When advancing the time computations, MACH2 splits the time step into substeps corresponding to important physical processes. Also, some of these properties are differenced implicitly with respect to time while the others are explicitly differenced. Thermal and equilibrium radiation diffusion, along with resistive diffusion and Lagrangian hydrodynamics, are implicitly differenced with respect to time. The Hall effect and convective transport processes are solved with explicit differencing. This time-splitting procedure relaxes the small time step requirement that is prominent in fully-explicit solvers.

Boundary conditions in MACH2 are applied explicitly to ghost cells outside the computational domain of interest. This allows the solver to integrate the same governing equations up to and including the system boundaries. Without the ghost cell technique, special relations and iterative techniques would be required along the edges of the computational domain.

The computational region generated by MACH2 is decomposed into a set of logically rectangular blocks (arrays) that are patched together to form a problem

domain that is discretized into hexahedral cells that form the computational grid. This grid is of the Arbitrary Lagrangian Eulerian (ALE) variety. A purely Eulerian grid is at rest in the laboratory frame, while a Lagrangian grid moves with the plasma flow. The Arbitrary grid moves with respect to the laboratory and fluid frames. This adaptive mesh generation optimizes smoothness, orthogonality, and the deviation from desired mesh concentrations.

Another important aspect of the MACH2 code is the use of Cray FORTRAN-77 pointers. This allows the dynamic allocation of memory at run time, which lowers the amount of computer system memory required to run a simulation. Also, sources of magnetic flux for the MHD simulations can be included in the form of currents produced by applied external circuits.

3. PLASMA THERMODYNAMIC RESULTS

3.1 CEA Results

The CEA code described in sections 2.4 and 2.4.1 is used to calculate plasma thermodynamic properties at low temperatures. Two plasma mixtures are considered in this study: one with only Teflon, and the other with a mixture of 39% Teflon and 61% copper (by mass). In the CEA computations, the temperature is varied from 500 to 6000 K at increments of 10 K. The resulting pressure, molecular weight, internal energy, and constant volume specific heat variations with temperature for both cases are plotted in Figures 3.1.1 – 3.1.4 for density of $1.0\text{e-}07 \text{ kg/m}^3$. Distributions for 10 kg/m^3 are given in Figures 3.1.5 – 3.1.8. Only the results for the two densities are displayed, since this adequately illustrates the overall characteristics of the thermodynamic calculations. The step-like patterns in the pressure, molecular weight, and internal energy curves at the lower density show the stages of molecular dissociation that occur as the temperature is increased. These characteristics are less evident at the higher density due to the requirement of higher temperatures to achieve the same level of dissociation. The lower dissociation explains the higher molecular weight at a given temperature. Also, the pressure and internal energy are lower due to fewer energetic atomic species in the plasma. The addition of copper to the plasma produces a higher molecular weight and, therefore, a lower pressure (at a given density and temperature) than the plasma consisting of only Teflon.

One important calculation that deserves special mention is a correction to the internal energies computed by CEA. The energies in the NASA equilibrium code are

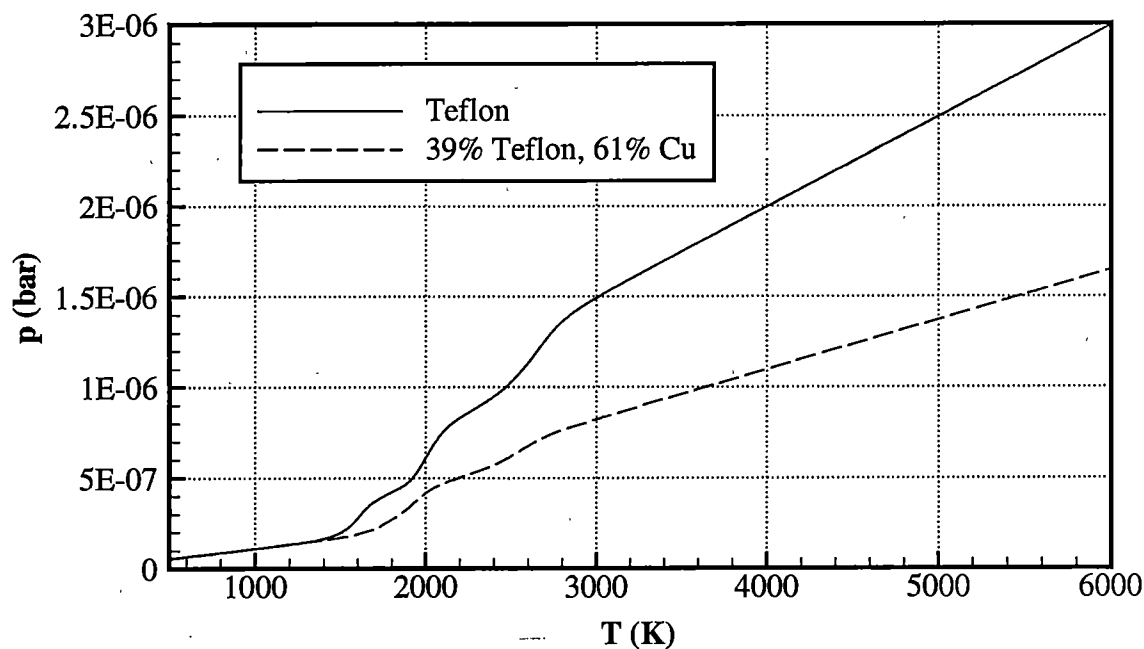


Figure 3.1.1: CEA Pressure Distribution for $\rho = 1.0e-07 \text{ kg/m}^3$

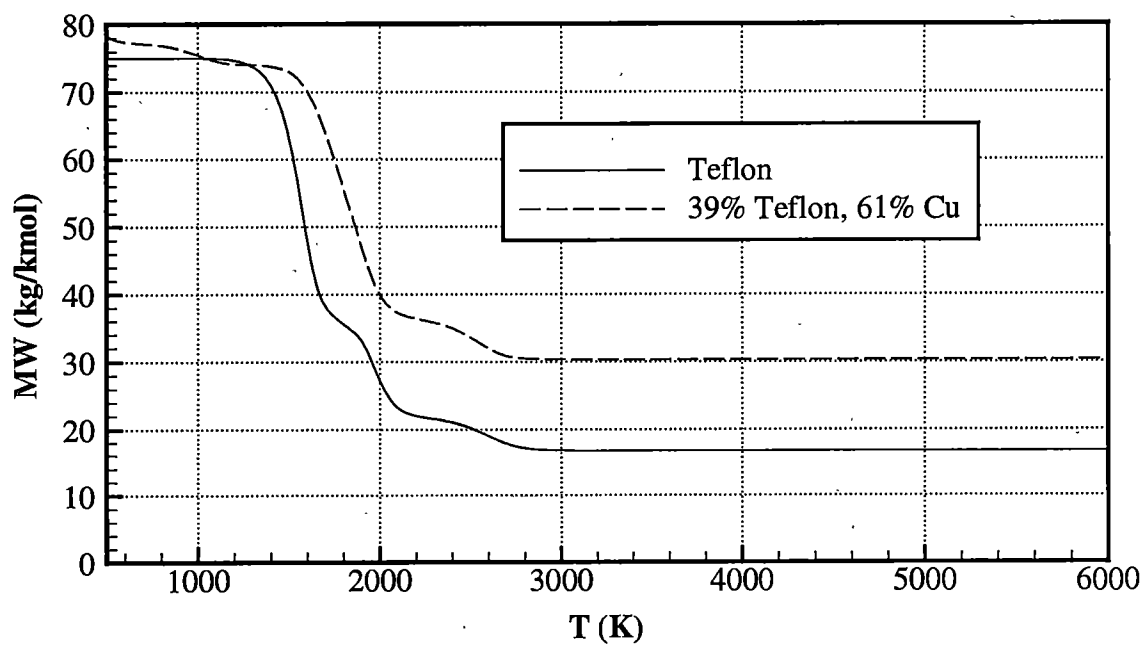


Figure 3.1.2: CEA MW Distribution for $\rho = 1.0e-07 \text{ kg/m}^3$

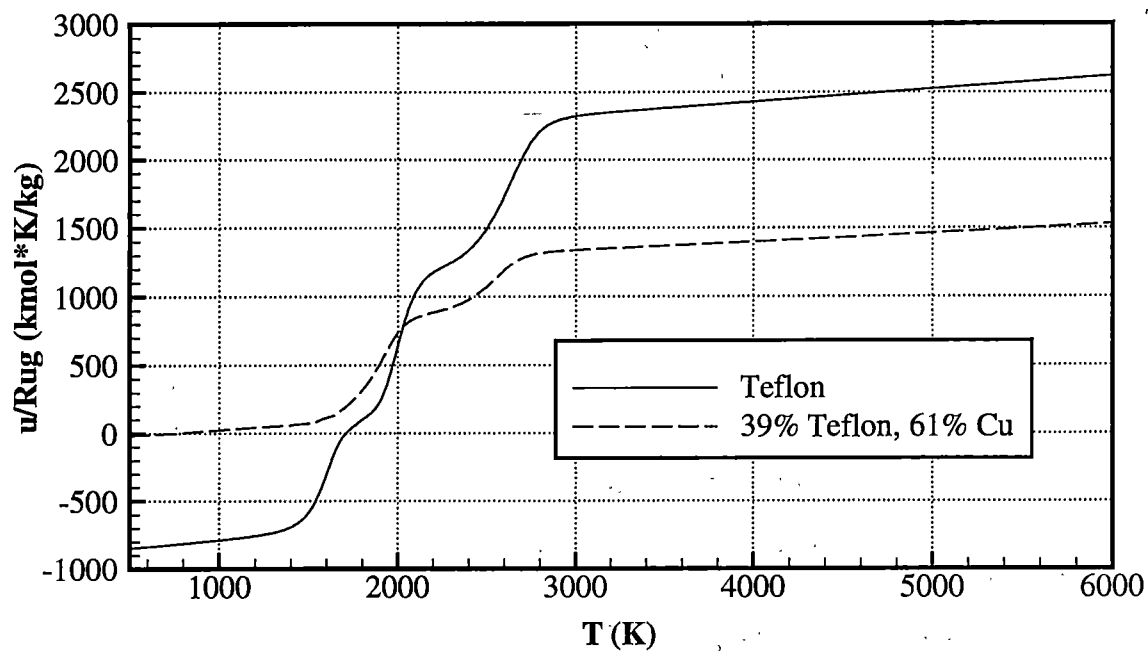


Figure 3.1.3: CEA u/Rug Distribution for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$

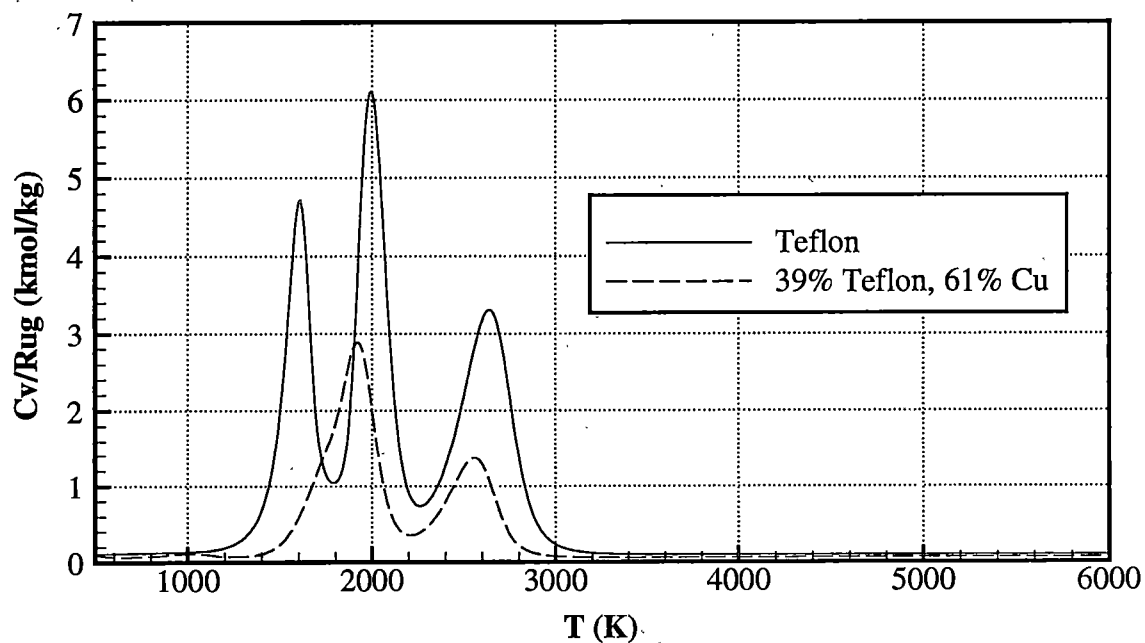


Figure 3.1.4: CEA Cv/Rug Distribution for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$

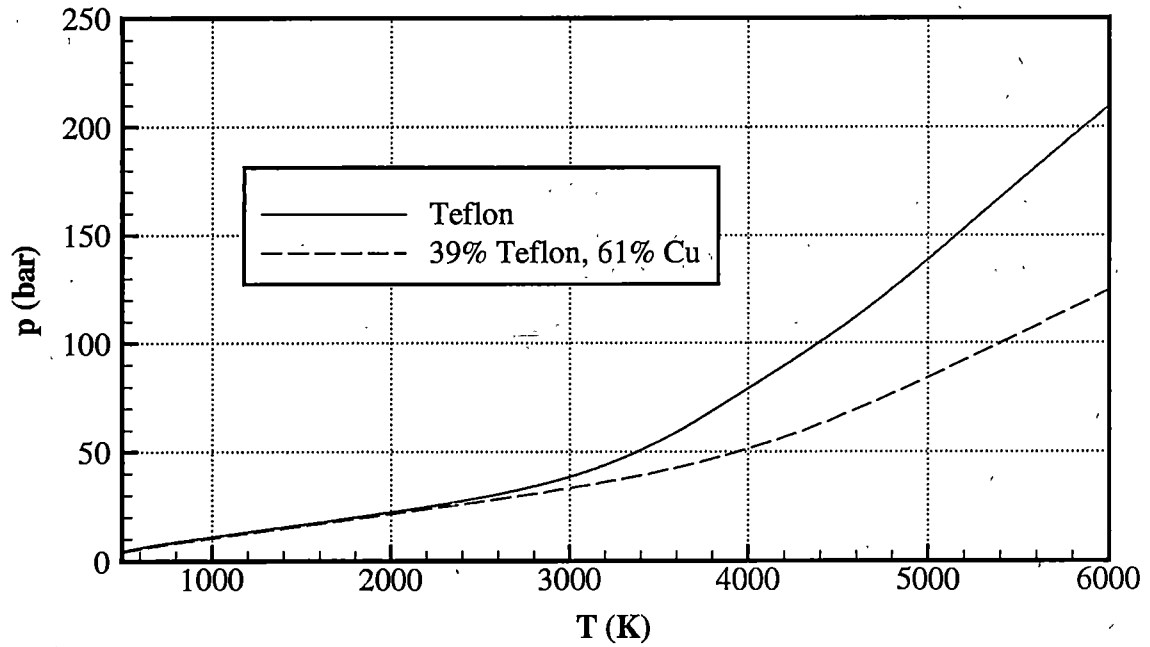


Figure 3.1.5: CEA Pressure Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

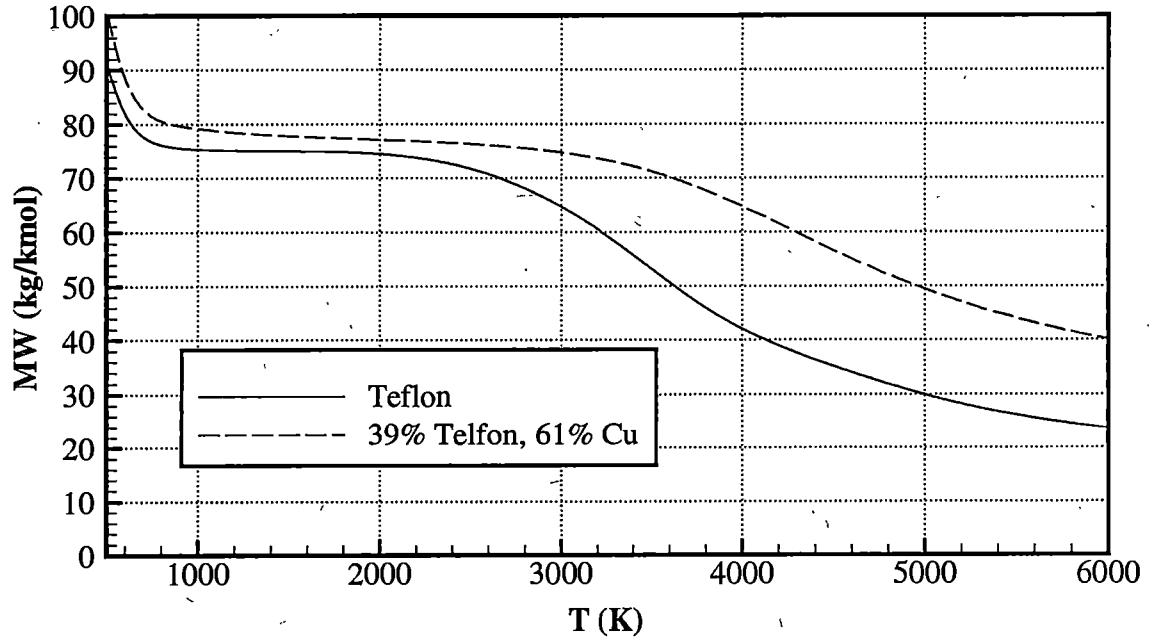


Figure 3.1.6: CEA MW Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

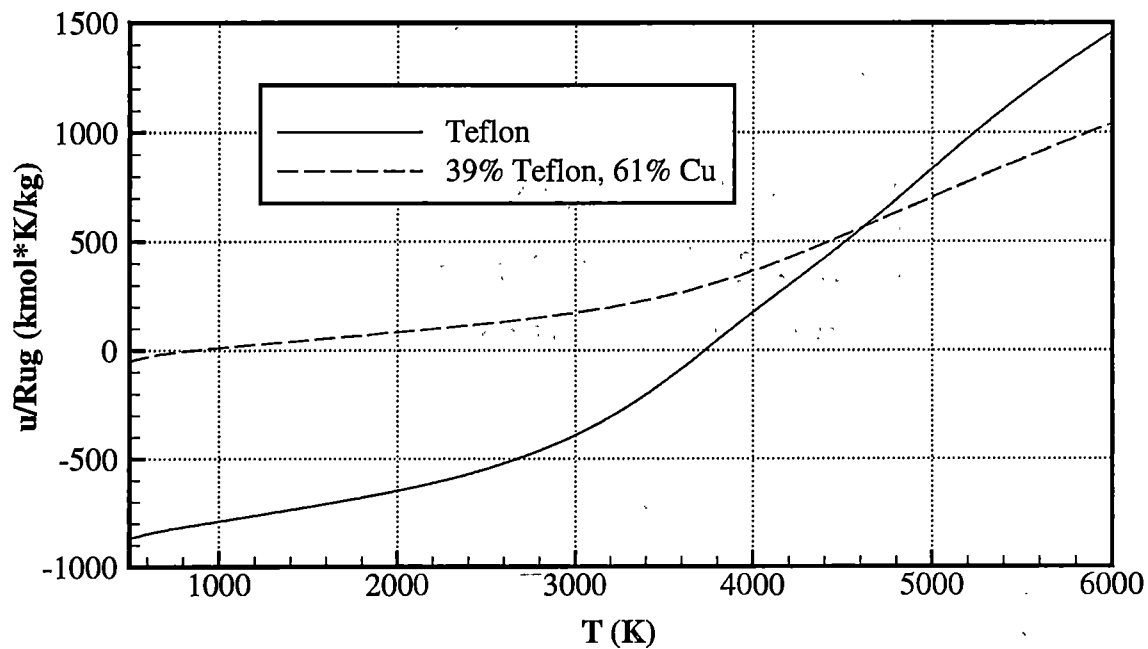


Figure 3.1.7: CEA u/Rug Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

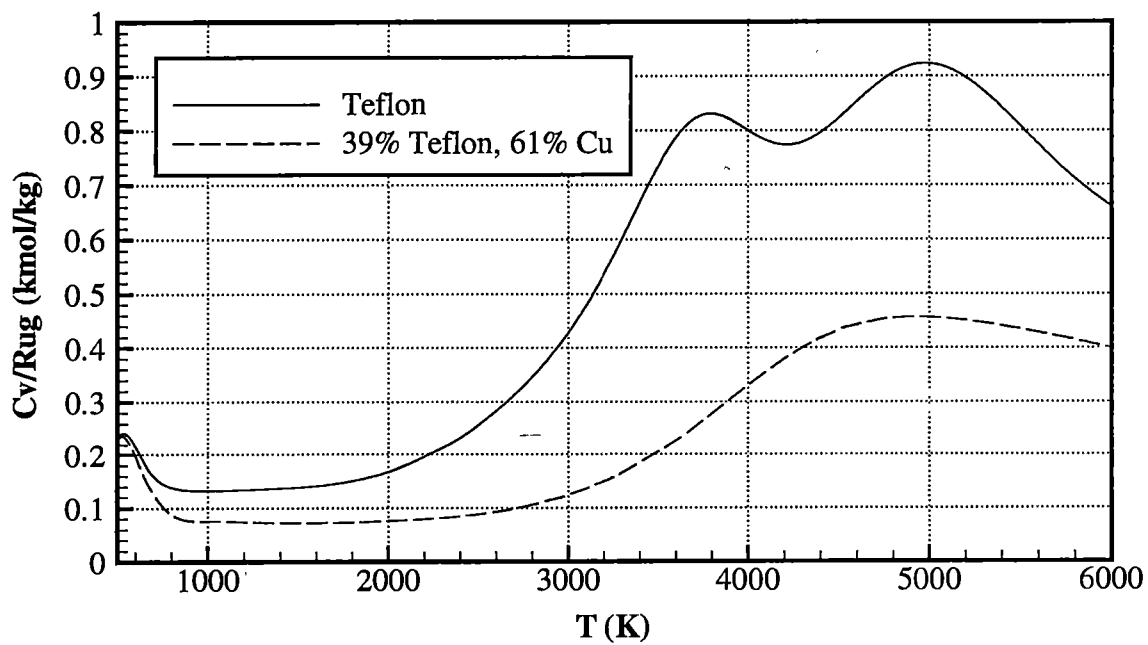


Figure 3.1.8: CEA Cv/Rug Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

referenced to a standard state at $p = 1$ atm and $T = 298.15$ K. However, the Saha iterated energies are referenced to zero at $T = 0$ K. Therefore, to maintain a consistent basis, the amount of energy required to dissociate the Teflon at the standard state and cool the products (C, F, and Cu) to 0 K is added to the CEA internal energies. The corresponding energy correction, divided by the universal gas constant, is 2058 kmol·K/kg for the Teflon plasma and 1185 kmol·K/kg for the Teflon-copper mixture.

3.2 Saha Results

For the higher temperatures considered in this study, the plasma properties are calculated using the method outlined in section 2.4.2. This procedure requires the initial mole fractions of the plasma constituents. A mass ratio of 39% Teflon and 61% copper translates into mole fractions of 0.2364 carbon, 0.4727 fluorine, and 0.2909 copper, which are input into the Saha iteration. For the Teflon only case, the ratio of fluorine to carbon atoms is 2 to 1, resulting in mole fractions of 0.3333 for carbon and 0.6667 for fluorine. Only atomic species and their first 4 ions are considered (C through C++++), and spectroscopic notation is adopted such that C1 corresponds to C, C2 represents C+, etc. The Saha generated thermodynamic results are evaluated from 3000 to 100,000 K at increments of 10 K.

The mole fraction variations with temperature for the considered densities are given in Figures 3.2.1 – 3.2.12, while the thermodynamic properties are plotted in Figures 3.2.13 – 3.2.20. The major differences in the results for the two densities are similar to those described above for the CEA output. However, at the low density, the plasma reaches the maximum allowed ionization before 60,000 K. At the higher density, the

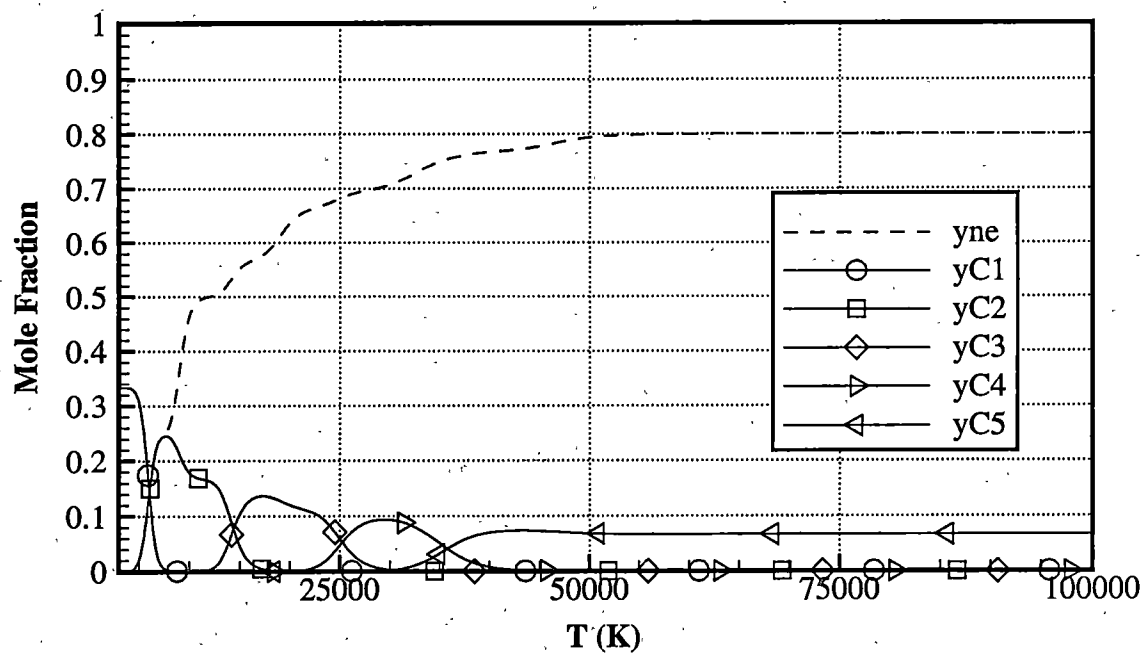


Figure 3.2.1: Mole Fractions of C for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$ (Teflon)

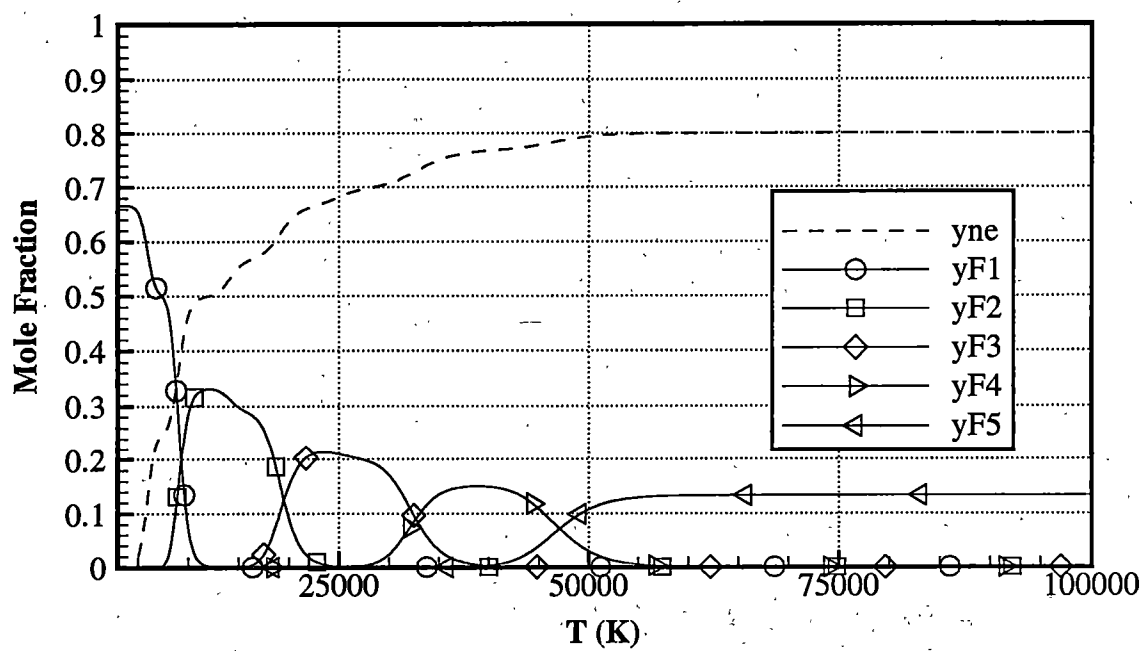


Figure 3.2.2: Mole Fractions of F for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$ (Teflon)

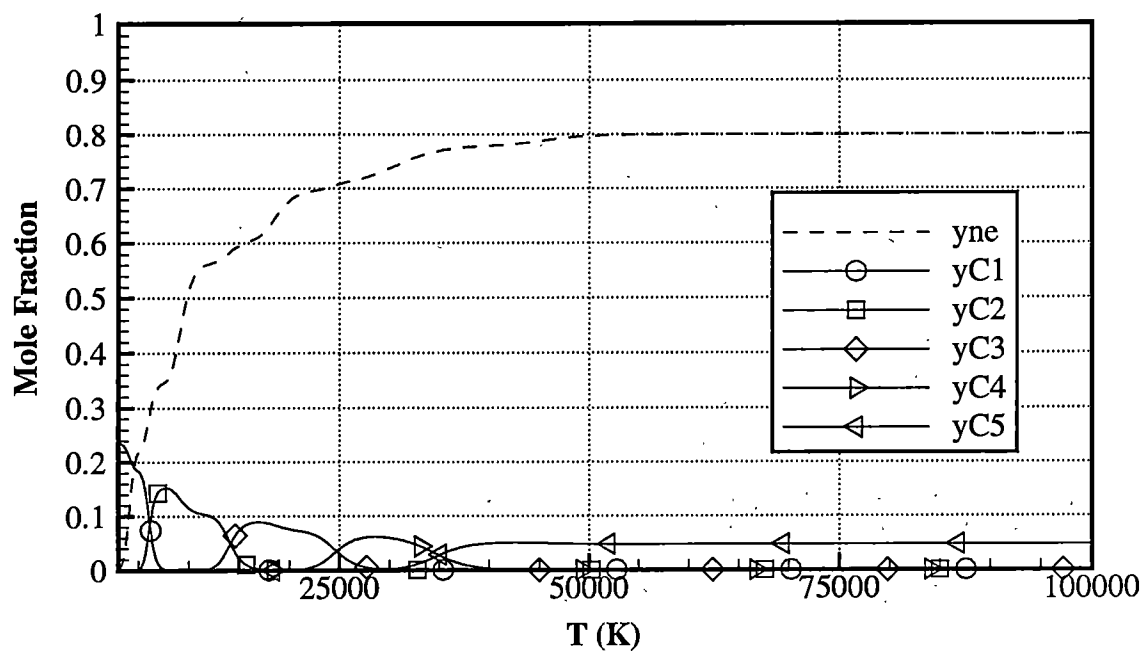


Figure 3.2.3: Mole Fractions of C for $\rho = 1.0 \times 10^{-7} \text{ kg/m}^3$ (39% Teflon, 61% Cu)

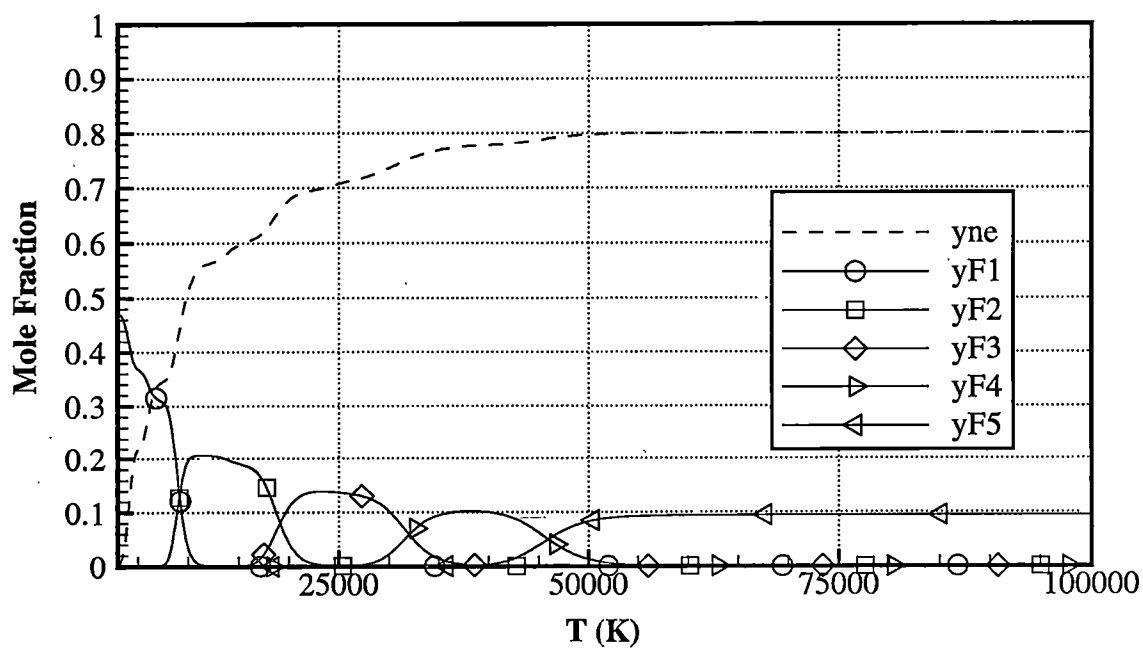


Figure 3.2.4: Mole Fractions of F for $\rho = 1.0 \times 10^{-7} \text{ kg/m}^3$ (39% Teflon, 61% Cu)

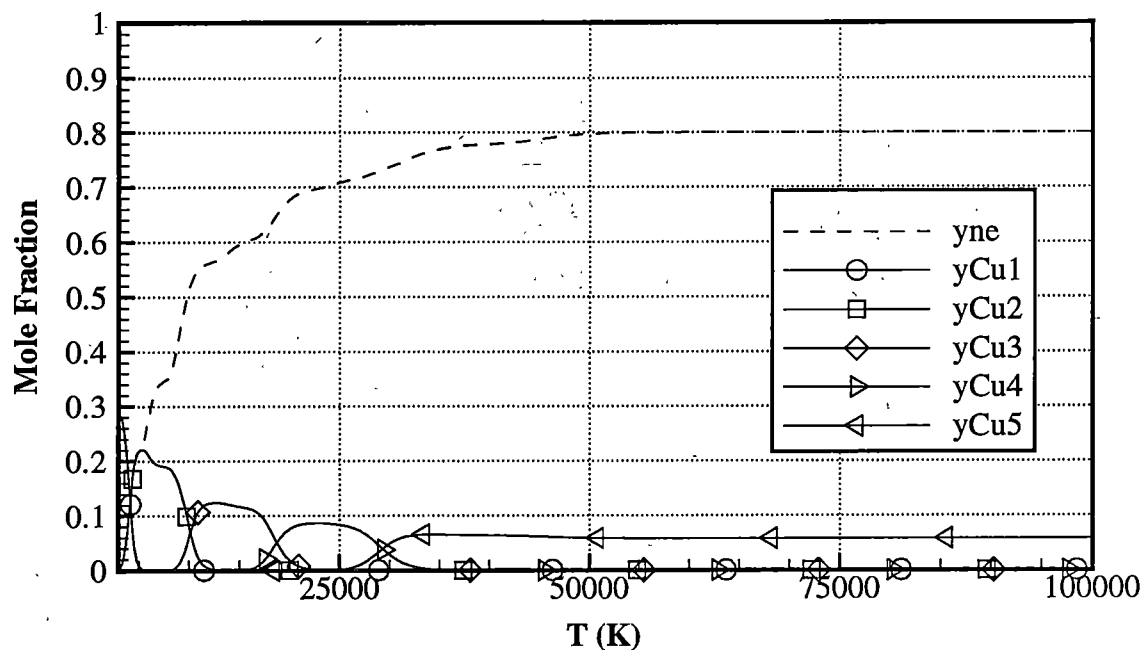


Figure 3.2.5: Mole Fractions of Cu for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$ (39% Teflon, 61% Cu)

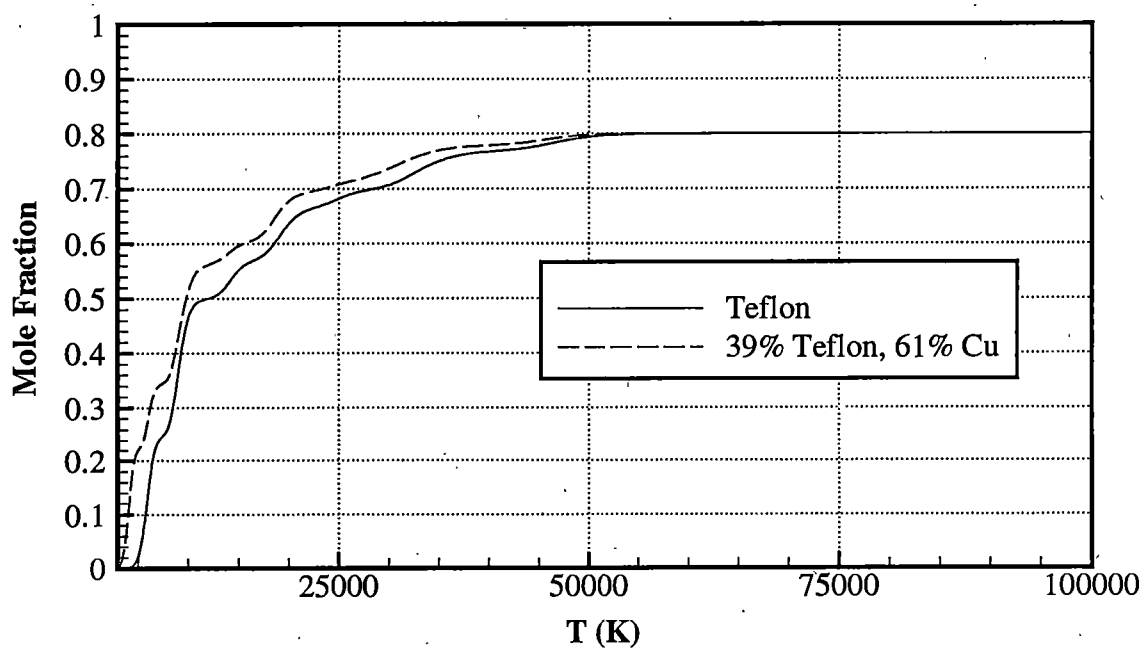


Figure 3.2.6: Electron Mole Fractions for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$

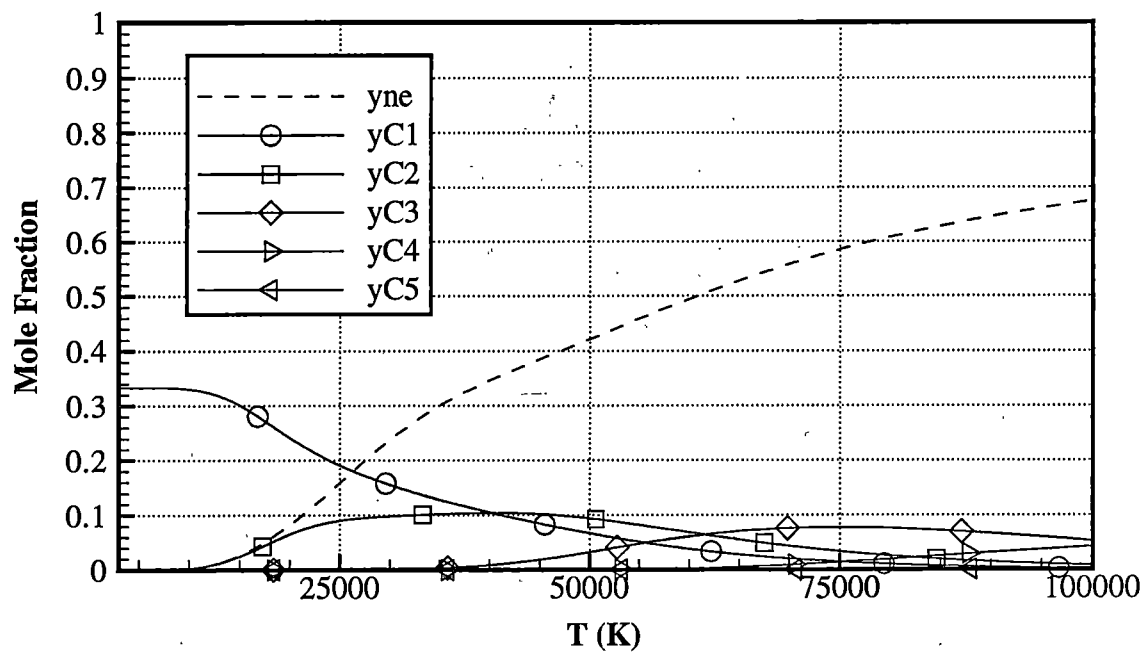


Figure 3.2.7: Mole Fractions of C for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$ (Teflon)

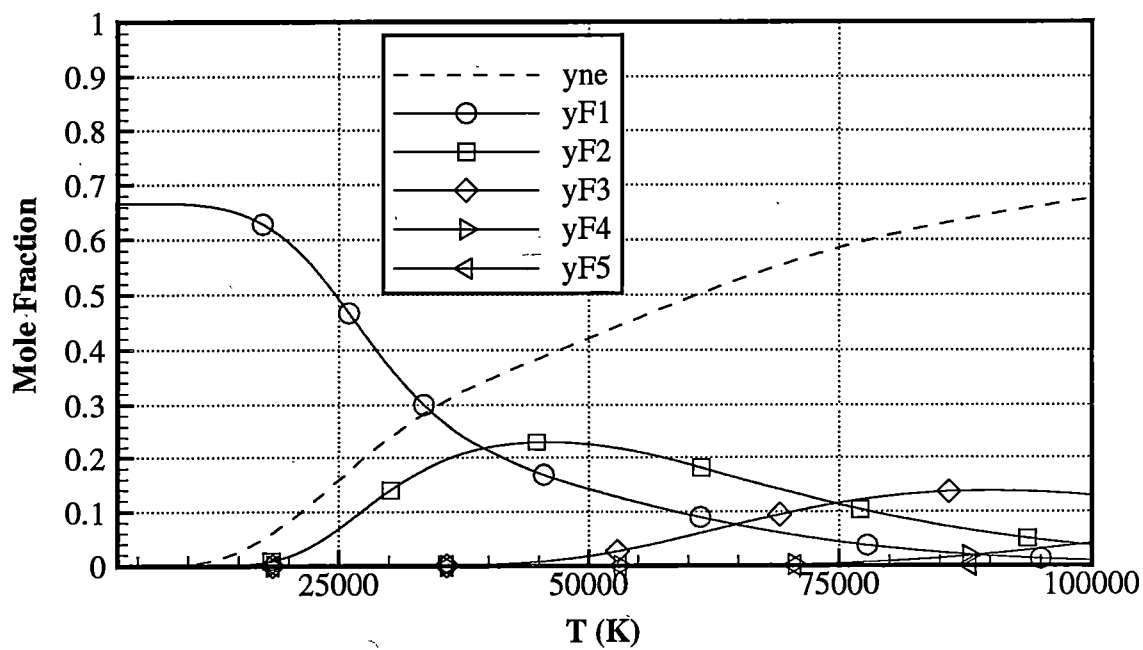


Figure 3.2.8: Mole Fractions of F for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$ (Teflon)

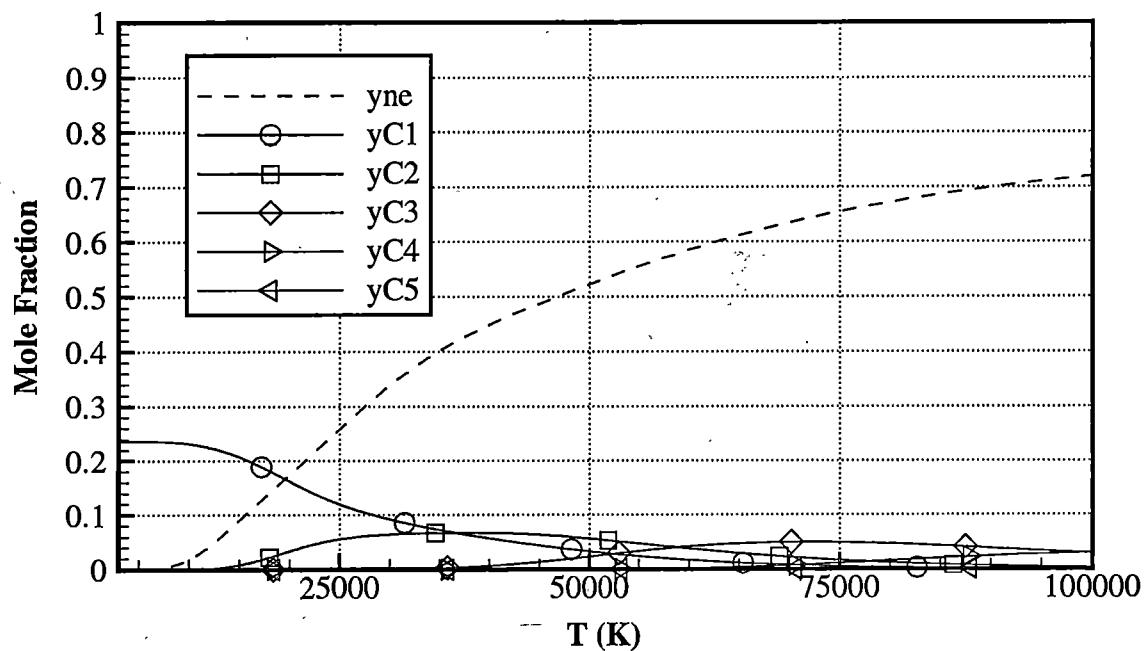


Figure 3.2.9: Mole Fractions of C for $\rho = 1.0 \times 10^1 \text{ kg/m}^3$ (39% Teflon, 61% Cu)

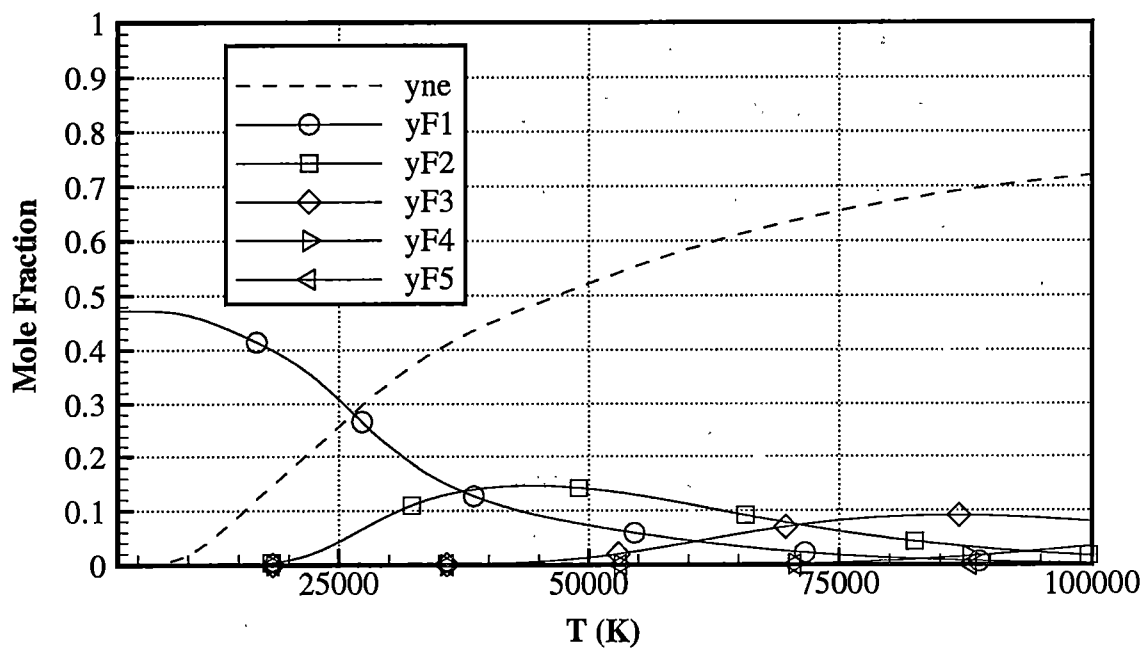


Figure 3.2.10: Mole Fractions of F for $\rho = 1.0 \times 10^1 \text{ kg/m}^3$ (39% Teflon, 61% Cu)

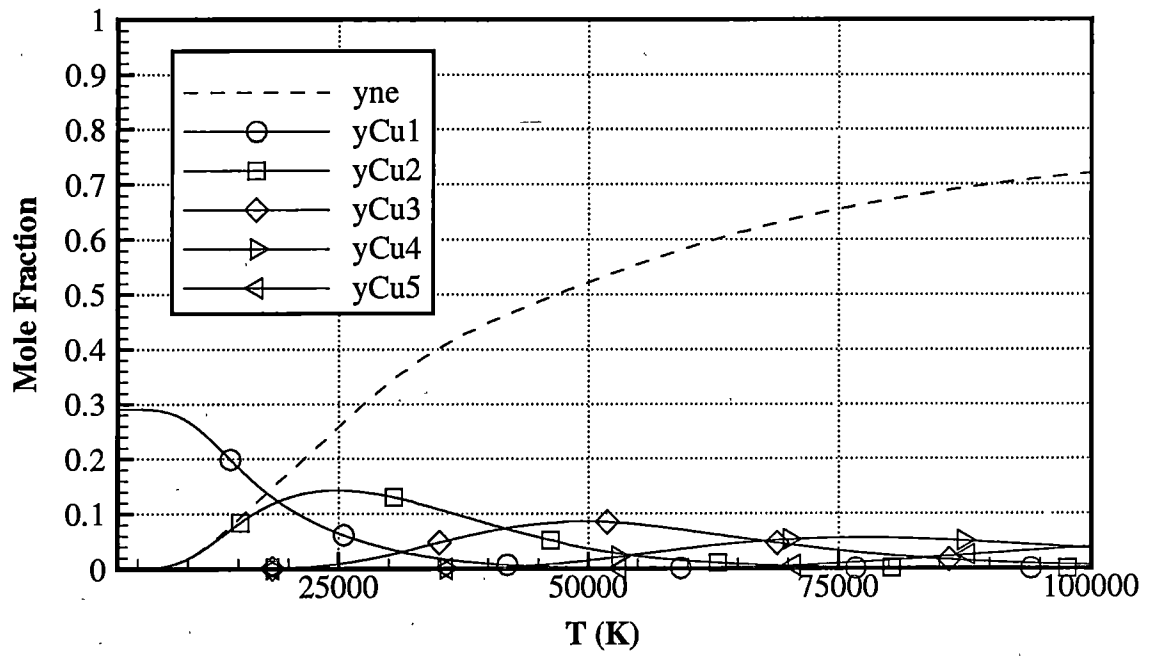


Figure 3.2.11: Mole Fractions of Cu for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$ (39% Teflon, 61% Cu)

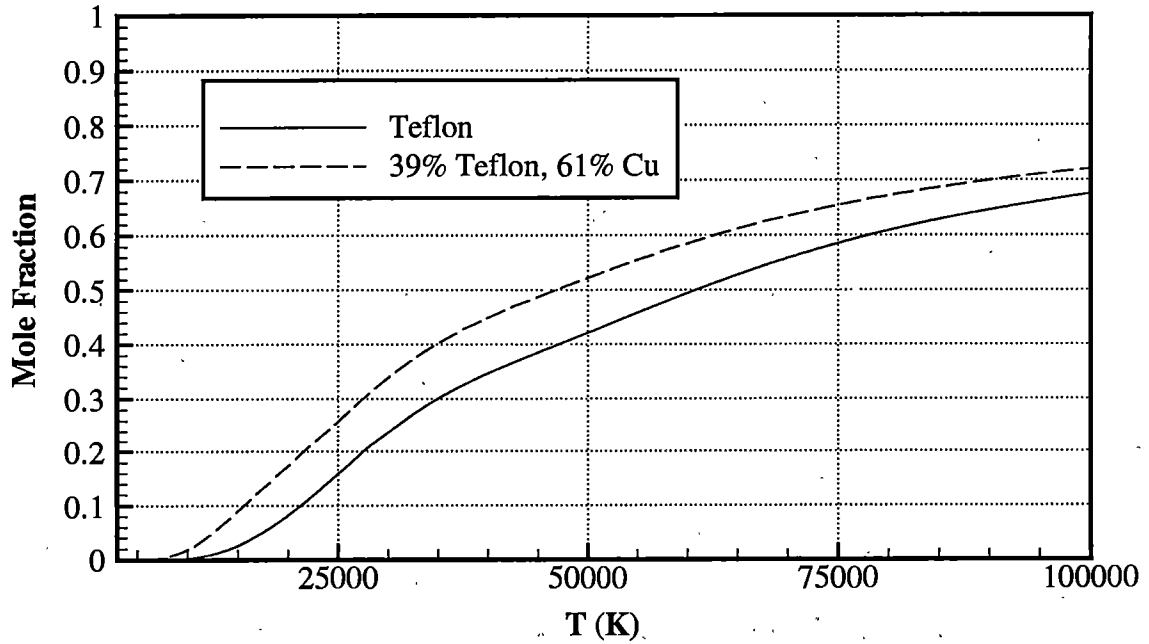


Figure 3.2.12: Electron Mole Fractions for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

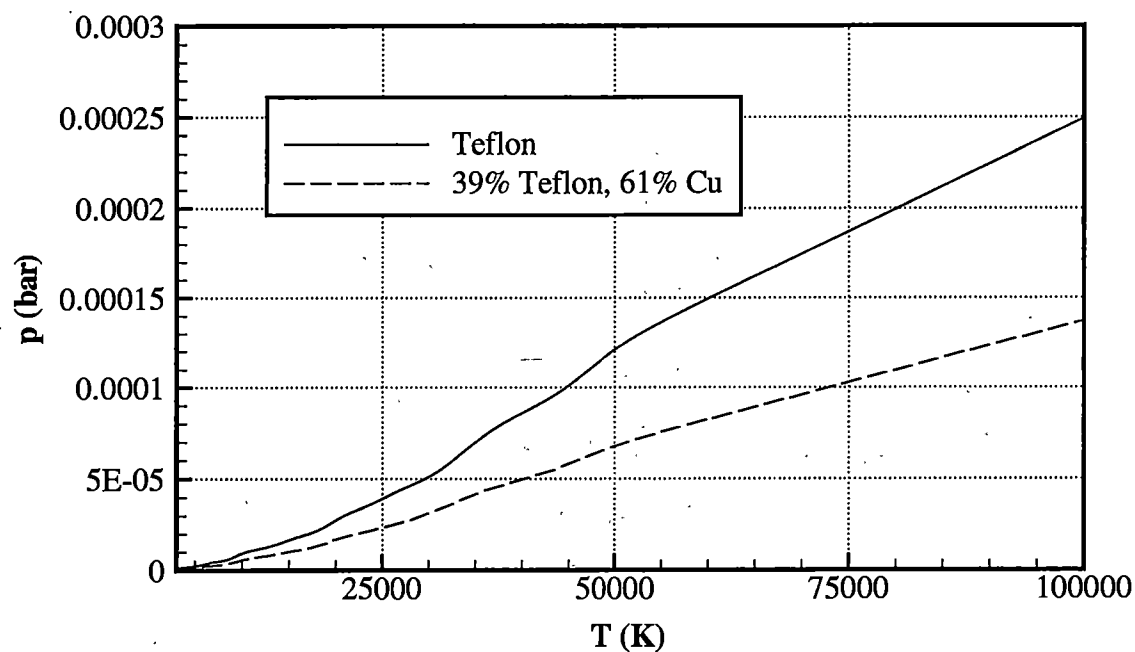


Figure 3.2.13: Saha Pressure Distribution for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$

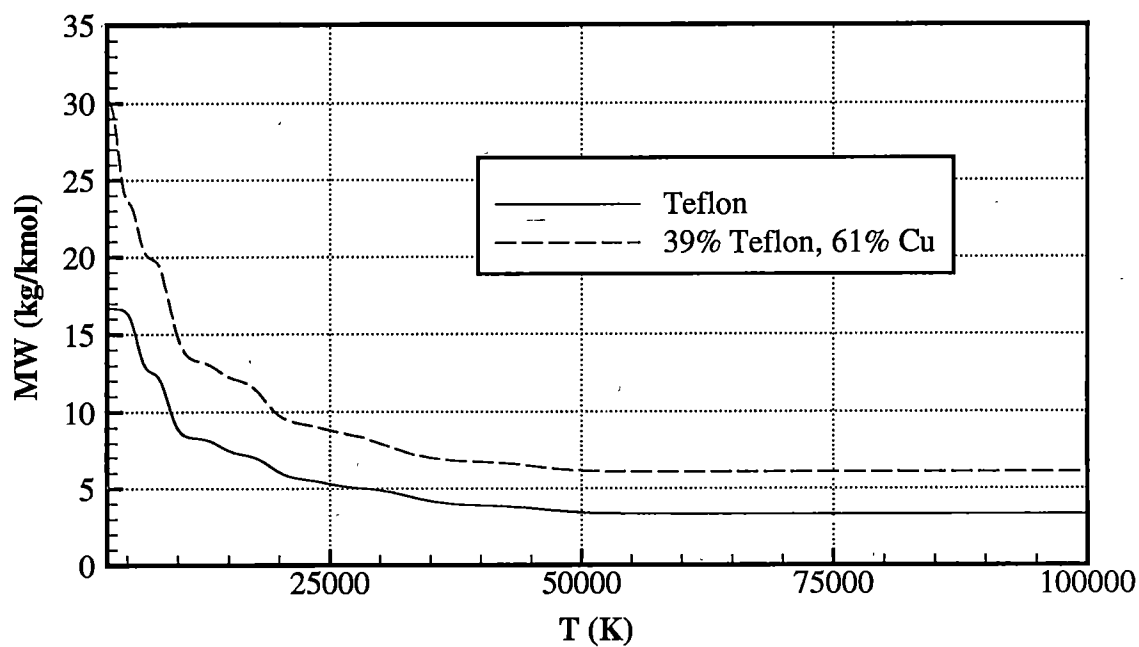


Figure 3.2.14: Saha MW Distribution for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$

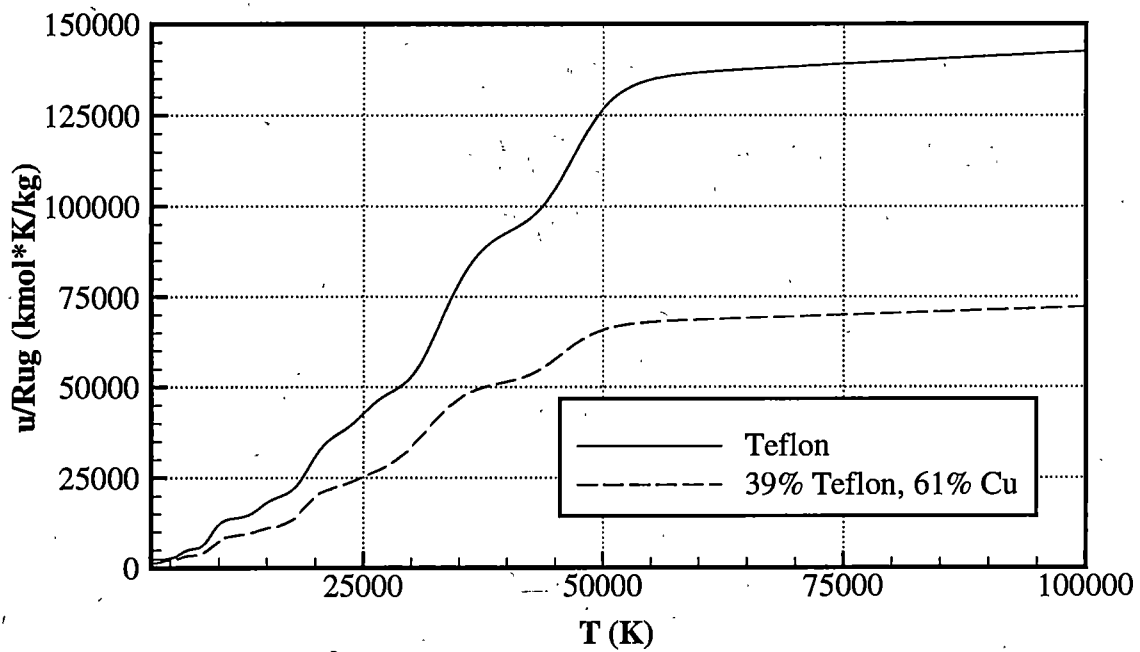


Figure 3.2.15: Saha u/Rug Distribution for $\rho = 1.0e-07 \text{ kg/m}^3$

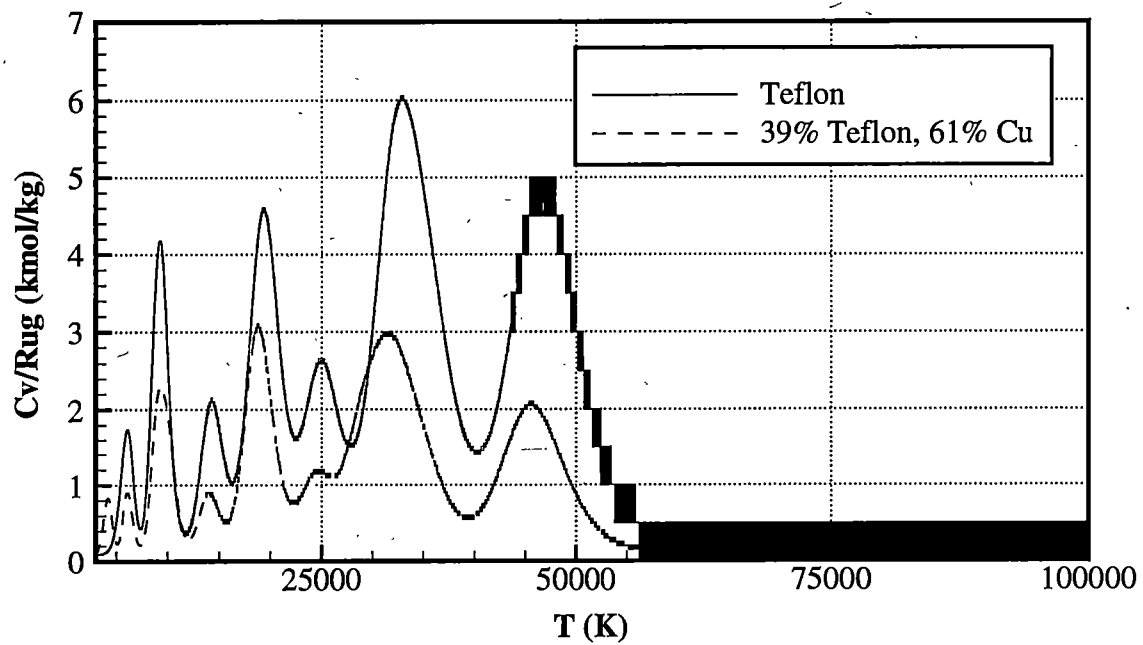


Figure 3.2.16: Saha Cv/Rug Distribution for $\rho = 1.0e-07 \text{ kg/m}^3$

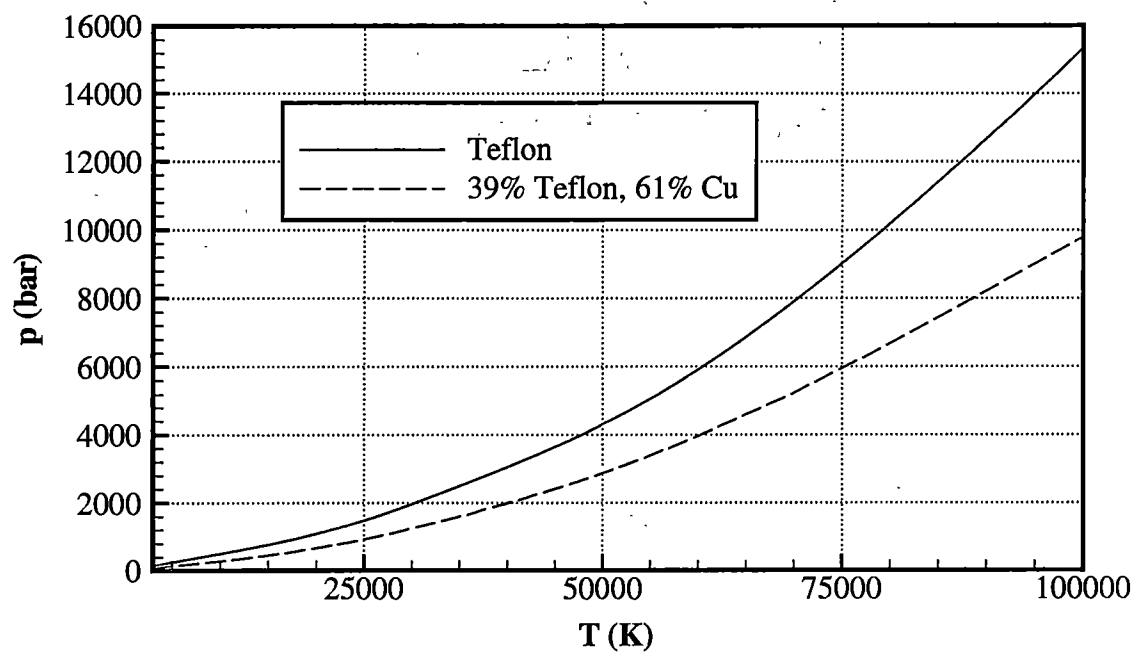


Figure 3.2.17: Saha Pressure Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

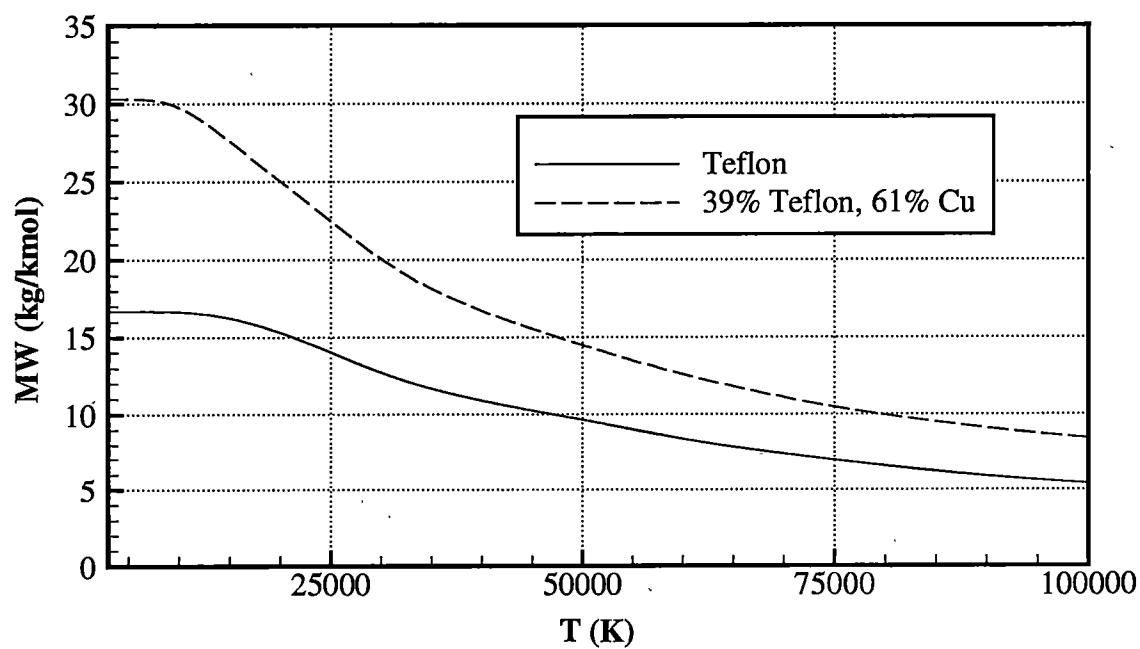


Figure 3.2.18: Saha MW Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

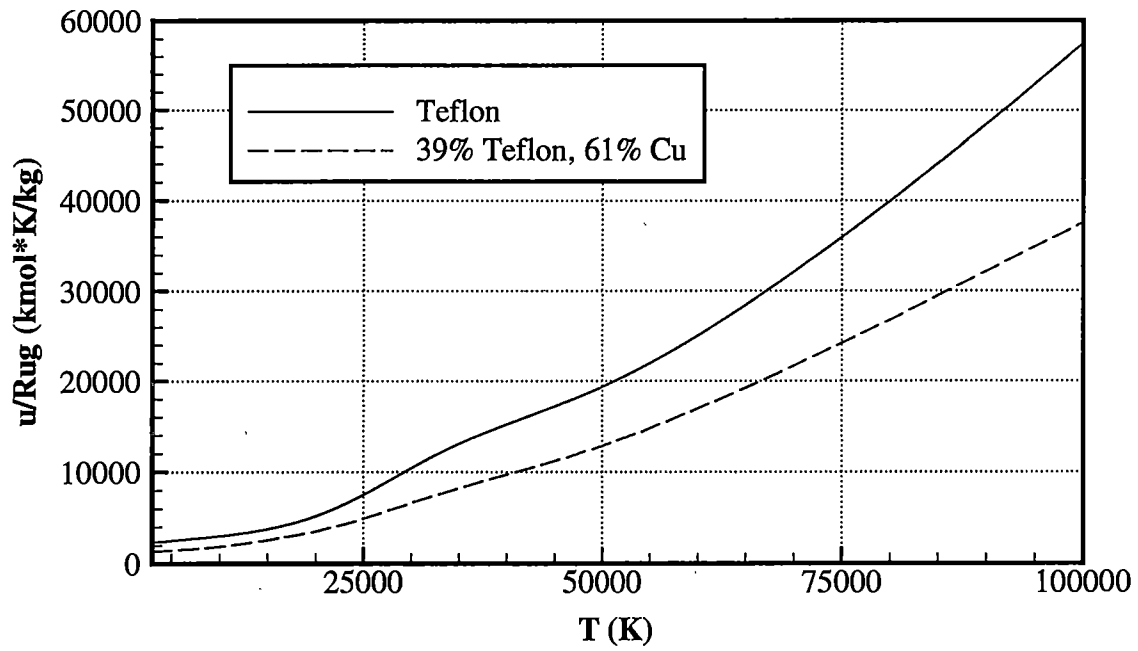


Figure 3.2.19: Saha u/Rug Distribution for $\rho = 1.0e+01 \text{ kg/m}^3$

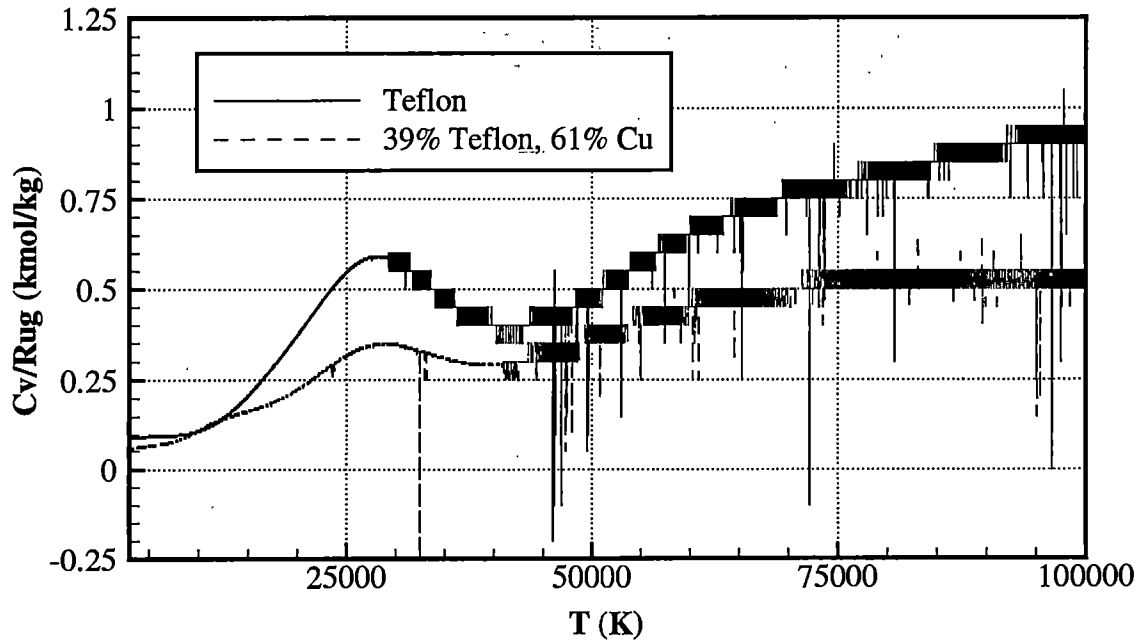


Figure 3.2.20: Saha Cv/Rug Distribution for $\rho = 1.0e+01 \text{ kg/m}^3$

plasma never reaches the fully ionized state, resulting in higher molecular weights and lower pressures and internal energies due to fewer highly energetic electrons (and ions) in the plasma. Figures 3.2.6 and 3.2.12 show the effect of adding copper into the plasma. Due to the lower ionization potential of copper, more electrons are liberated in the Teflon-copper mixture at a given density and temperature until the ionization limit is reached.

Note the severe noise in the C_v calculations, especially at the higher density. The partition function variations with temperature contain discrete jumps caused by the lowering of the ionization potential. This directly translates into jumps in the internal energies (see equation 2.4.16) that, in turn, create spikes in the numerical derivatives in (2.4.20). These spikes represent unreasonable changes in internal energy for small temperature differences and can create numerical difficulties while running MACH2. Therefore, the C_v data must be filtered to provide smooth interpolations between temperatures (or internal energies).

The best solution found to reduce the C_v noise is the use of a (1,1)-fold trimmed mean filter [29]. First, the internal energy data is run through the filter. Then, C_v is calculated according to (2.4.20). Finally, the resulting C_v data is also run through the filter. The smoothed C_v curves are shown in Figures 3.2.21 and 3.2.22. For the low density Teflon plasma, the internal energy data is run through an 81-point filter, and then the corresponding C_v data is run twice through the same filter. The Teflon-copper plasma data is processed in a similar manner, except that a 21-point filter is used. For the high density plasmas (both cases), the internal energy data is passed through a 101-point filter, and the C_v data is run through the filter 3 times. Additional smoothing is required

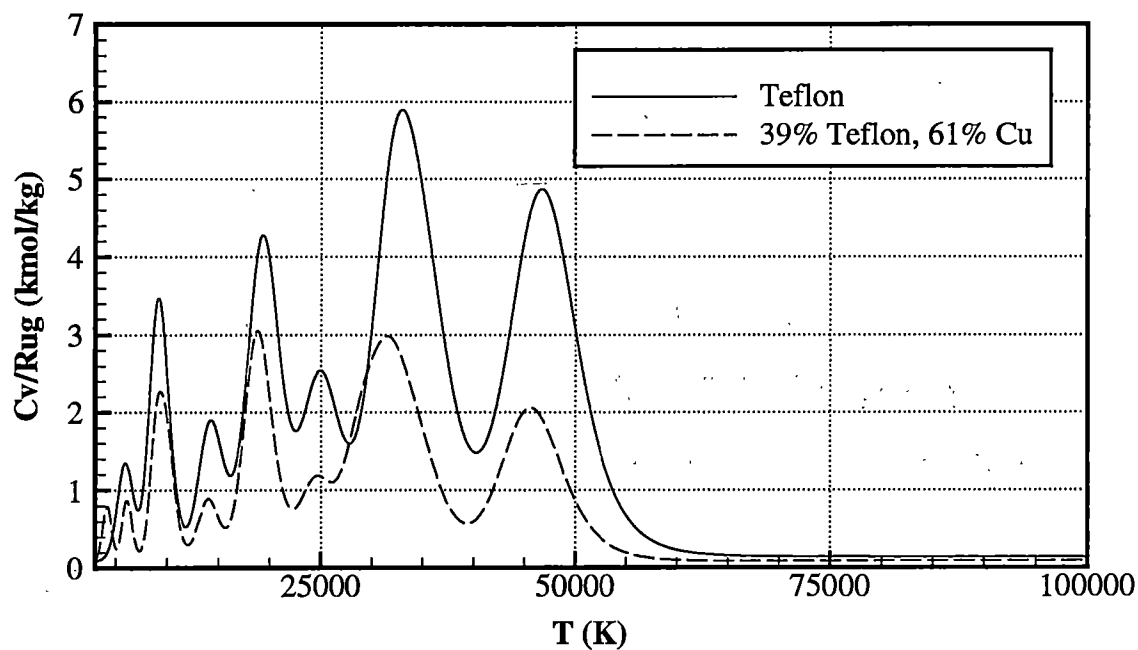


Figure 3.2.21: Filtered Saha C_v/R_{ug} for $\rho = 1.0e-07 \text{ kg/m}^3$

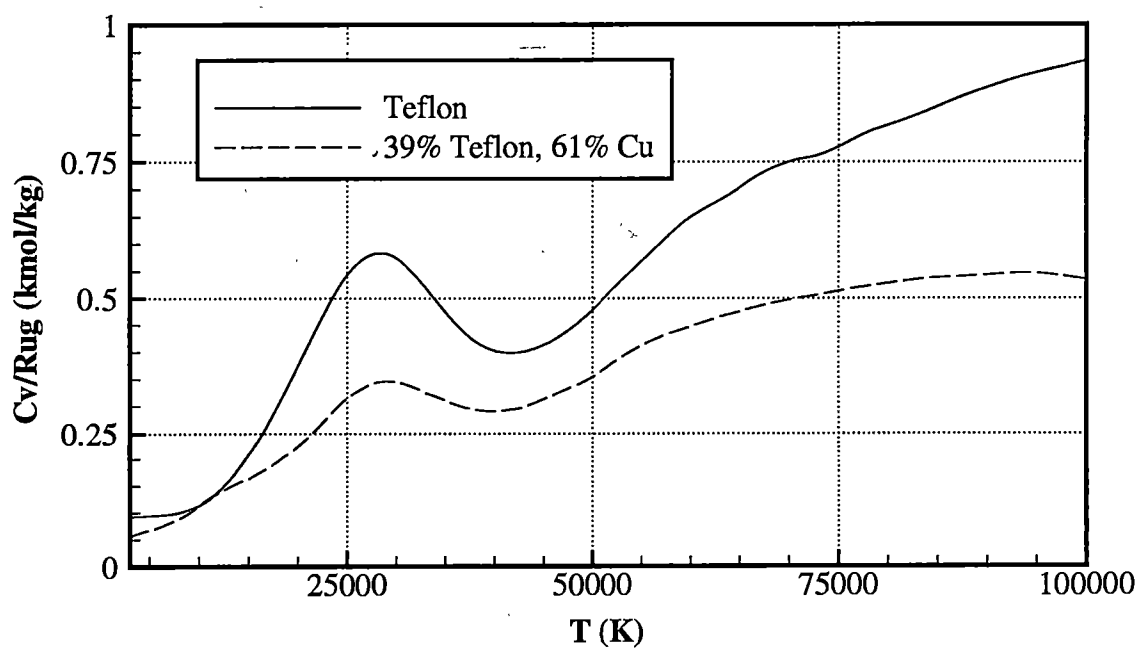


Figure 3.2.22: Filtered Saha C_v/R_{ug} for $\rho = 1.0e+01 \text{ kg/m}^3$

for the higher density plasmas. When necessary, the temperature range is broken into segments, and cubic polynomials are then fit through the C_v data over each segment, matching the values and slopes at the segment endpoints. 690-point segments are used at the highest density for both cases.

3.3 Blending Results

The blending method described in section 2.4.3 is used to combine the low and high temperature thermodynamic data, providing a smooth transition between the two regions. The blending temperature range is minimized to maintain as much physics as possible between the low-temperature CEA results and the high-temperature Saha calculations. To aid in this effort, the CEA and Saha results for C_v/R_u (R_u is the universal gas constant) at the intermediate temperatures are shown in Figures 3.3.1-3.3.4. Figures 3.3.1-2 give the results for the Teflon-only plasma at densities of $1.0\text{e-}07 \text{ kg/m}^3$ and 10 kg/m^3 , respectively, while Figures 3.3.3-4 show the Teflon-copper plasma variations.

In the low density CEA results, the Teflon-only gas fully-dissociates into its atomic constituents around 3200 K. This explains the excellent agreement between the CEA and Saha results at this temperature, since the Saha calculations at the lower temperatures start with a fully-dissociated, un-ionized gas. The low density results for the Teflon-copper plasma also show good agreement but do not match as well as the Teflon-only calculations. In this case, dissociation occurs at a higher temperature than in the Teflon-only gas. Also, ionization occurs at lower temperatures (see Figure 3.2.6) due to the easily-ionized copper. This explains the slight difference between the CEA and

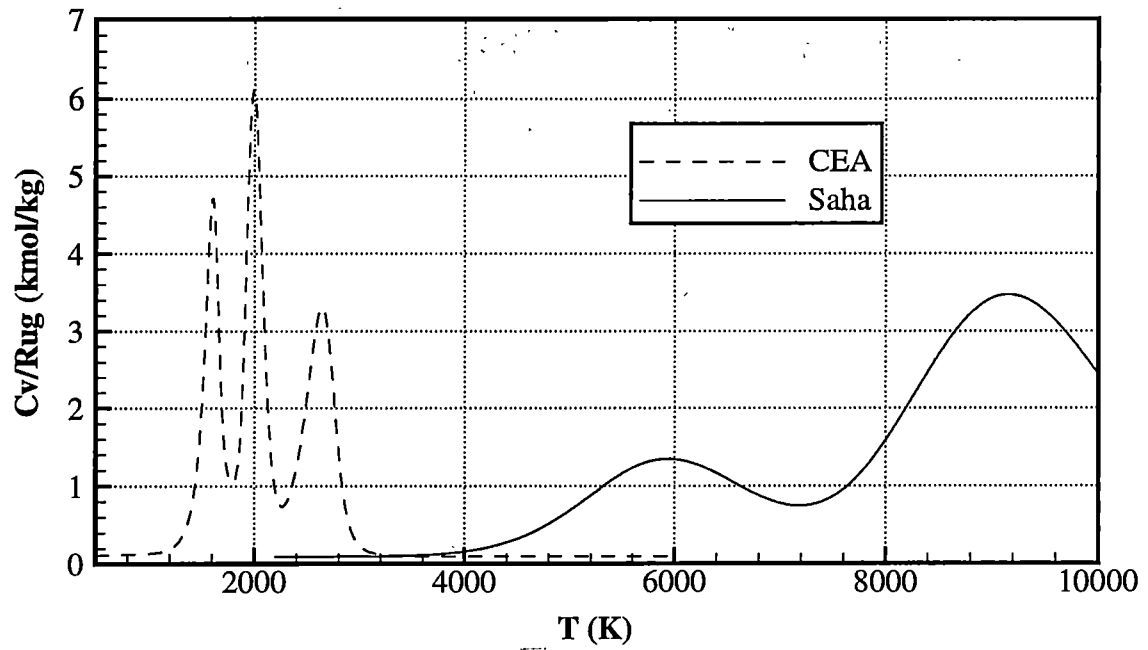


Figure 3.3.1: CEA and Saha Cv/Rug Results for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$ (Teflon)

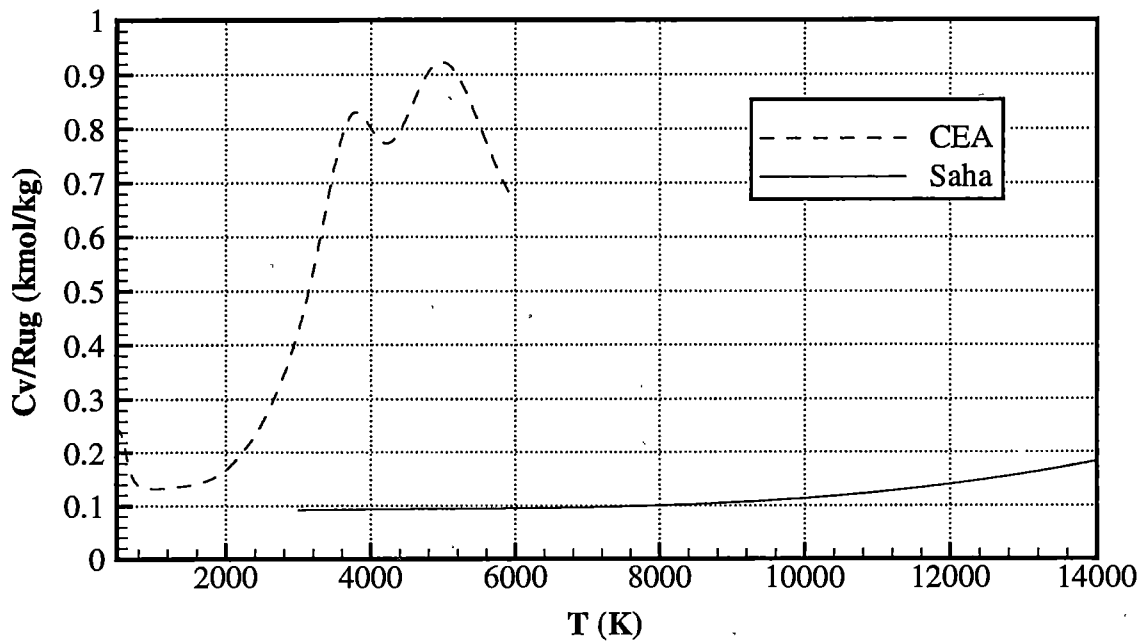


Figure 3.3.2: CEA and Saha Cv/Rug Results for $\rho = 1.0\text{e+}01 \text{ kg/m}^3$ (Teflon)

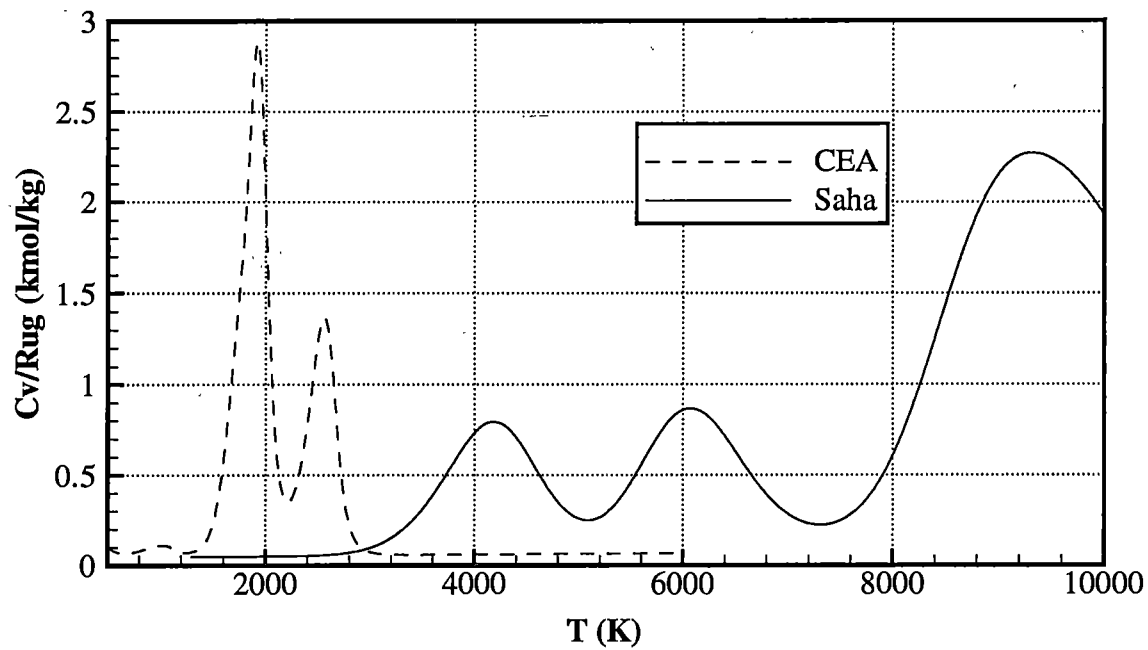


Figure 3.3.3: CEA and Saha Cv/Rug Results for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$ (Teflon-copper)

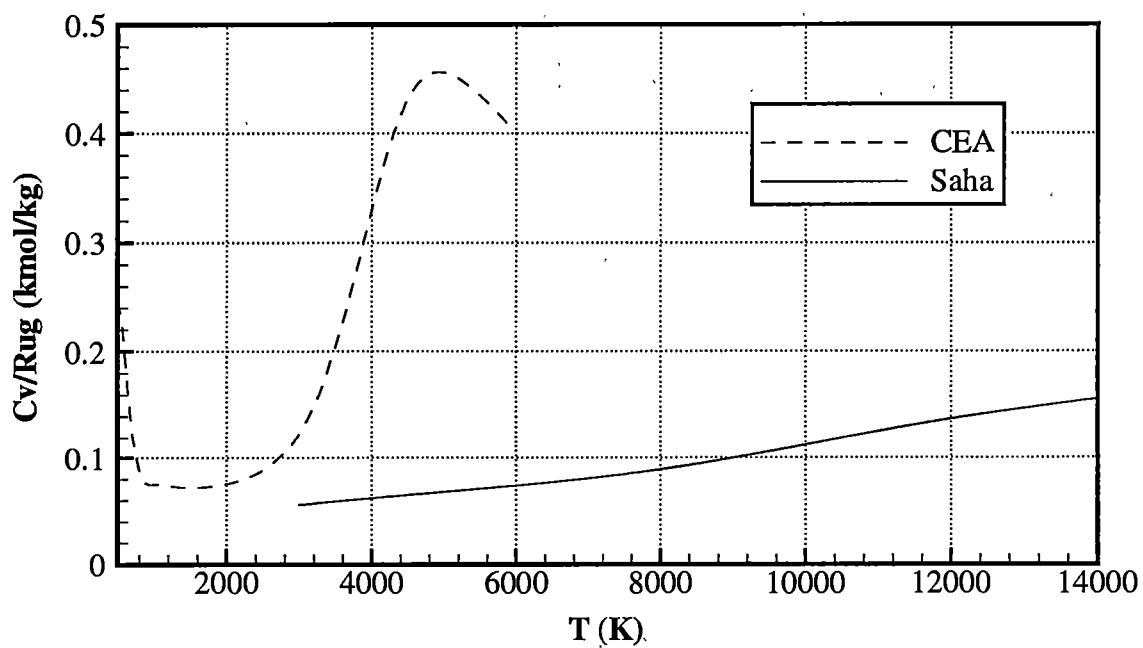


Figure 3.3.4: CEA and Saha Cv/Rug Results for $\rho = 1.0\text{e+}01 \text{ kg/m}^3$ (Teflon-copper)

Saha results at the intermediate temperatures.

The high density results represent the worst cases in blending the CEA and Saha calculations together. At this density, the Teflon-only and Teflon-copper gases are not fully-dissociated at 6000 K. Therefore, the results are blended from the CEA values close to the 6000 K limit to the Saha values corresponding to the temperatures at which dissociation is expected to be complete. This is the best approximation that can be made to determine the thermodynamic variables over the intermediate temperatures at the higher densities using the above-stated blending method; the CEA code does not contain the specie-data required to predict the dissociation of molecular species involving carbon, fluorine, and copper above temperatures of 6000 K.

The blended CEA and Saha thermodynamic properties at the low density are shown in Figures 3.3.5 – 3.3.9, and the high density results are shown in Figures 3.3.11 – 3.3.14. For the low density, the blending range selected for the Teflon plasma is 3300 K to 3500 K, while 2900 K to 3200 K is chosen for the Teflon-copper mixture. At this low density, the CEA and Saha results nearly coincide at an intermediate temperature (3400 K for Teflon and 3000 K for Teflon-copper), allowing a small blending region. Minimizing the blending range is desired to maintain the physics displayed in the data. Since no ionization occurs in the CEA runs, the electron mole fractions are blended from zero at a selected low temperature to the values given at the lowest temperature in the Saha results. For the Teflon plasma, the electron mole fraction blending range is 2000 K to 2200 K, and the Teflon and copper mixture range is 1120 K to 1200 K.

More care must be exhibited in blending the high density data. Due to incomplete dissociation in the CEA output, the CEA and Saha results do not match at intermediate

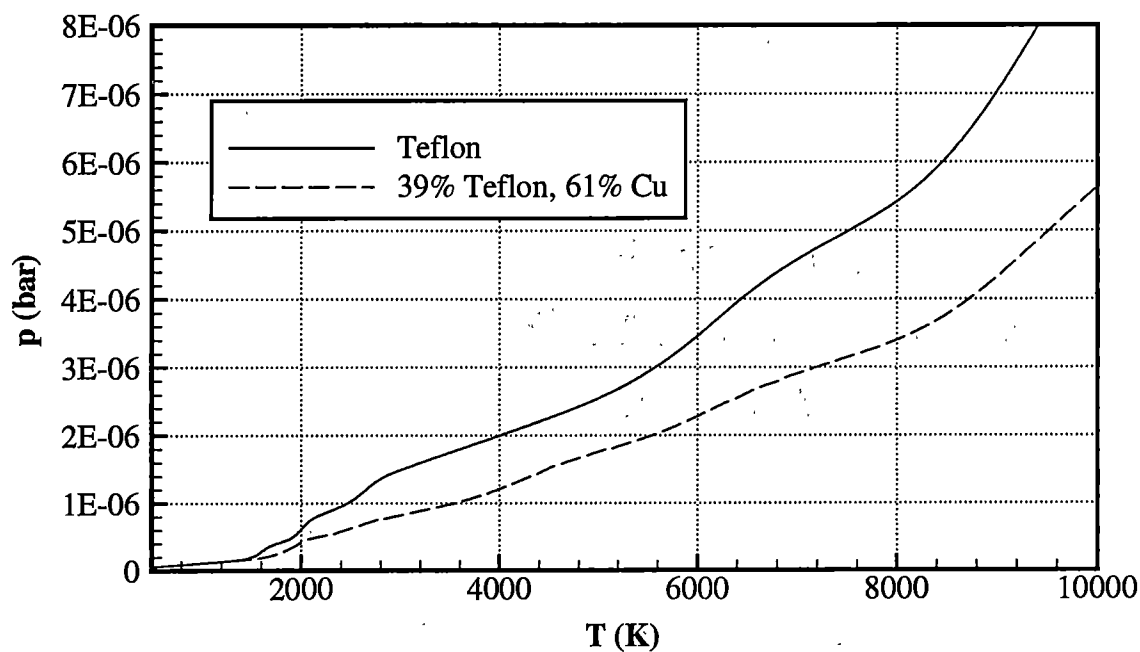


Figure 3.3.5: Blended p Distribution for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$

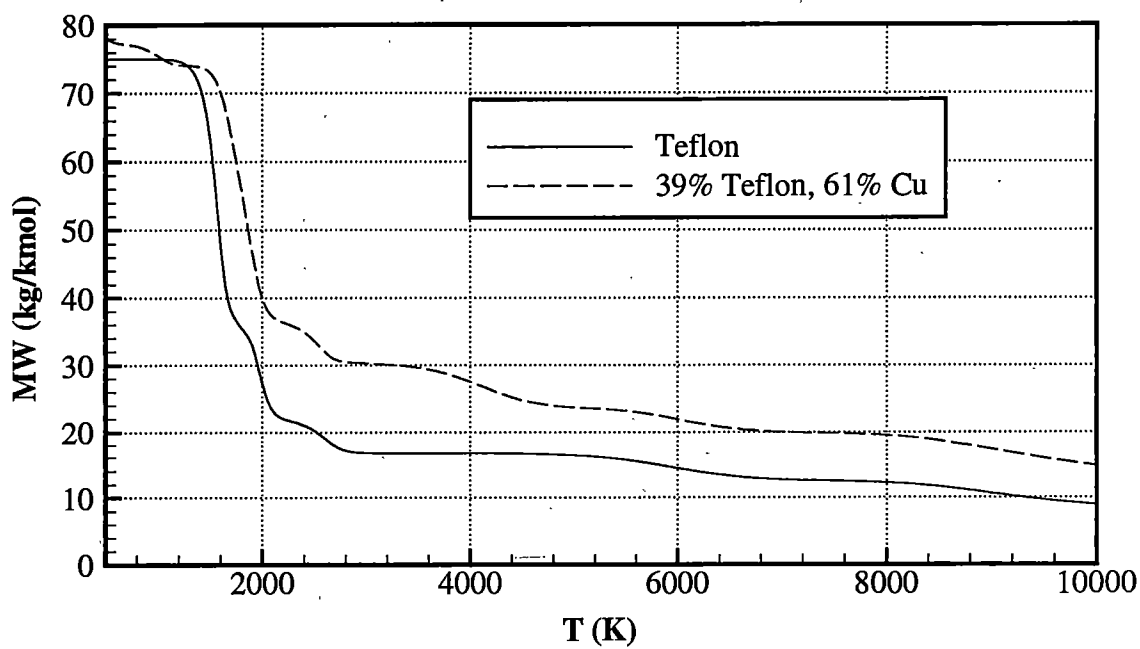


Figure 3.3.6: Blended MW Distribution for $\rho = 1.0\text{e-}07 \text{ kg/m}^3$

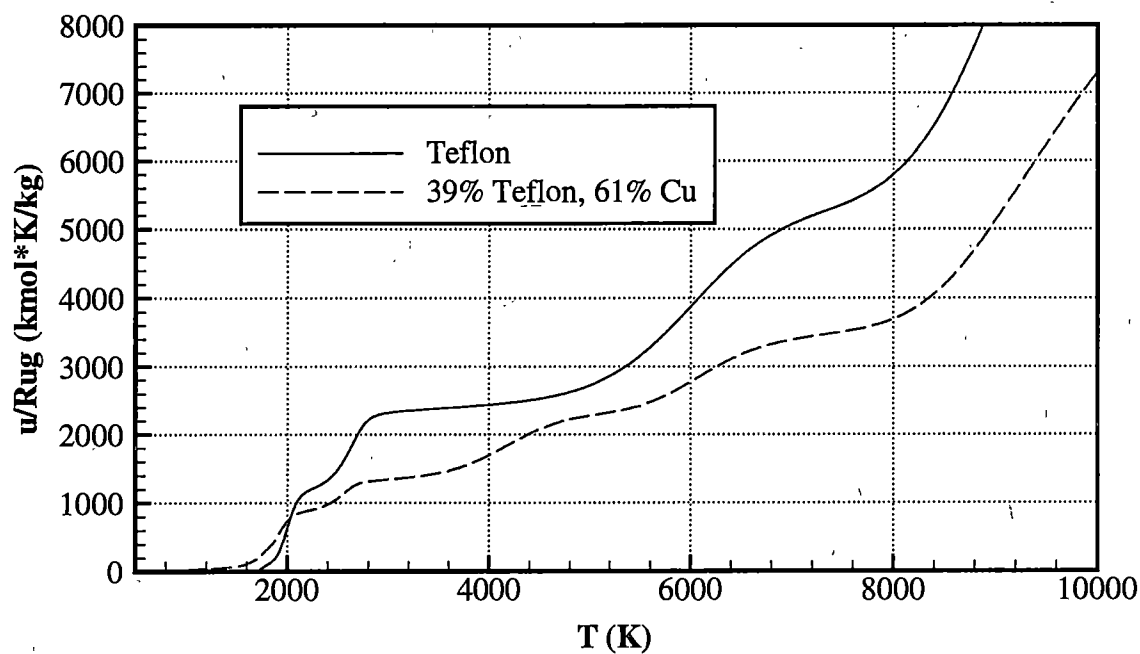


Figure 3.3.7: Blended u/Rug Distribution for $\rho = 1.0e-07 \text{ kg/m}^3$

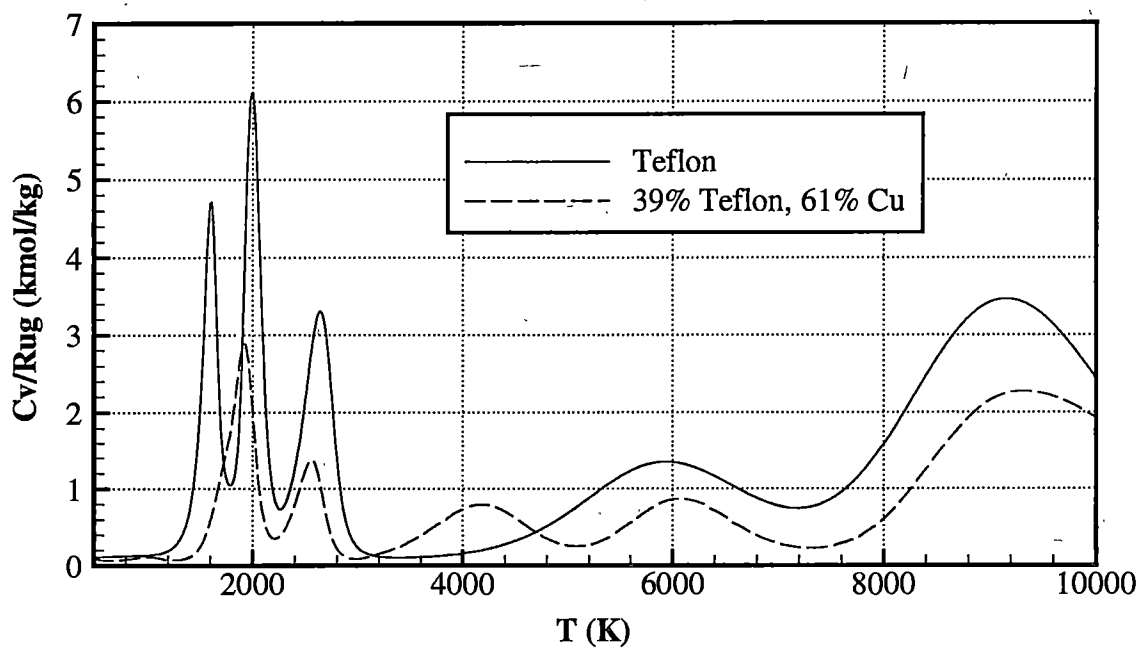


Figure 3.3.8: Blended Cv/Rug Distribution for $\rho = 1.0e-07 \text{ kg/m}^3$

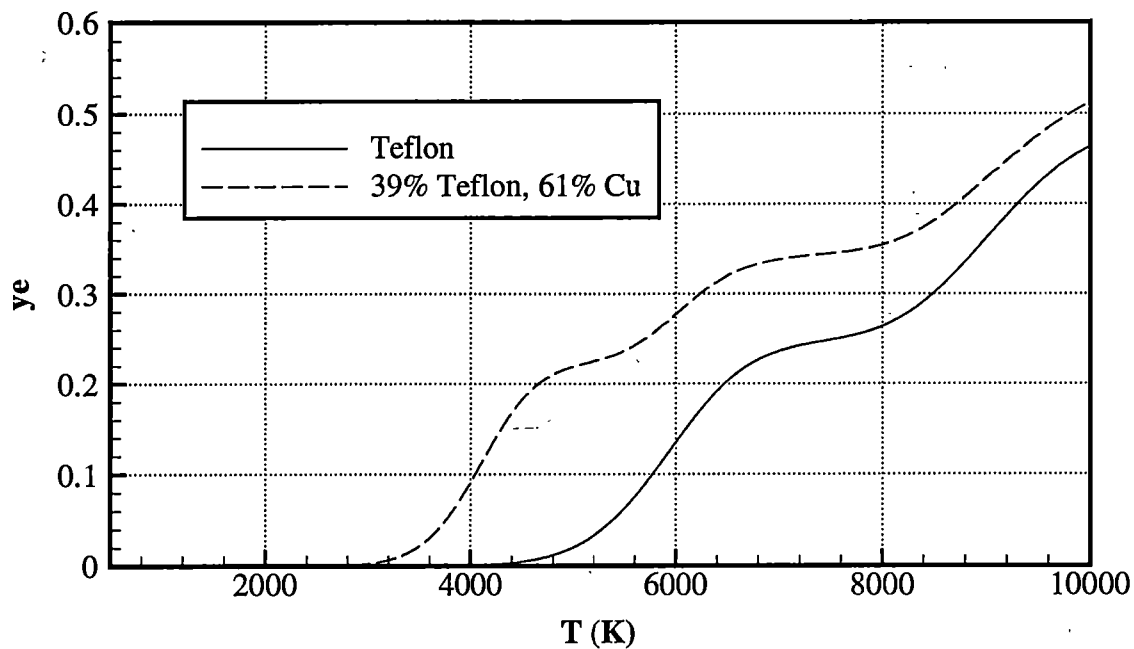


Figure 3.3.9: Blended ye Distribution for $\rho = 1.0e-07 \text{ kg/m}^3$

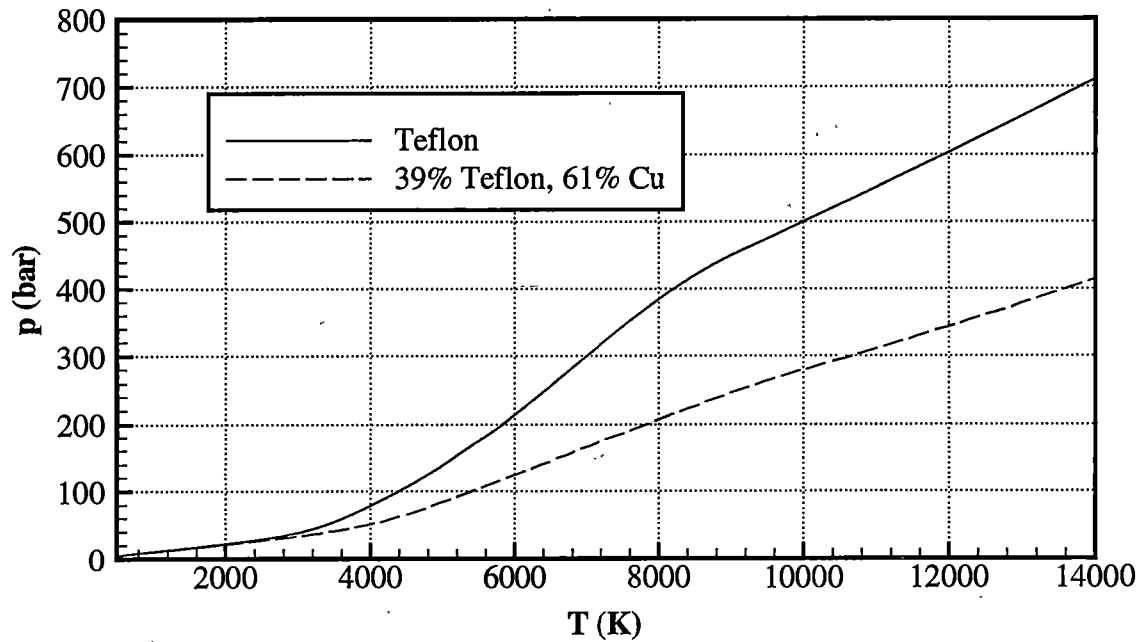


Figure 3.3.10: Blended p Distribution for $\rho = 1.0e+01 \text{ kg/m}^3$

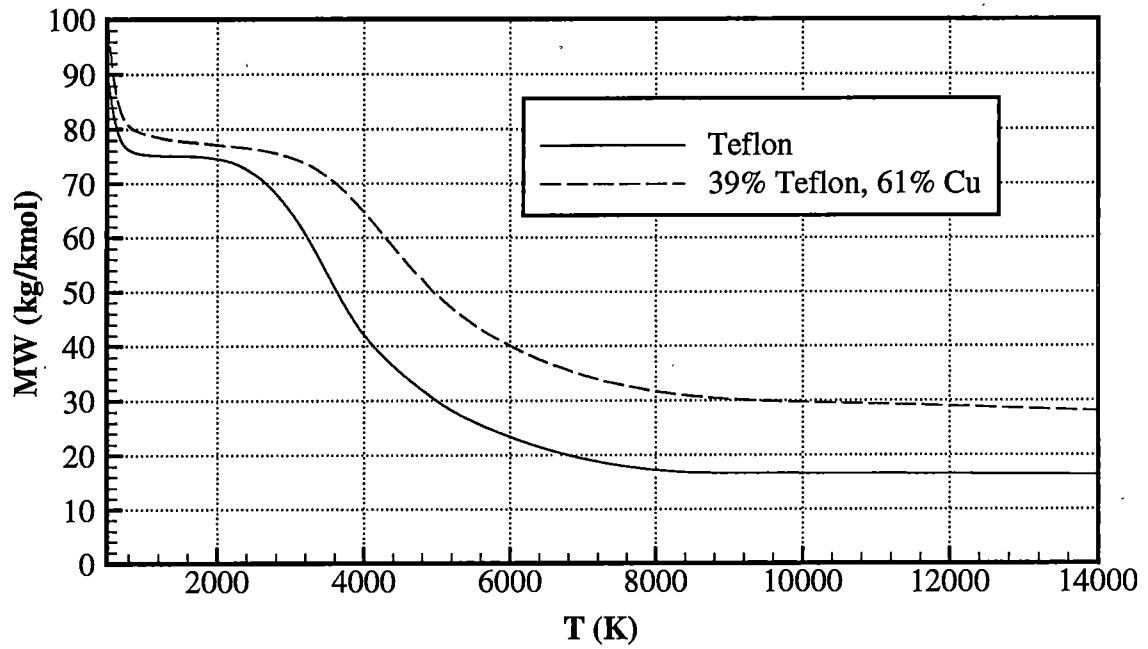


Figure 3.3.11: Blended MW Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

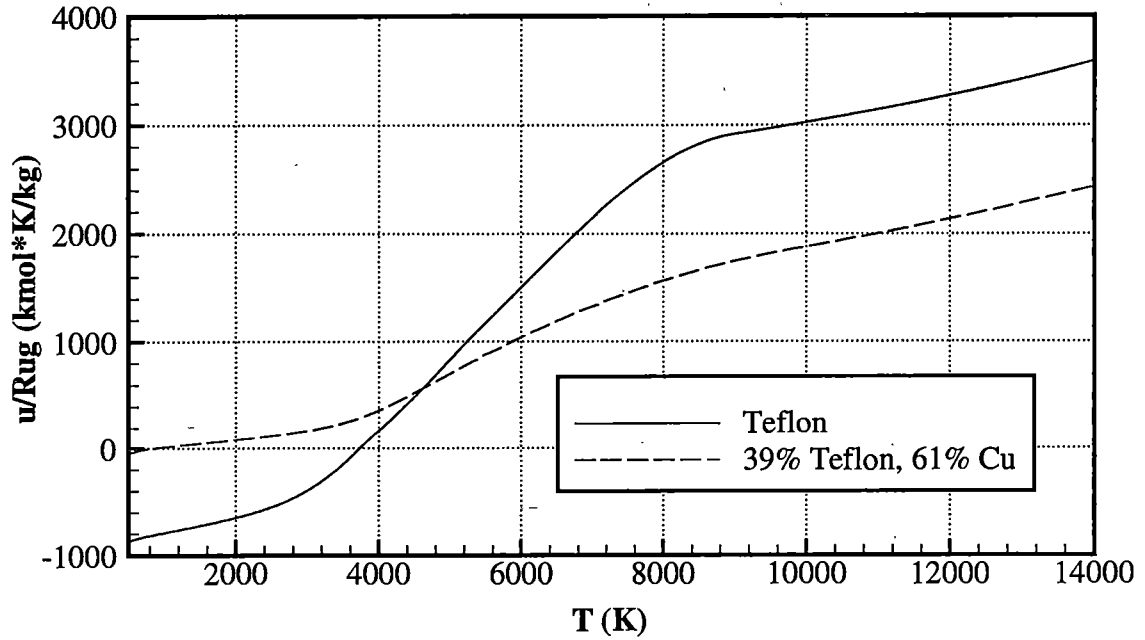


Figure 3.3.12: Blended u/Rug Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

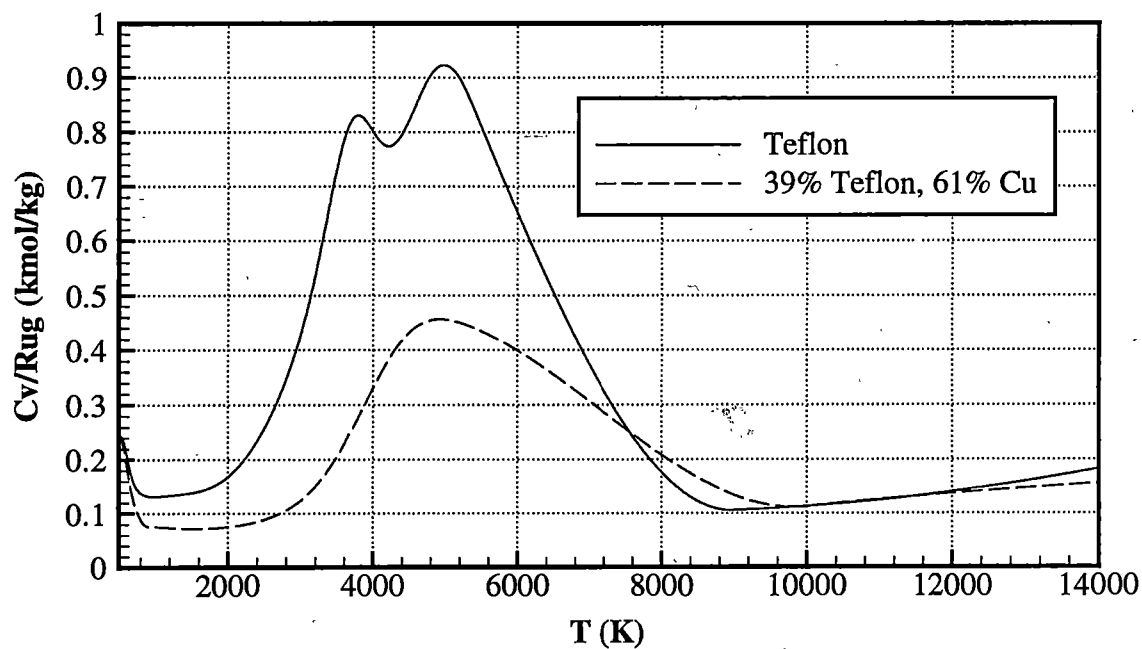


Figure 3.3.13: Blended Cv/Rug Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

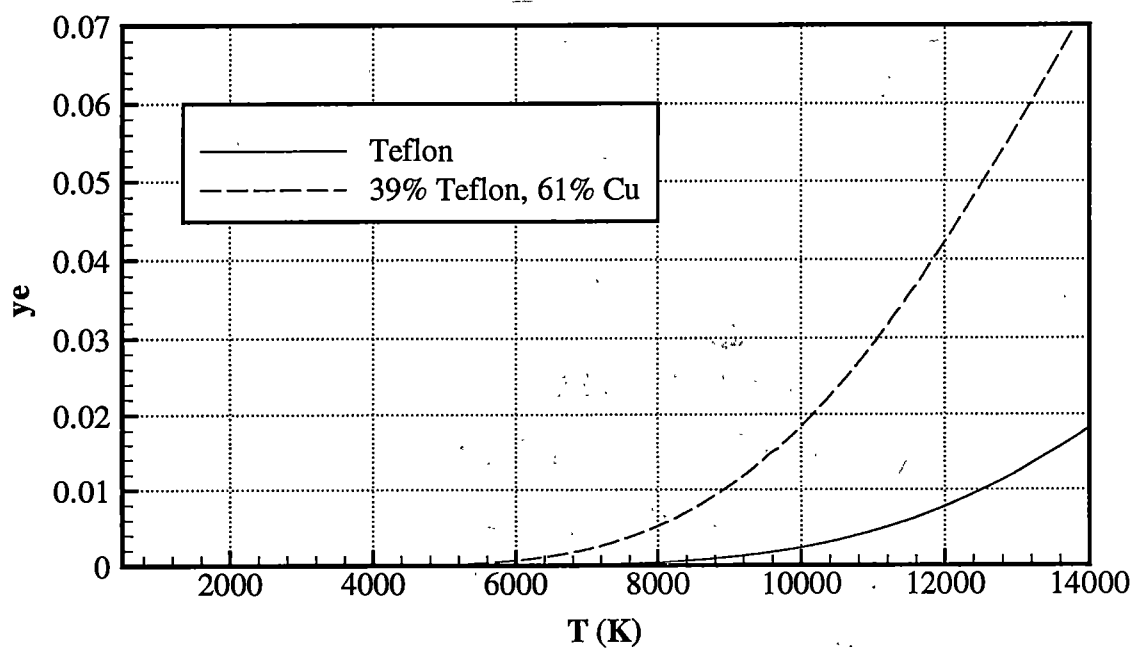


Figure 3.3.14: Blended ye Distribution for $\rho = 1.0\text{e}+01 \text{ kg/m}^3$

temperatures as well as in the low density case. Therefore, one must choose the upper limit of the blending to coincide with the point at which dissociation is expected to be complete, so that only atomic species are present in the output (consistent with the Saha results). The selected blending ranges of 5400 K to 9000 K for the Teflon plasma and 5900 to 10000 K for the Teflon-copper plasma allow smooth transitions without large overshoots in C_v close to the blending endpoints. The electron mole fractions are blended from 2740 K to 3000 K in the Teflon plasma and from 2600 K to 3000 K in the Teflon and copper mixture.

4. APPLICATION OF MACH2 TO THRUSTER GEOMETRIES

4.1 UTSI Cablegun

In conjunction with the numerical simulations of the cablegun, experimental work was also performed at UTSI to investigate the use of cableguns in plasma opening switches [30]. A two-beam, HeNe laser interferometer was used to measure electron number densities at the beam locations of 2.5 cm and 5.0 cm from the nozzle exit. This data was then used to determine the plasma velocities between the two beam locations.

A cablegun is operated by discharging the voltage stored in a capacitor across a solid propellant insulating two electrodes. This results in a dielectric breakdown between the electrodes, and the current discharge ablates the propellant and ionizes the gas, which is then expanded out the nozzle. Figure 4.1.1 shows a cutaway section of the UTSI cablegun after 300 shots (capacitor charged to 25 kV). Both the anode and cathode are copper. The dashed lines denote the physical boundaries prior to operation, while the solid lines show the eroded boundaries after 300 shots. A total mass loss of 83 mg from the cablegun was calculated by weighing the device before and after the 300 shots. This gives an average ablated mass of 2.7×10^{-7} kg per shot. As seen in the figure, the cathode is heavily eroded. The plasma constituents in the exhaust are determined by measuring the eroded volumes of the cathode and Teflon insulator. The resulting plasma contains 68% Teflon and 32% copper by volume or 39% Teflon and 61% copper by mass.

To investigate the cablegun performance without the copper in the plasma, another device was operated with a tungsten cathode, which has the same thickness as the copper cathode but has a flat end instead of the sharp cone. After 300 shots, no

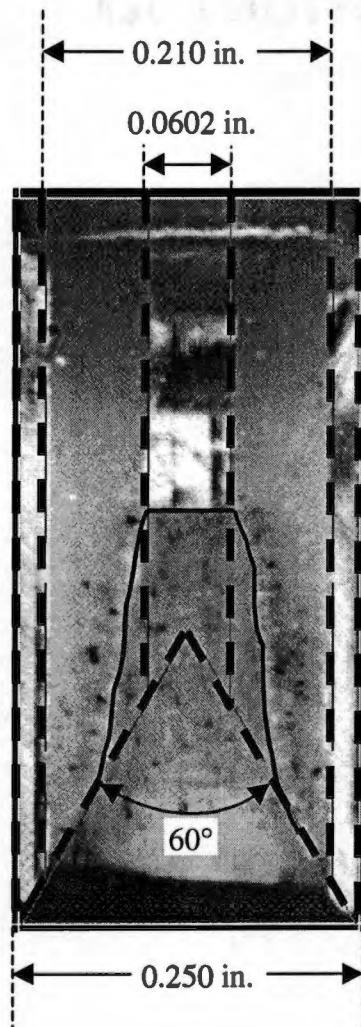


Figure 4.1.1: Cablegun Section with Copper Cathode after 300 25-kV Shots (Dotted Lines Denote Initial Boundaries, and Solid Lines Show Eroded Boundaries)

significant erosion of the tungsten was seen. Therefore, the observed ablated mass of 5.6×10^{-8} kg per shot contained only Teflon.

The computational domains for the cablegun simulations are divided into 9 blocks (shown in Figure 4.1.2). Due to the axial symmetry of the thruster, only half of the cross section needs to be input into MACH2. The grids are set up according to Table 4.1.1 and

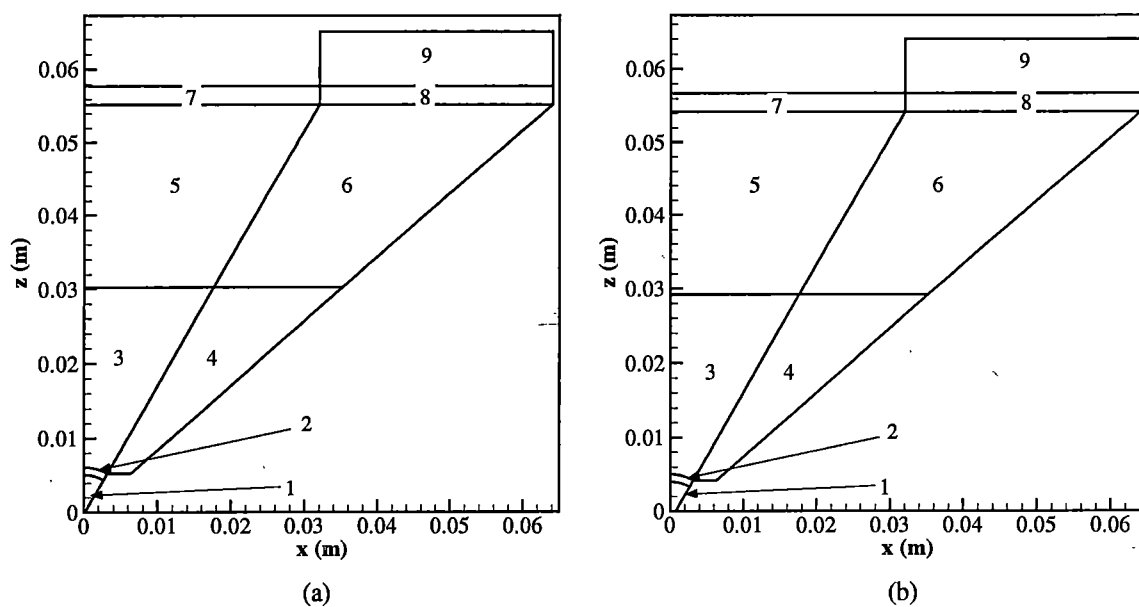


Figure 4.1.2: Computational Domains for the (a) Copper and (b) Tungsten Cathode Cableguns

Table 4.1.1: Number of Grid Cells per Block in Cablegun Simulations

Block #	# Cells in x-direction	# Cells in z-direction
1	16	48
2	16	4
3	16	48
4	16	48
5	16	32
6	16	32
7	16	10
8	16	10
9	16	16

are shown in Figure 4.1.3 for blocks 1 and 2, which correspond to the physical boundaries of the thrusters. The cathode is at the base of block 1, while the anode is along the sidewall in block 2. The Teflon boundary is along the right-hand side of block 1. Note that the base of the copper cathode is extended to more closely represent the tip of a cone. In the experimental setup mentioned above, two interferometer beams are used to determine line integral values of electron number density. These beams are located 2.5 cm and 5.0 cm from the thruster exit. Therefore, the plasma velocity between these locations can be determined by dividing the distance between the beams by the time difference between similar points in the interferometer data (such as the midpoint of the rise time of the first peak). The upper boundaries of blocks 3 and 4 correspond to the 2.5 cm beam, while the upper boundaries of 5 and 6 represent the 5.0 cm location.

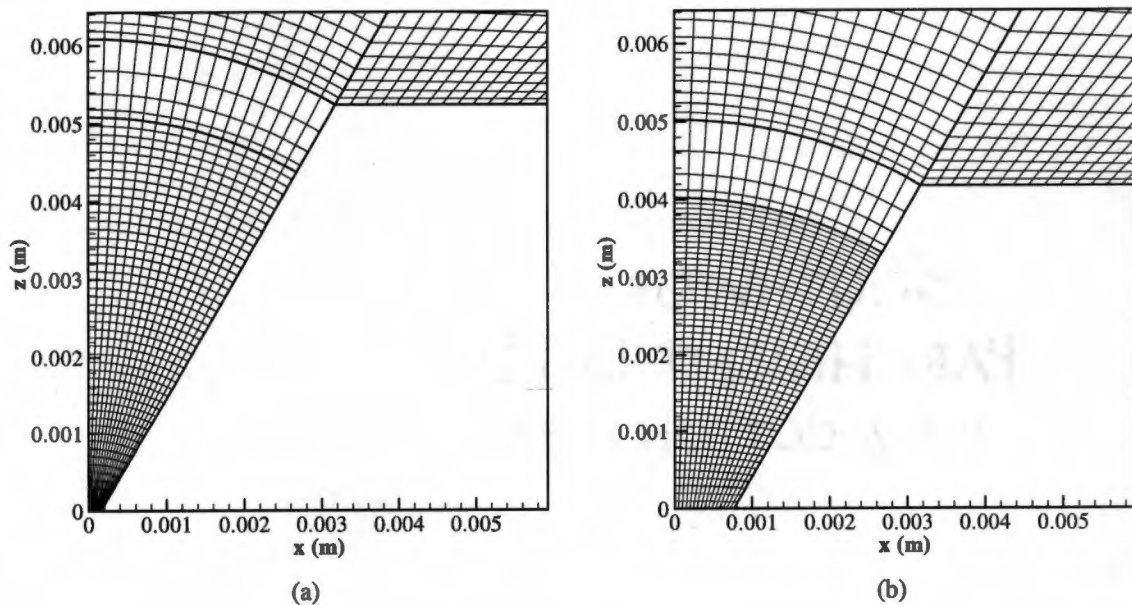


Figure 4.1.3: Computational Grids for the (a) Copper and (b) Tungsten Cathode Cableguns

The initial temperature in the entire computational domain is set to 0.025 eV, and the initial density is $3.2 \times 10^{-7} \text{ kg/m}^3$. The cablegun simulations consisting of only Teflon plasma are run for 15 μsec , while the simulations using 39% Teflon and 61% Cu run for 12 μsec . This provides ample time for the ablated mass to exit the thruster. The experimental circuit driving the cablegun is simulated with the RLC (resistor-inductor-capacitor) model in MACH2. In the experimental operation of the thruster, an initial voltage of -25 kV discharges across a $0.629 \mu\text{F}$ capacitor. The modeled circuit resistance and inductance are 0.24Ω and $4.0 \times 10^{-7} \text{ H}$, respectively. The current histories calculated by MACH2 for the two plasmas are compared to the experimental profile in Figure 4.1.4.

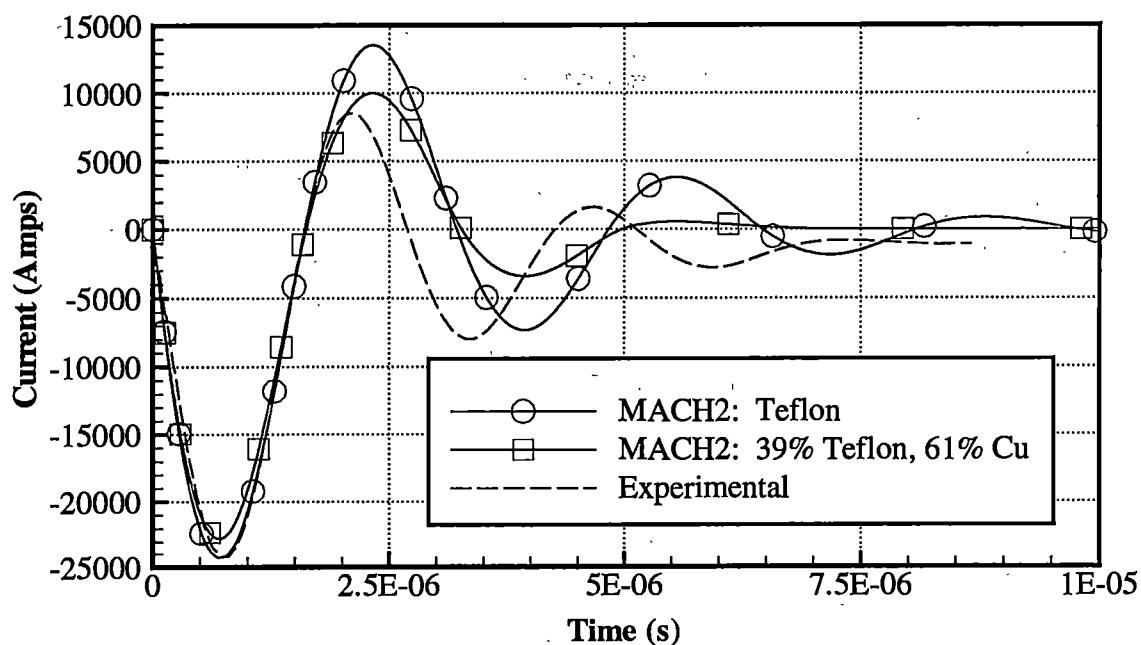


Figure 4.1.4: Modeled Current History in Cablegun Simulations

To accurately simulate the cablegun plasma using MACH2, two parameters are varied to adjust the mass of the ablated propellant and the velocity between the 2.5 cm and 5.0 cm locations. The simulations tend to over-estimate the temperature, which drives the plasma to unrealistic velocities. The plasma absorption coefficient, κ (discussed in section 2.3), is varied to control the radiation loss from each cell, which controls the cell temperature via the conservation of energy. Therefore, this indirectly controls the plasma velocity and directly effects the radiation to the wall and, hence, the amount of propellant that is ablated. The radiation to the wall is calculated using the elements in block 1 only. For simplicity in the Teflon-copper simulations, the copper is input along with the Teflon from the side wall. The propellant absorptivity, α , is used to provide the correct ablated mass for the chosen value of κ . MACH2 actually uses an absorption coefficient, denoted "fox", that is proportional to the plasma density of the emitting cell. This ad-hoc correction is based on the argument that the emission is proportional to the density of the radiating plasma.

Several simulations are performed with various values of "fox" and α and with thermal diffusion and the Hall current turned off. Figures 4.1.5 and 4.1.6 show the effects of increasing "fox" on the line integrals of electron density at the 2.5 cm and 5.0 cm locations, respectively, for the Teflon plasma. Figures 4.1.7 and 4.1.8 show the results for the Teflon-copper mixture. Also included are the experimental data obtained by Smith, Keefer, and Wright [30]. No error bars are given since the uncertainties are very small compared to the measured interferometer phase shifts [31]. Although the simulations do not agree quantitatively with the experimental interferometer data over the

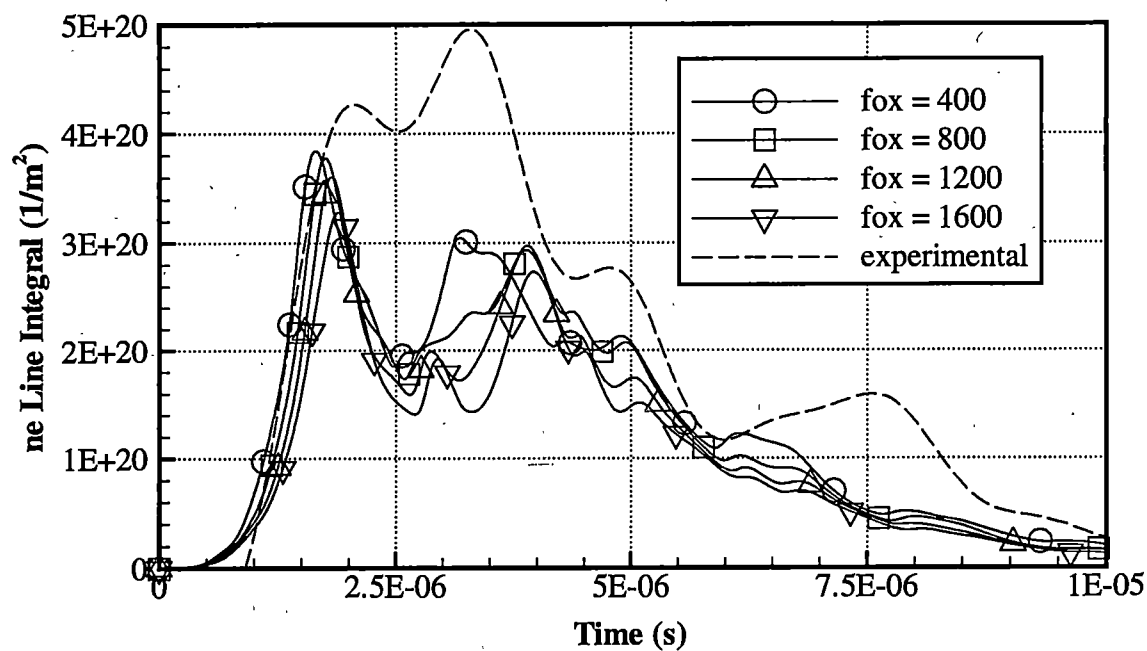


Figure 4.1.5: Cablegun ne Line Integrals for Teflon Plasma at 2.5 cm from Exit

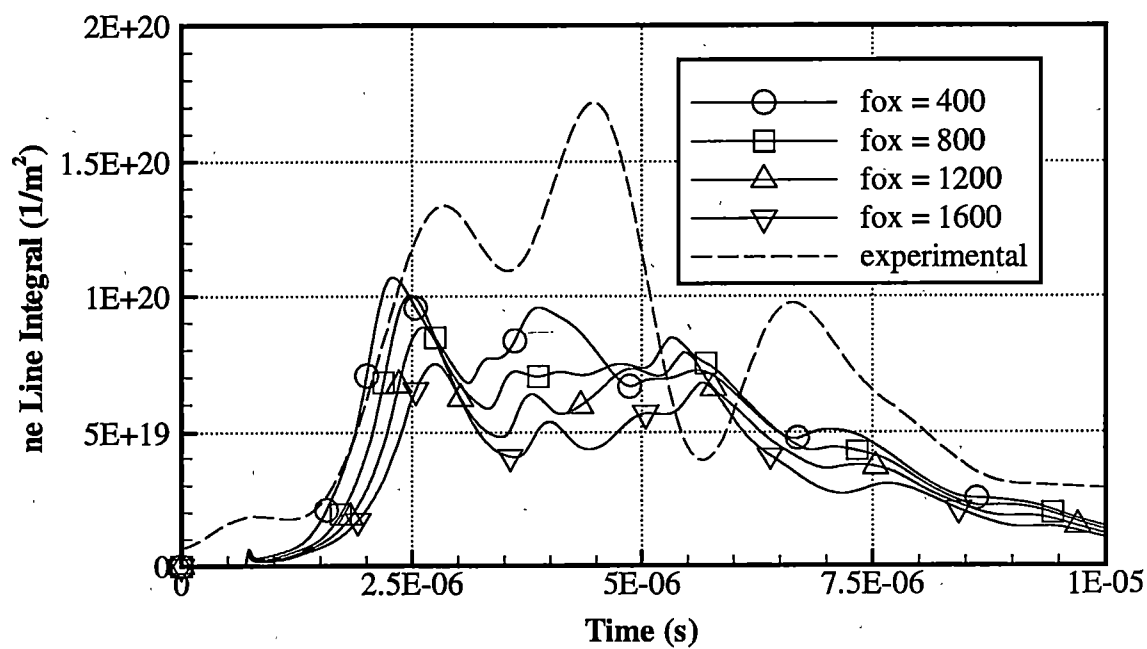


Figure 4.1.6: Cablegun ne Line Integrals for Teflon Plasma at 5.0 cm from Exit

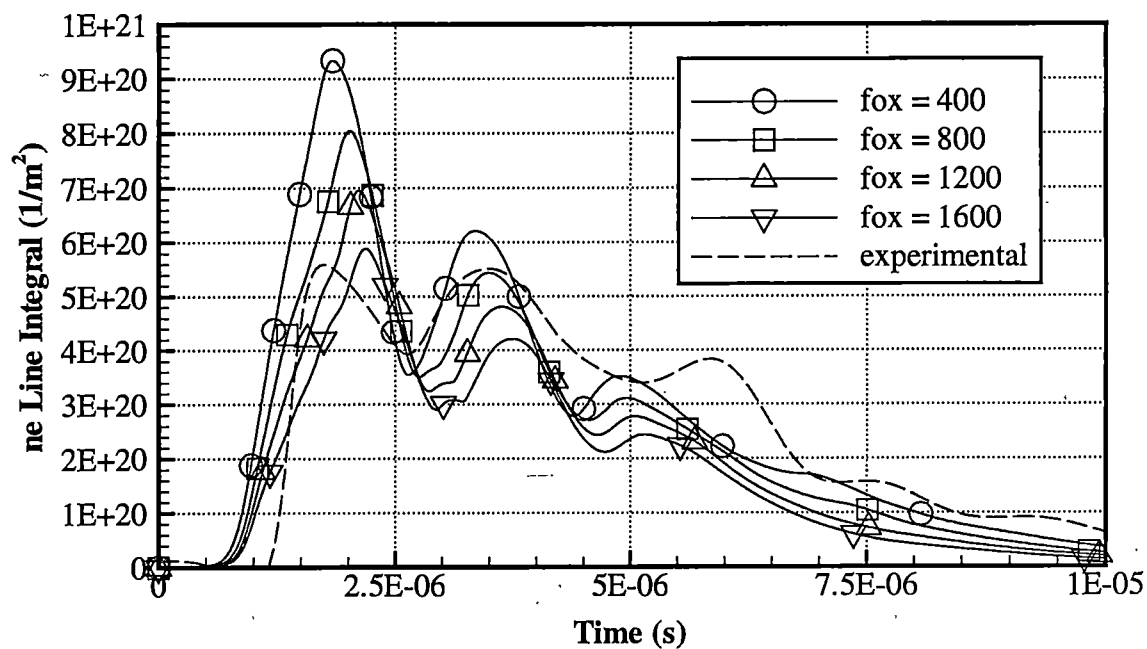


Figure 4.1.7: Cablegun ne Line Integrals for Teflon-copper Mixture at 2.5 cm from Exit

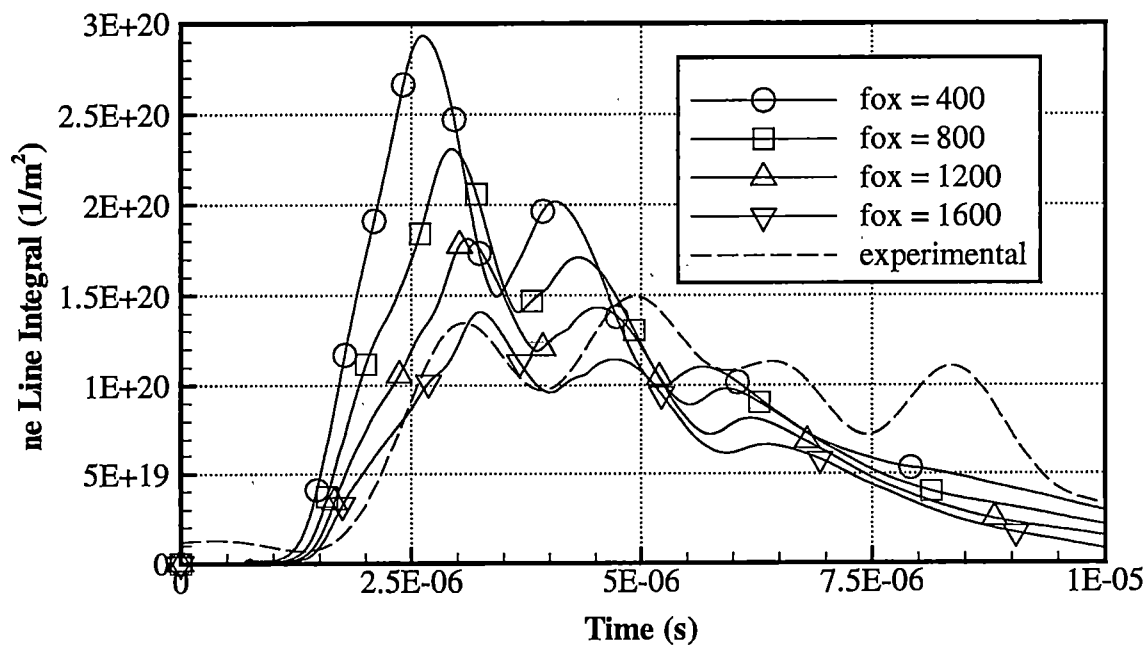


Figure 4.1.8: Cablegun ne Line Integrals for Teflon-copper Mixture at 5.0 cm from Exit

entire run time, they do show similar qualitative behavior, especially between the first three peaks. Surprising is the magnitude of the second experimental peak compared to the first. The second peak in the simulations may be lower due to the ionization limitation of 4 set in the equation of state tables, or the computational domain may not be big enough at the 2.5 cm beam location (however, this seems unlikely). As seen in the figures, increasing "fox" delays the rise time of the peaks. This is expected, since increasing "fox" also increases the radiation output from each grid cell, which reduces the temperature and, therefore, reduces the velocity of the plasma. Note that the inclusion of copper slows the plasma due to the higher molecular weight. Figure 4.1.9 shows the dependence of the propellant absorptivity (α) on "fox" that is required to maintain the observed mass loss for both plasmas. For most of the "fox" values used in the simulations, the corresponding α 's are lower for Teflon than for the plasma containing 39% Teflon and 61% copper. Lower velocities and electron temperatures exist in the Teflon-copper simulations. Therefore, the radiation to the wall is lower, requiring an increased value of α to obtain the required ablated mass. The slopes of the lines in Figure 4.1.9 are approximately equal with the Teflon-only α 's showing a slightly smaller dependence on "fox".

Using the procedure mentioned above for determining the plasma velocities from the interferometer data, the experimental velocities are calculated to be 30,440 m/s and 25,600 m/s for the Teflon and Teflon-copper plasmas, respectively. The simulated plasma velocities for the two mixtures are shown in Figure 4.1.10 for the range of "fox" values. At the lower values of "fox", the simulated velocities are over-predicted. "fox"

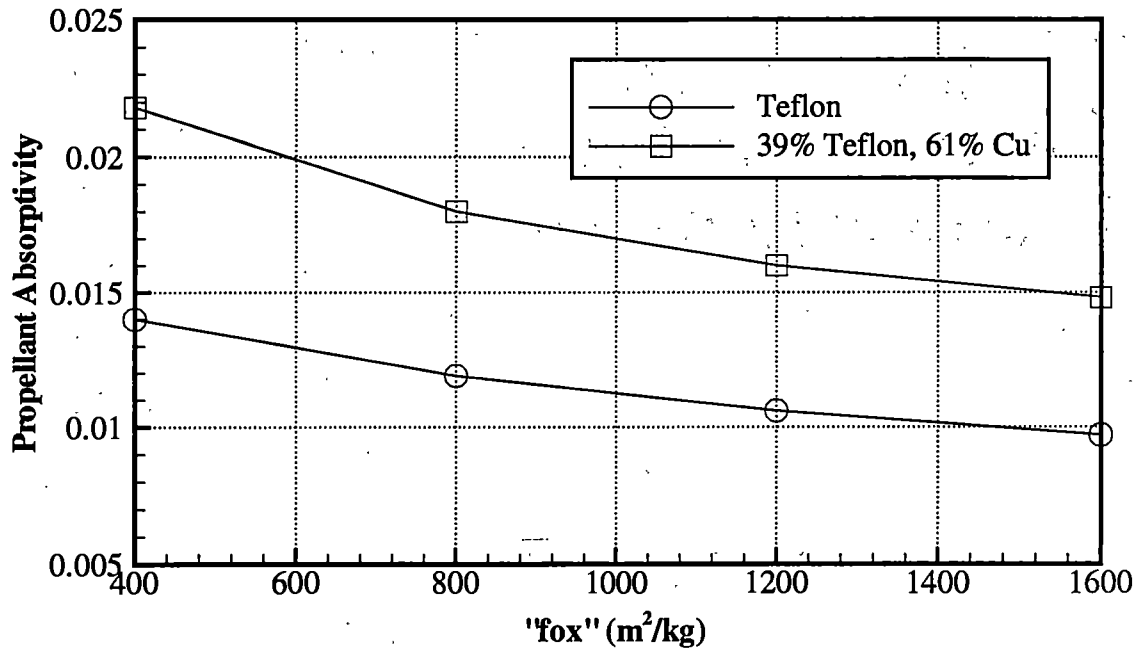


Figure 4.1.9: Cablegun Variation of Plasma Absorptivity (α) with "fox"

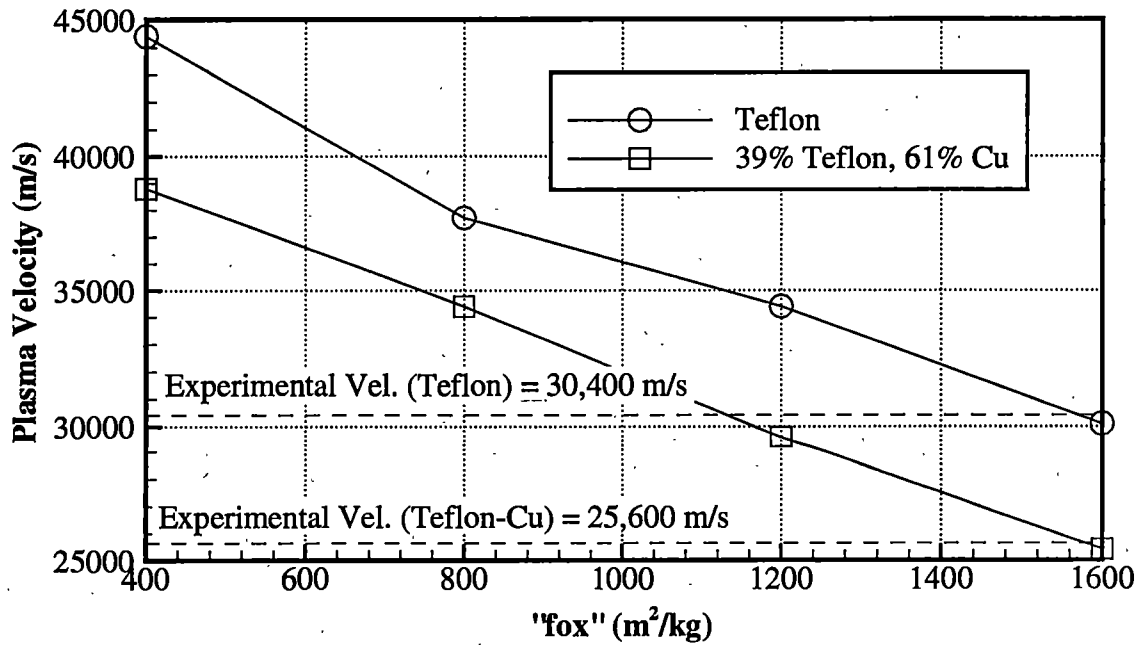
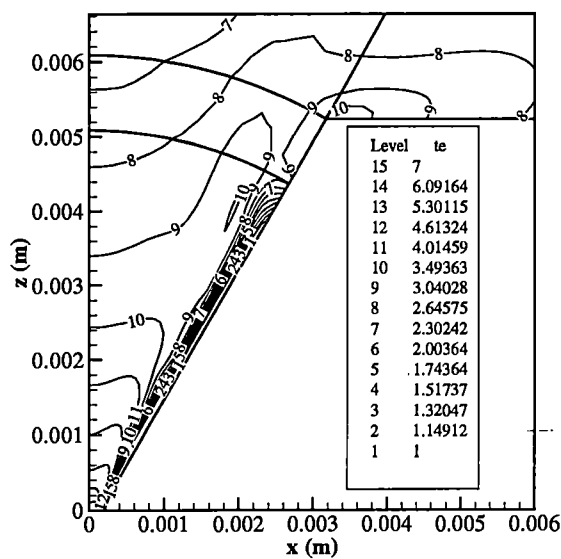


Figure 4.1.10: Cablegun Variation of Plasma Velocity with "fox"

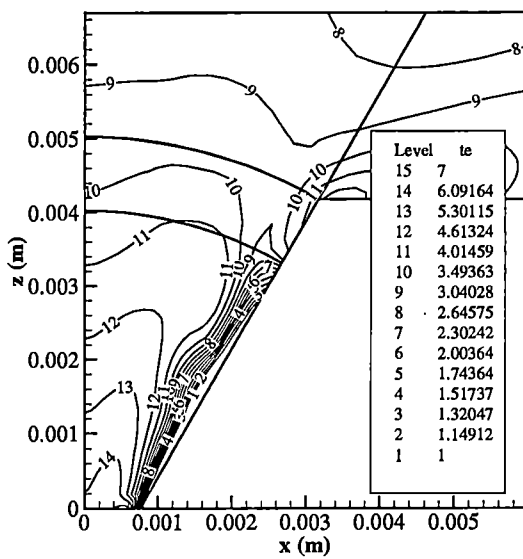
must be increased to 1600 for the experimental and simulated velocities to coincide.

Figures 4.1.11 and 4.1.12 show the temperature (in eV) and density (in kg/m^3) contours for the two cableguns at "fox" = 1600. The plasma temperature in the cablegun with the copper cathode ranges from 2.3 eV at the exit to above 7 eV at the base. The 7, 8, and 9 contour levels fall within the expected experimental temperature range of 2.2 to 2.8 eV (discussed in Chapter 5). The simulated temperatures in the Teflon-only case are higher, ranging from around 3.4 eV at the exit to above 7 eV at the cathode. The higher temperature in the tungsten cathode shot agrees with the higher velocities observed experimentally. Velocity streamlines are displayed in Figure 4.1.13 and show the plasma coming off the sidewall and then expanding out the nozzle exit. Part (a) of the figure shows the plasma moving away from the propellant surface. The close-grouping of streamlines represents the edge of the plasma jet. Therefore, the Teflon-copper plasma has not expanded as much as the Teflon-only plasma shown in part (b). This is supported by the density contours (kg/m^3) in Figure 4.1.12. The higher molecular weight of the Teflon-copper mixture results in the grouping of the streamlines lines close to the wall, which represents the higher density, slower moving material. The density is also higher due to the smaller base of the copper cathode.

Of special interest is the total impulse that the cableguns produce. The thrust output of the cableguns during a shot can be calculated by taking a control volume that coincides with the inner surface of the nozzle and base and surrounds the thruster. Assuming that the ambient pressure is much less than the pressure exerted on the nozzle and cathode surface, the thrust in the axial (or $\hat{k}$) direction at a particular time is given by:



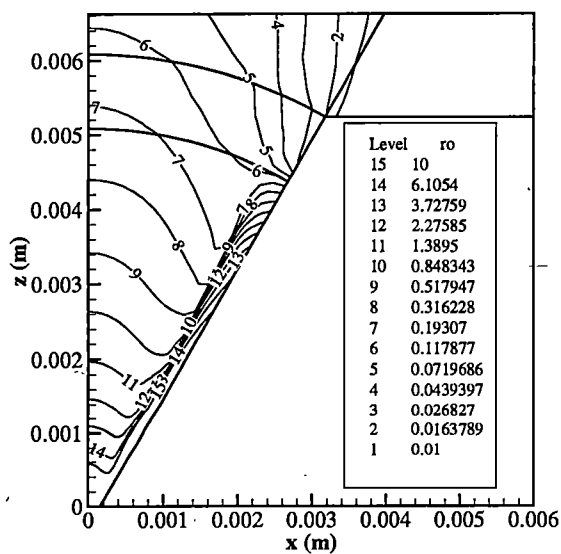
(a)



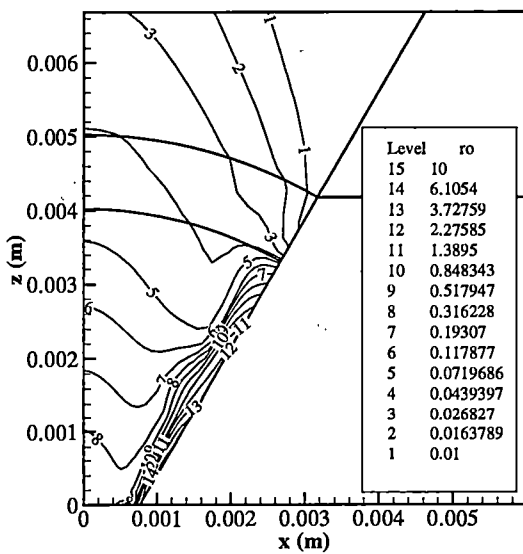
(b)

Figure 4.1.11: Cablegun Temperature Contours (eV) for (a) Copper and (b) Tungsten

Cathode Cableguns with "fox" = 1600 at $t = 1.0 \mu s$



(a)



(b)

Figure 4.1.12: Cablegun Density Contours (kg/m^3) for (a) Copper and (b) Tungsten

Cathode Cableguns with "fox" = 1600 at $t = 1.0 \mu s$

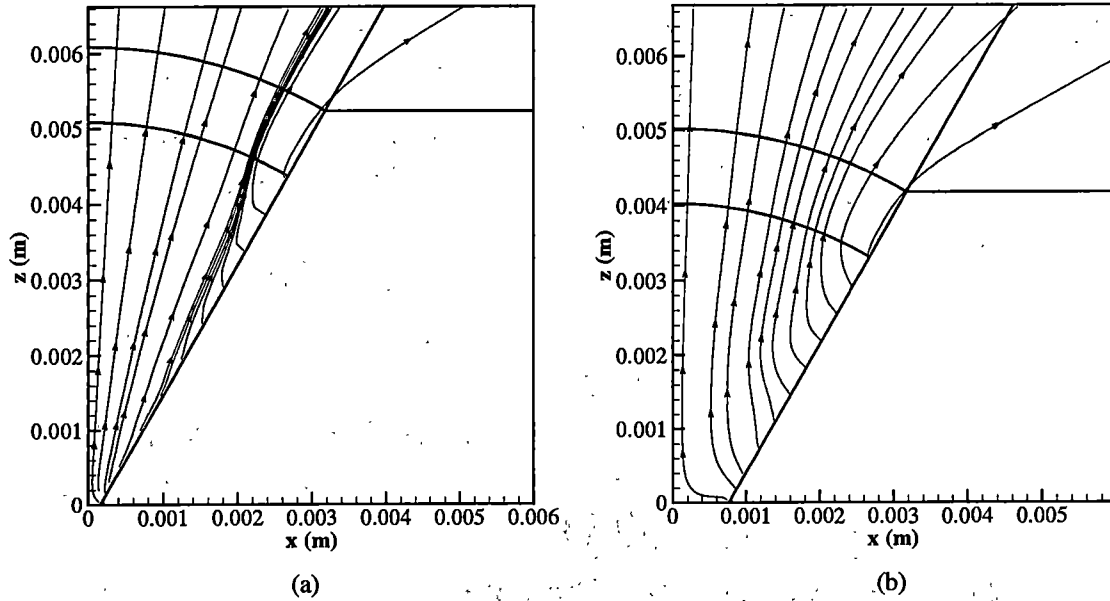


Figure 4.1.13: Cablegun Velocity Streamlines for (a) Copper and (b) Tungsten Cathode

Cableguns with "fox" = 1600 at $t = 1.0 \mu s$

$$F_T = \iint_{\text{anode}} p(\hat{k} \cdot \hat{n} dA) + \iint_{\text{teflon}} p(\hat{k} \cdot \hat{n} dA) + \iint_{\text{cathode}} p(\hat{k} \cdot \hat{n} dA) + \iiint_{\text{contained plasma}} (\vec{j} \times \vec{B}) \cdot \hat{k} dV \quad (4.1.1)$$

where the electromagnetic force on the external circuit is assumed to be equal to the electromagnetic force exerted on the plasma contained in the thruster. Integrating the thrust over the shot duration produces the desired value of impulse. Using this technique results in the thruster impulses shown in Figures 4.1.14-15 (for "fox" = 1600). The cablegun with the tungsten cathode produces a total impulse of $6.89e-04 \text{ N}\cdot\text{s}$. Dividing by the total weight of propellant expelled from the thruster gives an average specific impulse of 1254 s. The cablegun with the copper cathode produces a much higher impulse of $1.84e-03 \text{ N}\cdot\text{s}$ but operates at a lower specific impulse of 695 s; the addition of copper results in a much higher weight of propellant that is lost.

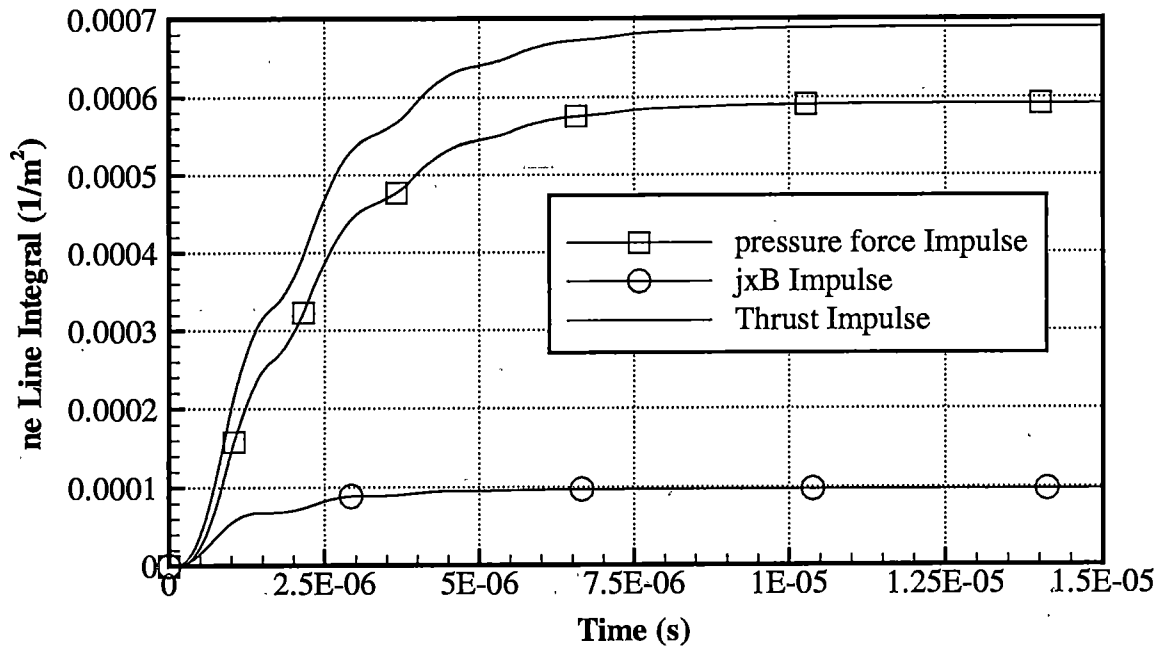


Figure 4.1.14: Simulated Impulse of the Teflon-only Cablegun

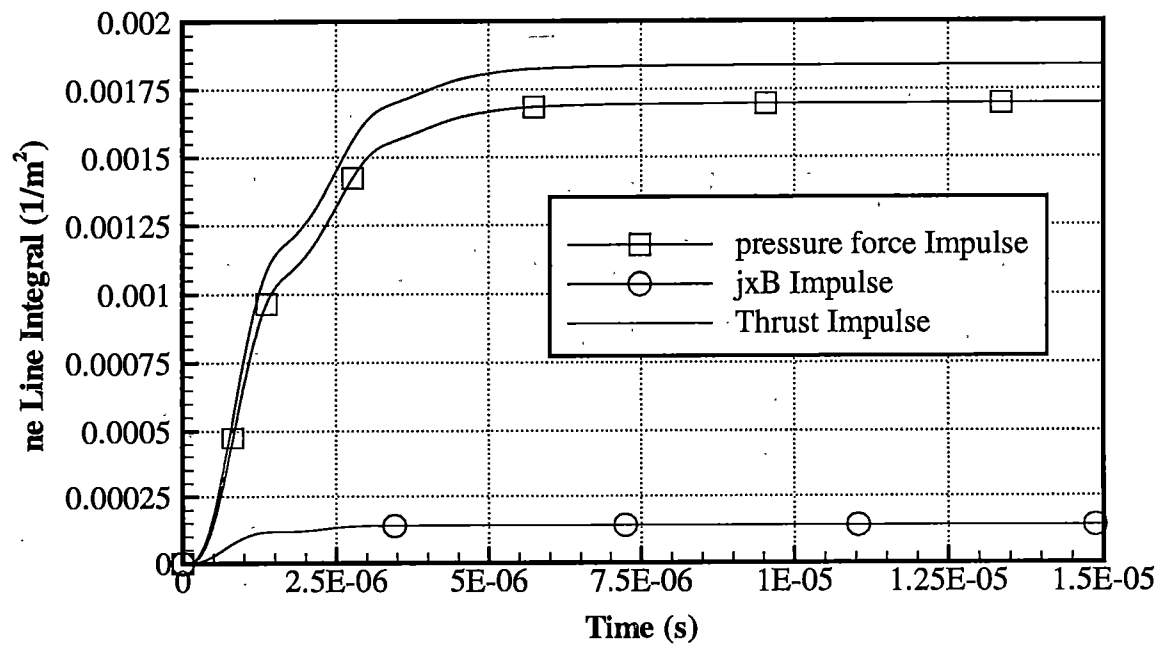


Figure 4.1.15: Simulated Impulse of the Teflon-copper Cablegun

Two physical processes accelerate the plasma in the thrusters: electromagnetic forces exerted on the plasma by the external circuit and the remaining electrothermal (or gas-dynamic) acceleration. The $\vec{j} \times \vec{B}$ term in equation (4.1.1) represents the electromagnetic contribution to the thrust, while the pressure forces give the force resulting from the electrothermal expansion of the plasma. The $\vec{j} \times \vec{B}$ contribution to the total impulse is $9.75\text{e-}05 \text{ N}\cdot\text{s}$ for the Teflon-only thruster and $1.41\text{e-}04 \text{ N}\cdot\text{s}$ for the Teflon-copper cablegun. Therefore, 14.2% of the impulse of the Teflon-only thruster is electromagnetic, and the electromagnetic contribution is only 7.7% in the cablegun containing 39% Teflon and 61% copper. During the early portions of the shots, the simulations show a low density, high velocity plasma jet exiting the thrusters. As seen in Figures 4.1.16 and 4.1.17, the electromagnetic fractions of the total thrust are higher during the early operation of the devices. This explains the high velocities achieved by the initial plasma pulse.

4.2 PPT-4

The PPT-4 thruster operates in a similar manner to the cablegun. In this device, the capacitor is rated at $8.0 \mu\text{F}$ and is charged to 1.5 kV . Figure 4.2.1 shows the computational domain, which is separated into 4 blocks. The electrodes are located at the base of block 1 and the side wall in block 3. The Teflon is located along the side wall in block 1, and the upper edge of block 3 represents the thruster exit. Blocks 4 and 5 are designed such that the upper edges are located at 2.5 cm and 5.0 cm , respectively, to provide velocity calculations consistent with the method used in the cablegun analysis.

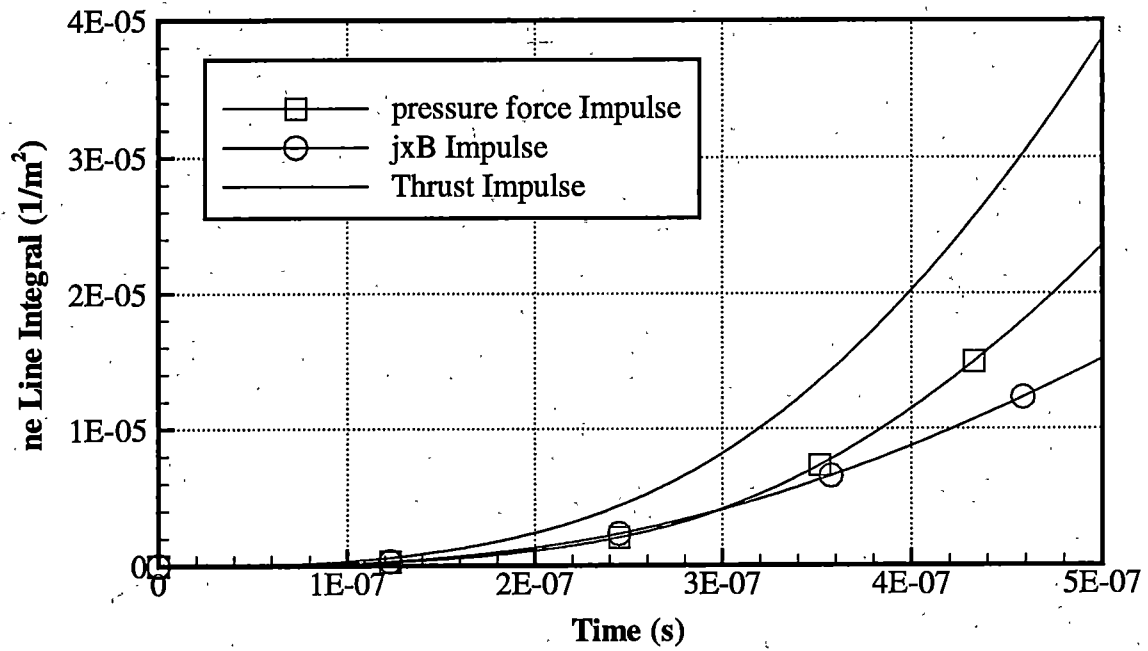


Figure 4.1.16: Simulated Impulse during early Operation of Teflon Cablegun

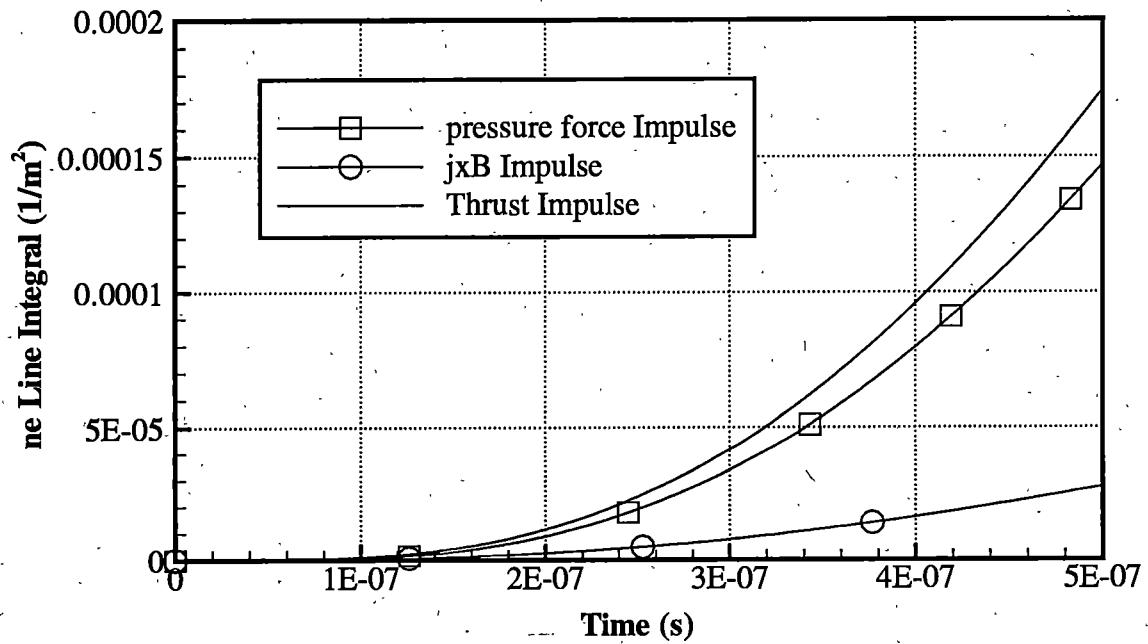


Figure 4.1.17: Simulated Impulse during early Operation of Teflon-copper Cablegun

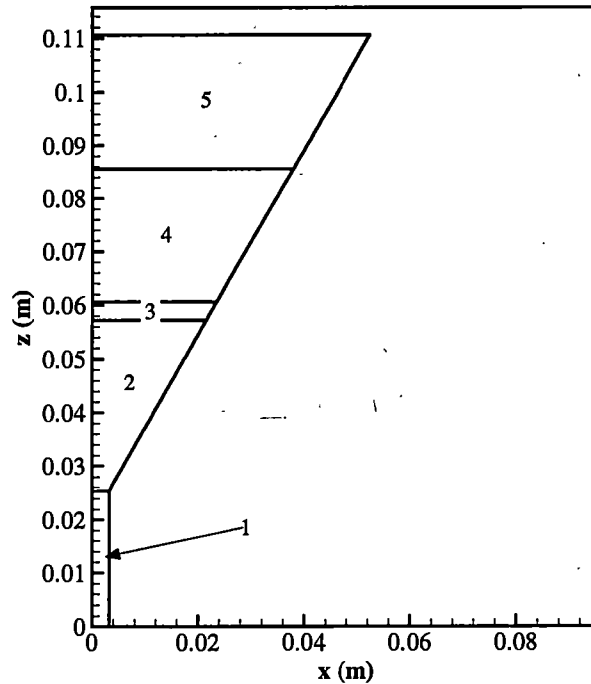


Figure 4.2.1: Computational Domain Used in the PPT-4 Simulations

The PPT-4 grid is set up according to Table 4.2.1, and a portion of the grid at the interface of blocks 1 and 2 is shown in Figure 4.2.2.

The initial temperature and density in the computational domain are set to the same values used in the cablegun simulations. The PPT-4 simulations are run for 150 μsec to provide sufficient time for the plasma to exit the thruster. The longer runtime is required due to the much larger size of PPT-4 compared to the cableguns. A first approximation to the initial resistance and inductance input into the MACH2 RLC circuit model are $4.90\text{e-}05 \, \Omega$ and $1.95\text{e-}07 \, \text{H}$, respectively. After $3.25 \, \mu\text{sec}$, the capacitance is set to $1.0\text{e+}25$, the resistance is set to $0.13 \, \Omega$, the inductance is set to $1.95\text{e-}07 \, \text{H}$, and the voltage is set to 0.0. This effectively shorts the capacitor and simulates the diode in the

Table 4.2.1: Number of Grid Cells per Block in PPT-4 Simulations

Block #	# Cells in x-direction	# Cells in z-direction
1	32	48
2	32	48
3	32	16
4	32	24
5	32	24

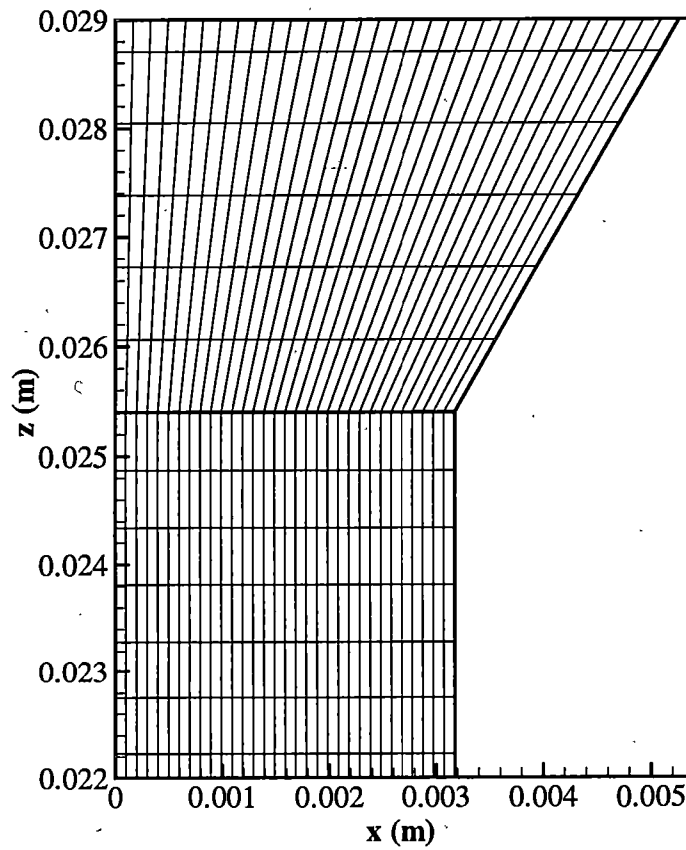


Figure 4.2.2: Computational Grid Used in the PPT-4 Simulations

experimental operation of the device [16]. The current history calculated by MACH2 during the PPT-4 simulation is shown in Figure 4.2.3. The currents achieved in the simulation are much less than the experimental values given in [16], which is approximated by the hand-drawn dashed line. Several attempts were made to increase the current by changing the resistance and inductance of the external RLC circuit, but no combination of R and L proved successful. The current deficit is believed to be caused by an over-prediction (by MACH2) of the voltage between the thruster electrodes. The error is less significant in the cablegun simulations, because the calculated port voltage represents only a small percentage of the initial voltage (25 kV) across the capacitor. The results are nevertheless presented, but a possible solution is recommended later in Chapter 6.

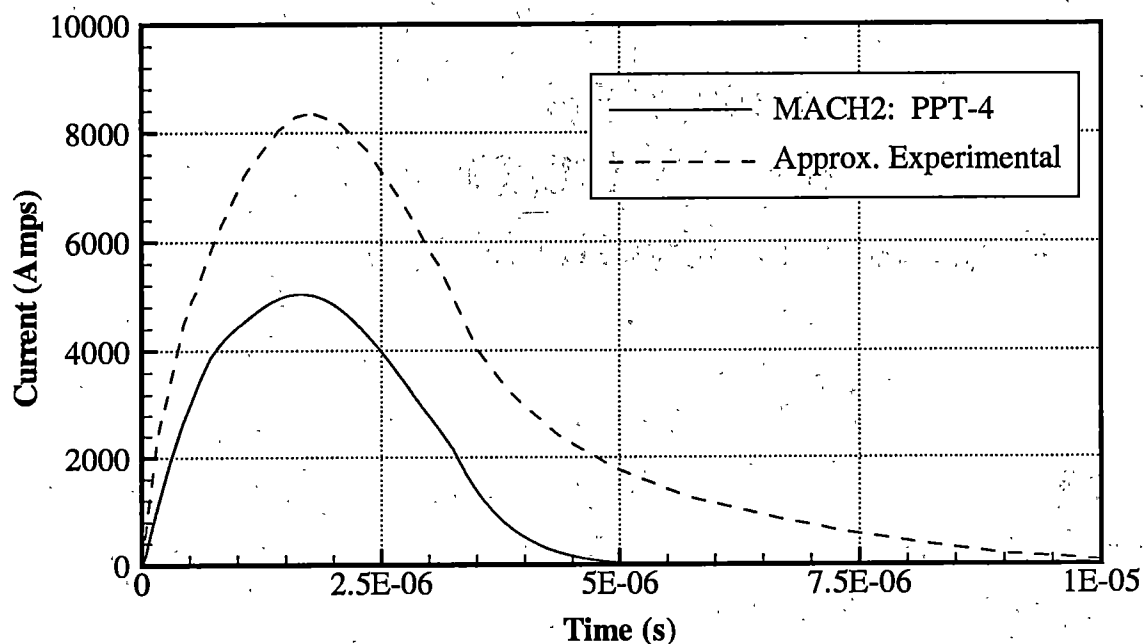


Figure 4.2.3: Modeled Current History in PPT-4 Simulations

The ablated mass per shot for the given 9 J of energy stored in the capacitor is $3.96\text{e-}08$ kg [16]. For a basis of comparison to the cablegun operation, line integrals of electron number density are calculated at 2.5 cm and 5.0 cm from the thruster exit for a simulation with "fox" set to 1600. The corresponding propellant absorptivity required to give the correct ablated mass is 0.0136, which falls between the cablegun values at that "fox" setting. Figures 4.2.4 and 4.2.5 show the electron number density line integrals for this simulation over the entire shot and at the beginning of the pulse, respectively. As seen in the figures, an initial low density plasma jet exits the thruster between 1 and 8 μs (during the current discharge), which is followed by a slower moving higher density plasma over the remainder of the shot. Using the same method as used in the cablegun simulations to estimate the plasma velocity between the 2.5 and 5.0 cm locations, the velocity is 34,341 m/s for the initial plasma discharge and 4950 m/s for the high density plasma.

The impulse calculations for the PPT-4 simulation are shown in Figures 4.2.6-7. The total impulse produced by the PPT-4 thruster over the entire shot is $2.03\text{e-}04$ N $\cdot$ s. The electrothermal and electromagnetic contributions are $1.95\text{e-}04$ N $\cdot$ s and $7.73\text{e-}06$ N $\cdot$ s, respectively. This corresponds to an electromagnetic contribution of 3.8% to the total impulse. This value is much lower than the 11% predicted in [16]. The small effect of electromagnetic portion of the total impulse is due to the very low density (and small mass) of the plasma that this mechanism accelerates. The bulk of the plasma is joule-heated and remains in the lower section of the thruster for a length of time much greater than the current pulse. This higher density plasma, which produces the majority of the impulse, then thermally expands out the thruster exit. The average specific impulse

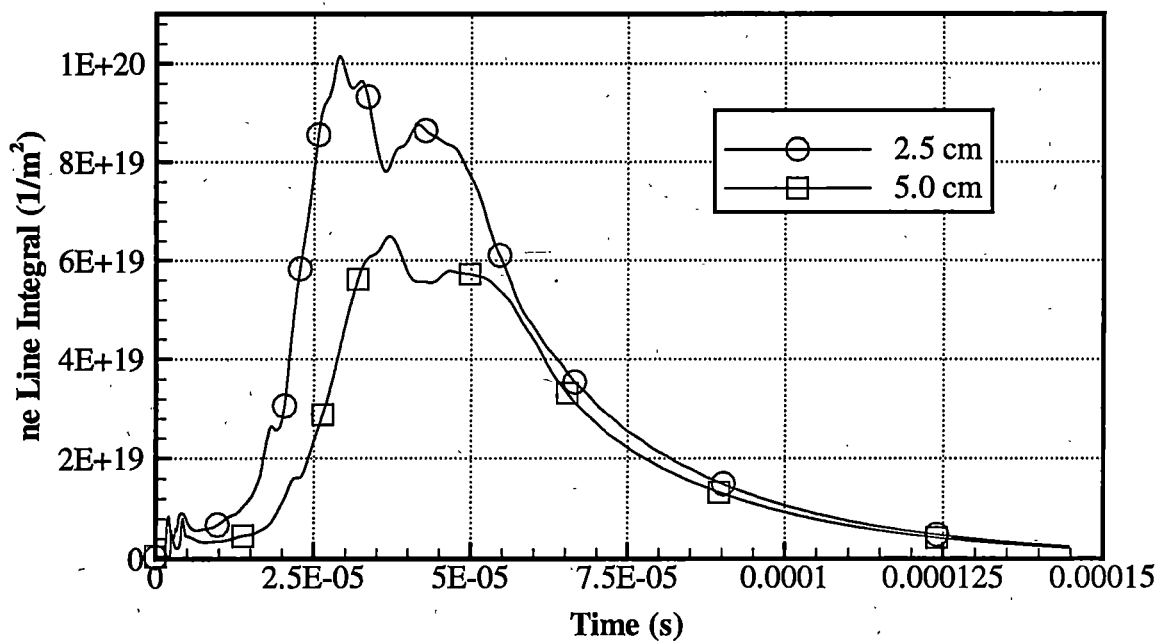


Figure 4.2.4: PPT-4 ne Line Integrals at 2.5 cm and 5.0 cm from Exit

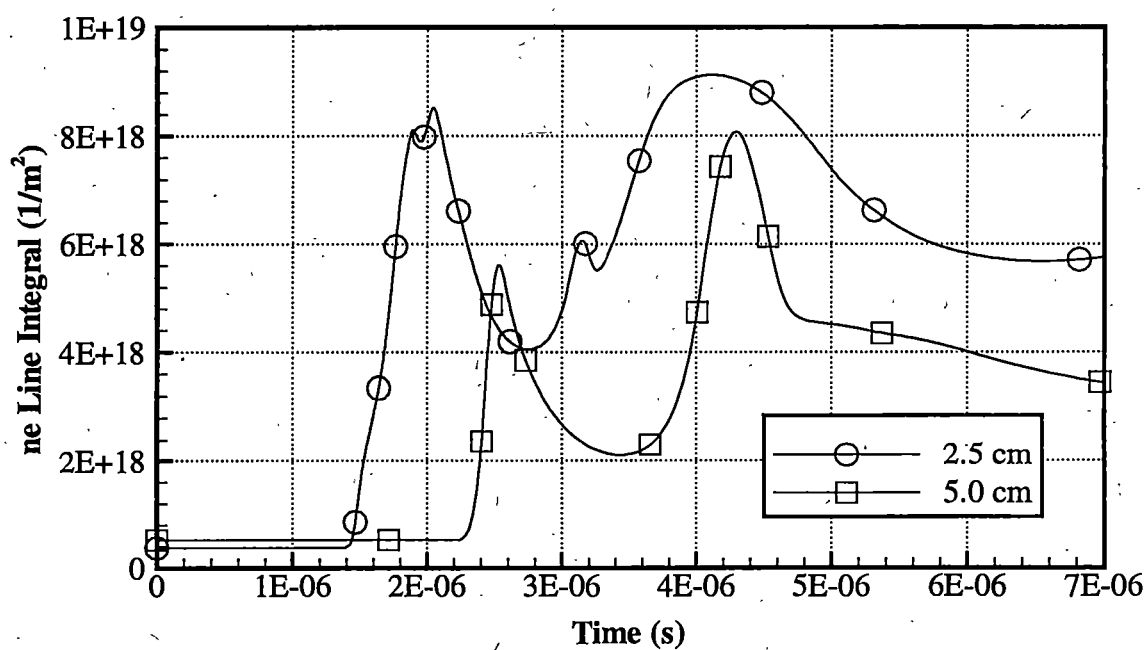


Figure 4.2.5: PPT-4 ne Line Integrals during early Operation

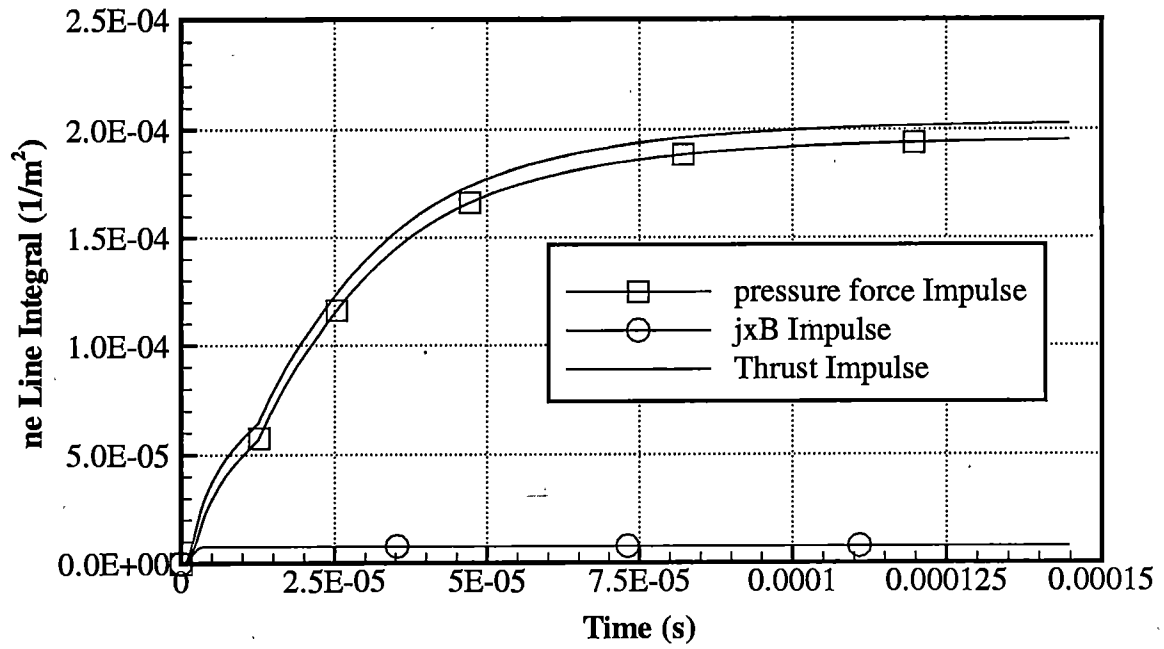


Figure 4.2.6: Simulated Impulse of PPT-4

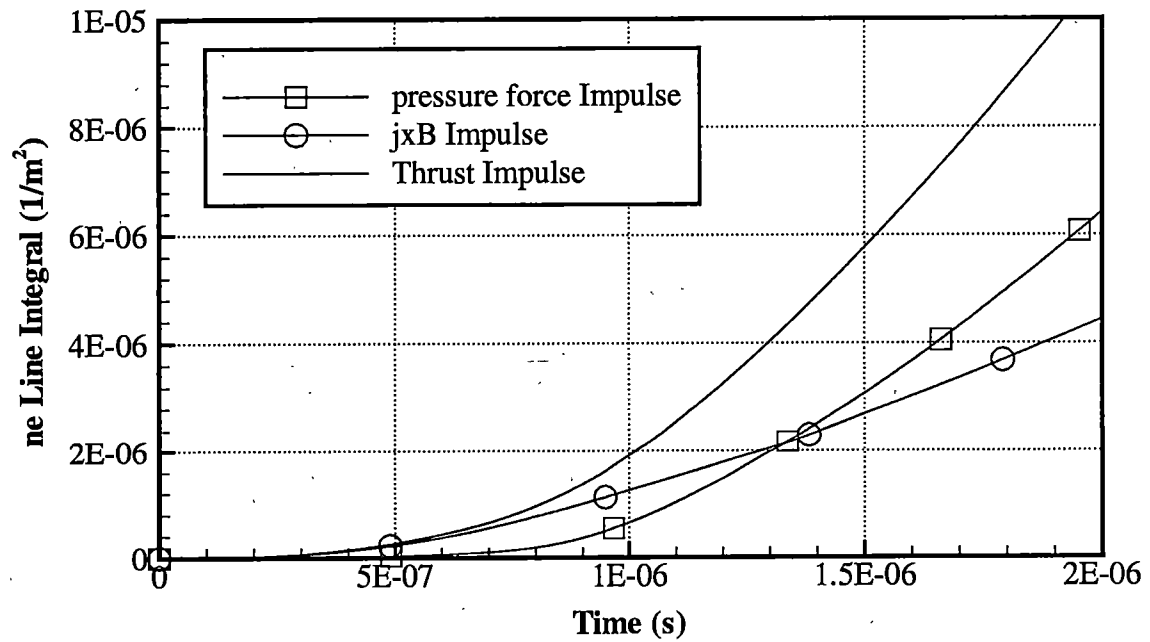


Figure 4.2.7: Simulated Impulse during early Operation of PPT-4

calculated by dividing the total impulse by the propellant weight loss is 521 s. This is only 3.3% different than the specific impulse based on the 4950 m/s velocity of the high density material between 2.5 cm and 5.0 cm from the thruster exit.

Temperature contours for the PPT-4 simulations at $t = 1 \mu\text{s}$ and $25 \mu\text{s}$ are given in Figures 4.2.8 and 4.2.9, respectively. The density contours are given in Figures 4.2.10 and 4.2.11. The times are chosen such that the early time contours capture the fast-moving, low density plasma, while the later time shows the higher density, slower-moving material. The temperatures at the early time are much higher than those seen at $25 \mu\text{s}$, which is consistent with the higher velocities observed initially. As seen in the density contours, a low density plasma jet exits along the center-line of the device. After $25 \mu\text{s}$ has elapsed, more of the higher density plasma has expanded throughout the nozzle. The plasma densities are much lower than those seen in the cablegun simulations. This is due to the much larger size of PPT-4.

Figure 4.2.12 shows the velocity streamlines at $1.0 \mu\text{s}$. Part (a) of the figure shows a re-circulation zone near the anode. This is the result of the build-up of plasma in block 1 during the early operation of the device. The flow coming off the bottom of the ablating Teflon boundary is blocked by plasma at the higher z -locations and is forced into the vortical pattern. The streamlines that terminate on the anode do not represent actual streamlines but are a consequence of the zero-velocity boundary condition set along the anode. The right-hand side of the figure shows the above-mentioned, clearly defined plasma jet traveling close to center-line of thruster. Figure 4.2.13 shows the streamlines after more of the plasma has expanded through the nozzle. Again, the streamlines

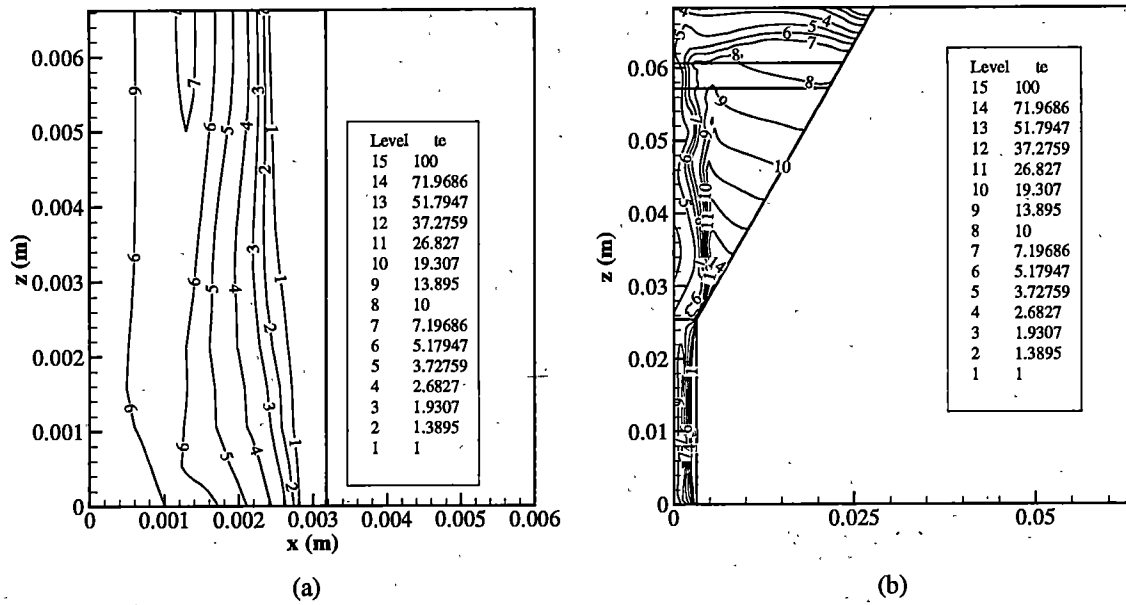


Figure 4.2.8: PPT-4 Temperature Contours (eV) for "fox" = 1600 at $t = 1.0 \mu s$

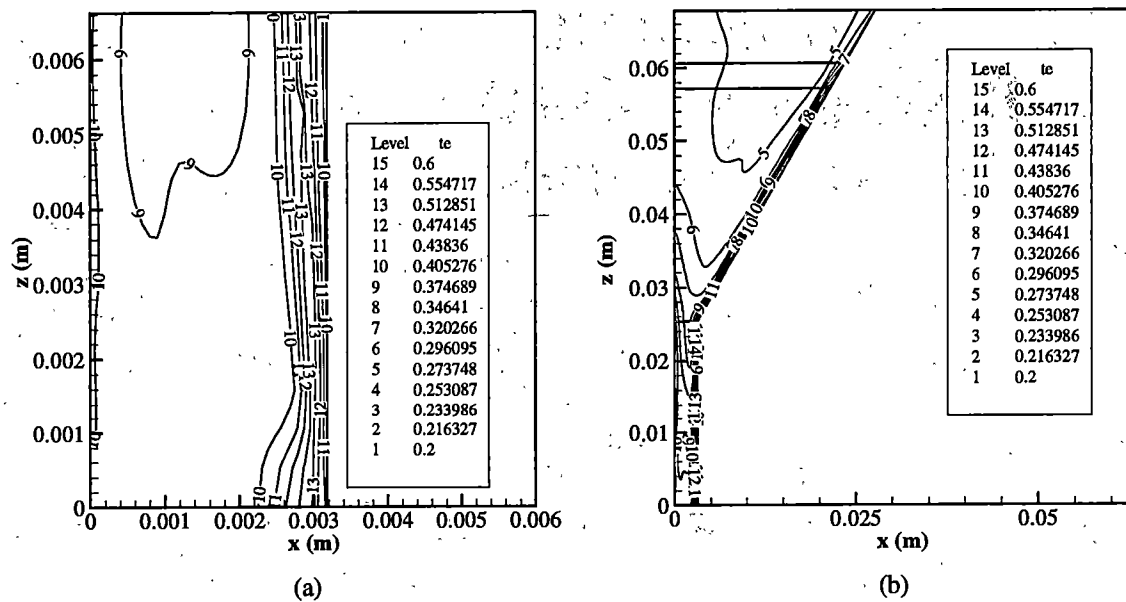


Figure 4.2.9: PPT-4 Temperature Contours (eV) for "fox" = 1600 at $t = 25 \mu s$

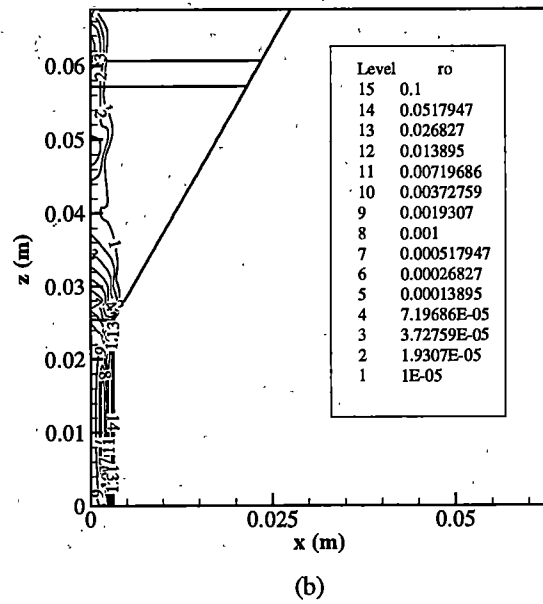
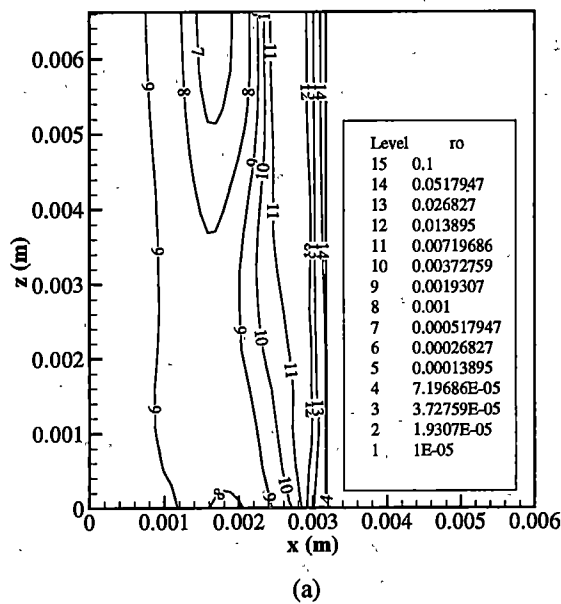


Figure 4.2.10: PPT-4 Density Contours (kg/m^3) for "fox" = 1600 at $t = 1.0 \mu\text{s}$

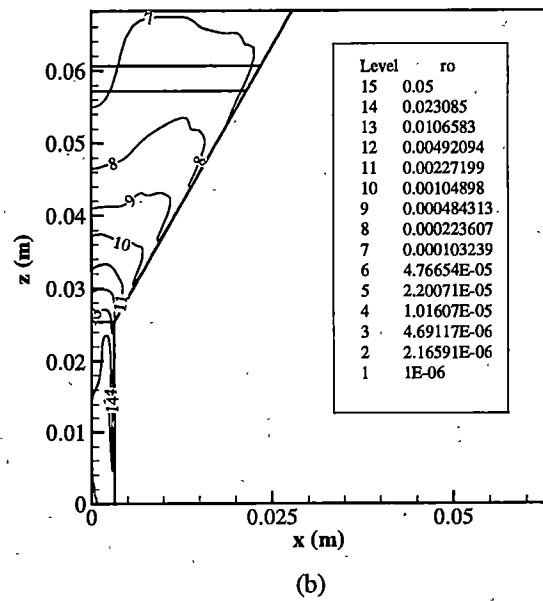
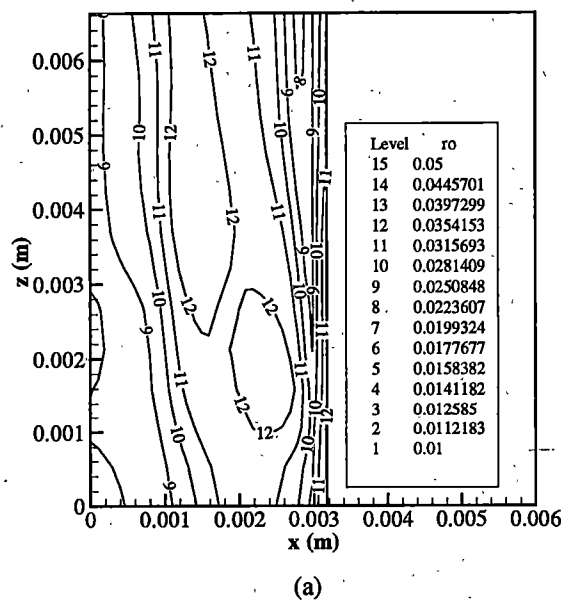


Figure 4.2.11: PPT-4 Density Contours (kg/m^3) for "fox" = 1600 at $t = 25 \mu\text{s}$

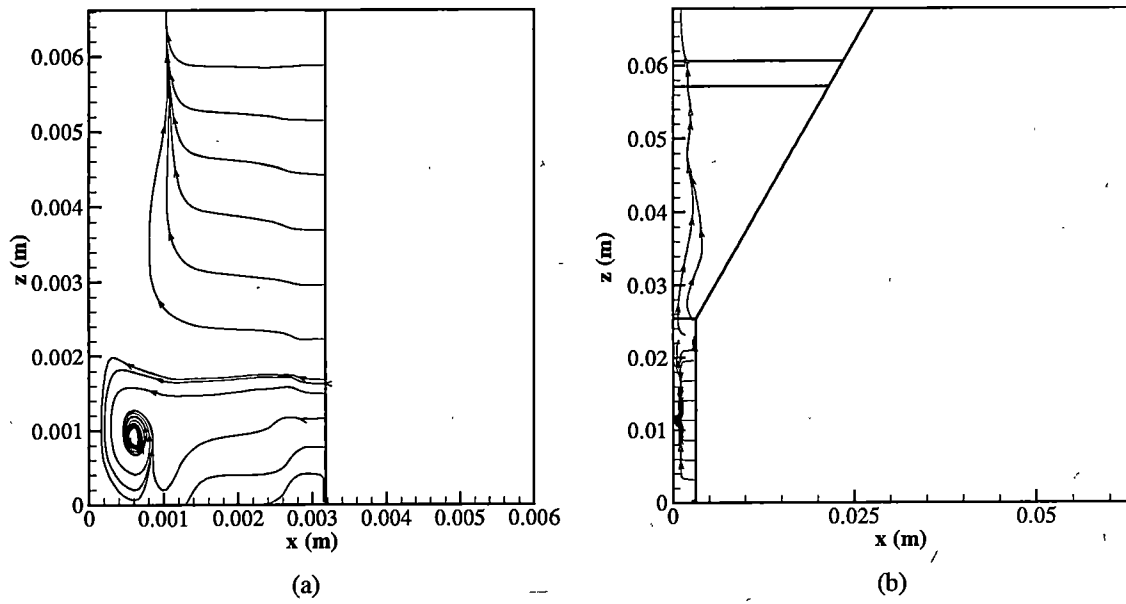


Figure 4.2.12: PPT-4 Velocity Streamlines for "fox" = 1600 at $t = 1.0 \mu\text{s}$.

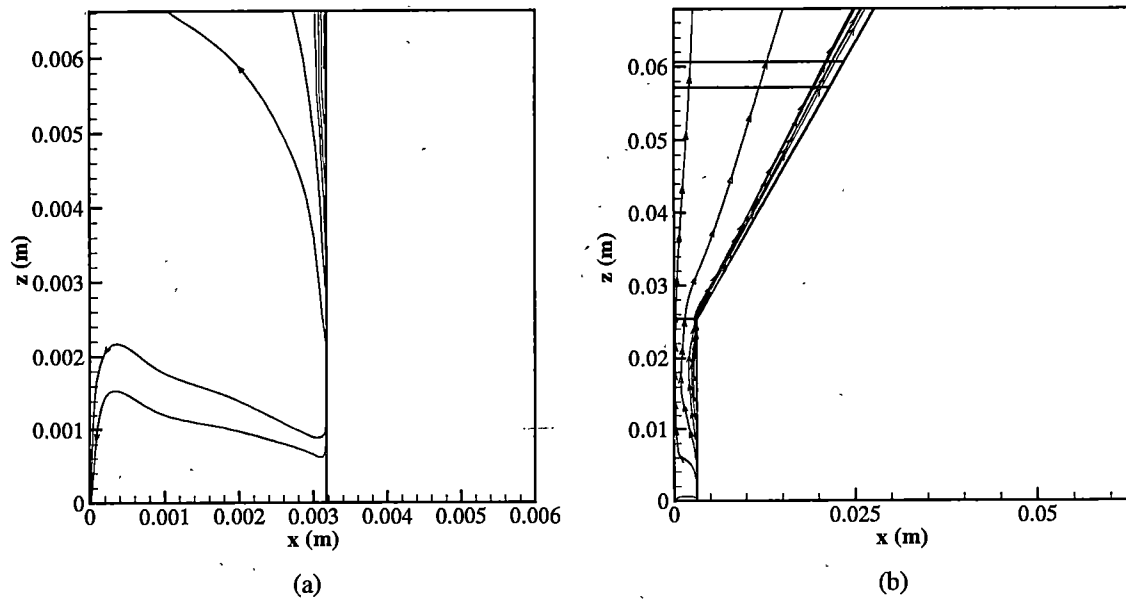


Figure 4.2.13: PPT-4 Velocity Streamlines for "fox" = 1600 at $t = 25 \mu\text{s}$.

directed toward the anode are the result of plasma mass lingering in the chamber which forces the lower material toward the anode.

5. PRELIMINARY CABLEGUN SPECTROSCOPIC ANALYSIS

The objective of this chapter is to identify the species present in the cablegun exhaust and to estimate the temperature of the ejected plasma by analyzing spectroscopic data taken during the experimental operation of the device. Species of interest include the ions of carbon and fluorine from the Teflon propellant along with copper from the cathode.

The spectroscopic data analyzed in this study give line intensities over a wavelength range from 330 nm to 450 nm and correspond to a 1200 groove grating in the spectrometer. The intensity curves over the observed wavelength range are shown in Figures 5.1-3. Several species transition lines are labeled using line data given in Striganov and Svetitskii [32]. To assist in the analysis, the relative ionization energies and observed lines are displayed in Figure 5.4 along with the Boltzmann electron energy distributions at various temperatures of interest.

No lines are observed for elemental carbon (C). However, spectral transitions are observed for C⁺ and F⁺ (given below) at energy levels greater than the carbon ionization potential (E_i) of 11.27 eV. This supports the expected result that all the ablated CF₂ is broken down into its constituent elements, which are then excited to their ionized states. A strong doublet transition for C⁺ ($E_i = 24.38$ eV) is observed at $\lambda = 426.5$ nm. These lines are the result of a transition from 20.95 eV to 18.05 eV. No transitions are observed for C⁺⁺ ($E_i = 47.9$ eV). This suggests that either no C⁺⁺ is present due to insufficient energy input into the plasma, or C⁺⁺ is in fact present with transitions occurring in the ultra-violet (UV) region. Within the experimental wavelength range, the expected

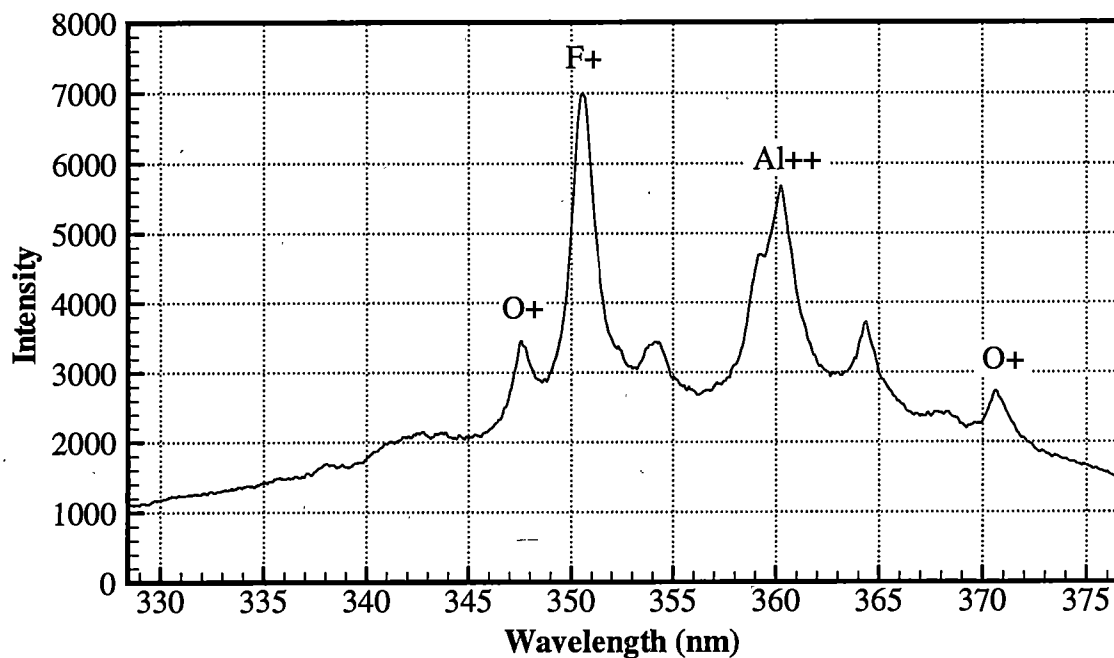


Figure 5.1: Spectral Data from 330-375 nm

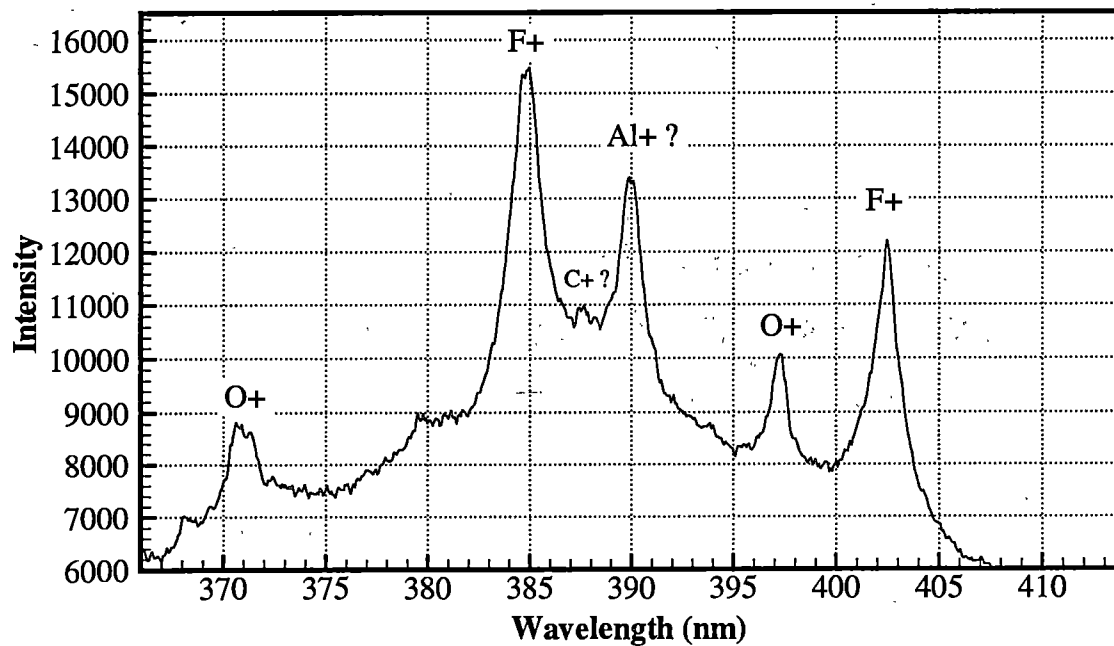


Figure 5.2: Spectral Data from 370-410 nm

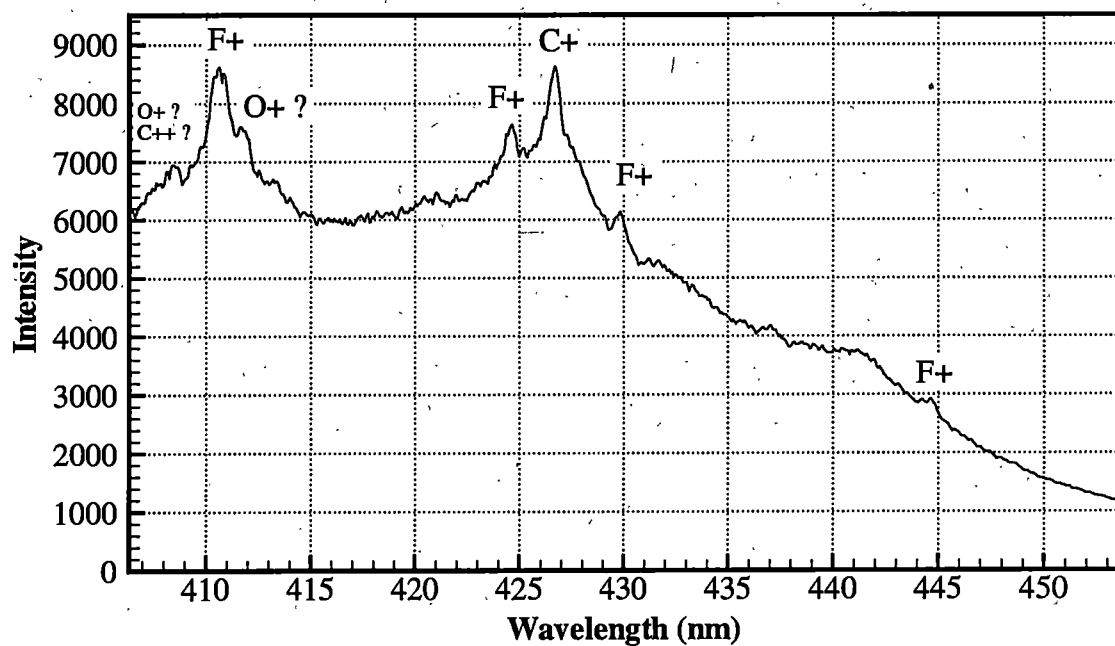


Figure 5.3: Spectral Data from 410-450 nm

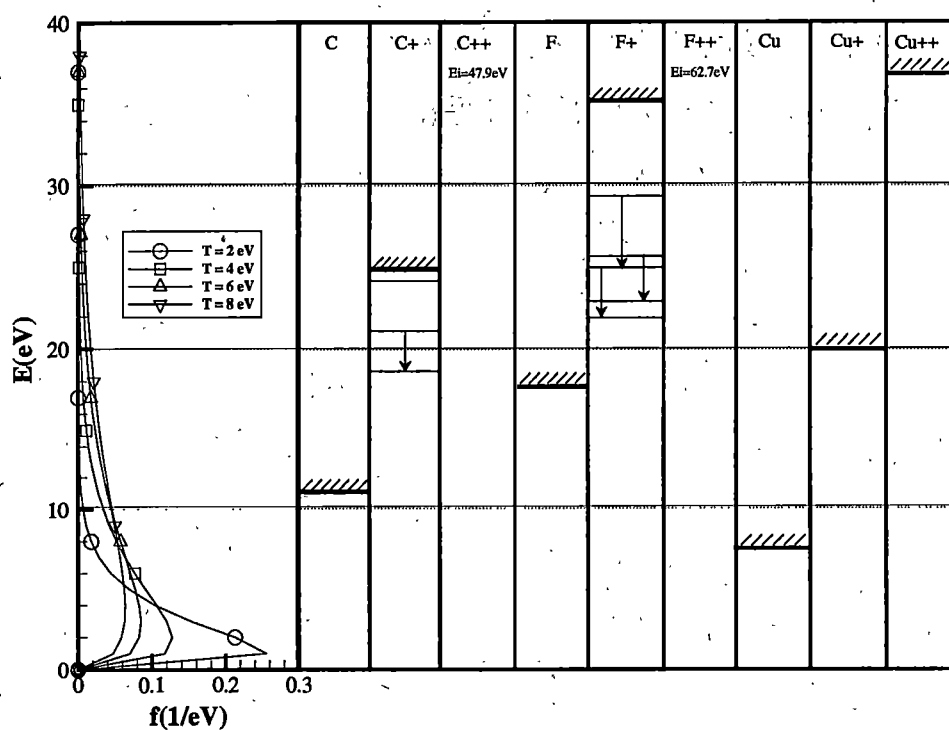


Figure 5.4: Energy Levels for Ions and Electrons

transition energy levels (40-50 eV) for C^{++} are too high to be significant; as shown in the electron energy distributions in Figure 5.4, a lower number of electrons are available at the higher energies. However, Striganov does list spectral lines for C^{++} in the UV range between energy levels similar to those observed for F^{+} (given below), which are above the ionization potential of C^{+} . Therefore, many of the C^{+} ions are most likely ionized to C^{++} in the plasma, and the transition lines exist in the UV range.

No lines are observed for F ($E_i = 17.4$ eV). However, strong F^{+} transitions are detected. Therefore, most of the fluorine atoms are ionized. Several spectral lines are observed for F^{+} ($E_i = 35.0$ eV), most prominently at wavelengths of 351 nm, 385 nm, and 402.5 nm. At $\lambda = 351$ nm, 8 lines exist that are transferring from 28.66 eV to 25.12 eV. A triplet state exists at 385 nm with transitions from 25.12 eV to 21.9 eV. The 402.5 nm peak represents another triplet state from 25.75 eV to 22.7 eV. No transitions are observed for F^{++} ($E_i = 62.7$ eV). Due to the high energy requirement in producing this ion, there is probably very little or no F^{++} in the plasma exhaust.

No atomic copper ($E_i = 7.73$ eV) lines are given in the observed spectra. C^{+} and F^{+} transitions exist in the 20 to 30 eV range, which is much greater than the 7.73 eV ionization potential of Cu. This suggests that most of the copper is ionized to Cu^{+} and/or Cu^{++} . No Cu^{+} ($E_i = 20.3$ eV) lines are given in Figures 5.1-3. However, several strong intensity lines of transitions between 15 and 18 eV are listed in Striganov over the wavelength range viewed in the experiment. Since we observed strong C^{+} and F^{+} lines at higher transition energies, it is likely that most of the ablated copper exists as Cu^{++} in the plasma. Like Cu^{+} , no Cu^{++} ($E_i = 36.83$ eV) lines are observed in the data, but Striganov lists several lines in the ultra-violet region that occur between the probable

energy transition levels (20-30 eV). Therefore, Cu^{++} is expected to exist in the cablegun exhaust even though no spectral lines are viewed experimentally.

Another atomic species to look for in the spectral analysis is neutral oxygen along with the excited states ($\text{O}:\text{Ei} = 13.62 \text{ eV}$; $\text{O}^+:\text{Ei} = 35.12 \text{ eV}$; $\text{O}^{++}:\text{Ei} = 54.95$). Oxygen exists in the vacuum chamber due to incomplete evacuation of the air and also due to air molecules trapped in the Teflon and copper materials during their formation. No neutral oxygen lines are observed, because the O atoms are ionized to O^+ . This explains the O^+ lines in the data with transitions between 22 eV and 32 eV. The spectrum data contain no O^{++} lines due to the larger energy requirements within the viewable range (33 eV – 50 eV), not to mention the 35.12 eV required to produce O^{++} from O^+ . However, a small amount of O^{++} (compared to O^+) may exist with spectral lines, corresponding to transitions between 23 and 37 eV, falling in the UV range.

From the ablated Teflon and copper, only C^+ and F^+ show up in the observed spectral range. A larger number of clearly defined F^+ lines are observed compared to C^+ . A possible explanation is that the F^+ transitions are occurring at higher energies than the ionization potential of C^+ , and, therefore, many C^+ ions may be excited to C^{++} . In addition, the plasma exhaust likely contains Cu^{++} due to the large amount of copper ablated from the cathode during testing; the corresponding Cu^{++} transitions occur within the energy ranges given for the F^+ and C^+ lines but are in the UV region outside the range of observation. Most of the fluorine exists as F^+ , while the copper is expected to be in the Cu^{++} state. All of the F, C, and Cu atoms (and most of the Cu^+) are excited to their higher ionic states, but not enough energy is input to the plasma to obtain significant amounts of F^{++} , C^{+++} , Cu^{+++} or higher energy ions.

The mole fractions of the plasma constituents are predicted by solving the Saha equations over a wide temperature range for an intermediate density of 0.01 kg/m^3 and are shown in Figure 5.5-7. Given the relative amounts of the ions resulting from the rudimentary spectral analysis and comparing to the Saha calculations, the temperature of the plasma is expected to fall within a lower limit of 2.2 eV and an upper limit of 2.8 eV. The lower limit is set by the conclusion that most of the F atoms are ionized, and the upper limit is set by the assumption that there are very few F^{++} ions in the plasma.

In the above rudimentary analysis of the cablegun spectrum, the plasma temperature is estimated to fall between 2.2 eV and 2.8 eV. Using this temperature range and the Saha-predicted temperature variations of the mole fractions given in Figures 5.5-7, the Teflon-copper plasma is expected to consist of similar amounts of C^+ and C^{++} . Most of the fluorine exists in the F^+ state, with a very small amount existing as F^{++} . Most of the copper in the plasma is expected to exist as Cu^{++} ; a small amount of the ablated copper may be singly ionized (Cu^+).

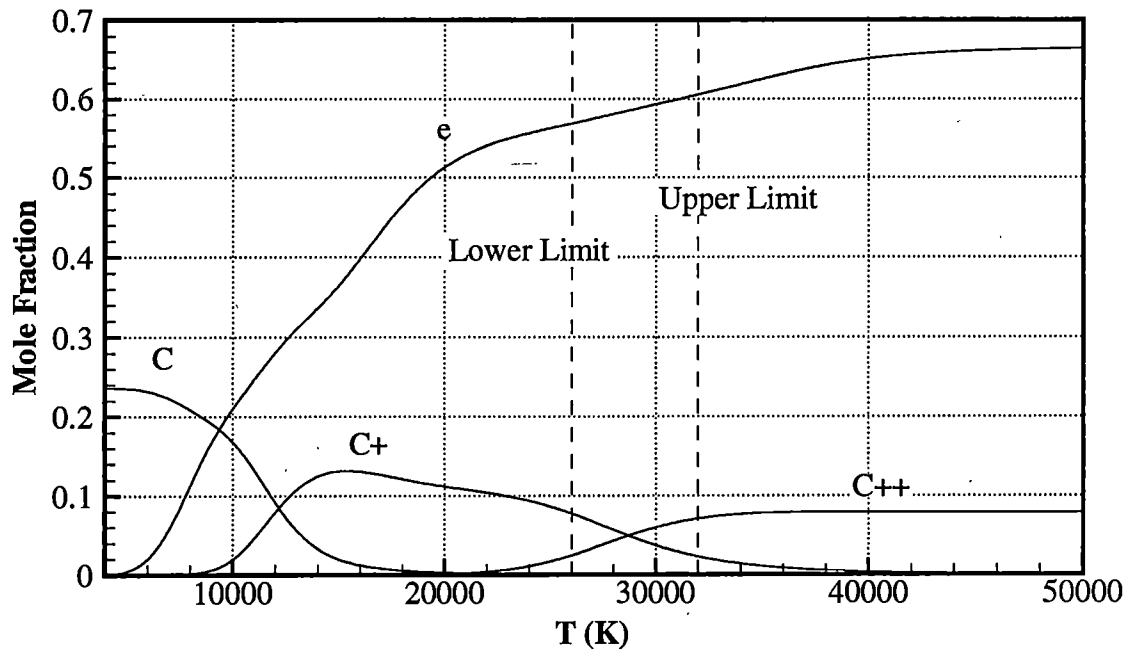


Figure 5.5: Saha Mole Fractions for C for $\rho = 0.01 \text{ kg/m}^3$ (39% Teflon, 61% Cu)

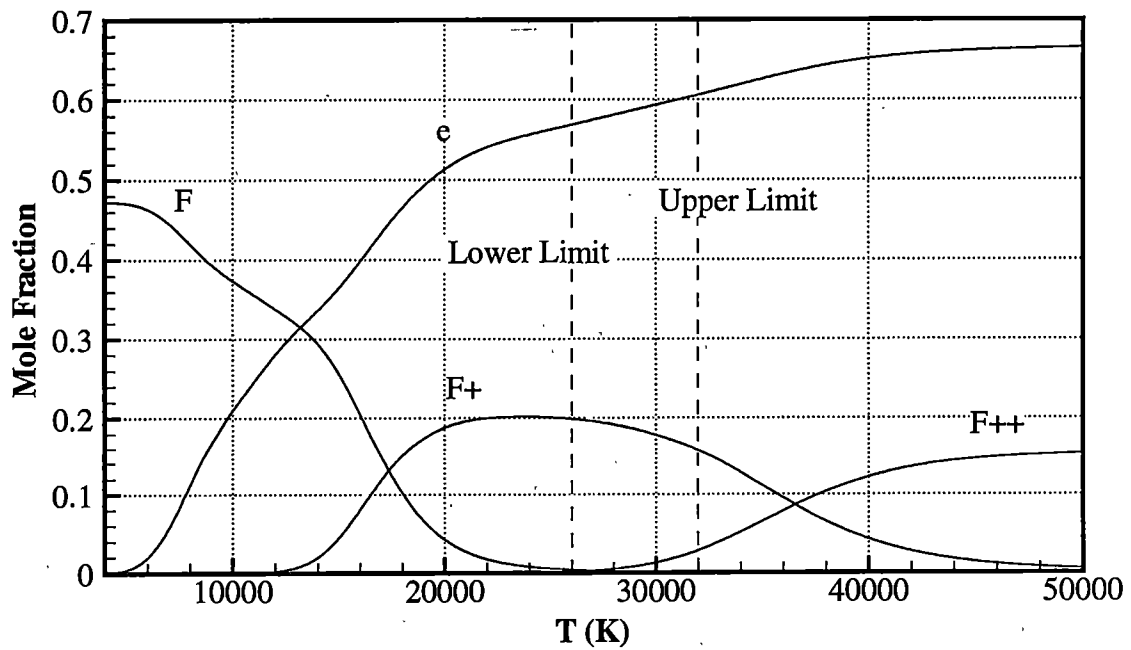


Figure 5.6: Saha Mole Fractions for F for $\rho = 0.01 \text{ kg/m}^3$ (39% Teflon, 61% Cu)

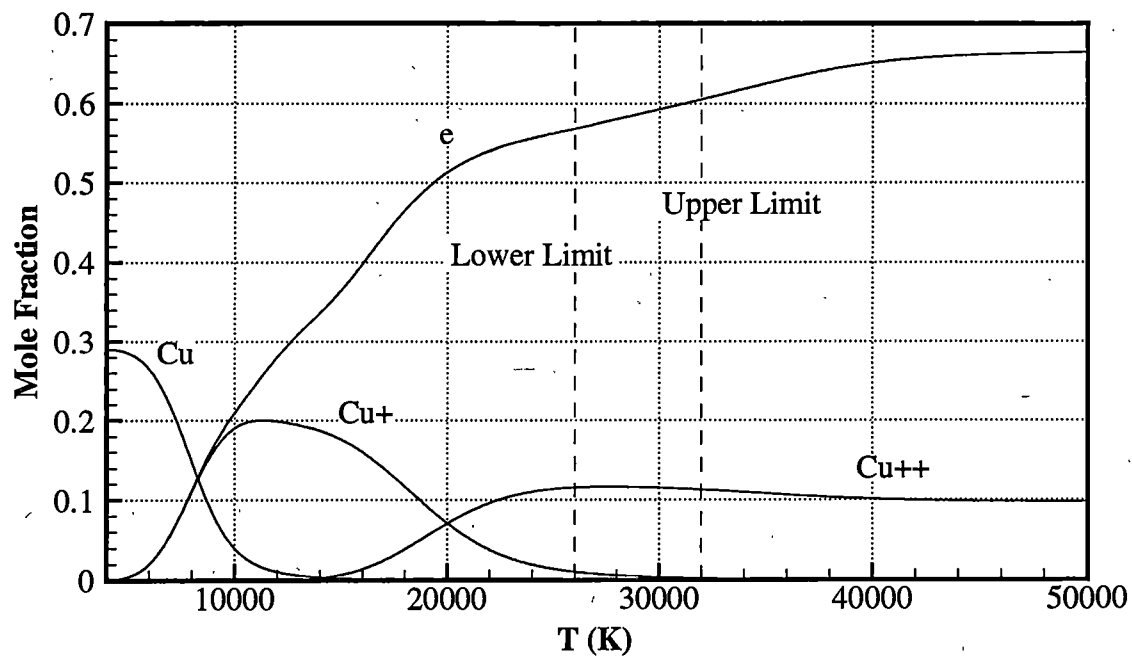


Figure 5.7: Saha Mole Fractions for Cu for $\rho = 0.01 \text{ kg/m}^3$ (39% Teflon, 61% Cu)

6. CONCLUSIONS AND RECOMMENDATIONS

Using the new radiation model and equation of state tables developed in this study, the MACH2 simulations of the cablegun and PPT-4 geometries capture the overall physics of the plasma flows in these devices. In both cablegun and PPT-4 geometries, the simulations show initial, low density and high velocity plasmas exiting the thrusters followed by higher density plasmas that move at lower velocities. The addition of copper into the cablegun plasma reduces the plasma exhaust velocity due to the higher molecular weight of the Teflon-copper mixture compared to the Teflon-only plasma.

An indicator of the overall performance of the cablegun and PPT-4 thrusters is the average specific impulse (I_{sp}), calculated by dividing the total impulse by the propellant weight required to produce that impulse. The higher the specific impulse, the less fuel is required to achieve the desired total impulse. Another design parameter is the total impulse divided by the energy stored in the capacitor (I_T/E_0). This is an indicator of which thruster provides the most impulse for a given energy input into the device (including circuit) and is proportional to the mass of the required power-supply. Table 6.1 gives a summary of the performance calculations for the thrusters analyzed in this study. The Teflon-only cablegun produces the highest specific impulse, while the PPT-4 outputs the most impulse for a given energy stored in the capacitor. Therefore, the Teflon-only cablegun is the preferred device if the propellant weight is the design parameter. However, if the power-supply mass is the overall design concern, PPT-4 is clearly the better choice.

Due to its large size and geometry, the PPT-4 device operates mainly as an

Table 6.1: Summary of Thruster Performance

Thruster	I_{sp} (s)	I_T/E_0 (s/m)
Cablegun: Teflon-copper	695	9.4e-06
Cablegun: Teflon-only	1254	3.5e-06
PPT-4	521	2.3e-05

electrothermal device (96.2%). The Teflon-copper cablegun produces an impulse which is 92.3% electrothermal, and the cablegun with the tungsten cathode produces an impulse with the highest electromagnetic fraction of 14.2%, which corresponds to a 85.8% gasdynamic contribution. The electromagnetic effect is smallest in the PPT-4 device, because the plasma continues to collect in the lower part of the thruster, long after the current goes to zero. Therefore, the bulk of the ejected plasma expands out the nozzle with no influence from electromagnetic forces. A possible area for improving the performance of the PPT-4 is designing a smaller device. A propellant cavity of similar geometry (but smaller) would still promote a pressure buildup while allowing the plasma to exit the cavity during the current pulse, thereby increasing the electromagnetic contribution to the thrust. Also, different nozzle geometries may provide better electrothermal acceleration of the plasma in both the cableguns and the PPT-4 thruster. Another possibility for increasing the performances of the thrusters is the use of polyethylene as the solid propellant. Recent studies at UTSI [33] have shown that using a propellant with a lower molecular weight increases the specific impulses achieved by cablegun geometries.

The thermodynamic properties developed in this dissertation show variations with temperature and density that comply with the expected physics of the problem, such as the relation of the properties to the formation of new ionic species in the plasma. Also, no problems are evident in the blending of the CEA and Saha results. However, one area that requires more attention is the filtering process performed on the C_v data.

Theoretically, a low-pass digital filter can be designed to more successfully remove the data noise while maintaining the overall physics in the results. For future work, the author suggests updating the CEA libraries with more accurate data for copper atoms and ions. Also, the Saha method outlined in this study can be modified to compute molecular as well as atomic species at the higher temperatures by adding the effects of molecular vibration and rotation to the partition functions [34].

Another area of recommended study is the radiation model developed in this dissertation. During the early phases of operation, the plasma densities are not high enough to produce the collisions necessary for a Maxwell-Boltzmann distribution. Therefore, the developed equilibrium radiation model and equation of state tables are not accurate during the early segment of the thruster shot. Low density models should be implemented during the initial phases of the thruster operation. The models developed in this study should be used when enough mass has ablated into the thruster volume to provide enough particle collisions to produce equilibrium.

An initial attempt was made to implement the thermal conduction capability in MACH2 with little success. A PPT-4 simulation was run with the "Spitzer" thermal diffusion model in MACH2, and the results were the same as those without thermal diffusion. This may be the result of an incorrectly implemented input file which caused

MACH2 to turn off thermal diffusion by default. By including thermal diffusion, the simulations would more accurately model the energy lost from the plasma to the thruster walls; Burton and Bushman [16] measured severe heat losses to the walls of the PPT-4 thruster. Including the effects of thermal conduction would lower the plasma temperature (which MACH2 tends to overestimate) and, therefore, would lower the "fox" required to achieve accurate plasma temperatures and velocities.

For future PPT-4 (and other devices with low, initial voltages across the capacitors) simulations, improvements must be made in the MACH2 calculation of the voltage across the electrodes. Currently, MACH2 calculates the port voltage using Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot \hat{n} dA \quad (6.1)$$

where the line integral on the left-hand side of equation (6.1) is calculated across the sidewalls in blocks 1, 2, and 3, across the upper boundary of block 3, down the centerline, and then across the bottom of block 1 (see Figure 4.2.1). The term on the right-hand side is the integral of the magnetic flux through the surface defined by the three blocks. The problem occurs in the line integrals across the upper boundary of block 3 and down the portion of the centerline where the currents are very weak and the resistivities are high due to extremely low plasma densities during the current pulse. A possible solution is to calculate an average port voltage (V_p) using a power-based method. The power flowing into the plasma at a given instant is determined by:

$$P = \frac{d}{dt} \iiint \frac{B^2}{2\mu_0} dV + \iiint \eta J^2 dV \quad (6.2)$$

If the first term on the right is small compared to the second, it may be possible to back out the port voltage from the calculated power ($P = IV_p$).

Out of curiosity, an attempt was made to increase the simulated PPT-4 current by raising the voltage (and energy stored) across the capacitor. Increasing the voltage to 2160 volts produces the current history shown in Figure 6.1, which is much closer to the approximated experimental distribution. The impulse calculated for this simulation increased to 2.5×10^{-4} N·s, which is only 7.4% different from the experimentally observed 2.7×10^{-4} N·s given in [16]. The computed electromagnetic contribution increased to 6.8% but is still much lower than the reported value of 14%.

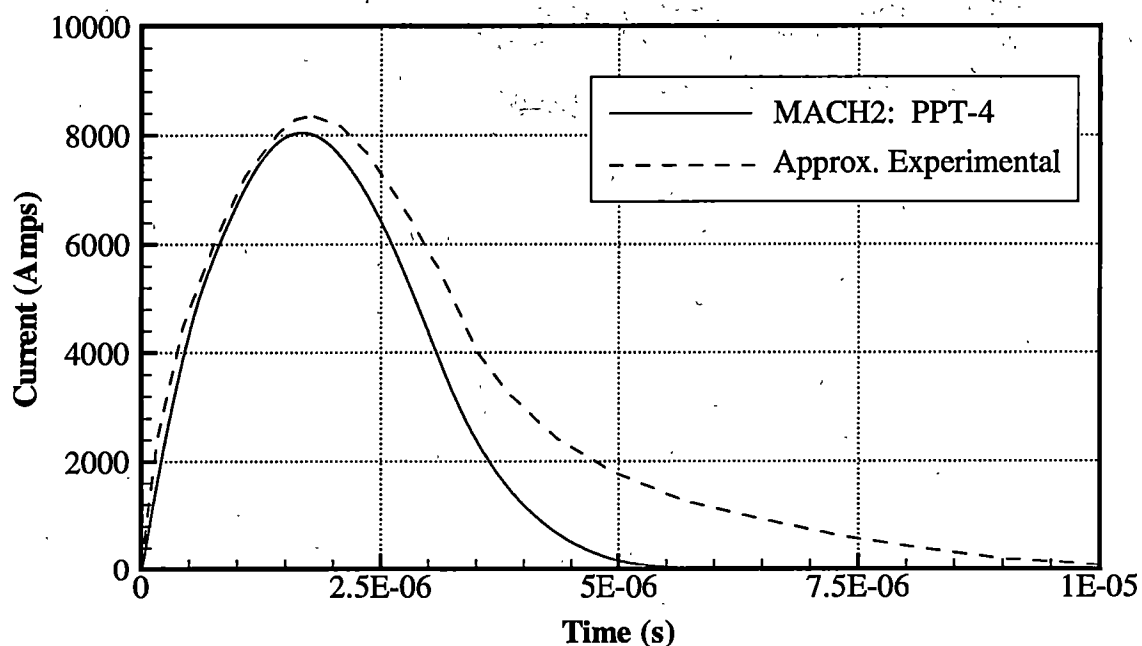


Figure 6.1: Simulated PPT-4 Current History for an Initial Voltage of 2160 Volts across the Capacitor

LIST OF REFERENCES

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1. Stuhlinger, E., "Origin and Early Development of Electric Propulsion Concepts", AIAA-91-3442, AIAA/NASA/OAI Conference on Advanced SEI Technologies, Cleveland, OH, September 4-6, 1991.
2. Jahn, R.G., Physics of Electric Propulsion, McGraw-Hill, Inc., New York, 1968.
3. Meckel, N.J., et.al., "Improved Pulsed Plasma Thruster Systems for Satellite Propulsion", AIAA-96-2735, 32nd. AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Lake Buena Vista, FL, July 1-3, 1996.
4. Goyer, J.R., Kortbawi, D., Childers, F.K., and Sincerny, P.S., "Low Jitter Operation of a Plasma Opening Switch", *Journal of Applied Physics*, Vol. 74, September 1993, pp. 4236-4238.
5. Solbes, A. and Vondra, R.J., "Performance Study of a Solid Fuel-Pulsed Electric Microthruster", *Journal of Spacecraft and Rockets*, Vol. 10, No. 6, June 1973, pp. 406-410.
6. Burton, R.L. and Turchi, P.J., "Pulsed Plasma Thruster", *Journal of Propulsion and Power*, Vol. 14, No. 5, September-October 1998, pp. 716-735.

7. Peterkin, R.E., Jr. and Frese, M. H., MACH: A Reference Manual-First Edition, Air Force Research Laboratory: Phillips Research Site, Kirtland AFB, NM., 1998.
8. Stuhlinger, E., "Electric Propulsion -- Past History and Future Prospects", AIAA-74-1081, AIAA/SAE 10th Propulsion Conference, San Diego, CA, October 21-23, 1974.
9. Oberth, H., Wege zur Raumschiffahrt, R. Oldenbourg KG, Munich, 1929.
10. Stuhlinger, E., "Possibilities of Electric Space Ship Propulsion", Ber. 5. Intern. Astronautical Kongress, Innsbruck, p. 81, August 1954.
11. Dornheim, M.A., "DS1 Asteroid Flyby Yields Uncertain Results", *Aviation Week & Space Technology*, Vol. 151, No. 5, August 2, 1999, p. 32.
12. Dunning, J. and Sankovic, J., "NASA's Electric Propulsion Program", AIAA-99-2161, 35th. AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Los Angeles, CA, June 20-24, 1999.
13. Garkusha, V., et.al., "Modern Status of Hall Thrusters Development in Russia", AIAA-99-2157, 35th. AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Los Angeles, CA, June 20-24, 1999.

14. Arrington, L.A., Haag, T.W., Pencil, E.J., and Meckel, N.J., "A Performance Comparison of Pulsed Plasma Thruster Electrode Configurations", 25th International Electric Propulsion Conference, IEPC-97-127, August 1997.
15. Vondra, R.J. and Thomassen, K.I., "A Flight Qualified Pulsed Electric Thruster for Satellite Control", AIAA-73-1067, AIAA 10th Electric Propulsion Conference, Lake Tahoe, NV, October 31 - November 2, 1973, 1973.
16. Burton, R.L. and Bushman, S.S., "Probe Measurements in a Coaxial Gasdynamic PPT", AIAA-99-2288, 35th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Los Angeles, CA, June 20-23, 1999.
17. Frese, M.H., "MACH2: A Two-Dimensional Magnetohydrodynamic Simulation Code for Complex Experimental Configurations", Prepared for: Weapons Laboratory/AWPP, Kirtland Air Force Base, NM 87117-6008, Contract F29601-86-C-0216, Subtask 03-07/03, CDRL A015 Section 4.6, March 1, 1990.
18. Siegel, R. and Howell, J.R., Thermal Radiation Heat Transfer, 3rd Ed., Hemisphere Publishing Co., Washington, 1992, pp. 66-72.
19. Modest, M.F., Radiative Heat Transfer, McGraw-Hill, Inc., New York, 1993, p. 312.

- 20 Gordon, S. and McBride, B.J., Computer Program for Calculation of Complex Chemical Equilibrium Compositions and Applications. 1. Analysis, NASARP-1311, 1994.
- 21 Marshall, R.A., "The Properties of Copper and Hydrogen Plasmas", Published at 80 Cliff Rd., Wollongon, NSW 2500, Australia, 1986.
- 22 Kelleher, D.E., NIST Standard Reference Database 61, Version 1, U.S. Department of Commerce, Gaithersberg, MD, 1995.
- 23 Vincenti, W.G. and Kruger C.H., Jr., Introduction to Physical Gas Dynamics, John Wiley and Sons, Inc., New York, 1965, p. 459.
- 24 Boulos, M.I., et. al., Thermal Plasmas: Fundamentals and Applications, Vol. 1, Plenum Press, New York, 1994, pp. 151-161.
- 25 Griem, H. R., Principles of Plasma Spectroscopy, Cambridge University Press, 1997, pp. 194-195.
- 26 Sember, V., "Study of Non-equilibrium Effects in Hydrogen-oxygen Thermal Plasma Jet, Heat and Mass Transfer under Plasma Conditions: Proceedings of the International Symposium on Heat and Mass Transfer under Plasma Conditions, Begell House, New York, 1995, pp. 145-146.

- 27 Burden, R.L. and Faires, J.D., Numerical Analysis, PWS-Kent Publishing Company, Boston, MA, 1989, p. 53.
- 28 Sontagg, R.E. and VanWylen, G.J., Fundamentals of Statistical Thermodynamics, Krieger Publishing Company, Malabar, Florida, 1985, pp. 86-87.
- 29 Astola, J. and Kuosmanen, P., Fundamentals of Nonlinear Digital Filtering, CRC Press, New York, 1997, pp. 52-53.
- 30 Smith, L.M., Keefer, D.R., and Wright, N.W., "Interferometric Investigation of a Cablegun Plasma Injector", Submitted to IEEE Transactions on Plasma Science, December 1999.
- 31 Keefer, D. R., Personal Communications, December 17, 1999.
- 32 Striganov, A.R. and Sventitskii, N.S., Tables of Spectral Lines of Neutral and Ionized Atoms, IFI/Plenum Data Corporation, 1968.
- 33 Walloschek, T.H., "Coaxial Thermal Pulsed Plasma Thrusters", Master of Science Thesis in Aerospace Engineering, The University of Tennessee, Knoxville, May 1998.
- 34 Turchi, P.J., "Transport Processes in Pulsed Plasma Thrusters", AFRL-SR-BL-TR-98-0265, Ohio State Univ. Research Foundation, Columbus, OH, 1997.

VITA

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