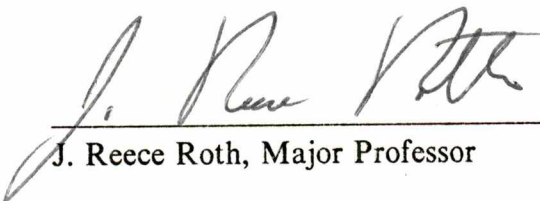
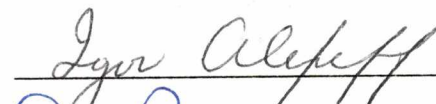
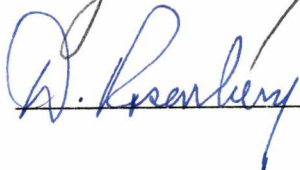


To the Graduate Council:

I am submitting herewith a thesis written by Larry R. Baylor entitled "Microwave Scattering as a Plasma Diagnostic in a Penning Discharge." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.


J. Reece Roth, Major Professor

We have read this thesis
and recommend its acceptance:

Accepted for the Council:


The Graduate School

✓

MICROWAVE SCATTERING AS A PLASMA DIAGNOSTIC
IN A PENNING DISCHARGE

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Larry R. Baylor

June 1984

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ABSTRACT

A microwave scattering apparatus consisting of 32.175-GHz Gunn diode in a conventional homodyne mixer configuration has been built to investigate plasma turbulence in a steady-state classical Penning discharge. The microwaves are incident on the plasma in the ordinary mode at a power level of 25 mW. The scattered microwave power is observed for scattering angles from 20 to 160° in a plane normal to the magnetic field axis. The scattered microwaves are fed to a balanced mixer where they are detected with crystal detectors. The detected signal is then displayed on a spectrum analyzer or fed into an analog-to-digital data handling system for analysis and correlation with other signals.

In these experiments the electron number densities ranged from 10^8 to $10^{10}/\text{cm}^3$, the magnetic field ranged from 0.1 to 0.4 T, the discharge current ranged up to 0.2 A, and the input power ranged from 50 to 400 W. The frequency spectrum, angular dependence, and scattering amplitudes were measured with the microwave scattering diagnostic. Data from the microwave scattering diagnostic were correlated with plasma operating conditions and with data from capacitive probes which measured electrostatic turbulence. The data indicate strong, low-frequency electrostatic turbulence in the plasma, caused by $\mathbf{E} \times \mathbf{B}$ drift.

TABLE OF CONTENTS

CHAPTER	PAGE
INTRODUCTION	1
I. SCATTERING THEORY	2
Scattering of Electromagnetic Radiation	2
Scattering from a Plasma	7
Scattered Spectral Density	9
II. PLASMA TURBULENCE	12
Plasma Theory	12
Penning Discharge Plasma	14
III. EXPERIMENTAL METHODS AND PROCEDURES	21
The Classical Penning Discharge	21
Microwave Scattering Diagnostic	25
Homodyne Detection	25
Microwave System	30
Data Collection	35
IV. DATA ANALYSIS	37
V. DISCUSSION OF DATA	56
VI. SUMMARY AND CONCLUSIONS	60
LIST OF REFERENCES	62
APPENDIXES	66
APPENDIX A. LIST OF SYMBOLS	67
APPENDIX B. PLASMA PHYSICS EQUATIONS	69
VITA	71

LIST OF FIGURES

FIGURE	PAGE
1.1 Scattering Geometry and Notation	6
2.1 Radial Electric Field in a Cylindrical Plasma	15
3.1 Penning Discharge Schematic	22
3.2 UT Classical Penning Discharge	23
3.3 Axial Magnetic Field of UT Classical Penning Discharge	24
3.4 Basic Homodyne System	26
3.5 Magic Tee with E Fields Shown	29
3.6 Microwave Scattering Experimental Setup	31
3.7 Microwave Scattering Geometry in Classical Penning Discharge	32
3.8 Square-law Characteristics of Typical 422A Crystal	34
4.1 Typical Frequency Spectrum of Plasma Turbulence	38
4.2 Dispersion Relation for Plasma Turbulence	40
4.3 Angular Dependence of Scattered Radiation Power	41
4.4 Comparison of Turbulence Frequency Spectra on Magnetic Field Strength	42
4.5 Frequency Spectra of Turbulence vs Magnetic Field Strength	43
4.6 Plasma Parameters as a Function of Magnetic Field Strength	44

FIGURE	PAGE
4.7 Dependence of Turbulence Frequency Spectra on Plasma Current	45
4.8 Frequency of Peak in Turbulent Spectra as a Function of Plasma Current	46
4.9 Plasma Parameters as a Function of Plasma Current	48
4.10 Dependence of Turbulence Frequency Spectra on Anode Voltage	49
4.11 Dependence on Peak Frequency in Turbulent Spectrum as a Function of Anode Voltage	50
4.12 Plasma Parameters as a Function of Anode Voltage	51
4.13 Microwave Scattering Signal Correlated with Capacitive Probe Signals	52
4.14 Correlation of Capacitive Probes at the Same Anode Location	54
4.15 Correlation of Capacitive Probes at Anode and Cathode Locations	55

INTRODUCTION

The scattering of microwave radiation from plasmas is a diagnostic that permits the study of electron density fluctuations in the plasma. The frequency spectrum of the scattered wave can give information about the frequency and wave number of collective electron number density fluctuations. To detect the correlated motion of the electrons, the wavelength of the incident wave must be large compared with the electron Debye distance. The amount of power scattered from thermal fluctuations in laboratory plasmas is small, therefore microwave scattering experiments are confined to the study of plasma fluctuations resulting from instabilities or driven waves.

The plasma in The University of Tennessee (UT) Plasma Science Laboratory's classical Penning discharge experiment is known to have a high level of turbulence and an electron number density in the range of 1×10^8 to $1 \times 10^{10}/\text{cm}^3$ and therefore lends itself to coherent scattering of 32-GHz microwaves. The turbulence in the plasma is believed to be primarily due to strong radial and axial electric fields and to beam-plasma interaction brought on by two interpenetrating beams resulting from electrons trapped in an electrostatic potential well.¹ The purpose of this thesis research was to develop a microwave scattering diagnostic and use it to determine additional information about the turbulence in the classical Penning discharge plasma.

CHAPTER I

SCATTERING THEORY

The theory describing the scattering of electromagnetic waves from plasmas has been derived by several authors.²⁻⁷ This theory describes the scattering from density fluctuations in a plasma and is important in understanding how microwave scattering can be used as a plasma diagnostic.

Scattering of Electromagnetic Radiation

Electromagnetic radiation is scattered by a charged particle since the particle is accelerated by oscillating fields and the accelerated charge itself reradiates electromagnetic radiation. The spectrum of electromagnetic radiation scattered by the charged particle depends on the mass, charge, position, and velocity of the particle. Electrons are more likely to scatter radiation than ions because the acceleration of ions is small in consequence of their much greater mass. The scattering from individual charged particles at low frequency ω , such that $\hbar\omega$ is much less than mc^2 , the rest mass of the particle, is called Thomson scattering.

A monochromatic wave incident on a charge q can be given by

$$E_i(r,t) = E_{io} e^{i[k_i \cdot r(t) - \omega_i t]} , \quad (1-1)$$

where $k_i = 2\pi/\lambda_i$, λ_i , and ω_i are the incident wave number, wavelength, and frequency, respectively. The general equation of motion for the charge is

$$\dot{\mathbf{v}} = q/m[\mathbf{E}_i + \mathbf{E}_o + \mathbf{v} \times (\mathbf{B}_i + \mathbf{B}_o)] + \frac{\mathbf{F}}{m} , \quad (1-2)$$

where $\mathbf{E}_o$ and $\mathbf{B}_o$ are externally applied fields and $\mathbf{F}$ is the result of collisions. By using Maxwell's equations (Appendix B) one can obtain the relation between current density $\mathbf{J}$ for a single charge and the electric field $\mathbf{E}$ where $\mathbf{J} = q\mathbf{v}(t)/V$,

$$\nabla \times (\nabla \times \mathbf{E}) + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu \frac{\partial \mathbf{J}}{\partial t} . \quad (1-3)$$

The solution of equation (1-3) is given by Jackson⁴ and can be simplified for low velocity charges, $v/c \ll 1$, to be

$$\mathbf{E}_s(R,t) = \frac{q}{4\pi\epsilon_o R c} [\hat{s} \times (\hat{s} \times \dot{\beta})] , \quad (1-4)$$

where $\beta = v/c$ and $\hat{s}$ is the scattered radiation direction. The scattered power per unit solid angle is given by Ref. 8 as

$$dP_s/d\Omega = R^2 \mathbf{S} \cdot \hat{s} = R^2 \frac{E^2}{\eta} , \quad (1-5)$$

where $\mathbf{S}$ is the Poynting vector and η is the wave impedance. By substitution from equation (1-4), the scattered power per unit solid angle can be written as

$$dP_s/d\Omega = \frac{q^2}{\eta(4\pi\epsilon_o c)^2} [\hat{s} \times (\hat{s} \times \dot{\beta})]^2 . \quad (1-6)$$

For a charge with low velocity the effect of the $\mathbf{B}_i$ field may be ignored and when the incident frequency is greater than the gyrofrequency $\omega_i \gg \omega_c$, the magnetic field $\mathbf{B}_o$ may be neglected in equation (1-2). If the incident frequency is much greater than the collision frequency $\omega_i \gg \nu$, the collision term $\mathbf{F}$ may also be neglected. Thus the equation of motion for the charge becomes

$$\dot{\mathbf{v}} = q/m \mathbf{E}_{io} e^{[\mathbf{k}_i \cdot \mathbf{r}(t) - \omega_i t]} . \quad (1-7)$$

The implicit time dependence of electron position $\mathbf{r}$ means that the scattered radiation will contain frequencies shifted from ω_i by amounts dependent on characteristics of the motion of the charge. Substitution into equation (1-4) yields for the scattered electric field,²

$$\mathbf{E}_s(R, t) = \frac{q^2}{4\pi\epsilon_o mc^2 R} [\hat{s} \times \hat{s} \times \mathbf{E}_{io}] \cos[k_s R - \omega t - (\mathbf{k}_s - \mathbf{k}_i) \cdot \mathbf{r}(0)] . \quad (1-8)$$

The time-averaged scattered power per unit solid angle from equations (1-5) and (1-8) is

$$\overline{P_s(R)} d\Omega = \frac{q^4 E_{io}^2}{2\eta(4\pi\epsilon_o mc^2)^2} [\hat{s} \times (\hat{s} \times \mathbf{E}_{io})]^2 . \quad (1-9)$$

The mass dependence indicates that scattering from ions can be neglected compared to that from electrons. If the incident radiation is polarized by an angle θ from the k_s, k_i plane, then

$$|\hat{s} \times (\hat{s} \times \mathbf{E}_{io})|^2 = 1 - \sin^2\theta \cos^2\phi . \quad (1-10)$$

The scattered power in equation (1-9) can also be written in terms of the incident power P_i as

$$\overline{P_s(R)} d\Omega = \frac{q^4}{(4\pi\epsilon_0 mc^2)^2 A} P_i , \quad (1-11)$$

where the incident power is

$$\overline{P_i} = \frac{E_{io}^2}{2\eta} A , \quad (1-12)$$

and A is the cross-sectional area of the incident beam of radiation.

In scattering problems it is convenient to use the shifted frequency ω and wavenumber k , which are defined as

$$\omega = \omega_s - \omega_i = (\mathbf{k}_s - \mathbf{k}_i) \cdot \mathbf{v} \quad (1-13)$$

$$\mathbf{k} = \mathbf{k}_s - \mathbf{k}_i , \quad (1-14)$$

where $\mathbf{k}_s$ and ω_s are the scattered wavenumber and frequency, respectively. From Figure 1.1 the scattering angle is denoted θ_s and by using the law of cosines it can be seen that

$$k = (k_s^2 + k_i^2 - 2k_s k_i \cos \theta_s)^{1/2} . \quad (1-15)$$

For the case where $v/c \ll 1$, $k_i = k_s$ and equation (1-15) can be written as

$$|k| = 2|k_i| \sin(\theta_s/2) \quad (1-16)$$

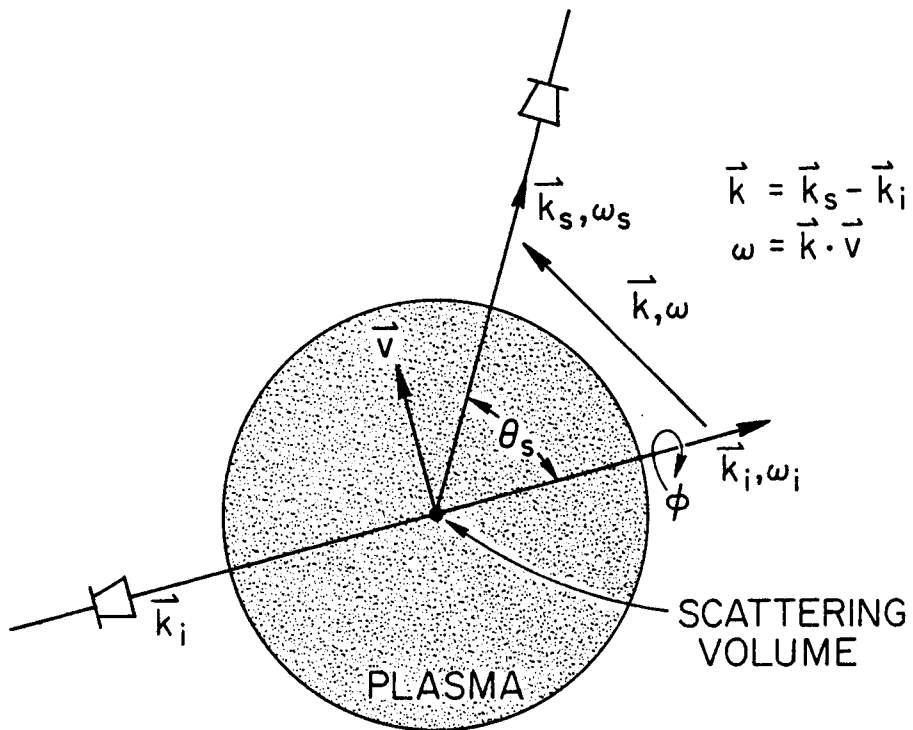


Figure 1.1. Scattering geometry and notation.

by using the trigonometric identity $1 - 2 \cos \theta = \sin^2 (\theta/2)$. This relation is equivalent to the Bragg condition for scattering.

Scattering from a Plasma

A plasma is a collection of electrons and positive ions which is approximately neutral and can be characterized by the following scale lengths:

$$\bar{n}^{1/3} \ll \lambda_D \ll L_p , \quad (1-17)$$

where n is the number density of charged particles and $n^{-1/3}$ is the average interparticle separation; λ_D is the Debye length, which is the distance over which the potential of a charge is shielded by neighboring charges; and L_p is the characteristic dimension of the plasma.

For an electromagnetic wave incident on a plasma with $\omega_i < \omega_{pe}$ and $\lambda_D \ll L_p$, the wave is cut off because charges can respond on a time scale $\sim 1/\omega_{pe}$ and are able to cancel the incident fields. Thus it is necessary for $\omega_i > \omega_{pe}$ in order for the incident wave to propagate in the plasma. For the case where $\omega_i < \omega_{pe}$ the wave number k_i is imaginary.² Substitution of the imaginary wave number into equation (1-1) yields a wave that is damped as it propagates into the plasma.

An incident electromagnetic wave that has fields small enough to not perturb the plasma will yield a total scattered electric field that is the vector sum of the scattered fields of the individual electrons within the scattering volume. The scattered power per unit solid angle can then be written as²

$$dP_s/d\Omega = \frac{R^2}{\eta} \left(\sum_{j=1}^N \mathbf{E}_{js} \sum_{l=1}^N \mathbf{E}_{ls} \right) . \quad (1-18)$$

By separating out terms where $j = 1$ and $j \neq 1$, equation (1-18) can be rewritten as

$$dP_s/d\Omega = \frac{R^2}{2\eta} NE_s^2 + \frac{R^2}{\eta} N(N-1)(\mathbf{E}_j \cdot \mathbf{E}_l) . \quad (1-19)$$

The first term in equation (1-19) can be considered to be incoherent scattering from N randomly distributed electrons. For incident wavelengths $\lambda_i \ll \lambda_D$, the wave sees electrons on a scale where they seem to be uncoupled. Thus the first term dominates. The significant features of incoherent scattering are that the scattered spectrum is representative of the electron velocity distribution and that the scattered power is proportional to N which determines the density, since $n = N/V$. This is the term which provides the basis of Thomson scattering diagnostics for plasmas.

The second term of equation (1-19) is zero if the motions of the electrons are completely uncorrelated since each E term has a negative counterpart that cancels. The term only makes a contribution when the electron motions are correlated. This coherent or collective scattering is largest when the incident wavelength is of the same scale as the Debye length, thus leading to cooperative scattering due to the Debye shielding effect. A useful parameter α , known as the Salpeter parameter,⁹ is defined as the incident wavelength divided by the Debye length. When $\alpha > 1$, the coherent scattering becomes the major scattering mechanism.

Plasma waves and turbulence lead to periodic density fluctuations which, due to the N^2 dependence of the coherent term, gives substantial enhancement

to the scattered power spectrum. An analogy between this situation and the Bragg scattering from a regular array of electrons in a crystal can be drawn. The individual scattered components are in phase in some directions and cancel in others. By crushing the crystal or eliminating the coherent motion in the plasma, the correlations between electron positions would be lost; consequently, there would be no coherence in the scattering, and only an incoherent spectrum would be obtained.

It is this form of coherent scattering that forms the basis of microwave and far infrared (FIR) scattering plasma diagnostics. Several investigations of plasma turbulence have been made by both microwave scattering and laser scattering for various plasma conditions and are described in the literature.¹⁰⁻¹⁴ These investigations show agreement with the scattering theory described above. Scattering diagnostics for plasmas have the capability to measure characteristics of plasma waves in relation to plasma heating experiments and also to obtain frequency distribution information from plasma turbulence resulting from beam-plasma interactions and other instabilities.

Scattered Spectral Density

For a plasma that is known to have electron fluctuations, the electron density is a function of the fluctuation wave number k and frequency ω . From equations (1-13) and (1-14), these quantities can be related to the incident and scattered radiation. The spectrum of the fluctuation density is written in terms of the spectral density function, which is defined as

$$S(\mathbf{k}, \omega) = \lim_{V, T \rightarrow \infty} \frac{1}{VT} \frac{\langle n_e(\mathbf{k}, \omega) n_e^*(\mathbf{k}, \omega) \rangle}{n_0} . \quad (1-20)$$

From ref. 1 it is shown that the scattered power is related to the spectral density function by

$$P(R, \omega_s) d\omega_s = \frac{P_i q^4}{A (4\pi\epsilon_0 m_e c^2)^2} d\Omega \frac{d\omega_s}{2\pi} [\hat{s} \times (\hat{s} \times \mathbf{E}_{io})]^2 NS(\mathbf{k}, \omega) . \quad (1-21)$$

In plasma diagnostics then, a measurement of the power spectrum of scattered waves gives the spectrum of correlated density fluctuations in the plasma $\langle \tilde{n}_e(\mathbf{k}, \omega) \rangle$. Therefore, an increase in density fluctuations can be expected to result in an increase in scattering from the plasma.

Additional information on how plasma properties determine the scattering can be realized by looking at the Fourier transform of $S(\mathbf{k}, \omega)$, which is called the density autocorrelation function,

$$\begin{aligned} C(\epsilon, \tau) &= \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{d\omega}{2\pi} e^{i(\omega\tau + \mathbf{k} \cdot \mathbf{x})} S(\mathbf{k}, \omega) \\ &= \lim_{V, T \rightarrow \infty} \frac{1}{VT} \int_V \int_T dx^3 dt \frac{\langle n_e(\mathbf{x}, t) n_e^*(\mathbf{x}, t) \rangle}{n_0} . \end{aligned} \quad (1-22)$$

The total scattering cross section is given by

$$S_T(\mathbf{k}) = \int_{-\infty}^{\infty} d\omega S(\mathbf{k}, \omega) , \quad (1-23)$$

and is of the order 1 for an equilibrium plasma. For an unstable or turbulent plasma, the total scattering cross section can be much greater than 1 due to the N^2 dependence of the fluctuation level in the coherent scattering. The total

scattering cross section can be separated into electron and ion components and written as

$$S_T(\mathbf{k}) = S_e(\mathbf{k}) + S_i(\mathbf{k}) \quad . \quad (1-24)$$

Detailed analytical expressions for each term in the above equation are found in ref. 15. If $\alpha > 1.5$, the collective effect of ions becomes strong and the second term of equation (1-24) is important. The width of the spectral distribution $S(\mathbf{k}, \omega)$ of the scattered radiation is of the order of the broadening of the ion thermal velocity. Therefore, ion temperature can be estimated from collective scattering with $\alpha > 1.5$.

CHAPTER II

PLASMA TURBULENCE

In this chapter we discuss the causes of density fluctuations in the plasma state by considering single particle effects and two interpenetrating fluid effects. The theories of these effects are then applied to the classical Penning discharge plasma to determine possible fluctuation mechanisms in this plasma.

Plasma Theory

A plasma can be described by what is known as the two fluid equations (Appendix B), which are actually separate conservation equations for the ions and the electrons. These equations—when linearized—describe wave phenomena over a wide range of frequencies. These phenomena include sound waves, electromagnetic waves, and electrostatic waves. Some of the kinds of waves that can propagate through a plasma are described below.

The first example of a wave in a plasma is the electron plasma wave. In an unmagnetized plasma with fixed density and infinite ion mass, the fluid equations can be solved¹⁶ to find the following wave frequency,

$$\omega^2 = \frac{n_0 e^2}{\epsilon_0 m_e} = \omega_{pe}^2, \quad (2-1)$$

which is the high-frequency electron plasma oscillation. Note that ω does not depend on the wave number k , so these waves are dispersionless and do not propagate. Similarly, it is possible to consider low-frequency waves by including

the ion continuity and force equations. By linearizing to first order and assuming quasineutrality, the ion-acoustic dispersion relation can be written as

$$\omega^2 = k^2 \frac{\gamma_e T_e + \gamma_i T_i}{m_i} . \quad (2-2)$$

These waves are similar to sound waves traveling through a gas.

In a uniform magnetized plasma there exists a variety of physical phenomena that cause waves. In looking for low-frequency waves propagating along B , the relevant fluid equations, without electron inertia, lead to the following dispersion relation for Alfvén waves:

$$\omega^2 = \frac{k^2 v_A^2}{1 + v_A^2/c^2} . \quad (2-3)$$

This is an acoustic dispersion relation for $v_A \ll c$, where the acoustic speed v_A is given by

$$v_A^2 = \frac{B_o^2}{\mu_o \rho_m} . \quad (2-4)$$

When a plasma does not consist of Maxwellian electrons and ions and the zero order velocities of components of the species differ, instabilities called streaming instabilities occur. For example, a plasma with stationary ions and electrons traveling with speed v_o can be analyzed with the fluid equations and Poisson's equation to lead to a dispersion relation¹⁶ of

$$\omega = kv_o \pm \omega_{pe} . \quad (2-5)$$

This relation leads to a range $-\omega_{pe}/v_o < k < \omega_{pe}/v_o$ of unstable wave numbers.

Among the known single particle effects that can cause instabilities in a plasma is the $\mathbf{E} \times \mathbf{B}$ drift wave. This type of instability develops when a radial electric field and axial magnetic field exist as shown in Figure 2.1. The drift velocity of an $\mathbf{E} \times \mathbf{B}$ drift is given by

$$v_D = E/B . \quad (2-6)$$

With the drift velocity defined above and the continuity equation (Appendix B), we can define the dispersion relation for electrostatic $\mathbf{E} \times \mathbf{B}$ drift waves as¹⁶

$$\omega = k v_D . \quad (2-7)$$

Penning Discharge Plasma

The UT classical Penning discharge creates a cylindrical plasma with a radius of approximately 5 cm. This plasma is in a uniform magnetic field and has both radial and axial electric fields. The axial electric field has been found with Langmuir probes to be largest in the region near the cathodes.¹⁷ For the plasma near the midplane, the electric and magnetic fields can be written as

$$\mathbf{E} = E_{or} \hat{r} + E_{oz} \hat{z} , \quad \mathbf{B} = B_o \hat{z} . \quad (2-8)$$

By using the fact that the plasma contains an externally imposed electric field, a zero order two-fluid analysis of the classical Penning discharge plasma velocities can be performed. This analysis assumes a collisionless plasma with no

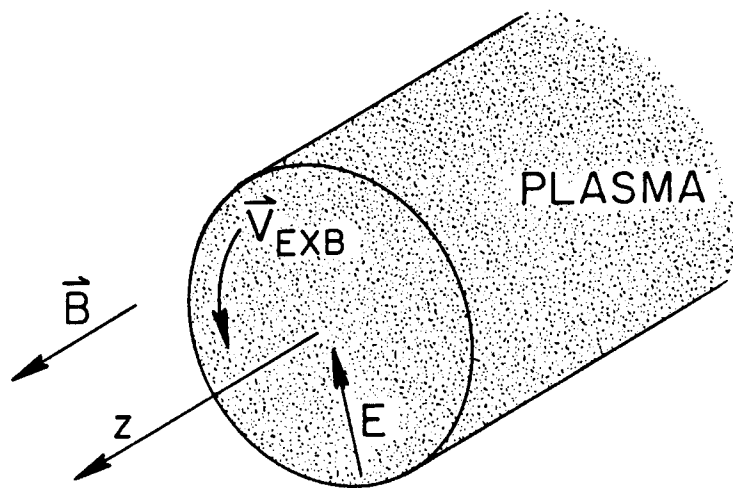


Figure 2.1. Radial electric field in a cylindrical plasma.

pressure gradients or time variations. The two fluid magnetohydrodynamic equations from Appendix B for this analysis are

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0 \quad (2-9)$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0 \quad (2-10)$$

$$\nabla \cdot \mathbf{E} = \frac{q(n_i - n_e)}{\epsilon_0} = \frac{\rho e}{\epsilon_0} = \frac{1}{r} \frac{\partial}{\partial r} (r E_{or}) + \frac{\partial E_{oz}}{\partial z} \quad (2-11)$$

$$\mathbf{J} = q(n_i \mathbf{v}_i - n_e \mathbf{v}_e) , \quad (2-12)$$

where the e and i subscripts are for the electrons and ions, respectively. The equations of motion using the Eulerian derivative for both the electrons and ions¹⁸ are

$$\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla) \mathbf{v}_i = \frac{q}{m_i} \mathbf{E} + \frac{q}{m_i} (\mathbf{v}_i \times \mathbf{B}) \quad (2-13)$$

$$\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \cdot \nabla) \mathbf{v}_e = \frac{-q}{m_e} \mathbf{E} - \frac{q}{m_e} (\mathbf{v}_e \times \mathbf{B}) . \quad (2-14)$$

The terms on the left side describe the macroscopic acceleration of a group of particles. The terms on the right represent the forces acting on the particle due

to its charge in an electric field and its velocity across a magnetic field. Note that terms for a pressure gradient of the species and a term for collisional interactions are left out.

By expanding the convective derivative in cylindrical coordinates, the momentum equations in the r and θ directions become

$$-\frac{v_\theta^2}{r} + v_r \frac{\partial v_r}{\partial r} = \frac{q}{m} E_{or} + v_\theta \omega_c, \quad r \text{ component} \quad (2-15)$$

$$\frac{v_r v_\theta}{r} + v_r \frac{\partial v_\theta}{\partial r} = -\frac{q}{m} E_\theta - v_r \omega_c, \quad \theta \text{ component} . \quad (2-16)$$

If we define

$$v_\theta = \omega r, \quad v_D = \frac{E_{or}}{B_o} = \omega_D r, \quad (2-17)$$

the θ component of the momentum equation becomes

$$v_r \left(\frac{\partial v_\theta}{\partial r} + r \frac{\partial \omega}{\partial r} \pm \omega_c \right) = 0. \quad (2-18)$$

Therefore, $v_r = 0$ and there is no radial dependence. The equation of motion then becomes

$$-\frac{v_\theta^2}{r} = \frac{q}{m} E_{or} + v_\theta \omega_c, \quad (2-19)$$

which can be written as the following quadratic equation,

$$\omega^2 \pm \omega\omega_c \pm \omega_c\omega_D , \quad (2-20)$$

where + refers to electrons and - to ions. By solving the quadratic by expanding the resulting square root, the solution for ω becomes

$$\omega = \frac{\omega_c}{2} \left(1 \pm 1 \mp \frac{2\omega_D}{\omega_c} \right) . \quad (2-21)$$

The low-frequency solution for equation (2-21) is then

$$\omega = \omega_D = \frac{E_{or}}{rB_o} , \quad (2-22)$$

which leads to an electric field and potential that are given by

$$E_{or} = E_o r , \quad \phi(r) = \phi(d) \left(\frac{r}{d} \right)^2 . \quad (2-23)$$

Another way to analyze the classical Penning discharge plasma is to look at the conservation equation for electrons. The change in density with time can be written as¹⁹

$$\frac{Dn}{Dt} = \frac{\partial n}{\partial t} + v \cdot \nabla n = \frac{\partial n}{\partial t} + \frac{1}{B_o r} \frac{\partial \phi}{\partial r} \frac{\partial n}{\partial \theta} . \quad (2-24)$$

If we now linearize to first order by assuming

$$n = n_o(r) + n(r)e^{i(l\theta - \omega t)} , \quad \phi = \phi(r) + \phi(r)e^{i(l\theta - \omega t)} , \quad (2-25)$$

where the plasma parameters are real parts of the complex quantities, we get from equation (2-24)

$$n \left(\omega - \frac{l v_D}{r} \right) = - \frac{l \phi}{B_o r} \frac{dn_o}{dr} . \quad (2-26)$$

Substitution into Poisson's equation ($\nabla^2 \phi = \rho$) gives

$$\left(\omega - \frac{l v_D}{r} \right) \left\{ \frac{1}{r} \frac{d}{dr} \left(r \frac{d\phi}{dr} \right) - \frac{l^2}{r^2} \phi \right\} = - \frac{q}{\epsilon_o} \frac{l \phi}{B_o r} \frac{dn_o}{dr} . \quad (2-27)$$

This equation suggests an interaction between potential ϕ and the radial density gradient in the plasma. If we assume that for some region in the plasma $dn_o/dr = 0$, then equation (2-26) reduces to

$$\omega = \frac{l v_D}{r} . \quad (2-28)$$

Similarly, equation (2-27) will reduce to

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\phi}{dr} \right) - \frac{l^2}{r^2} \phi = 0 . \quad (2-29)$$

For the fundamental ($l = 1$) mode, a general solution for ϕ is given by

$$\phi = \phi_o r^2 , \quad (2-30)$$

which gives the same r dependence to ϕ as the zero order analysis did in equation (2-23).

Both analyses lead to the same result—that the radial electric field must be of the form $E = E_0 r$, in order to lead to an $E \times B$ drift wave with frequency independent of radius. This radial drift wave, which occurs in charge sheets of finite width in a magnetized plasma, is an electrostatic instability known as the Diocotron (or slipping stream) instability.²⁰ The instability condition as shown is that the density distribution must have at least one maximum where $dn/dr = 0$. Additional physics can be included in these analyses to consider shear of drift waves from the interaction between adjacent charge sheets. Cap²⁰ should be consulted for further reference into the Diocotron instability. The results here indicate what may be somewhat intuitive—the classical Penning discharge plasma has the mechanism for creating strong azimuthal drift waves driven by $E \times B$ drift.

CHAPTER III

EXPERIMENTAL METHODS AND PROCEDURES

The experimental arrangement used for this investigation is shown schematically in Figure 3.1. The plasma is generated in a classical Penning discharge having a length of 60 cm and diameter of 10 cm. The microwave scattering apparatus was fixed in axial position due to the geometry constraint of the magnetic field coils. The details of the Penning discharge, the microwave scattering diagnostic, and data collection are described below.

The Classical Penning Discharge

The classical Penning discharge was first developed by F. M. Penning as a low pressure ion gage tube.²¹ It has subsequently been used to study hot ion production and electric-field-dominated plasmas by Roth and many others.²² The Penning discharge plasma is known to support high levels of electrostatic turbulence and emit broadband electromagnetic radiation.¹⁷

The classical Penning discharge experiment at UT, shown schematically in Figure 3.1 and pictorially in Figure 3.2, consists of a pyrex vacuum vessel inside eight fixed magnetic field coils which produce a uniform magnetic field up to 0.40 T. The axial magnetic field profile of the Penning discharge is shown in Figure 3.3. As can be seen from this profile, the magnetic field is essentially flat over the distance between the cathodes, which is the entire length of the plasma. The vacuum is produced in the experiment by a turbomolecular pump backed by a roughing pump. The base pressure in the discharge is 6×10^{-6} torr as measured by two ion gages.

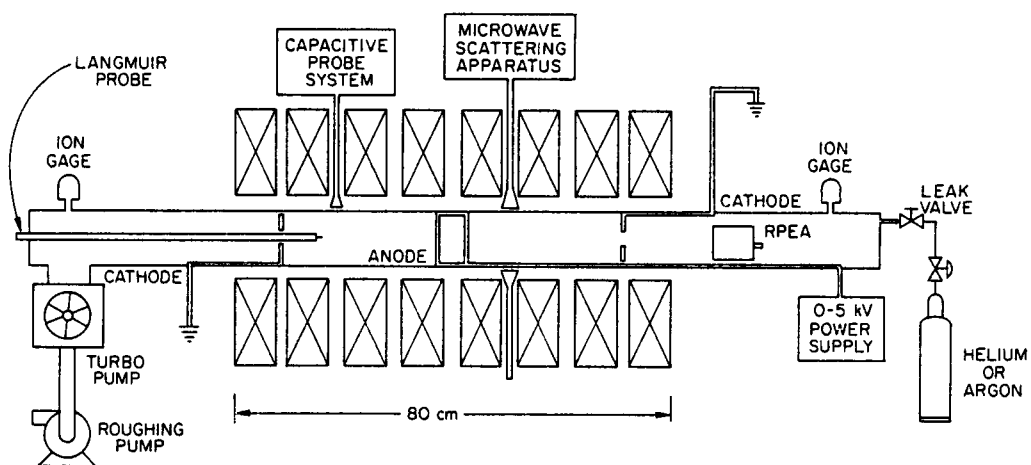


Figure 3.1. Penning discharge schematic.

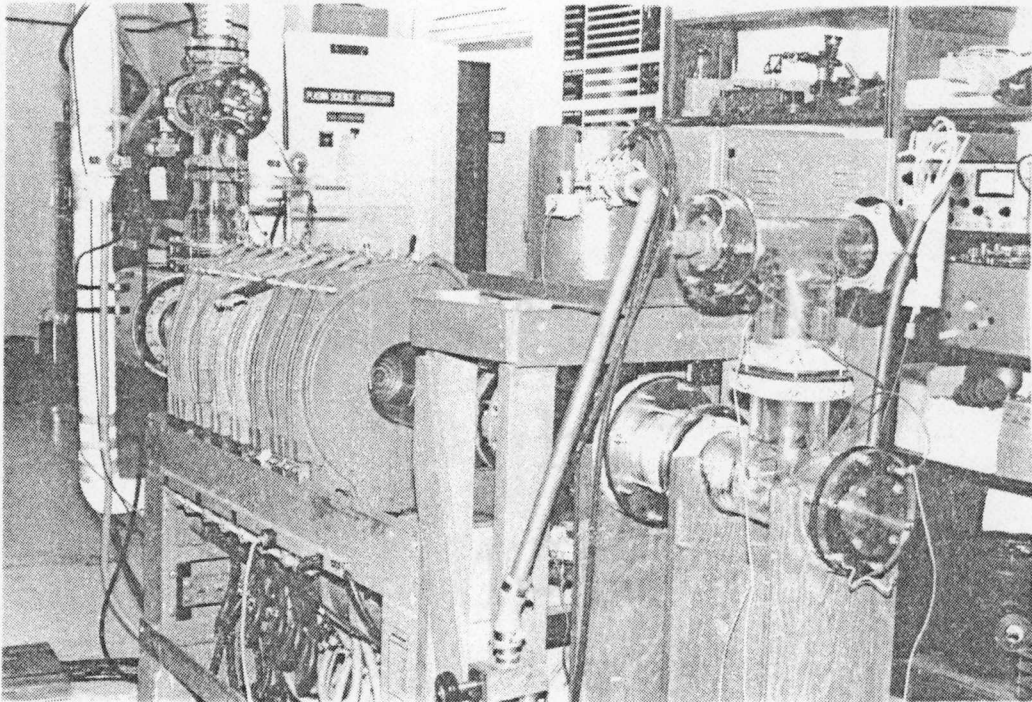


Figure 3.2. UT classical Penning discharge.

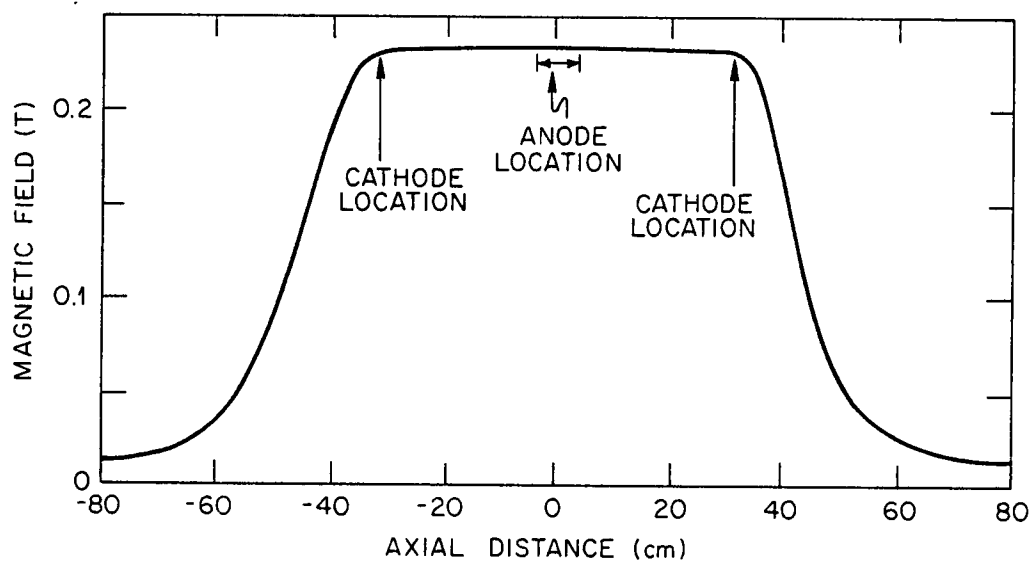


Figure 3.3. Axial magnetic field of UT classical Penning discharge.

The plasma is produced in the discharge by applying a high voltage to the anode, which breaks down the gas leaked into the vacuum vessel. For these experiments, argon and helium were used at background gage pressures of 1 to 7×10^{-4} torr. Plasma diagnostics used on the experiment include Langmuir probes, capacitive probes, and a retarding potential energy analyzer. Antennas are also used with a radio frequency (rf) spectrum analyzer to study rf emissions from the plasma.

Microwave Scattering Diagnostic

To investigate plasma turbulence in the Penning discharge, a microwave scattering diagnostic was developed to search for radial and azimuthal turbulence in the plasma. Axial turbulence is more difficult to detect with this diagnostic due to the geometry constraint of the fixed magnetic field coils external to the vacuum vessel. The microwave system uses homodyne coherent detection and is constructed of off-the-shelf components. A description of the diagnostic and its principle of operation appears below.

Homodyne Detection

Homodyne detection is a class of coherent detection in which an unmodulated carrier signal is mixed in a nonlinear device with a modulated signal having the same carrier frequency. The output signal contains amplitude as well as phase information. A homodyne mixer is simple to build, sensitive, and accurate over a wide dynamic range.²³ A basic homodyne system is shown in Figure 3.4

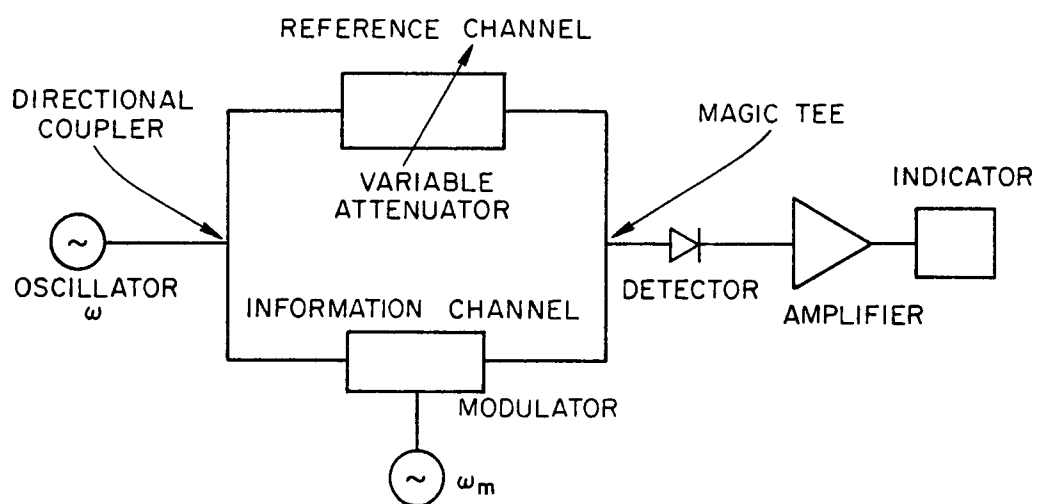


Figure 3.4. Basic homodyne system.

to illustrate the principle. In the homodyne system a reference channel of frequency ω is added to an information channel with a modulated signal also of frequency ω but with sideband frequencies of $\pm \omega_m$. The output is at the modulation frequency ω_m . Appropriate microwave hardware is used to isolate the channels and mix the two signals. A directional coupler and magic tee are typically used as shown. Crystal detectors with a square law response at low signal levels are typically used to detect the mixed output signal.

The reference channel carrier and modulated information channel signals can be represented by

$$E_r = Ae^{i\omega t}, \quad E_i = b(1 + m \cos \omega_m t)e^{i(\omega t + \phi)}, \quad (3-1)$$

where A and b are the amplitudes of the reference and information signals, respectively, and ϕ is the carrier phase difference between the two signals. The common factor $e^{i\omega t}$ can be suppressed, and the resulting sum of the two signals becomes

$$E_t = A + b(1 + m \cos \omega_m t) \cos \phi. \quad (3-2)$$

Nearly all microwave detecting devices respond to the input signal at low levels with a squared output. Considering the device as a square-law detector, the resultant output of the detector is proportional to

$$E_t^2 = A^2 + b^2(1 + m \cos \omega_m t)^2 + 2Ab(1 + m \cos \omega_m t) \cos \phi. \quad (3-3)$$

Considering only the terms containing the modulation frequency ω_m , the output of the detector at low amplitude becomes

$$E_o = kAbm \left(\frac{b}{A} + \cos \theta \right) \cos \omega_m t \quad . \quad (3-4)$$

If the carrier signal in the information channel is suppressed by a factor k_o ($0 < k_o < 1$), the resultant output of the detector becomes proportional to

$$E_o = kAbm \left(\frac{k_o b}{A} + \cos \theta \right) \cos \omega_m t \quad . \quad (3-5)$$

When k_o is small enough (good carrier suppression), the first term in equation (3-5) is negligible and can be neglected. The resultant output is then proportional to $b \cos \phi$, which is more attractive for phase measurements. Suppressing the carrier in the information signal also has the effect of increasing the upper dynamic range of b .

One method of suppressing the carrier in the information signal is to use a balanced mixer arrangement.^{23,24} The balanced mixer consists of a magic tee (shown in Figure 3.5) and two detectors.^{25,26} The magic tee divides the input in the E arm into fields of opposite phase that propagate in arms 3 and 4. The fields from the H arm are of the same phase. The detectors are then put on arms 3 and 4 and then subtracted from each other to give an output

$$E_o = kAbm \cos \phi \cos \omega_m t \quad . \quad (3-6)$$

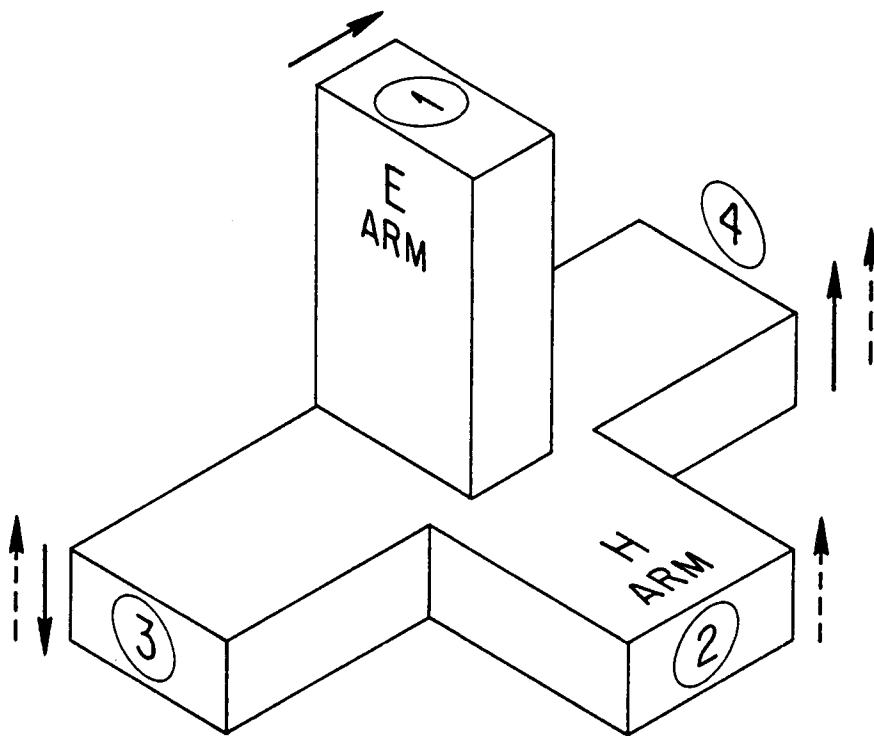


Figure 3.5. Magic tee with E fields shown.

The output is of the same form as equation (3-5) with the carrier suppressed. Balanced mixing therefore increases the dynamic range and phase sensitivity.

An additional advantage of balanced mixers is that any noise in the reference channel tends to be cancelled. The reference channel is connected to the H arm of the magic tee so that the noise outputs at arms 3 and 4 are in phase. When the two signals at port 3 and 4 are subtracted from each other, the noise signal from the reference channel is cancelled out.

Microwave System

A diagram of the microwave system used in the microwave scattering diagnostic is shown in Figure 3.6. The incident microwave power was polarized in the z direction so that propagation was in the ordinary mode. The scattering geometry is shown in more detail in Figure 3.7. From this figure it can be seen that the polarization angle ϕ is 90° . Thus from equation (1-10), the term $1 - \sin^2 \theta \cos^2 \phi$ becomes equal to 1.

The source of the microwave power used in the diagnostic was a Gunn diode with a measured output of 25 mW at 32.175 GHz. The frequency of the oscillator was measured with the wave meter in the reference channel. An isolator was located in front of the Gunn diode to protect it from reflected power. A 10-dB coupler was used to form the reference channel, which contained a phase shifter to adjust the phase of the carrier between the reference and information channels. The information channel uses a 20-dB gain horn to propagate microwaves into the center of the Penning discharge plasma through the vacuum vessel (pyrex glass). Flexible waveguide was used in the incident leg to enable the incident horn to move in angular displacement with respect to the

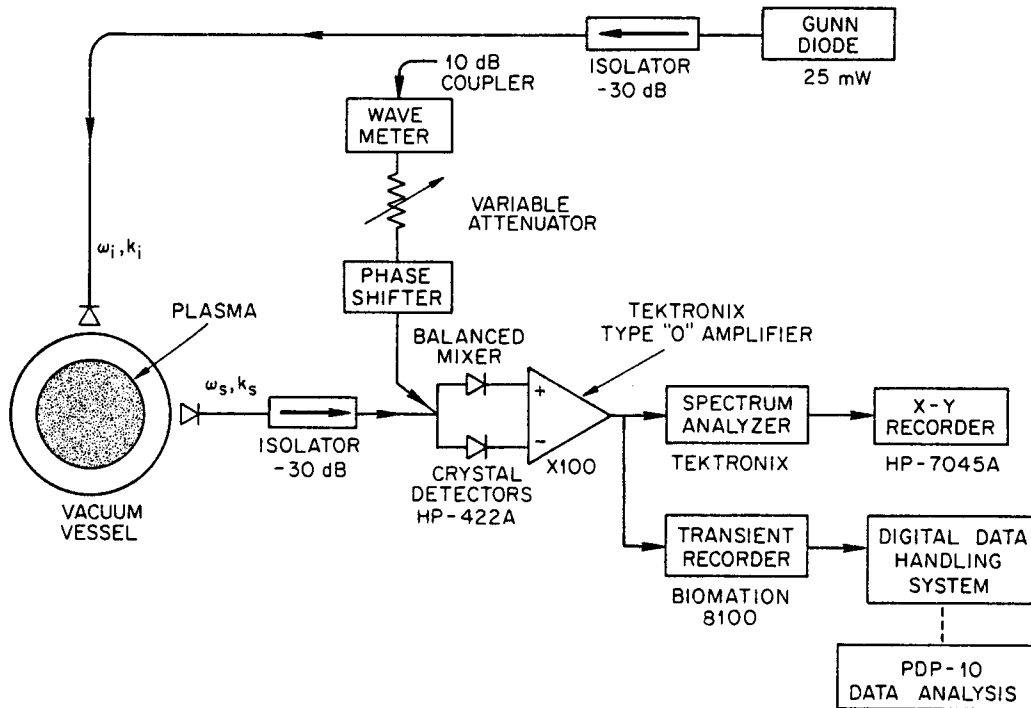


Figure 3.6. Microwave scattering experimental setup.

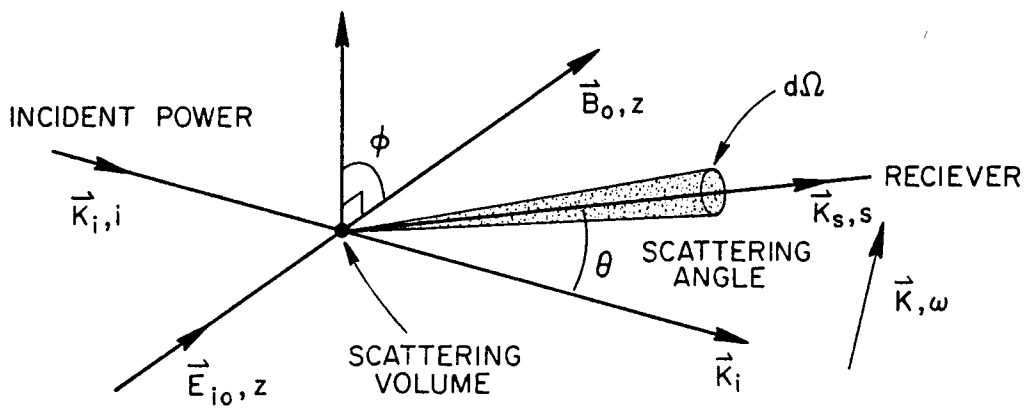


Figure 3.7. Microwave scattering geometry in classical Penning discharge.

receiving horn. The receiving horn was also a 20-dB gain horn that picks up microwave power scattered from the plasma. An isolator was used in the receiving leg to prevent reflections of scattered microwaves back into the plasma. Both the reference channel and the information channel signals were mixed in a balanced mixer to suppress the carrier and reduce any noise present in the reference signal.

The detectors used with the balanced mixer were model 422A crystal detectors with model 11523A matching load resistors manufactured by Hewlett-Packard Company.²⁷ The load resistance is factory-matched to the detector for optimum square-law response. The square-law response is specified to be less than 0.5 dB from ideal for low level up to 50-mV peak output. The typical response of the 422A is shown in Figure 3.8. For the ranges used in this research, the output can be considered square law with a sensitivity of 0.1 mV dc per μW . The signal from both detectors was input to a Tektronix Type 0 operational amplifier that was used in a differential mode with a gain of 100. Thus the output of the amplifier was carrier suppressed with reference channel noise reduced.

The glass vacuum vessel in the Penning discharge experiment has some effect on the propagation of microwaves, as one would expect. The relative dielectric strength of the pyrex glass is known to be 6.0 (Ref. 28). The impedance mismatch from the glass was calculated by using the following equation for a reflection coefficient for a perfect dielectric:

$$\Gamma = \frac{1 - \sqrt{\epsilon_r}}{1 + \sqrt{\epsilon_r}} \quad (3-7)$$

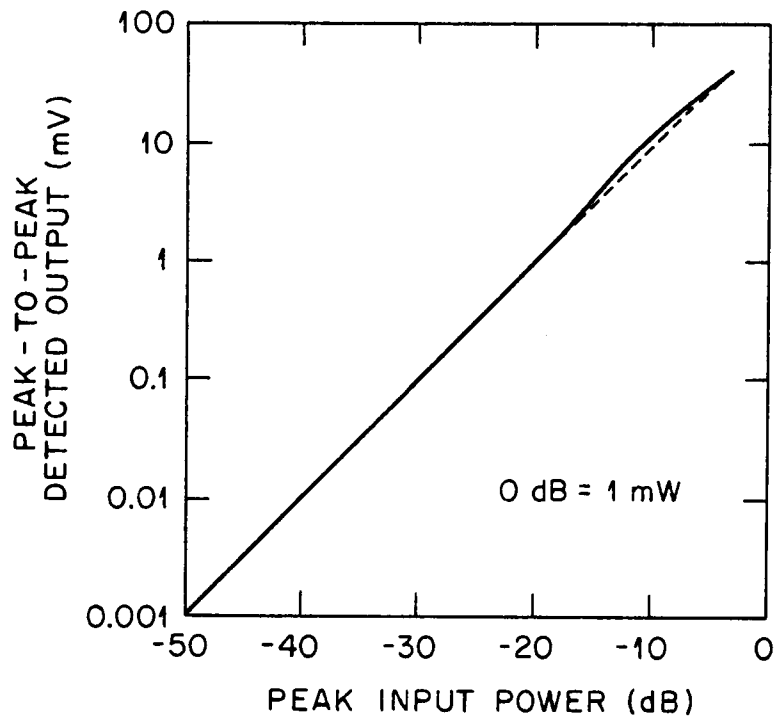


Figure 3.8. Square-law characteristics of typical 422A crystal. Source: Hewlett-Packard Co., "Operating Note: Crystal Detector 422A" (1972).

By using the following power transmission equation,

$$P_r/P_i = 1 - |\Gamma|^2, \quad (3-8)$$

it was calculated that 85% of the incident microwave power was transmitted through the glass. An improvement in impedance matching could be realized by using a 1.7-mm thickness of polystyrene ($\epsilon_r = 2.65$) between the air and glass interface to give a quarter-wave matching section. This impedance matching was never implemented due to sufficient scattering signal output. Since the scattered radiation from the plasma through the glass has the same impedance mismatch, the effect of the glass is to reduce the scattered microwave power by 70% over the theoretical value for the output power of the Gunn diode oscillator.

A phase shifter was put in the incident channel to compensate for reflected power standing waves reaching the reference channel. The microwave horns were focused on the center of the plasma for this experiment, thus covering a volume of 90 cm^3 since the horns have a 15° angle aperture.

Data Collection

The signal from the balanced mixer differential amplifier was analyzed by a Tektronix 1L5 spectrum analyzer²⁹ and by an analog-to-digital data handling system³⁰ for frequency content and correlation with capacitive probe signals. The spectrum analyzer produced a frequency spectrum plot on a CRT screen and on an X-Y recorder. The digital data system uses two Biomation 8100

transient digitizers to collect a sample of the scattered signal. The data are then transferred via magnetic tape to The University of Tennessee Computing Center PDP-10 computer where a signal analysis program,³¹ based on a fast Fourier transform (FFT) algorithm, produces auto power spectra, cross power spectra, coherence spectra, and phase spectra outputs. These outputs are then used to analyze the data.

In addition to the microwave scattering data that were collected, there were data from Langmuir probe and retarding potential energy analyzer diagnostics that were used to correlate scattering data with plasma conditions. Descriptions of these diagnostics and their data analysis are found in the literature.^{32,33}

CHAPTER IV

DATA ANALYSIS

Data were collected on The UT classical Penning discharge from the microwave scattering diagnostic and other diagnostics over the period from February 1984 through April 1984. An initial debugging period of several weeks was spent calibrating the spectrum analyzer and readying the Penning discharge. The digital data-handling system was brought on line independently by Mr. Peyman Dehkordi, a research assistant associated with the Air Force Office of Scientific Research contract at The UT Plasma Science Laboratory. This system was available for several runs of correlation studies between microwave scattering and capacitive probes.

The data from the microwave scattering diagnostic were taken with a low-frequency 1L5 spectrum analyzer and with the analog-to-digital data-handling system. The spectrum analyzer was used to take data to correlate with plasma conditions by recording the spectrum on an X-Y recorder. The digital data system was used to correlate microwave scattering data with capacitive probe signals. A typical power spectra of the plasma density fluctuations from the microwave scattering diagnostic is shown on a spectrum analyzer trace in Figure 4.1. The spectrum analyzer power spectra and the digitally computed power spectra are in good agreement for all the data taken.

The initial measurements made with the microwave scattering diagnostic were to determine the dispersion relation of the fluctuations detected. This was done by varying the scattering angle from 20 to 160°. By looking at equation (1-16), the dependence of wave number on scattering angle can be seen. Several

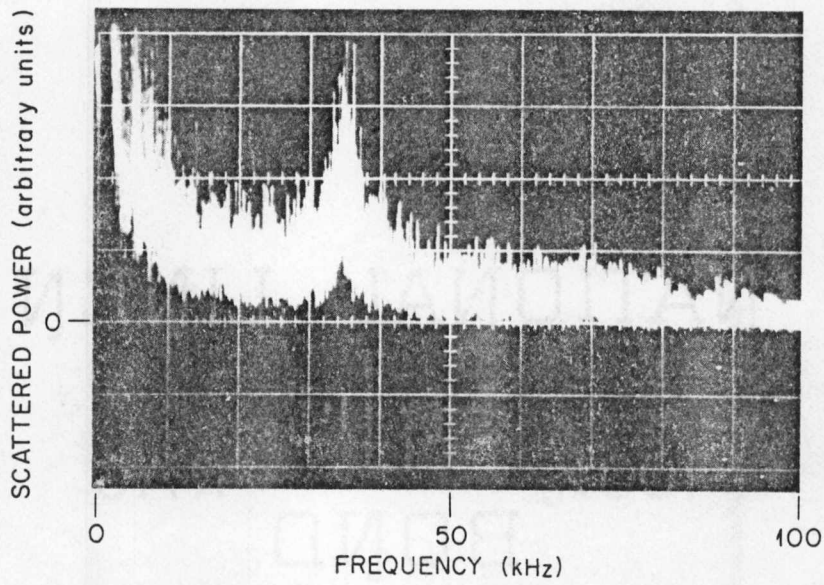


Figure 4.1. Typical frequency spectrum of plasma turbulence.

runs of dispersion data were taken, all with the same results. A plot of the dispersion relation for a typical run with constant plasma conditions is shown in Figure 4.2. The unexpected results show an essentially constant frequency. Also correlated with the scattering angle was the scattered power output. Figure 4.3 shows the scattered peak power output as a function of scattering angle.

Several runs of data were taken to determine correlations of the plasma fluctuations detected by the microwave scattering diagnostic with plasma parameters. Langmuir probes were used to measure plasma potential, electron temperature, and electron number density for the various plasma conditions investigated. There were three independent plasma parameters that were alterable on the classical Penning discharge. These were magnetic field, plasma current, and neutral pressure. By fixing the other two parameters, it was possible to determine the effect of each parameter on the plasma turbulence. In order to keep the plasma current constant and change the neutral pressure, it was necessary to change the anode voltage.

The effect of changing the magnetic field is shown in Figure 4.4. The frequency of the turbulence is seen to increase with magnetic field. The plot of Figure 4.5 shows that the change in frequency is linear with the magnetic field. An interesting detail this plot provides is the relation that the slopes of the three lines determined by these data are related by a multiple of two. The plasma parameter dependence on magnetic field is shown in the plots of Langmuir probe data in Figure 4.6.

Similar data were taken for different plasma currents. The effect of plasma current on the fluctuation spectrum is shown graphically in Figures 4.7 and 4.8.

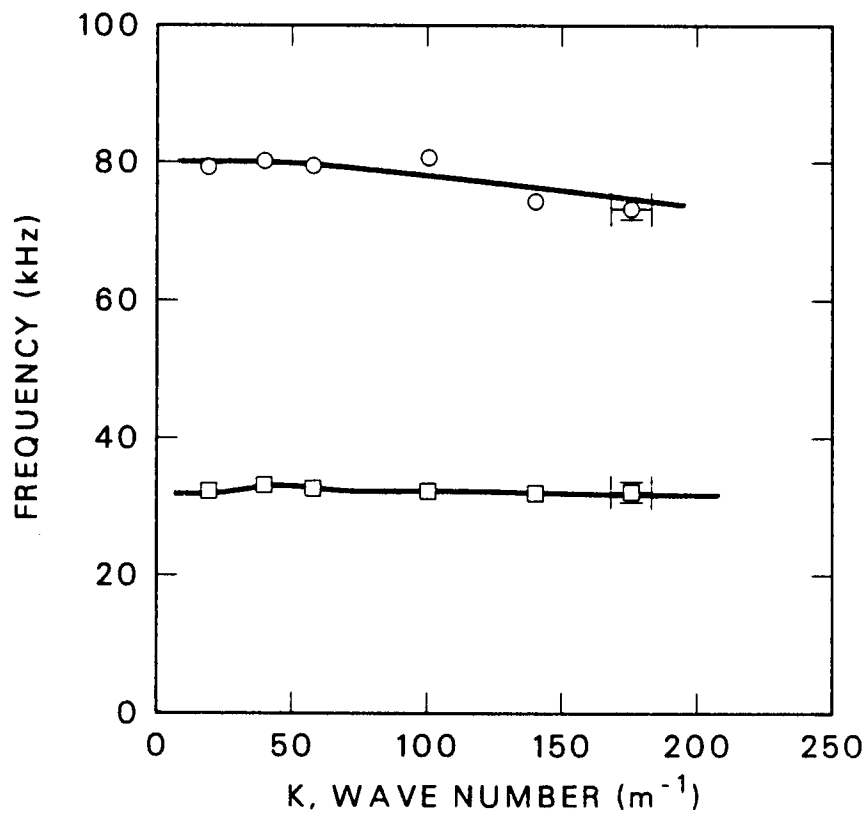


Figure 4.2. Dispersion relation for plasma turbulence.

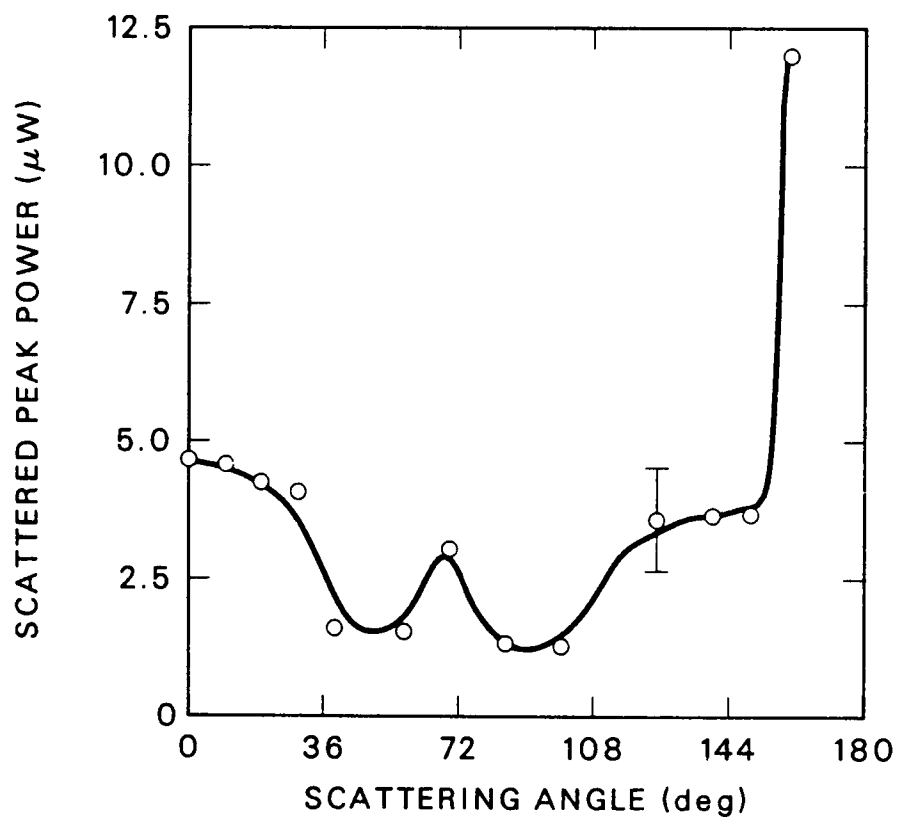


Figure 4.3. Angular dependence of scattered radiation power.

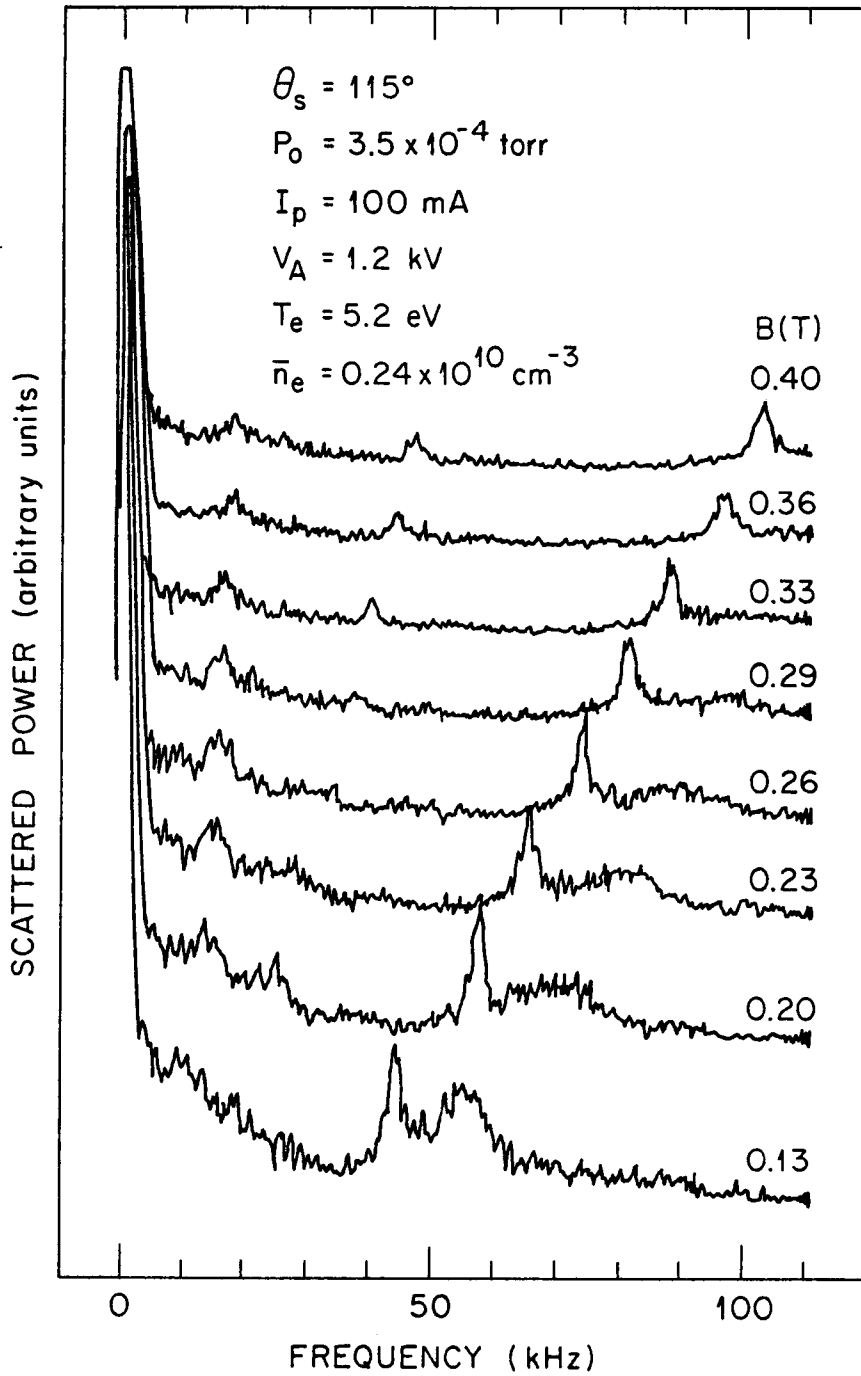


Figure 4.4. Comparison of turbulence frequency spectra on magnetic field strength.

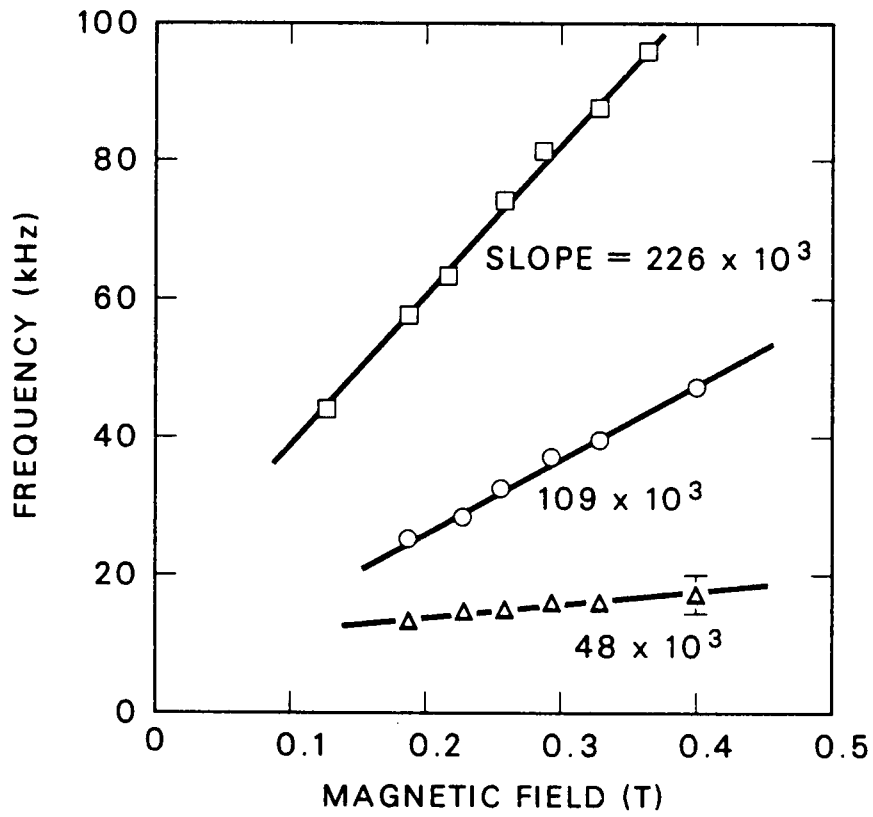


Figure 4.5. Frequency spectra of turbulence vs magnetic field strength.

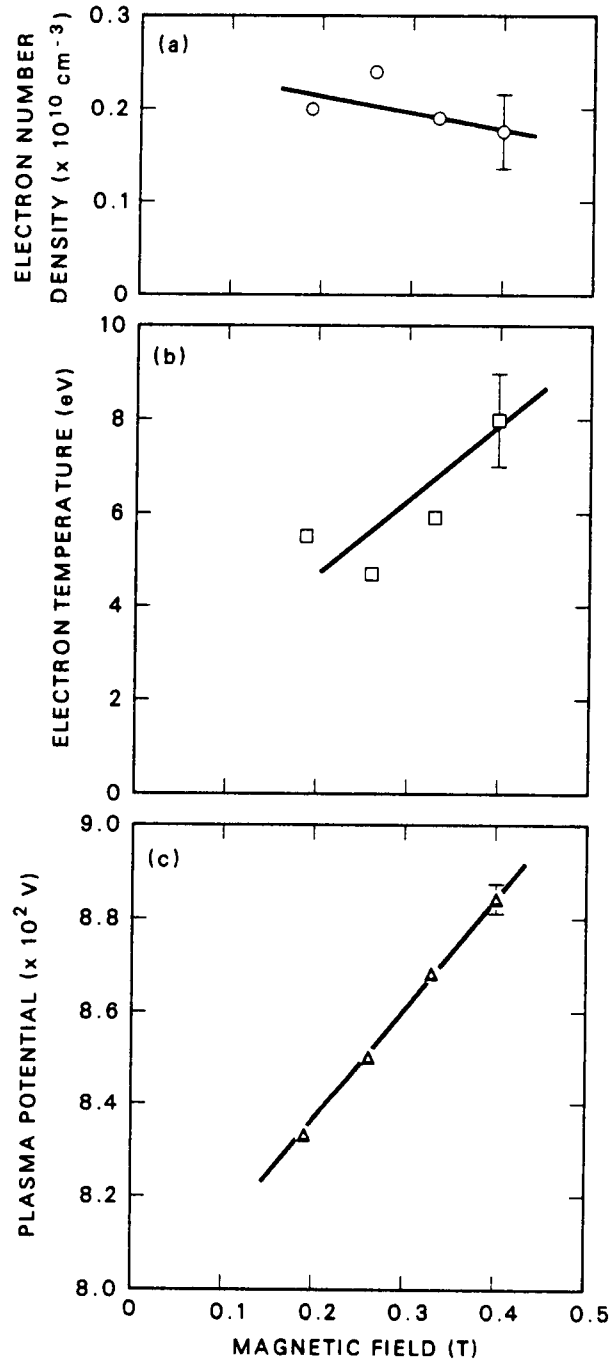


Figure 4.6. Plasma parameters as a function of magnetic field strength.

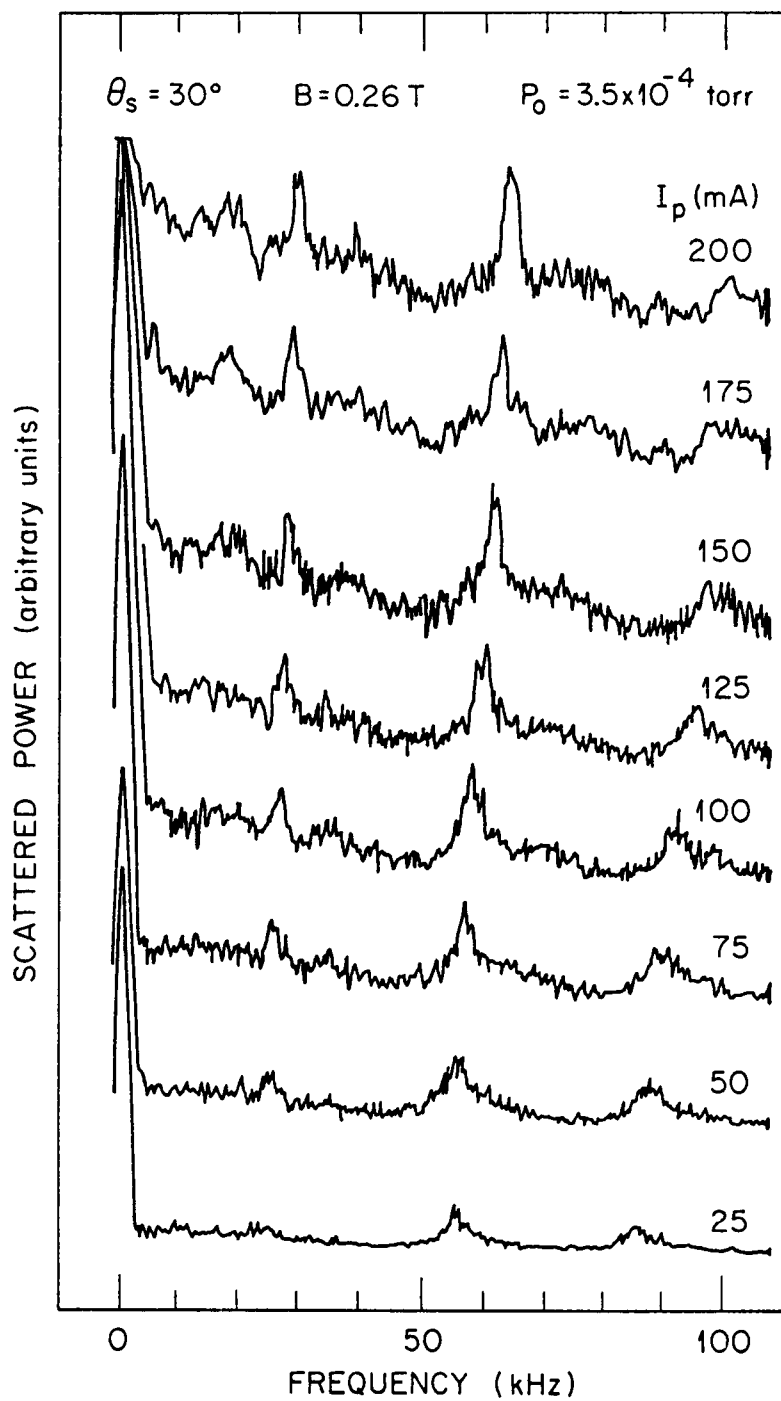


Figure 4.7. Dependence of turbulence frequency spectra on plasma current.

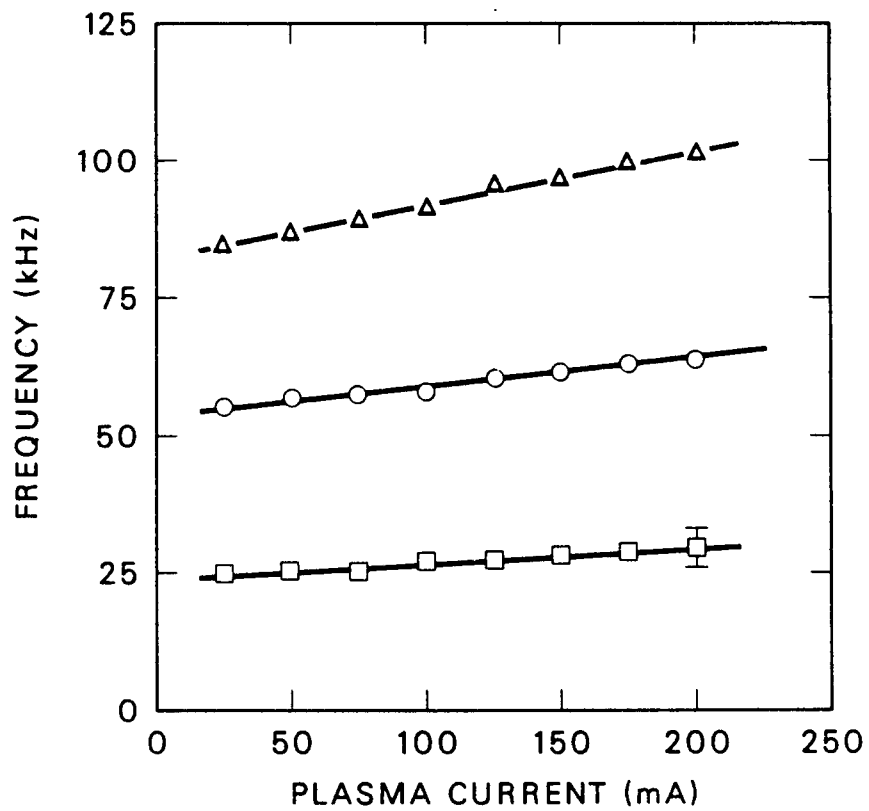


Figure 4.8. Frequency of peak in turbulent spectra as a function of plasma current.

The frequency of the fluctuations increases slightly with plasma current; however, the three peaks in the fluctuations spectrum can be seen to increase linearly at approximately the same rate. Langmuir probe data for these plasma conditions are shown in the plots in Figure 4.9.

Another set of data was taken to find the relation of the plasma fluctuations to different neutral pressures and, since the plasma current was kept constant, different anode voltages. The dependence of the fluctuations for different pressures and anode voltages is shown in Figures 4.10 and 4.11. These data show that the frequency increases as the anode voltage is increased and neutral pressure decreased. The change in plasma parameters for these different pressures and voltages is shown in Figure 4.12. As expected, the plasma potential can be seen to rise with increasing anode voltage.

In addition to correlating the plasma fluctuations with plasma operating conditions, there were some runs taken to correlate the microwave scattering signal and capacitive probe signals. The capacitive probes were placed at different axial locations and their signals correlated with the microwave scattering signal by the digital time series analysis computer program. The correlation at the scattering spectrum peaks was typically very good. An example of data taken with a capacitive probe at the same axial location as the microwave scattering diagnostic (one coil segment from the midplane) is shown in Figure 4.13. This indicates that the probe is looking at the same fluctuation in the plasma density that the microwave scattering is and that the fluctuation is electrostatic in nature.

As an attempt to measure the phase velocity of the turbulence in the plasma, two capacitive probes were placed at different azimuthal angles at the

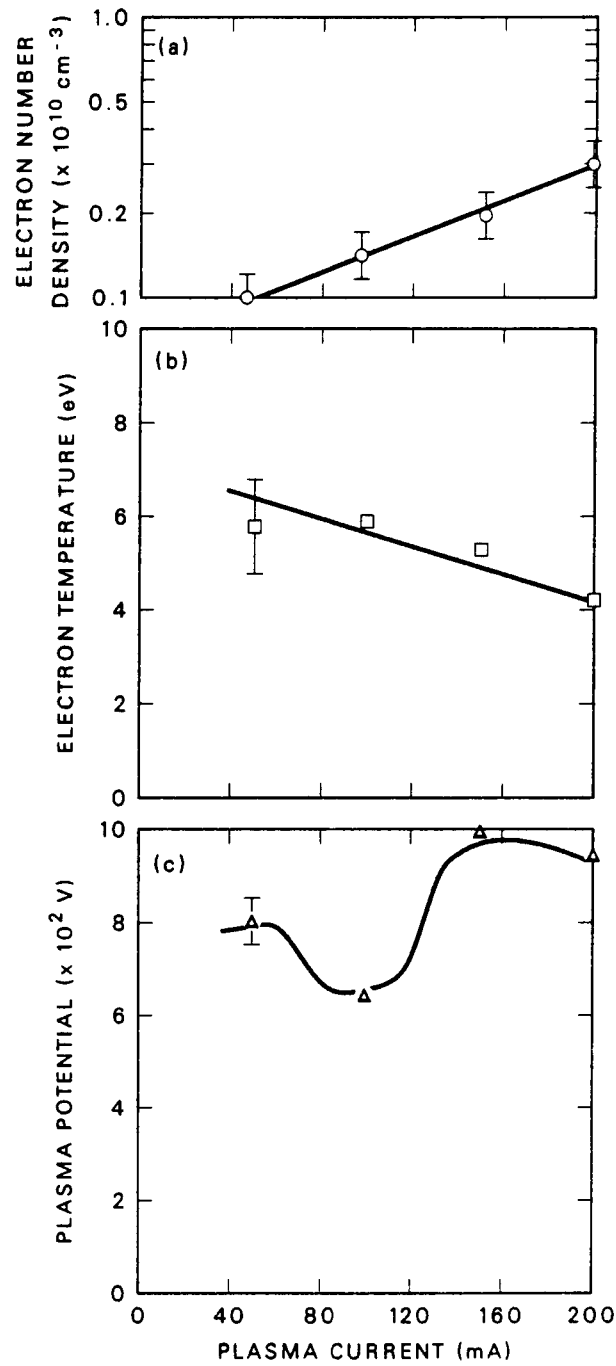


Figure 4.9. Plasma parameters as a function of plasma current. $B = 0.26 \text{ T}$, $P_0 = 3.5 \times 10^{-4} \text{ torr}$ helium.

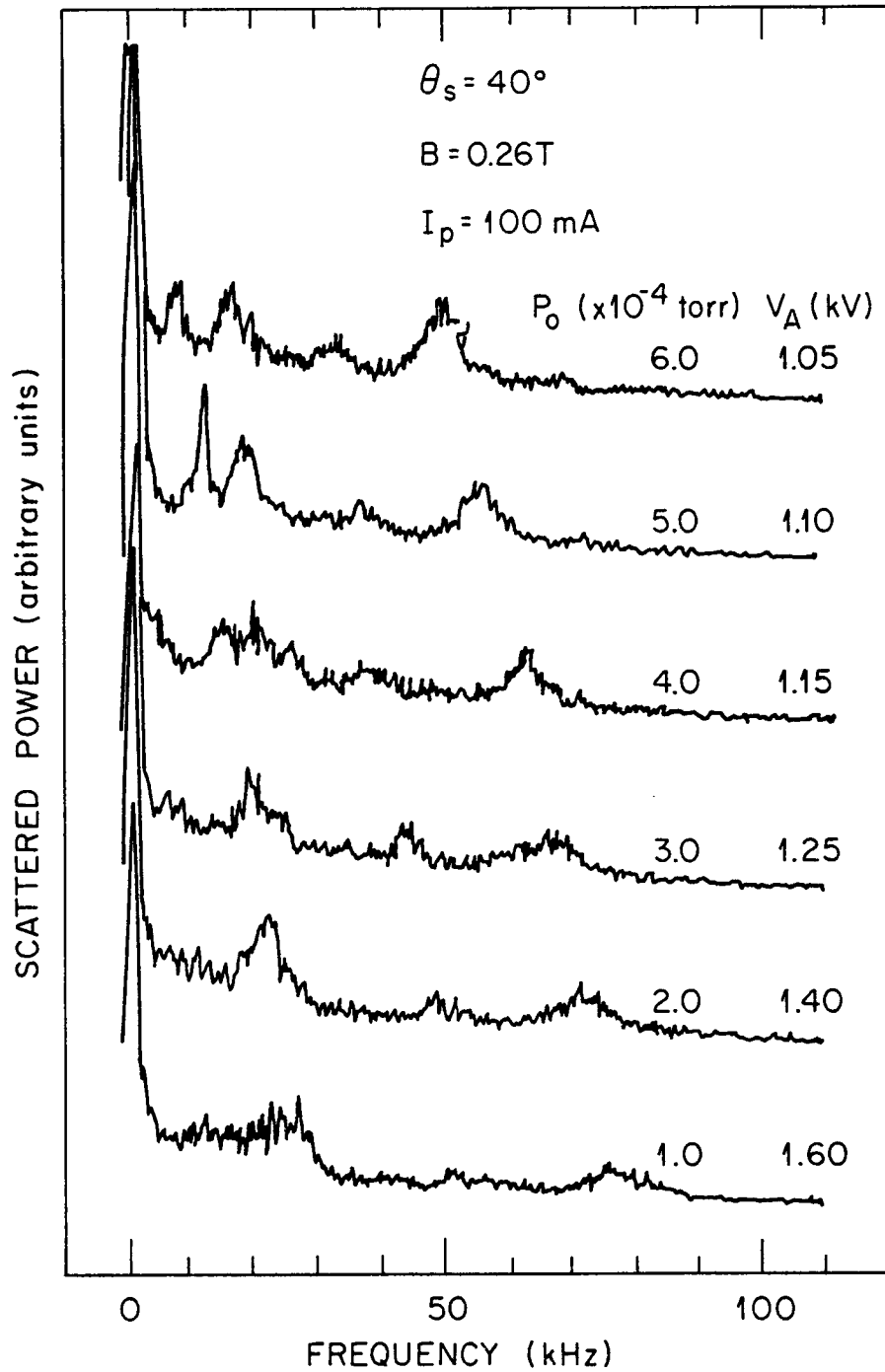


Figure 4.10. Dependence of turbulence frequency spectra on anode voltage.

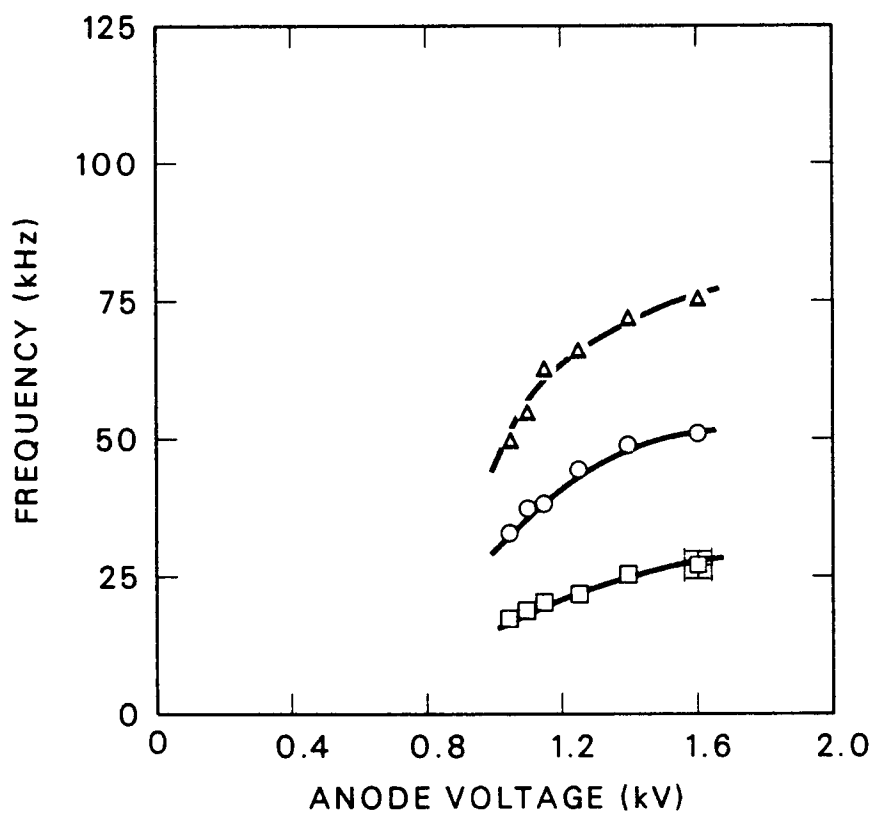


Figure 4.11. Dependence on peak frequency in turbulent spectrum as a function of anode voltage.

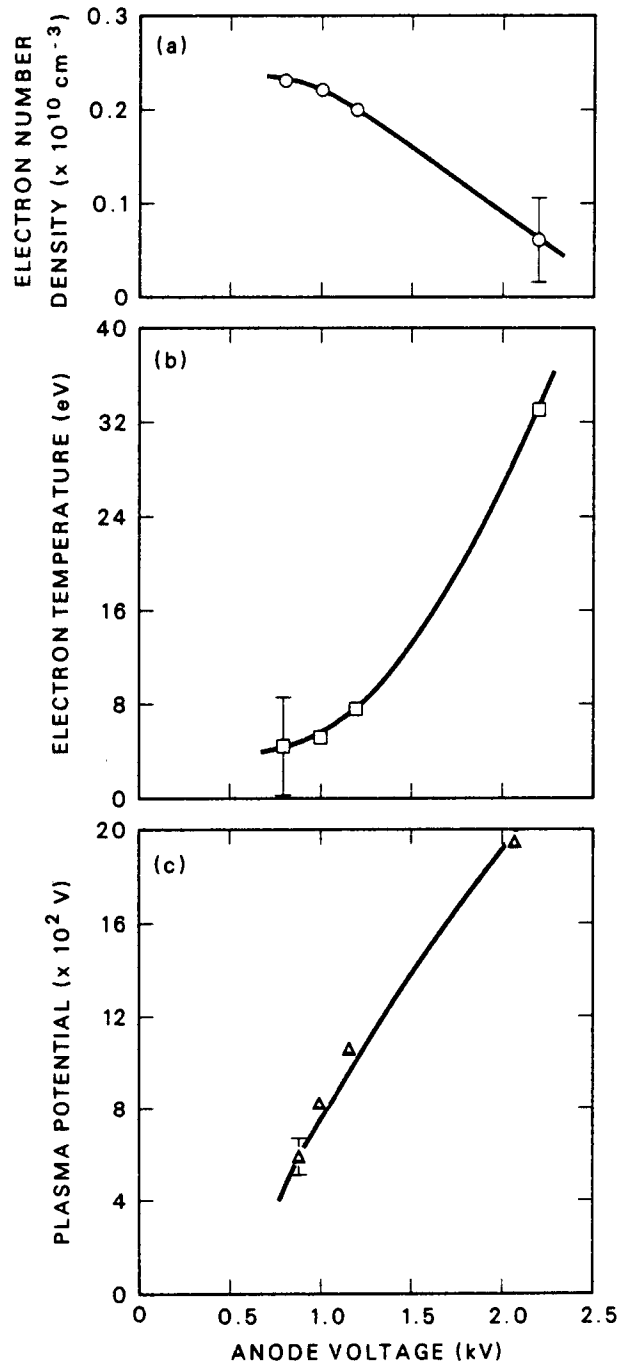


Figure 4.12. Plasma parameters as a function of anode voltage. $B = 0.26 \text{ T}$, $I_p = 100 \text{ mA}$ in helium.

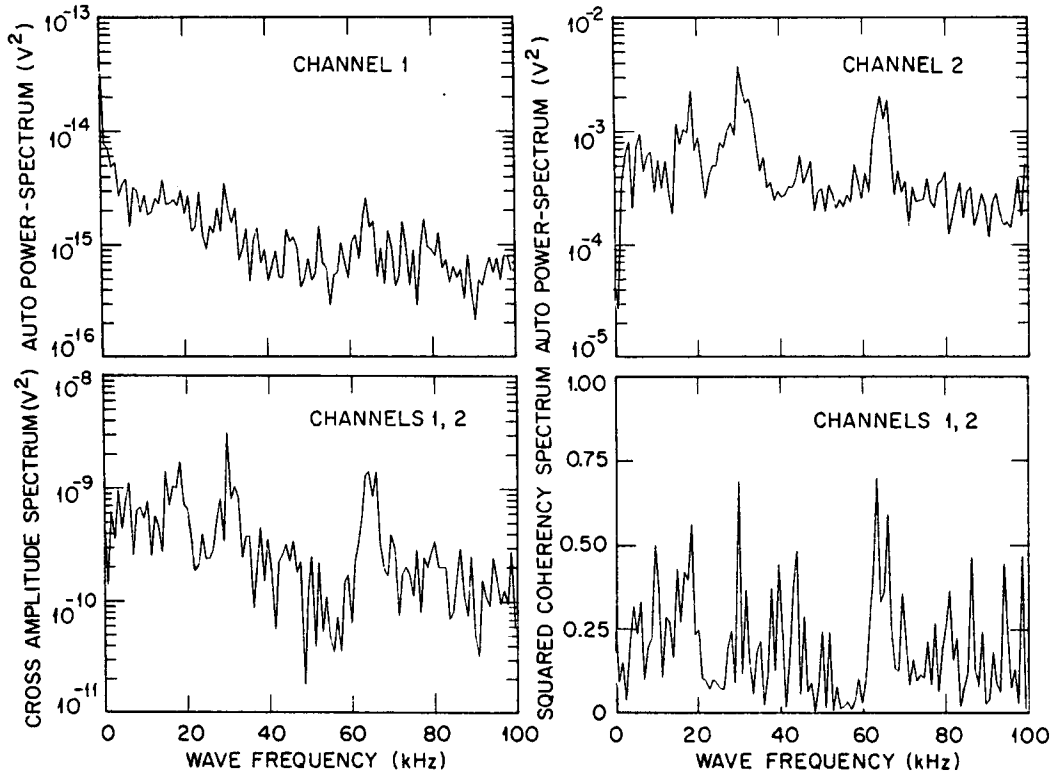


Figure 4.13. Microwave scattering signal correlated with capacitive probe signal. Channel 1 is the microwave scattering signal, and channel 2 is the capacitive probe signal. $B = 0.26$ T, $P_o = 3.5 \times 10^{-4}$ torr, $I_p = 100$ mA.

same axial location and their signals correlated by the digital time series computer analysis. The cross-power amplitude spectrum and squared coherency spectrum are shown for typical runs in Figures 4.14 and 4.15. The phase spectrum is not shown due to a phase shift problem encountered between the two Biomation waveform recorders.

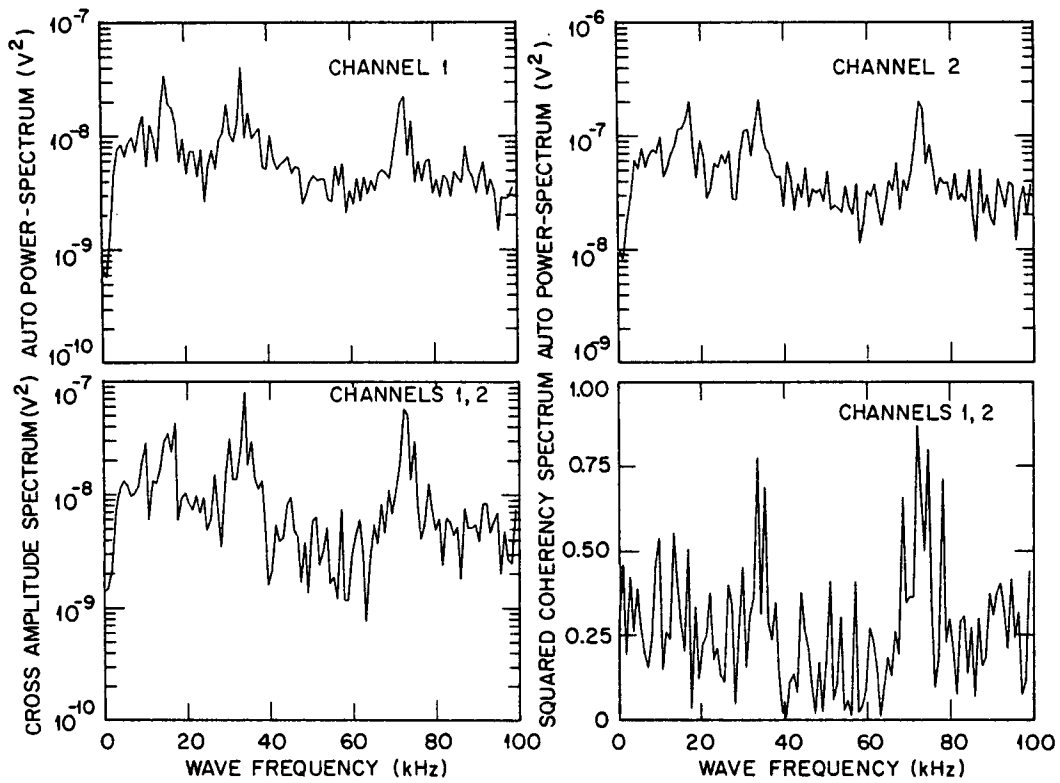


Figure 4.14. Correlation of capacitive probes at the same anode location.
 $B = 0.26 \text{ T}$, $P_o = 3.5 \times 10^{-4} \text{ torr}$, $I_p = 100 \text{ mA}$.

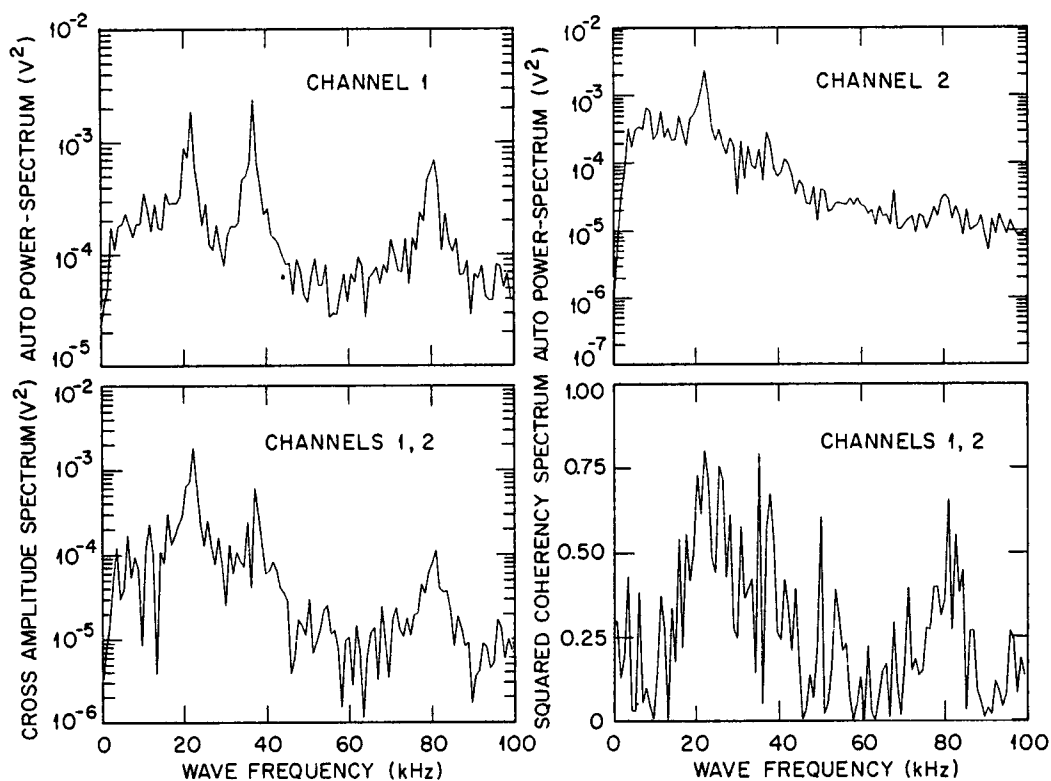


Figure 4.15. Correlation of capacitive probes at anode and cathode locations. Channel 1 is the probe at the anode, and channel 2 is the probe at the cathode. $B = 0.26$ T, $P_o = 3.5 \times 10^{-4}$ torr, $I_p = 100$ mA.

CHAPTER V

DISCUSSION OF DATA

The data presented in Chapter IV show that the microwave scattering diagnostic developed for this experiment is suitable for detecting density fluctuations in the classical Penning discharge plasma. The fluctuations found in the discharge plasma were previously undetected and, therefore, have yet to be fully analyzed. The indications are that the fluctuations are very strong since scattered power output is on the order of microwatts.

The frequency spectra data taken from the scattering diagnostic indicate low-frequency density fluctuations in the classical Penning discharge plasma. From these data it is easy to see that the fluctuations are not caused by either electron or ion plasma waves since the frequencies of the fluctuations are several orders of magnitude too low. Streaming instabilities are also not causing the measured fluctuations since the frequencies are so low. The strong dependence of the fluctuation frequency on magnetic field and the weak dependence of electron kinetic temperature on magnetic field rules out an acoustic type phenomena. An Alfvén wave would account for the strong magnetic field dependence of the fluctuation frequency; however, for the plasma parameters measured, the Alfvén velocity would approach the speed of light, thus ruling out this possibility. The frequency range of the fluctuations and the frequency dependence on plasma parameters does not eliminate the possibility of azimuthal $E \times B$ drift waves causing the measured frequency spectra.

The dispersion relation that is plotted in Figure 4.2 is quite unusual but may have a simple explanation. Since the scattering theory shows that the diagnostic in its azimuthal geometry is only sensitive to density fluctuations in the

radial or azimuthal direction, it is assumed that the fluctuations are azimuthal as derived in Chapter II. If an azimuthal drift wave exists in the plasma and has only one density maximum per azimuthal revolution, Bragg type scattering, which determines the wave number, would not have a scattering angle dependence. Since there would not be any adjacent wave crests, the comparison to scattering from layers in a crystal is not valid. It seems reasonable then that a constant fluctuation frequency would be seen independent of angle. Also, the poor spatial resolution makes determination of wave number in a specific location in the plasma difficult.

The scattered power output shown in Figure 4.3 can be used to determine an estimate of number density fluctuation by using equation (1-11). An abbreviated form of this equation can be written as

$$P_s = \frac{P_i}{A} r_o^2 V^2 \bar{n}_e^2 \Delta\Omega_r, \quad (5-1)$$

where $\Delta\Omega_r$ is the solid angle of the receiver horn (~ 0.02 sr) and r_o is the classical electron radius (2.8×10^{-13} cm). By plugging in the appropriate numbers for this equation, one finds that the fluctuation density $\bar{n}$ is of the order 10^{+8} cm $^{-3}$. This means that $\bar{n}_e/n_e$ is of the order 10^{-2} or 1%, which tells us that the plasma is indeed strongly turbulent.

The correlation of the density fluctuation data with plasma parameters does support the hypothesis that the fluctuation in the plasma density is an azimuthal $E \times B$ drift wave. The electric field dependence of the turbulence spectrum can be seen in Figure 4.10. As the anode voltage is increased, the

radial electric field increases as supported by the potential measurements (Figure 4.12). Therefore, one would expect the frequency to increase with increased anode voltage, and it does. The plasma current is not expected to have a large effect on the frequency spectrum, and it causes only a minor frequency shift, which is caused by the increased anode voltage associated with increased currents in the same neutral pressure background gas.

The magnetic field effect on the plasma density fluctuation is probably the most interesting and least understood phenomenon uncovered in this research. It is hypothesized that changing the magnetic field changes the plasma potential profiles significantly enough in magnitude to overcome the inverse B effect on the drift frequency. The fluctuation frequency linearity with B from Figure 4.5 indicates that the radial electric field varies as B . The plasma data taken with a Langmuir probe show that the plasma potential is affected by the magnetic field; without radial potential and density profiles, it is not possible to verify electric field dependence on magnetic field.

The capacitive probe data taken to correlate with the microwave scattering data show that the plasma disturbance seen by the probes has the same frequency spectra peaks as that seen by the microwave scattering (Figure 4.13). Since the probes are sensitive to electrostatic fluctuations, it seems reasonable to conclude that the fluctuations are electrostatic in nature. This supports the theory that an electrostatic drift wave is causing the fluctuations. The coherency of the signals from two capacitive probes near the anode is very strong (as shown in Figure 4.14), but the coherency is poor for one probe placed near the anode and the other near one of the cathodes (as shown in Figure 4.15). These results support the theory of radial electric fields causing the

drift waves since the radial field is strongest near the anode. Unfortunately, measurements of relative phase spectra between the two capacitive probes did not yield useful results due to a phase shift problem in the Biomation waveform recorders. From accurate phase measurements, it would be possible to determine phase velocities of the plasma density fluctuations.

All of the data taken show more than one maximum in the frequency spectrum. This can be caused by radial regions in the plasma with different electric field strengths or possibly by harmonics of the fluctuation frequency. Again, without radial plasma potential profiles it is difficult to examine these possibilities any further.

Some improvements in the microwave scattering diagnostic developed can be suggested from the data analysis. Poor spatial resolution greatly inhibited the wave vector determination. By placing an appropriate lens between the incident horn and plasma, a great increase in spatial resolution can be accomplished. The use of impedance matching material on the glass vacuum vessel can also increase the accuracy of power measurements.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Plasma turbulence caused by instabilities and beam-plasma interactions is an extremely important topic in the understanding of plasma physics. The classical Penning discharge is an example of an experimental plasma device whose plasma is turbulent due to electric fields and beam-plasma interactions and thus provides an opportunity to study these basic plasma phenomena. A useful diagnostic for studying turbulence in this plasma has been found to be microwave scattering. Information on the power spectra of low-frequency density fluctuations has been found, and correlation with capacitive probe measurements has given some insight into the nature of these fluctuations.

The study of fluctuation power spectra from microwave scattering of both naturally occurring and induced plasma turbulence has several applications relevant to fusion-grade plasmas.³⁴⁻³⁶ First, low-frequency microinstabilities in tokamak plasmas are thought to be responsible for anomalous transport. Second, rf heating of plasmas at the lower hybrid and ion cyclotron frequencies need studies of heating mechanisms and efficiencies. Several present fusion experiments use microwave scattering diagnostics with high-frequency sources (millimeter wave) such as extended interaction oscillators (EIOs) and carcinotrons (O-type backward wave oscillators).

Scattering from plasma waves as a diagnostic has the advantage of being a nonperturbing active diagnostic, and, since large enhancements in the scattered power above thermal levels can be obtained, lower power sources are needed than in Thomson scattering experiments. A wide frequency range of plasma

waves can be detected by scattering. With solid-state technology improvements, higher frequency power sources will become available, thus assuring that microwave scattering diagnostics will remain important in fusion plasma research.

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APPENDIXES

APPENDIX A

LIST OF SYMBOLS

A	Area
B	Magnetic flux density
c	Speed of light in vacuum; 2.998×10^8 m/s
E	Electric field
h	Planck's constant; 6.626×10^{-34} J·s
k	Wave number
L	Length
m	Mass
N	Number of particles
n	Number density
P_i	Incident power
P_s	Scattered power
q	Charge
R	Position
$S(\mathbf{k}, \omega)$	Spectral density function
t	Time
V	Volume
$\mathbf{v}$	Velocity
z	Coordinate
r	Radial coordinate
λ_D	Debye length
θ_s	Scattering angle

ϵ_r	Permittivity
μ_o	Permeability of free space; $4\pi \times 10^{-7}$ H/m
$d\Omega$	Element of a solid angle
ω	Angular frequency
ν	Collision frequency
$\emptyset$	Angle
ϕ	Plasma potential
$\lambda = 2\pi/k$	Wavelength
Γ	Reflection coefficient
ω_c	Gyrofrequency
η	Wave impedance
ω_{pe}	Electron plasma frequency
$\alpha = (k_i\lambda_D)^{-1}$	Salpeter parameter
ρ	Charge density
ρ_m	Mass density

APPENDIX B

PLASMA PHYSICS EQUATIONS

Two-fluid equations:

$$\left. \begin{aligned} \frac{dn_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) &= 0 \\ \frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) &= 0 \end{aligned} \right\} \text{equations of continuity}$$

$$\left. \begin{aligned} n_e m_e \frac{d\mathbf{v}_e}{dt} + \nabla \cdot \mathbf{P}_e + en_e (\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) &= \mathbf{R} \\ n_i m_i \frac{d\mathbf{v}_i}{dt} + \nabla \cdot \mathbf{P}_i - Zen_i (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) &= -\mathbf{R} \end{aligned} \right\} \text{equations of motion}$$

$$\mathbf{R} = \int d^3v C_{\alpha\beta} m_{\alpha} \mathbf{v} \quad \text{collision term}$$

Maxwell's equations:

$$\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= -\mu_o \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \\ \nabla \cdot \mathbf{E} &= \rho/\epsilon_o \end{aligned}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{where} \quad c = (\epsilon_o \mu_o)^{-1/2}$$

Lorentz force:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Gyrofrequency:

$$\omega_c = \frac{qB}{m}$$

Eulerian derivative:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

VITA

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