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**A REGULARIZED DATA FUSION APPROACH TO
MULTISENSOR SURFACE RECONSTRUCTION AND
EDGE DETECTION APPLICATIONS**

A Thesis

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Master of Science

Degree

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ABSTRACT

Enhancing a robotic system's inspection and manipulation capabilities requires the extraction of numerous features from an environment. To retrieve multiple properties necessitates multiple sensors since each sensing device is limited to the amount and type of data it can incorporate.

In this thesis, we examine the problem of extracting given features, such as object boundaries, from different sensing modalities for recognizing objects. The mathematical analysis will focus on the early vision problems of surface reconstruction and edge detection. Since these applications are characterized mathematically by ill-posed properties, regularization is proposed as the basic tool for restoring stability and well-posedness to these problems.

This thesis extends Salinas' [29] prior doctoral work on regularized fusion. A fusion functional for integrating data from different knowledge sources will be formulated to overcome the difficulties associated with using only one stabilizer. This new hybrid functional, containing first- and second-order stabilizers, will give rise to filtered, interpolated surfaces and good detection/localization properties for the edge detection scheme. The algorithm extends standard regularization theory by allowing multiple inputs of different sensory cues, constant and variable smoothing parameters, and a weighting factor associated with each data source. The computational aspects of the algorithm as well as its performance on real and synthetic images are evaluated.

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CHAPTER 1

INTRODUCTION

Many modern robotic systems require the use of multiple sensors because of one knowledge source's inability to provide all of the necessary information for inspection and manipulation tasks. Multisensor fusion combines the output from two or more sensors to form a reliable estimate of the environment which would be unattainable by any individual knowledge source. Using different sensory modules leads to a system that is more reliable, productive, and efficient in performing a given task. Multisensory robotic systems have applications in automatic target recognition (ATR) [28], automatic manufacturing, air traffic control systems, military intelligence gathering [17], and many other areas.

1.1 Advantages to Multisensor Fusion

The basic idea behind sensor fusion is to combine (or fuse) the output of two or more sensing devices for retrieval of a particular property from a known or unknown environment. For example, this property could be an object, such as a contaminated barrel located in a storage room for which a robot employing multiple sensors (video camera, range finder, sonar, and tactile sensor) would combine the output of two or more of these devices to locate the barrel. With

this illustration in mind, there are several distinct advantages to using multiple sensors [11, 17] :

- Individual sensors usually output a signal contaminated with noise and measurement errors that often leads to degraded or incomplete information. Multiple sensors can be used to determine the same property, but with the consensus of all sensors. This integration of redundant information can reduce sensor uncertainty and increase reliability.
- The output of a single sensor can be misleading, in which case another sensor could be used to resolve this deception. For instance, vision sensing does a poor job in scenes with shadows, so a range sensor could be added since it would have little difficulty in judging the distance or detecting an object.
- Every sensor is sensitive to different properties in the environment. For example, vision sensing is needed for object identification and recognition while range sensing provides depth measurements for guidance tasks, and touch sensing provides reassurance of a particular location. When all of these sensors are used in conjunction with one another, multiple properties can be extracted from a scene, which in turn provides a great deal more information than any single source could supply.

Based on these beneficial factors, data fusion plays a key role in applications where different sensing modules [4, 15] produce multiple properties (or characteristics) that must be fused for extracting useful features from an environment.

1.2 Data Fusion Techniques

Abidi *et. al* [17] defined *multisensor fusion* as the integration process in which there is an actual combination (or fusion) of different sources of sensory information into one representational format. With integration, a particular sensor performs a subtask or provides some type of information so the next sensor may proceed by using this knowledge to perform its own task.

There are several fusion strategies used for combining multiple data sources. The most simple case for a multisensor configuration is to fuse the data by averaging. In this method appropriate weights are assigned proportional to the reliability of each sensor. This simple heuristic method will have problems when a large number of sensors are used and the sensor interaction is more complex. Other heuristic methods include deciding and guiding. Deciding is the process of using one of the data sources during a certain time of the fusion process, while guiding is the use of one or more sensors to focus the attention of another sensor to a particular location in the scene. Another strategy for fusing multiple knowledge sources is based on Bayesian statistics where the central component is Bayes' theorem, which uses past knowledge about the occurrence of an event to infer the chances of a similar event happening in the future. The next fusion strategy is based on Dempster-Shafer theory which models sensor uncertainty and finds the occurrence of an event as a function of two or more assessments. Another technique is based on the use of fuzzy set theory to model uncertain measurements. In

these models, pixels are assigned a fuzzy membership value in which a defuzzification scheme is carried out to recover features from the combined fuzzy assessment. There are data fusion approaches that use pattern recognition principles, such as Pitman's Closeness and Complete Linkage Algorithm. This technique involves testing sensor data closeness, merging close sensor data, and optimizing the close sensor data. The final fusion strategy is based on analytical techniques that infer knowledge from two assessments by using some constraints on the evidence collected to infer the final judgment. The proposed hybrid energy functional can be placed into this later category. There are other sensor fusion methods that are not classified to a specific category. For instance, sensor measurements could be combined by simple *if-then* rules, or the logical *AND* operation.

A problem that must be addressed before any fusion technique can be implemented is the registration of the different sensors, data points. For instance, there must be some method for relating the data points from one sensor with the corresponding data points of another. This can easily be done between some sensors, for instance range and intensity data can be registered by using known fields of view, tilt, and pan angles. In the case of different fields of view the data from one sensor will not align with the corresponding data from another sensor. Another difficulty which must be taken into consideration is making sure the sensor measurements represent the same physical entity. In other words, a check needs to be made on the consistency of the sensor measurements. In our case, the fusion of the range and intensity data will be registered by the same field of view and have

a one-to-one correlation established by way of an edge map from each respective source.

1.3 Typical Multisensor Applications

This section illustrates two applications in which multiple sensors collaborate to obtain a particular property from an environment. The selection of sensors could vary depending upon the task requirement; however, the most common sensors include ultrasonic, range, and vision. Each sensor has unique characteristics which makes it more applicable in certain situations. We now illustrate two typical applications in which multisensor fusion is used in improving the overall efficiency, reliability, and accuracy of a robotic system in hazardous working environments.

Department of Energy (DOE) facilities throughout the United States have become littered with radioactive waste. It is now apparent that these sites must be cleaned for environmental purposes. Unfortunately, information relative to the specific contents of these sites is often incomplete. For buried waste, for instance, the location, depth, and distribution of the materials below the surface of each waste site must be determined prior to excavation. Hensel and Dalton [12] presented an approach for employing ground-penetrating radar (GPR) data in conjunction with Geomagnetic Anomaly Detection (GAD) data to obtain an accurate representation of the underground environment. The data from the suite of sensors was fused by an inversion methodology that provided an output containing

more detailed information than any single source could obtain. In conclusion, fusing the data from both sensors provided information characterizing the contents of the landfill (i.e., a steel drum, tank, pipe, etc.).

Many industrial robots are now equipped with multiple sensors for inspection and manipulation tasks. The reason is that many applications require the recognition and handling of complex objects, and one sensing modality cannot provide all of the information needed for the task. Abidi *et al.* [16] developed an approach for using multiple sensors in a nuclear environment to enhance safety, reliability, and productivity. The scene consisted of a six-degree-of-freedom industrial robot and a control panel simulating objects found in a nuclear power plant. The interchangeable end-effector consisted of a sensing module to which a number of sensors were added—vision, range, sound, proximity, force/torque, and touch—to enhance its inspection and manipulation capabilities. The idea was to remove the human operator and allow the robot to perform the desired tasks autonomously. Using its sensory cues, the robotic system mapped the workspace and performed several inspection and manipulation tasks, including reading digital commands, identifying a meter and reading its status, recognizing and manipulating a valve, turning switches on or off, and detecting an alarm. These applications show how a multisensing robotic system can be used in hazardous environments such as nuclear, military, chemical, and space. At the central part of every robotic system there is a need for acquiring, fusing, and interpreting multisensory data for performing a desired task quickly and efficiently.

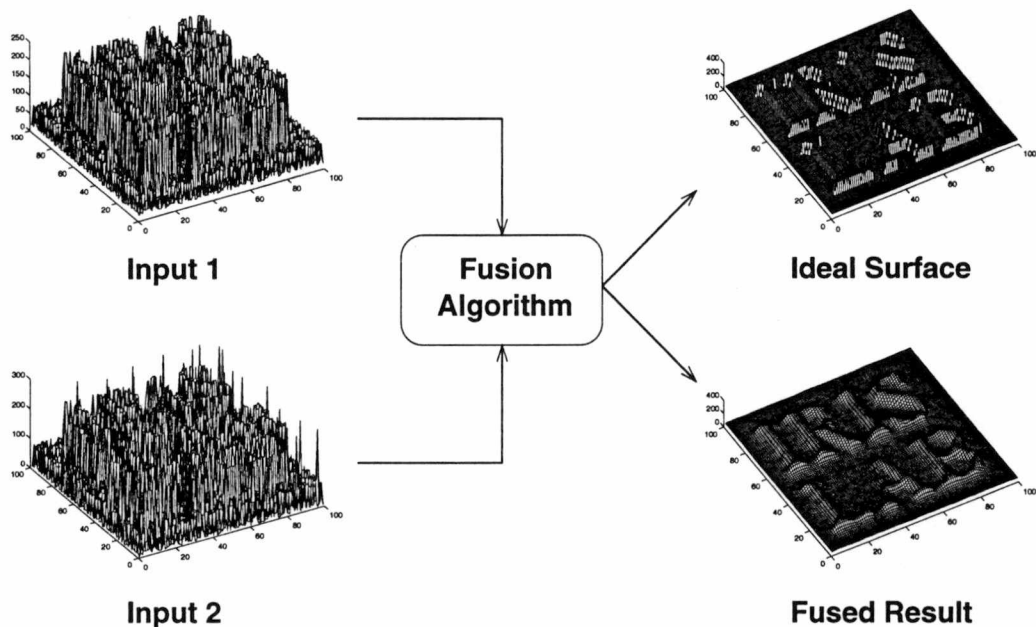


Figure 1.1: *A typical application for jointly reconstructing two inputs to achieve a noise-free output for the surface reconstruction scheme.*

1.4 Problem Statement

Identifying and locating object boundaries in a scene is the ultimate goal in many computer vision tasks. The objective of this thesis is to develop an extended version of standard regularization theory that allows multiple knowledge sources to be fused in order to aid the early vision problems of surface reconstruction and edge detection. Figure 1.1 is a good illustration of the surface reconstruction problem in which we are trying to achieve a jointly reconstructed noise-free output which still preserves edges. It has been established that multisensory fusion can increase reliability, productivity, and reduce sensor uncertainty by integrating multiple

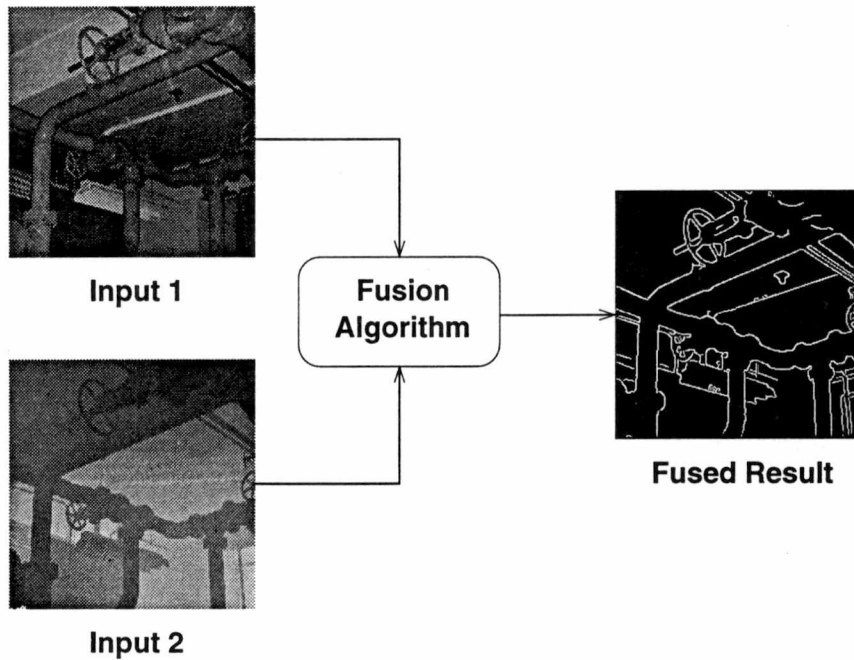


Figure 1.2: *A typical application for fusing range and intensity data to achieve a final edge map for object recognition purposes.*

properties from numerous sensing devices. The early vision problems of edge detection and surface reconstruction are ill-posed problems requiring a regularized approach for restoring stability in the final outcome. Figure 1.2 illustrates a typical fusion approach for combining range and intensity data into one representational format. This final output is an edge map which can be further processed to detect and recognize objects in a scene. Based on standard regularization theory we propose a hybrid energy functional which combines first- and second-order stabilizers for enhancing these early vision processes.

We will examine the hybrid functional for both 1-D and 2-D data with empha-

sis placed on variable regularization parameters instead of the traditional global approach found in standard regularization theory. This generalization arises from the fact that constant smoothing parameters tend to oversmooth the discontinuities which play an important role in many computer vision applications. The goal of the hybrid functional in relation to the surface reconstruction and edge detection problems is to provide a means for reconstructing a smooth, filtered surface from multiple noisy input sources to allow better edge detection capabilities.

1.5 Synopsis

This chapter provides an introduction to multisensor fusion and shows how many robotic systems benefit from using different sensing modules. On one hand, using one sensing component limits the performance of a robot's inspection and manipulation abilities, but on the other hand, using multiple sensors can provide data that was unavailable with a single source. The importance of data fusion is apparent, especially in robotic systems requiring full autonomous operation. If an industrial robot is substituted for a human observer at any workplace, it is, in effect, replacing an excellent multisensing organism; however, which will be more reliable, efficient, productive, and cost-effective in the future?

CHAPTER 2

BACKGROUND

This chapter provides background material for the early vision problems of surface reconstruction and edge detection in relation to various applications of standard regularization theory. First, Section 2.1 reviews the surface reconstruction problem. Then, Section 2.2 provides a thorough examination of regularized range and intensity data fusion.

2.1 Surface Reconstruction Techniques

This section introduces various approaches for applying regularization theory to the early vision problem of surface reconstruction. The goal is to reconstruct or interpolate a smooth surface of the 3-D world based on knowledge extracted from a 2-D image. The projection of a 3-D object to a 2-D Cartesian system results in the loss of a dimensionality, making the problem ill-posed. Regularization has become a necessary tool for solving this inversion problem and restoring well-posedness in the final solution.

We begin with a paper by Grimson [10] that illustrates a computational algorithm associated with the task of constructing complete surface specifications consistent with the information contained in the zero-crossings. He defined two

conditions: (1) *surface consistency constraint*, where the absence of zero-crossings constrains the possible surface shapes, and (2) “*no news is good news*”, which states a surface cannot change radically except when given information from the zero-crossings that states otherwise. He used these constraints for approximating a smooth surface for fitting the given input data. It is shown that the best functional is one which measures quadratic variation in surface orientation and is consistent in providing a smooth surface through the data. The functional proposed is the quadratic variation, assuming the surface is twice continuously differentiable:

$$\Phi(f) = \left\{ \int \int (f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2) dx dy \right\}^{1/2}. \quad (2.1)$$

Two algorithms were used for minimizing the potential energy functional: the gradient projection interpolation and the conjugate gradient approximation algorithm. His theory analyzed both synthetic and real data and results confirm his prediction that the quadratic variation functional produces surfaces that better agree with the input data.

Terzopoulos [30] proposed a nonquadratic stabilizing functional that allows for spatial control over the smoothing properties of the stabilizers. A class of multi-dimensional *controlled-continuity stabilizers*, composed of generalized spline kernels, is suggested for the regularization of inverse vision problems. Through regularization, a wide range of ill-posed visual reconstruction problems may be

reformulated as a variational principle: Find $u \in \mathcal{H}$ such that

$$\mathcal{E}(u) = \inf_{v \in \mathcal{H}} \mathcal{E}(v), \quad (2.2)$$

where the objective is to find the function u that minimizes $\mathcal{E}(v) = \mathcal{S}(v) + \mathcal{P}(v)$, where

$$\mathcal{S}(v) = \int \int_{\mathbb{R}^2} \rho(x, y) \{ [1 - \tau(x, y)] \underbrace{(v_x^2 + v_y^2)}_{T_1} + \tau(x, y) \underbrace{(v_{xx}^2 + 2v_{xy}^2 + v_{yy}^2)}_{T_2} \} dx dy$$

is a nonquadratic controlled-continuity stabilizer and

$$\mathcal{P}(v) = \sum_{i=1}^N \alpha_i (\mathcal{L}_i(v) - c_i)^2 \quad (2.3)$$

is a “penalty” term used for discrepancy purposes between admissible functions and the given data c_i . The terms $\mathcal{L}_i$ and α_i represent measurement functionals and nonnegative weights, respectively. The function u represents, in the case of surface reconstruction, a solution which can be expressed as a depth function. Terzopoulos illustrated that the term T_1 was proportional to the small deflection energy of a membrane, while T_2 corresponded to the bending energy of a thin plate. The proposed functional can be viewed as the combination of the thin plate and membrane spline kernels. The key to this approach is the spatial control of the thin plate model (affecting surface curvature) and the membrane model (affecting surface area). Standard regularization has been extended by using controlled-continuity constraints whose smoothness properties are regulated spatially to preserve visual discontinuities. This process benefits segmentation and reconstruction processes.

Blake and Zisserman [1] define visual reconstruction as the process of reducing visual data to stable descriptions. The term “visual data” refers to information

in various forms: raw intensity data, a depth map, optic flow, and sets of discrete data points. Their application includes using weak continuity constraints imposed by the General Non-Convexity (GNC) [1] algorithm, which has a natural edge detecting ability. First consider the simplest weak continuity problem for detecting step discontinuities (edges) in 1-D data. The aim is to construct a piecewise smooth 1-D function $u(x)$ which is a good fit to some data $d(x)$. This is achieved by modelling $u(x)$ as a “weak elastic string” and minimizing the total energy functional

$$E = \int_0^N (u - d)^2 dx + \lambda^2 \int_0^N u'^2 dx + P(x) , \quad (2.4)$$

where $P(x)$ is a penalty term and λ^2 is a measure of elasticity. The approach is, given $d(x)$, find the function $u(x)$ for which the total energy E is smallest. Using the “finite element method,” a conversion from the continuous to discrete form can be established.

$$E = \sum_0^N (u_i - d_i)^2 + \lambda^2 \sum_1^N (u_i - u_{i-1})^2 (1 - l_i) + \alpha \sum_1^N l_i , \quad (2.5)$$

where l_i is a “line process” used for breaking the elastic string. It was shown that the minimization of the sequence l_i could be performed separately, which allows the energy term to become

$$\min_{\{u_i\}} F = \sum_0^N (u_i - d_i)^2 + \sum_1^N g(u_i - u_{i-1}) . \quad (2.6)$$

Now we wish to minimize the above energy functional over the sequence u_i . This approach is quite difficult because the energy is nonconvex, thus yielding a system

u_i that may have numerous stable states, with each corresponding to a local minimum of energy. The GNC algorithm is used to find the global minimum by approximating a new energy functional which is convex and hence has only one local minimum.

Muraki *et al.* [21] proposed an algorithm for reconstructing a topographic shape from a real topographic map using regularization. They describe a method for reconstructing a 3-D surface from a contour line image by regularization and apply it to contour lines and altitude data extracted from a real topographic map. Their objective is to reconstruct detailed topographic information. Given altitude data, a surface is reconstructed as a solution of the problem of minimizing an energy functional

$$Energy = E_{smooth} + \lambda E_{data} , \quad (2.7)$$

where E_{smooth} represents a quadratic stabilizing functional imposing smoothness on a surface and E_{data} is a penalty functional characterizing the error between the observed data and surface. The constant λ is a weighting parameter adjusting the strength of conformity with the altitude data. A new constraint or penalty functional was added to the scheme, which established that points on the same contour line have the same height

$$E_{contour} = \sum_{i=1}^m \sum_{j=1}^n \left[\frac{\partial u}{\partial x} \Delta x_{(x,y)} + \frac{\partial u}{\partial y} \Delta y_{(x,y)} \right]^2 . \quad (2.8)$$

Hence, surface reconstruction by regularization, including contour line informa-

tion, can be done by finding a surface minimizing the energy functional

$$Energy = E_{smooth} + \lambda E_{data} + \mu E_{contour} , \quad (2.9)$$

where μ is a weighting parameter adjusting the strength of the contour line constraint. If given altitude data corresponding to geographic numerical information, a reconstructed shape from a contour line image can be approximated by minimizing an energy functional with its various constraints.

Gökmen and Li [6, 7, 8] illustrated an extension to standard regularization theory. They developed an edge detection and surface reconstruction algorithm in which the smoothness parameter (λ) is controlled spatially over the image space. This iteratively refined regularization method (IRR) outperformed standard regularization theory because it did not assume a fixed regularization parameter over the entire image space. It is clearly defined that the regularization parameter needs to be large in noisy or continuous regions (*e.g.* $\lambda = 10$ or higher), while small at or near discontinuities (*e.g.* $\lambda = [0.1 - 1.5]$). In other words, a small parameter preserves edges, while a larger value tends to oversmooth edges, in turn destroying crucial surface properties about the scene. The proposed energy functional consisted of a single membrane stabilizer with variable smoothing parameters,

$$E(u) = \int \int_{\mathbb{R}^2} \beta(u - d)^2 dx dy + \lambda \int \int_{\mathbb{R}^2} (u_x^2 + u_y^2) dx dy . \quad (2.10)$$

Using the finite difference method, this functional can be written in discrete form

as

$$E_d(u) = \sum_i \sum_j \beta_{i,j} (u_{i,j} - d_{i,j})^2 + \lambda \sum_i \sum_j [\underbrace{(u_{i,j} - u_{i-1,j})^2}_{S_x} + \underbrace{(u_{i,j} - u_{i,j-1})^2}_{S_y}] \quad (2.11)$$

where the function $\beta_{i,j}$ is 1 if a datum at point (i, j) exists. The stabilizer in this functional has two components: the first, S_x , imposes the smoothness constraint in the x direction, while S_y imposes smoothness in the y direction. Their final proposed energy functional consisted of both horizontal and vertical regularization parameters, $\lambda_{i,j}^h$ and $\lambda_{i,j}^v$,

$$\begin{aligned} E_d(u, \lambda^h, \lambda^v) = & \sum_i \sum_j \beta_{i,j} (u_{i,j} - d_{i,j})^2 \\ & + \sum_i \sum_j [\lambda_{i,j}^h (u_{i,j} - u_{i-1,j})^2 + \lambda_{i,j}^v (u_{i,j} - u_{i,j-1})^2]. \end{aligned} \quad (2.12)$$

The algorithm starts with an oversmoothed regularized solution and iteratively refines the surface around discontinuities by using the structure exhibited in the error signal. The algorithm was shown to satisfy their prior requirements of detecting true edges, noise filtering, efficient implementation, and applicability to sparse data. The performance of the IRR algorithm has successfully shown that it can be used as a discontinuity-preserving smoothing operator even for missing data points.

Girosi *et al.* [5] showed regularization networks encompass a much broader range of approximation schemes, including many of the popular general additive models used for interpolating surfaces. In particular they introduce new classes of smoothing functionals that lead to different classes of basis functions. The term *Generalized Regularization Networks* will be used to denote this broad class of

approximation schemes which follow from an extension of regularization theory. They illustrated the function that minimizes the energy functional has the form

$$f(\mathbf{x}) = \sum_{i=1}^N c_i G(\mathbf{x} - \mathbf{x}_i) + \sum_{\alpha=1}^k d_{\alpha} \psi_{\alpha}(\mathbf{x}) \quad (2.13)$$

where $\{\psi_{\alpha}\}_{\alpha=1}^k$ is a basis in the k -dimensional null space $\mathcal{N}$ and the coefficients d_{α} and c_i satisfy the following linear system:

$$(G + \lambda I)c + \Psi^T d = y. \quad (2.14)$$

A one hidden-layer neural network scheme was adapted for the approximation scheme of Eq. (2.13). They examined different multilayer networks with one hidden layer by using different classes of priors and associated smoothness functionals in a classical regularization principle. Three broad classes were examined: (1) radial basis functions that generalize into hyperbasis functions, (2) some tensor product splines, and (3) additive splines that generalize into schemes of the type of ridge approximation, hinge functions, and one-hidden-layer perceptrons. In summary, standard regularization has been extended by approximating its solution with a regularization network from different types of smoothing functionals.

Mizutani [18] extended standard regularization theory by developing a generalized smoothness constraint of a real-order stabilizer, which is composed of a generalized differential integral (GDI) operator. In recovering plausible surfaces it is not sufficient to use membrane or thin-plate energy stabilizers alone; therefore, other stabilizing functionals need to be considered. Their proposed stabilizing functional is constructed using the GDI operator. Its usefulness will

become apparent in the surface reconstruction problem. The GDI operator is defined as

$$D_x^\alpha f(x) \equiv \frac{d^\alpha f(x)}{dx^\alpha} = \int dk e^{sgn(k)i\pi\alpha/2} |k|^\alpha \tilde{f}(k) e^{ikx}, \quad (2.15)$$

where α is the *smoothness index*. This operator is defined as a pseudo-differential operator and yields the real-order derivatives and integrals of certain functions instead of yielding just the integer-order derivatives and integrals. For the surface reconstruction problem, we want to find the function $f(x)$ that minimizes the quadratic functional

$$J_1(f) = \sum_{n=1}^N (f(x_n) - g_n)^2 + \lambda \|D_x^\alpha f(x)\|^2. \quad (2.16)$$

This is for the 1-D case with N scattered data points g_n , and the smoothing parameter λ . Using Euler-Lagrange relations leads to a convolution filter R_1 for the reconstruction process. After some manipulation, the 1-D regularizing filter is found to be

$$R_1(x, \alpha, \lambda) = \sum_k \frac{1}{N + 2\pi\lambda|k|^{2\alpha}} e^{ikx}, \quad (2.17)$$

where the 1-D surface can be recovered using the convolution relation

$$f(x) = R_1(x, \alpha, \lambda) * g(x). \quad (2.18)$$

Another extension, the smoothness space, was proposed by Mizutani which controls continuities of functions so as to give any order of smoothness. The continuous stabilizer was able to reconstruct a surface that was more practical or ideal to

the original signal, while the thin-plate model delivered an oversmoothed version. A comparison of the continuous and thin-plate stabilizer reveals that this new method is far superior in reconstructing surfaces closer to the true surface from sparse noisy data.

Lovergine *et al.* [2] proposed an approach for extending the work of Gökmen and Li [8] by using the well-known model of a *weak thin-plate under tension* [31] for smoothing noisy range data. Their algorithm is applicable to the early vision problems of structure estimation and edge detection. The thin-plate model was modified by introducing continuous nonbinary line processes to locally control the “degree” of surface regularity. Continuity and differentiability of the surface were controlled by comparing the reconstructed surface with the initial data (sparse or dense). This information supplied an initial estimate to where the smoothing conditions should be weakened, especially near discontinuities, and also where they should be increased for noise removal. Their proposed energy functional consisted of both a “pure” membrane and thin-plate model

$$\begin{aligned} \frac{1}{2} \int \int_{\Omega} \rho(x, y) \tau(x, y) \underbrace{(f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2)}_{\text{thin-plate}} + [1 - \tau(x, y)] \underbrace{(f_x^2 + f_y^2)}_{\text{membrane}} \\ + \beta(x, y)(f - d)^2 dx dy, \end{aligned} \quad (2.19)$$

where the function d is the initial sparse data set with a weighting function $\beta(x, y)$. The functions $\rho(x, y)$ and $\tau(x, y)$ are line processes that control the degree of smoothing in the solution. In regularization theory, smoothness can be imposed by finding the solution f which minimizes the above energy functional [Eq. (2.19)].

This minimization approach yields a linear system of equations which can be solved by one of the iterative methods. Their regularization technique was applied to the visual surface reconstruction problem for noisy nondense data sets obtained from range finders. Test results indicated that for both synthetic and real range maps, each degraded with Gaussian noise, a completely reconstructed surface from sparse data was possible. The final solution consisted of an interpolated surface that was noise-free and still maintained sharp, distinctive edges in the solution.

Yaou and Chang [35] utilized the vector space decomposition (reconstruction) theory of the wavelet transform to analyze the multiresolution representation of the surface interpolation problem. Through this analysis, a technique was proposed which increases the convergence rate for minimizing the discrete formulation of the regularized functional through the use of a multiresolution basis transfer scheme based on wavelet theory. By applying the wavelet transform, the original interpolation basis is transformed into two sets of bases with larger supports while the admissible solution remains unchanged. With this basis transfer, a new set of nodal variables results, and an equivalent equation system with a better convergence condition can be solved. This wavelet approach is useful for various regularization problems because of its accelerated technique in approximating the solution. The results for this fast surface interpolation algorithm reveals that it can readily reconstruct a surface much faster than any of the iterative schemes. Pentland [22] also presented an approach to regularization by introducing orthogonal wavelet bases for preconditioning purposes. The most common iterative

algorithms have the drawback in that they require many iterations to produce a smooth interpolated surface; therefore, an approach using wavelets as preconditioning is used to give a good approximate solution in only two or three iterations.

2.2 Vision and Range Sensor Fusion

This section presents algorithms for fusing intensity and range data when the ability to reconstruct and recognize objects located in a scene (singulation tasks) is important. A vision sensor has the advantage of obtaining photometric characteristics such as shape, while the range sensor provides depth information for reassurance. Each sensor can be explicitly described as having its own unique characteristics and limitations. Results will reveal that integrating (or fusing) the two sensory devices leads to an output which clearly outperforms each of the individual sensors.

Our first paper begins with Rajapakse and Acharya [27] proposing a new regularized method for fusing different types of sensory data because a single set of visual data is not always sufficient for performing an intended vision task. The computational method presented can deal with fusion of higher-order discontinuities and with data sets belonging to different visual cues. The visual discontinuities under examination are the first-order differences of intensities (step edges) and the second-order differences of intensities (crease edges). Their energy functional

is given by

$$\mathcal{E}(v, \rho, \tau) = \mathcal{P}(v) + \mathcal{S}(v|\rho, \tau) + \mathcal{D}(\rho, \tau), \quad (2.20)$$

where ρ and τ represent the crease and the depth discontinuity functions, respectively. The disparity functional

$$\mathcal{P}(v) = \sum_i \sum_j \gamma_{i,j} (d_{i,j} - v_{i,j})^2 \quad (2.21)$$

represents the inequality between the reconstruction v and the visual data d . The controlled continuity stabilizer

$$\mathcal{S}(v|\rho, \tau) = \sum_i \sum_j (\tau_x \rho_x v_{xx}^2 + 2\tau_x \rho_x \tau_y \rho_y v_{xy}^2 + \tau_y \rho_y v_{yy}^2) + \tau_x (1 - \rho_x) v_x^2 + \tau_y (1 - \rho_y) v_y^2$$

provides the smoothness constraint for the visible surface and the functions τ and ρ denote step and crease discontinuities at a particular pixel (i, j) location. The discontinuity functional

$$\begin{aligned} \mathcal{D}(\rho, \tau) = & \sum_i \sum_j \alpha(\tau_{x,i,j})(1 - \tau_{x,i,j}) + \alpha(\tau_{y,i,j})(1 - \tau_{y,i,j}) \\ & + \beta(\rho_{x,i,j})(1 - \rho_{x,i,j}) + \beta(\rho_{y,i,j})(1 - \rho_{y,i,j}), \end{aligned} \quad (2.22)$$

where the functions α and β are *energy penalties* for the formation of step and crease discontinuities. The energy functional is minimized using the successive over-relaxation (SOR) method. This functional was examined on simulated range and intensity data contaminated with some degree of Gaussian noise. Results for these simulated test images indicates that the fusion approach is able to detect a high percentage of the true discontinuities while not marking false edges. The

contribution of the proposed method is its ability to detect step and crease discontinuities with different visual sources.

Another regularized fusion approach was examined by Abidi [17]. He established the notion of sensors as narrow passband filters that pass only partial information about the scene, with all of these pieces of data needing to be united to form a more accurate and complete description of the environment. For integrating multisensory information, he proposed two principles for guiding the fusion process. The first principle, *Knowledge Source Corroboration* (KSC), says a feature not detected by one sensor but detected by another will be included in the output. Every sensor has its own limitations; therefore, some type of sensor corroboration is necessary for providing information that one sensor cannot supply. The second principle, *Belief Enhancement/Withdrawal* (BEW), provides a means for securing the integrity of the initial data by using another sensor's measurements to resolve or influence conflict between inconsistent data. For instance, if sensor S_1 forms a belief about the occurrence of an event and sensor S_2 confirms this belief, then we say that the opinion enhances S_1 and the final outcome can contain either of the sensor's assessments. On the other hand, if S_2 's opinion contradicts S_1 's belief, then the upcoming result leads S_1 to withdraw its opinion. He developed a fusion functional for combining two fuzzy membership functions $\mu(\mathbf{x})$ and $\eta(\mathbf{x})$

$$\varphi(\mathbf{x}) = \mathcal{F}(\mu, \eta) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} c_{ij} \mu^i(\mathbf{x}) \eta^j(\mathbf{x}) , \quad (2.23)$$

where $\mathbf{x} = (x_1, x_2, \dots, x_n)'$ is a vector of dimension n and the c_{ij} 's are unknown

coefficients that must be determined. Note that the higher-order c_{ij} 's can be ignored and the remaining terms can be determined by imposing the proposed principles. To enforce the KSC principle, $\varphi(\mathbf{x})$ should be as near to its maximum value as possible. This can be accomplished by minimizing $(1 - \varphi)$ over Θ :

$$\min \left[\int \int \cdots \int_{\Theta} (1 - \varphi)^2 dx_1 dx_2 \cdots dx_n \right]. \quad (2.24)$$

At the same time the BEW principle can be enforced by

$$\min \left[\int \int \cdots \int_{\Theta} \|\nabla(\alpha\mu - \beta\eta)\|^2 dx_1 dx_2 \cdots dx_n \right], \quad (2.25)$$

where ∇ denotes $\partial/\partial\mathbf{x} = (\partial/\partial x_1, \partial/\partial x_2, \dots, \partial/\partial x_n)'$. Using these constraints he derives the energy functional

$$G = f^2 + \lambda^2 \sum_{i=1}^n (\alpha_{x_i}(\mu + \eta) + \alpha(\mu_{x_i} + \eta_{x_i}) - \eta_{x_i})^2, \quad (2.26)$$

where λ controls the emphasis of either principle. The goal is to find the solution $\alpha(\mathbf{x})$ which minimizes the energy functional (energy G is smallest). By using Euler-Lagrange methods, the solution can be minimized in its final form as

$$\varphi(\mathbf{x}) = \mathcal{F}(\mu, \eta) = \alpha(\mathbf{x})\mu(\mathbf{x}) + (1 - \alpha(\mathbf{x}))\eta(\mathbf{x}). \quad (2.27)$$

Some type of iterative method must be used for solving the linear system of equations resulting from the fusion functional. His analysis examined the *successive approximation* and *conjugate gradient* for successively approximating the solution. In conclusion, he presented a new technique for fusing different sensory cues modelled by fuzzy membership functions.

Not many regularization-based fusion techniques have been under investigation recently; therefore, we now illustrate other methods for combining range and intensity data. Our first illustration, Huntsberger and Jawerth [13], propose a sensor fusion technique based on the wavelet transform. The wavelet transform is a multiresolution method used to decompose images into detail and average channels. These channels maintain all of the image information, and sensor fusion logic operations can be done within the wavelet coefficient space. This paper presents an approach to sensor fusion that uses the wavelet coefficients directly in the integration process. Logic operations such as AND and OR are performed within the α and γ channels in either the uncompressed or compressed forms. The wavelet image decomposition is a type of multiresolution analysis that maintains both frequency and spatial information. The detail (γ) channels of the transform contain localized orientation information about the image at various scales and the average (α) channel contains general information at various scales. The standard wavelet decomposition of a general signal f is represented in terms of linear combinations of translated dilates of a single function ψ :

$$f(x) = \sum_n \sum_k c_{v,k} \psi_{v,k}(x) , \quad (2.28)$$

with $\psi_{v,k}(x) = 2^{v/2} \psi(2^v x - k)$ for integers v and k . The mapping that takes f into the corresponding coefficients $c_{v,k}$ is called the wavelet transform of f . A property of f is that it is defined on the whole plane or is double periodic. To satisfy this concept the original image f must be padded with zeros or allowed to continue periodically, but in either case border artifacts are introduced which degrade the

quality of a compressed image. Hence they propose the following biorthogonal wavelet algorithm to overcome this problem:

$$f(x) = \sum_{i \text{ finite}} \sum_v \sum_k \langle f, \tilde{\psi}_{v,k}^i \rangle \psi_{v,k}^i. \quad (2.29)$$

By allowing two different sets of building blocks, the theory becomes significantly more flexible and makes it easier to find wavelets with desirable properties such as symmetry, short filter lengths, and binary filter coefficients. Their fusion approach was tested on registered range and visible images from a cluttered laboratory environment. First, they generated the wavelet coefficients for the two images by the biorthogonal closed set algorithm and then applied the fusion logic operators of AND (min) and OR (max) to the data. Results for the OR fusion reveals its unique ability to supply the crisp edges which aid in segmentation.

Zhang and Wallace [36] presented a method for semantic labelling of edges and reconstruction of range data by fusion of registered range and intensity images. In this paper, they label the fused edge data by the subset

$$\Lambda = \{blade, fold, mark, no \ edge\}. \quad (2.30)$$

The detection and classification of discontinuities in the registered range and intensity data is accomplished in two stages. First, an initial classification scheme is obtained by filtering the registered data and by Bayesian estimation of the most likely label at each site. Second, these site labels are updated by an iterative relaxation process. The final edge classification and range reconstruction module are obtained using Bayesian estimation within coupled Markov random fields (MRF)

employing constraints of surface smoothness and edge continuity. This approach has two principal advantages in the context of object recognition. First, it makes use of both boundary- and surface-based descriptions, each of which can be used independently to describe the scene. Through their combination, a result can be achieved with a fuller description which takes advantage of the strengths and weaknesses of each method in isolation. Finally, each knowledge source has been used to modify the other through the MRF model, thus ensuring consistency in the fused description. The proposed technique is demonstrated on synthetic and real data sources obtained from an active laser range finder.

2.3 Synopsis

This chapter analyzed previous research in the areas of surface reconstruction, choosing the optimal regularization parameter for the variational energy functional, and range and intensity data fusion. This thesis will address these and other issues in the problem formulation, where emphasis will be placed on fusing multiple sensory inputs by extending regularization theory. This regularized fusion approach will (1) interpolate smooth surfaces for multiple noisy (dense or sparse data) inputs; (2) provide a means for fusing range and intensity data when the edges are extracted for object recognition tasks; and (3) examine the concept of choosing the optimal regularization (smoothing) parameter.

CHAPTER 3

REGULARIZED MULTISENSOR FUSION

This chapter analyzes the development of the hybrid energy functional and how it extends standard regularization theory by allowing (1) multiple inputs of different visual cues to be fused, (2) constant and variable smoothing parameters, and (3) the simultaneous use of first- and second-order stabilizers. The proposed formulation will be applied to the early vision problems of surface reconstruction (to dense and sparse data) and gray-level edge detection. The framework for this chapter is to describe standard regularization theory and how it solves the ill-posed problem for many computer vision applications. The formulation and implementation of the hybrid model will also be examined and discussed in detail.

Research conducted by Salinas [29] found the first-order stabilizer performs sufficiently in nonactive regions, meaning areas that contain no discontinuities and heavy concentrations of noise, by reconstructing a smooth filtered surface from the given noisy data. On the other hand, the second-order stabilizer performs adequately at or near discontinuities by interpolating a sharp, well-defined surface with some noticeable oscillations near edge boundaries. Therefore, based on the above knowledge we propose the hybrid energy functional, which filters the noisy data in continuous regions and reconstructs a surface with well-defined edges near

discontinuities for improving the detection and localization criteria required in the edge detection scheme.

3.1 Standard Regularization Theory

The computer vision problems of surface interpolation, edge detection, motion, structure estimation, and optical flow are ill-posed problems exhibiting solutions that are nonexistent, nonunique, or do not depend continuously on the data [25]. Numerical differentiation, or the detection of discontinuities, is ill-posed because its solution does not depend continuously on the data. Surface reconstruction is an inverse problem that occurs in the loss of a dimensionality during the projection from a 3-D world coordinate system to a 2-D image.

Poggio *et al.* [23, 24] showed that several early vision problems are mathematically ill-posed; therefore, regularization was proposed as the basic tool for restoring well-posed solutions [19, 20, 32]. The basic idea of regularization, as formulated by Tikhonov [33], is to restrict the space of acceptable solutions by choosing a function that minimizes an appropriate functional, such as

$$\|\mathbf{A}z - y\|^2 + \lambda \|\mathbf{P}z\|^2, \quad (3.1)$$

where $\mathbf{A}$ is an operator that samples the function z on a lattice, λ is the parameter that controls the degree of regularization of the solution and its closeness to the data, and $\mathbf{P}$ is the stabilizing functional that imposes the smoothness constraint. The minimized solution is represented in terms of approximating cubic splines

and is good since it has to be close to the data. A more general choice for $\mathbf{P}$ is Tikhonov's stabilizing functional, which contains a large range of derivatives

$$\|\mathbf{P}z\|^2 = \sum_{m=0}^p \int_{\mathfrak{R}} w_m(x) \left(\frac{d^m z(x)}{dx^m} \right)^2 dx, \quad (3.2)$$

where the function w_m is a prespecified weighting function. Let's examine a typical case for the stabilizing functional with parameters $w_1 = w_2 = 0$, $w_3 = 1$, and $p = 2$. This particular selection of values yields the second-order stabilizer

$$\|\mathbf{P}z\|^2 = \int_{\mathfrak{R}} (z''(x))^2 dx. \quad (3.3)$$

Using this stabilizer, the standard regularization method for the 1-D case can be stated as finding the approximate function z which minimizes the energy functional

$$E(z) = \sum_i \underbrace{(y(x_i) - z(x_i))^2}_{E_1} + \lambda \underbrace{\int (z''(x))^2 dx}_{E_2}, \quad (3.4)$$

where the term E_1 is a closeness measure of solution z to data y , and E_2 is a smoothness measure. The energy functional $E(z)$ can be thought of as a system of energy springs, Figure 3.1, in which the goal is to find the local minimum with the lowest energy. The compromise between the two terms, E_1 and E_2 , is controlled by the regularization parameter (λ). If λ is small, E_1 (faithfulness to the data) dominates and the resulting function $z(x)$ will be a close fit to the data $y(x)$. On the other hand, if λ is large, the stabilizer E_2 dominates and the function z will be a smooth, filtered version of the input. Here, smooth refers to a function which

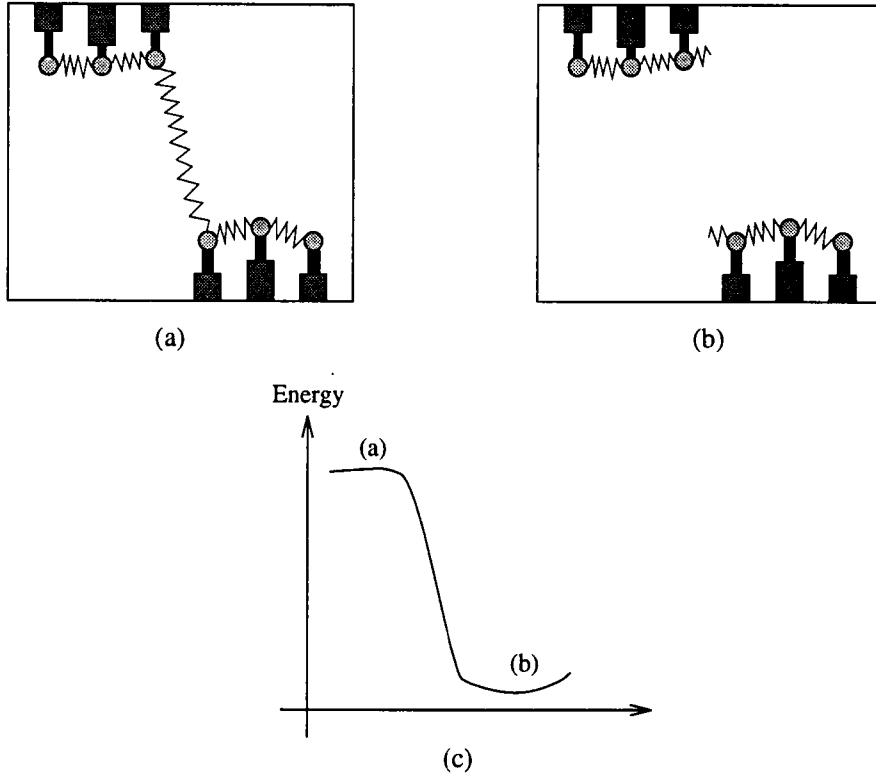


Figure 3.1: *The energy function can be regarded as a system of springs: (a) highest energy state, (b) lowest energy state, and (c) find the local minimum.*

has been low-pass filtered and appears to be smoother when compared to the original input. The smoothness constraint, E_2 (stabilizer), is justified by noting that the noiseless image has to be smooth in the sense that all its derivatives exist and are bounded. This constraint eliminates the noise caused during data acquisition and recovers the continuity of the solution, making the problem well-posed for the differentiation process. It should be noted that a first-order stabilizer could be constructed with weights $w_1 = 0$, $w_2 = 1$ and $p = 1$. The same properties mentioned for the second-order stabilizer also apply to the first-order stabilizer.

Standard regularization theory has many beneficial characteristics, especially in interpolating a surface through sparse data, but it does have a major drawback. Its global assumption of a constant smoothing parameter totally disregards discontinuities that are vital for surface reconstruction and edge detection applications. For instance, setting the smoothing parameter λ to a large value emphasizes regularization, yielding model solutions which are mostly noise-free, but virtually oversmooths all edges in the process. On the other hand, smaller values enable detection of discontinuities, but they also make the solutions model the original inputs closely. As a result, variable parameters are introduced to avoid the problem of filtering or oversmoothing discontinuities. The updating procedure for the regularization parameters will be based on knowledge extracted from both the structure of discontinuities exhibited in the error signal (obtained from the image data) and the previous regularized solution. This method will preserve edges by allowing a control parameter to be set at any discontinuity location. This special control parameter will force the regularization parameter to be small at areas near discontinuities and large in continuous regions. In conclusion, this approach will provide an output that filters noise and preserves edges while providing good detection and localization properties for edge detection applications.

3.2 Hybrid Functional

Based on standard regularization theory, we propose the hybrid energy functional which is a combination of both first- and second-order stabilizers. With this improved functional comes the benefits of noise tolerance and steeper, more distinct edges for the surface reconstruction process. Section 3.2.1 introduces the hybrid functional for the 1-D case and Section 3.2.2 examines the 2-D formulation. Section 3.3 develops an extension to the hybrid model and illustrates the minimization approach used for the first- and second-order stabilizers. The final section illustrates the various steps required for implementing the hybrid model.

3.2.1 One-Dimensional Case

Let's begin with the continuous form of the hybrid energy functional

$$\sum_{i=1}^m \int w_i (f(x) - d_i(x))^2 dx + \int [\lambda \underbrace{(f'(x))^2}_{FOS} + \beta \underbrace{(f''(x))^2}_{SOS}] dx \quad (3.5)$$

where the constant w_i represents the weight factor associated with each data source. It is obvious that when dealing with multiple inputs less weight is given to the noisier source. The function $d_i(x)$ represents m data inputs and the smoothing parameters λ and β are associated with the first- and second-order stabilizers, respectively. The stabilizers represent the amount of elastic energy stored when the energy functional is stretched. The constants λ and β are measures of "stretchability" or willingness to deform. Another form of the continuous case is for variable regularization parameters and is illustrated by a slight change in

Eq. (3.5)

$$\sum_{i=1}^m \int w_i (f(x) - d_i(x))^2 dx + \int [\lambda(x)(f'(x))^2 + \beta(x)(f''(x))^2] dx \quad (3.6)$$

where $\lambda(x)$ and $\beta(x)$ are now variable instead of constant. A discretized version of Eq. (3.5) and (3.6) can be established by means of the finite difference scheme, where

$$f_x \equiv f_i - f_{i-1}$$

and

$$f_{xx} \equiv f_{i-1} - 2f_i + f_{i+1} . \quad (3.7)$$

Using the above difference equations, the hybrid functional for the 1-D discrete case with constant smoothing parameters is

$$E_{DHC}(f) = \sum_{j=1}^m \sum_i w_j (f_i - d_j)^2 + \sum_i [\lambda (f_i - f_{i-1})^2 + \beta (f_{i-1} - 2f_i + f_{i+1})^2] . \quad (3.8)$$

The functional can also be formulated with variable smoothing terms which are better suited for preserving edges. When variable smoothing terms are included, the functional becomes

$$E_{DHV}(f) = \sum_{j=1}^m \sum_i w_j (f_i - d_j)^2 + \sum_i [\lambda_i (f_i - f_{i-1})^2 + \beta_i (f_{i-1} - 2f_i + f_{i+1})^2] \quad (3.9)$$

where the regularization parameters λ_i and β_i are variable. The functions $E_{DHC}(f)$ and $E_{DHV}(f)$ represent the discrete hybrid functional for constant and variable smoothing parameters, respectively.

Once the continuous functions $\lambda(x)$ and $\beta(x)$ are determined, the energy functional is strictly convex with a single local minimum. This eliminates the complexity of minimizing a nonconvex functional with multiple local minima where slow global minimization methods have to be used (*i.e.* graduated nonconvexity (GNC) or simulated annealing). The approach for minimizing the hybrid functional with variable and constant smoothing parameters is found by setting the first derivative to zero by

$$\frac{\partial E_{DH}}{\partial f_i} = 0 \quad (3.10)$$

where we are solving for the unknown function f_i that minimizes the energy functional associated with the hybrid stabilizer. The goal is to construct a smooth 1-D function f_i which is a good fit to the given data. Using this minimization approach the hybrid functional with constant smoothing parameters with two data sources and constant weights is

$$\begin{aligned} \frac{\partial E_{DHC}}{\partial f_i} = & (w_1 + w_2)f_i + 6\beta f_i + 2\lambda f_i - (w_1 d_1 + w_2 d_2) \\ & + \beta(f_{i-2} - 4f_{i-1} - 4f_{i+1} + f_{i+2}) - \lambda(f_{i-1} + f_{i+1}). \end{aligned} \quad (3.11)$$

A similar approach used for the variable smoothing parameters with two data sources and constant weights is given by

$$\begin{aligned} \frac{\partial E_{DHV}}{\partial f_i} = & (w_1 + w_2)f_i - w_1 d_1 - w_2 d_2 + \lambda_i(f_i - f_{i-1}) + \beta_i(-2f_{i-1} + 4f_i - 2f_{i+1}) \\ & + \lambda_{i+1}(f_i - f_{i+1}) + \beta_{i+1}(f_i - 2f_{i+1} + f_{i+2}) + \beta_{i-1}(f_{i-2} - 2f_{i-1} + f_i), \end{aligned}$$

or after rearranging,

$$\begin{aligned} \frac{\partial E_{DHV}}{\partial f_i} = & (w_1 + w_2 + \lambda_i + 4\beta_i + \lambda_{i+1} + \beta_{i+1} + \beta_{i-1})f_i - w_1 d_1 - w_2 d_2 - [\lambda_i f_{i-1} \\ & + 2\beta_i(f_{i-1} + f_{i+1}) + \lambda_{i+1} f_{i+1} + \beta_{i+1}(2f_{i+1} - f_{i+2}) + \beta_{i-1}(2f_{i-1} - f_{i-2})]. \end{aligned}$$

For a discrete signal $(x_1, x_2, \dots, x_N)$ this gives N linear simultaneous equations that can be written in the form

$$Au = b \tag{3.12}$$

where $A \in \mathbb{R}^{N \times N}$ is a symmetric quindagonal positive-definite matrix. A reasonable choice for finding the minimal solution is the iterative scheme successive over-relaxation (SOR) [3, 9, 14]. This iterative process starts with a trial solution vector $f^{(0)}$ and carries out an operation using A, b and $f^{(0)}$ to get a new vector $f^{(1)}$. This process is repeated until at the n^{th} stage the iterative process yields $f^{(n)}$ from A, b and $f^{(n-1)}$. It should be noted that the iterative procedure is terminated whenever the absolute error $|f^{(n+1)} - f^{(n)}| < \varepsilon$ reaches a specified threshold. The SOR method is an accelerated version of the Gauss-Seidel process and its general form is

$$f_i^{(n+1)} = f_i^{(n)} - \frac{\omega}{T} \frac{\partial E_{DH}}{\partial f_i}, \tag{3.13}$$

where T is a factor that has to be chosen to ensure convergence whenever $0 < \omega < 2$. For interior nodes, $T = w_1 + w_2 + \lambda_i + \lambda_{i+1} + 4\beta_i + \beta_{i+1} + \beta_{i-1}$, but would require modifications at or near boundary locations. Suppose we have a 1-D discrete signal $(x_1, x_2, x_3, \dots, x_n)$ where n is the number of elements in the 1-D

array. For this case, T would have to be modified for the boundary elements x_1 and x_2 and the last two discretized components x_{n-1} and x_n . This modification occurs as a result of minimizing the hybrid functional which introduces the functions f_{i-2} and f_{i+2} . These functions cannot be used at the beginning or end of the sampled signal because their values are nonexistent.

The modifications for the border conditions will be examined for each individual case. The x_1 (or left border) discrete location yields

$$\begin{aligned} \frac{\partial E_{DHV}}{\partial f_i} = & (w_1 + w_2 + \lambda_{i+1} + \beta_{i+1})f_i - w_1 d_1 - w_2 d_2 \\ & - [\beta_{i+1}(2f_{i+1} - f_{i+2}) + \lambda_{i+1}f_{i+1}], \end{aligned} \quad (3.14)$$

the x_2 location

$$\begin{aligned} \frac{\partial E_{DHV}}{\partial f_i} = & (w_1 + w_2 + 4\beta_i + \beta_{i+1} + \lambda_i + \lambda_{i+1})f_i - w_1 d_1 - w_2 d_2 \\ & - [\beta_i(2f_{i+1} - f_{i-1}) + \beta_{i+1}(2f_{i+1} - f_{i+2}) + \lambda_i f_{i-1} - \lambda_{i+1} f_{i+1}], \end{aligned} \quad (3.15)$$

the x_{n-1} location

$$\begin{aligned} \frac{\partial E_{DHV}}{\partial f_i} = & (w_1 + w_2 + 4\beta_i + \lambda_i + \lambda_{i+1})f_i - w_1 d_1 - w_2 d_2 \\ & - [\beta_{i-1}(2f_{i-1} - f_{i-2}) + 2\beta_i(f_{i-1} + f_{i+1}) + \lambda_i f_{i-1} - \lambda_{i+1} f_{i+1}], \end{aligned} \quad (3.16)$$

and the x_n (or right border) discrete location gives

$$\begin{aligned} \frac{\partial E_{DHV}}{\partial f_i} = & (w_1 + w_2 + \beta_{i-1} + \lambda_i)f_i - w_1 d_1 - w_2 d_2 \\ & - [\beta_{i-1}(2f_{i-1} - f_{i-2}) + \lambda_i f_{i-1}]. \end{aligned} \quad (3.17)$$

Results for the 1-D hybrid stabilizer with constant regularization parameters are illustrated in the next chapter.

3.2.2 Two-Dimensional Case

We now focus our attention to the 2-D continuous case

$$\sum_{i=1}^m \int \int w_i (f(\mathbf{x}) - d_i(\mathbf{x}))^2 d\mathbf{x} + \int [\lambda(\mathbf{x})(f'(\mathbf{x}))^2 + \beta(\mathbf{x})(f''(\mathbf{x}))^2] d\mathbf{x}, \quad (3.18)$$

where $\mathbf{x} = (x, y)$. This formulation can also be written as

$$\sum_{i=1}^m \int \int w_i (f(\mathbf{x}) - d_i(\mathbf{x}))^2 d\mathbf{x} + \int [\lambda(\mathbf{x})(f_x^2 + f_y^2) + \beta(\mathbf{x})(f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2)] d\mathbf{x},$$

where a discretized version can be achieved using the following finite difference equations

$$\begin{aligned} f_x &\equiv f_{i,j} - f_{i-1,j} \\ f_y &\equiv f_{i,j} - f_{i,j-1} \\ f_{xx} &\equiv f_{i-1,j} - 2f_{i,j} + f_{i+1,j} \\ f_{yy} &\equiv f_{i,j-1} - 2f_{i,j} + f_{i,j+1} \end{aligned}$$

and

$$f_{xy} \equiv f_{i,j} - f_{i+1,j} - f_{i,j+1} + f_{i+1,j+1}. \quad (3.19)$$

By using these difference equations the hybrid functional can be rewritten as

$$\begin{aligned} E_{DHC} &= \sum_{n=0}^m \sum_i \sum_j w_n (f_{i,j} - d_{n,i,j})^2 + \sum_i \sum_j \left[\underbrace{\lambda[(f_{i,j} - f_{i-1,j})^2 + (f_{i,j} - f_{i,j-1})^2]}_{S_1} \right. \\ &\quad + \underbrace{\beta[(f_{i-1,j} - 2f_{i,j} + f_{i+1,j})^2]}_{S_2} + \underbrace{2(f_{i,j} - f_{i+1,j} - f_{i,j+1} + f_{i+1,j+1})^2}_{S_3} \\ &\quad \left. + \underbrace{(f_{i,j-1} - 2f_{i,j} + f_{i,j+1})^2}_{S_4} \right], \end{aligned} \quad (3.20)$$

where m represents the number of inputs with a constant weighting factor w_n . (Note, we will denote the intensity of the data at the $(i, j)^{th}$ pixel site by $f_{i,j}$). To minimize the solution with respect to the function $f_{i,j}$ we must first take the discrete approximations for the terms S_1, S_2, S_3 , and S_4 in [Eq. (3.20)]. This results in the following discrete approximations:

$$S_1 = \lambda [(f_{i,j} - f_{i-1,j})^2 \quad (3.21-a)$$

$$+ (f_{i,j} - f_{i,j-1})^2 \quad (3.21-b)$$

$$+ (f_{i+1,j} - f_{i,j})^2 \quad (3.21-c)$$

$$+ (f_{i,j+1} - f_{i,j})^2], \quad (3.21-d)$$

$$S_2 = \beta [(f_{i-1,j} - 2f_{i,j} + f_{i+1,j})^2 \quad (3.22-a)$$

$$+ (f_{i,j} - 2f_{i+1,j} + f_{i+2,j})^2 \quad (3.22-b)$$

$$+ (f_{i-2,j} - 2f_{i-1,j} + f_{i,j})^2, \quad (3.22-c)$$

$$S_3 = 2 [(f_{i,j} - f_{i+1,j} - f_{i,j+1} + f_{i+1,j+1})^2 \quad (3.23-a)$$

$$+ (f_{i-1,j} - f_{i,j} - f_{i-1,j+1} + f_{i,j+1})^2 \quad (3.23-b)$$

$$+ (f_{i,j-1} - f_{i+1,j-1} - f_{i,j} + f_{i+1,j})^2 \quad (3.23-c)$$

$$+ (f_{i-1,j-1} - f_{i,j-1} - f_{i-1,j} + f_{i,j})^2], \quad (3.23-d)$$

and

$$S_4 = (f_{i,j-1} - 2f_{i,j} + f_{i,j+1})^2 \quad (3.24-a)$$

$$+ (f_{i,j} - 2f_{i,j+1} + f_{i,j+2})^2 \quad (3.24\text{-b})$$

$$+ (f_{i,j-2} - 2f_{i,j-1} + f_{i,j})^2 \Big] . \quad (3.24\text{-c})$$

The updating expression for the interior nodes would take the partial derivative with respect to $f_{i,j}$ for all the terms in S_1, S_2, S_3 , and S_4 yielding,

$$\begin{aligned} \sum_{k=1}^4 \frac{\partial S_k}{\partial f_{i,j}} = & \lambda(4f_{i,j} - f_{i-1,j} - f_{i,j-1} - f_{i+1,j} - f_{i,j+1}) + \beta(20f_{i,j} - 8f_{i-1,j} - 8f_{i+1,j} \\ & + f_{i+2,j} + f_{i-2,j} - 8f_{i,j+1} + 2f_{i+1,j+1} + 2f_{i-1,j+1} + 2f_{i+1,j-1} + 2f_{i-1,j-1} \\ & - 8f_{i,j-1} + f_{i,j+2} + f_{i,j-2}). \end{aligned} \quad (3.25)$$

Note, however, that the updating procedure would have to be modified for regions around edges and corners. For these nodes only a few terms would be required from the general equations S_1, S_2, S_3 , and S_4 . For an $N \times N$ image this discrete formulation results in a linear system of N^2 simultaneous equations of the form

$$Au = b \quad (3.26)$$

where $A \in \Re^{N^2 \times N^2}$ is a symmetric positive-definite matrix solvable by the iterative SOR method

$$f_{i,j}^{(n+1)} = f_{i,j}^{(n)} - \frac{w}{T} \frac{\partial E_{DH}}{\partial f_{i,j}}. \quad (3.27)$$

The 2-D regularization approach shares identical properties for the iterative SOR method as previously described for the 1-D case.

3.3 Extending the Hybrid Model

The hybrid model contains two different types of stabilizers, each of which has its very own unique characteristics and limitations. Salinas [29] examined the properties of the first- and second-order stabilizers. His analysis revealed that the first-order stabilizer tends to reconstruct surfaces with oversmoothed edges, while the second-order stabilizer interpolates surfaces with steeper, distinct edges and some noticeable oscillations. It was discovered that the first-order stabilizer, with its first order energy component, cannot detect crease discontinuities because its surface conforms smoothly at these instances. In order to detect step and crease edges simultaneously, a second-order surface with a high energy density must be used at these instances where the data is tightly curved. Another useful property of the first-order stabilizer was its ability to be noise tolerant. By examining the above properties, a hybrid functional which combined all aspects of the first- and second-order stabilizers was designed. With this in mind, a minimization approach for both first- and second-order stabilizers needs to be addressed.

3.3.1 First-Order Stabilizer Minimization

The procedure used for minimizing the first-order stabilizer and its associated energy functional will now be examined. First, let's begin with the first-order discrete energy functional

$$E = \sum_{n=0}^m \sum_i \sum_j w_n (f_{i,j} - d_{n,i,j})^2 + \lambda \sum_i \sum_j [(f_{i,j} - f_{i-1,j})^2 + (f_{i,j} - f_{i,j-1})^2], \quad (3.28)$$

where m represents the number of sources. We wish to find the function $f_{i,j}$ for which the total energy E is smallest. This is accomplished by taking the discrete approximations to Eq. (3.28) and finding its minimum

$$\begin{aligned} \frac{\partial E}{\partial f_{i,j}} = & (w_1 + \dots + w_n + 4\lambda)f_{i,j} - (w_1 d_{1,i,j} + \dots + w_n d_{n,i,j}) \\ & - \lambda(f_{i-1,j} + f_{i,j-1} + f_{i+1,j} + f_{i,j+1}). \end{aligned} \quad (3.29)$$

This results in a system of simultaneous linear equations that can be solved by the SOR method. For interior nodes, at least one pixel inside each edge, the update expression becomes

$$f_{i,j}^{(n+1)} = f_{i,j}^{(n)} - \omega \frac{(T)f_{i,j} - D - \lambda(f_{i-1,j}^{(n+1)} + f_{i,j-1}^{(n+1)} + f_{i+1,j}^{(n)} + f_{i,j+1}^{(n)})}{T} \quad (3.30)$$

where $T = w_1 + \dots + w_n + 4\lambda$ and $D = w_1 d_1 + \dots + w_n d_{n,i,j}$. It should be apparent that modifications are necessary at corners and along the array's borders at least one pixel away from each corner. These components can be found by realizing that only a few of the terms are needed from [Eq. (3.29)]. The corner templates are formulated by

$$f_{i,j}^{(n+1)} = f_{i,j}^{(n)} - \omega \frac{(T)f_{i,j} - w_1 d_1 + \dots + w_n d_{n,i,j} - \lambda(S_T)}{T} \quad (3.31)$$

with $T = w_1 + \dots + w_n + 2\lambda$ and

$$S_T = \begin{cases} f_{i+1,j} + f_{i,j+1} & NW \\ f_{i+1,j} + f_{i,j-1} & NE \\ f_{i,j-1} + f_{i-1,j} & SW \\ f_{i,j-1} + f_{i-1,j} & SE \end{cases} \quad (3.32)$$

for its respective corner. The edge templates have the same format as Eq. (3.31) except for some minor modifications with the variable $T = w_1 + \dots + w_n + 3\lambda$ and

$$S_T = \begin{cases} f_{i,j-1} + f_{i+1,j} + f_{i,j+1} & \text{North} \\ f_{i,j-1} + f_{i,j+1} + f_{i-1,j} & \text{South} \\ f_{i,j-1} + f_{i-1,j} + f_{i+1,j} & \text{East} \\ f_{i,j+1} + f_{i-1,j} + f_{i+1,j} & \text{West} \end{cases} \quad (3.33)$$

The SOR expressions used for minimizing the energy functional have been formulated for each region requiring modifications and are ready for implementation. Next it will be shown that the second-order stabilizer requires many more border modifications since it contains a second-order derivative.

3.3.2 Second-Order Stabilizer Minimization

The implementation details concerning the second-order stabilizer will now be addressed. The minimization is more complicated since it contains a second-order derivative and therefore requires more modifications at or near edge boundaries. Let's begin with the discrete second-order energy functional

$$E(f_{i,j}) = \sum_{n=0}^m \sum_i \sum_j w_n (f_{i,j} - d_{n,i,j})^2 + \beta \left[\sum_i \sum_j (f_{i-1} - 2f_{i,j} + f_{i+1,j})^2 + 2(f_{i,j} - f_{i+1,j} - f_{i,j+1} + f_{i+1,j+1})^2 + (f_{i,j-1} - 2f_{i,j} + f_{i,j+1})^2 \right], \quad (3.34)$$

where m represents the number of sources and w_n is a weighting factor associated with each input. Taking the discrete approximations for the terms in the energy functional and minimizing the result gives the SOR response for the interior nodes

(at least two pixels inside every border),

$$\begin{aligned}
f_{i,j}^{(n+1)} = f_{i,j}^{(n)} - \omega \bigg\{ & (w_1 + \dots + w_n + 20\beta)f_{i,j} - (w_1d_1 + \dots + w_nd_{n_{i,j}}) - \beta(8f_{i-1,j} \\
& + 8f_{i+1,j} - f_{i+2,j} - f_{i-2,j} + 8f_{i,j+1} - 2f_{i+1,j+1} - 2f_{i-1,j+1} - 2f_{i+1,j-1} \\
& - 2f_{i-1,j-1} + 8f_{i,j-1} - f_{i,j+2} - f_{i,j-2}) \bigg\} / (w_1 + \dots + w_n + 20\beta). \quad (3.35)
\end{aligned}$$

Now we will examine the various border modifications necessary for implementing the second-order stabilizer over every pixel in the image ($i = 0, \dots, N-1, j = 0, \dots, N-1$) realizing that its counterpart can be derived in a similar manner.

The NW corner is computed by

$$\begin{aligned}
f_{i,j}^{(n+1)} = f_{i,j}^{(n)} - \omega \bigg\{ & (w_1 + \dots + w_n + 4\beta)f_{i,j} - (w_1d_1 + \dots + w_nd_{n_{i,j}}) \\
& - \beta(4f_{i+1,j} + 4f_{i,j+1} - f_{i,j+2} - f_{i+2,j} - 2f_{i+1,j+1}) \bigg\} / (w_1 + \dots + w_n + 4\beta),
\end{aligned}$$

and the west edge (at least two pixels from each corner) is updated by

$$\begin{aligned}
f_{i,j}^{(n+1)} = f_{i,j}^{(n)} - \omega \bigg\{ & (w_1 + \dots + w_n + 11\beta)f_{i,j} - (w_1d_1 + \dots + w_nd_{n_{i,j}}) \\
& - \beta(6f_{i-1,j} + 6f_{i+1,j} - f_{i+2,j} - f_{i-2,j} + 6f_{i,j+1} - 2f_{i+1,j+1} - 2f_{i-1,j+1} \\
& + f_{i,j+2}) \bigg\} / (w_1 + \dots + w_n + 11\beta). \quad (3.36)
\end{aligned}$$

The next location of interest lies in pixels adjacent to the corners. For this particular case we examine the pixel below the NW corner ($i = 1, j = 0$):

$$\begin{aligned}
f_{i,j}^{(n+1)} = f_{i,j}^{(n)} - \omega \bigg\{ & (w_1 + \dots + w_n + 10\beta)f_{i,j} - (w_1d_1 + \dots + w_nd_{n_{i,j}}) \\
& - \beta(6f_{i+1,j} + 4f_{i-1,j} - f_{i+2,j} - f_{i,j+2} + 6f_{i,j+1} - 2f_{i-1,j+1} \\
& - 2f_{i+1,j+1}) \bigg\} / (w_1 + \dots + w_n + 10\beta). \quad (3.37)
\end{aligned}$$

The next modification will be the column of pixels located just inside the borders, but at least two pixels away from the opposite edge. A typical case ($i = 2, \dots, N - 3, j = 1$) gives

$$\begin{aligned} f_{i,j}^{(n+1)} = & f_{i,j}^{(n)} - \omega \left\{ (w_1 + \dots + w_n + 19\beta) f_{i,j} - (w_1 d_1 + \dots + w_n d_{n_{i,j}}) \right. \\ & - \beta (8f_{i+1,j} + 8f_{i-1,j} - f_{i-2,j} - f_{i+2,j} + 6f_{i,j-1} + 8f_{i,j+1} - f_{i,j+2} \\ & \left. - 2f_{i+1,j+1} - 2f_{i-1,j+1} - 2f_{i-1,j-1} - 2f_{i+1,j-1}) \right\} / (w_1 + \dots + w_n + 19\beta). \end{aligned}$$

The final border modification occurs at the ($i=1, j=1$) pixel just inside the NW corner and is represented by

$$\begin{aligned} f_{i,j}^{(n+1)} = & f_{i,j}^{(n)} - \omega \left\{ (w_1 + \dots + w_n + 18\beta) f_{i,j} - (w_1 d_1 + \dots + w_n d_{n_{i,j}}) \right. \\ & - \beta (8f_{i+1,j} + 6f_{i-1,j} - f_{i+2,j} + 6f_{i,j-1} + 8f_{i,j+1} - f_{i,j+2} - 2f_{i+1,j+1} \\ & \left. - 2f_{i-1,j+1} - 2f_{i-1,j-1} - 2f_{i+1,j-1}) \right\} / (w_1 + \dots + w_n + 18\beta). \quad (3.38) \end{aligned}$$

The update expressions for the second-order stabilizer have been established in their final form for selected regions with the assumption that their respective counterparts can be derived in a similar fashion.

3.3.3 Implementation of the Hybrid Energy Functional

The importance of the hybrid energy functional lies in its ability to implement the first- and second-order stabilizers simultaneously and in a unique convention. We now discuss a step-by-step procedure for implementing the extended hybrid

energy functional

$$\sum_{i=1}^m \int \int w_i (f(\mathbf{x}) - d_i(\mathbf{x}))^2 d\mathbf{x} + \int \left[[1 - \rho(\mathbf{x})] \lambda(\mathbf{x}) (f'(\mathbf{x}))^2 + \rho(\mathbf{x}) \beta(\mathbf{x}) (f''(\mathbf{x}))^2 \right] d\mathbf{x},$$

where $\mathbf{x} = (x, y)$. The new development is the addition of the function $\rho(x, y)$ and its ability to control the operation of the first- and second-order stabilizers. Once again, the procedure involves finding the approximate solution $f_{i,j}$ which minimizes the hybrid energy functional with a predetermined $\lambda(\mathbf{x})$ and $\beta(\mathbf{x})$. This minimization will take part by using the SOR method previously discussed. After performing one successful pass of the iterative method, a gradient operator [26] is applied to the first approximated regularized image. For edge detection purposes the magnitude of the gradient is important and is denoted by

$$\Delta f = \text{mag}(\Delta \mathbf{f}) = \sqrt{[G_x^2 + G_y^2]}. \quad (3.39)$$

The gradient technique was based on the Sobel operator masks which are defined by

$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \quad (3.40)$$

and

$$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7). \quad (3.41)$$

A threshold value will be determined from the gradient image that will allow the second-order stabilizer to be used at or near edges, while implementation of the first-order stabilizer remains distant or in continuous regions. Next, variable

parameters will be used instead of the traditional constant smoothing parameter. The approach used for updating the regularization parameters, $\lambda(\mathbf{x})$ and $\beta(\mathbf{x})$, is one developed by Gökmen and Li [8] and will be illustrated in the steps below with modifications for fusing multiple inputs. The following is a sequential list of steps required for implementing the hybrid energy functional with variable regularization parameters:

1. Initialize $\lambda(x, y)$ and $\beta(x, y)$ to a maximum value.
2. Find the solution $f_{i,j}$ that minimizes the energy functional by using the predetermined smoothing parameters.
3. After one successful pass of the SOR method, locate the edges using a gradient technique (Sobel operators) to select a threshold indicating where the first- and second-order stabilizers should be applied. (Note: This step is required only once in the entire process).
4. Compute the estimated noise variance [34] associated with M sources by

$$\sigma_M^2 = \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (f_{i,j} - d_{i,j})^2, \quad (3.42)$$

assuming an $N \times N$ image. Next, the variances are combined to form an estimate of the standard deviation

$$\sigma_{est} = \frac{1}{\sqrt{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \dots + \frac{1}{\sigma_M^2}}}. \quad (3.43)$$

5. Compute the error signal by

$$e(x, y) = f(x, y) - \frac{\frac{d_1}{\sigma_1^2} + \frac{d_2}{\sigma_2^2} + \cdots + \frac{d_M}{\sigma_M^2}}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \cdots + \frac{1}{\sigma_M^2}}. \quad (3.44)$$

6. Extract the control signal $c(x, y)$ by

$$c(x, y) = [[abs(e) * h] - \sigma_{est}] * h, \quad (3.45)$$

where $(*)$ represents convolution.

7. Update the smoothing functions $\lambda(x, y)$ and $\beta(x, y)$. If $c(x, y) > t_c$, where t_c is a threshold constant used for detecting edges in the control signal, and $\lambda_{old} > \lambda_{min}$ or $\beta_{old} > \beta_{min}$, then

$$\lambda_{new}(x, y) = (1 - e^{-c(x, y)/V_T})\lambda_{min} + (e^{-c(x, y)/V_T})\lambda_{old}$$

$$\beta_{new}(x, y) = (1 - e^{-c(x, y)/V_T})\beta_{min} + (e^{-c(x, y)/V_T})\beta_{old}$$

otherwise,

$$\lambda_{new}(x, y) = \lambda_{old}(x, y)$$

$$\beta_{new}(x, y) = \beta_{old}(x, y). \quad (3.46)$$

8. If no change occurs in either smoothing parameter, terminate iterations. Otherwise go back to Step 2 and repeat, skipping Step 3.

The goal of the above procedure is to use large regularization values in continuous regions for filtering noise and small values at or near discontinuities for the preservation of edges.

A description of the various steps involved in this approach will now be discussed. First, the algorithm starts with large regularization parameters $\lambda(\mathbf{x})$ and $\beta(\mathbf{x})$. After minimizing the hybrid functional with respect to the smoothing parameters chosen, the resulting function $f(\mathbf{x})$ is smoothed or its high frequency components are filtered. After this first successful iteration, the edges from the regularized solution are extracted using the gradient Sobel operators. From this gradient image, a threshold value is determined for applying the second-order stabilizer at or near discontinuities, while the first-order stabilizer remains in nonactivity regions for noise filtering. The error signal, the difference between regularized solution and the image data, is used for detecting the location of edges. This signal will return a large absolute value when it detects the existence of an edge and smaller values for nonexistent discontinuities. Next, the control signal is extracted from the error signal by first convolving the absolute value of the error signal with a 3×3 averaging filter. This supplies a signal having large positive values around edges. Subtracting σ_{est} from this signal and convolving with an averaging filter again creates large positive values around discontinuities, but smaller values in continuous regions. By using this approach, the control signal will have a large peak at an edge location and smaller, sometimes negative, values for nonactive areas. This iterative procedure in turn produces small regularization parameters near edges and large values in noisy continuous regions. The rule for updating $\lambda_{new}(x, y)$ and $\beta_{new}(x, y)$ is an exponentially decreasing function

with properties

$$\lim_{c(x,y) \rightarrow 0} \lambda_{new}(x, y) = \lambda_{old}(x, y)$$

and

$$\lim_{c(x,y) \rightarrow \infty} \lambda_{new}(x, y) = \lambda_{min} . \quad (3.47)$$

This states that when an edge is located the control signal will yield a large value, thereby allowing the chosen regularization parameter to be small. On the other hand, if the control signal is small the regularization parameter will be large, indicating a continuous region. It should be noted that the above rule also applies to updating $\beta_{new}(x, y)$. Finally, as iterations proceed, the regularization parameters are modified accordingly as being near an edge or in a continuous region. This, in turn, delivers an accurate estimate of the solution. The iterations are terminated when no changes in $\lambda(x, y)$ and $\beta(x, y)$ occur.

3.4 Synopsis

This chapter presents the formulation of the hybrid functional and the updating procedure employed to compute the most accurate approximation of the fused surface. The hybrid functional is illustrated for the cases of constant and variable smoothing parameters. It is necessary to use variable smoothing terms because a constant regularization parameter can oversmooth discontinuities. Due to this we used an approach in which variable regularization parameters are employed in

conjunction with the hybrid functional. This will allow smaller values for λ and β to be used at or near discontinuities while larger values are used in continuous regions. This realization was based on noise reduction in nonactive regions for large regularization parameters and preservation of edges for smaller values. In conclusion, the hybrid functional is an important tool for restoring ill-posed problems, such as surface reconstruction and edge detection, into well-posed solutions.

CHAPTER 4

EXPERIMENTAL RESULTS

This chapter provides results for the hybrid energy functional in relation to 1-D and 2-D data. The regularized technique will be applied to the early vision problems of surface reconstruction and gray-level edge detection. In developing a clear understanding of the hybrid functional it is better to start with the 1-D case and then proceed to 2-D experiments. Based on this knowledge, Section 4.1 examines the results for the 1-D analysis for both single and multiple inputs. After grasping the concepts associated with the 1-D hybrid functional the experiments are then conducted for its 2-D counterpart. Section 4.2 provides results for fusing two noise-free and noisy inputs for the surface reconstruction and edge detection applications. The surface interpolation process is conducted on dense and sparse data. The edge detection approach is applied to range and intensity data. Next, Section 4.3 analyzes how the first- and second-order stabilizers compare with the hybrid stabilizer under increasing levels of noise. Finally, Section 4.4 provides results and information on the computational efficiency of the hybrid fusion functional. From these experiments it will be concluded that the implementation of the hybrid stabilizer outperforms standard regularization with either first- or second-order stabilizers.

4.1 One-Dimensional Results for the Hybrid Functional

This section presents results for the regularized 1-D hybrid functional with one and two input sources. The goal is to construct a smooth 1-D function $f(x)$ for which the total energy is smallest and also a good fit to some given data $d(x)$. The hybrid model used for the 1-D case is

$$\sum_{i=1}^m \int w_i (f(x) - d_i(x))^2 dx + \int [\lambda \underbrace{(f'(x))^2}_{FOS} + \beta \underbrace{(f''(x))^2}_{SOS}] dx \quad (4.1)$$

where multiple inputs are allowed with constant smoothing parameters. Results will reveal that the hybrid energy functional approximates a function which models the true solution. The degree of smoothness depends upon the selection of the regularization parameters (λ) and (β). First we will analyze the hybrid for one input and then fuse two noisy input signals. Both cases will be examined for different types of inputs, each of which contains step and/or roof edges.

4.1.1 Case I: One Input Source

The analysis of the hybrid functional will be examined for one input ($m = 1$ in Eq. (4.1)) and fixed regularization parameters. By examining roof and step edges in the input we can see how the hybrid functional performs in modeling surfaces around these particular discontinuities. The first experiment, shown in Fig. 4.1, illustrates how different values of λ and β affect the regularized solution.

The input signal is shown as a roof on top of a pulse edge which contains roof and step discontinuities. It is quite evident from Figs. 4.1(a), (b), and (c) that as

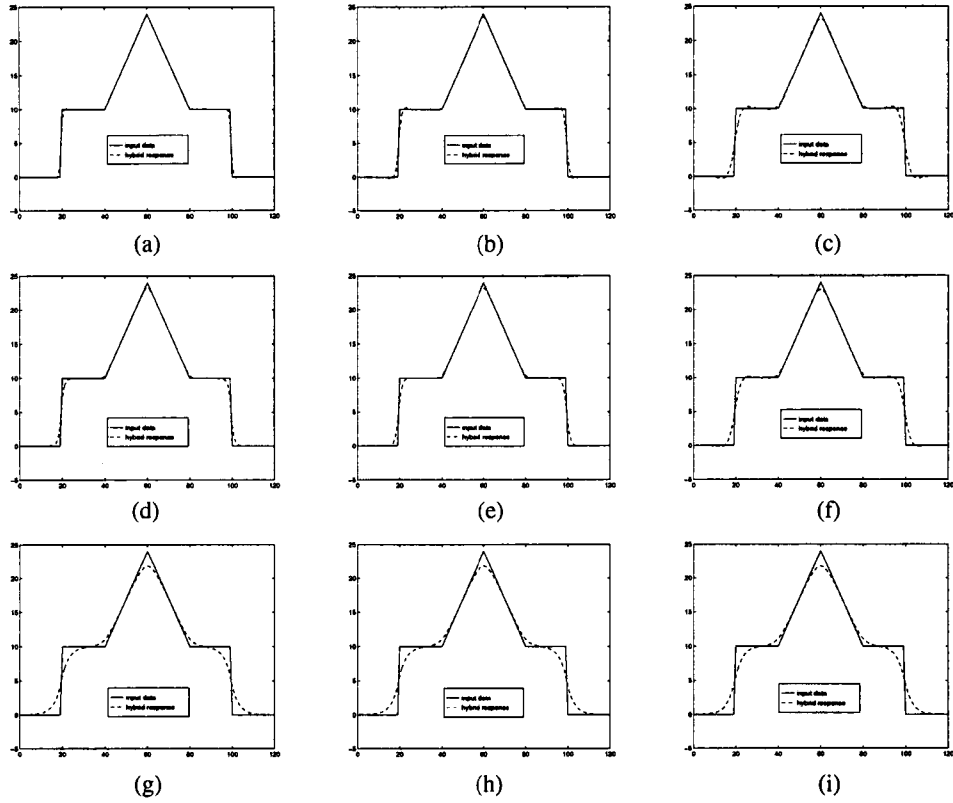


Figure 4.1: *Responses of the hybrid functional (first- and second-order stabilizers with fixed regularization parameters) under noise-free conditions: (a) $\lambda = \beta = 0.1$; (b) $\lambda = 0.1, \beta = 1.0$; (c) $\lambda = 0.1, \beta = 10.0$; (d) $\lambda = 1.0, \beta = 0.1$; (e) $\lambda = \beta = 1.0$, (f) $\lambda = 1.0, \beta = 10.0$; (g) $\lambda = 10.0, \beta = 0.1$; (h) $\lambda = 10.0, \beta = 1.0$; and (i) $\lambda = \beta = 10.0$.*

$\lambda = 0.1$ remains fixed and β varies from (0.1, 1.0, 10.0) the regularized response goes from a well-modeled solution to one that oscillates and oversmooths edges. As β increases, the second-order stabilizer becomes dominant, resulting in a function that contains oscillations. This characteristic exists for all inputs to be examined. Our next case of interest involves keeping $\lambda = 1.0$ fixed and varying β as previously stated. Figures 4.1(d), (e), and (f) illustrate this variation in the

smoothing parameters. The output reveals that the solution oscillates and is over-smoothed much faster as both regularization parameters become large. The final case examines fixing $\lambda = 10.0$ and varying (β) . The results conclude that with $\lambda = 10.0$ the solution has become saturated. This is apparent in Figs. 4.1(g), (h), and (i) where the value of β has little or no effect upon the approximated solution.

Note: The format used above for varying λ and β will be used in the upcoming results. This experiment has established three important concepts:

1. If λ and β are relatively small, the approximated solution tends to be sensitive to the data; therefore, the regularization parameters can be viewed as a characteristic length or scale for the curve-fitting process.
2. On the other hand, if λ and β are large, the interpolated surface will model the original data but will be somewhat oversmoothed.
3. In light of all of these facts, the hybrid functional is able to provide a good estimation of the original ideal input.

Using the same input as previously described, we would like to examine the effects of noise and how the hybrid functional reacts to such adverse conditions. The input is contaminated with zero-mean Gaussian noise with standard deviation of $[\sigma_n = 1.5]$. Examining Figs. 4.2(a), (b), and (c), one can conclude that for λ and β small the solution tends to model the given data. This can be seen in Fig. 4.2(a) in which the solution models the noisy signal very closely, but as β increases, the estimated surface tends to oscillate while filtering much of the addi-

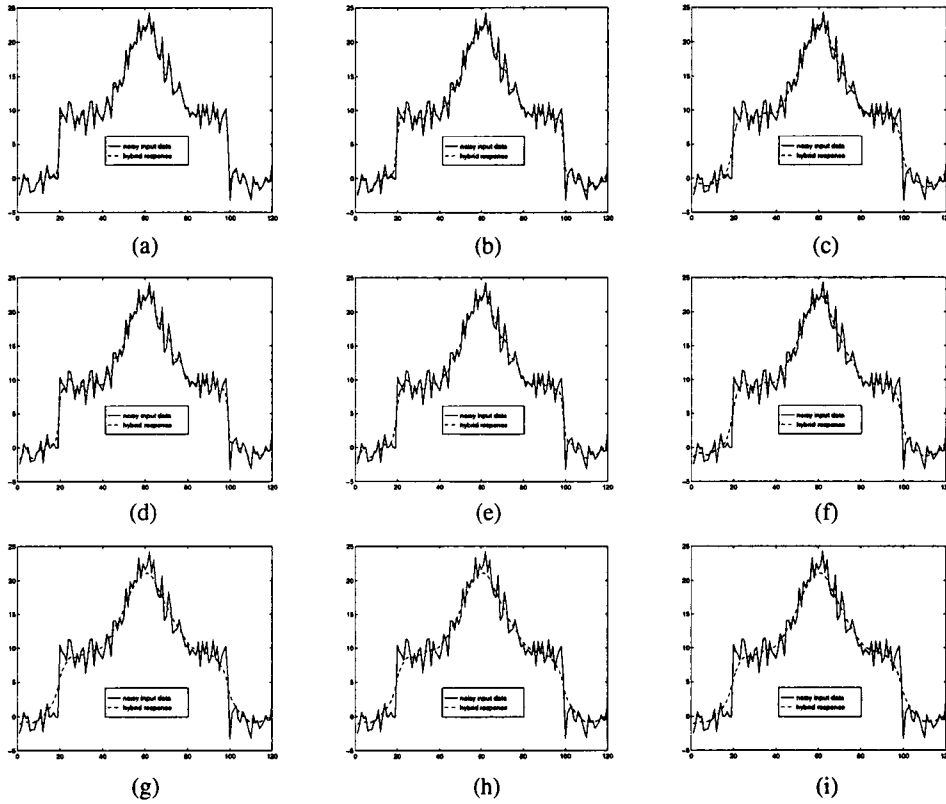


Figure 4.2: Responses of the hybrid functional (first- and second-order stabilizers with fixed regularization parameters) under noisy conditions: (a) $\lambda = \beta = 0.1$; (b) $\lambda = 0.1$, $\beta = 1.0$; (c) $\lambda = 0.1$, $\beta = 10.0$; (d) $\lambda = 1.0$, $\beta = 0.1$; (e) $\lambda = \beta = 1.0$; (f) $\lambda = 1.0$, $\beta = 10.0$; (g) $\lambda = 10.0$, $\beta = 0.1$; (h) $\lambda = 10.0$, $\beta = 1.0$; and (i) $\lambda = \beta = 10.0$.

tive noise. Next, as both regularization parameters increase in value, Figs. 4.2(g), (h), and (i), the solution reaches a saturation point in which the signal cannot be filtered or smoothed anymore. Consequently, the regularized solution gets smoother as both λ and β increase, in turn filtering much of the noise present in the signal, while smaller values force the solution to model the given input function. This experiment discovered benefits of regularization theory.

- Given a noisy input signal, the hybrid functional is able to interpolate a function that is quite immune to noise.
- Adjusting the regularization parameters allows the user to control the degree of smoothing in the final interpolant.

A major benefit of regularization theory is its ability to approximate surfaces through sparse data points. This approach will be examined in the 2-D results. Given a very low density of data points, it will be seen that the hybrid functional will provide a solution which closely models the ideal data set.

Our next input, Fig. 4.3, represents a step edge occurring at $n = 50$ in the sequence $\{x_n\}$. In a physical scene a step edge originates from occlusion boundaries of object surfaces (*e.g.* color or texture changes and shadows). Applying the regularized technique to the step input reveals similar results as discovered from the first experiment. When the regularization parameters, λ and β , vary from small to large the solution becomes smooth at or near discontinuities. Figures 4.3(b), (c), and (f) illustrate that when λ remains relatively small compared to β the solution becomes oscillatory. This behavior is a characteristic of the second-order stabilizer. Figures 4.3(g), (h), and (i) support the same conclusion: with λ large the solution becomes saturated to such a point that the first-order stabilizer dominates. Overall, this experiment examined the regularized method for estimating a solution which modeled a single step input. It was established that an oscillatory solution occurs when emphasis is placed on the second-order stabilizer, while an over-smoothed response occurs when the first-order stabilizer

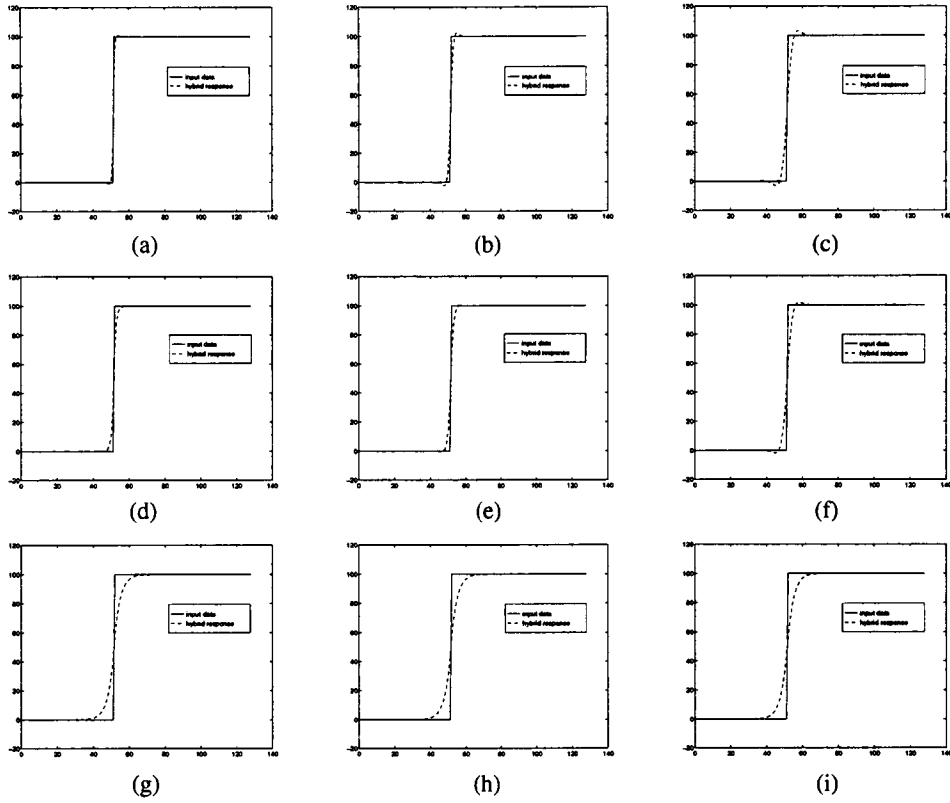


Figure 4.3: *Responses of the hybrid functional (first- and second-order stabilizers with fixed regularization parameters) under noise-free conditions: (a) $\lambda = \beta = 0.1$; (b) $\lambda = 0.1, \beta = 1.0$; (c) $\lambda = 0.1, \beta = 10.0$; (d) $\lambda = 1.0, \beta = 0.1$; (e) $\lambda = \beta = 1.0$; (f) $\lambda = 1.0, \beta = 10.0$; (g) $\lambda = 10.0, \beta = 0.1$; (h) $\lambda = 10.0, \beta = 1.0$; and (i) $\lambda = \beta = 10.0$.*

dominates.

What happens when the step input is contaminated with zero-mean Gaussian noise with standard deviation of $\sigma_n = 1.5$, as in Fig. 4.4? The results agree with previous characteristics for which large λ 's emphasize regularization and small λ 's emphasize closeness to the data. Though the input is noisy, the hybrid functional is still able to interpolate a smooth function that filters the noise and closely

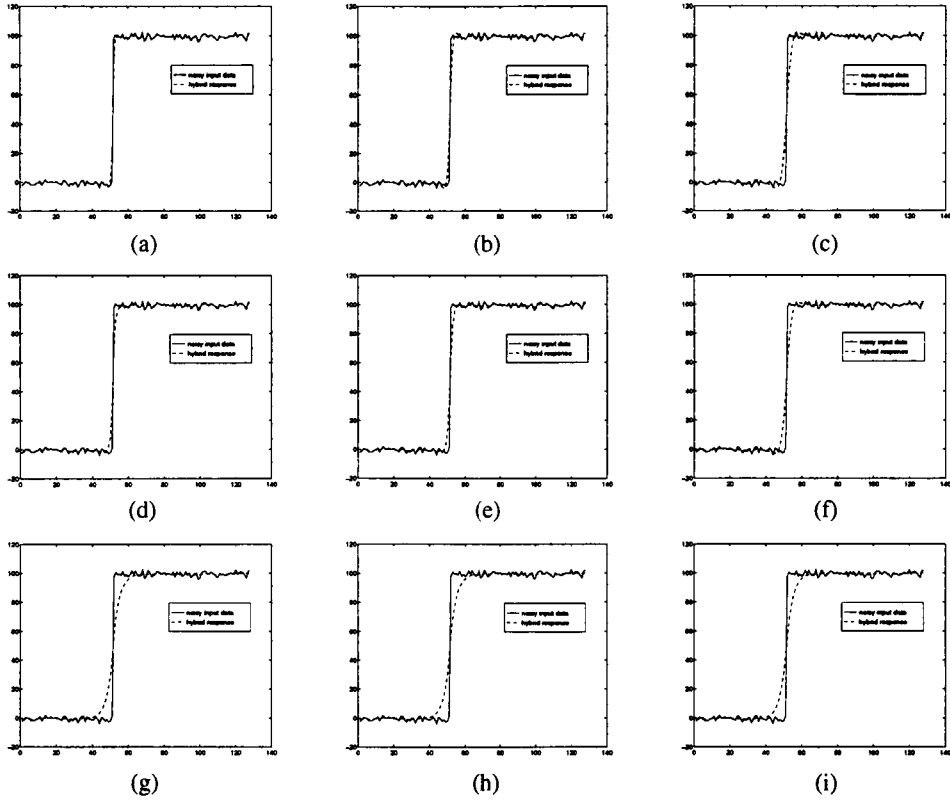


Figure 4.4: *Responses of the hybrid functional (first- and second-order stabilizers with fixed regularization parameters) under noisy conditions: (a) $\lambda = \beta = 0.1$; (b) $\lambda = 0.1$, $\beta = 1.0$; (c) $\lambda = 0.1$, $\beta = 10.0$; (d) $\lambda = 1.0$, $\beta = 0.1$; (e) $\lambda = \beta = 1.0$; (f) $\lambda = 1.0$, $\beta = 10.0$; (g) $\lambda = 10.0$, $\beta = 0.1$; (h) $\lambda = 10.0$, $\beta = 1.0$; and (i) $\lambda = \beta = 10.0$.*

resembles a step edge.

Analyzing the results for these single inputs has shown that regularization is indeed a good interpolating tool for noise-free and noisy data. The degree of smoothness was found to be controlled by the regularization parameters (λ) and (β). The importance of regularization lies in its ability to filter noise and in applications related to interpolating surfaces through sparse or dense data.

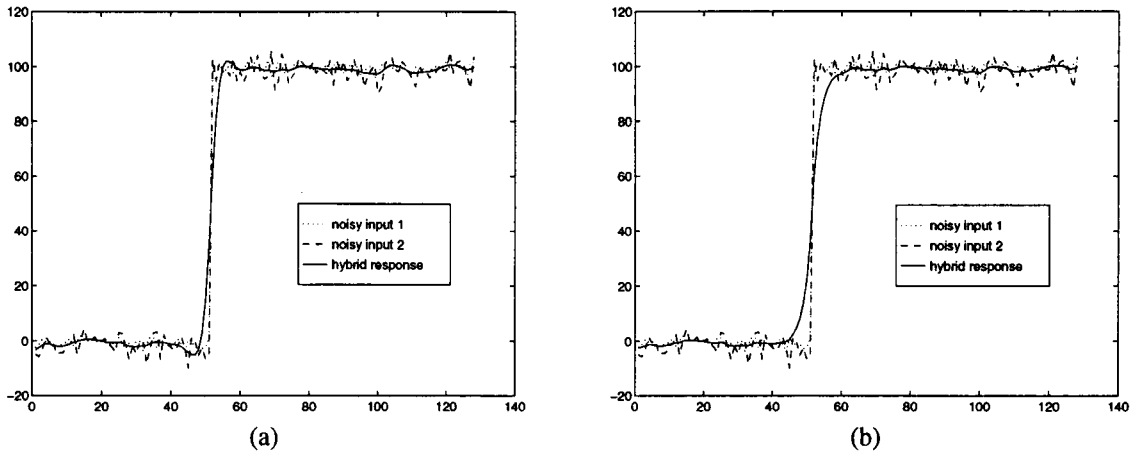


Figure 4.5: *1-D fusion with hybrid stabilizer using two noisy inputs, (a) $\lambda = 0.1$, $\beta = 10.0$ and (b) $\lambda = 10.0$, $\beta = 0.1$.*

4.1.2 Case II: Two Input Sources

So far we have examined the regularized technique for one input. Now we turn to a slightly more complicated situation by fusing two noisy input sources. The same energy functional given in Eq. (4.1) will be used except with $[m = 2]$. For the upcoming experiments, we are only analyzing noisy inputs, because in most practical situations the acquisition of any signal contains some amount of additive noise.

Our first experiment, Fig. 4.5, examines the fusion of two noisy step inputs, each contaminated with zero-mean Gaussian noise with standard deviations of $[\sigma_{1n} = 1.5]$ and $[\sigma_{2n} = 3.5]$. Figure 4.5(a) illustrates a case where $\lambda = 0.1$ and $\beta = 10.0$ with emphasis placed on the second-order stabilizer. For purposes of making the regularized response stand out, it is represented by the solid line in

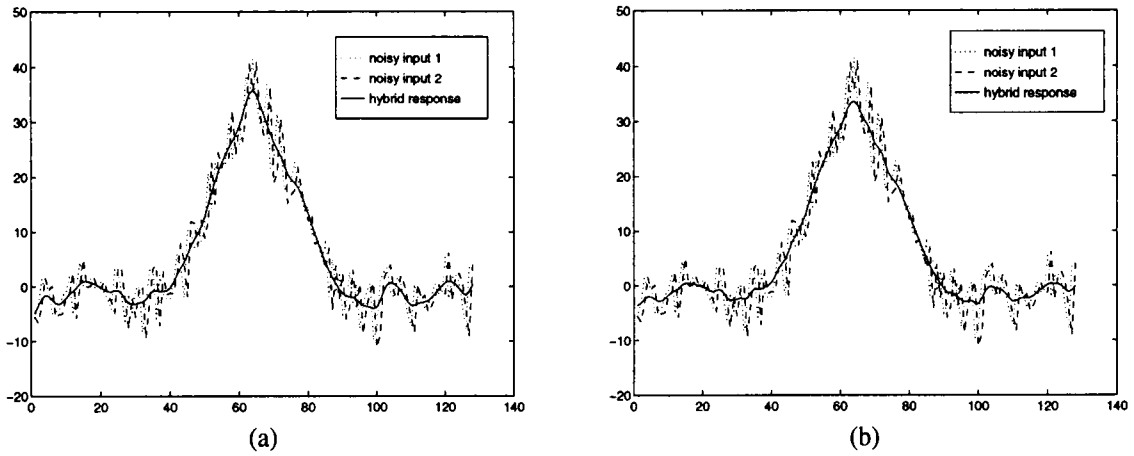


Figure 4.6: 1-D fusion with hybrid stabilizer using two noisy inputs, (a) $\lambda = 0.1$, $\beta = 10.0$ and (b) $\lambda = 10.0$, $\beta = 0.1$.

the upcoming graphs. As previously mentioned, the regularization parameters control the severity of the smoothing or filtering operation. Using a large β and small λ yields oscillations in the outcome because of the second-order stabilizer's nature. On the other hand, Fig. 4.5(b), with a large λ and small β emphasizes the first-order response, in turn providing an output that is greatly oversmoothed at or near discontinuities.

The next experiment, Fig. 4.6, contains two noisy roof inputs, [(Input 1) $\sigma_{1n} = 3.5$] and [(input 2) $\sigma_{2n} = 4.0$]. For the given noise levels, the regularized solution supplies an adequate approximation to the given data. Due to the selected noise levels upon the inputs, it is still noticeable that Fig. 4.6(a) contains more of an oscillatory sequence than that shown in Fig. 4.6(b). The next two experiments, Fig. 4.7, further emphasize the results already obtained. Given two noisy inputs

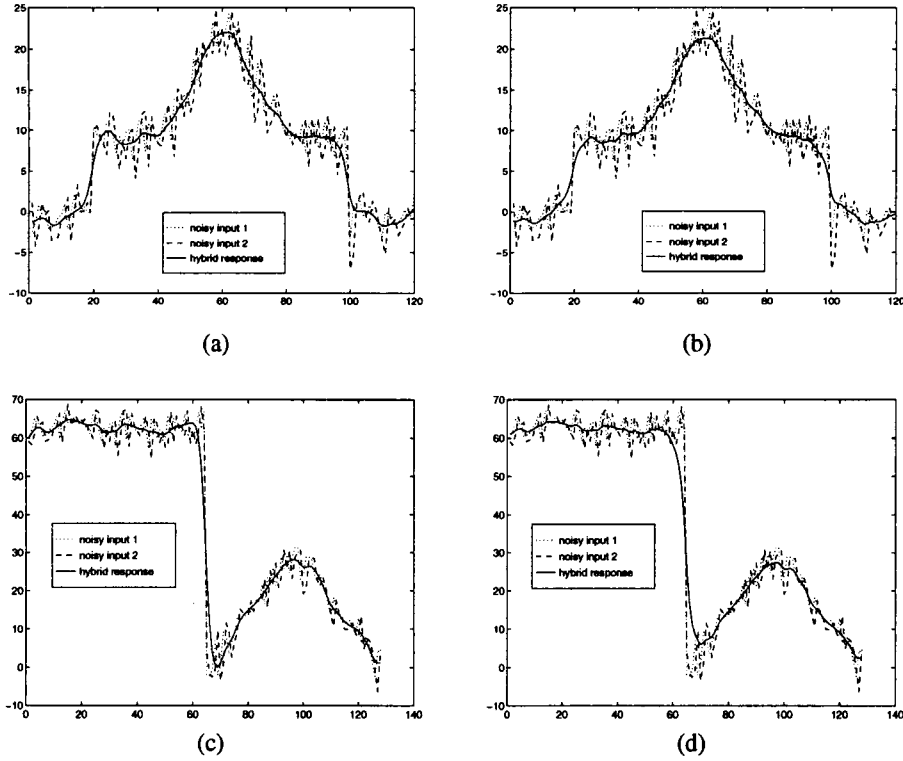


Figure 4.7: 1-D fusion with hybrid stabilizer using two noisy inputs, (a) $\lambda = 0.1$, $\beta = 10.0$; (b) $\lambda = 10.0$, $\beta = 0.1$; (c) $\lambda = 0.1$, $\beta = 10.0$; and (d) $\lambda = 10.0$, $\beta = 0.1$.

the regularized method is still able to interpret a smooth surface through the given data. If emphasis is placed on the second-order stabilizer, the output tends to have oscillations, while the first-order stabilizer provides an oversmoothed solution at edge boundaries. The theme behind these results is that by using regularization we can fuse two noisy inputs to obtain an output which is better than any single source.

The results for the 1-D case have been illustrated for one and two inputs. This section's objective was to develop an understanding of the regularized method for

1-D data before proceeding with the 2-D case. Our next building block concerns fusing 2-D noisy data that will be used in the surface reconstruction and edge detection applications.

4.2 Two-Dimensional Results for the Hybrid Functional

This section provides results for the hybrid functional when applied to 2-D data. The focus of this material will be directed toward the problems of multisensor surface reconstruction and fusing range and intensity data to obtain a feature edge map for recognizing objects. Section 4.2.1 begins by illustrating the need for variable regularization parameters and later supplies a comparison of the hybrid functional to the first- and second-order stabilizers. Next, Section 4.2.2 takes the regularized method one step further by fusing range and intensity data. Results will show that the hybrid energy functional clearly outperforms either the first- or second-order stabilizer.

4.2.1 Surface Reconstruction Results

This section presents results for the hybrid functional when applied to the fusion of two noisy inputs. Using the regularized fusion approach with the hybrid energy functional, we wish to minimize this energy term and solve for the best fit surface through the given data points. The concept will be applied to both dense and sparse data. The ability of regularization theory will become apparent in reconstructing smooth surfaces through sparse or dense data points. The energy

functionals examined in this section include the first-order response

$$\sum_{i=1}^m \int \int w_i(f(\mathbf{x}) - d_i(\mathbf{x}))^2 d\mathbf{x} + \int \lambda(\mathbf{x})(f_x^2 + f_y^2) d\mathbf{x}, \quad (4.2)$$

the second-order response

$$\sum_{i=1}^m \int \int w_i(f(\mathbf{x}) - d_i(\mathbf{x}))^2 d\mathbf{x} + \int \beta(\mathbf{x})(f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2) d\mathbf{x}, \quad (4.3)$$

and the hybrid response

$$\sum_{i=1}^m \int \int w_i(f(\mathbf{x}) - d_i(\mathbf{x}))^2 d\mathbf{x} + \int [\lambda(\mathbf{x})(f_x^2 + f_y^2) + \beta(\mathbf{x})(f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2)] d\mathbf{x}, \quad (4.4)$$

where $\mathbf{x} = (x, y)$ and the regularization parameters $\lambda(\mathbf{x})$ and $\beta(\mathbf{x})$ can be constant or variable.

Before experimenting with the fusion of dual inputs, we would like to examine the effects of constant and variable regularization parameters on the regularized process. Our first experiment, Fig. 4.8, illustrates the surface reconstruction of a single input with constant and variable smoothing parameters. Figure 4.8(a) is the “square” intensity image contaminated with zero-mean Gaussian noise with $\sigma_n = 10$. Figure 4.8(b) is the ideal surface for the given input. Figures 4.8(c), (e), and (g) represent surfaces reconstructed with constant smoothing parameters for the first-order, second-order, and hybrid stabilizers, respectively, whereas Figs. 4.8(d), (f), and (h) represent surfaces reconstructed with variable smoothing parameters in the order mentioned previously. This experiment was conducted to demonstrate the adverse effects of having a constant smoothing parameter throughout the entire image space. As established previously, the regularized

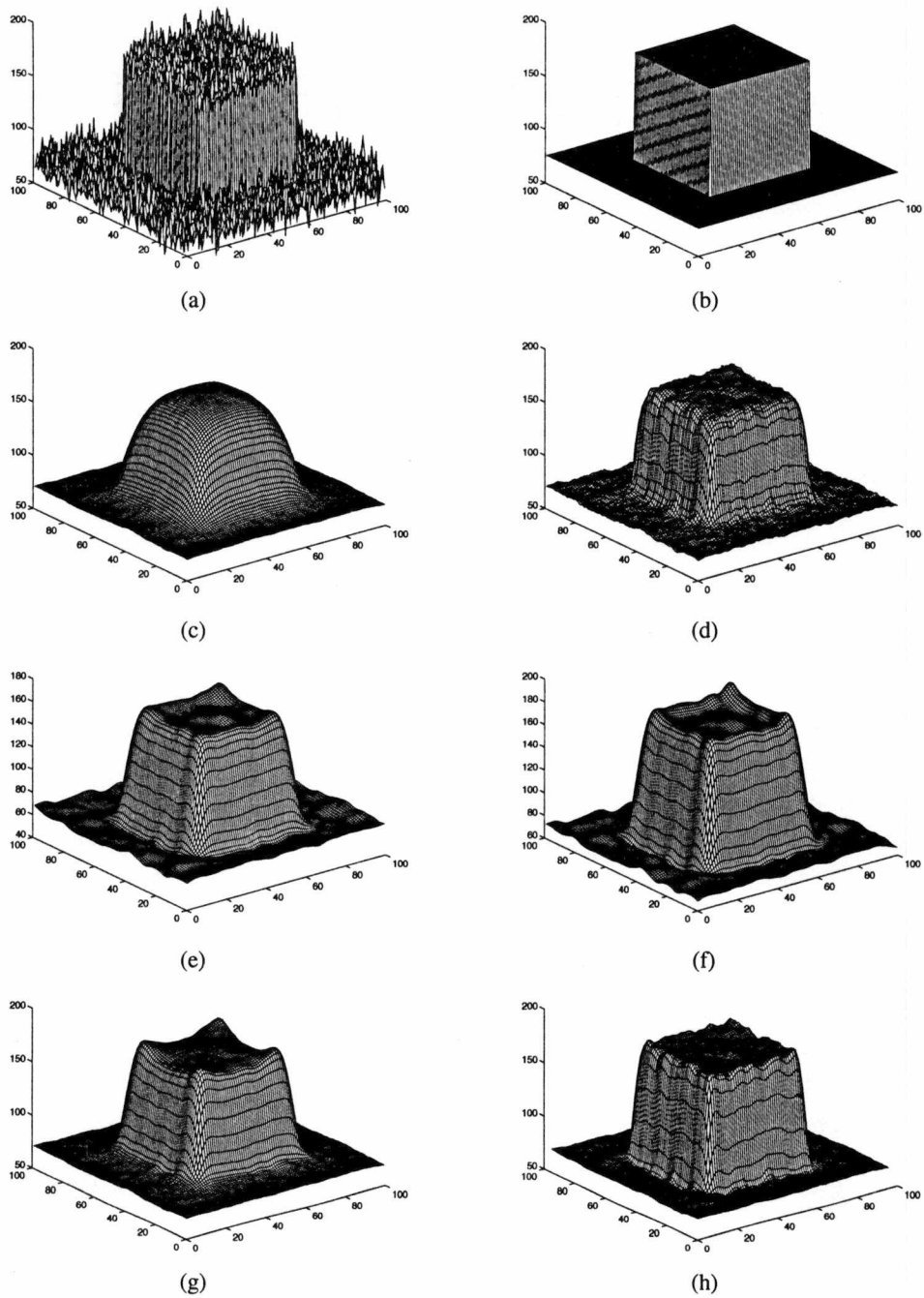


Figure 4.8: Surface reconstruction of square intensity image: (a) input, $\sigma_n = 10$; (b) ideal surface; (c) recovered surface, (constant parameter) first-order stabilizer; (d) (variable parameter) first-order stabilizer; (e) recovered surface, (constant parameter) second-order stabilizer; (f) (variable parameter) second-order stabilizer; (g) recovered surface, (constant parameters) hybrid stabilizer; and (h) (variable parameters) hybrid stabilizer.

solution using constant smoothing parameters for the first-order stabilizer tends to destroy or oversmooth edges, but filters much of the noise. Figure 4.8(c) illustrates the (FOS) with fixed regularization parameter $\lambda = 16$ and Fig. 4.8(d) gives the reconstructed surface for variable parameters $\lambda_{high} = 10$ and $\lambda_{low} = 0.9$. It is evident that variable regularization parameters deliver a model much closer to the ideal case. Recall that the variable parameters are updated by examining the control signal extracted from the error signal, which indicates whether or not an edge exists at a particular location.

The updating procedure is to use large regularization values to promote smoothness of the solution (resembling the convolution with a large filter size), while a small parameter forces the solution to be as close as possible to the image data (weakening the connection between successive pixels). Examining Figs. 4.8(e), and (f) reveals that the second-order stabilizer contains some noticeable oscillations for both constant and variable terms; however, it does reconstruct a surface with steeper, more defined, edges at discontinuities. Finally, examination of the hybrid model, Figs. 4.8(g) and (h), revealed that constant smoothing parameters, $\lambda = 5$ and $\beta = 1$, gave oscillations and an oversmoothed response. However, using variable parameters, $\lambda_{high} = 15$, $\lambda_{low} = 10$, $\beta_{high} = 1.0$, and $\beta_{low} = 0.7$, one can establish a result that removes noise and still maintains sharp, well-defined edges with very few oscillations. The goal of the hybrid functional is to model a first-order stabilizer in noisy areas away from any discontinuity and to use the second-order stabilizer at or near discontinuities with small variable

values. This is the reason for the initial selection of the regularization parameters (λ) and (β) .

Our next experiment, Fig. 4.9, illustrates the fusion of two noisy inputs and how the different energy functionals react. Figures 4.9(a) and (b) are the two noisy “square” image inputs with $[\sigma_{1n} = 5]$ and $[\sigma_{2n} = 10]$. Figure 4.9(c) gives the ideal case for the reconstruction process. Figure 4.9(d) provides the filtered response due to the first-order stabilizer, while Fig. 4.9(e) illustrates the second-order stabilizer. It is evident from this plot that the SOS provides oscillations and steeper edges in the output. The FOS has a difficult time interpolating surfaces at creases; therefore, the output is curved at any of these locations in the given data. Finally, Fig. 4.9(f) reveals the hybrid model for the fusion of the two inputs and it is found to interpolate a filtered surface which still preserves edge characteristics. The goal of the hybrid model is to use the SOS and FOS in active and nonactive regions, respectively. The term nonactive refers to regions containing no discontinuities. Therefore, after one pass of the fusion approach a gradient operator is used on the regularized image. From this image, a threshold value is chosen for implementing the SOS at or near discontinuities. The parameters for the final hybrid result were $\lambda_{high} = 40$, $\lambda_{low} = 30$, $\beta_{high} = 10$, and $\beta_{low} = 2$. The weights for the inputs were chosen as $w_1 = 0.7$ and $w_2 = 0.3$ where less weight was given to the noisier source. The threshold value for detecting where the SOS should be used was $t_{sos} = 80$ (intensity value). It can be seen from the hybrid result that it clearly outperforms either the first- or second-order stabilizer. In

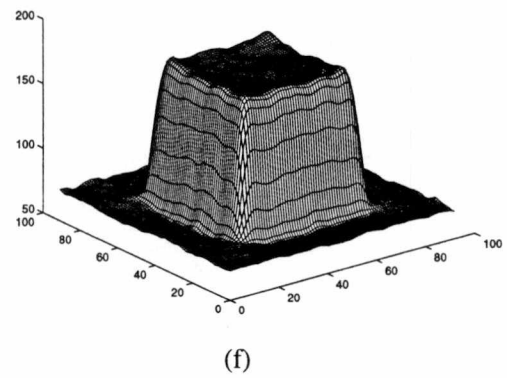
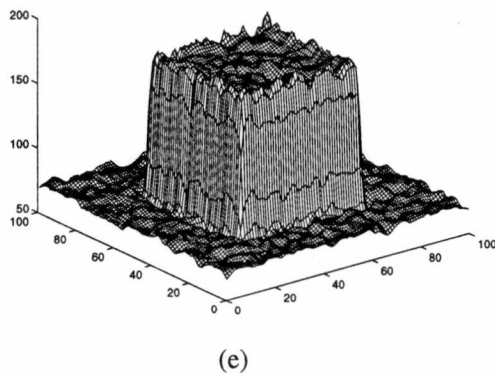
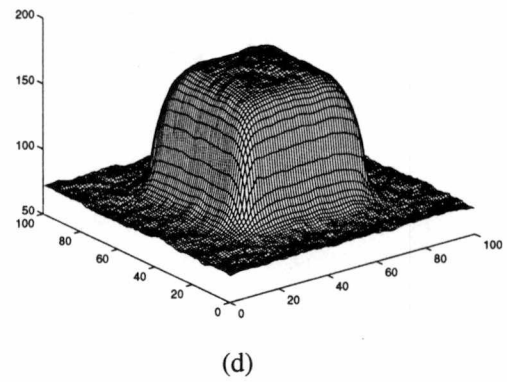
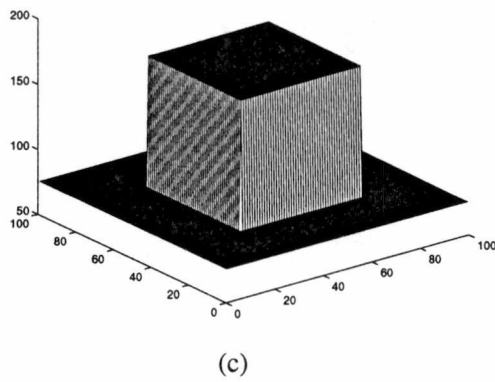
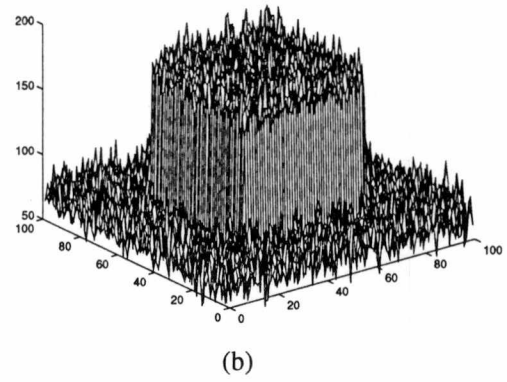
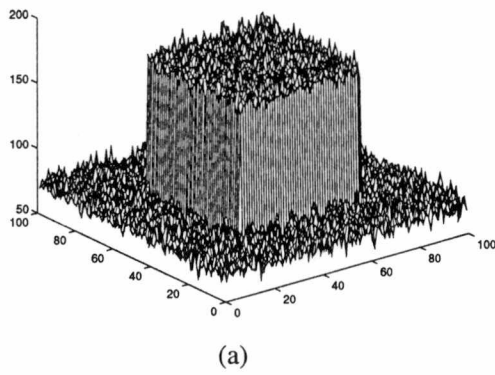


Figure 4.9: Surface reconstruction of square intensity image: (a) Input 1, $\sigma_n = 5$; (b) Input 2, $\sigma_n = 10$; (c) ideal surface; (d) recovered surface, first-order stabilizer; (e) recovered surface, second-order stabilizer; and (f) recovered surface, hybrid stabilizer.

conclusion, combining characteristics from both stabilizers reveals a unique way of filtering the noise while still preserving edge boundaries.

A previous experiment explored constant and variable smoothing parameters and results found the variable terms to be very beneficial. Here we will analyze another image, Fig. 4.10, called the “cake” intensity image. This experiment established similar results as before, in which the fixed regularization parameters oversmoothed edges and the variable regularization parameters managed to preserve discontinuities. The stabilizers performed identically as before with the FOS reconstructing a surface with oversmoothed edges and the SOS providing oscillations and well-defined edges in the regularized result. The parameters used in the above experiment were the same for both FOS and SOS cases, for the fixed case $\lambda = \beta = 16$, the variable case $\lambda_{high} = \beta_{high} = 10$ and $[\lambda_{low} = \beta_{low} = 0.9]$. The hybrid stabilizer consisted of $\lambda = 5$ and $\beta = 1$ for the fixed case and the variable case yielded $\lambda_{high} = 15$, $\lambda_{low} = 10$, $\beta_{high} = 2$, and $\beta_{low} = 0.7$ with a threshold at $t_{sos} = 57$ (intensity value) for the second-order stabilizer. A very noticeable feature in Fig. 4.10(h) is how the hybrid functional filters the noise in the bottom plane with few oscillations.

Our next surface reconstruction experiment involved fusing two noisy “cake” image inputs with Input 1, $\sigma_{1n} = 10$, and Input 2, $[\sigma_{2n} = 15]$. This experiment will support the fact that the hybrid stabilizer outperforms either the first- or second-order stabilizer. Figures 4.11(a) and (b) illustrate the two inputs and Fig. 4.11(c) gives the ideal case. The final result for the FOS, SOS, and hybrid

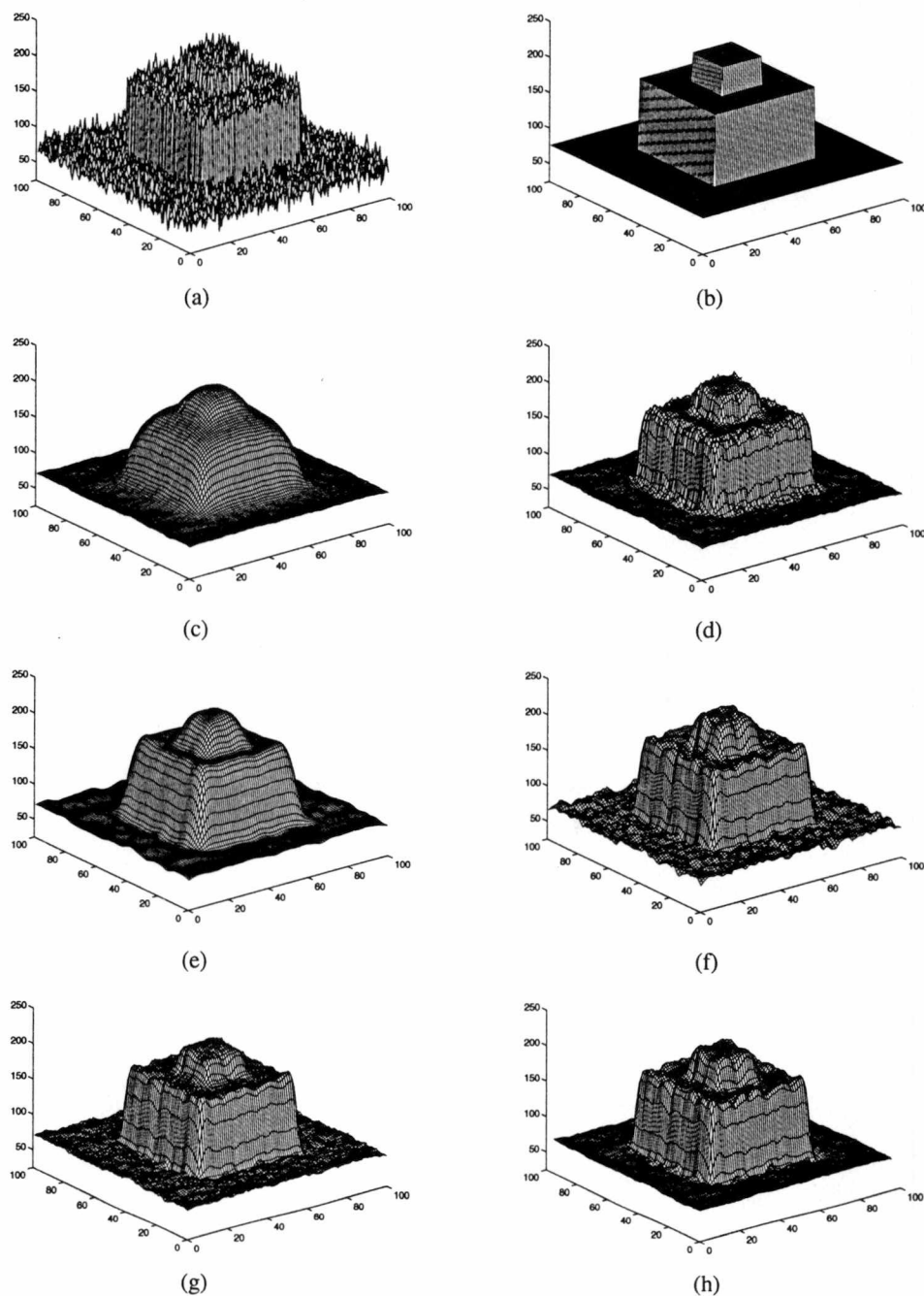
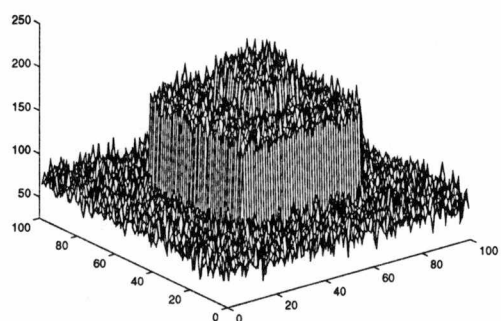
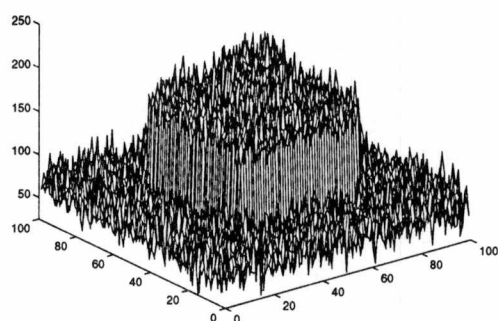


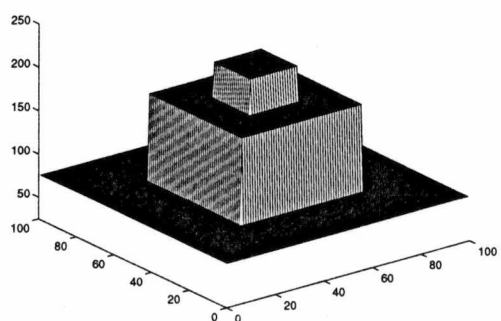
Figure 4.10: Surface reconstruction of cake intensity image: (a) input, $\sigma_n = 10$; (b) ideal surface; (c) recovered surface, (constant parameter) first-order stabilizer; (d) (variable parameter) first-order stabilizer; (e) recovered surface, (constant parameter) second-order stabilizer; (f) (variable parameter) second-order stabilizer; (g) recovered surface, (constant parameter) hybrid stabilizer; and (h) (variable parameter) hybrid stabilizer.



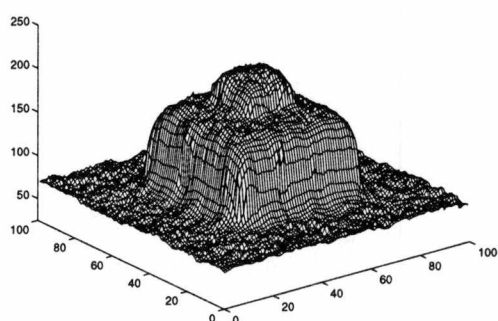
(a)



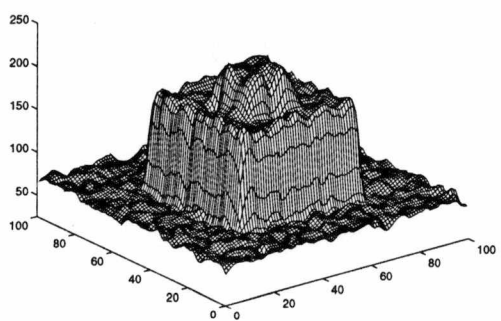
(b)



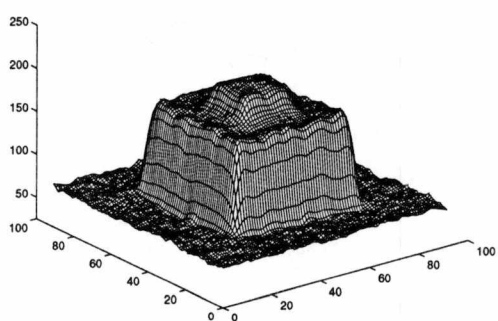
(c)



(d)



(e)



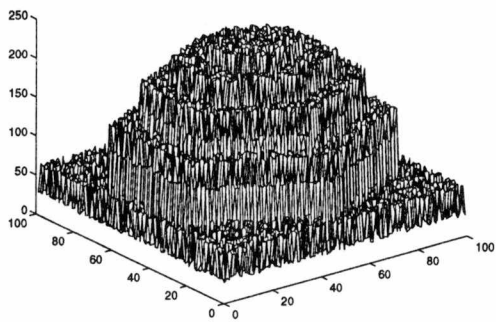
(f)

Figure 4.11: Surface reconstruction of cake intensity image: (a) Input 1, $\sigma_n = 10$; (b) Input 2, $\sigma_n = 15$; (c) ideal surface; (d) recovered surface, first-order stabilizer; (e) recovered surface, second-order stabilizer; and (f) recovered surface, hybrid stabilizer.

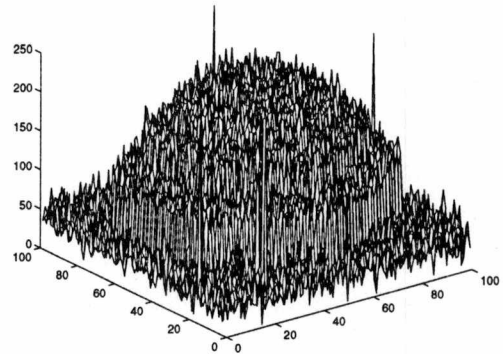
stabilizer can be seen in Figs. 4.11(d), (e), and (f), respectively. The results prove once again that the hybrid model yields a surface most similar to the ideal case. It can be concluded from this experiment that fusing noisy inputs by a regularized approach yields a filtered response which closely resembles the ideal case.

Next, we wanted to use the hybrid stabilizer on a more complicated image with many jump edges. Figure 4.12 illustrates the “hexagonal” intensity image with Input 1 (Fig. 4.12(a)) with noise concentration $\sigma_{1n} = 10$ and Input 2, Fig. 4.12(b), with noise concentration $[\sigma_{2n} = 15]$. Once again we fuse the images by applying the second-order stabilizer near discontinuities and the first-order stabilizer near noisy nonactive areas. It was found that a threshold value of $t_{sos} = 100$ was sufficient for applying the SOS near discontinuities. Figure 4.12(c) supplies the ideal case illustrating the various jump edges. Figures 4.12(d) and (e) illustrate the regularized response for first- and second-order stabilizers, respectively. Notice the oversmoothed edges for the FOS and the oscillatory surfaces for the SOS, especially near any edges. One can see from the hybrid response that it provided steep edges and flattened the tops and bottoms of the hexagonal structures to some degree.

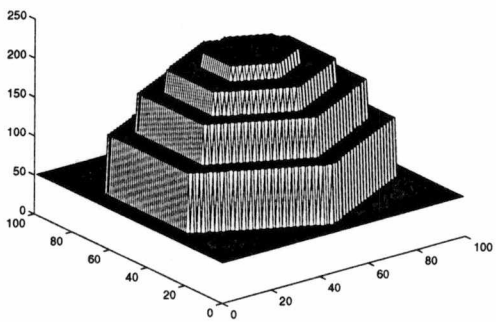
Our next surface reconstruction experiment involved fusing two noisy “cylindrical” intensity images with Input 1, $\sigma_{1n} = 10$, and Input 2, $\sigma_{2n} = 15$. Figures 4.13(a) and (b) supply the two inputs while Fig. 4.13(c) illustrates the ideal case. This particular input image is generated with four different sized cylinders stacked on top of one another. It will test how the stabilizers react to different



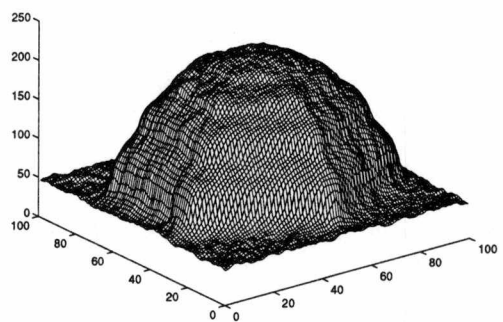
(a)



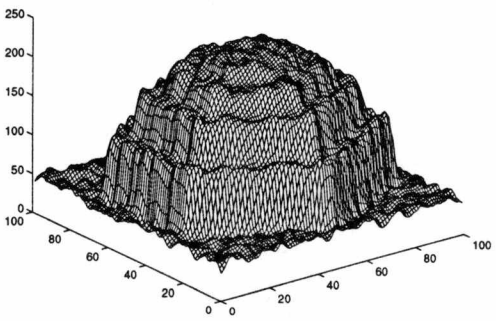
(b)



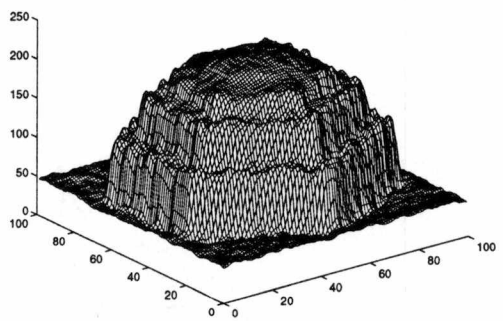
(c)



(d)



(e)



(f)

Figure 4.12: Surface reconstruction of hexagonal intensity image: (a) Input 1, $\sigma_n = 10$; (b) Input 2, $\sigma_n = 15$; (c) ideal surface; (d) recovered surface, first-order stabilizer; (e) recovered surface, second-order stabilizer; and (f) recovered surface, hybrid stabilizer.

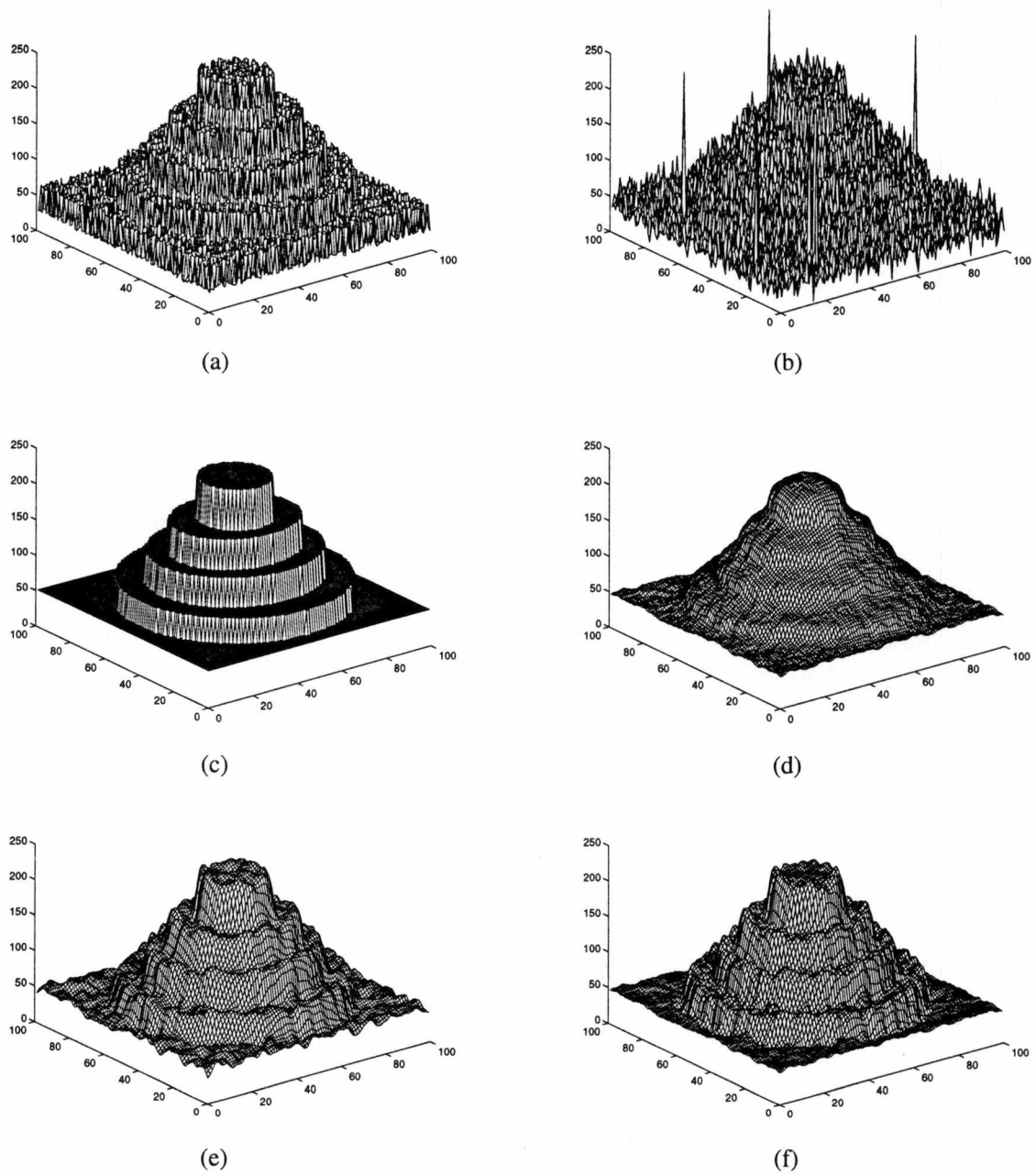
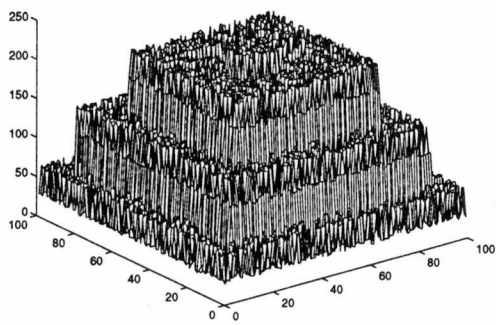


Figure 4.13: *Surface reconstruction of cylindrical intensity image: (a) Input 1, $\sigma_n = 10$; (b) Input 2, $\sigma_n = 15$; (c) ideal surface; (d) recovered surface, first-order stabilizer; (e) recovered surface, second-order stabilizer; and (f) recovered surface, hybrid stabilizer.*

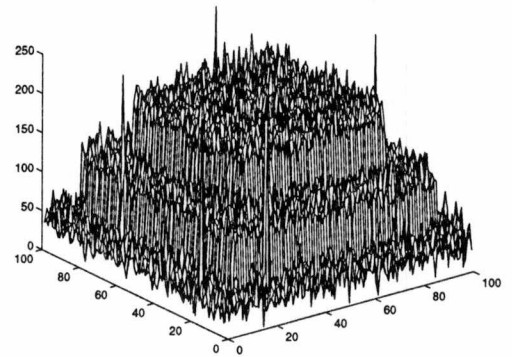
structured objects. The final results for the FOS, SOS, and hybrid stabilizer can be seen in Figs. 4.13(d), (e), and (f), respectively. Examining the different stabilizers reveals that the hybrid model provides the most complete reconstructed description of the noisy inputs.

Our final experiment, involving the fusion of dual noisy inputs, is illustrated in Fig. 4.14, with Input 1, $\sigma_{1n} = 10$, and Input 2, $\sigma_{2n} = 15$, given in Figs. 4.14(a) and (b), respectively. The ideal surface, Fig. 4.14(c), is constructed with two different sized blocks stacked on top of one another with a cylinder placed inside an indentation on the top block. This experiment tests how the different functionals react to blocks, cylinders, and indentions. Once again, the hybrid model, Fig. 4.14(f), outperforms the FOS, Fig. 4.14(d), and the SOS, Fig. 4.14(e), by reconstructing filtered surfaces and still preserving edge characteristics.

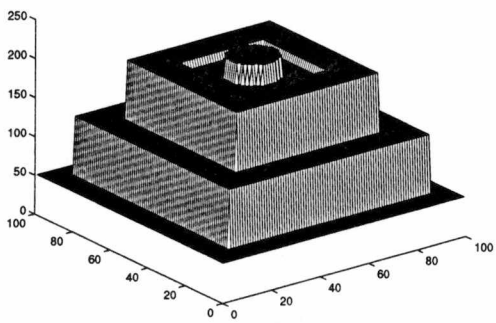
So far we have only tried the regularized approach for synthetic range images. Now we examine the use of range data for the surface reconstruction process. Figure 4.15 depicts two noisy range data sources with Input 1 shown in Fig. 4.15(a) with noise concentration $\sigma_{1n} = 10$ and Input 2 illustrated in Fig. 4.15(b) with noise concentration $[\sigma_{2n} = 20]$. The ideal surface depicting a “cup” is represented by the 3-D plot given in [Fig. 4.15(c)]. Figures 4.15(d) and (e) give the responses for the first- and second-order stabilizers, respectively. The FOS provides a response which is oversmoothed and the SOS supplies steeper edges but some noticeable oscillations located throughout. The hybrid response, Fig. 4.15(f), removes the heavy amounts of noise and delivers sharp, interpolated edges. Notice that the



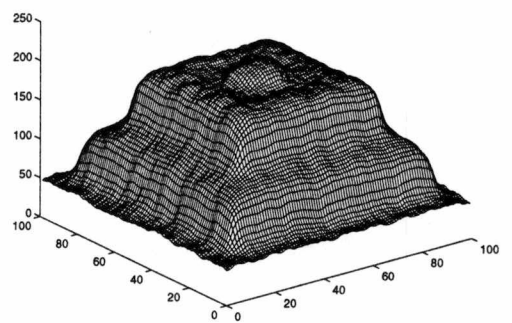
(a)



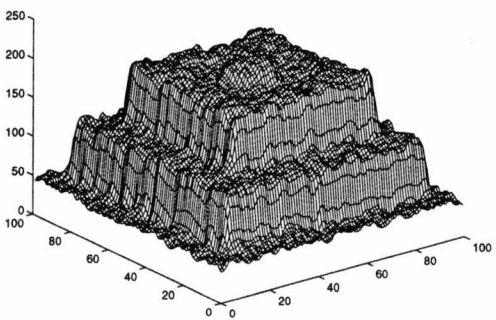
(b)



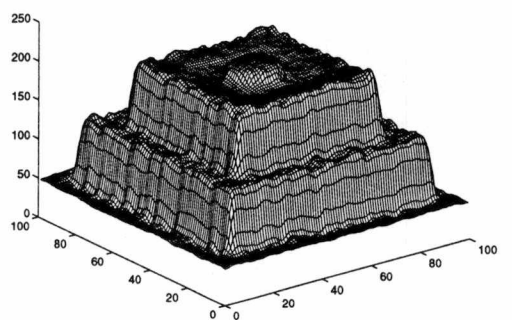
(c)



(d)

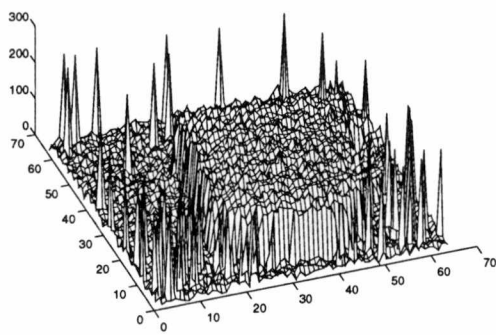


(e)

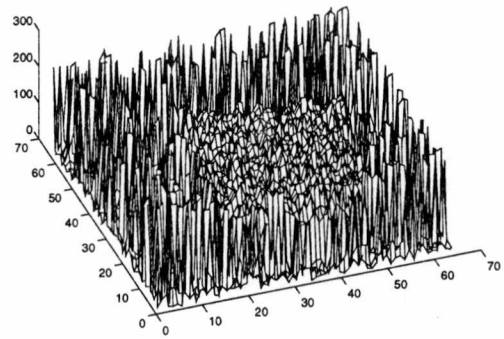


(f)

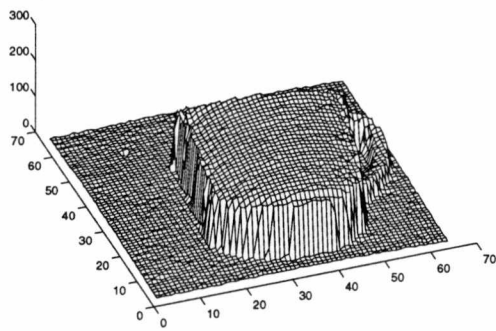
Figure 4.14: Surface reconstruction of pyramid intensity image: (a) Input 1, $\sigma_n = 10$; (b) Input 2, $\sigma_n = 15$; (c) ideal surface; (d) recovered surface, first-order stabilizer; (e) recovered surface, second-order stabilizer; and (f) recovered surface, hybrid stabilizer.



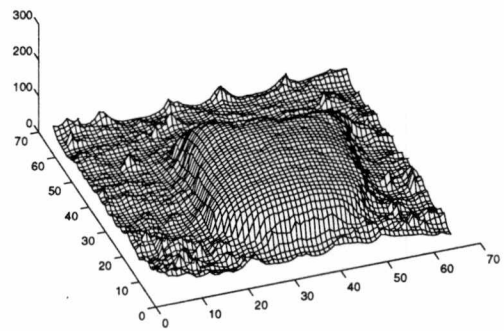
(a)



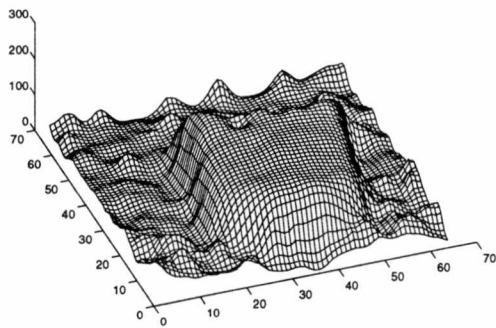
(b)



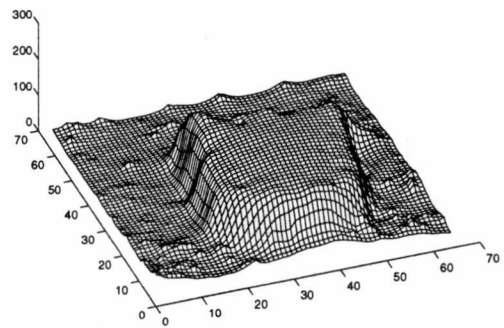
(c)



(d)



(e)

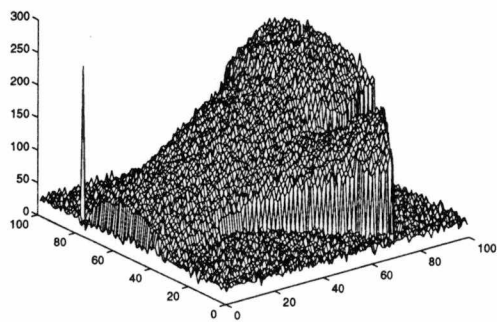


(f)

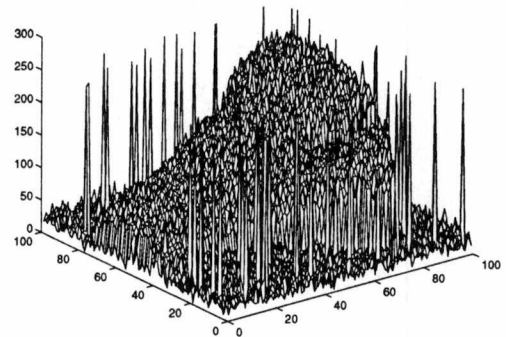
Figure 4.15: Surface reconstruction of cup range image: (a) Input 1, $\sigma_n = 10$; (b) Input 2, $\sigma_n = 20$; (c) ideal surface; (d) recovered surface, first-order stabilizer; (e) recovered surface, second-order stabilizer; and (f) recovered surface, hybrid stabilizer.

cup handle is not fully recovered for any of the stabilizers. The noise present in the two inputs totally consumes the handle, making it indistinguishable. Looking closely at the hybrid response, you can see that the top part of the handle in the upper right part of the cup is beginning to be reconstructed; however, the noise quickly took over.

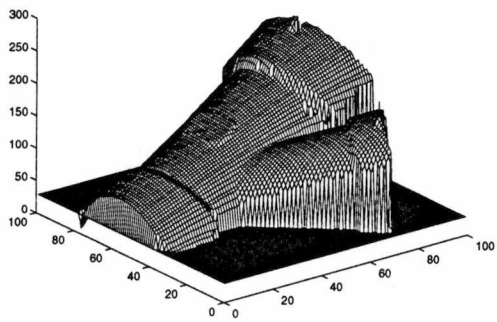
Our next surface reconstruction experiment, Fig. 4.16, examines the fusion of real range data for reconstructing a PCV pipe structure. Figure 4.16(a) depicts a wye-shaped model contaminated with zero-mean Gaussian noise of standard deviation $\sigma_n = 5$, and a second input contaminated with noise of $\sigma_n = 10$ [Fig. 4.16(b)]. The two inputs are jointly reconstructed using either a first-order stabilizer Fig. 4.16(d), second-order stabilizer Fig. 4.16(e), or a hybrid stabilizer [Fig. 4.16(f)]. The ideal (noise-free) surface is shown in Fig. 4.16(c) with some bad range values; especially in the top portion of the pipe where a gap is noticeable. The first-order stabilizer provided a somewhat smoother version of the ideal surface; however, note the reconstructed surface at the gap or crease where the bad range values were located. The first-order stabilizer provides a response which conforms smoothly at crease edges such as this, while the second-order stabilizer is able to fold or bend at these instances. In general, the second-order stabilizer provides better surfaces at edge boundaries, but creates very noticeable oscillations throughout the surface. The hybrid stabilizer reconstructs a model with fewer oscillations than the second-order response and a surface with well-preserved edge characteristics as found in the first-order reconstruction.



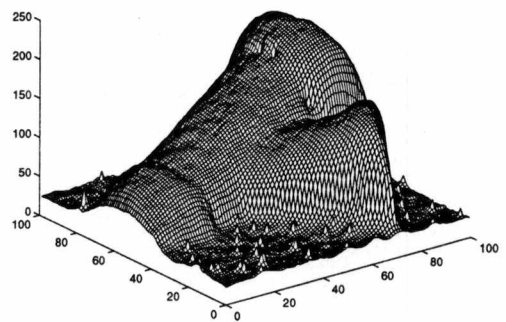
(a)



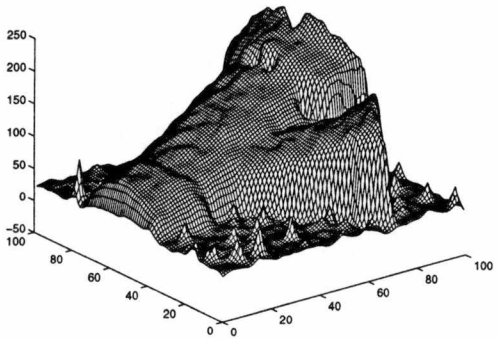
(b)



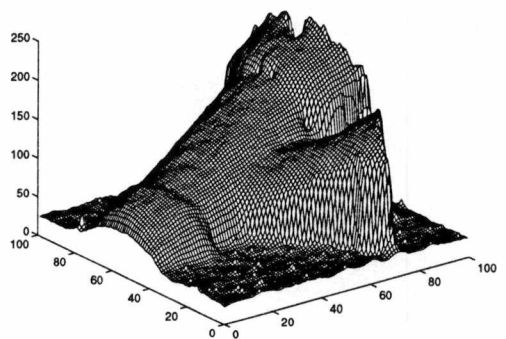
(c)



(d)



(e)

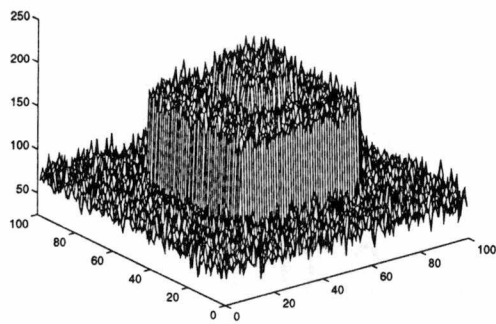


(f)

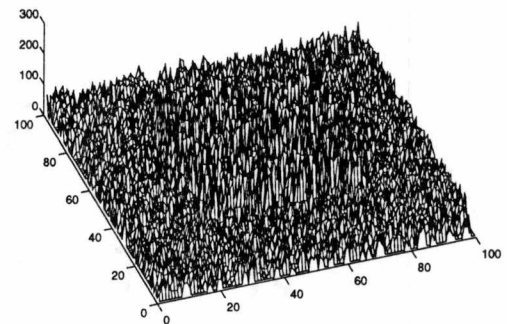
Figure 4.16: Surface reconstruction of real range data (image scanned by MSU PRIP Lab): (a) Input 1, $\sigma_n = 5$; (b) Input 2, $\sigma_n = 10$; (c) ideal surface; (d) recovered surface, first-order stabilizer; (e) recovered surface, second-order stabilizer; and (f) recovered surface, hybrid stabilizer.

Next we examine the regularized method on sparse data. As previously mentioned, regularization theory is unique in its ability to interpolate smooth surfaces through sparse data points. Our first illustration, Fig. 4.17, takes into account the use of a single input with sparse data at 30% density, Fig. 4.17(b), from the original “cake” input Fig. 4.17(a), $[\sigma_n = 10]$. Figure 4.17(c) reconstructs the interpolated surface using the first-order stabilizer. It should be apparent that this surface is recognizable but oversmoothed. The examination of the second-order and hybrid stabilizers reveals steeper edges in the output with fewer oscillations occurring in the latter. The three stabilizers are able to provide an approximated surface which closely resembles the original data. It should be noted, however, that the hybrid stabilizer provided the best response due to its removal of noise and preservation of edges. One drawback of the hybrid model for this case is oscillatory surfaces along the edges.

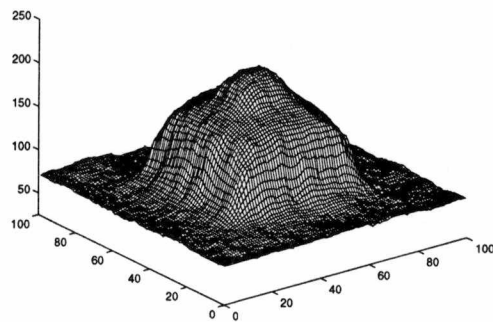
The next experiment conducted involved fusing two noisy sparse data inputs, Input 1, Fig. 4.18(a) with 20% density and Input 2, Fig. 4.18(b) with 30% density. For comparison purposes the ideal case for the “cake” image is shown in [Fig. 4.18(c)]. Using the hybrid stabilizer with constant smoothing terms the recovered surface, Fig. 4.18(d), resembled the ideal case to some degree. The recovered surface for the variable smoothing parameters is found in Fig. 4.18(e) with a trade-off occurring between noise removal and edge preservation. Examining Figs. 4.18(d) and (e), which one is better? Observations led to the conclusion that the variable case is better, because it has fewer oscillations along the edges



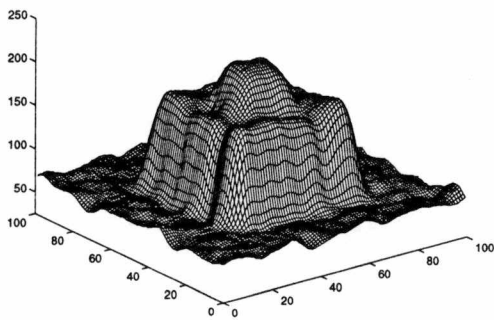
(a)



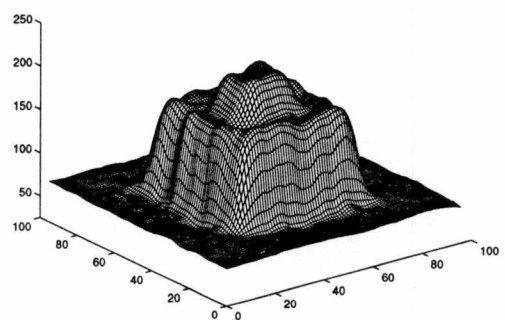
(b)



(c)



(d)



(e)

Figure 4.17: Surface reconstruction with sparse data from cake intensity image: (a) input data, $\sigma_n = 10$; (b) sparse data with 30% density; (c) recovered surface, first-order stabilizer; (d) recovered surface, second-order stabilizer; and (e) recovered surface, hybrid stabilizer.

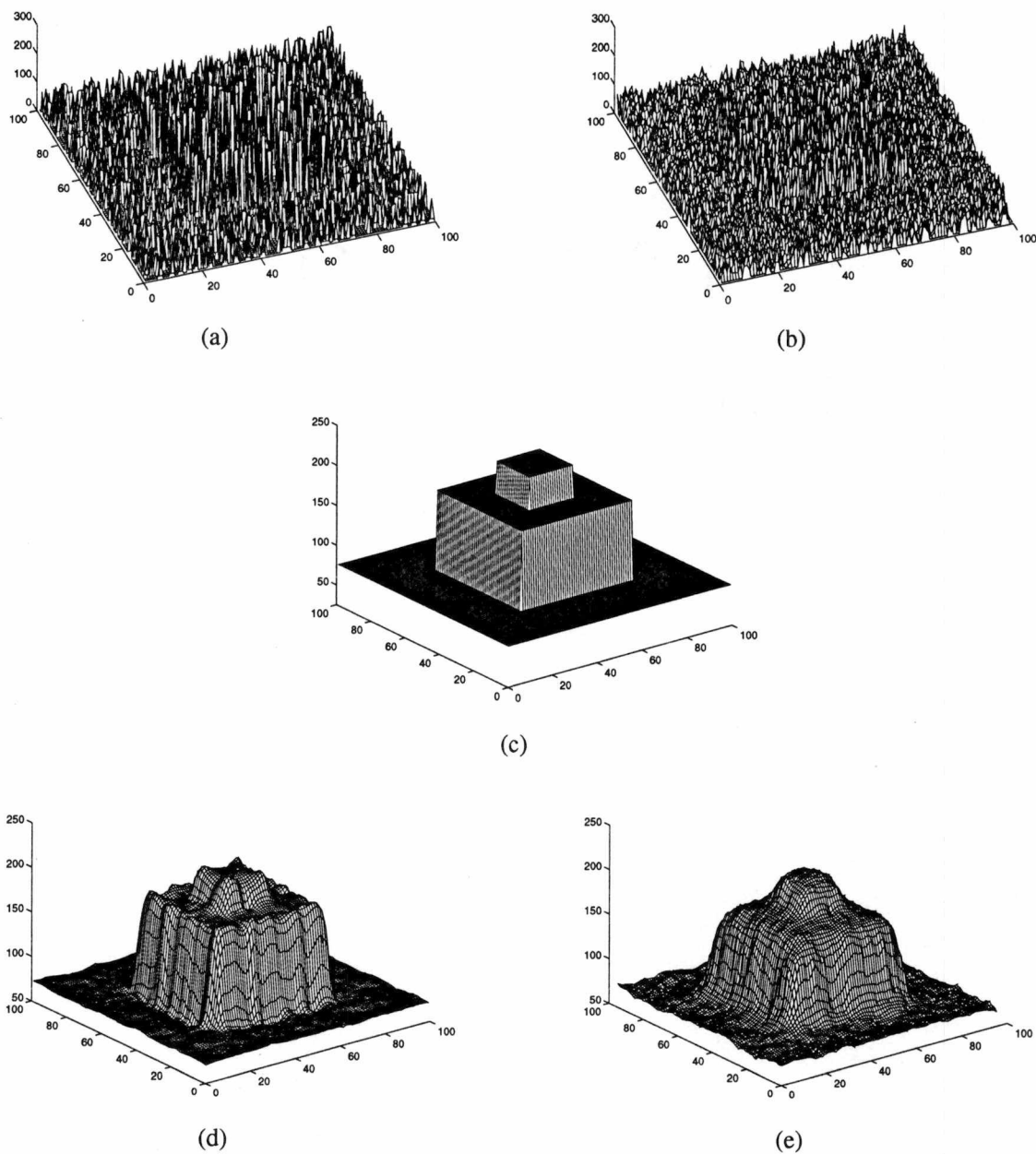


Figure 4.18: Surface reconstruction with two sparse inputs from cake intensity image: (a) sparse data Input 1 with 20% density; (b) sparse data Input 2 with 30% density; (c) ideal surface; (d) recovered surface, (constant parameters) hybrid stabilizer; and (e) recovered surface, (variable parameters) hybrid stabilizer.

and still maintained some sharp edges.

4.2.2 Gray-Level Edge Detection Results

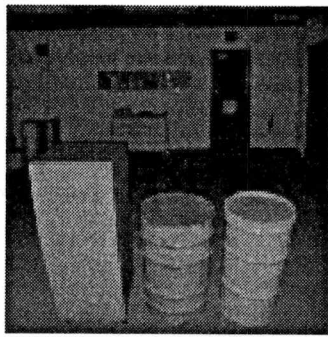
An important goal of early vision is to obtain a complete and meaningful description of a scene, including details of surface properties. Physical edges are one of the most important features since they correspond to object boundaries and can be used for detection purposes. A major difficulty in the edge detection approach occurs during the acquisition and sampling process when the data is often degraded by noise. Since numerical differentiation, the detection of edges, magnifies noise, this becomes an ill-posed process. Thus, some type of noise suppression method must be performed before the differentiation process. In our case, the smoothing operation is performed by minimizing the hybrid energy functional containing a smoothness constraint. Suppressing the noise allows good detection (marking all true edge points), but the localization (edges should be as close as possible to real edges) accuracy is reduced.

Regularized Range and Intensity Data Fusion

This section describes the results achieved using the new regularized method for fusing range and intensity data. For many practical cases, it is desirable to have an extra sensor that provides information another sensor can't. This combination of sources will deliver an output that contains more scene characteristics than a single source. For our regularized fusion process we assume the different visual

sensors are related to the same scene and their corresponding data points are perfectly registered in a pixel-by-pixel manner.

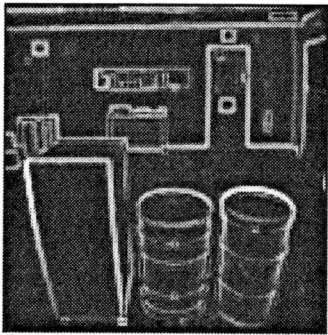
Our first experiment, Fig. 4.19, illustrates the two inputs used in the fusion process, Input 1 (intensity) in Fig. 4.19(a), and Input 2 (range) in Fig. 4.19(b). A vision source provides reflectance properties while a range sensor provides depth and shading information. For the range image a darker intensity value represents closeness and a lighter value represents objects farther away. There is very little detailed information in the range image about the box and barrels in the foreground. In fact, they appear as blobs with no discernible interior edges other than their silhouettes. Information about interior edges located on the objects is contained in the intensity or visible image. In order to fuse range and intensity data they must be in the same domain. "In the same domain" implies that the data being fused must have similar characteristics or possess a one-to-one correlation. For example, the approach used for the regularized fusion methodology is to apply a gradient operator to both the range and intensity images. The Sobel gradient operator was used, which allowed the detection of edges from the intensity image [Fig. 4.19(c)] and range image [Fig. 4.19(d)]. These two images correspond to their highlighted edges. The fusion process occurs from this data. Figure 4.19(e) is the fused result and Fig. 4.19(f) provides the final edge map taken from the regularized image. The final edge map was achieved by choosing a threshold value which maintained most of the edges. For this particular experiment the extraction of edges from the fused component was used for identifying objects in the scene.



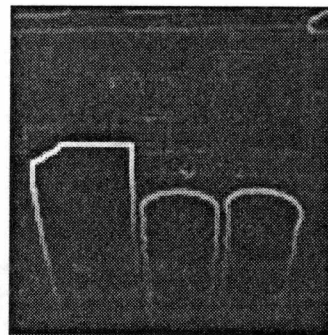
(a)



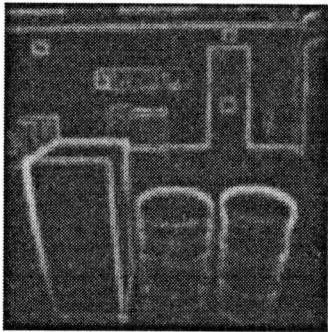
(b)



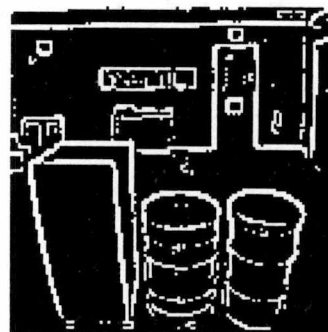
(c)



(d)



(e)



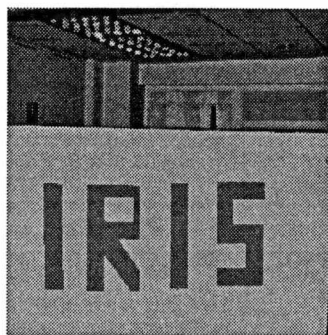
(f)

Figure 4.19: Regularized edge detection: (a) intensity image (Input 1), (b) range image (Input 2), (c) gradient of Input 1, (d) gradient of Input 2, (e) regularized response, and (f) final edge map.

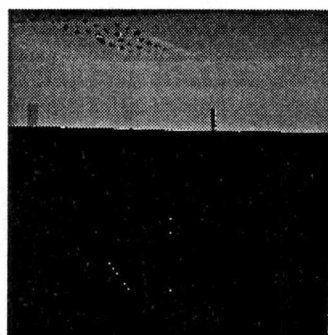
The parameters λ and β tend not only to be a characteristic length for smoothing continuous portions of discrete data, but they also exhibit a characteristic distance for interaction between pixels.

Our next experiment, Fig. 4.20, depicts a range and intensity image containing the Imaging, Robotics, and Intelligent Systems (IRIS) Laboratory name pinned against a flat bulletin board. Figures 4.20(a) and (b) contain the intensity and range images, respectively. Notice that the range image was unable to distinguish the depth disparity between the IRIS label and backboard. This is a classical example of how range and camera sensors differ in their capabilities. The intensity image contains much more detailed information than its counterpart; however, the range data reinforces the intensity data by collaborating with it. For instance, examining the gradient responses of the intensity Fig. 4.20(c) and range image, Fig. 4.20(d), reveals features apparent in one but not the other. The goal of the fusion algorithm is to extract all of the important features or characteristics from the range and intensity images which increases the success of future processing modules. One such module could be an algorithm for recognizing objects in the final edge map [Fig. 4.20(f)]. For this experiment, the regularized method was able to provide a noise-free response while correctly marking all relevant edge components belonging to important object features.

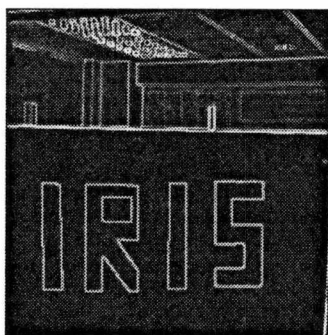
The next fusion illustration, Fig. 4.21, examines a real scene consisting of several pipes and pressure valves. The intensity image, Fig. 4.21(a), and the range image, Fig. 4.21(b), are fused for segmenting the scene into its respective parts.



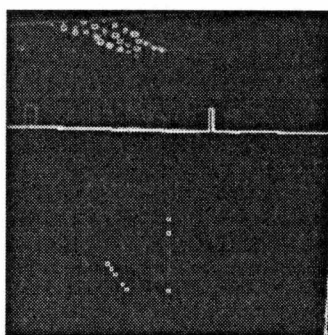
(a)



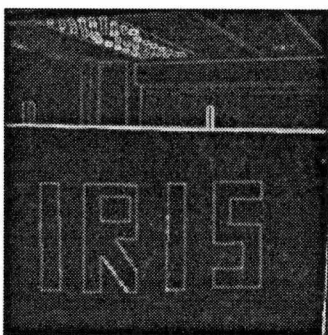
(b)



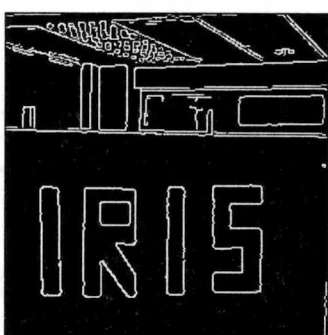
(c)



(d)

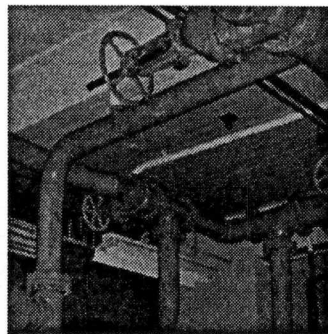


(e)



(f)

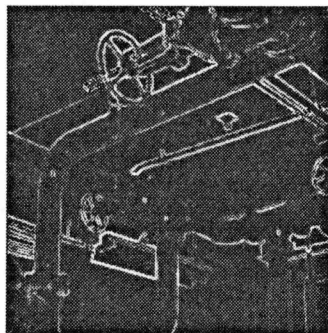
Figure 4.20: *Regularized edge detection: (a) intensity image (Input 1), (b) range image (Input 2), (c) gradient of Input 1, (d) gradient of Input 2, (e) regularized response, and (f) final edge map.*



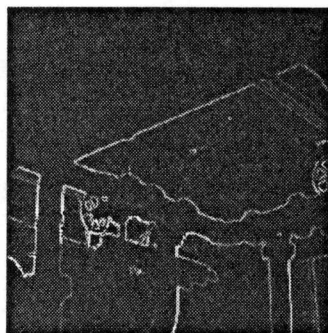
(a)



(b)



(c)



(d)



(e)

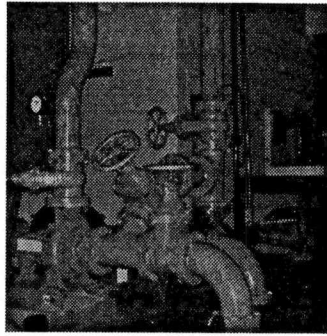


(f)

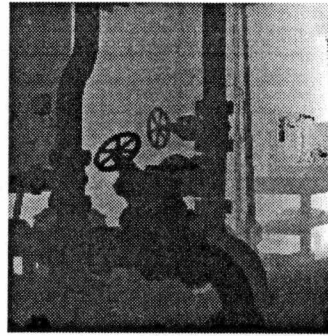
Figure 4.21: *Regularized edge detection: (a) intensity image (Input 1), (b) range image (Input 2), (c) gradient of Input 1, (d) gradient of Input 2, (e) regularized response, and (f) final edge map.*

For instance, the range and intensity sources work together to provide information unattainable by each. The two sources will provide a final edge map useful for segmenting the pipes, valves, and walls. Before integrating or combining the range and intensity images into one representational format, both types of data must share similar characteristics. Thus we apply a gradient technique to ensure that the range Fig. 4.21(d) and intensity Fig. 4.21(c) images are of the same entity. These edge maps are fused by the regularized approach to obtain the response shown in Fig. 4.21(e) where a threshold value is selected for obtaining the final edge map shown in [Fig. 4.21(f)]. Notice that information in the final output was extracted from both the range and intensity images. In particular, the intensity image contains a black pulley ring in the ceiling which does not appear in the range image; however, this feature is recognizable in the final fused output.

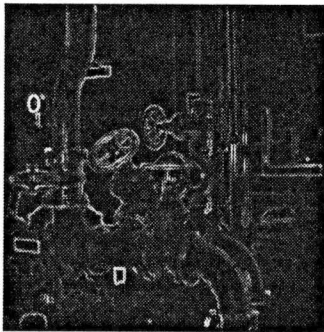
Our next experiment examines another scene, Fig. 4.22, consisting of pipes, pressure valves, and gauges. The intensity image, Fig. 4.22(a), provides the most detailed characteristics of the pipes and valves, while the range image, Fig. 4.22(b), provides a silhouette version of the scene based on depth measurements. Taking the gradient operation for the intensity, Fig. 4.22(c), and range image, Fig. 4.22(d), prepares the sensor data to be of the same entity. The two images are now suitable for the fusion process. The regularized fusion approach provides the response shown in Fig. 4.22(e) with the final edge map given in Fig. 4.22(f). Once again the final edge map contains features extracted from both the range and intensity sources. For instance, the interior details of the gauge located on the large pipe



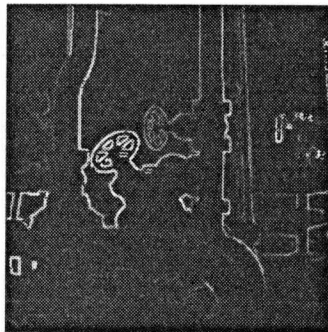
(a)



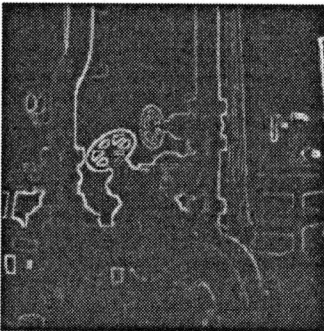
(b)



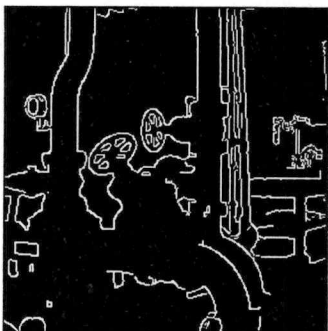
(c)



(d)



(e)

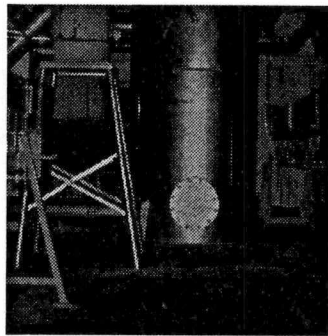


(f)

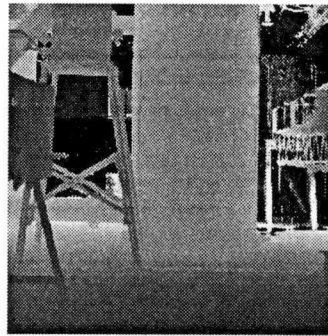
Figure 4.22: *Regularized edge detection: (a) intensity image (Input 1), (b) range image (Input 2), (c) gradient of Input 1, (d) gradient of Input 2, (e) regularized response, and (f) final edge map.*

in the far left portion of the intensity image are clearly not visible in the range image; however, these features do appear in the final fused response. The final output has segmented the scene into edge components, each of which could help identify and locate pipes, gauges, or pressure valves.

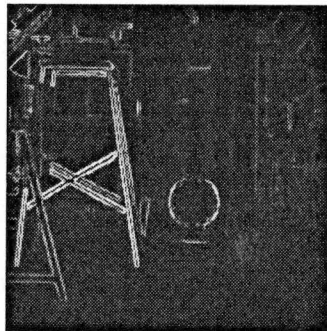
Our final fusion experiment, Fig. 4.23, illustrates another hydraulic scene consisting of pipes and a tank. The intensity, Fig. 4.23(a), and range image, Fig. 4.23(b), provide all photometric and depth properties required for the fusion process. Once again, each source's data is prepared to be of the same entity for input to the fusion algorithm. The goal is to recover every piece of vital edge information which could be useful for identifying and locating important features in a scene. Fusing data from two different sensing sources provides more detailed characteristics of a scene. Examining the gradient images for the intensity, Fig. 4.23(c), and range data, Fig. 4.23(d), reveals features apparent in one but not the other. For example, the large riveted pipe extending out from the tank is quite noticeable in the intensity image; however, the range image is not able to detect the depth disparity between the tank and the pipe. Sometimes the range profile might not contain any relevant edges to add to the intensity image; however, it can supply edge components which reinforce the visible data.



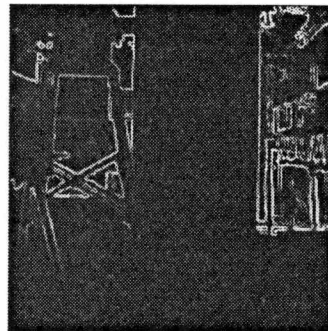
(a)



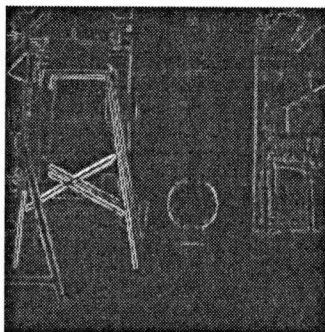
(b)



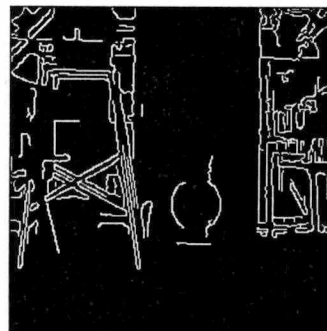
(c)



(d)



(e)



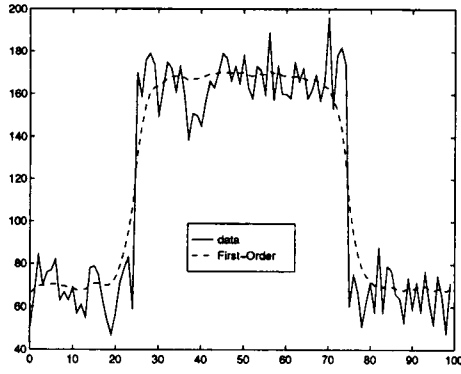
(f)

Figure 4.23: Regularized edge detection: (a) intensity image (Input 1), (b) range image (Input 2), (c) gradient of Input 1, (d) gradient of Input 2, (e) regularized response, and (f) final edge map.

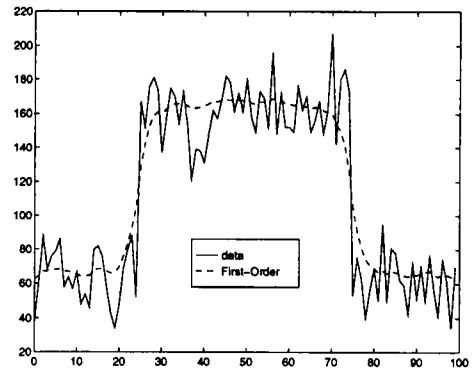
4.3 Noise Analysis

In this section we present experiments related to how the different energy functionals and their associated stabilizers perform under noisy conditions. A comparison will be made between the first-order, second-order, and hybrid stabilizers to indicate which one, if any, performs adequately in severe noise concentrations. The goal of this analysis is find the stabilizer which is noise tolerant and still preserves edges during the filtering process. For the upcoming experiments, a single row or slice of the image is extracted for further examination.

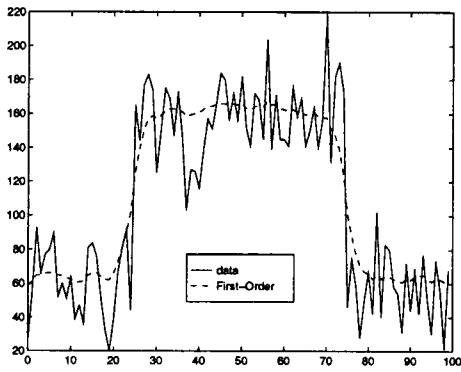
Our first experiment requires the simple “square” intensity image as the input where it is contaminated with zero-mean Gaussian noise of varying standard deviations from ($\sigma_n = 10$ to $\sigma_n = 35$ in increments of 5 units). From this noisy image we extracted a single slice or row. Figure 4.24 supplies the responses for the first-order stabilizer under varying amounts of noise. An examination of Figs. 4.24(a)–(f) reveals that the first-order response becomes erratic as the noise level increases. Increasing the standard deviation beyond $\sigma_n = 35$ causes the input signal to be degraded to such a high degree the output response becomes insufficient. Close attention should be brought to Figs. 4.24(a) and (b) since they provide information at a lower noise level. The same procedure was implemented for the second-order stabilizer, Fig. 4.25, and results reveal that it is far superior in reconstructing sharp edges, but as previously mentioned the surface is extremely oscillatory for increasing levels of noise. Finally, the same procedure



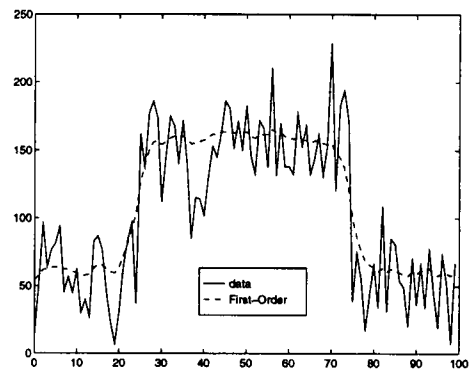
(a)



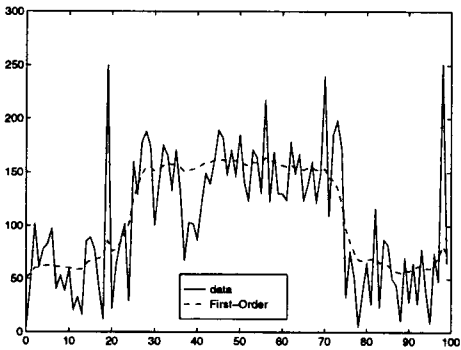
(b)



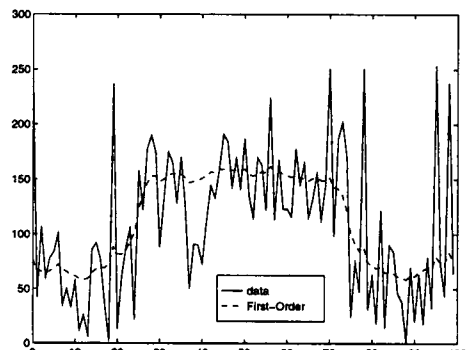
(c)



(d)



(e)



(f)

Figure 4.24: Response of the first-order stabilizer to increasing noise levels: (a) $\sigma_n = 10$, (b) $\sigma_n = 15$, (c) $\sigma_n = 20$, (d) $\sigma_n = 25$, (e) $\sigma_n = 30$, and (f) $\sigma_n = 35$.

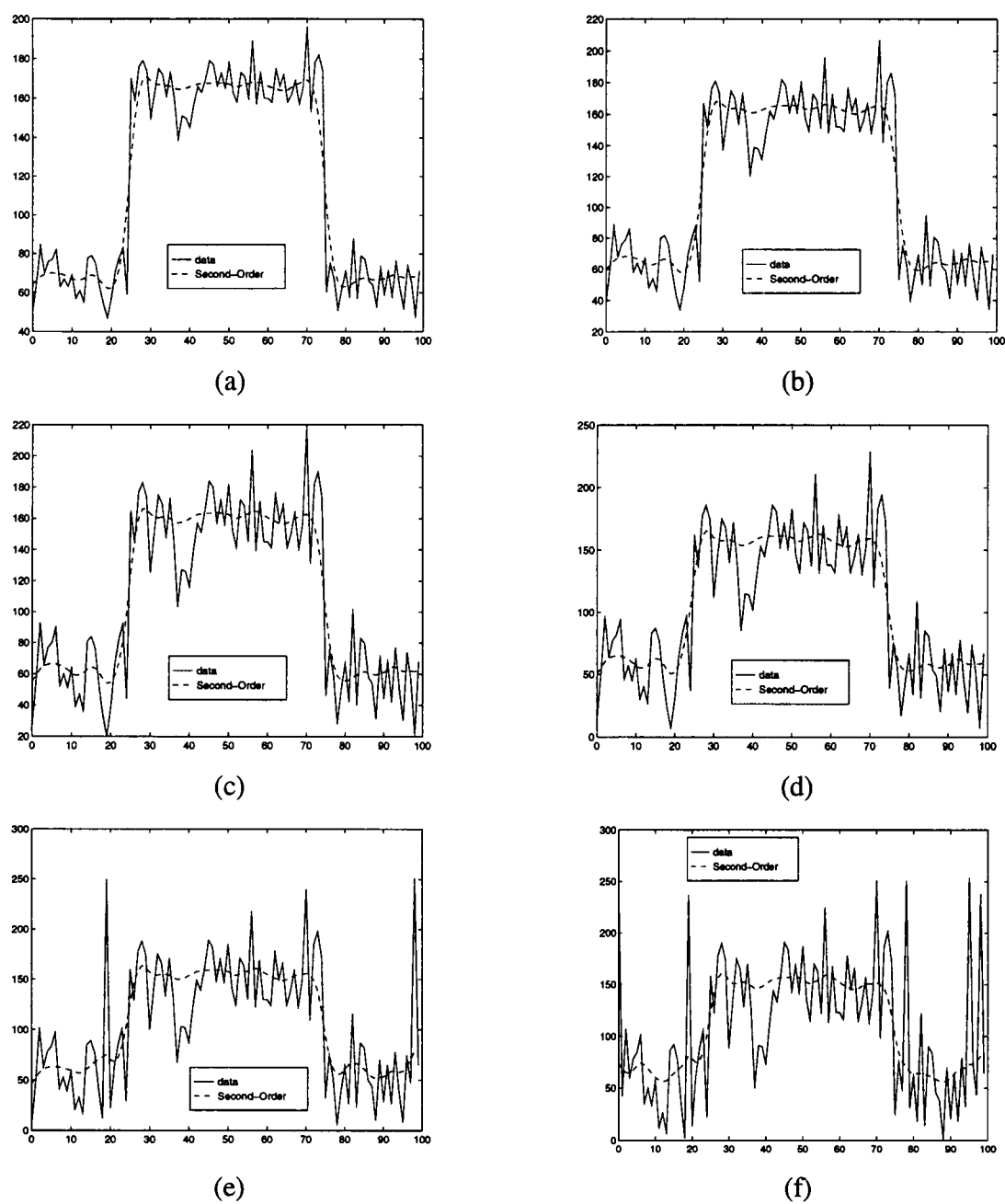


Figure 4.25: Response of the second-order stabilizer to increasing noise levels: (a) $\sigma_n = 10$, (b) $\sigma_n = 15$, (c) $\sigma_n = 20$, (d) $\sigma_n = 25$, (e) $\sigma_n = 30$, and (f) $\sigma_n = 35$.

was performed for the hybrid stabilizer, Fig. 4.26. Examination of Figs. 4.26(a)–(f) indicates its noise tolerance with increasing noise levels while still maintaining edge characteristics. Figure 4.26(a) illustrates a typical hybrid response in which the edges are extremely sharp. Notice that the signal on top is relatively flattened and filters much of the noisy signal.

Figure 4.27 compares the regularized responses of the different stabilizers for small and large regularization parameters. For either case the hybrid stabilizer is far superior to the first- and second-order stabilizers because of its noise removal and ability to preserve discontinuities. The reason for this dominance is that the hybrid model has the ability to control where the first- and second-order stabilizers are implemented. These experiments were conducted using the SOS in areas around discontinuities and the FOS vice versa. For this procedure we chose small regularization values for the SOS in order to preserve edges and large values for the FOS for noise filtering. In conclusion, we have shown the hybrid's ability to preserve edges while still maintaining its immunity to noise.

4.4 Computational Time Analysis

Many modern robotic systems require fast and efficient algorithms for computer vision applications. The efficiency of an algorithm can be related to how fast it solves a problem. Many industrial systems must rely on a quick algorithm for its fast-paced production requirements; therefore this section examines the

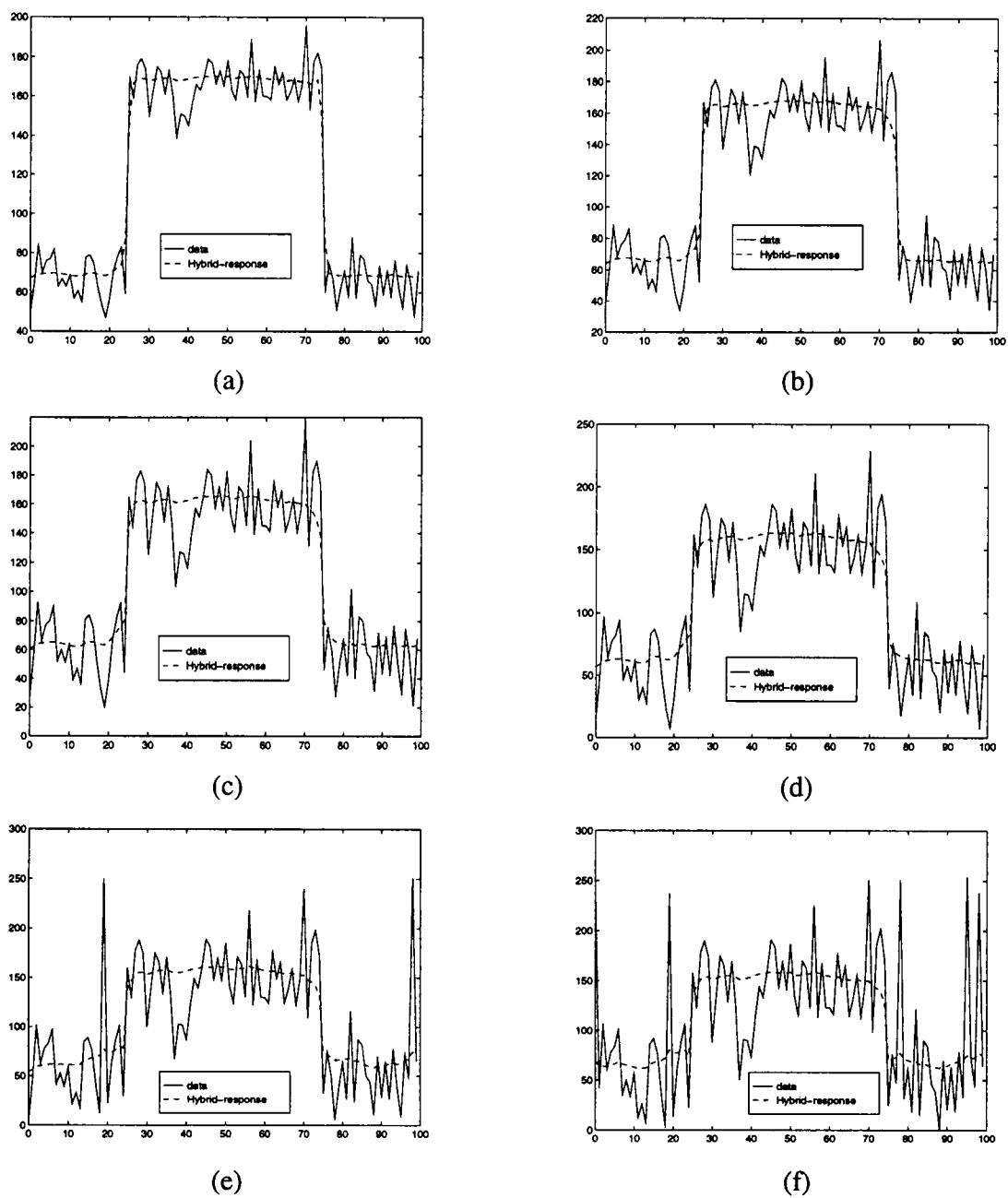
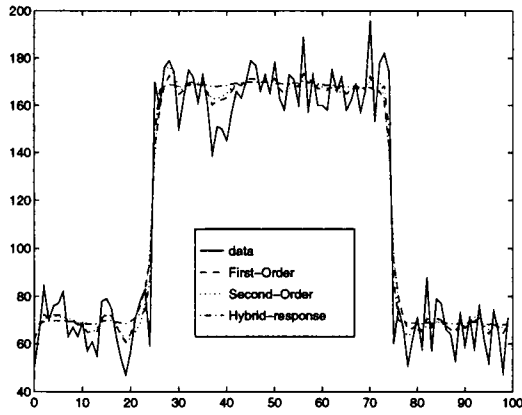
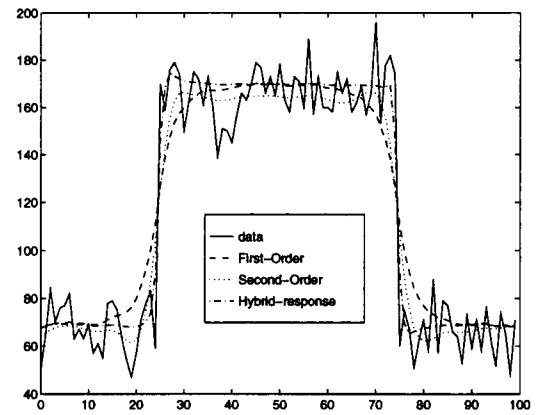


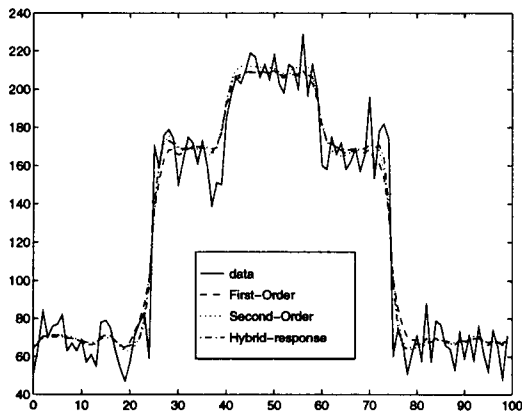
Figure 4.26: *Response of the hybrid stabilizer to increasing noise levels: (a) $\sigma_n = 10$, (b) $\sigma_n = 15$, (c) $\sigma_n = 20$, (d) $\sigma_n = 25$, (e) $\sigma_n = 30$, and (f) $\sigma_n = 35$.*



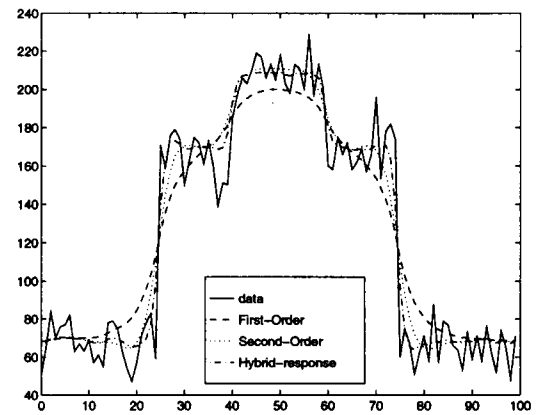
(a)



(b)



(c)



(d)

Figure 4.27: A comparison of the first-order, second-order, and hybrid stabilizers for a particular section of the square intensity image: (a) small and (b) large regularization parameters and also a cross-section of the cake intensity image: (c) small and (d) large regularization parameters.

computational time required for implementing the hybrid energy functional. The functional is minimized by iteratively solving for the solution using the successive over-relaxation (SOR) method. As with any iterative method, sometimes a large number of iterations are required before the algorithm can converge.

Examining the SOR procedure used for computing the local minimum of the energy functional we find the n^{th} iteration is

$$u_l^{(n+1)} = u_l^{(n)} - \omega \frac{1}{T_l} \frac{\partial F^{(p)}}{\partial u_l}, \quad (4.5)$$

where the speed of convergence is controlled by the “SOR parameter” ($0 < \omega < 2$).

The variable T_l is an upper bound on the second derivative

$$T_l \geq \frac{\partial^2 F^{(p)}}{\partial u_l^2} \quad \forall \mathbf{u}. \quad (4.6)$$

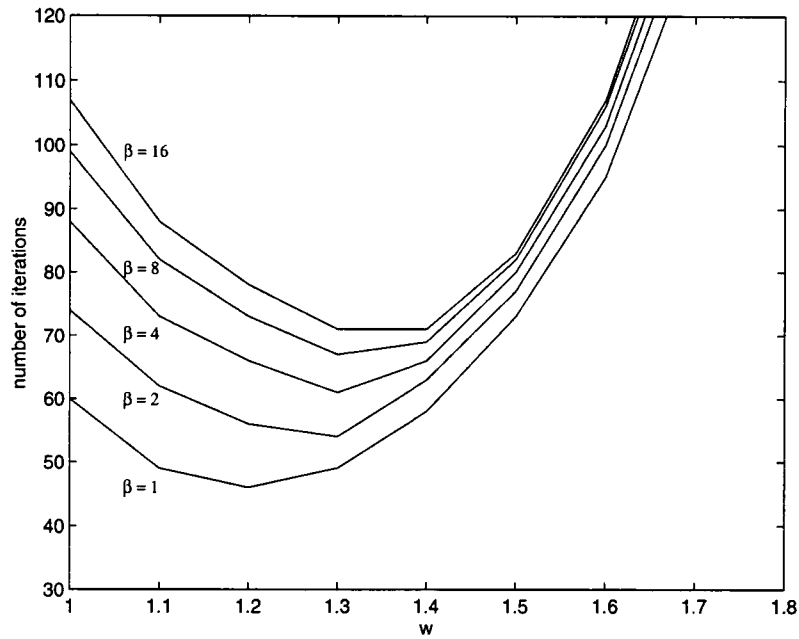
This approximation scheme is convergent for ($0 < \omega < 2$). A formula of relative importance is

$$\underbrace{\|\mathbf{u}^{(n)} - \mathbf{u}^{(\infty)}\|}_{Norm_1} \approx \gamma \underbrace{\|\mathbf{u}^{(n+1)} - \mathbf{u}^{(n)}\|}_{Norm_2}, \quad (4.7)$$

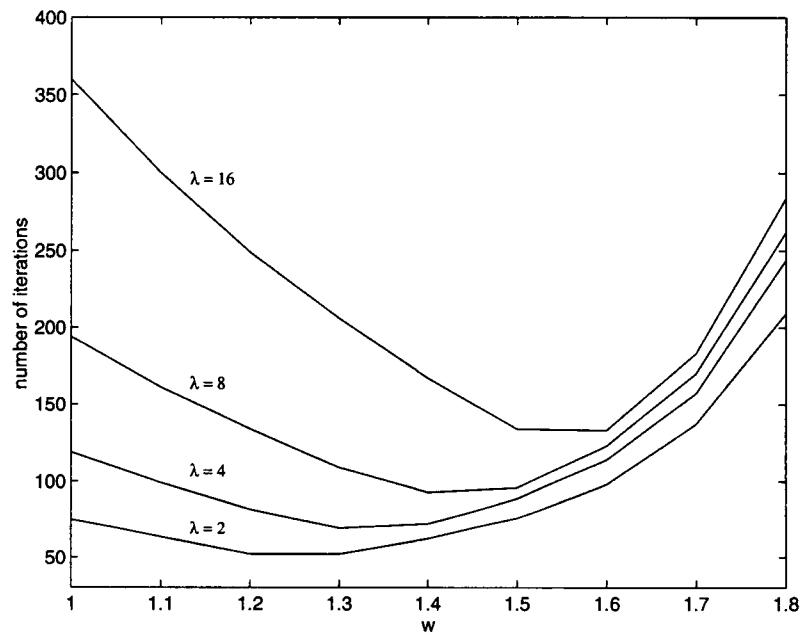
where γ is the decay-time for the algorithm, measured in iterations. The term $Norm_1$ is the absolute norm which measures the accuracy of the current $\mathbf{u}^{(n)}$ to the true solution and $Norm_2$ is the dynamic norm which indicates the progress made towards convergence. The optimal SOR parameter is ideally computed by

$$\omega = 2 \left(1 + \frac{\sqrt{1 + 2\gamma}}{1 + \gamma} \right)^{-1}, \quad (4.8)$$

and test results shown in Figure 4.28 confirm that this optimal value does indeed produce a faster convergence rate. The SOR method has a higher rate of conver-



(a)



(b)

Figure 4.28: Convergence of SOR method in relation to relaxation parameter (ω): (a) $\lambda = 1$ is fixed and β varies from 1, 2, 4, 8, and 16, and (b) $\beta = 1$ is fixed and λ varies 2, 4, 8, and 16.

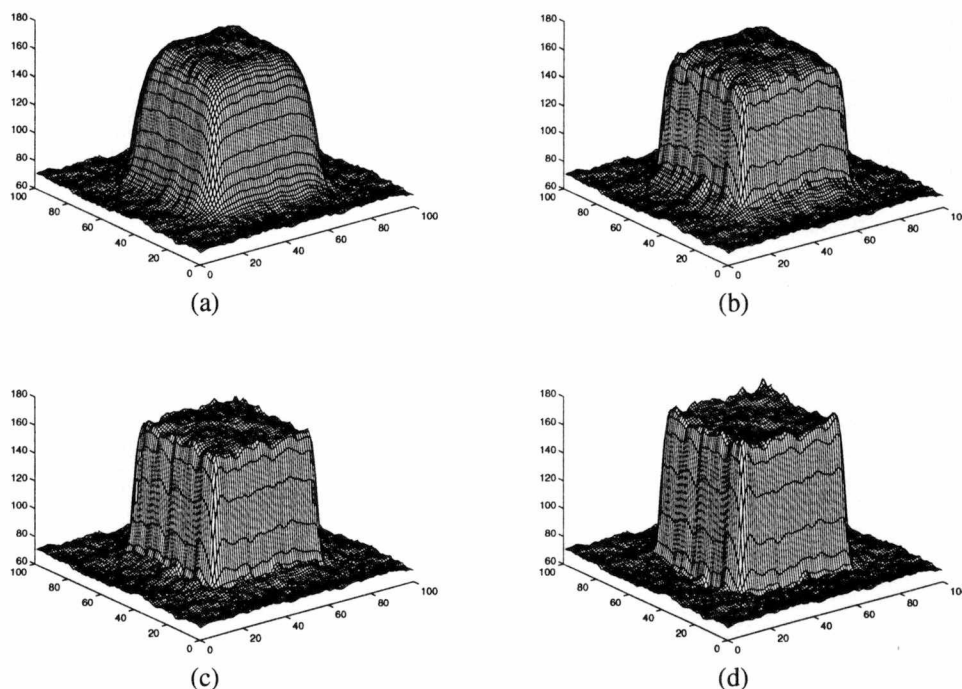


Figure 4.29: *Breakdown of iterations for SOR method: (a) 100 iterations, (b) 195 iterations, (c) 200 iterations, and (d) final surface at 382 iterations.*

gence when compared to other methods due to the fact that new values of $f_{i,j}$ are exploited immediately after they are available instead of using the values obtained in the previous iteration.

The examples we are about to demonstrate were implemented on a Unix-based platform Sun Sparc10 workstation. Our first illustration, Fig. 4.29, examines the computational efficiency of jointly reconstructing two noisy “square” intensity image inputs, each contaminated with zero-mean Gaussian noise with standard deviations of $\sigma_{1n} = 5$ and $\sigma_{2n} = 10$. The initial parameter selection for the

regularized approach is as follows ($\lambda_{high} = 15$, $\lambda_{low} = 10$, $\beta_{high} = 2$ and $\beta_{low} = 0.5$). Figure 4.29(a)–(d) supplies the reconstructed surfaces at different intervals involved in the iterative scheme. The final solution shown in Fig. 4.29(d) was achieved in 33.55 (sec) with 96.9% of the total computation time spent in the subroutine for iteratively solving the regularized response. It should be noted that the iterations could be stopped any time depending on the stopping criteria (absolute error) chosen by the user. For our experiments, $\varepsilon = 0.001$ was a normal stopping criteria.

The next experiment involved fusing two noisy “cake” intensity images, each contaminated with Gaussian noise of standard deviations $\sigma_{1n} = 10$ and $\sigma_{2n} = 15$. Once again, Figs. 4.30(a)–(d), show an interpolated surface at different intervals in the iterative process. This test involved the following initial parameters $\lambda_{high} = 10$, $\lambda_{low} = 5$, $\beta_{high} = 1.2$, and $\beta_{low} = 0.2$. The time required to achieve the final solution given in Fig. 4.30(d) is 13.51 (sec) with the majority of this time (94.8% of the computational time) spent minimizing the solution. It should be noted that minimizing a second-order functional requires more computational time than a first-order functional due to its second derivative nature.

These experiments demonstrate the convergence rate decreases as λ and β increase since larger values of these regularization parameters require the interaction between far apart pixels and propagation of information along this distance requires more iterations. In other words, increasing the regularization parameters enlarges the convolving mask to a wider pixel region.

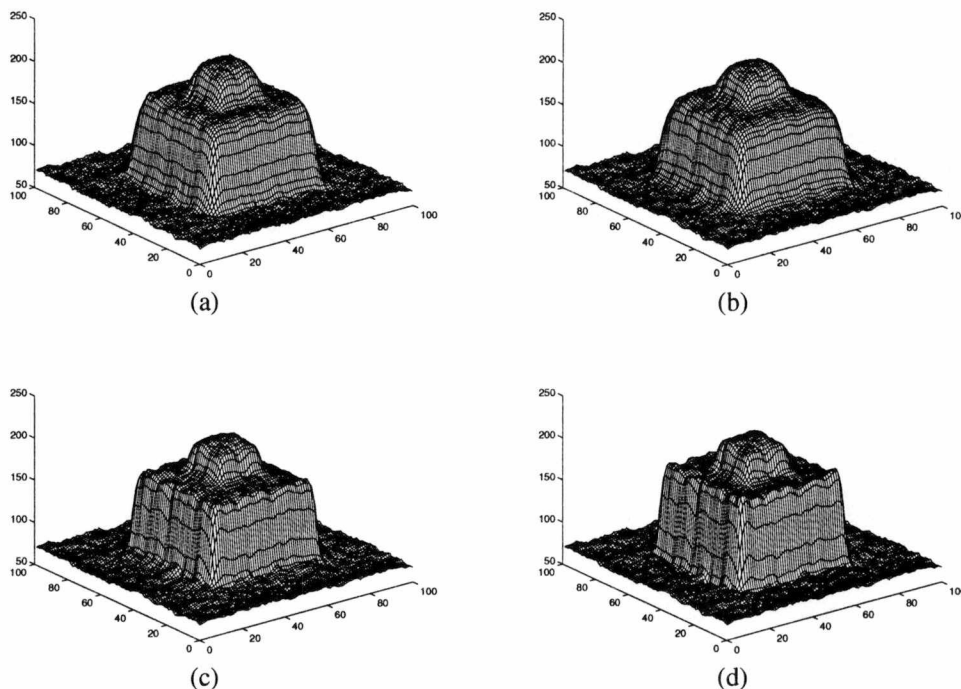


Figure 4.30: *Breakdown of iterations for SOR method: (a) 5 iterations, (b) 50 iterations, (c) 106 iterations, and (d) final surface at 215 iterations.*

4.5 Synopsis

This chapter illustrates results for the hybrid functional when applied to the early vision problems of surface reconstruction and gray-level edge detection. Tests conducted for the surface interpolation problem showed the hybrid's ability to filter noise while still preserving edge characteristics. Each experiment found the first-order stabilizer could oversmooth discontinuities (remove large concentrations of noise) and the second-order stabilizer could deliver an approximated

surface with sharp, well-defined edges and some oscillations. Based on these facts, the hybrid was the superior stabilizer since it combined aspects from both stabilizers. The regularized approach was especially suited for our edge detection scheme since it provided good detection properties. Good detection capabilities were achieved using large regularization values for removing the noise. This method supplied wide edges which were oversmoothed because of the large interaction between pixels. On the other hand, good localization characteristics were achieved using smaller values, because there is less interaction between pixels.

This chapter concluded with an analysis of the computational efficiency of the hybrid energy functional in relation to the regularization parameters (λ) and (β). It was found that the optimum relaxation parameter (ω) greatly influences the final outcome of the iterative SOR method by minimizing the number of iterations required. The computational time required for implementing the hybrid functional depends on the selection of the regularization parameters. A large smoothing parameter requires interaction between distant pixels, resulting in a large convolution (filtering) mask, while a smaller value requires less interaction between neighboring pixels. Therefore a small regularization parameter yields a convolving mask requiring fewer computations than a larger value, which increases the convergence rate.

CHAPTER 5

SUMMARY AND CONCLUSIONS

Restoring stability to ill-posed problems is a challenging task. Regularization offers an alternative for formulating the inverse problem to one that is well-posed. The early vision problems of surface reconstruction and edge detection exhibit mathematically ill-posed properties which must be transformed to well-posed problems by imposing physical constraints such as smoothness.

After introducing standard regularization theory, we encountered a basic problem. The smoothness parameter (λ) was imposed over the entire image space. This constraint disregards edge characteristics by oversmoothing regions not requiring such an operation. The goal is to achieve some type of controlling mechanism in order to vary the smoothing parameter in different situations. For instance, it is desirable to have a large regularization parameter in noisy continuous regions, whereas a small parameter would be sufficient at or near edges to avoid blurring object features or edge boundaries.

A thorough examination of the first- and second-order stabilizers influenced the development of a hybrid stabilizer. This new stabilizer provides characteristics from both first- and second-order stabilizer components. The hybrid's most noticeable features include an interpolated surface which is mostly noise-free and

still preserves discontinuities for edge detection purposes. These properties were accomplished by presenting a controlling operator for implementing the second-order stabilizer at or near discontinuities, and the first-order stabilizer in noisy homogeneous regions. The new regularized hybrid fusion algorithm was extended to allow for multiple inputs and variable (and constant) regularization parameters (λ) and (β). The inputs could be from different sensory devices as long as they were registered and possessed a one-to-one correlation in their respective outputs. A comparison of the different stabilizers revealed that the hybrid energy functional clearly outperformed all by providing a smooth, filtered surface robust to noise with sharp reconstructed surfaces, efficient implementation, and good detection and localization properties for edge detection purposes. Extensive testing on both synthetic and real images was performed for both the edge detection and surface reconstruction problems. Results for the gray-level edge detection shows this new analytical technique is able to provide an output containing information from both the range and intensity data sources which would be unattainable by a single source. In conclusion, multiple sensors allow extraction of multiple properties from a scene, which further enables a robotic system to operate autonomously.

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