A DYNAMIC PROGRAMMING MODEL FOR STRATEGIC MATERIALS

Franklin R. Shupp *

IT becomes immediately apparent in analyzing any national economy that some resources are scarce not only in the economic sense, but also in the more popular interpretation of that word. Because of international trade these shortages frequently impose no great hardship on the "have not" nation; for example, the absence of tea and coffee plantations in the United States does not prevent Americans from consuming prodigious quantities of these beverages. However, in the event of an hostility these resource scarcities can create a serious problem, especially if they happen to include some of the so-called strategic materials. It is with these materials, which are both vital to the country's defense and in short domestic supply, that this paper is concerned.

Some of the more publicized scarce resources which have been regarded as critical by the United States include: tin, uranium, manganese, rubber, quinine, diamonds, and possibly plants manufacturing heavy machinery and precision instruments.[1] The very heterogeneity of this group augurs strongly against finding a single solution or even a unique mode of attack to the problem of how best to assure an adequate supply of these resources under all political conditions. Nevertheless, while the details of any possible solution may vary enormously with the individual material, almost without exception the available policy measures can be fitted into one of the following categories:

1. Importation under all conditions
2. Subsidized domestic production and research
3. Stockpiling
4. Conservation
5. Substitution

These classes of policy measures are generally regarded as alternatives, but only infrequently is one pursued to the exclusion of the others. Generally, a considerable degree of interdependence exists among alternative measures and occasionally even *among* the various strategic materials themselves. Furthermore, within each category a large number of decisions must also be made before a policy can be formulated. For example, should a convoy with destroyer escort be employed or should an airlift be used? What is the optimal size of the stockpile? Do we invest in butene or butydiene rubber or both?

Both the interdependence and the complexity of the strategic materials problem suggest that some tools of operations research might prove useful in its analysis. However, the uncertainty in the political forecasts so dominates the problem that it virtually precludes the use of "linear" programming, especially if a quadratic objective function is also necessary. Nor is the problem amenable to analysis by game theory, since the major objective of any potential aggressor would be to win the war not merely to disrupt the flow of strategic materials.[2] On the other hand, if one is willing to accept a modicum of aggregation, dynamic programming can be applied quite effectively, and it is a model using this technique which is presented here.

* This research was undertaken while the author was associated with the Princeton Econometric Research Program and supported by a contract with the Office of Naval Research Nonr-1858 (02). The author benefited greatly from the criticisms and suggestions of Oskar Morgenstern, Harlan Mills, Stuart Dreyfus, Herman Karreman, and other members of the Program. All errors are the author's responsibility.

[1] These last two items are included because subsidies in the forms of high protective tariffs, quotas, and government contracts at bids substantially in excess of foreign offers have been granted on precisely these grounds, i.e., their strategic importance.

[2] This should not be construed to mean that the potential antagonist may not try to impede critical material flows as part of his over-all objective of winning the battle. On the other hand, if the stockpiled material is a potentially effective deterrent, e.g., hydrogen bombs, the whole complexion of the problem changes. The object then is to prevent an hostility, not to insure access to strategic materials. This new problem is beyond the scope of this paper, and perhaps is not at all amenable to any sort of quantitative analysis since it involves evaluating the cost of a war.

In order to both facilitate the exposition and to give some specificity to the argument a single mineral, manganese, is used to illustrate the model. Manganese qualifies as a strategic material since it is (1) in short domestic supply and (2) is vital in any war effort. With regards to the first criterion it is sufficient to observe that during the last decade despite the stimulus provided by huge government subsidies, U.S. production of commercial grade manganese ore averaged only 10 per cent of the annual consumption of roughly 2,000,000 tons. On the second count we note that manganese is an indispensable ingredient in the steel making process, where it is added in the form of a ferro or silico manganese alloy to impart both hardness and ductility to the finished steel.

As implied earlier there is no strategic materials problem unless some hostility either exists or is expected with some positive probability within the planning horizon. Fortunately, this does not necessitate a detailed forecast of the complex political milieu of the world for the planning period. Only those movements which cause significant changes in the demand for and the nondomestic supply of the strategic material under study need be forecast. In the case of manganese, the demand is a derived one and is contingent on the level of steel production which in turn fluctuates with the prevailing political climate.[3] Three possible political states, peace, cold war, and hot war, are here considered to have significant effects on the level of steel production and therefore on manganese demand. The supply side of the problem can be summarized by observing the existence of only four manganese producing regions. These are: U.S.S.R., India, West Africa, and South America. The accessibility of these ores to the United States in any year yields the supply constraint for manganese. Like the demand function, this is also dependent on the prevailing political situation.

In the next section the political forecast and the five policy measures as they impinge on the manganese situation are outlined in some detail. In the following section a model embracing the main characteristics of the problem is constructed, and in the concluding section results of the model are analyzed.

[3] Although steel production also fluctuates with the general level of business activity, no provision for these fluctuations is made in our model. However, since only sizable discrete variations in demand are permitted in the model, the above restriction is not too severe.

I. The Manganese Problem

As indicated above the supply of and the demand for manganese are related to the prevailing political conditions, and therefore, a forecast of these conditions for the entire planning horizon is essential. The length of this planning horizon has been arbitrarily chosen as ten years. We assume that only three possible levels of hostility, *i*) peace, *ii*) cold war, *iii*) hot war,[4] determine the demand, and four possible sources of manganese ore, *a*) U.S.S.R., India, West Africa, and South America, *b*) all except Russia, *c*) West Africa and South America, *d*) South America, yield the supply constraints. From these twelve demand-supply combinations are obtained; some of these sets are clearly impossible, for example (*i–d*), while others are extremely improbable, for example, (*iii–a*). Eliminating pairs in this manner reduces the number of feasible political states from twelve to five. These five sets are given in Table 1.

TABLE 1. — DEMAND AND SUPPLY CONSTRAINTS FOR MANGANESE ORE
(Net tons manganese metal)

Political States		Demand	Supply Constraints
y_1	*i–a*	800,000	1,600,000
y_2	*ii–b*	1,000,000	1,200,000
y_3	*ii–c*	1,000,000	800,000
y_4	*iii–b*	1,200,000	1,200,000
y_5	*iii–d*	1,200,000	600,000

The forecast of these five states for each year of the planning horizon given by the probability vector $S = \{s_1, s_2, s_3, s_4, s_5\}$ determines both the demand for and nondomestic availability of manganese. The strategic materials problem consists of meeting these demands from current

[4] The hot war assumed here is one of attrition. Other types of war negate the whole concept of strategic materials. The assumption does not imply that an all out hydrogen bomb war is impossible or even unlikely; only that in that event any strategic materials policy for manganese is useless. Paradoxical as it may seem, a quick hydrogen war is quite similar to peace from the strategic materials viewpoint.

nondomestic supplies and from other sources at the minimum cost consistent with the welfare of friendly nations. Before discussing these other sources let us consider some possible methods of introducing the political forecast. Three methods are discussed in this paper and these are somewhat arbitrarily labeled *deterministic*, *Markovian*, and *"tree forecast."* Since we shall be concerned with the varying conclusions resulting from using each of these three techniques, a paragraph describing each seems in order.

All of these forecasts mechanisms can be implemented by introducing a transition matrix Q whose elements are transition probabilities defined as:

$$q_{ij(t)} = P(y_{j(t+1)}|y_{i(t)})$$

i.e., if state y_i prevails in year t, then the probability that state y_j will prevail in year $t+1$ is equal to $q_{ij(t)}$. If these transitions probabilities are time independent, as in our Markovian forecast, then the subscript t is usually dropped. It follows then from the definition that

$$S_{t+1} = QS_t.$$

In the *deterministic* case we assume that the prevailing political states, y_i's, are known with certainty for each of the years in the planning horizon. (This need not imply clairvoyance since this "certainty" can be interpreted as either the most likely political situation or a weighted mean of a probabilistic forecast.) Certainty dictates that in each period only one of the elements of the probability vector S is positive (and equal to 1); this can be achieved for succeeding periods by introducing only one positive element in the transition matrix Q. The position of this unit element will vary over time in order to introduce in proper sequence each of the political states of the forecast. The deterministic forecast used in this study is shown by the solid line in Chart 1.

The transition matrix for the *Markovian case* is time independent and is given by the following matrix which very roughly characterizes the situation of the last fifty years.

$$Q = \begin{vmatrix} .7 & .2 & .1 & 0 & 0 \\ .1 & .6 & .2 & .1 & 0 \\ .1 & .2 & .4 & .1 & .2 \\ .4 & 0 & 0 & .4 & .2 \\ .5 & 0 & 0 & .2 & .3 \end{vmatrix}$$

The matrix is completely ergodic and converges to an S vector, $\{.39, .28, .16, .10, .07\}$. These are the steady state probabilities for the respective political states $\{y_1, y_2, y_3, y_4, y_5\}$. Since it seemed unrealistic to assume that the prevailing political climate was not known with certainty for the initial period, state y_2 was selected as the initial prevailing state for all computations. As a consequence instead of having steady-state properties throughout the planning horizon, the forecast exhibited transient behavior during the first five periods, after which nearly complete convergence obtained.

The *"tree forecast"* is a modified branch analysis similar to that shown in Chart 1, and requires that the transition matrix change from period to period. Furthermore, if the continuity of each projection (branch) is to be preserved, expansion of the matrix is necessary during those three periods when two or more projections meet, e.g., period 3. Alternatively, if continuity is not felt to be important a 5 x 5 transition matrix suffices.[5]

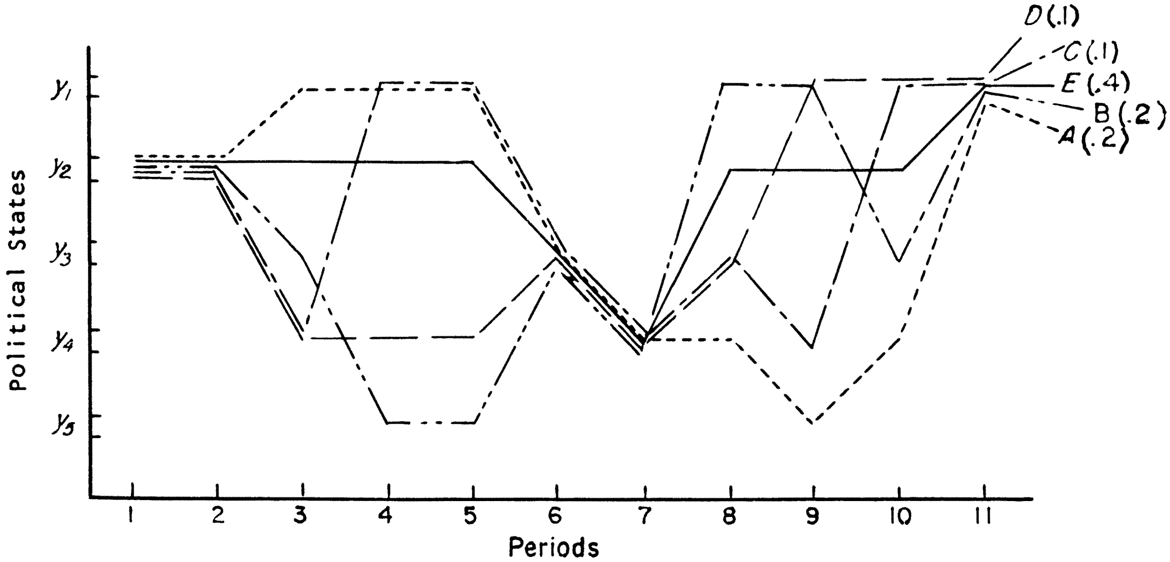
The forecast in Chart 1 has some interesting properties. Projection E not only is the most likely path, but it is also the weighted mean of all five paths.[6] It is this projection that is used for the deterministic forecast. Also over the ten year planning horizon, the average time in each of the five states is approximately equal to that obtained in the Markovian forecast with an initial political situation given by y_2.

We must still explore the available alternative policy measures before we can begin to construct a model. The first of these, which we have called "import under all conditions," is generally the most economical in either peace time or for a short hydrogen war. Under other political conditions it may not only be very expensive, but even impossible since severe limitations may be placed on the quantity of ore than can be imported. For example, it has been estimated that losses due to German submarine activity in World War II increased the

[5] Since a computer of very limited memory capacity was used for these calculations, this latter procedure was followed.

[6] Due to the limited number of possible political states the strange bunching during period 5 and 6 was necessitated if the mean path was also to be the modal path. This is a desirable characteristic for subsequent analysis.

Chart 1. — Possible Political Projections and Their Probabilities



price of manganese ore by tenfold.[7] Also prior to 1950 as much as 50 per cent of all manganese imports came from the U.S.S.R.; today none does. It follows immediately then that both the costs of and the constraints imposed on this alternative are largely a function of the prevailing political climate.

Subsidized domestic production is another possible policy which has been employed with mixed success. A 100 per cent import duty promulgated in 1922 did succeed in increasing domestic production to 17 per cent of consumption in 1925, but only one year later with the protective tariff still intact production reverted to 7 per cent. More recently the government has paid domestic producers two to three times the import price for ores to be stockpiled. Even these guaranteed high prices failed to raise domestic production above 15 per cent of current consumption. At the same time these policies virtually depleted all high grade ore reserves in the United States. Federally financed research projects to develop methods for beneficiating low grade domestic ores have been more successful. Several processes have proven to be technologically feasible in prototype sized plants, but none are competitive with imported ores.

Stockpiling needs little elaboration. The U.S. government has stockpiled approximately 6,000,000 tons of high grade foreign ore (a three-year supply) costing over $250,000,000 and some 1,750,000 tons of assorted quality domestic ores costing close to $100,000,000. The bulk of this stockpile was accumulated in the early 1950's during which time the c.i.f. Eastern Seaboard price increased more than 50 per cent.

A fourth policy to be considered is the substitution of manganese by some other material(s). The addition of manganese alloys in steel manufacturing accounts for approximately 97 per cent of all manganese consumed, and no other known alloy can impart both hardness and ductility to steel and at the same time serve as a scavenger to combine with sulphur. Titanium and molybdenum have been considered but neither do both jobs effectively. Furthermore, the supply of these metals is almost microscopic in relation to the quantities needed for this purpose, and they cost five to ten times as much as manganese alloys. Also during an emergency there exists a tendency to substitute manganese itself for nickel and chrome yielding on balance a zero or possible negative substitution.

The final category, conservation, is primarily a function of changes in steel practices and steel specifications. A thorough investigation by the American Iron and Steel Institute in

[7] C. B. Larson, internal paper, General Services Administration.

1949 indicated that savings of more than 5 per cent in either of these areas could not be realistically anticipated.[8] During the Korean conflict most steel makers implemented a program of increased ladle additions of alloy (cutting back furnace additions) to conserve manganese. This proved effective and the continuance of this program virtually precludes the possibility of further significant reductions. A technique for reclaiming manganese from steel slags has also been developed, but has proven uneconomical. Finally, it is quite unlikely that appreciable changes in steel specifications will be made by the Department of Defense in time of an emergency.

II. The Model

The manganese problem outlined in the preceding section can be incorporated in a dynamic programming model, the objective function of which minimizes the cost of securing adequate quantities of manganese to meet industrial demands. Essential to this model is a recursive relationship which utilizes the Markovian-like property that an optimal decision for a given state of the system is independent of how that state was reached. The state of the system itself can be characterized by one or more state variables, the level of which is usually influenced by activities in preceding periods, but which can be autonomously given. It is this latter type of state variable that is used to introduce the uncertain political forecast into our model.

The functional equation which illustrates both the recursive relationship of the cost function and the technique for incorporating a probabilistic forecast is given by:

$$f_{n-1}(x,y_i) = \underset{p \epsilon P}{Min}\left[R(x,y_i,p) + \sum_{j=1}^{5} q_{ij}\ f_n(x_p',y_j) \right]$$

Assuming that the planning horizon has N periods then:

$f_{n-1}(x,y_i)$ = total expected cost from the beginning of period $n - 1$ to the end of the planning horizon given that the current state of the system is prescribed by the endogenous state variable x (e.g., the level of the stockpile) and the exogenous state variable y_i (e.g., the prevailing political climate).

$R\ (x,y_i,p)$ = the cost in period $n - 1$ of pursuing a particular policy p from a finite set of feasible policies P given the system described by (x,y_i).

x_p' = the resulting state of the system at the beginning of period n given that policy p was employed in period $n - 1$.

The above equation can be solved for all combinations of the set of permissable values of x and y in the period $N - 1$, and then iteratively until the first period of the planning horizon is reached.[9] The necessity of a numerical approach as opposed to the analytic techniques of the Arrow-Harris-Marschak model [10] and its variants, is imposed by the complexity of the manganese problem. More specifically as can be seen in the succeeding model we note that:

1. There is more than one endogenous state variable, x, and furthermore one of these involves both nonrecurring capital investment costs and a time lag. Time lags are normally handled by backlogging orders but this is ruinous in a war situation.

2. The set of feasible policies P is determined by five unrelated constraints which could not be incorporated into a single criterion function.

3. The demand forecast is not visualized as being time invariant. This is partially a consequence of the necessarily short planning horizon.

The complete manganese model can be written in functional equation form as: [11]

$$f_n(x_s,x_r,x_d,y_i) = \text{Min}\Big[\ 1.00z_s + c_{ii}\ (z^c_i + z^s_i) + 21.54z_d + (24.00 + 24.60z_c)\ z_c + 59.14\triangle x_d + \delta \sum_{j=1}^{5} \{q_{ij}\ f_{n+1}(x_s + z^s_i - z_s,\ x_r - z_d,\ x_d + \triangle x_d,\ y_j)\}\Big]$$

[8] American Iron and Steel Institute, *Problems Involved in the Conservation of Manganese* (New York, 1949), 5.

[9] An authoritative discussion of the computational aspects of dynamic programming can be found in R. Bellman [1], 85–90.

[10] Kenneth Arrow, Theodore Harris, and Jacob Marshak "Optimal Inventory Policy," *Econometrica*, 19 (July 1951), 250–272.

[11] All costs are given in millions of dollars and each unit of x or z corresponds to the equivalent of 200,000 net tons of manganese metal.

for $i = (1,2, \ldots 5)$
subject to

$z_s \leqq x_s$
$z_d \leqq \min\ [x_d, x_r]$
$z_i^c + z_i^s \leqq \beta_i$
$z_i^c + z_s + z_d + z_c \geqq \gamma_i$
$c_{ii}\ z_i^s + 59.14\ \triangle x_d \leqq\ 125$[12]

where

$x_s = 0, 1, \ldots 15$ = level of stockpile
$x_r = 0, 1, \ldots 40$ = size of domestic reserves[13]
$x_d = 0, 1, \ldots 4$ = capacity of domestic beneficiating facilities, $\triangle x_d \geqq 0$
y_i for $i = (1, \ldots 5)$ = prevailing political state
$z_i^c = 0, 1 \ldots$ = quantity of manganese imported for consumption
$z_i^s = 0, 1 \ldots$ = quantity of manganese imported for stockpiling
$z_s = 0, 1 \ldots$ = quantity of manganese withdrawn from stockpile
$z_d = 0, 1 \ldots$ = quantity of manganese produced domestically
$z_c = 0, 1 \ldots$ = quantity of manganese deficit (obtained via conservation and substitution techniques)
q_{ij} = transition probabilities
δ = discount factor
β_i = import constraint given state i (Values given in Table 1)
γ_i = demand given state i (Values given in Table 1)
c_{ii} = import cost given the prevailing political situation, y_i
$c_{i1} = 18.65 + 1.51z_i$
$c_{i2} = 17.65 + 1.76z_i$
$c_{i3} = 15.67 + 2.88z_i$
$c_{i4} = 28.09 + 1.76z_i$
$c_{i5} = 37.41 + 9.00z_i$

A few comments on these and other costs may be in order. The import costs have been calculated from time series data of f.o.b. prices for each of the four major producing regions over the period 1947–1957. These data were deflated and a simple regression run to obtain the price elasticity. For states y_4 and y_5 during which an hostility is indicated, a war risk insurance premium is included. Since insurance is generally limited to the cargo (and not the ship) these costs contain a downward bias.

Both the domestic production operating costs (21.54) and capital costs (59.14) are based on an engineering study[14] of the Dean-Leute beneficiation process, which has been developed in a prototype size operation. The cost of ore withdrawn from the stockpile (1.00) is largely a transportation charge estimated by the late J. H. Crickett, while the penalty charges ($24.00 + 24.60z_c$) are estimated costs of conservation and substitution practices. The strongly positive quadratic cost reflects the limited quantities of ores which can be obtained by either of these two methods.

By properly manipulating the transition probability parameters each of the three types of forecasts (deterministic, Markovian, and "tree forecast") can be achieved. Using this model, optimal policies have been computed for each of the three forecast approaches, and the results are indicated in the succeeding section.

III. General Characteristics of the Solution

Using the deterministic forecasting technique, optimal policies for each of the five paths (A, B, C, D, E) of Chart 1 were calculated, and as anticipated substantial differences in the level of activities (z), state variables (x), and costs obtained for the various forecasts. To find a single best policy for all five paths, the payoff matrix given in Table 2 was constructed using the optimal policies for each of these forecasts as strategies and each forecast as a possible event. The events (forecasts) were assigned probabilities equal to those shown in Chart 1.

Of the five possible policies the C optimal policy yielded the minimum expected cost while the B optimal policy would be selected if the maximum criterion were employed. Both of these policies differ widely with respect to the level of domestic processing facilities constructed and/or the size of the stockpile to be accumulated from those values given by the E optimal policy (that of the weighted mean

[12] In the actual computations the first term in this constraint was eliminated. The assumption being that while Congress might limit annual funds for producing high cost beneficiating facilities, they would be freer with funds to purchase stockpiled goods which might later be resold. Obviously the model could be solved as written.

[13] Since in this instance this constraint is never binding it was eliminated from the computations thus reducing the dimensionality of the problem to $400 = (16 \times 5 \times 5)$.

[14] Singmaster and Breyer, *An Appraisal of Processes for Recovery of Manganese From Basic Open Hearth Slags* (New York, 1955).

TABLE 2. — PAYOFF MATRIX FOR FIVE SELECTED STRATEGIES [a]

Strategies	Events and Their Probabilities Forecast *A*	Forecast *B*	Forecast *C*	Forecast *D*	Forecast *E*
A Optimal Policy	1,148.7	1,507.1	1,300.5	1,573.3	1,194.0
B Optimal Policy	1,277.9	1,238.9	1,290.3	1,350.9	1,238.4
C Optimal Policy	1,260.0	1,380.3	1,131.5	1,389.4	1,185.8
D Optimal Policy	1,254.4	1,455.3	1,220.1	1,269.3	1,185.8
E Optimal Policy	1,191.9	1,531.5	1,211.3	1,355.8	1,075.6

[a] Costs are in millions of dollars.

path). These differences are shown in Table 3 and clearly demonstrate the shortcomings of using a weighted mean forecast.[15]

Although the *C* optimal policy yields the minimum expected cost of those five strategies considered, it may not be the best possible policy for the uncertain future given by Chart 1 for two reasons. First, since there are literally thousands of other possible strategies it is quite possible that a number of these would be less costly than the *C* optimal policy. Secondly, since each of the five strategies is an optimal policy for a deterministic forecast, it consists of a single sequence of decisions to be pursued irrespective of the prevailing political climate.[16]

TABLE 3. — COMPARISON OF POLICIES USING DIFFERENT DECISION CRITERIA

Change In State Variable	Weighted Mean Forecast (E optimal)	Best Policies [a] Expected Cost (Bayes) (C optimal)	Maximin (Wald) (B optimal)
Increase in Stockpile Level			
1st year	0	0	0
2nd year	0	400,000	0
Domestic Beneficiating Capacity Built			
1st year	200,000	400,000	400,000
2nd year	0	0	0
Expected Cost	$1,232,000,000	$1,228,000,000	$1,263,000,000

[a] Values are net tons of manganese metal content.

[15] In fact, because the weighted mean forecast is also the most likely forecast, the difference between the expected costs of the *C* and *E* optimal policies is only $4,000,000.

[16] Obvious modifications of these decisions were permitted in our calculations. E. g., if optimal policy *A* calls for importing more ore than forecast *C* requires for consumption, the excess can be transferred to the stockpile. Conversely, if consumption demands exceeds imports and domestic production, stockpile withdrawals are permitted.

But, given the uncertain political forecast which confronts us, we would expect that a more flexible policy (strategy), in which the decision is based on new information acquired as the political future unfolds, would result in a lower expected cost.

This second point forces us to distinguish between an optimal policy under certainty and one under uncertainty. In the former case an optimal policy consists of a single sequence of decisions, while in the latter case it consists of a large number of possible alternative sequences of decisions, which indicate for each period what decisions are to be made once the current prevailing political situation is known. This is shown in Table 4 which indicates the optimal policy for the first two of eleven periods using the Markovian forecast approach and the transition matrix given in Section I.

TABLE 4. — OPTIMAL POLICIES AND EXPECTED COSTS FOR THE MARKOVIAN FORECAST

Period and State	Policy (Activity Level)[a] Imports For Consumption	Domestic Production	Deficit Quantity	Stockpile Withdrawals	Imports For Stockpile	Expansion Of Domestic Capacity	Expected Cost [b]
1–y_2	10	0	0	0	0	4	1123
2–y_1	4	4	0	0	0	0	791
2–y_2	6	4	0	0	0	0	875
2–y_3	6	4	0	0	2	0	924
2–y_4	8	4	0	0	0	0	977
2–y_5	6	4	2	0	0	0	1055

[a] Values are in 100,000 net tons manganese metal contained in ore, and millions of dollars.
[b] Costs are in millions of dollars.

Since the political climate in period 1 is assumed to be known with certainty, a single set of decisions for that period suffices. However, since an uncertain period 2 is postulated, a different set of decisions is indicated for each prevailing state in that period. Table 5 presents the results obtained by using the "tree forecast" mechanism outlined in Section I. It differs from the results of the Markovian technique since two periods are assumed to be known with certainty. Note also that a small

TABLE 5. — OPTIMAL POLICIES AND EXPECTED COSTS FOR THE "TREE FORECAST"

Period and State	Policy (Activity Levels) [a] Imports For Consumption	Domestic Production	Deficit Quantity	Stockpile Withdrawals	Imports For Stockpile	Expansion Of Domestic Capacity	Expected Costs [b]
1–y_2	10	0	0	0	0	4	1162
2–y_2	6	4	0	0	2	0	924.2
3–y_1	2	4	0	2	0	0	704.6
3–y_2	4	4	0	2	0	0	768.3
3–y_3	6	4	0	0	2	4	945.5
3–y_4	6	4	0	2	0	0	863.3

[a] Values are in 100,000 net tons manganese metal contained in ore, and millions of dollars.
[b] Costs are in millions of dollars.

stockpile (200,000 net tons) is accumulated in this second period.

Since the "tree" forecasting technique yields a political forecast almost identical to that given in Chart 1, (and therefore, also to that of our payoff matrix), it is interesting to compare the above costs and policies with those indicated by the Bayes criterion in Table 3. During the first two periods, the two policies are similar except for the smaller stockpile accumulation in the "tree forecast" policy. However, an expected savings of $66,000,000 is obtained by pursuing the more flexible stochastic ("tree") policy.

The numerical approach used in the preceding computations spews out considerable quantities of data similar to that shown in Table 6. Simple decision rules can be easily derived by noting that movement across a row indicates the imputed value of a stockpile and down a column the imputed value of domestic beneficiation capacity.

Suppose, for example, we are in state y_2 with no stockpile and no beneficiating capacity. In this situation the imputed value of a 400,000 net ton stockpile is $1123 − $1063 or $60 (million). Likewise the imputed value of a 400,000 net ton capacity beneficiating plant equals $1123 − $975 or $148 (million). If the marginal cost of acquiring a 400,000 net tons of manganese is less than $60,000,000 a stockpile of this size should be accumulated, and if a plant with 400,000 net ton capacity can be constructed for less than $148,000,000 it should be built.

Using the proper coefficients from our model (c_{i2}) and noting that manganese for stockpiling is purchased *after* ore for consumption, the marginal cost is calculated at $77,540,000. Since this exceeds $60,000,000 this quantity of manganese should not be stockpiled. Likewise, we can calculate the capital cost of constructing a 400,000 net ton capacity plant, which is $2(59.14) =$ $118,280,000. Since this is less than $148,000,000 the plant should be built. (These results are consistent with those given in the first row of Table 4.)

If we always think in terms of a ten year planning horizon, similar decision rules can be derived merely by noting our current location in Table 6 and proceeding as above. A somewhat more complicated method for deriving decision rules must be employed when the results of the "tree forecast" model are used. In this event, the current *period* is just as important as the level of the current state variables, since the transition probabilities are not assumed to be time invariant. Consequently, a new table for each period is necessary.

TABLE 6. — SELECTED COST DATA [a] FROM THE PRINT OUT SHEET OF THE MARKOVIAN FORECAST MODEL (Millions of dollars)

Political Situation	Domestic Capacity (100,000 net tons manganese metal)	Level of Stockpile (100,000 net tons manganese metal as ore) 0	4	8	12	16	20
y_1	0	1012	956	903	853	802	753
	4	877	829	783	740	699	661
	8	832	803	781	768	764	765
y_2	0	1123	1063	1006	949	894	842
	4	975	919	866	817	770	726
	8	900	859	821	793	776	770
y_3	0	1200	1106	1036	972	913	855
	4	1019	950	885	828	777	729
	8	914	864	826	795	777	770
y_4	0	1209	1178	1097	1022	953	890
	4	1091	1010	935	867	804	747
	8	961	894	838	802	781	771
y_5	0	1522	1265	1122	1020	940	873
	4	1178	1036	933	853	789	734
	8	986	884	829	796	777	769

[a] Values are the expected costs of pursuing an optimal policy over the entire ten year planning horizon given the initial state variables indicated by the positon of the cost figure in the table.

The emphasis in this paper has been on the techniques employed and very little has been said concerning policy implications. This is as it should be since all results are based on a hypothetical forecast of the political future. Furthermore, the model did not take into account such factors as start-up troubles, which frequently plague a new operation such as the domestic beneficiation process. Nevertheless, it seems reasonable to infer that the government stockpile of both domestic and foreign ores containing almost 3,500,000 net tons of manganese metal greatly exceeds those levels which could be justified on purely economic grounds, since the largest stockpile indicated by any of our stochastic forecasts is 400,000 net tons and even the most pessimistic deterministic forecast (projection B) calls for a maximum stockpile of 1,200,000 net tons of metal.

REFERENCES

[1] Bellman, R., *Adaptive Control Processes* (Princeton University Press, 1961).

[2] Princeton Econometric Research Program, *Econometric Analysis of the United States Manganese Problem*, Part I, Research Memorandum No. 3 (1958).

[3] Princeton Econometric Research Program, *Econometric Analysis of the United States Manganese Problem*, Part II, Research Memorandum No. 16 (1960).

[4] Shupp, F. R., *Programming As A Guide To National Problems* (Ph.D. Thesis, Princeton University, 1960).