October 5, 1973

To the Graduate Council:

I am submitting herewith a thesis written by Yun-Ching Tsuei entitled "New Development on High Reliability Strapdown Platforms Using TDF Sensors." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

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Major Professor

We have read this thesis
and recommend its acceptance:

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Accepted for the Council:

Milton A. Smith
Vice Chancellor for
Graduate Studies and Research

NEW DEVELOPMENT ON HIGH RELIABILITY STRAPDOWN PLATFORMS USING TDF SENSORS

A Thesis

Presented to

the Graduate Council of

The University of Tennessee

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

---

by

Yun-Ching Tsuei

December 1973

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Appreciation is deeply expressed to the writer's parents, and his wife, for their constant encouragement during the period of the study.

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ABSTRACT

New development in high reliability strapdown navigation platform using two-degree-of-freedom sensors and redundancy concept is presented in this paper. The development is based on the assumption that each axis of the sensor can fail without affecting the remaining axis. The systems reliability is investigated, the optimum redundancy configurations are proposed, a technique of sensor performance management is developed for the proposed systems.

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CHAPTER I

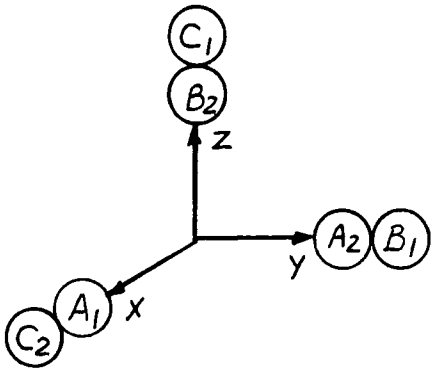
INTRODUCTION

In recent literature [1,2], a technique has been proposed by Hung and Doran to use the concept of sensor redundancy to improve the reliability of a strapdown navigation platform employing two-degree-of-freedom (TDF) sensors. Three important parts involved in the development of the system are: (1) the optimum configuration for sensor arrangement; (2) data reduction; and (3) the sensor performance management method.

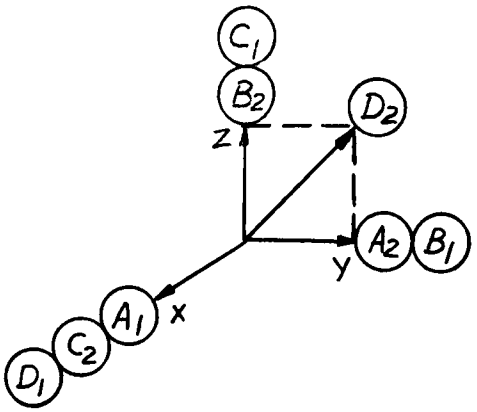
In Hung and Doran's development, it is assumed that when a TDF sensor fails, measurement from both of its output axes are erroneous. As pointed out in their paper, the validity of this assumption is debatable.

The optimum configuration is obtained based on three considerations, namely, the maximum system reliability, the effect of sensor orientation on measurement error due to sensor inaccuracy, and the simplicity of computation. With these considerations in mind, the optimum configurations for systems using three, four, five and six sensors are symbolically shown in Fig. 1.1. In this figure, the circles labeled A1 and A2 on the x and y axes, respectively, represent the directions of two sensitive axes of sensor A. Similar representation is used for all other sensors.

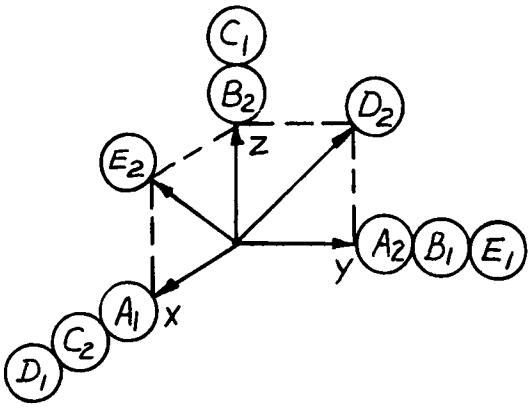
Because of the use of redundant sensors, redundant measurements are available, a data reduction method is needed to derive the desired navigation information from the set of redundant measurement data.



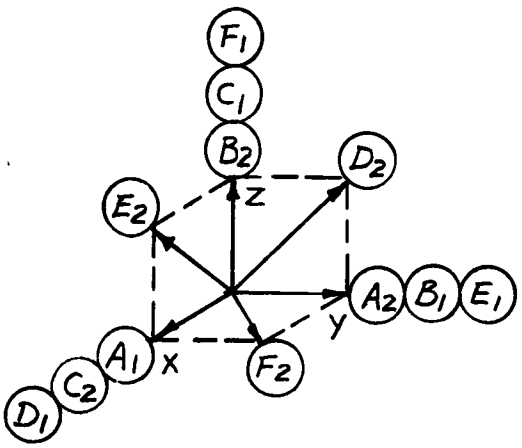
(a) 3-gyro system



(b) 4-gyro system



(c) 5-gyro system



(d) 6-gyro system

Fig. 1.1. Optimum system configuration.

The sensor performance management (SPM) involves four operations which are indispensable to any system utilizing the concept of redundancy. These operations are the failure detection, faulty sensor identification, and system reconfiguration. A fourth operation, sensor recalibration, can also be included to make maximum use of each sensor, therefore further enhance the total system reliability. Figures 1.2 and 1.3 show two SPM schemes. The failure detection is accomplished by the use of a set of parity check equations whose input are the measured data from sensors. The faulty sensor identification is done by using a truth table. The data reduction method used is the well known least-square method. By changing the parameters in the data reduction equations, the system reconfiguration is effected.

This paper proposes a solution to the same problem, but developed under a different assumption. Here, it is assumed that each axis of a TDF sensor can fail without affecting the other axis. Although the basic concepts used in this development are similar to the aforementioned development, the details are different. The optimum system configurations are no longer the ones shown in Fig. 1.1, the parity check equations are not the same, the truth tables are different, and the parameters in the data reduction formula are also different.

In the following chapters the system reliability based on the present assumption will be discussed. Next, the optimum redundancy configurations will be developed for the system using different number of TDF sensors. The associated measurement equations will also be given. Then, several data reduction methods will be presented and discussed. Finally, one concept of sensor performance management will be presented.

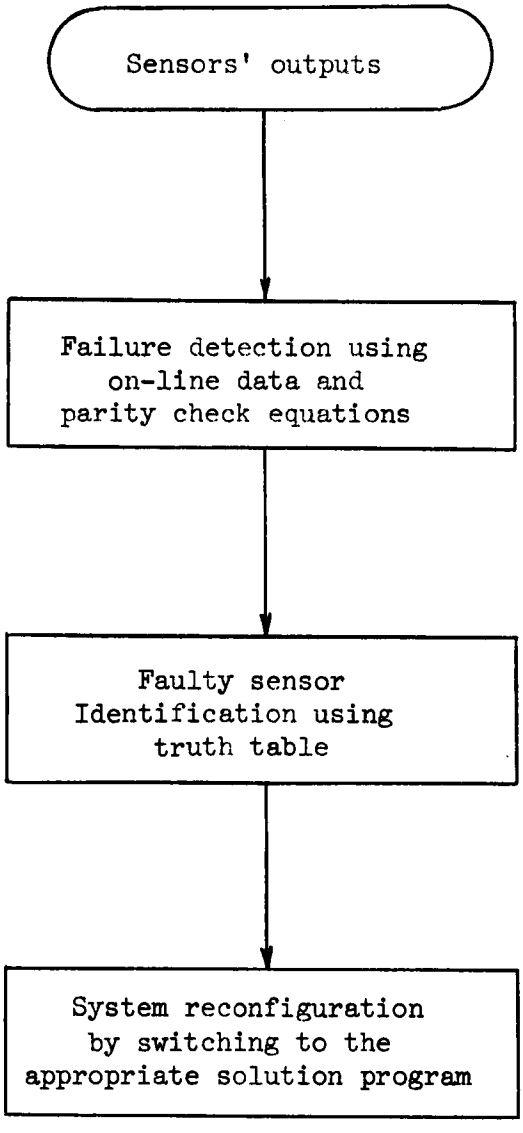


Fig. 1.2. The first SPM scheme.

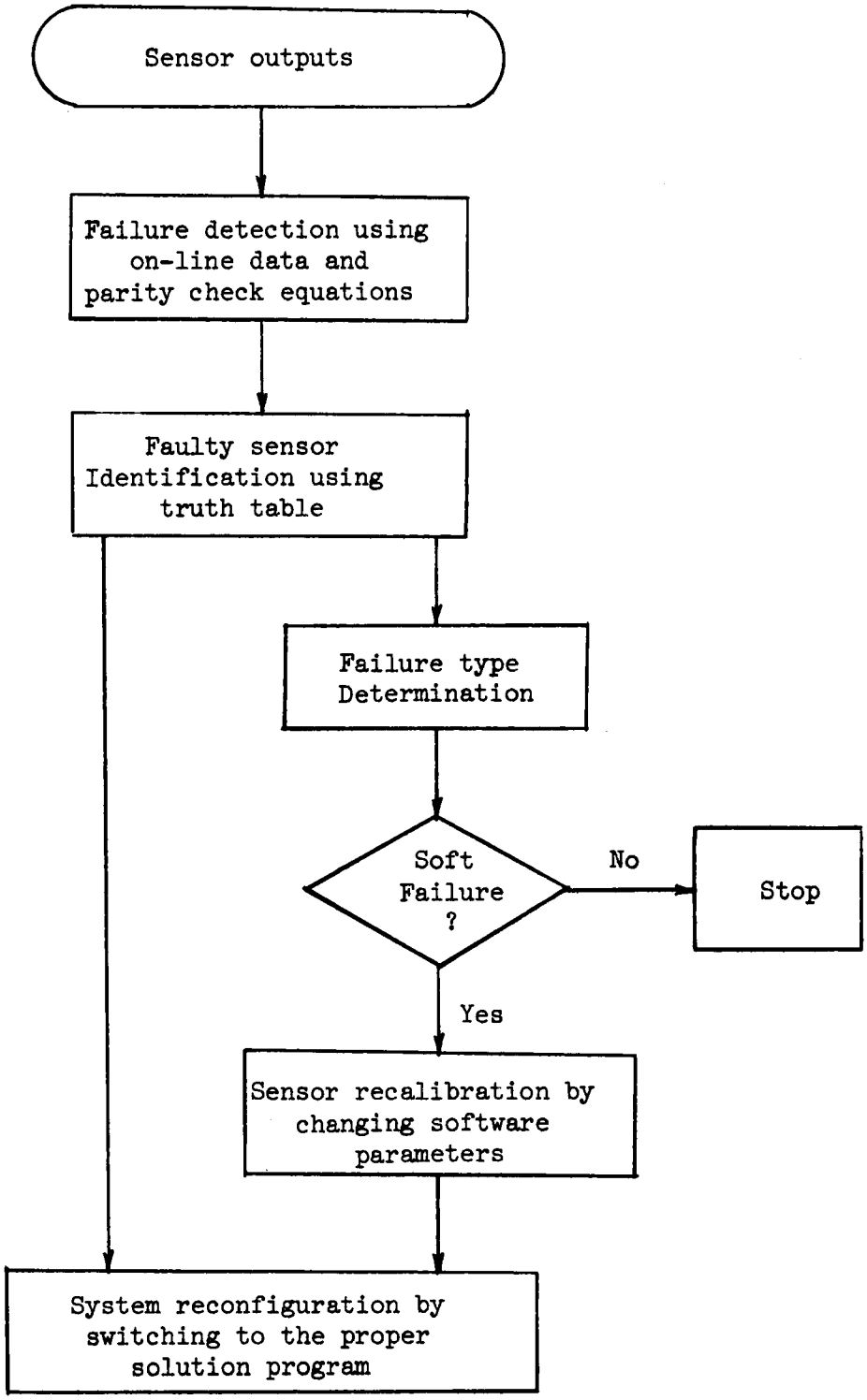


Fig. 1.3. The second SPM scheme.

The concepts will be described in detail and will be used in conjunction with the proposed optimum redundancy configurations. A reference section will be included at the end of the chapter for readers' convenience.

CHAPTER 2

SYSTEM RELIABILITY

Assume that all individual axes of the TDF sensors have equal reliability

$$r = e^{-\lambda t} \tag{2-1}$$

where λ is a positive real number and t represents time. Eq. (2-1) shows that reliability is a monotonically decreasing function of time.

Consider an attitude sensing system formed by n TDF gyros. Under the assumption that each axis of the gyro can fail without affecting the other axis, the n-gyro attitude sensing system will operate satisfactorily if only any 3 out of 2n axes are in good condition. The reliability of such a system is given by

$$\begin{aligned} R_n &= \binom{2n}{2n} r^{2n} + \binom{2n}{2n-1} r^{2n-1}(1-r) + \binom{2n}{2n-2} r^{2n-2}(1-r)^2 \\ &\quad + - - - - - - - + \binom{2n}{3} r^3 (1-r)^{2n-3}. \\ &= 1 - \binom{2n}{2} r^2 (1-r)^{2n-2} - \binom{2n}{1} r(1-r)^{2n-1} - \binom{2n}{0} (1-r)^{2n} \end{aligned} \tag{2-2}$$

This reliability is the same as that of an attitude sensing system using 2n single-degree-of-freedom (SDF) gyros having the same per axis reliability as shown in (2-1).

It is interesting to compare the system reliability obtained under present assumption to that obtained under an early assumption

that both axes of a TDF sensor fail simultaneously[1,2]. Under the latter assumption an attitude sensing system composed of n TDF gyros will operate satisfactorily if any 2 out of n gyros are in good condition. The reliability of such a system is

$$R_n^* = \binom{n}{n} r_1^n + \binom{n}{n-1} r_1^{n-1}(1-r_1) + - - - - - - - + \binom{n}{2} r_1^2 (1-r_1)^{n-2} \tag{2-3}$$

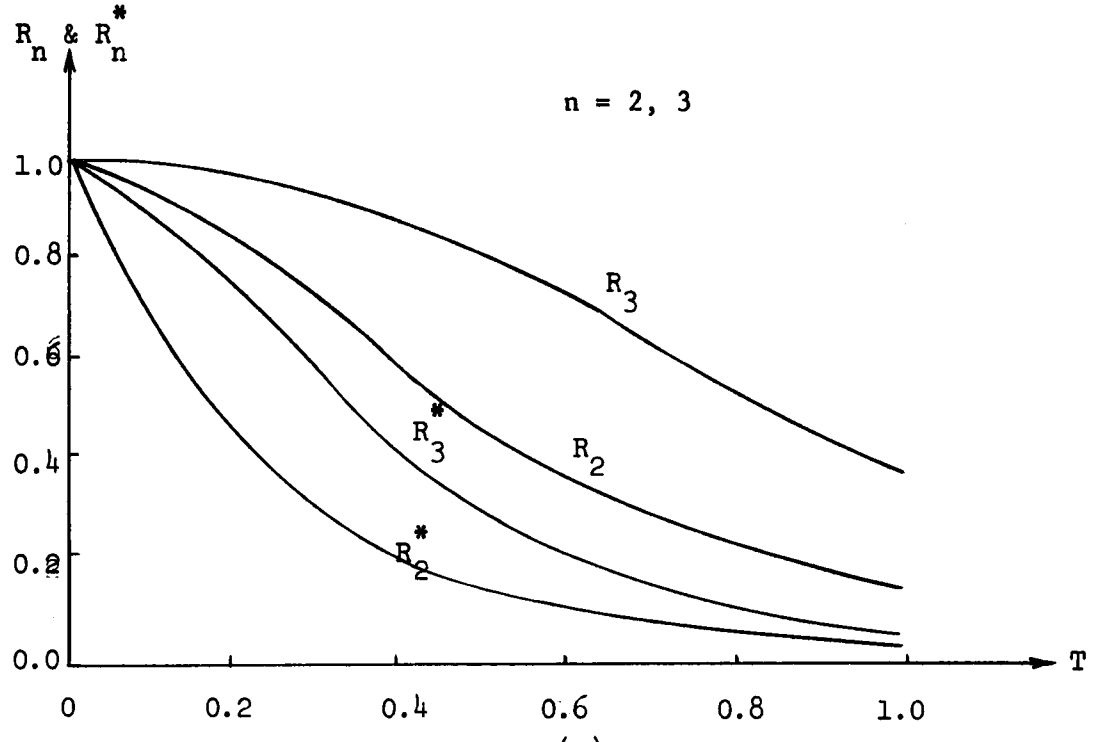
where r_1 is the reliability of each gyro, which is given by

$$r_1 = r^2 \tag{2-4}$$

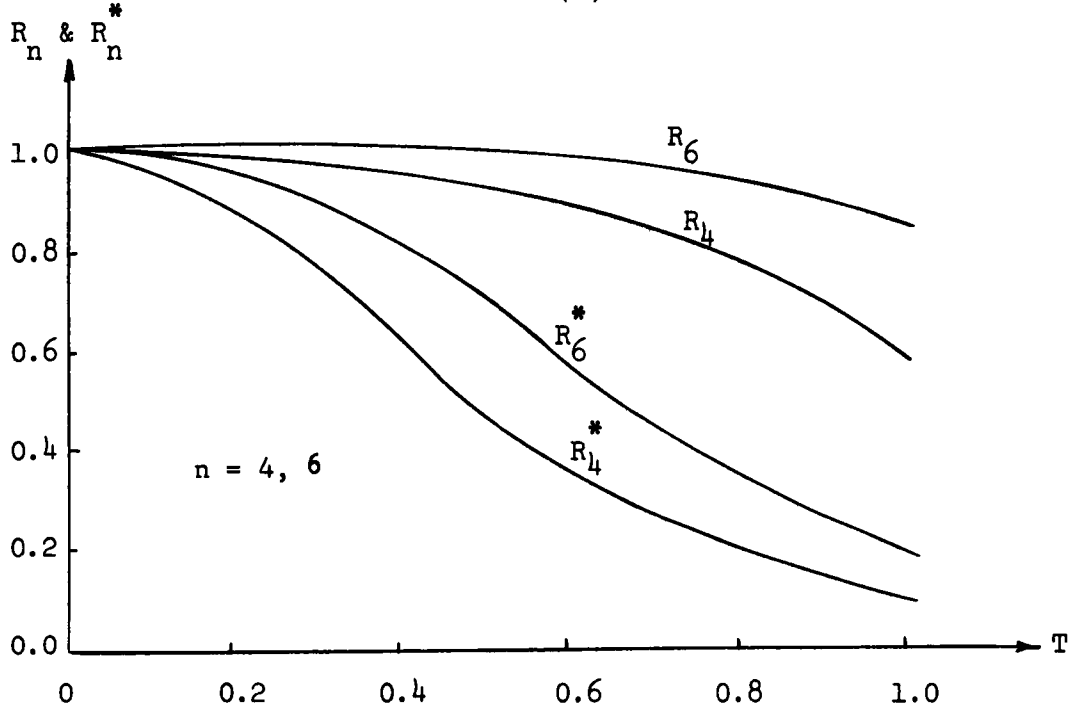
and r is given by (2-1). Substituting (2-4) into (2-3), yields

$$R_n^* = \binom{n}{n} r^{2n} + \binom{n}{n-1} r^{2n-2}(1-r^2) + - - - - - - - + \binom{n}{2} r^4 (1-r^2)^{n-2} \tag{2-5}$$

Fig. 2.1 depicts a plot of R_n and R_n^* for n equals 2, 3, 4, and 6. The associated data are listed in Table 2.1. It is clear that the reliability R_n^* is more conservative.



(a)



(b)

Fig. 2.1. Reliabilities R_n and R_n^*.

Table 2.1. Data for R_n and R_n^*.

n	R_n & R_n^*	λt					
		0	0.2	0.4	0.6	0.8	1.0
2	R_2	1	0.8473	0.5991	0.3890	0.2406	0.1442
	R_2^*	1	0.4493	0.2019	0.0907	0.0408	0.0183
3	R_3	1	0.9881	0.9034	0.7428	0.5571	0.3893
	R_3^*	1	0.7456	0.4243	0.2175	0.1058	0.0500
4	R_4	1	0.9993	0.9814	0.9104	0.7787	0.6141
	R_4^*	1	0.8921	0.6080	0.3504	0.1837	0.0911
6	R_6	1	1.0000	0.9995	0.9919	0.9574	0.8765
	R_6^*	1	0.9831	0.8356	0.5824	0.3493	0.1896

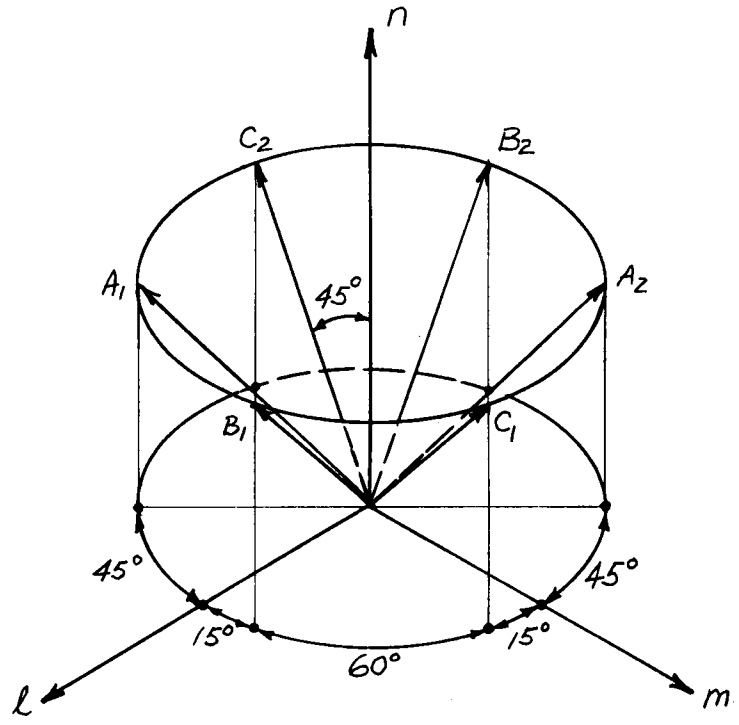
CHAPTER 3

OPTIMUM REDUNDANCY CONFIGURATIONS

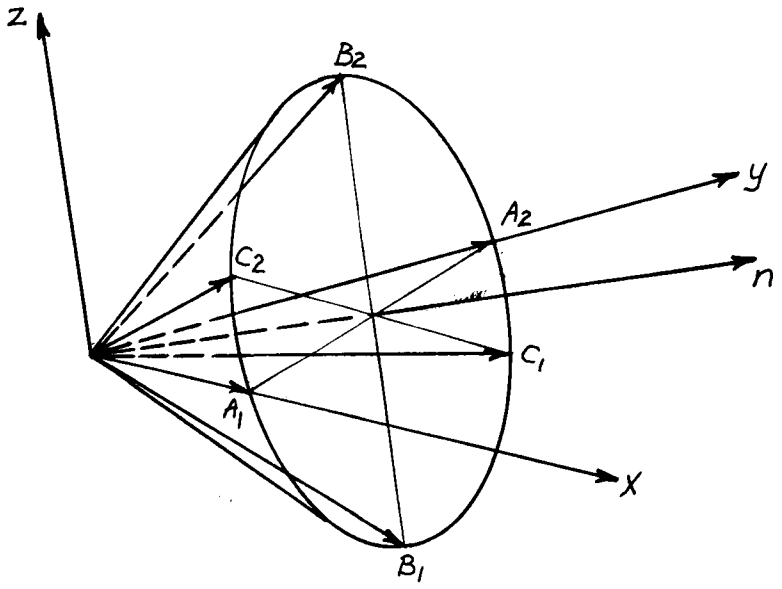
Under the condition that each sensor axis of a TDF sensor can fail without affecting the other axis, we can treat n TDF sensors as 2n SDF sensors except that there is an angular separation constraint of 90° between two axes of each sensor. Based on the considerations of maximum system reliability, the effect of sensor orientation on measurement error due to sensor inaccuracy, and the simplicity of computation, the optimum redundancy configuration is to arrange the 2n sensitive axes of n sensors to lie on the surface of a right circular cone having a cone angle of 90°.

There are several ways the circular cone can be oriented. Case A If the sensed information along one vehicle axis n is more important than that along the other two vehicle axes m and ℓ, it is desirable to line up the axial direction of the cone along the important vehicle axis. Fig. 3.1(a) illustrates this case where the most important vehicle axis n is taken to be the roll axis. Case B On the other hand, if we like to have the least computation requirement, it is desirable to place the two sensitive axes of one of the n sensors along two vehicle axes x and y. Thus the outputs of this sensor give directly the information along two vehicle axes without any computation. Fig. 3.1(b) illustrates this case. For a fixed cone position in space the relationship between (ℓ,m,n) and (x,y,z) coordinates are shown in Fig. 3.2 .

The arrangement of sensor axes for systems composed of 3, 4, 5,



(a) Case A



(b) Case B

Fig. 3.1. Optimum configurations for 3-sensor systems.

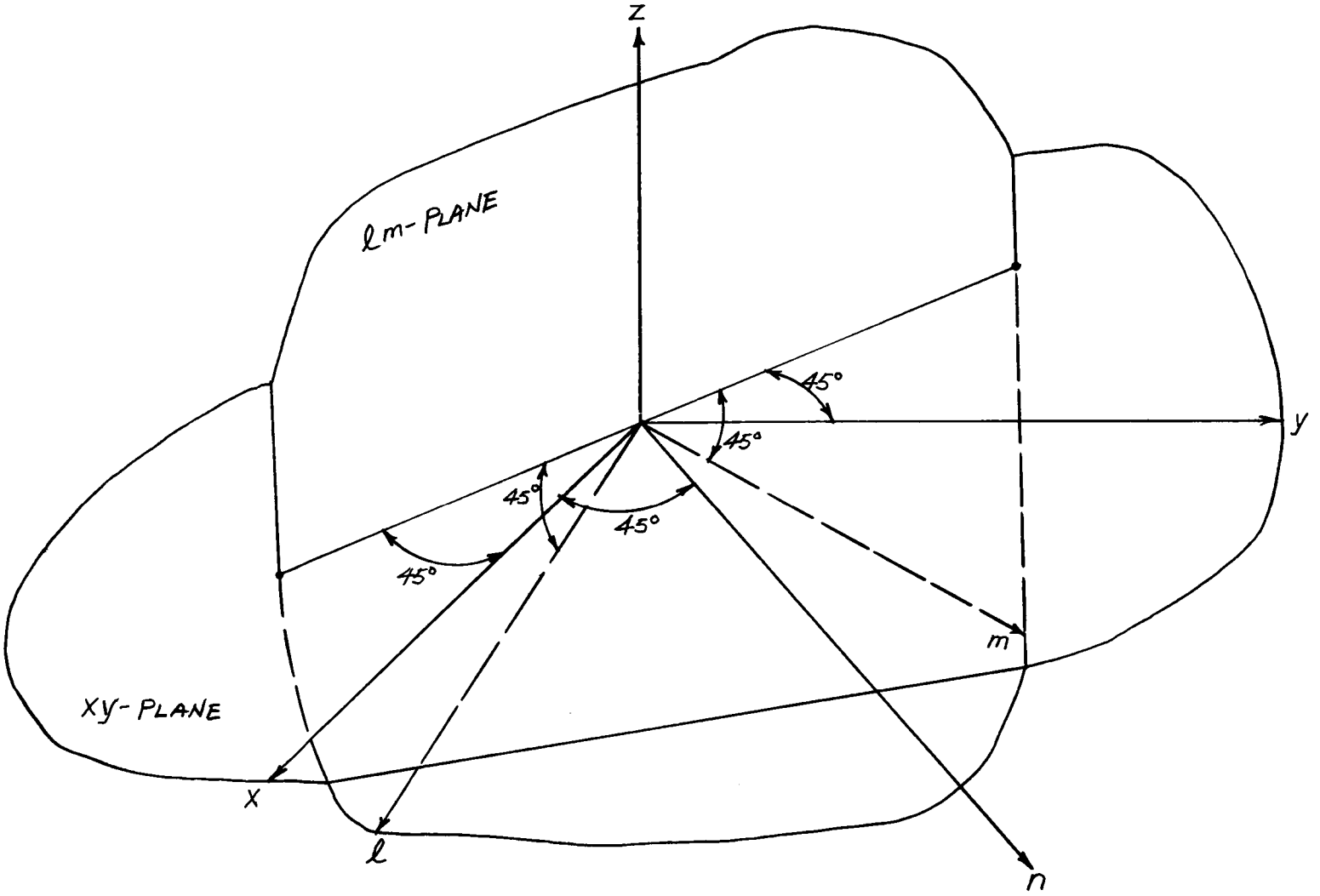


Fig. 3.2. Relationship between (ℓ,m,n) and (x,y,z) coordinates.

and 6 sensors will be given separately in the following together with their respective sensors' measurement equations. Both Cases A and B are presented.

3-Sensor System

For a system composed of 3 TDF sensors, either all gyros or all accelerometers, its Case A and Case B redundancy configurations are shown in Fig. 3.1. In the figure the vectors A_1 and A_2 represent the directions of the two sensitive axis of sensor A. Similar representation is used for sensors B and C.

The measurement equation for Case A can be obtained readily from Fig. 3.1(a), which is

$$\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \cos45\ \sin45 & -\cos45\ \sin45 & \cos45 \\ -\cos45\ \sin45 & \cos45\ \sin45 & \cos45 \\ \cos15\ \sin45 & \cos75\ \sin45 & \cos45 \\ -\cos15\ \sin45 & -\cos75\ \sin45 & \cos45 \\ \cos75\ \sin45 & \cos15\ \sin45 & \cos45 \\ -\cos75\ \sin45 & -\cos15\ \sin45 & \cos45 \end{bmatrix} \begin{bmatrix} \ell \\ m \\ n \end{bmatrix} \qquad (3\text{-}1)$$

where all angles are in degrees. Since

$$\left.\begin{aligned} \sin45 = \cos45 &= \frac{1}{\sqrt{2}} \\ \cos75 = \cos(30 + 45) &= \frac{\sqrt{6} - \sqrt{2}}{4} \\ \cos15 = \cos(45 - 30) &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}\right\}, \qquad (3\text{-}2)$$

(3-1) becomes

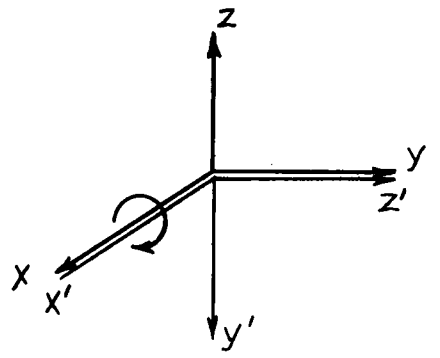
$$\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ C_1 \\ C_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ \frac{\sqrt{6}+\sqrt{2}}{4} & \frac{\sqrt{6}-\sqrt{2}}{4} & 1 \\ -\frac{\sqrt{6}+\sqrt{2}}{4} & -\frac{\sqrt{6}-\sqrt{2}}{4} & 1 \\ \frac{\sqrt{6}-\sqrt{2}}{4} & \frac{\sqrt{6}+\sqrt{2}}{4} & 1 \\ -\frac{\sqrt{6}-\sqrt{2}}{4} & -\frac{\sqrt{6}+\sqrt{2}}{4} & 1 \end{bmatrix} \begin{bmatrix} \ell \\ m \\ n \end{bmatrix} \qquad (3\text{-}3)$$

The measurement equation for Case B can be obtained from that for Case A by applying a linear transformation to (3-3). Let us establish the transformation between (ℓ,m,n) and (x,y,z) coordinate systems by successive Euler angle rotations as shown in Fig. 3.3. First, rotate the (x,y,z) about x axis clockwise by 90 degrees to give

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-90) & \sin(-90) \\ 0 & -\sin(-90) & \cos(-90) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad (3\text{-}4)$$

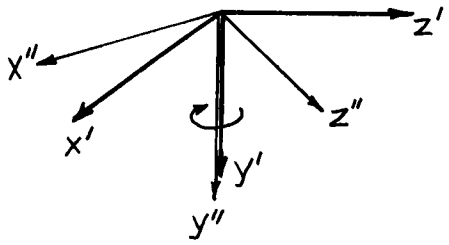
Next, rotate (x',y',z') about y' by 45 degrees counter-clockwise to give

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} \cos 45 & 0 & -\sin 45 \\ 0 & 1 & 0 \\ \sin 45 & 0 & \cos 45 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}. \qquad (3\text{-}5)$$



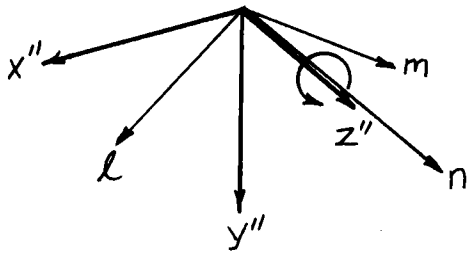
CW rotation about x.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 \\ 0 & +\sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



CCW rotation about y'.

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} \cos 45 & 0 & -\sin 45 \\ 0 & 1 & 0 \\ \sin 45 & 0 & \cos 45 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$



CCW rotation about z".

$$\begin{bmatrix} \ell \\ m \\ n \end{bmatrix} = \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}$$

Fig. 3.3. Transformation from (x,y,z) to (ℓ,m,n) coordinates.

Finally, rotate (x",y",z") about z" by 45° counter-clockwise to yield

$$\begin{bmatrix} \ell \\ m \\ n \end{bmatrix} = \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} . \qquad (3\text{-}6)$$

Combining the above three equations results in the transformation equation from (x,y,z) to (ℓ,m,n) coordinate system.

$$\begin{bmatrix} \ell \\ m \\ n \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad (3\text{-}7)$$

The measurement equation for Case B is obtained by substituting (3-7) into (3-3), giving

$$\underbrace{\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ C_1 \\ C_2 \end{bmatrix}}_{\underline{m}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{3}{4} & \frac{1}{4} & -\frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{3}{4} & -\frac{\sqrt{6}}{4} \\ \frac{3}{4} & \frac{1}{4} & \frac{\sqrt{6}}{4} \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\underline{r}} \qquad (3\text{-}8)$$

In general, measurement equations such as (3-3) and (3-8) are all in the form of.

$$\underline{m} = R\underline{r}. \tag{3-9}$$

where $\underline{m}$ and $\underline{r}$ are vectors and R is the measurement matrix. In the case of (3-8)

$$\underline{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ C_1 \\ C_2 \end{bmatrix}, \qquad \underline{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{3-10}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{3}{4} & \frac{1}{4} & -\frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{3}{4} & -\frac{\sqrt{6}}{4} \\ \frac{3}{4} & \frac{1}{4} & \frac{\sqrt{6}}{4} \end{bmatrix} \tag{3-11}$$

If the sensors considered are three TDF gyros, the measurement equations are obtained from (3-3) and (3-8) by replacing vectors

$$\begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ C_1 \\ C_2 \end{bmatrix}, \quad \begin{bmatrix} \ell \\ m \\ n \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

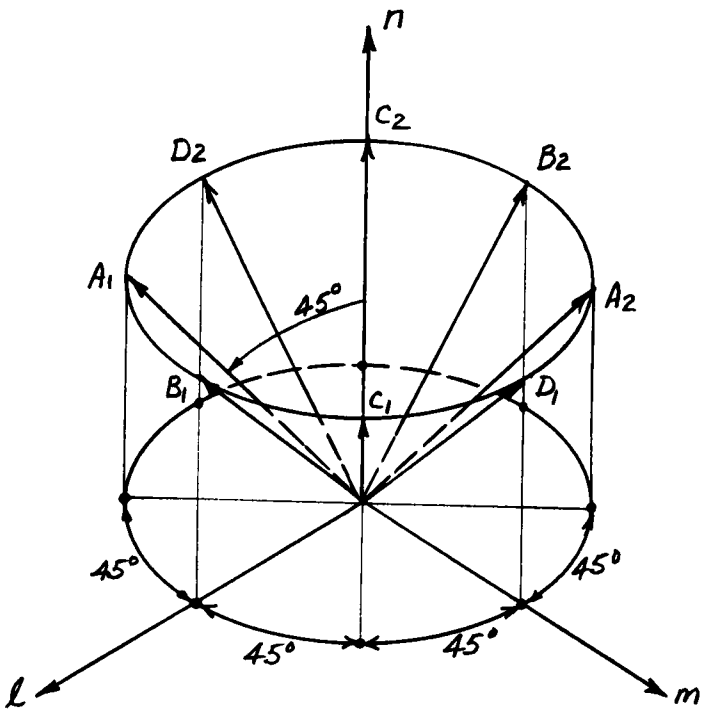
respectively by vectors

$$\begin{bmatrix} \omega_{A1} \\ \omega_{A2} \\ \omega_{B1} \\ \omega_{B2} \\ \omega_{C1} \\ \omega_{C2} \end{bmatrix}, \quad \begin{bmatrix} \omega_c \\ \omega_m \\ \omega_n \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix},$$

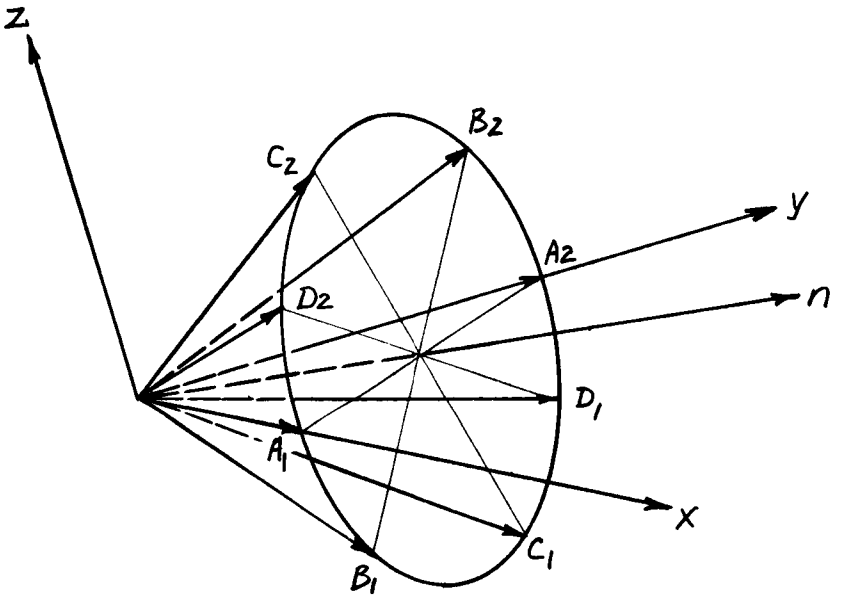
where ω represents the angular velocity. If the sensors are accelerometers, the corresponding notation is α rather than ω, and the same subscripts are used. α represents the acceleration.

4-sensor, 5-sensor, and 6-sensor Systems

The redundancy configuration for a system employing 4, 5, and 6 TDF sensors of the same kind are shown, respectively, in Figures 3.4, 3.5, and 3.6. The associated R matrices for the measurement equations are shown in pages 23, 24, and 25.

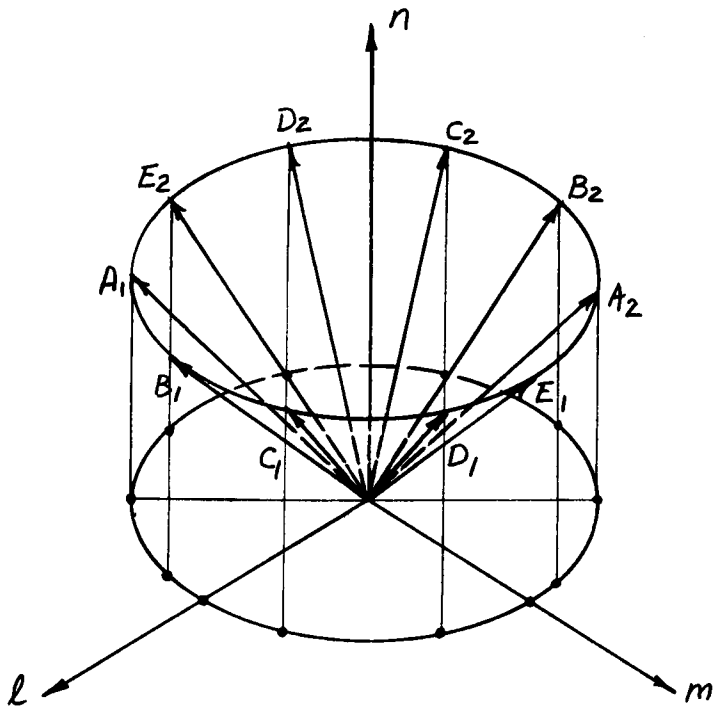


(a) Case A

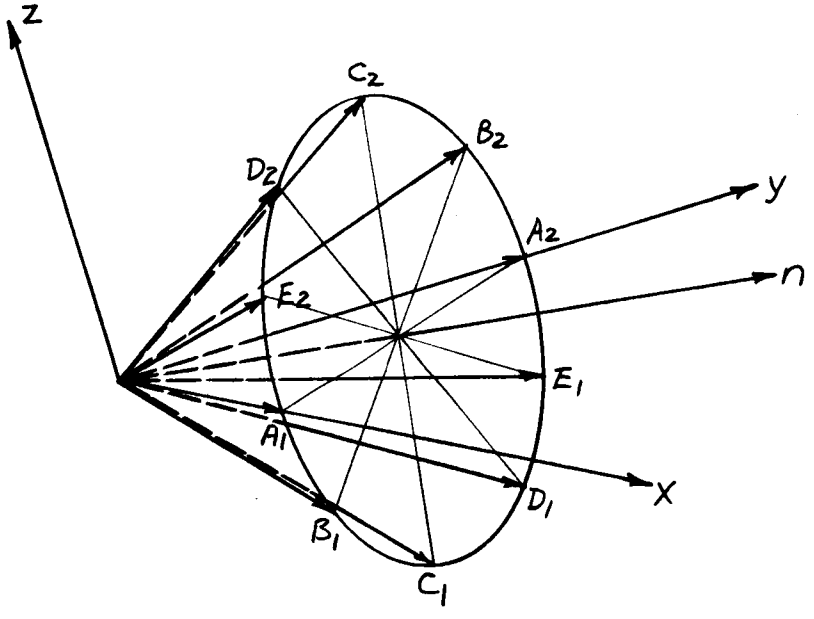


(b) Case B

Fig. 3.4. Optimum configurations for 4-sensor systems.

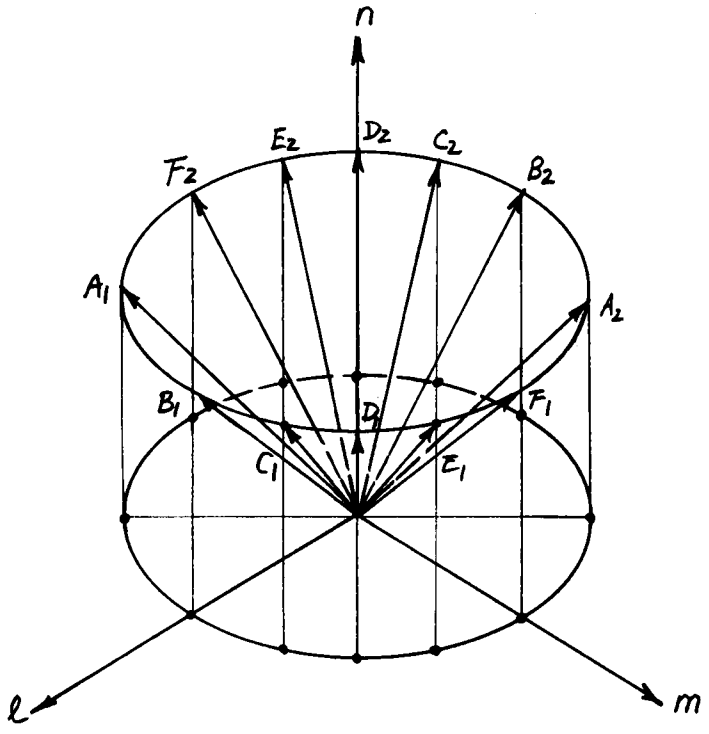


(a) Case A

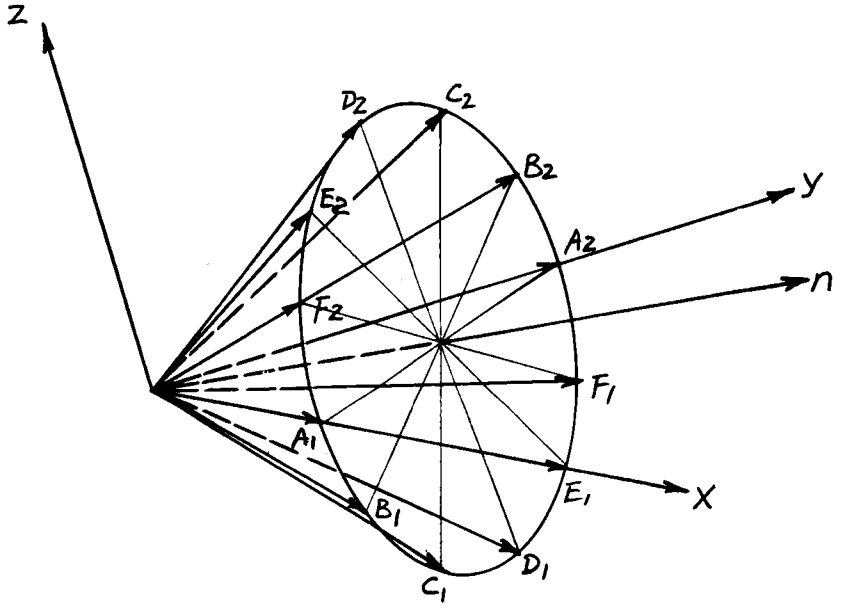


(b) Case B

Fig. 3.5. Optimum configurations for 5-sensor systems.



(a) Case A



(b) Case B

Fig. 3.6. Optimum configurations for 6-sensor system.

R Matrices for 4-sensor Systems

Case A:

$$R = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Case B:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{2+\sqrt{2}}{4} & \frac{2-\sqrt{2}}{4} & -\frac{1}{2} \\ \frac{2-\sqrt{2}}{4} & \frac{2+\sqrt{2}}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{2-\sqrt{2}}{4} & \frac{2+\sqrt{2}}{4} & -\frac{1}{2} \\ \frac{2+\sqrt{2}}{4} & \frac{2-\sqrt{2}}{4} & \frac{1}{2} \end{bmatrix}$$

R Matrices for 5-sensor Systems

Case A:

$$R = \begin{bmatrix} 0.5000 & -0.5000 & 0.7071 \\ -0.5000 & 0.5000 & 0.7071 \\ 0.6984 & -0.1106 & 0.7071 \\ -0.6984 & 0.1106 & 0.7071 \\ 0.6300 & -0.3210 & 0.7071 \\ -0.6300 & -0.3210 & 0.7071 \\ 0.3200 & 0.6300 & 0.7071 \\ -0.3200 & -0.6300 & 0.7071 \\ -0.1106 & 0.6984 & 0.7071 \\ 0.1106 & -0.6984 & 0.7071 \end{bmatrix}$$

Case B:

$$R = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \\ 0.9045 & 0.0955 & -0.4156 \\ 0.0955 & 0.9045 & 0.4156 \\ 0.6545 & 0.3455 & -0.6725 \\ 0.3455 & 0.6545 & 0.6725 \\ 0.3455 & 0.6545 & -0.6725 \\ 0.6545 & 0.3455 & 0.6725 \\ 0.0955 & 0.9045 & -0.4156 \\ 0.9045 & 0.0955 & 0.4156 \end{bmatrix}$$

R Matrices for 6-sensor Systems

Case A:

$$R = \begin{bmatrix} 0.5000 & -0.5000 & 0.7071 \\ -0.5000 & 0.5000 & 0.7071 \\ 0.6830 & -0.1830 & 0.7071 \\ -0.6830 & 0.1830 & 0.7071 \\ 0.6830 & 0.1830 & 0.7071 \\ -0.6830 & -0.1830 & 0.7071 \\ 0.5000 & 0.5000 & 0.7071 \\ -0.5000 & -0.5000 & 0.7071 \\ 0.1830 & 0.6830 & 0.7071 \\ -0.1830 & -0.6830 & 0.7071 \\ -0.1830 & 0.6830 & 0.7071 \\ 0.1830 & -0.6830 & 0.7071 \end{bmatrix}$$

Case B:

$$R = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \\ 0.9330 & 0.0670 & -0.3536 \\ 0.0670 & 0.9330 & 0.3536 \\ 0.7500 & 0.2500 & -0.6123 \\ 0.2500 & 0.7500 & 0.6123 \\ 0.5000 & 0.5000 & -0.7071 \\ 0.5000 & 0.5000 & 0.7071 \\ 0.2500 & 0.7500 & -0.6123 \\ 0.7500 & 0.2500 & 0.6123 \\ 0.0670 & 0.9330 & -0.3536 \\ 0.9330 & 0.0670 & 0.3536 \end{bmatrix}$$

CHAPTER 4

SENSOR PERFORMANCE MANAGEMENT TECHNIQUE

A high reliability navigation system employing a redundancy concept must be capable of "failure detection", "faulty sensor axis identification", "system reconfiguration", and "sensor recalibration". All of these functions can be handled by a centralized control called "sensor performance management", abbreviated SPM. This management task can be performed by a computer software[2].

When the system consists of n TDF sensors of the same kind and when each axis of a sensor can fail without affecting the remaining axis the total number of possible states for this n-sensor system is 2^{2n}. Since at least four good axes are needed to identify other bad axes and since the number of states having x good axes is $\binom{2n}{x}$, the number of states which allow faulty sensor identification is therefore $2^{2n} - \binom{2n}{0} - \binom{2n}{1} - \binom{2n}{3}$.

In this chapter, details of the SPM are developed for an attitude sensing system containing three TDF sensors. Systems having other number of sensors can be developed in like manner. The development is described in three separate parts, namely, data reduction, failure detection and faulty axis identification, and system reconfiguration.

DATA REDUCTION

In general a measurement equation has the form

$$\underline{m} = R\underline{b} + \underline{e} \qquad (4\text{-}1)$$

where R is the measurement matrix, $\underline{m}$ is a vector representing the measurements, $\underline{b}$ is a vector representing the desired navigation information, and $\underline{e}$ is a vector representing measurement noise. For the systems considered, the dimension of $\underline{m}$ is greater than that of $\underline{b}$. For an attitude sensing system, $\underline{b}$ is the attitude rate $\underline{\omega}$; for a linear motion sensing system, $\underline{b}$ is the acceleration $\underline{a}$. The purpose of data reduction is to obtain the best estimate of $\underline{b}$ from the measurement $\underline{m}$. It is assumed that the expected value of the measurement error is zero.

Two data reduction algorithms will be presented. They are the least-square algorithm and weighted least-square algorithm. Each of them will be discussed separately in the following.

Least-Square Algorithm

When the statistics of the measurement error $\underline{e}$ is not known, a least-square algorithm is probably the best one to use. Under this condition we are dealing with the equation

$$\underline{m} = R\hat{\underline{b}}, \tag{4-2}$$

which comes from (4-1) by omitting $\underline{e}$. Since R has more rows than columns, no simple $\underline{b}$ can be found to satisfy (4-2). Any selected $\underline{b}$ will result in a solution error

$$\underline{\varepsilon} = R\hat{\underline{b}} - \underline{m} \tag{4-3}$$

The least-square criterion is to choose the $\underline{b}$ which minimizes the scalar quantity

$$I = \underline{\varepsilon}^T\underline{\varepsilon} = (R\hat{\underline{b}} - \underline{m})^T(R\hat{\underline{b}} - \underline{m}). \tag{4-4}$$

Taking the gradient of (4-4) with respect to $\underline{b}$, setting the result to zero, and solving for $\underline{b}$, gives

$$\hat{\underline{b}} = (R^T R)^{-1} R^T \underline{m} \tag{4-5}$$

Eq. (4-5) is the least-square data reduction algorithm.

The error of the estimate is

$$\begin{aligned} \hat{\underline{b}} - \underline{b} &= (R^T R)^{-1} R^T \underline{m} - \underline{b} \\ &= (R^T R)^{-1} R^T \underline{e} \end{aligned} \tag{4-6}$$

The expected value of this error is zero due to the zero-mean nature of the measurement error $\underline{e}$. Therefore $\hat{\underline{b}}$ is an unbiased estimate of $\underline{b}$. The covariance matrix of the estimation error is

$$P = \left\langle (\hat{\underline{b}}-\underline{b})(\hat{\underline{b}}-\underline{b})^T \right\rangle = (R^T R)^{-1} R^T \left\langle \underline{e}\, \underline{e}^T \right\rangle R (R^T R)^{-1}.$$

Denote the covariance matrix of the measurement error by

$$\left\langle \underline{e}\, \underline{e}^T \right\rangle = \Lambda,$$

then

$$P = (R^T R)^{-1} R^T \Lambda R (R^T R)^{-1} \tag{4-7}$$

Weighted-Least-Square Algorithm

In the weighted-least-square method, the value of $\underline{b}$ is chosen to minimize a scalar quantity

$$\underline{\varepsilon}^T Q \underline{\varepsilon} = -(R\hat{\underline{b}}-\underline{m})^T Q (\underline{m}-R\hat{\underline{b}}) \tag{4-8}$$

where Q is a weighting matrix which is symmetric and positive definite.

Setting the gradient of (4-8) to zero and solving for $\underline{b}$, gives the estimate

$$\hat{\underline{b}} = (R^T Q R)^{-1} R^T Q \underline{m} \tag{4-9}$$

The error of the estimate in this case is

$$\begin{aligned} \hat{\underline{b}} - \underline{b} &= (R^T Q R)^{-1} R^T Q \underline{m} - \underline{b} \\ &= (R^T Q R)^{-1} R^T Q \underline{e} \ . \end{aligned} \tag{4-10}$$

Since $\langle e \rangle = 0$, therefore $\hat{\underline{b}}$ is again an unbiased estimate. The covariance matrix of the estimation error is

$$\begin{aligned} P_Q &= \left\langle (\hat{\underline{b}} - \underline{b})(\hat{\underline{b}} - \underline{b})^T \right\rangle \\ &= (R^T Q R)^{-1} R^T Q \langle e \ e^T \rangle Q R (R^T Q R)^{-1} \end{aligned} \tag{4-11}$$

It is well known[5,6] that the error covariance expressed by (4-11) is the least when the weighting matrix Q is taken to be the inverse of the covariance matrix of the measurement error. That is,

$$Q = \Lambda^{-1} \tag{4-12}$$

Substituting (4-12) into (4-9) gives the weighted-least-square algorithm

$$\hat{\underline{b}} = (R^T \Lambda^{-1} R)^{-1} R^T \Lambda^{-1} \underline{m}. \tag{4-13}$$

Substituting (4-12) into (4-11), gives

$$P_\Lambda = (R^T \Lambda^{-1} R)^{-1} \tag{4-14}$$

Note that the least-square method is a special case of the weighted-least-square method by letting Q = I, the unity matrix. Thus

$$P_\Lambda \leq P \tag{4-15}$$

which means that the difference $P - P_\Lambda$ is a positive semi-definite matrix.

DETECTION AND IDENTIFICATION

The function of failure detection is to find out if a failure has occurred somewhere in the system while the function of faulty axis identification is to identify the faulty sensor axis. For a system consists of three TDF sensors there is a total of $2^{2n} = 2^6 = 64$ possible states, including the cases when all axes are good, one axis is bad, two axes are bad, etc.

Parity Check Equations

"Parity check equations" are used to perform this task. Since any 3 axes can span the entire 3-dimensional real space any additional axes are redundant. In other words, any 4 vector measurements can be linearly combined to give a resultant of zero when these 4 sensors are all good. Let each parity check equation be a linear combination of measurements from 4 axes, which equals zero under normal condition. For a system having 6 axes, $\binom{6}{4} = 15$ such equations can be found. Occurrence of any 2 bad axes will make all equations non-zero except the one equation which does not contain the measurements of the 2 bad axes. This latter equation remains zero. Thus the 2 bad axes can be identified. Since each axis is involved in $\binom{5}{3} = 10$ parity check

equations, 10 of the 15 equations will be non-zero if only 1 axis goes bad. By checking which 10 of them are non-zero the single bad axis can be identified.

Now we proceed to generate a set of 15 parity check equations. The fact that 15 linear combinations of 6 measurements, each contains 4 measurements, are made zero can be represented by the relationship

$$C\underline{m} = CR\underline{\omega} = 0 \tag{4-16}$$

where C is a 15 x 6 matrix,

$$\underline{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ C_1 \\ C_2 \end{bmatrix}$$

and

$$\underline{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} .$$

Since $\underline{\omega}$ is arbitrary, we must have

$$CR = 0 \tag{4-17}$$

Consider that the i-th row of C is multiplied by R.

$$\sum_{j=1}^{6} c_{ij} a_{jk} = 0 \qquad (k = 1, 2, 3) \tag{4-18}$$

where c_{ij} for all j are elements of the i-th row of c, and a_{jk} for all j are elements of the k-th column of R. In (4-18) all a_{jk} are known but all c_{ij} are to be determined. Since each parity check equation is supposed to involve measurements from 4 sensor axes, therefore 2 out of 6 c_{ij}'s, for each i, should be set to zero.

Consider the first equation, i.e. i = 1. Let $c_{15} = c_{16} = 0$, implying that this equation involves measurements m_1, m_2, m_3 and m_4. Then (4-18) becomes

$$\sum_{\substack{j=1,\\2,3,4}} c_{ij} a_{jk} = 0 \qquad (k = 1, 2, 3) \tag{4-19}$$

Substituting the values of a_{jk}, which are in (3-11), into (4-19), gives

$$\left.\begin{aligned} c_{11} + \frac{3}{4} c_{13} + \frac{1}{4} c_{14} &= 0 \\ c_{12} + \frac{1}{4} c_{13} + \frac{3}{4} c_{14} &= 0 \\ -\frac{\sqrt{6}}{4} c_{13} + \frac{\sqrt{6}}{4} c_{14} &= 0 \end{aligned}\right\} \tag{4-20}$$

Hence $c_{13} = c_{14}$, $c_{11} = -c_{13} = -c_{14}$, $c_{12} = -c_{13} = -c_{14}$. Choose $c_{14} = -1$, the first row of c becomes

$$[1 \quad 1 \quad -1 \quad -1 \quad 0 \quad 0] \tag{4-21}$$

Consider one more case where i = 2, corresponding to the second row of c. Let $c_{24} = c_{26} = 0$, implying that measurement m_4 and m_6 are not involved in this parity check equation. Then (4-18) becomes

$$\sum_{\substack{j=1,\\2,3,5}} c_{ij} a_{jk} = 0 \qquad (k = 1, 2, 3). \tag{4-22}$$

Using the known values of a_{jk} in (4-22), we get

$$\left.\begin{aligned} c_{21} + \tfrac{3}{4} c_{23} + \tfrac{1}{4} c_{25} &= 0 \\ c_{22} + \tfrac{1}{4} c_{23} + \tfrac{3}{4} c_{25} &= 0 \\ -\tfrac{\sqrt{6}}{4} c_{23} + \tfrac{\sqrt{6}}{4} c_{25} &= 0 \end{aligned}\right\} \qquad (4\text{-}23)$$

Thus $c_{23} = -c_{25}$, $c_{21} = \frac{1}{2} c_{25}$, $c_{22} = -\frac{1}{2} c_{25}$. Let us choose $c_{25} = 2$, the second row of c becomes

$$[1 \quad -1 \quad -2 \quad 0 \quad 2 \quad 0] \qquad (4\text{-}24]$$

Continuing in this manner, all 15 rows of c can be determined, giving

$$C = \begin{bmatrix} 1 & 1 & -1 & -1 & 0 & 0 \\ 1 & -1 & -2 & 0 & 2 & 0 \\ 1 & 3 & 0 & -2 & -2 & 0 \\ 2 & 0 & -3 & -1 & 2 & 0 \\ 0 & -2 & -1 & 1 & 2 & 0 \\ 3 & 1 & -2 & 0 & 0 & -2 \\ 1 & -1 & 0 & 2 & 0 & -2 \\ 1 & 0 & -1 & 1 & 0 & -1 \\ 0 & 2 & 1 & 3 & 0 & -2 \\ 1 & 1 & 0 & 0 & -1 & -1 \\ 2 & 0 & -2 & 0 & 1 & -1 \\ 0 & 2 & 2 & 0 & -3 & -1 \\ 2 & 0 & 0 & 2 & -1 & -3 \\ 0 & 2 & 0 & -2 & -1 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{bmatrix} \qquad (4\text{-}25)$$

Using (4-25) the set of 15 parity check equations are given as follows.

$$
\begin{aligned}
&1.\quad m_1 + m_2 - m_3 - m_4 \simeq 0\\
&2.\quad m_1 - m_2 - 2m_3 + 2m_5 \simeq 0\\
&3.\quad m_1 + 3m_2 - 2m_4 - 2m_5 \simeq 0\\
&4.\quad 2m_1 - 3m_3 - m_4 + 2m_5 \simeq 0\\
&5.\quad -2m_2 - m_3 + m_4 + 2m_5 \simeq 0\\
&6.\quad 3m_1 + m_2 - 2m_3 - 2m_6 \simeq 0\\
&7.\quad m_1 - m_2 + 2m_4 - 2m_6 \simeq 0\\
&8.\quad m_1 - m_3 + m_4 - m_6 \simeq 0\\
&9.\quad 2m_2 + m_3 + 3m_4 - 2m_6 \simeq 0\\
&10.\quad m_1 + m_2 - m_5 - m_6 \simeq 0\\
&11.\quad 2m_1 - 2m_3 + m_5 - m_6 \simeq 0\\
&12.\quad 2m_2 + 2m_3 - 3m_5 - m_6 \simeq 0\\
&13.\quad 2m_1 + 2m_4 - m_5 - 3m_6 \simeq 0\\
&14.\quad 2m_2 - 2m_4 - m_5 + m_6 \simeq 0\\
&15.\quad m_3 + m_4 - m_5 - m_6 \simeq 0
\end{aligned}
$$

The "approximately equal" sign "$\simeq$" accounts for the normal measurement error.

The above development has used the assumption that no 2 faulty axes have identical errors; or, if they do have identical errors they do not occur at the same time.

Truth Table for Faulty Gyro Identification

Let us associate with the i-th parity check equation a binary valued quantity k_i. k_i equals 0 when the equation is satisfied and

equals 1 when not.

Suppose that the A_1 axis of sensor A is bad then all k_i equal 1 except k_5, k_9, k_{12}, k_{14}, and k_{15} which are equal 0. Next, suppose that two axes A_1 and B_2 are bad. That is, m_1 and m_4 are not consistant. Then all k_i equal to 1 except $k_{12} = 0$. A truth table can be constructed in this manner as shown in Table 4.1.

Examining Table 4.1 shows that there are 23 distinguishable cases. Twenty-two of them allow both failure detection and faulty axis identification. The last case which includes 42 states, where less than 4 axes are good, allow failure detection only but not the faulty axis identification.

SYSTEM RECONFIGURATION

The system reconfiguration need not be done by hardware modification. Instead, it is conveniently and rapidly done by changing the software program, which can be executed by a computer. When all sensor axes of the three TDF sensors are good the measurement matrix R is given by (3-11). Consider the case that axis A_1 is found faulty. Under this condition measurement $m_1 = A_1$ will not be used in the data reduction. This is done simply by eliminating the first row of the R matrix to give a new measurement matrix

$$R_1 = \begin{bmatrix} 0 & 1 & 0 \\ 3/4 & 1/4 & -\sqrt{6}/4 \\ 1/4 & 3/4 & \sqrt{6}/4 \\ 1/4 & 3/4 & -\sqrt{6}/4 \\ 3/4 & 1/4 & \sqrt{6}/4 \end{bmatrix} \qquad (4\text{-}26)$$

TABLE 4.1

Truth Table for Failure Detection and Faulty Axis Identification for a 3-sensor System.

Failed Axes	Parity Check Equations															Measure-ment Matrix
	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}	K_{11}	K_{12}	K_{13}	K_{14}	K_{15}	
None	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	R
A_1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	R_1
A_2	1	1	1	1	1	1	0	0	0	0	1	1	1	1	0	R_2
B_1	1	1	1	0	0	0	1	1	1	0	1	1	1	0	1	R_3
B_2	1	0	0	1	1	0	1	1	0	1	1	1	0	1	1	R_4
C_1	0	1	0	1	0	1	1	0	1	1	1	0	1	1	1	R_5
C_2	0	0	1	0	1	1	0	1	1	1	0	1	1	1	1	R_6
A_1 A_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	R_{12}
A_1 B_1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	R_{13}
A_1 B_2	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	R_{14}
A_1 C_1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	R_{15}
A_1 C_2	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	R_{16}
A_2 B_1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	R_{23}
A_2 B_2	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	R_{24}
A_2 C_1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	R_{25}
A_2 C_2	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	R_{26}
B_1 B_2	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	R_{34}
B_1 C_1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	R_{35}
B_1 C_2	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	R_{36}
B_2 C_1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	R_{45}
B_2 C_2	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	R_{46}
C_1 C_2	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	R_{56}
All Remaining States	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	None

In general one measurement m_i can be eliminated from the data reduction by removing the i-th row of the R to give a new measurement matrix R_i, and any two measurements m_i and m_j can be eliminated from the data reduction by removing both the i-th and j-th rows of R to give a new measurement matrix R_{ij}.

Irrespective of the system configuration, the data reduction algorithms are the same as before. Eq. (4-5) is used for the least-square data reduction and (4-9) is used for the weighted least-square data reduction. The only modification is that the new measurement matrix is used in place of R in these equations.

SENSOR RECALIBRATION

If a sensor axis has a soft failure, such as a bias error, a change of scale factor, or change of error variance, it can be recalibrated. The faulty axis is taken out of service by system reconfiguration and put into a fault determination mode. During this mode, measurement from this axis are compared with measurements from good axes to determine the type of fault which has occurred and the amount of correction needed. The correction is made by changing parameters in the software. The system is then reconfigurated once.

The functions of failure detection, faulty sensor axis identification, and system reconfiguration are indispensable in sensor performance management, but the function of sensor recalibration is not. The inclusion of the latter enhances the systems reliability at the expense of an increase in the computer requirement.

Several concepts are available for sensor recalibration, most of them require the use of statistical techniques. The details of these concepts will not be included here.

CHAPTER V

CONCLUSION

This paper presented a new development in the use of the redundant two-degree-of-freedom inertial sensors (gyros and/or accelerometers) for strapdown navigation platforms. The development is based on the assumption that two axes of a sensor can fail independently without affecting one another.

Optimum redundancy configurations for systems using three, four, five, and six sensors were described. The details of a sensor performance management technique were developed, which can perform the task of failure detection, faulty axis identification, and system reconfiguration. Two data reduction algorithms were proposed, namely, the least-square algorithm and the weighted-least-square algorithm.

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