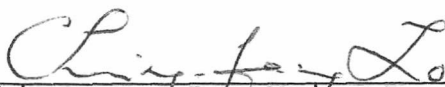
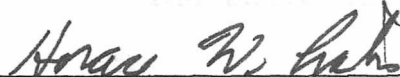


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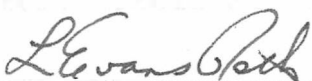
I am submitting herewith a thesis written by Werner Johann Anton Dahm entitled "A General Solution for Wind Tunnel Boundary-Induced Interference in Two-Dimensional Subsonic Flow." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.


Ching-Fang Lo, Major Professor

We have read this thesis
and recommend its acceptance:

Accepted for the Council:


Vice Chancellor
Graduate Studies and Research

A GENERAL SOLUTION FOR WIND TUNNEL BOUNDARY-INDUCED
INTERFERENCE IN TWO-DIMENSIONAL
SUBSONIC FLOW

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

This report is not approved for public release
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Station, Tennessee.

Werner Johann Anton Dahm

March 1981

3049616

ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to his major professor, Dr. Ching-Fang Lo, Engineering Supervisor in the Propulsion Wind Tunnel Facility at the Arnold Engineering Development Center and part-time Associate Professor of Aerospace Engineering at The University of Tennessee Space Institute, for his definition of the thesis problem and the numerous suggestions required for its completion, and for his support and encouragement during the author's studies. The author also wishes to thank Prof. Horace Crater, Associate Professor of Physics at The University of Tennessee Space Institute, for his help in the numerical procedure, and Prof. Mitsuru Kurosaka, Professor of Mechanical Engineering at The University of Tennessee Space Institute, for his review of the thesis and his direction during the author's studies.

Special thanks are due Mrs. Lorene Mottern for preparation of the figures, and Mrs. Jeannie Harper for preparation of the final manuscript.

The research reported herein was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), under Contract F40600-77-C-0003 with ARO, Inc., at the AEDC, Arnold Air Force Station, Tennessee. Reproduction to satisfy the requirements of the United States Government is authorized.

ABSTRACT

A general solution is presented for determination of wind tunnel boundary-induced interference in two-dimensional subsonic flow. The solution requires no a priori knowledge of the model geometry or tunnel wall characteristics, and instead, relates the interference to velocity measurements at a selected control surface within the tunnel. The solutions for the tunnel, free-air and interference fields are obtained using linearized subsonic theory with the Fourier transform technique. An analytical example dealing with the case of the wavy wall in the presence of a generalized tunnel wall boundary condition demonstrates the validity of the solutions in closed form. Numerical examples dealing with a variety of model geometries and tunnel wall characteristics demonstrate practical applications of the theory.

TABLE OF CONTENTS

CHAPTER		PAGE
I.	INTRODUCTION	1
	Wind Tunnel Boundary Interference.	1
	Background Information	1
	Statement of the Problem	4
II.	GENERAL ANALYSIS	6
	Governing Field Equations.	6
	Boundary Value Problem Formulations.	10
	Tunnel Field Solution.	17
	Free-Air Field Solution.	22
	Interference Field Solution.	31
III.	APPLICATION TO THE CLASSIC WAVY WALL MODEL WITH A GENERALIZED TUNNEL BOUNDARY CONDITION.	38
	General Description.	38
	Tunnel Field Solution.	40
	Free-Air Field Solution.	41
	Interference Field Solution.	45
IV.	NUMERICAL DEMONSTRATIONS	49
	Method of Calculation.	49
	Numerical Examples	55
V.	CONCLUDING REMARKS	61
	BIBLIOGRAPHY	62

CHAPTER	PAGE
APPENDIXES	65
A. FIGURES	66
B. INVERSE TRANSFORMS OF CERTAIN FUNCTIONS.	88
C. LIST OF PRINCIPLE SOLUTIONS.	91
VITA	94

LIST OF FIGURES

FIGURE		PAGE
1.	Schematic of the Physical Problem for Determining the Interference Field in a Wind Tunnel.	67
2.	Boundary Value Problem for the Symmetric Tunnel Field Potential in the Upper Half-Plane.	68
3.	Boundary Value Problem for the Asymmetric Tunnel Field Potential in the Upper Half-Plane.	69
4.	Boundary Value Problem for the Symmetric Free-Air Field Potential in the Upper Half-Plane.	70
5.	Boundary Value Problem for the Asymmetric Free-Air Field Potential in the Upper Half-Plane.	71
6.	Interior and Exterior Boundary Value Problems for the Symmetric Free-Air Potential in the Upper Half-Plane.	72
7.	Interior and Exterior Boundary Value Problems for the Asymmetric Free-Air Potential in the Upper Half-Plane.	73
8.	Boundary Value Problem for the Interference Field Solution.	74

FIGURE

PAGE

9.	Schematic of the Physical Problem for the Wavy Wall with a Generalized Boundary Condition	75
10.	Demonstration of the Numerical Procedure for Analytic Continuation	76
11.	Airfoil Surface Pressure Distributions Based on Eq. (109) for 12 Percent Thickness Parabolic Arc Airfoil, $M = 0.6$, $\alpha = 0^\circ$, $h/c = 0.5$	77
12.	Airfoil Surface Pressure Distributions Based on Eq. (109) for NACA 0012 Airfoil, $M = 0.6$, $\alpha = 0^\circ$, $h/c = 0.5$	80
13.	Airfoil Surface Pressure Distributions Based on Eq. (109) for 12 Percent Thickness Parabolic Arc Airfoil, $M = 0.6$, $\alpha = 1^\circ$, $h/c = 0.5$	83
14.	Airfoil Surface Pressure Distributions Based on Eq. (109) for NACA 0012 Airfoil, $M = 0.6$, $\alpha = 1^\circ$, $h/c = 0.5$	84
15.	Airfoil Surface Pressure Distributions Based on Eq. (109) for 12 Percent Thickness Parabolic Arc Airfoil, $M = 0.6$, $\alpha = 2^\circ$, $h/c = 0.5$	85
16.	Airfoil Surface Pressure Distributions Based on Eq. (109) and Exact Camber Modification for 12-Percent Thickness Parabolic Arc Airfoil, $M = 0.6$, $\alpha = 2^\circ$, $h/c = 0.5$	86

FIGURE

PAGE

17. Airfoil Surface Pressure Distributions Based on

Eq. (61) for NACA 0012 Airfoil,

 $M = 0.6$, $\alpha = 0^\circ$, $h/c = 0.5$ 87

NOMENCLATURE

a, b	Limits on range of x
c	Model chord
c_n	Tschebyscheff series coefficient
C_p	Pressure coefficient
$f(\hat{x}, h)$	Dummy function
$f_T(\xi, \pm h)$	Dummy function
$F(\hat{x})$	Equivalent profile thickness distribution
$g(z)$	Dummy function
$G(h')$	Defined in Eq. (119)
h	Nondimensional control surface height, H/c
$\tilde{h}$	Scaled nondimensional control surface height, βh
$\hat{h}$	Scaled nondimensional control surface height, $\frac{2}{(b-a)} h$
h'	Nondimensional tunnel boundary height
H	Tunnel boundary height
$H^+(y)$	Defined in Eq. (99)
$H^-(y)$	Defined in Eq. (100)
i	Pure imaginary constant
$I(x, \pm h)$	Defined in Eq. (167)
$J^+(y)$	Defined in Eq. (101)
$J^-(y)$	Defined in Eq. (102)
$L[\phi]$	Generalized linear operator
M_∞	Free-stream Mach number

N	Limit on Tschebyscheff series
p	Fourier transform variable
$\tilde{p}$	Scaled Fourier transform variable
u	Nondimensional streamwise perturbation velocity component, U/U_∞
U	Streamwise perturbation velocity component
U_∞	Free-stream velocity
$U_n(\hat{x})$	Tschebyscheff Type II polynomial
v	Nondimensional normal perturbation velocity component, V/U_∞
V	Normal perturbation velocity component
x	Nondimensional streamwise coordinate, X/c
$\tilde{x}$	Scaled nondimensional coordinate in Eq. (5)
$\hat{x}$	Scaled nondimensional coordinate in Eq. (155)
X	Streamwise Cartesian coordinate
y	Nondimensional normal coordinate, Y/c
$\tilde{y}$	Scaled nondimensional coordinate, βy
$\hat{y}$	Scaled nondimensional coordinate, $\frac{2}{(b-a)}y$
Y	Normal Cartesian coordinate
z	Complex variable $(\xi+i\eta)$
$\tilde{z}$	Defined in Eq. (13)
$\hat{z}$	Defined in Eq. (162)
α	Angle of incidence
β	Prandtl-Glauert factor, $(1-M_\infty^2)^{1/2}$
$\gamma(\tilde{x})$	Equivalent profile circulation distribution
ϵ	Amplitude of sinusoidal wavy wall

ϵ_N	Series truncation parameter
λ	Wavy wall wavelength parameter
ϕ	Nondimensional perturbation velocity potential, $\phi/U_\infty c$
Φ	Perturbation velocity potential
ξ	Dummy integration variable
$(\bar{})$	Quantity in Fourier transform plane
$(\vec{})$	Vector quantity
$(\prime)$	Derivative with respect to $\tilde{x}$
$(^+)$	Limit through positive values
$()$	Absolute value of the argument
$\text{Re}[\]$	Real part of the complex argument
$\text{Im}[\]$	Imaginary part of the complex argument
∇	Gradient operator
∇^2	Laplace operator

Subscripts

i	Interference field quantity
S, A	Symmetric and asymmetric components, respectively
T	Tunnel field quantity
∞	Free-air field quantity

CHAPTER I

INTRODUCTION

I. WIND TUNNEL BOUNDARY INTERFERENCE

Wind tunnel interference is defined to exist if the flow field about a model in a wind tunnel differs in any way from that about the same model in free air, provided the similarity parameters relevant to the flow problem are identical in both cases. In most situations, the principle contribution to the interference is the modification of the flow field induced by the flow constraint at the stream boundaries, an effect generally termed wind tunnel boundary interference.

In solid wall tunnels, the flow tangency condition at the tunnel walls alters the streamline pattern throughout the field, and consequently, induces potentially significant errors in the measured aerodynamic quantities. A similar situation exists in open-jet tunnels due to the constant pressure condition along the stream boundary, and in ventilated wall tunnels due to the mixed boundary condition at the tunnel walls.

II. BACKGROUND INFORMATION

The existence of boundary interference had been recognized even before the advent of wind tunnel testing

from water channels used in the design of optimum hull shapes for oceangoing vessels. Historically, analytical and semi-empirical correction techniques to separate the effects of boundary interference from measurements in a tunnel have concentrated on determining equivalent mathematical representations for the stream boundary conditions and deducing the consequent modifications in the flow field and on measured quantities in the wind tunnel.

Although rigorous determination of the modifications in the flow field due to boundary constraints is a complex nonlinear problem, it is generally assumed that the interference may be treated as a linear field quantity. Consequently, the net modifications in the flow field may be expressed as the sum of two independent components; velocity gradients induced in the streamwise direction due to the volume in the tunnel occupied by the model and its wake, generally termed blockage interference; and changes in the stream incidence associated with the circulation around the model, generally termed lift interference.

Analytical solutions for tunnel wall interference date back to 1919, when Prandtl's [1]¹ lifting line theory presented a method of estimating the lift interference of a finite wing at small lift in a closed circular tunnel. Theoretical and empirical studies over the next decade

¹Numbers in brackets refer to similarly numbered references in the Bibliography.

built a framework of interference correction methods based on the early work of Prandtl and Glauert, summarized by Glauert's [2] classic monograph in 1933. Theodorsen's [3] observation of the opposing interference effects associated with closed and open tunnel boundaries introduced the prospect that, through some judicious combination of these boundaries, an interference-free tunnel configuration might be achieved, leading to the advent of ventilated wall wind tunnels. Goethert's [4] monograph gave the first comprehensive treatment of wall interference in ventilated tunnels, while subsequent general theories for the correction of interference in such tunnels concentrated on replacing the mixed boundary condition at the wall with a single homogeneous condition over the entire wall. A number of correction methods employing such mathematical representations for ventilated wall tunnels are summarized by Pindzola and Lo [5], and by Garner et al. [6].

These classical methods require a definition of the tunnel wall characteristics and a mathematical representation of the test article. The definition of the wall characteristic is one of the obstacles to the routine application of classical wind tunnel theory to ventilated wall tunnels. Furthermore, the representation of the test model by certain combinations of singularities neglects all viscous effects. In 1978, Lo [7] presented an alternative to this classical approach by proposing to base interference assessments on measurements of flow variables at a control

surface near the tunnel wall inside the test section. Such a method offered the advantage of not requiring any equivalent mathematical representation for the tunnel wall characteristics or test article geometry. Subsequent analytical and numerical investigations [8, 9] proved that this method could be used to determine the interference distribution at the control surface.

III. STATEMENT OF THE PROBLEM

The investigation described herein is concerned with determining a general solution for the boundary-induced interference in two-dimensional subsonic flow based on flow variable measurements at a control surface in a wind tunnel. The governing flow equations for the tunnel, free-air and interference fields are solved subject to the boundary measurements using linearized subsonic theory with the Fourier transform technique. The interference solution presented, unlike the results obtained from classical wind tunnel theory, does not require any a priori knowledge of the tunnel wall characteristics or test article geometry.

The approach employed assumes that the model thickness and circulation distributions are identical in the tunnel and free-air cases. If the model geometry is interpreted as a potential equivalent profile including viscous effects, then it is reasonable to assume that the same equivalent profile exists in both cases. However, the

observed difference in the lift measured in a wind tunnel and in free-air implies a different circulation distribution between these two cases. Since the approach employed assumes the same distribution, the solution will contain an error, presumably small, in the correction for lift.

CHAPTER II

GENERAL ANALYSIS

I. GOVERNING FIELD EQUATIONS

The model is considered as a potential equivalent body including a thin, attached boundary layer with a downstream wake region. The flow field is defined as the area between two control surfaces in the tunnel stream located at some $Y = \pm H$ outside any wall boundary layer or open-jet mixing region at which velocity component measurements are made and between which the model is centered, as shown in Fig. 1, Appendix A.¹ Within this region, the flow is considered steady, two-dimensional, inviscid and irrotational. The assumption of potential flow is rigorous and consequently the tunnel velocity field may be defined in terms of the gradient of a scalar potential as

$$\vec{V}_T(X,Y) = \vec{\nabla} \Phi_T(X,Y) \quad (1)$$

Components of the velocity field in the tunnel may be derived from the potential by the relationships

$$\begin{aligned} U_T(X,Y) &= \beta \frac{\partial}{\partial X} \Phi_T(X,Y) \\ V_T(X,Y) &= \frac{\partial}{\partial Y} \Phi_T(X,Y) \end{aligned} \quad (2)$$

¹All figures appear in Appendix A.

It is assumed that the model satisfies small disturbance boundary conditions and that, consequently, squares and products of perturbation velocity components may be neglected in the equation of motion. The assumption is also made that the flow may be treated as a continuum and that the flow is subsonic throughout the field.

Under these restrictions, the field equation written in terms of the tunnel perturbation velocity potential is the well-known linearized equation of compressible flow,

$$\beta^2 \frac{\partial^2 \phi_T}{\partial X^2} + \frac{\partial^2 \phi_T}{\partial Y^2} = 0 , \quad (3)$$

where,

$$\beta = \sqrt{1 - M_\infty^2}$$

is the Prandtl-Glauert compressibility factor, and ϕ_T is the perturbation velocity potential of the flow field within the tunnel.

By assigning the free-stream velocity U_∞ and the model chord c or some other scale as the characteristic velocity and length, respectively, the mathematical system can be nondimensionalized as

$$x = \frac{X}{c} \quad y = \frac{Y}{c} \quad \phi_T = \frac{\phi_T}{U_\infty c} . \quad (4)$$

With the aid of the Prandtl-Glauert transformation, the y-dimension is scaled by the factor β to yield the scaled variables,

$$\tilde{x} = x \quad \tilde{y} = \beta y . \quad (5)$$

Writing Eq. (3) in terms of scaled nondimensional variables leads to the familiar Laplace equation,

$$\nabla^2 \phi_T(\tilde{x}, \tilde{y}) = \left(\frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{y}^2} \right) \phi_T(\tilde{x}, \tilde{y}) = 0 , \quad (6)$$

subject to the boundary conditions

$$\frac{\partial}{\partial \tilde{x}} \phi_T(\tilde{x}, \pm \tilde{h}) = u_T(\tilde{x}, \pm \tilde{h}) , \quad (7)$$

where $u_T(\tilde{x}, \pm \tilde{h})$ is a quantity measured at the control surface, and the condition that the potential be independent of $\tilde{x}$ at upstream and downstream infinity, namely,

$$\lim_{\tilde{x} \rightarrow \pm \infty} \frac{\partial}{\partial \tilde{x}} \phi_T(\tilde{x}, \tilde{y}) = 0 . \quad (8)$$

Owing to the linearity of this equation and the accompanying boundary conditions, the perturbation potential ϕ_T may be treated as the sum of two independent components; the potential for flow over the model in free-air, ϕ_∞ , and the potential for the difference between the tunnel and free-air potentials, presumably induced by the tunnel boundaries, defined as the interference potential ϕ_i ,

related as

$$\phi_T(\tilde{x}, \tilde{y}) = \phi_\infty(\tilde{x}, \tilde{y}) + \phi_i(\tilde{x}, \tilde{y}) . \quad (9)$$

The linearity of the Laplace operator allows the field equations for each of the free-air and interference flow fields to be written directly from Eq. (6) as

$$\left(\frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{y}^2} \right) \phi_\infty(\tilde{x}, \tilde{y}) = 0 \quad (10)$$

and

$$\left(\frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{y}^2} \right) \phi_i(\tilde{x}, \tilde{y}) = 0 . \quad (11)$$

The free-air field potential in Eq. (10) contains the model singularities and satisfies the vanishing condition at infinity, namely,

$$\lim_{|\tilde{z}| \rightarrow \infty} \phi_\infty(\tilde{x}, \tilde{y}) = 0 , \quad (12)$$

where,

$$|\tilde{z}| = \sqrt{\tilde{x}^2 + \tilde{y}^2} . \quad (13)$$

Boundary conditions on the interference field in Eq. (11) are obtained by imposing the vanishing condition on the derivative at upstream and downstream infinity,

$$\lim_{\tilde{x} \rightarrow \pm\infty} \frac{\partial}{\partial \tilde{x}} \phi_i(\tilde{x}, \tilde{y}) = 0 , \quad (14)$$

and specifying the interference at the control surfaces as

$$\phi_i(\tilde{x}, \pm\tilde{h}) = \phi_T(\tilde{x}, \pm\tilde{h}) - \phi_\infty(\tilde{x}, \pm\tilde{h}) . \quad (15)$$

II. BOUNDARY VALUE PROBLEM FORMULATIONS

The potentials for the tunnel and free-air fields as formulated in the previous section contain the model singularities. However, the linearity of the governing field equations and the accompanying boundary conditions for the tunnel, free-air and interference fields allows the potential in each to be separated into symmetric and asymmetric parts, defined for the tunnel field as

$$\phi_{T_S}(\tilde{x}, \pm\tilde{y}) = \frac{1}{2} \left[\phi_T(\tilde{x}, \pm\tilde{y}) + \phi_T(\tilde{x}, \mp\tilde{y}) \right]$$

and

(16)

$$\phi_{T_A}(\tilde{x}, \pm\tilde{y}) = \pm \frac{1}{2} \left[\phi_T(\tilde{x}, \pm\tilde{y}) - \phi_T(\tilde{x}, \mp\tilde{y}) \right] .$$

Each of the symmetric and asymmetric potentials also satisfies the Laplace equation, and perturbation velocity components may be derived from each as

$$u_{T_S}(\tilde{x}, \pm\tilde{y}) = \frac{1}{2} \left[u_T(\tilde{x}, \tilde{y}) + u_T(\tilde{x}, -\tilde{y}) \right]$$

$$v_{T_S}(\tilde{x}, \pm\tilde{y}) = \pm \frac{1}{2} \left[v_T(\tilde{x}, \tilde{y}) - v_T(\tilde{x}, -\tilde{y}) \right]$$

and

(17)

$$u_{T_A}(\tilde{x}, \pm\tilde{y}) = \pm \frac{1}{2} \left[u_T(\tilde{x}, \tilde{y}) - u_T(\tilde{x}, -\tilde{y}) \right]$$

$$v_{T_A}(\tilde{x}, \pm\tilde{y}) = \frac{1}{2} \left[v_T(\tilde{x}, \tilde{y}) + v_T(\tilde{x}, -\tilde{y}) \right]$$

Definitions for the symmetric and asymmetric velocity components in the free-air and interference fields are similar.

In each field, the symmetric potential represents the solution to the blockage problem; i.e., the solution including only the effects of thickness. The asymmetric potential represents the solution to the lifting problem; i.e., the solution including only the effects of camber and incidence in which the circulation is represented as a centerline distribution of vortices, $\gamma(\tilde{x})$. It must be emphasized that the thickness and circulation distributions representing the body never need to be specified and that, hence, no knowledge of the model geometry is required.

Solutions for each of the symmetric and asymmetric potentials in the tunnel, free-air and interference fields, need only be obtained in the upper half-plane of each field, since the lower half-plane solutions may be determined directly from the symmetric and asymmetric conditions in

Eq. (17). In a manner consistent with small disturbance theory, the boundary conditions for the model are applied in the limit approaching the centerline in each of the half-planes. The symmetric and asymmetric fields in the upper half-plane do not contain any singularities and solutions may be obtained using the Fourier transform technique.

The boundary value problem for the upper half-plane solution of the symmetric tunnel field potential is formulated by specifying the boundary conditions around the field shown in Fig. 2. The boundary condition at the control surface is the symmetric part of the measured streamwise perturbation velocity component, $u_{TS}(\tilde{x}, \tilde{h})$. Further, the condition that the potential is independent of $\tilde{x}$ at upstream and downstream infinity leads to the boundary conditions

$$\lim_{\tilde{x} \rightarrow \pm\infty} \frac{\partial}{\partial \tilde{x}} \phi_{TS}(\tilde{x}, \tilde{y}) = 0. \quad (18)$$

Finally, the flow tangency condition at the equivalent body is applied in the limit approaching the tunnel centerline, as

$$\lim_{\tilde{y} \rightarrow 0^+} v_{TS}(\tilde{x}, \tilde{y}) = F'(\tilde{x}). \quad (19)$$

The boundary value problem for the asymmetric tunnel field potential is formulated using the asymmetric

velocity component $u_{T_A}(\tilde{x}, \tilde{h})$ as the boundary condition at the control surface and the vanishing derivative condition at upstream and downstream infinity, and imposing the induced streamwise velocity from the circulation distribution in the limit approaching the centerline, namely,

$$\lim_{\tilde{y} \rightarrow 0^+} u_{T_A}(\tilde{x}, \tilde{y}) = \frac{1}{2} \gamma(\tilde{x}) , \quad (20)$$

leading to the problem shown in Fig. 3.

The boundary value problems for the symmetric and asymmetric free-air field potentials in Figs. 4 and 5 are formulated by specifying similar boundary conditions at the centerline as for the tunnel field, namely,

$$\lim_{\tilde{y} \rightarrow 0^+} v_{\infty S}(\tilde{x}, \tilde{y}) = F'(\tilde{x}) \quad (21)$$

$$\lim_{\tilde{y} \rightarrow 0^+} u_{\infty A}(\tilde{x}, \tilde{y}) = \frac{1}{2} \gamma(\tilde{x}) , \quad (22)$$

but removing the constraint at the control surface and imposing the vanishing condition at infinity as

$$\lim_{|\tilde{z}| \rightarrow \infty} \phi_{\infty S}(\tilde{x}, \tilde{y}) = 0 \quad (23)$$

and

$$\lim_{|\tilde{z}| \rightarrow \infty} \phi_{\infty A}(\tilde{x}, \tilde{y}) = 0 , \quad (24)$$

where,

$$|\tilde{z}| = \sqrt{\tilde{x}^2 + \tilde{y}^2} .$$

The boundary value problems for the tunnel, free-air and interference fields are solved in the complex Fourier transform plane, since the governing partial differential equations reduce to ordinary differential equations under the transformation. Defining $\bar{\phi}(\tilde{p}, \tilde{y})$ as the complex Fourier transform on $\tilde{x}$ of the potential $\phi(\tilde{x}, \tilde{y})$, related through the standard form,

$$\bar{\phi}(\tilde{p}, \tilde{y}) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} \phi(\tilde{x}, \tilde{y}) e^{i\tilde{p}\tilde{x}} d\tilde{x} , \quad (25)$$

transforms the Laplace equation to the form

$$-\tilde{p}^2 \bar{\phi}(\tilde{p}, \tilde{y}) + \frac{d^2}{d\tilde{y}^2} \bar{\phi}(\tilde{p}, \tilde{y}) = 0 , \quad (26)$$

which is an ordinary differential equation to which solutions may be readily formulated. Components of the velocity field are derived from the transformed potential by applying a complex Fourier transform on forms analogous to Eq. (2), resulting in

$$\begin{aligned} \bar{u}(\tilde{p}, \tilde{y}) &= -i\tilde{p}\bar{\phi}(\tilde{p}, \tilde{y}) \\ \bar{v}(\tilde{p}, \tilde{y}) &= \frac{d}{d\tilde{y}} \bar{\phi}(\tilde{p}, \tilde{y}) \end{aligned} , \quad (27)$$

provided

$$\lim_{\tilde{x} \rightarrow +\infty} e^{i\tilde{x}\tilde{p}} \phi(\tilde{x}, \tilde{y}) - \lim_{\tilde{x} \rightarrow -\infty} e^{i\tilde{x}\tilde{p}} \phi(\tilde{x}, \tilde{y}) = 0 . \quad (28)$$

The boundary value problems formulated in the physical plane may now be expressed in the transform plane.

Symmetric Tunnel Field Potential, $\bar{\phi}_{T_S}(\tilde{p}, \tilde{y})$

In the upper half-plane of the symmetric tunnel field, defined by $0 \leq \tilde{y} \leq \tilde{h}$, the governing field equation is the transformed Laplace equation of the form

$$-\tilde{p}^2 \bar{\phi}_{T_S}(\tilde{p}, \tilde{y}) + \frac{d^2}{d\tilde{y}^2} \bar{\phi}_{T_S}(\tilde{p}, \tilde{y}) = 0 , \quad (29)$$

subject to the boundary conditions

$$-i\tilde{p}\bar{\phi}_{T_S}(\tilde{p}, \tilde{h}) = \bar{u}_{T_S}(\tilde{p}, \tilde{h}) \quad (30)$$

and

$$\lim_{\tilde{y} \rightarrow 0^+} \frac{d}{d\tilde{y}} \bar{\phi}_{T_S}(\tilde{p}, \tilde{y}) = F'(\tilde{p}) . \quad (31)$$

Asymmetric Tunnel Field Potential, $\bar{\phi}_{T_A}(\tilde{p}, \tilde{y})$

The governing field equation in the upper half-plane of the asymmetric tunnel field is again a transformed Laplace equation of the form

$$-\tilde{p}^2 \bar{\phi}_{T_A}(\tilde{p}, \tilde{y}) + \frac{d^2}{d\tilde{y}^2} \bar{\phi}_{T_A}(\tilde{p}, \tilde{y}) = 0 , \quad (32)$$

subject to the boundary conditions

$$-i\tilde{p}\bar{\phi}_{T_A}(\tilde{p}, \tilde{h}) = \bar{u}_{T_A}(\tilde{p}, \tilde{h}) , \quad (33)$$

and

$$\lim_{\tilde{y} \rightarrow 0^+} -i\tilde{p}\bar{\phi}_{T_A}(\tilde{p}, \tilde{y}) = \frac{1}{2} \bar{\gamma}(\tilde{p}) \quad (34)$$

for

$$0 \leq \tilde{y} \leq \tilde{h} .$$

Symmetric Free-Air Field Potential, $\bar{\phi}_{\infty S}(\tilde{p}, \tilde{y})$

The field equation is again a transformed Laplace equation of the form

$$-\tilde{p}^2 \bar{\phi}_{\infty S}(\tilde{p}, \tilde{y}) + \frac{d^2}{d\tilde{y}^2} \bar{\phi}_{\infty S}(\tilde{p}, \tilde{y}) = 0 , \quad (35)$$

with the boundary conditions

$$\lim_{\tilde{y} \rightarrow \infty} \bar{\phi}_{\infty S}(\tilde{p}, \tilde{y}) = 0 \quad (36)$$

and

$$\lim_{\tilde{y} \rightarrow 0^+} \frac{d}{d\tilde{y}} \bar{\phi}_{\infty S}(\tilde{p}, \tilde{y}) = \bar{F}'(\tilde{p}) \quad (37)$$

for

$$\tilde{y} \geq 0 .$$

Asymmetric Free-Air Field Potential, $\bar{\phi}_{\infty A}(\tilde{p}, \tilde{y})$

The governing field equation is of the form

$$-\tilde{p}^2 \bar{\phi}_{\infty A}(\tilde{p}, \tilde{y}) + \frac{d^2}{d\tilde{y}^2} \bar{\phi}_{\infty A}(\tilde{p}, \tilde{y}) = 0 , \quad (38)$$

subject to the conditions

$$\lim_{\tilde{y} \rightarrow \infty} \bar{\phi}_{\infty A}(\tilde{p}, \tilde{y}) = 0 \quad (39)$$

and

$$\lim_{\tilde{y} \rightarrow 0^+} -i\tilde{p} \bar{\phi}_{\infty A}(\tilde{p}, \tilde{y}) = \frac{1}{2} \gamma(\tilde{p}) \quad (40)$$

for

$$\tilde{y} \geq 0 .$$

III. TUNNEL FIELD SOLUTION

The solution for the tunnel field potential is to be derived in a form requiring only the measured tunnel control surface quantities $u_T(x, \pm h)$ and $v_T(x, \pm h)$. The boundary value problems formulated in the previous section for the symmetric and asymmetric components of the potential in the upper half-plane are each solved separately, with terms involving the model geometry being algebraically eliminated. The solutions are extended to the lower half-plane using the symmetric and asymmetric conditions and combined to form the complete transformed

potential. Applying the inverse transform leads to the solution for the tunnel field potential expressed solely in terms of the control surface quantities.

Symmetric Tunnel Field Solution

Solving Eq. (29) for the symmetric tunnel field potential under the boundary conditions in Eqs. (30) and (31) and returning to nondimensional variables p and y yields the upper half-plane potential of the form

$$\bar{\phi}_{T_S}(p,y) = \frac{i}{p} \bar{u}_{T_S}(p,h) \frac{\cosh[p\beta y]}{\cosh[p\beta h]} - \frac{\bar{F}'(p)}{p\beta} \frac{\sinh[p\beta(h-y)]}{\cosh[p\beta h]} . \quad (41)$$

Velocity field components are determined from this potential using Eq. (27) as

$$\bar{u}_{T_S}(p,y) = \bar{u}_{T_S}(p,h) \frac{\cosh[p\beta y]}{\cosh[p\beta h]} + \frac{i}{\beta} \bar{F}'(p) \frac{\sinh[p\beta(h-y)]}{\cosh[p\beta h]} \quad (42)$$

and

$$\bar{v}_{T_S}(p,y) = i\beta \bar{u}_{T_S}(p,h) \frac{\sinh[p\beta y]}{\cosh[p\beta h]} + \bar{F}'(p) \frac{\cosh[p\beta(h-y)]}{\cosh[p\beta h]} . \quad (43)$$

Evaluating Eq. (43) at $y = h$ allows the transformed thickness distribution to be expressed as

$$\bar{F}'(p) = \bar{v}_{T_S}(p,h) \cosh[p\beta h] - i\beta \bar{u}_{T_S}(p,h) \sinh[p\beta h] , \quad (44)$$

which is the same as Eq. (10) of Lo and Sickles [9].

Substituting this result into Eqs. (42) and (43) to eliminate the model geometry leads to the upper half-plane

solution for the symmetric tunnel field velocity components as

$$\bar{u}_{TS}(p, y) = \bar{u}_{TS}(p, h) \cosh[p\beta(h-y)] + \frac{i}{\beta} \bar{v}_{TS}(p, h) \sinh[p\beta(h-y)] \quad (45)$$

and

$$\bar{v}_{TS}(p, y) = \bar{v}_{TS}(p, h) \cosh[p\beta(h-y)] - i\beta \bar{u}_{TS}(p, h) \sinh[p\beta(h-y)] . \quad (46)$$

Velocity field components in the lower half-plane $-h \leq y \leq 0$ are obtained directly from symmetry conditions as

$$\bar{u}_{TS}(p, -y) = \bar{u}_{TS}(p, -h) \cosh[p\beta(h-y)] - \frac{i}{\beta} \bar{v}_{TS}(p, -h) \sinh[p\beta(h-y)] \quad (47)$$

and

$$\bar{v}_{TS}(p, -y) = \bar{v}_{TS}(p, -h) \cosh[p\beta(h-y)] + i\beta \bar{u}_{TS}(p, -h) \sinh[p\beta(h-y)] . \quad (48)$$

Hence, components throughout the plane may be written as

$$\bar{u}_{TS}(p, \pm y) = \bar{u}_{TS}(p, \pm h) \cosh[p\beta(h-y)] \pm \frac{i}{\beta} \bar{v}_{TS}(p, \pm h) \sinh[p\beta(h-y)] \quad (49)$$

and

$$\bar{v}_{TS}(p, \pm y) = \bar{v}_{TS}(p, \pm h) \cosh[p\beta(h-y)] \mp i\beta \bar{u}_{TS}(p, \pm h) \sinh[p\beta(h-y)] , \quad (50)$$

where,

$$0 \leq y \leq h .$$

Asymmetric Tunnel Field Solution

Solving Eq. (32) for the symmetric tunnel field potential under the boundary conditions in Eqs. (33) and (34) in terms of nondimensional variables p and y leads to the form

$$\bar{\phi}_{T_A}(p,y) = \frac{i}{p} \bar{u}_{T_A}(p,h) \frac{\sinh[p\beta y]}{\sinh[p\beta h]} + \frac{1}{2} \bar{\gamma}(p) \frac{i}{p} \frac{\sinh[p\beta(h-y)]}{\sinh[p\beta h]} . \quad (51)$$

Components of the velocity field are derived from this potential as

$$\bar{u}_{T_A}(p,y) = \bar{u}_{T_A}(p,h) \frac{\sinh[p\beta y]}{\sinh[p\beta h]} + \frac{1}{2} \bar{\gamma}(p) \frac{\sinh[p\beta(h-y)]}{\sinh[p\beta h]} \quad (52)$$

and

$$\bar{v}_{T_A}(p,y) = i\beta \bar{u}_{T_A}(p,h) \frac{\cosh[p\beta y]}{\sinh[p\beta h]} - i\beta \frac{1}{2} \bar{\gamma}(p) \frac{\cosh[p\beta(h-y)]}{\sinh[p\beta h]} . \quad (53)$$

The circulation distribution $\bar{\gamma}(p)$ may be expressed in terms of control surface variables by evaluating Eq. (53) at $y = h$ as

$$\frac{1}{2} \bar{\gamma}(p) = \bar{u}_{T_A}(p,h) \cosh[p\beta h] + \frac{1}{\beta} \bar{v}_{T_A}(p,h) \sinh[p\beta h] , \quad (54)$$

which is the same as Eq. (13) of Lo and Sickles [9].

Substituting this result into Eqs. (52) and (53) to eliminate the model geometry yields the asymmetric tunnel field velocity components in the transformed plane as

$$\bar{u}_{T_A}(p,y) = \bar{u}_{T_A}(p,h) \cosh[p\beta(h-y)] + \frac{i}{\beta} \bar{v}_{T_A}(p,h) \sinh[p\beta(h-y)] \quad (55)$$

and

$$\bar{v}_{T_A}(p,y) = \bar{v}_{T_A}(p,h) \cosh[p\beta(h-y)] - i\beta \bar{u}_{T_A}(p,h) \sinh[p\beta(h-y)] . \quad (56)$$

Deriving the lower half-plane solutions from symmetry conditions allows asymmetric tunnel velocity components to be expressed throughout the plane as

$$\bar{u}_{T_A}(p,\pm y) = \bar{u}_{T_A}(p,\pm h) \cosh[p\beta(h-y)] \pm \frac{i}{\beta} \bar{v}_{T_A}(p,\pm h) \sinh[p\beta(h-y)] \quad (57)$$

and

$$\bar{v}_{T_A}(p,\pm y) = \bar{v}_{T_A}(p,\pm h) \cosh[p\beta(h-y)] \mp i\beta \bar{u}_{T_A}(p,\pm h) \sinh[p\beta(h-y)] \quad (58)$$

for

$$0 \leq y \leq h .$$

Combined Tunnel Field Solution

Combining the symmetric and asymmetric components of the tunnel velocity field, Eqs. (49) and (50), and Eqs. (57) and (58), results in complete solutions for the velocity components as

$$\bar{u}_T(p,\pm y) = \bar{u}_T(p,\pm h) \cosh[p\beta(h-y)] \pm \frac{i}{\beta} \bar{v}_T(p,\pm h) \sinh[p\beta(h-y)] \quad (59)$$

and

$$\bar{v}_T(p,\pm y) = \bar{v}_T(p,\pm h) \cosh[p\beta(h-y)] \mp i\beta \bar{u}_T(p,\pm h) \sinh[p\beta(h-y)] . \quad (60)$$

Applying an inverse transform on x to Eqs. (59) and (60), following the method outlined in Appendix B, leads to solutions in the physical plane for the tunnel field velocity components as

$$u_T(x, \pm y) = \operatorname{Re} [u_T(x + i\beta(h-y), \pm h)] \mp \frac{1}{\beta} \operatorname{Im} [v_T(x + i\beta(h-y), \pm h)] \quad (61)$$

and

$$v_T(x, \pm y) = \operatorname{Re} [v_T(x + i\beta(h-y), \pm h)] \pm \beta \operatorname{Im} [u_T(x + i\beta(h-y), \pm h)] \quad (62)$$

for

$$0 \leq y \leq h .$$

Equations (61) and (62) allow velocity components throughout the tunnel field to be obtained solely from the control surface flow variable measurements.

IV. FREE-AIR FIELD SOLUTION

In a similar manner as was done for the tunnel field potential, the free-air field potential is to be derived solely in terms of the measured tunnel quantities $u_T(x, \pm h)$ and $v_T(x, \pm h)$. However, unlike the tunnel field solution, obtaining the inverse transform of the resulting free-air solution is possible only at the control surfaces. Consequently, the boundary value problems previously formulated for the free-air potential are separated by the resulting free-air boundary condition at the control surface into an interior and an exterior region. Solutions

for the symmetric and asymmetric components in each region are derived and then combined to form the complete transformed potential for each region. Applying the inverse transform leads to the free-air solution in the interior and exterior regions expressed solely in terms of the measured tunnel control surface quantities.

Symmetric Free-Air Field Solution

Solving Eq. (35) for the symmetric free-air potential in the upper half-plane under the boundary conditions in Eqs. (36) and (37) leads to the form

$$\bar{\phi}_{\infty S}(p, y) = - \frac{1}{|p|^{\beta}} e^{-|p|^{\beta} y} \bar{F}'(p) , \quad (63)$$

from which the velocity field components are derived as

$$\bar{u}_{\infty S}(p, y) = \frac{i}{\beta} \frac{p}{|p|} e^{-|p|^{\beta} y} \bar{F}'(p) \quad (64)$$

and

$$\bar{v}_{\infty S}(p, y) = e^{-|p|^{\beta} y} \bar{F}'(p) . \quad (65)$$

Equations (64) and (65) are consistent with Eqs. (2a) and (2b) of Lo [7] for the case where $y = h$.

Deriving solutions in the lower half-plane from symmetry conditions and eliminating the model geometry by substituting the form of $\bar{F}'(p)$ from Eq. (44) into the forms above, using the relationship

$$\frac{p}{|p|} \sinh[p\beta h] = \sinh[|p|^{\beta} h] \quad (66)$$

leads to expressions for the symmetric free-air components in terms of tunnel variables as

$$\begin{aligned} \bar{u}_{\infty S}(p, \pm y) = e^{-|p|\beta y} \left[\sinh[|p|\beta h] \bar{u}_{TS}(p, \pm h) \right. \\ \left. \pm \frac{i}{\beta} \frac{p}{|p|} \cosh[p\beta h] \bar{v}_{TS}(p, \pm h) \right] \end{aligned} \quad (67)$$

and

$$\begin{aligned} \bar{v}_{\infty S}(p, \pm y) = \mp e^{-|p|\beta y} \left[i\beta \frac{p}{|p|} \sinh[|p|\beta h] \bar{u}_{TS}(p, \pm h) \right. \\ \left. \mp \cosh[p\beta h] \bar{v}_{TS}(p, \pm h) \right] \end{aligned} \quad (68)$$

when $y \geq 0$. Equations (67) and (68) agree with Eqs. (37) and (38) of Lo and Sickles [9] for the case where $y = h$.

Asymmetric Free-Air Field Solution

Solving for the asymmetric free-air field potential in Eq. (38) under the boundary conditions in Eqs. (39) and (40) yields the form

$$\bar{\phi}_{\infty A}(p, y) = \frac{i}{p} e^{-|p|\beta y} \frac{1}{2} \bar{\gamma}(p) \quad (69)$$

for $y \geq 0$.

Velocity field components in the upper half-plane are determined from this potential as

$$\bar{u}_{\infty A}(p, y) = e^{-|p|\beta y} \frac{1}{2} \bar{\gamma}(p) \quad (70)$$

and

$$\bar{v}_{\infty A}(p, y) = -i\beta \frac{p}{|p|} e^{-|p|\beta y} \frac{1}{2} \bar{\gamma}(p) , \quad (71)$$

which are consistent with Eqs. (14) and (15) of Lo and Sickles [9] for the case where $y = h$.

Deriving solutions in the lower half-plane from symmetry conditions and substituting the expression for $\bar{\gamma}(p)$ from Eq. (54) to eliminate the model geometry allows the free-air field velocity components to be expressed in terms of tunnel variables as

$$\begin{aligned} \bar{u}_{\infty A}(p, \pm y) = e^{-|p|\beta y} & \left[\cosh[p\beta h] \bar{u}_{TA}(p, \pm h) \right. \\ & \left. \pm \frac{i}{\beta} \frac{p}{|p|} \sinh[|p|\beta h] \bar{v}_{TA}(p, \pm h) \right] \end{aligned} \quad (72)$$

and

$$\begin{aligned} \bar{v}_{\infty A}(p, \pm y) = \mp e^{-|p|\beta y} & \left[i\beta \frac{p}{|p|} \cosh[p\beta h] \bar{u}_{TA}(p, \pm h) \right. \\ & \left. \mp \sinh[|p|\beta h] \bar{v}_{TA}(p, \pm h) \right] \end{aligned} \quad (73)$$

for $y \geq 0$. Equations (72) and (73) agree with Eqs. (39) and (40) of Lo and Sickles [9] for the case when $y = h$.

Combined Free-Air Field Solution

Combining symmetric and asymmetric components of the free-air velocity field, Eqs. (67) and (68), and Eqs. (72) and (73), and returning from symmetry variables using the relationships in Eq. (17) leads to the forms

$$\begin{aligned}\bar{u}_{\infty}(p, \pm y) &= \frac{1}{2} \bar{u}_T(p, \pm h) e^{-|p|\beta(y-h)} - \frac{1}{2} \bar{u}_T(p, \mp h) e^{-|p|\beta(y+h)} \\ &\quad \pm \frac{1}{2} \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, \pm h) e^{-|p|\beta(y-h)} \quad (74) \\ &\quad \mp \frac{1}{2} \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, \mp h) e^{-|p|\beta(y+h)}\end{aligned}$$

and

$$\begin{aligned}\bar{v}_{\infty}(p, \pm y) &= \frac{1}{2} \bar{v}_T(p, \pm h) e^{-|p|\beta(y-h)} - \frac{1}{2} \bar{v}_T(p, \mp h) e^{-|p|\beta(y+h)} \\ &\quad \mp \frac{1}{2} i\beta \frac{p}{|p|} \bar{u}_T(p, \pm h) e^{-|p|\beta(y-h)} \quad (75) \\ &\quad \pm \frac{1}{2} i\beta \frac{p}{|p|} \bar{u}_T(p, \mp h) e^{-|p|\beta(y+h)}\end{aligned}$$

In obtaining the Fourier inversion of Eqs. (74) and (75), the behavior of $\bar{u}_T(p, \pm h)$ and $\bar{v}_T(p, \pm h)$ may prevent the divergence of the resulting integrals for a limited range of $\beta(h \pm y)$. In general, however, the integrals remain finite only for $y = \pm h$. Obtaining the inversions under this restriction, the free-air components of the control surfaces may be related to the measured tunnel variables as

$$\begin{aligned}
u_{\infty}(x, \underline{+}h) = & \frac{1}{2} u_T(x, \underline{+}h) - \frac{\beta h}{\pi} \int_{-\infty}^{+\infty} \frac{u_T(\xi, \bar{+}h)}{(\xi-x)^2 + (2\beta h)^2} d\xi \\
& + \frac{1}{2\pi\beta} \oint_{-\infty}^{+\infty} \frac{v_T(\xi, \underline{+}h)}{\xi-x} d\xi \pm \frac{1}{2\pi\beta} \int_{-\infty}^{+\infty} \frac{v_T(\xi, \bar{+}h)}{(\xi-x)^2 + (2\beta h)^2} (\xi-x) d\xi
\end{aligned} \tag{76}$$

and

$$\begin{aligned}
v_{\infty}(x, \underline{+}h) = & \frac{1}{2} v_T(x, \underline{+}h) - \frac{\beta h}{\pi} \int_{-\infty}^{+\infty} \frac{v_T(\xi, \bar{+}h)}{(\xi-x)^2 + (2\beta h)^2} d\xi \\
& \pm \frac{\beta}{2\pi} \oint_{-\infty}^{+\infty} \frac{u_T(\xi, \underline{+}h)}{\xi-x} d\xi \mp \frac{\beta}{2\pi} \int_{-\infty}^{+\infty} \frac{u_T(\xi, \bar{+}h)}{(\xi-x)^2 + (2\beta h)^2} (\xi-x) d\xi .
\end{aligned} \tag{77}$$

Equations (76) and (77) are the same as Eqs. (43) and (44) of Lo and Sickles [9]. The singularity arising in these equations is treated using the Cauchy Principle Value.

With the aid of Eqs. (76) and (77), the boundary value problems for the free-air field in Figs. 4 and 5 may be divided into separate problems for an interior and an exterior region, separated by the additional boundary condition specified at the control surface, as shown in Figs. 6 and 7. The problems for the symmetric and asymmetric interior regions are now analogous to the symmetric and asymmetric tunnel field problems in Figs. 2 and 3, to which solutions have been presented in Eqs. (49), (50), (57) and

(58). Hence, solutions for the symmetric and asymmetric interior regions may be written directly from analogy with these results as

$$\bar{u}_{\infty S}(p, \pm y) = \bar{u}_{\infty S}(p, \pm h) \cosh[p\beta(h-y)] \pm \frac{1}{\beta} \bar{v}_{\infty S}(p, \pm h) \sinh[p\beta(h-y)] \quad (78)$$

$$\bar{v}_{\infty S}(p, \pm y) = \bar{v}_{\infty A}(p, \pm h) \cosh[p\beta(h-y)] \mp i\beta \bar{u}_{\infty S}(p, \pm h) \sinh[p\beta(h-y)] \quad (79)$$

and

$$\bar{u}_{\infty A}(p, \pm y) = \bar{u}_{\infty A}(p, \pm h) \cosh[p\beta(h-y)] \pm \frac{1}{\beta} \bar{v}_{\infty A}(p, \pm h) \sinh[p\beta(h-y)] \quad (80)$$

$$\bar{v}_{\infty A}(p, \pm y) = \bar{v}_{\infty A}(p, \pm h) \cosh[p\beta(h-y)] \mp i\beta \bar{u}_{\infty A}(p, \pm h) \sinh[p\beta(h-y)] \quad (81)$$

Combining symmetric and asymmetric parts and taking the inverse transform to the physical plane, again following the method in Appendix B, yields forms as

$$u_{\infty}(x, \pm y) = \operatorname{Re} [u_{\infty}(x+i\beta(h-y), \pm h)] \mp \frac{1}{\beta} \operatorname{Im} [v_{\infty}(x+i\beta(h-y), \pm h)] \quad (82)$$

and

$$v_{\infty}(x, \pm y) = \operatorname{Re} [v_{\infty}(x+i\beta(h-y), \pm h)] \pm \beta \operatorname{Im} [u_{\infty}(x+i\beta(h-y), \pm h)] \quad (83)$$

for the interior region $0 \leq y \leq h$.

In solving the problem for the exterior region, either of Eqs. (76) or (77) may be used to specify the boundary condition at the control surface. Solutions for the symmetric and asymmetric parts of the exterior free-air

potential for the case where $u_{\infty}(x,h)$ is specified are of the form

$$\bar{\phi}_{\infty S}(p,y) = \frac{i}{p} e^{-|p|\beta(y-h)} \bar{u}_{\infty S}(p,h) \quad (84)$$

and

$$\bar{\phi}_{\infty A}(p,y) = \frac{i}{p} e^{-|p|\beta(y-h)} \bar{u}_{\infty A}(p,h) \quad (85)$$

for $y \geq h$.

Combining the symmetric and asymmetric potentials and imposing the symmetry conditions allows velocity components in the upper and lower exterior free-air fields to be determined as

$$\bar{u}_{\infty}(p,\pm y) = \bar{u}_{\infty}(p,\pm h) e^{-|p|\beta(y-h)} \quad (86)$$

and

$$\bar{v}_{\infty}(p,\pm y) = \mp i\beta \frac{p}{|p|} e^{-|p|\beta(y-h)} \bar{u}_{\infty}(p,\pm h) \quad (87)$$

for $y \geq h$.

For the case where $v_{\infty}(x,h)$ is specified at the control surface, components in the exterior velocity field may be similarly determined as

$$\bar{u}_{\infty}(p,\pm y) = \pm \frac{i}{\beta} \frac{p}{|p|} \bar{v}_{\infty}(p,\pm h) e^{-|p|\beta(y-h)} \quad (88)$$

and

$$\bar{v}_{\infty}(p, \pm y) = \bar{v}_{\infty}(p, \pm h) e^{-|p|^{\beta}(y-h)} \quad (89)$$

for $y \geq h$.

Evaluating Eqs. (87) and (88) at $y = h$ yields relationships between free-air velocity components at the control surface of the forms

$$\bar{u}_{\infty}(p, \pm h) = \pm \frac{1}{\beta} \frac{p}{|p|} \bar{v}_{\infty}(p, \pm h) \quad (90)$$

and

$$\bar{v}_{\infty}(p, \pm h) = \mp i\beta \frac{p}{|p|} \bar{u}_{\infty}(p, \pm h) . \quad (91)$$

Equations (90) and (91) are the same as Eqs. (2) and (3) of Lo and Sickles [9].

Taking inverse transforms in Eqs. (87) and (88) allows velocity components in the exterior free-air region to be determined as

$$u_{\infty}(x, \pm y) = \pm \frac{1}{\pi\beta} \int_{-\infty}^{+\infty} v_{\infty}(\xi, \pm h) \frac{(x-\xi)}{(x-\xi)^2 + \beta^2(y-h)^2} d\xi \quad (92)$$

and

$$v_{\infty}(x, \pm y) = \mp \frac{\beta}{\pi} \int_{-\infty}^{+\infty} u_{\infty}(\xi, \pm h) \frac{(x-\xi)}{(x-\xi)^2 + \beta^2(y-h)^2} d\xi \quad (93)$$

for $|y| \geq h$, which reduce to the classic exterior functional

relationships when $y = h$, as in Eqs. (A1) and (A2) of Lo and Kraft [8]. The singularity arising for the case when $y = h$ is treated using the Cauchy Principle Value.

The interior field solutions in Eqs. (82) and (83) together with the exterior field solutions in Eqs. (92) and (93) allow the entire free-air velocity field to be determined. The free-air components at the control surface in these solutions are obtained solely from the control surface measurements in the tunnel using Eqs. (76) and (77). No information about the model geometry or tunnel wall characteristics is required.

V. INTERFERENCE FIELD SOLUTION

A solution for the interference field in terms of tunnel variables is possible by recognizing that the field contains no singularities, and thus considering the boundary value problem in Fig. 8. The boundary conditions at the control surfaces are expressed in transformed variables from Eq. (9) as

$$\bar{u}_i(p, \pm h) = \bar{u}_T(p, \pm h) - \bar{u}_\infty(p, \pm h) . \quad (94)$$

With the aid of Eq. (74), the boundary conditions may be expressed solely in terms of tunnel field variables as

$$\bar{u}_i(p, \pm h) = \frac{1}{2} \bar{u}_T(p, \pm h) + \frac{1}{2} \bar{u}_T(p, \mp h) e^{-2|p|\beta h} \quad (95)$$

$$\mp \frac{1}{2} \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, \pm h) \pm \frac{1}{2} \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, \mp h) e^{-2|p|\beta h}$$

The boundary value problem in Fig. 8 may be solved for the interference potential as

$$\bar{\phi}_i(p, y) = \frac{i}{p} \left[\bar{u}_i(p, h) \frac{\sinh p\beta(h+y)}{\sinh 2p\beta h} + \bar{u}_i(p, -h) \frac{\sinh p\beta(h-y)}{\sinh 2p\beta h} \right] \quad (96)$$

for $-h \leq y \leq h$. Velocity components in this region are derived from the potential as

$$\bar{u}_i(p, y) = \bar{u}_i(p, h) \frac{\sinh p\beta(h+y)}{\sinh 2p\beta h} + \bar{u}_i(p, -h) \frac{\sinh p\beta(h-y)}{\sinh 2p\beta h} \quad (97)$$

and

$$\bar{v}_i(p, y) = i\beta \bar{u}_i(p, h) \frac{\cosh p\beta(h+y)}{\sinh 2p\beta h} - i\beta \bar{u}_i(p, -h) \frac{\cosh p\beta(h-y)}{\sinh 2p\beta h} \quad (98)$$

Substituting the boundary conditions from Eq. (95) and defining the auxiliary functions

$$H^+(y) \equiv \frac{\sinh p\beta(h+y)}{\sinh 2p\beta h} \quad (99)$$

$$H^-(y) \equiv \frac{\sinh p\beta(h-y)}{\sinh 2p\beta h} \quad (100)$$

and

$$J^+(y) \equiv \frac{\cosh p\beta(h+y)}{\sinh 2p\beta h} \quad (101)$$

$$J^-(y) \equiv \frac{\cosh p\beta(h-y)}{\sinh 2p\beta h} \quad (102)$$

leads to expressions for the velocity components as

$$\begin{aligned} \bar{u}_1(p, y) = \frac{1}{2} & \left[\left[\bar{u}_T(p, h) - \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, h) \right] \left(H^+(y) + H^-(y) e^{-2|p|\beta h} \right) \right. \\ & \left. + \left[\bar{u}_T(p, -h) + \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, -h) \right] \left(H^-(y) + H^+(y) e^{-2|p|\beta h} \right) \right] \end{aligned} \quad (103)$$

and

$$\begin{aligned} \bar{v}_1(p, y) = \frac{i\beta}{2} & \left[\left[\bar{u}_T(p, h) - \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, h) \right] \left(J^+(y) - J^-(y) e^{-2|p|\beta h} \right) \right. \\ & \left. - \left[\bar{u}_T(p, -h) + \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, -h) \right] \left(J^-(y) - J^+(y) e^{-2|p|\beta h} \right) \right] \end{aligned} \quad (104)$$

Manipulating terms involving the auxiliary functions leads to the forms

$$\begin{aligned} \bar{u}_1(p, y) = \frac{1}{2} & \left[\left[\bar{u}_T(p, h) - \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, h) \right] e^{-|p|\beta(h-y)} \right. \\ & \left. + \left[\bar{u}_T(p, -h) + \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, -h) \right] e^{-|p|\beta(h+y)} \right] \end{aligned} \quad (105)$$

and

$$\bar{v}_i(p, y) = \frac{i\beta}{2} \left[\left[\bar{u}_T(p, h) - \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, h) \right] e^{-|p|\beta(h-y)} - \left[\bar{u}_T(p, -h) + \frac{i}{\beta} \frac{p}{|p|} \bar{v}_T(p, -h) \right] e^{-|p|\beta(h+y)} \right] \quad (106)$$

Applying an inverse Fourier transform on Eqs. (105) and (106) leads to solutions for the interference field based solely on measurements at the control surface in the tunnel field as

$$u_i(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left\{ \frac{\beta u_T(\xi, h)(h-y) + \frac{1}{\beta} v_T(\xi, h)(\xi-x)}{(\xi-x)^2 + \beta^2(h-y)^2} + \frac{\beta u_T(\xi, -h)(h+y) - \frac{1}{\beta} v_T(\xi, -h)(\xi-x)}{(\xi-x)^2 + \beta^2(h+y)^2} \right\} d\xi \quad (107)$$

and

$$v_i(x, y) = \frac{-1}{2\pi} \int_{-\infty}^{+\infty} \left\{ \frac{\beta u_T(\xi, h)(\xi-x) - \beta v_T(\xi, h)(h-y)}{(\xi-x)^2 + \beta^2(h-y)^2} - \frac{\beta u_T(\xi, -h)(\xi-x) + \beta v_T(\xi, -h)(h+y)}{(\xi-x)^2 + \beta^2(h+y)^2} \right\} d\xi \quad (108)$$

for $-h \leq y \leq h$. Equation (107) is consistent with Eq. (4) of Lo [7] for the case where $y = 0$.

The integrals in Eqs. (107) and (108) allow the entire interference field to be determined based solely on the control surface measurements in the tunnel. No information about the model geometry or wall characteristics is required.

Perhaps the most significant consequence of this interference solution is that these equations may be used to correct the effects of boundary interference on aerodynamic data measured in a wind tunnel. Using the definition for free-air variables in Eq. (7) and recalling the form of the first-order pressure coefficient consistent with small disturbance theory, the free-air pressure distribution is obtained from the measured distribution in the tunnel as

$$C_{P_{\infty}}(x, 0^{\pm}) = C_{P_T}(x, 0^{\pm}) + 2u_i(x, 0) \quad , \quad (109)$$

where,

$$\begin{aligned} u_i(x, 0) = & \frac{\beta h}{2\pi} \int_{-\infty}^{+\infty} \frac{[u_T(\xi, h) + u_T(\xi, -h)]}{(\xi - x)^2 + (\beta h)^2} d\xi \\ & + \frac{1}{2\pi\beta} \int_{-\infty}^{+\infty} \frac{[v_T(\xi, h) - v_T(\xi, -h)]}{(\xi - x)^2 + (\beta h)^2} (\xi - x) d\xi \end{aligned} \quad (110)$$

from Eq. (107) with $y = 0$.

The lift interference at the model, $v_i(x,0)$ from Eq. (108), may be interpreted as a local modification in the effective angle-of-attack. Since this local angle-of-attack modification will, in general, vary along the model, the pressure distribution obtained from Eq. (109) corresponds to a model in free-air having a different camber distribution than the model in the wind tunnel. However, this effect may be approximated by making a gross adjustment to the model incidence in free-air as

$$\alpha_\infty = \alpha_T - \alpha_i, \quad (111)$$

where,

$$\alpha_i = \int_0^1 v_i(x,0) dx \quad (112)$$

is the integral mean of the local angle-of-attack modifications, and

$$\begin{aligned} v_i(x,0) = & -\frac{\beta}{2\pi} \int_{-\infty}^{+\infty} \frac{[u_T(\xi,h) - u_T(\xi,-h)]}{(\xi-x)^2 + (\beta h)^2} (\xi-x) d\xi \\ & + \frac{\beta}{2\pi} \int_{-\infty}^{+\infty} \frac{[v_T(\xi,h) + v_T(\xi,-h)]}{(\xi-x)^2 + (\beta h)^2} d\xi \end{aligned} \quad (113)$$

from Eq. (108) with $y = 0$.

In applications where the pressure distribution correction in Eq. (109) is not practical, such as in wind tunnel tests where aerodynamic forces and moments are

measured, an approximation for this blockage correction may be determined as an adjustment to the Mach number in free-air, using isentropic relations, as

$$M_{\infty} = M_T + M_T \left(1 + \frac{\gamma-1}{2} M_T^2 \right) \epsilon_B , \quad (114)$$

where,

$$\epsilon_B = \int_0^1 u_i(x,0) dx \quad (115)$$

and $u_i(x,0)$ is obtained from Eq. (110).

Thus, in wind tunnel tests where pressure distributions are measured, the effects of blockage interference may be precisely corrected by adjusting the pressure distribution using Eq. (109). In tests where aerodynamic forces and moments are measured, the effects of blockage interference may be approximately corrected by adjusting the Mach number using Eq. (114). In either case, the effects of lift interference may be precisely corrected by introducing the appropriate camber distribution into the model geometry, or may be approximately corrected by adjusting the free-air angle-of-attack using Eq. (111).

CHAPTER III

APPLICATION TO THE CLASSIC WAVY WALL MODEL WITH A GENERALIZED TUNNEL BOUNDARY CONDITION

I. GENERAL DESCRIPTION

In the following analytical demonstration, the generalized solutions for the tunnel, free-air and interference fields developed in this investigation are applied to the classic problem of flow past a wave-shaped wall, due to Ackeret [10], to clarify the concepts involved in the investigation and to demonstrate the validity and generality of the solutions obtained. The example of the wavy wall is chosen since the resulting tunnel, free-air and interference fields reflect only the effects of thickness, and consequently, it should be expected that the solutions developed in this investigation will yield exact results. Also, arbitrary symmetric profile solutions can be built up by Fourier superposition from this simple example.

Consider the subsonic flow past a small disturbance boundary of sinusoidal shape described by

$$y = \epsilon \sin \lambda x \quad (116)$$

in the presence of a generalized linear homogeneous boundary condition located at some $y = h'$ satisfying the requirement

$$L [\phi_T(x, h')] = 0 , \quad (117)$$

as shown in Fig. 9. In the region $0 \leq y \leq h'$, the potential $\phi_T(x, y)$ represents the solution for flow over the wavy wall in a wind tunnel with some general linear homogeneous boundary condition. It is readily shown that the solution for this potential may be written as

$$\phi_T(x, y) = \frac{\varepsilon}{\beta} \left[\sinh \lambda \beta y - \frac{L(\sinh \lambda \beta h')}{L(\cosh \lambda \beta h')} \cosh \lambda \beta y \right] \cos \lambda x . \quad (118)$$

Defining

$$G(h') \equiv \frac{L(\sinh \lambda \beta h')}{L(\cosh \lambda \beta h')} \quad (119)$$

allows perturbation velocity components in the tunnel field to be derived from the potential in Eq. (118) as

$$u_T(x, y) = -\frac{\varepsilon \lambda}{\beta} [\sinh \lambda \beta y - G(h') \cosh \lambda \beta y] \sin \lambda x \quad (120)$$

and

$$v_T(x, y) = \varepsilon \lambda [\cosh \lambda \beta y - G(h') \sinh \lambda \beta y] \cos \lambda x . \quad (121)$$

Arbitrarily defining a control surface at some $y = h$, where $0 < h \leq h'$, allows the flow variables at the control surface to be written directly as

$$u_T(x, h) = -\frac{\varepsilon \lambda}{\beta} [\sinh \lambda \beta h - G(h') \cosh \lambda \beta h] \sin \lambda x \quad (122)$$

and

$$v_T(x,h) = \varepsilon\lambda [\cosh \lambda\beta h - G(h')\sinh \lambda\beta h] \cos \lambda x . \quad (123)$$

The generalized solutions for the tunnel, free-air and interference fields developed in this investigation will be evaluated based solely on the flow variables at the control surface in Eqs. (122) and (123). For the purposes of this analytical demonstration, all solutions will be obtained in closed form; however, in a practical application of the theory, only discrete measurements of the control surface variables are available, and consequently, the numerical techniques described in the next section would be used to obtain discretized solutions.

II. TUNNEL FIELD SOLUTION

The solutions presented in Eqs. (61) and (62) for the tunnel velocity field will be evaluated based on the control surface information. The analytic continuations of Eqs. (122) and (123) into the complex plane are of the forms

$$\begin{aligned} \operatorname{Re} [u_T(x+i\beta(h-y),h)] = \\ - \frac{\varepsilon\lambda}{\beta} [\sinh \lambda\beta h - G(h')\cosh \lambda\beta h] \cosh \lambda\beta(h-y) \sin \lambda x \quad (124) \end{aligned}$$

$$\begin{aligned} \text{Im} [u_T(x+i\beta(h-y), h)] = \\ - \frac{\varepsilon\lambda}{\beta} [\sinh \lambda\beta h - G(h') \cosh \lambda\beta h] \sinh \lambda\beta(h-y) \cos \lambda x \end{aligned} \quad (125)$$

and

$$\begin{aligned} \text{Re} [v_T(x+i\beta(h-y), h)] = \\ \varepsilon\lambda [\cosh \lambda\beta h - G(h') \sinh \lambda\beta h] \cosh \lambda\beta(h-y) \cos \lambda x \end{aligned} \quad (126)$$

$$\begin{aligned} \text{Im} [v_T(x+i\beta(h-y), h)] = \\ - \varepsilon\lambda [\cosh \lambda\beta h - G(h') \sinh \lambda\beta h] \sinh \lambda\beta(h-y) \sin \lambda x . \end{aligned} \quad (127)$$

Substituting into Eqs. (61) and (62) and reducing yields

$$u_T(x, y) = - \frac{\varepsilon\lambda}{\beta} [\sinh \lambda\beta y - G(h') \cosh \lambda\beta y] \sin \lambda x \quad (128)$$

and

$$v_T(x, y) = \varepsilon\lambda [\cosh \lambda\beta y - G(h') \sinh \lambda\beta y] \cos \lambda x , \quad (129)$$

which agree with the forms in Eqs. (120) and (121).

III. FREE-AIR FIELD SOLUTION

The free-air solution for flow over the wavy wall may be obtained directly from the tunnel solution in Eq. (118) by taking the limit as $h' \rightarrow \infty$ and recognizing that

$$\lim_{h' \rightarrow \infty} \frac{L(\sinh \lambda\beta h')}{L(\cosh \lambda\beta h')} = 1 \quad (130)$$

for all L . Hence, the free-air potential is of the form

$$\phi_{\infty}(x,y) = -\frac{\varepsilon}{\beta} e^{-\lambda\beta y} \cos \lambda x \quad (131)$$

for $y \geq 0$.

Velocity components in the free-air field are determined from this potential as

$$u_{\infty}(x,y) = \frac{\varepsilon\lambda}{\beta} e^{-\lambda\beta y} \sin \lambda x \quad (132)$$

and

$$v_{\infty}(x,y) = \varepsilon\lambda e^{-\lambda\beta y} \cos \lambda x . \quad (133)$$

A solution for the free-air field will be obtained based solely on the control surface information in the tunnel field and compared with the known solution in Eqs. (132) and (133).

The free-air components at the control surface will be derived by Eqs. (76) and (77) using the control surface flow variables in Eqs. (122) and (123) together with the symmetry condition. Substituting appropriately leads to the expressions

$$\begin{aligned} u_{\infty}(x,h) = & \frac{\varepsilon\lambda}{\beta} \left\{ -\frac{1}{2} [\sinh \lambda\beta h - G(h') \cosh \lambda\beta h] \sin \lambda x \right. \\ & + \frac{\beta h}{\pi} [\sinh \lambda\beta h - G(h') \cosh \lambda\beta h] \int_{-\infty}^{+\infty} \frac{\sin \lambda\xi}{(\xi-x)^2 + (2\beta h)^2} d\xi \\ & - \frac{1}{2\pi} [\cosh \lambda\beta h - G(h') \sinh \lambda\beta h] \int_{-\infty}^{+\infty} \frac{\cos \lambda\xi}{\xi-x} d\xi \\ & \left. - \frac{1}{2\pi} [\cosh \lambda\beta h - G(h') \sinh \lambda\beta h] \int_{-\infty}^{+\infty} \frac{\cos \lambda\xi}{(\xi-x)^2 + (2\beta h)^2} (\xi-x) d\xi \right\} \end{aligned} \quad (134)$$

and

$$\begin{aligned}
 v_{\infty}(x,h) = \varepsilon \lambda \left\{ \frac{1}{2} [\cosh \lambda \beta h - G(h') \sinh \lambda \beta h] \cos \lambda x \right. \\
 + \frac{\beta h}{\pi} [\cosh \lambda \beta h - G(h') \sinh \lambda \beta h] \int_{-\infty}^{+\infty} \frac{\cos \lambda \xi}{(\xi-x)^2 + (2\beta h)^2} d\xi \\
 - \frac{1}{2\pi} [\sinh \lambda \beta h - G(h') \cosh \lambda \beta h] \int_{-\infty}^{+\infty} \frac{\sin \lambda \xi}{\xi-x} d\xi \\
 \left. + \frac{1}{2\pi} [\sinh \lambda \beta h - G(h) \cosh \lambda \beta h] \int_{-\infty}^{+\infty} \frac{\sin \lambda \xi}{(\xi-x)^2 + (2\beta h)^2} (\xi-x) d\xi \right\} \quad (135)
 \end{aligned}$$

The integrals in Eqs. (134) and (135) are readily evaluated using the calculus of residues, resulting in expressions as

$$\begin{aligned}
 u_{\infty}(x,h) = \frac{1}{2} \frac{\varepsilon \lambda}{\beta} \left\{ \cosh \lambda \beta h [1 + G(h')] - \sinh \lambda \beta h [1 + G(h')] \right. \\
 + \sinh \lambda \beta h [1 - G(h')] e^{-2\lambda \beta h} \\
 \left. + \cosh \lambda \beta h [1 - G(h')] e^{-2\lambda \beta h} \right\} \sin \lambda x \quad (136)
 \end{aligned}$$

and

$$v_{\infty}(x,h) = \frac{1}{2} \varepsilon \lambda \left\{ \begin{aligned} & \cosh \lambda \beta h [1 + G(h')] - \sinh \lambda \beta h [1 + G(h')] \\ & + \sinh \lambda \beta h [1 - G(h')] e^{-2\lambda \beta h} \\ & + \cosh \lambda \beta h [1 - G(h')] e^{-2\lambda \beta h} \end{aligned} \right\} \cos \lambda x \quad (137)$$

Combining as necessary and reducing leads to the forms

$$u_{\infty}(x,h) = \frac{\varepsilon \lambda}{\beta} e^{-\lambda \beta h} \sin \lambda x \quad (138)$$

and

$$v_{\infty}(x,h) = \varepsilon \lambda e^{-\lambda \beta h} \cos \lambda x \quad (139)$$

These solutions for the free-air components at the control surface will be used to evaluate Eqs. (82) and (83) to obtain the interior free-air field solutions.

Analytically continuing Eqs. (138) and (139) into the complex plane leads to Real and Imaginary parts as

$$\operatorname{Re} [u_{\infty}(x+i\beta(h-y), h)] = \frac{\varepsilon \lambda}{\beta} e^{-\lambda \beta h} \cosh \lambda \beta (h-y) \sin \lambda x \quad (140)$$

$$\operatorname{Im} [u_{\infty}(x+i\beta(h-y), h)] = \frac{\varepsilon \lambda}{\beta} e^{-\lambda \beta h} \sinh \lambda \beta (h-y) \sin \lambda x \quad (141)$$

and

$$\operatorname{Re} [v_{\infty}(x+i\beta(h-y), h)] = \varepsilon \lambda e^{-\lambda \beta h} \cosh \lambda \beta (h-y) \cos \lambda x \quad (142)$$

$$\operatorname{Im} [v_{\infty}(x+i\beta(h-y), h)] = -\varepsilon \lambda e^{-\lambda \beta h} \sinh \lambda \beta (h-y) \cos \lambda x . \quad (143)$$

Substituting into Eqs. (82) and (83) and reducing yields

$$u_{\infty}(x, y) = \frac{\varepsilon \lambda}{\beta} e^{-\lambda \beta y} \sin \lambda x \quad (144)$$

and

$$v_{\infty}(x, y) = \varepsilon \lambda e^{-\lambda \beta y} \cos \lambda x , \quad (145)$$

which agree with the known solutions in Eqs. (127) and (128).

IV. INTERFERENCE FIELD SOLUTION

The solution for the interference field potential may be determined directly from the tunnel and free-air solutions in Eqs. (118) and (131) to yield

$$\phi_i(x, y) = -\frac{\varepsilon}{\beta} [G(h') - 1] \cosh \lambda \beta y \cos \lambda x \quad (146)$$

for $0 \leq y \leq h'$. Interference velocity components are determined from this potential as

$$u_i(x, y) = \frac{\varepsilon \lambda}{\beta} [G(h') - 1] \cosh \lambda \beta y \sin \lambda x \quad (147)$$

$$v_i(x,y) = - \epsilon \lambda [G(h')-1] \sinh \lambda \beta y \cos \lambda x . \quad (148)$$

Solutions for the interference field will be determined based entirely on control surface information in the tunnel field using the results developed in this investigation and compared with the known solution above.

The integrals for interference evaluation in Eqs. (107) and (108) provide a solution for the interference field directly in terms of the tunnel control surface variables. Substituting the control surface variables in Eqs. (122) and (123) into these integrals with the aid of the symmetric condition leads to the forms

$$\begin{aligned} u_i(x,y) = \frac{\epsilon \lambda}{2\pi} & \left\{ -(\sinh \lambda \beta h - G(h')) \cosh \lambda \beta h (h-y) \int_{-\infty}^{+\infty} \frac{\sin \lambda \xi}{(\xi-x)^2 + \beta^2 (h-y)^2} d\xi \right. \\ & + \frac{1}{\beta} (\cosh \lambda \beta h - G(h')) \sinh \lambda \beta h \int_{-\infty}^{+\infty} \frac{\cos \lambda \xi}{(\xi-x)^2 + \beta^2 (h-y)^2} (\xi-x) d\xi \\ & - (\sinh \lambda \beta h - G(h')) \cosh \lambda \beta h (h+y) \int_{-\infty}^{+\infty} \frac{\sin \lambda \xi}{(\xi-x)^2 + \beta^2 (h+y)^2} d\xi \\ & \left. + \frac{1}{\beta} (\cosh \lambda \beta h - G(h')) \sinh \lambda \beta h \int_{-\infty}^{+\infty} \frac{\cos \lambda \xi}{(\xi-x)^2 + \beta^2 (h+y)^2} (\xi-x) d\xi \right\} \quad (149) \end{aligned}$$

and

$$\begin{aligned}
 v_i(x,y) = \frac{\varepsilon\lambda}{2\pi} & \left\{ (\sinh \lambda\beta h - G(h')) \cosh \lambda\beta h \int_{-\infty}^{+\infty} \frac{\sin \lambda\xi}{(\xi-x)^2 + \beta^2 (h-y)^2} (\xi-x) d\xi \right. \\
 & + \beta (\cosh \lambda\beta h - G(h')) \sinh \lambda\beta h (h-y) \int_{-\infty}^{+\infty} \frac{\cos \lambda\xi}{(\xi-x)^2 + \beta^2 (h-y)^2} d\xi \\
 & + (\sinh \lambda\beta h - G(h')) \cosh \lambda\beta h \int_{-\infty}^{+\infty} \frac{\sin \lambda\xi}{(\xi-x)^2 + \beta^2 (h+y)^2} (\xi-x) d\xi \\
 & \left. + \beta (\cosh \lambda\beta h - G(h')) \sinh \lambda\beta h (h+y) \int_{-\infty}^{+\infty} \frac{\cos \lambda\xi}{(\xi-x)^2 + \beta^2 (h+y)^2} d\xi \right\} . \quad (150)
 \end{aligned}$$

Evaluating the integrals using residue calculus results in the expressions

$$\begin{aligned}
 u_i(x,y) = \frac{1}{2} \frac{\varepsilon\lambda}{\beta} & \left\{ \sinh \lambda\beta h \left[e^{-\lambda\beta (h-y)} (G(h')-1) + e^{-\lambda\beta (h+y)} (G(h')-1) \right] \right. \\
 & \left. + \cosh \lambda\beta h \left[e^{-\lambda\beta (h-y)} (G(h')-1) + e^{-\lambda\beta (h+y)} (G(h')-1) \right] \right\} \sin \lambda x \quad (151)
 \end{aligned}$$

and

$$\begin{aligned}
 v_i(x,y) = \frac{1}{2} \varepsilon\lambda & \left\{ \sinh \lambda\beta h \left[e^{-\lambda\beta (h-y)} (G(h')-1) - e^{-\lambda\beta (h+y)} (G(h')-1) \right] \right. \\
 & \left. + \cosh \lambda\beta h \left[e^{-\lambda\beta (h-y)} (G(h')-1) - e^{-\lambda\beta (h+y)} (G(h')-1) \right] \right\} \cos \lambda x \quad (152)
 \end{aligned}$$

Combining terms leads to the forms

$$u_i(x,y) = \frac{\varepsilon\lambda}{\beta} [G(h')-1] \cosh \lambda\beta y \sin \lambda x \quad (153)$$

and

$$v_i(x,y) = -\varepsilon\lambda [G(h')-1] \sinh \lambda\beta y \cos \lambda x , \quad (154)$$

which agree with the known solutions in Eqs. (147) and (148).

The methods developed in this investigation have led to exact solutions for the tunnel, free-air and interference fields for the case of the wavy wall in the presence of a generalized linear homogeneous boundary condition. It should be pointed out that the information about the model geometry and tunnel boundary condition was used only to obtain theoretical solutions for comparison. The actual field solutions were based solely on the control surface quantities in the tunnel field.

CHAPTER IV

NUMERICAL DEMONSTRATIONS

I. METHOD OF CALCULATION

Since in a practical application of the theory the control surface flow variable distributions $u_T(x, \pm h)$ and $v_T(x, \pm h)$ will be given in the form of discrete measured values rather than as closed form analytical expressions, the field solutions presented in this investigation will need to be evaluated using numerical techniques. In this section, numerical methods developed specifically for this purpose are presented and a variety of examples are treated using these methods.

The solutions for the tunnel and free-air fields in Eqs. (61) and (62), and Eqs. (82) and (83), involve analytically continuing the discrete representations for the control surface variables into the complex plane. The most successful approach to this involves forming a real-valued series expansion of orthogonal polynomials to represent the control surface distributions, and then evaluating the series for the appropriate complex argument.

Specifically, over some finite interval $a < x < b$, the discrete control surface distributions $u_T(x, \pm h)$ and $v_T(x, \pm h)$ are interpolated to a fine spacing using a third-order spline fit routine. The resulting distributions are

transformed to the interval of orthogonality - $1 < \hat{x} < 1$ for the Tschhebyscheff polynomials of the second kind through the linear transformation

$$\hat{x} = \frac{2}{(b-a)} x - \frac{(b+a)}{(b-a)} . \quad (155)$$

The Tschhebyscheff Type II polynomials are used for the expansion because the finite interval of orthogonality allows solutions to be determined over any segment on the x-axis and because the integrals for the series coefficients are nonsingular. Each of the control surface distributions is represented in a series as

$$f(\hat{x}, h) = \sum_{n=0}^{\infty} c_n U_n(\hat{x}) , \quad (156)$$

where,

$$c_n = \int_{-1}^{+1} f(\hat{x}, h) U_n(\hat{x}) \sqrt{1 - \hat{x}^2} d\hat{x} . \quad (157)$$

The polynomials $U_n(\hat{x})$ are generated from the recursion relationship

$$U_n(\hat{x}) = 2\hat{x} U_{n-1}(\hat{x}) - U_{n-2}(\hat{x}) \quad (158)$$

with the conditions

$$\begin{aligned} U_0(\hat{x}) &= 1 \\ U_1(\hat{x}) &= 2\hat{x} \end{aligned} \quad (159)$$

The integral for each of the series coefficients in Eq. (157) is treated using a simple trapezoidal quadrature technique.

The expansion of Eq. (156) is exact if the number of terms in the series is infinite; however, in practical applications the series must be truncated after a finite number of terms. A limit on the number of terms that can accurately represent the series arises from machine round-off error involved in the numerical integration, which eventually becomes of the same order as the coefficient value since the coefficients tend to zero for sufficiently large n and the round-off error remains constant. Consequently, a criterion is necessary to determine when the series is to be truncated.

From practical considerations, a logical criterion is to continue to add terms to the series as long as the resulting n -term series is a better approximation to the real-valued function than the previous $(n-1)$ -term series. In this criterion, the test to determine if the series has been improved by including the n -th term is to compare the integral of the n -term approximation with that of the original function. Specifically, a parameter ϵ_N is defined as

$$\epsilon_N \equiv \frac{\sqrt{\int_{-1}^{+1} \left[\sum_{n=0}^N c_n U_n(\hat{x}) - f(\hat{x}, h) \right]^2 d\hat{x}}}{\int_{-1}^{+1} f(\hat{x}, h) d\hat{x}} \quad (160)$$

and the series is terminated at (N-1) terms when

$$\left| \frac{\epsilon_N}{\epsilon_{N-1}} \right| > 1 . \quad (161)$$

This leads to the series expansion utilizing the maximum accuracy attainable with a given computer. In practical applications on an IBM 370/165 computer, the parameter ϵ_N typically reaches a minimum around 10^{-4} before it begins increasing, leading to a series of typically 10 to 20 terms.

The resulting series expansion for each of the control surface distributions is then evaluated for the complex variable

$$\hat{z} = \hat{x} + i\beta(\hat{h} - \hat{y}) \quad (162)$$

as

$$f(\hat{x} + i\beta(\hat{h} - \hat{y}), h) = \sum_{n=0}^N c_n U_n(\hat{x} + i\beta(\hat{h} - \hat{y})) , \quad (163)$$

where the coefficients are the same as for the real-valued series. Real and Imaginary parts of Eq. (163) are then

separated, either using the complex arithmetic capabilities of most modern computers, or by considering separate series for each from the recursion relationship in Eq. (158).

As a check on the validity of this numerical technique for evaluating the complex continuations of the control surface distributions, the method is applied to a test function for which the Real and Imaginary parts may be evaluated directly in closed form. In particular, the function

$$f(x,h) = e^{-\frac{1}{6}\left(x - \frac{1}{2}\right)^2} \quad (164)$$

on the interval $-6 < x < 6$ is used for this test since it is similar to the flow variables $u_T(x, \pm h)$ that will be encountered in practical applications. The closed form analytic continuations of this function leads to expressions for the complex parts as

$$\text{Re} [f(x+i\beta(h-y), h)] = e^{-\frac{1}{6} [x^2 - \beta^2 (h-y)^2 - \frac{1}{4}]} \cos x\beta(h-y) \quad (165)$$

$$\text{Im} [f(x+i\beta(h-y), h)] = e^{-\frac{1}{6} [x^2 - \beta^2 (h-y)^2 - \frac{1}{4}]} \sin x\beta(h-y) \quad (166)$$

Applying the numerical technique to this function with typical parameter values $\beta = 0.8$, $h = 0.5$, and $y = 0$ on the interval $-6 < x < 6$ leads to the numerical solutions for the Real and Imaginary parts in Fig. 10. The agreement between the computed complex parts and the values generated

by Eqs. (165) and (166) indicates that the numerical technique is, indeed, capable of analytically continuing the discrete representations for the control surface distributions into the complex plane.

The solutions for the free-air velocity components at the control surface in Eqs. (76) and (77) may be evaluated numerically using simple integration techniques. The nonsingular integrals are evaluated by interpolating the discrete flow variable distributions using a third-order spline fit and applying a simple trapezoidal quadrature routine.

The singular integrals of the form

$$I(x, \pm h) = \int_{-\infty}^{+\infty} \frac{f_T(\xi, \pm h)}{\xi - x} d\xi \quad (167)$$

are evaluated by treating the equivalent form

$$I(x, \pm h) = \int_{-\infty}^{+\infty} \frac{f_T(\xi, \pm h) - f_T(x, \pm h)}{\xi - x} d\xi + f_T(x, \pm h) \int_{-\infty}^{+\infty} \frac{d\xi}{\xi - x} . \quad (168)$$

The second term above represents the integral of an odd function over an even interval and hence, vanishes identically. The first integral is treated using a simple trapezoidal quadrature technique and judiciously avoiding the singular point to avoid machine overflow restrictions.

Finally, the interference field solutions in Eqs. (107) and (108) are evaluated by interpolating the discrete flow variable distributions and applying a simple trapezoidal quadrature technique. Evaluating Eq. (107) with $y = 0$ allows Eq. (109) to be evaluated for the interference-free pressure distribution.

The numerical techniques described are used to evaluate the solutions for the tunnel, free-air and interference fields based solely on discrete measurements of the flow variables $u_T(x, \pm h)$ and $v_T(x, \pm h)$ at the control surface. In the following section, these techniques are applied to a variety of numerical examples simulating practical applications of the theory.

II. NUMERICAL EXAMPLES

The results of numerical simulations based on the two-dimensional transonic small disturbance computer program TSFOIL [11] are presented in this section. The examples included deal with a variety of model geometries and tunnel boundary characteristics so as to establish the generality of the solutions.

The first example involves a 12-percent thickness biconvex circular arc airfoil at Mach number 0.6 and zero incidence in a wind tunnel with straight solid walls located at $h' = 0.75$. Solutions for the tunnel and free-air fields are obtained from the TSFOIL program. A comparison of the airfoil surface pressure distribution for

the tunnel and free-air cases in Fig. 11a indicates a significant level of interference at the model. Choosing the control surfaces at $h = \pm 0.5$ and using the distributions $u_T(x, \pm h)$ and $v_T(x, \pm h)$ computed by the TSFOIL program to evaluate Eq. (109) leads to the pressure distribution corrected for the effects of blockage interference in Fig. 11a. Since this example deals with a nonlifting (i.e., purely symmetric) case, the interference correction is exact and the free-air pressure distribution can be obtained directly from the measured distribution in the tunnel.

The second example deals with the same airfoil at the same conditions, but considers the case of an open-jet tunnel boundary condition. Solutions for the tunnel and free-air fields are again computed by the simulation program. The airfoil surface pressure distributions are shown in Fig. 11b and compared with the corrected distribution from Eq. (109), based solely on the control surface flow variable distributions in the tunnel. The interference correction is again seen to be exact for this purely symmetric case. The example shown in Fig. 11c also deals with the same airfoil at the same condition, but considers the case of a uniformly porous wall tunnel. The interference correction is again exact.

The three examples presented demonstrate that the solutions are independent of the tunnel wall characteristics.

In order to demonstrate the independence with regard to the model geometry, the examples are repeated in the same manner described, but now considering the case of an NACA 0012 airfoil. Results for the solid wall, open-jet and porous wall tunnel cases are presented in Figs. 12a, 12b and 12c, respectively. The interference correction is again exact in these nonlifting configurations.

In order to demonstrate the applicability of the interference solution in Eq. (109) to lifting configurations, examples are presented involving the biconvex and NACA 0012 airfoils at incidence. In Fig. 13, the biconvex airfoil is considered at Mach number 0.6 and 1-degree incidence in a tunnel with straight solid walls at $h' = 0.75$.

The resulting pressure distribution is first adjusted for the effects of blockage interference using the precise correction in Eq. (109). Finally, the effects of lift interference are approximately corrected by making a gross adjustment to the free-air angle-of-attack. Following the approach in Eq. (111) leads to an angle-of-attack of 1.10-degrees in the free-air case. The slight residual interference apparent in Fig. 13 is due in part to the fact that a gross correction for the lift interference was used, and in part to the error involved in the development of the interference solutions by assuming that the circulation in the tunnel and free-air cases is the same. Thus, the

pressure distribution in the wind tunnel at Mach number 0.6 and 1-degree model incidence is adjusted using Eq. (109) and then approximately corresponds to the distribution on the same airfoil in free-air at Mach number 0.6 and 1.10-degrees model incidence.

The same procedure is repeated for the case of the NACA 0012 airfoil at the same tunnel conditions in Fig. 14. In this case, the corrected pressure distribution in the tunnel approximately corresponds to that for a model in free-air at an incidence of 1.15 degrees.

The example shown in Fig. 15 considers the biconvex airfoil at Mach number 0.6 and 2-degrees incidence in a tunnel with straight solid walls. Again following the same procedure, the pressure distribution is precisely corrected for the effects of blockage interference using Eq. (109), and then approximately corrected for the lift interference by adjusting the free-air angle-of-attack to 2.21 degrees. The residual interference seen is due, in part, to the approximate correction for the lift interference.

In Fig. 16, the example of the biconvex airfoil at Mach number 0.6 and 2-degrees incidence is repeated, but in this case the exact correction for the lift interference is used. This amounts to modifying the geometry of the model in the tunnel by introducing a camber distribution related to the lift interference as

$$c(x) = \int_0^x v_i(x,0) dx .$$

The pressure distribution on the cambered wing in the tunnel at the original Mach number and incidence is then corrected for blockage effects using Eq. (109) and compared with the pressure distribution on the original wing at Mach number 0.6 and 2-degrees incidence in free-air. Since in this case both the blockage and lift corrections are precise, the residual error is due entirely to the assumption of equivalent circulation in the tunnel and free-air cases.

In order to demonstrate an application of the complex parts solutions for the tunnel and free-air fields in Eqs. (61) and (62), and Eqs. (82) and (83), the solution for the tunnel field is evaluated for the case of the NACA 0012 airfoil at Mach number 0.6 and zero incidence in a tunnel with straight solid walls. The control surface distributions $u_T(x, \pm h)$ and $v_T(x, \pm h)$ at $h = 0.5$ are interpolated to a fine spacing using a third-order spline fit routine and are analytically continued into the complex plane using the numerical technique described in the previous section. The real-valued series expansions for $u_T(x, \pm h)$ are each terminated after 14 terms and reach a minimum ϵ_N -value at $3(10^{-3})$, resulting in a real-valued series approximation that reproduces the original control

surface distribution to within 0.1 percent on the interval $-0.9 \leq \hat{x} \leq 0.9$. The expansions for $v_T(x, \pm h)$ are each terminated after 19 terms and reach a minimum ϵ_N -value at $5(10^{-4})$, leading to a series approximation accurate to within 0.02 percent over the same $\hat{x}$ -interval. Evaluating the complex series with $y = 0.001$ allows Eq. (61) to be evaluated for $u_T(x, 0^+)$ and compared with the solution obtained from the numerical simulation. The comparison appears in Fig. 17 and indicates a discrepancy with the TSFOIL solution.

Since the numerical method was capable of correctly evaluating the complex parts of the test function presented in Eq. (165), as well as a wide range of other test functions, it is concluded that the distribution $u_T(x, 0^+)$ obtained from the evaluation of Eq. (61) is, indeed, the correct analytic continuation of the control surface flow variable distributions. An analytical sensitivity investigation of Eq. (59), superimposing a high frequency sinusoidal error function onto the distribution $u_T(x, h)$ and $v_T(x, h)$, indicates that the analytic continuations may be dramatically altered by small errors in the control surface distributions. Errors introduced in these distributions during the TSFOIL solution routine are believed to be the source of the discrepancy in Fig. 17.

CHAPTER V

CONCLUDING REMARKS

The investigation described herein develops a general solution in closed form for wind tunnel boundary-induced interference in two-dimensional subsonic flow based solely on measured velocity components at a control surface in the tunnel. The solution presented, unlike results developed from classical wind tunnel theory, does not require any a priori knowledge of the tunnel wall characteristics or model geometry.

The approach employed leads to exact solutions for the interference in nonlifting cases. In situations involving lift, the interference solution developed will contain a small error in the correction for lift.

The example of the wavy wall with a generalized boundary condition demonstrates in closed form that the solution developed leads to exact results for nonlifting cases. The numerical examples presented, dealing with a variety of model geometries and tunnel wall characteristics, demonstrate not only the generality of the interference solution developed, but also the applicability of the solution for determining the interference in cases involving small lift.

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BIBLIOGRAPHY

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APPENDIXES

APPENDIX A

FIGURES

Figures 1 through 17 contain schematic diagrams of the boundary value problems for the tunnel, free-air and interference fields, in addition to results obtained by applying the interference solutions developed in this investigation to a variety of model geometries and tunnel boundary conditions.

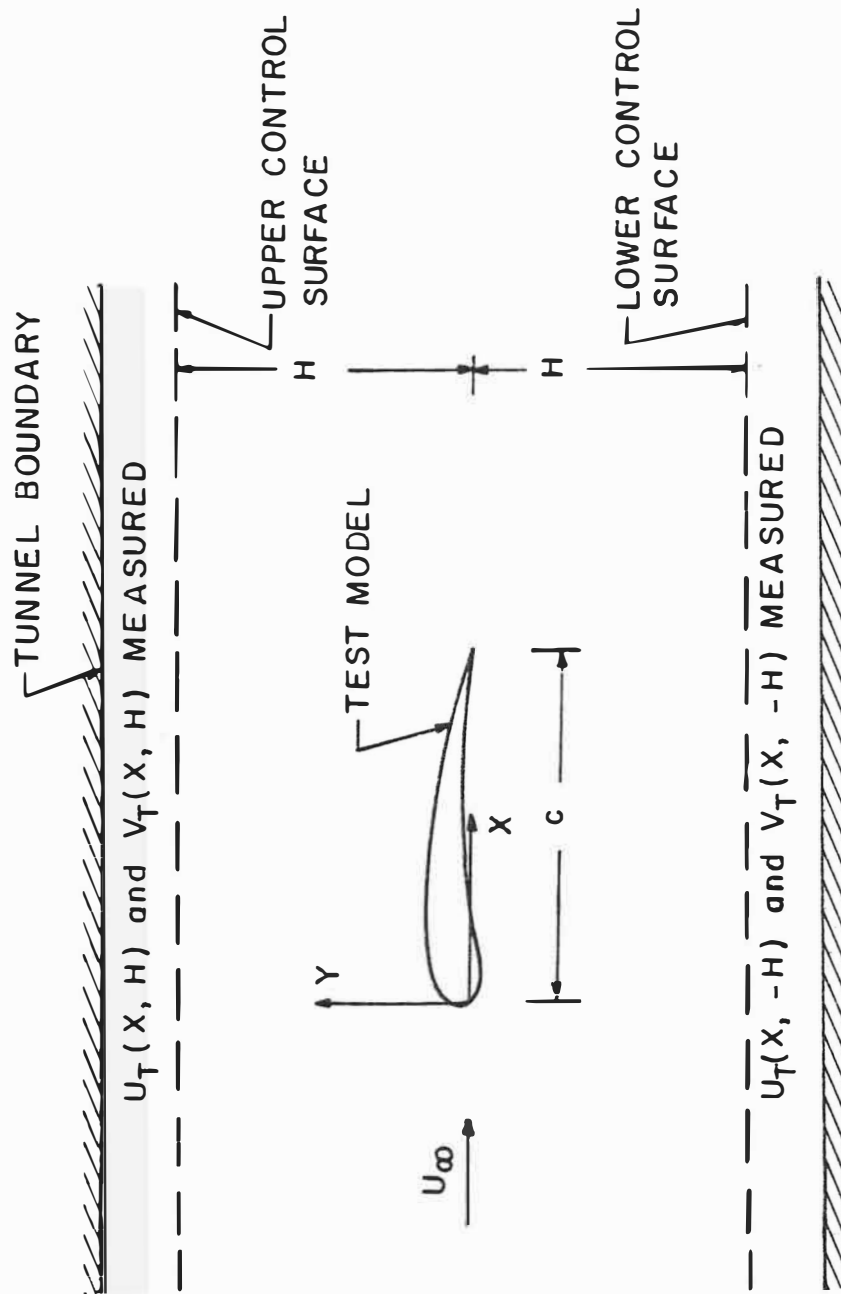


Figure 1. Schematic of the physical problem for determining the interference field in a wind tunnel.

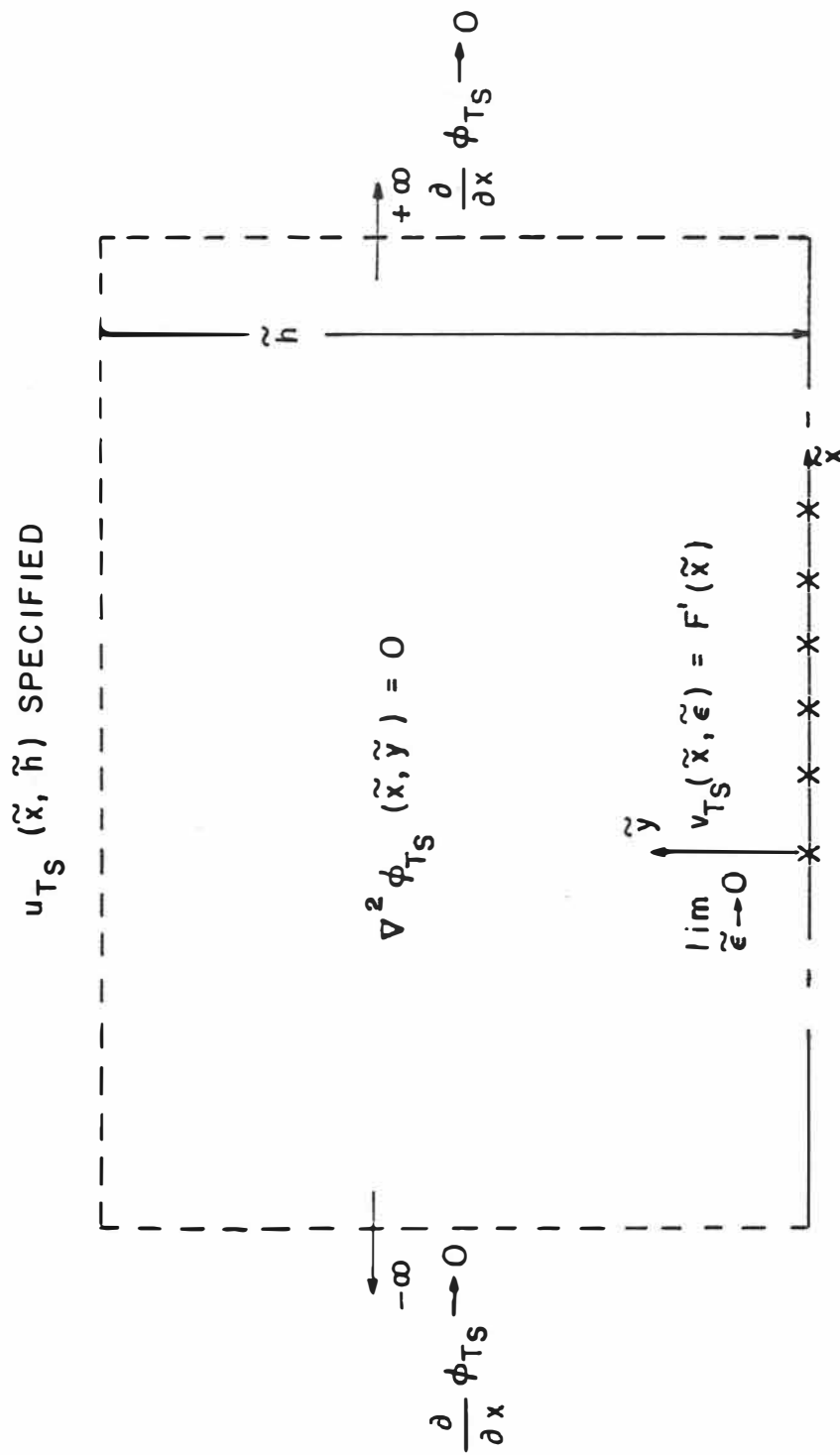


Figure 2. Boundary value problem for the symmetric tunnel field potential in the upper half-plane.

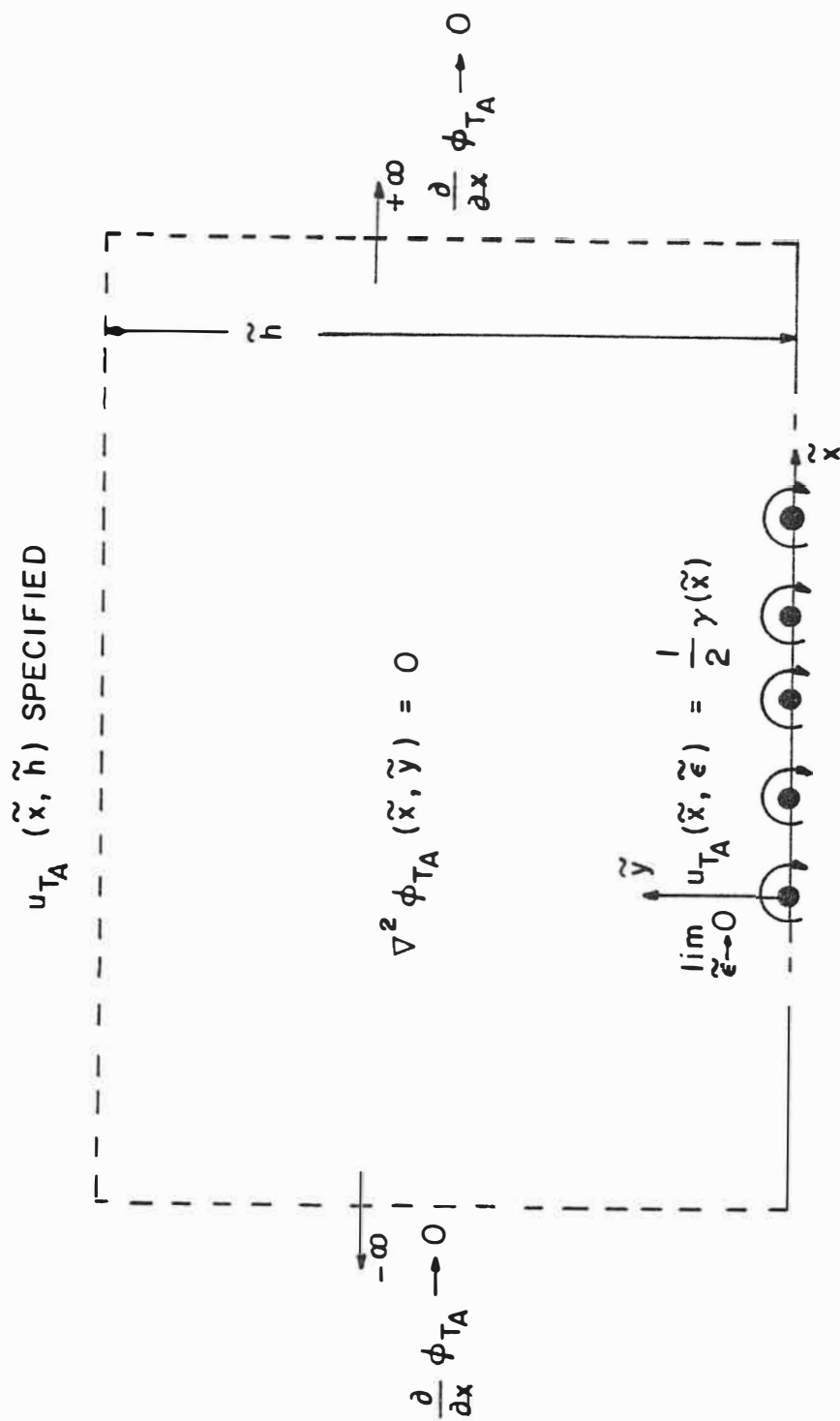


Figure 3. Boundary value problem for the asymmetric tunnel field potential in the upper half-plane.

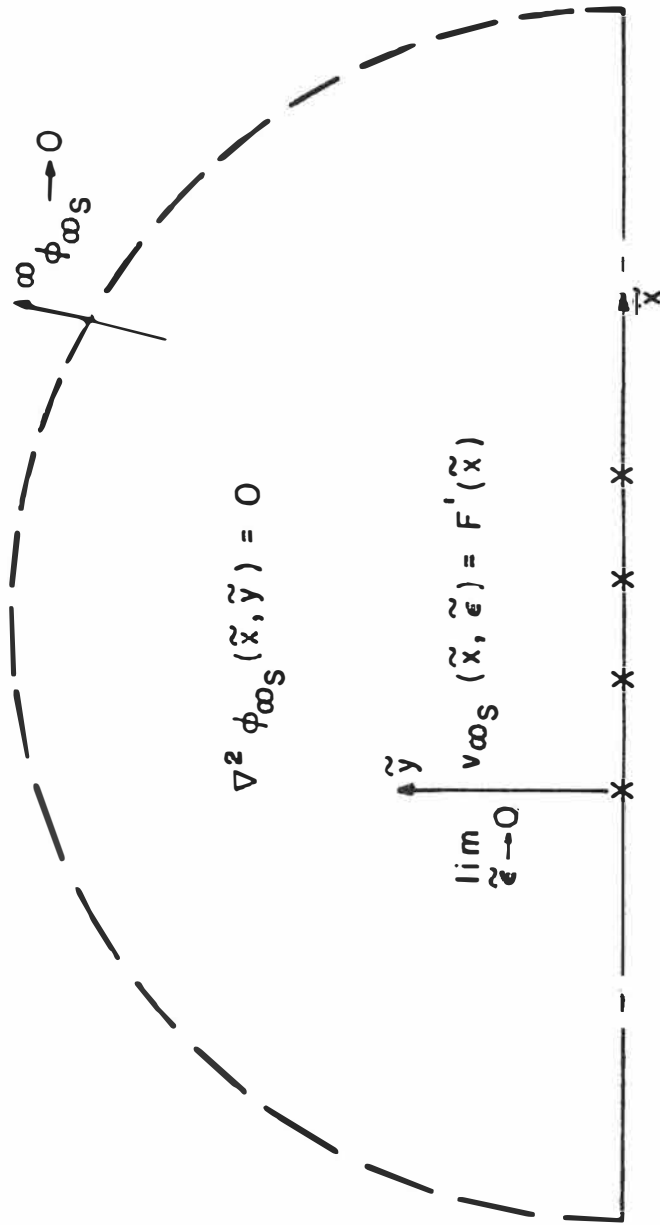


Figure 4. Boundary value problem for the symmetric free-air field potential in the upper half-plane.

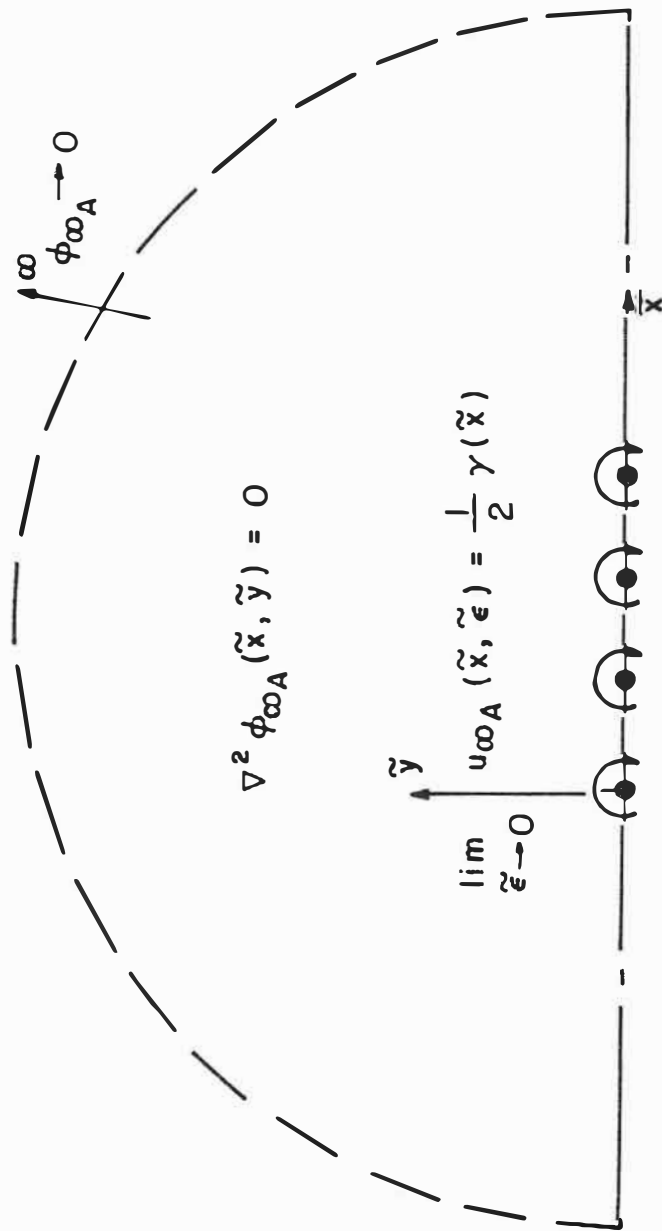


Figure 5. Boundary value problem for the asymmetric free-air field potential in the upper half-plane.

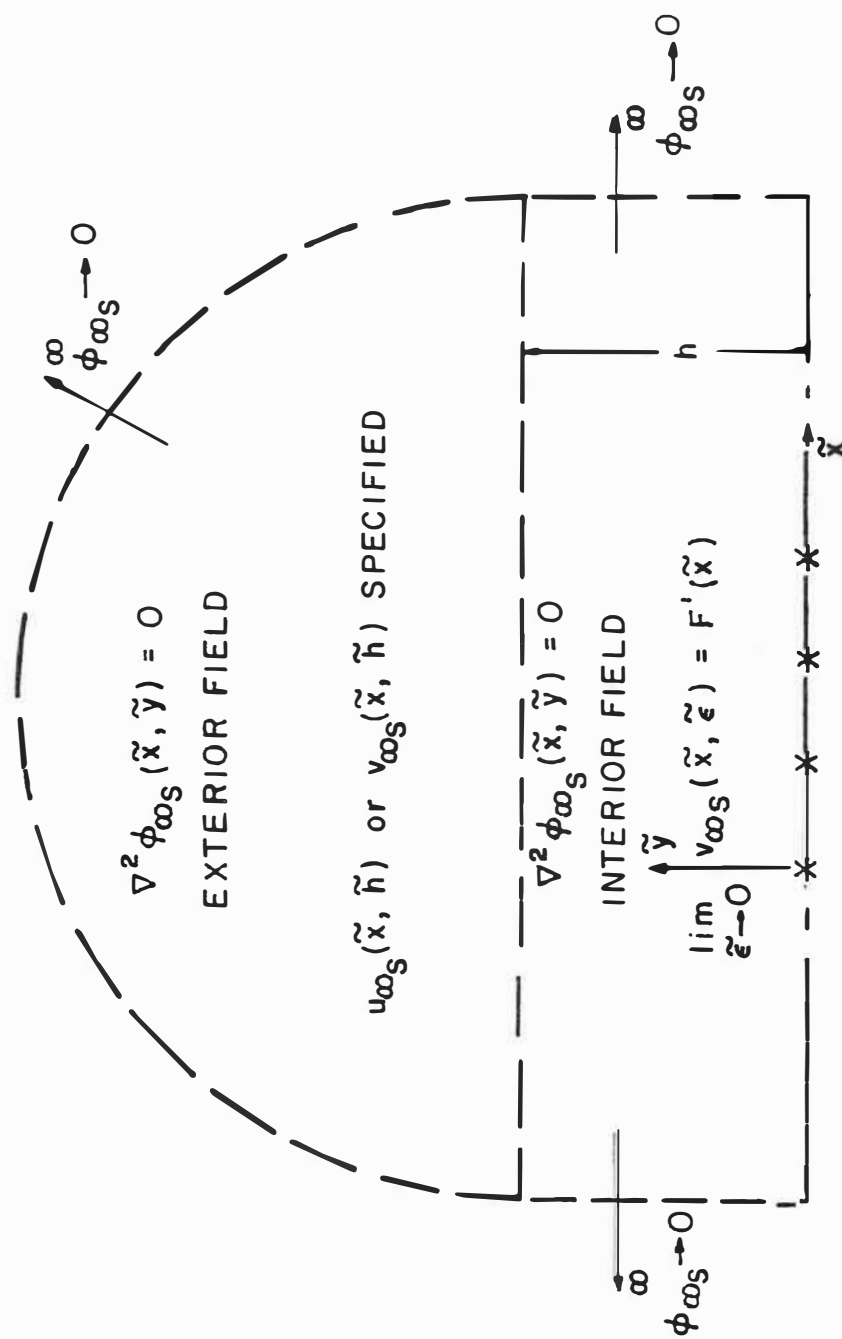


Figure 6. Interior and exterior boundary value problems for the symmetric free-air potential in the upper half-plane.

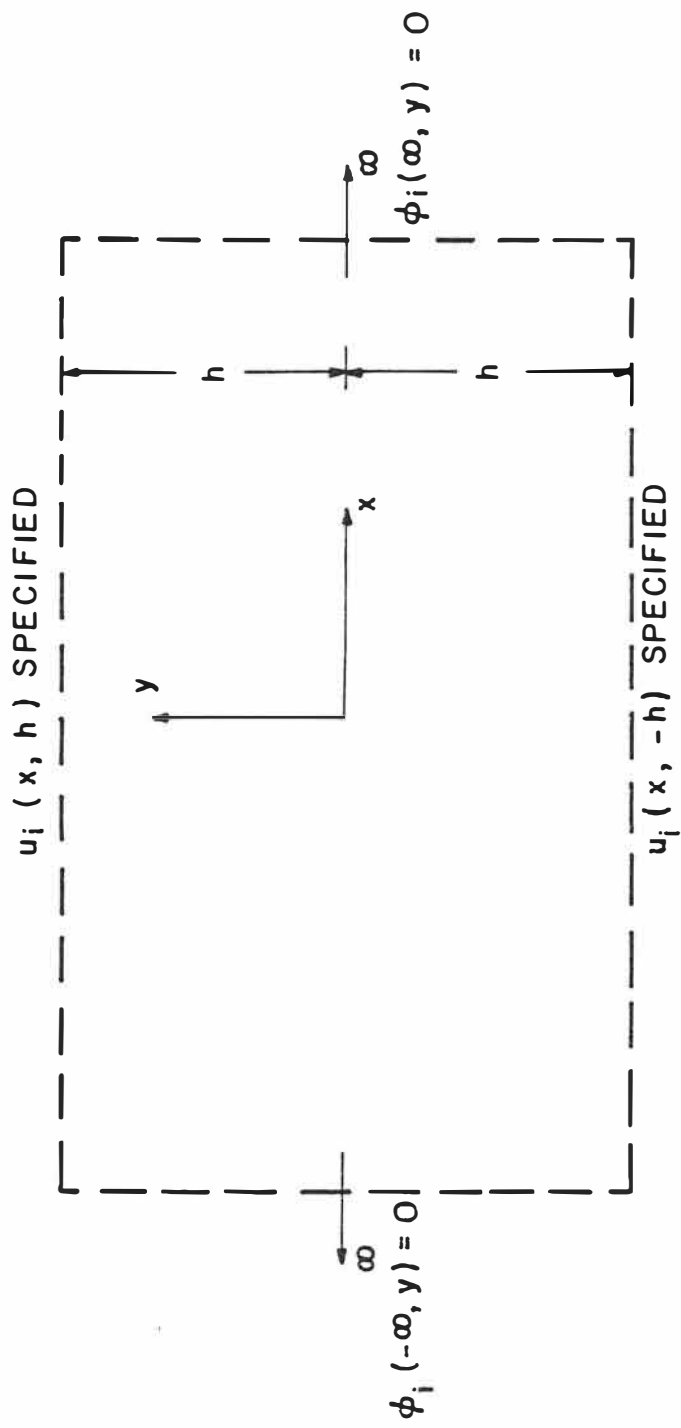


Figure 8. Boundary value problem for the interference field solution.

GENERALIZED BOUNDARY
CONDITION $L(\phi)_{y=h'} = 0$

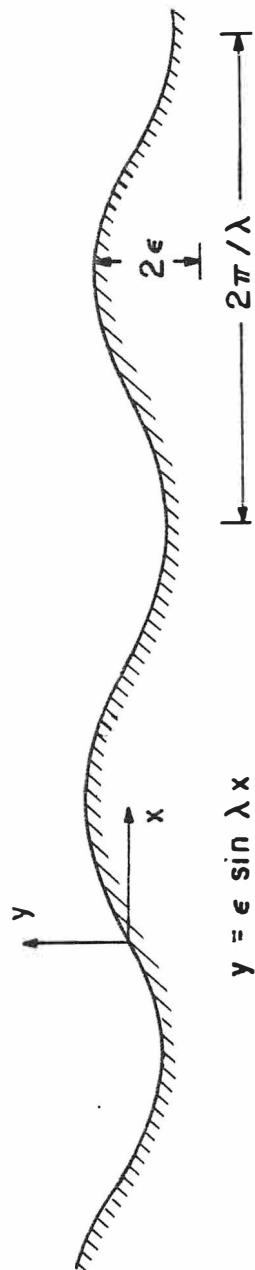


Figure 9. Schematic of the physical problem for the wavy wall with a generalized boundary condition.

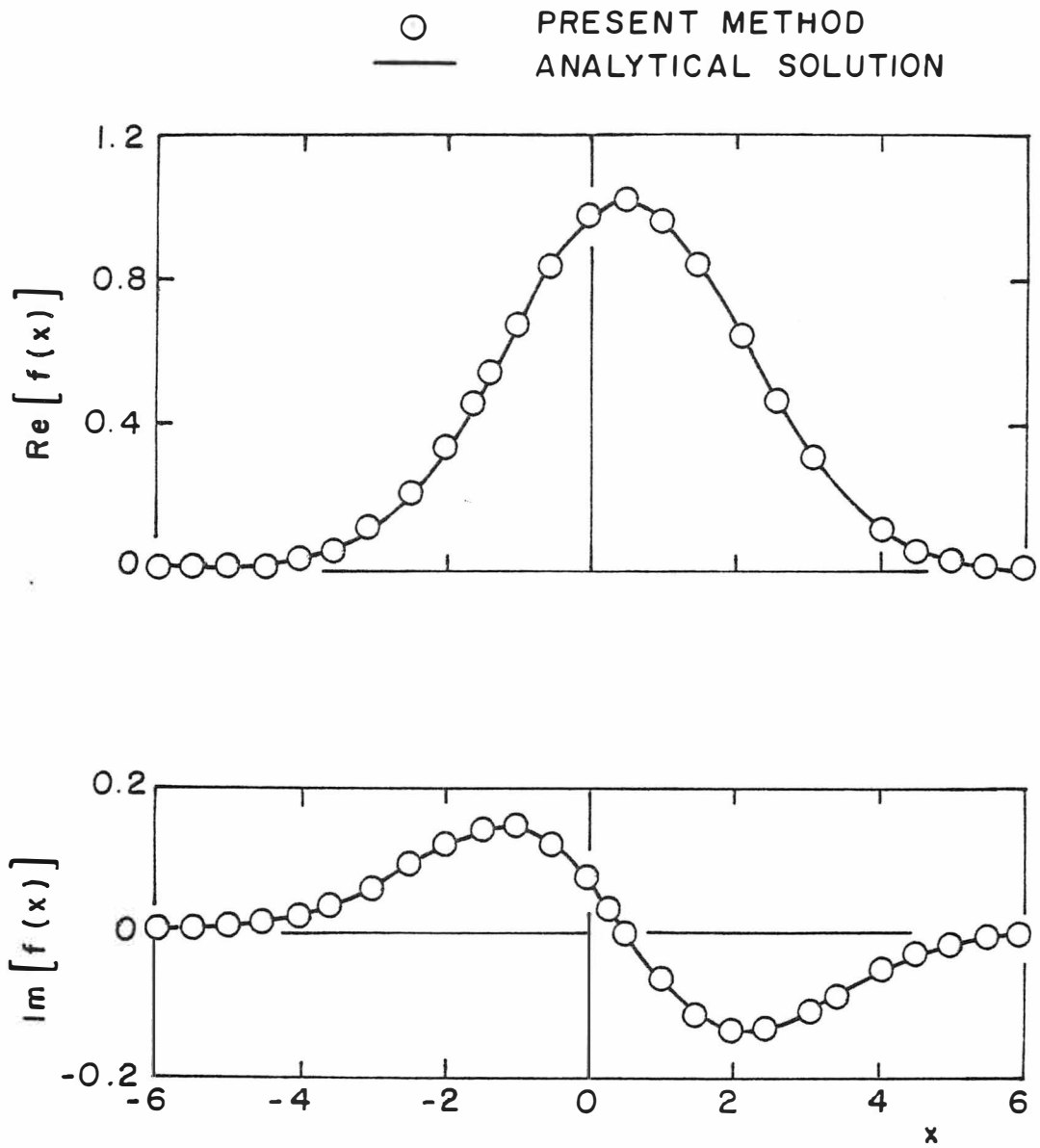
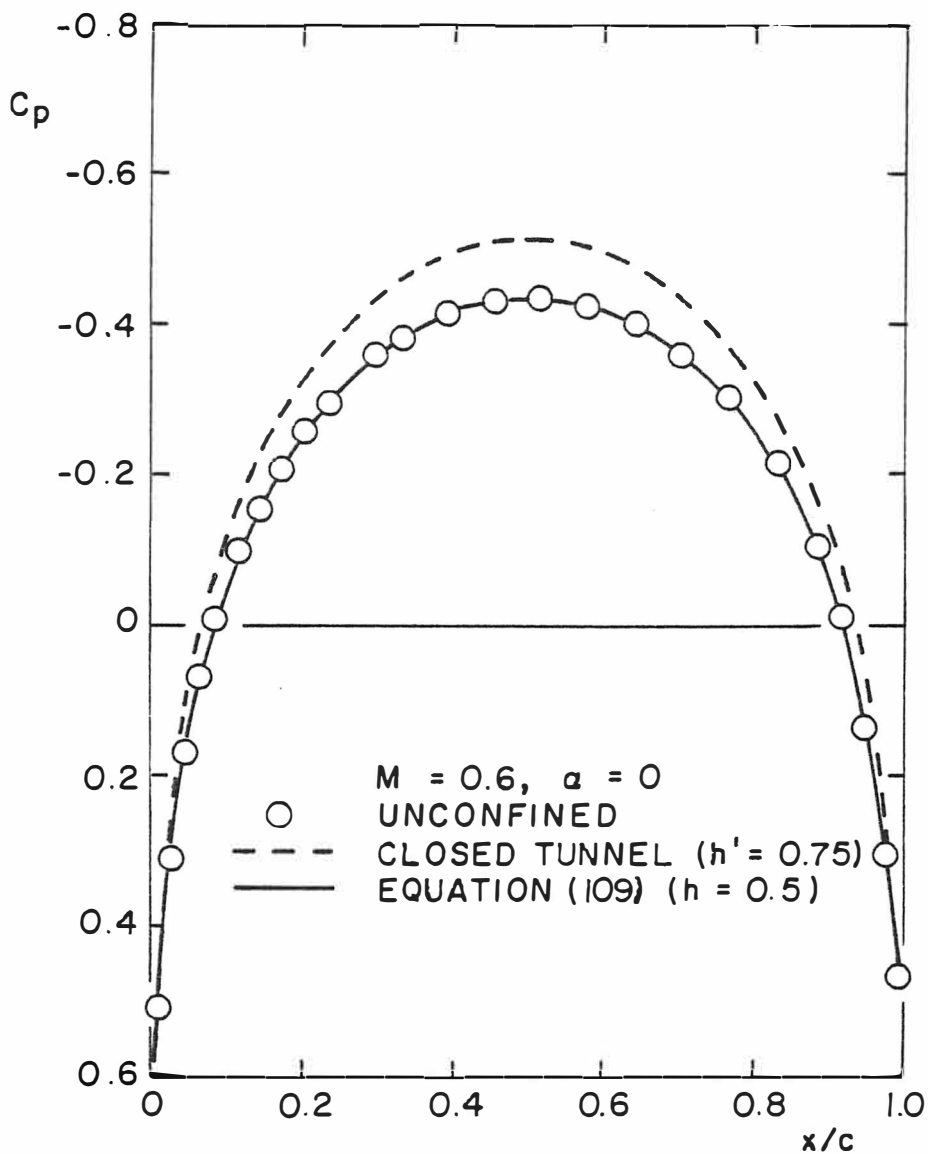
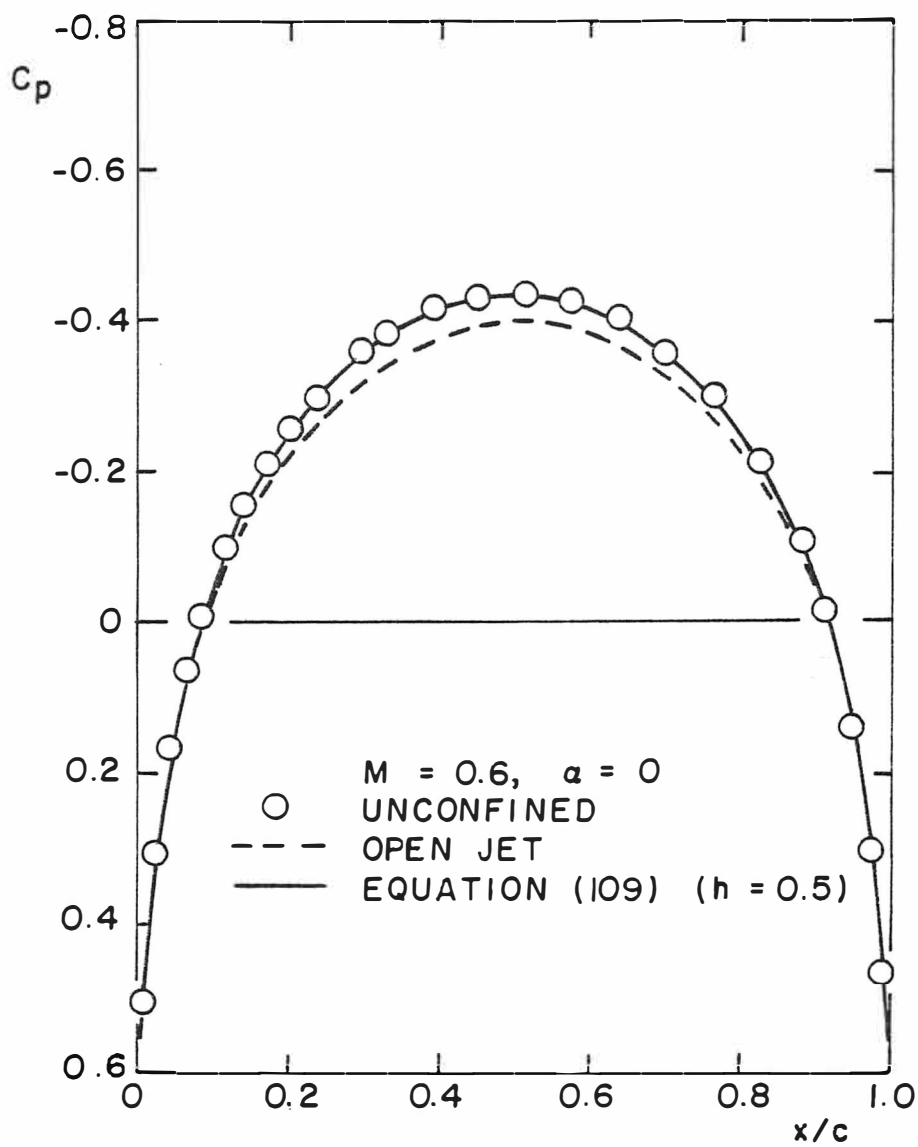


Figure 10. Demonstration of the numerical procedure for analytic continuation.



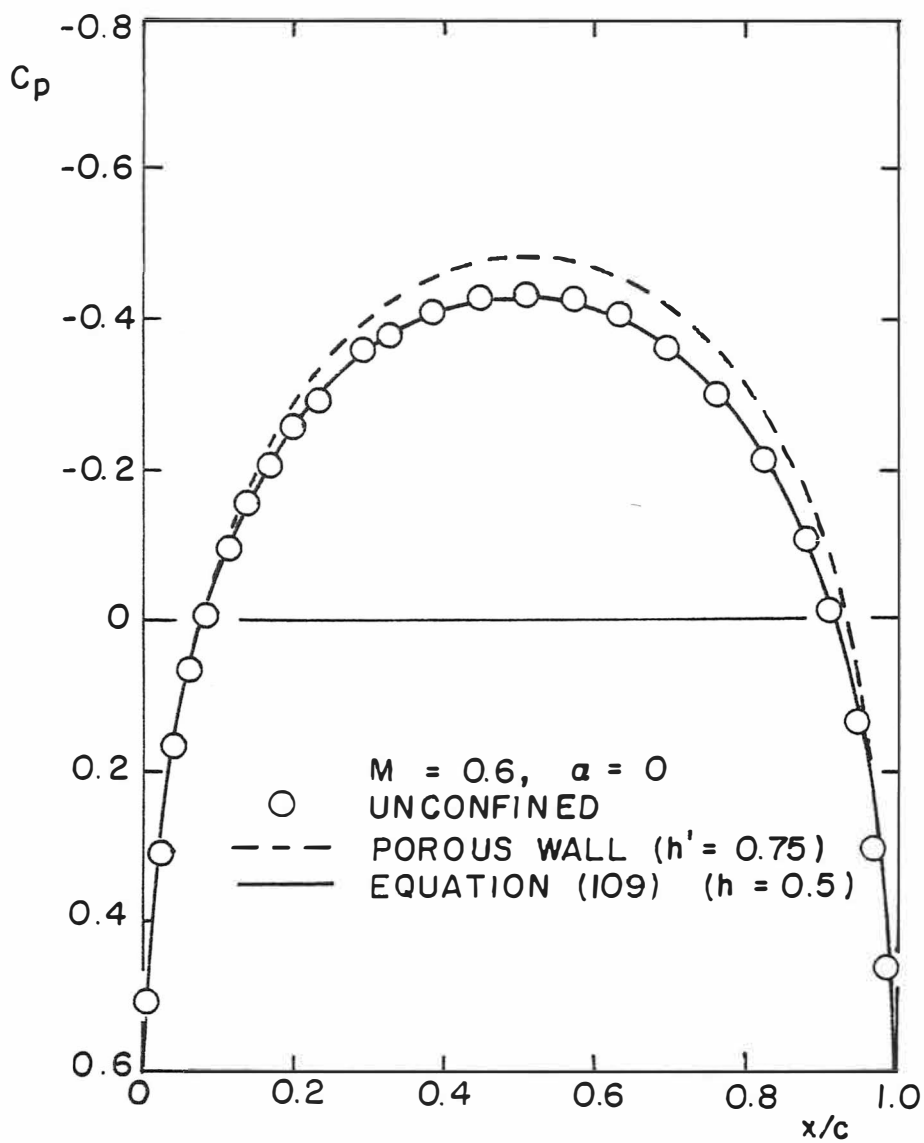
a. Closed tunnel

Figure 11. Airfoil surface pressure distributions based on Eq. (109) for 12 percent thickness parabolic arc airfoil, $M = 0.6$, $\alpha = 0^\circ$, $h/c = 0.5$.



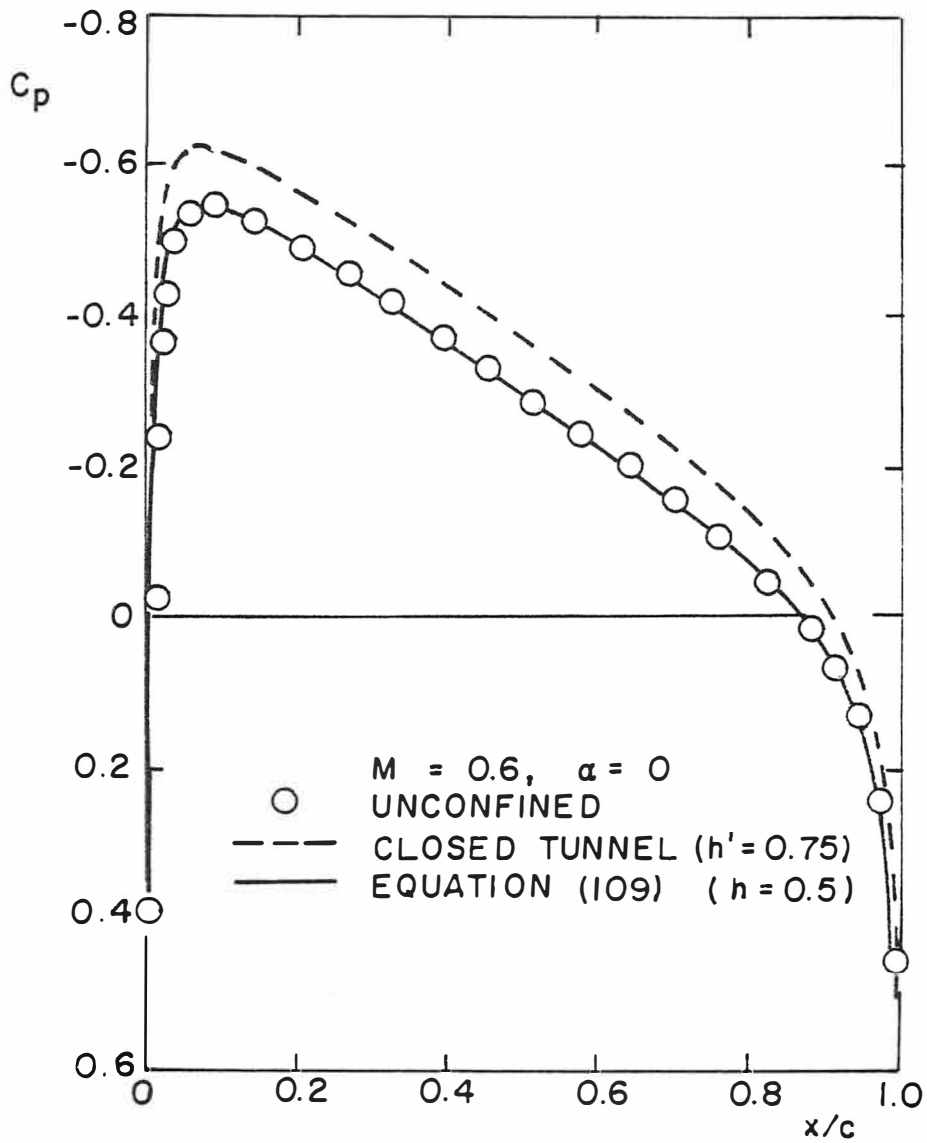
b. Open jet

Figure 11. (Continued)



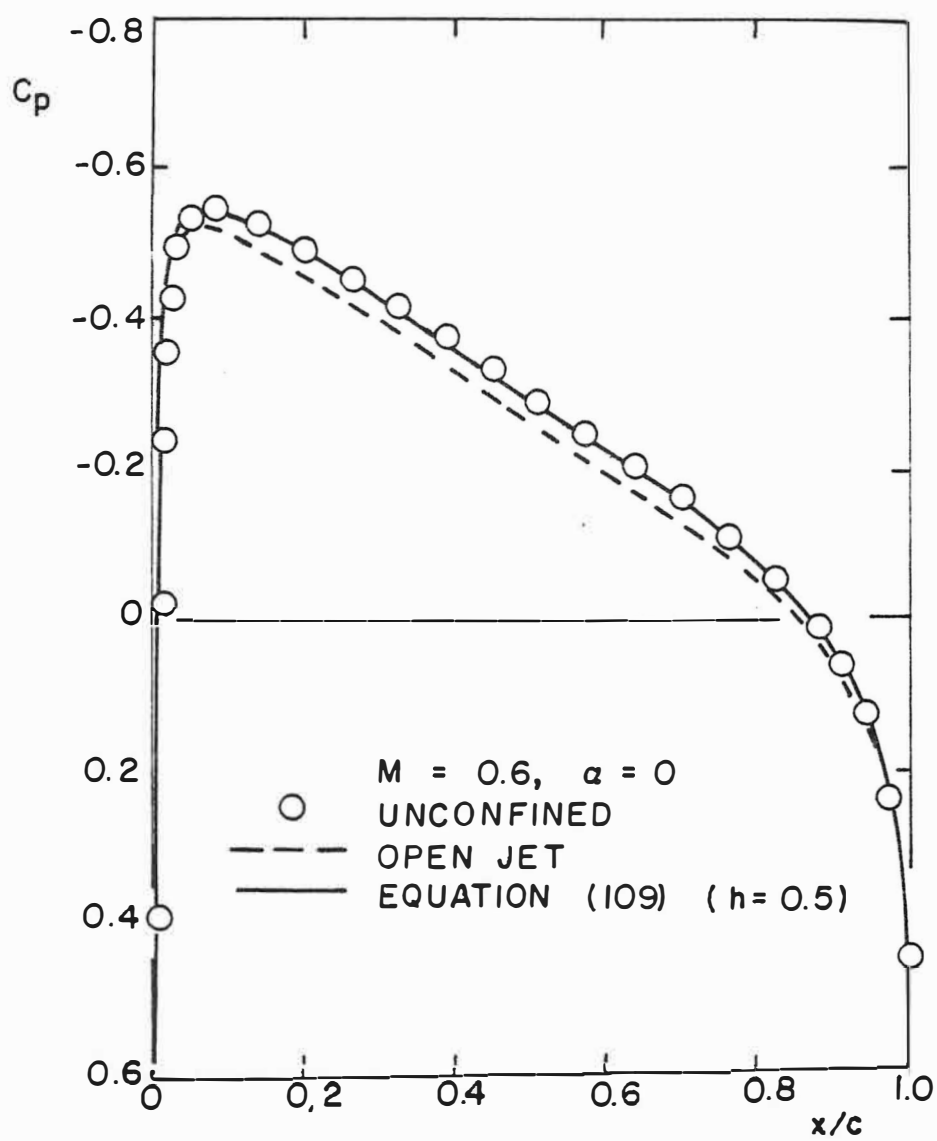
c. Porous wall

Figure 11. (Continued)



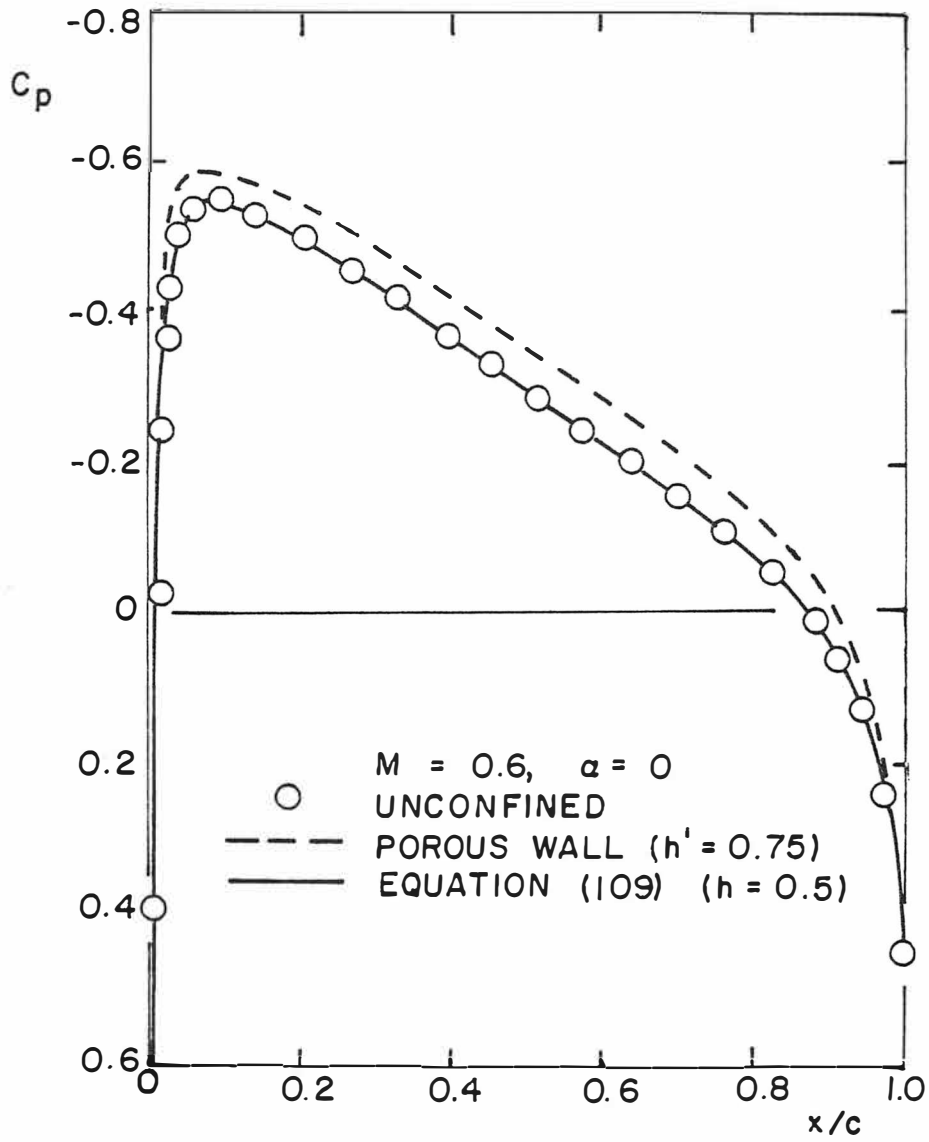
a. Closed tunnel

Figure 12. Airfoil surface pressure distributions based on Eq. (109) for NACA 0012 airfoil, $M = 0.6$, $\alpha = 0^\circ$, $h/c = 0.5$.



b. Open jet

Figure 12. (Continued)



c. Porous wall

Figure 12. (Continued)

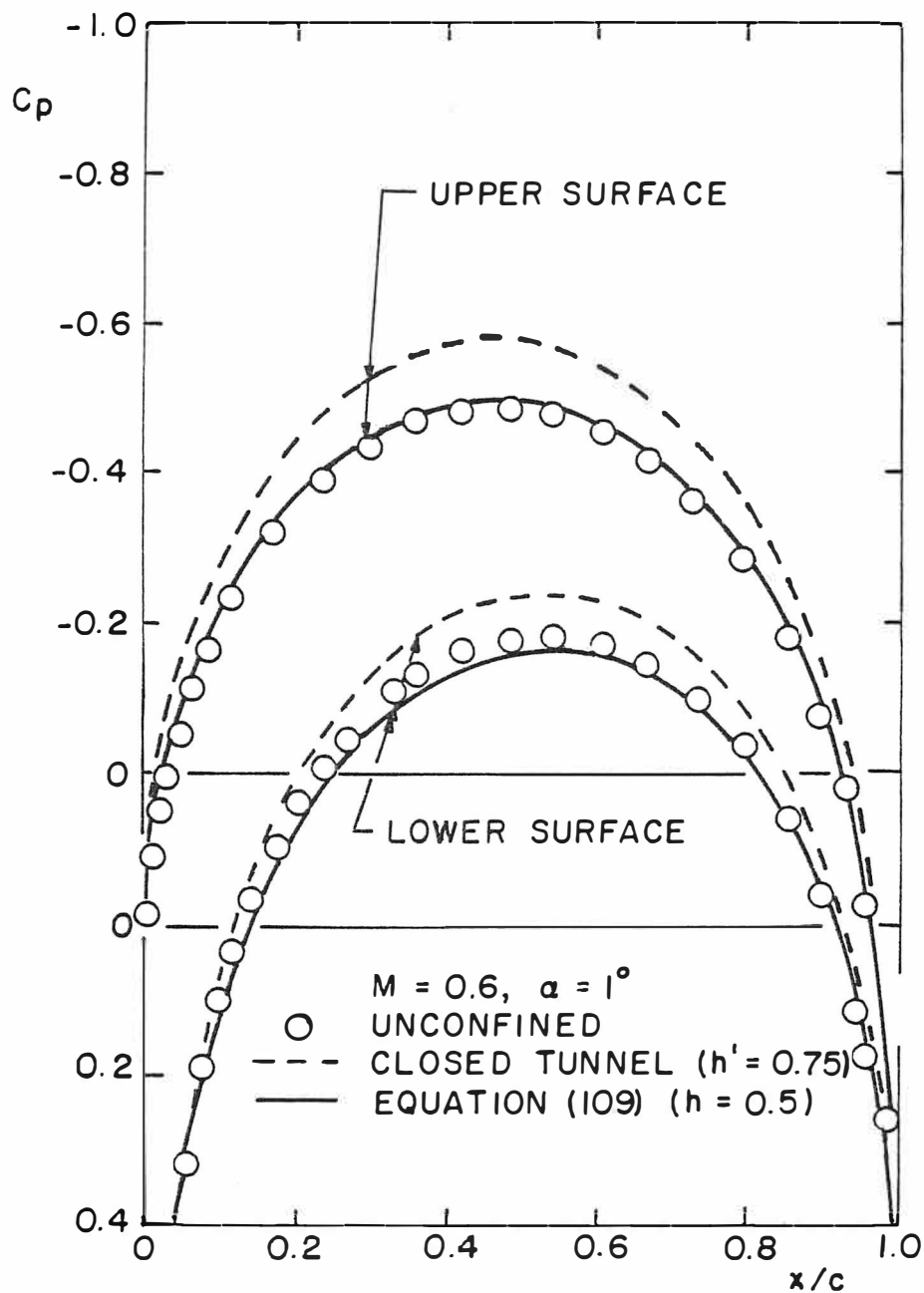


Figure 13. Airfoil surface pressure distributions based on Eq. (109) for 12 percent thickness parabolic arc airfoil, $M = 0.6$, $\alpha = 1^\circ$, $h/c = 0.5$.

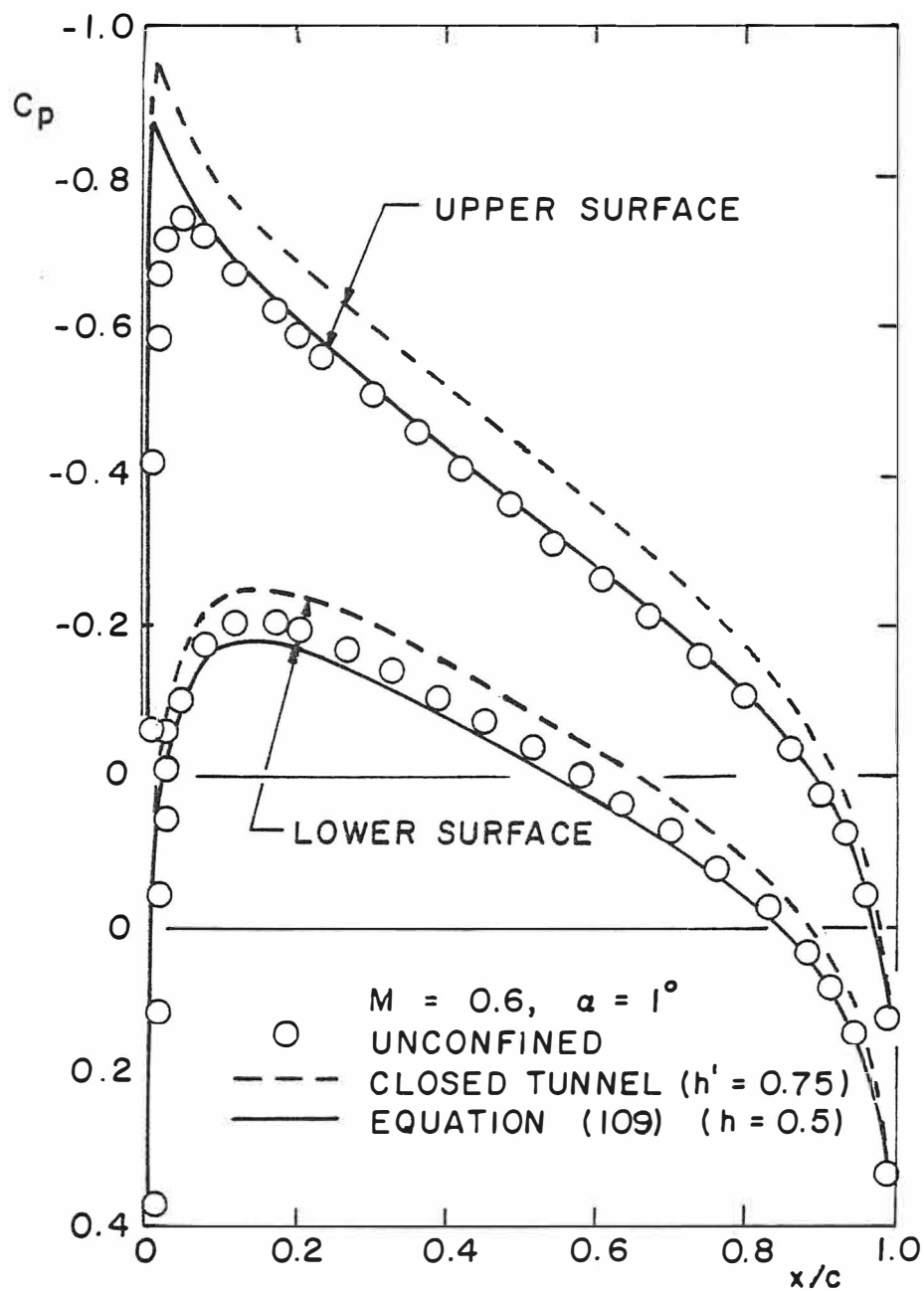


Figure 14. Airfoil surface pressure distributions based on Eq. (109) for NACA 0012 airfoil, $M = 0.6$, $\alpha = 1^\circ$, $h/c = 0.5$.

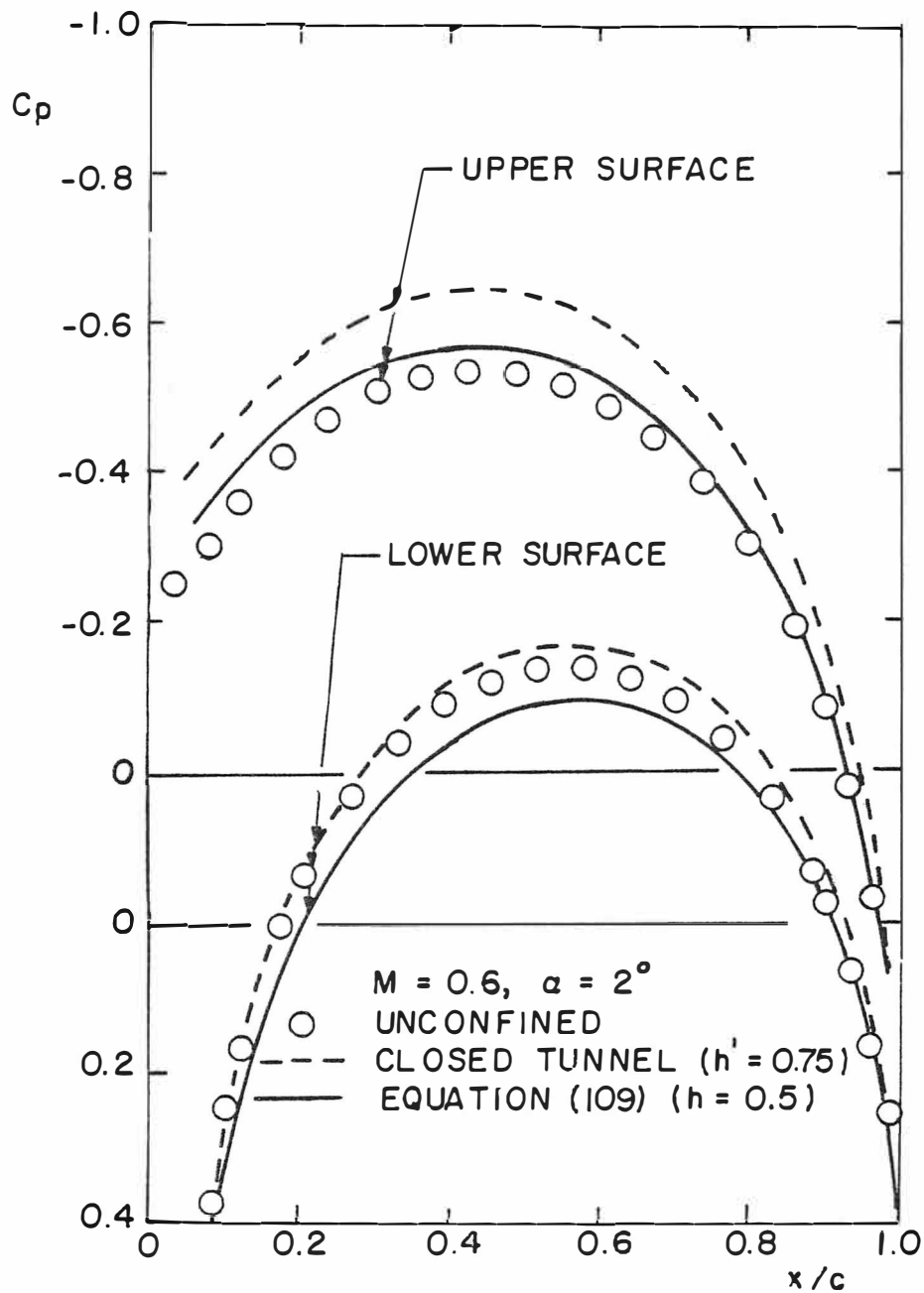


Figure 15. Airfoil surface pressure distributions based on Eq. (109) for 12 percent thickness parabolic arc airfoil, $M = 0.6$, $\alpha = 2^\circ$, $h/c = 0.5$.

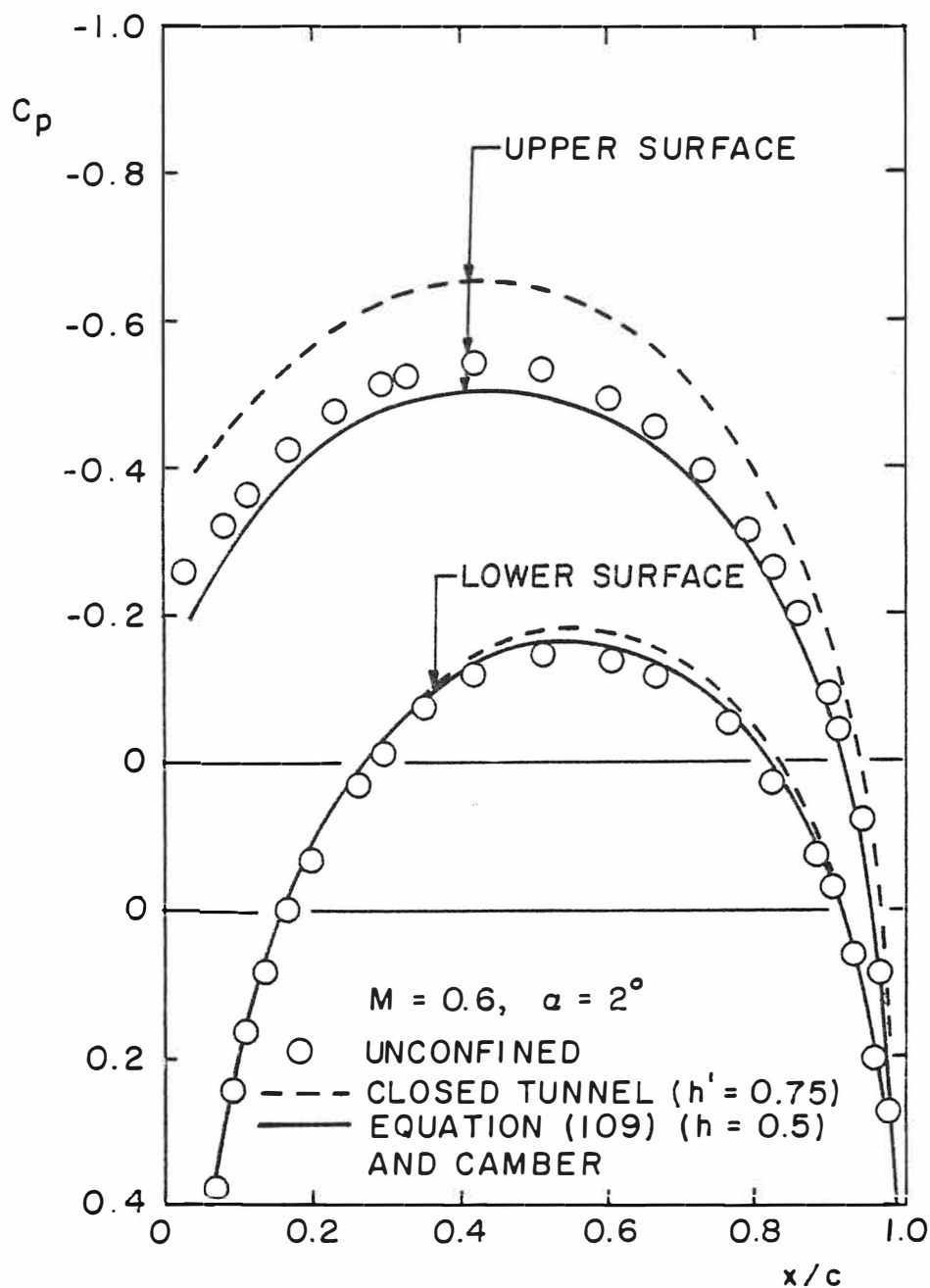


Figure 16. Airfoil surface pressure distributions based on Eq. (109) and exact camber modification for 12-percent thickness parabolic arc airfoil, $M = 0.6$, $\alpha = 2^\circ$, $h/c = 0.5$.

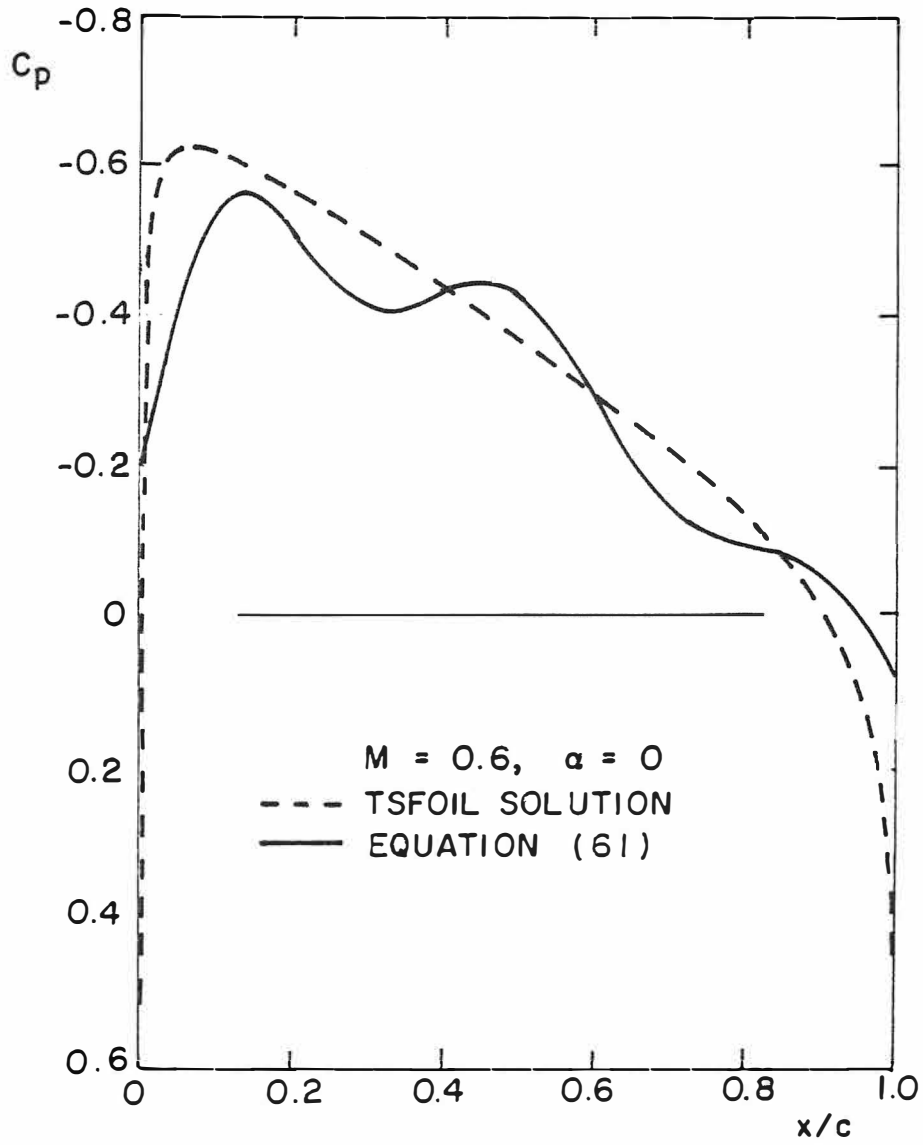


Figure 17. Airfoil surface pressure distributions based on Eq. (61) for NACA 0012 airfoil, $M = 0.6$, $\alpha = 0^\circ$, $h/c = 0.5$.

APPENDIX B

INVERSE TRANSFORMS OF CERTAIN FUNCTIONS

The inverse complex Fourier transform of a function $\bar{g}(p)$ is ordinarily defined for real z as

$$g(z) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} \bar{g}(p) e^{-ipz} dp . \quad (169)$$

The Wiener-Hopf technique [12] extends z to be complex, so that for $z = \xi + i\eta$,

$$\begin{aligned} g(\xi+i\eta) &= \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} \bar{g}(p) e^{-ip(\xi+i\eta)} dp \\ &= \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} [\bar{g}(p) e^{p\eta}] e^{-ip\xi} dp \end{aligned} \quad (170)$$

The behavior of $\bar{g}(p)$ may prevent the divergence of the integral in Eq. (170) for a limited range of η . In particular, if

$$\lim_{p \rightarrow \pm\infty} \bar{g}(p) e^{p\eta} \rightarrow 0 \quad (171)$$

for $\eta_1 < \eta < \eta_2$, then the inverse transform in Eq. (170) is analytic in the strip $\eta_1 < \eta < \eta_2$ in the complex plane. Hence, in this region of the plane

$$\frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} [\bar{g}(p) e^{p\eta}] e^{-ip\xi} dp = g(\xi + i\eta) . \quad (172)$$

This form may be applied to obtain the inverse transforms of certain functions.

Inverse of $\bar{g}(p, y) = \bar{u}(p) \cosh py$

The function $\bar{g}(p, y)$ may be alternatively written as

$$\bar{g}(p, y) = \frac{1}{2} \bar{u}(p) [e^{py} + e^{-py}] . \quad (173)$$

Applying an inverse transform to Eq. (173) leads to the form

$$g(x, y) = \frac{1}{2} \left\{ \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} [\bar{u}(p) e^{py}] e^{-ipx} dp + \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} [\bar{u}(p) e^{-py}] e^{-ipx} dp \right\} . \quad (174)$$

From analogy with Eq. (172), this is equivalent to

$$g(x, y) = \frac{1}{2} u(x + iy) + \frac{1}{2} u(x - iy) , \quad (175)$$

which may be alternatively expressed as

$$g(x, y) = \operatorname{Re}[u(x + iy)] . \quad (176)$$

Inverse of $\bar{g}(p,y) = \bar{u}(p)\sinh py$

Similarly, writing $\bar{g}(p,y)$ as

$$\bar{g}(p,y) = \frac{1}{2} \bar{u}(p) [e^{py} - e^{-py}] \quad (177)$$

leads to the inverse transforms as

$$g(x,y) = \frac{1}{2} u(x+iy) - \frac{1}{2} u(x-iy) , \quad (178)$$

which may be alternatively expressed in the form

$$g(x,y) = i \operatorname{Im} [u(x+iy)] . \quad (179)$$

APPENDIX C

LIST OF PRINCIPLE SOLUTIONS

Tunnel Field Solution, (Eqs. (61) and (62))

$$u_T(x, \pm y) = \operatorname{Re} \left[u_T(x + i\beta(h-y), \pm h) \right] \mp \frac{1}{\beta} \operatorname{Im} \left[v_T(x + i\beta(h-y), \pm h) \right]$$

$$v_T(x, \pm y) = \operatorname{Re} \left[v_T(x + i\beta(h-y), \pm h) \right] \pm \beta \operatorname{Im} \left[u_T(x + i\beta(h-y), \pm h) \right]$$

for $0 \leq y \leq h$.

Free-Air Control Surface Solution, Eqs. (76) and (77)

$$\begin{aligned} u_\infty(x, \pm h) = & \frac{1}{2} u_T(x, \pm h) - \frac{\beta h}{\pi} \int_{-\infty}^{+\infty} \frac{u_T(\xi, \mp h)}{(\xi-x)^2 + (2\beta h)^2} d\xi \\ & \mp \frac{1}{2\pi\beta} \oint_{-\infty}^{+\infty} \frac{v_T(\xi, \pm h)}{\xi-x} d\xi \pm \frac{1}{2\pi\beta} \int_{-\infty}^{+\infty} \frac{v_T(\xi, \mp h)}{(\xi-x)^2 + (2\beta h)^2} (\xi-x) d\xi . \end{aligned}$$

$$\begin{aligned} v_\infty(x, \pm h) = & \frac{1}{2} v_T(x, \pm h) - \frac{\beta h}{\pi} \int_{-\infty}^{+\infty} \frac{v_T(\xi, \mp h)}{(\xi-x)^2 + (2\beta h)^2} d\xi \\ & \pm \frac{\beta}{2\pi} \oint_{-\infty}^{+\infty} \frac{u_T(\xi, \pm h)}{\xi-x} d\xi \mp \frac{\beta}{2\pi} \int_{-\infty}^{+\infty} \frac{u_T(\xi, \mp h)}{(\xi-x)^2 + (2\beta h)^2} (\xi-x) d\xi . \end{aligned}$$

Interior Free-Air Field Solution, Eqs. (82) and (83)

$$u_{\infty}(x, \pm y) = \operatorname{Re} \left[u_{\infty}(x + i\beta(h-y), \pm h) \right] \mp \frac{1}{\beta} \operatorname{Im} \left[v_{\infty}(x + i\beta(h-y), \pm h) \right]$$

and

$$v_{\infty}(x, \pm y) = \operatorname{Re} \left[v_{\infty}(x + i\beta(h-y), \pm h) \right] \pm \beta \operatorname{Im} \left[u_{\infty}(x + i\beta(h-y), \pm h) \right]$$

for $0 \leq y \leq h$.

Exterior Free-Air Field Solution, Eqs. (92) and (93)

$$u_{\infty}(x, \pm y) = \pm \frac{1}{\pi\beta} \int_{-\infty}^{+\infty} v_{\infty}(\xi, \pm h) \frac{(x-\xi)}{(x-\xi)^2 + \beta^2(y-h)^2} d\xi$$

and

$$v_{\infty}(x, \pm y) = \mp \frac{\beta}{\pi} \int_{-\infty}^{+\infty} u_{\infty}(\xi, \pm h) \frac{(x-\xi)}{(x-\xi)^2 + \beta^2(y-h)^2} d\xi$$

for $|y| \geq h$.

Interference Field Solution, Eqs. (107) and (108)

$$u_i(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left\{ \frac{\beta u_T(\xi, h)(h-y) + \frac{1}{\beta} v_T(\xi, h)(\xi-x)}{(\xi-x)^2 + \beta^2(h-y)^2} + \frac{\beta u_T(\xi, -h)(h+y) - \frac{1}{\beta} v_T(\xi, -h)(\xi-x)}{(\xi-x)^2 + \beta^2(h+y)^2} \right\} d\xi$$

and

$$v_i(x, y) = \frac{-1}{2\pi} \int_{-\infty}^{+\infty} \left\{ \frac{\beta u_T(\xi, h)(\xi-x) - \beta v_T(\xi, h)(h-y)}{(\xi-x)^2 + \beta^2(h-y)^2} - \frac{\beta u_T(\xi, -h)(\xi-x) + \beta v_T(\xi, -h)(h+y)}{(\xi-x)^2 + \beta^2(h+y)^2} \right\} d\xi$$

for $-h \leq y \leq h$.

VITA

Werner Johann Anton Dahm was born in Huntsville, Alabama, on July 12, 1957. He attended a private elementary school in Huntsville and was graduated from Virgil I. Grissom High School in 1975. In November 1978, he graduated from the University of Alabama in Huntsville, with the degree Bachelor of Science in Engineering with a Mechanical Engineering concentration.

In January 1979, he entered The University of Tennessee Space Institute as a full-time graduate student on a research assistantship at the Propulsion Wind Tunnel Facility of the Arnold Engineering Development Center. One year later, he accepted a full-time position as a Research Engineer at the Arnold Engineering Development Center and continued his graduate studies at The University of Tennessee Space Institute on a part-time basis, receiving a Master of Science degree in Mechanical Engineering in March 1981.