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I am submitting herewith a dissertation written by Stephen C. Morse entitled "Declining Marginal Utility or Constant Marginal Utility in the Consumer Characteristics Model: A Box-Cox Analysis of the Hedonic Price Equation." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Agricultural Economics.

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DECLINING MARGINAL UTILITY OR CONSTANT MARGINAL UTILITY IN THE CONSUMER CHARACTERISTICS MODEL: A BOX-COX ANALYSIS OF THE HEDONIC PRICE EQUATION

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Steve Morse

December 1988

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ABSTRACT

Because of the increasing interest in nutritional attributes contained in food, hedonic price equations derived from the consumer characteristics model have been used to attempt to explain changing consumption patterns of consumers. Choice of functional form of the consumer hedonic price equation must be consistent with economic theory and show declining marginal utility for nutritional attributes. The Box-Cox methodology is used to search for the appropriate functional form of the hedonic price equation that is consistent with declining marginal utility of attributes. Past empirical work has used the linear functional form of the hedonic price equation based on pure statistical grounds and is shown to yield constant marginal utility of attributes and is inconsistent with economic theory.

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CHAPTER I

INTRODUCTION

The agricultural policy of the United States has been traditionally farm oriented with the main objectives to ensure a healthy and efficient farm economy. Consumer oriented agricultural policy has been aimed primarily at stabilizing food prices and ensuring adequate food supplies. In the Food and Agriculture Act of 1977, food production and distribution policies became more nutrient oriented recognizing nutritionists' findings that Americans do not live as long or as healthy lives as they could because of the choices of foods they eat. Insufficient nutrient intake and eating the wrong choices of food may lead to malnutrition. Government intervention via agricultural policy is hoped to improve the health status and well-being of the American public.

Numerous current government policies attempt to influence the nutrient intake of the U.S. population. Programs such as food stamps, school lunch and aid to families with dependent children focus on improving the quality of diets by imposing nutritional standards (Tarrant, 1980). Agricultural policy that affects market prices of food can affect food purchases. Policies such as price supports, acreage set-asides, marketing orders, food standards and grades change food prices and strongly affect the food choices available to consumers. To the extent that relative prices and incomes are affected, changes in food purchases occur and ultimately nutritional intake levels change.

Because of the new interest in the nutritional attributes contained in food, there has been the need to develop a conceptual framework to understand the households' food purchases and resulting nutritional intake. Recently, emphasis on the relationship among food prices, nutrient content and consumer preferences for nutrients have evolved. In the past, the importance of nutritional attributes on household food choices (preferences) has not been given much priority by U.S. researchers because "nutrition has very little market impact. Farmers do not produce nutrients, processors do not process nutrients, and consumers do not buy nutrients, except indirectly" (Timmer, 1978, p. 53). There does not exist an explicit market for nutrients. The demand for nutrients must be analyzed using the derived demand concept because in the model to be presented, consumers purchase food items for the purpose of consuming nutritional attributes contained in the item.

The familiar classical consumer demand theory uses market prices, income, and consumer preferences in explaining patterns of consumer demand for food items. In developing a theory of consumer demand for nutritional characteristics (attributes) contained in food items, one needs to know implicit prices of nutritional attributes contained in food items. That is, nutritional attributes are not sold separately on the market, and no observable market prices exist. Consumers receive these nutrients by purchasing products that contain them. Although there are direct ways to measure the quantity of nutrients available in foods consumed by the household (through tables of nutrient content of

various food items), there is no way to observe directly the valuation of nutrients consumed by the household.

Several models that attempt to estimate household demand for nutritional attributes have been developed in which measures of consumers' implicit prices (valuations) of nutrients have been estimated. These models include Houthakker (1951), Ladd and Suvannunt (1976), Lancaster (1966), and Theil (1952). Each of these models uses the idea that market goods possess desirable characteristics, such as nutrients (Lancaster), and obtain measures of consumers' implicit prices (valuations) of characteristics and seek to develop a theory for the demand for these characteristics.

The characteristics theory presents a consumer utility maximization problem and assumes that characteristics of market goods yield utility, not products themselves. A characteristic is defined as a component of a good that yields utility. For example an automobile may have the components of engine size and leg room among the many characteristics it contains. Among the many components of a steak, two characteristics are protein and fat. The consumer is interested in consuming the characteristics that goods possess because the characteristics yield utility not the goods themselves. The usual budget constraint defines the market good choices available to the consumer. A consumer purchases market goods to obtain the utility from the characteristics they contain. A hedonic price equation for each market good is derived that expresses this price as a function of the implicit prices of the characteristics as valued by the consumer. From this implicit

valuation, a demand function for these characteristics may be estimated for each consumer. The consumer task is to identify the optimal purchase of characteristics (via market goods) that maximizes his utility within the budget constraint.

The Problem

This dissertation is an effort to look into the structure of the hedonic price equation associated with constant marginal utility and to perform an empirical test for constant marginal utility. The main thesis of this paper is concerned with the marginal utilities of the characteristics contained in market goods in general and food products in particular. By comparison to the classical consumer demand theory optimizing conditions, the characteristics theory will be examined for appropriate optimizing conditions which are assumed to exist. A cornerstone of the theory is shown to be explicitly declining positive marginal utilities of characteristics. In food products, characteristics are generally referred to as attributes.

The existence of declining marginal utility of attributes in the characteristics model ensures that maximization of the utility function subject to the budget constraint will yield a unique solution. Failure to use a model that does not specify declining marginal utility of attributes can result in multiple solutions. The importance of identifying a unique solution in the characteristics model is crucial in deriving the demand function for attributes and predicting consumers' behavior when changes in the economic environment, such as income and

product prices occur. A model with multiple solutions reveals little or no information about consumer preferences and demand for attributes. Thus, for the characteristics theory to be useful in explaining household preferences for nutritional attributes, the model used must display declining marginal utility for attributes.

As will be shown, the key to determining if a characteristics model, such as the Ladd-Suvannunt model, exhibits declining marginal utility of attributes is the functional form used in the hedonic price equation of the model. A linear functional form of the hedonic price equation will be shown to yield marginal utilities of attributes which are independent of the level of a particular attribute consumed, thus displaying a constant marginal utility for attributes. This linear functional form of the hedonic price equation is used commonly in the literature [see Eastwood, Brooker and Terry (1986), Hager (1985), Houthakker (1951), Ladd and Suvannunt (1976), Lancaster (1966), Suvannunt (1973) and Theil (1952)], even though the theoretical framework supporting these models yields multiple nonunique solutions that do no conform to mathematical optimization and relevant economic theory.

Objectives and Procedures

The objectives of this study are

1. to provide an overview of relevant characteristic theory models,

2. to identify necessary and sufficient mathematical properties of the characteristics model which are consistent with economic theory,
3. to examine the importance of declining marginal utility of attributes in the characteristics model, and
4. to evaluate alternative functional forms of the hedonic price equation in terms of declining marginal utility of attributes which involves a search for the best fitting functional form of the hedonic price equation using Box-Cox methodology to identify the appropriate functional form and evaluating its significance with regard to mathematical optimization properties and economic theory.

The data used in achieving objective four are drawn from the cross-section sample of households from the U.S.D.A.'s Nationwide Food Consumption Survey of 1977-78.

This dissertation is divided into six additional chapters. Chapter II presents (1) the appropriate conditions for optimization of the classical consumer demand model and explains the importance of declining marginal utilities of products, (2) a survey of the literature concerning characteristics theory, and (3) a definition of the appropriate conditions required for optimization of the Ladd-Suvannunt characteristics model and the importance of declining marginal utility of characteristics in the model. In Chapter III the popular linear functional form of the hedonic price equation and points out why specification of this hedonic equation plays a crucial role in meeting

conditions for a unique solution are examined. In Chapter IV a general functional form of the hedonic equation is used in the Box-Cox procedure of variable transformation in order to identify the "correct" functional form. Here, the search for a functional form is explained for the maximum likelihood estimation procedure. Chapter V discusses the data source, variables used, and specification of the general functional form of the model used in the Box-Cox procedure. The nutritional attributes used in the analysis are identified in this section. Chapter VI presents the results of the Box-Cox procedure of searching for a "correct" functional form. Five functional forms are evaluated and compared to the "correct" functional form identified by the Box-Cox procedure. Chapter VII presents the summary and conclusions of the study.

CHAPTER II

MATHEMATICAL MODEL AND APPROPRIATE CONDITIONS FOR OPTIMIZATION OF THE CLASSICAL AND CHARACTERISTICS THEORY OF CONSUMER DEMAND

The Classical Theory of Consumer Demand

The classical consumer choice model of demand theory uses the idea that utility is realized from the consumption of goods and services purchased in the marketplace. The model assumes a one-period analysis of consumer choice when utility, prices of products, and the consumers budget constraint are known. The classical model reveals the quantities of products which are purchased to maximize utility. Intrilligator (1971) presents the classical economizing problem as a mathematical programming problem. The mathematical programming problem seeks to maximize an objective function (utility) subject to a set of constraints (prices and income). As will be shown, the mathematical model has certain properties which have meaningful economic interpretations.

When presented as a mathematical programming problem, the classical model is expressed as the consumers attempt to:

$$\text{Maximize } U = U(Q_1, Q_2, \ldots, Q_n) = U(\mathbf{Q}) \tag{2-1}$$

$$\text{subject to} \quad \sum_{i=1}^{n} P_i Q_i = I \,, \tag{2-2}$$

where U = consumer's utility,

Q_i = quantity of the ith product purchased,

P_i = price of the ith product,

I = consumer's income available in the one period analysis,

n = number of goods, and

$\mathbf{Q}$ = a vector in Euclidean n-space where $\mathbf{Q} = (Q_1, Q_2, \ldots Q_n)$.

The objective function (2-1) is assumed to be given and twice continuously differentiable, and (2-2) is a given continuous differentiable function of prices and quantities of products. The vector $\mathbf{Q}$ yields utility directly to the consumer. The parameters $P_1, \ldots, P_n$, and I are considered exogenous in the one-period choice problem.

The vector $\mathbf{Q}^s$ is a subset of the opportunity set or all possible combinations of goods, defined as $\mathbf{Q}$. The opportunity set $\mathbf{Q}^s$ contains several vectors of $\mathbf{Q}$ as members and only vectors of $\mathbf{Q}$ which are feasible solutions. The instrument vector $\mathbf{Q}^s$ is considered a feasible solution if it satisfies all the constraints of the problem. Therefore, the opportunity set $\mathbf{Q}^s$ is defined as the set of all feasible vectors $\mathbf{Q}$ which are candidates for the solution. From the opportunity set $\mathbf{Q}^s$ the consumer's problem is viewed as that of choosing one instrument vector $\mathbf{Q}$ (from all the possible vectors $\mathbf{Q}$ in the opportunity set) that will maximize the consumer's utility function.

Types of Maxima in the Classical Model

Global Maximum

Vector $\mathbf{Q}^*$ is a global maximum if it is feasible, and it yields a value of the utility function larger than or equal to that yielded by

any other feasible vector Q. That is: $Q^* \varepsilon Q^S$ and $U(Q^*) \geq U(Q)$ for all $Q \varepsilon Q^S$ where Q^S is a subset of Euclidean n-space.

Strict Global Maximum

The global maximum Q^* is a strict global maximum if the value of the utility function at Q^* is strictly larger than at any other point on the utility surface. That is: $U(Q^*) > U(Q)$ for all $Q \varepsilon Q^S$ and $Q \neq Q^*$. If Q^{**} and Q^* were each distinct strict global maxima, it holds from the definition of a strict global maximum that $U(Q^{**}) > U(Q^*)$ and $U(Q^*) > U(Q^{**})$ both of which obviously cannot be true. Therefore, a solution that is a strict global maximum is a unique solution.

Local Maximum

If feasible, the instrument vector Q^* is a local maximum as long as it yields a value of the utility function larger than or equal to that yielded by any other feasible vector sufficiently close to it. That is:

$$Q^* \varepsilon Q^S \text{ and } U(Q^*) \geq U(Q) \text{ for all } Q \varepsilon Q^S \varepsilon N_\varepsilon(Q^*)$$

where $N_\varepsilon(Q^*)$ is an ε minus neighborhood of Q^* for some positive ε, however small. The term $N_\varepsilon(Q^*)$ is defined as the set of all Q such that

$$|Q - Q^*| = \sqrt{\sum_{i=1}^{n} (Q_i - Q_i^*)^2} < \varepsilon \ .$$

Strict Local Maximum

The local maximum Q^* is a strict local maximum if the value of the utility function at Q^* is larger than that of any other feasible vector sufficiently close to it. That is Q^* is a strict local maximum if:

$$U(Q^*) > U(Q) \text{ for all } Q \varepsilon Q^S \_\_ N_\varepsilon(Q^*)$$

where $Q \neq Q^*$. The connection between a global and local maximum is that a global maximum is a local maximum. However, a local maximum is not necessarily a global maximum, as there could be other local maxima yielding a higher value of the utility function.

Local-Global Maximum

This theorem gives sufficient conditions for a local maximum to be a global maximum. If the opportunity set Q^S is a nonempty compact set that is convex and $U(Q)$ is a continuous function concave over Q, then a local maximum is a global maximum. This ensures that the set of all points at which the maximum is obtained is a convex set.

Compact and Convex Sets of the Classical Model

In the classical consumer behavior model, the opportunity set Q^S is considered a compact set when it is (1) closed and (2) bounded. For ease of illustration consider a model which consists of two goods (Q_1, Q_2) which the consumer may purchase. Figure 1 shows the model for two goods (Q_1, Q_2). The axes define the opportunity set Q, and the line AB shows the consumers budget constraint. The feasible opportunity set Q^S

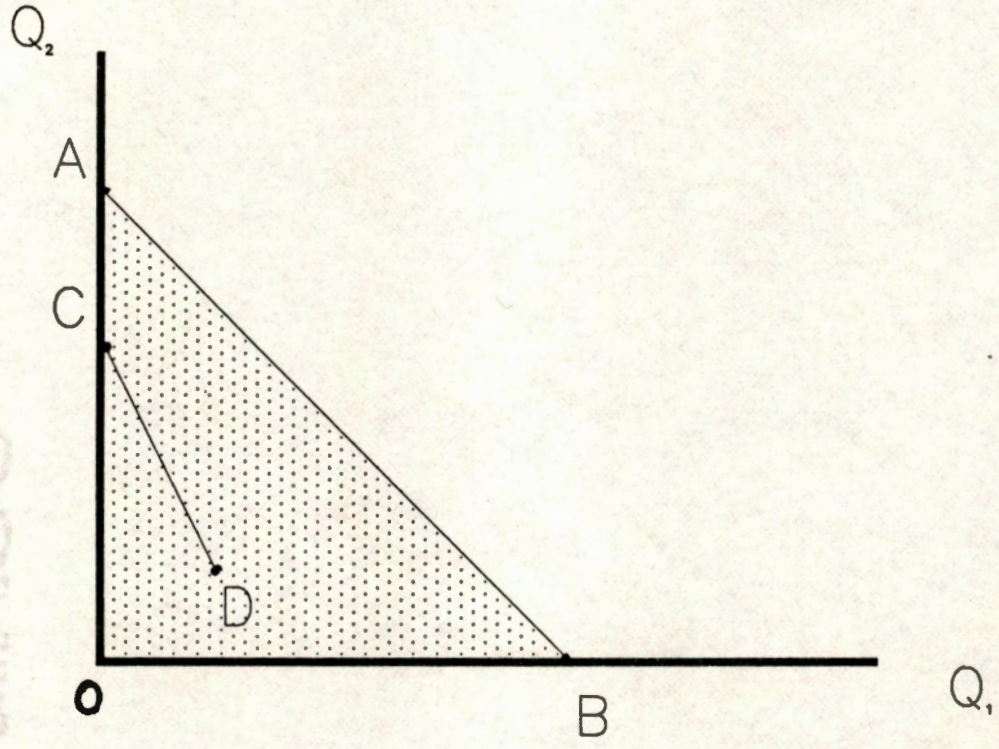


Figure 1. Hypothetical Closed and Bounded Opportunity Set for Classical Consumer Choice Model

is the shaded area OAB and is considered a closed set by definition since the closed set $\mathbf{Q}^S$ includes points along all of its boundaries of OAB. The set $\mathbf{Q}^S$ is bounded by definition since the shape of the opportunity set $\mathbf{Q}^S$ is definite--without open ends which rules out an undefined shape of $\mathbf{Q}^S$. Points along the budget constraint AB indicate those quantities of Q_1 and Q_2 purchased that exhaust the consumer's income. Points (vectors) not along AB do not use all the consumer's income but still satisfy the constraint of not exceeding the consumer's budget.

The definition of a convex set stipulates that, for any two points in the opportunity set, a line connecting these two points must also lie in the opportunity set. Arbitrary points, such as points C and D connected by line segment CD, show that the opportunity set $\mathbf{Q}^S$ is a convex set.

Uniqueness Theorem

When $U(\mathbf{Q})$ is strictly concave over $\mathbf{Q}$, and the opportunity set $\mathbf{Q}^S$ is convex, then a local maximum is a unique strict global maximum. This ensures there is one and only one solution.

Weierstrass Theorem

The Weierstrass theorem gives sufficient conditions for the existence of a global maximum. These conditions are:

1. The opportunity set $\mathbf{Q}^S$ is compact (i.e., closed and bounded),
2. $\mathbf{Q}^S$ is a nonempty set, and

3. the objective function $U(\mathbf{Q})$ is continuous over $\mathbf{Q}^s$. If these three conditions hold for the classical model, then $U(\mathbf{Q})$ possess a global maximum either on the interior or on the boundary of Q^s.

Graphical Illustrations of the Conditions in the Weierstrass Theorem and Uniqueness Theorem in the Classical Model

The classical model with a given budget constraint can be shown to possess the three conditions necessary for the Weierstrass theorem to hold and hence have a global maximum solution. Figure 2a shows the relationship among the levels of good j (Q_j) consumed and the utility function $U(Q_j)$. Assume that $Q_j \varepsilon \mathbf{Q}$, $\mathbf{Q} \varepsilon \mathbf{Q}^s$ and all other quantities of goods in $\mathbf{Q}$ are held constant.

The first condition of the Weierstrass Theorem defines the closed and bounded opportunity set of all Q_j^s as the horizontal line segment OQ_j^* in Figure 2a. This opportunity set of Q_j defines all Q_j which satisfies the budget constraint in the classical model, and this opportunity set is closed and bounded. The second condition is satisfied since OQ_j^* is a nonempty set. The objective function $U(Q_j)$ is a continuous function over the range OQ_j^* in Figure 2a which satisfies the third condition of the Weierstrass theorem.

Given the existence of a global maximum by the conditions of the Weierstrass theorem, we may guarantee the classical model yields a unique solution when the conditions of the Uniqueness theorem are satisfied. The Uniqueness theorem requires that the objective function $U(\mathbf{Q})$ is strictly concave over $\mathbf{Q}$ and the opportunity set $\mathbf{Q}^s$ is a convex

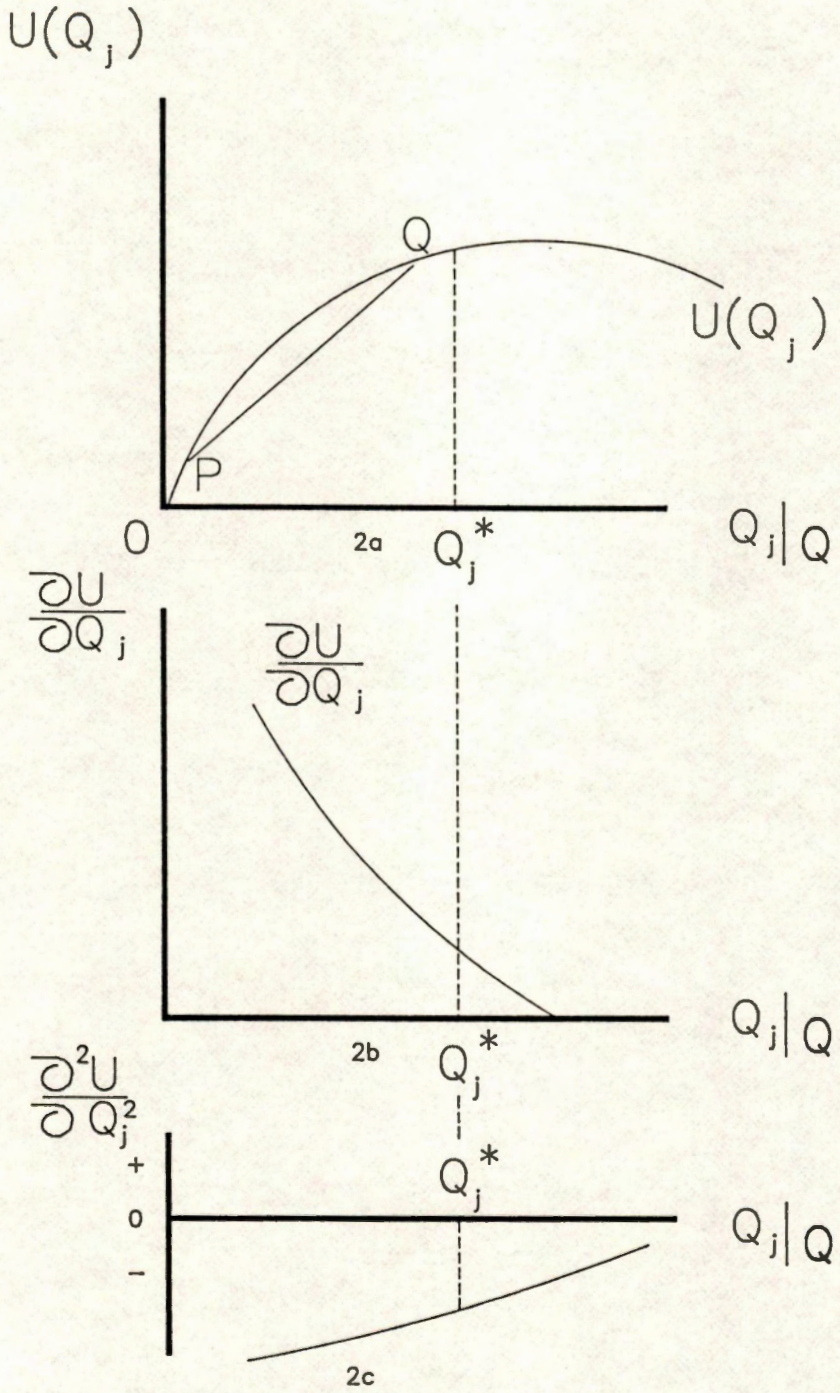


Figure 2. Functional Relationships of the Utility Function As Required by the Weierstrass and Uniqueness Theorems for the Classical Consumer Choice Model

set. In Figure 2a the utility function $U(Q_j)$ is defined as strictly concave if the chord connecting two points on the curve $U(Q_j)$ lies below the curve. The chord PQ lies below the utility function $U(Q_j)$ which implies the utility function $U(Q_j)$ is strictly concave over the opportunity set OQ_j^*. The only way the chord PQ can be below $U(Q_j)$ is when the marginal utility of Q_j is positive and $U(Q_j)$ is increasing at a decreasing rate. The second condition for Uniqueness requires that the opportunity set be a convex set. By inspection of Figure 2a, the opportunity set OQ_j^* is seen to be a convex set as a line connecting any two points in the set belongs to the set.

The Uniqueness theorem of the classical model implies additional properties of the model. This theorem states the utility function must be strictly concave. The slope of the strictly concave utility function $U(Q_j)$ in Figure 2a is $\partial U/\partial Q_j$, or the marginal utility of Q_j. Figure 2b shows that the slope of $U(Q_j)$ declines over the opportunity set OQ_j^*. Figure 2b and the function $\partial U/\partial Q_j$ show the slope of $U(Q_j)$ is declining, or marginal utility is positive and declining over the relevant range. Strict concavity of the utility function $U(Q_j)$ also implies the slope of the marginal utility function $\partial U/\partial Q_j$ is negative. The function $\partial^2 U/\partial Q_j^2$ in Figure 2c measures the direction of the slope of marginal utility in Figure 2b and is seen to be negative in the relevant range.

Unique Global Maximum Properties of the Classical Model

In summary, the Uniqueness theorem imposes the following properties for a unique solution in the classical model.

Property I

Within the relevant range of the opportunity set, marginal utilities are positive (nonsatiation) or

$$\frac{\partial U}{\partial Q_j} > 0 \qquad i = 1, \ldots, n .$$

Property II

Within the relevant range of the opportunity set, marginal utilities are declining or

$$\frac{\partial^2 U}{\partial Q_j^2} < 0 \qquad i = 1, \ldots, n .$$

Properties I and II of the marginal utility function are shown to exist in a unique solution to the classical model. Consider a two good world where $U = (Q_1, Q_2)$. Using (2-1) and (2-2), the Lagrangian function is

$$L = U(Q_1, Q_2) + \lambda \left(I - \sum_{i=1}^{2} P_i Q_i\right) , \tag{2-3}$$

which yields the first order conditions:

$$\frac{\partial L}{\partial Q_1} = \frac{\partial U}{\partial Q_1} - \lambda P_1 = 0 , \tag{2-4}$$

$$\frac{\partial L}{\partial Q_2} = \frac{\partial U}{\partial Q_2} - \lambda P_2 = 0 , \text{ and} \tag{2-5}$$

$$\frac{\partial L}{\partial \lambda} = I - \sum_{i=1}^{2} P_i Q_i = 0 \ . \qquad (2\text{-}6)$$

In (2-4) and (2-5) we know from Property I of the Uniqueness theorem that $\partial U/\partial Q_1$ and $\partial U/\partial Q_2$ are positive. Product prices P_1 and P_2 are positive since products are desirable. The marginal utility of money λ is assumed to be positive since the marginal utility of goods are positive.

Property I of the Uniqueness theorem satisfies the nonsatiation axiom in the classical consumer choice model. The nonsatiation axiom states that if the consumer's choice of bundle A contains no less of any good and more of some goods than bundle B, then A must be preferred to B. In terms of the utility function this axiom states:

$$A \geq B \text{ implies } U(A) \geq U(B), \text{ and if}$$

$$A \geq B \text{ and } A \neq B, \text{ then } U(A) > U(B).$$

This axiom requires the marginal utilities of goods to be positive. Thus, this axiom and Property I state that if bundle A is identical to bundle B except that A contained one more unit of Q_1 than B (Q_2 is the same in both bundles) then $U(A) > U(B)$.

Property II of the Uniqueness theorem can be described by using the Hessian matrix of (2-1). From (2-1) the Hessian matrix **H** is defined as:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 U}{\partial Q_1^2} & \frac{\partial^2 U}{\partial Q_1 \partial Q_2} \\ \frac{\partial^2 U}{\partial Q_2 \partial Q_1} & \frac{\partial^2 U}{\partial Q_2 \partial Q_1} \end{bmatrix}$$

Property II requires $\partial^2 U/\partial Q_1^2$ and $\partial^2 U/\partial Q_2^2$ to be negative. This is consistent with the conditions that **H** be a negative definite matrix, or that the principal minors of **H** alternate in sign beginning with the arbitrary $\partial^2 U/\partial Q_1^2 < 0$ in this example, and $[(\partial^2 U/\partial Q_1^2)\ (\partial^2 U/\partial Q_2^2) - (\partial^2 U/\partial Q_1 \partial Q_2)\ (\partial^2 U/\partial Q_2 \partial Q_1)] > 0$.

Slope and Shape of Indifference Curves

The local-global, Weierstrass and Uniqueness theorems of the classical model define certain properties of the utility surface and indifference curves. To determine the slope of an indifference curve, consider a two good world and utility function $U = U(Q_1, Q_2)$. The total differential of the utility function is

$$dU = \frac{\partial U}{\partial Q_1} dQ_1 + \frac{\partial U}{\partial Q_2} dQ_2 \quad , \tag{2-7}$$

which expresses the total change in utility caused by variations in Q_1 and Q_2. To determine the slope of the indifference curve in two dimensional space let the consumer move along one indifference curve by giving up some Q_2 in exchange for more Q_1. Since the change in utility along the indifference curve is zero, set $dU = 0$ in (2-7) which becomes

$$\frac{\partial U}{\partial Q_1} dQ_1 + \frac{\partial U}{\partial Q_2} dQ_2 = 0 \quad . \tag{2-8}$$

As the consumption of Q_1 increases by dQ_1, consumption of Q_2 decreases by dQ_2 along the indifference curve. The slope of the indifference curve is dQ_2/dQ_1, and by rearranging (2-8) this slope is the negative ratio of marginal utilities or

$$\frac{dQ_2}{dQ_1} = - \frac{\partial U/\partial Q_1}{\partial U/\partial Q_2} \quad . \tag{2-9}$$

Thus, the slope of an indifference curve in two dimensional space is negative because Property I of the Uniqueness theorem ensured positive marginal utilities.

The slope of the indifference curve is the ratio of marginal utilities of goods. In two dimensional space, the indifference curve would be a straight line with a constant slope throughout (like I_1 in Figure 3a) if marginal utilities were constant. However, property II of the Uniqueness theorem requires that the marginal utilities decline. Indifference curve I_1, in Figure 3a, is not possible when the Uniqueness theorem holds. By virtue of declining marginal utilities from property II, one can determine the shape of the indifference curve. Moving along the indifference curve and acquiring more Q_1 and less Q_2 by property II, $\partial U/\partial Q_1$ decreases as units of Q_1 are added and $\partial U/\partial Q_2$ increases as units of Q_2 are given up. The increasing relative abundance of Q_1 causes the numerator in (2-9) to decrease while the increasing scarcity of Q_2 causes the denominator in (2-9) to increase resulting in the absolute value of the slope becoming smaller. The resulting shape of (2-9) is shown in Figure 3b where the indifference curve I_2 is convex.

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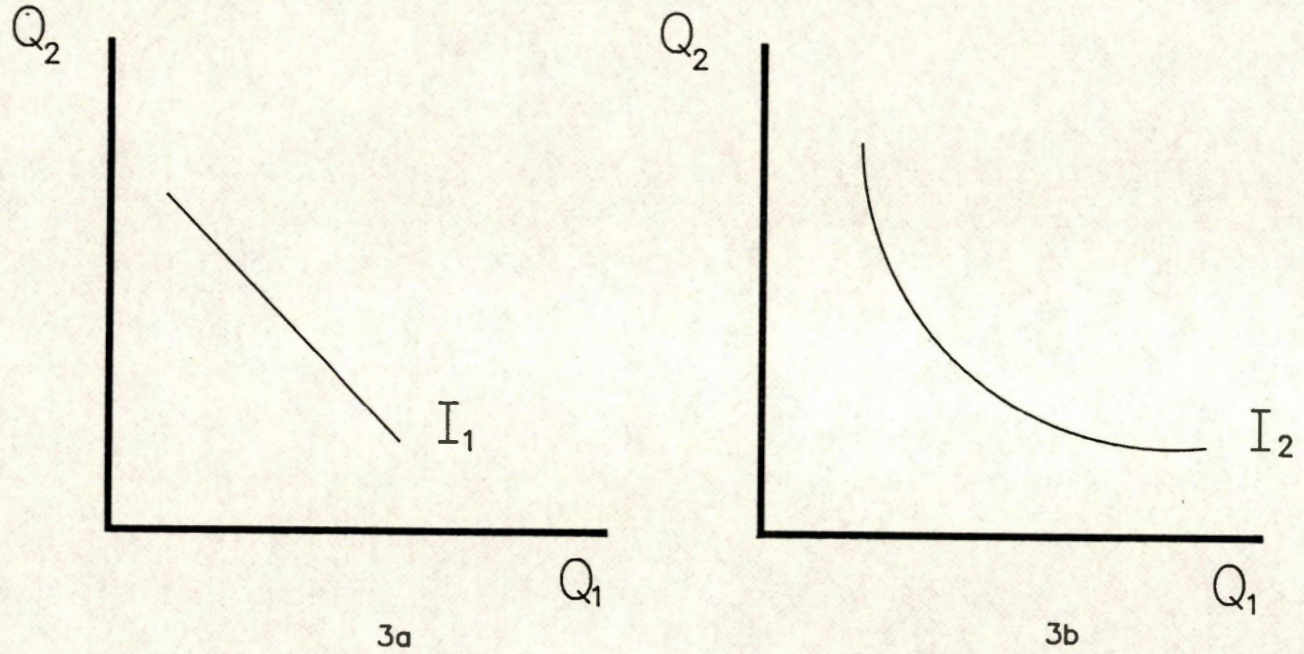


Figure 3. Indifference Curves for Utility Functions of the Classical Consumer Choice Model

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Property III

The classical model, which yields a unique solution, will have a utility surface that is convex, yielding indifference curves that are negatively sloped and convex.

When the local-global, Weierstrass, and Uniqueness theorems hold in the classical model, the utility surface is continuous, nondecreasing, and compact having positive marginal utilities and a negative definite Hessian. These conditions are depicted in a three dimensional diagram in Figure 4 where the two axies on the "floor" define the units of the goods, and the vertical axis is total utility. The utility surface is the shaded area where the surface rises as one moves outward from the origin. The height of the utility surface above the "floor" measures utility. For example the contour CC is an indifference curve whose height (utility) above the "floor" is measured as the vertical distance CC' and is constant throughout. Indifference curve CC represents the same total utility level CC' in its entire range. Indifference curve DD represents a higher level of total utility measured by the higher vertical distance DD'. When the vertical distance CC' is traced along the "floor," indifference curve C'C' portrays the traditional convex indifference curve in two dimensional space.

The importance of properties I, II and III of the Uniqueness theorem cannot be overemphasized. As the preceding discussion has shown, the Uniqueness theorem and its properties ensure indifference curves are convex, and a unique solution to the classical model exists. The

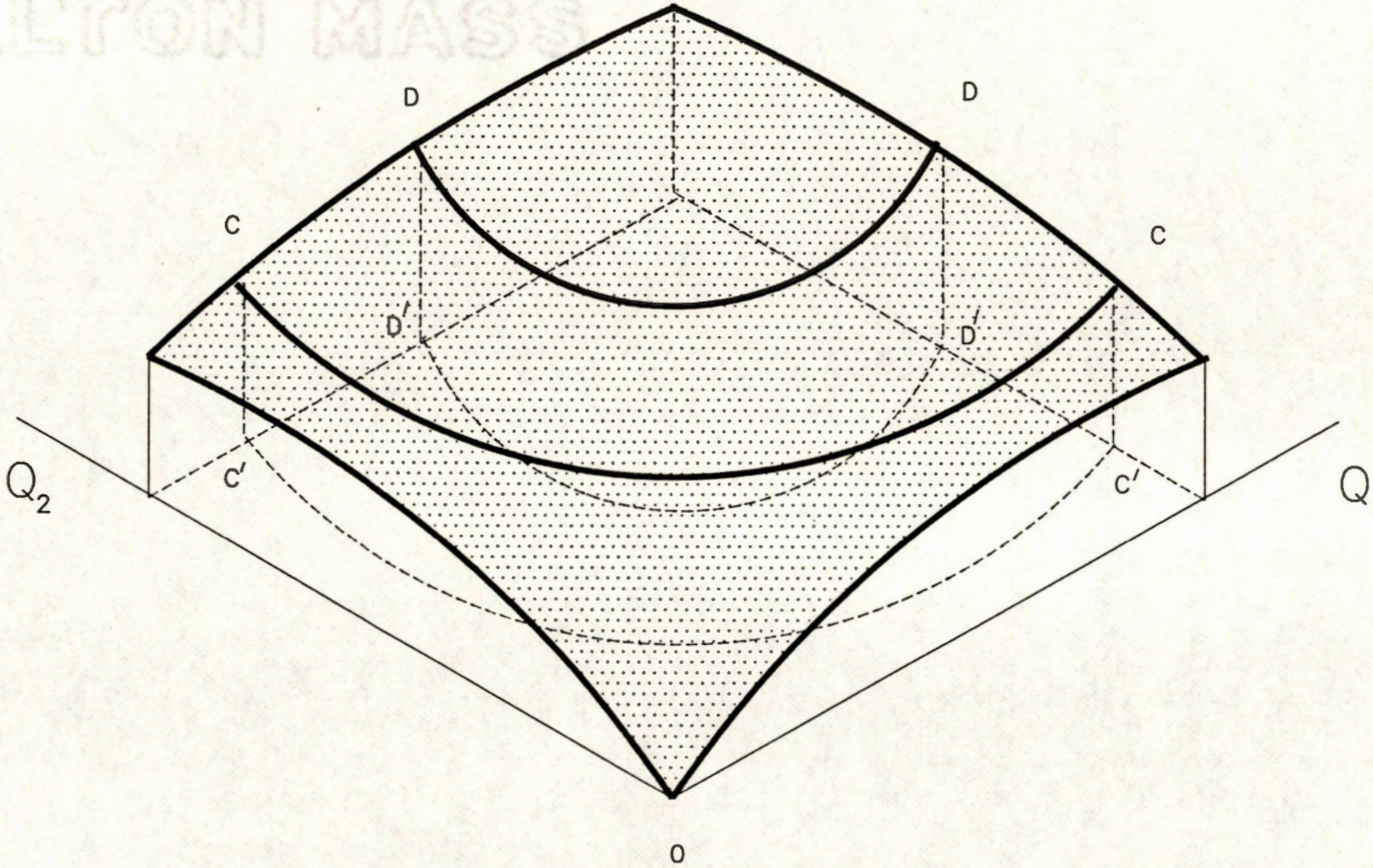


Figure 4. Utility Surface for Classical Consumer Choice Model for Two Goods Under Unique-Global Maximum Conditions

optimal consumption bundle is made explicit with only the knowledge of prices of goods and his fixed income. With this knowledge of preferences at specific prices and income level, prediction of consumer behavior when there are small changes in the price and income parameters is possible. Without a unique solution at given prices and income, the researcher knows little, and perhaps nothing, concerning preferences given these parameters and prediction of consumer choices when prices and income change will have little or no meaning to the researcher.

Summary of Conditions for a Unique Global Maximum in the Classical Model

The preceding discussion focused on appropriate necessary and sufficient conditions for the classical model to yield a unique maximum. When the local-global, Weierstrass, and Uniqueness theorems hold for the classical model, conditions that yield a unique maximum of the model are outlined in the Unique Global Maximum (UGM) conditions.

Summary of Unique Global Maximum Conditions

1. The utility function $U(\mathbf{Q})$ is continuous, strictly concave over Q_j for all $Q_j \varepsilon \mathbf{Q}$, $Q \varepsilon \mathbf{Q}^S$ where $\mathbf{Q}^S$ is closed, bounded, and non-empty.
2. Each good in $\mathbf{Q}^S$ yields positive marginal utility where

$$\frac{\partial U}{\partial Q_j} > 0 \quad j = 1, \ldots n \ ,$$

the nonsatiation axiom is satisfied.

3. The Hessian matrix is negative definite, and part of this condition is

$$\frac{\partial^2 U}{\partial Q_j^2} < 0 \quad j = 1, \ldots n \ .$$

The utility surface and indifference curves are strictly convex.

Derived Demand Models

The classical model has two special assumptions that make this analysis of consumer behavior unique. First, utility is generated directly by the consumer from the purchase and consumption of products whose prices are observable in the marketplace. Second, products purchased and consumed are generally thought to be final products in the sense that (1) products are not used as inputs to produce something else that generates utility, and (2) no component or characteristic that is contained in the product yields utility, only the entire product itself can provide utility.

In the section two models of consumer choice are discussed in which the special assumptions in the classical model are changed. The household production theory is presented first, then the characteristics theory of consumer choice is discussed. Both of the newer models share the same assumption that market products do not generate utility directly to the consumer, and that utility is generated by nonmarket items whose prices are unobservable in the marketplace.

The Household Production Theory

Becker (1965) and Michael and Becker (1973) developed the household production theory of consumer demand. This model combines market products and household time via production processes similar to that of a firm's production function to produce commodities which generate utility. This theory implies that all market products are desired as inputs into the production of commodities, thus the demands for market products are derived demands.

Utility in household production theory is a function of the nonmarket commodities Z_i ($i = 1, \ldots k$). Each final nonmarket commodity is a result of a production process combining market products Q_j ($j = 1, \ldots n$) and household production time t_i ($i = 1, \ldots k$). The members of the household's problem is to maximize $U = U(Z_1, Z_2, \ldots Z_k)$, subject to the production functions:

$$\begin{aligned} Z_1 &= Z_1 (Q_1, \ldots Q_n, t_1) \\ &\vdots \qquad\qquad \vdots \\ Z_k &= Z_k (Q_1, \ldots Q_n, t_k) \quad . \end{aligned}$$

The household chooses the combination of $Z_1 \ldots Z_k$ that maximizes utility. In this one period choice, a household has two additional constraints, the budget constraint and the time constraint.

The household's time in a given period is divided among (1) household production time t_i, (2) income earning time t_w, and (3) leisure time t_e.

$$\text{Total time } T = \sum_{i=1}^{K} t_i + t_w + t_e \quad .$$

Time that is not spent in household production or earning income is considered leisure time. When leisure time is combined with market goods, this combination enters implicitly in the utility function as a commodity.

The household's budget constraint limits the amount of market products that can be purchased. Given the prices of market products, P_i, the household's income, I, equals the households wage rate, w, multiplied by time spent at work, t_w, assuming the household members earn no income from savings and investment. Then

$$I = w \cdot t_w \quad ,$$

and assuming all household income is allocated to purchasing market products then:

$$I = \sum_{i=1}^{n} P_i Q_i \quad .$$

Household production theory shows how the household must allocate income and time most efficiently to maximize utility. Allocation of these two resources occurs simultaneously, as the household must decide how to allocate time to work while allocating income from work among market products. Decisions about quantities of market products to purchase are dependent upon how time is allocated, so both decisions are simultaneous in the given period of analysis.

The utility function is assumed to be increasing, concave and twice differentiable. With convex indifference curves used in the analysis,

convexity implies that the marginal utilities of commodities consumed through household production are positive and decreasing.

In contrast with the classical model, the demand for market products is a derived demand in which market products do not yield utility directly to the household. Unlike the classical model, household production theory has the ability to reveal the impact on consumer behavior of the number of family members in the household, of the importance of time as a constraint on household decision making, of quality changes among market products, and how new products enter the household production process.

The characteristics theory of consumer behavior is ideologically similar to the household production theory in that both theories use the idea that market products do not yield utility directly to the consumer. The main concept of the characteristics theory is that market products or services are viewed as a collection of characteristics or attributes. Market products themselves do not generate direct utility to the consumer. Rather, market products are desired because of the utility that their attributes generate. Thus, in the characteristics theory, demand for market products becomes a derived demand. This approach changes the basic relationships among variables in the classical model. While utility is a function of market products in the classical model, it becomes a function of the attributes provided by market products in the characteristics model.

Early Characteristics Theories

Early work in characteristics theory began with Waugh (1928, 1929) in which the quality attributes of vegetables were measured. He measured various dimensions of quality in cucumbers, asparagus, and tomatoes. From a time series of prices and quantities of these vegetables, Waugh used the linear regression technique to estimate quality prices of these vegetables. For measures of quality he collected data such as color, size, shape, and condition of locally grown vegetables in the Boston wholesale market. Examples of the final analysis of the quality-price relationship of asparagus included (1) each additional inch of green color per stalk added 34.45 cents to the price of one dozen standard bunches, and (2) each additional percentage variation in size of stalk decreased price by .76 cents per one dozen standard bunches (Waugh, 1929).

Similar studies followed that related product prices to physical characteristics of the product. Clark and Bressler (1938) attempted to explain that strawberry and egg prices are related to quality measured by the characteristics of size, condition, uniformity, color, and variety. Automobile prices were expressed as a linear combination of attributes in a number of studies (Court, 1939; Adelman and Griliches, 1961; Dhrymes, 1971; Ladd and Zober, 1977). Refrigerator prices are expressed as functions of height, weight, and freezer capacity by Dhrymes (1971). Horsepower and types of engine were related to tractor prices by Fettig (1963).

Application of the characteristics theory to the housing market by Witte, Sumka and Erekson (1979), Palmquist (1984), and King (1976) expresses housing prices as a function of physical characteristics of the home. These studies emphasize that product differentiation among housing units creates problems when characteristics other than physical ones (i.e., location, schools, air quality) enter the demand function for characteristics of housing. Freeman (1979) estimates demand for characteristics of property values. Specifically, the study isolates the environmental characteristics of air quality and develops a model in which property values reflect various levels of this characteristic.

Houthakker (1951-1952) and Theil (1951-1952) introduced qualities of products into the utility function with quantities of products. Houthakker assumed the problem facing the consumer is to maximize $U = U(Q_1, \ldots Q_n, v_1, \ldots, v_m)$ where v_i, $i = 1, \ldots, m$, is the quality variable assigned to product i. The model assigns only one quality variable per product. Houthakker divided the price of the product into $P_i = a_i + b_i v_i$ where a_i is the quantity price and b_i is the quality price paid per unit of quality. Thus, the budget constraint facing the consumer is

$$I = \sum_{i=1}^{n} Q_i (a_i + b_i v_i) \quad .$$

Houthakker assumed the utility surface to be increasing, convex in (Q_i, v_i) and twice differentiable. Thus, the marginal utility of each v_i is positive and decreasing as well as the marginal utility of Q_i.

Houthakker's model divided the consumer's decision into two steps: (1) the consumer first chooses the product to purchase and then (2) the consumer decides how much to spend on a variety where the variety is distinguished by the one quality variable v_i. Hanemann (1982) gave an excellent example by defining the product Q_i as wine and the sugar content of the wine as the quality variable v_i. Implicitly, the price per unit of wine is expressed as a function of its sugar content v_i in the equation $P_i = a_i + b_i v_i$. The model identifies products with different characteristics as the same product, only differing by the one variable quality. In this model the quality variable is not a discrete measurement and is allowed to range through a continuous spectrum.

Theil (1951-1952) introduced a model similar to Houthakker's model. Theil assume the consumer will maximize utility, that is:

$$\text{Maximize } U = U(Q_1, \ldots, Q_n, Z_1, \ldots, Z_n) \quad ,$$

subject to the budget constraint

$$I = \sum_{i=1}^{n} X_i \, P_i(Z_i) \quad ,$$

where Z_i is a vector of the qualities contained in the ith product.

Theil indicated that the price of a product is a good indication of its quality. Higher quality products will have higher prices than lower quality products. Thus, Theil expressed the price of a product as a function of the vector of quality characteristics it contains or $P_i = P_i(Z_i)$.

Cox and Wohlgenant (1986) use the Houthakker-Theil (H-T) model in identifying several problems when using cross-sectional household budget data. The H-T model is used to explain why prices reflect quality differences within the cross-section as well as the usual seasonal supply and demand variations. Analysis of the quality effects of price differences in cross sectional household data developed a conceptual framework of demand analysis when prices are nonconstant in cross-section data. The Theil model allows each variety of a product to be described by more than one quality variable, unlike the Houthakker model. Both models treat products with different quality characteristics as the same generalized product and assume a continuous spectrum of qualities. In a fashion analogous to that outlined in the classical model above, the H-T models have unique optimal solutions when the utility function satisfies the UGM conditions stated previously.

Lancaster's Characteristic Theory

The next major work on the characteristics model was by Lancaster (1966). In Lancaster's model the relationship between utility realized by the consumer and the purchased product involves two steps. First, there is a technical relationship among the product and the type and amount of characteristics contained in the product. This relationship is called the consumption technology. The second step is the link between characteristics and individual preferences of consumers. It is assumed that each consumer has a preference for a collection of characteristics that depends on the utility each characteristic yields. Preferences and demands for products are indirect, as the products serve

only as instruments that provide characteristics which yield utility directly to the consumer. Thus, product demand is a derived demand.

In Lancaster's model, as developed in a number of studies (Suvannunt, 1973; Ladd and Suvannunt, 1976; Ladd, 1978; LaFrance, 1983; Terry, 1985), a consumer attempts to maximize

$$U = U(X_1, X_2, \ldots, X_m)$$

subject to the budget constraint

$$I = \sum_{i=1}^{n} P_i Q_i \quad ,$$

and the following conditions:

$$X_j = \sum_{i=1}^{n} x_{ij} Q_i \quad ,$$

$$X_j \geq 0, \; x_{ij} \geq 0, \; Q_i \geq 0, \text{ and } P_i > 0 \quad .$$

where:

U = consumer's utility,

X_j = the total amount of the jth characteristic or attribute provided by the consumption of all products Q_i ($i = 1, \ldots n$),

x_{ij} = the quantity of the jth characteristic provided by one unit of the ith product,

m = the number of common characteristics in all products,

n = the number of products available,

P_i = the price of the ith product,

Q_i = the quantity of the ith product consumed, and

I = consumer's fixed income.

As in the classical model, this model assumes the utility function U is increasing and concave over all X_j. The utility surface is convex. The marginal rate of substitution between attributes is decreasing, thus we can further state the model exhibits positive decreasing marginal utility for attributes.

To illustrate Lancaster's model assume a four product, two attribute world. Each product, Chicken (C), Pork (P), Beef (B) and Lamb (L) are assumed to contain only two attributes where, X_1 is protein and X_2 is fat. These two attributes can be measured objectively in each product. This model is illustrated in Figure 5 where the horizontal and vertical axes show the various amounts of the two characteristics X_1 and X_2. The rays from the origin represent each of the four products C, P, B, and L. Each ray from the origin shows that the proportion of X_1 and X_2 the product contains is independent of the quantity of the product purchased. For example, no matter how many pounds of chicken are consumed, the ray C shows that X_1 and X_2 will be consumed in the same proportion per pound.

Assume the consumer has a fixed amount of income I to spend on the four products. If the consumer spends the entire income on chicken and none on the remaining three products, then point A represents how much chicken is purchased. Points B, C, and D are analogous to A in that B, C and D show how much of each product can be purchased with the fixed

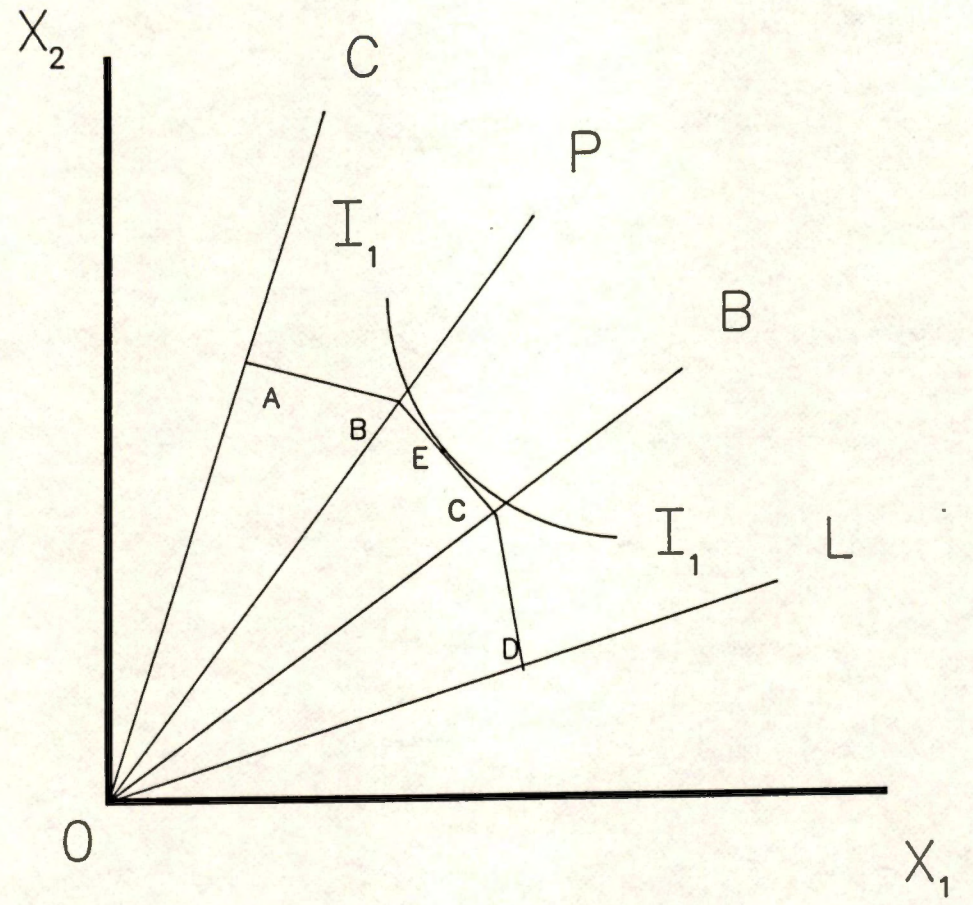


Figure 5. Utility Maximization in Lancaster's Consumer Characteristics Model

income. The locus ABCD represents the frontier of attainable combinations of X_1 and X_2 given the budget constraint I.

An indifference curve in the model is represented by I_1I_1. The slope of the indifference curve represents the negative ratio between the marginal utility of X_1 and marginal utility of X_2. By assuming the utility function is increasing, concave, and twice differentiable, one asserts that the marginal utility of each attribute is positive and decreasing, hence the indifference curve I_1I_1 is convex. These conditions will be examined in more detail later in the paper.

Lancaster's model allows the consumer to purchase combinations of products. For example, in Figure 5 the consumer will purchase a combination of P and B at point E along the frontier BC because point E is the combination of characteristics X_1 and X_2 that yields the maximum utility level at I_1I_1. If the consumer purchased all P at B or all B at C, either of these combinations of (X_1,X_2) at B or C would yield a lower utility level than the combination of attributes yields at point E.

This model makes it possible to assess the consumer's attempt to maximize utility without the prior knowledge of the individual's preferences for characteristics. The products the consumer chooses depends on how important the consumer's preferences for various characteristics are. In Figure 5, when the consumer purchased combination E of pork and beef, his primary purpose was to purchase X_1 and X_2. If the consumer had chosen a combination of characteristics represented by a point inside the frontier ABCD, one could conclude that the consumer has not maximized utility. This conclusion is reached with no prior knowledge

of the consumer's preferences for attributes, only with the aid of the consumption technology and the budget constraint.

Lancaster made three assumptions about the characteristics model that have been criticized (Ladd and Zober, 1977). These assumptions are

1. every characteristic yields nonnegative marginal utility (NNMU),
2. utility is independent of the distribution of characteristics among products (IDC), and
3. there is a linear consumption technology (LCT).

Hendler (1975) questions the NNMU assumption. He showed that if the NNMU assumption is violated then the efficiency frontier does not exist and concludes it is impossible to judge a consumer's efficiency without the knowledge of his preferences. For example in Figure 5, if characteristic X_1 yields negative marginal utility at any level of X_1 the indifference curve I_1I_1 would have a positive slope. In Lancaster's model, it is only necessary to know the consumption technology, prices of products and the budget constraint to judge the consumer's attempt to maximize utility. The underlying concept of convex indifference curves implies positive decreasing marginal utilities for characteristics and the assumption that consumers always prefer more to less characteristics. However with Hendler's violation of NNUU assumption, one must now know the consumer's preferences to judge the efficiency of his choices because now one is not sure of the tradeoff between the characteristic which yields positive marginal utility and the characteristic which yields negative marginal utility. Hendler noted that while one

characteristic may yield a negative marginal utility, the product itself must still yield net positive marginal utility.

The linear consumption technology (LCT) is questioned by Lucas (1975). The LCT assumption implies that if Q_1 units of a product contain x_{11} amounts of a characteristic, then twice as much Q_1 will contain twice as much x_{11}. In Lancaster's model, it is impossible to determine the consumer's efficiency if the consumption technology is not known. Lancaster's model with the LCT assumption does not yield predictive results when products are used whose characteristics are measured ordinally (i.e., music, comfort, etc.). The model is a very useful tool when products are used whose characteristics are measurable on a cardinal scale. For example, food products possess nutritional characteristics that permit cardinal measurements. For food nutrients the LCT assumption is quite appropriate since doubling the quantity of a food product will double the quantity of nutritional characteristics contained in the food products.

Ladd (1982) supposed Lancaster's IDC assumption is violated. The IDC assumption states that a consumer is indifferent among any number of combinations of products available in order to obtain a characteristic. If products A and B both contain one unit of characteristic X_1 per pound, the IDC assumption is not violated if the consumer is indifferent between products A and B in order to obtain 10 units of X_1 for example. However, if the consumer prefers product A to B in order to obtain 10 units of X_1, then the IDC is violated, and all available characteristics are not being considered in the consumer's decision. For the IDC

assumption to be appropriate, each point in the characteristic space represents a specific combination of characteristics and a unique level of utility. If the IDC assumption is violated, at least one point in two-dimensional characteristic space represents two (or more) levels of utility, each level being relevant for a different distribution of characteristics among products. Then to evaluate a consumer's efficiency, one must know the relationship of his utility and the distribution of characteristics consumed among the products purchased (Ladd, 1982, p. 27).

The Ladd-Suvannunt Characteristics Model

Suvannunt (1973) and Ladd and Suvannunt (1976) developed a model similar to Lancaster's model. Although the Ladd-Suvannunt (L-S) model uses the same idealology as the Lancaster model, the L-S model does not assume NNMU.

The L-S model assumes n products and m common attributes contained in the products. In addition the L-S model is structured to accommodate unique attributes not provided by other products. The utility function is

$$U = U(X_1, X_2, \ldots, X_m, X_{m+1}, \ldots, X_{m+n}) \quad , \qquad (2\text{-}10)$$

where:

X_j = the total quantity of the jth attribute obtained by a consumer from the consumption of all products, and

X_{m+i} = the total quantity of the unique attribute provided by the consumption of the ith product.

The level of one particular attribute consumed is a function of the quantity of products consumed Q_i and the input-output coefficients associated with one unit of each product or x_{ij}. The x_{ij}'s are exogenous to the consumer and considered fixed by the consumption technology. This is expressed as

$$X_j = f(Q_1, Q_2, \ldots, Q_n, x_{ij}, x_{ij}, \ldots, x_{nj}) \quad , \tag{2-11}$$

and

$$X_{m+i} = f(Q_i, x_{im+i}) \quad , \tag{2-12}$$

where X_j and X_{m+i} are endogenous to the consumer.

Substituting (2-11) and (2-12) into (2-10) gives the relationship

$$U = U(Q_1, Q_2, \ldots Q_n, x_{11}, \ldots x_{n1}, x_{12}, \ldots x_{n2}, x_{1m+1}, \ldots x_{nm+n}) \tag{2-13}$$

or

$$U = U(\cdot) \quad .$$

In this one-period analysis one may assume the consumer's fixed budget constraint is spent and defined as

$$I = \sum_{i-1}^{n} P_i Q_i \quad . \tag{2-14}$$

The relationships among U, Q, X, P and I in the L-S model are written to maximize (2-13) subject to (2-14) or

$$\text{Maximize } U = U(\cdot) \quad , \tag{2-15}$$

subject to

$$I = \sum_{i-1}^{n} P_i Q_i \quad . \tag{2-16}$$

Before the L-S model is fully developed and explained, distinctions between the H-T and the L-S model are explored. First, the H-T model assumes a continuous distribution of quality that is assigned to each product. In the L-S model products with differing levels of quality characteristics are treated as separate products. Second, in the H-T model quality is measured by one variable so products containing different characteristics are essentially the same goods only differing in the one variable quality. The L-S model assumes that the quality of the product depends on the level of many common and unique characteristics in the product. Third, in the H-T model the consumer maximizes utility using two-steps: (1) the consumer chooses the generalized product and (2) then the consumer decides how much to spend on a particular variety or brand of the generalized product based on the one quality variable he assigns to that brand. In the the L-S model the consumer maximizes utility by choosing the combination of quality characteristics available. The instruments for choosing this level in the L-S model are the quantities of products purchased.

The L-S model can be solved through creating the Lagrangian function

$$L = U(\cdot) + \lambda \left(I - \sum_{i=1}^{n} P_i Q_i\right) \tag{2-17}$$

Because the consumer alters the quantities of products purchased to vary the attributes obtained, the first order partials with respect to Q_i ($i = 1, \ldots, n$) are taken. The first order conditions are

$$\frac{\partial L}{\partial Q_i} = U_i - \lambda P_i = 0 \text{ , } (i = 1, \ldots n), \text{ and} \tag{2-18}$$

$$\frac{\partial L}{\partial \lambda} = I - \sum_{i=1}^{n} P_i Q_i = 0 \text{ ,} \tag{2-19}$$

where $U_i = \partial U(\cdot)/\partial Q_i$. Recalling (2-10), (2-11) and (2-12) one can rewrite U_i as

$$U_i = \sum_{j=1}^{m} \frac{\partial U}{\partial X_j} \cdot \frac{\partial X_j}{\partial Q_i} + \frac{\partial U}{\partial X_{m+i}} \frac{\partial X_{m+i}}{\partial Q_i} \quad (i = 1, \ldots, n) \tag{2-20}$$

Substituting (2-20) into (2-18) yields:

$$\frac{\partial L}{\partial Q_i} = \sum_{j=1}^{m} \frac{\partial U}{\partial X_j} \cdot \frac{\partial X_j}{\partial Q_i} + \frac{\partial U}{\partial X_{m+i}} \frac{\partial X_{m+i}}{\partial Q_i} - \lambda P_i = 0 \text{ .} \tag{2-21}$$

Solving for P_i

$$P_i = \frac{1}{\lambda} \sum_{j=1}^{m} \frac{\partial U}{\partial X_j} \frac{\partial X_j}{\partial Q_i} + \frac{\partial U}{\partial X_{m+i}} \frac{\partial X_{m+i}}{\partial Q_i} \text{ .} \tag{2-22}$$

The marginal utility for income is $\lambda = \partial U/\partial I$ so (2-22) becomes

$$P_i = \sum_{j=1}^{m} \frac{\partial X_j}{\partial Q_i} \left(\frac{\partial U}{\partial X_j} \frac{\partial I}{\partial U}\right) + \frac{\partial X_{m+i}}{\partial Q_i} \left(\frac{\partial U}{\partial X_{m+i}} \frac{\partial I}{\partial U}\right), \quad (i = 1, \ldots, n) \tag{2-23}$$

From (2-23) the term $\partial U/\partial X_j \cdot \partial I/\partial U$ can be rewritten as the ratio

$$\frac{\partial U/\partial X_j}{\partial U/\partial I},$$

which equals the marginal rate of substitution of income for the jth attribute. This ratio expresses the amount of income that is required to compensate the consumer for giving up one unit of X_j and remain at the same utility level. Define

$$\frac{\partial I}{\partial X_j} = \frac{\partial U}{\partial X_j} \cdot \frac{\partial I}{\partial U} \tag{2-24}$$

as the hedonic price of the jth attribute. The interpretation of the hedonic price of the jth attribute is that it represents the marginal implicit price of the jth attribute, or the price the consumer would be willing to pay for one additional unit of the jth attribute.

From (2-23) define

$$\frac{\partial I}{\partial X_{m+i}} = \frac{\partial U}{\partial X_{m+i}} \cdot \frac{\partial I}{\partial U}, \tag{2-25}$$

and assume that one unit of each product provides one unit of its own unique characteristic, then $\partial X_{m+i}/\partial Q_i = 1$, and one can rewrite (2-23) as

$$P_i = \sum_{j=1}^{m} \frac{\partial X_j}{\partial Q_i} \cdot \frac{\partial I}{\partial X_j} + \frac{\partial I}{\partial X_{m+i}} \quad (i = 1, \ldots, n). \tag{2-26}$$

The hedonic price equation for the unit price of product i is equation (2-26). This equation expresses the price of product i as the sum of the marginal amounts of each attribute of each product multiplied by the marginal implicit prices. This hedonic price equation for product i reveals the hypothesis that a particular product's price is related to the attributes it contains. Ladd and Suvannunt (1976) stated this was testable hypothesis, and they turned it into a linear structure by assuming

$$\frac{\partial X_j}{\partial Q_i} = x_{ij} \quad \text{and} \quad \frac{\partial I}{\partial X_j} = \beta_j \ . \tag{2-27}$$

Here x_{ij} is the quantity of the jth attribute contained in one unit of the ith product, and β_j is the marginal implicit price paid for the jth attribute. Therefore, the linear form of the hedonic price equation (2-26) is

$$P_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \ . \ . + \beta_m x_{im} + \beta_{m+i} \tag{2-28}$$

or

$$P_i = \sum_{j=1}^{m} \beta_j x_{ij} + \beta_{m+i} \ . \tag{2-29}$$

Once the hedonic price equation (2-29) is known for the representative consumer, the consumer's demand function for an individual attribute X_j is

$$X_j = f(P_1, P_2, \ldots, P_n, I, x_{11}, x_{12}, \ldots, x_{1m+1}, x_{21}, x_{22}, \ldots, x_{2m+2}, \ldots, x_{n1}, x_{n2}, \ldots, x_{nm+n}). \quad (2\text{-}30)$$

Recalling (2-29) that each P_i is expressed as a function of β_j and x_{ij}, and assuming producers do not vary attribute input-output coefficients x_{ij}, (2-30) can be rewritten as:

$$X_j = f(\beta_1, \beta_2, \ldots, \beta_m, \beta_{m+1}, \ldots, \beta_{m+2}, I) \quad (2\text{-}31)$$

Estimation of the attribute demand equation is usually with a linear form (e.g., Ladd and Suvannunt, 1976; Terry, 1985)

$$X_j = \gamma_0 + \gamma_1\hat{\beta}_1 + \gamma_2\hat{\beta}_2 + \ldots + \gamma_m\hat{\beta}_m + \gamma_{m+i}\ \hat{\beta}_{m+i} + \gamma_I I \quad (2\text{-}32)$$

where the $\hat{\beta}_j$'s and $\hat{\beta}_{m+i}$'s are the previously estimated implicit attribute prices from (2-29).

Estimating the demand for an attribute in the L-S model involves a two-step process. Following Terry (1985), assume the products in the L-S model have no unique attributes ($x_{i\ m+i} = 0$). That is, all attributes are common to all products. First regress the price of product i on all the attributes it contains as in (2-29). Then use these estimated implicit prices (β_j's) to estimate attribute demand in (2-32). However, as in any market exchange process, there is an identification problem between the demand and supply of attributes. The attribute demand equation in functional form (2-31) is only one side of

the market. There is also an attribute supply equation. The identification problem has been examined by Brown and Rosen (1982), Freeman (1979), Hanemann (1982), and Rosen (1974).

If the hedonic price equation is linear in attributes, then identification of (2-31) is possible, because producers face constant hedonic prices in the supply equation for attributes where $\partial P_i/\partial X_j$ = the constant β_j. When the exchange in the market occurs, the consumer's marginal willingness to pay for attribute j must equal producers constant hedonic price or

$$\frac{\partial P_i}{\partial X_j} = \frac{\partial I}{\partial X_j} = \text{constant } \beta_j \ . \qquad (2\text{-}33)$$

Therefore, (2-33) is independent of the quantities of attributes exchanged, and β_j enters the consumers demand equation (2-31), and no identification problem exists.

If the form of (2-26) is nonlinear in the attributes, the implicit price of an additional unit of the attribute is dependent on the quantity of the attribute exchanged in the market. This presents a special problem in identifying (2-31) because the marginal implicit price of attribute j is dependent on the quantity of j while (2-31) specifies that the level of j is dependent upon the price of j. Obviously, (2-31) cannot be identified under these circumstances because the level of j and its marginal implicit price cannot both be endogenous variables in (2-31). Identification of (2-31) is possible with a nonlinear hedonic price equation when the supply of the product is

assumed to be perfectly elastic at observed market prices. With this assumption of perfectly elastic supplies of products, the marginal implicit price for attribute j ($\partial I/\partial X_j$) does not vary with the level of j or the quantities of product consumed. The marginal implicit prices are exogenous variables that enter (2-31) and make the demand for attribute equation identifiable.

Hanemann (1982) pointed out that the identification problem in the H-T model can be avoided using the same assumption of perfectly elastic product supply curves. He recognized the demand function for attribute j and the hedonic price equation for j depend on the same quality and price variables. He noted that in the H-T model, with upward sloping supply curves where producer's marginal implicit price of attribute j is dependent on the amount of j and the amount of the product, identification of the attribute demand is impossible without the assumption of perfectly elastic product supply curves. Within the context of an individual consumer's demands for nutrients in the marketplace, the assumption of perfectly elastic supply is reasonable.

Mathematical Model and Appropriate Conditions for Optimization of the L-S Model

Classical consumer demand theory can be described as an economizing problem of allocating scarce resources among competing ends during a particular period of time. The preceding discussion emphasized that the classical economizing problem can be approached as a mathematical programming problem of determining the values of certain variables subject to a set of constraints, as to maximize a given function. When

presented as a mathematical programming problem, the classical model was shown to have certain mathematical properties which offered meaningful economic interpretations.

The L-S characteristics model can also be analyzed as an economizing problem and approached as a mathematical programming problem. Viewed as a mathematical problem, the L-S model seeks to determine the value of certain variables subject to a constraint while maximizing a given function. As a result of viewing the L-S model as a mathematical programming problem, various mathematical properties of the model emerge that have important economic meanings.

In the past, development and use of the L-S model has emphasized the economic nature of the maximization problem at hand, while making the assumption that the appropriate mathematical properties of optimization are satisfied. As stated earlier, mathematical properties have strict economic interpretations and recent use of the L-S model indicates some researchers have violated necessary mathematical properties, and thus their economic analysis of the problem is suspect.

This section approaches the L-S model as a mathematical programming problem which to the author's knowledge has not been presented in this fashion. Mathematical properties evolving from this approach will be given economic interpretation where appropriate.

When viewed as a mathematical programming problem, the L-S model is to

$$\text{maximize } U = U(\mathbf{X}) \qquad (2\text{-}34)$$

$$\text{subject to } I = \sum_{i=1}^{n} P_i Q_i \quad , \tag{2-35}$$

where **X** defines a vector in Euclidean m-space as:

$$\mathbf{X} = [X_1, X_2, \ldots, X_m] \ .$$

Assume m attributes and n products in the model. The instruments Q_i are used in the constraint function (2-35) because it is ultimately the products purchased which provide the vector **X** that yields utility. The parameters $P_1, \ldots, P_n$ and income I are exogenous in this one-period maximization problem.

The vector **X** is considered a feasible solution if it satisfies all the constraints of the problem. There may exist several vectors **X**, each with varying levels of $X_1, \ldots, X_n$ that may be candidates for the solution. The opportunity set $\mathbf{X}^S$ is defined as the set of all feasible vectors **X**. The problem is now viewed as that of choosing the vector **X** (from all the possible vectors **X** contained in $\mathbf{X}^S$) that will maximize the objective function U.

Types of Maxima in the L-S Model

Global Maximum

Define vector **X*** as a global maximum if it is feasible and it yields a value of the objective function larger than or equal to that yielded by any other feasible vector **X**. That is

$$\mathbf{X}^* \varepsilon \mathbf{X}^S \text{ and } U(\mathbf{X}^*) \geq U(\mathbf{X}) \text{ for all } \mathbf{X} \varepsilon \mathbf{X}^S \quad ,$$

where $\mathbf{X}^S$ is a subset of Euclidean m-space.

Strict Global Maximum

The global maximum $\mathbf{X}^*$ is a strict global maximum if the value of the utility function at $\mathbf{X}^*$ is strictly larger than that at any other point on the utility surface. That is

$$U(\mathbf{X}^*) > U(\mathbf{X}) \text{ for all } \mathbf{X} \varepsilon \mathbf{X}^S, \text{ and } \mathbf{X} \neq \mathbf{X}^* .$$

If $\mathbf{X}^{**}$ and $\mathbf{X}^*$ were each distinct strict global maxima, then from the definition of a strict global maximum

$$U(\mathbf{X}^{**}) > U(\mathbf{X}^*) \text{ and } U(\mathbf{X}^*) > U(\mathbf{X}^{**}) ,$$

both of which obviously cannot be true. Hence, a solution that is a strict global maximum is obviously a unique solution.

Local Maximum

If feasible, the instrument vector $\mathbf{X}^*$ is a local maximum as long as it yields a value of the utility function larger than or equal to that yielded by any other feasible vector sufficiently close to it. That is:

$$\mathbf{X}^* \varepsilon \mathbf{X}^S \text{ and } U(\mathbf{X}^*) \geq U(\mathbf{X}) \text{ for all } \mathbf{X} \varepsilon \mathbf{X}^S \quad N_\varepsilon(\mathbf{X}^*)$$

where $N_\varepsilon(\mathbf{X}^*)$ is an ε minus neighborhood of X^* for some positive ε, however small. Here the term $N_\varepsilon(\mathbf{X}^*)$ is defined as the set of all $\mathbf{X}$ such that

$$|X - X^*| = \sqrt{\sum_{i=1}^{m} (X_i - X_i^*)^2} < \varepsilon \ .$$

Strict Local Maximum

A local maximum X^* is a strict local maximum if the value of the utility function at X^* is larger than that of any other feasible vector sufficiently close to it. That is X^* is a strict local maximum if:

$$U(X^*) > U(X) \text{ for all } X \varepsilon X^S \quad N_\varepsilon(X^*) \ ,$$

where $X \neq X^*$.

The connection between a global and local maximum is that a global maximum is a local maximum. However, a local maximum is not necessarily a global maximum, as there could be other local maxima yielding a higher value of the utility function.

Local-Global Theorem

A local-global theorem gives sufficient conditions for a local maximum to be a global maximum. If the opportunity set X^S is a nonempty compact set that is convex and $U(X)$ is a continuous function that is concave over X, then a local maximum is a global maximum. This ensures that the set of points at which the maximum is obtained is convex.

Compact and Convex Sets of the L-S Model

The opportunity set X^S of the L-S model is considered compact when X^S is (1) closed and (2) bounded. For ease of illustration consider a

two attribute world with four products as depicted in Figure 6. The feasible opportunity set $\mathbf{X}^S$ is the shaded area OABCD and is considered closed since the set $\mathbf{X}^S$ includes all of its boundaries (i.e., the set includes the points on the boundary OABCD). The set is bounded since the shape of the shaded area is definite without open ends which rules out an infinite shape of $\mathbf{X}^S$. Here the frontier ABCD defines the maximum amount of each product the consumer is able to purchase with the given income in this one-period analysis.

The definition of a convex set stipulates that for any two points in the opportunity set, a line connecting these two points must also be in the opportunity set. Arbitrary points E and F connected by line segment EF prove that the opportunity set $\mathbf{X}^S$ is a convex set.

Uniqueness Theorem

When $U(\mathbf{X})$ is strictly concave over $\mathbf{X}$ and the opportunity set $\mathbf{X}^S$ is convex, then a local maximum is a unique strict global maximum.

Weierstrass Theorem

The Weierstrass Theorem gives conditions sufficient for the existence of a global maximum. These conditions are: (1) the opportunity set $\mathbf{X}^S$ is compact, (2) $\mathbf{X}^S$ is a nonempty set, and (3) the objective function $U(\mathbf{X})$ is continuous over $\mathbf{X}^S$. If these three conditions hold, then $U(\mathbf{X})$ possess a global maximum either on the interior or on the boundary of $\mathbf{X}^S$.

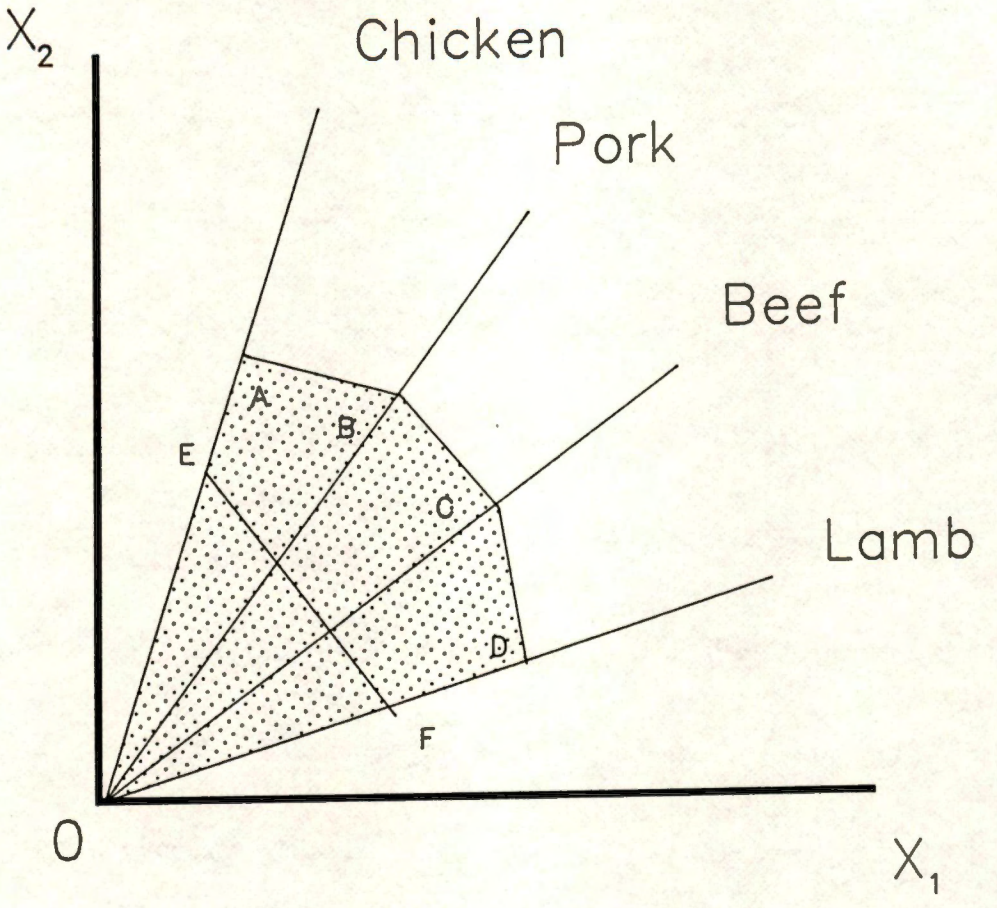


Figure 6. Hypothetical Closed and Bounded Opportunity Set for Consumer Characteristics Model

Graphical Illustration of Mathematical Properties of the Weierstrass and Uniqueness Theorems in the L-S Model

The first requirement of the Weierstrass theorem is that the opportunity set $\mathbf{X}^S$ be compact (i.e., closed and bounded). Figure 7a shows the relationship among X_j and the utility function $U(X_j)$ where the horizontal line segment OX_j^* defines the opportunity set of X_j. From the previous discussion of compact sets, it is seen that OX_j^* is closed and bounded. Since OX_j^* is a nonempty set, the second condition of the Weierstrass theorem is satisfied. The third condition maintains that the objective function $U = U(\mathbf{X})$ is a continuous function over the range OX_j^* as is true in Figure 7a. These three conditions are the sufficient conditions for the existence of a global maximum. We may guarantee the L-S maximum is a unique solution when conditions of the Uniqueness theorem are satisfied.

The first condition of the Uniqueness theorem requires the objective function $U(\mathbf{X})$ to be strictly concave over $\mathbf{X}$. Figure 7a shows the utility function $U(X_j)$ as strictly concave over OX_j^*, since the chord PQ lies below the function $U(X_j)$. The second condition for Uniqueness requires that the opportunity set be a convex set. By inspection of Figure 7a, the opportunity set OX_j^* is the convex, since a line connecting any two points in the set belongs to the set.

The Uniqueness theorem of the L-S model implies additional mathematical properties of the model that have significant economic meaning. The slope of the strictly concave function $U(X_j)$ in Figure 7a equals $\partial U/\partial X_j$, or the marginal utility of attribute X_j. The concave function

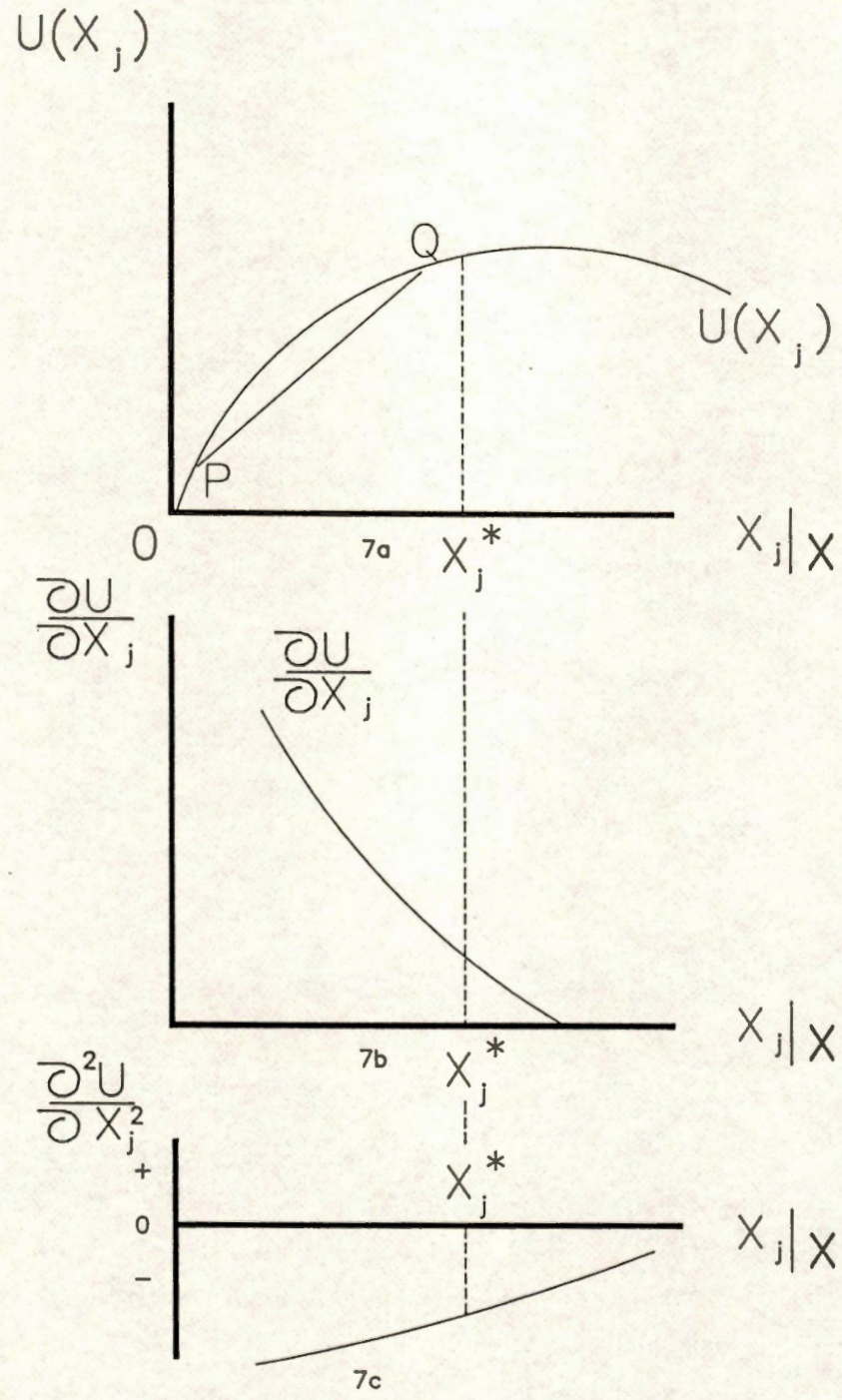


Figure 7. Functional Relationships of the Utility Function as Required by the Weierstrass and Uniqueness Theorems for the Consumer Characteristics Model

$U(X_j)$ increases at a decreasing rate. Figure 7b shows the marginal utility function $\partial U/\partial X_j$ as positive and declining over the relevant range. The function $\partial^2 U/\partial X_j^2$ in Figure 7c measures the direction of the slope of marginal utility in 7b and is seen to be negative in the relevant range.

In summary, the Uniqueness theorem imposes the following properties for a unique solution in the L-S model.

Property I

Within the range of the opportunity set, marginal utilities are positive, or $\partial U/\partial X_j > 0$, $i = 1, \ldots, n$.

Property II

Within the range of the opportunity set, marginal utilities are declining or $\partial^2 U/\partial X_j^2 < 0$, $i = 1, \ldots, n$.

Properties I and II can be shown to exist in the unique solution to the L-S model. Consider the L-S model presented earlier and recall the Lagrangian function (2-17)

$$L = U(\mathbf{X}) + \lambda(I - \sum_{i=1}^{n} P_i Q_i) \quad , \qquad \text{(2-17) (repeated)}$$

where the resulting first order conditions (2-21) are

$$\frac{\partial L}{\partial Q_i} = \sum_{j=1}^{m} \frac{\partial U}{\partial X_j} \cdot \frac{\partial X_j}{\partial Q_i} + \frac{\partial U}{\partial X_{m+i}} \frac{\partial X_{m+i}}{\partial Q_i} - \lambda P_i = 0 \quad \text{for } i = 1, \ldots, n \, . \qquad \text{(2-21) (repeated)}$$

In (2-21) we know from property I of the Uniqueness theorem that $\partial U/\partial X_j > 0$, and the unique attribute yields marginal utility or $\partial U/\partial X_{m+i} > 0$. Product prices $P_1, \ldots P_n$ are positive indicating products are desirable. The input-output coefficients $\partial X_j/\partial Q_i$ and $\partial X_{m+i}/\partial Q_i$ are positive since products Q_i are assumed to provide additional units of X_j and the unique attribute X_{m+i} if one exists. The marginal utility of money, λ, is positive since the marginal utilities of attributes are positive as increases in income will increase purchases of products and increase attribute consumption levels. The existence of positive marginal utilities of attributes satisfy the nonsatiation assumption applied to the L-S model.

Property II of the Uniqueness theorem can be described by using the Hessian matrix derived from (2-34) and is defined as **H** where:

$$\mathbf{H} = \begin{bmatrix} \dfrac{\partial^2 U}{\partial X_1^2} & \cdots & \dfrac{\partial^2 U}{\partial X_1 \partial X_m} \\ \dfrac{\partial^2 U}{\partial X_2 X_1} & \dfrac{\partial^2 U}{\partial X_2^2} & \dfrac{\partial^2 U}{\partial X_2 \partial X_m} \\ \vdots & & \vdots \\ \dfrac{\partial^2 U}{\partial X_m \partial X_1} & \cdots & \dfrac{\partial^2 U}{\partial X_m^2} \end{bmatrix}$$

Property II requires that **H** be negative definite. This holds if the principal minors alternate in sign starting with the arbitrary $\partial^2 U/\partial X_1^2$ being negative, or there is declining marginal utility.

Slope and Shape of Indifference Curves

The local-global, Weierstrass and Uniqueness theorems of the L-S model specify certain properties of the utility surface and indifference curves. Consider a two attribute world with no unique attributes X_{m+i}. The utility function is $U = U(X_1, X_2)$. To determine the slope of the indifference curve, take the total differential of $U = U(X_1, X_2)$ and get

$$dU = \frac{\partial U}{\partial X_1} dX_1 + \frac{\partial U}{\partial X_2} dX_2 = 0 , \qquad (2\text{-}36)$$

which expresses the total change in utility due to variations in X_1 and X_2. We set (2-36) equal to zero since the change in utility as one moves along an indifference curve is zero. As the consumption of X_1 increases by dX_1, consumption of X_2 decreases by dX_2 along the indifference curve. The slope of the indifference curve is dX_2/dX_1, and by solving for this slope in (2-36), the result is the negative ratio of marginal utilities or

$$\frac{dX_2}{dX_1} = - \frac{\partial U/\partial X_1}{\partial U/\partial X_2} . \qquad (2\text{-}37)$$

The slope of the indifference curve in two dimensional space is negative, as positive marginal utilities for attributes are ensured by property I of the Uniqueness theorem.

Ensured that indifference curves are negatively sloped by Property I, the specific shape of the indifference curve will be examined. In two dimensional space the indifference curve would be a straight line with a constant slope throughout (like I_1 in Figure 8a) if marginal

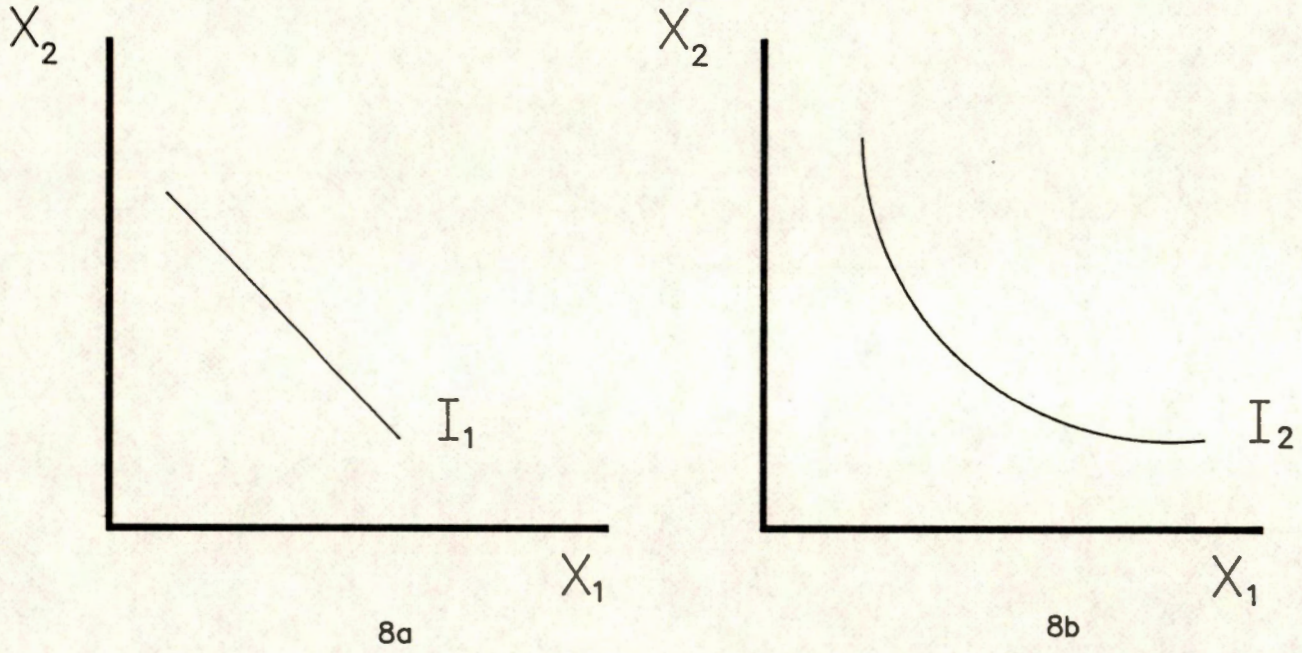


Figure 8. Indifference Curves for Utility Functions of the Consumer Characteristics Model

utilities were constant. However, property II of the Uniqueness theorem states that marginal utilities are not constant but declining. Indifference curves with a constant slope like I_1 in Figure 8a are not possible when the Uniqueness theorem holds in the L-S model.

By virtue of declining marginal utilities specified in property II, the slope of the indifference curve is found to be diminishing and convex. Moving along the indifference curve and acquiring more X_1 and less X_2, we see that $\partial U/\partial X_1$ decreases as units of X_1 are added, and $\partial U/\partial X_2$ increases as units of X_2 are given up. By property II more X_1 causes the numerator in (2-37) to decrease while less X_2 causes the denominator in (2-37) to increase. In Figure 8b, a movement downward and to the right results in the slope of the indifference curve I_2 to become smaller in absolute value by the relationship of marginal utilities described above. In Figure 8b, I_2 is seen to be convex in two dimensional space which leads to property III of the Uniqueness theorem.

Property III

The L-S model which yields a unique solution will have a convex utility surface generating indifference curves which are negatively sloped and convex.

When the local-global, Weierstrass and Uniqueness theorems hold for the L-S model, the utility surface is continuous, nondecreasing, compact having positive marginal utilities and a negative definite Hessian. These conditions are depicted in a three dimensional diagram similar to Figure 4 where the only difference is the axes on the "floor" measure units of attributes instead of products.

Figure 9 depicts optimization of the L-S model when the local-global, Weierstrass, and Uniqueness theorems hold assuming a two attribute, four good world in two dimensional space. Here the four products C, P, B and L contain common attributes X_1 and X_2 with no unique attributes provided by the products. The unique solution is found at point E where the indifference curve is tangent to the opportunity set frontier ABCD.

Summary of Unique Global Maximum Conditions in the L-S Model

As properties I, II and III of the Uniqueness theorem have demonstrated, convex indifference curves are crucial to generating a unique solution. Armed with the unique preferences at specific levels of prices and incomes, the researcher is able to analyze consumer choices when small changes in prices and income occur. Without a unique solution the researcher knows little concerning preferences at given prices and income levels, so analysis of changes in consumer choices where changes in prices and income occur will have little meaning. This is the importance of a unique solution.

This presentation summarizes the conditions that exist when optimization of the L-S model yields a unique global maximum. Assuming the local-global, Weierstrass and Uniqueness theorems hold, the L-S model will yield a UGM, and the following conditions will prevail in the model.

1. The utility function $U(\mathbf{X})$ is strictly concave over X_j for all $X_j \varepsilon \mathbf{X}$, $\mathbf{X} \varepsilon \mathbf{X}^S$ where $\mathbf{X}^S$ is a closed, bounded and nonempty set.

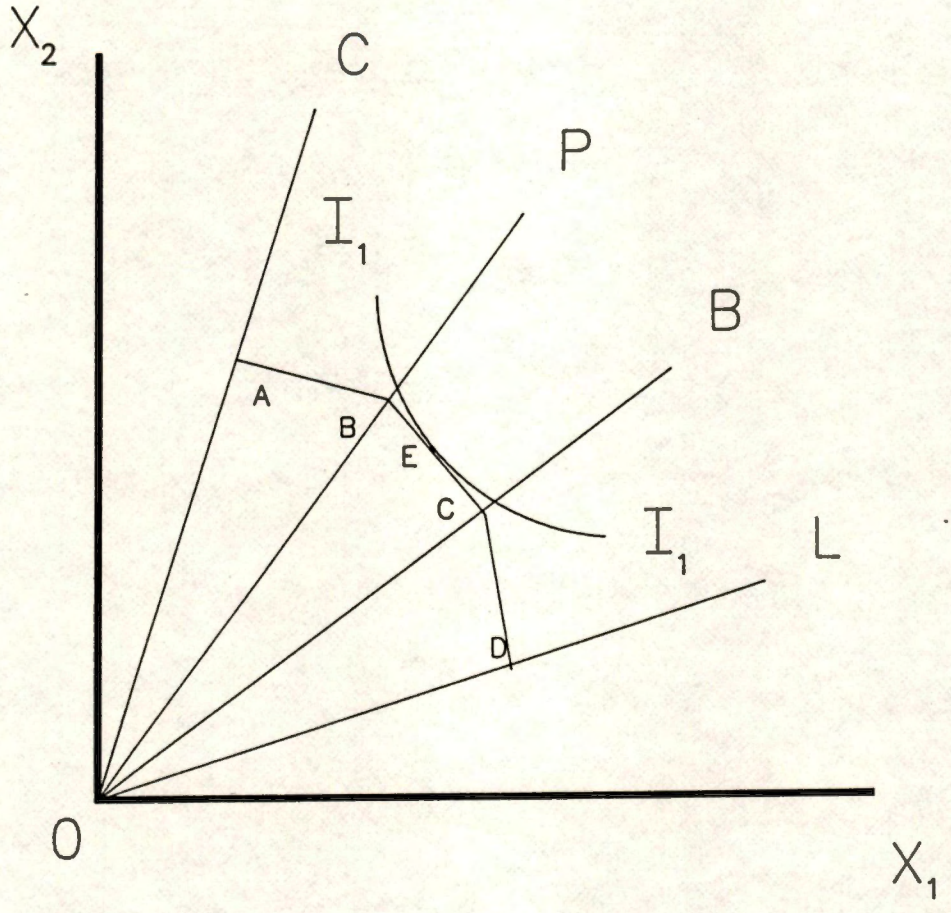


Figure 9. Utility Maximization in the Ladd-Suvannunt Consumer Characteristics Model

2. Each attribute in **X** yields positive marginal utility, the nonsatiation axion is satisfied, or $\partial U/\partial X_j > 0$, $j = 1 \ldots m$, which ensures negatively sloped indifference curves.
3. The Hessian matrix is negative definitive, including $\partial^2 U/\partial X_j^2 < 0$, $j = 1, \ldots m$, implying that the positive marginal utilities in condition (2) above are declining. This ensures indifference curves are strictly convex.

CHAPTER III

A CLOSER LOOK AT THE LINEAR HEDONIC PRICE EQUATION

In this section the linear form of the hedonic price equation of the L-S model will be examined to see if the linear form satisfies the UGM conditions for a unique global maximum. In the L-S model the hedonic price equation (2-28) is shown in the commonly used linear form (without a unique attribute of each product),

$$P_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_m x_{im} \qquad \text{(2-28) (repeated)}$$

or

$$P_i = \sum_{j=1}^{m} \beta_j x_{ij} \qquad \text{(2-29) (repeated)}$$

The major questions at hand are (1) generally does the linear form of the hedonic price equation of the L-S model satisfy needed conditions for a unique solution and (2) more specifically does the linear form assume constant or declining marginal utility for attributes plus strictly convex indifference curves.

Closer inspection of (2-28) shows that the relationship between the price of good i and the attributes contained in product i is linear. The marginal implicit price of the jth attribute, β_j, is a constant. Recall what β_j represents. From the derivation of the hedonic price equation for product i recall (2-23).

$$P_i = \sum_{j=1}^{m} \frac{\partial X_j}{\partial Q_i} \left(\frac{\partial U}{\partial X_j} \frac{\partial I}{\partial U}\right) \text{ (without a unique attribute for i)} \quad \text{(2-23) (repeated)}$$

and the term

$$\beta_j = \frac{\partial U}{\partial X_j} \frac{\partial I}{\partial U} \quad . \qquad (3\text{-}1)$$

With a positive constant β_j, two statements of what is causing β_j to be constant can be made. First, both terms of β_j (i.e., $\partial U/\partial X_j$ and $\partial I/\partial U$) are changing in such a way as to keep the ratio constant. The second possibility is that both components of β_j in (3-1) are constant. This implies that $\partial I/\partial U$ is a constant (the inverse of the marginal utility of income), and that $\partial U/\partial X_j$ is also a constant (the marginal utility of the jth attribute).

Consider the first possibility where both terms of (3-1) are assumed to change in a fashion that keeps β_j constant. This implies that, with positive decreasing marginal utility of attribute X_j, the term $\partial U/\partial X_j$ will decrease and will be exactly off-set by an identical increase in the inverse of the marginal utility of income $\partial I/\partial U$. The term $1/\partial U/\partial I$ must increase by the same amount as $\partial U/\partial X_j$ decreases, therefore, the denominator $\partial U/\partial I$ (marginal utility of money) must decrease which means changing money income. However, in the L-S model money income is given and the marginal utility of income is considered constant and zero since income is unchanged. Marshall (1920) introduced the idea of constant marginal utility of income in the derivation of classical demand curves where he assumed that even changes in the prices

of products did not change real income for products that formed a small portion of the consumer's budget. Friedman (1949) argues that the type of demand curve Marshall had in mind was one in which real income was held constant, not money income. By adoption of the assumptions of perfectly elastic supply curves for products in the L-S model and fixed money income, both Marshall's and Friedman's interpretations of a constant marginal utility of income (money income or real income) imply that the term $1/\partial U/\partial I$ cannot increase to offset declining marginal utility of attributes so β_j will remain constant. Thus, the first possibility of both terms of (3-1) changing for a constant β_j is not considered to occur.

The second possibility of explaining the constant β_j in the linear hedonic equation is that both terms of (3-1) are constants. With the acceptance of a constant marginal utility of money as proposed by Marshall and Friedman, the remaining term $\partial U/\partial X_j$ must be constant. If $\partial U/\partial X_j$ is constant, then an attribute does not exhibit declining marginal utility. The L-S model using a linear functional form of the hedonic price equation cannot yield a unique solution because previously mentioned properties of the model for a unique maximum are positive diminishing marginal utilities for attributes and a negative definite Hessian matrix.

In a linear hedonic price equation for the L-S model, we wish to know if the constancy of the marginal utility of attributes satisfy the UGM conditions of the L-S model required for a unique solution. For convenience assume the L-S model is a two attribute model with four

products, a fixed LCT, and a given budget constraint. To determine if the necessary theorems are satisfied, consider the utility function $U(X_1|X_2)$ where attribute X_2 remains constant. Figure 10b shows the function $\partial U/\partial X_1 = k$, a constant, and $k > 0$. The total utility function $U(X_1|X_2)$ is found by the integral

$$\int k dx_1 = k \int 1 dx_1 = kx_1 + c \ ,$$

and the total utility function is $U(kX_1|X_2)$ and shown in Figure 10a. The derivative of $\partial U/\partial X_1 = k$ with respect to X_1 is $\partial^2 K/\partial X_1^2 = 0$. This is not consistent with the Hessian being negative definite and is shown in Figure 10c to be zero over the range of opportunity set X_1.

Given the L-S model described with constant marginal utility of attributes, we must examine the resulting utility functions shown in Figures 10a, b and c to determine if the necessary conditions are satisfied for a unique global maximum.

UGM conditions for the L-S model specify the mathematical properties the model must exhibit for a unique solution to exist. The following is an analysis of the linear hedonic model in order to determine if the UGM conditions are satisfied.

The first UGM condition is not satisfied since in Figure 10a the utility function $U(X_1)$ is not strictly concave. The second UGM condition is satisfied since Figure 10b depicts the marginal utility function $\partial U/\partial X_1$ as positive. The third UGM condition states that the Hessian matrix is negative definite, including $\partial^2 U/\partial X_1^2 < 0$. From

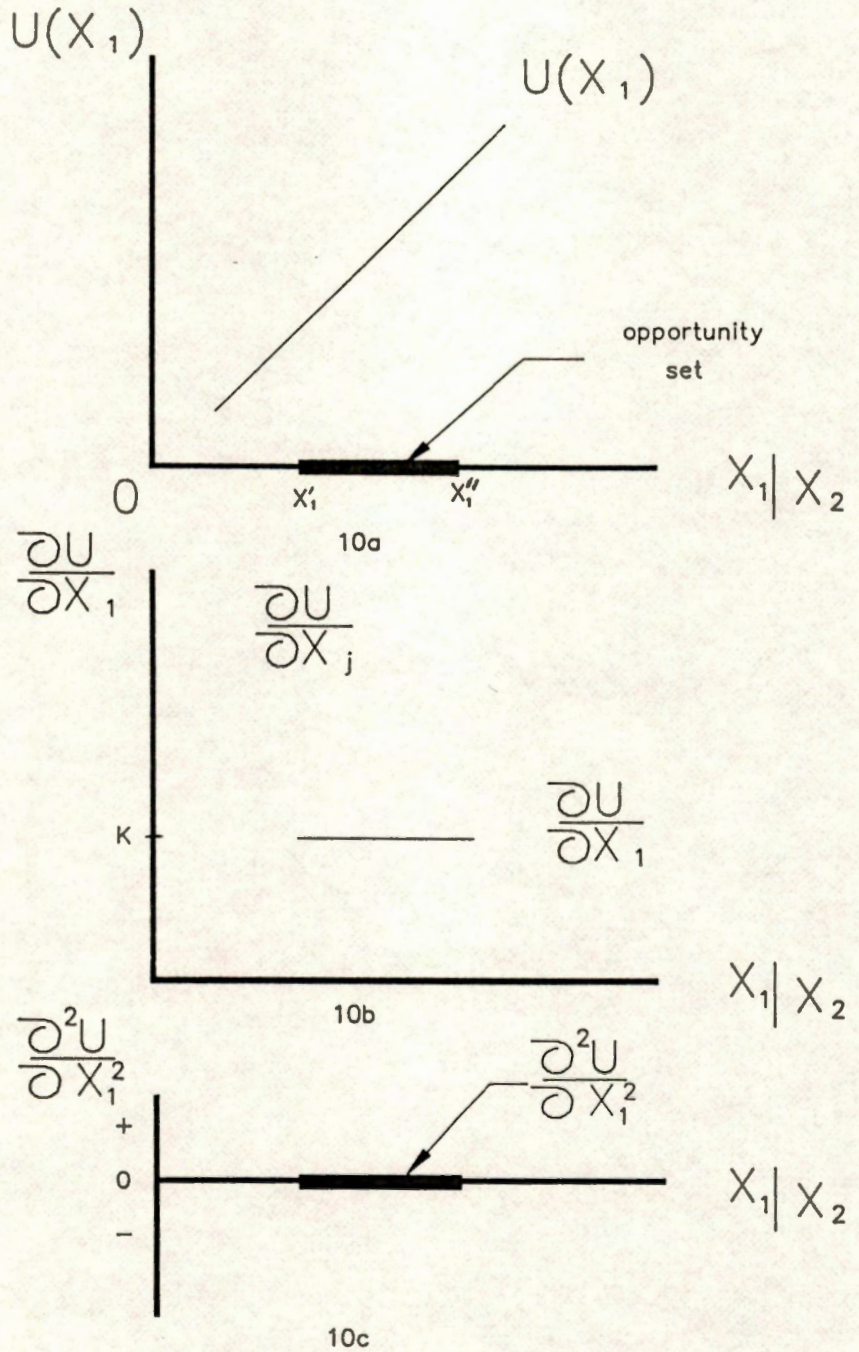


Figure 10. Functional Relationships of the Utility Function and a Linear Hedonic Price Equation of the Consumer Characteristics Model

Figure 10c the function $\partial^2 U/\partial X_1^2$ is not negative which reflects the condition seen in 10b where marginal utility of X_1 is constant.

As a result of the first and third UGM conditions being violated by the linear hedonic price equation, marginal utilities in the two attribute models are constants. From the previous discussion concerning the slope of the indifference curve (2-37), constant marginal utilities generate indifference curves with a constant slope like I_1 in Figure 8a.

To reenforce the idea that the linear hedonic price equation generates nonconvex indifference curves consider the linear hedonic price equation (2-29) for the two attribute four product world (without unique attributes). Recall

$$P_i = \sum_{j=1}^{m} x_{ij}\beta_j \qquad (i = C, P, B, L) \quad , \qquad \begin{matrix}(2\text{-}29)\\(\text{repeated})\end{matrix}$$

and rewrite (2-29) in the matrix form

$$\begin{bmatrix} P_C \\ P_P \\ P_B \\ P_L \end{bmatrix} = \begin{bmatrix} x_{C1} & x_{C2} \\ x_{P1} & x_{P2} \\ x_{B1} & x_{B2} \\ x_{L1} & x_{L2} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \quad . \qquad (3\text{-}2)$$

For the example consider the hedonic price for chicken

$$P_C = x_{C1}\beta_1 + x_{C2}\beta_2 \quad . \qquad (3\text{-}3)$$

From (3-1)

$$\beta_1 = \frac{\partial U}{\partial X_1} \cdot \frac{\partial I}{\partial U} \quad \text{and} \quad \beta_2 = \frac{\partial U}{\partial X_2} \cdot \frac{\partial I}{\partial U}$$

where $\partial U/\partial X_1$, $\partial U/\partial X_2$ and $\partial I/\partial U$ are positive constants. Therefore for X_1 and X_2 in each of the four products

$$\frac{\partial U}{\partial X_1} = k_1,\ k_1 > 0 \text{ and } \frac{\partial^2 U}{\partial X_1^2} = 0 \quad , \text{ and}$$

$$\frac{\partial U}{\partial X_2} = k_2,\ k_2 > 0 \text{ and } \frac{\partial^2 U}{\partial X_2^2} = 0 \quad .$$

From the slope of the indifference curve (2-37),

$$\frac{dQ_2}{dQ_1} = -\frac{k_1}{k_2} \tag{3-3}$$

which generates straight line indifference curves.

Figure 11 depicts the L-S model when the first and third conditions of the UGM conditions are violated which generate straight line indifference curves discussed above. In Figure 11, a unique solution does not exist with the slope of the indifference curve being equal to the slope of the opportunity set frontier at all points along BC.

With the solutions generated in Figure 11, the researcher actually does not know anything specific concerning preferences at prescribed prices and incomes. The demand for attributes has no meaning as it is crucial for a specific bundle of attributes to be preferred at certain prices and income level for demand theory to have any analytical

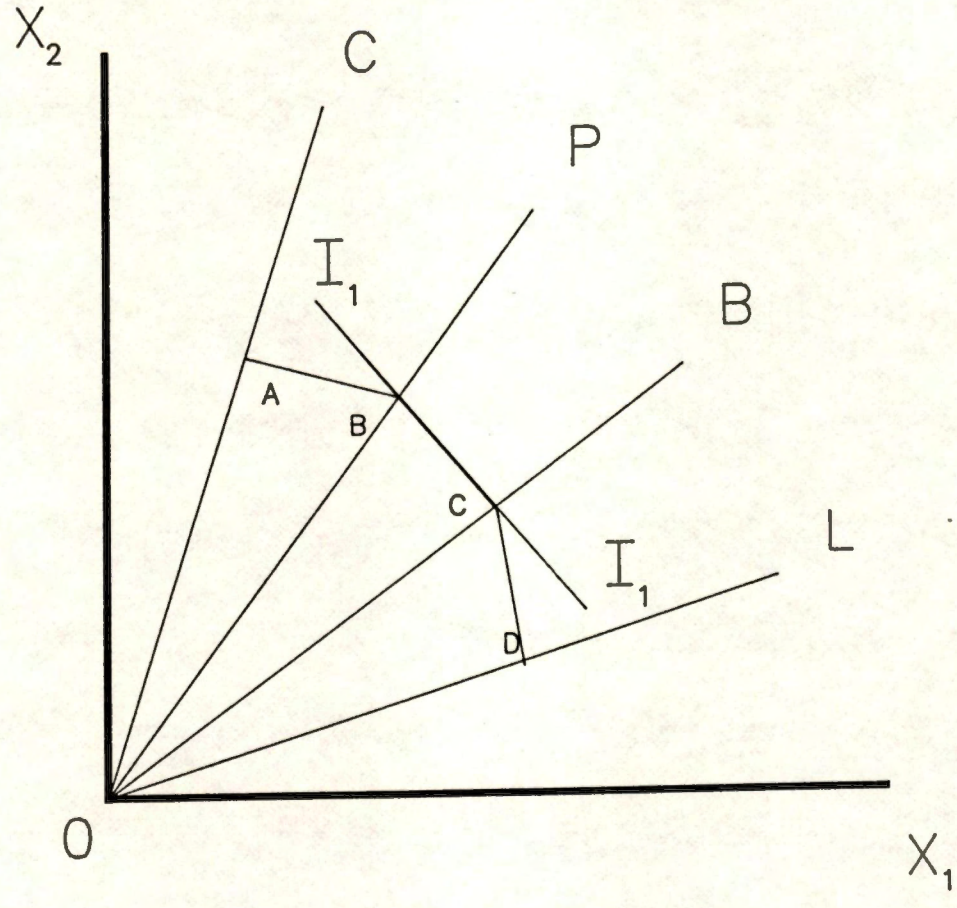


Figure 11. Utility Maximization of Consumer Characteristics Model With a Linear Hedonic Price Equation

capability. This example should emphasize the importance of the specification of the L-S model in order to satisfy the UGM conditions.

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CHAPTER IV

A GENERAL FUNCTIONAL FORM OF THE HEDONIC PRICE EQUATION AND ESTIMATION PROCEDURES

In the L-S characteristics model, a crucial step in deriving a unique solution is the choice of the functional form of the hedonic price equation. The previous discussion explains why the choice of the linear hedonic equation is inadequate and does not meet Properties I, II, and III and UGM conditions 1, 2, and 3 needed for unique optimization. Most empirical work on the functional form of the hedonic equation has concentrated on justifying alternative functional forms on purely statistical grounds with no regard to economic theory. Previous studies concerning hedonic functional form analysis assume economic theory provides no justification for any specific functional form. Halvorsen and Pollakowski (1981, p. 37) state,

> A hedonic price equation is a reduced-form equation reflecting both supply and demand influences. Therefore, the appropriate functional form for the hedonic equation cannot in general be specified on theoretical grounds. The lack of a firm theoretical basis for the choice of functional form is unfortunate since the results obtained using the hedonic approach often depend critically on the functional form used.

Jorden et al. (1985, p. 141) confirm this belief by writing, "Economic theory provides no guidance to correct specification of the functional form for hedonic equations. Consequently, the appropriate functional form cannot be specified on theoretical grounds." Also see Rosen (1974) and Bender et al. (1980).

Chapter II provides a detailed mathematical model and appropriate conditions for economic optimization of the L-S model. From the optimization of the L-S model, Properties I-III and UGM conditions 1-3 were specified which provided critical guidelines leading to ideas of specification of the functional form of the hedonic equations. Unlike Halvorsen and Pollakowski, Jordan et al., Rosen and Bender, this author believes that the previously mentioned Properties and UGM conditions of the L-S model provide an adequate theoretical economic basis for the specification of the functional form of the hedonic equations. The failure to use the guidelines of Properties I-III and the UGM conditions of the optimized L-S model is surprising since these properties are very similar to those used in the classical model.

Actually, the Properties and UGM conditions of the L-S model do not lead directly to a certain functional form, but provide guidelines that allow some functional forms to be eliminated. For example, Chapter III provided evidence that eliminated the linear form based on the constant marginal utilities of attributes this form yielded. Thus, the Properties and the UGM conditions that specify declining marginal utilities provided the theoretical basis for the elimination of the widely used linear functional form of the hedonic price equation.

The previous studies that searched for functional forms based purely on statistical grounds used the methodology developed by Box and Cox (1964). The introduction of the Box-Cox transformation (BCT) of variables in order to determine the appropriate functional form is appealing on statistical grounds, but ignores the restrictions that

economic theory may place on the model. The Box-Cox methodology has been used in a number of studies, for instance in macroeconomics (White, 1972; Spitzer, 1976; Zarembka, 1968, 1974); and in demand analysis (Chang, 1977a, 1980); (Blaylock, 1983) and in interstate migrations (Goss and Chang, 1983). There is also a Fourier flexible form that is used with a repetitive series of data. However, its computational demands are not suitable for this cross-section analysis (see Chalfant and Gallant, 1985; Smallwood and Blaylock, 1986).

With regard to searching for functional forms of hedonic equations, the Box-Cox methodology has been used in several areas. Goodman (1978) rejected the linear functional form for housing prices and characteristics and used a multiplicative form. Bender et al. (1980) searched for a functional form for the price of clean air and air quality. Halverson and Pollakowski (1981) provide a complete analysis of the statistical criteria for choosing a functional form of the hedonic equation and point out that there is no theoretical ground on which to base the choice of a functional form. Milton et al. (1984) develop a flexible functional form for amenity valuations of coastal land in Florida. Jordan et al. (1985) attempt to estimate the relationship between price and quality characteristics of tomatoes and found no specific functional form to be significant. While the Box-Cox methodology has been used in the transformation of the hedonic equations in these studies, none of the studies has concentrated on the theoretical grounds for choosing the functional form of hedonic equations in the context of consumer demand for nutritional attributes.

Functional Forms of the Hedonic Price Equation Yielding Declining Marginal Utility for Attributes

The remaining objective of this research is to engage in a search for an appropriate functional form of the hedonic price equation that satisfies the previously mentioned conditions for a unique global maximum to exist in the consumer characteristics model. The specific criteria of interest in this study is finding a function form of the hedonic price equation that has the capability of yielding positive declining marginal utility for nutritional attributes.

The requirement of positive declining marginal utility in the hedonic price equation can be represented by many explicit functional forms. The basic criterion is for marginal utility to be positive and fall as the quantity of each nutrient increases, or $\partial U/\partial X_j$ needs to reflect this condition.

Recall the general form of the hedonic price equation (2-23), assuming no unique attributes. The market price the consumer is willing to pay, P_i, is determined by three derivatives, and based upon the UGM conditions as they apply to foods, the hedonic price equation must display certain properties, explained below.

First, the LCT assumption specifies that $\partial X_j/\partial Q_i = x_{ij}$. And x_{ij} must be greater than or equal to zero.

Second, Marshall (1920) and Friedman (1949) state that the marginal utility of income (money income or real income) is constant for small changes in expenditures. For individual foods, this leads to $\partial I/\partial U$ being constant in equation (2-23). This derivative is positive.

Third, Properties I and II of the UGM conditions for the characteristic model show that the marginal utilities, $\partial U/\partial X_j$ are positive and declining.

Taken together, the above mentioned characteristics of the three derivatives in (2-23) should yield a hedonic price equation with positive P_i where this P_i increases at a decreasing rate with X_j. Among the functional forms that meet the criteria are the reciprocal, doubl-log, semi-log, and log-reciprocal forms of the hedonic price equation. For example, consider one nutrient X_1 and one food product i. The corresponding relationship between P_i and X_1 (and the level of all other nutrients do not change) can be shown in Figure 12 for these functional forms of the hedonic price equation. The next section outlines the Box-Cox regression technique and shows these four functional forms are special cases within the estimation technique.

The Box-Cox Methodology

Box and Cox suggest that the transformation of variables in a general functional form framework can be used to estimate the best relationship between the dependent and independent variables. Application of the Box-Cox methodology in this paper involves using a general functional form for the hedonic price equation and using the maximum likelihood method in order to estimate parameters that appear in the general functional form.

A single parameter general functional form of the hedonic price equation can be expressed as:

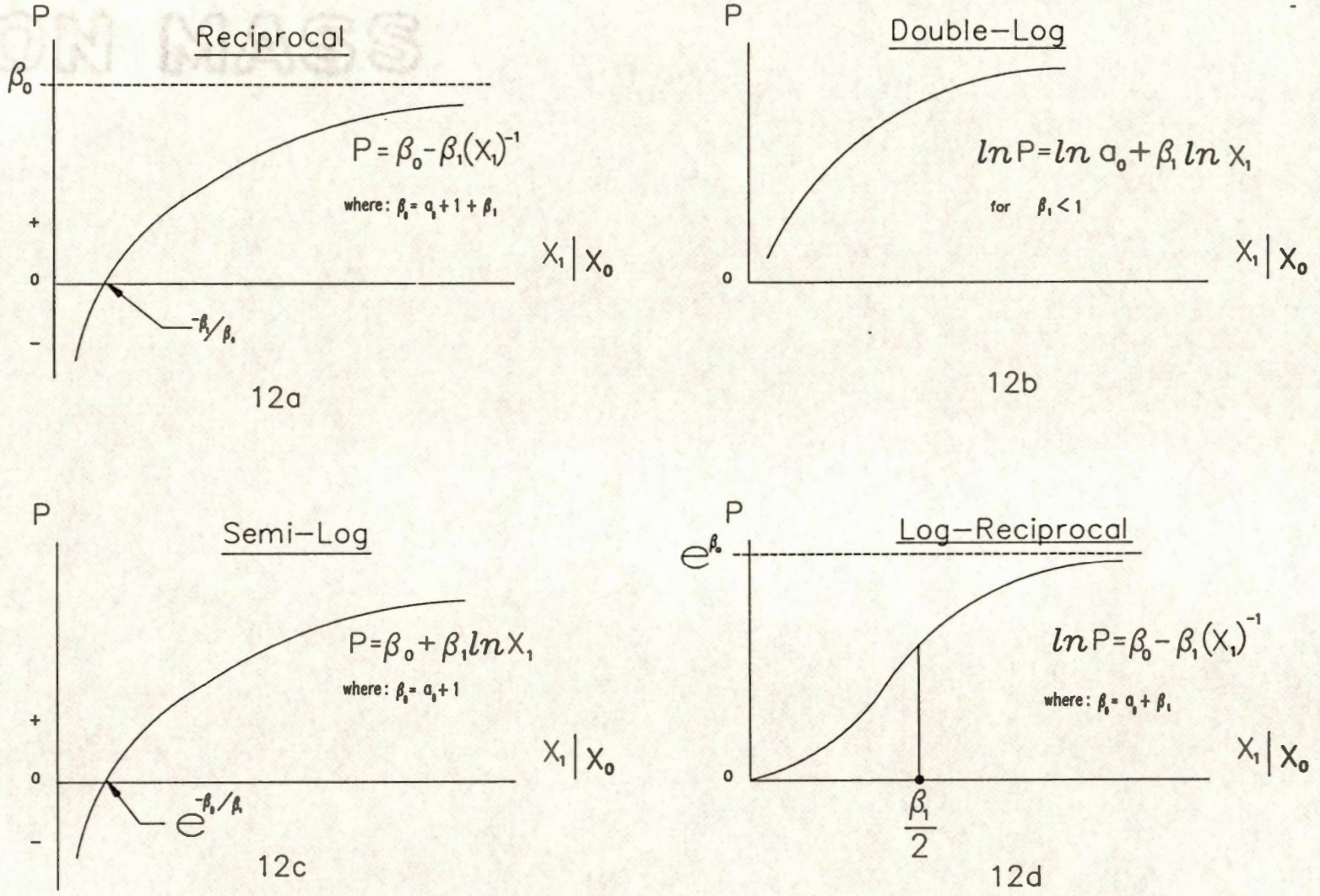


Figure 12. Functional Forms of the Hedonic Price Equation That Show Declining Marginal Utility for Attribute X_1.

$$P_i^* = a_0 + \beta_1 x_{i1}^* + \beta_2 x_{i2}^* + \ldots + \beta_m x_{im}^* \qquad (4\text{-}1)$$

where the variables with an asterisk are transformed variables by λ defined as:

$$P_i^* = (P_i^{\lambda} - 1)/\lambda \quad \text{and} \quad x_{ij}^* = (x_{ij}^{\lambda} - 1)/\lambda$$

The λ term represents a transformation parameter to be determined. When λ takes on different values, then equation (4-1) takes on varying functional forms. For instance it is obvious that if $\lambda = 1$, equation (4-1) is a linear form. Kmenta (1971) shows that (4-1) approaches a log form when λ approaches zero. Equation (4-1) represents a single transformation parameter (λ) of the Box-Cox representation of the hedonic price equation.

The general functional form of the hedonic price equation can be represented as a double transformation parameter of the Box-Cox methodology. Consider the general functional form of the hedonic price equation to be

$$P_i^* = a_0 + \beta_1 x_{i1}^{**} + \beta_2 x_{i2}^{**} + \ldots + \beta_m x_{im}^{**} . \qquad (4\text{-}2)$$

The variables P_i and x_{ij} are transformed variables by λ and μ and defined as:

$$P_i^* = (P_i^{\lambda} - 1)/\lambda \qquad (4\text{-}3)$$

$$x_{ij}^{**} = (x_{ij}^{\mu} - 1)/\mu \qquad (4\text{-}4)$$

The terms λ and μ are functional form to be estimated. By using the maximum likelihood method suggested by Box and Cox (1964) to estimate λ and μ in the general functional form, the most appropriate functional form for the specified set of variables can be selected. The following forms of the hedonic equation will exist when λ and μ have the restrictive values, were ℓn denotes the natural log. Box and Cox state when the transformation parameter in (4-3) and (4-4) take on the value of zero (i.e., $\lambda = 0$ and $\mu = 0$), rather than being undefined, the transformed variable is the natural log value. That is, when $\lambda = 0$ then $P_i^{*} = \ell n P_i$ and when $\mu = 0$, $x_{ij}^{**} = \ell n x_{ij}$.

λ, μ	Functional form of Hedonic Price Equation	
$\lambda = 1, \mu = 1$	$P_i = \beta_0 + \sum_{j=1}^{m} \beta_j x_{ij}$ (linear)	(4-5)
	where $\beta_0 = a_0 + 1 - \sum_{j=1}^{m} \beta_j$	
$\lambda = 0, \mu = 0$	$\ell_n(P_i) = \ell_n a_0 + \sum_{j=1}^{m} \beta_j \ell_n(x_{ij})$ (double-log)	(4-6)
$\lambda = 1, \mu = 0$	$P_i = \beta_0 + \sum_{j=1}^{m} \beta_j \ell_n(x_{ij})$ (semi-log)	(4-7)
	where $\beta_0 = a_0 + 1$	

$$\lambda = 1,\ \mu = -1 \quad P_i = \beta_0 - \sum_{j=1}^{m} \beta_j x_{ij}^{-1} \text{ (reciprocal)} \tag{4-8}$$

$$\text{where } \beta_0 = a_0 + 1 + \sum_{j=1}^{m} \beta_j$$

$$\lambda = 0,\ \mu = -1 \quad \ln(P_i) = \beta_0 - \sum_{j=1}^{m} \beta_j x_{ij}^{-1} \text{ (log-reciprocal)} \tag{4-9}$$

$$\text{where } \beta_0 = a_0 + \sum_{j=1}^{m} \beta_j$$

As the previous discussion in this chapter and Figure 12 show, a hedonic price equation that satisfied the conditions for a unique global maximum must show P_i increasing at a decreasing rate with x_{ij}. Thus, the slope of the hedonic price equation ($\partial P_i / \partial X_{ij}$) must be positive, decline, and approach zero. These relationships of the hedonic price equation will be satisfied when (1) $\partial P_i / \partial x_{ij} > 0$, $j = 1, \ldots, m$, and (2) the Hessian matrix of (4-2) is negative definite, including $\partial^2 P_i / \partial x_{ij}^2 < 0$, $j = 1, \ldots, m$.

By differentiating (4-2) with respect to an attribute x_{ij}, the first and second order partial derivatives and their required values can be expressed in terms of β_j, P_i, x_{ij}, λ, and μ. These are defined as the General Hedonic Price Equation (GHPE) conditions, and are comparable to those of Goss and Chang (1983).

GHPE Conditions

1. $\dfrac{\partial P_i}{\partial x_{ij}} = \beta_j \, (P_i^{1-\lambda})(x_{ij}^{\mu-1}) > 0$, and

$$2.\quad \frac{\partial^2 P_i}{\partial x_{ij}^2} = \beta_j\,(\mu-1)(P_i^{1-\lambda})(x_{ij}^{\mu-2})$$

$$+\,\beta_j^{\,2}\,(x_{ij}^{\mu-1})^2(P_i^{1-2\lambda})(1-\lambda) < 0 \quad .$$

Inspection of these conditions indicates the values of β_j, λ and μ affect the sign of either GHPE condition since the variables P_i ans x_{ij} are positive. Thus, interest centers on the effects of β_j, λ and μ on the first and second order derivatives. A Box-Cox flexible functional form of the hedonic price equation will be consistent with delining marginal utility of attributes when GHPE conditions (1) and (2) are satisfied.

In order to satisfy GHPE conditions (1) and (2) the following requirements are defined for parameters β_j, λ and μ. From GHPE condition (2) let

$$z = \beta_j\,(\mu-1)(P_i^{1-\lambda})(x_{ij}^{\mu-2}), \text{ and}$$

$$w = \beta_j^{\,2}(x_{ij}^{\mu-1})^2(P_i^{1-2\lambda})(1-\lambda) \quad .$$

GHPE Parameter Requirements

Requirement 1. $\beta_j > 0$, $\mu < 1$, $\lambda \geq 1$; $\mu \leq 1$, $\lambda > 1$, or

Requirement 2. $\beta_j > 0$, $\mu < 1$, $\lambda < 1$, and $w < |z|$, or

Requirement 3. $\beta_j > 0$, $\mu > 1$, $\lambda > 1$, and $z < |w|$.

The linear form (4-5) is not acceptable since $\lambda = 1$, $\mu = 1$ and these parameters do not meet the parameter requirements of the GHPE. The semi-log (4-7) and the reciprocal (4-8) functional forms are acceptable since their values of λ and μ satisfy GHPE parameter requirements one. The double-log (4-6) and log-reciprocal (4-9) forms are acceptable only if the conditions of z and w are satisfied in GHPE requirement two. Thus, the linear functional form is not consistent declining marginal utility of attributes while the remaining four functional forms are acceptable if GHPE parameter requirements are satisfied.

Overall then, the Box-Cox relationship has been shown to have a subset of parameter values that are consistent with economic theory. The empirical issue is one of whether estimated Box-Cox hedonic price equations fall within this subset.

Maximum Likelihood Estimation

Estimating the Box-Cox model of equation (4-2) requires the use of maximum likelihood techniques. The general hedonic equation (4-2) is modified to include a random error term e_i that is assumed to be normally, independently and identically, distributed with zero mean and constant variance σ^2, ($e_i \sim N(o,\sigma^2)$), for a given value of λ and μ. With this modification (4-2) becomes:

$$P_i^* = a_0 + \beta_1 x_{i1}^{**} + \beta_2 x_{i2}^{**} + \ldots + \beta_m x_{im}^{**} + e_i. \quad (4\text{-}10)$$

In equation (4-10), P_i^* assumed to be normally distributed with a mean of

$$a_0 + \beta_1 x_{i1}^{**} + \beta_2 x_{i2}^{**} + \ldots + \beta_m x_{im}^{**} ,$$

and variance σ^2. The likelihood function corresponding to (4-10) is given by:

$$L = 2(\pi\sigma^2)^{-n/2} \exp[(-\frac{1}{2\sigma^2}) \sum_{i=1}^{n} (P_i^* - a_0 - \sum_{j=1}^{m} \beta_j x_{ij}^{**})^2 (P_1 \cdot P_2 \ldots \cdot P_n)^{\lambda-1}]$$

The estimation of equation (4-11) can be accomplished by maximizing the natural logarithm of the likelihood function denoted as ℓnL or:

$$\ell_n L = -\frac{n}{2} \ell_n \sigma^2 - \frac{n}{2} \ell_n 2\pi - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (P_i^* - a_0 - \sum_{j=1}^{m} \beta_j x_{ij}^{**})^2$$

$$+ (\lambda - 1) \sum_{i=1}^{n} \ell_n P_i . \qquad (4\text{-}12)$$

Notice that the term (-n/2 ℓn2π) in equation (4-12) is a constant and irrelevant to the estimation process of the parameters and can be eliminated. Thus, equation (4-12) is rewritten as:

$$\ell_n L = -\frac{n}{2} \ell_n \sigma^2 - \frac{1}{2\sigma^2} \sum_{i=1}^{m} (P_i^* - a_0 - \sum_{j=1}^{m} \beta_j x_{ij}^{**})^2$$

$$+ (\lambda-1) \sum_{i=1}^{n} \ell_n P_i . \qquad (4\text{-}13)$$

where (i = 1, . . . , n) and (j = 1, . . . , m).

Recall (4-3) and (4-4) and note that for any given values of λ and μ there exists the maximum likelihood estimates of σ^2, a_o, β_1, β_2, . . . β_m. To estimate these parameters for given values of λ and μ, equate the partial derivatives of ℓnL with respect to σ^2, a_o, β_1, β_2, . . . β_m and set each to zero. Solving for each unknown parameter yields the estimates:

$$\hat{\beta} = (x'x)^{-1}(x'P) \quad , \tag{4-14}$$

$$a_0 = \frac{\left[\sum_{i=1}^{n} P_i^{*} - \sum_{j=1}^{m} \beta_j x_{ij}^{**}\right]}{n} \quad , \text{ and} \tag{4-15}$$

$$\hat{\sigma}^2(\lambda,\mu) = \frac{1}{n} \sum_{i=1}^{n} (P_i^{*} - \hat{a}_0 - \sum_{j=1}^{m} \hat{\beta}_j x_{ij}^{**})^2 \quad . \tag{4-16}$$

where:

$$x = \begin{bmatrix} x_{1,1}^{**} & x_{1,2}^{**} & \cdots & x_{1,m}^{**} \\ x_{2,1}^{**} & x_{2,2}^{**} & \cdots & x_{2,m}^{**} \\ \vdots & \vdots & & \vdots \\ x_{n,1}^{**} & x_{n,2}^{**} & \cdots & x_{n,m}^{**} \end{bmatrix} \quad ,$$

$$(\mathbf{x'x}) = \begin{bmatrix} \Sigma x_{i1}^{**2} & \Sigma x_{i1}^{**}x_{i2}^{**} & \cdots & \Sigma x_{i1}^{**}x_{im}^{**} \\ \Sigma x_{i2}^{**}x_{i1}^{**} & \Sigma x_{i2}^{**2} & \cdots & \Sigma x_{i2}^{**}x_{im}^{**} \\ \Sigma x_{i3}^{**}x_{i1}^{**} & \Sigma x_{i3}^{**}x_{i2}^{**} & \cdots & \Sigma x_{i3}^{**}x_{im}^{**} \\ \vdots & \vdots & & \vdots \\ \Sigma x_{nm}^{**}x_{i1}^{**} & \Sigma x_{nm}^{**}x_{i2}^{**} & \cdots & \Sigma x_{nm}^{**2} \end{bmatrix},$$

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_m \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} P_1^* \\ P_2^* \\ \vdots \\ P_n^* \end{bmatrix}, \quad (\mathbf{X'P}) = \begin{bmatrix} \Sigma x_{i1}^{**} P_i^* \\ \Sigma x_{i2}^{**} P_i^* \\ \vdots \\ \Sigma x_{nm}^{**} P_n^* \end{bmatrix} \text{ and}$$

the estimates $\hat{\sigma}^2$, $\hat{A}_o$ $\hat{\beta}_1$, $\hat{\beta}_2$ . . . $\hat{\beta}_M$ are found by choosing alternative values of λ and μ and regressing P_i^* on x_{ij}^{**}'s. This is a conditional regression as these estimated parameters are conditionsl on the values of λ and μ chosen.

By substituting the value of $\hat{\sigma}^2(\lambda,\mu)$, from (4-16) for σ^2 in (4-13), one can rewrite (4-13) as:

$$\ell_n L(\lambda,\mu) = -\frac{n}{2}\ell_n \hat{\sigma}^2(\lambda,\mu) + (\lambda - 1) \sum_{i=1}^{n} \ell_n P_i \,. \qquad (4\text{-}17)$$

Equation (4-17) is the likelihood function to maximize.

By substitution, (4-17) can be shown to be an explicit function of the parameters to be estimated. By using equations (4-3) and (4-4) we can rewrite equation (4-16) as:

$$\hat{\sigma}^2(\lambda,\mu) = \frac{\sum_{i=1}^{n} \left(\frac{(P_i^{\lambda} - 1)}{\lambda} - \hat{a}_o - \sum_{j=1}^{m} \hat{\beta}_j \frac{(x_{ij}^{\mu} - 1)}{\mu} \right)^2}{n} \qquad (4\text{-}18)$$

Substituting $\hat{\sigma}^2$ in (4-18) for $\hat{\sigma}^2$ in (4-17) one gets:

$$\ell_n L(\lambda,\mu) = -\frac{n}{2}\ell_n \sum_{i=1}^{n} \left\{ \left(\frac{(P_i^{\lambda} - 1)}{\lambda} - \hat{a}_o - \sum_{j=1}^{m} \hat{\beta}_j \frac{(x_{ij}^{\mu} - 1)}{\mu} \right)^2 \Big/ n \right\}$$

$$+ (\lambda - 1) \sum_{i=1}^{n} \ell_n P_i \quad . \qquad (4\text{-}19)$$

Note that (4-17) equals (4-19). To maximize (4-19), one must find the values of λ and μ which do so. In particular, by choosing alternative values of λ and μ over a reasonable interval and regression P_i^* on x_{ij}^{**}'s, we can identify the transformation parameters $\hat{\lambda}$ and $\hat{\mu}$ which yield the largest values of (4-19). For each given value of λ and μ considered, there exists distinct values of $\hat{\sigma}^2$, $\hat{a}_o$, $\hat{\beta}_0$ . . . $\hat{\beta}_m$ which maximizes (4-19).

Test for Functional Form

The value of the transformation parameters λ and μ that maximize (4-17) will identify the functional form of the hedonic price equation that is appropriate. If the likelihood function is maximized when $\lambda = \hat{\lambda}$

and $\mu = \hat{\mu}$, then we wish to know if $\hat{\lambda}$ and $\hat{\mu}$ are significantly different or the same as those values of λ and μ associated with those functional forms in equations (4-5) through (4-9). The functional form of the general hedonic price equation (4-2) will depend on the values of $\hat{\lambda}$ and $\hat{\mu}$. When $\hat{\lambda}$ and $\hat{\mu}$ are calculated, the next step is to use a test of significance to determine if $\hat{\lambda}$ and $\hat{\mu}$ are significantly different from those values of λ and μ associated with each functional form and ultimately which functional form provides a reasonable fit to the data.

In this analysis, the unrestricted model will be defined as the hedonic price equation (4-2) in which the double transformation parameters λ and μ are equal to $\hat{\lambda}$ and $\hat{\mu}$, which maximize the likelihood function (4-17). The value of the maximum likelihood function at $\hat{\lambda}$ and $\hat{\mu}$ is called $\ell nL(\hat{\lambda},\hat{\mu})$ for the unrestricted model. The restricted models of the hedonic price equation are defined as the models that result when λ and μ are restricted to the values that prevail for each functional form. For example, $\lambda = 1$ and $\mu = 1$ in the restricted linear model. The following functional forms will be considered relative to the unrestricted model. Special attention is given to them because of their popularity in applied research and their theoretical properties.

Model ID	λ,μ	Functional Form	Restricted Likelihood Function
Model 1a	$\lambda = 1, \mu = 1$	linear	ℓnL (1a)
Model 1b	$\lambda = 0, \mu = 0$	double-log	ℓnL (1b)
Model 1c	$\lambda = 1, \mu = 0$	semi-log	ℓnL (1c)
Model 1d	$\lambda = 1, \mu = -1$	reciprocal	ℓnL (1d)
Model 1e	$\lambda = 0, \mu = -1$	log-reciprocal	ℓnL (1e)

The likelihood ratio method by Box and Cox (1964) is used to test the null hypothesis that no significant difference exists between the unrestricted model (4-2) and the restricted models of this hedonic equation. The results of the test of significance will reveal the appropriate functional form (if any) that supports the theoretical model. The likelihood ratio is defined as:

$$\Phi = \frac{L(\hat{\lambda},\hat{\mu})}{L(\lambda,\mu)}$$

where $\ell nL(\lambda,\mu)$ is the restricted likelihood function which corresponds to models 1a-1e and $\ell nL\lambda(\hat{\lambda},\hat{\mu})$ refers to the unrestricted likelihood function using $\hat{\lambda}$ and $\hat{\mu}$. Thiel (1971) has shown that minus twice the natural logarithm of the likelihood ratio is approximately distributed as a chi-square (χ^2) with the degrees of freedom equal to the number of restricted parameters in the model. Box and Cox (1964) also indicate that $a(1-\alpha)$ confidence interval for $\hat{\lambda},\hat{\mu}$ can be calculated as a chi-square distribution so that

$$2[\ell n\ L(\hat{\lambda},\hat{\mu}) - \ell n\ L(\lambda,\ \mu)] \sim \chi^2$$

with two degrees of freedom. In order to identify a $(1-\alpha)$ confidence level, we must first identify the appropriate confidence region for λ and μ. This confidence region consists of all values of λ and μ which satisfy the following inequality:

$$\ell n\ L(\hat{\lambda},\hat{\mu}) - \ell n\ L(\lambda,\mu) < \frac{1}{2} X^2\ \nu\ (\alpha) \qquad (4\text{-}20)$$

Equation 4-20 will be used to test the null hypothesis that the correct functional form of the hedonic price equation is one of the functional forms outlined in models 1a-1e. For this analysis where ν equals 2 (the number of degrees of freedom), a 95 percent confidence level ($\alpha = .05$) will define the confidence region of λ and μ that satisfies the following inequality:

$$\ell n\ L(\hat{\lambda},\hat{\mu}) - \ell n\ L(\lambda,\mu)_2 < \frac{1}{2} X^2\ (.05) = 2.995 \qquad (4\text{-}21)$$

CHAPTER V

THE DATA SOURCE AND VARIABLES USED

The Data Source

The best raw data available to estimate the functional form of the general hedonic price equation (4-2) are from the Nationwide Food Consumption Survey of 1977-78. This survey was conducted under the direction of the U.S. Department of Agriculture. The survey was performed over a 12-month period, and during each 3-month period an average of 3,750 households were surveyed.

In the 3-month period representing Spring of 1977, about 3,300 households were surveyed in the contiguous United States. Deleted from the survey were data from 1,075 households because of crucial missing data. The remaining households from the West, North Central, Northeast, and South regions comprised 401, 600, 490 and 727 households per region, respectively.

The methodology used to collect the data on household food consumption in the 1977-78 survey did more than just provide the purchases of food by the household. The survey sought to measure the food "used" by the household during the survey period. Noting that during a limited time period, food purchases may not be equal to food consumed, this survey attempted to measure food consumption at the level at which foods are served as consumption. The survey contains the cost of food used and the quantity used. Thus by dividing the cost by the quantity used we have the per unit calculation of price paid for food

used at the household level. Fluctuations in food purchases may occur from week to week because of sales, changes in food storage capacity and seasonal buying habits.

One of the major features that makes this survey attractive to use is that it contains nutritional information that permits the conversion of food items into nutritional components. Therefore, the methodology that measures food consumption rather than food purchased provides the best raw data available to estimate the functional form of the general hedonic equation (4-2).

For a household surveyed, an interview with the person identified as most responsible for food planning and preparation was performed. This person was contacted seven days prior to the actual interview and asked to keep grocery receipts, shopping lists, menus, labels or anything else which would help them recall the food consumed in the seven day period. During the initial interview seven days later, the interviewers recorded the kind, form, quantity and cost of food and beverages actually consumed in the household during the seven days prior to the interview date. Other data also collected at this time included food produced at home, the number of meals eaten at home and away by household members, food eaten by guests, household income, race, education and participation in government food programs.

The resulting data should be interpreted as economic consumption rather than total physiological consumption. That is, we recognize that the data represent the allocation of income for consumption and not the consumption for purposes of the operation of the vital process of living

organisms. Thus, it is recognized that some food could be discarded in the kitchen and at the table causing the consumption of nutrients to be overestimated. However, the consumption of nutrients could also be underestimated because the survey did not account for household's nutrient supplements (vitamins and minerals). Before food nutritional values were applied to food quantities consumed, estimated loss of nutritional attributes due to cooking were deducted (U.S. Department of Agriculture, 1982).

The Variables Used

There are 14 nutritional attributes identified in the data. These 14 nutritional attributes are protein, fat, carbohydrates, calcium, iron, magnesium, phosphorus, vitamin A, thiamin, riboflavin, niacin, vitamin B_6, vitamin B_{12}, and vitamin C.

The Aggregation of Nutrients

If all 14 nutrients are used in the estimation of equation (4-2), two problems arise. First there is a multicollinearity problem among nutrients, and second, all 14 nutrients were not contained in each food.

For the high degree of multicollinearity among these 14 nutrients, Terry (1985, pp. 92-93) wrote that for the same data set used, relatively high pair-wise correlations exist for phosphorus and protein (.56), phosphorus and calcium (.59), vitamin A and vitamin B_{12} (.55), thiamin and niacin (.48), iron and thiamin (.50), vitamin B_6 and riboflavin (.58), vitamin B_6 and niacin (.47), thiamin and vitamin B_6 (.68), niacin and magnesium (.75), and niacin and riboflavin (.62).

Thus, there is some concern as to the limitations of the regression procedure for the full set of attributes.

The multicollinearity problem of nutritional attributes has been avoided by the aggregation of nutrients (Terry, 1985; Eastwood, Gray and Brooker, 1986). Their methodology is followed to remedy the multicollinearity problem in this study. Two new variables were created by aggregation. A new "vitamin B" variable was created by summing the milligrams of all those vitamins in the B-complex group--that is thiamin, riboflavin, niacin, vitamin B_6 and vitamin B_{12}. Similarly, a new variable called "minerals" was generated by taking the sums of milligrams of calcium, iron, magnesium and phosphorus. Thus, this aggregation makes the number of available nutrients in the data set equal to seven. These seven nutrients are vitamin B, minerals, vitamin A, vitamin C, carbohydrates, fat, and protein. This aggregation is consistent with the view that consumers are concerned with broader groups of nutrients.

The second problem arises because each of the seven remaining nutrients is not present in each food item purchased by the households. For example, meat products and eggs contain zero grams of carbohydrates. Analysis of the foods purchased also reveals that many foods have no vitamin A or vitamin C nutrients. The problem arises when the nutrient x_{ij} equals zero in equation (4-4). The resulting transformation in equation (4-4) is undefined when x_{ij} equals zero.

The problem of having some nutrients (x_{ij}'s) entered as zero was resolved by omitting vitamin A and aggregating remaining nutrients.

Inspection of the data found that vitamin A was absent from a large number of foods. The option of aggregating vitamin A with other vitamins was discarded on the basis that vitamin A is the only vitamin measured in International Units and aggregation into remaining vitamins measured in milligrams would be inappropriate. Thus, the vitamin A nutrient was omitted from the data. The omission is consistent with the results of Eastwood, Gray and Brooker (1986) and the view that consumers are not very knowledgeable about the presence of specific vitamins in foods.

Vitamin C was found to be the next nutrient most commonly absent from foods. A new nutrient called "vitamin BC" was formed by the summation of vitamin B and vitamin C, whose measurement in milligrams allowed this aggregation. The next commonly absent nutrient seemed to be equally shared by carbohydrates, fat and protein. In its place, their linear combination called "food energy," also available in the data set, was used. This aggregation allowed many foods to remain in the data set that would have been eliminated on the basis of one missing nutrient. The variable identified as minerals was retained and found to be positive in all foods.

The aggregations of nutrients leave three nutrients to be considered in the analysis. These are: (1) vitamin B and C, (2) minerals, and (3) food energy. As required by the Box and Cox methodology, all observations for the transformation equations (4-3) and (4-4) are positive. The work by Weimer (1980) suggests that consumers notice broader groups of nutrients as opposed to those identified in smaller groups. Eastwood, Gray and Brooker (1986) suggest that this

aggregation of nutritional attributes addresses the argument that many foods do not have nutritional labeling, making consumers unaware of nutritional content. The point of view taken here is that consumers make food choices based on broader groups of nutrients represented by the three specified as a result of aggregation.

Model Specification

The general hedonic price equation (4-2) for a particular household h can be written as:

$$P_i^{h*} = \beta_0^h + \beta_1^h x_{i1}^{**} + \beta_2^h x_{i2}^{**} + \beta_3^h x_{i3}^{**} + \varepsilon^h \qquad (5\text{-}1)$$

where: β_j^h = the household level parameters to be estimated,

ε^h = error term,

j = 1 = vitamin BC,

j = 2 = minerals, and

j = 3 = food energy.

Regional differences in food consumption led to more than one household from each region being selected. In order to minimize computer costs, the number of households and the number of food items sampled were quite limited. Therefore, the food consumption data for eight households were used in the analysis. Two households from each region (West, North Central, Northeast and South) were selected at random from the subsample of households in each of the regions as defined above. Households that purchased 45 to 70 food items during the survey week were drawn. The reason this range of food items purchased

was selected is because this number of foods is felt to represent more closely the average number of purchased food items for the typical household. Some households reported purchasing up to 140 food items, but this was considered to reflect purchases made for one month as opposed to purchases for one week. Similarly, those households that purchased less than 45 food items were not considered because it was felt they were consuming a large portion of foods from stocks. For each household selected, the problem of zero value aggregated nutrients was quite small. In each case only two to four food items had to be deleted. The relatively small number of observations involved suggests the omission introduced minimal sample selection bias. The following households which purchased the specified number of food items (Table 1) were chosen from each region.

Table 1. Full Sample of Households By Region and Number of Food Items Purchased Per Household, Spring 1977.

Household ID	Region	Number of Food Items Purchased
1A	1	63
1B	1	66
2A	2	59
2B	2	53
3A	3	73
3B	3	47
4A	4	59
4B	4	59
Full Sample	--	Total = 479

Region 1 = North East
2 = North Central
3 = South
4 = West

The general hedonic equation for one arbitrary household is defined here. For example, consider household 1A from the full sample. This household used 63 food items, and the general hedonic equation can be written to reflect this household:

$$\mathbf{P}^{1A*} = \mathbf{x}^{**}\hat{\boldsymbol{\beta}}^{1A} \tag{5-2}$$

where:

$$\mathbf{P}^{1A*} = \begin{bmatrix} P_1^{h*} \\ P_2^{h*} \\ \cdot \\ \cdot \\ \cdot \\ P_{63}^{h*} \end{bmatrix},$$

$$\mathbf{x}^{**} = \begin{bmatrix} x_{1,1}^{**} & x_{1,2}^{**} & x_{1,3}^{**} \\ x_{2,1}^{**} & x_{2,2}^{**} & x_{2,3}^{**} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ x_{63,1}^{**} & x_{63,2}^{**} & x_{63,3}^{**} \end{bmatrix}, \text{ and}$$

$$\hat{\boldsymbol{\beta}}^{1A} = \begin{bmatrix} \hat{\beta}_1^{1A} \\ \hat{\beta}_2^{1A} \\ \hat{\beta}_3^{1A} \end{bmatrix}.$$

The general hedonic price equation for all eight representative households is the same as equation (5-2). This "full sample" represents all food items purchased by the eight households. This full sample includes 479 food items. Therefore the dimensions for equation (5-2) for the full model would be:

$$P^{*}_{479,1} = X^{**}_{479,3} \hat{\beta}_{3,1} .$$

Equation (5-2) for each household will be tested for the appropriate functional form using the Box-Cox procedure previously described. In addition, the full sample will be tested.

CHAPTER VI

RESULTS OF THE BOX-COX ANALYSIS

The Transformation Variables and Regression Equations

For each household selected and described earlier and for the full sample described, equation (5-2) was tested for the appropriate functional form using the Box-Cox procedure developed by Chang (1977b). The data for each household (P_i^h and x_{ij}) were transformed according to equations (4-3) and (4-4) by λ's and μ's ranging from 1.6 to -1.6 at intervals of 0.10. The grid search, which involves calculating least squares regression of P_i^{h*} on x_{i1}^{**}, x_{i2}^{**} and x_{i3}^{**}, was performed on each set of the transformed data.

With λ and μ taking on values between 1.6 and -1.6 at 0.1 intervals, there are 33 values that each λ and μ may assume (including 0.0). With these two values changing in the transformation (4-2), the total number of regression equations calculated was therefore 33^2 = 1,089. These 1,089 regression equations were calculated for hedonic equation (5-2) that represents each household and, also, the full sample represented by (5-2). These 1,089 regression equations represented every combination of λ and μ possible when both values range from 1.6 to -1.6 at 0.1 intervals.

Included in this range of λ and μ are the theoretical values that yield declining marginal utility of attributes described in Chapter IV. If the maximum likelihood estimates of $\hat{\lambda}$ and $\hat{\mu}$ are within these constraints, their values yield a solution to utility maximization that

is a unique global maximum. If the maximum likelihood extimates of $\hat{\lambda}$ and $\hat{\mu}$ lie near the bounds of 1.6 and -1.6, then a larger range of λ and μ will be considered for the grid search process.

The 1,089 regression equations can be divided into 33 groups. Each group contains 33 equations in which each equation represents the value of λ held constant and a specific value of μ between the range 1.6 to -1.6 at 0.1 intervals. For example, group 1 in Table 2 is represented

Table 2. Intervals and Two Groups Used in Grid Search for Box-Cox Transformation Parameters of the Hedonic Price Equation.

	Group 1		Group 2		
	λ	μ	λ	μ	
	1.6	1.6	1.5	1.6	
	1.6	1.5	1.5	1.5	
	1.6	1.4	1.5	1.4	
33	:	:	:	:	33
equations	:	:	:	:	equations
	1.6	0.0	1.5	0.0	
	:	:	:	:	
	:	:	:	:	
	1.6	-1.6	1.5	-1.6	

as the group of equations in which λ is held constant at 1.6 and μ changes at 0.1 intervals between 1.6 and -1.6. Group 1 contains 33 regression equations. For each equation, the value of the likelihood function $[\ell_n L_{max}(\lambda,\mu)]$ was calculated by using equation (4-19). Group 2 represents the 33 equations in which λ is held constant at 1.5 and μ changes 33 times.

Maximum Likelihood Results

In Table 3 the values of the restricted maximum likelihood function $\ell nL(\lambda,\mu)$ of each functional form are presented for each household and the full sample. These are values of (4-19) when λ and μ assume the values associated with models 1a-1e. For example, the $\ell nL(\lambda,\mu)$ for the linear functional form (model 1a) is the value of (4-19) when $\lambda = 1$ and $\mu = 1$ for each household, or 44.875. Also, in Table 3 is the maximum likelihood function for the unrestricted model for each household and the full sample or $\ell nL(\hat{\lambda},\hat{\mu})$. The values of $\hat{\lambda}$ and $\hat{\mu}$ are associated with $\ell nL(\hat{\lambda},\hat{\mu})$ for each household and the full sample.

Tests for Significance of Functional Forms

The question to be answered now: are the point estimates of the various functional forms significantly different from those associated with the maximum likelihood estimates? The models were tested for each household and the full sample. The null hypotheses were that the functional form is linear (H_0: $\lambda = 1.0$, $\mu = 1.0$), double-log (H_0: $\lambda = 0.0$, $\mu = 0.0$), semi-log (H_0: $\lambda = 1.0$, $\mu = 0.0$), reciprocal (H_0: $\lambda = 1.0$, $\mu = -1.0$), or log-reciprocal (H_0: $\lambda = 0.0$, $\mu = -1.0$). The alternative hypothesis was H_1: $(\hat{\lambda},\hat{\mu})$ where $\hat{\lambda}$ and $\hat{\mu}$ are values of $\ell nL(\hat{\lambda},\hat{\mu})$ for each household and the full sample. For example, the alternative hypothesis for household 1A is H_1: $\hat{\lambda} = 0.2$, $\hat{\mu} = 0.0$, since these values maximize the unrestricted maximum likelihood function $\ell nL(\hat{\lambda},\hat{\mu})$ for household 1A. The values of $\ell nL(\hat{\lambda},\hat{\mu})$ and $(\hat{\lambda},\hat{\mu})$ are shown in the last two columns of Table 3. For household 1A, the values of

Table 3. Values of the Restricted and Unrestricted Maximum Likelihood Functions of the Hedonic Price Equation for Each Household and the Full Sample.

	Restricted					Unrestricted	
House ID	ℓnL(1.0,1.0) linear	ℓnL(0.0,0.0) double-log	ℓnL(1.0,0.0) semi-log	ℓnL(1.0,-1.0) reciprocal	ℓnL(0.0,-1.0) log-reciprocal	ℓnL($\hat{\lambda}$,$\hat{\mu}$)	($\hat{\lambda}$,$\hat{\mu}$)
1A	44.875	61.776*	49.858	43.030	52.981	63.099	(0.2,0.0)
1B	-22.152	37.053*	-27.425	-30.848	26.563	38.611	(-0.1,0.1)
2A	6.621	48.851*	7.240	5.918	44.777	49.664	(-0.1,0.1)
2B	36.463	46.037*	38.348	34.766	42.716**	46.722	(0.2,0.0)
3A	32.543	57.237*	30.763	25.024	49.746	58.289	(0.0,0.3)
3B	5.665	44.335**	6.481	6.497	43.579**	47.483	(-0.4,0.0)
4A	17.985	46.331*	16.131	11.584	42.695**	46.442	(0.0,0.1)
4B	35.762	64.017*	36.240	33.418	62.666*	64.590	(-0.1,0.6)
Full Sample	75.473	396.645*	78.559	58.266	358.016	397.815	(-0.1,0.1)

*Significant functional form at .05 level.

**Significant functional form at .01 level.

$\ell nL(\lambda,\mu)$ for each functional form is subtracted from the value of $\ell nL(\hat{\lambda},\hat{\mu})$ in accordance with (4-21). In order to test for a linear functional form in household 1A, the H_0:($\lambda = 1.0$, $\mu = 1.0$) and H_1: ($\hat{\lambda} = 0.2$, $\hat{\mu} = 0.0$) were defined. From Table 3, for household 1A, $\ell nL(1.0,1.0) = 44.875$ and $\ell nL(0.2,0.0) = 63.099$. When these two values were substituted into (4-21), it was clear that the difference between the restricted and unrestricted models would have to be less than the critical value of 2.995 for the null hypothesis to be accepted at the .05 level of significance. The null hypothesis that the hedonic price equation is linear for household 1A was rejected at the .05 level of significance.

To show another test of significance for functional form in household 1A, consider testing the double-log functional form. Here we have H_0:($\lambda = 0.0$, $\mu = 0.0$) and H_1:($\hat{\lambda} = 0.2$, $\hat{\mu} = 0.0$). When $\ell nL(0.0,0.0)$ and $\ell nL(0.2,0.0)$ are applied to (4-21), the difference in the two maximum likelihood functions was less than the critical value of 2.995, and we conclude that the null hypothesis that the double log form of the hedonic price equation for household 1A cannot be rejected at the .05 level. In Table 3 each functional form associated with $\ell nL(\lambda,\mu)$ designated with a superscript * was significant at the .05 level for each household and the full sample. Those values of $\ell nL(\lambda,\mu)$ with superscript ** were those whose difference with $\ell nL(\hat{\lambda},\hat{\mu})$ were less than the critical value 4.605 at the .01 level of significance.

The results fail to reject the null hypothesis for the double-log functional form for each household and the full sample at the .05 level.

Thus for each household and the fullsample, the double-log functional form is not significantly different from the unrestricted maximum likelihood functional form. Note that the double-log functional form is consistent across all regions.

At the .05 level, the results fail to reject the null hypothesis for the log-reciprocal form for household 4B. For households 2B, 3B, and 4A the results fail to reject the null hypotheses for the log-reciprocal form at the .01 level. The log-reciprocal functional form is not consistently rejected across all regions and is rejected for the full sample.

Analysis of the Maximum Likelihood Estimates

For each household and the full sample, Table 4 shows the coefficients, R^2, t-values and values of $\hat{\lambda}$ and $\hat{\mu}$ that represent the maximum likelihood estimates. The significance of each coefficient is shown at the .05 level.

The relevance of $\hat{\lambda}$ and $\hat{\mu}$ for declining marginal utility is discussed below. Since interest centers on this issue, interpretation of the nutrient coefficients is treated briefly, given the values of $\hat{\lambda}$ and $\hat{\mu}$. Vitamin BC always has a coefficient that is not significantly different from zero. Both minerals and food energy have positive significant coefficients with one insignificant exception for each. These significant coefficients lead to inferences that the respective nutrients have positive declining marginal implicit valuations.

Table 4. Regression Coefficients, R^2, t-Values and Values of the Maximum Likelihood Parameters $(\hat{\lambda},\hat{u})$ for Each Household and the Full Sample, Spring 1977.

House ID	$\hat{\beta}_0$ intercept	$\hat{\beta}_1$ vitamin BC	$\hat{\beta}_2$ minerals	$\hat{\beta}_3$ food energy	R^2	$(\hat{\lambda},\hat{u})$
1A	-3.277 (-6.428)*	0.076 (1.415)	0.213 (4.035)*	0.220 (3.206)*	.397	(0.2, 0.0)
1B	-3.081 (-5.861)*	0.014 (0.344)	0.191 (4.819)*	0.115 (2.431)*	.397	(-0.1, 0.1)
2A	-2.764 (-4.936)*	-0.005 (-0.107)	0.123 (2.746)*	0.113 (1.995)*	.243	(-0.1, 0.1)
2B	-2.318 (-3.875)*	-0.066 (-1.027)	0.148 (2.116)*	0.185 (2.423)*	.225	(0.2, 0.0)
3A	-1.937 (-5.935)*	0.033 (1.382)	0.039 (3.701)*	0.027 (2.258)*	.306	(0.0, 0.3)
3B	-3.550 (-3.456)*	0.075 (0.795)	0.215 (2.150)*	0.174 (1.294)	.118	(-0.4, 0.0)
4A	-2.886 (-4.652)*	-0.025 (-0.471)	0.120 (2.882)*	0.152 (2.476)*	.274	(0.0, 0.1)
4B	-1.40 (-5.777)*	+0.008 (+1.208)	0.002 (1.312)	0.004 (1.991)*	.094	(-0.1, 0.6)
Full Sample	-2.709 (-13.876)*	0.013 (0.728)	0.127 (8.219)*	0.112 (6.125)*	.267	(-0.1, 0.1)

*Significant at .05 level.

Values in parentheses are t-values.

Analysis of the R^2 of the maximum likelihood estimates shows that both households in he North Central regin have higher R^2 values than any other regin and the full sample. For these households, both R^2 values equal .397, thus over one-third of the variation in market prices can be explained by variations in three nutritional attributes.

Inferences for Marginal Utility

From Table 4, the issue arises if the maximum likelihood parameters estimated from the Box-Cox transformation yields parameter values consistent with those required for declining marginal utility. Specifically, do the $\hat{\beta}_j$, $\hat{\lambda}$ and $\hat{\mu}$ values fall within the range needed to satisfy GHPE conditions (1) and (2). Inspection of $\hat{\lambda}$ and $\hat{\mu}$ in Table 4 shows that all these values for each household and the full sample fall within GHPE requirement two. To determine if the additional condition that $w < |z|$ in GHPE requirement two is satisfied, the following substitutions in z and w are made.

$$z^h = \hat{\beta}_j^h \, (\hat{\partial}^h - 1)(\bar{P}^h)^{1-\hat{\lambda}^h}(\bar{X}_j^h)^{\hat{\ell}^h - 2}$$

$$w^h = (\hat{\beta}_j^h)^2(\bar{X}_j^h \, \hat{\mu}^h - 1)^2(\bar{P}^h)^{1-(2)\hat{\lambda}^h}(1-\lambda^h)$$

where:

$\hat{\beta}_j^h$ = coefficient for attribute j from household h,

$\hat{\mu}^h$ = estimated $\hat{\mu}$ for household h,

$\hat{\lambda}^h$ = estimated $\hat{\lambda}$ for household h,

$\bar{P}^h$ = mean price per pound of food products consumed by household n,

$\bar{X}_j^h$ = mean quantity of attribute j consumed by household h from all products.

When the appropriate evaluations of z and w are made for the attributes of each household and the full sample, the results show that each household and the full sample shows declining marginal utility for minerals and food energy. The attribute vitamin BC showed declining marginal utility in the full sample and each of the households except three (i.e., 2A, 2B, and 4A). Thus, the Box-Cox technique generated functional forms of the hedonic price equation that is consistent with economic theory and showed declining marginal utility for the majority of attributes in this sample.

CHAPTER VII

SUMMARY AND CONCLUSIONS

Summary

Traditional economic theory and the classical consumer choice model use the idea that utility is from the consumption of goods and services purchased in the marketplace. Most followers of the classical theory of consumer behavior define the "consumer problem" as how to allocate a given budget (income) over the set of all possible goods and services in order to maximize utility. Because goods and services themselves yield utility, the quantities purchased reveal how the budget is allocated to maximize utility. When the classical "consumer problem" is presented as a mathematical programming problem, the resulting utility maximizing solution has certain mathematical properties with clear economic implications. One specific economic interpretation of these mathematical properties is that goods and services yield positive declining marginal utility. Without this property and theoretical condition, the classical consumer demand model is unable to yield a unique maximum solution to the "consumer problem."

The classical model of consumer demand is not the only model that attempts to explain consumer behavior. Recently, the consumer characteristics theory has been introduced in order to explain the consumer utility maximization problem. The main change from the classical consumer theory is that in the characteristics theory it is

assumed that characteristics of market product yield utility, not products themselves. A characteristic is defined as a component of a product like engine size and leg room in an automobile or protein and fat in food products. Here, consumers purchase market products in order to obtain utility from the characteristics they contain. The "consumer problem" in the characteristics theory seeks to maximize utility by allocating the fixed budget (income) over the set of all possible characteristics (via purchasing products that contain the characteristics). The purchased products that maximize utility in this model reveal the quantities of each characteristic chosen by the consumer. A hedonic price equation for each market product is derived that expresses the product price as a function of the implicit prices of the characteristics as valued by consumers.

When the characteristics theory is presented as a mathematical programming model, the resulting utility maximizing solution has mathematical properties with economic interpretations for characteristics. As shown in the discussion, these properties imply that characteristics yield positive declining marginal utilities. The analysis shows that a characteristics theory that yields declining marginal utility of characteristics has implications regarding the functional form specified for the hedonic price equation. Thus, using a functional form of the hedonic price equation that is unable to show declining marginal, utilities of characteristics is inconsistent with a unique maximum solution in the characteristics model.

Past empirical work using hedonic price equations justified using certain functional forms based on statistical grounds with no regard to economic theory. In a seminal article by Zvi Griliches (1961), he states that there is no a priori reason to expect the relationship in the hedonic equation to be in any particular fashion. In justifying using the semilogarithmic form, he based his choice on convenience and inspection of the data. Recent studies [Rosen (1974); Bender et al. (1980); Halvorsen and Pollakowski (1981)] concerning hedonic functional forms conform with Jorden et al. (1985, p. 141) who write, "Economic theory provides no guidance for the correct functional form for hedonic equations." The main purpose of this paper is to investigate the hedonic price equation and the theoretical grounds of declining marginal utility in the characteristics model.

The particular hedonic price equation investigated is for food products where characteristics are the nutritional attributes contained in food. Having established that a characteristics model with food and nutritional attributes will show positive declining marginal utilities for attributes when a unique maximum is found, we search for a functional form of the hedonic equation that shows declining marginal utilities.

In the discussion, it was shown that a hedonic price equation with declining marginal utilities is concave over the values of one attribute. Further, it was shown that the hedonic price equation will increase at a decreasing rate over the range of one attribute while the quantities of other attributes are held constant. The Box-Cox

methodology is used to search for an appropriate functional form of the hedonic price equation that satisfies the conditions outlined for a unique global maximum and thus declining marginal utilities. Four functional forms were identified that show declining marginal utility. The double-log, log-reciprocal, semi-log, and the reciprocal functional forms were shown to yield concave hedonic price functions and thus declining marginal utility of attributes.

The Box-Cox technique was used with data on food prices and consumption for households' taken in a survey conducted by the USDA in Spring 1977. The Nationwide Food Consumption Survey of 1977-78 allowed for the empirical test for functional form using the Box-Cox technique. The procedure used the maximum likelihood method to test the four previously mentioned functional forms and the linear functional form for the "correct" form to use with these data. Unlike the four functional forms included because they showed the mathematical and declining marginal utility criteria, the linear functional form was tested because of its popularity and frequent past use in hedonic models.

The data represented four regions of the United States. Two households from each region were selected at random. The functional form of the hedonic equation for each household was tested as well as the overall form for the pooled sample of all eight households. Each functional form was tested for significance for the model and data used.

A general hedonic price equation consistent with economic theory and showing declining marginal utility of attributes was defined in terms of the parameters β_j, μ and λ. With the necessary and sufficient

conditions for the hedonic price equation to show declining marginal utility, one only has to evaluate the choice of functional form of the hedonic price equation based on he values of these three parameters.

As a result of using the Box-Cox maximum likelihood estimation technique, the parameters $\hat{\mu}$, $\hat{\lambda}$ and $\hat{\beta}_j$ are estimated in the general hedonic price equation. Evaluation of the estimated Box-Cox equations were found to generate parameter values of $\hat{\beta}_j$, $\hat{\lambda}$ and $\hat{\mu}$ that fell within the range consistent with economic theory and declining marginal utility for attributes. One final point is that the linear hedonic price equation does not possess the necessary parameter values to be consistent with economic theory and thus is not acceptable.

Conclusions

Previous empirical work on the functional form of hedonic price equations have not based their choice of functional form on economic theory. Reasons given for choosing certain functional forms have been based on their simplicity, inspection of the data, convenience, statistical properties, and their easily interpreted results. As Halvorsen and Pollakowski (1981, p. 37) state, "The lack of a firm theoretical basis for the choice of functional form is unfortunate since the results obtained using the hedonic approach often depend critically on the functional form used."

When the consumer characteristics theory is viewed as a mathematical programming problem in which one seeks a unique maximum solution, specific mathematical optimization properties are seen to be consistent

with economic theory. As a result of examination of three basic mathematical properties, a firm foundation of economic theory can be found for the hedonic price equation. Namely, one must choose a functional form of the hedonic price equation that is consistent with the necessary and sufficient conditions for optimization in the model. In the characteristic theory the functional form of the hedonic price equation must show positive declining marginal utility for attributes. Many past studies have not considered this theoretical basis for choosing functional form. As a result of this study, future empirical work with characteristic theory should have a better theoretical basis on which to choose the appropriate functional form for the hedonic price equation.

REFERENCES

REFERENCES

Adelman, Irma and Zvi Griliches. 1961. "On An Index of Quality Change." Journal of American Statistics Association. 56: 535-548.

Becker, Gary S. 1965. "A Theory of the Allocation of Time." The Economic Journal. 75: 493-517.

Bender, B., J. Gronberg, and H. Huang. 1980. "Choice of Functional Form and Demand for Air Quality." Review of Econimics and Statistics. 62: 638-643.

Blaylock, James R. 1983. "Functional Form and Error Term in Engel Analysis." Unpublished doctoral dissertation, University of California at Berkeley.

Box, G. E. P. and D. R. Cox. 1964. "An Analysis of Transformations." Journal of the Royal Statistical Society, series B. 26: 211-243.

Brown, James N., and Havey S. Rosen. 1982. "On the Estimatino of Structural Hedonic Price Models," Econometrica. 50: 765-768.

Chalfant, James A. and Ronald A. Gallant. 1985.. "Estimating Substitution Elasticities With the Fourier Cost Function: Some Monte Carlo Results." Journal of Econometrics. 28: 205-222.

Chang, H. S. 1977a. "Functional Forms and the Demand for Meat in the United States." The Review of Economics and Statistics. 59: 355-359.

Chang, H. S. 1977b. "A Computer Program for Box-Cox Transformation and Estimation Technique." Econometrica. 45: No. 7: pp. 1741.

Chang, H. S. 1980. "Functional Forms and the Demand for Meat in the United States: A Reply." The Review of Economics and Statistics. 62: 148-150.

Clark, C. B. and R. G. Bressler. 1938. "Prices as Related to Quality on the Connecticut Strawberry Auctions." Agriculture Experiment Station Bulletin No. 227, University of Connecticut. Storrs, Connecticut.

Court, A. T. 1939. "Hedonic Price Indexes with Automotive Examples." The Dynamics of Automobile Demand. General Motors Corporation. New York.

Cox, Thomas L., and Michael K. Wohlgenant. 1986. "Price and Quality Effects in Cross-Sectional Demand Analysis." American Journal of Agricultural Economics. 68: 908-919.

Dhrymes, P. J. 1971. "Price and Quality Changes in Consumer Capital Goods: An Empirical Study." Price Indexes and Quality Changes: Studies in New Methods of Measurement. Zvi Griliches, editor. Harvard University Press. Cambridge, MA.

Eastwood, David B., John R. Brooker and Danny E. Terry. 1986. "Household Nutrient Demand: Use of Characteristics Theory and a Common Attribute Model." Southern Journal of Agricultural Economics, 20, No. 2.

Eastwood, David B., M. D. Gray and J. R. Brooker. 1986. "Socioeconomic Factors Affecting the Marginal Implicit Prices of Food Nutrients." The University of Tennessee Agriculture Experiment Station Research Bulletin No. 649, Knoxville, TN.

Edwards, S. R. and D. G. Anderson. 1984. "Land Use Conflicts in the Coastal Zone: An Approach for the Analysis of the Opportunity Cost of Protecting Coastal Resources." Journal of Northeastern Agricultural Economics Council. 13: 73-81.

Fettig, Lyle P. 1963. "Adjusting Farm Tractor Prices for Quality Changes, 1950-1962." Journal of Farm Economics. 45: 599-611.

Freeman, A. M., III. 1979. "Hedonic Prices, Property Values and Measuring Environmental Benefits: A Survey of the Issues." Scandinavian Journal of Economics. 81: 154-173.

Friedman, Milton. 1949. "The Marshallian Demand Curve." Journal of Political Economy. LVII: 463-495.

Goodman, Allen C. 1978. "Hedonic Prices, Price Indices, and Housing Markets." Journal of Urban Economics. 5: 471-484.

Griliches, Zvi. 1961. "Hedonic Price Indexes for Automobiles: An Econometric Analysis of Quality Change." Price Indexes and Quality Change: Studies in New Methods of Measurement. Edited by Zvi Griliches. Howard University Press. Cambridge, MA. 1971.

Gross, Ernest and Hui S. Chang. 1983. "Change sin Elasticities of Interstate Migration: Implication of Alternative Functional Forms." Journal of Regional Science. 23-2: 223-231.

Hager, Christine J. 1985. "Demand for Nutrient and Non-nutrient Components in Household Purchases of Red Meat, Poultry and Fish Products Using a Hedonic Approach." Unpublished doctoral dissertation, North Carolina State University. University Microfilms, Ann Arbor, Michigan.

Halrorsen, Robert and Henry O. Pollakowski. 1981. "Choice of Functional Form for Hedonic Price Equations." Journal of Urban Economics. 10: 37-49.

Hanemann, Michael W. 1982. "Quality and Demand Analysis." New Directions in Econometric Modeling and Forecasting in U.S. Agriculture. Edited by Gordon C. Rausser. Elsevier Science Publishing Company, New York. 1982.

Hendler, Reuben. 1975. "Lancaster's New Approach to Consumer Demand and Its Limitations." The American Economic Review. 65: 194-199.

Houthakker, H. S. 1951-1952. "Compensated Changes in Quantities and Qualities Consumed." Review of Economic Studies. 19: 155-164.

Hu, Teh-wei. 1982. Econometrics: An Introductory Analysis. University Park Press, Baltimore.

Intrilligator, Michael D. 1971. Mathematical Optimization and Economic Theory. Prentice-Hall Inc., Englewood Cliffs, N.J.

Johnston, J. 1984. Econometric Methods. McGraw-Hill, New York..

Jordan, Jeffery J., R. L. Shewfelt and W. C. Hurst. 1985. "Estimating Implicit Marginal Prices of Quality Characteristics of Tomatoes." Southern Journal of Agricultural Economics. December. 139-146.

King, Arthur T. 1976. "The Demand for Housing: A Lancastrian Approach." Southern Economic Journal. 43: 1077-87.

Kmenta, Jan. 1971. Elements of Econometrics. Macmillan Publishing Company, New York.

Ladd, George W. 1978. "Research on Product Characteristics: Models, Applications and Measures." Agriculture and Home Economics Experiment Station. Iowa State University of Science and Technology Research Bulletin No. 584, Amers, Iowa.

Ladd, George W. 1982. "Survey of Promising Developments in Demand Analysis: Economics of Product Characteristics." New Direction in Econometric Modeling and Forecasting in U.S. Agriculture. Edited by Gordon C. Rausser. Elsevier Science Publishing Company, New York.

Ladd, George W., and Veraphol Suvannunt. 1976. "A Model of Consumer Goods Characteristics." American Journal of Agricultural Economics. 58: 504-510.

Ladd, George W., and Martin Zober. 1977. "A Model of Consumer Reaction to Product Characteristics." Journal of Consumer Research. 4: 89-101.

Lafrance, Jeffrey Thomas. 1983. "The Economics of Nutrient Content and Consumer Demand for Food." Unpublished doctoral dissertation, University of California-Berkeley. University Microfilsm, Ann Arbor, Michigan.

Lancaster, Kelvin. 1966. "A New Approach to Consumer Theory." Journal of Political Economy. 74: 132-157.

Lucas, Robert E. B. 1975. "Hedonic Price Functions." Economic Inquiry. 13: 157-178.

Marshall, Alfred. 1920. The Principles of Economics; An Introductory Volume. Macmillan, London, England.

Michael, Robert T., and Gary S. Becker. 1973. "On the New Theory of Consumer Behavior." Swedish Journal of Economics. 75: 378-396.

Milon, J. W., J. Gressel, and D. Mulkey. 1984. "Hedonic Amenity Valuation and Functional Form Specification." Land Economics. 6: 378-387.

Palmquist, Raymond B. 1984. "Estimating the Demand for the Characteristics of Housing." Review of Economics and Statistics. 66: 394-404.

Rosen, Sherwin. 1974. "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition." Journal of Political Economy.

Smallwood, David M. and James R. Blaylock. 1986. "Forecasting Performance of Models Using the Box-Cox Transformation." United States Department of Agriculture. Agricultural Economics Research. 38: No. 4: 14-24.

Spitzer, J. J. 1976. "The Demand for Money, the Liquidity Trap and Functional Forms." International Economic Review. 17: 220-227.

Suvannunt, Veraphol. 1973. "Measurement of Quantities and Prices of Product Qualities." Unpublished doctoral dissertation, Iowa State University. University Microfilms, Ann Arbor, Michigan.

Tarrant, John R. 1980. Food Policies. John Wiley and Sons, New York.

Terry, Danny E. 1985. "An Evaluation of Characteristics Theory: Implicit Prices and the Demand for Nutritional Attributes." Unpublished doctoral dissertation, The University of Tennessee, Knoxville.

Timmer, C. Peter. 1978. "The Equitable Distribution of Domestic Food Aid." Agricultural-Food Policy Review: Proceedings of Five Food Policy Seminars. USDA, ESCS, ESCS-AFPR-2, No. 2: 51-57.

Theil, Henri. 1952-53. "Qualities, Prices and Budget Enquiries." Review of Economic Studies. 19: 129-147.

Theil, Henri. 1971. Principles of Econometrics. New York: John Wiley and Sons, Inc. 396-397.

U.S. Department of Agriculture. 1982. Consumer Nutrition Center, Human Nutrition Information Service. Food Consumption: Households in the United States, Spring 1977. Nationwide Food Consumption Survey 1977-78. Report No. H-1. U.S. Government Printing Office, Washington, D.C.

Waugh, Frederick V. 1928. "Quality Factors Influencing Vegetable Prices." Journal of Farm Economics. 10: 185-196.

Waugh, Frederick V. 1929. Quality as a Determinant of Vegetable Prices. Columbia University Press, New York.

Weimer, John. 1980. "Nutritional Labeling: The Unresolved Issues." National Food Review. July: 20-23.

White, K. J. 1972. "Estimation of the Liquidity Trap with a Generalized Functional Form." Econometrica. 40: 193-199.

Witte, Ann D., Howard J. Sumka and Homer Erekson. 1979. "An Estimation of a Structural Hedonic Price Model of the Housing Market: An Application of Rosen's Theory of Implicit Markets." Econometrica. 47: 1151-1173.

Zarembka, P. 1968. "Functional Form in the Demand for Money." Journal of the American Statistical Association. 63: 502-511.

Zarembka, P. 1974. "Transformation of Variables in Econometrics." Frontiers in Econometrics, P. Zarembka, editor. Academic Press, New York.

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