

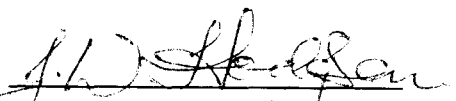
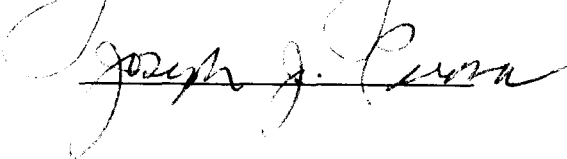
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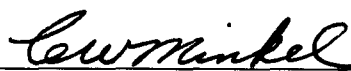
I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Mechanical Engineering.


Edward G. Keshock, Major Professor

We have read this dissertation
and recommend its acceptance :

Accepted for the Council:


Vice Provost
and Dean of The Graduate School

**MEASUREMENTS AND ANALYTICAL MODELING
OF HEAT TRANSFER IN FLOWING
GRANULAR MEDIA**

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Mohamed M. Alsharif

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ABSTRACT

An investigation was conducted of the heat transfer characteristics of a variety of flowing particulate solid media . Five different granular materials were studied in a vertical gravity driven flow through an electrically heated circular stainless steel tube test section. From measurements of wall temperature distribution, particle bulk inlet temperature , radial temperature distribution of the particle flow at the test section exit, and mass flow rate,determination of local and average heat transfer coefficients along the test section were made. Comparison of the experimental results with predictions of various models appearing in the literature were made. Agreement existed with the modified versions of the packet model. However, for correlations based upon and compared with only limited data, agreement was not satisfactory.

Consequently a more physically realistic theoretical model was developed to improve the agreement between experimental results and analytical predictions. The packet theory of heat transfer of Mickley was modified by accounting for the presence of the wall in affecting the local voidage. A model of the heat transfer processes within the media adjacent to the wall was developed that takes into account the variation of the thermophysical properties of the media in that region. A simple numerical solution to the resulting energy equation was obtained. The theory was found to correctly predict the trend of the measurements of the present study as well as the experimental data of controlled residence time available in the literature. Comparing present experimental

measurements with predictions from the model (in terms of Nusselt number versus Fourier number), the maximum difference observed was 39 percent, over a Fourier number range from 0.001 to 20. The average difference observed was estimated to be about 12.6 percent.

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NOMENCLATURE

A	area of heat transfer surface
A_r	Archimeds number $[gd^3\rho_g(\rho_s-\rho_g)]/\mu_g^2$
B	gradient matrix
C	capacitance matrix
C_c	calorimeter calibration constant
c_p	specific heat
D	material property matrix
d	particle mean diameter
d_i	diameter of particles in given size range
E	rate of energy transfer by convection across a control surface
e	calorimeter millivolt output
F	global force vector
h_i	instantaneous packet heat transfer coefficient
h_χ	local time average heat transfer coefficient, Equation (3-20)
h_m	surface mean heat transfer coefficient
h_x	local time average heat transfer coefficient, Equation (3-23)
$h_{\max}$	maximum flowing packed bed to surface heat transfer coefficient
I	variational form
K	conductance matrix
k	thermal conductivity
k_g	thermal conductivity of gas
k_o	bulk effective thermal conductivity
k_s	thermal conductivity of solid particles

L	test section length
m	mass flow rate
N	shape function matrix
n	defined by Equation (3-7-d)
Num_g	surface mean Nusselt number based on gas conductivity
Num_o	surface mean Nusselt number based on bed conductivity
Nux_g	local time average Nusselt number based on gas conductivity
Nux_o	local time average Nusselt number based on bed conductivity
Nu^*	modified Nusselt number
Pe_L^*	modified Peclet number
Pe_L	Peclet number
Q	net power input to the test section
q	rate of energy transfer by conduction across a control surface
q'	heat input per unit length of wire heat source
q''	heat flux
q_c	energy released in the calorimeter
R	test section inner radius
R_o	radius of stationary cylinder of granular media
r	radial distance from the center of the tube
T	temperature
T_c	temperature at longitudinal axis of stationary cylinder of granular media after time t
T_i	inlet temperature to test section or initial temperature of stationary cylinder of granular media
T_L	radial temperature at exit
T_{mL}	mean exit temperature
T_{mx}	mean temperature at x

T_s	surface temperature of stationary cylinder of granular media
T_{wx}	wall temperature at x
T_x	temperature profile at x
t	time
V	mean velocity in axial direction
V_s	volume of particulate solid
V_t	total volume
w_i	weight fraction in a given size interval
x	axial distance from the inlet section

Greek Symbols

α	effective thermal diffusivity
α	defined by Equation (3-7-b)
γ	defined by Equations (3-7-e) and (3-7-f)
Δ	increment
θ	defined by Equation (3-7-c)
τ	packet age
$\psi(\tau)$	frequency of occurrence of packets of age τ
ε	voidage
ε_o	bulk voidage
π	3.14159
Σ	summation
ρ	density

Subscripts

e	element
---	---------

g	gas
r	radial direction
s	solid
x	axial direction

Superscripts

e	element
T	transpose

1. INTRODUCTION

In recent years, there has been an increased interest in the thermal behavior of granular solids because of many relatively new industrial applications. Industries involving particulate solids being immersed or carried in a "continuous" fluid phase include chemical, petroleum, metallurgical, and food and pharmaceuticals process industries. Many fluidized bed operations are physically similar in nature, as in drying, coating, granulation, coal combustion and gasification processes. Also contact dominated flows are found to occur in certain industrial heat exchange equipment designed to heat, cool or dry granular materials.

In many cases of cooling or heating granular solids it is necessary to keep the solids physically separated from the cooling or heating medium. One method of accomplishing the desired heat exchange in such cases is to allow the solid to flow in a settled condition through a single vertical pipe which is jacketed with the heating or cooling fluid. Units of this type are inexpensive, simple and cheap to operate, and are readily available. However, in packed or fluidized bed systems the design of efficient heat transfer surfaces immersed in the bed is a subject which has received much study in recent years, but yet needs additional investigation. A better understanding of the heat transfer processes involved is still required. Thus, the general intention of the present investigation is to examine the heat transfer characteristics of gravity flowing particle beds and the dependence on different flow conditions and design parameters.

Among the many recent studies of heat transfer in granular media is the work by Botterill and Desai [1] and Denloye and Botterill [2], They have used a flowing packed bed to model the heat transfer behavior of a freely fluidized bed of particles (i.e. one having no net aggregate velocity). In [1] it was stated that the advantage of making experimental tests in a flowing packed bed was that more precise control of particle residence time was possible than that obtained in a freely fluidized bed. In another relatively recent study, Braun and Durrant [3] used sand in a gas-to-sand heat exchanger and a sand-to-water steam boiler. Although the use of phase change materials in energy storage systems is attractive because of their high latent heat of fusion, the utilization of sand as an energy storage material offers a number of significant advantages over conventional molten salt or metal liquid systems as summarized in [3]. Braun and Sarikelle [4] have also proposed sand as a novel thermal energy storage material, citing drawbacks in using molten salts; the principal advantage of a medium such as sand or silica is the high temperature levels attainable in such a working system.

Thermal analyses of the performance of equipment transferring heat to or from granular materials have been hampered by a lack of data and theoretical models for the convective heat transfer coefficients in flowing particle beds. Braun and Durrant [3], in their analysis of a gas-to-sand heat exchanger, mentioned that the range of the sand-side heat transfer coefficient is somewhat uncertain and the results reported in literature vary widely from one researcher to another. Spelt, Brennen and Sabersky [5] stated that the general problem of the flow of granular materials is extremely complex and not yet well understood. They also stated that the energy transport mechanisms and processes require further insight and investigation. Sullivan and Sabersky [6] concluded that the previously published heat transfer data indicate that the graininess of the media can, under certain conditions, directly and substantially influence convective heat transfer. Yet no definite parameter characterizing the importance of the granular nature of the

material is generally specified. Glicksman and Decker [7] reported that although there have been numerous studies of heat transfer in fluidized beds, there remain a number of unresolved questions concerning the heat transfer mechanisms. Few warnings are made concerning the limits of applicability of predictions or correlations based upon the popular homogeneous packet model. Furthermore Glicksman and Decker [7] reported that little effort has been directed at characterizing the nature of particle packing at immersed surfaces.

Heat transfer and thermal properties of a flowing granular medium, such as sand, appear to be of fundamental interest and value. The effects of particle size, ordering and thermal properties, average channel velocity and heating surface geometry appear to warrant further study.

The particular type of flow to be considered in this study is that in which the adjacent material particles are in physical contact and the interstitial fluid moves with it passively. In such a contact dominated flow the heat is generally transferred by convection between a solid surface and the granules which move relative to that surface.

The main objectives of the present research are:

- 1) To obtain a fundamental understanding of the heat transfer and flow characteristics of granular flowing media.
- 2) To generate data describing the thermal behavior and characteristics of granular media.
- 3) To develop a physical and analytical model of the heat transfer behavior of flowing granular materials that will be capable of providing accurate predictions and design criteria for practical systems.

To approach the mentioned objectives, the relevant literature pertaining to experimental and theoretical aspects of heat transfer to granular media were reviewed. An experimental and analytical study was undertaken and is described in the following

sections. The results obtained are then used to test existing correlations for their generality and accuracy.

2. LITERATURE SURVEY

2.1 INTRODUCTION

In this section previous experimental and theoretical analyses of convective heat transfer to flowing granular media will be reviewed. Mechanisms previously postulated for the heat transfer phenomena between a bed and a surface will be summarized. Also, available formulas and techniques for predictions of effective thermal conductivity and voidage fraction distribution near the surface are presented and discussed.

2.2 PREVIOUS EXPERIMENTAL AND THEORETICAL STUDY

The heat transfer rates between a heated solid surface and moving or fluidized beds of solid particles are much higher than for single phase gaseous flow. In order to explain this phenomenon and predict heat transfer rates for design purposes, several investigators have studied this problem experimentally and analytically.

One of the earliest investigations (1948) includes an experimental and theoretical study by Brinn, Friedman, and Gluckert [8]. They examined the factors which influence the heat transfer rates from beds of sand moving downward through vertical steam-jacketed tubes, assuming rodlike flow; Design charts were developed by them for use in predicting heat transfer rates to solids flowing in a settled condition inside vertical pipes.

Experimental measurements were also made of the heat transfer from constant temperature plates immersed in rotating packed beds of granular material by Dunskey, Zabrodsky, and Tamarin [9]. They found an increase in the average heat transfer coefficient with a decrease in the residence time or a decrease in the particle diameter. Measured values of the heat transfer coefficient were 70 percent lower than that predicted from a model based on considering heat transfer through a gas layer between a single solid sphere at constant temperature and a plane surface in contact with it.

Another experimental study, by Harakas and Beatty [10], considered an electrically heated surface immersed in a bed of fine particles (0.003 mm - 0.375 mm) contained in a rotating trough. Heat transfer coefficients were measured for a moving bed of solid particles with various interstitial gases. Velocities of the bed relative to the heat transfer surface were varied between 1.8 to 14.0 m/min. Comparison with coefficients predicted from a simple homogeneous conduction model, however, indicated discrepancies and a failure of the model as the effective thermal conductivity of the bed decreased and as the particle size increased.

In an experimental study of: (a) a pressurized system of solid particles flowing as a packed bed past a hot surface and (b) a pressurized fluidized bed by Botterill and Desai [1] it was found that the static pressure had little effect on the rates of heat transfer for flowing packed beds but an important effect on the heat transfer rates to fluidized beds of larger, closely sized particles. The maximum coefficients of heat transfer in the fluidized bed tests were generally lower by about 25 percent from those obtained in the flowing packed bed except for large, closely sized material. They concluded

No adequate model for the prediction of bed to surface heat transfer coefficient is available because the magnitude of the relevant parameters and their variations with the fluidizing conditions are not known. Until the effects of the particle packing density and their replacement rates near the transfer surface can be included, models are only likely to be useful as a basis for a posterior fitting to the experimental data.

An early theoretical model was developed by Mickley and Fairbanks [11] in which a group, or "packet", of particles was considered to be in contact with the heat transfer surface for a short residence time, then to be replaced by a fresh packet of particles from the bulk of the bed. They assumed that the thermal properties of the packets were the same as those of the quiescent bed, and derived an expression for the local instantaneous heat transfer coefficient. To support the proposed heat transfer mechanism they conduct an experimental work where heat transfer measurements were made on the same solid with several different fluidizing gases.

Botterill and William [12] introduced a simple model of conductive heat transfer to a single spherical particle immersed in a gas which comes into contact with a hot surface (at time zero). Then, by considering the number of particles in contact with a unit surface area, the overall heat transfer coefficient between the surface and the bed was estimated as a function of particle residence time at the surface. Predictions of the single particle model were lower than the experimental values. Later Botterill, Butt, Cain, and Redish [13] extended the model to consider heat transfer into a two-particle-depth layer in contact with the surface. Discrepancies between their experimental measurements and the theoretical predictions from the model were consistent with the presence of a gas film between the wall and the particles of the order of 10 percent of the particle diameter. Regarding the question of whether such gaps had any physical reality or not, Botterill [14] speculated that they did, but that it is difficult to distinguish to what extent their magnitude may differ between a flowing packed bed system and a freely fluidized bed.

Baskakov [15] introduced an additional contact resistance to the Mickley and Fairbanks model [11] to allow for the effect of the resistance to heat transfer in the region of increased voidage adjacent to the surface acting in series with the thermal resistance of the packets. The thickness of the gas layer was chosen to be approximately equal to the particle radius. Gelperin and Einstein [16] developed this approach further and

derived expressions for the overall heat transfer coefficient for a flowing granular medium in contact with a heated surface as a function of residence time, and particle and voidage thermal conductivity.

Denloye and Botterill [2] utilized the experimental heat transfer results obtained from a flowing packed bed to allow further tests of the modified packet model of heat transfer. They found that the flowing packed bed-to-surface heat transfer coefficient increased with decreasing particle residence time, with decreasing particle size, and with increasing gas thermal conductivity. They interpreted these results with the aid of a modified version of the Mickley and Fairbanks model [11]. The modified Mickley and Fairbanks model incorporated an additional wall heat transfer resistance that was given in terms of the particle diameter and the effective thermal conductivity near the wall. An empirical correlation was developed in terms of the mean particle diameter, gas conductivity and Archimedes number. In a later study, Botterill and Denloye [17] proposed a theoretical model to describe the heat transfer between an immersed heater and unrestrained packed beds. The bed was divided into two regions; (a) a region of increased voidage close to the heat transfer surface and (b) the region outside this. The voidage and gas velocity were assumed to be constant in each region. Heat transfer was assumed to take place by steady state conduction, and effective thermal conductivities were used to calculate the heat transfer in both regions. Their model overpredicted the heat transfer coefficient for particles of mean size less than 0.34 mm, but with discrepancies being less than 25 percent in general. With large particles there was an increasing discrepancy between the experimental results and the predictions of the model, with the predictions being consistently low. They suggested that the key factor in bed-to-surface heat transfer is the local voidage and the particle behavior immediately adjacent to the heat transfer surface.

Another experimental and analytical study by Sullivan and Sabersky [6], focuses upon the heat transfer from a flat plate immersed in a flowing granular medium. They showed that their experimental results corresponded well to a model in which the granular material was represented by a continuum connected to the heating surface by a special contact resistance, which was imagined to result from pure conduction through a thin gas layer, whose presence was postulated on the basis of their experimental data. The thickness of the gas layer was thus "determined" empirically. Its resistance was found to be approximately equivalent to an air gap of $1/10$ of a particle diameter thickness. The correlation derived from this model agreed well with their experimental results. Their results indicated that the magnitude of the heat transfer in this configuration may vary over a large range, depending on the magnitude of the Peclet number encountered.

Spelt, et al. [5] investigated convection heat transfer for two granular materials flowing along an inclined chute. For each material and for a given depth of flow, it was found that the heat transfer coefficient at first increases with velocity, then reaches a maximum and decreases as the velocity increases further. They found that the curves for smaller depths fall below those for the larger depth flows. These unexpected results clearly indicate the complexity of the heat transfer processes in a flowing granular material. Spelt, et al. related this behavior to changes in the packing density of the material caused by the flow field. The relation between velocity, density, and regimes of inclined granular chute flow has been studied recently by Campell, Brennen and Sabersky [18].

Schlunder [19] proposed a model for heat transfer between packed, agitated and fluidized beds and immersed surfaces. Within this model, the particulate material may absorb the heat supplied from the rigid wall in the sensible as well as in the latent mode. The pressure of the interstitial gas may vary from vacuum to pressures above

atmospheric. The heat transfer mechanism from the heated surface to the adjacent particles was described by: (1) wall-to-particle heat transfer by conduction, and (2) heat convection by particle motion. The combination of these mechanisms in the model leads to the introduction of several dimensionless groups, that reduce the number of variables as well as anticipating the structure of empirically based correlations. Parameters such as physical properties of the gas and solid, gas pressure and temperature, particle density, void fraction, etc. were taken into consideration. Comparison of the theory with numerous experimental data taken from various sources was found to agree within 30 percent. Predictions of heat transfer coefficients from this model (which apply to contact heating or cooling of packed beds either resting on or sliding past a stationary heated surface) agrees fairly well with the correlation by Sullivan and Sabersky [6]. Schlunder pointed out that the correlation given by Sullivan and Sabersky [6], however, does not correctly describe the effect of the particle diameter. Moreover the empirical parameter employed in the Sullivan-Sabersky correlation does not at all take into account the influence of the gas pressure, while Schlunder's studies have shown gas pressure to have a strong effect on the average heat transfer coefficient. (Botterill and Desai [1] reported that the static pressure had an important effect on the heat transfer rates to fluidized beds of large closely sized particles, but a little effect on the rates of heat transfer for flowing packed beds).

Nietert and Khalik [20] determined the heat transfer characteristics of gravity flowing particle beds at controlled rates through an electrically heated tube. The local and average convective heat transfer coefficients were evaluated as a function of the average bed velocity, particle size and heat flux. They concluded, from the results obtained by operating the experiment at different heat flux levels, that there is a strong dependence of the heat transfer coefficient on the physical properties of the particles. They indicated that additional experiments over a wide range of variables are required to

obtain a generalized Nusselt type correlation for the heat transfer coefficients in gravity flowing particle beds.

Studies of the characteristics of wall-to-bed heat transfer in a vertical tube using a fluidized particle bed were performed by Izumi and Yamashita [21]. Their vertical tube was heated electrically so as to maintain a uniform heat flux condition. Particles of glass beads were fluidized by air flowing upward. The local Nusselt number in the vertical direction was measured for different particle diameters, initial bed height, and air velocity.

Burolla, Hruby and Steele [22] investigated the heat transfer characteristics of a solar collector. They indicated that particle absorptivity, particle diameter, mass flow rate and wall temperature are important parameters indicating the final particle temperature. A simple model describing system efficiency is used to predict the effect of these parameters on the energy absorbed and the final exit temperature of the solid particles.

Decker and Glicksman [23,24] proposed a physical model of the particle-to-surface heat transfer in the emulsion phase* of a fluidized bed. They concluded that the heat transfer between the bed particles and an immersed surface is dependent on the thermal time constant of the particles and the average particle residence time on the surface. When the residence time is much greater than the time constant, the surface resistance is unimportant and a continuum model for heat transfer in the emulsion is appropriate. When the time constant is larger than the residence time, the thermal resistance between the surface must be considered and a continuum model is inappropriate. In the region of close contact between particles and the surface, a realistic model of the detailed surface geometry near points of contact must be included. When a representative model of surface roughness is assumed, the conductive heat transfer is

* Regions of high solid density.

found to be insensitive to thermophysical properties of the particles and only modestly influenced by the level of roughness. They failed make a direct comparison of results presented by their analysis and other experimental findings because of the absence of a detailed knowledge of the contact zone geometry.

In [24], Decker and Glicksman considered and studied heat transfer to immersed surface in large particle fluidized beds. Two mechanisms of particle heat transfer at low temperatures were assumed to occur, namely (a) conduction heat transfer near the point of contact between particles and the heated surface, and (b) convection augmentation due to lateral mixing of the gas in the large voids between individual particles in the emulsion. Glicksman and Decker [7] also introduced a simplified physical model of heat transfer from a fluidized bed to an immersed surface. They concluded that packet models using continuum properties are reasonable only when particle diameters are less than 0.3 to 0.6 mm. For particles with diameters greater than 0.6 mm they stated that the thermal gradient extends only one or two particle diameters into the bed. They suggested that a discrete analysis of heat transfer to individual particles is necessary for such large particles. In the simulation of particle packing at the heat transfer surface, assuming random packing, they point out that irregular shaped particles will cover a smaller fraction of the surface than will spherical particles. This is consistent with their numerical estimates of the heat transfer coefficients for spherical particles being 50 percent greater than for irregular particles. The contribution of radiation to the total heat flux of high temperature beds is demonstrated to be a function of fluid mechanical variables as well as particle size and bed and surface temperature.

Another more recent study by Gloski, Glicksman, and Decker [25], focuses upon measuring the heat transfer between a surface and particles of diameters between 0.65 and 1.0 mm for both fixed and fluidized beds. An experimental apparatus was developed which permitted measurements of the thermal resistance between a surface

and adjacent particles during a rapid transient process. Large heat transfer coefficients were observed in the initial 20 milliseconds of contact while those during an ensuing period of about 80 milliseconds were much lower and nearly invariant with time; After that, the coefficients began to decline. They reported that this behavior is most consistent with a discrete particle model in which initial heat transfer occurs through the asperities in direct contact with the surface, followed by heat transfer through gas layers in areas adjacent to the contact points. The results obtained for fixed and fluidized beds at vertical heat transfer surfaces were virtually identical. They concluded from the absence of any significant difference between the packed bed and fluidized bed results that it is not necessary to postulate an intervening gas layer between the contact points on the surface and the particles of a fluidized bed. They further reported that one dimensional models reflecting the local voidage distribution may be inadequate since they neglect the inhomogeneous distribution of thermal properties; Consequently, the numerical complexities of such models may not be warranted in light of this uncertainty. They call for further work in order to determine the influence of particle sphericity and the distribution of particle size.

Heat transfer in a large particle diameter fluidized beds (greater than 1 mm) with immersed surfaces have been studied by Ganzha, Upadhyay, and Saxena [26]. The model which they propose neglects radiation and regards the heat transfer coefficient as composed of gas conduction and convection components. They assumed that particles on the heat transfer surface are arranged in an orthorhombic arrangement. They further assumed that particles can be replaced by equivalent cylinders having the same volume as the actual particles and of equal height and diameter. The conduction contribution is calculated by computing the unsteady state heat transfer rate from the surface to the first layer of particles through the gas film (or lens) enclosed between the two. The convection contribution is obtained by assuming that a turbulent boundary layer on the

heat transfer surface is continuously disrupted by the solid particles, and by considering the analogy of this process with that of a flat plate immersed in a solids-free turbulent gas stream. It was found that the theoretical model predictions for the total heat transfer coefficient are in good agreement with the available experimental data on large particle systems. Comments and discussion of the paper by Ganzha, et al. [26] is presented by Kubie [27].

Yoshida, Tamura, and Kunii [28] measured effective thermal conductivity in the radial direction and the apparent film coefficient at the wall surface in moving beds of glass beads and sand. Solid particles descended downwards in a vertical tube heated externally by means of condensing steam, while air was passed upwards through the moving bed. They obtained empirical equations for the effective radial thermal conductivity and the wall film heat transfer coefficient.

Takahashi and Takeuchi [29] investigated the characteristics of the axial temperature distribution in a moving bed in which air was forced countercurrently to the gravity flow of solid particles. Both the fluid and solids were heated by an electric heater outside the tube. The axial temperature curve obtained usually had a maximum in a particular axial position.

A model for the mechanism of heat transfer from the surface of body fixed in a fluidized bed is proposed by Tasutomi and Yokota [30]. The emulsion phase was assumed to be a composite solid consisting of two layers. The first layer contacts the surface; Its void fraction is higher on account of a wall effect, while its thickness is assumed to be approximately equal to the radius of the fluidized particles. The second layer is considered to have a void fraction equal to that at minimum fluidization; It exists beyond the first layer and is assumed to have an infinite thickness. Their equation for the heat transfer coefficient is derived considering unsteady conduction from the constant temperature surface to the solids.

Kubie and Broughton [31] proposed a model for surface-to-bed heat transfer analogous to the variable wall properties approach long used in single phase heat transfer. In their model packets of material from the bed are swept to the heat transfer surface, where transient conduction occurs for the time of packet residence. Allowance is made for the variation of the void fraction within a distance of one particle diameter from the surface. The voidage variation near the surface is described from simple geometrical considerations. This important feature of the model permits the bulk density and thermal conductivity to vary according to some specified voidage distribution function in the vicinity of the wall. It was found that for the voidage distribution assumed to exist in a bed of uniform spherical particles, the thermophysical properties of the bulk changed very rapidly with distance from the surface. They concluded that the derived model agrees well with all controlled residence time data available in the literature. Further, their model was reported to require no physically unjustified concepts in order to produce agreement with experimental data. By contrast, none of the available models of heat transfer in fluidized beds which they examined were capable of producing acceptable agreement with experimental data over the full range of data without the introduction of some form of semiempirical approximation. They observe that while approximations such as (a) a gas film, (b) a region of reduced voidage or (c) a finite penetration depth, all have obvious physical significance, they are ad hoc expedients rather than solutions to the real problem of behavior close to a surface.

A modified packet model which accounts for the change of void fraction near the surface, was introduced by Ozkaynak and J.Chen [32]. Heat transfer coefficients from a vertical tube in fluidized beds were predicted by measuring emulsion packet residence times. They found that the packet model by Mickley and Fairbanks [11] was satisfactory for small particles of different physical and thermal properties but overpredicted the heat

transfer coefficients for large particles. Their modified packet model was found to successfully predict heat transfer coefficients for both small and large particles and for particles with different physical and thermal properties. Also Chandran and J.Chen [33] proposed a model for local heat transfer between a gas fluidized bed and a submerged tube based on combined dense phase and lean phase transport. The heat transfer process during dense phase contact at the tube surface is modeled by a packet renewal mechanism, while the transfer process during lean phase contact is modeled by a fluid convection mechanism. The model predictions show good agreement with experimental local and average heat transfer coefficient data for horizontal tubes.

Another study by Chandran and J.Chen [34] investigated the influence of the wall on transient conduction into packed media. Since surfaces immersed in moving packed beds and fluidized beds frequently experience a renewal type of contact with the contained media, the heat transfer during the surface renewal process was modeled by them as a transient conduction process. They maintained that this is permissible provided the contribution due to thermal radiation is negligible. They developed a simple closed form relation for the heat transfer coefficient which is applicable for use in design purposes.

Another theoretical study by P.Chen and Pei [35] was developed to describe the heat transfer between a fluidized bed and an immersed surface, based on the definition of a two phase thermal boundary layer around the surface. In considering the vigorous particle motion and the increasing bed voidage, two correlations for predicting the maximum heat transfer coefficient and the optimum flow rate were obtained. These correlations agree well over a wide range of conditions with the data reported in the literature.

A computer model for a hot fluidized bed was developed by Syamlal and Gidaspow [36]. They used a hydrodynamic model of fluidization, utilizing the continuum approach

to model fluidized bed heat transfer and the principals of conservation of mass, momentum and energy. Predictions of the heat transfer coefficients were obtained by numerical solution of the established set of nonlinear partial differential equations. They agree with measurements reported by Ozkaynak and J.Chen [32] within the accuracy of the estimated thermal conductivity of particulate aggregate solids.

2.3 OVERVIEW OF PHENOMENOLOGY OF BASIC MODELS

A review of the models proposed in the literature for heat transfer between a moving bed and a surface are given in Gelperin and Einstein [16], Botterill [14], Gutfinger and Abuaf [37], and Saxena, et al. [38]. Generally, the various models may be grouped into three basic types, as will now be summarized.

2.3.1 Film Model

The first category of models is based on a fluid film theory [37]. In this model heat transfer is assumed to occur through a hypothetical fluid film in contact with the wall to the particles moving downward outside this film; The thermal resistance of the fluid film is affected by the solid particles passing through it. As this kind of model does not take into account the influence of particle properties, the subsequent formulas are usually not in good agreement with the experimental results, as reported by Chen and Pei [35]. Also Gutfinger and Abuaf [37] indicted that because of not taking into account the influence of the solid particles on the heat transfer phenomena, the mechanism proposed cannot be considered to be complete.

2.3.2 Single Particle Model

The second category of models is based on the transient heat conduction process occurring between the immersed surface and a single particle [14]. Predictions of these models generally deviate greatly from experimental data gathered from closely controlled residence time [31]. This shortcoming was removed by introducing a hypothetical gas film between the particle and the surface [2,12,14] This model is acceptable at low Fourier numbers only [31]. To improve the model additional layers of particles were considered in an extended analyses [13,14].

2.3.3 Emulsion Phase Models

In this type of model a packet of constant voidage emulsion phase is considered to be swept into contact with the heat transfer surface for a period of time during which heat is transferred by non-steady conduction [11]. Good predictions have been obtained at large values of the Fourier number [2,31] while at small values the model fails [31,38]. Models based on this approach have been refined, however, in order to extend their validity to low Fourier numbers [16]. The method used is to introduce a time independent contact resistance at the bed surface interface to account for the increased voidage in the vicinity of the surface [2,15]. The evaluation of this contact resistance is based upon empirical considerations which enable a good data fit to be obtained. A recent modification to the packet theory has been introduced by Kubie [31] that accounts for the presence of the surface and its effect on the local voidage, and hence alters the local thermophysical properties. The variation of the voidage was assumed to be confined to the plane normal to the wall.

2.4 EFFECTIVE THERMAL CONDUCTIVITY

In the development of the analytical methods for the design of equipment for gas-solid system, the proper values of the effective thermal conductivities must be estimated. The granular medium in general consists of solid particles of various shapes and sizes in physical contact with each other, and voids of more complicated shape. A knowledge of the effective thermal conductivity of such composite materials is important for the thermal analyses of such systems. Mickley and Fairbanks [11] modeled the thermal properties of bulk material as a homogeneous mixture of the solid and fluid properties. It was shown [15,17] that the contact resistance changes due to variations with effective conductivity of the bulk material in the immediate vicinity of the wall. A number of experimental measurements and many formulae are available in the literature to predict the thermal conductivity of two component solid-gas granular systems. Generally the thermal conductivity of the bed is expressed as a function of the thermal conductivity of the gas and solid phases, the void fraction, particle size, and the pressure of the system. A review of the major published theoretical correlations of thermal conductivity of granular material and a number of proposed heat transfer models are summarized by Crane, Vachon, and Khader [39]. Both experimental techniques and theoretical models for evaluating the effective thermal conductivity of low conductivity materials are given in Pratt [40].

Effective thermal conductivity models for granular systems are generally classified as being either Ohm's law models or flux law models. Correlations for the effective thermal conductivity based on flux law models may be derived from solution of the heat conduction equation for a given system geometry. Ohm's law models assume parallel or series distribution of the components according to their arrangements with respect to the direction of the flow of heat. The effective thermal conductivity is measured utilizing

the basic standard steady state methods (guarded hot plate) and the dynamic methods (line source and probe).

Brinn et al. [8] and Mickley and Fairbanks [11] measured the effective thermal conductivity of granular media by a method similar to that described by Waddams [41], which involves measuring the unsteady radial heat flow between the surface and axis of a cylinder of material. Yagi and Kunii [42,43] performed experiments and obtained theoretical formula for effective thermal conductivities in packed beds. Willhite, Kunii, and Smith [44] and Kunii and Smith [45], derived equations for predicting the effective thermal conductivity of beds of unconsolidated and consolidated particles containing stagnant fluid; Comparison with the available experimental data indicated that their equations are satisfactory for fluids and solid particles of both high and low thermal conductivities. In [44] experimental results showed that the measured stagnant conductivity also satisfactorily represented heat transfer for flowing conditions.

An analytical study was performed by Yovanovich [46] for predicting the apparent thermal conductivity of beds of uniform diameter glass spheres. His mathematical model considered the thermal conduction resistance within the spheres, together with the conduction resistance of an effective gas gap thickness. The model is valid for gas pressures ranging from vacuum conditions to atmospheric.

Abrahamsen and Geldart [47] studied the effective thermal conductivity of fine powders fluidized with different gases. They concluded that in the range studied, the effective radial thermal conductivity does not appear to be a function of gas velocity and is most sensitive to the gas phase thermal conductivity .

The validity of existing thermal conductivity correlations for solid-gas mixtures have been examined by Kuzay [48]. He also examined the effects of porosity and radiation at higher temperatures. Dixon [49] derived formulas for the effective axial and radial thermal conductivities of packed beds from a simple thermal resistance model.

Also Yagi, Kunii, and Wakao [50] showed that the axial effective thermal conductivities in packed beds coincide with the radial conductivity when the gas flow rate approaches zero. Vortmeyer and Adam [51] analytically correlated the axial thermal conductivity at low gas flow rates. The results they obtained were in agreement with the previous work of Yagi et. al. [50]. A model of the effective thermal conductivity and diffusivity of a packed bed with stagnant fluid was proposed by Jaguaribe [52]. The model requires only the void fraction and the physical properties of the fluid and solid as input; No empirical or theoretical model parameters were required.

Pande, Saxena and Chaudhary [53] measured the thermal conductivity of loose granular materials using the thermal probe method. Interstitial air pressure was varied from 0.1 to 1800 mm of mercury. A line source technique was utilized by Keshock [54] for measuring the effective thermal conductivity and thermal diffusivity of porous materials in several gaseous environments. An error analysis of the line-source technique was performed in order to estimate the resulting uncertainty of the thermal conductivity and thermal diffusivity determinations.

2.5 VOIDAGE VARIATION NEAR A CONSTRAINING SURFACE

When considering heat transfer between the particles in a packed or a fluidized bed and a surface immersed within the bed, it is necessary to know the porosity distribution at the surface across which the heat transfer occurs. Though the average void fraction within a bed may be known, the existence of a constraining surface will alter the particle packing, and hence the solid cross-sectional area, in the vicinity of that surface.

The variation of local voidage of packed beds in the vicinity of a constraining wall has been investigated by several workers. Roblee, Baird, and Tirney [55] measured the

radial variation of void fraction in randomly packed beds of spheres, cylinders, and other configurations. They utilized paraffin to fill the bed after packing and then allowed it to solidify. Results for spheres and cylinders of diameters equal to their height were such that the porosity was a minimum at one particle radius from the wall, with alternate maximums and minimums occurring at successive particle radii. The amplitude of the cycling decreased as distance from the wall increased, but significant differences were found beyond three particle diameters from the wall.

Benenati and Brosilow [56] investigated void fraction distribution in beds of uniform spheres. It was found that the radial variation of void fraction varies from unity at the wall to about 38 percent in the interior of the bed. The bed-to-particle diameter ratios were found to vary from 2.6 to infinity. Containing walls of concave and convex curvatures were studied. In another experimental work by Korolev and Syromyatnikov [57], the porosity of fixed and fluidized beds near the wall were investigated by means of x-radioscopy. According to their experimental results, the effect of the wall on the packing of the particles diminishes with increasing distance from the wall. A formula of the local voidage variation was derived in terms of the particle diameter, mean porosity, and the x-ray beam width.

Kubie et al. [31] described the local voidage variation in the vicinity of a constraining wall by a simple geometrical function for a distance of up to one particle diameter from the surface. Pilla [58] performed an experimental investigation in a two dimensional packed bed. He concluded that the geometrical function given by Kubie [31] is a very good approximation for distances of up to one particle diameter from the wall.

3. HEAT TRANSFER ANALYSIS

3.1 INTRODUCTION

An efficient approach for determining the rate of heat transfer associated with flowing granular media in smooth tubes for a uniform wall heat flux boundary condition is developed in this chapter.

The key to this proposed approach lies in the use of proper equations to evaluate the thermophysical properties of the medium in the critical region adjacent to the heat transfer surface, i.e thermal conductivity, density, and specific heat; The voidage in this region must also be properly evaluated. Then, an efficient numerical finite element technique is used to solve the energy equation. This boundary layer type concept, which takes into account the influence of the surface on the local packing of a bed of granular material, was first introduced by Kubie and Broughton [31]. The behavior of the particles in the core of the flow field is treated as homogeneous in the present analysis.

Predictions are obtained for local time average and surface mean heat transfer coefficients for different flowing granular materials of different interstitial gases. The results of this suggested method are compared with predictions based on other theoretical methods as well as with available experimental data.

3.2 BASIS OF PROPOSED MODEL

For predictions of heat transfer coefficient of surfaces immersed in moving packed beds there exist two distinct general approaches, the homogeneous approach by Mickley et al. [11], and the discrete particle model by Botterill et al. [12]. Many investigators have suggested modifications to the basic models cited above so as to achieve good agreement with experimental data. The homogeneous model [11] has been modified recently by Kubie [31] to take into account the presence of the surface and its effect on the thermophysical properties of the bulk material in the immediate vicinity of the wall.

In the present model the packet theory of heat transfer is modified to take into account the presence of the surface and its effect on the local voidage. This is done by introducing a property boundary layer concept introduced by Kubie et al. [31] that has been found by them to be physically justified. The bed is considered as consisting of a region of higher voidage within a one particle-diameter distance from the cylindrical heat transfer surface and a core region which constitutes the bulk of the bed.

The model is based on the following assumptions:

- 1) Transient heat transfer takes place into a disk of the granular media flowing in contact with the hot surface (see Figure 3-1-a)*.
- 2) The region adjacent to the wall is characterized by greater voidage, which alters the local effective or composite thermophysical properties, such as density, specific heat, and thermal conductivity.
- 3) The variation of voidage is confined to within one particle diameter from the wall within any plane perpendicular to the channel's longitudinal axis.

* All Figures may be found in Appendix A

- 4) The core of the bed has a constant voidage and the temperatures of the gas and solid particles at any given point in the bed are of negligible difference because of the relatively high exposed particle surface area.
- 5) The interparticle heat transfer due to thermal radiation is negligible. (It was reported by Yagi and Kunii [42] that radiation heat transfer must be considered when the mean temperature of the packed bed is higher than 400 °C. In the present analysis the bed temperature is assumed to be much less than 400 °C. Also, in the experimental phase of the present study the maximum temperature level of the flowing media was about 100 °C).
- 6) A constant heat flux is maintained on the tube surface.

3.3 FORMULATION OF THE MODEL

By taking a ring shaped differential volume of length dx , thickness dr , and radius r as shown in Figure 3-1-b and making the assumptions of steady flow, no internal heat generation, no chemical reaction, and neglecting work done by body and surface forces, the energy equation reduced to

$$E_r + q_r + E_x + q_x = E_{r+dr} + q_{r+dr} + E_{x+dx} + q_{x+dx} \quad (3-1-a)$$

where E is the rate of energy convected with the bulk granular motion across the control surface and q is the rate of energy transferred across the control surface by conduction. The convective terms in the radial direction, E_r and E_{r+dr} , are neglected since there is no mass flow in that direction. To elaborate upon this point, results of an experimental study of downward flowing particulate media by Davis and cited by Zenz and Othmer [59] are shown in Figures 3-1-c and 3-1-d. The experiments were carried out in a thin,

rectangular bin, with glass sides, permitting observation of the flow through a colored layer of particles. A photograph was obtained by using one piece of film exposed at 60-second intervals for 9 minutes without moving the camera. The solid lines in Figures 3-1-c and 3-1-d, taken from Zenz and Othmer [59], represents the flow contours at each exposure. The dashed lines connecting the large colored pieces were drawn to show the paths followed by the particles in down flow. Also visualisation experiments by Brinn, et al. [8] and Sullivan [60] indicated that the flow of granular material in a vertical channel is essentially rectilinear.

Additionally, it is assumed that no velocity fluctuations occur in the radial direction, analogous to those that might occur in the turbulent flow of liquids or gases, equation 3-1-a reduces to

$$q_r - q_{r+dr} + q_x - q_{x+dx} = E_{x+dx} - E_x \quad (3-1-b)$$

by noting that

$$q_{x+dx} = q_x + \frac{d}{dx} (q_x) dx \quad (3-1-c)$$

and a similar expression for q_{r+dr} and E_{x+dx} and by utilizing Fourier's law of conduction for q_x and q_r along with $E_x = \rho V c_p T$, equation 3-1-b leads to the following partial differential equation

$$\frac{1}{r} \frac{\partial}{\partial r} [k(r) r \frac{\partial T}{\partial r}] + \frac{\partial}{\partial x} [k(x) \frac{\partial T}{\partial x}] = \rho(r) c_p(r) V \frac{\partial T}{\partial x} \quad (3-1-d)$$

The right side of this equation expresses the net rate at which heat is transferred by convection in the differential element owing to the flow into and out of the element faces in the direction parallel to the flow channel longitudinal axis. The left side represents the net rate at which heat is transferred by conduction in the radial and axial directions.

Visual experiments by Brinn, et al. [8] and Sullivan [60] not only indicated that the flow of granular material in a vertical channel is essentially rectilinear, but also the particles adjacent to the smooth wall slide along the wall at approximately the same speed as the bulk flow. Thus the velocity, V , in Equation (3-1-c) may be assumed constant. Equation (3-1-c) is subject to the the following boundary conditions:

at the hot surface

$$r = R, \text{ and } 0 < x < L$$

$$-k(r) \frac{\partial T}{\partial r} = q'' \quad (3-2)$$

at the centerline

$$r = 0, \text{ and } 0 < x < L$$

$$\frac{\partial T}{\partial r} = 0 \quad (3-3)$$

at the inlet section

$$x = 0, \text{ and } 0 < r < R$$

$$T = T_i \quad (3-4)$$

at the outlet section

$$x = L$$

$$2\pi R L q'' = mc_p (T_{mL} - T_i) \quad (3-5-a)$$

$$T_{mL} = \frac{2}{R^2} \int_0^R T_L r \, dr \quad (3-5-b)$$

In order to solve Equation (3-1-c) with the boundary conditions expressed by Equations (3-2)-(3-5) the functions $k(r)$, $k(x)$, and $\rho(r) c_p(r)$ must be specified. The evaluation of these functions thus requires a knowledge of the voidage variation in the vicinity of the surface.

3.3.1 Voidage Variation

The variation of local voidage of packed beds in the vicinity of a constraining wall has been investigated by several workers as summarized in section 2.5. The geometrical function proposed by Kubie [31] for the voidage variation within one particle diameter from the wall is recommended by Pillai [58] and has been used by Chandran and Chen [34]. This function; Drived on the basis of simple geometrical considerations of single sphere in contact with plane surface, is utilized in the present analysis. Thus,

$$\varepsilon(r) = 1 - 3(1 - \varepsilon_0) \left[\frac{R-r}{d} - \frac{2}{3} \frac{(R-r)^2}{d^2} \right] \quad (3-6-a)$$

$$\text{for} \quad (R-d) < r < R$$

$$\text{and} \quad \varepsilon(r) = \varepsilon_0 \quad (3-6-b)$$

$$\text{for} \quad r < (R-d)$$

3.3.2 Effective Thermal Conductivity

The effective thermal conductivity can be calculated from one of the models presented in section 2.4. Crane et al. [39] recommended the equation proposed by Willhite et al. [44]. Willhite indicated that their proposed equation satisfactorily represented heat transfer for flowing conditions. This equation is also valid in the region of high voidage as reported by Kubie [31] and Botterill [17]. It is as follows:

$$\frac{k_o}{k_g} = 1 + (1 - \epsilon) (1 - k_g/k_s) + (\gamma/\alpha) (1 - k_g/k_s)^2 (1 - \epsilon) \quad (3-7-a)$$

where

$$\alpha = 0.5 \left\{ (1 - k_g/k_s)^2 \frac{\sin^2 \theta}{\ln [k_s/k_g - (k_s/k_g - 1) \cos \theta] - (1 - k_g/k_s)(1 - \cos \theta)} \right\} \quad (3-7-b)$$

$$\sin^2 \theta = 1/n \quad (3-7-c)$$

$$n = 3 (5.01 - 8.42 \epsilon_o) / (1.91 - 1.91 \epsilon_o) \quad (3-7-d)$$

For nonspherical or high conductivity spherical particles

$$\gamma = 0.5 \quad (3-7-e)$$

while for spherical low conductivity particles,

$$\gamma = 2/3 \quad (3-7-f)$$

The voidage ϵ will be evaluated here by Equations (3-6-a) and (3-6-b).

As pointed out in section 2.4 the axial effective thermal conductivity coincides with the radial conductivity for stagnant interstitial gas.

3.3.3 Heat Capacity

The effective heat capacity in the region adjacent to the wall within one particle diameter is taken here as a weighted average of the solid and gas phases, i.e.

$$\rho(r) c_p(r) = (\rho c_p)_s (1-\epsilon(r)) + (\rho c_p)_g \epsilon(r) \quad (3-8-a)$$

The heat capacity of the gas phase can be neglected compared to that of the solid matter, so that

$$\rho(r) c_p(r) = (\rho c_p)_s (1-\epsilon(r)) \quad (3-8-b)$$

where $\epsilon(r)$ is specified by Equation (3-6-a). In the bulk of the bed, Equation (3-8-a) or (3-8-b) is utilized with $\epsilon(r)$ being specified by Equation (3-6-b).

3.4 THEORETICAL HEAT TRANSFER PREDICTIONS

Calculations of the heat transfer coefficient are obtained by solving equations (3-1-d)-(3-5) numerically using the proper equations specifying the properties of the medium in the region of high voidage adjacent to the surface as well as in the bulk of the flow field. Either finite difference or finite element methods can be utilized to

solve the partial differential equation and its boundary conditions. However, Gladwell and Wait [61] reported that the finite element method enjoys the advantages of (1) ease in modelling complex two and three dimensional geometries, and (2) its computation strategies, based on matrix handling procedures, can be highly automated.

In the present study the finite element technique is utilized to solve the governing equations of the present flow field problem.

3.4.1 Finite Element Analysis

The objective of this section is to determine the temperature distribution within a medium of granular material flowing under the action of gravity having a uniform velocity V inside a vertical channel of circular cross section. A constant heat flux in the radial direction is maintained on the surface of the channel over the length L in the axial direction.

Instead of seeking a finite element solution of equation (3-1-d) in the domain shown in Figure 3-1-e, the uniform flow velocity assumption for the interior region will be inserted in equation (3-1-d).

Since the particles move at a uniform constant velocity V , the x coordinate, $x=Vt$, is used as a time scale. Then, equation (3-1-c) becomes

$$\frac{1}{r} \frac{\partial}{\partial r} [k(r) r \frac{\partial T}{\partial r}] + \frac{\partial}{\partial x} [k(x) \frac{\partial T}{\partial x}] = \rho(r) c_p(r) \frac{\partial T}{\partial t} \quad (3-9)$$

The associated boundary conditions are given by equations (3-2)-(3-5). The initial condition is

$$T = T_i \quad (3-10)$$

at time

$$t = 0, \quad 0 < x < L, \quad \text{and} \quad 0 < r < R$$

Consider a disc with an axial thickness Δx . The finite element idealization is shown in Figure 3-1-a, where the disc media is replaced by triangular ring elements. The equivalent variational form of equation (3-9) and the associated boundary and initial condition are given by many authors in the literature. Rao [62] and Segerlind [63] give the functional I as:

$$I = 0.5 \int_V [k(r) r (\partial T / \partial r)^2 + k(x) r (\partial T / \partial x)^2 + 2\rho(r) c_p(r) (\partial T / \partial t) r T] dV + \int_S r q T ds \quad (3-11)$$

The finite element equations are obtained by using the variational approach which minimizes the functional I and gives

$$[C] \partial T / \partial t + [K] \{T\} + \{F\} = 0 \quad (3-12)$$

where $[C]$ is referred to as the capacitance matrix, $[K]$ is the conductance matrix, and $\{F\}$ is the global force vector. They are related to the element capacitance matrix $[C^e]$, element conductance matrix $[K^e]$, and element force vector $\{F^e\}$ by

$$[C] = \sum_e [C^e] \quad (3-13-a)$$

$$[K] = \sum_e [K^e] \quad (3-13-b)$$

$$\{F\} = \sum_e \{F^e\} \quad (3-13-c)$$

and

$$[C^e] = \int_v \rho c_p [N]^T [N] dv \quad (3-14-a)$$

$$[K^e] = \int_v [B]^T [D] [B] dv \quad (3-14-b)$$

$$\{F^e\} = \int_s q [N]^T ds \quad (3-14-c)$$

where $[N]$ is the shape function matrix, $[B]$ is the gradient matrix, and $[D]$ is the material property matrix. In the present analysis, a linear triangular axisymmetric finite element is used for which the shape function matrix $[N]$, and the gradient matrix $[B]$ are given in Appendix B. The material property matrix $[D]$, which is a function of thermal conductivity, is also given in Appendix B. All of the integrals in equations (3-14-a,b,c) are evaluated over a single element. Expressions for matrices $[C^e]$, $[K^e]$, and $\{F^e\}$ are given in Appendix B. Contributions from all the elements in the finite element mesh are assembled to give the overall global matrices $[C]$, $[K]$, and $\{F\}$.

Equation (3-12) represents a system of first order linear differential equations. There are two popular procedures for solving this system of equations to obtain the variation of the values of $\{T\}$ at all nodal points over a period of time. The first procedure is to approximate the time derivative using a central finite difference scheme. The alternate procedure is to use finite elements defined in the time domain [63]. The finite difference technique is used in the present case. The first derivative of the nodal values between two points in time is

$$d\{T\}/dt = 1/\Delta t (\{T\}_1 - \{T\}_o) \quad (3-15)$$

also $\{T\}$ and $\{F\}$ in equation (3-12) are evaluated at the mid-point of the time interval as

$$\{T\} = (\{T\}_1 + \{T\}_o) / 2 \quad (3-16)$$

and

$$\{F\} = (\{F\}_1 + \{F\}_o) / 2 \quad (3-17)$$

Then equation (3-12) gives

$$(2/\Delta t [C] + [K]) \{T\}_1 = (2/\Delta t [C] - [K]) \{T\}_o - (\{F\}_1 + \{F\}_o) \quad (3-18)$$

Equation (3-18) can be solved to yield the nodal values at time $t+\Delta t$ given the nodal values at time t .

A FORTRAN program is listed in Appendix C for the solution of the present heat transfer problem. Subroutines HEATAX, DECOMP, and SOLVE are taken from Rao [62] and modified to solve the transient problem defined by Equation (3-18). Input data regarding element geometry, material properties, and boundary conditions are provided in the main program. In the subroutine HEATAX a loop over all elements is used to construct for each element in turn (1) the element conductance matrix, (2) the element capacitance matrix, and (3) the element force vector. These element matrices are then assembled directly into global conductance and capacitance matrices and a global load

vector. The subroutines DECOMP and SOLVE are then used to solve the resulting system of linear equations

$$[A]\{T\}_j = \{b\}_j \quad (3-19-a)$$

where

$$[A] = (2/\Delta t [C] + [K]) \quad (3-19-b)$$

and

$$\{b\}_j = (2/\Delta t [C] - [K])\{T\}_{j-1} - (\{F\}_j + \{F\}_{j-1}) \quad (3-19-c)$$

The subroutine DECOMP decomposes the matrix $[A]$ into $[U]^T[U]$ and the elements of the upper triangular matrix $[U]$ are stored in the array A itself. The Choleski method [62] is utilized in the subroutine SOLVE to solve equation (3-19-a) by using the decomposed coefficient matrix $[A]$. At a given time step Δt and known $\{T\}_0$ equation (3-18) reduces to the form of (3-19-a) and can be solved to get $\{T\}_1$. Then $\{T\}_1$ values are utilized to solve equation (3-18) for $\{T\}_2$ and so on. Considering a disc of granular media flowing over a heated length L with a uniform velocity V , the total time scale, L/V , is subdivided into a number of time increments, Δt , such that $\Delta x (=V\Delta t)$ is equal to 0.1 mm at any specified flowing velocity. The distance Δx will be taken as the length of the finite element side on the solid boundary as shown in Figure 3-1-a.

In the homogeneous packet theory by Mickley and Fairbanks [11] a local time average heat transfer coefficient, h_x , and a surface mean heat transfer coefficient, h_m , are defined as

$$h_{\chi} = \int_0^{\infty} h_i \psi(\tau) d\tau \quad (3-20)$$

and

$$h_m = 1/A \int_A h_{\chi} dA \quad (3-21)$$

where τ is the time during which a packet has been in contact with the surface, $\psi(\tau)$ is the frequency of occurrence in time of packets having age τ ; That is, over along period of time, the fraction of the total time during which the surface is in contact with packets of ages ranging between τ and $\tau+d\tau$ is $\psi(\tau)d\tau$. The instantaneous packet heat transfer coefficient is given by h_i , and A is the area of the heat transfer surface. For a specified surface heat flux Kubie and Broughton [31] defined the time mean heat transfer coefficient as

$$1/h_{\chi} = 1/\tau \int_0^{\tau} (1/h_i) dt \quad (3-22)$$

The flow situation considered here, which models the actual flow, studied in the experimental phase of the present study, resembles a "slug flow" of a solid past a surface as discussed by Mickley [11]. For such a type of flow the age of all packets is always $\tau = L/V$ and $\psi(\tau)=0$, except for $\psi(L/V)$, which is infinite, so that $\psi(L/V)d\tau = 1$ and the local time average heat transfer coefficient of the present analysis can be defined as

$$h_x = q''/(T_{wx} - T_{mx}) \quad (3-23)$$

where T_{wx} is the wall temperature at x (or at $t=x/V$) and T_{mx} is the mean disc temperature

defined by

$$T_{mx} = 2/R^2 \int_0^R T_x r dr \quad (3-24)$$

and $T_x = T_x(r)$ is the temperature profile at x . The surface mean heat transfer coefficient can be calculated from equation (3-21) or from an equation similar to equation (3-22) by replacing h_x with h_i . The local time average Nusselt number based on bed or gas thermal conductivities and particle mean diameter, d , can be defined as

$$Nux_g = h_x d / k_g \quad (3-25)$$

$$Nux_o = h_x d / k_o \quad (3-26)$$

Similarly, the surface mean Nusselt numbers are given by

$$Num_g = h_m d / k_g \quad (3-27)$$

$$Num_o = h_m d / k_o \quad (3-28)$$

Predictions of the local time-average heat transfer coefficient and surface mean heat transfer coefficient vs axial distance, x , are obtained by the present analysis for flowing beds of copper/air ($k_o/k_g = .531/.026 = 19.73$), glass/carbon dioxide ($k_o/k_g = .100/.016 = 6.27$), and glass/helium ($k_o/k_g = .354/.148 = 2.39$); Results are shown in Figures 3-2 and 3-3. Also, predictions of the present analytical model of local time-averaged and mean surface Nusselt numbers based on effective bed and gas conductivity are shown in Figures 3-4, 3-5, 3-6, and 3-7 respectively.

3.4.2 Comparison With Available Models And Experimental Data

Comparisons of the local time averaged Nusselt number ($h_x d/k_g$) obtained by the present method with (1) the two-particle model [13], (2) the contact resistance model [2], and (3) Mickley's packet model [11] are shown on Figures 3-8 and 3-9. These Figures for copper/air and sand/air beds are taken from Denloye and Botterill [2]. Also shown are experimental measurements of different particle diameters. It can be seen from Figures 3-8 and 3-9 that agreement between the experimental results of [2] and present method is closed.

Figure 3-10 shows predictions of the present method of local time averaged Nusselt number vs Fourier number for a steel shot/air flowing bed. Predictions obtained from Schlunder's approximate solution [19] and Sullivan's correlation [6] are shown for comparison. Numerical solutions of Kubie [31] and a homogeneous continuum model of Mickley [11] are also shown.

In Figures 3-11 and 3-12 predictions of the present method for glass beads/air and copper shot/air beds, along with predictions of Kubie [31] and Mickley [11] are shown. In addition, data by Butt, Hampshire, Smith, Cain, and Desai are shown for comparison in Figures 3-10, 3-11, and 3-12. These three Figures are taken from Kubie [31]. In the range of Fourier numbers for the data, both Kubie [31] and present models are equally valid.

Predictions of the present method for the local time averaged and the surface mean Nusselt numbers of a glass/carbon dioxide bed ($k_o/k_g = 6.27$) are shown in Figure 3-13. Four other different models cited by Gloski [25] are shown for comparison, namely (1) the numerical solution of Chandran, (2) a homogeneous emulsion with surface resistance, (3) a single layer of particles, and (4) Mickley's analysis. At large

Fourier numbers (large residence time) Mickley's provides an upper bound as well as an asymptote approached by the other continuum models including the present model.

Figure 3-14 shows local time-average Nusselt numbers predicted by the present method in comparison with the Gloski [25] model and Kubie's [31] numerical solution. Also, data of Antonishin [31] for moderate to large Fourier numbers cited by Kubie are shown for comparison. The agreement of the three models are approximately the same in the range of Fourier numbers for the data given.

Local time averaged Nusselt numbers obtained by the present method for a sand/helium bed ($k_o/k_g=4.15$) are shown in Figure 3-15. Predictions of Chandran, those of the Gloski [25] model, and experimental measurements are also shown for comparison. Agreement between the experimental results and present method is quite valid.

Surface mean Nusselt number predicted by the present method are compared with data by Dunskey et al. [9] and predictions of the Gloski model [25] in Figure 3-16. It can be seen that the numerical solution of the present method lies reasonably close to the measurements of [9] and Gloski model [25]. Figures 3-13 to 3-16 are taken from Gloski et al. [25].

4. EXPERIMENTAL INVESTIGATIONS

4.1 INTRODUCTION

Measurements of thermophysical properties of five test media chosen for investigation are given in this section. Three of these media, namely steel shot and two sizes of glass beads, consisted of hard spherical particles with a narrow distribution of diameters about the mean. The other two materials are (1) common fine-grained sand of relatively non-uniform size distribution, and (2) Ottawa sand of a more uniform size distribution. Both types of sand are generally comprised of irregularly shaped particles.

The design and construction of the thermal loop test facility used in this study will be described in this chapter. Additionally, measurements obtained of heat transfer coefficients for all test media flowing through the vertical test section are presented in this section. The measurements obtained are compared with theoretical predictions of the method described in the previous chapter as well as with available experimental data and predictions based on other models appearing in the literature.

4.2 THERMOPHYSICAL PROPERTIES MEASUREMENTS

Before conducting any of the test transfer studies per se, measurements were first made of (1) density, (2) void fraction, (3) particle size distribution, (4) thermal conductivity, (5) thermal diffusivity, and (6) specific heat of all the materials selected for this experimental study. These measurements are described in the following sections.

4.2.1 Density

Bulk densities of a stationary bed of each of the granular media were obtained by weighing the known volume of a sample of the material contained in a graduated cylinder. Both critical state density and dense state densities were measured. The critical state is that obtained by capping the end of the graduated cylinder and causing the material to flow to the other end by slowly inverting the cylinder. The dense state is obtained by repeated striking of the cylinder until no further compaction is observed. The values obtained are presented in table 4-1.*

4.2.2 Void Fraction

The bed void fraction, ϵ_o , was evaluated by using the expression

$$\epsilon_o = (V_t - V_s) / V_t \quad (4-1)$$

where V_t and V_s are the total bed and particulate volumes respectively. The volume V_t is measured by means of graduated cylinder, while V_s is measured using a known volume of suitable liquid to fill the spaces between the particles within the bed. Void fractions for both the critical and dense states were measured. The values obtained are presented in table 4-1.

4.2.3 Particle Size Distribution

The mean particle size of each bed material was measured using a set of standard sized screens. Screen mesh numbers ranged from 8 (2362 microns) to 200 (74

* All tables may be found in appendix D.

microns). To estimate the mean particle diameter, numerous mean diameter expressions have been defined in the literature depending upon the use to which the material is being adapted. Zenz and Othmer [59] presented a summary of the defining equations of the arithmetic, geometric, logarithmic, harmonic mean, and median diameters. Also, in [59] defining expressions for the length, surface, volume, weight and volume-surface mean diameters are given. The volume-surface mean diameter equation has been suggested by Kunii and Levenspiel [64] for situations in which the sieving method is utilized for determining the particle size and distribution. Other investigators [65,66] have recommended its use in other applications also. In the present study the mean particle diameter was calculated from the following equation for the volume-surface mean:

$$d = 1 / \sum_i (w/d)_i \tag{4-2}$$

where w_i is the weight fraction in a given size interval and d_i is the diameter of the particles in a particular size range. The volume-surface diameter defines the total surface of a unit weight of a granular media of mixed particle size. Figures 4-1-a to 4-1-e illustrate the measured results. The values of the mean particle diameter obtained are also presented in table 4-1.

4.2.4 Thermal Conductivity

The line source technique, described and utilized in [54], was employed to measure thermal conductivity values of the porous media employed in the present tests because of its relative simplicity and speed. A similar apparatus was designed and fabricated and used in the present study; It is illustrated schematically in Figure 4-2.

The test media were poured into a container of rectangular parallelepiped shape, 15

cm by 15 cm by 20 cm. A line source of heat was generated within the test sample by passing 400 ma of current through a 0.31 mm manganin heater wire, using a d-c power supply (HP 6216 A model). A chromel-alumel thermocouple of 30 gauge was wrapped about the heater wire and connected to a potentiometric chart recorder (Tekman Electronics Ltd., TE 220 series) for recording the transient temperature behaviour.

The transient temperature rise within an infinitely long thin heat source embedded in an infinite homogeneous medium initially at equilibrium is given by [54] and [40] as

$$T = q' / 2\pi k_o \int_{\beta}^{\infty} [\text{Exp}(-y^2) / y] dy \quad (4-3)$$

where $\beta^2 = r_o^2 / 4\alpha t$, k_o = thermal conductivity of the medium, q' = heat input per unit length of the wire, and y is a dummy variable. The integral expression of Equation (4-3) may be evaluated by an infinite series, so that the temperature is given by

$$T = q' / 2\pi k_o [-v/2 - \ln\beta + \beta^2 / 2 (1!) - \beta^4 / 4 (2!)..] \quad (4-4)$$

where v = Euler's constant = 0.5772156. Since $(r_o^2 / 4\alpha t)$ is small (< 0.0256) the temperature change between two times t_1 and t_2 may be approximated by

$$T_2 - T_1 = (q' / 4\pi k_o) \ln(t_2/t_1) \quad (4-5)$$

Thus a plot of temperature rise versus the logarithm of time gives a straight line with a slope of $q'/4\pi k_o$. Knowing q' and the slope, k_o may be calculated as

$$k_o = q' / (4\pi (\text{slope})) \quad (4-6)$$

Figures 4-3-a to 4-3-e show the recorded temperature-time history of the heater wire when imbedded within a sample of each of the test media. A plot of the temperature rise versus the logarithm of time for the five chosen granular materials are shown in Figure 4-4. Figure 4-5, taken from Kuzay [48] shows a comparison of the thermal conductivity values obtained here with some of the existing correlations and available data at a porosity of $\epsilon_o=0.40$. Also, values calculated from the Willhite and Kunii [44] correlation, Equation (3-7), are shown in Figure 4-5. The values obtained are also presented in Table 4-1.

4.2.5 Thermal Diffusivity

Measurements of thermal diffusivity, α , were made using the method described by Waddams [41] and Brinn, et al [8], in which the transient heating of a cylinder of granular media of radius R_o and length $2L$ ($2L > 4R_o$) at initial temperature T_i is measured.

The apparatus is shown in Figure 4-6. One 30-gauge chromel-alumel thermocouple was located on the tube surface to measure the surface temperature T_s , and another at the center of the cylinder to measure the variation of the center line temperature, T_c , with time t . Atmospheric steam was admitted into the jacket surrounding the test medium using a Scott boiler, model no. 9058. The temperature-time history of the center line of each of the test media was recorded by a Tekman Electronics TE 220 series potentiometric chart recorder as shown in Figures 4-7-a to 4-7-e.

The theoretical equation relating $\alpha t/R_o^2$ and $(T_s-T_c)/(T_s-T_i)$ is given by Waddams [41]

$$\frac{T_s - T_c}{T_s - T_i} = \frac{8}{\pi} \frac{[\sum_i \text{Exp}(-\alpha \zeta_i^2 t / R_o^2) / \zeta_i J_1(\zeta_i)]}{[\sum_{\text{odd } j} \text{Exp}(\alpha j^2 \pi^2 t / 4L^2) (-1)^{(j-1)/2} / j]} \quad (4-7)$$

where $J_1(\zeta_i)$ is the Bessel function of the first order and ζ_i is the i th root of equation $J_0(\zeta)=0$ and j and i are positive integers. Equation (4-7) is shown plotted in Figure 4-8, from which the slope is determined to have a value of -5.72. Plots of $(T_s - T_c)/(T_s - T_i)$ versus time, t , for the five testing materials are shown in Figure 4-9. The negative slope of each curve (straight line part) is set equal to $5.72(\alpha/R_o^2)$, so that α may be calculated from

$$\alpha = (\text{slope}) R_o^2 / 5.72 \quad (4-8)$$

The values obtained are presented in table 4-1.

4.2.6 Specific Heat

Specific heat values may be obtained from the measured values of thermal diffusivity α , thermal conductivity k_o , and bulk density ρ from the expression

$$c_p = k_o / \rho \alpha \quad (4-9)$$

It was particularly necessary to know the specific heat accurately, because this quantity is used to calculate the mean temperature of the granular material at different stations in the flow field.

The specific heats were measured by utilizing Thermonetics Seebeck Envelop Calorimeter (SEC) C-11 120-5-MC; No 101 model. The principle of the SEC is

based on the fact that all of the heat flow in the calorimeter must pass through its walls, where temperature gradient sensors are located. Thus the calorimeter envelope integrates the total heat flow in the system on either an instantaneous or long term basis. The calorimeter was calibrated to relate the heat flow (watts) to the calorimeter millivolt output signal by a calibration constant, C_c determined by accurately measuring the calorimeter millivolt output signal, e , resulting when a constant and measured (I^2R) heating rate, q_c , is released in the calorimeter using the calibration heater supplied with the calorimeter. The constant is given by the equation

$$q_c = C_c e \quad (4-10)$$

To measure the specific heat the unloaded calorimeter that has come to thermal equilibrium (zero millivolt output) at a temperature level T_1 , is raised to a different temperature level, T_2 . The heat flow versus time trace as the transient process proceeds from the initial steady state condition to the final steady state condition was obtained with an Omega millivolt recording potentiometer (Model No. 595, Three Channel). The area under this curve is equal to the heat added to the calorimeter and its empty specimen container for the superimposed temperature perturbation. Next, the calorimeter is loaded with the granular material sample to be investigated and again exposed to the original temperature level, T_1 and allowed to come to equilibrium. Then the loaded SEC is exposed to the temperature level, T_2 , allowed to reach equilibrium again, and the corresponding transient heat flow-time trace recorded. These curves are shown in Figures 4-10-a to 4-10-e. The area under a given curve is equal to the heat added to the calorimeter, its specimen container and the sample. Subtraction of the areas under the two transient traces yields the desired heat added to the sample. The specific heat, c_p , is determined by substituting the measured heat given to the sample, q , the sample

weight, m , and the temperature difference for the thermal perturbation, T_2-T_1 , into the equation

$$q = mc_p(T_2-T_1) \quad (4-11)$$

The results obtained for $T_1=50\text{ }^\circ\text{C}$ and $T_2=150\text{ }^\circ\text{C}$ are summarized in Table 4-1. Also the percent difference between the specific heat values measured and those calculated from Equation (4-9) are given in Table 4-1.

4.3 HEAT TRANSFER MEASUREMENTS

4.3.1 *Experimental Apparatus*

The flow apparatus, shown schematically in Figure 4-11, consists of five basic parts, from top to bottom: the suction fan with suitable size storage tank; the supply hopper; the test section; the regulating valve; and the heat exchanger.

The particles flow by gravity from the supply hopper through an entrance region 32 cm in length before entering the test section. The supply hopper has a considerably larger volume than the test section and enables continuous flow to persist in the vertical tube until steady state heating conditions are attained.

The test section is a stainless steel tube, 27 cm long with a 1.44 cm inside diameter and .07 cm wall thickness. Heat was added to the test section by an insulated electric heating element wrapped about the outside surface of the flow channel so as to maintain a uniform heat flux condition. The heating element was fabricated with nichrome wire and had a resistance of 41.2 ohms. A 1-140 volt variable transformer

was connected in parallel with the heater and a constant A.C. power supply. The test section and the entrance region were thermally insulated from the outside environment by a cylindrical composite wall consisting of an outer layer of fiber glass and an inner layer of high temperature organic impregnated fibrous glass. The entrance of the tube is mounted in a sleeve to allow the test section to expand freely within 5 mm in the vertical direction when it is heated. The test section is shown in Figure 4-12.

The particle flow rate through the test section was controlled by means of a sliding cone valve assembly located at the lower end of the tube exit region. The particles were continuously cooled through a high aspect-ratio rectangular cross-section heat exchanger and returned to the upper storage tank by a large suction fan located on the top of the tank.

The system as described is capable of maintaining a continuous velocity in the channel between approximately 2 and 23 cm per second, the exact limits depending on the particular material used in the flow. The lower limit on the velocity results from the inability of the flow valve to pass a steady flow without clogging at very low flow rates; The upper limit results from the difficulty in maintaining the maximum flow rate for a period of time long enough to reach steady state.

4.3.2 Measurement Techniques

The wall temperature distribution along the flow direction was measured by means of 30-gauge chromel-alumel thermocouples that were spot welded onto the outer tube surface. The heat losses in the axial and radial directions were estimated by measuring the tube surface temperature 1.5 cm before the beginning of the heated section and at the outer edge of the insulating material. A thermocouple that could be moved radially into the flowing medium was used to measure the particle inlet bulk temperature

at the inlet of the test section. These temperatures were recorded on Omega (three-channel) and Tekman (two-channel) chart recorders.

A standard voltmeter and ammeter were used to measure the AC voltage across the heating element and the current passing through it. This measurement, coupled with the known heater resistance was used to calculate the power consumed by the heater. The power input to the test section was calculated by subtracting the losses in the axial and the radial directions. The percentage losses were determined to vary between 1.5 percent to 3.5 percent.

The radial temperature distribution of the particles at the test section exit was measured by means of a chromel-alumel thermocouple wire mounted on a micrometer assembly placed upstream of the flow control valve. The thermocouple output was recorded on Tekman chart recorder. Figures 4-13-a to 4-13-c show an output chart of surface, inlet, and exit temperature of one run of glass beads of mean diameter 2.07 mm.

The mass flow rate through the channel was determined by weighing the net mass output from the channel in a fixed time interval. The mass flow rate could be converted to velocity, knowing the density of the flowing material and the known channel cross-sectional area. This process was repeated so that an average value of the velocity was determined.

The wall temperature distribution, power input, particle inlet temperature, and flow rate were used to determine the local and average heat transfer coefficients along the test section.

4.3.3 Experimental Procedure

Initially the complete inventory of particles is placed in the supply hopper. The heat exchanger cooling water is turned on, the suction fan is started, and the control

valve adjusted to its fully open position. After initiation of the flow, the power supply is turned on and the variable transformer adjusted to obtain the desired power input to the heating element (100 volts, corresponding ammeter reading 2.2 am). The particle radial exit temperature profile probe is initially placed flush with the tube wall and the axial wall temperatures monitored until steady state conditions are reached. The exit temperature profile is then measured by moving the micrometer-mounted probe at approximately 1 mm radial intervals. The flow rate for these conditions is then measured. The flow control valve is then changed to the next lower flow rate and the temperature balancing process repeated. A total of seven to ten flow rates were examined for the same material. The loop was thoroughly cleaned before the next test material was studied.

4.4 EXPERIMENTAL RESULTS

The mean temperature distribution, T_{mx} , of the granular media along the flow direction has been calculated by an energy balance over the test section using the known power input, granular flow rate, and inlet temperature. The local heat transfer coefficient, h_x , at specific axial location was then calculated using equations (4-12) and (4-13)

$$q'' = Q / 2\pi RL \quad (4-12)$$

$$h_x = q'' / (T_{wx} - T_{mx}) \quad (4-13)$$

where R is the inside radius of the tube, L its length, and Q is the net power input to the tube (equal to the electric power input minus the losses in the radial and the axial directions). The losses in the radial direction were estimated by knowing the thermal

conductivity and the temperature at the inner and outer surfaces of the insulating material. Similarly the losses in the axial direction were calculated knowing the tube surface temperature at different locations and its thermal conductivity.

An analysis of the present experimental data to estimate errors, precision, and the general validity of experimental measurements is given in Appendix F. The approach used was to perform an uncertainty analysis [67] similar to that described and used by J.P.Holman [68] and Keshock [54]. The summary of the results of this analysis is given in Table 4-2 in appendix D.

Figure 4-14 shows the measured local heat transfer coefficient along the axial direction of the test section when each of the five media flows at an approximate mean velocity 0.07 m/s. The smaller size particles of glass beads and local sand, have higher values of local heat transfer coefficient than the larger particle media.

The overall average heat transfer coefficient is calculated using the logarithmic mean temperature difference between the wall and the granular mean temperatures at inlet and exit of the test section.

Figures 4-15-a and 4-15-b show the effect of particle size and granular bulk velocity on the average heat transfer coefficient. The average heat transfer coefficient increases with decreasing particle size and with increasing average bulk velocity.

Local and mean Nusselt numbers based on particle mean diameter and bed or gas conductivity are calculated utilizing equations (3-25) to (3-28). Measurements of Nusselt number, $h_m d/k_g$, of the present study for the five media tested are compared with experimental data and the theoretical correlation obtained by Sullivan and Sabersky [6] in Figure 4-16. Sullivan and Sabersky's correlation resulted in an expression relating the Nusselt number to a function of Peclet number, given in equation (4-14-a)

$$Nu^* = \frac{1}{0.085 + 0.5 (\pi/Pe_L^*)^{0.5}} \quad (4-14-a)$$

where

$$Nu^* = h_m d / k_g \quad (4-14-b)$$

$$Pe_L^* = (k_o / k_g)^2 (d/L)^2 Pe_L \quad (4-14-c)$$

$$Pe_L = VL / \alpha \quad (4-14-d)$$

Although their Nusselt-Peclet correlation curve is seen to pass reasonably close to most of their data points, Figure 4-16 appears to reveal an inability of their model to successfully correlate data for a variety of heat transfer media.

The average heat transfer coefficient data obtained in the present study for local sand and Ottawa sand has been placed into the form of Nusselt numbers and are presented in Figure 4-17, superimposed over the results obtained for sand/air experiments by Denloye and Botterill [2]. Their results are compared with a modified Mickley-Fairbanks model (additional resistance to heat transfer at the transfer surface is included). It is seen that the present data fall reasonably close to the modified Mickley model predictions. Predictions of the present theoretical method for sand/air flowing media are shown for comparison.

Also Denloye and Botterill correlated the maximum value of the flowing bed coefficient by plotting the corresponding Nusselt number, $(h_{\max} d / k_g)$, against the Archimedes number. Their correlation expressed by equation (4-15)

$$h_{\max} d / k_g = 1.283 Ar^{0.162} \quad 10^3 < Ar < 10^6 \quad (4-15)$$

is shown in Figure 4-18. The maximum average heat transfer coefficient obtained for the test media in the present study are shown for comparison. Agreement between correlation and measurements is close.

Another comparison has also been made with the predictions of the Kubie and Broughton [31]. In their model they attempted to improve upon the Mickley-Fairbanks model by taking into account the variation of thermal properties within a fluid packet in the region of the surface. Their numerical predictions for glass/air and steel shot/air media are shown in Figures 4-19 and 4-20. The present test results are compared with those predicted by Kubie-Broughton and Mickley-Fairbanks. Also, predictions of the present analytical method and Schlunder correlation [19] for glass/air flowing media are shown in Figure 4-19. In addition, data by Butt, Hampshire, Smith, Cain, and Desai [1] are shown for comparison. Predictions of the present analytical method for a steel shot/air bed as well as predictions of Schlunder's approximate solution [19] and Sullivan's [6] correlation are shown in Figure 4-20 for comparison.

5. DISCUSSION OF RESULTS

The experiments were carried out with media having the range of particle sizes given in table 4-1 (approximately from 0.26 to 2.07 mm) using air as the stagnant fluid medium in the flowing packed bed. Bed flow velocities from 0.02 to 0.23 m/s, giving packet residence times at the heat transfer surface between 1 and 13.5 second.

The effective thermal conductivity, k_o , was measured for the static packed bed and compared with some of the existing correlations and available data in Figure 4-5. It was not possible to know or measure the mean voidage of the granular media while flowing in order to estimate the effective thermal conductivity and bulk density. To elaborate on this matter, all of the solid particles except those adjacent to the wall (see visual results of [8]) are falling vertically under the action of the force of gravity. The particles are not being subjected to a co-current or countercurrent injection of a gaseous phase that would tend to disperse, entrain and "fluidize" the particulate media. Thus, the static packed bed values was used. However, the type of flow in the present study is considered to be a contact dominated flow, in which the adjacent material particles are in physical contact with each other, while the interstitial fluid moves with the particles passively. Thus it was anticipated that utilizing the static packed bed values of effective thermal conductivity and bulk density would be reasonably accurate for the contact dominated flow occurring within the heated flow channels.

In addition, Willhite et al. [44] have indicated that their proposed equation for the effective thermal conductivity, based upon experimental results, satisfactorily represented heat transfer for flowing gas conditions. They reported that their measured effective

thermal conductivity, when the direction of heat flow is perpendicular to the fluid motion, was found to be independent of fluid flow rate for a modified Reynold's number (dG/μ_g where G is the mass flux flow rate) range from 0 to 6.6. Furthermore, they reported that the effect of any turbulence in the fluid is negligible. This equation is also valid in the region of high voidage as reported by Kubie [31] and Botterill [17]. Similar equations have also been utilized by Botterill [2], Colakyan and Levenspiel [65] in their heat transfer analysis of flowing packed beds.

The specific heats of the granular test media were measured directly by utilizing a Thermochemical Seebach Envelope calorimeter. Calculated values were also obtained using the measured values of thermal diffusivity, thermal conductivity, and bulk density and Equation (4-9). The percentage difference of the specific heat values obtained by the two methods was less than 5% for the all media tested except for Ottawa sand, where it was 20%. An average value of the specific heat of Ottawa sand and measured values of the other materials were utilized in the present study.

Figure 4-15-a and 4-15-b display the average heat transfer coefficients versus linear velocity for 0.26 mm and 0.71 mm sand particles and 0.69 mm and 2.06 mm glass beads spheres. As seen from both Figures, the average heat transfer coefficients for each size of particles increase with increasing velocity (an accompanying decrease in the particle residence time) and approached limiting maximum values. The rate of increase, however, is greater at lower solids velocities than at higher velocities. For the large 0.71 mm Ottawa sand and 2.07 mm glass beads, the heat transfer coefficient increases slower with increasing velocity. It can also be noted that, for the same linear granular media velocity, lower heat transfer coefficients are obtained with the large size particles. The same trend, i.e., lower heat transfer coefficient for increasing particle size, and higher heat transfer coefficient for increasing solids velocity have also been observed by several other investigators [1, 2, 6, 14, 20, 65].

Figures 4-14 and 4-15-b show local and average heat transfer coefficients for 1.09 mm steel shot. In previous studies of Botterill et al. [13], Botterill and Desai [1], and Harakas and Beatty [10], it was found that flowing packed beds of metals (copper-steel) gave higher heat transfer coefficients. Because of the limited kinds of test media used here, the trend observed is not especially clear; Only the data of 1.09 mm steel shot can be compared here to that of 0.69 mm glass beads (i.e. of smaller diameter) in Figure 4-15-b. Also it was noticed that the steel shot utilized in the present study was covered with an oxidized microlayer on the outer surface of the individual particles (pellets), which may significantly affect the thermal conductivity of the bed. Even with these considerations, Figure 4-15-b displays higher values of the heat transfer coefficient for steel shot than that of 0.69 glass beads for a bed velocity up to 0.06 m/s.

It is clear from the data plots of Figures 4-16, 4-17, 4-19, and 4-20 that existing methods of predicting heat transfer coefficients in the channel flow of particulate media are not sufficiently accurate. One of the intentions of the present study was to obtain heat transfer data for a wide variety of particulate media in order to test the existing predictive correlations. The results, plotted using the suggested correlation developed by Sullivan and Sabersky [6], appears to be that their proposed correlation is not sufficiently accurate.

Reasons for such inadequacy obviously lie in the simplification made in modelling the actual heat transfer and flow processes. For example, the ordered, square array of particles in the Sullivan and Sabersky model [6] is an idealization that becomes more unrealistic for (a) irregularly shaped particles, and (b) particles having a wide distribution of sizes. Also, the equivalent gas layer thickness of 1/10 of a particle diameter used to properly account for conduction through the gaseous phase is of empirical origin, suggesting an inadequacy in the modelling of the actual gaseous phase conduction process.

In order to investigate the effect of the thermophysical properties of the gas and the solid of flowing granular media; Predictions of the local time-averaged heat transfer coefficients and surface mean heat transfer coefficients obtained by the present analysis for flowing beds of copper/air, glass/carbon dioxide, and glass/helium are shown in Figures 3-2 and 3-3. The effect of the interstitial gas thermal conductivity on the heat transfer rate can be determined by comparing the relative heat transfer coefficients of the glass/helium and glass/carbon dioxide beds shown in Figures 3-2 and 3-3. It is clear that beds of higher gas thermal conductivities gives higher heat transfer coefficients.

Also the effect of the thermal properties of the solid on the heat transfer of the system is noticed from comparing the relative heat transfer coefficients of the copper/air and glass/helium beds. Even though the helium is of higher thermal conductivity than the air, the copper/air flowing bed gives higher heat transfer coefficients than the glass/helium bed. This illustrates that the solid particles of high thermal conductivity constitute the dominant contribution to the heat transfer coefficients.

The effect of gas thermal conductivity and solid thermal properties cannot be directly noticed if the predictions of the heat transfer coefficients are plotted in terms of Nusselt number (based on bed or gas effective thermal conductivities) as shown in Figures 3-4 to 3-7.

The predictions for heat transfer coefficients by the present model (for flowing copper/air and sand/air beds) are compared with (1) the two-particle model [13], (2) the contact resistance model [2], and (3) Mickleys packet model [11] on Figures 3-8 and 3-9. Also in these Figures are shown experimental results for copper/air and sand/air flowing beds [2]. it can be seen from Figure 3-8 that the two-particle model overpredicts the coefficients of heat transfer for t/d^2 values less than $2 \times 10^7 \text{ s/m}^2$ and under predicts the coefficients for values greater than this. By introducing a gas gap of thickness equal to 10% of the particle diameter the contact resistance model reduces the discrepancy at short

resistance times but increases it at longer ones. Finally, the predictions of the packet model for copper/air and sand/air systems overestimate the experimental results. It can be seen from Figures 3-8 and 3-9 that the agreement between the present model and the experimental results is quite good.

The correlations of Sullivan and Sabersky [6], Schlunder [19] and the predictions of Mickley's method [11] and Kubie numerical model [31] are compared with Nusselt number predictions of the present model for a steel/air system in Figure 3-10. The present method's predictions and the schlunder correlation are in good agreement with the experimental data of Butt and Hampshier (presented by Kubie [31]). While the Sullivan correlation underpredicts the experimental heat transfer coefficient data, Mickley's model [11] overpredicts the values.

The predictions of the present method for glass/air and copper/air flowing beds are compared with experimental data cited by Kubie [31] in Figures 3-11, 3-12, and 3-14. Over the range of Forier numbers for the experimental data, both the Kubie [31] and present models compare with the data equally well.

Figure 3-13 compares four models of the wall-to-bed heat transfer. The Mickley [11] homogeneous model provides an upper bound as well as an asymptote approached by the other models at large residence time (Fourier number greater than 10). Chandran's predictions [25] are almost the same as those of the present method for the local heat transfer coefficient. Gloski's [25] method gives constant Nusselt number for a value of Fourier number less than about 0.02.

A test of the present model is made by comparing its oredictions of local Nusselt numbers with the experimental measurements of Gloski [25], which span the range of Fourier numbers from 0.5 down to about 0.001. The comparison is shown in Figure 3-15, where the Gloski [25] and Chandran [25] models are also shown for comparison. The predictions of the present model are in very good agreement with the experimental

data over this range of small values of Fourier number. Results at low Fourier number levels provide another indication of the validity of the method. The very good agreement of the present method's predictions with data at low Fourier numbers, as shown in Figure 3-15, is a clear indication of the validity of the proposed model. It was reported by Gloski [25] that at high Fourier numbers it is impossible to properly evaluate the relative accuracy of the various models, since they all yield similar results.

Predictions of the mean Nusselt number by the present analytical method are shown in Figures 4-17, 4-19 and 4-20 for sand/air, glass/air and steel/air flowing beds. The prediction is shown in comparison with the experimental measurements of the present study. The minimum and maximum percentage differences between the experimental measurements and theoretical predictions are -1% and -39% for local sand, 0% and 0% for Ottawa sand, 0% and -8% for small size glass beads, 15% and 32% for glass beads of large size and -3% and -28% for steel shot. The percentage difference is defined by $((N_{ex}-N_{th})/0.5(N_{ex}+N_{th})) \times 100$ where N_{ex} and N_{th} are the experimental and theoretical Nusselt numbers. Quite good agreement between predicted and experimental values are obtained for Ottawa sand ($d=0.71$ mm) and for glass beads ($d=0.69$ mm). The maximum percentage difference between the measured and theoretically predicted values of Nusselt number for steel/air flowing media used in the present study is 28%. The low measured values might be due to an oxide film on the surface of the particles as mentioned previously.

For the local sand measurements ($d=0.26$ mm) the heat transfer coefficient values are a maximum of 39% below the theoretical predictions. For the glass beads of mean diameter 2.07 mm, the measurements are above the theoretical predictions a maximum of 32%.

In the case of local sand the higher heat transfer predictions (compared to the experimental values) might be due to utilizing the measured mean voidage of the bed in

the static state in the Willhite [44] equation to estimate the effective thermal conductivity and density of the flowing bed. Perhaps the mean voidage under flowing conditions may be of greater importance for local sand, whose particle size distribution somewhat different from the other media in that it is very nearly uniform over a wide range of particle diameters. This larger range of size distribution may result in greater relative "packing" in a static state as compared with the flowing state.

In the case of large diameter glass beads (2.07 mm) the predictions are consistently lower than the measured values. This may suggest that near the wall region significant particle rotation could occur (as compared with small particle diameters) so that radial mixing could thereby be generated. This effect seems even more likely considering that the diameter of the heated tube is only 14.4 mm compared to the particle diameter of 2.07 mm.

In a recent study by Colakyan and Levenspiel [65] a model for heat transfer between moving bed of solids and immersed cylinders is proposed, and a predictive correlation is given. Their correlation is also compared with the experimental measurements of the present study. A maximum percentage difference of about 65% above the measured values is seen for local sand, Ottawa sand, small glass beads, and large glass beads. For the steel shot data their correlation is 104% above the measured values.

Figure 4-18 shows comparison of the measured maximum heat transfer coefficient with a correlation given by Botterill [2]. Agreement between measured values and the correlation is quite good.

From a broader point of view, however, it is very satisfying to see the existence of good agreement between the present theory and the controlled residence time data available in the literature. The previous models of heat transfer in flowing granular media introduced some form of semi-empirical approximation (such as gas-film) to produce acceptable agreement with experimental data. The model developed here uses no assumed

film properties or empirical approximation. The numerical solution of the present analytical analysis is straight forward and simple enough to be easily utilized.

6. CONCLUSIONS AND RECOMMENDATIONS

A heat transfer loop was constructed and used to determine the local and average heat transfer coefficient of gravity flowing granular media as a function of the average flowing velocity and particle size for a constant heat flux heating condition. The results obtained were compared with predictions of various models appearing in the literature. The correlation of Sullivan and Sabersky [6] in the form of a modified Nusselt-Peclet number relationship, was found to be inadequate in predicting all of the results obtained in the present study. Also, the recent correlation of Colakyan and Levenspiel [65], which assumes the existence of a gas layer between the heat transfer surface and the first row of particles, failed to correlate the present data. Therefore, an improved model was developed as part of the present study.

The model obtained was shown to agree well with the data of controlled residence time of flowing media available in the literature. Also, the theoretical model successfully correlates data measured well into the low Fourier number range. (It was reported by Gloski [25] that successful predictions in the low Fourier number range is a true test for a model.)

The predictions of the present model have also been found to agree well with the numerical method predictions of Kubie and Broughton [31] as well as those of Chandran and Chen [34]. However, the present model is simple to use by comparison and can be easily applied to different flow geometries and heating conditions.

Finally, it is important to note that in the development of the model, no physically unjustified concepts (as the gas layer of $1/10$ of particle diameter) are required. The

basic intent of the model was to modify the packet theory of heat transfer by accounting for the presence of the wall and its effect on the local voidage by introducing a realistic variation in the thermophysical properties of media adjacent to the wall. In effect, the model acknowledges and accounts for the particle behavior immediately adjacent to the heat transfer surface being the key factor in bed-to-surface heat transfer.

To develop the model further it would be particularly valuable to have more information about possible particle rotation and induced radial mixing in the near wall region, in addition to that radial mixing that may occur elsewhere within the bulk of the bed. Also, information about the voidage of a flowing bed and its relation to thermal transport properties appears to be a desirable goal for future research.

Additional experiments over a wide range of variables may also be desirable. Among these variables are particle size distribution and sphericity (wide and narrow particle size distribution), thermophysical properties of the solid particles, tube diameter and geometry, heating conditions (e.g. uniform wall temperature, linear variation of heat flux, etc.) Also, the effect of thermal radiation at high heat flux levels and the effect of different pressure levels of the gas-solid system should be investigated.

Finally, development of a generalized Nusselt-type heat transfer coefficient correlation for gravity flowing particle beds should be a major future goal.

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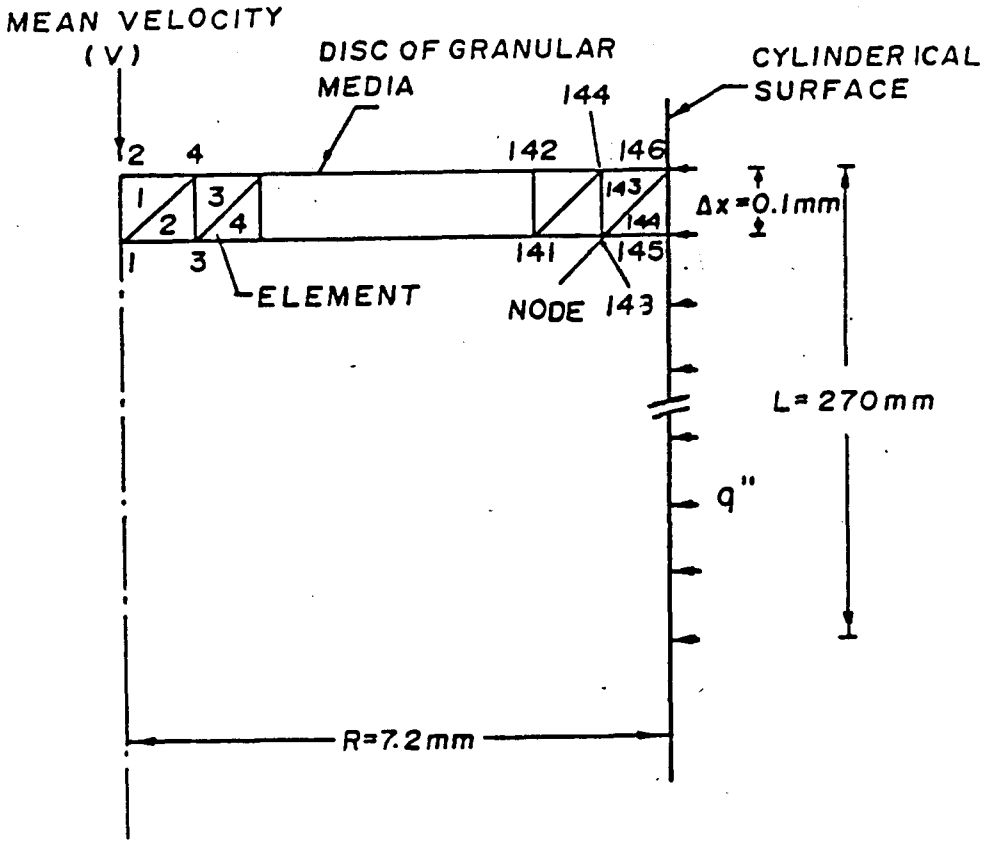
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APPENDIXES

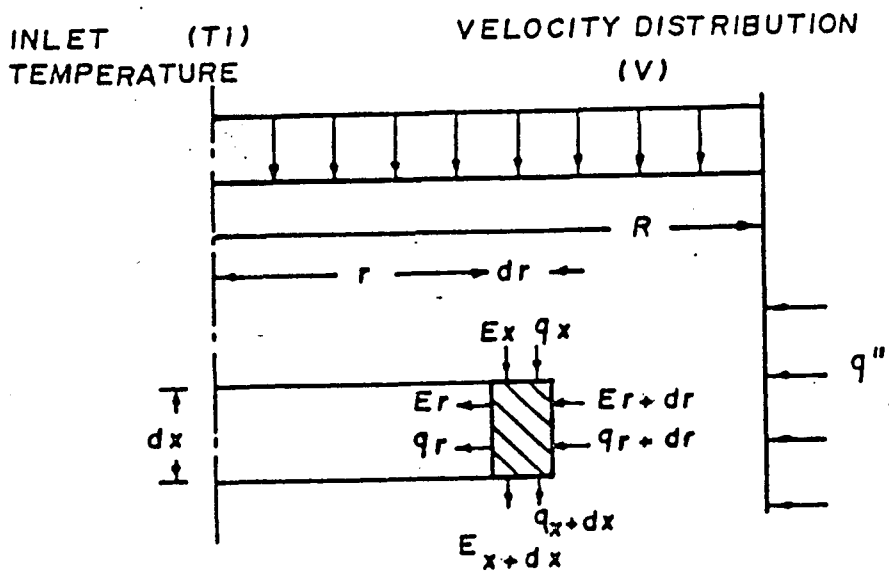
APPENDIX A

FIGURES



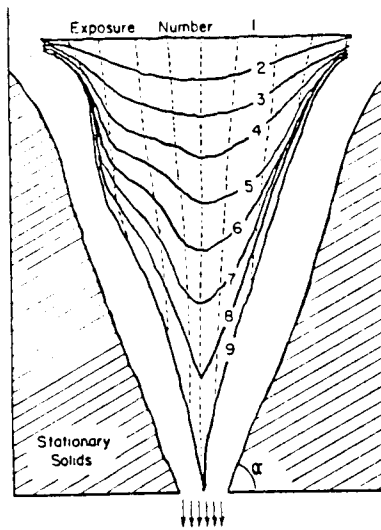
- a. Disc of the granular media flowing in contact with the hot surface.

Figure 3-1. Gravity flowing particulate media in a vertical tube.



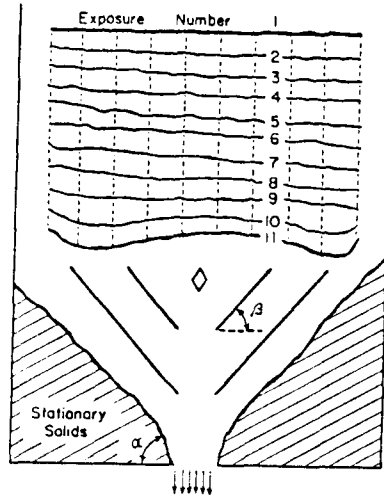
- b. Illustration of heat balance used for driving the energy equation.

Figure 3-1. (continued)



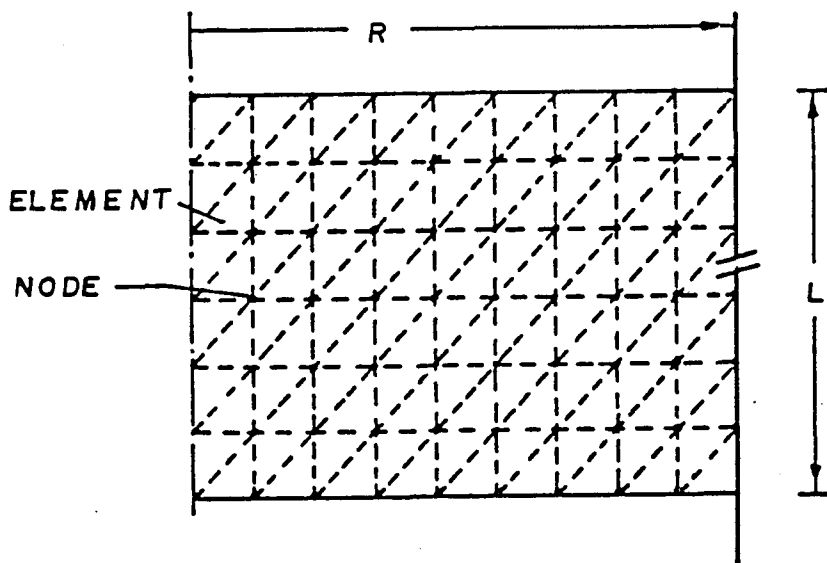
- c. Sketch of superimposed consecutive photographs of bin during solids discharge showing flow paths and profiles.

Figure 3-1. (continued)



- d. Sketch of superimposed consecutive photographs of discharge from baffled bin showing solids flow paths and profiles.

Figure 3-1. (continued)



e. Division of the domain into elements.

Figure 3-1. (continued)

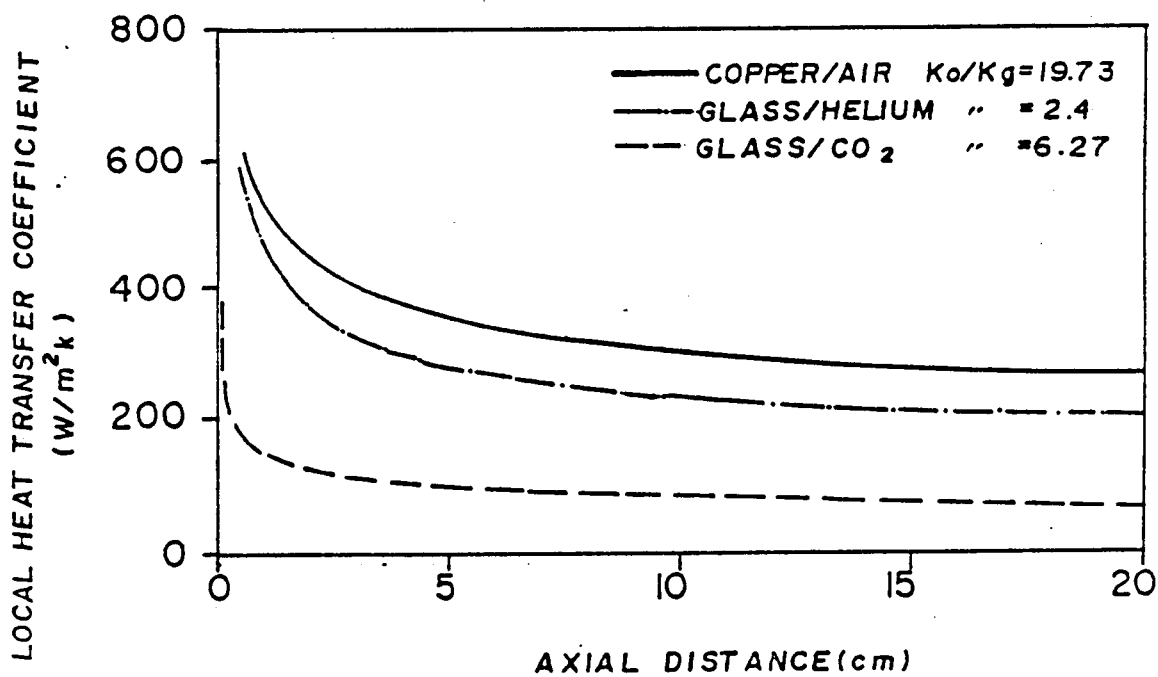


Figure 3-2. Local time-average heat transfer coefficient vs. axial distance.

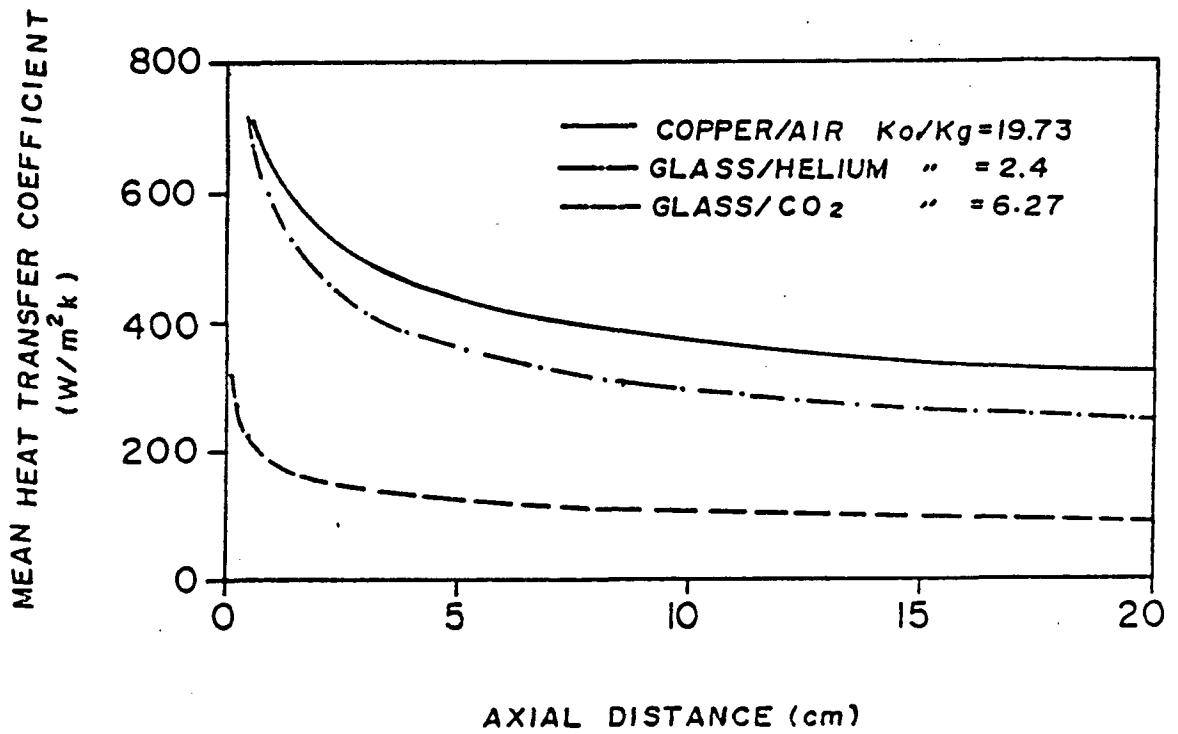


Figure 3-3. Mean heat transfer coefficient vs. axial distance.

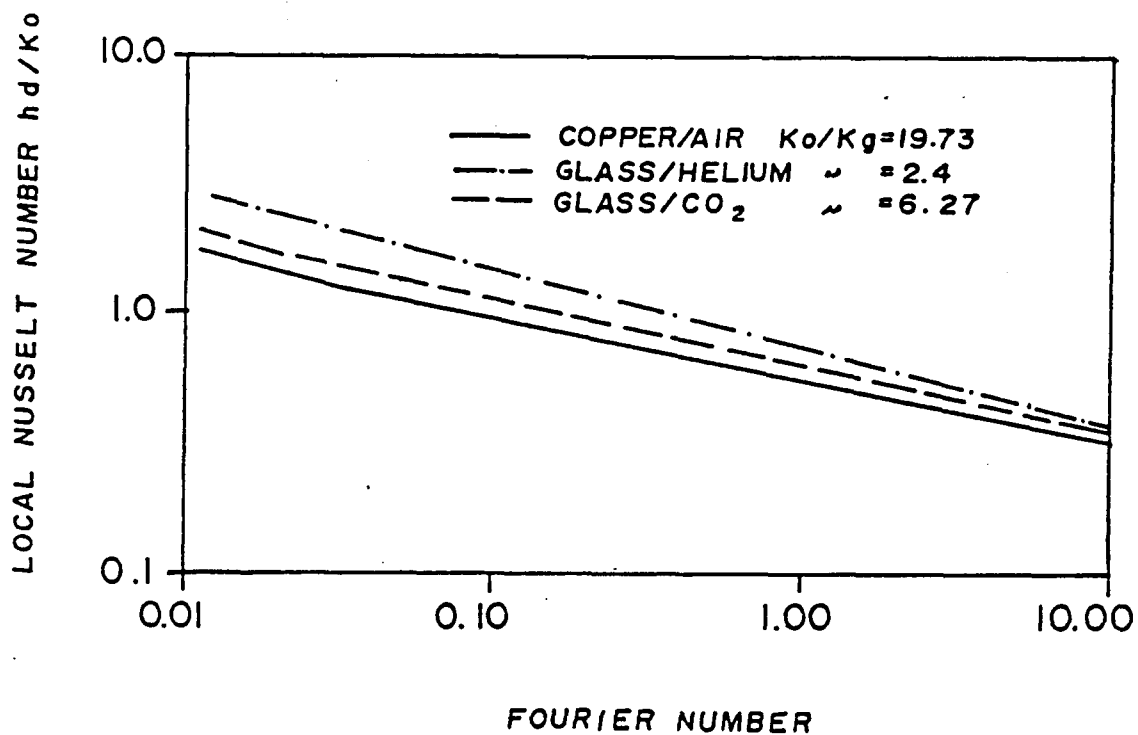


Figure 3-4. Local time-average Nusselt number based on bed conductivity vs. Fourier number.

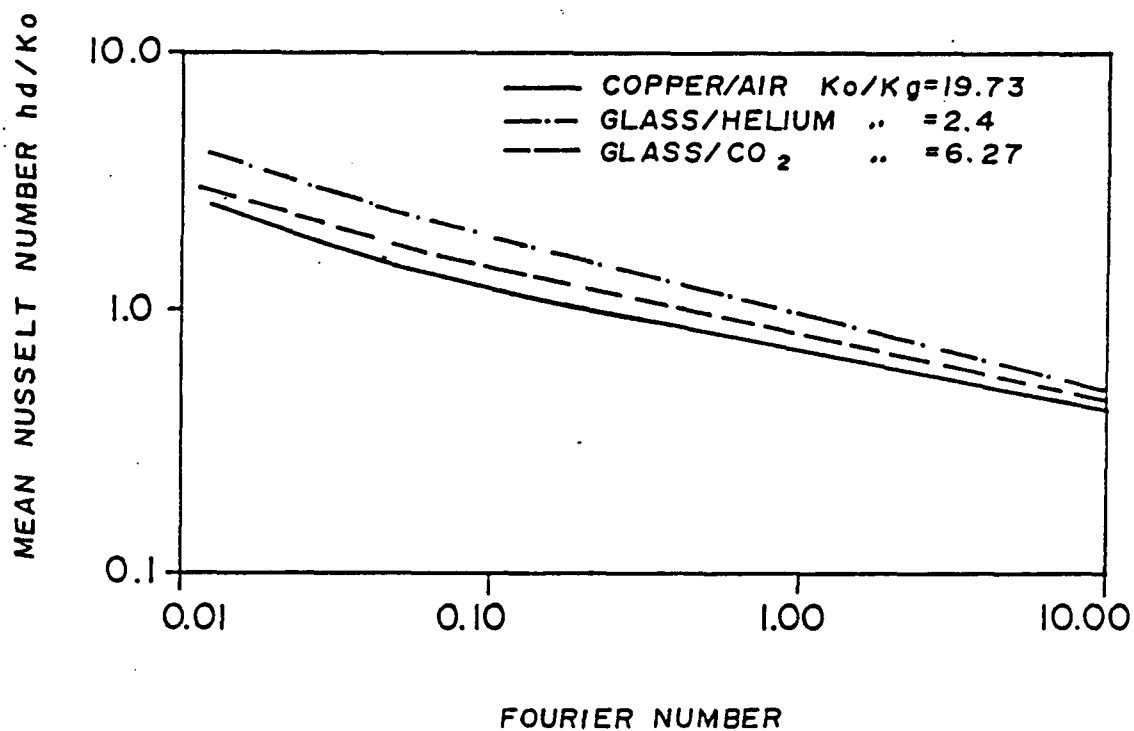


Figure 3-5. Mean Nusselt number based on bed conductivity vs. Fourier number.

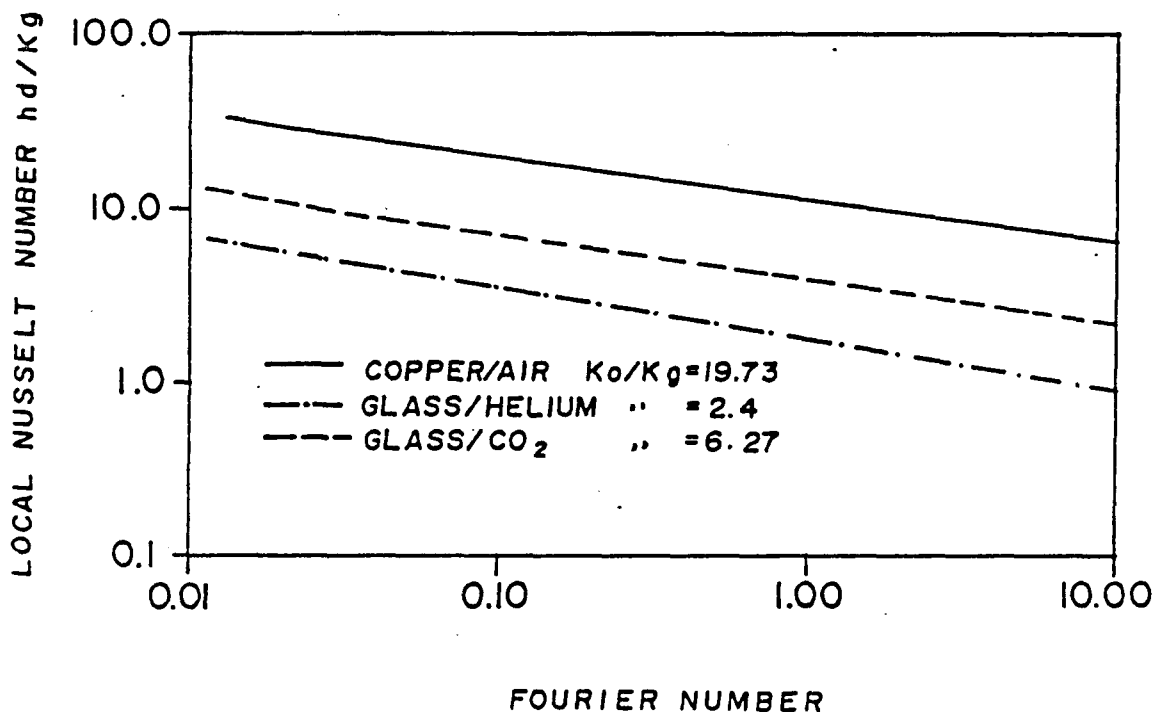


Figure 3-6. Local Nusselt number based on gas conductivity vs. Fourier number.

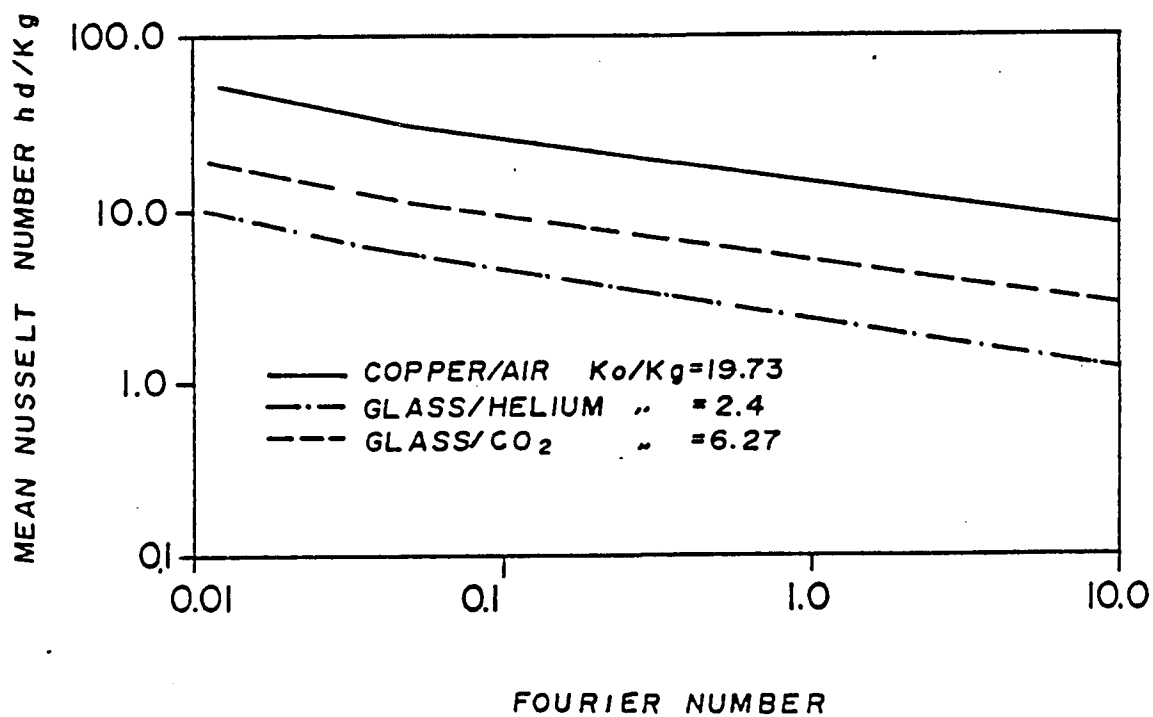


Figure 3-7. Mean Nusselt number based on gas conductivity vs. Fourier number.

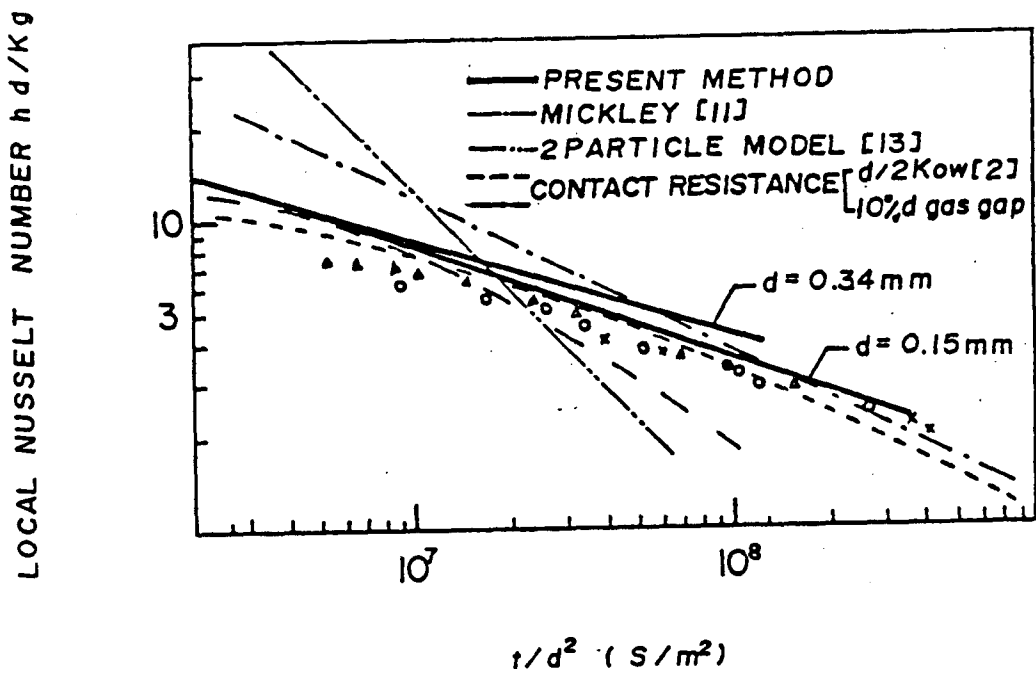


Figure 3-8. Comparison of present method with contact resistance model [2], two particle model [13], and Mickley's model [11] for local heat transfer in the copper/air system.

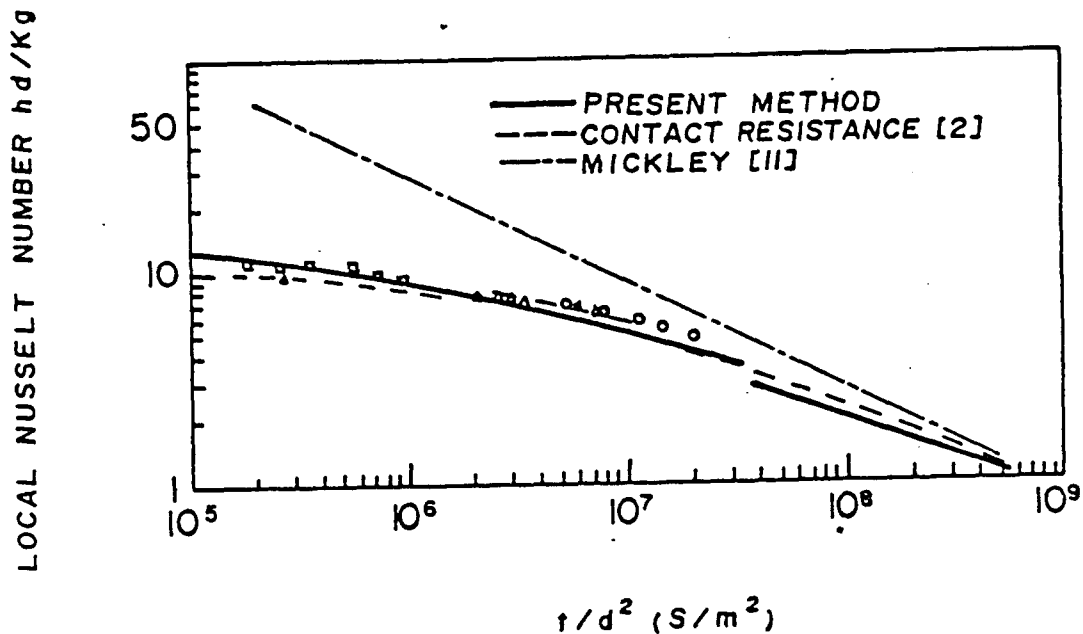
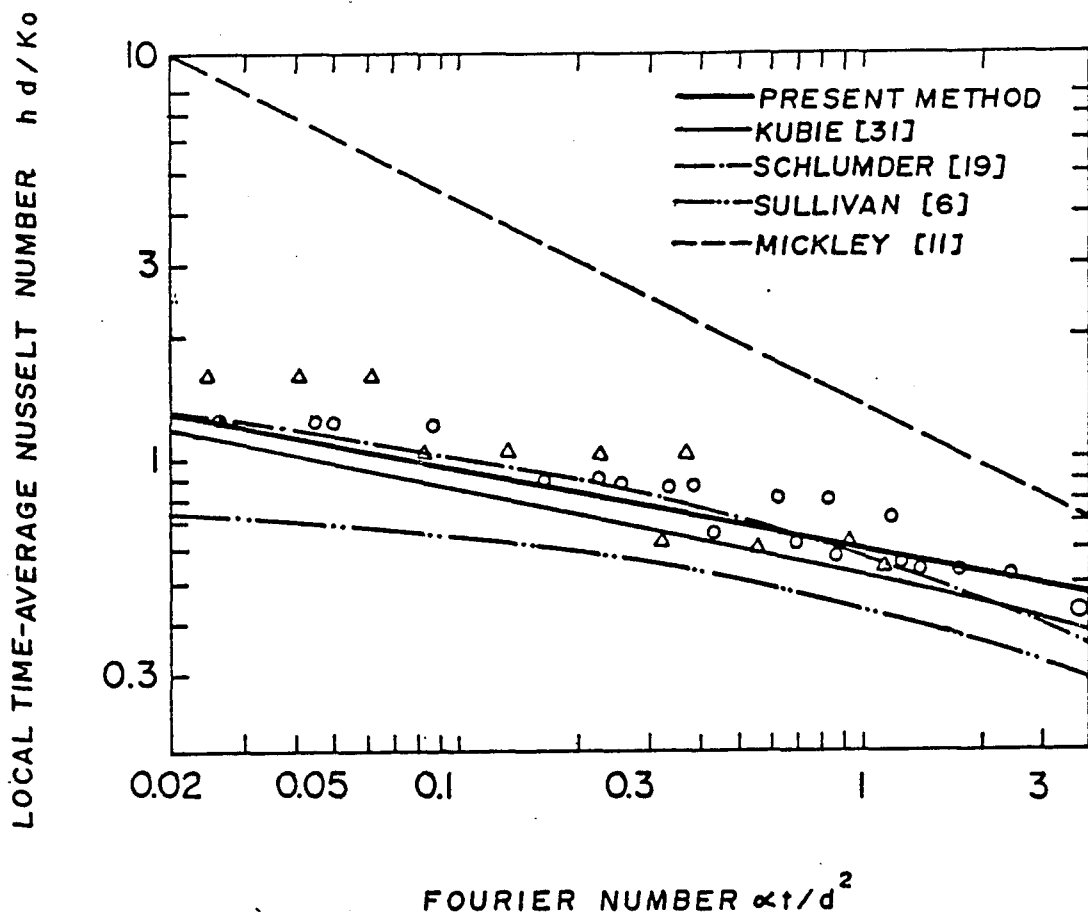


Figure 3-9. Comparison of present method with Mickley and contact resistance models for local heat transfer in sand/air system.



$\Delta, \circ$ DATA FROM [31]

Figure 3-10. Comparison of present method with other theoretical predictions for local time-average heat transfer in steel/air system.

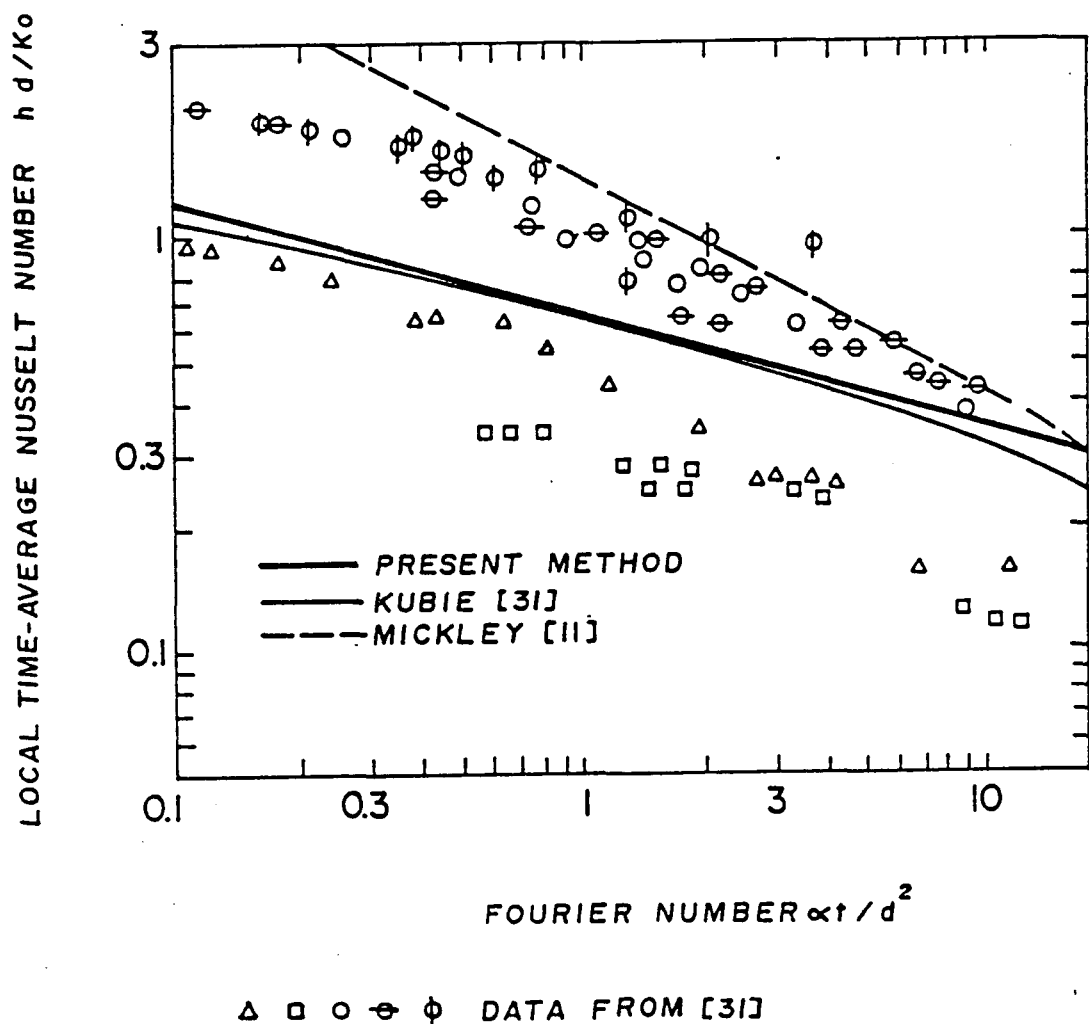


Figure 3-11. Comparison of present method with predictions by Kubie [31], Mickley [11], and data from [31] for local time-average heat transfer in the glass/air system.

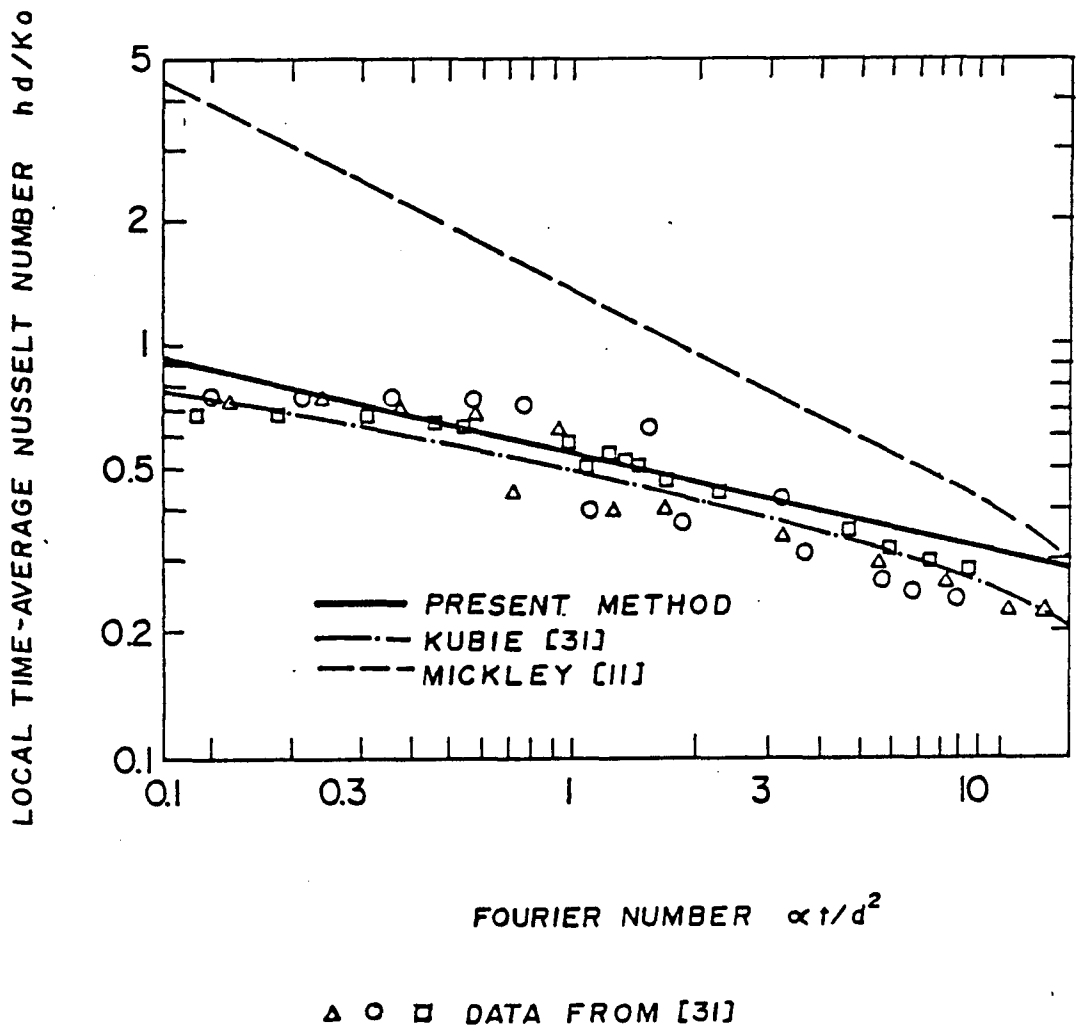


Figure 3-12.

Comparison of present method with predictions by Kubie [31], Mickley [11], and data from [31] for local time-average heat transfer in the copper/air system.

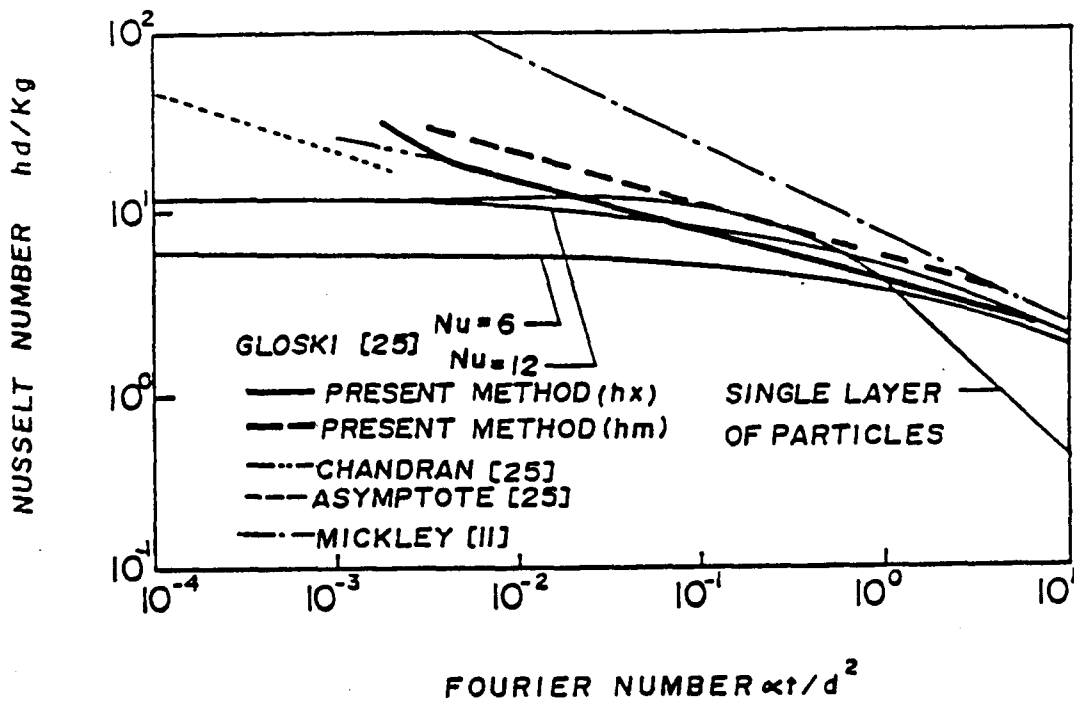


Figure 3-13. Comparison of present method ($k_o/k_g=6.27$) with various models ($k_o/k_g=6.03$) of the time averaged Nusselt number.

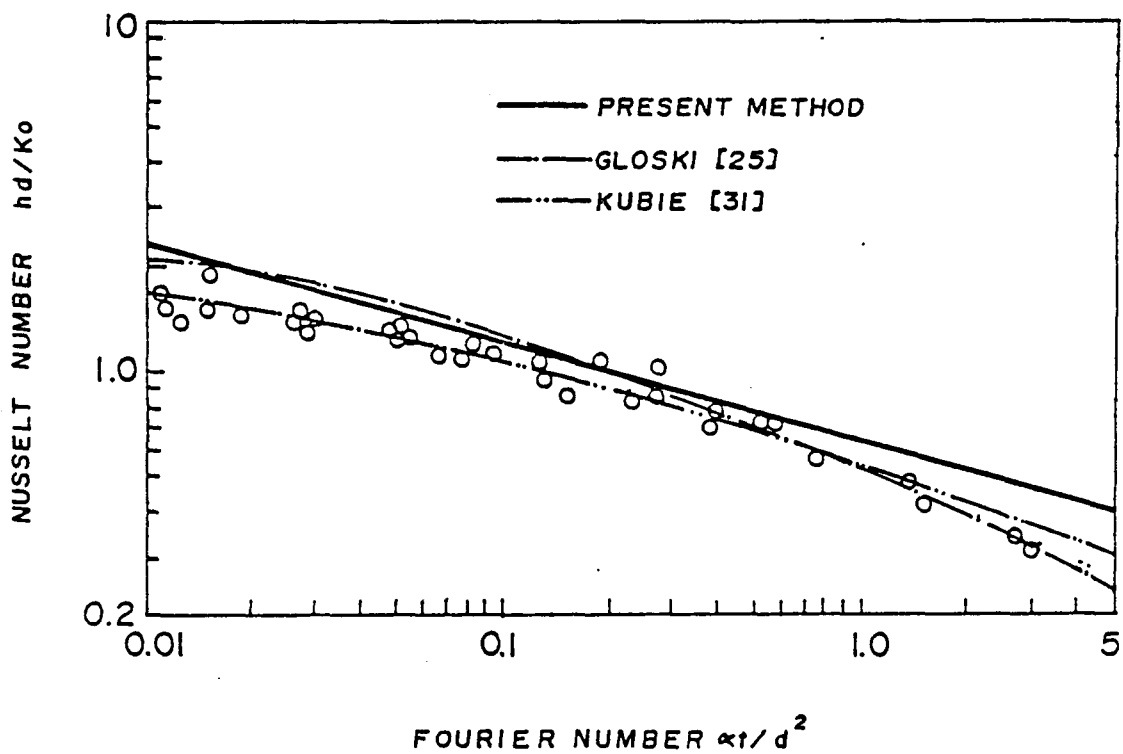


Figure 3-14. Comparison of present method with predictions by Gloski [25] and Kubie [31] for local time-average Nusselt number.

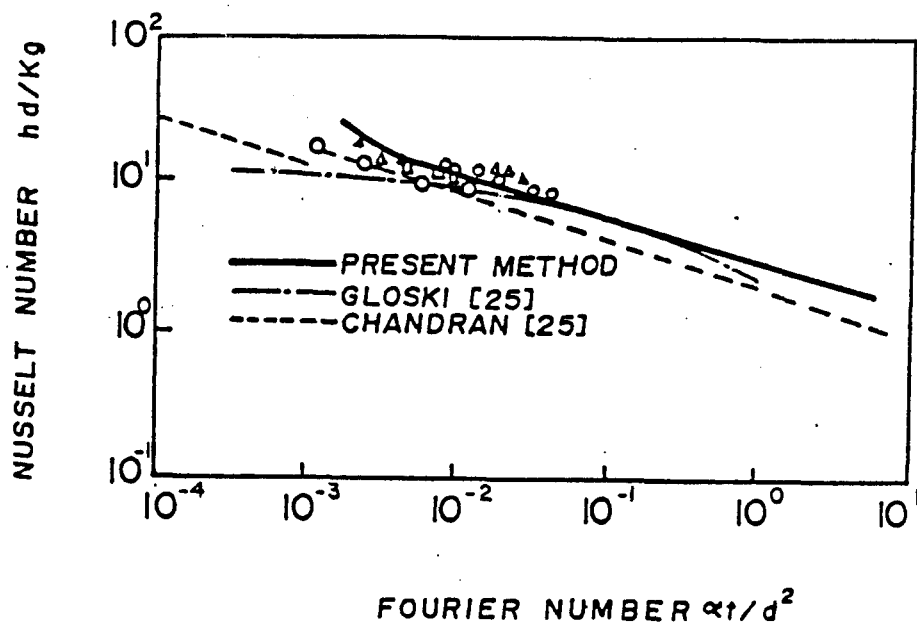


Figure 3-15. Comparison of present method ($k_0/k_g=4.15$) with predictions and data by Gloski [25] for local time-average Nusselt number.

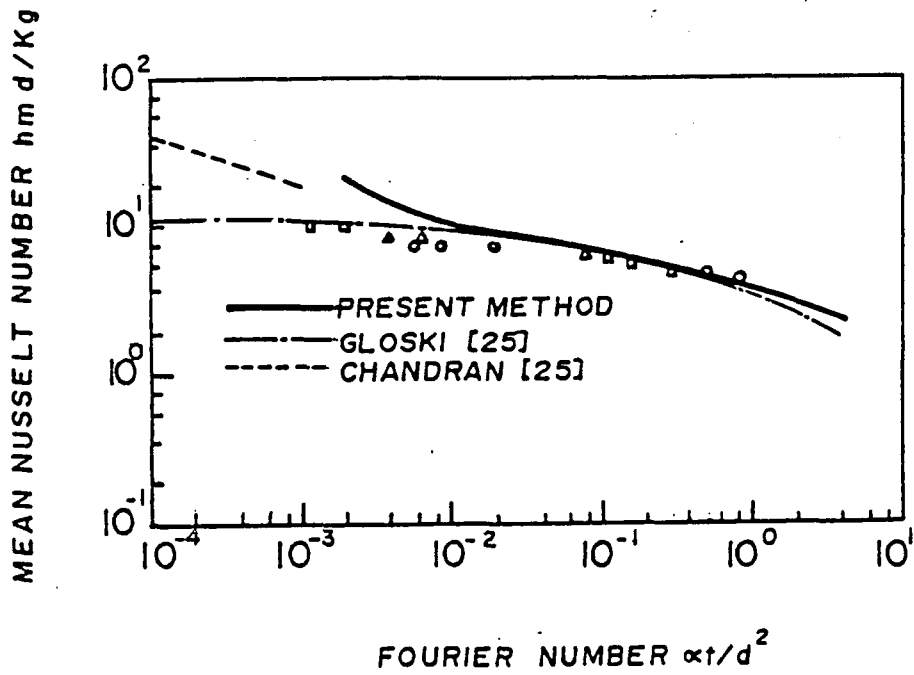
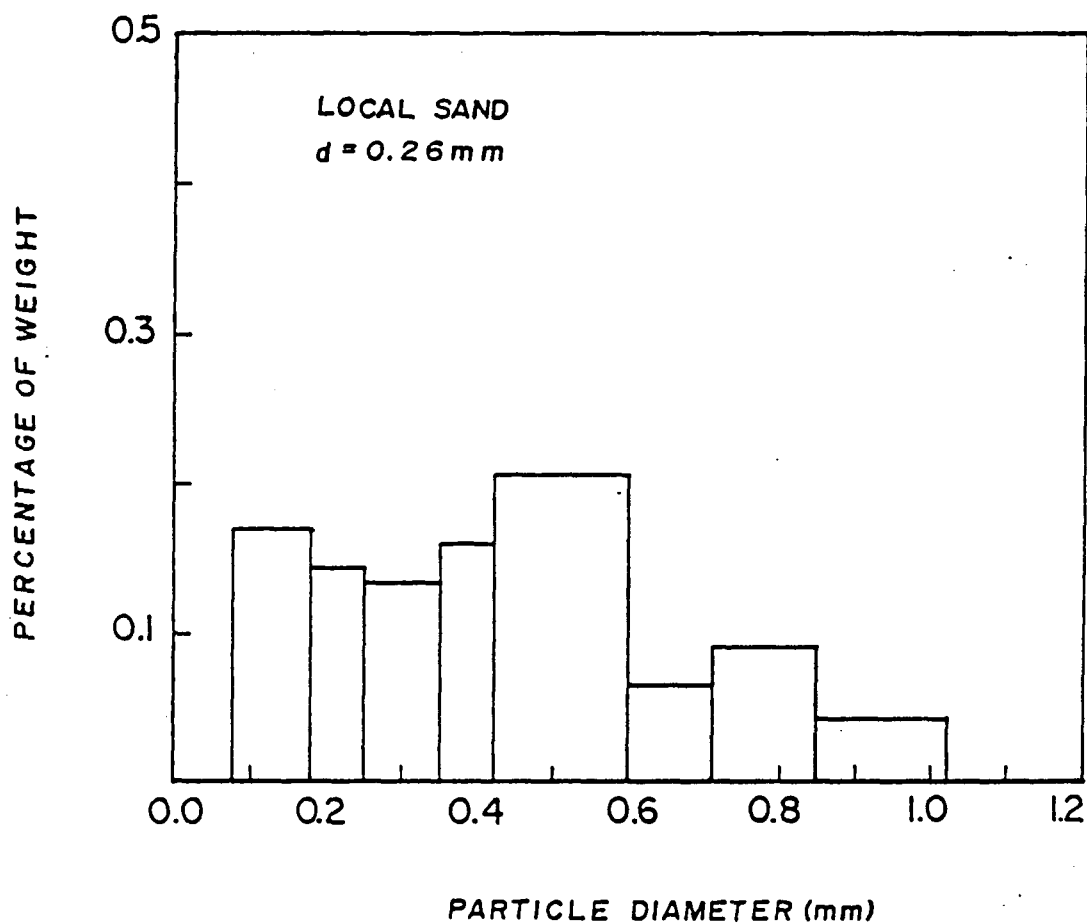
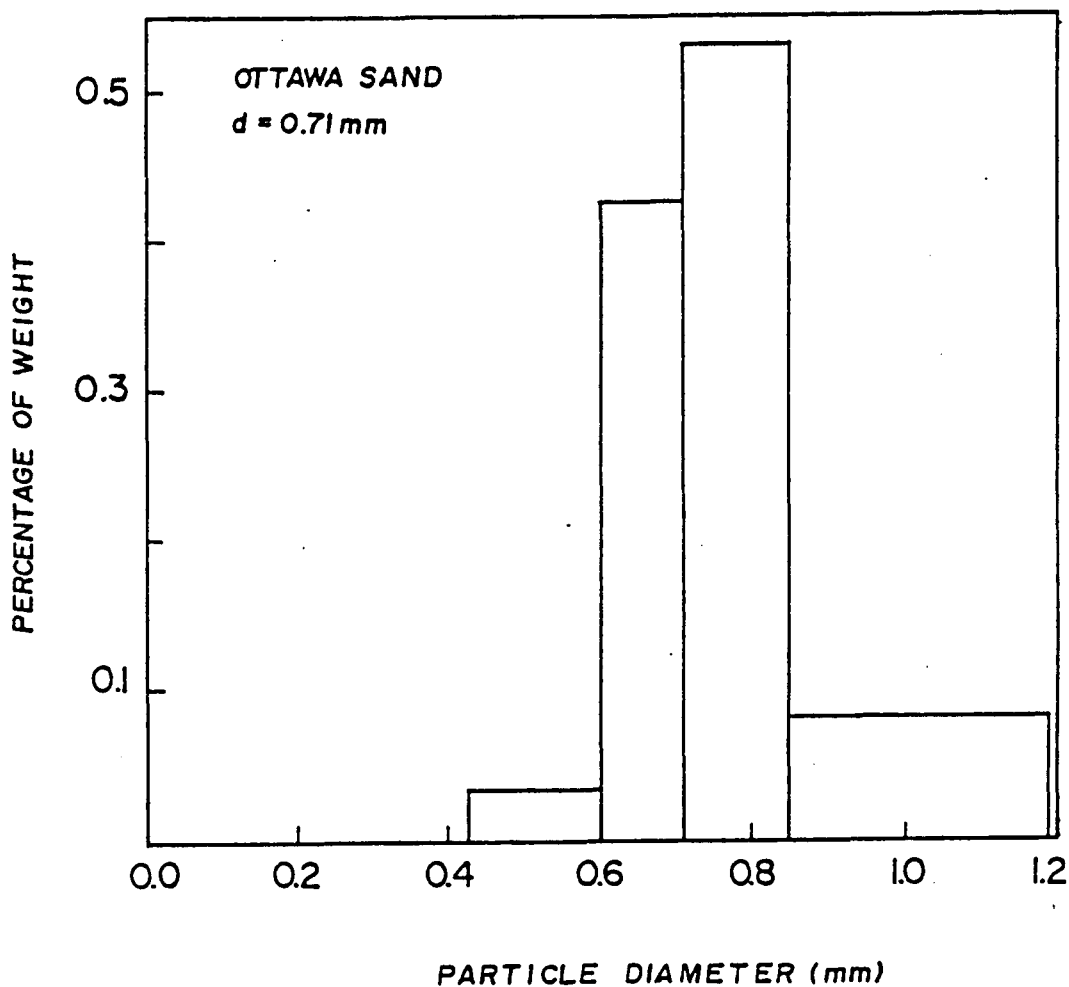


Figure 3-16. Comparison of present method with predictions by Gloski [25] and data by Dunskey [9] for mean Nusselt number.



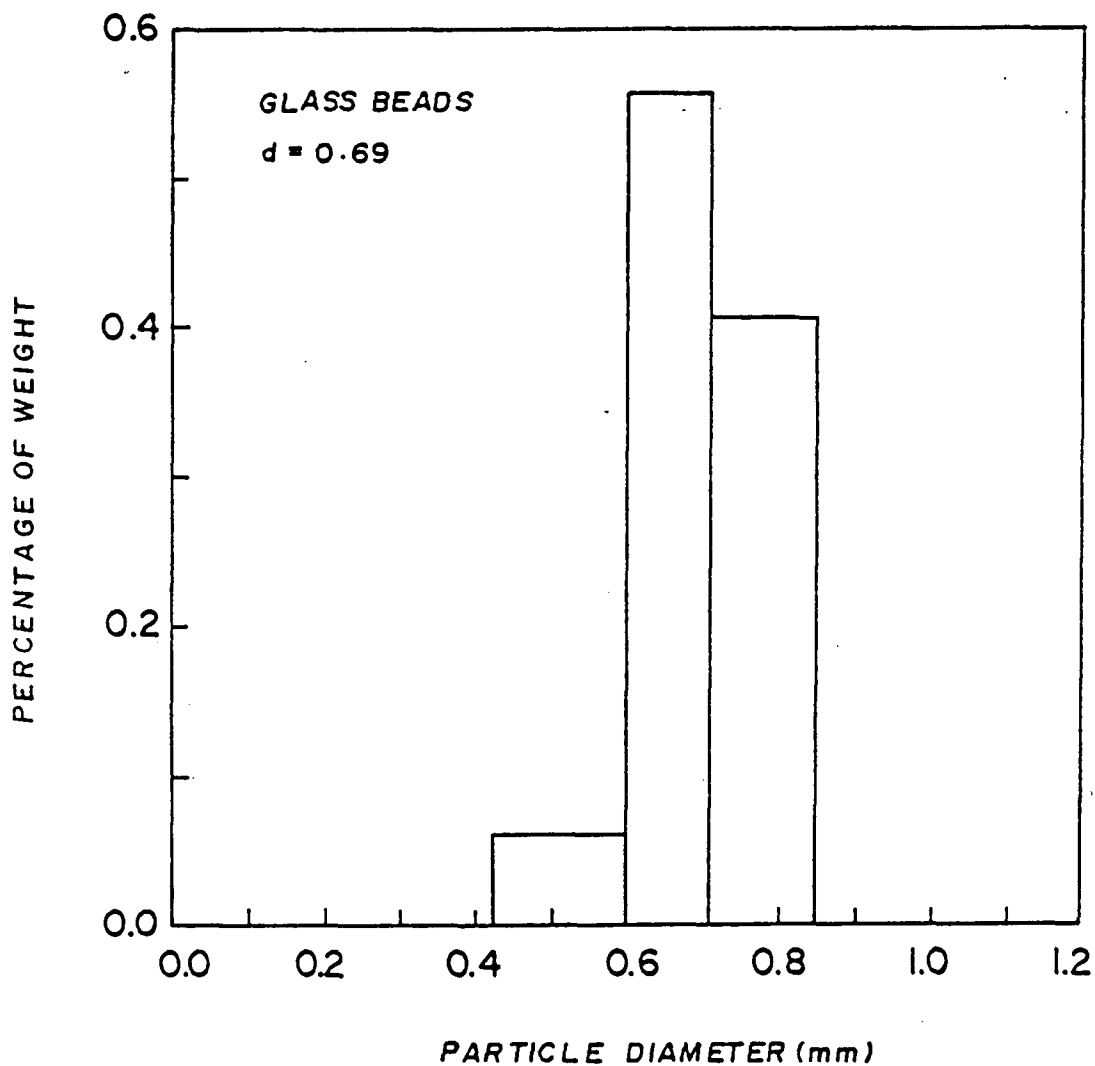
a. Particle size distribution for local sand.

Figure 4-1. Particle size distribution.



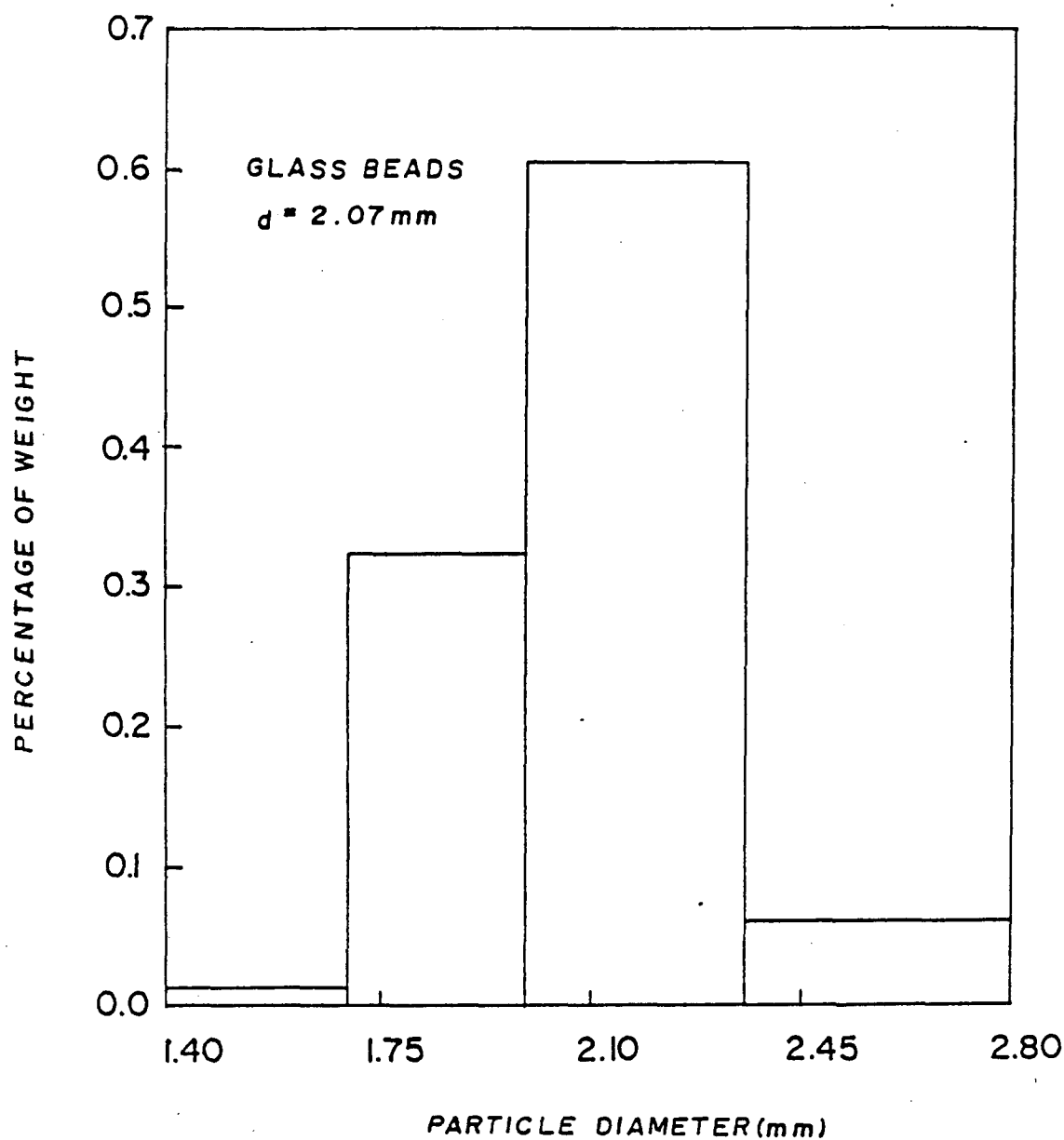
b. Particle size discrete distribution for Ottawa sand.

Figure 4-1. (continued)



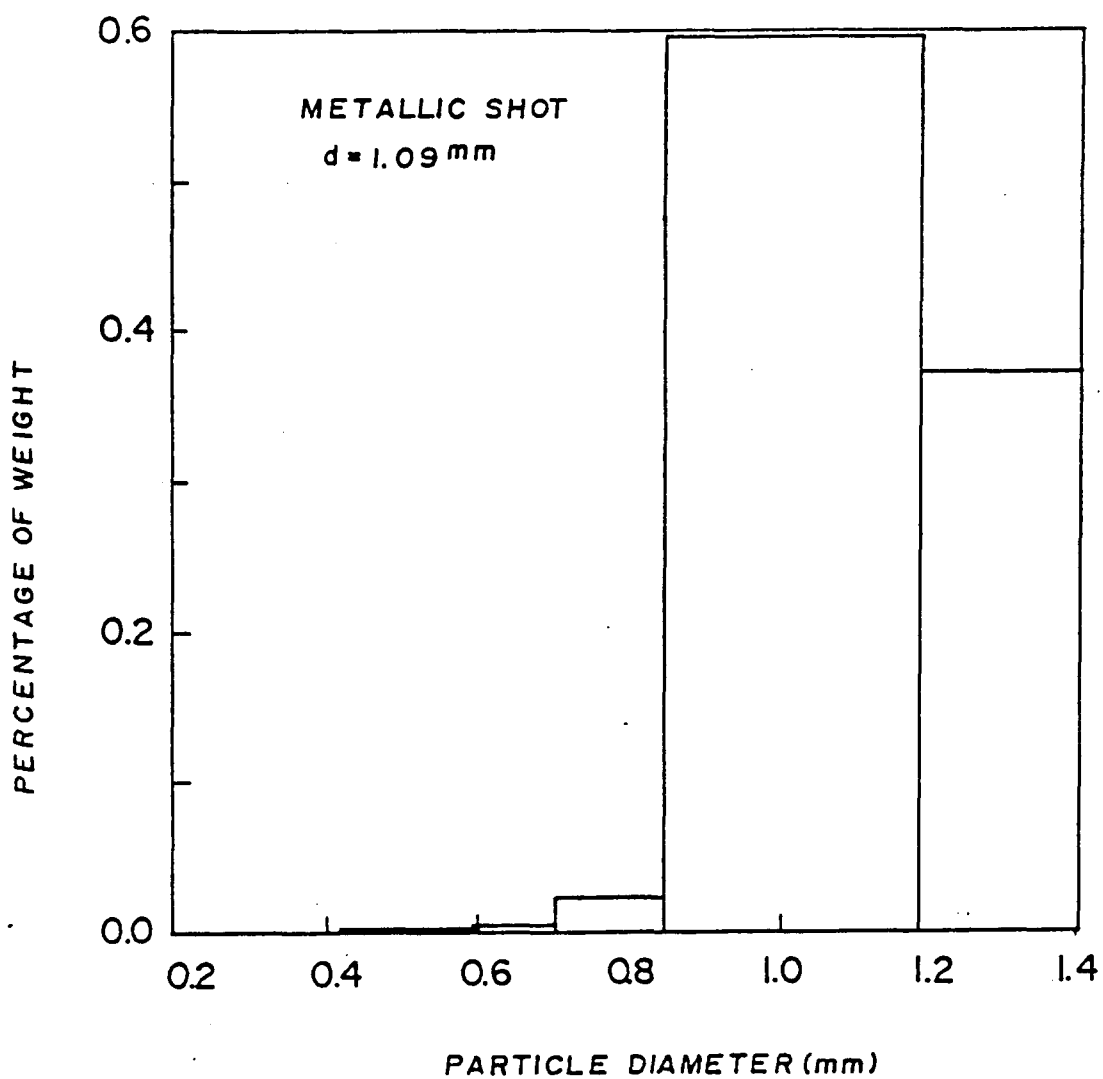
- c. Particle size discrete distribution for small size glass beads.

Figure 4-1. (continued)



- d. Particle size discrete distribution for large size glass beads.

Figure 4-1. (continued)



e. Particle size discrete distribution for metallic shot.

Figure 4-1. (continued)

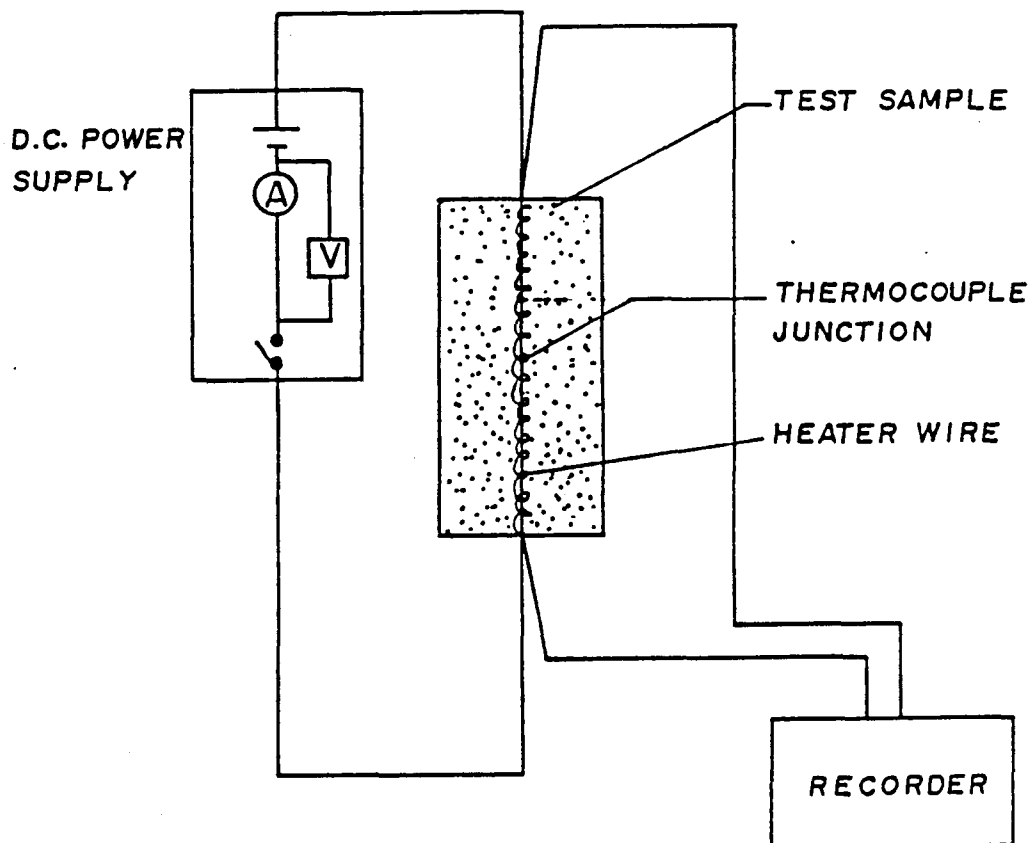
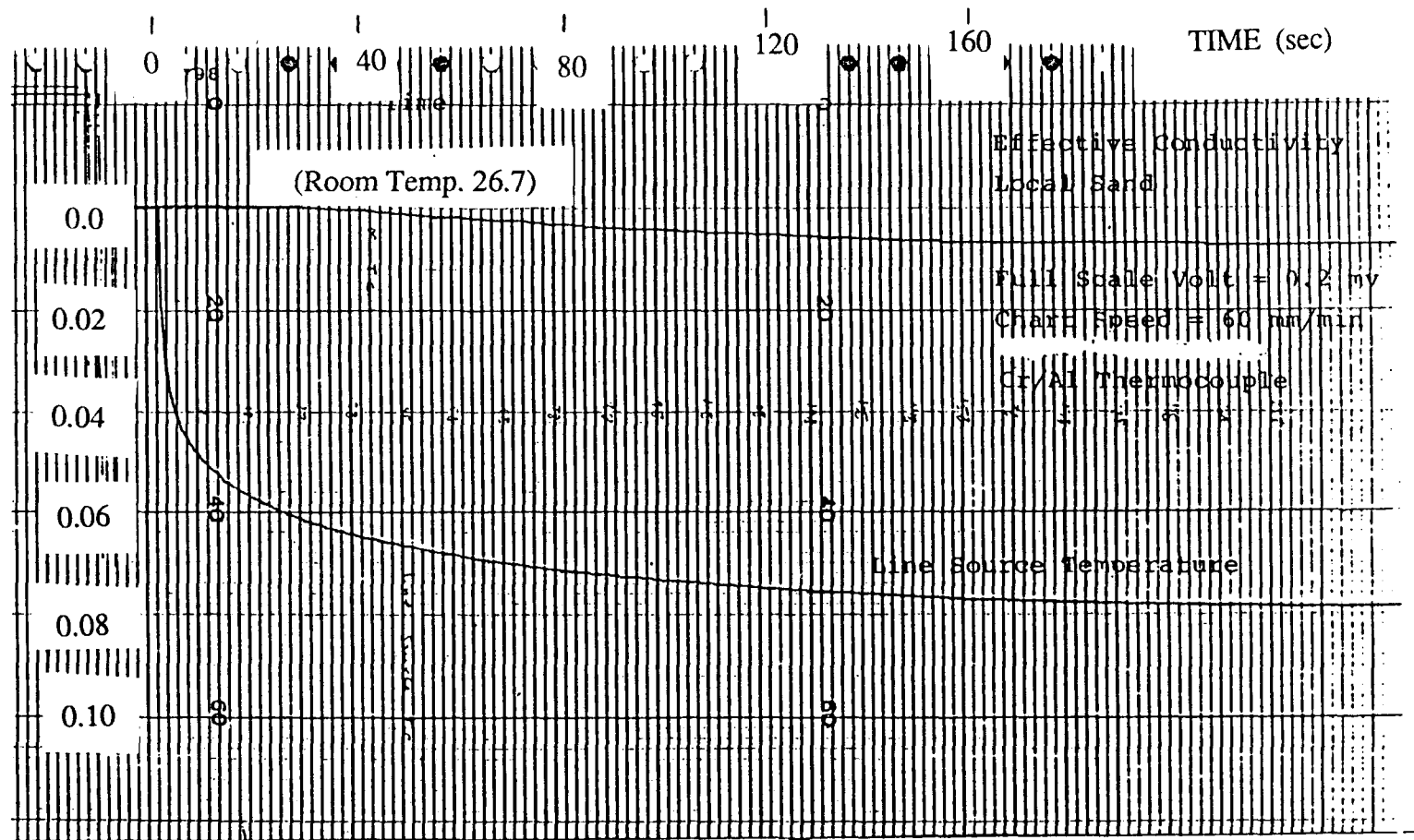


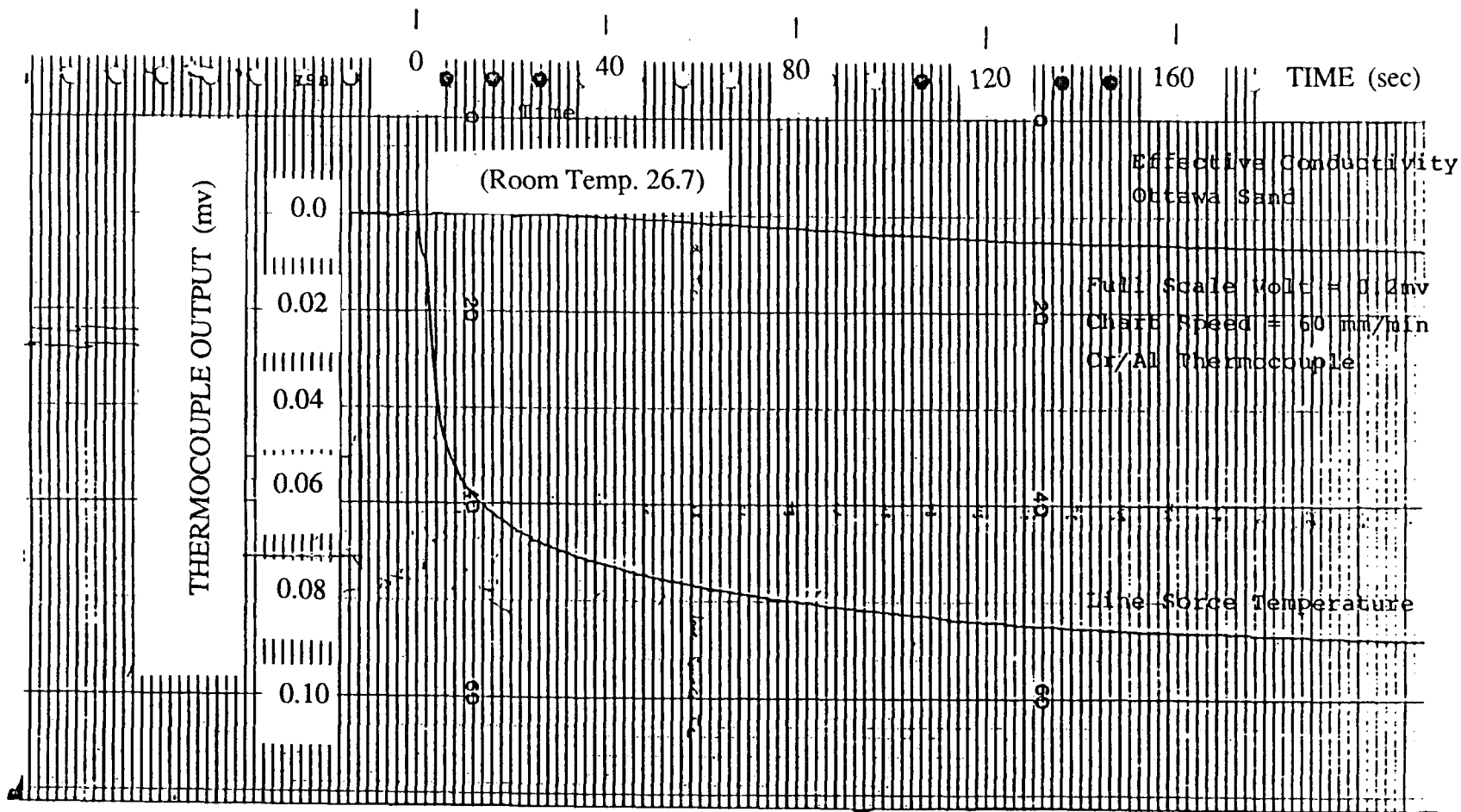
Figure 4-2. Schematic diagram of apparatus used for measuring thermal conductivity.

THERMOCOUPLE OUTPUT (mv)



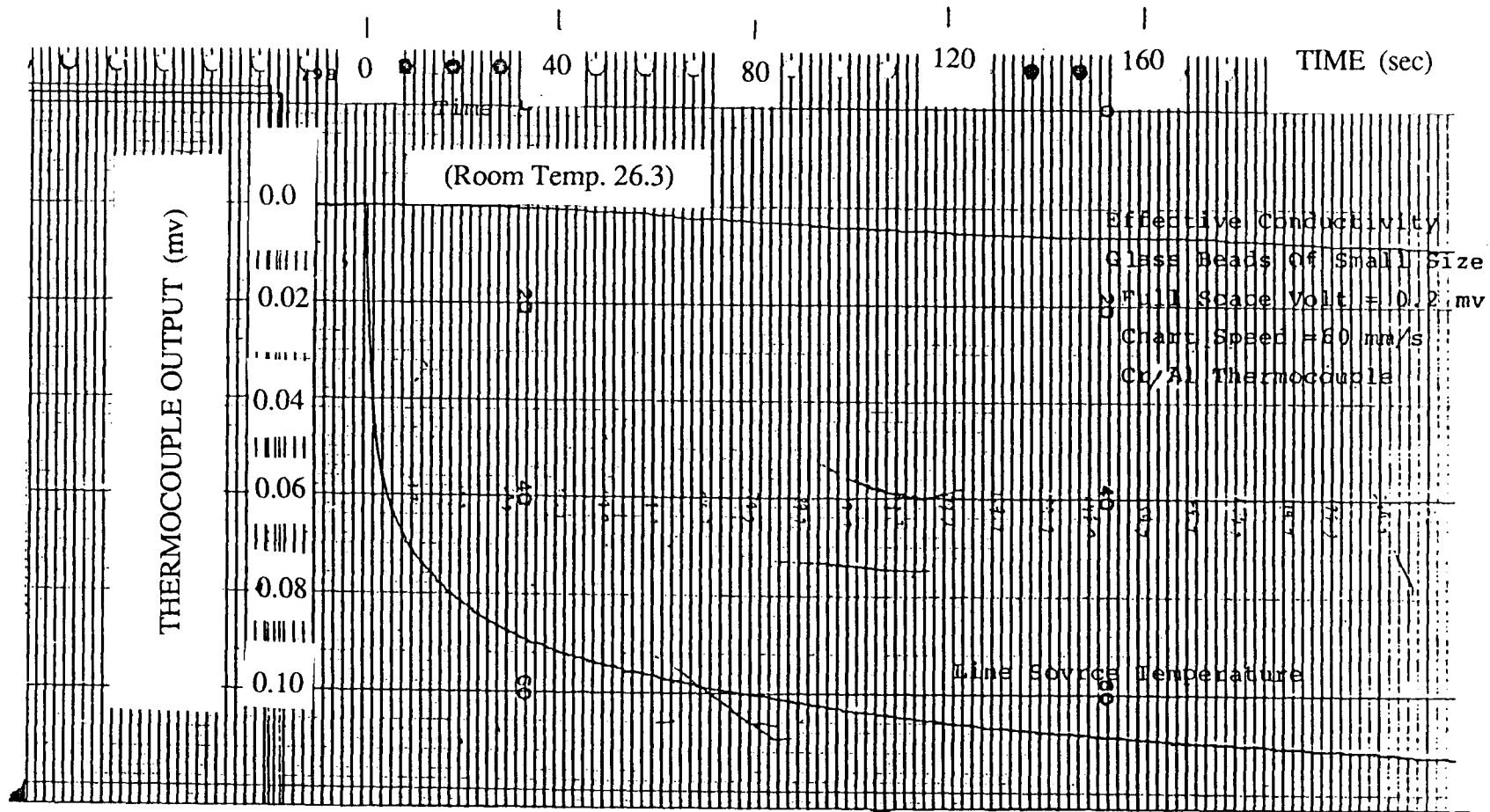
a. Recorded temperature-time history for local sand.

Figure 4-3. Effective thermal conductivity.



b. Recorded temperature-time history for Ottawa sand.

Figure 4-3. (continued)



c. Recorded temperature-time history for small size glass beads.

Figure 4-3. (continued)

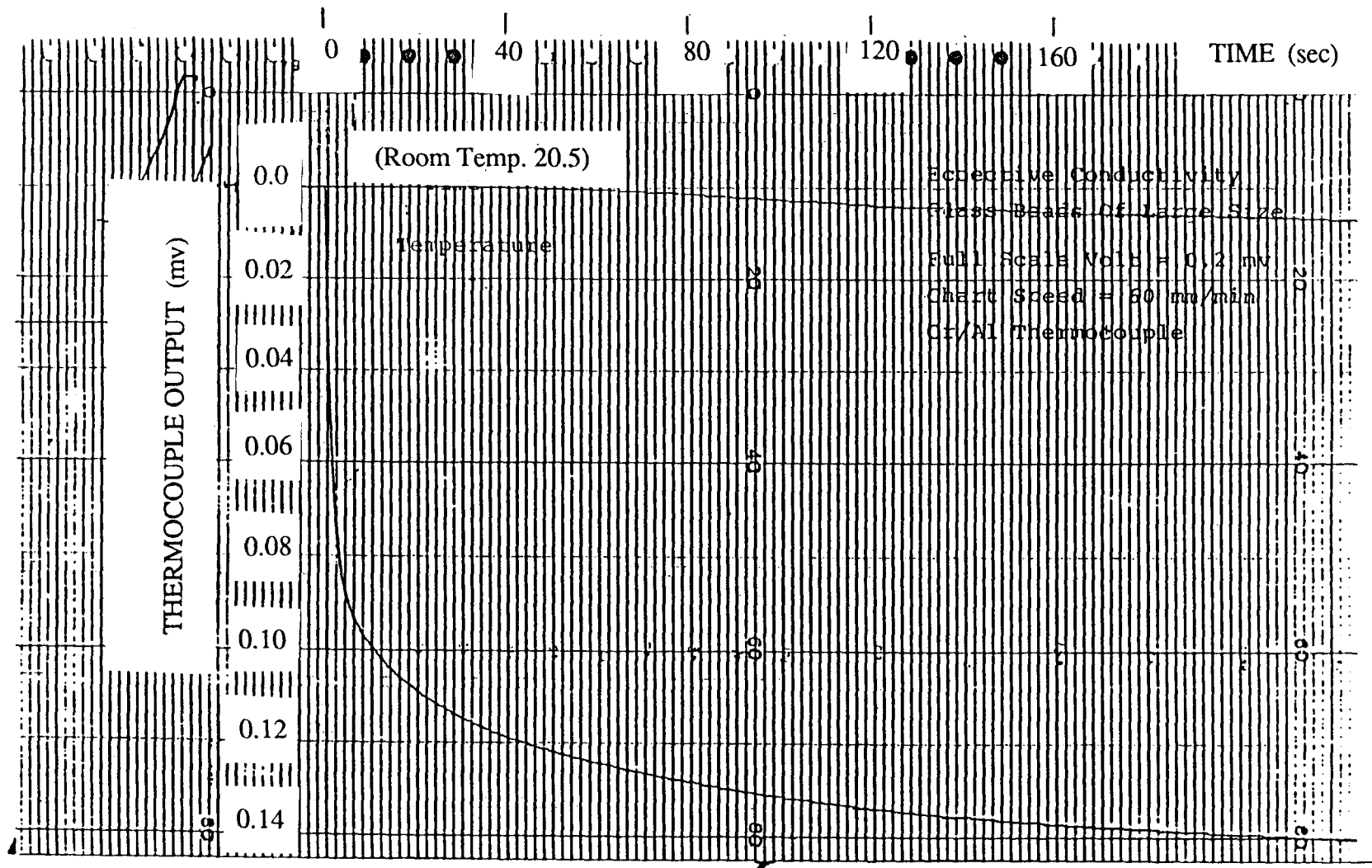
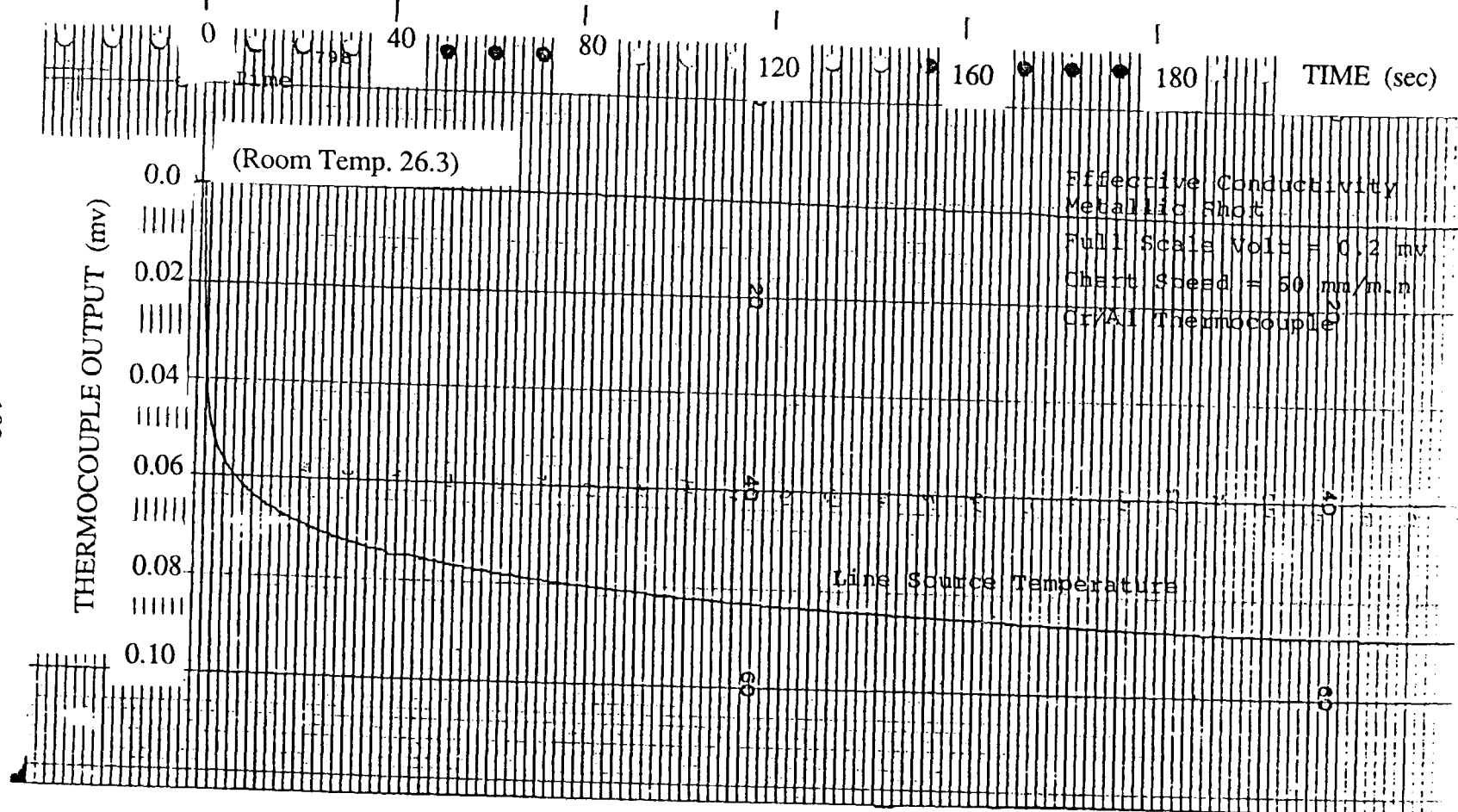


Figure 4-3. (continued) d. Recorded temperature-time history for large size glass beads.



e. Recorded temperature-time history for metallic shot.

Figure 4-3. (continued)

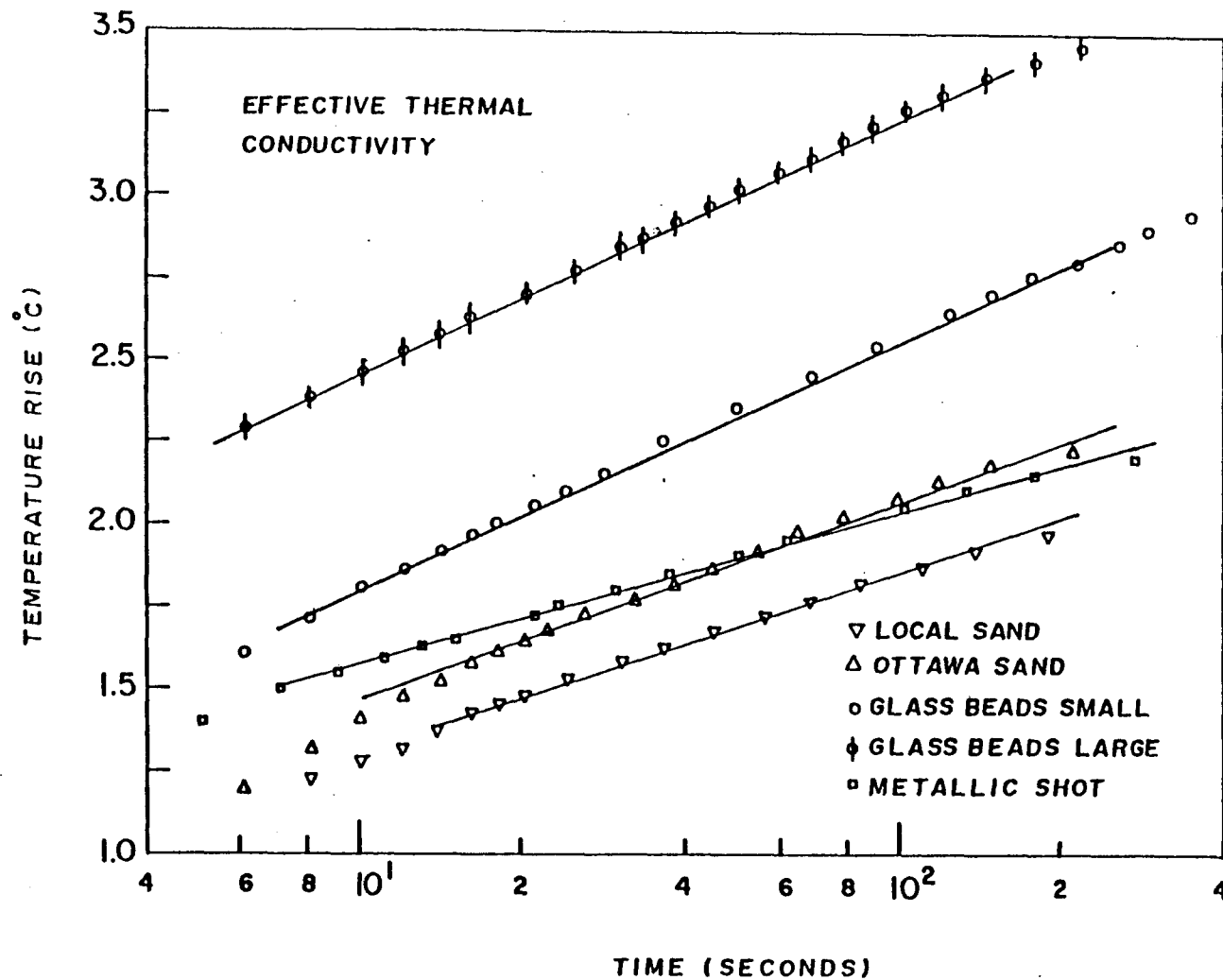


Figure 4-4. Transient temperature rise within the line source embedded in granular media.

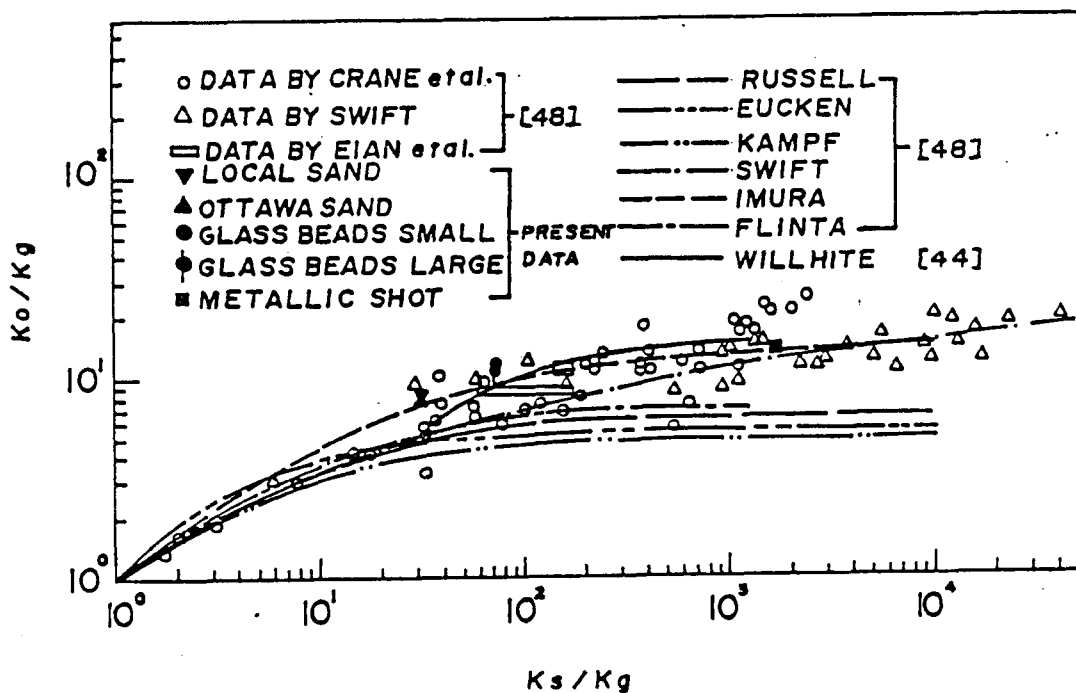


Figure 4-5. Comparison of some of the existing correlations and data for $\epsilon_o \approx 0.40$ with present measurements.

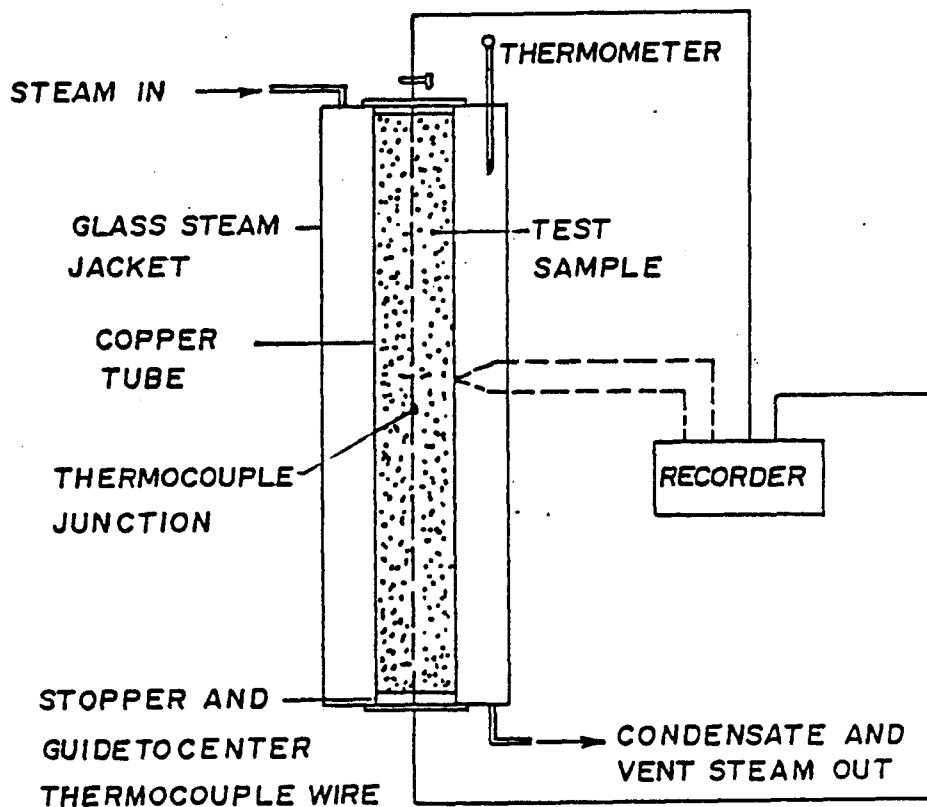
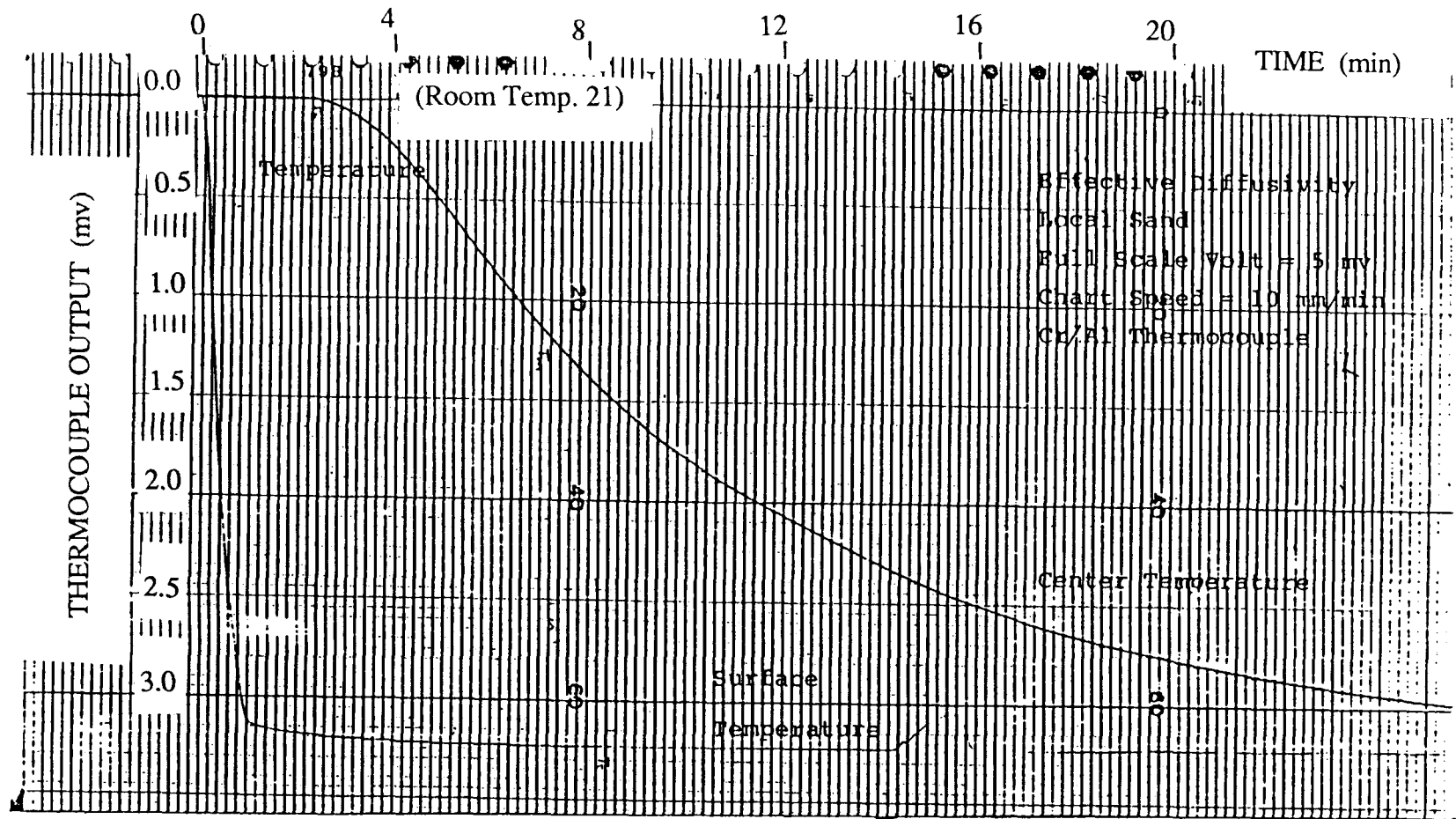
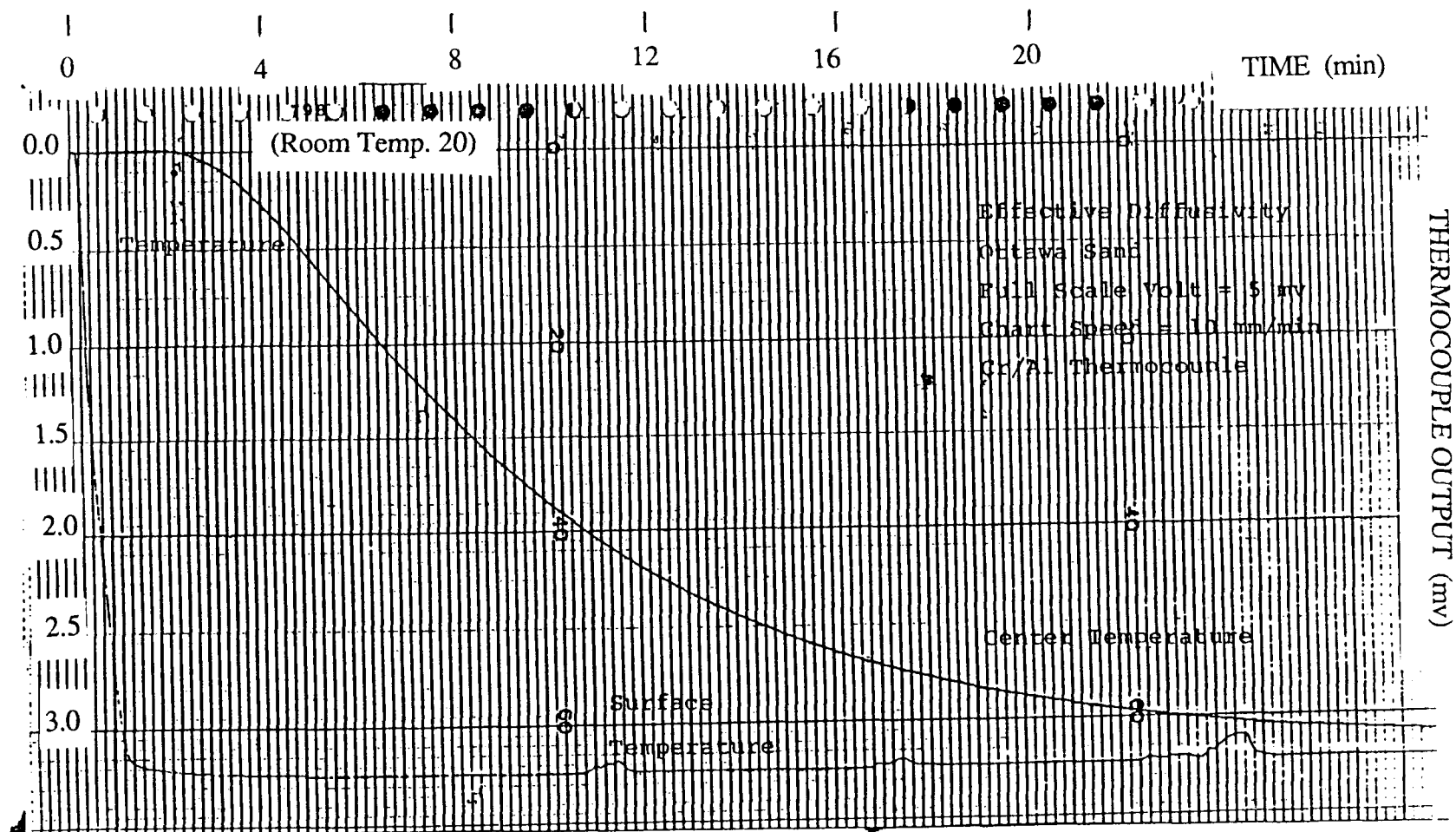


Figure 4-6. Schematic diagram of apparatus used for measuring thermal diffusivity.



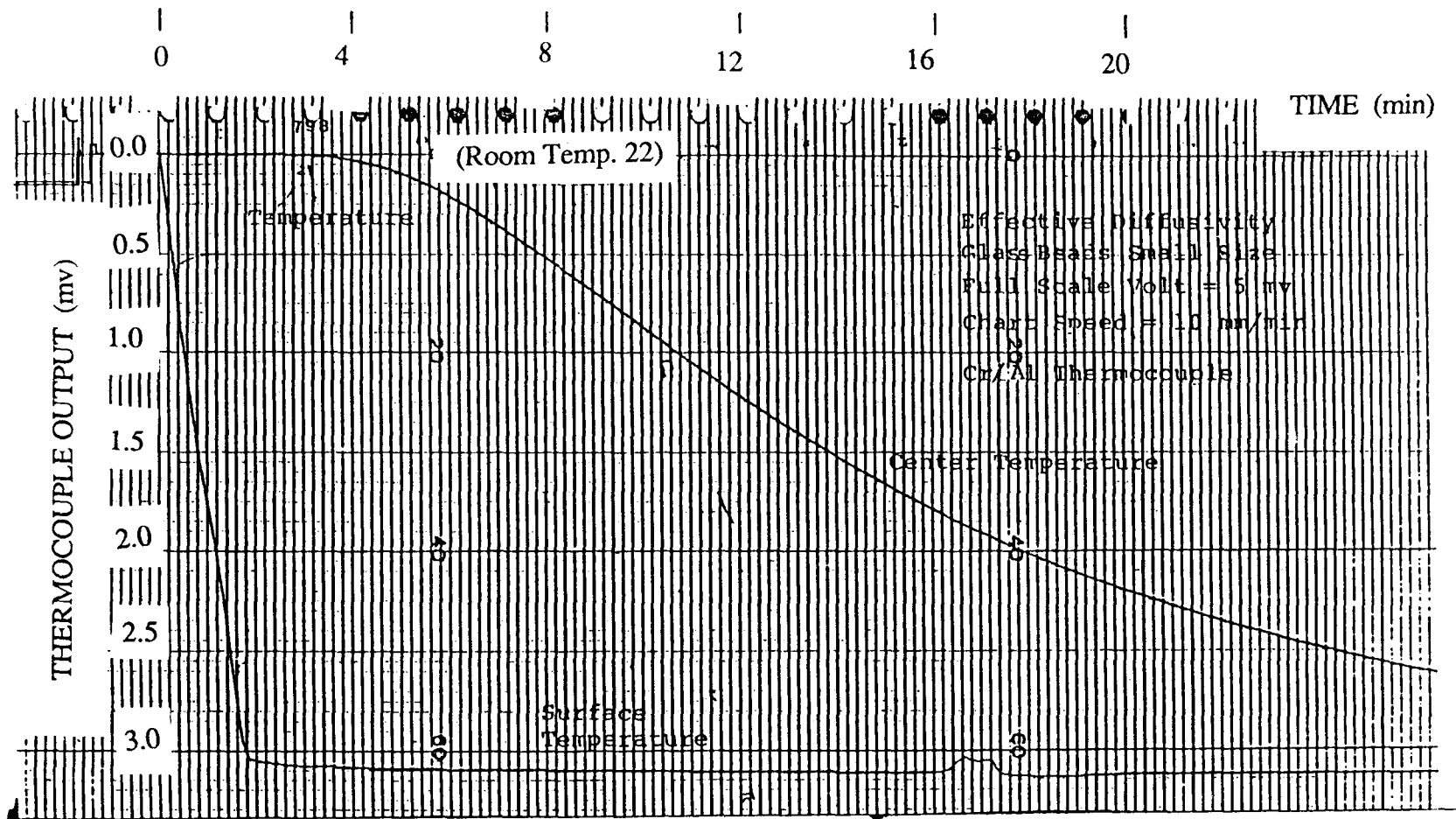
a. Recorded temperature-time history for local sand.

Figure 4-7. Effective thermal diffusivity.

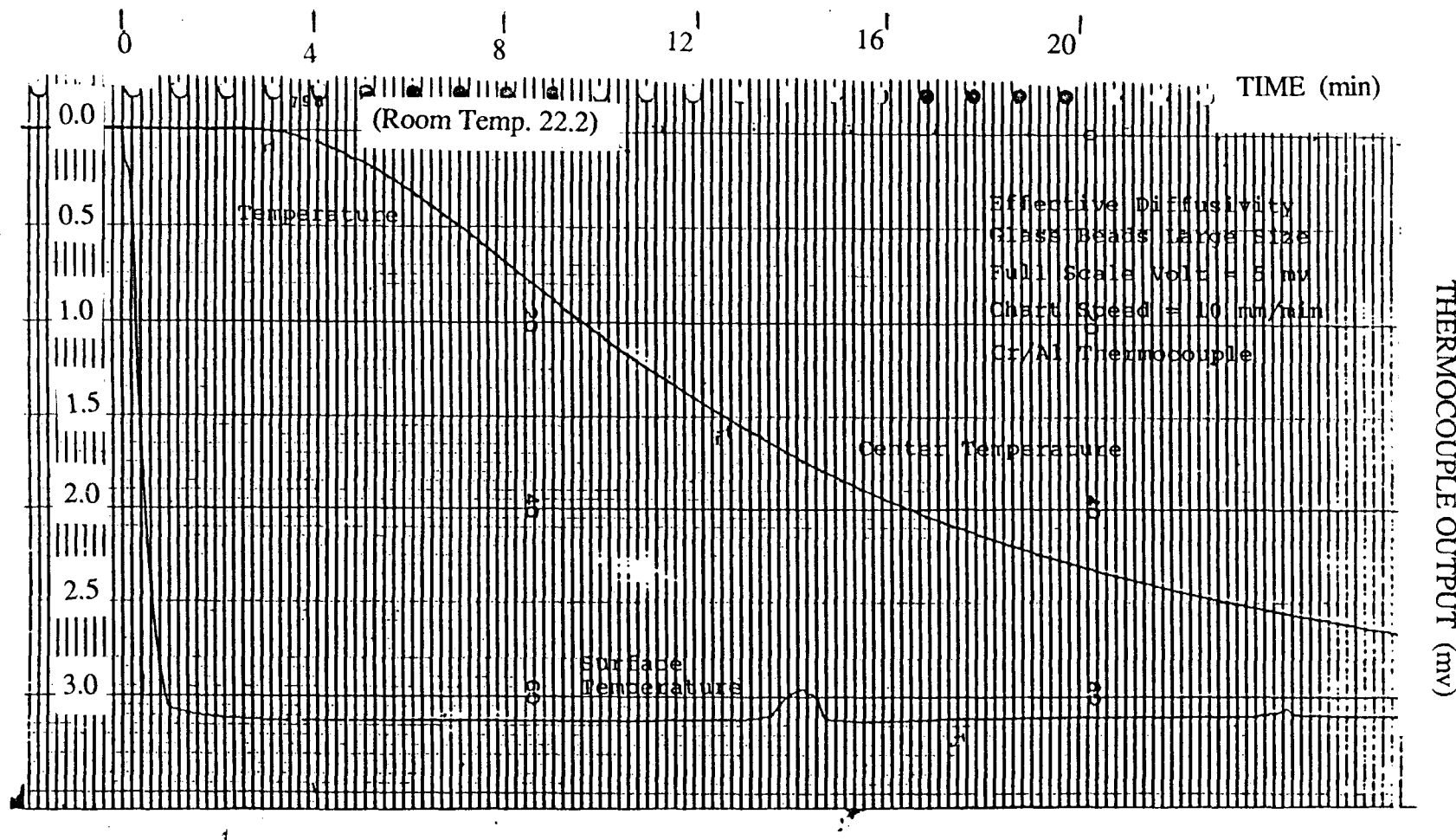


b. Recorded temperature-time history for Ottawa sand.

Figure 4-7. (continued)

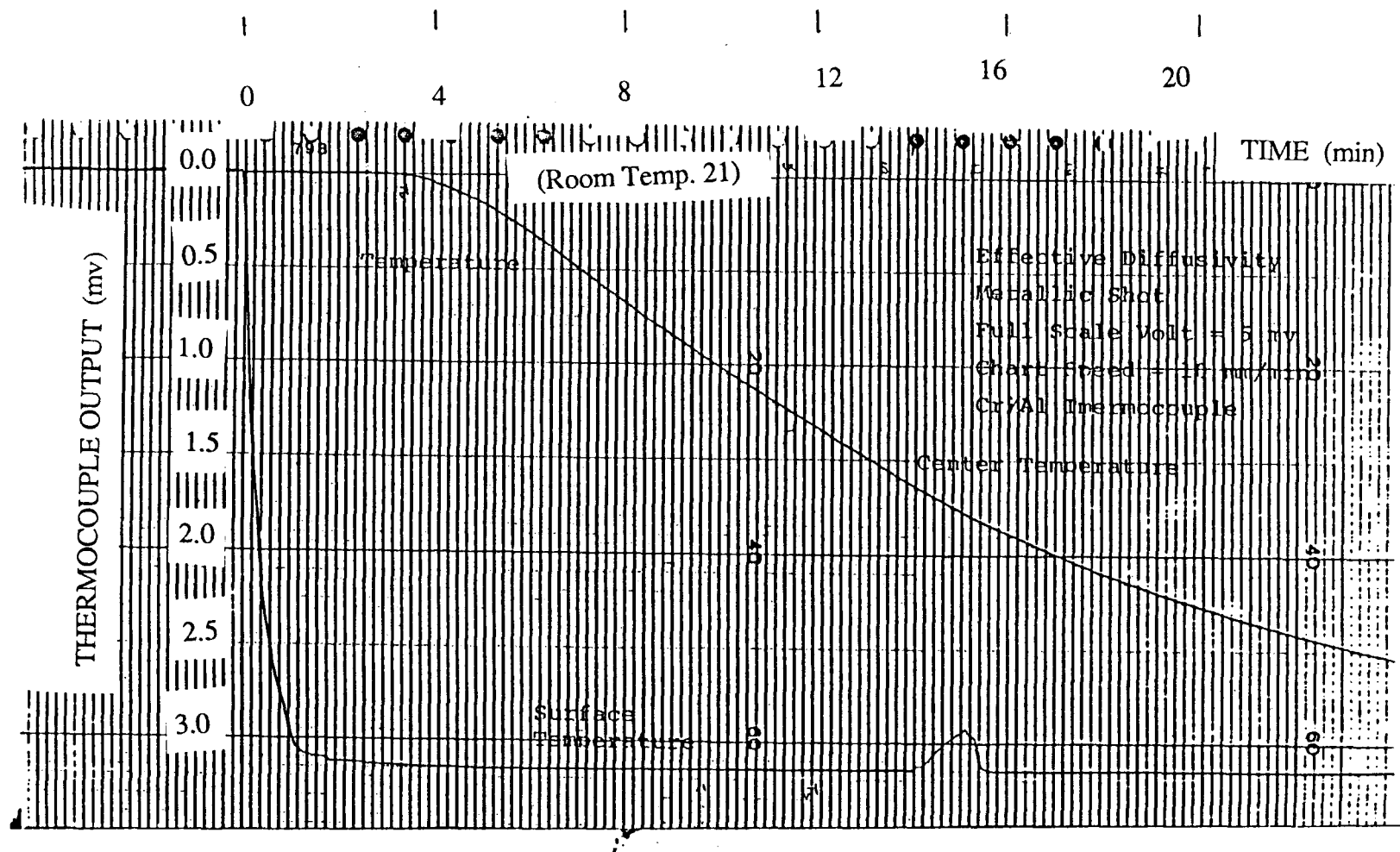


c. Recorded temperature-time history for small glass beads.



d. Recorded temperature-time history for large glass beads.

Figure 4-7. (continued)



e. Recorded temperature-time history for metallic shot.

Figure 4-7. (continued)

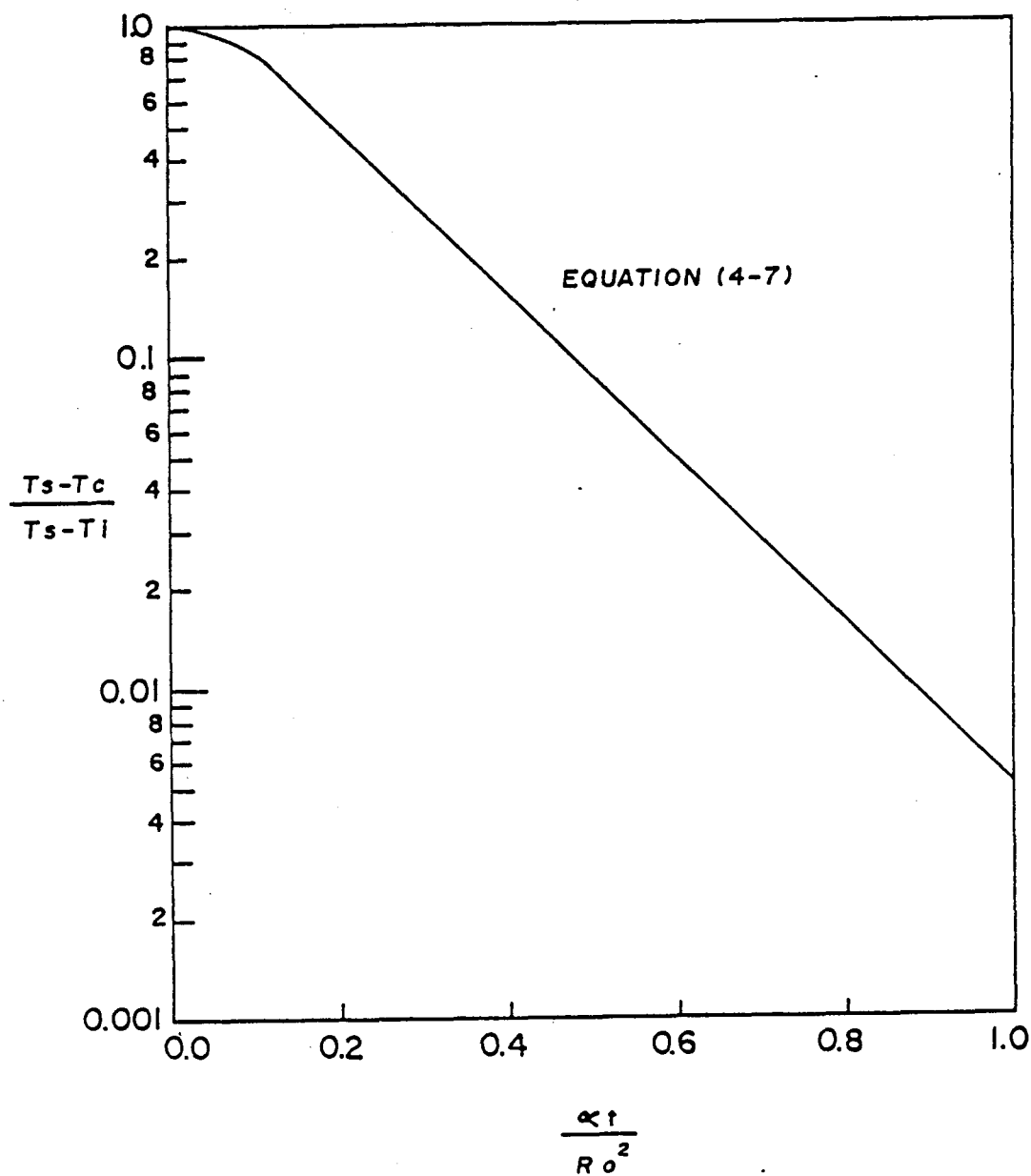


Figure 4-8. Plotting of equation (4-7)

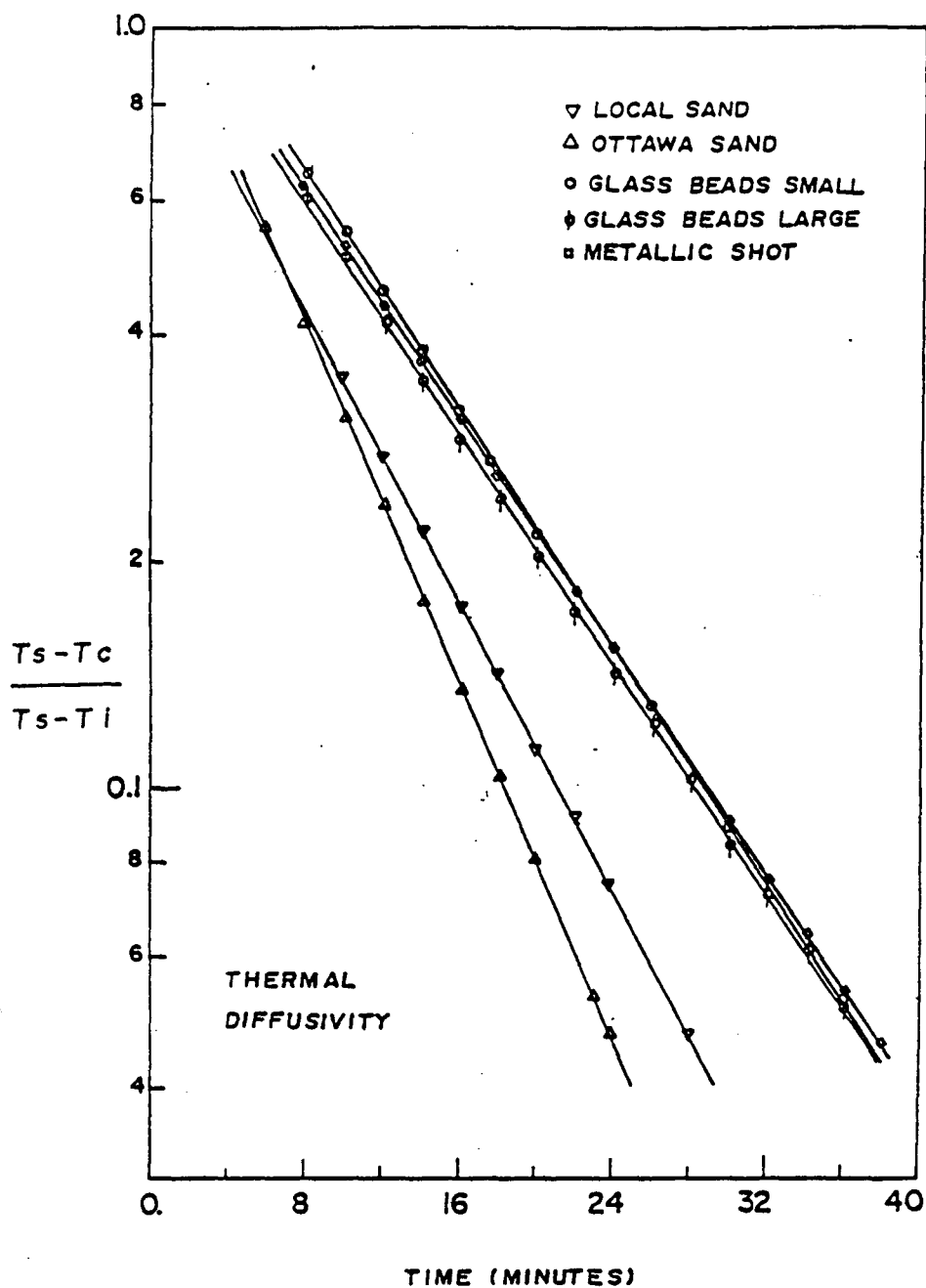
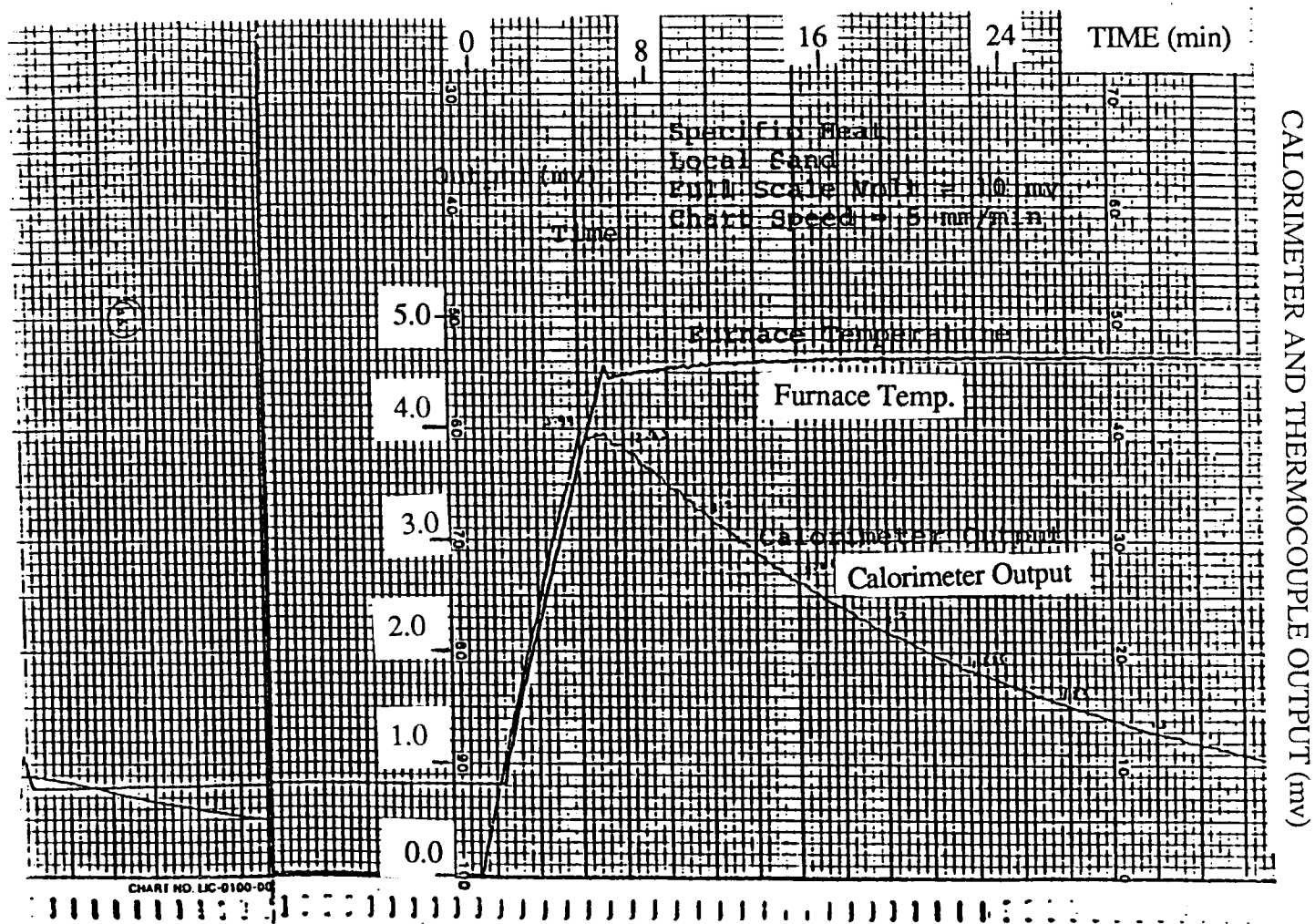
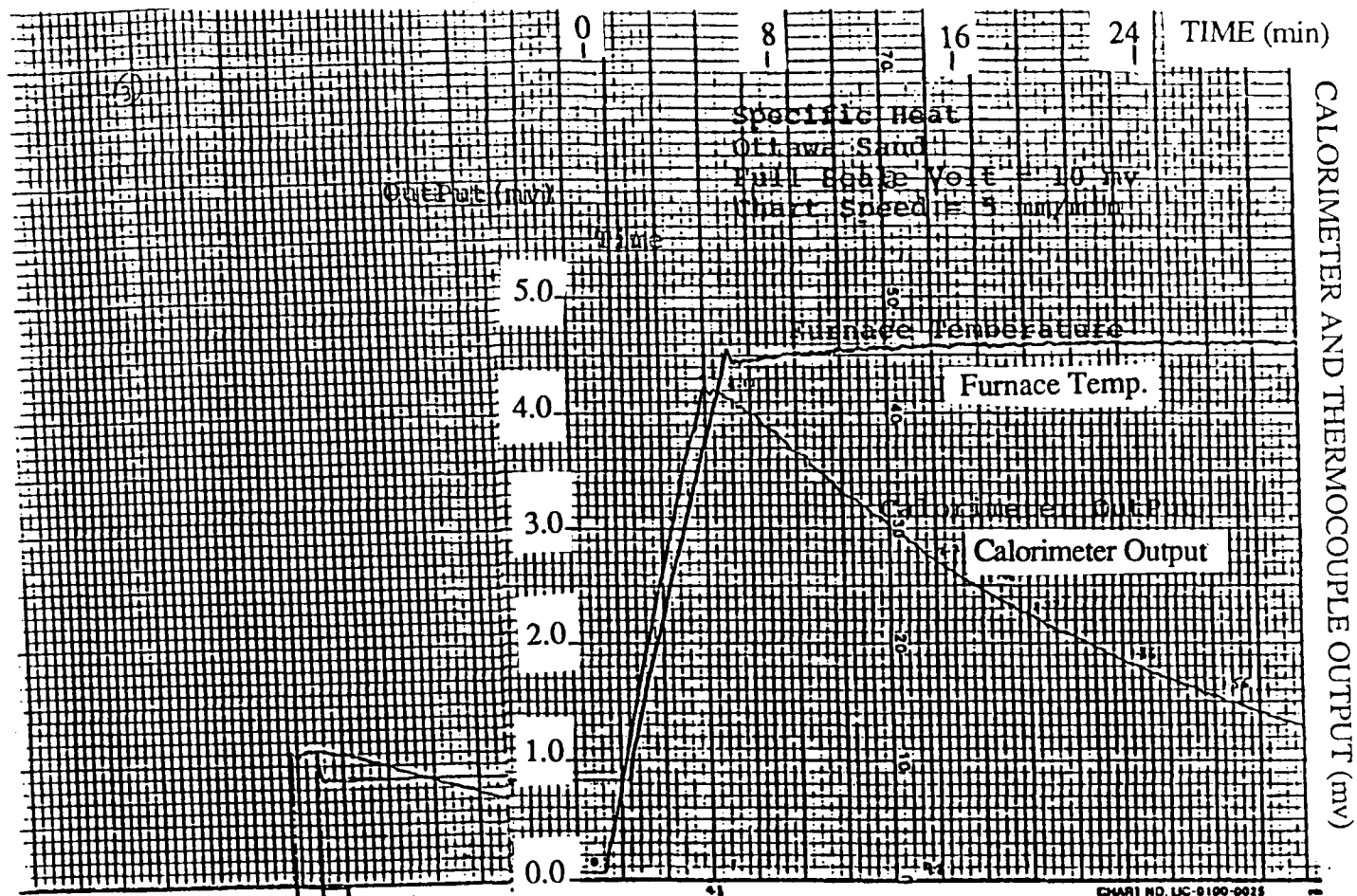


Figure 4-9. Variation in center temperature with time for granular media.



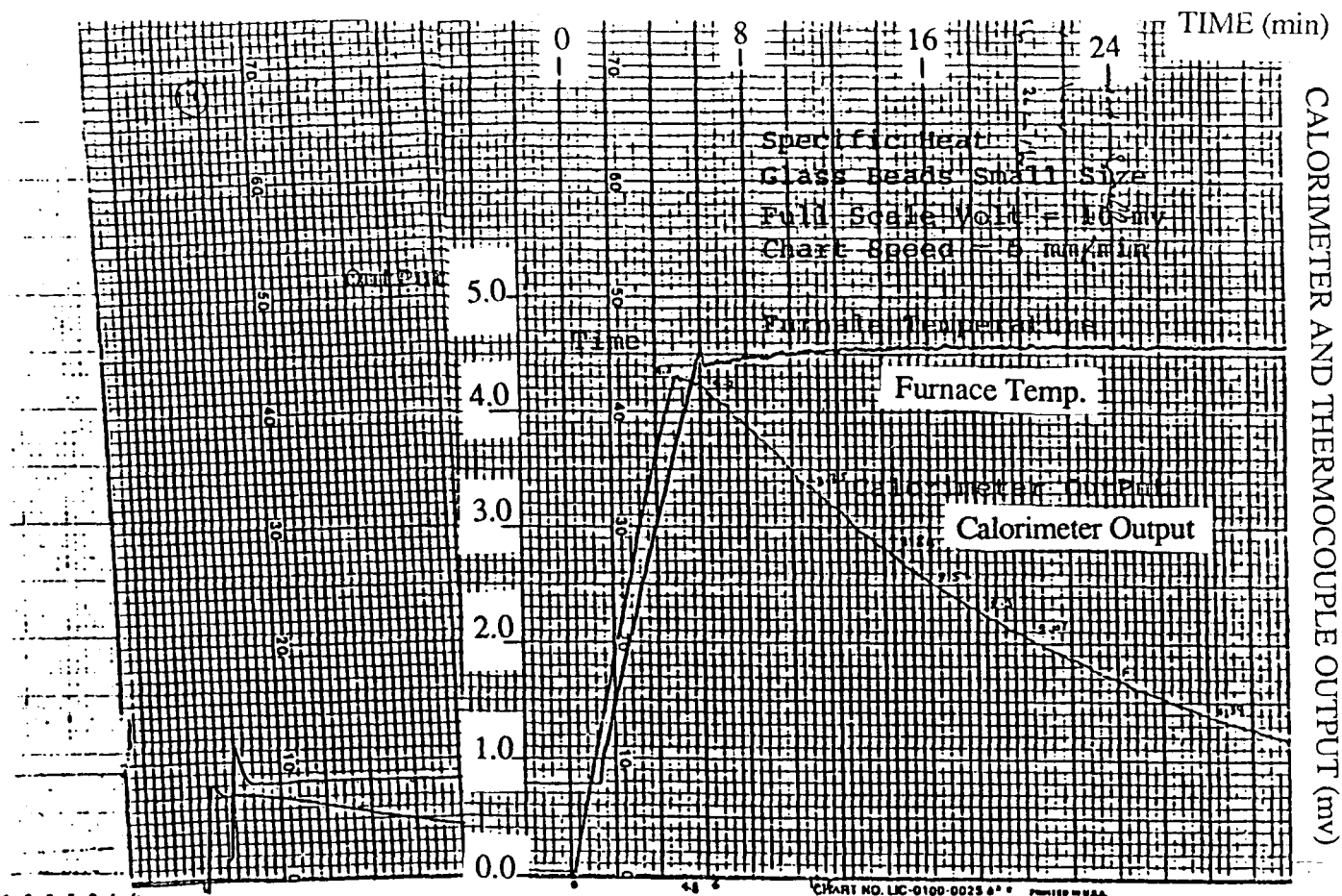
a. Recorded calorimeter output history for local sand.

Figure 4-10. Specific heat.

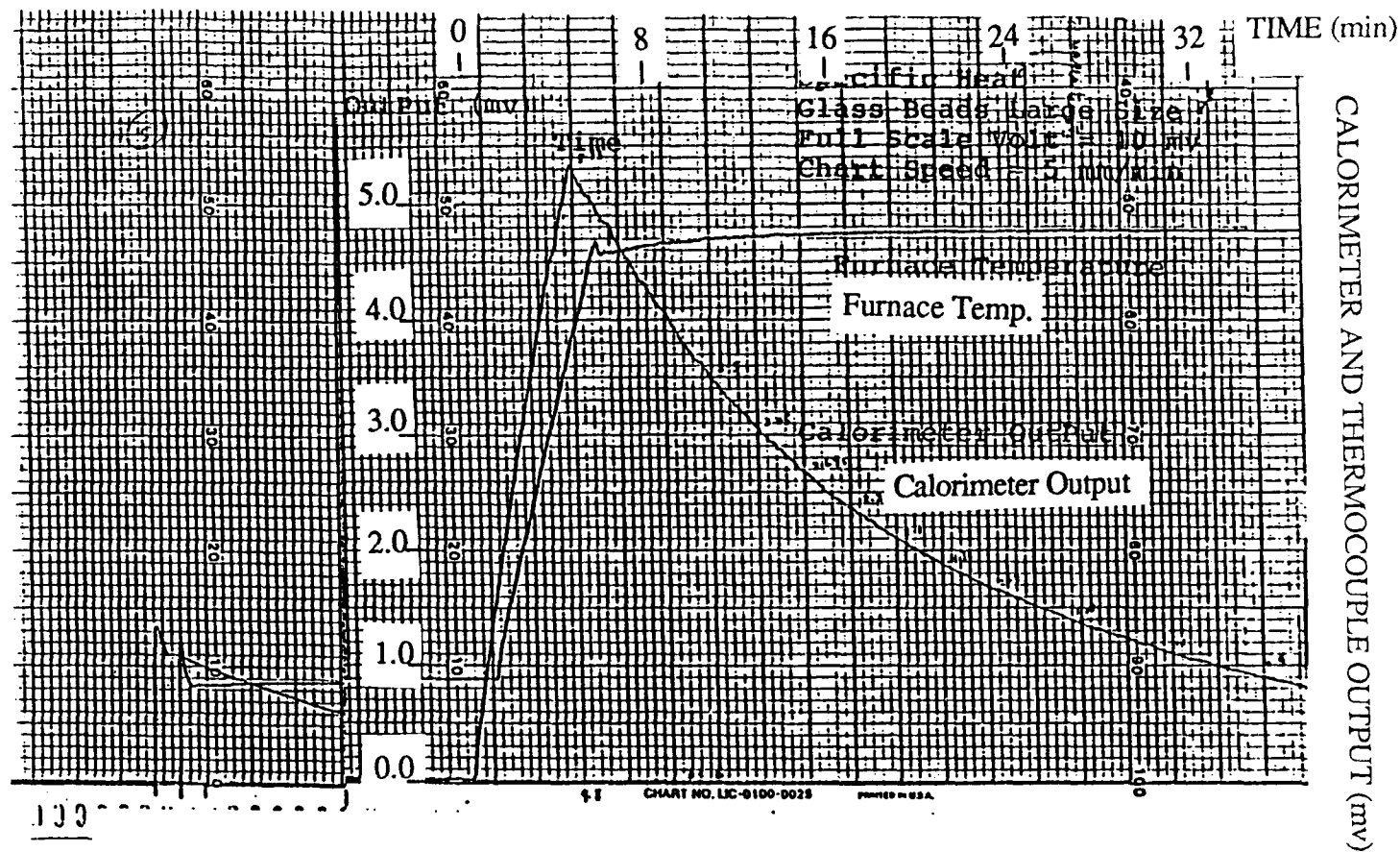


b. Recorded calorimeter output history for Ottawa sand.

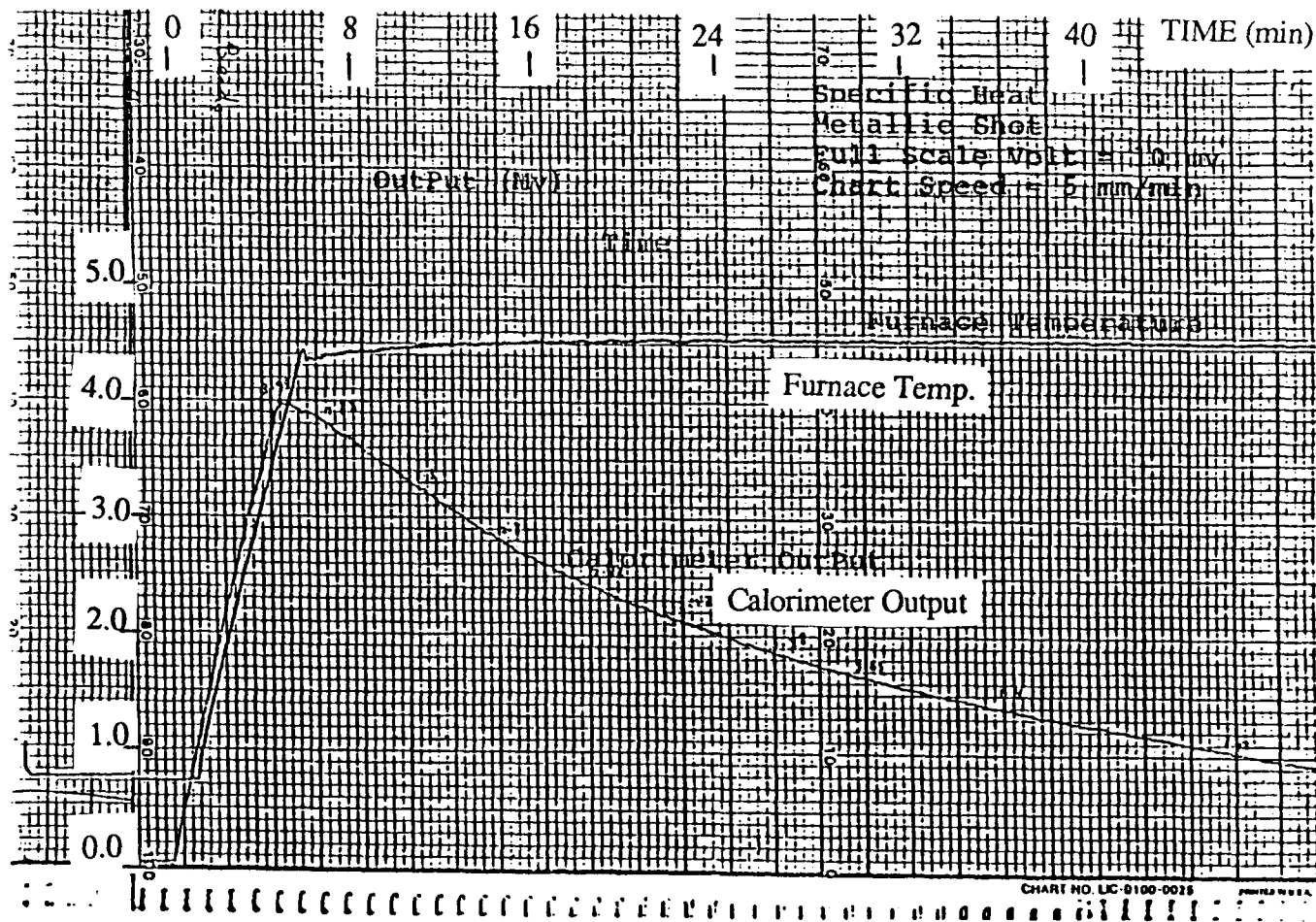
Figure 4-10. (continued)



c. Recorded calorimeter output history for small glass beads.



d. Recorded calorimeter output history for large glass beads.



CALORIMETER AND THERMOCOUPLE OUTPUT (mv)

e. Recorded calorimeter output history for metallic shot.

Figure 4-10. (continued)

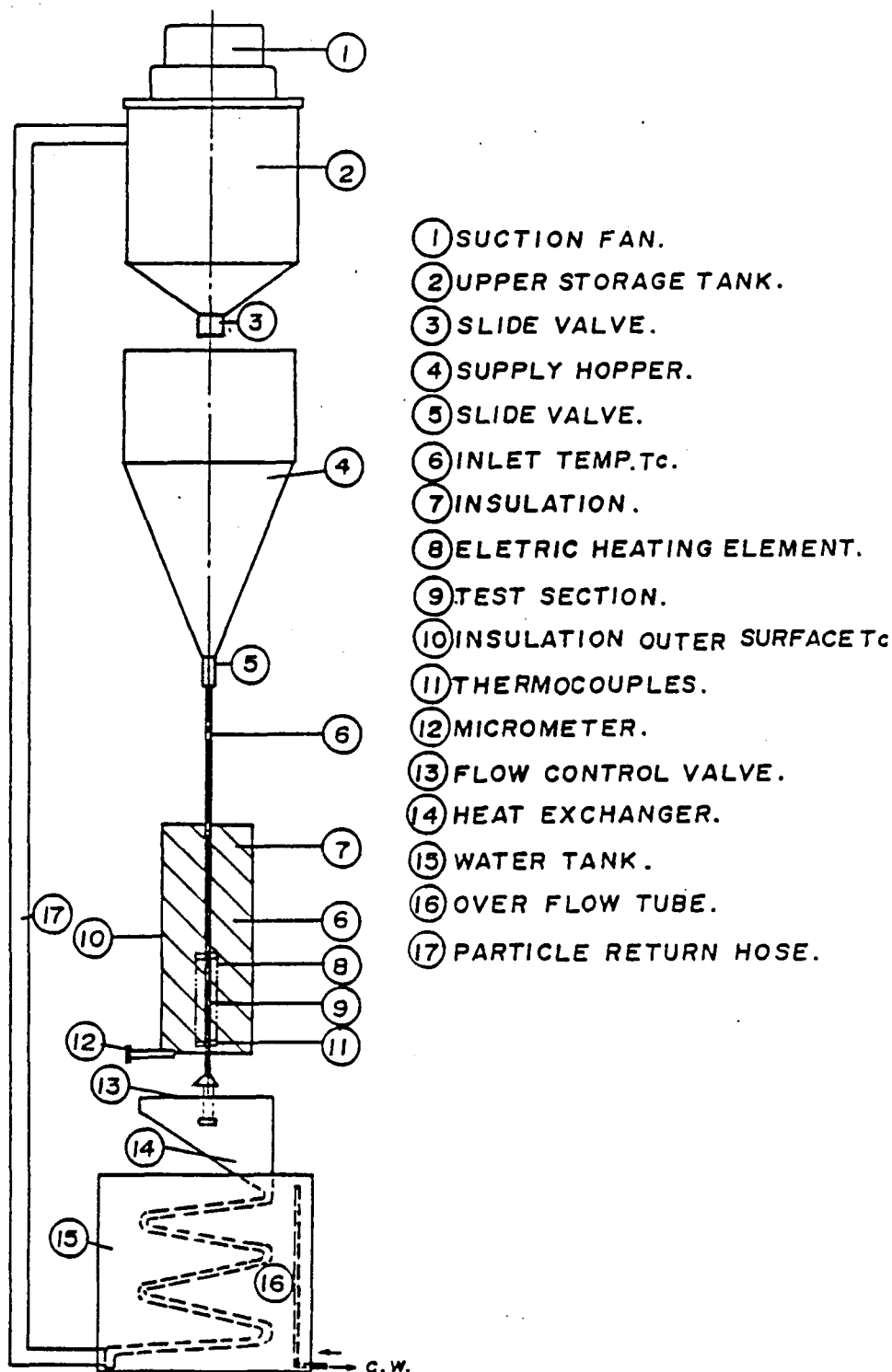


Figure 4-11. Schematic diagram of the test loop.

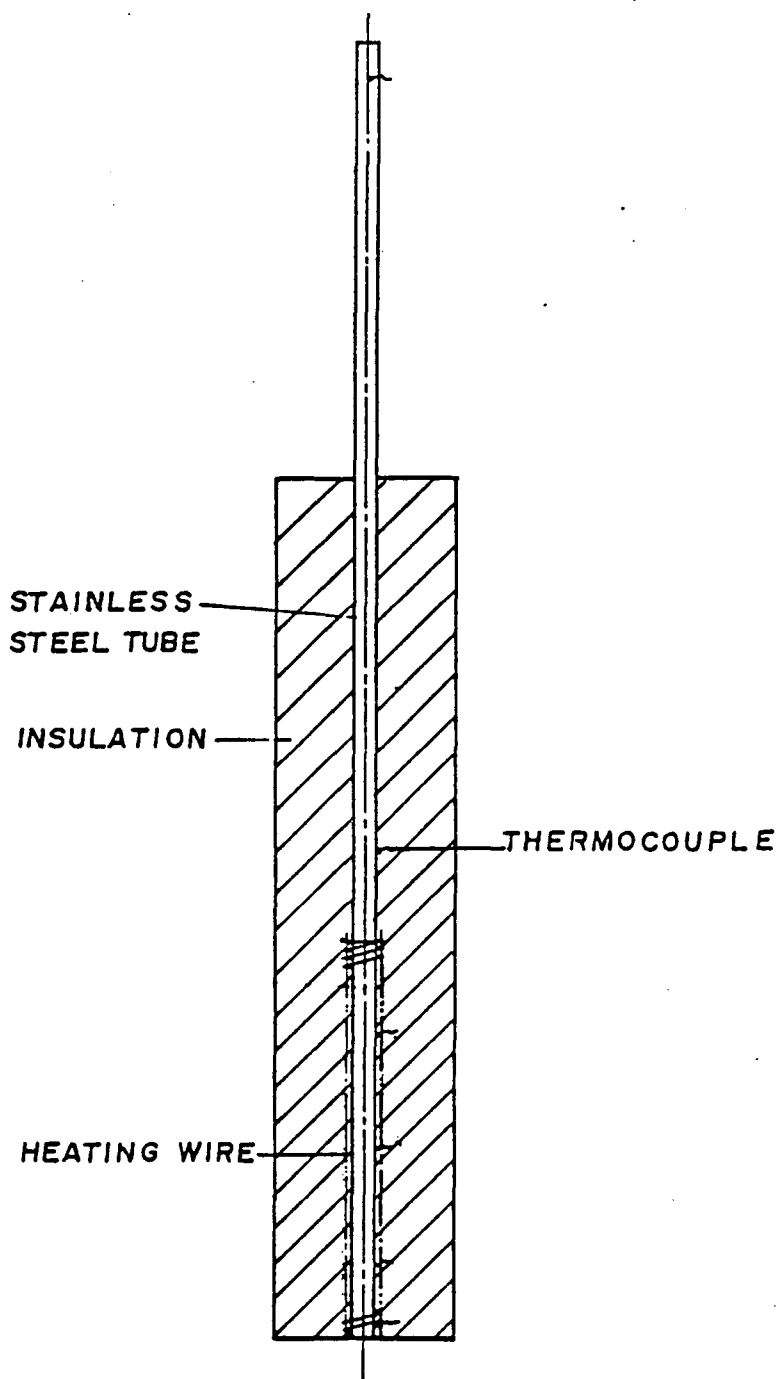
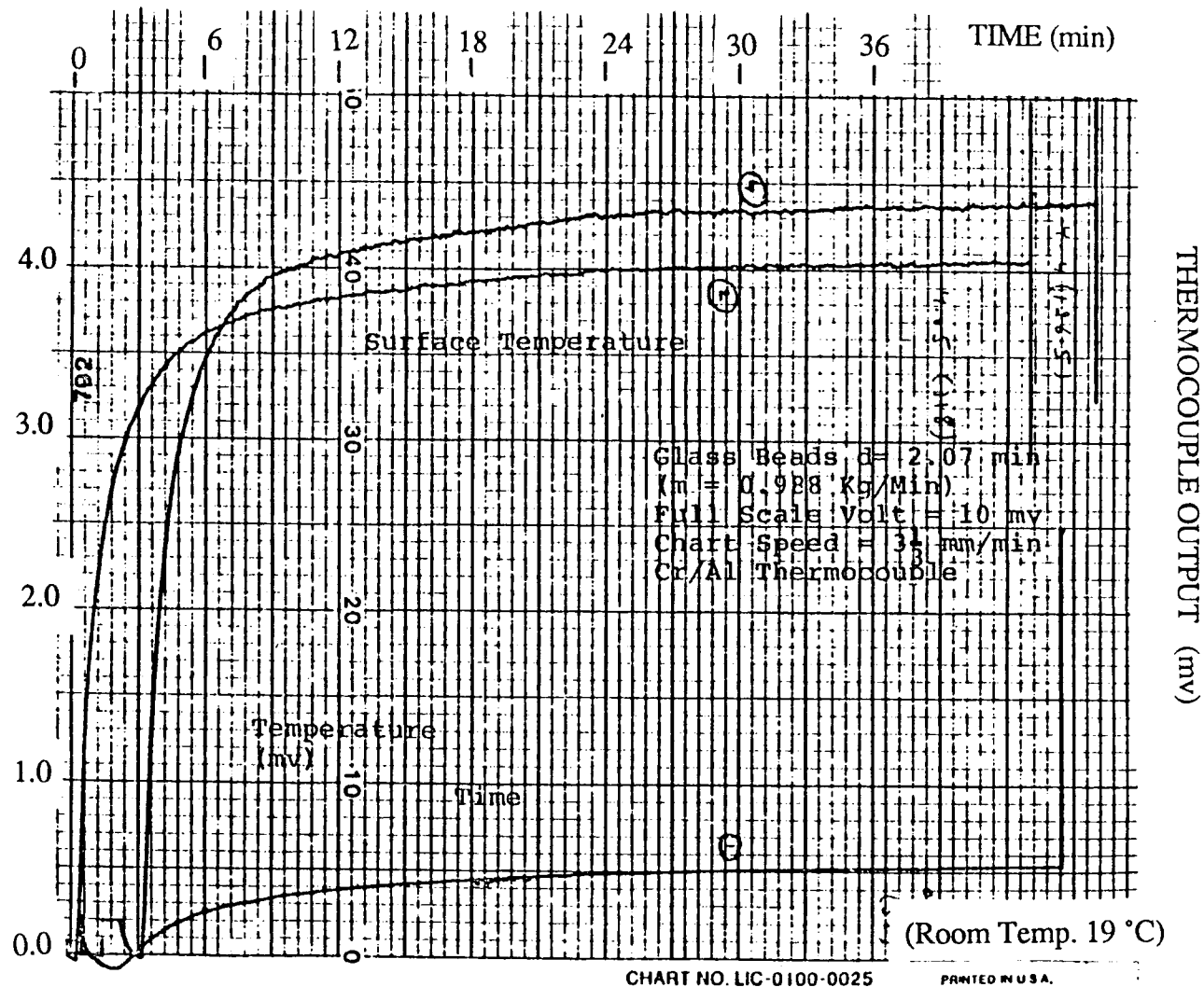


Figure 4-12. Schematic diagram of the test section.



a. Recorded tube surface temperature-time history for flowing glass beads.

Figure 4-13. Flowing glass beads $d=2.07$ mm.

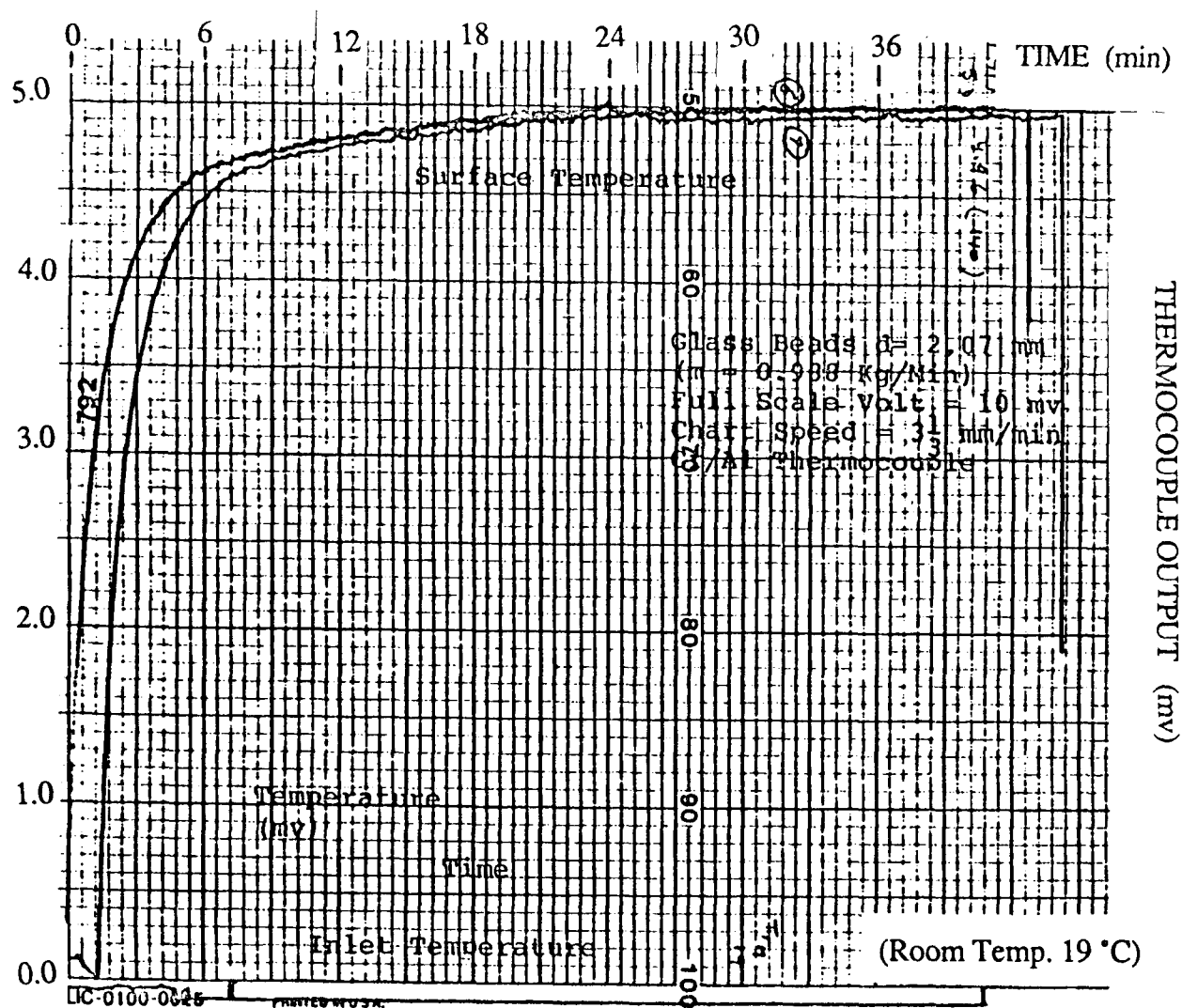
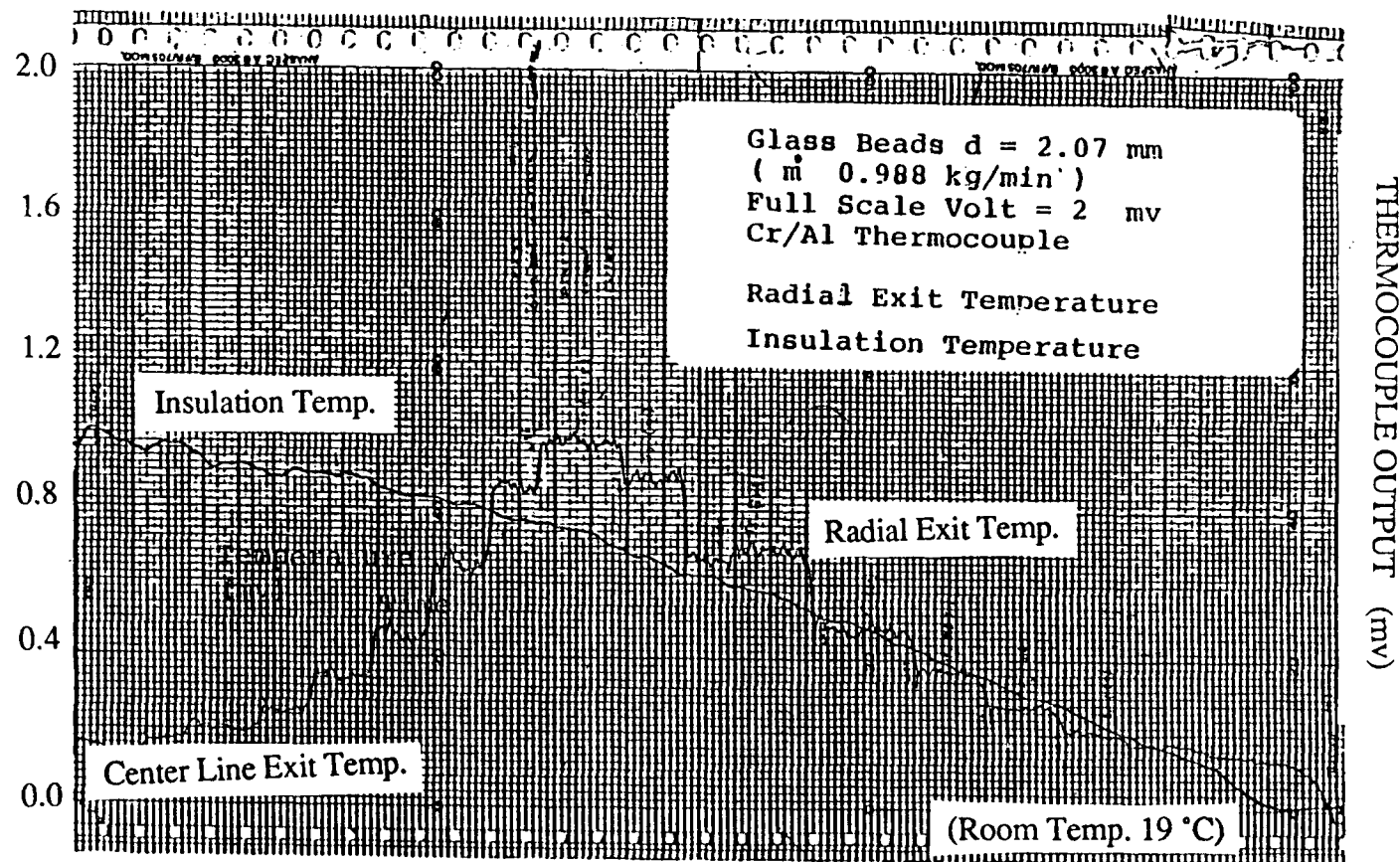


Figure 4-13. (continued)



c. Recorded radial exit temperature glass beads $d=2.07 \text{ mm}$.

Figure 4-13. (continued)

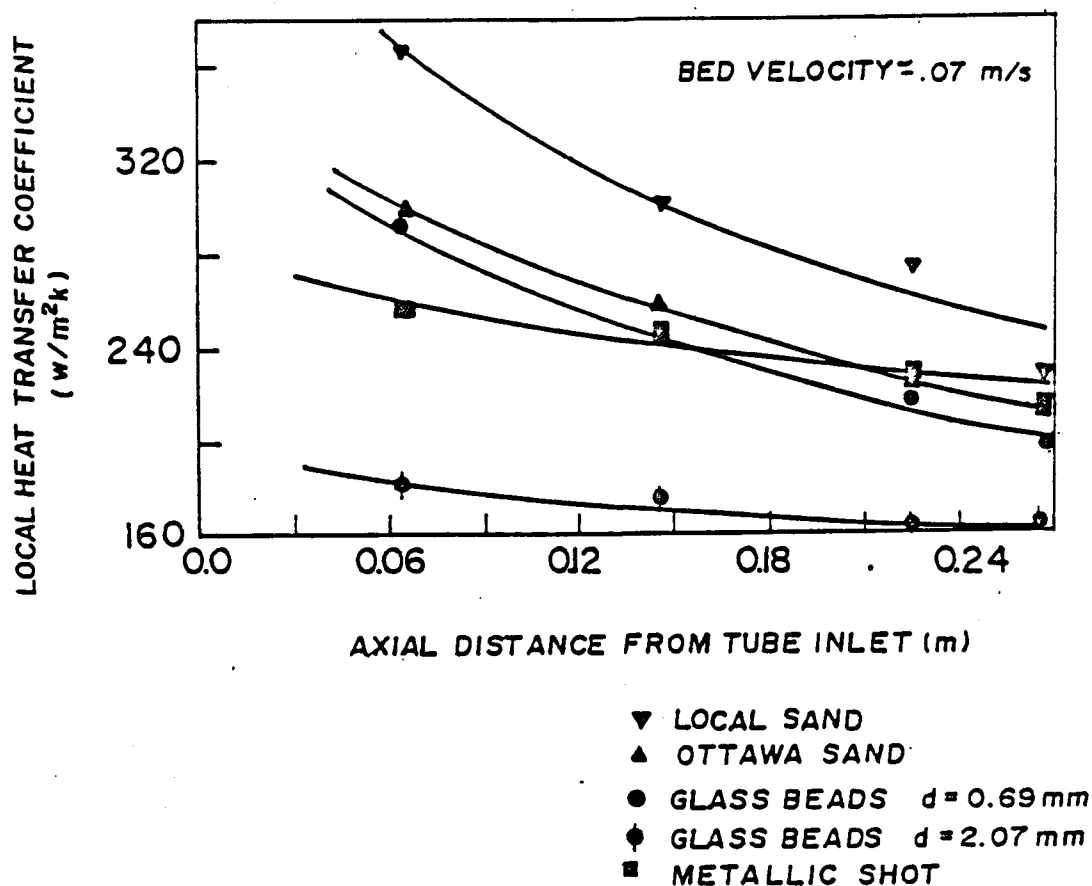
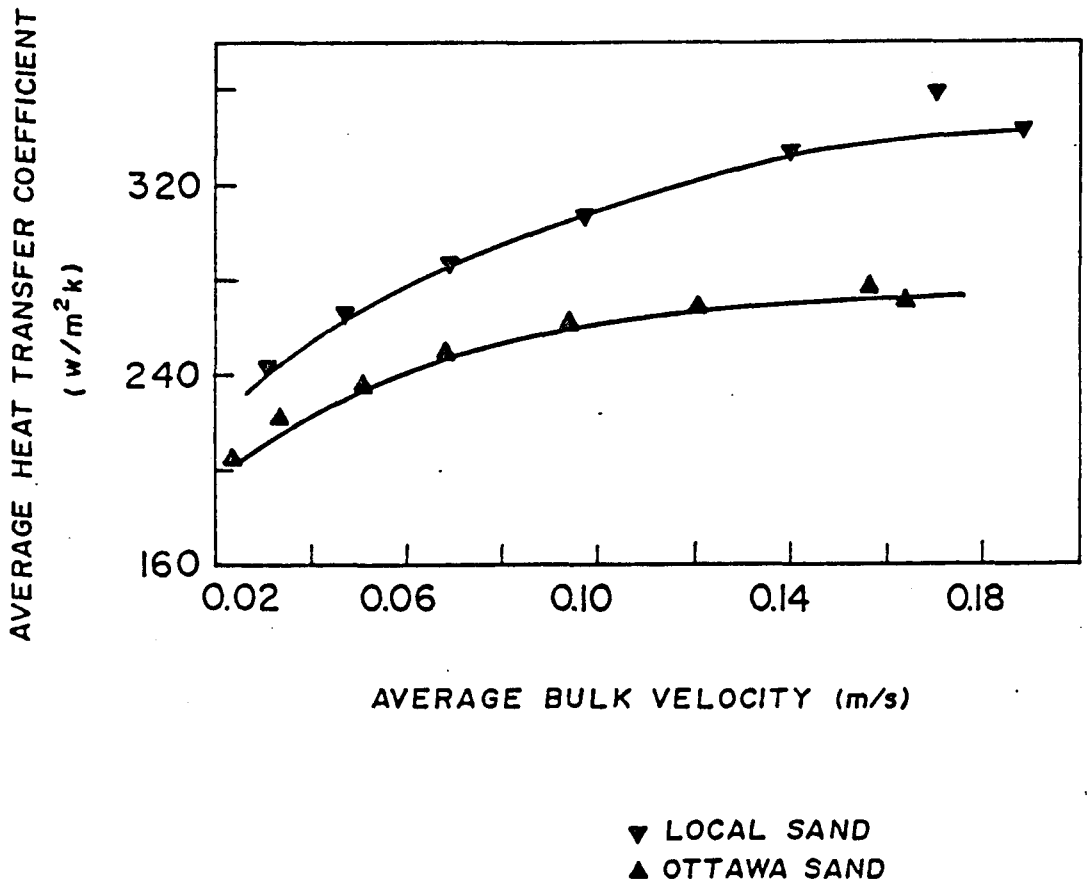
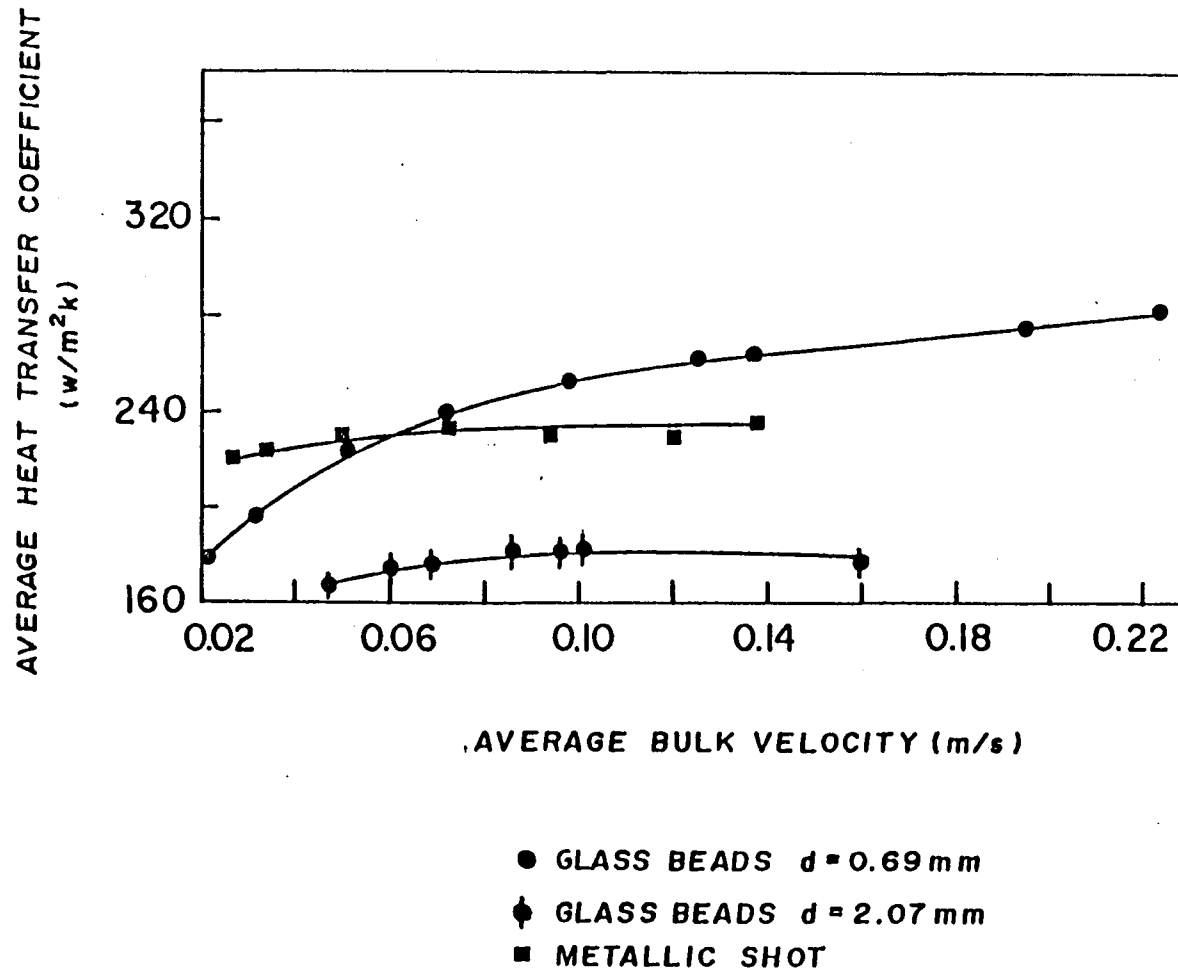


Figure 4-14. Local heat transfer coefficient along axial distance from tube inlet for the tested granular media.



- a. Average heat transfer coefficient vs. bulk velocity for local sand and Ottawa sand.

Figure 4-15. Average heat transfer coefficient.



b. Average heat transfer coefficient vs. bulk velocity for small glass beads, large glass beads, and metallic shot.

Figure 4-15: (continued)

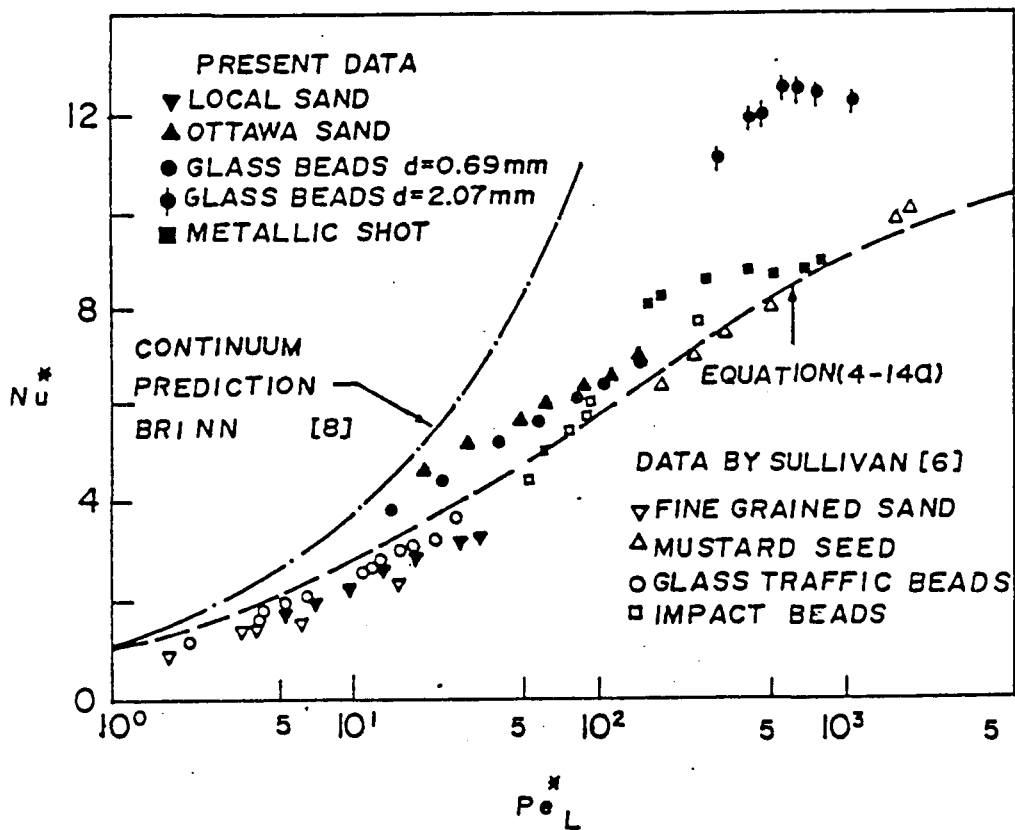
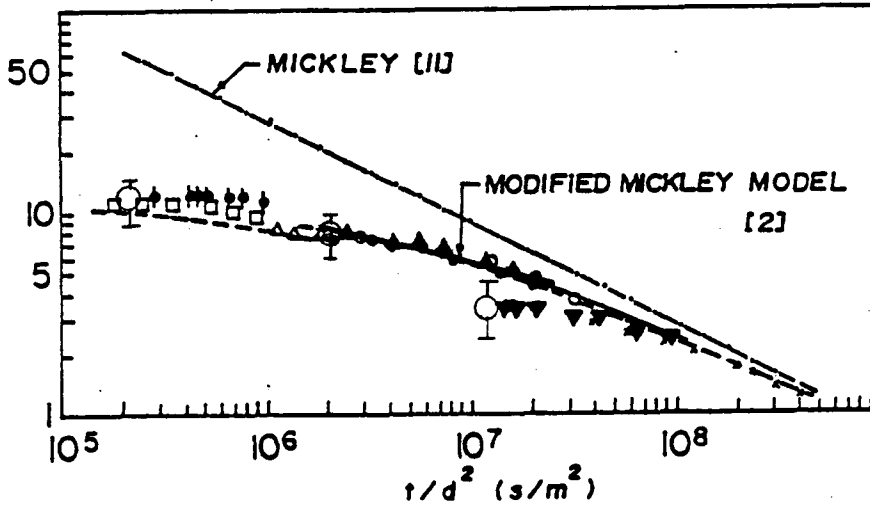


Figure 4-16. Modified Nusselt number versus modified Peclet number by Sullivan [6] compared with measurements for all materials tested.

AVERAGE NUSSELT NUMBER $h d / K g$



DATA BY [2]

- x SAND $d = 0.16 \text{ mm}$
- SAND $d = 0.59 \text{ mm}$
- △ SAND $d = 0.6 - 1.4 \text{ mm}$
- SAND $d = 2.37 \text{ mm}$

PRESENT DATA

- ▼ LOCAL SAND $d = 0.26 \text{ mm}$
- ▲ OTTAWA SAND $d = 0.71 \text{ mm}$
- GLASS BEADS $d = 0.69 \text{ mm}$
- GLASS BEADS $d = 2.07 \text{ mm}$

Figure 4-17.

Present measurements and data by Denloye [2] compared with Mickley [11] model and modified Mickley model [2].
 _____ Present Method

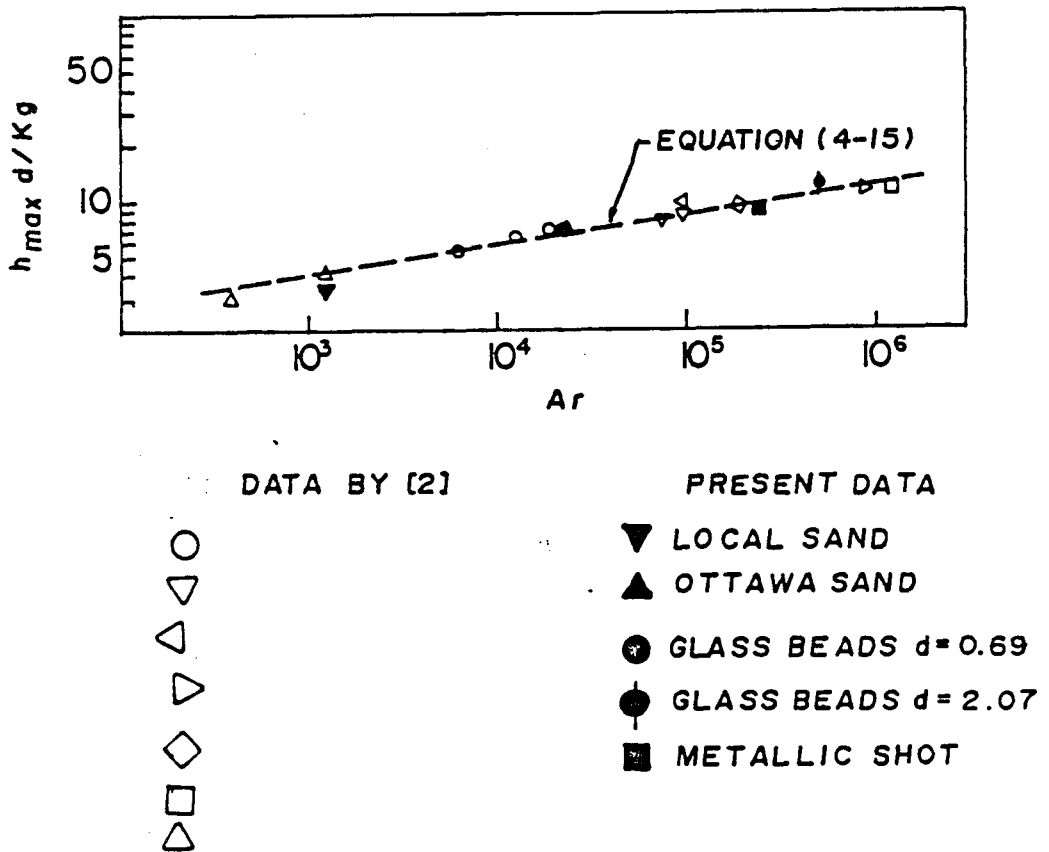


Figure 4-18. Correlation of maximum flowing bed to surface coefficients against Archimedes number by [2].

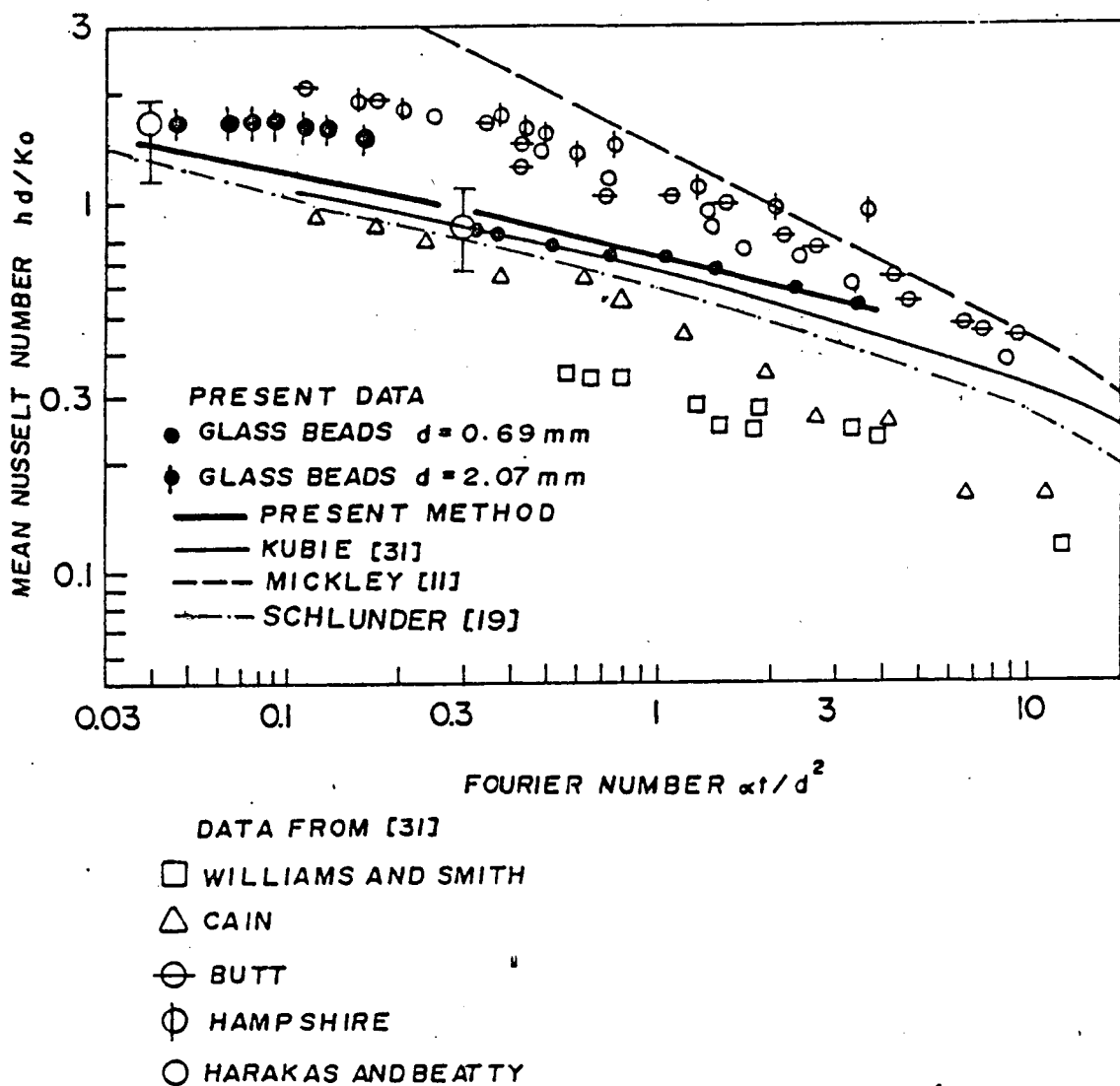


Figure 4-19. Comparison of present experimental data and theory with predictions by Kubie [31], Mickley [11], and data from [31] for mean heat transfer in the glass/air system.

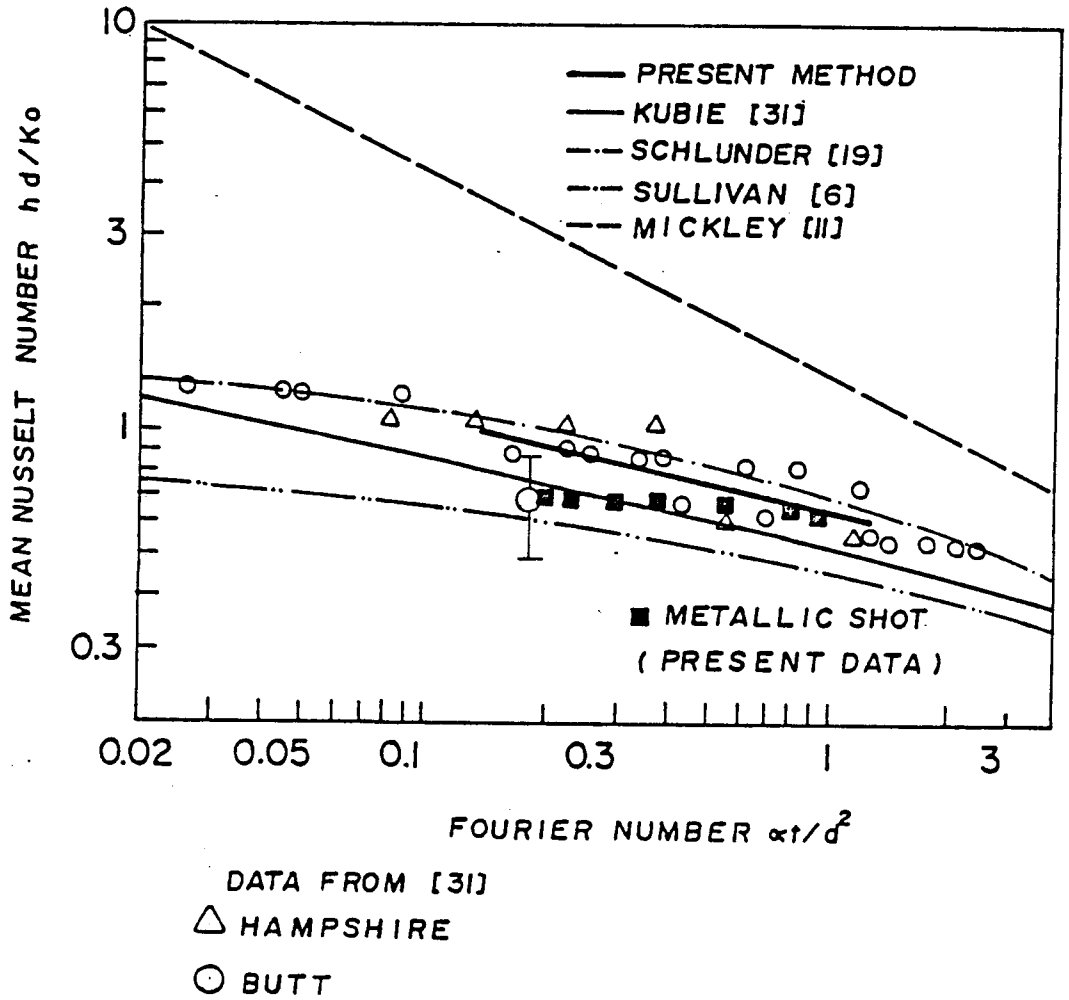
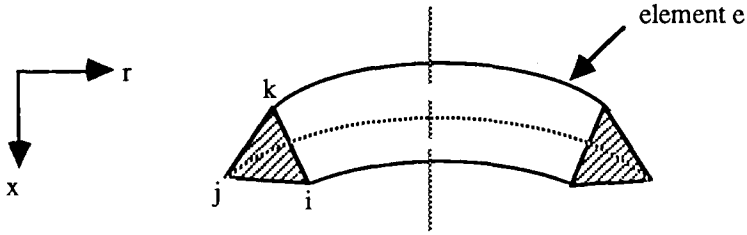


Figure 4-20.

Comparison of present experimental data and theory with other theoretical predictions and data for mean heat transfer in the steel/air system.

APPENDIX B
FINITE ELEMENT MATRICES

FINITE ELEMENTS MATRICES



Triangular ring element

The shape function matrix $[N]$ is given by

$$[N] = [N_i \ N_j \ N_k],$$

$$N_i = L_1, \ N_j = L_2, \ N_k = L_3 \quad (B-1)$$

where L_1 , L_2 , and L_3 are related to the global cylindrical coordinates (r, x) of nodes i , j , and k as:

$$\begin{Bmatrix} 1 \\ r \\ x \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ r_i & r_j & r_k \\ x_i & x_j & x_k \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \\ L_3 \end{Bmatrix} \quad (B-2)$$

or, equivalently,

$$\begin{Bmatrix} L_1 \\ L_2 \\ L_3 \end{Bmatrix} = (1/2A^e) \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{Bmatrix} 1 \\ r \\ x \end{Bmatrix} \quad (B-3)$$

where

$$\begin{aligned} a_1 &= r_j x_k - r_k x_j \\ a_2 &= r_k x_i - r_i x_k \\ a_3 &= r_i x_j - r_j x_i \\ b_1 &= x_j - x_k \\ b_2 &= x_k - x_i \\ b_3 &= x_i - x_j \\ c_1 &= r_k - r_j \\ c_2 &= r_i - r_k \\ c_3 &= r_j - r_i \end{aligned} \quad (B-4)$$

and A^e = area of the triangle ijk

$$= 0.5 [r_i (x_j - x_k) + r_j (x_k - x_i) + r_k (x_i - x_j)] \quad (B-5)$$

The gradient matrix [B] is given by

$$\begin{aligned} [B] &= \begin{bmatrix} \partial N_i / \partial r & \partial N_j / \partial r & \partial N_k / \partial r \\ \partial N_i / \partial x & \partial N_j / \partial x & \partial N_k / \partial x \end{bmatrix} \\ &= (1/2A^e) \begin{bmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \end{aligned} \quad (B-6)$$

The material property matrix [D] is given by

$$[D] = \begin{bmatrix} rk_r & 0 \\ 0 & rk_x \end{bmatrix} \quad (B-7)$$

The capacitance matrix, $[C^e]$, is defined by equation (3-14-a) as

$$[C^e] = \int_v \rho c_p [N]^T [N] dv \quad (3-14-a)$$

By expressing dv as $2\pi r dA$, where dA is the differential area of the triangle ijk , and $[N]$ by equation (B-1);

$$[C^e] = (\rho c_p)^e 2\pi \int_{A^e} \begin{bmatrix} L_1^2 & L_1 L_2 & L_1 L_3 \\ L_1 L_2 & L_2^2 & L_2 L_3 \\ L_1 L_3 & L_2 L_3 & L_3^2 \end{bmatrix} (r_i L_1 + r_j L_2 + r_k L_3) dA \quad (B-8)$$

where r has been substituted as

$$r = r_i L_1 + r_j L_2 + r_k L_3 \quad (B-9)$$

By using the integration formula for natural coordinates, namely,

$$\int_{A^e} L_1^a L_2^b L_3^c dA = \frac{a! b! c!}{(a+b+c+2)!} 2A^e \quad (B-10)$$

Equation (B-8) will be

$$[C^e] = \frac{\pi (\rho c_p)^e A^e}{30} \begin{bmatrix} (6r_i+2r_j+2r_k) & (2r_i+2r_j+r_k) & (2r_i+r_j+2r_k) \\ (2r_i+6r_j+2r_k) & (r_i+2r_j+2r_k) & (2r_i+2r_j+6r_k) \\ \text{symmetric} & & \end{bmatrix} \quad (B-11)$$

The conductance matrix, $[K^e]$, is defined by equation (3-14-b), by writing $dV = 2\pi r \, dA$,

$$[K^e] = 2\pi \int_V [B]^T [D] [B] dA \quad (3-14-b)$$

utilizing equations (B-6), (B-7), (B-9), and (B-10)

$$[K^e] = \frac{\pi k_r R^2}{2A^e} \begin{bmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3^2 \end{bmatrix} + \frac{\pi k_x R^2}{2A_e} \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_3 & c_2 c_3 & c_3^2 \end{bmatrix} \quad (B-12)$$

where

$$R^2 = \int_{A^e} r^2 dA = 1/12 (r_i \ r_j \ r_k) \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{Bmatrix} r_i \\ r_j \\ r_k \end{Bmatrix} \quad (B-13)$$

For simplicity, if the centroidal values of rk_r and rk_x , namely, $r_c k_r$ and $r_c k_x$ in place of rk_r and rk_x respectively in matrix $[D]$, equation (B-12) gives an approximate expression for $[K^e]$ with R^2 replaced by r_c^2

where

$$r_c = (r_i + r_j + r_k)/3 \quad (B-14)$$

The force vector matrix, $\{F^e\}$, is defined by equation (3-14-c) as

$$\{F^e\} = \int_s q [N]^T ds \quad (3-14-c)$$

utilizing equations (B-1) to (B-5), $ds = 2\pi r ds$, and the integration formula

$$\int_{s_i}^{s_j} L_1^a L_2^b ds = (s_j - s_i) \frac{a! b!}{(a+b+1)!} \quad (B-15)$$

Equation (3-14-c) gives

$$\{F^e\} = \begin{cases} \frac{\pi q s_{ji}}{3} \begin{Bmatrix} (2r_i + r_j) \\ (r_i + 2r_j) \\ 0 \end{Bmatrix} & \text{if the edge } ij \text{ lies on } s \\ \frac{\pi q s_{kj}}{3} \begin{Bmatrix} 0 \\ (2r_j + r_k) \\ (r_j + 2r_k) \end{Bmatrix} & \text{if the edge } jk \text{ lies on } s \\ \frac{\pi q s_{ik}}{3} \begin{Bmatrix} 0 \\ (2r_i + r_k) \\ (r_i + 2r_k) \end{Bmatrix} & \text{if the edge } k_i \text{ lies on } s \end{cases} \quad (B-16)$$

APPENDIX C

COMPUTER PROGRAM OF ANALYTICAL ANALYSES

```

C-----
C
C
C      FLOWING GRANULAR MEDIA; FINITE ELEMENT ANALYSIS
C
C-----

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C=====
C      DETERMINATION OF TEMPERATURE DISTRIBUTION IN AXISYMMETRIC FIELD
C      PROBLEM.
C=====

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```

      INTEGER EDGE(144)
      REAL NKR(146), NKZ(146), LMTD, NUSD, NUB, NUIB, NUIG, NUMZB, NUMZG
      DIMENSION LOC(144,3), R(146), Z(146), TS(146), TINF(144), H(144)
      & , Q(144), QD(144), GS(146,4), TEMP(146,1), TO(146), A(144)
      & , ROC(146)
      & , TR(73), RD(73), FUN(73)
      & , VELO(9), IPRNT(17), TMD(17), DIS(17), HIS(17), HMZ(17)
      COMMON /B1/DTIM
      DATA NE, NN, NB/144, 146, 4/
      DATA IPRNT /10, 20, 30, 40, 50, 60, 80, 100, 200, 300, 400, 500,
      &          650, 1450, 2250, 2650, 2700/
      DATA VELO /.001, .01, .1, .2, .5, 1., 2., 5., 10./
      HTF=17600.
      TIN=20.
      DLTZ=0.1E-3

```

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C-----PHYSICAL AND THERMAL PROPERTIES OF MATERIALS.

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C
C
C      3  WRITE(6,123)
C123  FORMAT(1H1,30X,'--- GLASS BEADS OF MEAN DIAMETER 0.69 MM ---')
C      D=0.69E-3
C      THKS=0.850
C      CPS=765.
C      DNSS=2700.
C      VOID=0.41
C      GAMA=0.5
C      GO TO 7
C
C      4  WRITE(6,124)
C124  FORMAT(1H1,30X,'--- OTAWA SAND OF MEAN DIAMETER 0.71 MM ---')
C      D=0.71E-3
C      THKS=1.87
C      CPS=860.
C      DNSS=2600.
C      VOID=0.41
C      GAMA=2./3.
C      GO TO 7

```

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C-----SPECIFICATION OF TEST SECTION.

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FET00010
FET00020
FET00030
FET00040
FET00050
FET00060
FET00070
FET00080
FET00090
FET00100
FET00110
FET00120
FET00130
FET00140
FET00150
FET00160
FET00170
FET00180
FET00190
FET00200
FET00210
FET00220
FET00230
FET00240
FET00250
FET00260
FET00270
FET00280
FET00290
FET00300
FET00310
FET00320
FET00330
FET00340
FET00350
FET00360
FET00370
FET00380
FET00390
FET00400
FET00410
FET00420
FET00430
FET00440
FET00450
FET00460
FET00470
FET00480
FET00490
FET00500
FET00510
FET00520
FET00530
FET00540
FET00550

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7  DI=14.4E-3
   OR=DI/2.
   DO=15.87E-3
   XAB=27.E-2
   XA2=6.5E-2
   XA4=14.5E-2
   XA6=22.5E-2
   XA7=26.5E-2
   X12=8.0E-2
C
C-----PROPERTIES OF GAS (AIR).
C
C   THKG=0.026
C   DNSG=1.19
C   CPG=1008.
C
C-----PROPERTIES OF GAS (CO2).
C
C   THKG=0.016
C   CPG=850.
C   DNSG=1.79
C
C-----PROPERTIES OF GAS (HELIUM)
C
C   THKG=0.148
C   CPG=5200
C   DNSG=.165
C
C-----LOCALIZATION OF GLOBAL NODES TO EACH ELEMENT.
C
C   LOC(1,1)=1
C   LOC(1,2)=4
C   LOC(1,3)=2
C   LOC(2,1)=4
C   LOC(2,2)=1
C   LOC(2,3)=3
C   DO 10 J=1,3
C   DO 10 I=3,NE
C   JJ=I-2
C   LOC(I,J)=LOC(JJ,J)+2
10  CONTINUE
C   WRITE(6,92)
C   DO 15 I=1,NE
C   WRITE(6,93) I, LOC(1,1), LOC(1,2), LOC(1,3)
C 15  CONTINUE
C 92  FORMAT(1H1,5X, ' -- ELEMENT NO. AND ITS GLOBAL NODE NO. -- ',//)
C 93  FORMAT(5X,4(14,5X))
C
C----- SPECIFYING THE REDIAL AND AXIAL COORDINATES OF NODES.
C
C   R(1)=0.0
C   R(2)=0.0
C   DO 20 I=3,NN,2
C   JJ=I-2
C   JK=I+1

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FET00560
FET00570
FET00580
FET00590
FET00600
FET00610
FET00620
FET00630
FET00640
FET00650
FET00660
FET00670
FET00680
FET00690
FET00700
FET00710
FET00720
FET00730
FET00740
FET00750
FET00760
FET00770
FET00780
FET00790
FET00800
FET00810
FET00820
FET00830
FET00840
FET00850
FET00860
FET00870
FET00880
FET00890
FET00900
FET00910
FET00920
FET00930
FET00940
FET00950
FET00960
FET00970
FET00980
FET00990
FET01000
FET01010
FET01020
FET01030
FET01040
FET01050
FET01060
FET01070
FET01080
FET01090
FET01100

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      R(I)=R(JJ)+0.0001
      R(JK)=R(I)
20    CONTINUE
      DO 30 I=1,NN,2
      Z(I)=0.0
      KK=I+1
      Z(KK)=0.0001
30    CONTINUE
C
C----- SPECIFYING NODE AND ELEMENT BOUNDARY CONDITIONS.
C
      DO 40 I=1,NN
40    TS(I)=-1.0E+6
      C      EDG(I)=1 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 1-2
      C      EDG(I)=2 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 2-3
      C      EDG(I)=3 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 3-1
      C      EDG(I)=4 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 1-2 AND 2-3
      C      EDG(I)=5 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 2-3 AND 3-1
      C      EDG(I)=6 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 3-1 AND 1-2
      DO 50 I=1,NE
50    EDGE(I)=0.0
      EDGE(144)=3
      DO 60 I=1,NE
      Q(I)=0.0
      QD(I)=0.0
      H(I)=0.0
      TINF(I)=0.0
60    CONTINUE
C
C----- SPECIFYING THERMAL CONDUCTIVITIES AND HTF. B.C.
C
      DO 300 IV=1,9
      C
      VEL=VELO(IV)
      Q(144)=-HTF
      DO 65 I=1,NN
      TO(I)=0.0
      TO(I)=TIM
65    CONTINUE
      DTIM=DLTZ/VEL
      TIM=XAB/VEL
      NTIM=TIM/DTIM+1
      TIME=DTIM
      IC=1
      SN=3.0*((5.01-8.42*VOID)/(1.91-1.91*VOID))
      SIN2=1.0/SN
      COS=SQRT(1.0-1.0/SN)
      ROCB=DNSS*CPS*(1.0-VOID)+DNSG*CPS*VOID
      C      ROCB=DNSS*CPS*(1.0-VOID)
      DO 85 ITIM=1,NTIM
      T1=0.5*(1.0-THKG/THKS)**2*SIN2
      T2=ALOG(THKS/THKG-(THKS/THKG-1.0)*COS)-(1.0-THKG/THKS)*(1.0-COS)
      ALBA=T1/T2
      C      T3=THK/THKG-1.0-(1.-VOID)*(1.0-THKG/THKS)
      C      T4=(1.-VOID)*(1.0-THKG/THKS)**2

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FETO1110
 FETO1120
 FETO1130
 FETO1140
 FETO1150
 FETO1160
 FETO1170
 FETO1180
 FETO1190
 FETO1200
 FETO1210
 FETO1220
 FETO1230
 FETO1240
 FETO1250
 FETO1260
 FETO1270
 FETO1280
 FETO1290
 FETO1300
 FETO1310
 FETO1320
 FETO1330
 FETO1340
 FETO1350
 FETO1360
 FETO1370
 FETO1380
 FETO1390
 FETO1400
 FETO1410
 FETO1420
 FETO1430
 FETO1440
 FETO1450
 FETO1460
 FETO1470
 FETO1480
 FETO1490
 FETO1500
 FETO1510
 FETO1520
 FETO1530
 FETO1540
 FETO1550
 FETO1560
 FETO1570
 FETO1580
 FETO1590
 FETO1600
 FETO1610
 FETO1620
 FETO1630
 FETO1640
 FETO1650

C	GAMA=ALBA*T3/T4	FET01660
	DO 35 I=1,NN	FET01670
	WRGB=0.0072-D	FET01680
	IF(R(I).GE. WRGB) GO TO 36	FET01690
	ROC(I)=ROCB	FET01700
	THKB=THKG*(1.+(1.-VOID)*(1.-THKG/THKS)+GAMA/ALBA*	FET01710
	& (1.-THKG/THKS)**2*(1.-VOID))	FET01720
	NKR(I)=THKB	FET01730
	NKZ(I)=NKR(I)	FET01740
	GO TO 35	FET01750
36	XOD=(0.0072-R(I))/D	FET01760
	VOIDX=1.0-3.0*(1.0-VOID)*(XOD-(2./3.)*XOD**2)	FET01770
	ROC(I)=DNSS*CPS*(1.0-VOIDX)+DNSS*CPG*VOIDX	FET01780
C	ROC(I)=DNSS*CPS*(1.0-VOIDX)	FET01790
C		FET01800
C	-----EQ. {8} WILLHITE, KUNII & SMITH A.I. CH. E. J V. 8 NO. 3 P. 340 JULY, 1962	FET01810
C	NKR(I)=THKG*(1.+(1.-VOIDX)*(1.-THKG/THKS)+GAMA/ALBA*	FET01820
	& (1.-THKG/THKS)**2*(1.-VOIDX))	FET01830
	NKZ(I)=NKR(I)	FET01840
35	CONTINUE	FET01850
C		FET01860
C	----- SOLUTION AND PRINT OUT OF DIFFERENT VARIABLES.	FET01870
C		FET01880
	CALL HEATAX (LOC,R,Z,NN,NE,NB,Q,QD,H,TINF,TS,EDGE,GS	FET01890
	& ,TEMP,TO,A,ROC,NKR,NKZ)	FET01900
	DLTR=OR/(NE/2)	FET01910
	NM1=NN-1	FET01920
	TWAL=(TEMP(NN,1)+TEMP(NM1,1))/2.0	FET01930
	DO 210 J=1,NM1,2	FET01940
	IJ=(J+1)/2	FET01950
	TR(IJ)=(TEMP(J,1)+TEMP(J+1,1))/2.0	FET01960
	RD(IJ)=(R(J)+R(J+1))/2.0	FET01970
	FUN(IJ)=TR(IJ)*RD(IJ)	FET01980
210	CONTINUE	FET01990
	NH=NN/2	FET02000
	NHM1=NH-1	FET02010
	SUN=0.0	FET02020
	DO 220 K=2,NHM1	FET02030
	SUN=SUN+FUN(K)	FET02040
220	CONTINUE	FET02050
	ANT=(DLTR/2.0)*(FUN(1)+FUN(NH)+2.0*SUN)	FET02060
	TMEAN=(2.0/(OR*OR))*ANT	FET02070
	TFILM=(TMEAN+TWAL)/2.	FET02080
	HZ=HTF/(TWAL-TMEAN)	FET02090
	NU1B=HZ*D/THKB	FET02100
	NU1G=HZ*D/THKG	FET02110
	ALFA=THKB/ROCB	FET02120
	THKBOG=THKB/THKG	FET02130
	FOR1=ALFA*TIME/(D**2)	FET02140
	TIME=TIME+DTIM	FET02150
	TIMEP=TIME-DTIM	FET02160
	DIST=TIMEP*VEL	FET02170
	IF(ITIM.NE. IPRNT(IC)) GO TO 85	FET02180
	DIS(IC)=DIST	FET02190
		FET02200

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97  FORMAT(/      , ' MEAN HEAT TR.CO.(W/M2K) = ', F10.3,/      FET02760
&      , ' MEAN NUSSELT NO.(HMD/KG)= ', F10.3,/      FET02770
&      , ' MDEFIED PECLET NO. = ', F10.3,/      FET02780
&      , ' MEAN NUSSELT NO.(HMD/KB)= ', F10.4,/      FET02790
&      , ' MEAN FOURIER NO.(FTA/D2)= ', F10.4,/      FET02800
&      , ' RESD.TIME/D**2 (SEC/M2 )= ', E15.7,/      FET02810
98  FORMAT(3X,'GAS COND. =', F8.5,6X,'ALFA =', E15.7,6X,'ALBA =', F7.3/) FET02820
99  FORMAT(3X,'BED COND. =', F8.5,6X,'THKB/THKG =', F7.3/) FET02830
    STOP FET02840
    END FET02850
===== FET02860
C FET02870
C SUBROUTINE HEATAX FET02880
C FET02890
C===== FET02900
C SUBROUTINE HEATAX (LOC,R,Z,NN,NE,NB,Q,QD,H,TINF,TS,EDGE,GS FET02910
& , PLOAD,TO,A,ROC,NKR,NKZ) FET02920
  INTEGER EDGE(NE) FET02930
  REAL NKR(NN),NKZ(NN) FET02940
  DIMENSION GS(NN,NB),PLOAD(NN,1),LOC(NE,3),R(NN),Z(NN),Q(NE),QD(NE) FET02950
& , H(NE),TINF(NE),A(NE),GK1(3,3),GK2(3,3),P1(3),P2(3),P3(3),TS(NN) FET02960
& , GK3(3,3),B(3,3),TOE(3),P4(3),TO(NN),ROC(NN) FET02970
  COMMON /B1/DTIM FET02980
  DOUBLE PRECISION DIFF(1) FET02990
  PI=3.1416 FET03000
  DO 10 I=1,NN FET03010
    PLOAD(I,1)=0.0 FET03020
    DO 10 J=1,NB FET03030
      GS(I,J)=0.0 FET03040
10  CONTINUE FET03050
    DO 100 I=1,NE FET03060
      N1=LOC(I,1) FET03070
      N2=LOC(I,2) FET03080
      N3=LOC(I,3) FET03090
      R1=R(N1) FET03100
      RJ=R(N2) FET03110
      RK=R(N3) FET03120
      Z1=Z(N1) FET03130
      ZJ=Z(N2) FET03140
      ZK=Z(N3) FET03150
      EKRI=NKR(N1) FET03160
      EKRJ=NKR(N2) FET03170
      EKRK=NKR(N3) FET03180
      EKZI=NKZ(N1) FET03190
      EKZJ=NKZ(N2) FET03200
      EKZK=NKZ(N3) FET03210
      ROCI=ROC(N1) FET03220
      ROCJ=ROC(N2) FET03230
      ROCK=ROC(N3) FET03240
      B1=ZJ-ZK FET03250
      B2=ZK-ZI FET03260
      B3=ZI-ZJ FET03270
      C1=RK-RJ FET03280
      C2=R1-RK FET03290
      C3=RJ-R1 FET03300

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A(1)=(R1*(ZJ-ZK)+RJ*(ZK-ZI)+RK*(ZI-ZJ))/2.0
A(1)=ABS(A(1))
RC=(R1+RJ+RK)/3.0
ROCE=(ROC1+ROCJ+ROCK)/3.0
EKR=(EKRI+EKRJ+EKRK)/3.0
EKZ=(EKZI+EKZJ+EKZK)/3.0
GK1(1,1)=EKR*B1**2+EKZ*C1**2
GK1(1,2)=EKR*B1*B2+EKZ*C1*C2
GK1(1,3)=EKR*B1*B3+EKZ*C1*C3
GK1(2,2)=EKR*B2**2+EKZ*C2**2
GK1(2,3)=EKR*B2*B3+EKZ*C2*C3
GK1(3,3)=EKR*B3**2+EKZ*C3**2
GK1(2,1)=GK1(1,2)
GK1(3,1)=GK1(1,3)
GK1(3,2)=GK1(2,3)
DO 20 M=1,3
DO 20 N=1,3
GK1(M,N)=PI*RC*   GK1(M,N)/(2.0*A(1))
GK2(M,N)=0.0
GK3(M,N)=0.0
B(M,N)=0.0
20 CONTINUE
D=PI*A(1)*ROCE/30.0
GK3(1,1)=D*(6.0*R1+2.0*RJ+2.0*RK)
GK3(1,2)=D*(2.0*R1+2.0*RJ+   RK)
GK3(1,3)=D*(2.0*R1+   RJ+2.0*RK)
GK3(2,2)=D*(2.0*R1+6.0*RJ+2.0*RK)
GK3(2,3)=D*(   R1+2.0*RJ+2.0*RK)
GK3(3,3)=D*(2.0*R1+2.0*RJ+6.0*RK)
GK3(2,1)=GK3(1,2)
GK3(3,1)=GK3(1,3)
GK3(3,2)=GK3(2,3)
DO 30 M=1,3
P1(M)=0.0
P2(M)=0.0
P3(M)=0.0
P4(M)=0.0
30 TOE(M)=0.0
SUM=PI*RC*QD(1)*A(1)/6.0
P1(1)=SUM*(2.0*R1+RJ+RK)
P1(2)=SUM*(R1+2.0*RJ+RK)
P1(3)=SUM*(R1+RJ+2.0*RK)
40 IF (EDGE(1) .EQ. 0.0) GO TO 80
IF (EDGE(1) .EQ. 1 .OR. EDGE(1) .EQ. 4 .OR. EDGE(1) .EQ. 5) GO TO
& 70
IF (EDGE(1) .EQ. 2 .OR. EDGE(1) .EQ. 6) GO TO 60
50 SIK=SQRT((R1-RK)**2+(ZI-ZK)**2)
CON=PI*H(1)*SIK/6.0
GK2(1,1)=GK2(1,1)+CON*(3.0*R1+RK)
GK2(1,3)=GK2(1,3)+CON*(R1+RK)
GK2(3,1)=GK2(3,1)+CON*(R1+RK)
GK2(3,3)=GK2(3,3)+CON*(R1+3.0*RK)
P2(1)=P2(1)+(P1*Q(1)*SIK/3.0)*(2.0*R1+RK)
P2(3)=P2(3)+(P1*Q(1)*SIK/3.0)*(R1+2.0*RK)
P3(1)=P3(1)+(P1*H(1)*TINF(1)*SIK/3.0)*(2.0*R1+RK)

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FET03310
FET03320
FET03330
FET03340
FET03350
FET03360
FET03370
FET03380
FET03390
FET03400
FET03410
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FET03600
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FET03680
FET03690
FET03700
FET03710
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FET03760
FET03770
FET03780
FET03790
FET03800
FET03810
FET03820
FET03830
FET03840
FET03850

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P3(3)=P3(3)+(PI*H(1)*TINF(1)*SIK/3.0)*(R1+2.0*RK)
GO TO 80
60 SKJ=SQRT((RK-RJ)**2+(ZK-ZJ)**2)
CON=PI*H(1)*SKJ/6.0
GK2(2,2)=GK2(2,2)+CON*(3.0*RJ+RK)
GK2(2,3)=GK2(2,3)+CON*(RJ+RK)
GK2(3,2)=GK2(3,2)+CON*(RJ+RK)
GK2(3,3)=GK2(3,3)+CON*(RJ+3.0*RK)
P2(2)=P2(2)+(PI*Q(1)*SKJ/3.0)*(2.0*RJ+RK)
P2(3)=P2(3)+(PI*Q(1)*SKJ/3.0)*(RJ+2.0*RK)
P3(2)=P3(2)+(PI*H(1)*TINF(1)*SKJ/3.0)*(2.0*RJ+RK)
P3(3)=P3(3)+(PI*H(1)*TINF(1)*SKJ/3.0)*(RJ+2.0*RK)
IF (EDGE(1) .EQ. 6) GO TO 50
GO TO 80
70 SJ1=SQRT((RJ-R1)**2+(ZJ-Z1)**2)
CON=PI*H(1)*SJ1/6.0
GK2(1,1)=GK2(1,1)+CON*(3.0*R1+RJ)
GK2(1,2)=GK2(1,2)+CON*(R1+RJ)
GK2(2,1)=GK2(2,1)+CON*(R1+RJ)
GK2(2,2)=GK2(2,2)+CON*(R1+3.0*RJ)
P2(1)=P2(1)+(PI*Q(1)*SJ1/3.0)*(2.0*R1+RJ)
P2(2)=P2(2)+(PI*Q(1)*SJ1/3.0)*(R1+2.0*RJ)
P3(1)=P3(1)+(PI*H(1)*TINF(1)*SJ1/3.0)*(2.0*R1+RJ)
P3(2)=P3(2)+(PI*H(1)*TINF(1)*SJ1/3.0)*(R1+2.0*RJ)
IF (EDGE(1)-4) 80,60,50
80 CONTINUE
DO 85 J=1,3
DO 85 K=1,3
B(J,K)=B(J,K)-GK1(J,K)-GK2(J,K)+(2.0/DTIM)*GK3(J,K)
85 CONTINUE
DO 87 M=1,3
IM=LOC(1,M)
TOE(M)=TO(IM)
87 CONTINUE
CALL MULT(3,3,1,B,TOE,P4)
DO 90 M=1,3
IM=LOC(1,M)
PLOAD(IM,1)=PLOAD(IM,1)+2.*P1(M)-2.*P2(M)+2.*P3(M)+P4(M)
DO 90 N=1,3
IN=LOC(1,N)-IM+1
IF (IN .LE. 0) GO TO 90
GS(IM,IN)=GS(IM,IN)+GK1(M,N)+GK2(M,N)+(2.0/DTIM)*GK3(M,N)
90 CONTINUE
100 CONTINUE
DO 110 M=1,NN
IF (TS(M) .LT. -1.0E+5) GO TO 110
GS(M,1)=GS(M,1)*1.0E+9
PLOAD(M,1)=TS(M)*GS(M,1)
110 CONTINUE
CALL DECOMP (NN,NB,GS)
CALL SOLVE (NN,NB,1,GS,PLOAD,DIFF)
DO 120 K=1,NN
TO(K)=0.0
TO(K)=PLOAD(K,1)
120 CONTINUE

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FETO3860
FETO3870
FETO3880
FETO3890
FETO3900
FETO3910
FETO3920
FETO3930
FETO3940
FETO3950
FETO3960
FETO3970
FETO3980
FETO3990
FETO4000
FETO4010
FETO4020
FETO4030
FETO4040
FETO4050
FETO4060
FETO4070
FETO4080
FETO4090
FETO4100
FETO4110
FETO4120
FETO4130
FETO4140
FETO4150
FETO4160
FETO4170
FETO4180
FETO4190
FETO4200
FETO4210
FETO4220
FETO4230
FETO4240
FETO4250
FETO4260
FETO4270
FETO4280
FETO4290
FETO4300
FETO4310
FETO4320
FETO4330
FETO4340
FETO4350
FETO4360
FETO4370
FETO4380
FETO4390
FETO4400

	RETURN	FET04410
	END	FET04420
C=====		FET04430
C		FET04440
C	SUBROUTINE DECOMP	FET04450
C=====		FET04460
C		FET04470
	SUBROUTINE DECOMP(N,NB,A)	FET04480
	DIMENSION A(N,NB)	FET04490
	DOUBLE PRECISION DIFF	FET04500
	A(1,1)=SQRT(A(1,1))	FET04510
	DO 5 K=2,NB	FET04520
5	A(1,K)=A(1,K)/A(1,1)	FET04530
	DO 25 K=2,N	FET04540
	KP1=K+1	FET04550
	KM1=K-1	FET04560
	DIFF=A(K,1)	FET04570
	DO 10 JP=1,KM1	FET04580
	ICOL=K+1-JP	FET04590
	IF (ICOL.GT. NB) GO TO 10	FET04600
	DIFF=DIFF-A(JP,ICOL)*A(JP,ICOL)	FET04610
10	CONTINUE	FET04620
	A(K,1)=DSQRT(DIFF)	FET04630
	DO 20 J=2,NB	FET04640
	IF (K+J-1.GT. N) GO TO 25	FET04650
	DIFF=A(K,J)	FET04660
	DO 15 JP=1,KM1	FET04670
	ICOL=K+1-JP	FET04680
	JCOL=K+J-JP	FET04690
	IF (JCOL.GT. NB) GO TO 15	FET04700
	IF (ICOL.GT. NB) GO TO 15	FET04710
	DIFF=DIFF-A(JP,ICOL)*A(JP,JCOL)	FET04720
15	CONTINUE	FET04730
20	A(K,J)=DIFF/A(K,1)	FET04740
25	CONTINUE	FET04750
	RETURN	FET04760
	END	FET04770
C=====		FET04780
C		FET04790
C	SUBROUTINE SOLVE	FET04800
C=====		FET04810
C		FET04820
	SUBROUTINE SOLVE(N,NB,M,A,B,DIFF)	FET04830
	DIMENSION A(N,NB),B(N,M)	FET04840
	DOUBLE PRECISION DIFF(M)	FET04850
	DO 5 J=1,M	FET04860
5	B(1,J)=B(1,J)/A(1,1)	FET04870
	DO 30 I=2,N	FET04880
	DO 10 J=1,M	FET04890
10	DIFF(J)=B(1,J)	FET04900
	DO 20 K=2,NB	FET04910
	IROW=I+1-K	FET04920
	IF (IROW.LT. 1) GO TO 20	FET04930
	ICOL=I+1-IROW	FET04940
	IF (ICOL.GT. NB) GO TO 20	FET04950

	DO 15 J=1,M	FET04960
15	DIFF(J)=DIFF(J)-A(IROW,ICOL)*B(IROW,J)	FET04970
20	CONTINUE	FET04980
	DO 25 J=1,M	FET04990
25	B(I,J)=DIFF(J)/A(I,1)	FET05000
30	CONTINUE	FET05010
	DO 35 J=1,M	FET05020
35	B(N,J)=B(N,J)/A(N,1)	FET05030
	DO 60 I1=2,N	FET05040
	I=N+1-I1	FET05050
	DO 40 J=1,M	FET05060
40	DIFF(J)=B(I,J)	FET05070
	DO 50 K=2,NB	FET05080
	IK=I-1+K	FET05090
	IF (IK.GT. N) GO TO 50	FET05100
	DO 45 J=1,M	FET05110
45	DIFF(J)=DIFF(J)-A(I,K)*B(IK,J)	FET05120
50	CONTINUE	FET05130
	DO 55 J=1,M	FET05140
55	B(I,J)=DIFF(J)/A(I,1)	FET05150
60	CONTINUE	FET05160
	RETURN	FET05170
	END	FET05180
C=====		FET05190
C		FET05200
C	SUBROUTINE MULT	FET05210
C		FET05220
C=====		FET05230
	SUBROUTINE MULT(N,M,K,A,B,C)	FET05240
	DIMENSION A(N,M),B(M,K),C(N,K)	FET05250
	DO 10 I=1,N	FET05260
	DO 10 J=1,K	FET05270
	C(I,J)=0.0	FET05280
	DO 10 L=1,M	FET05290
	C(I,J)=C(I,J)+A(I,L)*B(L,J)	FET05300
10	CONTINUE	FET05310
	RETURN	FET05320
	END	FET05330

APPENDIX D

TABLES

Table 4-1. Measured Values of Thermophysical Properties of Test Media

Measured Property	Medium				
	Local sand	Ottawa sand	Metallic beads	Glass beads	Glass beads
Mean particle size (mm)	0.26	0.71	1.09	0.69	2.07
Bulk density (kg/m ³)					
Critical state	1600	1570	4450	1460	1460
Dense State	1800	1680	4710	1550	1550
Solid density (kg/m ³)	2620	2610	7500	2440	2446
Void Ratio ($V_t - V_s$)/ V_t					
Critical state	0.39	0.41	0.41	0.40	0.41
Dense state	0.32	0.35	0.38	0.36	0.37
Thermal conductivity (W/m ² °K)					
Critical state	0.312	0.286	0.369	0.226	0.220
Thermal diffusivity (m /s ²)	0.207 x 10 ⁻⁶	0.250 x 10 ⁻⁶	0.162 x 10 ⁻⁶	0.170 x 10 ⁻⁶	0.163 x 10 ⁻⁶
Specific heat, (J/kg °K)					
Equation (4-9)	942	728	512	907	924
Measured (SEC)	933	895	486	861	915
Percent difference	1	-20	5	5	1

Table 4-2. Summary of Uncertainty Analysis Results

Property	Medium				
	Local sand	Ottawa sand	Metallic beads	Glass beads	Glass beads
Mean diameter using [59]:					
Volume-surface mean	0.26	0.71	1.09	0.69	2.07
Arithmetic mean	0.16	0.69	1.05	0.68	2.03
Weight mean (mm)	0.41	0.72	1.11	0.70	2.09
Standard Deviation					
(mm)	0.128	0.085	0.139	0.075	0.207
(%)	49	12	12.7	10.8	10
Volume-surface mean diameter (mm)	$0.26 \pm \begin{smallmatrix} 0.13 \\ 0.06 \end{smallmatrix}$	0.71 ± 0.17	1.09 ± 0.28	0.69 ± 0.15	2.07 ± 0.41
Maximum	0.39	0.88	1.37	0.84	2.48
Minimum	0.195	0.54	0.81	0.54	1.66
Particle size range (mm)	0.074-1.19	0.42-1.19	0.42-1.4	0.42-0.85	1.4-2.8
Estimated uncertainty of:					
Measured heat transfer coefficient (%)	± 5.1	± 4.9	± 4.4	± 5.0	± 4.0
Nusselt number					
(based on k_g) (%)	$\begin{smallmatrix} +51 \\ -26 \end{smallmatrix}$	± 24.6	± 26.4	± 23.2	± 21
(based on k_o) (%)	$\begin{smallmatrix} +52 \\ -28.5 \end{smallmatrix}$	± 26.8	± 28.7	± 25.7	± 25.5

APPENDIX E

COMPUTER PROGRAM FOR EXPERIMENTAL STUDY

```

C-----
C THIS PROGRAM DESIGNED FOR PROCESSING THE EXPERIMENTAL DATA
C MEASUREMENTS OF PROJECT ME/PORMEDIA/87 "THERMAL CHARACTERISTICS
C OF FLOWING GRANULAR MATERIALS".
C
C VALUES OF K CORRESPONDS TO DIFFERENT TYPES OF MATERIALS :
C K=1 LOCAL SAND , K=2 GLASS BEADS D2.07 , K=3 GLASS BEADS D0.69 ,
C K=4 OTTAWA SAND , K=5 METALLIC BEADS .
C-----
      REAL KINS,MAS,LMTD6,LMTD7,NUSD,L6,L7,L,MS(9),NUS(9),KPA,NUM(9)
      DIMENSION VL(9),HF(9),QL(9),E(9),H(9),PEC(9),RT(9),FOR(9)
      DIMENSION H1(9),H1D(9),H2(9),H2D(9),H3(9),H3D(9),H4(9),H4D(9)
      &,H5(9),H5D(9),H6(9),H6D(9)
      DATA PF/0.91/
      DO 30 KK=1,5
      READ(5,*)K,NR
      GO TO(1,2,3,4,5),K
      WRITE(6,112)
112  FORMAT(1X,'*** VALUE OF K OUTSIDE THE RANGE')
C
C*****PHYSICAL AND THERMAL PROPERTIES OF MATERIALS.
C
C*****LOCAL SAND.
C
      1  WRITE(6,121)
121  FORMAT(1H1,30X,'*** LOCAL SAND OF MEAN DIAMETER 0.26 MM ***')
      D=0.26E-3
      DNS=1.6E3
      DNSS=2.62E3
      VOID=0.39
      THK=0.312
      ALF=0.207E-6
      CP=942
      GO TO 7
C
C*****GLASS BEADS OF D=2.07
C
      2  WRITE(6,122)
122  FORMAT(1H1,30X,'*** GLASS BEADS OF MEAN DIAMETER 2.07 MM ***')
      D=2.07E-3
      DNS=1.46E3
      DNSS=2.44E3
      VOID=0.41
      THK=0.22
      ALF=0.163E-6
      CP=924
      GO TO 7
C
C*****GLASS BEADS OF D=0.69
C
      3  WRITE(6,123)
123  FORMAT(1H1,30X,'*** GLASS BEADS OF MEAN DIAMETER 0.69 MM ***')
      D=0.69E-3
      DNS=1.46E3
      DNSS=2.44E3

```

EXP00010
 EXP00020
 EXP00030
 EXP00040
 EXP00050
 EXP00060
 EXP00070
 EXP00080
 EXP00090
 EXP00100
 EXP00110
 EXP00120
 EXP00130
 EXP00140
 EXP00150
 EXP00160
 EXP00170
 EXP00180
 EXP00190
 EXP00200
 EXP00210
 EXP00220
 EXP00230
 EXP00240
 EXP00250
 EXP00260
 EXP00270
 EXP00280
 EXP00290
 EXP00300
 EXP00310
 EXP00320
 EXP00330
 EXP00340
 EXP00350
 EXP00360
 EXP00370
 EXP00380
 EXP00390
 EXP00400
 EXP00410
 EXP00420
 EXP00430
 EXP00440
 EXP00450
 EXP00460
 EXP00470
 EXP00480
 EXP00490
 EXP00500
 EXP00510
 EXP00520
 EXP00530
 EXP00540
 EXP00550

VOID=0.4	EXP00560
THK=0.226	EXP00570
ALF=0.17E-6	EXP00580
CP=907	EXP00590
GO TO 7	EXP00600
C	EXP00610
C*****OTTAWA SAND.	EXP00620
C	EXP00630
4 WRITE(6,124)	EXP00640
124 FORMAT(1H1,30X,'*** OTTAWA SAND OF MEAN DIAMETER 0.71 MM ***')	EXP00650
D=0.71E-3	EXP00660
DNS=1.57E3	EXP00670
DNSS=2.61E3	EXP00680
VOID=0.41	EXP00690
THK=0.286	EXP00700
ALF=0.25E-6	EXP00710
CP=728.	EXP00720
CB CP=895.	EXP00730
CC CP=950.	EXP00740
GO TO 7	EXP00750
C	EXP00760
C*****METALLIC BEADS.	EXP00770
C	EXP00780
5 WRITE(6,125)	EXP00790
125 FORMAT(1H1,30X,'*** METALLIC BEADS OF MEAN DIAMETER 1.09 MM ***')	EXP00800
D=1.09E-3	EXP00810
DNS=4.45E3	EXP00820
DNSS=7.5E3	EXP00830
VOID=0.41	EXP00840
THK=0.369	EXP00850
ALF=0.162E-6	EXP00860
CP=512.	EXP00870
CB CP=486.	EXP00880
CC CP=516	EXP00890
C	EXP00900
C*****SPECIFICATION OF TEST SECTION.	EXP00910
C	EXP00920
7 DI=14.4E-3	EXP00930
DO=15.87E-3	EXP00940
XAB=27.E-2	EXP00950
XA2=6.5E-2	EXP00960
XA4=14.5E-2	EXP00970
XA6=22.5E-2	EXP00980
XA7=26.5E-2	EXP00990
X12=8.0E-2	EXP01000
PI=3.14159	EXP01010
A=(PI/4.)*(DI**2)	EXP01020
SA=PI*DI*XAB	EXP01030
AN=(PI/4.)*(DO**2-DI**2)	EXP01040
STK1=14.9	EXP01050
STK2=16.6	EXP01060
DKST=(STK2-STK1)/100.	EXP01070
C	EXP01080
C*****RADIAL DISTANCES INPUT.	EXP01090
C	EXP01100

```

RO=D1/2.
R1=1.E-3
R2=2.E-3
R3=3.E-3
R4=4.E-3
R5=5.E-3
R6=6.E-3
R7=RO
RR1=R1/RO
RR2=R2/RO
RR3=R3/RO
RR4=R4/RO
RR5=R5/RO
RR6=R6/RO
RR7=R7/RO
C
C*****SPECIFICATION OF INSULATION.
C
DINS=11.44E-2
KINS=0.07
C
C*****PROPERTIES OF GAS (AIR) AT 300&400 DEG K.
C
AK1=0.0263
AK2=0.0338
DKA=(AK2-AK1)/100.
C
C*****HEAT TRANSFER CALCULATION.
C
DO 10 I=1,NR
READ(5,*)TBIN,VLT,AMP,MAS
READ(5,*)TS1,TS2,TS4,TS6,TS7,TINS
READ(5,*)TR1,TR2,TR3,TR4,TR5,TR6,TR7
TRAV=TR7-(TR7-TR6)*RR6**2-(TR6-TR5)*RR5**2-(TR5-TR4)*RR4**2
&
-(TR4-TR3)*RR3**2-(TR3-TR2)*RR2**2-(TR2-TR1)*RR1**2
MS(1)=MAS
VEL=(MAS/60.)/(DNS*A)
VL(1)=VEL
RT27=(XA7-XA2)/VEL
RT26=(XA6-XA2)/VEL
E27=RT27/(D**2)
E26=RT26/(D**2)
RT(1)=RT27
E(1)=E27
FOR(1)=ALF*E(1)
C
C*****LOSSES AND HEAT FLUX CALCULATIONS.
C
TSAV=(TS2+TS4+TS6+TS7)/4.
QLR=2.*PI*XAB*KINS*(TSAV-TINS)/ALOG(DINS/DO)
STK=STK1+DKST*((TS1+TS2)/2.+273.-300.)
QLX=STK*AN*(TS2-TS1)/X12
PWR=VLT*AMP
QLT=QLR+QLX
QLTP=(QLT/PWR)*100.

```

EXP01110
EXP01120
EXP01130
EXP01140
EXP01150
EXP01160
EXP01170
EXP01180
EXP01190
EXP01200
EXP01210
EXP01220
EXP01230
EXP01240
EXP01250
EXP01260
EXP01270
EXP01280
EXP01290
EXP01300
EXP01310
EXP01320
EXP01330
EXP01340
EXP01350
EXP01360
EXP01370
EXP01380
EXP01390
EXP01400
EXP01410
EXP01420
EXP01430
EXP01440
EXP01450
EXP01460
EXP01470
EXP01480
EXP01490
EXP01500
EXP01510
EXP01520
EXP01530
EXP01540
EXP01550
EXP01560
EXP01570
EXP01580
EXP01590
EXP01600
EXP01610
EXP01620
EXP01630
EXP01640
EXP01650

```

QT=PWR-QLT
HFLX=QT/(PI*DI*XAB)
HF(1)=HFLX
QL(1)=QLTP
C
C*****BULK TEMPERATURE CALCULATION.
C
CC=QT/((MAS/60.)*CP*XAB)
TB2=TBIN+CC*XA2
TB4=TBIN+CC*XA4
TB6=TBIN+CC*XA6
TB7=TBIN+CC*XA7
C
C*****LOCAL HEAT TRANSFER COEFFICIENT.
C
HX2=HFLX/(TS2-TB2)
HX4=HFLX/(TS4-TB4)
HX6=HFLX/(TS6-TB6)
HX7=HFLX/(TS7-TB7)
C
C*****LOGARITHMIC MEAN HEAT TRANSFER COEFFICIENT.
C
LMTD7=((TS7-TB7)-(TS2-TB2))/ALOG((TS7-TB7)/(TS2-TB2))
LMTD6=((TS6-TB6)-(TS2-TB2))/ALOG((TS6-TB6)/(TS2-TB2))
HM7=HFLX/LMTD7
HM6=HFLX/LMTD6
HM=HM7
H(1)=HM
C
C*****MODIFIED NUSSELT AND PECLET NUMBERS.
C
TBM=(TB2+TB4+TB6+TB7)/4.
TPROP=(TSAV+TBM)/2.+273.
GK=AK1+DKA*(TPROP-300.)
NUSD=(HM*D)/GK
NUS(1)=NUSD
NUM(1)=(HM*D)/THK
L7=XA7-XA2
L6=XA6-XA2
L=L7
PESL=(THK/GK)**2*(D/L)**2*(VEL*L/ALF)
PEC(1)=PESL
C
C*****H.T.MODEL OF W.N.SULLIVAN AND R.H.SABERSKY
C*****[REF.J.HEAT MASS TRANSFER. VOL.18 PP.97-107]
C
KPA=0.055
H6(1)=(GK/D)/((KPA+0.5*SQRT(PI/PESL))
H6D(1)=((H6(1)-H(1))/H(1))*100.
C
C*****H.T.MODEL OF MICKLEY & FAIRBANKS FOR THE CASE OF HIGBIE TYPE
C*****SOLIDS FLOW WITHOUT CONTACT RESISTANCE.
C*****[REF.POWDER TECHNOLOGY, 30 (1981)175-184 Q.E.J.J.M.HOELEN]
C
H1(1)=2.*SQRT((THK*DNS*CP)/(PI*RT27))

```

EXP01660
 EXP01670
 EXP01680
 EXP01690
 EXP01700
 EXP01710
 EXP01720
 EXP01730
 EXP01740
 EXP01750
 EXP01760
 EXP01770
 EXP01780
 EXP01790
 EXP01800
 EXP01810
 EXP01820
 EXP01830
 EXP01840
 EXP01850
 EXP01860
 EXP01870
 EXP01880
 EXP01890
 EXP01900
 EXP01910
 EXP01920
 EXP01930
 EXP01940
 EXP01950
 EXP01960
 EXP01970
 EXP01980
 EXP01990
 EXP02000
 EXP02010
 EXP02020
 EXP02030
 EXP02040
 EXP02050
 EXP02060
 EXP02070
 EXP02080
 EXP02090
 EXP02100
 EXP02110
 EXP02120
 EXP02130
 EXP02140
 EXP02150
 EXP02160
 EXP02170
 EXP02180
 EXP02190
 EXP02200

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      H1D(1)=((H1(1)-H(1))/H(1))*100.                                EXP02210
C                                                                           EXP02220
C*****H.T.MODEL OF MICKLEY & FAIRBANKS FOR THE CASE OF HIGBIE TYPE    EXP02230
C*****SOLIDS FLOW WITH CONTACT RESISTANCE.                             EXP02240
C*****{REF.POWDER TECHNOLOGY, 30 (1981)175-184 Q.E.J.J.M.HOELEN}      EXP02250
C                                                                           EXP02260
      RW=(D/2.)/(0.7*THK)                                              EXP02270
      H2(1)=1./(RW+0.5*SQR(T(PI*RT27/(THK*DNS*CP))))                  EXP02280
      H2D(1)=((H2(1)-H(1))/H(1))*100.                                  EXP02290
C                                                                           EXP02300
C*****H.T.MODEL OF MICKLEY & FAIRBANKS FOR THE CASE OF HIGBIE TYPE    EXP02310
C*****SOLIDS FLOW WITH CONTACT RESISTANCE.                             EXP02320
C*****{REF.CH.ENGNG.SC., 1977,VOL.32,P461 A.O.O.DENLOYE&J.S.M.BOTTERILL} EXP02330
C                                                                           EXP02340
      RW=(D/2.)/(0.7*THK)                                              EXP02350
      RA=1./(2.*SQR(T(THK*DNS*CP)/(PI*RT27)))                          EXP02360
      H3(1)=1./RA*(1.-(RW/(2.*RA))*ALOG(1.+2.*RA/RW))                  EXP02370
      H3D(1)=((H3(1)-H(1))/H(1))*100.                                  EXP02380
C                                                                           EXP02390
C*****H.T.MODEL OF MICKLEY & FAIRBANKS FOR THE CASE OF HIGBIE TYPE    EXP02400
C*****SOLIDS FLOW WITH CONTACT RESISTANCE.                             EXP02410
C*****{REF.N.I.GELPERIN&V.G.EINSTEIN P481 1971 FLUIDIZATION H4 FOR    EXP02420
C***** CONSTANT HETFLUX AND H5 FOR CONST. TEMP. SURFACES.)           EXP02430
C                                                                           EXP02440
      RW=(D/2.)/(0.7*THK)                                              EXP02450
      RA=1./(2.*SQR(T(THK*DNS*CP)/(PI*RT27)))                          EXP02460
      RR=2.*RA                                                         EXP02470
      WOR=RW/RR                                                         EXP02480
      ROW=1./WOR                                                         EXP02490
      H4(1)=PI/RR*(1.-(WOR*PI/2.)*ALOG(1.+2.*ROW/PI))                  EXP02500
      H4D(1)=((H4(1)-H(1))/H(1))*100.                                  EXP02510
      H5(1)=2./RR*(1.-(WOR*PI/2.)*(1.-EXP((ROW/SQR(T(PI))**2)*ERFC(ROW/
&SQR(T(PI))))                                                         EXP02520
      H5D(1)=((H5(1)-H(1))/H(1))*100.                                  EXP02530
      IF (.NE.1) WRITE(6,118)                                           EXP02540
      FORMAT(1H1)                                                         EXP02550
118  WRITE(6,100)MAS                                                     EXP02560
      WRITE(6,101)VEL                                                     EXP02570
      WRITE(6,102)HFLX                                                    EXP02580
      WRITE(6,103)QLTP                                                    EXP02590
      WRITE(6,104)TBIN                                                    EXP02600
      WRITE(6,105)TS1                                                     EXP02610
100  FORMAT(/,5X,'MASS FLOW RATE (KG/MIN)                               = ',F6.4)                   EXP02620
101  FORMAT(/,5X,'MEAN VELOCITY (M/SEC)                                = ',F6.4)                   EXP02630
102  FORMAT(/,5X,'HEAT FLUX (WATT/M**2)                                = ',F9.2)                   EXP02640
103  FORMAT(/,5X,'TOTAL LOSSES (PERCENT)                                = ',F6.2)                   EXP02650
104  FORMAT(/,5X,'INLET BULK TEMP.(DEG C)                              = ',F6.2)                   EXP02660
105  FORMAT(/,5X,'WALL TEMP. TS1 (DEG C)                                = ',F6.2)                   EXP02670
      WRITE(6,106)E27                                                     EXP02680
106  FORMAT(/,5X,'RES.D. TIME/D**2 (SEC/M**2)                           = ',E15.7)                   EXP02690
      WRITE(6,107)HM                                                       EXP02700
107  FORMAT(/,5X,'MEAN H.T. COEFF. (W/M**2DEGK)                        = ',F6.2)                   EXP02710
      WRITE(6,108)NUSD                                                     EXP02720
108  FORMAT(/,5X,'MODIFIED NUSSELT NO.                                  = ',F6.3)                   EXP02730
      WRITE(6,109)PESL                                                    EXP02740
                                                                           EXP02750

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```

109 FORMAT(/,5X,'MODIFIED PECTLET NO.' = ',F7.6) EXP02760
WRITE(6,210)GK EXP02770
210 FORMAT(/,5X,'GAS(AIR) TH. CONDUCTIVITY' = ',F7.4) EXP02780
WRITE(6,110) EXP02790
110 FORMAT(////) EXP02800
WRITE(6,111)TS2,TB2,HX2 EXP02810
WRITE(6,111)TS4,TB4,HX4 EXP02820
WRITE(6,111)TS6,TB6,HX6 EXP02830
WRITE(6,111)TS7,TB7,HX7 EXP02840
111 FORMAT(/,5X,3(10X,F6.2)) EXP02850
WRITE(6,113)TR1,TR2,TR3,TR4,TR5,TR6,TR7 EXP02860
113 FORMAT(///,5X,7(5X,F6.2)) EXP02870
WRITE(6,114)RR1,RR2,RR3,RR4,RR5,RR6,RR7 EXP02880
114 FORMAT(/,5X,7(5X,F6.2)) EXP02890
WRITE(6,115)TRAV EXP02900
115 FORMAT(/,5X,'MEAN OUTLET TEMP. (DEG C)' = ',F6.2) EXP02910
10 CONTINUE EXP02920
WRITE(6,116) EXP02930
116 FORMAT(1H1,////,30X,'***SUMMARY OF RESULTS***') EXP02940
DO 20 J=1,NR EXP02950
WRITE(6,117)MS(J),VL(J),HF(J),QL(J),E(J),H(J),NUS(J),PEC(J),RT(J) EXP02960
&.FOR(J),NUM(J) EXP02970
C WRITE(6,117)MS(J),H(J),H1(J),H2(J), EXP02980
C &H3(J),H4(J),H5(J),H6(J),H1D(J),H2D(J),H3D(J),H4D(J),H5D(J),H6D(J) EXP02990
20 CONTINUE EXP03000
117 FORMAT(/,1X,F6.4,1X,F6.4,1X,F9.2,1X,F6.2,1X,E15.7,1X,6(F7.3,1X)) EXP03010
C117 FORMAT(/,1X,F6.4,1X,13(F6.1,1X)) EXP03020
30 CONTINUE EXP03030
STOP EXP03040
END EXP03050

```

APPENDIX F
UNCERTAINTY ANALYSIS

UNCERTAINTY ANALYSIS

In an experiment requiring measurements of several quantities, each of which has an error or "uncertainty" associated with it, the total error that propagates into the final result may be estimated by the expression [54,67,68]

$$e_E^2 = \sum_{n=1}^i \left[\left(\frac{\partial F}{\partial n} \right) e_n \right]^2 \quad (F-1)$$

where F is a function of n independent variables having errors e_n . For repeated measurements of the variables, it is assumed that errors are normally distributed about the true value.

The "function" whose resultant error is to be estimated here is the Nusselt number, given by

$$Nu = hd/k \quad (F-2)$$

Utilizing the above expression for e_E

$$\begin{aligned} e_{Nu}^2 &= \left(\frac{\partial Nu}{\partial h} e_h \right)^2 + \left(\frac{\partial Nu}{\partial d} e_d \right)^2 + \left(\frac{\partial Nu}{\partial k} e_k \right)^2 \\ &= \left(\frac{d}{k} e_h \right)^2 + \left(\frac{h}{k} e_d \right)^2 + \left(\frac{hd}{k^2} e_k \right)^2 \end{aligned} \quad (F-3)$$

where e_h and e_k may be estimated as follow

$$h = q''/(T_w - T_m)$$

$$\begin{aligned} e_h^2 &= \left(\frac{\partial h}{\partial q''} e_{q''}\right)^2 + \left(\frac{\partial h}{\partial T_w} e_{T_w}\right)^2 + \left(\frac{\partial h}{\partial T_m} e_{T_m}\right)^2 \\ &= \left(\frac{1}{T_w - T_m} e_{q''}\right)^2 + \left(\frac{q''}{(T_w - T_m)^2} e_{T_w}\right)^2 + \left(\frac{q''}{(T_w - T_m)^2} e_{T_m}\right)^2 \end{aligned} \quad (F-4)$$

where $e_{q''}$, e_{T_w} , and e_{T_m} may be estimated as follows.

Since $q'' = (VI - \text{Loss})/\pi DL$

$$\begin{aligned} e_{q''}^2 &= \left(\frac{I}{\pi DL} e_V\right)^2 + \left(\frac{V}{\pi DL} e_I\right)^2 + \left(\frac{VI - \text{Loss}}{\pi D^2 L} e_D\right)^2 \\ &\quad + \left(\frac{VI - \text{Loss}}{\pi DL^2} e_L\right)^2 + \left(\frac{1}{\pi DL} e_{\text{Loss}}\right)^2 \end{aligned} \quad (F-5)$$

By substituting into equation (F-5) the measured values of V, I, D, L and Losses together with the estimated errors of each of these quantities, $e_{q''}$ was calculated.

Similarly, since

$$T_m = T_i + q'' \pi D x / mc_p$$

$$e_{Tm}^2 = (e_{Ti})^2 + \left(\frac{\pi D x}{m c_p} e_{q''} \right)^2 + \left(\frac{\pi q'' x}{m c_p} e_D \right)^2 + \left(\frac{\pi q'' D}{m c_p} e_x \right)^2 \\ + \left(\frac{q'' \pi D x}{m^2 c_p} e_m \right)^2 + \left(\frac{q'' \pi D x}{m c_p^2} e_{cp} \right)^2 \quad (F-6)$$

where m is the mass flow rate given by

$$m = M/t$$

and

$$e_m^2 = (1/t e_M)^2 + (M/t^2 e_t)^2 \quad (F-7)$$

The error quantities e_{Tw} , e_{Ti} , and e_{cp} were estimated from the accuracy of the temperature measuring devices employed. Alternately, e_h may then be calculated from equation (F-4).

When the Nusselt number is based on the bed effective thermal conductivity, k_o , an expression for e_{ko} is obtained similar to that derived by [54],

$$e_{ko}^2 = \frac{1}{16\pi^2} \frac{(\ln t/t_1)^2}{(T - T_1)^2} [I^2 e_R^2 + 4 I^2 R^2 e_I^2] \\ + \frac{q'^2}{16\pi^2} \frac{(\ln t/t_1)^2}{(T - T_1)^2} [e_T^2 + e_{T1}^2] \\ + \frac{q'^2}{16\pi^2} \frac{1}{(T - T_1)^2} \left[\frac{1}{t_1^2} e_{t1}^2 + \frac{1}{t_2^2} e_{t2}^2 \right] \quad (F-8)$$

when k_g is utilized e_{kg} was estimated based on the error associated with the gas film temperature measurements.

Finally, e_d was estimated by using the standard deviation formula [68]

$$\sigma = \left[\sum_{i=1}^N (d_i - d)^2 / N \right]^{1/2} \quad (F-9)$$

where $d \pm 2\sigma$ was taken as the limit of 95 percent of the measurements to lie within. The exceptional case was that for local sand, where its size measurements are not normally distributed, a procedure proposed by Keshock [69] was utilized to estimate the mean particle diameter limits as $d+\sigma$ and $d-0.5\sigma$. Volume-surface mean diameters were calculated assuming that 50% errors were made in the particle diameter ranges on either end of the size spectrum (Figure 4-1-a). In one extreme case, the, 50% more particles were assumed in the largest size range, while at the same time 50% fewer particles were assumed for the smallest size range. This permitted calculating an "extreme" volume-surface mean diameter on the large diameter side of the spectrum. A similar procedure was used to determine the "minimum" reasonable diameter associated with errors in measuring particle distribution size. These extreme values were roughly $+\sigma$ and -0.3σ about the mean value.

Values e_h , e_d , and e_k may then be substituted into (F-3) to obtain an estimate of e_{Nu} . The uncertainties in d , h , and Nu for the test media utilized in the present study were estimated and given in Table 4-2 Appendix D.

The preceding errors or uncertainties assigned take into account instrumentation accuracy, errors due to lack of resolution, human and general system losses, heat leaks, etc.

VITA

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After three years as a lecturer in the Mechanical Engineering Department at KFUPM, he accepted a teaching assistantship at the University of Tennessee, Knoxville, USA in January 1983. He received the Doctor of Philosophy degree with a major in Mechanical Engineering in March 1987.

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