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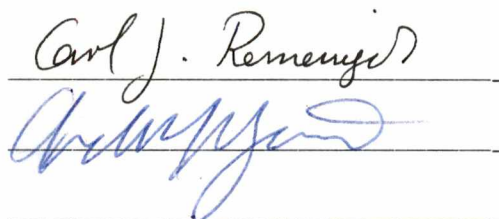
I am submitting herewith a thesis written by Subrata Roy entitled "An Efficient Finite Element Methodology for Hyperthermia Analysis". I have examined the final copy of this thesis for form and content and recommend that it be accepted for the degree of Master's of Science, with a major in Engineering Science.



A. J. Baker, Ph.D

Major Professor

We have read this thesis
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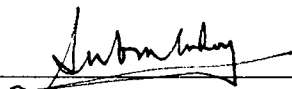
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April 24, 1990

**AN EFFICIENT FINITE ELEMENT METHODOLOGY
FOR HYPERTHERMIA ANALYSIS**

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Subrata Roy

May 1990

To
my parents

ACKNOWLEDGEMENTS

I would like to express my special appreciation to my major professor, Dr. A. J. Baker. His guidance and enthusiasm has made me excited about my work. I must acknowledge the other committee members, Dr. C. J. Remenyik and Dr. A. J. Milligan, for their genuine help and concern during the period of thesis writing. Dr. Masood Panjehpour, whose Ph.D. dissertation work I have extrapolated, has always come forward with suggestions and references for my thesis work. I would also like to thank Leif Wilkening and Giuseppe Iannelli for their consistent support as a colleague and willingness to provide assistance during numerous computer crises. My parents in India gave me continuous inspiration over the telephone and in the mail from thousands of miles away and I must say that worked. I express my sincere gratitude for financial assistance provided by the University of Tennessee, Knoxville, Graduate School, to support my completion of Master's Degree.

ABSTRACT

Hyperthermia is a longstanding clinical treatment for malignant tumors. The protocol involves uniform localized heating both spatially and temporally. This thesis project develops and verifies a cost-effective finite element approximate numerical methodology to solve the unsteady three-dimensional bio-heat transfer equation governing hyperthermia with great efficiency and to a defined degree of accuracy. Mesh density covering five levels of refinement are explored and found to be a key issue in accurately predicting the temperature distribution over the treatment region. The application of water cooled needles along with multiple electrodes has been found to promote a more homogeneous temperature distribution over the malignant tissue. For normal tissue interface heat flux convection boundary condition can also yeild a more realistic simulation for the hyperthermia treatment modality.

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CHAPTER I

INTRODUCTION

Hyperthermia is a cancer treatment technique using heat to destroy malignant cells. It has been clinically determined that cancer cells exposed to high temperature lose their ability to reproduce and hence, the tumor regresses. Hyperthermia, known to radiosensitize cells, has better effect when it is combined with radiotherapy. Since most tumors are poorly vascularized the intratumor environment tends to be acidic. But pH has little effect on the radiosensitivity of a cell. However, hyperthermia is more effective in low pH. An important advantage of this modality is oxygenation which occurs due to hyperthermia treatment procedure. The increase in blood flow within the tumor results in more effective transport of oxygen to its cells. Since oxygenated cells, as compared to hypoxic (oxygen deficient) ones, are more susceptible to radiation, heat can increase the cell killing effect of radiation. Hyperthermia also enhances the effect of treatment when applied simultaneously with chemotherapy. Heat condition increases the perfusion (non-directional capillary level blood flow) rate and the permeability of cells. Hence, more effective transport of chemotherapy drugs can be achieved by using a simultaneous hyperthermia treatment. These two together is now believed to produce very critical synergistic effect. Again, perfusion rate in tumor is smaller than the nearby normal tissue. During hyperthermia treatment this generates a localized high temperature

gradient between the tumor and its surrounding normal tissue to enhance killing of malignant cells.

There are several ways to deliver hyperthermia.

1) Regional systems, where heat is applied to a larger region of the body to treat an unknown volume of tumor. Concentric coil and annular phased arrays are among regional hyperthermia systems.

2) Localized superficial systems, used when the location and size of the tumor is exactly known and is easily accessible from the surface. Microwave applicators, ultrasound applicators, capacitive RF electrodes and inductive pancake coils are examples of localized superficial systems.

3) Interstitial systems, used when thermal energy can be directly delivered to the tumor mass. Interstitial RF needles and interstitial microwave antennae are among the interstitial hyperthermia applicators.

One of the most difficult aspects of such treatments is the determination of temperature profiles actually induced in the treatment region. Since temperature measurements can be made only at a limited number of locations, it is necessary to accurately predict this overall temperature distribution due to the specific heating modality. This task becomes even more difficult in realistic situations such as the irregular geometry of the normal and tumor tissue, and the non-uniform perfusion and thermal properties of the treatment and surrounding regions. A discussion of prior research on this subject is presented in Chapter II of this thesis.

The objective of this thesis is to establish and document an efficient computa-

tional algorithm for predicting three-dimensional temperature distributions during RF interstitial hyperthermia treatment of malignant tissue. At the first level of complexity, the simulation of the heat flow in living tissue involves solution of the bio-heat transfer equation, suitably modified to account for the external heat source due to hyperthermia modality. A finite element theory is developed to respond to this need, which possesses no fundamental limitation on useable mesh density. The application of a banded numerical algebra procedure in this simulation provides a very cost economic computer solution. Coupled with an efficient and viable way to refine three-dimensional grids, one can numerically verify the reasonable mesh density required to predict temperature distributions in the treatment region to a specified accuracy. This thesis presents and details this computational solution methodology application to the bio-heat transfer equation.

CHAPTER II

PRIOR RESEARCH

(1) Case Study :

Hyperthermia is not a recent breakthrough in the medical sciences. An ancient cancer treatment modality involved heat therapy using a red hot iron. In the beginning of this century, Westermarck [39] observed remarkably good effects by circulating hot water over uterine tumors. In 1927 rat tumors has been treated with hyperthermia [40]. It has been scientifically established that high temperature has direct lethal effect on living cells. Dewey, et al. [9] showed that all cells survive at 37°C . However, 90 percent of the cells cannot withstand a temperature at 41.5°C for 5 hours. Cells die drastically (99.99 percent) for another 1°C increase in temperature. Although hyperthermia may be used alone to treat malignant tumors it is ususally applied in combination with radiotherapy and/or chemotherapy [.]-[.] for improved performance. It is necessary to apply the two modalities almost at the same time [.]-[.]. The exact cause for such synergism is still unknown. Otherwise, the effect of heat to increase radio- or chemo-sensitivity of the cell is completely lost and does not serve any purpose.

Several experiments by Hahn, et al. [12] and Gerner, et al. [10] confirm that hyperthermia has a direct effect to improve perfusion (non-directional blood flow in tissue) and cell permeability, and hence the transport of chemical drugs into the

tissue. Researchers believe that the synergistic effect of the combination of heat and drug is due to increased perfusion or higher cell membrane permeability. In 1978, Bleehen, et al. [4]-[5] reported that sublethal exposure of hypoxic (oxygen deficient) cells to nitro-compounds made them supersensitive to heat. Experiments have shown that there is little effect of pH on radiosensitivity of a cell. However, many reports have also stated an enhanced effect of hyperthermia in relatively low pH environments [15],[17]. In 1981, Freeman, et al., recorded an increased X-irradiation in acidic medium using hyperthermia together with radio-therapy.

The distinction between the thermosensitivity of a normal tissue and a malignant tissue has not yet been established. But Corry, et al. [8] and Overgaard [22] have documented that a thermal dose, non-toxic to a normal tissue, could be hazardous to a cancerous growth. This may be due to the nutritionally deprived and acidic intratumor microenvironment. Gulliono and Grantham [11] found the blood flow in tumor tissue as low as 2 to 15 percent of similar normal tissue. In 1985 Pence and Song [25] studied the effect of hyperthermia on normal tissue blood flow. According to their report, the ratio of perfusion in a mouse skin and a rat skin before and after local heating at $43.5^{\circ}\text{C} \times 30 \text{ min}$ and $42.0^{\circ}\text{C} \times 60 \text{ min}$, respectively, ranges from 0.05 to 1.0. Song, et al. [28]-[30] observed that temperature increment has direct relation to the rise in blood flow in a normal tissue. Blood flow increases exponentially after a certain temperature and goes on increasing until a critical temperature is reached. Vascular damage causes a subsequent decrease of blood flow.

Hyperthermia destroys the malignant tissue in different ways. Hall [14] has shown heat as an inducer of chain bio-chemical reaction in the nucleus of the cell, causing its thermal death. Strauss [31] and Sugaar [36] confirmed that dead cancer cells released antigens to initiate an immune system response against the living tumor cells. Thus hyperthermia appears to induce dead tumor cells to kill other malignant cells. Some researchers report that heat destroys the intratumor microvascular network. This is because, they believe, unlike normal tissue capillary, the capillaries in the tumor tissue lack thermal receptors and do not dilate to remove excess heat.

(2) Computational Research :

Blood circulation is the most vital cooling mechanism of the tissue. In hyperthermia, non-uniform blood flow can introduce non-uniform temperature distributions in the treatment region. The response of cells to different temperature levels varies widely. Hence, a treatment protocol taking into account the effect of blood flow variation within the treatment volume is necessary. Several researchers have studied the mathematical modeling of hyperthermia modality in this regard.

In 1987, Sullivan, et al. [37] published the Finite-Difference Time-Domain (FDTD) scheme to evaluate the power deposition patterns inside the human body due to an incident electromagnetic wave. This scheme requires a computer storage of $(3N)^2$ and computation time in the order of $(3N)^3$ for N equal to the number of cells. Paulsen, et al. [24] developed an algorithm using a boundary element formulation of the Maxwell equation to solve the electric field distribution in a three dimen-

sional homogeneous model of man. Pennes [26] formulated the bio-heat transfer equation, introducing an equilibrium constant which accounted for the equilibration of the blood temperature as it travels through the capillary vessels. Recent results of Chen and Holmes [7] can be further extended to prove this equilibrium constant as unity.

Uzunoglu and Nikita [38] analytically estimated the temperature distribution inside tissue heated by interstitial RF electrodes using a Green's functions technique to solve the bio-heat transfer equation. Strobehn, et al. [32]-[35] formulated, both numerically and analytically, the bio-heat transfer equation to predict the temperature profiles in hyperthermia modality. In 1989, Charny and Levin [6] developed a realistic three-dimensional model of a human lower leg using a commercial finite element program. In exploring bio-heat transfer, they subjected the muscle and skin tissue elements to both a local and central thermoregulatory mechanism. The computer simulations of hyperthermia revealed that a tumor would be preferentially heated relative to the surrounding normal muscle tissue.

A major original contribution of Panjehpour [23] was development and verification of a mathematical solution model of the three-dimensional bio-heat transfer equation for steady state temperature distribution using a Galerkin finite element algorithm in a Cartesian form computer simulation. His code allowed quantitative simulation applications using $M = 3 \times 3 \times 3$ and $5 \times 5 \times 5$ three dimensional uniform meshes. The practical clinical procedure requires the handling of more complex geometries and boundary conditions, and simulation analyses using far in

excess of the prototypical 5^3 mesh density to accurately predict detailed thermal profiles. Panjehpour used a sparse matrix solution methodology adopting a direct Gaussian elimination procedure with complete pivoting for solving the steady state problem statement. The scheme total memory requirement for the system matrix of an N^3 mesh is on the order of $N^2(N + 1)$, while the number of computations to solution is order of $(N + 1)^4$. Hence, the solution cost of a finer mesh simulation rapidly becomes intractable.

CHAPTER III

THEORETICAL DEVELOPMENT

(1) The Bio-heat Transfer Equation :

The bio-heat transfer equation established by Pennes [26] to model a hyperthermia procedure is a partial differential equation of the form

$$\rho_t c_t \left(\frac{\partial T}{\partial t} \right) = \nabla \cdot (k_t \nabla T) + Q_{bl} + Q_m + Q_e \quad (1)$$

The following variables describe tissue properties,

ρ_t is density,

c_t is heat capacity,

k_t is thermal conductivity.

The temperature $T(\mathbf{x}, t)$, is the dependent variable and the remaining terms define the energy fluxes as

Q_{bl} heat transfer rate to the muscle from the blood,

Q_m heat generation rate due to metabolism, and

Q_e heat generation rate due to an external source (for hyperthermia)

and may be functions of time t . The heat transfer rate model to the muscle from blood, Q_{bl} , can be expressed as [23]

$$Q_{bl} = P \rho_t \rho_{bl} c_{bl} \mu (T_a - T) = \beta (T_a - T) \rho_t c_t \quad (2)$$

where,

P is the *perfusion* (non-directional capillary level blood flow rate,

$cm^3/min/100gm$ of muscle),

ρ_{bl} is the density of blood,

c_{bl} is the heat capacity of blood,

T_a is the arterial blood temperature,

μ is heat exchange effectiveness between tissue and venous blood.

For complete equilibrium, $\mu = 1$ and

$$\beta = P\rho_{bl}c_{bl}\mu/c_t.$$

The basic assumptions in establishing the bio-heat transfer equation (1) are: (i) the tissue is homogeneous; (ii) the perfusion is constant, uniform and nondirectional; (iii) the thermal conductivity of the tissue is a uniform scalar, i.e. an isotropic conductor; and (iv) there are only two internal sources of heat. These are metabolic heat generation, which is considered to be constant, and heat transferred from the blood to the tissue, which occurs at the capillary level. The applicability of this equation assumes that the blood temperature reaches equilibrium with the tissue and there is no significant heat transfer in the arterial tree prior to the microcirculation.

For the present study, the bio-heat transfer equation (1) is simplified by neglecting the metabolic heat generation and heat transfer effects due to blood flow. The model equation is then,

$$\rho_t c_t \left(\frac{\partial T}{\partial t} \right) = \nabla \cdot (k_t \nabla T) + Q_e \quad (3)$$

Including the applicable boundary conditions, the governing equation system can

be rewritten as

$$\begin{aligned}
L(T) &= \frac{\partial T(x_i, t)}{\partial t} - \nabla \cdot (\alpha \nabla T) - s(x_i) = 0 & , on \quad \Omega \subset \mathbb{R}^3 \\
l(T) &= \alpha \nabla T \cdot \hat{n} + h(T - T_r) = 0 & , on \quad \partial\Omega_1 \\
l(T) &= \alpha \nabla T \cdot \hat{n} + q_n = 0 & , on \quad \partial\Omega_2 \\
T(x_t, t) &= T_b(t) & , on \quad \partial\Omega_3
\end{aligned} \tag{4}$$

In equation (4), h , T_r and q_n correspond to boundary heat flux data, both variable and fixed, and T_b is any fixed temperature. Further, $\alpha = \left(\frac{k_t}{\rho_t c_t} \right)$ now denotes the thermal diffusivity, and the boundary of the three dimensional problem domain Ω is the *union* of segments, i.e., $\partial\Omega = \partial\Omega_1 \cup \partial\Omega_2 \cup \partial\Omega_3$, completely encompassing Ω . In addition, an initial temperature distribution $T(x_i, t=0) = T_0(x_i)$ is required to close system (4).

(2) Formulation of a Weak Statement :

The classical calculus defines an interpolation of any function $T(\mathbf{x})$ as the sum of the products of known functions of spatial variables $\phi_i(\mathbf{x})$ and unknown expansion coefficients c_i . The Taylor (or power) series form is,

$$T(\mathbf{x}) \approx T^N(\mathbf{x}) = \sum_{i=1}^N c_i \phi_i(\mathbf{x}), \quad for \ 1 \leq i \leq N. \tag{5}$$

where the superscript notation $T^N(\mathbf{x})$ represents there are N terms in the approximation. Finding the suitably general *set of trial functions* $\phi_i(\mathbf{x})$ is the major challenge in utilizing (5). Using a superposition principle, a function of different variables can also be expressed as the aggregate of products of the seperated variables (assuming the variables are indeed separable).

The approximation T^N of a particular function T generally involves an inexactness. This error e^N , for the N term approximation (5), is

$$e^N \equiv T(\mathbf{x}) - T^N(\mathbf{x}) \quad (6)$$

A measure of this error (6) on Ω can be written as

$$L(e^N) = L(T) - L(T^N) = -L(T^N) \quad (7)$$

An intelligent approach to obtain the most accurate approximate solution to (4) is to reduce this error measure to its minimum in an overall integrated sense. The corresponding mathematical expression is termed a *weak statement* and the general form is

$$\int_{\Omega} \omega(\mathbf{x}) L(T^N) d\mathbf{x} \equiv 0, \quad \forall \omega(\mathbf{x}). \quad (8)$$

Equation (8) is the general expression for a "weighted residual method" and $\omega(\mathbf{x})$ is called the *weight* or *test* function.

Two fundamental theories following the above approach, to construct an approximate solution, are a Rayleigh-Ritz variational statement and Galerkin weak statement. In contrast to the calculus of a function, the theory and practice of these finite element formulations are based on the variational calculus, which is concerned with the extremum values of a functional, i.e., a function of functions.

For the Galerkin definition, each weight function $\omega(\mathbf{x})$ is defined as identical to the appropriate trial function $\phi_i(\mathbf{x})$, hence

$$\omega(\mathbf{x}) \Rightarrow \omega_i(\mathbf{x}) \equiv \phi_i(\mathbf{x}), \quad \forall 1 \leq i \leq N \quad (9)$$

Use of (9) constitutes the requirement that the measure of the governing equation solution approximation error be *orthogonal* to the approximation trial space ϕ_i , for $1 \leq i \leq N$, on domain $\Omega \subset \mathbb{R}^n$. Realizing that both $T(\mathbf{x})$ and $T^N(\mathbf{x})$ are surfaces spanning the domain Ω , orthogonal infers that the distribution of the *distance* between these two surfaces is the absolute smallest it can be.

The Galerkin weak statement for the unsteady bioheat transfer error equation system is

$$\begin{aligned}
 WS^N &\equiv - \int_{\Omega} \phi_i L(e^N) d\tau = \int_{\Omega} \phi_i L(T^N) d\tau = 0 \quad , for \ 1 \leq i \leq N \\
 &= \int_{\Omega} \phi_i \left(\frac{\partial T^N}{\partial t} - \frac{\partial}{\partial x_j} \left(k \frac{\partial T^N}{\partial x_j} \right) - s \right) d\tau \\
 &= \int_{\Omega} \left(\phi_i \frac{\partial T^N}{\partial t} + \frac{\partial \phi_i}{\partial x_j} \left(k \frac{\partial T^N}{\partial x_j} \right) - \phi_i s \right) d\tau \\
 &\quad - \oint_{\partial\Omega} \phi_i k \frac{\partial T^N}{\partial n} d\sigma
 \end{aligned} \tag{10}$$

where $\partial\Omega$ is the boundary of Ω and $\hat{n}$ is the outward pointing unit vector normal to $\partial\Omega$.

The elementary mathematical properties that any member of ϕ_i must possess is a non-vanishing first spatial derivative $\left(\frac{\partial \phi_i}{\partial \mathbf{x}} \right)$, and the product of this derivative with all other trial space member first spatial derivatives $\left(\frac{\partial \phi_i}{\partial \mathbf{x}} \right) \left(\frac{\partial \phi_j}{\partial \mathbf{x}} \right)$ must be integrable on Ω . Specifically, only the first derivative of $T^N(\mathbf{x})$ is required to exist in the weak statement (WS), hence, this Galerkin statement is called *weak* because it has weakened the differentiability condition that the approximation T^N must satisfy.

The introduction of a discretization Ω^h of Ω is a central concept of the finite element method. This greatly simplifies construction of the trial function set $\phi_i(\mathbf{x})$ as well as evaluation of integrals in the weak statement (10). It is important to note that a simple *sum* of the generic element contributions $T_e(\mathbf{x}, t)$ over the number of discretized elements, does not yield the approximate solution $T^h(\mathbf{x}, t)$ since a double contribution would occur on every finite element boundary. Hence, introduction of the non-overlapping finite element discretization concept for establishing the trial function set $\phi_i(\mathbf{x})$ is an important step in this procedure. The corresponding equation statement replacement for (5) is

$$T^N(\mathbf{x}, t) \Rightarrow T^h(\mathbf{x}, t) = \bigcup_{e=1}^M T_e(\mathbf{x}, t) = \bigcup_{e=1}^M \{N_k(\mathbf{x})\}^T \{Q(t)\}_e \quad (11)$$

In equation (11), $\bigcup$ is the *union*, the non-overlapping sum on the elements of the discretized solution domain $\Omega^h = \bigcup \Omega_e$. For definition of time-dependence to occur in $\{Q(t)\}_e$, the element nodal data for the dependent variable $\{N_k(\mathbf{x})\}$ remains the traditional finite element basis function set, typically constituted of polynomials complete to degree k corresponding to the Lagrange interpolant of the trial function ϕ_j . The index k thus specifies the interpolation degree of the element, e.g., linear ($k=1$), quadratic ($k=2$), and cubic ($k=3$).

A particular set of normalized monomials ζ_i , $1 \leq i \leq n+1$, invariant with Ω_e , plays a central role in development of finite element basis families. For example, see Baker and Pepper [2], for a one dimensional analysis these monomials are,

$$\zeta_1 \equiv \left(1 - \frac{\bar{x}}{l_e}\right)$$

$$\zeta_2 \equiv \left(\frac{\bar{x}}{l_e} \right) \quad (12)$$

where $\bar{x}$ is the local coordinate with origin at left end of Ω_e , i.e., XL , (Fig.1). Hence, on a one dimensional element Ω_e , the linear finite element approximation for state variable becomes

$$T_e(\bar{x}) = \left(1 - \frac{\bar{x}}{l_e} \right) QL + \left(\frac{\bar{x}}{l_e} \right) QR \quad (13)$$

where, for a generic element of length l_e , $QL \equiv T_e(\bar{x} = 0)$, and $QR \equiv T_e(\bar{x} = l_e)$.

Combining (12) and (13) yields

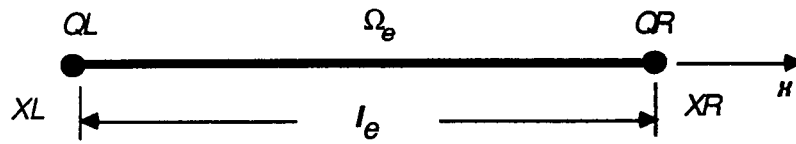
$$\begin{aligned} T_e(\bar{x}) &= \zeta_1 QL + \zeta_2 QR \\ &= \{\zeta_1, \zeta_2\} \begin{Bmatrix} QL \\ QR \end{Bmatrix} = \{\zeta_1, \zeta_2\} \{Q\}_e \\ &\equiv \{N_1\}^T \{Q\}_e \end{aligned} \quad (14)$$

Using (11) - (14), $\{N_k(\zeta_i)\}$ for one dimension is obviously

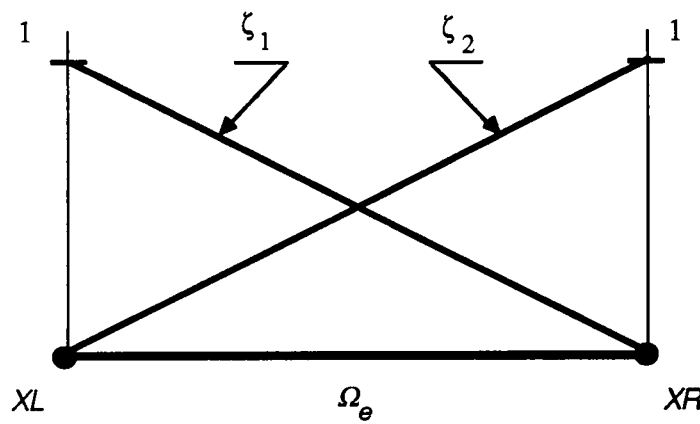
$$\{N_{k=1}\} \equiv \begin{Bmatrix} \zeta_1 \\ \zeta_2 \end{Bmatrix} \quad (15)$$

For the bioheat transfer problem domain, three dimensional hexahedron elements are employed. The intrinsic coordinate system spanning the hexahedron Ω_e is the *tensor product* system η_i , rather than the natural coordinate system ζ_i . The finite element procedure *assembles* (11) as the *union* over the elements Ω_e of the mesh Ω^h of the finite element basis definition

$$T_e(\mathbf{x}, t) \equiv \{N_k^+(\eta_i)\}^T \{Q(t)\}_e, \text{ for } 1 \leq i \leq n \quad (16)$$



a) Generic Element



b) Linear Basis ζ_i

Figure 1. Linear Basis One Dimensional ζ_i .

(a) Generic element

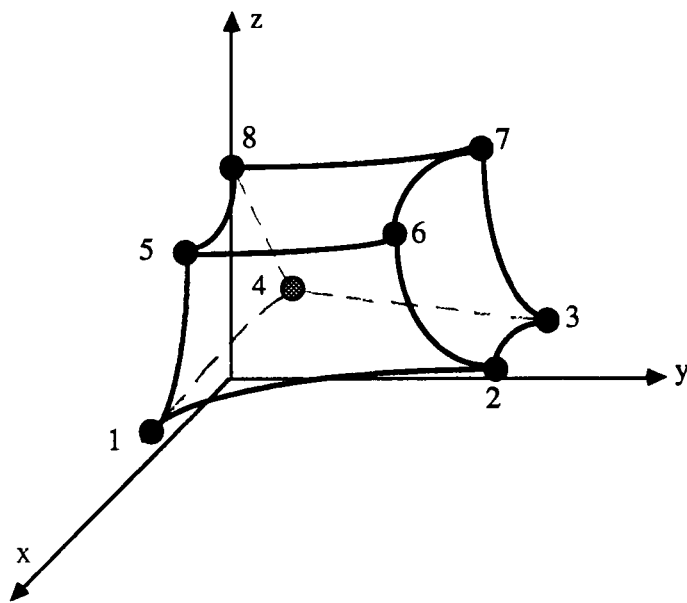
(b) Linear basis functions ζ_i

The superscript '+' distinguishes the tensor product basis formulation of (16) from the natural coordinate form (14). The $k=1$ basis $\{N_1^+\}$ must involve polynomials expressible in the global coordinate system x_i with powers no higher than unity. For a local affine transformation, the centroidal η_i coordinate system in Cartesian components (Fig. 2) is defined as

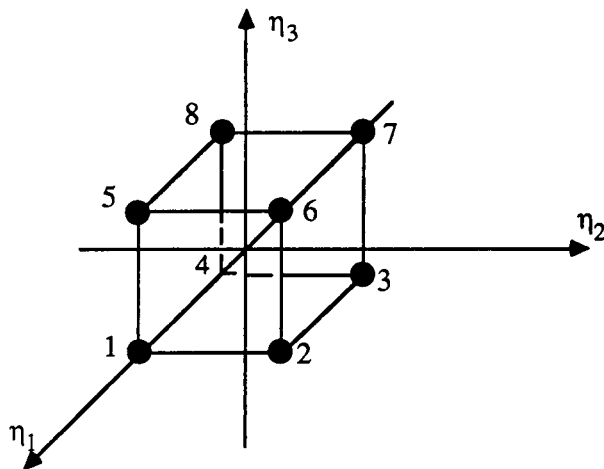
$$\begin{aligned}\eta_1 &\equiv \frac{x - X_{cn}}{a} \\ \eta_2 &\equiv \frac{y - Y_{cn}}{b} \\ \eta_3 &\equiv \frac{z - Z_{cn}}{c}\end{aligned}\tag{17}$$

In (17), (X_{cn}, Y_{cn}, Z_{cn}) is the global coordinate of the centroid of Ω_e , and the measures of Ω_e , see Fig. 2, are conventionally taken as $2a$, $2b$, and $2c$. So, the local intrinsic coordinate system η_i spans $(-1$ to $+1)$ on Ω_e (Fig. 2). The specification $k=1$ in (16) denotes the *trilinear* basis definition. For three dimensional Ω_e the elements of $\{N_1^+(\eta_i)\}$ are

$$\{N_1^+(\eta_i)\} = \frac{1}{8} \left\{ \begin{array}{l} (1 - \eta_1)(1 - \eta_2)(1 - \eta_3) \\ (1 + \eta_1)(1 - \eta_2)(1 - \eta_3) \\ (1 + \eta_1)(1 + \eta_2)(1 - \eta_3) \\ (1 - \eta_1)(1 + \eta_2)(1 - \eta_3) \\ (1 - \eta_1)(1 - \eta_2)(1 + \eta_3) \\ (1 + \eta_1)(1 - \eta_2)(1 + \eta_3) \\ (1 + \eta_1)(1 + \eta_2)(1 + \eta_3) \\ (1 - \eta_1)(1 + \eta_2)(1 + \eta_3) \end{array} \right\}\tag{18}$$



a) Global Reference Frame



b) Local Reference Frame

Figure 2. Intrinsic Coordinate System

In Figure 2, the nodal plane (1,2,3,4) is spanned by (η_1, η_2) with $\eta_3 = -1$, with the η_i -origin at the geometric centroid of Ω_e . The inverse coordinate transformation on Ω_e , using tensor notation, is $(x_j)_e = \{N_1^+(\eta_i)\}^T \{XJ\}_e$, for $1 \leq (i, j = J) \leq N$. It is easy to verify (18) using definition (16).

$$\begin{aligned}
T_e(x = X_1, y = Y_1, z = Z_1) &\equiv T_e(\eta_1 = -1, \eta_2 = -1, \eta_3 = -1) \\
&= \{N_1^+(\eta_1 = -1, \eta_2 = -1, \eta_3 = -1)\}^T \{Q\}_e \\
&= \frac{1}{8} \{8, 0, 0, 0, 0, 0, 0, 0\} \\
&\quad \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, Q_8\}^T \\
&\equiv Q_1
\end{aligned} \tag{19}$$

With this choice, note that the basis functions are not an element property [1] and also not a function of time. So,

$$\frac{\partial T_e(\mathbf{x}, t)}{\partial t} = \{N_k(\mathbf{x})\}^T \frac{d}{dt} \{Q(t)\}_e = \{N_k(\mathbf{x})\}^T \{Q(t)\}'_e \tag{20}$$

where superscript prime (') denotes the ordinary (time) derivative of the column matrix of nodal variables $\{Q\}_e$.

Following (10) - (20), a matrix statement for a discretization Ω^h of the problem domain with a uniform or non-uniform mesh can be developed. Using S_e to denote the element matrix summation process of *assembly*, amounting to rowwise addition in the global matrix statement, (10) produces

$$\begin{aligned}
WS^h = & S_e \left[\int_{\Omega} \{N_k\} \{N_k\}^T \{Q\}'_e d\tau + \int_{\Omega} \nabla \{N_k\} \cdot k_e \nabla \{N_k\}^T \{Q\}_e d\tau \right. \\
& - \int_{\Omega} \{N_k\} \{N_k\}^T \{S\}_e d\tau + \int_{\partial\Omega_1} \{N_k\} h_e \{N_k\}^T \{Q - TR\}_e d\sigma \\
& \left. + \int_{\partial\Omega_2} \{N_k\} \{N_k\}^T \{QN\}_e d\sigma - \int_{\partial\Omega_3} \{N_k\} k_e \nabla T_e \cdot \hat{n} d\sigma \right] \quad (21)
\end{aligned}$$

where,

$\nabla \equiv \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$, is the differential operator,

subscript e denotes element property,

superscript T denotes transpose of a column matrix $\{\cdot\}$,

$\{QN\}_e$ is the formal normal component of the state variable nodal array

$\{Q\}$ on

the boundary $\partial\Omega$ of the domain Ω , and

$\{TR\}_e$ is the reference temperature array of the convective flux on Ω .

Independent of the dimension n of Ω , and for general variation of physical properties and heat flux data, the weak statement finite element algorithm (21) yields a set of ordinary differential equations written on time of the form, Baker [1]

$$\begin{aligned}
WS^h = & S_e ([M]_e \{Q(t)\}'_e + [K + H]_e \{Q(t)\}_e - \{b(t)\}_e) \\
= & [M] \{Q\}' + [K + H] \{Q\} - \{b\} = \{0\} \quad (22)
\end{aligned}$$

The second line in (22) represents the assembled global form of the finite element weak statement algorithm where:

$\{Q(t)\}$ contains the time-dependent nodal values of temperature

approximation on $\Omega^h \cup \partial\Omega^h$;

$\{Q(t)\}' = \frac{d\{Q\}}{dt}$ is the ordinary time derivative;

$[M]$ is a square matrix representing the distribution of thermal capacity on the discretization of the problem domain;

$[K]$ is a square matrix involving all diffusion effects;

$[H]$ is a matrix involving all non-homogeneous boundary heat convection effects;

$\{b(t)\}$ is a column matrix containing all non-homogeneous applied heat source *data* on $\Omega^h \cup \partial\Omega^h$.

The specific form of the element matrices defined in the first line of (22) are

$$\begin{aligned}
[M]_e &= \int_{\Omega} \{N_k\} \{N_k\}^T d\tau \\
[K]_e &= \int_{\Omega} k_e \nabla \{N_k\} \cdot \nabla \{N_k\}^T d\tau \\
[H]_e &= \int_{\partial\Omega_1 \cup \partial\Omega_2 \cap \partial\Omega_e} \{N_k\} h_e \{N_k\}^T d\sigma \\
\{b(t)\} &= \int_{\Omega} \{N_k\} \{N_k\}^T d\tau \{S\}_e
\end{aligned} \tag{23}$$

Thus the assembled global matrix form coupled with time derivative balances thermal conduction, boundary heat convection and source term effects at any time t .

WS^h in (22) allows one to evaluate the rate at which $\{Q(t)\}$ changes with time knowing its value at time t^n . Using a Taylor series expansion, the time-dependent

state variable $\{Q\}$ at t^{n+1} can be expressed as

$$\{Q\}^{n+1} = \{Q\}^n + \Delta t \left(\theta \frac{d\{Q\}^{n+1}}{dt} + (1 - \theta) \frac{d\{Q\}^n}{dt} + O(\Delta t^2) \right), \text{ for } 0 \leq \theta \leq 1 \quad (24)$$

where $\{Q\}^n = \{Q(t^n)\}$ and θ , the implicitness factor, defines the specific selected Taylor series, i.e., *forward* Euler ($\theta=0$), *trapezoidal* rule ($\theta=0.5$), and *backwards* Euler ($\theta=1$). Equation (22) can be rewritten as

$$\frac{d\{Q\}}{dt} = [M]^{-1}(-[K + H]\{Q\} + \{b\}) \quad (25)$$

Substituting (25) into (24) yields

$$\begin{aligned} \{Q\}^{n+1} &= \{Q\}^n + \Delta t \theta [M]^{-1} (-[K + H]\{Q\}^{n+1} + \{b\}^{n+1}) \\ &\quad + \Delta t (1 - \theta) [M]^{-1} (-[K + H]\{Q\}^n + \{b\}^n), \text{ for } 0 \leq \theta \leq 1 \end{aligned} \quad (26)$$

Multiplication of (26) by $[M]$ produces

$$\begin{aligned} [M] (\{Q\}^{n+1} - \{Q\}^n) &= \Delta t \theta (-[K + H]\{Q\}^{n+1} + \{b\}^{n+1}) \\ &\quad + \Delta t (1 - \theta) (-[K + H]\{Q\}^n + \{b\}^n) \end{aligned} \quad (27)$$

since $[M][M]^{-1} = [I]$, the identity matrix. Now, introducing the definition $\{\Delta Q\}^{n+1} \equiv (\{Q\}^{n+1} - \{Q\}^n)$, as the change in $\{Q\}$ over the time interval Δt , and carrying the right hand side non-data terms multiplied by θ to the left side (27) yields a global matrix statement

$$\begin{aligned} ([M] + \theta \Delta t [K + H]) \{\Delta Q\}^{n+1} &= -\Delta t [K + H]\{Q\}^n \\ &\quad + \Delta t (\theta \{b\}^{n+1} + (1 - \theta)\{b\}^n) \end{aligned} \quad (28)$$

Assuming the data in $\{b\}$ are time independent, the assembly form of the steady state algorithm is

$$S_e([M]_e + \theta \Delta t [K + H]_e) \{\Delta Q\}_e + \Delta t \{R\}_e^n = \{0\} \quad (29)$$

where,

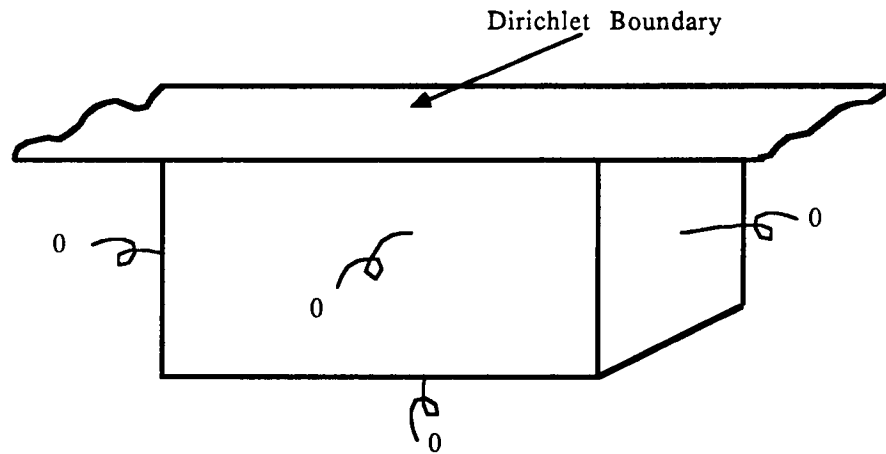
$$\{R\}_e^n = [K + H]_e \{Q\}_e^n - \{b\}_e^n \quad (30)$$

is called the *residual* matrix of the algorithm. The unknown in (28)-(30) is $\{\Delta Q\}^{n+1}$ which can be solved from the data matrix $\{b\}$ and the *known* solution $\{Q\}^n$ at the former time station.

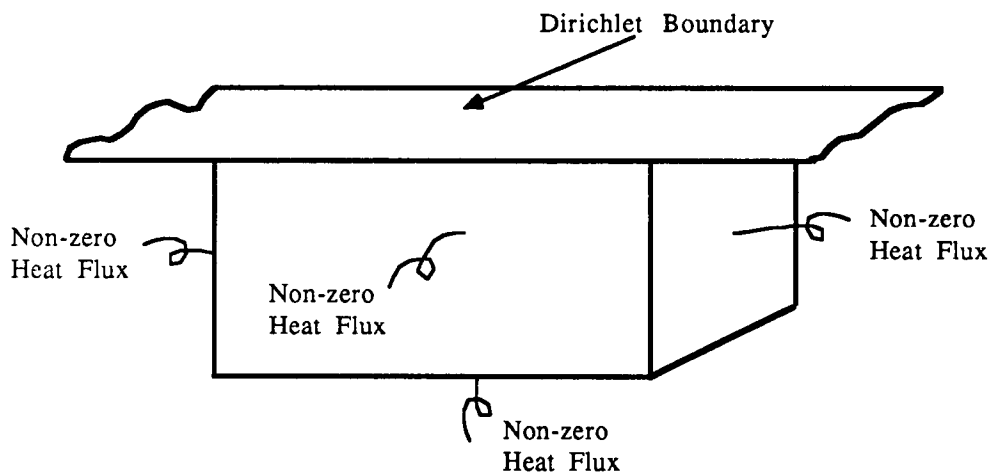
(3) Boundary Conditions :

The governing equation of our problem statement in the symmetric Galerkin weak statement formulation (10) involves integrals over the generic finite element domain Ω_e which are easy to evaluate. The other terms contain boundary fluxes which for n dimensional finite element domain can be evaluated over an $(n-1)$ dimensional boundary. For accuracy of the approximate solution, the boundary integrals are precisely calculated.

The temperature of the *top* surface of the simulation domain was taken as Dirichlet (*fixed*) data by Panjehpour [23], which is a close approximation to the ambient value that can be measured. Assuming the treatment volume is small enough, compared to its surrounding tissue region, the resistance to heat flux due to conduction in directions normal to the five remaining domain surfaces may be taken as negligible in comparison to that through the top surface (Figure 3).



a) No Tissue Interface Boundary Flux
(Adiabatic Condition)



b) Neumann Tissue Interface Boundary Condition

Figure 3. Boundary Conditions of the Simulation Domain.

It has also been experimentally established that the majority of heat removal in normal tissue is due to blood flow (perfusion). Following these reasonings, Panjehpour [23] neglected the convective heat flux through the five domain surfaces, and hence, in his solution methodology the corresponding boundary heat fluxes were zero, i.e. *adiabatic*. His results [23] indicate the zero flux assumption is valid only at a low power delivery in the treatment region. To avoid this limitation, an arbitrary Neumann condition at external boundaries of the treatment volume has been introduced in some simulations in this thesis project. However, the main intention is to simulate the hyperthermia modality temperature distribution using an advanced mathematical solution methodology admitting use of dense meshes.

(4) Numerical Linear Algebra Solution Procedures :

If a direct solution procedure is adopted, the global matrix statement (28) will require the effective inversion of the $m \times m$ square matrix $([M] + \theta\Delta t [K + H])$, where m is the total number of nodes in the discretized problem domain. As the mesh density increases, direct solution methodology computation time increases as the fourth power of m . This certainly becomes intractable for the problem under study where we are dealing with $\sim 10^5$ nodes. The key technical aspect of this thesis is the implementation of the replacement of direct solution for the global matrix statement (28) with a grid sweeping iterative algorithm where lexicographic partitions of the domain are solved in sequence. There are two different formulation techniques available with this implementation.

Since the diffusion and boundary convection contributions can each be resolved

into components parallel to the global coordinate system x_i , $1 \leq i \leq n$ (dimension of Ω), equation (29) can be reexpressed as

$$S_e([M]_e + \theta\Delta t[KX + KY + KZ + HX + HY + HZ]_e)\{\Delta Q\}_e + \Delta t\{R\}_e^n = \{0\} \quad (31)$$

Upon assembly over all Ω_e of Ω^h , (31) becomes

$$([M] + \theta\Delta t[KX + KY + KZ + HX + HY + HZ])\{\Delta Q\} = -\Delta t\{R\}^n \quad (32)$$

where all *data* terms, i.e., those multiplied by Δt in (30), have been stored in the residual $\{R\}^n$.

Letting $[A]$ denote the system matrix in (32), the resolution yields

$$\begin{aligned} [A] &\equiv [M] + \theta\Delta t[KX + KY + KZ + HX + HY + HZ] \\ &\simeq [AX][AY][AZ] \end{aligned} \quad (33)$$

The product $[AX][AY][AZ]$ in (33) is truly an approximation and it involves a truncation error of the order $O(\theta\Delta t)^2$. (It is important to note that at this point *no approximation* is assumed in forming $\{R\}$.) This error order is same as that for the forward or backwards Euler integration procedures. Hence,

$$[A]\{\Delta Q\} \simeq [AX][AY][AZ]\{\Delta Q\} = -\Delta t\{R\}^n + O(\Delta t^2) \quad (34)$$

Since the element mass matrix $[M]_e$ can be exactly resolved into three different directions, i.e., $[M]_e = [MX]_e[MY]_e[MZ]_e$, an excellent approximation for the factored matrices defined in (34) is

$$[AX] \equiv S_e([AX]_e) = S_e([MX]_e + \theta\Delta t[KX + HX]_e)$$

$$\begin{aligned}
[AY] &\equiv S_e([AY]_e) = S_e([MY]_e + \theta\Delta t[KY + HY]_e) \\
[AZ] &\equiv S_e([AZ]_e) = S_e([MZ]_e + \theta\Delta t[KZ + HZ]_e)
\end{aligned} \tag{35}$$

A. Time-Split (TS) Algorithm :

In (35) all contributions are formed using one dimensional element matrices. In the time-split procedure [41], $\{R\}^n$ is also approximately formed via contributions resolved parallel to the global coordinate system x_i . Defining

$$\begin{aligned}
\{RX\}_e &= ([KX]_e + [HX]_e) \{Q\}_e - \{b_x\}_e \\
\{RY\}_e &= ([KY]_e + [HY]_e) \{Q\}_e - \{b_y\}_e \\
\{RZ\}_e &= ([KZ]_e + [HZ]_e) \{Q\}_e - \{b_z\}_e
\end{aligned} \tag{36}$$

the linear basis one dimensional master matrix library from Baker and Pepper [eq. 8.26] can be used to derive appropriate residuals in the x_i directions. The resultant approximation can be formed, if wanted, as $\{R\} \simeq S_e(\{RX\}_e + \{RY\}_e + \{RZ\}_e)$. However, for the *time-splitting* (TS), finite-difference algorithm of Yanenko (1971), the resolved approximations are always coupled with the resolved matrix factorization on node rows/columns (as appropriate), and the multi-dimensional residual assembly is never formed.

For solution ordering parallel to X, Y, Z , sequentially, the matrix statement equation (34) is replaced by the TS solution sequence,

$$\begin{aligned}
[AX]\{\Delta QX\} &= -\Delta t \frac{1}{3} \{RX(Q)^n\} \\
\{Q\}^{n+\frac{1}{3}} &\equiv \{Q\}^n + \{\Delta QX\}
\end{aligned}$$

$$\begin{aligned}
[AY]\{\Delta QY\} &= -\Delta t \frac{1}{3} \{RY(Q)^{n+\frac{1}{3}}\} \\
\{Q\}^{n+\frac{2}{3}} &\equiv \{Q\}^{n+\frac{1}{3}} + \{\Delta QY\} \\
[AZ]\{\Delta QZ\} &= -\Delta t \frac{1}{3} \{RZ(Q)^{n+\frac{2}{3}}\} \\
\{Q\}^{n+1} &\equiv \{Q\}^{n+\frac{2}{3}} + \{\Delta QZ\}
\end{aligned} \tag{37}$$

First, TS utilizes the intermediate solution data array to update the dependent variable at the one-third time interval. Then, $\{Q\}^{n+\frac{1}{3}}$ is used for a directional residual evaluation during the second solution sweep sequence and $\{Q\}^{n+\frac{2}{3}}$ is formed and used for another directional residual evaluation during the third solution sweep.

B. Full Residual (FR) Method :

It is important to note that the TS procedure (37) assembles the factor matrix and the one dimensional approximate residual in the same direction. An alternative form requires the full-dimensional (non-split) $\{R\}^n$ to be used to improve accuracy in the more general situation. This form constitutes the outer (tensor) product factorization (35) with full dimensional residual evaluation. The computational form is

$$\begin{aligned}
[AX]\{P\} &= -\Delta t \{R\}^n \\
[AY]\{S\} &= \{P\} \\
[AZ]\{\delta Q\} &= \{S\} \\
\{Q\}^{n+1} &= \{Q\}^n + \{\delta Q\}
\end{aligned} \tag{38}$$

where $\{P\}$ and $\{S\}$ are (column matrix) arrays of *intermediate* data.

C. General Geometries :

The bio-heat transfer benchmark problem statement that has been analyzed by Panjehpour [23] strictly involved hexahedron geometries. In reality, the geometry of the tissue is of more general form. The upgraded *LEARN.FE* code can very well be extended to handle such general geometries.

Generalized mesh finite element algorithm involves forming transformation jacobians $[J]$ in the left hand side matrices in (38), i.e.,

$$[J] \equiv \left[\frac{\partial \eta_i}{\partial x_j} \right] = \left[\frac{\partial x_j}{\partial \eta_i} \right]^{-1} \quad (39)$$

An example shows the calculation complexity of these jacobians. Using elementary matrix algebra the elements of the required jacobian (two dimensional element for simplicity) are then

$$\begin{aligned} [J]_e &\equiv \left[\frac{\partial x_j}{\partial \eta_i} \right]_e^{-1} = \left[\begin{array}{cc} \frac{\partial x_1}{\partial \eta_1} & \frac{\partial x_1}{\partial \eta_2} \\ \frac{\partial x_2}{\partial \eta_1} & \frac{\partial x_2}{\partial \eta_2} \end{array} \right]_e^{-1} \\ &= \frac{1}{\det[\]_e} \left[\begin{array}{cc} \frac{\partial x_2}{\partial \eta_2} & \frac{\partial x_1}{\partial \eta_2} \\ \frac{\partial x_2}{\partial \eta_1} & \frac{\partial x_1}{\partial \eta_1} \end{array} \right]_e \end{aligned} \quad (40)$$

where

$$\begin{aligned} \det_e &= \det \left[\begin{array}{cc} \frac{\partial x_1}{\partial \eta_1} & \frac{\partial x_1}{\partial \eta_2} \\ \frac{\partial x_2}{\partial \eta_1} & \frac{\partial x_2}{\partial \eta_2} \end{array} \right]_e \\ &= \left(\frac{\partial x_1}{\partial \eta_1} \right)_e \left(\frac{\partial x_2}{\partial \eta_2} \right)_e - \left(\frac{\partial x_1}{\partial \eta_2} \right)_e \left(\frac{\partial x_2}{\partial \eta_1} \right)_e \end{aligned} \quad (41)$$

For three dimensional element equation (40) becomes

$$[J]_e \equiv \left[\frac{\partial x_j}{\partial \eta_i} \right]_e^{-1} = \begin{bmatrix} \frac{\partial x_1}{\partial \eta_1}, & \frac{\partial x_1}{\partial \eta_2}, & \frac{\partial x_1}{\partial \eta_3} \\ \frac{\partial x_2}{\partial \eta_1}, & \frac{\partial x_2}{\partial \eta_2}, & \frac{\partial x_2}{\partial \eta_3} \\ \frac{\partial x_3}{\partial \eta_1}, & \frac{\partial x_3}{\partial \eta_2}, & \frac{\partial x_3}{\partial \eta_3} \end{bmatrix}_e^{-1} \quad (42)$$

Hence,

$$[J]_e \equiv \frac{1}{\det_e} \begin{bmatrix} \left| \begin{array}{cc} \frac{\partial x_2}{\partial \eta_2} & \frac{\partial x_2}{\partial \eta_3} \\ \frac{\partial x_3}{\partial \eta_2} & \frac{\partial x_3}{\partial \eta_3} \end{array} \right| & \left| \begin{array}{cc} \frac{\partial x_3}{\partial \eta_1} & \frac{\partial x_3}{\partial \eta_3} \\ \frac{\partial x_2}{\partial \eta_1} & \frac{\partial x_2}{\partial \eta_3} \end{array} \right| & \left| \begin{array}{cc} \frac{\partial x_2}{\partial \eta_1} & \frac{\partial x_2}{\partial \eta_2} \\ \frac{\partial x_3}{\partial \eta_1} & \frac{\partial x_3}{\partial \eta_2} \end{array} \right| \\ \left| \begin{array}{cc} \frac{\partial x_3}{\partial \eta_2} & \frac{\partial x_3}{\partial \eta_3} \\ \frac{\partial x_1}{\partial \eta_2} & \frac{\partial x_1}{\partial \eta_3} \end{array} \right| & \left| \begin{array}{cc} \frac{\partial x_1}{\partial \eta_1} & \frac{\partial x_1}{\partial \eta_3} \\ \frac{\partial x_3}{\partial \eta_1} & \frac{\partial x_3}{\partial \eta_2} \end{array} \right| & \left| \begin{array}{cc} \frac{\partial x_3}{\partial \eta_1} & \frac{\partial x_3}{\partial \eta_2} \\ \frac{\partial x_1}{\partial \eta_1} & \frac{\partial x_1}{\partial \eta_2} \end{array} \right| \\ \left| \begin{array}{cc} \frac{\partial x_1}{\partial \eta_2} & \frac{\partial x_1}{\partial \eta_3} \\ \frac{\partial x_2}{\partial \eta_2} & \frac{\partial x_2}{\partial \eta_3} \end{array} \right| & \left| \begin{array}{cc} \frac{\partial x_2}{\partial \eta_1} & \frac{\partial x_2}{\partial \eta_3} \\ \frac{\partial x_1}{\partial \eta_1} & \frac{\partial x_1}{\partial \eta_2} \end{array} \right| & \left| \begin{array}{cc} \frac{\partial x_1}{\partial \eta_1} & \frac{\partial x_1}{\partial \eta_2} \\ \frac{\partial x_2}{\partial \eta_1} & \frac{\partial x_2}{\partial \eta_2} \end{array} \right| \end{bmatrix}_e$$

$$= \frac{1}{\det_e} \begin{bmatrix} ETA11, & ETA21, & ETA31 \\ ETA12, & ETA22, & ETA32 \\ ETA13, & ETA23, & ETA33 \end{bmatrix}_e \quad (43)$$

where $\det_e = \text{determinant of } \left[\frac{\partial x_i}{\partial \eta_j} \right]_e$.

For $i=1$ and $j=1$

$$\frac{\partial x_1}{\partial \eta_1} = \frac{\partial}{\partial \eta_1} \{N_1^+(\eta_i)\}^T \{X1\}_e \equiv \{X1\}_e^T \frac{\partial}{\partial \eta_1} \{N_1^+\} \quad (44)$$

Evaluating $\frac{\partial}{\partial \eta_1} \{N_1^+\}$ from (18) equation (44) yields

$$\begin{aligned} \frac{\partial x_1}{\partial \eta_1} &= \frac{1}{8} ((1 - \eta_2)(1 - \eta_3)(X_2 - X_1)_e + (1 + \eta_2)(1 - \eta_3)(X_3 - X_4)_e \\ &\quad + (1 - \eta_2)(1 + \eta_3)(X_6 - X_5)_e + (1 + \eta_2)(1 + \eta_3)(X_7 - X_8)_e) \end{aligned} \quad (45)$$

The element-average (centroidal) value of the linear interpolation occurs for $\eta_1 \equiv \eta_2 \equiv \eta_3 \equiv 0$, whereupon we are lead to define the element average metric set as $ETAIJ_e$. For the linear tensor product basis $\{N_1^+\}$, the finite element domain $\mathfrak{R}_e^3$ possesses only vertex nodes $\{XI\}_e \equiv \{XI, YI, ZI, \quad 1 \leq I \leq 8\}$. For the definition given in (44), the elements on the *trace* of $\left[\frac{\partial \eta_k}{\partial x_j}\right]$, evaluated at the centroid ($\eta_i = 0, 1 \leq i \leq 3$) of $\mathfrak{R}_e^3$ are

$$\begin{aligned}\{ETA11\}_e^T &= \frac{1}{4}\{Y41Z51, Y32Z62, Y32Z73, Y41Z84, \\ &\quad Y85Z51, Y76Z62, Y76Z73, Y85Z84\}_e \\ \{ETA22\}_e^T &= \frac{1}{4}\{X21Z51, X21Z62, X34Z73, X34Z84, \\ &\quad X65Z51, X65Z62, X78Z73, X78Z84\}_e \\ \{ETA33\}_e^T &= \frac{1}{4}\{X21Y41, X21Y32, X34Y32, X34Y41, \\ &\quad X65Y85, X65Y76, X78Y76, X78Y85\}_e\end{aligned}\quad (46)$$

For rectangular grid, $\{ETAKJ\}_e = \{0\}$ for $K \neq J$, and the notation in (46) is defined as

$$Yab Zcd \equiv (Ya - Yb)(Zc - Zd), \quad for \quad 1 \leq a, b, c, d \leq 8 \quad (47)$$

Using the same definition (47), the elements of $\{DET\}_e \equiv \overline{det}_e$ evaluated at the origin of the domain $\mathfrak{R}_e^3$ are

$$\begin{aligned}\{DET\}_e^T &= \frac{1}{64}\{X21Y41Z51, X21Y32Z62, \\ &\quad X34Y32Z73, X34Y41Z84, \\ &\quad X65Y85Z51, X65Y76Z62, \\ &\quad X78Y76Z73, X78Y85Z84\}_e\end{aligned}$$

$$X78Y76Z73, X78Y85Z84\}_e \quad (48)$$

In this thesis project, the *residual* $\{R\}^n$ has been handled in two different ways, i.e., (a) $\{R\}^n$ has been split parallel to three intrinsic (η_i coordinate directions, equation (37), and (b) $\{R\}^n$ has been formed fully, equation (38).

CHAPTER IV

RESULTS AND DISCUSSION

A. A Benchmark Problem :

The *LEARN.FE* computer code (Baker and Pepper [2]) was upgraded in [3] to handle equations (31)-(44). The benchmark simulation is based on the experiment reported in Panjehpour [23]. A 10 MHz sinusoidal signal of peak-to-peak 22.6 volts is assumed between two electrodes. The resultant heat source is clustered towards the center of the simulation hexahedron. Despite knowing that the initial condition $T_0(\mathbf{x})$ has no effect on the steady-state solution, a uniform initial temperature of 33.3°C was used to approximate the actual situation *in vivo*. The new code is capable of predicting time-accurate unsteady solutions; however, the comparisons reported herein are restricted to steady state solutions for data reported by Panjehpour [23]. The temperature of the outer (*top*) surface of the simulation domain is Dirichlet (*fixed*) at 33.3°C which is a close approximation to the ambient value that can be measured. It is further assumed that the heat flux through the other five external surfaces due to conduction heat transfer is negligible compared to the heat removed by blood flow. Hence, the heat flux Neumann boundary condition for the remaining five external surfaces is zero, i.e. *adiabatic*, (Fig. 3). Panjehpour [23] indicates that the zero heat flux assumption is valid only at a low power delivery to the treatment region.

(1) Accuracy :

Using the *TS* finite element procedure, solutions on four uniform discretizations of the prototypical problem domain were established. The solution meshes are $M = 6^3$, 12^3 , 24^3 , and 48^3 right rectangular hexahedron elements, respectively. The computed steady state temperature distributions are summarized in Figures B1,B3-B5¹ in terms of planar isotherm distribution and perspective surface plots of temperature distribution on three mesh planes common to all discretizations. For reference, the solution on the $M = 5^3$ mesh from [23] is included as Figure B2¹. This result compares qualitatively with the current $M = 6^3$ mesh solution. All plots are scaled with respect to the most accurate solution of the $M = 48^3$ mesh. Each plane presented is located at the same elevation in the problem domain, and the differences in computed temperature levels in each profile are very evident. A summary comparison of normalized planar isotherm and perspective surface profiles for the four different mesh solutions are given in Figure B6¹. There is a sharp rise in peak-to base temperature difference as the mesh density increases from 6^3 to 48^3 . This can be noticed in gradual growth of the bell shaped surface plots for different mesh size. The variation in peak-to-base temperature for the mid-zplane in $M = 48^3$ mesh is apporximately 20 percent more than that in the $M = 6^3$ mesh. This reflects the numerical approximation inaccuracy associated with the coarse mesh solutions.

This is a most important issue with numerical solutions to this problem. It

¹Figures are located in Appendix B

has been observed [9] that if cancerous cells are maintained at 41.5°C for 5 hours, approximately 10 percent will survive. Conversely, if the temperature is increased by only one degree to 42.5°C , a much greater efficiency results with almost 99.99 percent of the cells dying. Hence, a fraction of a degree improvement in predicted temperature accuracy can exert significant impact on design of a hyperthermia treatment procedure. Figure B7¹ summarizes the prediction accuracy improvement at the location of the maximum temperature in the three common observation planes. The $M = 24^3$ mesh solution appears accurate to within 0.2°C assuming the $M = 48^3$ mesh solution is genuinely accurate.

The surface temperature distribution for the benchmark test case in three different planes, namely # 2, #4 and #6, for $M = 6^3$ mesh is plotted in actual scale in Figure B8¹. There is a small rise in temperature distribution at the center of each plane and in the mid-plane, (# 4), this rise is maximum. It is important to note that the average distribution in #4 is about 37°C while for planes #2 and #6 it is approximately 34°C and 37.5°C respectively.

(2) Efficiency :

The upgraded *LEARN.FE* computer code solves the finite element algorithm matrix system (28) using the highly efficient time-split (*TS*) numerical linear algebra procedure (31)-(45). The memory requirement for the *TS* matrix solution procedure is only $3n$ for an n^3 nodal mesh. The number of matrix solution computations in each grid-sweep algorithm step is approximately $3n^2$, and there are three grid sweeps required for every solution step. Note that, although *TS* (37) is utilizing

the directional residual evaluation, while FR (38) is dealing with the full residual right hand side, both TS and FR use the same number of algebraic steps in their respective solution algorithm and have the same left hand side matrix data. Hence, they have the same operation count. The memory requirement for the matrix solution procedure in FR is also $3n$ for an n^3 nodal mesh.

In comparison, in his research project, Panjehpour [23] adopted a Gaussian elimination matrix solution procedure with complete pivoting for directly solving the steady state problem statement. This direct scheme possesses an inherent limitation regarding application to denser mesh solutions. The total memory requirement for the system matrix of an n^3 nodal mesh is order $n^2(n+2)$, while the number of computations to the solution is order $(n+1)^4$. Hence for $n \approx 50$ (the $M = 48^2$ mesh), $\sim 10^5$ words of memory storage for the jacobian $[A]$ in (33) would be required in comparison to the current ~ 150 for TS and/or FR in (37-38). Further, the direct solution operation count is $\sim 6 \times 10^6$ for $n = 48$, while for the TS and/or FR method the comparison count is $\sim 7 \times 10^3$ for each time step. However, it is necessary to mention that the number of timesteps required to reach the steady state solution for $M = 6^3$ mesh is ~ 400 . This number is drastically reduced, $\sim O(10)$, for calculations involving higher mesh densities by using the solution on the coarser mesh, e.g. $M = 6^3$, as an initial condition to the next denser mesh, e.g. $M = 12^3$. A comparison of the CPU computer time requirement for the direct solution procedure and the TS algorithm is shown in Figure B9¹. For the small mesh size the direct solver takes less CPU time than does the TS solver. As the

number of nodes involved in the computation increases, a steep rise in CPU time is recorded for the direct solution procedure. Conversely, the slope of the CPU time curve for *TS* method changes negligibly. Hence, CPU time requirement for *TS* becomes much smaller than that for the direct solver as the mesh density increases. The pertinence of this developed numerical algebra procedure to the hyperthermia problem class is thus very apparent.

B. Localized Cooling :

The benchmark problem is a simple solution of bell shaped temperature distribution with its axis of symmetry along the center of zplanes of the simulation domain. Evidently, this kind of nonhomogeneous temperature distribution cannot heat the entire region of malignant tissue uniformly. Since, the peak temperature in any zplane for the benchmark simulation is at its center, a moderation of the temperature non-homogeneity in the simulation region can be achieved by insertion of a cooled needle (carrying liquid nitrogen, or cold water) at that location.

For this thesis the cooled needle is approximated by a line of fixed (i.e. *Dirichlet*) nodal temperatures running vertically through the center of each *z*-plane of the simulation hexahedron (Figure 4). The top surface boundary plane remains as a *Dirichlet* plane with temperature held at $33.3^{\circ}C$. The other five boundaries are first considered as adiabatic, (i.e. no boundary heat flux), and then a non-homogeneous heat flux *Neumann* boundary condition is imposed on them. Like the benchmark case, a uniform temperature of 33.3° is the initial condition for the simulation.

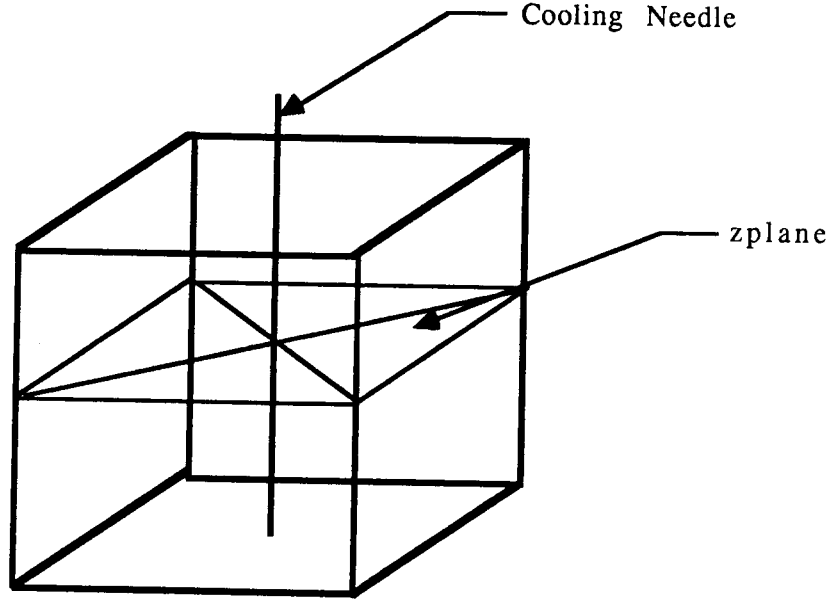


Figure 4. Location of Cooling Needle in a Simulation Domain

(1) Cooled Needle Simulation :

A. Constant Temperature :

For the first computational experiment, the temperature of the simulated cooled needle was set at a uniform constant, $T_{needle}(z) = 33.3^{\circ}C$. The energy input due to the line electric potential is kept at the same value as the benchmark test case. The formulation of the *TS* algorithm (37) can create directional bias as it resolves the component of the *Residual Matrix* in the directions parallel to the global coordinate system and iterates, sequentially. This does not affect the benchmark test case due to the simplicity of the problem statement. However, for the study of the

constant temperature needle, a noticeable asymmetry emerges in the *TS* algorithm simulation solution, Figure B10¹. Moreover, a node-to-node fluctuation in the *z*-plane temperature distribution occurs that yeilds a undesirably non-homogeneous error. Since the *FR* formulation (38) does not resolve the residual into its directional component this asymmetry would not occur in a *FR* solution. The comparison of the *TS* and *FR* simulation results for a $6 \times 6 \times 6$ cubic mesh for this problem is summarized in Figure B10¹. Although the planer isotherm plots for both *TS* and *FR* look symmetric, the plane graph in Figure B11¹ clearly demonstrates the nonsymmetric variation of temperature in the two perpendicular mesh directions lying on the same plane. Hence, for all subsequent simulations, the full residual *FR* algorithm has been employed. The comparison study of CPU time for the *TS* and *FR* algorithms has been plotted as a line graph in Figure B12¹. The *FR* algorithm needs negligibly less computation time for the solution procedure.

The planar steady state *FR* temperature distributions for the $M = 6 \times 6 \times 6$ mesh with constant temperature cooled needle are plotted for various *z*planes in Figure B13¹. Note that a locally nonhomogeneous temperature distribution still dominates the solution. This is due to fixing the central node column temperature to the constant $T_{needle} = 33.3^{\circ}C$, which is well below the resultant surrounding tissue temperature for the selected energy input. The following computational experiment clarifies the issue.

B. Variable Temperature :

The previous experiments has shown that the temperature at any point inside

the tissue increases with its distance from the fixed temperature surface (five other surfaces are adiabatic at this point). Hence, $T_{needle}(z) = c$, c being the constant temperature, would not produce a practical solution. An improved simulation would use a reasonable temperature profile in the z -direction for fixing the temperature of the needle, i.e., $T_{needle}(z) = \psi(z)$, $\psi(z)$ being taken as a linear function of z . In this experiment, the fixed temperature of the central node has been taken as 38.6, 40.5, 42.4°C for z planes #2, #4 and #6 respectively.

This improves the steady state solution homogeneity to a reasonable degree. The temperature inside the tissue is also elevated to a higher value although the solution symmetry is absolutely preserved. The corresponding results has been plotted in Figure B14¹ as surface profiles for different z planes. A summary comparison of variable temperature cooled needle and constant temperature needle is shown in Figure B15¹. There is almost 20 percent temperature increament in all the z planes. This confirms the pertinence of variable Dirichlet temperature of the needle. The variation of temperature distribution with different mesh size is plotted as a line graph in Figure B16¹. There remains a noticeable radial variation of temperature, however, the maximum difference at any location is about 0.4°C, for the two different meshes.

(2) Non-homogeneous Heat Flux Boundary Condition :

As mentioned earlier the vanishing boundary flux assumption is valid only at a low power delivery to the treatment volume. The malignant tissue is generally surrounded by normal tissue where perfusion plays an important role in reduc-

ing local temperature rise (above ambient) due to the neighborhood hyperthermia treatment. Hence, there is a temperature difference between the tumor tissue and a normal tissue during hyperthermia treatment, that causes a non-zero heat flux across the external boundaries of the treatment region. These surfaces have so far been considered as adiabatic. For an improved simulation, a non-zero heat flux (*Neumann*) boundary condition has been introduced on these five external boundaries of the treatment hexahedron. The assumed initial condition of uniform 33.3°C remains for the whole region.

It is also apparent that for most practical *in vivo* situations, the benchmark cubical treatment domain is not representative. The boundary of the surrounding standard tissue is also not regular, and it may have nonplanar surfaces. The rectangular parallelepiped configuration ($2 \times 1 \times 1$, Figure 5) has been introduced as an intermediate step to this general geometrical domain requirement. The selected grid is shown in Figure 6. Since there is a fixed temperature node at the center, better refinement of the mesh is necessary near that point in order to capture the temperature distribution in the surrounding region.

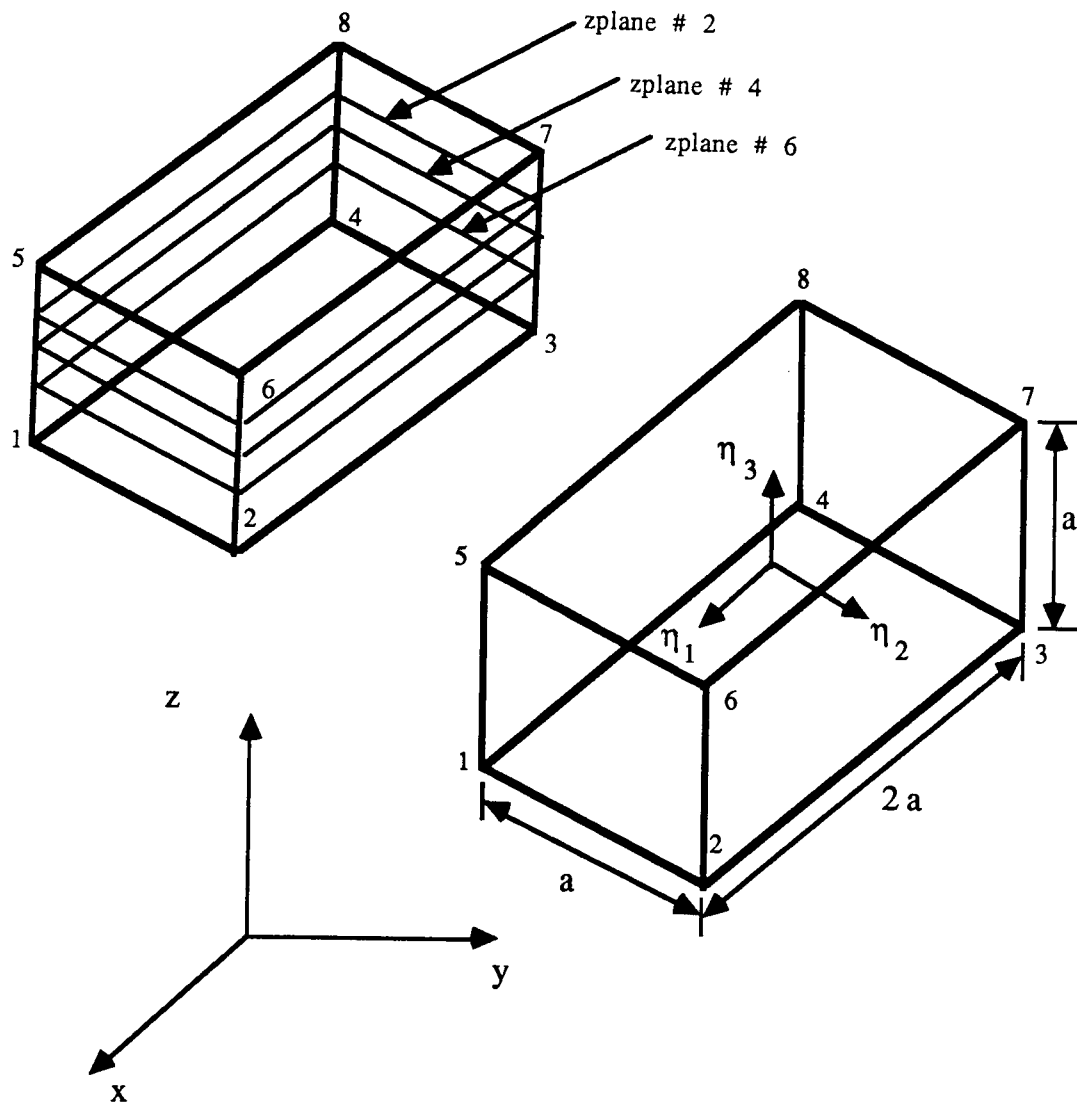
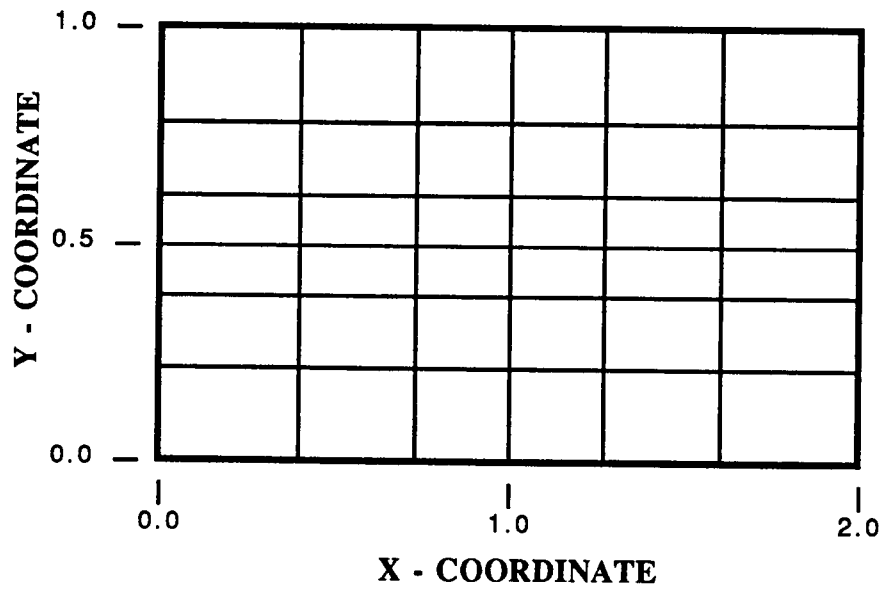
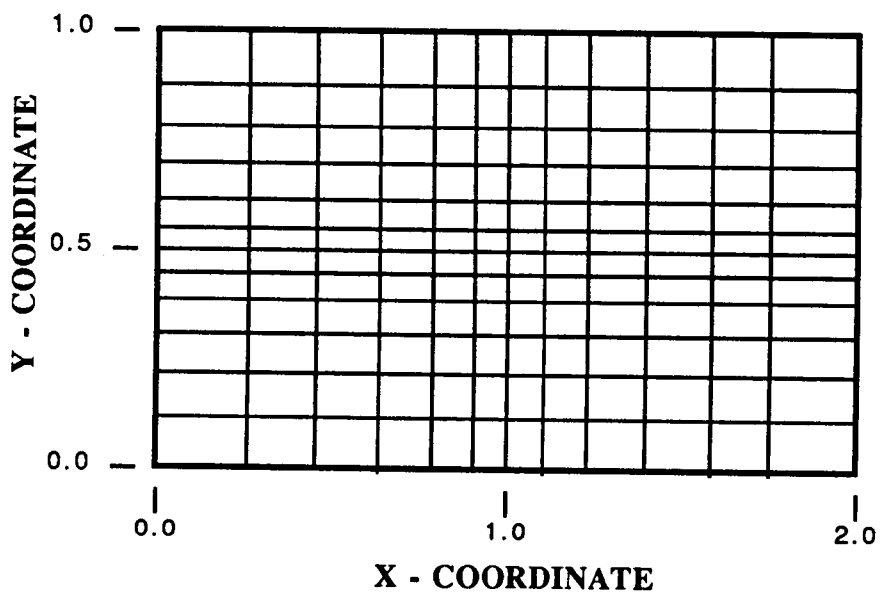


Figure 5. Rectangular Parallelepiped Simulation Domain ($2 \times 1 \times 1$)



a) 6 x 6 x 6 mesh



b) 12 x 12 x 12 mesh

Figure 6. The zplaner mesh of the $2 \times 1 \times 1$ simulation parallelepiped

a) $6 \times 6 \times 6$, b) $12 \times 12 \times 12$

The steady state results of these simulations are summarized in Figure B17-B20¹. An interesting development in the solution surface plots can be observed. Finer mesh discretization essentially reduces the error fluctuations to negligible magnitude, Figure B19-B20¹ hence the homogeneity of the solution simulation is improved. The introduction of the variable Dirichlet cooling needle along with the non-zero boundary heat flux in a parallelepiped domain has increased the test problem intricacies to a great extent. This produces some fluctuations in the solution, both at the corner nodes and at the central nodes of the various zplanes, at steady state solutions for lower mesh density (e.g 12×12), Figure B18¹. There still remains some fluctuations around the central node of the zplane of 24^3 mesh (Figure B19¹) while the temperature distribution has become oscillation free and nearly homogeneous for the 48^3 mesh (Figure B20¹) solution.

The distribution of source due to application of line electric potential has been illustrated in Figures B21-B23¹. It is evident that $6 \times 6 \times 6$ mesh is too coarse to capture the details of the distribution. The more the grid is refined, the better is the definition of the source and hence, the solution temperature distribution. This confirms that sufficient mesh resolution is absolutely necessary for simulation accuracy.

(3) Application of Multiple Pairs of Electrodes :

The requirement of uniform heat distribution for a better hyperthermia treatment modality has led this thesis research towards increasing the number of electrode pairs in the tissue volume in order to generate an improved uniform distribu-

tion of energy inside the tissue. Simulation experiments has been done with four pairs of electrodes, equidistant from the center of the hexahedral tissue, carrying a 10 MHz sinusoidal signal of peak-to-peak 30.4 volts. The surface plots of the temperature distribution for $M = 6 \times 6 \times 6$ mesh and $M = 48 \times 48 \times 48$ mesh for this simulation has been illustrated in Figure B24-B25¹. This result is an excellent improvement in the temperature distribution of the treatment tissue. In $M = 48^3$ mesh the homogeneous temperature distribution is $\sim 42^\circ C$ in almost all z-planes. This is desirable in hyperthermia treatment procedure.

Since the perfusion rate of the surrounding normal tissue is high enough there will be sufficient convective heat transfer at the boundary of the hypethermia treatment region in order to cool down the surrounding normal tissue. Hence, use of multiple pairs of electrode for hyperthermia is not expected to affect the normal tissue for reasonable electrode heating levels. Note that at the corners of z-plane surface plots (Figure B23¹) the solution does not remain smooth. This may be due to the incorrect way of handling the Jacobian in solution procedure at those singular points.

CHAPTER V

CONCLUSIONS

As stated in details in Chapter II, half a degree accuracy of the modality temperature can have a significant influence on efficacy of hyperthermia treatment. Hence, sufficient mesh resolution in a numerical model is absolutely essential for accurate simulation. Its numerical convergence of has also been confirmed.

The results have shown that a heat flux neumann boundary condition is realistic for normal tissue interface. Application of a water cooled needle along with multiple electrodes promotes a more homogeneous temperature distribution over the malignant tissue ensuing an improved hyperthermia treatment. This thesis has established the contruction of finite element procedures to satisfy the accuracy constraint and design practical *in vivo* situations. Mathematical versatility is preserved, enabling extension to a generalized hexahedral domain.

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APPENDICES

Appendix A :

CODE EXECUTION PROCEDURES

The updated source code for running three-dimensional hyperthermia simulations is named *LEARN6.FE\$D*. It is command name driven. The theoretical formulation of this code and its numerical algebra procedures has been discussed in details in Chapter III. The input file of *LEARN6.FE\$D* is called *FE.DAT*. The output data after computation is stored in *FE.OUT*.

(1) INPUT DATA STRUCTURE:

LEARN6.FE\$D is command name driven. There are several command names in the input file, namely, *TITL*, *NEUM*, *Diri*, *INTE* etc. The command name *TITLE* allows one to specify an appropriate name for a test problem. Command name *TYPE* contains all integer data informations required for the code to handle the problem definition. The basis (linear, quadratic or cubic) for discretization, the dimension of the test case, the number of nodes per discretized element, the number of progression ratios, and the type of the element shape are generally included in this command. *PRIN* controls output data presentation. A three dimensional element can be of different shapes, namely, rectangular parallelopiped, tetrahedron, planer-faced hexahedron, and curved-faced hexahedron. The details of the grid dimensions are given in *GRID*. This contains all basic data for the macro element including the extreme coordinates and the progression ratios in each direction.

The next command name, *MATL*, defines the properties, e.g., thermal conduc-

tivity, of the test material. *SORC* contains the data regarding energy generation inside to the problem domain. In the thesis test case, it involves the description of the location of the electrodes inside the tissue and their voltage difference(s). Command name *INIT* is required for the unsteady time integration solver. It involves the nodal initial conditions (*data*) for the unknown variable at time zero. Both *NEUM* and *Diri* supply the boundary informations for the code. *NEUMann* governs the boundary heat flux, while *Dirichlet* contains the fixed temperature boundary data. Command name *INTE* turns on the transient solver to handle a time evolving unsteady equation. It contains data for all the parameters involved in time integration procedure.

An example of *FE.DAT* is given below.

```

TITL
**** Experiment with a Cooled Needle T(c), Non-zero Boundary Heat Flux ****
                      **** Mesh = 6x6x6, V = 1x1x1 ****

TYPE   [ K  N  NNODEL  NPR  NTRAN  NAXI ]
       1   3   8 2 0    0

PRIN   [ NBUG(1) NBUG(2) NBUG(3) NBUG(4) ]
       1   1   0 2 100 0

GRID   [ NPR ( XI(*) XF(*) NPR(*) ) NEM(NPR,*) ]
       0. 0.5 0.8 1.0 1.25
       3 3
       0. 0.5 0.8 1.0 1.25
       3 3
       0. 0.5 0.8 1.0 1.25
       3 3

MATL
       1   2. 2.
       2   2. 2.

SORC   [NSORC IFUNC]
       2   3
       0.4 0.6
       0.4 0.6
       0.4 0.6
       17.0
       -17.0
       4.0

INIT   [* * INT INDCT]
       1 0 2 0
       33.3

NEUM
       0 0 5
       2
       -1
       20.
       3
       -1
       20.
       4
       -1
       20.
       5
       -1
       20.
       6
       -1
       20.

DIRI
       6 1 1
       74 123 172 221 270 319
       6 33.3
       1
       -1
       33.3

INTE TSTOP TSTEP THETA EPSILN QDMAX TSTEP1 NTSMAX
      3.0 0.05 0.5 0.005 1.0 0.0001 500

STOP

```

Some of the notations used for setting the input file.

K.....Finite Element Basis Degree

N.....Dimension of Problem Domain (N=1,2 or 3)

NAXI.....(0,1) Switch for Regular/Axisymmetric 1D problem

NPR.....Number of Macro Discretization Progression Ratios

NNODEL.....Number of Nodes per Element

NFIX.....Number of Dirichlet Boundary Nodes

JFIX(*)....Dirichlet Boundary Node Numbers Keyed to Data

NTRAN.....Index Defining Problem Geometry

= 0 = Rectangular Geometry

= 1 = General Straight-sided Quadrilateral Geometry

= 2 = Parabolic Curved-sided Quadrilateral Geometry

= 3 = Region Between Two Concentric Circular Arcs

NBUG(*).....Output Print Control

(1)... =X= Print after every X Code Passes

(2)... =1= Print SMAT and F

(3)... =1= Print Element Nodal Connection Array

(4)... =0= Print Floating Point Format

=1= Print Exponential Format

=2= Print Integer Format

=3= Print Floating Point & Integer

(5)...=X, Transient Output Printed every X Timesteps

(6) =1 Code Debug Parameter, Prints every Time-step

in Format: CAUTION : produces large Output file

IFUNC.....Source Distribution Mode Selector

≤ 0 = User Specification

=1=Sine Function

=2=Gaussian Distribution

=3=Line Electric Potential

INT.....Initial Condition Switch

INDCT.....Grid Information Indicator, automated interpolation for higher
mesh density

(2) OUTPUT DATA:

A sample of the output file for $6 \times 6 \times 6$ hexahedral domain has been shown below. It is important to note that for higher mesh density the array sizes of the dimensional variables of the code has to be changed accordingly.

An example of the output file *FE.OUT* is given here.

```

*****
C F D L CODE: LEARN FINITE ELEMENTS
*****

TITL: **** Experiment with a Cooled Needle T(c), Non-zero Boundary Heat Flux
      K      N NNODEL      NPR      NTRAN      NAXI
      1      3      8      2      0      0
PRIN:
NBUG1  NBUG2  NBUG3  NBUG4  NBUG5  NBUG6
      1      1      0      2      100      0
      N      NPR
      1      2

      XI      XF1      PR1      XF2      PR2      XF3      PR3
      0.0000      0.5000      0.8000      1.0000      1.2500
NEM1  NEM2  NEM3
      3      3
      N      NPR
      2      2

      XI      XF1      PR1      XF2      PR2      XF3      PR3
      0.0000      0.5000      0.8000      1.0000      1.2500
NEM1  NEM2  NEM3
      3      3
      N      NPR
      3      2

      XI      XF1      PR1      XF2      PR2      XF3      PR3
      0.0000      0.5000      0.8000      1.0000      1.2500
NEM1  NEM2  NEM3
      3      3

MATL:  NODAL CONDUCTIVITIES
      1  0.20000E+01  0.20000E+01
      2  0.20000E+01  0.20000E+01

SORC:  SIMULATED ENERGY SOURCE DUE TO LINE ELECTRIC POTENTIAL

EXPONENT:  0
ZPLANE:  1
PRNTIT:  ALL VALUES ZERO IN THIS PLANE
ZPLANE:  2
      0      0      0      0      0      0      0
      0      0      0      0      0      0      0
      0      0      0      88      0      0      0
      0      0      88      94      88      0      0
      0      0      0      88      0      0      0
      0      0      0      0      0      0      0
      0      0      0      0      0      0      0
ZPLANE:  3
      0      0      0      0      0      0      0
      0      0      0      88      0      0      0
      0      0      109      117      109      0      0
      0      88      117      126      117      88      0
      0      0      109      117      109      0      0
      0      0      0      88      0      0      0
      0      0      0      0      0      0      0
ZPLANE:  4
      0      0      0      0      0      0      0
      0      0      88      94      88      0      0
      0      88      117      126      117      88      0
      0      94      126      136      126      94      0

```

Continuation of output file *FE.OUT*

```

      0  88  117  126  117  88  0
      0  0  88  94  88  0  0
      0  0  0  0  0  0  0
ZPLANE: 5
      0  0  0  0  0  0  0
      0  0  0  88  0  0  0
      0  0  109  117  109  0  0
      0  88  117  126  117  88  0
      0  0  109  117  109  0  0
      0  0  0  88  0  0  0
      0  0  0  0  0  0  0
ZPLANE: 6
      0  0  0  0  0  0  0
      0  0  0  0  0  0  0
      0  0  0  88  0  0  0
      0  0  88  94  88  0  0
      0  0  0  88  0  0  0
      0  0  0  0  0  0  0
      0  0  0  0  0  0  0
ZPLANE: 7
PRNTIT: ALL VALUES ZERO IN THIS PLANE

```

```

INIT: NFIX: 1
*****
EXPONENT: -1
ZPLANE: 1
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
ZPLANE: 2
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
ZPLANE: 3
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
ZPLANE: 4
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
ZPLANE: 5
333 333 333 333 333 333 333
333 333 333 333 333 333 333

```

Continuation of output file *FE.OUT*

```

333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
ZPLANE: 6
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
ZPLANE: 7
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
NEUM: NFIX: 0
EXPONENT: -1
ZPLANE: 1
200 200 200 200 200 200 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 200 200 200 200 200 200
ZPLANE: 2
200 200 200 200 200 200 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 200 200 200 200 200 200
ZPLANE: 3
200 200 200 200 200 200 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 200 200 200 200 200 200
ZPLANE: 4
200 200 200 200 200 200 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 200 200 200 200 200 200
ZPLANE: 5
200 200 200 200 200 200 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200

```

Continuation of output file *FE.OUT*

```

200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 200 200 200 200 200 200
ZPLANE: 6
200 200 200 200 200 200 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 0 0 0 0 0 200
200 200 200 200 200 200 200
ZPLANE: 7
200 200 200 200 200 200 200
200 200 200 200 200 200 200
200 200 200 200 200 200 200
200 200 200 200 200 200 200
200 200 200 200 200 200 200
200 200 200 200 200 200 200
200 200 200 200 200 200 200

```

```

DIRI: NFIX: 6
JFIX: 74 123 172 221 270 319
33.30000 33.30000 33.30000 33.30000 33.30000 33.30000

```

EXPONENT: -1

```

ZPLANE: 1
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333

```

```

ZPLANE: 2
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333

```

```

ZPLANE: 3
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333

```

```

ZPLANE: 4
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333

```

```

ZPLANE: 5
333 333 333 333 333 333 333
333 333 333 333 333 333 333

```

Continuation of output file *FE.OUT*

```

333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
ZPLANE: 6
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
ZPLANE: 7
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333

```

```

INTE
      TSTOP      TSTEP      THETA      EPSILN      QDMAX      TSTEPI NTSM
0.30000E+01 0.50000E-01 0.50000E+00 0.50000E-02 0.10000E+01 0.10000E-03 500
WARNING: NODE      1 BOTH DIRI AND INIT. VALUE= 0.33300E+02
EPSILN = 4.99999999E-03
NDOFS =      343
DQMAX = 3.3047434E-02 QDMAX = 1.000000
NDOFS =      343
DQMAX = 3.2082777E-02 QDMAX = 1.000000
*****
NDOFS =      343
DQMAX = 7.8125149E-03 QDMAX = 1.000000
NDOFS =      343
DQMAX = 7.0904568E-03 QDMAX = 1.000000
NDOFS =      343
DQMAX = 6.4259544E-03 QDMAX = 1.000000
NDOFS =      343
DQMAX = 5.8604218E-03 QDMAX = 1.000000
NDOFS =      343
DQMAX = 5.6508826E-03 QDMAX = 1.000000
NDOFS =      343
DQMAX = 5.5256784E-03 QDMAX = 1.000000
NDOFS =      343
DQMAX = 5.3883083E-03 QDMAX = 1.000000
NDOFS =      343
DQMAX = 5.2401219E-03 QDMAX = 1.000000
NDOFS =      343
DQMAX = 5.0839270E-03 QDMAX = 1.000000
NDOFS =      343
DQMAX = 4.9229376E-03 QDMAX = 1.000000
INTE: REACHED STEADY STATE
NORMAL TERMINATION: DQMAX .LE. EPSILN
      TIME      # STEPS
      3.01943      357+65

INTE: NODAL SOLUTION
EXPONENT: -1
ZPLANE: 1

```

Continuation of output file *FE.OUT*

```

333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333
333 333 333 333 333 333 333

```

ZPLANE: 2

```

358 350 349 349 349 350 358
352 350 349 349 349 350 352
350 350 348 346 348 350 350
350 350 347 333 347 350 350
350 350 348 346 348 350 350
352 350 349 349 349 350 352
358 350 349 349 349 350 358

```

ZPLANE: 3

```

371 362 362 362 362 362 371
365 363 362 364 362 363 365
363 364 361 357 361 364 363
364 365 358 333 358 365 364
363 364 361 357 361 364 363
365 363 362 364 362 363 365
371 362 362 362 362 362 371

```

ZPLANE: 4

```

378 370 369 370 369 370 378
373 370 371 371 371 370 373
371 372 368 362 368 372 371
372 373 364 333 364 373 372
371 372 368 362 368 372 371
373 370 371 371 371 370 373
378 370 369 370 369 370 378

```

ZPLANE: 5

```

384 375 373 373 373 375 384
378 375 373 374 373 375 378
376 375 369 364 369 375 376
376 376 365 333 365 376 376
376 375 369 364 369 375 376
378 375 373 374 373 375 378
384 375 373 373 373 375 384

```

ZPLANE: 6

```

390 380 377 376 377 380 390
384 378 374 373 374 378 384
382 376 367 362 367 376 382
381 376 363 333 363 376 381
382 376 367 362 367 376 382
384 378 374 373 374 378 384
390 380 377 376 377 380 390

```

ZPLANE: 7

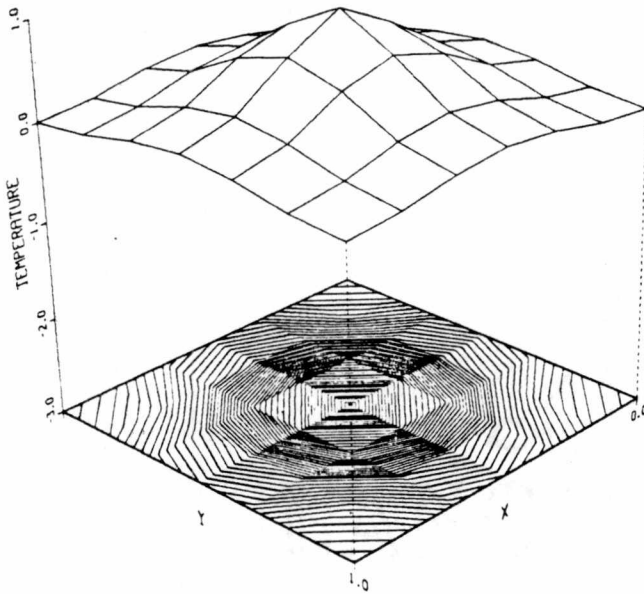
```

394 385 381 380 381 385 394
392 380 375 373 375 380 392
390 377 366 359 366 377 390
390 376 361 333 361 376 390
390 377 366 359 366 377 390
392 380 375 373 375 380 392
394 385 381 380 381 385 394

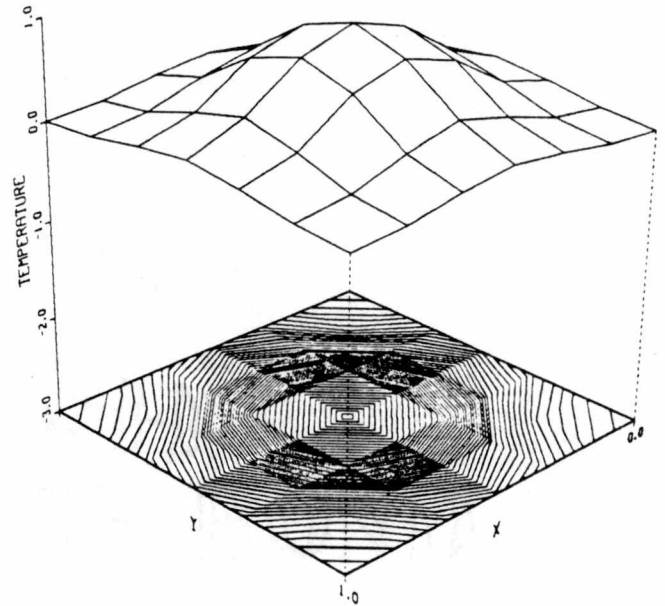
```

STOP

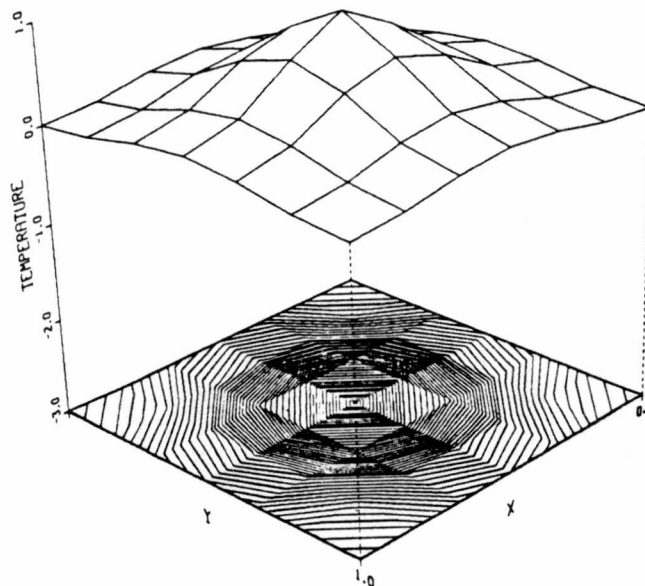
Appendix B : FIGURES



a) z-plane A: #2



b) z-plane B: #4



c) z-plane C: #6

Figure B1. Surface and Contour of Steady State Normalized Temperature Distribution for the Benchmark Case of Panjehpour: Uniform $M = 6 \times 6 \times 6$ mesh

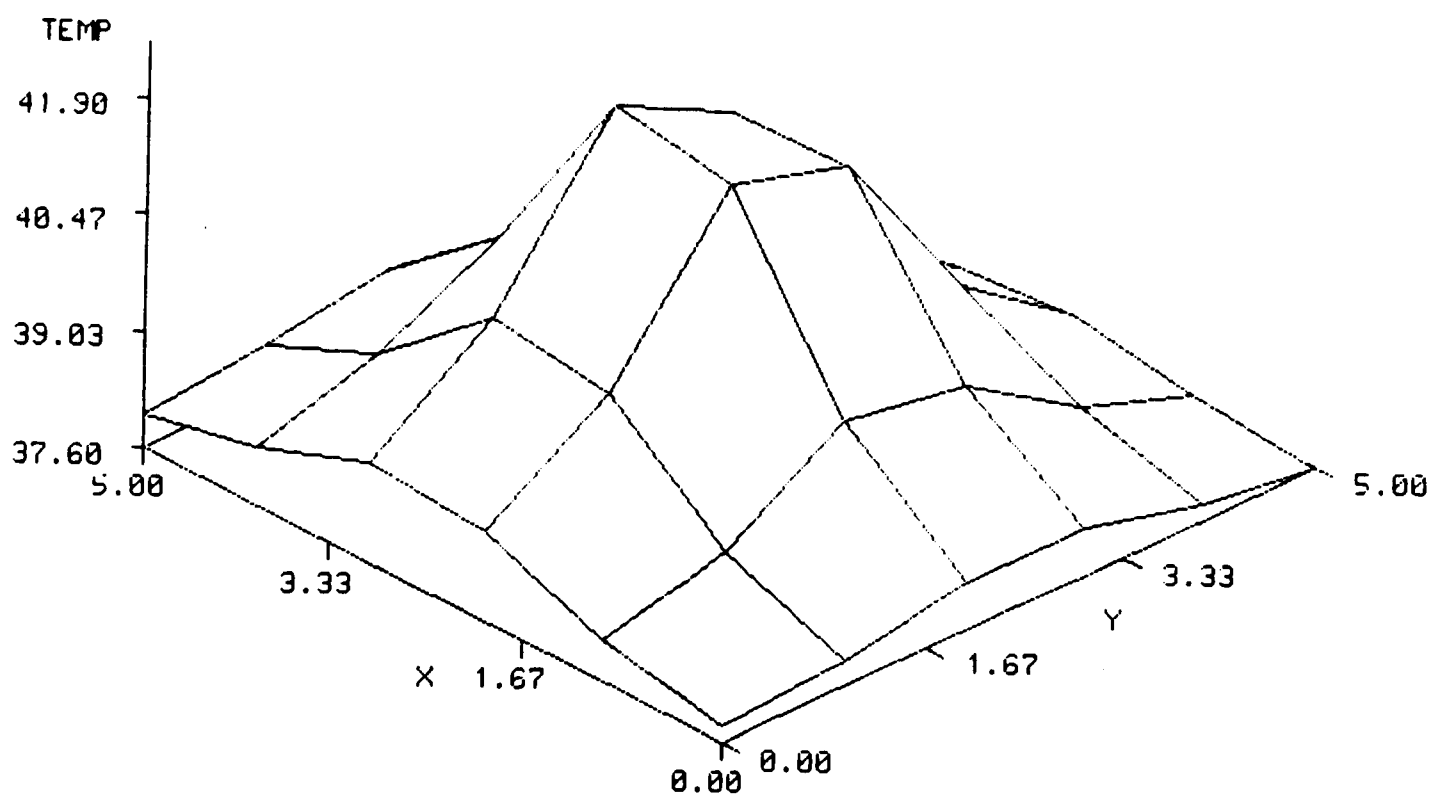
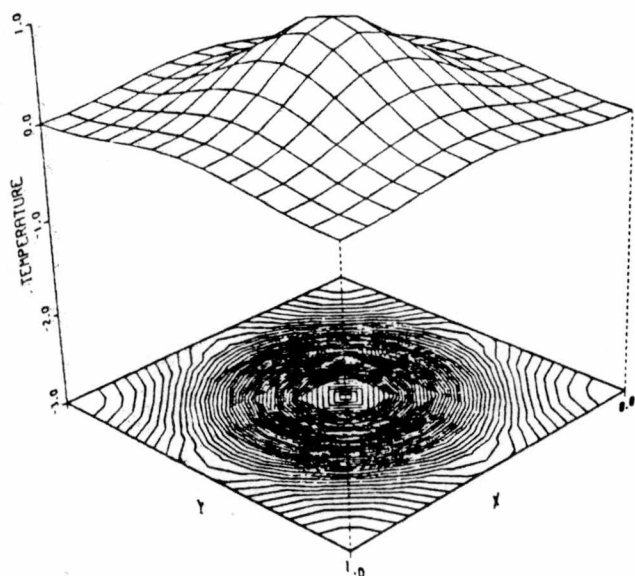
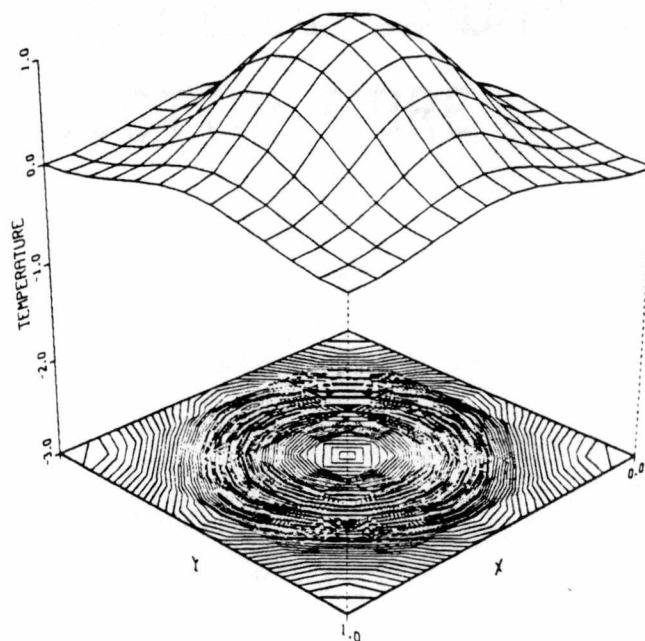


Figure B2. Surface profile of Temperature Distribution in z-plane #4,

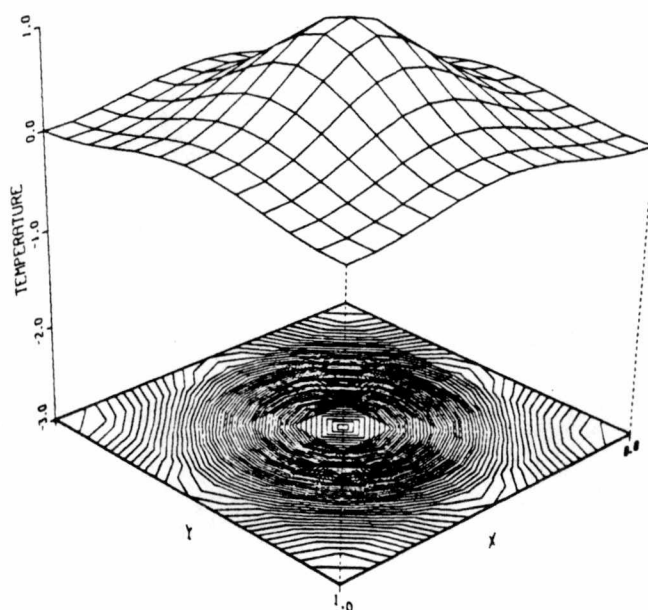
$M = 5 \times 5 \times 5$, from Panjehpour [23]



a) z-plane A: #3

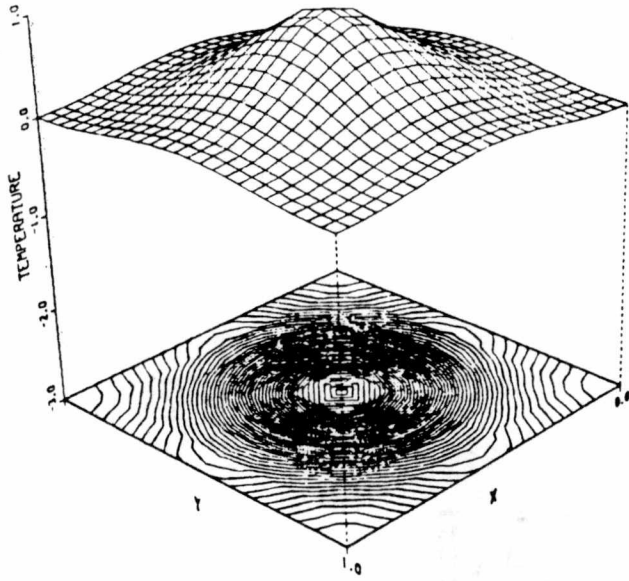


b) z-plane B: #7

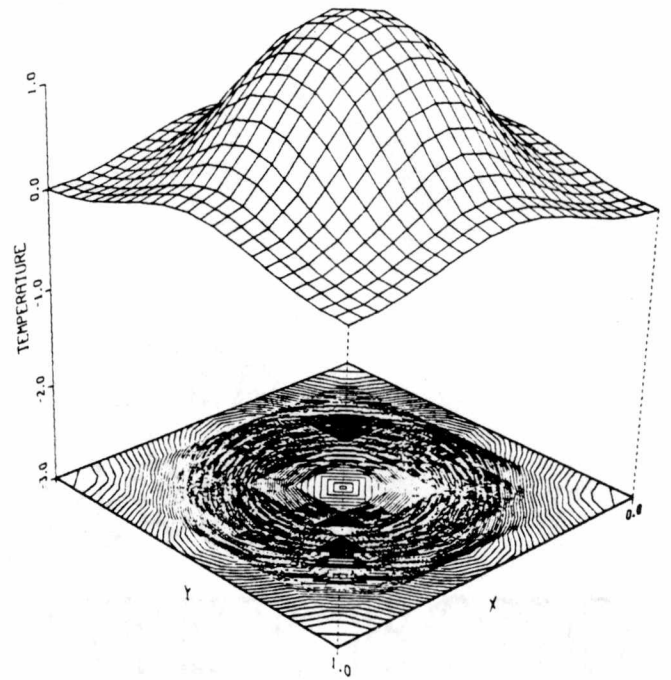


c) z-plane C: #11

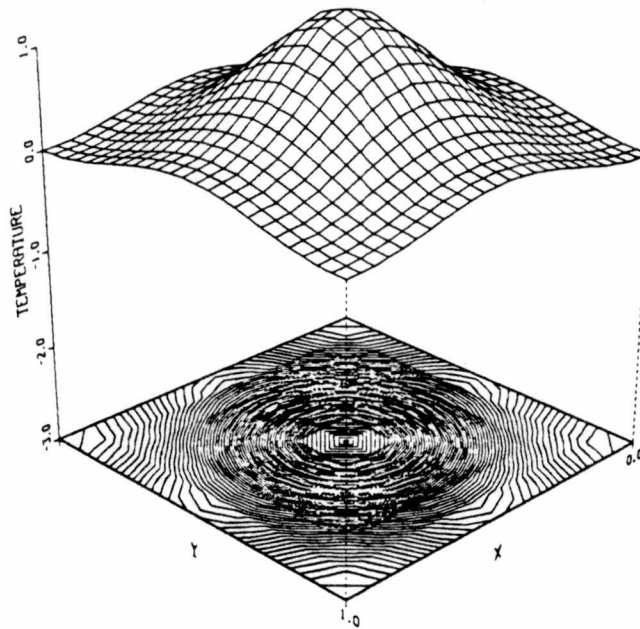
Figure B3. Surface and Contour of Steady State Normalized Temperature Distribution for the Benchmark Case of Panjehpour: Uniform $M = 12 \times 12 \times 12$ mesh



a) z-plane A: #5

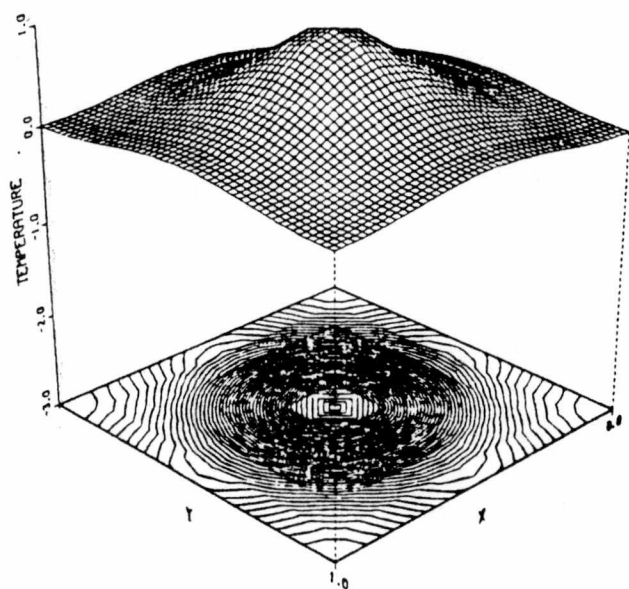


b) z-plane B: #13

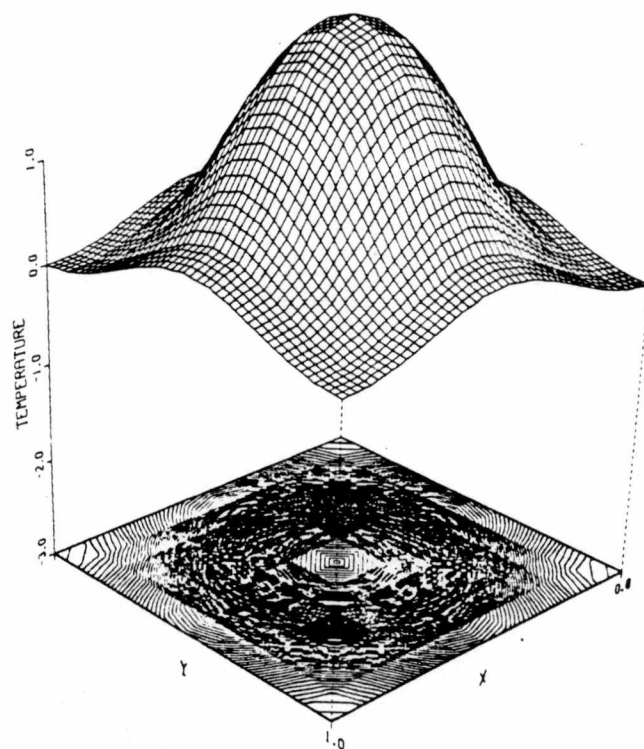


c) z-plane C: #21

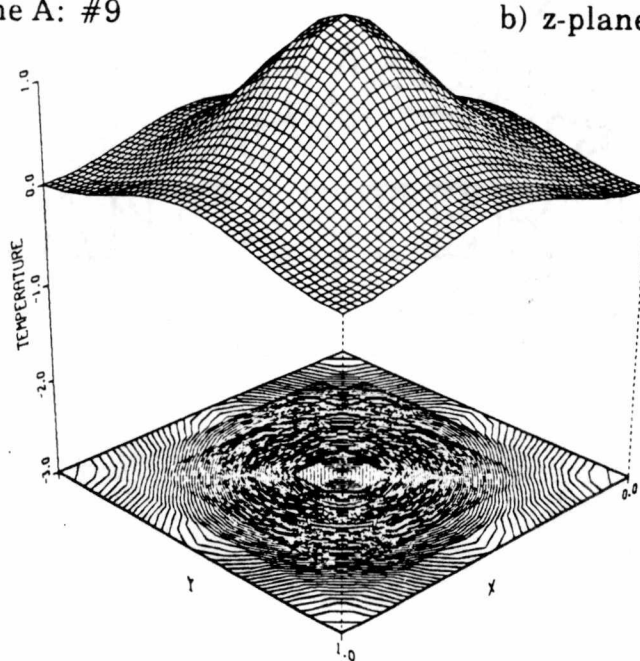
Figure B4. Surface and Contour of Steady State Normalized Temperature Distribution for the Benchmark Case of Panjehpour: Uniform $M = 24 \times 24 \times 24$ mesh



a) z-plane A: #9

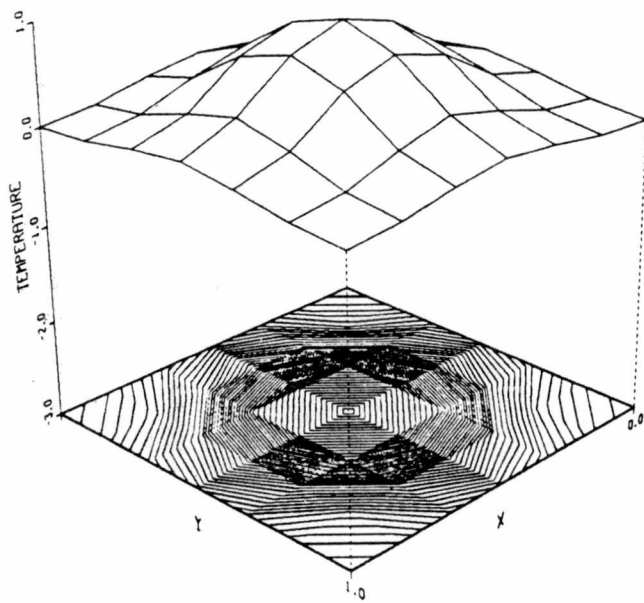


b) z-plane B: #25

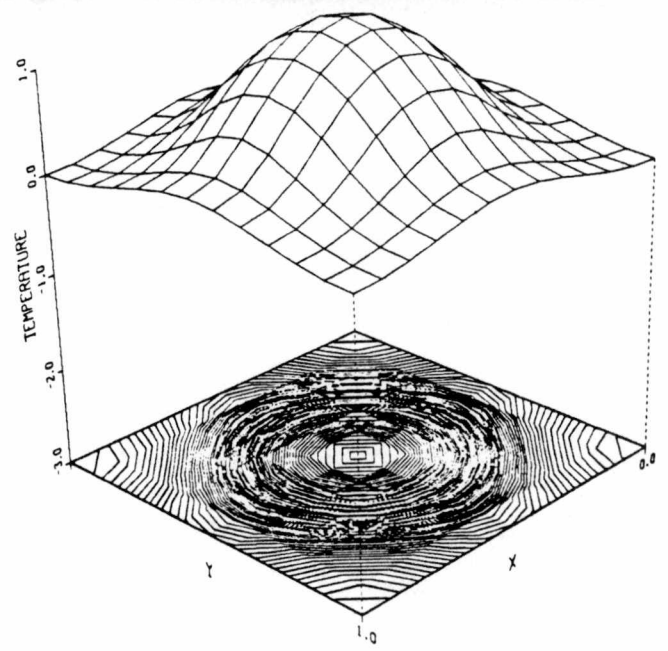


c) z-plane C: #41

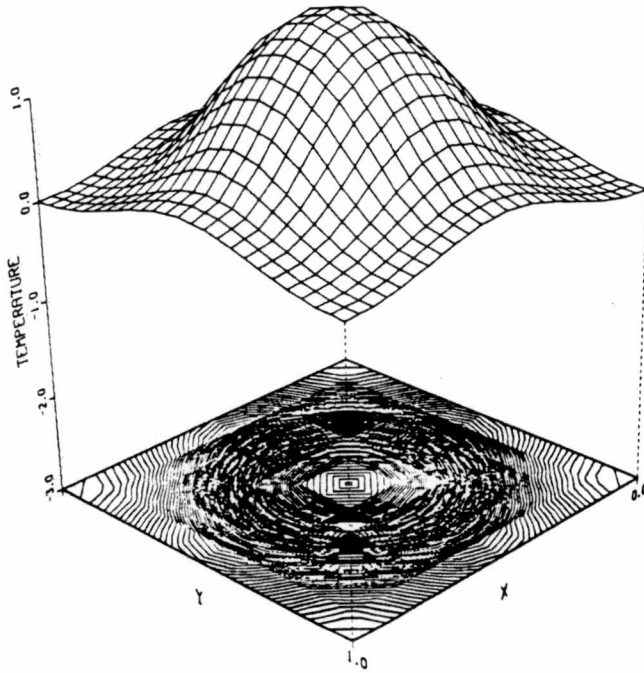
Figure B5. Surface and Contour of Steady State Normalized Temperature Distribution for the Benchmark Case of Panjehpour: Uniform $M = 48 \times 48 \times 48$ mesh



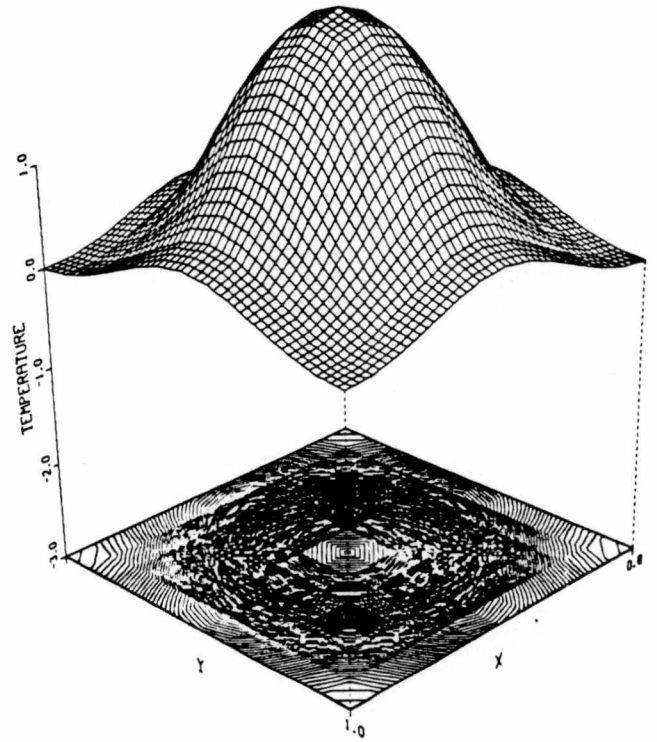
a) $M=6 \times 6 \times 6$



b) $M=12 \times 12 \times 12$



c) $M=24 \times 24 \times 24$



d) $M=48 \times 48 \times 48$

Figure B6. Summary Comparison of Surface and Contour Normalized Temperature Distribution for Four Uniform Mesh Refinement, Mid-zplane. a) $6 \times 6 \times 6$, b) $12 \times 12 \times 12$, c) $24 \times 24 \times 24$, d) $48 \times 48 \times 48$

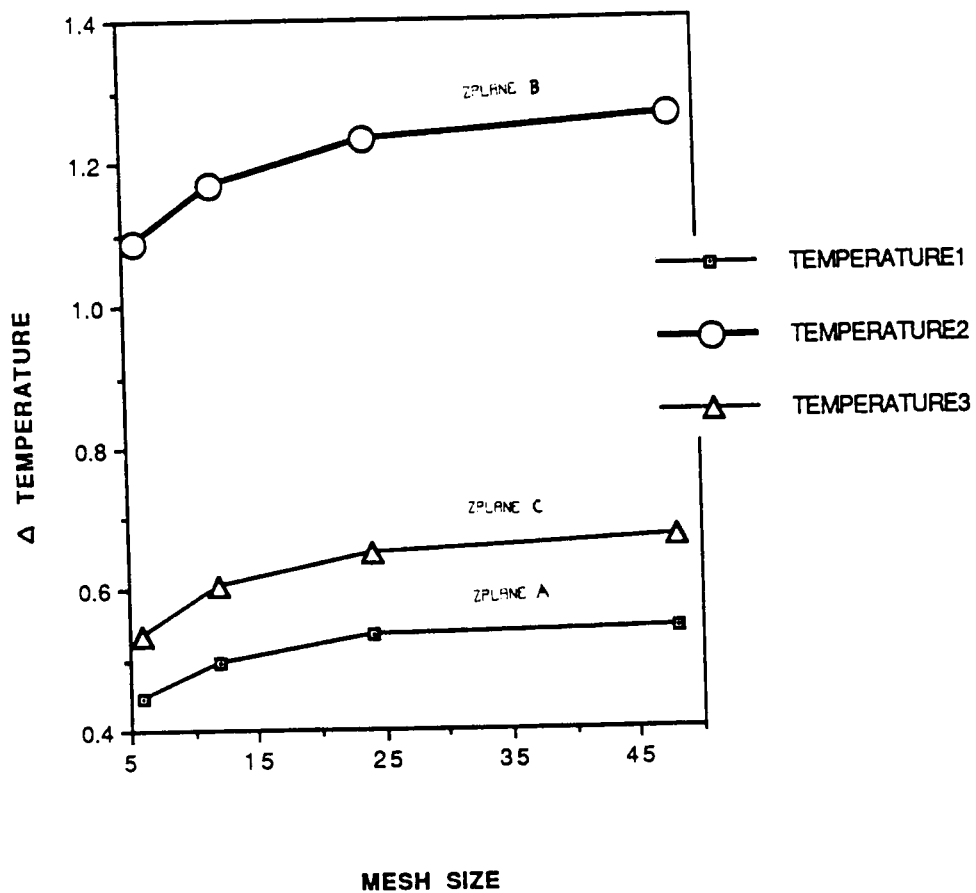
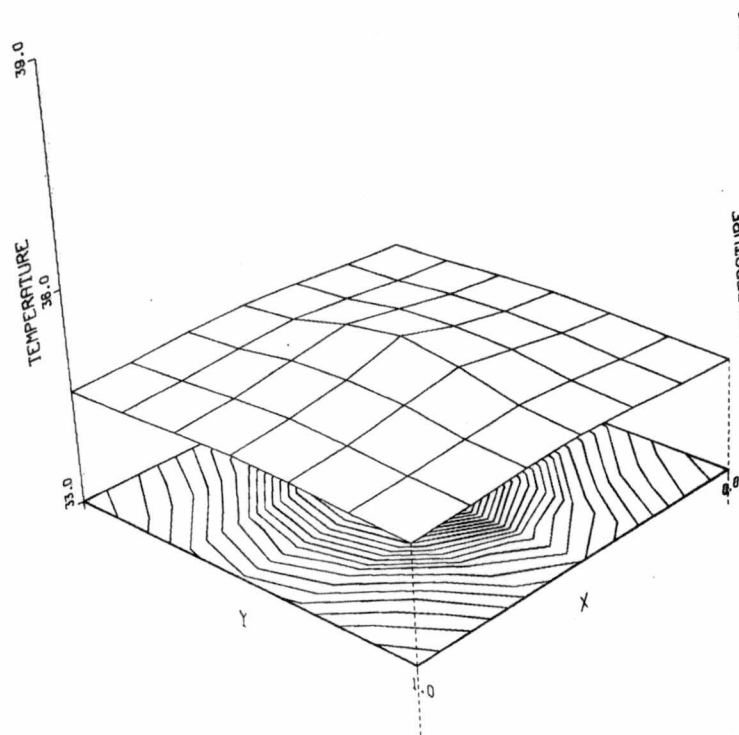
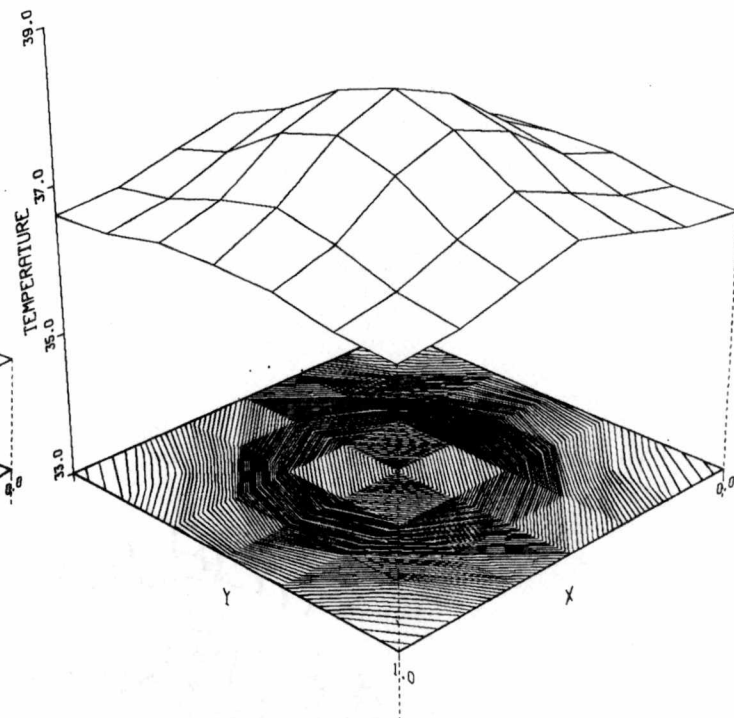


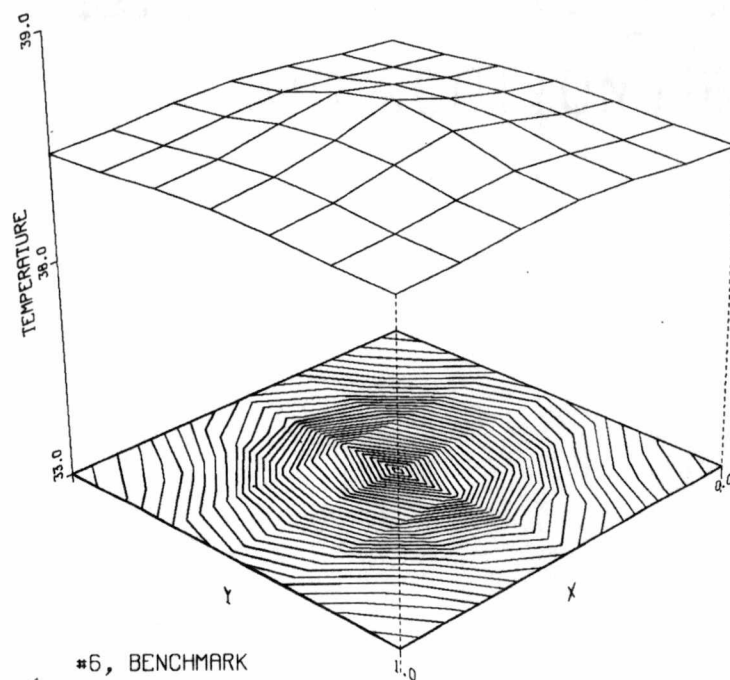
Figure B7. Accuracy and Convergence Study of Temperature vs. Mesh Density
for the Benchmark Test Problem



#2, BENCHMARK



#4, BENCHMARK



#6, BENCHMARK

Figure B8. Surface Temperature Distributions in Actual Scale for Benchmark Test

Case, mesh $6 \times 6 \times 6$: a) zplane # 2, b) zplane # 4, c) zplane # 6

CPU vs. Number of Nodes for 2D Mesh

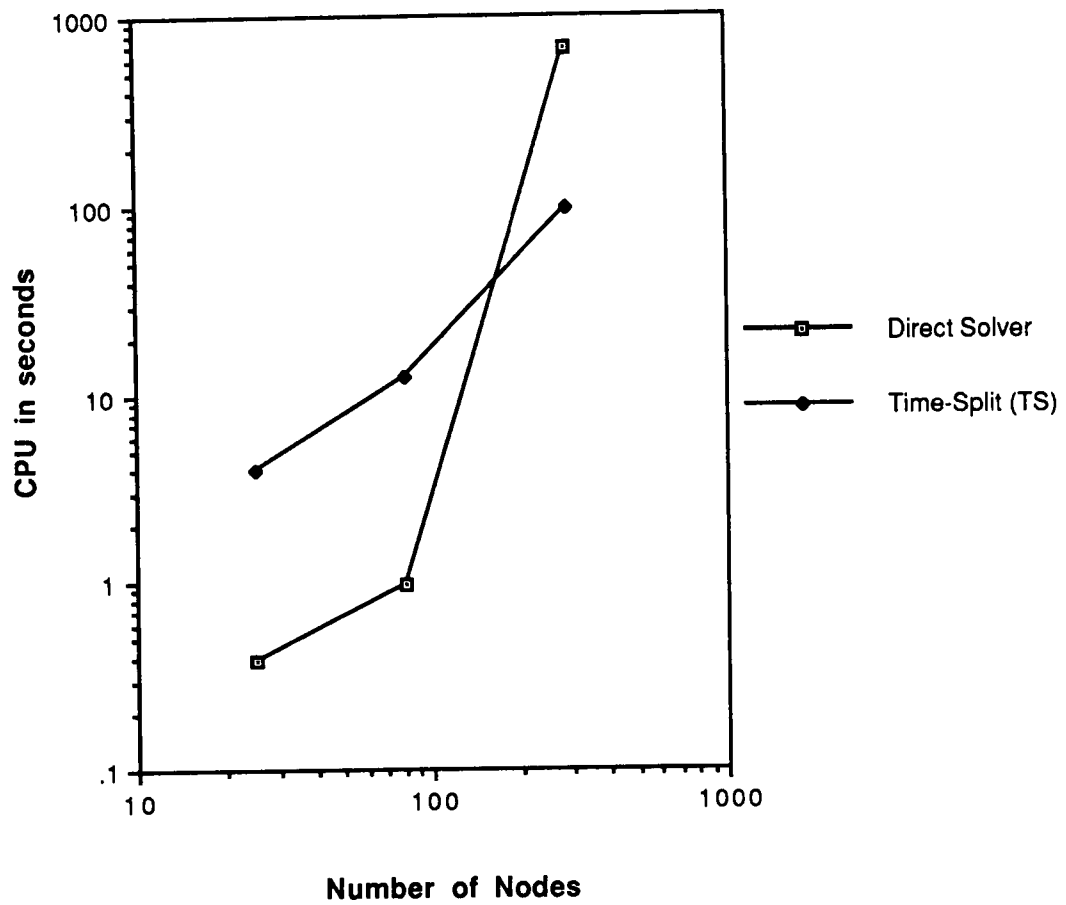


Figure B9. Comparison Study of CPU Time for Direct Solution vs. *TS* solver

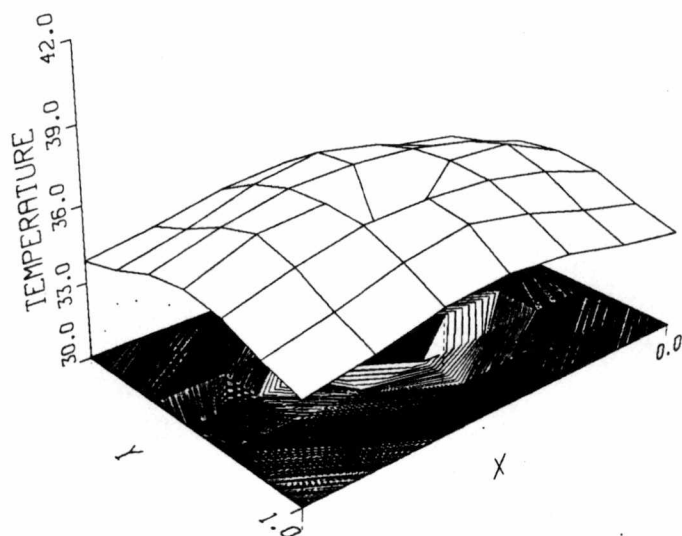
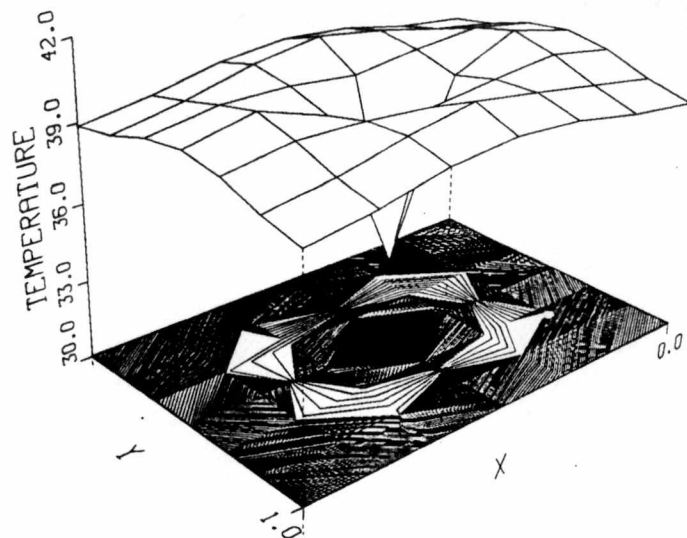


Figure B10. Comparison Study of Time-Split (*TS*) and Full-Residual (*FR*) Algorithm; Surface and Contour Temperature Distributions, *z*-plane #4, $M = 6 \times 6 \times 6 \times$ mesh, a) *TS* solution, b) *FR* solution

Temperature Plots for TS and FR Method

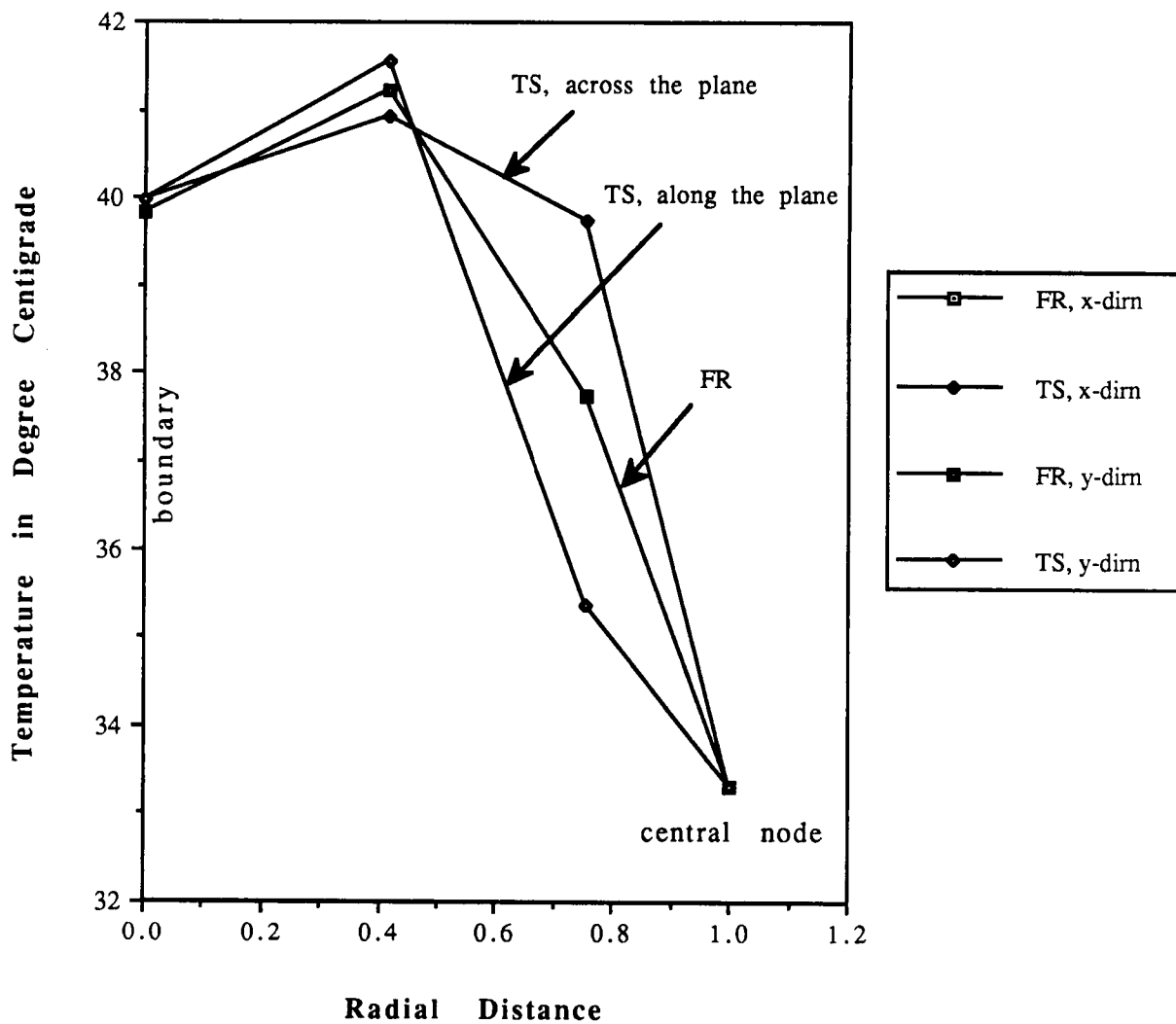


Figure B11. Comparison Study for *TS* solution vs. *FR* solution: Temperature

Plots for *z*-plane #4, $M = 6 \times 6 \times 6$

CPU vs. Number of Nodes for 3D Mesh

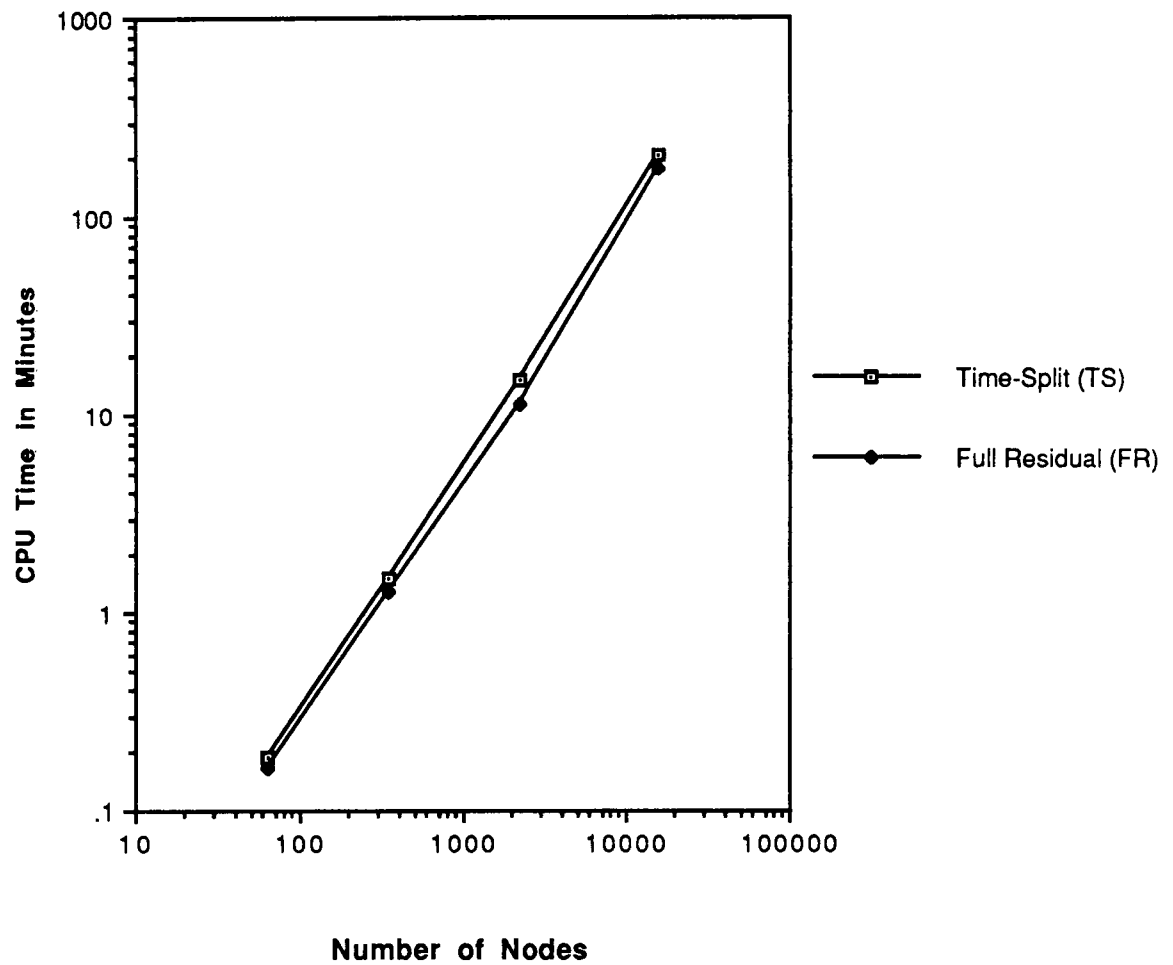
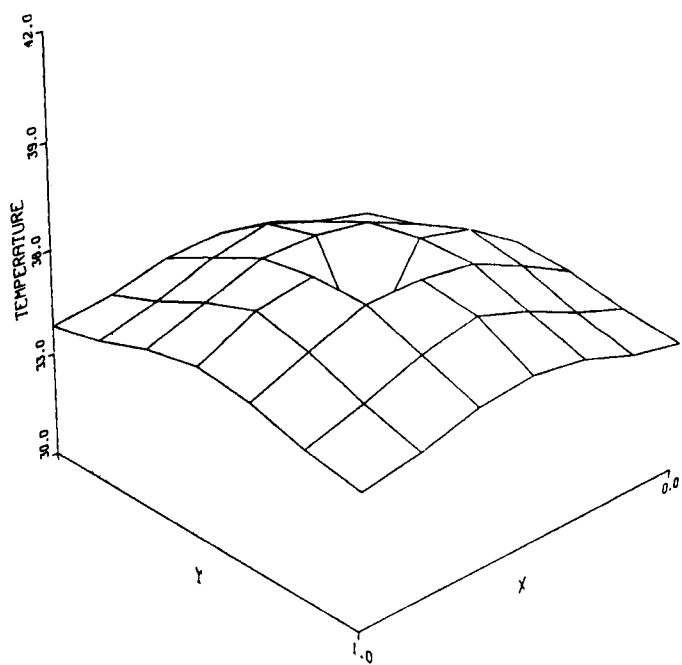
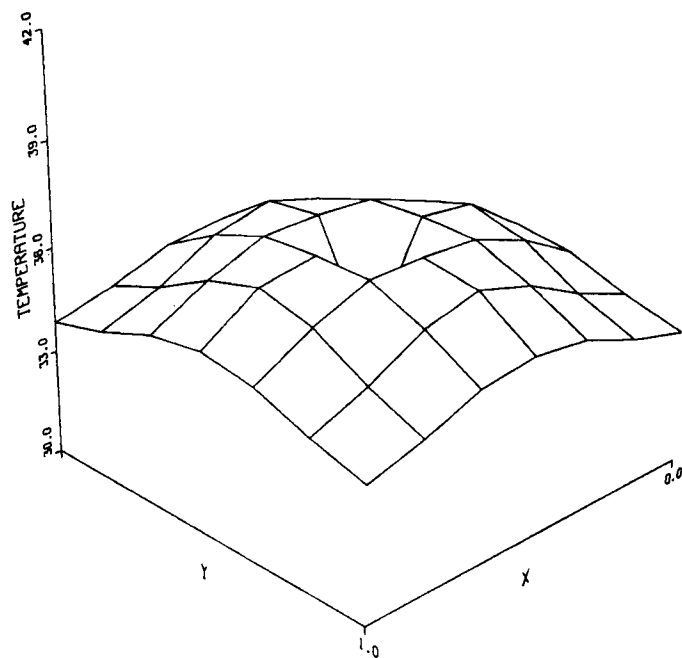


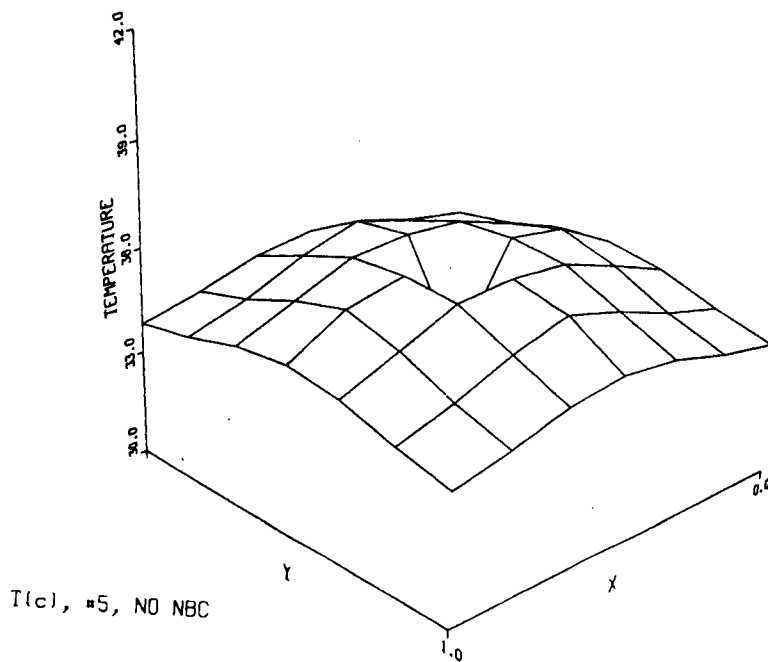
Figure B12. Comparison Study of CPU Time vs. Mesh Size in different algorithm for Three Dimensional Computations



T(c), #3, NO NBC

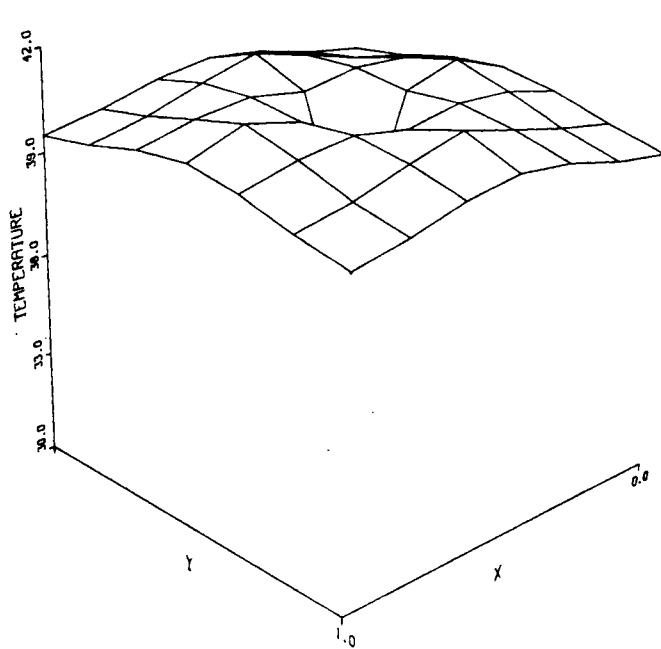


T(c), #4, NO NBC

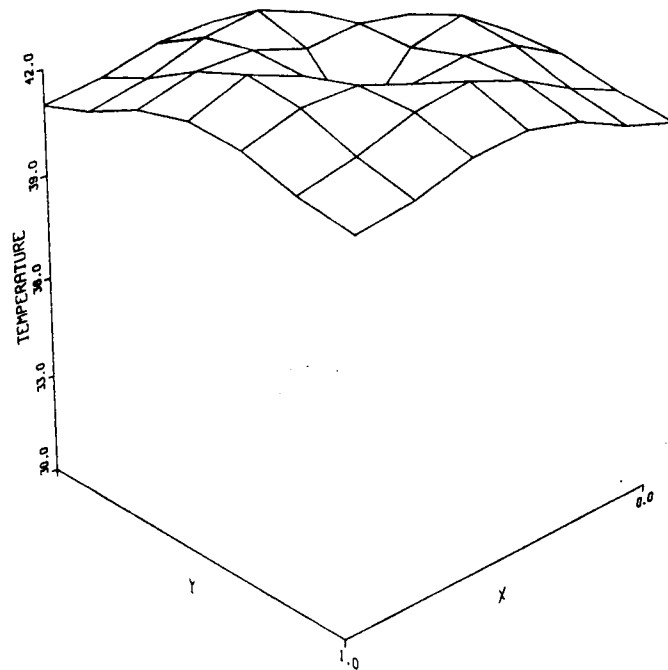


T(c), #5, NO NBC

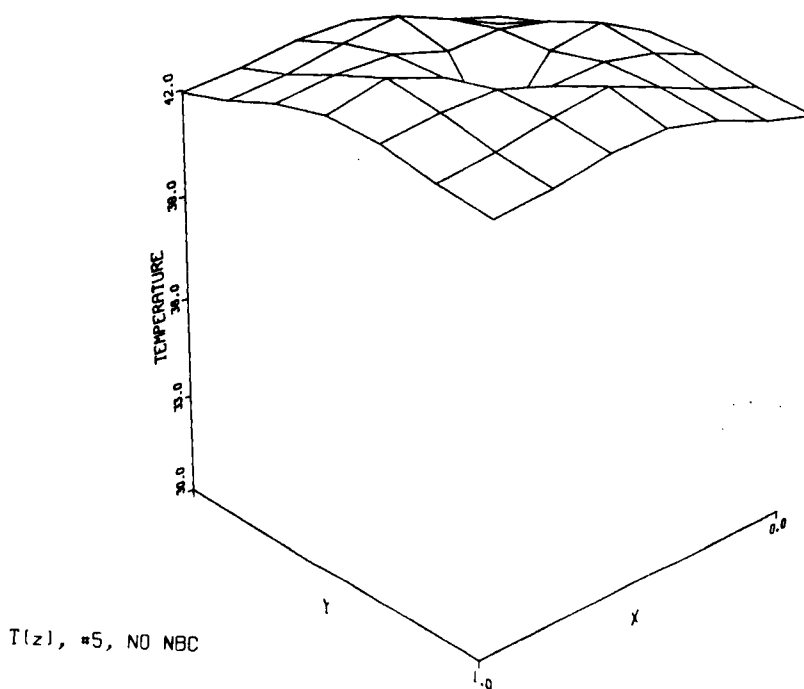
Figure B13. Surface Temperature Distributions for Constant Temperature Cooling Needle in various z-planes of a $M = 6 \times 6 \times 6$ mesh, a) zplane # 3, b) zplane # 4, c) zplane # 5



$T(z)$, #3, NO NBC

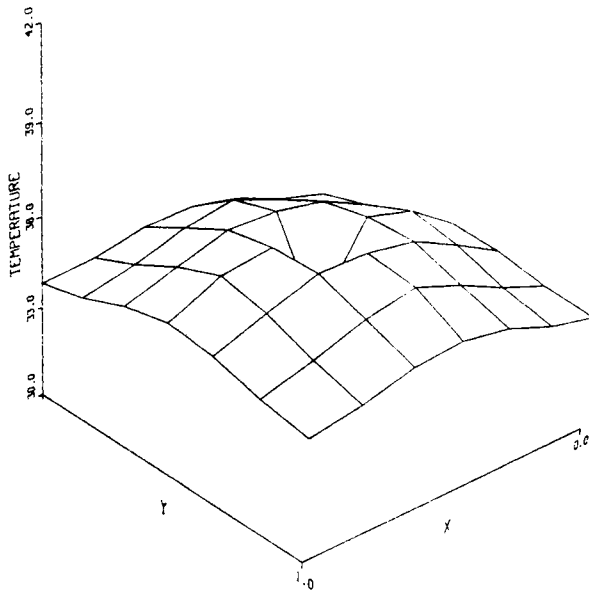


$T(z)$, #4, NO NBC

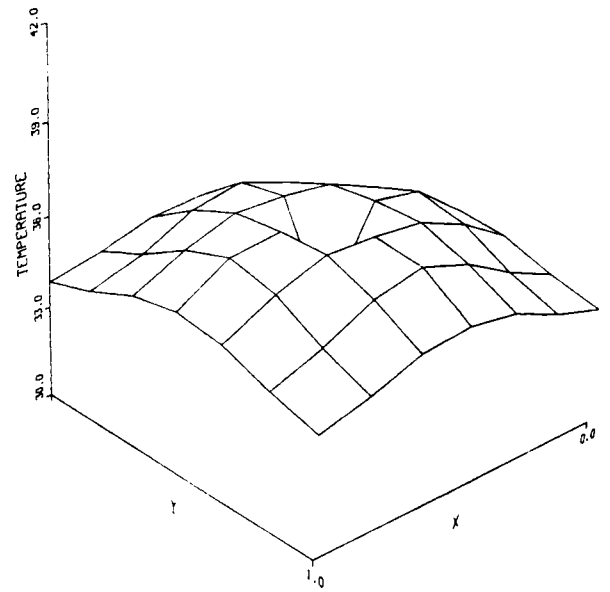


$T(z)$, #5, NO NBC

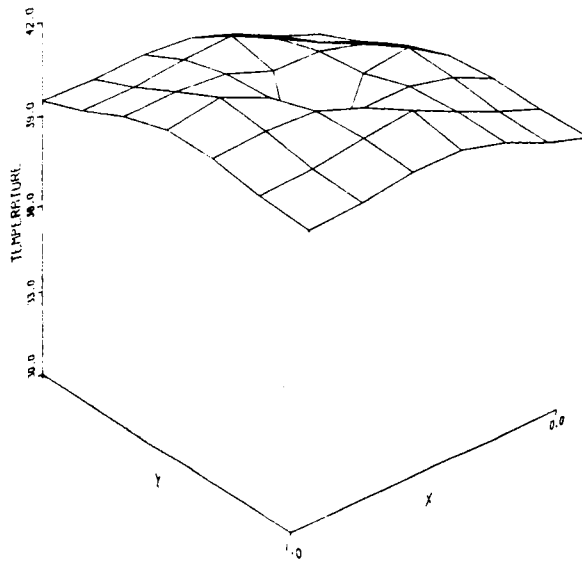
Figure B14. Surface Temperature Distributions for Variable Dirichlet needle in Various Zplanes of a $6 \times 6 \times 6$ mesh, a) zplane # 3, b) zplane # 4, c) zplane # 5



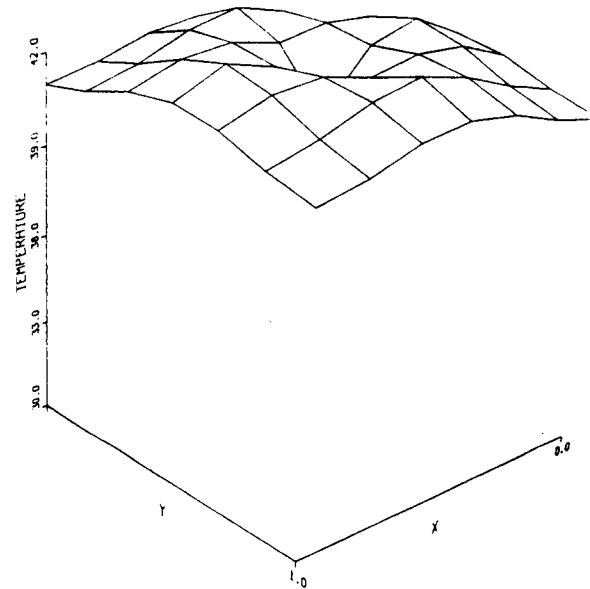
$T_{needle}(c)$ zplane # 3



$T_{needle}(c)$ zplane # 4



$T_{needle}(z)$ zplane # 3



$T_{needle}(z)$ zplane # 4

Figure B15. Comparison of Variable Temperature Needle $T_{needle}(z)$ vs. Constant Temperature Needle $T_{needle}(const)$, $M = 6 \times 6 \times 6$ mesh : a) $T_{needle}(c)$ zplane # 3, b) $T_{needle}(c)$ zplane # 4, c) $T_{needle}(z)$ zplane # 3, d) $T_{needle}(z)$ zplane # 4

Mid - zplane

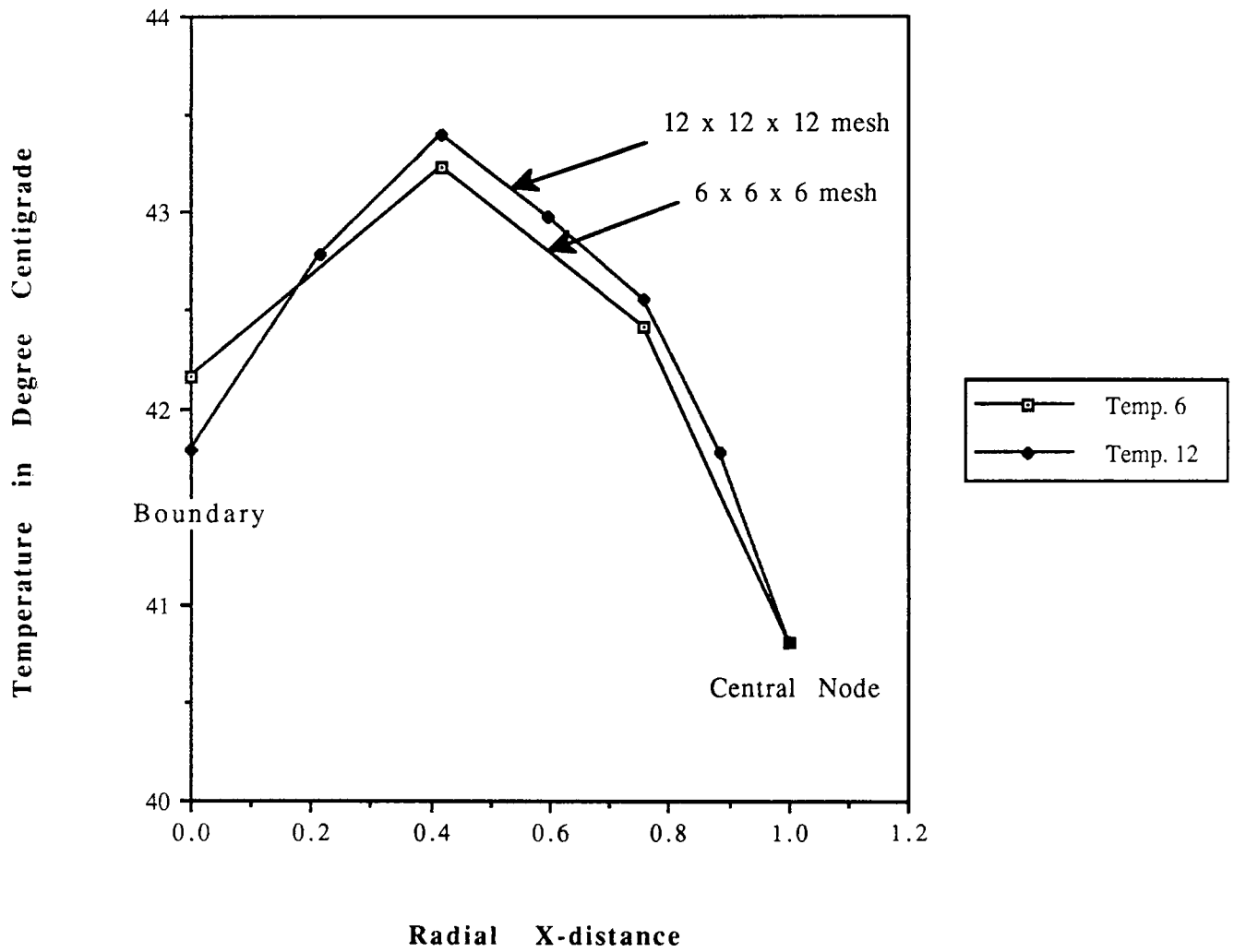


Figure B16. Temperature vs. Radial distance Plots for various meshes

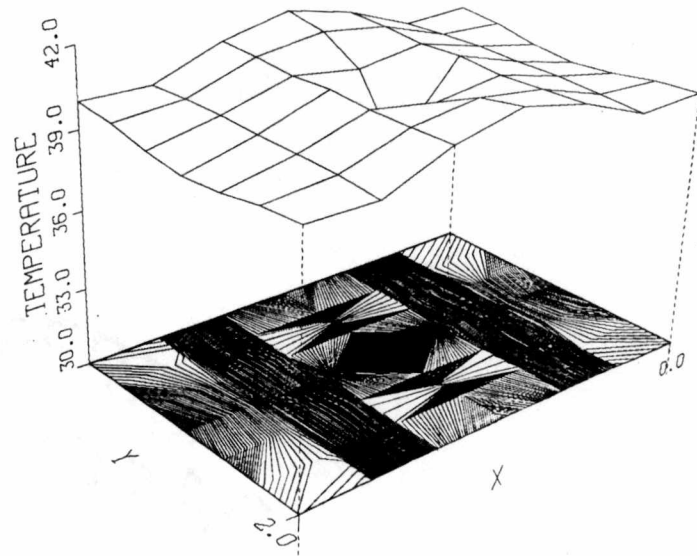
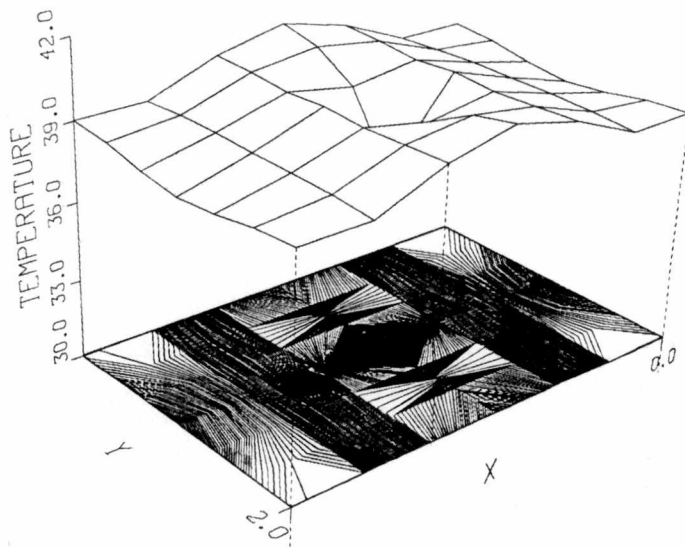
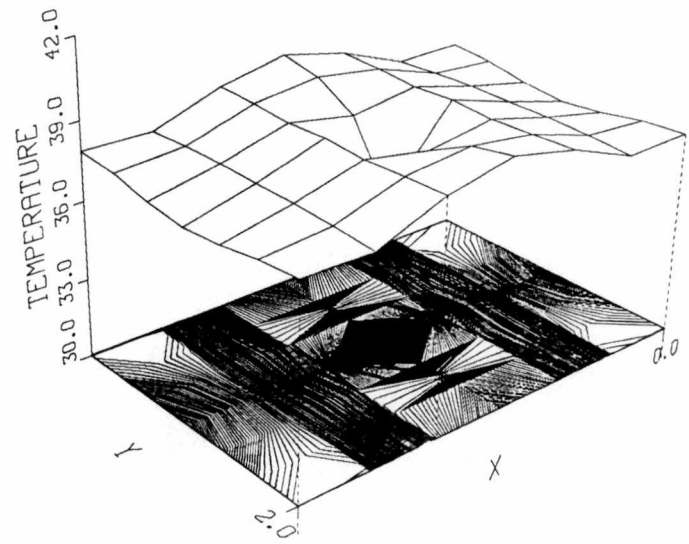
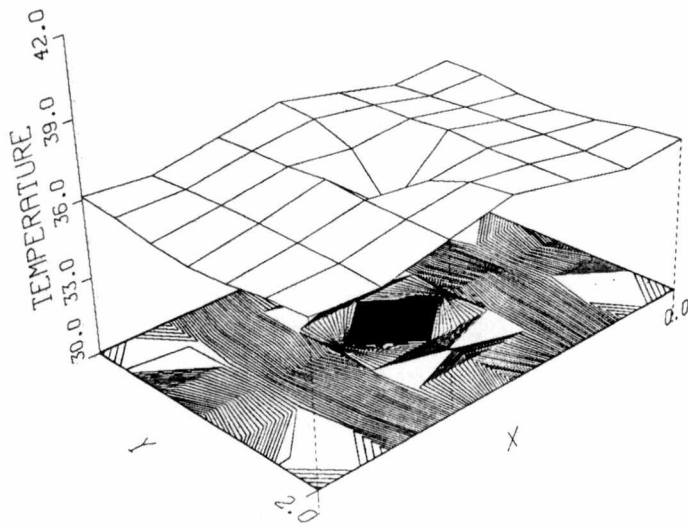
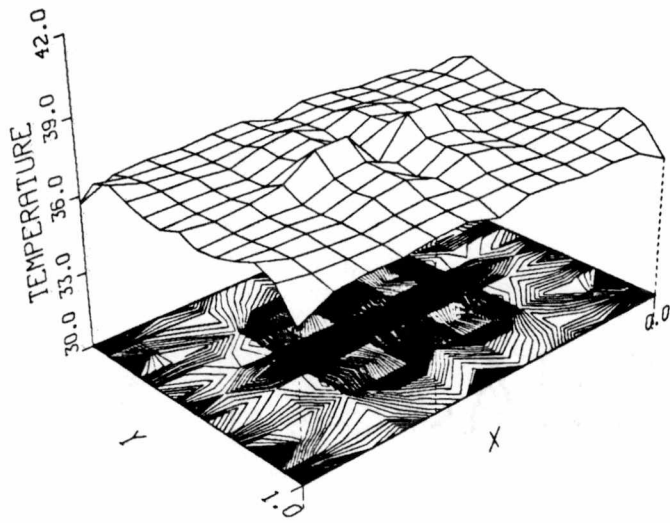
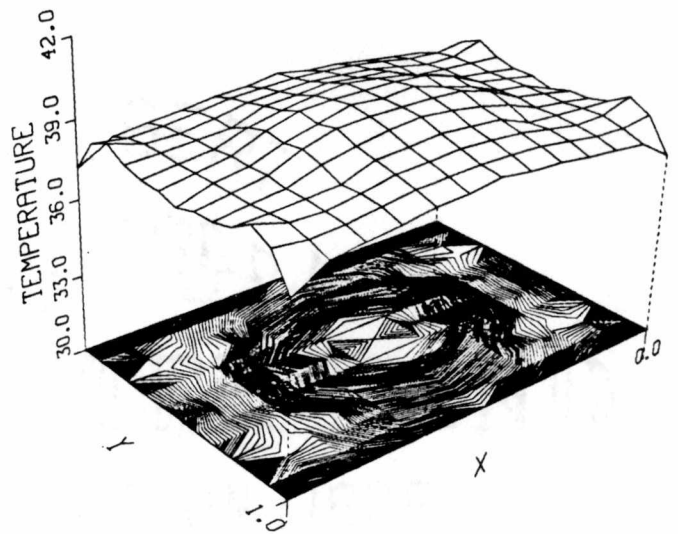


Figure B17. Surface and Contour Plots for Steady State Temperature Distribution for Variable Dirichlet Needle $T_{needle}(z)$ in a Parallelopiped Simulation Domain with Boundary Heat Flux (*Neumann*), for $6 \times 6 \times 6$ mesh : a) zplane # 2, b) zplane # 3, c) zplane # 4, d) zplane # 5



a) zplane # 3



b) zplane # 5

c) zplane # 7

d) zplane # 9

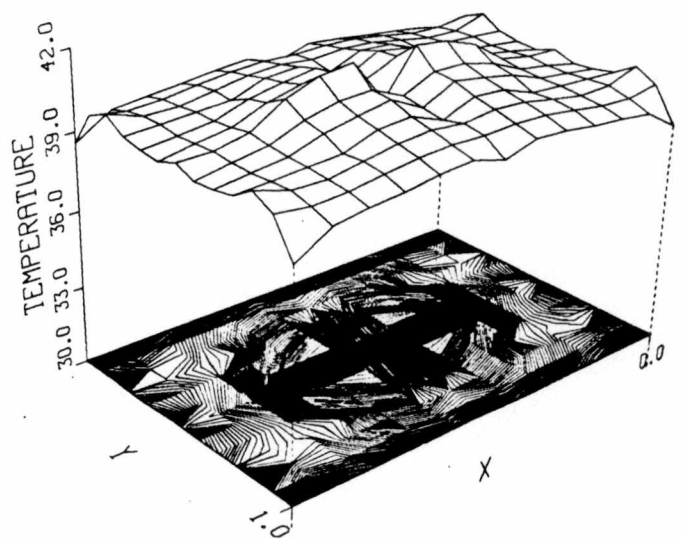
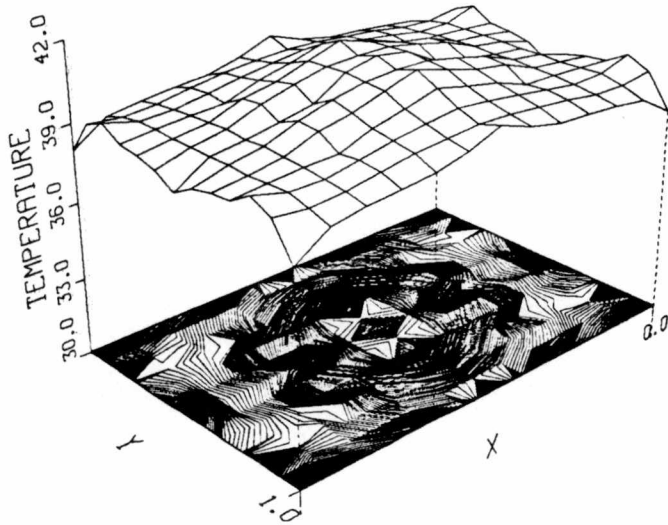
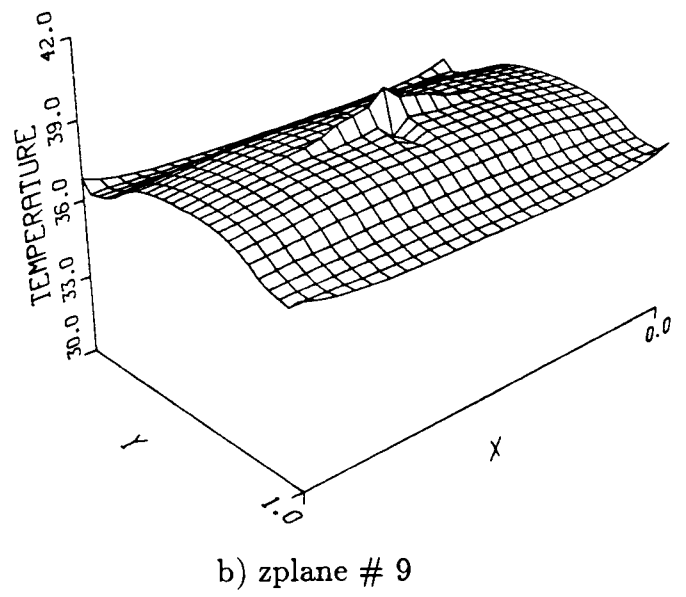
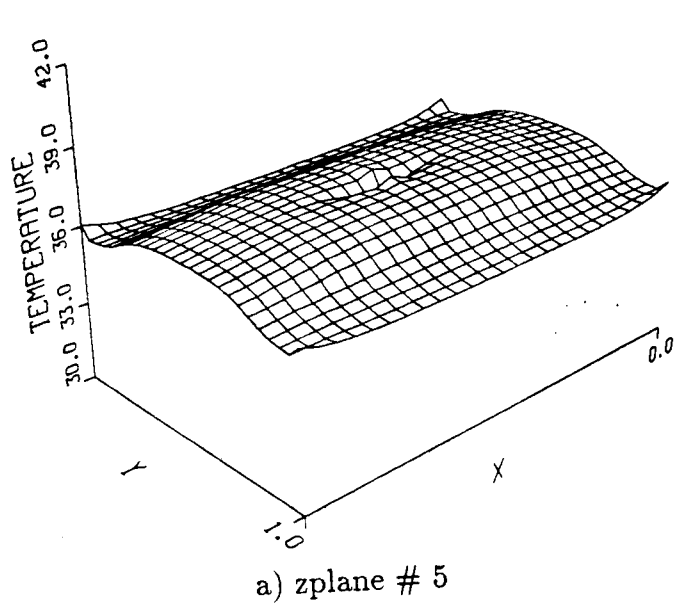


Figure B18. Surface and Contour Plots for Steady State Temperature Distribution for Variable Dirichlet Needle $T_{needle}(z)$ in a Parallelopiped Simulation Domain with Boundary Heat Flux (*Neumann*), for $12 \times 12 \times 12$ mesh : a) zplane # 3, b) zplane # 5, c) zplane # 7, d) zplane # 9



c) zplane # 13

d) zplane # 17

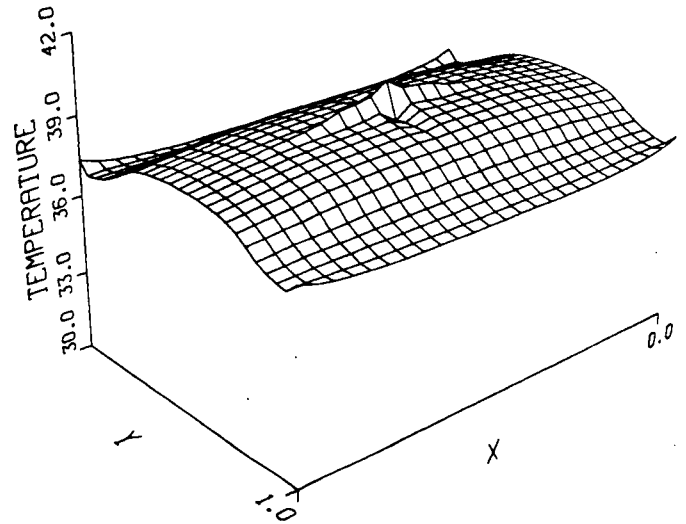
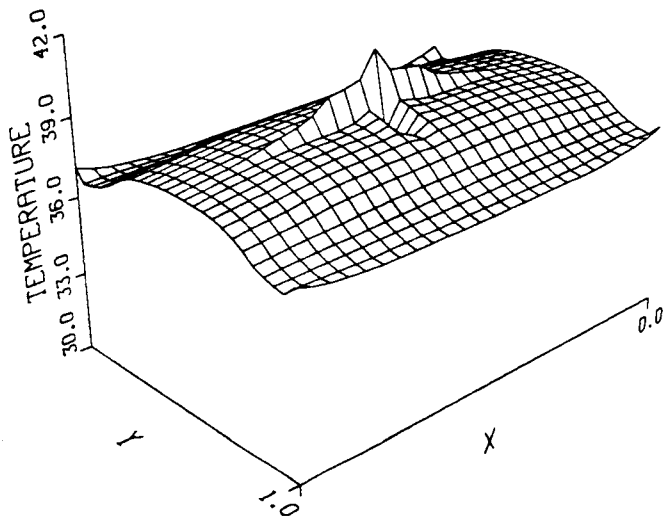
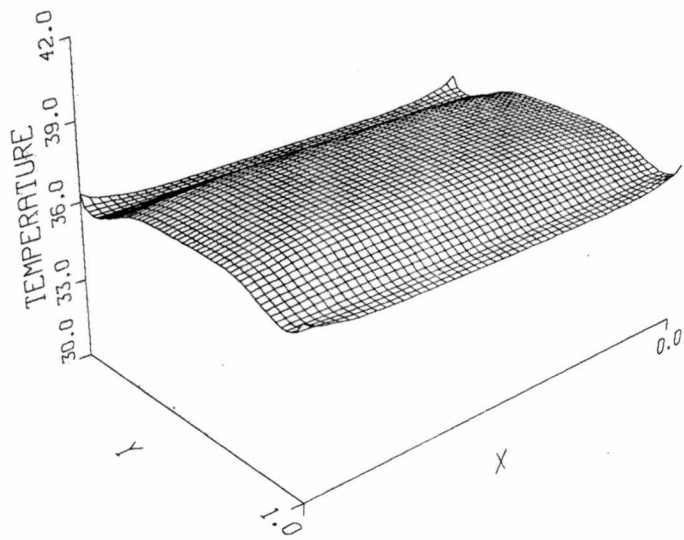
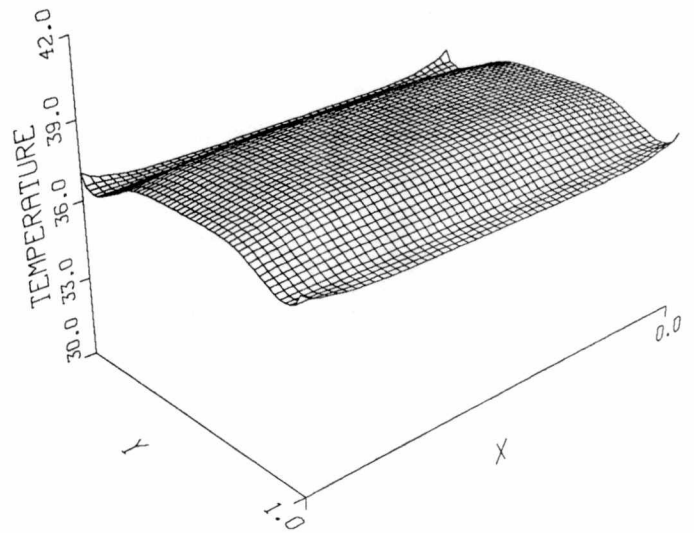


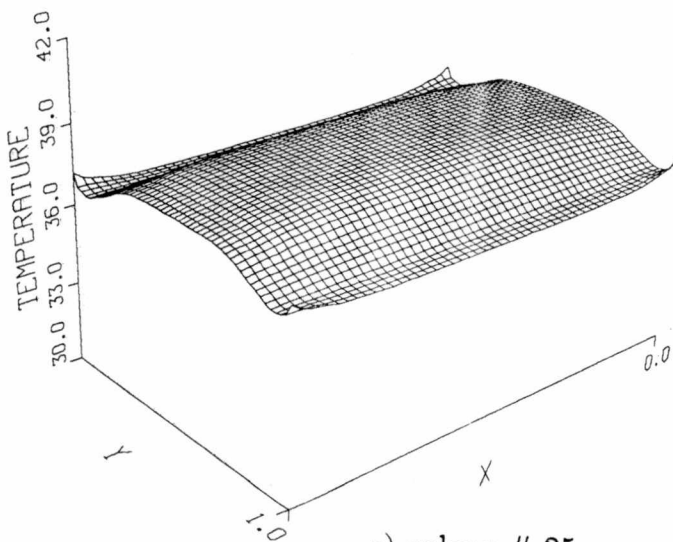
Figure B19. Surface Plots for Steady State Temperature Distribution in a Parallelopiped Simulation Domain for Variable Dirichlet Needle $T_{needle}(z)$ with Boundary Flux (*Neumann*), for $24 \times 24 \times 24$ mesh : a) zplane # 5, b) zplane # 9, c) zplane # 13, d) zplane # 17



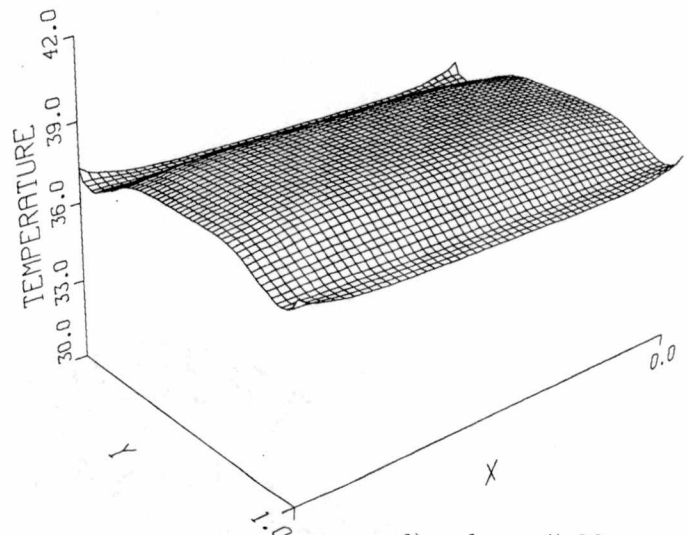
a) zplane # 9



b) zplane # 17



c) zplane # 25



d) zplane # 33

Figure B20. Surface Plots for Steady State Temperature Distribution in a Parallelopiped Simulation Domain for Variable Dirichlet Needle $T_{needle}(z)$ with Boundary Flux (*Neumann*), for $48 \times 48 \times 48$ mesh : a) zplane # 9, b) zplane # 17, c) zplane # 25, d) zplane # 33

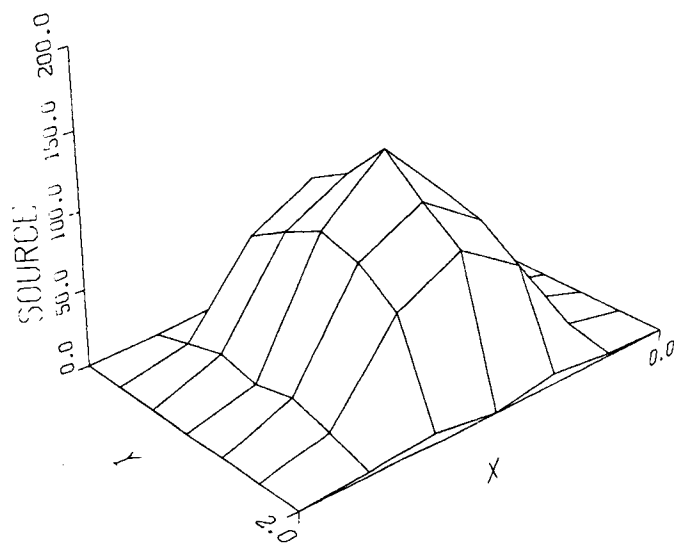
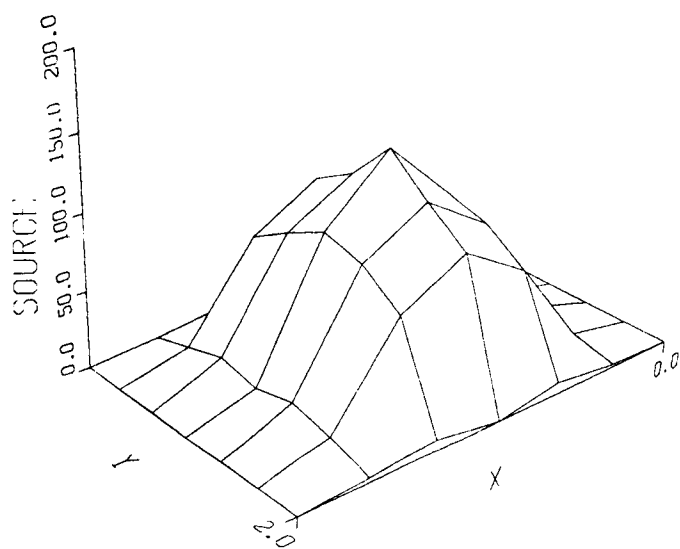
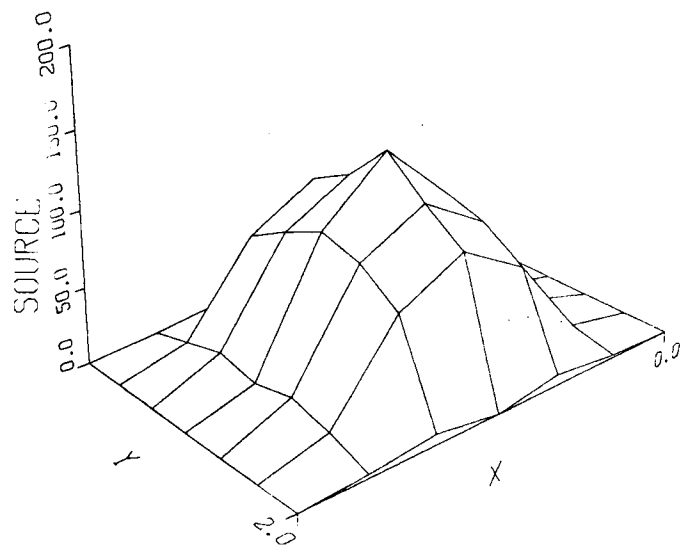
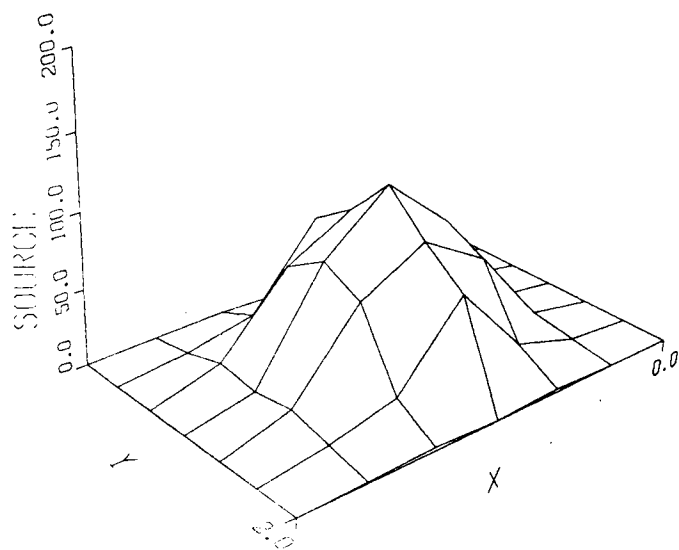


Figure B21. Surface Plots for the Source Distribution in a Parallelopiped
 Simulation Domain for $6 \times 6 \times 6$ mesh : a) zplane # 2, b) zplane # 3, c) zplane #
 4, d) zplane # 5

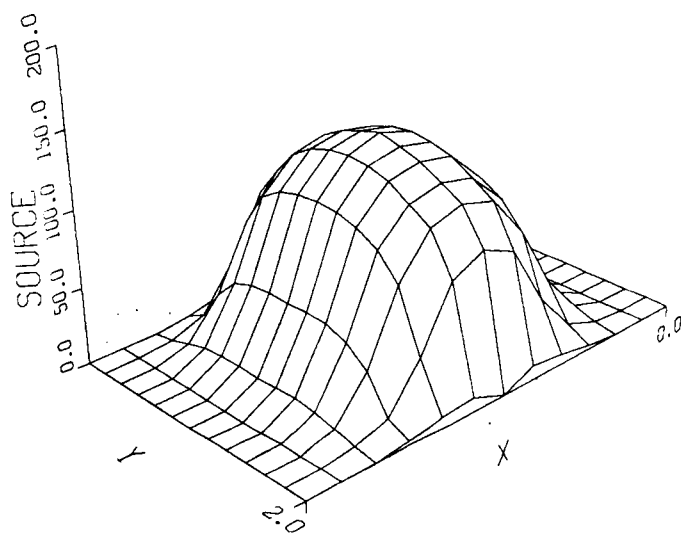
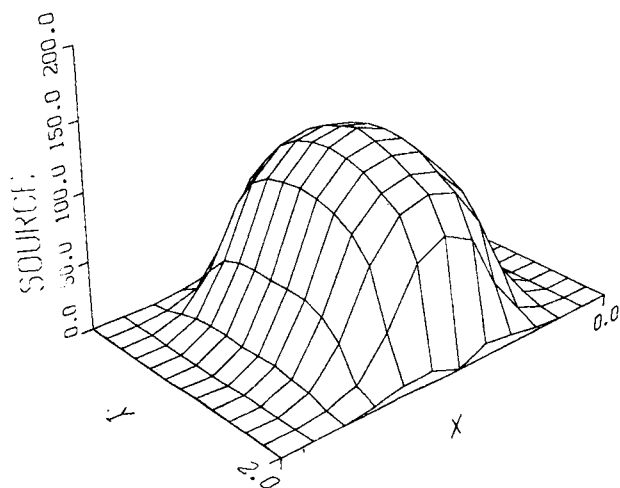
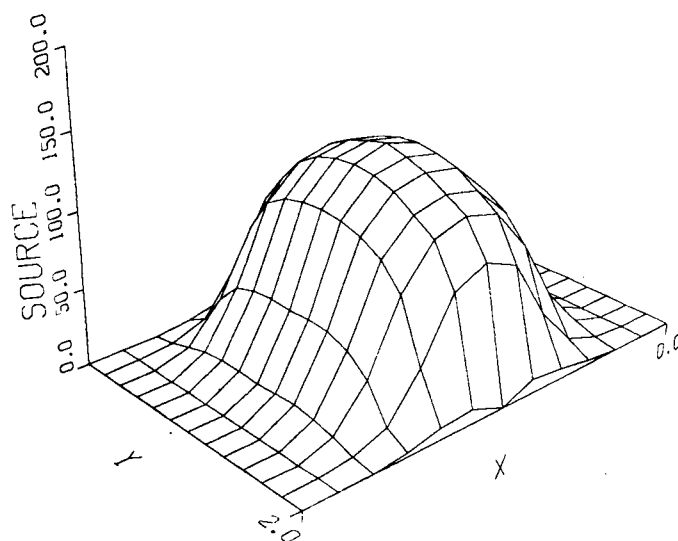
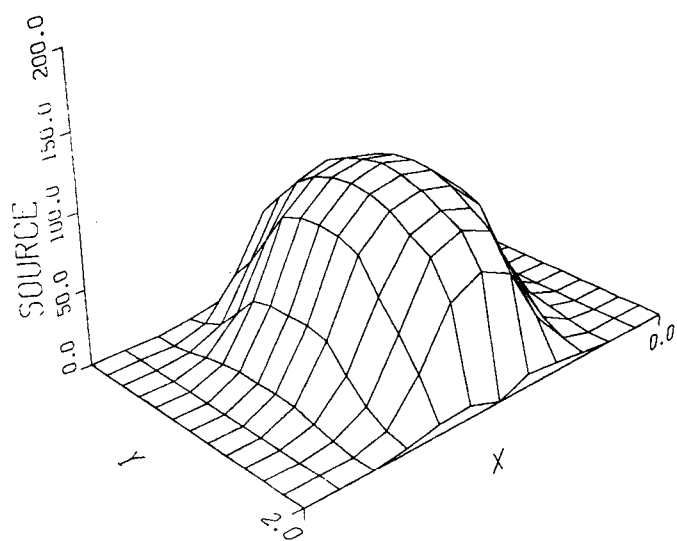


Figure B22. Surface Plots for the Source Distribution in a Parallelopiped Simulation Domain for $12 \times 12 \times 12$ mesh : a) zplane # 3, b) zplane # 5, c) zplane # 7, d) zplane # 9

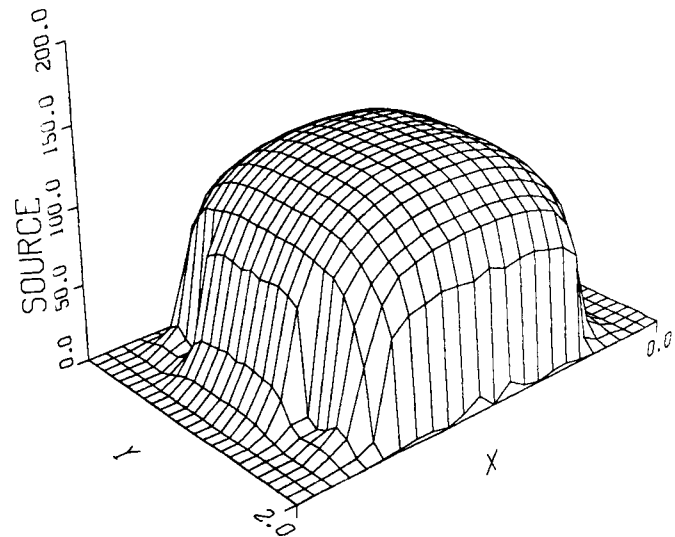
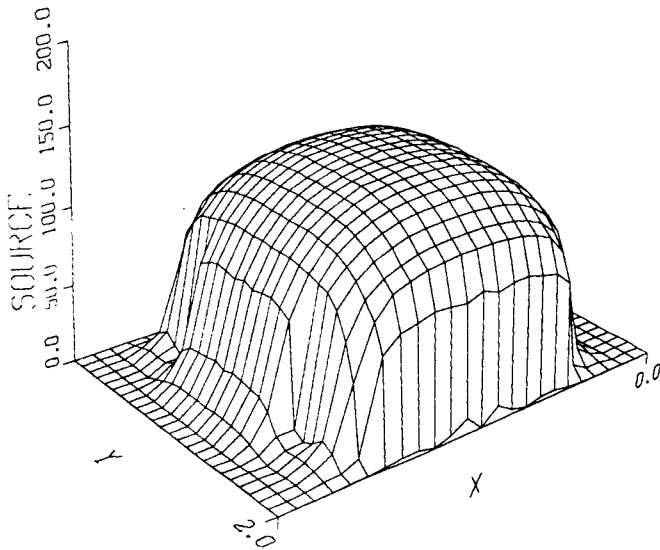
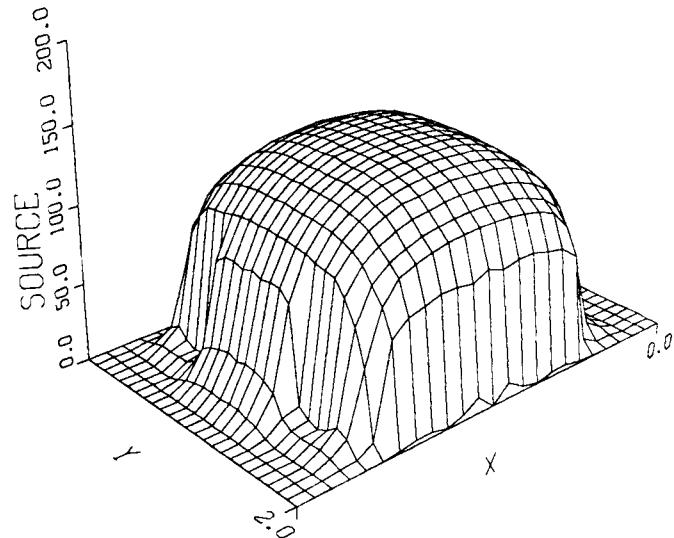
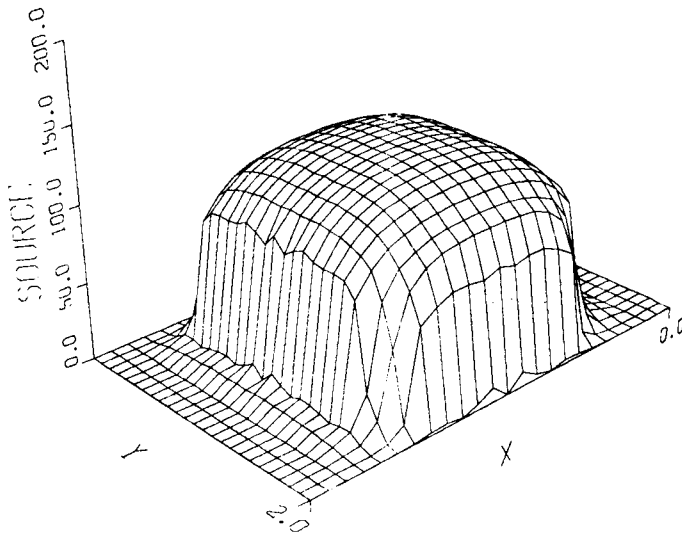


Figure B23. Surface Plots for the Source Distribution in a Parallelopiped Simulation Domain for $24 \times 24 \times 24$ mesh : a) zplane # 5, b) zplane # 9, c) zplane # 13, d) zplane # 17

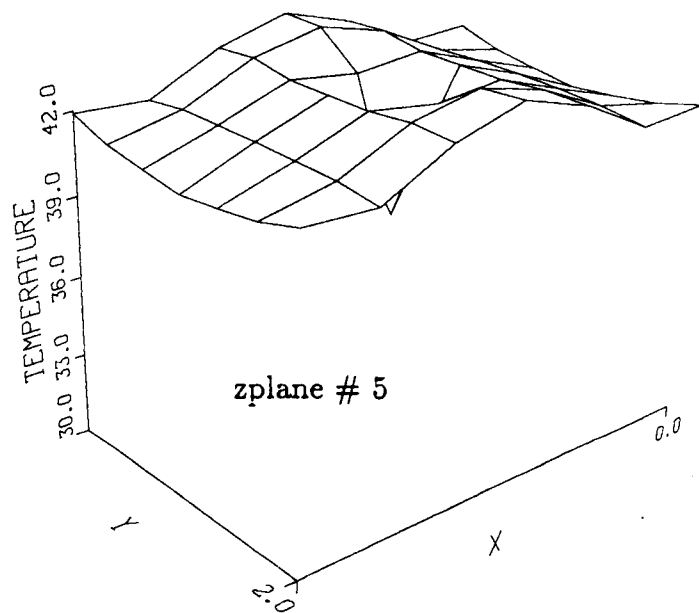
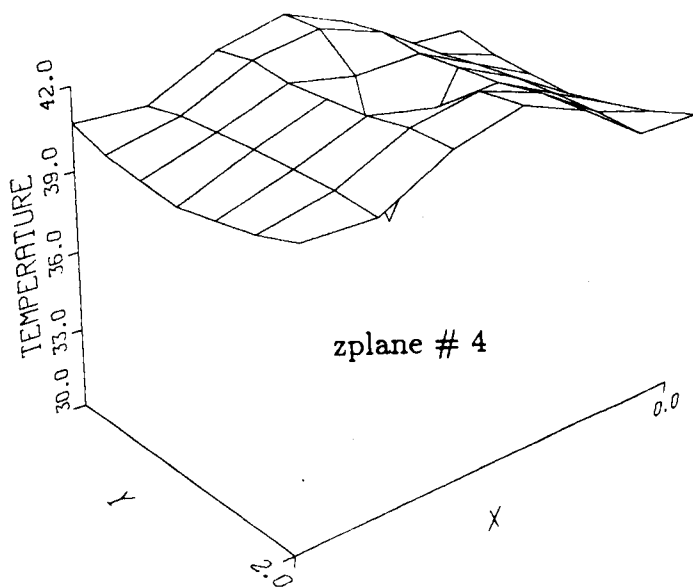
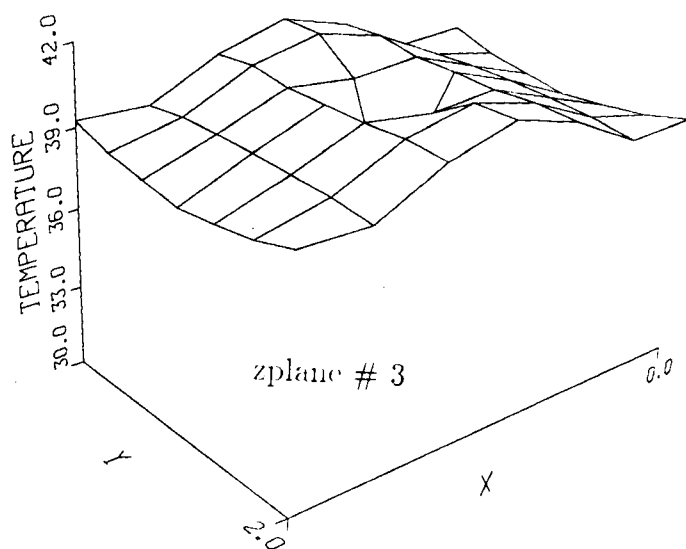
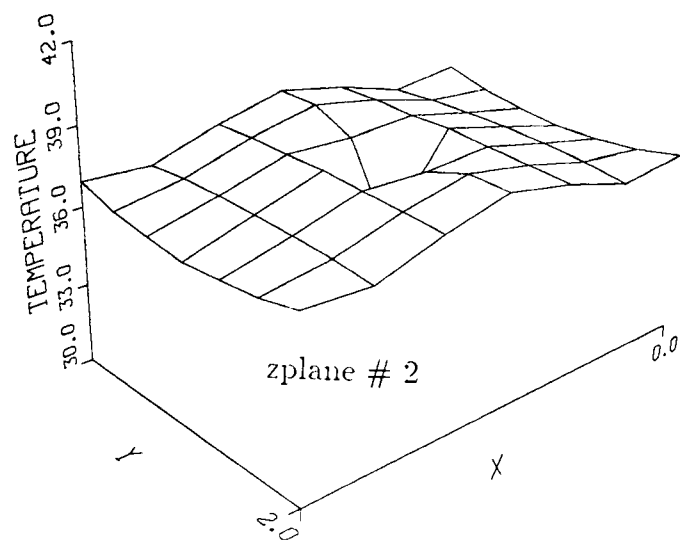


Figure B24. Surface Plots for Parallelepiped Simulation Domain with Multiple

Pairs of Electrodes, Mesh $6 \times 6 \times 6$

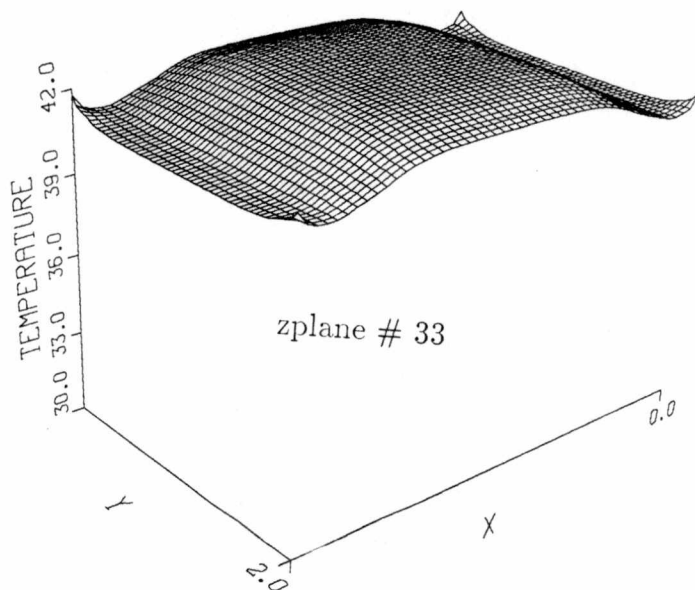
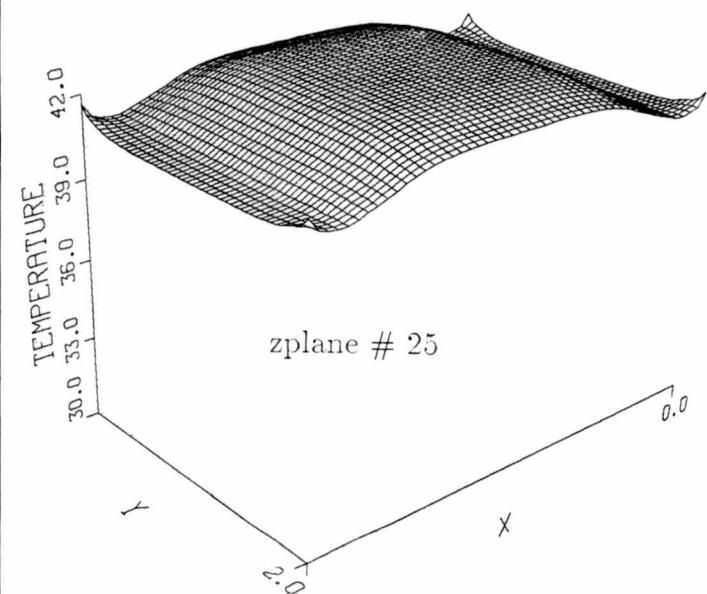
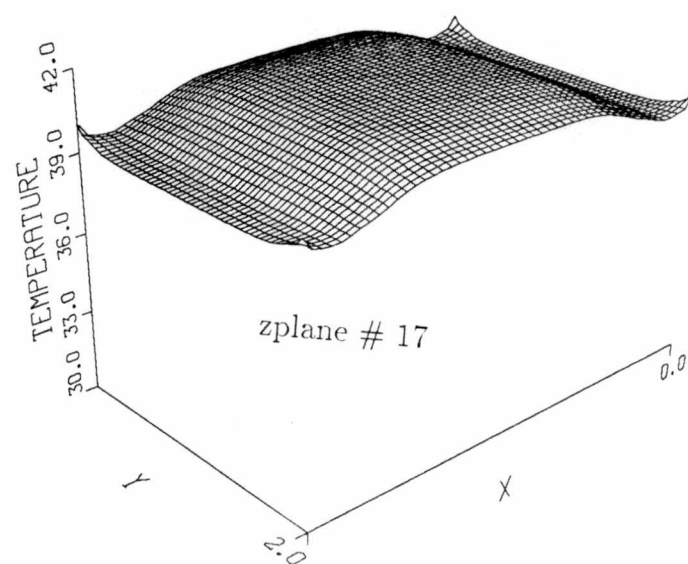
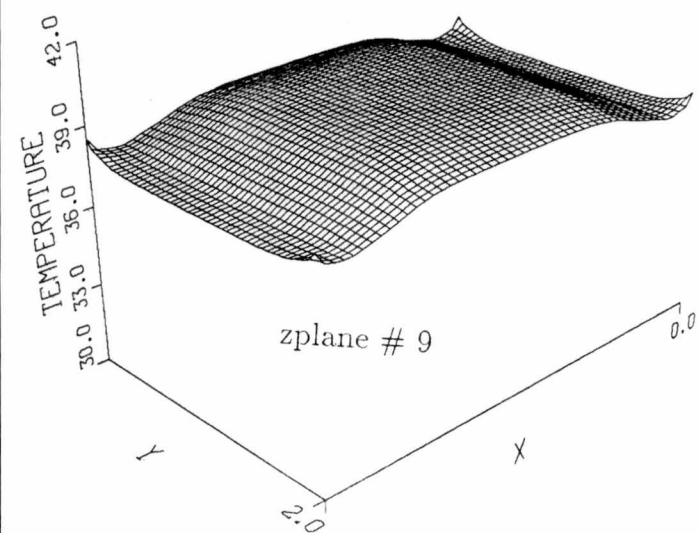


Figure B25. Surface Plots for Parallelopiped Simulation Domain with Multiple
Pairs of Electrodes: Mesh $48 \times 48 \times 48$

VITA

Subrata Roy (সুব্রত রায়) was born in Calcutta, India on September 14, 1962. He attended elementary schools in that city and was graduated from Jagadbandhu Institution in June 1978. The following August he entered St. Xavier's College, and passed Higher Secondary examination in July 1980. In August 1980 he got admission in the Engineering School of *Jadavpur University*, Calcutta, where he received a Bachelor of Mechanical Engineering (*B.M.E*) degree in July 1984.

After a four-year tour of industry with a private pump manufacturing company in Calcutta, in the fall of 1988, he accepted a graduate assistantship at *The University of Tennessee, Knoxville, USA*, and began study toward a Master of Science degree in *Engineering Science* with an emphasis in Computational Fluid Dynamics (CFD). He received that degree in May 1990.

The author is a National Scholar of his country. His undergraduate research work has been selected and published as a paper entitled *Computer Aided Water Turbine Design (CAWTDEN)* in the proceedings of 4th Canadian CAD/CAM & Robotics Conference organized by CIM in Toronto on June 1985. He has a reputation in extra-curricular activity too. He was a free-lance journalist in famous regional dailies and served as a *casual artist* in All India Radio and Door-darshan (TV), Calcutta. Recently, he has been elected as the Secretary of the India Student Association (ISA) in Knoxville.

Mr. Roy is now enrolled for the Ph.D. program in the department of Engineering Science and Mechanics, University of Tennessee, Knoxville.