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I am submitting herewith a dissertation written by Howard Douglas Sanders entitled "A Process Oriented Approach to Repairable Systems Reliability." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Ph.D., with a major in Management Science.

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A Process Oriented Approach  
to Repairable System Reliability

A Dissertation  
Presented for the  
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Howard Douglas Sanders  
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## **ABSTRACT**

This research strove to address concerns with the use of stochastic processes to describe repairable system reliability. An alternative method for examining repairable system reliability data, based on a strategy that directs and modifies the usage of traditional tools, is presented.

This alternate approach attempts to tie the reliability of repairable systems to sources of variation in the engineering and manufacturing processes in a cost effective manner that is easily communicated to engineers and managers.

## TABLE OF CONTENTS

| CHAPTER                                                                                                 | PAGE |
|---------------------------------------------------------------------------------------------------------|------|
| 1. INTRODUCTION                                                                                         | 1    |
| 2. LITERATURE REVIEW                                                                                    | 4    |
| 3. ORGANIZATION, QUESTIONS, & AVAILABLE DATA                                                            | 53   |
| 4. COMMENT ON THE SITUATIONAL USEFULNESS OF THE<br>REPAIRABLE SYSTEM MODELS & PROPOSED DATA<br>ANALYSIS | 62   |
| 5. HISTORICAL DATA ANALYSIS                                                                             | 96   |
| 6. CONCLUSIONS AND RECOMMENDATIONS                                                                      | 149  |
| BIBLIOGRAPHY                                                                                            | 175  |
| APPENDIX                                                                                                | 182  |
| VITA                                                                                                    | 201  |

## LIST OF TABLES

| Tables                                                   | Page |
|----------------------------------------------------------|------|
| 1. Linear Fits to USDS MCRF                              | 101  |
| 2. Component Removals in the USDS Data                   | 109  |
| 3. Percentages of Component Removals in the USDS Data    | 111  |
| 4. Multiple Removals in the USDS Data                    | 116  |
| 5. AX Component Removal, USDS and Return Parts Databases | 129  |
| 6. Failure Modes for AX Return Components                | 132  |
| 7. BX Component Removal, USDS and Return Parts Databases | 135  |
| 8. Failure Modes for BX Return Components                | 141  |
| 9. CX Component Removal, USDS and Return Parts Databases | 144  |
| 10. Failure Modes for CX Return Components               | 145  |
| 11. AX RAT Results                                       | 184  |
| 12. BX RAT Results                                       | 185  |
| 13. CX RAT Results                                       | 187  |
| 14. AX Runs Up And Down Test                             | 190  |
| 15. BX Runs Up And Down Test                             | 192  |

|                              |     |
|------------------------------|-----|
| 16. CX Runs Up And Down Test | 193 |
| 17. AX Median Runs Tests     | 195 |
| 18. AX Median Runs Tests II  | 196 |
| 19. BX Median Runs Tests     | 197 |
| 20. BX Median Runs Tests II  | 198 |
| 21. CX Median Runs Tests     | 199 |
| 22. CX Median Runs Tests II  | 200 |

## LIST OF FIGURES

| Figure                                                    | Page |
|-----------------------------------------------------------|------|
| 1. Times to Failure of Nonrepairable Components           | 7    |
| 2. Nonrepairable Components And Calendar Time             | 7    |
| 3. Repairable System Failures                             | 10   |
| 4. Cumulative Failures for a Repairable System            | 11   |
| 5. $M(t)$ and $u(t)$                                      | 12   |
| 6. Natural Estimator of $N(t)$                            | 13   |
| 7. Relationship Between $Y$ and $X$ in a Point Process    | 14   |
| 8. Truncation & Censoring                                 | 18   |
| 9. The Bathtub curve and Drenick's Theorem                | 29   |
| 10. Superimposed Renewal Processes                        | 34   |
| 11. The Bathtub Shaped Curve                              | 79   |
| 12. A Single Component System - $t$ and System Age        | 80   |
| 13. A Single Component System                             | 82   |
| 14. A Two Socket SP System                                | 82   |
| 15. Cumulative Failures vs. System Age                    | 84   |
| 16. Hypothetical Cumulative Failure vs. System Age Curves | 85   |
| 17. Multiple Repairable Systems                           | 86   |

|                                                                         |     |
|-------------------------------------------------------------------------|-----|
| 18. Multiple Repairable Systems II                                      | 88  |
| 19. Mean Cumulative Failures vs. System Age                             | 89  |
| 20. Rate of Occurrence of Failures                                      | 90  |
| 21. Example Pareto Chart                                                | 92  |
| 22. Another View of the Bathtub Shaped Curve                            | 93  |
| 23. Changing Failure Modes Across System Life                           | 95  |
| 24. MCRF USDS Data For Three Generations of System                      | 100 |
| 25. Cumulative Failures Versus System Age - USDS Data                   | 103 |
| 26. AX1 and AX2 Cumulative Failures in USDS Data                        | 105 |
| 27. MCRF CT USDS Data                                                   | 107 |
| 28. MCRF AX and MCRF AX Components                                      | 113 |
| 29. MCRF BX and MCRF BX Components                                      | 114 |
| 30. MCRF CX and MCRF CX Components                                      | 115 |
| 31. Individual System Cumulative Failures vs. System Age, AX Systems    | 118 |
| 32. Individual System Cumulative Failures vs. System Age, BX Systems    | 119 |
| 33. Individual System Cumulative Failures versus System Age, CX Systems | 120 |
| 34. AX MCRF, Return parts Database                                      | 131 |

|                                                                                                            |     |
|------------------------------------------------------------------------------------------------------------|-----|
| 35. AX Cumulative Failures versus System Age, Return Parts                                                 | 134 |
| 36. BX MCRF, Return parts Database                                                                         | 138 |
| 37. Individual System Cumulative Failures versus System Age and MCRF vs. System Age, Return Parts Database | 139 |
| 38. Component A BX System Returns                                                                          | 143 |
| 39. CX Cumulative Failures versus System Age, Return Parts                                                 | 147 |
| 40. CX Cumulative Failures versus System Age, Returns                                                      | 148 |
| 41. A Typical Return Process                                                                               | 159 |
| 42. A Typical Return Process and Issues                                                                    | 160 |
| 43. Spare Parts Processing                                                                                 | 161 |
| 44. Most Problematic BX System - Cumulative Returns                                                        | 167 |
| 45. Cumulative Failures for A Problematic System                                                           | 168 |
| 46. MCRF for A Components for Generation CX                                                                | 172 |

# **CHAPTER 1**

## **INTRODUCTION**

The competitive nature of business today requires that an organization's products and services meet or exceed continually evolving customer expectations. This lofty goal requires the continuous improvement of manufacturing, engineering, and service processes.

Reliability metrics such as Mean Time Between Failures are frequently used to monitor reliability; an important facet of the performance of an organization's product in the customer's environment. Unfortunately, typical advice regarding the analysis of reliability data is not useful to organizations interested in improving processes.

An example is the use of stochastic processes to describe repairable system reliability. First, although the stochastic process

models are useful when it is desired to characterize the times between failures in the past, common usage generally does not provide information to begin to tie the reliability of the system to sources of variation in the engineering or manufacturing processes, and so do not indicate possible directions that might be useful in alternating (improving) the future reliability of the system. Second, the esoteric nature of the methods does not facilitate process improvement. Third, in a typical fiscally constrained business environment, it is not clear that the development and maintenance of stochastic process models is the best use of limited resources. And lastly, it is rarely communicated that the gathering and reporting of reliability data is part of a process itself.

An alternative method for examining reliability data is based on a strategy that recognizes these concerns and so directs and modifies the usage of traditional repairable system reliability tools. This dissertation demonstrates the philosophy and methodology with particular emphasis on one organization.

In chapter 2 we survey and illustrate the literature on systems reliability. Chapter 3 is a description of the organization and the available data and phrases the questions of interest. In chapter 4 we comment on the available methods given the organizational needs and data. In chapter 5 we analyze the available data and develop questions for further exploration. Chapter 6 further illuminates some of these questions and provides direction for appropriate work on appropriate processes.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **Repairable Systems and NonRepairable Systems**

Probabilistic reliability models, with some overlap, can be divided into those models concerning the study of repairable systems and those concerning the study of nonrepairable systems. Nonrepairable system models concern themselves with the time to occurrence of an event such as a failure of a mechanical item or death of a biological organism for a particular group(s) or population(s). Repairable systems are repaired rather than replaced and so the models generally concern themselves with the multiple failures of individual systems over time. Repairable systems may be composed of only nonrepairable components, only repairable components, or of both nonrepairable and repairable components.

## Terminology

In this paper we will refer often to systems, components, and sockets. From this point on when we use component, it will mean a nonrepaired item that is replaced upon failure. A socket will consist of a position, the component mounted in that position, and the interconnections that connect the component with the remainder of the system. A system is the collection or assembly of all sockets.

## Notation and Illustration

In this section, we first define the notation necessary to discuss the literature associated with system reliability.

We use  $f(t)$  and  $F(t)$  to denote the density and distribution function of the nonreparable component lifetime  $T$ . Let  $\bar{F}(t) = 1 - F(t)$  be the survival or reliability function. It gives the probability that the component will operate without failure for mission time  $t$ . We call

$\gamma(t) = \frac{f(t)}{F(t)} = \frac{d}{dt} [-\log F(t)]$  the force of mortality (FOM) of the

lifetime distribution or the failure rate of the lifetime distribution or the hazard rate of the lifetime distribution. If  $\gamma(t)$  does not decrease, then F and T are said to have increasing force of mortality or IFR for increasing failure rate. Decreasing force of mortality or decreasing failure rate (DFR) are similarly defined.

We use Figure 1 to illustrate nonrepairable component reliability. The horizontal axis represents component age. Lifetimes for each of five components (also called parts) labeled A, B, C, D, and E are represented by the length of a corresponding line. Each line tracks component age from 'turn-on' or '0' to failure.

Generally components are turned on at different points in calendar time as illustrated in Figure 2. Calendar time is typically different than component age. In general, calendar time is not considered in the simple analysis of nonrepairable system reliability data. A practical consideration (when components are 'turned on' at different points in calendar time) is to capture that information and reorder the failures by component age.

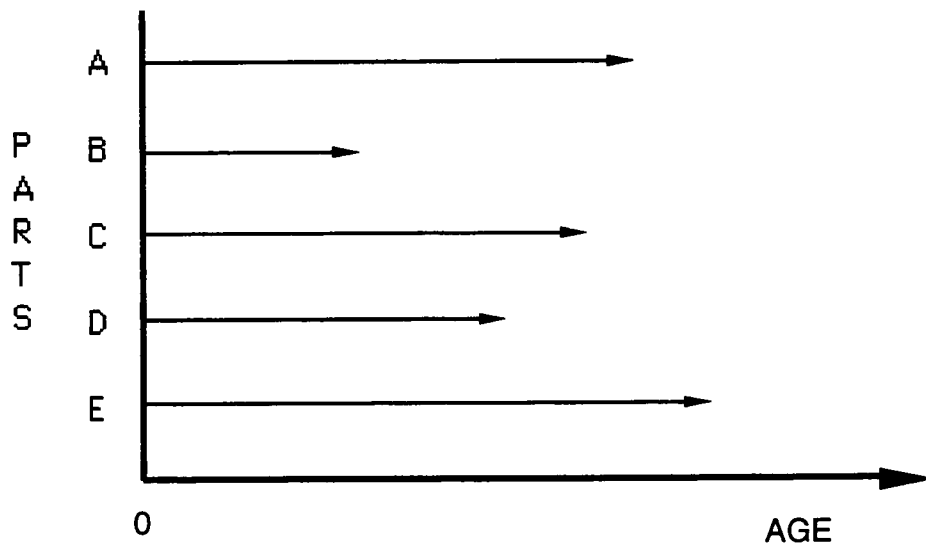


Figure 1: Times to Failure of Nonrepairable Components

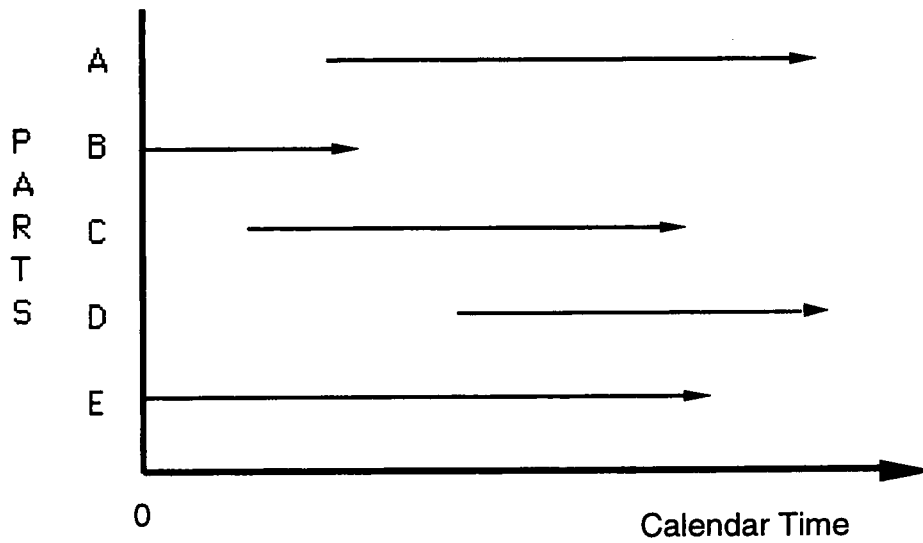


Figure 2: Nonrepairable Components and Calendar Time

The natural estimator of  $f(t)$  is  $\hat{f}(t) = \left(\frac{1}{n}\right) \left(\frac{n(t) - n(t + \Delta t)}{\Delta t}\right)$  where  $n$  is the number of units observed and  $n(t)$  is the number of units functioning at  $t$  (part age). Usually  $n = n(0)$ . The natural estimator of  $\gamma(t)$  is  $\hat{\gamma}(t) = \frac{\hat{f}(t)}{\hat{F}(t)} = \left(1 - \left(\frac{n - n(t)}{n}\right)\right)^{-1} \left(\frac{1}{n}\right) \left(\frac{n(t) - n(t + \Delta t)}{\Delta t}\right) = \left(\frac{1}{n(t)}\right) \left(\frac{n(t) - n(t + \Delta t)}{\Delta t}\right)$ . (The natural estimator of  $F(t)$  is  $F_n(t) = \frac{n - n(t)}{n} = \frac{\text{number of failure times} \leq t}{n}$  and by previous notation  $\hat{F}(t) = 1 - F(t)$ .) We observe that in the case of  $\gamma(t)$  the estimated probability of failure in  $(t + \Delta t)$  is 'normalized' by  $\left(\frac{1}{n(t)}\right)$ . For  $\hat{f}(t)$  the number of failures that occur in the interval is proportional to the total number of units observed rather than the number of units that are still functioning at  $t$ .

Repairable system's reliability are often modeled by point processes where each failure is an event in a series. Let  $N(t)$ , the number of failures in  $(0, t]$ , be a point process. Then  $\{N(t)\}$  is a collection of random variables, each random variable mapped to a point  $t$  on the

positive line such that  $N(t_2) - N(t_1) = N(t_1, t_2)$  is with probability one a finite nonnegative integer for all  $t_2 > t_1 \geq 0$ . We will assume that  $N(0) = 0$ .

In the repairable system models we will discuss,  $t$  is system operating time with repair assumed instantaneous. In the case where repair is not assumed instantaneous, the failure process still unfolds in calendar time, that is the first failure occurs before the second failure in calendar time, the second before the third and so forth. This is in contrast to the analysis of nonrepairable component reliability data where component age is generally different than calendar time.

Figure 3 illustrates failures in time for a repairable system. Here we assume that repair is instantaneous with continual operation of the system so that system age  $t$  is also calendar time. In Figure 3 an 'X' represents a system failure.

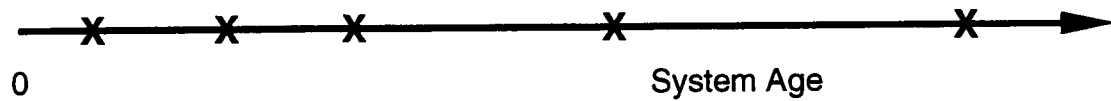


Figure 3: Repairable System Failures

We let  $M(t) \equiv \text{Expected } N(t)$ .  $M(t)$  is a nondecreasing right continuous function. If  $M(t)$ , the expected number of failures in  $(0, t]$ , is differentiable, we let  $u(t) = M'(t)$ . We call  $u(t)$  the failure rate of the process or The Rate Of Occurance of Failures (ROCOF). The ROCOF is the instantaneous rate of change of the expected number of failures with respect to time. (A point process where  $M(t)$  is continuous is a regular point process).

A second function of interest for failure processes is the intensity:

$$\lambda(t) = \lim_{\Delta t \Rightarrow 0} \frac{P[N(t + \Delta t) - N(t) \geq 1]}{\Delta t} \equiv \lim_{\Delta t \Rightarrow 0} \frac{P[N(t, t + \Delta t) \geq 1]}{\Delta t}. \text{ If}$$

$\lambda(t)$  exists, then  $\lambda(t)\Delta t$  is approximately the probability of failure in

the interval  $(t, t + \Delta t)$ . If simultaneous failures cannot occur, the process is regular or orderly and  $\lambda(t) = u(t)$ .

The observed cumulative number of failures for a repairable system is illustrated in Figure 4.

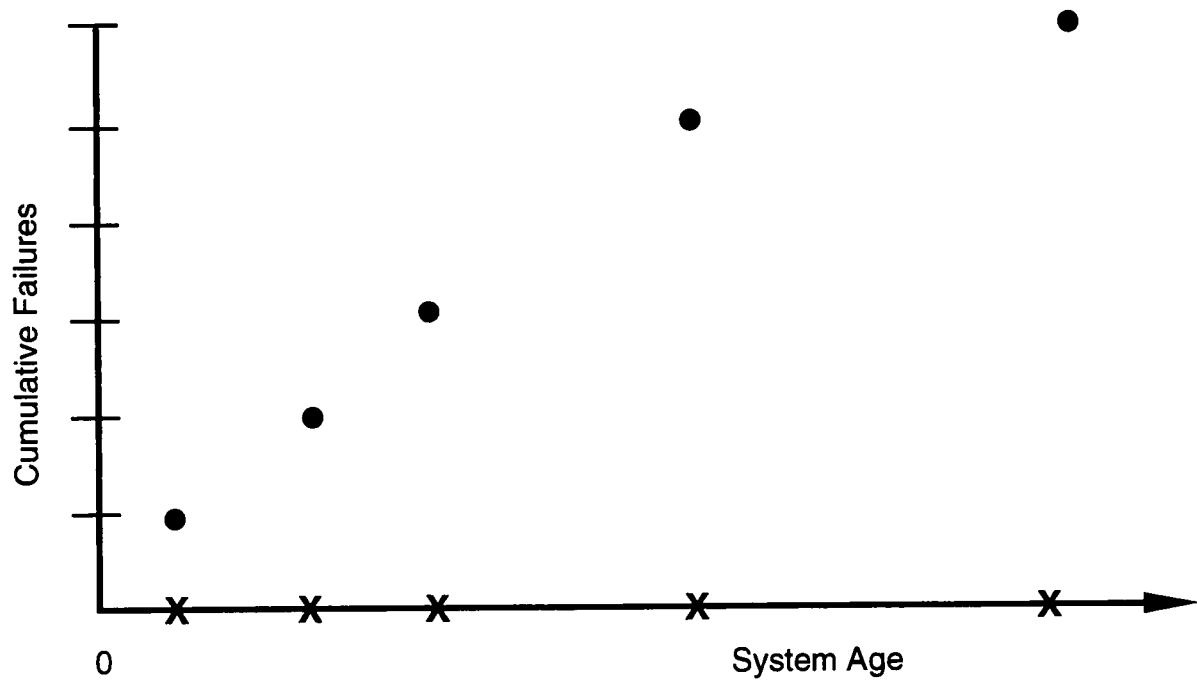


Figure 4: Cumulative Failures for a Repairable System

The expected number of cumulative failures in  $t$ ,  $M(t) = EN(t)$ , assumed continuous, is illustrated in Figure 5. The ROCOF are tangent lines to  $M(t)$  and so the interpretation as the rate of change of the expected number of failures. Again,  $u(t)\Delta t$  is the probability that a failure occurs in  $(t, t + \Delta t)$ .

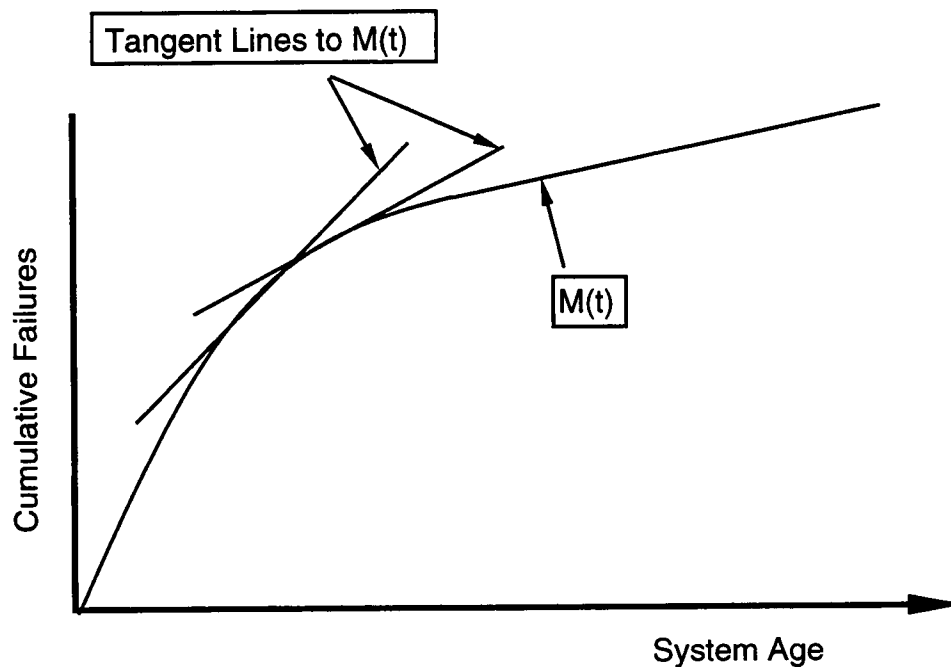


Figure 5:  $M(t)$  and  $u(t)$

The natural estimator of  $u(t)$  is  $\frac{N(t, t + \Delta t)}{\Delta t}$ . We compare this estimator to  $h(t) = \left( \frac{1}{n(t)} \right) \left( \frac{n(t) - n(t + \Delta t)}{\Delta t} \right)$  and observe that  $t$  indexes calendar time in  $\hat{u}(t)$  and component age in  $\hat{f}(t)$  and  $h(t)$  and that the estimator of  $u(t)$  is not tempered by conditional survival to  $t$  as is  $h(t)$ . The natural estimator of  $N(t)$  is illustrated in Figure 6.

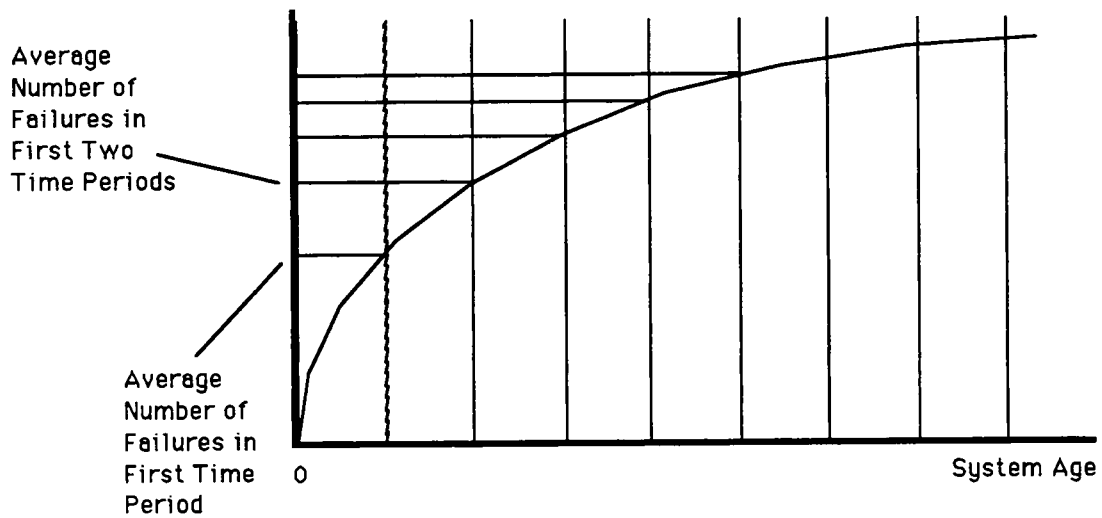


Figure 6: Natural Estimator of  $N(t)$

Rather than a random number of failures in an interval of time, we can consider a fixed number of failures in a random interval of time.

Let  $Y_n$  be the successive failure times, and  $Y_n$  has distribution function  $G_n(t)$  and density  $g_n(t)$ . Note that  $P[N(t) \geq n] = P[Y_n \leq t] = G_n(t)$  the distribution function of  $Y_n$ . Then  $P[N(t) = n] = P[N(t) \geq n] -$

$$P[N(t) \geq n+1] = G_n(t) - G_{n+1}(t). \quad \text{And } M(t) = \sum_{n=0}^{\infty} n P[N(t) = n] = \sum_{n=1}^{\infty} G_n(t),$$

so  $M'(t) = u(t) = \sum_{n=1}^{\infty} g_n(t)$  a third expression of the ROCOF. The times

between failures (TBF's) are defined by  $X_1 = Y_1, X_2 = Y_2 - Y_1, \dots, X_i =$

$Y_i - Y_{i-1}$ . These variables are clarified by Figure 7.

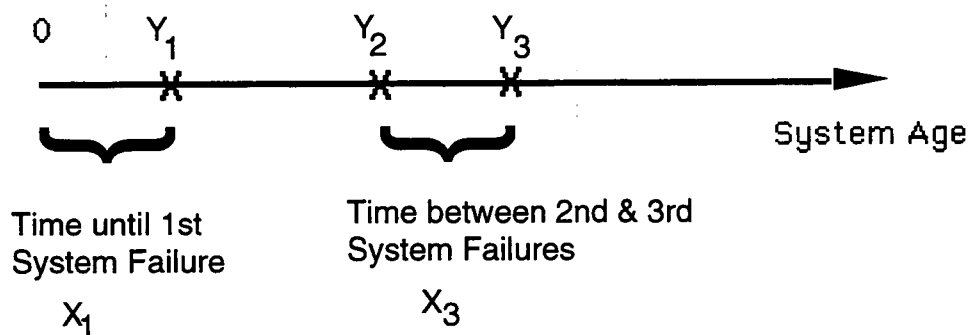


Figure 7: Relationship Between  $Y$  and  $X$  in a Point Process

A 'Mean Time Between Failures' (MTBF) statistics is the cumulative mean time between failures (CMTBF) =  $t/M(t)$ . Another MTBF statistic is the expected waiting time to failure from arbitrary fixed time  $t$  where the waiting time to failure from arbitrary fixed time  $t$  or forward recurrence time is  $W_t = Y_{N(t)+1} - t$ . Another is the expected instantaneous time between failure ( $E[ITBF]$ ),  $E[t: W_t | N(t-, t) \geq 1]$ , or  $E[W_t \text{ given a failure occurs at time } t]$ , and  $E[X_i]$ .

The distribution function of  $W_t$  is  $K_t(w) = P[N(t, t+w) \geq 1]$  so that

$$E[Y_{N(t)+1} - t] = \int_0^\infty P[W > w] dw = \int_0^\infty 1 - P[N(t, t+w) \geq 1] dw = \int_0^\infty P[N(t, t+w) = 0] dw. \quad \text{We observe that } \lambda(t) =$$

$$\lim_{\Delta t \rightarrow 0} \frac{P[N(t, t+\Delta t) \geq 1]}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{P[0 < W_t \leq \Delta t]}{\Delta t} = k_t(0) \quad \text{the}$$

density function of time to failure evaluated at 0.

The distribution function of ITBF is  $L_t(w) =$

$$\lim_{\Delta t \rightarrow 0} P[W(t) \leq w | N(t-\Delta t, t) \geq 1]. \quad \text{We observe that ITBF differs from } X_{i+1}$$

only when  $i$  at  $t$  is unknown.

Typically reliability growth is defined as the tendency for TBFs to get longer as the system ages and reliability deterioration is defined as the tendency for TBFs to get shorter.

A point process has independent increments if for all  $t_1 < s_1 \leq t_2 < s_2 \leq \dots \leq t_n \leq s_n$  points on the time line and all  $n$ ,  $P[N(t_1, s_1) = k_1, \dots, N(t_n, s_n) = k_n] = \prod_{i=1}^n P[N(t_i, s_i) = k_i]$ . Equivalently,  $N(t_1) - N(0), \dots$

$N(t_n) - N(t_{n-1})$  are independent random variables and therefore the number of failures in an interval is not influenced by the number of failures in an earlier non-overlapping interval.

A process is said to be homogeneous or have stationary increments if for each  $(t_1, t_2)$  and  $\Delta t > 0$ , the distribution of  $N(t_1, t_2)$  is the same as  $N(t_1 + \Delta t, t_2 + \Delta t)$ . Equivalently,  $P[N(t, t + \Delta t) = k]$  does not depend on  $t$ .

## **Truncation and Censoring**

Right Truncation refers to the situation where information on the number and times of failures past some age are unknown. Similarly, left truncation refers to the situation where information on the number and times of failures before some age are unknown. Right censoring is the situation where the number of failures past a certain age are known, but the failure times are unknown. Left censoring is the situation where the number of failures before a certain age are known, but the failure times are unknown. Truncation and censoring may refer to either repairable or nonrepairable units.

Figure 8 illustrates the difference in truncation and censoring for nonrepairable components. An 'X' on the time line corresponds to failure age of a sample component. In Figure 8A, there is no censoring or truncation. Figure 8B illustrates right censoring. An 'O' on the time line corresponds to the removal of a sample unit from the study (the unit had not failed at the time of removal). Right censoring may be due to the retirement of a component from a fleet

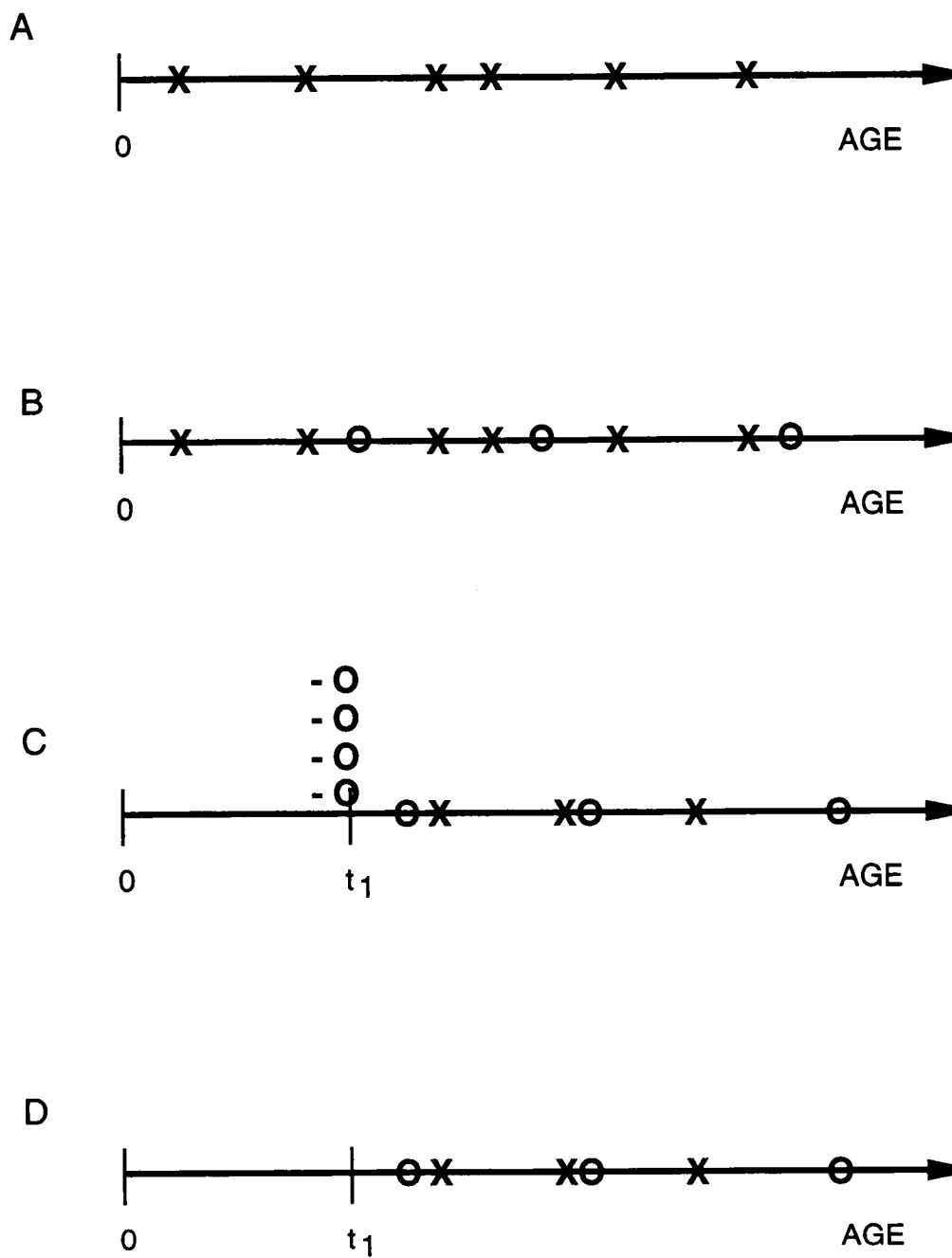


Figure 8: Truncation & Censoring

of components or the analysis of data while the component is still functioning.

In Figure 8C, an '-O' represents left censored data. The units are not observed until  $t_1$  where four are discovered to have failed. Although the number of units that have failed before  $t_1$  are known, the exact failure ages are not known. In Figure 8D the entire failure history before  $t_1$  is unknown (both the dates and numbers of failures). The data are left truncated.

It is worth noting that Figure 8 can illustrate truncation & censoring for a repairable system if 'X's' and "O's' represent failure times for a single system.

### **Renewal Processes**

A renewal process is a sequence of random variables  $\{Y_1, Y_2, \dots\}$  of the form  $Y_n = X_1 + X_2 + \dots + X_n$  where  $\{X_1, X_2, \dots\}$  are independent and identically distributed (iid) with  $F(x)$ . Thus, a repairable system

which can be described by a renewal process postulates a "good as new" (GAN) or "same as new" condition for the system after repair.

Thompson develops (1969) and discusses (1981) properties of renewal processes. For instance  $L_t(w) = F(w)$  and  $E(ITBF) = E(X_i) =$

$$\int_0^\infty \bar{F}(w) dw = \theta \quad \text{say.} \quad \text{Renewal processes are asymptotically}$$

stationary, so  $M(t) \rightarrow t/q$ ,  $u(t) \rightarrow \theta^{-1}$ ,  $k_t(w) \rightarrow F(w)/q$ .

A process with independent increments satisfying  $P[N(t, t + \Delta x) = k] = p(k; \lambda x)$ ; where  $p(k; \lambda) = \lambda^k (k!)^{-1} \exp(-\lambda)$ ;  $k = 0, 1, \dots$  the frequency function of the Poisson distribution, with  $0 < \lambda < \infty$ , is said to be a homogeneous poisson process (HPP). Equivalently, a process is HPP if and only if (iff) times between failures are independent with common exponential distribution:  $P[X_i \leq x] = 1 - \exp(-\lambda x)$ ,  $x \geq 0$ . Or the HPP is the process that has stationary and independent increments with no probability of simultaneous failures. The HPP is a special case of a renewal process.

If a renewal process is a HPP, then  $M(t) = \lambda t$ ,  $u(t) = \lambda(t) = \lambda$  and  $E(X_i) = \lambda^{-1}$ . Furthermore,  $K_t(w) = 1 - \exp(-\lambda w)$ ;  $t, w > 0$  and  $EW_t = \lambda^{-1}$ . And as  $t \rightarrow \infty$   $\hat{CMTBF} = t/N(t) \rightarrow \lambda^{-1}$ .

An assumption of a renewal process (in particular an HPP because of these properties), greatly eases estimation of system reliability metrics. For instance, the prediction of future reliability (MTBF) is straightforward for HPP. The military handbook, MIL-STD-757 assumes that successive failures of a system are IID exponential and provides techniques for the estimation of reliability parameters such as MTBF based on this assumption.

If a renewal process is assumed, all of the statistical techniques and models used for survival analysis or lifetime data are applicable to system TBFs. In addition, if the TBFs are exponential, the probabilistic and statistical techniques for HPPs are also available for the study of repairable system reliability.

One of the pivotal papers in repairable system study is Proschan (1963). Proschan, implicitly assuming a renewal process, analyzes time between failure data for 13 jet airplane air conditioning systems. The original intent of the study was to make predictions and decisions for the fleet of airplanes rather than for single airplane air conditioning systems.

Proschan shows a fraction surviving versus operating time plot fitted with the exponential survival probability yields evidence that the pooled data has a decreasing failure rate. Proschan then tests the individual airplane data for trend with Mann's Test and examines the combined individual test results with Fisher's Procedure. There is no evidence of trend among successive time between failures. The examination of individual airplane data shows no evidence of lack of fit for an exponential distribution.

Proschan proves that an analysis of pooled mean time to failures where the multiple system copies have differing nonincreasing failure rates will yield an incorrect conclusion of an overall

decreasing hazard rate. This observation along with the use of Mann's test for trend for the chronologically ordered failure times were major contributions to the field of repairable system reliability.

There have been many published papers on renewal processes for repairable systems since Proschan's paper. Papers that significantly advance the area include the following. Brown and Proschan (1983) introduce a model under the assumption that repair restores the system to "good as new" with probability  $p$  and is otherwise repaired to the condition immediately prior to failure. The Brown and Proschan (BP) model assumes that the quality of repair depends on factors external to the system and not system age. Block, Borges, and Savits (1985) extend the BP model to the case where the probability of perfect repair is dependent on age at failure. Whitaker and Samaniego (1988) examine estimation and inference problems for the BP model.

Follmann & Goldberg (1988) address the situation where multiple system copies have differing failure rates. They assume a renewal process but present a model that allows heterogeneity between system copies. They assume that the TBFs of each system follows a Weibull distribution each with an identical Weibull shape parameter. The Weibull scale parameter for each system is assumed to be a random sample from a gamma distribution. The scale parameters model the heterogeneity between system copies and the shape parameter models the hazard rate for each system copy. This methodology avoids the spurious conclusion of a decreasing hazard rate.

Denby and Vardi (1985) develop alternate estimators for renewal processes. Tortorella (1989) discusses a method for comparing two renewal processes. More articles analyzing repairable system data assuming renewal processes are also available [de la Mare (1978)], but the main philosophies and contributions are captured by these authors.

Nelson (1990) discusses the hazard plotting of left truncated data. He carefully explains the difference between left censored and left truncated data. The method assumes that combined samples cover age zero through the last right censoring age. The distribution can not be estimated over periods not covered. The method assumes random sampling. Failure, truncation, and censoring dates are assumed known. Actually the method provides an estimate of the renewal function since data from the truncated distribution are assumed to have the truncation age. Therefore parts that have been replaced in the sample bias the estimate towards longer life.

Thompson (1981) and Asher & Feingold (1984) discuss the problems associated with the ad hoc assumption of a renewal process, particularly a HPP. For instance, the HPP cannot be used to model system reliability growth or deterioration because of the constant failure rate. Therefore a HPP can not be used to determine policies for maintenance or trade in's. These authors recommend the use of a trend test before assuming a renewal process. They caution that a

prior application of a trend test does not check the independence of TBFs'.

### **Nonhomogeneous Poisson Processes**

Again following Thompson (1981), a point process with independent Poisson distributed increments,  $P[N(t, t+x) = k] = p[k; a(t, x)]$ ;  $k = 0, 1, \dots$ , where necessarily  $a(t, x) = EN(t, t+x) = M(t+x) - M(t)$ , is called a Poisson process. If  $M(t) = \lambda t$ , the process is homogeneous, and the process is nonhomogeneous if  $M(t) \neq \lambda t$ .

The use of Nonhomogeneous Poisson Processes (NHPP) to model repairable system reliability may be an extension of the Duane Postulate. According to Thompson (1981), the Duane Postulate is a deterministic one based on Duane's observation that plots of  $\log N(t)/t$  versus  $\log$  cumulative  $t$  are frequently linear. This implies that  $N(t) \approx \lambda t^\beta$  with  $\beta > 0$ . Since  $N(t) \Rightarrow M(t)$  for large  $t$  for Poisson processes,  $M(t) = \lambda t^\beta$  and  $u(t) = \lambda \beta t^{\beta-1}$ . We discuss this model further later in the monogram.

Asher (1968) published a pivotal paper in the use of NHPP to model repairable system reliability. Asher does not discuss Duane's Postulate, but bases his paper on two arguments. The first is that reparable systems consist of many parts, components, and interconnections. He implicitly assumes a series system so that the failure of one of these components or parts results in system failure. Intuitively, since in most cases system repair consists of repairing only one component or part or at least only a small percentage of the system, the overall system failure rate will remain much the same after repair than as before failure. Models that assume the reliability of the system is unchanged by failure and repair are called minimal repair or "bad-as-old" (BAO) or "same as old" models. A BAO model does not preclude system reliability deterioration or growth as a function of operating time.

Asher's second heuristic argument is that based on Drenick's Theorem and the well known Bathtub curve. Drenick's Theorem states that whatever the form of the ROCOF in the early life of a

complex repairable system, the ROCOF will tend towards a constant rate as the cumulative operating time of the system becomes large enough. Intuitively, this is because parts fail and are replaced at random times and once a large enough number of failures has taken place, the parts comprising the system have a random mixture of ages.

The Bathtub Curve is the well known assumption that parts of interest have a decreasing  $\gamma(t)$  in early life, a constant  $\gamma(t)$  in middle life, and an increasing  $\gamma(t)$  late in life due to wearout. Asher notes the conflict of Drenick's Theorem and the bathtub curve in the repairable system case (note, in general  $\gamma(t) \neq \text{ROCOF}$ ) and rejects the bathtub curve assumption for repairable systems on a logical basis. Figure 9 illustrates the bathtub curve and Drenick's Theorem.

While acknowledging Drenick's Theorem, Asher argues that most repairable systems never get old enough for the failure rate to be constant. So Asher argues that in the early and intermediate life of

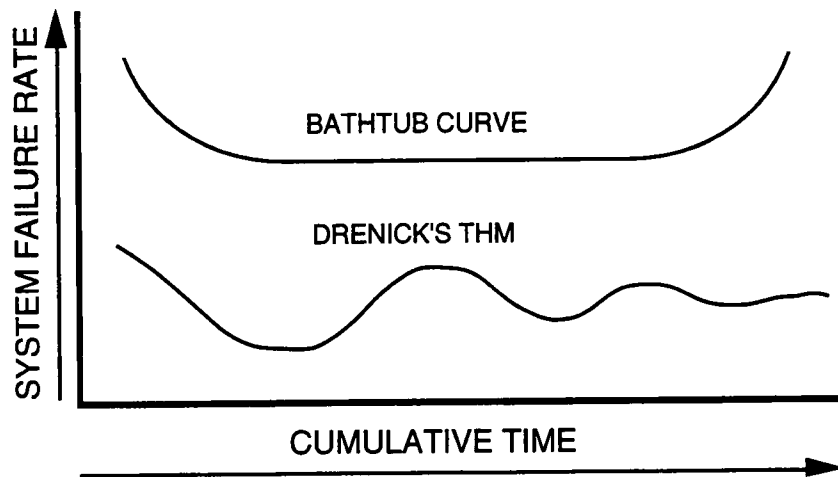


Figure 9: The Bathtub curve and Drenick's Theorem

a repairable system (possibly the entire useful life of the product), it is incorrect to assume a priori a renewal process. Ascher believes the first step in analyzing repairable system data should be a statistical test for trend. Evidence of a trend would support Ascher's conviction that for most repairable systems each of the successive times to failure could be considered to have a different lifetime distribution. There would possibly be dependency between distributions for early times to failure and distributions for later times to failure.

Ascher encourages a BAO model for fitting complex repairable systems reliability data before the constant failure rate is reached under Drenick's Theorem. He believes that the easiest and simplest BAO model to use is the NHPP, even though this BAO model assumes that all system parts including the repaired or replaced part have system age. He recommends that this model be used unless the assumptions can be invalidated.

Asher expects that a ROCOF of the form  $u(t) = \lambda + \alpha\beta^t$  is a good choice in many cases. This  $u(t)$  can be used to model increasing, decreasing, and constant ROCOFs. For instance, if  $0 < \beta < 1$  and  $\alpha < 0$  then  $u(t)$  models an increasing ROCOF. Asher cautions that the rejection of the goodness of fit for this failure rate form does not reject the BAO hypothesis, merely this form of a NHPP.

The use of NHPP in repairable system reliability is also encouraged by Crow. Crow (1990) discusses his development of the Weibull or power law NHPP process as published in a series of articles from 1972 through 1990. Crow states that his model is a stochastic

version of Duane's Postulate. Crow allows the failure rate of the process to vary with time:  $u(t) = \lambda\beta\tau^{\beta-1}$  with  $t > 0$ ,  $\beta > 0$ ,  $\lambda > 0$ . This NHPP has  $E[N(t)] = M(t) = \lambda t^\beta$ . If  $\beta = 1$ , we have the HPP which is a special case of the Weibull process and is perhaps a boundary between renewal processes and minimal repair processes. For  $\beta > 1$ ,  $u(t)$  is increasing and therefore the TBFs are stochastically decreasing. For  $\beta < 1$ , the TBFs are stochastically increasing. (It is worth noting that Crow recommends that an analysis be performed for each individual system, and he observes that evidence that TBFs are stochastically increasing for an individual system does not imply reliability growth. Improvement pertaining only to the reliability of an individual system does not address the reliability of future systems.)

Crow has developed estimation procedures, confidence intervals, and pooling techniques for his model. For instance if the process is time truncated on the right at  $T$  with  $n$  failures, then the MLEs

derived from the likelihood function  $L = (\alpha\beta)^n \left( \prod_{i=1}^n Y_i \right)^{\beta-1} \exp(-\lambda T^\beta)$  are

$$\hat{\beta} = \frac{n}{\sum_{i=1}^n \ln\left(\frac{T}{Y_i}\right)}, \hat{\lambda} = \frac{n}{T^{\hat{\beta}}}. \text{ Where } n \geq 1 \text{ and } 0 \leq y_1 \leq \dots y_n \leq T. \text{ Similarly if}$$

the processes is failure truncated on the right at  $y_n$ , the MLEs are

$$\hat{\beta} = \frac{n}{\sum_{i=1}^n \ln\left(\frac{Y_n}{Y_i}\right)}, \hat{\lambda} = \frac{n}{Y_n^{\hat{\beta}}}.$$

Cox and Lewis (1966) propose  $u(t) = \exp(\alpha + \beta t)$  as the form of the failure rate in a NHPP. Lawless (1982) and Cox and Lewis develop maximum likelihood tests for this form to determine whether there is evidence of a time trend (assuming this failure rate model is correct). This simple form allows for monotone trends in the failure rate. Estimation of the model parameters is straightforward.

Thompson (1981) and Leitnaker (1993) discuss the fact that the ROCOF for a NHPP (and all Poisson processes) is entirely determined by the distribution of the time to first failure. Thompson also observes that time to failure,  $W_t$ , is stochastically decreasing in  $t$  if and only if  $u(t)$  is increasing. Note that while the  $X_i$ s are not iid, the NHPP still has independent increments.

## **Superimposed Renewal Processes**

The models discussed to this point assume that the failure patterns of the repairable system of interest are being examined. Another philosophy is that the system's failure patterns can be explained by modeling the reliability behavior of the system components and then combining the results. A superimposed renewal process (SRP) model assumes a series system, that components fail independently, that the times between failures of the components in particular sockets are iid sequences, and that failure of a part or component in a socket results in a new replacement.

In Figure 10, a sixteen socket repairable system is illustrated. The line corresponding to a particular socket represents the failure history of that socket. An arrow depicts a component failure and subsequent replacement with a new component. We observe that with this model, system failures are 'explained' by component failures.

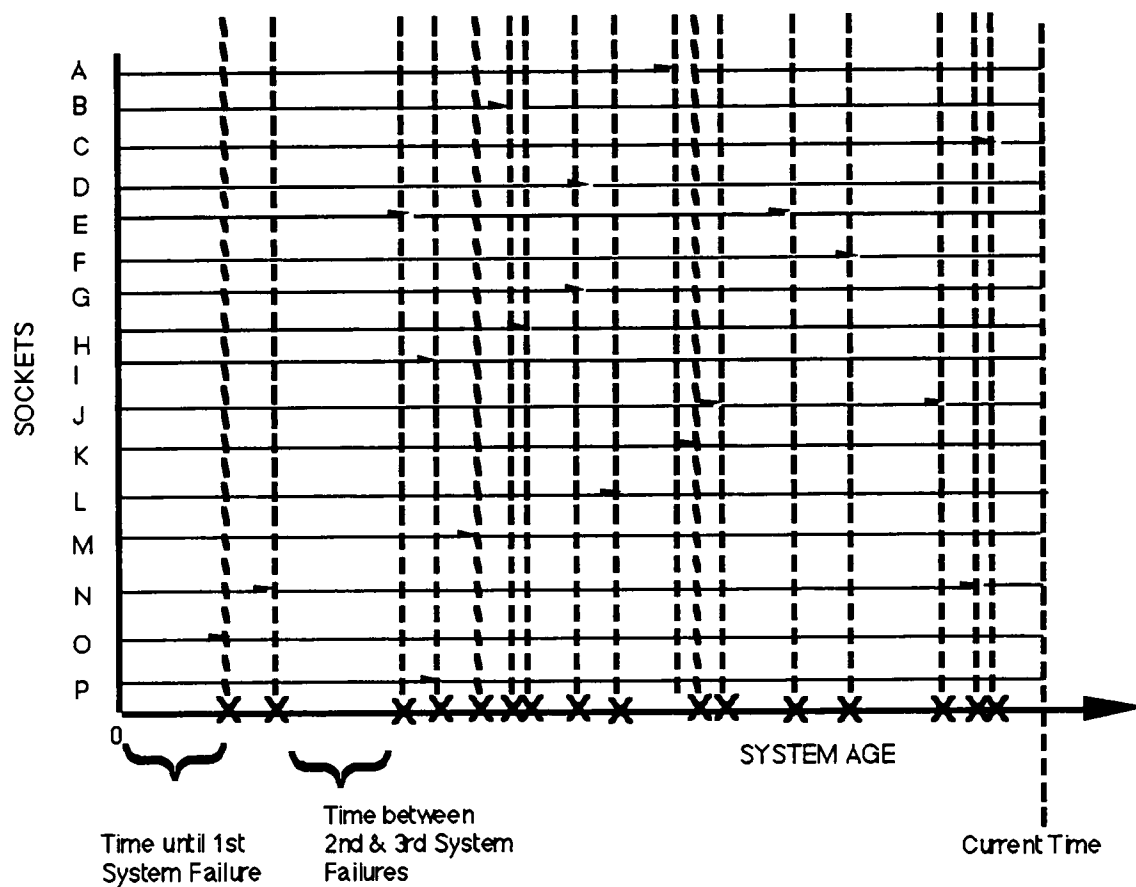


Figure 10: Superimposed Renewal Processes

Drenick's Theorem, discussed by Asher (1968), develops properties of the SRP as  $t \Rightarrow \infty$  for large  $n$  where  $n$  is the number of renewal processes to be superimposed. In particular, SRPs tend towards a constant as  $t \Rightarrow \infty$ . Correction terms are available for finite  $n < 200$  (Cox 1962).

Blumenthal, Greenwood, and Herbach (BGH) discuss properties of SRP when  $n$  and  $t$  are  $\ll \infty$  in three papers (1971, 1973, 1976). They demonstrate that while asymptotically  $EL_t(w) = \lambda = \sum_{i=1}^n \lambda_i$  where  $\lambda_i$  are  $EL_{it}(w_i)$ , that because of the slow convergence of the SRP to a HPP, in most practical situations this is not true. They compare  $W_t$  for the BAO and SRP and show that if  $F(t)$  is IFR for the components that  $W_{t \text{ BAO}}$  is stochastically less than  $W_{t \text{ SRP}}$ . They discuss the fact that "locally" in time the superimposed process appears to be Poisson. Ascher (1984) believes that "locally" in time could be the entire useful life of the product. In short, there are not analytic expressions for the transient properties of superimposed renewal

processes, but models such as the NHPP can be used to approximate system behavior during the transient period.

Moltoft (1990) & Rimestad (1990) discuss the methods developed by themselves, Rimestad, Jensen, and Klaergaard. They use SRP philosophy, but their methodology allows one to work in both directions; from component level to system level and from system level to component level. The former is useful for reliability prediction and the latter for examination of field data.

They also assume that a failure to the system is due to a failure of a component in a socket, and that subsequent repair is due to placement of a new component with an identical lifetime distribution as the failed component in the socket. These assumptions allow one to treat each socket position as a renewal process, so that in each socket, the mean cumulative number of failures as a function of operating time is determined by  $m(t) = t / \mu + d + \varepsilon(t)$ . Here,  $\mu$  is the mean value of the TBF of the component including its interconnections in the socket;  $d$  is a constant:

$d = \frac{\sigma^2 - \mu^2}{2\mu^2}$  , and  $\varepsilon(t)$  is a transient term that approaches 0 as  $t$  increases. When this transient term is zero, a plot of  $m(t)$  versus time is a straight line with slope  $1/\mu$ , and  $d$  as the intercept of  $m(t)$ . The time period in which  $m(t)$  follows the asymptotic function,  $m_a(t) = t/\mu + d$ , is the steady state period. The failure intensity is the derivative with respect to  $t$  of  $m(t)$ . The asymptotic value of the failure rate,  $I = 1/\mu$ , is approached as  $t$  increases no matter what the distributional form of the TBFs for the socket. It is clearly not correct to quote a constant failure rate for a socket until after the transient period.

This method is applied to the system level by again assuming a series system so that any component failure results in system failure and that simultaneous component failures can not occur. Then the pattern of system failures may be modeled as a SRP. In this case,  $M_{\text{system}}(t) = M_s(t) = \sum_{i=1}^k m_i(t)$  where  $k$  is the number of sockets in the system.

This theory can be used in system reliability prediction if there is information on the component reliability. It can be used to study field reliability by producing a  $M_s(t)$  graph and component  $m(t)$  graphs as necessary to explain system reliability. To predict system reliability before field introduction, component  $m(t)$  graphs can be summed.

Moltoft observes that three parameters are needed to characterize component populations:  $p$  - the percentage of flawed components,  $I$  - the steady state intensity, and  $T$  - the failure free period.

### **Mixture & SRP Models**

Hanson & Thyregod (1990) begin by explaining SRP philosophy for repairable systems in the same fashion and with the same terminology as Moltoft & Rimestad, but quickly observe that the steady state for  $M_s(t)$  corresponds to the situation where all components in all sockets have been replaced several times and note that this situation is unlikely in real-world applications. (We note

that this is BGH's observation.) They argue that the seemingly transient behavior of repairable systems is likely due to the infant mortality of some components. Thereafter they concern themselves with the behavior of systems where some percentage of the components are 'weak' perhaps due to manufacturing flaws.

Their method assumes a system composed of  $n$  equivalent sockets where each socket is filled with a component from either a 'standard' or 'weak' population. The argument is that this is not a restrictive assumption since a system may be evaluated by considering multiple subsystems in this fashion. Components from the two populations are assumed to have exponentially distributed lifetimes with parameters  $\lambda_0$  and  $\lambda_1$  respectively. The mixed lifetime distribution pdf is then  $f(t) = (1-\pi)\lambda_0\exp(-\lambda_0 t) + (\pi)\lambda_1\exp(-\lambda_1 t)$  where  $\pi$  is the probability that a randomly selected component is 'weak'. They then develop methods for modeling repairable system reliability assuming this heterogeneity in the component population.

This discussion and methods are expanded by Hanson, Thyregod, & Arjas (1991). The authors, specifically note the work of Blumenthal, Greenwood, and Herbach (1973), and observe that simple analytic expressions for the transient properties of SRP are not available. Then SRP models are difficult to use for system reliability until asymptotic steady state behavior is attained. The authors note that the NHPP may be used to model the improvement of repairable system reliability early in system life, but that this improvement is rather mysterious. The authors explain this improvement by commenting that component quality populations are heterogeneous. The weaker population of components tend to fail early in system life and since that population is less numerous than the standard population, weak components tend to be replaced by standard components, thereby leading to a stronger system.

Hanson, Thyregod, & Arjas (1991) note that their methodology considers only the case where part quality is directly identified with its failure propensity and in which there is no aging. This

assumption allows them to use exponential distributions to model the part heterogeneity. Then the lifetimes of the components in a socket are a mixture of exponential distributions and so then the common force of mortality in the socket is decreasing (following Proschan). The authors then show that the counting process in such a socket must have a decreasing ROCOF and so the counting process itself must be concave (this depends on the assumption that a new part is in the socket at time  $t = 0$ .) The authors note that the SRP ROCOF is the sum of the socket ROCOF and must therefore be decreasing.

Due to the difficulty in estimating the large number of parameters that result from a mixture of more than two exponential distributions, the authors recommend a sparse parameterization (a mixture of two exponential distributions). Let  $\lambda_0$  and  $\lambda_1$  be the exponential parameter for the 'standard' and 'weak' populations respectively with  $\lambda_0 < \lambda_1$ . Then the life distribution is:  $F(t) = (\pi)(1 - \exp(-\lambda_0 t)) + (1 - \pi)(1 - \exp(-\lambda_1 t))$   $t > 0$  where  $\pi$  is the

probability that a randomly selected component is 'standard'. The force of mortality is then

$$h(t) = \frac{\pi\lambda_0 \exp(-\lambda_0 t) + (1-\pi)\lambda_1 \exp(-\lambda_1 t)}{\pi \exp(-\lambda_0 t) + (1-\pi) \exp(-\lambda_1 t)} \quad t \geq 0 \quad \text{which is decreasing.}$$

The renewal rate for a socket is then

$$v(t) = C_0 + (C_1 - C_0) \exp(-C_2 t) \quad t \geq 0 \quad \text{where} \quad C_0 = \mu^{-1} = \left( \frac{\pi}{\lambda_0} + \frac{1-\pi}{\lambda_1} \right)^{-1},$$

$$C_1 = \pi\lambda_0 + (1-\pi)\lambda_1, \text{ and } C_2 = \pi\lambda_1 + (1-\pi)\lambda_0.$$

The authors assume  $\lambda_0 < \lambda_1$  so that  $C_1 > C_0$  and  $v(0) = C_1$  and  $v(\text{infinity}) = C_0$  with  $v(t) - v(\text{infinity})$  decreasing exponentially fast to 0 as  $t$  approaches infinity. If a system consists of  $n$  sockets each with the same part-life length distributions then the ROCOF for the SRP is  $v_{\text{SRP}}(t) = n(C_0 + (C_1 - C_0) \exp(-C_2 t)) \quad t \geq 0$  with expected

$$\text{number of failures } M_{\text{SRP}}(t) = n \left( C_0 t + \frac{(C_1 - C_0)}{C_2} (1 - \exp(-C_2 t)) \right) \quad t \geq 0$$

Estimation of  $(C_0, C_1, C_2)$  estimates the limiting ROCOF for a socket, the initial ROCOF, and the the speed of reliability improvement. The estimation is based on the maximum likelihood principle.

If only aggregated data is available from the SRP then the log likelihood is not tractable. In this case, the authors recommend approximating the SRP with a NHPP. If the number of failures is less than the number of sockets then an assumption that each socket has experienced at most one failure would allow the use of the likelihood above with  $N_i(t)$  being either 0 or 1. The authors note that this model will not allow system deterioration.

Moltoft (1994) categorizes field failures of components into three groups. Those failures due to inadequate design, those failures due to inherited flaws, and those failures due to induced flaws. Inherited flaws are those due to the (supplier) manufacturing processes of the piece parts composing the components. Induced flaws are those caused by the manufacturing and handling process of the system manufacturer. Improper ESD procedures, bending of wires, high solder temperatures, etc. are examples. Design flaws are those due to the inability of a design to perform certain tasks. Moltoft believes that the majority of flaws are induced and that this

type of flaw tends to manifest itself early in system life. Inherent flaws are the next largest category and the author believes that they tend to show themselves after a year or so of operation. Design flaws, in the authors opinion, are infrequent causes of failure. The author finishes by proposing an approach similar to that of Hanson, Thyregod, & Arjas.

### **Nonparametric Models**

Nelson (1988) presents a Nonparametric Method for analyzing system repair data. The method provides an estimate of the Mean Cumulative Cost (Repair) Function MCCF (MCRF) and plots those means against system age. The resulting graph is helpful in evaluating whether fleet costs or repairs increase or decrease as a function of system age. The method can also be used to begin to compare different samples and predict future the number of future system repairs.

Nelson's MCRF or MCCF is  $\sum_t \frac{\text{Number of Events}(t)}{\text{Number of Systems}(t)}$  where  $t$  is

system age. System age may be any appropriate unit of system

usage. This estimate may include all types of system repairs or particular types of system repairs. We observe that this is the same estimator Moltoft and Rimestad use for their SRP model.

This estimate is unbiased for either a finite or infinite fleet and under right or left censoring. The proof depends on an assumption that each of the  $N$  systems in the fleet has a cumulative repair history through the greatest censoring age, that the  $N$  censoring ages are randomly assigned to the  $N$  systems, and that the  $N!$  possible random assignments has the same  $1/N!$  probability. It is assumed that the true mean exists and is finite.

It is worth noting that not assumed is that the systems have statistically independent histories, that systems or the fleet have independent increments, or that components, repairs, or failure modes are independent. Clearly a specific parametric curve is not assumed. Nor even that the same stochastic process generated each system history.

Nelson observes that it is useful to imagine a smooth curve through the staircase function generated by his estimate (the true MCRF would be a smooth curve). If a continuous curve is imagined then a derivative can also be imagined. The method does not assume such curves exist.

Nelson (1995) develops the variance of the MCRF [Var(MCRF)]. The assumptions to develop Var(MCRF) are the same as those for the estimate of the MCRF except that an infinite population is assumed, the sampling distribution of  $\hat{MCRF}(t)$  is approximated by a normal, and the population variances and covariances for the terms used in calculating  $\hat{MCRF}(t)$  are assumed to exist. If there is no censoring of the systems then the estimate of MCRF at age  $t$  is approximately normal by the Central Limit Theorem with the true mean of MCRF( $t$ ) and a true variance of  $\sigma^2/(\text{the number of systems in the sample})$ . Hence the usual estimate of the variance,  $s^2$ , may be used to calculate 2-sided confidence intervals for MCRF( $t$ ):  $\bar{Y} \pm K_c \left( \frac{s_t^2}{N} \right)^{1/2}$

where  $\bar{Y}$  is  $\hat{MCRF}(t)$ ,  $s_t^2$  is the usual unbiased sample estimate of  $(Y_1, \dots, Y_N)$ , and  $K_c$  is a standard normal percentile. The theory is modified for censored data by using the usual formula for the variance of a sum:  $\sum_t \hat{MCRF}(t)$ . The algorithm is somewhat complicated by the population autocorrelation between incremental failures in intervals  $i$  and  $j$  where  $i \neq j$ . The usefulness of the method is restricted by the infinite population assumption. This assumption is relaxed in Robinson (1995).

Robinson modifies the Nelson estimate of the  $\text{Var}(\text{MCRF})$  for finite

populations: 
$$\text{Var}(\hat{\text{MCRF}}) = \sum_{j=1}^m \left(1 - \frac{n_j}{N}\right) \left(\frac{s_{jj}}{n_j}\right) + 2 \sum_{j=1}^{m-1} \frac{\left(1 - \frac{n_j}{N}\right)}{n_j} \sum_{k=j+1}^m s_{jk} \text{ where } N$$

is the number of systems in the population,  $n$  is the number of systems in the sample,  $j$  and  $k$  index the number of observation times,  $m$  is the total number of observation times,  $1 \leq j \leq k \leq m$ , and a matrix exists with a row for each system and a column for each time the process is observed with each element the number of

failures for a particular system in a time period. Nelson's formula is identical with  $N = \infty$ . This formula is a direct application of sampling theory and so Robinson credits W. G. Cochran.

### **Articles Discussing Multiple Point Process Models**

Ansell and Philips (1989) discuss the use of stochastic point processes to model repairable systems. They present portions of the analysis of three datasets and comment on the role of reliability analysis. The authors believe that the purpose of any reliability study is reliability assessment.

The five general classes of point process models Ansell and Philips mention are the HPP, the NHPP, the branching Poisson process, the RP, and SRP. In particular the authors spend a great deal of time discussing methods useful when a RP is the model chosen. These methods include total time on test graphs, probability plots to determine appropriate lifetime distributions, Cox's proportional hazards model, common stress failure models, and common summary statistics of the hazard function for RP.

This literature review, in general, follows Thompson's (1981) notation. Thompson's contributions are: an overview of the various point process models, definitions of various statistics used to estimate 'MTBF', properties of those statistics under assumptions of the various models, and a discussion concerning the point process models' usefulness and the role of the statistician in reliability investigations. (Statisticians) 'need to delve into the scientific and engineering origins in order to discover the real statistical problems. Reliability is not just actuarial science recast in an engineering setting; in some cases force of mortality may not even be to the point.....The various models examined in this paper will sometimes prove useful..'.

### **Criticism of Current Practices**

Several authors have criticized current practice with respect to the use of statistical models to quantify system reliability.

O'Conner (1993) states that "quality and reliability engineering is concerned with the prevention of failures, not primarily with predicting or counting them." He discusses what he believes to be a fallacy, the mathematical relationship of time to failure rate. He does believe that mathematical methods such as regression, proportional hazards, and so forth can be useful for providing clues to engineers trying to solve reliability problems or to forecast overhaul needs; but he absolutely does not believe in stochastic modeling for system reliability. "The authors ... make all necessary simplifying assumptions,...It is very hard to find practicing engineers and managers who make use of this material, yet the flow is endless. It seems to be a manifestation of the 'Gold Effect', by which a specialist discipline continues down paths laid by its devotees, and does not tolerate dissent or new ideas. Similar habits have lead to the disenchantment widely held in relation to operations research and the social sciences. Engineers and statisticians working to support them should know better."

O'Conner recommends systematic use of pragmatic methods such as statistical process control (SPC), the design of experiments (DOE), and the Deming approach. He recommends "...Eliminate all methods that distract from the pursuit of excellence. Statisticians working in the Quality and Reliability field should review their work against this criterion."

Met (1992) discusses Duane's original intent which was to measure and predict reliability through the development stage of complex equipment. The deterministic curve described the rate at which learning would occur. The learning would be accomplished through a "test, analyze, and fix" (TAAF) program. Met questions the usefulness of the probabilistic models that appear to have grown from Duane's postulate, especially given that reliability growth may be due to intervention through a TAAF program or similiar methodology.

In a discussion of Ansell and Philips, Newton (1989) points out that the role of the reliability engineer should be more than that of

assessing current reliability. In the author's view any statistical analysis should be to help eliminate causes of failure. Bendell (1989) discussing the same paper makes similar comments. In his view, the engineer wants glimpses of what may be causality whereas the statisticians like estimation and prediction. Bendell believes that identification of failure modes is likely more important than quantitative assessment. Sherwin (1989) comments; 'Reliability is an engineering problem in which statistics are helpful - not a statistical problem per se'.

## **CHAPTER 3**

### **ORGANIZATION, QUESTIONS, & AVAILABLE DATA**

#### **Company and Product Description**

The manufacturer is a worldwide leader in the provision of products and services for the medical imaging market. The manufacturer's technology is marketed and serviced worldwide by an international corporation.

The industry and technology are dynamic, and the products are characterized by frequent design changes and application innovations.

The industry is extremely competitive and therefore the organization is not identified for proprietary reasons. However despite the confidentiality required by the situation, the data are not transformed. We are grateful to the organization for the use of the data and for the opportunity to learn.

## **Quality and Reliability History**

The organization is structured in typical fashion with Research and Development (R&D) and Manufacturing Divisions. The Quality Control (QC) function (reporting to the Quality and Regulatory Leader), is responsible for incoming and in-process inspection. Organizational focus has been on furthering technology rather than optimizing manufacturing quality or field reliability. The QC department is utilized as a filter to remove excessive product variations from production before final assembly.

## **Motivation for Reliability Study**

Historically, these particular medical imaging devices have been a research tool primarily used to obtain disease process information. This organization dominated the small market with rapidly introduced technology advances. Today, as the number clinical applications grows and the cost of the technology declines, these devices are becoming viable clinical tools. One result is a new market segment, another is increasing competition in the marketplace.

The 'clinical application' customer appears to have differing expectations from the 'research' customer. The initial 'research' customers were more interested in the technology and tended to accept the equipment reliability as a given. In fact, it was not (and for that segment of the market is not) unusual for the customer to replace defective parts or modify software without assistance from either the service organization or the manufacturer. In contrast, while the expanding number of 'clinical' customers value superior and rapidly introduced technology, they also expect ever increasing quality and reliability. Anecdotal evidence suggests that this customer segment's reliability expectations are due to their experiences with other complex medical devices produced by various manufacturers some of which are entering this market.

This organization is still the world leader in technological performance, but the reliability of the product may need to be increased. Unfortunately reliability has not been systematically examined. Data are required for ongoing assessment of field

reliability, but primarily for analytic and systemic improvement. This is true for past, current, and future generations of units. Of course reliability information will be useful for other business, marketing, manufacturing, and engineering needs. And since the devices are labeled as clinical diagnostic tools the manufacturer is obligated to comply with Food and Drug Administration (FDA) requirements. The intent of the FDA requirements is that the manufacturer understand product reliability. This includes the ability to detect any quality problems or deterioration of reliability that might occur in the field.

### **Constraints**

The organization is severely resource constrained. This impacts the ability of the organization to implement additional data collection systems. Further, there will not be statistical technical resources available to 'interpret' data. In addition to their current activities, the existing Quality and Regulatory personnel will be the initial handlers and interpreters of reliability data.

## **Questions and Requirements**

The organizational leadership determined that current quality and reliability issues are of concern. So in the future, along with the current emphasis on technological advancement, designs will have increased reliability as a goal.

So, system reliability must be continually characterized in a manner that is able to detect changes in system reliability, but more importantly provides clues as to the causal factors concerning failure. Additional requirements are that the methodology for characterization must recognize business constraints. In practical terms, it must effectively utilize available data on an existing PC spreadsheet. The methodology should not be labor intensive. Tasks such as monthly updating should be quick and easy. Results should be communicated in a medium that engineers and managers will readily understand. Graphical tools as they are available are an example. The only need for predicative methods is with respect to the logistical task of planning safety stock for spare parts. Analysis should specifically address various generations of units.

## **Available Data**

We have data available from two sources. We have one year of field data for the subset of units installed in the United States (US) and returned parts data.

From July 1993 through June 1994, we had access to United States Data System (USDS) data. USDS was the database through which the International Corporation tracked service calls for their products. Our US units were tracked the last year that this database was active. In general, parts are removed the day of the service call. A record in this database contains:

MODEL NUMBER (identification of the generation of unit),

SITE NAME (identification of the individual unit),

PART NUMBER USED (the part identification of the part replaced),

ARR DATE (date of service call),

PART NAME (Verbal description of part replaced).

A return part record includes the following fields:

P/N (Part number identification of returned part)

DESCRIPTION (English identification of returned part)

SERIAL # (Serial # for returned part when available and when used)

DISP DATE (Date part returned to manufacturer)

SITE INFO (Site identification)

COMPLAINT (English description of part problem, when available)

FINDING (English description of part malfunction findings, when available)

COMMENTS (Any additional information, when available)

FMC1 (Failure codes, when available).

There are some difficulties with the databases. The USDS database covers only those units in the US, whereas the system is sold worldwide. This database does not include the newest systems. Further, in most cases the data is truncated on both right and left, in the best case on the right.

The return part database includes only parts valued at greater than \$100.00. Parts of lesser dollar value are capable of causing system downtime. Return data is available for fiscal years 1992, 1993 and 1994. Before fiscal 1992, parts were repaired or scrapped but records were not maintained. (Fiscal years begin with October and end with September.) In general, in neither database are actual part lifetimes traceable.

The return part database evolved over this three year period and records are incomplete. Failure codes are not available for all data. Further, European sites return failed parts to a central warehouse location. Here the parts are stockpiled and returned to the manufacturer in batches. This process does not retain individual part failure data or site information. Rest of World (ROW) data, data from sites located outside the US and Europe, are similarly incomplete. Even in the US, there is always lag time between the time a part fails and the time it is received and processed at the manufacturer. This lag time ranges from several days to over a year in an extreme case. A sample average for US sites was a month

(although some parts never show up). The lag time situation for Europe and ROW is worse than the domestic situation. European and ROW spares inventory reduction efforts have resulted in good, never used spares being returned in the same batch as failed parts without sufficient labeling to differentiate the two classes. As a general statement the tracability of parts returned from US sites is somewhat better than that for Europe and ROW. An estimate of total returns for any particular part type is impossible.

## **CHAPTER 4**

# **COMMENT ON THE SITUATIONAL USEFULNESS OF THE REPAIRABLE SYSTEM MODELS & PROPOSED DATA ANALYSIS**

### **Role of the Statistician & Statistical Methods**

We first comment on the role of the statistician in industry. The statistician does not produce product and often is not intimately familiar with the processes or technology. The statistician assists managers, engineers, and operators in the acquisition and development of process and product knowledge through the selection, modification, and teaching of appropriate statistical and process improvement methodologies.

For instance when properly applied, Statistical Process Control (SPC) has been useful in industry. SPC focuses attention on the process, provides information on the process average and variation,

and most importantly encourages the development of knowledge of sources of variation and their effects on the average and variation of the process output characteristics. The statistical techniques are easily accessible to managers and engineers and does not hinder this development of process knowledge.

Statistical design of experiment techniques have been available for 75 years but with some exceptions were not widely accepted in industry until Taguchi popularized them. Arguably it may be said that while Taguchi did not invent either the statistical theory or methodology, he did speak to engineers with language and methods that did not overly complicate the supporting mathematics and in a format that encouraged process and product knowledge development.

There are some vocabulary and mathematical techniques for a practitioner to learn with both Taguchi and SPC methods, but they are not overwhelming. And the focus of the methodology is the practitioner's area of concern whether it be scientific or empirical

knowledge of the product or process for engineering and management purposes.

The intent of statistical process control has been described by Sanders, Leitnaker, and Sanders (1995) as identifying assignable causes, often by changing sampling and measurement strategy, learning the root causes of those assignable events, and then removing or permanently nullifying the effects of those causes. In effect, developing empirical understanding of the causal structure of products and processes. The acquisition and analysis of reliability data should also support this task with appropriate methodologies.

### **The Probabilistic Models**

We learned in the literature review that the NHPP model is usually adequate for probabilistic modeling of repairable system failures. Still, Patrick O'Conner believes that the physics of most electronic components are such that there are no intrinsic failure mechanisms effective within normal operation. Rather the observed hazard rate

(or ROCOF) is a historical comment on the results of many physical, chemical, and human processes.

The modeling of the manifestation of historical events is inadequate for our needs. Describing the past, no matter how well, does not generally help with altering the future. We want to alter the future through the prevention of future failures. This implies that we have interest in the causal factors impacting system reliability.

Authors such as Hanson, Thyregod, Arjas, and Moltoft have observed that most populations of electronic components contain a small percentage of defective or inferior parts due to variations in those upstream processes, but that the majority of parts are not inferior or defective. Moltoft's categorization of field failures of components into those failures due to inadequate design, those failures due to inherited flaws, and those failures due to induced flaws is particularly insightful. But again, the literature on repairable systems focuses on the historical pattern of events as a function of time alone and not on those physical, chemical, and

human processes responsible for the variation that results in component and therefore system failure.

Further, the assumptions associated with the majority of the probabilistic models are untenable. For instance merely from an intuitive point of view given the typical numerous process and engineering changes that occur over a product's lifetime, independent and/or stationary increments between failures are unlikely. (The fact that Nelson's nonparametric estimate does not require multiple assumptions is attractive).

The repairable systems literature, by and large, assumes that the complete failure pattern of interest is available. Our data is at best left and/or right truncated and more typically incomplete in an unknowable fashion.

Also, these esoteric models are usually so theoretically elegant and complex that they not useful, that is insightful, to non-statisticians. This complexity does not lend itself to the main

purpose of the reliability study, that is to enable managers and engineers to understand the field reliability of the systems and most importantly to provide clues as to the causal structure of field failures.

### **Difficulties Formulating a Definition of Failure**

Regardless of the tact taken to model, assess, predict, or improve reliability, a definition of failure is required. Of course, almost any pronouncement of failure for most repairable systems is arguable. This is true in our particular case. Seldom does one of these units abruptly cease to function due to say, hardware failure. More commonly, efficacy slowly decreases as one or more components drift outside of specification or fail.

Even if it is agreed that a failure has indeed occurred, rapid categorization of a failure as due to hardware or software failure or operator (in)competence is difficult. Unhappily, field failures just cannot always be placed in a category. An apparent hardware failure will be a software failure. Similarly, apparent software failures

are potentially user error, due to misapplication or user modification, or frequently are truly hardware failures. True software malfunctions typically only concern one area of application - the unit will function perfectly for other tasks.

In our search for a definition of failure an examination of software failures is worthwhile. Historically, a customer determination of software malfunction was handled in one of several ways. For those software 'failures' that were an euphemism for 'it won't do what I want it to do' the code was frequently modified by the (research) customer themselves. Often, the customer would directly call a programmer for assistance in modifying or 'improving' the code. Assistance was generally affably provided. This type of complaint might be categorized as misapplication. Enough people interested in a particular application or modification of the software eventually resulted in a new software release. Other software failures were handled by rebooting the system sometimes after hardware replacement.

We have chosen not to specifically include software failures in our definition of system failure. Because of their deterministic nature, true software failures do not lend themselves to statistical or probabilistic methods. The time between software failures in the field is not a rate that characterizes software improvement or deterioration, but is more a statement of the frequency of attempts at certain types of applications. Of course, knowledge of the number and type of errors, and rate of correction of errors per software release is useful for improvement activities. We are currently constructing a software complaint database to address this area of concern. This database will be used to examine software reliability growth before and after field release. It will be a major tool in improvement activities of current and future generations of software. Since many software complaints are due to misapplication, the nature, type, and frequency of misapplication errors will assist in the projection of future customer needs.

Operator competence is also not specifically examined here. Operator competence is a function of knowledge, experience,

equipment manuals, application, and user interface and is difficult to model even if data are available. To maximize customer value this area of reliability must be understood. Historically, operator issues were handled in a manner similar to software reliability. As with software reliability, processes and measures are being developed that will address this facet of reliability. For instance, technical advisory groups will periodically include operators and address such topics as ease of use and clarity of instruction. Customer satisfaction surveys will be used to gather further information.

### **Definition of Failure**

Because of the realistic concerns discussed above, developing an operational definition of failure is difficult, and most definitions of failure are likely to be somewhat ambiguous and easy to criticize. This is due in part because the field service engineer is not always able to categorize the failure, in part because the categories of software, operator, and hardware may interact or are not mutually exclusive, and in part due to the available data acquisition

processes. Regardless we proceed and define a failure as any system performance issue that results in component replacement(s). Clearly, as noted in the discussions of software and operator reliability, this definition is incomplete and does not eliminate all ambiguities (and subsequent discussions) with failure definition.

### **Advantages and Disadvantages of the Definition**

Happily or unhappily, anecdotal evidence suggests that a typical field service engineer's response to a customer complaint is to replace hardware. (Discussions with field service engineers indicate that they believe that the majority of system failures are due to individual component failure.) Regardless, the root cause of removal of some components will be either software or operator issues (possibly misdiagnosed by the service engineer). But if a failure code from the analysis of the removed hardware shows no defect found, this information will provide opportunity for improvement although not of the hardware. Likely those hardware failures misdiagnosed as due to software or operator will be

eventually discovered when the system continues to malfunction. At that time the faulty hardware will be replaced.

A failure code of no defect found will also provide the opportunity to determine whether other causes of removal unrelated to 'true' component reliability such as improper connections, incorrect handling, or the like are sources of variation that have a negative impact on system reliability. If so, their frequency can be reduced or eliminated by proper action.

Likewise, those replacements due to unplanned system occurrences such as power surges (due say to improper system installation), are sources of variation that can be studied and eliminated.

So, the advantages of this definition of system failure include the fact that the data is available to some degree, that it incorporates the most common reaction to system failures, and that it provides the opportunity to investigate causal factors not directly related to component reliability. There is also the high probability that the

availability and completeness of this type of data can be quickly and cheaply improved (see chapter 6).

To completely understand the disadvantages and some of the difficulties with this definition of failure, we will consider some other possible scenarios surrounding the replacement of components. We begin by observing that some of those potential sources of variation for system reliability beyond those located in the manufacturing or design arena are possible and would be difficult to identify. Consider the hypothetical situation where part A is no longer performing within specification, but has not completely failed. The cost conscious site (perhaps one without a service contract) may elect to relocate that part within the system to a location less crucial to the efficacy of the unit rather than purchase a replacement. Another site may elect to immediately replace any part or component the instant it is no longer within specification. (A discussion of specification relevancy is not included here.) Many people in the industry believe that length of experience with the technology affects this type of decision. This hypothesis states

that sites with long experience with these systems ordinarily understand that the majority of parts singularly contribute little to the efficacy of the unit, whereas a less experienced staff typically is greatly concerned at any indication that a component is no longer functioning within specification.

In a similar fashion, possible differences exist between service representatives. Various service representatives may have differing opinions on when to ignore a part's specifications, relocate a part, or replace a part. It is also possible to damage surrounding parts when replacing parts. The type of application may impact the site's or service representative's decision to replace or relocate a component.

These types of differences between sites and service representatives should be at least partially captured by an examination of the rate of component removal and the corresponding failure modes. Unfortunately, this type of investigation is no doubt difficult.

In summary, until that time when we are able to monitor unit efficacy and/or other data acquisition processes are implemented, our definition of system failure is arguably the best given the current data and the data likely to be available in the near future. On the positive side, hardware reliability is a major element of the equipment reliability, particularly for those customers primarily involved with routine tasks. And to repeat, understanding the dangers inherent in evaluating processes only 'where the light is on,' the organization is designing processes to measure, understand, and improve software reliability and operator competence. The hardware reliability information will support the future studies of software and operator quality and reliability.

### **Proposed Data Analysis**

Arguably, the most important basic concepts for an organization involved in reliability analysis to understand is the difference between system age, component age, and calendar time; the fact that the system ROCOF is likely not constant; that failure modes may change over time; and that continuous improvement of the

manufacturing and engineering processes requires continuous improvement of the reliability data acquisition and analysis processes. The data analysis we use addresses the first three of these elements and takes advantage of properties of the Nelson estimator. We supplement the analysis with nontraditional usage of Pareto Charts. The process improvement aspects are discussed in Chapter 6.

### **Nelson Mean Cumulative Functions**

As we discussed, the difficulties in fitting a statistical model to repairable systems data include the validation of accompanying assumptions associated with any mathematical model. The Nelson Mean Cumulative Functions as shown in the literature chapter are not burdened by the majority of these assumptions. An additional benefit is that the graphical display of this estimate is intuitively interpretable. We use the Nelson Estimates as described by Nelson to explore and display system reliability data. We use the same estimator in a fashion similar to that described by Moltoft and

Rimestad to explore and display component data. We later describe the methodology used to explain the estimator.

### **Pareto Chart Usage**

We are interested in causes of system failure both within and between generations of units. We investigate these differences with Pareto Charts. Within generation we are particularly curious about the pattern of causes of system failure as units age. Since our definition of system failure is component removal, we investigate which components are removed across system age. Traditional use of Pareto charts does not include a time element. We modify the usage of Pareto Charts to specifically investigate differences across time. A second layer of analysis tracks failure modes of components across system age. Pareto Charts are easily generated and interpreted. When they are used in the fashion we suggest they are an informative supplement to the Nelson Estimates.

## **Explanation of The Methodology Within an Organization\_**

The intent of the data analysis is to provide a view of system reliability to both engineers and managers. This view includes an understanding of the level of system reliability and insight to what is typically failing. This nonmathematically intense methodology directs work in the engineering, manufacturing, and management organizations.

We begin with the Nelson estimator. The intent and interpretation of the Nelson estimator are intuitively understandable to engineers and managers once explained. The discussion will not take long in an engineering organization. Here we outline the explanation we use.

It has been our experience that the 'bathtub' shaped curve for nonrepairable components is frequently a familiar concept and perhaps the only thing many engineers 'know' of reliability data analysis. We begin our discussion with this curve. Figure 11 is of the bathtub shaped curve with the familiar labeling of the 'infant mortality', 'random mortality', and 'wear out' periods. We clarify

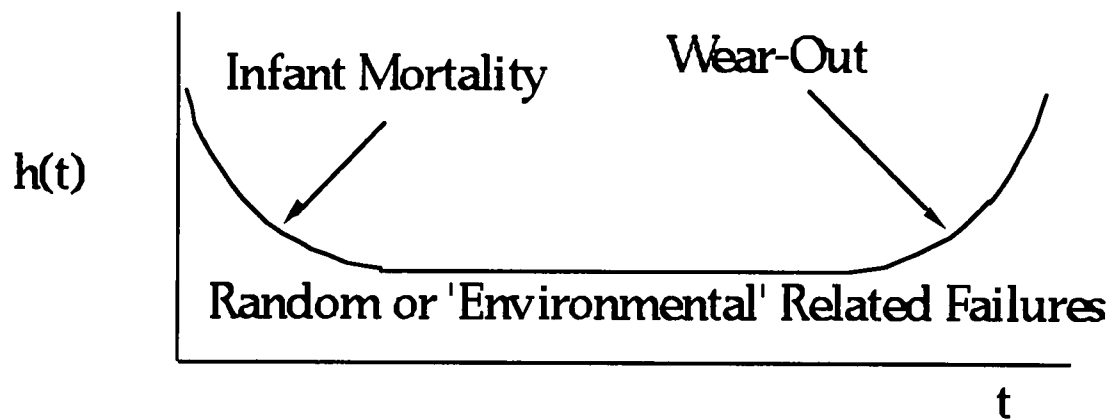


Figure 11: The Bathtub Shaped Curve

the fact that  $h(t)$  is conditional on survival to  $t$  by introducing the natural estimator of  $h(t)$  which is  $\left(\frac{1}{n(t)}\right)\left(\frac{n(t) - n(t + \Delta t)}{\Delta t}\right)$  where  $n(t)$  is the number of components unfailed at  $t$ .

Figure 12 is useful to demonstrate that component age  $t$  is different than system age. We ask the audience to imagine a system composed of a single socket within which components are replaced as they fail. A good example is a single socket lighting system on a backporch. When a lightbulb burns out it is replaced with another lightbulb. It is apparent that component age is different than

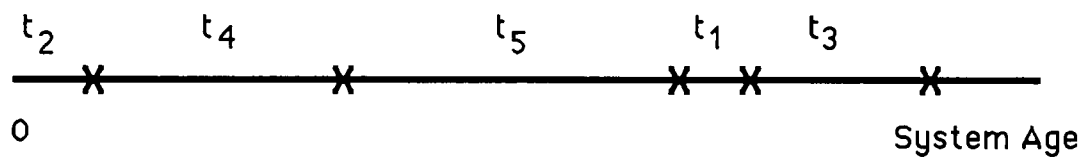


Figure 12: A Single Component System -  $t$  and System Age

system age. We might quickly show how  $h(t)$  would be calculated in this case. We have the illustrated component lifetimes labeled from shortest to longest ( $t_1$  through  $t_5$ ) to facilitate this calculation. We start to introduce the idea that information on component life does not necessarily provide understanding into the system reliability experience. Figure 13 shows a system where the sequence of times between failures is decreasing and one where the sequence of times between failures is increasing. Calculation of  $h(t)$  in these two situations would yield identical results.

We introduce a two socket system with Figure 14. This figure as shown is incomplete. The figure is completed as the conversation develops. The heavy line at the bottom of the figure is used to represent system age and reliability. We assume a series system so that a component failure necessarily results in a system failure. We complete the sequence of failures on the line representing system life with the usual 'X's. We then label the system interarrival times. It is usually clear that the usual reliability

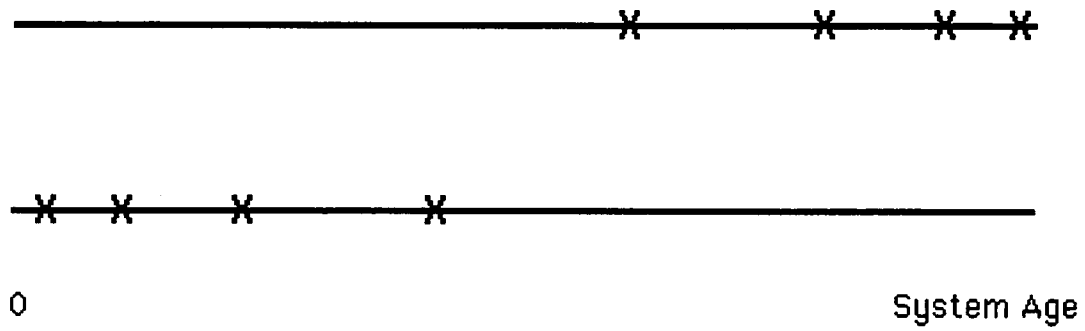


Figure 13: A Single Component System

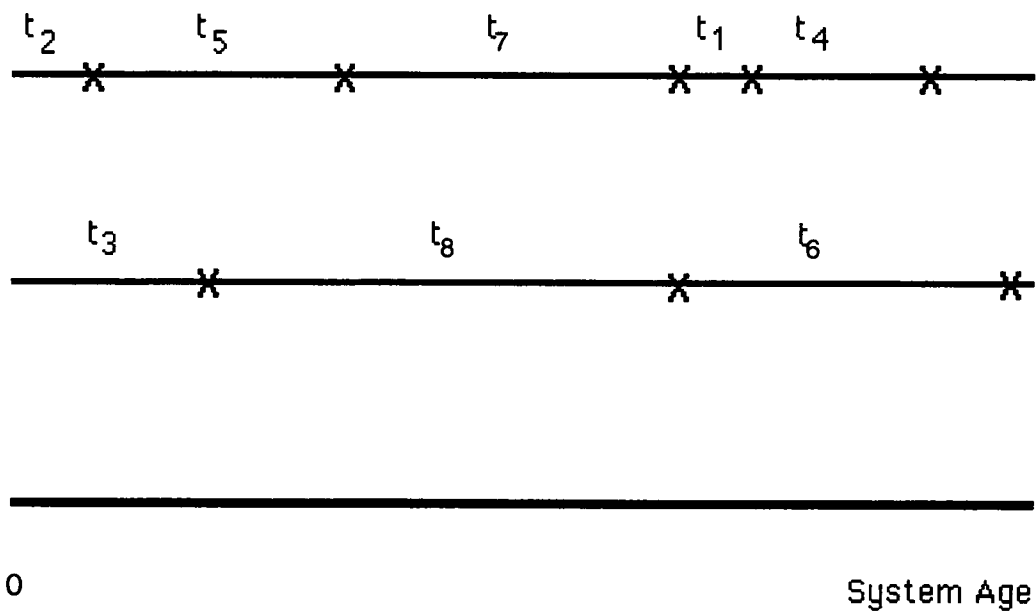


Figure 14: A Two Socket SP System

methods must be modified to deal with this situation, especially when there is no visibility of the individual component lifetimes.

Figures 15 and 16 are used to explain basic concepts for one repairable system. In Figure 15 it is easy to see that as time between system failures increases that a cumulative failures vs. system age plot flattens out (the overall curve is concave). Figure 16A shows a hypothetical smooth curve in this situation. Similarly, in Figure 16B a hypothetical convex curve is used to illustrate decreasing time between failures.

In Figure 17 the case where there are multiple repairable systems is introduced. The concept of multiple repairable systems is deliberately introduced with calendar time to illustrate the difference between calendar time and system age and therefore to show the need for maintenance of records of system age. System ages are represented by the length of the lines representing systems. As usual, 'Xs' represent failures.

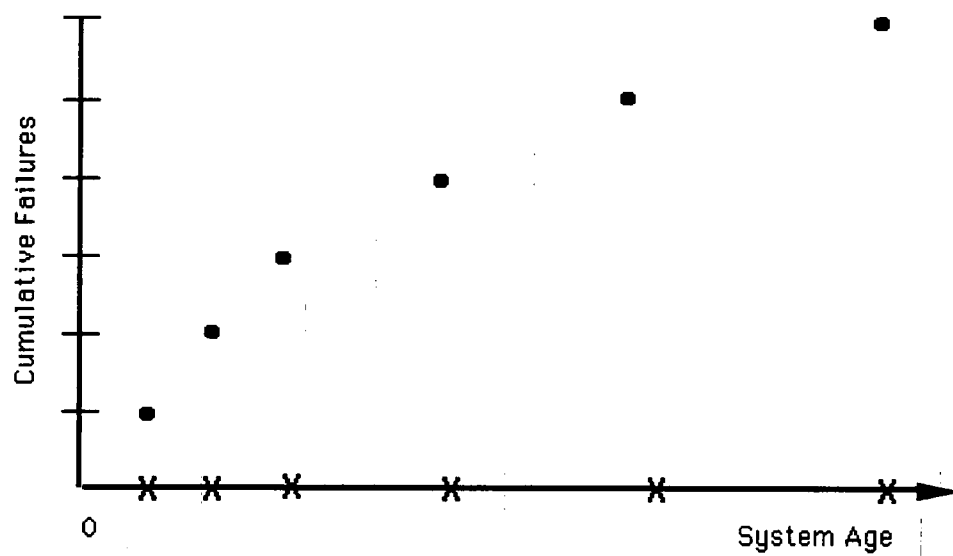
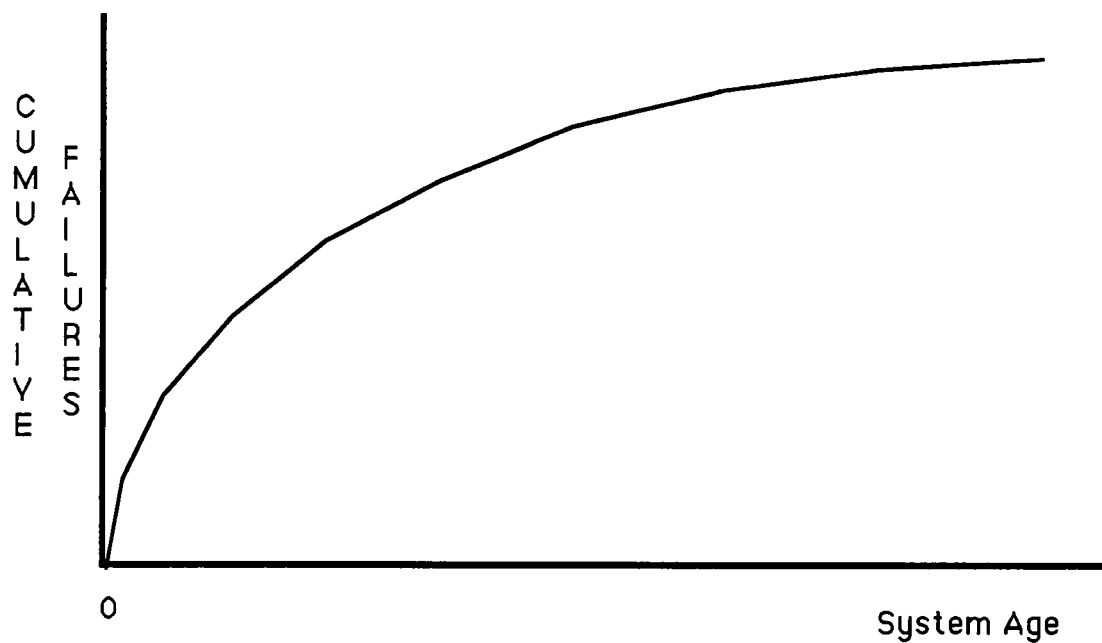


Figure 15: Cumulative Failures vs. System Age

A



B

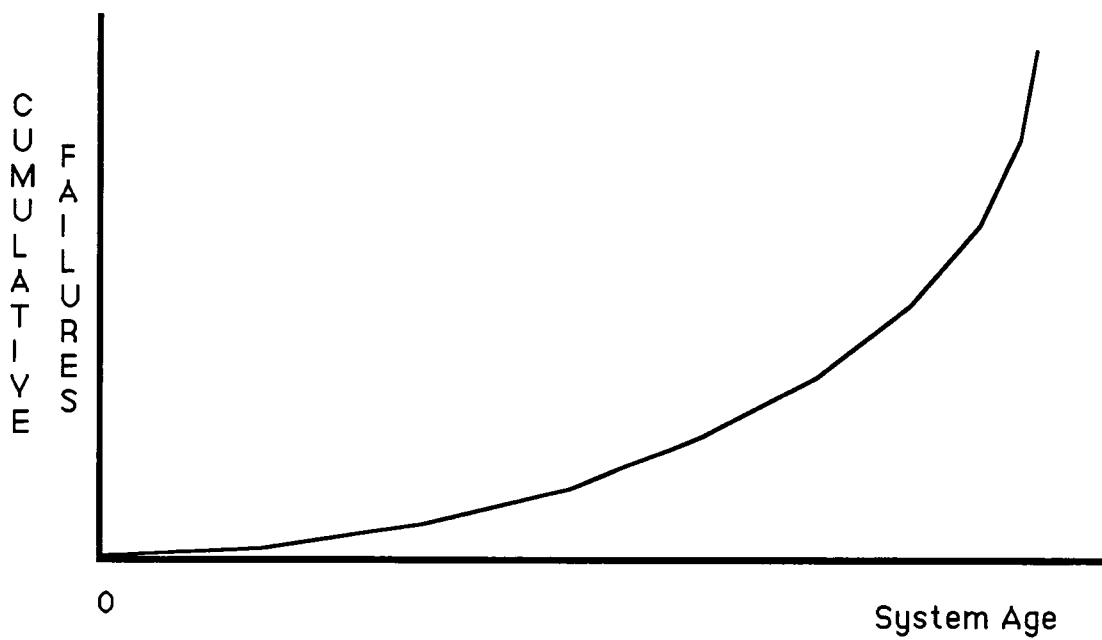


Figure 16: Hypothetical Cumulative Failure vs. System Age Curves

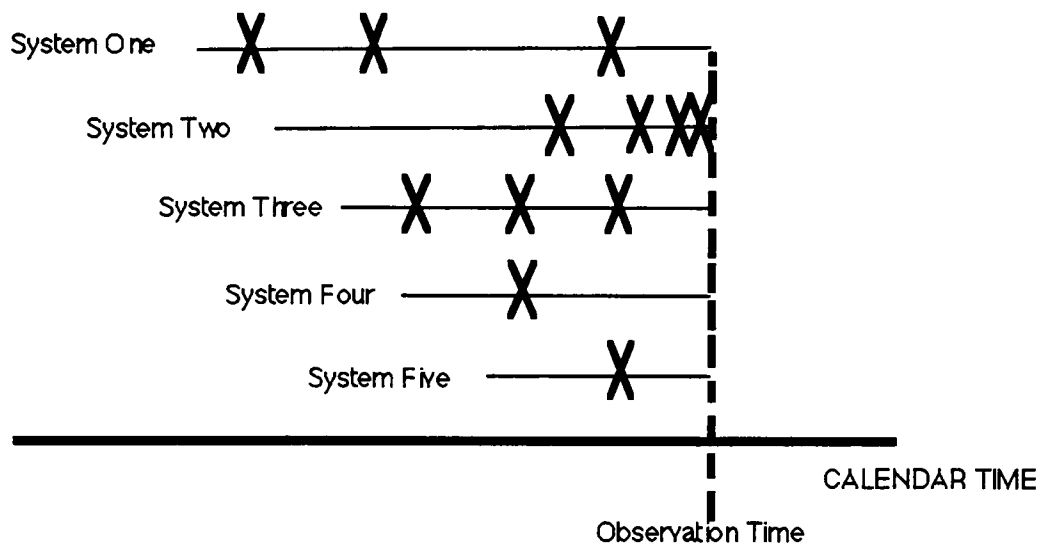


Figure 17: Multiple Repairable Systems

Figure 18 allows us to discuss the concept of average failures across system age expressed as average number of failures per unit per unit time. Of course, the nature of the data available or the needs of the organization will dictate unit time.

Then we combine the concept of an average modulated by time with that of cumulative failures using as a visual aid Figure 19. People are usually comfortable with labeling this curve and following convention we call it  $M(t)$ . We may or may not distinguish between an estimate and an expected value.

At this point in the conversation we want to introduce the Rate Of Occurrence Of Failures (ROCOF) and discuss the fact that the ROCOF may well not be constant. We supplement this claim with Figure 20. Clearly the estimate of  $u(t)$  is changing over time in this figure. We may introduce the estimator of  $u(t)$ :  $\frac{N(t, t + \Delta t)}{\Delta t}$  with  $\Delta t$  constant. We specifically mention that reporting a average 'failure rate' is almost always incorrect.

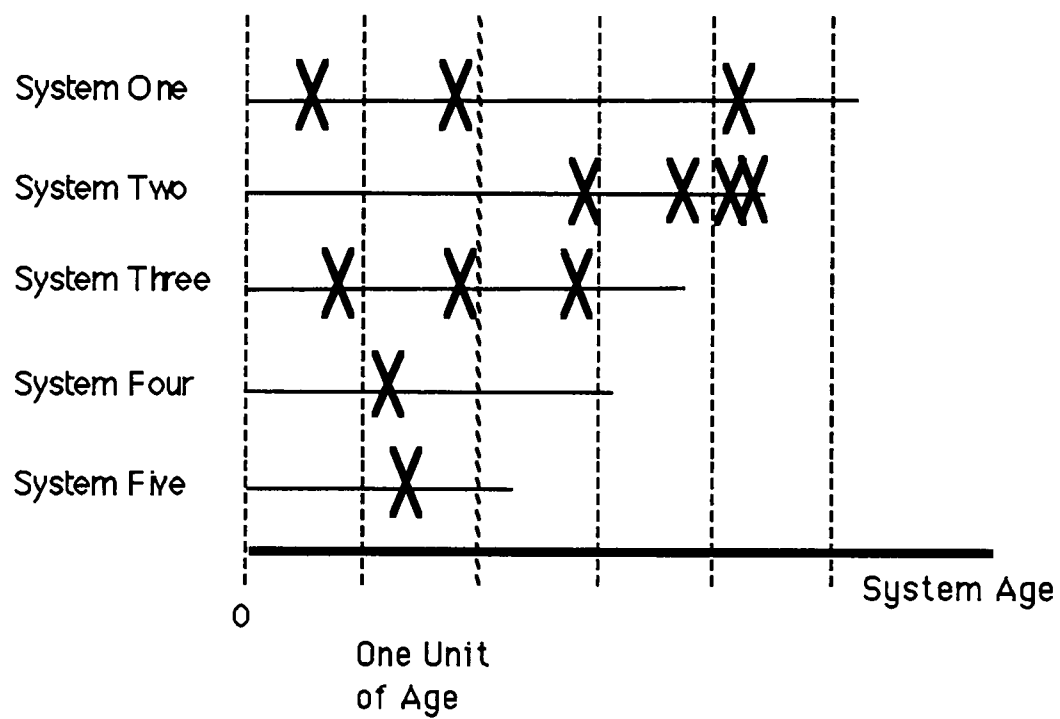


Figure 18: Multiple Repairable Systems II

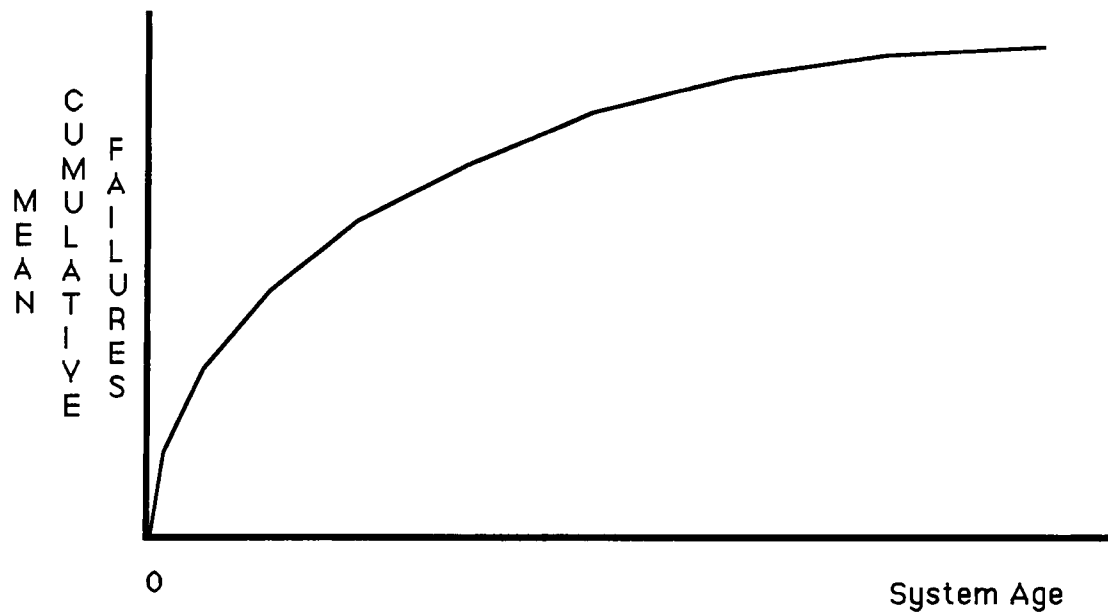


Figure 19: Mean Cumulative Failures vs. System Age

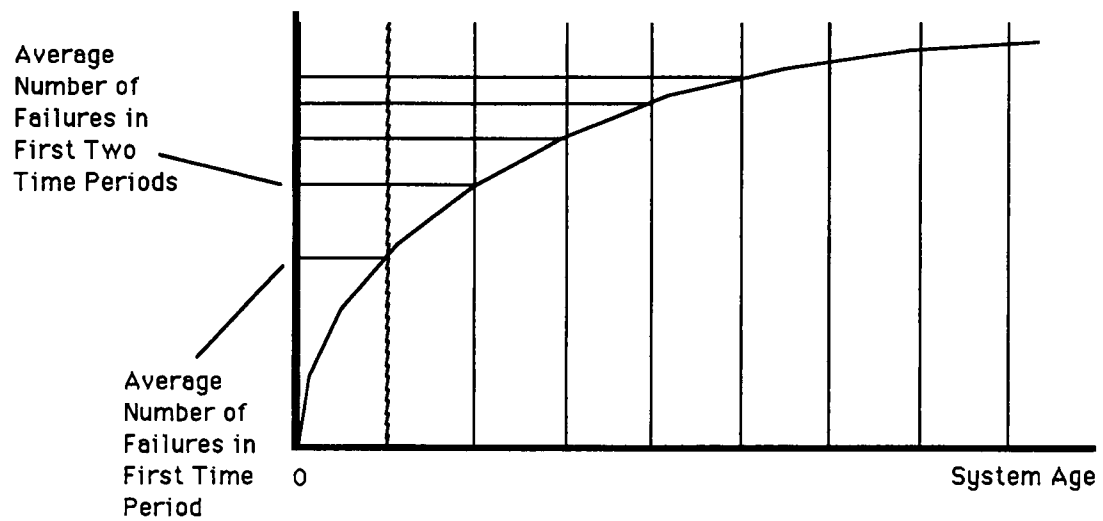


Figure 20: Rate of Occurrence of Failures

Thus, we have made the point that system age, component age, and calendar time are three different things. We illustrate that once a MCRF curve for a particular component is estimated that an easy method of forecasting spares is to merely create a column labeled 'number of systems' beside the graph and another labeled spares prediction. This column is the product of 'number of systems' in each time unit by the historical average number of components replaced during that time unit.

We then want to mention that information on failure modes is necessary to improve the reliability of systems in the field. We use an illustration of a Pareto Chart as illustrated in Figure 21. The categories are deliberately left blank and are filled in during the discussion with failure modes relevant to the audience.

We discuss the fact that these categories may well be types of components replaced. Once we have made this distinction we return to the bathtub curve for nonrepairable components. We do modify the curve as shown in Figure 22. We point out that understanding the

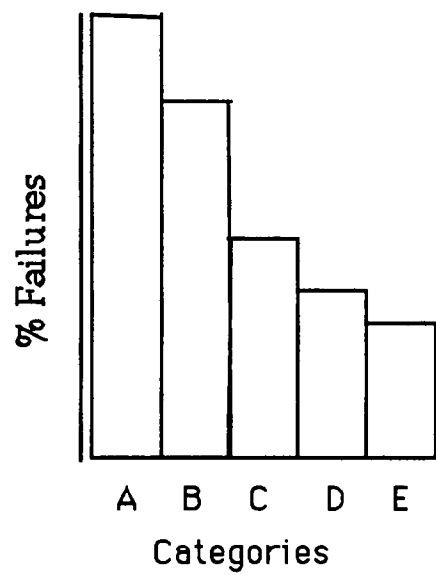


Figure 21: Example Pareto Chart

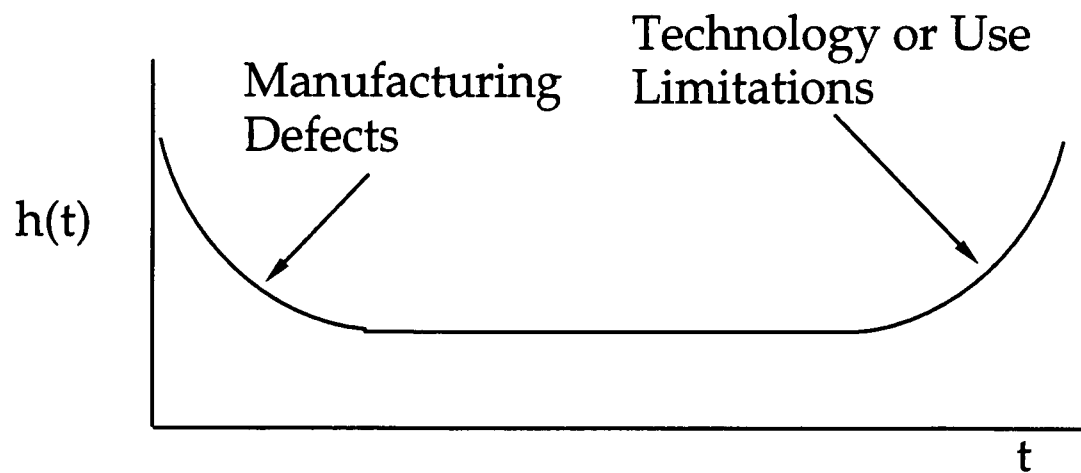


Figure 22: Another View of the Bathtub Shaped Curve

changing failure modes across component life will provide opportunity for process improvement in both manufacturing and engineering. If infant mortality is due to the manufacturing processes, then failure modes in that period of life will perhaps provide insight into the causal factors. Likewise, failure modes in the 'random' period will perhaps provide insight into either the technology or the environment in which the product has to operate. Similarly, understanding wear out failure modes will perhaps provide further insight into the underlying physics governing current technology.

We finish the discussion with Figure 23. Here we combine the ideas of changing failure modes across time with the cumulative failures vs. system age plots for repairable systems.

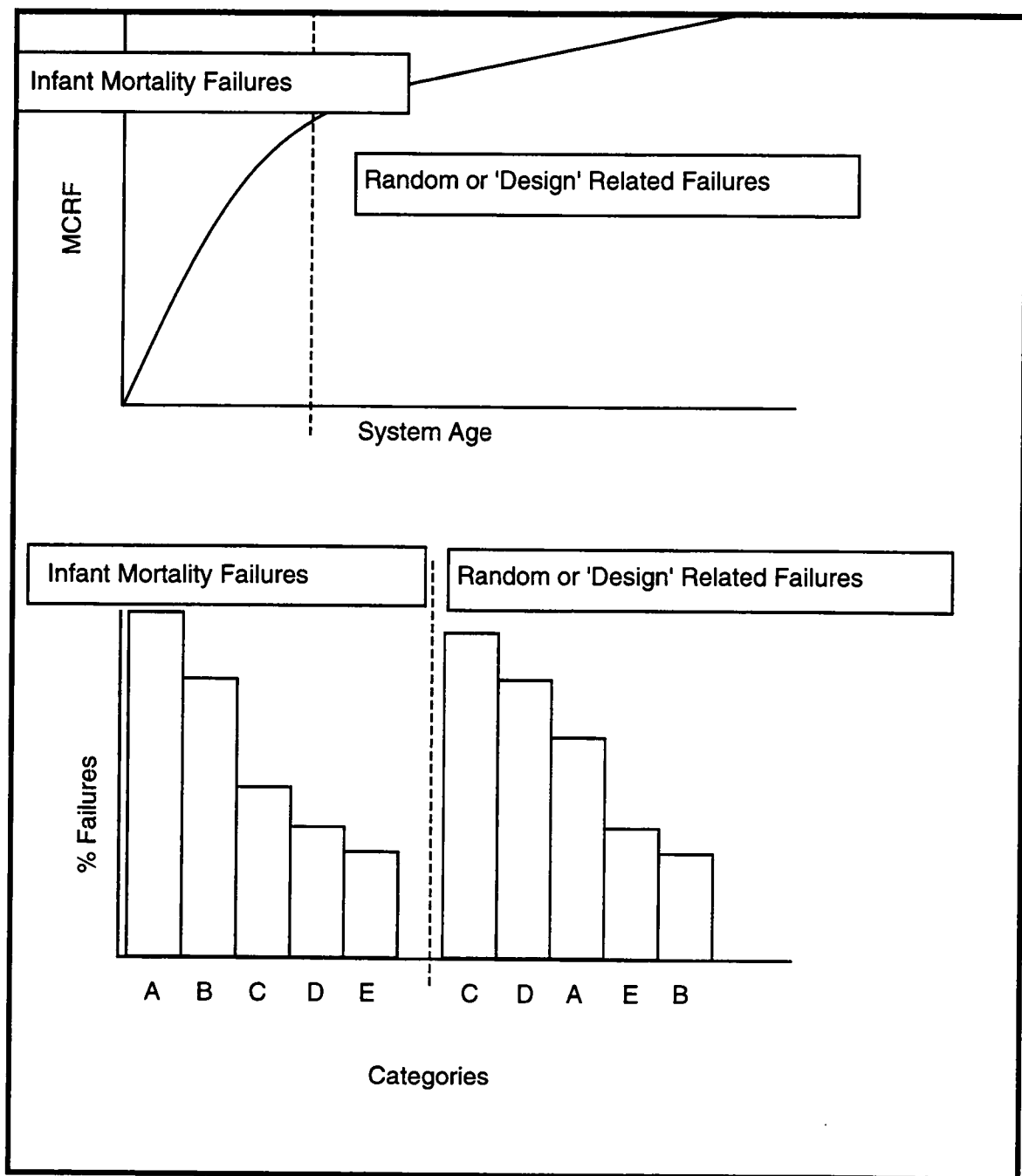


Figure 23: Changing Failure Modes Across System Life

## **CHAPTER 5**

### **HISTORICAL DATA ANALYSIS**

#### **Introduction**

The current process for obtaining and interpreting reliability data evolved out of several business needs. There was the need to provide spare parts to the international organization that serviced systems, there was the need to salvage returned components for subcomponents to be used in building spare parts, there was the need for financial information as it pertained to warrantied and nonwarrantied returns (who paid for the spare parts), there was the increasingly enforced requirement by the FDA that reliability be understood, and eventually the recognition that those spare parts might be useful in terms of understanding.

There are no formal measures on the performance of these processes. There is evidence that they are not optimized. Symptoms are alarmingly high spare parts inventory, slow turn around time on

replacement spare parts (one estimate is that 75% of the international corporation's inventory of component A's is in the return parts process), a large portion of the planning department's burdened expenses are in processing paperwork for return parts, and there is no information on field reliability beyond the anecdotal.

It was because of this last issue that we began querying the international corporation and so discovered their United States Data System database. Unfortunately, we only had access to this database in the last year of its existence. Previous data were not available.

The USDS and existing return part data are examined for information and to further understand how to best meet the requirements of the organization. The USDS data is examined first. Our definition of system failure is component removal and we are subsequently interested in failure modes across system age for components.

## **USDS DATA**

The USDS is the best data available with respect to the accuracy of the failure date and the completeness of the data. The USDS data covers one calendar year and the data is truncated on both the left and the right.

The data covers three generations of systems. We are interested in failure patterns both within and between generation of system. We are curious about the pattern of causes of system failure as systems age and differences in those patterns between systems and generations. We use Pareto Charts on the USDS data to supplement the Nelson's estimator's in order to explore these questions. The conclusions from this data analysis may be useful as a control (validation) on the analysis and conclusions drawn from the remainder of the data.

## **Nelson's MCRF**

The first step in our examination of data is to graphically display the three generations of unit; AX, BX, and CX covered by the USDS

data with The Nelson Mean Cumulative Replacement Functions (MCRF). There are 6 AX systems with data for system ages from 1395 days to 2409 days, 15 BX systems with data for system ages from 314 days to 1429 days, and there are 6 CX systems covering system ages from 0 to 825 days. The MCRF reports the average cumulative number of replacements per day (as a function of system age) for systems in the USDS sample. We then use the same estimator to display the average cumulative number of replacements per day for component types with the greatest propensity to cause system failure. The USDS MCRF by generation are displayed in Figure 24.

Visual inspection of the MCRFs does not suggest any abrupt changes in ROCOF. Statistical tests for trend are reported later, but in order to get a feel for the frequency of replacements, straight lines are fitted to the MCRFs. The resulting estimates of the average replacement rates observed for each system type per month and year of age are reported in Table 1.

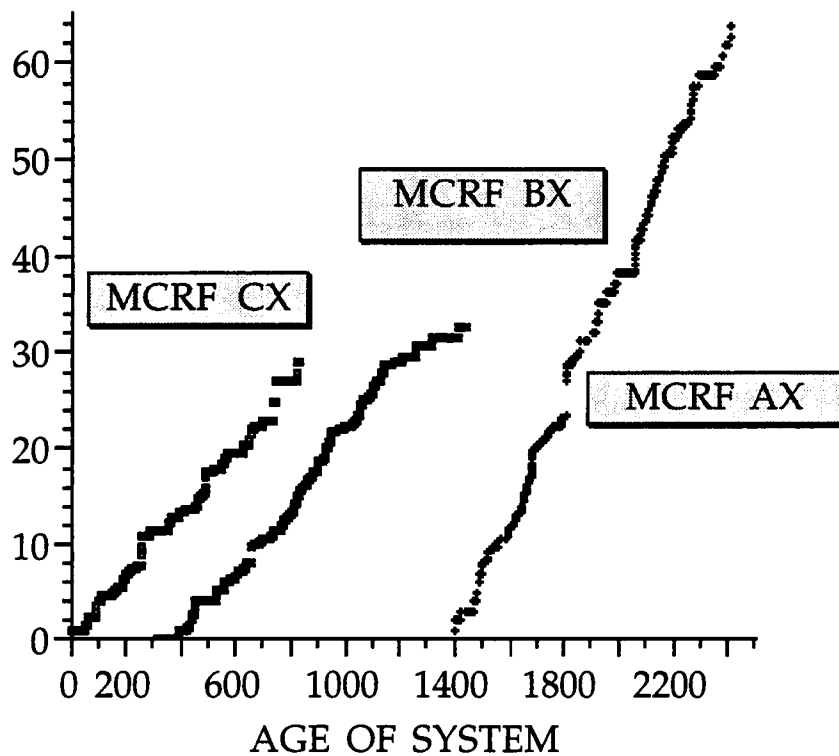


Figure 24 : MCRF USDS Data For Three Generations of System

Table 1: Linear Fits to USDS MCRF

| System Type | Average Replacement<br>per Month of Age | Average Replacement<br>per Year of Age |
|-------------|-----------------------------------------|----------------------------------------|
| AX          | 1.9                                     | 23.2                                   |
| BX          | 1.1                                     | 12.8                                   |
| CX          | 1.0                                     | 12.2                                   |

Because of the implied HPP, these rates are not to be used as an estimate of ROCOF. However, these estimates are useful. For instance, are the 'apparent' differences in slope between generation of unit due to sampling variation, age, generational differences, or some other unknown factor?

To provide further insight into the importance of age as a surrogate explanatory variable for system reliability and to explore within generational differences, the number of failures in the USDS observation year are plotted against system age at end of the observation year in Figure 25.

In Figure 25, AX generation units are designated by large squares, BX generation units are designated by small squares, and CX generation units are designated by circles.

Figure 25 shows that age effects are somewhat confounded with unit generation. However, within generation, age does not appear to be an important surrogate explanatory variable. This is of interest

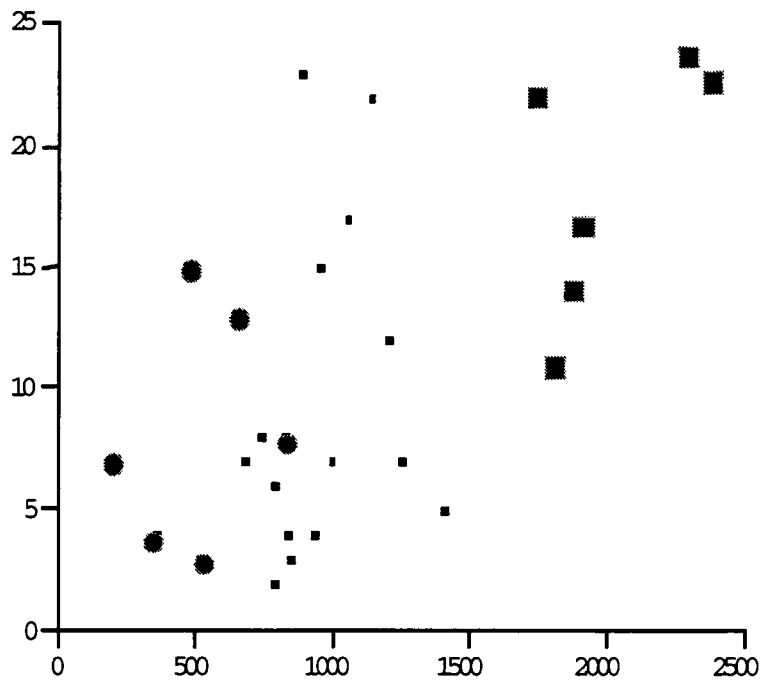


Figure 25: Cumulative Failures Versus System Age - USDS Data

to leadership, manufacturing, and engineering and has implications in modeling the pattern of system failures.

Only the AX generation of design had easily identifiable within generation major design changes, AX1 and AX2. The AX2 design was intended to improve the efficacy of the systems. Figure 26 could indicate that age is an important surrogate explanatory variable in reliability explanation for the AX1 systems and that there are differences in AX1 and AX2 systems. The small sample size makes it difficult to render such judgments.

A straight line fit to the AX1 and AX2 MCRFs reports that AX1 has an increase in cumulative failures of .0647 per day while the AX2 has a an increase in cumulative failures of .0727 per day. This result is, given the sample size, not very conclusive.

### **Calendar Time**

We modify Nelson's method to graphically examine the USDS data across calendar time (MCRF CT). This modification supports the

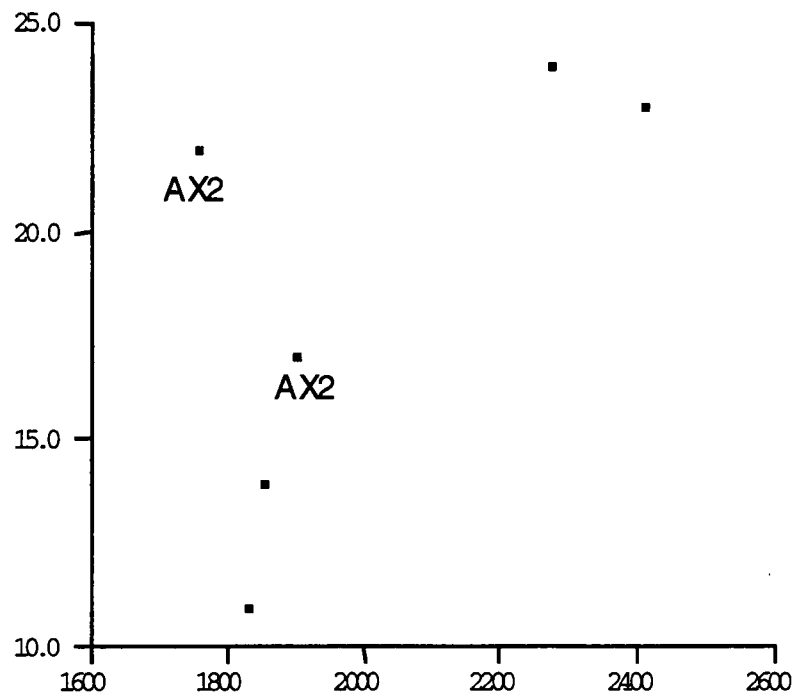


Figure 26: AX1 and AX2 Cumulative Failures in USDS Data

development of knowledge concerning age as a surrogate variable in explaining reliability. It encourages the questioning of the impact of changing policies, practices, or events not associated with system age on reliability (or on hardware removal from systems). And it will develop our understanding of the customer reliability experience during the sample year. Because the method implies a HPP, failures per year per system from this modification of Nelson's method are not estimates of ROCOFs. Visual examination of Figure 27 shows one possible change in the AX curve at 313 days into the observation year. Reexamination of the AX MCRF implies a similar abrupt change in the MCRF at 1804 days of age. Further investigation is discussed later.

### **Discussion of Nelson's MCRFs**

A summary of the exploratory work to this point follows. Differences in slopes imply differing ROCOFs for the three generations of systems. However, differences within generation of system beyond those explainable by the surrogate variable age are apparent. A modification of Nelson's method to examine average

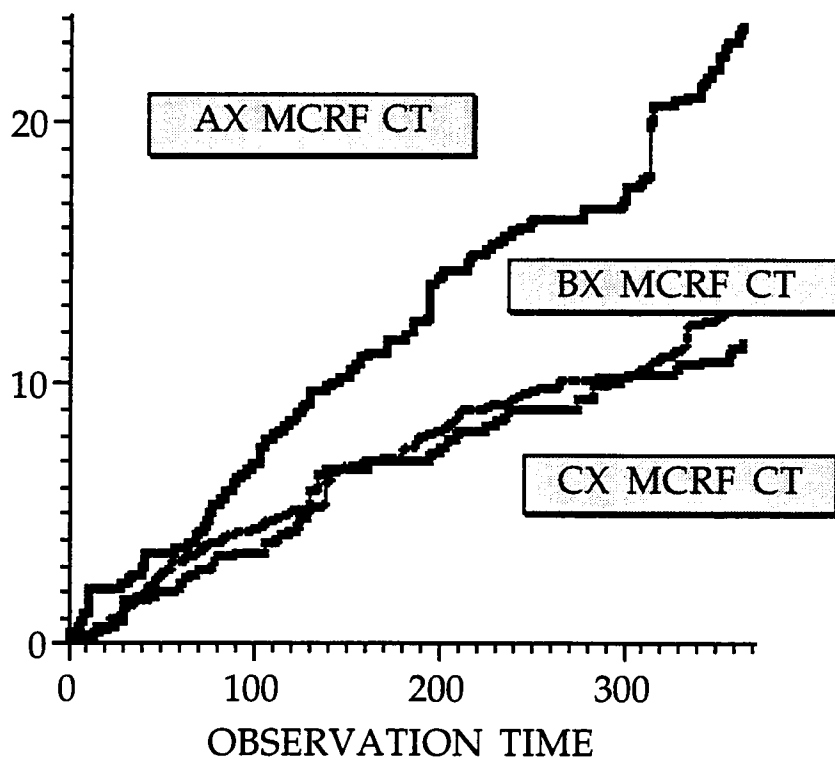


Figure 27: MCRF CT USDS Data

removal rates across the observation time showed an obvious discontinuity in the AX generation that should be investigated further.

### **What Components Were Replaced?**

The specific hardware components that were replaced upon system failure are of interest. Were components removed in the same relative percentages across systems and generations? What was the rate of replacement for the various component types? Was the rate of replacement for any component increasing? Were the removals due to component failure or was the symptom that lead to component removal due to software or operator or misapplication. The answers to these questions could provide insight to the causal factors associated with ROCOFs.

Table 2 details which components were replaced on the systems in the USDS sample. The categories indicate components with comparable functionality for the different unit generations. Category 'C' is other. The category 'B<sub>i</sub>';  $i = 1, 2, 3$ ; for unit types CX

Table 2: Component Removals in the USDS Data

| AX Systems     |        | BX Systems     |        | CX Systems     |        |
|----------------|--------|----------------|--------|----------------|--------|
| Component Type | Number | Component Type | Number | Component Type | Number |
| A              | 71     | A              | 81     | A              | 21     |
| B              | 45     | B              | 24     | B              | 19     |
| C              | 26     | B1             | 7      | B1             | ?      |
|                |        | B2             | 11     | B2             | ?      |
|                |        | B3             | 6      | B3             | ?      |
|                |        | C              | 92     | C              | 29     |

and BX indicate the three components that replaced one component in the AX generation. There is no visibility of these subcomponents in the CX data. The numbers in Table 2 reflect actual quantity replaced and have not been normalized for number of systems in the sample. A system failure may result in multiple component replacements.

Whether differences in the relative removal rate are of practical or statistical significance is of interest. The apparent differences can be statistically evaluated with a  $\chi^2$  test. We use the AX distribution of components replaced as the expected distribution and compare both of the later generation's distributions of component replacements to that of the AX. The percentages of component type removed for the USDS data are shown in Table 3.

There is statistical evidence that the distribution of components replaced is changing from generation to generation of unit (both p-values are close to 0). The apparent decreasing percentage of component A is desirable from an organizational viewpoint. Also of interest is whether there are within generation differences in the

Table 3: Percentages of Component Removals in the USDS Data

| AX Systems     |            | BX Systems     |            | CX Systems     |            |
|----------------|------------|----------------|------------|----------------|------------|
| Component Type | Percentage | Component Type | Percentage | Component Type | Percentage |
| A              | 50         | A              | 41         | A              | 30         |
| B              | 31.7       | B              | 12         | B              | 28         |
| C              | 18.3       | C              | 47         | C              | 42         |

distribution of component replacements. This question will be addressed with other sources of data due to the small amount of information available in the USDS sample. The removal over time for the various component types is shown in Figures 28, 29, and 30. Small changes in these curves may not be indicative of the true situation due to the small data available. However, the possible abrupt shift apparent in the AX MCRF is also evident in the AX MCRF for component A.

### **Multiple Removals**

The data is examined for day 313 of the observation period to determine which site(s) had the 'large' number of replacements for component A. This work reveals that 11 component As were replaced on one system at 1804 days of age. The conditional distribution for number of components replaced given at least one component is replaced on a particular age is examined for the USDS data in Table 4. Ages are not shown in the table although the tendency for multiple components to be replaced was examined for correlation with age. A small number of the multiple replacements

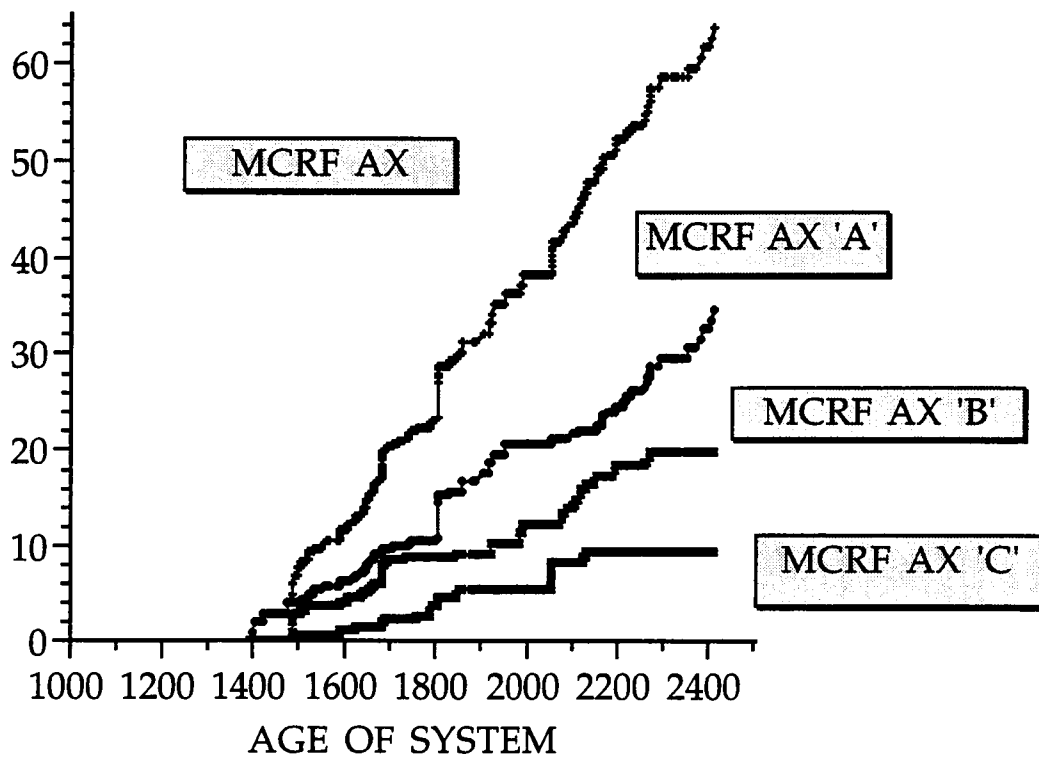


Figure 28: MCRF AX and MCRF AX Components

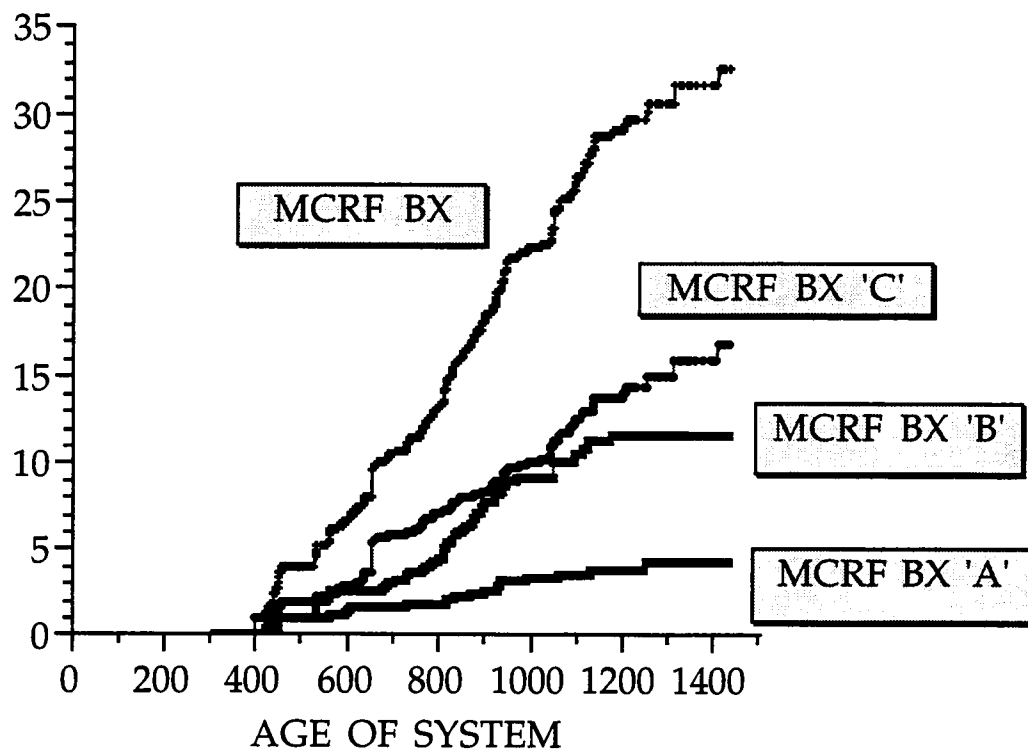


Figure 29: MCRF BX and MCRF BX Components

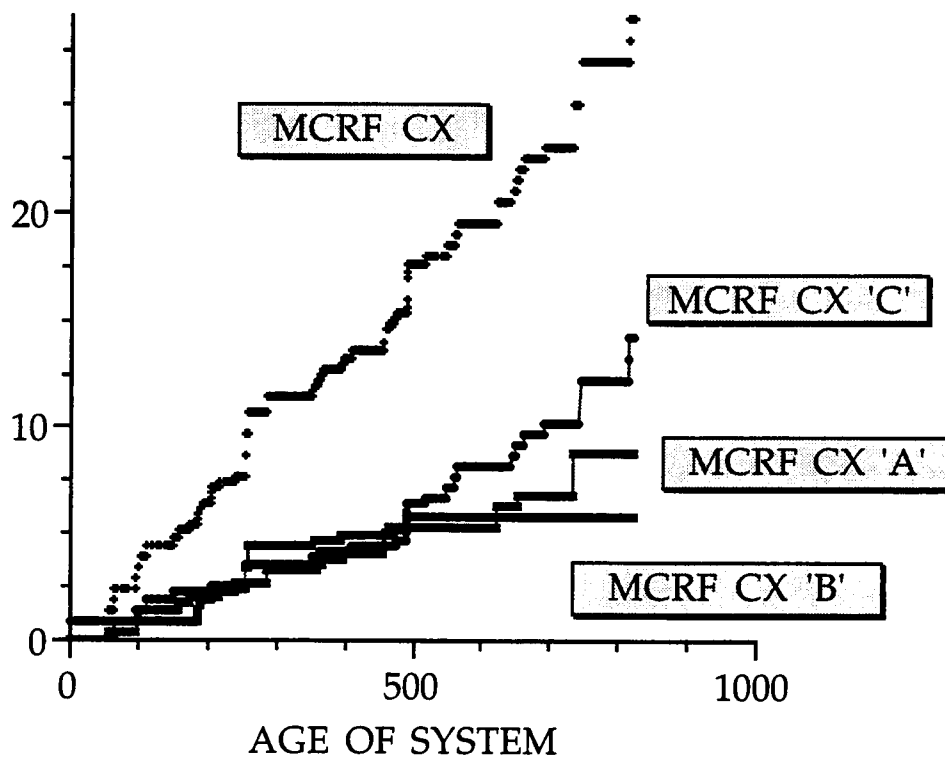


Figure 30: MCRF CX and MCRF CX Components

Table 4: Multiple Removals in the USDS Data

| # Removals | AX  | BX  | CX |
|------------|-----|-----|----|
| 1          | 121 | 136 | 47 |
| 2          | 5   | 16  | 5  |
| 3          |     | 1   | 4  |
| 4          |     | 2   |    |
| 5          |     | 1   |    |
| 11         | 1   |     |    |
| 13         |     | 1   |    |

are for different systems. For instance, one of the 5 instances of AX multiple replacements of '2' occurred due to single replacements on two systems. To provide further insight into multiple replacements, cumulative plots of failures against system age are shown in Figures 31, 32, and 33.

Of the AX removals, 13.3% are multiple in nature (4 of the 6 systems had multiple removals. However, of BX removals 31% are multiple in nature (8 of 15 systems had multiple removals). The CX systems are similar with 31.8% of removals multiple in nature (4 of 6 systems had multiple removals)

Again the within generation variation in total number of removals during the USDS observed year is apparent. For instance, in the CX generation two systems are more rapidly accumulating failures than the remaining four systems.

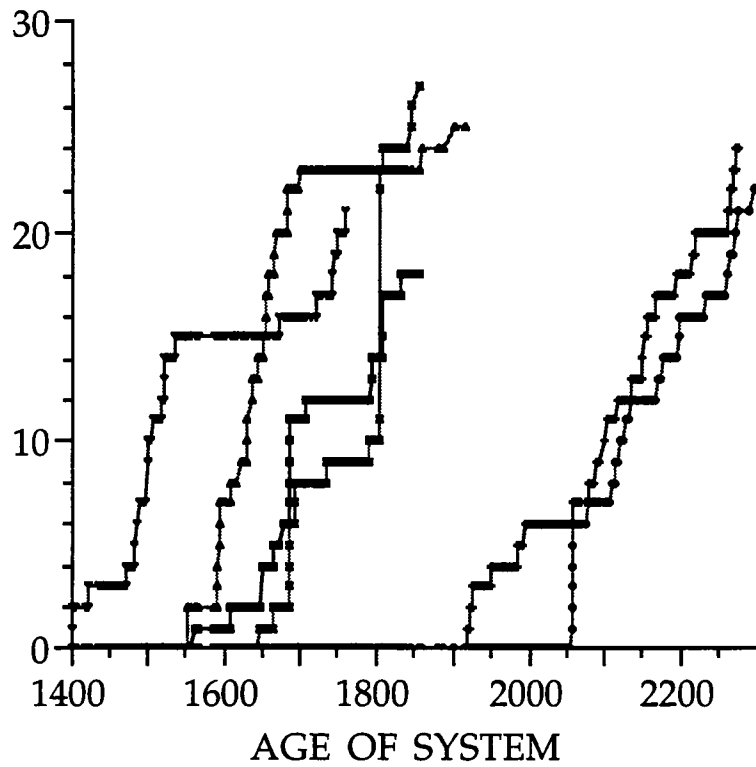


Figure 31: Individual System Cumulative Failures vs. System Age, AX  
Systems

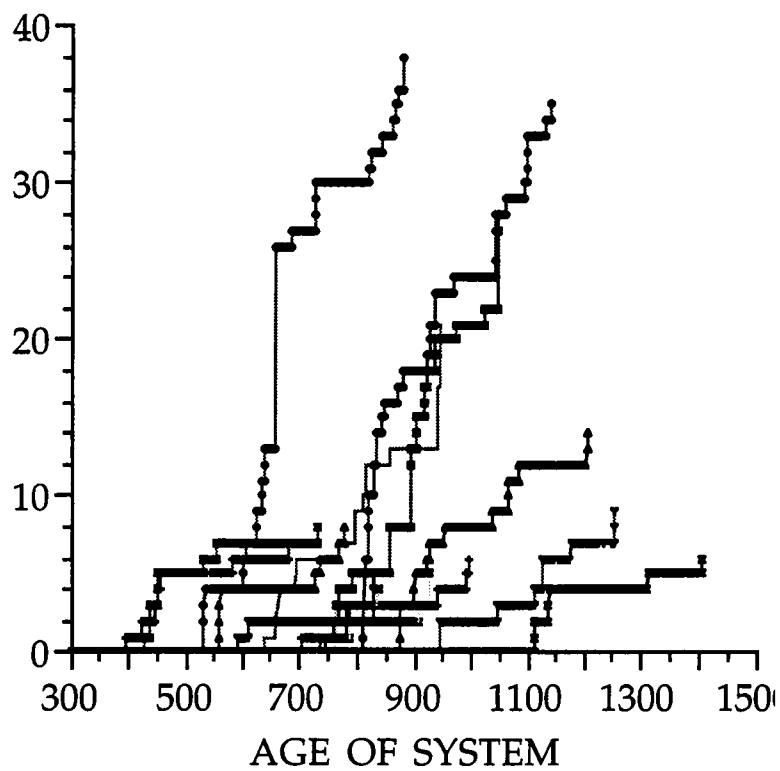


Figure 32: Individual System Cumulative Failures vs. System Age, BX  
Systems

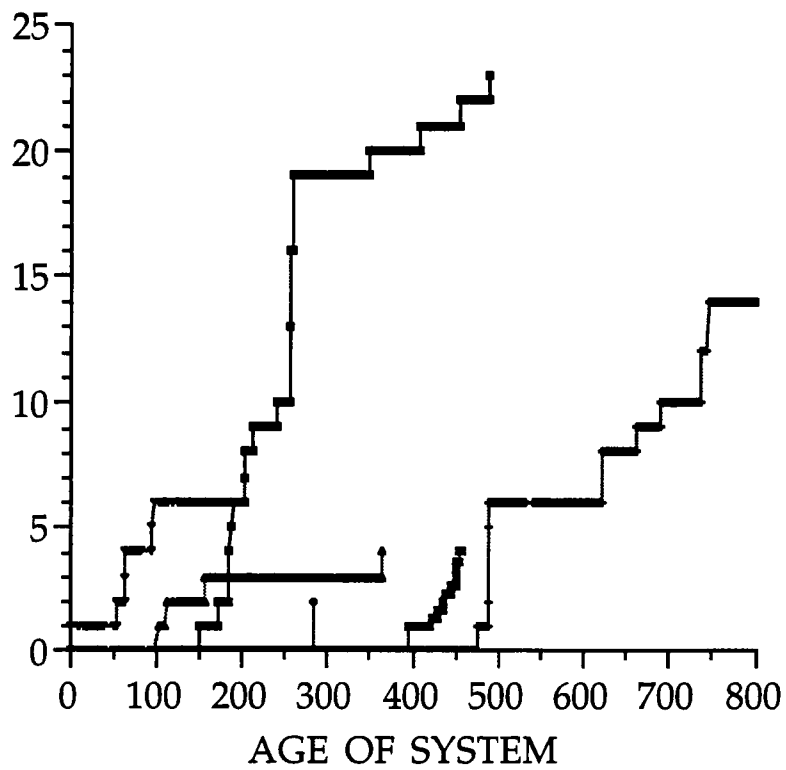


Figure 33: Individual System Cumulative Failures versus System Age, CX Systems

## **Statistical Trend & Dependency Tests**

In Appendix A formal statistical tests for trend and dependency are applied to the USDS data. In statistical terms we are testing for nonidentical (marginal) distributions and nonindependence of interarrival times respectively. In practical terms the trend tests allow us to formally investigate whether the ROCOF are still increasing and the dependency tests examine the data for other patterns. These other patterns, if present, may provide clues to causal structure.

The statistical trend test incorporates multiple removals by assigning a '0' to interarrival times when there are multiple removals in one day. This test provides evidence that the BX generation ROCOF is deteriorating. There is no evidence that either the AX or CX ROCOF is deteriorating.

The tests for dependency treat each removal event as an event whether there are multiple replacements or not. There is evidence that interarrival times are not independent in either the AX or BX

generations. There is no evidence of dependency in the CX generation. From a probabilistic view this implies models more complex than the NHPP are required to properly model the failure patterns of the AX and BX generation of system. Since the statistical trend tests assume a HPP, from a statistical view the trend tests may be invalid. From a practical view this may well lead to an investigation concerning service policies and practices. For instance short interarrival times following short interarrival times may indicate incorrect diagnosis's or that field service engineers could damage the systems when making repairs thus necessitating the need for a quick return.

### **Summary of USDS Data Analysis**

One year of reliability data for a subset of all systems was collected in the USDS dataset. The replacement rate data was presented as a function of age for each system type with MCRFs. While visual examination of these overall curves did not show obvious signs of increasing failure rates, combined results of statistical trend tests on individual system replacements provide

evidence that the AX and BX generation replacement rate is either increasing or is not a random process. There is no evidence that the CX generation replacement rate has increased. This possibility is a business concern. Two other issues of business and engineering concern were uncovered in this data analysis. First, multiple removals are occurring within the sample of systems covered by the USDS data. Second, there are apparently large differences in replacement rates within generation of system.

Possible differences in hardware replacement rates due to factors associated with age are partially confounded with system type. The first generation of unit, the AX, have the highest replacement rate, followed in turn by the second generation BX, with the newest generation CX having the lowest replacement rate. What may be most interesting and insightful is that the BX generation (the most data available in this sample) has the most within generation spread of number of system failures (hardware replacements). Possible differences may be explained by service rather than performance, differences in usage, or could indicate manufacturing variations.

These possibilities and other explanations for differences need to be further explored.

Differences between the generations for the percentages of which components are replaced may provide insight into this work. We will investigate differences of type of component replaced within each generation with data from the return parts database.

### **Directions for Further Work**

When did the failure rates for the BXs increase (are they increasing)? Why is there evidence that the AX replacement rates are not random? What happened on day 313 of the observation period? Are multiple removals due to environmental issues or manufacturing issues? What are the differences in hardware replacement rates within generation and how do those differences compare to apparent differences between generations? Can any differences be explained with engineering knowledge? Is the USDS data representative of the unit population at large? These questions and similar ones need to be addressed. A mechanism for

ongoing analytic understanding is still required and the understanding of what will be required is increasing.

### **Repair Data**

The nature of the USDS data permits two options to nonparametrically estimate and compare the survival curves for a particular component. First we could use the component MCRF as an estimate of the survival curve. This is arguably a viable alternative since less than 10% of components failed during this period. For instance, there are either 96 or 128 Component As on a BX system or approximately 1856 Component As functioning at any one time in the systems included in the USDS data. Less than 10% of these actually were replaced during the observation year. A second alternative is to assume that each component has system age (no socket had multiple failures) and use survival curve estimation methods for left truncated data. These two alternatives do not provide an estimate of component reliability across all system ages. For example, BX component A data is truncated before 346 days of age. Both alternatives merely restate the results of the initial

examination of the USDS data. We need information on a broader range of data.

Another possibility is the use of repair records from returns database to augment the USDS data. There are difficulties with this approach. We have the same limited system set because of the difficulties discussed in return components data description. Further complicating matters is that the exact failure dates are not known, merely the date of reception at the manufacturer. Fortunately, some of these difficulties can be mitigated by the fact that these are US systems which increases the probability that searches of paper records will yield fruitful information. For instance information for which site a component was replaced is sometimes available. This would allow us to track, say a particular component A, through the Component A technology and manufacturing lab which did keep some partial records on return parts.

Another issue was whether to use system age as a returned component's age. (Some replacements could be in sockets where a

component(s) had already been replaced). Use of system age for a replaced component would result in estimating the MCRF or renewal rate rather than the component survival curve. We choose to calculate the renewal functions.

Before beginning the discussion of return component data we reiterate that the best return component information available is for US systems due to traceability and other issues. In some cases, a component returned from a US system may have enough associated information to allow backtracking through paper records or databases scattered through various manufacturing and engineering labs. This exercise may enable us to understand if the component actually failed (or at least whether that component was replaced), from which system, and to determine an approximate replacement date.

For the purposes of initial analysis, the return components database for US data is organized into two major segments. The first segment contains information from fiscal years 1992/1993 and the second from fiscal year 1994. In fiscal year 1994 there is a small

improvement in the availability of information. For instance, for fiscal years 1992/1993 there are almost no records of failure modes. In fiscal year 1994, failure mode information is increasingly common. Traceability is still an issue in 1994, although slightly improved.

There is more overall data available for the BX generation of systems than for the CX or AX systems. This is primarily due to the fact that there are more BX systems than any other and that they are not the oldest systems. Two of the AX systems and one of the CX systems that included in the USDS data are not traceable in the return part database.

### **AX Returns**

The number and type of component replacements for the AX systems in the USDS database and in the return parts database for fiscal years 1992/93 and 1994 are shown in Table 5. Only four of the six systems are traceable in the return parts database.

Table 5: AX Component Removal, USDS and Return Parts Databases

| USDS Data      |        | Fiscal 92-93   |        | Fiscal 94      |        |
|----------------|--------|----------------|--------|----------------|--------|
| Compt.<br>Type | Number | Compt.<br>Type | Number | Compt.<br>Type | Number |
| A              | 71     | A              | 67     | A              | 41     |
| B              | 45     | B              | 56     | B              | 24     |
| C              | 26     | C              | 21     | C              | 8      |

The MCRF and MCRF for components A, B, and C for the four AX systems in the return parts database are shown in figure 34.

The failure modes that are available are summarized in Table 6. A code of 00 represents no defect found. Failure modes of 81 and 84 are known to occur with component A for this generation. Interesting is that of all components with failure modes recorded that only one is 'no defect found'. All of the failure codes reported are for fiscal year 1994. Of the component A's 20 of 41 have failure codes reported, component B's 18 of 24, component C's 7 of 8. The reason for failure code data scarcity is due less to lack of discipline in entering the data and more to do with the length of time the component is in the queue to be analyzed. In fact, what data is available is due to some individual Herculean effort. A recent complaint from the servicing organization was that 75% of their inventory of components A and B was in the manufacturer's inventory and they have neither the parts nor the credit for them!

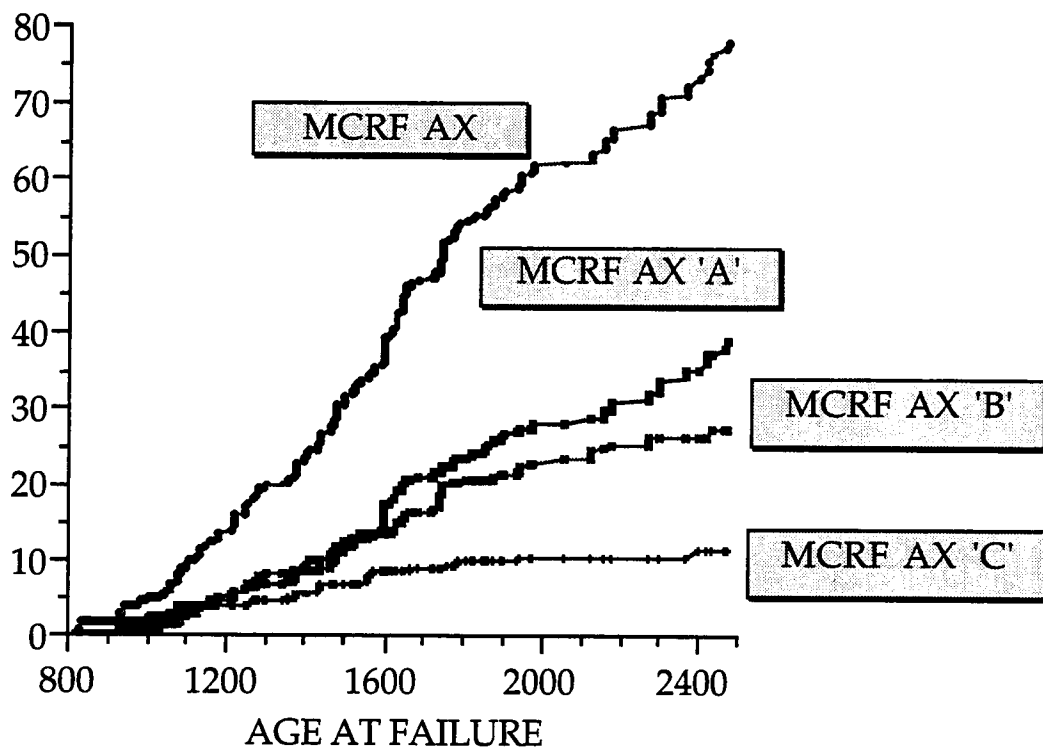


Figure 34: AX MCRF, Return parts Database

Table 6: Failure Modes for AX Return Components

| System | Comp. | Failure | Number |
|--------|-------|---------|--------|
| 93X    | A     | -       | 88     |
| 93X    | A     | 81      | 9      |
| 93X    | A     | 82      | 1      |
| 93X    | A     | 84      | 9      |
| 93X    | A     | 87      | 1      |
| 93X    | B     | -       | 74     |
| 93X    | B     | 00      | 1      |
| 93X    | B     | 11      | 2      |
| 93X    | B     | 28      | 1      |
| 93X    | B     | 70      | 1      |
| 93X    | B     | 97      | 1      |
| 93X    | C     | -       | 28     |
| 93X    | C     | 25      | 1      |

The cumulative failures versus system age for individual AX systems is shown in Figure 35. No obvious differences in system ROCOF is apparent.

We continue the examination of returns data with category C or 'other'. We examine these data from several viewpoints. First the components are identified to the quality and regulatory manager for determination of safety and efficacy issues (none are identified). Second the data are examined for large differences in return rates. No component in the 'other' category accounts for more than 3% of total returns. These categories are compared from 1992/93 to 1994. There are no indications of statistically significant increase/decrease in any particular component in category C.

### **BX Returns**

The number and type of component replacements for the US BX systems in the USDS database and the number of returns for the same set of systems for fiscal years 1992/93 and 1994 are shown in Table 7.

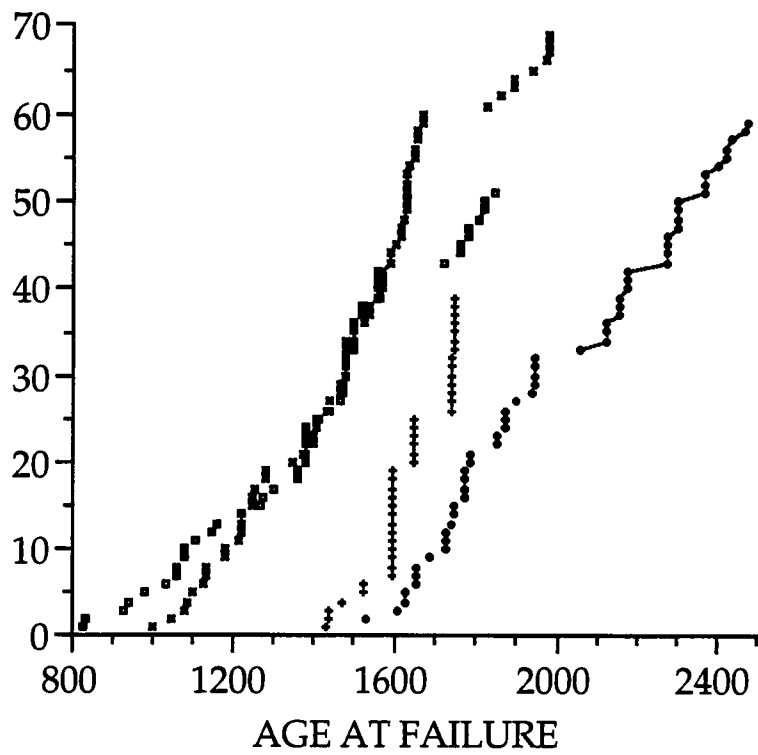


Figure 35: AX Cumulative Failures versus System Age, Return Parts

Table 7: BX Component Removal, USDS and Return Parts Databases

| USDS Data      |        | Fiscal 92-93   |        | Fiscal 94      |        |
|----------------|--------|----------------|--------|----------------|--------|
| Component Type | Number | Component Type | Number | Component Type | Number |
| A              | 81     | A              | 109    | A              | 80     |
| B              | 24     | B              | 37     | B              | 24     |
| C              | 92     | C              | 264    | C              | 129    |

A  $\chi^2$  test of homogeneity of the 1992/93 and the 1994 data do not provide conclusive evidence that percentages of type of component returned has changed (p-value = .07). However, even though not found to be statistically significant, it is noted that the percentage of component A's replaced has increased while that of C has decreased. Because the problems in traceability has resulted in a reduced sample size and since component A is so important to system efficacy, it is felt that further investigation of component A is warranted.

However, while the Component A's are an increasing percentage of BX hardware replacements in the returns database (a function of calendar time) there is no evidence that they are failing more often as a function of system age. Also, the analysis of the USDS data did seem to indicate that differences in overall ROCOF between BX systems obscured age as an important surrogate variable.

We continue the examination of returns data with category C or 'other'. We examine these data from several viewpoints. First the

components are identified to the quality and regulatory manager for determination of safety and efficacy issues (none are identified). Second the data are examined for large differences in return rates. No component in the 'other' category accounts for more than 4% of total returns. These categories are compared from 1992/93 to 1994. There are no indications of statistically significant increase/decrease in any particular component in category C.

Visual examination of Figure 36, the BX MCRF for returns, does not indicate an increasing ROCOF overall or with respect to component A or B. But, the MCRF does not provide insight to the experiences of individual sites. Since we have been compiling evidence that differences in individual BX systems may obscure overall age related replacement patterns, we examine the possible differences in ROCOF for the BX systems with cumulative failures versus system age plots for return data in Figure 37.

The solid line is the overall MCRF with individual systems using a unique marker. The individual markers are not distinct in this

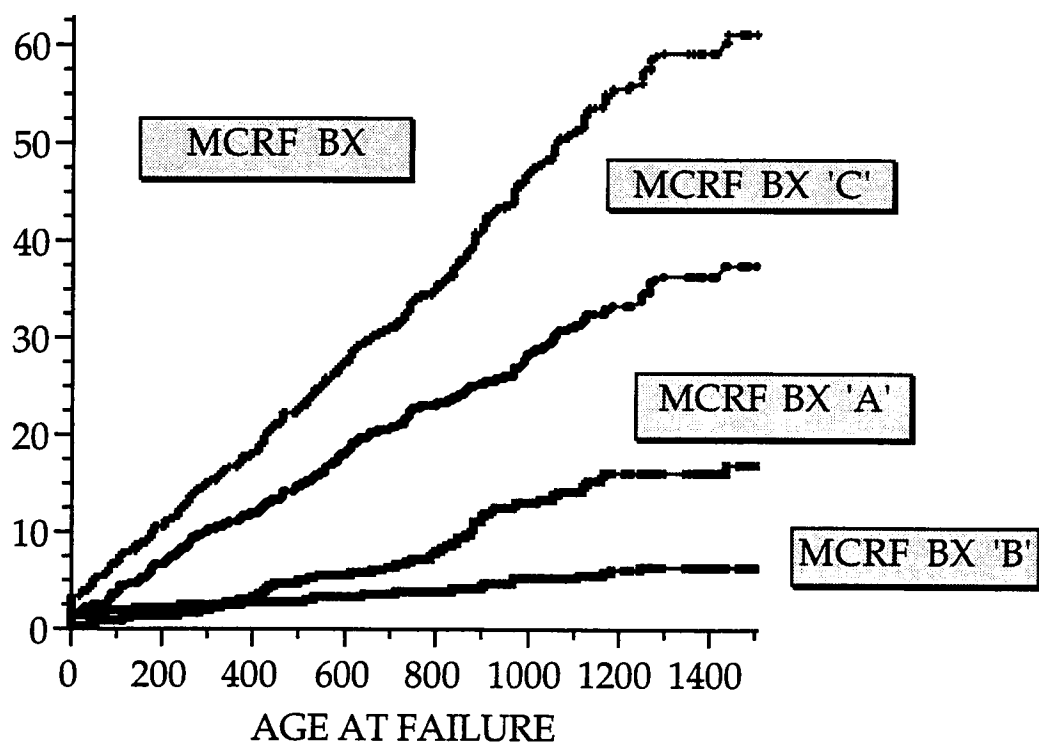


Figure 36: BX MCRF, Return parts Database

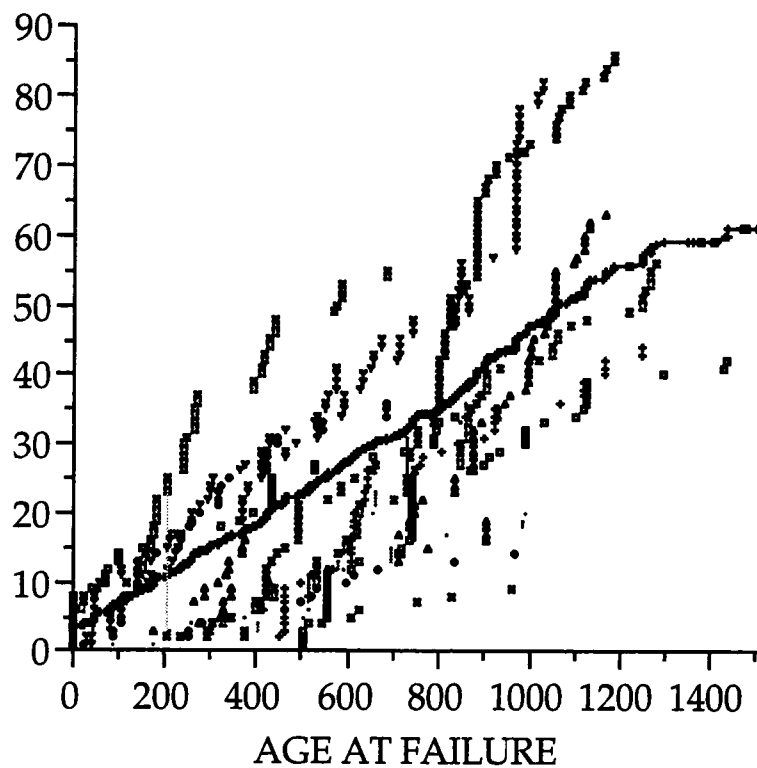


Figure 37: Individual System Cumulative Failures versus System Age  
and MCRF vs. System Age, Return Parts Database

picture. In practice colored markers are used to distinguish individual systems. The vertical lines are indicative of the batch processing of return components (for a given system). Of course as discussed in the USDS data, multiple components are sometimes replaced on individual days. Batch processing of return components prevents rapid organizational assimilation of reliability data, increases inventory, increases repair and overhaul cycles, and hides multiple component removals. Multiple replacement of components is characteristic of infrequent service, poorly trained service personnel, abrupt system failures due perhaps to environmental conditions, or would indicate a batch of components with manufacturing difficulties.

Table 8 shows available failure modes. Again, for a majority of returns, failure modes are unavailable. Also again, through a tremendous effort, failure codes are available for the majority of the US systems for fiscal year 1994 returns. (There is not failure mode information on 18 fiscal year 1994 component A returns, 10 component B returns, and 53 component C returns). The 33 'no

Table 8: Failure Modes for BX Return Components

| System | Comp. | Failure | Number |
|--------|-------|---------|--------|
| BX     | A     | -       | 127    |
| BX     | A     | 80      | 9      |
| BX     | A     | 81      | 23     |
| BX     | A     | 83      | 2      |
| BX     | A     | 84      | 27     |
| BX     | A     | 87      | 1      |
| BX     | B     | -       | 47     |
| BX     | B     | 00      | 8      |
| BX     | B     | 01      | 1      |
| BX     | B     | 28      | 3      |
| BX     | B     | 30      | 2      |
| BX     | C     | -       | 317    |
| BX     | C     | 00      | 33     |
| BX     | C     | 13      | 1      |
| BX     | C     | 14      | 1      |
| BX     | C     | 24      | 5      |
| BX     | C     | 28      | 2      |
| BX     | C     | 33      | 1      |
| BX     | C     | 50      | 1      |
| BX     | C     | 70      | 21     |
| BX     | C     | 72      | 1      |
| BX     | C     | 80      | 7      |
| BX     | C     | 81      | 1      |
| BX     | C     | 96      | 1      |
| BX     | C     | 99      | 1      |

defect found's for component C is at least partially due to an inability to test at a detail level the majority of C components. More alarming, is that all of the component As tested are true failures. The fact that 8 of 15 component Bs tested recorded 'no defect found' is cause for investigation.

Because of the concern over component A, individual cumulative failure versus system age plots are shown in Figure 38 for component A for the BX generation of unit. Again, the within generation differences are apparent.

## **CX GENERATION**

The number and type of component replacements for the US CX systems in the USDS database and the number of returns for the same set of systems for fiscal years 1992/93 and 1994 are shown in Table 9.

The failure codes available are reported in Table 10 . For this set of systems, 10 of 17 component As for fiscal year 1994 are reported, 9

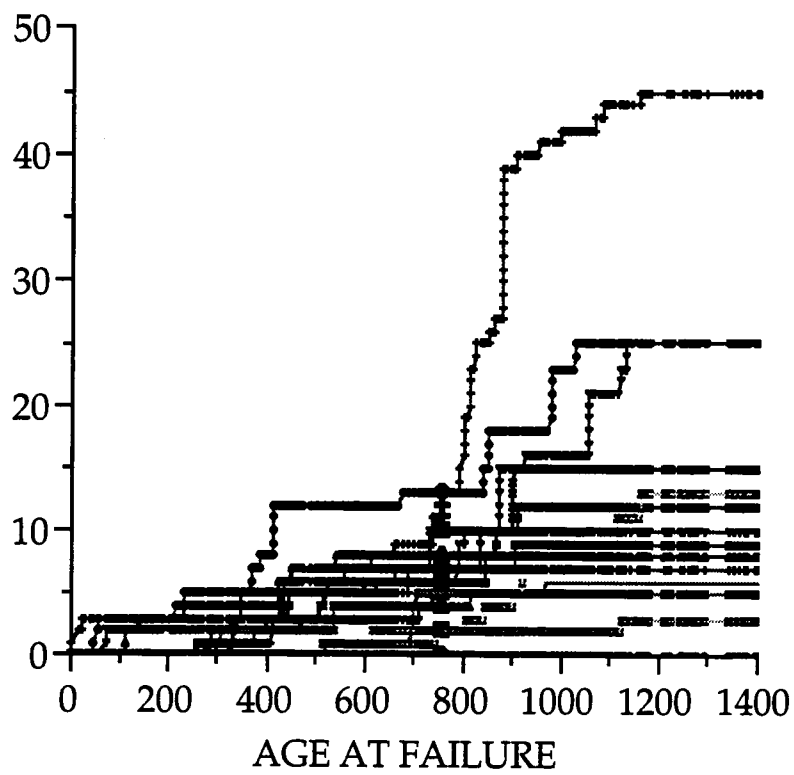


Figure 38: Component A BX System Returns

Table 9: CX Component Removal, USDS and Return Parts Databases

| USDS Data         |        | Fiscal 92-93      |        | Fiscal 94         |        |
|-------------------|--------|-------------------|--------|-------------------|--------|
| Component<br>Type | Number | Component<br>Type | Number | Component<br>Type | Number |
| A                 | 21     | A                 | 28     | A                 | 17     |
| B                 | 19     | B                 | 20     | B                 | 9      |
| C                 | 29     | C                 | 43     | C                 | 33     |

Table 10: Failure Modes for CX Return Components

| System | Comp. | Failure | Number |
|--------|-------|---------|--------|
| 92X    | A     | -       | 38     |
| 92X    | A     | 81      | 5      |
| 92X    | A     | 84      | 1      |
| 92X    | A     | 88      | 1      |
| 92X    | B     | -       | 20     |
| 92X    | B     | 00      | 3      |
| 92X    | B     | 28      | 1      |
| 92X    | B     | 70      | 4      |
| 92X    | B     | 80      | 1      |
| 92X    | C     | -       | 52     |
| 92X    | C     | 00      | 9      |
| 92X    | C     | 13      | 1      |
| 92X    | C     | 25      | 1      |
| 92X    | C     | 35      | 2      |
| 92X    | C     | 40      | 2      |
| 92X    | C     | 70      | 8      |
| 92X    | C     | 80      | 1      |

of 9 component Bs, and 24 of 33 component Cs. Once again, all of component A returns are truly failures. Again, components Cs failures are examined by the Regulatory Officer for safety and efficacy issues.

Figure 39 is of the CX MCRF calculated from the return data. In Figure 40 the cumulative failures for individual systems are shown. (Again, while markers for individual systems are difficult to differentiate here, in practice color markers are used). The cumulative traceable failures for this subset of US systems are 3, 21, 25, 43, and 58. Interestingly, while the average cumulative failures for these systems are not unlike that observed in the USDS data, we are receiving verbal complaints concerning the reliability of these systems. The complaints seem to center on component A.

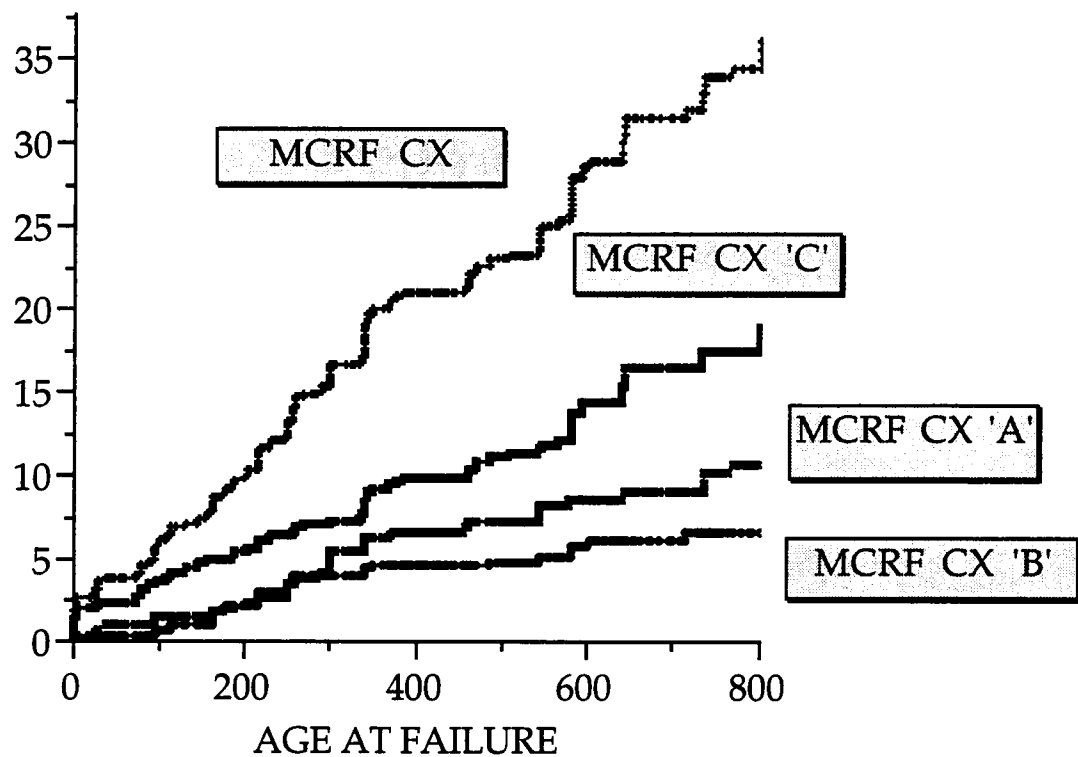


Figure 39: CX Cumulative Failures versus System Age, Return Parts

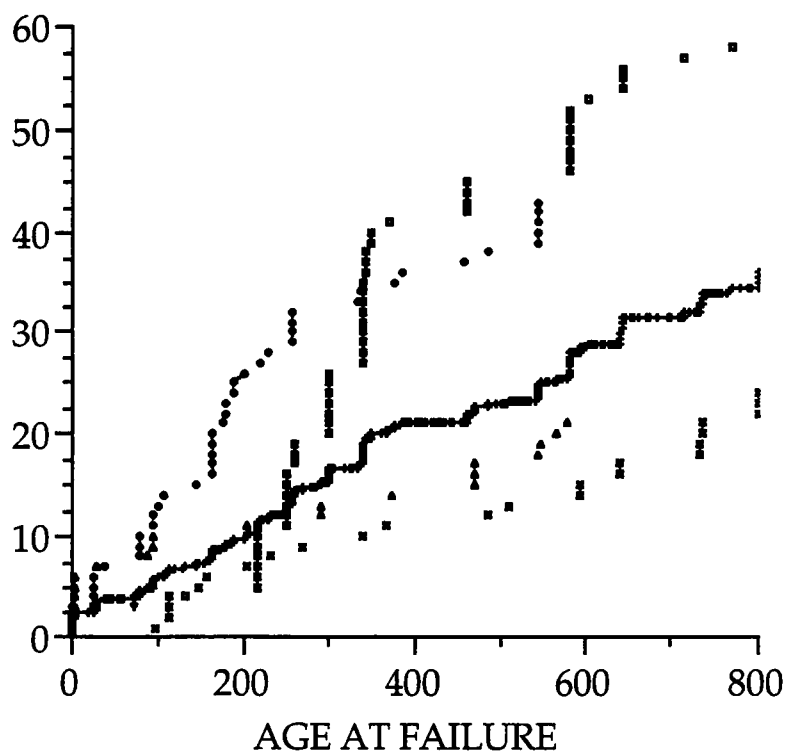


Figure 40: CX Cumulative Failures versus System Age, Returns

## CHAPTER 6

### CONCLUSIONS AND RECOMMENDATIONS

#### Introduction

Typical reliability analysis does not recognize the differences in repairable systems and nonrepairable systems. An analysis of repairable system data as if they originated from nonrepairable systems can be misleading. If the analyst does acknowledge the differences inherent in repairable system reliability data, then it is probable that a complex stochastic model will be applied to the pattern of failures. In either case, the assumption that statisticians make when presented a set of reliability data is that the primary intent is to model failures. There may be no one available to provide advice otherwise. Unfortunately, this implicit assumption is usually wrong. At a minimum, the intent of the analysis should be to provide ongoing information concerning an important aspect of customer value to engineers, manufacturing, and management in a manner that is understandable, enhances insight into causal structures, and that provides information when the

pattern of reliability failures changes due to either deliberate or unintentional changes to the product, manufacturing process, or customer environment. If this work is done in a manner that recognizes the increasing competitiveness of business and so seeks to optimize the process of providing and interpreting reliability information to the organization, so much the better.

### **Repairable Systems**

The recognition of whether items are repairable systems must occur before the data analysis. Methods for examination of repairable system reliability data are different than those methods used to examine nonrepairable system data (except in rare cases). The easiest tools to use to understand and to communicate reliability information are graphical. Plots of cumulative failures for individual repairable system(s) and of the mean cumulative failures across a fleet of repairable systems against system age are a good choice for repairable systems.

## **Conscious Choice of Data**

If multiple 'identical' components are used on repairable systems, decisions are made on the amount of information to be kept on each component. These should be conscious decisions. Socket information, date of system installation, age of components, characterizations of manufacturing processes and of design configurations, and so forth are potentially useful information in the examination of reliability data.

Data on component failures are required for estimation of lifetime distributions. Deliberate tradeoffs between the amount and kind of data collected and the time and expense involved are understandable. An organization may choose to track only system information for failed components. This conserves resources or computer space for failure mode information. In these situations, collections of components may be viewed as subsystems and repairable system techniques used to analyze the data (estimate the renewal function). Usually, an organization should elect to examine both failed components and systems. The reliability processes should generate and provide information on failure modes.

## **Different Types of Time**

The determination of whether a system or collection of components are repairable systems or are to be considered repairable systems will have implications concerning which measure(s) of time will be tracked. The operating time until failure by component age is used to estimate lifetime distributions for populations of nonrepairable items. For repairable systems the time between each system failure and the sequence of failure times is of vital interest. Here system age must be kept. The sequence of failures is used to determine whether times between failures are increasing or decreasing. Of course, reliability growth or deterioration is of interest to the customer. The pattern of system failures can also be examined across calendar time to determine the impact of changing practices and policies on component replacement activities. System age does not have to be time, but can be any measure of usage that makes sense in an engineering or customer value context.

## **Measures of Reliability**

The RCOOF can be estimated for a particular system or summarized for a fleet of systems. It may be useful to use a subscript for

system age:  $\text{ROCOF}(t)$ . This emphasizes that  $\text{ROCOF}$  is not usually constant. For most engineering applications, reporting a system  $\text{ROCOF}(t)$  may also be incomplete. This is a metric averaged across differing manufacturing processes and environmental usage conditions. More insight may be available by viewing the cumulative failures versus system age graphs for individual systems and the MCRF for the fleet of systems. This motivates attempts to understand and subsequently manage the reasons responsible for any differences in system reliability.

### **Definition of Failure**

Usually a definition of system failure includes the identification of a discrete point in time as the moment of failure. This is potentially a gross oversimplification. Repairable systems are more likely to suffer degradation of performance over time rather than abruptly fail. Manufacturers either neglect to consider a data collection system that would allow proper collection of degradation data or choose not to design such a system because of internal constraints or additional product costs. It should be a conscious decision. With either choice, a clear definition of failure, relevant

to customer expectations, that can consistently be applied is required. Idealistically, this definition would consider customer expectations concerning system performance.

### **Probabilistic Models**

It is known that, even if a particular stochastic model is an appropriate choice after a repairable system has reached steady state behavior, there will not likely be an equation that exactly models the transient behavior in early system life. In many cases, stochastic models such as the NHPP can be used to approximate the chaotic nature of early system life. It is hard for engineers and managers to use such models to provide insight into causal structures. Such insight is obtained by graphical techniques.

The transient behavior of system reliability in the early life stages can be explained logically or mathematically. For instance, many authors believe that the frequently observed early reliability growth is due in large part to the heterogeneous nature of subsystems and components. The literature mostly concerns itself with the case of

heterogeneous components within a system and not with heterogeneous components across multiple systems.

### **Our Situation**

In most organizations there will not be the opportunity to create from a vacuity a process that obtains, analyses, and reports system reliability. Rather there will exist multiple processes, both formal and informal, each developed or evolved for reasons perhaps lost to the organization that might as a by-product supply some reliability information.

Many of these processes will exist merely for financial purposes. (Arguably the majority of financial processes merely trace transactions and do not add value to the organization). Others are patches to compensate for special or common cause events. Some are designed to optimize particular organizational or functional area metrics. From the reliability analyst point of view, some of these processes will be dysfunctional. There will be gaps between these processes. They will not be aligned from a business strategy and

customer value view point. This is an accurate portrayal of our initial situation.

We will communicate some specific process deficiencies and product reliability problems. Before doing so, we again note that the organization is the leader of technological provision in this highly specialized area. The reason for disclosure is not to criticize the organization, but to communicate how processes in areas not previously addressed are being improved or might be improved.

This communication is done with a Discovery, Assessment, and Recommendations and Action format. The "Discovery" portion of each topic is a sentence or short paragraph descriptive of a process deficiency or product reliability problem indicated by the attempt to gather certain information, shortcomings of available or gathered data, or the analysis of data. The "Assessment" is an elaboration and expansion of that description that addresses broader topics such as whether the process deficiencies or product reliability problem will continue to be a issue given manufacturing changes or process improvements or whether there are or will be resources available to

commit to the deficiencies. The "Recommendations and Actions" discusses the specific process changes and business decisions made or recommended that positively impact the issue at hand.

### **Process Changes**

Discovery: There was not ongoing provision of reliability information to the organization.

Assessment: The first investigation into reliability data indicated that there was not a process or processes in place to obtain, analyze, and communicate reliability information to the organization. This motivated the discovery and use of the USDS database. There were several formal and informal processes where there was some information available. These processes included the (external) return parts process and the (internal) return parts processing.

The issues with both of these processes were quick compilation of return part data, the analysis of that data, and communication of that information. The organization has decreasing resources. There

are certainly not enough resources to have an on-line feedback system. Therefore the information from returned components is critical to ongoing understanding of reliability.

Figure 41 shows a high level view of the return process. Some issues that were associated with component return are shown in Figure 42. There are suspected differences in ROCOF(t) when a site has a service contract. There is the question of whether good components are damaged or not all defective components are removed during a service call. We know that not all information is captured when a component is removed or if it is captured it may be lost during at some point in the return process. Many times data on removal, site information, symptoms, and possible explanations for multiple removals is not captured or is lost. It is again worth noting that this process does not belong to the manufacturer, but to the servicing organization. The other issue of concern is the (un)timeliness of return.

Similar issues exist with the return parts processing. A top level flowchart of this process is shown in Figure 43. Diagnosis of this

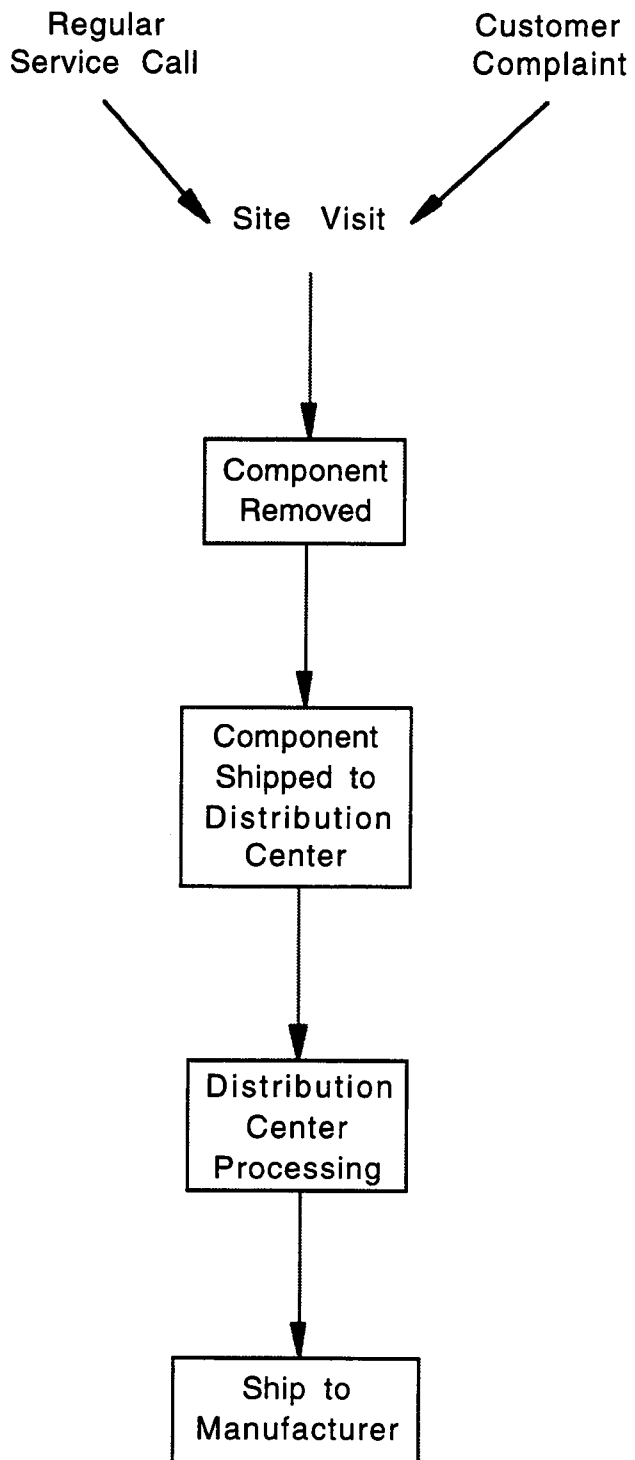


Figure 41: A Typical Return Process

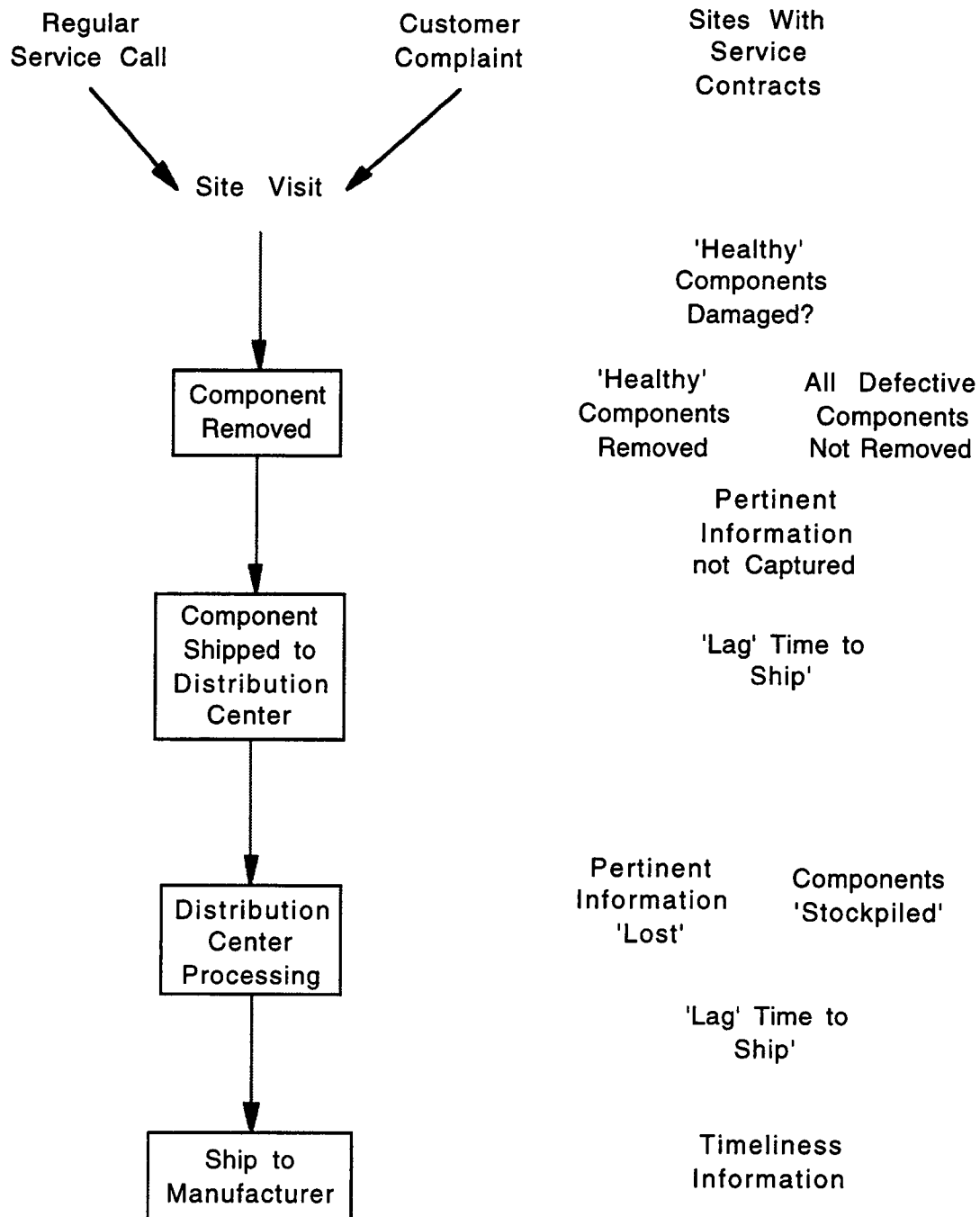


Figure 42: A Typical Return Process and Issues

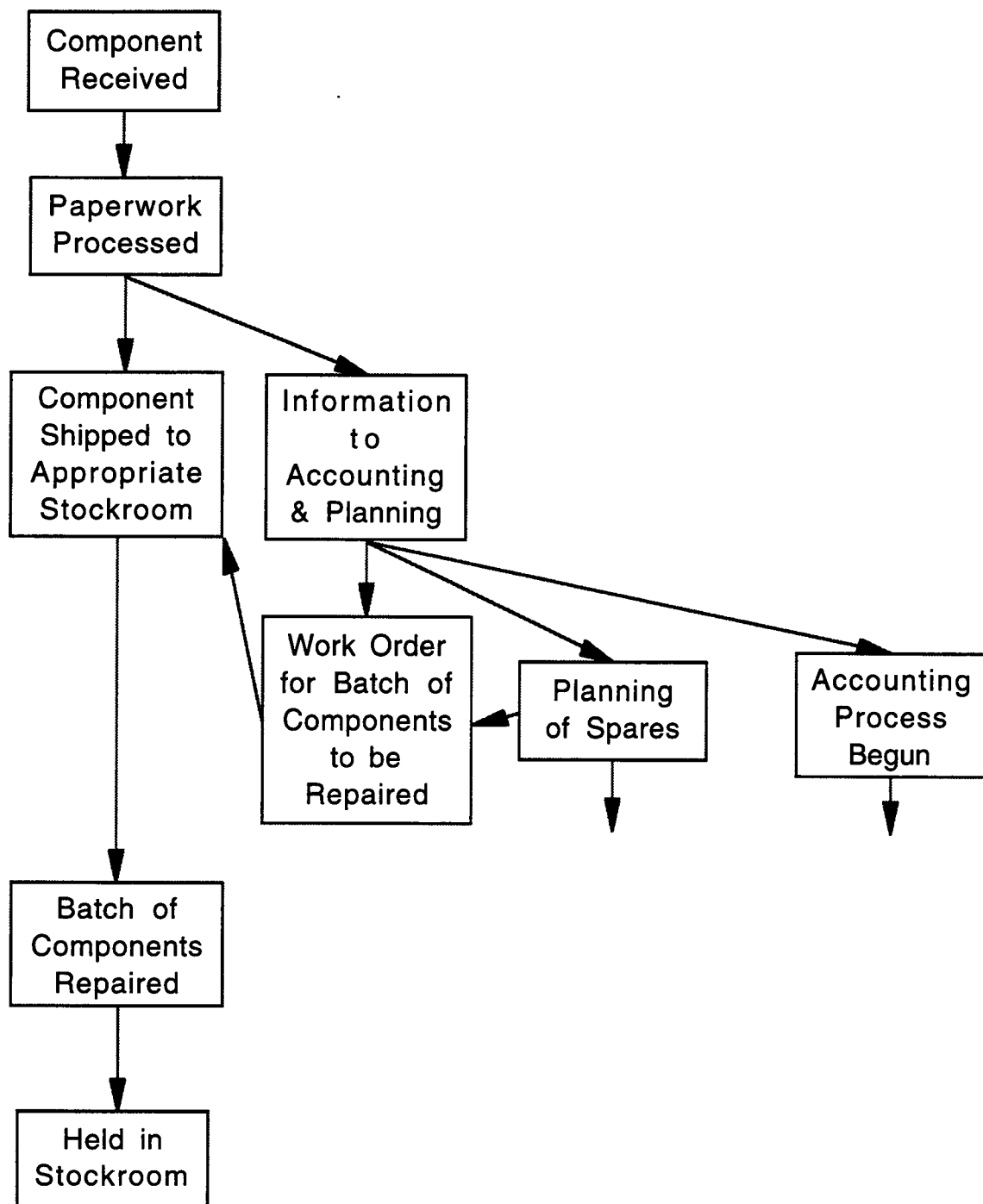


Figure 43: Spare Parts Processing

process with respect to the needs for understanding reliability revealed that cycle time for a returned component was lengthy, that failure mode information was not being captured, and that possible information available to the organization was not being compiled and disseminated.

Recommendations and Actions: The organization has implemented a process whereby failure modes and available information on components and systems is entered into an Excel database. At appropriate time intervals the data are downloaded into a JMP spreadsheet where individual cumulative plots versus system age, MCRF plots, and Pareto Charts are updated automatically. These charts are then reviewed by the Quality and Regulatory Manager who initiates investigative action when appropriate. The process has been reviewed by the FDA and found to meet or exceed all regulatory requirements. The reliability information is also communicated graphically to management on a regular interval.

The internal timeliness issue has been addressed by weekly releases from the stockroom of all returned components into the appropriate

test locations or labs. Eventually a process whereby certain returned components are delivered straight from receiving to the appropriate test location will be implemented.

The external return process is being addressed in similar manner. A process has been proposed that will expedite a removed component straight from the removal site to the manufacturer. Already implemented is a modification to the return process that captures system and date of removal information. The improvements to the internal return parts processing allow the organization to quickly react to multiple removals or other alarming situations. Several options now exist for initial exploration of such a situation. The field service engineer may be directly contacted or the service records be requested. Either option allows exploration of symptoms and increases the probability that causal factors may be discovered.

Discovery: There are multiple removals from systems.

Assessment: Because failure mode analysis shows that most removals are of failed components, multiple removals may be

indicative of atypical batches of components or environmental stresses. Unfortunately, there was not the ability to track particular components back into the manufacturing processes and as discussed the timeliness and incompleteness of information precluded the possibility to quickly respond to multiple removals in order to understand possible causal factors due to environmental or usage conditions.

Recommendations and Actions: The organization has resolved future manufacturing tractability issues with the implementation and maintenance of a Device History Log. The ability to respond quickly to multiple removals is dependent on rapid acquisition of knowledge of the multiple removal event along with site information. The improved process discussed in the previous section addresses the speed of acquisition of information. Additionally, possible multiple removals will generate a complaint in the Complaint Process which initiates investigation.

Discovery: The appearance that some of the BX generation of unit have a higher ROCOF than other units with higher component A ROCOF being of particular concern.

Assessment: On an ongoing basis, with the process changes discussed earlier, this type of issue will be more rapidly identifiable, manufacturing issues will be more readily traceable, and solutions therefore implemented.

Recommendations and Actions: The BX system with the most failures in the USDS database (38) had a one time removal of 13 C components for unknown reasons. This event is no longer traceable, but anecdotal evidence suggests that this was an attempt to replace two components. The service technician either was incorrectly installing replacements or the replacements were defective upon arrival at the site. Multiple attempts were made to replace the two components with functioning replacements.

The system with the second most removals in the USDS database also has had the most components traceable in the return parts

database. The cumulative returns plot vs. system age plot for this second system is shown in Figure 44. Since the majority of those failures are due to part A (45 of 85), the cumulative part A failures are also shown. Anecdotal evidence is that an aged substandard batch of subcomponents was used in the construction of the component A's for this system. While hard evidence to support this conjecture is missing, 12 of 19 component A's with failure modes recorded do not provide evidence to the contrary.

There is evidence to support a similar claim concerning the BX unit with the second most failures in the return parts database. A batch of subcomponents with nominally similar performance, but different configuration was used to construct component A's for this system. The cumulative failures versus system age plot is shown in Figure 45. We observe that our new process of reliability monitoring would have indicated that there were problems with this system early in system life.

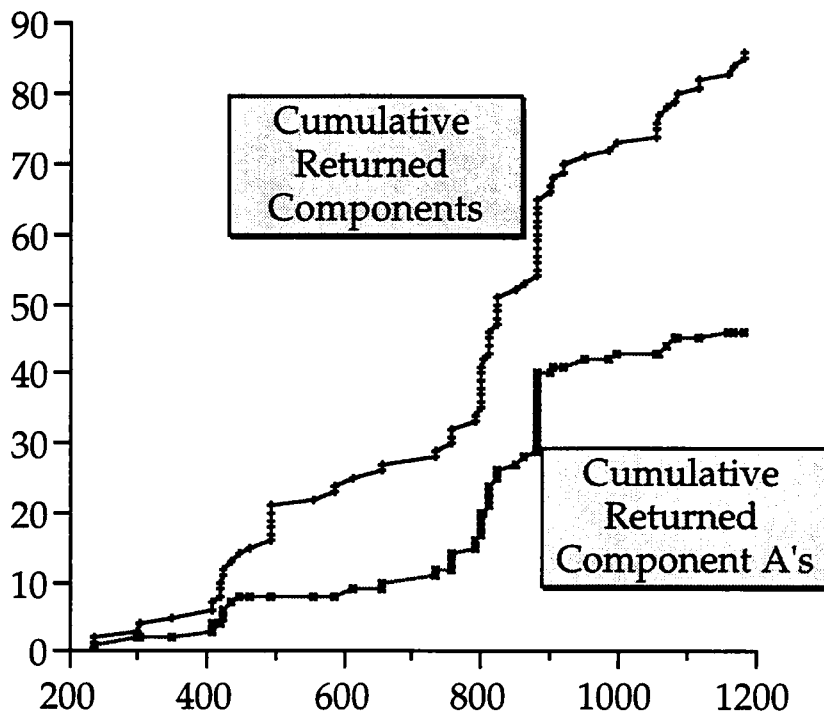


Figure 44: Most Problematic BX System - Cumulative Returns

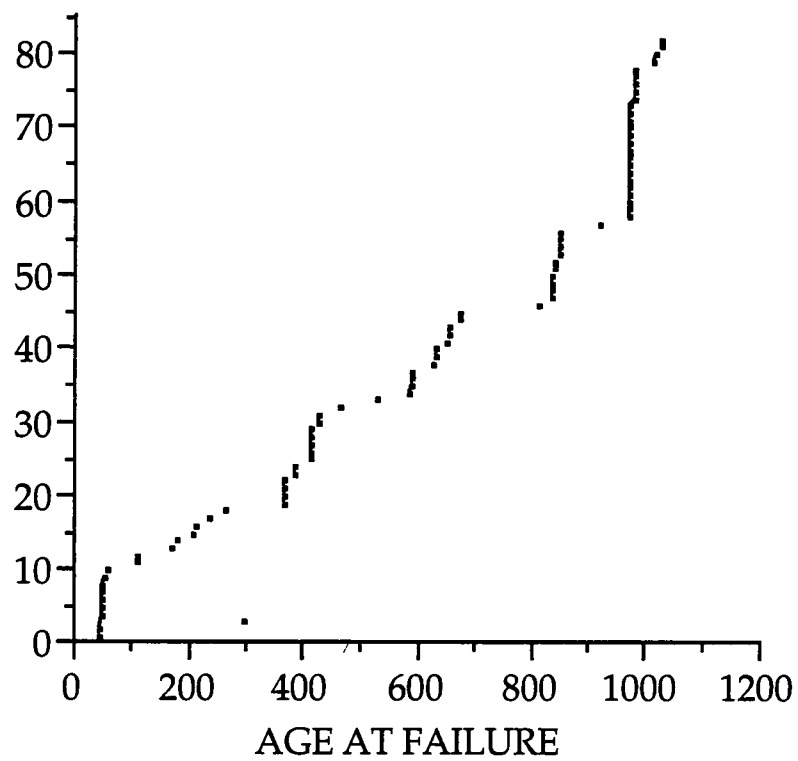


Figure 45: Cumulative Failures for A Problematic System

Discovery: A large portion of the available inventory of components A and B for generation AX is in the manufacturer's possession.

Assessment: At the current rate of removal there is enough inventory for several years of replacement for the fleet of systems. Neither the ROCOF(t), the failure modes, or possible causal mechanisms of those failure modes are indicative of a wear out portion of a hazard curve (although we would not necessarily be aware of this situation only by examining the ROCOF). The engineering assessment is that there is no additional information to be obtained from more components with the same failure mechanism.

Recommendations and Actions: Since the organization is operating under fiscal and resource constraints, it was determined that analysis of returned AX components is no longer required. Cumulative graphs of removals will continue. If there is any indication of an increasing ROCOF(t) then analysis will be resumed. Multiple removal events will also initiate failure mode investigation.

Discovery: The most prevalent failure mode for generation BX component A's is indicative of certain conditions from which the component should be protected. Engineering examination of the returned components indicates deterioration of a rubber subcomponent designed to protect component A. The rubber subcomponents used are from widely differing batches and wear-out is possible at widely varying ages. The subcomponents are very inexpensive and a decision for field inspection with replacement if required was made.

Discovery: The current process (although not completely implemented) does provides information to engineering and management. Included in this data are MCRF and failure modes for component A for generation CX.

Assessment: Engineering assessment of failure modes for component A for generation CX indicates opportunity for improvements (identification of causal structure).

Recommendations and Actions: Subsequent systems are deployed with the new configuration of component. Figure 46 shows the MCRF before and after the new configuration. The multiple replacement event at 341 days for the new configuration was also quickly understood. Corrective action was required for that system, but not across the fleet.

## **The Future**

The processes that connect the manufacturer to the customer need to continue to be strengthened. These include the transfer of customer reliability experiences via component removal information. This process will not provide all information on customer expectations and values that is necessary to guide and direct organizational activity. The software complaint database under development is an advance in the right direction. However, more information concerning customer expectations and values is required. Customer surveys and Technical Advisory Groups are an additional avenue to obtain customer information and have been implemented.

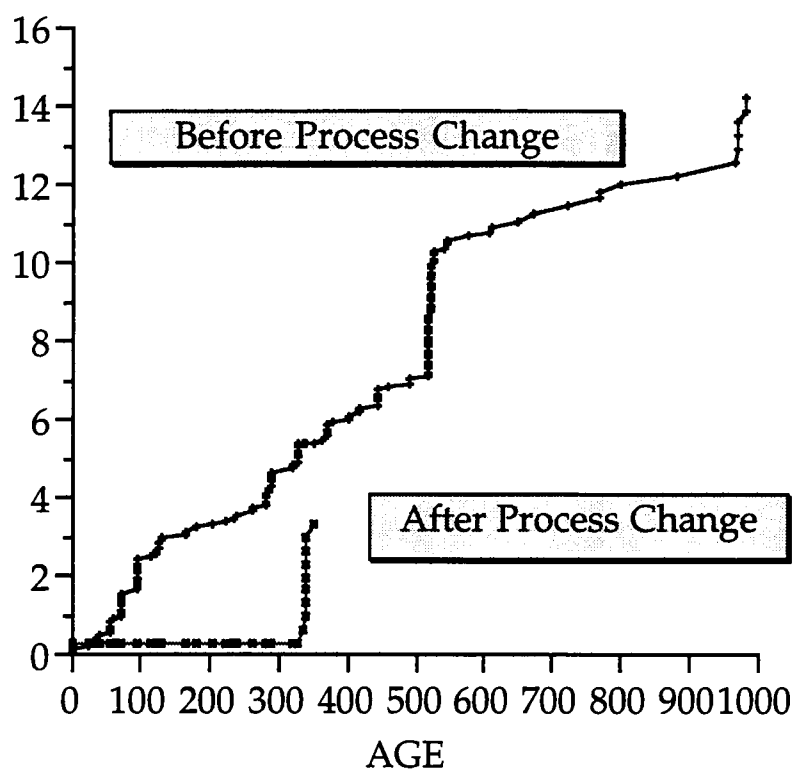


Figure 46: MCRF for A Components for Generation CX

Information obtained from these processes is an input into continual improvement of the reliability processes. For instance, it is becoming apparent that a customer description of increased reliability implies more than hardware reliability. Specifically, clinical sites are more concerned with patient throughput rates rather than hardware reliability per se.

It has been proposed that a metric of system availability or patient throughput (for operating days) be used to manage systems in accordance with customer values. This metric could be reported and communicated with Mean Cumulative and Individual Cumulative Graphs. This is a potentially more powerful metric than component removals because there are multiple reasons for variable patient throughput rate at a site. System downtime during the operating day due to component replacement would contribute to a decrease in throughput rates and increased variability in throughput. However, ease of interface between system operator and software also might impact throughput rates. Similarly, software efficiency and efficacy of first time patient passage also impact throughput and availability.

These metrics would allow an organizational understanding of a broadened system's performance to customer values. The broadened system would include not only the medical equipment discussed in this dissertation, but also the entire process of readying and evaluating the patient. This broadened scope allows identification of opportunities to increase customer value.

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## **APPENDIX**

## **Statistical Tests**

We examine the USDS data statistically with the Reverse Arrangement Test, the Runs Up And Down Test and the Median Runs Test.

### **Monotonic Trend**

To formally determine whether trends over the year of USDS data are present, The Reverse Arrangement Test (RAT) developed by Kendall and tabled by Mann is applied. The RAT test a null hypothesis of a renewal process against an alternative of a monotonic trend in ROCOF. The p values are reported in Table 11. Multiple parts replaced on the same day result in one or interval lengths of '0'.

When the test results are pooled with the method of Fisher, the resulting p-value is 0.1237 which indicates no overall evidence of trend.

The BX generation RAT results are shown in Table 12.

Table 11: AX RAT Results

| System | # Failures | Sum Reversals | p-value |
|--------|------------|---------------|---------|
| AX1    | 27         | 210           | 0.93    |
| AX2    | 24         | 111           | 0.09    |
| AX3    | 25         | 169.5         | 0.82    |
| AX4    | 27         | 136           | 0.05    |
| AX5    | 18         | 58.5          | 0.09    |
| AX6    | 21         | 100.5         | 0.40    |

Table 12: BX RAT Results

| System | # Failures | Sum Reversals | p-value |
|--------|------------|---------------|---------|
| BX1    | 6          | 8             | 0.64    |
| BX2    | 9          | 12.5          | 0.98    |
| BX3    | 14         | 51.5          | 0.76    |
| BX4    | 35         | 349           | 0.93    |
| BX5    | 28         | 155.5         | 0.1     |
| BX6    | 4          | 2             | 0.375   |
| BX7    | 6          | 3             | 0.068   |
| BX8    | 21         | 61            | 0       |
| BX9    | 5          | 1             | 0.042   |
| BX10   | 5          | 2             | 0.117   |
| BX11   | 8          | 16.5          | 0.76    |
| BX12   | 38         | 353.5         | 0.53    |
| BX13   | 3          | 0             | 0.167   |
| BX14   | 8          | 16.5          | 0.76    |
| BX15   | 7          | 8             | 0.089   |

The p-value from the results pooled via Fisher's method is 0.03, which indicates that taken collectively the failure patterns of the BX generation of scanner are worsening.

The CX generation RAT results are shown in Table 13. The p-value from the results pooled via Fisher's method is 0.5761. There is no evidence of increasing TBF's.

### **Discussion of RAT Results**

Consider that Fisher's method of pooling is designed for tests based on continuous distributions. This fact does not significantly impact the pooled calculations for the AX generation due to the close approximation of a continuous space by a variate with roughly a normal distribution. However, the BX and CX pooled RAT probabilities are impacted. In such cases, where one or more of the tests to be pooled are based on a discontinuous distribution, Fisher's transformation may be regarded as an upper bound for the true probability of the pooled tests.

Table 13: CX RAT Results

| System | # Failures | Sum Reversals | p-value |
|--------|------------|---------------|---------|
| CX1    | 16         | 57            | 0.4109  |
| CX2    | 15         | 46.5          | 0.2931  |
| CX3    | 4          | 3             | 0.625   |
| CX4    | 23         | 121.5         | 0.4061  |
| CX5    | 4          | 3             | 0.625   |
| CX6    | 7          | 8             | 0.281   |

The collective RAT p-value for the BX generation indicates that the ROCOF may still be deteriorating. The results of collective RAT p-value for the AX generation do not contradict this hypothesis. The p-value of .1203 is not overwhelming especially given the sample size and interval length. We do not want the MCRFs for the CX and future generations to follow the implied hardware replacement pattern as the BXs and AXs.

### **Other Forms of Dependency**

Mann's test, under both the null and alternative hypothesis', assumes that the interarrival times are independent. Whether the interarrival times can be considered independent is important for certain statistical tests and probabilistic models. From an engineering and business viewpoint, the question of dependency is also important. Field service engineers may damage the units when making repairs, thus leading to short interarrival times after repairs. Or, sites with dedicated service representatives tend to fail on a regular basis due somehow to the service representatives' presence. Unfortunately, we do not know which sites have dedicated

site reps. If there is evidence of too regular a pattern of part replacements, then we can provide that information to the service organization.

The Runs Up And Down Test is used to test differences of successive interarrival times between visits resulting in one or more parts replacements. If  $X_{i+1} < X_i$ , a "0" is recorded. Likewise if  $X_{i+1} > X_i$ , then "1" is recorded. A small number of runs indicates a trend. A large number of runs is an indication of serial dependency (perhaps indicative of too regular a pattern of part replacement activity by the service representatives). In this case, the suggested dependency would be further investigated with data from other sources. Ties are not considered and the first left-truncated interarrival time is not used in the calculations. The p-values for this test are obtained from Edgington (1961). Interarrival times are times between part replacements regardless of the number of parts replaced. The p-values for the AX generation are reported in Table 14.

Table 14: AX Runs Up And Down Test

| System | Interarrival Times | Runs | $P(r \leq \text{Runs})$ |
|--------|--------------------|------|-------------------------|
| AX1    | 21                 | 14   | .6707                   |
| AX2    | 23                 | 11   | .0356                   |
| AX3    | 16                 | 7    | .0367                   |
| AX4    | 13                 | 9    | .7942                   |
| AX5    | 10                 | 4    | .0633                   |
| AX6    | 19                 | 9    | .0523                   |

The p-value from Fisher's method of pooling is 0.0109 indicating evidence of that the interarrival times are not independent. The p-values for the BX generation are reported in Table 15.

The p-value from Fisher's method of pooling is 0.9908. There is perhaps evidence of too many runs, indicating lack of independence. The CX tests results are shown in Table 16.

The p-value from Fisher's method of pooling is 0.5097. There is no evidence of dependency.

### **Discussion of The Runs Up And Down Test Results**

The results of the Runs Up And Down Tests provide evidence that the intervals between site visits resulting in part replacement are not statistically independent. It is interesting that that for the AX generation of units there is evidence of too few runs and that for the BX generation of units evidence of too many runs. Whether this difference is due to characteristics of the units themselves, or of

Table 15: BX Runs Up And Down Test

| System | Activities | Runs | $P(r \leq \text{Runs})$ |
|--------|------------|------|-------------------------|
| BX1    | 4          | 3    | 1                       |
| BX2    | 5          | 2    | .25                     |
| BX3    | 11         | 7    | .6460                   |
| BX4    | 21         | 16   | .9409                   |
| BX5    | 16         | 9    | .2975                   |
| BX6    | 2          | 1    | 1                       |
| BX7    | 5          | 3    | .7333                   |
| BX8    | 13         | 7    | .2749                   |
| BX9    | 3          | 2    | 1                       |
| BX10   | 3          | 2    | 1                       |
| BX11   | 5          | 4    | .8306                   |
| BX12   | 20         | 12   | .3873                   |
| BX13   | 1          | 0    | -                       |
| BX14   | 7          | 6    | 1                       |
| BX15   | 6          | 3    | .4139                   |

Table 16: CX Runs Up And Down Test

| System | Activities | Runs | $P(r \leq \text{Runs})$ |
|--------|------------|------|-------------------------|
| CX1    | 7          | 5    | .8921                   |
| CX2    | 13         | 6    | .0964                   |
| CX3    | 2          | 1    | 1                       |
| CX4    | 14         | 7    | .1534                   |
| CX5    | 3          | 1    | .3333                   |
| CX6    | 6          | 4    | .8306                   |

the maintenance policies, or some other unknown factor is unanswered at this point.

### **Median test**

The Runs Up And Down Test is the best nonparametric test for detecting autocorrelation trends. Other types of dependency are best detected with a Median Runs Test. If an interarrival time is below the median for a particular unit then a "-1" is recorded. A "1" is recorded if the interarrival time is greater than the median. The number of points above and below the median ( $N_1$  and  $N_2$  respectively), the number of runs, and the p-values are reported. The median test is done in two fashions. First, multiple parts replaced in a single visit result in one or more '0's being used in the calculation of the median of the interarrival times. Second, interarrival times are calculated between part replacements, even if a site visit results in multiple parts replaced. The test results are shown in Tables 17, 18, 19, 20, 21, and 22. No additional forms of dependency are noted.

Table 17: AX Median Runs Tests

| System | Failures | Runs | N1 | N2 | $P(R \leq \text{Obs}R)$ |
|--------|----------|------|----|----|-------------------------|
| AX1    | 27       | 11   | 13 | 13 | .1566                   |
| AX2    | 24       | 11   | 10 | 11 | .5                      |
| AX3    | 25       | 14   | 12 | 12 | .4211                   |
| AX4    | 27       | 5    | 13 | 13 | .00019                  |
| AX5    | 18       | 9    | 8  | 8  | .5952                   |
| AX6    | 21       | 10   | 10 | 10 | .4141                   |

Table 18: AX Median Runs Tests II

| System | Failures | Runs | N1 | N2 | $P(R \leq \text{Obs}R)$ |
|--------|----------|------|----|----|-------------------------|
| AX1    | 22       | 10   | 10 | 8  | .6209                   |
| AX2    | 24       | 11   | 10 | 11 | .5                      |
| AX3    | 17       | 5    | 7  | 6  | .1212                   |
| AX4    | 14       | 6    | 6  | 6  | .3918                   |
| AX5    | 11       | 5    | 5  | 5  | .3571                   |
| AX6    | 20       | 8    | 9  | 9  | .2380                   |

Table 19: BX Median Runs Tests

| System | Failures | Runs | N1 | N2 | P( $R \leq \text{Obs}R$ ) |
|--------|----------|------|----|----|---------------------------|
| BX1    | 6        | 2    | 2  | 2  | .3333                     |
| BX2    | 9        | 5    | 4  | 4  | .6286                     |
| BX3    | 14       | 9    | 6  | 6  | .9329                     |
| BX4    | 35       | 16   | 16 | 16 | .4311                     |
| BX5    | 28       | 10   | 13 | 13 | .08118                    |
| BX6    | 4        | 2    | 1  | 1  | ---                       |
| BX7    | 6        | 4    | 2  | 2  | .1                        |
| BX8    | 21       | 6    | 9  | 9  | .04447                    |
| BX9    | 5        | 2    | 2  | 2  | .3333                     |
| BX10   | 5        | 2    | 2  | 2  | .3333                     |
| BX11   | 8        | 2    | 3  | 3  | .1                        |
| BX12   | 38       | 7    | 18 | 15 | .0001351                  |
| BX13   | 3        | 2    | 1  | 1  | ----                      |
| BX14   | 8        | 2    | 3  | 3  | .1                        |
| BX15   | 7        | 3    | 3  | 3  | .3                        |

Table 20: BX Median Runs Tests II

| System | Failures | Runs | N1 | N2 | P( $R \leq \text{Obs}R$ ) |
|--------|----------|------|----|----|---------------------------|
| BX1    | 5        | 2    | 2  | 2  | .3333                     |
| BX2    | 6        | 3    | 2  | 2  | .6667                     |
| BX3    | 12       | 7    | 5  | 5  | .8333                     |
| BX4    | 22       | 8    | 9  | 9  | .2380                     |
| BX5    | 17       | 8    | 8  | 8  | .4048                     |
| BX6    | 3        | 2    | 1  | 1  | -----                     |
| BX7    | 6        | 4    | 2  | 2  | 1                         |
| BX8    | 14       | 6    | 6  | 6  | .3918                     |
| BX9    | 4        | 2    | 1  | 1  | -----                     |
| BX10   | 4        | 2    | 1  | 1  | -----                     |
| BX11   | 6        | 4    | 2  | 2  | 1                         |
| BX12   | 23       | 12   | 11 | 11 | .5900                     |
| BX13   | 2        | 0    | 0  | 0  | -----                     |
| BX14   | 8        | 2    | 3  | 3  | .1                        |
| BX15   | 7        | 3    | 3  | 3  | .3                        |

Table 21: CX Median Runs Tests

| System | Failures | Runs | N1 | N2 | $P(R \leq \text{Obs}R)$ |
|--------|----------|------|----|----|-------------------------|
| CX1    | 16       | 1    | 7  | 0  | -----                   |
| CX2    | 15       | 7    | 6  | 6  | .6082                   |
| CX3    | 4        | 2    | 1  | 1  | -----                   |
| CX4    | 23       | 9    | 11 | 11 | .1349                   |
| CX5    | 4        | 2    | 1  | 1  | -----                   |
| CX6    | 7        | 5    | 3  | 3  | .9                      |

Table 22: CX Median Runs Tests II

| System | Failures | Runs | N1 | N2 | P( $R \leq \text{Obs}R$ ) |
|--------|----------|------|----|----|---------------------------|
| CX1    | 8        | 6    | 3  | 3  | 1                         |
| CX2    | 14       | 7    | 6  | 5  | .7381                     |
| CX3    | 3        | 2    | 1  | 1  | -----                     |
| CX4    | 15       | 5    | 7  | 7  | .0775                     |
| CX5    | 4        | 2    | 1  | 1  | -----                     |
| CX6    | 7        | 5    | 3  | 3  | .9                        |

## VITA

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