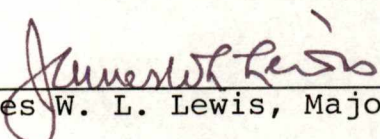
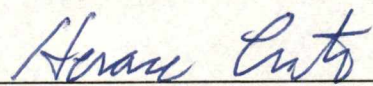


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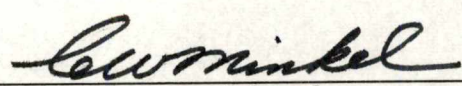
I am submitting herewith a thesis written by Michael Steven Smith entitled "Processes Occurring in a Pressure-Driven Shock Tube." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Physics.


James W. L. Lewis, Major Professor

We have read this thesis
and recommend its acceptance:

Accepted for the Council:


The Graduate School

PROCESSES OCCURRING IN A PRESSURE-DRIVEN
SHOCK TUBE

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Michael Steven Smith

August 1984

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I am especially grateful to my wife, Karen, and daughters, Alissa and Rhianna, for encouragement and patience.

ABSTRACT

Calculations involving the processes occurring within a pressure-driven shock tube are described in this work. Ideal incident and reflected shock-wave theory are reviewed. Nonequilibrium vibrational relaxation and laminar boundary layer theory are discussed.

A numerical integration of the incident and reflected shock conservation equations is performed. Results are presented for diatomic nitrogen (N_2) gas over the incident shock Mach number range of $3 \leq M_s \leq 6$ and over the temperature range of $800 \leq T \leq 4,000$ K. It is shown that, as a result of the rapid vibrational excitation in the reflected shock region, the reflected shock front, once fully excited, accelerates as it propagates into the oncoming incident shock-wave of fractional vibrational excitation.

This analysis was accompanied by a shock tube experiment which was performed prior to the completion of these calculations. The shock tube system was designed and constructed to provide a high-temperature, local-thermal-equilibrium gas sample for the determination of Raman scattering cross sections. The shock tube was designed for operation over the temperature range of $800 \leq T \leq 4,000$ K and the number density range of $0.2 \leq n \leq 2.1$ amagat. The fabricated steel shock tube was 6.98-m long with a 6.35-cm inner diameter; the driven/driver section length ratio was 1.88.

The initial demonstration of a laser Raman shock tube system for high-temperature cross section and calibration measurements was accomplished. The incident Mach number range of 4 to 5 was studied with the He driver gas and N_2 as the driven species using a seamless steel shock tube. The Raman spectra obtained with the seamless steel tube revealed excessively high contributions to the scattering signal due to effects of rust particles which had been scrubbed from the tube wall. To minimize this problem a stainless steel liner was inserted into the seamless steel tube, and these effects were reduced to small corrections to the Raman signal. Vibrational Raman spectra of shock-heated N_2 were acquired using a frequency-doubled ruby laser, spectrometric dispersion, and a single photomultiplier tube (PMT) detection channel. The results were acquired on separate shots at two wavelengths of the vibrational Raman Stokes band, and T_v was determined. The results of these measurements yielded a mean vibrational temperature $\bar{T}_v = 2,110 \pm 220$ K. Heat-transfer gauges were used to determine the shock speed u_s and incident Mach number M_1 , which was found to be 4.96 ± 0.04 . The incident gas temperature corresponding to $M_1 = 4.96$ was approximately $T_2 = 1,600$ K, and the discrepancy between T_2 and $\bar{T}_v$ is attributed to observation in the early portion of the reflected shock region rather than the latter phase of the incident shock.

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CHAPTER I

INTRODUCTION

Laser scattering techniques, such as spontaneous and nonlinear Raman scattering, have previously been successfully employed in the diagnostics of gas flow field and combustion environments. Local species number density measurements as well as the energy distribution or enthalpy of the rotational and the individual vibrational modes of molecular species may be performed using these methods. The knowledge of the differential scattering cross section, $d\sigma/d\Omega$, is essential for quantitative determinations of number density and the energy distribution.

It is necessary to provide a gas sample with known density and temperature over the range of interest of these parameters. The temperature range of interest for the various applications of Raman scattering is quite broad and spans the nominal range of 20 to 4,000 K. Calibration and cross-section measurements for the 20 to 1,200 K range can be performed using a combination of experimental methods including expansion flow fields, cryogenically cooled gases, resistance-heated furnaces, and compression ignition engines which are operated in a noncombusting

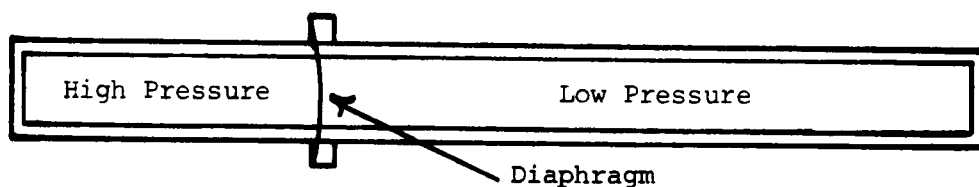
mode [1].¹ However, for the higher temperature range (1,200 to 4,000 K) the choices are less abundant.

Even though a variety of techniques exists to heat gases, for cross-section measurements it is essential that the gas be in a state of local thermal equilibrium (LTE), at least with respect to the modes of molecular motion involved in the Raman process. Because of this, the available methods for providing the LTE gas sample are reduced to essentially two: the flat-flame burner [2, pp. 168-170] and the shock tube [3, pp. 6-7]. Lapp and co-workers [4] have successfully used the burner method for a variety of cross-section measurements of gases which occur normally as a result of the combustion process. However, there are features of this approach which contraindicate its use for general cross-section measurements. First, the temperature range of the flat-flame burner is inadequate because 3,000 K cannot be obtained routinely. Second, unless the species of interest is a normal combustion product of the flame reactants, the selection of which determines the flame temperature, the species must be injected into the flame with a concentration sufficient for the cross-section measurement. However, this very injection can alter significantly the

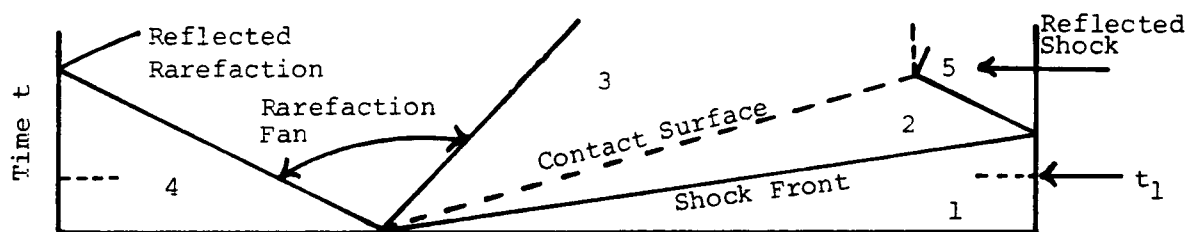
¹Numbers in brackets refer to similarly numbered references in the List of References.

properties of the flame, thereby invalidating its status as a standard. Additional flame calibrations will then be required for each concentration level of the injected species, and the precision of the cross-section measurement is degraded. Moreover, it is possible that the injected species will experience chemical reactions with the flame species, further complicating the task. Finally, for a burner of reasonable size, the magnitude of the spatial gradients of the flame properties is a serious consideration, and, quite likely, these gradients result in severe restrictions in the experimental design of the cross-section measurements.

The shock tube alternative is a relatively simple laboratory tool for the production of high-temperature, LTE gas samples. The conventional pressure-driven shock tube consists of a long tube of uniform cross section and, as illustrated in Figure 1.1(a), the tube is divided into two parts by a thin diaphragm. The driver section is filled with a light-molecular-weight gas such as hydrogen (H_2) or helium (He) at a high pressure. The driven section is filled with the experimental low-pressure gas whose properties are being studied. For ideal operation, the desired high-temperature gas sample can be provided over a range of temperatures simply by varying the ratio of the initial driver-driven gas pressures [3, pp. 18-21]. Also,



(a) Conventional Shock Tube



(b) (x-t) Diagram

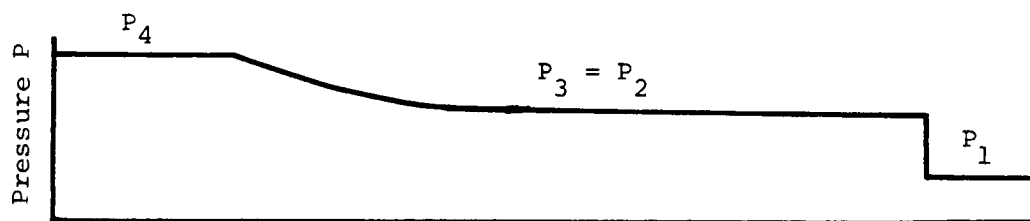
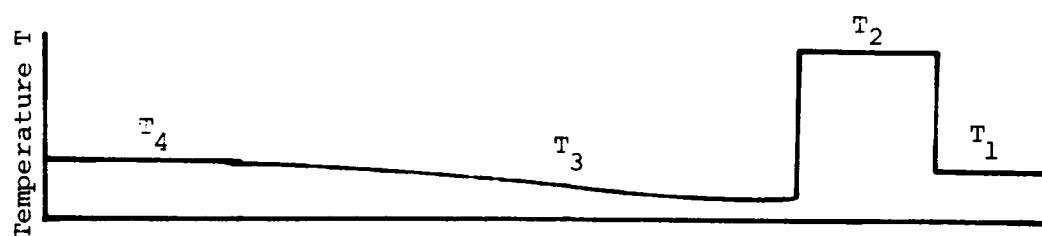
(c) Pressure Distribution along Tube at Time t_1 (d) Temperature Distribution at Time t_1

Figure 1.1. Simple Shock Tube and Thermodynamic Processes.

the shocked gas density can be varied while the shocked gas temperature is held constant by simultaneously varying the driven gas pressure and the ratio of the driver-driven gas pressures. Additional advantages of the technique include the capability of operation of the shock tube with the species of interest as a pure gas sample, unlike the combustion source, and, using a reasonable diameter shock tube, measurements can be performed, in principle, with an LTE gas sample without spatial gradients.

The diaphragm is removed, by either increasing the driver gas pressure until a natural burst occurs, or puncturing it with a fast-operating plunger mechanism. After the burst, a compression wave forms which produces a propagating shock front throughout the test gas region. As the front moves down the tube, an expansion, or rarefaction, wave moves with the local speed of sound back into the driver gas. The rarefaction front creates a gradual and continuous decrease in the pressure of the driver gas. This decrease is commonly referred to as a rarefaction fan. The driver gas and the experimental gas are in contact at the contact surface, whose arrival at the test station marks the end of the shock-heated test gas region. The development of the shock front, contact surface, and rarefaction fan are shown in Figure 1.1(b). The pressure and temperature distributions along the tube at specified

time, t_1 , are shown in Figures 1.1(c) and 1.1(d), respectively. The subscripts denote the following: 1 refers to the conditions initially throughout the experimental gas; 2 refers to conditions within the shocked gas, which is the region between the shock front and the contact surface; 3 refers to conditions between the contact surface and the rarefaction fan; 4 refers to the initial driver-gas conditions; 5 refers to conditions within the reflected shock wave, which is created when a closed-ended tube of proper length is used. The reflected gas will be at a much higher temperature, T_5 , and pressure, p_5 , due to the propagation of the reflected wave into the oncoming-shock heated or incident shock wave. Utilization of the reflected wave thus affords an even greater opportunity for the study of high-temperature phenomena.

The major disadvantage of the use of shock tubes for cross-section measurements has been the comparatively short run-times and data acquisition periods available for measurement. For example, 10 to 50 μsec represents a typical interval of time available for measurements in the incident shock region; operation of the shock tube in the reflected mode can increase this interval by over an order of magnitude [3, pp. 66-67]. Regardless of the mode of operation of the shock tube, data-acquisition times of 50 μsec to 1 msec are quite short in comparison to the time

available for continuous combustion sources. Clearly, the availability of pulsed laser sources lessens the severity of the short run-time disadvantage, and the use of a Q-switched laser source with a nominal 10-nsec pulse width easily achieves the required temporal resolution. However, the flow duration disadvantage is not eliminated because of the methods normally employed for spectral dispersion and subsequent detection. Specifically, detection and registration are desired for the spectrally resolved scattered radiation for broad wavelength regions, but practical requirements for the minimum acceptable signal-to-noise ratio (S/N) of Raman scattering and the typical pulse duration and repetition rate of the laser source prevent the use of rapid-scan spectrometers. The utilization of single or multiple photomultiplier tubes (PMT) with a fixed wavelength setting of the spectrometer is, of course, possible [1]. However, using this approach to cover the desired broad spectral region and to simultaneously provide the spectral resolution necessary for the cross-section measurements requires the use of multiple shock tube-laser source firings in conjunction with variation in the central wavelength of the spectrometer. Consequently, because of the shot-to-shot variations in both laser and shock tube parameters, the resulting cross-section data can be expected to exhibit larger than

necessary experimental uncertainty values. As a result of these anticipated difficulties, the use of shock tube flows for Raman scattering studies has not been actively pursued.

However, the relatively recent improvement and development of image intensifier detectors provides exactly the needed detection method. With this type of detector, both broad-band and high spectral resolution detection are possible for 10-nsec pulse sources. This development now provides the capability of cross-section measurements, using single laser firings, thus removing a major obstacle in the use of the shock tube for cross-section measurements.

This work presents details of the processes occurring within a pressure-driven shock tube. Chapter II presents a brief review of incident and reflected shock-wave theory. Shock relations are developed from the mass, momentum, and energy conservation equations. Chapter III considers the effects on shock tube performance of vibrational relaxation and boundary layer effects. Additionally, the effect of low concentration levels of water vapor on the vibrational relaxation of both the incident and reflected waves is discussed. Laminar boundary layer effects upon the incident region are included.

Chapter IV presents a computer simulation of the aforementioned processes. The effects of vibrational relaxation and boundary layers are presented graphically.

Chapter V considers a shock tube-Raman scattering system experiment participated in by the author [5]. The experiment, performed before the relaxational and boundary layer calculations were developed, is analyzed and the need for the inclusion of these effects is shown.

Chapter VI presents a summary and the conclusions of this investigation.

CHAPTER II

IDEAL SHOCK TUBE THEORY

Incident Shock-Wave Theory

Particle motion throughout the shocked gas region is most easily studied in a system of coordinates which assumes that the shock front is fixed. Such a system is illustrated in Figure 2.1. As can be seen, the relationship between the laboratory and shock-fixed velocities is given by:

$$u_1 = u_s - v_1$$

and

$$u_2 = u_s - v_2 ,$$

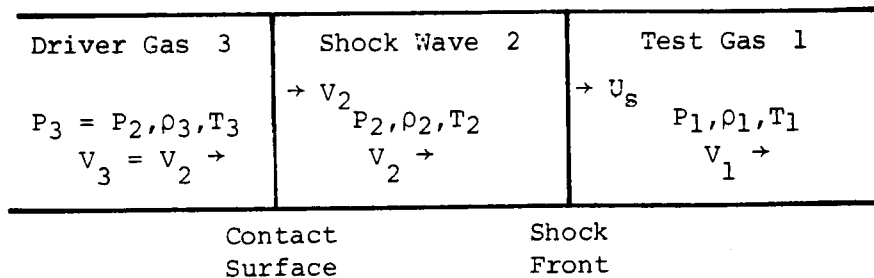
where u_s is the shock-front velocity relative to the shock tube; v_i and u_i represent the laboratory and shock-fixed gas molecule velocities, respectively.

In the shock-fixed system, the mass, momentum, and energy conservation equations, respectively, are:

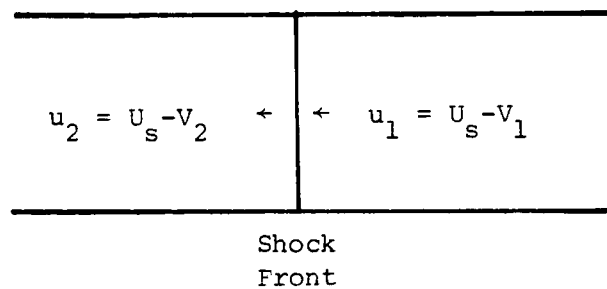
$$\rho_1 u_1 = \rho_2 u_2 , \tag{1}$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 , \tag{2}$$

$$h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2} u_2^2 , \tag{3}$$



(a) Laboratory-Fixed Coordinates



(b) Shock-Fixed Coordinates

Figure 2.1. Gas Parameters Associated with a Shock Wave in the Two Basic Coordinate Systems.

where ρ_i , p_i , and h_i are the density, pressure, and specific enthalpy, respectively, of the gas in the specified regions. Equation (3) assumes no heat loss to the tube wall, which is a valid assumption [3, p. 14], since the hot flow generally lasts for only a few hundred microseconds.

Casting the specific enthalpy, h_i , in terms of the ratio of the specific heats at constant pressure and constant volume, c_p and c_v , respectively, leads to

$$h_i = \frac{\gamma_i}{\gamma_i - 1} RT_i , \quad (4)$$

where $\gamma_i = c_{pi}/c_{vi}$, and R is the specific gas constant, $R = \hat{R}/M$, where $\hat{R}$ is the universal gas constant and M is the gram molecular weight. Using this result, along with the equation of state for an ideal gas,

$$p_i = \rho_i RT_i , \quad (5)$$

the following relationships can be developed [6, pp. 26-31] using the previously defined Equations (1), (2), and (3):

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma - 1} , \quad (6)$$

$$\frac{T_2}{T_1} = \left(\gamma M_1^2 - \frac{\gamma-1}{2} \right) \frac{\left(\frac{\gamma-1}{2} M_1^2 + 1 \right)}{\left(\frac{\gamma+1}{2} M_1 \right)^2}, \quad (7)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{((\gamma-1)M_1^2-2)}, \quad (8)$$

where γ_i becomes γ since vibrational excitation is not assumed in the ideal theory, and $M_1 = u_1/a_1$ is the incident Mach number, where a_1 is the local speed of sound in the test gas. Thus, if the initial conditions are known, and a measurement of the shock speed, u_s , is made, the parameters within the shocked region may be determined.

Equations (6), (7), and (8) apply only in the case that γ is constant. For a real molecular gas, the temperature dependence of γ must be considered.

Figure 2.2 illustrates the shock speed u_s and the pressure, temperature, and density ratios as a function of incident Mach number for a gas with $\gamma=7/5$.

Reflected Shock-Wave Theory

If the incident shock parameters are known the reflected properties may be calculated. Figure 2.3 illustrates a coordinate system in which it is assumed that the reflected shock front is fixed. As can be seen, the incident shock-heated gas molecules travel into the reflected shock front with a relative velocity

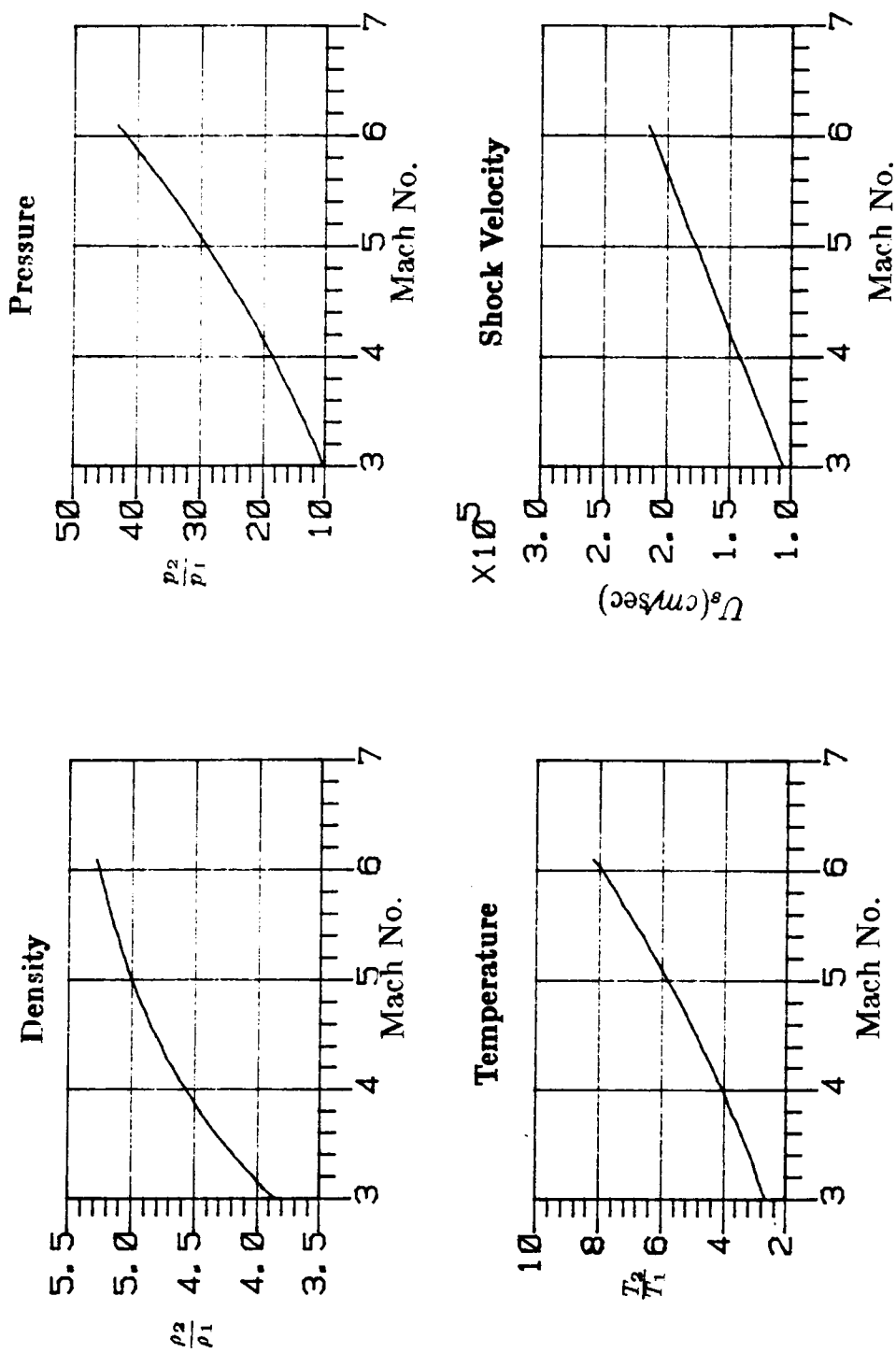
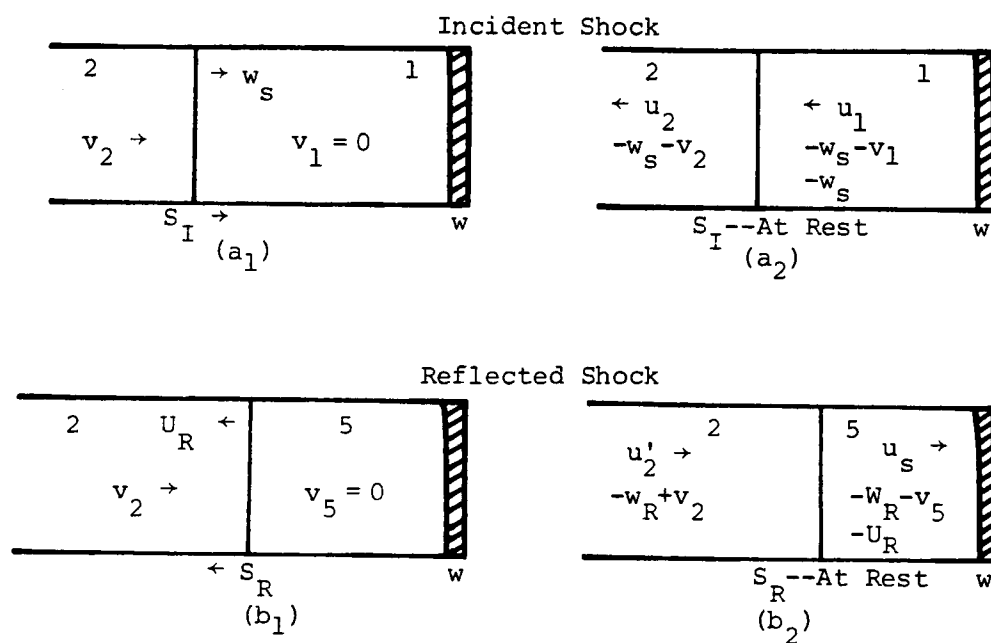


Figure 2.2. Ideal Incident Shock Parameter Ratios as a Function of Incident Mach Number for $\gamma = 7/5$.



(a₁) and (b₁) are as measured in laboratory-fixed coordinates.
 (a₂) and (b₂) are as measured in shock-fixed coordinates.

Figure 2.3. Reflected Fixed Coordinates.

$$u_2' = u_R + v_2 = u_R + u_1 - u_2 ,$$

and the particle velocity within the reflected shock is given by

$$u_5 = u_R - v_5 = u_R ;$$

but, since the reflected front is fixed, $v_5=0$. As before, the reflected shock Mach number M_5 can be defined as $M_5=u_2'/a_2$. Similarly, conservation of mass, momentum, and energy leads to the following:

$$\rho_5 u_5 = \rho_2 (u_1 - u_2 + u_5) , \quad (9)$$

$$p_5 + \rho_5 u_5^2 = p_2 + \rho_2 (u_1 - u_2 + u_5)^2 , \quad (10)$$

$$h_5 + \frac{1}{2} u_5^2 = h_2 + \frac{1}{2} (u_1 - u_2 + u_5)^2 . \quad (11)$$

Using methods [6, pp. 52-56] similar to those for the previous section, the ideal reflected shock parameter ratios are given by the following:

$$\frac{p_5}{p_1} = \left[\frac{[2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)} \right] \left[\frac{[(3\gamma - 1)M_1^2 - 2(\gamma - 1)]}{[(\gamma - 1)M_1^2 + 2]} \right] , \quad (12)$$

$$\frac{T_5}{T_1} = [2(\gamma - 1)M_1^2 + (3 - \gamma)] \left[\frac{(3\gamma - 1)M_1^2 - 2(\gamma - 1)}{(\gamma + 1)^2 M_1^2} \right] , \quad (13)$$

$$\frac{\rho_5}{\rho_1} = \frac{\frac{p_5}{p_1}}{\frac{T_5}{T_1}} . \quad (14)$$

Figure 2.4 illustrates the variation of the ratios p_5/p_2 , T_5/T_2 , and ρ_5/ρ_2 with incident Mach number, M_1 , for a gas with $\gamma=7/5$.

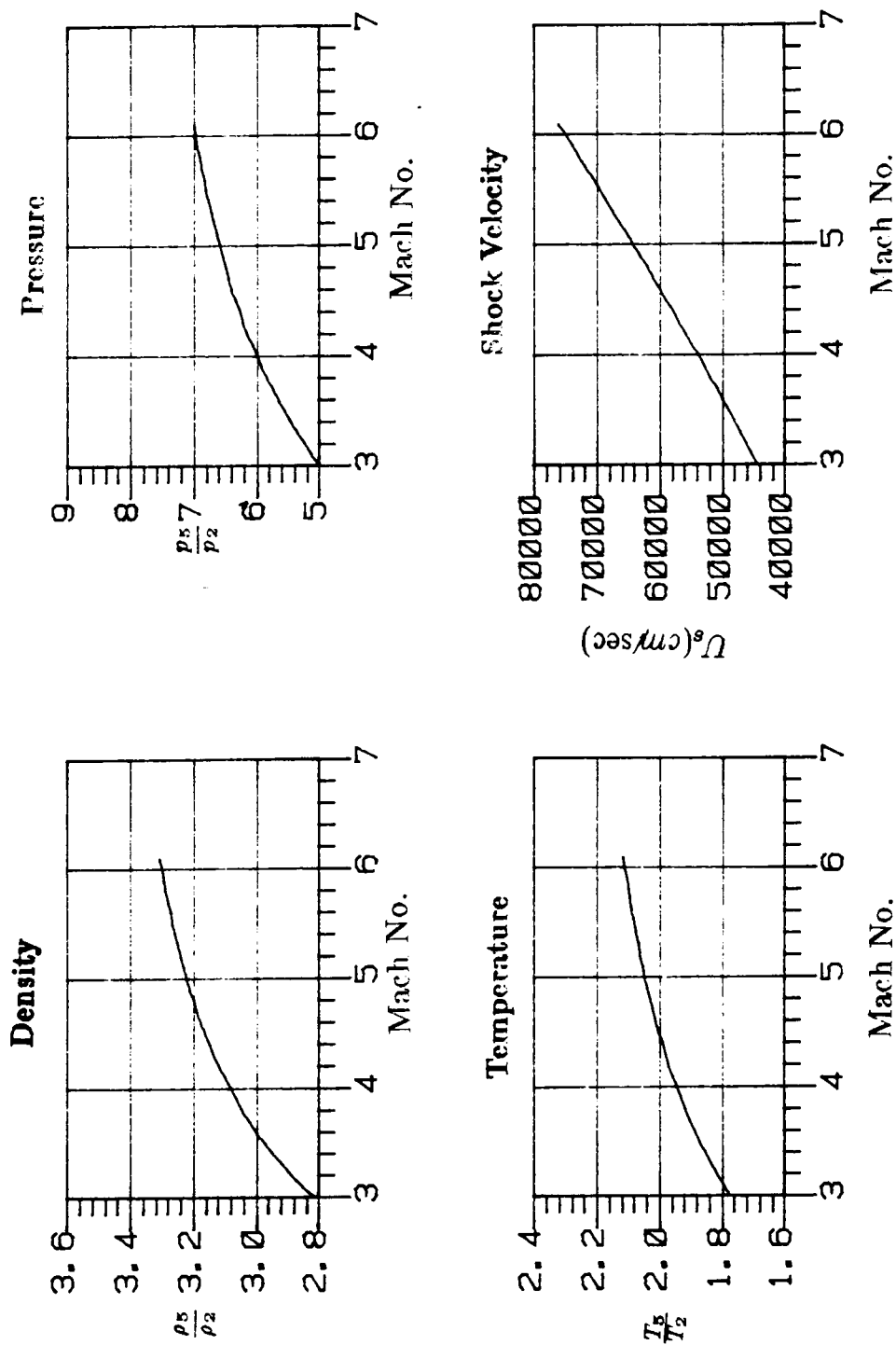


Figure 2.4. Ideal Reflected Shock Parameter Ratios as a Function of Incident Mach Number for $\gamma = 7/5$.

CHAPTER III

NONEQUILIBRIUM PROCESSES

Vibrational Relaxation

Ideal shock tube theory must be modified if the gas undergoes a nonequilibrium process such as vibrational relaxation or chemical dissociation. For this situation, the flow properties may be calculated by integrating the differential conservation equations.

For the incident shock, differentiation of the mass equation gives

$$u_1 \frac{d\rho_1}{dx} + \rho_1 \frac{du_1}{dx} = u_2 \frac{d\rho_2}{dx} + \rho_2 \frac{du_2}{dx} . \quad (15)$$

Similarly, for momentum conservation,

$$\frac{dp_1}{dx} + u_1^2 \frac{d\rho_1}{dx} + 2\rho_1 u_1 \frac{du_1}{dx} = \frac{dp_2}{dx} + u_2^2 \frac{d\rho_2}{dx} + 2\rho_2 u_2 \frac{du_2}{dx} . \quad (16)$$

The energy conservation equation yields,

$$\frac{dh_1}{dx} + u_1 \frac{du_1}{dx} = \frac{dh_2}{dx} + u_2 \frac{du_2}{dx} . \quad (17)$$

In most cases, region 1 contains a static gas of constant thermodynamic properties. Thus, the above

relations may be expressed in terms of the region 2 properties as follows:

$$\frac{d\rho_2}{dx} = - \frac{\rho_2}{u_2} \frac{du_2}{dx} , \quad (18)$$

$$\frac{dp_2}{dx} = - \rho_2 u_2 \frac{du_2}{dx} , \quad (19)$$

$$\frac{dh_2}{dx} + u_2 \frac{du_2}{dx} = 0 . \quad (20)$$

The specific enthalpy, h_i , may be determined from the defining equation [3, p. 15],

$$h_i = e_i + RT_i , \quad (21)$$

where e_i is the specific internal energy. Differentiating Equation (21) leads to

$$\frac{dh_i}{dx} = \frac{de_i}{dx} + R \frac{dT_2}{dx} + T_2 \frac{dR}{dx} , \quad (22)$$

where

$$\frac{dR}{dx} = - \frac{R}{M^2} \frac{dM}{dx} = 0 .$$

If the shocked gas is undergoing molecular dissociation, M may be expressed as $M = \sum_i X_i M_i$, where X_i is the mole fraction of species i . Thus, by casting M in

differential form, a method of tracking the individual constituents of the gas exists. An example is the dissociation of ammonia gas, NH_3 , into its constituent species, such as NH_2 , N_2 , H_2 , N_2H_3 , and N_2H_4 .

The specific internal energy,

$$e_i = \frac{e_{\text{int}}}{M} ,$$

may be expressed in terms of the individual contributions from the translational, rotational, and vibrational motion as

$$e_{2\text{int}} = e_{2\text{trans}} + e_{2\text{rot}} + e_{2\text{vib}} , \quad (23)$$

where, for a diatomic molecule,

$$e_{\text{trans}} = \frac{3}{2} RT_{\text{trans}} \quad \text{and} \quad e_{\text{rot}} = RT_{\text{trans}} ,$$

where it is assumed that the translational and rotational motion are in equilibrium with one another. This is a valid assumption, since rotational motion requires only a few collisions to equilibrate with translational motion [7, pp. 1-5], and this occurs within a few mean-free-paths' travel of the molecule. It cannot, in general, be assumed that the vibrational motion is in equilibrium with the rotational or translational motion, since many more collisions are required [7, pp. 1-5]. This model

assumes a harmonic oscillator potential and only vibrational-translational excitation is considered. It has been shown [8] that vibrational-vibrational collisions may be ignored for vibrational relaxation processes behind gasdynamic shocks.

Assuming initial static conditions, the internal energy derivative may be written, with the appropriate substitutions for e_{trans} and e_{rot} , as,

$$\frac{de_2}{dx} = \frac{3}{2} R \frac{dT_2}{dx} + R \frac{dT_2}{dx} + \frac{de_{2vib}}{dx}, \quad (24)$$

where $T_2 = T_{2trans}$. The translational temperature may be obtained from the equation of state,

$$p_2 = \rho_2 R T_2. \quad (25)$$

The vibrational energy derivative, de_{2vib}/dx , may be obtained using the principle of detailed balancing [9, pp. 198-206] and simple harmonic oscillator partition function theory. This is outlined in Appendix A. The end result is

$$\frac{de_{2vib}}{dx} = - \frac{e_{2vib} - e_{2vib}^0}{\tau u_2}, \quad (26)$$

where $e_{2\text{vib}}^0$ is the vibrational energy at the equilibrium or translational temperature, T_2 , and τ is the vibrational relaxation time which is determined from experimental kinetic-rate data. For this work diatomic nitrogen, N_2 , was used as the experimental gas. The temperature variation of τ was assumed to be that given by Anderson [10, p. 127],

$$\log_{10}(\tau p) = 93T^{-1/3} - 4.61(\mu\text{sec-atm}) , \quad (27)$$

where p is the pressure of the gas in atmospheres, and T is the local translational temperature in Kelvin.

In the event that there are impurities present within the experimental gas, the relaxation process will generally occur at a faster rate. It can be shown [7, pp. 30-32] that, for example, for trace H_2O vapor of mole fraction C in an H_2O/N_2 mixture, the vibrational relaxation time for N_2 is given by

$$\frac{1}{\tau} = \frac{C}{\tau_{N_2-H_2O}} + \frac{1-C}{\tau_{N_2-N_2}} , \quad (28)$$

where $\tau_{N_2-N_2}$ is the previously defined pure N_2 relaxation time, and $\tau_{N_2-H_2O}$ is the de-excitation time of an N_2 molecule in otherwise pure sample of H_2O vapor. Again, from Anderson [10, p. 128],

$$\log_{10}(p_{\tau_{\text{H}_2\text{O}-\text{N}_2}}) = 27.65T^{-1/3} - 3.2415(\mu\text{sec-atm}) . \quad (29)$$

Equations (18), (19), (20), (22), and (24) may be cast in terms of Equations (25) and (26), thus forming the basis for a numerical integration, the results of which are presented in Chapter IV of this study.

Reflected, ideal shock tube theory must also be modified if nonequilibrium processes are occurring. As before, the reflected flow conservation equations must be cast into differential form. Differentiating Equations (9), (10), and (11) leads to, respectively,

$$\rho_5 \frac{du_5}{dx^*} + u_5 \frac{d\rho_5}{dx^*} = \rho_2 \frac{du'_2}{dx^*} + u'_2 \frac{d\rho_2}{dx^*} , \quad (30)$$

$$\begin{aligned} \frac{dp_5}{dx^*} + 2u_5\rho_5 \frac{du_5}{dx^*} + u_5^2 \frac{d\rho_5}{dx^*} &= \frac{dp_2}{dx^*} + 2u'_2\rho_2 \frac{du'_2}{dx^*} \\ &+ u'^2_2 \frac{d\rho_2}{dx^*} , \end{aligned} \quad (31)$$

$$\frac{dh_5}{dx^*} + u_5 \frac{du_5}{dx^*} = \frac{dh_2}{dx^*} + u'_2 \frac{du'_2}{dx^*} , \quad (32)$$

where, as before, $u'_2 = u_1 - u_2 + u_5$, and x^* refers to reflected-fixed coordinates. The correlation between shock-fixed and reflected-fixed coordinates is discussed in Chapter IV.

The reflected equivalents of Equations (22) and (24) are, respectively,

$$\frac{dh_5}{dx^*} = \frac{de_5}{dx^*} + R \frac{dT_5}{dx^*} + T_5 \frac{dR}{dx^*} \quad (33)$$

and

$$\frac{de_5}{dx^*} = \frac{3}{2} R \frac{dT_5}{dx^*} + R \frac{dT_5}{dx^*} + \frac{de_{5vib}}{dx^*}, \quad (34)$$

where, as before,

$$\frac{de_{5vib}}{dx^*} = - \frac{e_{5vib}^0}{\tau u_5} \quad (35)$$

and

$$p_5 = \rho_5 R T_5. \quad (36)$$

Again, using the relaxation times of Anderson [10, p. 128], Equations (30), (31), (32), (33), and (34) may be cast in terms of Equations (35) and (36) and the previously defined incident shock differential set, thus creating the basis for a reflected numerical integration, presented with results in Chapter IV.

Boundary Layer Effects

Ideal shock tube theory predicts a continuous increase in the separation of the shock front and the contact surface as the shock front's distance from the diaphragm increases. However, due to the tube wall, a more

slowly moving and colder layer of gas exists between the hot flow and the cooler wall. This phenomenon is commonly referred to as the laminar boundary layer effect and has been investigated by many authors, the most often-quoted being Mirels [11,12].

The most pronounced effect of the boundary layer is that it can significantly shorten the calculated ideal test time in a small bore or low-pressure tube. This is because mass is lost to the boundary layer, causing the shock front and contact surface to reach a maximum separation. Once this separation has been reached, the two surfaces move with the same constant velocity.

Assuming maximum separation has been reached, the mass conservation equation may be written, using the Mirels method [11,12], which assumes inviscid flow equivalent to a channel flow having an areal distribution defined by

$$\frac{\rho_2 u_2}{\rho_{2s} u_{2s}} = \frac{A_{2s}}{A_2} ,$$

as

$$\rho_2 u_2 = \rho_{2s} u_{2s} \left(1 - \left(\frac{x}{x_{lm}} \right)^{1/2} \right) , \quad (37)$$

as illustrated in Figure 3.1. The subscript 2s refers to the properties immediately behind the shock front, x is the distance of the particle behind the front, and x_{lm} is the limiting separation of the shock front and contact

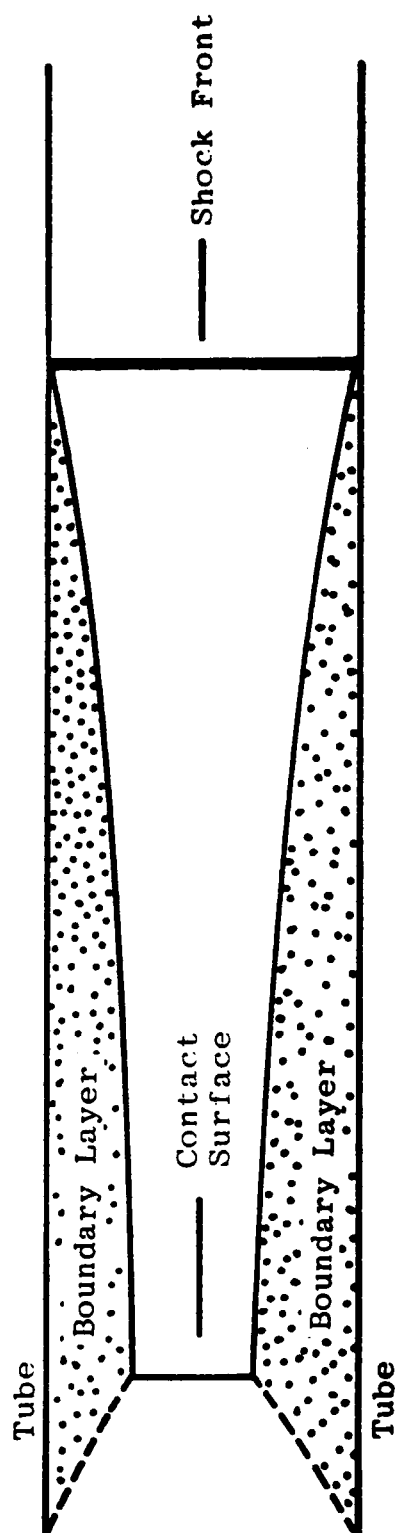


Figure 3.1. Boundary Layer in Shocked Gas.

surface. Also,

$$\rho_2 u_{2s} = \rho_1 u_s, \quad (38)$$

which is just the flux of mass entering the shock wave.

The ideal test time, t_ℓ , is the laboratory-measured time between the passage of the shock front and the measurement of a property of gas at an observation location. For the ideal case this time is given by

$$t_\ell = \int_0^x \frac{dl'}{u_s} = \frac{x}{u_s}, \quad (39)$$

where u_s is the shock-front speed and x is the distance behind the shock front. The particle time, t_p , is the time required for a particle to pass from the shock front to the distance x downstream of the shock and is given by

$$t_p = \int_0^x \frac{dl'}{u_2}. \quad (40)$$

Thus, for uniform incident-shock conditions,

$$t_p = \frac{t_\ell \int_0^x \frac{dl'}{u_2}}{\frac{x}{W_s}} = t_\ell \frac{\rho_2}{\rho_1}, \quad (41)$$

where complete vibrational excitation has been assumed for the gas in region 2. Otherwise, the integral over x must

be evaluated numerically. In any case, however, the particle time is significantly greater than the lab time.

In order to predict the effects of the wall boundary layer upon the flow properties throughout the shocked region, Equation (37) must be cast in to differential form; i.e.,

$$\frac{d\rho_2}{dx} = - \frac{\rho_2 s u_{2s}}{2u_2} (xx_{lm})^{1/2} = \frac{\rho_2}{u_2} \frac{du_2}{dx} . \quad (42)$$

Equation (42), when substituted for Equation (18), page 20, and coupled with the other differential equations of the set, forms the basis for predicting the boundary layer-corrected flow properties. The particle time, t_{pbl} , is given by

$$t_{pbl} = \int_0^l \frac{dx}{u_2} ,$$

where u_{2bl} is obtained from the solution of the u_{2bl} derivative, as outlined in Chapter IV of this investigation. For uniform conditions in region 2, assuming full vibrational excitation, it can be shown [13] that

$$\frac{t_{pbl}}{t_p} = - 2 \frac{x_{lm}}{x} \left(\frac{x}{x_{lm}} \right)^{1/2} + x_{lm} \left(1 - \left(\frac{x}{x_{lm}} \right)^{1/2} \right) , \quad (43)$$

where t_p is the uncorrected particle time. The ratio $t_{pb\ell}/t_p$ is shown as a function of $x/x_{\ell m}$ in Figure 3.2, and it is seen that the correction can be significant. As will be seen in Chapter IV, the boundary layer causes some significant changes to the ideally expected flow properties.

The above results depend upon a value for the maximum separation distance, $x_{\ell m}$. A method [11] for establishing $x_{\ell m}$ is outlined in Appendix B.

Another interesting situation occurs when the shock tube is not of sufficient length for the limiting separation, $x_{\ell m}$, to be reached. When this happens, the length, x_ℓ , of the shocked gas arriving at the observation station must be known. This calculation [14] is outlined in Appendix C.

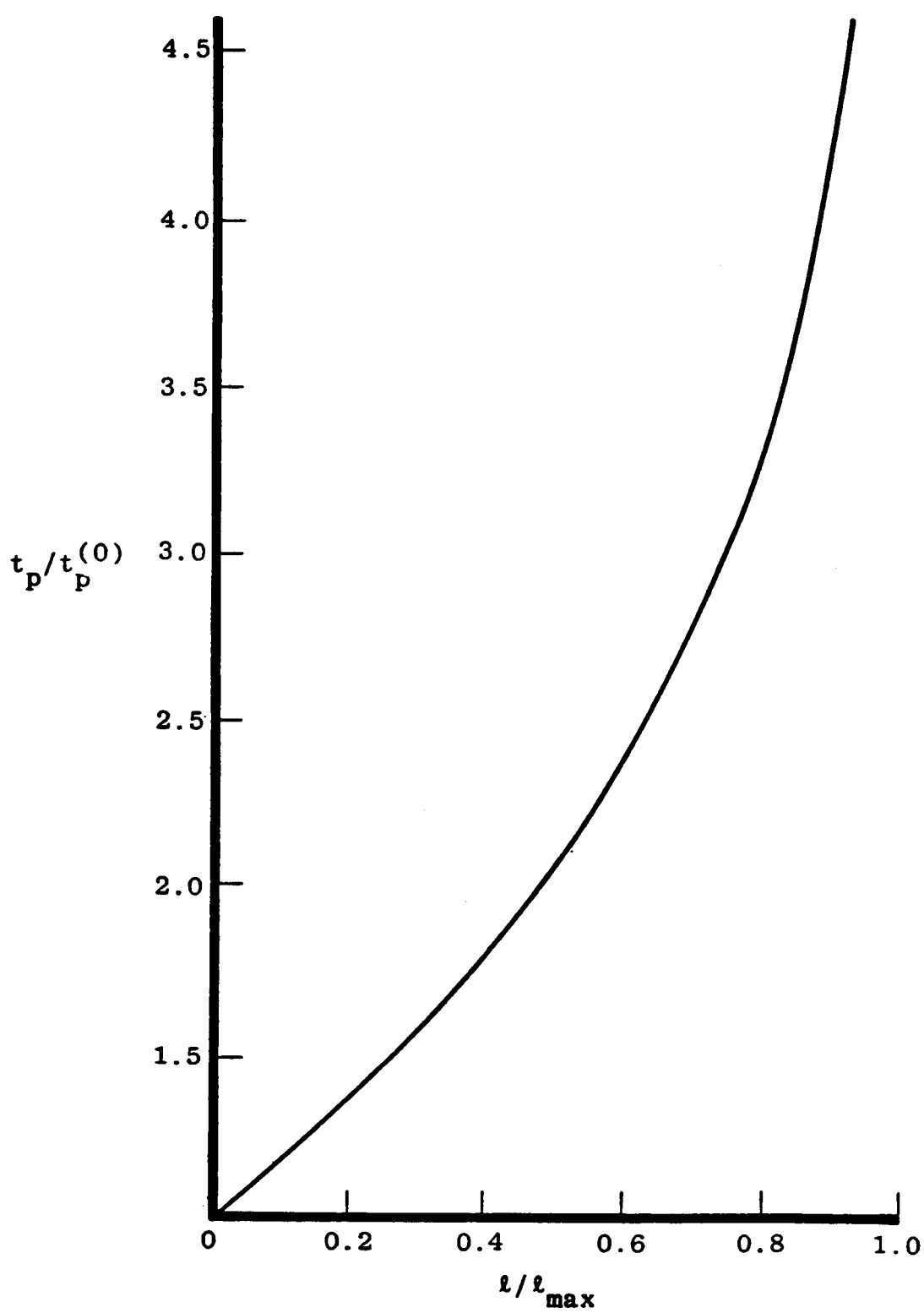


Figure 3.2. Particle Time as a Function of Shock Distance.

CHAPTER IV

COMPUTER CALCULATIONS

A numerical integration has been performed, using the equations from Chapter III. The incident shock relations used for the shock tube flow without boundary layer effects, after being case in terms of Equation (26), are given by

$$\frac{de_{2vib}}{dx} = - \frac{e_{2vib} - e_{2vib}^0}{\tau u_2} , \quad (44)$$

$$\frac{du_2}{dx} = \frac{\frac{de_{2vib}}{dx}}{\frac{5}{2} u_2 - \frac{7}{2} \frac{p_2}{\rho_2 u_2}} , \quad (45)$$

$$\frac{dp_2}{dx} = - \rho_2 u_2 \frac{du_2}{dx} , \quad (46)$$

$$\frac{dt_p}{dx} = \frac{1}{u_2} , \quad (47)$$

$$T_2 = \frac{p_2}{R \rho_2} , \quad (48)$$

$$T_{2vib} = \frac{\theta_v}{\ln(1 + \frac{R \theta_v}{e_{2vib}})} , \quad (49)$$

where t_p is the particle time of flight, and T_{2vib} is the vibrational temperature; T_{2vib} was obtained from the vibrational partition function by assuming local thermal vibrational equilibrium. This implies that there is a Boltzmann probability distribution function for the vibrational population density, which is approximately true since the vibrational-vibrational equilibrium rate is much faster than the vibrational-translational rate.

The incident shock relations used for the boundary-layer-corrected case were Equations (44), (46), (47), (48), (49), and

$$\frac{du_2}{dx} = \frac{(\frac{7}{4})p_2\rho_{2s}u_{2s}}{u_2\rho_2x_{lm}^{1/2} + \frac{de_{2vib}}{dx}}, \quad (50)$$

where $\rho_{2s}u_{2s} = \rho_1u_s = \text{constant}$. As mentioned in Chapter III, the particle time, t_{pbl} , is calculable, once u_2 is known, by integrating

$$t_{pbl} = \int_0^{\ell} \frac{dx}{u_2}. \quad (51)$$

Initial conditions for the incident cases were calculated using the ideal shock ratios of Chapter II.

For the reflected region, the following equations were integrated:

$$\frac{de_{5vib}}{dx^*} = \frac{-1}{\tau u_5} (e_{5vib} - e_{5vib}^0) , \quad (52)$$

$$\frac{du_5}{dx^*} = \frac{\frac{de_{5vib}}{dx} + A + B + C}{D} , \quad (53)$$

where

$$A = u_2' \frac{du_2}{dx^*} - \frac{dh_2}{dx^*} - u_2' \frac{du_1}{dx^*} ,$$

$$B = -\left(\frac{7}{2}\right)\left(\frac{p_5}{\rho_5}\right)\left[\frac{1}{u_5}\left(\rho_2 \frac{du_1}{dx^*} - \rho_2 \frac{du_2}{dx^*} + u_2' \frac{d\rho_2}{dx^*}\right)\right] ,$$

$$C = \left(\frac{7}{2}\right)\left(\frac{1}{\rho_5}\right)\left[\frac{dp_2}{dx^*} + 2u_2'\rho_2\left(\frac{du_1}{dx^*} - \frac{du_2}{dx^*}\right) + u_2'^2 \frac{d\rho_2}{dx^*} - u_5\left(\rho_2 \frac{du_1}{dx^*} - \rho_2 \frac{du_2}{dx^*} + u_2' \frac{d\rho_2}{dx^*}\right)\right] ,$$

$$D = [u_2' - u_5 - \frac{7}{2}\left(\frac{p_5}{\rho_5 u_5}\right)(\rho_5 - \rho_2) - \frac{7u_2'^2}{\rho_5} + 7u_5 - \frac{7}{2} \frac{u_5}{\rho_5}(\rho_5 - \rho_2)] ;$$

$$\begin{aligned} \frac{dp_5}{dx^*} &= \frac{dp_2}{dx^*} + 2u_2'\rho_2\left(\frac{du_5}{dx^*} - \frac{du_2}{dx^*} + \frac{du_1}{dx^*}\right) + u_2'^2 \frac{d\rho_2}{dx^*} \\ &\quad - 2u_5\rho_5 \frac{du_5}{dx^*} + F , \end{aligned} \quad (55)$$

where

$$F = -u_5 \left[\rho_2 \frac{du_1}{dx^*} - \rho_2 \frac{du_2}{dx^*} + u_2' \frac{d\rho_2}{dx^*} \right] + u_5 [\rho_5 - \rho_2] \frac{du_5}{dx^*} ;$$

$$\frac{dt_p}{dx^*} = \frac{1}{u_5} , \quad (56)$$

$$T_5 = \frac{p_5}{R\rho_5} , \quad (57)$$

$$T_{5vib} = \frac{\theta_v}{\ln(1 + \frac{R\theta_v}{e_{5vib}})} , \quad (58)$$

where T_{5vib} is the reflected vibrational temperature. Again, initial conditions for the reflected shock flow were calculated using the reflected ideal shock ratios of Chapter II.

For the reflected calculation, the incident derivatives are used as well as the reflected derivatives. This presents a problem in that the incident relations are specified in reflected fixed coordinates. A relation between the two is needed. This is accomplished as follows: The incident flow is moving across the reflected fixed-front with a relative velocity $u_2' = u_5 - u_2 + u_1$, where as in the incident case, the flow is being processed at a relative velocity u_2 . Thus, relating the two situations to a common time,

$$\frac{dx}{u_2} = \frac{dx^*}{u_5 + u_1 - u_2} ,$$

which implies that

$$dx^* = dx \left(\frac{u_5 + u_1 - u_2}{u_2} \right) , \quad (59)$$

as illustrated in Figure 4.1. The relation is applied to the de_{2vib}/dx equation, Equation (26), and is thus propagated throughout the other incident equations when the reflected calculation is performed.

The differential equation-solving code utilized was that of Gear [15, pp. 187-193]. These calculations were performed subsequent to an experiment to measure the vibrational temperature of diatomic nitrogen, N_2 . Details of the experiment are presented in Chapter V.

Figures 4.2(a), (b), (c), and (d) illustrate the effects of adding small concentrations of H_2O vapor to diatomic N_2 . Shown are the velocity, density, pressure, and translational temperature profiles behind a Mach 4 incident shock. It will be noted in Figure 4.2(a) that equilibrium has not been achieved at the time of arrival of the contact surface, which occurs at about 80 cm. However, as shown in Figures 4.2(b), (c), and (d), the H_2O addition greatly enhances the vibrational equilibration.

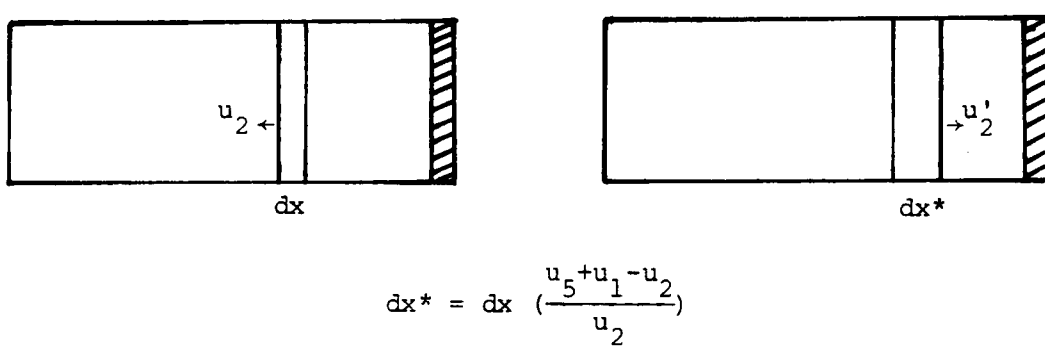
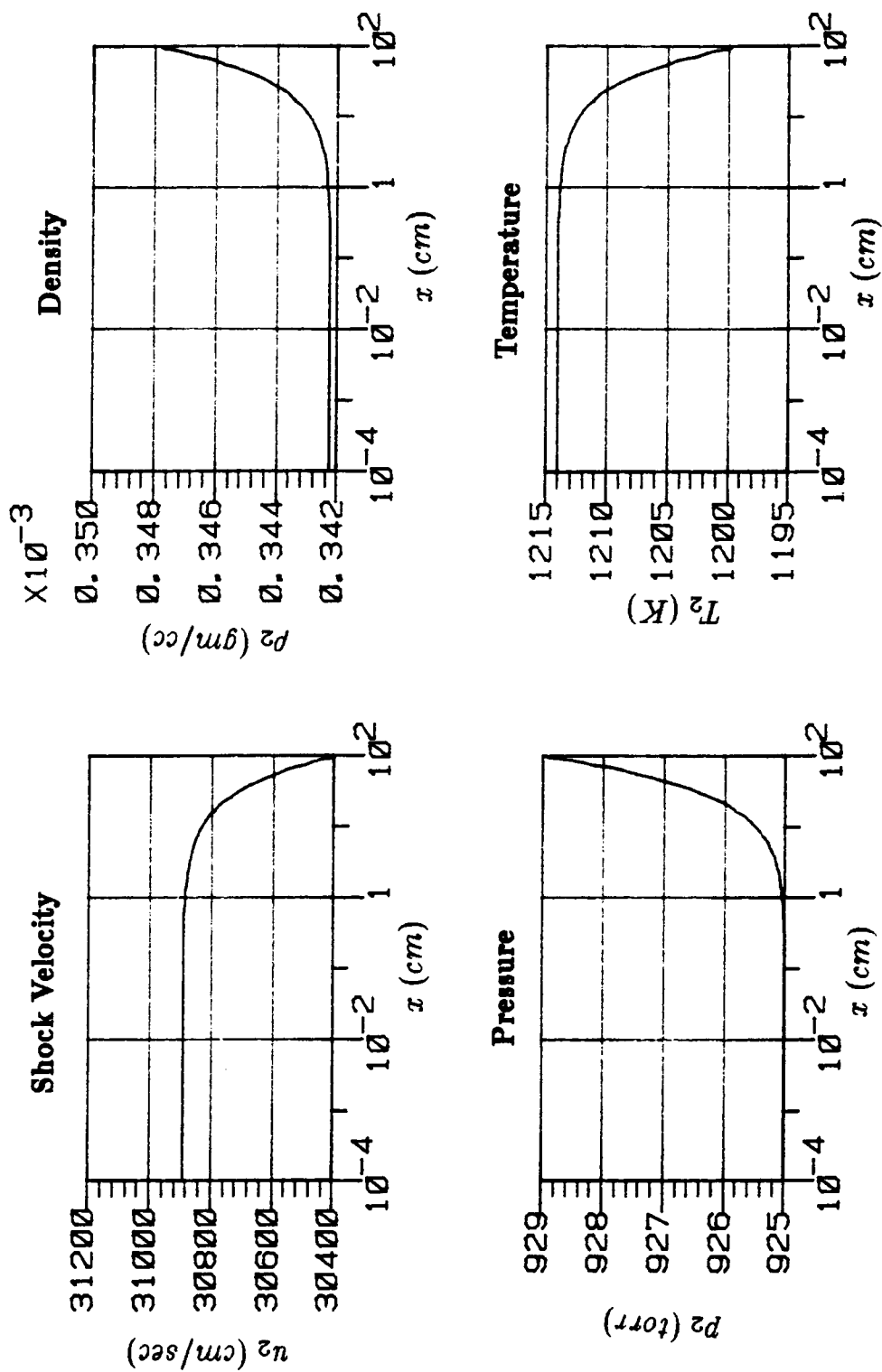
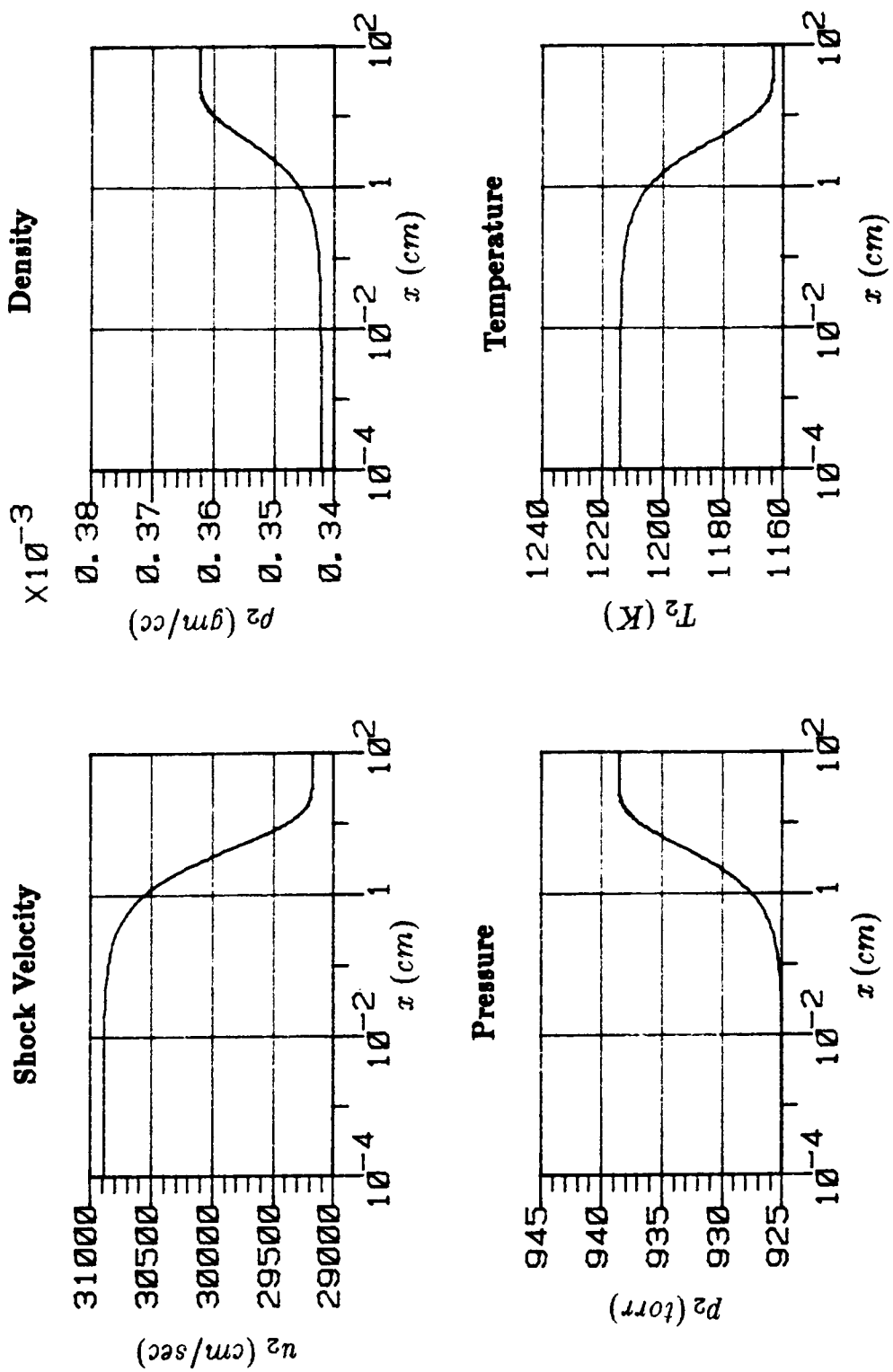


Figure 4.1. Correlation Between Incident-Fixed and Reflected-Fixed Coordinates.



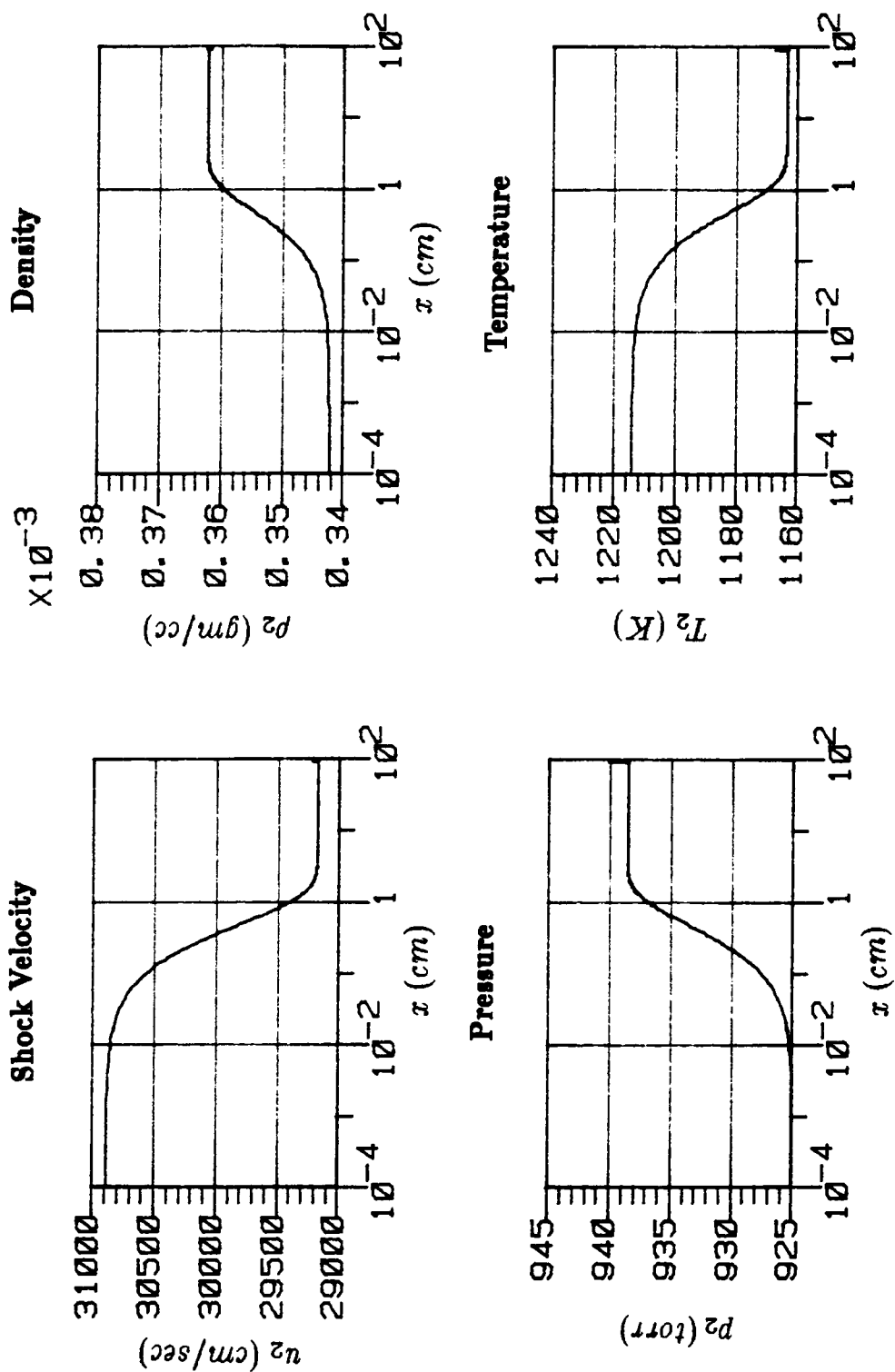
(a) $x_{H_2O} = 0.0$

Figure 4.2. Mach 4 Incident Shock Parameters versus Distance Behind Front.



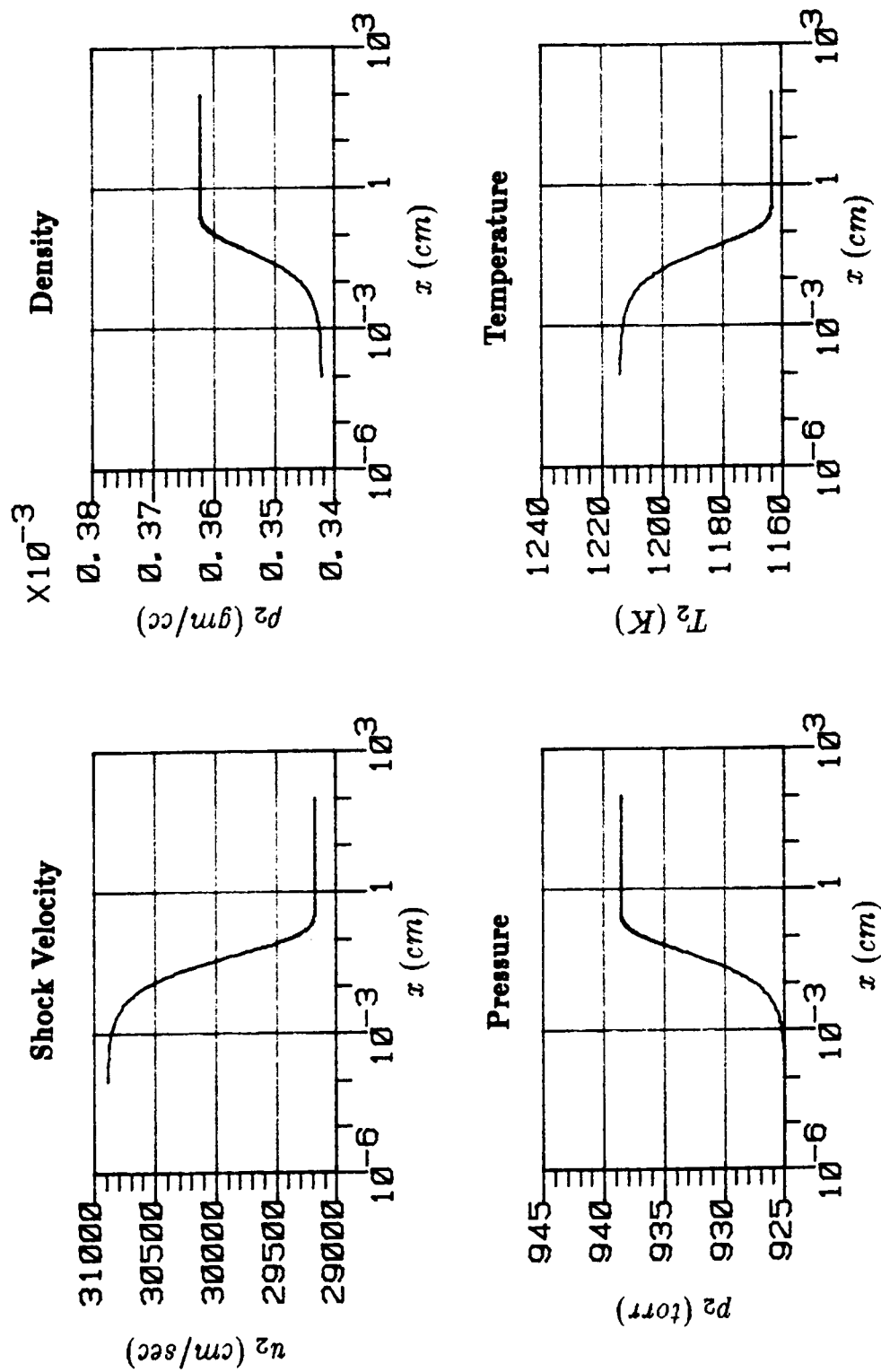
(b) $x_{H_2O} = 0.001$

Figure 4.2. (Continued)



(c) $x_{H_2O} = 0.01$

Figure 4.2. (Continued)



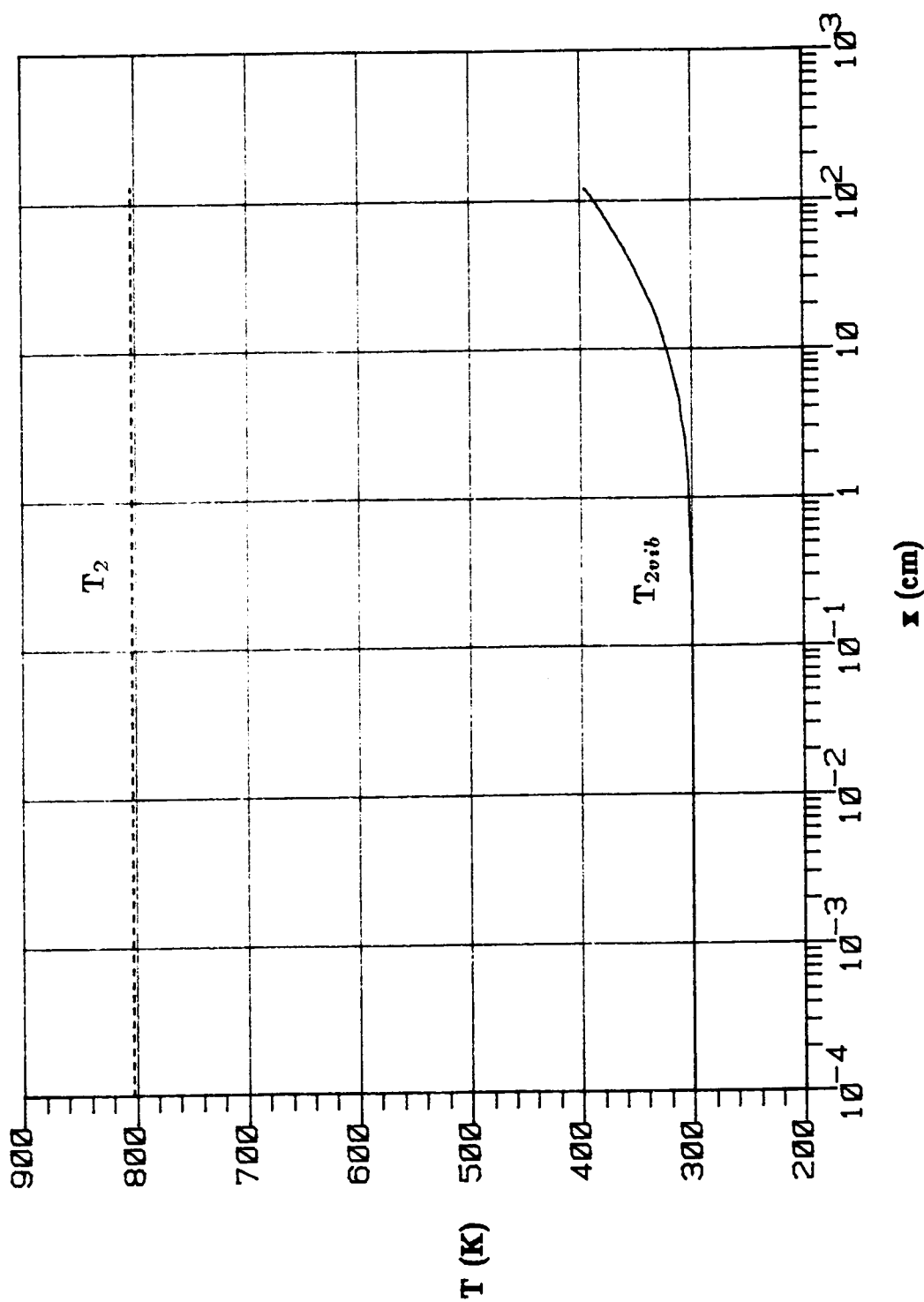
(d) $x_{H_2O} = 0.1$

Figure 4.2. (Continued)

Figures 4.3(a), (b), (c), and (d) illustrate the approach to vibrational equilibrium of shocks of Mach numbers 3, 4, 5, and 6. Shown are T_2 and T_{2vib} . It is observed that higher shock velocities reach equilibrium much faster due to the higher gas temperatures and the larger amount of energy present in the form of translational kinetic energy. This is especially obvious in Figures 4.4(a), (b), (c), and (d), which illustrate the reflected approach to vibrational equilibrium for shocks of Mach numbers 3, 4, 5, and 6.

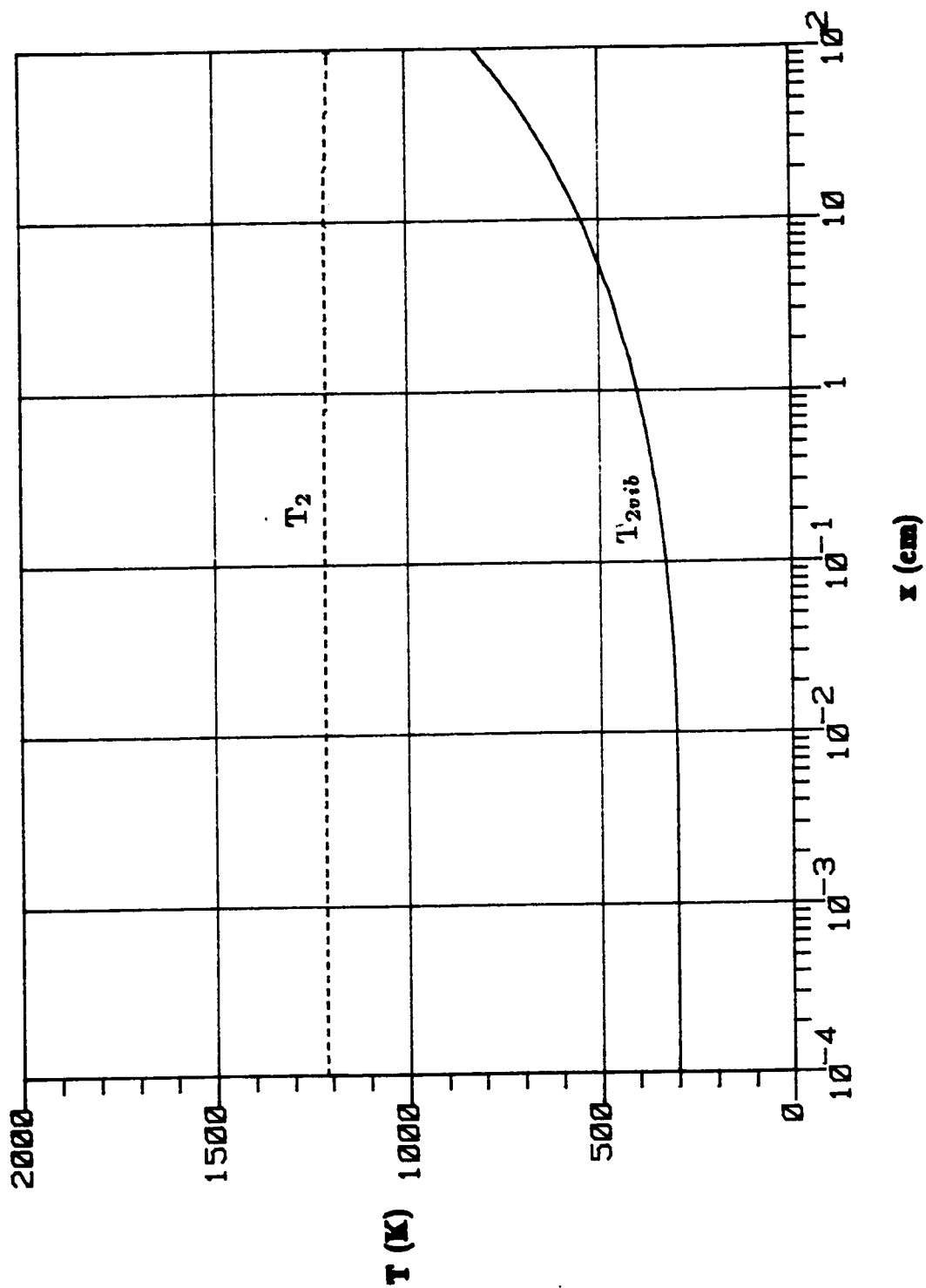
Figure 4.5 illustrates the reflected shock velocity profiles for Mach 3, 4, 5, and 6 shocks. It is noted that equilibrium is achieved much more rapidly in the reflected wave than in the incident case. Figure 4.6 shows u_5 , ρ_5 , p_5 , and T_5 behind a relaxing reflected wave for an incident Mach number of 4. As is noted in Figures 4.5 and 4.6, the reflected wave decelerates as the reflected vibrational energy approaches equilibrium and then accelerates due to the incoming unrelaxed incident shock. This is especially obvious in the p_5 profile of Figure 4.6. This phenomenon has been observed by Toennies and Greene [16].

Figure 4.7 shows the vibrational temperature profile as it should be seen from an observation point on the experimental tube described in Chapter V. It is noted that the reflected wave arrives at the observation window long



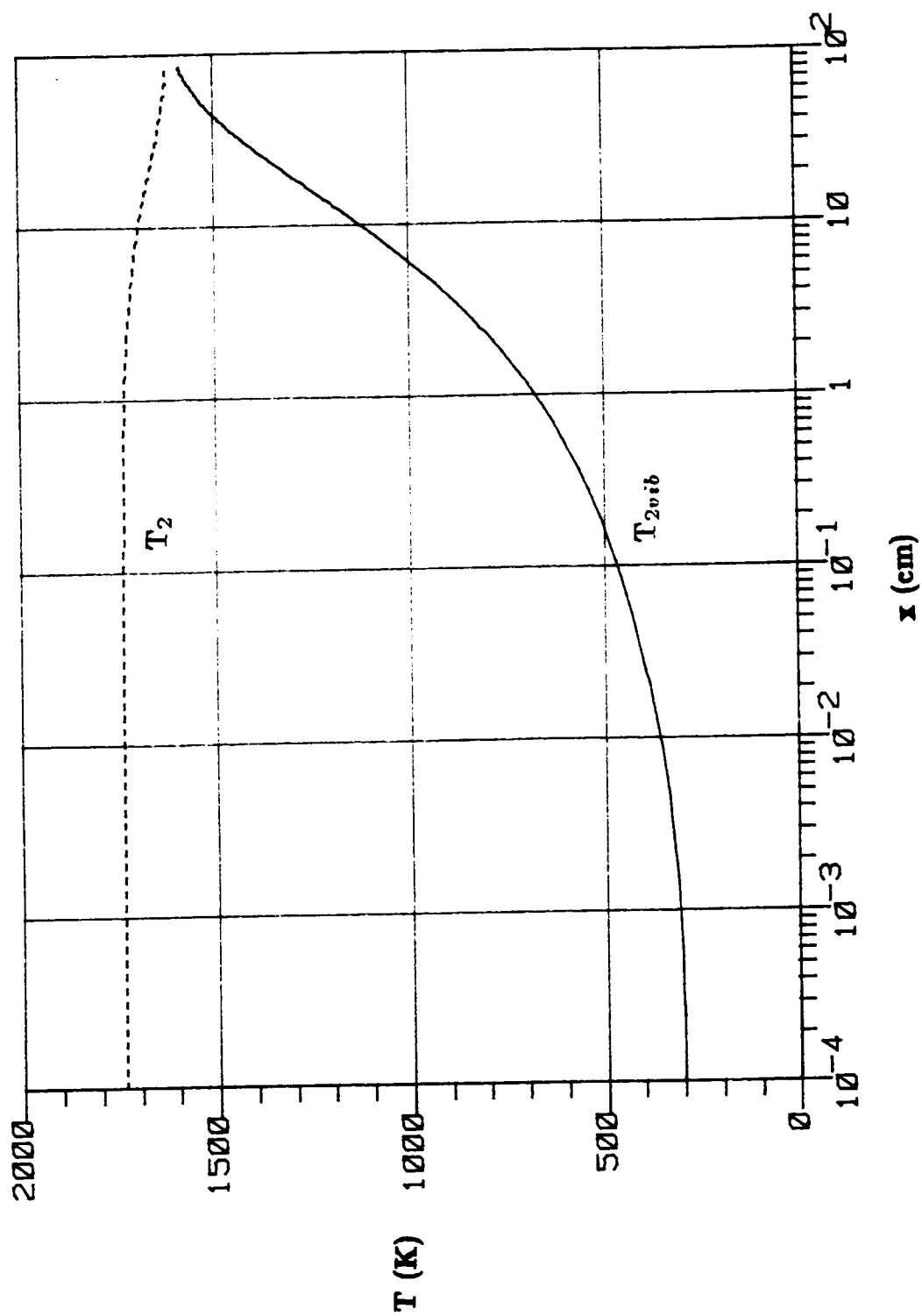
(a) Mach 3 Incident Shock

Figure 4.3. Incident Temperatures T_2 and T_{2vib} versus x .



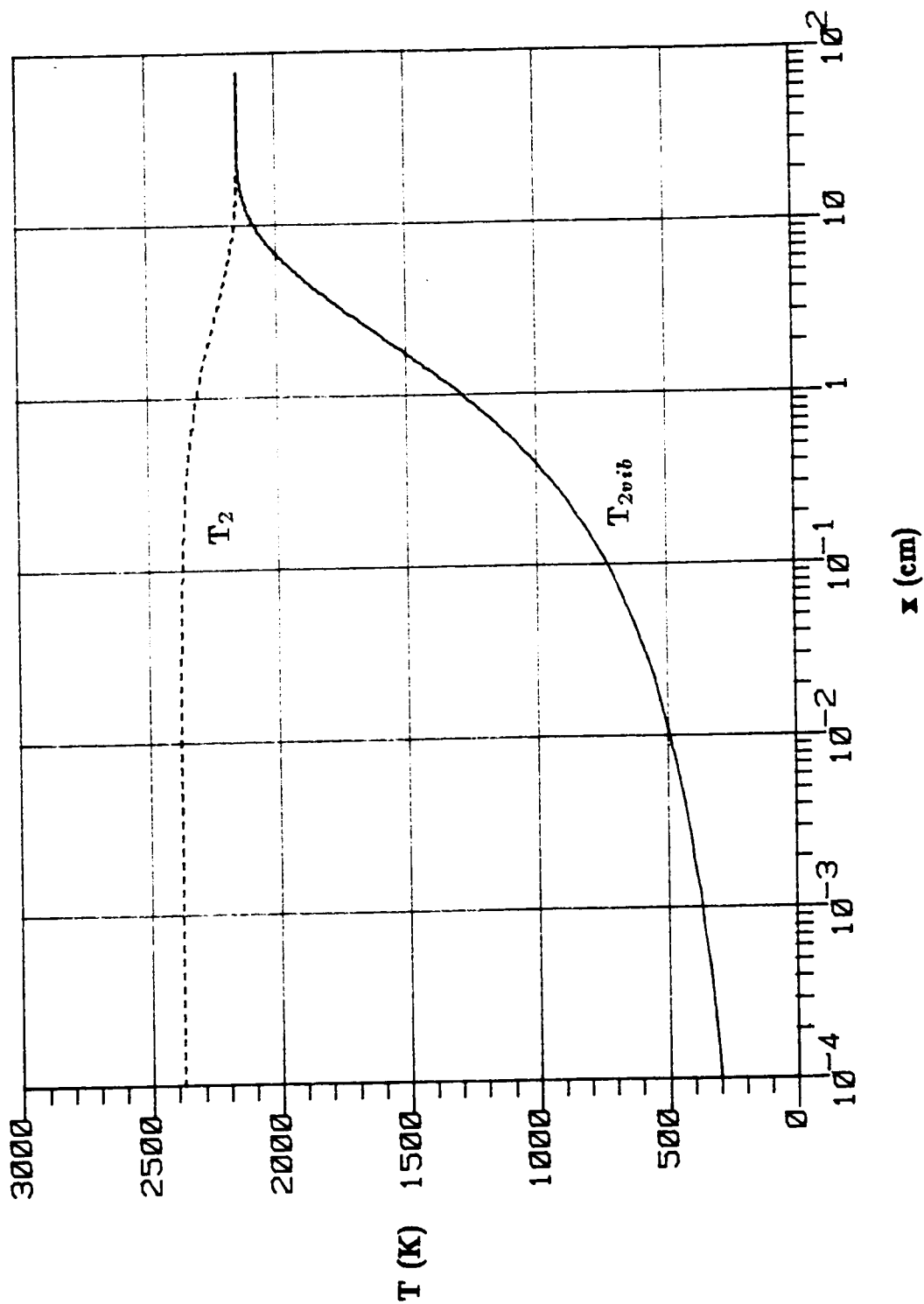
(b) Mach 4 Incident Shock

Figure 4.3. (Continued)



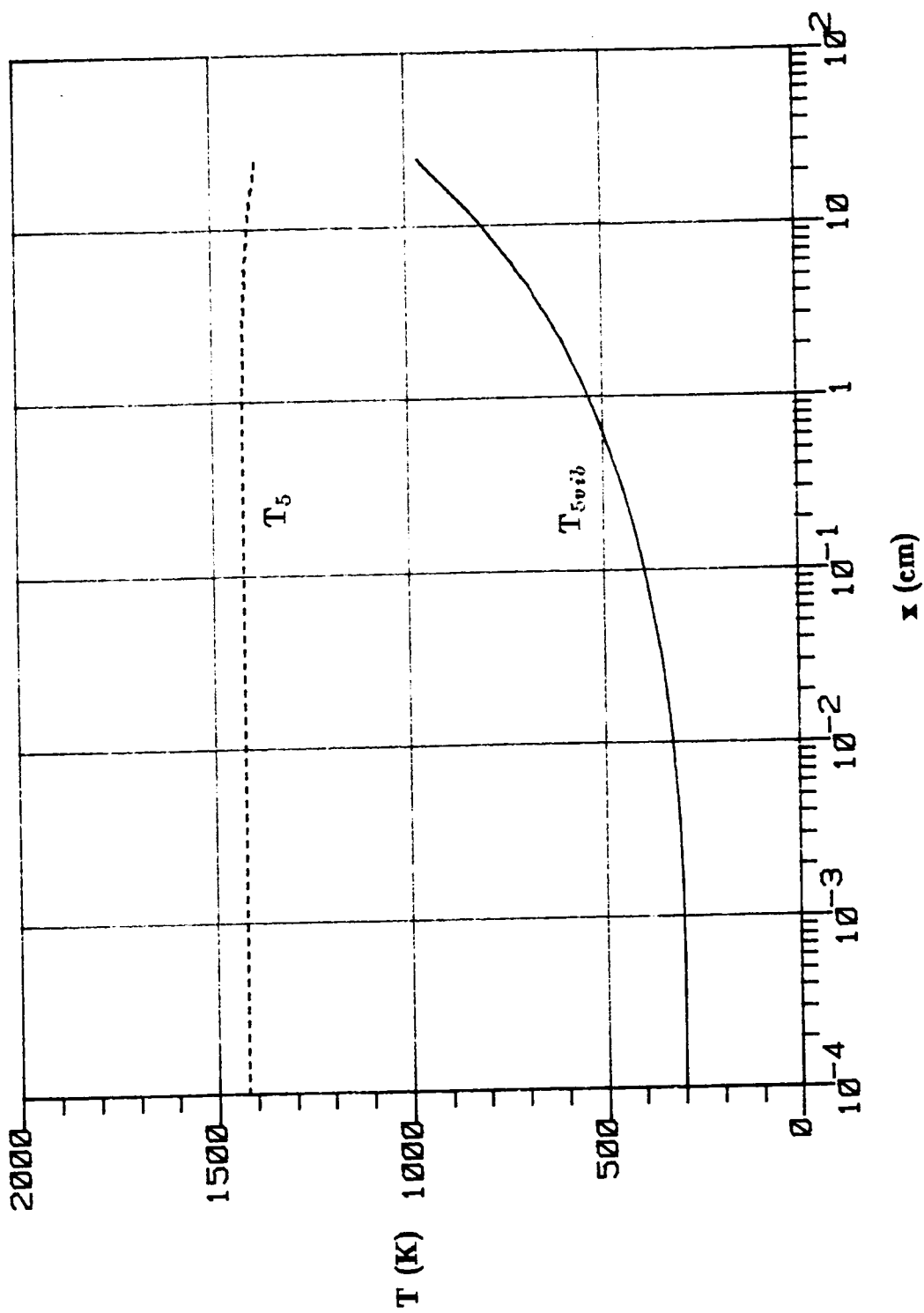
(c) Mach 5 Incident Shock

Figure 4.3. (Continued)



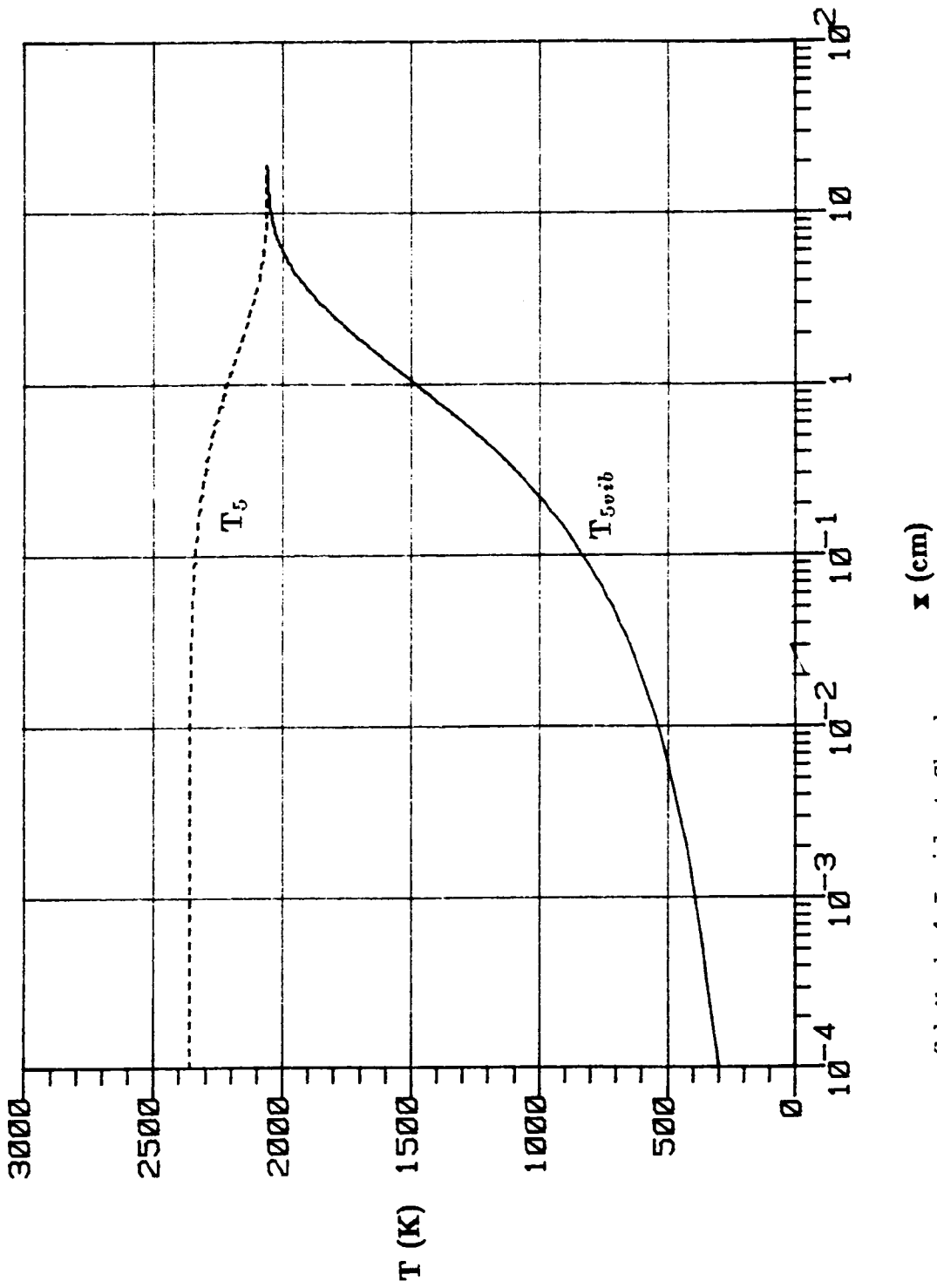
(d) Mach 6 Incident Shock

Figure 4.3. (Continued)



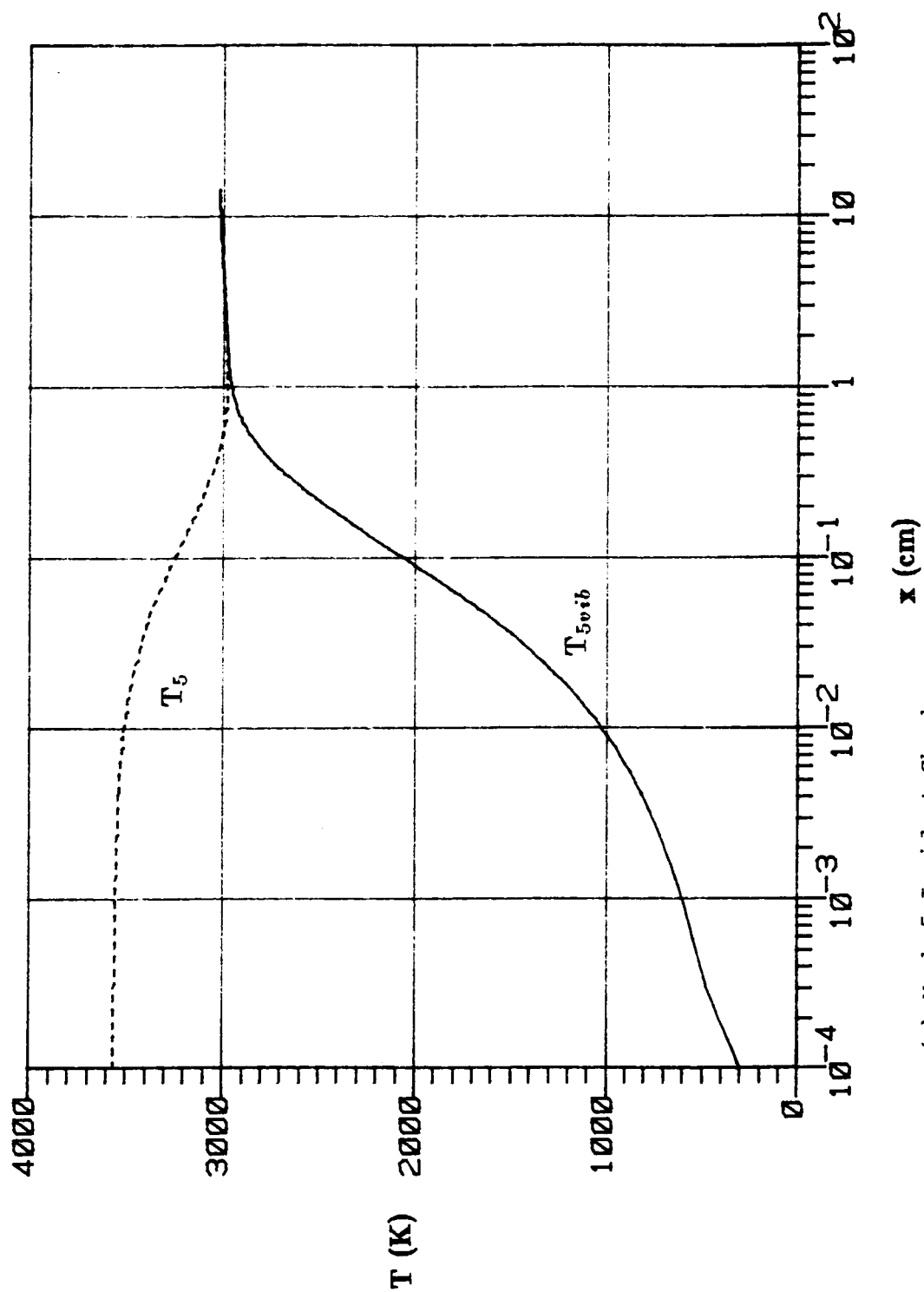
(a) Mach 3 Incident Shock

Figure 4.4. Reflected Temperatures T_5 and T_{5vib} versus x^* .



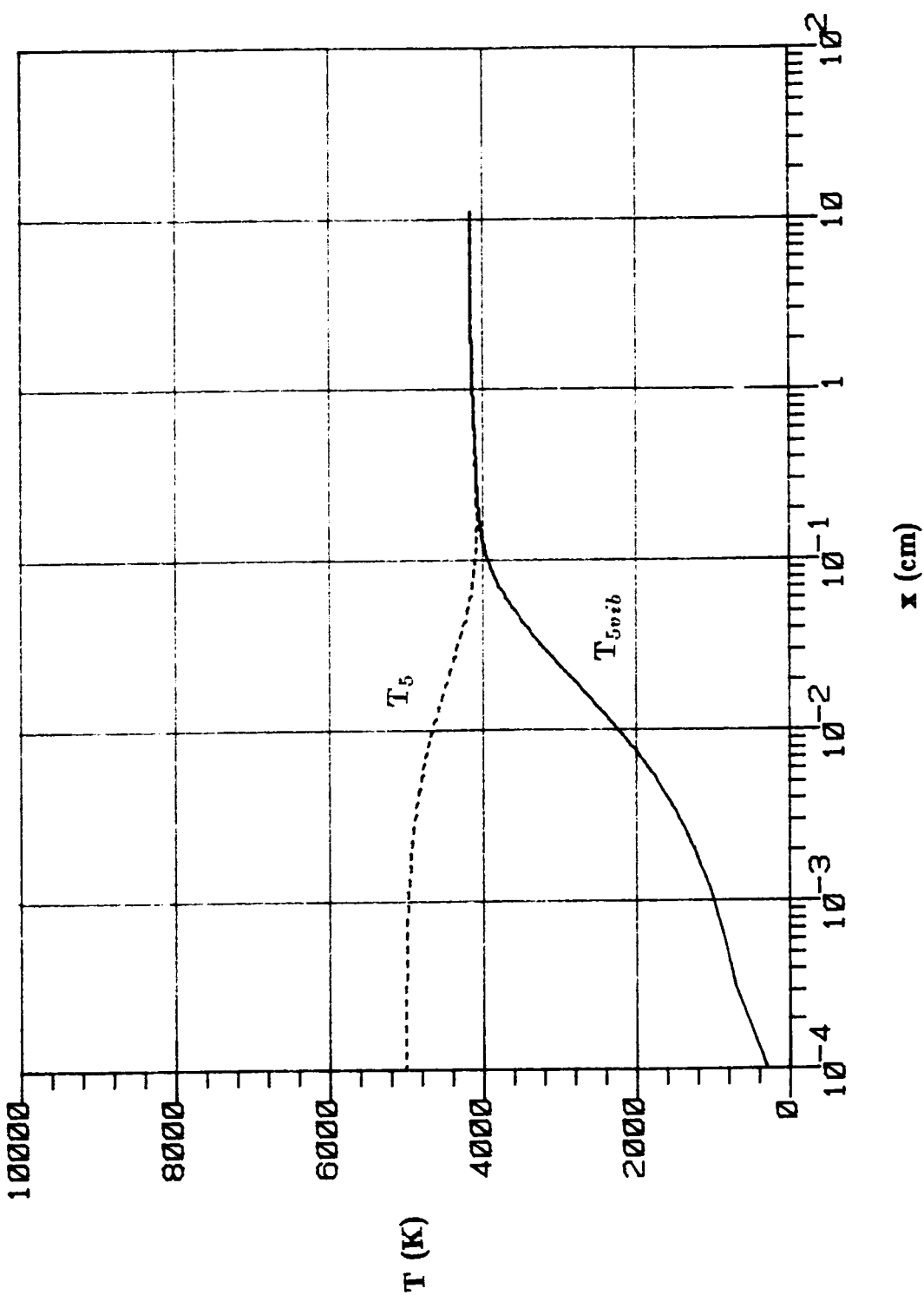
(b) Mach 4 Incident Shock

Figure 4.4. (Continued)



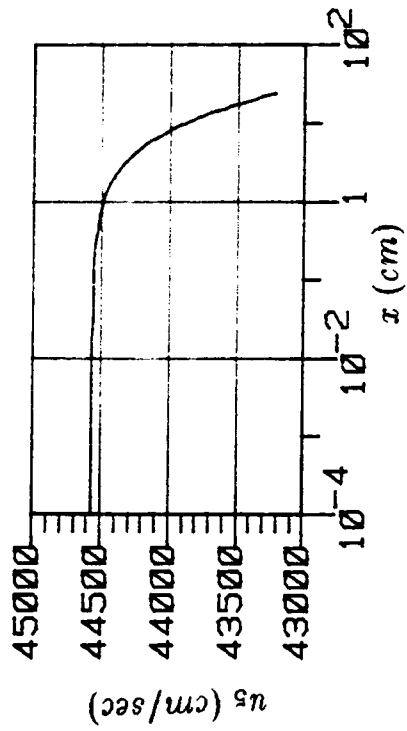
(c) Mach 5 Incident Shock

Figure 4.4. (Continued)

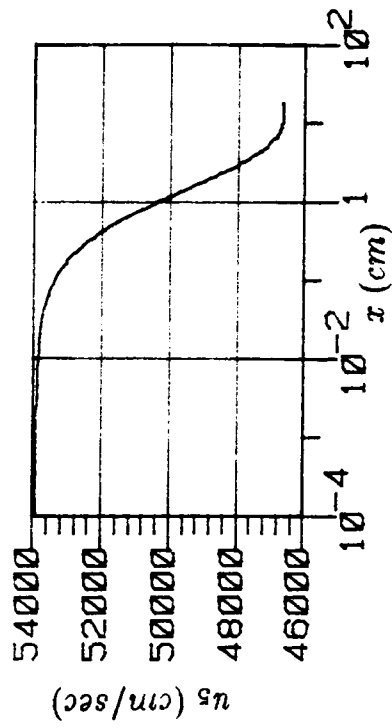


(d) Mach 6 Incident Shock

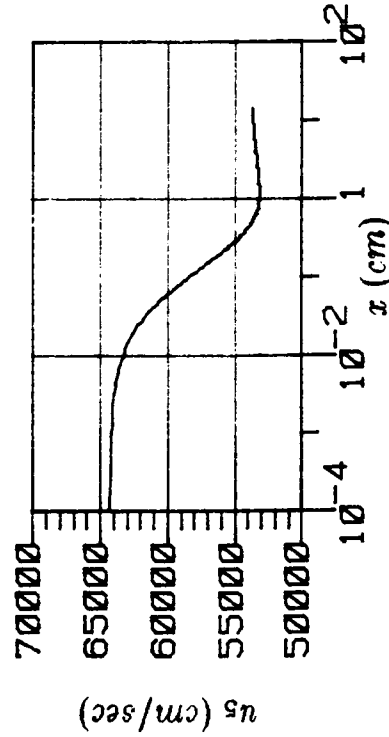
Figure 4.4. (Continued)



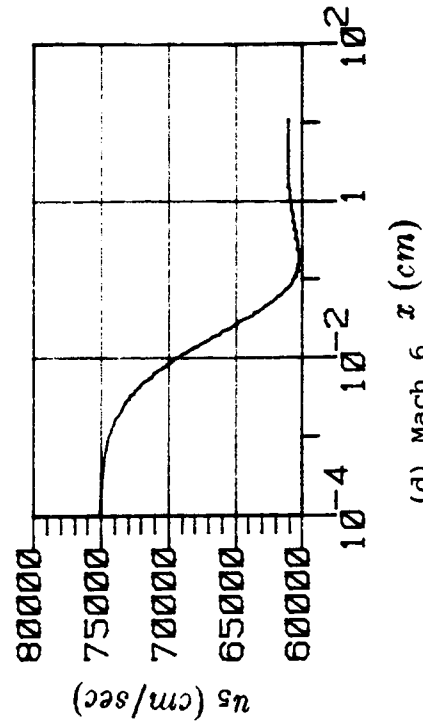
(a) Mach 3



(b) Mach 4



(c) Mach 5



(d) Mach 6

Figure 4.5. Reflected Shock Velocity Profiles.

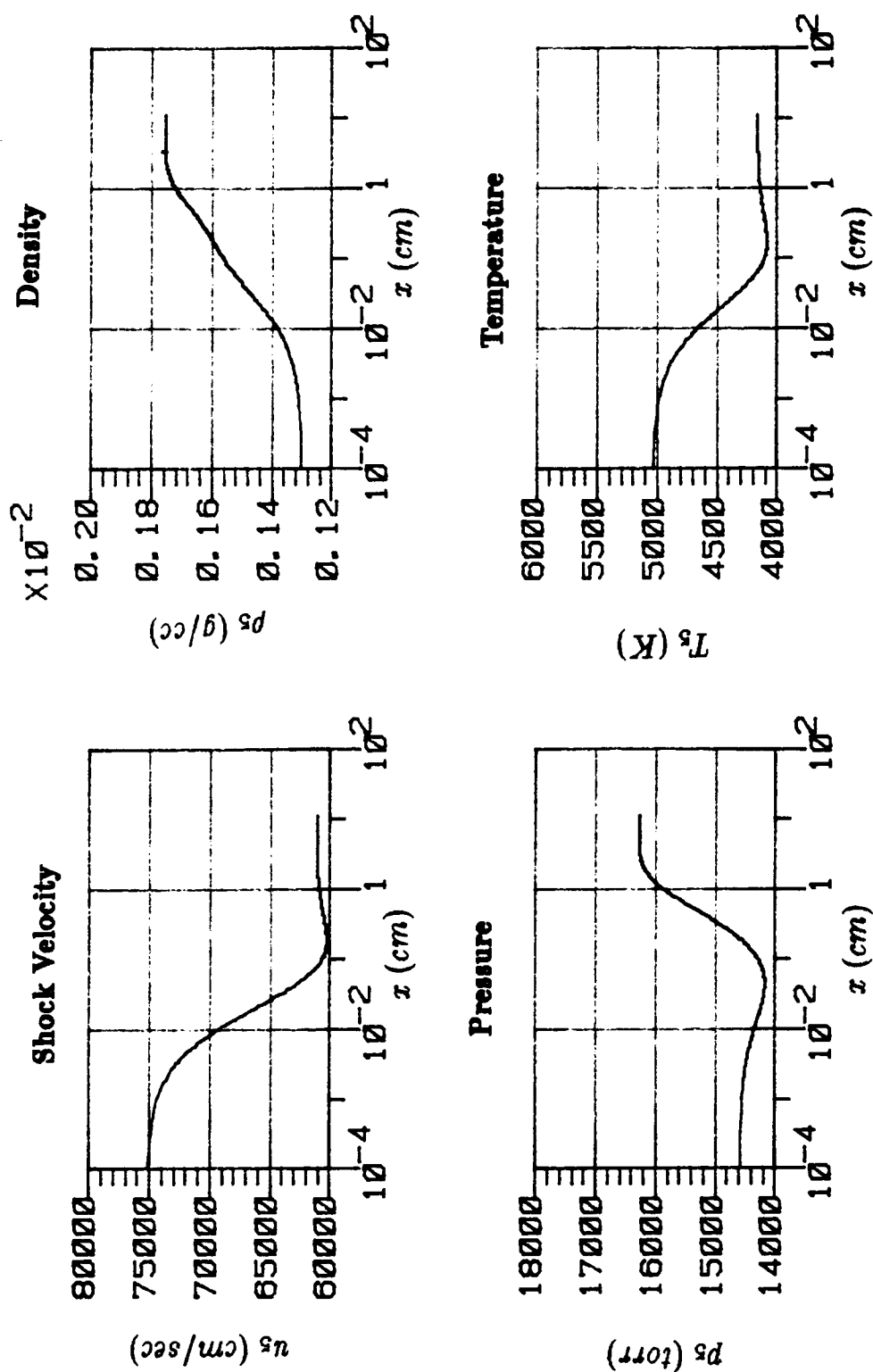
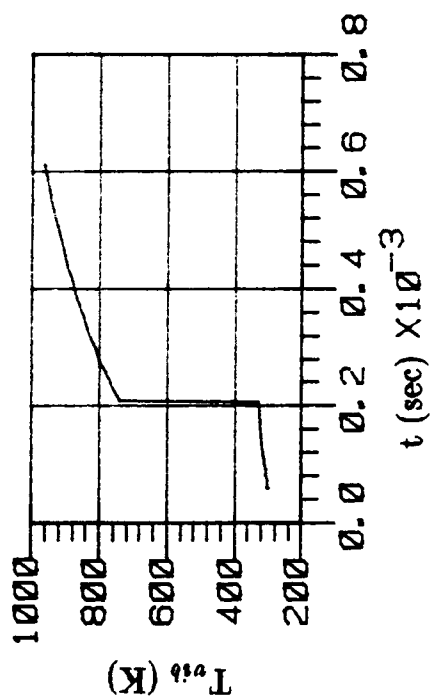
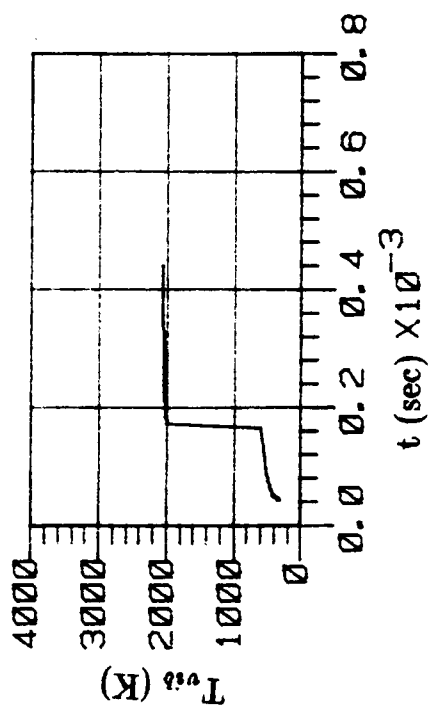


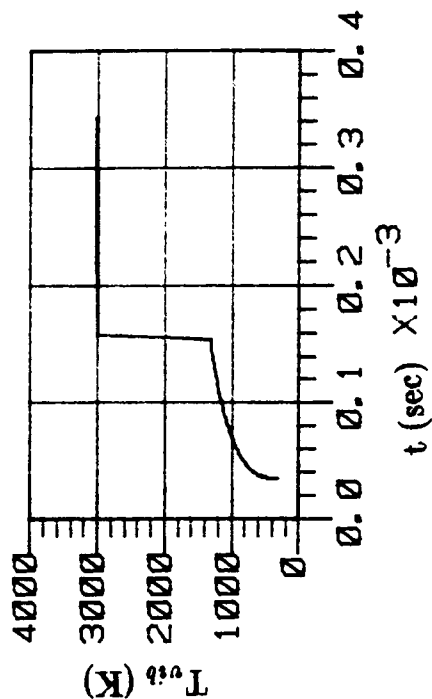
Figure 4.6. Reflected Velocity, Density, Pressure, and Temperature Profiles for Mach 4 Incident Shock.



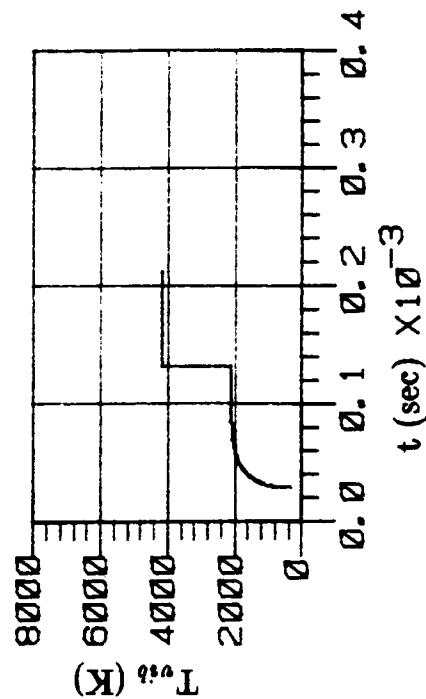
(a) Mach 3



(b) Mach 4



(c) Mach 5



(d) Mach 6

Figure 4.7. Vibrational Temperature in Laboratory Frame.

before the incident wave has reached vibrational equilibrium. However, notice that the reflected front is essentially in equilibrium at the time of its arrival, and that an observation time of several hundred microseconds is available, thus affording an ideal situation for the measurement of the reflected-shock thermodynamic properties.

Figure 4.8 shows the variation of the boundary layer separation distance $x_{\ell m}$ as a function of pressure p_1 for Mach numbers of 3, 4, 5, and 6. It is observed that the flow time is significantly shortened for lower pressures. Figure 4.8 is based on the diameter of the experimental tube of Chapter V. Figure 4.9 shows $x_{\ell m}$ as a function of tube diameter and pressure. It is noted that for low pressures and small bore tubes, the boundary layer almost completely renders any measurements useless.

Figure 4.10 shows x_ℓ , the length of shocked gas in a tube of insufficient length to reach $x_{\ell m}$ as a function of p_1 for the experimental tube to be described in Chapter V.

Figure 4.11 is a verification that, in the case of infinitely fast and slow vibrational excitation times, the vibrational temperature profiles proceed in step-function manner to the correct values.

The preceding calculations were performed subsequent to a laser Raman scattering shock tube experiment [5]. The calculations indicate that incident shock temperature

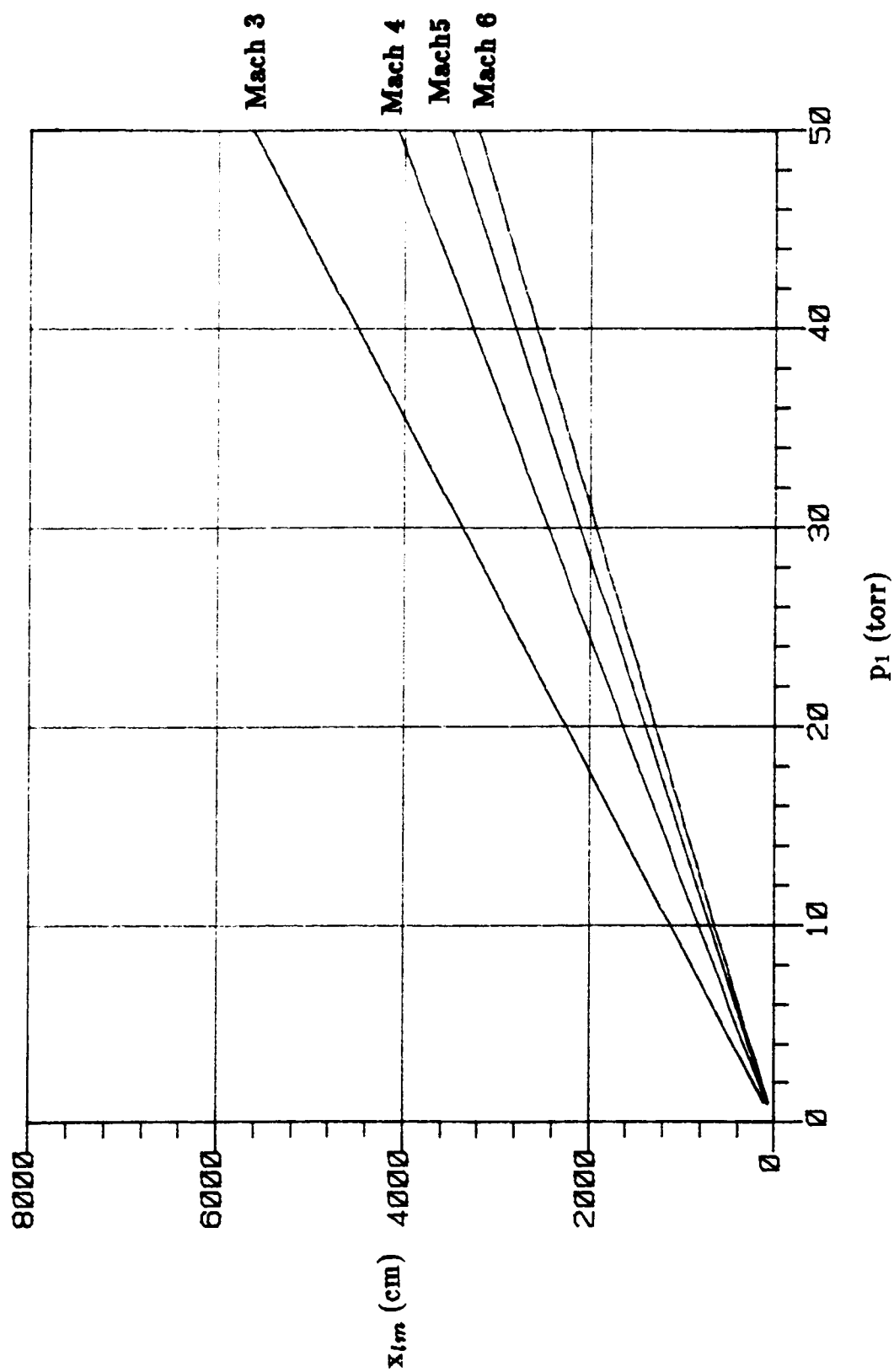


Figure 4.8. Maximum Separation x_{lm} versus p_1 for Mach 3, 4, 5, and 6 Shocks.

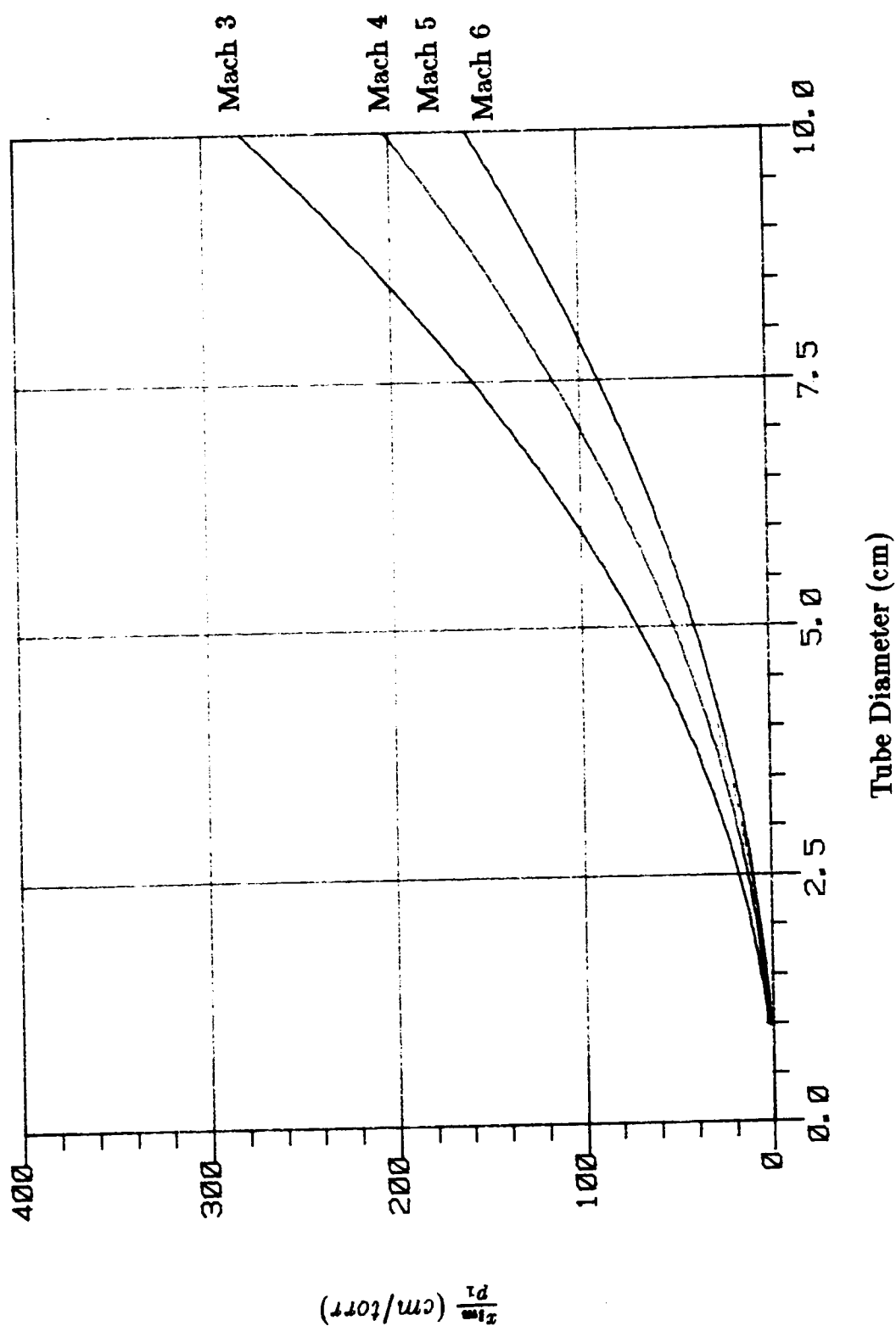


Figure 4.9. Maximum Separation x_{lm} versus Tube Diameter and p_1 .

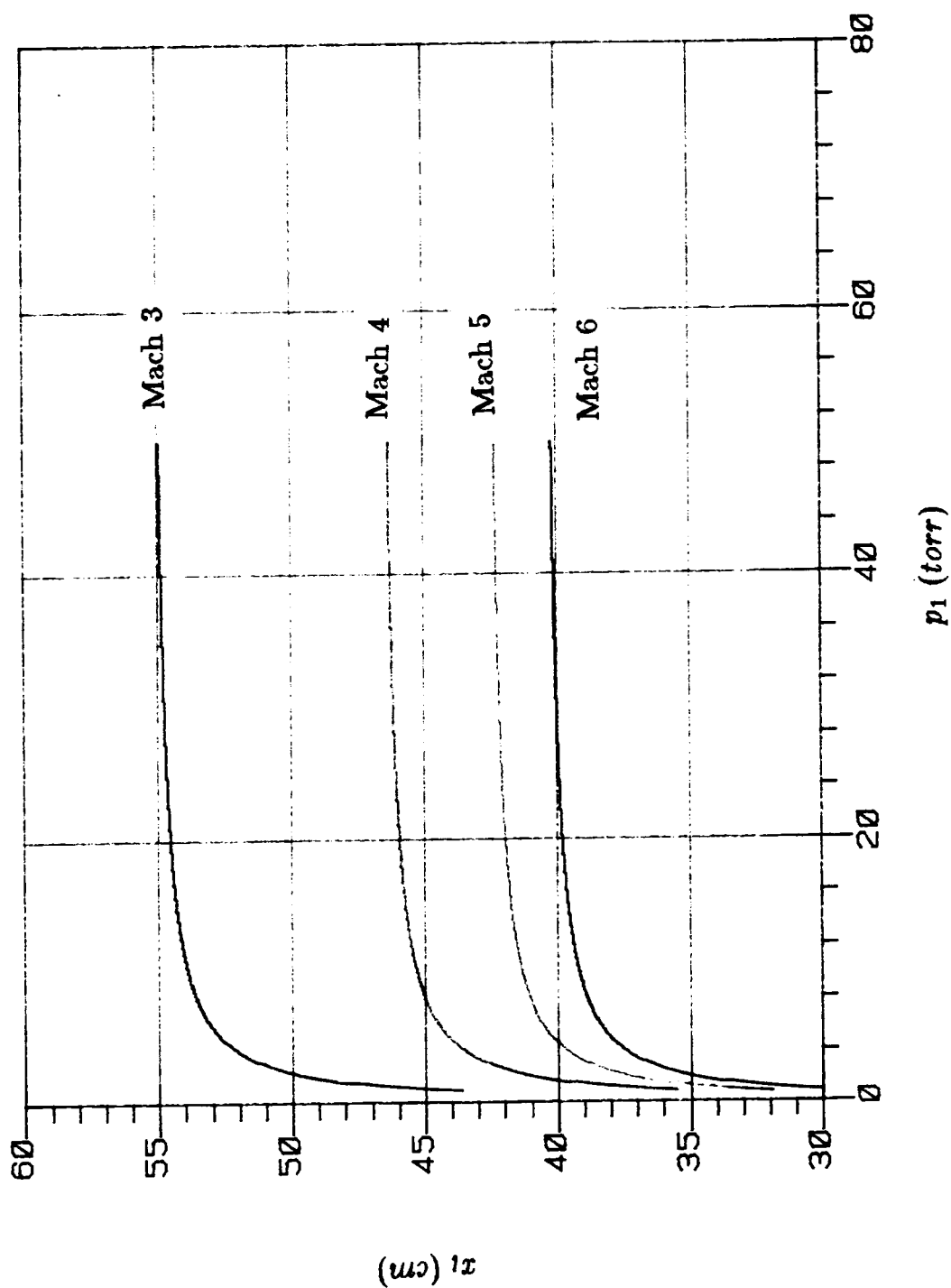


Figure 4.10. Shocked Gas Length x_0 versus p_1 for Mach 3, 4, 5, and 6 Shocks.

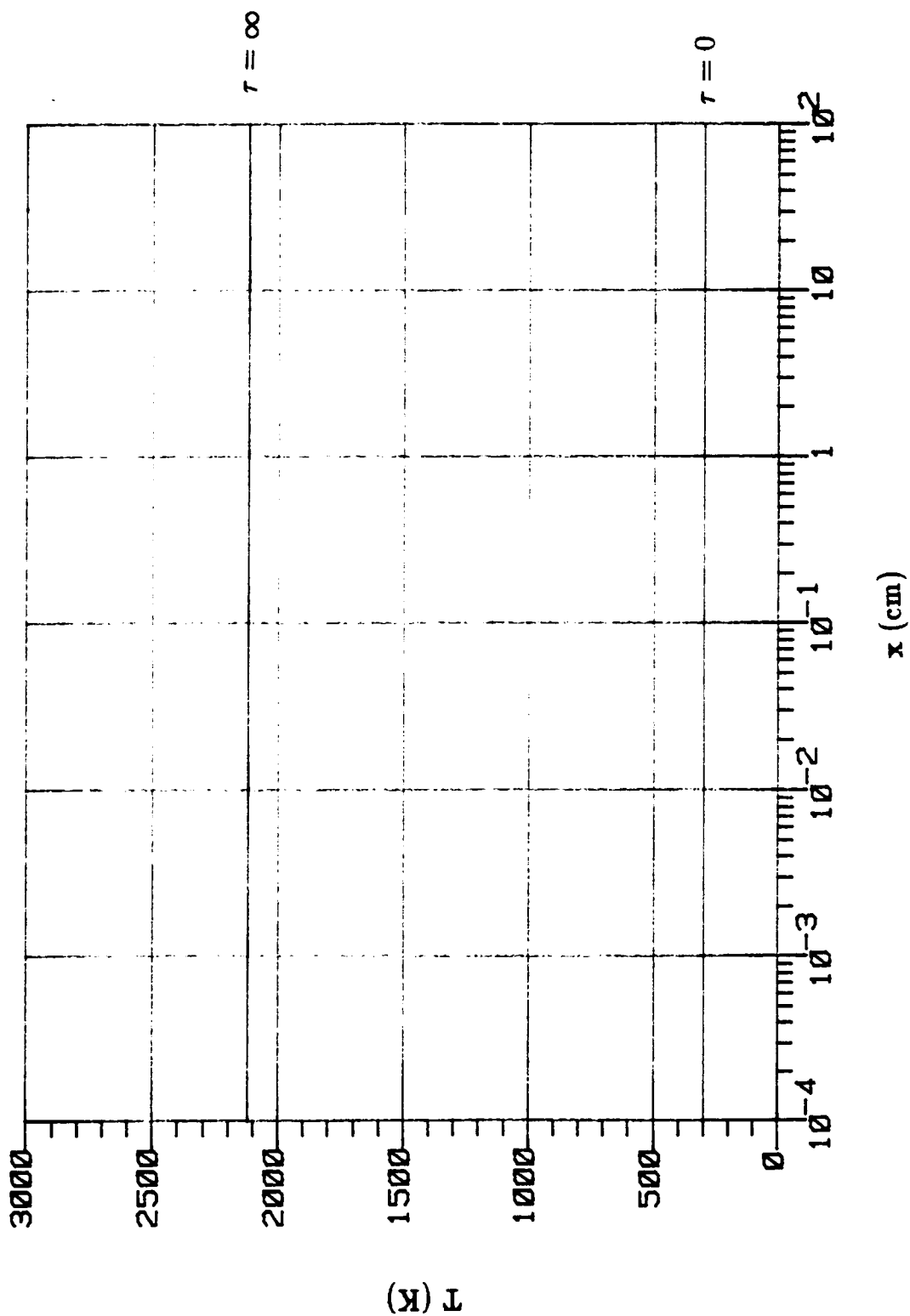


Figure 4.11. Limit Check of Tvib for Infinitely Fast and Slow Vibrational Excitation Times, Mach 6 Shock.

measurements would probably be difficult due to the unrelaxed vibrational temperature and the short incident flow duration. This was indeed the case, as is described in Chapter V.

CHAPTER V

SHOCK TUBE EXPERIMENT

Shock Tube Configuration

A shock tube system was designed and constructed to provide a high-temperature, local-thermal-equilibrium gas sample for the determination of Raman scattering cross sections [5]. The shock tube was designed for operation over the temperature range of $800 \leq T \leq 4,000$ K and the number density range of $0.2 \leq n \leq 2.1$ amagat. The fabricated steel shock tube was 6.98-m long with a 6.35-cm inner diameter. The driven/driver section length ratio was 1.88. Figure 5.1 is a sketch of the shock tube and the mounting installation. Details of the design and construction are presented in Reference [5].

Before the initial operation of the shock tube, corrosion was discovered on the interior wall of the tube. Initial operation of the shock tube in conjunction with the laser scattering system (described in the section to follow entitled "Laser System and Observation Cell") showed that the particulate content of the shocked gas was excessively large for Raman scattering studies. A representative oscilloscope trace showing the particle incandescence is illustrated in Figure 5.2. Annular liners of 304 stainless steel were inserted into both the driver and driven

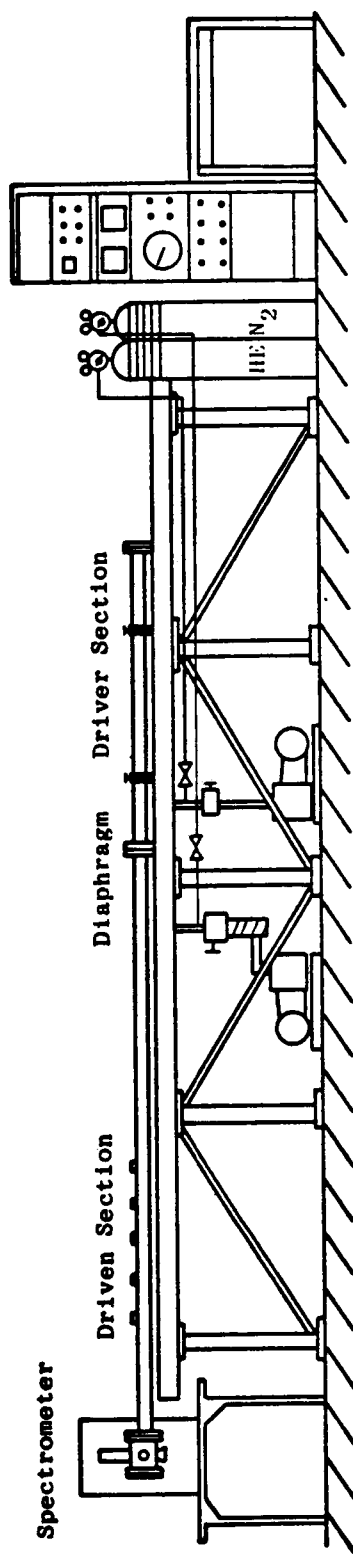


Figure 5.1. Shock Tube and Mounting Installation.

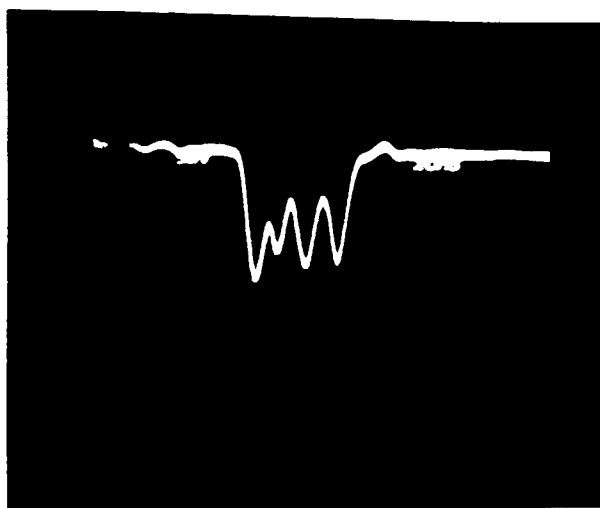


Figure 5.2. Oscilloscope Trace of Typical Particle Incandescence.

sections of the shock tube in an effort to eliminate the problem of corrosion and to minimize particulate scrubbing from the wall. Dimensions and details of the installation of the liner are found in Reference [5].

The diaphragm was ruptured by the conventional pressure burst method and, alternatively, to achieve greater shock reproducibility, by the use of a plunger device. With the plunger system, the driver section can be filled to a pressure which is slightly less than the burst pressure. The driver gas can then be stabilized, and the diaphragm plunger can be activated. Mylar and annealed or scribed aluminum were used as diaphragm materials.

The shock tube was equipped with mechanical pumps and an oil diffusion pump for fast removal of post-shock gases. Using these pumps, a pressure of approximately 10^{-5} torr was obtainable in both the driver and driven sections. The diffusion-pump cold-trap was cooled to -30°C by a one-half-ton refrigerator to minimize oil backstreaming, and, as a result, the rate of outgassing was approximately 2 m torr/min.

Conventional Instrumentation

The previously presented ideal shock relations show that in order to predict the shocked-gas parameters, an accurate measurement of the initial pressure and temperature

conditions and the incident shock speed u_s must be obtained. The initial pressures were measured using calibrated Tabor gauges to an accuracy of 0.1 percent. The pressure gauges and other conventional instrumentation used are listed in Table 1.

A Kistler gauge was located at the end of the driven tube. The gauge was calibrated to obtain full-scale readout at 300 psia (20.4 atm) dynamic pressure. An accuracy of approximately 5 percent of full-scale with a 10- μ sec rise time was expected.

The incident shock speed was measured with a series of platinum heat transfer (HT) gauges which were installed upstream of the test section. A representative HT gauge is illustrated in Figure 5.3. The HT gauges were spaced 20.5-cm apart, with the last gauge 1-m from the end of the tube. Construction of the HT gauges is outlined in Reference [5]. In order to achieve maximum time resolution, the platinum strips were oriented perpendicular to the shock flow. Measurement of the transit time of the shock front between the HT gauges yields the shock speed. The uncertainty in the time interval due to the width of the strips was about 0.2 percent.

Table 1. Conventional Instrumentation

Type	Nominal Range	Parameter Measured
Pressure Transducers		
Tabor	$<2,000$ psia (136 atm)	Driver Gas Pressure
Tabor	<5 psia (0.35 atm)	Test Gas Pressure
Kistler	<300 psia (20 atm)	Shocked Gas Pressure
Low-Pressure Gauges		
Thermal Conductivity (2)	$>10^{-2}$ torr	Pumping Line Pressures
Wallace and Tiernan	$<10^{-3}$ torr	Test Section Pressure
Ion Gauge	<100 torr	Test Section Pressure
Thermocouples		
Copper-Constantan (2)	± 250 to 300 K	Driver and Driven Section Temperatures
Thermistor	± 250 to 300 K	Test Section Temperatures
High-Pressure Gauges		
Bourdon	$<2,000$ psia (136 atm)	Driver Gas Pressure
Bourdon	<600 psia (41 atm)	Driven Gas Pressure

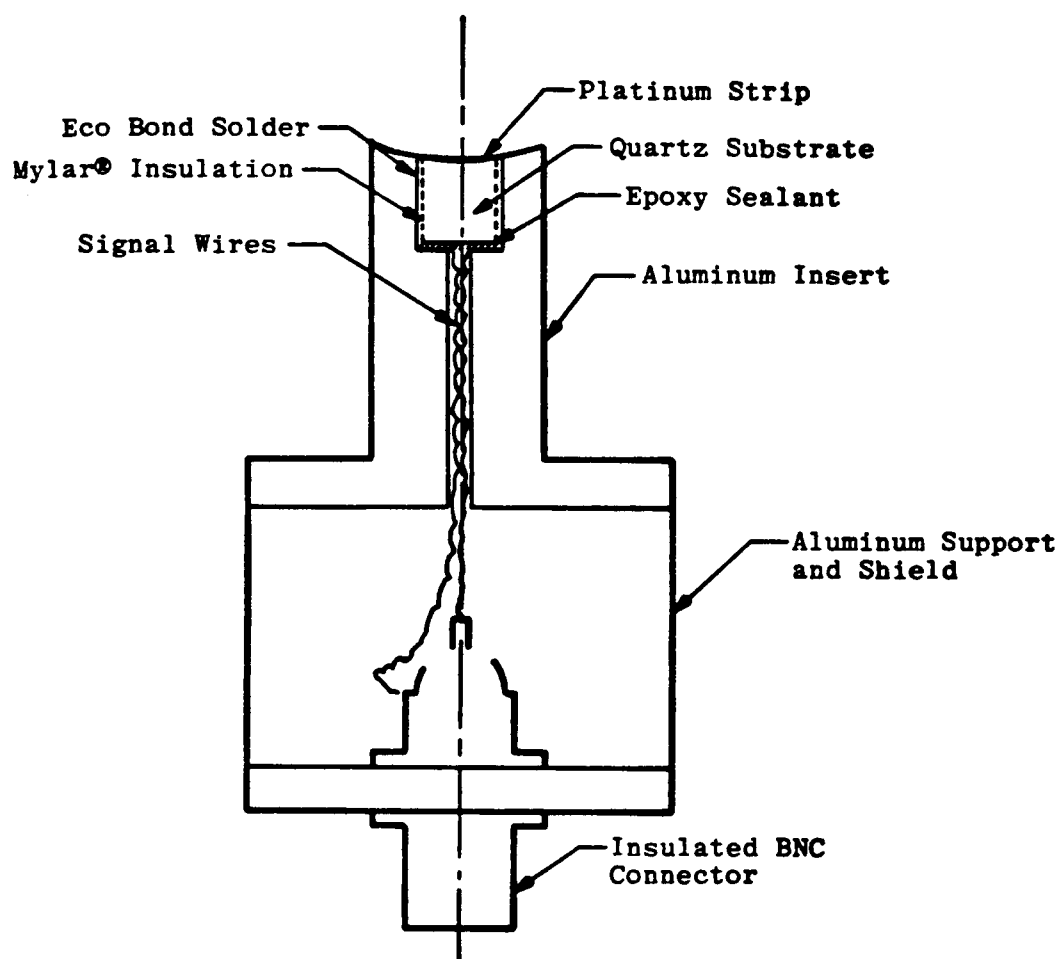


Figure 5.3. Platinum Heat-Transfer Gauge Insert.

Laser System and Observation Cell

In order to perform Raman cross-section measurements in both the incident and reflected modes, a Raman scattering observation cell was attached to the end of the driven section, as shown in Figure 5.4. The aluminum cell included four optical ports, each of which contained 3.49-cm-diameter quartz windows of 6.35-mm thickness.

Figure 5.4 illustrates the shock tube test cell, laser, and spectrometer configuration. The laser beam was injected vertically downward into the observation volume, and the incident beam was collected and dissipated by a beam-collector assembly which was located opposite the injection optics port. The laser source was a Model 620 Holobeam pulsed ruby laser with an output at 694.3 nm with a nominal pulse width and energy of 650 μ sec and 36 J, respectively, in the conventional mode of operation. A Pockels cell Q-switch provided the capability for operation with a pulse width of 25 nsec and a pulse energy of 3 J. As previously stated, particulates in the shocked gas resulted in laser-heated particulate incandescence. This problem can be significantly reduced, as previously demonstrated [1], by frequency-doubling the laser beam. As a result of the doubled frequency, the Raman scattering signal is shifted to the blue-region tail of the near-blackbody particulate incandescence spectrum. This yields

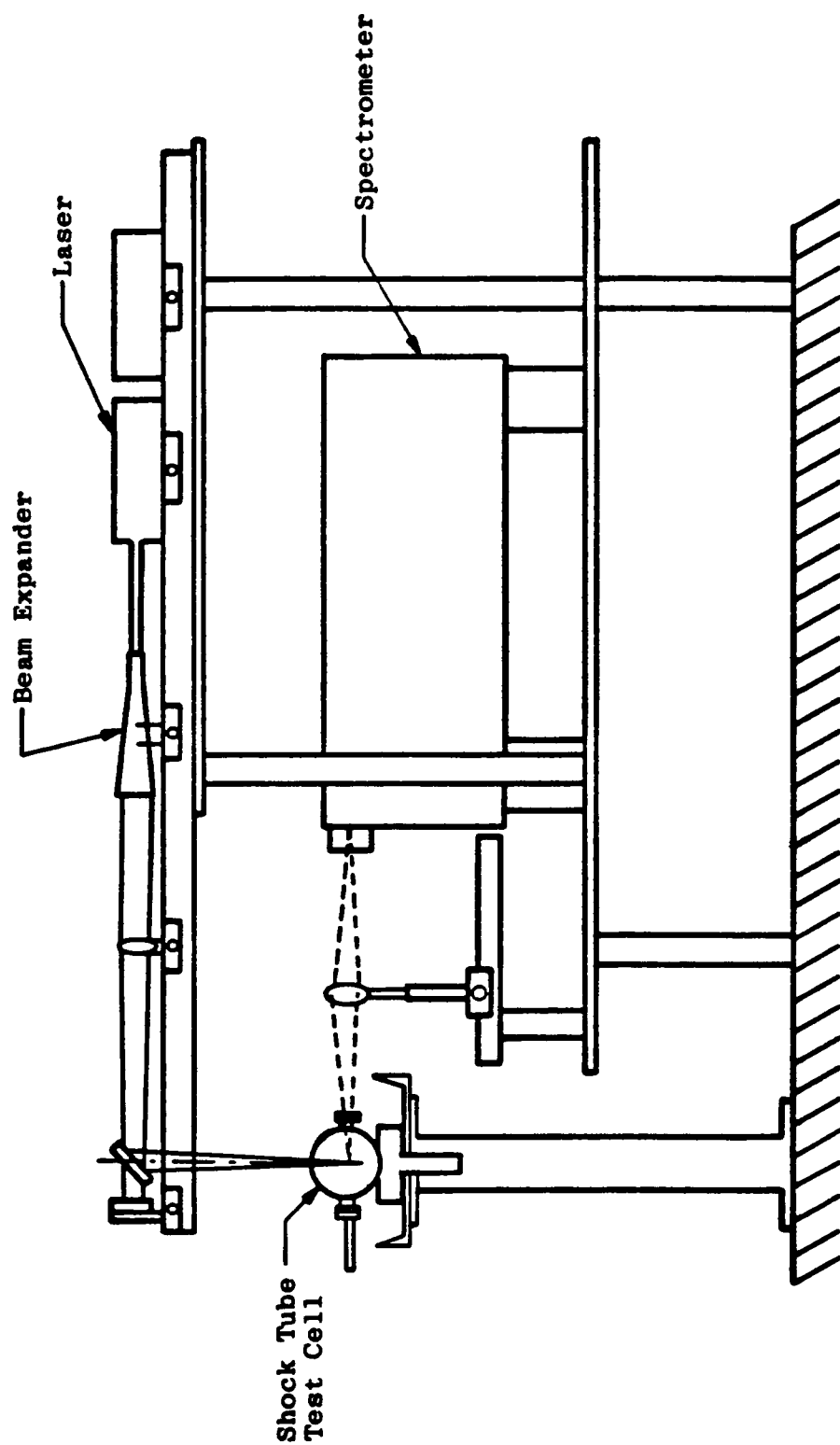


Figure 5.4. Shock Tube Test Cell, Laser, and Spectrometer Configuration.

an increase in the signal-to-incandescence noise of approximately two orders of magnitude. This wavelength shift was accomplished with a rubidium-dihydrogen-arsenate (RDA) frequency-doubling crystal, which converted the ruby-laser beam output from 694.3 nm to 347.15 nm with approximately 20 percent efficiency.

Initially, the laser was operated at an output power of approximately 500 MW which produced a frequency-doubled output power of approximately 100 MW. The 0.635-cm-diameter beam at 347.15 nm was focused onto the centerline of the shock tube using a lens system with an effective focal length of 7.62 cm. The focal volume was approximately cylindrical with dimensions of 1-mm-diameter and 6-mm-length, yielding an approximate power-areal-density at the scattering volume of 10^{10} W/cm². The scattered radiation was focused onto the 400- μ m-wide entrance slit of an 0.85- μ m focal length double spectrometer which had a typical reciprocal dispersion of 0.44 nm/mm. A 400- μ m-wide exit slit was used to pass the dispersed radiation onto the window of an RCA 31034A photomultiplier tube (PMT) which was operated at 2,000 V. The data was acquired and registered by using a gated integrator and CRT display with photographic recording. More complete details of the laser scattering system are found in References [17-21].

CHAPTER VI

RESULTS AND CONCLUSIONS

Because of reporting requirements, the shock tube experiment was performed prior to the completion of the theoretical analysis of Chapter III and the calculations of Chapter IV. As a consequence, only ideal shock tube calculations were used in the prediction of the initial parameters and the timing of the laser firing. As will be seen in this section, there were significant errors in the timing of the laser pulse and thus in the measurement of the gas vibrational temperature, T_{vib} , as a result of this omission.

Results

Temperature measurements were performed to demonstrate the operation of the shock tube and its associated systems. The driver gas used was He, and N_2 was used as the driven gas because of its well-known and comparatively simple Raman scattering profile. Two spectral locations of the Stokes vibration-rotation band of N_2 were observed. The vibrational temperature T_{vib} was determined by using a spectral bandpass such that T_{vib} measurements were insensitive to the value of the rotational temperature T_R .

The shock tube was operated over the incident shock Mach number range of approximately 4 to 5, and the shock speed u_s was measured using the HT gauges previously described. Erratic performance was encountered for one of the HT gauges and for the end-wall pressure gauge, and data were acquired without their use. This resulted in less accuracy in the measurement of the shock speed u_s . Sample data of the time history of the incident shock front are shown in Figure 6.1. A series of four shots was performed for a single Raman scattering measurement of T_v of N_2 . Two Raman data shots were performed at wavelengths λ_0 and λ_1 , and two background shots were performed at the wavelength λ_B . As an example of a series of such shock tube firings, from the data of Figure 6.1 and the data of the two additional shots it was found that the mean value of this data set was $u_s = (1.706 \pm 0.014) \times 10^3$ m/sec. Thus, the shock speed can be measured using the HT gauge method to an imprecision level of about 1 percent. The temperature ratio, T_2/T_1 , which is calculated from Equation (7), varies for a series of such shots by approximately 1.5 to 2.0 percent for the ideal gas case where γ is assumed constant. For a single shot, the uncertainty in T_2/T_1 is approximately 0.5 to 1.0 percent. For this series of shots, the mean incident Mach number, M_1 , is found to be $M_1 = 4.96 \pm 0.04$.

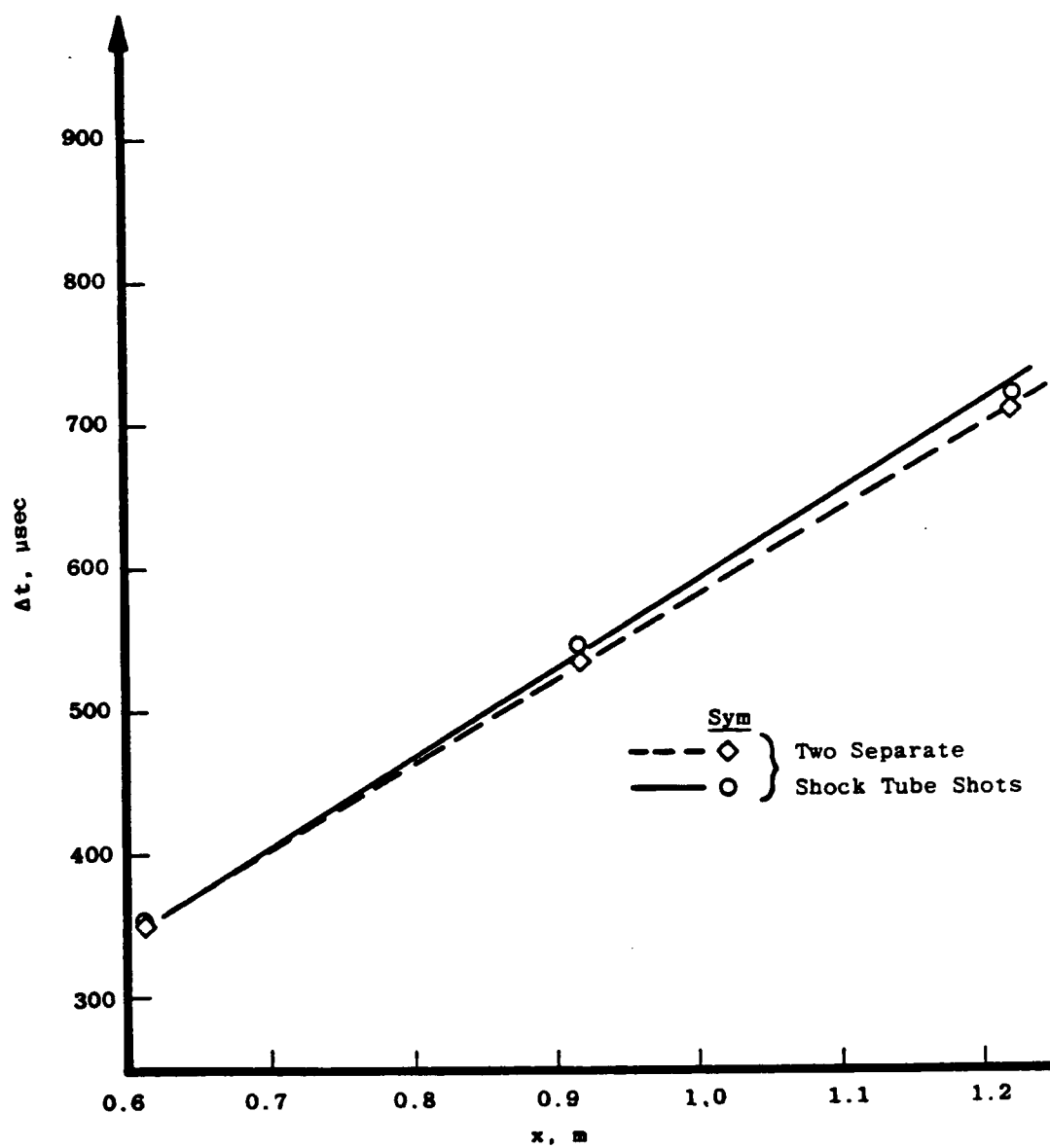


Figure 6.1. Shock Front Time-Distance Results.

This corresponds to an incident shock temperature T_2 of approximately 1,600 K.

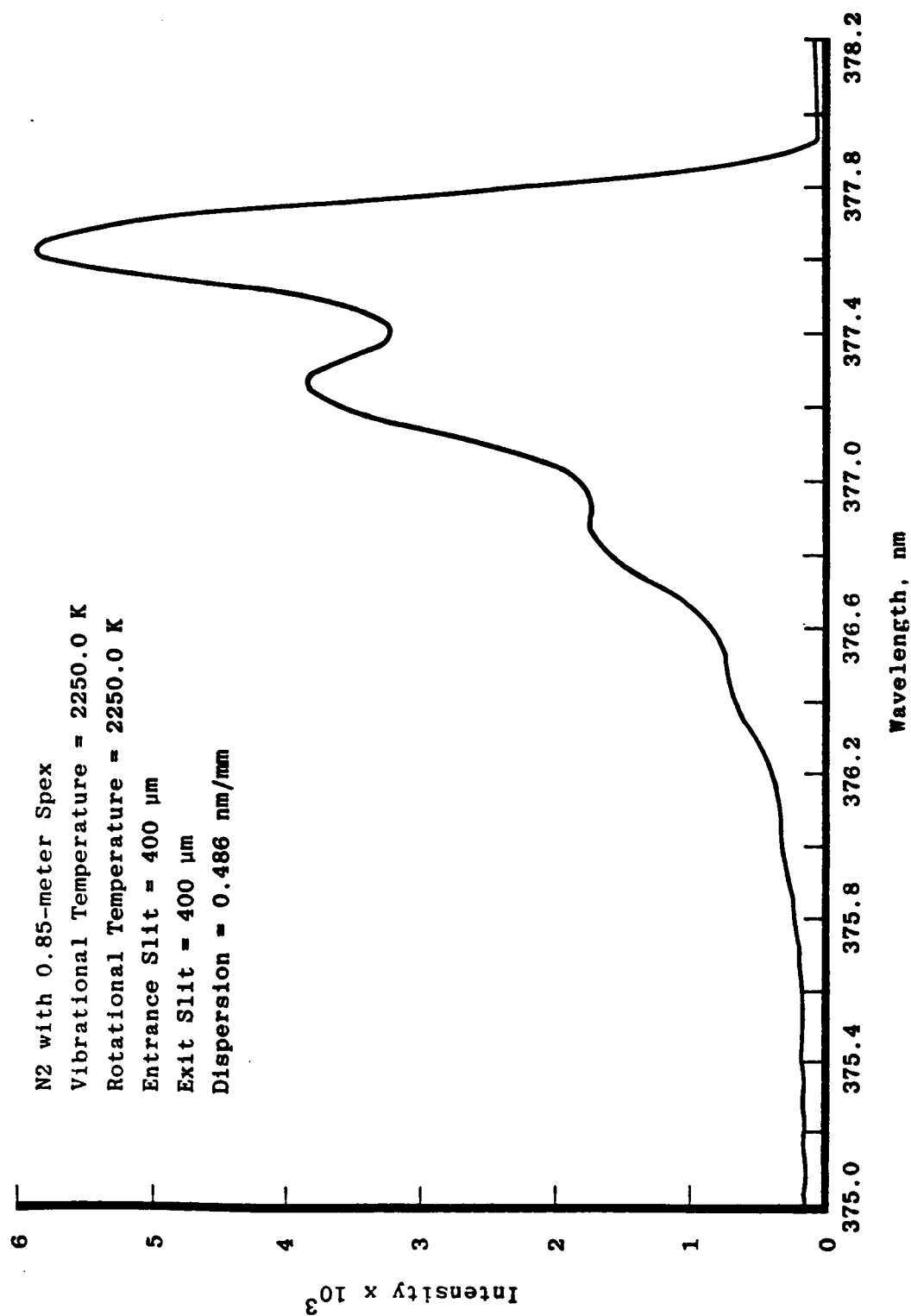
Proper timing of the laser pulse was seriously impaired due to the malfunctioning end-wall pressure gauge. For a shock tube in which the laser scattering volume is relatively far from the tube end-wall, the timing of the laser firing and the placement of the laser measurement within the incident shock wave becomes a simple task. The laser firing time can be established by measuring the time of passage of the shock front past one of the HT gauges and setting an appropriate delay time. However, for a shock tube designed primarily for reflected shock wave studies, the shock tube end-wall is very close to the observation volume, and the arrival time of the incident shock at the end-wall becomes very critical. Once it is known that the incident shock front has arrived, the reflected shock speed, u_R , is obtained for an ideal gas from [3, p. 6]:

$$\frac{u_R}{u_s} = \frac{[2 + (\frac{2}{\gamma-1})(\frac{p_1}{p_2})]}{[\frac{\gamma+1}{(\gamma-1)} - \frac{p_2}{p_1}]} . \quad (60)$$

This result for u_R can be used to determine the time of arrival of the reflected shock front at the observation

station. For N_2 , for which γ varies from $7/5$ in the incident shock to approximately 1.3 in the reflected shock, for $p_2/p_1=30$, $u_R/u_s=0.363$ and 0.291 for $\gamma=1.4$ and 1.3, respectively. It was the intention of these measurements to observe the latter portion of the incident shock front, which would have been achieved if the reflected shock were behaving as a fully vibrationally excited $\gamma=9/7$ molecule. If, however, the vibrationally relaxing N_2 gas were behaving as a $\gamma=7/5$ molecule at the instant of the laser firing, the early portion of the reflected shock would have been viewed. This is due to the rather large (20 percent) difference in u_R/u_s for the two behaviors. This uncertainty is sufficiently large that it could cause a large error in the measurement of the vibrational temperature. Accurate end-wall timing would have removed much of this uncertainty.

Vibrational Raman scattering measurements were performed using a calibrated single PMT channel in conjunction with the apparatus described in a previous section. To determine the wavelength or spectral locations of these measurements, Raman spectral profiles were calculated over a range of temperatures, and Figure 6.2 shows the results of this calculation for the selected temperature of 2,250 K for a 0.85-m focal length double spectrometer which was operated with 0.4-mm entrance and exit slit widths,

Figure 6.2. Raman Spectral Profile for N₂.

a 1,200 groove/mm grating and with a reciprocal linear dispersion of 0.44 nm/mm. For the range of temperatures anticipated, measurements were performed at the wavelengths of $\lambda_0=377.66$ nm and $\lambda_1=377.26$ nm, corresponding approximately to the peaks of Raman scattering from the $v=0$ and $v=1$ N_2 vibrational energy levels, respectively. The scattering signal levels at λ_0 and λ_1 , $S(\lambda_0)$, and $S(\lambda_1)$, respectively, are shown in Figure 6.3 as a function of the temperature, and, for the results shown in both Figures 6.2 and 6.3, it was assumed that the N_2 gas sample exhibited local thermal equilibrium; i.e., $T=T_v$ was assumed. Evident in Figure 6.3 is the depletion in number density population of the $v=0$ level and the increase in population of the $v=1$ level as the temperature increases. Figure 6.4 shows the ratio $R_v=S(\lambda_1)/S(\lambda_0)$ as a function of temperature, and the approximately linear dependence of R_v on T_v is to be noted.

Raman scattering data were acquired at the wavelength-calibrated spectral locations of λ_0 and λ_1 . Successive shock tube firings were required for these measurements, and a series of shots was performed to obtain data at λ_0 , λ_1 , and the background radiation location $\lambda_B=378.3$ nm where only background and particulate radiation components existed. These background data indicated signals of approximately 10 percent of the

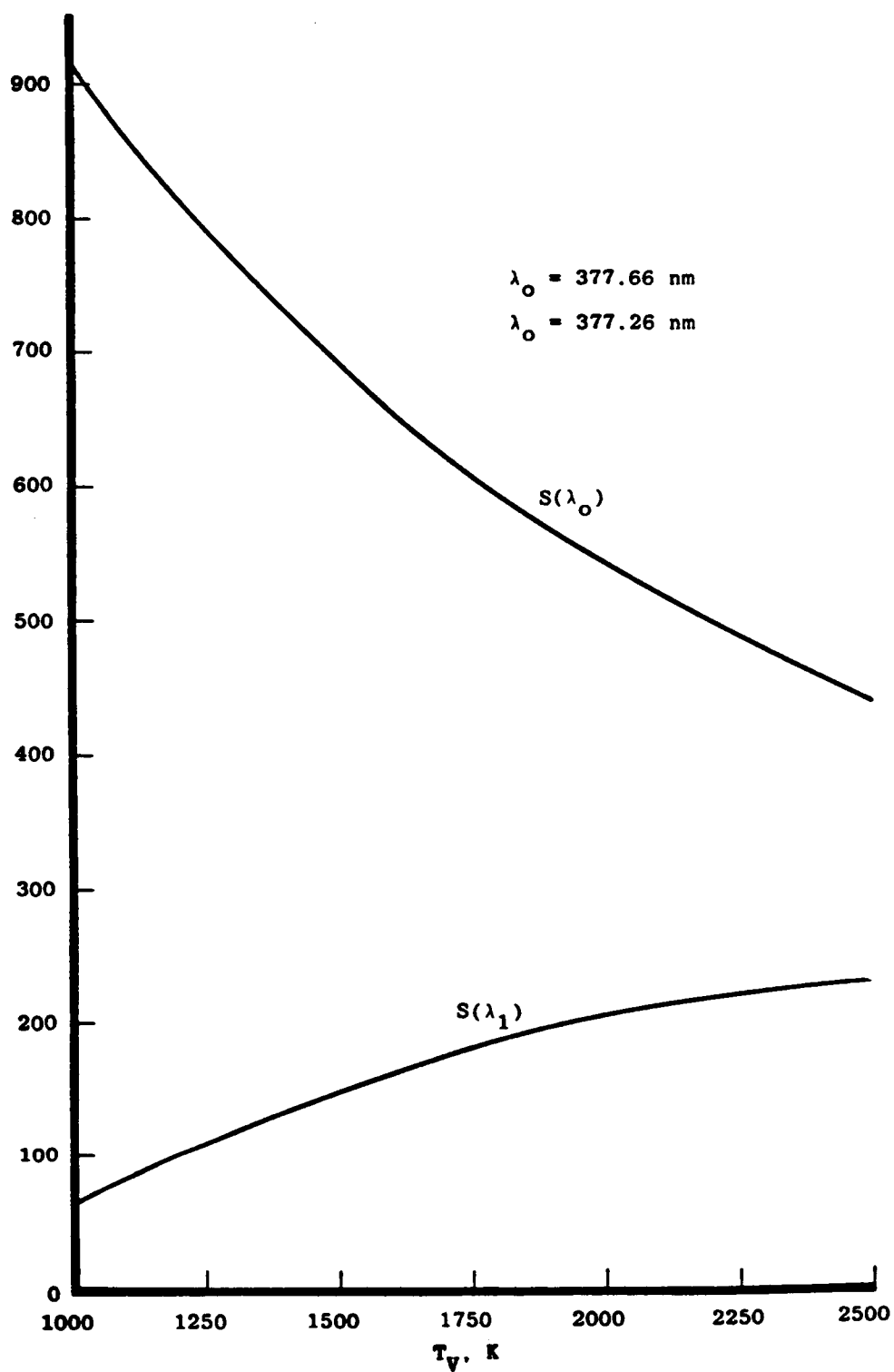


Figure 6.3. Variation of Raman Signal Levels with Vibrational Temperature T_{vib} .

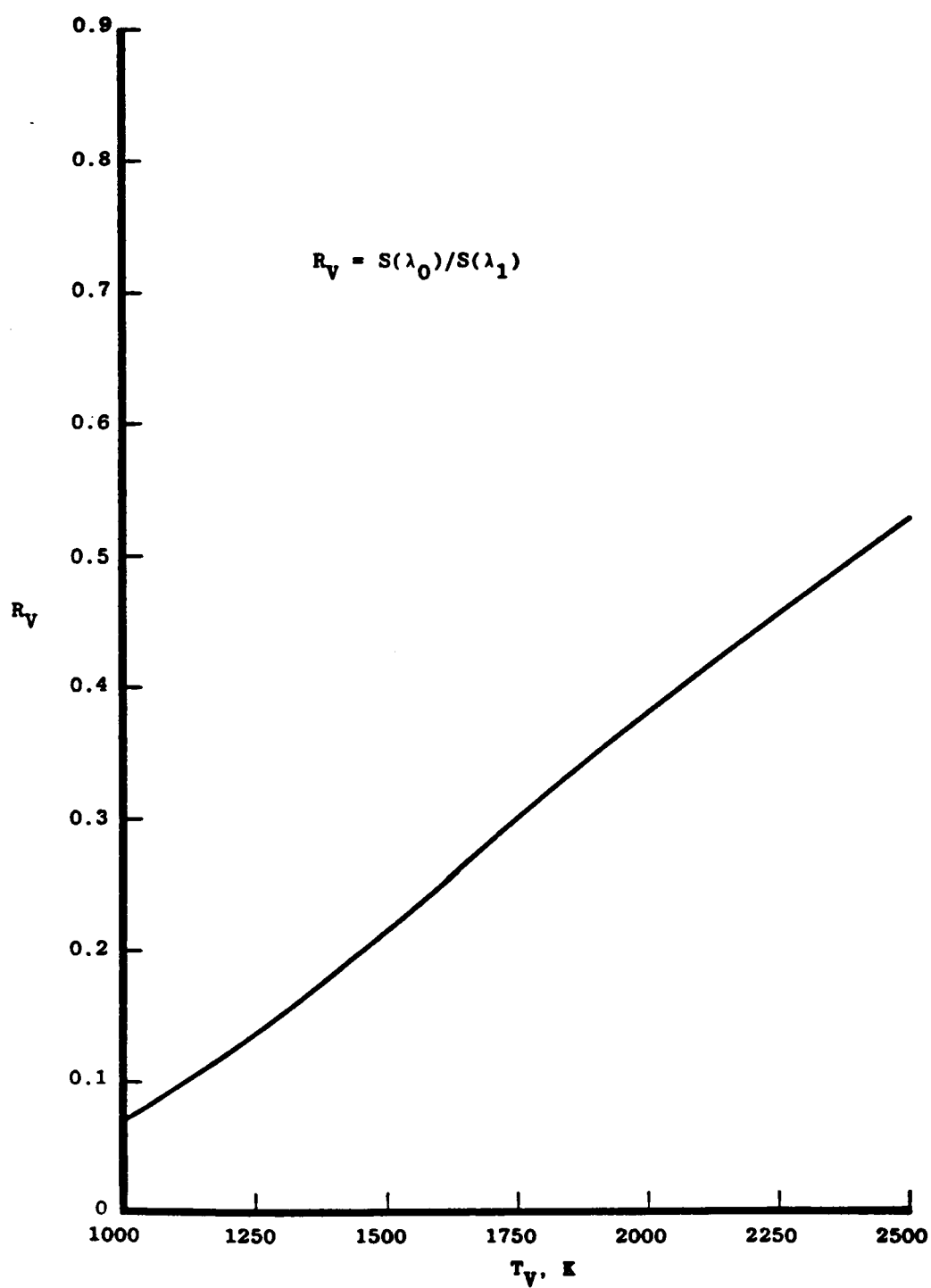


Figure 6.4. Ratio R_V of Raman Signals versus T_V .

magnitude of the Raman signals, and simple subtraction was used to correct the Raman signals for this background continuum which was assumed to vary little over the spectral band pass of the Raman measurements. Figure 6.5 shows typical oscilloscope traces for the Raman signals at λ_0 and λ_1 , and the nominal 20 nsec width and signal levels, which exceed 20 mV, are to be noted. For a series of measurements corresponding to $M_1 = 4.96 \pm 0.04$, the values of T_{vib} were determined, and the results were $T_{vib} = 2,110 \pm 220$ K, where the uncertainty value of 220 K represents the standard deviation of the sample.

The measured value of T_{vib} exceeds the gas temperature $T_2 = 1,600$ K which was determined by u_s measurements, and the approximate 20-percent difference exceeds the estimated uncertainty in both T_{vib} and T_2 , the latter of which is on the order of 2 to 3 percent. The most likely cause of the discrepancy between T_{vib} and T_2 is that measurements of T_{vib} were performed early in the reflected shock region where T_{vib} is increasing to its final value, which is significantly larger than both 1,600 K and 2,110 K. Finally, the approximately 10-percent error in T_{vib} is reasonably consistent with the overall uncertainty of about 7.5 percent for $S(\lambda_0)$ and $S(\lambda_1)$.

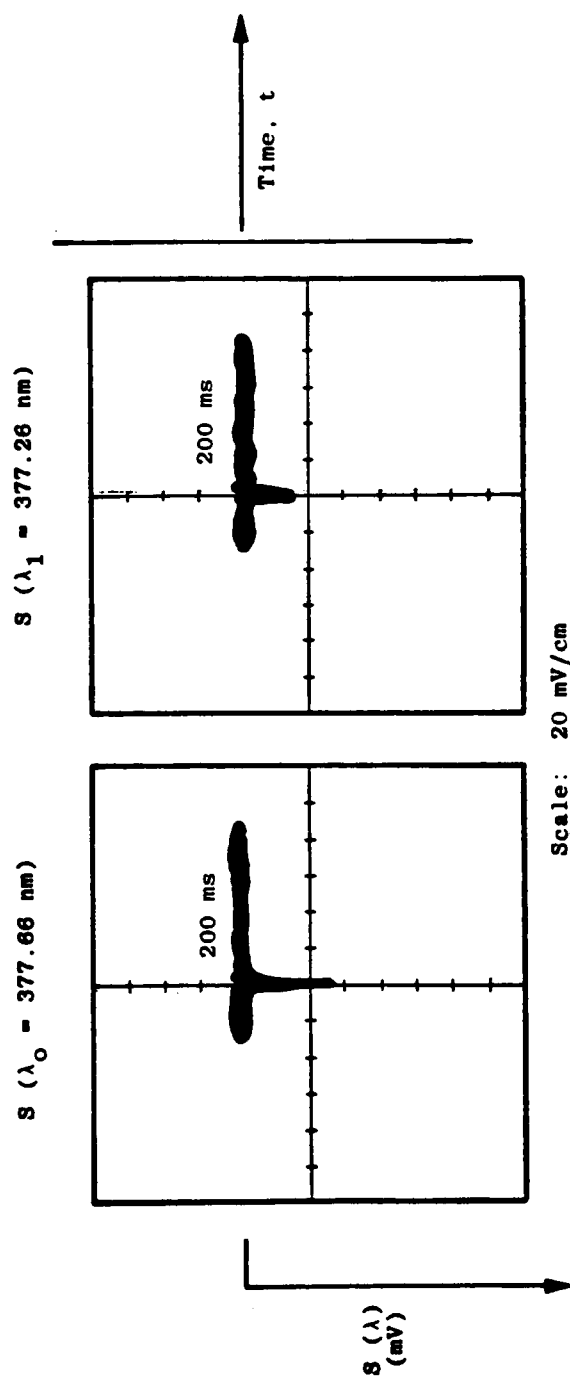


Figure 6.5. Oscilloscope Traces of Raman Signals $S(\lambda_0)$ and $S(\lambda_1)$.

Conclusions

The demonstration of the laser Raman-shock tube system has been accomplished, and the ability to perform Raman measurements in a shock tube environment has been shown. The necessity for operation with a stainless steel interior wall, rather than seamless steel, was shown conclusively, as was the comparatively low levels of particulate-related scattering signals with the stainless steel surface. The vibrational temperature T_{vib} was found to be $2,110 \pm 220$ K for a series of measurements. These results were acquired using a single PMT channel for detection, and successive laser-shock tube shots were required to obtain the two signals required for the T_{vib} determining ratio. One or more background shots were necessary to correct for the background radiation effects. The uncertainty in T_{vib} could be lessened by performing at least two of these measurements during a single firing of the laser and shock tube. It is believed that uncertainty levels of ± 5 percent are obtainable in this manner.

The incident shock speed was determined using heat-transfer gauges, and it was found that the shock speed u_s could be measured with an imprecision of less than approximately 1 percent and that shot-to-shot repeatability, as reflected by u_s data, was on the order

of 1 to 2 percent. Improvements in diaphragm rupture techniques could lessen this uncertainty. The shock Mach number M_1 was found to be 4.96 ± 0.04 for the series of T_{vib} measurements and this value of M_1 corresponds to an ideal gas temperature of approximately 1,600 K. The necessity for accurate diagnostics of the reflected shock pressure and speed was made clear during the course of this study. Failure to obtain such data resulted in an uncertainty in the location within the shock of the laser scattering data. The discrepancy between $T_2 = 1,600$ K and $T_{vib} = 2,110$ K is suggested to arise from the acquisition of laser data to have occurred in the higher temperature reflected shock region than in the incident shock region. Further evidence that the measurements were made in the early portions of the reflected shock exists in the calculations of Chapter IV which predicted a very slowly relaxing incident shock followed by a much faster relaxation of the reflected shock wave. If the observation had been made within the incident wave, T_{vib} would have very probably been much lower than 1,600 K.

Use of the laser-shock tube system for accurate cross-section and calibration measurements will require making the relatively minor and previously mentioned experimental improvements in the system and incorporating into the theoretical analysis procedure the computations

presented in Chapters III and IV. With the aid of these calculations and the experimental improvements, vibrational rate processes could very easily be studied in the pressure-driven shock tube. Moreover, other nonequilibrium processes, such as dissociation and ionization can most likely be observed. Thus, the pressure-driven, Raman scattering shock tube system, carefully constructed, can be an extremely useful high-temperature gas laboratory.

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LIST OF REFERENCES

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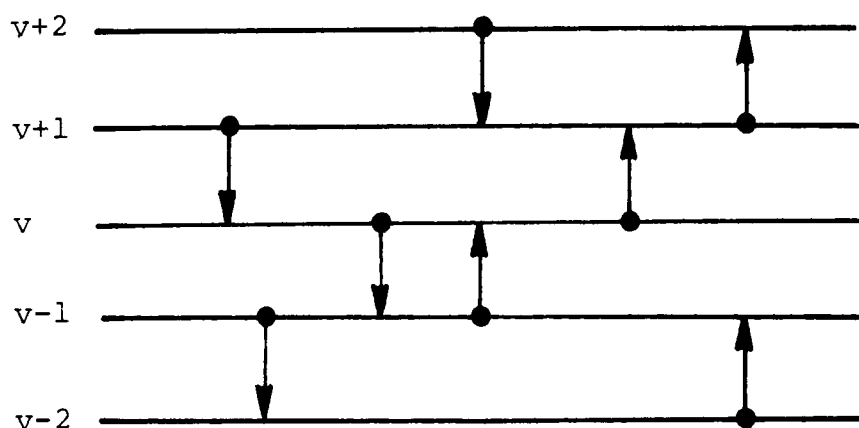
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APPENDICES

APPENDIX A

DETAILED BALANCING

To determine the vibrational energy derivative, de_{vib}/dx , consider the following model of energy transfer processes:



For a vibrational level v , using the selection rule for vibrational transitions, $\Delta v = \pm 1$, the number of depletion transitions for level v is given by

$$\text{number of depletion transitions} = k_{v,v+1} n_v + k_{v,v-1} n_v ,$$

where $k_{v,v+1}$ and $k_{v,v-1}$ are the occurrence rates for transitions from level v to levels $v+1$ and $v-1$, respectively; i.e., $k_{v,v+1}$ and $k_{v,v-1}$ are directly proportional to the

respective transition probabilities. Similarly, the number of additive transitions for level v is given by

$$\text{number of addition transitions} = k_{v-1,v}n_{v-1} + k_{v+1,v}n_{v+1} ,$$

where $k_{v-1,v}$ and $k_{v+1,v}$ are the transition probabilities for transitions into level v from level $v-1$ and $v+1$, respectively. Thus, for level v , the time rate of change of the number of molecules is given by

$$\frac{dn_v}{dt} = k_{v+1,v}n_{v+1} + k_{v-1,v}n_{v-1} - (k_{v,v+1}n_v + k_{v,v-1}n_v) . \quad (\text{A.1})$$

In the case $v=0$,

$$\frac{dn_0}{dt} = k_{10}n_1 - k_{01}n_0 ,$$

as $n_{-1}=0$.

At equilibrium,

$$\frac{dn_0^0}{dt} = 0 ,$$

which leads to

$$k_{10}n_1^0 - k_{01}n_0^0 = 0 \quad \text{or} \quad \frac{k_{10}}{k_{01}} = \frac{n_0^0}{n_1^0} . \quad (\text{A.2})$$

By similar methods,

$$\begin{aligned} \frac{dn_v^0}{dt} = 0 = & (k_{v+1,v}n_{v+1} - k_{v,v+1}n_v) \\ & + (k_{v-1,v}n_{v-1} - k_{v,v-1}n_v) . \end{aligned} \quad (\text{A.3})$$

By the principle of detailed balancing, the parenthetical quantities must separately vanish, leading to

$$\frac{k_{v+1,v}}{k_{v,v+1}} = \frac{n_v}{n_{v+1}} \quad \text{and} \quad \frac{k_{v+1,v}}{k_{v,v-1}} = \frac{n_v}{n_{v+1}} \quad (\text{A.4})$$

But, from statistical mechanics,

$$\frac{n_v}{n_T} = \frac{e^{\frac{-\eta_v}{kT}}}{q_{\text{vib}}} , \quad (\text{A.5})$$

where η_v is the vibrational energy and q_{vib} is the vibrational partition function.

At equilibrium,

$$\frac{n_v^0}{n_T} = \frac{\exp[-(v + \frac{1}{2})\frac{h\nu}{kT}]}{q_{\text{vib}}} , \quad (\text{A.6})$$

or,

$$\frac{n_v^0}{n_T} = e^{\frac{-v\theta_v}{T}} (1 - e^{\frac{-\theta_v}{T}}) , \quad (\text{A.7})$$

where $\theta_v = h\nu/k$, h is Planck's constant, ν is the molecular vibrational frequency, and k is Boltzmann's constant. In the above, use has been made of the simple harmonic oscillator partition function,

$$q_v = \frac{e^{-\frac{\theta_v}{2T}}}{(1 - e^{-\frac{\theta_v}{T}})} . \quad (\text{A.8})$$

Similarly,

$$\frac{n_{v+1}^0}{n_T} = e^{-\frac{(v+1)\theta_v}{T}} (1 - e^{-\frac{\theta_v}{T}}) . \quad (\text{A.9})$$

Dividing by n_v/n_T leads to

$$\frac{n_{v+1}^0}{n_v^0} = e^{-\frac{\theta_v}{T}} = \frac{n_v^0}{n_{v-1}^0} = K . \quad (\text{A.10})$$

Thus, using the results that

$$\frac{k_{v,v+1}}{k_{v+1,v}} = \frac{n_{v+1}}{n_v}$$

implies that

$$\frac{k_{v,v+1}}{k_{v+1,v}} = \frac{n_{v+1}}{n_v} = K = \frac{k_{v-1,v}}{k_{v,v-1}} = e^{-\frac{\theta_v}{T}} , \quad (\text{A.11})$$

which implies that $k_{01}=Kk_{10}$, where the equilibrium subscript has been dropped. Thus, $k_{v,v+1}=Kk_{v+1,v}$ and $k_{v-1,v}=Kk_{v,v-1}$.

From simple harmonic oscillator dipole transition moments [9, pp. 198-206], it can be shown that

$$\frac{k_{v+1,v}}{k_{v,v-1}} = \frac{(v+1)}{v} \quad \text{implies} \quad k_{v+1,v} = \frac{(v+1)}{v} k_{v,v-1} , \quad (\text{A.12})$$

or, more generally,

$$k_{v+1,v} = \left[\frac{(v+1)}{(v-j)} \right] k_{v-j,v-j-1} . \quad (\text{A.13})$$

This result leads to the general result, when substituted into the dn_v/dt equation and rearranged,

$$\frac{dn_v}{dt} = k_{v+1,v}(n_{v+1}-Kn_v) + k_{v-1,v}[Kn_{v-1}-n_v] , \quad (\text{A.14})$$

or,

$$\frac{dn_v}{dt} = k_{10}[(v+1)n_{v+1} - [K+v(1+K)n_v + vKn_{v-1}]] . \quad (\text{A.15})$$

This gives a number-density relaxation equation, which can be expressed as an energy relaxation also.

Expressing all vibrational energies, η_v , relative to the $v=0$ level, leads to $\eta_v=v\hbar\nu$. Substituting into Equation (A.15) leads to

$$\frac{dE_v}{dt} = h\nu k_{10} \sum_{v=1}^{\infty} [Kv^2 n_{v-1} - v^2 n_v - K(v+1)vn_v + v(v+1)n_{v+1}] , \quad (A.16)$$

where E_v is the total vibrational energy, and

$$\sum_0^{\infty} n_v \frac{dn_v}{dt} = \sum_1^{\infty} n_v \frac{dn_v}{dt} = \frac{dE_v}{dt} . \quad (A.17)$$

The fourth term within the bracket may be rewritten as

$$\sum_1^{\infty} v(v+1)n_v = \sum_0^{\infty} v(v-1)n_v = \sum_0^{\infty} (v^2 - v)n_v . \quad (A.18)$$

The second term can be expressed as $\sum_0^{\infty} -v^1 n_v$. Summing the second and fourth term yields $-\sum_0^{\infty} vn_v$. Summing the first and third yields $K\sum_0^{\infty} (v+1)n_v$. Substitution leads to

$$\frac{dE_v}{dt} = h\nu k_{10} [K \sum_0^{\infty} (v+1)n_v - \sum_0^{\infty} vn_v] \quad (A.19a)$$

or

$$\frac{dE_v}{dt} = -k_{10} [(1-K)E_v - h\nu Kn_T] . \quad (A.19b)$$

This may be rewritten as

$$\frac{dE_v}{dt} = -k_{10}(1-K) \left[E_v - \frac{K}{1-K} h\nu n_T \right] . \quad (A.20)$$

Previously it was shown that

$$K = e^{\frac{-\theta_v}{T}} ;$$

thus,

$$\left(\frac{K}{1-K}\right) h\nu n_T = \left(\frac{h\nu e^{\frac{-\theta_v}{T}}}{1 - e^{\frac{-\theta_v}{T}}}\right) n_T = E_v^0 , \quad (\text{A.21})$$

which implies

$$\frac{dE_v}{dt} = -k_{10}(1-K)[E_v - E_v^0] = -\frac{1}{\tau}[E_v - E_v^0] , \quad (\text{A.22})$$

where

$$\frac{1}{\tau} = k_{10}(1 - e^{\frac{-\theta_v}{T}}) \quad \text{or} \quad \frac{k_{10}}{k_{01}} = e^{\frac{\theta_v}{T}} .$$

The result can be expressed as de_{vib}/dt by noting that E_v^0 can be expressed as a molar energy E_v^0 ,

$$E_v^0 = R\theta_v \frac{e^{\frac{-\theta_v}{T}}}{1 - e^{\frac{-\theta_v}{T}}} . \quad (\text{A.23})$$

The end result is

$$\frac{de_{\text{vib}}}{dt} = \frac{-1}{\tau}[e_{\text{vib}} - e_{\text{vib}}^0] \quad \text{or} \quad \frac{de_{\text{vib}}}{dx} = \frac{-1}{\tau u}[e_{\text{vib}} - e_{\text{vib}}^0] . \quad (\text{A.24})$$

APPENDIX B

MAXIMUM SHOCK-FRONT-CONTACT-SURFACE
SEPARATION DISTANCE

The separation distance between the contact surface and shock front has been experimentally shown to reach a maximum value. Once this separation has been reached, the shock front and contact surface move with the same constant velocity. For a tube of uniform cross section, the flow is thus steady in a shock-fixed coordinate system. Using Mirels' [11,12] notation, the shock front can be viewed as a wall moving with velocity u_w , which is equal to the shock velocity in laboratory coordinates. This situation is shown diagrammatically in Figure 3.1, page 27. The shock is located at $x_\ell = 0$, and the contact surface at $x_\ell = x_{\ell m}$. The upstream conditions are denoted by the subscript ∞ , with $u_w = u_\infty$. Conditions within the shocked gas region are denoted by the subscript e, and immediately downstream of the shock front by the double subscript eo.

According to Roshko [14], the flow rate of mass through the shock is given by

$$\frac{dm_s}{dt} = \rho e u_{eo} A, \quad (B.1)$$

where ρ indicates density and A is the cross-sectional area. Roshko assumed a thin boundary layer with the flow in the

boundary layer at the contact surface given by

$$\frac{dm_c}{dt} = L\rho_{w,o}(u_w - u_{e,o})\delta_R, \quad (B.2)$$

in which L is the tube perimeter, δ_R is a characteristic boundary layer thickness at x_{lm} , $\rho_{w,o}$ is a characteristic boundary layer density, and $u_w - u_{e,o}$ is a characteristic boundary layer velocity. The boundary layer thickness, δ_R is also given by Roshko as

$$\delta_R = \beta \left(\frac{v_{w,o} x_{lm}}{u_w - u_{e,o}} \right)^{1/2}, \quad (B.3)$$

where β is a constant that must be found from an exact solution of the boundary layer development, and $v_{w,o}$ is a characteristic boundary layer viscosity. Mirels combined Equations (B.1) and (B.2), yielding, after some rearrangement,

$$x_{lm} = \frac{d^2}{16\beta^2} \left(\frac{\rho_{e,o}}{\rho_{w,o}} \right)^2 \left(\frac{u_{e,o}}{u_w - u_{e,o}} \right) \left(\frac{u_{e,o}}{v_{w,o}} \right), \quad (B.4)$$

in which d is the hydraulic diameter. Mirels assumed that the temperature upstream of the shock is at a standard condition, $T_\infty = T_{st}$, the speed of sound $a_\infty = a_{st}$, and $\mu_\infty = \mu_{st}$. Mirels also assumed that the wall temperature remains at its initial value, $T_w = T_{st}$. Using the conservation of

mass, $\rho_\infty u_w = (\rho_e u_e)_o$, and substituting into Equation (B.4), yields,

$$\frac{p_{st}}{p_\infty} \frac{x_{lm}}{d^2} = \frac{1}{16\beta^2} \frac{p_\infty}{p_{e,o}} \frac{WM_s}{(W-1)} \left(\frac{\rho a}{\mu}\right)_{st}, \quad (B.5)$$

in which $W = u_w/u_{e,o}$, $M_s = u_w/a_\infty$, and p_{st} is a standard pressure (one atmosphere in appropriate units).

Examination of Equation (B.5) shows that for a given Mach number M_s , x_{lm} depends on the product $p_\infty d^2$. Thus, for a small diameter tube, or for a low initial test gas pressure, the actual test time may be very short.

Also, from Equation (B.5) it is noted that an accurate estimate of β must be obtained in order for the separation distance x_{lm} to be calculable. Estimates of β have been presented by Mirels using the concept of local similarity. The form for β used in the present study is that of β_1 given by Mirels as

$$\beta = \beta_1 = 1.59 C_{e,o}^{0.37} \left[1 + \left(\frac{1.796 + 0.802W}{ZW - 1} \right) \right], \quad (B.6)$$

where $z = \gamma + 1/\gamma - 1$ for an ideal shock and Z is replaced by W for a strong nonideal shock ($W > Z$); $C_{e,o} = \left(\frac{\rho_e \mu_e}{\rho_w \mu_w} \right)_o$ is a constant which corrects for the variable $\rho\mu$ in the boundary layer. Values for $C_{e,o}$ are taken from Tables 1 and 2 of the Mirels paper [11].

With Equation (B.6) as an estimate for β , the maximum separation distance $x_{\ell m}$ may now be calculated from Equation (B.5). The standard condition $(\frac{\rho a}{\mu})_{st}$ was taken to be $p_{st}=1$ (atm) (in cgs units) yielding

$$(\frac{\rho a}{\mu})_{st} = 2.30 \times 10^5 \text{ cm}^{-1} (\text{air}) = 2.35 \times 10^5 (\text{cm})^{-1} (\text{N}_2) . \quad (\text{B.7})$$

Figure 4.8, page 55, illustrates the variation of the maximum separation distance, $x_{\ell m}$, with initial test gas pressure p_1 at various flow Mach numbers M_s . Figure 4.9, page 56, shows the variation of the maximum separation $x_{\ell m}$ with tube diameter, d .

APPENDIX C

LIMITATIONS DUE TO THE TUBE LENGTH

As is noted in Figures 3.1 and 3.2, pages 27 and 31, respectively, maximum separation distance, $x_{\ell m}$, may not be reached for a large diameter or high test gas pressure shock tube due to length restrictions. This will result in a decrease in the maximum test time. Therefore, the separation distance as a function of the distance x_s from the diaphragm is needed in order to predict the actual test time available.

This problem has been discussed thoroughly by Mirels [11,12]. He assumed that, since the flow is steady, the mass flow through the shock is the same as the mass flow at any point x_ℓ along the centerline of the flow. Continuity implies

$$A(\rho_e u_e)_0 = A\rho_e u_e + L \int_0^\infty (\rho u - \rho_e u_e) dy, \quad (C.1)$$

as illustrated in Figure 3.1. In this equation it is assumed that the boundary layer thickness is much smaller than the tube diameter d . This implies that the integrand in Equation (C.1) is zero everywhere except very near the tube wall. Mirels defines

$$\delta = \left(\frac{4}{d}\right) \left[\frac{\rho_e u_e}{(\rho_e u_e)_o} \right] (-\delta^*) , \quad (C.2)$$

where δ^* is the local boundary layer thickness,

$$\delta^* = \int_0^\infty \left(1 - \frac{\rho_u}{\rho_e u_e}\right) dy . \quad (C.3)$$

Here, δ is the ratio of the mass flow through the boundary layer to the mass flowing through the shock front. This implies that δ varies from 0, at $x_\ell=0$, to 1, at $x_\ell=x_{\ell m}$. From Equations (C.1) and (C.2), thus,

$$\delta = 1 - \frac{\rho_e u_e}{(\rho_e u_e)_o} . \quad (C.4)$$

This result can be used to develop an approximation for x as follows: Neglecting variations in the free-stream density, the second term in Equation (C.4) can be written simply as $u_e/u_{e,o}$. Mirels, using parametric equations, writes this as $u_e/u_{e,o} = dT/dX$, where $x = u_{e,o} t / x_{\ell m} = x_s / W x_{\ell m}$, and $T = x_\ell / x_{\ell m}$. Mirels notes that since δ is the ratio of the excess mass flow through the boundary layer at x_ℓ to the mass flow through the boundary layer at $x_{\ell m}$, and since, for strong shocks, the excess mass flow in the boundary layer grows as $x_\ell^{1/2}$, then $\delta = \left(\frac{x_\ell}{x_{\ell m}}\right)^{1/2} = T^{1/2}$. Thus, Equation (C.4) can be written as:

$$T^{1/2} = 1 - \frac{dT}{dX} , \quad (C.5)$$

with solution

$$-\frac{1}{2}X = \ln(1-T^{1/2}) + T^{1/2} . \quad (C.6)$$

Equation (C.6) can be solved for $x_\ell/x_{\ell m}$, and thus x_ℓ is determined for a specific length and diameter tube. Figure 4.10, page 57, shows the variation of x_ℓ with initial pressure p_1 for a range of flow Mach numbers assuming a 2.5-inch diameter and 4.26-meter diaphragm-to-test cell distance, x_s .

VITA

Michael Steven Smith was born in Elizabethton, Tennessee, on July 17, 1954. He received a Bachelor of Science degree in Physics from East Tennessee State University in June 1976. Subsequently he was employed as a Non-Destructive Assay Specialist by Nuclear Fuel Services, Inc., in Erwin, Tennessee. In February 1979, he accepted a position as a Quality Assurance Engineer with the Fuels Engineering group of Westinghouse Hanford Company in Richland, Washington.

In the fall of 1979 he accepted a graduate research assistantship with the Advanced Test Diagnostics group at Arnold Engineering Development Center, Tullahoma, Tennessee, and began study toward a Master's degree at The University of Tennessee Space Institute. He became a Technical Sales Engineer with Pulcir, Inc., in Oak Ridge, Tennessee, in April 1982, and in August 1983 he returned to The University of Tennessee Space Institute as a graduate research assistant with the Applied Physics Research Group. He was awarded the Master of Science degree in Physics in August 1984.

Following graduation Mr. Smith will continue his graduate studies for the Doctor of Philosophy degree in Physics.