May 23, 1973

To the Graduate Council:

I am submitting herewith a thesis written by Behrad Nakhai entitled "Fluid Flow Measurement Employing Temperature Fluctuations." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Nuclear Engineering.

Major Professor

We have read this thesis and recommend its acceptance:

Accepted for the Council:

Vice Chancellor for
Graduate Studies and Research

FLUID FLOW MEASUREMENT EMPLOYING TEMPERATURE FLUCTUATIONS

A Thesis

Presented to

the Graduate Council of

The University of Tennessee

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

---

by

Behrad Nakhai

August 1973

ACKNOWLEDGMENT

This work is dedicated to A., the man who has inspired my soul and to my parents who have patiently made me what I am.

I would like to extend my appreciation to my committee members, especially Dr. T. W. Kerlin who served as my advisor. My special thanks go to Mr. Jimmy Smith who helped in every phase of the experiments and built the necessary instruments.

My deepest gratitude goes to my wife who patiently endured this long lasting task and shared the efforts.

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ABSTRACT

Theoretically, any periodic or random signal that is transported by a flowing fluid could be analyzed for fluid flow measurements. In this study, temperature fluctuations were used to determine the volumetric flow rate of water flowing through a pipe. Signals, detected by thermocouples at two points in the flow stream, were crosscorrelated using different delay times. The delay time that gave maximum correlation was taken as the transit time of fluid between the thermocouples. The technique proved to be satisfactory and the uncertainties observed were due to finite sampling interval, since the peak of the crosscorrelation function occurred at the same sampling point in each run (for a particular flow), and hence no information was available on statistical uncertainties.

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CHAPTER I

INTRODUCTION

The purpose of this study is to demonstrate the use of inherent temperature fluctuations in a flowing stream for flow measurements by means of the Transit Time Flowmeter (TTF) technique. Theoretically, any periodic or random signal that is transported by a flowing fluid could be analyzed for fluid flow measurement. The TTF technique is based on detecting random temperature fluctuations at two points in the flow stream. Signals are crosscorrelated using different delay time. As it is explained below, the delay time that gives maximum correlation is the transit time of fluid between the detectors.

The correlation function is defined as:

$$C_{ij}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f_i(t) f_j(t+\tau) dt \qquad (1)$$

where

$$C_{ij}(\tau) = \begin{cases} \text{autocorrelation function} & \text{if } i = j \\ \text{crosscorrelation function} & \text{if } i \neq j \end{cases}$$

It is well known (1, 2, 3)[1] that the autocorrelation function has its maximum value at $\tau = 0$.

---

[1]The numbers in parentheses correspond to the references at the end of the report.

To simplify the explanation of the use of temperature crosscorrelation measurements, let us consider two signals which are time-shifted versions of the same signal (Figure 1). $f_j(t)$ is the same signal as $f_i(t)$ except for a time delay, τ_t. If τ_t was equal to zero, the signals would be identical and the maximum value of the crosscorrelation function would be at $\tau_t = 0$. If the two signals $f_i(t)$ and $f_j(t)$ are identical but displaced τ_t seconds relative to one another, the peak of the crosscorrelation function would no longer occur at $\tau = 0$ but rather at $\tau = \tau_t$ (Figure 2).

Suppose $f_i(t)$ represent the response of a thermocouple to temperature fluctuations at a point in the flow stream. Also, let $f_j(t)$ be the response of another thermocouple placed downstream at a distance L from the first thermocouple. Now, suppose that the stream is made up of small packets of fluid and that the packets each have temperatures that differ slightly from the average fluid temperature. In this case, $f_j(t)$, the response of the downstream thermocouple, is identical (assuming slug flow and no heat transfer) to $f_i(t)$, the response of the upstream thermocouple, except for the time lag τ_t, which is equal to the transit time of the packets of fluid between the two thermocouples. Consequently, the crosscorrelation function between these signals looks like the crosscorrelation function described above (Figure 2), and has its maximum value at τ_t. If the flow is turbulent the velocity profile across the channel is nearly constant and the lag time, τ_t, is the transport time of fluid between the thermocouples, hence

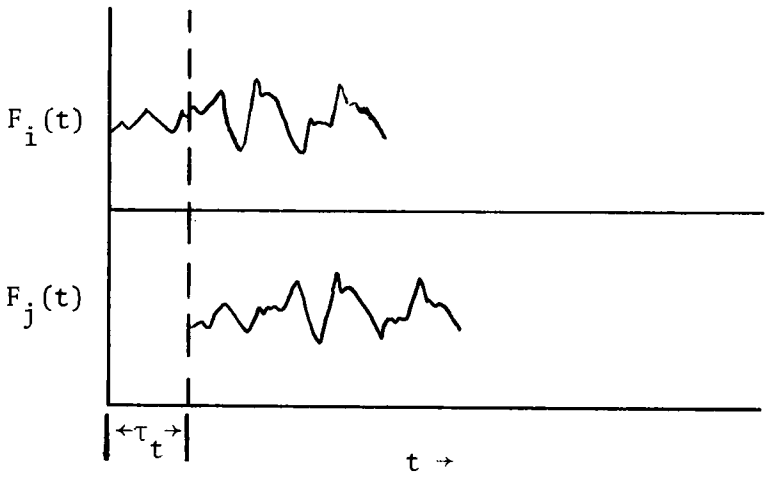


Figure 1. A Random Function and the Same Function Displaced by τ_t.

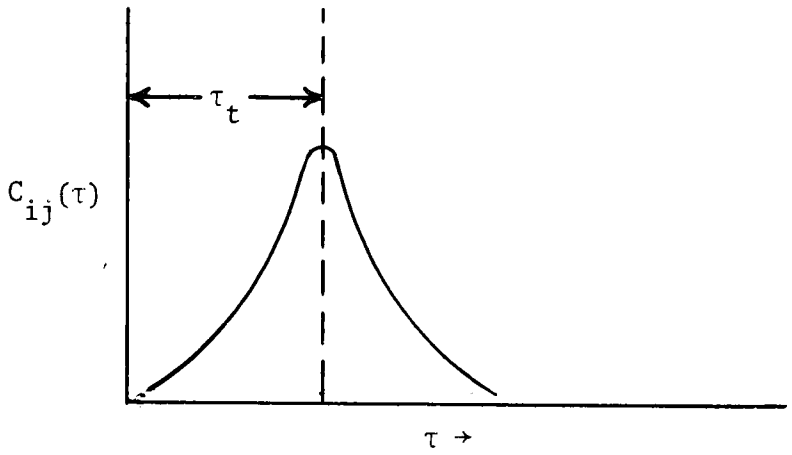


Figure 2. The Crosscorrelation Function Between $F_i(t)$ and $F_j(t)$.

$$\tau_t = \frac{L}{V} \tag{2}$$

where V is the stream velocity.

In the actual case, turbulent mixing and thermal diffusion tend to decrease the peak correlation (1) and broaden the peak.

CHAPTER II

REVIEW OF PRIOR INVESTIGATIONS

The feasibility of the TTF technique for fluid flow measurement has been investigated by several researchers. As their results show, the TTF technique is a feasible technique for flow measurement, and both its accuracy and simplicity in implementation are confirmed.

Bentley and Dawson (4) have used thermocouples as sensors and have analyzed their data in the frequency domain by means of a cross spectrum analyzer.

Randall (1) performed his analysis in the real time domain with the aid of an analog crosscorrelator. He employed three different sensors: (1) thermocouples, (2) thermocouple fin assemblies, and (3) conductivity electrodes.

Termaat (5) performed his experiments on the Dodewaard Nuclear Power Plant. The reactor was a 50-MWe Boiling Water Reactor with natural circulation. He used 30 pairs of fast response thermocouples to detect temperature fluctuations in the recirculating reactor water and a multipoint polarity coincidence correlator to form the crosscorrelation function. His results show the method to be very useful in the determination of local fluid velocities, velocity profiles and total recirculation flow.

D. Pallagi and S. Horanyi of the Central Research Institute for Physics of Hungary (6) employed correlation methods for out-of-core

detection of bulk-boiling and for in-core coolant flow determination.

In recent studies (7), it has been noted that, in cases where the fluid-to-channel wall heat transfer coefficient is high, the transit time of fluid measured by the standard TTF technique, does not agree with the transit time measured by the other techniques. This effect is due to the elongation of the energy flow path between thermocouples. If the fluid has a high heat transfer coefficient, an appreciable amount of its thermal energy is transferred to the pipe walls temporarily and later transferred back to the fluid giving an effective path length for energy transport that is longer than the flow path length. As a consequence, the crosscorrelation function peaks at a delay time that is larger than the true delay time. Hence, the TTF technique which is based on detecting the temperature fluctuations of fluid would indicate a delay time that is too long. Successful application of the TTF technique will require the development of new experimental procedures or interpretation methods.

CHAPTER III

THEORY

A. General Theory

The ensemble average[1] of the product of a time dependent function and another function of time displaced τ seconds is called "The Correlation Function" and is given by:

$$C_{ij}(\tau) = \overline{f_i(t) f_j(t+\tau)} \tag{3}$$

where

τ = searching or scanning parameter (lag time)

$$C_{ij}(\tau) = \begin{cases} \text{autocorrelation function} & \text{if } i = j \\ \text{crosscorrelation function} & \text{if } i \neq j \end{cases}$$

For ergodic processes,[2] the ensemble averages and time averages are identical, and the correlation function could also be written as:

---

[1] The average value of the collection of a set of sample functions is called the ensemble average of that set.

[2] Ergodic processes are processes where the complete statistics can be determined from any one sample function (8).

$$C_{ij}(\tau) = \overline{f_i(t)f_j(t+\tau)} = \lim_{T \to \infty} \frac{1}{T} \int_0^T f_i(t)f_j(t+\tau)dt \qquad (4)$$

To get a feel for the magnitude of $C_{ij}(\tau)$ associated with the correlated and uncorrelated waveforms, often the normalized cross-correlation function defined by

$$\rho_{ij}(\tau) = \frac{C_{ij}(\tau)}{[C_{ii}(0)\ C_{jj}(0)]^{\frac{1}{2}}} \leq 1 \qquad (5)$$

is formed. Values of the magnitude of $\rho_{ij}(\tau)$ near unity, e.g. 0.8 or 0.9 indicate a high degree of correlation between the signals, while values below 0.3 indicate a very poor correlation (9).

B. Theory as Adopted in this Analysis

When dealing with discrete functions, as was the case for this study because a digital computer was used for data analysis, the integral in the crosscorrelation function could be replaced by a summation as follows:

$$C_{12}(\tau) = \frac{1}{N} \sum_{k=1}^{N} f_1(k)\ f_2(k+\tau) \qquad (6)$$

which is a form of the crosscorrelation for discrete functions.

To make the study and comparison of the results easier, the crosscorrelation functions are normalized to a maximum of unity. This is done by dividing each term of the crosscorrelation function by the maximum value of the function. The normalized crosscorrelation function is then given by:

$$NC_{12}(\tau) = \frac{C_{12}(\tau)}{\text{Max } C_{12}(\tau)} \tag{7}$$

where

Max $C_{12}(\tau)$ = the maximum value of the crosscorrelation function.

$NC_{12}(\tau)$ = normalized crosscorrelation function.

CHAPTER IV

SYSTEM AND INSTRUMENTATION

Initially, this study was performed on the heat transfer loop of the Nuclear Engineering Department of The University of Tennessee. The loop is a closed system containing distilled water. The major components of this system are (Figure 3):

1. Centrifugal pump
2. Flowmeter
3. Resistance heater
4. Test section (a piece of stainless steel pipe 6 feet long and 1 inch in diameter)
5. Heat exchanger

Thermocouples were mounted 2½ feet apart in-stream along the test section.

During the course of experiments performed on this rig, the behavior of the crosscorrelation function was observed to be periodic rather than the expected bell shape, as it is explained previously (see Figure 2, page 3). The periodicity was later explained to be due to the electrical field (60 HZ noise) induced by the heater. As it is shown in Appendix A, the crosscorrelation function between two periodic functions of equal frequency is periodic with the same period, and is zero for functions with different periods. For this system, the crosscorrelation function obtained was actually the supperposition of the crosscorrelation function between the thermocouple outputs and that of the 60 HZ noise. Theory

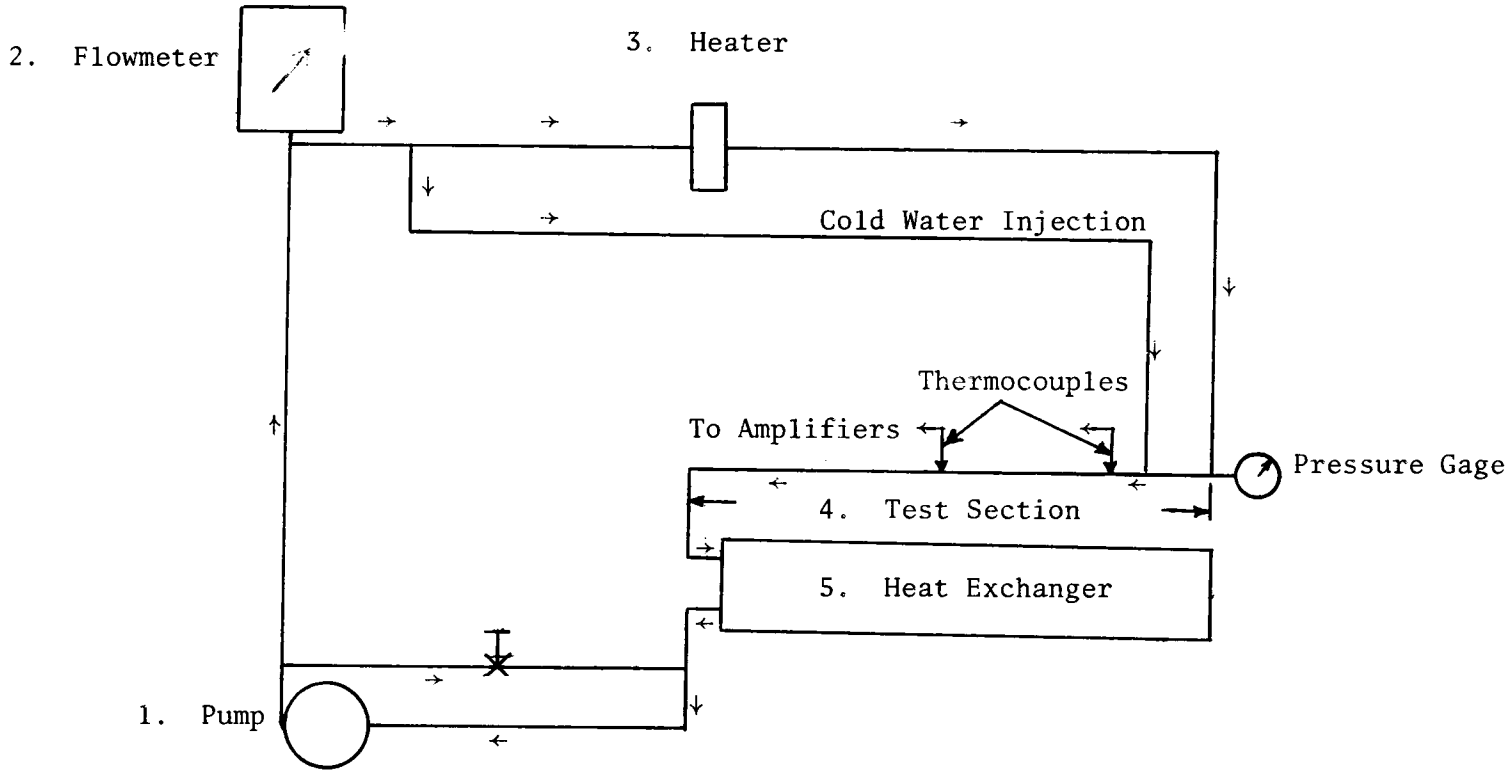


Figure 3. Schematic Diagram of the Heat Transfer Loop.

indicates that the desired crosscorrelation function and the transit time of the fluid could be deduced from the obtained crosscorrelation function by proper filtering.

However it was decided to modify the system to avoid the 60 HZ background rather than use a filtering method. The modified system and its components are described below.

Figure 4 shows the schematic diagram of the system. The main components of the system are:

A. Sensors

B. Test section

C. Amplifiers

C. Analog computer

E. Digital mini computer utilized with analog to digital convertor and pulse generator.

A short description of each component follows.

A. Sensors

Iron-constantan thermocouples (Figure 5) were used as sensors to measure the temperature fluctuations of the fluid. Thermocouples spaced 2½ feet apart were inserted to the center line (approximately) of the test pipe. In order to study the effect of time constants of the thermocouples on flow measurements, three different sizes of thermocouples were used. In this report, the thermocouples are referred to as thermocouple size one, thermocouple size two, and thermocouple size three (see Table 1).

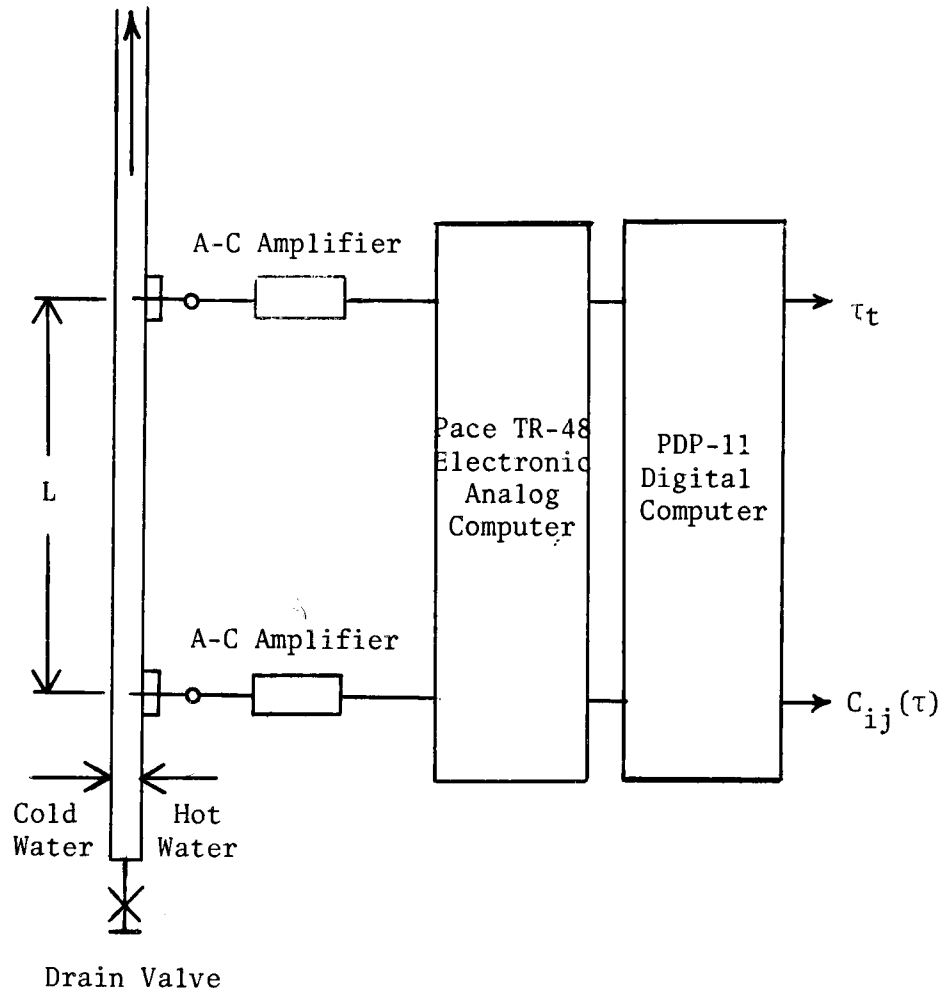


Figure 4. Schematic Diagram of the System.

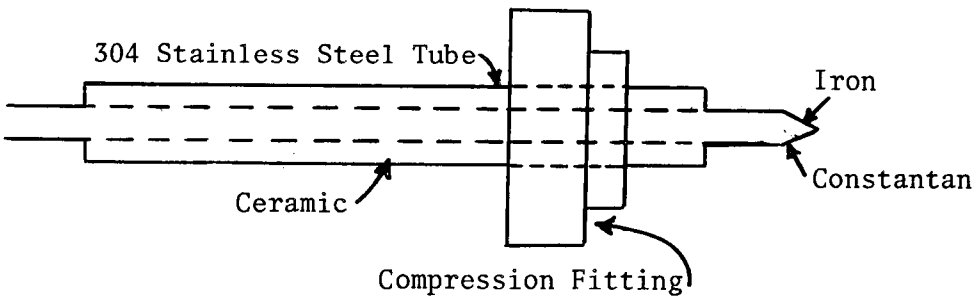


Figure 5. Schematic Diagram of Thermocouple.

TABLE 1

Thermocouple Specifications

Thermocouple Size	Sheath Diameter (in.)†	Wall Thickness (in.)†	Diameter of Wires (in.)†
1	0.125	0.018	0.020
2	0.188	0.025	0.032
3	0.250	0.036	0.040

Thermocouples: Iron constantan type J (ISA symbol) sheath material 304 SS.

†From "The Omega Temperature Measurement Handbook," Omega Engineering, Inc., Stamford, Conn., 06907.

B. Test Apparatus

Hot and cold water reached the test section[1] through rubber hoses, mixed at the bottom of the pipe and exited from the top. The pipe was electrically insulated from the rack and other instruments. Two blocks of stainless steel were welded (2.5 feet apart from their centerlines) on the pipe, and milled through to provide proper housing for the compression fittings through which the thermocoupes were inserted. The schematic diagram of the test section is shown in Figure 4.

C. Amplifiers

Two identical Princeton Applied Research amplifiers model PAR™ 113 were used for the purpose of blocking the D-C components and amplifying the A-C components of the signals. The low and high frequency cutoffs were set at 0.03 HZ and 30 HZ respectively. The gains on both amplifiers were set at 10,000.

D. Analog Computer

In the initial phase of the experiments, the analog to digital converter on the PDP-11 could only accept positive signals between 0 and 10 volts. Therefore a Pace TR-48 electronic analog computer was used to bias the output signals of the amplifiers such that their amplitudes stayed within 0 and 10 volts. Later, the analog to digital converter was modified to accept signals between -10 and +10 volts. Then, the

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[1] The piece of stainless steel pipe was 6 feet long and 1 inch in diameter.

analog computer was left in the system to protect the analog to digital converter on the PDP-11.

E. Digital Computer

Data were analyzed on a PDP-11, a digital mini computer utilized with an analog to digital converter and a pulse generator used as a timer. The continuous signals from the amplifiers, after passing through the analog computer, were sampled at the rate R (varied between 20 and 50 samples per second in different runs) and were stored in the digital computer, which was programmed in the BASIC language to calculate the crosscorrelation function. The following is a summary of what the program does. Details of the complete program appear in Appendix B.

The program computes the average value of each signal and takes the difference between the signal and its average value as the new function. Since the signals were discrete, they could be presented as $F_1(i)$ and $F_2(i)$ for $i=1,2,\ldots,N$, with N being the number of points sampled and stored. The crosscorrelation function between the two signals is computed by:

$$C_{12}(j) = \sum_{i=1}^{N1} F_1(i)\ F_2(i+j) \qquad \text{for } j=0,1,\ldots,k \qquad (8)$$

where

$N1 = N-k \quad , \quad k<N$

N = number of data points stored for each signal

k = maximum delay time for which the crosscorrelation function is computed.

After computing $C_{12}(j)$ for j=0,1,...,k, the computer finds the j for which $C_{12}(j)$ has a maximum. This j, represented by J1, divided by the sampling rate, R, is the transit time of fluid between the thermocouples. The diameter of the pipe, D, and the distance separating the thermocouples, L, are also given to the computer. The computer then computes the flow rate using the following equation, noting that $\tau_t = \frac{J1}{R}$:

$$Q_E = AV = \frac{\pi D^2}{4}\left[\frac{L}{J1/R}\right] = \frac{\pi}{4}(D^2L)\frac{1}{J1/R} \tag{9}$$

where

A = cross sectional area of the pipe

V = stream velocity

D = pipe diameter

L = distance separating the thermocouples

and compares it with

$$Q_T = \frac{1.34}{T_f} \tag{10}$$

where Q_T is the volumetric flow rate of the fluid measured using a 10-gallon bucket and a stopwatch. 1.34 is the volume of the bucket in cubic feet and T_f is the time required to fill it in seconds.

CHAPTER V

PROCEDURE

Since a larger flow was available for cold water, cold water was taken as the main stream and a hot water stream was introduced as a noise promoter. Both hot and cold water entered from the bottom of the pipe and exited from the top to the measuring tank.

For each setting of the control valves at least three sets of data corresponding to the three sets of thermocouples were taken. At the same setting, the flow rate was measured by means of a 10-gallon bucket and a stopwatch. To minimize the statistical uncertainties, several measurements of the time required to fill the bucket were performed and their average value was taken as the "filling time", T_f, by which the flow rate was calculated. The program calculates the flow rate using 10 gallons as the volume which takes T_f seconds to be filled, and later compares this with the flow rate obtained by the TTF technique.

Signals from thermocouples were sent to the amplifiers through short coaxial cables. The A-C mode of amplifier operation was used with 0.03 HZ as the low frequency cutoff and 30 HZ as the high frequency cutoff; the gain on the amplifier was set to maximum at 10,000. Since the computer laboratory was remote from the laboratory in which the equipment for this experiment was located, signals from the amplifiers were sent through long wires to the analog computer.

Outputs of the analog computer were sampled and stored in the digital computer through the analog to digital converter. The data, representing the temperature fluctuations detected by the thermocouples, were analyzed, as it is mentioned in part E of the previous section, and the results were printed by the teletype for further studies.

CHAPTER VI

ANALYSIS AND DISCUSSION OF THE RESULTS

The delay time which yielded the maximum value of the crosscorrelation function was taken as the transit time of fluid between the thermocouples. Since the peak of the crosscorrelation function occurred at the same sampling point in each run (for a particular flow) there is no information available on statistical uncertainties. The uncertainties cited here are due to the finite sampling interval.

The maximum value of the normalized crosscorrelation function, $\rho_{12}(\tau)$, defined by equation 5 varied between 0.7 and 0.99 for different runs, which indicated a good correlation between the signals at that delay time.

To study the effect and importance of different parameters involved in the flow measurement by the TTF technique, the experiments were divided into three parts where in each part certain parameters or factors were examined.

Part I. Consistency of the technique

Part II. Dependence of results on length of data record

Part III. The effect of the thermocouple response on the accuracy of the technique

Part I. Consistency of the technique. The first phase of the study was devoted to investigating the consistency of repeated flow measurements.

Each of the three thermocouple sizes was used. Thermocouple sizes, parameters and other specifications pertaining to Part I are shown in Table 2.

The results obtained in the eight tests performed in this part were identical. The TTF technique measured a value of 0.52 ± 0.02 sec. (Figures 6 through 13 and Table 2) the TTF technique was consistent in measuring the transit time of the fluid, in the sense that it measured the same value for the transit time in the eight tests without any observable uncertainty between the measured values. The curves of the crosscorrelation functions, however, were not as smooth as the desired bell shape (see Figures 6 through 13). These deviations were due to the statistical nature of the analysis and do not affect the performance of the technique, since the peak was readily observable.

Part II. Dependence of results on length of data record. The second phase of the experiment was intended to determine whether and how much the flow measurements were affected by the number of terms, N1, chosen to approximate the series representing the crosscorrelation function.

Several tests for different sizes of thermocouples and different values for N1 were performed. As it is shown in Table 3, the transit times measured by the TTF technique were not affected for different N1 (see Table 3 and Figures 6, 8, 14 and 15). The only apparent effect of N1 was on the shape of the curves of the crosscorrelation functions (Figures 6, 8, 14 and 15) As the value of N1 increased, the magnitude of the peaks and valleys on both sides of the peak corresponding to the

TABLE 2

Specifications and Parameters of Part I of the Experiments

Run	Figure	Thermo-couple Size	R[1] Pulses Sec	N1[2]	t1[3] (sec)	τ_t[4] (sec) B&W[5]	τ_t[4] (sec) TTF[6]	Q[7] (cu. ft/sec) B&W	Q[7] (cu. ft/sec) TTF
1	6	1	42.5	30	2	.49	.52±.02	.027	.026
2	7	1	42.5	30	2	.49	.52±.02	.027	.026
3	8	2	42.5	50	2.5	.49	.52±.02	.027	.026
4	9	2	42.5	50	2.5	.49	.52±.02	.027	.026
5	10	2	42.5	50	2.5	.49	.52±.02	.027	.026
6	11	2	42.5	50	2.5	.49	.52±.02	.027	.026
7	12	3	42.5	30	2	.49	.52±.02	.027	.026
8	13	3	42.5	30	2	.49	.52±.02	.027	.026

[1] R = sampling rate.

[2] N1 = length of data record.

[3] t1 = sampling period.

[4] τ_t = transit time of fluid between the thermocouples.

[5] B&W = bucket and watch.

[6] TTF = transit time flowmeter technique.

[7] Q = volumetric flow rate.

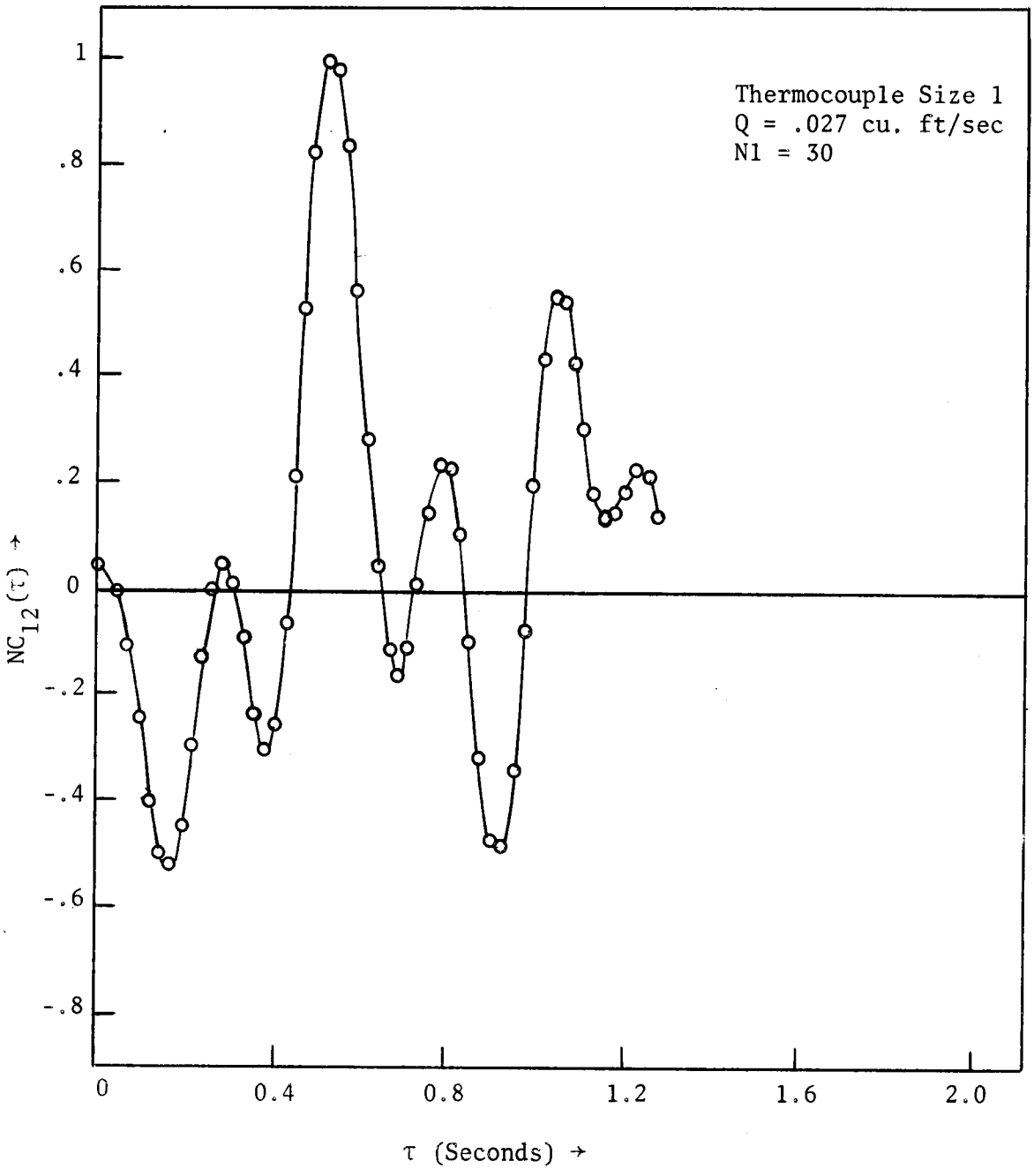


Figure 6. Crosscorrelation Function for Run 1.

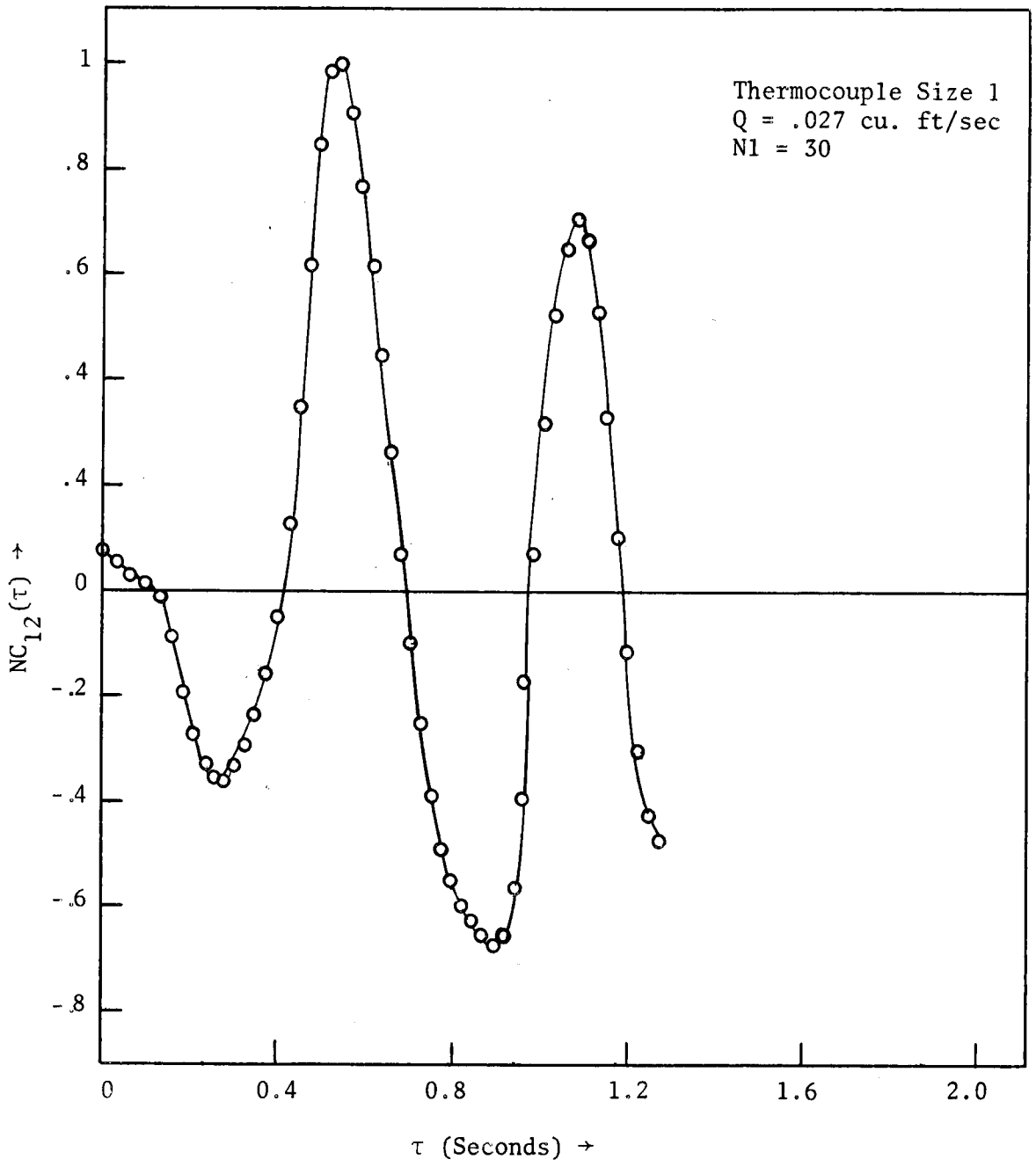


Figure 7. Crosscorrelation Function for Run 2.

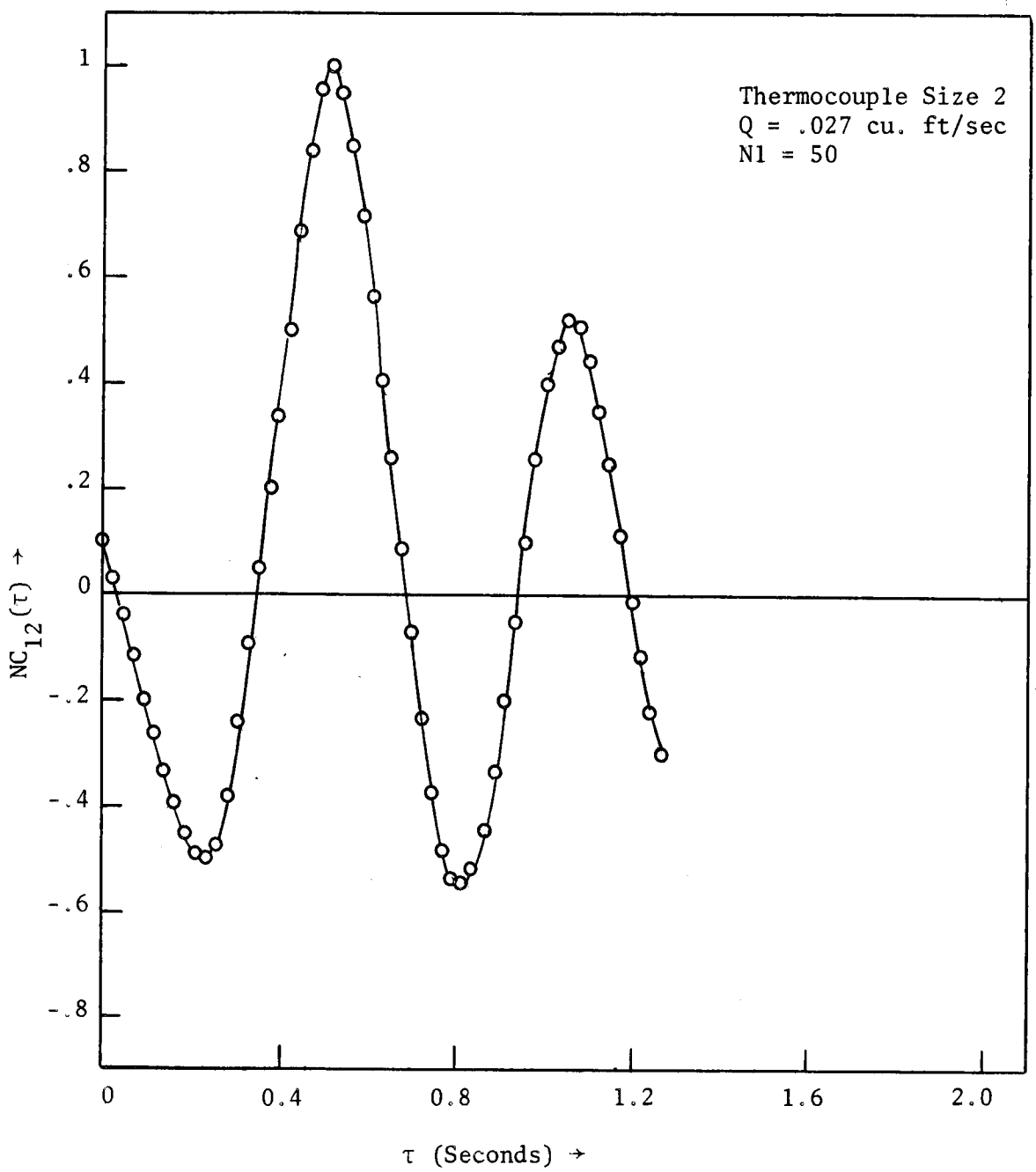


Figure 8. Crosscorrelation Function for Run 3.

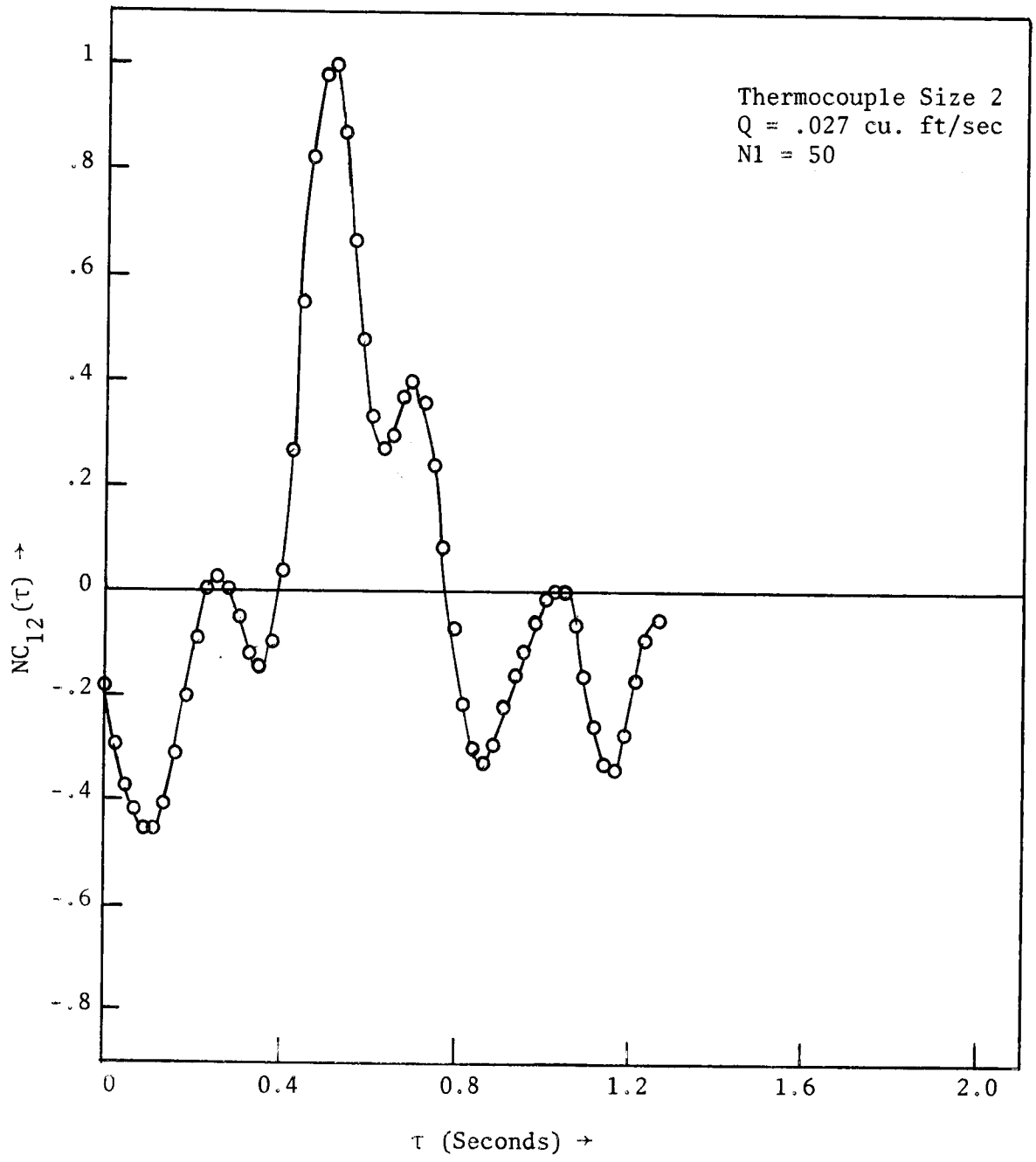


Figure 9. Crosscorrelation Function for Run 4.

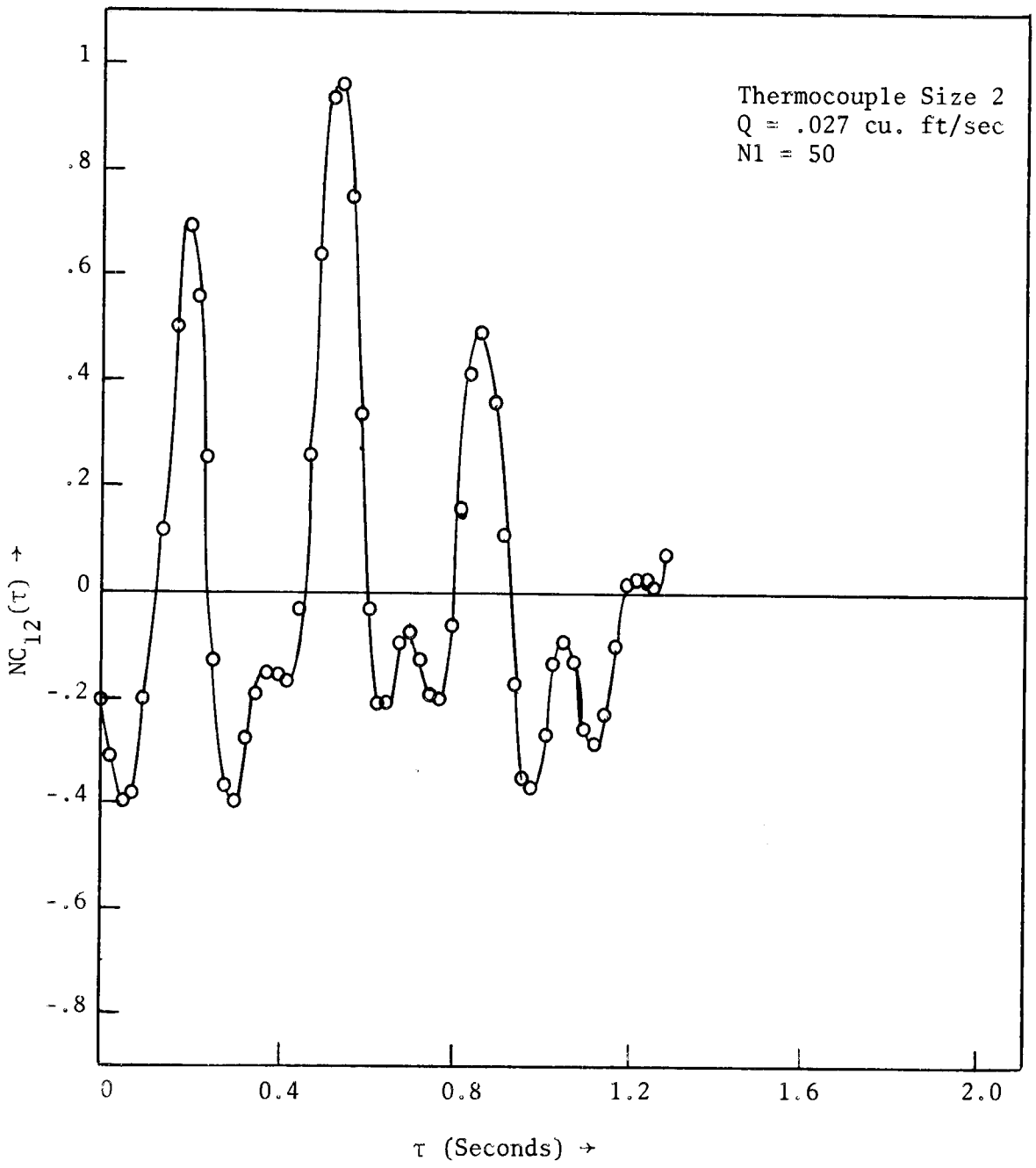


Figure 10. Crosscorrelation Function for Run 5.

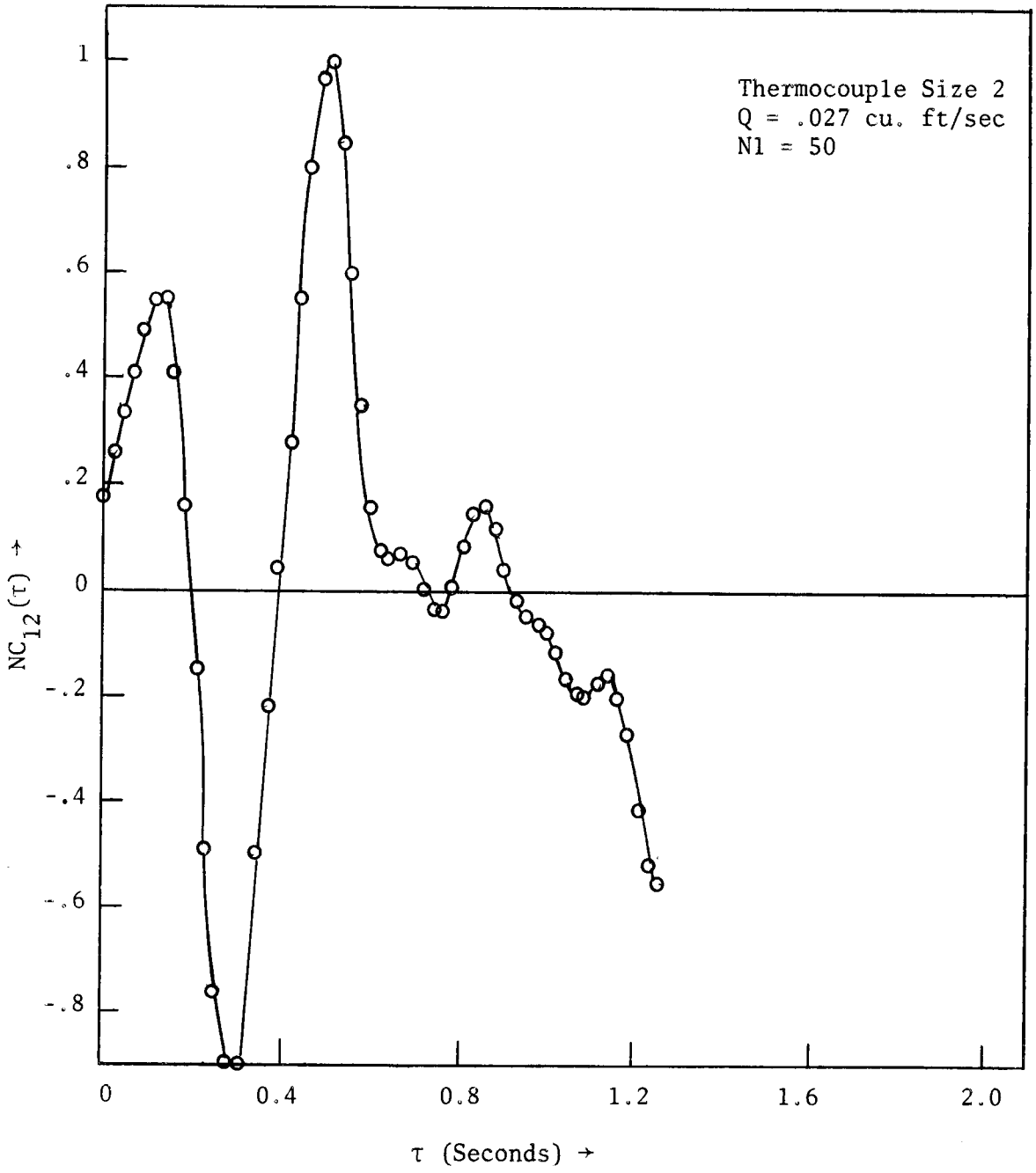


Figure 11. Crosscorrelation Function for Run 6.

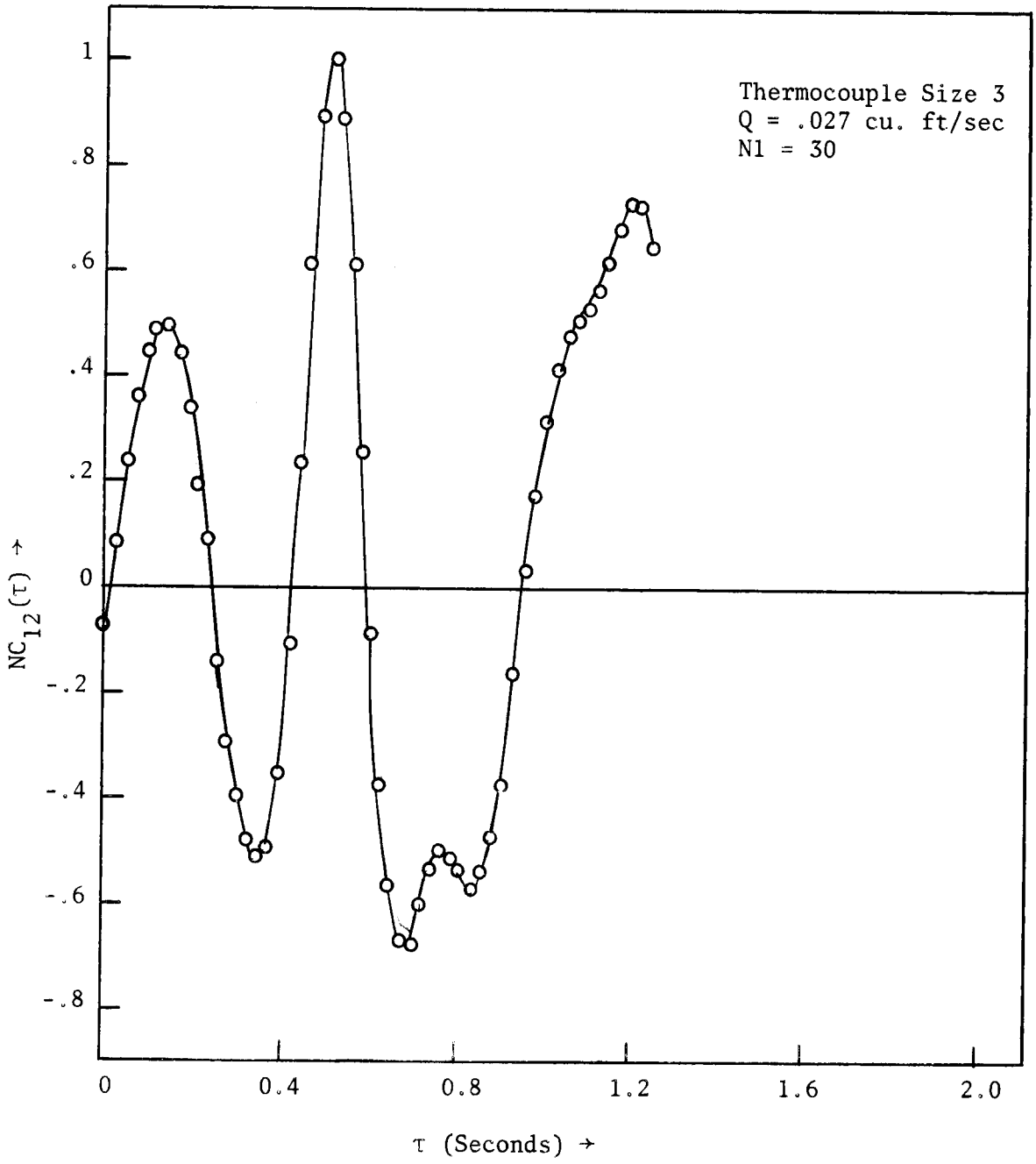


Figure 12. Crosscorrelation Function for Run 7.

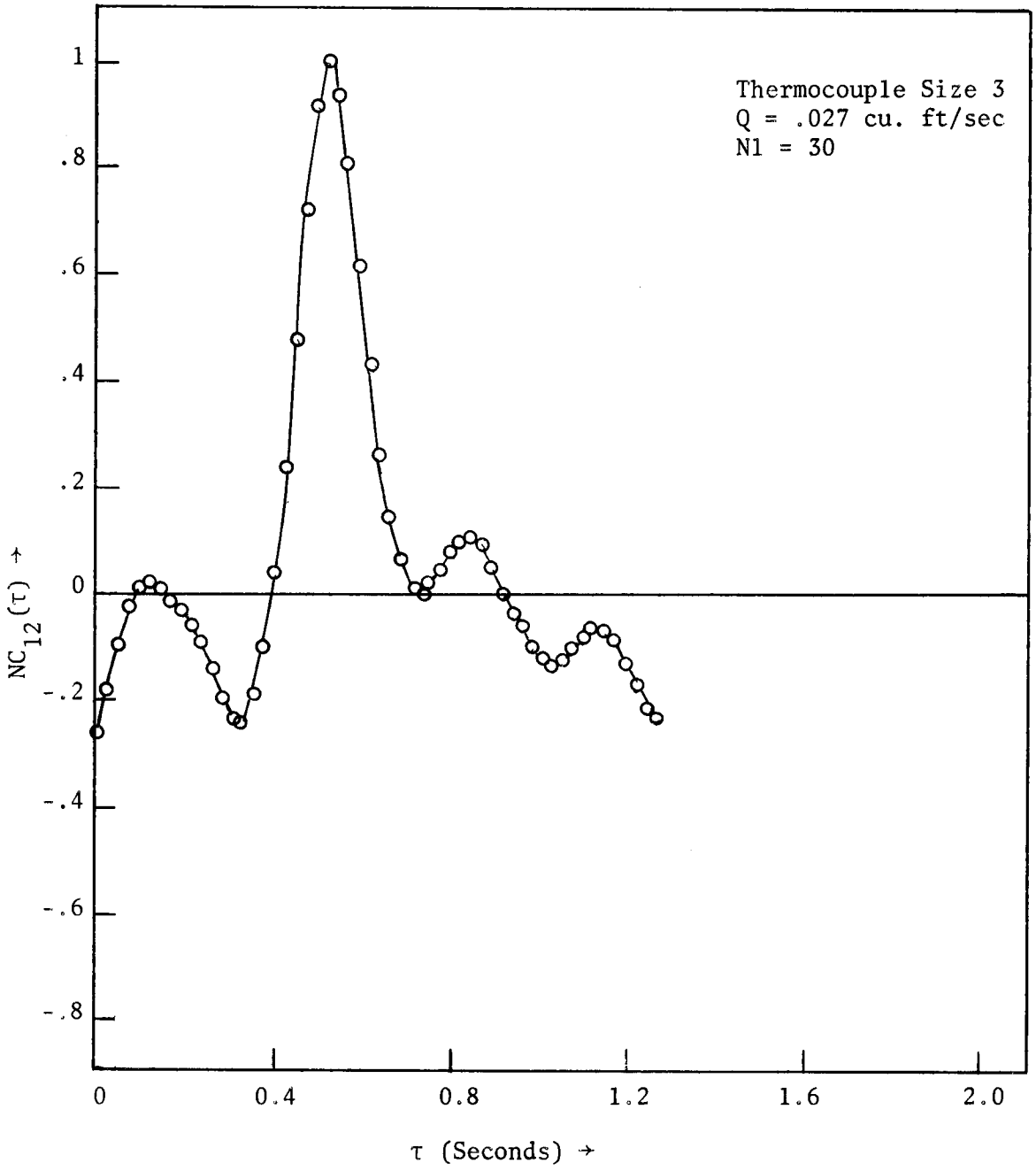


Figure 13. Crosscorrelation Function for Run 8.

TABLE 3

Specifications and Parameters of Part II of the Experiments

Run	Figure	Thermo-couple Size	R[1] Pulses/Sec	N1[2]	t1[3] (sec)	τ_t[4] (sec) B&W[5]	τ_t[4] (sec) TTF[6]	Q[7] (cu. ft/sec) B&W	Q[7] (cu. ft/sec) TTF
1,2	6,7	1	42.5	30	2	.49	.52±.02	.027	.026
9	14	1	42.5	200	6	.49	.52±.02	.027	.026
11	16	2	42.5	30	2	.49	.52±.02	.027	.026
3-6	8-11	2	42.5	50	2.5	.49	.52±.02	.027	.026
10	15	2	50	200	5.1	.49	.52±.02	.027	.026

[1] R = sampling rate.

[2] N1 = length of data record.

[3] t1 = sampling period.

[4] τ_t = transit time of fluid between the thermocouples.

[5] B&W = bucket and watch.

[6] TTF = transit time flowmeter technique.

[7] Q = volumetric flow rate.

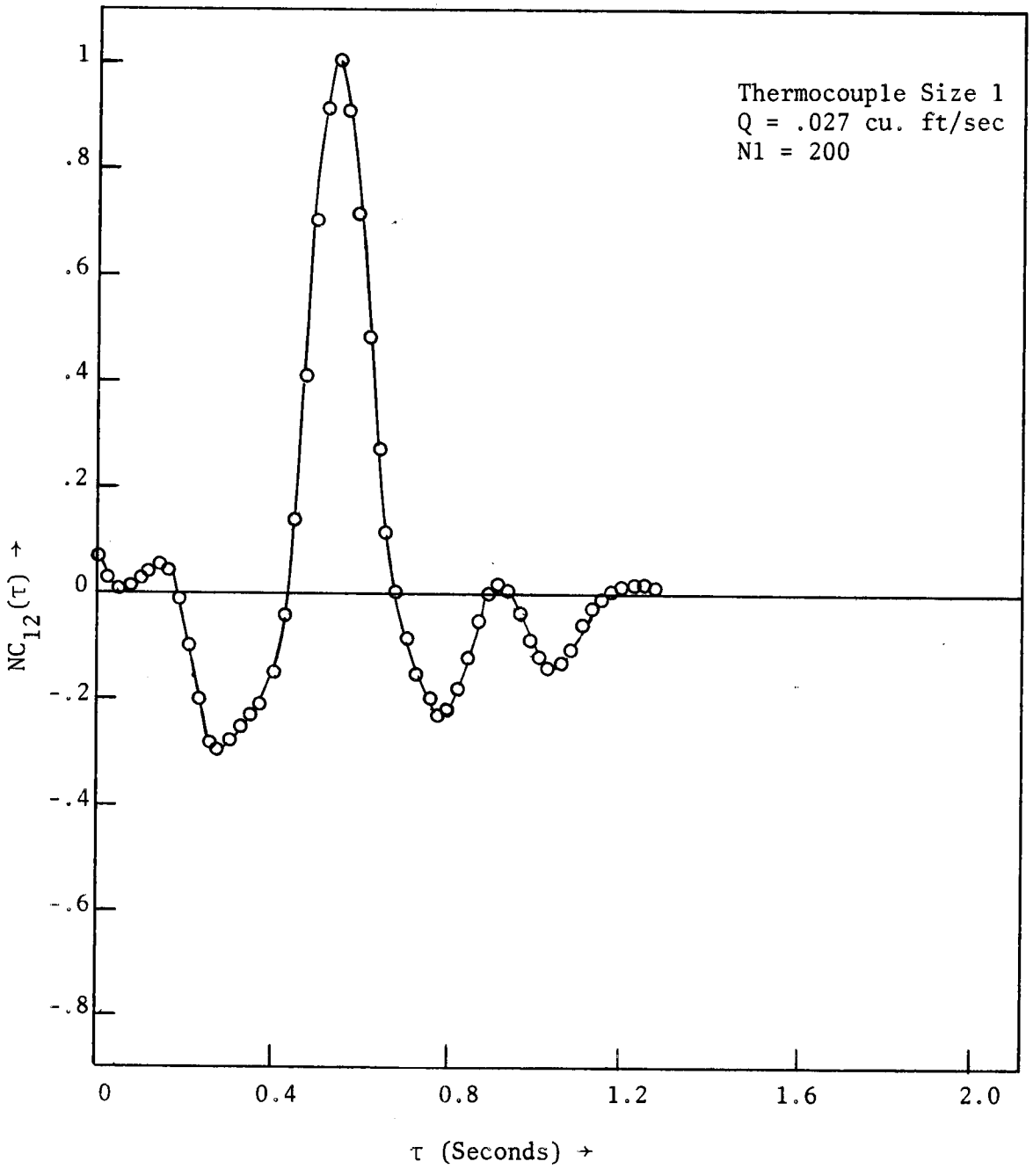


Figure 14. Crosscorrelation Function for Run 9.

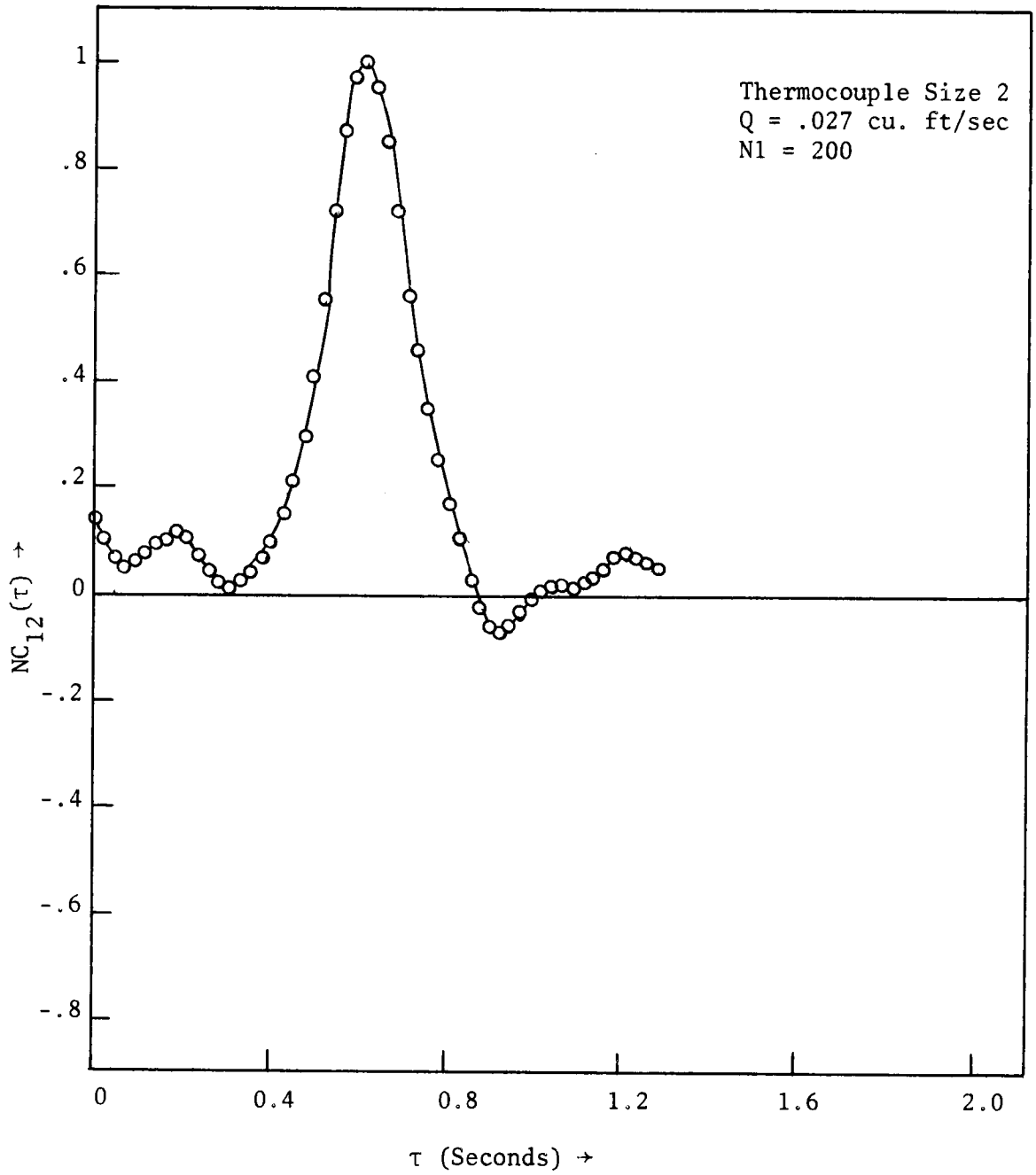


Figure 15. Crosscorrelation Function for Run 10.

transit time decreased (Figures 6, 14, 15 and 16). This behavior was expected and was due to the statistics involved in the analysis.

As the value of N1 increases, more fictitious peaks and valleys average to zero, and the peak corresponding to the transit time builds up. (Figures 6, 14, 15 and 16). In the limit, the curve approaches the bell shape (Figure 2, page 3).

In the TTF technique, the event of interest is the largest peak and not the shape of the crosscorrelation function, unless the fictitious peaks are, in magnitude, comparable to the largest peak so that the desired peak is not realizable.

Part III. The effect of the thermocouple response on the accuracy of the technique. As it is stated in the introduction, one of the objectives of this study was to determine the effect of the size of the thermocouple in the flow measurement using the TTF technique. For thermocouples of the same material, the time constant is a function of the size of the thermocouple junction, which normally depends on the size of the thermocouple wires. In this part of the study, the three different thermocouples described in previous sections were utilized to study the dependence of the technique on the size (and therefore, the time constant) of the thermocouple.

Part III consisted of three sections, where in each section several tests were performed for a fixed flow rate, each with a different set of thermocouples.

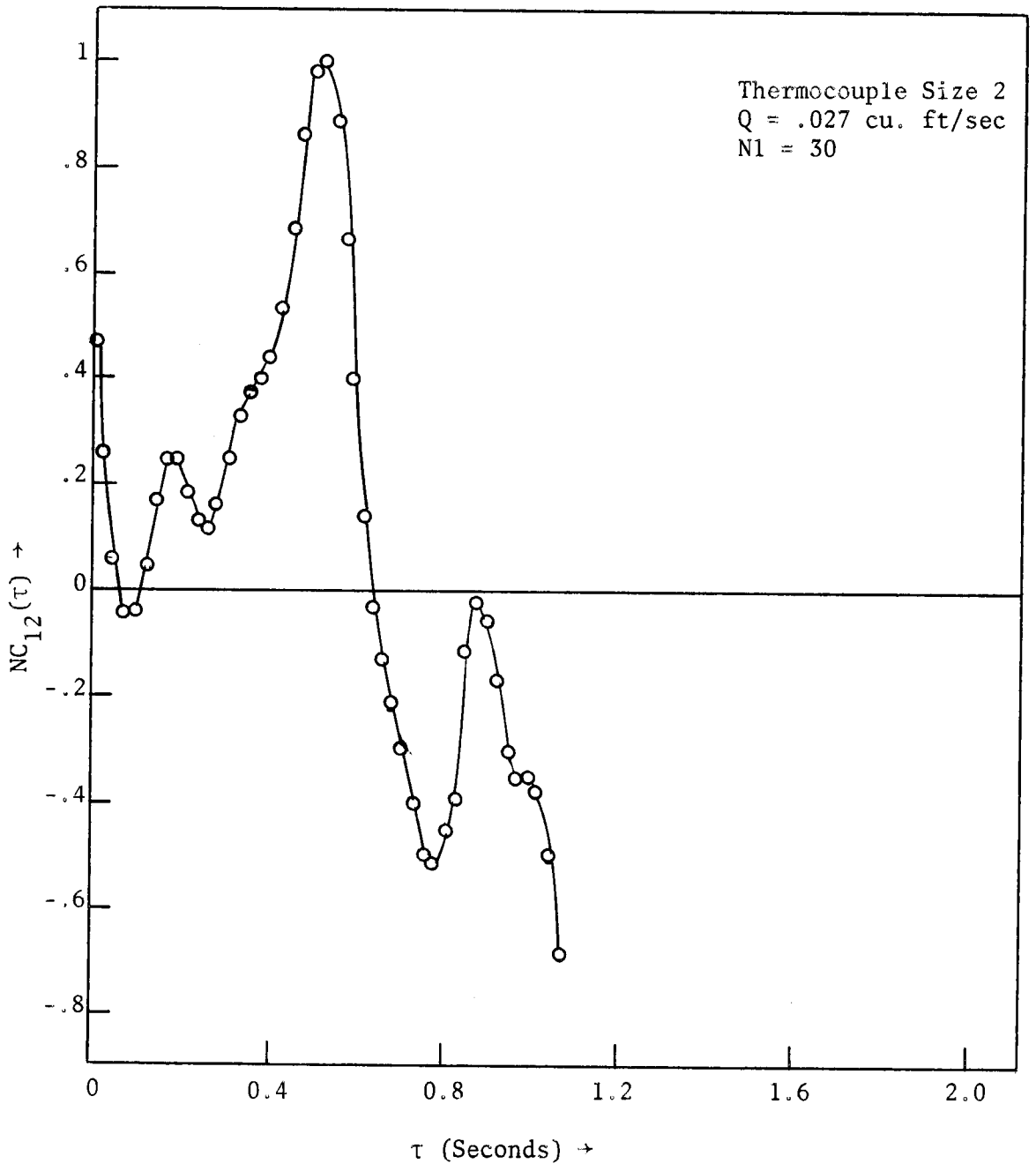


Figure 16. Crosscorrelation Function for Run 11.

Section A. In the first section, the flow rate was set at 0.039 cubic feet per second with the corresponding transit time of 0.34 second. The transit time measured by the TTF technique with any of the three sizes of thermocouples was 0.36 ± 0.02 second (see Table 4 and Figures 17 and 18).

Section B. In section B, the flow rate was decreased to 0.027 cubic feet per second with a transit time of 0.49 second. For the three different sets of thermocouples, the TTF technique measured a value of 0.52 ± 0.02 second for the transit time (see Table 4 and Figures 6 through 16).

Section C. The flow was further decreased to 0.0079 cubic feet per second with a transit time of 1.7 seconds. The value of the transit time measured by the TTF technique was identical (1.75 ± 0.05 second) for the three different sizes of thermocouples (see Table 4 and Figures 19 through 21).

As the above tests imply, the size of the thermocouples used in this study had no effect on the flow measurements.

TABLE 4

Specifications and Parameters of Part III of the Experiments

Section	Run	Figure	Thermo-couple Size	R[1] Pulses/Sec	N1[2]	t1[3] (Sec)	τ_t[4] (sec) B&W[5]	τ_t[4] (sec) TTF[6]	Q[7] (cu. ft/sec) B&W	Q[7] (cu. ft/sec) TTF
A	12	17	1	50	30	1.7	.34	.36±.02	.039	.037
	13	18	3	50	30	1.7	.34	.36±.02	.039	.037
B	1,2	6,7	1	42.5	30	2	.49	.52±.02	.027	.026
	9	14	1	42.5	200	6	.49	.52±.02	.027	.026
	3-6	8-11	2	42.5	50	2.5	.49	.52±.02	.027	.026
	10	15	2	50	200	5.1	.49	.52±.02	.027	.026
	11	16	2	42.5	30	2	.49	.52±.02	.027	.026
	7,8	12,13	3	42.5	30	2	.49	.52±.02	.027	.026
C	14	19	1	20	70	6.25	1.70	1.75±.05	.0079	.0077
	15	20	2	20	70	6.25	1.70	1.75±.05	.0079	.0077
	16	21	3	20	70	6.25	1.70	1.75±.05	.0079	.0077

[1] R = sampling rate.

[2] N1 = length of data record.

[3] t1 = sampling period.

[4] τ_t = transit time of fluid between the thermocouples.

[5] B&W = bucket and watch.

[6] TTF = transit time flowmeter technique.

[7] Q = volumetric flow rate.

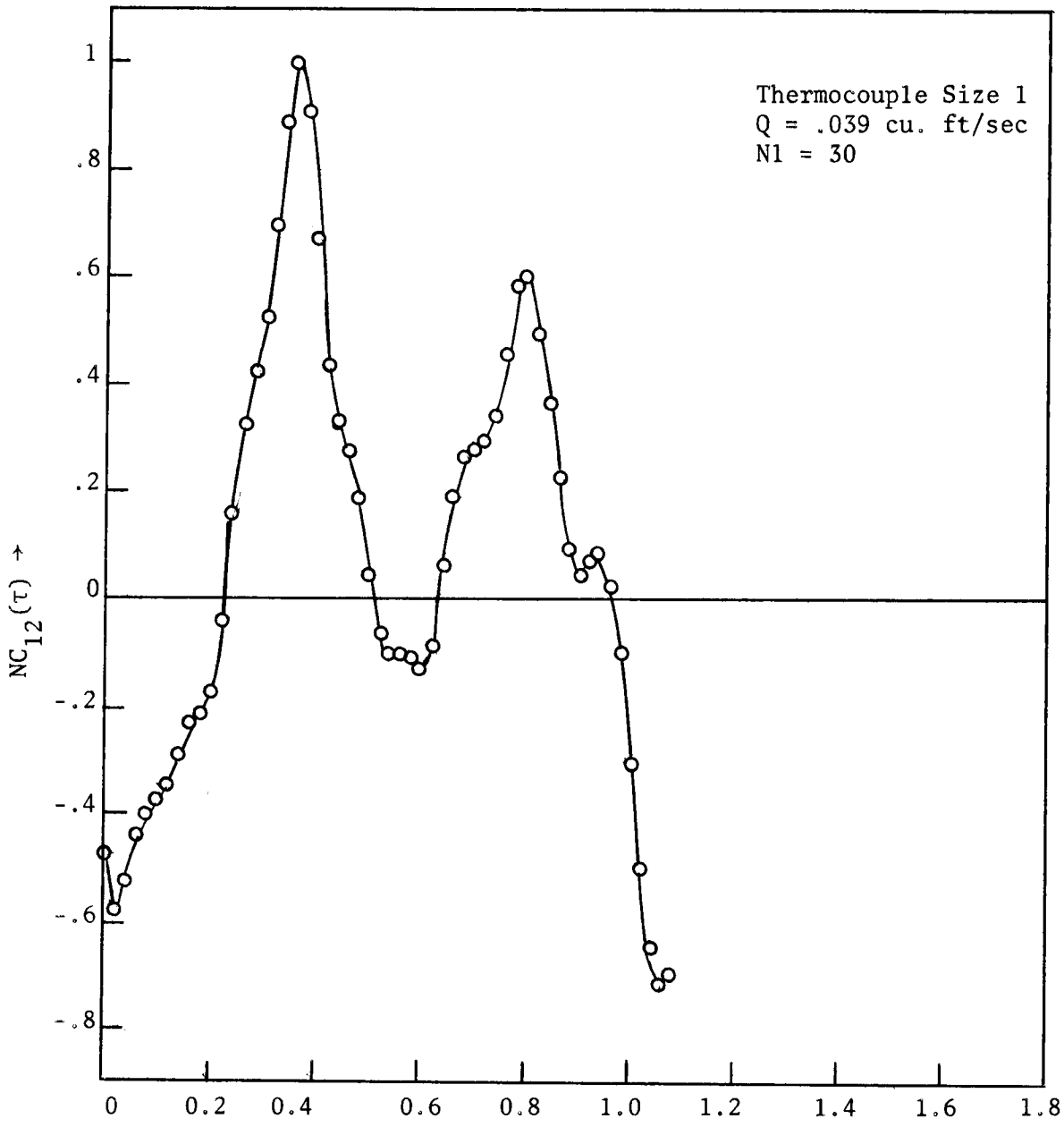


Figure 17. Crosscorrelation Function for Run 12.

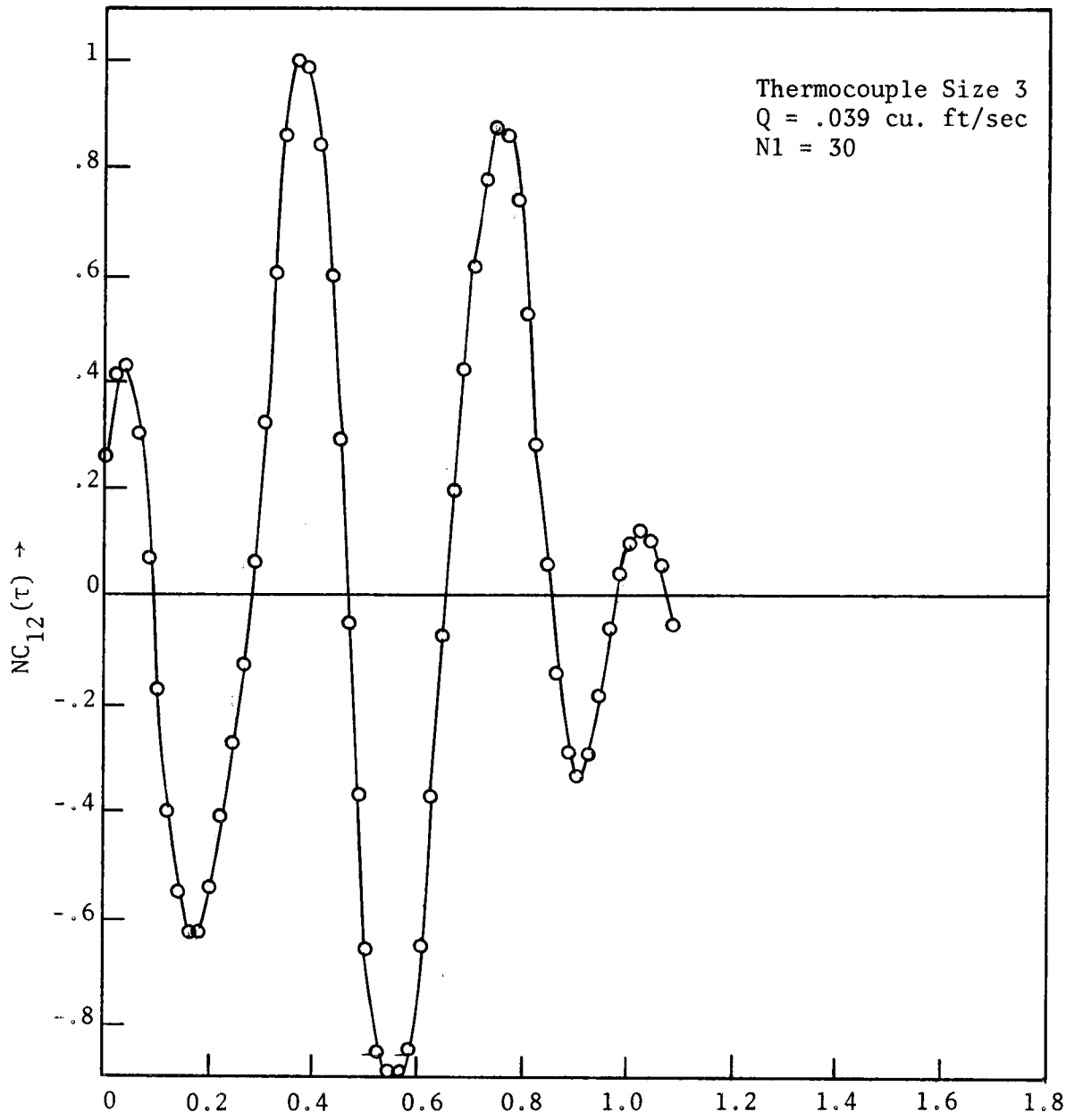


Figure 18. Crosscorrelation Function for Run 13.

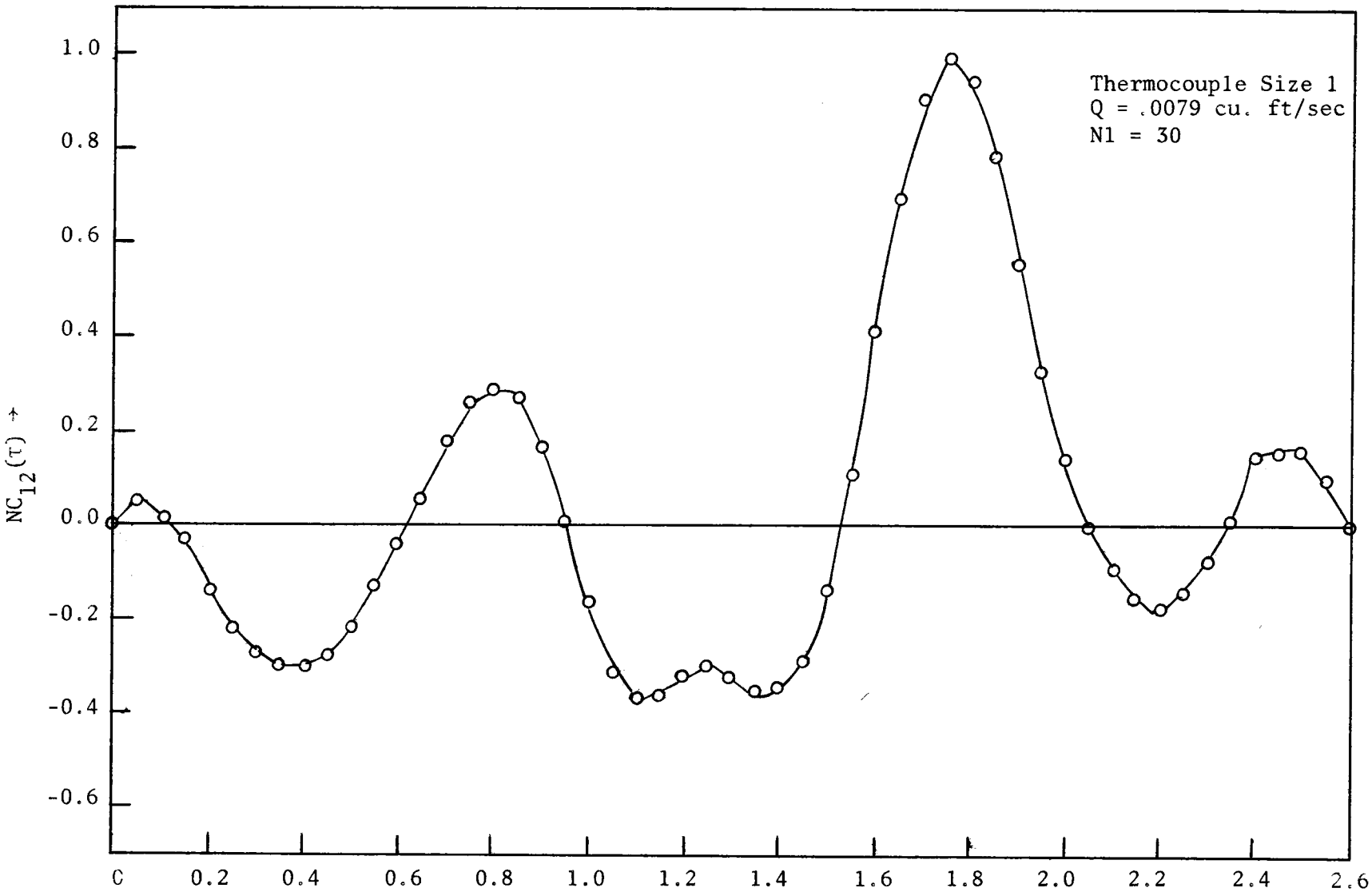


Figure 19. Crosscorrelation Function for Run 14.

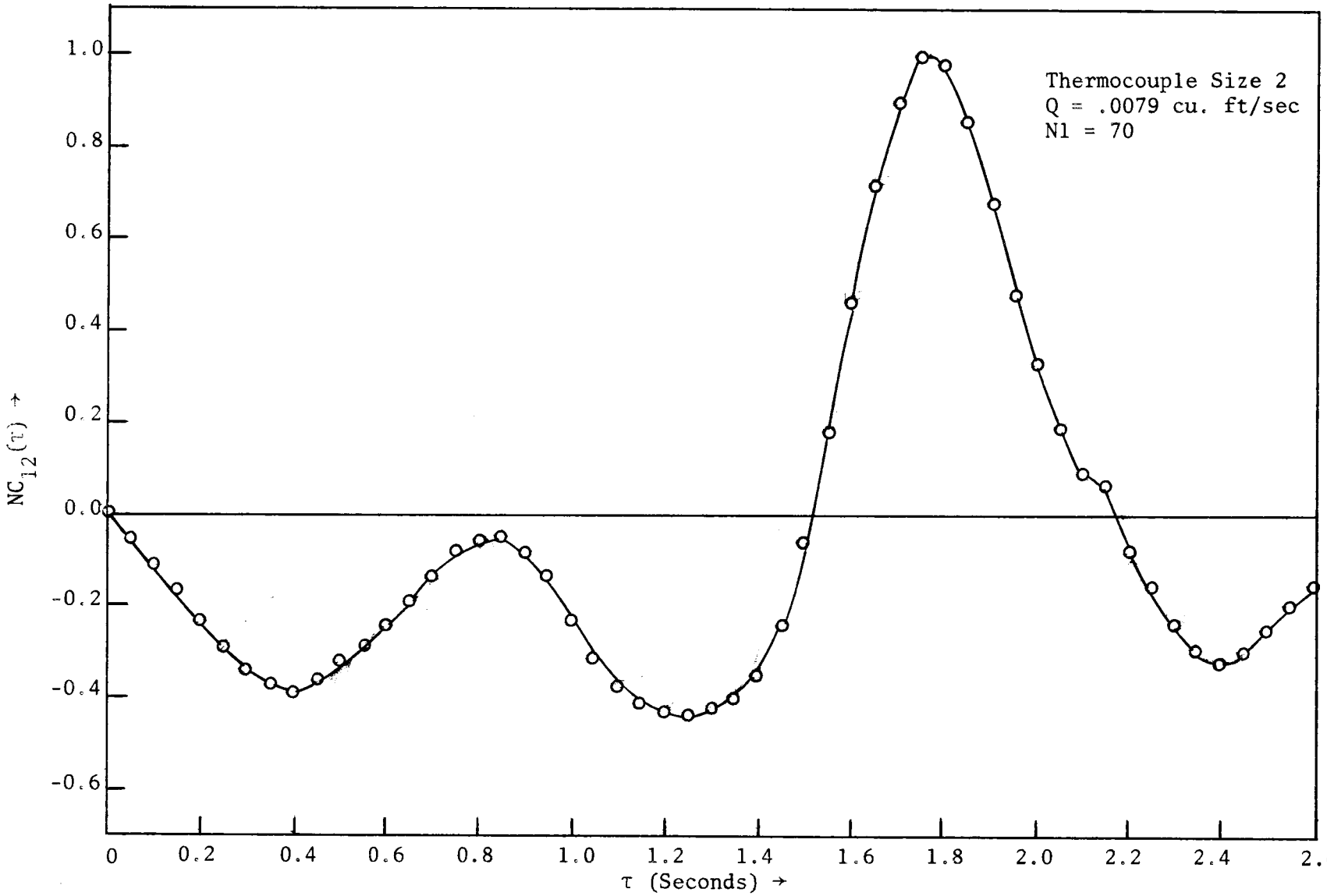


Figure 20. Crosscorrelation Function for Run 15.

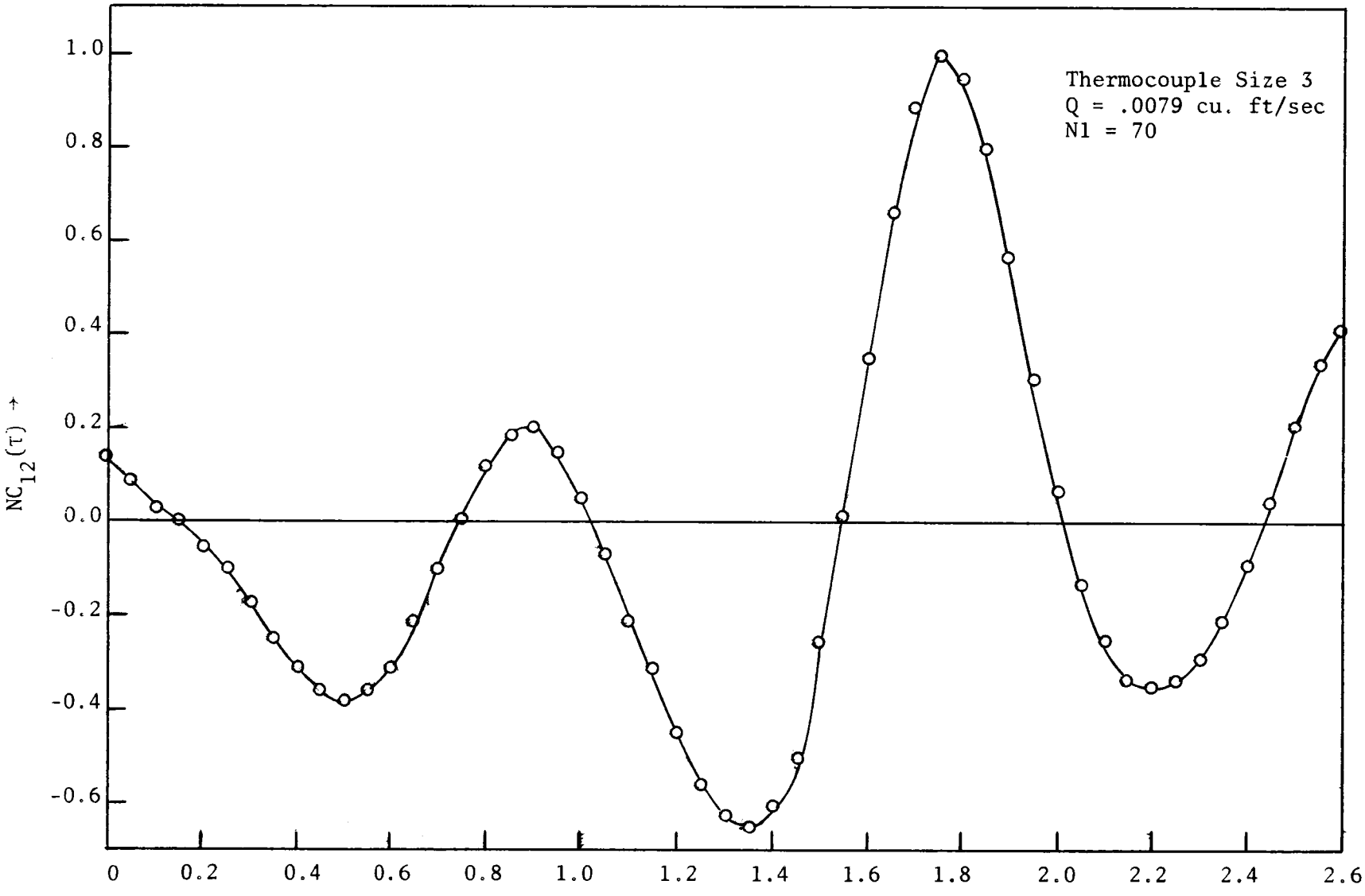


Figure 21. Crosscorrelation Function for Run 16.

CHAPTER VII

CONCLUSION

Based on the results of Parts I, II, and III, the Transit Time Flowmeter technique proves to be a feasible technique in measuring the volumetric flow rate of water in pipes.

Some of the advantages and disadvantages of the technique and also of the system and analysis are given briefly below.

Advantages. The advantages of the TTF technique over other techniques are: negligible pressure drop,; simplicity of the sensors; small space requirement; no requirement of nozzles, moving parts or magnetic materials; and direct determination of fluid velocity based on absolute measurements of distance and time.

Disadvantages. Implementation of the TTF technique for fluid flow measurement requires some sort of computing machine. This need could be viewed as a disadvantage of the TTF technique. If a mini computer is used and the program to analyze the data is written in a language other than the machine language, as was the case for this study, the data analysis takes quite a long time, depending upon the number of terms in the series, N1, approximating the crosscorrelation function and the maximum delay time (k) desired. On the average, for 200 terms and 55 delay terms it takes approximately 1½ hour. The same program, on the time sharing computer (IBM 360), would probably take about 5 minutes. If the

program is written in machine language rather than the Basic language, the execution time on the PDP-11 could probably be on the order of a few seconds.

If the existing thermocouples in a reactor system are used as the sensors in the fluid flow measurements by the TTF technique, their relatively large time constants (several times larger than the ones used in this study) could be encountered as a disadvantage of the technique. Because, as the time constant increases, it tends to flatten the cross-correlation function (7), and this might lead to the loss of a readily detectable peak corresponding to the transit time of fluid between the thermocouples.

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REFERENCES

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APPENDICES

APPENDIX A

TREATMENT OF NOISE AND ITS EFFECT ON THE CORRELATION FUNCTION

The purpose of this section is to demonstrate the effect of noise in the calculation of the crosscorrelation function and to show the means of extracting the desired information from the noise polluted function. This is done for two types of noise: (A) random noise, and (B) periodic noise.

A. Random noise. Let $n_k(t)$ be the random noise added to the functions $f_k(t)$ ($k = i, j$). Then the new functions, designated by $f'_k(t)$ can be written as:

$$f'_k(t) = f_k(t) + n_k(t) \qquad k = i, j \qquad \text{(A-1)}$$

the crosscorrelation function $C'_{ij}(\tau)$ between $f'_i(t)$ and $f'_j(t)$ is then given by:

$$C'_{ij}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f'_i(t) f'_j(t+\tau)\,dt \qquad \text{(A-2)}$$

Substituting for $f'_k(t)$ and performing the multiplications yields:

$$C'_{ij}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T [f_i(t) + n_i(t)]\,[f_j(t+\tau) + n_j(t+\tau)]\,dt \qquad \text{(A-3)}$$

$$= \lim_{T \to \infty} \left[\frac{1}{T} \int_0^T f_i(t) f_i(t+\tau) dt + \frac{1}{T} \int_0^T f_i(t) n_j(t+\tau) dt \right.$$

(A-4)

$$\left. + \frac{1}{T} \int_0^T n_i(t) f_j(t+\tau) dt + \frac{1}{T} \int_0^T n_i(t) n_j(t+\tau) dt \right]$$

the first term in the right-hand side of equation (A-4) is recognized as the crosscorrelation function, $C_{ij}(\tau)$, between the original functions. As it is known from the crosscorrelation theory, the second, third, and the fourth terms of equation (A-4), being the crosscorrelation function between uncorrelated random functions, are identically zero. Hence

$$C'_{ij}(\tau) = C_{ij}(\tau) \tag{A-5}$$

where

$$C_{ij}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f_i(t) f_j(t+\tau) dt \tag{A-6}$$

In practice, since $T \neq \infty$, the last three terms of equation (A-4) do not vanish, but rather yield a function which causes the peaks and valleys on both sides of the main peak. This effect can be very well observed in Figures 6 through 21 (pages 23-30, 32, 33, 35 and 38-42) in which the number of points was varied. The reduction of the peaks and valleys that occurs when the number of points increases is apparent.

B. Sinusoidal noise. Suppose the functions are sinusoids and are given by:

$$f_k(t) = A_k \sin w_k t \qquad k = i, j \tag{A-7}$$

the crosscorrelation function between $f_i(t)$ and $f_j(t)$ is:

$$\begin{aligned} C_{ij}(\tau) &= \lim_{T \to \infty} \frac{1}{T} \int_0^T A_i \sin w_i t \; A_j \sin w_j (t+\tau) \; dt \\ &= A_i A_j \lim_{T \to \infty} \frac{1}{T} \int_0^T \sin w_i(t) \; \sin w_j(t+\tau) \; dt \end{aligned} \tag{A-8}$$

after expanding $\sin w_j(t+\tau)$, and rearranging the terms in equation (A-8):

$$\begin{aligned} C_{ij}(\tau) = A_i A_j \Bigg(& \cos w_j \tau \left[\lim_{T \to \infty} \frac{1}{T} \int_0^T \sin w_i t \; \sin w_j t \, dt \right] \\ & + \sin w_j \tau \left[\lim_{T \to \infty} \frac{1}{T} \int_0^T \sin w_i t \; \cos w_j t \; dt \right] \Bigg) \end{aligned} \tag{A-9}$$

if we let $T = 2n\pi + \delta$, where n is an integer and $\delta < 2\pi$, the integrals inside the brackets can be written as:

$$\int_0^{2n\pi} \sin w_i t \sin w_j t \, dt + \int_{2n\pi}^{2n\pi+\delta} \sin w_i t \sin w_j t \, dt \qquad \text{(A-10)}$$

and

$$\int_0^{2n\pi} \sin w_i t \cos w_j t \, dt + \int_{2n\pi}^{2n\pi+\delta} \sin w_i t \cos w_j t \, dt \qquad \text{(A-11)}$$

if n is large, the contribution of the second terms of equations (A-10) and (A-11) are insignificant, hence using the orthogonality properties to evaluate equations (A-10) and (A-11), equation (A-9) can be written as:

$$C_{ij}(\tau) = A \cos w\tau \qquad \text{(A-12)}$$

where

$$A = \frac{A_i A_j}{2} \qquad \text{(A-13)}$$

and

$$w = w_i = w_j$$

Consequently the crosscorrelation function is a cosine function with the same frequency as the noise function and with an amplitude of A.

Now suppose that a periodic function is added to each of the original function, i.e.

$$f'_{\ell}(t) = f_{\ell}(t) + A_{\ell} \sin w_{\ell} t \qquad \ell = i, j \qquad \text{(A-14)}$$

writing the crosscorrelation function between f_i (t) and f_j (t) yields:

$$C'_{ij}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f'_i(t) f'_j(t+\tau) dt \qquad \text{(A-15)}$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_0^T [f_i(t) + A_i \sin w_i t] [f_i(t+\tau) + A_j \sin w_j(t+\tau)] dt$$

After performing the intermediate operations:

$$C'_{ij}(\tau) = \lim_{T \to \infty} \left[\frac{1}{T} \int_0^T f_i(t) f_j(t+\tau) dt + \frac{A_j}{T} \int_0^T f_i(t) \sin w_j(t+\tau) dt \right. \qquad \text{(A-16)}$$

$$\left. + \frac{A_i}{T} \int_0^T f_i(t+\tau) \sin w_i t \, dt + \frac{A_i A_j}{T} \int_0^T \sin w_i t \sin w_i(t+\tau) dt \right]$$

the first term of the right-hand side of equation (A-16) is recognized as $C_{ij}(\tau)$, the crosscorrelation function between the original functions,

the second and the third terms are equal to zero and the last term is equal to A Cos w τ, therefore,

$$C'_{ij}(\tau) = C_{ij}(\tau) + A \text{ Cos } w \tau \qquad \text{(A-17)}$$

where A, and w are defined previously.

If the frequency w is known, then the term A Cos w τ can be filtered numerically during the computation. It is apparent from equation (A-17) that the peak of the crosscorrelation function is not displaced, therefore if the peak is the only event of interest, as in the TTF technique, the desired information can be obtained even without filtering the periodic part of the $C'_{ij}(\tau)$ if the periodic component does not swamp the desired signal.

APPENDIX B

THE COMPUTER PROGRAM

```
ØØ1  PRINT "RUN NUMBER & DATE";:INPUT RØ,R1,R2,R3
ØØ2  PRINT "DIAMETER OF THERMOCOUPLE WIRES (IN.), D=";:INPUT D
ØØ5  PRINT
Ø1Ø  DIM X(255,1),Z(255)
Ø12  LET W=9Ø.25-5.5
Ø14  LET RØ=62.4
Ø15  PRINT
Ø16  LET VØ=W/RØ
Ø2Ø  PRINT "TIME REQUIRED TO FILL THE BUCKET IN SECONDS,T1=";
Ø22  INPUT T1
Ø25  LET Q1=VØ/T1
Ø26  LET Q=448.83*Q1
Ø3Ø  PRINT "SAMPLING RATE IN PULSES/SECOND, R=";:INPUT R
Ø5Ø  LET H=Ø
Ø6Ø  LET G=Ø
Ø65  LET L=2.5
Ø7Ø  LET P=55
Ø8Ø  LET N=85
Ø9Ø  LET M=N
1ØØ  LET B=1Ø2.4
1Ø2  LET B2=B↑2
1Ø5  LET D=Ø.99/12
1Ø6  LET D2=D*D
11Ø  LET P1=3.14159
112  LET A=P1*D2/4
114  LET V=Q1/A
116  LET T=L/V
12Ø  LET 0=EXF(1,1,4,X(Ø,Ø))
121  PRINT
122  PRINT "VOLUMETRIC FLOW RATE IN CU.FT/SECOND, Q1="Q1
123  FOR I=1 TO N
125  LET G=G+X(I,1)/B
126  LET X1=G/N
127  LET H=H+X(I,Ø)/B
128  LET Y1=H/N
13Ø  NEXT I
14Ø  PRINT "VELOCITY OF FLUID IN FT/SECOND, V=";V
15Ø  PRINT "TRANSIT TIME IN SECOND, T=";T
16Ø  PRINT "A VALUE OF 1 SHOULD BE OBTAINED FOR NORMALIZED CROSS-"
17Ø  PRINT "CORRELATION FUNCTION,I.E., NC12(TAU)=1 @ TAU="T
18Ø  PRINT "WHERE NC12(TAU)=C12(TAU)/MAX[C12(TAU)]."
2ØØ  PRINT
222  PRINT "AVERAGE VALUE OF X(I),  X1=";X1
223  PRINT "AVERAGE VALUE OF Y(I),  Y1=";Y1
224  PRINT "NUMBER OF TERMS OF SERIES WHICH APPROXIMATES THE"
225  PRINT "CROSS CORRELATION INTEGRAL, N1=";N-P
226  PRINT
```

```
227  PRINT "J","TAU","  X(J)","  Y(J)"," C12(TAU) "
228  PRINT "---","-----","----------","----------","----------"
23Ø  LET Z4=Ø
235  LET Z6=Ø
24Ø  FOR I=1 TO N-P
245  LET C4=(X1-X(I,1)/B)*(X1-X(I,1)/B)
25Ø  LET C6=(Y1-X(I,Ø)/B)*(Y1-X(I,Ø)/B)
252  LET Z4=Z4+C4
254  LET Z6=Z6+C6
256  NEXT I
258  LET Z7=SQR(Z4*Z6)
260  FOR J=1 TO P
265  PRINT
267  LET Z1=Ø
27Ø  FOR I=1 TO N
275  LET K=I+J-1
278  IF K>M GO TO 294
28Ø  LET C=(X1-X(I,1)/B)*(Y1-X(K,Ø)/B)
285  LET Z1=Z1+C
287  IF I<N-P THEN 293
289  IF I>N-P THEN 294
291  LET Z(J)=Z1
292  PRINT J-1,(J-1)/R,X(J,1)/B,X(J,Ø)/B,Z1
293  NEXT I
294  NEXT J
295  PRINT:PRINT "================================================"
296  PRINT "C11(Ø)="Z4;" C22(Ø)="Z6;" SQR[C11(Ø)*C22(Ø)]="Z7
297  PRINT "================================================":PRINT
298  LET Q9=-1ØØØ
299  FOR J=1 TO P
3ØØ  LET QØ=Q9
3Ø2  IF Z(K)<QØ THEN 3Ø5
3Ø3  LET Q9=Z(J)
3Ø4  LET J1=J-1
3Ø5  NEXT J
3Ø6  PRINT "MAXIMUM VALUE OF CROSS CORRELATION FUNCTION,"
3Ø7  PRINT "MAX[C12(TAU)]=";Q9;" @ TAU=";J1/R
3Ø8  PRINT "NORC12(TAU)=C12(TAU)/SQR[C11(Ø)*C22(Ø)]":PRINT
3Ø9  PRINT " J","  TAU","NC12(TAU)","NORC12(TAU)"
31Ø  PRINT "-------------------------------------------------------"
312  FOR J=1 TO P
314  LET Z=Z(J)/Q9
316  LET ZØ=Z(J)/Z7
318  PRINT J-1,(J-1)/R,Z,ZØ
319  NEXT J
32Ø  LET J2=J1/R
321  PRINT:PRINT
322  PRINT "  Q","CU.FT/SEC,","GPM"
```

```
323  LET Q5=A*L/J2
324  PRINT "*********************************************"
325  LET Q6=448.83*Q5
326  PRINT "THEORETICAL",Q1,Q
327  PRINT "EXPERIMENTAL",Q5,Q6
329  PRINT "%ERROR BASED ON EXPERIMENTAL VALUE",ABS((Q1-Q5)/Q5)*1ØØ
332  PRINT "%ERROR BASED ON THEORETICAL VALUE",ABS((Q1-Q5)/Q1)*1ØØ
338  PRINT "*********************************************"
339  PRINT " I","  X(I)","  X2(I)","  Y(I)","  Y2(I)"
34Ø  PRINT "---","--------","--------","--------","--------"
35Ø  FOR I=1 TO N
36Ø  PRINT I,X(I,1)/B,X1-X(I,1)/B,X(I,Ø)/B,Y1-X(I,Ø)/B
37Ø  NEXT I
38Ø  PRINT
39Ø  PRINT
4ØØ  PRINT
5ØØ  END
```

VITA

Behrad Nakhai was born on February 15, 1947, in Tehran, Iran. He attended École Saint Louis for six years and then transferred to Andisheh High School where he graduated in 1965. He entered The University of Tennessee in the spring of 1966 and received the Bachelor of Science degree in Nuclear Engineering in 1969. Since then he has been studying for the Master of Science degree and the Doctor of Philosophy degree in Nuclear Engineering. He received his Master of Science degree in August, 1973.

He is married to the former Ms. Mandana Momtaz from Tehran, Iran, who graduated from The University of Tennessee in the spring of 1973 majoring in English.