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# Polarimetry Studies for the NPDGamma Experiment at the SNS

A thesis submitted in partial fulfillment  
for the Master of Science degree  
Department of Physics and Astronomy  
University of Tennessee, Knoxville

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*This work is dedicated to my parents and to LCTC.*

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# Abstract

The NPDGamma collaboration aims to measure a parity-violating directional gamma-ray asymmetry from the capture of polarized cold neutrons on protons. The asymmetry, predicted to be on the order of 50 ppb, is a result of the weak interaction between mesons and nucleons during the neutron capture reaction. The experiment is designed to make a statistics limited measurement of the asymmetry at the level of about 20%. The experiment will be carried out at the Fundamental neutron Physics Beamline (FnPB) at the Spallation Neutron Source (SNS). The neutron beam is polarized by a super-mirror polarizer. A 9.5 G magnetic field guides the neutron spin to a liquid para-hydrogen target, where the neutrons capture forming a deuteron and a 2.2 MeV gamma-ray. The gamma-rays will be detected by a cylindrical array of CsI scintillating crystals. The beam polarization is reversed with the aid of a resonant RF Spin Rotator (RFSR). This thesis considers different methods for determining the beam polarization and efficiency of the RFSR.

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# Chapter 1

## Theory and Motivation

The weak interaction is the only force known to violate mirror-symmetry (parity). Hadrons, being composites of quarks, participate in all fundamental interactions (gravitational, electromagnetic, strong, and weak). Weak interactions involving leptons (which do not interact strongly) have been well characterized by high-energy experiments see [60]. The weak boson ( $W^\pm$  and  $Z$ ) masses are on the order of 100 GeV; thus the relative strength of the weak interaction to the strong or electromagnetic force is small; and since the range of the weak bosons is about  $10^{-18}\text{m}$ , quarks can only interact weakly when they are sufficiently close. The study of hadronic weak interactions (HWI) is vital as it provides a probe of strong quark-quark correlations within the hadron, but has proven experimentally challenging.

Under normal conditions in the present universe, hadrons are usually bound together in nuclei. Parity-violation (PV) has been observed in semi-leptonic decays of medium-mass nuclei such as  $^{60}\text{Co}$ . Also, hadronic PV in atomic transitions has been seen, due to many-body enhancement effects. In low-mass nuclei, where a microscopic theory is able to predict the size of PV, no convincing evidence of PV has been observed, due mainly to experiments not having the required sensitivity.

The simplest nuclear reaction,  $\vec{n} + p \rightarrow d + \gamma$ , at low energy, provides an ideal lab to study hadronic weak interactions and test models; by measuring the parity-violating correlation (the gamma-ray asymmetry  $A_\gamma$ ) between neutron spin and gamma momentum in the capture of polarized neutrons on protons.

In this chapter a brief history of PV is mentioned and an introduction to the theory of the expected gamma-ray asymmetry is presented.

## 1.1 Parity and its Violation in Weak Interactions

A parity operation reverses the sign of all spatial coordinates. When an observable is invariant under a parity operation, the observable is parity-even (e.g. axial vectors like the magnetic field and angular momentum, as well as scalars). When a parity operation reverses the sign of an observable, that quantity is parity-odd (e.g. polar vectors like position, momentum, and the electric field; and pseudo-scalars like the scalar product of a vector and axial vector). For a while, parity symmetry was long thought to be a fundamental discrete symmetry of nature. Since the gravitational and electromagnetic interactions are invariant under a spatial inversion, it was assumed that strong and weak nuclear forces should obey the same invariance principle. Further investigations showed this not to be the case for the weak interaction.

### 1.1.1 Brief History of PV in Nuclear Physics

In the 1930s Fermi proposed a theory of weak interactions to explain  $\beta$ -decay, and Yukawa developed the theory of meson-exchange to explain the strong nuclear force [22, 61]. Yukawa's theory postulated massive bosons, later determined to be pions ( $\pi^{\pm,0}$ ), as force carriers of the strong interaction; thereby, explaining the short-range (about 1 fm) behavior of the nuclear force. Even with the advent of Quantum Chromodynamics (QCD) this approach is a useful descriptive theory. Parity is conserved in the strong/color force, as it is a purely vector interaction; there is no mixing of parity-even/odd functions that would give rise to parity-violating observables. Fermi's theory is a four fermion contact interaction, that also conserves parity. Purcell and Ramsey, in 1950, pointed out that parity conservation had not been experimentally verified in nuclear/particle physics [48].

In 1954 Chinowsky and Steinberger [15] showed that the pions must have negative parity, thus they are pseudo-scalar particles. This led to a problem in explaining kaon ( $K^+$ ) decays. It was observed that  $K^+$ s decayed in two ways

$$\begin{aligned} K^+ &\rightarrow \pi^+ \pi^+ \pi^- \\ K^+ &\rightarrow \pi^+ \pi^0. \end{aligned}$$

If parity were conserved in nature, assuming no angular momentum between the pions, the following relationship would hold

$$\begin{aligned}\mathbb{P}(\pi^+\pi^0) &= \mathbb{P}K^+ = \mathbb{P}(\pi^+\pi^+\pi^-) \\ \Rightarrow (-1)(-1) &= (-1)(-1)(-1) \\ 1 &= -1*\end{aligned}$$

A common belief at the time was that there were two distinct particles with identical lifetimes and masses. One decayed to three pions, while the other, decayed to two; these were known as  $\tau$  and  $\theta$ , respectively.

In 1956, inspired by the  $\tau$ - $\theta$  puzzle, Lee and Yang [34] showed that PV in the weak interaction was not well constrained, and suggested experimental searches. Wu et al. [59] took up this challenge and in 1957 the first clear evidence of PV in the nucleus was observed in the distribution of electrons from the  $\beta$ -decay of  $^{60}\text{Co}$ . Immediately following this result Garwin et al. [25] found evidence of PV in the leptonic decay of pions.

In 1958 Feynman and Gell-Mann published their V – A current-current theory of the weak interaction to account for PV [23]. In the 1960s two major theoretical developments were the accurate methods of calculating parity violating observables from symmetries of the V – A theory [11] and the Weinberg-Salam standard model of electroweak interactions [56, 51]. It was also in this decade that the first evidence of hadronic PV was observed in the 472 keV transition in  $^{181}\text{Ta}$  [36] and in the capture of polarized neutrons on  $^{113}\text{Cd}$  [1]; though this latter result came under scrutiny [43].

By the end of the 1970s the first searches for PV arising from electron nucleus neutral currents in atomic transitions had been carried out [16] and were in disagreement with results from neutrino neutral current studies [9]. Also, in 1972 Lobashov et al. [37] reported a large gamma-ray circular polarization ( $P_\gamma = -1.30 \pm 0.45 \cdot 10^{-6}$ ) in the capture of thermal neutrons in water (a fraction of this signal is due to  $\vec{n} + p \rightarrow d + \gamma$ ); suggesting that hadronic weak interactions are not well characterized by the standard model. This led Cavaignac et al. [14] in 1977 to carry out the first search for parity violating neutral currents in the  $\vec{n} + p \rightarrow d + \gamma$  reaction using a liquid hydrogen target, by looking for an asymmetry in the distribution of gamma-rays emitted with respect to neutron spin. Some of these issues remain unsettled; but by the 1980s a new theoretical framework had been introduced.

In 1980 Desplanques, Donoghue, and Holstein (DDH) published calculations that related parity violating observables to weak meson-nucleon ( $mN$ , with  $m = \pi, \rho, \omega$ ) coupling constants [20]  $H_m^{\Delta I}$  (where  $\Delta I$  is the isospin

change of the reaction)<sup>1</sup>; this will be discussed in some detail later. This framework inspired a host of experiments to measure these coupling constants, such as:

1. Searches to determine the analyzing power for longitudinally polarized protons in  $\vec{p} + p$  scattering; which is sensitive primarily to  $\rho N$  coupling; but, depending on energy, also to the  $\omega N$  coupling.
2. Measurements of the circular polarization ( $P_\gamma$ ) of photons in the 1080 MeV transition of  $^{18}\text{F}$ ; sensitive to  $\pi N$  coupling.

Interestingly, nuclear anapole moment searches in atomic transitions, primarily in  $^{133}\text{Cs}$ , can also be related to these meson-nucleon coupling constants. These transitions are predominantly sensitive to  $\pi N$  coupling, but also to  $\rho N$  and  $\omega N$  couplings.

A review, in 1985, of the  $^{18}\text{F}$  null results [2] showed that the data imply a small value of the  $\pi N$  coupling constant as compared with the DDH best value of 50 ppb. In 1997 Wood et al. found evidence of an anapole moment in the  $^{133}\text{Cs}$  system [58]; when this was integrated with DDH [24], it suggested that the  $\pi N$  coupling constant was a factor of seven larger than the upper limit set by the results of [2]. In the next section, we delve into the theory related to  $A_\gamma$  in the  $\vec{n} + p \rightarrow d + \gamma$  reaction, to gain an understanding of how the NPDGamma experiment can resolve some of this confusion.

## 1.2 The Weak NN Interaction

The strong nuclear force can be modeled as the exchange of mesons between nucleons. The deuteron is known to have only a single weakly bound state, thus only the lightest mesons need be considered; these are the pions ( $\pi^\pm, \pi^0$ ).<sup>2</sup> Barton showed that, so far as CP is conserved,<sup>3</sup> spinless

---

<sup>1</sup>The small mass difference between the neutron and proton allow them to be considered, in the strong interaction, a single two-state system (the nucleon); similar to the electron which is observed to be either in a spin-up or spin-down state. Thus the proton and neutron form a doublet in an abstract pseudo-spin or isospin space. For a detailed discussion of isospin see for example [10, 13].

<sup>2</sup>However, it is noted that the heavy mesons contribute about 10% of the nuclear force in the deuteron

<sup>3</sup>The combined discrete symmetry of charge-conjugation and parity reversal, CP, was once thought to be conserved; it is now known that, to a small degree, the SM violates CP as well.

neutral currents cannot lead to PV [8]. Thus, only  $\pi^\pm$  exchange will dominate the hadronic weak interaction. The mass of these particles is about 140 MeV, implying the range is about 1.4 fm. Looking at the empirical formula ( $r = 1.2A^{1/3}\text{fm}$ , [57]) we expect the dueteron to have a radius of about 1.5 fm, in good agreement with the range of the  $\pi^\pm$ . To first order we are only interested in single pion exchange (see figure 1.1). We note that this interaction switches the isospin of each of the nucleons, hence this is  $\Delta I = 1$  transition.

In the Standard Model (SM) the weak force is also characterized by the exchange of massive particles ( $W^\pm$  and  $Z$ ). The masses of these particles are on the order of 100 GeV, hence the ranges are on the order of  $10^{-3}$  fm. Due to  $2\pi$  and  $\omega$  exchange, at a distance of  $\lesssim 0.2$  fm the strong force becomes repulsive [13]; thus contact interactions between nucleons are suppressed. In this context there can only be a Weak coupling between an exchanged pion and a nucleon. The authors in [2] showed that the charged weak current is suppressed by the Cabibo angle in the  $\Delta I = 1$  channel; meaning that PV in the two nucleon system is primarily due to  $Z$  exchange between quarks in a  $\pi N$  vertex.

An estimate of the relative coupling strengths between the weak and strong interactions can be gained from the following example: The relationship between a particle's lifetime and the coupling strength of the interaction responsible for its decay can be found in many particle physics textbooks (e.g. [32]); the relationship shows

$$\tau \propto 1/\alpha^2,$$

where  $\alpha$  is the coupling strength. We compare a weak flavor changing decay to a strong flavor conserving decay, whose reaction products are the same.

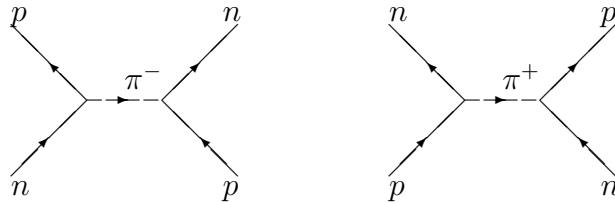


Figure 1.1:  $\pi^\pm$  exchange in the deuteron

$$\begin{aligned}\Sigma^+ &\rightarrow p + \pi^0 \\ \Delta^+ &\rightarrow p + \pi^0\end{aligned}$$

The lifetimes are  $8.0 \cdot 10^{-11}\text{s}$  and  $5.6 \cdot 10^{-24}\text{s}$ , respectively. From the above we find the relative strength of the weak to the strong coupling to be

$$\frac{\alpha_W}{\alpha_S} \propto \sqrt{\frac{\tau_\Delta}{\tau_\Sigma}} = 2.7 \cdot 10^{-7}$$

We conclude that the natural size of the weak contribution to the  $\pi\text{N}$  coupling is on the order of a few parts per million (ppm). Hence, in the following we introduce the weak potential as a perturbation to the strong hamiltonian.

### 1.2.1 The Weak Nucleon Potential

In the context of pion exchange the strong interaction hamiltonian can be written as

$$H_s = ig\bar{N}\gamma_5(\vec{\tau} \cdot \vec{\phi})N, \quad (1.1)$$

where  $\vec{\tau} \propto (\sigma_1, \sigma_2, \sigma_3)$  (the  $\sigma_i$  are the Pauli matrices),  $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$  (the  $\gamma^i$ s are the Dirac matrices), and  $g \simeq 0.08$  ([10]) is the strong interaction coupling strength. This leads to the one pion exchange NN potential

$$V_s = \left(\frac{g}{m_\pi}\right)^2 \tau_1 \cdot \tau_2 (\sigma_1 \cdot p_1)(\sigma_2 \cdot p_2) \frac{e^{-m_\pi r}}{4\pi r}. \quad (1.2)$$

Where  $m_\pi$  is the pion mass,  $r = |\vec{r}_1 - \vec{r}_2|$  is the distance between the nucleons, and  $p_i = \partial_{r_i}$ .

Naively, one might write down the contact V – A weak interaction between nucleons like so

$$H_w \propto (\bar{\psi}_p \gamma_\mu (1 - \gamma_5) \psi_n) (\bar{\psi}_n \gamma_\mu (1 - \gamma_5) \psi_p). \quad (1.3)$$

This is incorrect, as discussed above, nucleonic contact interactions are suppressed; and the weak bosons have finite range. A proper hamiltonian, at the microscopic level, requires the electroweak interaction between quarks; as done in [20]. There the perturbative weak hamiltonian ( $H_w$ ) has been isolated into parity conserving (PC) and parity non-conserving (PNC) terms<sup>4</sup>

$$H_w = H_{PC} + H_{PNC}. \quad (1.4)$$

---

<sup>4</sup>This is ultimately allowed by the assumption that one nucleon emits a meson through a weak interaction, which is then absorbed by another nucleon through a strong interaction

The total (strong plus weak) hamiltonian of the NN system may be written as

$$H = H_s + H_w. \quad (1.5)$$

From perturbation theory it is known that the perturbed nucleon eigenstates ( $\psi$ ) are the sum of the unperturbed strong eigenstates ( $\psi_0$ ) with the weak eigenstates ( $\psi_1$ );

$$\psi = \psi_0 + a\psi_1. \quad (1.6)$$

The weak mixing coefficient is given by

$$a = \frac{\psi_1^\dagger H_w \psi_0}{\Delta E}; \quad (1.7)$$

and  $\Delta E$  is the difference in energy between the two parity states.

Surrendering to arguments in [20, 26] the  $\pi$ NN weak potential ( $V_w$ ) can be calculated by using the Lippmann–Schwinger equation: The Green’s function

$$G = \frac{1}{E - H_s \pm i\epsilon} \quad (1.8)$$

is inserted into the transfer–matrix ( $T$ ) between final and initial nucleon pairs,

$$\langle f|T|i\rangle = \langle f|H_w|i\rangle + \langle f|H_wGH_w|i\rangle + \dots \quad (1.9)$$

Keeping only the 1<sup>st</sup> order non–vanishing terms this becomes

$$\langle f|H_w|i\rangle = \langle f|H_{PC}\frac{1}{E - H_s}H_{PNC}|i\rangle + \langle f|H_{PNC}\frac{1}{E - H_s}H_{PC}|i\rangle. \quad (1.10)$$

DDH give  $H_{PC}$  and  $H_{PNC}$ , in the context of one pion exchange, as

$$H_{PC} = ig_\pi \bar{N} \gamma_5 (\tau_+ \phi_- + \tau_- \phi_+) N \quad (1.11)$$

$$H_{PNC} = \frac{h_\pi}{\sqrt{2}} \bar{N} (\tau_+ \phi_- - \tau_- \phi_+) N. \quad (1.12)$$

Here  $g_\pi \simeq (G_F F_\pi)/2\sqrt{2} \simeq 3.8 \cdot 10^{-8}$  ( $G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$  is the Fermi constant and  $F_\pi = 0.094 \text{ GeV}$  is the pion decay constant)[62],  $\phi_{-(+)}$  represents the creation of a  $\pi^{-(+)}$  or correspondingly the destruction of a  $\pi^{+(-)}$ ,  $\tau_{-(+)}$  is the isospin transformation matrix. We have suppressed a

term involving  $\tau_z\phi_0$ , as  $\pi_0$  is being ignored. Plugging these into equation 1.10 and fourier transforming into position space (see [26]) yields

$$V_w(r) = i \frac{H_\pi^1}{M_N} (\tau_{1+}\tau_{2-} - \tau_{1-}\tau_{2+}) (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[ (\vec{p}_1 - \vec{p}_2), \frac{e^{-m_\pi r}}{4\pi r} \right], \quad (1.13)$$

where  $H_\pi^1 = (g_\pi h_\pi)/\sqrt{32}$  is the weak  $\pi$ N coupling strength. The main difference between the strong and weak  $\pi$ NN potentials is the coupling between the isospin of the nucleons; the strong potential has a scalar while the weak potential has a psuedo-scalar coupling. In the next sub-section it is shown how this potential can be related to a gamma-ray asymmetry in the  $\vec{n} + p \rightarrow d + \gamma$  reaction.

### 1.2.2 EM Transitions and the Gamma Distribution

When the neutron captures on the proton, electromagnetic transitions between the unbound and bound (deuteron) states lead to the emission of a gamma-ray. In the language of partial-waves, the unbound state of the neutron and proton is predominantly S-wave; that is the orbital angular momentum ( $L_i$ ) of the system is zero. From the previous subsection it is known that to leading order,  $V_w$  perturbatively adds P-waves whose isospin differs by one unit from that of the original S-waves. Using the spectroscopic notation  $|^{2s+1}L_J, I\rangle$ , where  $I$  is the isospin of the state and  $J$  is the vector sum of the orbital and spin angular momentum, the unbound state with  $J_i = 0$ , is written as

$$|^1S_0, 1\rangle.$$

Knowing the deuteron ground state (see below), the unbound states with  $J_i = 1$  are given by

$$0.98|^3S_1, 0\rangle, 0.2|^3D_1, 0\rangle, \text{ and } a_1|^3P_1, 1\rangle.$$

The subindex on  $a$  indicates that this coefficient comes from a  $\Delta I = 1$  transition.

The properties of the deuteron are summarized in [10]. Its one bound state is predominantly S-wave, though the tensor component of the strong interaction adds a quadrupole moment (D-wave). The total angular momentum is  $J_d = 1$ . Again, we assert that  $V_w$  perturbatively adds the appropriate P-waves; hence,

$$|d\rangle = 0.98|{}^3S_1, 0\rangle + 0.2|{}^3D_1, 0\rangle + a_1|{}^3P_1, 1\rangle.$$

The electromagnetic transition operator ( $H_{EM}$ ) may be split into electric and magnetic components, we write these simply as  $E(l)$  and  $M(l)$  respectively. To leading order only dipole transitions will contribute as higher multipoles are suppressed by a factor of  $(kr)^l$ ; and since  $\mathbb{P}(E(l)) = (-1)^l$  and  $\mathbb{P}(M(l)) = (-1)^{l+1}$  the 1<sup>st</sup> order parity violating term comes from  $E(1)$  (the electric-dipole transition). Thus only  $\Delta L = 1$  transitions will contribute to PV. It is noted that electro-magnetic transitions between states with the same isospin lead to vanishing matrix elements [29]. Putting this together there will be one matrix element in the transition  $|{}^1S_0, 1\rangle \rightarrow |d\rangle$ , which is

$$0.98\langle{}^3S_1, 0|M(1)|{}^1S_0, 1\rangle$$

this is a PC transition and produces no asymmetry. Scattering from the  $J_i = 1$  to the  $|d\rangle$  state leads to the following PNC matrix elements

$$\begin{aligned} &0.98a_1\langle{}^3P_1, 1|E(1)|{}^3S_1, 0\rangle \\ &0.98a_1\langle{}^3S_1, 0|E(1)|{}^3P_1, 1\rangle \\ &0.20a_1\langle{}^3D_1, 0|E(1)|{}^3P_1, 1\rangle \\ &0.20a_1\langle{}^3P_1, 1|E(1)|{}^3D_1, 0\rangle \end{aligned}$$

By applying Fermi's golden rule,

$$\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle f|V_{EM}|i\rangle|^2 \rho \propto d\sigma, \quad (1.14)$$

and using Wigner D-functions to obtain arbitrary gamma directions<sup>5</sup> the results given in [26] may be expanded to include the D-wave; we find

$$\frac{d\sigma}{d\Omega} \simeq \frac{1}{4\pi} \left( 1 - 4\sqrt{2} \cos \theta_{p_\gamma, \sigma_n} \frac{\langle E1 \rangle + \langle E1 \rangle'}{\langle M1 \rangle} \right), \quad (1.15)$$

where  $\langle M1 \rangle$ ,  $\langle E1 \rangle$ , and  $\langle E1 \rangle'$  are given by

---

<sup>5</sup>The D-functions allow one to determine the rotation between the  $\vec{L}$  of the field radiation with respect to the  $\vec{L}$  of the  $np$  system; since the gamma has helicity  $\mu' = \pm 1$  and the neutron is polarized, the Euler angle in the D-function is the angle between gamma momentum ( $p_\gamma$ ) and neutron spin ( $\sigma_n$ ). We also note that the initial unbound state for a polarized beam of neutrons incident on an unpolarized target is a coherent superposition of both the  $J_i = 0$  and  $J_i = 1$  states [26, 39]; and the helicity of the gamma-ray must satisfy the relation  $\mu' = M_{J_f} - M_{J_i}$

$$\begin{aligned}
\langle M1 \rangle &= 0.98 \int d^3r \langle {}^3S_1, 0 | M(1) | {}^1S_0, 1 \rangle \\
\langle E1 \rangle &= 0.98a_1 \int d^3r (\langle {}^3P_1, 1 | E(1) | {}^3S_1, 0 \rangle + \langle {}^3S_1, 0 | E(1) | {}^3P_1, 1 \rangle) \\
\langle E1 \rangle' &= 0.20a_1 \int d^3r (\langle {}^3P_1, 1 | E(1) | {}^3D_1, 0 \rangle + \langle {}^3D_1, 0 | E(1) | {}^3P_1, 1 \rangle).
\end{aligned}$$

Using states as given by [10], we calculate  $|\langle E1 \rangle'|/|\langle E1 \rangle| \sim 10\%$ . The DDH model (to be discussed below) is not sensitive to an effect on this order; hence we define the gamma asymmetry ( $A_\gamma$ ) simply as

$$A_\gamma \simeq -4\sqrt{2} \frac{|\langle E1 \rangle|}{|\langle M1 \rangle|}. \quad (1.16)$$

When the calculations are carried out, it is found that  $A_\gamma \sim -0.045H_\pi^1$ . Thus, a measured gamma-ray asymmetry in  $\vec{n} + p \rightarrow d + \gamma$  is a direct measure of the  $\pi N$  coupling constant in the parity violating weak potential.

### 1.2.3 DDH Framework and EFT

The authors of [20] made an attempt at a unified treatment of hadronic PV in the Standard Model. They began by relating the group  $SU(6)_w$  to the quark model.  $SU(6)_w$  is a Lorentz invariant generalization of  $SU(6)$ . The indices of  $SU(6)_w$  can be related to the three lightest quarks in the following way:  $1 = u \uparrow, 2 = u \downarrow \dots 6 = s \downarrow$ ; where  $\uparrow\downarrow$  represents the spin-up or spin-down state of the quark. By using the electroweak model, DDH explicitly includes both charged-current ( $W^\pm$ ) and neutral-current ( $Z$ ) interactions between quarks.

Hyperon decay data were used to constrain couplings in weak  $N \rightarrow N'm$  vertices; from there the effective NN weak potential is constructed. There are large uncertainties in the estimation of the  $mN$  weak coupling constants. Thus, what the DDH model predicts is a reasonable range of values for the constants. For the case of  $\pi^\pm$  exchange, the predicted range is  $H_\pi^1 = 0 - 1.55 \cdot 10^{-6}$  with a “best value” of  $H_\pi^1 = 6.2 \cdot 10^{-7}$ , which implies that  $|A_\gamma| \simeq 3 \cdot 10^{-8}$ .

Recently Effective Field Theory (EFT) has found application in the study of HWI. A comparison of EFT and DDH (single pion exchange) approaches is illustrated in figure 1.2. From the diagrams it is seen that EFT is applicable for a range of scales; to name of few: There is the weak scale  $\Lambda_w \propto \sqrt{G_F} \simeq 250$  GeV, the hadron scale  $\Lambda_H \propto m_p \simeq 1$  GeV (this is also about the same scale that the strong force becomes repulsive due to  $\rho$  and  $\omega$  exchange), and

the “pion scale” derived from the fact that the pion mass ( $m_\pi$ ) and decay constant ( $F_\pi$ ) are both on the order of 100 MeV.

In the EFT framework, S and P wave mixing is parameterized by low energy amplitudes with constant coefficients. These constants are related to parameters in general parity violating short-range, medium-range, and long-range potentials that are linearly dependent on momentum transfer. These parameters are related to DDH couplings and to experimental observables. When all this is done, the expected gamma-ray asymmetry is still predicted to be on the order of 50 ppb. For a more detailed introduction to EFT see [62, 35, 49].

In the next chapter the NPDGamma apparatus is described in some detail, and it is explained how it enables an accurate measurement of  $A_\gamma$ .

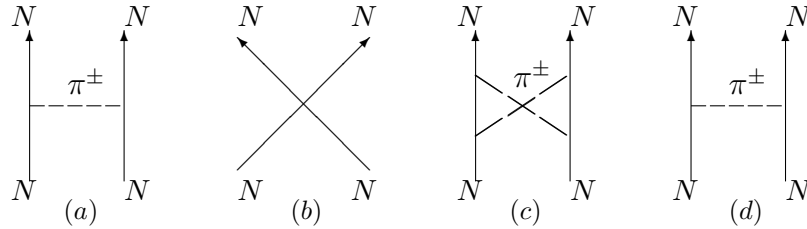


Figure 1.2: Each diagram has one parity violating vertex (unlabeled). *a* represents the DDH  $\pi$ NN interaction; *b*, *c*, and *d* are short-range, medium-range, and long-range NN interactions in EFT. This figure is based off of a figure in [49]

## Chapter 2

# Experimental Apparatus

To make a statistics limited measurement on the order of 10–20% of  $A_\gamma \sim 5 \cdot 10^{-8}$ , systematic effects that produce false asymmetries, should be  $\ll 10^{-8}$ . The statistical error must be  $\lesssim 1 \cdot 10^{-8}$ ; according to counting statistics about  $10^{16}$  gamma-rays must be detected, thus about the same amount of neutrons must capture on protons. These constraints help define the experimental components. The requirements to measure  $A_\gamma$  are: a high flux polarized neutron beam, a proton-rich target that will not depolarize the beam, high-efficiency gamma detectors, and a means of comparing the gamma flux with respect to neutron spin without introducing systematic effects (this is done by fast spin reversal of the neutron beam).

A model of the NPDGamma experiment is shown in figure 2.1. At the Fundamental neutron Physics Beamline (FnPB) at the Spallation Neutron Source (SNS), cold neutrons from a liquid hydrogen ( $\text{LH}_2$ ) moderator are transported by a super-mirror guide to a super-mirror polarizer. The polarizer absorbs one of the neutron spin states, rendering a beam with a net polarization. A magnetic guide field defines the axis of quantization and preserves beam polarization between the polarizer and the target. The neutrons enter the Radio-Frequency Spin Rotator (RFSR)<sup>1</sup>; where the polarization is reversed (when the RFSR is on) or left alone (when the RFSR is off). Finally, the neutrons enter the  $\text{LH}_2$  target. When a neutron is captured by a proton, a deuteron and a 2.2 MeV gamma-ray are formed. The gammas are detected by an array of 48 CsI crystals in a cylindrical formation giving about  $3\pi$  sr coverage of the  $\text{LH}_2$  vessel. The gamma-rays produce ionization

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<sup>1</sup>An audio frequency (30 kHz) is used; but, for historical reasons is referred to as “RF.”

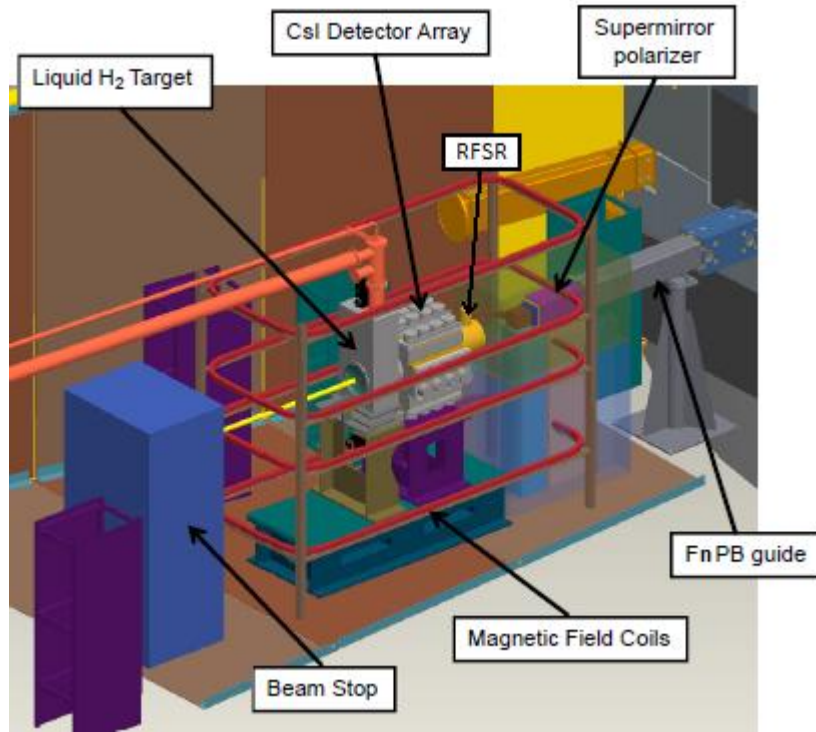


Figure 2.1: Model of the NPDGamma experiment at FnPB. Original image taken from [47]

in the CsI crystals, which then emit scintillation light. The light is detected by vacuum photo diodes (VPDs) that create a current signal. The current signal is sent to preamplifiers, and then read by the ADC in the Data Acquisition system (DAQ). The above components will be elaborated on in this chapter.

## 2.1 Neutron Production at FnPB

Neutron production at the SNS begins with a beam of hydrogen ions ( $H^-$ ) that are accelerated down a 0.3 km long LINAC (made of both normal and superconducting segments) to 1 GeV; the beam current is about 1 mA. The  $H^-$  beam is passed through a foil that strips the electrons off, and passes the protons to an accumulator ring where the beam is spooled around about 1000

times before the protons are delivered to a liquid mercury (LHg) target in a  $1\mu\text{s}$  long pulse; the proton pulses impinge on the LHg target every 17 ms (i.e. a pulse repetition rate of 60 Hz). The Hg nuclei undergo spallation, releasing tens of neutrons per proton. The neutrons enter a  $\text{LH}_2$  moderator where they lose energy to the hydrogen and are then delivered to the Fundamental neutron Physics Beamline (FnPB).

When in operation, the accelerated proton beam deposits approximately 1 MW of power into the LHg target. The LHg is housed in a thimble that is exposed to the proton beam,<sup>2</sup> that circulates about  $1.4\text{m}^3$  of mercury (about 20 tons [30]) through a cooling system designed to dissipate heat up to 2 MW. For more details about the SNS and its operation, please see [44, 30]

### 2.1.1 Super–Mirror Guide

Neutrons are transported to the experimental area by super–mirror guides. The neutron guides at FnPB have a cross–sectional area of  $10\times 12\text{ cm}^2$ ; chosen to match the dimensions of the moderator. Neutrons leaving the moderator are emitted isotropically. Upon entering the guide this geometric dependence is reduced (i.e. if the guide were completely lossless the intensity would remain constant from the entrance to the exit of the guide).

The cold guide starts about 1 m away from the moderator exit with a core segment that is about 1.3 m long. Following the core guide, neutrons enter a five channel bender that is approximately 5 m long with a radius of curvature on the order of 50 m. The bender blocks the line of sight to the moderator and thus helps shield the experiment from high energy neutrons and gamma–rays. The channels in the bender are composed of four equally spaced vertical vanes.<sup>3</sup> After the bender, neutrons are transported down a straight section of  $m = 3.6$ <sup>4</sup> super–mirror guide that extends about 8 m; ending approximately 15 m away from the moderator, in the experimental area. Along the length of this guide are the primary and secondary shutters, four chopper housings, and the  $8.9\text{ \AA}$  intercalated graphite monochromator for the UCN line. For further details please see [31].

As the guide and polarizer operate on similar principles, an understanding of neutron optics is instructive. We start with a partial wave analysis of plane

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<sup>2</sup>We note that if the SNS ran continuously for one year, the thimble would have been exposed to beam for about 30 minutes.

<sup>3</sup>Channels are required as the critical angle is on the order of a few milliradians.

<sup>4</sup>The quantity  $m$  is defined below.

waves scattering from a nucleus at the origin, we have

$$\Psi(r, \theta, \phi) = e^{ikz} + f(\theta, \phi) \frac{e^{ikr}}{r}. \quad (2.1)$$

Where  $z$  is the direction of the incoming plane waves,  $k = 2\pi/\lambda$ , with  $\lambda$  on the order of angstroms for cold neutrons,  $r$  is the separation between the neutron and the scattering site, and  $f(\theta, \phi)$  is the scattering amplitude. S-wave scattering dominates the process,<sup>5</sup> hence  $f(\theta, \phi) \rightarrow f$ .

The nucleus can be approximated by a several MeV deep square-well, with a range on the order of a few fm. Hence, the neutron's wavelength, in the well, is on the order of 1 fm; while outside of the potential well the neutron's wavelength is a few Å. Far from the nucleus ( $r \gg 1$ ) the plane waves behave as

$$e^{ikz} \rightarrow \frac{\sin kr}{kr}. \quad (2.2)$$

The wavefunction is

$$\frac{e^{ikr} - e^{-ikr}}{2ik} + fe^{ikr} = r\Psi(k, r) = u(k, r). \quad (2.3)$$

Far from the scattering site the Schrodinger equation for the radial wavefunction,  $u(k, r)$ , is just the Helmholtz equation, which has solutions  $u(k, r) = C \sin(kr + \delta)$ ; where  $C$  and  $\delta$ <sup>6</sup> are determined by boundary conditions. If  $C = e^{i\delta}/k$ , then

$$f = e^{i\delta} \frac{\sin \delta}{k}.$$

If the phase shift is small then, to first order in  $\delta$ ,  $f = \delta/k = -a$ . Since the differential scattering cross-section is given by

$$\frac{d\sigma}{d\Omega} = |f|^2,$$

Then the total coherent scattering cross-section is

$$\sigma = 4\pi a^2.$$

---

<sup>5</sup>Using a classical argument, if a particle approaches a hard sphere with radius  $r$  at an impact parameter greater than  $r$  no scattering occurs. Since  $l = \hbar kr$  then partial waves with  $l/\hbar > kr$  do not scatter (i.e. if  $l \gg kr$  then the phase shift of the outgoing wave is zero). For more details see [57]

<sup>6</sup>It can be shown that for low energy scattering  $\delta = -ka$ , where  $a$  is the scattering length [33]

Thus, far from the nucleus, the outgoing wavefunction is that of a particle scattering from a hard sphere with radius  $a$ .

A cold neutron, with  $\lambda$  on the order of a few Å, interacts with a homogeneous material (of a single element); whose nuclei are rigidly fixed with a spacing also on the order of angstroms; experiences several such scattering lengths. Assuming that the scattering does not significantly change the incident wave (i.e. the wave incident on each nucleus,  $\Psi_j(r')$ , is the same as the incident wave on the entire sample,  $\Psi(r')$ ,) the total wavefunction becomes.

$$\Psi(r) = e^{i\vec{k}_0 \cdot \vec{r}} - \int \Psi(r') \frac{e^{ik|r-r'|}}{|r-r'|} Na(r') d^3r'. \quad (2.4)$$

Following the example of Golub et al. [28], we operate on both sides by  $(-\hbar^2/2m_n)(\nabla^2 + k_0^2)$ , which yields

$$\frac{-\hbar^2}{2m_n} \nabla^2 \Psi(r) + \frac{2\pi\hbar^2}{m_n} Na \Psi(r) = \frac{(\hbar k_0)^2}{2m_n} \Psi(r) = E \Psi(r). \quad (2.5)$$

If  $a > 0$ , this is the Schrodinger equation for scattering from a step potential ( $V$ ) given by

$$V = \frac{2\pi\hbar^2}{m_n} Na. \quad (2.6)$$

The index of refraction ( $n$ ) for the material is given by the ratio of the neutron momentum in the material to the free neutron far from the guide walls:

$$n = \sqrt{1 - \frac{V}{E}} \simeq 1 - \frac{Na}{2\pi} \lambda^2. \quad (2.7)$$

From Snell's law the critical angle for total external reflection from the surface of the guide is

$$\cos \theta_c = \frac{n_2}{n_1}. \quad (2.8)$$

Letting  $n_1 = 1$ , for small angles we find

$$\theta_c \simeq \sqrt{\frac{Na}{\pi}} \lambda. \quad (2.9)$$

From [28] we find that for nickel-58  $Na = 1.3 \cdot 10^{-5} \text{ Å}^{-2}$ . Thus a neutron of four angstroms (around the peak flux at FnPB) will have a critical angle of  $\theta_c \simeq 8 \text{ mrad}$ .

Adding alternating layers of Ni and Ti,<sup>7</sup> for example, with a characteristic spacing of  $d \simeq \hbar/p_{\perp}$  allows neutrons (with the appropriate momentum perpendicular to the guide surface,  $p_{\perp}$ ) to Bragg reflect off the multilayers; thereby extending the critical angle (i.e.  $\theta_c \rightarrow m\theta_c$ ). Presently, super-mirrors with  $m = 3-4$  are commercially available. Varying the layer spacing allows neutrons of different  $p_{\perp}$  to also be Bragg reflected.

The neutron intensity ( $N_f(\lambda)$ ) exiting the guide is given by the input intensity ( $N_i(\lambda)$ ) scaled by geometrical properties of the guide (e.g. the length of the guide, its radius of curvature, and its effective solid angle); Maier-Leibnitz et al. [41] have determined these scaling factors for straight and curved guides. A detailed simulation of the spectra exiting the guide was done by Huffman [31]. Figure 2.2 shows this simulated spectra and the measured neutron flux at FnPB, as a function of wavelength in angstroms. The figure shows the simulated spectra to be in good agreement with the measured spectra.

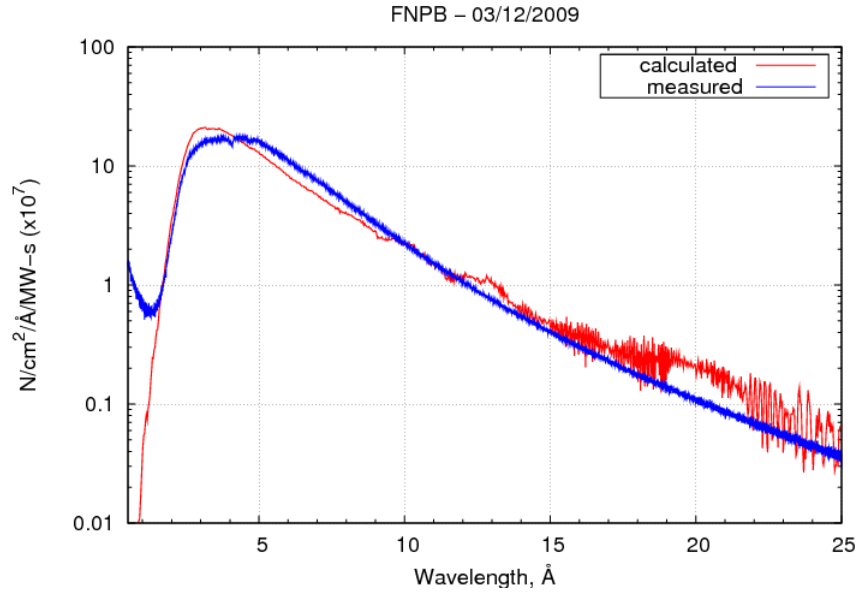


Figure 2.2: Simulated and measured neutron flux exiting the guide at FnPB. Image taken from [45]

<sup>7</sup>The scattering cross-section for <sup>58</sup>Ni is about 26 b, while for Ti it is about 4 b; the absorption cross-section for both is about 5 b; see [19]

### 2.1.2 Frame Definition Choppers

Frame definition choppers make it possible to select only part of the neutron spectrum in a given pulse. This helps to control for experimental errors; for example, it allows for effective neutron spin reversal. A pair of choppers have been phased to prevent overlap of slow neutrons from a given pulse with fast neutrons from the subsequent pulse.

The choppers are aluminum disks covered in  $^{10}\text{B}$  paint<sup>8</sup> excluding a wedge with an opening angle  $\theta$ . The choppers rotate at a frequency  $f = 60\text{Hz}$  (as this is the pulse rate of the accelerator).

A solution to the chopper opening angles can be found in the following way: A neutron beam travels down the guide, where at a distance  $x$  away from the moderator it encounters the chopper. At a distance  $L > x$  away from the moderator the beam enters the  $\text{LH}_2$  target. An illustration of the position of the neutrons as a function of time is presented in figure 2.3; from the figure it is seen that by similar triangles the following relation holds

$$\theta = \frac{2\pi x}{L}; \quad (2.10)$$

Assuming that the opening and closing times are short as compared with the time ( $\tau$ ) that the chopper is open we find

$$\tau = \frac{\theta}{2\pi f}. \quad (2.11)$$

FnpB has chopper housings at 5.5 m, 7.5 m, 9.0 m, and 10.5 m away from the moderator; the  $\text{LH}_2$  target will be about 18 m from the moderator. An estimate of the optimal opening angle for choppers 1 and 2 is below:

$$\theta_1 = \frac{2\pi 5.5}{18} \simeq 1.92\text{rad} \simeq 110^\circ, \quad (2.12)$$

$$\theta_2 = \frac{2\pi 7.5}{18} \simeq 2.62\text{rad} \simeq 150^\circ. \quad (2.13)$$

Mahurin and Crawford in [38] made a detailed study of various chopper configurations and opening angles and found that two counter-rotating choppers, with the smallest opening angles that disallow wrap-around neutrons, have the optimal opening angles of  $\theta_1 \simeq 132^\circ$  and  $\theta_2 \simeq 167^\circ$ ; in rough agreement with the above estimates.

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<sup>8</sup>Boron-10 has an absorption cross-section of about  $\gtrsim 3800\text{b}$ , while the scattering cross-section is a few barns.

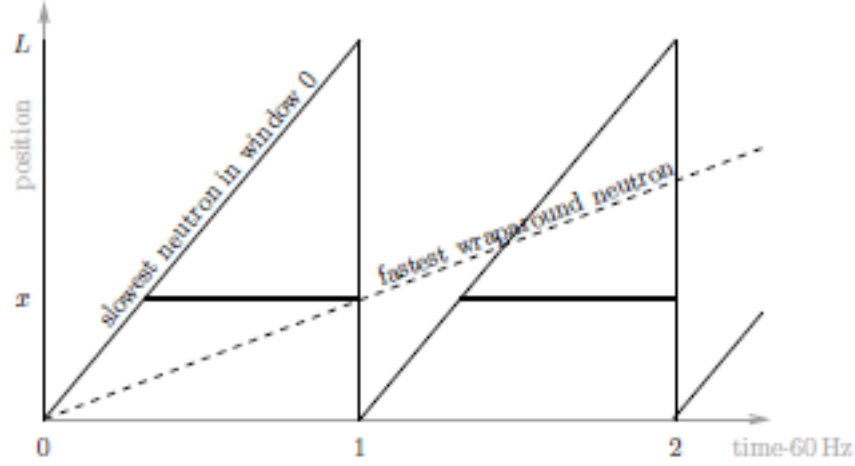


Figure 2.3: Plot of neutron position (relative to the moderator exit) vs time (in 60 Hz intervals). Wrap-around neutrons are eliminated by installing a second chopper downstream. Image from [39].

## 2.2 Super-Mirror Polarizer

The first super-mirrors were developed by Mezei for neutron polarization [42]; their more common use as spin-independent guides occurred later. In the case of the polarizers the multi-layers are interspersed with magnetic material (e.g. iron). The device sits in a large uniform magnetic field, on the order of a few hundred Gauss, that magnetizes the multi-layers. The neutron magnetic moment ( $\mu_n$ ) interacts with the effective field, leading to a Zeeman splitting of the optical potential; thus the critical angle is modified in the following way:

$$\theta_c = \lambda \sqrt{\frac{Na}{\pi} \mp \frac{2m_n}{h^2} \vec{\mu}_n \cdot \vec{B}_{eff}}. \quad (2.14)$$

Thus the polarizer can be crafted such that one spin-state has  $\theta_c \ll 1$ , and will penetrate the multi-layers and be absorbed by the substrate;<sup>9</sup> while the other spin-state has  $\theta_c \simeq \sqrt{2}\theta_{c0}$ , where  $\theta_{c0}$  is the critical angle in the absence of magnetic fields.

<sup>9</sup>Borated glass is a typical substrate; the wrong spin-state neutrons are captured by the boron and emit a single 0.5 MeV gamma-ray; iron in the multilayers can also become activated. This requires the polarizer to be enclosed in shielding.

The polarizer at FnPB is a 43 cm long bender, with a radius of curvature of about 11 m (the bend ensures that every neutron makes a bounce, leading to a high beam polarization). It is 10 cm wide by 12 cm tall (to match the cross-section of the guide), and has 45 channels. The concave surfaces of the vanes will have  $m = 3$  coating; and the convex surfaces have  $m = 2$ . The expected transmission will be about 30%; neutron beam polarization will be  $\gtrsim 95\%$ . For more details please see the design studies done by Crawford [17].

### 2.2.1 Compensation of the Polarizer Field

The polarizer is encased in a magnetic yoke that is composed of two magnets; a schematic is shown in figure 2.4. The inner magnet provides the required field for beam polarization. It is made of 12 pairs of NdFeB stacks; each stack is about 1.5 cm wide, 2 cm long, and 14 cm tall. The outer magnet's field is oriented so as to compensate the fringing field and gradient of the inner magnet. The outer magnet is composed of 6 pairs of NdFeB stacks; each is 2.5 cm wide, 2 cm long, and 25 cm tall. The top and bottom of each magnet is made of high grade Armco iron plates; the compensation magnet's plates are 1 cm thick and about 29 cm wide by 50 cm long, while the inner magnet's plates are 0.8 cm thick and about 50 cm long by 16.5 cm wide. Balascuta has shown that the field in the region of the polarizer is above 300 G for this design [6]; which satisfies the requirement for polarization coming from the manufacturer [47].

Shimming strips will be added to the sides of the compensation magnet to help minimize field gradients in the region of the experiment that can lead to beam depolarization in the RFSR and/or Stern–Gerlach steering of the beam in the target. First order numerical calculations (included in the appendix) show that these effects should be negligible for this design.

## 2.3 Magnetic Guide Field

A magnetic guide field, of about 9.5 G, defines the quantization axis (the vertical direction in this experiment). The field is created by four racetrack coils that are about 3 m long and 1.5 m wide. The coils are stacked vertically, such that the top and bottom coils are 2 m apart; these each have a total current of about 1 kA. The middle coils are separated by a vertical distance of 0.5 m and have a total current of about 400 A. See [6] for more details.

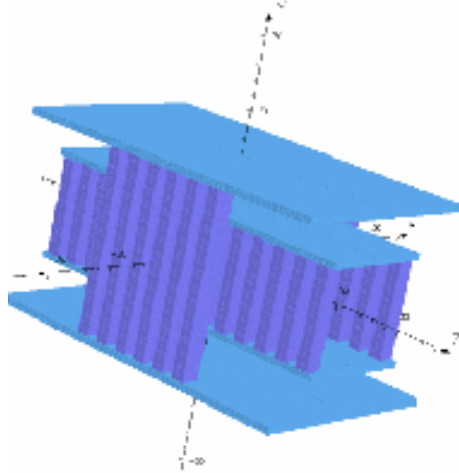


Figure 2.4: Schematic of the double yoke design, to compensate the field of the polarizer. Image originally from [6].

The guide field will be shimmed to satisfy the uniformity requirement set by the Stern–Gerlach asymmetry. Vertical field gradients must be kept at approximately the level of 1 mG/cm to ensure that this false asymmetry is below the level of 1 ppb.

## 2.4 Resonant RF Spin Rotator

The efficiency of the gamma-ray detectors (to be discussed later) will slowly vary over time due to activation, from being exposed to the neutron beam, and temperature fluctuations; and it is very unlikely that opposing detectors will behave identically over a period of time required to perform the measurement. That is, the difference in the gamma-yields ( $Y$ ) between a detector pair will lead to an asymmetry

$$A = \frac{Y_U - Y_D}{Y_U + Y_D}, \quad (2.15)$$

not wholly dependent on the parity violating asymmetry,  $A_\gamma$ . Also, the asymmetry cannot be determined, at the required level of precision, by measuring the signal in the  $i^{th}$  detector and the corresponding signal for the the reversed

spin-state in the same detector,

$$A = \frac{Y_i^\uparrow - Y_i^\downarrow}{Y_i^\uparrow + Y_i^\downarrow}, \quad (2.16)$$

because pulse-to-pulse fluctuations in the beam current will lead to a false asymmetry. The way around these potential sources of systematic errors is by frequent spin reversal of the neutron beam, from which the raw asymmetry is constructed,

$$A_{raw} = \frac{Y_U^\uparrow - Y_D^\uparrow - Y_U^\downarrow + Y_D^\downarrow}{Y_U^\uparrow + Y_D^\uparrow + Y_U^\downarrow + Y_D^\downarrow}. \quad (2.17)$$

From here,  $A_\gamma$  can be extracted (see [26] for more details).

To achieve the desired spin reversal a radio-frequency spin rotator (RFSR) is used. The RFSR must satisfy the following requirements: Reversal of the spin of all neutrons in the energy range of interest (about 2 meV to 13 meV), with high efficiency. The AC fields must not introduce stray field gradients that could steer the beam, nor can the fields couple to any other component; and the AC field must be able to be turned on and off in a time frame of about 1 ms.

The RFSR is a 30 cm long solenoid with a 30 cm diameter, housed in a 40 cm long, 40 cm diameter aluminum can; the Al housing is about 2 mm thick [12]. To minimize electronic pick-up in other circuits the load in the power circuit is held constant. To turn the RFSR off, the current is switched to an impedance matched dummy load. The RFSR is turned off ( $\uparrow$ ) and on ( $\downarrow$ ) for consecutive pulses in an eight step spin sequence ( $\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow$ ), designed to cancel quadratic drifts in detector gains that take place on a time scale of 100 ms.

### 2.4.1 Field of the RFSR

The expectation values of the neutron's quantum mechanical spin projections will follow the same dynamics as a classical magnetic moment. Hence a discussion of a magnetic moment that arises from a classical angular momentum is instructive.

The neutrons gyromagnetic ratio is about  $\gamma_n/2\pi \simeq 2.9$  kHz/G, since the guide field is about 9.5 G the Larmor frequency is about  $f_L \simeq 28$  kHz. The RFSR produces a field oscillating at the Larmor frequency along the beam

axis ( $Z$ ). The total field in the lab frame is

$$B_{lab} = B_0 \hat{Y} + B_{rf} \cos(\omega_L t) \hat{Z}, \quad (2.18)$$

where  $B_0$  is the vertical guide field,  $B_{rf}$  is the rf amplitude, and  $\omega_L$  is the angular Larmor frequency. The RF field can be rewritten as

$$B_{rf}(t) = \frac{B_{rf}}{2} \left( \cos(\omega_L t) \hat{Z} + \sin(\omega_L t) \hat{X} + \cos(\omega_L t) \hat{Z} - \sin(\omega_L t) \hat{X} \right). \quad (2.19)$$

The rotating field is brought to rest by transforming to a frame, rotating at a frequency of  $\omega_L$ . The total field in the rotating frame is

$$B_{rot} = \left( B_0 - \frac{\omega_L}{\gamma_n} \right) \hat{Y} + \frac{B_{rf}}{2} (1 + \cos(2\omega_L t)) \hat{Z}_r - \sin(2\omega_L t) \hat{X}_r. \quad (2.20)$$

The subindex ( $r$ ) indicates these coordinates are in the rotating frame. The counter-rotating field in this rotating frame is oscillating at twice the Larmor frequency, and thus is off resonance. The counter-rotating field makes the magnetic moment “wobble” back and forth without significantly disturbing the rotation of the neutron spin [26]; hence to first order we neglect it. The effective field then, at resonance in the rotating frame is

$$B_{eff} = \frac{B_{rf}}{2} \hat{Z}_r. \quad (2.21)$$

The precession angle, about this field, is given by

$$\theta = \pi \gamma_n B_{rf} \Delta t, \quad (2.22)$$

where  $\Delta t$  is the time the neutron spends in the RFSR:

$$\Delta t = L \frac{m_n}{h} \lambda,$$

here  $L$  is the length the neutron traverses through the RFSR (0.3 m),  $m_n$  is the neutron’s mass,  $h$  is Plank’s constant, and  $\lambda$  is the neutron’s wavelength. Thus to induce a  $\pi$  flip of the neutron spin, the amplitude,  $B_{rf}$ , is varied over the neutron time of flight (tof), or correspondingly  $\lambda$ ;

$$B_{rf}(\lambda) = \frac{h}{\gamma_n m_n L} \frac{1}{\lambda} \simeq \frac{7.2 \cdot 10^{-15} T m}{\lambda}. \quad (2.23)$$

For a 4Åneutron, the required RF amplitude is  $B_{rf} \sim 0.2$  G. In practice, a 1/tof waveform from a function generator defines the envelope for the AC field; this function gets triggered every proton pulse.

Further details about the RFSR and its efficiency (measured at LANSCE), can be found in [52]. The issue of determining the efficiency of the RFSR at FnPB is discussed in the next chapter.

## 2.5 Liquid Hydrogen Target

The  $\bar{n} + p \rightarrow d + \gamma$  reaction requires a dense unbound-proton rich material. From [19], for thermal neutrons on hydrogen, the total scattering cross-section (which is dominated by incoherent scattering) is about 80 b, while the absorption cross-section is about 1/3 b. Qualitatively, we expect that most hydrogen rich materials (e.g. methylene  $\text{CH}_2$ , methane  $\text{CH}_4$ , or water  $\text{H}_2\text{O}$ ) will depolarize a neutron beam, as spin-incoherent scattering is much more likely than coherent scattering or capture.

Molecular hydrogen is covalently bonded; from the Exclusion Principle the two electrons filling the  $\Sigma$ -orbital have their spins anti-aligned. The proton spins may be aligned in a triplet state, ortho-hydrogen ( $\text{oH}_2$ ), or anti-aligned in a singlet state, para-hydrogen ( $\text{pH}_2$ ). The singlet state, with  $S = L = 0$ , is the ground-state; the lowest energy triplet state,  $S = L = 1$ , is higher in energy by 15 meV, due to the rotational degree of freedom. In  $\text{LH}_2$  at 20 K, at equilibrium, the concentration of  $\text{oH}_2$  to  $\text{pH}_2$  is on the order of  $10^{-3}$  [39]. Since angular momentum is conserved, neutrons interacting with  $\text{LpH}_2$  can spin-flip only if  $\text{pH}_2 \rightarrow \text{oH}_2$ . But, conservation of energy forbids this unless the neutron can deliver 15 meV to induce the transition  $L = 0 \rightarrow 1$ .

The flux at FnPB is peaked at an energy of a few meV, hence spin-flip scattering is suppressed in  $\text{LpH}_2$  and coherent scattering and absorption both have cross-sections on the order of a few barns. Neutron beam depolarization will be small in the target.

The target vessel is an aluminum cylinder that is about 30 cm long and 30 cm in diameter. Its walls are about 3.8 mm thick and the windows are about 1.5 mm thick [47]; simulations show that about 60% of incident neutrons will capture in the target [26]. A schematic of the target cryostat with  $\text{LpH}_2$  vessel is shown in figure 2.5.

The target is filled as follows: Hydrogen gas flows through a gas purifier to filter out water and particulates. Then  $\text{H}_2$  goes through a  $\text{LN}_2$  purifier pre-cooler, which filters out hydrocarbons and cools the gas to 120 K. The  $\text{H}_2$  passes through a pre-heat-exchanger where it is cooled to 80 K. Then  $\text{H}_2$  condenses on copper fins attached to a 15 K cold head of the cryo-cooler. The hydrogen drips into a ferrous hydroxide powder; this is to catalyze the ortho to para conversion.<sup>10</sup> As the heat of conversion is greater than the heat

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<sup>10</sup>The dominant mode for spontaneous  $\text{oH}_2 \rightarrow \text{pH}_2$  conversion is by ortho-ortho scattering. The rate of this process depends on the square of the ortho concentration. Thus, small concentrations  $\text{oH}_2$  are stable [39]

of vaporization, the  $\text{H}_2$  will tend to boil off until the conversion is nearly complete. Next the  $\text{LH}_2$  flows to the target vessel; hydrogen that boils off is directed to the condenser, where it is reliquified. The process continues until the vessel is filled. The temperature is kept around 15-18 K, and monitored by a pressure transducer and thermometers. A control system keeps the target stable, and prevents the formation of bubbles; which could create a vertical density gradient in the  $\text{LpH}_2$  that can lead to a false asymmetry. For more details see, for example, [12] and [39].

## 2.6 CsI Detector Array and Data Acquisition System

In the NPDGamma experiment, photo-multiplier tubes (PMTs) cannot be used as part of the gamma-ray detectors, because the charge multiplication is very sensitive to magnetic field variations. Instead, CsI crystal scintillators are attached to vacuum photo-diodes (VPDs).

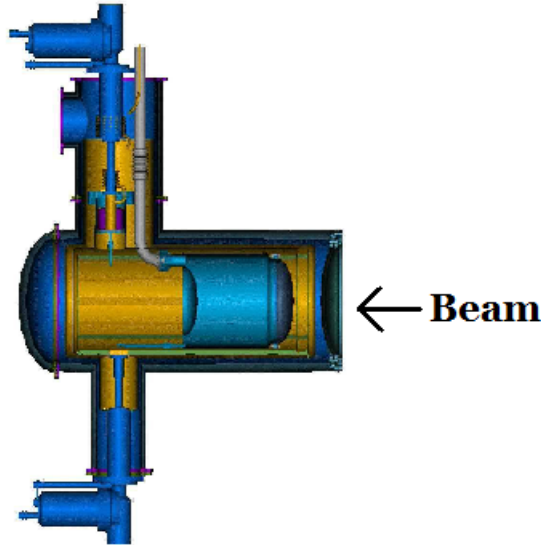


Figure 2.5: Schematic of the  $\text{LpH}_2$  target cryostat (with cryo-coolers at the top and bottom) and the target vessel. Original image from [26]

The detector array is shown in figure 2.6; it is composed of four rings, each with 12 CsI(Tl) crystals; the 48 crystals arranged in this cylindrical formation give about  $3\pi$  sr coverage of the LpH<sub>2</sub> vessel. CsI is well suited to detect the 2.2 MeV gamma-rays because of its high density, about 5 g/cm<sup>3</sup> [12]. The mean free path of the gamma-ray in CsI(Tl) is about 5 cm; to attenuate 95% of the gamma-rays, the detector's size is about three mean free paths (i.e. the dimension of each crystal is (15 cm)<sup>3</sup>). Also, CsI(Tl) is very radiation hard, even with doses of up to 50 kRad [12]. Thus the crystal's transparency should not be significantly diminished over time; preventing a decrease in the detected light which would lead to decreased sensitivity in a given detector.

The dominant form of the scintillation process arises when an electron-hole pair recombine forming an excited molecular state that decays through photoemission. Another form comes from phosphorescence which is when the electron-hole pair are in a metastable state that can take several  $\mu$ s to recombine. This effect can account for about one third of the total light emission [12]. Each crystal is viewed by a VPD. Scintillation light from each gamma-ray creates about 500 photoelectrons in the cathode [12]. The current output from the VPD is converted to a voltage using a I-V preamplifier that is attached to the back of each detector. The signal is sent to an op-amp that sums and averages all the signals. If the counting rates in all detector elements are equal it is sufficient to subtract this average from each signal. Finally, the sums and differences are digitized using an ADC, and stored. The digitizers sample the sum and difference signals at a rate on the order of 50 kHz during one neutron pulse [26], to get time-of-flight (tof) information. The digitizers are in a VME crate where they store the data on memory that is shared with a VME controller (the transient digitizers also have their own memory and controller). The memory is split into two buffers that hold data from alternate pulses. The inactive buffer is read by the VME host which combines the data with tof information and spin direction. After some data compression, the total event information is written to tape (through a SCSI interface) and to RAID array [47].

From the flux measurement (figure 2.2) we expect about  $10^{10}$  n/s exiting the guide; the polarizer transmits about 30% and 60% of these capture in the target, the detector array views about 75% of the target and each detector attenuates 95% of the emitted gammas. Thus, we estimate that about  $10^9$   $\gamma$ /s will be detected; hence the detector is operated in current mode. Further details on the detector array and DAQ, are in [12, 39, 26].

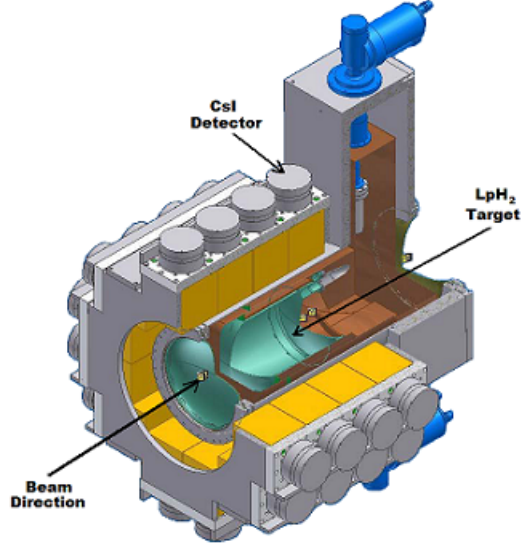


Figure 2.6: Schematic cut-out of the CsI detector array with the  $\text{LpH}_2$  target in place. Original image from [18]

## 2.7 Beam Monitors

The beam monitors are wire ionization chambers filled with about 1 atm of  $\text{CO}_2$  and about 0.004 atm of  $^3\text{He}$ . The gas is contained in a cylindrical volume with an outside diameter of 25.4 cm; the aluminum windows are about 0.10 cm thick and are separated by a distance of about 3.38 cm. There are three planes of wires, perpendicular to the beam. The two outer wire planes each contain 23 wires to generate a HV bias of about -1 kV. The middle wire plane has 24 sense wires connected to ground through a preamplifier.<sup>11</sup>

The output signal is produced when a neutron, passing through the monitor, undergoes the capture reaction:  $n + ^3\text{He} \rightarrow p + ^3\text{H}$ , which has a Q-value of 765 keV. The proton and triton lose energy by ionizing the gas. The negative ions are collected by the sense wires, thus producing a current signal which is converted to a voltage and amplified by the preamp. The voltage is proportional to the neutron capture rate ( $N_{\text{cap}}(\lambda)$ ), which in turn is proportional to the incident neutron flux ( $N_i(\lambda)$ ) by

$$N_{\text{cap}}(\lambda) = N_i(\lambda)(1 - e^{-\sigma(\lambda)nd}). \quad (2.24)$$

<sup>11</sup>These stated dimensions come from [40]

Where  $\sigma(\lambda)$  is the energy dependent capture cross-section for neutrons on  $^3\text{He}$ , and  $nd$  is the “thickness” of the  $^3\text{He}$  gas; these will be defined in the following chapter. The output voltage is sent through a low-pass filter to eliminate high frequency noise. The filter has a time constant on the order of  $100\ \mu\text{s}$  (this is about the time scale of ion collection). The voltages are then sent to the DAQ. For more details on the operation of beam monitors, see for example [27].

# Chapter 3

## Polarimetry Methods

The gamma-ray differential cross-section is related to the parity-violating gamma-ray asymmetry ( $A_\gamma$ ) in the following way (see equation 1.15);

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{4\pi} (1 + A_\gamma \cdot \cos \theta_{s_n, p_\gamma}) . \quad (3.1)$$

Correction factors that modify the differential cross-section and thus the gamma yield ( $Y$ ) of a given detector can be written as

$$Y_i(\lambda) = \frac{V_i(\lambda)}{4\pi} [1 + A_\gamma g_i(\theta, \phi) P_n(\lambda) \epsilon(\lambda) S(\lambda)] h(Z, \lambda) f_i(\theta, \phi). \quad (3.2)$$

Where  $V_i$  is the output signal in the  $i^{th}$  detector,  $\lambda$  is the neutron's wavelength,  $g_i(\theta, \phi)$  is a geometry factor,  $P_n(\lambda)$  is neutron beam polarization,  $\epsilon(\lambda)$  is RFSR efficiency of reversing  $P_n$ , and  $S(\lambda)$  is the spin-flip scattering probability (depolarization). The quantity  $h(Z, \lambda)$  is the capture centroid along the beam axis ( $Z$ ); it is not evenly distributed along  $Z$  and hence represents fluctuations along the beam axis. A detector is at a particular  $\theta$  and  $\phi$  relative to a capture location (or equivalently a gamma-ray source). In addition gamma-rays emitted at different angles can deposit different amounts of energy in a given detector, or even among two different detectors. These angular beam fluctuations are represented by the detection efficiency,  $f_i(\theta, \phi)$ . For a detailed derivation see [26].

Systematic uncertainties are added in quadrature to the statistical uncertainty, giving the total uncertainty of the experiment. The goal is to make a statistics limited measurement on the order of 10–20%. Systematic errors

should be controlled at the 1% level; that is

$$\left(\frac{\sigma_{A_\gamma}}{A_\gamma}\right)_{sys} = \sqrt{\left(\frac{\sigma_{P_n}}{P_n}\right)^2 + \left(\frac{\sigma_\epsilon}{\epsilon}\right)^2 + \left(\frac{\sigma_G}{G}\right)^2 + \left(\frac{\sigma_S}{S}\right)^2} \lesssim \text{a few \%}. \quad (3.3)$$

The authors of [26, 21] point out that the depolarization uncertainty is estimated to be on the order of a few percent and the uncertainty in the total geometry factor ( $G$ ) is better than 1%. Thus  $P_n$  and  $\epsilon$  need only be known at the level of 1–2%. Given the flux of FnPB ( $\sim 10^8$  n/cm<sup>2</sup>/s), the uncertainty of  $P_n$  will not be dominated by statistics; instead systematic uncertainties will dominate the error of the polarization measurement.

Zimmer et al. [63, 64] showed how the polarization of a neutron beam can be determined by taking relative measurements of the neutron flux ( $N$ ) through a polarized <sup>3</sup>He analyzer (with polarization  $P_3$ ), by reversing  $P_n$  with the aid of a well characterized high-efficiency Spin-Flipper (SF). The SF is located downstream of a polarizer; the analyzer is downstream of the SF and a beam monitor measures the transmitted flux. It is noted that either  $P_n$  may be reversed by the use of the RFSR (discussed in chapter 2) or  $P_3$  may be reversed by adiabatic fast passage (AFP). In both cases, assuming that either  $P_n$  or  $P_3$  can be reversed with 100% efficiency,  $P_n$  is determined from measured neutron intensities in the following way:

$$P_n = \frac{N_{aligned} - N_{anti-aligned}}{N_{aligned} + N_{anti-aligned}} \frac{1}{A}. \quad (3.4)$$

The subindex on  $N$  indicates whether  $P_n$  and  $P_3$  are aligned or anti-aligned with each other.<sup>1</sup> The analyzing power ( $A$ ) of the <sup>3</sup>He cell is given by

$$A = \sqrt{1 - \left(\frac{2N_{P_3=0}}{N_{aligned} + N_{anti-aligned}}\right)^2}. \quad (3.5)$$

$N_{P_3=0}$  is the neutron intensity after passage through the analyzer for  $P_3 = 0$ . If the efficiency of reversing either  $P_n$  or  $P_3$  is very close to 100%, the sums of the aligned and anti-aligned signals will approximate the signal of an unpolarized beam. From there  $P_n$  can be determined without any direct knowledge of  $P_3$  or other properties of the analyzer. These relative transmission methods are insensitive to uncertainties in  $P_3$  which usually dominates the experimental uncertainty in determining  $P_n$ .

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<sup>1</sup>Note that  $P_n$  and  $P_3$  are always along the guide field direction.

In the following, the method for determining  $P_n$  as proposed in [63] is described; the method uses the RFSR to reverse  $P_n$ . The alternative method using AFP on the analyzer to reverse  $P_3$ , is also discussed. First, we begin with a discussion of the analyzer.

### 3.1 The $^3\text{He}$ Analyzer

Figure 3.1 shows a picture of the analyzer cell. The analyzer is a cylindrical cell made of GE-180 glass, that will be filled with  $^3\text{He}$ . This type of glass was selected because it is non-borated (this minimizes neutron attenuation through the glass and radiation through neutron capture) and for its hardness (to minimize  $^3\text{He}$  leakage). The cylindrical shape of the cell was designed to ensure that every neutron passing through the cell experiences the same  $^3\text{He}$  thickness (e.g. if the cell were spherical we would need to integrate over its cross-section to make an accurate transmission measurement, see [21] for details); this can be confirmed by neutron transmission. Helium-3 is ideal for its spin-dependent cross-section.



Figure 3.1: A picture of the  $^3\text{He}$  analyzer cell, connected to the filling station at the SNS.

The helium gas will have a number density ( $n$ ), a length that the neutron beam will traverse ( $d$ ),<sup>2</sup> and a time-dependent polarization  $P_3(t)$ . Safety considerations at the SNS require that the pressure in the cell be no more than a couple atmospheres; the length of the cell is about 5 cm, hence the column thickness will be about,  $nd \simeq 10 \text{ atm}\cdot\text{cm}$ .<sup>3</sup>

The neutron absorption cross-section ( $\sigma$ ) on  $^3\text{He}$  is highly spin-dependent, due to the fact that  $n + ^3\text{He}$  goes to a virtual  $^4\text{He}$  state with angular momentum  $J = 0$  [46]. Thus, from the exclusion principle, if the neutron spin is anti-aligned with the spin of the  $^3\text{He}$  it will be absorbed; while if the spins are aligned the neutron will scatter. Hence if the helium is polarized, we may approximate the absorption cross-section as

$$\sigma_{\pm} \simeq \sigma(\lambda)(1 \mp P_3).$$

Here  $\sigma_{\pm}$  represents the cross-section for neutrons, whose spins are parallel (+) or anti-parallel (-) with  $P_3$ . The cross-section follows the  $1/v$  law [13], thus for a given neutron wavelength ( $\lambda$ ), we have

$$\sigma(\lambda) = \frac{\sigma_0}{\lambda_0} \lambda.$$

Using thermal neutrons as a benchmark,  $\lambda_0 \simeq 2 \text{ \AA}$  and  $\sigma_0 \simeq 5333 \text{ b}$ .

The neutron transmittance ( $T_{\pm}$ , where  $\pm$  indicates neutron spin aligned or anti-aligned with  $P_3$ ) through the helium takes the following form

$$T_{\pm} = e^{-\frac{\sigma_0}{\lambda_0} nd \lambda (1 \mp P_3)}. \quad (3.6)$$

The analyzing power ( $A$ ) of the cell is the polarization an unpolarized neutron beam acquires after transmission through the cell;

$$A = \frac{T_+ - T_-}{T_+ + T_-} = \frac{e^{\frac{\sigma_0}{\lambda_0} nd \lambda P_3} - e^{-\frac{\sigma_0}{\lambda_0} nd \lambda P_3}}{e^{\frac{\sigma_0}{\lambda_0} nd \lambda P_3} + e^{-\frac{\sigma_0}{\lambda_0} nd \lambda P_3}} = \tanh \left( \frac{\sigma_0}{\lambda_0} nd \lambda P_3 \right). \quad (3.7)$$

The quantity

$$x = \frac{\sigma_0}{\lambda_0} nd \lambda P_3$$

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<sup>2</sup>The product,  $nd$  we call the column thickness, and may be given in units of  $\text{amg}\cdot\text{cm} \sim \text{atm}\cdot\text{cm}$

<sup>3</sup>The reasons for having a large column thickness will be discussed below.

is the opacity of the cell.

The uncertainty of the analyzing power, which dominates the uncertainty in  $P_n$ , has the following form [64]

$$\frac{\sigma_{P_n}}{P_n} \simeq \frac{\sigma_A}{A} = \frac{2x}{\sinh(2x)} \cdot \frac{\sigma_x}{x}, \quad (3.8)$$

where  $x$  is the opacity. Complete ignorance of the cell's opacity (i.e.  $\sigma_x/x = 1$ ) implies that  $\sigma_{P_n}/P_n \sim 2x/\sinh(2x)$ . Letting  $P_3 = 0.6$  (a typical  $^3\text{He}$  polarization), we estimate the uncertainty of  $P_n$  to be a few percent for 4 Å neutrons traversing a 10 atm·cm cell, while for a 5 atm·cm cell the uncertainty is about 30%. Hence an analyzer with a large opacity is ideal for this measurement.<sup>4</sup>

If the neutron beam has an initial polarization ( $P_n$ ) then the transmitances become

$$T_{\pm} = (1 \pm P_n)e^{-\frac{\sigma_0}{\lambda_0}nd\lambda(1 \mp P_3)}. \quad (3.9)$$

Thus, the total transmittance is

$$T = T_+ + T_- = e^{-\frac{\sigma_0}{\lambda_0}nd\lambda} \left( \cosh \left( \frac{\sigma_0}{\lambda_0}nd\lambda P_3 \right) + P_n \sinh \left( \frac{\sigma_0}{\lambda_0}nd\lambda P_3 \right) \right). \quad (3.10)$$

This relationship can be confirmed by noting that if  $P_n = P_3 = \pm 1$  then  $T = 1$ , as is required.

### 3.1.1 SEOP

The analyzer will be polarized through Spin Exchange Optical Pumping (SEOP). Several reviews of this method can be found in [54, 3, 21]. Below we present a brief summary of SEOP.

The analyzer cell will be filled with about 2 atm of  $^3\text{He}$  at room temperature (about  $10^{20}$ – $10^{21}$  atoms), a “drop” of Rb (approximately  $10^{14}$ – $10^{15}$  atoms), and about 0.1 atm of  $\text{N}_2$  gas (about  $10^{19}$  atoms); the  $\text{N}_2$  acts as a quencher. During the optical pumping process the Rb must have a vapor density on the order of  $10^{14}$  atoms/cc [21], hence a temperature on the order of 450 K is required (this leads to a helium pressure of  $\sim 3$  atm during pumping).

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<sup>4</sup>A completely opaque cell will lead to 0 systematic uncertainty but 100% statistical uncertainty, as neutrons will not be transmitted; a balance must be reached.

Figure 3.2 shows a schematic of the SEOP process. Left circularly polarized laser light, with a wavelength of 795 nm, excites spin-down atoms from the  $5S_{1/2}$  state to the spin-up  $5P_{1/2}$  state. Collisional mixing brings the spin-up and spin-down sublevels of the excited atoms into equilibrium. Collisions between Rb atoms and  $N_2$  molecules then quench the excited Rb atoms down to the spin-up and spin-down ground state sublevels with about equal probability. Each atom in the initial population has an electron spin of  $-\hbar/2$ , while the average spin of the final population is 0; hence each photon deposits an angular momentum of  $\hbar/2$  to the Rb; the remainder goes to heating the vapor [54]. Binary collisions between the Rb and  $^3\text{He}$  allow the hyperfine interaction to transfer the spin from the Rb atom's  $5P_{1/2}^\uparrow$  state to the  $^3\text{He}$  nucleus; the cross-section for this is about 2 b.

The  $P_3$  saturation depends on the Rb polarization ( $P_{Rb}$ ), the spin exchange rate ( $\gamma_{se}$ ), the Rb density ( $N_{Rb}$ ), and the room temperature helium relaxation rate ( $\Gamma \simeq 10$  b) [4]:

$$P_{3sat} = P_{Rb} \frac{\gamma_{se} N_{Rb}}{1.33\gamma_{se} N_{Rb} + \Gamma}. \quad (3.11)$$

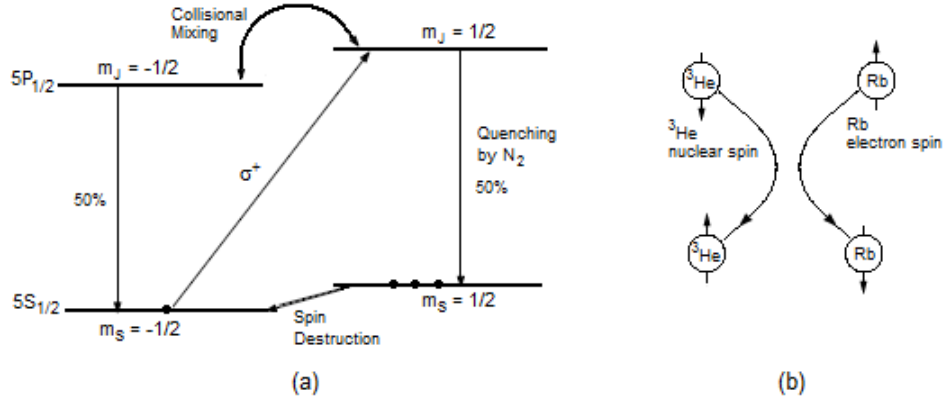


Figure 3.2: a) Schematic level diagram for optically pumping Rb with left circularly polarized light ( $\sigma^+$ ). b) Conceptual picture of spin exchange between Rb and  $^3\text{He}$ . Image inspired by figures in [54, 55]

If  $\Gamma \ll \gamma_{se} N_{Rb}$ , then  $P_{3sat} \simeq 0.75 P_{Rb}$ .<sup>5</sup> Once the polarization is saturated, the cell is cooled and the optical pumping is stopped,  $P_3$  will decay with a characteristic decay time ( $T_1$ ). The decay time is dominated by dipole–dipole interactions between the helium atoms (in the high pressure limit) and collisions with the cell walls; these relaxation rates are given by  $\Gamma$ . Hence we find:

$$P_3(t) = P_{3sat} e^{-t/T_1}. \quad (3.12)$$

Currently, typical cell decay times are on the order of hundreds of hours [4].

The  $^3\text{He}$  analyzer is ideal for neutron polarimetry; and, in this experiment, it will also be used to tune the RFSR. In the next section the issue of tuning the RFSR and determining its efficiency, without using AFP to reverse  $P_3$ , is discussed along with the method for determining  $P_n$  using the tuned RFSR.

## 3.2 Determining $\epsilon(\lambda, X, Y)$ and $P_n(\lambda, X, Y)$ Without Reversing $P_3$

Figure 3.3 depicts an experimental set–up with a super–mirror polarizer followed downstream by the RFSR, then a polarized  $^3\text{He}$  analyzer, and finally a beam monitor. This set–up allows a determination of RFSR efficiency ( $\epsilon$ ), and neutron beam polarization ( $P_n$ ) that does not involve adiabatic fast passage to reverse the  $^3\text{He}$  polarization ( $P_3$ );  $P_3$  is maintained by the same magnetic guide field used to preserve  $P_n$ . Note, that in the figure, we have suppressed showing the beam collimator. Collimation is required for an accurate map of  $\epsilon(\lambda, X, Y)$  and  $P_n(\lambda, X, Y)$ ; otherwise stray neutrons may enter the analyzer from the side and produce a signal.

### 3.2.1 RFSR Efficiency Measurements

We define the  $X$ –axis to be beam–left, the  $Y$ –axis is the vertical direction, and the beam axis ( $Z$ ) pierces the  $XY$  plane at the point  $(X, Y) = (0, 0)$ . We define the point  $Z = 0$  to be the center of the analyzer cell. Several points  $(X, Y, 0)$  will be selected over which to measure  $\epsilon(\lambda, X, Y)$  and  $P_n(\lambda, X, Y)$ . Neglecting the issue of beam divergence, the neutron beam is about 10 cm wide and 12 cm tall at the exit of the super–mirror polarizer. Since the

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<sup>5</sup>It is a current topic of investigation why there is a factor of 1.33 in front of  $\gamma_{se} N_{Rb}$  in the denominator, rather than unity [4]

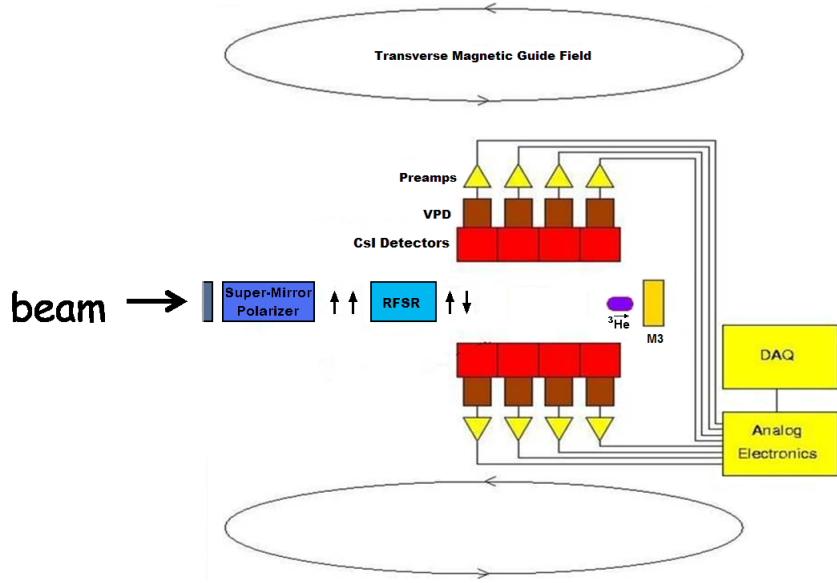


Figure 3.3: Conceptual schematic of the apparatus for the proposed method for measuring  $P_n(\lambda, X, Y)$  and  $\epsilon(\lambda, X, Y)$ , without reversing  $P_3$  by AFP. Image inspired by a figure from [47]

analyzer has a radius of about 1 cm; then nine locations, that will give a broad map of  $\epsilon$  and  $P_n$  across the beam, are the pairs  $(X, Y)$ ; where  $X = -4, 0, 4\text{cm}$  and  $Y = -5, 0, 5\text{cm}$ .

The RFSR can be tuned to be 100% efficient [52] over a small radius around the beam axis equal to the RFSR field axis. For the sake of argument we specify that the RFSR be tuned to be 100% efficient on the beam axis  $(0,0)$ . In any case five neutron intensity measurements can be made at each location. Assuming that the transmission through the glass is spin-independent; wavelength dependence is not a concern, as relative transmission measurements will be made per energy bin and hence will be canceled; the neutron yields exiting the analyzer are<sup>6</sup>

$$N_{\uparrow,\uparrow}(\lambda), N_{\rightarrow,\uparrow}(\lambda), N_{\downarrow,\uparrow}(\lambda), N_{\leftarrow,\uparrow}(\lambda), \text{ and } N'_{\uparrow,\uparrow}(\lambda).$$

These respectively represent neutron yields entering the beam monitor after the RFSR has delivered 0,  $\pi/2$ ,  $\pi$ ,  $3\pi/2$ , and  $2\pi$  pulses to the beam. Figure

<sup>6</sup>In our notation the first and second subindices respectively represent the directions of  $P_n$  and  $P_3$  with respect to the guide field.

3.4 depicts the rotated polarization vectors after each rf pulse; for the sake of argument we have assumed that the net result of the RFSR is to slightly under rotate  $P_n$ .

From figure 3.4 it is seen that the various spin rotations lead to beam polarizations that are reduced by factors: 1,  $\alpha$ ,  $\epsilon$ ,  $\beta$ , and  $\gamma$ ; respectively. Note that  $\epsilon$  is the RFSR efficiency for reversing  $P_n$ , by rotating the spin direction by  $\pi$ . Using the small angle approximation these factors can be written all in terms of  $\alpha \ll 1$ ; we have: 1,  $\alpha$ ,  $2\alpha^2 - 1$ ,  $-3\alpha$ , and  $1 - 8\alpha^2$ . This is confirmed by noting that if the spins are not under rotated then  $\alpha = 0$  and  $\epsilon = -1$  as desired.

If  $N_{\uparrow,\uparrow}(\lambda) = N_{\uparrow,\uparrow}^*(\lambda)$ , then within statistical error we can assert that the RFSR is perfectly tuned. That is, from figure 3.4, if  $1-\gamma = 0$  then  $1 = 1-8\theta^2$ , whose solution is  $\theta = 0$ . This can also be confirmed by the following relation:

$$N_{\uparrow,\uparrow}(\lambda) + N_{\downarrow,\uparrow}(\lambda) = N_{\rightarrow,\uparrow}(\lambda) + N_{\leftarrow,\uparrow}(\lambda);$$

implying that  $1 - \cos(2\theta) = \sin \theta - \sin(3\theta)$ . Assuming  $\theta \ll 1$  we have  $2\theta^2 + 2\theta = 0$ ; whose only solution, under our assumptions, is  $\theta = 0$ .

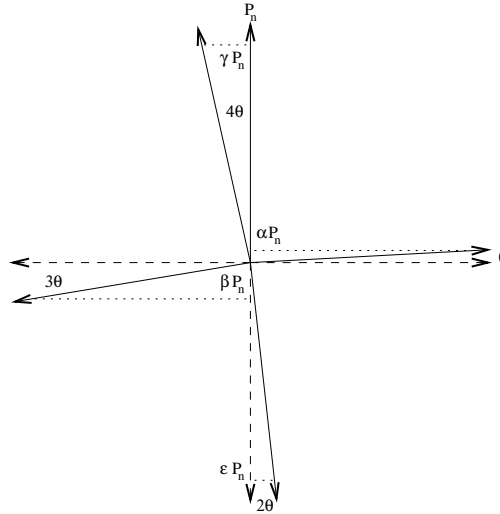


Figure 3.4: Conceptual picture showing how a non-ideal RFSR rotates  $P_n$ , leading to a reduced polarization of the beam, given by the projection of the new polarization onto the initial polarization axis. Dashed lines show the ideal rotation of  $P_n$

The signal to noise ratio must be  $\gg 1$ . Figure 3.5 shows plots of the expected spectra exiting the analyzer ( $N_{\uparrow,\uparrow}(\lambda)$  and  $N_{\downarrow,\uparrow}(\lambda)$ ).<sup>7</sup> The figure shows that we are limited by  $N_{\downarrow,\uparrow}(\lambda)$ ; so long as the background is  $\lesssim 500$  counts per energy bin per second (over the energy window) then the error due to noise should be washed out. If this is the case then the time required to make the five neutron intensity measurements, such that the statistical error is at the 1% level in each energy bin, will be on the order of a few minutes. So long as the percent difference of  $P_3$  is not significant over this time interval, then the measured neutron intensities can be compared, as described above. If  $P_3$  does not vary by more than, say, 1% we find

$$1 - e^{-t/T_1} \lesssim 0.01,$$

which implies that if the time interval to accomplish these measurements ( $t$ ) is on the order of 10 minutes, then the polarization decay time ( $T_1$ ) of the analyzer cell must be on the order of 20 hours. As previously stated, it is presently not uncommon to produce cells with  $T_1 \simeq 100$  hrs.

Once the RFSR has been tuned on axis, the analyzer and monitor can be moved to the next point ( $X, Y$ ) and the measurements repeated at the new location. Hence to make a scan over the beam, at the desired nine locations, will take on the order of 0.5 – 1 hrs. This means that if  $T_1 \simeq 50$  hrs, then  $P_3$  will have changed by a couple percent. This allows us to repeat the measurements to ensure that, at a given location, we get consistent results for  $P_n(\lambda, X, Y)$  and  $\epsilon(\lambda, X, Y)$ .

To go more indepth regarding the determination of  $\epsilon$ , we write out some of the neutron intensities in full detail:

$$N_{\uparrow\uparrow}(\lambda) = N_0(\lambda) e^{-\frac{\sigma_0}{\lambda_0} nd\lambda} \left[ \cosh \left( \frac{\sigma_0}{\lambda_0} ndP_3\lambda \right) + P_n(\lambda) \sinh \left( \frac{\sigma_0}{\lambda_0} ndP_3\lambda \right) \right], \quad (3.13)$$

$$N_{\downarrow\uparrow}(\lambda) = N_0(\lambda) e^{-\frac{\sigma_0}{\lambda_0} nd\lambda} \left( \cosh \left( \frac{\sigma_0}{\lambda_0} ndP_3\lambda \right) + \epsilon P_n(\lambda) \sinh \left( \frac{\sigma_0}{\lambda_0} ndP_3\lambda \right) \right), \quad (3.14)$$

---

<sup>7</sup>We used the root analysis package [50] with the n-tuple generated from [31] and a C-program (originally written by Crawford [53]) the author has modified to take into account beam polarization. The model assumes  $\epsilon = 0.98$  [52] and a reasonable value of  $P_3 = 0.6$ ; the model also takes into account the choppers with optimal opening angles and beam collimators

$$N_{\rightarrow\uparrow}(\lambda) = N_0(\lambda)e^{-\frac{\sigma_0}{\lambda_0}nd\lambda} \left( \cosh \left( \frac{\sigma_0}{\lambda_0}ndP_3\lambda \right) + \alpha P_n(\lambda) \sinh \left( \frac{\sigma_0}{\lambda_0}ndP_3\lambda \right) \right). \quad (3.15)$$

From these we may construct the following ratio:

$$R_\alpha = \frac{N_{\uparrow\uparrow}(\lambda) - N_{\downarrow\uparrow}(\lambda)}{N_{\uparrow\uparrow}(\lambda) - N_{\rightarrow\uparrow}(\lambda)} = \frac{1 - \epsilon}{1 - \alpha} \simeq \frac{2 - 2\alpha^2}{1 - \alpha}. \quad (3.16)$$

This can be solved by

$$\alpha = \frac{R_\alpha - \sqrt{R_\alpha^2 - 8(R_\alpha - 2)}}{4}. \quad (3.17)$$

In the ideal case  $P_n = P_3 = \epsilon = 1$ , then  $R_\alpha = 2$  and  $\alpha = 0$ . Finally, the efficiency can be determined by  $\epsilon = 2\alpha^2 - 1$ .

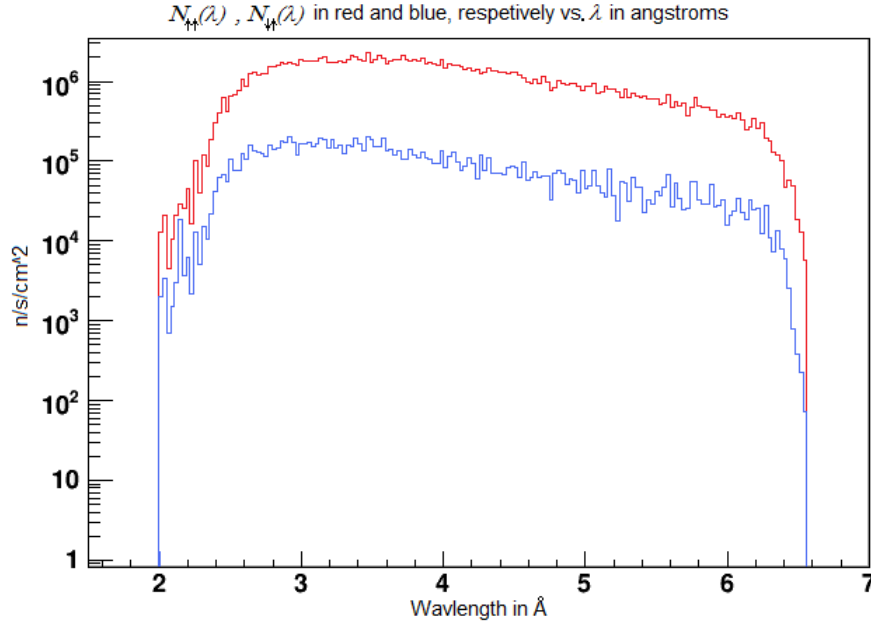


Figure 3.5: Plot of the expected spectra exiting the analyzer, for  $N_{\uparrow,\uparrow}(\lambda)$  in red, and  $N_{\downarrow,\uparrow}(\lambda)$  in blue, vs  $\lambda$  in  $\text{\AA}$ .

### 3.2.2 Polarimetry Method Using the Characterized RFSR and $^3\text{He}$ Analyzer

The experimental apparatus for this procedure is depicted in Figure 3.3. For  $P_3(t + \delta t) - P_3(t) \ll 1\%$ , over the time scale  $(\delta t)$  of the following three measurements, we may assume  $P_3(t) \rightarrow P_3$ . At a given location  $(X, Y)$ , we have three spectral measurements:

If  $P_3$  and  $P_n$  are aligned with the guide field, the neutron spectrum exiting the analyzer will be

$$N_{\uparrow\uparrow}(\lambda) = N_0(\lambda)e^{-\frac{\sigma_0}{\lambda_0}nd\lambda} \left[ \cosh\left(\frac{\sigma_0}{\lambda_0}ndP_3\lambda\right) + P_n(\lambda) \sinh\left(\frac{\sigma_0}{\lambda_0}ndP_3\lambda\right) \right]. \quad (3.18)$$

When  $P_3$  is aligned and  $P_n$  is anti-aligned (i.e.  $P_n \rightarrow -\epsilon P_n$ , the -1 has been factored out of  $\epsilon$ ), the spectrum will look like

$$N_{\downarrow\uparrow}(\lambda) = N_0(\lambda)e^{-\frac{\sigma_0}{\lambda_0}nd\lambda} \left( \cosh\left(\frac{\sigma_0}{\lambda_0}ndP_3\lambda\right) - \epsilon P_n(\lambda) \sinh\left(\frac{\sigma_0}{\lambda_0}ndP_3\lambda\right) \right). \quad (3.19)$$

Finally, when  $P_3 = 0^8$ ,  $P_n$  may be aligned or anti-aligned, the count rates are

$$N_{\uparrow 0}(\lambda) = N_0(\lambda)e^{-\frac{\sigma_0}{\lambda_0}nd\lambda}. \quad (3.20)$$

Letting  $\epsilon \rightarrow 1$ , we may write

$$\cosh^{-1}\left(\frac{N_{\uparrow\uparrow}(\lambda) + N_{\downarrow\uparrow}(\lambda)}{2N_{\uparrow 0}(\lambda)}\right) = \frac{\sigma_0}{\lambda_0}ndP_3\lambda, \quad (3.21)$$

hence

$$P_n(\lambda) = \frac{N_{\uparrow\uparrow}(\lambda) - N_{\downarrow\uparrow}(\lambda)}{2N_{\uparrow 0}(\lambda) \sinh\left(\cosh^{-1}\left(\frac{N_{\uparrow\uparrow}(\lambda) + N_{\downarrow\uparrow}(\lambda)}{2N_{\uparrow 0}(\lambda)}\right)\right)}. \quad (3.22)$$

We may also write  $P_n(\lambda)$  as

$$P_n(\lambda) = \frac{N_{\uparrow\uparrow}(\lambda) - N_{\downarrow\uparrow}(\lambda)}{N_{\uparrow\uparrow}(\lambda) + N_{\downarrow\uparrow}(\lambda)} \left( 1 - \left( \frac{2N_{\uparrow 0}(\lambda)}{N_{\uparrow\uparrow}(\lambda) + N_{\downarrow\uparrow}(\lambda)} \right)^2 \right)^{-1/2}. \quad (3.23)$$

---

<sup>8</sup>It will be known that  $P_3 = 0$  when  $N_{\uparrow\uparrow}(\lambda) = N_{\downarrow\uparrow}(\lambda)$ .

Assuming that  $\epsilon \lesssim 1$  but that the value is known with high precision (at the level of 1%), we find that

$$\cosh\left(\frac{\sigma_0}{\lambda_0}ndP_3\lambda\right) = \frac{N_{\uparrow\uparrow}(\lambda) + N_{\downarrow\uparrow}(\lambda)}{2N_{\uparrow 0}(\lambda)} - \frac{1 - \epsilon}{1 + \epsilon} \frac{N_{\uparrow\uparrow}(\lambda) - N_{\downarrow\uparrow}(\lambda)}{2N_{\uparrow 0}(\lambda)}. \quad (3.24)$$

Thus we have

$$P_n(\lambda) = \frac{N_{\uparrow\uparrow}(\lambda) - N_{\downarrow\uparrow}(\lambda)}{2N_{\uparrow 0}(\lambda)(1 + \epsilon) \sinh\left(\frac{\sigma_0}{\lambda_0}ndP_3\lambda\right)}; \quad (3.25)$$

where the opacity  $x = \frac{\sigma_0}{\lambda_0}ndP_3\lambda$  is determined from eq. 3.24. We also may write

$$P_n(\lambda) = \frac{N_{\uparrow\uparrow}(\lambda) - N_{\downarrow\uparrow}(\lambda)}{\epsilon N_{\uparrow\uparrow}(\lambda) + N_{\downarrow\uparrow}(\lambda)} \left(1 - \frac{1}{\cosh^2\left(\frac{\sigma_0}{\lambda_0}ndP_3\lambda\right)}\right)^{-1/2}. \quad (3.26)$$

Again, the opacity is determined from equation 3.24.

The fact that  $P_3$  is time-dependent allows validation of this method. The beam polarization coming out of the super-mirror polarizer should not vary over time. Hence measurements made over long time intervals ( $\Delta t$ ), such that

$$P_3(t) > P_3(t + \Delta t) > P_3(t + 2\Delta t) > \dots,$$

must give consistent results for  $P_n(\lambda)$  within experimental and statistical errors. Indeed the neutron intensities, for each spin configuration (aligned or anti-aligned with the guide field), over long times can be fit to a decaying exponential function to determine  $T_1$ . These two values must be in agreement with each other as well as an independent measurement of  $T_1$  done offline.

### 3.3 Determing $\epsilon(\lambda, X, Y)$ and $P_n(\lambda, X, Y)$ , by Reversing $P_3$ through AFP

As in the previous method, the LH<sub>2</sub> target is replaced with a polarized <sup>3</sup>He analyzer. However, now there are RF-coils to perform adiabatic fast passage (AFP) to reverse  $P_3$ , and pick-up coils to measure the NMR signal after an

AFP flip; the RFSR is not used to determine  $P_n$ .<sup>9</sup> Figure 3.6 depicts the experimental set-up.

### 3.3.1 Determining AFP Efficiency

Reversing  $P_3$  requires the addition of AFP-NMR coils. The AFP coils can be split-pair Helmholtz coils whose axis is aligned with the  $X$  axis. The diameter of the AFP coils would be about 15 cm, to ensure that the field is homogeneous within the region of the analyzer. The NMR pick-up coils would also be a split-pair design, wound around the analyzer near the windows (i.e. the axis of the pick-up coils is along  $Z$ ).

Like the RFSR, AFP operates on the principles of NMR. In the frame rotating with frequency  $\omega(t)$ , the effective field is given by

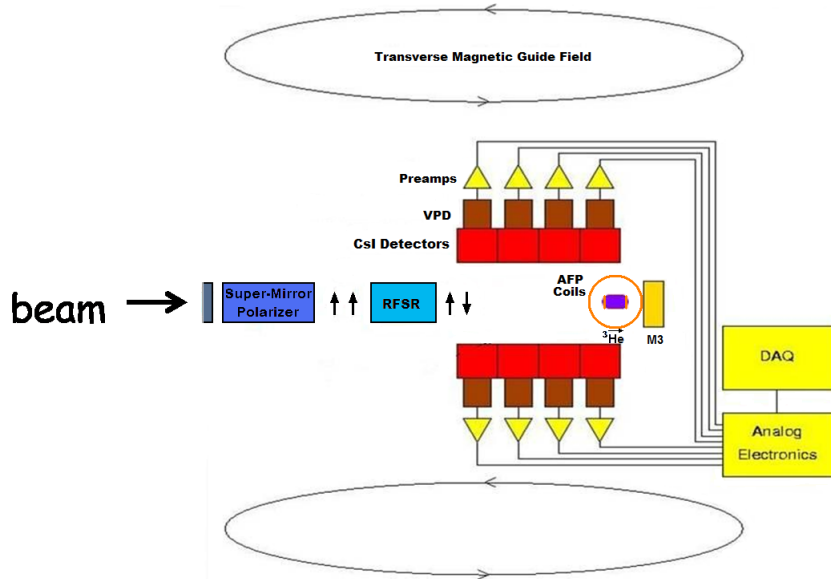


Figure 3.6: Conceptual schematic of the apparatus for the proposed method for measuring  $P_n(\lambda, X, Y)$  and  $\epsilon(\lambda, X, Y)$ , by reversing  $P_3$  through AFP.

<sup>9</sup>The RF field amplitude (of the AFP coils) must be perpendicular with the guide field; we set the axis of the RF field to be in the beam-left ( $X$ ) direction. The axis of the NMR pick-up coils must be perpendicular with both the guide field and the RF field

$$B_{eff} = \left( B_0 - \frac{\omega(t)}{\gamma_3} \right) \hat{Y} + \frac{B_{afp}}{2} \hat{X}_r. \quad (3.27)$$

The frequency of the AFP field ( $\omega(t)$ ) is swept past the Larmor frequency at a rate that satisfies the adiabaticity condition

$$D \frac{|\nabla B_0|^2}{B_{afp}^2} \ll \frac{\gamma_3 \omega(t)}{B_{afp}} \ll \gamma_3 B_{afp}; \quad (3.28)$$

where  $D$  is the  $^3\text{He}$  diffusion constant,  $\omega(t)$  is the frequency of the AFP field at a given time,  $B_{afp}$  is the amplitude of the field,  $\gamma_3$  is the  $^3\text{He}$  gyromagnetic ratio, and  $\nabla B_0$  is the gradient of the guide field.

Figure 3.7 shows a schematic of how the effective field in the rotating frame changes by varying  $\omega(t)$ . The pick-up coils register small changes in the net magnetic field coming from the reversed magnetization of the  $^3\text{He}$ .

Finding the average efficiency ( $\kappa$ ) of AFP reversal of  $P_3$  requires that the time scale to perform many AFP flips is short compared with the decay time of the analyzer ( $t_{AFP} \ll T_1$ ). Then  $P_3$  can be flipped many times until there is a significant reduction in the initial NMR signal. If the AFP reversal is very near 100%, many such flips will be required, and thus  $\kappa$  can be found with high precision. For example, if 1000 flips are required to reduce  $P_3$  to 90% of its initial value, with  $t_{AFP} = 0.1$  hrs and  $T_1 = 100$  hrs, the “natural”  $^3\text{He}$  polarization loss will be  $\lesssim 0.1\%$  and  $\kappa \simeq 99.99\%$ . Since  $\sigma_\kappa \sim 1/\sqrt{1000} \simeq 3.2\%$ , the uncertainty for this measurement is

$$\frac{\sigma_\kappa}{\kappa} \sim 3.2\%.$$

### 3.3.2 Polarimetry Method Using AFP to Reverse $P_3$

Again, at a given  $P_3$  and  $(X, Y)$ , three spectral measurements can be made:

If  $P_3$  and  $P_n$  are aligned with the guide field, the spectrum is

$$N_{\uparrow\uparrow}(\lambda) = N_0(\lambda) e^{-\frac{\sigma_0}{\lambda_0} nd\lambda} \left[ \cosh \left( \frac{\sigma_0}{\lambda_0} ndP_3\lambda \right) + P_n(\lambda) \sinh \left( \frac{\sigma_0}{\lambda_0} ndP_3\lambda \right) \right]. \quad (3.29)$$

If  $P_n$  is aligned and  $P_3$  is anti-aligned (i.e.  $P_3 \rightarrow -\kappa P_3$ ), the spectrum will be

$$N_{\uparrow\downarrow}(\lambda) = N_0(\lambda) e^{-\frac{\sigma_0}{\lambda_0} nd\lambda} \left[ \cosh \left( \frac{\sigma_0}{\lambda_0} nd\kappa P_3\lambda \right) - P_n(\lambda) \sinh \left( \frac{\sigma_0}{\lambda_0} nd\kappa P_3\lambda \right) \right]. \quad (3.30)$$

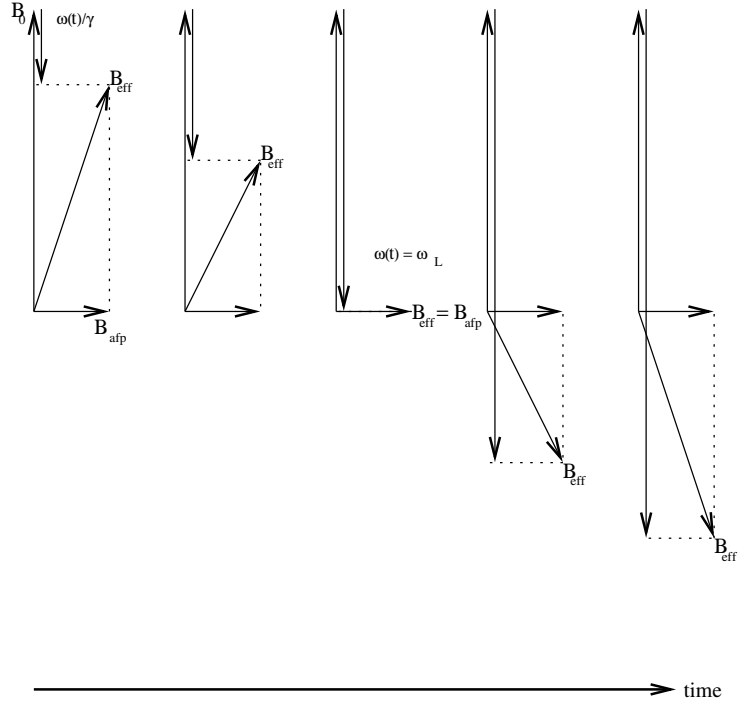


Figure 3.7: Schematic showing how the effective field in the rotating frame can be reversed by sweeping through the resonance condition. If the sweeping-rate is sufficiently slow, then  $P_3$  adiabatically follows the effective field. When the AFP coils are switched off,  $P_3$  remains anti-aligned with the guide field.

When  $P_3 = 0^{10}$  ( $P_n$  is aligned) the yield is

$$N_{\uparrow 0}(\lambda) = N_0(\lambda) e^{-\frac{\sigma_0}{\lambda_0} n d \lambda}. \quad (3.31)$$

Assuming  $\kappa \rightarrow 1$ , we have

$$P_n(\lambda) = \frac{N_{\uparrow\uparrow}(\lambda) - N_{\uparrow\downarrow}(\lambda)}{2N_{\uparrow 0}(\lambda) \sinh \left( \cosh^{-1} \left( \frac{N_{\uparrow\uparrow}(\lambda) + N_{\uparrow\downarrow}(\lambda)}{2N_{\uparrow 0}(\lambda)} \right) \right)}. \quad (3.32)$$

We may also write

$$P_n(\lambda) = \frac{N_{\uparrow\uparrow}(\lambda) - N_{\uparrow\downarrow}(\lambda)}{N_{\uparrow\uparrow}(\lambda) + N_{\uparrow\downarrow}(\lambda)} \left( 1 - \left( \frac{2N_{\uparrow 0}(\lambda)}{N_{\uparrow\uparrow}(\lambda) + N_{\uparrow\downarrow}(\lambda)} \right)^2 \right)^{-1/2} \quad (3.33)$$

For the case when the AFP efficiency is not 100%, but is known at the level of 1% or better, this method does not allow an analytic solution for the opacity. Instead it is found that

$$N_{\uparrow\downarrow}(\lambda) \cdot \sinh \left( \frac{\sigma_0}{\lambda_0} n d P_3 \lambda \right) + N_{\uparrow\uparrow}(\lambda) \cdot \sinh \left( \frac{\sigma_0}{\lambda_0} n d P_3 \lambda \kappa \right) - \sinh \left( \frac{\sigma_0}{\lambda_0} n d P_3 \lambda (1 + \kappa) \right) = 0. \quad (3.34)$$

Thus the opacity can be found numerically. Figure 3.8 shows a simulation of the neutron spectra for  $N_{\uparrow\uparrow}(\lambda)$  and  $N_{\uparrow\downarrow}(\lambda)$ . In this case we have left the RFSR alone and reversed  $P_3 = 0.6$  with  $\kappa = 0.999$  (a mildly optimistic value for AFP efficiency [5]); the results are in good agreement with those shown in figure 3.5.

As an example, let us assume that for a given energy bin we have  $N_{\uparrow,\uparrow}(\lambda) \sim 10^6 \text{ n/s}$  and  $N_{\uparrow,\downarrow}(\lambda) \sim 10^5 \text{ n/s}$ . If  $\kappa = 99.9\%$ , then from Eq. 3.34 we find the opacity to be 13.91. Once the opacity is found then  $P_n(\lambda)$  can be determined by

$$P_n(\lambda) = \frac{\frac{N_{\uparrow\uparrow}(\lambda)}{N_{\uparrow 0}} - \cosh \left( \frac{\sigma_0}{\lambda_0} n d P_3 \lambda \right)}{\sinh \left( \frac{\sigma_0}{\lambda_0} n d P_3 \lambda \right)}; \quad (3.35)$$

where the opacity is determined from eq. 3.34.

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<sup>10</sup>In this case,  $P_3 = 0$  when  $N_{\uparrow\uparrow}(\lambda) = N_{\uparrow\downarrow}(\lambda)$  and the NMR coils should pick-up no magnetization from the  $^3\text{He}$ .

The efficiency of the RFSR can also be found using this AFP method. From the above, we construct the ratio

$$R_{afp} = \frac{N_{\uparrow,\uparrow}(\lambda) - N_{\downarrow,\uparrow}(\lambda)}{N_{\uparrow,\uparrow}(\lambda) - N_{\uparrow,\downarrow}(\lambda)}. \quad (3.36)$$

If the AFP flip is 100% efficient then we have

$$\epsilon = 1 - 2R_{afp}. \quad (3.37)$$

If  $\kappa \lesssim 1$  but known with high precision, and if  $P_n$  has been found, then  $\epsilon$  can be determined numerically by

$$R_{afp} = \frac{(1 - \epsilon)P_n \sinh(x)}{\cosh(x) - \cosh(\kappa x) + P_n(\sinh(x) + \sinh(\kappa x))}; \quad (3.38)$$

the opacity,  $x$ , is determined from Eq. 3.34.

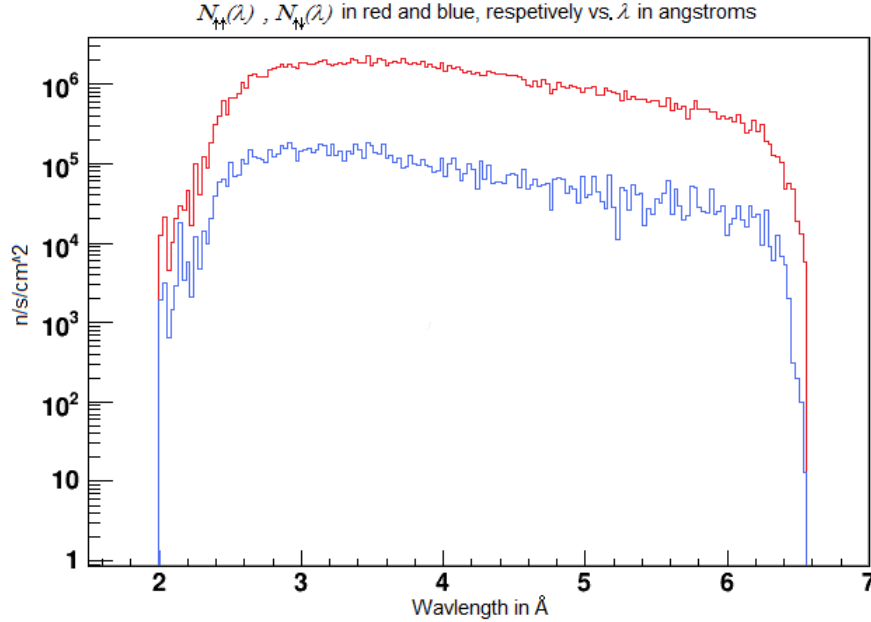


Figure 3.8: Plot of the expected spectra exiting the analyzer, for  $N_{\uparrow,\uparrow}(\lambda)$  in red, and  $N_{\uparrow,\downarrow}(\lambda)$  in blue, vs  $\lambda$  in  $\text{\AA}$ .

As in the method not involving  $P_3$  reversal, the fact that the helium polarization varies over long time intervals is an advantage for this method. However, it is important to note that  $P_3(t)$  depends on  $T_1$  and the AFP efficiency ( $\kappa$ ). Since this method relies on precise knowledge of  $\kappa$ , it is important to be able to separate these two polarization losses;  $T_1$  must be determined by an independent measurement. Once it is shown that many AFP reversals can be made over a time scale ( $\delta t$ ) such that  $P_3(t + \delta t) \simeq P_3(t)$ , then  $\kappa$  can be determined with high precision as was explained in the previous subsection.

### 3.4 Comparison of Methods

Both methods discussed offer a means of finding  $P_n(\lambda, X, Y)$  and  $\epsilon(\lambda, X, Y)$ , at the level of a few percent. In both methods, to determine these quantities at the desired level of precision, each spectral measurement must be determined at the level of 1% or better. It will be easy to control statistical uncertainties at this level, given the high flux of FnPB. Determining the opacity through these relative transmission measurements, coupled with an optically dense analyzer, allows for the control of systematic uncertainties at a comparable level of precision.

The first method (not involving AFP) only uses components of the NPDGamma apparatus, excluding the  $\text{LpH}_2$  target, with the addition of a  $^3\text{He}$  analyzer. While the second method is similar, it also requires AFP-NMR coils to be constructed. For this reason, and the fact that the AFP method has no analytic expression for the opacity of the analyzer when the reversal of  $P_3$  is not 100% efficient, we find the first method to be simpler.

## Chapter 4

## Conclusions

The NPDGamma experiment is ready to start commissioning at the fundamental neutron physics beamline (FnPB) at the SNS in early 2010. The experiment is designed to search for an up-down parity violating gamma-ray asymmetry in the capture of a polarized beam of neutrons on protons in a liquid para-hydrogen target. In this thesis two independent methods for determining neutron beam polarization ( $P_n$ ) and resonant RF spin rotator efficiency ( $\epsilon$ ) have been discussed. It has been shown that either method will suffice to measure these quantities at the desired level of a couple percent. The option that does not use adiabatic fast passage (AFP), to reverse the polarization ( $P_3$ ) of the  $^3\text{He}$  analyzer, is simplest and is therefore the recommended method for the measurements.

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# Appendix A

In this appendix we make first order numerical estimates for Stern–Gerlach steering of the neutron beam and beam depolarization in the RFSR, primarily due to the fringing field of the super–mirror polarizer’s magnetic yoke.

## A.1 Stern-Gerlach Steering

The Stern-Gerlach effect arises from the coupling of the neutron magnetic moment  $\mu_n$  to the gradients of the total magnetic field of the experiment (guide field, fringing field of the polarizer, etc.).

$$F = \vec{\mu}_n \cdot \vec{\nabla} B$$

In the presence of magnetic field gradients ( $\partial B_Y / \partial Y$ ) neutron trajectories will be deflected up or down. This results in a change of the solid-angle of a given CsI detector and can lead to a false up-down asymmetry.

We define the coordinates such that  $X$  is beam left,  $Y$  is the vertical, and  $Z$  is the beam axis. Far from the super–mirror polarizer, the magnetic field produced by the polarizer’s yoke approximates that of a dipole. Vertical gradients cause neutrons of opposing spins to be steered in opposite directions by a distance  $d$ .<sup>1</sup> Hence a given detector’s geometric efficiency  $E$  (i.e. the effective detector solid angle) depends on whether the neutron’s spin is  $\uparrow$  or  $\downarrow$  like so:

$$E_{\uparrow} = \frac{A}{(L - d)^2} \text{ or } E_{\downarrow} = \frac{A}{(L + d)^2}; \quad (\text{A.1})$$

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<sup>1</sup>These gradients will be largest at the top or bottom of the beam.

where  $A = 225 \text{ cm}^2$  is the area of the detector and  $L = 16 \text{ cm}$  is the distance the 2.2 MeV gamma-ray would travel in the absence of steering. These changes in the solid angle lead to the following asymmetry

$$E_{\uparrow} - E_{\downarrow} = \frac{A}{L^2} \left[ \left(1 - \frac{d}{L}\right)^{-2} - \left(1 + \frac{d}{L}\right)^{-2} \right] \simeq \frac{4Ad}{L^3}. \quad (\text{A.2})$$

For this asymmetry to be  $\lesssim 10^{-9}$ , then  $d \lesssim 1 \text{ \AA}$ , and the field gradient everywhere must be kept  $\lesssim 1 \text{ mG/cm}$ . It is noted that if the fringing field of the polarizer is symmetric vertically and if the beam and detector array are perfectly aligned with the field, this effect only produces beam “breathing.” The Stern–Gerlach effect leads to a false asymmetry if some component of the apparatus (e.g the magnetic shielding) is not perfectly aligned.

Figures A.1 and A.2 depict maps of the net displacement and Stern–Gerlach asymmetry that the neutron would experience at different positions on the  $XY$ -plane. The Stern–Gerlach steering was modeled using simulated field values from [7]. The total field, including the guide coils and the fringing field of the polarizer, was calculated in the region between the exit of the RFSR and the expected center of the  $\text{LpH}_2$  target. The  $XY$ -plane was divided into an  $N \times N$  grid and the field gradients were tabulated by linear interpolation of the field values. The equations of motion for transverse displacement of the neutron was integrated as the neutron moved downstream. Figure A.3 shows the total Stern–Gerlach steering as calculated for a 9 cm diameter beam. The center of the beam circle was displaced along the vertical axis ( $Y$ ) in 5 mm steps.

As is seen in figure A.3 the integrated asymmetry across the beam face is about  $\lesssim 1\%$  of the predicted gamma-ray asymmetry. The surface plots are not symmetric along  $Y$ , due to the fact that the magnetic shielding is not symmetric about the experiment. Note also that these calculations implicitly assume that the neutron trajectory moves no more than 1 cm in the  $X$  or  $Y$  directions. A more realistic neutron transport analysis, that takes into account the large beam divergence exiting the polarizer, is required.

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<sup>2</sup>We have assumed that capture occurs at the top of the beam, 6cm above the  $ZX$  plane

## A.2 Depolarization in the RFSR

Depolarization of the neutron beam arises from magnetic field gradients in the region of the RFSR. A reduction of beam polarization,  $P_n$ , decreases the sensitivity of the parity-violating gamma-ray asymmetry measurement by

$$\sigma_{A_\gamma} \sim \frac{1}{P_n \sqrt{N}},$$

where  $N$  is the number of gamma-rays detected. A small vertical gradient changes the resonance field in the rotating frame as follows

$$B_{eff} = \frac{B_{rf}}{2} \hat{X}_r + \frac{\partial B'_Y}{\partial Y} \Delta Y \hat{Y}. \quad (\text{A.3})$$

The components form a triangle such that

$$\frac{2\Delta B'_Y}{B_{rf}} = \tan \theta \simeq \theta. \quad (\text{A.4})$$

The reversed polarization in the presence of such a gradient is scaled by  $\cos 2\theta$ , for small angles then the depolarization ( $f$ ) is

$$f \simeq 2\theta^2.$$

If  $B_{rf}(\lambda) = 0.2$  G and  $\Delta B'_Y = 10$  mG, then there is a 2% loss of polarization for neutrons of the “correct” wavelength.

Again, far from the polarizer, the magnetic yoke’s field approximates that of a dipole. Looking at only variations in the vertical direction, we may write

$$B'_Y = B'_0 - \chi Y^2. \quad (\text{A.5})$$

Where  $B'_0$  is the dipole field strength on axis. The total depolarization can be found by

$$f_{tot} = \frac{2 \int \theta^2(Y) dY}{\int dY} = \frac{8}{B_{rf}^2 Y_{max}} \int (B'_0 - \chi Y^2)^2 dY \quad (\text{A.6})$$

Solving this equation and minimizing with respect to the on-axis field strength leads to the following expression for the depolarization:

$$f = \frac{32}{45} \left( \frac{\Delta B'_Y}{B_{rf}} \right)^2, \quad (\text{A.7})$$

where  $\Delta B'_Y = |B'_Y(Y = 0) - B'_Y(Y = Y_{max})|$ .

Figure A.5 shows a map of the net depolarization that the neutron would experience at different positions on the  $XY$ -plane. The depolarization was modeled from field data by [7]. The total field, including the guide coils and the fringing field of the polarizer, was calculated in the region the RFSR. Once more the  $XY$ -plane was divided into an  $N \times N$  grid and the depolarization was tabulated based on the above equations. Figure A.6 shows the total depolarization as calculated for a 9 cm diameter beam. The center of the beam circle was displaced along the vertical axis ( $Y$ ) in 5 mm steps.

As is seen in figure A.6 the integrated depolarization across the beam face is about  $\lesssim 1\%$ . Again, these calculations implicitly assume that the neutron trajectory moves no more than 1 cm in the  $X$  or  $Y$  directions. The estimate is conservative in that it is assumed the beam is depolarized over the entire length of the RFSR. The surface map is not symmetric along  $Y$ , because the magnetic shielding is not symmetric around the experiment. A more realistic analysis must be done that solves the spin equations of motion for neutrons traveling on realistic trajectories, and includes realistic fields from the RFSR.

The simulated field data (from [7]) and analysis codes can be found at [45].

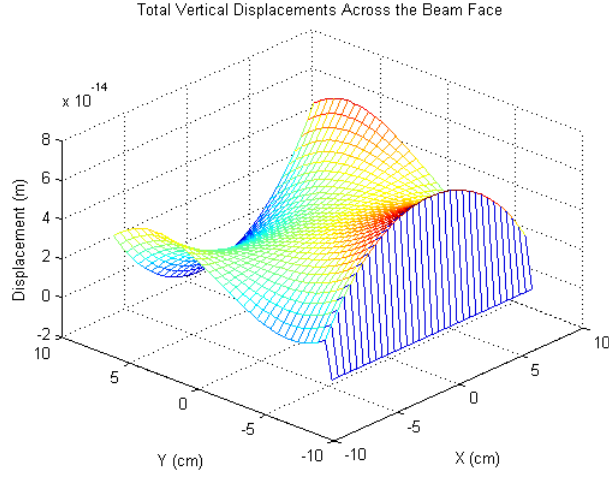


Figure A.1: The net displacement a neutron experiences traveling from the exit of the RFSR to the center of the target, across the  $XY$  plane.

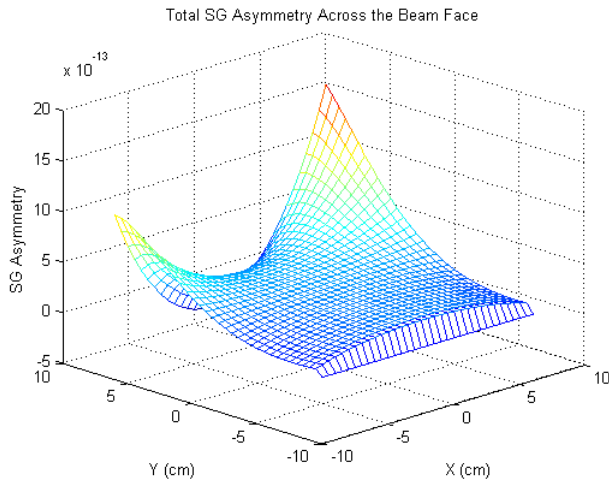


Figure A.2: The asymmetry produced, given the net displacement the neutron would experience, across the  $XY$  plane.

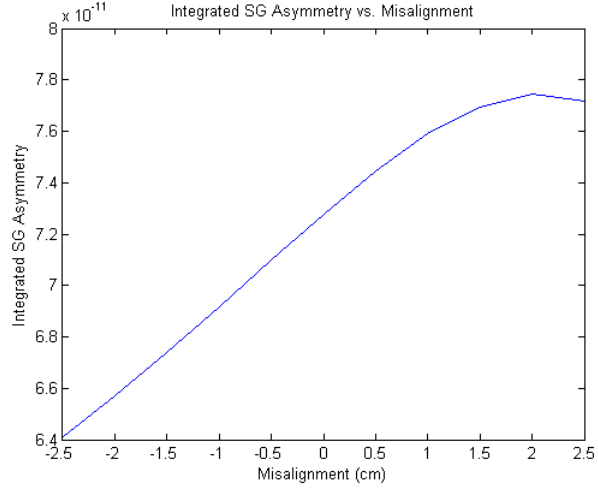


Figure A.3: The integrated asymmetry due to Stern–Gerlach steering for a 9 cm diameter beam as a function of misalignment between the beam axis and magnetic field axis, in the vertical direction.

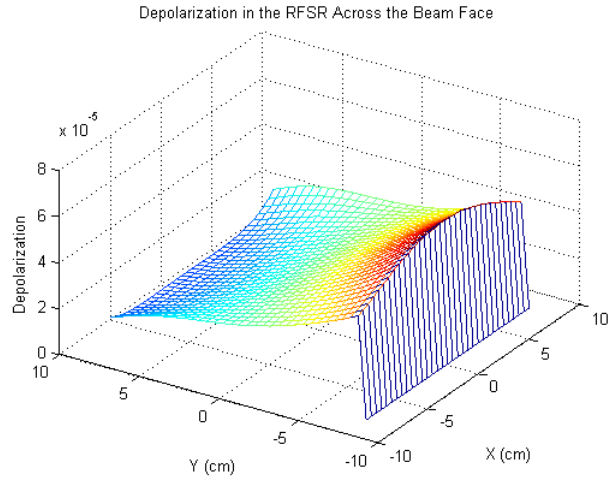


Figure A.4: Beam depolarization in the RFSR, across the  $XY$  plane.

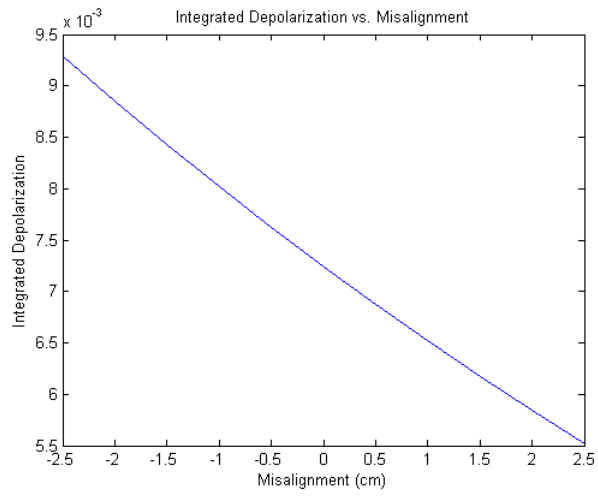


Figure A.5: Integrated depolarization of a 9 cm diameter beam as a function of misalignment between the beam axis and magnetic field axis, in the vertical direction.

# Vita

M. Jon Dadras was born and raised in Colorado. He recieved his B.A. in Physics and Mathematics from CU–Boulder in 2004. As an undergrad he did research in experimental condensed matter physics and theoretical gravity/astrophysics; he finds experimental nuclear/particle physics to be a happy medium.