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THE IMPACT OF 1960-65 CONDITIONS ON NONFARM INCOME, FARM EMPLOYMENT, AND MOBILITY, BY RACE AND SEX IN TENNESSEE

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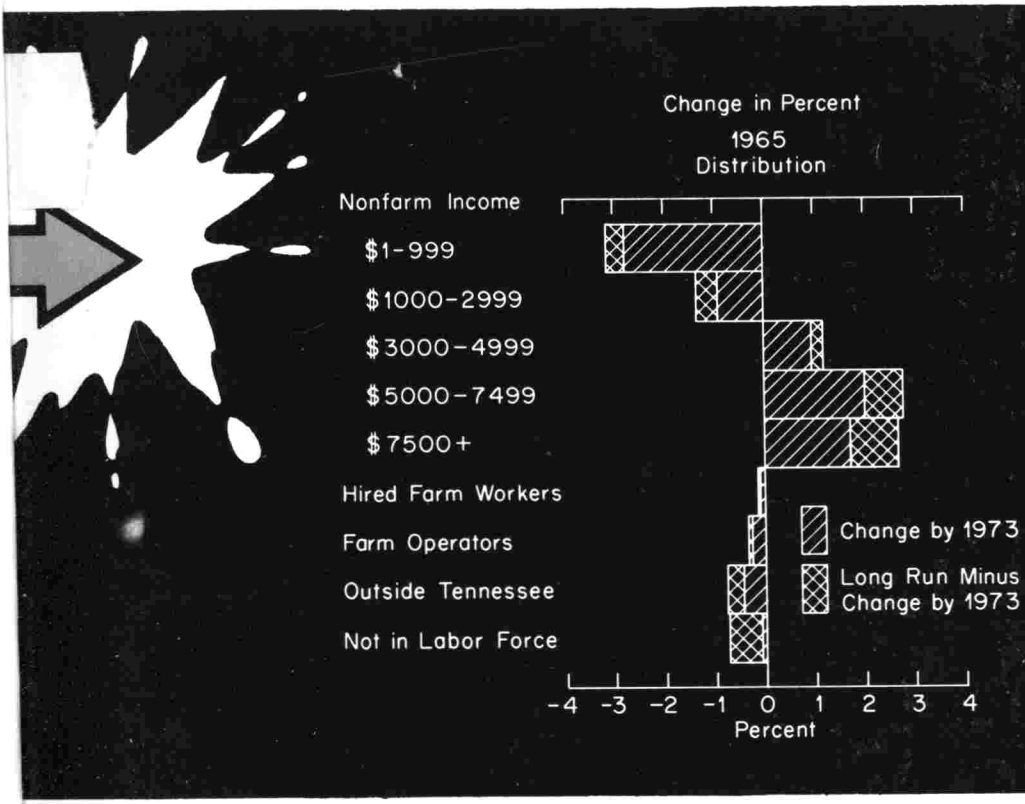

by L. L. Bauer and C. B. Sappington

The University of Tennessee
Agricultural Experiment Station
John A. Ewing, Dean
Knoxville

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SUMMARY

In this report, a mathematical technique, Markov analysis, was used to project percentage distributions of nonfarm incomes, and of farm operators and workers into the future. The data used were the Tennessee portion of the Social Security Administration's Continuous Work History Sample which is a 1% sample of all those covered under "Old Age Survivors Disability Insurance" during 1960-65.

In the analysis, a long-run distribution was estimated. This is what can be expected to exist after all the factors influencing the distribution of incomes and jobs during the 1960-65 period have had enough time to be completely felt, and the distribution stabilizes. This distribution was compared with what was estimated to have existed in 1965, and one projected to 1973 for all people in the sample, and for four race-sex classifications. From these comparisons, conclusions could be drawn about relative changes and rates of changes among the groups.

The basic conclusions were:

1) The real income of nonfarm workers will improve through time.

2) The real income of nonfarm workers by race-sex groups will remain best for white males followed in order by black males, white females, and black females.

3) The changes in the real income of nonfarm workers will be about the same for three of the race-sex groups. The exception is black females who will show relatively little improvement.

4) The rate of change in the distributions will be about the same for all race-sex groups.

5) The number of hired farm workers and farm operators will decrease rather substantially and rapidly.

6) More blacks will be unemployed, temporarily or permanently, in the future, although the increase in numbers will not be large.

7) There will be a slight net migration of nonfarm workers into Tennessee.

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THE IMPACT OF 1960-65 CONDITIONS ON NONFARM INCOME, FARM EMPLOYMENT, AND MOBILITY, BY RACE AND SEX IN TENNESSEE*

by
L. L. Bauer and C. B. Sappington**

OBJECTIVES

The primary objective of this study was to estimate future income distributions for nonfarm workers in Tennessee and to compare the estimated future distributions for the four race-sex classifications, white-male, black-male, white-female, and black-female. Specifically, the objective was to determine if the income distributions for the four race-sex groups can be expected to become more nearly equal or more divergent than during 1960-65, given that the factors affecting income during that period remain unchanged. Part of this objective was also to estimate the relative rates of change in the income distributions of the four race-sex categories.

The second objective was to estimate for all people and for the four race-sex classifications, the degree and rate of change of the numbers engaged in farming, either as farm operators or hired farm workers, given that the 1960-65 conditions influencing these numbers remained unchanged.

The third objective was to estimate for all people and for the four race-sex classifications, the degree and rate of change in the numbers of people who, having worked in Tennessee at least 1 year, will be working in another state during another year.

MARKOV ANALYSIS

Given the objective of projecting income distributions into the future, it was necessary to find an analytical technique that would accomplish this with some precision. Simplicity was also considered as being desirable. The mathematical technique of Markov analysis meets these criteria quite well.

A brief description of the concept, the necessary form of the data, and how our particular data were handled will be presented in this and the following sections. A more complete and rigorous mathematical description of Markov analysis is presented in the Appendix.

*The data used in this publication were processed by the Tennessee Valley Authority from data tapes furnished by the Social Security Administration. The authors and the TVA personnel involved did not at any time have access to nor did they receive any information relating to specific individuals or reporting units.

**Assistant Professor and Associate Professor of Agricultural Economics, respectively, The University of Tennessee, Knoxville.

The basic concept of this technique revolves around probabilities or likelihoods of moving from one class or category to another. Specifically, in this study, the interest was in the probability of changing from one income class to another or from one job classification to another.

What was needed for this analysis was a transition probability table of the form:

		1961	
		I_1	I_2
1960	I_1	p_{11}	p_{12}
	I_2	p_{21}	p_{22}

where I_1 and I_2 are two mutually exclusive income classes, and the p_{ij} elements, called transition probabilities, are the probability of an individual who was in income class i in 1960 being in income class j in 1961. The elements in each row of the table add to one. If we let n_1 be the number of people with income I_1 in 1960 and n_2 the number of people who had income I_2 in 1960, then $p_{11}n_1$ will be the number of people who had income I_1 in 1960 and remained in the same class in 1961. The number $p_{12}n_1$ will be those who move from income class I_1 to I_2 between 1960 and 1961.

From the above transition probability table, estimates can be made of the probabilities of an individual having a particular income at various points of time in the future. For our example, the "probability tree" in Figure 1 can be constructed. If an individual starts with income I_1 in 1960, the probability is $p_{11}\ p_{11} + p_{12}\ p_{21}$ that he will be in income class I_1 in 1962 and $p_{11}\ p_{12} + p_{12}\ p_{22}$ that he will be in income class I_2 in 1962. The same thing can be done for the lower half of the diagram and the four branches each ending in I_1 and I_2 can be summed to find the probability of being in each income class in 1962. For I_1 the probability is:

$$\underbrace{(p_{11}\ p_{11} + p_{12}\ p_{21})}_{\text{started in } I_1} + \underbrace{(p_{21}\ p_{11} + p_{22}\ p_{21})}_{\text{started in } I_2}$$

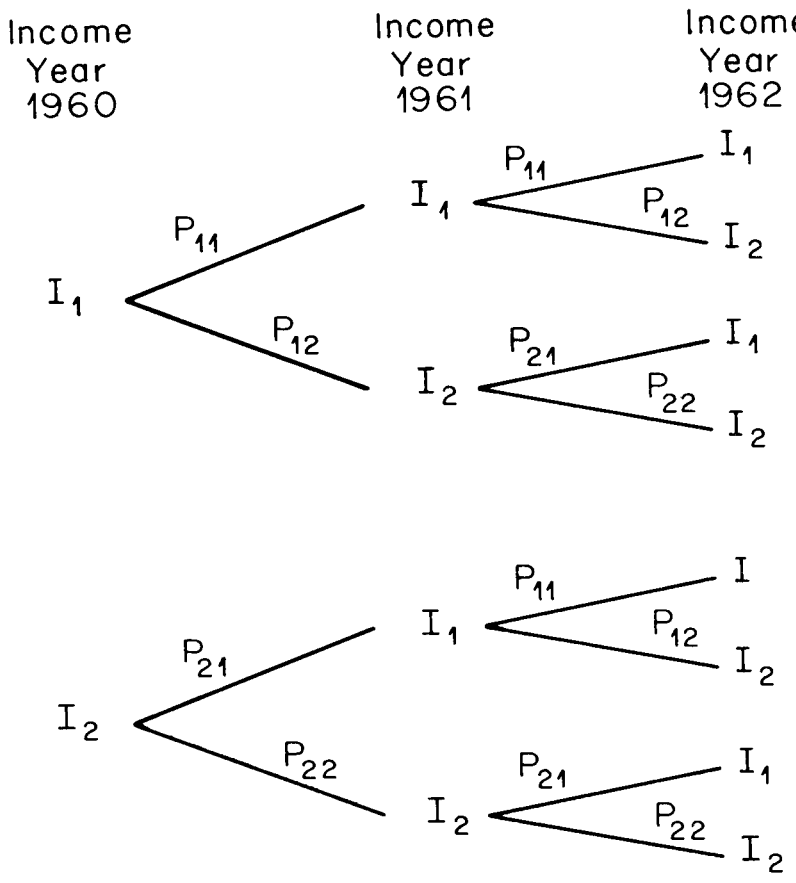


Figure 1. Probability tree for probability table example.

For I_2:

$$(p_{11}\ p_{12} + p_{12}\ p_{22}) + (p_{21}\ p_{12} + p_{22}\ p_{22})$$

started in I_1 started in I_2

The projected numbers of people in each income class in 1962 are:

In I_1:

$$n_1(p_{11}\ p_{11} + p_{12}\ p_{21}) + n_2(p_{21}\ p_{11} + p_{22}\ p_{21})$$

and in I_2:

$$n_1(p_{11}\ p_{12} + p_{12}\ p_{22}) + n_2(p_{21}\ p_{12} + p_{22}\ p_{22})$$

If the probabilities remain constant, the percentage of persons in the income class in any particular year can be easily computed. It is also a feature of the Markov process that with the passage of each year the income distributions approach closer and closer to a long-run distribution. This distribution can also be calculated quite easily and represents the distribution toward which incomes are moving.

DEFINITIONS

In order to estimate the transition probabilities, it was necessary to define the classes or categories into which the sample could be divided. The definitions used are:

Income: Income as reported is for individual wages or earnings; that is, the figures do not represent family income nor do they include investment income. All incomes are in 1959 dollars.

Classification of Employment: Employed persons were first classified into three categories: nonfarm workers, hired farm workers, and farm operators. Persons were assigned to an employment category according to the major source of income or wages.

Two additional categories were necessary to insure that each person in the sample would be included in the analysis each year. These were "not in labor force" and "outside Tennessee" and are defined in the following paragraphs.

Not in Labor Force: This category includes those people who had no reported income during a particular year, but who did have income in Tennessee sometime between 1960 and 1965.

Outside Tennessee: This category includes those people who have been employed in Tennessee but then move to a job outside the

state. For example, if a person had a job in Tennessee in 1960 and moved to a job in Alabama in 1961, he would be counted as "Outside Tennessee" in 1961.

If a person worked in a Tennessee nonfarm job, he was put into one of five income classes. Those who were not so classified were put into one of the other four categories; farm operator, hired farm worker, not in labor force, or outside Tennessee. The distributions of people among these nine categories are called **income-job distributions.**

Race: The data classify persons as "negro" and "non-negro". This report uses "black" and "white," respectively, since only about 0.1% of the Tennessee population is neither white nor negro.

DATA[1]

The data used are from the U. S. Social Security Administration's 1% sample of "Old Age and Survivors Disability Insurance" (OASDI) which is used by the Administration in its Continuous Work History Sample. The sample is continuous in that the same individuals are in the sample every year. The Tennessee Valley Authority has access to this OASDI data and the subsample actually used in this study was supplied by them. To have been included in the subsample, a person must have earned the major source of his wages or income for at least 1 year from 1960 through 1965 from a business or farm located in Tennessee. Only those persons reporting income to the Social Security Administration were included.

The data covered the years 1960 through 1965. In each of the 6 years, there were 24,512 persons distributed among the nine income-job classifications. The data summaries supplied by TVA indicated, for example, the number of nonfarm workers in the lowest income class in 1960, and for 1961, how many of these remained in this same income class, how many moved to a higher class, and how many moved to each of the four other job classifications. Similar data were supplied for each of the nine classifications for the 1960-61 transition period. The data also indicated the breakdown of these numbers by race and sex. Similar information was supplied for each pair of subsequent years.

ESTIMATION OF THE TRANSITIONAL PROBABILITIES

The estimate of the transitional probability of people moving from class i to class j was estimated by:

[1]The data used in this study is the same as was used in: Sappington, C. B. and Larry L. Bauer, **Income and Mobility of Tennessee Farm and Nonfarm Workers, 1960-1965,** Tennessee Agricultural Experiment Station Bulletin 471. The emphasis in that study was analysis of changes in income-job distributions during the 1960-1965 period, while the emphasis in this study is the projection of income-job distributions into the future.

$$p_{ij} = \frac{\sum_{t=1}^{5} A_{ijt}}{\sum_{j=1}^{9} \sum_{t=1}^{5} A_{ijt}}$$

i = income or job classification in 1 year

j = income or job classification in the next year

t = transition periods

A_{ijt} = number of persons moving from class i to j in transition period t.

For example, if we want p_{12}, the probability of moving from class 1 to class 2, we have

$$p_{12} = \frac{\sum_{t=1}^{5} A_{12t}}{\sum_{j=1}^{9} \sum_{t=1}^{5} A_{1jt}}$$

or in words, the total number of people moving from class 1 to 2 over the five periods covered divided by the total number of people who were ever in class 1 during the five periods covered.

A requirement of Markov analysis is many observations in order to estimate reliable probability tables. In the data used here, there were 24,512 people in the total sample, so the estimated probabilities can be looked upon with considerable confidence. These 24,512 persons were in one job classification or another in each of the years involved. The probability estimates are made by following the movements through time of these people from one of the nine income-job classifications to another.

As an illustration of how the transition probabilities were estimated, Table 1 shows the total class-to-class movements for all people for all five transition periods. Each of the 24,512 individuals in the sample is counted five times, that is, once each transition period. The two numbers used to estimate each p_{ij} can be found in this table. For example, the table indicates that there was a total of 6,907 persons during the five transition periods who were observed to have been in the income class $1-999 one year and still in the same class the next.

Table 1. Total class-to-class movements of all people by income-job classification, Tennessee, 1960-65*

From	To: Nonfarm income $1-999	$1,000-2,999	$3,000-4,999	$5,000-7,499	$7,500+	Hired farm workers	Farm operators	Outside Tennessee	Not in labor force	Total
Nonfarm Income										
$1-999	6,907	3,395	235	31	4	92	101	1,581	3,878	16,224
1,000-2,999	2,137	13,256	2,837	193	23	44	61	1,367	996	20,914
3,000-4,999	240	1,394	9,020	1,626	129	7	21	684	181	13,302
5,000-7,499	30	133	830	5,298	792	0	10	300	48	7,441
7,500+	13	27	123	355	2,930	0	11	214	45	3,718
Hired farm workers	82	83	15	2	0	691	13	117	140	1,143
Farm operators	82	75	18	10	8	12	1,563	44	299	2,111
Outside Tennessee	1,167	1,405	777	377	210	47	32	21,252	1,664	26,931
Not in labor force	5,745	1,422	145	46	63	259	244	2,479	20,373	30,776

*Each of the 24,512 persons in the sample is counted five times in these figures.

If a person remained in this class for all 6 years of the data, he accounted for 5 of the 6,907 observed. The number in the next column of the first row indicates that a total of 3,395 persons were observed to have moved from the $1-999 income during 1 year to the $1,000-2,999 class the next year. The numbers in the first nine columns of Table 1 are then the numerators of their respective estimated transitional probabilities, and the number in the last, or total column of the table is the denominator for the transitional probabilities in each respective row. For example:

$$p_{11} = \frac{6,907}{16,224} = .426$$

$$p_{12} = \frac{3,395}{16,224} = .209$$

These two numbers appear in the first row of Table 2, which is the estimated income-job classification transition probability table for all people in the sample. The first row of that table indicates that for every 1,000 nonfarm workers earning $1-999 in any year, an estimated 426 of these same workers will earn between $1 and $999 next year; 209 are estimated to move into the income class of $1,000-2,999; 14 to the $3,000-4,999 income class; and so forth across the row to the last figure which indicates that 239 of these same 1,000 people are estimated to report no income in the next year. This last figure appears to be quite large if we were speaking about those who need to work for a living. It should be mentioned, therefore, that this class would include, for example, a high school student who works a few hours a month in the library 1 year but does not work the following year. Because of this large propensity not to work next year, this lowest income class is thought to include many "inners and outers" in the labor force.

A statistical test was performed on Table 2, the result of which was that it is not 95% certain that the transition probabilities differ from year to year. The analysis was, therefore, continued on the assumption that the probabilities were stable. Details of this test are given in the Appendix.

The transition probability tables for the four race-sex groups are given in Appendix Tables 1-4.

ANALYSIS

For each of the groups, all people and the four race-sex groups, two future income-job distributions were estimated, one for 1973 and a long-run distribution. The long-run distributions are estimates of the equilibrium toward which each respective group is moving, given no changes in the factors affecting income between 1960-65 and the time

Table 2. Income-job classification transition probability table, all people, Tennessee, 1960-65

	Nonfarm income					Hired farm workers	Farm operators	Outside Tennessee	Not in labor force
	$1-999	$1,000-2,999	$3,000-4,999	$5,000-7,499	$7,500 +				
Nonfarm income									
$1-999	.426	.209	.014	.002	.000	.006	.006	.098	.239
1,000-2,999	.102	.634	.136	.009	.001	.002	.003	.065	.048
3,000-4,999	.018	.105	.678	.122	.010	.001	.002	.051	.013
5,000-7,499	.004	.018	.112	.712	.107	.000	.001	.040	.006
7,500+	.003	.007	.033	.096	.788	.000	.003	.058	.012
Hired farm workers	.072	.073	.013	.002	.000	.605	.011	.102	.122
Farm operators	.039	.035	.009	.005	.004	.006	.740	.021	.141
Outside Tennessee	.043	.052	.029	.014	.008	.002	.001	.789	.062
Not in labor force	.187	.046	.005	.001	.002	.008	.008	.081	.662

when the long-run is reached. By comparing these two estimated distributions with what was estimated to have existed in 1965, we can get some indications of the relative income-job changes among groups and how rapidly these changes are occurring.

RESULTS

All Tennessee Workers

The estimated 1965, 1973, and long-run income-job distributions for all Tennessee workers are shown in Table 3. The original data indicated that 13.82%, or 138 of every 1,000 Tennessee nonfarm workers in our sample, earned less than $1,000. If the transition probabilities remain stable, the estimate is that the comparable figure for 1973 will be 11.03%, or 110 out of every 1,000 in our sample. The comparable long-run estimate is 10.64%, or 106 out of every 1,000. It can be concluded from these figures that most of the long-run adjustment will have occurred by 1973. The long-run projections indicate that the number of people in this income class will eventually drop by 3.18%, or 32 per 1,000. The estimated distribution for 1973 indicates that by then the number in this class will have decreased by 2.79%, or 28 per 1,000. This decrease of 28 per 1,000 represents 88% of the total decrease of 32.

Taken as a whole, the estimated changes in the income distribution for nonfarm workers look favorable: fewer are in the income classes below $3,000 and more in the classes above $3,000.

Table 3 indicates that there will be a decrease in the number of hired farm workers and that this adjustment will be essentially complete by 1973. Even though the absolute number of hired farm workers is small, the estimated 1973 distribution indicates that there will be slightly less than 8 hired farm workers per 1,000 population compared to almost 9 per 1,000 in 1965. In other words, 1 out of every 9 hired farm jobs that existed in 1965 will have disappeared by 1973. After 1973, the changes will be of substantially lesser magnitude.

The situation for farm operators is similar to that for hired farm workers. The 1965 distribution indicates that 16 of every 1,000 in the sample were farm operators. By 1973, the number is estimated to be about 13 per 1,000. The long-run estimate differs little from that for 1973.

The last two classes in the distributions, as mentioned previously, were added to conform to the requirements of Markov analysis and their interpretation is less clear than for the other income-job classes. The outside Tennessee figures for 1965, 1973, and for the long-run indicate a small net migration of workers into Tennessee. The "not in labor force" class shows very little change, although the figures do

Table 3. Estimated 1965, 1973, and long-run income-job distribution for all workers, Tennessee

	Estimated 1965	Estimated 1973	Estimated long-run
	(%)	(%)	(%)
Nonfarm income			
$1-999	13.82	11.03	10.64
1,000-2,999	17.77	16.83	16.41
3,000-4,999	12.93	13.85	14.08
5,000-7,499	7.36	9.35	10.10
7,500+	4.26	5.95	6.91
Nonfarm workers (total)	(56.14)	(57.01)	(58.14)
Hired farm workers	.88	.78	.74
Farm operators	1.61	1.34	1.28
Outside Tennessee	24.62	24.20	23.84
Not in labor force	16.75	16.67	16.00

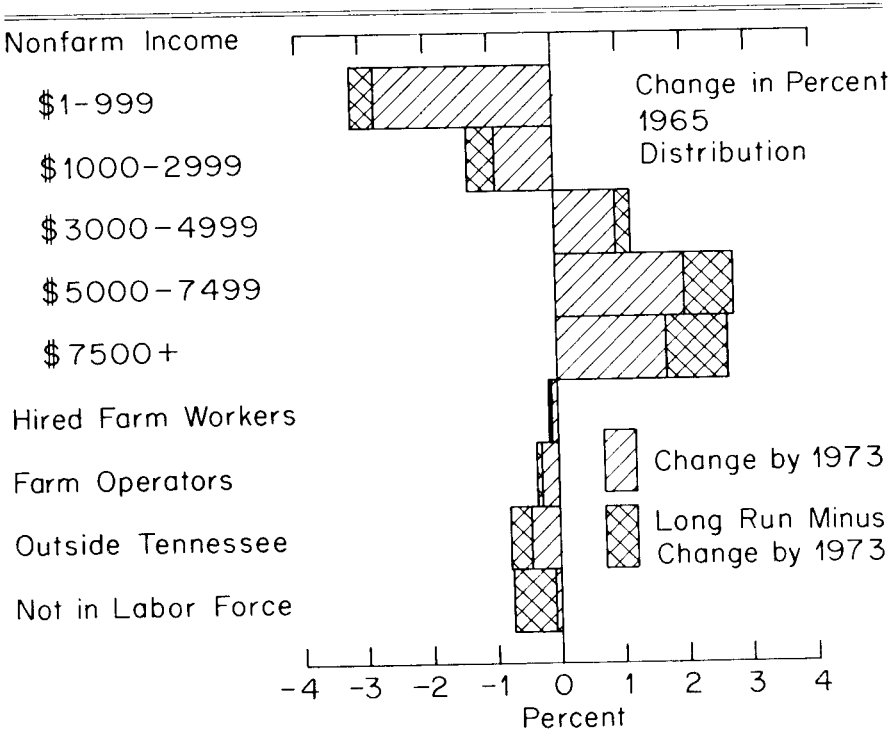


Figure 2. Estimated change in percentage distribution from 1965, by income-job classification, all workers, Tennessee, 1973 and long run.

indicate that a higher percentage of people will be working in the future than in 1965.

Figure 2 shows the relative rate at which movement from the 1965 to the long-run distribution is taking place. The 1973 and the long-run percentage changes from 1965 are graphed. The top bar, for example, shows the projected long-run decrease of 3.18% in the numbers of nonfarm workers in the lowest income class and the projected 2.79% projected decrease by 1973. The speed of the adjustment is indicated by the nearness of the 1973 and the long-run projections. Figure 2 indicates that the speed of adjustment is considerably slower in the nonfarm income classes over $5,000. The adjustments by hired farm workers and farm operators seem quite rapid, while the last two classes show a relative slow rate of change.

Race-Sex Groups

The estimated income-job distributions for 1965, 1973, and the long-run for each of the four race-sex groups are shown in Appendix Tables 5-8. The indication there is that the predicted changes differ very little among the four groups; that is, each of the race-sex groups show the same kind of changes that were indicated when the entire sample was analyzed; a more favorable income distribution, fewer hired farm workers and farm operators, and a small decrease in the number working outside Tennessee. The only difference of any significance is that the category of "not in labor force" indicates a decrease in the future for whites, and an increase for blacks.

The graphs showing the rate of change for the race-sex groups are Appendix Figures 1-4. There are only small differences in the four, indicating that the rates of change in the income-job distributions for the race-sex groups are about the same.

Figures 3-5 summarize the information found in Appendix Tables 5-8. Figure 3 compares the 1965 income-job distribution for the four race-sex groups, Figure 4 the 1973 distribution, and Figure 5 the long-run distribution. In each case, it is evident that the nonfarm income distribution for white males is best, in that a higher percentage of the individuals have high incomes. It is also evident that the income distribution for black female nonfarm workers is the worst of the four groups.

The figures also show that relatively speaking, most farm operators are white males and most hired farm workers are black males. The "outside Tennessee" class indicates that males are more mobile than females while the "not in labor force" class indicates that females tend to be "inners and outers" more than males, as might be expected.

A comparison of Figures 3-5 indicates only very slight differences among the race-sex groups. The estimated income-job distributions

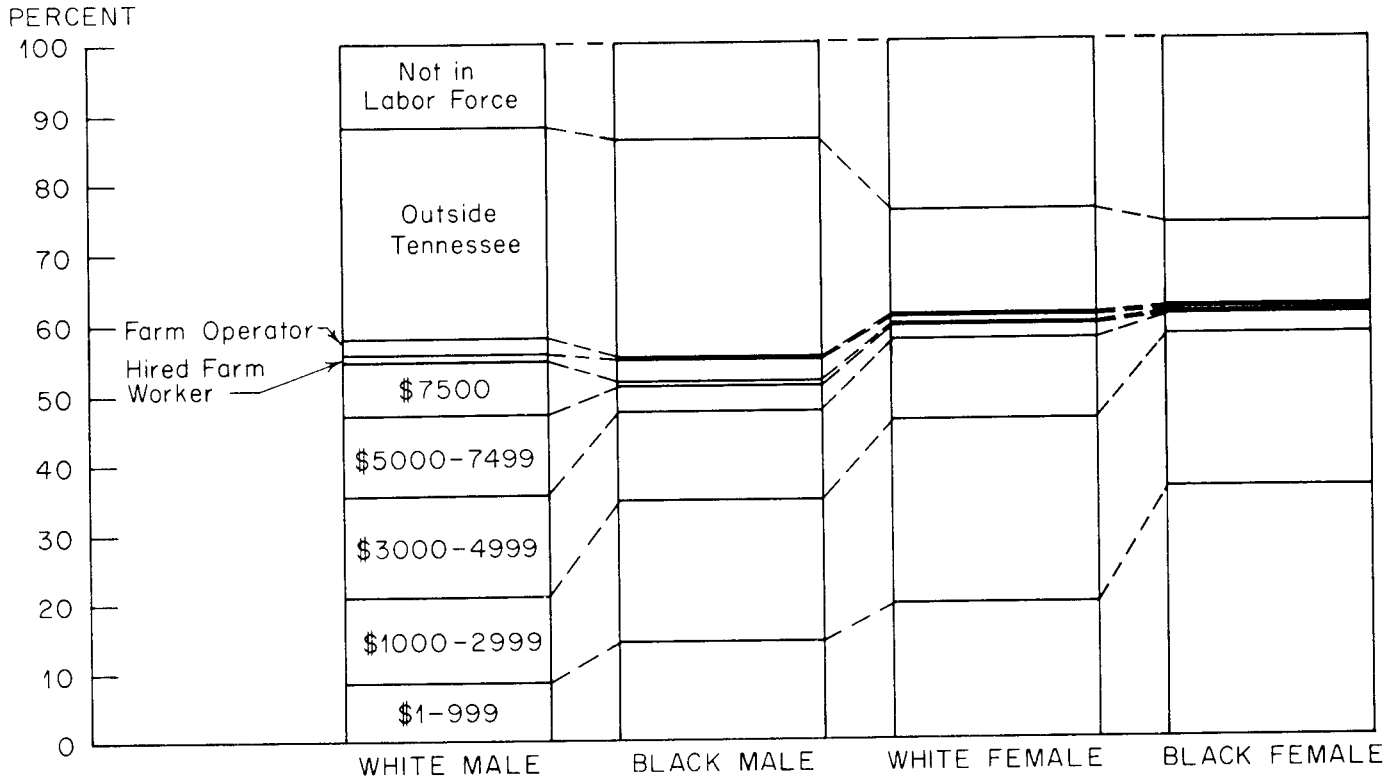


Figure 3. Estimated income-job distribution by race-sex groups, Tennessee, 1965.

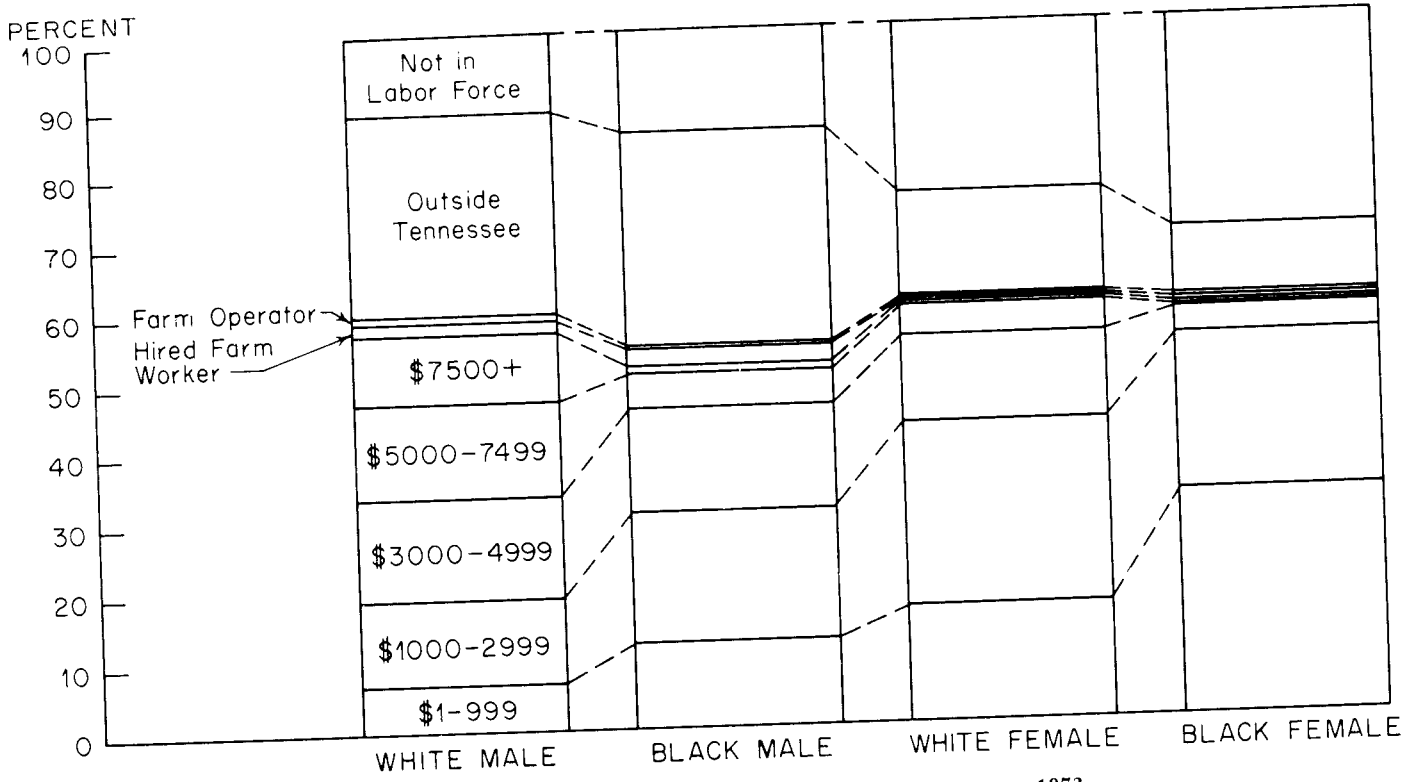


Figure 4. Estimated income-job distribution, by race-sex category, Tennessee, 1973.

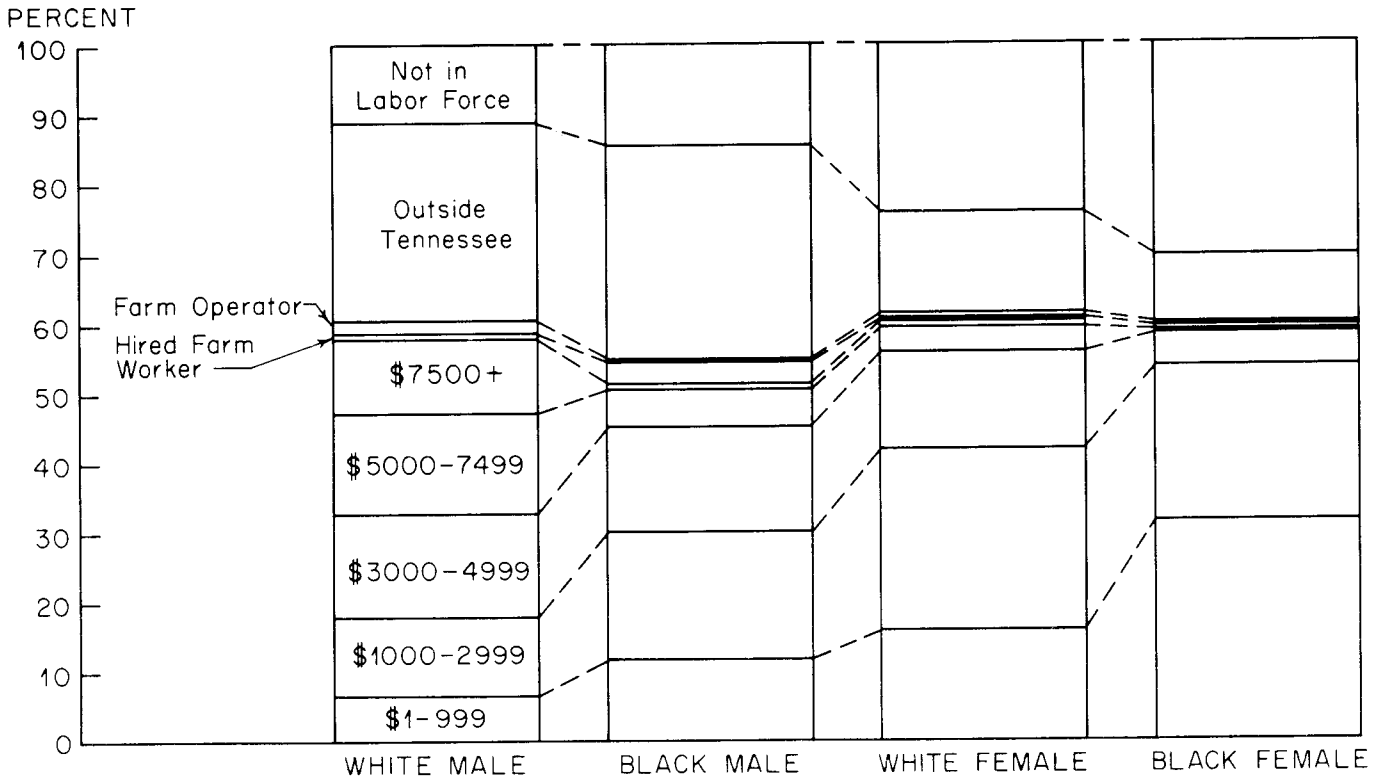


Figure 5. Estimated income-job distribution by race-sex category, Tennessee, long run.

indicate that changes are about the same and at the same rate for all four groups.

The median income for each group for each estimated distribution was estimated. They are presented in Table 4. These give another indication of the magnitude and relative speed of the changes in nonfarm income. The magnitudes of the changes in the estimated median incomes from 1965 to the long-run are $596 for white males, $422 for black males, $304 for white females, and $71 for black females. On a percentage basis, the incomes of black males, white males, and white females can be expected to increase by about the same percent, while the percentage increase for black females is much lower. The rate of change seems to be about equal for all four groups.

CONCLUSIONS

This study was done under the rather severe assumption that the estimated transitional probabilities would remain stable indefinitely. The fact that there is statistical evidence of stability from 1960 to 1965 does not imply stability beyond 1965. However, in the absence of some startling economic development, stability until 1973 seems to be reasonable. Stability forever does not seem reasonable. For these reasons, the estimated long-run distributions should not be interpreted as actual predictions, but rather as estimates which are consistent for all race-sex groups and, thus, standards of measurement. The estimated 1973 distributions come much closer to being an actual prediction.

With these reservations in mind, several conclusions present themselves:

1) The real income of nonfarm workers will improve through time.

2) The real income of nonfarm workers by sex-race groups will remain best for white males followed in order by black males, white females, and black females.

3) The changes in the real income of nonfarm workers will be about the same for three of the race-sex groups. The exception is black females who will show relatively little improvement.

4) The rate of change in the distributions will be about the same for all race-sex groups.

5) The number of hired farm workers and farm operators will decrease rather substantially and rapidly.

6) More blacks will be unemployed, temporarily or permanently, in the future, although the increase in numbers will not be large.

7) There will be a slight net migration of nonfarm workers into Tennessee.

Table 4. Estimated 1965, 1973, and long-run median incomes for nonfarm workers, by race-sex groups, Tennessee

Race-sex group	Median income		
	1965 estimate	1973 estimate	Long-run estimate
White male	$3,882	$4,338	$4,478
Black male	2,148	2,464	2,570
White female	1,799	2,051	2,103
Black female	860	911	931

APPENDIX

MATHEMATICAL STATEMENT OF MARKOV ANALYSIS

Data Required

Markov chain analysis requires detailed data on a large number of micro units— people, farms, or firms — over time. The data must be such that each micro unit can be located in some category such as income class, size of farm, or county of residence at any one time. These categories are called states and they must be mutually exclusive and exhaustive; that is, each micro unit must be in one and only one state at any one time. The data must indicate the number of these micro units moving from one state to another from one time period to the next.

In this study the micro units are people, the states are income-job classes, and the time unit of observation is 1 year.

The Algebra of the Markov Chain Process

In any 1 year there are a certain number of people in each income class. This number represents a certain percentage of the population under discussion.

Let this percentage in class i for year 1 be noted w_{il}. These people in w_{il} have a probability of moving into class j in year 2 given by p_{ij} and a probability of not moving of p_{ii}. That is:

$$w_{11}p_{11} + w_{21}p_{21} + w_{31}p_{31} + \ldots + w_{n1}p_{n1} = w_{12}$$

$$w_{11}p_{12} + w_{21}p_{22} + w_{31}p_{32} + \ldots + w_{n1}p_{n2} = w_{22}$$

$$\vdots$$

$$w_{11}p_{1n} + w_{21}p_{2n} + w_{31}p_{3n} + \ldots + w_{n1}p_{nn} = w_{n2}$$

or in matrix notation:

$$W_1 P = W_2$$

where:

$$W_1 = [w_{11}\ w_{21}\ w_{31}\ \ldots\ w_{n1}]$$

$$W_2 = [w_{12}\ w_{22}\ w_{32}\ \ldots\ w_{n2}]$$

and $P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{1n} \\ p_{31} & p_{32} & p_{33} & \cdots & p_{1n} \\ \vdots & & & & \vdots \\ p_{n1} & & & & p_{nn} \end{bmatrix}$

Similarly,

$$W_2 P = W_3$$

which can be written as:

$$(W_1 P)\, P = W_3$$

$$W_1 P^2 = W_3$$

Therefore in this study the 1973 distributions were estimated by:

$$W_{1965}\, P^8 = W_{1973}$$

To find the estimate of the long-run distributions, a distribution of the population is sought such that with the passage of 1 more year, there is no change in distribution by income class; that is:

$$W_t P = W_t$$

or

$$\begin{aligned}
&w_{1t}p_{11} + w_{2t}p_{21} + w_{3t}p_{31} + \cdots + w_{nt}p_{n1} = w_{1t} \\
&w_{1t}p_{12} + w_{2t}p_{22} + w_{3t}p_{32} + \cdots + w_{nt}p_{n2} = w_{2t} \\
&\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
&w_{1t}p_{1n} + w_{2t}p_{2n} + w_{3t}p_{3n} + \cdots + w_{nt}p_{nn} = w_{nt}
\end{aligned}$$

This series of equations is to be solved for the unknown w's. There are n equations with n unknowns; one of the equations is, however, redundant. If all of the equations above excepting one were known, then the excepted one would also be known. For example:

$$p_{11} = 1 - (p_{12} + p_{13} + p_{14} + \ldots + p_{1n})$$
$$p_{21} = 1 - (p_{22} + p_{23} + p_{24} + \ldots + p_{2n})$$
$$\vdots$$
$$p_{n1} = 1 - (p_{n2} + p_{n3} + p_{n4} + \ldots + p_{nn})$$

Therefore, solution of $W_t P = W_t$ is not possible directly since there are n unknowns but only n-1 linearly independent equations. Another linearly independent equation is needed. Since the elements of W_t are percentages and since the states are exhaustive, the sum of the elements must be 100% or 1. That is,

$$w_{1t} + w_{2t} + w_{3t} + \ldots + w_{nt} = 1$$

This equation is substituted for any one of the equations of $W_t P = W_t$ yielding n linearly independent equations in n unknowns.

If this new equation is substituted for the top equation and like terms are gathered together, the set of simultaneous equations is:

$$w_{1t} + w_{2t} + w_{3t} + \ldots + w_{nt} = 1$$
$$w_{1t}p_{12} + w_{2t}(p_{22}-1) + w_{3t}p_{32} + \ldots + w_{nt}p_{nt} = 0$$
$$\vdots$$
$$w_{1t}p_{1n} + w_{2t}p_{2n} + w_{3t}p_{3n} + \ldots + w_{nt}(p_{nn}-1) = 0$$

These equations can be solved for the unknown w's in the usual manner.

These equations can also be presented for solution in matrix form as:

$$[w_{1t}\ w_{2t}\ w_{3t}\ \ldots\ w_{nt}] \begin{bmatrix} 1 & p_{12} & p_{13} & \ldots & p_{1n} \\ 1 & (p_{22}-1) & p_{23} & \ldots & p_{2n} \\ 1 & p_{32} & (p_{33}-1) & \ldots & p_{3n} \\ \vdots & & & & \\ 1 & p_{n2} & p_{n3} & \ldots & (p_{nn}-1) \end{bmatrix} =$$

$$= [1\ \ 0\ \ 0\ \ldots\ 0]$$

Let $\overset{o}{P}$ represent the above n by n matrix. Finding $[\overset{o}{P}]^{-1}$ and post multiplying both sides by this inverse yields

$$W_t = [1 \quad 0 \quad 0 \ldots 0] \quad [\overset{o}{P}]^{-1}$$

or W_t is the top row of the inverted matrix $\overset{o}{P}$.

Procedure

From the preceding, it is clear that if the matrix P is known,[1] solution is relatively simple. Estimates of the elements of P are made from the data. Each p_{ij} is estimated by adding the people who, for the whole time period of the data, moved from i to j divided by the number of people who were ever in state i.

Let A_{ijt} represent the number of people moving from i to j in one time period, then

$$p_{ij} = \frac{\sum_{t=1}^{5} A_{ijt}}{\sum_{j=1}^{n} \sum_{t=1}^{5} A_{ijt}}$$

Each p_{ij} has a range of 0 to 1, that is, $0 \leq p_{ij} \leq 1$ and the sum of any row of $\overset{o}{P}$ must be equal to 1.

The estimate of the P matrix is altered to $\overset{o}{P}$ and inverted by a computer program. Care must be taken to read the correct row of the inverted matrix which is determined by the arbitrary selection of the one equation removed during the alteration process.

STATISTICAL TEST OF TRANSITION PROBABILITY TABLE

To test the null hypothesis that the true transition probabilities are stationary, the following test statistic[2] was calculated:

$$-2 \log \lambda = 2 \left(\sum_{i=1}^{9} \sum_{j=1}^{9} \sum_{t=1}^{5} A_{ijt} p_{ijt} - \sum_{i=1}^{9} \sum_{j=1}^{9} \sum_{t=1}^{5} A_{ijt} p_{ij} \right)$$

[1] This matrix must have certain other characteristics which are not dwelt on here.

[2] T. W. Anderson and L. A. Goodman, "Statistical Inference in Markov Chains," **Ann. Math. Stat.** 28:89-110, 1957, p. 101.

where:

p_{ij} is as defined in the text of this report, and

$$p_{ijt} = \frac{A_{ijt}}{\sum_{j=1}^{9} A_{ijt}}$$

In words, p_{ijt} is an estimate of a transition probability between two consecutive years and the p_{ij} is, in a sense, the average transition probability over all years included in the data. The statistic $-2 \log \lambda$ is distributed as a Chi-square with (T-1) (N-1)N degrees of freedom where T is the number of transition periods, five in our case, and N is the number of classes in the transitional probability table, nine in our case. The degrees of freedom in this case is therefore 288. For degrees of freedom this large, the critical value can be approximated by the following formula:[3]

$$X^2 = v\left(1 - \frac{2}{9v} + Z_{\alpha}\sqrt{\frac{2}{9v}}\right)^3$$

where:

α = percentile, 95% in our case
v = degrees of freedom, 288 in our case
Z = normal deviate, 1.65 in our case

The estimated critical value was about 540 and the calculated test statistic was about 520. The hypothesis of stationary transition probabilities therefore could not be rejected with $P \geq .95$.

[3]W. J. Dixon and F. J. Massey, Jr., **Introduction to Statistical Analysis,** Second Edition, McGraw-Hill, New York, 1957, p. 385.

Appendix Table 1. Income-job classification transition probability table for white male workers, Tennessee, 1960-65

	Nonfarm income					Hired farm workers	Farm operators	Outside Tennessee	Not in labor force
	$1-999	$1,000-2,999	$3,000-4,999	$5,000-7,499	$7,500+				
Nonfarm income									
$1-999	.360	.231	.027	.005	.001	.010	.016	.144	.206
1,000-2,999	.086	.544	.194	.020	.003	.004	.006	.094	.049
3,000-4,999	.016	.094	.641	.155	.014	.001	.002	.061	.016
5,000-7,499	.004	.017	.104	.712	.113	.000	.002	.042	.006
7,500+	.003	.007	.033	.094	.789	.000	.003	.059	.012
Hired farm workers	.077	.082	.020	.003	.000	.568	.019	.119	.112
Farm operators	.041	.036	.009	.005	.004	.006	.743	.021	.135
Outside Tennessee	.033	.048	.034	.019	.011	.002	.002	.807	.044
Not in labor force	.161	.044	.008	.003	.005	.012	.016	.107	.644

Appendix Table 2. Income-job classification transition probability table for black male workers, Tennessee, 1960-65

	Nonfarm income					Hired farm workers	Farm operators	Outside Tennessee	Not in labor force
	$1-999	$1,000-2,999	$3,000-4,999	$5,000-7,499	$7,500 +				
Nonfarm income									
$1-999	.436	.204	.008	.000	.001	.014	.003	.121	.213
1,000-2,999	.108	.672	.124	.003	.000	.006	.000	.058	.029
3,000-4,999	.019	.112	.735	.087	.001	.000	.001	.039	.006
5,000-7,499	.008	.013	.199	.693	.058	.000	.000	.025	.004
7,500+	.000	.000	.000	.200	.734	.000	.000	.033	.033
Hired farm workers	.060	.072	.000	.000	.000	.675	.000	.087	.106
Farm operators	.016	.066	.000	.000	.000	.016	.738	.016	.148
Outside Tennessee	.048	.041	.014	.003	.001	.003	.000	.837	.053
Not in labor force	.178	.034	.003	.000	.000	.026	.003	.100	.656

Appendix Table 3. Income-job classification transition probability table for white female workers, Tennessee, 1960-65

	Nonfarm income					Hired farm workers	Farm operators	Outside Tennessee	Not in labor force
	$1-999	$1,000-2,999	$3,000-4,999	$5,000-7,499	$7,500+				
Nonfarm income									
$1-999	.424	.217	.009	.000	.000	.001	.001	.070	.278
1,000-2,999	.108	.699	.094	.001	. 000	.000	.001	.044	.053
3,000-4,999	.023	.129	.745	.058	.002	000	.000	.033	.010
5,000-7,499	.006	.025	.159	.723	.053	.000	.000	.025	.009
7,500+	.020	.010	.029	.127	.774	.000	.000	.020	.020
Hired farm workers	.054	.022	.011	.000	.000	.619	.000	.033	.261
Farm operators	.022	.022	.007	.007	.000	.000	.712	.014	.216
Outside Tennessee	.073	.077	.021	.002	.001	.000	.000	.709	.117
Not in labor force	.204	.055	.003	.000	.000	.002	.002	.056	.678

Appendix Table 4. Income-job classification transition probability table for black female workers, Tennessee, 1960-65

	Nonfarm income				Hired farm workers	Farm operators	Outside Tennessee	Not in labor force
	$1-999	$1,000-2,999	$3,000-4,999	$5,000+**				
Nonfarm income								
$1-999	.614	.124	.001	.000	.000	.000	.037	.224
1,000-2,999	.173	.746	.031	.000	.000	.000	.022	.028
3,000-4,999	.019	.126	.804	.025	.000	.000	.013	.013
5,000+	.000	.067	.200	.733	.000	.000	.000	.000
Hired farm workers	.125	.031	.000	.000	.625	.000	.094	.125
Farm operators	.000	.000	.000	.000	.000	.700	.000	.300
Outside Tennessee	.094	.027	.008	.002	.002	.000	.667	.200
Not in labor force	.239	.027	.000	.000	.005	.001	.052	.676

**The two higher income classes had to be combined for Markov analysis.

Appendix Table 5. Estimated 1965, 1973, and long-run income-job distributions for white male workers, Tennessee

	Estimated 1965	Estimated 1973	Estimated long-run
	(%)	(%)	(%)
Nonfarm income			
$1-999	8.57	6.75	6.52
1,000-2,999	12.12	11.68	11.41
3,000-4,999	14.70	14.76	14.83
5,000-7,499	11.57	13.76	14.39
7,500+	7.39	9.67	10.66
Nonfarm workers (total)	(54.35)	(56.62)	(57.81)
Hired farm workers	.91	.79	.76
Farm operators	2.68	2.12	1.99
Outside Tennessee	30.23	28.90	28.33
Not in labor force	11.83	11.57	11.11

Appendix Table 6. Estimated 1965, 1973, and long-run income-job distributions for black male workers, Tennessee

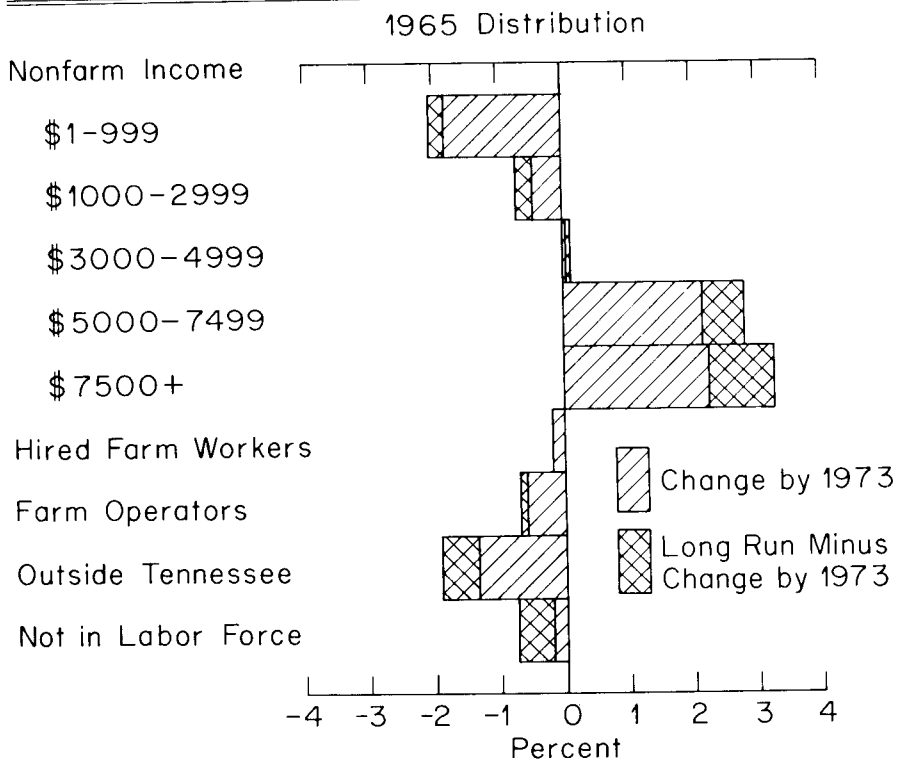
	Estimated 1965	Estimated 1973	Estimated long-run
	(%)	(%)	(%)
Nonfarm income			
$1-999	14.11	11.95	11.62
1,000-2,999	20.15	18.84	18.48
3,000-4,999	12.78	14.53	15.04
5,000-7,499	3.68	5.09	5.68
7,500+	.67	1.19	1.45
Nonfarm workers (total)	(51.39)	(51.60)	(52.27)
Hired farm workers	3.20	2.42	2.32
Farm operators	.40	.37	.36
Outside Tennessee	31.29	30.89	30.29
Not in labor force	13.72	14.72	14.76

Appendix Table 7. Estimated 1965, 1973, and long-run income-job distributions for white female workers, Tennessee

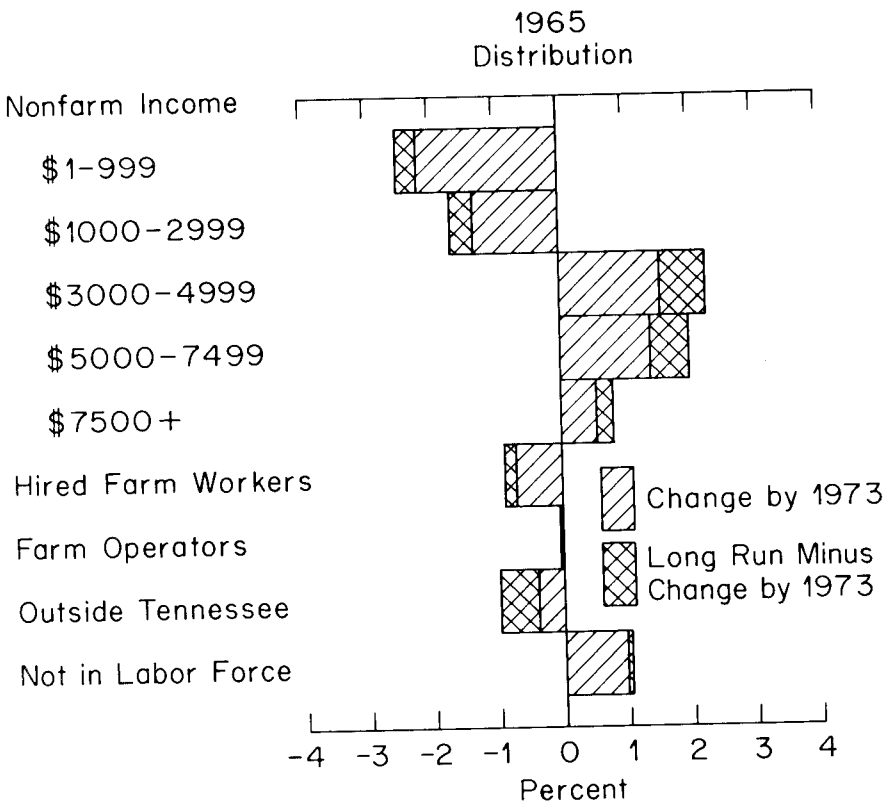
	Estimated 1965	Estimated 1973	Estimated long-run
	(%)	(%)	(%)
Nonfarm income			
$1-999	19.42	16.28	15.99
1,000-2,999	26.35	26.22	26.06
3,000-4,999	11.51	13.53	14.04
5,000-7,499	2.16	3.28	3.62
7,500+	.48	.81	1.04
Nonfarm workers (total)	(59.92)	(60.12)	(60.75)
Hired farm workers	.20	.17	.17
Farm operators	.32	.32	.31
Outside Tennessee	14.79	14.62	14.47
Not in labor force	24.77	24.77	24.30

Appendix Table 8. Estimated 1965, 1973, and long-run income-job distributions for black female workers, Tennessee

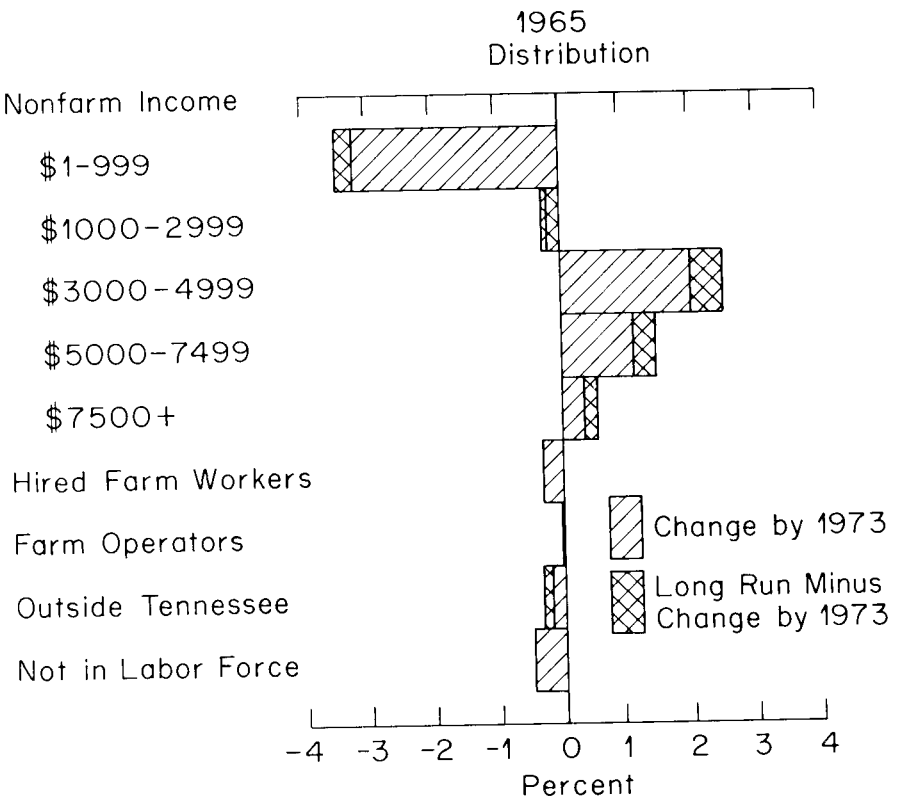
	Estimated 1965	Estimated 1973	Estimated long-run
	(%)	(%)	(%)
Nonfarm income			
$1-999	35.39	32.10	31.62
1,000-2,999	22.34	22.23	22.22
3,000-4,999	2.89	4.10	4.60
5,000-7,499			
7,500+	.31	.43	.51
Nonfarm workers (total)	(60.93)	(58.86)	(58.95)
Hired farm workers	.55	.46	.46
Farm operators	.08	.10	.10
Outside Tennessee	11.88	10.14	10.05
Not in labor force	26.56	30.44	30.44



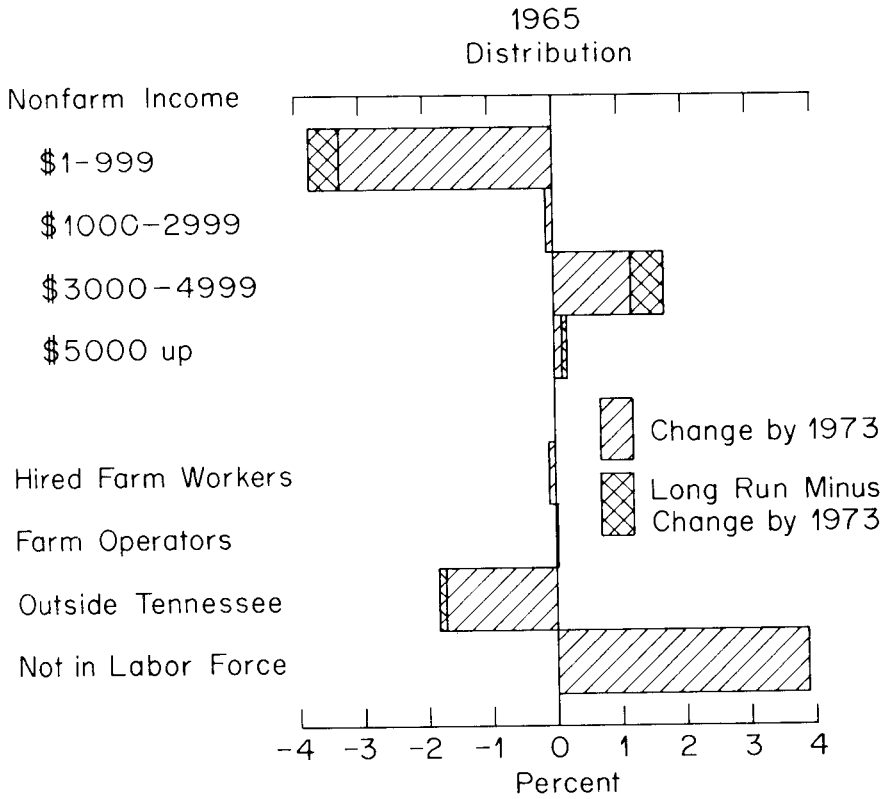
Appendix Figure 1. Estimated change in percentage distribution from 1965 by income-job classification, white male workers, Tennessee, 1973 and long run.



Appendix Figure 2. **Estimated change in percentage distribution from 1965 by income-job classification, black male workers, Tennessee, 1971 and long run.**



Appendix Figure 3. Estimated change in percentage distribution from 1965, by income-job classification, white female workers, Tennessee, 1973 and long run.



Appendix Figure 4. Estimated change in percentage distribution from 1965, by income-job classification, black female workers, Tennessee, 1973 and long run.

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