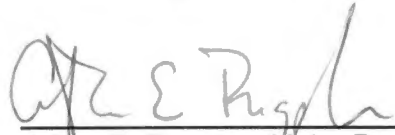
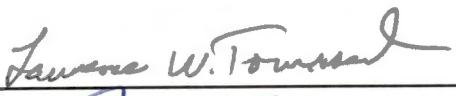


To the Graduate Council:

I am submitting herewith a thesis written by William David Pointer entitled "Evaluation of Low Pressure Leak Flows in Metallic Vacuum Vessels for Detection Systems Using Helium Dissolved in a External Water Cooling Jacket." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Nuclear Engineering.


Arthur E. Ruggles, Major Professor

We have read this thesis and
Recommend its acceptance:




Accepted for the Council:


Associate Vice Chancellor and
Dean of the Graduate School

**EVALUATION OF LOW PRESSURE LEAK FLOWS IN METALLIC
VACUUM VESSELS FOR DETECTION SYSTEMS USING HELIUM
DISSOLVED IN AN EXTERNAL WATER COOLING JACKET**

A thesis submitted for the

Master of Science Degree

The University of Tennessee, Knoxville

W. David Pointer

August 1999

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ABSTRACT

Vacuum containment vessels are often utilized in industrial and research applications to isolate equipment and experiments from the surrounding environment. The development of leaks in these vessels may result in damage to internal components and facility down time. The inclusion of a vessel integrity monitoring system (VIMS) in such a facility allows detection of leak formation, characterization of leak size, and provides a tool for the evaluation of the risk for significant damage to the facility with continued operation.

The proposed vessel integrity monitoring system (VIMS) evaluated herein utilizes helium dissolved in a water jacket surrounding a vacuum vessel for the detection of leaks. The water jacket and vacuum vessel that are considered in these analyses are part of a thermal management and isolation system for a cold neutron moderator in the High Flux Isotope Reactor. However, the results of the theoretical and computational evaluations are of use to a variety of similar low pressure applications. The concentration of helium in the vacuum vessel is monitored using commercially available Residual Gas Analyzer (RGA) technology to detect leaks as they develop. Conservative system simulations indicate that using existing technology a leak as small as 5×10^{-6} m can be detected in less than 3 hours when the vacuum vessel volume is 29 liters or less, demonstrating the feasibility of the proposed system.

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NOMENCLATURE

Symbols

A_{HT}	Area available for heat transfer (m^2)
A_{xs}	Cross-sectional area (m^2)
c_p	Specific heat ($J/kg\cdot K$)
C_j	Concentration of species j
D	Diameter (m)
D_{ij}	Diffusion coefficient for material i in material j
f	Friction factor
g	Gravitational acceleration ($9.81\ m/s^2$)
G	Mass flux ($kg/m^2\cdot s$)
h	Enthalpy (J/kg)
h	Heat transfer coefficient ($J/m^2\cdot K$)
h_{fg}	Latent heat of vaporization (J/kg)
J	Diffusive transport rate (atoms/s)
k	Thermal conductivity ($W/m\cdot K$)
K	Discharge coefficient (0.61 for sharp edge)
K_j	Henry's coefficient for gas species j
L	Axial Length (m)
m	Mass (kg)
M	Molecular weight ($kg/mole$)
N	Number density (number/ m^3)
N_a	Avagadro's number (6.022×10^{23} /mole)
p	Pressure (Pa, atm, or Torr)

Pr	Prandtl number ($Pr = hD/k$)
q	Power (W)
q''	Heat flux (W/m^2)
Q	Molecular transport rate (molecules/sec)
r	Radial position from centerline (m)
r_o	Radius of channel (m)
R	Ideal gas constant
Re	Reynolds number ($Re = \rho vD/\mu$)
S	Suppression factor
S	Slip ratio ($S = v_v/v_l$)
t	Time (s)
T	Temperature ($^{\circ}C$ or K)
V	Volume (m^3)
v	Velocity (m/s)
$\bar{v}$	Channel average velocity (m/s)
$\langle v \rangle$	Expected value of the velocity (m/s)
W_s	Work (W)
x	Flow quality
x	Vertical coordinate
X_{tt}	Martinelli factor
z	Axial position from inlet (m)

Greek Symbols

α	Void fraction
α	Transmission probability

δ	Boundary layer or development layer thickness (m)
λ_{mfp}	Particle mean free path (m)
μ	Dynamic viscosity (N/m-s)
θ	Angle between flow direction and vector normal to earth's surface
ρ	Density (kg/m ³)
σ	Surface tension (N/m)
τ	Sheer stress (N/m ²)

Subscripts

bulk	Value at infinite distance from source of perturbation
c	Convective
CRIT	Corresponding to critical condition
EXIT	Corresponding to exit condition
f	Fluid
g	Gas
l	Liquid
o	Initial or limiting value
wall	Value at the wall of the channel
2 ϕ	Two-phase

CHAPTER 1

INTRODUCTION

Vacuum containment vessels cooled by circulating water jackets are often utilized in research and industrial applications where thermal isolation of equipment or experiments from the influences of the surrounding environment is desirable. The development of leaks in these vessels can result in costly down time for the facility and damage to internal components in addition to the costs associated with the repair or replacement of the vessel itself. The inclusion of a vessel integrity monitoring system to detect leak formation and estimate the size of the leak provides a tool for evaluation of the risk associated with continued operation and the level of maintenance required.

Specific Application

The High Flux Isotope Reactor (HFIR) Cold Neutron Source facility is a cryogenic neutron beam port ("Neutron Scattering at the High Flux Isotope Reactor," 1997; *High Flux Isotope Reactor Safety Analysis Report*, 1999). Cold neutrons are neutrons with long wavelengths that are often used in neutron scattering research. The neutrons are slowed to this wavelength in a cryogenic chamber surrounded by a vacuum containment vessel. A high quality vacuum must be maintained in order to reduce the convective and conductive heat transfer between the cryostat and other structures within the Cold Neutron Source. Additionally, gases in the vacuum vessel may freeze on the surface of the cryostat if a high-quality vacuum is not maintained. A high-quality vacuum is also necessary to ensure that the cold neutrons are not absorbed before reaching the experiment station. A vessel integrity monitoring system would provide information about the quality of the vacuum within the Cold Neutron Source assembly and provide a valuable tool to evaluate the ability of the Cold Neutron Source to operate safely. This *in situ* knowledge could also enhance availability by aiding in the scheduling of repairs and maintenance.

System Description

The Cold Neutron Source assembly consists of a liquid hydrogen filled cryostat and related liquid hydrogen transfer lines surrounded by a vacuum containment vessel as illustrated in Figure 1.1. The vacuum containment vessel serves three primary purposes. First, the vessel serves as vacuum boundary for the high quality vacuum that must be maintained for cold neutron production. Second, the vessel serves as a containment vessel in case of cryostat failure to prevent the release of hydrogen into the core. Third, the vessel serves as a pressure boundary to protect the cryostat and related systems from the water in the core and reactor pool.

Since the Cold Neutron Source is located within the beryllium reflector surrounding the reactor core, the neutron flux passing through that portion of the vacuum vessel is large. Consequently, significant thermal energy is generated within the vessel volume by neutrons interacting with the vessel material. A candidate heat transfer system to remove the heat generated in the vacuum containment vessel utilizes water that is circulated outside the vessel, in the gap between the vessel and the beam tube as shown in Figure 1.1.

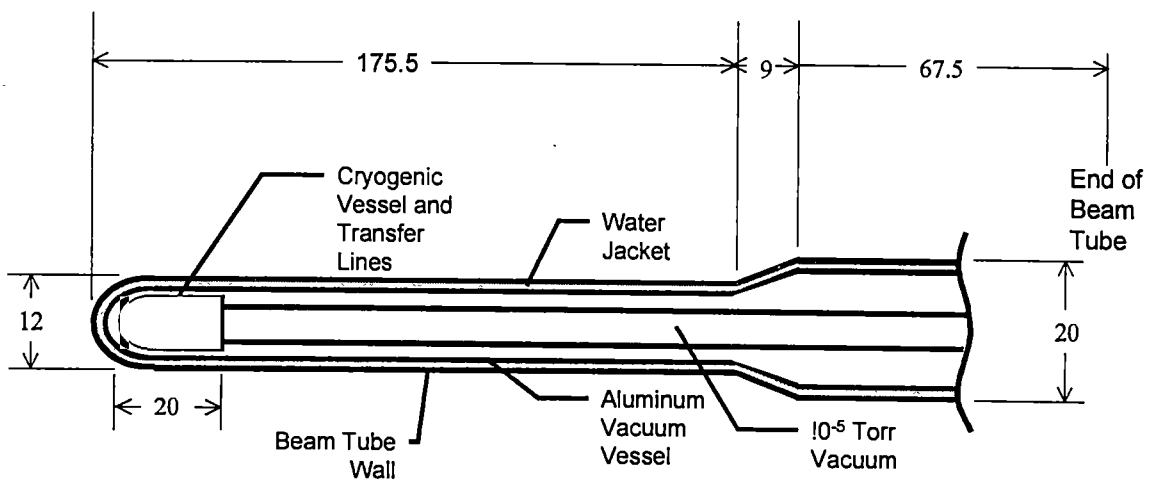


Figure 1.1 Idealized Cold Neutron Source Geometry (Sectional View of Cylindrical Vessel, Dimensions in cm, Not to Scale).

The design of a vessel integrity monitoring system for a water-cooled vacuum vessel is complicated by the difficulty associated with detecting the changes in the concentration of water vapor in the vacuum vessel. Water vapor in the vacuum vessel will tend to freeze on the cryostat and hydrogen delivery systems. Therefore, the small changes in water vapor concentration in the vacuum containment vessel resulting from leak formation may not be detectable. To enhance the ability of a monitoring system to detect the development of small leaks, the vessel may be surrounded with a gas with a lower saturation temperature and background concentration, such as helium. However, low-pressure helium is not an adequate coolant for systems with large heat loads. In these cases, the dissolution of helium gas in the cooling water provides both the necessary heat transfer characteristics and desirable detection characteristics.

The proposed Vessel Integrity Monitoring System for the Cold Neutron Source application would utilize helium gas dissolved in the external water cooling jacket. The theoretical analyses of the idealized leak flows that are included herein provide the basis for evaluation of the feasibility of the proposed system.

Boundary Conditions

The operating temperature and pressure on each side of the vacuum boundary is prescribed by engineering and practical constraints on system operation. A very high quality vacuum must be maintained to minimize the loss of cold neutrons through collisions with other particles in the Cold Neutron Source assembly and to minimize heat transfer between the cryostat and warm surfaces within the vacuum vessel. The vacuum pressure that is to be maintained in the Cold Neutron Source assembly is approximately 10^{-5} torr, or 10^{-3} Pa and the operating temperature of the cryostat is near that of liquid hydrogen, approximately -253°C , or 20 K. Consequently, water entering the vacuum vessel in liquid phase will evaporate as it leaves the leak but may freeze on the cryogenic systems within the vessel.

Ideally, the cooling water jacket surrounding the vacuum vessel should operate near atmospheric pressure, approximately 101325 Pa, to minimize material stresses in the vessel walls. The desired operating temperature for both the vacuum containment vessel and the cooling system is approximately 40°C , or 313.15 K.

The concentration of helium in the water is directly proportional to the pressure in the water jacket. As the pressure increases, the helium concentration in the water increases. The concentration of helium in the water jacket is calculated using Henry's law, which is given by:

$$K_j = \frac{p_j}{C_j} \quad 1.1$$

The value of Henry's coefficient for helium dissolved in water, K_{He} , at 40 °C is $1.23 \times 10^4 \text{ MPa} / \frac{\text{moleHe}}{\text{moleH}_2\text{O}}$ (*International Critical Tables*, 1928). At this temperature and pressure, the saturation concentration of helium that can be dissolved in water is one helium atom per 1.21×10^5 water molecules.

CHAPTER 2

LEAK IDEALIZATIONS

Geometric Idealizations

The geometric configuration of a leak in a metallic vessel is often highly irregular and may change dramatically as the leak develops. The diameter of the leak might change as the fluid moves through the channel. Furthermore, the direction of the leak flow might be highly varied within the leak.

For the purpose of these analyses, the leak is idealized as a cylindrical channel with a constant cross-sectional area that connects two large volumes. In order to insure that results from this idealization are conservative, the leak diameters considered in these analyses are assumed to represent the minimum diameter of a given leak path configuration.

Since the hydraulic diameter of a leak may be quite small, the roughness of the channel wall might significantly enhance the irregularity of the leak. For simplicity, the idealized leak is assumed smooth-walled. Again, in order to insure conservative approximation of the actual leak flow rates, the leak diameters considered are assumed to represent the minimum diameter encountered by the fluid as a result of surface roughness.

Furthermore, leaks might form labyrinthine channels with many changes in direction. For the purpose of these analyses, the idealized leak follows a straight path from the water jacket surrounding the vacuum vessel. A range of leak lengths is considered, the straight-through case and the tangential case as illustrated in Figure 2.1.

The idealized leak may have any orientation with respect to the direction of gravitational acceleration. The average pressure gradient within the leak, given by $\frac{p_{in} - p_{out}}{L}$, is on the

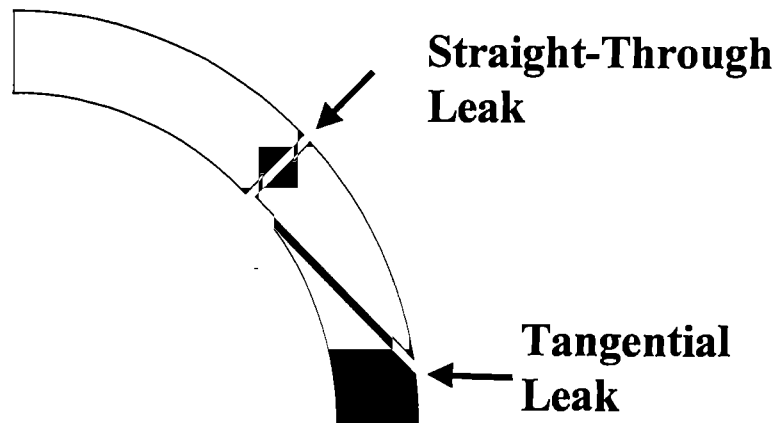


Figure 2.1 Range of Leak Lengths.

order of 100 MPa/m for the application of interest. Thus, the total pressure gradient is many times greater than the pressure gradient resulting from gravity, about one kPa/m, and the effects of gravitational acceleration are negligible.

In summary, the idealized leak is a smooth-walled horizontal cylindrical channel with a constant cross-sectional area for flow as shown in Figure 2.2. The channel connects two large plena that exhibit constant thermophysical and thermodynamic properties. The fluid transport rates calculated for the idealized leak represent only a conservative approximation of the actual leak rates that is useful in characterizing the performance of the leak detection system.

Thermodynamic Idealizations

For the Cold Neutron Source, the water jacket is required as a consequence of neutron interactions with the vacuum vessel itself. The neutron flux in the in the Cold Neutron Source beam tube is on the order of 1×10^9 neutrons per square centimeter per second (Blosser and Thomas, 1961). The flux reaching the cold neutron source is predominately thermal in nature since neutrons must travel through the beryllium reflector of the reactor in order to enter the beam tube. The macroscopic total cross-section for thermal neutrons incident on an aluminum target is 0.099 cm^{-1} (Duderstadt, 1976). Consequently, the

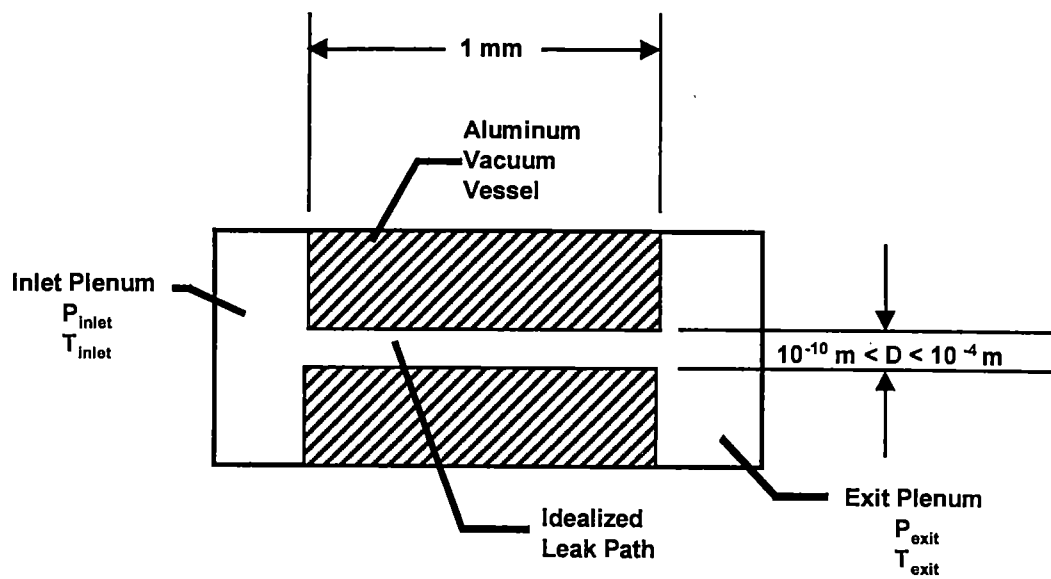


Figure 2.2 Idealized Leak Geometry.

neutron reaction rate in the vacuum vessel is on the order of 1×10^8 interactions per cubic centimeter per second. Furthermore, the neutron interactions with materials surrounding the vacuum vessel and the vessel itself will produce many secondary particles that also interact with the vacuum boundary. Most of the energy deposited in the vacuum vessel by the interactions of neutrons and secondary particles with the vessel material is converted to thermal energy, resulting in a volumetric heat generation rate on the order of 1 MW/m^3 in the vacuum boundary. This volumetric heat generation rate gives rise to a temperature difference of $3 \times 10^{-3} \text{ }^\circ\text{C}$ across the boundary if the side facing the vacuum is considered adiabatic and the wet surface is assumed to have a constant temperature of $40 \text{ }^\circ\text{C}$. The heat transfer coefficient to the water is greater than $1000 \text{ W/m}^2\text{-}^\circ\text{C}$, resulting in a temperature difference of less than $0.36 \text{ }^\circ\text{C}$ between the cooling jacket bulk temperature and the temperature of the vacuum boundary. The expected temperature profile for the idealized leak is shown in Figure 2.3.

In order to evaluate the transport of fluid through a channel, the thermodynamic path that the fluid will follow must be determined. The thermodynamic path that is followed often determines the behavior of the fluid properties as the fluid moves through the channel. As a result, the calculated transport rate is dependent on the path chosen for the evaluation. The thermodynamic path describes changes in the total energy of the fluid as it moves through the channel. The energy balance for a fluid element moving along a streamline in the leak flow may be written as

$$\Delta h + g \Delta z + \Delta \left(\frac{v^2}{2} \right) = q - W_s \quad 2.1$$

where Δh is the enthalpy of the fluid, q is the energy absorbed by the fluid in the form of heat transferred from the wall. W_s is the work done on the fluid and $g \Delta z$ is the change in the potential energy of the fluid due to movement through the gravitational potential field. The change in potential energy due to gravity is negligible in these evaluations.

For small channels surrounded by large conductive masses, heat exchange with the surrounding medium is often sufficient to maintain a constant temperature in the fluid,

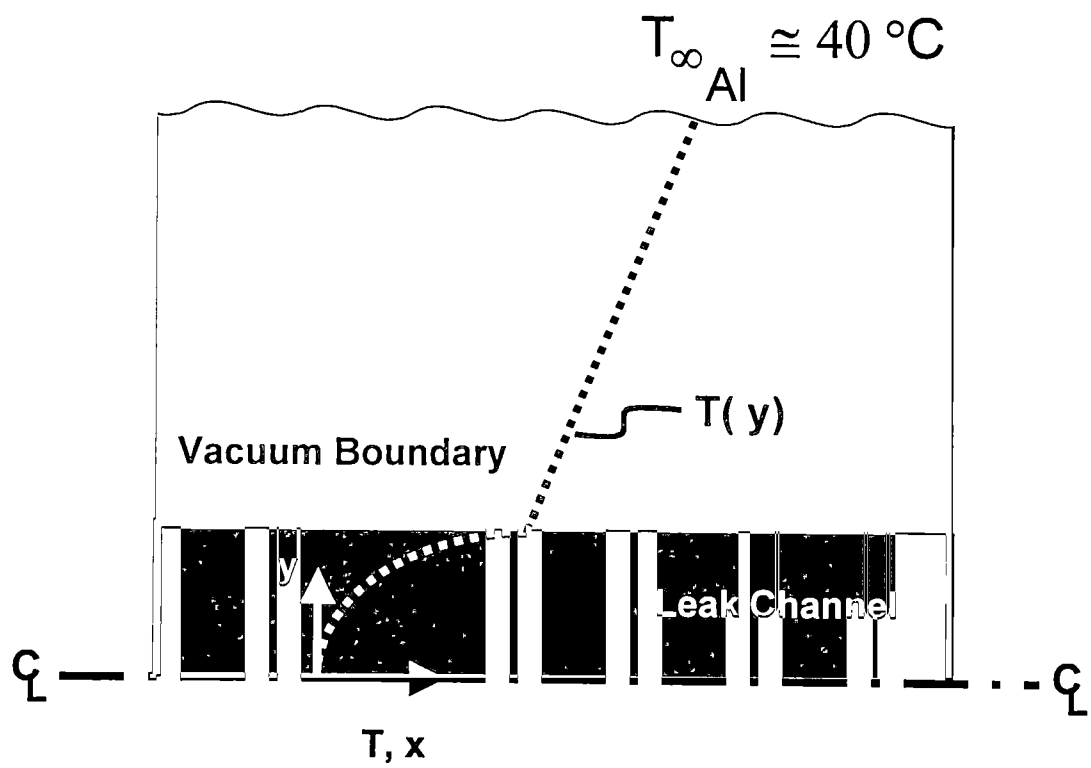


Figure 2.3 Temperature Profile in a Heated Channel.

establishing an isothermal thermodynamic path in the channel. For an isothermal path to be maintained, convective heat transfer to the fluid in the channel must be sufficient to overcome fluid cooling resulting from vapor formation or fluid expansion. Furthermore, conduction in the surrounding medium must be sufficient to supply the necessary heat flux at the wall of the channel. In order to validate the isothermal path for the Cold Neutron Source application, bulk temperature gradients, enthalpy gradients, convective heat transfer to the fluid, and conductive heat transfer in the metal boundary are evaluated for typical leak flows. Since leaks may exhibit single-phase liquid flow, two-phase flow, and single-phase vapor flow, each flow regime must be considered in the evaluation of the thermodynamic path.

Single-Phase Liquid Flow

In the cooling jacket of the Cold Neutron Source, the temperature of the water in the vicinity of the wall is approximately equal to the temperature of the vacuum vessel itself. Since the temperature of the liquid at the leak inlet is equal to the temperature of the vacuum vessel, the enthalpy of the liquid will not increase due to heat addition as the flow moves through the leak. The temperature of the fluid will not decrease unless the fluid expands as it moves through the channel, resulting in a net change in the enthalpy of the fluid. For incompressible single-phase liquid flows, the total change in enthalpy for the liquid in the channel is attributable to the pV work done by the wall. The total amount of this work per unit mass leak flow is approximated by the difference between the inlet pressure and the liquid saturation pressure multiplied by the specific volume of the liquid. The change in the enthalpy of the flow attributable to wall friction is approximately 95 J/kg which corresponds to a temperature change of $2.2 \times 10^{-2} \text{ }^{\circ}\text{C}$. Since the heat transfer coefficient to the liquid in the single-phase leak is greater than 1000 W/m^2 , the difference between the leak flow bulk temperature and the vacuum boundary temperature is even less than the predicted wall superheat, and the effects of external heating can be neglected. Therefore, isothermal thermodynamic path is valid for single-phase liquid leak flows.

Two-Phase Flow

For leaks in the Cold Neutron Source vacuum vessel that exhibit two-phase flow, the liquid flashes to vapor as it moves through the leak. The energy required to produce the vapor

must come from the internal energy of the liquid phase if the channel is adiabatic. The change in the internal energy of the liquid results in a decrease in fluid temperature in the absence of external heating. If the fluid is pure liquid at the inlet and evaporates or flashes completely within the channel so that the fluid is pure vapor at the exit, the internal energy change in the liquid is approximately equal to the latent heat of vaporization, h_{fg} . The latent heat of vaporization for a leak flow in this application is approximately 2.4×10^6 J/kg.

In order for the two-phase leak flow to follow an isothermal thermodynamic path, the energy absorbed by the fluid from the surrounding material as a result of convective heat transfer must be equal to the change in internal energy of the liquid in the adiabatic case. The typical mass transport rate for a leak in which two-phase flows might develop is on the order of 10^{-10} kg/s. This flow rate corresponds to a leak with a diameter of approximately 10^{-4} m. Under these conditions, the heat flux is given by

$$q'' = \frac{h_{fg} \dot{V}}{A_{HT}} \quad 2.2$$

Therefore, the fluid must absorb a heat flux of approximately 760 W/m^2 from the surrounding medium for the isothermal thermodynamic path to be valid.

Convective heat transfer from the channel wall can be evaluated using the expression

$$q'' = h(T_{\text{wall}} - T_{\text{bulk}}) \quad 2.3$$

where the heat transfer coefficient, h , is determined from the energy balance for the channel or from engineering models. The heat transfer coefficient for two-phase flows is often calculated as the sum of a term resulting from nucleate boiling, h_{NB} , and a term resulting from forced convective heat transfer, h_c as proposed by Chen (1966)

$$h_{2\phi} = h_{NB} + h_c \quad 2.4$$

The convective term may be calculated using the Dittus-Boelter correlation:

$$h_c = 0.023 (Re_{2\phi})^{0.8} (Pr)^{0.4} \left(\frac{k_{2\phi}}{D} \right). \quad 2.5$$

The Dittus-Boelter correlation for two-phase flow may be simplified using

$$F = \left(\frac{Re_{2\phi}}{Re} \right)^{0.8} = \left[\frac{Re_{2\phi} \mu_1}{G(1-x)D} \right]^{0.8} \quad 2.6$$

to write (Tang and Tong, 1997)

$$h_c = 0.023 F \left[\frac{G(1-x)D}{\mu_1} \right]^{0.8} \left(\frac{\mu c_p}{k} \right)_1^{0.4} \left(\frac{k_1}{D} \right). \quad 2.7$$

In this form, F is the only unknown and may be determined as a function of the Martinelli factor, X_{tt} , which is given by (Martinelli, 1948)

$$\frac{1}{X_{tt}} = \left(\frac{x}{1-x} \right)^{0.9} \left(\frac{\rho_l}{\rho_g} \right)^{0.5} \left(\frac{\mu_g}{\mu_l} \right)^{0.1}. \quad 2.8$$

For values of the Martinelli factor less than 10, the value of F is given by

$$F = 2.35 \left(0.213 + \frac{1}{X_{tt}} \right)^{0.763}. \quad 2.9$$

For values of the Martinelli factor greater than 10, the value of F is unity. (Chen, 1966)
The nucleate boiling component of the heat transfer coefficient is calculated using a form of the Forster-Zuber relation (1955),

$$h_{NB} = 0.00122 \left[\frac{k_l^{0.79} c_{pl}^{0.45} \rho_l^{0.49}}{\sigma^{0.5} \mu_l^{0.29} h_{fg}^{0.24} \rho_g^{0.24}} \right] (\Delta T_o)^{0.24} (\Delta p_o)^{0.75} \quad 2.10$$

The mean superheat of the fluid, ΔT_o , is difficult to determine and is generally lower than the wall superheat, ΔT_{sat} . The Forster-Zuber relation was originally developed for pool boiling for which this difference between the two superheat values is negligible. However, the difference cannot be neglected for forced-convection boiling (Tong and Tang, 1997). To address this difference, Chen (1966) defined the ratio of the mean superheat to the wall superheat as a suppression factor, S , which is given by

$$S = \left(\frac{\Delta T_o}{\Delta T_{sat}} \right)^{0.99} = \left(\frac{\Delta T_o}{\Delta T_{sat}} \right)^{0.24} \left(\frac{\Delta p_o}{\Delta p_{sat}} \right)^{0.75} \quad 2.11$$

Thus, Equation 2.10 can be written as

$$h_{NB} = 0.00122 S \left[\frac{k_l^{0.79} c_{pl}^{0.45} \rho_l^{0.49}}{\sigma^{0.5} \mu_l^{0.29} h_{fg}^{0.24} \rho_g^{0.24}} \right] (\Delta T_{sat})^{0.24} (\Delta p_{sat})^{0.75} \quad 2.12$$

The value of Chen's suppression factor ranges from unity at high velocities to zero at low velocities, and is calculated using (Chen, 1966)

$$S = \left[\frac{k_l}{(h_{lo}) F \delta} \right] \left[1 - \exp \left(\frac{-h_{lo} F \delta}{k_l} \right) \right], \quad 2.13$$

where h_{lo} is the heat transfer coefficient from the single-phase liquid case, $h_{lo} = 3.66 k_l / D$, and δ is the thickness of the growth region. This thickness is expressed by (Tong and Tang, 1997)

$$\delta = 0.041 \left(\frac{\sigma}{g(\rho_l - \rho_v)} \right)^{0.5}.$$

2.14

Since the bubble departure diameter might approach the leak diameter in some leaks, film evaporation processes might be more probable than flow boiling. However, Chen's model for the heat transfer coefficient of two-phase flows provides a conservative approximation in this case. The heat transfer coefficient for two-phase flashing or boiling flows typically falls in the range of 2,500 – 50,000 W/m²-°C. (Todreas and Kazimi, 1993)

In order for the isothermal thermodynamic path to be valid, the total heat flux corresponding to convective heat transfer must be on the order of 760 W/m². Using the minimum expected value of the heat transfer coefficient, 2,500 W/m²-°C, the minimum wall superheat required to maintain the temperature of the flow is 0.3 °C. Since the wall superheat is small, than the bulk flow temperature is not much affected, and convective heat transfer is capable of supplying the energy needed to offset the temperature change in the fluid resulting from vapor formation. However, conductive heat transfer within the metal vacuum vessel must be sufficient to move the required heat to the wall of the channel.

For aluminum with an average thermal conductivity of about 180 W/m¹-°C, the thermal gradient required to provide a heat flux of 760 W/m² at the channel wall is approximately 3.8 °C/m. The vessel boundary is approximately 10⁻³ m thick, and therefore the characteristic length for conduction in the vessel boundary is on the order of 10⁻³m, indicating that the temperature difference required to maintain the necessary heat flux at the channel wall is on the order of 10⁻² °C. Thus, conductive heat transfer is sufficient to provide the required heat flux. Therefore, the thermodynamic path of the leak can be considered isothermal.

Single-Phase Gas Flow

In some cases, leaks in the Cold Neutron Source vacuum vessel may exhibit single-phase gas or vapor flow. In the adiabatic case, vapor production at the liquid vapor interface results in a decrease in the internal energy of the liquid. As a result, the fluid cools as

vapor is formed. As the vapor expands through the channel, the enthalpy of the vapor decreases, resulting in further decrease in temperature. For an isothermal path to be maintained in this case, the heat flux must be sufficient to replace not only the internal energy lost by the liquid as a result of vapor formation, but also the enthalpy lost by the single-phase vapor as it expands through the channel. The energy lost by the liquid as a result of the state change is equal to the latent heat of vaporization, 2.4×10^6 J/kg. The enthalpy loss resulting from expansion of the gas through the leak is on the order of 7.4×10^4 J/kg. Therefore the total energy that the heat flux must supply to maintain the isothermal path is on the order of 2.5×10^6 J/kg.

In order to maintain an isothermal thermodynamic path in leaks exhibiting single-phase gas flow, convective heat transfer must be sufficient to offset the temperature change in the fluid resulting from vapor production and expansion. The typical leak in the Cold Neutron Source vacuum vessel for this flow regime has a diameter of approximately 10^{-6} m and a mass flow rate on the order of 10^{-12} kg/s. Therefore the heat flux required to replace the enthalpy loss of the fluid is approximately of 795 W/m^2 .

The heat transfer coefficient for a single-phase vapor flow with constant wall temperature can be calculated using (Todreas and Kazimi, 1993)

$$\text{Nu} = \frac{hD}{k} = 3.66. \quad 2.15$$

For the given channel conditions, the heat transfer coefficient for the single-phase gas flow case is approximately $732 \text{ W/m}^2\text{-}^\circ\text{C}$. Therefore the wall superheat required to maintain a constant temperature in the flow is 1.1°C . Since the wall superheat is small, the bulk temperature of the fluid is not much affected, and convective heat transfer is sufficient to maintain a constant temperature in the leak. Therefore, the isothermal thermodynamic path is valid if conductive heat transfer within the surrounding metal is sufficient to supply the required heat flux.

The thermal conductivity of the aluminum is approximately $180 \text{ W/m-}^\circ\text{C}$, as in the two-phase case. In order for conductive heat transfer to provide a heat flux of 795 W/m^2 at the wall of the channel, a temperature gradient of approximately 4.4°C/m must be established

in the metal. The characteristic length for conduction in the vacuum boundary is on the order of 10^{-3} m. Therefore the temperature difference in the vacuum boundary resulting from conduction is on the order of 10^{-2} °C. Thus, conductive heat transfer is sufficient to provide the necessary heat flux, and the isothermal thermodynamic path is also valid for single-phase gas flows.

For this application, an isothermal thermodynamic path is appropriate for all of the expected flow regimes.

CHAPTER 3

LEAK FLOW PHYSICS

A variety of flow phenomena are possible in microchannels and leaks as a result of the typically small diameters. In some cases, the surface tension and wettability characteristics of the fluid may prevent the liquid from entering the leak, resulting in the development of an interface and a single-phase vapor flow. In other cases, favorable wettability characteristics may result in capillary flows. Furthermore, for leaks with very small diameters, the mean free path of the molecules may be greater than the diameter of the leak. In this case, the molecules are as likely to collide with the wall as each other and must be treated as molecular flow. The determination of the dominant flow phenomena is further complicated by the vacuum condition at the leak exit, which draws the water below saturation and may result in flashing near or at the exit. Thus, two-phase flows are possible when liquid penetrates the leak.

Lines of Demarcation between Flow Phenomena

In many cases, the range of conditions for which the various flow phenomena may be expected to develop in a leak in the vacuum vessel may be clearly defined. However, in some cases no one phenomena will clearly dominate for a given set of conditions, and a conservative approach is taken by evaluating the mass flow rate that gives conservative performance predictions for the leak detection system.

For relatively large leaks, those with diameters on the order 10^{-4} m to 10^{-5} m, the liquid coolant penetrates the leak and a laminar single-phase liquid flow develops. The bulk pressure decreases as the liquid flows through the leak, as a result of shear stresses acting on the fluid at the channel wall. When the exit pressure is sufficiently low and the temperature of the fluid is constant, the pressure of the fluid must decrease below the saturation pressure of the liquid. As the pressure falls below this limit, the liquid begins to flash to vapor, and a saturated two-phase flashing flow develops, as shown in Figure 3.1.

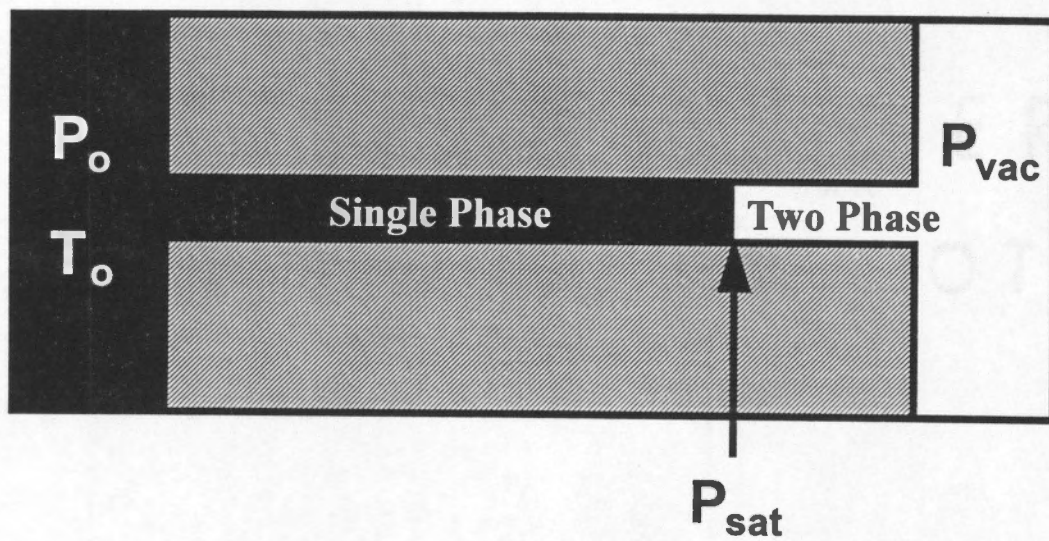


Figure 3.1 Idealized Large Leak.

As a result of vapor production, compressibility is introduced into the fluid. The flow of compressible fluids is often limited by the ability of the fluid to propagate pressure information back toward the inlet through the flow. As the velocity of the flow approaches the sound speed in the compressible fluid, further decreases in the exit pressure will no longer result in increased acceleration of the flow. At the sound speed, the fluid is traveling at the same velocity as the propagation velocity of the pressure wave resulting from the pressure decrease. Since the fluid and pressure waves are traveling in opposite directions and at the same speed, the pressure wave cannot travel beyond the exit of the channel to initiate the acceleration of the fluid. This phenomenon is commonly referred to as critical flow. The limiting mass transport rate may be attributable to either single-phase viscous flow or two-phase flashing flow. Therefore, both phenomena must be considered, and the more limiting mass flow rate used in the evaluation of system performance.

For small leaks, the surface tension of the liquid can prevent the liquid from penetrating the leak. As a result of the surface tension, a gas-liquid interface forms near the inlet of the leak, as shown in Figure 3.2. Fluid molecules then enter the leak through a combination of evaporation and diffusion at the interface, developing a single-phase vapor flow in the leak. Like the single-phase liquid system, the mass transport rate associated with the single-phase vapor flow may be limited by the viscous shear stresses acting on the fluid at the channel wall or by the critical flow rate determined by the sound speed associated with the fluid. Again, both limits must be considered, and the more conservative mass transport rate is used for the evaluation of the system performance.

Another transport phenomenon that might be observed in smaller leaks is capillary flow similar to the flows often found in porous media. In this case, the liquid-surface wettability promotes the flow of liquid through the channel. As with the single-phase liquid flows previously considered, the liquid might flash within the leak resulting in a two-phase flow, or a vapor-liquid interface may develop at or near the channel exit where the liquid evaporates into the vacuum vessel. As a result of the significant difference in the density, gas flows result in lower mass flow rates than liquid flows in channels of the same size. Again, the conservative mass transport rate is used to provide a quantitative evaluation of leak detection system performance. Therefore, the surface tension induced interface

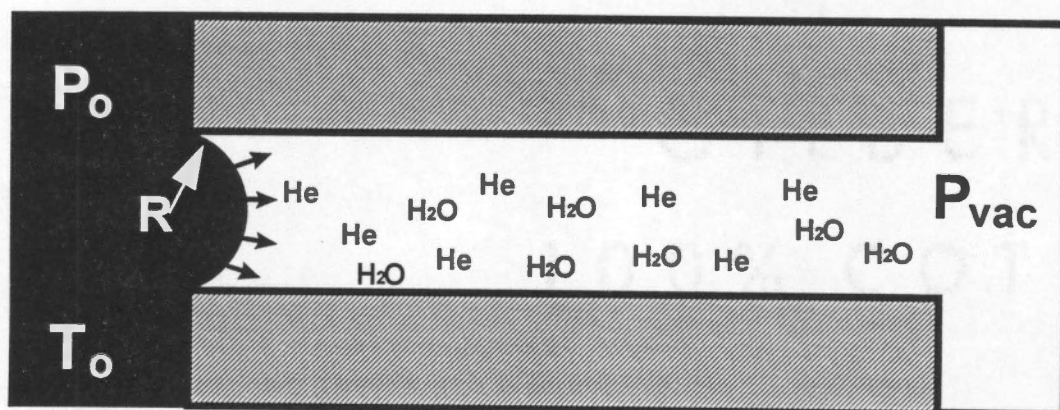


Figure 3.2 Idealized Small Leak.

scenario is the more conservative case and is used in these evaluations. Capillary flows are not considered.

In leaks that exhibit vapor flow, the expected mean free path of a vapor molecule may be greater than the leak diameter. As the mean free path of the molecule approaches the diameter of the channel, the molecule is as likely to interact with the wall as with other molecules of fluid. Since conventional continuum mechanics models are based on the assumption that fluid particles interact with each other more frequently than with the wall, those models cannot be used in this case. Statistical or stochastic mechanics must be used to evaluate the mass transport under these conditions. This flow phenomenon is often called molecular flow. The mean free path of a vapor molecule is primarily dependent on the pressure and temperature of the fluid (Hecht, 1990)

As the leak diameter approaches the size of the grains and crystal structures within the surrounding material, the molecules no longer flow freely through the leak. As the fluid molecules begin to interact more frequently with the atoms around them, the dominant transport phenomenon shifts from a clearly defined flow path toward molecular diffusion. When molecular diffusion is dominant, the fluid molecules follow a path determined by the momentum exchanged in collisions with atoms in the crystal lattice. Molecular diffusion rates are determined as function of the properties governing the interaction of the fluid molecules with the atomic components of the crystal lattice and the temperature of the system. The diffusion coefficient for helium through aluminium is near zero. (Schmittmann and Zia, 1995) However the atomic displacements resulting from the high neutron flux through the aluminum may enhance species diffusion slightly.

Viscous Liquid Flow

Relatively large leaks, i.e. leaks with diameters on the order of 10^{-3} m to 10^{-5} m, the liquid coolant penetrates the leak and flows freely toward the outlet until the bulk pressure falls below the saturation pressure of the liquid. Consequently, a substantial portion of the flow length may be dominated by single-phase liquid flow. Single-phase liquid flows may exhibit smooth streamline behavior, i.e., laminar flow, or turbulent motion with the formation of vortices within the flow field. The Reynolds number, given by

$$Re = \frac{\rho \bar{v} D}{\mu}, \quad 3.1$$

is a unitless quantity often used to provide a demarcation between turbulent and laminar flow. For fully developed laminar flow in a tube, the Reynolds number that corresponds to the onset of turbulence is 2300, although fully turbulent conditions may not develop until much larger Reynolds numbers are achieved. For leaks in the vacuum vessel smaller than 10^{-4} m, the flow may be considered laminar since the Reynolds numbers associated with these flows are on the order of unity. Leaks larger than 10^{-4} m in diameter may exhibit turbulent flow.

As the fluid enters the leak and makes contact with the surface, the shear forces acting on the fluid affect the velocity of the fluid near the wall. As a result, a boundary layer develops as the liquid flows through the leak, as shown in Figure 3.3. As the boundary layer develops, the inviscid flow region shrinks until the boundary layer merges at the centerline of the leak. When the boundary layer merges, the viscous effects extend across the entire flow cross-section. As a result of the shear stress being distributed across the entire cross-section the velocity profile no longer changes, and the flow is considered fully developed. For fully developed laminar flow in a pipe, the velocity profile is given by

$$v(r) = -\frac{1}{4\mu} \left(\frac{dp}{dz} \right) r_o^2 \left[1 - \left(\frac{r}{r_o} \right)^2 \right] \quad 3.2$$

Therefore, the fully developed velocity profile for laminar flow in a cylindrical pipe is parabolic. (Coulson and Richardson, 1996)

The effects of the viscous shear stresses acting on the fluid which cause this parabolic distribution are often lumped into the Moody friction factor for the purpose of calculating

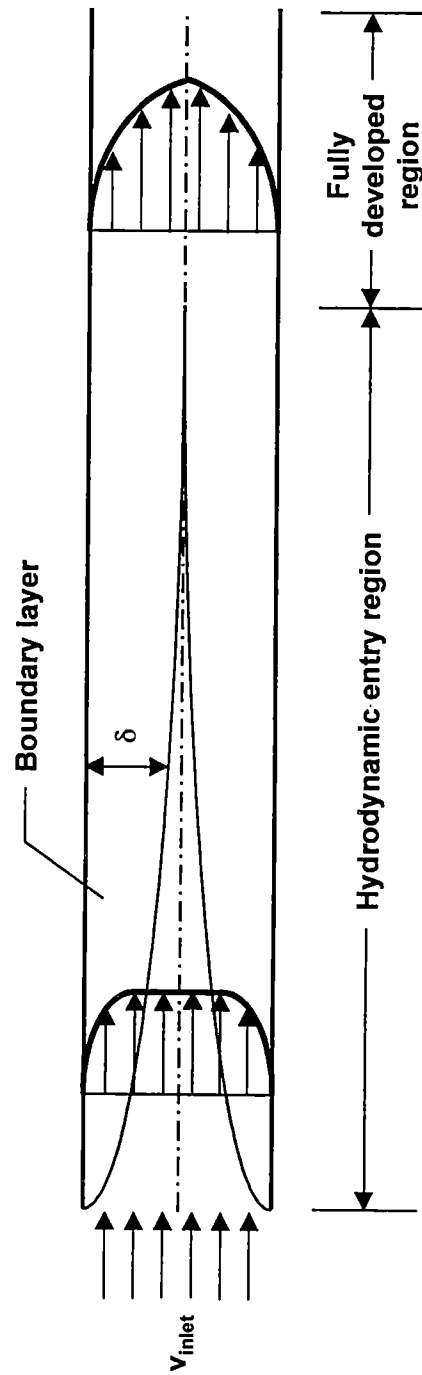


Figure 3.3 Boundary Layer Development in a Smooth-Walled Channel.

pressure drops or flow rates for fully developed flows in tubes. The Moody friction factor is a dimensionless parameter given by

$$f = \frac{-(dp/dz)D}{\rho \bar{v}^2 / 2} . \quad 3.3$$

Using the definition of the mean velocity and the velocity profile for laminar flow, the Moody friction factor can be written as

$$f = \frac{64}{Re} . \quad 3.4$$

Noting that the friction factor is constant for fully developed flow, the pressure drop in the fully developed flow between position z_1 and z_2 is given by

$$\Delta p = - \int_{p_1}^{p_2} dp = f \frac{\rho \bar{v}^2}{2D} \int_{x_1}^{x_2} dx = f \frac{\rho \bar{v}^2}{2D} (x_1 - x_2) . \quad 3.5$$

It should be noted, however, that this equation accounts only for the viscous forces acting on the fluid and neglects other losses such as those resulting from changes in potential energy and those resulting from flow geometries which exhibit losses greater than those in a cylindrical pipe. The compressibility of the fluid is also neglected in this formulation.

Compressibility results in changes in velocity as the fluid expands through the channel. The pressure drop across the channel resulting from this acceleration in an inviscid flow is given by the Bernoulli equation,

$$p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) . \quad 3.6$$

In viscous flows, the contribution of the frictional losses, given by Equation 3.5, must also

be included. In this case, the Bernoulli equation is modified to

$$p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) + \frac{1}{2} \rho \bar{v}^2 \frac{L}{D} f \quad 3.7$$

In order to incorporate the effects of gravitational acceleration, the term $\rho g \Delta x$ is added to Equation 3.7. The complete modified Bernoulli equation also includes a form loss term that accounts for changes in channel geometry. The form loss term is given by $\rho \bar{v}^2 C_{\text{form}}/2$, where C_{form} is the form loss coefficient, which has been determined empirically for various changes in geometry. Thus the fully modified Bernoulli equation is given by (Coulson and Richardson, 1996)

$$p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho g \Delta x + \frac{1}{2} \rho \bar{v}^2 \frac{L}{D} f + \frac{1}{2} \rho \bar{v}^2 C_{\text{form}} \quad 3.8$$

However, for single-phase liquid the compressibility contribution is generally negligible. For incompressible flow in a channel of constant cross-section, the pressure drop is given by

$$p_1 - p_2 = \rho g \Delta x + \frac{1}{2} \rho \bar{v}^2 \frac{L}{D} f + \frac{1}{2} \rho \bar{v}^2 C_{\text{form}} \quad 3.9$$

where the thermophysical properties are evaluated at the inlet conditions.

Critical Flashing Flow

As the pressure falls below the saturation pressure, the liquid might begin to flash to vapor, introducing compressibility into the flow. The flow rate of a compressible fluid in a pipe cannot exceed some maximum critical value for given upstream conditions. The maximum flow rate results from "choking" of the flow. A flow is choked when a disturbance occurring downstream of some section of the flow cannot propagate upstream of this section. For example, a decrease in pressure at the outlet cannot affect the flow

parameters upstream of the critical section. Therefore, lowering the outlet pressure cannot increase the flow rate. For flows that can be considered one-dimensional, its velocity in the choked section is equal to the propagation velocity of small pressure disturbances, i.e. the sound speed in the fluid.

For some leaks on the order 10^{-6} m in diameter, surface tension might be sufficient to prevent bubble development on the wall and suppress vapor formation in the channel. If this is the case, the single-phase vapor flow dominates throughout the channel and the liquid flashes in the vacuum vessel itself. While the single-phase liquid flow model should be utilized to model this phenomenon, the conservative flow rates used for the range of diameters that may exhibit critical flashing flow are also conservative for leaks in this range in which flashing is suppressed.

Two-Phase Flow Models

The pressure gradient is often calculated from the momentum balance for a two-phase mixture flowing in the axial direction of a one-dimensional channel. The momentum balance is generally expressed as

$$\frac{\partial}{\partial t}(G_{2\phi} A_{xs}) + \frac{\partial}{\partial z} \left(\frac{G_{2\phi}^2 A_{xs}}{\rho_{2\phi}^+} \right) = - \frac{\partial}{\partial z} (p A_{xs}) - \int_{P_{xs}} \tau_w dP_{xs} - \rho_{2\phi} g \cos \theta A_{xs}. \quad 3.10$$

For steady state flows in constant area channels, Equation 3.10 may be simplified to

$$- \frac{dp}{dz} = \frac{d}{dz} \left(\frac{G_{2\phi}^2}{\rho_{2\phi}^+} \right) + \frac{1}{A_{xs}} \int_{P_{xs}} \tau_w dP_{xs} + \rho_{2\phi} g \cos \theta. \quad 3.11$$

Equation 3.11 describes the static pressure gradient in the channel as the sum of three components resulting from acceleration, friction and gravity.

$$\left(\frac{dp}{dz} \right)_{acc} = \frac{d}{dz} \left(\frac{G_{2\phi}^2}{\rho_{2\phi}^+} \right) \quad 3.12$$

$$\left(\frac{dp}{dz}\right)_{\text{fric}} = \frac{1}{A_{xs} P_{xs}} \int \tau_w dP_{xs} = \frac{\bar{\tau}_w P_{xs}}{A_{xs}} \quad 3.13$$

$$\left(\frac{dp}{dz}\right)_{\text{gravity}} = \rho_{2\phi} g \cos \theta \quad 3.14$$

The pressure drop is then evaluated by integrating the components of the pressure gradient equation over the length of the channel. The resulting pressure drop equation is also the sum of three terms representing the acceleration, friction, and gravitational contributions. These contributions are given by

$$\Delta p_{\text{acc}} = \left(\frac{G_{2\phi}^2}{\rho_{2\phi}^+}\right)_{\text{out}} - \left(\frac{G_{2\phi}^2}{\rho_{2\phi}^+}\right)_{\text{in}}, \quad 3.15$$

$$\Delta p_{\text{fric}} = \int_{z_1}^{z_2} \frac{\bar{\tau}_w P_{xs}}{A_{xs}} dz, \quad 3.16$$

and

$$\Delta p_{\text{gravity}} = \int_{z_1}^{z_2} \rho_{2\phi} g \cos \theta dz. \quad 3.17$$

The gravitational contribution is straightforward and the acceleration contribution can be easily obtained if the dynamic density, $\rho_{2\phi}^+$, can be evaluated. The dynamic density is a function of the flow quality if the momentum of each phase is written in terms of the mass flux.

$$\frac{G_{2\phi}^2}{\rho_{2\phi}^+} = (\rho_v \alpha v_v^2) + (\rho_l [1 - \alpha] v_l^2). \quad 3.18$$

Since the flow quality can be defined by

$$xG_{2\phi} = (\rho_v \alpha v_v) \quad 3.19$$

and

$$(1-x)G_{2\phi} = (\rho_l [1-\alpha] v_l), \quad 3.20$$

Equation 3.18 can be rewritten as

$$\frac{G_{2\phi}^2}{\rho_{2\phi}^+} = \frac{x^2 G_{2\phi}^2}{c_v (\rho_v \alpha)} + \frac{(1-x)^2 G_{2\phi}^2}{c_l (\rho_l [1-\alpha])} \quad 3.21$$

where the specific heat of the vapor is given by

$$c_v \equiv \frac{(\rho_v \alpha v_v)^2}{(\rho_v \alpha v_v^2)(\rho_v \alpha)} \quad 3.22$$

and the specific heat of the liquid is given by

$$c_l \equiv \frac{(\rho_l [1-\alpha] v_l)^2}{(\rho_l [1-\alpha] v_l^2)(\rho_l [1-\alpha])} \quad 3.23$$

Assuming a uniform radial velocity profile for each of the two phases so that $c_v = c_l = 1.0$, the dynamic density of the two-phase mixture is given by

$$\frac{1}{\rho_{2\phi}^+} = \frac{x^2}{\{\rho_v \alpha\}} + \frac{(1-x^2)}{\{\rho_l (1-\alpha)\}} \quad 3.24$$

The friction contribution is not as easily obtained and is often simplified to a form similar to the single-phase flow model using a two-phase friction factor. (Coulson and Richardson, 1996)

$$\left(\frac{dp}{dz} \right)_{\text{fric}} \equiv \frac{f_{2\phi}}{D_e} \left[\frac{G_{2\phi}^2}{2\rho_{2\phi}^+} \right]. \quad 3.25$$

The components of the pressure gradient equation can often be further simplified, if thermodynamic equilibrium or equal phase velocities are assumed. Two simplified models based on such assumptions are commonly used – the Homogenous Equilibrium Model (HEM) and the Separated Flow Model.

Homogeneous Equilibrium Model

The equation used to evaluate the pressure gradient can be simplified to the Homogeneous Equilibrium Model if the two phases are assumed to have equal velocities and to be in thermodynamic equilibrium. In this case the velocities of the two phases are equal to a single two-phase velocity, i.e., $v_v = v_l = v_{2\phi}$. Then, by substituting into Equation 3.24 from Equations 3.19, 3.20 and 3.21, the dynamic density can be written as

$$\frac{1}{\rho_{2\phi}^+} = \frac{x(\rho_g \alpha)v_{2\phi}}{(\rho_g \alpha)G_{2\phi}} + \frac{(1-x)(\rho_g (1-\alpha))v_{2\phi}}{(\rho_g (1-\alpha))G_{2\phi}}. \quad 3.26$$

Simplifying,

$$\frac{1}{\rho_{2\phi}^+} = \frac{xv_{2\phi} + (1-x)v_{2\phi}}{G_{2\phi}} = \frac{v_{2\phi}}{G_{2\phi}} = \frac{1}{\rho_{2\phi}}. \quad 3.27$$

Thus, for the HEM model, the dynamic density is equivalent to the static density, $\rho_{2\phi}^+ = \rho_{2\phi}$.

The equivalence of the dynamic and static densities in the homogeneous model can be used to simplify the components of the pressure gradient equation. Expanding on the equivalence of the two densities, the dynamic density can be written as

$$\frac{1}{\rho_{2\phi}^+} = \frac{1}{\rho_{2\phi}} = \frac{v_{2\phi}}{G_{2\phi}} = \frac{\alpha v_{2\phi} + (1-\alpha)v_{2\phi}}{G_{2\phi}}. \quad 3.28$$

Using the definition of the flow quality (Equations 3.19 and 3.20), the dynamic density can be expressed as

$$\frac{1}{\rho_{2\phi}^+} = \frac{xG_{2\phi}/\rho_g + (1-x)G_{2\phi}/\rho_l}{G_{2\phi}} = \frac{x}{\rho_g} + \frac{1-x}{\rho_l}. \quad 3.29$$

Using this definition of the dynamic density the acceleration component of the pressure can be rewritten as

$$\left(\frac{dp}{dz}\right)_{\text{accel}} = G_{2\phi}^2 \frac{d}{dz} \left[\frac{1}{\rho_l} + \left(\frac{1}{\rho_v} - \frac{1}{\rho_l} \right) x \right], \quad 3.30$$

which can be expanded to

$$\left(\frac{dp}{dz}\right)_{\text{accel}} = G_{2\phi}^2 \left[\frac{dv_l}{dz} + x \left(\frac{dv_g}{dz} - \frac{dv_l}{dz} \right) + (v_g - v_l) \frac{dx}{dz} \right]. \quad 3.31$$

If the liquid phase is assumed to be incompressible, Equation 3.31 can be simplified to

$$\left(\frac{dp}{dz}\right)_{\text{accel}} = G_{2\phi}^2 \left(x \frac{\partial v_g}{\partial p} \frac{dp}{dz} + v_{fg} \frac{dx}{dz} \right). \quad 3.32$$

For the HEM model, the friction contribution to the pressure gradient can be simplified to

$$\left(\frac{dp}{dz}\right)_{\text{fric}} = \frac{f_{2\phi}}{D_e} \left(\frac{G_{2\phi}^2}{2\rho_{2\phi}} \right). \quad 3.33$$

Combining the gravitational component and the simplified acceleration and friction components, the total pressure gradient is expressed by

$$\left(\frac{dp}{dz}\right)_{\text{HEM}} = - \frac{\frac{f_{2\phi}}{D_e} \left(\frac{G_{2\phi}^2}{2\rho_m} \right) + G_{2\phi}^2 v_{fg} \frac{dx}{dz} + \rho_m g \cos \theta}{\left(1 + G_{2\phi}^2 x \frac{\partial v_g}{\partial p} \right)}. \quad 3.34$$

The two-phase friction multiplier is generally approximated as some function of the single-phase liquid multiplier. (Todreas and Kazimi, 1993)

Separated Flow Model

In contrast to the Homogeneous Equilibrium Model which specifies that both the temperatures and velocities of the two phases be in equilibrium, the Separated Flow Model requires that only the temperatures of the two phases be in equilibrium. In this case, the velocities of the two phases are not equal, allowing the application of this model beyond the scope of the HEM model.

If the compressibility of the liquid is neglected and the densities and velocities of the two phases are assumed to be constant in the radial direction, the acceleration term from the general model (Equation 3.11) may be simplified using the general definition of the dynamic density (Equation 3.17) such that

$$\frac{dp}{dz}_{\text{accel}} = G_{2\phi}^2 \frac{d}{dz} \left[\frac{(1-x)^2 v_l}{(1-\alpha)} + \frac{x^2 v_v}{\alpha} \right]. \quad 3.35$$

Application of the chain rule to Equation 3.35 yields

$$\begin{aligned} \frac{dp}{dz_{\text{accel}}} = & G_{2\phi}^2 \left[-\frac{2(1-x)v_l}{(1-\alpha)} + \frac{2xv_v}{\alpha} \right] \frac{dx}{dz} \\ & + G_{2\phi}^2 \left[\frac{(1-x)^2 v_l}{(1-\alpha)^2} - \frac{x^2 v_v}{\alpha^2} \right] \frac{d\alpha}{dz} + G_{2\phi}^2 \frac{x^2}{\alpha} \frac{\partial v_v}{\partial p} \frac{dp}{dz}. \end{aligned} \quad 3.36$$

Substituting the acceleration component described by Equation 3.36 in place of the general acceleration component in the two-phase pressure gradient model and simplifying the result produces

$$\begin{aligned} \frac{dp}{dz_{\text{SEP}}} = & \left[1 + G_{2\phi}^2 \frac{x^2}{\alpha} \frac{\partial v_v}{\partial p} \right]^{-1} \\ & \cdot \left[\frac{f_{2\phi}}{D_e} \left(\frac{G_{2\phi}^2}{2\rho_l} \right) + G_{2\phi}^2 \left(\frac{2xv_v}{\alpha} - \frac{2(1-x)v_l}{1-\alpha} \right) \frac{dx}{dz} \right. \\ & \left. + G_{2\phi}^2 \left(\frac{(1-x)^2}{(1-\alpha)^2} v_l - \frac{x^2}{\alpha^2} v_v \right) \frac{d\alpha}{dz} + \rho_{2\phi} g \cos \theta \right]. \end{aligned} \quad 3.37$$

The Separated Flow Model can be further simplified at high pressures, since the compressibility of the gas phase may be neglected under those conditions, i.e., $\frac{\partial v_v}{\partial p} \approx 0$.

However, in this case critical flow rate prediction models are not possible. (Todreas and Kazimi, 1993)

Two-Phase Critical Flow Models

The critical condition requires that the system exhibit a constant mass flow rate with respect to the downstream pressure. Therefore, the critical mass flow rate is prescribed by the condition that

$$\frac{dm}{dp} = 0. \quad 3.38$$

An alternate interpretation of this condition is that the term $\partial p / \partial z$ in the momentum balance must become infinite at some location in the channel.

Thermal Equilibrium Models

For the HEM model (Equation 3.34), the pressure gradient is represented as a quotient. This representation results from the additional pressure gradient dependence in the acceleration component. In order to satisfy the critical condition, the denominator of the HEM model must approach zero. In this case,

$$\left(1 + G_{2\phi}^2 x \frac{\partial v_v}{\partial p} \right) = 0. \quad 3.39$$

As a result the critical mass flux is given by

$$(G_{2\phi})_{\text{crit}} = \sqrt{-\frac{1}{x} \frac{\partial p}{\partial v_v}}. \quad 3.40$$

Similarly, the denominator of the Separated Flow Model must approach zero in order to satisfy the critical condition. Therefore, the critical flow rate for the Separated Flow Model is derived from

$$\left(1 + G_{2\phi}^2 \frac{x^2}{\alpha} \frac{\partial v_v}{\partial p} \right) = 0. \quad 3.41$$

Thus, the critical flow rate is given by

$$(G_{2\phi})_{\text{crit}} = \sqrt{-\frac{\alpha}{x^2} \frac{\partial p}{\partial v_v}}. \quad 3.42$$

These models of the critical flow rate, while accurate, are not particularly useful since the independent parameters are not easily determined. Consequently, an alternate representation of these models is more commonly used. This representation is derived from the enthalpy balance of the two-phase mixture as it expands under thermal equilibrium. The enthalpy of the mixture is given by

$$h_0 = xh_v + (1-x)h_l + x \frac{v_v^2}{2} + (1-x) \frac{v_l^2}{2}. \quad 3.43$$

Substituting the definition of the mass flux, the enthalpy equation can be rewritten as

$$h_0 = xh_v + (1-x)h_l + x^3 \frac{G_{2\phi}^2}{2\rho_v^2} + (1-x)^3 \frac{G_{2\phi}^2}{2\rho_l^2}. \quad 3.44$$

Solving for the two-phase mass flux results in the slip model, given by

$$G_{2\phi} = (\rho''') \sqrt{2[h_0 - xh_v - (1-x)h_l]}, \quad 3.45$$

where the multiplier ρ''' is given by

$$\rho''' = \left\{ \left[\frac{x}{\rho_v} + \frac{(1-x)S}{\rho_l} \right] \left[x + \frac{1-x}{S^2} \right]^{1/2} \right\}^{-1} \quad 3.46$$

and S is the slip ratio, which is defined by $S = v_v/v_l$. The two-phase slip model describes the mass flux in a general two-phase system if the thermophysical properties

and the slip ratio are known. The slip model predicts the critical mass flux for systems in thermal equilibrium if the thermophysical properties of the system are evaluated at the critical pressure.

The critical pressure is the exit pressure beyond which further decreases in exit pressure does not accelerate the flow for a given inlet pressure. The critical pressure is proportional to the inlet pressure and is evaluated using the critical pressure ratio, p_{critical}/p_0 . The critical pressure ratio is generally considered a pure function of the length-to-diameter ratio of a channel. The critical pressure ratio is shown as a function of the length-to-diameter ratio in Figure 3.4. (Wallis and Collier, 1967)

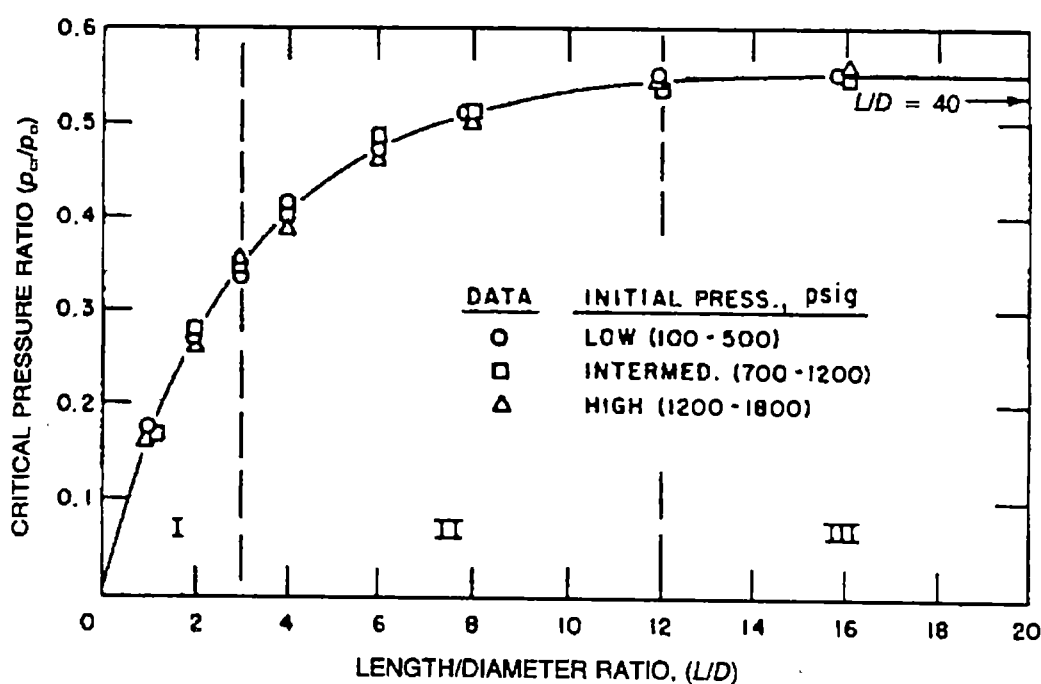


Figure 3.4 Critical Pressure Ratio (Taken from Fauske, 1965).

Since the critical pressure and the thermophysical properties are generally known for a critical system, the only unknown in the model for the critical mass flux is the slip ratio. In the case of the equal phase velocities, the slip ratio is unity. This case is equivalent to the HEM model. Two other formulations of the slip ratio are commonly employed – the Moody model and the Fauske model. The Moody model maximizes the kinetic energy of the two-phase mixture with respect to the slip ratio. The Moody model is derived from

$$\frac{\partial}{\partial S} \left[\frac{x v_v^2}{2} + \frac{(1-x)v_l^2}{2} \right] = 0, \quad 3.47$$

and the Moody slip ratio is given by (Moody, 1966)

$$S = \left(\frac{\rho_l}{\rho_v} \right)^{1/3}. \quad 3.48$$

The Fauske model maximizes the flow momentum with respect to the slip ratio, and is derived from

$$\frac{\partial}{\partial S} [x v_v + (1-x)v_l] = 0, \quad 3.49$$

where the Fauske slip ratio is given by (Fauske, 1965)

$$S = \left(\frac{\rho_l}{\rho_v} \right)^{1/2}. \quad 3.50$$

The performance of the various equilibrium critical flow models is uncertain. While one model may predict the critical mass flux well for one data set, another model may perform well for another data set. In general, the HEM model under-predicts the flow rate, the Moody model over predicts the flow rate, and the Fauske model falls somewhere between the other two.

Thermal Nonequilibrium Models

In many cases, the thermal equilibrium assumption is not appropriate. For flows that do not involve subcooling or noncondensable gases, the development length for thermal equilibrium is on the order of 10 - 40 times the diameter. The actual development length is dependent on many parameters such as interfacial area and heat transfer characteristics.

The presence of subcooling or noncondensable gases might increase the development length. Assuming that frictional losses are not significant for short channels, Fauske (1985) proposed that the critical mass flux in nonequilibrium conditions could be expressed as

$$G_{cr} = \frac{h_{fg}}{v_{fg}} \sqrt{\frac{1}{NTc_f}} \quad 3.51$$

where

$$N = \frac{h_{fg}^2}{2\Delta p \rho_f K^2 v_{fg}^2 Tc_f} + 10L \quad 3.52$$

For channels with lengths greater than 0.1 m, the value of N is prescribed as unity. When the thermophysical properties are evaluated at the inlet pressure, Fauske's nonequilibrium model is known as the Equilibrium Rate Model (ERM).

Viscous Compressible Flow

In the case of single-phase vapor flow, the fluid must be considered fully compressible. As with viscous liquid flow, the Bernoulli equation, given by

$$p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2). \quad 3.7$$

is used to relate the pressure drop and the fluid velocity. As a result of the fluid compressibility, the thermophysical properties of the fluid cannot be considered constant and the thermophysical properties must be considered as functions of axial position in the channel. Rather than treating the fluid properties as continuous functions of position, a more practical approach is the development of a discretized form of the modified Bernoulli equation. In the discretized form, the fluid properties may be treated as constant within a given discrete element. As a simplification, the contributions of changes in channel

geometry are neglected in this development. Therefore, this development is limited to channels with constant cross-section.

The discrete form of the modified Bernoulli equation divides the channel into a number of discrete elements, n . Then the channel is evaluated as a series of short channels with near constant properties. The resulting n equations are given by

$$\begin{aligned} p_1 - p_2 &= \frac{\rho_1}{2} (v_2 - v_1)^2 + \frac{1}{2} \rho_1 \bar{v}^2 \frac{L_1}{D} f_1 \\ &\vdots \\ p_{\text{exit}} - p_n &= \frac{\rho_n}{2} (v_{\text{exit}} - v_n)^2 + \frac{1}{2} \rho_n \bar{v}^2 \frac{L_n}{D} f_n \end{aligned} \quad 3.53$$

where $\rho_1 = \rho @ p_1$ and $f_1 = f @ p_1$, etc.

For low pressures, the ideal gas law may be used to evaluate the density of the water vapor. The ideal gas law is given by

$$\rho_i = \frac{p_i}{RT} \quad 3.54$$

where R is the ideal gas constant. Furthermore, the friction factor may be simplified to

$$f = \frac{64\mu}{\rho v D} \quad 3.55$$

if fully developed laminar flow is assumed.

Continuity is invoked to develop the relationships necessary to solve the system of equations. Continuity requires that

$$\dot{m}_1 = \dot{m}_2 = \dots = \dot{m}_n = \dot{m}_{\text{exit}} . \quad 3.56$$

The mass flow rate in each discrete element is approximated as the product of the density in that element, the cross-sectional area of that element, and the exit velocity in that element. The continuity relation can be rewritten in the form

$$\rho_1 A_{XS} v_2 = \dots = \rho_{n-1} A_{XS} v_n = \rho_n A_{XS} v_{exit} . \quad 3.57$$

It should be noted that the mass flow rate is more appropriately related to the average velocity, but it is assumed that the channel is divided into a sufficient number of elements so that the difference in the average velocity and the exit velocity in a given element is small. Since the cross-section of the channel is assumed constant and the density is approximated using the ideal gas law, the continuity relation becomes

$$\frac{p_1}{RT} v_2 = \dots = \frac{p_{n-1}}{RT} v_n = \frac{p_n}{RT} v_{exit} . \quad 3.58$$

In an isothermal flow, this equality can be reduced to

$$p_1 v_2 = \dots = p_{n-1} v_n = p_n v_{exit} . \quad 3.59$$

Using Equation 3.59, the system of equations can be reduced to

$$\begin{aligned} p_1 - p_2 &= \frac{p_1}{2RT} v_2^2 + \frac{4\mu L v_2}{D^2} \\ p_2 - p_3 &= \frac{p_2}{2RT} \left[\left(\frac{p_1}{p_2} v_2 \right)^2 - v_2^2 \right] + \frac{4\mu L \left(\frac{p_1}{p_2} v_2 + v_2 \right)}{D^2} \\ &\vdots \\ p_n - p_{exit} &= \frac{p_n}{2RT} \left[\left(\frac{p_1}{p_n} v_2 \right)^2 - \left(\frac{p_1}{p_{n-1}} v_2 \right)^2 \right] + \frac{4\mu L \left(\frac{p_1}{p_n} v_2 + \frac{p_1}{p_{n-1}} v_2 \right)}{D^2} \end{aligned} \quad 3.60$$

Using this form of the discretized Bernoulli equation, the viscous vapor mass flux can be determined if the inlet and outlet pressure and the fluid properties are known.

Critical Single Phase Vapor Flow

For single-phase compressible flow in a horizontal channel of constant cross-sectional area and length dL , the energy balance is expressed by

$$-\frac{G^2}{\rho^3} d\rho + \frac{1}{\rho} dp + 4\left(\frac{R}{\rho v^2}\right) \frac{G^2}{\rho^2} \frac{dL}{D} = 0. \quad 3.61$$

The compressible friction factor $(R/\rho v^2)$ is a function of the Reynolds number and the roughness of the channel surface. The Reynolds number is given by $Re = GD/\mu$. Since the mass flux and diameter are constant in the channel, the Reynolds number is a function of the viscosity, which is only a slight function of temperature and pressure. As a result, the Reynolds number and the friction factor can be regarded as constant, allowing the integration of Equation 3.61 over the length of the channel to obtain

$$-G^2 \ln \frac{\rho_1}{\rho_2} + \int_{p_1}^{p_2} \rho dp + 4\left(\frac{R}{\rho v^2}\right) \frac{L}{D} G^2 = 0. \quad 3.62$$

For the isothermal expansion of an ideal gas, the energy balance becomes

$$-G^2 \ln \frac{p_2}{p_1} + \rho_1 \frac{p_2^2 - p_1^2}{2p_1} + 4\left(\frac{R}{\rho v^2}\right) \frac{L}{D} G^2 = 0. \quad 3.63$$

Simplifying Equation 3.63 to

$$-\ln \frac{p_2}{p_1} + \frac{\rho_1}{G^2} \frac{p_2^2 - p_1^2}{2p_1} + 4\left(\frac{R}{\rho v^2}\right) \frac{L}{D} = 0 \quad 3.64$$

and differentiating with respect to the exit pressure, p_2 , yields

$$-\frac{p_1}{p_2} \frac{1}{p_1} + \frac{p_1}{G^2} \frac{p_2}{p_1} - \rho_1 \frac{p_2^2 - p_1^2}{p_1} \frac{1}{G^3 A_{xs}} \frac{dG}{dp_2} = 0. \quad 3.65$$

When $dG/dp_2 = 0$, the flow rate is maximum, and the critical condition is satisfied. Using the critical condition to solve for the critical mass flux gives the result

$$G_{crit}^2 = p_{exit} \rho_{exit}. \quad 3.66$$

Noting that the exit density is given by

$$\rho_{exit} = \rho_{in} \frac{p_{exit}}{p_{in}}. \quad 3.67$$

the critical mass flux can be simplified to

$$G_{crit} = p_{crit} \sqrt{\frac{\rho_{in}}{p_{in}}}. \quad 3.68$$

Since the inlet conditions are known, the only unknown is the critical pressure. For single-phase critical gas flow, the critical pressure can be approximated using a critical pressure ratio of about 0.5.

Molecular Flow

As the mean free path of the fluid molecules approaches the diameter of the channel defining the flow, the usual rules from continuum mechanics that govern fluid behavior no longer apply. The molecules comprising the fluid begin to react more often with the wall surrounding them than with each other. As a result, additional degrees of freedom are introduced into the system and the expected behavior of the bulk flow is no longer defined

by volume and time averaged fluid properties. However, the expected behavior of a single molecule can be defined. Consequently, the flow of molecules traveling through the channel can be defined statistically.

The expected molecular flow through a small opening in the wall of a pressurized container opening to vacuum can be defined based on the probability of a molecule within the volume entering the hole. This probability is proportional to the size of the hole and the rate at which the molecules are traveling. Knudsen (1909) defined the molecular flow rate through the opening as

$$Q = \frac{1}{4} N_f \langle v \rangle A_{xs} . \quad 3.69$$

As a result of the low probability of molecule-molecule collisions with respect to the probability of molecule-wall collisions, this expression can be extended to two containers with different gas number densities (and thus different pressures in an isothermal system) that are connected by an orifice between them. In this case, the net molecular flow rate is given by the difference of the flow from container 1 to container 2 and the flow from container 2 to container 1. The molecular flow rate for two containers connected by an orifice is given by

$$Q = \frac{1}{4} N_1 \langle v \rangle A_{xs} - \frac{1}{4} N_2 \langle v \rangle A_{xs} = \frac{1}{4} \langle v \rangle A_{xs} (N_2 - N_1) . \quad 3.70$$

If the orifice connecting the two containers is extended to a straight tube, the probability of a molecule traveling from one container to the other decreases. The molecule might now interact with the wall of the connecting tube and be reflected back into its original container.

The probability of the molecule traveling through the tube after it enters the tube is defined as the Clausing transmission probability (Clausing, 1932). This parameter is a geometry-specific constant with a value less than unity. For long tubes with diameters small enough for molecules to undergo elastic wall collisions, the transmission probability is given by

$$\alpha = \frac{1}{1 + 3L/4D}. \quad 3.71$$

The molecular flow rate for two connected containers can be generally expressed as

$$Q = \frac{1}{4} \alpha \langle v \rangle A_{xs} (N_2 - N_1). \quad 3.72$$

For two containers connected by a straight tube the molecular flow rate becomes

$$Q = \frac{1}{4} \frac{1}{1 + 3L/4D} \langle v \rangle A_{xs} (N_2 - N_1). \quad 3.73$$

Using the definition of the number density, $N = \rho N_a / M$, and the ideal gas law, $\rho = p/RT$, the general transmission equation can be rewritten as

$$Q = \frac{1}{4} \alpha \langle v \rangle A_{xs} \frac{N_a}{MRT} (p_1 - p_2) \quad 3.74$$

For geometries other than long straight tubes, the transmission probability is not as clearly defined. More sophisticated methods, often involving Monte Carlo simulation, are required for the analysis of molecular flows in more complex geometries.

The velocity distribution of the gas molecules at a given temperature is treated as a Maxwell-Boltzmann distribution. As a result, the expected or most probable velocity for a gas molecule at a given temperature is approximated by (Hecht, 1990)

$$\langle v \rangle = \left(\frac{2kT}{m} \right)^{1/2} = \left(\frac{2RT}{M} \right)^{1/2}. \quad 3.75$$

After substituting this expression into Equation 3.74, the molecular flow rate can be calculated as a function of leak size, temperature, and the pressure drop across the pressure boundary.

Molecular Diffusion

In many ways, molecular diffusion is similar to molecular flow. In the case of molecular flow, a clearly defined channel must exist. Furthermore, the molecular exchange through the channel is incited by the total pressure difference across the boundary in molecular flow. In molecular diffusion, the exchange of molecules occurs when fluid molecules travel through the boundary by displacing atoms present in the crystal structure of the boundary. Concentration gradients or partial pressure gradients drive this exchange across the boundary rather than total pressure gradients.

In its simplest form, the transport of fluid molecules across the boundary by molecular diffusion is an extension of Fick's law of diffusion, which is given by

$$J_i = -D_{ij} \frac{dC_i}{dz} . \quad 3.76$$

Using Fick's Law, the molecular transport rate can be evaluated as a function of the concentration gradient across the boundary and the empirically determined diffusion coefficient for fluid *i* in material *j*. (Coulson and Richardson, 1996)

CHAPTER 4

LEAK FLOW ANALYSIS

In order to quantify the expected performance of the proposed Cold Neutron Source vessel integrity monitoring system, a conservative approach is developed. Leak rates must be evaluated for leak diameters that the system should be capable of detecting. Consequently, all of the leak flow regimes discussed in the previous chapter must be considered in the performance evaluation. The algorithm used to determine the appropriate flow regime for a given set of leak characteristics and to insure a conservative evaluation of the systems performance is illustrated in Figure 4.1. First, the lines of demarcation that separate the flow regimes are evaluated. Then, the appropriate engineering models are used to evaluate the mass flow rate of fluid through the leak. The helium transport rate is then calculated for each model used, and the smallest transport rate is used as a conservative approximation of the actual transport rate for the given leak characteristics.

Lines of Demarcation

The inlet pressure, the diameter of the leak, and the thermophysical properties of the fluid prescribe the type of flow that can be expected in the leak. For the prescribed conditions in the Cold Neutron Source vacuum vessel, all of the flow regimes discussed in the previous chapter may occur.

For leaks between 10^{-4} m and 10^{-6} m in diameter, the liquid coolant in the water jacket will flow freely through the leak since surface tension effects are not sufficient to stop the flow of liquid and establish a liquid-vapor interface. As the liquid travels through the leak, the local pressure decreases as a result of sheer stresses at the wall. When the liquid reaches the saturation pressure, 7.38 kPa at 40 °C, water vapor might form in the flow. The addition of compressibility to the flow allows the development of critical flows. The critical pressure corresponding to an inlet pressure of 7.38 kPa is 4 kPa. Since the exit pressure, 10^{-3} Pa, is well below the critical pressure, the critical flow rate may be the limiting flow rate in larger leaks.

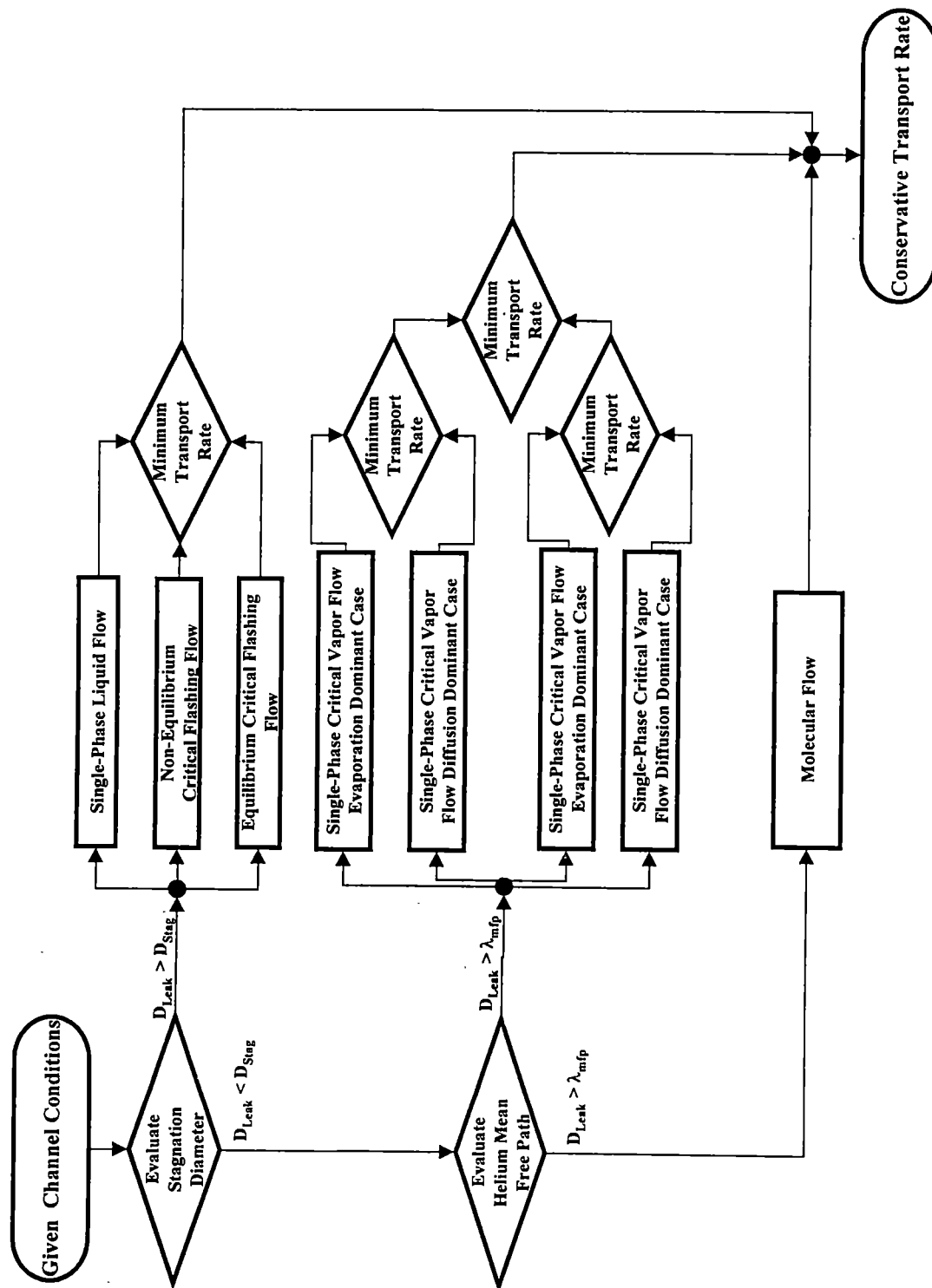


Figure 4.1 Leak Rate Evaluation Algorithm.

For leaks smaller than some critical diameter, a liquid-gas interface develops at the entrance of the leak as shown in Figure 3.2. The pressure across the liquid-vapor interface is given by

$$p_l - p_g = \frac{4\sigma}{D} \quad 4.1$$

When the calculated pressure difference for a given diameter is less than the inlet pressure, liquid freely enters the leak. When the difference is greater than the inlet pressure, surface tension is sufficient to prevent free liquid penetration. For the purpose of these analyses, a non-wetting leak wall is assumed to provide a conservative leak rate prediction. Therefore, the development of a liquid-vapor interface near the leak inlet is assumed.

The surface tension of water at atmospheric pressure and 40 °C is approximately 0.07 N/m. For leaks with diameters smaller than 10^{-5} m, the pressure difference across the surface tension induced liquid-vapor interface at the leak inlet is greater than 0.145 MPa. Since the inlet pressure is 0.101 MPa, a stable liquid-vapor interface develops for leaks with diameters on the order of 10^{-6} m and smaller. For leaks larger than 10^{-6} m in diameter, the liquid freely penetrates the leak and viscous liquid or critical flashing flows develop.

The mean free path length of the molecules of gas or vapor can be used to establish a line of demarcation between the realms of continuum and quantum, or statistical, mechanics. The mean free path of a fluid molecule is proportional to the fluid temperature and the fluid pressure. The line of demarcation is illustrated in Figure 4.2, which shows the pressure at which the helium mean free path is equal to the diameter of a given leak. For the conditions expected in the Cold Neutron Source, leaks with diameters smaller than 10^{-6} m in diameter are expected to exhibit molecular flow.

As the leak diameter approaches the size of the crystal structures within the metal of the vacuum vessel (i.e., 10^{-10} m), the transport phenomena shift from flow phenomena to molecular transport phenomena. The diffusion coefficients for both helium and water

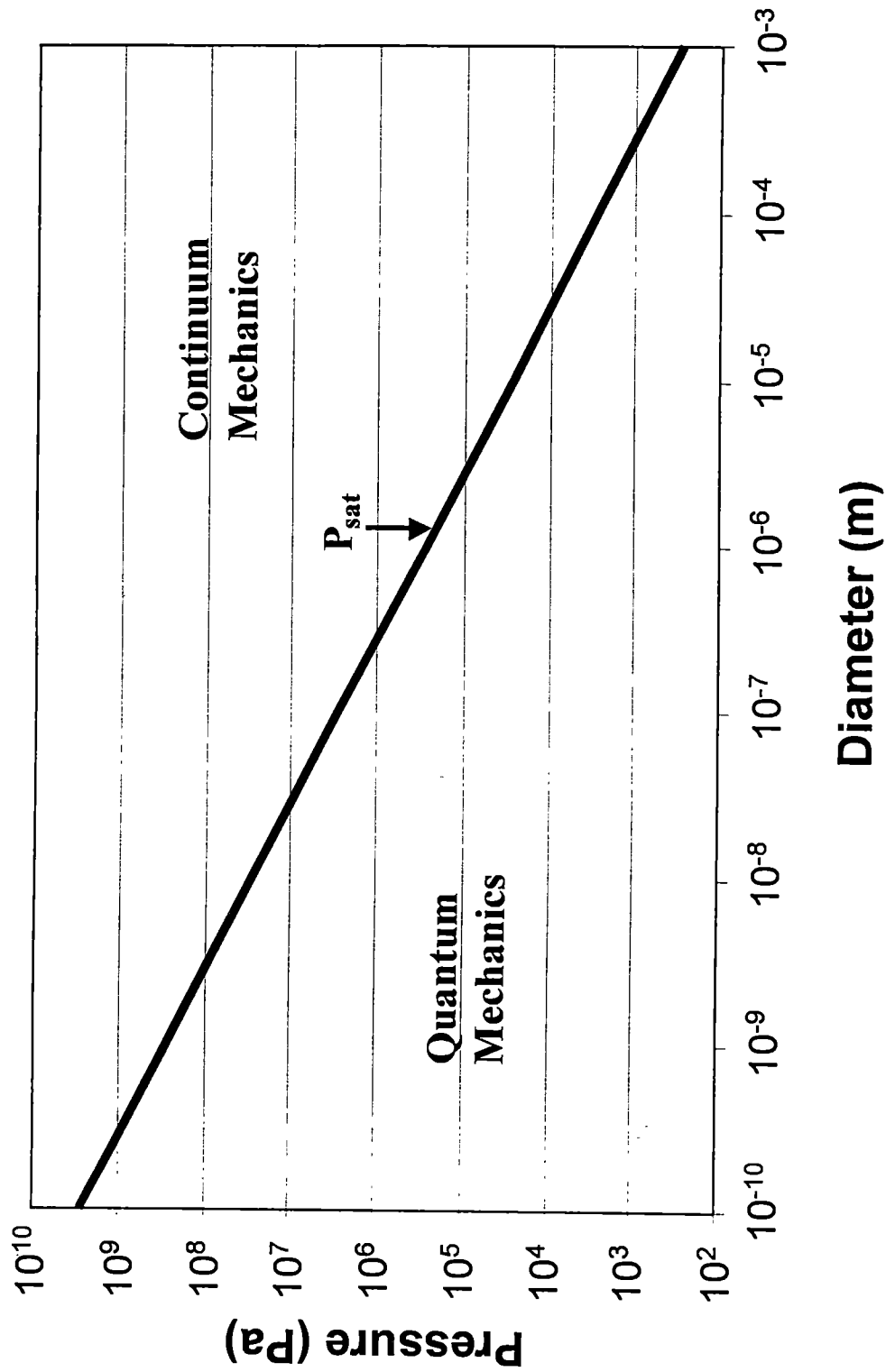


Figure 4.2 Pressure at which the Molecular Mean Free Path Equals the Leak Diameter.

vapor in aluminum are approximately zero. Therefore, the contributions of molecular diffusion to the buildup of helium in the vacuum vessel are neglected, and the minimum diameter considered in these analyses is 10^{-10} m.

Free Liquid Penetration

In the cases where liquid freely enters the leak, viscous liquid flows and critical flashing might develop as the fluid flows toward the vacuum maintained in the vessel. In order to insure that the evaluation of the detection system effectiveness is conservative, both flow regimes are considered for these large leaks. The smaller transport rate is used as a conservative estimate for determining the rate of helium buildup in the vacuum vessel.

Assumptions and Boundary Conditions

For leaks exhibiting free liquid flows, the channel may be divided into two segments. In the first segment, a single-phase viscous liquid flow develops. As the fluid moves through the channel the pressure decreases. When the pressure reaches the saturation pressure the liquid might begin to flash to vapor. In this second segment, a two-phase flow develops. Since the saturation pressure is approximately 7.38 kPa at 40 °C and the exit pressure is on the order of 10^{-3} Pa, the inlet pressure to exit pressure ratio is significantly greater than the critical pressure ratio of approximately 0.5 for L/D less than 10. Therefore, critical two-phase flow can be expected in the second segment. In these analyses, the helium gas is assumed to remain in solution.

In the viscous liquid flow segment of the leak, the leak is treated as a smooth-walled horizontal pipe with a constant circular cross-section. The inlet pressure is equal to the bulk pressure of the fluid in the water jacket, 0.101 MPa. As a result of the isothermal thermodynamic path, the flow exhibits a constant temperature of 40 °C. Since the viscous liquid flow segment ends at the point where vapor production begins, the exit pressure for the segment is the saturation pressure for water at 40 °C, or 7.38 kPa. At that pressure, the fluid can no longer be considered an incompressible liquid and must be treated as a two-phase compressible fluid.

In the critical two-phase flow segment of the leak, the leak is again treated as two large vessels connected by a horizontal smooth walled pipe of constant cross-section. The inlet

of this segment is equivalent to the exit of the first segment. Therefore, the inlet pressure is the saturation pressure of water at 40 °C, or 7.38 kPa. The exit pressure in this case is the vacuum pressure of 10^{-3} Pa maintained in the vacuum vessel.

For water vapor, the critical pressure ratio is roughly 0.5 for $L/D > 10$, and the critical pressure for this segment is approximately 3.7 kPa. Since the critical pressure is well above the exit pressure, critical flow is expected. However, since the pressure drop in the critical flow segment is limited to 3.7 kPa, the rate of vapor production in the channel is also limited. As a result, the majority of the phase change may actually occur after discharge into the vacuum vessel rather than within the leak channel.

Transport Rate Modeling

The limiting transport rate of dissolved helium through a leak exhibiting free liquid penetration may occur in the segment where viscous liquid flow dominates or in the segment where critical flashing flow dominates. The helium transport rate is evaluated for each flow regime independently, and the lowest transport rate for each regime provides a conservative basis for the prediction of the effectiveness of a vessel integrity monitoring system based on the detection of helium in the vacuum vessel.

Viscous Liquid Flow

The single-phase viscous liquid flow segment of the leak is evaluated using the modified Bernoulli equation given by

$$p_1 - p_2 = \rho g \Delta x + \frac{1}{2} \rho \bar{v}^2 \frac{L}{D} f + \frac{1}{2} \rho \bar{v}^2 C_{\text{form}} \quad 3.9$$

However, for this application the contribution of the gravitational losses to the total pressure loss is negligible as shown in Chapter 2. Furthermore, the form loss coefficient for this application, on the order of 1.5, is much less than the term $\frac{L}{D} f$ as a result of the small diameters considered here. Therefore, the Bernoulli equation is further simplified to

$$p_1 - p_2 = \frac{1}{2} \rho \bar{v}^2 \frac{L}{D} f \quad 4.2$$

For smooth-walled systems exhibiting fully developed laminar flow, the friction factor can be expressed as

$$f = \frac{64 \mu}{\rho v D} \quad 4.3$$

The mass flux of water into the vacuum vessel is calculated using the modified Bernoulli equation for given vessel conditions and the friction factor defined by Equation 4.3.

For the conditions expected in the Cold Neutron Source vacuum vessel, the mass flux of water into the vacuum vessel is evaluated for leak diameters ranging from 10^{-6} m to 10^{-3} m. A range of leak lengths that represents the range bounded by the tangential and straight-through leak geometries is considered for each diameter evaluated. From the water mass flux, the helium transport rate is calculated using a saturated helium concentration of 2.8×10^{20} helium atoms per kilogram of fluid at 40 °C and a pressure of 0.101 MPa.

Critical Flashing Flow

The two-phase portion of the flow through large leaks clearly requires more complex modeling than the single-phase portion. Whether thermal equilibrium will be established in a channel or non-equilibrium effects will dominate is often unclear. Both cases are considered in these analyses, and the smaller mass flux is used as a conservative estimate in evaluating the effectiveness of the detection system.

Equilibrium flow may occur for length to diameter ratios greater than 10. In this case, the equilibrium critical flow models developed from the two-phase flow models are used to evaluate the mass flux. For the purpose of these analyses, the slip model and Fauske's slip ratio are used to approximate the equilibrium mass flux. The slip model is given by

$$G_{2\phi} = (\rho'') \sqrt{2[h_0 - x h_v - (1-x)h_f]}, \quad 3.45$$

where the multiplier , ρ''' ,is given by

$$\rho''' = \left\{ \left[\frac{x}{\rho_v} + \frac{(1-x)S}{\rho_l} \right] \left[x + \frac{1-x}{S^2} \right]^{1/2} \right\}^{-1} \quad 3.46$$

Fauske's model for the slip ratio, given by $S = (\rho_l/\rho_v)^{1/2}$, is used as a best estimate for the slip ratio because its values fall between the Moody and HEM values. For the slip model all fluid properties are evaluated at the critical pressure.

In order to allow for non-equilibrium effects, the Equilibrium Rate Model was also used to evaluate the mass flow rate in the compressible segment. The Equilibrium Rate Model is given by

$$G_{cr} = \frac{h_{fg}}{v_{fg}} \sqrt{\frac{1}{NTc_f}} \quad 3.51$$

where,

$$N = \frac{h_{fg}^2}{2\Delta p \rho_f K^2 v_{fg}^2 Tc_f} + 10L \quad 3.52$$

In this case, all fluid properties are evaluated at the inlet conditions. For both models the helium transport rate is evaluated using a saturated helium concentration of 2.8×10^{20} helium atoms per kilogram of fluid.

Surface Tension Induced Stationary Liquid-Vapor Interface Development

For leaks with hydraulic diameters less than approximately 10^{-5} m, the surface tension of the fluid is sufficient to prevent the flow of liquid through the leak. However, water vapor and helium continue to enter the leak as a result of evaporation and diffusion at the liquid-vapor interface. It is not immediately clear which process will dominate at the interface. Therefore, the mass flux is evaluated for both the evaporation-dominant and diffusion-

dominant cases in order to bound the actual transport rate at the interface. The more conservative assumption is used in the evaluation of the detection capabilities of the vessel integrity monitoring system.

Assumptions and Boundary Conditions

The inlet conditions for the surface tension induced interface case are dependent on the process by which vapor and gas are entering the channel through the liquid vapor interface. If the diffusion process dominates the transport of fluid across the liquid-vapor interface, the helium reaches some equilibrium concentration at the interface. If the evaporation process is dominant, the relative concentrations of helium and water maintain an equilibrium value.

Diffusion-Dominant Case

The diffusion dominant case represents one extreme in which helium diffuses across the liquid-vapor interface to establish an equilibrium concentration. The gas flow may then be evaluated as a pure helium flow with an inlet pressure equal to the saturation pressure of the helium in the helium-water solution. For a water jacket temperature of 40 °C and pressure of 0.101 MPa, the saturation pressure of the dissolved helium is 94.2 kPa. The critical pressure, evaluated using the critical pressure ratio, is 45.86 kPa. Since the exit pressure is much less than the critical pressure, the possibility of critical flow in the channel must be considered for the diffusion-dominant case. It should be noted that the diffusion-dominant condition occurs only when the flow rate through the leak is less than the molecular exchange rate at the interface, allowing the buildup of significant concentrations of dissolved gas at the interface.

Evaporation-Dominant Case

In the evaporation-dominant case the relative concentrations of helium and water in the gaseous phase are identical to the relative concentrations in the liquid phase. For evaporation at the interface to occur, the pressure on the vapor side of the interface must be at the saturation pressure for the fluid. The inlet pressure for the single-phase gas flow is equal to the saturation pressure of the fluid, 7.38 kPa. Since the helium atom to water molecule ratio is approximately 8.24×10^{-6} , the fluid properties of the water can be

expected to dominate those of the helium. The density of the mixture is approximately 0.5 kg/m^3 , and all other thermodynamic properties are equivalent to those of the water vapor.

These two cases bound the flow rate through the leak because they represent the two extreme inlet conditions. In reality, the leak flow rate falls between these two extreme cases. In order to evaluate the effectiveness of the vessel integrity monitoring system, the more conservative of the two cases is used to determine the helium buildup rate in the Cold Neutron Source vacuum vessel.

Transport Rate Modeling

The flow of the vapor beyond the liquid-vapor interface may be limited either by the viscous stresses acting on the fluid or by the critical velocity prescribed by the critical pressure. While the channel is not divided into two segments as in the liquid case, both phenomena can occur in the same channel. In some cases, the viscous stresses may be sufficient to prevent the vapor from being accelerated to the critical velocity. As with the inlet conditions, the more conservative case provides the baseline transport rates used to evaluate the performance of the vessel integrity monitoring system.

Critical Vapor Flow

The small leak is first treated as a compressible vapor flow that has been accelerated to its critical velocity. The critical mass flux is evaluated for both the diffusion-dominant and evaporation-dominant inlet conditions using the critical mass flux equation (Equation 3.68)

$$G_{\text{crit}} = p_{\text{crit}} \sqrt{\frac{\rho_{\text{in}}}{p_{\text{in}}}}. \quad 3.68$$

The helium transport rate is evaluated using the helium concentration prescribed by the choice of inlet conditions. For the diffusion-dominant case, the flow is considered a pure helium flow. For the evaporation-dominant case, the flow is considered a water vapor flow with some helium content. In this case, the water vapor mass flux is calculated, and the helium transport rate is determined using a helium saturation concentration of 2.8×10^{20}

atoms of helium per kilogram of water vapor – the saturation concentration for helium in liquid H₂O at 40 °C.

Viscous Compressible Flow

In order to determine if the viscous stresses acting on the fluid are sufficient to prevent the flow from reaching the critical flow rate, the leak flow is treated as a compressible viscous flow. The leak channel is divided into four segments, as shown in Figure 4.3, and Equation set 3.60 is solved to determine the leak flow rate.

$$\begin{aligned}
 p_1 - p_2 &= \frac{p_1}{2RT} v_2^2 + \frac{4\mu L v_2}{D^2} \\
 p_2 - p_3 &= \frac{p_2}{2RT} \left[\left(\frac{p_1}{p_2} v_2 \right)^2 - v_2^2 \right] + \frac{4\mu L \left(\frac{p_1}{p_2} v_2 + v_2 \right)}{D^2} \\
 &\vdots \\
 p_n - p_{\text{exit}} &= \frac{p_n}{2RT} \left[\left(\frac{p_1}{p_n} v_2 \right)^2 - \left(\frac{p_1}{p_{n-1}} v_2 \right)^2 \right] + \frac{4\mu L \left(\frac{p_1}{p_n} v_2 + \frac{p_1}{p_{n-1}} v_2 \right)}{D^2}
 \end{aligned} \tag{3.60}$$

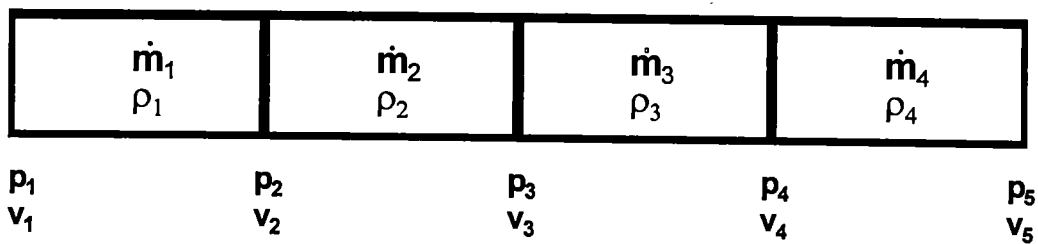


Figure 4.3 Viscous Vapor Model Nodalization.

When applied to the four-segment case, Equation set 3.60 becomes

$$\begin{aligned}
 p_1 - p_2 &= \frac{p_1}{2RT} v_2^2 + \frac{4\mu L v_2}{D^2} \\
 p_2 - p_3 &= \frac{p_2}{2RT} \left(\frac{p_1}{p_2} v_2 - v_2 \right)^2 + \frac{4\mu L \left(\frac{p_1}{p_2} v_2 + v_2 \right)}{D^2} \\
 p_3 - p_4 &= \frac{p_3}{2RT} \left(\frac{p_1}{p_3} v_2 - \frac{p_1}{p_2} v_2 \right)^2 + \frac{4\mu L \left(\frac{p_1}{p_3} v_2 + \frac{p_1}{p_2} v_2 \right)}{D^2} \\
 p_4 - p_{\text{ext}} &= \frac{p_4}{2RT} \left(\frac{p_1}{p_4} v_2 - \frac{p_1}{p_3} v_2 \right)^2 + \frac{4\mu L \left(\frac{p_1}{p_4} v_2 + \frac{p_1}{p_3} v_2 \right)}{D^2}.
 \end{aligned}
 \tag{4.4}$$

In this set of four equations, only four parameters are unknown: p_2 , p_3 , p_4 , and v_2 . Therefore, the mass flux through the leak can be determined algebraically. The helium transport rate is then determined using the appropriate helium content for the chosen boundary conditions, i.e., evaporation-dominant or diffusion-dominant.

Statistical Mechanics

For leaks with diameters less than 10^{-6} m, the penetration of the liquid coolant continues to be prevented by surface tension effects. Leaks of this diameter are sufficiently small that the probability of a given fluid molecule colliding with the wall of the channel is greater than the probability of that molecule colliding with another fluid molecule. Therefore, classical continuum mechanics models are no longer effective in the prediction of transport phenomena. These leak flows are evaluated using the statistically based molecular flow models.

Assumptions and Boundary Conditions

Since the molecular flow regime is a continuation of the condition in which surface tension prevents the penetration of the liquid coolant, the inlet conditions are bounded by the two extreme cases of evaporation-dominant and diffusion-dominant exchange at the interface.

As a result of the relatively low helium concentrations present, the water vapor properties approximate those of the saturated helium-water mixture in the evaporation-dominant case. Since the diffusion-dominant case implies the development of a nearly pure helium flow, the helium properties approximate those of the mixture in this case. Again, the actual inlet conditions fall somewhere between the two extremes, but the more conservative of these cases forms a basis for the evaluation of the performance of the vessel integrity monitoring system.

Transport Rate Modeling

Since the leak diameter is on the order of the molecular mean free path and continuum mechanics no longer apply, the leak flow rate is dependent on the results of interactions between the wall and the molecules of the fluid rather than the propagation of pressure waves through the fluid. Therefore, critical flow limits do not apply to the flow of the mixture of water vapor and helium through leaks of this size. The transport rate is evaluated using the molecular flow rate model given by

$$Q = \frac{1}{4} \alpha \langle v \rangle A_{xs} \frac{N_a}{MRT} (p_1 - p_2). \quad 3.74$$

The expected velocity, approximated using the Maxwell-Boltzmann velocity distribution for an ideal gas, is given by

$$\langle v \rangle = \left(\frac{2kT}{m} \right)^{1/2} = \left(\frac{2RT}{M} \right)^{1/2}. \quad 3.75$$

The helium transport rate is calculated directly in the case of the diffusion-dominant boundary condition. In the evaporation-dominant case, the water vapor transport rate is calculated and the helium transport rate is evaluated based on a helium concentration of 2.8×10^{20} atoms per kilogram of the helium-water mixture.

Molecular Diffusion

As the leak diameter approaches the size of the molecules making up the walls and the fluid, molecular diffusion becomes the dominant transport phenomenon. The diffusion coefficient for helium in aluminum alloys is approximately equal to zero. Therefore substantial amounts of helium do not diffuse through the material. Hence, molecular diffusion is not a significant contributor to the buildup of helium in the vacuum vessel for the Cold Neutron Source application. For the purpose of these analyses, the minimum diameter of interest is greater than 10^{-10} m, where molecular diffusion clearly becomes the primary transport phenomenon.

CHAPTER 5

LEAK FLOW RESULTS

The helium transport rate is a function of the prescribed channel inlet and exit conditions, the channel dimensions, and the expected flow regime. The following results provide conservative estimates of the helium transport rate into the Cold Neutron Source vacuum vessel as a function of leak diameter for nominal system conditions. These transport rates provide a conservative basis for the evaluation of the performance of the proposed vessel integrity monitoring system. Careful selection of flow models that have been demonstrated to provide good results for similar applications insures the applicability of these results to the evaluation of the feasibility of the proposed VIMS.

Free Liquid Penetration

For leaks exhibiting free liquid penetration, helium transport rates were calculated for channels with lengths of 1 and 10 mm and diameters of 10^{-4} , 10^{-5} , and 10^{-6} m. The length increments represent the two extreme geometries resulting from tangential and straight-through leak paths as shown in Figure 2.1. The three diametric increments represent the maximum range of application for the models developed around free liquid penetration. For this range of application, the helium transport rates are calculated for viscous liquid flows and for critical flashing flows.

Viscous Liquid Flow

For this case, the helium transport rate is evaluated by first determining the mass flow rate of water through the leak and then using a saturated helium concentration to determine the helium content of the fluid flowing through the leak. The results of these calculations are shown in Table 5.1 for straight-through and tangential leak orientations.

Critical Flashing Flow

In the case of critical flashing flow, the two-phase flow is first treated as a separated flow in thermal equilibrium. Critical mass flow rates for pure water are calculated using the

Table 5.1 Viscous Liquid Model Leak Rates.

(a) Straight-Through Leak (Length = 10^{-3} m).

Diameter (m)	Mixture Mass Flow Rate (kg/s)	Helium Transport Rate (Atoms/sec)
1 E - 4	3.5 E - 10	9.8 E + 10
1 E - 5	3.5 E - 14	9.8 E + 6
1 E - 6	3.5 E - 18	9.8 E + 2

(b) Tangential Leak (Length = 10^{-2} m).

Diameter (m)	Mixture Mass Flow Rate (kg/s)	Helium Transport Rate (Atoms/sec)
1 E - 4	3.5 E - 11	9.8 E + 9
1 E - 5	3.5 E - 15	9.8 E + 5
1 E - 6	3.5 E - 19	9.8 E + 1

separated flow model, and the expected relative helium concentration is used to extrapolate the helium transport rate from those values. Results of this calculation are shown in Table 5.2.

The equilibrium rate model was used to evaluate the same range of leak geometries for non-equilibrium critical flashing flow. As in the previous calculations, the mass flow rate of the water is calculated and the helium transport rate extrapolated from those results. The results from these calculations are given in Table 5.3.

The transport rates for the non-equilibrium case are more limiting than the equilibrium case. However, for leaks smaller than 10^{-4} m, the helium transport rate is more restricted in the viscous flow segment of the leak than the critical flow segment. The viscous liquid flow calculations are used to provide a conservative estimate of the transport rates for leaks with diameters within the range of these calculations.

Surface Tension Induced Stationary Liquid-Vapor Interface Development

For leaks exhibiting surface tension induced liquid-vapor interfaces, the inlet conditions can be bounded by the evaporation-dominant and diffusion-dominant conditions. For each of these conditions, critical and viscous gas flow models are used to provide estimates of the mass flow rate through the leak. Leak diameters ranging from 10^{-6} m to 10^{-10} m were considered for the idealized straight-through leak geometry.

Diffusion-Dominant Case

In the diffusion-dominant case, the leak flow is treated as a pure helium flow, with the gas pressure equal to 94.2 kPa at the liquid-vapor interface, as established in chapter 3. For the purpose of these calculations, the properties of the helium dominate those of any entrained water vapor. The critical flow rates for the diffusion-dominant case are shown in Table 5.4. The flow rates calculated using the viscous vapor flow model are shown in Table 5.5. The helium transport rate is calculated using an atomic concentration of 1.5×10^{26} atoms per kilogram of helium.

For comparison, the helium transport rates for the diffusion-dominant case are shown in Figure 5.1 as a function of leak size for both the critical vapor flow model and the viscous

Table 5.2 Separated Critical Flow Model Leak Rates.

Diameter (m)	Mixture Mass Flow Rate (kg/s)	Helium Transport Rate (Atoms/sec)
1 E - 4	6.4 E - 4	1.8 E + 17
1 E - 5	6.4 E - 6	1.8 E + 15
1 E - 6	6.4 E - 8	1.8 E + 13

Table 5.3 Equilibrium Rate Model Leak Rates.

Diameter (m)	Mixture Mass Flow Rate (kg/s)	Helium Transport Rate (Atoms/sec)
1 E - 4	1.8 E - 8	5.1 E + 12
1 E - 5	1.8 E - 10	5.1 E + 10
1 E - 6	1.8 E - 12	5.1 E + 8

Table 5.4 Critical Vapor Flow Model Leak Rates – Diffusion-Dominant Case.

Diameter (m)	Mixture Mass Flow Rate (kg/s)	Helium Transport Rate (Atoms/sec)
1 E – 6	4.5 E – 11	6.7 E + 15
1 E – 7	4.5 E – 13	6.7 E + 13
1 E – 8	4.5 E – 15	6.7 E + 11
1 E – 9	4.5 E – 17	6.7 E + 9
1 E – 10	4.5 E – 19	6.7 E + 7

Table 5.5 Viscous Vapor Flow Model Leak Rates – Diffusion-Dominant Case.

Diameter (m)	Mixture Mass Flow Rate (kg/s)	Helium Transport Rate (Atoms/sec)
1 E – 6	3.9 E – 10	5.9 E + 16
1 E – 7	3.9 E – 12	5.9 E + 14
1 E – 8	3.9 E – 14	5.9 E + 12
E – 9	3.9 E – 16	5.9 E + 10
1 E – 10	3.9 E – 18	5.9 E + 8

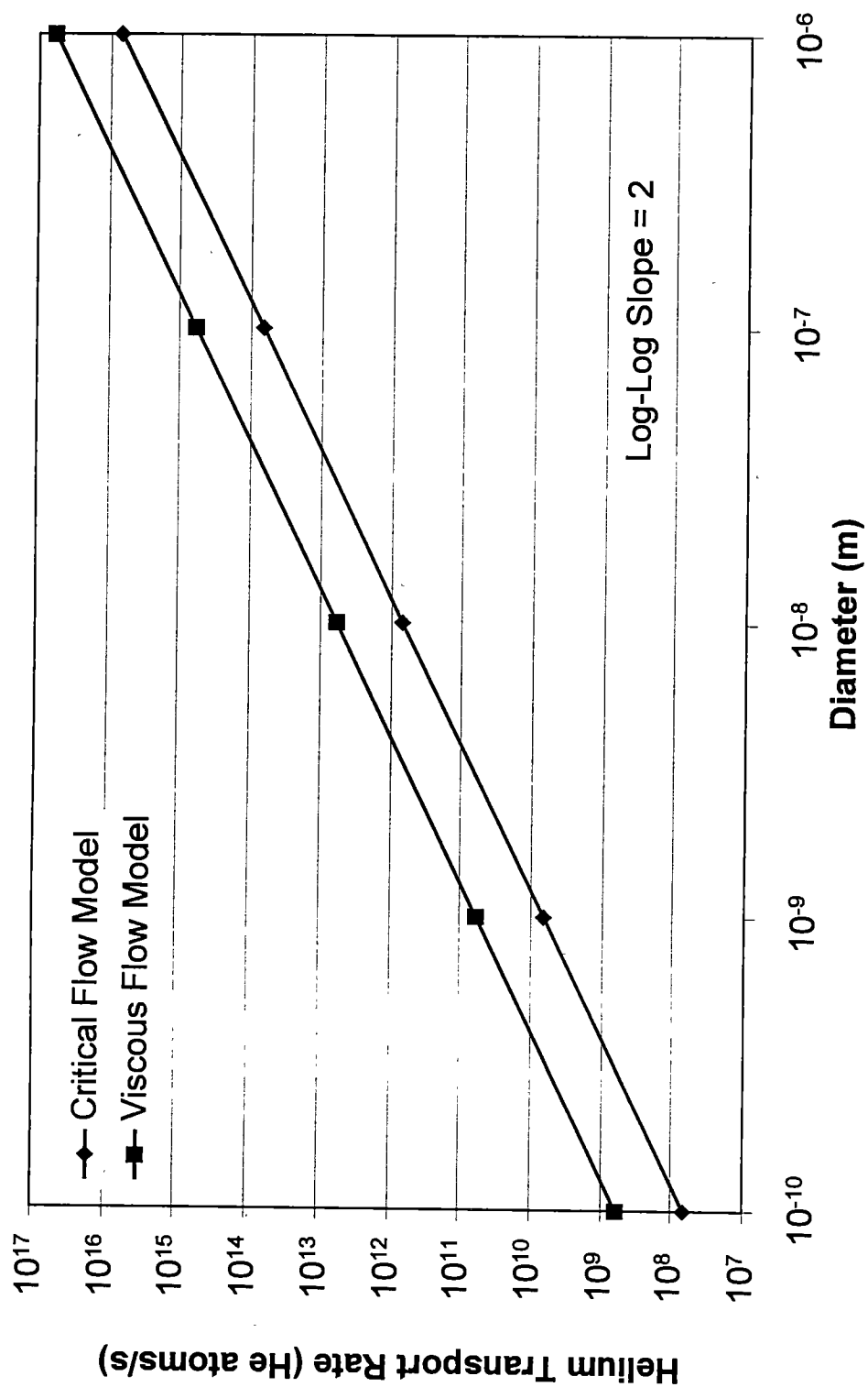


Figure 5.1 Helium Transport Rate In Leaks with Stationary Liquid-Vapor Interface
– Diffusion-Dominant Case.

vapor flow model. The critical flow limit is lower than the flow rate limit resulting from viscous shear stresses over the total range of diameters considered for the diffusion-dominant inlet condition. Therefore, the critical flow limit is assumed to represent a conservative estimate of the helium transport rate for the diffusion-dominant case.

Evaporation-Dominant Case

For the evaporation dominant case, the leak flow is treated as a pure water vapor flow, ignoring the effects of the entrained helium. The critical flow model is used to evaluate the critical limit for this case. Both the calculated flow rates and the resulting helium transport rates are shown in Table 5.6, where the helium transport rate is calculated using the saturated helium concentration of 2.8×10^{20} helium atoms per kg of mixture.

In order to evaluate the effects of shear stresses at the wall, the flow rate is evaluated using the modified Bernoulli equation. The calculated flow rates and the resulting helium transport rates are shown in Table 5.7. Again, the saturated helium concentration of 2.8×10^{20} helium atoms per kg of mixture is used to extrapolate the helium transport rate from the calculated flow rate.

The helium transport rate for the evaporation-dominant case is shown Figure 5.2 as a function of leak diameter for both the critical flow and viscous flow regimes. The critical flow model predicts slower transport rates than those predicted by the viscous flow model for all diameters considered. Clearly the helium transport rate is limited by the critical flow limit for leaks of this size. Therefore, the critical flow model calculations are used as a conservative estimate of the helium transport rate for the evaporation-dominant case.

In order to provide a conservative analysis of the performance of the vessel integrity monitoring system, the more limiting helium transport rate from the diffusion-dominant and evaporation-dominant cases is used as a conservative approximation of the transport rate in leaks exhibiting interface development. In the evaluation of the effectiveness of the vessel integrity monitoring system, the evaporation-dominant case provides a conservative estimate for the helium transport rates in leaks where interface development occurs.

Table 5.6 Critical Vapor Flow Model Leak Rates – Evaporation-Dominant Case.

Diameter (m)	Mixture Mass Flow Rate (kg/s)	Helium Transport Rate (Atoms/sec)
1 E – 6	8.0 E – 12	2.2 E + 9
1 E – 7	8.0 E – 14	2.2 E + 7
1 E – 8	8.0 E – 16	2.2 E + 5
1 E – 9	8.0 E – 18	2.2 E + 3
1 E – 10	8.0 E – 20	2.2 E + 1

Table 5.7 Viscous Vapor Flow Model Leak Rates – Evaporation-Dominant Case.

Diameter (m)	Mixture Mass Flow Rate (kg/s)	Helium Transport Rate (Atoms/sec)
1 E – 6	3.0 E – 11	8.4 E + 9
1 E – 7	3.0 E – 13	8.4 E + 7
1 E – 8	3.0 E – 15	8.4 E + 5
1 E – 9	3.0 E – 17	8.4 E + 3
1 E – 10	3.0 E – 19	8.4 E + 1

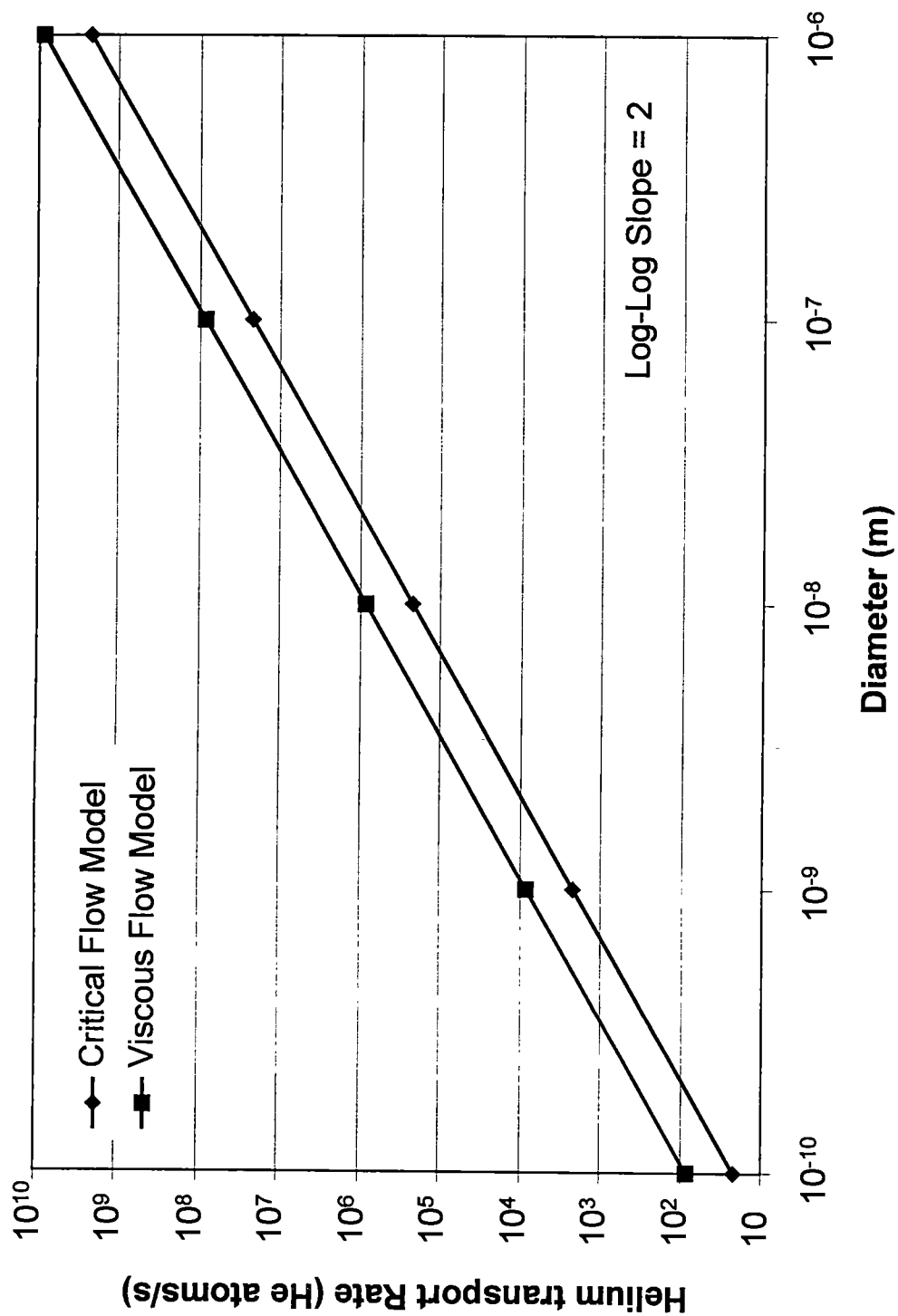


Figure 5.2 Helium Transport Rate In Leaks with Stationary Liquid-Vapor Interface
– Evaporation-Dominant Case.

Statistical Mechanics

The Knudsen-Clausing molecular flow model is used to evaluate the flow of pure helium in the diffusion-dominant case and pure water vapor in the evaporation-dominant case. Results for the diffusion-dominant and evaporation dominant cases are shown in Table 5.8 and Table 5.9, respectively. For comparison, the helium transport rates for the two cases are shown in Figure 5.3 as a function of leak diameter. Also included in Figure 5.3 are the helium transport rates resulting from the evaporation-dominant critical flow rate calculations. It is easily seen that for leaks less than 10^{-6} m in diameter, the evaporation-dominant molecular flow calculations provide the most conservative estimate of the helium transport rate.

Table 5.8 Molecular Flow Model Helium Transport Rates – Diffusion-Dominant Case.

Diameter (m)	Helium Transport Rate (Atoms/sec)
1 E – 6	1.15 E + 15
1 E – 7	1.15 E + 12
1 E – 8	1.15 E + 9
1 E – 9	1.15 E + 6
1 E – 10	1.15 E + 3

Table 5.9 Molecular Flow Model Helium Transport Rates – Evaporation-Dominant Case.

Diameter (m)	Helium Transport Rate (Atoms/sec)
1 E – 6	3.37 E + 8
1 E – 7	3.37 E + 5
1 E – 8	3.37 E + 2
1 E – 9	3.37 E – 1
1 E – 10	3.37 E – 4

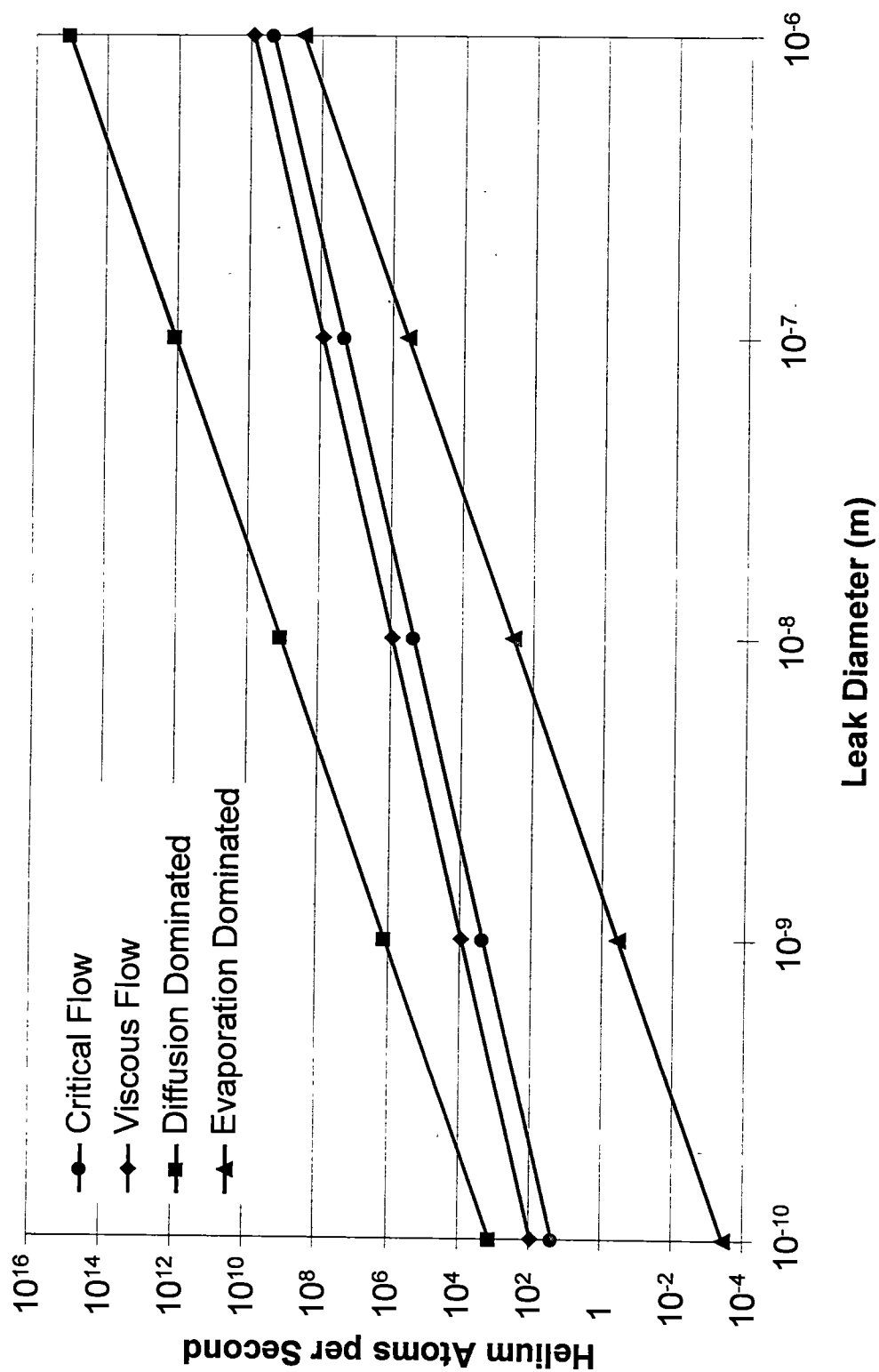


Figure 5.3 Helium Molecular Transport Rates. (Shown with Results from Critical and Viscous Flow Evaluations for Comparison)

CHAPTER 6

LEAK DETECTION CAPABILITY

The sensitivity of a leak monitoring system based on the detection of dissolved gasses is limited by the sensitivity of the detector. As the cooling jacket fluid leaks into the vacuum vessel, the dissolved gas leaves the solution. The partial pressure of the dissolved gas increases at a constant rate determined primarily by the leak geometry. The leak detection system will not indicate the presence of the leak until the partial pressure of the dissolved gas reaches the minimum detection limit of the gas detection equipment. Therefore, the effectiveness of the system for leak detection is quantified by the time required to detect a leak of a given diameter.

Gas Detection Technology

Current gas detection technology relies primarily on the use of Residual Gas Analyzer (RGA) systems. RGA's are essentially portable mass spectrometers that can identify the concentration of various atomic species. Unlike full-scale mass spectrometers which are designed to detect a full range of atomic species, RGA's are usually capable of detecting only a few select species.

Two variations of the RGA technology are commonly available. The more common utilizes a Faraday cup type detector, and the other is based on a collimated electron multiplier (CEM) type detector. The primary advantages of the Faraday cup type system are its higher operating pressure and temperature. This type of RGA can be utilized for vacuum systems operating at up to 10^{-4} Torr (10^{-2} Pa) and up to 250 °C. The CEM-based RGA is limited to systems which operate at less than 10^{-5} Torr (10^{-3} Pa) and 100 °C. The CEM-based RGA offers better detection characteristics however. The minimum detectable partial pressure for the CEM-type RGA is roughly 5×10^{-14} Torr versus 5×10^{-11} Torr for the Faraday cup variation. (Stanford Research Systems, 1997)

VIMS Detection Capability

Since the operating conditions of the Cold Neutron Source vacuum vessel are within the operation range of the CEM-type RGA, the additional detection capability of that system configuration is desirable. For the 22.9-liter Cold Neutron Source vacuum vessel, which operates at a temperature near the temperature of liquid hydrogen (approximately 20 K or -235 °C), a minimum detectable partial pressure of 5×10^{-14} Torr is equivalent to a minimum detectable population of 6.9×10^8 helium atoms. This minimum detectable concentration is used to evaluate the expected detection capabilities of the vessel integrity monitoring system.

The primary criterion that determines the efficiency of the system is the time required for detection of a leak of a given diameter. Using the most conservative helium transport rates from viscous liquid flow calculations, evaporation-dominant critical flow calculations, and evaporation-dominant molecular flow calculations, a conservative estimate of the time required to detect a leak can be determined. The minimum leak detection time is shown in Figure 6.1 as a function of leak diameter for straight-through leaks.

As the leak diameter approaches 2.76×10^{-6} m, the surface tension of the liquid begins to prevent the free penetration of liquid into the leak. As a result a liquid-vapor interface develops at the leak inlet. At this point, critical gas flows begin to dominate the other flow regimes. The evaporation-dominant case inlet conditions provide a conservative estimate of transport rates for leaks of this size. The minimum detection time decreases for leaks of this size since transport rates increase slightly when gas flow begins to replace liquid flow. This increase occurs because the viscous losses in the vapor flow do not prevent the fluid from reaching the critical flow limit as they do in the liquid case.

As the leak diameter approaches the mean free path of the helium atoms in the channel, which is approximately 1.35×10^{-7} m, the molecules begin to interact with the wall with greater frequency, and molecular flow begins to dominate. The detection time in the molecular flow range is greater than the detection time in the critical vapor flow range. This discontinuity again results from the inability of the fluid to reach the critical flow limit. In the molecular flow range, the detection time increases with decreasing diameter until the diameter becomes so small that molecular flow can no longer occur, and molecular

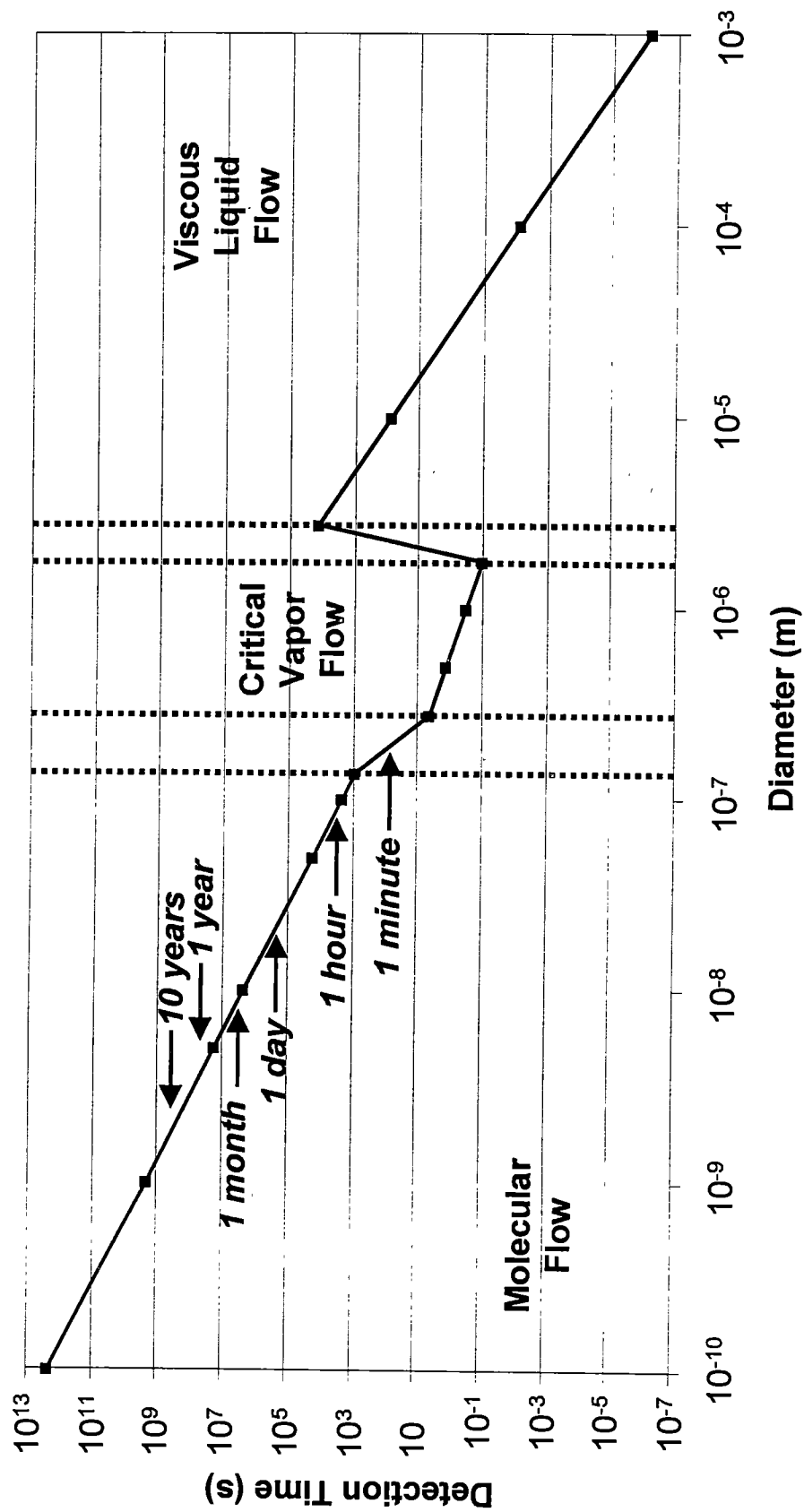


Figure 6.1 Leak Detection Time as a Function of Leak Diameter.

diffusion is the only available transport process. Since helium does not diffuse through aluminum and its alloys, the detection time for helium concentrations resulting from molecular diffusion approaches infinity.

CHAPTER 7

SENSITIVITY TO PRESSURE

The rate of helium transport across the vacuum boundary is dependent on the water jacket pressure. Increasing the pressure in the water jacket may enhance detection system performance. In order to determine the effects of increasing water jacket pressure, helium transport rates are calculated for water jacket pressures ranging from 1 atm to 20 atm (0.101325 MPa to 2.02650 MPa).

Helium Concentrations

The concentration of helium in the water is directly related to the pressure in the water jacket. As the pressure increases, the helium concentration in the water increases. The concentration of helium in the water jacket is calculated using Henry's law. The saturation helium concentration for various water jacket pressures is shown in Table 7.1.

Viscous Liquid Flow

For relatively large leaks in which liquid freely penetrates the leak, viscous liquid flow is the limiting transport process. When viscous liquid flow dominates, increases in water jacket pressure result in increased mass flow rates through the leak. As a result of the increased flow rate and the increased helium content, the rate which helium is transported into the vessel increases. Mass flow rates and helium transport rates are shown in Table 7.2 as a function of leak diameter and water jacket pressure. The helium transport rates are calculated using the helium concentrations given in Table 7.1.

Critical Vapor Flow

For smaller leaks ($< 10^{-6}$ m), surface tension prevents the flow of liquid through the leak, and a liquid vapor interface develops near the leak inlet. When the pressure differential across the liquid-vapor interface resulting from surface tension is greater than the inlet pressure, the liquid no longer flows through the leak. The increase in water jacket

Table 7.1 Saturation Helium Concentration for Various Pressures.

Jacket Pressure (atm)	Saturated Helium Concentration (atoms/ water molecule)
1	8.24 E – 6
5	4.12 E – 5
10	8.24 E – 5
20	1.65 E – 4

Table 7.2 Viscous Liquid Flow Model Leak Rates.

	Helium Transport Rate (He atoms/s)			
Diameter (m)	1 atm	5 atm	10 atm	20 atm
1 E - 4	9.8 E 10	2.5 E 12	1.0 E 13	4.1 E 13
1 E - 5	9.8 E 6	2.5 E 8	1.0 E 9	4.1 E 9
1 E - 6	9.8 E 2	2.5 E 4	1.0 E 5	4.1 E 5

pressure moves the boundary between the liquid flow case and the diffusion/evaporation case to smaller leak diameters, with the diameter for liquid penetration inversely proportional to the inlet pressure.

The diffusion-dominant and evaporation-dominant cases are both affected by the increase in water jacket pressure. The evaporation-dominant case uses the saturation pressure of water at the fluid temperature as the inlet pressure. The mass flow rate does not change with increasing water jacket pressure, but the change in pressure results in an increase in the saturation helium concentration in the water jacket coolant. As a result, the helium flow rate for the evaporation-dominant case increases with increasing water jacket pressure. The diffusion-dominant case uses the saturation pressure of the helium in the liquid solution as the inlet pressure. Therefore, the mass flow rate increases with increasing water jacket pressure. The transport rate for the evaporation-dominant case is significantly lower than the diffusion-dominant flow rate. The evaporation-dominant case is used as a conservative estimate of the critical vapor flow rate when evaluating the effectiveness of the leak detection system. Figure 7.1 shows the helium flow rate as a function of leak diameter, water jacket pressure, and assumed inlet conditions for the critical vapor flow regime.

Molecular Flow

For leak diameters on the order of the mean free path of the fluid molecules, the transport rate is evaluated using the Knudsen-Clausing molecular flow model. The evaporation-dominant case continues to provide the most conservative estimate of the helium transport rate. For the evaporation-dominant case, the mean free path remains constant as the water jacket pressure increases since the inlet pressure is the saturation pressure of the water at 40 °C. Helium transport rates for the molecular flow regime are shown in Table 7.3 as a function of leak diameter and water jacket pressure. The pressure dependence of the helium transport rate in this regime is again a result of the pressure dependence of the saturation helium concentration.

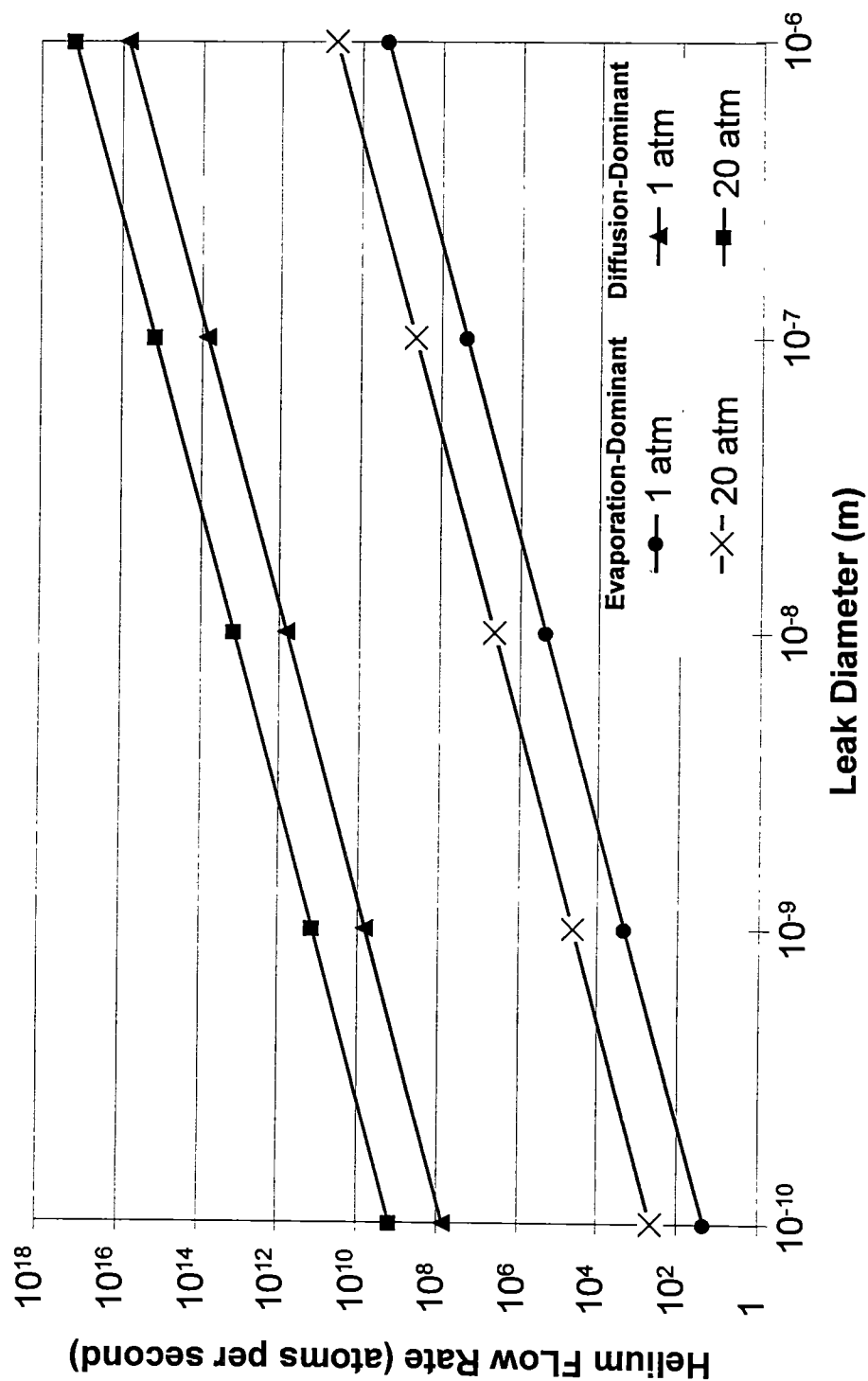


Figure 7.1 Critical Vapor Flow Model Transport Rates.

Table 7.3 Molecular Flow Model Transport Rates.

	Helium Transport Rate (He atoms/s)			
Diameter (m)	1 atm	5 atm	10 atm	20 atm
1 E - 6	3.37 E 8	1.69 E 9	3.37 E 9	6.75 E 9
1 E - 7	3.37 E 5	1.69 E 6	3.37 E 6	6.75 E 6
1 E - 8	3.37 E 2	1.69 E 3	3.37 E 3	6.75 E 3
1 E - 9	3.37 E - 1	1.69 E 0	3.37 E 0	6.75 E 0
1 E - 10	3.37 E - 4	1.69 E - 3	3.37 E - 3	6.75 E - 3

Detection Capability

The effectiveness of the vessel integrity monitoring system for leak detection is quantified by the time required to detect a leak of a given diameter. Figure 7.2 shows the detection time as a function of leak diameter, assuming a CEM-type residual gas analyzer is used for detection. For relatively large leaks, viscous fluid flow is assumed to dominate. For smaller leaks the evaporation-dominant case is used as conservative estimate of the actual flow rate for both the critical vapor flow regime and the molecular flow regime. As the water jacket pressure increases, the range in which critical vapor flow can occur is reduced. For water jacket pressures approaching 20 atm (2.0265 MPa), the viscous liquid flow region transitions directly to the molecular flow region when the liquid-vapor interface develops. Slight inflections in the curves in the transition region result from the functional fit and do not represent aberrations in the calculated data.

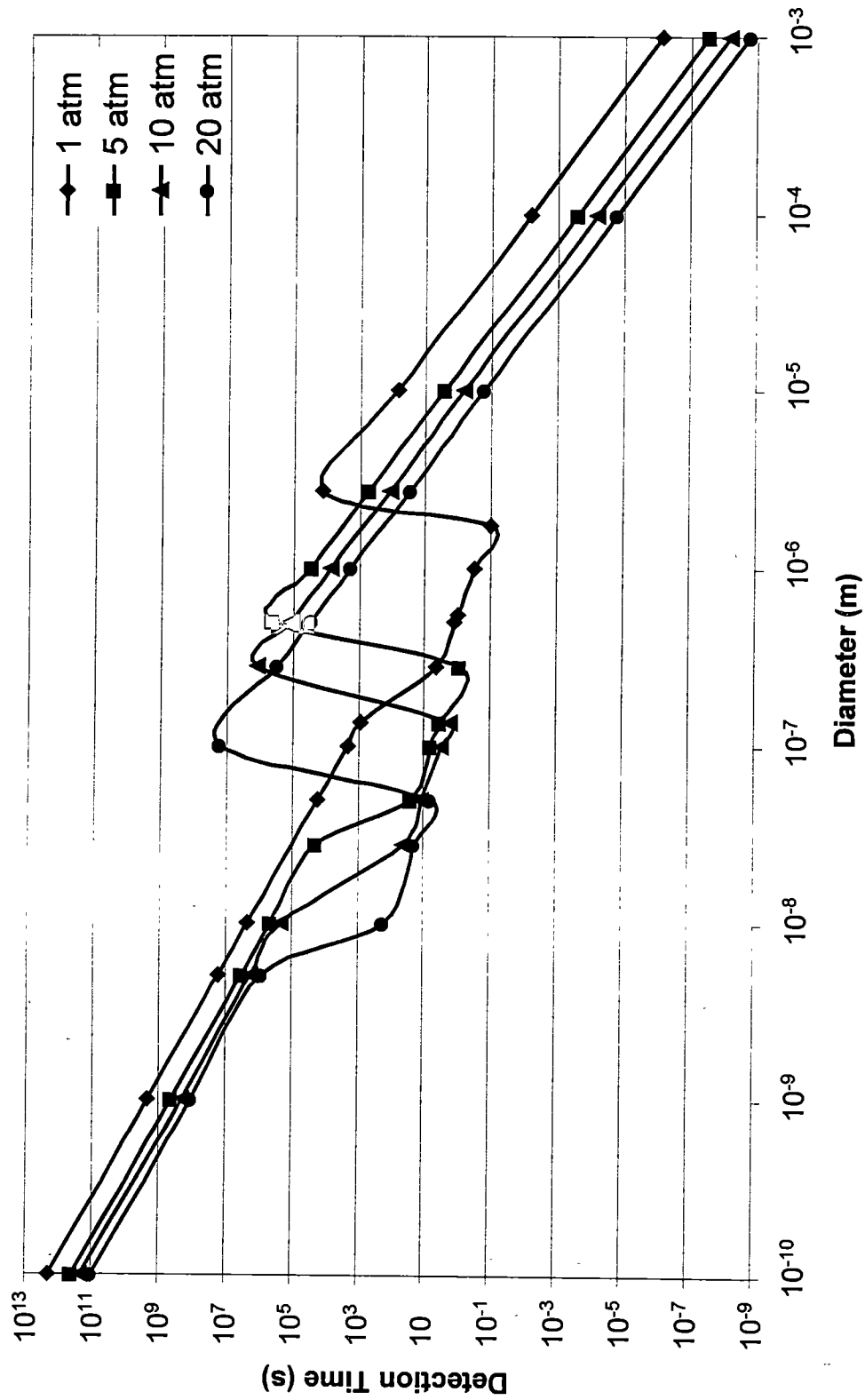


Figure 7.2 Pressure Sensitivity of Minimum Detection Time for Various Leak Diameters.

CHAPTER 8

CONCLUSION

The detection of leaks in vacuum vessels is critical to the operation of a wide variety of commercial and scientific equipment. The use of gas detection systems to monitor for leak development by surrounding the vacuum vessel with a cover jacket of helium or other gases with low background concentrations within the vessel is common. However, the cover gas might not provide adequate heat transfer characteristics to satisfy cooling requirements for the vacuum vessel in some applications. This study shows that dissolving the desired cover gas in a water jacket that surrounds the vacuum vessel provides a highly sensitive leak detection system for these applications.

The proposed vessel integrity monitoring system, which utilizes helium-saturated water to detect leaks in the vacuum vessel of the Cold Neutron Source, provides a sensitive instrument to detect sub-micron-sized leaks long before they endanger the integrity of the vacuum vessel. Three flow regimes provide a conservative estimate of the rate of helium transport into the vacuum vessel over a range of diameters from 10^{-3} to 10^{-10} meters: viscous liquid flow, evaporation-dominant critical gas flow, and evaporation-dominant molecular gas flow. The conservative flow rate is used to determine the time required for helium concentration in the vacuum vessel to reach detectable levels. While a leak in the Cold Neutron Source vacuum vessel that is larger than 10^{-7} m can be detected in less than thirty minutes, smaller leaks can require significantly more time to detect. A leak of diameter 10^{-8} m has a minimum detection time of approximately 20 days. Increasing the water jacket pressure can increase the sensitivity of the vessel integrity monitoring system, but increasing the water jacket pressure by factor of 10 reduces the time required to detect a leak of a given size by only a factor of 10 in most cases.

The leak types modeled here represent a small subset of the range of leak types that might appear in a vacuum pressure boundary. Leaks could have tapered cross sections or present a labyrinthine path for the flow. The idealized leak types considered serve to

provide basic information on the viability of the leak detection technique and have helped to clarify the flow physics important to the function of such a leak detection system.

CHAPTER 9

RECOMMENDATIONS FOR FUTURE WORK

In this study, the dissolved gas is assumed to remain in solution throughout the leak in the cases where free liquid penetration occurs. In reality, the gas might dissociate from the solution as the pressure decreases within the flow channel. Therefore, the effects of helium gas bubbles should be considered in addition to the water vapor bubbles that might develop when the fluid reaches the saturation pressure.

Furthermore, the transport rate across the liquid-vapor interface that develops in small leaks as a result of surface tension effects was approximated by the bounding evaporation-dominant and diffusion-dominant cases. An effort should be made to more accurately determine the transport rate at the interface and link that rate to the gas flow calculations.

These additional studies would provide a clearer picture of the flow physics in the transition region between viscous liquid flow and molecular flow. Additional knowledge of the dominant physics in this region could enhance the ability of the vessel integrity monitoring system to characterize leaks that may develop in the vacuum vessel.

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VITA

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