

NAVY SUPPLY SYSTEM RESEARCH -
OBJECTIVES AND PLANS

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M A T H E M A T I C A

92-A Nassau Street

Princeton, New Jersey

Subsidiary of **MARKET RESEARCH CORPORATION OF AMERICA**

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Summary	1
Introduction	1
Logistic Structures in the Navy Supply System	2
Product-Class Inventory Systems	5
Mathematical Formulations of Product-Class Inventory Systems	
Harlan D. Mills	
Comments for Decision	
September 1959	
Integrated Distribution Systems	8
Mathematical Formulations of Integrated Distribution Systems	
Fixed Cost Transportation Problems	
Military Supply	
Contract Nonr 2928(00)	
Bureau of Supplies and Accounts	
United States Navy	
Decentralized Ordering Systems	10
Mathematical Formulations of Decentralized Ordering Systems	
6035-10	
An Evaluative Survey of Inventory Decision Models	
Stability Problems in Decentralized Ordering Systems	

Contents

	Page
Summary	i
Introduction	1
Logistic Structures in the Navy Supply System . .	2
Product-Class Inventory Systems	5
Mathematical Formulations of Product- Class Inventory Systems	
Criteria for Decentralizing Decision Procedures	
Integrated Distributing Systems	8
Mathematical Formulations of Integrated Distributing Systems	
Fixed Cost Transportation Problems	
Military Worth of Inventory	
Decentralized Ordering Systems	10
Mathematical Formulations of Decentralized Ordering Systems	
An Evaluative Survey of Inventory Decision Models	
Stability Problems in Decentralized Ordering Systems	

SUMMARY

This Report formulates objectives and plans for logistics research by MATHEMATICA in the Navy Supply System, particularly with regard to its multi-echelon and large scale systems characteristics. In this formulation, problems of logistic policy in the Navy Supply System are divided into three areas, namely

1. problems of operating logistic subsystems on a centralized basis, by means of overall systems information and criteria
2. problems of operating logistic subsystems on a decentralized basis, by means of multi-echelon structures employing localized information and criteria at each point in such structures
3. problems of organizing and synthesizing logistic subsystems of types 1 and 2 above into effective large scale logistic systems.

These problem areas are then further decomposed into projects which make up the present research program under way at MATHEMATICA.

INTRODUCTION

In this Study of Multi-echelon Supply System Models, MATHEMATICA seeks to discover, analyze, and assess, at advanced research levels, effective decision processes in the Navy Supply System. It recognizes two major facts in the situation. First, in order to be of ultimate practical interest, effective decision processes must have certain inherent simplicities, for use on a varied and repetitive scale. For this reason, broad concepts and methods of generating good decisions are of greater interest than sharp techniques for finding optimal decisions. Second, the Navy Supply System, as a classic and continuing example of a logistic operation, has the benefit of much intelligence and experience, including the particularly effective learning process of an ongoing organization. Progress on the basis of common sense and arithmetic, while perhaps possible, is likely to be marginal in the face of such a background. If easy answers, or major improvements, do appear possible on the basis of elementary reasoning, it may be well to examine their origins more thoroughly in regard to the realities of the Navy Supply System. Because of this, mathematical research of a high order of sophistication and ingenuity seems required to provide real hope of substantial progress.

Initial work in the Study is directed at formulating practical objectives and identifying scientific resources for an effective research program. This program is considered in logistic terms; the usual scientific liberties have been taken in abstracting an area of research out of a practical situation. Thus problems of personnel, morale, security, and even materials handling, are ignored in order to better concentrate on problems of inventory control, procurement, redistribution, etc.

This Report describes research objectives and plans developed to date for this Study. The next Section formulates certain basic concepts and perspectives of the Navy Supply System within which the remainder of the Report is developed. The remaining Sections deal with three research areas so formulated.

LOGISTIC STRUCTURES IN THE NAVY SUPPLY SYSTEM

This Study is concerned with decision processes of the Navy Supply System which order the procurement, distribution, redistribution, disposal, and release of materiel to consuming Navy units. These decision processes operate under a wide variety of rapidly changing conditions of information, criteria, and analytical resources. They face such difficulties as many decision points, wide geographic dispersion, appreciable intransit inventories, alternative routes of supply, alternative modes of procurement and disposal, and uncertainty of demands by consuming Navy units. The Study seeks, first of all, a perspective and overall understanding of the totality of these decision processes, and their role in the organization and operation of the Navy Supply System. With such a structure, specific problems can be abstracted for concentrated study, and specific results related back to a more coherent whole for assessment and implementation.

In its practical operation, a large scale system, such as the Navy Supply System, is necessarily managed by means of a spectrum of centralized and decentralized decision processes. Where the importance of the item and amounts involved seem to warrant it, and where information processing loads allow, decisions may be made centrally, with the entire System's situation under consideration. On the other hand, where information processing problems seem to outweigh the benefits of using total systems criteria, decentralized decisions, based on local information and local criteria may expedite the operation of the System. In practice, many situations fall between these extremes, with some subsystems criteria being employed and decisions made on a partially decentralized basis. Thus the complexity and scale of operation of the Navy Supply System call for an overall formulation, within which specific logistic problems of both centralized and decentralized origins can be abstracted and synthesized.

For convenience, the operations of the Navy Supply System will be most grossly structured by items, or classes of items, of materiel it handles. With respect to each item, the Navy Supply System is physically embodied in what

we shall call a Product-Class Inventory System (of inventory points) - all locations in Navy jurisdiction storing the given item (from the largest depot to a spare fuse box). In turn, a Product-Class Inventory System is, usually, a synthesis of several subsystems, of two general types, namely

1. a single Integrated Distribution (sub)System of major inventory points (called reporting points), whose inventories are managed centrally, on the basis of over-all systems information and criteria, and
2. a set of Decentralized Ordering (sub)Systems, which organize the remaining inventory points (called ordering points) into multi-echelon supply systems originating from reporting points, each such system being managed throughout in a decentralized manner, on the basis of local information and criteria at its individual ordering points.

The reporting points play dual roles in these latter two types of systems - as consumers in the Integrated Distributing System and suppliers to Decentralized Ordering Systems - and thereby synthesize these subsystems into an overall Product-Class Inventory System.

In this general structure of operations in the Navy Supply System, two classes of problems arise:

- 1) How to organize a Product-Class Inventory System.
What overall pattern of information and control is to be used in the System, and
- 2) How to operate a Product-Class Inventory System.
Given an organization of Integrated Distributing and Decentralized Ordering Systems, how to construct effective decision processes in them.

The latter problem can be decomposed in turn into two subproblems:

- 2a) How to operate an Integrated Distributing System.
How to construct effective centralized decision processes.

2b) How to operate a Decentralized Ordering System.

How to construct structures of effective decentralized decision processes.

These three final class of problems 1, 2a, 2b, furnish the organization of the remainder of this Report and the research program of the Study. Each class gives rise to rather different technical problems.

1. Product-Class Inventory Systems. The organization of Product-Class Inventory Systems is the least well-defined class of problems formulated, with little or no theoretical precedent. As will appear, the most relevant mathematical resources available seem to be those related to the development of extensive games in game theory - further determination awaits progress in formulating problems and possibilities.

2a. Integrated Distributing Systems. The programming of interdependent activities appears to be the most relevant mathematical concept in constructing decision processes in Integrated Distributing System. Beginning with the simplest linear transportation problems, realism seems to require adding nonlinearities, programming under uncertainties, and up to time horizons, and concepts of implicit valuations of dynamic programming.

2b. Decentralized Ordering Systems. The cascading and aggregation of servo-statistical properties of decision procedures seems to be the most effective point of view in Decentralized Ordering Systems. These Systems are studied with dual purposes, namely 1) to construct effective decision processes, and 2) to characterize consumption requirements in Integrated Distributing Systems. Techniques of stochastic processes and dynamic programming appear to offer greatest promise.

PRODUCT-CLASS INVENTORY SYSTEMS

The problem of organizing Product-Class Inventory Systems, particularly questions of where burdens of centralized information processing and systems criteria outweigh their benefits, and decentralized decision procedures in fact expedite operations, is a very broad one. Although it is being resolved, in one way or another every day, at the practical level in the Navy Supply System, little scientific research relates to it. Rather, the overwhelming bulk of research in inventory theory, logistics and related matters, is concerned only with how to operate pieces of an already well organized system. The question of whether a decision procedure is to be a centralized or decentralized one is usually taken as already determined and the problems deal solely with how to convert a given stream of information into an effective sequence of operating decisions within a relatively narrow organizational framework.

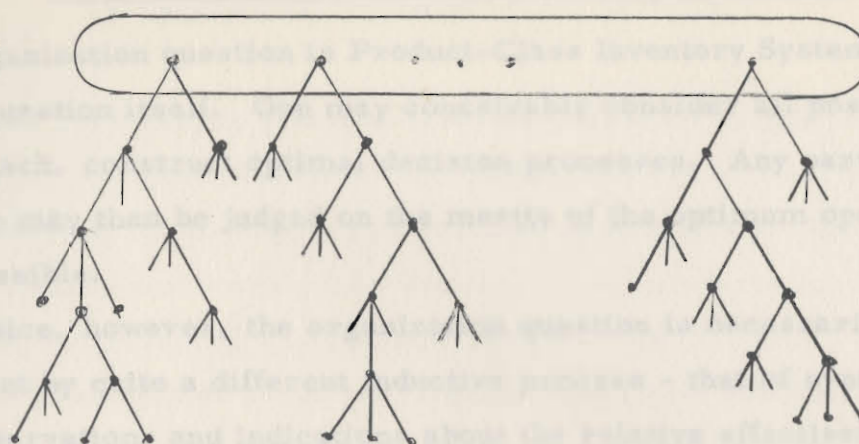
In recognition of the importance of these organization problems, in addition to operational problems of the Navy Supply System, this Study singles out the formulation and investigation of Product-Class Inventory Systems, regarded as syntheses of their centralized (Integrated Distributing) and decentralized (Decentralized Ordering) subsystems. Research is planned along the following fronts.

Mathematical Formulations of Product-Class Inventory Systems

The decomposition of Product-Class Inventory Systems into an Integrated Distributing System and a set of Decentralized Ordering Systems can be described in graphic form as a "forest" of "trees", each tree representing a multi-echelon Decentralized Ordering System, and the roots of the trees, encircled, forming the Integrated Distributing System, as illustrated on the next page.

Reporting
points:

Ordering
points:



Navy users

This decomposition visualizes only gross organization characteristics - completely centralized or completely decentralized operations - whereas a whole spectrum of possibilities in fact exists. The mathematical paraphernalia of graphs, sets, etc., as used above provides potentials for sharper and more comprehensive descriptions of the organization of Product-Class Inventory Systems. The opportunities here seem similar to those in the description of games in extensive form, where the mathematization of a familiar but more or less ill-defined set of competitive phenomena have lead to much crisper and surer understandings.

This Study plans to further investigate possibilities and potentials of mathematical descriptions of large scale logistic systems, especially with regard to organization, information patterns, and decision criteria.

Criteria for Decentralizing Decision Procedures

The organization question in Product-Class Inventory Systems is, in principle, a systems question itself. One may conceivably consider all possible organizations, and within each, construct optimal decision processes. Any particular form of organization may then be judged on the merits of the optimum operating performance it makes possible.

In practice, however, the organization question is necessarily resolved to a great extent by quite a different inductive process - that of synthesizing many intuitive observations and indications about the relative effectiveness of centralized and decentralized decision procedures in various part of the Product-Class Inventory System. Thus, whereas in concept, one might wish to treat a Product-Class Inventory System only in its entirety, in practice it is convenient to develop criteria for dealing with its components, a few at a time, in approximating an overall systems viewpoint. In particular, the question often arises as to whether a given inventory point should be regarded as part of a centralized subsystem, or be decentralized, with the attendant benefits in lower information processing loads. By and large, such questions have not been well stated, and few mathematical problems have been abstracted from them. This Study plans to initiate work in this direction, with the beginning emphasis on 1) formulating concepts and problems, 2) surveying and adapting related concepts from the general literature of logistics, and 3) assessing research techniques.

INTEGRATED DISTRIBUTING SYSTEMS

Integrated Distributing Systems of major inventory points pose the most nearly classical systems problems of balancing individual interdependent activities in the best interest of total systems performance. Much of the literature of operations research is applicable, particularly the literature of mathematical programming and activity analysis. In this area, the "transportation problem" has been studied to the point where singularly effective techniques exist to exploit its special structure. Much of the work planned in Integrated Distributing Systems seeks to take advantage of this resource, in

- 1) formulating the general problems of procurement, disposal and redistribution as much in the format of transportation problems as possible, and
- 2) extending techniques for solving transportation problems to more complex cost structures.

In addition, the question of military worth of inventories, and conversely, the evaluation of the effect of stockouts and unfilled orders is also being investigated in its own right. Research is planned along the following lines.

Mathematical Formulations of Integrated Distributing Systems

One of the simplest problems which can be formulated in an Integrated Distributing System is that of pure redistribution of an item from reporting points with excesses to points with deficiencies, say on a criteria of least transportation cost. The realities of the Navy Supply System require further complexities in the form of

- 1) procurement and disposal activities;
- 2) time requirements for transportation, procurement, and disposal activities;
- 3) time dimensions in materiel requirements - recognizing distinctions among immediate, intermediate, and long range requirements;
- 4) uncertainties and contingencies in materiel requirements.

This Study seeks to introduce these additional complexities in such ways as to retain as much of the structure of transportation problems as possible, in order to exploit the special properties and techniques that obtain.

Fixed Cost Transportation Problems

The ordinary linear transportation problem allows very effective computational procedures; yet, in many cases realism demands the introduction of a fixed cost, due to using a route or a facility at all, in addition to a linear (unit) transportation or procurement cost. While recent developments have made solutions to these fixed cost transportation problems theoretically available, such solutions are still quite expensive computationally in comparison with linear problems.

Exploratory research in this Study has already produced a promising technique for obtaining approximate solutions to these fixed cost problems with no more computational effort than is required for linear problems. In addition, upper and lower bounds are obtained for the exact solution, to check the approximation.

Military Worth of Inventory

At some point, in every decision process, whether implicitly or explicitly recognized, the question of the military worth of inventories arises. This question is intimately tied to uncertainties - were there no uncertainties, the problem would be conceptually trivial. Two initial types of exploration appear worth while in this connection, namely

1. An investigation of the applicability of certain concepts of utility under uncertainty - the von Neumann-Morgenstern utility theory - which may give rise to operational means of developing additive value scales for inventories, and
2. A census of various measures of supply failure, of such conditions as out-of-stock, unfilled orders, etc., and the development of logical relationships which exist among them.

DECENTRALIZED ORDERING SYSTEMS

Decentralized Ordering Systems occupy a paradoxical role as objects of research in that 1) most of classical inventory theory is concerned with the problems of components of Decentralized Ordering Systems, while 2) practically nothing is said in inventory theory about the operation of such Systems as entities. A major problem in Decentralized Ordering Systems is that of understanding how very many interrelated decision processes, each based on local information and local criteria, aggregate into a total system operation. This understanding is required to 1) evaluate the performance of alternative structures of decision processes in Decentralized Ordering Systems, and 2) understand the general statistical character of the requirements which Decentralized Ordering Systems place on their source reporting points. In recognition of these kinds of problems, research is planned in the following areas.

Mathematical Formulations of Decentralized Ordering Systems

The internal structure of a Decentralized Ordering System gives rise to a complex collection of materiel conservation relationships. These relationships, in conjunction with decision processes employed in the System, in turn, determine statistical relationships among measures of activities and positions in these points - that is, relationships among means, variances, autocorrelations, etc. of demands on, inventories at, or in intransit to, orders placed by, etc., various ordering points. The view of such Decentralized Ordering Systems as complex servo-statistical processes seems most promising in finding logical ways of aggregating discrete, microscopic local descriptions into stochastic, macroscopic global descriptions of their operations.

This Study seeks to formulate the operation of Decentralized Ordering Systems from this servo-statistical perspective, in order to exploit potentially relevant literature, stated in other terms, in servo-mechanics, communication theory, stochastic processes, etc.

An Evaluative Survey of Inventory Decision Models

The general lack of global perspectives and the oftentimes embarrassing riches of local viewpoints of operations in Decentralized Ordering Systems is, itself, a comment on the relative immaturity of Inventory Theory. It turns out occasionally that two viewpoints, or decision models, while apparently concerned with the same realities, give conflicting evidence in decision problems. These conflicts are usually difficult to resolve because incompatibilities between the models make comparisons difficult or impossible.

In the absence of sufficient conceptual frameworks to resolve and eliminate such difficulties, descriptions and characterizations of various general types of decision models in as comparable terms as possible, even on a necessarily ad hoc basis, can serve useful purposes in 1) furnishing policy makers and decision makers in Decentralized Ordering Systems with additional perspectives and insights, and 2) possibly suggesting, via inductive associations, new conceptual structures for resolving comparative problems at more fundamental levels. This Study plans to make comparative studies of decision models in inventory theory literature as seems appropriate to these ends.

Stability Problems in Decentralized Ordering Systems

Properly speaking, operating a Decentralized Ordering System on a decentralized basis is a total systems problem, and all aspects of the activity contribute in interrelated ways to the total performance. However, as a practical matter, it appears that such problems can be decomposed to a good degree of approximation into 1) "problems of level" and 2) "problems of stability". That is, into questions of 1) what average values of inventories, orders, etc. should be, and 2) how to maintain as much stability as possible under these average value targets. It is planned to handle questions of the first type, by and large, in the general considerations of Decentralized Ordering Systems. But questions of stability seem involved enough to single out for specific attention.

Since Decentralized Ordering Systems are operated on a decentralized basis the stability referred to is necessarily statistical stability. For example, given certain statistical demands on an ordering point, it can be shown that the use of certain rather standard decision procedures will result in variabilities in these demands being amplified in the orders placed on the next echelon. Further analysis permits the determination of decision processes which dampen these variabilities as demands within the System are propagated from one echelon to the next. This Study plans to formulate and classify decision processes with regard to their performance in dampening variation between echelons, and to characterize the ultimate statistical loads which multi-echelon structures of decision processes place on a source reporting point.

