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FUZZY DESIGN PROCEDURE

FOR SINGLE ATTRIBUTE SAMPLING PLAN:

DECISION-MAKING IN A FUZZY ENVIRONMENT

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Sung-Ho Son

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ABSTRACT

This paper presents an alternative design procedure for solving single attribute sampling plans using fuzzy set theory. This design attempts to properly represent risk conditions (traditionally given in a classical mathematical model) using a fuzzy mathematical model in an attempt to illustrate the value of fuzzy set theory to Industrial Engineering applications.

This proposed design utilizes, as its base conditions, a conventional single attribute sampling plan developed by A. Hald [11,12]. Specifically, the proposed design attempts to generate sample size requirements (which can be compared to sample size requirements by Hald) to determine the value of applying fuzzy set theory to single attribute sampling plans similar to sampling plans introduced by H. Ohta and H. Ichihashi [13] (which is based on fuzzy set membership functions). However, the proposed design modifies Ohta & Ichihashi's membership functions of the producer's and the consumer's risks to be more representative of membership functions intended by Hald.

This research involves developing a computer program to ease the implementation of the design procedure by performing most of the tedious and iterative calculations involved in obtaining the solution.

The proposed design procedure is applied to a range of

problems which are then presented and compared to results obtained by applying Hald's conventional design. This illustrates an alternative approach in obtaining a comparable qualitative-based solution to Hald's conventional, quantitative-based solution, in terms of the number of samples required. The research illustrates that all solution space in Hald's solution are equal, if not superior, to the proposed design. However, further research is needed to estimate membership functions for upper tolerance limits for both risk conditions and the means of interpreting the results of fuzzy analysis.

TABLE OF CONTENTS

CHAPTER		PAGE
1.	INTRODUCTION	1
	1.1 PURPOSE OF STUDY	1
	1.2 PROBLEM STATEMENT	3
	1.3 SUMMARY	5
2.	LITERATURE REVIEW	7
	2.1 FUZZY SET THEORY AND ITS RELEVANCE TO INDUSTRIAL ENGINEERING	7
	2.2 CURRENT APPLICATION USES IN INDUSTRIAL ENGINEERING USING FUZZY SET THEORY	11
3.	FUZZY METHODOLOGIES	13
	3.1 CLASSICAL SETS	13
	3.2 FUZZY SETS	16
	3.3 FUZZY GOALS, CONSTRAINTS, AND DECISIONS	24
4.	SINGLE ATTRIBUTE SAMPLING PLAN	28
	4.1 SAMPLING PLAN	28
	4.2 OPERATING CHARACTERISTIC CURVE	31
	4.3 CONSUMER'S AND PRODUCER'S RISKS	36
	4.4 DESIGN OF SAMPLING PLAN	37

5. ADAPTATION TO FUZZY METHODOLOGIES 41

5.1 CONVENTIONAL DESIGN 41

5.2 FUZZY DESIGN 44

5.3 PROPOSED FUZZY DESIGN 48

6. RESULT AND CONCLUSION 53

6.1 NUMERICAL EXAMPLE 53

6.2 DISCUSSION 63

6.3 CONCLUSION 67

LIST OF REFERENCES 70

APPENDIXES 74

A 75

B 76

C 77

D 80

VITA 84

LIST OF FIGURES

FIGURE		PAGE
1.1	Thesis Overview	6
3.1	Set A Contained in U, With Element x Being a Nonmember of Set A	15
3.2	Membership Function of A, $\mu_A(x)$	19
3.3	Representation of Normality and Convexity	20
3.4	Intersection of Fuzzy Sets A and B	23
3.5	Union of Fuzzy Sets A and B	23
3.6	Definition of Decision D	27
3.7	Definition of Decision D Given C_1, C_2, C_3 and G_1, G_2, G_3	27
4.1	OC Curve With (n=76,c=2) Sampling Plan	31
4.2	Ideal OC Curve for Figure 3.1	34
4.3	OC Curves With Various Sampling Plan	35
4.4	Two Specific Points on OC Curve	39
5.1	Membership Function, $\mu_A(\alpha')$	45
5.2	Membership Function, $\mu_B(\beta')$	46
5.3	Proposed Membership Function, $\mu_A(\alpha')$	49
5.4	Proposed Membership Function, $\mu_B(\beta')$	50
5.5	Membership Function of, $\mu_n(n)$	51
6.1	Illustration of Equation (5.1)	56
6.2	Illustration of Equation (5.2)	56
6.3	Intersection of A, B, and n for c=3	61

6.4 Intersection of A, B, and n for c=2 61

6.5 Intersection of A, B, and n for c=1 62

6.6 Intersection of A, B, and n for All c 62

LIST OF TABLES

TABLE		PAGE
2.1	Various Problems in Today's Industry	10
4.1	Probability of Acceptance (n=76, c=2)	33
6.1	Membership Function **A**, $\mu_A(n)$	58
6.2	Membership Function **B**, $\mu_B(n)$	59
6.3	Examples Comparing Hald's (H) and Proposed (P) Methods for Determining Single Attribute Sampling Plan	64
6.4	Determining Sampling Plan (α=0.05, β=0.1) ..	66

CHAPTER 1

INTRODUCTION

System complexity, due to advances in science and technology, are in direct correlation with the complexity of the decision-making requirements in Industrial Engineering (IE). Specifically, decisions have become increasingly uncertain, and hence more difficult to analyze. According to Bellman & Zadeh [2],

> "... the decision-making problems now take place in a fuzzy environment in which the goals, the constraints, and the corresponding possible actions are represented by linguistic terms such as *about 10* or *approximately between 10 and 20*."

One method IE's can utilize to address uncertainty (in IE related problems) is fuzzy set theory, which has not been investigated to any significant level. Hence, this research applies fuzzy set theory to a standard IE problem: a single attribute sampling plan.

1.1 PURPOSE OF STUDY

The decision to accept or reject an individual lot in many manufacturing settings is often based on the result of

inspecting a random sample of items drawn from the lot. More specifically, the lot in question is accepted if the number of defective items (in a random sample size drawn from the lot) is less than or equal to an acceptance number; otherwise, the lot is rejected. The determination of the sample size n and the acceptance number c is what is known as a ***sampling plan***.

Usually, a sampling plan meets two key points on the Operating Characteristic (OC) curve in order to provide protection on a given lot for both the producer and the consumer. These two points are known as the ***producer's risk*** (risk that a good lot could be rejected) and the ***consumer's risk*** (risk that a bad lot could be accepted).

An exact sampling plan is ideally defined when the producer's risk and the consumer's risk are exactly equal to certain specified points. However, the problem is that the sample size n and the acceptance number c must be integers. Thus, determining a single attribute sampling plan is not likely when using an exact sampling plan.

The purpose of this research is to present a design procedure that determines n and c for the above sampling plan by relaxing the producer's and the consumer's risks. This relaxation is done by applying linguistic terms, such as ***approximately α*** and ***approximately β***, on risk conditions and properly representing these conditions as membership functions. Thus, by using fuzzy set theory, this proposed approach attempts to obtain a comparable qualitative-based

solution to Hald's traditional quantitative-based solution.

1.2 PROBLEM STATEMENT

The following are the standard notations and terminologies used in a sampling plan:

p_1 = fraction defective corresponding to an acceptable quality
p_2 = fraction defective corresponding to an unsatisfactory quality level, where $p_1 < p_2$
$P(p_1)$ = probability of accepting a lot with p_1 fraction defective
α = $1 - P(p_1)$, producer's risk
β = $P(p_2)$, consumer's risk
n = sample size
c = acceptance number

The conventional sampling plan (formulated by Hald) restricts conditions given in the exact sampling plan as follows:

Determine a sampling plan (n,c) so that,
$P(p_1) \geq (1-\alpha)$,
$P(p_2) \leq \beta$,
and c is as small as possible.
Where $p_1 < p_2$ and $1-\alpha > \beta$.

Solution to this conventional sampling plan leads to a

uniquely determined value of c and interval values of n (upper and lower values of n). In this conventional design, a large sample size is needlessly sought and the issue of reducing the sample size is completely avoided.

In general, reducing the sample size in any inspection process would mean less handling of the product, fewer personnel, and less time involved in inspection activities resulting in reduced product cost. Thus, reduction of sample size is associated with cost reduction in manufacturing settings. Therefore, the goal of minimizing the sample size should be a primary concern for IE's in determining a sampling plan.

Recently, Ohta & Ichihashi [13] presented a design procedure which considers the sampling plan based on membership functions using fuzzy set theory. This formulated fuzzy design is, however, inconsistent with the risks defined by Hald and does not explicitly consider minimization of the sample size.

In this research, a modified version of Ohta & Ichihashi's design procedure is being proposed to properly represent Hald's procedure with an objective to reduce the sample size in lot inspections. The conditions on the producer's risk, $P(p_1) \geq 1-\alpha$, and the consumer's risk, $P(p_2) \leq \beta$, are relaxed using membership functions. This proposed design procedure offers an alternative solution based on fuzzy set theory.

1.3 SUMMARY

An overview of this thesis is illustrated in Figure 1.1. In chapter 2, the development and applications of fuzzy set theory in IE are reviewed. In chapter 3, the concepts of fuzzy set theory are discussed. Chapter 4 details the design of sampling plans. Chapter 5 utilizes both existing design procedures (by Hald and Ohta & Ichihashi) for a new design procedure. Finally, in chapter 6, numerical examples are presented based on a computer program. This program eliminates the tedious and iterative calculations involved in obtaining solutions. The results of this proposed design are then compared with Hald's solutions and conclusions derived. Associated recommendations are then presented.

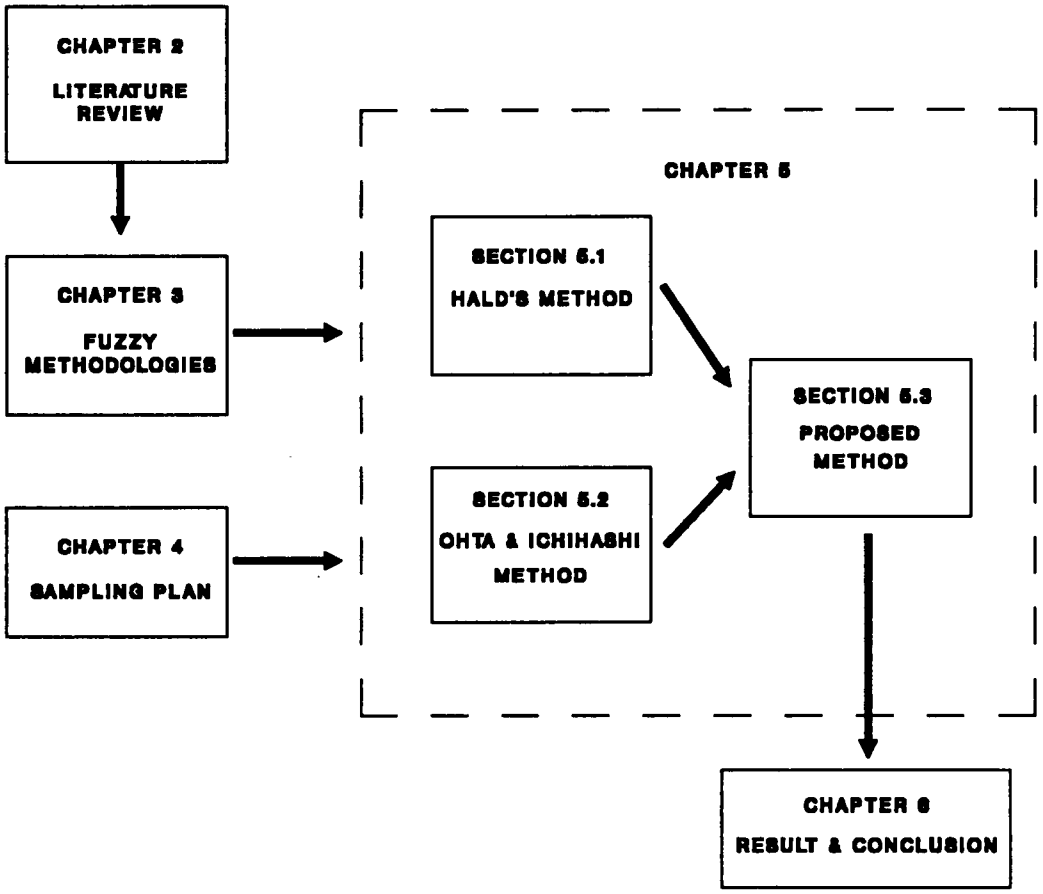


Figure 1.1 Thesis Overview

CHAPTER 2

LITERATURE REVIEW

2.1 FUZZY SET THEORY AND ITS RELEVANCE TO INDUSTRIAL ENGINEERING

Fuzzy set theory is a mathematical concept proposed by L.A. Zadeh [1] in 1965. This theory was born under the shadow of Artificial Intelligence (AI) technology and is currently one of the rapidly developing frontiers of research and applications [4,5,7,14].

The original intent of this theory was to improve the relationship between the human and the computer. Zadeh was awed by the fact that humans posess special characteristics that enable them to learn in an uncertain and fuzzy environment. In other words, humans have the ability to arrive at a decision based on imprecise and qualitative data, in contrast to formal mathematics and formal logic which demand precise and quantitative data. Thus, according to Ragade & Gupta [3], fuzzy set theory is defined as

> "... a body of concepts and techniques that keeps track of the precise mathematical form of human thought processes, which in many ways are imprecise and ambiguous by the classical mathematical standards."

But, why should an IE be concerned with fuzzy set theory? In order to answer this question, one must understand that computers have become an integral part of IE's systems approach to problem-solving. The potential of using fuzzy set theory could make computers emulate the reasoning, problem-solving, creativity, and planning behaviors of humans so they can solve problems that are too large or too complex to be solved with traditional techniques.

According to Terano, Asai, & Sugeno [5], the outstanding feature of fuzzy set theory is the ability to express the amount of ambiguity in human thinking and subjectivity (including natural language) in a comparatively undistorted manner. As a result, fuzzy set theory becomes very suitable for problems with complexity surrounded by uncertainties such as those addressed by IE's. From Terano, et al. [5], these problems are divided into three types of systems:

Machine systems: The first problem with machine systems is that of giving machines a high level of intelligence. At present, intelligence is expressed in terms of propositions in knowledge engineering where recognition, judgement, evaluation, and inference are being imitated. However, human thinking includes illogical elements like intuition and inspiration, and it is virtually impossible to express these in terms of conventional two-valued logic.

Human systems: Human systems are the building of human models. In contrast to machine systems, human systems try to introduce scientific methodology into the complicated problems of man and society. Because recent technological development has come very suddenly, social customs and institutions have not been able to keep up, and here and there distortion has arisen. Information technology, especially, is connected with human psychological activity, and has a great influence. In this case, research into the human side of things is important, but up to now modeling has been an application of either extremely abstract written descriptions or pure mathematical models based on unnatural assumptions.

Human/Machine systems: The human/machine systems use both people and machines in collaboration to get work done that cannot be done by either alone. In these kinds of systems, the communication between human and machine is extremely important, and if the communication is poor, even the most fabulous machine will not help. The most important problem to be solved is the simplification of the input process from people to machines. Up to now, people have worked by bringing themselves to the level of machines, but in a high-level human/machine system, the level of the machines must be able to understand natural language. Another problem is the communication from the

machines to people, and one would hope for information furnished in a way that stimulates man's creativity and judgement.

From Terano, et al. [5], concrete examples of the various problems described above are shown below in Table 2.1 .

Table 2.1 Various Problems in Today's Industry

Machine Systems	Human Systems	Human/Machine Systems
Process Control	Human Reliability Model	Data Processing
Production Management	Cognitive Psychology	Expert Systems
Transportation Operations	Thinking/Behavior Model	Optimization Planning
Maintenance Systems	Risk Assessment	Development Planning
Automatic Operations	Human Relations Structures	Quality Evaluation
Picture/Voice Recognition	Market Selection Model	Management Decision-making
Intelligent Robot	Social Psychology	Knowledge Bases
		Data Bases

2.2 CURRENT APPLICATION USES IN INDUSTRIAL ENGINEERING USING FUZZY SET THEORY

Currently, the literature research reveals only a few applications of fuzzy set theory in IE. These sample applications are by F. Choobineh & V. Jain [17], M. Etschmaier [18], J. Clymer, P. Corey, & J. Gardner [19], and Thomas L. Ward [20].

Choobineh & Jain [17] presented a procedure that improved the utilization of the knowledge of an expert in setting the operating parameter in a manufacturing process. This procedure utilized a membership function (based on fuzzy set theory) for every input parameter of an electrochemical machining (ECM) process. This allowed the process planner to increase his/her capability to better analyze the process variations and to arrive at optimum value of machining parameters. By doing so, proper operating conditions was selected, and thus led to higher work-piece quality and lower operating cost.

Etschmaier [18] presented fuzzy controls for maintenance scheduling in transportation systems. In this system, a computer generated instructions to control the daily operations of an airline. Fuzzy controls were used to notify human operators of predetermined maintenance tasks needed. This led to minimizing the total maintenance workload for the maintenance shop. This illustrated how fuzzy controls can be

applied to a human/machine interactive environment.

Clymer, Corey, & Gardner [19] applied fuzzy logic concepts to mimic human control of an airport. The goal was to incorporate control into a discrete event, nonlinear air traffic simulation. It showed that optimal control can be achieved by manipulating the object and rule confidence factors (given an appropriate set of rules). This was done by translating a human expert's verbal fuzzy advise into machine-usable certainty factor form, and fuzzy logic provided a systematic manipulation of these certainty factors in much the same way a human operator would in reaching conclusions. The result showed that this discrete airport controller optimized its own effectiveness.

Finally, Ward [20] described an application of fuzzy set theory to engineering economic analysis where the cash flows are modeled as fuzzy numbers rather than crisp numbers. Ward presented relatively straight forward extensions of traditional engineering economic techniques such as capital budgeting, rate of return analysis, etc. However, further research was suggested for estimating cash flow membership functions and the consideration of means of interpreting the fuzzy analysis results.

CHAPTER 3

FUZZY METHODOLOGIES

3.1 CLASSICAL SETS

This section introduces the concepts of classical set theory which are necessary for understanding fuzzy set theory.

According to Goldberg [8], any well defined collection of distinct objects, in which nothing special is assumed about the nature of individual objects, is called a *set*. The objects contained in a set are called *elements* or *members* of a set. If x is an element of given set A, then it is defined as:

$$x \in A. \tag{3.1}$$

If x is not an element of set A, then it is defined as:

$$x \notin A. \tag{3.2}$$

If every element of set A also belongs to another given set B, then A is called a *subset of* B or is said to be *contained in* B. This is defined as:

$$A \subset B \quad \text{or} \quad B \supset A. \tag{3.3}$$

Two sets are *equal* if, and only if, each is contained in the other; that is:

$$A = B, \text{ iff } A \subseteq B \text{ and } A \supseteq B. \tag{3.4}$$

A set is specified by either *listing* its elements or by *stating* its properties. This characterizes the elements of a set. For example, if set A consists of the numbers 1, 5, 10, 15, and 20, then it is listed as:

$$A = \{1, 5, 10, 15, 20\}.$$

If set B consists of positive integers less than 10, then it is stated as:

$$B = \{x: x \text{ is a positive integer}, x < 10\}.$$

Unless otherwise stated, all sets under consideration are assumed to be subsets of some fixed set called the *universal set* U, for example, sets A and B are contained in U. That is:

$$A, B \subseteq U. \tag{3.5}$$

The process by which, elements from the universal set U can be determined to be either *members* or *nonmembers* of a certain subset are defined by the *characteristic function* χ.

For example, given set A, this function assigns a value of 1 or 0 to every $x \in U$ such that:

$$\chi_A(x) = \begin{cases} 1, & \text{iff } x \in A \\ 0, & \text{iff } x \notin A. \end{cases} \tag{3.6}$$

Therefore, the characteristic function χ maps each element of universal set U to a set containing values of 0 and 1. Thus, a distinction can be made between an element as a member which belongs to set A, and as a nonmember which does not. This is indicated by:

$$\chi_A: U \rightarrow \{0, 1\}. \tag{3.7}$$

An abstract representation of set A contained in universal set U, in which element x is a nonmember of set A, is shown in Figure 3.1.

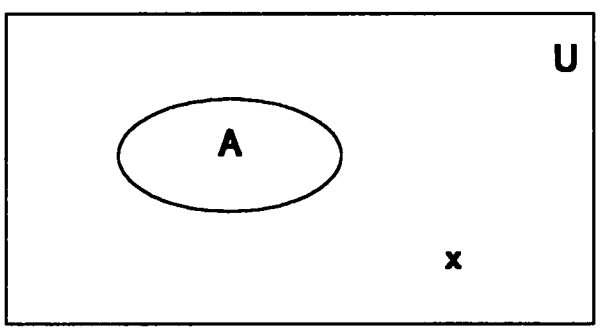


Figure 3.1 Set A Contained in U, With Element x Being a Nonmember of Set A

In Figure 3.1, the boundary that surrounds set A is *sharp* and *precise*. This means that an element is a member of set A if, and only if, its characteristic function is 1; otherwise it is not. Sometimes, this is referred to as a *crisp boundary* of a *crisp set* [5,7].

In set operations, according to Goldberg [8], the *union* of set A and set B is denoted by $A \cup B$. This is the set of elements that belong to A *or* B. This is defined as:

$$A \cup B = \{x: x \in A \quad or \quad x \in B\}. \tag{3.8}$$

The *intersection* of sets A and B is denoted by $A \cap B$. This is the set of elements that belong to both A *and* B. This is defined as:

$$A \cap B = \{x: x \in A \quad and \quad x \in B\}. \tag{3.9}$$

3.2 FUZZY SETS

As mentioned in the previous section, the characteristic function χ, of classical sets, assigns value of 1 or 0 to each element x in universal set U. By performing the above step, it discriminates each element as a member (those that belong to a set) and as a nonmember (those which do not).

Zadeh extends this concept such that the values assigned are no longer limited (to 1 or 0), but are continuous (between

0 and 1). The *membership function* μ is characterized as such; values assigned to all elements fall within a continuous range (between 0 and 1) and are indicated by the membership grade of these elements. Large and small values indicate high and low degrees of set membership, respectively. A set that is defined by such a membership function is called a *fuzzy set*.

In words according to Kandel [6],

> "... a fuzzy set is a class of objects or elements in which there is no sharp boundary between those objects that belong to the class (set) and those that do not."

By Zadeh's [1] definition, a fuzzy set **A** (Note: fuzzy sets are in bold characters) contained in *universe of discourse* X, where X = {x} is defined as a collection of elements generically denoted by x, is a set of ordered pairs written as:

$$\mathbf{A} = \{x, \mu_{\mathbf{A}}(x)\}, \quad x \in X. \tag{3.10}$$

The membership function $\mu_{\mathbf{A}}(x)$ is termed as the grade of membership of x in fuzzy set **A**, and is written as:

$$\mu_{\mathbf{A}}: X \rightarrow [0,1]. \tag{3.11}$$

The interval [0,1] is where the membership values lies, with 0 representing the lowest grade of membership and 1 representing the highest grade of membership.

The basic assumption is that a fuzzy set, despite the *imprecision* of its boundaries, can be defined *precisely* by associating each element x with a membership value (between 0 and 1). For example, let universe of discourse X be defined as the real line **R** and fuzzy set **A** as numbers which are *much greater than 10*. Then, one can give a subjective characterization of **A** by specifying $\mu_A(x)$ as a function on **R**. Thus, the values representing membership function $\mu_A(x)$ could be characterized and written as:

$$\mu_A(0) = 0$$

$$\mu_A(1) = 0$$

$$\vdots$$

$$\mu_A(15) = 0.05$$

$$\vdots$$

$$\mu_A(20) = 0.10$$

$$\vdots$$

$$\mu_A(100) = 1.00 \ .$$

A graphical representation of fuzzy set **A**, in this example, would resemble Figure 3.2.

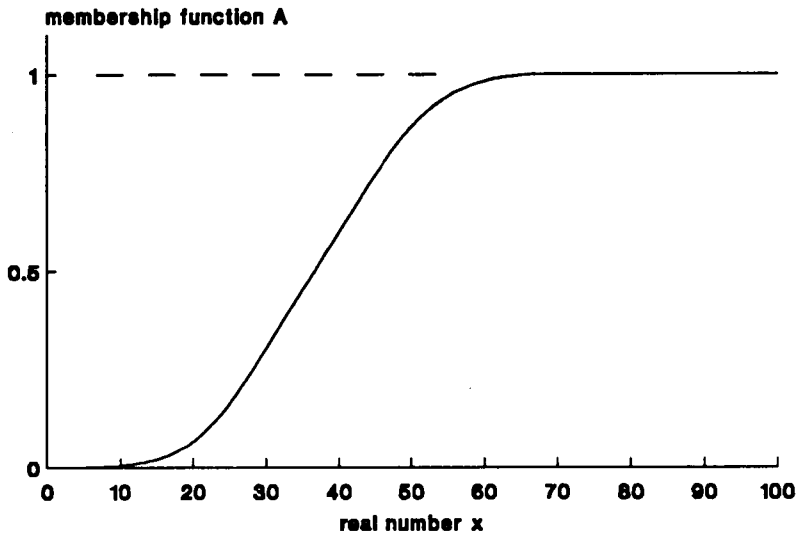


Figure 3.2 Membership Function of **A**, $\mu_{\mathbf{A}}(x)$

When X is an interval of real numbers and is continuous, fuzzy sets are defined and written as: (Note: this is not a division, but a way of representing ordered pairs.)

$$\mathbf{A} = \int_X \frac{\mu_{\mathbf{A}}(x)}{x}. \tag{3.12}$$

But when X is discrete and finite, the equation (3.12) becomes:

$$\mathbf{A} = \sum_{i=1}^{n} \frac{\mu_{\mathbf{A}}(x_i)}{x_i}. \tag{3.13}$$

More generally, assume that x_i is an element of fuzzy set **A** and $\mu_{\mathbf{A}}(x_i)$ is its grade of membership in **A**. Then **A** is written as: (Note: '+' symbol means 'or', the union of these sets is found by connecting them with a '+')

$$\mathbf{A} = \frac{\mu_{\mathbf{A}}(x_1)}{x_1} + \frac{\mu_{\mathbf{A}}(x_2)}{x_2} + \ldots + \frac{\mu_{\mathbf{A}}(x_n)}{x_n}. \qquad (3.14)$$

According to Zadeh and Ikonomopoulos [1,7], a fuzzy set is said to be *normal* and *convex*. That is, fuzzy set **A** is normal if, and only if, the maximum of $\mu_{\mathbf{A}}(x)=1$. In other words, the supreme (maximum) of $\mu_{\mathbf{A}}(x)$ over X is unity. Normality and convexity are graphically shown in Figure 3.3.

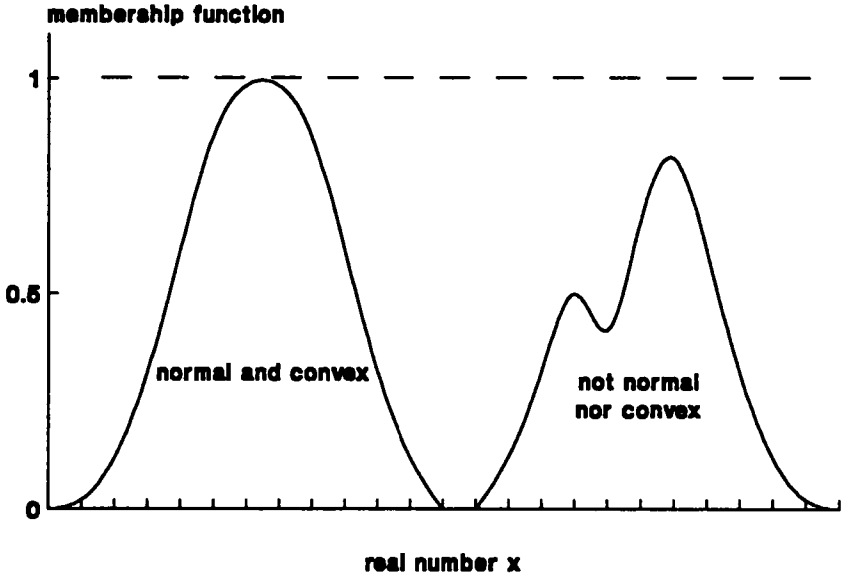


Figure 3.3 Representation of Normality and Convexity

Next, basic operations performed on fuzzy sets will be discussed according to Klir & Folger and Terano, et al. [4,5]. Two fuzzy sets, **A** and **B**, are *equal* if, and only if, their membership functions are equal. That is:

$$\mathbf{A} = \mathbf{B}, \text{ iff } \mu_{\mathbf{A}}(x) = \mu_{\mathbf{B}}(x); \text{ for all } x \in X. \quad (3.15)$$

A fuzzy set **A** is *contained* in fuzzy set **B** if, and only if, the membership function of **A** is less than or equal to the membership function of **B**. That is:

$$\mathbf{A} \subseteq \mathbf{B}, \text{ iff } \mu_{\mathbf{A}}(x) \leq \mu_{\mathbf{B}}(x); \quad x \in X. \quad (3.16)$$

The *intersection* of fuzzy sets **A** and **B** is denoted by $\mathbf{A} \cap \mathbf{B}$ and is defined as the largest fuzzy set contained in both **A** and **B**. The membership function of $\mathbf{A} \cap \mathbf{B}$ is given as:

$$\mu_{\mathbf{A}\cap\mathbf{B}}(x) = \min\{\mu_{\mathbf{A}}(x), \mu_{\mathbf{B}}(x)\}; \quad x \in X. \quad (3.17)$$

The *union* of fuzzy sets **A** and **B** is denoted by $\mathbf{A} \cup \mathbf{B}$ and is defined as the smallest fuzzy set contained in both **A** and **B**. The membership function of $\mathbf{A} \cup \mathbf{B}$ is given as:

$$\mu_{\mathbf{A}\cup\mathbf{B}}(x) = \max\{\mu_{\mathbf{A}}(x), \mu_{\mathbf{B}}(x)\}; \quad x \in X. \quad (3.18)$$

For example, assume that the universe of discourse is given as X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. If we let fuzzy set **A** be *approximately equal to 5*, and fuzzy set **B** be *approximately equal to 7*, then, using equation (3.14), fuzzy sets **A** and **B** can be subjectively defined as:

$$\mathbf{A} = 0.1/3 + 0.6/4 + 1/5 + 0.6/6 + 0.1/7$$
$$\mathbf{B} = 0.1/5 + 0.6/6 + 1/7 + 0.6/8 + 0.1/9.$$

Using equation (3.17), the intersection of fuzzy sets **A** and **B** is:

$$\mathbf{A} \cap \mathbf{B} = \min\{1,0.1\}/5 + \min\{0.6,0.6\}/6 + \min\{0.1,1\}/7$$
$$= 0.1/5 + 0.6/6 + 0.1/7.$$

Using equation (3.18), the union of fuzzy sets **A** and **B** is:

$$\mathbf{A} \cup \mathbf{B} = \max\{0.1,0\}/3 + \max\{0.6,0\}/4 + \max\{1,0.1\}/5 + \max\{0.6,0.6\}/6 + \max\{0.1,1\}/7 + \max\{0,0.6\}/8 + \max(0,0.1)/9$$
$$= 0.1/3 + 0.6/4 + 1/5 + 0.6/6 + 1/7 + 0.6/8 + 0.1/9.$$

These examples are represented respectively in Figure 3.4 and Figure 3.5. The areas under bold lines are the intersection of **A** and **B** and the union of **A** and **B**.

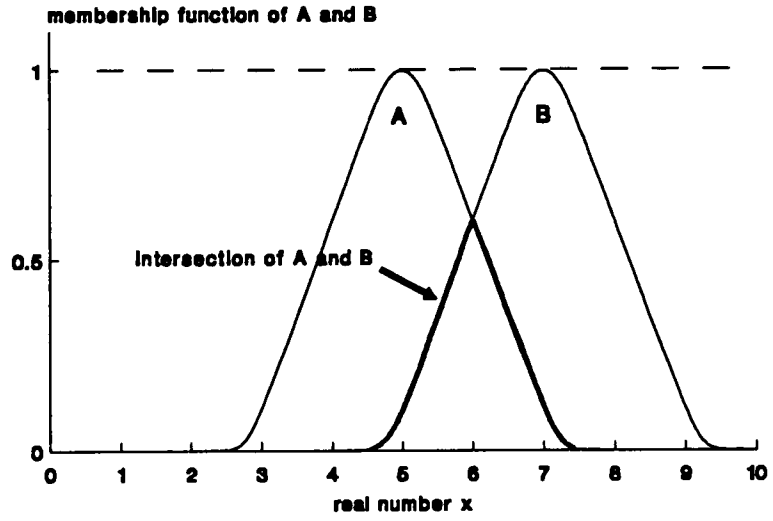


Figure 3.4 Intersection of Fuzzy Sets **A** and **B**

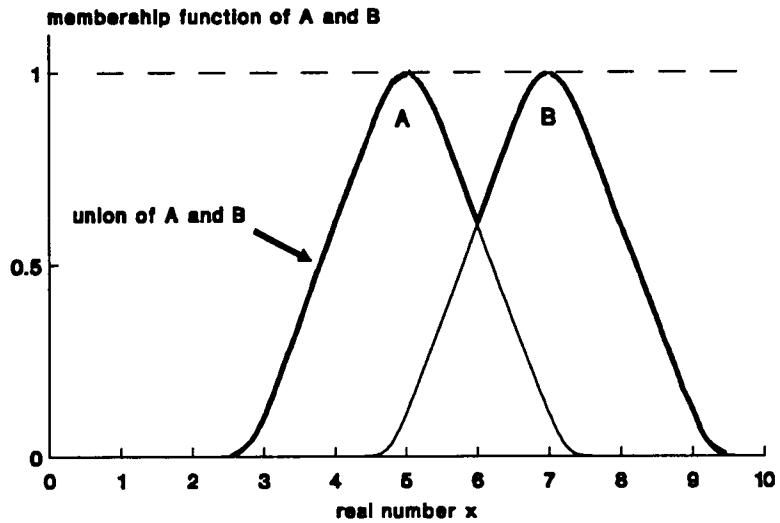


Figure 3.5 Union of Fuzzy Sets **A** and **B**

3.3 FUZZY GOALS, CONSTRAINTS, AND DECISIONS

The concepts of fuzzy sets allow a decision-making process a broader perspective compared to the conventional decision-making. Decision-making in a fuzzy environment implies that the goals and the constraints constitute boundaries which are not sharply defined and therefore fuzzy in nature. Fuzzy set theory allows precisely defining fuzzy goals and fuzzy constraints in a domain characterized by membership functions. Thus according to Bellman & Zadeh [2], a fuzzy decision may be viewed as an intersection of the given goals and constraints.

Perhaps the most important feature in decision-making in a fuzzy environment is the symmetry of the goals and constraints. This makes it possible to correlate decisions to goals and constraints. More specifically, if X={x} represents a set of alternatives, then, a *fuzzy goal* in X will be identified as a fuzzy set **G** in X. For example, if X=**R** (real numbers), then the fuzzy goal **G** expressed in words such as *x is substantially larger than 20* may be represented by a fuzzy set in **R**, whose membership function μ_G is given subjectively. Similarly, a *fuzzy constraint* in X is defined to be a fuzzy set **C** in X. For example, the constraint *x should be approximately between 10 and 20* could be represented by a fuzzy set **C**, whose membership function μ_C is also attained subjectively. An important aspect is that both the goal and

the constraint are defined as fuzzy sets in the space of alternatives.

The goals and the constraint can be treated identically in the formulation of a decision process. This implies that fuzzy goal **G** and the fuzzy constraint **C** can be considered as a combined effect on the choice of alternatives, which may be represented by the intersection **G** $\cap$ **C**. Therefore, using equation (3.17), the membership function of this intersection is given as:

$$\mu_{G \cap C}(x) = \min\{\mu_G(x), \mu_C(x)\}, \quad x \in X. \tag{3.19}$$

A decision is basically defined as a choice or a set of choices drawn from the available alternatives. Therefore, a *fuzzy decision* **D** is defined as the fuzzy set of alternatives resulting from the intersection of the goal and the constraint. For example, assume that we are given a fuzzy goal **G** and a fuzzy constraint **C** in a space of alternatives X. Then, **G** and **C** are combined to form a decision **D**, which is a fuzzy set resulting from intersection of **G** and **C**. This is defined as:

$$\mathbf{D} = \mathbf{G} \cap \mathbf{C} \tag{3.20}$$

More generally, suppose that we have n goals ($\mathbf{G}_1$, $\mathbf{G}_2$, ..., $\mathbf{G}_n$) and m constraints ($\mathbf{C}_1$, $\mathbf{C}_2$, ..., $\mathbf{C}_m$). Then, the resultant

decision is the intersection of given goals and given constraints. That is:

$$\mathbf{D} = \mathbf{G}_1 \cap \mathbf{G}_2 \cap \ldots \cap \mathbf{G}_n \cap \mathbf{C}_1 \cap \mathbf{C}_2 \cap \ldots \cap \mathbf{C}_m. \qquad (3.21)$$

These definitions of decision **D** are illustrated in Figure 3.6 and Figure 3.7, respectively. Notice that the goals $\mathbf{G}_n$ and the constraints $\mathbf{C}_m$ enter into the expression for **D** in exactly the same way.

In chapter 5, these concepts will be used to identify the role of goals and constraints. Thus the proposed design in this research will be formulated in similar manner.

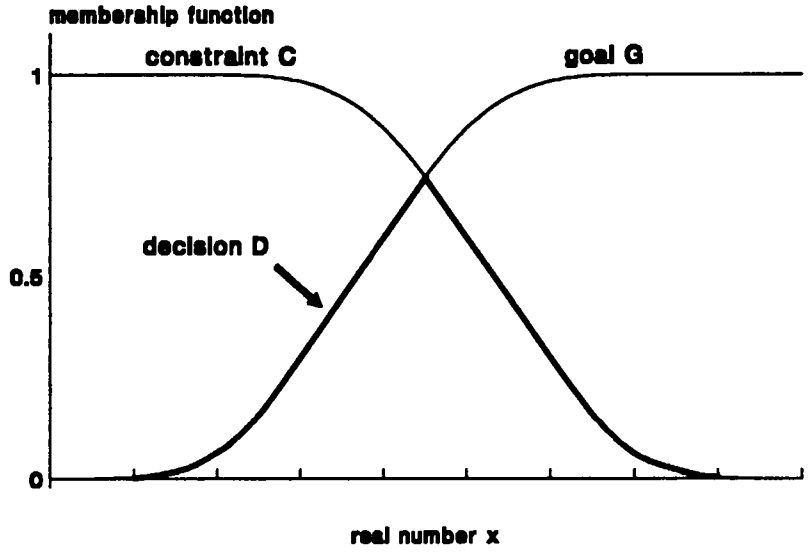


Figure 3.6 Definition of Decision D

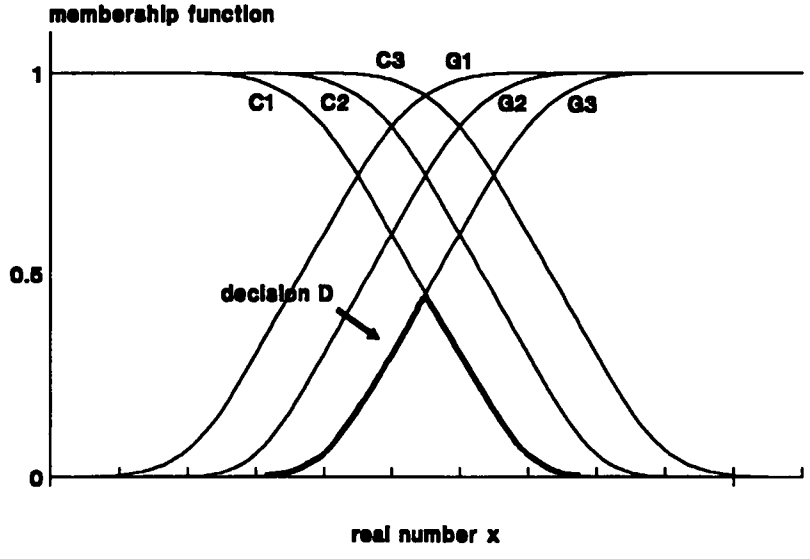


Figure 3.7 Definition of Decision D, Given C_1, C_2, C_3 and G_1, G_2, G_3

CHAPTER 4

SINGLE ATTRIBUTE SAMPLING PLAN

4.1 SAMPLING PLAN

General definitions of a sampling plan are discussed in this section. Montgomery [9] states that a sampling plan is typically used to accept or to reject the lot in manufacturing settings. A typical example is where a manufacturer receives a shipment from a vendor, from which a random sample is taken and some quality characteristics in the sample are inspected. This provides the basis to either accept or reject the lot. However, there are other uses for such a sampling method. For example, a manufacturer frequently samples and inspects its own product at various stages of production to ensure quality. Accepted lots are sent forward for further processing, while rejected lots are routed to rework or scrap.

A sampling plan is most likely to be utilized (given the following conditions):

(1) When 100% inspection is too costly.
(2) When there are too many items to be inspected and the error rate of human inspection is high.
(3) When 100% inspection severely impacts scheduling production.
(4) When testing is destructive.

Comparing sampling plans with 100% inspection, the sampling plans demonstrate the following advantages:

(1) Less costly.
(2) Fewer personnel are required in inspection activities.
(3) Tends to reduce inspection errors.
(4) Reduced handling of the product, thus limiting damage.
(5) Applicable to destructive testing.
(6) Rejecting the entire lot as opposed to returning only defective items to provide stronger motivation for the vendor quality improvements.

However, sampling plans have the following disadvantages as identified below:

(1) Risks of accepting *bad* lots and rejecting *good* lots.
(2) Less information is generated about the product or about the process that manufactured the product.
(3) Requires planning and documentation.

There are important considerations in lot inspection. One of these considerations is that larger lots are preferred over smaller ones. Another consideration is that lots should maintain homogeneity. Also, units selected for inspection should be chosen at random, such that, they are representative of the lot.

According to Nachlas & Kim [21], sampling plan for

attributes is an established technique for obtaining quality protection in acquired goods. One major classification of sampling plans is by attributes and variables. Attributes are quality characteristics that are expressed on a *yes* or *no* basis. A single sampling plan is a procedure in which one sample of n units is selected at random from the lot. A combination of these would result in a single attribute sampling plan.

A sampling plan consists of a sample size n, an acceptance number c, and d defective items. The procedure for a sampling plan operates as follows:

(1) Select n items at random from the lot.
(2) If d defective items found in n items are equal to or less than c, then accept the lot.
(3) If d defective items found in n items are greater than c, then reject the lot.

For example; suppose that a lot of size N has been submitted for inspection. The sampling plan is defined by the sample size n and the acceptance number c. Thus if the lot size is 8,000 (N=8,000), then the sampling plan is defined as 76 (sample size) and 2 (acceptance number); that is, n=76 and c=2. This implies that from the lot size of 8,000, a random sample of 76 units is inspected and d defective items are observed. If the number of observed defectives, d, is less than or equal to 2, then the lot will be accepted. If the

number of observed defectives, d, is greater than 2, then the lot will be rejected.

4.2 OPERATING CHARACTERISTIC CURVE

The Operating Characteristic (OC) curve [9,10] is an important performance measurement in a sampling plan. This curve plots the probability of accepting the lot versus the lot fraction defective, thus representing the probability that a lot submitted with a certain fraction defective will be either accepted or rejected. A graphical representation of an OC curve, with sampling plan (n=76,c=2) is shown below in Figure 4.1.

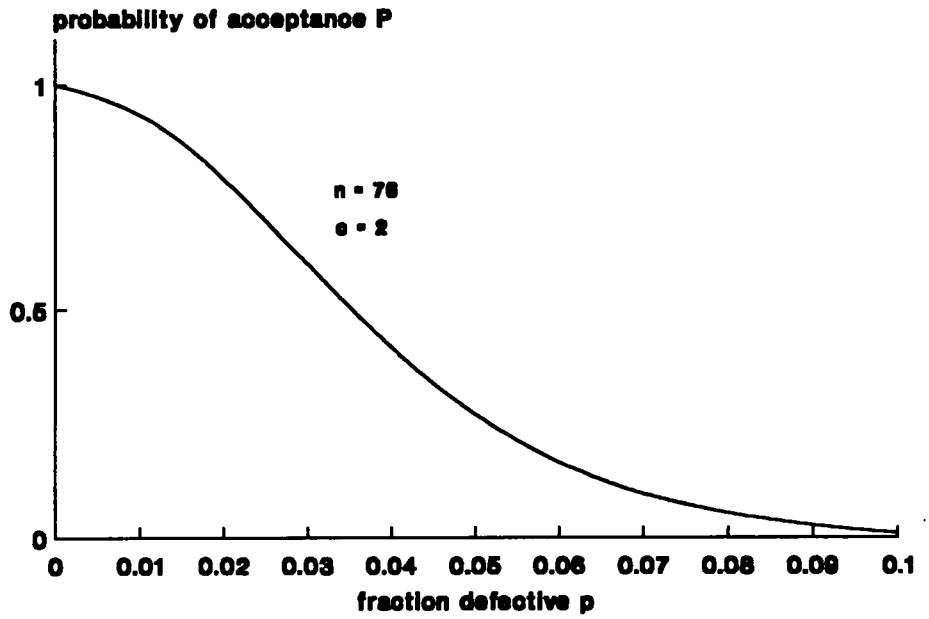


Figure 4.1 OC Curve With (n=76, c=2) Sampling Plan

The points on the OC curve are obtained in a simple manner. Suppose that the lot size N is large (infinite). Under this condition, the distribution of d defective items in a random sample of n items is binomial with parameters n and p, where p is the fraction of defective items in the lot. An equivalent way to conceptualize this is to draw lots of N items at random from a infinite process, and then to draw random samples of n from these lots. Sampling from the lot in this manner is the equivalent to sampling directly from the process. Montgomery [9] states that the probability of observing exactly d defective items is defined as:

$$P(d) = \binom{n}{d} p^d (1-p)^{n-d}. \qquad (4.1)$$

Then, the probability of acceptance (a point on the OC curve) is simply the probability d less than or equal to c. Using equation (4.1), this is defined as:

$$P(d \leq c) = \sum_{d=0}^{c} \binom{n}{d} p^d (1-p)^{n-d}. \qquad (4.2)$$

An OC curve is developed by evaluating equation (4.2) for various values for p. For example, when the sampling plan is n=76 and c=2 (referring back to Figure 4.1) and the lot fraction defective is p=0.01, then the probability of

acceptance is calculated as:

$$P(d \leq 2) = \sum_{d=0}^{2} \binom{76}{d} (0.01)^d (0.99)^{76-d}$$

$$= 0.4659 + 0.3576 + 0.1355$$
$$= 0.9590$$

Table 4.1 shown below represents the values for Figure 4.1.

Table 4.1
Probability of Acceptance (n=76, c=2)

Fraction Defective p	Probability of Acceptance $P(p)$
.01	.9590
.02	.8051
.03	.6002
.04	.4096
.05	.2615
.06	.1584
.07	.0920
.08	.0516

A sampling plan that discriminates perfectly between good and bad lots would have a OC curve that is illustrated in Figure 4.2. In this case, the OC curve remains at a probability of acceptance of 1 until a quality considered bad is reached. At this point, the curve drops to a probability of acceptance of 0. Then the curve remains at 0, for all lot fraction defectives greater than the undesirable level. If such a sampling plan could be employed, all lots with poor quality would be rejected, and all lots with good quality would be accepted.

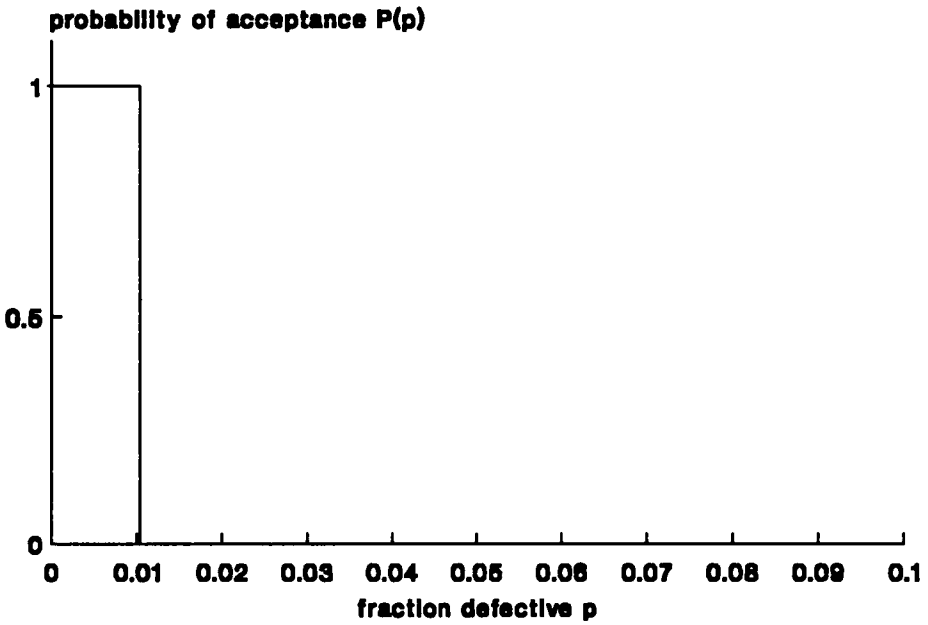


Figure 4.2 Ideal OC Curve for Figure 3.1

In theory, this ideal OC curve could be realized by 100% inspection, as long as the inspection was error free. But, the ideal OC curve is practically unattainable. However, the ideal OC curve shape can be approached by increasing the sample size. This is illustrated in Figure 4.3 (Note, the acceptance number c also changes). Thus the precision with which a sampling plan differentiates between good and bad lots increases with the sample size. The greater the slope of the OC curve, the greater the discriminatory power.

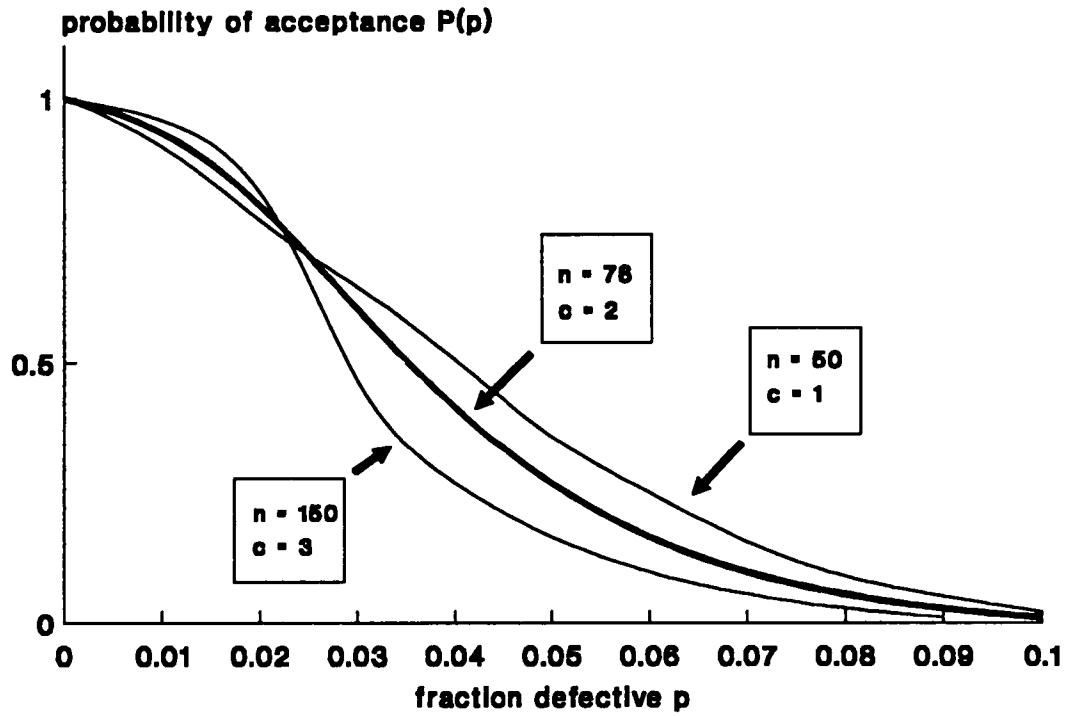


Figure 4.3 OC Curves With Various Sampling Plans

Thus the OC curve shows the discriminatory power of a sampling plan. For example, if the lots are 2% defective in the sampling plan (n=76, c=2), the probability of acceptance is approximately 0.8 (given in Table 4.1). This implies that given 100 lots from a process which manufactures 2% defective product, the producer and the consumer will agree to accept 80 lots and reject 20 lots.

Also, the OC curve changes as the acceptance number changes. Such as, when the acceptance number is decreased then the OC curve is shifted to the left. However, this does not significantly change the shape of the OC curve.

4.3 CONSUMER'S AND PRODUCER'S RISKS

The producer is interested in knowing the level of lot or process quality that would yield a high probability of acceptance. For example, the producer might be interested in the 0.96 probability of acceptance on the OC curve. This would indicate the level of process fallout that could be experienced and still have a 96% chance that the lots would be accepted. Conversely, the consumer is interested in the other end of the OC curve, that is, the process quality which will yield a low probability of acceptance.

Montgomery [9] states that the producer establishes a sampling plan with reference to an Acceptable Quality Level or AQL. The AQL represents the poorest level of quality for

the producer that the consumer would consider to be acceptable as a process average. (Note: AQL is a property of the producer's manufacturing process and is not a property of the sampling plan.) The producer designs the sampling procedure so that the OC curve gives a high probability of acceptance at the AQL. AQL is simply a standard against which to judge the lots and is not intended to be a specification on the product nor the production process.

The consumer will be interested in the other end of the OC curve, in the protection that is obtained for individual lots of poor quality. In such a case, according to Montgomery [9], the consumer may establish a Lot Tolerance Percent Defective or LTPD. The LTPD is the poorest level of quality that the consumer is willing to accept in an individual lot. LTPD is not a characteristic of the sampling plan, but is a level of lot quality specified by the consumer.

4.4 DESIGN OF SAMPLING PLAN

The following are the standard notations and terminologies used in a sampling plan:

p_1 = fraction defective corresponding to an acceptable quality

p_2 = fraction defective corresponding to an unsatisfactory quality level, where $p_1 < p_2$

$P(p_i)$ = probability of accepting a lot with p_i fraction defective

α = 1 - $P(p_1)$, producer's risk

β = $P(p_2)$, consumer's risk

n = sample size

c = acceptance number

A common approach to the design of an sampling plan is to require that the OC curve pass through two designated points, such as the AQL and LTPD points. The OC curve illustrates the probability of acceptance, which depends upon fraction defective in a lot. The objective is to construct a sampling plan such that the probability of acceptance is $1-\alpha$ for lots with fraction defective p_1, and the probability of acceptance is β for lots with fraction defective p_2. (Note: p_1 is also known as the fraction defective corresponding to an acceptable quality and p_2 as the fraction defective corresponding to an unsatisfactory quality level.)

Assuming that the binomial distribution sampling is appropriate (from section 4.2), it can be illustrated that the sample size n and the acceptance number c are the solutions to the following equations:

$$P(p_1) = 1-\alpha = \sum_{d=0}^{c} \binom{n}{d} p_1^d (1-p_1)^{n-d} . \qquad (4.3)$$

$$P(p_2) = \beta = \sum_{d=0}^{c} \binom{n}{d} p_2^d (1-p_2)^{n-d} . \qquad (4.4)$$

These equations were obtained by the two points on the OC curve using the binomial distribution (Montgomery [9]). The fact is that the two simultaneous equations are nonlinear, and there is no simple and direct solution. These two specific points are shown in Figure 4.4 for sampling plan of (n=76,c=2).

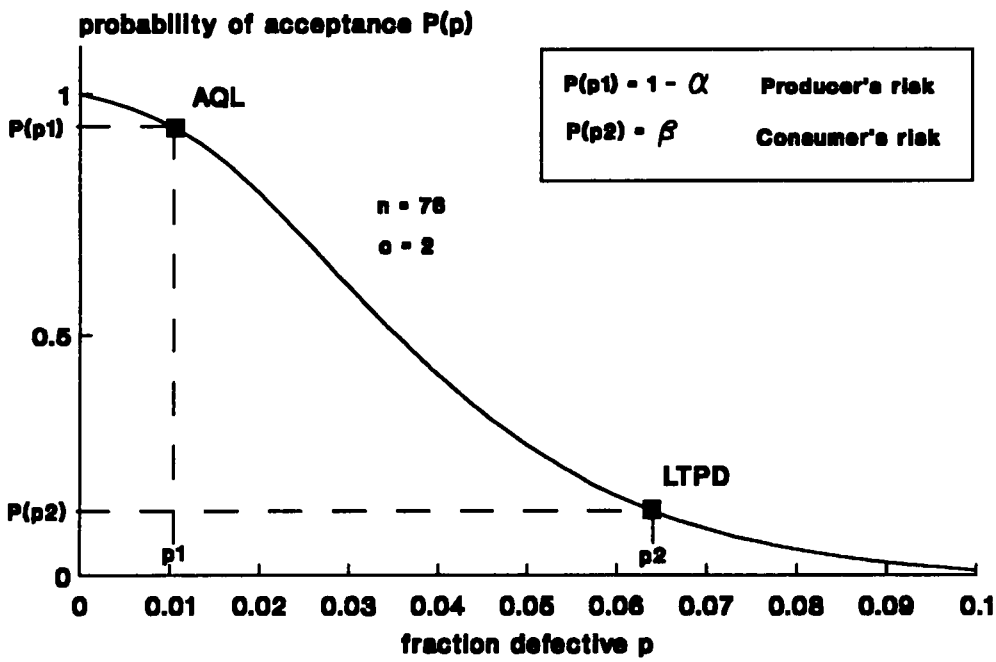


Figure 4.4 Two Specific Points on an OC Curve

While any two points on the OC curve could be used to define the sampling plan, according to Montgomery [9], it is customary in many industries to use the AQL and LTPD points for this purpose. Thus, levels of lot quality are specified as $P(p_1)$ = AQL and $P(p_2)$ = LTPD and corresponding points on the OC curve are usually referred to as the producer's risk and consumer's risk, respectively. That is:

$$P(p_1) = 1-\alpha, \textit{ producer's risk.} \tag{4.5}$$

$$P(p_2) = \beta \quad , \textit{ consumer's risk.} \tag{4.6}$$

The values of α, β, p_1, and p_2 are usually specified by mutual agreement between the producer and the consumer.

CHAPTER 5

ADAPTATION TO FUZZY METHODOLOGIES

Now that the concepts of single attribute sampling plan and fuzzy methodologies have been presented in the previous two chapters, the stated design procedures for this sampling plan are presented in this chapter. This chapter will introduce a conventional design procedure by Hald [11,12] and a fuzzy design procedure by Ohta & Ichihashi [13]. These are the basis for the proposed design procedure in the latter part of the chapter.

5.1 CONVENTIONAL DESIGN

The conventional design procedure, developed by Hald, assumes that the distribution of lot quality is binomial (this is explained in chapter 4). It is widely accepted that Poisson distribution gives a good approximation of the binomial distribution. Thus, the conditions of equations (4.3) and (4.4) can be expressed by using $\lambda(n,c,p)$ functions of the Poisson distribution as shown in equation (5.1). The expansion of $\lambda(n,c,p)$ is given by Molenaar [15], which is shown in equation (5.2).

$$P(p_1) \approx \sum_{d=0}^{c} \frac{e^{-\lambda(n,c,p_1)}\ \lambda^d(n,c,p_1)}{d!} \tag{5.1}$$

$$P(p_2) \approx \sum_{d=0}^{c} \frac{e^{-\lambda(n,c,p_2)}\ \lambda^d(n,c,p_2)}{d!}$$

where, $\lambda(n,c,p_1) = g(n,c)\ h(p_1) = \left(n-\frac{c}{2}\right)\left(\frac{2p_1}{2-p_1}\right)$ (5.2)

$$\lambda(n,c,p_2) = g(n,c)\ h(p_2) = \left(n-\frac{c}{2}\right)\left(\frac{2p_2}{2-p_2}\right).$$

According to Hald [12], using the relation between the Poisson and the χ^2 distribution, the conditions on consumer's and producer's risks are approximately equivalent to: (Note: percentage points of the χ^2 distribution is obtained from [16], with degrees of freedom $\nu = 2c + 2$.)

$$P(p_1) = \chi^2_{1-\alpha,2c+2} \geq 2\lambda(n,c,p_1) = 2g(n,c)\ h(p_1) \tag{5.3}$$

$$P(p_2) = \chi^2_{\beta,2c+2} \leq 2\lambda(n,c,p_2) = 2g(n,c)\ h(p_2). \tag{5.4}$$

Next, the auxiliary functions are introduced by Hald [12]. Auxiliary function r is the result of combining both equations in (5.3) and (5.4), and is defined as:

$$r = \frac{h(p_2)}{h(p_1)} = \frac{p_2(2-p_1)}{p_1(2-p_2)} \tag{5.5}$$

Similarly, auxiliary function R(c) is defined as:

$$R(c) = \frac{P(p_2)}{P(p_1)} = \frac{\chi^2_{\beta,2c+2}}{\chi^2_{1-\alpha,2c+2}} \tag{5.6}$$

When given α and β, the auxiliary function $R(c)$ is monotonically decreasing in c. Thus, the acceptance number c is determined as the smallest integer satisfying equations (5.3) and (5.4) with respect to r and R(c). That is:

$$r \geq R(c) \tag{5.7}$$

By solving equations (5.2), (5.3), and (5.4) with respect to the sample size n, these equations become:

$$\chi^2_{\beta,2c+2}\left(\frac{2-p_2}{4p_2}\right) + \frac{c}{2} \leq n \leq \chi^2_{1-\alpha,2c+2}\left(\frac{2-p_1}{4p_1}\right) + \frac{c}{2} \tag{5.8}$$

Therefore, the acceptance number c is determined by equation (5.7), and the sample size n is all integers satisfying equation (5.8). This will result in a sampling plan which satisfies all the conditions of the given risks. If the n interval obtained from equation (5.8) has no integer, the value of c must be increased one by one, until an integer is obtained.

To determine this sampling plan, first the acceptance number c is located in the table of the χ^2 distribution. Having found the acceptance number, iterative calculations are required to obtain the interval (upper and lower n) of sample sizes. Thus, a large sample size is needlessly sought and the issue of reducing the sample size is not mentioned.

5.2 FUZZY DESIGN

Recently, Ohta & Ichihashi [13] presented a paper describing a procedure for relaxing the conditions on the risks by the membership functions using fuzzy set theory. The conditions on equations (4.3) and (4.4) are redefined as:

$$\alpha'(n,c) \triangleq 1 - P(p_1) \approx \alpha \tag{5.9}$$

$$\beta'(n,c) \triangleq P(p_2) \approx \beta \tag{5.10}$$

The symbol '$\approx$' stands for an *approximate relation* and is classified as a *fuzzy restriction* associated with *fuzzy*

variables [5,6], with labels of the type: *approximately equal to* β or *approximately between* β_l *and* β_u. Thus, in a simplified manner, it is reasonable to make use of values α, α_l, α_u ($\alpha_l \leq \alpha \leq \alpha_u$) as the producer's risk, and values β, β_l, β_u ($\beta_l \leq \beta \leq \beta_u$) as the consumer's risk. The values α and β would indicate the most appropriate values, and the interval values of $[\alpha_l, \alpha_u]$ and $[\beta_l, \beta_u]$ would indicate the lower and upper limits of tolerance.

From this point of view, it seems natural to accept a membership function $\mu_A(\alpha')$ (illustrated below in Figure 5.1) for the fuzzy subset **A** of the set of producer's risk.

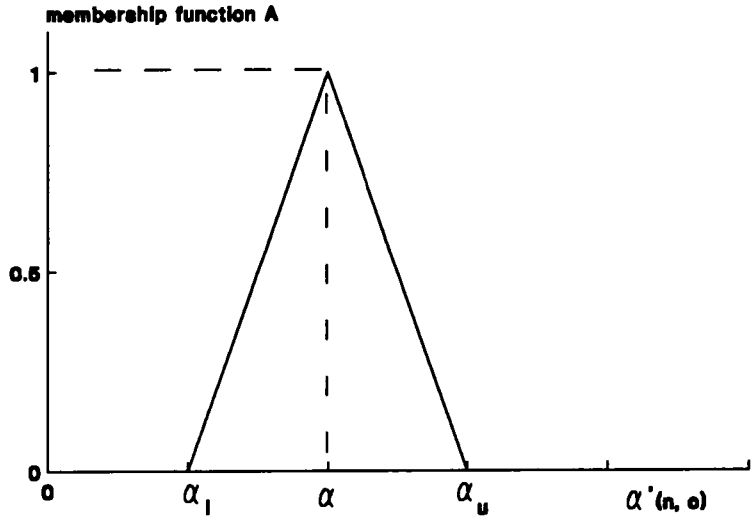


Figure 5.1 Membership Function, $\mu_A(\alpha')$

Figure 5.1 is defined below in equation (5.11).

$$\mu_{\tilde{A}}(\alpha') = \begin{cases} \dfrac{\alpha' - \alpha_l}{\alpha - \alpha_l}, & \text{for } \alpha' \in [\alpha_l, \alpha) \\ \dfrac{\alpha_u - \alpha'}{\alpha_u - \alpha}, & \text{for } \alpha' \in (\alpha, \alpha_u] \\ 0, & \text{for } \alpha' \notin [\alpha_l, \alpha_u]. \end{cases} \tag{5.11}$$

Similarly, the fuzzy subset **B** of the set of consumer's risk, a membership function $\mu_B(\beta')$ is accepted (illustrated below in Figure 5.2).

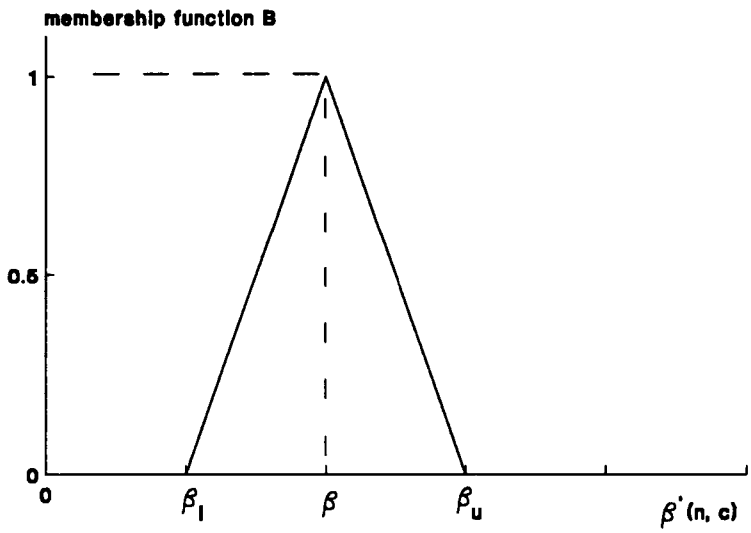


Figure 5.2 Membership Function, $\mu_B(\beta')$

Figure 5.2 is defined below in equation (5.12).

$$\mu_B(\beta') = \begin{cases} \dfrac{\beta' - \beta_l}{\beta - \beta_l}, & \text{for } \beta' \in [\beta_l, \beta) \\ \dfrac{\beta_u - \beta'}{\beta_u - \beta}, & \text{for } \beta' \in (\beta, \beta_u] \\ 0, & \text{for } \beta' \notin [\beta_l, \beta_u]. \end{cases} \tag{5.12}$$

Thus, by using the membership functions of sets **A** and **B**, the conventional problem is redefined as the following:

> Determine the sampling plan (n, c) so that,
> $\min\{\mu_A(\alpha'), \mu_B(\beta')\}$ is maximized
> and c is as small as possible.
> Where $p_1 < p_2$ and $1 - \alpha' > \beta'$.

This design is, however, inconsistent with the risks defined by Hald. This is due to the grades of membership functions, $\mu_A(\alpha')$ and $\mu_B(\beta')$ (shown in Figures 5.1 and 5.2). Both are monotonically decreasing as α' and β' are decreasing (when $\alpha'<\alpha$ and $\beta'<\beta$). This is inconsistent because, if the actual risks become smaller, then the grades of satisfaction of the producer and consumer become worse. Also, this design does not explicitly consider minimizing the sample size.

5.3 PROPOSED FUZZY DESIGN

The fuzzy design in the previous section is now enhanced in order to minimize (reduce) the sample size n. Also, the grades of membership functions $\mu_A(\alpha')$ and $\mu_B(\beta')$ are modified in order to be consistent with the risk conditions given by Hald. Therefore, using equations (5.9) and (5.10), the following formulation is proposed:

$$\alpha'(n,c) \triangleq 1 - P(p_1) \lesssim \alpha \tag{5.13}$$

$$\beta'(n,c) \triangleq P(p_2) \lesssim \beta \tag{5.14}$$

where '$\lesssim$' stands for fuzzy relation representing *approximately less or equal to* conditions.

Then the proposed membership functions $\mu_A(\alpha')$ and $\mu_B(\beta')$ are as follows.

The membership function $\mu_A(\alpha')$ is defined as

$$\mu_A(\alpha') = \begin{cases} 1 \quad , & \text{iff} \quad \alpha' < \alpha \\ \dfrac{\alpha_u - \alpha'}{\alpha_u - \alpha} \quad , & \text{iff} \quad \alpha \le \alpha' \le \alpha_u \\ 0 \quad , & \text{iff} \quad \alpha_u < \alpha'. \end{cases} \tag{5.15}$$

and the membership function $\mu_B(\beta')$ is defined as

$$\mu_B(\beta') = \begin{cases} 1 \quad , & \text{iff} \quad \beta' < \beta \\ \dfrac{\beta_u - \beta'}{\beta_u - \beta} \quad , & \text{iff} \quad \beta \le \beta' \le \beta_u \\ 0 \quad , & \text{iff} \quad \beta_u < \beta'. \end{cases} \tag{5.16}$$

Membership functions, $\mu_A(\alpha')$ and $\mu_B(\beta')$, are illustrated in Figure 5.3 and Figure 5.4, respectively.

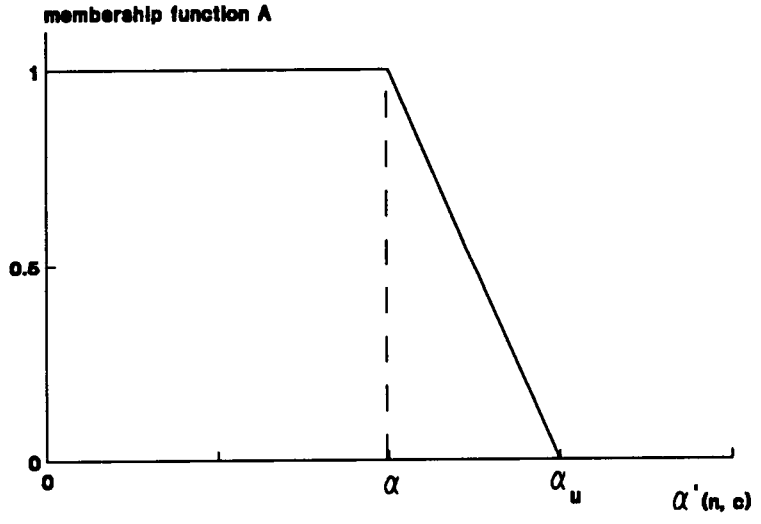


Figure 5.3 Proposed Membership Function, $\mu_A(\alpha')$

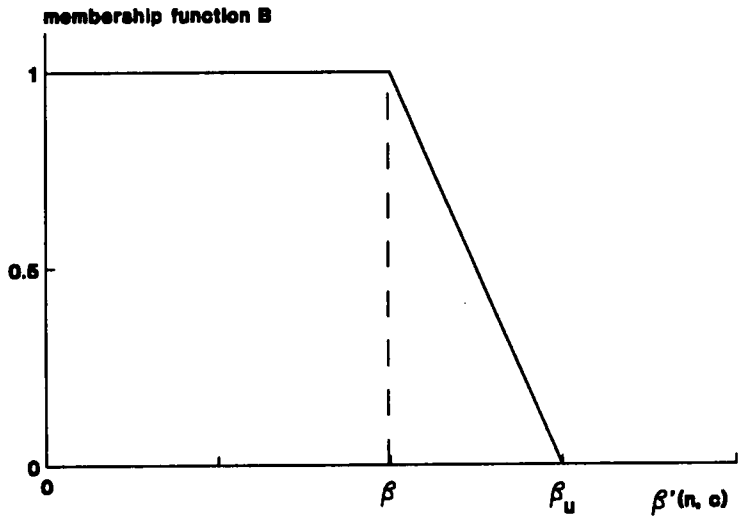


Figure 5.4 Proposed Membership Function, $\mu_B(\beta')$

According to Ohta & Ichihashi [13], the membership function $\mu_n(n)$, which represents the grade of satisfaction for the sample size, is monotonically decreasing as n increases. Thus, membership function $\mu_n(n)$ is given as the following:

$$\mu_n(n) = \begin{cases} \dfrac{n_u - n}{n_u}, & \text{iff} \quad n < n_u \\ 0, & \text{iff} \quad n \geq n_u. \end{cases} \qquad (5.17)$$

n_u is the upper tolerance limit of sample size n; hence n should be smaller than n_u. It is recommended to select n_u to be less than N/10, when using the binomial distribution. The membership function $\mu_n(n)$ is illustrated in Figure 5.5.

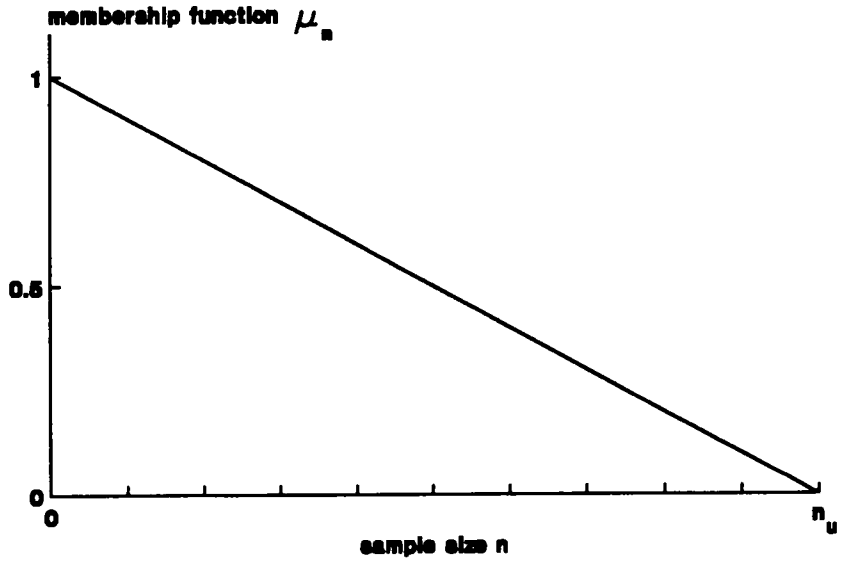


Figure 5.5 Membership Function, $\mu_n(n)$

The proposed fuzzy formulation in equations (5.13) and (5.14) can be rewritten as the following problem:

Determine the sampling plan (n, c) so that,
$\min\{\mu_A(\alpha'), \mu_B(\beta'), \mu_n(n)\}$ is maximized
and c is as small as possible.
Where $p_1 < p_2$ and $1 - \alpha' > \beta'$.

The decision **D** is then determined using equation (3.20) as:

$$\mathbf{D} = \bigcup_{c=i} \sum (\mathbf{A}_i \cap \mathbf{B}_i) \cap \mathbf{n}, \quad \text{for each c.} \qquad (5.18)$$

Sample size n and acceptance number c are obtained from the highest corresponding membership function value $\mu_D(n)$. Thus, this proposed fuzzy procedure minimizes the sample size by utilizing fuzzy membership functions.

CHAPTER 6

RESULT AND CONCLUSION

In this chapter, numerical examples are developed using the proposed fuzzy approach adapted in section 5.3. These examples are problems specified by Hald and represent a range of such problems. To show the reader the understanding of this proposed fuzzy design procedure, example A is selected and presented in section 6.1 to illustrate the mechanism in resolving the problem. The results are summarized and then compared to the results from Hald's approach in section 6.2. The conclusions are presented in section 6.3.

6.1 NUMERICAL EXAMPLE

In example A, α, β, p_1, and p_2 are given as:

$$p_1 = 0.01,\ p_2 = 0.06,\ \alpha = 0.05,\ \text{and}\ \beta = 0.10.$$

Hald's [11] solution to this example is n = [110,138] for c=3.

Before the fuzzy approach is presented, it is necessary to specify the upper tolerance limits for α, β, and n. These are given as:

$$\alpha_u = 0.09,\ \beta_u = 0.15,\ \text{and}\ n_u = 500.$$

Next, auxiliary functions are used to find the initial acceptance number c according to equation (5.7). From equation (5.5), the auxiliary function r is found. That is:

$$r = \frac{0.06(2 - 0.01)}{0.01(2 - 0.06)} = 6.1546$$

Then, using χ^2 values given in [16] with degrees of freedom ν=2c+2, R(c) is obtained from equation (5.6) until the condition in equation (5.7) is met. That is:

$$R(c) = \begin{cases} 44.885 & , \text{for } \nu = 2 \\ 10.946 & , \text{for } \nu = 4 \\ 6.509 & , \text{for } \nu = 6 \\ 4.890 & , \text{for } \nu = 8 \end{cases}$$

Thus, at $\nu = 8$, the condition is met and the acceptance number is $c = 3$.

The design procedure begins by accepting membership functions $\mu_A(\alpha')$ for fuzzy set **A**, from equation (5.17), and $\mu_B(\beta')$ for fuzzy set **B**, from equation (5.18). These membership functions are defined as:

$$\mu_{\tilde{A}}(\alpha') = \begin{cases} 1 & , \text{ iff } \alpha' < 0.05 \\ \dfrac{0.09 - \alpha'}{0.04} & , \text{ iff } 0.05 \le \alpha' \le 0.09 \\ 0 & , \text{ iff } 0.09 < \alpha'. \end{cases} \quad (6.1)$$

$$\mu_{\tilde{A}}(\beta') = \begin{cases} 1 & , \text{ iff } \beta' < 0.10 \\ \dfrac{0.15 - \beta'}{0.05} & , \text{ iff } 0.10 \le \beta' \le 0.15 \\ 0 & , \text{ iff } 0.15 < \beta'. \end{cases} \quad (6.2)$$

These membership functions are graphically illustrated in Figures 6.1 and 6.2, respectively.

It is indicated in equations (5.13) and (5.14) that α' and β' are functions of sample size n and acceptance number c. They are defined as $1-P(p_1)$ and $P(p_2)$, respectively. Therefore, we can rewrite equations (6.1) and (6.2) as:

$$\mu_{\tilde{A}}(\alpha') = \begin{cases} 1 & , \text{ iff } 0.95 < P(p_1) \\ \dfrac{P(p_1) - 0.91}{0.04} & , \text{ iff } 0.91 \le P(p_1) \le 0.95 \\ 0 & , \text{ iff } P(p_1) < 0.91 . \end{cases} \quad (6.3)$$

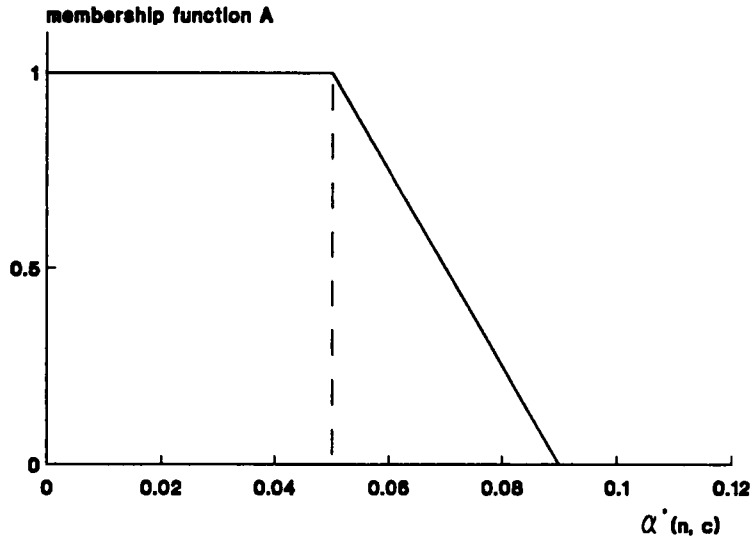


Figure 6.1 Illustration of Equation (6.1)

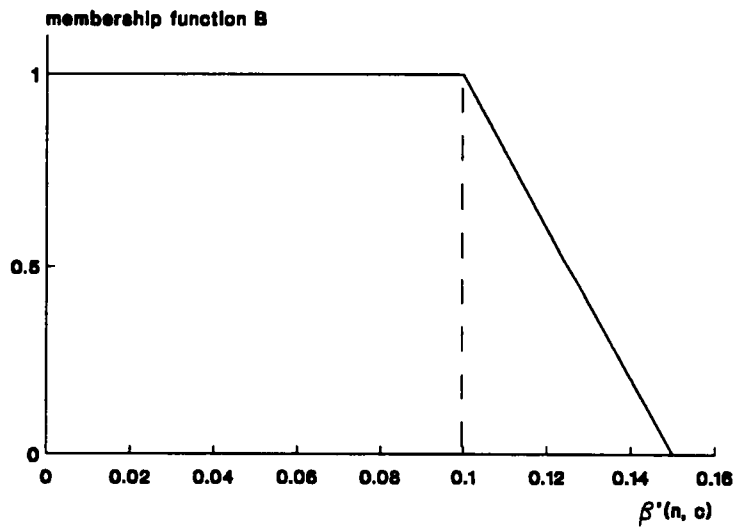


Figure 6.2 Illustration of Equation (6.2)

$$\mu_B(\beta') = \begin{cases} 1 & , \text{ iff } P(p_2) < 0.10 \\ \dfrac{0.15 - P(p_2)}{0.05} & , \text{ iff } 0.10 \le P(p_2) \le 0.15 \\ 0 & , \text{ iff } 0.15 < P(p_2) \ . \end{cases} \quad (6.4)$$

$P(p_1)$ and $P(p_2)$ can be obtained respectively from equations (4.3) and (4.4) with respect to sample size n for each specified acceptance number c. Values for equations (6.3) and (6.4) are shown in Table 6.1 and Table 6.2. These values are obtained from mathematical programs in appendix A for equation (6.3) and appendix B for equation (6.4).

For the membership function $\mu_n(n)$, the upper tolerance limit for sample size n is given as $n_u = 500$. Thus, from equation (5.17), the membership function $\mu_n(n)$ is defined as:

$$\mu_n(n) = \begin{cases} \dfrac{500 - n}{500} & , \quad \text{iff} \quad n < 500 \\ 0 & , \quad \text{iff} \quad n \ge 500. \end{cases} \quad (6.5)$$

This membership function is illustrated in Figure 5.5 with n_u=500.

Table 6.1 Membership Function **A**, $\mu_A(n)$

Sample size n	$\mu_A(n)$		
	for c=3	for c=2	for c=1
35	1	1	1
40	1	1	0.7316
45	1	1	0.3840
50	1	1	0.0141
55	1	1	0
60	1	1	0
65	1	1	0
70	1	1	0
75	1	1	0
80	1	1	0
85	1	0.9004	0
90	1	0.7014	0
95	1	0.4897	0
100	1	0.2657	0
105	1	0.0300	0
110	1	0	0
115	1	0	0
140	0.9296	0	0
145	0.7852	0	0
150	0.6327	0	0
155	0.4722	0	0
160	0.3040	0	0
165	0.1281	0	0
170	0	0	0
175	0	0	0

Table 6.2 Membership Function B, $\mu_B(n)$

Sample size n	$\mu_B(n)$		
	for c=3	for c=2	for c=1
54	0	0	0
56	0	0	0.1389
58	0	0	0.4014
60	0	0	0.6415
62	0	0	0.8610
64	0	0	1
66	0	0	1
68	0	0	1
70	0	0	1
72	0	0	1
74	0	0	1
76	0	0	1
78	0	0.0799	1
80	0	0.3111	1
82	0	0.5260	1
84	0	0.7255	1
86	0	0.9105	1
88	0	1	1
90	0	1	1
100	0.1395	1	1
102	0.3437	1	1
104	0.5352	1	1
106	0.7146	1	1
108	0.8825	1	1
110	1	1	1
112	1	1	1

The acceptance number is determined to be c = 3, $\mu_A(\alpha')$ and $\mu_B(\beta')$ are respectively obtained from equations (6.3) and (6.4). This is illustrated in Figure 6.3. Decision **D** is illustrated as the intersection of **A**, **B**, and **n**, which is indicated by a bold line. This is defined using equation (5.18) as:

$$\mathbf{D} = (\mathbf{A} \cap \mathbf{B}) \cap \mathbf{n} \quad , \text{ for } c = 3 \tag{6.6}$$

The membership functions of this fuzzy set **D** is written as:

$$\mu_D(n) = \min \{\mu_A(\alpha'), \mu_B(\beta'), \mu_n(n)\} \tag{6.7}$$

Similarly, Figure 6.4 illustrates the results for c = 2 and Figure 6.5 illustrates the results for c = 1. Notice in Figure 6.5, the intersection of μ_A and μ_B does not exist. Therefore, the decision for c = 1 is null and does not exist. Also, since the highest membership value of decision **D** is desired, it is not necessary to calculate membership values for c that is greater than 3. This is not needed since μ_n is decreasing as the sample size n increases.

An overall graphical illustration is shown in Figure 6.6 for c = 3, c = 2, and c = 1. The solution to the sampling plan is obtained by the highest membership value in the intersection, which is the decision **D** line.

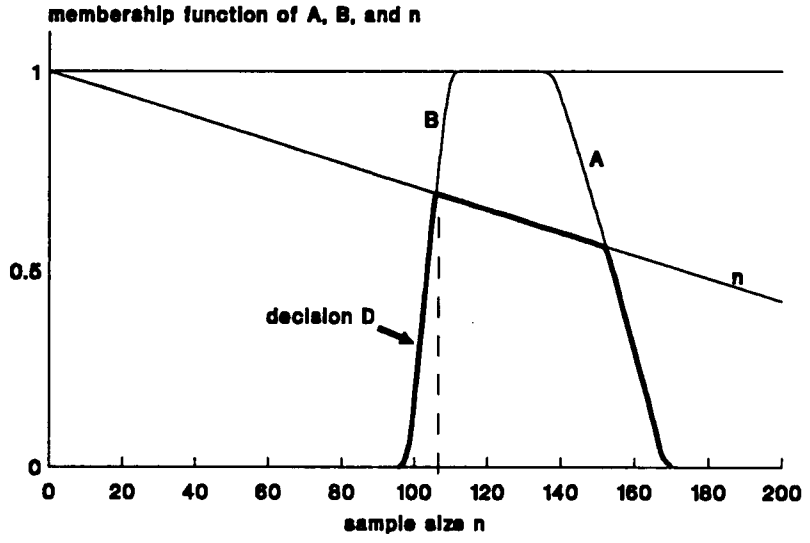


Figure 6.3 Intersection of **A**, **B**, and **n** for c=3

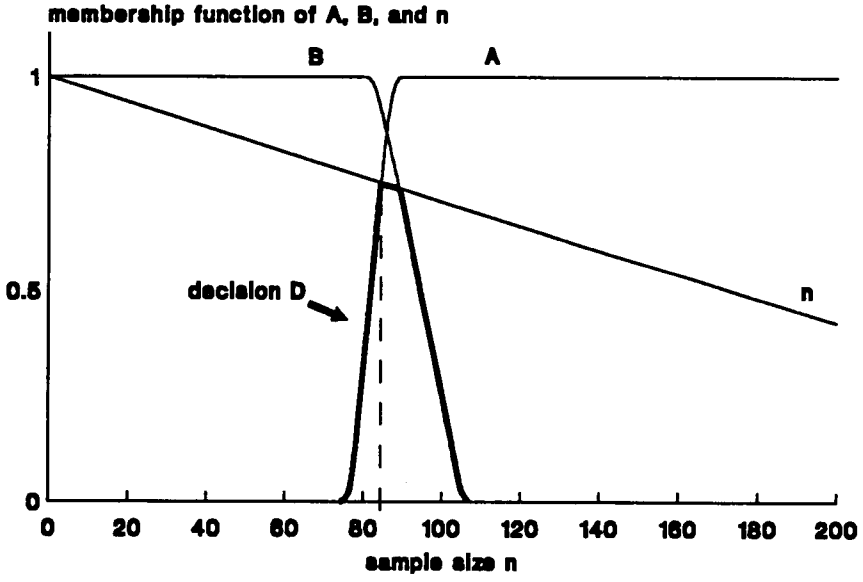


Figure 6.4 Intersection of **A**, **B**, and **n** for c=2

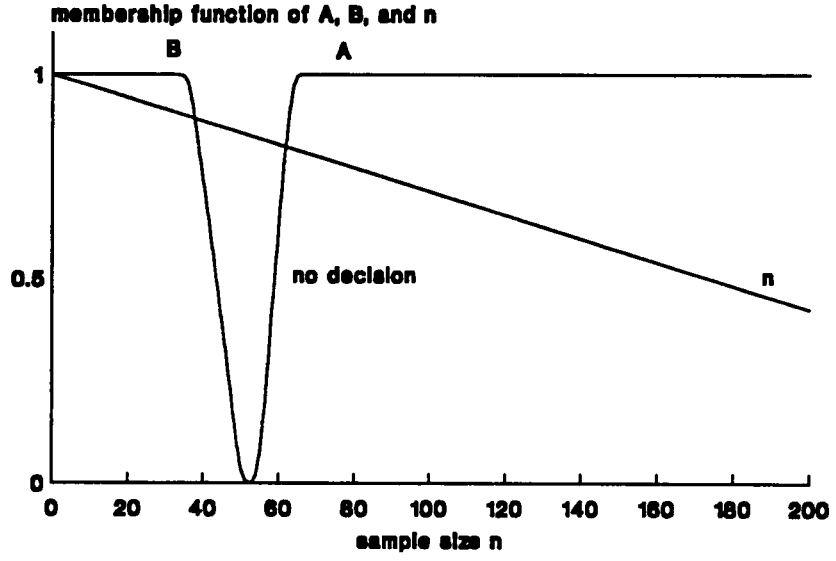


Figure 6.5 Intersection of **A**, **B**, and **n** for c=1

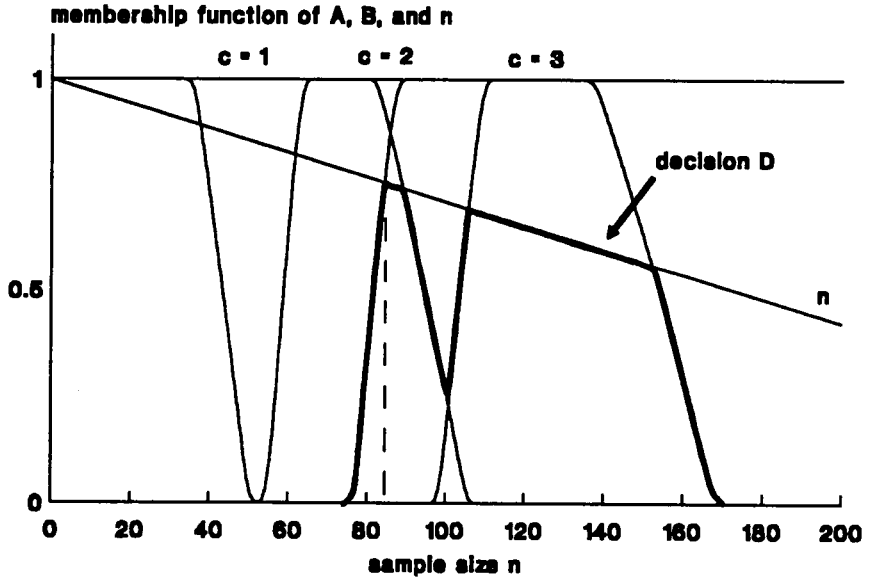


Figure 6.6 Intersection of **A**, **B**, and **n** for All c

From Figure 6.6, sampling plan of n = 86 and c = 2 is obtained for the membership value of 0.828. Thus, in example A, sample size was decreased by 24 and acceptance number was decreased by 1. The actual producer's risk is α'=0.056 (which is less than α_u=0.09) and the actual consumer's risk is β'=0.104 (which is less than β_u=0.15). The results of other examples are summarized in Table 6.3. A mathematical program has been developed to perform the iterative calculations in this design procedure. This program is written in BASICA and is presented in Appendix C. A sample run for this example is presented in Appendix D.

6.2 DISCUSSION

Since the proposed method relaxes the conditions on producer's and consumer's risks by giving upper tolerance limits (α_U and β_U), it is questionable to directly compare the proposed method's solution with Hald's conventional solution. For example, when α=0.09 and β=0.15 (upper tolerance limits for α and β in the numerical example A), Hald's solution is c=2 and n=[80,104]. Then the actual producer's risk becomes α=0.0466 (which is less than α=0.09) and the actual consumer's risk becomes β=0.1344 (which is less than β=0.15). Thus a problem exists in interpreting the amount of relaxation given in the producer's and consumer's risks. That is, the proper determination of the upper tolerance limits for α and β.

Table 6.3 Examples Comparing Hald's (H) and Proposed (P) Methods for Determining Single Attribute Sampling Plan

EXAMPLE	A			B		
α β p_1 p_2	0.050 0.100 0.010 0.060			0.005 0.010 0.010 0.060		
α_u β_u n_u	0.090 0.150 500			0.008 0.025 500		
METHOD	H		P	H		P
c	3		2	8		7
n	110	138	86	286	316	247
actual α	0.025	0.051	0.056	0.003	0.005	0.004
actual β	0.098	0.031	0.104	0.010	0.003	0.017

EXAMPLE	C			D		
α β p_1 p_2	0.025 0.050 0.010 0.050			0.010 0.001 0.010 0.100		
α_u β_u n_u	0.045 0.070 500			0.020 0.003 500		
METHOD	H		P	H		P
c	5		4	5		4
n	208	222	176	159	181	137
actual α	0.019	0.025	0.033	0.006	0.010	0.013
actual β	0.049	0.032	0.058	0.001	0.000	0.002

To analyze this problem, different sampling plans for example A are examined. These are presented in Table 6.4 (values obtained from program presented in appendix C). Notice, in Table 6.4, sample sizes n and acceptance numbers c generally tend to decrease as α_U and β_U increase.

A sampling plan then could be selected from Table 6.4 which satisfies the conditions defining Hald's. One sampling plan is n=110 and c=3, where the actual α=0.025 (which is less than α=0.05) and the actual β=0.098 (which is less than β=0.10). Thus, a sampling plan of n=110 and c=3 could be selected with $\alpha_U \leq 0.08$ and $\beta_U \leq 0.105$. This gives an identical sampling plan like Hald's.

Another significant finding is that when the conventional formulation is adopted (Hald's), both the actual risks of α and β do not exceed their specified values (as seen in Table 6.3). That is, according to Hald, the smaller the values for both actual risks represent a better condition. However, the requirement to reduce both actual risks conflicts with the requirement to reduce sample sizes in inspection. That is, utilizing Hald's method to determine the sample size will result in controlling either the producer's risk (α) or the consumer's risk (β) at the design level. Thus, in Hald's conventional method, a decision is made either:

Table 6.4 determining sampling plan with α=0.05 and β=0.1

β_U	α_U				
	.0550	.0600	.0700	.0800	.0900
.1050	n=110 c=3	n=110 c=3	n=110 c=3	n=110 c=3	n=87 c=2
.1100	n=109 c=3	n=109 c=3	n=109 c=3	n=109 c=3	n=87 c=2
.1200	n=109 c=3	n=109 c=3	n=109 c=3	n=109 c=3	n=87 c=2
.1300	n=108 c=3	n=108 c=3	n=108 c=3	n=86 c=2	n=86 c=2
.1400	n=108 c=3	n=108 c=3	n=108 c=3	n=86 c=2	n=86 c=2
.1500	n=107 c=3	n=107 c=3	n=85 c=2	n=85 c=2	n=86 c=2

(1) to control α (producer's risk) close to its design value and choose the sample size such that the actual value for β (consumer's risk) is smaller than the design value, or

(2) to control β (consumer's risk) close to its design value and choose the sample size such that the actual value for α (producer's risk) is smaller than the design value.

However, as indicated in the design of a single sampling attribute plan (see Section 4.4), one would like the producer's risk to equal α and the consumer's risk to equal β. By over-viewing the results of Hald's solution in Table 6.3, the conventional method simply does not satisfy the exact sampling plan's design values. However, the proposed fuzzy method does come close in achieving this goal. That is, in Table 6.3, all actual α and β values in the proposed design seem to be approximately equal to the given design values of α and β. While the actual α and β values in Hald's design seem to be approximately equal to the given design value of α (when looking at the upper sample size n) and approximately equal to the given design value of β (when looking at the lower sample size n). Thus, the proposed design gives strength to both the consumer's risk and the producer's risk by being approximately equal to both design values.

6.3 CONCLUSION

A design procedure for the single attribute sampling plan based on fuzzy set theory has been presented. In this research, the proposed fuzzy design was improved by explicitly considering the minimization of the sample size and by following Hald's risk conditions. The proposed solution (n=86 and c=2) results in 24 sample size reduction and 1 acceptance number reduction when compared to Hald's solution (n=[110,138]

and c=3) in example A. However, the proposed fuzzy design does not intend to replace the probabilistic methods by any means, as problems still remain to be resolved. One such problem is the question of assigning upper tolerance limit values for membership functions of α, β, and n.

But the significance of this paper lies in the usage of membership functions, in fuzzy set theory, to define and to represent the conditions of the producer's risk, the consumer's risk, and of the sample size. By using these fuzzy methodologies, goals and constraints become identical in the formulation of a decision process. Thus, getting rid of the imbalance between goals and constraints, results in having only one objective function. Also, the proposed design gives strength to both risks by being approximately equal to both specified α and β design values.

However, direct comparison of the proposed fuzzy design with Hald's has to be done with caution. The proposed fuzzy design incorporates qualitative measures of the design factors compared to the conventional design (Hald's) which incorporated discrete quantitative measures. It is simply two different design approaches and the results have some different meanings. A feasible range of the upper tolerance limits (α_U, β_U, n_U) in the proposed fuzzy design must be clearly understood and accepted by both the consumer and the producer. Thus, some problems exist in interpreting the information provided by these results.

Also, it appears that fuzzy methodologies can become a more significant tool in resolving IE type problems. Similar to the manner in which fuzzy set theory was applied in this paper to a single attribute sampling plan, alternative solution techniques need to be examined in other IE application areas (Engineering Economy, Decision Analysis, Expert Systems, etc.).

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APPENDIXES

Appendix A program for calculating the membership function, $\mu_A(n)$

```
10    CLS
20    CLEAR
30    '  **********************
40          Variables
50    '  **********************
60    INPUT "enter alpha ...  ",ALPHA
70    INPUT "enter upper alpha ...  ",ALPHAU
80    INPUT "enter p1 ...  ",P1
90    INPUT "enter acceptance number c ...  ",C
100   INPUT "enter the range of values for the sample size n      ...  ",A,B
110   PRINT:PRINT:PRINT:PRINT,"sample size n",,"membership          function
      A"
120   PRINT,"------------",,"----------------------"
130   '       ***************************************************
140   '       membership function respect to n : uA(n)
150   '       ***************************************************
160   DIM UA(400)
170   FOR N=A TO B STEP 1
180         FOR K=0 TO C
190         NFACT=1
200         KFACT=1
210         IF K<1 THEN 300
220               FOR X=N TO (N-K) STEP -1
230               NFACT=NFACT*X
240               NEXT X
250         NFACT=NFACT/(X+1)
260         IF K<2 THEN 300
270               FOR X=1 TO K
280               KFACT=KFACT*X
290               NEXT X
300         Y=Y+(NFACT/KFACT)*(P1^K)*(1-P1)^(N-K)
310         NEXT K
320   UA(N)=(ALPHAU-1+Y)/(ALPHAU-ALPHA)
330   IF (1-Y)<=ALPHA THEN UA(N)=1
340   IF (1-Y)>AHPLAU THEN UA(N)=0
350   PRINT, N,, USING"#.####";UA(N)
360   Y=0
370   NEXT N
380   END
```

Appendix B program for calculating the membership function, $\mu_B(n)$

```
10    CLS
20    CLEAR
30    '  **********************
40          Variables
50    '  **********************
60    INPUT "enter beta ...  ",BETA
70    INPUT "enter upper beta ...  ",BETAU
80    INPUT "enter p2 ...  ",P2
90    INPUT "enter acceptance number c ...  ",C
100   INPUT "enter the range of values for the sample size n      ...  ",A,B
110   PRINT:PRINT:PRINT:PRINT,"sample size n",,"membership          function
      B"
120   PRINT,"------------",,"--------------------"
130   '       ***************************************************
140   '       membership function respect to n : uB(n)
150   '       ***************************************************
160   DIM UB(400)
170   FOR N=A TO B STEP 1
180         FOR K=0 TO C
190         NFACT=1
200         KFACT=1
210         IF K<1 THEN 300
220               FOR X=N TO (N-K) STEP -1
230               NFACT=NFACT*X
240               NEXT X
250         NFACT=NFACT/(X+1)
260         IF K<2 THEN 300
270               FOR X=1 TO K
280               KFACT=KFACT*X
290               NEXT X
300         Y=Y+(NFACT/KFACT)*(P2^K)*(1-P2)^(N-K)
310         NEXT K
320   UB(N)=(BETAU-1+Y)/(BETAU-BETA)
330   IF (1-Y)<=BETA THEN UB(N)=1
340   IF (1-Y)>BETAU THEN UB(N)=0
350   PRINT, N,, USING"#.####";UB(N)
360   Y=0
370   NEXT N
380   END
```

Appendix C program for determining single attribute sampling plan using the proposed design procedure

```
10    CLS
20    CLEAR
30    '      ********************
40    '         Variables
50    '      ********************
60    DIM R(25), UB(500), UN(500), D(500), MAXN(25), MAXV(25)
70    PRINT SPC(10)"_____________________________________________________"
80    PRINT TAB(17)"Single Attribute Sampling Plan: Enter Data"
90    INPUT "                    Producer's Risk: alpha= ",ALPHA
100   INPUT "          Upper Producer's Risk: alpha_u= ",ALPHAU
110   INPUT "                    Consumer's Risk: beta= ",BETA
120   INPUT "          Upper Consumer's Risk: beta_u=",BETAU
130   PRINT "              Fraction Defective"
140   INPUT "                (Acceptable Level): p1= ",p1
150   PRINT "              Fraction Defective"
160   INPUT "              (Unacceptable Level): p2= ",p2
165   INPUT "                Upper Sample Size: n_u= ",NU
170   PRINT SPC(10)"_____________________________________________________"
180   PRINT:PRINT:PRINT
190   INPUT "                              press ENTER to continue ... ",A$
200   CLS
210   HP1=2*P1/(2-P1)
220   HP2=2*P2/(2-P2)
230   R=HP2/HP1
240   '**************************************
250   ' membership function: uA(alpha')
260   '**************************************
270   FOR ALPHAp=ALHPA TO ALPHAU STEP 0.005
280   UA(ALPHAp)=(ALPHAU-ALPHAp)/(ALPHAU-ALPHA)
290   NEXT ALPHAp
300   '**************************************
310   ' membership function: uB(beta')
320   '**************************************
330   FOR BETAp=BETA TO BETAU STEP 0.005
340   UA(BETAp)=(BETAU-BETAp)/(BETAU-BETA)
350   NEXT BETAp
360   FOR C=0 TO 50
370   PRINT "_____________________________________________________"
380   PRINT "              Selecting Acceptance Number c"
390   PRINT "      The Following Condition must be satisfied:"
```

```
400   PRINT "                    r >= R(c; alpha,beta)"
410   PRINT "              Where: r= ";USING "##.####";R
420   PRINT "              And:  d.f.= ";2*C+2;" 1-alpha= ";1-ALPHA;"     beta=
      ";BETA
430   PRINT "_____________________________________________________"
440   INPUT "      ENTER Chi-square Value for (1-alpha)...
      ",CHISQUAREa
445   PRINT "      ENTER Chi-square Value for beta...",CHISQUAREb
450   R(C)=CHISQUAREb/CHISQUAREa
460   PRINT:PRINT "R(c)= " USING "##.####";R(C)
470   IF R>=R(C) THEN 530
480   PRINT:PRINT "      ***** condition NOT satisfied *****"
490   PRINT "_____________________________________________________"
500   INPUT "              press ENTER to continue... ",A$
510   CLS
520   NEXT C
530   PRINT:PRINT "      ***** condition satisfied [c=";C;"] *****"
540   PRINT "_____________________________________________________"
550   INPUT "              press ENTER to continue... ",A$
560   CLS
570   PRINT "One Moment Please... "
580   '*****************************************
590   ' membership functions calculations
600   '*****************************************
610   FOR CMAX=C TO 1 STEP -1
620         FOR N=6 TO NU
630               FOR K=O TO CMAX
640               NFACT=1
650               KFACT=1
660               IF K<1 THEN 750
670                     FOR X=N TO (N-K) STEP -1
680                     NFACT=NFACT*X
690                     NEXT X
700               NFACT=NFACT/(X+1)
710               IF K<2 THEN 750
720                     FOR X=1 TO K
730                     KFACT=KFACT*X
740                     NEXT X
750               Y1=Y1+(NFACT/KFACT)*(P1^K)*(1-P1)^(N-K)
760               Y2=Y2+(NFACT/KFACT)*(P2^K)*(1-P2)^(N-K)
770               NEXT K
780         UA(N)=(ALPHAU-1+Y1)/(ALPHAU-ALPHA)
```

```
790         UB(N)=(BETAU-Y2)/(BETAU-BETA)
800         UN(N)=1-(N/NU)
810         IF (1-Y1)<=ALPHA THEN UA(N)=1
820         IF Y2<=BETA THEN UB(N)=1
830         IF (1-Y1)>ALPHAU THEN UA(N)=0
840         IF Y2>BETAU THEN UB(N)=0
850         IF UA(N)<UB(N) AND UA(N)<UN(N) THEN D(N)=UA(N)
860         IF UB(N)<UA(N) AND UB(N)<UN(N) THEN D(N)=UB(N)
870         IF UN(N)<UA(N) AND UN(N)<UB(N) THEN D(N)=UN(N)
880         IF D(N)>D(N-1) THEN MAXN(CMAX)=N
890         IF D(N)>D(N-1) THEN MAXV(CMAX)=D(N)
900         Y1=0
910         Y2=0
920         NEXT N
930   IF MAXV(CMAX)>MAXV(CMAX+1) THEN VMAX=MAXV(CMAX)
940   IF MAXV(CMAX)>MAXV(CMAX+1) THEN NMAX=MAXN(CMAX)
950   IF MAXV(CMAX)>MAXV(CMAX+1) THEN MAXC=CMAX
960   PRINT:PRINT "Acceptance Number c = ";CMAX
970   PRINT "Sample Size n = ";MAXN(CMAX)
980   PRINT "Membership Value u(n) = ";MAXV(CMAX)
990   PRINT:PRINT "One Moment Please ..."
1000  NEXT CMAX
1010  PRINT:INPUT "press ENTER for the solution... ";A$
1020  PRINT:PRINT:PRINT
1030  PRINT SPC(10)"_____"
1040  PRINT TAB(10)" Solution to Single Attribute Sampling      Plan:   Enter
      Data"
1050  PRINT "                    Producer's Risk: alpha= ",ALPHA
1060  PRINT "          Upper Producer's Risk: alpha_u= ",ALPHAU
1070  PRINT "                    Consumer's Risk: beta= ",BETA
1080  PRINT "          Upper Consumer's Risk: beta_u=",BETAU
1090  PRINT "              Fraction Defective"
1100  PRINT "                (Acceptable Level): p1= ",p1
1110  PRINT "              Fraction Defective"
1120  PRINT "              (Unacceptable Level): p2= ",p2
1130  PRINT "                 Upper Sample Size: n_u= ",NU
1140  PRINT SPC(10)"_____"
1150  PRINT:PRINT "Acceptance Number c = ";MAXC
1160  PRINT "Sample Size n = ";NMAX
1170  PRINT "Membership Value u(n) = ";VMAX
1180  PRINT SPC(10)"_____"
1190  END
```

Appendix D computer run example of given sampling plan using the proposed design procedure

---

Single Attribute Sampling Plan: Enter Data

Producer's Risk: alpha = 0.05
Upper Producer's Risk: alpha_u = 0.09
Consumer's Risk: beta = 0.1
Upper Consumer's Risk: beta_u = 0.15
Fraction Defective
(Acceptable Level): p1 = 0.01
Fraction Defective
(Unacceptable Level): p2 = 0.06
Upper Sample Size: n_u = 500

---

press ENTER to continue ...

Screen 1

Selecting Acceptance Number c
The Following Condition Must Be Satisfied:

r >= R(c; alpha,beta)

Where: r = 6.1546

d.f. = 2 1-alpha = 0.95 beta = 0.1

Enter Chi-square Value for (1-alpha)... 0.1026
Enter Chi-square Value for beta... 4.6052

R(c) = 44.8850

********* condition NOT satisfied *********

---

press ENTER to continue...

Screen 2

Selecting Acceptance Number c
The Following Condition Must Be Satisfied:

r >= R(c; alpha,beta)

Where: r = 6.1546

d.f. = 4 1-alpha = 0.95 beta = 0.1

Enter Chi-square Value for (1-alpha)... 0.7107
Enter Chi-square Value for beta... 7.7794

R(c) = 10.9461

********* condition NOT satisfied *********

---

press ENTER to continue...

Screen 3

Selecting Acceptance Number c
The Following Condition Must Be Satisfied:

r >= R(c; alpha,beta)

Where: r = 6.1546

d.f. = 6 1-alpha = 0.95 beta = 0.1

Enter Chi-square Value for (1-alpha)... 1.6354
Enter Chi-square Value for beta... 10.6446

R(c) = 6.5089

********* condition NOT satisfied *********

---

press ENTER to continue...

Screen 4

Selecting Acceptance Number c
The Following Condition Must Be Satisfied:

r >= R(c; alpha,beta)

Where: r = 6.1546

d.f. = 8 1-alpha = 0.95 beta = 0.1

Enter Chi-square Value for (1-alpha)... 2.7326
Enter Chi-square Value for beta... 13.3616

R(c) = 4.8897

******** condition satisfied [c=3] ********

---

press ENTER to continue...

Screen 5

One Moment Please... Acceptance Number c = 3
Sample Size n = 107
Membership Value u(n) = 0.786

One Moment Please... Acceptance Number c = 2
Sample Size n = 86
Membership Value u(n) = 0.828

One Moment Please... Acceptance Number c = 1
Sample Size n = 0
Membership Value u(n) = 0

press ENTER for the solution...

Screen 6

Solution to Single Attribute Sampling Plan

Producer's Risk:	alpha = 0.05
Upper Producer's Risk:	alpha_u = 0.09
Consumer's Risk:	beta = 0.1
Upper Consumer's Risk:	beta_u = 0.15
Fraction Defective (Acceptable Level):	p1 = 0.01
Fraction Defective (Unacceptable Level):	p2 = 0.06
Upper Sample Size:	n_u = 500

Acceptance Number c = 2
Sample Size n = 86
Membership Value u(n) = 0.828

Screen 7

VITA

Sung-ho Son was born in Choon-Chun, Republic of Korea, on January 11, 1966. He and his family came to United States in 1975. He attended public schools in Nashville, Tennessee and was graduated from Hillsboro High School in June 1984. The following January he entered The University of Tennessee at Knoxville, and in May 1989 he received a Bachelor of Science Degree in Electrical and Computer Engineering. He entered the Graduate School at The University of Tennessee at Knoxville in June 1991 and received a Master of Science Degree in Industrial Engineering in December 1993.