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To the Committee on Graduate Study:

I am submitting to you a thesis written by John Charles Harden entitled "Direct and Semidirect Products of Semigroups." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Arts with a major in Mathematics.

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DIRECT AND SEMIDIRECT PRODUCTS OF SEMIGROUPS

A THESIS

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DIRECT AND SEMIDIRECT PRODUCTS OF SEMIGROUPS

0. Introduction

The purpose of this thesis is to exhibit the direct products and the semidirect products of semigroups of orders two, three, and four. Section I consists of the definitions necessary for the proofs and computations which appear in the first five sections. In Section 2 theorems on direct products of semigroups of any order are proved. A method of writing by inspection the multiplication table of the direct product of two finite semigroups is presented in Section 3. Section 4 consists of tables listing the direct products of semigroups of order two with semigroups of orders two, three, and four and a table listing the direct products of semigroups of order three with semigroups of order three. In Section 5 is considered the problem of describing the multiplication table of the direct product of two semigroups S and T by adjoining rows and columns to the multiplication table of the direct product of subsemigroups contained in S and T respectively. The direct products of those third and fourth order semigroups which are not so described are listed in Section 5. In Section 6, semidirect products of semigroups are defined, some theorems concerning them are proved, and examples are cited to show that some of the theorems on direct products are not necessarily true for semidirect products. A method of writing by inspection the multiplication table of the semidirect product of a finite semigroup and the semigroup of its endomorphisms is exhibited in Section 7. The endomorphisms of the semigroups of orders two and three, the automorphisms of the semigroups of order four, and the semidirect products of orders two, three, and four

are listed in Section 8.

As an appendix ~~we include~~ a listing, compiled jointly by the author and Messrs. K. S. Carman and E. E. Posey, of the semigroups of orders two, three, and four. From time to time we cite examples by number from this list; the digit to the left of the decimal point of each such number is the order of the semigroup, and the number following the decimal point gives the lineal position of the semigroup among all those of that order.

1. Definitions

Definition 1.1. A semigroup is a system consisting of a non-empty set S of elements and a single-valued binary operation such that (1) to every ordered pair (a, b) of elements of S the operation associates an unique element c of S , usually called the product of a and b (we write $a \cdot b = c$ or $ab = c$), and (2) for any elements a, b, c of S , $a(bc) = (ab)c$. A semigroup is commutative if, for all elements a, b of S , $ab = ba$. The number of elements in a finite semigroup is the order of the semigroup.

Definition 1.2. A subsemigroup of a semigroup S is a subset of S which is itself a semigroup. Thus S itself is a subsemigroup; all other subsemigroups of S are called proper subsemigroups.

Definition 1.3. An element e of a semigroup S is a left identity element of S if, for every element a of S , $ea = a$. An element h of S is a right identity element of S if, for every element a of S , $ah = a$. An element of S which is both a left identity element and a right identity element is called a two-sided identity element, or simply an identity element, of S . A semigroup may have more than one left

identity element or right identity element, but if it has at least one left identity element e and at least one right identity element h , then $h = eh = e$, whence the semigroup has exactly one two-sided identity element and no other left or right identity element.

Definition 1.4. An element c of a semigroup S is a left constant element of S if there is an element $d \in S$ such that $ca = d$ for all $a \in S$. An element k of S is a right constant element of S if there is an element $g \in S$ such that $ak = g$ for all $a \in S$. An element of S which is both a left constant element and a right constant element is called a two-sided constant element. A semigroup may have more than one left constant or right constant element, but if it has at least one left constant element c such that $ca = d$ for all $a \in S$ and at least one right constant element k such that $ak = g$ for all $a \in S$, then $g = ck = d$.

Definition 1.5. A left constant element c of a semigroup S is a left zero element of S if $ca = c$ for all $a \in S$. A right constant element k of S is a right zero element of S if $ak = k$ for all $a \in S$. An element of S which is both a left zero element and a right zero element is called a two-sided zero element. A semigroup may have more than one left zero element or right zero element, but if it has at least one left zero element and one right zero element then by Definition 1.4 it has one and only one two-sided zero element and no other left or right zero element.

Lemma 1.1. If S is a semigroup containing a left (right) constant element c such that $ca = c'$ ($ac = c'$), then c' is a left (right) zero element for S .

Proof. Let $b \in S$. Then $c'b = (ca)b = c(ab) = c'$.

Lemma 1.2. If S is a semigroup containing a left (right) zero element z , then for any $a \in S$, the product of a and z in either order is a left (right) zero element for S .

Proof. Let $b \in S$. Then $(az)b = a(zb) = az$, whence az is a left zero element of S ; and by hypothesis $za = z$, a left zero element of S .

Definition 1.6. If a semigroup S contains a two-sided zero element z , an element a is a left zero divisor if there exists an element $b \in S$, distinct from z , such that $ab = z$, and an element a is said to be a right zero divisor if there exists an element $b \in S$, distinct from z , such that $ba = z$.

Definition 1.7. A semigroup S is a null semigroup if there is an element $z \in S$ such that $ab = z$ for all $a, b \in S$. The element z is a two-sided zero element of S .

Definition 1.8. A non-empty set A of elements contained in a semigroup S is a left ideal of S if $SA \subseteq A$. A non-empty subset B of S is a right ideal of S if $BS \subseteq B$. If a set of elements A is both a left ideal and a right ideal, it is a two-sided ideal.

Definition 1.9. An element a of a semigroup is idempotent if $aa = a$.

Definition 1.10. Two semigroups S and T are isomorphic if there exists a one-to-one correspondence $s \longleftrightarrow t$ between their elements which preserves semigroup multiplication, i.e., which is such that if $s \longleftrightarrow t$ and $s' \longleftrightarrow t'$, then $ss' \longleftrightarrow tt'$.

Definition 1.11. A semigroup S is a group if (1) there exists at least one left identity element $e \in S$, and (2) for every element $a \in S$,

there exists at least one element a^{-1} (called a left inverse of a) such that $a^{-1}a = e$. It is well known¹ that under these assumptions e is a two-sided identity element (and is therefore unique) and that $a^{-1}a = aa^{-1}$ and a^{-1} is unique.

Definition 1.12. The direct product of a semigroup S and a semigroup T (we write $S \times T$) is the system consisting of all ordered pairs (s, t) , where $s \in S$ and $t \in T$, and the operation $(s, t)(s', t') = (ss', tt')$. The ordered pairs (s, t) and (s', t') are equal if and only if $s = s'$ and $t = t'$. The order of $S \times T$, if S and T are of finite order, is obviously the product of the orders of S and T .

2. Theorems on direct products of semigroups of any order

Theorem 2.1. The system $S \times T$ is a semigroup.

Proof. Let (a, b) , (c, d) , (h, f) be arbitrary elements of $S \times T$. Then $(a, b)(c, d) = (ac, bd)$. Since $ac \in S$ and $bd \in T$, $(ac, bd) \in S \times T$; hence multiplication is well defined in $S \times T$.

Furthermore,

$$\begin{aligned} [(a, b)(c, d)](h, f) &= (ac, bd)(h, f) &= [(ac)h, (bd)f] \\ &= [a(ch), b(df)] &= (a, b)(ch, df) \\ &= (a, b)[(c, d)(h, f)], \end{aligned}$$

whence the multiplication defined in $S \times T$ is associative.

Theorem 2.2. An element $(a, b) \in S \times T$ is idempotent if and only if a is an idempotent element of S and b is an idempotent element of T .

Proof. (1) Let $(a, b)(a, b) = (a, b)$. Then

¹H. Zassenhaus, Lehrbuch der Gruppentheorie (Leipzig and Berlin: B. G. Teubner, 1937), p. 2.

$$(a, b) = (a, b)(a, b) = (aa, bb) ,$$

whence $a = aa$ and $b = bb$. (2) Let $a = aa$ and $b = bb$. Then $(a, b) = (aa, bb) = (a, b)(a, b)$.

Theorem 2.3. An element $(a, b) \in S \times T$ is a left (right) identity element of $S \times T$ if and only if a is a left (right) identity element of S and b is a left (right) identity element of T .

Proof. (1) Let (x, y) be an arbitrary element of $S \times T$, and let $(a, b)(x, y) = (x, y)$. Then $(x, y) = (a, b)(x, y) = (ax, by)$, whence $x = ax$ and $y = by$. (2) Let x and y be arbitrary elements of S and T , respectively, and let $ax = x$ and $by = y$. Then $(a, b)(x, y) = (ax, by) = (x, y)$.

Theorem 2.4. An element $(a, b) \in S \times T$ is a left (right) constant element of $S \times T$ if and only if a is a left (right) constant element of S and b is a left (right) constant element of T .

Proof. (1) Suppose $(a, b)(x, y) = (c, d)$ for some fixed element (c, d) and all elements (x, y) of $S \times T$. Then

$$(ax, by) = (a, b)(x, y) = (c, d) ,$$

whence $ax = c$ and $by = d$. (2) Suppose that $ax = c$ for some fixed element $c \in S$ and all elements $x \in S$, and that $by = d$ for some fixed element $d \in T$ and all elements $y \in T$. Then

$$(a, b)(x, y) = (ax, by) = (c, d) .$$

Corollary. An element $(a, b) \in S \times T$ is a left (right) zero element of $S \times T$ if and only if a is a left (right) zero element of S and b is a left (right) zero element of T .

If S_1 is a subset of a semigroup S and T_1 is a subset of a semigroup T , we shall denote by (S_1, T_1) the subset of $S \times T$

consisting of all elements (x, y) such that $x \in S_1$, $y \in T_1$. The product (written $M \cdot N$ or MN) of two subsets M and N of a semi-group is defined to be the set of all products mn , where $m \in M$ and $n \in N$. Hence the product of two subsets (S_1, T_1) and (S_2, T_2) of $S \times T$ is the set of all products $(s_1, t_1)(s_2, t_2)$, where

$(s_1, t_1) \in (S_1, T_1)$ and $(s_2, t_2) \in (S_2, T_2)$; but

$$(s_1, t_1)(s_2, t_2) = (s_1s_2, t_1t_2),$$

whence the product $(S_1, T_1)(S_2, T_2)$ is just the set (S_1S_2, T_1T_2) of all elements (x, y) of $S \times T$ such that $x = s_1s_2$, $y = t_1t_2$, where $s_1 \in S_1$, $s_2 \in S_2$, $t_1 \in T_1$ and $t_2 \in T_2$. We note that in this notation the set of all elements of $S \times T$ is written (S, T) .

Theorem 2.5. A subset (S_1, T_1) of $S \times T$ is a left (right) ideal if and only if S_1 is a left (right) ideal of S and T_1 is a left (right) ideal of T .

Proof. (1) Suppose $(S, T)(S_1, T_1) \subseteq (S_1, T_1)$. Then $(SS_1, TT_1) = (S, T)(S_1, T_1) \subseteq (S_1, T_1)$, whence $SS_1 \subseteq S_1$ and $TT_1 \subseteq T_1$. (2) Suppose $SS_1 \subseteq S_1$ and $TT_1 \subseteq T$. Then

$$(S, T)(S_1, T_1) = (SS_1, TT_1) \subseteq (S_1, T_1).$$

Theorem 2.6. A subset (S_1, T_1) of $S \times T$ is a subsemigroup of $S \times T$ if and only if S_1 is a subsemigroup of S and T_1 is a subsemigroup of T .

Proof. (1) Let (S_1, T_1) be a subsemigroup of $S \times T$. Then $(S_1S_1, T_1T_1) = (S_1, T_1)(S_1, T_1) \subseteq (S_1, T_1)$, whence $S_1S_1 \subseteq S_1$ and $T_1T_1 \subseteq T_1$. (2) Let S_1 be a subsemigroup of S and T_1 be a subsemigroup of T . Then $(S_1, T_1)(S_1, T_1) = (S_1S_1, T_1T_1) \subseteq (S_1, T_1)$.

Theorem 2.7. Two elements (a, b) and (c, d) of $S \times T$ commute if and only if a and c commute and b and d commute.

Proof. (1) Suppose $(a, b)(c, d) = (c, d)(a, b)$. Then

$$(ac, bd) = (a, b)(c, d) = (c, d)(a, b) = (ca, db),$$

whence $ac = ca$ and $bd = db$. (2) Suppose $ac = ca$ and $bd = db$.

Then $(a, b)(c, d) = (ac, bd) = (ca, db) = (c, d)(a, b)$.

Corollary. The semigroup $S \times T$ is commutative if and only if S is commutative and T is commutative.

Theorem 2.8. The semigroup $S \times T$ is a group if and only if the semigroups S and T are both groups.

Proof. (1) Let S and T be groups. Let e be the identity element of S , and let h be the identity element of T ; then, by Theorem 2.3, (e, h) is an identity element of $S \times T$. Let (x, y) be an arbitrary element of $S \times T$, let x^{-1} be the inverse of x in S , and let y^{-1} be the inverse of y in T . Then

$$(x^{-1}, y^{-1})(x, y) = (x^{-1}x, y^{-1}y) = (e, h).$$

Therefore, by Definition 1.11, $S \times T$ is a group. (2) Let $S \times T$ be a group, let (e, h) be its identity element, and let (x, y) be an arbitrary element of $S \times T$. Then $(ex, hy) = (e, h)(x, y) = (x, y)$, whence $ex = x$ and $hy = y$. Let (w, z) be the inverse of the arbitrary element (x, y) . Then $(wx, zy) = (w, z)(x, y) = (e, h)$, whence $wx = e$ and $zy = h$. Hence, by Definition 1.11, S and T are groups.

Our next two theorems establish the commutativity and associativity of direct multiplication.

Theorem 2.9. The semigroup $S \times T$ is isomorphic to the semigroup $T \times S$.

Proof. Let an arbitrary element (x, y) of $S \times T$ correspond to the element (y, x) of $T \times S$; this correspondence is obviously biunique, and we shall express such correspondences now and in the sequel by writing $(x, y) \longleftrightarrow (y, x)$. Let (a, b) and (c, d) be arbitrary elements of $S \times T$. Then $(a, b)(c, d) = (ac, bd) \longleftrightarrow (bd, ac) = (b, a)(d, c)$, so that the correspondence is an isomorphism.

Theorem 2.10. The semigroup $(S \times T) \times U$ is isomorphic to the semigroup $S \times (T \times U)$.

Proof. Let $[(x, y), u] \longleftrightarrow [x, (y, u)]$; let $[(a, b), c]$ and $[(d, f), g]$ be arbitrary elements of $(S \times T) \times U$. Then

$$\begin{aligned} & [(a, b), c] \cdot [(d, f), g] \\ &= [(a, b)(d, f), cg] \\ &= [(ad, bf), cg] \longleftrightarrow [ad, (bf, cg)] \\ &= [ad, (b, c)(f, g)] \\ &= [a, (b, c)] \cdot [d, (f, g)] . \end{aligned}$$

3. Computation of direct products of finite semigroups

Definition 3.1. For positive integers i, j, t , where $j = 1, 2, \dots, t$, the ordered pair (i, j) will denote the positive integer $j + (i - 1)t$.

Lemma 3.1. The ordered pairs (i, j) and (i', j') are equal if and only if $i = i'$ and $j = j'$.

Proof. (1) Let $(i, j) = (i', j')$, i.e.,

$$j + (i - 1)t = j' + (i' - 1)t .$$

If $j \neq j'$, without loss of generality we may assume $j > j'$.

Then $j - j' = (i' - i)t$. Since $j - j' > 0$ and $t > 0$, $i' - i > 0$.

Then $t \geq j > j - j' = (i' - i)t \geq t$. This is a contradiction; hence

$j = j'$. But then $(i - 1)t = (i' - 1)t$, whence $i = i'$. (2) Let $i = i'$ and $j = j'$. Then

$$(i, j) = j + (i - 1)t = j' + (i' - 1)t = (i', j') .$$

Let S be a semigroup whose elements are x_i ($i = 1, 2, \dots, s$) , and let T be a semigroup whose elements are y_j ($j = 1, 2, \dots, t$) . The elements of $S \times T$ are then (x_i, y_j) ($i = 1, 2, \dots, s$; $j = 1, 2, \dots, t$) . Let $(x_i, y_j) = u_{(i,j)} = u_{j+(i-1)t}$. By Lemma 3.1,

$u_{j+(i-1)t} = u_{j'+(i'-1)t}$ if and only if $i = i'$ and $j = j'$, and,

by Definition 1.12, $(x_i, y_j) = (x_{i'}, y_{j'})$ if and only if $i = i'$ and

$j = j'$; hence the symbols $u_{j+(i-1)t}$ correspond biuniquely to the

elements (x_i, y_j) of $S \times T$. Consider an arbitrary block of the multiplication table of $S \times T$ when the elements are ordered as:

$$(x_1, y_1), (x_1, y_2), \dots, (x_1, y_t), (x_2, y_1), (x_2, y_2), \dots, (x_2, y_t), \dots, (x_s, y_t) .$$

	(x_r, y_1)	(x_r, y_2)	$\dots$	(x_r, y_t)	
(x_p, y_1)	$(x_p x_r, y_1 y_1)$	$(x_p x_r, y_1 y_2)$	$\dots$	$(x_p x_r, y_1 y_t)$	— — —
(x_p, y_2)	$(x_p x_r, y_2 y_1)$	$(x_p x_r, y_2 y_2)$	$\dots$	$(x_p x_r, y_2 y_t)$	
(x_p, y_3)	$(x_p x_r, y_3 y_1)$	$(x_p x_r, y_3 y_2)$	$\dots$	$(x_p x_r, y_3 y_t)$	
$\vdots$	$\vdots$	$\vdots$		$\vdots$	
(x_p, y_t)	$(x_p x_r, y_t y_1)$	$(x_p x_r, y_t y_2)$	$\dots$	$(x_p x_r, y_t y_t)$	— — —

Figure 3.1

The product $x_p x_r$ can be read from the multiplication table of S . The subscript on y in the i -row and j -column in this arbitrary block is identical with the subscript on y in the i -row and j -column of the multiplication table of T . This block, and hence the whole multiplication table, can be written in the u -notation by inspection.

Example 3.1.

		x_1	x_2	x_3																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
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Although the symbol u_k ($k = j + (i - 1)t$) represents an unique element (x_i, y_j) in the direct product of two given semigroups, the same u_k may of course represent a different ordered pair in some other direct product. For example, in Example 3.1 the symbol u_7 was used for the ordered pair (x_3, y_1) , while in Example 3.2 u_7 was used for the ordered pair (x_2, y_3) . Henceforth we shall omit the letters x , y , and u and use only the subscripts, keeping in mind that the resulting multiplication tables do not represent multiplication of numbers in the ordinary sense.

4. Direct products: 2 x 2, 2 x 3, 2 x 4, and 3 x 3

The ten distinct direct products of the four semigroups of order two with themselves are semigroups of order four. Since the semigroups of orders two, three, and four are listed in the appendix, it is not necessary to exhibit the multiplication tables of these ten semigroups here. These ten direct products are:

2.1 x 2.1	is	4.11
2.1 x 2.2	is isomorphic to	4.15
2.1 x 2.3	is anti-isomorphic to	4.65
2.1 x 2.4	is isomorphic to	4.31
2.2 x 2.2	is	4.49
2.2 x 2.3	is	4.66
2.2 x 2.4	is isomorphic to	4.9
2.3 x 2.3	is	4.101
2.3 x 2.4	is isomorphic to	4.78
2.4 x 2.4	is	4.55

The seventy-two distinct direct products of the four semigroups of order two with the eighteen semigroups of order three are listed in Table I; the 484 distinct direct products of the four semigroups of order two with the 121 semigroups of order four are listed in Table II. The 171 distinct direct products of the eighteen semigroups of order three with the eighteen semigroups of order three are listed in Table III.

TABLE I
DIRECT PRODUCTS: 2 x 3

2.1 x 3.1	2.1 x 3.2	2.1 x 3.3	2.1 x 3.4	2.1 x 3.5
1 2 2 4 5 5	1 1 1 4 4 4	1 2 3 4 5 6	1 1 3 4 4 6	1 1 3 4 4 6
2 1 1 5 4 4	1 1 1 4 4 4	2 3 1 5 6 4	1 1 3 4 4 6	1 2 3 4 5 6
2 1 1 5 4 4	1 1 2 4 4 5	3 1 2 6 4 5	3 3 1 6 6 4	3 3 1 6 6 4
4 5 5 1 2 2	4 4 4 1 1 1	4 5 6 1 2 3	4 4 6 1 1 3	4 4 6 1 1 3
5 4 4 2 1 1	4 4 4 1 1 1	5 6 4 2 3 1	4 4 6 1 1 3	4 5 6 1 2 3
5 4 4 2 1 1	4 4 5 1 1 2	6 4 5 3 1 2	6 6 4 3 3 1	6 6 4 3 3 1
2.1 x 3.6	2.1 x 3.7	2.1 x 3.8	2.1 x 3.9	2.1 x 3.10
1 1 1 4 4 4	1 1 1 4 4 4	1 1 1 4 4 4	1 1 1 4 4 4	1 1 3 4 4 6
1 1 1 4 4 4	1 2 1 4 5 4	1 1 2 4 4 5	2 2 2 5 5 5	1 1 3 4 4 6
1 1 3 4 4 6	1 3 1 4 6 4	1 2 3 4 5 6	1 1 1 4 4 4	3 3 3 6 6 6
4 4 4 1 1 1	4 4 4 1 1 1	4 4 4 1 1 1	4 4 4 1 1 1	4 4 6 1 1 3
4 4 4 1 1 1	4 5 4 1 2 1	4 4 5 1 1 2	5 5 5 2 2 2	4 4 6 1 1 3
4 4 6 1 1 3	4 6 4 1 3 1	4 5 6 1 2 3	4 4 4 1 1 1	6 6 6 3 3 3
2.1 x 3.11	2.1 x 3.12	2.1 x 3.13	2.1 x 3.14	2.1 x 3.15
1 2 3 4 5 6	1 1 1 4 4 4	1 1 1 4 4 4	1 1 1 4 4 4	1 1 3 4 4 6
2 1 3 5 4 6	2 2 2 5 5 5	2 2 2 5 5 5	2 2 2 5 5 5	2 2 3 5 5 6
3 3 3 6 6 6	1 1 3 4 4 6	1 2 3 4 5 6	3 3 3 6 6 6	3 3 3 6 6 6
4 5 6 1 2 3	4 4 4 1 1 1	4 4 4 1 1 1	4 4 4 1 1 1	4 4 6 1 1 3
5 4 6 2 1 3	5 5 5 2 2 2	5 5 5 2 2 2	5 5 5 2 2 2	5 5 6 2 2 3
6 6 6 3 3 3	4 4 6 1 1 3	4 5 6 1 2 3	6 6 6 3 3 3	6 6 6 3 3 3
2.1 x 3.16	2.1 x 3.17	2.1 x 3.18	2.2 x 3.1	2.2 x 3.2
1 1 1 4 4 4	1 1 1 4 4 4	1 1 1 4 4 4	1 2 2 1 2 2	1 1 1 1 1 1
1 2 2 4 5 5	1 2 1 4 5 4	1 1 1 4 4 4	2 1 1 2 1 1	1 1 1 1 1 1
1 2 3 4 5 6	1 1 3 4 4 6	1 1 1 4 4 4	2 1 1 2 1 1	1 1 2 1 1 2
4 4 4 1 1 1	4 4 4 1 1 1	4 4 4 1 1 1	1 2 2 4 5 5	1 1 1 4 4 4
4 5 5 1 2 2	4 5 4 1 2 1	4 4 4 1 1 1	2 1 1 5 4 4	1 1 1 4 4 4
4 5 6 1 2 3	4 4 6 1 1 3	4 4 4 1 1 1	2 1 1 5 4 4	1 1 2 4 4 5
2.2 x 3.3	2.2 x 3.4	2.2 x 3.5	2.2 x 3.6	2.2 x 3.7
1 2 3 1 2 3	1 1 3 1 1 3	1 1 3 1 1 3	1 1 1 1 1 1	1 1 1 1 1 1
2 3 1 2 3 1	1 1 3 1 1 3	1 2 3 1 2 3	1 1 1 1 1 1	1 2 1 1 2 1
3 1 2 3 1 2	3 3 1 3 3 1	3 3 1 3 3 1	1 1 3 1 1 3	1 3 1 1 3 1
1 2 3 4 5 6	1 1 3 4 4 6	1 1 3 4 4 6	1 1 1 4 4 4	1 1 1 4 4 4
2 3 1 5 6 4	1 1 3 4 4 6	1 2 3 4 5 6	1 1 1 4 4 4	1 2 1 4 5 4
3 1 2 6 4 5	3 3 1 6 6 4	3 3 1 6 6 4	1 1 3 4 4 6	1 3 1 4 6 4

TABLE I
DIRECT PRODUCTS: 2 x 3 (continued)

2.2 x 3.8	2.2 x 3.9	2.2 x 3.10	2.2 x 3.11	2.2 x 3.12
1 1 1 1 1 1	1 1 1 1 1 1	1 1 3 1 1 3	1 2 3 1 2 3	1 1 1 1 1 1
1 1 2 1 1 2	2 2 2 2 2 2	1 1 3 1 1 3	2 1 3 2 1 3	2 2 2 2 2 2
1 2 3 1 2 3	1 1 1 1 1 1	3 3 3 3 3 3	3 3 3 3 3 3	1 1 3 1 1 3
1 1 1 4 4 4	1 1 1 4 4 4	1 1 3 4 4 6	1 2 3 4 5 6	1 1 1 4 4 4
1 1 2 4 4 5	2 2 2 5 5 5	1 1 3 4 4 6	2 1 3 5 4 6	2 2 2 5 5 5
1 2 3 4 5 6	1 1 1 4 4 4	3 3 3 6 6 6	3 3 3 6 6 6	1 1 3 4 4 6
2.2 x 3.13	2.2 x 3.14	2.2 x 3.15	2.2 x 3.16	2.2 x 3.17
1 1 1 1 1 1	1 1 1 1 1 1	1 1 3 1 1 3	1 1 1 1 1 1	1 1 1 1 1 1
2 2 2 2 2 2	2 2 2 2 2 2	2 2 3 2 2 3	1 2 2 1 2 2	1 2 1 1 2 1
1 2 3 1 2 3	3 3 3 3 3 3	3 3 3 3 3 3	1 2 3 1 2 3	1 1 3 1 1 3
1 1 1 4 4 4	1 1 1 4 4 4	1 1 3 4 4 6	1 1 1 4 4 4	1 1 1 4 4 4
2 2 2 5 5 5	2 2 2 5 5 5	2 2 3 5 5 6	1 2 2 4 5 5	1 2 1 4 5 4
1 2 3 4 5 6	3 3 3 6 6 6	3 3 3 6 6 6	1 2 3 4 5 6	1 1 3 4 4 6
2.2 x 3.18	2.3 x 3.1	2.3 x 3.2	2.3 x 3.3	2.3 x 3.4
1 1 1 1 1 1	1 2 2 1 2 2	1 1 1 1 1 1	1 2 3 1 2 3	1 1 3 1 1 3
1 1 1 1 1 1	2 1 1 2 1 1	1 1 1 1 1 1	2 3 1 2 3 1	1 1 3 1 1 3
1 1 1 1 1 1	2 1 1 2 1 1	1 1 2 1 1 2	3 1 2 3 1 2	3 3 1 3 3 1
1 1 1 4 4 4	4 5 5 4 5 5	4 4 4 4 4 4	4 5 6 4 5 6	4 4 6 4 4 6
1 1 1 4 4 4	5 4 4 5 4 4	4 4 4 4 4 4	5 6 4 5 6 4	4 4 6 4 4 6
1 1 1 4 4 4	5 4 4 5 4 4	4 4 5 4 4 5	6 4 5 6 4 5	6 6 4 6 6 4
2.3 x 3.5	2.3 x 3.6	2.3 x 3.7	2.3 x 3.8	2.3 x 3.9
1 1 3 1 1 3	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
1 2 3 1 2 3	1 1 1 1 1 1	1 2 1 1 2 1	1 1 2 1 1 2	2 2 2 2 2 2
3 3 1 3 3 1	1 1 3 1 1 3	1 3 1 1 3 1	1 2 3 1 2 3	1 1 1 1 1 1
4 4 6 4 4 6	4 4 4 4 4 4	4 4 4 4 4 4	4 4 4 4 4 4	4 4 4 4 4 4
4 5 6 4 5 6	4 4 4 4 4 4	4 5 4 4 5 4	4 4 5 4 4 5	5 5 5 5 5 5
6 6 4 6 6 4	4 4 6 4 4 6	4 6 4 4 6 4	4 5 6 4 5 6	4 4 4 4 4 4
2.3 x 3.10	2.3 x 3.11	2.3 x 3.12	2.3 x 3.13	2.3 x 3.14
1 1 3 1 1 3	1 2 3 1 2 3	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
1 1 3 1 1 3	2 1 3 2 1 3	2 2 2 2 2 2	2 2 2 2 2 2	2 2 2 2 2 2
3 3 3 3 3 3	3 3 3 3 3 3	1 1 3 1 1 3	1 2 3 1 2 3	3 3 3 3 3 3
4 4 6 4 4 6	4 5 6 4 5 6	4 4 4 4 4 4	4 4 4 4 4 4	4 4 4 4 4 4
4 4 6 4 4 6	5 4 6 5 4 6	5 5 5 5 5 5	5 5 5 5 5 5	5 5 5 5 5 5
6 6 6 6 6 6	6 6 6 6 6 6	4 4 6 4 4 6	4 5 6 4 5 6	6 6 6 6 6 6

TABLE I
DIRECT PRODUCTS: 2 x 3 (continued)

2.3 x 3.15	2.3 x 3.16	2.3 x 3.17	2.3 x 3.18	2.4 x 3.1
1 1 3 1 1 3	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1	1 2 2 1 2 2
2 2 3 2 2 3	1 2 2 1 2 2	1 2 1 1 2 1	1 1 1 1 1 1	2 1 1 2 1 1
3 3 3 3 3 3	1 2 3 1 2 3	1 1 3 1 1 3	1 1 1 1 1 1	2 1 1 2 1 1
4 4 6 4 4 6	4 4 4 4 4 4	4 4 4 4 4 4	4 4 4 4 4 4	1 2 2 1 2 2
5 5 6 5 5 6	4 5 5 4 5 5	4 5 4 4 5 4	4 4 4 4 4 4	2 1 1 2 1 1
6 6 6 6 6 6	4 5 6 4 5 6	4 4 6 4 4 6	4 4 4 4 4 4	2 1 1 2 1 1
2.4 x 3.2	2.4 x 3.3	2.4 x 3.4	2.4 x 3.5	2.4 x 3.6
1 1 1 1 1 1	1 2 3 1 2 3	1 1 3 1 1 3	1 1 3 1 1 3	1 1 1 1 1 1
1 1 1 1 1 1	2 3 1 2 3 1	1 1 3 1 1 3	1 2 3 1 2 3	1 1 1 1 1 1
1 1 2 1 1 2	3 1 2 3 1 2	3 3 1 3 3 1	3 3 1 3 3 1	1 1 3 1 1 3
1 1 1 1 1 1	1 2 3 1 2 3	1 1 3 1 1 3	1 1 3 1 1 3	1 1 1 1 1 1
1 1 1 1 1 1	2 3 1 2 3 1	1 1 3 1 1 3	1 2 3 1 2 3	1 1 1 1 1 1
1 1 2 1 1 2	3 1 2 3 1 2	3 3 1 3 3 1	3 3 1 3 3 1	1 1 3 1 1 3
2.4 x 3.7	2.4 x 3.8	2.4 x 3.9	2.4 x 3.10	2.4 x 3.11
1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1	1 1 3 1 1 3	1 2 3 1 2 3
1 2 1 1 2 1	1 1 2 1 1 2	2 2 2 2 2 2	1 1 3 1 1 3	2 1 3 2 1 3
1 3 1 1 3 1	1 2 3 1 2 3	1 1 1 1 1 1	3 3 3 3 3 3	3 3 3 3 3 3
1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1	1 1 3 1 1 3	1 2 3 1 2 3
1 2 1 1 2 1	1 1 2 1 1 2	2 2 2 2 2 2	1 1 3 1 1 3	2 1 3 2 1 3
1 3 1 1 3 1	1 2 3 1 2 3	1 1 1 1 1 1	3 3 3 3 3 3	3 3 3 3 3 3
2.4 x 3.12	2.4 x 3.13	2.4 x 3.14	2.4 x 3.15	2.4 x 3.16
1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1	1 1 3 1 1 3	1 1 1 1 1 1
2 2 2 2 2 2	2 2 2 2 2 2	2 2 2 2 2 2	2 2 3 2 2 3	1 2 2 1 2 2
1 1 3 1 1 3	1 2 3 1 2 3	3 3 3 3 3 3	3 3 3 3 3 3	1 2 3 1 2 3
1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1	1 1 3 1 1 3	1 1 1 1 1 1
2 2 2 2 2 2	2 2 2 2 2 2	2 2 2 2 2 2	2 2 3 2 2 3	1 2 2 1 2 2
1 1 3 1 1 3	1 2 3 1 2 3	3 3 3 3 3 3	3 3 3 3 3 3	1 2 3 1 2 3
2.4 x 3.17	2.4 x 3.18			
1 1 1 1 1 1	1 1 1 1 1 1			
1 2 1 1 2 1	1 1 1 1 1 1			
1 1 3 1 1 3	1 1 1 1 1 1			
1 1 1 1 1 1	1 1 1 1 1 1			
1 2 1 1 2 1	1 1 1 1 1 1			
1 1 3 1 1 3	1 1 1 1 1 1			

TABLE II
DIRECT PRODUCTS: 2 x 4

2.1 x 4.1	2.1 x 4.2	2.1 x 4.3	2.1 x 4.4
1 1 1 1 5 5 5 5 1 2 2 2 5 6 6 6 1 2 3 3 5 6 7 7 1 2 3 3 5 6 7 7 5 5 5 5 1 1 1 1 5 6 6 6 1 2 2 2 5 6 7 7 1 2 3 3 5 6 7 7 1 2 3 3	1 1 1 1 5 5 5 5 1 2 1 1 5 6 5 5 1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5 5 5 5 5 1 1 1 1 5 6 5 5 1 2 1 1 5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1	1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5 1 1 2 2 5 5 6 6 1 1 2 2 5 5 6 6 5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1 5 5 6 6 1 1 2 2 5 5 6 6 1 1 2 2	1 1 1 1 5 5 5 5 1 2 1 4 5 6 5 8 1 1 1 1 5 5 5 5 1 4 1 1 5 8 5 5 5 5 5 5 1 1 1 1 5 6 5 8 1 2 1 4 5 5 5 5 1 1 1 1 5 8 5 5 1 4 1 1
2.1 x 4.5	2.1 x 4.6	2.1 x 4.7	2.1 x 4.8
1 1 1 1 5 5 5 5 1 2 1 2 5 6 5 6 1 1 1 1 5 5 5 5 1 2 1 4 5 6 5 8 5 5 5 5 1 1 1 1 5 6 5 6 1 2 1 2 5 5 5 5 1 1 1 1 5 6 5 8 1 2 1 4	1 1 1 1 5 5 5 5 1 2 1 1 5 6 5 5 1 1 1 1 5 5 5 5 1 1 1 3 5 5 5 7 5 5 5 5 1 1 1 1 5 6 5 5 1 2 1 1 5 5 5 5 1 1 1 1 5 5 5 7 1 1 1 3	1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 2 5 5 5 6 1 1 2 2 5 5 6 6 5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 6 1 1 1 2 5 5 6 6 1 1 2 2	1 1 1 1 5 5 5 5 1 1 1 3 5 5 5 7 1 1 1 1 5 5 5 5 1 3 1 2 5 7 5 6 5 5 5 5 1 1 1 1 5 5 5 7 1 1 1 3 5 5 5 5 1 1 1 1 5 7 5 6 1 3 1 2
2.1 x 4.9	2.1 x 4.10	2.1 x 4.11	2.1 x 4.12
1 1 1 1 5 5 5 5 1 2 1 2 5 6 5 6 1 1 1 1 5 5 5 5 1 2 1 2 5 6 5 6 5 5 5 5 1 1 1 1 5 6 5 6 1 2 1 2 5 5 5 5 1 1 1 1 5 6 5 6 1 2 1 2	1 2 3 4 5 6 7 8 2 1 4 3 6 5 8 7 3 4 2 1 7 8 6 5 4 3 1 2 8 7 5 6 5 6 7 8 1 2 3 4 6 5 8 7 2 1 4 3 7 8 6 5 3 4 2 1 8 7 5 6 4 3 1 2	1 2 3 4 5 6 7 8 2 1 4 3 6 5 8 7 3 4 1 2 7 8 5 6 4 3 2 1 8 7 6 5 5 6 7 8 1 2 3 4 6 5 8 7 2 1 4 3 7 8 5 6 3 4 1 2 8 7 6 5 4 3 2 1	1 2 3 4 5 6 7 8 2 3 1 4 6 7 5 8 3 1 2 4 7 5 6 8 4 4 4 4 8 8 8 8 5 6 7 8 1 2 3 4 6 7 5 8 2 3 1 4 7 5 6 8 3 1 2 4 8 8 8 8 4 4 4 4
2.1 x 4.13	2.1 x 4.14	2.1 x 4.15	2.1 x 4.16
1 2 3 1 5 6 7 5 2 3 1 2 6 7 5 6 3 1 2 3 7 5 6 7 1 2 3 4 5 6 7 8 5 6 7 5 1 2 3 1 6 7 5 6 2 3 1 2 7 5 6 7 3 1 2 3 5 6 7 8 1 2 3 4	1 2 1 1 5 6 5 5 2 1 2 2 6 5 6 6 1 2 3 4 5 6 7 8 1 2 4 3 5 6 8 7 5 6 5 5 1 2 1 1 6 5 6 6 2 1 2 2 5 6 7 8 1 2 3 4 5 6 8 7 1 2 4 3	1 2 1 2 5 6 5 6 2 1 2 1 6 5 6 5 1 2 3 4 5 6 7 8 2 1 4 3 6 5 8 7 5 6 5 6 1 2 1 2 6 5 6 5 2 1 2 1 5 6 7 8 1 2 3 4 6 5 8 7 2 1 4 3	1 2 1 1 5 6 5 5 2 1 2 2 6 5 6 6 1 2 3 1 5 6 7 5 1 2 1 4 5 6 5 8 5 6 5 5 1 2 1 1 6 5 6 6 2 1 2 2 5 6 7 5 1 2 3 1 5 6 5 8 1 2 1 4

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.1 x 4.17	2.1 x 4.18	2.1 x 4.19	2.1 x 4.20
1 2 1 1 5 6 5 5	1 2 3 1 5 6 7 5	1 2 3 3 5 6 7 7	1 2 3 4 5 6 7 8
2 1 2 2 6 5 6 6	2 1 3 2 6 5 7 6	2 1 3 3 6 5 7 7	2 1 3 4 6 5 7 8
1 2 3 4 5 6 7 8	3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7	3 3 3 4 7 7 7 8
1 2 4 4 5 6 8 8	1 2 3 4 5 6 7 8	3 3 3 4 7 7 7 8	4 4 4 4 8 8 8 8
5 6 5 5 1 2 1 1	5 6 7 5 1 2 3 1	5 6 7 7 1 2 3 3	5 6 7 8 1 2 3 4
6 5 6 6 2 1 2 2	6 5 7 5 2 1 3 2	6 5 7 7 2 1 3 3	6 5 7 8 2 1 3 4
5 6 7 8 1 2 3 4	7 7 7 7 3 3 3 3	7 7 7 7 3 3 3 3	7 7 7 8 3 3 3 4
5 6 8 8 1 2 4 4	5 6 7 8 1 2 3 4	7 7 7 8 3 3 3 4	8 8 8 8 4 4 4 4
2.1 x 4.21	2.1 x 4.22	2.1 x 4.23	2.1 x 4.24
1 1 1 4 5 5 5 8	1 1 1 4 5 5 5 8	1 1 1 4 5 5 5 8	1 1 3 3 5 5 7 7
1 2 3 4 5 6 7 8	1 2 1 4 5 6 5 8	1 2 1 4 5 6 5 8	1 2 3 4 5 6 7 8
1 3 1 4 5 7 5 8	1 1 1 4 5 5 5 8	1 1 3 4 5 5 7 8	3 3 1 1 7 7 5 5
4 4 4 1 8 8 8 5	4 4 4 1 8 8 8 5	4 4 4 4 8 8 8 8	3 4 1 1 7 8 5 5
5 5 5 8 1 1 1 4	5 5 5 8 1 1 1 4	5 5 5 8 1 1 1 4	5 5 7 7 1 1 3 3
5 6 7 8 1 2 3 4	5 6 5 8 1 2 1 4	5 6 5 8 1 2 1 4	5 6 7 8 1 2 3 4
5 7 5 8 1 3 1 4	5 5 5 8 1 1 1 4	5 5 7 8 1 1 3 4	7 7 5 5 3 3 1 1
8 8 8 5 4 4 4 1	8 8 8 5 4 4 4 1	8 8 8 8 4 4 4 4	7 8 5 5 3 4 1 1
2.1 x 4.25	2.1 x 4.26	2.1 x 4.27	2.1 x 4.28
1 2 1 4 5 6 5 8	1 2 4 4 5 6 8 8	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	1 2 1 4 5 6 5 8	1 2 3 4 5 6 7 8
1 2 1 4 5 6 5 8	4 2 1 1 8 6 5 5	1 1 1 1 5 5 5 5	1 3 1 3 5 7 5 7
4 2 4 1 8 6 8 5	4 2 1 1 8 6 5 5	1 4 1 2 5 8 5 6	1 4 3 2 5 8 7 6
5 6 5 8 1 2 1 4	5 6 8 8 1 2 4 4	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2	5 6 5 8 1 2 1 4	5 6 7 8 1 2 3 4
5 6 5 8 1 2 1 4	8 6 5 5 4 2 1 1	5 5 5 5 1 1 1 1	5 7 5 7 1 3 1 3
8 6 8 5 4 2 4 1	8 6 5 5 4 2 1 1	5 8 5 6 1 4 1 2	5 8 7 6 1 4 3 2
2.1 x 4.29	2.1 x 4.30	2.1 x 4.31	2.1 x 4.32
1 1 3 1 5 5 7 5	1 2 1 1 5 6 5 5	1 2 1 2 5 6 5 6	1 2 2 2 5 6 6 6
1 2 3 2 5 6 7 6	2 1 2 2 6 5 6 6	2 1 2 1 6 5 6 5	2 1 1 1 6 5 5 5
3 3 1 3 7 7 5 7	1 2 1 1 5 6 5 5	1 2 1 2 5 6 5 6	2 1 1 1 6 5 5 5
1 2 3 2 5 6 7 6	1 2 1 1 5 6 5 5	2 1 2 1 6 5 6 5	2 1 1 1 6 5 5 5
5 5 7 5 1 1 3 1	5 6 5 5 1 2 1 1	5 6 5 6 1 2 1 2	5 6 6 6 1 2 2 2
5 6 7 6 1 2 3 2	6 5 6 6 2 1 2 2	6 5 6 5 2 1 2 1	6 5 5 5 2 1 1 1
7 7 5 7 3 3 1 3	5 6 5 5 1 2 1 1	5 6 5 6 1 2 1 2	6 5 5 5 2 1 1 1
5 6 7 6 1 2 3 2	5 6 5 5 1 2 1 1	6 5 6 5 2 1 2 1	6 5 5 5 2 1 1 1

TABLE II

DIRECT PRODUCTS: 2×4 (continued)

2.1 x 4.33	2.1 x 4.34	2.1 x 4.35	2.1 x 4.36
1 1 3 1 5 5 7 5	1 1 3 3 5 5 7 7	1 2 3 3 5 6 7 7	1 1 1 1 5 5 5 5
1 1 3 1 5 5 7 5	1 1 3 3 5 5 7 7	2 3 1 1 6 7 5 5	1 2 3 3 5 6 7 7
3 3 1 3 7 7 5 7	3 3 1 1 7 7 5 5	3 1 2 2 7 5 6 6	1 3 1 1 5 7 5 5
1 1 3 2 5 5 7 6	3 3 1 2 7 7 5 6	3 1 2 2 7 5 6 6	1 3 1 1 5 7 5 5
5 5 7 5 1 1 3 1	5 5 7 7 1 1 3 3	5 6 7 7 1 2 3 3	5 5 5 5 1 1 1 1
5 5 7 5 1 1 3 1	5 5 7 7 1 1 3 3	6 7 5 5 2 3 1 1	5 6 7 7 1 2 3 3
7 7 5 7 3 3 1 3	7 7 5 5 3 3 1 1	7 5 6 6 3 1 2 2	5 7 5 5 1 3 1 1
5 5 7 6 1 1 3 2	7 7 5 6 3 3 1 2	7 5 6 6 3 1 2 2	5 7 5 5 1 3 1 1
2.1 x 4.37	2.1 x 4.38	2.1 x 4.39	2.1 x 4.40
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 3 4 5 6 7 8	1 2 3 2 5 6 7 6	1 2 3 4 5 6 7 8	1 2 2 2 5 6 6 6
1 3 1 1 5 7 5 5	1 3 1 3 5 7 5 7	1 3 1 1 5 7 5 5	1 2 2 2 5 6 6 6
1 4 1 1 5 8 5 5	1 2 3 2 5 6 7 6	1 4 1 3 5 8 5 7	1 2 2 2 5 6 6 6
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
5 6 7 8 1 2 3 4	5 6 7 6 1 2 3 2	5 6 7 8 1 2 3 4	5 6 6 6 1 2 2 2
5 7 5 5 1 3 1 1	5 7 5 7 1 3 1 3	5 7 5 5 1 3 1 1	5 6 6 6 1 2 2 2
5 8 5 5 1 4 1 1	5 6 7 6 1 2 3 2	5 8 5 7 1 4 1 3	5 6 6 6 1 2 2 2
2.1 x 4.41	2.1 x 4.42	2.1 x 4.43	2.1 x 4.44
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 2 2 5 6 6 6	1 2 2 2 5 6 6 6	1 2 2 1 5 6 6 5	1 2 2 1 5 6 6 5
1 2 2 2 5 6 6 6	1 2 3 3 5 6 7 7	1 2 3 1 5 6 7 5	1 2 3 4 5 6 7 8
1 2 2 3 5 6 6 7	1 2 3 4 5 6 7 8	1 1 1 4 5 5 5 8	1 1 4 1 5 5 8 5
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
5 6 6 6 1 2 2 2	5 6 6 6 1 2 2 2	5 6 6 5 1 2 2 1	5 6 6 5 1 2 2 1
5 6 6 6 1 2 2 2	5 6 7 7 1 2 3 3	5 6 7 5 1 2 3 1	5 6 7 8 1 2 3 4
5 6 6 7 1 2 2 3	5 6 7 8 1 2 3 4	5 5 5 8 1 1 1 4	5 5 8 5 1 1 4 1
2.1 x 4.45	2.1 x 4.46	2.1 x 4.47	2.1 x 4.48
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 2 4 5 6 6 8	1 2 2 2 5 6 6 6	1 2 2 2 5 6 6 6	1 2 1 1 5 6 5 5
1 2 3 4 5 6 7 8	1 2 3 2 5 6 7 6	1 2 3 4 5 6 7 8	1 1 3 1 5 5 7 5
1 4 4 1 5 8 8 5	1 2 2 2 5 6 6 6	1 2 4 2 5 6 8 6	1 1 1 4 5 5 5 8
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
5 6 6 8 1 2 2 4	5 6 6 6 1 2 2 2	5 6 6 6 1 2 2 2	5 6 5 5 1 2 1 1
5 6 7 8 1 2 3 4	5 6 7 6 1 2 3 2	5 6 7 8 1 2 3 4	5 5 7 5 1 1 3 1
5 8 8 5 1 4 4 1	5 6 6 6 1 2 2 2	5 6 8 6 1 2 4 2	5 5 5 8 1 1 1 4

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.1 x 4.49	2.1 x 4.50	2.1 x 4.51	2.1 x 4.52
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 1 2 5 6 5 6	1 2 1 1 5 6 5 5	1 2 1 1 5 6 5 5	1 2 1 2 5 6 5 6
1 1 3 3 5 5 7 7	1 1 3 1 5 5 7 5	1 1 3 4 5 5 7 8	1 1 3 1 5 5 7 5
1 2 3 4 5 6 7 8	1 1 1 1 5 5 5 5	1 1 4 1 5 5 8 5	1 2 1 2 5 6 5 6
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
5 6 5 6 1 2 1 2	5 6 5 5 1 2 1 1	5 6 5 5 1 2 1 1	5 6 5 6 1 2 1 2
5 5 7 7 1 1 3 3	5 5 7 5 1 1 3 1	5 5 7 8 1 1 3 4	5 5 7 5 1 1 3 1
5 6 7 8 1 2 3 4	5 5 5 5 1 1 1 1	5 5 8 5 1 1 4 1	5 6 5 6 1 2 1 2
2.1 x 4.53	2.1 x 4.54	2.1 x 4.55	2.1 x 4.56
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8
1 1 1 2 5 5 5 6	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 2 4 5 5 6 8
1 1 2 1 5 5 6 5	1 1 1 2 5 5 5 6	1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 8 1 1 1 4
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 8 1 1 1 4
5 5 5 6 1 1 1 2	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 6 8 1 1 2 4
5 5 6 5 1 1 2 1	5 5 5 6 1 1 1 2	5 5 5 5 1 1 1 1	5 5 5 8 1 1 1 4
2.1 x 4.57	2.1 x 4.58	2.1 x 4.59	2.1 x 4.60
1 1 1 4 5 5 5 8	1 1 3 4 5 5 7 8	1 1 1 1 5 5 5 5	1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8	1 1 3 4 5 5 7 8	2 2 2 2 6 6 6 6	2 1 3 4 6 5 7 8
1 3 2 4 5 7 6 8	3 3 3 4 7 7 7 8	1 1 1 2 5 5 5 6	3 3 3 4 7 7 7 8
1 1 1 4 5 5 5 8	3 3 3 4 7 7 7 8	4 4 4 4 8 8 8 8	4 4 3 4 8 8 7 8
5 5 5 8 1 1 1 4	5 5 7 8 1 1 3 4	5 5 5 5 1 1 1 1	5 6 7 8 1 2 3 4
5 6 7 8 1 2 3 4	5 5 7 8 1 1 3 4	6 6 6 6 2 2 2 2	6 5 7 8 2 1 3 4
5 7 6 8 1 3 2 4	7 7 7 8 3 3 3 4	5 5 5 6 1 1 1 2	7 7 7 8 3 3 3 4
5 5 5 8 1 1 1 4	7 7 7 8 3 3 3 4	8 8 8 8 4 4 4 4	8 8 7 8 4 4 3 4
2.1 x 4.61	2.1 x 4.62	2.1 x 4.63	2.1 x 4.64
1 1 3 4 5 5 7 8	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 3 3 5 5 7 7
1 1 3 4 5 5 7 8	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 4 4 6 6 8 8
3 3 3 4 7 7 7 8	3 3 3 3 7 7 7 7	1 2 3 4 5 6 7 8	1 1 3 3 5 5 7 7
4 4 3 4 8 8 7 8	1 2 2 4 5 6 6 8	2 1 4 3 6 5 8 7	2 2 4 4 6 6 8 8
5 5 7 8 1 1 3 4	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 7 7 1 1 3 3
5 5 7 8 1 1 3 4	6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2	6 6 8 8 2 2 4 4
7 7 7 8 3 3 3 4	7 7 7 7 3 3 3 3	5 6 7 8 1 2 3 4	5 5 7 7 1 1 3 3
8 8 7 8 4 4 3 4	5 6 6 8 1 2 2 4	6 5 8 7 2 1 4 3	6 6 8 8 2 2 4 4

TABLE II

DIRECT PRODUCTS: 2 x 4 (continued)

2.1 x 4.65	2.1 x 4.66	2.1 x 4.67	2.1 x 4.68
1 2 3 4 5 6 7 8	1 1 1 1 5 5 5 5	1 1 3 3 5 5 7 7	1 1 3 3 5 5 7 7
2 1 4 3 6 5 8 7	2 2 2 2 6 6 6 6	2 2 3 3 6 6 7 7	2 2 3 3 6 6 7 7
1 2 3 4 5 6 7 8	1 1 3 3 5 5 7 7	3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7
2 1 4 3 6 5 8 7	2 2 4 4 6 6 8 8	4 4 4 4 8 8 8 8	4 4 3 3 8 8 7 7
5 6 7 8 1 2 3 4	5 5 5 5 1 1 1 1	5 5 7 7 1 1 3 3	5 5 7 7 1 1 3 3
6 5 8 7 2 1 4 3	6 6 6 6 2 2 2 2	6 6 7 7 2 2 3 3	6 6 7 7 2 2 3 3
5 6 7 8 1 2 3 4	5 5 7 7 1 1 3 3	7 7 7 7 3 3 3 3	7 7 7 7 3 3 3 3
6 5 8 7 2 1 4 3	6 6 8 8 2 2 4 4	8 8 8 8 4 4 4 4	8 8 7 7 4 4 3 3
2.1 x 4.69	2.1 x 4.70	2.1 x 4.71	2.1 x 4.72
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 3 4 5 5 7 8
1 1 2 2 5 5 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 3 4 6 6 7 8
1 2 3 4 5 6 7 8	3 3 3 3 7 7 7 7	1 1 1 1 5 5 5 5	3 3 3 4 7 7 7 8
1 2 3 4 5 6 7 8	3 3 3 4 7 7 7 8	4 4 4 4 8 8 8 8	3 3 3 4 7 7 7 8
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 7 8 1 1 3 4
5 5 6 6 1 1 2 2	6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2	6 6 7 8 2 2 3 4
5 6 7 8 1 2 3 4	7 7 7 7 3 3 3 3	5 5 5 5 1 1 1 1	7 7 7 8 3 3 3 4
5 6 7 8 1 2 3 4	7 7 7 8 3 3 3 4	8 8 8 8 4 4 4 4	7 7 7 8 3 3 3 4
2.1 x 4.73	2.1 x 4.74	2.1 x 4.75	2.1 x 4.76
1 1 3 3 5 5 7 7	1 1 3 4 5 5 7 8	1 1 3 3 5 5 7 7	1 1 1 1 5 5 5 5
2 2 3 3 6 6 7 7	2 2 3 4 6 6 7 8	2 2 3 3 6 6 7 7	1 1 1 3 5 5 5 7
3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7	1 1 1 3 5 5 5 7
3 3 3 4 7 7 7 8	3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7	1 2 3 4 5 6 7 8
5 5 7 7 1 1 3 3	5 5 7 8 1 1 3 4	5 5 7 7 1 1 3 3	5 5 5 5 1 1 1 1
6 6 7 7 2 2 3 3	6 6 7 8 2 2 3 4	6 6 7 7 2 2 3 3	5 5 5 7 1 1 1 3
7 7 7 7 3 3 3 3	7 7 7 7 3 3 3 3	7 7 7 7 3 3 3 3	5 5 5 7 1 1 1 3
7 7 7 8 3 3 3 4	7 7 7 7 3 3 3 3	7 7 7 7 3 3 3 3	5 6 7 8 1 2 3 4
2.1 x 4.77	2.1 x 4.78	2.1 x 4.79	2.1 x 4.80
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 1 1 1 5 5 5 5	2 2 2 2 6 6 6 6	1 1 1 4 5 5 5 8	2 2 2 4 6 6 6 8
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
5 5 5 5 1 1 1 1	6 6 6 6 2 2 2 2	5 5 5 8 1 1 1 4	6 6 6 8 2 2 2 4

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.1 x 4.81	2.1 x 4.82	2.1 x 4.83	2.1 x 4.84
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8
2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	1 2 1 4 5 6 5 8
1 1 1 3 5 5 5 7	1 2 3 1 5 6 7 5	1 1 1 3 5 5 5 7	1 3 1 4 5 7 5 8
2 2 2 4 6 6 6 8	1 1 1 1 5 5 5 5	1 2 1 4 5 6 5 8	1 1 1 4 5 5 5 8
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 8 1 1 1 4
6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2	5 6 5 8 1 2 1 4
5 5 5 7 1 1 1 3	5 6 7 5 1 2 3 1	5 5 5 7 1 1 1 3	5 7 5 8 1 3 1 4
6 6 6 8 2 2 2 4	5 5 5 5 1 1 1 1	5 6 5 8 1 2 1 4	5 5 5 8 1 1 1 4
2.1 x 4.85	2.1 x 4.86	2.1 x 4.87	2.1 x 4.88
1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6	2 2 2 4 6 6 6 8	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6
1 1 1 3 5 5 5 7	1 1 1 4 5 5 5 8	1 1 1 1 5 5 5 5	1 1 1 3 5 5 5 7
1 1 3 4 5 5 7 8	4 4 4 4 8 8 8 8	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8
5 5 5 5 1 1 1 1	5 5 5 8 1 1 1 4	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
6 6 6 6 2 2 2 2	6 6 6 8 2 2 2 4	6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2
5 5 5 7 1 1 1 3	5 5 5 8 1 1 1 4	5 5 5 5 1 1 1 1	5 5 5 7 1 1 1 3
5 5 7 8 1 1 3 4	8 8 8 8 4 4 4 4	5 6 7 8 1 2 3 4	5 6 7 8 1 2 3 4
2.1 x 4.89	2.1 x 4.90	2.1 x 4.91	2.1 x 4.92
1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6	2 2 2 4 6 6 6 8	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6
1 1 1 3 5 5 5 7	1 1 3 4 5 5 7 8	1 1 3 1 5 5 7 5	1 1 3 1 5 5 7 5
1 1 1 4 5 5 5 8	4 4 4 4 8 8 8 8	1 1 1 4 5 5 5 8	2 2 2 4 6 6 6 8
5 5 5 5 1 1 1 1	5 5 5 8 1 1 1 4	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
6 6 6 6 2 2 2 2	6 6 6 8 2 2 2 4	6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2
5 5 5 7 1 1 1 3	5 5 7 8 1 1 3 4	5 5 7 5 1 1 3 1	5 5 7 5 1 1 3 1
5 5 5 8 1 1 1 4	8 8 8 8 4 4 4 4	5 5 5 8 1 1 1 4	6 6 6 8 2 2 2 4
2.1 x 4.93	2.1 x 4.94	2.1 x 4.95	2.1 x 4.96
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6
1 2 3 1 5 6 7 5	1 1 3 3 5 5 7 7	1 1 3 3 5 5 7 7	3 3 3 3 7 7 7 7
1 1 1 4 5 5 5 8	1 1 3 4 5 5 7 8	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1
6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2	6 6 6 6 2 2 2 2
5 6 7 5 1 2 3 1	5 5 7 7 1 1 3 3	5 5 7 7 1 1 3 3	7 7 7 7 3 3 3 3
5 5 5 8 1 1 1 4	5 5 7 8 1 1 3 4	5 6 7 8 1 2 3 4	5 6 7 8 1 2 3 4

TABLE II

DIRECT PRODUCTS: 2 x 4 (continued)

2.1 x 4.97	2.1 x 4.98	2.1 x 4.99	2.1 x 4.100
1 1 1 1 5 5 5 5 2 2 2 2 6 6 6 6 1 2 3 3 5 6 7 7 1 2 3 4 5 6 7 8 5 5 5 5 1 1 1 1 6 6 6 6 2 2 2 2 5 6 7 7 1 2 3 3 5 6 7 8 1 2 3 4	1 1 3 4 5 5 7 8 2 2 3 4 6 6 7 8 3 3 3 4 7 7 7 8 4 4 3 4 8 8 7 8 5 5 7 8 1 1 3 4 6 6 7 8 2 2 3 4 7 7 7 8 3 3 3 4 8 8 7 8 4 4 3 4	1 1 1 4 5 5 5 8 2 2 2 4 6 6 6 8 1 2 3 4 5 6 7 8 4 4 4 4 8 8 8 8 5 5 5 8 1 1 1 4 6 6 6 8 2 2 2 4 5 6 7 8 1 2 3 4 8 8 8 8 4 4 4 4	1 1 3 4 5 5 7 8 2 2 3 4 6 6 7 8 3 3 3 3 7 7 7 7 4 4 4 4 8 8 8 8 5 5 7 8 1 1 3 4 6 6 7 8 2 2 3 4 7 7 7 7 3 3 3 3 8 8 8 8 4 4 4 4
2.1 x 4.101	2.1 x 4.102	2.1 x 4.103	2.1 x 4.104
1 1 1 1 5 5 5 5 2 2 2 2 6 6 6 6 3 3 3 3 7 7 7 7 4 4 4 4 8 8 8 8 5 5 5 5 1 1 1 1 6 6 6 6 2 2 2 2 7 7 7 7 3 3 3 3 8 8 8 8 4 4 4 4	1 1 1 4 5 5 5 8 2 2 2 4 6 6 6 8 3 3 3 4 7 7 7 8 4 4 4 4 8 8 8 8 5 5 5 8 1 1 1 4 6 6 6 8 2 2 2 4 7 7 7 8 3 3 3 4 8 8 8 8 4 4 4 4	1 1 3 4 5 5 7 8 2 2 3 4 6 6 7 8 3 3 3 4 7 7 7 8 4 4 4 3 8 8 8 7 5 5 7 8 1 1 3 4 6 6 7 8 2 2 3 4 7 7 7 8 3 3 3 4 8 8 8 7 4 4 4 3	1 1 3 4 5 5 7 8 2 2 3 4 6 6 7 8 3 3 3 4 7 7 7 8 4 4 4 4 8 8 8 8 5 5 7 8 1 1 3 4 6 6 7 8 2 2 3 4 7 7 7 8 3 3 3 4 8 8 8 8 4 4 4 4
2.1 x 4.105	2.1 x 4.106	2.1 x 4.107	2.1 x 4.108
1 1 3 4 5 5 7 8 1 1 3 4 5 5 7 8 3 3 3 3 7 7 7 7 3 3 3 3 7 7 7 7 5 5 7 8 1 1 3 4 5 5 7 8 1 1 3 4 7 7 7 7 3 3 3 3 7 7 7 7 3 3 3 3	1 2 3 4 5 6 7 8 2 1 3 4 6 5 7 8 3 3 3 3 7 7 7 7 3 3 3 3 7 7 7 7 5 6 7 8 1 2 3 4 6 5 7 8 2 1 3 4 7 7 7 7 3 3 3 3 7 7 7 7 3 3 3 3	1 1 1 4 5 5 5 8 1 2 1 4 5 6 5 8 1 3 1 4 5 7 5 8 4 4 4 1 8 8 8 5 5 5 5 8 1 1 1 4 5 6 5 8 1 2 1 4 5 7 5 8 1 3 1 4 8 8 8 5 4 4 4 1	1 1 1 1 5 5 5 5 1 2 1 1 5 6 5 5 1 3 1 1 5 7 5 5 1 3 1 1 5 7 5 5 5 5 5 5 1 1 1 1 5 6 5 5 1 2 1 1 5 7 5 5 1 3 1 1 5 7 5 5 1 3 1 1
2.1 x 4.109	2.1 x 4.110	2.1 x 4.111	2.1 x 4.112
1 1 1 1 5 5 5 5 1 2 1 1 5 6 5 5 1 3 1 1 5 7 5 5 1 1 1 1 5 5 5 5 5 5 5 5 1 1 1 1 5 6 5 5 1 2 1 1 5 7 5 5 1 3 1 1 5 5 5 5 1 1 1 1	1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5 1 2 3 4 5 6 7 8 5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1 5 6 7 8 1 2 3 4	1 1 1 1 5 5 5 5 1 2 1 4 5 6 5 8 1 3 1 1 5 7 5 5 1 1 1 1 5 5 5 5 5 5 5 5 1 1 1 1 5 6 5 8 1 2 1 4 5 7 5 5 1 3 1 1 5 5 5 5 1 1 1 1	1 1 1 1 5 5 5 5 1 1 1 2 5 5 5 6 1 1 1 1 5 5 5 5 1 2 3 4 5 6 7 8 5 5 5 5 1 1 1 1 5 5 5 6 1 1 1 2 5 5 5 5 1 1 1 1 5 6 7 8 1 2 3 4

TABLE II

DIRECT PRODUCTS: 2 x 4 (continued)

2.1 x 4.113	2.1 x 4.114	2.1 x 4.115	2.1 x 4.116
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8
1 2 1 1 5 6 5 5	1 2 1 1 5 6 5 5	1 2 1 2 5 6 5 6	1 2 1 4 5 6 5 8
1 3 1 1 5 7 5 5	1 3 1 1 5 7 5 5	1 3 1 3 5 7 5 7	1 3 1 4 5 7 5 8
1 3 3 4 5 7 7 8	1 1 3 4 5 5 7 8	1 2 1 4 5 6 5 8	4 4 4 4 8 8 8 8
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 8 1 1 1 4
5 6 5 5 1 2 1 1	5 6 5 5 1 2 1 1	5 6 5 6 1 2 1 2	5 6 5 8 1 2 1 4
5 7 5 5 1 3 1 1	5 7 5 5 1 3 1 1	5 7 5 7 1 3 1 3	5 7 5 8 1 3 1 4
5 7 7 8 1 3 3 4	5 5 7 8 1 1 3 4	5 6 5 8 1 2 1 4	8 8 8 8 4 4 4 4
2.1 x 4.117	2.1 x 118	2.1 x 119	2.1 x 4.120
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 2 2 1 5 6 6 5
1 2 1 1 5 6 5 5	1 2 1 2 5 6 5 6	1 1 1 2 5 5 5 6	2 1 1 2 6 5 5 6
1 3 1 1 5 7 5 5	1 3 1 3 5 7 5 7	1 1 3 3 5 5 7 7	2 1 1 2 6 5 5 6
1 1 1 4 5 5 5 8	1 2 3 4 5 6 7 8	1 1 3 4 5 5 7 8	1 2 3 4 5 6 7 8
5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 5 5 5 1 1 1 1	5 6 6 5 1 2 2 1
5 6 5 5 1 2 1 1	5 6 5 6 1 2 1 2	5 5 5 6 1 1 1 2	6 5 5 6 2 1 1 2
5 7 5 5 1 3 1 1	5 7 5 7 1 3 1 3	5 5 7 7 1 1 3 3	6 5 5 6 2 1 1 2
5 5 5 8 1 1 1 4	5 6 7 8 1 2 3 4	5 5 7 8 1 1 3 4	5 6 7 8 1 2 3 4
2.1 x 4.121	2.2 x 4.1	2.2 x 4.2	2.2 x 4.3
1 1 1 1 5 5 5 5	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 1 1 1 5 5 5 5	1 2 2 2 1 2 2 2	1 2 1 1 1 2 1 1	1 1 1 1 1 1 1 1
1 1 1 2 5 5 5 6	1 2 3 3 1 2 3 3	1 1 1 1 1 1 1 1	1 1 2 2 1 1 2 2
1 1 1 1 5 5 5 5	1 2 3 3 1 2 3 3	1 1 1 1 1 1 1 1	1 1 2 2 1 1 2 2
5 5 5 5 1 1 1 1	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
5 5 5 5 1 1 1 1	1 2 2 2 5 6 6 6	1 2 1 1 5 6 5 5	1 1 1 1 5 5 5 5
5 5 5 6 1 1 1 2	1 2 3 3 5 6 7 7	1 1 1 1 5 5 5 5	1 1 2 2 5 5 6 6
5 5 5 5 1 1 1 1	1 2 3 3 5 6 7 7	1 1 1 1 5 5 5 5	1 1 2 2 5 5 6 6
2.2 x 4.4	2.2 x 4.5	2.2 x 4.6	2.2 x 4.7
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 1 4 1 2 1 4	1 2 1 2 1 2 1 2	1 2 1 1 1 2 1 1	1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 2 1 1 1 2
1 4 1 1 1 4 1 1	1 2 1 4 1 2 1 4	1 1 1 3 1 1 1 3	1 1 2 2 1 1 2 2
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 1 4 5 6 5 8	1 2 1 2 5 6 5 6	1 2 1 1 5 6 5 5	1 1 1 1 5 5 5 5
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 2 5 5 5 6
1 4 1 1 5 8 5 5	1 2 1 4 5 6 5 8	1 1 1 3 5 5 5 7	1 1 2 2 5 5 6 6

TABLE II

DIRECT PRODUCTS: 2×4 (continued)

2.2 x 4.8	2.2 x 4.9	2.2 x 4.10	2.2 x 4.11
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 2 3 4 1 2 3 4	1 2 3 4 1 2 3 4
1 1 1 3 1 1 1 3	1 2 1 2 1 2 1 2	2 1 4 3 2 1 4 3	2 1 4 3 2 1 4 3
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	3 4 2 1 3 4 2 1	3 4 1 2 3 4 1 2
1 3 1 2 1 3 1 2	1 2 1 2 1 2 1 2	4 3 1 2 4 3 1 2	4 3 2 1 4 3 2 1
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8
1 1 1 3 5 5 5 5	1 2 1 2 5 6 5 6	2 1 4 3 6 5 8 7	2 1 4 3 6 5 8 7
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	3 4 2 1 7 8 6 5	3 4 1 2 7 8 5 6
1 3 1 2 5 7 5 6	1 2 1 2 5 6 5 6	4 3 1 2 8 7 5 6	4 3 2 1 8 7 6 5
2.2 x 4.12	2.2 x 4.13	2.2 x 4.14	2.2 x 4.15
1 2 3 4 1 2 3 4	1 2 3 1 1 2 3 1	1 2 1 1 1 2 1 1	1 2 1 2 1 2 1 2
2 3 1 4 2 3 1 4	2 3 1 2 2 3 1 2	2 1 2 2 2 1 2 2	2 1 2 1 2 1 2 1
3 1 2 4 3 1 2 4	3 1 2 3 3 1 2 3	1 2 3 4 1 2 3 4	1 2 3 4 1 2 3 4
4 4 4 4 4 4 4 4	1 2 3 4 1 2 3 4	1 2 4 3 1 2 4 3	2 1 4 3 2 1 4 3
1 2 3 4 5 6 7 8	1 2 3 1 5 6 7 5	1 2 1 1 5 6 5 5	1 2 1 2 5 6 5 6
2 3 1 4 6 7 5 8	2 3 1 2 6 7 5 6	2 1 2 2 6 5 6 6	2 1 2 1 6 5 6 5
3 1 2 4 7 5 6 8	3 1 2 3 7 5 6 7	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8
4 4 4 4 8 8 8 8	1 2 3 4 5 6 7 8	1 2 4 3 5 6 8 7	2 1 4 3 6 5 8 7
2.2 x 4.16	2.2 x 4.17	2.2 x 4.18	2.2 x 4.19
1 2 1 1 1 2 1 1	1 2 1 1 1 2 1 1	1 2 3 1 1 2 3 1	1 2 3 3 1 2 3 3
2 1 2 2 2 1 2 2	2 1 2 2 2 1 2 2	2 1 3 2 2 1 3 2	2 1 3 3 2 1 3 3
1 2 3 1 1 2 3 1	1 2 3 4 1 2 3 4	3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3
1 2 1 4 1 2 1 4	1 2 4 4 1 2 4 4	1 2 3 4 1 2 3 4	3 3 3 4 3 3 3 4
1 2 1 1 5 6 5 5	1 2 1 1 5 6 5 5	1 2 3 1 5 6 7 5	1 2 3 3 5 6 7 7
2 1 2 2 6 5 6 6	2 1 2 2 6 5 6 6	2 1 3 2 6 5 7 6	2 1 3 3 6 5 7 7
1 2 3 1 5 6 7 5	1 2 3 4 5 6 7 8	3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7
1 2 1 4 5 6 5 8	1 2 4 4 5 6 8 8	1 2 3 4 5 6 7 8	3 3 3 4 7 7 7 8
2.2 x 4.20	2.2 x 4.21	2.2 x 4.22	2.2 x 4.23
1 2 3 4 1 2 3 4	1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4
2 1 3 4 2 1 3 4	1 2 3 4 1 2 3 4	1 2 1 4 1 2 1 4	1 2 1 4 1 2 1 4
3 3 3 4 3 3 3 4	1 3 1 4 1 3 1 4	1 1 1 4 1 1 1 4	1 1 3 4 1 1 3 4
4 4 4 4 4 4 4 4	4 4 4 1 4 4 4 1	4 4 4 1 4 4 4 1	4 4 4 4 4 4 4 4
1 2 3 4 5 6 7 8	1 1 1 4 5 5 5 8	1 1 1 4 5 5 5 8	1 1 1 4 5 5 5 8
2 1 3 4 6 5 7 8	1 2 3 4 5 6 7 8	1 2 1 4 5 6 5 8	1 2 1 4 5 6 5 8
3 3 3 4 7 7 7 8	1 3 1 4 5 7 5 8	1 1 1 4 5 5 5 8	1 1 3 4 5 5 7 8
4 4 4 4 8 8 8 8	4 4 4 1 8 8 8 5	4 4 4 1 8 8 8 5	4 4 4 4 8 8 8 8

TABLE II
DIRECT PRODUCTS: 2×4 (continued)

2.2 x 4.24	2.2 x 4.25	2.2 x 4.26	2.2 x 4.27
1 1 3 3 1 1 3 3	1 2 1 4 1 2 1 4	1 2 4 4 1 2 4 4	1 1 1 1 1 1 1 1
1 2 3 4 1 2 3 4	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	1 2 1 4 1 2 1 4
3 3 1 1 3 3 1 1	1 2 1 4 1 2 1 4	4 2 1 1 4 2 1 1	1 1 1 1 1 1 1 1
3 4 1 1 3 4 1 1	4 2 4 1 4 2 4 1	4 2 1 1 4 2 1 1	1 4 1 2 1 4 1 2
1 1 3 3 5 5 7 7	1 2 1 4 5 6 5 8	1 2 4 4 5 6 8 8	1 1 1 1 5 5 5 5
1 2 3 4 5 6 7 8	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	1 2 1 4 5 6 5 8
3 3 1 1 7 7 5 5	1 2 1 4 5 6 5 8	4 2 1 1 8 6 5 5	1 1 1 1 5 5 5 5
3 4 1 1 7 8 5 5	4 2 4 1 8 6 8 5	4 2 1 1 8 6 5 5	1 4 1 2 5 8 5 6
2.2 x 4.28	2.2 x 4.29	2.2 x 4.30	2.2 x 4.31
1 1 1 1 1 1 1 1	1 1 3 1 1 1 3 1	1 2 1 1 1 2 1 1	1 2 1 2 1 2 1 2
1 2 3 4 1 2 3 4	1 2 3 2 1 2 3 2	2 1 2 2 2 1 2 2	2 1 2 1 2 1 2 1
1 3 1 3 1 3 1 3	3 3 1 3 3 3 1 3	1 2 1 1 1 2 1 1	1 2 1 2 1 2 1 2
1 4 3 2 1 4 3 2	1 2 3 2 1 2 3 2	1 2 1 1 1 2 1 1	2 1 2 1 2 1 2 1
1 1 1 1 5 5 5 5	1 1 3 1 5 5 7 5	1 2 1 1 5 6 5 5	1 2 1 2 5 6 5 6
1 2 3 4 5 6 7 8	1 2 3 2 5 6 7 6	2 1 2 2 6 5 6 6	2 1 2 1 6 5 6 5
1 3 1 3 5 7 5 7	3 3 1 3 7 7 5 7	1 2 1 1 5 6 5 5	1 2 1 2 5 6 5 6
1 4 3 2 5 8 7 6	1 2 3 2 5 6 7 5	1 2 1 1 5 6 5 5	2 1 2 1 6 5 6 5
2.2 x 4.32	2.2 x 4.33	2.2 x 4.34	2.2 x 4.35
1 2 2 2 1 2 2 2	1 1 3 1 1 1 3 1	1 1 3 3 1 1 3 3	1 2 3 3 1 2 3 3
2 1 1 1 2 1 1 1	1 1 3 1 1 1 3 1	1 1 3 3 1 1 3 3	2 3 1 1 2 3 1 1
2 1 1 1 2 1 1 1	3 3 1 3 3 3 1 3	3 3 1 1 3 3 1 1	3 1 2 2 3 1 2 2
2 1 1 1 2 1 1 1	1 1 3 2 1 1 3 2	3 3 1 2 3 3 1 2	3 1 2 2 3 1 2 2
1 2 2 2 5 6 6 6	1 1 3 1 5 5 7 5	1 1 3 3 5 5 7 7	1 2 3 3 5 6 7 7
2 1 1 1 6 5 5 5	1 1 3 1 5 5 7 5	1 1 3 3 5 5 7 7	2 3 1 1 6 7 5 5
2 1 1 1 6 5 5 5	3 3 1 3 7 7 5 7	3 3 1 1 7 7 5 5	3 1 2 2 7 5 6 6
2 1 1 1 6 5 5 5	1 1 3 2 5 5 7 6	3 3 1 2 7 7 5 6	3 1 2 2 7 5 6 6
2.2 x 4.36	2.2 x 4.37	2.2 x 4.38	2.2 x 4.39
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 3 3 1 2 3 3	1 2 3 4 1 2 3 4	1 2 3 2 1 2 3 2	1 2 3 4 1 2 3 4
1 3 1 1 1 3 1 1	1 3 1 1 1 3 1 1	1 3 1 3 1 3 1 3	1 3 1 1 1 3 1 1
1 3 1 1 1 3 1 1	1 4 1 1 1 4 1 1	1 2 3 2 1 2 3 2	1 4 1 3 1 4 1 3
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 3 3 5 6 7 7	1 2 3 4 5 6 7 8	1 2 3 2 5 6 7 6	1 2 3 4 5 6 7 8
1 3 1 1 5 7 5 5	1 3 1 1 5 7 5 5	1 3 1 3 5 7 5 7	1 3 1 1 5 7 5 5
1 3 1 1 5 7 5 5	1 4 1 1 5 8 5 5	1 2 3 2 5 6 7 6	1 4 1 3 5 8 5 7

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.2 x 4.40	2.2 x 4.41	2.2 x 4.42	2.2 x 4.43
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 2 2 1 2 2 2	1 2 2 2 1 2 2 2	1 2 2 2 1 2 2 2	1 2 2 1 1 2 2 1
1 2 2 2 1 2 2 2	1 2 2 2 1 2 2 2	1 2 3 3 1 2 3 3	1 2 3 1 1 2 3 1
1 2 2 2 1 2 2 2	1 2 2 3 1 2 2 3	1 2 3 4 1 2 3 4	1 1 1 4 1 1 1 4
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 2 2 5 6 6 6	1 2 2 2 5 6 6 6	1 2 2 2 5 6 6 6	1 2 2 1 5 6 6 5
1 2 2 2 5 6 6 6	1 2 2 2 5 6 6 6	1 2 3 3 5 6 7 7	1 2 3 1 5 6 7 5
1 2 2 2 5 6 6 6	1 2 2 3 5 6 6 7	1 2 3 4 5 6 7 8	1 1 1 4 5 5 5 8
2.2 x 4.44	2.2 x 4.45	2.2 x 4.46	2.2 x 4.47
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 2 1 1 2 2 1	1 2 2 4 1 2 2 4	1 2 2 2 1 2 2 2	1 2 2 2 1 2 2 2
1 2 3 4 1 2 3 4	1 2 3 4 1 2 3 4	1 2 3 2 1 2 3 2	1 2 3 4 1 2 3 4
1 1 4 1 1 1 4 1	1 4 4 1 1 1 4 1	1 2 2 2 1 2 2 2	1 2 4 2 1 2 4 2
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 2 1 5 6 6 5	1 2 2 4 5 6 6 8	1 2 2 2 5 6 6 6	1 2 2 2 5 6 6 6
1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8	1 2 3 2 5 6 7 6	1 2 3 4 5 6 7 8
1 1 4 1 5 5 8 5	1 4 4 1 5 8 8 5	1 2 2 2 5 6 6 6	1 2 4 2 5 6 8 6
2.2 x 4.48	2.2 x 4.49	2.2 x 4.50	2.2 x 4.51
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 1 1 1 2 1 1	1 2 1 2 1 2 1 2	1 2 1 1 1 2 1 1	1 2 1 1 1 2 1 1
1 1 3 1 1 1 3 1	1 1 3 3 1 1 3 3	1 1 3 1 1 1 3 1	1 1 3 4 1 1 3 4
1 1 1 4 1 1 1 4	1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1	1 1 4 1 1 1 4 1
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 1 1 5 6 5 5	1 2 1 2 5 6 5 6	1 2 1 1 5 6 5 5	1 2 1 1 5 6 5 5
1 1 3 1 5 5 7 5	1 1 3 3 5 5 7 7	1 1 3 1 5 5 7 5	1 1 3 4 5 5 7 8
1 1 1 4 5 5 5 8	1 2 3 4 5 6 7 8	1 1 1 1 5 5 5 5	1 1 4 1 5 5 8 5
2.2 x 4.52	2.2 x 4.53	2.2 x 4.54	2.2 x 4.55
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 1 2 1 2 1 2	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 1 3 1 1 1 3 1	1 1 1 2 1 1 1 2	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 1 2 1 2 1 2	1 1 2 1 1 1 2 1	1 1 1 2 1 1 1 2	1 1 1 1 1 1 1 1
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 1 2 5 6 5 6	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 1 3 1 5 5 7 5	1 1 1 2 5 5 5 6	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 1 2 5 6 5 6	1 1 2 1 5 5 6 5	1 1 1 2 5 5 5 6	1 1 1 1 5 5 5 5

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.2 x 4.56	2.2 x 4.57	2.2 x 4.58	2.2 x 4.59
1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4	1 1 3 4 1 1 3 4	1 1 1 1 1 1 1 1
1 1 1 4 1 1 1 4	1 2 3 4 1 2 3 4	1 1 3 4 1 1 3 4	2 2 2 2 2 2 2 2
1 1 2 4 1 1 2 4	1 3 2 4 1 3 2 4	3 3 3 4 3 3 3 4	1 1 1 2 1 1 1 2
1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4	3 3 3 4 3 3 3 4	4 4 4 4 4 4 4 4
1 1 1 4 5 5 5 8	1 1 1 4 5 5 5 8	1 1 3 4 5 5 7 8	1 1 1 1 5 5 5 5
1 1 1 4 5 5 5 8	1 2 3 4 5 6 7 8	1 1 3 4 5 5 7 8	2 2 2 2 6 6 6 6
1 1 2 4 5 5 6 8	1 3 2 4 5 7 6 8	3 3 3 4 7 7 7 8	1 1 1 2 5 5 5 6
1 1 1 4 5 5 5 8	1 1 1 4 5 5 5 8	3 3 3 4 7 7 7 8	4 4 4 4 8 8 8 8
2.2 x 4.60	2.2 x 4.61	2.2 x 4.62	2.2 x 4.63
1 2 3 4 1 2 3 4	1 1 3 4 1 1 3 4	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 1 3 4 2 1 3 4	1 1 3 4 1 1 3 4	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2
3 3 3 4 3 3 3 4	3 3 3 4 3 3 3 4	3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4
4 4 3 4 4 4 3 4	4 4 3 4 4 4 3 4	1 2 2 4 1 2 2 4	2 1 4 3 2 1 4 3
1 2 3 4 5 6 7 8	1 1 3 4 5 5 7 8	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
2 1 3 4 6 5 7 8	1 1 3 4 5 5 7 8	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6
3 3 3 4 7 7 7 8	3 3 3 4 7 7 7 8	3 3 3 3 7 7 7 7	1 2 3 4 5 6 7 8
4 4 3 4 8 8 7 8	4 4 3 4 8 8 7 8	1 2 2 4 5 6 6 8	2 1 4 3 6 5 8 7
2.2 x 4.64	2.2 x 4.65	2.2 x 4.66	2.2 x 4.67
1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1	1 1 3 3 1 1 3 3
2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3	2 2 2 2 2 2 2 2	2 2 3 3 2 2 3 3
1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4	1 1 3 3 1 1 3 3	3 3 3 3 3 3 3 3
2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3	2 2 4 4 2 2 4 4	4 4 4 4 4 4 4 4
1 1 3 3 5 5 7 7	1 2 3 4 5 6 7 8	1 1 1 1 5 5 5 5	1 1 3 3 5 5 7 7
2 2 4 4 6 6 8 8	2 1 4 3 6 5 8 7	2 2 2 2 6 6 6 6	2 2 3 3 6 6 7 7
1 1 3 3 5 5 7 7	1 2 3 4 5 6 7 8	1 1 3 3 5 5 7 7	3 3 3 3 7 7 7 7
2 2 4 4 6 6 8 8	2 1 4 3 6 5 8 7	2 2 4 4 6 6 8 8	4 4 4 4 8 8 8 8
2.2 x 4.68	2.2 x 4.69	2.2 x 4.70	2.2 x 4.71
1 1 3 3 1 1 3 3	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 2 3 3 2 2 3 3	1 1 2 2 1 1 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4	3 3 3 3 3 3 3 3	1 1 1 1 1 1 1 1
4 4 3 3 4 4 3 3	1 2 3 4 1 2 3 4	3 3 3 4 3 3 3 4	4 4 4 4 4 4 4 4
1 1 3 3 5 5 7 7	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
2 2 3 3 6 6 7 7	1 1 2 2 5 5 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6
3 3 3 3 7 7 7 7	1 2 3 4 5 6 7 8	3 3 3 3 7 7 7 7	1 1 1 1 5 5 5 5
4 4 3 3 8 8 7 7	1 2 3 4 5 6 7 8	3 3 3 4 7 7 7 8	4 4 4 4 8 8 8 8

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.2 x 4.72	2.2 x 4.73	2.2 x 4.74	2.2 x 4.75
1 1 3 4 1 1 3 4	1 1 3 3 1 1 3 3	1 1 3 4 1 1 3 4	1 1 3 3 1 1 3 3
2 2 3 4 2 2 3 4	2 2 3 3 2 2 3 3	2 2 3 4 2 2 3 4	2 2 3 3 2 2 3 3
3 3 3 4 3 3 3 4	3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3
3 3 3 4 3 3 3 4	3 3 3 4 3 3 3 4	3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3
1 1 3 4 5 5 7 8	1 1 3 3 5 5 7 7	1 1 3 4 5 5 7 8	1 1 3 3 5 5 7 7
2 2 3 4 6 6 7 8	2 2 3 3 6 6 7 7	2 2 3 4 6 6 7 8	2 2 3 3 6 6 7 7
3 3 3 4 7 7 7 8	3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7
3 3 3 4 7 7 7 8	3 3 3 4 7 7 7 8	3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7
2.2 x 4.76	2.2 x 4.77	2.2 x 4.78	2.2 x 4.79
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 1 1 3 1 1 1 3	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2
1 1 1 3 1 1 1 3	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1	2 2 2 2 2 2 2 2	1 1 1 4 1 1 1 4
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 1 1 3 5 5 5 7	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6
1 1 1 3 5 5 5 7	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
1 2 3 4 5 6 7 8	1 1 1 1 5 5 5 5	2 2 2 2 6 6 6 6	1 1 1 4 5 5 5 8
2.2 x 4.80	2.2 x 4.81	2.2 x 4.82	2.2 x 4.83
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1	1 1 1 3 1 1 1 3	1 2 3 1 1 2 3 1	1 1 1 3 1 1 1 3
2 2 2 4 2 2 2 4	2 2 2 4 2 2 2 4	1 1 1 1 1 1 1 1	1 2 1 4 1 2 1 4
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6
1 1 1 1 5 5 5 5	1 1 1 3 5 5 5 7	1 2 3 1 5 6 7 5	1 1 1 3 5 5 5 7
2 2 2 4 6 6 6 8	2 2 2 4 6 6 6 8	1 1 1 1 5 5 5 5	1 2 1 4 5 6 5 8
2.2 x 4.84	2.2 x 4.85	2.2 x 4.86	2.2 x 4.87
1 1 1 4 1 1 1 4	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 1 1 1 1 1
1 2 1 4 1 2 1 4	2 2 2 2 2 2 2 2	2 2 2 4 2 2 2 4	2 2 2 2 2 2 2 2
1 3 1 4 1 3 1 4	1 1 1 3 1 1 1 3	1 1 1 4 1 1 1 4	1 1 1 1 1 1 1 1
1 1 1 4 1 1 1 4	1 1 3 4 1 1 3 4	4 4 4 4 4 4 4 4	1 2 3 4 1 2 3 4
1 1 1 4 5 5 5 8	1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8	1 1 1 1 5 5 5 5
1 2 1 4 5 6 5 8	2 2 2 2 6 6 6 6	2 2 2 4 6 6 6 8	2 2 2 2 6 6 6 6
1 3 1 4 5 7 5 8	1 1 1 3 5 5 5 7	1 1 1 4 5 5 5 8	1 1 1 1 5 5 5 5
1 1 1 4 5 5 5 8	1 1 3 4 5 5 7 8	4 4 4 4 8 8 8 8	1 2 3 4 5 6 7 8

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.2 x 4.88	2.2 x 4.89	2.2 x 4.90	2.2 x 4.91
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 4 2 2 2 4	2 2 2 2 2 2 2 2
1 1 1 3 1 1 1 3	1 1 1 3 1 1 1 3	1 1 3 4 1 1 3 4	1 1 3 1 1 1 3 1
1 2 3 4 1 2 3 4	1 1 1 4 1 1 1 4	4 4 4 4 4 4 4 4	1 1 1 4 1 1 1 4
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8	1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 4 6 6 6 8	2 2 2 2 6 6 6 6
1 1 1 3 5 5 5 7	1 1 1 3 5 5 5 7	1 1 3 4 5 5 7 8	1 1 3 1 5 5 7 5
1 2 3 4 5 6 7 8	1 1 1 4 5 5 5 8	4 4 4 4 8 8 8 8	1 1 1 4 5 5 5 8
2.2 x 4.92	2.2 x 4.93	2.2 x 4.94	2.2 x 4.95
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2
1 1 3 1 1 1 3 1	1 2 3 1 1 2 3 1	1 1 3 3 1 1 3 3	1 1 3 3 1 1 3 3
2 2 2 4 2 2 2 4	1 1 1 4 1 1 1 4	1 1 3 4 1 1 3 4	1 2 3 4 1 2 3 4
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6
1 1 3 1 5 5 7 5	1 2 3 1 5 6 7 5	1 1 3 3 5 5 7 7	1 1 3 3 5 5 7 7
2 2 2 4 6 6 6 8	1 1 1 4 5 5 5 8	1 1 3 4 5 5 7 8	1 2 3 4 5 6 7 8
2.2 x 4.96	2.2 x 4.97	2.2 x 4.98	2.2 x 4.99
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 3 4 1 1 3 4	1 1 1 4 1 1 1 4
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 3 4 2 2 3 4	2 2 2 4 2 2 2 4
3 3 3 3 3 3 3 3	1 2 3 3 1 2 3 3	3 3 3 4 3 3 3 4	1 2 3 4 1 2 3 4
1 2 3 4 1 2 3 4	1 2 3 4 1 2 3 4	4 4 3 4 4 4 3 4	4 4 4 4 4 4 4 4
1 1 1 1 5 5 5 5	1 1 1 1 5 5 5 5	1 1 3 4 5 5 7 8	1 1 1 4 5 5 5 8
2 2 2 2 6 6 6 6	2 2 2 2 6 6 6 6	2 2 3 4 6 6 7 8	2 2 2 4 6 6 6 8
3 3 3 3 7 7 7 7	1 2 3 3 5 6 7 7	3 3 3 4 7 7 7 8	1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8	4 4 3 4 8 8 7 8	4 4 4 4 8 8 8 8
2.2 x 4.100	2.2 x 4.101	2.2 x 4.102	2.2 x 4.103
1 1 3 4 1 1 3 4	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 3 4 1 1 3 4
2 2 3 4 2 2 3 4	2 2 2 2 2 2 2 2	2 2 2 4 2 2 2 4	2 2 3 4 2 2 3 4
3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3	3 3 3 4 3 3 3 4	3 3 3 4 3 3 3 4
4 4 4 4 4 4 4 4	4 4 4 4 4 4 4 4	4 4 4 4 4 4 4 4	4 4 4 3 4 4 4 3
1 1 3 4 5 5 7 8	1 1 1 1 5 5 5 5	1 1 1 4 5 5 5 8	1 1 3 4 5 5 7 8
2 2 3 4 6 6 7 8	2 2 2 2 6 6 6 6	2 2 2 4 6 6 6 8	2 2 3 4 6 6 7 8
3 3 3 3 7 7 7 7	3 3 3 3 7 7 7 7	3 3 3 4 7 7 7 8	3 3 3 4 7 7 7 8
4 4 4 4 8 8 8 8	4 4 4 4 8 8 8 8	4 4 4 4 8 8 8 8	4 4 4 3 8 8 8 7

TABLE II
DIRECT PRODUCTS: 2×4 (continued)

2.2×4.104	2.2×4.105	2.2×4.106	2.2×4.107
1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4 1 1 3 4 5 5 7 8 2 2 3 4 6 6 7 8 3 3 3 4 7 7 7 8 4 4 4 4 8 8 8 8	1 1 3 4 1 1 3 4 1 1 3 4 1 1 3 4 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 1 1 3 4 5 5 7 8 1 1 3 4 5 5 7 8 3 3 3 3 7 7 7 7 3 3 3 3 7 7 7 7	1 2 3 4 1 2 3 4 2 1 3 4 2 1 3 4 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 1 2 3 4 5 6 7 8 2 1 3 4 6 5 7 8 3 3 3 3 7 7 7 7 3 3 3 3 7 7 7 7	1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 3 1 4 1 3 1 4 4 4 4 1 4 4 4 1 1 1 1 4 5 5 5 8 1 2 1 4 5 6 5 8 1 3 1 4 5 7 5 8 4 4 4 1 8 8 8 5
2.2×4.108	2.2×4.109	2.2×4.110	2.2×4.111
1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 1 1 1 1 1 5 5 5 5 1 2 1 1 5 6 5 5 1 3 1 1 5 7 5 5 1 3 1 1 5 7 5 5	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 5 5 5 5 1 2 1 1 5 6 5 5 1 3 1 1 5 7 5 5 1 1 1 1 5 5 5 5	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5 1 2 3 4 5 6 7 8	1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 5 5 5 5 1 2 1 4 5 6 5 8 1 3 1 1 5 7 5 5 1 1 1 1 5 5 5 5
2.2×4.112	2.2×4.113	2.2×4.114	2.2×4.115
1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 1 1 1 5 5 5 5 1 1 1 2 5 5 5 6 1 1 1 1 5 5 5 5 1 2 3 4 5 6 7 8	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 3 3 4 1 3 3 4 1 1 1 1 5 5 5 5 1 2 1 1 5 6 5 5 1 3 1 1 5 7 5 5 1 3 3 4 5 7 7 8	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 3 4 1 1 3 4 1 1 1 1 5 5 5 5 1 2 1 1 5 6 5 5 1 3 1 1 5 7 5 5 1 1 3 4 5 5 7 8	1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 3 1 3 1 3 1 3 1 2 1 4 1 2 1 4 1 1 1 1 5 5 5 5 1 2 1 2 5 6 5 6 1 3 1 3 5 7 5 7 1 2 1 4 5 6 5 8
2.2×4.116	2.2×4.117	2.2×4.118	2.2×4.119
1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 3 1 4 1 3 1 4 4 4 4 4 4 4 4 4 1 1 1 4 5 5 5 8 1 2 1 4 5 6 5 8 1 3 1 4 5 7 5 8 4 4 4 4 8 8 8 8	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 1 4 1 1 1 4 1 1 1 1 5 5 5 5 1 2 1 1 5 6 5 5 1 3 1 1 5 7 5 5 1 1 1 4 5 5 5 8	1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 3 1 3 1 3 1 3 1 2 3 4 1 2 3 4 1 1 1 1 5 5 5 5 1 2 1 2 5 6 5 6 1 3 1 3 5 7 5 7 1 2 3 4 5 6 7 8	1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 3 3 1 1 3 3 1 1 3 4 1 1 3 4 1 1 1 1 5 5 5 5 1 1 1 2 5 5 5 6 1 1 3 3 5 5 7 7 1 1 3 4 5 5 7 8

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.2 x 4.120	2.2 x 4.121	2.3 x 4.1	2.3 x 4.2
1 2 2 1 1 2 2 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 1 1 2 2 1 1 2	1 1 1 1 1 1 1 1	1 2 2 2 1 2 2 2	1 2 1 1 1 2 1 1
2 1 1 2 2 1 1 2	1 1 1 2 1 1 1 2	1 2 3 3 1 2 3 3	1 1 1 1 1 1 1 1
1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1	1 2 3 3 1 2 3 3	1 1 1 1 1 1 1 1
1 2 2 1 5 6 6 5	1 1 1 1 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5
2 1 1 2 6 5 5 6	1 1 1 1 5 5 5 5	5 6 6 6 5 6 6 6	5 6 5 5 5 6 5 5
2 1 1 2 6 5 5 6	1 1 1 2 5 5 5 6	5 6 7 7 5 6 7 7	5 5 5 5 5 5 5 5
1 2 3 4 5 6 7 8	1 1 1 1 5 5 5 5	5 6 7 7 5 6 7 7	5 5 5 5 5 5 5 5
2.3 x 4.3	2.3 x 4.4	2.3 x 4.5	2.3 x 4.6
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1	1 2 1 4 1 2 1 4	1 2 1 2 1 2 1 2	1 2 1 1 1 2 1 1
1 1 2 2 1 1 2 2	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 1 2 2 1 1 2 2	1 4 1 1 1 4 1 1	1 2 1 4 1 2 1 4	1 1 1 3 1 1 1 3
5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5
5 5 5 5 5 5 5 5	5 6 5 8 5 6 5 8	5 6 5 6 5 6 5 6	5 6 5 5 5 6 5 5
5 5 6 6 5 5 6 6	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5
5 5 6 6 5 5 6 6	5 8 5 5 5 8 5 5	5 6 5 8 5 6 5 8	5 5 5 7 5 5 5 7
2.3 x 4.7	2.3 x 4.8	2.3 x 4.9	2.3 x 4.10
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 2 3 4 1 2 3 4
1 1 1 1 1 1 1 1	1 1 1 3 1 1 1 3	1 2 1 2 1 2 1 2	2 1 4 3 2 1 4 3
1 1 1 2 1 1 1 2	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	3 4 2 1 3 4 2 1
1 1 2 2 1 1 2 2	1 3 1 2 1 3 1 2	1 2 1 2 1 2 1 2	4 3 1 2 4 3 1 2
5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 6 7 8 5 6 7 8
5 5 5 5 5 5 5 5	5 5 5 7 5 5 5 7	5 6 5 6 5 6 5 6	6 5 8 7 6 5 8 7
5 5 5 6 5 5 5 6	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	7 8 6 5 7 8 6 5
5 5 6 6 5 5 6 6	5 7 5 6 5 7 5 6	5 6 5 6 5 6 5 6	8 7 5 6 8 7 5 6
2.3 x 4.11	2.3 x 4.12	2.3 x 4.13	2.3 x 4.14
1 2 3 4 1 2 3 4	1 2 3 4 1 2 3 4	1 2 3 1 1 2 3 1	1 2 1 1 1 2 1 1
2 1 4 3 2 1 4 3	2 3 1 4 2 3 1 4	2 3 1 2 2 3 1 2	2 1 2 2 2 1 2 2
3 4 1 2 3 4 1 2	3 1 2 4 3 1 2 4	3 1 2 3 3 1 2 3	1 2 3 4 1 2 3 4
4 3 2 1 4 3 2 1	4 4 4 4 4 4 4 4	1 2 3 4 1 2 3 4	1 2 4 3 1 2 4 3
5 6 7 8 5 6 7 8	5 6 7 8 5 6 7 8	5 6 7 5 5 6 7 5	5 6 5 5 5 6 5 5
6 5 8 7 6 5 8 7	6 7 5 8 6 7 5 8	6 7 5 6 6 7 5 6	6 5 6 6 6 5 6 6
7 8 5 6 7 8 5 6	7 5 6 8 7 5 6 8	7 5 6 7 7 5 6 7	5 6 7 8 5 6 7 8
8 7 6 5 8 7 6 5	8 8 8 8 8 8 8 8	5 6 7 8 5 6 7 8	5 6 8 7 5 6 8 7

TABLE II

DIRECT PRODUCTS: 2 x 4 (continued)

2.3 x 4.15	2.3 x 4.16	2.3 x 4.17	2.3 x 4.18
1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1 1 2 3 4 1 2 3 4 2 1 4 3 2 1 4 3 5 6 5 6 5 6 5 6 6 5 6 5 6 5 6 5 5 6 7 8 5 6 7 8 6 5 8 7 6 5 8 7	1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 3 1 1 2 3 1 1 2 1 4 1 2 1 4 5 6 5 5 5 6 5 5 6 5 6 6 6 5 6 6 5 6 7 5 5 6 7 5 5 6 5 8 5 6 5 8	1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 3 4 1 2 3 4 1 2 4 4 1 2 4 4 5 6 5 5 5 6 5 5 6 5 6 6 6 5 6 6 5 6 7 8 5 6 7 8 5 6 8 8 5 6 8 8	1 2 3 1 1 2 3 1 2 1 3 2 2 1 3 2 3 3 3 3 3 3 3 3 1 2 3 4 1 2 3 4 5 6 7 5 5 6 7 5 6 5 7 6 6 5 7 6 7 7 7 7 7 7 7 7 5 6 7 8 5 6 7 8
2.3 x 4.19	2.3 x 4.20	2.3 x 4.21	2.3 x 4.22
1 2 3 3 1 2 3 3 2 1 3 3 2 1 3 3 3 3 3 3 3 3 3 3 3 3 3 4 3 3 3 4 5 6 7 7 5 6 7 7 6 5 7 7 6 5 7 7 7 7 7 7 7 7 7 7 7 7 7 8 7 7 7 8	1 2 3 4 1 2 3 4 2 1 3 4 2 1 3 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4 5 6 7 8 5 6 7 8 6 5 7 8 6 5 7 8 7 7 7 8 7 7 7 8 8 8 8 8 8 8 8 8	1 1 1 4 1 1 1 4 1 2 3 4 1 2 3 4 1 3 1 4 1 3 1 4 4 4 4 1 4 4 4 1 5 5 5 8 5 5 5 8 5 6 7 8 5 6 7 8 5 7 5 8 5 7 5 8 8 8 8 5 8 8 8 5	1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 1 1 4 1 1 1 4 4 4 4 1 4 4 4 1 5 5 5 8 5 5 5 8 5 6 5 8 5 6 5 8 5 5 5 8 5 5 5 8 8 8 8 5 8 8 8 5
2.3 x 4.23	2.3 x 4.24	2.3 x 4.25	2.3 x 4.26
1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 1 3 4 1 1 3 4 4 4 4 4 4 4 4 4 5 5 5 8 5 5 5 8 5 6 5 8 5 6 5 8 5 5 7 8 5 5 7 8 8 8 8 8 8 8 8 8	1 1 3 3 1 1 3 3 1 2 3 4 1 2 3 4 3 3 1 1 3 3 1 1 3 4 1 1 3 4 1 1 5 5 7 7 5 5 7 7 5 6 7 8 5 6 7 8 7 7 5 5 7 7 5 5 7 8 5 5 7 8 5 5	1 2 1 4 1 2 1 4 2 2 2 2 2 2 2 2 1 2 1 4 1 2 1 4 4 2 4 1 4 2 4 1 5 6 5 8 5 6 5 8 6 6 6 6 6 6 6 6 5 6 5 8 5 6 5 8 8 6 8 5 8 6 8 5	1 2 4 4 1 2 4 4 2 2 2 2 2 2 2 2 4 2 1 1 4 2 1 1 4 2 1 1 4 2 1 1 5 6 8 8 5 6 8 8 6 6 6 6 6 6 6 6 8 6 5 5 8 6 5 5 8 6 5 5 8 6 5 5
2.3 x 4.27	2.3 x 4.28	2.3 x 4.29	2.3 x 4.30
1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 1 1 1 1 1 1 1 1 4 1 2 1 4 1 2 5 5 5 5 5 5 5 5 5 6 5 8 5 6 5 8 5 5 5 5 5 5 5 5 5 8 5 6 5 8 5 6	1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 3 1 3 1 3 1 3 1 4 3 2 1 4 3 2 5 5 5 5 5 5 5 5 5 6 7 8 5 6 7 8 5 7 5 7 5 7 5 7 5 8 7 6 5 8 7 6	1 1 3 1 1 1 3 1 1 2 3 2 1 2 3 2 3 3 1 3 3 3 1 3 1 2 3 2 1 2 3 2 5 5 7 5 5 5 7 5 5 6 7 6 5 6 7 6 7 7 5 7 7 7 5 7 5 6 7 6 5 6 7 6	1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 5 6 5 5 5 6 5 5 6 5 6 6 6 5 6 6 5 6 5 5 5 6 5 5 5 6 5 5 5 6 5 5

TABLE II
DIRECT PRODUCTS: 2×4 (continued)

2.3 x 4.31	2.3 x 4.32	2.3 x 4.33	2.3 x 4.34
1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1 1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1 5 6 5 6 5 6 5 6 6 5 6 5 6 5 6 5 5 6 5 6 5 6 5 6 6 5 6 5 6 5 6 5	1 2 2 2 1 2 2 2 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 5 6 6 6 5 6 6 6 6 5 5 5 6 5 5 5 6 5 5 5 6 5 5 5 6 5 5 5 6 5 5 5	1 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 3 3 1 3 3 3 1 3 1 1 3 2 1 1 3 2 5 5 7 5 5 5 7 5 5 5 7 5 5 5 7 5 7 7 5 7 7 7 5 7 5 5 7 6 5 5 7 6	1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3 3 3 1 1 3 3 1 1 3 3 1 2 3 3 1 2 5 5 7 7 5 5 7 7 5 5 7 7 5 5 7 7 7 7 5 5 7 7 5 5 7 7 5 6 7 7 5 6
2.3 x 4.35	2.3 x 4.36	2.3 x 4.37	2.3 x 4.38
1 2 3 3 1 2 3 3 2 3 1 1 2 3 1 1 3 1 2 2 3 1 2 2 3 1 2 2 3 1 2 2 5 6 7 7 5 6 7 7 6 7 5 5 6 7 5 5 7 5 6 6 7 5 6 6 7 5 6 6 7 5 6 6	1 1 1 1 1 1 1 1 1 2 3 3 1 2 3 3 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 1 5 5 5 5 5 5 5 5 5 6 7 7 5 6 7 7 5 7 5 5 5 7 5 5 5 7 5 5 5 7 5 5	1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 3 1 1 1 3 1 1 1 4 1 1 1 4 1 1 5 5 5 5 5 5 5 5 5 6 7 8 5 6 7 8 5 7 5 5 5 7 5 5 5 8 5 5 5 8 5 5	1 1 1 1 1 1 1 1 1 2 3 2 1 2 3 2 1 3 1 3 1 3 1 3 1 2 3 2 1 2 3 2 5 5 5 5 5 5 5 5 5 6 7 6 5 6 7 6 5 7 5 7 5 7 5 7 5 6 7 6 5 6 7 6
2.3 x 4.39	2.3 x 4.40	2.3 x 4.41	2.3 x 4.42
1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 3 1 1 1 3 1 1 1 4 1 3 1 4 1 3 5 5 5 5 5 5 5 5 5 6 7 8 5 6 7 8 5 7 5 5 5 7 5 5 5 8 5 7 5 8 5 7	1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 5 5 5 5 5 5 5 5 5 6 6 6 5 6 6 6 5 6 6 6 5 6 6 6 5 6 6 6 5 6 6 6	1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 3 1 2 2 3 5 5 5 5 5 5 5 5 5 6 6 6 5 6 6 6 5 6 6 6 5 6 6 6 5 6 6 7 5 6 6 7	1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 3 1 2 3 3 1 2 3 4 1 2 3 4 5 5 5 5 5 5 5 5 5 6 6 6 5 6 6 6 5 6 7 7 5 6 7 7 5 6 7 8 5 6 7 8
2.3 x 4.43	2.3 x 4.44	2.3 x 4.45	2.3 x 4.46
1 1 1 1 1 1 1 1 1 2 2 1 1 2 2 1 1 2 3 1 1 2 3 1 1 1 1 4 1 1 1 4 5 5 5 5 5 5 5 5 5 6 6 5 5 6 6 5 5 6 7 5 5 6 7 5 5 5 5 8 5 5 5 8	1 1 1 1 1 1 1 1 1 2 2 1 1 2 2 1 1 2 3 4 1 2 3 4 1 1 4 1 1 1 4 1 5 5 5 5 5 5 5 5 5 6 6 5 5 6 6 5 5 6 7 8 5 6 7 8 5 5 8 5 5 5 8 5	1 1 1 1 1 1 1 1 1 2 2 4 1 2 2 4 1 2 3 4 1 2 3 4 1 4 4 1 1 4 4 1 5 5 5 5 5 5 5 5 5 6 6 8 5 6 6 8 5 6 7 8 5 6 7 8 5 8 8 5 5 8 8 5	1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 2 1 2 3 2 1 2 2 2 1 2 2 2 5 5 5 5 5 5 5 5 5 6 6 6 5 6 6 6 5 6 7 6 5 6 7 6 5 6 6 6 5 6 6 6

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.3 x 4.47	2.3 x 4.48	2.3 x 4.49	2.3 x 4.50
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 2 2 1 2 2 2	1 2 1 1 1 2 1 1	1 2 1 2 1 2 1 2	1 2 1 1 1 2 1 1
1 2 3 4 1 2 3 4	1 1 3 1 1 1 3 1	1 1 3 3 1 1 3 3	1 1 3 1 1 1 3 1
1 2 4 2 1 2 4 2	1 1 1 4 1 1 1 4	1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1
5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5
5 6 6 6 5 6 6 6	5 6 5 5 5 6 5 5	5 6 5 6 5 6 5 6	5 6 5 5 5 6 5 5
5 6 7 8 5 6 7 8	5 5 7 5 5 5 7 5	5 5 7 7 5 5 7 7	5 5 7 5 5 5 7 5
5 6 8 6 5 6 8 6	5 5 5 8 5 5 5 8	5 6 7 8 5 6 7 8	5 5 5 5 5 5 5 5
2.3 x 4.51	2.3 x 4.52	2.3 x 4.53	2.3 x 4.54
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 2 1 1 1 2 1 1	1 2 1 2 1 2 1 2	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
1 1 3 4 1 1 3 4	1 1 3 1 1 1 3 1	1 1 1 2 1 1 1 2	1 1 1 1 1 1 1 1
1 1 4 1 1 1 4 1	1 2 1 2 1 2 1 2	1 1 2 1 1 1 2 1	1 1 1 2 1 1 1 2
5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5
5 6 5 5 5 6 5 5	5 6 5 6 5 6 5 6	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5
5 5 7 8 5 5 7 8	5 5 7 5 5 5 7 5	5 5 5 6 5 5 5 6	5 5 5 5 5 5 5 5
5 5 8 5 5 5 8 5	5 6 5 6 5 6 5 6	5 5 6 5 5 5 6 5	5 5 5 6 5 5 5 6
2.3 x 4.55	2.3 x 4.56	2.3 x 4.57	2.3 x 4.58
1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4	1 1 3 4 1 1 3 4
1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 2 3 4 1 2 3 4	1 1 3 4 1 1 3 4
1 1 1 1 1 1 1 1	1 1 2 4 1 1 2 4	1 3 2 4 1 3 2 4	3 3 3 4 3 3 3 4
1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4	3 3 3 4 3 3 3 4
5 5 5 5 5 5 5 5	5 5 5 8 5 5 5 8	5 5 5 8 5 5 5 8	5 5 7 8 5 5 7 8
5 5 5 5 5 5 5 5	5 5 5 8 5 5 5 8	5 6 7 8 5 6 7 8	5 5 7 8 5 5 7 8
5 5 5 5 5 5 5 5	5 5 6 8 5 5 6 8	5 7 6 8 5 7 6 8	7 7 7 8 7 7 7 8
5 5 5 5 5 5 5 5	5 5 5 8 5 5 5 8	5 5 5 8 5 5 5 8	7 7 7 8 7 7 7 8
2.3 x 4.59	2.3 x 4.60	2.3 x 4.61	2.3 x 4.62
1 1 1 1 1 1 1 1	1 2 3 4 1 2 3 4	1 1 3 4 1 1 3 4	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 1 3 4 2 1 3 4	1 1 3 4 1 1 3 4	2 2 2 2 2 2 2 2
1 1 1 2 1 1 1 2	3 3 3 4 3 3 3 4	3 3 3 4 3 3 3 4	3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4	4 4 3 4 4 4 3 4	4 4 3 4 4 4 3 4	1 2 2 4 1 2 2 4
5 5 5 5 5 5 5 5	5 6 7 8 5 6 7 8	5 5 7 8 5 5 7 8	5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6	6 5 7 8 6 5 7 8	5 5 7 8 5 5 7 8	6 6 6 6 6 6 6 6
5 5 5 6 5 5 5 6	7 7 7 8 7 7 7 8	7 7 7 8 7 7 7 8	7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8	8 8 7 8 8 8 7 8	8 8 7 8 8 8 7 8	5 6 6 8 5 6 6 8

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.3 x 4.63	2.3 x 4.64	2.3 x 4.65	2.3 x 4.66
1 1 1 1 1 1 1 1	1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3	2 2 2 2 2 2 2 2
1 2 3 4 1 2 3 4	1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4	1 1 3 3 1 1 3 3
2 1 4 3 2 1 4 3	2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3	2 2 4 4 2 2 4 4
5 5 5 5 5 5 5 5	5 5 7 7 5 5 7 7	5 6 7 8 5 6 7 8	5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6	6 6 8 8 6 6 8 8	6 5 8 7 6 5 8 7	6 6 6 6 6 6 6 6
5 6 7 8 5 6 7 8	5 5 7 7 5 5 7 7	5 6 7 8 5 6 7 8	5 5 7 7 5 5 7 7
6 5 8 7 6 5 8 7	6 6 8 8 6 6 8 8	6 5 8 7 6 5 8 7	6 6 8 8 6 6 8 8
2.3 x 4.67	2.3 x 4.68	2.3 x 4.69	2.3 x 4.70
1 1 3 3 1 1 3 3	1 1 3 3 1 1 3 3	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 2 3 3 2 2 3 3	2 2 3 3 2 2 3 3	1 1 2 2 1 1 2 2	2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4	3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4	4 4 3 3 4 4 3 3	1 2 3 4 1 2 3 4	3 3 3 4 3 3 3 4
5 5 7 7 5 5 7 7	5 5 7 7 5 5 7 7	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5
6 6 7 7 6 6 7 7	6 6 7 7 6 6 7 7	5 5 6 6 5 5 6 6	6 6 6 6 6 6 6 6
7 7 7 7 7 7 7 7	7 7 7 7 7 7 7 7	5 6 7 8 5 6 7 8	7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8	8 8 7 7 8 8 7 7	5 6 7 8 5 6 7 8	7 7 7 8 7 7 7 8
2.3 x 4.71	2.3 x 4.72	2.3 x 4.73	2.3 x 4.74
1 1 1 1 1 1 1 1	1 1 3 4 1 1 3 4	1 1 3 3 1 1 3 3	1 1 3 4 1 1 3 4
2 2 2 2 2 2 2 2	2 2 3 4 2 2 3 4	2 2 3 3 2 2 3 3	2 2 3 4 2 2 3 4
1 1 1 1 1 1 1 1	3 3 3 4 3 3 3 4	3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4	3 3 3 4 3 3 3 4	3 3 3 4 3 3 3 4	3 3 3 3 3 3 3 3
5 5 5 5 5 5 5 5	5 5 7 8 5 5 7 8	5 5 7 7 5 5 7 7	5 5 7 8 5 5 7 8
6 6 6 6 6 6 6 6	6 6 7 8 6 6 7 8	6 6 7 7 6 6 7 7	6 6 7 8 6 6 7 8
5 5 5 5 5 5 5 5	7 7 7 8 7 7 7 8	7 7 7 7 7 7 7 7	7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8	7 7 7 8 7 7 7 8	7 7 7 8 7 7 7 8	7 7 7 7 7 7 7 7
2.3 x 4.75	2.3 x 4.76	2.3 x 4.77	2.3 x 4.78
1 1 3 3 1 1 3 3	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 2 3 3 2 2 3 3	1 1 1 3 1 1 1 3	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3	1 1 1 3 1 1 1 3	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1	2 2 2 2 2 2 2 2
5 5 7 7 5 5 7 7	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5
6 6 7 7 6 6 7 7	5 5 5 7 5 5 5 7	6 6 6 6 6 6 6 6	6 6 6 6 6 6 6 6
7 7 7 7 7 7 7 7	5 5 5 7 5 5 5 7	5 5 5 5 5 5 5 5	5 5 5 5 5 5 5 5
7 7 7 7 7 7 7 7	5 6 7 8 5 6 7 8	5 5 5 5 5 5 5 5	6 6 6 6 6 6 6 6

TABLE II
DIRECT PRODUCTS: 2 x 3 (continued)

2.3 x 4.79	2.3 x 4.80	2.3 x 4.81	2.3 x 4.82
1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 4 1 1 1 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 5 5 5 5 5 5 5 5 5 8 5 5 5 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 2 2 2 4 2 2 2 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 5 5 5 5 5 5 6 6 6 8 6 6 6 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 3 1 1 1 3 2 2 2 4 2 2 2 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 5 7 5 5 5 7 6 6 6 8 6 6 6 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 2 3 1 1 2 3 1 1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 6 7 5 5 6 7 5 5 5 5 5 5 5 5 5
2.3 x 4.83	2.3 x 4.84	2.3 x 4.85	2.3 x 4.86
1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 3 1 1 1 3 1 2 1 4 1 2 1 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 5 7 5 5 5 7 5 6 5 8 5 6 5 8	1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 3 1 4 1 3 1 4 1 1 1 4 1 1 1 4 5 5 5 8 5 5 5 8 5 6 5 8 5 6 5 8 5 7 5 8 5 7 5 8 5 5 5 8 5 5 5 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 3 1 1 1 3 1 1 3 4 1 1 3 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 5 7 5 5 5 7 5 5 7 8 5 5 7 8	1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 1 1 1 4 1 1 1 4 4 4 4 4 4 4 4 4 5 5 5 8 5 5 5 8 6 6 6 8 6 6 6 8 5 5 5 8 5 5 5 8 8 8 8 8 8 8 8 8
2.3 x 4.87	2.3 x 4.88	2.3 x 4.89	2.3 x 4.90
1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 5 5 5 5 5 5 5 6 7 8 5 6 7 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 3 1 1 1 3 1 2 3 4 1 2 3 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 5 7 5 5 5 7 5 6 7 8 5 6 7 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 3 1 1 1 3 1 1 1 4 1 1 1 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 5 7 5 5 5 7 5 5 5 8 5 5 5 8	1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 1 1 3 4 1 1 3 4 4 4 4 4 4 4 4 4 5 5 5 8 5 5 5 8 6 6 6 8 6 6 6 8 5 5 7 8 5 5 7 8 8 8 8 8 8 8 8 8
2.3 x 4.91	2.3 x 4.92	2.3 x 4.93	2.3 x 4.94
1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 1 1 1 3 1 1 1 1 4 1 1 1 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 7 5 5 5 7 5 5 5 5 8 5 5 5 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 1 1 1 3 1 2 2 2 4 2 2 2 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 7 5 5 5 7 5 6 6 6 8 6 6 6 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 2 3 1 1 2 3 1 1 1 1 4 1 1 1 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 6 7 5 5 6 7 5 5 5 5 8 5 5 5 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 3 1 1 3 3 1 1 3 4 1 1 3 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 7 7 5 5 7 7 5 5 7 8 5 5 7 8

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.3 x 4.95	2.3 x 4.96	2.3 x 4.97	2.3 x 4.98
1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 3 1 1 3 3 1 2 3 4 1 2 3 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 5 5 7 7 5 5 7 7 5 6 7 8 5 6 7 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 1 2 3 4 1 2 3 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 7 7 7 7 7 7 7 7 5 6 7 8 5 6 7 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 2 3 3 1 2 3 3 1 2 3 4 1 2 3 4 4 4 4 4 4 4 4 4 6 6 6 6 6 6 6 6 5 6 7 7 5 6 7 7 5 6 7 8 5 6 7 8	1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 3 4 4 4 3 4 5 5 7 8 5 5 7 8 6 6 7 8 6 6 7 8 7 7 7 8 7 7 7 8 8 8 7 8 8 8 7 8
2.3 x 4.99	2.3 x 4.100	2.3 x 4.101	2.3 x 4.102
1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 1 2 3 4 1 2 3 4 4 4 4 4 4 4 4 4 5 5 5 8 5 5 5 8 6 6 6 8 6 6 6 8 5 6 7 8 5 6 7 8 8 8 8 8 8 8 8 8	1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 5 5 7 8 5 5 7 8 6 6 7 8 6 6 7 8 7 7 7 7 7 7 7 7 8 8 8 8 8 8 8 8	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 7 7 7 7 7 7 7 7 8 8 8 8 8 8 8 8	1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4 5 5 5 8 5 5 5 8 6 6 6 8 6 6 6 8 7 7 7 8 7 7 7 8 8 8 8 8 8 8 8 8
2.3 x 4.103	2.3 x 4.104	2.3 x 4.105	2.3 x 4.106
1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 4 3 4 4 4 3 5 5 7 8 5 5 7 8 6 6 7 8 6 6 7 8 7 7 7 8 7 7 7 8 8 8 8 7 8 8 8 7	1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4 5 5 7 8 5 5 7 8 6 6 7 8 6 6 7 8 7 7 7 8 7 7 7 8 8 8 8 8 8 8 8 8	1 1 3 4 1 1 3 4 1 1 3 4 1 1 3 4 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 5 5 7 8 5 5 7 8 5 5 7 8 5 5 7 8 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	1 2 3 4 1 2 3 4 2 1 3 4 2 1 3 4 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 5 6 7 8 5 6 7 8 6 5 7 8 6 5 7 8 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7
2.3 x 4.107	2.3 x 4.108	2.3 x 4.109	2.3 x 4.110
1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 3 1 4 1 3 1 4 4 4 4 1 4 4 4 1 5 5 5 8 5 5 5 8 5 6 5 8 5 6 5 8 5 7 5 8 5 7 5 8 8 8 8 5 8 8 8 5	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 1 5 5 5 5 5 5 5 5 5 6 5 5 5 6 5 5 5 7 5 5 5 7 5 5 5 7 5 5 5 7 5 5	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 5 6 5 5 5 6 5 5 5 7 5 5 5 7 5 5 5 5 5 5 5 5 5 5	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 6 7 8 5 6 7 8

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.3 x 4.111	2.3 x 4.112	2.3 x 4.113	2.3 x 4.114
1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 5 6 5 8 5 6 5 8 5 7 5 5 5 7 5 5 5 5 5 5 5 5 5 5	1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 5 5 5 5 5 5 5 5 5 5 5 6 5 5 5 6 5 5 5 5 5 5 5 5 5 6 7 8 5 6 7 8	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 3 3 4 1 3 3 4 5 5 5 5 5 5 5 5 5 6 5 5 5 6 5 5 5 7 5 5 5 7 5 5 5 7 7 8 5 7 7 8	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 3 4 1 1 3 4 5 5 5 5 5 5 5 5 5 6 5 5 5 6 5 5 5 7 5 5 5 7 5 5 5 5 7 8 5 5 7 8
2.3 x 4.115	2.3 x 4.116	2.3 x 4.117	2.3 x 4.118
1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 3 1 3 1 3 1 3 1 2 1 4 1 2 1 4 5 5 5 5 5 5 5 5 5 6 5 6 5 6 5 6 5 7 5 7 5 7 5 7 5 6 5 8 5 6 5 8	1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 3 1 4 1 3 1 4 4 4 4 4 4 4 4 4 5 5 5 8 5 5 5 8 5 6 5 8 5 6 5 8 5 7 5 8 5 7 5 8 8 8 8 8 8 8 8 8	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 1 4 1 1 1 4 5 5 5 5 5 5 5 5 5 6 5 5 5 6 5 5 5 7 5 5 5 7 5 5 5 5 5 8 5 5 5 8	1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 3 1 3 1 3 1 3 1 2 3 4 1 2 3 4 5 5 5 5 5 5 5 5 5 6 5 6 5 6 5 6 5 7 5 7 5 7 5 7 5 6 7 8 5 6 7 8
2.3 x 4.119	2.3 x 4.120	2.3 x 4.121	2.4 x 4.1
1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 3 3 1 1 3 3 1 1 3 4 1 1 3 4 5 5 5 5 5 5 5 5 5 5 5 6 5 5 5 6 5 5 7 7 5 5 7 7 5 5 7 8 5 5 7 8	1 2 2 1 1 2 2 1 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 1 2 3 4 1 2 3 4 5 6 6 5 5 6 6 5 6 5 5 6 6 5 5 6 6 5 5 6 6 5 5 6 5 6 7 8 5 6 7 8	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 6 5 5 5 6 5 5 5 5 5 5 5 5	1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 3 1 2 3 3 1 2 3 3 1 2 3 3 1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 3 1 2 3 3 1 2 3 3 1 2 3 3
2.4 x 4.2	2.4 x 4.3	2.4 x 4.4	2.4 x 4.5
1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2	1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 1 1 1 1 1 1 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 1 1 1 1 1 1 1 1 4 1 1 1 4 1 1	1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4

TABLE II
DIRECT PRODUCTS: 2×4 (continued)

2.4×4.6	2.4×4.7	2.4×4.8	2.4×4.9
1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 3	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 2 2 1 1 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 2 2 1 1 2 2	1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 3 1 2 1 3 1 2 1 1 1 1 1 1 1 1 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 3 1 2 1 3 1 2	1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2
2.4×4.10	2.4×4.11	2.4×4.12	2.4×4.13
1 2 3 4 1 2 3 4 2 1 4 3 2 1 4 3 3 4 2 1 3 4 2 1 4 3 1 2 4 3 1 2 1 2 3 4 1 2 3 4 2 1 4 3 2 1 4 3 3 4 2 1 3 4 2 1 4 3 1 2 4 3 1 2	1 2 3 4 1 2 3 4 2 1 4 3 2 1 4 3 3 4 1 2 3 4 1 2 4 3 2 1 4 3 2 1 1 2 3 4 1 2 3 4 2 1 4 3 2 1 4 3 3 4 1 2 3 4 1 2 4 3 2 1 4 3 2 1	1 2 3 4 1 2 3 4 2 3 1 4 2 3 1 4 3 1 2 4 3 1 2 4 4 4 4 4 4 4 4 4 1 2 3 4 1 2 3 4 2 3 1 4 2 3 1 4 3 1 2 4 3 1 2 4 4 4 4 4 4 4 4 4	1 2 3 1 1 2 3 1 2 3 1 2 2 3 1 2 3 1 2 3 3 1 2 3 1 2 3 4 1 2 3 4 1 2 3 1 1 2 3 1 2 3 1 2 2 3 1 2 3 1 2 3 3 1 2 3 1 2 3 4 1 2 3 4
2.4×4.14	2.4×4.15	2.4×4.16	2.4×4.17
1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 3 4 1 2 3 4 1 2 4 3 1 2 4 3 1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 3 4 1 2 3 4 1 2 4 3 1 2 4 3	1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1 1 2 3 4 1 2 3 4 2 1 4 3 2 1 4 3 1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1 1 2 3 4 1 2 3 4 2 1 4 3 2 1 4 3	1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 3 1 1 2 3 1 1 2 1 4 1 2 1 4 1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 3 1 1 2 3 1 1 2 1 4 1 2 1 4	1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 3 4 1 2 3 4 1 2 4 4 1 2 4 4 1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 3 4 1 2 3 4 1 2 4 4 1 2 4 4
2.4×4.18	2.4×4.19	2.4×4.20	2.4×4.21
1 2 3 1 1 2 3 1 2 1 3 2 2 1 3 2 3 3 3 3 3 3 3 3 1 2 3 4 1 2 3 4 1 2 3 1 1 2 3 1 2 1 3 2 2 1 3 2 3 3 3 3 3 3 3 3 1 2 3 4 1 2 3 4	1 2 3 3 1 2 3 3 2 1 3 3 2 1 3 3 3 3 3 3 3 3 3 3 3 3 3 4 3 3 3 4 1 2 3 3 1 2 3 3 2 1 3 3 2 1 3 3 3 3 3 3 3 3 3 3 3 3 3 4 3 3 3 4	1 2 3 4 1 2 3 4 2 1 3 4 2 1 3 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4 1 2 3 4 1 2 3 4 2 1 3 4 2 1 3 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4	1 1 1 4 1 1 1 4 1 2 3 4 1 2 3 4 1 3 1 4 1 3 1 4 4 4 4 1 4 4 4 1 1 1 1 4 1 1 1 4 1 2 3 4 1 2 3 4 1 3 1 4 1 3 1 4 4 4 4 1 4 4 4 1

TABLE II

DIRECT PRODUCTS: 2×4 (continued)

2.4×4.22	2.4×4.23	2.4×4.24	2.4×4.25
1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 1 1 4 1 1 1 4 4 4 4 1 4 4 4 1 1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 1 1 4 1 1 1 4 4 4 4 1 4 4 4 1	1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 1 3 4 1 1 3 4 4 4 4 4 4 4 4 4 1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 1 3 4 1 1 3 4 4 4 4 4 4 4 4 4	1 1 3 3 1 1 3 3 1 2 3 4 1 2 3 4 3 3 1 1 3 3 1 1 3 4 1 1 3 4 1 1 1 1 3 3 1 1 3 3 1 2 3 4 1 2 3 4 3 3 1 1 3 3 1 1 3 4 1 1 3 4 1 1	1 2 1 4 1 2 1 4 2 2 2 2 2 2 2 2 1 2 1 4 1 2 1 4 4 2 4 1 4 2 4 1 1 2 1 4 1 2 1 4 2 2 2 2 2 2 2 2 1 2 1 4 1 2 1 4 4 2 4 1 4 2 4 1
2.4×4.26	2.4×4.27	2.4×4.28	2.4×4.29
1 2 4 4 1 2 4 4 2 2 2 2 2 2 2 2 4 2 1 1 4 2 1 1 4 2 1 1 4 2 1 1 1 2 4 4 1 2 4 4 2 2 2 2 2 2 2 2 4 2 1 1 4 2 1 1 4 2 1 1 4 2 1 1	1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 1 1 1 1 1 1 1 1 4 1 2 1 4 1 2 1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 1 1 1 1 1 1 1 1 4 1 2 1 4 1 2	1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 3 1 3 1 3 1 3 1 4 3 2 1 4 3 2 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 3 1 3 1 3 1 3 1 4 3 2 1 4 3 2	1 1 3 1 1 1 3 1 1 2 3 2 1 2 3 2 3 3 1 3 3 3 1 3 1 2 3 2 1 2 3 2 1 1 3 1 1 1 3 1 1 2 3 2 1 2 3 2 3 3 1 3 3 3 1 3 1 2 3 2 1 2 3 2
2.4×4.30	2.4×4.31	2.4×4.32	2.4×4.33
1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 2 1 2 2 2 1 2 2 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1	1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1 1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1 1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1 1 2 1 2 1 2 1 2 2 1 2 1 2 1 2 1	1 2 2 2 1 2 2 2 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 1 2 2 2 1 2 2 2 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1	1 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 3 3 1 3 3 3 1 3 1 1 3 2 1 1 3 2 1 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 3 3 1 3 3 3 1 3 1 1 3 2 1 1 3 2
2.4×4.34	2.4×4.35	2.4×4.36	2.4×4.37
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3 3 3 1 1 3 3 1 1 3 3 1 2 3 3 1 2 1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3 3 3 1 1 3 3 1 1 3 3 1 2 3 3 1 2	1 2 3 3 1 2 3 3 2 3 1 1 2 3 1 1 3 1 2 2 3 1 2 2 3 1 2 2 3 1 2 2 1 2 3 3 1 2 3 3 2 3 1 1 2 3 1 1 3 1 2 2 3 1 2 2 3 1 2 2 3 1 2 2	1 1 1 1 1 1 1 1 1 2 3 3 1 2 3 3 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 2 3 3 1 2 3 3 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 1	1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 3 1 1 1 3 1 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 3 1 1 1 3 1 1 1 4 1 1 1 4 1 1

TABLE II
DIRECT PRODUCTS: 2×4 (continued)

2.4×4.38	2.4×4.39	2.4×4.40	2.4×4.41
1 1 1 1 1 1 1 1 1 2 3 2 1 2 3 2 1 3 1 3 1 3 1 3 1 2 3 2 1 2 3 2 1 1 1 1 1 1 1 1 1 2 3 2 1 2 3 2 1 3 1 3 1 3 1 3 1 2 3 2 1 2 3 2	1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 3 1 1 1 3 1 1 1 4 1 3 1 4 1 3 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 3 1 1 1 3 1 1 1 4 1 3 1 4 1 3	1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2	1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 3 1 2 2 3 1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 3 1 2 2 3
2.4×4.42	2.4×4.43	2.4×4.44	2.4×4.45
1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 3 1 2 3 3 1 2 3 4 1 2 3 4 1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 3 1 2 3 3 1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1 1 2 2 1 1 2 2 1 1 2 3 1 1 2 3 1 1 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 1 2 2 1 1 2 2 1 1 2 3 1 1 2 3 1 1 1 1 4 1 1 1 4	1 1 1 1 1 1 1 1 1 2 2 1 1 2 2 1 1 2 3 4 1 2 3 4 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 1 1 2 2 1 1 2 2 1 1 2 3 4 1 2 3 4 1 1 4 1 1 1 4 1	1 1 1 1 1 1 1 1 1 2 2 4 1 2 2 4 1 2 3 4 1 2 3 4 1 4 4 1 1 4 4 1 1 1 1 1 1 1 1 1 1 2 2 4 1 2 2 4 1 2 3 4 1 2 3 4 1 4 4 1 1 4 4 1
2.4×4.46	2.4×4.47	2.4×4.48	2.4×4.49
1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 2 1 2 3 2 1 2 2 2 1 2 2 2 1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 2 1 2 3 2 1 2 2 2 1 2 2 2	1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 4 1 2 3 4 1 2 4 2 1 2 4 2 1 1 1 1 1 1 1 1 1 2 2 2 1 2 2 2 1 2 3 4 1 2 3 4 1 2 4 2 1 2 4 2	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 3 1 1 1 3 1 1 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 3 1 1 1 3 1 1 1 1 4 1 1 1 4	1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 1 3 3 1 1 3 3 1 2 3 4 1 2 3 4 1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 1 3 3 1 1 3 3 1 2 3 4 1 2 3 4
2.4×4.50	2.4×4.51	2.4×4.52	2.4×4.53
1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 3 4 1 1 3 4 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 3 4 1 1 3 4 1 1 4 1 1 1 4 1	1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 1 3 1 1 1 3 1 1 2 1 2 1 2 1 2 1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 1 3 1 1 1 3 1 1 2 1 2 1 2 1 2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 2 1 1 1 2 1

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.4 x 4.54	2.4 x 4.55	2.4 x 4.56	2.4 x 4.57
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 2 3 4 1 2 3 4
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 2 4 1 1 2 4	1 3 2 4 1 3 2 4
1 1 1 2 1 1 1 2	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 2 3 4 1 2 3 4
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 2 4 1 1 2 4	1 3 2 4 1 3 2 4
1 1 1 2 1 1 1 2	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 4 1 1 1 4
2.4 x 4.58	2.4 x 4.59	2.4 x 4.60	2.4 x 4.61
1 1 3 4 1 1 3 4	1 1 1 1 1 1 1 1	1 2 3 4 1 2 3 4	1 1 3 4 1 1 3 4
1 1 3 4 1 1 3 4	2 2 2 2 2 2 2 2	2 1 3 4 2 1 3 4	1 1 3 4 1 1 3 4
3 3 3 4 3 3 3 4	1 1 1 2 1 1 1 2	3 3 3 4 3 3 3 4	3 3 3 4 3 3 3 4
3 3 3 4 3 3 3 4	4 4 4 4 4 4 4 4	4 4 3 4 4 4 3 4	4 4 3 4 4 4 3 4
1 1 3 4 1 1 3 4	1 1 1 1 1 1 1 1	1 2 3 4 1 2 3 4	1 1 3 4 1 1 3 4
1 1 3 4 1 1 3 4	2 2 2 2 2 2 2 2	2 1 3 4 2 1 3 4	1 1 3 4 1 1 3 4
3 3 3 4 3 3 3 4	1 1 1 2 1 1 1 2	3 3 3 4 3 3 3 4	3 3 3 4 3 3 3 4
3 3 3 4 3 3 3 4	4 4 4 4 4 4 4 4	4 4 3 4 4 4 3 4	4 4 3 4 4 4 3 4
2.4 x 4.62	2.4 x 4.63	2.4 x 4.64	2.4 x 4.65
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3
3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4	1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4
1 2 2 4 1 2 2 4	2 1 4 3 2 1 4 3	2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3
3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4	1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4
1 2 2 4 1 2 2 4	2 1 4 3 2 1 4 3	2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3
2.4 x 4.66	2.4 x 4.67	2.4 x 4.68	2.4 x 4.69
1 1 1 1 1 1 1 1	1 1 3 3 1 1 3 3	1 1 3 3 1 1 3 3	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 3 3 2 2 3 3	2 2 3 3 2 2 3 3	1 1 2 2 1 1 2 2
1 1 3 3 1 1 3 3	3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4
2 2 4 4 2 2 4 4	4 4 4 4 4 4 4 4	4 4 3 3 4 4 3 3	1 2 3 4 1 2 3 4
1 1 1 1 1 1 1 1	1 1 3 3 1 1 3 3	1 1 3 3 1 1 3 3	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 3 3 2 2 3 3	2 2 3 3 2 2 3 3	1 1 2 2 1 1 2 2
1 1 3 3 1 1 3 3	3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4
2 2 4 4 2 2 4 4	4 4 4 4 4 4 4 4	4 4 3 3 4 4 3 3	1 2 3 4 1 2 3 4

TABLE II
DIRECT PRODUCTS: 2 x 4 (continued)

2.4 x 4.70	2.4 x 4.71	2.4 x 4.72	2.4 x 4.73
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 3 4 1 1 3 4	1 1 3 3 1 1 3 3
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 3 4 2 2 3 4	2 2 3 3 2 2 3 3
3 3 3 3 3 3 3 3	1 1 1 1 1 1 1 1	3 3 3 4 3 3 3 4	3 3 3 3 3 3 3 3
3 3 3 4 3 3 3 4	4 4 4 4 4 4 4 4	3 3 3 4 3 3 3 4	3 3 3 4 3 3 3 4
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 3 4 1 1 3 4	1 1 3 3 1 1 3 3
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 3 4 2 2 3 4	2 2 3 3 2 2 3 3
3 3 3 3 3 3 3 3	1 1 1 1 1 1 1 1	3 3 3 4 3 3 3 4	3 3 3 3 3 3 3 3
3 3 3 4 3 3 3 4	4 4 4 4 4 4 4 4	3 3 3 4 3 3 3 4	3 3 3 4 3 3 3 4
2.4 x 4.74	2.4 x 4.75	2.4 x 4.76	2.4 x 4.77
1 1 3 4 1 1 3 4	1 1 3 3 1 1 3 3	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 2 3 4 2 2 3 4	2 2 3 3 2 2 3 3	1 1 1 3 1 1 1 3	2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3	1 1 1 3 1 1 1 3	1 1 1 1 1 1 1 1
3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1
1 1 3 4 1 1 3 4	1 1 3 3 1 1 3 3	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 2 3 4 2 2 3 4	2 2 3 3 2 2 3 3	1 1 1 3 1 1 1 3	2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3	1 1 1 3 1 1 1 3	1 1 1 1 1 1 1 1
3 3 3 3 3 3 3 3	3 3 3 3 3 3 3 3	1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1
2.4 x 4.78	2.4 x 4.79	2.4 x 4.80	2.4 x 4.81
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 3 1 1 1 3
2 2 2 2 2 2 2 2	1 1 1 4 1 1 1 4	2 2 2 4 2 2 2 4	2 2 2 4 2 2 2 4
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 3 1 1 1 3
2 2 2 2 2 2 2 2	1 1 1 4 1 1 1 4	2 2 2 4 2 2 2 4	2 2 2 4 2 2 2 4
2.4 x 4.82	2.4 x 4.83	2.4 x 4.84	2.4 x 4.85
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	1 2 1 4 1 2 1 4	2 2 2 2 2 2 2 2
1 2 3 1 1 2 3 1	1 1 1 3 1 1 1 3	1 3 1 4 1 3 1 4	1 1 1 3 1 1 1 3
1 1 1 1 1 1 1 1	1 2 1 4 1 2 1 4	1 1 1 4 1 1 1 4	1 1 3 4 1 1 3 4
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 1 1 4 1 1 1 4	1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2	1 2 1 4 1 2 1 4	2 2 2 2 2 2 2 2
1 2 3 1 1 2 3 1	1 1 1 3 1 1 1 3	1 3 1 4 1 3 1 4	1 1 1 3 1 1 1 3
1 1 1 1 1 1 1 1	1 2 1 4 1 2 1 4	1 1 1 4 1 1 1 4	1 1 3 4 1 1 3 4

TABLE II

DIRECT PRODUCTS: 2 x 4 (continued)

2.4 x 4.86	2.4 x 4.87	2.4 x 4.88	2.4 x 4.89
1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 1 1 1 4 1 1 1 4 4 4 4 4 4 4 4 4 1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 1 1 1 4 1 1 1 4 4 4 4 4 4 4 4 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 3 1 1 1 3 1 2 3 4 1 2 3 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 3 1 1 1 3 1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 3 1 1 1 3 1 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 1 3 1 1 1 3 1 1 1 4 1 1 1 4
2.4 x 4.90	2.4 x 4.91	2.4 x 4.92	2.4 x 4.93
1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 1 1 3 4 1 1 3 4 4 4 4 4 4 4 4 4 1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 1 1 3 4 1 1 3 4 4 4 4 4 4 4 4 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 1 1 1 3 1 1 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 1 1 1 3 1 1 1 1 4 1 1 1 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 1 1 1 3 1 2 2 2 4 2 2 2 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 1 1 1 3 1 2 2 2 4 2 2 2 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 2 3 1 1 2 3 1 1 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 2 3 1 1 2 3 1 1 1 1 4 1 1 1 4
2.4 x 4.94	2.4 x 4.95	2.4 x 4.96	2.4 x 4.97
1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 3 1 1 3 3 1 1 3 4 1 1 3 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 3 1 1 3 3 1 1 3 4 1 1 3 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 3 1 1 3 3 1 2 3 4 1 2 3 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 1 3 3 1 1 3 3 1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 1 2 3 4 1 2 3 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 2 3 3 1 2 3 3 1 2 3 4 1 2 3 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 1 2 3 3 1 2 3 3 1 2 3 4 1 2 3 4
2.4 x 4.98	2.4 x 4.99	2.4 x 4.100	2.4 x 4.101
1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 3 4 4 4 3 4 1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 3 4 4 4 3 4	1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 1 2 3 4 1 2 3 4 4 4 4 4 4 4 4 4 1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 1 2 3 4 1 2 3 4 4 4 4 4 4 4 4 4	1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4	1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4

TABLE II
DIRECT PRODUCTS: 2×4 (continued)

2.4×4.102	2.4×4.103	2.4×4.104	2.4×4.105
1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4 1 1 1 4 1 1 1 4 2 2 2 4 2 2 2 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4	1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 4 3 4 4 4 3 1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 4 3 4 4 4 3	1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4 1 1 3 4 1 1 3 4 2 2 3 4 2 2 3 4 3 3 3 4 3 3 3 4 4 4 4 4 4 4 4 4	1 1 3 4 1 1 3 4 1 1 3 4 1 1 3 4 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 1 1 3 4 1 1 3 4 1 1 3 4 1 1 3 4 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
2.4×4.106	2.4×4.107	2.4×4.108	2.4×4.109
1 2 3 4 1 2 3 4 2 1 3 4 2 1 3 4 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 1 2 3 4 1 2 3 4 2 1 3 4 2 1 3 4 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 3 1 4 1 3 1 4 4 4 4 1 4 4 4 1 1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 3 1 4 1 3 1 4 4 4 4 1 4 4 4 1	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1 1	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1
2.4×4.110	2.4×4.111	2.4×4.112	2.4×4.113
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 4 1 2 1 4 1 3 1 1 1 3 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 1 1 1 1 1 1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 3 3 4 1 3 3 4 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 3 3 4 1 3 3 4
2.4×4.114	2.4×4.115	2.4×4.116	2.4×4.117
1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 3 4 1 1 3 4 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 3 4 1 1 3 4	1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 3 1 3 1 3 1 3 1 2 1 4 1 2 1 4 1 1 1 1 1 1 1 1 1 2 1 2 1 2 1 2 1 3 1 3 1 3 1 3 1 2 1 4 1 2 1 4	1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 3 1 4 1 3 1 4 4 4 4 4 4 4 4 4 1 1 1 4 1 1 1 4 1 2 1 4 1 2 1 4 1 3 1 4 1 3 1 4 4 4 4 4 4 4 4 4	1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 1 4 1 1 1 4 1 1 1 1 1 1 1 1 1 2 1 1 1 2 1 1 1 3 1 1 1 3 1 1 1 1 1 4 1 1 1 4

TABLE II

DIRECT PRODUCTS: 2×4 (continued)

2.4×4.118	2.4×4.119	2.4×4.120	2.4×4.121
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 2 2 1 1 2 2 1	1 1 1 1 1 1 1 1
1 2 1 2 1 2 1 2	1 1 1 2 1 1 1 2	2 1 1 2 2 1 1 2	1 1 1 1 1 1 1 1
1 3 1 3 1 3 1 3	1 1 3 3 1 1 3 3	2 1 1 2 2 1 1 2	1 1 1 2 1 1 1 2
1 2 3 4 1 2 3 4	1 1 3 4 1 1 3 4	1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1	1 2 2 1 1 2 2 1	1 1 1 1 1 1 1 1
1 2 1 2 1 2 1 2	1 1 1 2 1 1 1 2	2 1 1 2 2 1 1 2	1 1 1 1 1 1 1 1
1 3 1 3 1 3 1 3	1 1 3 3 1 1 3 3	2 1 1 2 2 1 1 2	1 1 1 2 1 1 1 2
1 2 3 4 1 2 3 4	1 1 3 4 1 1 3 4	1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1

TABLE III
DIRECT PRODUCTS: 3 x 3

3.1 x 3.1

```

1 2 2 4 5 5 4 5 5
2 1 1 5 4 4 5 4 4
2 1 1 5 4 4 5 4 4
4 5 5 1 2 2 1 2 2
5 4 4 2 1 1 2 1 1
5 4 4 2 1 1 2 1 1
4 5 5 1 2 2 1 2 2
5 4 4 2 1 1 2 1 1
5 4 4 2 1 1 2 1 1

```

3.1 x 3.2

```

1 1 1 4 4 4 4 4 4
1 1 1 4 4 4 4 4 4
1 1 2 4 4 5 4 4 5
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1
4 4 5 1 1 2 1 1 2
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1
4 4 5 1 1 2 1 1 2

```

3.1 x 3.3

```

1 2 3 4 5 6 4 5 6
2 3 1 5 6 4 5 6 4
3 1 2 6 4 5 6 4 5
4 5 6 1 2 3 1 2 3
5 6 4 2 3 1 2 3 1
6 4 5 3 1 2 3 1 2
4 5 6 1 2 3 1 2 3
5 6 4 2 3 1 2 3 1
6 4 5 3 1 2 3 1 2

```

3.1 x 3.4

```

1 1 3 4 4 6 4 4 6
1 1 3 4 4 6 4 4 6
3 3 1 6 6 4 6 6 4
4 4 6 1 1 3 1 1 3
4 4 6 1 1 3 1 1 3
6 6 4 3 3 1 3 3 1
4 4 6 1 1 3 1 1 3
4 4 6 1 1 3 1 1 3
6 6 4 3 3 1 3 3 1

```

3.1 x 3.5

```

1 1 3 4 4 6 4 4 6
1 2 3 4 5 6 4 5 6
3 3 1 6 6 4 6 6 4
4 4 6 1 1 3 1 1 3
4 5 6 1 2 3 1 2 3
6 6 4 3 3 1 3 3 1
4 4 6 1 1 3 1 1 3
4 5 6 1 2 3 1 2 3
6 6 4 1 1 3 1 1 3

```

3.1 x 3.6

```

1 1 1 4 4 4 4 4 4
1 1 1 4 4 4 4 4 4
1 1 3 4 4 6 4 4 6
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1
4 4 6 1 1 3 1 1 3
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1
4 4 6 1 1 3 1 1 3

```

3.1 x 3.7

```

1 1 1 4 4 4 4 4 4
1 2 1 4 5 4 4 5 4
1 3 1 4 6 4 4 6 4
4 4 4 1 1 1 1 1 1
4 5 4 1 2 1 1 2 1
4 6 4 1 3 1 1 3 1
4 4 4 1 1 1 1 1 1
4 5 4 1 2 1 1 2 1
4 6 4 1 3 1 1 3 1

```

3.1 x 3.8

```

1 1 1 4 4 4 4 4 4
1 1 2 4 4 5 4 4 5
1 2 3 4 5 6 4 5 6
4 4 4 1 1 1 1 1 1
4 4 5 1 1 2 1 1 2
4 5 6 1 2 3 1 2 3
4 4 4 1 1 1 1 1 1
4 4 5 1 1 2 1 1 2
4 5 6 1 2 3 1 2 3

```

3.1 x 3.9

```

1 1 1 4 4 4 4 4 4
2 2 2 5 5 5 5 5 5
1 1 1 4 4 4 4 4 4
4 4 4 1 1 1 1 1 1
5 5 5 2 2 2 2 2 2
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1
5 5 5 2 2 2 2 2 2
4 4 4 1 1 1 1 1 1

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.1 x 3.10

```

1 1 3 4 4 6 4 4 6
1 1 3 4 4 6 4 4 6
3 3 3 6 6 6 6 6 6
4 4 6 1 1 3 1 1 3
4 4 6 1 1 4 1 1 4
6 6 6 3 3 3 3 3 3
4 4 6 1 1 3 1 1 3
4 4 6 1 1 3 1 1 3
6 6 6 3 3 3 3 3 3

```

3.1 x 3.11

```

1 2 3 4 5 6 4 5 6
2 1 3 5 4 6 5 4 6
3 3 3 6 6 6 6 6 6
4 5 6 1 2 3 1 2 3
5 4 6 2 1 3 2 1 3
6 6 6 3 3 3 3 3 3
4 5 6 1 2 3 1 2 3
5 4 6 2 1 3 2 1 3
6 6 6 3 3 3 3 3 3

```

3.1 x 3.12

```

1 1 1 4 4 4 4 4 4
2 2 2 5 5 5 5 5 5
1 1 3 4 4 6 4 4 6
4 4 4 1 1 1 1 1 1
5 5 5 2 2 2 2 2 2
4 4 6 1 1 3 1 1 3
4 4 4 1 1 1 1 1 1
5 5 5 2 2 2 2 2 2
4 4 6 1 1 3 1 1 3

```

3.1 x 3.13

```

1 1 1 4 4 4 4 4 4
2 2 2 5 5 5 5 5 5
1 2 3 4 5 6 4 5 6
4 4 4 1 1 1 1 1 1
5 5 5 2 2 2 2 2 2
4 5 6 1 2 3 1 2 3
4 4 4 1 1 1 1 1 1
5 5 5 2 2 2 2 2 2
4 5 6 1 2 3 1 2 3

```

3.1 x 3.14

```

1 1 1 4 4 4 4 4 4
2 2 2 5 5 5 5 5 5
3 3 3 6 6 6 6 6 6
4 4 4 1 1 1 1 1 1
5 5 5 2 2 2 2 2 2
6 6 6 3 3 3 3 3 3
4 4 4 1 1 1 1 1 1
5 5 5 2 2 2 2 2 2
6 6 6 3 3 3 3 3 3

```

3.1 x 3.15

```

1 1 3 4 4 4 6 4 6
2 2 3 5 5 5 6 5 6
3 3 3 6 6 6 6 6 6
4 4 6 1 1 3 1 1 3
5 5 6 2 2 3 2 2 3
6 6 6 3 3 3 3 3 3
4 4 6 1 1 3 1 1 3
5 5 6 2 2 3 2 2 3
6 6 6 3 3 3 3 3 3

```

3.1 x 3.16

```

1 1 1 4 4 4 4 4 4
1 2 2 4 5 5 4 5 5
1 2 3 4 5 6 4 5 6
4 4 4 1 1 1 1 1 1
4 5 5 1 2 2 1 2 2
4 5 6 1 2 3 1 2 3
4 4 4 1 1 1 1 1 1
4 5 5 1 2 2 1 2 2
4 5 6 1 2 3 1 2 3

```

3.1 x 3.17

```

1 1 1 4 4 4 4 4 4
1 2 1 4 5 4 4 5 4
1 1 3 4 4 6 4 4 6
4 4 4 1 1 1 1 1 1
4 5 4 1 2 1 1 2 1
4 4 6 1 1 3 1 1 3
4 4 4 1 1 1 1 1 1
4 5 4 1 2 1 1 2 1
4 4 6 1 1 3 1 1 3

```

3.1 x 3.18

```

1 1 1 4 4 4 4 4 4
1 1 1 4 4 4 4 4 4
1 1 1 4 4 4 4 4 4
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1
4 4 4 1 1 1 1 1 1

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.2 x 3.2

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 2 1 1 2 1 1 2
2 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 2 1 1 2 1 1 2
1 1 1 1 1 1 4 4 4
1 1 1 1 1 1 4 4 4
1 1 2 1 1 2 4 4 5

```

3.2 x 3.3

```

1 2 3 1 2 3 1 2 3
2 3 1 2 3 1 2 3 1
3 1 2 3 1 2 3 1 2
1 2 3 1 2 3 1 2 3
2 3 1 2 3 1 2 3 1
3 1 2 3 1 2 3 1 2
1 2 3 1 2 3 4 5 6
2 3 1 2 3 1 5 6 4
3 1 2 3 1 2 6 4 5

```

3.2 x 3.4

```

1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 1 3 3 1 3 3 1
1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 1 3 3 1 3 3 1
1 1 3 1 1 3 4 4 6
1 1 3 1 1 3 4 4 6
3 3 1 3 3 1 6 6 4

```

3.2 x 3.5

```

1 1 3 1 1 3 1 1 3
1 2 3 1 2 3 1 2 3
3 3 1 3 3 1 3 3 1
1 1 3 1 1 3 1 1 3
1 2 3 1 2 3 1 2 3
3 3 1 3 3 1 3 3 1
1 1 3 1 1 3 4 4 6
1 2 3 1 2 3 4 5 6
3 3 1 3 3 1 6 6 4

```

3.2 x 3.6

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 4 4 4
1 1 1 1 1 1 4 4 4
1 1 3 1 1 3 4 4 6

```

3.2 x 3.7

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 3 1 1 3 1 1 3 1
1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 3 1 1 3 1 1 3 1
1 1 1 1 1 1 4 4 4
1 2 1 1 2 1 4 5 4
1 3 1 1 3 1 4 6 4

```

3.2 x 3.8

```

1 1 1 1 1 1 1 1 1
1 1 2 1 1 2 1 1 2
1 2 3 1 2 3 1 2 3
1 1 3 1 1 3 1 1 3
1 1 2 1 1 2 1 1 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 4 4 4
1 1 2 1 1 2 4 4 5
1 2 3 1 2 3 4 5 6

```

3.2 x 3.9

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 4 4 4
2 2 2 2 2 2 5 5 5
1 1 1 1 1 1 4 4 4

```

3.2 x 3.10

```

1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 4 4 6
1 1 3 1 1 3 4 4 6
3 3 3 3 3 3 6 6 6

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.2 x 3.11

```

1 2 3 1 2 3 1 2 3
2 1 3 2 1 3 2 1 3
3 3 3 3 3 3 3 3 3
1 2 3 1 2 3 1 2 3
2 1 3 2 1 3 2 1 3
3 3 3 3 3 3 3 3 3
1 2 3 1 2 3 4 5 6
2 1 3 2 1 3 5 4 6
3 3 3 3 3 3 6 6 6

```

3.2 x 3.12

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 4 4 4
2 2 2 2 2 2 5 5 5
1 1 3 1 1 3 4 4 6

```

3.2 x 3.13

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 4 4 4
2 2 2 2 2 2 5 5 5
1 2 3 1 2 3 4 5 6

```

3.2 x 3.14

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
1 1 1 1 1 1 4 4 4
2 2 2 2 2 2 5 5 5
3 3 3 3 3 3 6 6 6

```

3.2 x 3.15

```

1 1 3 1 1 3 1 1 3
2 2 3 2 2 3 2 2 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 1 1 3
2 2 3 2 2 3 2 2 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 4 4 6
2 2 3 2 2 3 5 5 6
3 3 3 3 3 3 6 6 6

```

3.2 x 3.16

```

1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 4 4 4
1 2 2 1 2 2 4 5 5
1 2 3 1 2 3 4 5 6

```

3.2 x 3.17

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 4 4 4
1 2 1 1 2 1 4 5 4
1 1 3 1 1 3 4 4 6

```

3.2 x 3.18

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 4 4 4
1 1 1 1 1 1 4 4 4
1 1 1 1 1 1 4 4 4

```

3.3 x 3.3

```

1 2 3 4 5 6 7 8 9
2 3 1 5 6 4 8 9 7
3 1 2 6 4 5 9 7 8
4 5 6 7 8 9 1 2 3
5 6 4 8 9 7 2 3 1
6 4 5 9 7 8 3 1 2
7 8 9 1 2 3 4 5 6
8 9 7 2 3 1 5 6 4
9 7 8 3 1 2 6 4 5

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.3 x 3.4

```

1 1 3 4 4 6 7 7 9
1 1 3 4 4 6 7 7 9
3 3 1 6 6 4 9 9 7
4 4 6 7 7 9 1 1 3
4 4 6 7 7 9 1 1 3
6 6 4 9 9 7 3 3 1
7 7 9 1 1 3 4 4 6
7 7 9 1 1 3 4 4 6
9 9 7 3 3 1 6 6 4

```

3.3 x 3.5

```

1 1 3 4 4 6 7 7 9
1 2 3 4 5 6 7 8 9
3 3 1 6 6 4 9 9 7
4 4 6 7 7 9 1 1 3
4 5 6 7 8 9 1 2 3
6 6 4 9 9 7 3 3 1
7 7 9 1 1 3 4 4 6
7 8 9 1 2 3 4 5 6
9 9 7 3 3 1 6 6 4

```

3.3 x 3.6

```

1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
1 1 3 4 4 6 7 7 9
4 4 4 7 7 7 1 1 1
4 4 4 7 7 7 1 1 1
4 4 6 7 7 9 1 1 3
7 7 7 1 1 1 4 4 4
7 7 7 1 1 1 4 4 4
7 7 9 1 1 3 4 4 6

```

3.3 x 3.7

```

1 1 1 4 4 4 7 7 7
1 2 1 4 5 4 7 8 7
1 3 1 4 6 4 7 9 7
4 4 4 7 7 7 1 1 1
4 5 4 7 8 7 1 2 1
4 6 4 7 9 7 1 3 1
7 7 7 1 1 1 4 4 4
7 8 7 1 2 1 4 5 4
7 9 7 1 3 1 4 6 4

```

3.3 x 3.8

```

1 1 1 4 4 4 7 7 7
1 1 2 4 4 5 7 7 8
1 2 3 4 5 6 7 8 9
4 4 4 7 7 7 1 1 1
4 4 5 7 7 8 1 1 2
4 5 6 7 8 9 1 2 3
7 7 7 1 1 1 4 4 4
7 7 8 1 1 2 4 4 5
7 8 9 1 2 3 4 5 6

```

3.3 x 3.9

```

1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 1 1 4 4 4 7 7 7
4 4 4 7 7 7 1 1 1
5 5 5 8 8 8 2 2 2
4 4 4 7 7 7 1 1 1
7 7 7 1 1 1 4 4 4
8 8 8 2 2 2 5 5 5
7 7 7 1 1 1 4 4 4

```

3.3 x 3.10

```

1 1 3 4 4 6 7 7 9
1 1 3 4 4 6 7 7 9
3 3 3 6 6 6 9 9 9
4 4 6 7 7 9 1 1 3
4 4 6 7 7 9 1 1 3
6 6 6 9 9 9 3 3 3
7 7 9 1 1 3 4 4 6
7 7 9 1 1 3 4 4 6
9 9 9 3 3 3 6 6 6

```

3.3 x 3.11

```

1 2 3 4 5 6 7 8 9
2 1 3 5 4 6 8 7 9
3 3 3 6 6 6 9 9 9
4 5 6 7 8 9 1 2 3
5 4 6 8 7 9 2 1 3
6 6 6 9 9 9 3 3 3
7 8 9 1 2 3 4 5 6
8 7 9 2 1 3 5 4 6
9 9 9 3 3 3 6 6 6

```

3.3 x 3.12

```

1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 1 3 4 4 6 7 7 9
4 4 4 7 7 7 1 1 1
5 5 5 8 8 8 2 2 2
4 4 6 7 7 9 1 1 3
7 7 7 1 1 1 4 4 4
8 8 8 2 2 2 5 5 5
7 7 9 1 1 3 4 4 6

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.3 x 3.13

```

1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 2 3 4 5 6 7 8 9
4 4 4 7 7 7 1 1 1
5 5 5 8 8 8 2 2 2
4 5 6 7 8 9 1 2 3
7 7 7 1 1 1 4 4 4
8 8 8 2 2 2 5 5 5
7 8 9 1 2 3 4 5 6

```

3.3 x 3.14

```

1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
3 3 3 6 6 6 9 9 9
4 4 4 7 7 7 1 1 1
5 5 5 8 8 8 2 2 2
6 6 6 9 9 9 3 3 3
7 7 7 1 1 1 4 4 4
8 8 8 2 2 2 5 5 5
9 9 9 3 3 3 6 6 6

```

3.3 x 3.15

```

1 1 3 4 4 6 7 7 9
2 2 3 5 5 6 8 8 9
3 3 3 6 6 6 9 9 9
4 4 6 7 7 9 1 1 3
5 5 6 8 8 9 2 2 3
6 6 6 9 9 9 3 3 3
7 7 9 1 1 3 4 4 6
8 8 9 2 2 3 5 5 6
9 9 9 3 3 3 6 6 6

```

3.3 x 3.16

```

1 1 1 4 4 4 7 7 7
1 2 2 4 5 5 7 8 8
1 2 3 4 5 6 7 8 9
4 4 4 7 7 7 1 1 1
4 5 5 7 8 8 1 2 2
4 5 6 7 8 9 1 2 3
7 7 7 1 1 1 4 4 4
7 8 8 1 2 2 4 5 5
7 8 9 1 2 3 4 5 6

```

3.3 x 3.17

```

1 1 1 4 4 4 7 7 7
1 2 1 4 5 4 7 8 7
1 1 3 4 4 6 7 7 9
4 4 4 7 7 7 1 1 1
4 5 4 7 8 7 1 2 1
4 4 6 7 7 9 1 1 3
7 7 7 1 1 1 4 4 4
7 8 7 1 2 1 4 5 4
7 7 9 1 1 3 4 4 6

```

3.3 x 3.18

```

1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
4 4 4 7 7 7 1 1 1
4 4 4 7 7 7 1 1 1
4 4 4 7 7 7 1 1 1
7 7 7 1 1 1 4 4 4
7 7 7 1 1 1 4 4 4
7 7 7 1 1 1 4 4 4

```

3.4 x 3.4

```

1 1 3 1 1 3 7 7 9
1 1 3 1 1 3 7 7 9
3 3 1 3 3 1 9 9 7
1 1 3 1 1 3 7 7 9
1 1 3 1 1 3 7 7 9
3 3 1 3 3 1 9 9 7
7 7 9 7 7 9 1 1 3
7 7 9 7 7 9 1 1 3
9 9 7 9 9 7 3 3 1

```

3.4 x 3.5

```

1 1 3 1 1 3 7 7 9
1 2 3 1 2 3 7 8 9
3 3 1 3 3 1 9 9 7
1 1 3 1 1 3 7 7 9
1 2 3 1 2 3 7 8 9
3 3 1 3 3 1 9 9 7
7 7 9 7 7 9 1 1 3
7 8 9 7 8 9 1 2 3
9 9 7 9 9 7 3 3 1

```

3.4 x 3.6

```

1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 3 1 1 3 7 7 9
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 3 1 1 3 7 7 9
7 7 7 7 7 7 1 1 1
7 7 7 7 7 7 1 1 1
7 7 9 7 7 9 1 1 3

```

TABLE III
DIRECT PRODUCTS: 3×3 (continued)

3.4 x 3.7

1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 9
1 3 1 1 3 1 7 9 7
1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 9
1 3 1 1 3 1 7 9 7
7 7 7 7 7 7 1 1 1
7 8 9 7 8 9 1 2 1
7 9 7 7 9 7 1 3 1

3.4 x 3.8

1 1 1 1 1 1 7 7 7
1 1 2 1 1 2 7 7 8
1 2 3 1 2 3 7 8 9
1 1 1 1 1 1 7 7 7
1 1 2 1 1 2 7 7 8
1 2 3 1 2 3 7 8 9
7 7 7 7 7 7 1 1 1
7 7 8 7 7 8 1 1 2
7 8 9 7 8 9 1 2 3

3.4 x 3.9

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 1 1 1 1 7 7 7
7 7 7 7 7 7 1 1 1
8 8 8 8 8 8 2 2 2
7 7 7 7 7 7 1 1 1

3.4 x 3.10

1 1 3 1 1 3 7 7 9
1 1 3 1 1 3 7 7 9
3 3 3 3 3 3 9 9 9
1 1 3 1 1 3 7 7 9
1 1 3 1 1 3 7 7 9
3 3 3 3 3 3 9 9 9
7 7 9 7 7 9 1 1 3
7 7 9 7 7 9 1 1 3
9 9 9 9 9 9 3 3 3

3.4 x 3.11

1 2 3 1 2 3 7 8 9
2 1 3 2 1 3 8 7 9
3 3 3 3 3 3 9 9 9
1 2 3 1 2 3 7 8 9
2 1 3 2 1 3 8 7 9
3 3 3 3 3 3 9 9 9
7 8 9 7 8 9 1 2 3
8 7 9 8 7 9 2 1 3
9 9 9 9 9 9 3 3 3

3.4 x 3.12

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 3 1 1 3 7 7 9
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 3 1 1 3 7 7 9
7 7 7 7 7 7 1 1 1
8 8 8 8 8 8 2 2 2
7 7 9 7 7 9 1 1 3

3.4 x 3.13

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 2 3 1 2 3 7 8 9
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 2 3 1 2 3 7 8 9
7 7 7 7 7 7 1 1 1
8 8 8 8 8 8 2 2 2
7 8 9 7 8 9 1 2 3

3.4 x 3.14

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
3 3 3 3 3 3 9 9 9
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
3 3 3 3 3 3 9 9 9
7 7 7 7 7 7 1 1 1
8 8 8 8 8 8 2 2 2
9 9 9 9 9 9 3 3 3

3.4 x 3.15

1 1 3 1 1 3 7 7 9
2 2 3 2 2 3 8 8 9
3 3 3 3 3 3 9 9 9
1 1 3 1 1 3 7 7 9
2 2 3 2 2 3 8 8 9
3 3 3 3 3 3 9 9 9
7 7 9 7 7 9 1 1 3
8 8 9 8 8 9 2 2 3
9 9 9 9 9 9 3 3 3

TABLE III

DIRECT PRODUCTS: 3×3 (continued) 3.4×3.16

```

1 1 1 1 1 1 7 7 7
1 2 2 1 2 2 7 8 8
1 2 3 1 2 3 7 8 9
1 1 1 1 1 1 7 7 7
1 2 2 1 2 2 7 8 9
1 2 3 1 2 3 7 8 9
7 7 7 7 7 7 1 1 1
7 8 8 7 8 8 1 2 2
7 8 9 7 8 9 1 2 3

```

 3.4×3.17

```

1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 7
1 1 3 1 1 3 7 7 9
1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 7
1 1 3 1 1 3 7 7 9
7 7 7 7 7 7 1 1 1
7 8 7 7 8 7 1 2 1
7 7 9 7 7 9 1 1 3

```

 3.4×3.18

```

1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
7 7 7 7 7 7 1 1 1
7 7 7 7 7 7 1 1 1
7 7 7 7 7 7 1 1 1

```

 3.5×3.5

```

1 1 3 1 1 3 7 7 9
1 2 3 1 2 3 7 8 9
3 3 1 3 3 1 9 9 7
1 1 3 4 4 6 7 7 9
1 2 3 4 5 6 7 8 9
3 3 1 6 6 4 9 9 7
7 7 9 7 7 9 1 1 3
7 8 9 7 8 9 1 2 3
9 9 7 9 9 7 3 3 1

```

 3.5×3.6

```

1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 3 1 1 3 7 7 9
1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
1 1 3 4 4 6 7 7 9
7 7 7 7 7 7 1 1 1
7 7 7 7 7 7 1 1 1
7 7 9 7 7 9 1 1 3

```

 3.5×3.7

```

1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 7
1 3 1 1 3 1 7 9 7
1 1 1 4 4 4 7 7 7
1 2 1 4 5 4 7 8 7
1 3 1 4 6 4 7 9 7
7 7 7 7 7 7 1 1 1
7 8 7 7 8 7 1 2 1
7 9 7 7 9 7 1 3 1

```

 3.5×3.8

```

1 1 1 1 1 1 7 7 7
1 1 2 1 1 2 7 7 8
1 2 3 1 2 3 7 8 9
1 1 1 4 4 4 7 7 7
1 1 2 4 4 6 7 7 8
1 2 3 4 5 6 7 8 9
7 7 7 7 7 7 1 1 1
7 7 8 7 7 8 1 1 2
7 8 9 7 8 9 1 2 3

```

 3.5×3.9

```

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 1 1 1 1 7 7 7
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 1 1 4 4 4 7 7 7
7 7 7 7 7 7 1 1 1
8 8 8 8 8 8 2 2 2
7 7 7 7 7 7 1 1 1

```

 3.5×3.10

```

1 1 3 1 1 3 7 7 9
1 1 3 1 1 3 7 7 9
3 3 3 3 3 3 9 9 9
1 1 3 4 4 6 7 7 9
1 1 3 4 4 6 7 7 9
3 3 3 6 6 6 9 9 9
7 7 9 7 7 9 1 1 3
7 7 9 7 7 9 1 1 3
9 9 9 9 9 9 3 3 3

```

TABLE III
DIRECT PRODUCTS: 3 x 3 (continued)

3.5 x 3.11

```

1 2 3 1 2 3 7 8 9
2 1 3 2 1 3 8 7 9
3 3 3 3 3 3 9 9 9
1 2 3 4 5 6 7 8 9
2 1 3 5 4 6 8 7 9
3 3 3 6 6 6 9 9 9
7 8 9 7 8 9 1 2 3
8 7 9 8 7 9 2 1 3
9 9 9 9 9 9 3 3 3

```

3.5 x 3.12

```

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 3 1 1 3 7 7 9
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 1 3 4 4 6 7 7 9
7 7 7 7 7 7 1 1 1
8 8 8 8 8 8 2 2 2
7 7 9 7 7 9 1 1 3

```

3.5 x 3.13

```

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 2 3 1 2 3 7 8 9
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 2 3 4 5 6 7 8 9
7 7 7 7 7 7 1 1 1
8 8 8 8 8 8 2 2 2
7 8 9 7 8 9 1 2 3

```

3.5 x 3.14

```

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
3 3 3 3 3 3 9 9 9
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
3 3 3 6 6 6 9 9 9
7 7 7 7 7 7 1 1 1
8 8 8 8 8 8 2 2 2
9 9 9 9 9 9 3 3 3

```

3.5 x 3.15

```

1 1 3 1 1 3 7 7 9
2 2 3 2 2 3 8 8 9
3 3 3 3 3 3 9 9 9
1 1 3 4 4 6 7 7 9
2 2 3 5 5 6 8 8 9
3 3 3 6 6 6 9 9 9
7 7 9 7 7 9 1 1 3
8 8 9 8 8 9 2 2 3
9 9 9 9 9 9 3 3 3

```

3.5 x 3.16

```

1 1 1 1 1 1 7 7 7
1 2 2 1 2 2 7 8 8
1 2 3 1 2 3 7 8 9
1 1 1 4 4 4 7 7 7
1 2 2 4 5 5 7 8 8
1 2 3 4 5 6 7 8 9
7 7 7 7 7 7 1 1 1
7 8 8 7 8 8 1 2 2
7 8 9 7 8 9 1 2 3

```

3.5 x 3.17

```

1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 7
1 1 3 1 1 3 7 7 9
1 1 1 4 4 4 7 7 7
1 2 1 4 5 4 7 8 7
1 1 3 4 4 6 7 7 9
7 7 7 7 7 7 1 1 1
7 8 7 7 8 7 1 2 1
7 7 9 7 7 9 1 1 3

```

3.5 x 3.18

```

1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
7 7 7 7 7 7 1 1 1
7 7 7 7 7 7 1 1 1
7 7 7 7 7 7 1 1 1

```

3.6 x 3.6

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 3 1 1 3 7 7 9

```

TABLE III
DIRECT PRODUCTS: 3 x 3 (continued)

3.6 x 3.7

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 3 1 1 3 1 1 3 1
1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 3 1 1 3 1 1 3 1
1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 7
1 3 1 1 3 1 7 9 7

```

3.6 x 3.8

```

1 1 1 1 1 1 1 1 1
1 1 2 1 1 2 1 1 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 1 1 1
1 1 2 1 1 2 1 1 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 7 7 7
1 1 2 1 1 2 7 7 8
1 2 3 1 2 3 7 8 9

```

3.6 x 3.9

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 1 1 1 1 7 7 7

```

3.6 x 3.10

```

1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 7 7 9
1 1 3 1 1 3 7 7 9
3 3 3 3 3 3 9 9 9

```

3.6 x 3.11

```

1 2 3 1 2 3 1 2 3
2 1 3 2 1 3 2 1 3
3 3 3 3 3 3 3 3 3
1 2 3 1 2 3 1 2 3
2 1 3 2 1 3 2 1 3
3 3 3 3 3 3 3 3 3
1 2 3 1 2 3 7 8 9
2 1 3 2 1 3 8 7 9
3 3 3 3 3 3 9 9 9

```

3.6 x 3.12

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 3 1 1 3 7 7 9

```

3.6 x 3.13

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 2 3 1 2 3 7 8 9

```

3.6 x 3.14

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
3 3 3 3 3 3 9 9 9

```

3.6 x 3.15

```

1 1 3 1 1 3 1 1 3
2 2 3 2 2 3 2 2 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 1 1 3
2 2 3 2 2 3 2 2 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 7 7 9
2 2 3 2 2 3 8 8 9
3 3 3 3 3 3 9 9 9

```

TABLE III
DIRECT PRODUCTS: 3 x 3 (continued)

3.6 x 3.16

```

1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 7 7 7
1 2 2 1 2 2 7 8 8
1 2 3 1 2 3 7 8 9

```

3.6 x 3.17

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 7
1 1 3 1 1 3 7 7 9

```

3.6 x 3.18

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7

```

3.7 x 3.7

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 3 1 1 3 1 1 3 1
1 1 1 4 4 4 1 1 1
1 2 1 4 5 4 1 2 1
1 3 1 4 6 4 1 3 1
1 1 1 7 7 7 1 1 1
1 2 1 7 8 7 1 2 1
1 3 1 7 9 7 1 3 1

```

3.7 x 3.8

```

1 1 1 1 1 1 1 1 1
1 1 2 1 1 2 1 1 2
1 2 3 1 2 3 1 2 3
1 1 1 4 4 4 1 1 1
1 1 2 4 4 5 1 1 2
1 2 3 4 5 6 1 2 3
1 1 1 7 7 7 1 1 1
1 1 2 7 7 8 1 1 2
1 2 3 7 8 9 1 2 3

```

3.7 x 3.9

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1 1
1 1 1 4 4 4 1 1 1
2 2 2 5 5 5 2 2 2
1 1 1 4 4 4 1 1 1
1 1 1 7 7 7 1 1 1
2 2 2 8 8 8 2 2 2
1 1 1 7 7 7 1 1 1

```

3.7 x 3.10

```

1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 3 3 3 3 3 3 3
1 1 3 4 4 6 1 1 3
1 1 3 4 4 6 1 1 3
3 3 3 6 6 6 3 3 3
1 1 3 7 7 9 1 1 3
1 1 3 7 7 9 1 1 3
3 3 3 9 9 9 3 3 3

```

3.7 x 3.11

```

1 2 3 1 2 3 1 2 3
2 1 3 2 1 3 2 1 3
3 3 3 3 3 3 3 3 3
1 2 3 4 5 6 1 2 3
2 1 3 5 4 6 2 1 3
3 3 3 6 6 6 3 3 3
1 2 3 7 8 9 1 2 3
2 1 3 8 7 9 2 1 3
3 3 3 9 9 9 3 3 3

```

3.7 x 3.12

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 3 1 1 3 1 1 3
1 1 1 4 4 4 1 1 1
2 2 2 5 5 5 2 2 2
1 1 3 4 4 6 1 1 3
1 1 1 7 7 7 1 1 1
2 2 2 8 8 8 2 2 2
1 1 3 7 7 9 1 1 3

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.7 x 3.13

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3
1 1 1 4 4 4 1 1 1
2 2 2 5 5 5 2 2 2
1 2 3 4 5 6 1 2 3
1 1 1 7 7 7 1 1 1
2 2 2 8 8 8 2 2 2
1 2 3 7 8 9 1 2 3

```

3.7 x 3.14

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
1 1 1 4 4 4 1 1 1
2 2 2 5 5 5 2 2 2
3 3 3 6 6 6 3 3 3
1 1 1 7 7 7 1 1 1
2 2 2 8 8 8 2 2 2
3 3 3 9 9 9 3 3 3

```

3.7 x 3.15

```

1 1 3 1 1 3 1 1 3
2 2 3 2 2 3 2 2 3
3 3 3 3 3 3 3 3 3
1 1 3 4 4 6 1 1 3
2 2 3 5 5 6 2 2 3
3 3 3 6 6 6 3 3 3
1 1 3 7 7 9 1 1 3
2 2 3 8 8 9 2 2 3
3 3 3 9 9 9 3 3 3

```

3.7 x 3.16

```

1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
1 1 1 4 4 4 1 1 1
1 2 2 4 5 5 1 2 2
1 2 3 4 5 6 1 2 3
1 1 1 7 7 7 1 1 1
1 2 2 7 8 8 1 2 2
1 2 3 7 8 9 1 2 3

```

3.7 x 3.17

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
1 1 1 4 4 4 1 1 1
1 2 1 4 5 4 1 2 1
1 1 3 4 4 6 1 1 3
1 1 1 7 7 7 1 1 1
1 2 1 7 8 7 1 2 1
1 1 3 7 7 9 1 1 3

```

3.7 x 3.18

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 4 4 4 1 1 1
1 1 1 4 4 4 1 1 1
1 1 1 4 4 4 1 1 1
1 1 1 7 7 7 1 1 1
1 1 1 7 7 7 1 1 1
1 1 1 7 7 7 1 1 1

```

3.8 x 3.8

```

1 1 1 1 1 1 1 1 1
1 1 2 1 1 2 1 1 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 4 4 4
1 1 2 1 1 2 4 4 5
1 2 3 1 2 3 4 5 6
1 1 1 4 4 4 7 7 7
1 1 2 4 4 5 7 7 8
1 2 3 4 5 6 7 8 9

```

3.8 x 3.9

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 4 4 4
2 2 2 2 2 2 5 5 5
1 1 1 1 1 1 4 4 4
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 1 1 4 4 4 7 7 7

```

3.8 x 3.10

```

1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 4 4 6
1 1 3 1 1 3 4 4 6
3 3 3 3 3 3 6 6 6
1 1 3 4 4 6 7 7 9
1 1 3 4 4 6 7 7 9
3 3 3 6 6 6 9 9 9

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.8 x 3.11

```

1 2 3 1 2 3 1 2 3
2 1 3 2 1 3 2 1 3
3 3 3 3 3 3 3 3 3
1 2 3 1 2 3 4 5 6
2 1 3 2 1 3 5 4 6
3 3 3 3 3 3 6 6 6
1 2 3 4 5 6 7 8 9
2 1 3 5 4 6 8 7 9
3 3 3 6 6 6 9 9 9

```

3.8 x 3.12

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 4 4 4
2 2 2 2 2 2 5 5 5
1 1 3 1 1 3 4 4 6
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 1 3 4 4 6 7 7 9

```

3.8 x 3.13

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 4 4 4
2 2 2 2 2 2 5 5 5
1 2 3 1 2 3 4 5 6
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 2 3 4 5 6 7 8 9

```

3.8 x 3.14

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
1 1 1 1 1 1 4 4 4
2 2 2 2 2 2 5 5 5
3 3 3 3 3 3 6 6 6
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
3 3 3 6 6 6 9 9 9

```

3.8 x 3.15

```

1 1 3 1 1 3 1 1 3
2 2 3 2 2 3 2 2 3
3 3 3 3 3 3 3 3 3
1 1 3 1 1 3 4 4 6
2 2 3 2 2 3 5 5 6
3 3 3 3 3 3 6 6 6
1 1 3 4 4 6 7 7 9
2 2 3 5 5 6 8 8 9
3 3 3 6 6 6 9 9 9

```

3.8 x 3.16

```

1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
1 1 1 1 1 1 4 4 4
1 2 2 1 2 2 4 5 5
1 2 3 1 2 3 4 5 6
1 1 1 4 4 4 7 7 7
1 2 2 4 5 5 7 8 8
1 2 3 4 5 6 7 8 9

```

3.8 x 3.17

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
1 1 1 1 1 1 4 4 4
1 2 1 1 2 1 4 5 4
1 1 3 1 1 3 4 4 6
1 1 1 4 4 4 7 7 7
1 2 1 4 5 4 7 8 7
1 1 3 4 4 6 7 7 9

```

3.8 x 3.18

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 4 4 4
1 1 1 1 1 1 4 4 4
1 1 1 1 1 1 4 4 4
1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7

```

3.9 x 3.9

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1 1
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 1 1 1 1 1 1 1

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.9 x 3.10

```

1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 3 3 3 3 3 3 3
4 4 6 4 4 6 4 4 6
4 4 6 4 4 6 4 4 6
6 6 6 6 6 6 6 6 6
1 1 3 1 1 3 1 1 3
1 1 3 1 1 3 1 1 3
3 3 3 3 3 3 3 3 3

```

3.9 x 3.11

```

1 2 3 1 2 3 1 2 3
2 1 3 2 1 3 2 1 3
3 3 3 3 3 3 3 3 3
4 5 6 4 5 6 4 5 6
5 4 6 5 4 6 5 4 6
6 6 6 6 6 6 6 6 6
1 2 3 1 2 3 1 2 3
2 1 3 2 1 3 2 1 3
3 3 3 3 3 3 3 3 3

```

3.9 x 3.12

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 3 1 1 3 1 1 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
4 4 6 4 4 6 4 4 6
1 1 1 1 1 1 1 1 1
  2 2 2 2 2 2 2 2 2
1 1 3 1 1 3 1 1 3

```

3.9 x 3.13

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
4 5 6 4 5 6 4 5 6
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3

```

3.9 x 3.14

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6 6
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3

```

3.9 x 3.15

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6 6
1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3

```

3.9 x 3.16

```

1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
4 4 4 4 4 4 4 4 4
4 5 5 4 5 5 4 5 5
4 5 6 4 5 6 4 5 6
1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3

```

3.9 x 3.17

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
4 4 4 4 4 4 4 4 4
4 5 4 4 5 4 4 5 4
4 4 6 4 4 6 4 4 6
1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3

```

3.9 x 3.18

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1

```

TABLE III
DIRECT PRODUCTS: 3 x 3 (continued)

3.10 x 3.10

```

1 1 3 1 1 3 7 7 9
1 1 3 1 1 3 7 7 9
3 3 3 3 3 3 9 9 9
1 1 3 1 1 3 7 7 9
1 1 3 1 1 3 7 7 9
3 3 3 3 3 3 9 9 9
7 7 9 7 7 9 7 7 9
7 7 9 7 7 9 7 7 9
9 9 9 9 9 9 9 9 9

```

3.10 x 3.11

```

1 2 3 1 2 3 7 8 9
2 1 3 2 1 3 8 7 9
3 3 3 3 3 3 9 9 9
1 2 3 1 2 3 7 8 9
2 1 3 2 1 3 8 7 9
3 3 3 3 3 3 9 9 9
7 8 9 7 8 9 7 8 9
8 7 9 8 7 9 8 7 9
9 9 9 9 9 9 9 9 9

```

3.10 x 3.12

```

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 3 1 1 3 7 7 9
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 3 1 1 3 7 7 9
7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8
7 7 9 7 7 9 7 7 9

```

3.10 x 3.13

```

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 2 3 1 2 3 7 8 9
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 2 3 1 2 3 7 8 9
7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8
7 8 9 7 8 9 7 8 9

```

3.10 x 3.14

```

1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
3 3 3 3 3 3 9 9 9
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
3 3 3 3 3 3 9 9 9
7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8
9 9 9 9 9 9 9 9 9

```

3.10 x 3.15

```

1 1 3 1 1 3 7 7 9
2 2 3 2 2 3 7 7 9
3 3 3 3 3 3 9 9 9
1 1 3 1 1 3 7 7 9
2 2 3 2 2 3 7 7 9
3 3 3 3 3 3 9 9 9
7 7 9 7 7 9 7 7 9
7 7 9 7 7 9 7 7 9
9 9 9 9 9 9 9 9 9

```

3.10 x 3.16

```

1 1 1 1 1 1 7 7 7
1 2 2 1 2 2 7 8 8
1 2 3 1 2 3 7 8 9
1 1 1 1 1 1 7 7 7
1 2 2 1 2 2 7 8 8
1 2 3 1 2 3 7 8 9
7 7 7 7 7 7 7 7 7
7 8 8 7 8 8 7 8 8
7 8 9 7 8 9 7 8 9

```

3.10 x 3.17

```

1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 9
1 1 3 1 1 3 7 7 9
1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 9
1 1 3 1 1 3 7 7 9
7 7 7 7 7 7 7 7 7
7 8 9 7 8 9 7 8 9
7 7 9 7 7 9 7 7 9

```

3.10 x 3.18

```

1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
7 7 7 7 7 7 7 7 7
7 7 7 7 7 7 7 7 7
7 7 7 7 7 7 7 7 7

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.11 x 3.11

```

1 2 3 4 5 6 7 8 9
2 1 3 5 4 6 8 7 9
3 3 3 6 6 6 9 9 9
4 5 6 1 2 3 7 8 9
5 4 6 2 1 3 8 7 9
6 6 6 3 3 3 9 9 9
7 8 9 7 8 9 7 8 9
8 7 9 8 7 9 8 7 9
9 9 9 9 9 9 9 9 9

```

3.11 x 3.12

```

1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 1 3 4 4 6 7 7 9
4 4 4 1 1 1 7 7 7
5 5 5 2 2 2 8 8 8
4 4 6 1 1 3 7 7 9
7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8
7 7 9 7 7 9 7 7 9

```

3.11 x 3.13

```

1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 2 3 4 5 6 7 8 9
4 4 4 1 1 1 7 7 7
5 5 5 2 2 2 8 8 8
4 5 6 1 2 3 7 8 9
7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8
7 8 9 7 8 9 7 8 9

```

3.11 x 3.14

```

1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
3 3 3 6 6 6 9 9 9
4 4 4 1 1 1 7 7 7
5 5 5 2 2 2 8 8 8
6 6 6 3 3 3 9 9 9
7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8
9 9 9 9 9 9 9 9 9

```

3.11 x 3.15

```

1 1 3 4 4 6 7 7 9
2 2 3 5 5 6 8 8 9
3 3 3 6 6 6 9 9 9
4 4 6 1 1 3 7 7 9
5 5 6 2 2 3 8 8 9
6 6 6 3 3 3 9 9 9
7 7 9 7 7 9 7 7 9
8 8 9 8 8 9 8 8 9
9 9 9 9 9 9 9 9 9

```

3.11 x 3.16

```

1 1 1 4 4 4 7 7 7
1 2 2 4 5 5 7 8 8
1 2 3 4 5 6 7 8 9
4 4 4 1 1 1 7 7 7
4 5 5 1 2 2 7 8 8
4 5 6 1 2 3 7 8 9
7 7 7 7 7 7 7 7 7
7 8 8 7 8 8 7 8 8
7 8 9 7 8 9 7 8 9

```

3.11 x 3.17

```

1 1 1 4 4 4 7 7 7
1 2 1 4 5 4 7 8 7
1 1 3 4 4 6 7 7 9
4 4 4 1 1 1 7 7 7
4 5 4 1 2 1 7 8 7
4 4 6 1 1 3 7 7 9
7 7 7 7 7 7 7 7 7
7 8 7 7 8 7 7 8 7
7 7 9 7 7 9 7 7 9

```

3.11 x 3.18

```

1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
4 4 4 1 1 1 7 7 7
4 4 4 1 1 1 7 7 7
4 4 4 1 1 1 7 7 7
7 7 7 7 7 7 7 7 7
7 7 7 7 7 7 7 7 7
7 7 7 7 7 7 7 7 7

```

3.12 x 3.12

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 1 3 1 1 3 1 1 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
4 4 6 4 4 6 4 4 6
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 1 3 1 1 3 7 7 9

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.12 x 3.13

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
4 5 6 4 5 6 4 5 6
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
1 2 3 1 2 3 7 8 9

```

3.12 x 3.14

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6 6
1 1 1 1 1 1 7 7 7
2 2 2 2 2 2 8 8 8
3 3 3 3 3 3 9 9 9

```

3.12 x 3.15

```

1 1 3 1 1 3 1 1 3
2 2 3 2 2 3 2 2 3
3 3 3 3 3 3 3 3 3
4 4 6 4 4 6 4 4 6
5 5 6 5 5 6 5 5 6
6 6 6 6 6 6 6 6 6
1 1 3 1 1 3 7 7 9
2 2 3 2 2 3 8 8 9
3 3 3 3 3 3 9 9 9

```

3.12 x 3.16

```

1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
4 4 4 4 4 4 4 4 4
4 5 5 4 5 5 4 5 5
4 5 6 4 5 6 4 5 6
1 1 1 1 1 1 7 7 7
1 2 2 1 2 2 7 8 8
1 2 3 1 2 3 7 8 9

```

3.12 x 3.17

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
4 4 4 4 4 4 4 4 4
4 5 4 4 5 4 4 5 4
4 4 6 4 4 6 4 4 6
1 1 1 1 1 1 7 7 7
1 2 1 1 2 1 7 8 7
1 1 3 1 1 3 7 7 9

```

3.12 x 3.18

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7
1 1 1 1 1 1 7 7 7

```

3.13 x 3.13

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
1 2 3 1 2 3 1 2 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
4 5 6 4 5 6 4 5 6
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
1 2 3 4 5 6 7 8 9

```

3.13 x 3.14

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6 6
1 1 1 4 4 4 7 7 7
2 2 2 5 5 5 8 8 8
3 3 3 6 6 6 9 9 9

```

3.13 x 3.15

```

1 1 3 1 1 3 1 1 3
2 2 3 2 2 3 2 2 3
3 3 3 3 3 3 3 3 3
4 4 6 4 4 6 4 4 6
5 5 6 5 5 6 5 5 6
6 6 6 6 6 6 6 6 6
1 1 3 4 4 6 7 7 9
2 2 3 5 5 6 8 8 9
3 3 3 6 6 6 9 9 9

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.13 x 3.16

```

1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
4 4 4 4 4 4 4 4 4
4 5 5 4 5 5 4 5 5
4 5 6 4 5 6 4 5 6
1 1 1 4 4 4 7 7 7
1 2 2 4 5 5 7 8 8
1 2 3 4 5 6 7 8 9

```

3.13 x 3.17

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
4 4 4 4 4 4 4 4 4
4 5 4 4 5 4 4 5 4
4 4 6 4 4 6 4 4 6
1 1 1 4 4 4 7 7 7
1 2 1 4 5 4 7 8 7
1 1 3 4 4 6 7 7 9

```

3.13 x 3.18

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4
1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7
1 1 1 4 4 4 7 7 7

```

3.14 x 3.14

```

1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3 3
4 4 4 4 4 4 4 4 4
5 5 5 5 5 5 5 5 5
6 6 6 6 6 6 6 6 6
7 7 7 7 7 7 7 7 7
8 8 8 8 8 8 8 8 8
9 9 9 9 9 9 9 9 9

```

3.14 x 3.15

```

1 1 3 1 1 3 1 1 3
2 2 3 2 2 3 2 2 3
3 3 3 3 3 3 3 3 3
4 4 6 4 4 6 4 4 6
5 5 6 5 5 6 5 5 6
6 6 6 6 6 6 6 6 6
7 7 9 7 7 9 7 7 9
8 8 9 8 8 9 8 8 9
9 9 9 9 9 9 9 9 9

```

3.14 x 3.16

```

1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2
1 2 3 1 2 3 1 2 3
4 4 4 4 4 4 4 4 4
4 5 5 4 5 5 4 5 5
4 5 6 4 5 6 4 5 6
7 7 7 7 7 7 7 7 7
7 8 8 7 8 8 7 8 8
7 8 9 7 8 9 7 8 9

```

3.14 x 3.17

```

1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1
1 1 3 1 1 3 1 1 3
4 4 4 4 4 4 4 4 4
4 5 4 4 5 4 4 5 4
4 4 6 4 4 6 4 4 6
7 7 7 7 7 7 7 7 7
7 8 7 7 8 7 7 8 7
7 7 9 7 7 9 7 7 9

```

3.14 x 3.18

```

1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1
4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4
4 4 4 4 4 4 4 4 4
7 7 7 7 7 7 7 7 7
7 7 7 7 7 7 7 7 7
7 7 7 7 7 7 7 7 7

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3.15 x 3.15

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1 1 3 1 1 3 7 7 9
2 2 3 2 2 3 8 8 9
3 3 3 3 3 3 9 9 9
4 4 6 4 4 6 7 7 9
5 5 6 5 5 6 8 8 9
6 6 6 6 6 6 9 9 9
7 7 9 7 7 9 7 7 9
8 8 9 8 8 9 8 8 9
9 9 9 9 9 9 9 9 9

```

TABLE III

DIRECT PRODUCTS: 3 x 3 (continued)

3.15 x 3.16	3.15 x 3.17	3.15 x 3.18
1 1 1 1 1 1 7 7 7	1 1 1 1 1 1 7 7 7	1 1 1 1 1 1 7 7 7
1 2 2 1 2 2 7 8 8	1 2 1 1 2 1 7 8 7	1 1 1 1 1 1 7 7 7
1 2 3 1 2 3 7 8 9	1 1 3 1 1 3 7 7 9	1 1 1 1 1 1 7 7 7
4 4 4 4 4 4 7 7 7	4 4 4 4 4 4 7 7 7	4 4 4 4 4 4 7 7 7
4 5 5 4 5 5 7 8 8	4 5 4 4 5 4 7 8 7	4 4 4 4 4 4 7 7 7
4 5 6 4 5 6 7 8 9	4 4 6 4 4 6 7 7 9	4 4 4 4 4 4 7 7 7
7 7 7 7 7 7 7 7 7	7 7 7 7 7 7 7 7 7	7 7 7 7 7 7 7 7 7
7 8 8 7 8 3 7 8 8	7 8 7 7 8 7 7 8 7	7 7 7 7 7 7 7 7 7
7 8 9 7 8 9 7 8 9	7 7 9 7 7 9 7 7 9	7 7 7 7 7 7 7 7 7
3.16 x 3.16	3.16 x 3.17	3.16 x 3.18
1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
1 2 2 1 2 2 1 2 2	1 2 1 1 2 1 1 2 1	1 1 1 1 1 1 1 1 1
1 2 3 1 2 3 1 2 3	1 1 3 1 1 3 1 1 3	1 1 1 1 1 1 1 1 1
1 1 1 4 4 4 4 4 4	1 1 1 4 4 4 4 4 4	1 1 1 4 4 4 4 4 4
1 2 2 4 5 5 4 5 5	1 2 1 4 5 4 4 5 4	1 1 1 4 4 4 4 4 4
1 2 3 4 5 6 4 5 6	1 1 3 4 4 6 4 4 6	1 1 1 4 4 4 4 4 4
1 1 1 4 4 4 7 7 7	1 1 1 4 4 4 7 7 7	1 1 1 4 4 4 7 7 7
1 2 2 4 5 5 7 8 8	1 2 1 4 5 4 7 8 7	1 1 1 4 4 4 7 7 7
1 2 3 4 5 6 7 8 9	1 1 3 4 4 6 7 7 9	1 1 1 4 4 4 7 7 7
3.17 x 3.17	3.17 x 3.18	3.18 x 3.18
1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 1 2 1	1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
1 1 3 1 1 3 1 1 3	1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1
1 1 1 4 4 4 1 1 1	1 1 1 4 4 4 1 1 1	1 1 1 1 1 1 1 1 1
1 2 1 4 5 4 1 2 1	1 1 1 4 4 4 1 1 1	1 1 1 1 1 1 1 1 1
1 1 3 4 4 6 1 1 3	1 1 1 4 4 4 1 1 1	1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 7 7 7	1 1 1 1 1 1 7 7 7	1 1 1 1 1 1 1 1 1
1 2 1 1 2 1 7 8 7	1 1 1 1 1 1 7 7 7	1 1 1 1 1 1 1 1 1
1 1 3 1 1 3 7 7 9	1 1 1 1 1 1 7 7 7	1 1 1 1 1 1 1 1 1

5. Direct products: 3×4 and 4×4

Since there are 2,178 possible distinct direct products of the semigroups of order three with the semigroups of order four and 7381 possible distinct direct products of the semigroups of order four with the semigroups of order four, it is desirable to find some method of describing the multiplication tables of these semigroups rather than listing them. Consider semigroups S and T whose elements are x_i ($i = 1, 2, \dots, s$) and y_j ($j = 1, 2, \dots, t$), respectively. The elements of $S \times T$ are (x_i, y_j) ($i = 1, 2, \dots, s$; $j = 1, 2, \dots, t$). Consider a semigroup S^* whose elements are x_i ($i = 0, 1, 2, \dots, s$) and in which the x_i for $i = 1, 2, \dots, s$ are identical with the $x_i \in S$ for $i = 1, 2, \dots, s$. The elements of $S^* \times T$ are (x_i, y_j) ($i = 0, 1, 2, \dots, s$; $j = 1, 2, \dots, t$). Clearly, the elements (x_i, y_j) for $i = 1, 2, \dots, s$ and $j = 1, 2, \dots, t$ are common to both $S \times T$ and $S^* \times T$. Since $S \times T$ is a semigroup, $S \times T$ is a subsemigroup of $S^* \times T$. If we can provide a sufficiently simple way to describe the products $(x_0, y_j)(x_i, y_j)$ and $(x_i, y_j)(x_0, y_j)$ for $i = 1, 2, \dots, s$ and $j = 1, 2, \dots, t$, then we shall be able to write the multiplication table of $S^* \times T$ from the multiplication table of $S \times T$. Before we can attempt this, however, we must be able to form a semigroup S^* from S by adding an element x_0 to S . The following are some of the ways in which semigroups can be formed by the addition of an element to a semigroup:

I. Let S be a semigroup containing a two-sided zero element z . Adjoin to S an element x , and define $ax = xa = x$ for all $a \in S$. Clearly, the multiplication so defined is associative; we wish so to define x^2 as to preserve associativity, and hence must have $(xx)a = x(xa) = xx = (ax)x = a(xx)$ for all $a \in S$. If $x^2 \in S$, then $x^2 = z$, for a semigroup can have only one two-sided zero element, and if $x^2 \notin S$, then $x^2 = x$. In the latter case, a new semigroup has been formed by the adjunction to S of a zero element x . In the former case, the resulting system has no zero element, and is a semigroup as we may show by verifying the associativity of multiplication for products involving x^2 :

$$\begin{aligned} x &= xz = x(xx) = (xx)x = zx = x \\ z &= xx = x(xa) = (xx)a = za = z && \text{for all } a \in S \\ z &= az = a(xx) = (ax)x = xx = z && \text{for all } a \in S \\ xx &= x(ax) = (xa)x = xx && \text{for all } a \in S \end{aligned}$$

II. Let S be a semigroup containing a two sided identity element e . Adjoin to S an element x , and define $ax = xa = a$ for all $a \in S$. Clearly, the multiplication so defined is associative; we wish so to define x^2 as to preserve associativity, and hence must have $(xx)a = x(xa) = xa = a = ax = (ax)x = a(xx)$ for all $a \in S$. If $x^2 \in S$, then $x^2 = e$, for a semigroup can have only one two-sided identity element, and if $x^2 \notin S$, then $x^2 = x$. In the latter case, a new semigroup has been formed by the adjunction to S of an identity element x . In the former case, the resulting system has no identity element, and is a semigroup as we may show by verifying the associativity of multiplication for products involving x^2 :

$$e = ex = (xx)x = x(xx) = xe = e$$

$$a = ea = (xx)a = x(xa) = xa = a \quad \text{for all } a \in S$$

$$a = ax = (ax)x = a(xx) = ae = a \quad \text{for all } a \in S$$

III. Let S be a semigroup containing a two-sided zero element z . Adjoin to S an element x , and define $ax = xa = z$ for all $a \in S$. Clearly, the multiplication so defined is associative; we wish so to define x^2 as to preserve associativity, and hence must have $(xx)a = x(xa) = xz = z = zx = (ax)x = a(xx)$ for all $a \in S$. If $x^2 \in S$, then $x^2 = z$ or $x^2 = p$ where $pa = ap = z$ for all $a \in S$; and if $x^2 \notin S$, then $x^2 = x$. If $x^2 = z$, a new semigroup has been formed by the adjunction to S of a two-sided constant element where the product of any element and the constant element is an element which is a two-sided zero in S . If $x^2 = x$, the resulting system is a semigroup as we may show by verifying the associativity of multiplication for products involving x^2 :

$$xx = x(xx) = (xx)x = xx$$

$$z = xz = x(xa) = (xx)a = xa = z \quad \text{for all } a \in S$$

$$z = ax = a(xx) = (ax)x = zx = z \quad \text{for all } a \in S$$

If $x^2 = p$ where $pa = ap = z$ for all $a \in S$ and where $p \in S$, the resulting system is a semigroup as we may show by verifying the associativity of multiplication for products involving x^2 :

$$z = xp = x(xx) = (xx)x = px = z$$

$$z = ap = a(xx) = (ax)x = zx = z \quad \text{for all } a \in S$$

$$z = xz = x(xa) = (xx)a = pa = z \quad \text{for all } a \in S$$

The fourth order semigroups to which this latter case applies may all be described in some other manner.

IV. Let S be a semigroup containing a two-sided zero element z . Adjoin to S an element x , and define $ax = x$ for all $a \in S$, $xx = x$, and $xa = z$ for all $a \in S$. The resulting system is a semigroup as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{aligned} x &= (qr)x = q(rx) = qx = x && \text{for all } q, r \text{ in the system} \\ z &= xa = (xx)a = x(xa) = xz = z && \text{for all } a \in S \\ z &= zb = (xa)b = x(ab) = z && \text{for all } a, b \in S \\ z &= xb = (ax)b = a(xb) = az = z && \text{for all } a, b \in S \end{aligned}$$

V. Let S be a semigroup containing a two-sided zero element z and in which there are no zero divisors except z . Adjoin to S an element x , and define $ax = x$ for all $a \in S$, $xx = x$, $xz = z$, and $xb = x$ for all b which are not zero divisors. The resulting system is a semigroup as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{aligned} x &= (qr)x = q(rx) = qx = x && \text{for all } q, r \text{ in the system} \\ xz &= (xx)z = x(xz) = xz \\ x &= xb = (xx)b = x(xb) = xx = x && \text{for all } b \text{ which are not zero divisors} \\ z &= za = (xz)a = x(za) = xz = z && \text{for all } a \in S \\ z &= xz = (xb)z = x(bz) = xz = z && \text{for all } b \text{ which are not zero divisors} \\ x &= xb' = (xb)b' = x(bb') = x && \text{for all } b, b' \text{ which are not zero divisors} \\ z &= xz = (ax)z = a(xz) = az = z && \text{for all } a \in S \\ x &= xb = (ax)b = a(xb) = ax = x && \text{for all } a \in S, \text{ for all } b \text{ which are not zero divisors} \end{aligned}$$

VI. Let S be a semigroup containing a two-sided zero element z and in which there are no zero divisors except z . Adjoin to S an element x , and define $ax = z$ for all $a \in S$, $xx = z$, $xz = z$, and $xb = x$ for all b which are not zero divisors. The resulting system is a semigroup as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{aligned} z &= (qr)x = q(rx) = qz = z && \text{for all } q, r \text{ in the system} \\ z &= zb = (xx)b = x(xb) = xx = z && \text{for all } b \text{ which are not zero divisors and which are contained in } S \\ x &= xb' = (xb)b' = x(bb') = x && \text{for all } b, b' \text{ which are not zero divisors and which are contained in } S \\ z &= zb' = (bx)b' = b(xb') = bx = z && \text{for all } b, b' \text{ which are not zero divisors and which are contained in } S \end{aligned}$$

VII. Let S be a semigroup containing a left zero element z , be such that $bz = z$ for all elements b which are not left zero elements, and be such that $da \neq z, e$ for all $a \in S$ where d is any element which is neither a left zero element nor a left identity element and where z' is any left zero element distinct from z . Adjoin to S an element x , and define $xa = x$ for all $a \in S$, $xx = x$, $wx = w$ for all left zero elements w , $ex = x$ for all left identity elements $e \in S$, and $dx = z$ for all elements d which are neither left zero elements nor left identity elements. The resulting system is a semigroup as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{aligned} x &= x(qr) = (xq)r = xr = x && \text{for all } q, r \text{ in the system} \\ w &= w(qr) = (wq)r = wr = w && \text{for all left zero elements } w \text{ and for all } q, r \text{ in the system} \\ qr &= e(qr) = (eq)r = qr && \text{for all left identity elements } e \text{ and for all } q, r \text{ in the system} \end{aligned}$$

$z = dx = d(xq) = (dx)q = zq = z$ for all d which are neither left zero elements nor left identity elements and for all q in the system

By Lemma 1.2 $dw = d(wq) = (dw)q$ for all d which are neither left zero elements nor left identity elements, for all left zero elements w , and for all q contained in the system

$z = dx = d(ex) = (de)x = z$ for all d which are neither left zero elements nor left identity elements

$z = dz = d(d'x) = (dd')x = z$ for all d which are neither left zero elements nor left identity elements

VIII. Let S be a semigroup containing a left zero element z and in which every element is either a left zero element or a left constant element. Adjoin to S an element x , and define $xa = z$ for all $a \in S$, $z'x = z'$ for all left zero elements z' , $cx = c'$ for all left constant elements c (where $ca = c'$ for all $a \in S$), and (i) $xx = x$ or (ii) $xx = z$. The resulting systems are semigroups as we may show by verifying the associativity of multiplication for products involving x :

$$(i) \quad xx = x(xx) = (xx)x = xx$$

$$z = xz = x(xa) = (xx)a = xa = z \quad \text{for all } a \in S$$

$$z = x(ax) = (xa)x = zx = z \quad \text{for all } a \in S$$

$ax = a(xx) = (ax)x$ for all $a \in S$, for if a is a left zero element $a = ax = a(xx) = (ax)x = ax = a$, and if a is not a left zero element it is a left constant element, hence, by Lemma 1.1, $ax = (ax)x$.

$$z = x(ad) = (xa)d = zd = z \quad \text{for all } a, d \in S$$

By Lemma 1.1, $z' = a(xd) = (ax)d = z'd = z'$ for some zero z' and for all $a, d \in S$

By Lemma 1.1, $z' = a(dx) = (ad)x = z'x = z'$ for some zero z' and for all $a, d \in S$

$$(ii) \quad z = x(qr) = (xq)r = zr = z \quad \text{for all } q, r \text{ in the system}$$

By Lemma 1.1 $z' = q(xr) = (qx)r = z'r = z'$ for some zero z' and for all q, r in the system

By Lemma 1.1, $z' = q(rx) = (qr)x = z'x = z'$ for some zero z' and for all q, r in the system

IX. Let S be a semigroup in which each element is either a left zero element or a left constant element. Adjoin to S an element x , and define $ax = a$ for all $a \in S$, $xx = x$, and $xa = z_1$ for all $a \in S$ where z_1 is a fixed left zero element. The resulting system is a semigroup as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{aligned} qr &= (qr)x = q(rx) = qr && \text{for all } q, r \text{ in the system} \\ z_1 &= xa = (xx)a = x(xa) = xz_1 = z_1 && \text{for all } a \in S \\ z_1 &= z_1d = (xa)d = x(ad) = z_1 && \text{for all } a, d \in S \\ ad &= (ax)d = a(xd) = az_1 && \text{for all } a, d \in S \end{aligned}$$

X. Let S be a null semigroup. Adjoin an element x to S , and define $px = m$, p fixed, $p \in S$, $m \neq p \neq z$ where z is the two-sided zero in S , $bx = b$ for all $b \neq p$, $xx = x$, and (i) $xa = a$ for all $a \in S$ or (ii) $xp = m$ and $xb = b$ for all $b \neq p$. The resulting systems form a semigroup as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{aligned} \text{(ii)} \quad & xx = x(xx) = (xx)x = xx \\ 1 \quad & m = px = p(xx) = (px)x = mx = m \\ 2 \quad & bx = b(xx) = (bx)x = bx && \text{for all } b \neq p \\ & m = xm = x(xp) = (xx)p = xp = m \\ & b = xb = x(xb) = (xx)b = xb = b && \text{for all } b \neq p \\ & m = xm = x(px) = (xp)x = mx = m \\ & b = xb = x(bx) = (xb)x = bx = b && \text{for all } b \neq p \\ & z = xz = x(ab) = (xa)b = z && \text{for all } a, b \in S \end{aligned}$$

$$3 \quad z = a(xb) = (ax)b = z \quad \text{for all } a, b \in S$$

$$4 \quad z = a(bx) = (ab)x = zx = z \quad \text{for all } a, b \in S$$

(i) 1, 2, 3, 4 of (ii)

XI. Let S be a semigroup containing left zero elements z_1 and z_2 and in which every other element is either a left zero element or a left constant element c such that $ca = z_1$ for all $a \in S$. Adjoin to S an element x , and define $ax = a$ for all $a \in S$, $xx = x$, $xc = z_1$ for all left constant elements c , $xz_1 = z_1$, and $xw = z_2$ for all left zero elements $w \neq z_2$. The resulting system is a semigroup as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{aligned} qr &= (qr)x = q(rx) = qr && \text{for all } q, r \text{ in the system} \\ z_1 &= xc = (xx)c = x(xc) = xz_1 = z_1 && \text{for all left constant elements } c \\ z_1 &= xz_1 = (xx)z_1 = x(xz_1) = xz_1 = z_1 \\ z_2 &= xw = (xx)w = x(xw) = xz_2 = z_2 && \text{for all left zero elements } w \neq z_2 \\ z_1 &= z_1a = (xc)a = x(ca) = xz_1 = z_1 && \text{for all } a \in S \text{ and for all left constant elements } c \\ z_1 &= z_1a = (xz_1)a = x(z_1a) = xz_1 = z_1 && \text{for all } a \in S \\ z_2 &= z_2a = (xw)a = x(wa) = xw = z_2 && \text{for all } a \in S \text{ and for all zero elements } w \neq z_1 \\ ab &= (ax)b = a(xb) && \text{for all } a, b \in S \end{aligned}$$

XII. Let S be a semigroup containing a right identity element e and such that $ab = z$ for all $b \neq e$ and for all $a \in S$. Adjoin to S an element x , and define $ex = x$, $bx = xb = z$ for all $b \neq e$ where $b \in S$, and (i) $xe = z$ and $xx = z$ or (ii) $xe = x$ and $xx = x$. The resulting systems are semigroups as we may show by verifying the associativity of multiplication for products involving x :

	$z = x(qr) = (xq)r = zr = z$	for all q, r in the system
	$z = ez = e(xq) = (ex)q = xq = z$	for all q in the system
	$z = bz = b(xq) = (bx)q = zq = z$	for all $b \neq e$ in the system and for all q in the system
	$ex = e(ex) = (ee)x = ex$	
	$z = ez = e(bx) = (eb)x = zx = z$	for all $b \neq e$, $b \in S$
	$z = ax = a(ex) = (ae)x = ax = z$	for all $a \in S$
	$z = bz = b(b'z) = (bb')z = b''z = x$	for all $b, b' \neq e$, $b, b' \in S$
(ii)	$xx = x(xx) = (xx)x = xx$	
	$x = xx = x(xe) = (xx)e = xe = x$	
	$z = xz = x(xb) = (xx)b = xb = z$	for all $b \neq e$, $b \in S$
	$x = ex = e(xx) = (ex)x = xx = x$	
	$z = bx = b(xx) = (bx)x = zx = z$	for all $b \neq e$, $b \in S$
	$x = xe = x(ee) = (xe)e = xe = x$	
	$z = xz = x(eb) = (xe)b = xb = z$	for all $b \neq e$, $b \in S$
	$z = xb = x(be) = (xb)e = ze = z$	
	$z = xz = x(ab) = (xa)b = zb = z$	for all $a, b \neq e$, $a, b \in S$
	$x = ex = e(xe) = (ex)e = xe = x$	
	$z = ez = e(xb) = (ex)b = xb = z$	for all $b \neq e$, $b \in S$
	$z = bx = b(xe) = (bx)e = ze = z$	for all $b \neq e$, $b \in S$
	$z = bz = b(xa) = (bx)a = za = z$	for all $a, b \neq e$, $a, b \in S$
	$ex = e(ex) = (ee)x = ex$	
	$z = ez = e(bx) = (eb)x = zx = z$	for all $b \neq e$, $b \in S$
	$z = bx = b(ex) = (be)x = bx = z$	for all $b \neq e$, $b \in S$
	$z = az = a(bx) = (ab)x = zx = z$	for all $a, b \neq e$, $b \in S$

XIII. Let S be a semigroup containing a two-sided zero element z and in which every element is either a left identity element or a left constant element. Adjoin to S an element x , and define $xa = z$ for all $a \in S$, $xx = z$, $px = m$ for some fixed element $p \neq m$, z where $ma = z$ for all $a \in S$, and $bx = z$ for all $b \neq p$. The resulting system is a semigroup as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{aligned}
 z &= x(qr) = (xq)r = zr = z && \text{for all } q, r \text{ in the system} \\
 z &= pz = p(xx) = (px)x = mx = z \\
 z &= bz = b(xx) = (bx)x = zx = z && \text{for all } b \neq p \\
 z &= pz = p(xa) = (px)a = ma = z && \text{for all } a \in S \\
 z &= bz = b(xa) = (bx)a = za = z && \text{for all } b \neq p \text{ and for all } a \in S \\
 px &= e(px) = (ep)x = px \\
 z &= am = a(px) = (ap)x = zx = z && \text{for all } a \neq e, \\
 bx &= e(bx) = (eb)x = bx && \text{for all } b \neq p \\
 z &= az = a(bx) = (ab)x = zx = z && \text{for all } a \neq e, \text{ for all } b \neq p
 \end{aligned}$$

XIV. Let S be a null semigroup. Adjoin to S an element x , and define $px = xp = m$ for some fixed $p \neq z$ where $m \neq p, x$, $bx = xb = z$ for all $b \neq p, x$, and (i) $xx = z$ or (ii) $xx = p$ or (iii) $xx = m$. The resulting systems are semigroups as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{aligned}
 \text{(i)} \quad z &= zx = (xx)x = x(xx) = xz = z \\
 z &= zp = (xx)p = x(xp) = xm = z \\
 z &= zb = (xx)b = x(xb) = xz = z && \text{for all } b \neq p \\
 z &= mx = (xp)x = x(px) = xm = z \\
 z &= zx = (xb)x = x(bx) = xz = z && \text{for all } b \neq p
 \end{aligned}$$

$$\begin{aligned}
& z = mx = (px)x = p(xx) = pz = z \\
& z = zx = (bx)x = b(xx) = bz = z && \text{for all } b \neq p \\
& z = ma = (xp)a = x(pa) = xz = z && \text{for all } a \in S \\
& z = za = (xb)a = x(ba) = xz = z && \text{for all } b \neq p, \\
& \quad z = (ax)d = a(xd) = z && \text{for all } a \in S \\
& \quad \quad \quad \text{for all } a, d \in S \\
& z = zx = (ad)x = a(dx) = z && \text{for all } a, d \in S \\
(ii) \quad & m = px = (xx)x = x(xx) = xp = m \\
& z = pp = (xx)p = x(xp) = xm = z \\
& z = pb = (xx)b = x(xb) = xz = z && \text{for all } b \neq p, \\
& \quad \quad \quad b \in S \\
& z = mx = (xp)x = x(px) = xm = z \\
& z = zx = (xb)x = x(bx) = xz = z && \text{for all } b \neq p, \\
& \quad \quad \quad b \in S \\
& z = mx = (px)x = p(xx) = pp = z \\
& z = zx = (bx)x = b(xx) = bp = z && \text{for all } b \neq p, \\
& \quad \quad \quad b \in S \\
& \quad z = (xa)d = x(ad) = xz = z && \text{for all } a, d \in S \\
& \quad \quad \quad z = (ax)d = a(xd) = z && \text{for all } a, d \in S \\
& z = zx = (ab)x = a(bx) = z \\
(iii) \quad & z = mx = (xx)x = x(xx) = xm = z \\
& z = mp = (xx)p = x(xp) = xm = z \\
& z = mb = (xx)b = x(xb) = xz = z && \text{for all } b \neq p, \\
& \quad \quad \quad b \in S \\
& z = mx = (xp)x = x(px) = xm = z \\
& z = zx = (xb)x = x(bx) = xz = z && \text{for all } b \neq p, \\
& \quad \quad \quad b \in S \\
& z = mx = (px)x = p(xx) = pm = z \\
& z = zx = (bx)x = b(xx) = bm = z && \text{for all } b \neq p, \\
& \quad \quad \quad b \in S \\
& \quad z = (xa)d = x(ad) = xz = z && \text{for all } a, d \in S \\
& \quad \quad \quad z = (ax)d = a(xd) = z \\
& z = zx = (ad)x = a(dx) = z
\end{aligned}$$

XV. Let S be a semigroup containing a two-sided identity element e and in which every other element is a left zero element. Adjoin to S an element x , and define $xa = z$ for all $a \in S$ where z is a fixed left zero element, $ex = z$, $z'x = z'$ for all left zero elements z' , and (i) $xx = z$ or (ii) $xx = x$. The resulting systems are semigroups as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{array}{ll}
 \text{(i)} & z = x(qr) = (xq)r = zr = z \quad \text{for all } q, r \text{ in the system} \\
 & z = ez = e(xq) = (ex)q = zq = z \quad \text{for all } q \text{ in the system} \\
 & ax = e(ax) = (ea)x = ax \quad \text{for all } a \in S \\
 & z' = z'(qr) = (z'q)r = z'r = z' \quad \text{for all } q, r \text{ in the system} \\
 \text{(ii)} & xx = x(xx) = (xx)x = xx \\
 & z = xz = x(xa) = (xx)a = xa = z \quad \text{for all } a \in S \\
 & z = x(ax) = (xa)x = zx = z \quad \text{for all } a \in S \\
 & z = ex = e(xx) = (ex)x = zx = z \\
 & z = ex = e(xa) = (ex)a = za = z \quad \text{for all } a \in S \\
 & ax = e(ax) = (ea)x = ax \quad \text{for all } a \in S \\
 & z' = z'(qr) = (z'q)r = z'r = z' \quad \text{for all } q, r \text{ in the system}
 \end{array}$$

XVI. Let S be any semigroup containing a right identity element e and in which $ab = z$ for all $b \neq e$ and for all $a \in S$. Adjoin to S an element x , and define $ax = z$ for all $a \in S$, $xx = x$, $xp = p$ for all $p \neq e$ and (i) $xe = z$ or (ii) $xe = p_1$ for some fixed p_1 . The resulting systems are semigroups, as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{array}{ll}
 \text{(i)} & xx = (xx)x = x(xx) = xx \\
 & z = xe = (xx)e = x(xe) = xz = z
 \end{array}$$

$$\begin{aligned}
z &= xz = (xx)z = x(xz) = xz = z \\
p &= xp = (xx)z = x(xp) = xp = p && \text{for all } p \text{ defined above} \\
z &= zx = (ax)x = a(xx) = ax = x && \text{for all } a \in S \\
z &= (xa)x = x(ax) = xz = z && \text{for all } a \in S \\
z &= (ab)x = a(bx) = az = z && \text{for all } a, b \in S \\
z &= ze = (ax)e = a(xe) = az = z && \text{for all } a \in S \\
z &= zz = (ax)z = a(xz) = az = z && \text{for all } a \in S \\
z &= zp = (ax)p = a(xp) = ap = z && \text{for all } a \in S \\
z &= ze = (xe)e = x(ee) = xe = z \\
z &= zz = (xe)z = x(ez) = xz = z \\
z &= zp = (xe)p = x(ep) = xz = z \\
z &= za = (xz)a = x(za) = xz = z && \text{for all } a \in S \\
p &= pe = (xp)e = x(pe) = xp = p \\
z &= pz = (xp)z = x(pz) = xz = z \\
z &= pp = (xp)p = x(pp) = xz = z \\
(ii) \quad &xx = (xx)x = x(xx) = xx \\
z &= zx = (ax)x = a(xx) = ax = z && \text{for all } a \in S \\
z &= (xa)x = x(ax) = xz = z && \text{for all } a \in S \\
p_1 &= xe = (xx)e = x(xe) = xp_1 = p_1 \\
p &= xp = (xx)p = x(xp) = xp = p \\
z &= (ab)z = a(bz) = az = z && \text{for all } a, b \text{ in the system} \\
z &= (ab)x = a(bx) = az = z && \text{for all } a, b \in S \\
z &= ze = (ax)e = a(ex) = ap_1 = z && \text{for all } a \in S \\
z &= zp = (ax)p = a(xp) = ap = z \\
p_1 &= p_1e = (xe)e = x(ee) = xe = p_1 \\
z &= p_1p = (xe)p = x(ep) = xz = z
\end{aligned}$$

$$p = pe = (xp)e = x(pe) = xp = p$$

$$z = pp = (xp)p = x(pp) = xz = z$$

XVII. Let S be a semigroup containing a two-sided identity element e and in which $ab = z \neq e$ for all $a, b \neq e$. Adjoin an element x to S and define $ex = x$, $ax = xa = a$ for all $a \neq e$, and (i) $xe = x$ and $xx = e$ or (ii) $xe = e$ and $xx = x$. The resulting systems are semigroups as we may show by verifying the associativity of multiplication for products involving x :

$$\begin{array}{ll}
 \text{(i)} & \begin{array}{l}
 x = xe = x(xx) = (xx)x = ex = x \\
 e = xx = x(xe) = (xx)e = ee = e \\
 a = xa = x(xa) = (xx)a = ea = a & \text{for all } a \neq e, \\
 e = ee = e(xx) = (ex)x = xx = e & a \in S \\
 a = ae = a(xx) = (ax)x = ax = a & \text{for all } a \neq e, \\
 xa = x(ea) = (xe)a = xa & a \in S \\
 b = xb = x(be) = (xb)e = be = b & \text{for all } b \neq e, \\
 z = xz = x(ab) = (xa)b = ab = z & b \in S \\
 x = ex = e(xe) = (ex)e = xe = x & \text{for all } b \neq e, \\
 a = ax = a(xe) = (ax)e = ae = a & a, b \in S \\
 a = ea = e(xa) = (ex)a = xa = a & a \neq e, \\
 ab = a(xb) = (ax)b = ab & a \in S \\
 ax = a(ex) = (ae)x = ax & \text{for all } a \neq e, \\
 b = eb = e(bx) = (eb)x = bx = b & a, b \in S \\
 z = ab = a(bx) = (ab)x = zx = z & \text{for all } a \neq e, \\
 & a, b \in S
 \end{array} \\
 \text{(ii)} & \begin{array}{l}
 xx = x(xx) = (xx)x = xx \\
 x = ex = e(xx) = (ex)x = xx = x \\
 ax = a(xx) = (ax)x = ax & \text{for all } a \neq e, \\
 & a \in S
 \end{array}
 \end{array}$$

$$\begin{array}{ll}
xa = x(xa) = (xx)a = xa & \text{for all } a \in S \\
ab = x(ab) = (xa)b = ab & \text{for all } a, b \in S \\
a = ea = e(xa) = (ex)a = xa = a & \text{for all } a \in S \\
ba = b(xa) = (bx)a = ba & \text{for all } b \neq e, \\
& b \in S, \text{ for all } a \in S \\
ax = a(ex) = (ae)x = ax & \text{for all } a \in S \\
b = eb = e(bx) = (eb)x = bx = b & \text{for all } b \neq e, \\
& b \in S \\
z = ab = a(bx) = (ab)x = zx = z & \text{for all } a, b \neq e
\end{array}$$

Let S be any semigroup whose elements are x_i ($i = 1, 2, \dots, s$);
let T be any semigroup whose elements are y_j ($j = 1, 2, \dots, t$).

Order the elements of $S \times T$ as:

$$(x_1, y_1), (x_1, y_2), \dots, (x_1, y_t), (x_2, y_1), (x_2, y_2), \dots, (x_2, y_t), \dots, (x_s, y_t).$$

Let S^* be a semigroup obtained from S by adjoining an element x_0 to S ; let T^* be a semigroup obtained from T by adjoining an element y_0 to T . Order the elements of $S^* \times T^*$ as:

$$(x_0, y_0), (x_0, y_1), \dots, (x_0, y_t), (x_1, y_1), (x_1, y_2), \dots, (x_1, y_t), \dots, (x_s, y_t).$$

Define the (f, g) block of the multiplication table of $S^* \times T^*$ to be the set of products $(x_f, y_j)(x_g, y_i)$ (f, g fixed; $i, j = 0, 1, \dots, t$).

	(x_0, y_0)	(x_0, y_1)	...	(x_0, y_t)	...	(x_g, y_0)	(x_g, y_1)	...	(x_g, y_t)	.	.	.
(x_0, y_0)	$(x_0 x_0, y_0 y_0)$	$(x_0 x_0, y_0 y_1)$	...	$(x_0 x_0, y_0 y_t)$		$(x_0 x_g, y_0 y_0)$	$(x_0 x_g, y_0 y_1)$	...	$(x_0 x_g, y_0 y_t)$			
(x_0, y_1)	$(x_0 x_0, y_1 y_0)$	$(x_0 x_0, y_1 y_1)$	...	$(x_0 x_0, y_1 y_t)$		$(x_0 x_g, y_1 y_0)$	$(x_0 x_g, y_1 y_1)$	...	$(x_0 x_g, y_1 y_t)$			
⋮	⋮	⋮		⋮		⋮	⋮		⋮			
(x_0, y_t)	$(x_0 x_0, y_t y_0)$	$(x_0 x_0, y_t y_1)$	...	$(x_0 x_0, y_t y_t)$		$(x_0 x_g, y_t y_0)$	$(x_0 x_g, y_t y_1)$	...	$(x_0 x_g, y_t y_t)$			
⋮	⋮	⋮		⋮		⋮	⋮		⋮			
(x_f, y_0)	$(x_f x_0, y_0 y_0)$	$(x_f x_0, y_0 y_1)$	...	$(x_f x_0, y_0 y_t)$		$(x_f x_g, y_0 y_0)$	$(x_f x_g, y_0 y_1)$	...	$(x_f x_g, y_0 y_t)$			
(x_f, y_1)	$(x_f x_0, y_1 y_0)$	$(x_f x_0, y_1 y_1)$	...	$(x_f x_0, y_1 y_t)$		$(x_f x_g, y_1 y_0)$	$(x_f x_g, y_1 y_1)$	...	$(x_f x_g, y_1 y_t)$			
⋮	⋮	⋮		⋮		⋮	⋮		⋮			
(x_f, y_t)	$(x_f x_0, y_t y_0)$	$(x_f x_0, y_t y_1)$	...	$(x_f x_0, y_t y_t)$		$(x_f x_g, y_t y_0)$	$(x_f x_g, y_t y_1)$	...	$(x_f x_g, y_t y_t)$			

Figure 5.1.

The multiplication table of $S^* \times T^*$

- (i) has for its element in the j -row and k -column of the $(0, 0)$ block $(x_0 x_0, y_j y_k)$; has its $(0, f)$ block the same as the $(0, 0)$ block except that $x_0 x_0$ is replaced by $x_0 x_f$; has its $(f, 0)$ block the same as the $(0, 0)$ block except that $x_0 x_0$ is replaced by $x_f x_0$;
- (ii) has for its (f, g) block ($f > 0$ and $g > 0$) the (f, g) block of $S \times T$ plus an additional row, the 0-row, and an additional column, the 0-column. The element in the 0-row and j -column ($j = 0, 1, \dots, t$) is $(x_f x_g, y_0 y_j)$, and the element in the j -row and 0-column is $(x_f x_g, y_j y_0)$.

In particular, if

I. For semigroups S and T ,

- A. $x_0 x_i = x_i x_0 = x_0$ for all $x_i \in S$ and
 - 1. $x_0 x_0 = z$ where z is a two sided zero element of S or
 - 2. $x_0 x_0 = x_0$,
- B. $y_0 y_j = y_j y_0 = y_0$ for all $y_j \in T$ and
 - 1. $y_0 y_0 = w$ where w is a two-sided zero element of T or
 - 2. $y_0 y_0 = y_0$,

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-1 :

- (i) The element in the j -row and k -column of the $(0, 0)$ block is $(z, y_j y_k)$. For $f > 0$, the $(f, 0)$ block and the $(0, f)$ block are the same as the $(0, 0)$ block except that z is replaced by x_0 .

- (ii) The element in the 0-row and 0-column of the (f, g) block is $(x_f x_g, w)$. For $j > 0$, the element in the 0-row and j -column and in the j -row and 0-column of the (f, g) block is $(x_f x_g, y_0)$.

(b) under A-2 and B-2 :

- (i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. The $(f, 0)$ block and the $(0, f)$ block are identical with the $(0, 0)$ block.
- (ii) The elements in the 0-row and in the 0-column of the (f, g) block are $(x_f x_g, y_0)$.

Clearly, if we wish the multiplication table of $S^* \times T$, then we omit the 0-row and 0-column of each block of $S^* \times T^*$.

The semigroups of order four which may be obtained from the semigroups of order three by the adjunction of a fourth element are:

- (1') by A-1 or B-1, 4.14, 4.16, 4.17, 4.21, 4.22, 4.29, 4.30, 4.33, 4.103, and 4.107.
- (2') by A-2 or B-2, 4.1, 4.12, 4.18, 4.20, 4.23, 4.25, 4.26, 4.40, 4.41, 4.42, 4.46, 4.47, 4.86, 4.90, 4.99, 4.102, 4.104, and 4.116.

Thus, from (a) and Table III we may obtain the multiplication table of the direct product of any two semigroups listed under (1'). Similarly, from (b) and Table III we may obtain the multiplication table of the direct product of any two semigroups listed under (2'). We may obtain the multiplication table of the direct product of any semigroup listed under (1') with any semigroup listed under (2') by first taking as the description of this multiplication table a-i and b-ii and then, with the use of

Table III, proceeding in the same manner as in the two previous cases. Henceforth, we shall list as (1'), (2'), and (3') those fourth order semigroups which may be obtained from the semigroups of order three by the adjunction of a fourth element by A-1 or B-1, A-2 or B-2, and A-3 or B-3, respectively.

II. For semigroups S and T

A. $x_0 x_i = x_i x_0 = x_i$ for all $x_i \in S$ and

1. $x_0 x_0 = e$ where e is a two-sided identity element of S or
2. $x_0 x_0 = x_0$,

B. $y_0 y_j = y_j y_0 = y_j$ for all $y_j \in T$ and

1. $y_0 y_0 = h$ where h is a two-sided identity element of T or
2. $y_0 y_0 = y_0$,

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-1 :

- (i) The element in the j -row and k -column of the $(0, 0)$ block is $(e, y_j y_k)$. For $f > 0$, the $(f, 0)$ block and the $(0, f)$ block are the same as the $(0, 0)$ block except that e is replaced by x_f .
- (ii) The element in the 0 -row and 0 -column of the (f, g) block is $(x_f x_g, h)$. For $j > 0$, the element in the 0 -row and j -column and in the j -row and 0 -column of the (f, g) block is $(x_f x_g, y_j)$.

(b) under A-2 and B-2 :

- (i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. The $(f, 0)$ block and the

$(0, f)$ block are the same as the $(0, 0)$ block except that

x_0 is replaced by x_f .

(ii) The element in the 0-row and j -column and in the j -row

and 0-column of the (f, g) block is $(x_f x_g, y_j)$.

(1') 4.25, 4.38.

(2') 4.13, 4.17, 4.18, 4.21, 4.24, 4.37, 4.39, 4.42, 4.44, 4.45,
4.47, 4.49, 4.88, 4.95, 4.97, 4.99, 4.118.

III. For semigroups S and T containing two-sided zero elements z and w respectively,

A. $x_0 x_i = x_i x_0 = z$ for all $x_i \in S$ and

1. $x_0 x_0 = z$ or

2. $x_0 x_0 = x_0$,

B. $y_0 y_j = y_j y_0 = w$ for all $y_j \in T$ and

1. $y_0 y_0 = w$ or

2. $y_0 y_0 = y_0$,

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-1 :

(i) The element in the j -row and k -column of the $(0, 0)$ block is $(z, y_j y_k)$. The $(f, 0)$ block and the $(0, f)$ block are identical with the $(0, 0)$ block.

(ii) The elements in the 0-row and the elements in the 0-column of the (f, g) block are $(x_f x_g, w)$.

(b) under A-2 and B-2 :

(i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. For $f > 0$, the $(f, 0)$

block and the $(0, f)$ block are the same as the $(0, 0)$ block except that x_0 is replaced by z .

(ii) The element in the 0-row and 0-column of the (f, g) block is $(x_f x_g, y_0)$. For $j > 0$, the element in the 0-row and j -column and in the j -row and 0-column of the (f, g) block is $(x_f x_g, w)$.

(1') 4.2, 4.4, 4.5, 4.9, 4.27, 4.50, 4.54, 4.55, 4.75, 4.109.

(2') 4.2, 4.6, 4.19, 4.43, 4.48, 4.50, 4.51, 4.52, 4.73, 4.117.

IV. For semigroups S and T containing two-sided zero elements z and w respectively,

A. $x_i x_0 = x_0$ for all $x_i \in S$, $x_0 x_0 = x_0$, and $x_0 x_i = z$ for all $x_i \in S$,

B. $y_j y_0 = y_0$ for all $y_j \in S$, $y_0 y_0 = y_0$, and $y_0 y_j = w$ for all $y_j \in T$,

the description of the multiplication table of $S^* \times T^*$ is:

(i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. For $f > 0$, the $(0, f)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by z . The $(f, 0)$ block is identical with the $(0, 0)$ block.

(ii) The elements in the 0-column of the (f, g) block are $(x_f x_g, y_0)$. For $j > 0$, the element in the 0-row and j -column is $(x_f x_g, w)$.

(1') 4.56, 4.57, 4.58, 4.67, 4.72, 4.77, 4.84, 4.85, 4.91, 4.94.

V. For semigroups S and T containing two-sided zero elements z and w respectively, and in which there are no zero divisors except z and w respectively,

A. $x_i x_0 = x_0$ for all $x_i \in S$, $x_0 x_0 = x_0$, $x_0 z = z$, and

$x_0 x_i = x_i$ for all x_i which are not zero divisors,

B. $y_j y_0 = y_0$ for all $y_j \in T$, $y_0 y_0 = y_0$, $y_0 w = w$, and

$y_0 y_j = y_j$ for all y_j which are not zero divisors,

the description of the multiplication table of $S^* \times T^*$ is:

(i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. The $(f, 0)$ block is identical with the $(0, 0)$ block. If $x_f \neq z$, the $(0, f)$ block is identical with the $(0, 0)$ block. If $x_f = z$, the $(0, f)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by $x_f = z$.

(ii) The elements in the 0-column of the (f, g) block are $(x_f x_g, y_0)$. If $y_j \neq w$, the element in the 0-row and j -column of the (f, g) block is $(x_f x_g, y_j)$. If $y_j = w$, the element in the 0-row and j -column of the (f, g) block is $(x_f x_g, w)$.

(1') 4.60, 4.61, 4.97, 4.98, 4.100.

VI. For semigroups S and T containing two-sided zero elements a and w respectively and in which there are no zero divisors except z and w respectively,

A. $x_i x_0 = z$ for all $x_i \in S$, $x_0 x_0 = z$, $x_0 z = z$ and

$x_0 x_i = x_0$ for all x_i which are not zero divisors,

B. $y_j y_0 = w$ for all $y_j \in T$, $y_0 y_0 = w$, $y_0 w = w$, and

$y_0 y_j = y_0$ for all y_j which are not zero divisors,

the description of the multiplication table of $S^* \times T^*$ is:

- (i) The element in the j -row and k -column of the $(0, 0)$ block is $(z, y_i y_k)$. The $(f, 0)$ block is identical with the $(0, 0)$ block. For $f > 0$, the $(0, f)$ block is identical with the $(0, 0)$ block if $x_f = z$ and is the same as the $(0, 0)$ block except that z is replaced by x_0 if $x_f \neq z$.
- (ii) The elements in the 0-column of the (f, g) block are $(x_f x_g, w)$. For $j > 0$, the element in the 0-row and j -column is $(x_f x_g, w)$ if $y_j = w$ and is $(x_f x_g, y_0)$ if $y_j \neq w$.

(1') 4.68, 4.74, 4.105, 4.106, 4.115.

VII. For a semigroup containing a left zero element z , such that $bz = z$ for all elements b which are not left zero elements, such that $da \neq z'$, e for all $a \in S$ where d is any element which is neither a left zero element nor a left identity element e and where z' is any left zero element distinct from z ; for a semigroup T containing a left zero element w , such that $gw = w$ for all elements g which are not left zero elements, such that $hf = w'$, e' for all $f \in T$ where h is any element which is neither a left identity element e' nor a left zero element and where w' is any left zero element distinct from w ;

A. $x_0 x_i = x_0$ for all $x_i \in S$, $x_0 x_0 = x_0$, $x_f x_0 = x_f$ if x_f is a left zero element, $ex_0 = x_0$ for all left identity elements e , $x_f x_0 = z$ for all elements x_f which are neither

left zero elements nor left identity elements,

- B. $y_0 y_j = y_0$ for all $y_j \in S$, $y_0 y_0 = y_0$, $y_j y_0 = y_j$ if y_j is a left zero element, $e' y_0 = y_0$ for all left identity elements e' , $y_j y_0 = w$ for all elements y_j which are neither left zero elements nor left identity elements,

the description of the multiplication table of $S^* \times T^*$ is:

- (i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. The $(0, f)$ block is identical with the $(0, 0)$ block. The $(f, 0)$ block is identical with the $(0, 0)$ block if $x_f = e$; is the same as the $(0, 0)$ block except that x_0 is replaced by x_f if x_f is a left zero element; is the same as the $(0, 0)$ block except that x_0 is replaced by z if x_f is neither a left zero element nor a left identity element.
- (ii) The elements in the 0-row of the (f, g) block are $(x_f x_g, y_0)$. The element in the j -row and 0-column is $(x_f x_g, y_j)$ if y_j is a left zero element, is $(x_f x_g, y_0)$ if $y_j = e'$, and is $(x_f x_g, w)$ if y_j is neither a left zero element nor a left identity element.

- (1') 4.56, 4.58, 4.67, 4.71, 4.77, 4.79, 4.87, 4.88, 4.89, 4.91, 4.62, 4.95, 4.96, 4.98, 4.101, 4.102.

VIII. For semigroups S and T containing left zero elements z and w respectively, and in which every element is either a left zero element or a left constant element,

- A. $x_0 x_i = z$ for all $x_i \in S$, $x_f x_0 = x_f$ if x_f is a left zero element of S , $x_f x_0 = x_f'$ if x_f is a left constant element

of S (where $x_f x_i = x_f^i$ for all $x_i \in S$) , and

$$1. x_0 x_0 = x_0 \text{ or}$$

$$2. x_0 x_0 = z ,$$

B. $y_0 y_j = w$ for all $y_j \in T$, $y_j y_0 = y_j$ if y_j is a left zero element of T , $y_j y_0 = y_j^i$ if y_j is a left constant element of T where $y_j y_k = y_j^i$ for all $y_k \in T$) , and

$$1. y_0 y_0 = y_0 \text{ or}$$

$$2. y_0 y_0 = w ,$$

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-1 :

(i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. For $f > 0$ the $(0, f)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by z . The $(f, 0)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by x_f if x_f is a left zero element and by x_f^i if x_f is a left constant element.

(ii) The element in the 0 -row and 0 -column of the (f, g) block is $(x_f x_g, y_0)$. For $j > 0$, the element in the 0 -row and j -column of the (f, g) block is $(x_f x_g, w)$. The element in the j -row and 0 -column of the (f, g) block is $(x_f x_g, y_j)$ if y_j is a left zero element and is $(x_f x_g, y_j^i)$ if y_j is a left constant element.

(b) under A-2 and B-2 :

(i) The element in the j -row and k -column of the $(0, 0)$ block is $(z, y_j y_k)$. The $(0, f)$ block is identical with the

$(0, 0)$ block. The $(f, 0)$ block is the same as the $(0, 0)$ block except that z is replaced by x_f if x_f is a left zero element and by x'_f if x_f is a left constant element.

(ii) The elements in the 0-row of the (f, g) block are

$(x_f x_g, w)$. The element in the j -row and 0-column of the (f, g) block is $(x_f x_g, y_j)$ if y_j is a left zero element and is $(x_f x_g, y'_j)$ if y_j is a left constant element.

(i) 4.2, 4.70, 4.79, 4.80.

(ii) 4.55, 4.71, 4.77, 4.78.

IX. For semigroups S and T in which every element is either a left zero or a left constant element,

A. $x_i x_0 = x_i$ for all $x_i \in S^*$ and $x_0 x_i = x^*$ for all $x_i \in S$ where x^* is a fixed left zero element,

B. $y_j y_0 = y_j$ for all $y_j \in T^*$ and $y_0 y_j = y^*$ for all $y_j \in T$ where y^* is a fixed left zero element,

the description of the multiplication table of $S^* \times T^*$ is:

(i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. For $f > 0$, the $(0, f)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by x^* . The $(f, 0)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by x_f .

(ii) The element in the j -row and 0-column of the (f, g) block is $(x_f x_g, y_j)$. For $j > 0$, the element in the 0-row and j -column is $(x_f x_g, y^*)$.

(1') 4.70, 4.81, 4.89, 4.110.

X. For semigroups S and T in which every element is a zero divisor,

A. $px_0 = m$ for a fixed p , $m \neq p \neq z$ where z is the two-sided zero element of S , $x_1x_0 = x_1$ for all $x_1 \neq p$, $x_1 \in S$, $x_0x_0 = x_0$, and

1. $x_0x_i = x_i$ or

2. $x_0p = m$ and $x_0x_i = x_i$ for all $x_i \neq p$,

B. $ty_0 = n$ for a fixed t , $n \neq t \neq w$ where w is the two-sided zero element of T , $y_jy_0 = y_j$ for all $y_j \neq t$, $y_j \in T$. $y_0y_0 = y_0$, and

1. $y_0y_j = y_j$ or

2. $y_0t = n$ and $y_0y_j = y_j$ for all $y_j \neq t$,

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-1 :

(i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is (x_0, y_jy_k) . The $(0, f)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by x_f . The $(f, 0)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by m if $x_f = p$ and by x_f if $x_f \neq p$.

(ii) The element in the 0 -row and j -column of the (f, g) block, is (x_fx_g, y_j) . The element in the j -row and 0 -column is (x_fx_g, n) if $y_j = t$ and is (x_fx_g, y_j) if $y_j \neq t$.

(b) under A-2 and B-2 :

(i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is (x_0, y_jy_k) . The $(0, f)$ block and the $(f, 0)$ block are the same as the $(0, 0)$ block except that

x_0 is replaced by m if $x_f = p$ and by x_f if $x_f \neq p$.

- (ii) The element in the 0-row and 0-column of the (f, g) block is $(x_f x_g, y_0)$. The element in the 0-row and j -column and in the j -row and 0-column is $(x_f x_g, n)$ if $y_j = t$ and is $(x_f x_g, y_j)$ if $y_j \neq t$.

(1') 4.36, 4.76, 4.112, 4.110.

- XI. For a semigroup containing left zero elements z_1 and z_2 and in which every other element is either a left zero element or a left constant element c such that $ca = z_1$ for all $a \in S$; for a semigroup T containing left zero elements w_1 and w_2 and in which every other element is either a left zero element or a left constant element k such that $ka = w_1$ for all $a \in T$;
- A. $x_i x_0 = x_i$ for all $x_i \in S$, $x_0 x_0 = x_0$, $x_0 x_i = z_1$ if x_i is a left constant element, $x_0 z_1 = z_1$, and $x_0 z' = z_2$ for all left zero elements $z' \neq z$,
- B. $y_j y_0 = y_j$ for all $y_j \in T$, $y_0 y_0 = y_0$, $y_0 y_j = w_1$ if y_j is a left constant element, $y_0 w_1 = w_1$, and $y_0 w' = w_2$ for all left zero elements $w' \neq w_1$,

the description of the multiplication table of $S^* \times T^*$ is:

- (i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. The $(f, 0)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by x_f . The $(0, f)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by z_1 if x_f is either z_1 or a left constant element and is

replaced by z_2 if otherwise.

- (ii) The element in the j -row and 0-column of the (f, g) block is $(x_f x_g, y_j)$. The element in the 0-row and j -column of the (f, g) block is $(x_f x_g, w_1)$ if y_j is either w_1 or a left constant element and is $(x_f x_g, w_2)$ if y_j is a left zero element distinct from w_1 .

(1') 4.62, 4.83.

XII. For a semigroup containing a right identity element e and in which $ab = z$ for all $b \neq e$ and for all $a \in S$; for a semigroup T containing a right identity element h and in which $ab = w$ for all $b \neq h$ and for all $a \in T$;

A. $ex_0 = x_0$, $x_i x_0 = x_0 x_i = z$ for all $x_i \neq e$ and $x_i \in S$, and

1. $x_0 e = z$ and $x_0 x_0 = z$ or

2. $x_0 e = x_0$ and $x_0 x_0 = x_0$,

B. $hy_0 = y_0$, $y_j y_0 = y_0 y_j = w$ for all $y_j \neq h$ and $y_j \in T$, and

1. $y_0 h = w$ and $y_0 y_0 = w$ or

2. $y_0 h = y_0$ and $y_0 y_0 = y_0$,

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-1:

- (i) The element in the j -row and k -column of the $(0, 0)$ block is $(z, y_j y_k)$. The $(0, f)$ block is identical with the $(0, 0)$ block. If $x_f \neq e$, the $(f, 0)$ block is identical with the $(0, 0)$ block; if $x_f = e$, the $(f, 0)$ block is the same as the $(0, 0)$ block except that z is replaced by x_0 .

(ii) The elements in the 0-row of the (f, g) block are

$(x_f x_g, w)$. The element in the j -row and 0-column of the (f, g) block is $(x_f x_g, y_0)$ if $y_j = h$ and is $(x_f x_g, w)$ if $y_j \neq h$.

(b) under A-2 and B-2 :

(i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. For $f > 0$, the $(0, f)$ block and the $(f, 0)$ block is identical with the $(0, 0)$ block if $x_f = e$ and is the same as the $(0, 0)$ block except x_0 is replaced by z if $x_f \neq e$.

(ii) The element in the 0-row and 0-column of the (f, g) block is $(x_f x_g, y_0)$. For $j > 0$, the element in the j -row and 0-column and in the 0-row and j -column is $(x_f x_g, y_0)$ if $y_j = h$ and is $(x_f x_g, w)$ if $y_j \neq h$.

(1') 4.111.

(2') 4.119.

XIII. For semigroups S and T containing two-sided zero z and w respectively, and in which every other element is either a left identity or a left constant element,

A. $x_0 x_i = z$ for all $x_i \in S$, $x_0 x_0 = z$, $px_0 = m$ for some fixed $p \neq m$, z where $mx_i = z$ for all $x_i \in S$ and $x_i x_0 = z$ for all $x_i \neq p$,

B. $y_0 y_i = w$ for all $y_i \in T$, $y_0 y_0 = w$, $ty_0 = n$ for some fixed $t \neq n$, w where $ny_i = w$ for all $y_i \in T$ and $y_i y_0 = w$ for all $y_i \neq t$.

the description of the multiplication table of $S^* \times T^*$ is:

- (i) The element in the j -row and k -column of the $(0, 0)$ block is $(z, y_j y_k)$. The $(0, f)$ block is identical with the $(0, 0)$ block. The $(f, 0)$ block is identical with the $(0, 0)$ block if $x_f \neq p$ and is the same as the $(0, 0)$ block except that z is replaced by m if $x_f = p$.
- (ii) The elements in the 0 -row of the (f, g) block are $(x_f x_g, w)$. The element in the j -row and 0 -column of the (f, g) block is $(x_f x_g, n)$ if $y_j = t$ and is $(x_f x_g, w)$ if $y_j \neq t$.

(1') 4.108, 4.121, 4.110.

XIV. For semigroups S and T in which every element is a zero divisor,

- A. $px_0 = x_0p = m$ for a fixed $p \neq z$ where z is the two-sided zero element of S and where $m \neq p$, x_0 , $x_i x_0 = x_0 x_i = z$ for all $x_i \neq p$ where $x_i \in S$, and

1. $x_0 x_0 = z$ or

2. $x_0 x_0 = p$

3. $x_0 x_0 = m$,

- B. $ty_0 = y_0 t = n$ for a fixed $t \neq w$ where w is the two-sided zero element of T and where $n \neq t$, y_0 , $y_j y_0 = y_0 y_j = w$ for all $y_j \neq t$ where $y_j \in T$, and

1. $y_0 y_0 = w$ or

2. $y_0 y_0 = t$

3. $y_0 y_0 = n$,

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-1 :

(i) The element in the j -row and k -column of the $(0, 0)$ block is $(z, y_j y_k)$. The $(f, 0)$ block is identical with the $(0, 0)$ block if $x_f \neq p$ and is the same as the $(0, 0)$ block except that z is replaced by m if $x_f = p$.

(ii) The element in the 0 -row and 0 -column of the (f, g) block is $(x_f x_g, w)$. The element in the 0 -row and j -column and in the j -row and 0 -column is $(x_f x_g, w)$ if $y_j \neq t$ and is $(x_f x_g, n)$ if $y_j = t$.

(b) under A-2 and B-2 :

(i) The same as a-i except that in the $(0, 0)$ block z is replaced by p .

(ii) The same as a-ii except that the element in the 0 -row and 0 -column of the (f, g) block is $(x_f x_g, t)$.

(c) under A-3 and B-3 :

(i) The same as a-i except that in the $(0, 0)$ block z is replaced by m .

(ii) The same as a-ii except that the element in the 0 -row and 0 -column of the (f, g) block is $(x_f x_g, n)$.

(1') 4.53, 4.55, (2') 4.8, 4.54, (3') 4.7.

XV. For semigroups S and T containing two-sided identity elements e and h respectively, and in which every other element is a left zero element,

A. $x_0 x_1 = z$ where z is a fixed left zero element, $ex_0 = z$

$x_f x_0 = x_f$ if x_f is a left zero element, and

1. $x_0 x_0 = z$

2. $x_0 x_0 = x_0$,

B. $y_0 y_j = w$ where w is a fixed left zero element, $h y_0 = w$,

$y_j y_0 = y_j$ if y_j is a left zero element, and

1. $y_0 y_0 = w$

2. $y_0 y_0 = y_0$,

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-1 :

(i) The element in the j -row and k -column of the $(0, 0)$

block is $(z, y_j y_k)$. The $(0, f)$ block is identical

with the $(0, 0)$ block. For $f > 0$, the $(f, 0)$ block

is identical with the $(0, 0)$ block if $x_f = e$ and is the

same as the $(0, 0)$ block except that z is replaced by

x_f if $x_f \neq e$.

(ii) The elements in the 0-row of the (f, g) block are

$(x_f x_g, w)$. For $j > 0$, the element in the j -row and

0-column of the (f, g) block is $(x_f x_g, w)$ if $y_j = h$

and is $(x_f x_g, y_j)$ if $y_j \neq h$.

(b) under A-2 and B-2 :

(i) The element in the j -row and k -column of the $(0, 0)$ block,

a subsemigroup, is $(x_0, y_j y_k)$. For $f > 0$, the $(0, f)$

block is the same as the $(0, 0)$ block except that x_0 is

replaced by z . The $(f, 0)$ block is the same as the

$(0, 0)$ block except that x_0 is replaced by x_f if

$x_f \neq e$ and is identical with the $(0, 0)$ block if $x_f = e$.

- (ii) The element in the 0-row and 0-column of the (f, g) block is $(x_f x_g, y_0)$. For $j > 0$, the element in the 0-row and j -column is $(x_f x_g, w)$. The element in the j -row and 0-column of the (f, g) block is $(x_f x_g, y_j)$ if $y_j \neq h$ and is $(x_f x_g, w)$ if $y_j = h$.

(1') 4.82.

(2') 4.93.

XVI. For a semigroup S containing a right identity element e and in which $ab = z$ for all $b \neq e$ and for all $a \in S$; for a semigroup T containing a right identity element h and in which $ab = w$ for all $b \neq h$ and for all $a \in T$;

A. $x_i x_0 = z$ for all $x_i \in S$, $x_0 x_0 = x_0$, $x_0 x_i = x_i$ for all $x_i \neq e$, and

1. $x_0 e = z$ or

2. $x_0 e = x_i'$ where x_i' is fixed and $x_i' \neq e$.

B. $y_j y_0 = w$ for all $y_j \in S$, $y_0 y_0 = y_0$, $y_0 y_j = y_j$ for all $y_j \neq h$, and

1. $y_0 h = w$ or

2. $y_0 h = y_j'$ where y_j' is fixed and $y_j' \neq h$.

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-1:

- (i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. The $(0, f)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by x_f .

if $x_f \neq e$ and by z if $x_f = e$. The $(f, 0)$ block ($f > 0$) is the same as the $(0, 0)$ block except that x_f is replaced by z .

- (ii) The element in the 0-row and 0-column of the (f, g) block is $(x_f x_g, y_0)$. The element in the j -row ($j > 0$) and 0-column of the (f, g) block is $(x_f x_g, w)$. The element in the 0-row and j -column is $(x_f x_g, w)$ if $y_j = h$ and is $(x_f x_g, y_j)$ if $y_j \neq h$.

(b) under A-2 and B-2 :

- (i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. The $(0, f)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by x_f' if $x_f = e$ and by x_f if $x_f \neq e$. The $(f, 0)$ block ($f > 0$) is the same as the $(0, 0)$ block except that x_0 is replaced by z .

- (ii) The element in the 0-row and 0-column of the (f, g) block is $(x_f x_g, y_0)$. The element in the j -row ($j > 0$) and 0-column of the (f, g) block is $(x_f x_g, w)$. The element in the 0-row and j -column of the (f, g) block is $(x_f x_g, y_j)$ if $y_j = h$ and is $(x_f x_g, y_j)$ if $y_j \neq h$.

(1') 4.113.

(2') 4.114.

XVII. For a semigroup S containing a two-sided identity element e and in which $ab = z \neq e$ for all $a, b \neq e$; for a semigroup T containing a two-sided identity element h and in which $ab = z \neq e$

for all $a, b \neq h$;

A. $ex_0 = x_0$, $x_i x_0 = x_0 x_i = x_i$ for all $x_i \neq e$, and

1. $x_0 e = x_0$ and $x_0 x_0 = e$ or

2. $x_0 e = e$ and $x_0 x_0 = x_0$,

B. $hy_0 = y_0$, $y_j y_0 = y_0 y_j = y_j$ for all $y_j \neq h$, and

1. $y_0 h = y_0$ and $y_0 y_0 = h$ or

2. $y_0 h = h$ and $y_0 y_0 = y_0$,

the description of the multiplication table of $S^* \times T^*$ is:

(a) under A-1 and B-2 :

(i) The element in the j -row and k -column of the $(0, 0)$ block is $(e, y_j y_k)$. The $(0, f)$ block and the $(f, 0)$ block ($f > 0$) is the same as the $(0, 0)$ block except that e is replaced by x_f if $x_f \neq e$ and by x_0 if $x_f = e$.

(ii) The element in the 0 -row and 0 -column of the (f, g) block is $(x_f x_g, h)$. The element in the j -row and 0 -column and in the 0 -row and j -column is $(x_f x_g, y_j)$ if $y_j \neq h$ and is $(x_f x_g, y_0)$ if $y_j = h$.

(b) under A-2 and B-2 :

(i) The element in the j -row and k -column of the $(0, 0)$ block, a subsemigroup, is $(x_0, y_j y_k)$. The $(f, 0)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by x_f if $x_f \neq e$ and is identical with the $(0, 0)$ block if $x_f = e$. The $(0, f)$ block is the same as the $(0, 0)$ block except that x_0 is replaced by x_f .

(ii) The element in the 0 -row and 0 -column of the (f, g) block

is $(x_f x_g, y_0)$. The element in the 0-row and j-column of the (f, g) block is $(x_f x_g, y_j)$. The element in the j-row and 0-column of the (f, g) block is $(x_f x_g, y_j)$ if $y_j = h$ and is $(x_f x_g, y_0)$ if $y_j \neq h$.

We may obtain the description multiplication table of the direct product of any semigroup listed under III-1' with any semigroup listed under VIII-2' by taking as the description III-a-i and VIII-b-ii. In a similar manner, we may obtain the multiplication tables of 2133 of the 2178 possible distinct direct products of the semigroups of order three with the semigroups of order four and 7261 of the 7381 possible distinct direct products of the semigroups of order four with the semigroups of order four.

The multiplication tables of the direct products of the remaining semigroups are listed in Table IV and Table V.

TABLE IV

DIRECT PRODUCTS: 3×4 3.1×4.3

1	1	1	1	5	5	5	5	5	5	5	5
1	1	1	1	5	5	5	5	5	5	5	5
1	1	2	2	5	5	6	6	5	5	6	6
1	1	2	2	5	5	6	6	5	5	6	6
5	5	5	5	1	1	1	1	1	1	1	1
5	5	5	5	1	1	1	1	1	1	1	1
5	5	6	6	1	1	2	2	1	1	2	2
5	5	6	6	1	1	2	2	1	1	2	2
5	5	5	5	1	1	1	1	1	1	1	1
5	5	5	5	1	1	1	1	1	1	1	1
5	5	6	6	1	1	2	2	1	1	2	2
5	5	6	6	1	1	2	2	1	1	2	2

 3.1×4.11

1	2	3	4	5	6	7	8	5	6	7	8
2	1	4	3	6	5	8	7	6	5	8	7
3	4	1	2	7	8	5	6	7	8	5	6
4	3	2	1	8	7	6	5	8	7	6	5
5	6	7	8	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3
7	8	5	6	3	4	1	2	3	4	1	2
8	7	6	5	4	3	2	1	4	3	2	1
5	6	7	8	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3
7	8	5	6	3	4	1	2	3	4	1	2
8	7	6	5	4	3	2	1	4	3	2	1

 3.1×4.31

1	2	1	2	5	6	5	6	5	6	5	6
2	1	2	1	6	5	6	5	6	5	6	5
1	2	1	2	5	6	5	6	5	6	5	6
2	1	2	1	6	5	6	5	6	5	6	5
5	6	5	6	1	2	1	2	1	2	1	2
6	5	6	5	2	1	2	1	2	1	2	1
5	6	5	6	1	2	1	2	1	2	1	2
6	5	6	5	2	1	2	1	2	1	2	1
5	6	5	6	1	2	1	2	1	2	1	2
6	5	6	5	2	1	2	1	2	1	2	1
5	6	5	6	1	2	1	2	1	2	1	2
6	5	6	5	2	1	2	1	2	1	2	1

 3.1×4.10

1	2	3	4	5	6	7	8	5	6	7	8
2	1	4	3	6	5	8	7	6	5	8	7
3	4	2	1	7	8	6	5	7	8	6	5
4	3	1	2	8	7	5	6	8	7	5	6
5	6	7	8	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3
7	8	6	5	3	4	2	1	3	4	2	1
7	8	5	6	4	3	1	2	4	3	1	2
5	6	7	8	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3
7	8	6	5	3	4	2	1	3	4	2	1
7	8	5	6	4	3	1	2	4	3	1	2

 3.1×4.15

1	2	1	2	5	6	5	6	5	6	5	6
2	1	2	1	6	5	6	5	6	5	6	5
1	2	3	4	5	6	7	8	5	6	7	8
2	1	4	3	6	5	8	7	6	5	8	7
5	6	5	6	1	2	1	2	1	2	1	2
6	5	6	5	2	1	2	1	2	1	2	1
5	6	7	8	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3
5	6	5	6	1	2	1	2	1	2	1	2
6	5	6	5	2	1	2	1	2	1	2	1
5	6	7	8	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3

 3.1×4.32

1	2	2	2	5	6	6	6	5	6	6	6
2	1	1	1	6	5	5	5	6	5	5	5
2	1	1	1	6	5	5	5	6	5	5	5
2	1	1	1	6	5	5	5	6	5	5	5
5	6	6	6	1	2	2	2	1	2	2	2
6	5	5	5	2	1	1	1	2	1	1	1
6	5	5	5	2	1	1	1	2	1	1	1
6	5	5	5	2	1	1	1	2	1	1	1
5	6	6	6	1	2	2	2	1	2	2	2
6	5	5	5	2	1	1	1	2	1	1	1
6	5	5	5	2	1	1	1	2	1	1	1
6	5	5	5	2	1	1	1	2	1	1	1

TABLE IV

DIRECT PRODUCTS: 3 x 4 (continued)

3.1 x 4.34

```

1 1 3 3 5 5 7 7 5 5 7 7
1 1 3 3 5 5 7 7 5 5 7 7
3 3 1 1 7 7 5 5 7 7 5 5
3 3 1 1 7 7 5 5 7 7 5 5
5 5 7 7 1 1 3 3 1 1 3 3
5 5 7 7 1 1 3 3 1 1 3 3
7 7 5 5 3 3 1 1 3 3 1 1
7 7 5 5 3 3 1 1 3 3 1 1
5 5 7 7 1 1 3 3 1 1 3 3
5 5 7 7 1 1 3 3 1 1 3 3
7 7 5 5 3 3 1 1 3 3 1 1
7 7 5 5 3 3 1 1 3 3 1 1
7 7 5 5 3 3 1 1 3 3 1 1
7 7 5 5 3 3 1 1 3 3 1 1

```

3.1 x 4.35

```

1 2 3 3 5 6 7 7 5 6 7 7
2 3 1 1 6 7 5 5 6 7 5 5
3 1 2 2 7 5 6 6 7 5 6 6
3 1 2 2 7 5 6 6 7 5 6 6
5 6 7 7 1 2 3 3 1 2 3 3
6 7 5 5 2 3 1 1 2 3 1 1
7 5 6 6 3 1 2 2 3 1 2 2
7 5 6 6 3 1 2 2 3 1 2 2
5 6 7 7 1 2 3 3 1 2 3 3
6 7 5 5 2 3 1 1 2 3 1 1
7 5 6 6 3 1 2 2 3 1 2 2
7 5 6 6 3 1 2 2 3 1 2 2

```

3.1 x 4.59

```

1 1 1 1 5 5 5 5 5 5 5 5
2 2 2 2 6 6 6 6 6 6 6 6
1 1 1 2 5 5 5 6 5 5 5 6
4 4 4 4 8 8 8 8 8 8 8 8
5 5 5 5 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2
5 5 5 6 1 1 1 2 1 1 1 2
8 8 8 8 4 4 4 4 4 4 4 4
5 5 5 5 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2
5 5 5 6 1 1 1 2 1 1 1 2
8 8 8 8 4 4 4 4 4 4 4 4

```

3.1 x 4.63

```

1 1 1 1 5 5 5 5 5 5 5 5
2 2 2 2 6 6 6 6 6 6 6 6
1 2 3 4 5 6 7 8 5 6 7 8
2 1 4 3 6 5 8 7 6 5 8 7
5 5 5 5 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2
5 6 7 8 1 2 3 4 1 2 3 4
6 5 8 7 2 1 4 3 2 1 4 3
5 5 5 5 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2
5 6 7 8 1 2 3 4 1 2 3 4
6 5 8 7 2 1 4 3 2 1 4 3

```

3.1 x 4.64

```

1 1 3 3 5 5 7 7 5 5 7 7
2 2 4 4 6 6 8 8 6 6 8 8
1 1 3 3 5 5 7 7 5 5 7 7
2 2 4 4 6 6 8 8 6 6 8 8
5 5 7 7 1 1 3 3 1 1 3 3
6 6 8 8 2 2 4 4 2 2 4 4
5 5 7 7 1 1 3 3 1 1 3 3
6 6 8 8 2 2 4 4 2 2 4 4
5 5 7 7 1 1 3 3 1 1 3 3
6 6 8 8 2 2 4 4 2 2 4 4
5 5 7 7 1 1 3 3 1 1 3 3
6 6 8 8 2 2 4 4 2 2 4 4

```

3.1 x 4.65

```

1 2 3 4 5 6 7 8 5 6 7 8
2 1 4 3 6 5 8 7 6 5 8 7
1 2 3 4 5 6 7 8 5 6 7 8
2 1 4 3 6 5 8 7 6 5 8 7
5 6 7 8 1 2 3 4 1 2 3 4
6 5 8 7 2 1 4 3 2 1 4 3
5 6 7 8 1 2 3 4 1 2 3 4
6 5 8 7 2 1 4 3 2 1 4 3
5 6 7 8 1 2 3 4 1 2 3 4
6 5 8 7 2 1 4 3 2 1 4 3
5 6 7 8 1 2 3 4 1 2 3 4
6 5 8 7 2 1 4 3 2 1 4 3

```

TABLE IV

DIRECT PRODUCTS: 3 x 4 (continued)

3.1 x 4.66

```

1 1 1 1 5 5 5 5 5 5 5
2 2 2 2 6 6 6 6 6 6 6
1 1 3 3 5 5 7 7 5 5 7 7
2 2 4 4 6 6 8 8 6 6 8 8
5 5 5 5 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2
5 5 7 7 1 1 3 3 1 1 3 3
6 6 8 8 2 2 4 4 2 2 4 4
5 5 5 5 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2
5 5 7 7 1 1 3 3 1 1 3 3
6 6 8 8 2 2 4 4 2 2 4 4

```

3.1 x 4.120

```

1 2 2 1 5 6 6 5 5 6 6 5
2 1 1 2 6 5 5 6 6 5 5 6
2 1 1 2 6 5 5 6 6 5 5 6
1 2 3 4 5 6 7 8 5 6 7 8
5 6 6 5 1 2 2 1 1 2 2 1
6 5 5 6 2 1 1 2 2 1 1 2
6 5 5 6 2 1 1 2 2 1 1 2
5 6 7 8 1 2 3 4 1 2 3 4
5 6 6 5 1 2 2 1 1 2 2 1
6 5 5 6 2 1 1 2 2 1 1 2
6 5 5 6 2 1 1 2 2 1 1 2
5 6 7 8 1 2 3 4 1 2 3 4

```

3.2 x 4.10

```

1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3
3 4 2 1 3 4 2 1 3 4 2 1
4 3 1 2 4 3 1 2 4 3 1 2
1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3
3 4 2 1 3 4 2 1 3 4 2 1
4 3 1 2 4 3 1 2 4 3 1 2
1 2 3 4 1 2 3 4 5 6 7 8
2 1 4 3 2 1 4 3 6 5 8 7
3 4 2 1 3 4 2 1 7 8 6 5
4 3 1 2 4 3 1 2 8 7 5 6

```

3.1 x 4.92

```

1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2
1 1 3 1 1 1 3 1 1 1 3 1
2 2 2 4 2 2 2 4 2 2 2 4
1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2
1 1 3 1 1 1 3 1 1 1 3 1
2 2 2 4 2 2 2 4 2 2 2 4
1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2
1 1 3 1 1 1 3 1 1 1 3 1
2 2 2 4 2 2 2 4 2 2 2 4

```

3.2 x 4.3

```

1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1
1 1 2 2 1 1 2 2 1 1 2 2
1 1 2 2 1 1 2 2 1 1 2 2
1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1
1 1 2 2 1 1 2 2 1 1 2 2
1 1 2 2 1 1 2 2 1 1 2 2
1 1 1 1 1 1 1 1 5 5 5 5
1 1 1 1 1 1 1 1 5 5 5 5
1 1 2 2 1 1 2 2 5 5 6 6
1 1 2 2 1 1 2 2 5 5 6 6

```

3.2 x 4.11

```

1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3
3 4 1 2 3 4 1 2 3 4 1 2
4 3 2 1 4 3 2 1 4 3 2 1
1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3
3 4 1 2 3 4 1 2 3 4 1 2
4 3 2 1 4 3 2 1 4 3 2 1
1 2 3 4 1 2 3 4 5 6 7 8
2 1 4 3 2 1 4 3 6 5 8 7
3 4 1 2 3 4 1 2 7 8 5 6
4 3 2 1 4 3 2 1 8 7 6 5

```

TABLE IV

DIRECT PRODUCTS: 3×4 (continued)

3.2 x 4.15	3.2 x 4.31
1 2 1 2 1 2 1 2 1 2 1 2	1 2 1 2 1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1 2 1 2 1	2 1 2 1 2 1 2 1 2 1 2 1
1 2 3 4 1 2 3 4 1 2 3 4	1 2 1 2 1 2 1 2 1 2 1 2
2 1 4 3 2 1 4 3 2 1 4 3	2 1 2 1 2 1 2 1 2 1 2 1
1 2 1 2 1 2 1 2 1 2 1 2	1 2 1 2 1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1 2 1 2 1	2 1 2 1 2 1 2 1 2 1 2 1
1 2 3 4 1 2 3 4 1 2 3 4	1 2 1 2 1 2 1 2 1 2 1 2
2 1 4 3 2 1 4 3 2 1 4 3	2 1 2 1 2 1 2 1 2 1 2 1
1 2 1 2 1 2 1 2 5 6 5 6	1 2 1 2 1 2 1 2 5 6 5 6
2 1 2 1 2 1 2 1 6 5 6 5	2 1 2 1 2 1 2 1 6 5 6 5
1 2 3 4 1 2 3 4 5 6 7 8	1 2 1 2 1 2 1 2 5 6 5 6
2 1 4 3 2 1 4 3 6 5 8 7	2 1 2 1 2 1 2 1 6 5 6 5
3.2 x 4.32	3.2 x 4.34
1 2 2 2 1 2 2 2 1 2 2 2	1 1 3 3 1 1 3 3 1 1 3 3
2 1 1 1 2 1 1 1 2 1 1 1	1 1 3 3 1 1 3 3 1 1 3 3
2 1 1 1 2 1 1 1 2 1 1 1	3 3 1 1 3 3 1 1 3 3 1 1
2 1 1 1 2 1 1 1 2 1 1 1	3 3 1 2 3 3 1 2 3 3 1 2
1 2 2 2 1 2 2 2 1 2 2 2	1 1 3 3 1 1 3 3 1 1 3 3
2 1 1 1 2 1 1 1 2 1 1 1	1 1 3 3 1 1 3 3 1 1 3 3
2 1 1 1 2 1 1 1 2 1 1 1	3 3 1 1 3 3 1 1 3 3 1 1
2 1 1 1 2 1 1 1 2 1 1 1	3 3 1 2 3 3 1 2 3 3 1 2
1 2 2 2 1 2 2 2 5 6 6 6	1 1 3 3 1 1 3 3 5 5 7 7
2 1 1 1 2 1 1 1 6 5 5 5	1 1 3 3 1 1 3 3 5 5 7 7
2 1 1 1 2 1 1 1 6 5 5 5	3 3 1 1 3 3 1 1 7 7 5 5
2 1 1 1 2 1 1 1 6 5 5 5	3 3 1 2 3 3 1 2 7 7 5 6
3.2 x 4.35	3.2 x 4.59
1 2 3 3 1 2 3 3 1 2 3 3	1 1 1 1 1 1 1 1 1 1 1 1
2 3 1 1 2 3 1 1 2 3 1 1	2 2 2 2 2 2 2 2 2 2 2 2
3 1 2 2 3 1 2 2 3 1 2 2	1 1 1 2 1 1 1 2 1 1 1 2
3 1 2 2 3 1 2 2 3 1 2 2	4 4 4 4 4 4 4 4 4 4 4 4
1 2 3 3 1 2 3 3 1 2 3 3	1 1 1 1 1 1 1 1 1 1 1 1
2 3 1 1 2 3 1 1 2 3 1 1	2 2 2 2 2 2 2 2 2 2 2 2
3 1 2 2 3 1 2 2 3 1 2 2	1 1 1 2 1 1 1 2 1 1 1 2
3 1 2 2 3 1 2 2 3 1 2 2	4 4 4 4 4 4 4 4 4 4 4 4
1 2 3 3 1 2 3 3 5 6 7 7	1 1 1 1 1 1 1 1 5 5 5 5
2 3 1 1 2 3 1 1 6 7 5 5	2 2 2 2 2 2 2 2 6 6 6 6
3 1 2 2 3 1 2 2 7 5 6 6	1 1 1 2 1 1 1 2 5 5 5 6
3 1 2 2 3 1 2 2 7 5 6 6	4 4 4 4 4 4 4 4 8 8 8 8

TABLE IV
DIRECT PRODUCTS: 3×4 (continued)

3.2 x 4.63	3.2 x 4.64
1 1 1 1 1 1 1 1 1 1 1 1	1 1 3 3 1 1 3 3 1 1 3 3
2 2 2 2 2 2 2 2 2 2 2 2	2 2 4 4 2 2 4 4 2 2 4 4
1 2 3 4 1 2 3 4 1 2 3 4	1 1 3 3 1 1 3 3 1 1 3 3
2 1 4 3 2 1 4 3 2 1 4 3	2 2 4 4 2 2 4 4 2 2 4 4
1 1 1 1 1 1 1 1 1 1 1 1	1 1 3 3 1 1 3 3 1 1 3 3
2 2 2 2 2 2 2 2 2 2 2 2	2 2 4 4 2 2 4 4 2 2 4 4
1 2 3 4 1 2 3 4 1 2 3 4	1 1 3 3 1 1 3 3 1 1 3 3
2 1 4 3 2 1 4 3 2 1 4 3	2 2 4 4 2 2 4 4 2 2 4 4
1 1 1 1 1 1 1 1 5 5 5 5	1 1 3 3 1 1 3 3 5 5 7 7
2 2 2 2 2 2 2 2 6 6 6 6	2 2 4 4 2 2 4 4 6 6 8 8
1 2 3 4 1 2 3 4 5 6 7 8	1 1 3 3 1 1 3 3 5 5 7 7
2 1 4 3 2 1 4 3 6 5 8 7	2 2 4 4 2 2 4 4 6 6 8 8
3.2 x 4.65	3.2 x 4.66
1 2 3 4 1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1 1 1 1 1
2 1 4 3 2 1 4 3 2 1 4 3	2 2 2 2 2 2 2 2 2 2 2 2
1 2 3 4 1 2 3 4 1 2 3 4	1 1 3 3 1 1 3 3 1 1 3 3
2 1 4 3 2 1 4 3 2 1 4 3	2 2 4 4 2 2 4 4 2 2 4 4
1 2 3 4 1 2 3 4 1 2 3 4	1 1 1 1 1 1 1 1 1 1 1 1
2 1 4 3 2 1 4 3 2 1 4 3	2 2 2 2 2 2 2 2 2 2 2 2
1 2 3 4 1 2 3 4 1 2 3 4	1 1 3 3 1 1 3 3 1 1 3 3
2 1 4 3 2 1 4 3 2 1 4 3	2 2 4 4 2 2 4 4 2 2 4 4
1 2 3 4 1 2 3 4 5 6 7 8	1 1 1 1 1 1 1 1 5 5 5 5
2 1 4 3 2 1 4 3 6 5 8 7	2 2 2 2 2 2 2 2 6 6 6 6
1 2 3 4 1 2 3 4 5 6 7 8	1 1 3 3 1 1 3 3 5 5 7 7
2 1 4 3 2 1 4 3 6 5 8 7	2 2 4 4 2 2 4 4 6 6 8 8
3.2 x 4.92	3.2 x 4.120
1 1 1 1 1 1 1 1 1 1 1 1	1 2 2 1 1 2 2 1 1 2 2 1
2 2 2 2 2 2 2 2 2 2 2 2	2 1 1 2 2 1 1 2 2 1 1 2
1 1 3 1 1 1 3 1 1 1 3 1	2 1 1 2 2 1 1 2 2 1 1 2
2 2 2 4 2 2 2 4 2 2 2 4	1 2 3 4 1 2 3 4 1 2 3 4
1 1 1 1 1 1 1 1 1 1 1 1	1 2 2 1 1 2 2 1 1 2 2 1
2 2 2 2 2 2 2 2 2 2 2 2	2 1 1 2 2 1 1 2 2 1 1 2
1 1 3 1 1 1 3 1 1 1 3 1	2 1 1 2 2 1 1 2 2 1 1 2
2 2 2 4 2 2 2 4 2 2 2 4	1 2 3 4 1 2 3 4 1 2 3 4
1 1 1 1 1 1 1 1 5 5 5 5	1 2 2 1 1 2 2 1 5 6 6 5
2 2 2 2 2 2 2 2 6 6 6 6	2 1 1 2 2 1 1 2 6 5 5 6
1 1 3 1 1 1 3 1 5 5 7 5	2 1 1 2 2 1 1 2 6 5 5 6
2 2 2 4 2 2 2 4 6 6 6 8	1 2 3 4 1 2 3 4 5 6 7 8

TABLE IV

DIRECT PRODUCTS: 3 x 4 (continued)

3.3 x 4.3

1	1	1	1	5	5	5	5	9	9	9	9
1	1	1	1	5	5	5	5	9	9	9	9
1	1	2	2	5	5	6	6	9	9	10	10
1	1	2	2	5	5	6	6	9	9	10	10
5	5	5	5	9	9	9	9	1	1	1	1
5	5	5	5	9	9	9	9	1	1	1	1
5	5	6	6	9	9	10	10	1	1	2	2
5	5	6	6	9	9	10	10	1	1	2	2
9	9	9	9	1	1	1	1	5	5	5	5
9	9	9	9	1	1	1	1	5	5	5	5
9	9	10	10	1	1	2	2	5	5	6	6
9	9	10	10	1	1	2	2	5	5	6	6

3.3 x 4.10

1	2	3	4	5	6	7	8	9	10	11	12
2	1	4	3	6	5	8	7	10	9	12	11
3	4	2	1	7	8	6	5	11	12	10	9
4	3	1	2	8	7	5	6	12	11	9	10
5	6	7	8	9	10	11	12	1	2	3	4
6	5	8	7	10	9	12	11	2	1	4	3
7	8	6	5	11	12	10	9	3	4	2	1
8	7	5	6	12	11	9	10	4	3	1	2
9	10	11	12	1	2	3	4	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7
11	12	10	9	3	4	2	1	7	8	6	5
12	11	9	10	4	3	1	2	8	7	5	6

3.3 x 4.11

1	2	3	4	5	6	7	8	9	10	11	12
2	1	4	3	6	5	8	7	10	9	12	11
3	4	1	2	7	8	5	6	11	12	9	10
4	3	2	1	8	7	6	5	12	11	10	9
5	6	7	8	9	10	11	12	1	2	3	4
6	5	8	7	10	9	12	11	2	1	4	3
7	8	5	6	11	12	9	10	3	4	1	2
8	7	6	5	12	11	10	9	4	3	2	1
9	10	11	12	1	2	3	4	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7
11	12	9	10	3	4	1	2	7	8	5	6
12	11	10	9	4	3	2	1	8	7	6	5

3.3 x 4.15

1	2	1	2	5	6	5	6	9	10	9	10
2	1	2	1	6	5	6	5	10	9	10	9
1	2	3	4	5	6	7	8	9	10	11	12
2	1	4	3	6	5	8	7	10	9	12	11
5	6	5	6	9	10	9	10	1	2	1	2
6	5	6	5	10	9	10	9	2	1	2	1
5	6	7	8	9	10	11	12	1	2	3	4
6	5	8	7	10	9	12	11	2	1	4	3
9	10	9	10	1	2	1	2	5	6	5	6
10	9	10	9	2	1	2	1	6	5	6	5
9	10	11	12	1	2	3	4	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7

3.3 x 4.31

1	2	1	2	5	6	5	6	9	10	9	10
2	1	2	1	6	5	6	5	10	9	10	9
1	2	1	2	5	6	5	6	9	10	9	10
2	1	2	1	6	5	6	5	10	9	10	9
5	6	5	6	9	10	9	10	1	2	1	2
6	5	6	5	10	9	10	9	2	1	2	1
5	6	5	6	9	10	9	10	1	2	1	2
6	5	6	5	10	9	10	9	2	1	2	1
9	10	9	10	1	2	1	2	5	6	5	6
10	9	10	9	2	1	2	1	6	5	6	5
9	10	9	10	1	2	1	2	5	6	5	6
10	9	10	9	2	1	2	1	6	5	6	5

3.3 x 4.32

1	2	2	2	5	6	6	6	9	10	10	10
2	1	1	1	6	5	5	5	10	9	9	9
2	1	1	1	6	5	5	5	10	9	9	9
2	1	1	1	6	5	5	5	10	9	9	9
5	6	6	6	9	10	10	10	1	2	2	2
6	5	5	5	10	9	9	9	2	1	1	1
6	5	5	5	10	9	9	9	2	1	1	1
6	5	5	5	10	9	9	9	2	1	1	1
9	10	10	10	1	2	2	2	5	6	6	6
10	9	9	9	2	1	1	1	6	5	5	5
10	9	9	9	2	1	1	1	6	5	5	5
10	9	9	9	2	1	1	1	6	5	5	5

TABLE IV

DIRECT PRODUCTS: 3 x 4 (continued)

3.3 x 4.34

1	1	3	3	5	5	7	7	9	9	11	11
1	1	3	3	5	5	7	7	9	9	11	11
3	3	1	1	7	7	5	5	11	11	9	9
3	3	1	2	7	7	5	6	11	11	9	10
5	5	7	7	9	9	11	11	1	1	3	3
5	5	7	7	9	9	11	11	1	1	3	3
7	7	5	5	11	11	9	9	3	3	1	1
7	7	5	6	11	11	9	10	3	3	1	2
9	9	11	11	1	1	3	3	5	5	7	7
9	9	11	11	1	1	3	3	5	5	7	7
11	11	9	9	3	3	1	1	7	7	5	5
11	11	9	10	3	3	1	2	7	7	5	6

3.3 x 4.35

1	2	3	3	5	6	7	7	9	10	11	11
2	3	1	1	6	7	5	5	10	11	9	9
3	1	2	2	7	5	6	6	11	9	10	10
3	1	2	2	7	5	6	6	11	9	10	10
5	6	7	7	9	10	11	11	1	2	3	3
6	7	5	5	10	11	9	9	2	3	1	1
7	5	6	6	11	9	10	10	3	1	2	2
7	5	6	6	11	9	10	10	3	1	2	2
9	10	11	11	1	2	3	3	5	6	7	7
10	11	9	9	2	3	1	1	6	7	5	5
11	9	10	10	3	1	2	2	7	5	6	6
11	9	10	10	3	1	2	2	7	5	6	6

3.3 x 4.59

1	1	1	1	5	5	5	5	9	9	9	9
2	2	2	2	6	6	6	6	10	10	10	10
1	1	1	2	5	5	5	6	9	9	9	10
4	4	4	4	8	8	8	8	12	12	12	12
5	5	5	5	9	9	9	9	1	1	1	1
6	6	6	6	10	10	10	10	2	2	2	2
5	5	5	6	9	9	9	10	1	1	1	2
8	8	8	8	12	12	12	12	4	4	4	4
9	9	9	9	1	1	1	1	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6
9	9	9	10	1	1	1	2	5	5	5	6
12	12	12	12	4	4	4	4	8	8	8	8

3.3 x 4.63

1	1	1	1	5	5	5	5	9	9	9	9
2	2	2	2	6	6	6	6	10	10	10	10
1	2	3	4	5	6	7	8	9	10	11	12
2	1	4	3	6	5	8	7	10	9	12	11
5	5	5	5	9	9	9	9	1	1	1	1
6	6	6	6	10	10	10	10	2	2	2	2
5	6	7	8	9	10	11	12	1	2	3	4
6	5	8	7	10	9	12	11	2	1	4	3
9	9	9	9	1	1	1	1	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6
9	10	11	12	1	2	3	4	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7

3.3 x 4.64

1	1	3	3	5	5	7	7	9	9	11	11
2	2	4	4	6	6	8	8	10	10	12	12
1	1	3	3	5	5	7	7	9	9	11	11
2	2	4	4	6	6	8	8	10	10	12	12
5	5	7	7	9	9	11	11	1	1	3	3
6	6	8	8	10	10	12	12	2	2	4	4
5	5	7	7	9	9	11	11	1	1	3	3
6	6	8	8	10	10	12	12	2	2	4	4
9	9	11	11	1	1	3	3	5	5	7	7
10	10	12	12	2	2	4	4	6	6	8	8
9	9	11	11	1	1	3	3	5	5	7	7
10	10	12	12	2	2	4	4	6	6	8	8

3.3 x 4.65

1	2	3	4	5	6	7	8	9	10	11	12
2	1	4	3	6	5	8	7	10	9	12	11
1	2	3	4	5	6	7	8	9	10	11	12
2	1	4	3	6	5	8	7	10	9	12	11
5	6	7	8	9	10	11	12	1	2	3	4
6	5	8	7	10	9	12	11	2	1	4	3
5	6	7	8	9	10	11	12	1	2	3	4
6	5	8	7	10	9	12	11	2	1	4	3
9	10	11	12	1	2	3	4	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7
9	10	11	12	1	2	3	4	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7

TABLE IV

DIRECT PRODUCTS: 3 x 4 (continued)

3.3 x 4.66												3.3 x 4.92											
1	1	1	1	5	5	5	5	9	9	9	9	1	1	1	1	5	5	5	5	9	9	9	9
2	2	2	2	6	6	6	6	10	10	10	10	2	2	2	2	6	6	6	6	10	10	10	10
1	1	3	3	5	5	7	7	9	9	11	11	1	1	3	1	5	5	7	5	9	9	11	9
2	2	4	4	6	6	8	8	10	10	12	12	2	2	4	2	6	6	8	6	10	10	12	12
5	5	5	5	9	9	9	9	1	1	1	1	5	5	5	5	9	9	9	9	1	1	1	1
6	6	6	6	10	10	10	10	2	2	2	2	6	6	6	6	10	10	10	10	2	2	2	2
5	5	7	7	9	9	11	11	1	1	3	3	5	5	7	5	9	9	11	9	1	1	3	1
6	6	8	8	10	10	12	12	2	2	4	4	6	6	8	10	10	10	12	2	2	2	4	
9	9	9	9	1	1	1	1	5	5	5	5	9	9	9	9	1	1	1	1	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6	10	10	10	10	2	2	2	2	6	6	6	6
9	9	11	11	1	1	3	3	5	5	7	7	9	9	11	9	1	1	3	1	5	5	7	5
10	10	12	12	2	2	4	4	6	6	8	8	10	10	12	12	2	2	2	4	6	6	8	8
3.3 x 4.120																							
1	2	2	1	5	6	6	5	9	10	10	9	1	2	2	1	5	6	6	5	9	10	10	9
2	1	1	2	6	5	5	6	10	9	9	10	2	1	1	2	6	5	5	6	10	9	9	10
2	1	1	2	6	5	5	6	10	9	9	10	2	1	1	2	6	5	5	6	10	9	9	10
1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12
5	6	6	5	9	10	10	9	1	2	2	1	5	6	6	5	9	10	10	9	1	2	2	1
6	5	5	6	10	9	9	10	2	1	1	2	6	5	5	6	10	9	9	10	2	1	1	2
6	5	5	6	10	9	9	10	2	1	1	2	6	5	5	6	10	9	9	10	2	1	1	2
5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4
9	10	10	9	1	2	2	1	5	6	6	5	9	10	10	9	1	2	2	1	5	6	6	5
10	9	9	10	2	1	1	2	6	5	5	6	10	9	9	10	2	1	1	2	6	5	5	6
10	9	9	10	2	1	1	2	6	5	5	6	10	9	9	10	2	1	1	2	6	5	5	6
9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8

TABLE V

DIRECT PRODUCTS: 4×4 4.3×4.3

```

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2
1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2
1 1 2 2 1 1 2 2 1 1 2 2 1 1 2 2
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5
1 1 2 2 1 1 2 2 5 5 6 6 5 5 6 6
1 1 2 2 1 1 2 2 5 5 6 6 5 5 6 6
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5
1 1 2 2 1 1 2 2 5 5 6 6 5 5 6 6
1 1 2 2 1 1 2 2 5 5 6 6 5 5 6 6

```

 4.3×4.10

```

1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
3 4 2 1 3 4 2 1 3 4 2 1 3 4 2 1
4 3 1 2 4 3 1 2 4 3 1 2 4 3 1 2
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
3 4 2 1 3 4 2 1 3 4 2 1 3 4 2 1
4 3 1 2 4 3 1 2 4 3 1 2 4 3 1 2
1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
3 4 2 1 3 4 2 1 7 8 6 5 7 8 6 5
4 3 1 2 4 3 1 2 8 7 5 6 8 7 5 6
1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
3 4 2 1 3 4 2 1 7 8 6 5 7 8 6 5
4 3 1 2 4 3 1 2 8 7 5 6 8 7 5 6

```

 4.3×4.11

```

1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2
4 3 2 1 4 3 2 1 4 3 2 1 4 3 2 1
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2
4 3 2 1 4 3 2 1 4 3 2 1 4 3 2 1
1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
3 4 1 2 3 4 1 2 7 8 5 6 7 8 5 6
4 3 2 1 4 3 2 1 8 7 6 5 8 7 6 5
1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
3 4 1 2 3 4 1 2 7 8 5 6 7 8 5 6
4 3 2 1 4 3 2 1 8 7 6 5 8 7 6 5

```

 4.3×4.15

```

1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
1 2 1 2 1 2 1 2 5 6 5 6 5 6 5 6
2 1 2 1 2 1 2 1 6 5 6 5 6 5 6 5
1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
1 2 1 2 1 2 1 2 5 6 5 6 5 6 5 6
2 1 2 1 2 1 2 1 6 5 6 5 6 5 6 5
1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7

```

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.3 x 4.31

```

1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
1 2 1 2 1 2 1 2 5 6 5 6 5 6 5 6
2 1 2 1 2 1 2 1 6 5 6 5 6 5 6 5
1 2 1 2 1 2 1 2 5 6 5 6 5 6 5 6
2 1 2 1 2 1 2 1 6 5 6 5 6 5 6 5
1 2 1 2 1 2 1 2 5 6 5 6 5 6 5 6
2 1 2 1 2 1 2 1 6 5 6 5 6 5 6 5
1 2 1 2 1 2 1 2 5 6 5 6 5 6 5 6
2 1 2 1 2 1 2 1 6 5 6 5 6 5 6 5
1 2 1 2 1 2 1 2 5 6 5 6 5 6 5 6
2 1 2 1 2 1 2 1 6 5 6 5 6 5 6 5

```

4.3 x 4.32

```

1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2
2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1
2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1
2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1
1 2 2 2 1 2 2 2 1 2 2 2 1 2 2 2
2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1
2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1
2 1 1 1 2 1 1 1 2 1 1 1 2 1 1 1
1 2 2 2 1 2 2 2 5 6 6 6 5 6 6 6
2 1 1 1 2 1 1 1 6 5 5 5 6 5 5 5
2 1 1 1 2 1 1 1 6 5 5 5 6 5 5 5
2 1 1 1 2 1 1 1 6 5 5 5 6 5 5 5
1 2 2 2 1 2 2 2 5 6 6 6 5 6 6 6
2 1 1 1 2 1 1 1 6 5 5 5 6 5 5 5
2 1 1 1 2 1 1 1 6 5 5 5 6 5 5 5
2 1 1 1 2 1 1 1 6 5 5 5 6 5 5 5

```

4.3 x 4.34

```

1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3
3 3 1 1 3 3 1 1 3 3 1 1 3 3 1 1
3 3 1 2 3 3 1 2 3 3 1 2 3 3 1 2
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3
3 3 1 1 3 3 1 1 3 3 1 1 3 3 1 1
3 3 1 2 3 3 1 2 3 3 1 2 3 3 1 2
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7
3 3 1 1 3 3 1 1 7 7 5 5 7 7 5 5
3 3 1 2 3 3 1 2 7 7 5 6 7 7 5 6
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7
3 3 1 1 3 3 1 1 7 7 5 5 7 7 5 5
3 3 1 2 3 3 1 2 7 7 5 6 7 7 5 6

```

4.3 x 4.35

```

1 2 3 3 1 2 3 3 1 2 3 3 1 2 3 3
2 3 1 1 2 3 1 1 2 3 1 1 2 3 1 1
3 1 2 2 3 1 2 2 3 1 2 2 3 1 2 2
3 1 2 2 3 1 2 2 3 1 2 2 3 1 2 2
1 2 3 3 1 2 3 3 1 2 3 3 1 2 3 3
2 3 1 1 2 3 1 1 2 3 1 1 2 3 1 1
3 1 2 2 3 1 2 2 3 1 2 2 3 1 2 2
3 1 2 2 3 1 2 2 3 1 2 2 3 1 2 2
1 2 3 3 1 2 3 3 5 6 7 7 5 6 7 7
2 3 1 1 2 3 1 1 6 7 5 5 6 7 5 5
3 1 2 2 3 1 2 2 7 5 6 6 7 5 6 6
3 1 2 2 3 1 2 2 7 5 6 6 7 5 6 6
1 2 3 3 1 2 3 3 5 6 7 7 5 6 7 7
2 3 1 1 2 3 1 1 6 7 5 5 6 7 5 5
3 1 2 2 3 1 2 2 7 5 6 6 7 5 6 6
3 1 2 2 3 1 2 2 7 5 6 6 7 5 6 6

```

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.3 x 4.59	4.3 x 4.63
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2	1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
1 1 1 2 1 1 1 2 1 1 1 2 1 1 1 2	1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5	1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5
2 2 2 2 2 2 2 2 6 6 6 6 6 6 6 6	2 2 2 2 2 2 2 2 6 6 6 6 6 6 6 6
1 1 1 2 1 1 1 2 5 5 5 6 5 5 5 6	1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
4 4 4 4 4 4 4 4 8 8 8 8 8 8 8 8	2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5	1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5
2 2 2 2 2 2 2 2 6 6 6 6 6 6 6 6	2 2 2 2 2 2 2 2 6 6 6 6 6 6 6 6
1 1 1 2 1 1 1 2 5 5 5 6 5 5 5 6	1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
4 4 4 4 4 4 4 4 8 8 8 8 8 8 8 8	2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
4.3 x 4.64	4.3 x 4.65
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3	1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4	2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7	1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 2 4 4 2 2 4 4 6 6 8 8 6 6 8 8	2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7	1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 2 4 4 2 2 4 4 6 6 8 8 6 6 8 8	2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7	1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 2 4 4 2 2 4 4 6 6 8 8 6 6 8 8	2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7	1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8
2 2 4 4 2 2 4 4 6 6 8 8 6 6 8 8	2 1 4 3 2 1 4 3 6 5 8 7 6 5 8 7

TABLE V
DIRECT PRODUCTS: 4 x 4 (continued)

4.3 x 4.66	4.3 x 4.92
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3	1 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1
2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4	2 2 2 4 2 2 2 4 2 2 2 4 2 2 2 4
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
1 1 3 3 1 1 3 3 1 1 3 3 1 1 3 3	1 1 3 1 1 1 3 1 1 1 3 1 1 1 3 1
2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4	2 2 2 4 2 2 2 4 2 2 2 4 2 2 2 4
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5	1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5
2 2 2 2 2 2 2 2 6 6 6 6 6 6 6 6	2 2 2 2 2 2 2 2 6 6 6 6 6 6 6 6
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7	1 1 3 1 1 1 3 1 5 5 7 5 5 5 7 5
2 2 4 4 2 2 4 4 6 6 8 8 6 6 8 8	2 2 2 4 2 2 2 4 6 6 8 6 6 6 8 8
1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5	1 1 1 1 1 1 1 1 5 5 5 5 5 5 5 5
2 2 2 2 2 2 2 2 6 6 6 6 6 6 6 6	2 2 2 2 2 2 2 2 6 6 6 6 6 6 6 6
1 1 3 3 1 1 3 3 5 5 7 7 5 5 7 7	1 1 3 1 1 1 3 1 5 5 7 5 5 5 7 5
2 2 4 4 2 2 4 4 6 6 8 8 6 6 8 8	2 2 2 4 2 2 2 4 6 6 8 6 6 6 8 8
4.3 x 4.120	
1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1	
2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2	
2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2	
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4	
1 2 2 1 1 2 2 1 1 2 2 1 1 2 2 1	
2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2	
2 1 1 2 2 1 1 2 2 1 1 2 2 1 1 2	
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4	
1 2 2 1 1 2 2 1 5 6 6 5 5 6 6 5	
2 1 1 2 2 1 1 2 6 5 5 6 6 5 5 6	
2 1 1 2 2 1 1 2 6 5 5 6 6 5 5 6	
1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8	
1 2 2 1 1 2 2 1 5 6 6 5 5 6 6 5	
2 1 1 2 2 1 1 2 6 5 5 6 6 5 5 6	
2 1 1 2 2 1 1 2 6 5 5 6 6 5 5 6	
1 2 3 4 1 2 3 4 5 6 7 8 5 6 7 8	

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.10×4.10

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
3	4	2	1	7	8	6	5	11	12	10	9	15	16	14	13
4	3	1	2	8	7	5	6	12	11	9	10	16	15	13	14
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
7	8	6	5	3	4	2	1	15	16	14	13	11	12	10	9
8	7	5	6	4	3	1	2	16	15	13	14	12	11	9	10
9	10	11	12	13	14	15	16	5	6	7	8	1	2	3	4
10	9	12	11	14	13	16	15	6	5	8	7	2	1	4	3
11	12	10	9	15	16	14	13	7	8	6	5	3	4	2	1
12	11	9	10	16	15	13	14	8	7	5	6	4	3	1	2
13	14	15	16	9	10	11	12	1	2	3	4	5	6	7	8
14	13	16	15	10	9	12	11	2	1	4	3	6	5	8	7
15	16	14	13	11	12	10	9	3	4	2	1	7	8	6	5
16	15	13	14	12	11	9	10	4	3	1	2	8	7	5	6

 4.10×4.11

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
3	4	1	2	7	8	5	6	11	12	9	10	15	16	13	14
4	3	2	1	8	7	6	5	12	11	10	9	16	15	14	13
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
7	8	5	6	3	4	1	2	15	16	13	14	11	12	9	10
8	7	6	5	4	3	2	1	16	15	14	13	12	11	10	9
9	10	11	12	13	14	15	16	5	6	7	8	1	2	3	4
10	9	12	11	14	13	16	15	6	5	8	7	2	1	4	3
11	12	9	10	15	16	13	14	7	8	5	6	3	4	1	2
12	11	10	9	16	15	14	13	8	7	6	5	4	3	2	1
13	14	15	16	9	10	11	12	1	2	3	4	5	6	7	8
14	13	16	15	10	9	12	11	2	1	4	3	6	5	8	7
15	16	13	14	11	12	9	10	3	4	1	2	7	8	5	6
16	15	14	13	12	11	10	9	4	3	2	1	8	7	6	5

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.10×4.15

1	2	1	2	5	6	5	6	9	10	9	10	13	14	13	14
2	1	2	1	6	5	6	5	10	9	10	9	14	13	14	13
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	6	5	6	1	2	1	2	13	14	13	14	9	10	9	10
6	5	6	5	2	1	2	1	14	13	14	13	10	9	10	9
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
9	10	9	10	13	14	13	14	5	6	5	6	1	2	1	2
10	9	10	9	14	13	14	13	6	5	6	5	2	1	2	1
9	10	11	12	13	14	15	16	5	6	7	8	1	2	3	4
10	9	12	11	14	13	16	15	6	5	8	7	2	1	4	3
13	14	13	14	9	10	9	10	1	2	1	2	5	6	5	6
14	13	14	13	10	9	10	9	2	1	2	1	6	5	6	5
13	14	15	16	9	10	11	12	1	2	3	4	5	6	7	8
14	13	16	15	10	9	12	11	2	1	4	3	6	5	8	7

 4.10×4.31

1	2	1	2	5	6	5	6	9	10	9	10	13	14	13	14
2	1	2	1	6	5	6	5	10	9	10	9	14	13	14	13
1	2	1	2	5	6	5	6	9	10	9	10	13	14	13	14
2	1	2	1	6	5	6	5	10	9	10	9	14	13	14	13
5	6	5	6	1	2	1	2	13	14	13	14	9	10	9	10
6	5	6	5	2	1	2	1	14	13	14	13	10	9	10	9
5	6	5	6	1	2	1	2	13	14	13	14	9	10	9	10
6	5	6	5	2	1	2	1	14	13	14	13	10	9	10	9
9	10	9	10	13	14	13	14	5	6	5	6	1	2	1	2
10	9	10	9	14	13	14	13	6	5	6	5	2	1	2	1
9	10	9	10	13	14	13	14	5	6	5	6	1	2	1	2
10	9	10	9	14	13	14	13	6	5	6	5	2	1	2	1
13	14	13	14	9	10	9	10	1	2	1	2	5	6	5	6
14	13	14	13	10	9	10	9	2	1	2	1	6	5	6	5
13	14	13	14	9	10	9	10	1	2	1	2	5	6	5	6
14	13	14	13	10	9	10	9	2	1	2	1	6	5	6	5

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.10×4.32

1	2	2	2	5	6	6	6	9	10	10	10	13	14	14	14
2	1	1	1	6	5	5	5	10	9	9	9	14	13	13	13
2	1	1	1	6	5	5	5	10	9	9	9	14	13	13	13
2	1	1	1	6	5	5	5	10	9	9	9	14	13	13	13
5	6	6	6	1	2	2	2	13	14	14	14	9	10	10	10
6	5	5	5	2	1	1	1	14	13	13	13	10	9	9	9
6	5	5	5	2	1	1	1	14	13	13	13	10	9	9	9
6	5	5	5	2	1	1	1	14	13	13	13	10	9	9	9
9	10	10	10	13	14	14	14	5	6	6	6	1	2	2	2
10	9	9	9	14	13	13	13	6	5	5	5	2	1	1	1
10	9	9	9	14	13	13	13	6	5	5	5	2	1	1	1
10	9	9	9	14	13	13	13	6	5	5	5	2	1	1	1
13	14	14	14	9	10	10	10	1	2	2	2	5	6	6	6
14	13	13	13	10	9	9	9	2	1	1	1	6	5	5	5
14	13	13	13	10	9	9	9	2	1	1	1	6	5	5	5
14	13	13	13	10	9	9	9	2	1	1	1	6	5	5	5

 4.10×4.34

1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
3	3	1	1	7	7	5	5	11	11	9	9	15	15	13	13
3	3	1	2	7	7	5	6	11	11	9	10	15	15	13	14
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
7	7	5	5	3	3	1	1	15	15	13	13	11	11	9	9
7	7	5	6	3	3	1	2	15	15	13	14	11	11	9	10
9	9	11	11	13	13	15	15	5	5	7	7	1	1	3	3
9	9	11	11	13	13	15	15	5	5	7	7	1	1	3	3
11	11	9	9	15	15	13	13	7	7	5	5	3	3	1	1
11	11	9	10	15	15	13	14	7	7	5	6	3	3	1	2
13	13	15	15	9	9	11	11	1	1	3	3	5	5	7	7
13	13	15	15	9	9	11	11	1	1	3	3	5	5	7	7
15	15	13	13	1	1	9	9	3	3	1	1	7	7	5	5
15	15	13	14	11	11	9	10	3	3	1	2	7	7	5	6

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.10 x 4.35

1	2	3	3	5	6	7	7	9	10	11	11	13	14	15	15
2	3	1	1	6	7	5	5	10	11	9	9	14	15	13	13
3	1	2	2	7	5	6	6	11	9	10	10	15	13	14	14
3	1	2	2	7	5	6	6	11	9	10	10	15	13	14	14
5	6	7	7	1	2	3	3	13	14	15	15	9	10	11	11
6	7	5	5	2	3	1	1	14	15	13	13	10	11	9	9
7	5	6	6	3	1	2	2	15	13	14	14	11	9	10	10
7	5	6	6	3	1	2	2	15	13	14	14	11	9	10	10
9	10	11	11	13	14	15	15	5	6	7	7	1	2	3	3
10	11	9	9	14	15	13	13	6	7	5	5	2	3	1	1
11	9	10	10	15	13	14	14	7	5	6	6	3	1	2	2
11	9	10	10	15	13	14	14	7	5	6	6	3	1	2	2
13	14	15	15	9	10	11	11	1	2	3	3	5	6	7	7
14	15	13	13	10	11	9	9	2	3	1	1	6	7	5	5
15	13	14	14	11	9	10	10	3	1	2	2	7	5	6	6
15	13	14	14	11	9	10	10	3	1	2	2	7	5	6	6

4.10 x 4.59

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	1	2	5	5	5	6	9	9	9	10	13	13	13	14
4	4	4	4	8	8	8	8	12	12	12	12	16	16	16	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	5	6	1	1	1	2	13	13	13	14	9	9	9	10
8	8	8	8	4	4	4	4	16	16	16	16	12	12	12	12
9	9	9	9	13	13	13	13	5	5	5	5	1	1	1	1
10	10	10	10	14	14	14	14	6	6	6	6	2	2	2	2
9	9	9	10	13	13	13	14	5	5	5	6	1	1	1	2
12	12	12	12	16	16	16	16	8	8	8	8	4	4	4	4
13	13	13	13	9	9	9	9	1	1	1	1	5	5	5	5
14	14	14	14	10	10	10	10	2	2	2	2	6	6	6	6
13	13	13	14	9	9	9	10	1	1	1	2	5	5	5	6
16	16	16	16	12	12	12	12	4	4	4	4	8	8	8	8

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.10×4.63

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
9	9	9	9	13	13	13	13	5	5	5	5	1	1	1	1
10	10	10	10	14	14	14	14	6	6	6	6	2	2	2	2
9	10	11	12	13	14	15	16	5	6	7	8	1	2	3	4
10	9	12	11	14	13	16	15	6	5	8	7	2	1	4	3
13	13	13	13	9	9	9	9	1	1	1	1	5	5	5	5
14	14	14	14	10	10	10	10	2	2	2	2	6	6	6	6
13	14	15	16	9	10	11	12	1	2	3	4	5	6	7	8
14	13	16	15	10	9	12	11	2	1	4	3	6	5	8	7

 4.10×4.64

1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12
9	9	11	11	13	13	15	15	5	5	7	7	1	1	3	3
10	10	12	12	14	14	16	16	6	6	8	8	2	2	4	4
9	9	1	1	13	13	15	15	5	5	7	7	1	1	3	3
10	10	12	12	14	14	16	16	6	6	8	8	2	2	4	4
13	13	15	15	9	9	11	11	1	1	3	3	5	5	7	7
14	14	16	16	10	10	12	12	2	2	4	4	6	6	8	8
13	13	15	15	9	9	11	11	1	1	3	3	5	5	7	7
14	14	16	16	10	10	12	12	2	2	4	4	6	6	8	8

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.10×4.65

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
9	10	11	12	13	14	15	16	5	6	7	8	1	2	3	4
10	9	12	11	14	13	16	15	6	5	8	7	2	1	4	3
9	10	11	12	13	14	15	16	5	6	7	8	1	2	3	4
10	9	12	11	14	13	16	15	6	5	8	7	2	1	4	3
13	14	15	16	9	10	11	12	1	2	3	4	5	6	7	8
14	13	16	15	10	9	12	11	2	1	4	3	6	5	8	7
13	14	15	16	9	10	11	12	1	2	3	4	5	6	7	8
14	13	16	15	10	9	12	11	2	1	4	3	6	5	8	7

 4.10×4.66

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12
9	9	9	9	13	13	13	13	5	5	5	5	1	1	1	1
10	10	10	10	14	14	14	14	6	6	6	6	2	2	2	2
9	9	11	11	13	13	15	15	5	5	7	7	1	1	3	3
10	10	12	12	14	14	16	16	6	6	8	8	2	2	4	4
13	13	13	13	9	9	9	9	1	1	1	1	5	5	5	5
14	14	14	14	10	10	10	10	2	2	2	2	6	6	6	6
13	13	15	15	9	9	11	11	1	1	3	3	5	5	7	7
14	14	16	16	10	10	12	12	2	2	4	4	6	6	8	8

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.10 x 4.92

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	1	5	5	7	5	9	9	11	9	13	13	15	13
2	2	2	4	6	6	6	8	10	10	10	12	14	14	14	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	5	1	1	3	1	13	13	15	13	9	9	11	9
6	6	6	8	2	2	2	2	14	14	14	16	10	10	10	12
9	9	9	9	13	13	13	13	5	5	5	5	1	1	1	1
10	10	10	10	14	14	14	14	6	6	6	6	2	2	2	2
9	9	11	9	13	13	15	13	5	5	7	5	1	1	3	1
10	10	10	12	14	14	14	16	6	6	6	8	2	2	2	2
13	13	13	13	9	9	9	9	1	1	1	1	5	5	5	5
14	14	14	14	10	10	10	10	2	2	2	2	6	6	6	6
13	13	15	13	9	9	11	9	1	1	3	1	5	5	7	5
14	14	14	16	10	10	10	12	2	2	2	2	6	6	6	8

4.10 x 4.120

1	2	2	1	5	6	6	5	9	10	10	9	13	14	14	13
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	6	5	1	1	2	1	13	14	14	13	9	10	10	9
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
9	10	10	9	13	14	14	13	5	6	6	5	1	1	2	1
10	9	9	10	14	13	13	14	6	5	5	6	2	1	1	2
10	9	9	10	14	13	13	14	6	5	5	6	2	1	1	2
9	10	11	12	13	14	15	16	5	6	7	8	1	2	3	4
13	14	14	13	9	10	10	9	1	1	2	1	5	6	6	5
14	13	13	14	10	9	9	10	2	1	1	2	6	5	5	6
14	13	13	14	10	9	9	10	2	1	1	2	6	5	5	6
13	14	15	16	9	10	11	12	1	2	3	4	5	6	7	8

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.11×4.11

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
3	4	1	2	7	8	5	6	11	12	9	10	15	16	13	14
4	3	2	1	8	7	6	5	12	11	10	9	16	15	14	13
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
7	8	5	6	3	4	1	2	15	16	13	14	11	12	9	10
8	7	6	5	4	3	2	1	16	15	14	13	12	11	10	9
9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
10	9	12	11	14	13	16	15	2	1	4	3	6	5	8	7
11	12	9	10	15	16	13	14	3	4	1	2	7	8	5	6
12	11	10	9	16	15	14	13	4	3	2	1	8	7	6	5
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4
14	13	16	15	10	9	12	11	6	5	8	7	2	1	4	3
15	16	13	14	11	12	9	10	7	8	5	6	3	4	1	2
16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

 4.11×4.15

1	2	1	2	5	6	5	6	9	10	9	10	13	14	13	14
2	1	2	1	6	5	6	5	10	9	10	9	14	13	14	13
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	6	5	6	1	2	1	2	13	14	13	14	9	10	9	10
6	5	6	5	2	1	2	1	14	13	14	13	10	9	10	9
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
9	10	9	10	13	14	13	14	1	2	1	2	5	6	5	6
10	9	10	9	14	13	14	13	2	1	2	1	6	5	6	5
9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
10	9	12	11	14	13	16	15	2	1	4	3	6	5	8	7
13	14	13	14	9	10	9	10	5	6	5	6	1	2	1	2
14	13	14	13	10	9	10	9	6	5	6	5	2	1	2	1
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4
14	13	16	15	10	9	12	11	6	5	8	7	2	1	4	3

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.11×4.31

1	2	1	2	5	6	5	6	9	10	9	10	13	14	13	14
2	1	2	1	6	5	6	5	10	9	10	9	14	13	14	13
1	2	1	2	5	6	5	6	9	10	9	10	13	14	13	14
2	1	2	1	6	5	6	5	10	9	10	9	14	13	14	13
5	6	5	6	1	2	1	2	13	14	13	14	9	10	9	10
6	5	6	5	2	1	2	1	14	13	14	13	10	9	10	9
5	6	5	6	1	2	1	2	13	14	13	14	9	10	9	10
6	5	6	5	2	1	2	1	14	13	14	13	10	9	10	9
9	10	9	10	13	14	13	14	1	2	1	2	5	6	5	6
10	9	10	9	14	13	14	13	2	1	2	1	6	5	6	5
9	10	9	10	13	14	13	14	1	2	1	2	5	6	5	6
10	9	10	9	14	13	14	13	2	1	2	1	6	5	6	5
13	14	13	14	9	10	9	10	5	6	5	6	1	2	1	2
14	13	14	13	10	9	10	9	6	5	6	5	2	1	2	1
13	14	13	14	9	10	9	10	5	6	5	6	1	2	1	2
14	13	14	13	10	9	10	9	6	5	6	5	2	1	2	1

 4.11×4.32

1	2	2	2	5	6	6	6	9	10	10	10	13	14	14	14
2	1	1	1	6	5	5	5	10	9	9	9	14	13	13	13
2	1	1	1	6	5	5	5	10	9	9	9	14	13	13	13
2	1	1	1	6	5	5	5	10	9	9	9	14	13	13	13
5	6	6	6	1	2	2	2	13	14	14	14	9	10	10	10
6	5	5	5	2	1	1	1	14	13	13	13	10	9	9	9
6	5	5	5	2	1	1	1	14	13	13	13	10	9	9	9
6	5	5	5	2	1	1	1	14	13	13	13	10	9	9	9
9	10	10	10	13	14	14	14	1	2	2	2	5	6	6	6
10	9	9	9	14	13	13	13	2	1	1	1	6	5	5	5
10	9	9	9	14	13	13	13	2	1	1	1	6	5	5	5
10	9	9	9	14	13	13	13	2	1	1	1	6	5	5	5
13	14	14	14	9	10	10	10	5	6	6	6	1	2	2	2
14	13	13	13	10	9	9	9	6	5	5	5	2	1	1	1
14	13	13	13	10	9	9	9	6	5	5	5	2	1	1	1
14	13	13	13	10	9	9	9	6	5	5	5	2	1	1	1

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.11×4.34

1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
3	3	1	1	7	7	5	5	11	11	9	9	15	15	13	13
3	3	1	2	7	7	5	6	11	11	9	10	15	15	13	14
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
7	7	5	5	3	3	1	1	15	15	13	13	11	11	9	9
7	7	5	6	3	3	1	2	15	15	13	14	11	11	9	10
9	9	11	11	13	13	15	15	1	1	3	3	5	5	7	7
9	9	11	11	13	13	15	15	1	1	3	3	5	5	7	7
11	11	9	9	15	15	13	13	3	3	1	1	7	7	5	5
11	11	9	10	15	15	13	14	3	3	1	2	7	7	5	6
13	13	15	15	9	9	11	11	5	5	7	7	1	1	3	3
13	13	15	15	9	9	11	11	5	5	7	7	1	1	3	3
15	15	13	13	11	11	9	9	7	7	5	5	3	3	1	1
15	15	13	14	11	11	9	10	7	7	5	6	3	3	1	2

 4.11×4.35

1	2	3	3	5	6	7	7	9	10	11	11	13	14	15	15
2	3	1	1	6	7	5	5	10	11	9	9	14	15	13	13
3	1	2	2	7	5	6	6	11	9	10	10	15	13	14	14
3	1	2	2	7	5	6	6	11	9	10	10	15	13	14	14
5	6	7	7	1	2	3	3	13	14	15	15	9	10	11	11
6	7	5	5	2	3	1	1	14	15	13	13	10	11	9	9
7	5	6	6	3	1	2	2	15	13	14	14	11	9	10	10
7	5	6	6	3	1	2	2	15	13	14	14	11	9	10	10
9	10	11	11	13	14	15	15	1	2	3	3	5	6	7	7
10	11	9	9	14	15	13	13	2	3	1	1	6	7	5	5
11	9	10	10	15	13	14	14	3	1	2	2	7	5	6	6
11	9	10	10	15	13	14	14	3	1	2	2	7	5	6	6
13	14	15	15	9	10	11	11	5	6	7	7	1	2	3	3
14	15	13	13	10	11	9	9	6	7	5	5	2	3	1	1
15	13	14	14	11	9	10	10	7	5	6	6	3	1	2	2
15	13	14	14	11	9	10	10	7	5	6	6	3	1	2	2

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.11×4.59

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	1	2	5	5	5	6	9	9	9	10	13	13	13	14
4	4	4	4	8	8	8	8	12	12	12	12	16	16	16	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	5	6	1	1	1	2	13	13	13	14	9	9	9	10
8	8	8	8	4	4	4	4	16	16	16	16	12	12	12	12
9	9	9	9	13	13	13	13	1	1	1	1	5	5	5	5
10	10	10	10	14	14	14	14	2	2	2	2	6	6	6	6
9	9	9	10	13	13	13	14	1	1	1	2	5	5	5	6
12	12	12	12	16	16	16	16	4	4	4	4	8	8	8	8
13	13	13	13	9	9	9	9	5	5	5	5	1	1	1	1
14	14	14	14	10	10	10	10	6	6	6	6	2	2	2	2
13	13	13	14	9	9	9	10	5	5	5	6	1	1	1	2
16	16	16	16	12	12	12	12	8	8	8	8	4	4	4	4

 4.11×4.63

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
9	9	9	9	13	13	13	13	1	1	1	1	5	5	5	5
10	10	10	10	14	14	14	14	2	2	2	2	6	6	6	6
9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
10	9	12	11	14	13	16	15	2	1	4	3	6	5	8	7
13	13	13	13	9	9	9	9	5	5	5	5	1	1	1	1
14	14	14	14	10	10	10	10	6	6	6	6	2	2	2	2
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4
14	13	16	15	10	9	12	11	6	5	8	7	2	1	4	3

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.11×4.64

1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12
9	9	11	11	13	13	15	15	1	1	3	3	5	5	7	7
10	10	12	12	14	14	16	16	2	2	4	4	6	6	8	8
9	9	11	11	13	13	15	15	1	1	3	3	5	5	7	7
10	10	12	12	14	14	16	16	2	2	4	4	6	6	8	8
13	13	15	15	9	9	11	11	5	5	7	7	1	1	3	3
14	14	16	16	10	10	12	12	6	6	8	8	2	2	4	4
13	13	15	15	9	9	11	11	5	5	7	7	1	1	3	3
14	14	16	16	10	10	12	12	6	6	8	8	2	2	4	4

 4.11×4.65

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
10	9	12	11	14	13	16	15	2	1	4	3	6	5	8	7
9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
10	9	12	11	14	13	16	15	2	1	4	3	6	5	8	7
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4
14	13	16	15	10	9	12	11	6	5	8	7	2	1	4	3
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4
14	13	16	15	10	9	12	11	6	5	8	7	2	1	4	3

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.11 x 4.66

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12
9	9	9	9	13	13	13	13	1	1	1	1	5	5	5	5
10	10	10	10	14	14	14	14	2	2	2	2	6	6	6	6
9	9	11	11	13	13	15	15	1	1	3	3	5	5	7	7
10	10	12	12	14	14	16	16	2	2	4	4	6	6	8	8
13	13	13	13	9	9	9	9	5	5	5	5	1	1	1	1
14	14	14	14	10	10	10	10	6	6	6	6	2	2	2	2
13	13	15	15	9	9	11	11	5	5	7	7	1	1	3	3
14	14	16	16	10	10	12	12	6	6	8	8	2	2	4	4

4.11 x 4.92

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	1	5	5	7	5	9	9	11	9	13	13	15	13
2	2	2	4	6	6	6	10	10	10	10	12	14	14	14	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	5	1	1	3	1	13	13	15	13	9	9	11	9
6	6	6	10	2	2	2	4	14	14	14	16	10	10	10	12
9	9	9	9	13	13	13	13	1	1	1	1	5	5	5	5
10	10	10	10	14	14	14	14	2	2	2	2	6	6	6	6
9	9	11	9	13	13	15	13	1	1	3	1	5	5	7	5
10	10	10	12	14	14	14	16	2	2	2	4	6	6	6	8
13	13	13	13	9	9	9	9	5	5	5	5	1	1	1	1
14	14	14	14	10	10	10	10	6	6	6	6	2	2	2	2
13	13	15	13	9	9	11	9	5	5	7	5	1	1	3	1
14	14	14	16	10	10	10	12	6	6	6	8	2	2	2	4

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.11×4.120

1	2	2	1	5	6	6	5	9	10	10	9	13	14	14	13
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	6	5	1	2	2	1	13	14	14	13	9	10	10	9
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
9	10	10	9	13	14	14	13	1	2	2	1	5	6	6	5
10	9	9	10	14	13	13	14	2	1	1	2	6	5	5	6
10	9	9	10	14	13	13	14	2	1	1	2	6	5	5	6
9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
13	14	14	13	9	10	10	9	5	6	6	5	1	2	2	1
14	13	13	14	10	9	9	10	6	5	5	6	2	1	1	2
14	13	13	14	10	9	9	10	6	5	5	6	2	1	1	2
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4

 4.15×4.15

1	2	1	2	5	6	5	6	1	2	1	2	5	6	5	6
2	1	2	1	6	5	6	5	2	1	2	1	6	5	6	5
1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
2	1	4	3	6	5	8	7	2	1	4	3	6	5	8	7
5	6	5	6	1	2	1	2	5	6	5	6	1	2	1	2
6	5	6	5	2	1	2	1	6	5	6	5	2	1	2	1
5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
6	5	8	7	2	1	4	3	6	5	8	7	2	1	4	3
1	2	1	2	5	6	5	6	9	10	9	10	13	14	13	14
2	1	2	1	6	5	6	5	10	9	10	9	14	13	14	13
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	6	5	6	1	2	1	2	13	14	13	14	9	10	9	10
6	5	6	5	2	1	2	1	14	13	14	13	10	9	10	9
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.15×4.31

1	2	1	2	5	6	5	6	1	2	1	2	5	6	5	6
2	1	2	1	6	5	6	5	2	1	2	1	6	5	6	5
1	2	1	2	5	6	5	6	1	2	1	2	5	6	5	6
2	1	2	1	6	5	6	5	2	1	2	1	6	5	6	5
5	6	5	6	1	2	1	2	5	6	5	6	1	2	1	2
6	5	6	5	2	1	2	1	6	5	6	5	2	1	2	1
5	6	5	6	1	2	1	2	5	6	5	6	1	2	1	2
6	5	6	5	2	1	2	1	6	5	6	5	2	1	2	1
1	2	1	2	5	6	5	6	9	10	9	10	13	14	13	14
2	1	2	1	6	5	6	5	10	9	10	9	14	13	14	13
1	2	1	2	5	6	5	6	9	10	9	10	13	14	13	14
2	1	2	1	6	5	6	5	10	9	10	9	14	13	14	13
5	6	5	6	1	2	1	2	13	14	13	14	9	10	9	10
6	5	6	5	2	1	2	1	14	13	14	13	10	9	10	9
5	6	5	6	1	2	1	2	13	14	13	14	9	10	9	10
6	5	6	5	2	1	2	1	14	13	14	13	10	9	10	9

 4.15×4.32

1	2	2	2	5	6	6	6	1	2	2	2	5	6	6	6
2	1	1	1	6	5	5	5	2	1	1	1	6	5	5	5
2	1	1	1	6	5	5	5	2	1	1	1	6	5	5	5
2	1	1	1	6	5	5	5	2	1	1	1	6	5	5	5
5	6	6	6	1	2	2	2	5	6	6	6	1	2	2	2
6	5	5	5	2	1	1	1	6	5	5	5	2	1	1	1
6	5	5	5	2	1	1	1	6	5	5	5	2	1	1	1
6	5	5	5	2	1	1	1	6	5	5	5	2	1	1	1
1	2	2	2	5	6	6	6	9	10	10	10	13	14	14	14
2	1	1	1	6	5	5	5	10	9	9	9	14	13	13	13
2	1	1	1	6	5	5	5	10	9	9	9	14	13	13	13
2	1	1	1	6	5	5	5	10	9	9	9	14	13	13	13
5	6	6	6	1	2	2	2	13	14	14	14	9	10	10	10
6	5	5	5	2	1	1	1	14	13	13	13	10	9	9	9
6	5	5	5	2	1	1	1	14	13	13	13	10	9	9	9
6	5	5	5	2	1	1	1	14	13	13	13	10	9	9	9

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.15 x 4.34

1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
3	3	1	1	7	7	5	5	3	3	1	1	7	7	5	5
3	3	1	2	7	7	5	6	3	3	1	2	7	7	5	6
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
7	7	5	5	3	3	1	1	7	7	5	5	3	3	1	1
7	7	5	6	3	3	1	2	7	7	5	6	3	3	1	2
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
3	3	1	1	7	7	5	5	11	11	9	9	15	15	13	13
3	3	1	2	7	7	5	6	11	11	9	10	15	15	13	14
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
7	7	5	5	3	3	1	1	15	15	13	13	11	11	9	9
7	7	5	6	3	3	1	2	15	15	13	14	11	11	9	10

4.15 x 4.35

1	2	3	3	5	6	7	7	1	2	3	3	5	6	7	7
2	3	1	1	6	7	5	5	2	3	1	1	6	7	5	5
3	1	2	2	7	5	6	6	3	1	2	2	7	5	6	6
3	1	2	2	7	5	6	6	3	1	2	2	7	5	6	6
5	6	7	7	1	2	3	3	5	6	7	7	1	2	3	3
6	7	5	5	2	3	1	1	6	7	5	5	2	3	1	1
7	5	6	6	3	1	2	2	7	5	6	6	3	1	2	2
7	5	6	6	3	1	2	2	7	5	6	6	3	1	2	2
1	2	3	3	5	6	7	7	9	10	11	11	13	14	15	15
2	3	1	1	6	7	5	5	10	11	9	9	14	15	13	13
3	1	2	2	7	5	6	6	11	9	10	10	15	13	14	14
3	1	2	2	7	5	6	6	11	9	10	10	15	13	14	14
5	6	7	7	1	2	3	3	13	14	15	15	9	10	11	11
6	7	5	5	2	3	1	1	14	15	13	13	10	11	9	9
7	5	6	6	3	1	2	2	15	13	14	14	11	9	10	10
7	5	6	6	3	1	2	2	15	13	14	14	11	9	10	10

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.15×4.59

1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
2	2	2	2	6	6	6	6	2	2	2	2	6	6	6	6
1	1	1	2	5	5	5	6	1	1	1	2	5	5	5	6
4	4	4	4	8	8	8	8	4	4	4	4	8	8	8	8
5	5	5	5	1	1	1	1	5	5	5	5	1	1	1	1
6	6	6	6	2	2	2	2	6	6	6	6	2	2	2	2
5	5	5	6	1	1	1	2	5	5	5	6	1	1	1	2
8	8	8	8	4	4	4	4	8	8	8	8	4	4	4	4
1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	1	2	5	5	5	6	9	9	9	10	13	13	13	14
4	4	4	4	8	8	8	8	12	12	12	12	16	16	16	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	5	6	1	1	1	2	13	13	13	14	9	9	9	10
8	8	8	8	4	4	4	4	16	16	16	16	12	12	12	12

 4.15×4.63

1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
2	2	2	2	6	6	6	6	2	2	2	2	6	6	6	6
1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
2	1	4	3	6	5	8	7	2	1	4	3	6	5	8	7
5	5	5	5	1	1	1	1	5	5	5	5	1	1	1	1
6	6	6	6	2	2	2	2	6	6	6	6	2	2	2	2
5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
6	5	8	7	2	1	4	3	6	5	8	7	2	1	4	3
1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.15×4.64

1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
2	2	4	4	6	6	8	8	2	2	4	4	6	6	8	8
1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
2	2	4	4	6	6	8	8	2	2	4	4	6	6	8	8
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
6	6	8	8	2	2	4	4	6	6	8	8	2	2	4	4
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
6	6	8	8	2	2	4	4	6	6	8	8	2	2	4	4
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12

 4.15×4.65

1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
2	1	4	3	6	5	8	7	2	1	4	3	6	5	8	7
1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
2	1	4	3	6	5	8	7	2	1	4	3	6	5	8	7
5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
6	5	8	7	2	1	4	3	6	5	8	7	2	1	4	3
5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
6	5	8	7	2	1	4	3	6	5	8	7	2	1	4	3
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.15 x 4.66

1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
2	2	2	2	6	6	6	6	2	2	2	2	6	6	6	6
1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
2	2	4	4	6	6	8	8	2	2	4	4	6	6	8	8
5	5	5	5	1	1	1	1	5	5	5	5	1	1	1	1
6	6	6	6	2	2	2	2	6	6	6	6	2	2	2	2
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
6	6	8	8	2	2	4	4	6	6	8	8	2	2	4	4
1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12

4.15 x 4.92

1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
2	2	2	2	6	6	6	6	2	2	2	2	6	6	6	6
1	1	3	1	5	5	7	5	1	1	3	1	5	5	7	5
2	2	2	4	6	6	6	8	2	2	2	4	6	6	6	8
5	5	5	5	1	1	1	1	5	5	5	5	1	1	1	1
6	6	6	6	2	2	2	2	6	6	6	6	2	2	2	2
5	5	7	5	1	1	3	1	5	5	7	5	1	1	3	1
6	6	6	8	2	2	4	2	6	6	6	8	2	2	4	2
1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	1	5	5	7	5	9	9	11	9	13	13	15	13
2	2	2	4	6	6	6	8	10	10	10	12	14	14	14	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	5	1	1	3	1	13	13	15	13	9	9	11	9
6	6	6	8	2	2	4	2	14	14	16	14	10	10	10	12

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.15×4.120

1	2	2	1	5	6	6	5	1	2	2	1	5	6	6	5
2	1	1	2	6	5	5	6	2	1	1	2	6	5	5	6
2	1	1	2	6	5	5	6	2	1	1	2	6	5	5	6
1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
5	6	6	5	1	2	2	1	5	6	6	5	1	2	2	1
6	5	5	6	2	1	1	2	6	5	5	6	2	1	1	2
6	5	5	6	2	1	1	2	6	5	5	6	2	1	1	2
5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
1	2	2	1	5	6	6	5	9	10	10	9	13	14	14	13
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	6	5	1	2	2	1	13	14	14	13	9	10	10	9
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12

 4.31×4.31

1	2	1	2	5	6	5	6	1	2	1	2	5	6	5	6
2	1	2	1	6	5	6	5	2	1	2	1	6	5	6	5
1	2	1	2	5	6	5	6	1	2	1	2	5	6	5	6
2	1	2	1	6	5	6	5	2	1	2	1	6	5	6	5
5	6	5	6	1	2	1	2	5	6	5	6	1	2	1	2
6	5	6	5	2	1	2	1	6	5	6	5	2	1	2	1
5	6	5	6	1	2	1	2	5	6	5	6	1	2	1	2
6	5	6	5	2	1	2	1	6	5	6	5	2	1	2	1
1	2	1	2	5	6	5	6	1	2	1	2	5	6	5	6
2	1	2	1	6	5	6	5	2	1	2	1	6	5	6	5
1	2	1	2	5	6	5	6	1	2	1	2	5	6	5	6
2	1	2	1	6	5	6	5	2	1	2	1	6	5	6	5
5	6	5	6	1	2	1	2	5	6	5	6	1	2	1	2
6	5	6	5	2	1	2	1	6	5	6	5	2	1	2	1
5	6	5	6	1	2	1	2	5	6	5	6	1	2	1	2
6	5	6	5	2	1	2	1	6	5	6	5	2	1	2	1

 4.31×4.32

1	2	2	2	5	6	6	6	1	2	2	2	5	6	6	6
2	1	1	1	6	5	5	5	2	1	1	1	6	5	5	5
2	1	1	1	6	5	5	5	2	1	1	1	6	5	5	5
2	1	1	1	6	5	5	5	2	1	1	1	6	5	5	5
5	6	6	6	1	2	2	2	5	6	6	6	1	2	2	2
6	5	5	5	2	1	1	1	6	5	5	5	2	1	1	1
6	5	5	5	2	1	1	1	6	5	5	5	2	1	1	1
6	5	5	5	2	1	1	1	6	5	5	5	2	1	1	1
1	2	2	2	5	6	6	6	1	2	2	2	5	6	6	6
2	1	1	1	6	5	5	5	2	1	1	1	6	5	5	5
2	1	1	1	6	5	5	5	2	1	1	1	6	5	5	5
2	1	1	1	6	5	5	5	2	1	1	1	6	5	5	5
5	6	6	6	1	2	2	2	5	6	6	6	1	2	2	2
6	5	5	5	2	1	1	1	6	5	5	5	2	1	1	1
6	5	5	5	2	1	1	1	6	5	5	5	2	1	1	1
6	5	5	5	2	1	1	1	6	5	5	5	2	1	1	1

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.31×4.34

```

1 1 3 3 5 5 7 7 1 1 3 3 5 5 7 7
1 1 3 3 5 5 7 7 1 1 3 3 5 5 7 7
3 3 1 1 7 7 5 5 3 3 1 1 7 7 5 5
3 3 1 2 7 7 5 6 3 3 1 2 7 7 5 6
5 5 7 7 1 1 3 3 5 5 7 7 1 1 3 3
5 5 7 7 1 1 3 3 5 5 7 7 1 1 3 3
7 7 5 5 3 3 1 1 7 7 5 5 3 3 1 1
7 7 5 6 3 3 1 2 7 7 5 6 3 3 1 2
1 1 3 3 5 5 7 7 1 1 3 3 5 5 7 7
1 1 3 3 5 5 7 7 1 1 3 3 5 5 7 7
3 3 1 1 7 7 5 5 3 3 1 1 7 7 5 5
3 3 1 2 7 7 5 6 3 3 1 2 7 7 5 6
5 5 7 7 1 1 3 3 5 5 7 7 1 1 3 3
5 5 7 7 1 1 3 3 5 5 7 7 1 1 3 3
7 7 5 5 3 3 1 1 7 7 5 5 3 3 1 1
7 7 5 6 3 3 1 2 7 7 5 6 3 3 1 2

```

 4.31×4.35

```

1 2 3 3 5 6 7 7 1 2 3 3 5 6 7 7
2 3 1 1 6 7 5 5 2 3 1 1 6 7 5 5
3 1 2 2 7 5 6 6 3 1 2 2 7 5 6 6
3 1 2 2 7 5 6 6 3 1 2 2 7 5 6 6
5 6 7 7 1 2 3 3 5 6 7 7 1 2 3 3
6 7 5 5 2 3 1 1 6 7 5 5 2 3 1 1
7 5 6 6 3 1 2 2 7 5 6 6 3 1 2 2
7 5 6 6 3 1 2 2 7 5 6 6 3 1 2 2
1 2 3 3 5 6 7 7 1 2 3 3 5 6 7 7
2 3 1 1 6 7 5 5 2 3 1 1 6 7 5 5
3 1 2 2 7 5 6 6 3 1 2 2 7 5 6 6
3 1 2 2 7 5 6 6 3 1 2 2 7 5 6 6
5 6 7 7 1 2 3 3 5 6 7 7 1 2 3 3
6 7 5 5 2 3 1 1 6 7 5 5 2 3 1 1
7 5 6 6 3 1 2 2 7 5 6 6 3 1 2 2
7 5 6 6 3 1 2 2 7 5 6 6 3 1 2 2

```

 4.31×4.59

```

1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6 2 2 2 2 6 6 6 6
1 1 1 2 5 5 5 6 1 1 1 2 5 5 5 6
4 4 4 4 8 8 8 8 4 4 4 4 8 8 8 8
5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1
6 6 6 6 2 2 2 2 6 6 6 6 2 2 2 2
5 5 5 6 1 1 1 2 5 5 5 6 1 1 1 2
8 8 8 8 4 4 4 4 8 8 8 8 4 4 4 4
1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6 2 2 2 2 6 6 6 6
1 1 1 2 5 5 5 6 1 1 1 2 5 5 5 6
4 4 4 4 8 8 8 8 4 4 4 4 8 8 8 8
5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1
6 6 6 6 2 2 2 2 6 6 6 6 2 2 2 2
5 5 5 6 1 1 1 2 5 5 5 6 1 1 1 2
8 8 8 8 4 4 4 4 8 8 8 8 4 4 4 4

```

 4.31×4.63

```

1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6 2 2 2 2 6 6 6 6
1 2 3 4 5 6 7 8 1 2 3 4 5 6 7 8
2 1 4 3 6 5 8 7 2 1 4 3 6 5 8 7
5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1
6 6 6 6 2 2 2 2 6 6 6 6 2 2 2 2
5 6 7 8 1 2 3 4 5 6 7 8 1 2 3 4
6 5 8 7 2 1 4 3 6 5 8 7 2 1 4 3
1 1 1 1 5 5 5 5 1 1 1 1 5 5 5 5
2 2 2 2 6 6 6 6 2 2 2 2 6 6 6 6
1 2 3 4 5 6 7 8 1 2 3 4 5 6 7 8
2 1 4 3 6 5 8 7 2 1 4 3 6 5 8 7
5 5 5 5 1 1 1 1 5 5 5 5 1 1 1 1
6 6 6 6 2 2 2 2 6 6 6 6 2 2 2 2
5 6 7 8 1 2 3 4 5 6 7 8 1 2 3 4
6 5 8 7 2 1 4 3 6 5 8 7 2 1 4 3

```

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.31×4.64

1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
2	2	4	4	6	6	8	8	2	2	4	4	6	6	8	8
1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
2	2	4	4	6	6	8	8	2	2	4	4	6	6	8	8
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
6	6	8	8	2	2	4	4	6	6	8	8	2	2	4	4
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
6	6	8	8	2	2	4	4	6	6	8	8	2	2	4	4
1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
2	2	4	4	6	6	8	8	2	2	4	4	6	6	8	8
1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
2	2	4	4	6	6	8	8	2	2	4	4	6	6	8	8
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
6	6	8	8	2	2	4	4	6	6	8	8	2	2	4	4
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
6	6	8	8	2	2	4	4	6	6	8	8	2	2	4	4

 4.31×4.65

1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
2	1	4	3	6	5	8	7	2	1	4	3	6	5	8	7
1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
2	1	4	3	6	5	8	7	2	1	4	3	6	5	8	7
5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
6	5	8	7	2	1	4	3	6	5	8	7	2	1	4	3
5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
6	5	8	7	2	1	4	3	6	5	8	7	2	1	4	3
1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
2	1	4	3	6	5	8	7	2	1	4	3	6	5	8	7
1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
2	1	4	3	6	5	8	7	2	1	4	3	6	5	8	7
5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
6	5	8	7	2	1	4	3	6	5	8	7	2	1	4	3
5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
6	5	8	7	2	1	4	3	6	5	8	7	2	1	4	3

 4.31×4.66

1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
2	2	2	2	6	6	6	6	2	2	2	2	6	6	6	6
1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
2	2	4	4	6	6	8	8	2	2	4	4	6	6	8	8
5	5	5	5	1	1	1	1	5	5	5	5	1	1	1	1
6	6	6	6	2	2	2	2	6	6	6	6	2	2	2	2
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
6	6	8	8	2	2	4	4	6	6	8	8	2	2	4	4
1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
2	2	2	2	6	6	6	6	2	2	2	2	6	6	6	6
1	1	3	3	5	5	7	7	1	1	3	3	5	5	7	7
2	2	4	4	6	6	8	8	2	2	4	4	6	6	8	8
5	5	5	5	1	1	1	1	5	5	5	5	1	1	1	1
6	6	6	6	2	2	2	2	6	6	6	6	2	2	2	2
5	5	7	7	1	1	3	3	5	5	7	7	1	1	3	3
6	6	8	8	2	2	4	4	6	6	8	8	2	2	4	4

 4.31×4.92

1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
2	2	2	2	6	6	6	6	2	2	2	2	6	6	6	6
1	1	3	1	5	5	7	5	1	1	3	1	5	5	7	5
2	2	2	4	6	6	6	8	2	2	2	4	6	6	6	8
5	5	5	5	1	1	1	1	5	5	5	5	1	1	1	1
6	6	6	6	2	2	2	2	6	6	6	6	2	2	2	2
5	5	7	5	1	1	3	1	5	5	7	5	1	1	3	1
6	6	6	8	2	2	2	4	6	6	6	8	2	2	2	4
1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5
2	2	2	2	6	6	6	6	2	2	2	2	6	6	6	6
1	1	3	1	5	5	7	5	1	1	3	1	5	5	7	5
2	2	2	4	6	6	6	8	2	2	2	4	6	6	6	8
5	5	5	5	1	1	1	1	5	5	5	5	1	1	1	1
6	6	6	6	2	2	2	2	6	6	6	6	2	2	2	2
5	5	7	5	1	1	3	1	5	5	7	5	1	1	3	1
6	6	6	8	2	2	2	4	6	6	6	8	2	2	2	4

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.31×4.120

```

1 2 2 1 5 6 6 5 1 2 2 1 5 6 6 5
2 1 1 2 6 5 5 6 2 1 1 2 6 5 5 6
2 1 1 2 6 5 5 6 2 1 1 2 6 5 5 6
1 2 3 4 5 6 7 8 1 2 3 4 5 6 7 8
5 6 6 5 1 2 2 1 5 6 6 5 1 2 2 1
6 5 5 6 2 1 1 2 6 5 5 6 2 1 1 2
6 5 5 6 2 1 1 2 6 5 5 6 2 1 1 2
5 6 7 8 1 2 3 4 5 6 7 8 1 2 3 4
1 2 2 1 5 6 6 5 1 2 2 1 5 6 6 5
2 1 1 2 6 5 5 6 2 1 1 2 6 5 5 6
2 1 1 2 6 5 5 6 2 1 1 2 6 5 5 6
1 2 3 4 5 6 7 8 1 2 3 4 5 6 7 8
5 6 6 5 1 2 2 1 5 6 6 5 1 2 2 1
6 5 5 6 2 1 1 2 6 5 5 6 2 1 1 2
6 5 5 6 2 1 1 2 6 5 5 6 2 1 1 2
5 6 7 8 1 2 3 4 5 6 7 8 1 2 3 4

```

 4.32×4.32

```

1 2 2 2 5 6 6 6 5 6 6 6 5 6 6 6
2 1 1 1 6 5 5 5 6 5 5 5 6 5 5 5
2 1 1 1 6 5 5 5 6 5 5 5 6 5 5 5
2 1 1 1 6 5 5 5 6 5 5 5 6 5 5 5
5 6 6 6 1 2 2 2 1 2 2 2 1 2 2 2
6 5 5 5 2 1 1 1 2 1 1 1 2 1 1 1
6 5 5 5 2 1 1 1 2 1 1 1 2 1 1 1
6 5 5 5 2 1 1 1 2 1 1 1 2 1 1 1
5 6 6 6 1 2 2 2 1 2 2 2 1 2 2 2
6 5 5 5 2 1 1 1 2 1 1 1 2 1 1 1
6 5 5 5 2 1 1 1 2 1 1 1 2 1 1 1
6 5 5 5 2 1 1 1 2 1 1 1 2 1 1 1
5 6 6 6 1 2 2 2 1 2 2 2 1 2 2 2
6 5 5 5 2 1 1 1 2 1 1 1 2 1 1 1
6 5 5 5 2 1 1 1 2 1 1 1 2 1 1 1
6 5 5 5 2 1 1 1 2 1 1 1 2 1 1 1

```

 4.32×4.34

```

1 1 3 3 5 5 7 7 5 5 7 7 5 5 7 7
1 1 3 3 5 5 7 7 5 5 7 7 5 5 7 7
3 3 1 1 7 7 5 5 7 7 5 5 7 7 5 5
3 3 1 1 7 7 5 5 7 7 5 5 7 7 5 5
5 5 7 7 1 1 3 3 1 1 3 3 1 1 3 3
5 5 7 7 1 1 3 3 1 1 3 3 1 1 3 3
7 7 5 5 3 3 1 1 3 3 1 1 3 3 1 1
7 7 5 5 3 3 1 1 3 3 1 1 3 3 1 1
5 5 7 7 1 1 3 3 1 1 3 3 1 1 3 3
5 5 7 7 1 1 3 3 1 1 3 3 1 1 3 3
7 7 5 5 3 3 1 1 3 3 1 1 3 3 1 1
7 7 5 5 3 3 1 1 3 3 1 1 3 3 1 1
5 5 7 7 1 1 3 3 1 1 3 3 1 1 3 3
5 5 7 7 1 1 3 3 1 1 3 3 1 1 3 3
7 7 5 5 3 3 1 1 3 3 1 1 3 3 1 1
7 7 5 5 3 3 1 1 3 3 1 1 3 3 1 1

```

 4.32×4.35

```

1 2 3 3 5 6 7 7 5 6 7 7 5 6 7 7
2 3 1 1 6 7 5 5 6 7 5 5 6 7 5 5
3 1 2 2 7 5 6 6 7 5 6 6 7 5 6 6
3 1 2 2 7 5 6 6 7 5 6 6 7 5 6 6
5 6 7 7 1 2 3 3 1 2 3 3 1 2 3 3
6 7 5 5 2 3 1 1 2 3 1 1 2 3 1 1
7 5 6 6 3 1 2 2 3 1 2 2 3 1 2 2
7 5 6 6 3 1 2 2 3 1 2 2 3 1 2 2
5 6 7 7 1 2 3 3 1 2 3 3 1 2 3 3
6 7 5 5 2 3 1 1 2 3 1 1 2 3 1 1
7 5 6 6 3 1 2 2 3 1 2 2 3 1 2 2
7 5 6 6 3 1 2 2 3 1 2 2 3 1 2 2
5 6 7 7 1 2 3 3 1 2 3 3 1 2 3 3
6 7 5 5 2 3 1 1 2 3 1 1 2 3 1 1
7 5 6 6 3 1 2 2 3 1 2 2 3 1 2 2
7 5 6 6 3 1 2 2 3 1 2 2 3 1 2 2

```

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.32×4.59

1	1	1	1	5	5	5	5	5	5	5	5	5	5	5	5
2	2	2	2	6	6	6	6	6	6	6	6	6	6	6	6
1	1	1	2	5	5	5	6	5	5	5	6	5	5	5	6
4	4	4	4	8	8	8	8	8	8	8	8	8	8	8	8
5	5	5	5	1	1	1	1	1	1	1	1	1	1	1	1
6	6	6	6	2	2	2	2	2	2	2	2	2	2	2	2
5	5	5	6	1	1	1	2	1	1	1	2	1	1	1	2
8	8	8	8	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	1	1	1	1	1	1	1	1	1	1	1	1
6	6	6	6	2	2	2	2	2	2	2	2	2	2	2	2
5	5	5	6	1	1	1	2	1	1	1	2	1	1	1	2
8	8	8	8	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	1	1	1	1	1	1	1	1	1	1	1	1
6	6	6	6	2	2	2	2	2	2	2	2	2	2	2	2
5	5	5	6	1	1	1	2	1	1	1	2	1	1	1	2
8	8	8	8	4	4	4	4	4	4	4	4	4	4	4	4

 4.32×4.63

1	1	1	1	5	5	5	5	5	5	5	5	5	5	5	5
2	2	2	2	6	6	6	6	6	6	6	6	6	6	6	6
1	2	3	4	5	6	7	8	5	6	7	8	5	6	7	8
2	1	4	3	6	5	8	7	6	5	8	7	6	5	8	7
5	5	5	5	1	1	1	1	1	1	1	1	1	1	1	1
6	6	6	6	2	2	2	2	2	2	2	2	2	2	2	2
5	6	7	8	1	2	3	4	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3	2	1	4	3
5	5	5	5	1	1	1	1	1	1	1	1	1	1	1	1
6	6	6	6	2	2	2	2	2	2	2	2	2	2	2	2
5	6	7	8	1	2	3	4	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3	2	1	4	3
5	5	5	5	1	1	1	1	1	1	1	1	1	1	1	1
6	6	6	6	2	2	2	2	2	2	2	2	2	2	2	2
5	6	7	8	1	2	3	4	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3	2	1	4	3

 4.32×4.64

1	1	3	3	5	5	7	7	5	5	7	7	5	5	7	7
2	2	4	4	6	6	8	8	6	6	8	8	6	6	8	8
1	1	3	3	5	5	7	7	5	5	7	7	5	5	7	7
2	2	4	4	6	6	8	8	6	6	8	8	6	6	8	8
5	5	7	7	1	1	3	3	1	1	3	3	1	1	3	3
6	6	8	8	2	2	4	4	2	2	4	4	2	2	4	4
5	5	7	7	1	1	3	3	1	1	3	3	1	1	3	3
6	6	8	8	2	2	4	4	2	2	4	4	2	2	4	4
5	5	7	7	1	1	3	3	1	1	3	3	1	1	3	3
6	6	8	8	2	2	4	4	2	2	4	4	2	2	4	4
5	5	7	7	1	1	3	3	1	1	3	3	1	1	3	3
6	6	8	8	2	2	4	4	2	2	4	4	2	2	4	4
5	5	7	7	1	1	3	3	1	1	3	3	1	1	3	3
6	6	8	8	2	2	4	4	2	2	4	4	2	2	4	4

 4.32×4.65

1	2	3	4	5	6	7	8	5	6	7	8	5	6	7	8
2	1	4	3	6	5	8	7	6	5	8	7	6	5	8	7
1	2	3	4	5	6	7	8	5	6	7	8	5	6	7	8
2	1	4	3	6	5	8	7	6	5	8	7	6	5	8	7
5	6	7	8	1	2	3	4	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3	2	1	4	3
5	6	7	8	1	2	3	4	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3	2	1	4	3
5	6	7	8	1	2	3	4	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3	2	1	4	3
5	6	7	8	1	2	3	4	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3	2	1	4	3
5	6	7	8	1	2	3	4	1	2	3	4	1	2	3	4
6	5	8	7	2	1	4	3	2	1	4	3	2	1	4	3

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.32 x 4.66

```

1 1 1 1 5 5 5 5 5 5 5 5 5 5 5
2 2 2 2 6 6 6 6 6 6 6 6 6 6 6
1 1 3 3 5 5 7 7 5 5 7 7 5 5 7 7
2 2 4 4 6 6 8 8 6 6 8 8 6 6 8 8
5 5 5 5 1 1 1 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2 2 2 2
5 5 7 7 1 1 3 3 1 1 3 3 1 1 3 3
6 6 8 8 2 2 4 4 2 2 4 4 2 2 4 4
5 5 5 5 1 1 1 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2 2 2 2
5 5 7 7 1 1 3 3 1 1 3 3 1 1 3 3
6 6 8 8 2 2 4 4 2 2 4 4 2 2 4 4
5 5 5 5 1 1 1 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2 2 2 2
5 5 7 7 1 1 3 3 1 1 3 3 1 1 3 3
6 6 8 8 2 2 4 4 2 2 4 4 2 2 4 4

```

4.32 x 4.92

```

1 1 1 1 5 5 5 5 5 5 5 5 5 5 5
2 2 2 2 6 6 6 6 6 6 6 6 6 6 6
1 1 3 1 5 5 7 5 5 5 7 5 5 5 7 5
2 2 2 4 6 6 8 6 6 6 8 6 6 6 8
5 5 5 5 1 1 1 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2 2 2 2
5 5 7 5 1 1 3 1 1 1 3 1 1 1 3 1
6 6 6 8 2 2 2 4 2 2 2 4 2 2 2 4
5 5 5 5 1 1 1 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2 2 2 2
5 5 7 5 1 1 3 1 1 1 3 1 1 1 3 1
6 6 6 8 2 2 2 4 2 2 2 4 2 2 2 4
5 5 5 5 1 1 1 1 1 1 1 1 1 1 1
6 6 6 6 2 2 2 2 2 2 2 2 2 2 2
5 5 7 5 1 1 3 1 1 1 3 1 1 1 3 1
6 6 6 8 2 2 2 4 2 2 2 4 2 2 2 4

```

4.32 x 4.120

```

1 2 2 1 5 6 6 5 5 6 6 5 5 6 6 5
2 1 1 2 6 5 5 6 6 5 5 6 6 5 5 6
2 1 1 2 6 5 5 6 6 5 5 6 6 5 5 6
1 2 3 4 5 6 7 8 5 6 7 8 5 6 7 8
5 6 6 5 1 2 2 1 1 2 2 1 1 2 2 1
6 5 5 6 2 1 1 2 2 1 1 2 2 1 1 2
6 5 5 6 2 1 1 2 2 1 1 2 2 1 1 2
5 6 7 8 1 2 3 4 1 2 3 4 1 2 3 4
5 6 6 5 1 2 2 1 1 2 2 1 1 2 2 1
6 5 5 6 2 1 1 2 2 1 1 2 2 1 1 2
6 5 5 6 2 1 1 2 2 1 1 2 2 1 1 2
5 6 7 8 1 2 3 4 1 2 3 4 1 2 3 4
5 6 6 5 1 2 2 1 1 2 2 1 1 2 2 1
6 5 5 6 2 1 1 2 2 1 1 2 2 1 1 2
6 5 5 6 2 1 1 2 2 1 1 2 2 1 1 2
5 6 7 8 1 2 3 4 1 2 3 4 1 2 3 4

```

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.34×4.34

1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
3	3	1	1	3	3	1	1	11	11	9	9	11	11	9	9
3	3	1	2	3	3	1	2	11	11	9	10	11	11	9	10
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
3	3	1	1	3	3	1	1	11	11	9	9	11	11	9	9
3	3	1	2	3	3	1	2	11	11	9	10	11	11	9	10
9	9	11	11	9	9	11	11	1	1	3	3	1	1	3	3
9	9	11	11	9	9	11	11	1	1	3	3	1	1	3	3
11	11	9	9	11	11	9	9	3	3	1	1	3	3	1	1
11	11	9	10	11	11	9	10	3	3	1	2	3	3	1	2
9	9	11	11	9	9	11	11	1	1	3	3	5	5	7	7
9	9	11	11	9	9	11	11	1	1	3	3	5	5	7	7
11	11	9	9	11	11	9	9	3	3	1	1	7	7	5	5
11	11	9	10	11	11	9	10	3	3	1	2	7	7	5	6

 4.34×4.35

1	2	3	3	1	2	3	3	9	9	11	11	9	9	11	11
2	3	1	1	2	3	1	1	9	9	11	11	9	9	11	11
3	1	2	2	3	1	2	2	11	11	9	9	11	11	9	9
3	1	2	2	3	1	2	2	11	11	9	10	11	11	9	10
1	2	3	3	1	2	3	3	9	9	11	11	9	9	11	11
2	3	1	1	2	3	1	1	9	9	11	11	9	9	11	11
3	1	2	2	3	1	2	2	11	11	9	9	11	11	9	9
3	1	2	2	3	1	2	2	11	11	9	10	11	11	9	10
9	9	11	11	9	9	11	11	1	2	3	3	1	2	3	3
9	9	11	11	9	9	11	11	2	3	1	1	2	3	1	1
11	11	9	9	11	11	9	9	3	1	2	2	3	1	2	2
11	11	9	10	11	11	9	10	3	1	2	2	3	1	2	2
9	9	11	11	9	9	11	11	1	2	3	3	5	6	7	7
9	9	11	11	9	9	11	11	2	3	1	1	6	7	5	5
11	11	9	9	11	11	9	9	3	1	2	2	7	5	6	6
11	11	9	10	11	11	9	10	3	1	2	2	7	5	6	6

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.34×4.59

1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	1	2	1	1	1	2	9	9	9	10	9	9	10
4	4	4	4	4	4	4	4	12	12	12	12	12	12	12
1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	1	2	1	1	1	2	9	9	9	10	9	9	10
4	4	4	4	4	4	4	4	12	12	12	12	12	12	12
9	9	9	9	9	9	9	9	1	1	1	1	1	1	1
10	10	10	10	10	10	10	10	2	2	2	2	2	2	2
9	9	9	10	9	9	9	10	1	1	1	2	1	1	2
12	12	12	12	12	12	12	12	4	4	4	4	4	4	4
9	9	9	9	9	9	9	9	1	1	1	1	5	5	5
10	10	10	10	10	10	10	10	2	2	2	2	6	6	6
9	9	9	10	9	9	9	10	1	1	1	2	5	5	6
12	12	12	12	12	12	12	12	4	4	4	4	9	9	9

 4.34×4.63

1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12
1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12
9	9	9	9	9	9	9	9	1	1	1	1	1	1	1
10	10	10	10	10	10	10	10	2	2	2	2	2	2	2
9	10	11	12	9	10	11	12	1	2	3	4	1	2	3
10	9	12	11	10	9	11	12	2	1	4	3	2	1	4
9	9	9	9	9	9	9	9	1	1	1	1	5	5	5
10	10	10	10	10	10	10	10	2	2	2	2	6	6	6
9	10	11	12	9	10	11	12	1	2	3	4	5	6	7
10	9	12	11	10	9	12	11	2	1	4	3	6	5	8

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.34×4.64

1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12	12
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12	12
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12	12
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12	12
9	9	11	11	9	9	11	11	1	1	3	3	1	1	3	3
10	10	12	12	10	10	12	12	2	2	4	4	2	2	4	4
9	9	11	11	9	9	11	11	1	1	3	3	1	1	3	3
10	10	12	12	10	10	12	12	2	2	4	4	2	2	4	4
9	9	11	11	9	9	11	11	1	1	3	3	5	5	7	7
10	10	12	12	10	10	12	12	2	2	4	4	6	6	8	8
9	9	11	11	9	9	11	11	1	1	3	3	5	5	7	7
10	10	12	12	10	10	12	12	2	2	4	4	6	6	8	8

 4.34×4.65

1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
9	10	11	12	9	10	11	12	1	2	3	4	1	2	3	4
10	9	12	11	10	9	12	11	2	1	4	3	2	1	4	3
9	10	11	12	9	10	11	12	1	2	3	4	1	2	3	4
10	9	12	11	10	9	12	11	2	1	4	3	2	1	4	3
9	10	11	12	9	10	11	12	1	2	3	4	5	6	7	8
10	9	12	11	10	9	12	11	2	1	4	3	6	5	8	7
9	10	11	12	9	10	11	12	1	2	3	4	5	6	7	8
10	9	12	11	10	9	12	11	2	1	4	3	6	5	8	7

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.34×4.66

1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	3	3	1	1	3	3	9	9	10	10	9	9	10
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12
1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	3	3	1	1	3	3	9	9	10	10	9	9	10
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12
9	9	9	9	9	9	9	9	1	1	1	1	1	1	1
10	10	10	10	10	10	10	10	2	2	2	2	2	2	2
9	9	10	10	9	9	10	10	1	1	3	3	1	1	3
10	10	12	12	10	10	12	12	2	2	4	4	2	2	4
9	9	9	9	9	9	9	9	1	1	1	1	5	5	5
10	10	10	10	10	10	10	10	2	2	2	2	6	6	6
9	9	10	10	9	9	10	10	1	1	3	3	5	5	7
10	10	12	12	10	10	12	12	2	2	4	4	6	6	8

 4.34×4.92

1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	3	1	1	1	3	1	9	9	11	9	9	9	11
2	2	2	4	2	2	2	4	10	10	10	12	10	10	12
1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	3	1	1	1	3	1	9	9	11	9	9	9	11
2	2	2	4	2	2	2	4	10	10	10	12	10	10	12
9	9	9	9	9	9	9	9	1	1	1	1	1	1	1
10	10	10	10	10	10	10	10	2	2	2	2	2	2	2
9	9	11	9	9	9	11	9	1	1	3	1	1	1	3
10	10	10	12	10	10	10	12	2	2	2	4	2	2	4
9	9	9	9	9	9	9	9	1	1	1	1	5	5	5
10	10	10	10	10	10	10	10	2	2	2	2	6	6	6
9	9	11	9	9	9	11	9	1	1	3	1	5	5	7
10	10	10	12	10	10	10	12	2	2	2	4	6	6	8

TABLE V

DIRECT PRODUCTS: 4×4 (continued) $4_0 34 \times 4_0 120$

1	2	2	1	1	2	2	1	9	10	10	9	9	10	10	9
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
1	2	2	1	1	2	2	1	9	10	10	9	9	10	10	9
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
9	10	10	9	9	10	10	9	1	2	2	1	1	2	2	1
10	9	9	10	10	9	9	10	2	1	1	2	2	1	1	2
10	9	9	10	10	9	9	10	2	1	1	2	2	1	1	2
9	10	11	12	9	10	11	12	1	2	3	4	1	2	3	4
9	10	10	9	9	10	10	9	1	2	2	1	5	6	6	5
10	9	9	10	10	9	9	10	2	1	1	2	6	5	5	6
10	9	9	10	10	9	9	10	2	1	1	2	6	5	5	6
9	10	11	12	9	10	11	12	1	2	3	4	5	6	7	8

 $4_0 35 \times 4_0 35$

1	2	3	3	5	6	7	7	9	10	11	11	9	10	11	11
2	3	1	1	6	7	5	5	10	11	9	9	10	11	9	9
3	1	2	2	7	5	6	6	11	9	10	10	11	9	10	10
3	1	2	2	7	5	6	6	11	9	10	10	11	9	10	10
5	6	7	7	9	10	11	11	1	2	3	3	1	2	3	3
6	7	5	5	10	11	9	9	2	3	1	1	2	3	1	1
7	5	6	6	11	9	10	10	3	1	2	2	3	1	2	2
7	5	6	6	11	9	10	10	3	1	2	2	3	1	2	2
9	10	11	11	1	2	3	3	5	6	7	7	5	6	7	7
10	11	9	9	2	3	1	1	6	7	5	5	6	7	5	5
11	9	10	10	3	1	2	2	7	5	6	6	7	5	6	6
11	9	10	10	3	1	2	2	7	5	6	6	7	5	6	6
9	10	11	11	1	2	3	3	5	6	7	7	5	6	7	7
10	11	9	9	2	3	1	1	6	7	5	5	6	7	5	5
11	9	10	10	3	1	2	2	7	5	6	6	7	5	6	6
11	9	10	10	3	1	2	2	7	5	6	6	7	5	6	6

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.35×4.59

1	1	1	1	5	5	5	5	9	9	9	9	9	9	9	9	9
2	2	2	2	6	6	6	6	10	10	10	10	10	10	10	10	10
1	1	1	2	5	5	5	6	9	9	9	10	9	9	9	9	10
4	4	4	4	8	8	8	8	12	12	12	12	12	12	12	12	12
5	5	5	5	9	9	9	9	1	1	1	1	1	1	1	1	1
6	6	6	6	10	10	10	10	2	2	2	2	2	2	2	2	2
5	5	5	6	9	9	9	10	1	1	1	2	1	1	1	1	2
8	8	8	8	12	12	12	12	4	4	4	4	4	4	4	4	4
9	9	9	9	1	1	1	1	5	5	5	5	5	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6	6	6	6	6	6
9	9	9	10	1	1	1	2	5	5	5	6	5	5	5	5	6
12	12	12	12	4	4	4	4	8	8	8	8	8	8	8	8	8
9	9	9	9	1	1	1	1	5	5	5	5	5	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6	6	6	6	6	6
9	9	9	10	1	1	1	2	5	5	5	6	5	5	5	5	6
12	12	12	12	4	4	4	4	8	8	8	8	8	8	8	8	8

 4.35×4.63

1	1	1	1	5	5	5	5	9	9	9	9	9	9	9	9	9
2	2	2	2	6	6	6	6	10	10	10	10	10	10	10	10	10
1	2	3	4	5	6	7	8	9	10	11	12	9	10	11	12	12
2	1	4	3	6	5	8	7	10	9	12	11	10	9	12	11	11
5	5	5	5	9	9	9	9	1	1	1	1	1	1	1	1	1
6	6	6	6	10	10	10	10	2	2	2	2	2	2	2	2	2
5	6	7	8	9	10	11	12	1	2	3	4	1	2	3	4	4
6	5	8	7	10	9	12	11	2	1	4	3	2	1	4	3	3
9	9	9	9	1	1	1	1	5	5	5	5	5	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6	6	6	6	6	6
9	10	11	12	1	2	3	4	5	6	7	8	5	6	7	8	8
10	9	12	11	2	1	4	3	6	5	8	7	6	5	8	7	7
9	9	9	9	1	1	1	1	5	5	5	5	5	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6	6	6	6	6	6
9	10	11	12	1	2	3	4	5	6	7	8	5	6	7	8	8

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.35 x 4.64

1	1	3	3	5	5	7	7	9	9	11	11	9	9	11	11
2	2	4	4	6	6	8	8	10	10	12	12	10	10	12	12
1	1	3	3	5	5	7	7	9	9	11	11	9	9	11	11
2	2	4	4	6	6	8	8	10	10	12	12	10	10	12	12
5	5	7	7	9	9	11	11	1	1	3	3	1	1	3	3
6	6	8	8	10	10	12	12	2	2	4	4	2	2	4	4
5	5	7	7	9	9	11	11	1	1	3	3	1	1	3	3
6	6	8	8	10	10	12	12	2	2	4	4	2	2	4	4
1	1	3	3	9	9	11	11	5	5	7	7	5	5	7	7
2	2	4	4	10	10	12	12	6	6	8	8	6	6	8	8
1	1	3	3	9	9	11	11	5	5	7	7	5	5	7	7
2	2	4	4	10	10	12	12	6	6	8	8	6	6	8	8
1	1	3	3	9	9	11	11	5	5	7	7	5	5	7	7
2	2	4	4	10	10	12	12	6	6	8	8	6	6	8	8

4.35 x 4.65

1	2	3	4	5	6	7	8	9	10	11	12	9	10	11	12
2	1	4	3	6	5	8	7	10	9	12	11	10	9	12	11
1	2	3	4	5	6	7	8	9	10	11	12	9	10	11	12
2	1	4	3	6	5	8	7	10	9	12	11	10	9	12	11
5	6	7	8	9	10	11	12	1	2	3	4	1	2	3	4
6	5	8	7	10	9	12	11	2	1	4	3	2	1	4	3
5	6	7	8	9	10	11	12	1	2	3	4	1	2	3	4
6	5	8	7	10	9	12	11	2	1	4	3	2	1	4	3
9	10	11	12	1	2	3	4	5	6	7	8	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7	6	5	8	7
9	10	11	12	1	2	3	4	5	6	7	8	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7	6	5	8	7
9	10	11	12	1	2	3	4	5	6	7	8	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7	6	5	8	7
9	10	11	12	1	2	3	4	5	6	7	8	5	6	7	8
10	9	12	11	2	1	4	3	6	5	8	7	6	5	8	7

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.35 x 4.66

1	1	1	1	5	5	5	5	9	9	9	9	9	9	9	9
2	2	2	2	6	6	6	6	10	10	10	10	10	10	10	10
1	1	3	3	5	5	7	7	9	9	11	11	9	9	11	11
2	2	4	4	6	6	8	8	10	10	12	12	10	10	12	12
5	5	5	5	9	9	9	9	1	1	1	1	1	1	1	1
6	6	6	6	10	10	10	10	2	2	2	2	2	2	2	2
5	5	7	7	9	9	11	11	1	1	3	3	1	1	3	3
6	6	8	8	10	10	12	12	2	2	4	4	2	2	4	4
9	9	9	9	1	1	1	1	5	5	5	5	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6	6	6	6	6
9	9	11	11	1	1	3	3	5	5	7	7	5	5	7	7
10	10	12	12	2	2	4	4	6	6	8	8	6	6	8	8
9	9	9	9	1	1	1	1	5	5	5	5	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6	6	6	6	6
9	9	11	11	1	1	3	3	5	5	7	7	5	5	7	7
10	10	12	12	2	2	4	4	6	6	8	8	6	6	8	8

4.35 x 4.92

1	1	1	1	5	5	5	5	9	9	9	9	9	9	9	9
2	2	2	2	6	6	6	6	10	10	10	10	10	10	10	10
1	1	3	1	5	5	7	5	9	9	11	9	9	9	11	9
2	2	2	4	6	6	6	8	10	10	10	12	10	10	10	12
5	5	5	5	9	9	9	9	1	1	1	1	1	1	1	1
6	6	6	6	10	10	10	10	2	2	2	2	2	2	2	2
5	5	7	5	9	9	11	9	1	1	3	1	1	1	3	1
6	6	6	8	10	10	10	12	2	2	2	4	2	2	2	4
9	9	9	9	1	1	1	1	5	5	5	5	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6	6	6	6	6
9	9	11	9	1	1	3	1	5	5	7	5	5	5	7	5
10	10	10	12	2	2	2	4	6	6	6	8	6	6	6	8
9	9	9	9	1	1	1	1	5	5	5	5	5	5	5	5
10	10	10	10	2	2	2	2	6	6	6	6	6	6	6	6
9	9	11	9	1	1	3	1	5	5	7	5	5	5	7	5
10	10	10	12	2	2	2	4	6	6	6	8	6	6	6	8

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

$$4.35 \times 4.120$$

1	2	2	1	5	6	6	5	9	10	10	9	9	10	10	9
2	1	1	2	6	5	5	6	10	9	9	10	10	9	9	10
2	1	1	2	6	5	5	6	10	9	9	10	10	9	9	10
1	2	3	4	5	6	7	8	9	10	11	12	9	10	11	12
5	6	6	5	9	10	10	9	1	2	2	1	1	2	2	1
6	5	5	6	10	9	9	10	2	1	1	2	2	1	1	2
6	5	5	6	10	9	9	10	2	1	1	2	2	1	1	2
5	6	7	8	9	10	11	12	1	2	3	4	1	2	3	4
9	10	10	9	1	2	2	1	5	6	6	5	5	6	6	5
10	9	9	10	2	1	1	2	6	5	5	6	6	5	5	6
10	9	9	10	2	1	1	2	6	5	5	6	6	5	5	6
9	10	11	12	1	2	3	4	5	6	7	8	5	6	7	8
9	10	10	9	1	2	2	1	5	6	6	5	5	6	6	5
10	9	9	10	2	1	1	2	6	5	5	6	6	5	5	6
10	9	9	10	2	1	1	2	6	5	5	6	6	5	5	6
9	10	11	12	1	2	3	4	5	6	7	8	5	6	7	8

 4.59×4.59 [illegible]

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.59 x 4.63

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
6	5	8	7	6	5	8	7	6	5	8	7	6	5	8	7
1	1	1	1	1	1	1	1	1	1	1	1	5	5	5	5
2	2	2	2	2	2	2	2	2	2	2	2	6	6	6	6
1	2	3	4	1	2	3	4	1	2	3	4	5	6	7	8
2	1	4	3	2	1	4	3	2	1	4	3	6	5	8	7
13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
13	14	15	16	13	14	15	16	13	14	15	16	13	14	15	16
14	13	16	15	14	13	16	15	14	13	16	15	14	13	16	15

4.59 x 4.64

1	1	3	3	1	1	3	3	1	1	3	3	1	1	3	3
2	2	4	4	2	2	4	4	2	2	4	4	2	2	4	4
1	1	3	3	1	1	3	3	1	1	3	3	1	1	3	3
2	2	4	4	2	2	4	4	2	2	4	4	2	2	4	4
5	5	7	7	5	5	7	7	5	5	7	7	5	5	7	7
6	6	8	8	6	6	8	8	6	6	8	8	6	6	8	8
5	5	7	7	5	5	7	7	5	5	7	7	5	5	7	7
6	6	8	8	6	6	8	8	6	6	8	8	6	6	8	8
1	1	3	3	1	1	3	3	1	1	3	3	5	5	7	7
2	2	4	4	2	2	4	4	2	2	4	4	6	6	8	8
1	1	3	3	1	1	3	3	1	1	3	3	5	5	7	7
2	2	4	4	2	2	4	4	2	2	4	4	6	6	8	8
13	13	15	15	13	13	15	15	13	13	15	15	13	13	15	15
14	14	16	16	14	14	16	16	14	14	16	16	14	14	16	16
13	13	15	15	13	13	15	15	13	13	15	15	13	13	15	15
14	14	16	16	14	14	16	16	14	14	16	16	14	14	16	16

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.59×4.65

1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
6	5	8	7	6	5	8	7	6	5	8	7	6	5	8	7
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
6	5	8	7	6	5	8	7	6	5	8	7	6	5	8	7
1	2	3	4	1	2	3	4	1	2	3	4	5	6	7	8
2	1	4	3	2	1	4	3	2	1	4	3	6	5	8	7
1	2	3	4	1	2	3	4	1	2	3	4	5	6	7	8
2	1	4	3	2	1	4	3	2	1	4	3	6	5	8	7
13	14	15	16	13	14	15	16	13	14	15	16	13	14	15	16
14	13	16	15	14	13	16	15	14	13	16	15	14	13	16	15
13	14	15	16	13	14	15	16	13	14	15	16	13	14	15	16
14	13	16	15	14	13	16	15	14	13	16	15	14	13	16	15

 4.59×4.66

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	3	3	1	1	3	3	1	1	3	3	1	1	3	3
2	2	4	4	2	2	4	4	2	2	4	4	2	2	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	5	7	7	5	5	7	7	5	5	7	7	5	5	7	7
6	6	8	8	6	6	8	8	6	6	8	8	6	6	8	8
1	1	1	1	1	1	1	1	1	1	1	1	5	5	5	5
2	2	2	2	2	2	2	2	2	2	2	2	6	6	6	6
1	1	3	3	1	1	3	3	1	1	3	3	5	5	7	7
2	2	4	4	2	2	4	4	2	2	4	4	6	6	8	8
13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
13	13	15	15	13	13	15	15	13	13	15	15	13	13	15	15
14	14	16	16	14	14	16	16	14	14	16	16	14	14	16	16

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.59×4.92

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	3	1	1	1	3	1	1	1	3	1	1	1	3	1
2	2	2	4	2	2	2	4	2	2	2	4	2	2	2	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	5	7	5	5	5	7	5	5	5	7	5	5	5	7	5
6	6	6	8	6	6	6	8	6	6	6	8	6	6	6	8
1	1	1	1	1	1	1	1	1	1	1	1	5	5	5	5
2	2	2	2	2	2	2	2	2	2	2	2	6	6	6	6
1	1	3	1	1	1	3	1	1	1	3	1	5	5	7	5
2	2	2	4	2	2	2	4	2	2	2	4	6	6	6	8
13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
13	13	15	13	13	13	15	13	13	13	15	13	13	13	15	13
14	14	14	16	14	14	14	16	14	14	14	16	14	14	14	16

 4.59×4.120

1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
5	6	6	5	5	6	6	5	5	6	6	5	5	6	6	5
6	5	5	6	6	5	5	6	6	5	5	6	6	5	5	6
6	5	5	6	6	5	5	6	6	5	5	6	6	5	5	6
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
1	2	3	4	1	2	3	4	1	2	3	4	5	6	7	8
13	14	14	13	13	14	14	13	13	14	14	13	13	14	14	13
14	13	13	14	14	13	13	14	14	13	13	14	14	13	13	14
14	13	13	14	14	13	13	14	14	13	13	14	14	13	13	14
13	14	15	16	13	14	15	16	13	14	15	16	13	14	15	16

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.63×4.63

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3
2	1	4	3	2	1	4	3	2	1	4	3	2	1	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7
6	5	8	7	6	5	8	7	6	5	8	7	6	5	8
1	1	1	1	5	5	5	5	9	9	9	9	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12

 4.63×4.64

1	1	3	3	1	1	3	3	1	1	3	3	1	1	3
2	2	4	4	2	2	4	4	2	2	4	4	2	2	4
1	1	3	3	1	1	3	3	1	1	3	3	1	1	3
2	2	4	4	2	2	4	4	2	2	4	4	2	2	4
5	5	7	7	5	5	7	7	5	5	7	7	5	5	7
6	6	8	8	6	6	8	8	6	6	8	8	6	6	8
5	5	7	7	5	5	7	7	5	5	7	7	5	5	7
6	6	8	8	6	6	8	8	6	6	8	8	6	6	8
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.63×4.65

1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
2	1	4	3	2	1	4	3	2	1	4	3	2	1	4	3
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
6	5	8	7	6	5	8	7	6	5	8	7	6	5	8	7
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
6	5	8	7	6	5	8	7	6	5	8	7	6	5	8	7
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11

 4.63×4.66

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	3	3	1	1	3	3	1	1	3	3	1	1	3	3
2	2	4	4	2	2	4	4	2	2	4	4	2	2	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	5	7	7	5	5	7	7	5	5	7	7	5	5	7	7
6	6	8	8	6	6	8	8	6	6	8	8	6	6	8	8
1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.63×4.92

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	3	1	1	1	3	1	1	1	3	1	1	1	3	1
2	2	2	4	2	2	2	4	2	2	2	4	2	2	2	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	5	7	5	5	5	7	5	5	5	7	5	5	5	7	5
6	6	6	8	6	6	6	8	6	6	6	8	6	6	6	8
1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	1	5	5	7	5	9	9	11	9	13	13	15	13
2	2	2	4	6	6	6	8	10	10	10	12	14	14	14	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	5	1	1	3	1	13	13	15	13	9	9	11	9
6	6	6	8	2	2	2	4	14	14	14	16	10	10	10	12

 4.63×4.120

1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
5	6	6	5	5	6	6	5	5	6	6	5	5	6	6	5
6	5	5	6	6	5	5	6	6	5	5	6	6	5	5	6
6	5	5	6	6	5	5	6	6	5	5	6	6	5	5	6
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
1	2	2	1	5	6	6	5	9	10	10	9	13	14	14	13
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	6	5	1	2	2	1	13	14	14	13	9	10	10	9
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.64×4.64

1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12	12
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12	12
5	5	7	7	5	5	7	7	13	13	15	15	13	13	15	15
6	6	8	8	6	6	8	8	14	14	16	16	14	14	16	16
5	5	7	7	5	5	7	7	13	13	15	15	13	13	15	15
6	6	8	8	6	6	8	8	14	14	16	16	14	14	16	16
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12	12
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12	12
5	5	7	7	5	5	7	7	13	13	15	15	13	13	15	15
6	6	8	8	6	6	8	8	14	14	16	16	14	14	16	16
5	5	7	7	5	5	7	7	13	13	15	15	13	13	15	15
6	6	8	8	6	6	8	8	14	14	16	16	14	14	16	16

 4.64×4.65

1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	14	13	16	15
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	14	13	16	15
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	13	10	9	12	11
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	14	13	16	15
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	14	13	16	15

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.64×4.66

1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12
5	5	5	5	5	5	5	5	13	13	13	13	13	13	13
6	6	6	6	6	6	6	6	14	14	14	14	14	14	14
5	5	7	7	5	5	7	7	13	13	15	15	13	13	15
6	6	10	10	6	6	10	10	14	14	16	16	14	14	16
1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12
5	5	5	5	5	5	5	5	13	13	13	13	13	13	13
6	6	6	6	6	6	6	6	14	14	14	14	14	14	14
5	5	7	7	5	5	7	7	13	13	15	15	13	13	15
6	6	10	10	6	6	10	10	14	14	16	16	14	14	16

 4.64×4.92

1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	3	1	1	1	3	1	9	9	11	9	9	9	11
2	2	2	4	2	2	2	4	10	10	10	12	10	10	12
5	5	5	5	5	5	5	5	13	13	13	13	13	13	13
6	6	6	6	6	6	6	6	14	14	14	14	14	14	14
5	5	7	5	5	5	7	5	13	13	15	13	13	13	15
6	6	6	8	6	6	6	8	14	14	14	16	14	14	16
1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	3	1	1	1	3	1	9	9	11	9	9	9	11
2	2	2	4	2	2	2	4	10	10	10	12	10	10	12
5	5	5	5	5	5	5	5	13	13	13	13	13	13	13
6	6	6	6	6	6	6	6	14	14	14	14	14	14	14
5	5	7	5	5	5	7	5	13	13	15	13	13	13	15
6	6	6	8	6	6	6	8	14	14	14	16	14	14	16

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.64×4.120

1	2	2	1	1	2	2	1	9	10	10	9	9	10	10	9
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
5	6	6	5	5	6	6	5	13	14	14	13	13	14	14	13
6	5	5	6	6	5	5	6	14	13	13	14	14	13	13	14
6	5	5	6	6	5	5	6	14	13	13	14	14	13	13	14
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16
1	2	2	1	1	2	2	1	9	10	10	9	9	10	10	9
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
5	6	6	5	5	6	6	5	13	14	14	13	13	14	14	13
6	5	5	6	6	5	5	6	14	13	13	14	14	13	13	14
6	5	5	6	6	5	5	6	14	13	13	14	14	13	13	14
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16

 4.65×4.65

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.65 x 4.66

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12
1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	3	5	5	7	7	9	9	11	11	13	13	15	15
2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	7	1	1	3	3	13	13	15	15	9	9	11	11
6	6	8	8	2	2	4	4	14	14	16	16	10	10	12	12

4.65 x 4.92

1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	1	5	5	7	5	9	9	11	9	13	13	15	13
2	2	2	4	6	6	6	8	10	10	10	12	14	14	14	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	5	1	1	3	1	13	13	15	13	9	9	11	9
6	6	6	8	2	2	2	4	14	14	14	16	10	10	10	12
1	1	1	1	5	5	5	5	9	9	9	9	13	13	13	13
2	2	2	2	6	6	6	6	10	10	10	10	14	14	14	14
1	1	3	1	5	5	7	5	9	9	11	9	13	13	15	13
2	2	2	4	6	6	6	8	10	10	10	12	14	14	14	16
5	5	5	5	1	1	1	1	13	13	13	13	9	9	9	9
6	6	6	6	2	2	2	2	14	14	14	14	10	10	10	10
5	5	7	5	1	1	3	1	13	13	15	13	9	9	11	9
6	6	6	8	2	2	2	4	14	14	14	16	10	10	10	12

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.65×4.120

1	2	2	1	5	6	6	5	9	10	10	9	13	14	14	13
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	6	5	1	2	2	1	13	14	14	13	9	10	10	9
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
1	2	2	1	5	6	6	5	9	10	10	9	13	14	14	13
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	6	6	5	1	2	2	1	13	14	14	13	9	10	10	9
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
6	5	5	6	2	1	1	2	14	13	13	14	10	9	9	10
5	6	7	8	1	2	3	4	13	14	15	15	9	10	11	12

 4.66×4.66

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	3	3	1	1	3	3	1	1	3	3	1	1	3	3
2	2	4	4	2	2	4	4	2	2	4	4	2	2	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	5	7	7	5	5	7	7	5	5	7	7	5	5	7	7
6	6	8	8	6	6	8	8	6	6	8	8	6	6	8	8
1	1	1	1	1	1	1	1	9	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10	10
1	1	3	3	1	1	3	3	9	9	11	11	9	9	11	11
2	2	4	4	2	2	4	4	10	10	12	12	10	10	12	12
5	5	5	5	5	5	5	5	13	13	13	13	13	13	13	13
6	6	6	6	6	6	6	6	14	14	14	14	14	14	14	14
5	5	7	7	5	5	7	7	13	13	15	15	13	13	15	15
6	6	8	8	6	6	8	8	14	14	16	16	14	14	16	16

TABLE V

DIRECT PRODUCTS: 4 x 4 (continued)

4.66 x 4.92

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	3	1	1	1	3	1	1	1	3	1	1	1	3
2	2	2	4	2	2	2	4	2	2	2	4	2	2	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	5	7	5	5	5	7	5	5	5	7	5	5	5	7
6	6	6	8	6	6	6	8	6	6	6	8	6	6	8
1	1	1	1	1	1	1	1	9	9	9	9	9	9	9
2	2	2	2	2	2	2	2	10	10	10	10	10	10	10
1	1	3	1	1	1	3	1	9	9	11	9	9	9	11
2	2	2	4	2	2	2	4	10	10	10	12	10	10	10
5	5	5	5	5	5	5	5	13	13	13	13	13	13	13
6	6	6	6	6	6	6	6	14	14	14	14	14	14	14
5	5	7	5	5	5	7	5	13	13	15	13	13	13	15
6	6	6	8	6	6	6	8	14	14	14	16	14	14	14

4.66 x 4.120

1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
5	6	6	5	5	6	6	5	5	6	6	5	5	6	6	5
6	5	5	6	6	5	5	6	6	5	5	6	6	5	5	6
6	5	5	6	6	5	5	6	6	5	5	6	6	5	5	6
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
1	2	2	1	1	2	2	1	9	10	10	9	9	10	10	9
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
2	1	1	2	2	1	1	2	10	9	9	10	10	9	9	10
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
5	6	6	5	5	6	6	5	13	14	14	13	13	14	14	13
6	5	5	6	6	5	5	6	14	13	13	14	14	13	13	14
6	5	5	6	6	5	5	6	14	13	13	14	14	13	13	14
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16

TABLE V

DIRECT PRODUCTS: 4×4 (continued) 4.92×4.92

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	3	1	1	1	3	1	1	1	3	1	1	3	1
2	2	2	4	2	2	2	4	2	2	2	4	2	2	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
5	5	7	5	5	5	7	5	5	5	7	5	5	7	5
6	6	6	8	6	6	6	8	6	6	6	8	6	6	8
1	1	1	1	1	1	1	1	9	9	9	9	1	1	1
2	2	2	2	2	2	2	2	10	10	10	10	2	2	2
1	1	3	1	1	1	3	1	9	9	11	9	1	1	3
2	2	2	4	2	2	2	4	10	10	10	12	2	2	4
5	5	5	5	5	5	5	5	5	5	5	5	13	13	13
6	6	6	6	6	6	6	6	6	6	6	6	14	14	14
5	5	7	5	5	5	7	5	5	5	7	5	13	13	15
6	6	6	8	6	6	6	8	6	6	6	8	14	14	16

 4.92×4.120

1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
5	6	6	5	5	6	6	5	5	6	6	5	5	6	6	5
6	5	5	6	6	5	5	6	6	5	5	6	6	5	5	6
6	5	5	6	6	5	5	6	6	5	5	6	6	5	5	6
5	6	7	8	5	6	7	8	5	6	7	8	5	6	7	8
1	2	2	1	1	2	2	1	9	10	10	9	1	2	2	1
2	1	1	2	2	1	1	2	10	9	9	10	2	1	1	2
2	1	1	2	2	1	1	2	10	9	9	10	2	1	1	2
1	2	3	4	1	2	3	4	9	10	11	12	1	2	3	4
5	6	6	5	5	6	6	5	5	6	6	5	13	14	14	13
6	5	5	6	6	5	5	6	6	5	5	6	14	13	13	14
6	5	5	6	6	5	5	6	6	5	5	6	14	13	13	14
5	6	7	8	5	6	7	8	5	6	7	8	13	14	15	16

TABLE V

DIRECT PRODUCTS: 4×4 (continued)

4.120 x 4.120

1	2	2	1	5	6	6	5	5	6	6	5	1	2	2	1
2	1	1	2	6	5	5	6	6	5	5	6	2	1	1	2
2	1	1	2	6	5	5	6	6	5	5	6	2	1	1	2
1	2	3	4	5	6	7	8	5	6	7	8	1	2	3	4
5	6	6	5	1	2	2	1	1	2	2	1	5	6	6	5
6	5	5	6	2	1	1	2	2	1	1	2	6	5	5	6
6	5	5	6	2	1	1	2	2	1	1	2	6	5	5	6
5	6	7	8	1	2	3	4	1	2	3	4	5	6	7	8
5	6	6	5	1	2	2	1	1	2	2	1	5	6	6	5
6	5	5	6	2	1	1	2	2	1	1	2	6	5	5	6
6	5	5	6	2	1	1	2	2	1	1	2	6	5	5	6
5	6	7	8	1	2	3	4	1	2	3	4	5	6	7	8
1	2	2	1	5	6	6	5	9	10	10	9	13	14	14	13
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
2	1	1	2	6	5	5	6	10	9	9	10	14	13	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

6. Semidirect products

Definition 6.1. An endomorphism ϕ of a semigroup S is a mapping (= single-valued transformation) of S into S such that multiplication is preserved, i.e., such that $(ab)\phi = a\phi \circ b\phi$ for any elements a and b of S . An endomorphism is called an automorphism if it is biunique (one-to-one) and onto, i.e., if every element of S is the image of one and only one element of S under the endomorphism. The product $\phi_1 \phi_2$ of two mappings of a set into itself is defined to be the mapping which has the same effect as ϕ_1 followed by ϕ_2 ; i.e., if x is any element of the set and $x\phi_1 = y$ and $y\phi_2 = z$ then $x(\phi_1 \phi_2) = z$. If the set is a semigroup S and ϕ_1 and ϕ_2 are endomorphisms of S , then $\phi_1 \phi_2$ is an endomorphism of S ; for, if a and b are any elements of S ,

$$\begin{aligned} (ab)(\phi_1 \phi_2) &= [(ab)\phi_1]\phi_2 \\ (a\phi_1 \circ b\phi_2)\phi &= (a\phi_1)\phi_2 \circ (b\phi_1)\phi_2 \\ &= a(\phi_1 \phi_2) \circ b(\phi_1 \phi_2) \end{aligned}$$

Whenever we refer to a semigroup of endomorphisms, we shall always understand that the multiplication in the semigroup is that defined above.

Definition 6.2. The semidirect product of a semigroup K (whose operation is symbolized by $+$) and a semigroup U (whose operation is symbolized by $\circ$) of distinct endomorphisms of K is the system consisting of all ordered pairs (x, y) , where $x \in K$ and $y \in U$, and the operation $(x, y) * (x', y') = (x + yx', y \circ y')$, where yx' is the image of x' under the endomorphism y . By virtue of Definition 6.1, $(y \circ y')x = y(y'x)$. For the semidirect product of

K and U we write $K \otimes U$.

Theorem 6.1. The system $K \otimes U$, where K is a semigroup and U is a semigroup of endomorphisms of K , is a semigroup.

Proof. Let (a, b) , (c, d) , and (h, f) be arbitrary elements of $K \otimes U$. Then $(a, b) * (c, d) = (a + bc, b \circ d)$. Since b is an endomorphism of K , bc is an element of K ; hence $a + bc$ is an element of K . Both b and d are elements of U , hence $b \circ d \in U$. Therefore $(a + bc, b \circ d) \in K \otimes U$. Furthermore,

$$\begin{aligned} & [(a, b) * (c, d)] * (h, f) \\ &= (a + bc, b \circ d) * (h, f) \\ &= [(a + bc) + (b \circ d)h, (b \circ d) \circ f] \\ &= \{a + [bc + b(dh)], b \circ (d \circ f)\} = [a + b(c + dh), b \circ (d \circ f)] \\ &= (a, b) * (c + dh, d \circ f) \\ &= (a, b) * [(c, d) * (h, f)] . \end{aligned}$$

Theorem 6.2. If K is a semigroup and U is a semigroup of endomorphisms of K , an element $(a, b) \in K \otimes U$ is a left identity element of $K \otimes U$ if and only if a is a left identity element of K and b is the identity automorphism.

Proof. (1) Let a be a left identity element of K and let b be the identity automorphism. Let (c, d) be an arbitrary element of $K \otimes U$. Then $(a, b) * (c, d) = (a + bc, b \circ d) = (a + c, d) = (c, d)$.
(2) Let (a, b) be a left identity element of $K \otimes U$ and let (c, d) be an arbitrary element of $K \otimes U$. Then

$$(a + bc, b \circ d) = (a, b) * (c, d) = (c, d) .$$

Since this implies that $b \circ d = d$, b must be the identity automorphism, whence $c = a + bc = a + c$, and a is therefore a left identity

element of K .

This theorem is not necessarily true for right identity elements of K .

Example 6.1.

	x_1	x_2	x_3		x_1	x_2	x_3		y_1	y_2	y_3
x_1	x_1	x_1	x_1	y_1	x_1	x_2	x_3	y_1	y_1	y_2	y_3
x_2	x_1	x_1	x_2	y_2	x_1	x_1	x_3	y_2	y_2	y_2	y_3
x_3	x_1	x_1	x_3	y_3	x_1	x_1	x_1	y_3	y_3	y_3	y_3

$$(x_2, y_3) * (x_3, y_1) = (x_2 + y_3 x_3, y_3 \circ y_1) \quad (x_2 + x_1, y_3) = (x_1, y_3)$$

Theorem 6.3. If K is a semigroup, K_1 is a right ideal of K , U is a semigroup of endomorphisms of K , and U_1 is a right ideal of U , then the set of elements (K_1, U_1) is a right ideal of $K \hat{\otimes} U$.

Proof. We are given that $K_1 + K \subseteq K_1$ and $U_1 \circ U \subseteq U_1$.

$$\begin{aligned} & (K_1, U_1) \hat{\otimes} (K, U) \\ &= (K_1 + U_1 K, U_1 \circ U) \subseteq (K_1 + K, U_1 \circ U) \\ &\subseteq (K_1, U_1) . \end{aligned}$$

This theorem is not necessarily true if "left" is substituted for "right." In fact, it need not then be true even if U is a group of automorphisms of K .

Example 6.2.

	x_1	x_2	x_3		x_1	x_2	x_3		y_1	y_2
x_1	x_1	x_1	x_1	y_1	x_1	x_2	x_3	y_1	y_1	y_2
x_2	x_1	x_2	x_3	y_2	x_1	x_3	x_2	y_2	y_2	y_1
x_3	x_1	x_2	x_3							

$$K_1 \equiv [x_1, x_2] \quad U_1 \equiv [y_1, y_2]$$

$$(x_2, y_2) * (x_2, y_2) = (x_2 + y_2 x_2, y_2 \circ y_2) = (x_2 + x_3, y_1) = (x_3, y_1)$$

Theorem 6.4. If k is a left constant element of K and c is a left constant element of U , then the element (k, c) is a left constant element of $K \otimes U$.

Proof. Since k is a left constant element of K , there is an element $d \in K$ such that, for all $x \in K$, $k + x = d$; since c is a left constant element of U , there is an element $b \in U$ such that, for all $y \in U$, $c \circ y = b$. Then

$$(k, c) * (x, y) = (k + cx, c \circ y) = (d, b)$$

for all $(x, y) \in K \otimes U$.

Corollary. If k is a left zero element of K and c is a left zero element of U , then the element (k, c) is a left zero element of $K \otimes U$.

Example 6.3.

$$K \equiv \begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline x_1 & x_1 & x_1 & x_1 & x_1 \\ x_2 & x_1 & x_1 & x_1 & x_1 \\ x_3 & x_3 & x_3 & x_3 & x_3 \\ x_4 & x_4 & x_4 & x_4 & x_4 \end{array} \quad U_1 \equiv \begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline y_1 & x_1 & x_2 & x_1 & x_2 \\ y_2 & x_1 & x_2 & x_1 & x_1 \end{array} \quad U \equiv \begin{array}{c|cc} & y_1 & y_2 \\ \hline y_1 & y_2 & y_2 \\ y_2 & y_2 & y_2 \end{array}$$

$$x_2 + x_1 = x_1$$

$$x_1 + x_1 = x_1$$

$$y_1 \circ y_j = y_2$$

$$y_2 \circ y_j = y_2$$

$$(x_2, y_1) * (x_1, y_j) = (x_1, y_2)$$

$$(x_1, y_2) * (x_1, y_j) = (x_1, y_2)$$

Theorem 6.4 is not necessarily true if "right" is substituted for "left."

Example 6.4.

$$\begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \\ \hline x_1 \quad x_1 \quad x_1 \\ x_2 \quad x_1 \quad x_2 \\ x_3 \quad x_1 \quad x_3 \end{array} \\
 K \equiv
 \end{array}
 \begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \\ \hline x_1 \quad x_2 \quad x_3 \\ x_1 \quad x_1 \quad x_3 \\ x_1 \quad x_1 \quad x_1 \\ y_4 \quad x_3 \quad x_3 \quad x_3 \end{array} \\
 y_1 \quad y_2 \quad y_3 \quad y_4
 \end{array}
 \begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \\ \hline x_1 \quad x_2 \quad x_3 \\ x_1 \quad x_1 \quad x_3 \\ x_1 \quad x_1 \quad x_1 \\ x_3 \quad x_3 \quad x_3 \end{array} \\
 U \equiv
 \end{array}
 \begin{array}{c}
 \begin{array}{c} y_1 \quad y_2 \quad y_3 \quad y_4 \\ \hline y_1 \quad y_2 \quad y_3 \quad y_4 \\ y_2 \quad y_2 \quad y_3 \quad y_4 \\ y_3 \quad y_4 \quad y_3 \quad y_4 \\ y_4 \quad y_4 \quad y_3 \quad y_4 \end{array}
 \end{array}$$

$$x_1 + x_2 = x_1 \quad y_j y_4 = y_4$$

$$(x_3, y_4) * (x_2, y_4) = (x_3 + y_4 x_2, y_4 \circ y_4) = (x_3 + x_3, y_4) = (x_3, y_4)$$

$$x_1 + x_1 = x_1 \quad y_j y_4 = y_4$$

$$(x_3, y_4) * (x_1, y_4) = (x_3 + y_4 x_1, y_4 \circ y_4) = (x_3 + x_3, y_4) = (x_3, y_4) .$$

Given a commutative group K and a commutative group U , where U is a group of automorphisms of K , $K \otimes U$ is not necessarily commutative.

Example 6.5.

$$\begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \quad x_4 \\ \hline x_1 \quad x_2 \quad x_3 \quad x_4 \\ x_2 \quad x_1 \quad x_4 \quad x_3 \\ x_3 \quad x_4 \quad x_1 \quad x_2 \\ x_4 \quad x_3 \quad x_2 \quad x_1 \end{array} \\
 K \equiv
 \end{array}
 \begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \quad x_4 \\ \hline x_1 \quad x_2 \quad x_3 \quad x_4 \\ x_1 \quad x_2 \quad x_4 \quad x_3 \end{array} \\
 y_1 \quad y_2
 \end{array}
 \begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \quad x_4 \\ \hline x_1 \quad x_2 \quad x_3 \quad x_4 \\ x_1 \quad x_2 \quad x_4 \quad x_3 \end{array} \\
 U \equiv
 \end{array}
 \begin{array}{c}
 \begin{array}{c} y_1 \quad y_2 \\ \hline y_1 \quad y_2 \\ y_2 \quad y_1 \end{array}
 \end{array}$$

$$(x_1, y_2) * (x_3, y_1) = (x_1 + y_2 x_3, y_2 \circ y_1) = (x_1 + x_4, y_2 \circ y_1) = (x_4, y_2)$$

$$(x_3, y_1) * (x_1, y_2) = (x_3 + y_1 x_1, y_1 \circ y_2) = (x_3 + x_1, y_1 \circ y_2) = (x_3, y_2)$$

Let x be an idempotent element of K and let y be an idempotent element of U ; the (x, y) is not necessarily an idempotent element of $K \otimes U$.

Example 6.6.

$$\begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \\ \hline x_1 \quad x_2 \quad x_3 \\ x_2 \quad x_1 \quad x_3 \\ x_3 \quad x_1 \quad x_2 \end{array} \\
 K \equiv \begin{array}{c} x_1 \quad x_2 \quad x_3 \\ \hline x_1 \quad x_2 \quad x_3 \\ x_1 \quad x_2 \quad x_3 \\ x_1 \quad x_2 \quad x_3 \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \\ \hline x_1 \quad x_2 \quad x_3 \\ x_1 \quad x_3 \quad x_2 \\ x_1 \quad x_1 \quad x_1 \end{array} \\
 y_1 \quad y_2 \quad y_3
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{c} y_1 \quad y_2 \quad y_3 \\ \hline y_1 \quad y_2 \quad y_3 \\ y_2 \quad y_1 \quad y_3 \\ y_3 \quad y_3 \quad y_3 \end{array} \\
 U \equiv \begin{array}{c} y_1 \quad y_2 \quad y_3 \\ \hline y_1 \quad y_2 \quad y_3 \\ y_2 \quad y_1 \quad y_3 \\ y_3 \quad y_3 \quad y_3 \end{array}
 \end{array}$$

$(x_2, y_3) \times (x_2, y_3) = (x_2, y_3 \circ y_3) = (x_2, y_3)$

If K_1 is a subsemigroup of a semigroup K and U_1 is a subsemigroup of a semigroup U of endomorphisms of K , (K_1, U_1) is not necessarily a subsemigroup of (K, U) . In fact, (K_1, U_1) need not be a subsemigroup even if K is a group, K_1 a subgroup, U a group of automorphisms of K , and U_1 a subgroup of U .

Example 6.7.

$$\begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \quad x_4 \\ \hline x_1 \quad x_2 \quad x_3 \quad x_4 \\ x_2 \quad x_1 \quad x_4 \quad x_3 \\ x_3 \quad x_4 \quad x_1 \quad x_2 \\ x_4 \quad x_3 \quad x_2 \quad x_1 \end{array} \\
 K \equiv \begin{array}{c} x_1 \quad x_2 \quad x_3 \quad x_4 \\ \hline x_1 \quad x_2 \quad x_3 \quad x_4 \\ x_2 \quad x_1 \quad x_4 \quad x_3 \\ x_3 \quad x_4 \quad x_1 \quad x_2 \\ x_4 \quad x_3 \quad x_2 \quad x_1 \end{array}
 \end{array}
 \quad
 \begin{array}{c}
 \begin{array}{c} x_1 \quad x_2 \quad x_3 \quad x_4 \\ \hline x_1 \quad x_2 \quad x_3 \quad x_4 \\ x_1 \quad x_2 \quad x_4 \quad x_3 \\ x_1 \quad x_4 \quad x_3 \quad x_2 \\ x_1 \quad x_3 \quad x_2 \quad x_4 \\ x_1 \quad x_3 \quad x_4 \quad x_2 \\ x_1 \quad x_4 \quad x_2 \quad x_3 \end{array} \\
 y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6
 \end{array}$$

$$U \cong \begin{array}{c|cccccc} & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \\ \hline y_1 & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \\ y_2 & y_2 & y_1 & y_6 & y_5 & y_4 & y_3 \\ y_3 & y_3 & y_5 & y_1 & y_6 & y_2 & y_4 \\ y_4 & y_4 & y_6 & y_5 & y_1 & y_3 & y_2 \\ y_5 & y_5 & y_3 & y_4 & y_2 & y_6 & y_1 \\ y_6 & y_6 & y_4 & y_2 & y_3 & y_1 & y_5 \end{array}$$

$$K_1 \cong [x_1, x_2]$$

$$U_1 = U \text{ or } U_1 [y_1, y_4]$$

$$\begin{aligned} & (x_1, y_4) * (x_2, y_1) \\ &= (x_1 + x_2 y_4, y_4 \circ y_1) = (x_1 \quad x_3, y_4 \circ y_1) \\ &= (x_3, y_4) \\ & \notin (K_1, U_1) \end{aligned}$$

Of course, if the subsemigroup U_1 is a semigroup of endomorphisms of K_1 , then $(K_1, U_1) = K_1 \otimes U_1$, a subsemigroup of $K \otimes U$.

The operation of semidirect multiplication of semigroups, unlike that of direct multiplication (see Theorem 2.9), need not be commutative. Given a semigroup K and a semigroup U such that each is isomorphic to a semigroup of endomorphisms of the other, $K \otimes U$ is not necessarily isomorphic to $U \otimes K$.

Example 6.8.

		x_1	x_2	x_3		x_1	x_2	x_3	
	x_1	x_1	x_2	x_3		y_1	x_1	x_2	x_3
$K \equiv$	x_2	x_1	x_2	x_3		y_2	x_1	x_3	x_2
	x_3	x_1	x_2	x_3		y_3	x_1	x_1	x_1
						y_4	x_2	x_2	x_2
						y_5	x_3	x_3	x_3

	y_1	y_2	y_3	y_4	y_5		y_1	y_2	y_3	y_4	y_5
$U \equiv$	y_1	y_1	y_2	y_3	y_4	y_5	y_1	y_3	y_3	y_3	y_3
	y_2	y_2	y_1	y_3	y_4	y_5	y_2	y_4	y_4	y_4	y_4
	y_3	y_3	y_3	y_3	y_4	y_5	y_3	y_5	y_5	y_5	y_5
	y_4	y_4	y_5	y_3	y_4	y_5					
	y_5	y_5	y_4	y_3	y_4	y_5					

The element (x_1, y_1) of $K \hat{\otimes} U$ is a left identity element:

$$(x_1, y_1) * (x_i, y_j) = (x_1 + y_1 x_i, y_1 \circ y_j) = (x_1 + x_i, y_j) = (x_i, y_j) .$$

But there is no left identity element in $U \hat{\otimes} K$:

$$(y_j, x_1) * (y_1, x_p) = (y_j + x_1 y_1, x_1 \circ x_p) = (y_j + y_3, x_p) = (y_3, x_p) ,$$

$$(y_j, x_2) * (y_1, x_p) = (y_j + x_2 y_1, x_2 \circ x_p) = (y_j + y_4, x_p) = (y_4, x_p) ,$$

$$(y_j, x_3) * (y_1, x_p) = (y_j + x_3 y_1, x_3 \circ x_p) = (y_j + y_5, x_p) = (y_5, x_p) .$$

Commutativity of semidirect multiplication need not obtain even if each semigroup is a group and is isomorphic to a group of automorphisms of the other. For example, let

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
$K \equiv$	x_1	x_1	x_2	x_3	x_4	y_1	x_1	x_2	x_3
	x_2	x_2	x_1	x_4	x_3	y_2	x_1	x_2	x_4
	x_3	x_3	x_4	x_1	x_2	y_3	x_1	x_4	x_3
	x_4	x_4	x_3	x_2	x_1	y_4	x_1	x_3	x_2
						y_5	x_1	x_3	x_4
						y_6	x_1	x_4	x_2

	y_1	y_2	y_3	y_4	y_5	y_6		y_1	y_2	y_3	y_4	y_5	y_6
$U \equiv$	y_1	y_2	y_3	y_4	y_5	y_6	x_1	y_1	y_2	y_3	y_4	y_5	y_6
	y_2	y_1	y_6	y_5	y_4	y_3	x_2	y_1	y_3	y_2	y_4	y_5	y_6
	y_3	y_5	y_1	y_6	y_2	y_4	x_3	y_1	y_2	y_3	y_4	y_5	y_6
	y_4	y_6	y_5	y_1	y_3	y_2	x_4	y_1	y_3	y_2	y_4	y_6	y_5
	y_5	y_3	y_4	y_2	y_6	y_1							
	y_6	y_4	y_2	y_3	y_1	y_5							

As the reader may verify, $U \otimes K$ contains elements $a = (y_5, x_1)$, $b = (y_6, x_1)$, $c = (y_3, x_2)$, such that $a^2 = b$, $b^2 = a$, and $c^2 = a$; but $K \otimes U$ contains no such set of three elements.

7. Computation of semidirect products of finite semigroups

Let K be a semigroup whose elements are x_i ($i = 1, 2, \dots, k$) and let U be a semigroup of endomorphisms of K whose elements are y_j ($j = 1, 2, \dots, u$). As in the case of direct products, let $(x_i, y_j) = z_{j+(i-1)u}$. When the elements of $K \otimes U$ are ordered as $(x_1, y_1), (x_1, y_2), \dots, (x_1, y_u), (x_2, y_1), (x_2, y_2), \dots, (x_k, y_u)$, consider an arbitrary row of an arbitrary block of the multiplication table of $K \otimes U$:

	(x_k, y_1)	(x_k, y_2)	...	(x_k, y_u)
(x_i, y_j)	$(x_i + y_j x_k, y_j \cdot y_1)$	$(x_i + y_j x_k, y_j y_2)$	...	$(x_i + y_j x_k, y_j \cdot y_u)$

Let $x_p = y_j x_k$ (which we may find in the list of endomorphisms). Let $x_r = x_i + x_p$ (which we may find in the multiplication table of K).

Then the arbitrary row becomes:

$$(x_i, y_j) \left| \begin{array}{cccc} (x_k, y_1) & (x_k, y_2) & (x_k, y_3) & \dots & (x_k, y_u) \\ (x_r, y_j \circ y_1) & (x_r, y_j \circ y_2) & (x_r, y_j \circ y_3) & \dots & (x_r, y_j \circ y_u) \end{array} \right.$$

The y components of this row are exactly the j -row in multiplication table of K . Therefore we may write this row (and consequently the whole table) by inspection.

Example 6.9.

	x_1	x_2	x_3		x_1	x_2	x_3		y_1	y_2	y_3	y_4
x_1	x_1	x_1	x_1	y_1	x_1	x_2	x_3	y_1	y_1	y_2	y_3	y_4
$K \equiv x_2$	x_1	x_1	x_2	y_2	x_1	x_1	x_3	y_2	y_2	y_2	y_3	y_4
x_3	x_1	x_1	x_3	y_3	x_1	x_1	x_1	y_3	y_3	y_3	y_3	y_4
				y_4	x_3	x_3	x_3	y_4	y_4	y_4	y_3	y_4

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}	z_{11}	z_{12}
z_1	z_1	z_2	z_3	z_4	z_1	z_2	z_3	z_4	z_1	z_2	z_3	z_4
z_2	z_2	z_2	z_3	z_4	z_2	z_2	z_3	z_4	z_2	z_2	z_3	z_4
z_3	z_3	z_3	z_3	z_4	z_3	z_3	z_3	z_4	z_3	z_3	z_3	z_4
z_4	z_4	z_4	z_3	z_4	z_4	z_4	z_3	z_4	z_4	z_4	z_3	z_4
z_5	z_1	z_2	z_3	z_4	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
z_6	z_2	z_2	z_3	z_4	z_2	z_2	z_3	z_4	z_6	z_6	z_7	z_8
z_7	z_3	z_3	z_3	z_4	z_3	z_3	z_3	z_4	z_3	z_3	z_3	z_4
z_8	z_8	z_8	z_7	z_8	z_8	z_8	z_7	z_8	z_8	z_8	z_7	z_8
z_9	z_1	z_2	z_3	z_4	z_1	z_2	z_3	z_4	z_9	z_{10}	z_{11}	z_{12}
z_{10}	z_2	z_2	z_3	z_4	z_2	z_2	z_3	z_4	z_{10}	z_{10}	z_{11}	z_{12}
z_{11}	z_3	z_3	z_3	z_4	z_3	z_3	z_3	z_4	z_3	z_3	z_3	z_4
z_{12}	z_{12}	z_{12}	z_{11}	z_{12}	z_{12}	z_{12}	z_{11}	z_{12}	z_{12}	z_{12}	z_{11}	z_{12}

$K \otimes U$

It is well known² that every mapping of a semigroup K onto itself is an endomorphism if and only if every element of K is a left (right) zero element. In this case the multiplication table of the semidirect product is easily described. Let K be a semigroup in which every element is a left zero element. Let $y_i \in U$ ($i = 1, 2, \dots, t$) be endomorphisms of K . Consider an arbitrary block of the multiplication table of $K \otimes U$.

	(x_q, y_1)	(x_q, y_2)	$\dots$	(x_q, y_t)
(x_j, y_1)	$(x_j + y_1 x_q, y_1 \circ y_1)$	$(x_j + y_1 x_q, y_1 \circ y_2)$	$\dots$	$(x_j + y_1 x_q, y_1 \circ y_t)$
(x_j, y_2)	$(x_j + y_2 x_q, y_2 \circ y_1)$	$(x_j + y_2 x_q, y_2 \circ y_2)$	$\dots$	$(x_j + y_2 x_q, y_2 \circ y_t)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
(x_j, y_t)	$(x_j + y_t x_q, y_t \circ y_1)$	$(x_j + y_t x_q, y_t \circ y_2)$	$\dots$	$(x_j + y_t x_q, y_t \circ y_t)$

Since x_j is a left zero element this arbitrary block becomes:

	(x_q, y_1)
(x_j, y_1)	$(x_j, y_1 \circ y_1)$
(x_j, y_2)	$(x_j, y_2 \circ y_1)$
$\vdots$	$\vdots$
(x_j, y_t)	$(x_j, y_t \circ y_1)$

The y subscripts in this block are identical with the y subscripts in the multiplication table of U . Hence this block (and thus the whole table) can be written by inspection.

²E. E. Posey, "Endomorphisms and Translations of Semigroups," (Master's Thesis, University of Tennessee, 1949).

The endomorphisms of the second and third order semigroups are listed in Table VI and Table VII respectively. The automorphisms of the fourth order semigroups are listed in Table VIII.

Since the semidirect product of two semigroups has meaning only when one of the semigroups is isomorphic to a semigroup of endomorphisms of the other, we employ the notation $y \circ z \otimes U$ with the understanding that U refers to a semigroup of endomorphisms of the semigroup of order y in lineal position z as listed in the appendix. The semidirect products of the semigroups of orders two and three with their semigroups of endomorphisms are listed in Table IX and Table X respectively. The semidirect product of the semigroups of order four with their semigroups of automorphisms are listed in Table XI.

TABLE VI
ENDOMORPHISMS OF SECOND ORDER SEMIGROUPS

2.1	2.2
$\begin{array}{c} 1 \quad \frac{1 \ 2}{1 \ 2} \\ 2 \quad 1 \ 1 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 2 \end{array}$	$\begin{array}{c} 1 \quad \frac{1 \ 2}{1 \ 2} \\ 2 \quad 1 \ 1 \\ 3 \quad 2 \ 2 \end{array} \quad U = \begin{array}{c} 1 \ 2 \ 3 \\ 2 \ 2 \ 3 \\ 3 \ 2 \ 2 \end{array}$
2.3	2.4
$\begin{array}{c} 1 \quad \frac{1 \ 2}{1 \ 2} \\ 2 \quad 2 \ 1 \\ 3 \quad 1 \ 1 \\ 4 \quad 2 \ 2 \end{array} \quad U = \begin{array}{c} 1 \ 2 \ 3 \ 4 \\ 2 \ 1 \ 3 \ 4 \\ 3 \ 4 \ 3 \ 4 \\ 4 \ 3 \ 3 \ 4 \end{array}$	$\begin{array}{c} 1 \quad \frac{1 \ 2}{1 \ 2} \\ 2 \quad 1 \ 1 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 2 \end{array}$

TABLE VII
ENDOMORPHISMS OF THIRD ORDER SEMIGROUPS

3.1	3.2
$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3}{1 \ 2 \ 3} \\ 2 \quad 1 \ 1 \ 1 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 2 \end{array}$	$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3}{1 \ 2 \ 3} \\ 2 \quad 1 \ 1 \ 2 \\ 3 \quad 1 \ 1 \ 1 \end{array} \quad U = \begin{array}{c} 1 \ 2 \ 3 \\ 2 \ 3 \ 3 \\ 3 \ 3 \ 3 \end{array}$
3.3	3.4
$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3}{1 \ 2 \ 3} \\ 2 \quad 1 \ 3 \ 2 \\ 3 \quad 1 \ 1 \ 1 \end{array} \quad U = \begin{array}{c} 1 \ 2 \ 1 \\ 2 \ 1 \ 1 \\ 1 \ 1 \ 1 \end{array}$	$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3}{1 \ 2 \ 3} \\ 2 \quad 1 \ 1 \ 1 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 2 \end{array}$

TABLE VII

ENDOMORPHISMS OF THIRD ORDER SEMIGROUPS

$$U = \begin{array}{ccccc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{array}{ccccc} \hline 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 1 & & \\ 1 & 1 & 3 & & \\ 1 & 1 & 1 & & \\ 3 & 3 & 3 & & \end{array} \end{array}$$
$$U = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 3 & 3 \end{pmatrix}$$
$$U = \begin{array}{c} \begin{array}{ccc} & \underline{1} & \underline{2} & \underline{3} \\ 1 & 1 & 2 & 3 \\ 2 & 1 & 1 & 3 \\ 3 & 1 & 1 & 1 \\ 4 & 3 & 3 & 3 \end{array} \end{array}$$
$$U = \begin{array}{c|cccccccc} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{matrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{matrix} \end{array}$$

TABLE VIII
AUTOMORPHISMS OF FOURTH ORDER SEMIGROUPS

4.2

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 2 \ 4 \ 3 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.3

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 2 \ 4 \ 3 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.10

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 2 \ 4 \ 3 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.11

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 2 \ 4 \ 3 \\ 3 \quad 1 \ 3 \ 2 \ 4 \\ 4 \quad 1 \ 4 \ 2 \ 3 \\ 5 \quad 1 \ 3 \ 4 \ 2 \\ 6 \quad 1 \ 4 \ 3 \ 2 \end{array} \quad U = \begin{array}{c} 1 \ 2 \ 3 \ 4 \ 5 \ 6 \\ 2 \ 1 \ 5 \ 6 \ 3 \ 4 \\ 3 \ 4 \ 1 \ 2 \ 6 \ 5 \\ 4 \ 3 \ 6 \ 5 \ 1 \ 2 \\ 5 \ 6 \ 2 \ 1 \ 4 \ 3 \\ 6 \ 5 \ 4 \ 3 \ 2 \ 1 \end{array}$$

4.12

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 3 \ 2 \ 4 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.13

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 3 \ 2 \ 4 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.16

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 2 \ 4 \ 3 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.23

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 3 \ 2 \ 4 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.30

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 2 \ 4 \ 3 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.32

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 2 \ 4 \ 3 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

TABLE VIII

AUTOMORPHISMS OF FOURTH ORDER SEMIGROUPS (continued)

4.37

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 2 & 4 & 3
 \end{array} \\
 U = \begin{array}{cc} 1 & 2 \\ 2 & 1 \end{array}
 \end{array}$$

4.40

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 2 & 4 & 3
 \end{array} \\
 U = \begin{array}{cc} 1 & 2 \\ 2 & 1 \end{array}
 \end{array}$$

4.48

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 2 & 4 & 3 \\
 3 & 1 & 3 & 2 & 4 \\
 4 & 1 & 4 & 2 & 3 \\
 5 & 1 & 3 & 4 & 2 \\
 6 & 1 & 4 & 3 & 2
 \end{array} \\
 U = \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 6 & 3 & 4 \\ 3 & 4 & 1 & 2 & 6 & 5 \\ 4 & 3 & 6 & 5 & 1 & 2 \\ 5 & 6 & 2 & 1 & 4 & 3 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{array}
 \end{array}$$

4.49

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 3 & 2 & 4
 \end{array} \\
 U = \begin{array}{cc} 1 & 2 \\ 2 & 1 \end{array}
 \end{array}$$

4.50

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 3 & 2 & 4
 \end{array} \\
 U = \begin{array}{cc} 1 & 2 \\ 2 & 1 \end{array}
 \end{array}$$

4.53

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 2 & 4 & 3
 \end{array} \\
 U = \begin{array}{cc} 1 & 2 \\ 2 & 1 \end{array}
 \end{array}$$

4.55

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 2 & 4 & 3 \\
 3 & 1 & 3 & 2 & 4 \\
 4 & 1 & 4 & 2 & 3 \\
 5 & 1 & 3 & 4 & 2 \\
 6 & 1 & 4 & 3 & 2
 \end{array} \\
 U = \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 6 & 3 & 4 \\ 3 & 4 & 1 & 2 & 6 & 5 \\ 4 & 3 & 6 & 5 & 1 & 2 \\ 5 & 6 & 2 & 1 & 4 & 3 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{array}
 \end{array}$$

4.60

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 2 & 4 & 3
 \end{array} \\
 U = \begin{array}{cc} 1 & 2 \\ 2 & 1 \end{array}
 \end{array}$$

TABLE VIII

AUTOMORPHISMS OF FOURTH ORDER SEMIGROUPS (continued)

4.61

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 2 \ 4 \ 3 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.63

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 2 \ 1 \ 3 \ 4 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.64

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 2 \ 1 \ 4 \ 3 \\ 3 \quad 3 \ 4 \ 1 \ 2 \\ 4 \quad 4 \ 3 \ 2 \ 1 \end{array} \quad U = \begin{array}{c} 1 \ 2 \ 3 \ 4 \\ 2 \ 1 \ 4 \ 3 \\ 3 \ 4 \ 1 \ 2 \\ 4 \ 3 \ 2 \ 1 \end{array}$$

4.65

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 3 \ 4 \ 1 \ 2 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.66

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 2 \ 1 \ 4 \ 3 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.67

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 2 \ 1 \ 3 \ 4 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.68

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 2 \ 1 \ 3 \ 4 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.69

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 2 \ 4 \ 3 \end{array} \quad U = \begin{array}{c} 1 \ 2 \\ 2 \ 1 \end{array}$$

4.70

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 3 \ 2 \ 4 \\ 3 \quad 3 \ 2 \ 1 \ 4 \\ 4 \quad 2 \ 1 \ 3 \ 4 \\ 5 \quad 2 \ 3 \ 1 \ 4 \\ 6 \quad 3 \ 1 \ 2 \ 4 \end{array} \quad U = \begin{array}{c} 1 \ 2 \ 3 \ 4 \ 5 \ 6 \\ 2 \ 1 \ 6 \ 5 \ 4 \ 3 \\ 3 \ 5 \ 1 \ 6 \ 2 \ 4 \\ 4 \ 6 \ 5 \ 1 \ 3 \ 2 \\ 5 \ 3 \ 4 \ 2 \ 6 \ 1 \\ 6 \ 4 \ 2 \ 3 \ 1 \ 5 \end{array}$$

4.71

$$\begin{array}{c} 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\ 2 \quad 1 \ 4 \ 3 \ 2 \\ 3 \quad 4 \ 2 \ 3 \ 1 \\ 4 \quad 2 \ 1 \ 3 \ 4 \\ 5 \quad 2 \ 4 \ 3 \ 1 \\ 6 \quad 4 \ 1 \ 3 \ 2 \end{array} \quad U = \begin{array}{c} 1 \ 2 \ 3 \ 4 \ 5 \ 6 \\ 2 \ 1 \ 6 \ 5 \ 4 \ 3 \\ 3 \ 5 \ 1 \ 6 \ 2 \ 4 \\ 4 \ 6 \ 5 \ 1 \ 3 \ 2 \\ 5 \ 3 \ 4 \ 2 \ 6 \ 1 \\ 6 \ 4 \ 2 \ 3 \ 1 \ 5 \end{array}$$

TABLE VIII

AUTOMORPHISMS OF FOURTH ORDER SEMIGROUPS (continued)

4.72

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 1 \ 2 \ 4 \ 3 \\
 3 \quad 2 \ 1 \ 3 \ 4 \\
 4 \quad 2 \ 1 \ 4 \ 3
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \ 3 \ 4 \\
 2 \ 1 \ 4 \ 3 \\
 3 \ 4 \ 1 \ 2 \\
 4 \ 3 \ 2 \ 1
 \end{array}$$

4.73

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 2 \ 1 \ 3 \ 4
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \\
 2 \ 1
 \end{array}$$

4.74

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 2 \ 1 \ 3 \ 4
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \\
 2 \ 1
 \end{array}$$

4.75

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 2 \ 1 \ 3 \ 4
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \\
 2 \ 1
 \end{array}$$

4.77

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 1 \ 2 \ 4 \ 3
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \\
 2 \ 1
 \end{array}$$

4.78

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 2 \ 1 \ 4 \ 3
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \\
 2 \ 1
 \end{array}$$

4.91

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 1 \ 2 \ 4 \ 3
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \\
 2 \ 1
 \end{array}$$

4.92

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 2 \ 1 \ 4 \ 3
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \\
 2 \ 1
 \end{array}$$

4.96

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 1 \ 3 \ 2 \ 4 \\
 3 \quad 3 \ 2 \ 1 \ 4 \\
 4 \quad 2 \ 1 \ 3 \ 4 \\
 5 \quad 2 \ 3 \ 1 \ 4 \\
 6 \quad 3 \ 1 \ 2 \ 4
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \ 3 \ 4 \ 5 \ 6 \\
 2 \ 1 \ 6 \ 5 \ 4 \ 3 \\
 3 \ 5 \ 1 \ 6 \ 2 \ 4 \\
 4 \ 6 \ 5 \ 1 \ 3 \ 2 \\
 5 \ 3 \ 4 \ 2 \ 6 \ 1 \\
 6 \ 4 \ 2 \ 3 \ 1 \ 5
 \end{array}$$

4.97

$$\begin{array}{c}
 1 \quad \frac{1 \ 2 \ 3 \ 4}{1 \ 2 \ 3 \ 4} \\
 2 \quad 2 \ 1 \ 3 \ 4
 \end{array}
 \quad U = \quad
 \begin{array}{c}
 1 \ 2 \\
 2 \ 1
 \end{array}$$

TABLE VIII

AUTOMORPHISMS OF FOURTH ORDER SEMIGROUPS (continued)

4.98

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 2 & 4 & 3 \\
 3 & 2 & 1 & 3 & 4 \\
 4 & 2 & 1 & 4 & 3
 \end{array} \\
 U = \begin{array}{cccc}
 1 & 2 & 3 & 4 \\
 2 & 1 & 4 & 3 \\
 3 & 4 & 1 & 2 \\
 4 & 3 & 2 & 1
 \end{array}
 \end{array}$$

4.100

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 2 & 4 & 3 \\
 3 & 2 & 1 & 3 & 4 \\
 4 & 2 & 1 & 4 & 3
 \end{array} \\
 U = \begin{array}{cccc}
 1 & 2 & 3 & 4 \\
 2 & 1 & 4 & 3 \\
 3 & 4 & 1 & 2 \\
 4 & 3 & 2 & 1
 \end{array}
 \end{array}$$

4.102

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 1 & 3 & 2 & 4 \\
 3 & 3 & 2 & 1 & 4 \\
 4 & 2 & 1 & 3 & 4 \\
 5 & 2 & 3 & 1 & 4 \\
 6 & 3 & 1 & 2 & 4
 \end{array} \\
 U = \begin{array}{cccccc}
 1 & 2 & 3 & 4 & 5 & 6 \\
 2 & 1 & 6 & 5 & 4 & 3 \\
 3 & 5 & 1 & 6 & 2 & 4 \\
 4 & 6 & 5 & 1 & 3 & 2 \\
 5 & 3 & 4 & 2 & 6 & 1 \\
 6 & 4 & 2 & 3 & 1 & 5
 \end{array}
 \end{array}$$

4.103

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 2 & 1 & 3 & 4
 \end{array} \\
 U = \begin{array}{cc}
 1 & 2 \\
 2 & 1
 \end{array}
 \end{array}$$

4.104

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 2 & 2 & 1 & 3 & 4
 \end{array} \\
 U = \begin{array}{cc}
 1 & 2 \\
 2 & 1
 \end{array}
 \end{array}$$

4.110

$$\begin{array}{c}
 \begin{array}{cccc}
 & 1 & 2 & 3 & 4 \\
 1 & 1 & 2 & 3 & 4 \\
 1 & 1 & 3 & 2 & 4
 \end{array} \\
 U = \begin{array}{cc}
 1 & 2 \\
 2 & 1
 \end{array}
 \end{array}$$

TABLE X
SEMDIRECT PRODUCTS: ORDER THREE

3.6 $\otimes$ U

1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
2	2	4	4	5	2	2	4	4	5	2	2	4	4	5
3	4	3	4	5	3	4	3	4	5	3	4	3	4	5
4	4	4	4	5	4	4	4	4	5	4	4	4	4	5
5	4	5	4	5	5	4	5	4	5	5	4	5	4	5
1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
2	2	4	4	5	2	2	4	4	5	2	2	4	4	5
3	4	3	4	5	3	4	3	4	5	3	4	3	4	5
4	4	4	4	5	4	4	4	4	5	4	4	4	4	5
5	4	5	4	5	5	4	5	4	5	5	4	5	4	5
1	2	3	4	5	1	2	3	4	5	11	12	13	14	15
2	2	4	4	5	2	2	4	4	5	2	2	4	4	5
3	4	3	4	5	3	4	3	4	5	13	14	13	14	15
4	4	4	4	5	4	4	4	4	5	4	4	4	4	5
15	14	15	14	15	15	14	15	14	15	15	14	15	14	15

3.7 $\otimes$ U

1	2	3	4	1	2	3	4	1	2	3	4
2	2	3	4	2	2	3	4	2	2	3	4
3	3	3	4	3	3	3	4	3	3	3	4
4	4	3	4	4	4	3	4	4	4	3	4
1	2	3	4	5	6	7	8	1	2	3	4
2	2	3	4	6	6	7	8	2	2	3	4
3	3	3	4	3	3	3	4	3	3	3	4
8	8	7	8	8	8	7	8	8	8	7	8
1	2	3	4	9	10	11	12	1	2	3	4
2	2	3	4	10	10	11	12	2	2	3	4
3	3	3	4	3	3	3	4	3	3	3	4
12	12	11	12	12	12	11	12	12	12	11	12

TABLE X
SEMIDIRECT PRODUCTS: ORDER THREE

3.8 $\otimes$ U

1	2	3	4	1	2	3	4	1	2	3	4
2	2	3	4	2	2	3	4	2	2	3	4
3	3	3	4	3	3	3	4	3	3	3	4
4	4	3	4	4	4	3	4	4	4	3	4
1	2	3	4	1	2	3	4	5	6	7	8
2	2	3	4	2	2	3	4	6	6	7	8
3	3	3	4	3	3	3	4	3	3	3	4
8	8	7	8	8	8	7	8	8	8	7	8
1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	1	2	3	4	10	10	11	12
3	3	3	4	3	3	3	4	3	3	3	4
12	12	11	12	12	12	11	12	12	12	11	12

3.9 $\otimes$ U

1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	2	3	5	5	6	2	2	3	5	5	6	2	2	3	5	5	6
3	3	2	5	5	6	3	3	2	5	5	6	3	3	2	5	5	6
4	5	6	4	5	6	4	5	6	4	5	6	4	5	6	4	5	6
5	5	6	5	5	6	5	5	6	5	5	6	5	5	6	5	5	6
6	6	5	5	5	6	6	6	5	5	5	6	6	6	5	5	5	6
7	8	9	10	11	12	7	8	9	10	11	12	7	8	9	10	11	12
8	8	9	11	11	12	8	8	9	11	11	12	8	8	9	11	11	12
9	9	8	11	11	12	9	9	8	11	11	12	9	9	8	11	11	12
10	11	12	10	11	12	10	11	12	10	11	12	10	11	12	10	11	12
11	11	12	11	11	12	11	11	12	11	11	12	11	11	12	11	11	12
12	12	11	11	11	12	12	12	11	11	11	12	12	12	11	11	11	12
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	2	3	5	5	6	2	2	3	5	5	6	2	2	3	5	5	6
3	3	2	5	5	6	3	3	2	5	5	6	3	3	2	5	5	6
4	5	6	4	5	6	4	5	6	4	5	6	4	5	6	4	5	6
5	5	6	5	5	6	5	5	6	5	5	6	5	5	6	5	5	6
6	6	5	5	5	6	6	6	5	5	5	6	6	6	5	5	5	6

TABLE X
SEMIDIRECT PRODUCTS: ORDER THREE

3.10 $\otimes$ U

1	2	3	4	1	2	3	4	9	10	11	12
2	2	3	4	2	2	3	4	10	10	11	12
3	3	3	4	3	3	3	4	3	3	3	4
12	12	11	12	12	12	11	12	12	12	11	12
1	2	3	4	1	2	3	4	9	10	11	12
2	2	3	4	2	2	3	4	10	10	11	12
3	3	3	4	3	3	3	4	3	3	3	4
12	12	11	12	12	12	11	12	12	12	11	12
9	10	11	12	9	10	11	12	9	10	11	12
10	10	11	12	10	10	11	12	10	10	11	12
11	11	11	12	11	11	11	12	11	11	11	12
12	12	11	12	12	12	11	12	12	12	11	12

3.11 $\otimes$ U

1	2	3	4	5	6	7	8	9	10	11	12
2	2	3	4	2	2	3	4	10	10	11	12
3	3	3	4	3	3	3	4	3	3	3	4
12	12	11	12	12	12	11	12	12	12	11	12
5	6	7	8	1	2	3	4	9	10	11	12
6	6	7	8	6	6	7	8	10	10	11	12
7	7	7	8	7	7	7	8	7	7	7	8
12	12	11	12	12	12	11	12	12	12	11	12
9	10	11	12	9	10	11	12	9	10	11	12
10	10	11	12	10	10	11	12	10	10	11	12
11	11	11	12	11	11	11	12	11	11	11	12
12	12	11	12	12	12	11	12	12	12	11	12

TABLE X
SEMIDIRECT PRODUCTS: ORDER THREE

3.12 $\otimes$ U

1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
2	2	3	5	5	6	7	2	2	3	5	5	6	7	2	2	3	5	5	6	7
3	3	2	5	5	6	7	3	3	2	5	5	6	7	3	3	2	5	5	6	7
4	5	6	4	5	6	7	4	5	6	4	5	6	7	4	5	6	4	5	6	7
5	5	6	5	5	6	7	5	5	6	5	5	6	7	5	5	6	5	5	6	7
6	6	5	5	5	6	7	6	6	5	5	5	6	7	6	6	5	5	5	6	7
7	5	6	7	5	6	7	7	5	6	7	5	6	7	7	5	6	7	5	6	7
8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14
9	9	10	12	12	13	14	9	9	10	12	12	13	14	9	9	10	12	12	13	14
10	10	9	12	12	13	14	10	10	9	12	12	13	14	10	10	9	12	12	13	14
11	12	13	11	12	13	14	11	12	13	11	12	13	14	11	12	13	11	12	13	14
12	12	13	12	12	13	14	12	12	13	12	12	13	14	12	12	13	12	12	13	14
13	13	12	12	12	13	14	13	13	12	12	12	13	14	13	13	12	12	12	13	14
14	12	13	14	12	13	14	14	12	13	14	12	13	14	14	12	13	14	12	13	14
1	2	3	4	5	6	7	1	2	3	4	5	6	7	15	16	17	18	19	20	21
2	2	3	5	5	6	7	2	2	3	5	5	6	7	2	2	3	5	5	6	7
3	3	2	5	5	6	7	3	3	2	5	5	6	7	3	3	2	5	5	6	7
4	5	6	4	5	6	7	4	5	6	4	5	6	7	18	19	20	18	19	20	21
5	5	6	5	5	6	7	5	5	6	5	5	6	7	5	5	6	5	5	6	7
6	6	5	5	5	6	7	6	6	5	5	5	6	7	6	6	5	5	5	6	7
21	19	20	21	19	20	21	21	19	20	21	19	20	21	21	19	20	21	19	20	21

3.13 $\otimes$ U

1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
2	1	3	4	5	6	7	2	1	3	4	5	6	7	2	1	3	4	5	6	7
3	4	3	4	5	6	7	3	4	3	4	5	6	7	3	4	3	4	5	6	7
4	3	3	4	5	6	7	4	3	3	4	5	6	7	4	3	3	4	5	6	7
5	6	5	6	5	6	7	5	6	5	6	5	6	7	5	6	5	6	5	6	7
6	5	5	6	5	6	7	6	5	5	6	5	6	7	6	5	5	6	5	6	7
7	7	7	7	5	6	7	7	7	7	7	5	6	7	7	7	7	7	5	6	7
8	9	10	11	12	13	14	8	9	10	11	12	13	14	8	9	10	11	12	13	14
9	8	10	11	12	13	14	9	8	10	11	12	13	14	9	8	10	11	12	13	14
10	11	10	11	12	13	14	10	11	10	11	12	13	14	10	11	10	11	12	13	14
11	10	10	11	12	13	14	11	10	10	11	12	13	14	11	10	10	11	12	13	14
12	13	12	13	12	13	14	12	13	12	13	12	13	14	12	13	12	13	12	13	14
13	12	12	13	12	13	14	13	12	12	13	12	13	14	13	12	12	13	12	13	14
14	14	14	14	12	13	14	14	14	14	14	12	13	14	14	14	14	14	12	13	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
9	8	10	11	12	13	14	2	1	3	4	5	6	7	16	15	17	18	19	20	21
3	4	3	4	5	6	7	3	4	3	4	5	6	7	17	18	17	18	19	20	21
11	10	10	11	12	13	14	11	10	10	11	12	13	14	18	17	17	18	19	20	21
5	6	5	6	5	6	7	5	6	5	6	5	6	7	5	6	5	6	5	6	7
13	12	12	13	12	13	14	13	12	12	13	12	13	14	13	12	12	13	12	13	14
21	21	21	21	19	20	21	21	21	21	21	19	20	21	21	21	21	21	19	20	21

TABLE X
SEMIDIRECT PRODUCTS: ORDER THREE

3.15 $\otimes$ U

1	2	3	4	5	6	7	1	2	3	4	5	6	7	15	16	17	18	19	20	21
2	1	3	4	5	6	7	2	1	3	4	5	6	7	16	15	17	18	19	20	21
3	4	3	4	5	6	7	3	4	3	4	5	6	7	17	18	17	18	19	20	21
4	3	3	4	5	6	7	4	3	3	4	5	6	7	18	17	17	18	19	20	21
5	6	5	6	5	6	7	5	6	5	6	5	6	7	5	6	5	6	5	6	7
6	5	5	6	5	6	7	6	5	5	6	5	6	7	6	5	5	6	5	6	7
21	21	21	21	19	20	21	21	21	21	21	19	20	21	21	21	21	21	19	20	21
8	9	10	11	12	13	14	8	9	10	11	12	13	14	15	16	17	18	19	20	21
9	8	10	11	12	13	14	9	8	10	11	12	13	14	16	15	17	18	19	20	21
10	11	10	11	12	13	14	10	11	10	11	12	13	14	17	18	17	18	19	20	21
11	10	10	11	12	13	14	11	10	10	11	12	13	14	18	17	17	18	19	20	21
12	13	12	13	12	13	14	12	13	12	13	12	13	14	12	13	12	13	12	13	14
13	12	12	13	12	13	14	13	12	12	13	12	13	14	13	12	12	13	12	13	14
21	21	21	21	19	20	21	21	21	21	21	19	20	21	21	21	21	21	19	20	21
15	16	17	18	19	20	21	15	16	17	18	19	20	21	15	16	17	18	19	20	21
16	15	17	18	19	20	21	16	15	17	18	19	20	21	16	15	17	18	19	20	21
17	18	17	18	19	20	21	17	18	17	18	19	20	21	17	18	17	18	19	20	21
18	17	17	18	19	20	21	18	17	17	18	19	20	21	18	17	17	18	19	20	21
19	20	19	20	19	20	21	19	20	19	20	19	20	21	19	20	19	20	19	20	21
20	19	19	20	19	20	21	20	19	19	20	19	20	21	20	19	19	20	19	20	21

TABLE IX

SEMIDIRECT PRODUCTS: ORDER THREE

3.16 (x) U

1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
2	2	8	4	8	9	7	8	9	10	2	2	8	4	8	9	7	8	9	10	2	2	8	4	8	9	7	8	9	10
3	3	8	5	8	9	6	8	9	10	3	3	8	5	8	9	6	8	9	10	3	3	8	5	8	9	6	8	9	10
4	2	2	4	4	7	7	8	9	10	4	2	2	4	4	7	7	8	9	10	4	2	2	4	4	7	7	8	9	10
5	3	3	5	5	6	6	8	9	10	5	3	3	5	5	6	6	8	9	10	5	3	3	5	5	6	6	8	9	10
6	9	3	10	5	6	10	8	9	10	6	9	3	10	5	6	10	8	9	10	6	9	3	10	5	6	10	8	9	10
7	9	2	10	4	7	10	8	9	10	7	9	2	10	4	7	10	8	9	10	7	9	2	10	4	7	10	8	9	10
8	8	8	8	8	9	9	8	9	10	8	8	8	8	8	9	9	8	9	10	8	8	8	8	8	9	9	8	9	10
9	9	8	10	8	9	10	8	9	10	9	9	8	10	8	9	10	8	9	10	9	9	8	10	8	9	10	8	9	10
10	10	9	10	10	10	10	8	9	10	10	10	9	10	10	10	10	8	9	10	10	10	9	10	10	10	10	8	9	10
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	11	12	13	14	15	16	17	18	19	20
2	2	8	4	8	9	7	8	9	10	12	12	18	14	18	19	17	18	19	20	12	12	18	14	18	19	17	18	19	20
3	3	8	5	8	9	6	8	9	10	3	3	8	5	8	9	6	8	9	10	13	13	18	15	18	19	16	18	19	20
4	2	2	4	4	7	7	8	9	10	14	12	12	14	14	17	17	18	19	20	14	12	12	14	14	17	17	18	19	20
5	3	3	5	5	6	6	8	9	10	5	3	3	5	5	6	6	8	9	10	15	13	13	15	15	16	16	18	19	20
16	19	13	20	15	16	20	18	19	20	16	19	13	20	15	16	20	18	19	20	16	19	13	20	15	16	20	18	19	20
17	19	12	20	14	17	20	18	19	20	17	19	12	20	14	17	20	18	19	20	17	19	12	20	14	17	20	18	19	20
8	8	8	8	8	9	9	8	9	10	8	8	8	8	8	9	9	8	9	10	8	8	8	8	8	9	9	8	9	10
19	19	18	20	18	19	20	18	19	20	19	19	18	20	18	19	20	18	19	20	19	19	18	20	18	19	20	18	19	20
20	20	19	20	20	20	20	18	19	20	20	20	19	20	20	20	20	18	19	20	20	20	19	20	20	20	20	18	19	20
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
2	2	8	4	8	9	7	8	9	10	12	12	18	14	18	19	17	18	19	20	12	12	18	14	18	19	17	18	19	20
3	3	8	5	8	9	6	8	9	10	3	3	8	5	8	9	6	8	9	10	13	13	18	15	18	19	16	18	19	20
4	2	2	4	4	7	7	8	9	10	24	22	22	24	24	27	27	28	28	30	24	22	22	24	24	27	27	28	29	30
5	3	3	5	5	6	6	8	9	10	5	3	3	5	5	6	6	8	9	10	25	23	23	25	25	26	26	28	29	30
16	19	13	20	15	16	20	18	19	20	16	19	13	20	25	16	20	18	19	20	26	29	23	30	25	26	30	28	29	30
17	19	12	20	14	17	20	18	19	20	27	29	22	30	24	27	30	28	29	30	27	29	22	30	24	27	30	28	29	30
8	8	8	8	8	9	9	8	9	10	8	8	8	8	8	9	9	8	9	10	8	8	8	8	8	9	9	8	9	10
19	19	18	20	18	19	20	18	19	20	19	19	18	20	18	19	20	18	19	20	19	19	18	20	18	19	20	18	19	20
30	30	29	30	30	30	30	28	29	30	30	30	29	30	30	30	30	28	29	30	30	30	29	30	30	30	30	28	29	30

TABLE IX

SEMIDIRECT PRODUCTS: ORDER THREE

3.17 $\otimes$ U

1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
2	1	4	3	6	5	7	8	9	2	1	4	3	6	5	7	8	9	2	1	4	3	6	5	7	8	9
3	6	3	7	7	6	7	8	9	3	6	3	7	7	6	7	8	9	3	6	3	7	7	6	7	8	9
4	5	4	7	7	5	7	8	9	4	5	4	7	7	5	7	8	9	4	5	4	7	7	5	7	8	9
5	4	7	4	5	7	7	8	9	5	4	7	4	5	7	7	8	9	5	4	7	4	5	7	7	8	9
6	3	7	3	6	7	7	8	9	6	3	7	3	6	7	7	8	9	6	3	7	3	6	7	7	8	9
7	7	7	7	7	7	7	8	9	7	7	7	7	7	7	7	8	9	7	7	7	7	7	7	7	8	9
8	8	8	7	7	9	7	8	9	8	8	8	7	7	9	7	8	9	8	8	8	7	7	9	7	8	9
9	9	7	8	9	7	7	8	9	9	9	7	8	9	7	7	8	9	9	9	7	8	9	7	7	8	9
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	1	2	3	4	5	6	7	8	9
2	1	4	3	6	5	7	8	9	2	1	4	3	6	5	7	8	9	11	10	13	12	15	14	16	17	18
3	6	3	7	7	6	7	8	9	12	15	12	16	16	15	16	17	18	3	6	3	7	7	6	7	8	9
4	5	4	7	7	5	7	8	9	4	5	4	7	7	5	7	8	9	13	14	13	16	16	14	16	17	18
5	4	7	4	5	7	7	8	9	5	4	7	4	5	7	7	8	9	5	4	7	4	5	7	7	8	9
6	3	7	3	6	7	7	8	9	6	3	7	3	6	7	7	8	9	6	3	7	3	6	7	7	8	9
7	7	7	7	7	7	7	8	9	7	7	7	7	7	7	7	8	9	7	7	7	7	7	7	7	8	9
17	17	17	16	16	18	16	17	18	17	17	17	16	16	18	16	17	18	17	17	17	16	16	18	16	17	18
9	9	7	8	9	7	7	8	9	9	9	7	8	9	7	7	8	9	9	9	7	8	9	7	7	8	9
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	19	20	21	22	23	24	25	26	27
2	1	4	3	6	5	7	8	9	20	19	22	21	24	23	25	26	27	2	1	4	3	6	5	7	8	9
3	6	3	7	7	6	7	8	9	3	6	3	7	7	6	7	8	9	3	6	3	7	7	6	7	8	9
4	5	4	7	7	5	7	8	9	4	5	4	7	7	5	7	8	9	4	5	4	7	7	5	7	8	9
5	4	7	4	5	7	7	8	9	5	4	7	4	5	7	7	8	9	23	22	25	22	23	25	25	26	27
6	3	7	3	6	7	7	8	9	24	21	25	21	24	25	25	26	27	6	3	7	3	6	7	7	8	9
7	7	7	7	7	7	7	8	9	7	7	7	7	7	7	7	8	9	7	7	7	7	7	7	7	8	9
8	8	8	7	7	9	7	8	9	8	8	8	7	7	9	7	8	9	8	8	8	7	7	9	7	8	9
27	27	25	26	27	25	25	26	27	27	27	25	26	27	25	25	26	27	27	27	25	26	27	25	25	26	27

SEMIDIRECT PRODUCTS: ORDER THREE

1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
2	1	3	5	4	6	8	7	9	2	1	3	5	4	6	8	7	9	2	1	3	5	4	6	8	7	9
3	6	3	3	9	6	9	6	9	3	6	3	3	9	6	9	6	9	3	6	3	3	9	6	9	6	9
4	8	4	4	9	8	9	8	9	4	8	4	4	9	8	9	8	9	4	8	4	4	9	8	9	8	9
5	7	5	5	9	7	9	7	9	5	7	5	5	9	7	9	7	9	5	7	5	5	9	7	9	7	9
6	3	3	9	3	6	6	9	9	6	3	3	9	3	6	6	9	9	6	3	3	9	3	6	6	9	9
7	5	5	9	5	7	7	9	9	7	5	5	9	5	7	7	9	9	7	5	5	9	5	7	7	9	9
8	4	4	9	4	8	8	9	9	8	4	4	9	4	8	8	9	9	8	4	4	9	4	8	8	9	9
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
2	1	3	5	4	6	8	7	9	2	1	3	5	4	6	8	7	9	2	1	3	5	4	6	8	7	9
3	6	3	3	9	6	9	6	9	3	6	3	3	9	6	9	6	9	3	6	3	3	9	6	9	6	9
4	8	4	4	9	8	9	8	9	4	8	4	4	9	8	9	8	9	4	8	4	4	9	8	9	8	9
5	7	5	5	9	7	9	7	9	5	7	5	5	9	7	9	7	9	5	7	5	5	9	7	9	7	9
6	3	3	9	3	6	6	9	9	6	3	3	9	3	6	6	9	9	6	3	3	9	3	6	6	9	9
7	5	5	9	5	7	7	9	9	7	5	5	9	5	7	7	9	9	7	5	5	9	5	7	7	9	9
8	4	4	9	4	8	8	9	9	8	4	4	9	4	8	8	9	9	8	4	4	9	4	8	8	9	9
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	
1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
2	1	3	5	4	6	8	7	9	2	1	3	5	4	6	8	7	9	2	1	3	5	4	6	8	7	9
3	6	3	3	9	6	9	6	9	3	6	3	3	9	6	9	6	9	3	6	3	3	9	6	9	6	9
4	8	4	4	9	8	9	8	9	4	8	4	4	9	8	9	8	9	4	8	4	4	9	8	9	8	9
5	7	5	5	9	7	9	7	9	5	7	5	5	9	7	9	7	9	5	7	5	5	9	7	9	7	9
6	3	3	6	3	6	6	9	9	6	3	3	6	3	6	6	9	9	6	3	3	6	3	6	6	9	9
7	5	5	9	5	7	7	9	9	7	5	5	9	5	7	7	9	9	7	5							

TABLE XI
SEMIDIRECT PRODUCTS: ORDER FOUR

4.2 $\otimes$ U	4.3 $\otimes$ U	4.10 $\otimes$ U
1 2 1 2 1 2 1 2	1 2 1 2 1 2 1 2	1 2 3 4 5 6 7 8
2 1 2 1 2 1 2 1	2 1 2 1 2 1 2 1	2 1 4 3 8 7 6 5
1 2 3 4 1 2 1 2	1 2 1 2 1 2 1 2	3 4 1 2 7 8 5 6
2 1 4 3 2 1 2 1	2 1 2 1 2 1 2 1	4 3 2 1 6 5 8 7
1 2 1 2 1 2 1 2	1 2 1 2 3 4 3 4	5 6 7 8 3 4 1 2
2 1 2 1 2 1 2 1	2 1 2 1 4 3 4 3	6 5 8 7 2 1 4 3
1 2 1 2 1 2 1 2	1 2 1 2 3 4 3 4	7 8 5 6 1 2 3 4
2 1 2 1 2 1 2 1	2 1 2 1 4 3 4 3	8 7 6 5 4 3 2 1
4.11 $\otimes$ U		
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24		
2 1 5 6 3 4 8 7 11 12 9 10 20 19 23 24 21 22 14 13 17 18 15 16		
3 4 1 2 6 5 15 16 13 14 18 17 9 10 7 8 12 11 21 22 19 20 24 23		
4 3 6 5 1 2 22 21 24 23 19 20 10 9 12 11 7 8 16 15 18 17 13 15		
5 6 2 1 4 3 17 18 14 13 16 15 23 24 20 19 22 21 11 12 8 7 10 9		
6 5 4 3 2 1 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7		
7 8 9 10 11 12 1 2 3 4 5 6 19 20 21 22 23 24 13 14 15 16 17 18		
8 7 11 12 9 10 2 1 5 6 3 4 14 13 17 18 15 16 20 19 23 24 21 22		
9 10 7 8 12 11 21 22 19 20 24 23 3 4 1 2 6 5 15 16 13 14 18 17		
10 9 12 11 7 8 16 15 18 17 13 15 4 3 6 5 1 2 22 21 24 23 19 20		
11 12 8 7 10 9 23 24 20 19 22 21 17 18 14 13 16 15 5 6 2 1 4 3		
12 11 10 9 8 7 18 17 16 15 14 13 24 23 22 21 20 19 6 5 4 3 2 1		
13 14 15 16 17 18 19 20 21 22 23 24 1 2 3 4 5 6 7 8 9 10 11 12		
14 13 17 18 15 16 20 19 23 24 21 22 8 7 11 12 9 10 2 1 5 6 3 4		
15 16 13 14 18 17 3 4 1 2 6 5 21 22 19 20 24 23 9 10 7 8 12 11		
16 15 18 17 13 15 10 9 12 11 7 8 22 21 24 23 19 20 4 3 6 5 1 2		
17 18 14 13 16 15 5 6 2 1 4 3 11 12 8 7 10 9 23 24 20 19 22 21		
18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 24 23 22 21 20 19		
19 20 21 22 23 24 13 14 15 16 17 18 7 8 9 10 11 12 1 2 3 4 5 6		
20 19 23 24 21 22 14 13 17 18 15 16 2 1 5 6 3 4 8 7 11 12 9 10		
21 22 19 20 24 23 9 10 7 8 12 11 15 16 13 14 18 17 3 4 1 2 6 5		
22 21 24 23 19 20 4 3 6 5 1 2 16 15 18 17 13 15 10 9 12 11 7 8		
23 24 20 19 22 21 11 12 8 7 10 9 5 6 2 1 4 3 17 18 14 13 16 15		
24 23 22 21 20 19 6 5 4 3 2 1 12 11 10 9 8 7 18 17 16 15 14 13		

TABLE XI
SEMIDIRECT PRODUCTS: ORDER FOUR

4.12 $\otimes$ U

```

1 2 3 4 5 6 7 8
2 1 6 5 4 3 8 7
3 4 5 6 1 2 7 8
4 3 2 1 6 5 8 7
5 6 1 2 3 4 7 8
6 5 4 3 2 1 8 7
7 8 7 8 7 8 7 8
8 7 8 7 8 7 8 7

```

4.13 $\otimes$ U

```

1 2 3 4 5 6 1 2
2 1 6 5 4 3 2 1
3 4 5 6 1 2 3 4
4 3 2 1 6 5 4 3
5 6 1 2 3 4 5 6
6 5 4 3 2 1 6 5
1 2 3 4 5 6 7 8
2 1 6 5 4 3 8 7

```

4.16 $\otimes$ U

```

1 2 3 4 1 2 1 2
2 1 4 3 2 1 2 1
3 4 1 2 3 4 3 4
4 3 2 1 4 3 4 3
1 2 3 4 5 6 1 2
2 1 4 3 2 1 6 5
1 2 3 4 1 2 7 8
2 1 4 3 8 7 2 1

```

4.23 $\otimes$ U

```

1 2 1 2 1 2 7 8
2 1 2 1 2 1 8 7
1 2 3 4 1 2 7 8
2 1 2 1 4 3 8 7
1 2 1 2 5 6 7 8
2 1 6 5 2 1 8 7
7 8 7 8 7 8 7 8
8 7 8 7 8 7 8 7

```

4.30 $\otimes$ U

```

1 2 3 4 1 2 1 2
2 1 4 3 2 1 2 1
3 4 1 2 3 4 3 4
4 3 2 1 4 3 4 3
1 2 3 4 1 2 1 2
2 1 4 3 2 1 2 1
1 2 3 4 1 2 1 2
2 1 4 3 2 1 2 1

```

4.32 $\otimes$ U

```

1 2 3 4 3 4 3 4
2 1 4 3 4 3 4 3
3 4 1 2 1 2 1 2
4 3 2 1 2 1 2 1
3 4 1 2 1 2 1 2
4 3 2 1 2 1 2 1
3 4 1 2 1 2 1 2
4 3 2 1 2 1 2 1

```

4.37 $\otimes$ U

```

1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1
1 2 3 4 5 6 7 8
2 1 4 3 8 7 6 5
1 2 5 6 1 2 1 2
2 1 6 5 2 1 2 1
1 2 7 8 1 2 1 2
2 1 8 7 2 1 2 1

```

4.40 $\otimes$ U

```

1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1
1 2 3 4 3 4 3 4
2 1 4 3 4 3 4 3
1 2 3 4 3 4 3 4
2 1 4 3 4 3 4 3
1 2 3 4 3 4 3 4
2 1 4 3 4 3 4 3

```

TABLE XI
SEMIDIRECT PRODUCTS: ORDER FOUR

4.48 $\otimes$ U

1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	5	6	3	4	2	1	5	6	3	4	2	1	5	6	3	4	2	1	5	6	3	4
3	4	1	2	6	5	3	4	1	2	6	5	3	4	1	2	6	5	3	4	1	2	6	5
4	3	6	5	1	2	4	3	6	5	1	2	4	3	6	5	1	2	4	3	6	5	1	2
5	6	2	1	4	3	5	6	2	1	4	3	5	6	2	1	4	3	5	6	2	1	4	3
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1
1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	1	2	3	4	5	6
2	1	5	6	3	4	8	7	11	12	9	10	2	1	5	6	3	4	2	1	5	6	3	4
3	4	1	2	6	5	3	4	1	2	6	5	9	10	7	8	12	11	3	4	1	2	6	5
4	3	6	5	1	2	4	3	6	5	1	2	10	9	12	11	7	8	4	3	6	5	1	2
5	6	2	1	4	3	5	6	2	1	4	3	5	6	2	1	4	3	11	12	8	7	10	9
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1	12	11	10	9	8	7
1	2	3	4	5	6	1	2	3	4	5	6	13	14	15	16	17	18	1	2	3	4	5	6
2	1	5	6	3	4	2	1	5	6	3	4	2	1	5	6	3	4	14	13	17	18	15	16
3	4	1	2	6	5	15	16	13	14	18	17	3	4	1	2	6	5	3	4	1	2	6	5
4	3	6	5	1	2	4	3	6	5	1	2	4	3	6	5	1	2	16	15	18	17	13	14
5	6	2	1	4	3	17	18	14	13	16	15	5	6	2	1	4	3	5	6	2	1	4	3
6	5	4	3	2	1	6	5	4	3	2	1	18	17	16	15	14	13	6	5	4	3	2	1
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	19	20	21	22	23	24
2	1	5	6	3	4	2	1	5	6	3	4	20	19	23	24	21	22	2	1	5	6	3	4
3	4	1	2	6	5	3	4	1	2	6	5	3	4	1	2	6	5	21	22	19	20	24	23
4	3	6	5	1	2	22	21	24	23	19	20	4	3	6	5	1	2	4	3	6	5	1	2
5	6	2	1	4	3	5	6	2	1	4	3	23	24	20	19	22	21	5	6	2	1	4	3
6	5	4	3	2	1	24	23	22	21	20	19	6	5	4	3	2	1	6	5	4	3	2	1

4.49 $\otimes$ U

1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1
1	2	3	4	1	2	3	4
2	1	2	1	4	3	4	3
1	2	1	2	5	6	5	6
2	1	6	5	2	1	6	5
1	2	3	4	5	6	7	8
2	1	6	5	4	3	8	7

4.50 $\otimes$ U

1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1
1	2	3	4	1	2	1	2
2	1	2	1	4	3	2	1
1	2	1	2	5	6	1	2
2	1	6	5	2	1	2	1
1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1

4.53 $\otimes$ U

1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1
1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1
1	2	1	2	1	2	3	4
2	1	2	1	4	3	2	1
1	2	1	2	3	4	1	2
2	1	2	1	2	1	4	3

TABLE XI
SEMIDIRECT PRODUCTS: ORDER FOUR

4.55 $\otimes$ U

1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	5	6	3	4	2	1	5	6	3	4	2	1	5	6	3	4
3	4	1	2	6	5	3	4	1	2	6	5	3	4	1	2	6	5
4	3	6	5	1	2	4	3	6	5	1	2	4	3	6	5	1	2
5	6	2	1	4	3	5	6	2	1	4	3	5	6	2	1	4	3
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	5	6	3	4	2	1	5	6	3	4	2	1	5	6	3	4
3	4	1	2	6	5	3	4	1	2	6	5	3	4	1	2	6	5
4	3	6	5	1	2	4	3	6	5	1	2	4	3	6	5	1	2
5	6	2	1	4	3	5	6	2	1	4	3	5	6	2	1	4	3
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	5	6	3	4	2	1	5	6	3	4	2	1	5	6	3	4
3	4	1	2	6	5	3	4	1	2	6	5	3	4	1	2	6	5
4	3	6	5	1	2	4	3	6	5	1	2	4	3	6	5	1	2
5	6	2	1	4	3	5	6	2	1	4	3	5	6	2	1	4	3
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	5	6	3	4	2	1	5	6	3	4	2	1	5	6	3	4
3	4	1	2	6	5	3	4	1	2	6	5	3	4	1	2	6	5
4	3	6	5	1	2	4	3	6	5	1	2	4	3	6	5	1	2
5	6	2	1	4	3	5	6	2	1	4	3	5	6	2	1	4	3
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	5	6	3	4	2	1	5	6	3	4	2	1	5	6	3	4
3	4	1	2	6	5	3	4	1	2	6	5	3	4	1	2	6	5
4	3	6	5	1	2	4	3	6	5	1	2	4	3	6	5	1	2
5	6	2	1	4	3	5	6	2	1	4	3	5	6	2	1	4	3
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	5	6	3	4	2	1	5	6	3	4	2	1	5	6	3	4
3	4	1	2	6	5	3	4	1	2	6	5	3	4	1	2	6	5
4	3	6	5	1	2	4	3	6	5	1	2	4	3	6	5	1	2
5	6	2	1	4	3	5	6	2	1	4	3	5	6	2	1	4	3
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1

4.60 $\otimes$ U

1	2	3	4	5	6	7	8
2	1	4	3	8	7	6	5
3	4	1	2	5	6	7	8
4	3	2	1	8	7	6	5
5	6	5	6	5	6	7	8
6	5	6	5	8	7	6	5
7	8	7	8	5	6	7	8
8	7	8	7	8	7	6	5

4.61 $\otimes$ U

1	2	1	2	5	6	7	8
2	1	2	1	8	7	6	5
1	2	1	2	5	6	7	8
2	1	2	1	8	7	6	5
5	6	5	6	5	6	7	8
6	5	6	5	8	7	6	5
7	8	7	8	5	6	7	8
8	7	8	7	8	7	6	5

4.63 $\otimes$ U

1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1
3	4	3	4	3	4	3	4
4	3	4	3	4	3	4	3
1	2	3	4	5	6	7	8
4	3	2	1	6	5	8	7
3	4	1	2	7	8	5	6
2	1	4	3	8	7	6	5

TABLE XI
SEMIDIRECT PRODUCTS: ORDER FOUR

4.64 $\otimes$ U															
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
11	12	9	10	11	12	9	10	3	4	1	2	3	4	1	2
12	11	10	9	12	11	10	9	4	3	2	1	4	3	2	1
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	14	13	16	15
15	16	13	14	15	16	13	14	7	8	5	6	7	8	5	6
16	15	14	13	16	15	14	13	8	7	6	5	8	7	6	5
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
11	12	9	10	11	12	9	10	3	4	1	2	3	4	1	2
12	11	10	9	12	11	10	9	4	3	2	1	4	3	2	1
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	14	13	16	15
15	16	13	14	15	16	13	14	7	8	5	6	7	8	5	6
16	15	14	13	16	15	14	13	8	7	6	5	8	7	6	5
1	2	3	4	1	2	3	4	9	10	11	12	9	10	11	12
2	1	4	3	2	1	4	3	10	9	12	11	10	9	12	11
11	12	9	10	11	12	9	10	3	4	1	2	3	4	1	2
12	11	10	9	12	11	10	9	4	3	2	1	4	3	2	1
5	6	7	8	5	6	7	8	13	14	15	16	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	14	13	16	15
15	16	13	14	15	16	13	14	7	8	5	6	7	8	5	6
16	15	14	13	16	15	14	13	8	7	6	5	8	7	6	5

4.65 $\otimes$ U	4.66 $\otimes$ U	4.67 $\otimes$ U
1 2 3 4 5 6 7 8	1 2 1 2 1 2 1 2	1 2 1 2 5 6 5 6
6 5 8 7 2 1 4 3	2 1 2 1 2 1 2 1	2 1 2 1 6 5 6 5
3 4 1 2 7 8 5 6	3 4 3 4 3 4 3 4	3 4 3 4 5 6 5 6
8 7 6 5 4 3 2 1	4 3 4 3 4 3 4 3	4 3 4 3 6 5 6 5
1 2 3 4 5 6 7 8	1 2 1 2 5 6 5 6	5 6 5 6 5 6 5 6
6 5 8 7 2 1 4 3	2 1 2 1 6 5 6 5	6 5 6 5 6 5 6 5
3 4 1 2 7 8 5 6	3 4 3 4 7 8 7 8	7 8 7 8 7 8 7 8
8 7 6 5 4 3 2 1	4 3 4 3 8 7 8 7	8 7 8 7 8 7 8 7

4.68 $\otimes$ U	4.69 $\otimes$ U
1 2 1 2 5 6 5 6	1 2 1 2 1 2 1 2
2 1 2 1 6 5 6 5	2 1 2 1 2 1 2 1
3 4 3 4 5 6 5 6	1 2 1 2 3 4 3 4
4 3 4 3 6 5 6 5	2 1 2 1 4 3 4 3
5 6 5 6 5 6 5 6	1 2 3 4 5 6 7 8
6 5 6 5 6 5 6 5	2 1 4 3 8 7 6 5
7 8 7 8 5 6 5 6	1 2 3 4 5 6 7 8
8 7 8 7 6 5 6 5	2 1 4 3 8 7 6 5

TABLE XI
SEMIDIRECT PRODUCTS: ORDER FOUR

4.70 $\otimes$ U

1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3
3	5	1	6	2	4	3	5	1	6	2	4	3	5	1	6	2	4	3	5	1	6	2	4
4	6	5	1	3	2	4	6	5	1	3	2	4	6	5	1	3	2	4	6	5	1	3	2
5	3	4	2	6	1	5	3	4	2	6	1	5	3	4	2	6	1	5	3	4	2	6	1
6	4	2	3	1	5	6	4	2	3	1	5	6	4	2	3	1	5	6	4	2	3	1	5
7	8	9	10	11	12	7	8	9	10	11	12	7	8	9	10	11	12	7	8	9	10	11	12
8	7	12	11	10	9	8	7	12	11	10	9	8	7	12	11	10	9	8	7	12	11	10	9
9	11	7	12	8	10	9	11	7	12	8	10	9	11	7	12	8	10	9	11	7	12	8	10
10	12	11	7	9	8	10	12	11	7	9	8	10	12	11	7	9	8	10	12	11	7	9	8
11	9	10	8	12	7	11	9	10	8	12	7	11	9	10	8	12	7	11	9	10	8	12	7
12	10	8	9	7	11	12	10	8	9	7	11	12	10	8	9	7	11	12	10	8	9	7	11
13	14	15	16	17	18	13	14	15	16	17	18	13	14	15	16	17	18	13	14	15	16	17	18
14	13	18	17	16	15	14	13	18	17	16	15	14	13	18	17	16	15	14	13	18	17	16	15
15	17	13	18	14	16	15	17	13	18	14	16	15	17	13	18	14	16	15	17	13	18	14	16
16	18	17	13	15	14	16	18	17	13	15	14	16	18	17	13	15	14	16	18	17	13	15	14
17	15	16	14	18	13	17	15	16	14	18	13	17	15	16	14	18	13	17	15	16	14	18	13
18	16	14	15	13	17	18	16	14	15	13	17	18	16	14	15	13	17	18	16	14	15	13	17
13	14	15	16	17	18	13	14	15	16	17	18	13	14	15	16	17	18	19	20	21	22	23	24
14	13	18	17	16	15	14	13	18	17	16	15	14	13	18	17	16	15	20	19	24	23	22	21
15	17	13	18	14	16	15	17	13	18	14	16	15	17	13	18	14	16	21	23	19	24	20	22
16	18	17	13	15	14	16	18	17	13	15	14	16	18	17	13	15	14	22	24	23	19	21	20
17	15	16	14	18	13	17	15	16	14	18	13	17	15	16	14	18	13	23	21	22	20	24	19
18	16	14	15	13	17	18	16	14	15	13	17	18	16	14	15	13	17	24	22	20	21	19	23

TABLE X
SEMIDIRECT PRODUCTS: ORDER FOUR

4.71 $\otimes$ U

1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3
3	5	1	6	2	4	3	5	1	6	2	4	3	5	1	6	2	4	3	5	1	6	2	4
4	6	5	1	3	2	4	6	5	1	3	2	4	6	5	1	3	2	4	6	5	1	3	2
5	3	4	2	6	1	5	3	4	2	6	1	5	3	4	2	6	1	5	3	4	2	6	1
6	4	2	3	1	5	6	4	2	3	1	5	6	4	2	3	1	5	6	4	2	3	1	5
7	8	9	10	11	12	7	8	9	10	11	12	7	8	9	10	11	12	7	8	9	10	11	12
8	7	12	11	10	9	8	7	12	11	10	9	8	7	12	11	10	9	8	7	12	11	10	9
9	11	7	12	8	10	9	11	7	12	8	10	9	11	7	12	8	10	9	11	7	12	8	10
10	12	11	7	9	8	10	12	11	7	9	8	10	12	11	7	9	8	10	12	11	7	9	8
11	9	10	8	12	7	11	9	10	8	12	7	11	9	10	8	12	7	11	9	10	8	12	7
12	10	8	9	7	11	12	10	8	9	7	11	12	10	8	9	7	11	12	10	8	9	7	11
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3
3	5	1	6	2	4	3	5	1	6	2	4	3	5	1	6	2	4	3	5	1	6	2	4
4	6	5	1	3	2	4	6	5	1	3	2	4	6	5	1	3	2	4	6	5	1	3	2
5	3	4	2	6	1	5	3	4	2	6	1	5	3	4	2	6	1	5	3	4	2	6	1
6	4	2	3	1	5	6	4	2	3	1	5	6	4	2	3	1	5	6	4	2	3	1	5
19	20	21	22	23	24	19	20	21	22	23	24	19	20	21	22	23	24	19	20	21	22	23	24
20	19	24	23	22	21	20	29	24	23	22	21	20	29	24	23	22	21	20	19	24	23	22	21
21	23	19	24	20	22	21	23	19	24	20	22	21	23	19	24	20	22	21	23	19	24	20	22
22	24	23	19	21	20	22	24	23	19	21	20	22	24	23	19	21	20	22	24	23	19	21	20
23	21	22	20	24	19	23	21	22	20	24	19	23	21	22	20	24	19	23	21	22	20	24	19
24	22	20	21	19	23	24	22	20	21	19	23	24	22	20	21	19	23	24	22	20	21	19	23

4.72 $\otimes$ U

1	2	3	4	1	2	3	4	9	10	11	12	13	14	15	16
2	1	4	3	2	1	4	3	14	13	16	15	10	9	12	11
3	4	1	2	3	4	1	2	11	12	9	10	15	16	13	14
4	3	2	1	4	3	2	1	16	15	14	13	12	11	10	9
5	6	7	8	5	6	7	8	9	10	11	12	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	10	9	12	11
7	8	5	6	7	8	5	6	11	12	9	10	15	16	13	14
8	7	6	5	8	7	6	5	16	15	14	13	12	11	10	9
9	10	11	12	9	10	11	12	9	10	11	12	13	14	15	16
10	9	12	11	10	9	12	11	14	13	16	15	10	9	12	11
11	12	9	10	11	12	9	10	11	12	9	10	15	16	13	14
12	11	10	9	12	11	10	9	16	15	14	13	12	11	10	9
9	10	11	12	9	10	11	12	9	10	11	12	13	14	15	16
10	9	12	11	10	9	12	11	14	13	16	15	10	9	12	11
11	12	9	10	11	12	9	10	11	12	9	10	15	16	13	14
12	11	10	9	12	11	10	9	16	15	14	13	12	11	10	9

TABLE X
SEMIDIRECT PRODUCTS: ORDER FOUR

4.73 $\otimes$ U

1 2 1 2 5 6 5 6
2 1 2 1 6 5 6 5
3 4 3 4 5 6 5 6
4 3 4 3 6 5 6 5
5 6 5 6 5 6 5 6
6 5 6 5 6 5 6 5
5 6 5 6 5 6 7 8
6 5 6 5 6 5 8 7

4.74 $\otimes$ U

1 2 1 2 5 6 7 8
2 1 2 1 6 5 8 7
3 4 3 4 5 6 7 8
4 3 4 3 6 5 8 7
5 6 5 6 5 6 5 6
6 5 6 5 6 5 6 5
5 6 5 6 5 6 5 6
6 5 6 5 6 5 6 5

4.75 $\otimes$ U

1 2 1 2 5 6 5 6
2 1 2 1 6 5 6 5
3 4 3 4 5 6 5 6
4 3 4 3 6 5 6 5
5 6 5 6 5 6 5 6
6 5 6 5 6 5 6 5
5 6 5 6 5 6 5 6
6 5 6 5 6 5 6 5

4.77 $\otimes$ U

1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1
3 4 3 4 3 4 3 4
4 3 4 3 4 3 4 3
1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1
1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1

4.78 $\otimes$ U

1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1
3 4 3 4 3 4 3 4
4 3 4 3 4 3 4 3
1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1
3 4 3 4 3 4 3 4
4 3 4 3 4 3 4 3

4.91 $\otimes$ U

1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1
3 4 3 4 3 4 3 4
4 3 4 3 4 3 4 3
1 2 1 2 5 6 1 2
2 1 2 1 2 1 6 5
1 2 1 2 1 2 7 8
2 1 2 1 8 7 2 1

4.92 $\otimes$ U

1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1
3 4 3 4 3 4 3 4
4 3 4 3 4 3 4 3
1 2 1 2 5 6 1 2
2 1 2 1 2 1 6 5
3 4 3 4 3 4 7 8
4 3 4 3 8 7 4 3

4.97 $\otimes$ U

1 2 1 2 1 2 1 2
2 1 2 1 2 1 2 1
3 4 3 4 3 4 3 4
4 3 4 3 4 3 4 3
1 2 3 4 5 6 5 6
4 3 2 1 6 5 6 5
1 2 3 4 5 6 7 8
4 3 2 1 6 5 8 7

TABLE X
SEMIDIRECT PRODUCTS: ORDER FOUR

4.96 $\otimes$ U

1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3
3	5	1	6	2	4	3	5	1	6	2	4	3	5	1	6	2	4	3	5	1	6	2	4
4	6	5	1	3	2	4	6	5	1	3	2	4	6	5	1	3	2	4	6	5	1	3	2
5	3	4	2	6	1	5	3	4	2	6	1	5	3	4	2	6	1	5	3	4	2	6	1
6	4	2	3	5	1	6	4	2	3	5	1	6	4	2	3	5	1	6	4	2	3	5	1
7	8	9	10	11	12	7	8	9	10	11	12	7	8	9	10	11	12	7	8	9	10	11	12
8	7	12	11	10	9	8	7	12	11	10	9	8	7	12	11	10	9	8	7	12	11	10	9
9	11	7	12	8	10	9	11	7	12	8	10	9	11	7	12	8	10	9	11	7	12	8	10
10	12	11	7	9	8	10	12	11	7	9	8	10	12	11	7	9	8	10	12	11	7	9	8
11	9	10	8	12	7	11	9	10	8	12	7	11	9	10	8	12	7	11	9	10	8	12	7
12	10	8	9	7	11	12	10	8	9	7	11	12	10	8	9	7	11	12	10	8	9	7	11
13	14	15	16	17	18	13	14	15	16	17	18	13	14	15	16	17	18	13	14	15	16	17	18
14	13	18	17	16	15	14	13	18	17	16	15	14	13	18	17	16	15	14	13	18	17	16	15
15	17	13	18	14	16	15	17	13	18	14	16	15	17	13	18	14	16	15	17	13	18	14	16
16	18	17	13	15	14	16	18	17	13	15	14	16	18	17	13	15	14	16	18	17	13	15	14
17	15	16	14	18	13	17	15	16	14	18	13	17	15	16	14	18	13	17	15	16	14	18	13
18	16	14	15	13	17	18	16	14	15	13	17	18	16	14	15	13	17	18	16	14	15	13	17
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2	1	6	5	4	3	14	13	18	17	16	15	8	7	12	11	10	9	20	19	24	23	22	21
15	17	13	18	14	16	9	11	7	12	8	10	3	5	1	6	2	4	21	23	19	24	20	22
10	12	11	7	9	8	4	6	5	1	3	2	16	18	17	13	15	14	16	18	17	13	15	14
11	9	10	8	12	7	17	15	16	14	18	13	5	3	4	2	6	1	23	21	22	20	24	19
18	16	14	15	13	17	6	4	2	3	1	5	12	10	8	9	7	11	24	22	20	21	19	23

TABLE X
SEMIDIRECT PRODUCTS: ORDER FOUR

4.98 $\otimes$ U

1	2	3	4	1	2	3	4	9	10	11	12	13	14	15	16
2	1	4	3	2	1	4	3	14	13	16	15	10	9	12	11
3	4	1	2	3	4	1	2	11	12	9	10	15	16	13	14
4	3	2	1	4	3	2	1	16	15	14	13	12	11	10	9
5	6	7	8	5	6	7	8	9	10	11	12	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	10	9	12	11
7	8	5	6	7	8	5	6	11	12	9	10	15	16	13	14
8	7	6	5	8	7	6	5	16	15	14	13	12	11	10	9
9	10	11	12	9	10	11	12	9	10	11	12	13	14	15	16
10	9	12	11	10	9	12	11	14	13	16	15	10	9	12	11
11	12	9	10	11	12	9	10	11	12	9	10	15	16	13	14
12	11	10	9	12	11	10	9	16	15	14	13	12	11	10	9
13	14	15	16	13	14	15	16	9	10	11	12	13	14	15	16
14	13	16	15	14	13	16	15	14	13	16	15	10	9	12	11
15	16	13	14	15	16	13	14	11	12	9	10	15	16	13	14
16	15	14	13	16	15	14	13	16	15	14	13	12	11	10	9

4.100 $\otimes$ U

1	2	3	4	1	2	3	4	9	10	11	12	13	14	15	16
2	1	4	3	2	1	4	3	14	13	16	15	10	9	12	11
3	4	1	2	3	4	1	2	11	12	9	10	15	16	13	14
4	3	2	1	4	3	2	1	16	15	14	13	12	11	10	9
5	6	7	8	5	6	7	8	9	10	11	12	13	14	15	16
6	5	8	7	6	5	8	7	14	13	16	15	10	9	12	11
7	8	5	6	7	8	5	6	11	12	9	10	15	16	13	14
8	7	6	5	8	7	6	5	16	15	14	13	12	11	10	9
9	10	11	12	9	10	11	12	9	10	11	12	9	10	11	12
10	9	12	11	10	9	12	11	10	9	12	11	10	9	12	11
11	12	9	10	11	12	9	10	11	12	9	10	11	12	9	10
12	11	10	9	12	11	10	9	12	11	10	9	12	11	10	9
13	14	15	16	13	14	15	16	13	14	15	16	13	14	15	16
14	13	16	15	14	13	16	15	14	13	16	15	14	13	16	15
15	16	13	14	15	16	13	14	15	16	13	14	15	16	13	14
16	15	14	13	16	15	14	13	16	15	14	13	16	15	14	13

TABLE X
SEMDIRECT PRODUCTS: ORDER FOUR

4.102 $\otimes$ U

1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	19	20	21	22	23	24
2	1	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	20	19	24	23	22	21
3	5	1	6	2	4	3	5	1	6	2	4	3	5	1	6	2	4	21	23	19	24	20	22
4	6	5	1	3	2	4	6	5	1	3	2	4	6	5	1	3	2	22	24	23	19	21	20
5	3	4	2	6	1	5	3	4	2	6	1	5	3	4	2	6	1	23	21	22	20	24	19
6	4	2	3	1	5	6	4	2	3	1	5	6	4	2	3	1	5	24	22	20	21	19	23
7	8	9	10	11	12	7	8	9	10	11	12	7	8	9	10	11	12	19	20	21	22	23	24
8	7	12	11	10	9	8	7	12	11	10	9	8	7	12	11	10	9	20	19	24	23	22	21
9	11	7	12	8	10	9	11	7	12	8	10	9	11	7	12	8	10	21	23	19	24	20	22
10	12	11	7	9	8	10	12	11	7	9	8	10	12	11	7	9	8	22	24	23	19	21	20
11	9	10	8	12	7	11	9	10	8	12	7	11	9	10	8	12	7	23	21	22	20	24	19
12	10	8	9	7	11	12	10	8	9	7	11	12	10	8	9	7	11	24	22	20	21	19	23
13	14	15	16	17	18	13	14	15	16	17	18	13	14	15	16	17	18	19	20	21	22	23	24
14	13	18	17	16	15	14	13	18	17	16	15	14	13	18	17	16	15	20	19	24	23	22	21
15	17	13	18	14	16	15	17	13	18	16	14	15	17	13	18	14	16	21	23	19	24	20	22
16	18	17	13	15	14	16	18	17	13	15	14	16	18	17	13	15	14	22	24	23	19	21	20
17	15	16	14	18	13	17	15	16	14	18	13	17	15	16	14	18	13	23	21	22	20	24	19
18	16	14	15	13	17	18	16	14	15	13	17	18	16	14	15	13	17	24	22	20	21	19	23
19	20	21	22	23	24	19	20	21	22	23	24	19	20	21	22	23	24	19	20	21	22	23	24
20	19	24	23	22	21	20	19	24	23	22	21	20	19	24	23	22	21	20	19	24	23	22	21
21	23	19	24	20	22	21	23	19	24	20	22	21	23	19	24	20	22	21	23	19	24	20	22
22	24	23	19	21	20	22	24	23	19	21	20	22	24	23	19	21	20	22	24	23	19	21	20
23	21	22	20	24	19	23	21	22	20	24	19	23	21	22	20	24	19	23	21	22	20	24	19
24	22	20	21	19	23	24	22	20	21	19	23	24	22	20	21	19	23	24	22	20	21	19	23

TABLE X
SEMIDIRECT PRODUCTS: ORDER FOUR

4.103 $\otimes$ U	4.104 $\otimes$ U	4.110 $\otimes$ U
1 2 1 2 5 6 7 8	1 2 1 2 5 6 7 8	1 2 1 2 1 2 1 2
2 1 2 1 6 5 8 7	2 1 2 1 6 5 8 7	2 1 2 1 2 1 2 1
3 4 3 4 5 6 7 8	3 4 3 4 5 6 7 8	1 2 1 2 1 2 1 2
4 3 4 3 6 5 8 7	4 3 4 3 6 5 8 7	2 1 2 1 2 1 2 1
5 6 5 6 5 6 7 8	5 6 5 6 5 6 7 8	1 2 1 2 1 2 1 2
6 5 6 5 6 5 8 7	6 5 6 5 6 5 8 7	2 1 2 1 2 1 2 1
7 8 7 8 7 8 5 6	7 8 7 8 7 8 7 8	1 2 3 4 5 6 7 8
8 7 8 7 8 7 6 5	8 7 8 7 8 7 8 7	2 1 6 5 4 3 8 7

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BIBLIOGRAPHY

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APPENDIX

APPENDIX

In this appendix, compiled jointly by Messrs. K. S. Carman, J. C. Harden, and E. E. Posey, are listed what the authors believe to be all distinct semigroups of orders two, three, and four, and their proper sub-semigroups. Two semigroups are regarded as distinct if no isomorphism (or anti-isomorphism) can be set up between them.

Commutative semigroups of these orders (under the name of "finite ova") were listed by A. R. Poole in his dissertation,¹ but were omitted from the published version.² We have constructed independently all semigroups of orders two and three (non-commutative as well as commutative), and our list of commutative semigroups of those orders agrees perfectly with Poole's.

In addition to constructing the non-commutative semigroups of order four, we have examined Poole's list of commutative semigroups of that order, and have found what appear to be a few errors. It should be pointed out that the copy of Poole's thesis available to us was a carbon copy borrowed from the library of the California Institute of Technology, with the subscripts in the semigroup multiplication tables filled in by hand, so that it is entirely possible that the errors noted do not occur in the original dissertation; indeed, some of these errors are obviously merely mistakes in copying. We did not construct the commutative

¹A. R. Poole, "Finite Ova," (Doctoral Dissertation, California Institute of Technology, 1935).

²A. R. Poole, "Finite Ova," The American Journal of Mathematics, 59:23-32, 1937.

semigroups of order four independently, and cannot claim that our examination of Poole's list was sufficiently exhaustive to preclude the possibility that it contains errors other than those noted below.

Poole's R_{421} appears as:

$$u_1 \quad u_1 \quad u_1 \quad u_1$$

$$u_1 \quad u_2 \quad u_3 \quad u_2$$

$$u_1 \quad u_3 \quad u_1 \quad u_3$$

$$u_1 \quad u_2 \quad u_1 \quad u_2$$

which is non-commutative since $u_1 u_3 \neq u_3 u_1$. An examination of associativity shows that if $u_1 u_3 = u_1$ then the system is not a semigroup regardless of what the product $u_3 u_1$ may be. Changing $u_1 u_3$ from u_1 to u_3 , we have a commutative semigroup, distinct from all others in Poole's list, which is given in our list as No. 4.38.

The multiplication table of Poole's R_{430} is identical with that of his R_{417} ; this semigroup appears in our list as No. 4.9. Changing a single subscript yields a new commutative semigroup, No. 4.5 in our list:

R_{417}, R_{430}

4.5

$$u_1 \quad u_1 \quad u_1 \quad u_1$$

$$u_1 \quad u_1 \quad u_1 \quad u_1$$

$$u_1 \quad u_2 \quad u_1 \quad u_2$$

$$u_1 \quad u_2 \quad u_1 \quad u_2$$

$$u_1 \quad u_1 \quad u_1 \quad u_1$$

$$u_1 \quad u_1 \quad u_1 \quad u_1$$

$$u_1 \quad u_2 \quad u_1 \quad u_2$$

$$u_1 \quad u_2 \quad u_1 \quad u_2$$

Poole's T_{42} and T_{43} are identical, and appear in our list as No. 4.22. Two changes produce a new commutative semigroup, our No. 4.23:

T 4 ₂ , T 4 ₃				4.23			
u ₁	u ₁	u ₁	u ₄	u ₁	u ₁	u ₁	u ₄
u ₁	u ₂	u ₁	u ₄	u ₁	u ₂	u ₁	u ₄
u ₁	u ₁	u ₁	u ₄	u ₁	u ₁	u ₃	u ₄
u ₁	u ₄	u ₄	u ₁	u ₁	u ₄	u ₄	u ₁

Multiplication table 3.4₁₅ in Poole's list fails to satisfy the associativity condition, but can be so modified as to become a commutative semigroup apparently distinct from all others in that list; we include it as No. 4.3:

R 4 ₁₅				4.3			
u ₁	u ₁	u ₁	u ₁	u ₁	u ₁	u ₁	u ₁
u ₁	u ₂	u ₁	u ₁	u ₁	u ₁	u ₁	u ₁
u ₁	u ₁	u ₁	u ₂	u ₁	u ₁	u ₂	u ₂
u ₁	u ₁	u ₂	u ₁	u ₁	u ₁	u ₂	u ₂

Poole's semigroups R 4₂ and R 4₃ seem to be isomorphic; this semigroup appears in our list as No. 4.43. Poole's tables and an isomorphism between them are exhibited below:

R 4 ₂				R 4 ₃			
u ₁	u ₁	u ₁	u ₁	u ₁ ↔ u ₁	u ₁	u ₁	u ₁
u ₁	u ₂	u ₁	u ₁	u ₂ ↔ u ₄	u ₁	u ₂	u ₁
u ₁	u ₁	u ₃	u ₃	u ₃ ↔ u ₂	u ₁	u ₁	u ₃
u ₁	u ₁	u ₃	u ₄	u ₄ ↔ u ₃	u ₁	u ₁	u ₄

In Table Ia are listed the four semigroups of order two, of which only No. 2.3 is non-commutative. Table IIa contains the six non-commutative semigroups of order three (Nos. 3.7, 3.9, 3.12, 3.13, 3.14, 3.15).

and the twelve commutative semigroups of order three, the latter being identical (to within isomorphism) with Poole's ova of order three. In Table IIIa we list the 55 commutative semigroups of order four (Nos. 4.1-4.55) and the 66 non-commutative semigroups of order four (Nos. 4.56 - 4.121).

In Table IVa, the columns are headed by the reference numbers of the semigroups of order two, and the rows are labeled with the reference numbers of the semigroups of order three. If No. 3.z ($z = 1, \dots, 18$) contains k subsemigroups isomorphic to No. 2.y ($y = 1, \dots, 4$), the letter k is placed at the intersection of the row 3.z and the column 2.y. Table Va exhibits similarly the subsemigroups of orders two and three of the semigroups of order four. Subsemigroups of order one are simply the idempotent elements of a semigroup, and are not listed separately since they are visible immediately upon inspection of the multiplication table of the semigroup.

It would be futile to expect tables of this sort to be entirely free from errors. The authors have checked their work repeatedly, and have rectified several mistakes, but others have doubtless escaped their notice. They will appreciate receiving corrections from users of the tables.

TABLE Ia

SEMIGROUPS OF ORDER TWO

2.1	2.2	2.3	2.4
1 2 2 1	1 1 1 2	1 1 2 2	1 1 1 1

TABLE IIa

SEMIGROUPS OF ORDER THREE

3.1	3.2	3.3	3.4	3.5	3.6
1 2 2 2 1 1 2 1 1	1 1 1 1 1 1 1 1 2	1 2 3 2 3 1 3 1 2	1 1 3 1 1 3 3 3 1	1 1 3 1 2 3 3 3 1	1 1 1 1 1 1 1 1 3
3.7	3.8	3.9	3.10	3.11	3.12
1 1 1 1 2 1 1 3 1	1 1 1 1 1 2 1 2 3	1 1 1 2 2 2 1 1 1	1 1 3 1 1 3 3 3 3	1 2 3 2 1 3 3 3 3	1 1 1 2 2 2 1 1 3
3.13	3.14	3.15	3.16	3.17	3.18
1 1 1 2 2 2 1 2 3	1 1 1 2 2 2 3 3 3	1 1 3 2 2 3 3 3 3	1 1 1 1 2 2 1 2 3	1 1 1 1 2 1 1 1 3	1 1 1 1 1 1 1 1 1

TABLE IIIa

SEMIGROUPS OF ORDER FOUR

4.1	4.2	4.3	4.4	4.5	4.6	4.7
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
1 2 2 2	1 2 1 1	1 1 1 1	1 2 1 4	1 2 1 2	1 2 1 1	1 1 1 1
1 2 3 3	1 1 1 1	1 1 2 2	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 2
1 2 3 3	1 1 1 1	1 1 2 2	1 4 1 1	1 2 1 4	1 1 1 3	1 1 2 2
4.8	4.9	4.10	4.11	4.12	4.13	4.14
1 1 1 1	1 1 1 1	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 1	1 2 1 1
1 1 1 3	1 2 1 2	2 1 4 3	2 1 4 3	2 3 1 4	2 3 1 2	2 1 2 2
1 1 1 1	1 1 1 1	3 4 2 1	3 4 1 2	3 1 2 4	3 1 2 3	1 2 3 4
1 3 1 2	1 2 1 2	4 3 1 2	4 3 2 1	4 4 4 4	1 2 3 4	1 2 4 3
4.15	4.16	4.17	4.18	4.19	4.20	4.21
1 2 1 2	1 2 1 1	1 2 1 1	1 2 3 1	1 2 3 3	1 2 3 4	1 1 1 4
2 1 2 1	2 1 2 2	2 1 2 2	2 1 3 2	2 1 3 3	2 1 3 4	1 2 3 4
1 2 3 4	1 2 3 1	1 2 3 4	3 3 3 3	3 3 3 3	3 3 3 4	1 3 1 4
2 1 4 3	1 2 1 4	1 2 4 4	1 2 3 4	3 3 3 4	4 4 4 4	4 4 4 1
4.22	4.23	4.24	4.25	4.26	4.27	4.28
1 1 1 4	1 1 1 4	1 1 3 3	1 2 1 4	1 2 4 4	1 1 1 1	1 1 1 1
1 2 1 4	1 2 1 4	1 2 3 4	2 2 2 2	2 2 2 2	1 2 1 4	1 2 3 4
1 1 1 4	1 1 3 4	3 3 1 1	1 2 1 4	4 2 1 1	1 1 1 1	1 3 1 3
4 4 4 1	4 4 4 4	3 4 1 1	4 2 4 1	4 2 1 1	1 4 1 2	1 4 3 2
4.29	4.30	4.31	4.32	4.33	4.34	4.35
1 1 3 1	1 2 1 1	1 2 1 2	1 2 2 2	1 1 3 1	1 1 3 3	1 2 3 3
1 2 3 2	2 1 2 2	2 1 2 1	2 1 1 1	1 1 3 1	1 1 3 3	2 3 1 1
3 3 1 3	1 2 1 1	1 2 1 2	2 1 1 1	3 3 1 3	3 3 1 1	3 1 2 2
1 2 3 2	1 2 1 1	2 1 2 1	2 1 1 1	1 1 3 2	3 3 1 2	3 1 2 2
4.36	4.37	4.38	4.39	4.40	4.41	4.42
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
1 2 3 3	1 2 3 4	1 2 3 2	1 2 3 4	1 2 2 2	1 2 2 2	1 2 2 2
1 3 1 1	1 3 1 1	1 3 1 3	1 3 1 1	1 2 2 2	1 2 2 2	1 2 3 3
1 3 1 1	1 4 1 1	1 2 3 2	1 4 1 3	1 2 2 2	1 2 2 3	1 2 3 4

TABLE IIIa

SEMGROUPS OF ORDER FOUR (Continued)

4.43	4.44	4.45	4.46	4.47	4.48	4.49
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
1 2 2 1	1 2 2 1	1 2 2 4	1 2 2 2	1 2 2 2	1 2 1 1	1 2 1 2
1 2 3 1	1 2 3 4	1 2 3 4	1 2 3 2	1 2 3 4	1 1 3 1	1 1 3 3
1 1 1 4	1 1 4 1	1 4 4 1	1 2 2 2	1 2 4 2	1 1 1 4	1 2 3 4
4.50	4.51	4.52	4.53	4.54	4.55	4.56
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 4
1 2 1 1	1 2 1 1	1 2 1 2	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 4
1 1 3 1	1 1 3 4	1 1 3 1	1 1 1 2	1 1 1 1	1 1 1 1	1 1 2 4
1 1 1 1	1 1 4 1	1 2 1 2	1 1 2 1	1 1 1 2	1 1 1 1	1 1 1 4
4.57	4.58	4.59	4.60	4.61	4.62	4.63
1 1 1 4	1 1 3 4	1 1 1 1	1 2 3 4	1 1 3 4	1 1 1 1	1 1 1 1
1 2 3 4	1 1 3 4	2 2 2 2	2 1 3 4	1 1 3 4	2 2 2 2	2 2 2 2
1 3 2 4	3 3 3 4	1 1 1 2	3 3 3 4	3 3 3 4	3 3 3 3	1 2 3 4
1 1 1 4	3 3 3 4	4 4 4 4	4 4 3 4	4 4 3 4	1 2 2 4	2 1 4 3
4.64	4.65	4.66	4.67	4.68	4.69	4.70
1 1 3 3	1 2 3 4	1 1 1 1	1 1 3 3	1 1 3 3	1 1 1 1	1 1 1 1
2 2 4 4	2 1 4 3	2 2 2 2	2 2 3 3	2 2 3 3	1 1 2 2	2 2 2 2
1 1 3 3	1 2 3 4	1 1 3 3	3 3 3 3	3 3 3 3	1 2 3 4	3 3 3 3
2 2 4 4	2 1 4 3	2 2 4 4	4 4 4 4	4 4 3 3	1 2 3 4	3 3 3 4
4.71	4.72	4.73	4.74	4.75	4.76	4.77
1 1 1 1	1 1 3 4	1 1 3 3	1 1 3 4	1 1 3 3	1 1 1 1	1 1 1 1
2 2 2 2	2 2 3 4	2 2 3 3	2 2 3 4	2 2 3 3	1 1 1 3	2 2 2 2
1 1 1 1	3 3 3 4	3 3 3 3	3 3 3 3	3 3 3 3	1 1 1 3	1 1 1 1
4 4 4 4	3 3 3 4	3 3 3 4	3 3 3 3	3 3 3 3	1 2 3 4	1 1 1 1
4.78	4.79	4.80	4.81	4.82	4.83	4.84
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 4
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	1 2 1 4
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 3	1 2 3 1	1 1 1 3	1 3 1 4
2 2 2 2	1 1 1 4	2 2 2 4	2 2 2 4	1 1 1 1	1 2 1 4	1 1 1 4

TABLE IIIa

SEMIGROUPS OF ORDER FOUR (Continued)

4.85	4.86	4.87	4.88	4.89	4.90	4.91
1 1 1 1	1 1 1 4	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 4	1 1 1 1
2 2 2 2	2 2 2 4	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 4	2 2 2 2
1 1 1 3	1 1 1 4	1 1 1 1	1 1 1 3	1 1 1 3	1 1 3 4	1 1 3 1
1 1 3 4	4 4 4 4	1 2 3 4	1 2 3 4	1 1 1 4	4 4 4 4	1 1 1 4
4.92	4.93	4.94	4.95	4.96	4.97	4.98
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 3 4
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 3 4
1 1 3 1	1 2 3 1	1 1 3 3	1 1 3 3	3 3 3 3	1 2 3 3	3 3 3 4
2 2 2 4	1 1 1 4	1 1 3 4	1 2 3 4	1 2 3 4	1 2 3 4	4 4 3 4
4.99	4.100	4.101	4.102	4.103	4.104	4.105
1 1 1 4	1 1 3 4	1 1 1 1	1 1 1 4	1 1 3 4	1 1 3 4	1 1 3 4
2 2 2 4	2 2 3 4	2 2 2 2	2 2 2 4	2 2 3 4	2 2 3 4	1 1 3 4
1 2 3 4	3 3 3 3	3 3 3 3	3 3 3 4	3 3 3 4	3 3 3 4	3 3 3 3
4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 3	4 4 4 4	3 3 3 3
4.106	4.107	4.108	4.109	4.110	4.111	4.112
1 2 3 4	1 1 1 4	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
2 1 3 4	1 2 1 4	1 2 1 1	1 2 1 1	1 1 1 1	1 2 1 4	1 1 1 2
3 3 3 3	1 3 1 4	1 3 1 1	1 3 1 1	1 1 1 1	1 3 1 1	1 1 1 1
3 3 3 3	4 4 4 1	1 3 1 1	1 1 1 1	1 2 3 4	1 1 1 1	1 2 3 4
4.113	4.114	4.115	4.116	4.117	4.118	4.119
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 4	1 1 1 1	1 1 1 1	1 1 1 1
1 2 1 1	1 2 1 1	1 2 1 2	1 2 1 4	1 2 1 1	1 2 1 2	1 1 1 2
1 3 1 1	1 3 1 1	1 3 1 3	1 3 1 4	1 3 1 1	1 3 1 3	1 1 3 3
1 3 3 4	1 1 3 4	1 2 1 4	4 4 4 4	1 1 1 4	1 2 3 4	1 1 3 4
4.120	4.121					
1 2 2 1	1 1 1 1					
2 1 1 2	1 1 1 1					
2 1 1 2	1 1 1 2					
1 2 3 4	1 1 1 1					

TABLE IVa

PROPER SUBSEMIGROUPS OF SEMIGROUPS OF ORDER THREE

	2.1	2.2	2.3	2.4
3.1	1			
3.2				1
3.3				
3.4	1			1
3.5	1	1		
3.6		1		1
3.7		1		1
3.8		1		1
3.9			1	1
3.10		1		1
3.11	1	1		
3.12		1	1	
3.13		2	1	
3.14			3	
3.15		2	1	
3.16		3		
3.17		2		
3.18				2

TABLE Va

PROPER SUBSEMIGROUPS OF SEMIGROUPS OF ORDER FOUR

4.	2.				3.																	
	1	2	3	4	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1		3		1										2						1		
2		1		2						2												1
3				1		2																
4		1		2						1		1										1
5		3		1						2										1		
6		1		1		1				1												
7				2		1																1
8				2																		1
9		1		2						1				1								
10	1																					
11	2																					
12		1					1															
13		1					1															
14	2								1						1							
15	2								1													
16	1	2							2												1	
17	1	3							2											1		
18	1	3							1						1					1		
19	1	3													1						1	
20	1	3													2					1		
21	1	1		1				1	1			1										
22	1	1		1				1	1	1												
23		5																		2	1	
24	1	1			1				1													
25	1	1		1				1						1	1							
26	1	1			1										1							
27		1		1						1					1							
28		1		1								1			1							
29	1	1							1					1								
30	1			2				2														1
31	1			1	1			1														
32	1				2																	
33	1			1		1		1														
34	1			1				1														
35							1															
36		1		2								1										1
37		1		2								1										1
38		1		2								1		1								
39		1		1		1						1										
40		1		2										2								1
41		1		1		1								1								

TABLE Va

PROPER SUBSEMIGROUPS OF SEMIGROUPS OF ORDER FOUR
(Continued)

4.	2.				3.																	
	1	2	3	4	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
42		6																		4		
43		4																		1	2	
44		3								1		1								1		
45		3										2								1		
46		3								1				1						1		
47		3		1								1		1						1		
48		3																			3	
49		4																		2	1	
50		2		1						2											1	
51		2		1						1		1									1	
52		2												1							1	
53				3																		2
54				2		1																1
55				3																		3
56			1	1		1							1									
57	1	1	1												1	1						
58		1	1	1										1		1						
59			3	1									1					1				
60	1	1	1												2		1					
61		2	1	1										2			1					
62		2	3													1	1	1				
63	1	2	1														1					
64			3																			
65	2		1																			
66		1	2													2						
67		2	2													2				1		
68		2	1	1							2									1		
69		2	1	1								2								1		
70		1	3													2		1				
71			3	1									2					1				
72		1	2													2				1		
73		3	1																	1		2
74		2	1	1							2									1		
75		2	1	1						2										1		
76		1		2								1										1
77			1	2									2									1
78			1	2									2									
79		1	1	1						1			1			1						
80		1	1	1									1			1						
81		1	1	1									1			1						
82		2	1	1		1							1				1					

TABLE Va

PROPER SUBSEMGROUPS OF SEMIGROUPS OF ORDER FOUR
(Continued)

4.	2.				3.																	
	1	2	3	4	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
83		2	1	1							1		1				1					
84		1	1								1		1			1						
85		1	1	1								1	1			1						
86		2	1	1									1	1					1			
87		2	1	1							1		1				1					
88		2	1	1								1	1				1					
89		1	1	1							1		1			1						
90		4	1													1			1	1		
91		2	1													2					1	
92		2	1													2						
93		3	1													1	1			1		
94		3	1													2				1		
95		4	1													1	1			1		
96		3	3														3	1				
97		5	1														2			2		
98		4	2														2		2			
99		5	1														1		1	2		
100		4	2															2	2			
101			6															4				
102		3	3															1	3			
103	1	2	1						2											1		
104		5	1																2	2		
105		1		2							1				1							
106	1	1		1							1					1						
107	1	1		1				1	1		1											
108		1		2							1											1
109		2		1						1	1										1	
110		1		2							2											1
111		1		2							2											1
112		1		2							1	1										1
113		2		1							2											
114		2		1							2											1
115		3		1							1									2		
116		3		1							1				1					1		
117		2		1						1	1										1	
118		3		1							1	1								1		
119		3		1							1	1								1		
120	1	1			1				1													
121			3																			2