

Graphic Representation of Exotic Nuclear Shapes in the Pasta Phase of Matter in Neutron Stars

An Honors Thesis

Presented for the

Chancellor's Honors Program

The University of Tennessee Knoxville

Mark Alexander Randolph Kaltenborn

May 2014

Acknowledgement:

I would like to thank everyone in the Astrophysics group at UT/ORNL for helping me along with my undergraduate research. In particular, I would like to thank my supervisor Jirina Stone for all her help and encouragement, Reuben Budiardja for his help with the programming and optimization with the programs, Helena Pais for originally bringing me into the project for her PhD studies [26], and Will Newton for his help with the project. I would also like to thank all of those in the Physics department here at UT for a wonderful four years of study.

This research project used resources of the Oak Ridge Leadership Computing Facility, located in the National Center for Computational Sciences at Oak Ridge National Laboratory, which is supported by the Office of Science of the Department of Energy under Contract No. DE-AC05-00OR22725. It was supported by an INCITE grant AST005: Multidimensional Simulations of Core-Collapse Supernovae, PI: Anthony Mezzacappa, Oak Ridge National Laboratory.

Abstract:

As one of the most catastrophic events in the entire Universe, Core-Collapse Supernovae (CCSN) present major challenges to theoretical astrophysics. The pressures and temperatures involved in stars are also some of the most extreme pressures and temperatures known to man. Since it is impossible to recreate these conditions in laboratories, programming of astrophysical models is necessary in order to understand these events. The Equation of State is the most significant input to understanding these processes, along with pressure, energy density, and temperature. The regional focus of

the CCSN matter is the transitional region between homogeneous and inhomogeneous phases. The nuclear structures experience variations from spherical configurations to more exotic, ‘nuclear pasta,’ forms, consisting of rods, slabs, cylindrical voids, and spherical voids. Utilizing a three-dimensional, finite temperature Hartree-Fock +BCS (3DHF) with the Skyrme interaction model to study the inhomogeneous nuclear matter, we have been calculating the nuclear pasta phase and determining the phase transition between pasta and uniform matter. Since nuclear matter properties depend on effective nucleon-nucleon relations in the model, we used four different parameterizations of Skyrme interactions, NRAPR, QMC700, Sly4, and SkM*. For each of these interactions, we calculated free-energy density and pressure, as well as other important properties in the pasta region of the neutron star. The data analyzed was for densities ranging from 0.01 to 0.12 fm⁻³ and at temperature $T = 0$ MeV, representing temperatures in neutron stars and pre-supernovae iron core matter, and proton-neutron ratios from 0.01 to 0.15. The data has determined that transitions occur naturally between the phases of pasta configurations without any need for thermodynamic manipulations. However, the exact transition points between pasta phases are hard to pinpoint with certainty at this stage and will be a subject of future research. For future work, we will continue to study the properties of the pasta as a function of the proton/neutron fraction and the chosen model of the nuclear interaction corresponding to neutron star matter.

Content:

01)	Introduction.....	5
02)	Core-Collapse Supernovae and Neutron Stars.....	6
03)	History of Neutron Stars.....	7
04)	Pulsars and Neutron Stars.....	9
05)	Life of Large Mass Stars.....	10
06)	Death of Large Stars.....	12
07)	Neutron Stars.....	15
08)	Pasta Phase.....	24
09)	Equation of State.....	26
10)	Hartree-Fock Approximation.....	26
11)	Results.....	31
12)	Conclusions.....	91
13)	Future Work.....	92
14)	Bibliography.....	93

Introduction:

There are complex forms of nuclear matter that affect many astrophysical and nuclear physics phenomena. In these forms, the density approaches the central density of heavy nuclei (0.16 fm^{-3}), and temperatures are less than 20 MeV. This nuclear matter critically affects the physics in neutron stars and core-collapse supernovae. In laboratories, we can achieve high densities to study the physical properties of nuclear matter through use of particle accelerators and neutron-rich beams, but true study of these dense complex structures is based solely on theoretical models.

Here we concentrate on neutron stars (NS) and core-collapse supernovae (CCSN), some of the most intriguing occurrences in the universe. The physics involved in these explosions and the resulting neutron stars varies greatly, from properties of atomic and subatomic particles on the small scale to gravity on the large scale. The key microscopic input into CCSN model simulations is the equation of state, or EoS, connecting the pressure of stellar matter to its energy density and temperature, which are, in turn, determined by its composition and modeling of interactions between its components. The composition of CCSN matter changes with increasing density and temperature. At low densities, an inhomogeneous phase exists, made up of discrete heavy nuclei immersed in a sea of single nucleons (predominantly neutrons), light nuclei (deuterium, tritium, helions, α -particles), electrons, and potentially a degenerate gas of trapped neutrinos. At higher density and temperature, a homogeneous phase evolves, consisting of nucleons, leptons, heavy baryons, mesons, and possibly quarks.

Core-Collapse Supernovae and Neutron Stars:

A neutron star is a type of stellar remnant that can result from the gravitational collapse of a massive star during supernovae events. It is believed that such stars are comprised almost entirely of neutrons, which are subatomic particles without electrical charge and with a slightly larger mass than protons. Neutron stars are hot in the earliest stages of formation, but quickly cool off in terms of stellar temperatures. Neutron stars are supported against further collapse by quantum degeneracy pressure of the neutrons that comprise the star. The 'typical' neutron star has a mass of $1.35 \sim 2.0$ solar masses with a corresponding radius of about 12 kilometers. The term 'typical' is used loosely because the study of what is typical is still in its infancy, thus the sizes and radii collected from observations of pulsars could be fairly unusual. For comparison, one could look to the nearest star, our sun. The sun's radius is approximately 60,000 times that of a neutron star. The neutron star has an increasing density similar to that of Earth as one approaches the center, but magnitudes of the density and pressure are massive in comparison. Predicted densities of $3.7 \times 10^{17} \sim 5.9 \times 10^{17} \text{ kg/m}^3$ are expected to be seen in the neutron star. The density of the neutron star makes its environment an interesting place for a high-density physicist to study.

The neutron star packs the amount of matter found in our sun into an area the size of a city. In order to envision the density of the material in the star, imagine that you could pack all of humanity into a volume the size of a sugar cube, there you would find a density similar to what is found in the star. Neutron stars also have the strongest magnetic fields in the known universe [6]. The magnetic field is trillions of times stronger than that of the Earth's magnetic field. The extremes found in neutron stars

afford physicists, and other scientists, unique glimpses into an area of physics that would be only dreamed about otherwise. So, given the extreme stellar smallness, it is easy to see that neutron stars are a very complex and only relatively recently discovered astronomical phenomena.

History of Neutron Stars:

Soon after the discovery of the neutron by Sir James Chadwick in 1932, a physicist, by the name of Landau, first theorized neutron stars [10]. Landau suggested that neutron stars could be supported by neutron degeneracy pressure, much like electron degeneracy pressure is found in white dwarfs. The first well-known paper to reference neutron stars was published by Walter Baade and Fritz Zwicky in 1934, which first suggested that neutron stars could be the remnants of supernovae [1]. In 1939, another physicist by the name of Tolman produced work theorizing the structure of neutron stars [24]. He did this by employing the relativistic equations of stellar structures following Einstein's equations of general relativity. His work determined that there would be a limiting mass for stars such as the neutron star, which was interestingly close to the Chandrasekhar mass limit, the mass limit for a white dwarf that exists in Newtonian Gravity.

In 1939, physicists Oppenheimer and Volkoff were the first to tackle the physical structure of neutron stars [15]. From numerous papers, their radius and maximum mass were estimated to be approximately ten kilometers and $\frac{3}{4}$ of a solar mass. At this time, it was also hypothesized that the magnetic fields of 10^{12} gauss would be produced in the formation of these neutron stars. They then proposed that neutron stars were

likely rotating very rapidly in order to have these magnetic fields. These high fields led physicist Franco Pacini to predict that a rotating magnetized neutron star would emit radio waves [17]. This idea supported a phenomenon in the Crab nebula, where the slowing of the expansion of the nebula was not acting as expected. Momentum, energy emitted from a rotating neutron star, was being shared with the nebula, resulting in a misunderstood occurrence.

At the time, little to no work had been done involving the search for neutron stars, even after the publication of the papers previously mentioned. It was not until the serendipitous discovery of the first neutron star, thirty-five years after the publication of Baade and Zwicky's paper that physicists seriously searched for the stars. Before this discovery, physicists and astronomers were unsure of what exactly they should be looking for, given that all they had to go on was that the knowledge that these stars were comprised of closely packed degenerate neutrons. Therefore, these stars could not produce energy on their own, but only could radiate the energy they had as a result of their creation over millions of years through the slow process of photon diffusion [6]. This process is similar to that of white dwarfs, except that neutron stars would be much smaller, so it would be nearly impossible to view the stars optically through a telescope.

A Cambridge professor, Anthony Hewish, designed a radio telescope and other equipment with a short time response and an extended observing routine for the sole purpose of studying scintillation of point radio sources such as quasars. Unknown to Hewish, the specific attributes of his telescope and other equipment were exactly what was required to make an important discovery in the field of neutron star physics.

Jocelyn Bell, a student of Anthony Hewish, noticed the first indications of a persistent periodic source measured to have a pulse of 1.337 seconds. In 1967, Hewish and Bell published a paper stating that they had detected a very small source of pulsed radio signal lying outside the solar system, presumably a compact star, either a white dwarf or neutron star [9]. This was the discovery of the first pulsar. Today, over a thousand pulsars have been discovered.

Pulsars and Neutron Stars:

Pulsars, called as such because of the sources of periodic signals of extreme timing stability, are the astrophysical objects that are presumed to correspond to neutron stars. Pulsars are observed under a variety of circumstances. They are typically found independently, but can often be found in binary orbit with another star, in X-ray binary systems, in γ -ray bursters, and in soft γ -ray repeaters. The pulsation of the signal is the manifestation of a cone of radiation of small angular width emitted along the magnetic axis, which is fixed in a rotating neutron star and beamed in our direction at each revolution.

Although pulsars are assumed to be neutron stars, the term ‘pulsar’ is used to define astrophysical objects that have the property of pulsed radiation emission; therefore, pulsars and neutron stars are not equivalent. ‘Neutron star’ is used to define the theoretical object, independent of its observation as a pulsar, or for a very compact star that is not observed by its pulsed radiation, but instead by other means. In other words, almost all pulsars are believed to be neutron stars, but not all neutron stars are pulsars.

Life of Large Mass Stars:

In order to understand more about neutron stars, it is necessary to have a good understanding of the formation of stars. The evolution of stars from birth to death is not only fundamental to all life in the universe, but it is also essential for the formation of neutron stars. Neutron stars are believed to develop out of the death of a super red giant star.

Stars are formed from clouds of interstellar gas, consisting of mostly molecular hydrogen and interstellar dust. One such example of these cloud clusters is the Horsehead Nebula in Orion. Most of the gas is cold at 10K, although some regions are as hot as 2000K. The clouds of gas and other dust compress on and condense on themselves to form “stars that are more massive than a few M_s [solar masses],” and most “are observed to form in small groups in the densest regions of the clouds” [6]. The motion of a given star often suggests the gravitational influence of several nearby stars. Approximately half of all stars are in binaries, two stars that orbit one another.

The understanding of the formation of stars is rudimentary at best. Although, it is known that the important factors in the formation of stars are gravity, dust, gas pressure, rotation, magnetic fields, winds, radiation from nearby young stars, and radiative shock. Over time, the thermal pressure in the clouds decreases, leading to the inevitable collapse of the denser parts of the cloud, which forms stars.

At some time during this process, a critical mass is reached and the cloud collapses towards its center under the influence of the gravitational force. This gravitational energy is transformed into heat as the star collapses. Energy loss by radiation at the protostar’s surface causes further slow contraction and heating until the

core temperature rises to the ignition point for fusing hydrogen into helium ($T \approx 10^7 \text{K}$). At this point, fusion becomes the main energy source for the star and the thermal and radiation pressure balance for millions to billions of years. At this point the protostar becomes an actual star. The star will remain in a constant state as long as the thermonuclear fusion process continues, and fusion will only end “when iron, the most bound nuclear species, is reached. Beyond iron, fusion is no longer exothermic. Nuclei in the region of iron are referred to as the iron peak nuclei because of their higher binding than other nuclei” (see figure 1) [6].

Fusion begins by burning hydrogen. Once the hydrogen is spent, the star will start burning helium, which was formed by hydrogen fusion. As the helium burns, a carbon core is formed. This carbon core will only provide energy for a few thousand years, which is an incredibly short time for the life of the star. Gamma rays in the core cause neutrino pairs, and the loss of these neutrino pairs cause the stages to progress more rapidly. Oxygen is burned in a year and silicon in a week, and so on until, in the case of large stars, the fusion reaction reaches iron. At the point of exhaustion of each elemental fuel, the core contracts further and further until the appropriate temperature for ignition is obtained for the next step in the chain reaction.

As the duration of the nuclear fusion stage of the star comes to a close, the next stage of the star commences. The next stage in evolution is determined by the star's solar mass, indicated by M_s . Stars, those with at least eight solar masses, end their lives either through that of a neutron star or that of a black hole. Lighter stars, those with masses smaller than eight solar masses, end their lives as white dwarfs. Regardless of

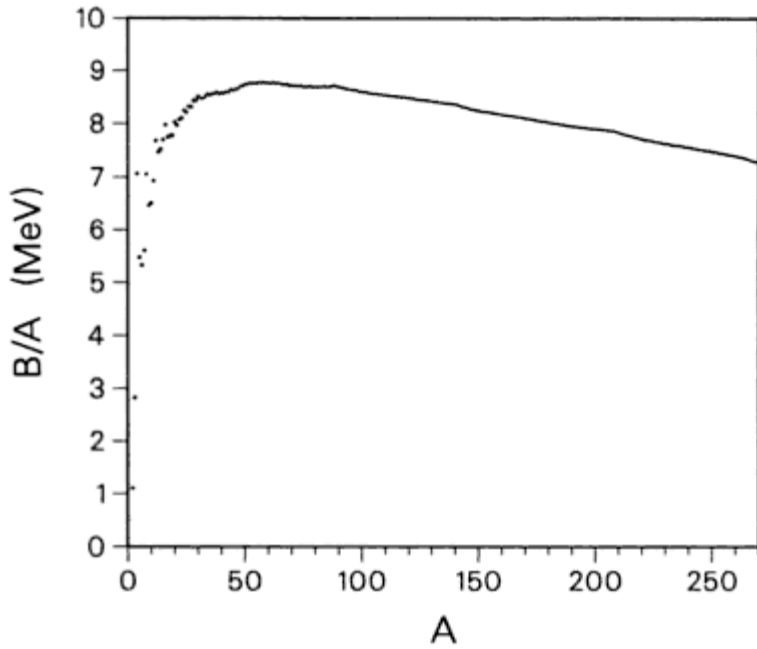


Figure 1: Binding energy as a function of baryon number showing the peak in binding at iron. [6]

the type of star and how it ends its life, the duration of its life is determined by the gravity of its mass.

Death of Large Stars:

Because the evolution of white dwarfs is vastly different from that of black holes and neutron stars, the final stages of heavier stars will be the main focus of this paper. Stars larger than eight solar masses evolve rapidly when compared to stars of less mass. The process of fusion, which is the main power source for stars during their lifetime, continues until it reaches the endpoint of what is exothermically possible. At this point, the large star becomes a super red giant. The core of the super red giant is composed almost exclusively of iron and has a radius of only several thousand kilometers. This is exceptionally small compared to the radius of the whole star, which is greater than 10^8

km. The core is only supported against collapse by the pressure of degenerate nonrelativistic electrons [6]. As the outer cores of the star continue to burn, iron is added to the core's mass. With the immense pressure from gravity, the core is crushed to such a density that the electrons become relativistic. The relativistic electrons have a much lesser capacity to equalize the pressure in the core of the star. Once this has taken place, the core has reached its maximum possible mass, referred to as the Chandrasekhar mass.

At this point in its life, the star begins to go through an extremely energetic change. Within a second, the core of the star implodes, attaining a temperature of approximately 10^{11}K . The density of the imploding core is understood to be constrained by thermalized electrons and neutrinos. As their Fermi energies—a concept in quantum mechanics usually referring to the energy difference between the highest and lowest occupied single-particle states, in a quantum system of non-interacting fermions at absolute zero temperature—increase, the short-range repulsion between the nucleons resists further compression.

During the implosion, the “collapsing material that falls in towards the core [is] rebounded by the stiffened core, sending out a shock wave originating somewhere in the core interior” [6]. This shockwave stops a few hundred kilometers from the stellar center. The material outside of the core is no longer supported by the core and begins to decompress. All of this material begins to fall in, but stalls at the shockwave front. A bubble region is formed between the high-density core and the accreting shock front. In a complex and little understood process, a fraction, less than one percent, of the core's gravitational energy is transported to the accretion front. This incredibly small fraction

provides the critical kinetic energy required for the ejection of all but the core of the progenitor star, in a process that is popularly known as a supernova explosion. A calculation of the energy released by the core is $\sim 10^{53}$ ergs.

For those stellar evolutions that end in a supernova explosion, the hot collapsed core, or *protoneutron* star, with a temperature of tens of MeV, loses its trapped neutrinos over an interval of some seconds and cools to an MeV or less. At that point, the collapsed core has reached its equilibrium composition of neutrinos, protons, hyperons, leptons, and possibly quarks. Thus is born a *neutron star* [with a] radius [of] about ten kilometers and [an] average density 10^{14} times greater than that of Earth. The star continues to cool for millions of years by the slow diffusion of protons to the surface and their radiation into space. [6]

In some unknown fraction of the massive collapsing star cases, the explosion of the in-falling material fails to expel enough material or fails to happen at all. This failure to expel material results in the progenitor star continuing to collapse into the eventual formation of a black hole equal to the mass of the presupernova star. This formation occurs because there is a “maximum mass, called the Oppenheimer-Volkoff mass limit that can be sustained against gravitational collapse by the pressure of degenerate neutrons and their repulsive interaction” [6].

Once isolated neutron stars are formed, they will live on practically unchanged forever. They will slowly cool off on timescale of the life of the universe, their magnetic fields will disappear, the rotation of the pulsars will slowly come to a stop, and then the star will effectively become invisible, disappearing entirely.

The evolution of stars was paramount in the formation of not only our planet, but also human beings. In the Big Bang, only elements up to Lithium were formed. Elements up to iron were formed in the thermonuclear reactions caused by the evolution of massive stars; heavier elements were formed in the last few days of the life of presupernova stars, and the heaviest elements were formed in supernova explosions.

Neutron Stars:

Neutron stars are compact stars that have densities close to the limit of a collapse into a black hole. The center of the neutron star is composed of matter that is a few times the density of nuclear matter, while its surface is composed of iron, a difference of 14 orders of magnitude. The size of the neutron star is on the order of 10 kilometers, and its total mass, the integral of the energy density over its volume, is several solar masses. The range of densities in neutron stars is vast, with the central density of a neutron star approximated to be $\sim 10^{15}$ g/cm³, multiple times that of nuclear density [12]. The density continues to decrease until it reaches the surface, where the surface density is substantially lower than that of the central density: a few grams per cubic centimeter.

The matter on the edge of the neutron star, where the pressure is essentially zero, is comprised of iron, where the iron exists essentially as it does on Earth. The temperature of the surface of the neutron star is still very high by earthly standards. It most likely ranges from 10^5 to 10^7 Kelvin. Since the pressure increases so rapidly towards the center of the star, the form of matter expected to exist on the surface of the star is very thin (see figure 2). This holds for most neutron stars, excluding the very

light stars. The lighter neutron stars are much larger in size than the more massive neutron stars because they are much less compacted by the force of gravity. The low-density crust of the light neutron stars is very thick, assumed to be around tens of kilometers in thickness. However, the existence of neutron stars that have these properties is probably not realized in nature [6].

The minimum limit of the lightest neutron star is calculated to be around $0.1 M_{\odot}$. Theory states that existing neutron stars could exist anywhere in the range from this limiting minimum to the limiting mass of the Equation of State (EoS - for definition see below) of nature. In actuality, this minimum cannot exist so low, due to the fact that the gravitational binding energy of the lightest neutron stars proposed is of the wrong sign. The mass equivalent number of nucleons dispersed at infinity is actually less than the gravitational energy. This does not mean that these minimum mass stars could not exist stably if somehow formed, but the formation of the star by compaction would not release energy [6]. Without the release of energy, the expulsion of most of the progenitor star could not happen. This means that there is a much higher lower limiting mass of neutron stars powered by a supernova explosion.

The maximum mass possible in the neutron star cannot exceed the mass of an EoS corresponding to the causal limit—the speed of sound in a medium is less than the speed of light—increased slightly if the rotational frequency is high. This causal limit is approximately $3 M_{\odot}$. However, the actual upper limit is much lower. With knowledge of nuclear matter properties, the limiting mass falls more in the range of 1.4 - $2.4 M_{\odot}$. This depends on the EoS and on high-density phenomena such as hyperonization and phase transitions (e.g., quark deconfinement and Bose condensation).

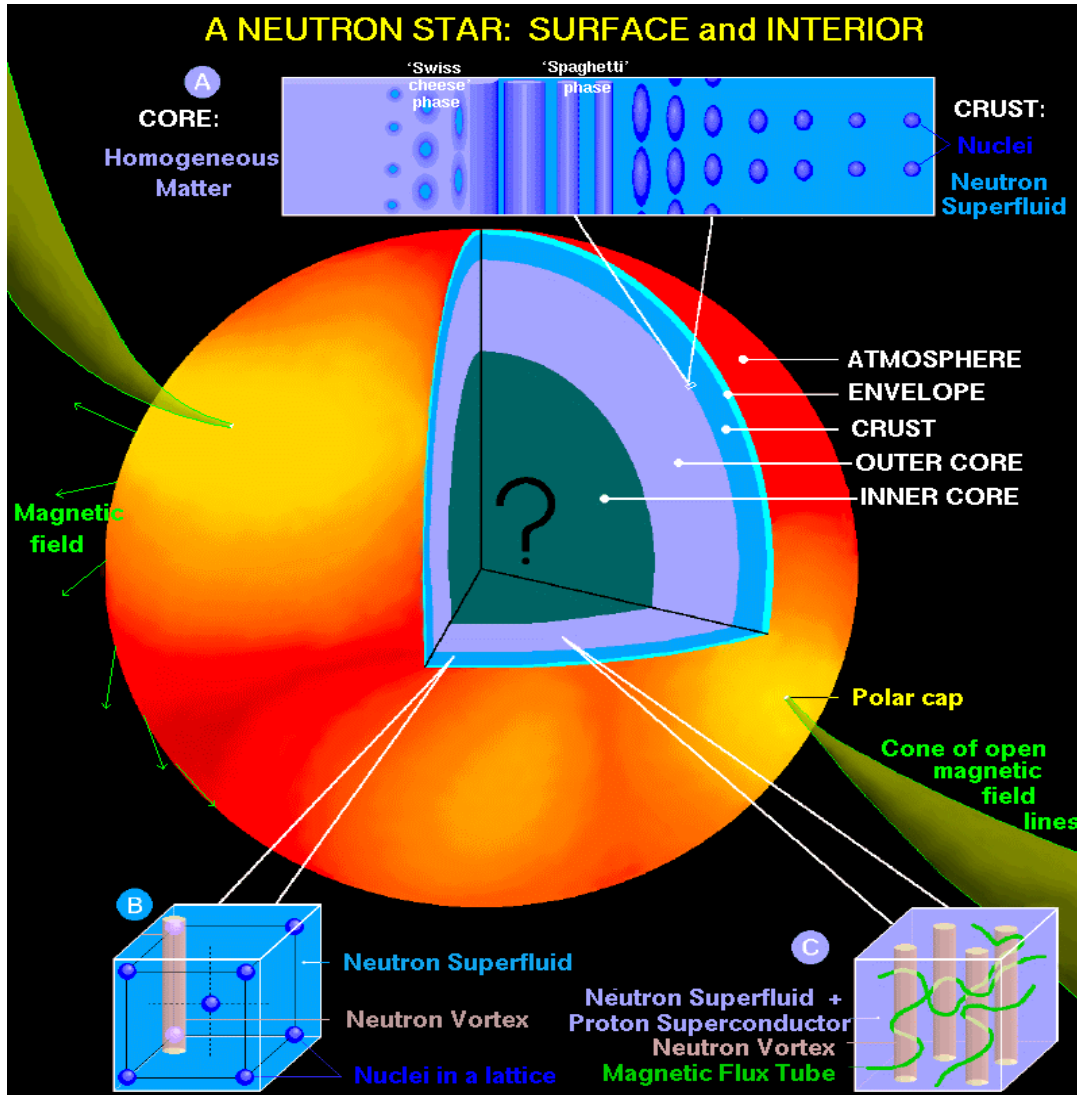


Figure 2: Illustration of the anatomy of a neutron star. [12]

When considering the particular way that neutron stars are created in supernovae, the limiting mass is approximately the Chandrasekhar mass. The core collapse that triggers the supernova explosion depends on the Chandrasekhar limiting mass, which is found to be 1.4 to 1.5 $M_{\odot}$. Since this Chandrasekhar limiting mass depends on lepton fractions in the core, the size of the neutron star depends, albeit weakly, on the mass of the presupernova star [6]. This establishes that the EoS of nature must support stars at least as massive as the Chandrasekhar mass, otherwise, no

neutron star could possibly exist—core collapse would lead directly to black holes. So, observed masses of neutron stars may not depend on the limiting mass corresponding to the EoS of nature, but rather, the mass of neutron stars depends directly on the astrophysical means that are possible to create the stars.

The matter of neutron stars is not bound by the nuclear force, but instead by gravity. The nuclear force is the strong force, but is short ranged, only acting on its nearby neighbors. The gravitational force is long ranged and acts on all mass-energy. For large and dense objects, the gravitational force becomes the binding force.

The matter in neutron stars has important similarities and differences from nuclear matter. Both nuclear matter and neutron star matter is composed of baryons and the densities are the same within an order of magnitude. One difference is that nuclei tend to be symmetric in isospin, whereas neutron stars are very asymmetric. Another difference is that strangeness and lepton number are not conserved in astrophysical objects. Strangeness would not be conserved in stable nuclei either, but it is not energetically favorable to have hyperons in the ground state because their masses exceed the nucleon mass by more than the Fermi energy (~ 30 MeV) of the nucleons in a nucleus. Nuclear reactions are so fast ($\sim 10^{-22}$ s) that strangeness is conserved on their timescale [6]. So the matter studied in nuclei or their reactions has a zero net strangeness, whereas neutron stars can, and almost certainly do, contain hyperons and have a net strangeness.

Through relativistic nuclear field theory, a connection is made between the nuclear matter and neutron star matter. This theory describes symmetric nuclear matter and the matter produced in high-energy collisions when the field equations are

solved subject to the constraints of charge symmetry, strangeness conservation, and neutron star matter, when the field equations, supplemented by those leptons, are solved subject to the constraints of charge neutrality and generalized beta equilibrium without conservations of strangeness. With this theory, it is easy to characterize the neutron star matter EoS by the compression modulus and other properties of symmetric matter to which the coupling constants are fixed.

Contrary to what their name implies, neutron stars are not comprised solely of neutrons as was first proposed. The stars must be charge neutral, but being comprised solely of neutrons is not the lowest energy state of dense neutral matter. For reasons of chemical potential and isospin energy symmetry, neutron star matter is very complex in composition, and the Lagrangian, a function that summarizes the dynamics of the system, used in nuclear field theory has to be generalized to include these complications. When taking the general manner for equilibrium for a low density, the charge-neutral matter is almost pure in neutrons, with an equal number of protons and electrons. As the density increases, the electron Fermi energy increases to the muon mass, and then muons, an unstable subatomic particle of the same class as an electron, begin to populate the matter [6]. Hyperon thresholds are met as densities increase to three times nuclear density. Hyperons become very important to understand when studying high-density neutron stars.

Neutron stars are also comprised of hadronic matter at the lowest energy state consistent with charge neutrality. From this understanding, it is obvious how rich in baryon species neutron stars actually are. This complex and varying matter in these stars are simply referred to as neutron star matter.

In order to understand the phenomena that are allowed to occur in neutron stars, one must have understanding of certain general principles. Within ten principles, considerable insight can be gathered into the possible constitution of neutron stars and phase transitions that occur at varying densities. The Lorentz covariance, general relativity, causal EoS, microscopic stability known as Le Chatelier's principle, Baryon and electric charge conservation, Pauli principle, generalized beta equilibrium, phase equilibrium, asymptotic freedom of quarks, and properties of matter at saturation density are all required to understand the inner workings of neutron stars [6]. By applying these principles to specific theorems of matter, models explaining the workings of these stars will be greater understood.

The matter at the surface of the star is of little importance to its mass and radius because it is so thin. The edge of the star has zero pressure, which results in a molten pool of material. Since Fe^{56} is the lowest energy state of hadronic matter, the outer material is essentially entirely comprised of this iron in a molten state. Pressure in the star rises rapidly with the distance from the surface, and the resultant high degree of ionization will lead to the formation of a Coulomb lattice. Deeper into the star, the atoms will be compressed due to the higher pressure. This results in the nuclear spacing being reduced so that there is no room for normal atomic structure. Until the density reaches the neutron drip density, the nuclear forces will hold the nuclei together as individual entities. In this state, the nuclei are embedded in an electron sea.

From a deeper understanding of the central density, it can be determined what would happen if a neutron star goes below or above the limiting mass of the stars. Below the minimum mass of the neutron stars, the fundamental vibrational mode is

unstable. It is known that low mass neutron stars are unbounded, which means that, theoretically, configurations immediately below the onset of positive slope for the neutron star branch are unstable to radial oscillations that destroy them by dispersal. For more massive stars, the configurations are gravitationally bound. If a neutron star below the limiting mass were to accrete matter adiabatically so that it surpasses this limiting mass, the fundamental vibrational mode would destroy it. The oscillations grow in amplitude rapidly and would elevate the central density such that a central region would fall within the Schwarzschild radius, which would result in the formation of a black hole.

The mass-radius relationship for neutron stars is the most important graphical representation. This relationship is uniquely related to the underlying EoS. As far as our understanding is now, both mass and radius are not known for a particular neutron star. Masses can be determined if the neutron star is in a binary orbit with a companion star, while the radius, in theory, can be determined through the measurement of the Doppler shift of known spectral transitions or of the photon produced by the annihilation of electron pairs (mass-radius relationship for neutron stars). As previously mentioned, the relation of mass to the radius of a neutron star at first glance is counter-intuitive (see figure 3 and 4 for mass-radius relations). For low-mass stars, the gravitational attraction is relatively weak, so the particular low-mass star is large and diffuse. With high-mass stars, the gravitational force is much stronger resulting in a much more compact star with a smaller radius. Different models of what matter comprises the neutron star result in different maximum limits. Neutron stars composed entirely of neutrons give a higher maximum mass for similar radii; the maximum mass is $\sim 2.4 M_{\odot}$

at radius ~ 12 kilometers. When protons and leptons are taken into consideration for the composition of the matter, the maximum mass at different radii is decreased. When hyperons are considered in the composition of neutron star matter, the upper limit of mass is again decreased; the maximum mass being $1.5 M_{\text{S}}$ at a radius of ~ 11 kilometers. With the addition of the more strange matter, the EoS is softened, resulting in the changes observed in the models.

To understand the limiting masses of neutron stars, one must understand the softening and stiffening properties of the EoS. This limiting mass depends on the compressibility of the matter comprising the neutron star, which is detailed in the EoS. A soft EoS is more easily compressed than a stiff equation of state. An EoS is said to be stiffer than another if the pressure at every energy density is greater for the former state than the latter. So, the stiffer the equation of state, the larger the limiting mass can be before the collapse of the star. Taking an extremely stiff EoS, the limiting mass of the star is just over $3 M_{\text{S}}$. Conversely, when a very soft equation of state is taken, the limiting mass is found to be $\sim 0.7 M_{\text{S}}$. The actual limiting mass in nature must fall between these two values. The limiting mass of neutron stars is so elusive due to the lack of a precise equation of state of nature. The existence of hyperons in the matter of the neutron star is what softens the EoS. Sharing the baryon number among many species lowers both energy and pressure.

These stars, that have radii comparable to that of many of our cities and contain masses comparable to our Sun, deserve to be studied in depth. Invaluable knowledge can be gathered from the exotic composition of matter comprised under the severe and exclusive conditions created in neutron stars. From the study of these tiny stellar

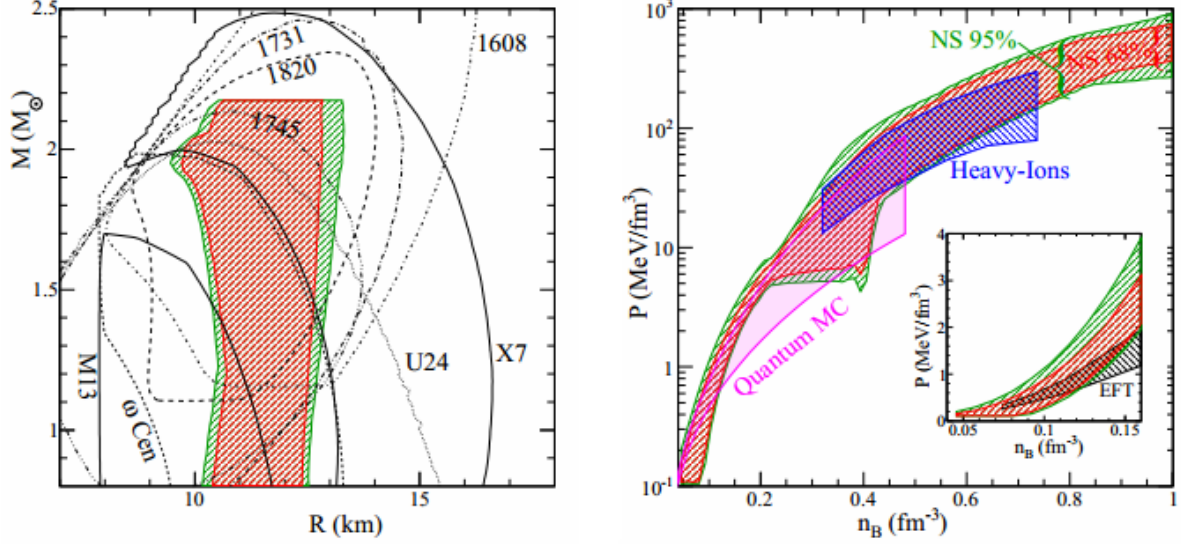


Figure 3 (a) and (b): (a) A comparison of the predicted M - R relation with the observations. The shaded regions outline the 68% and 95% confidences for the M - R relation; these include variations in the EOS model and the modifications to the data set, but not the more speculative scenarios. The lines give the 95% confidence regions for the eight neutron stars in our data set. (b) The predicted pressure as a function of baryon density of neutron-star matter as obtained from astrophysical observations. [21]

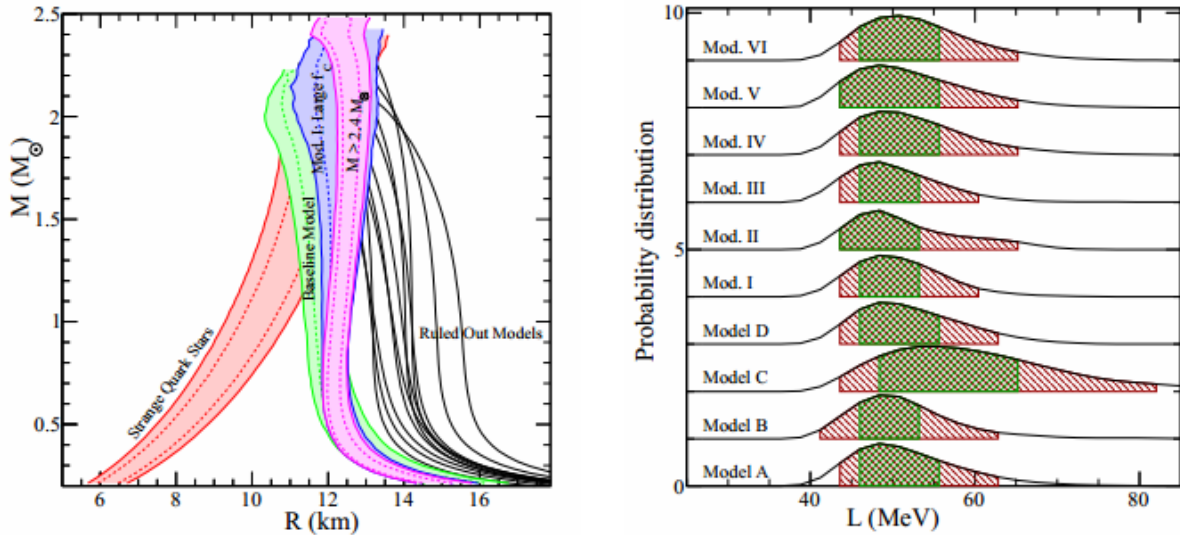


Figure 4 (a) and (b): (a) Predicted M - R relations for different EOS models and data interpretations. Proceeding from back to front, the red contours and probability distributions are for strange quark stars. Next are green contours, which correspond to the baseline model, and the magenta results are those assuming a larger maximum mass to accommodate a mass of 2.4 solar masses for B1957+20. Finally, the black lines are the 10 Skyrme models from Stone et al.. (b) The limits on the density derivative of the symmetry energy, L . The single-hatched (red) regions show the 95% confidence limits and the double-hatched (green) regions show the 68% confidence limits. [21]

objects, scientists can not only learn more about the forms of stars, but also the uncommon particles that are only found in these stars or high-speed collisions achieved in particle accelerators. The existence of these high-density cosmic occurrences presents a unique realm for high-density physicists to study.

Pasta Phase:

The transitional regions between homogeneous and inhomogeneous phases are some of the most exotic phases in the neutron stars and CCSN. As the density and temperature increase, heavy quasi-nuclei structures are formed, which undergo a series of changes from spherical to exotic forms: rods, slabs, cylindrical holes, and bubbles, all referred to as “nuclear pasta” (see figure 5). This is an extension of the trend toward heavier, more neutron-rich nuclei that occurs during the earlier phases of core collapse. This process is mainly caused by the competition between surface tension and the Coulomb repulsion of closely spaced heavy nuclei. This not only occurs in CCSN matter but, for example, also at the transitional region between the crust and core of neutron stars.

The different forms of pasta appear from densities $0.01\text{--}0.1\text{ fm}^{-3}$. The pasta phase is the ground-state configuration if it minimizes the free energy (i.e., if the free energy per particle is lower than that of the free energy in a homogeneous configuration for the same density, then the pasta phase is the ground state).

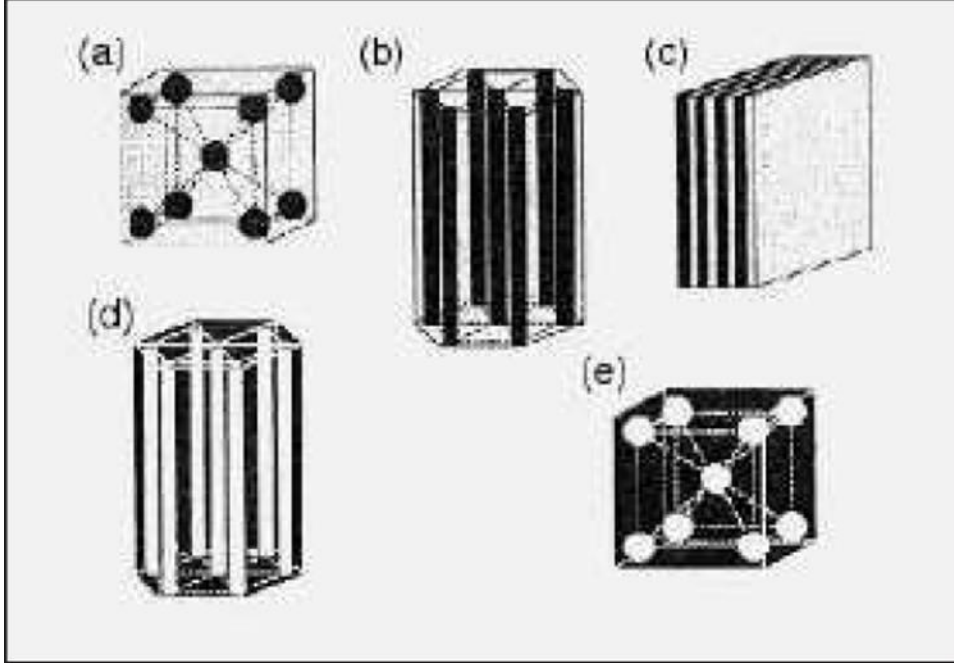


Figure 5: Nuclear Pasta: (a) spherical nuclei (gnocchi); (b) cylindrical nuclei (spaghetti); (c) slab-like nuclei (lasagna); (d) tube (penne); (e) spherical bubbles (Swiss cheese). [16]

The main goals of our research were to calculate the self-consistent nuclear pasta phase and determine the phase transitions between different pasta phases and uniform matter. We used a finite temperature 3D-Hartree-Fock method (3DHF) with several models for the effective density-dependent Skyrme interaction on an extended grid of three variable parameters, temperature, T , particle number density ρ , and the proton/neutron ratio in the matter y_p . We used $0.01 < \rho < 0.12 \text{ fm}^{-3}$, $T = 0 \text{ MeV}$ and $y_p = 0.05, 0.10, \& 0.15$ to study specific properties of the pasta phase, namely the threshold density of its appearance, a density at which it is dissolved into a uniform matter, the sequence of pasta formations as a function of y_p , and the model of the nuclear interaction chosen in the Hartree-Fock calculation. We selected four different interactions, SkM* (Bartel et al., 1982 [2]), SLy4 (Chabanat et al., 1998 [4]), NRAPR (Steiner et al., 2005 [22]), and QMC700 (Guichon et al., 2006 [8]).

Equation of State:

One of the most important parts to modeling the supernovae and neutron star matter is the equation of state, or EoS. The EoS relates the pressure of matter to its density and temperature, $P = P(\rho, T)$. Once the EoS is known for a system, one can ascertain all the other equilibrium thermodynamic properties. From the first law of thermodynamics, the EoS can be derived from the energy-density of the system $\epsilon = \epsilon(\rho, T)$. From this we get

$$P(\rho, T) = \rho^2 \frac{d(\epsilon/\rho)}{d\rho} \quad (1)$$

By calculating the specific total energy of the particular system, the EoS can be obtained [13]. Supernovae and neutron star matter is dominated by nucleons and their interactions. In order to get the total energy for these systems, one must start by modeling the nuclear force for the system. One of the future goals of study of the pasta phase is its incorporation into the EoS, which forms an important input for the simulation of core-collapse supernovae and neutron stars. To date, none of the EoS used in these simulations include a fully developed self-consistent contribution from a pasta phase.

Hartree-Fock Approximation:

We used three-dimensional Hartree-Fock (3DHF) approximation to calculate the nuclear mean field with a phenomenological density dependent Skyrme model for the nuclear force with BCS pairing. The non-relativistic phenomenological effective

interaction was first written down by Skyrme [19], from whom the interaction gets its name. It became widely used in nuclear physics following its application to the calculation of finite nuclei by Vautherin and Brink [25]. It is chosen for the simplicity with which it may be applied to calculations at the Hartree-Fock level of approximation. However, in principle, any phenomenological effective interaction, relativistic ones included, may be used within our theoretical framework.

The Skyrme interaction takes the form of an effective two body potential between particles i and j . Its form is based on an expansion of the matrix elements of a two body potential in momentum space up to second order:

$$\begin{aligned} \hat{v}_{\text{Skyrme}}(\mathbf{r}_i, \mathbf{r}_j) = & t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{2} t_1(1 + x_1 P_\sigma) [\mathbf{k}_{ij}^2 \delta(\mathbf{r}_i - \mathbf{r}_j) + \delta(\mathbf{r}_i - \mathbf{r}_j) \mathbf{k}_{ij}^2] \\ & + t_2(1 + x_2 P_\sigma) \mathbf{k}_{ij} \cdot \delta(\mathbf{r}_i - \mathbf{r}_j) \mathbf{k}_{ij} + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^3(\bar{\mathbf{r}}) \delta(\mathbf{r}_i - \mathbf{r}_j) \\ & + i t_4 \mathbf{k}_{ij} \delta(\mathbf{r}_i - \mathbf{r}_j) (\hat{\sigma}_i + \hat{\sigma}_j) \times \mathbf{k}_{ij} \end{aligned} \quad (2)$$

where ρ is the matter density, $\mathbf{k}_{ij} \equiv -\frac{1}{2}i(\nabla_i - \nabla_j)$ is the relative wave-vector,

$P_\sigma = \frac{1}{2}(1 + \hat{\sigma}_i \cdot \hat{\sigma}_j)$ is the spin exchange operator, $\hat{\sigma}$ is the vector of Pauli spin matrices,

and $\bar{\mathbf{r}} = \frac{\mathbf{r}_i + \mathbf{r}_j}{2}$ [13]. It is to be understood that any operator to the left of a delta function

operates to the left and those to the right operate to the right. The interaction has the parameters $t_0, t_1, t_2, t_3, t_4, x_0, x_1, x_2, x_3$, and α to be adjusted so that the interaction describes a certain set of nuclear properties accurately.

The full computational formalism of the 3DHF method used in this work has been developed by Newton [13] and Stone (Newton and Stone, 2009 [14]). The full solution of the Schrodinger equation for N particles in the system is currently beyond

any computer power available. The HF method offers an approximation in which the N -body Schrodinger equation is transformed into a system of N one-body Schrodinger equations, which can be solved self-consistently. The ground state wave function of a many-body system is approximated by a single Slater determinant $|\Phi\rangle = |\varphi_{1q}, \varphi_{2q}, \dots\rangle$ where φ_{iq} are single particle wave-functions of the i^{th} particle and $q = p, n$, instead of the linear combination of N -Slater determinants. Φ is found by the minimization of the expectation value of the Hamiltonian of the system $\delta E_{\text{Skyrme}}[\Phi] = \delta\langle\Phi|\hat{H}|\Phi\rangle = 0$, where E_{Skyrme} is the energy density functional (Stone and Reinhard, 2007 [23]). Minimization with respect to φ_{iq} yields a system of single particle equations with a one-body HF potential u_q :

$$\left[-\frac{\hbar^2}{2m_q^*}\nabla + u_q(\mathbf{r})\right]\varphi_{i,q}(\mathbf{r}) = \epsilon_{i,q}\varphi_{i,q}(\mathbf{r}) \quad (3)$$

where $\epsilon_{i,q}$ are single particle energies and m_q^* is the effective mass. The single particle potentials are expressed in terms of the parameters x_j and t_j of the Skyrme interaction (see below) as

$$\begin{aligned} u_q = & t_0 \left(1 + \frac{1}{2}x_0\right)\rho - t_0 \left(\frac{1}{2} + x_0\right)\rho_q \\ & + \frac{1}{12}t_3\rho^\alpha \left[(2 + \alpha)\left(1 + \frac{1}{2}x_3\right)\rho - 2\left(\frac{1}{2}x_3\right)\rho_q - \alpha\left(\frac{1}{2} + x_3\right)\frac{\rho_p^2 + \rho_n^2}{\rho}\right] \\ & + \frac{1}{4}\left[t_1\left(1 + \frac{1}{2}x_1\right) + t_2\left(1 + \frac{1}{2}x_2\right)\right]\tau - \frac{1}{4}\left[t_1\left(\frac{1}{2} + x_1\right) - t_2\left(\frac{1}{2} + x_2\right)\right]\tau_q \\ & - \frac{1}{8}\left[3t_1\left(1 + \frac{1}{2}x_1\right) - t_2\left(1 + \frac{1}{2}x_2\right)\right]\nabla^2\rho + \frac{1}{8}\left[3t_1\left(\frac{1}{2} + x_1\right) + t_2\left(\frac{1}{2} + x_2\right)\right]\nabla^2\rho_q \\ & - \frac{1}{2}t_4(\nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{J}_q) \end{aligned} \quad (4)$$

The effective mass is given as

$$\frac{\hbar^2}{2m_q^*} = \frac{\hbar^2}{2m_q} + \frac{1}{4} \left[t_1 \left(1 + \frac{1}{2} x_1 \right) + t_2 \left(1 + \frac{1}{2} x_2 \right) \right] \rho - \frac{1}{4} \left[t_1 \left(\frac{1}{2} + x_1 \right) - t_2 \left(\frac{1}{2} + x_2 \right) \right] \rho_q \quad (5)$$

Here $\rho = \rho_n + \rho_p$ are the nucleon densities, $\tau = \tau_n + \tau_p$ are kinetic energy densities, and $J = J_n + J_p$ are the spin currents. The many parameters of the Skyrme interaction have to be fitted to experimental data. The parameters are correlated and, in principle, there are infinite number of the parameter sets, which can be fitted to the experimental data on finite nuclei and infinite nuclear matter. Recently, Dutra et al.. [5] explored the performance of 240 Skyrme parameter sets against a number of constraints related to properties on nuclear matter. They found very few parameter sets that satisfied all the constraints. We have chosen two of those, NRAPR and QMC700. In addition, we used two more traditional parameter sets, SkM* and Sly4, to allow a comparison against previous calculations [14] and Magierski and Heenen [11].

In the calculation, it is assumed that at a given density and temperature matter is arranged in a periodic structure throughout a sufficiently large region of space for a unit cell to be identified. As a result, only one unit cell must be calculated in order to obtain the bulk and microscopic properties of the matter. The calculation is performed in cubic cells with periodic boundary conditions and assuming reflection symmetry across the three Cartesian axes. Only shapes with cubic symmetry are allowed. The required reflection symmetry allows us to get solutions only in one octant of the unit cell, which significantly reduces the computational computer time.

It is expected that the absolute minimum of the free energy of a cell containing A nucleons is not going to be particularly pronounced, and there will be a host of local minima separated by relatively small energy differences. In order to systematically survey the ‘shape space’ of all nuclear configurations of interest, the quadrupole moment of the neutron density distributions has been parameterized, and those parameters have been constrained. It is assumed that the proton distribution closely follows that of the neutrons.

Calculations performed in a finite cell can be a source of unwanted shell effects. Magierski, Heenen, Newton, and Stone explored the shell effects, which can be of two sorts:

(i) Spurious, arising in analogy with a Fermi gas in a box, caused by the discretization of the physical space due to the finite computational volume. These effects will manifest themselves usually at higher densities and temperatures when a large number of nucleons are unbound but are not limited to these conditions.

(ii) Physical, due to a combination of the shell energies of bound nucleons and unbound neutrons scattered by the bound nucleons, which are characterized by more rapid fluctuations in nucleon number, typically at lower densities, temperatures, and values of A .

The distinction between the form and occurrence of the two types of shell effects is encouraging as it allows for their easy identification. In such a situation where the shell effects are purely spurious, the physical value of the free energy density is not the minimum, but that value to which the free energy occurs tends to be at high A -values.

The minimum of the free energy density in a cell at a given particle number density, temperature, and a proton fraction is sought as a function of 3 free parameters: the number of particles in the cell (determining the cell size) and the parameters of the quadrupole moment of the neutron distribution β and γ . Each minimization takes approximately 12 hours of the CPU time on Cray XT5/XK6 machine and is performed in a trivially parallel mode, typically using 45,000 processors or more in one run.

Results:

Pais and Stone [18] used the 3DHFEOS model to examine the pasta phase in supernova matter at a fixed proton/neutron $y_p = 0.3$ and the temperature range of $2 < T < 10$ MeV. The main goal of our research was to extend the study properties of the pasta phase as a function of decreasing y_p , at $T = 0$ MeV. This scenario is approaching the region of y_p relevant for neutron stars. However, in variance with the supernova matter, neutron stars are in beta-equilibrium, which determines y_p corresponding to the equilibrium condition. This y_p can be found by analysis of data for a sequence of y_p and looking for a minimum of the total energy density as a function of y_p . Our focus was on $T = 0$ MeV, $y_p = 0.05, 0.10, 0.15$, $\rho = 0.0100 \sim 0.1200$ fm⁻³ in steps of 0.01 (or until the transition to homogeneous matter), $300 < A < 1200$ in steps of 20, for all four different nucleon-nucleon interactions NRAPR, QMC700, Sly4, and SkM*. This data will be used for finding the minimum y_p and the corresponding density, which would describe the beta-equilibrium situation in neutron stars.

We present in figures 6-60 results of the calculation of the A -dependence of the free energy density for all densities and interactions considered in this work depicted by

solid points. We can see a large amplitude low frequency scatter at low A , which we interpret as physical shell effects that die away with increasing A . The lower amplitude large frequency oscillations, which persist to higher A , can be attributed to the spurious shell effects. It is interesting to observe that their amplitude also decreases with increasing A and that they appear even at zero temperature and relatively low densities. The dashed curves represent an interpolated fit over the oscillations to help to fit the A -value corresponding to the minimum value of the free energy density. The best handling of the spurious shell effects is a complicated process and still under development.

However, the preliminary results look promising, as illustrated in figures 61 -63. Here, the evolution of the spatial neutron density distribution in a unit shell is visualized. Each unit cell corresponds to a fixed total neutron particle number density that is shown, taken at the minimum free energy density for each value A . We obtained an expected sequence of shapes from spherical to uniform matter through rods, slabs, tubes, and bubbles. A comparison of the figures shows that there is a systematic decrease in the range of densities within which the pasta phases appear with decreasing y_p , which was not observed before. Some dependence on the chosen Skyrme interaction in the model can be also observed, but a more detailed calculation with a finer particle number density mesh would be required to make a definite conclusion.

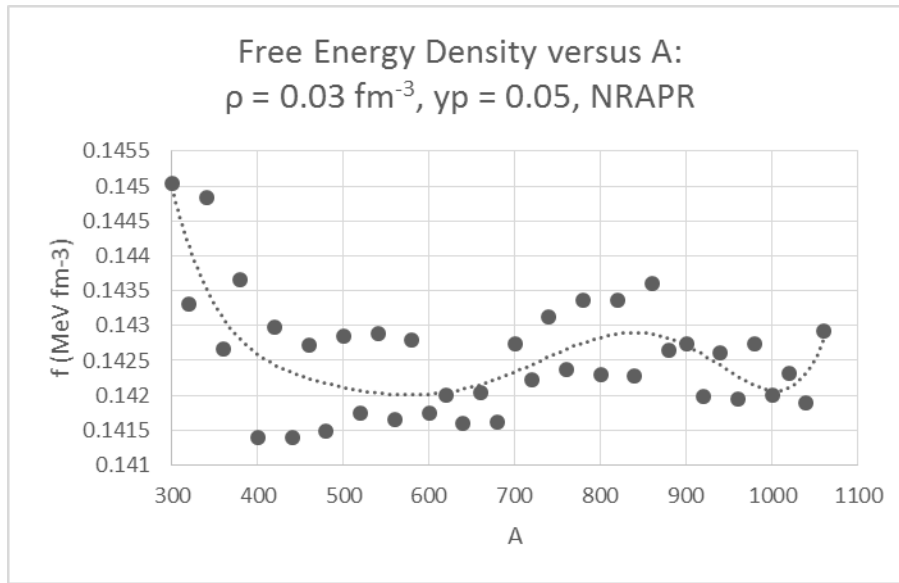
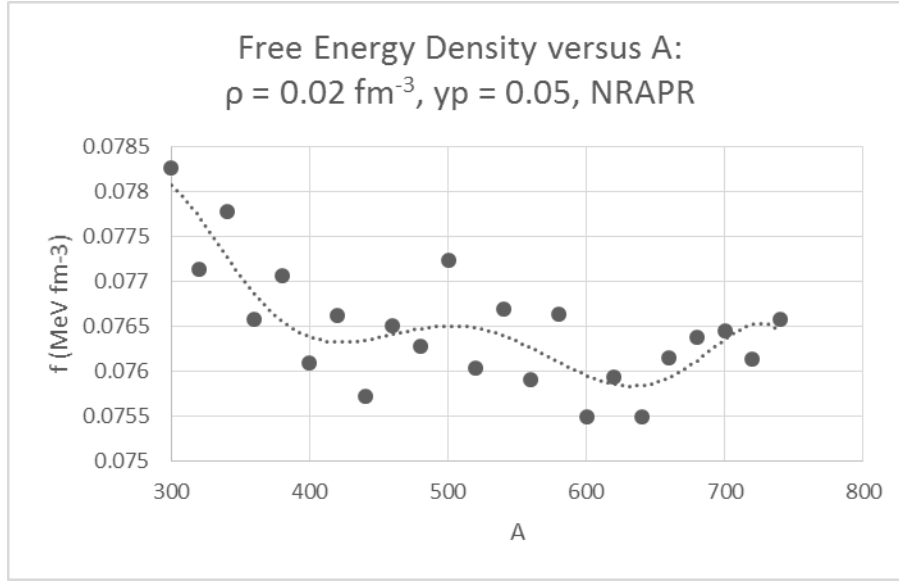


Figure 6: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.02$ (top), 0.03 (bottom) fm^{-3}).

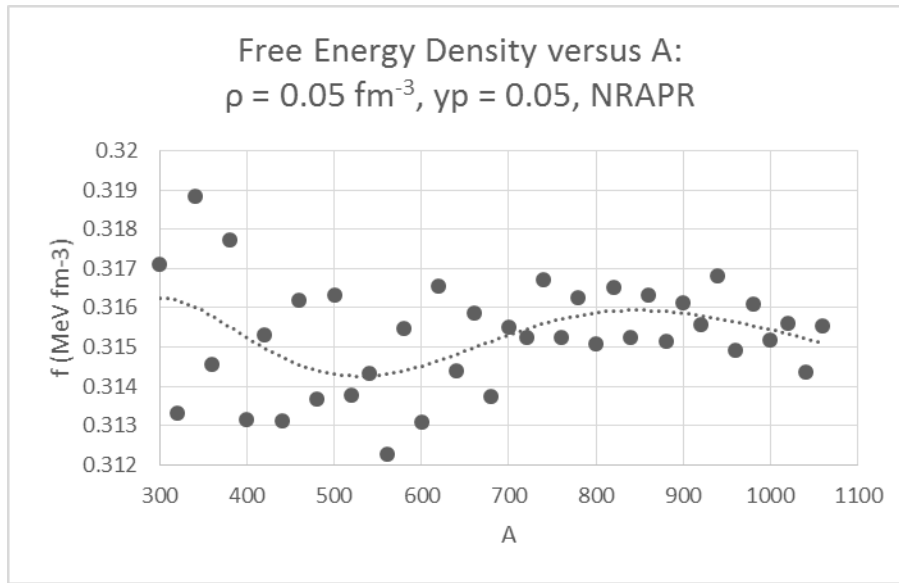
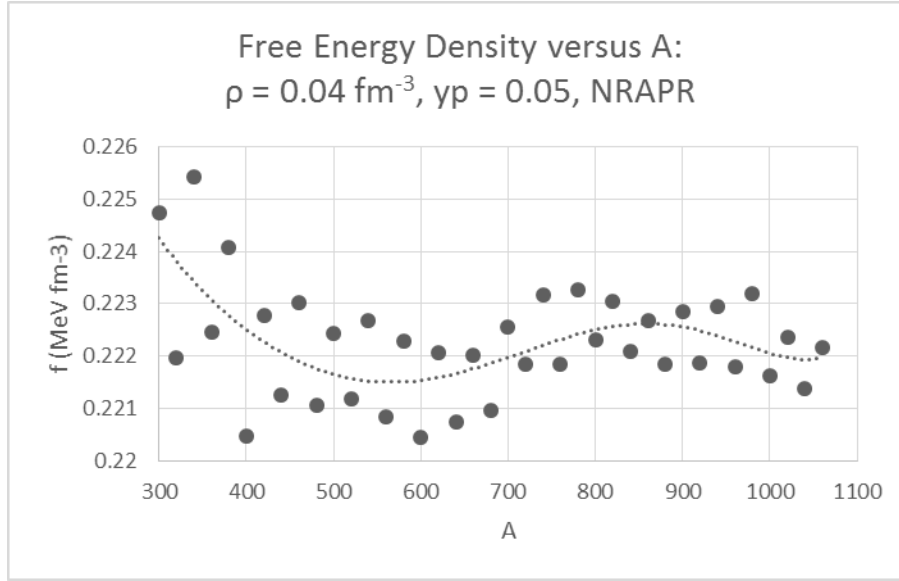


Figure 7: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.04$ (top), 0.05 (bottom) fm^{-3}).

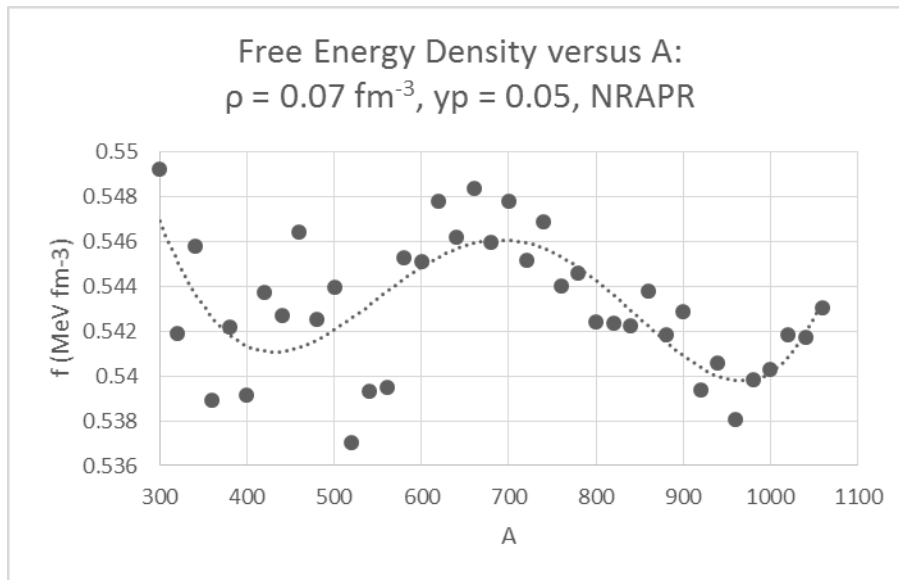
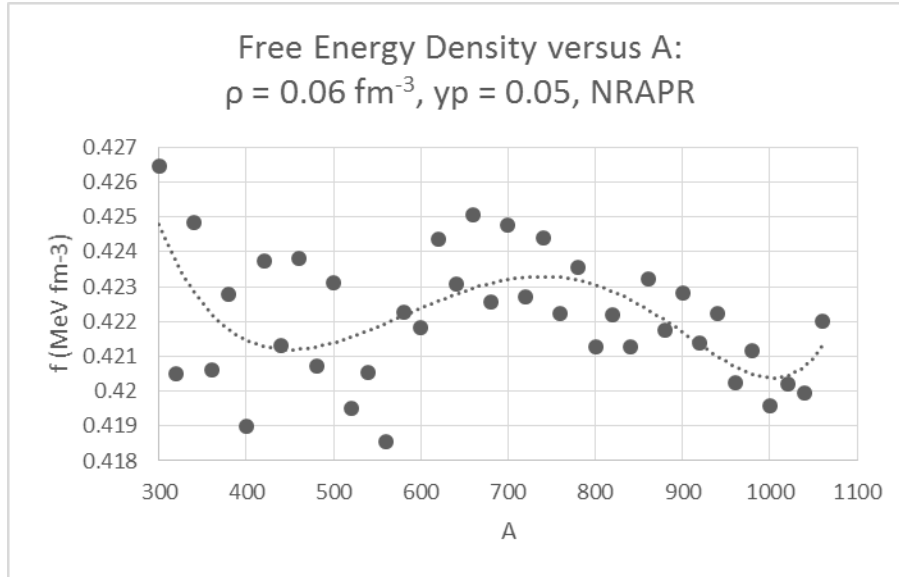


Figure 8: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.06$ (top), 0.07 (bottom) fm^{-3}).

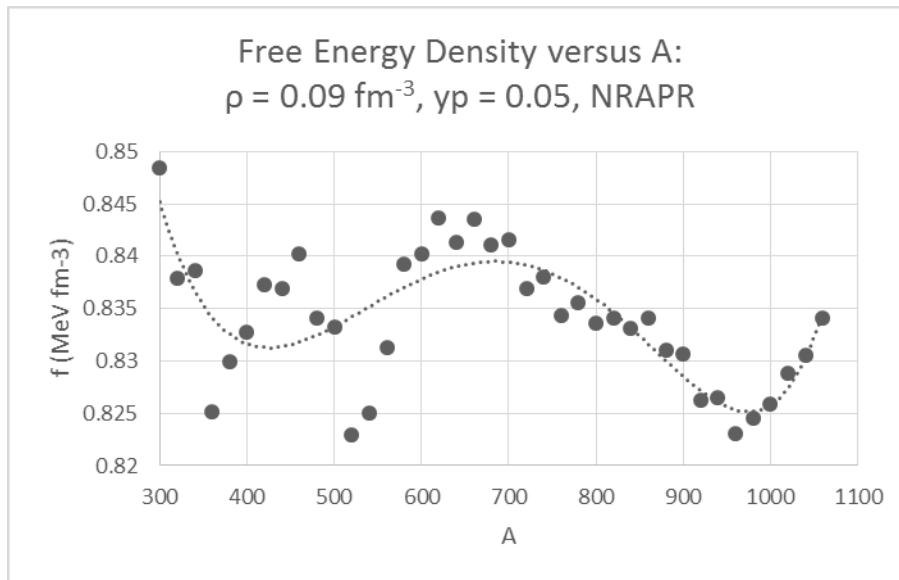
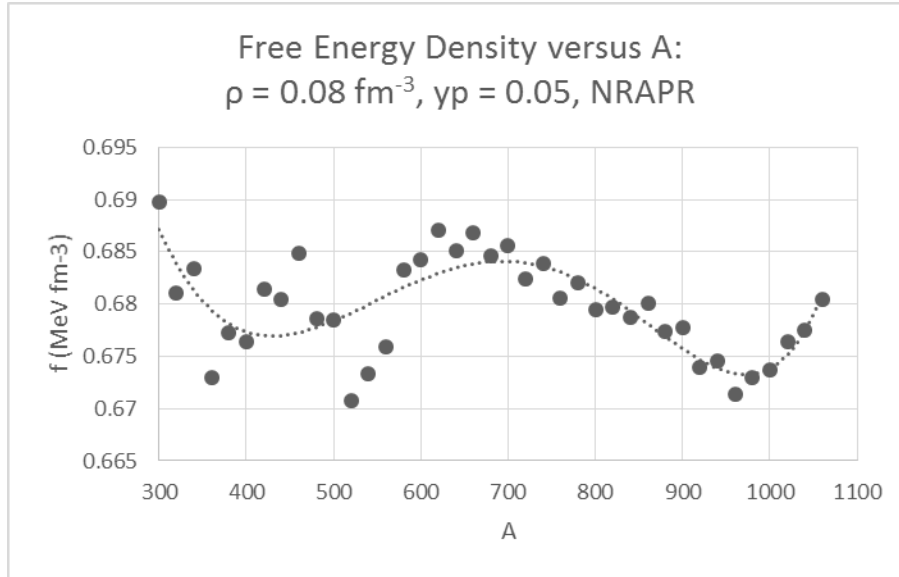


Figure 9: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.08$ (top), 0.09 (bottom) fm^{-3}).

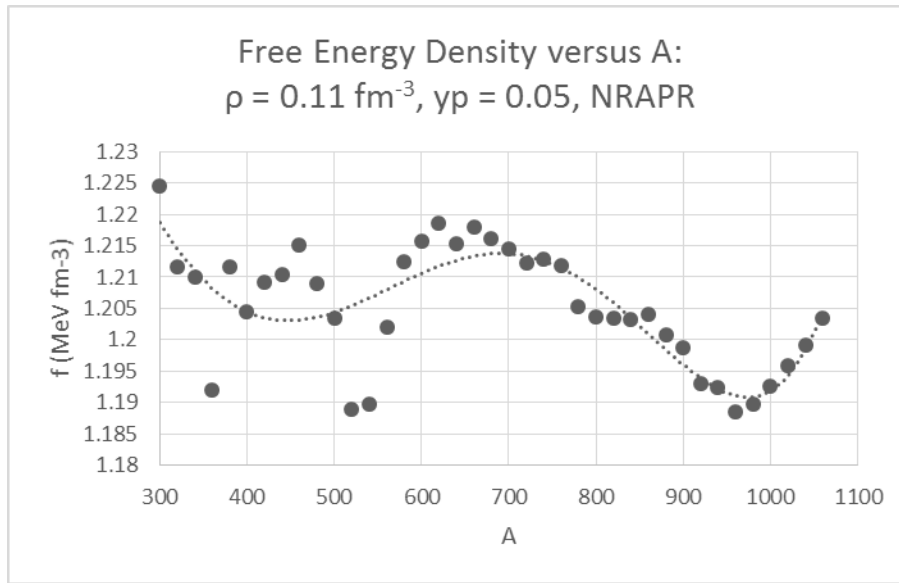
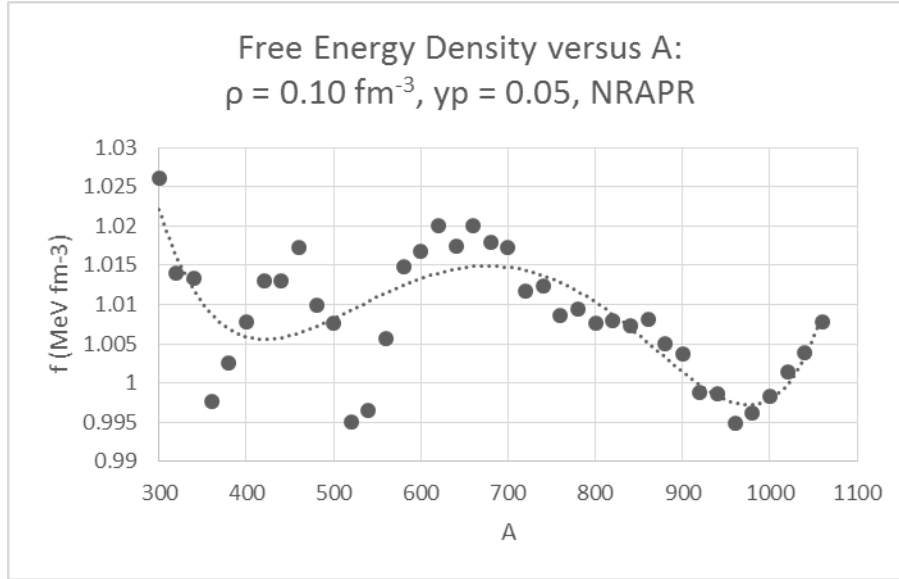


Figure 10: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.10$ (top), 0.11 (bottom) fm^{-3}).

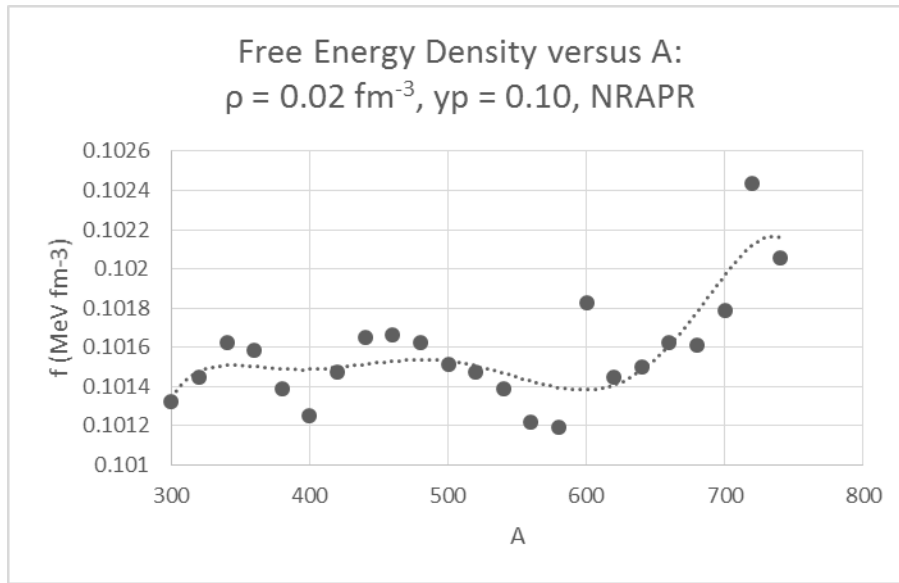
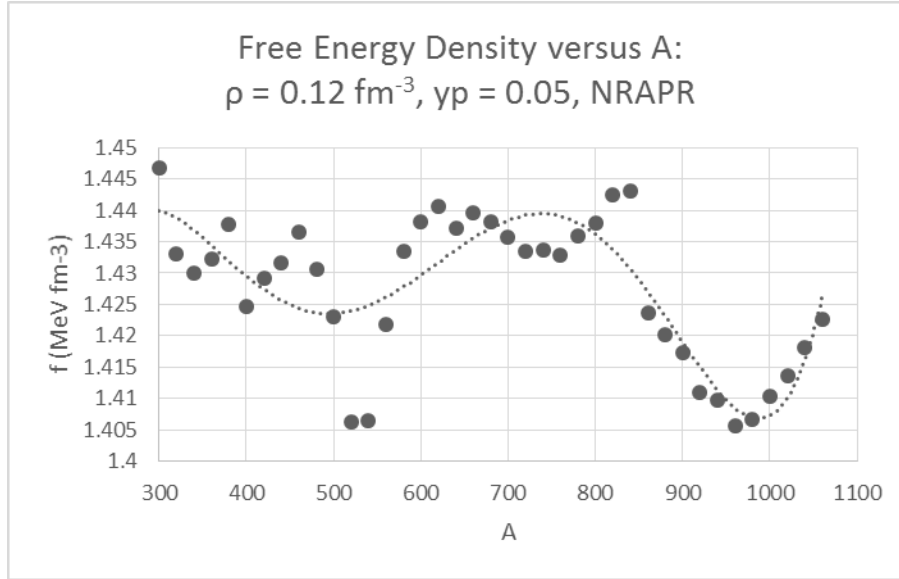


Figure 11: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.05$ (top), 0.10 (bottom), $\rho = 0.12$ (top), 0.02 (bottom) fm^{-3}).

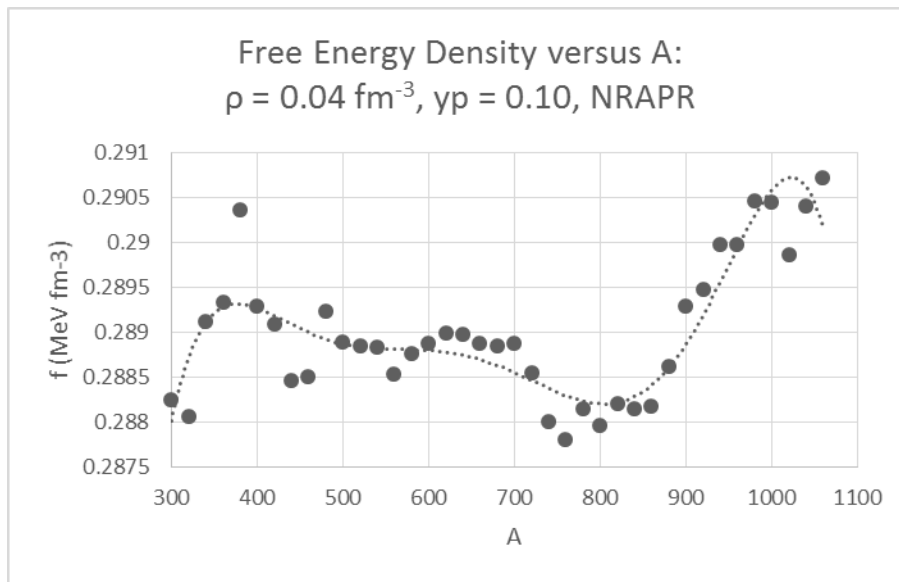
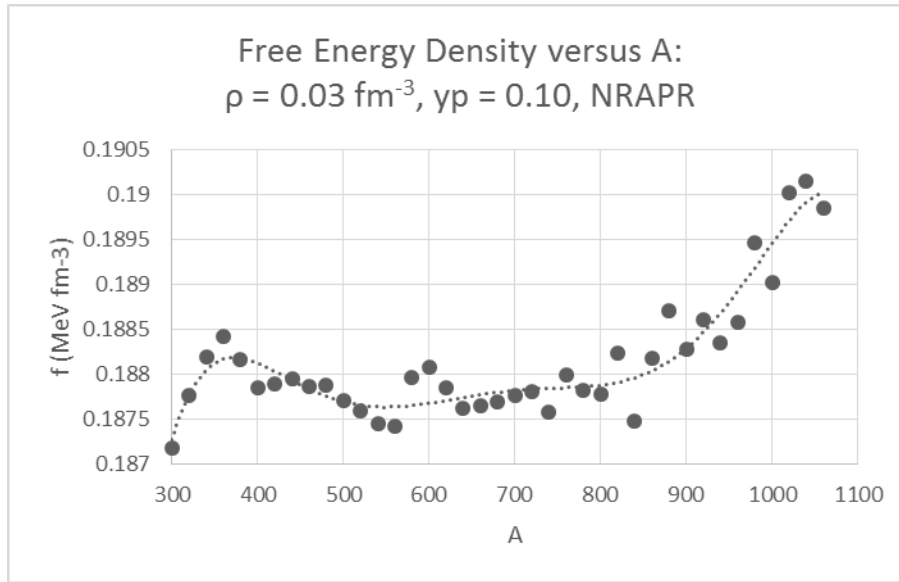


Figure 12: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.03$ (top), 0.04 (bottom) fm^{-3}).

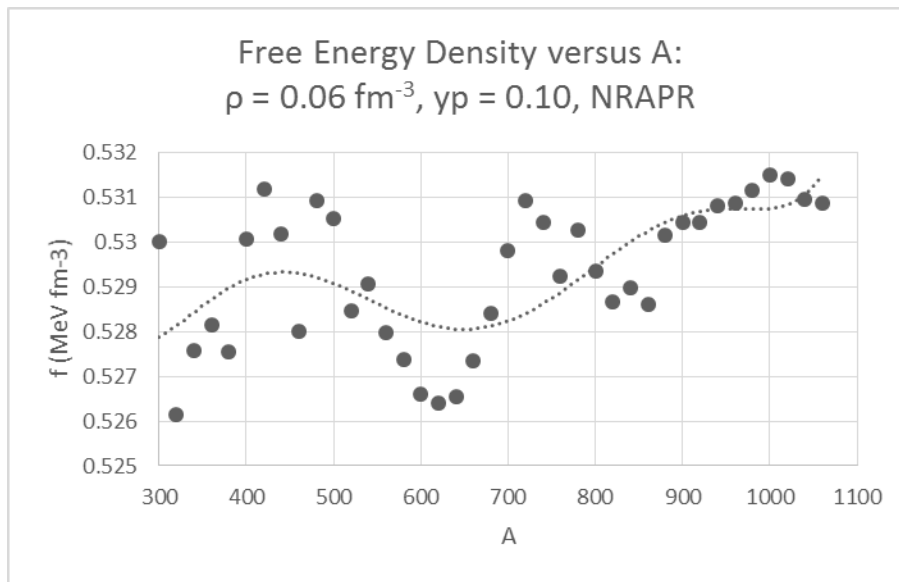
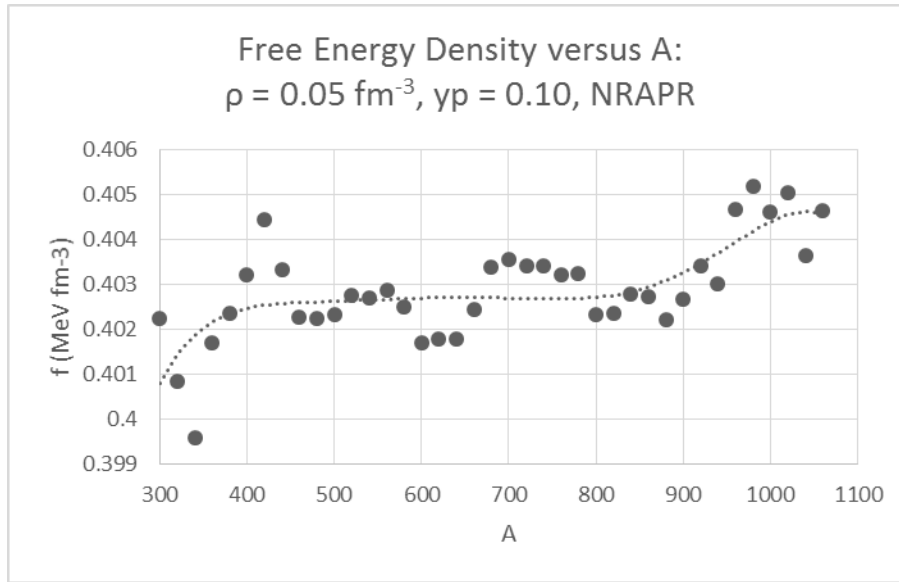


Figure 13: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.05$ (top), 0.06 (bottom) fm^{-3}).

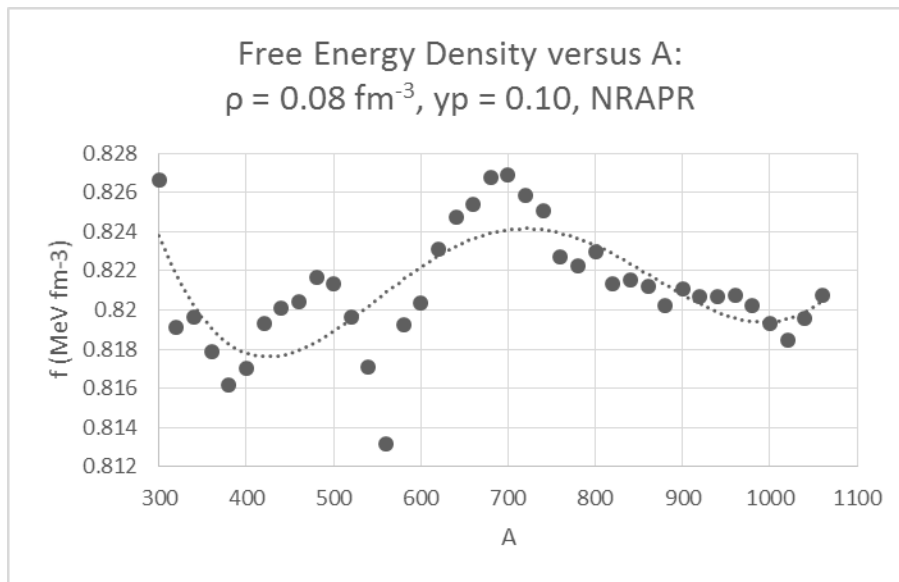
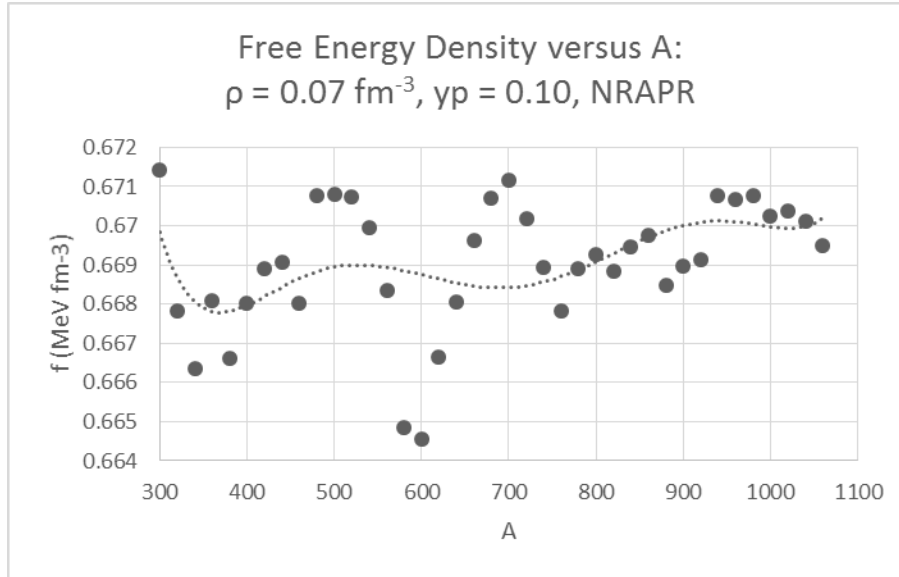


Figure 14: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.07$ (top), 0.08 (bottom) fm^{-3}).

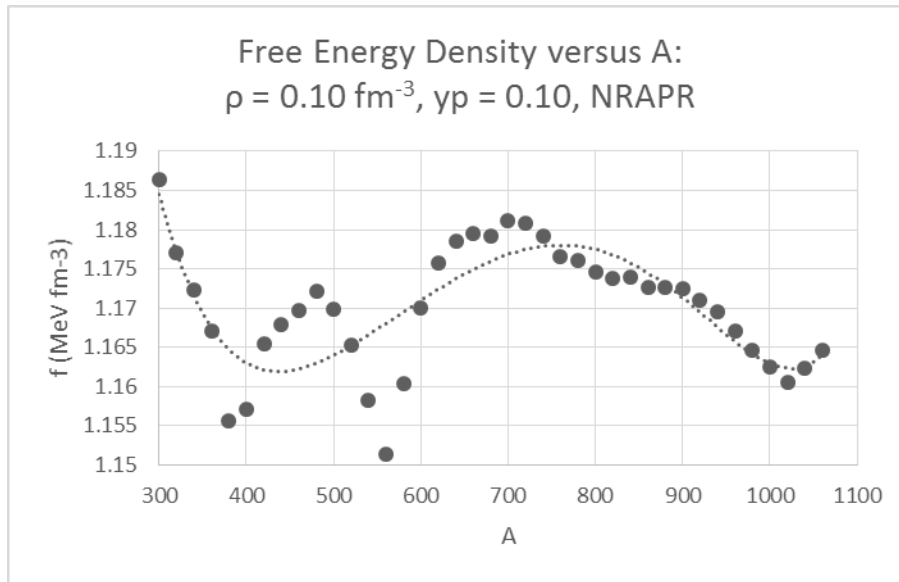
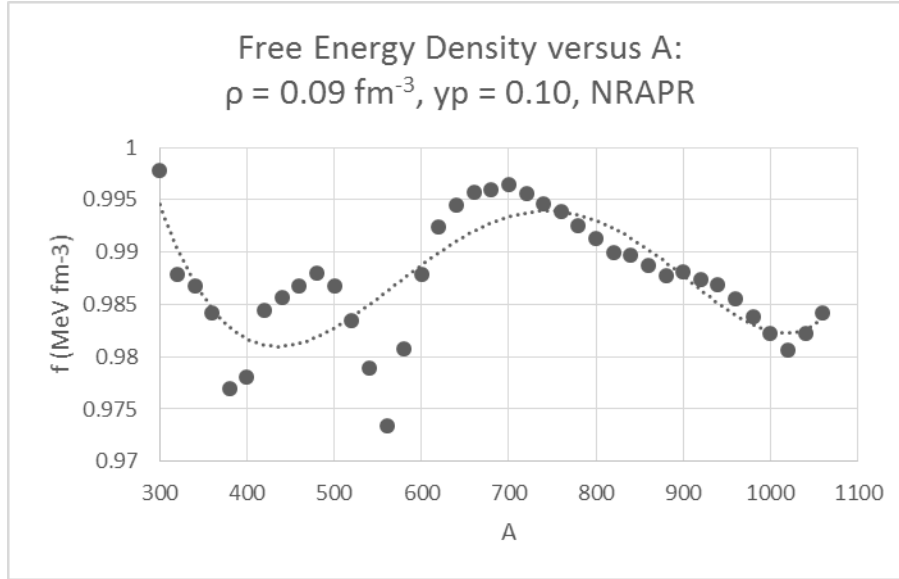


Figure 15: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.09$ (top), 0.10 (bottom) fm^{-3}).

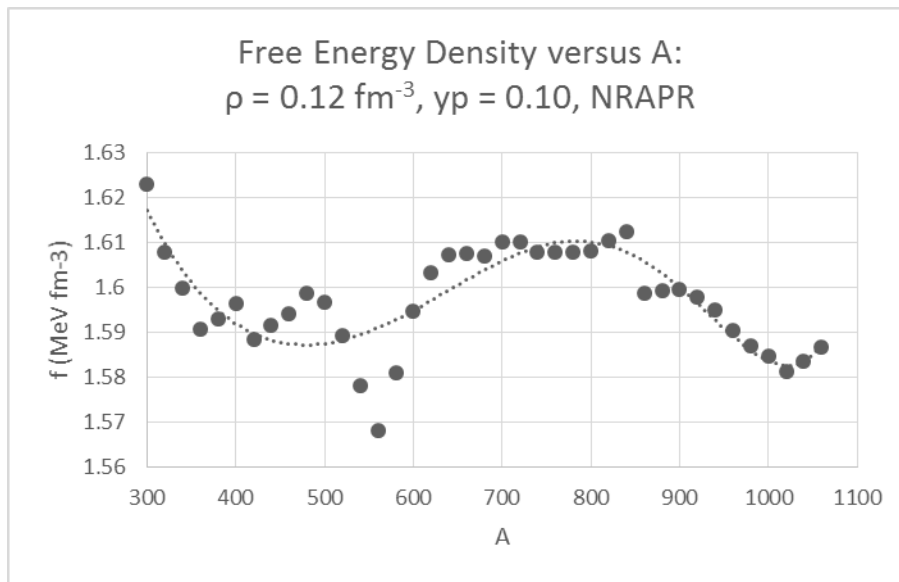
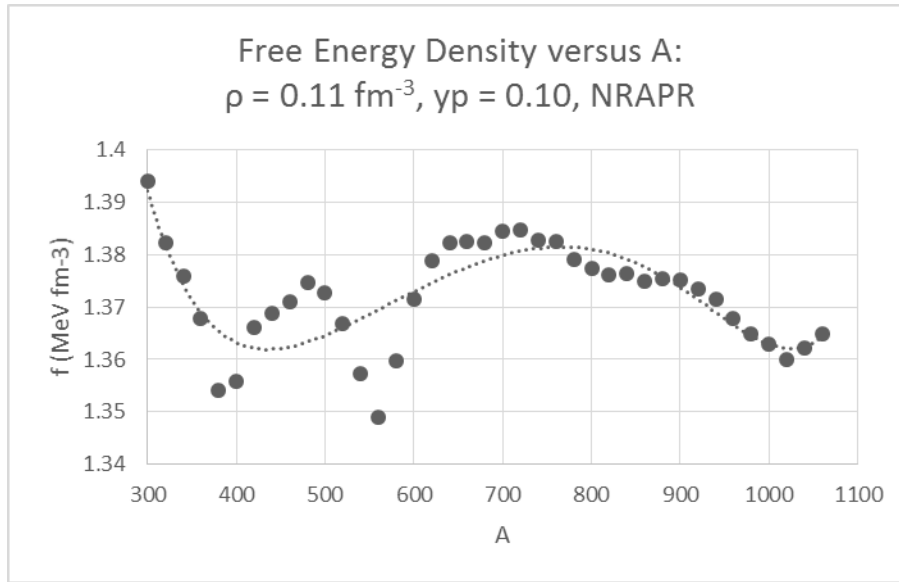


Figure 16: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.11$ (top), 0.12 (bottom) fm^{-3}).

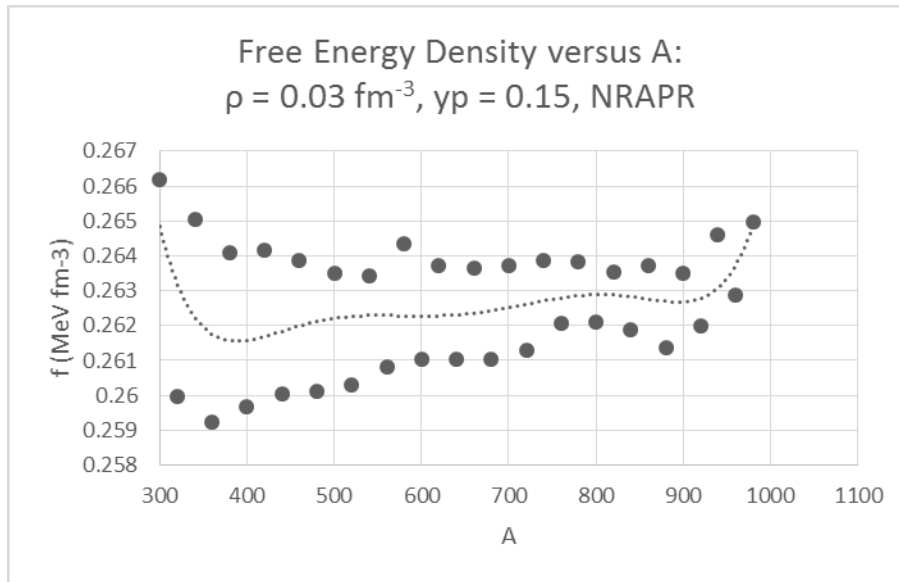
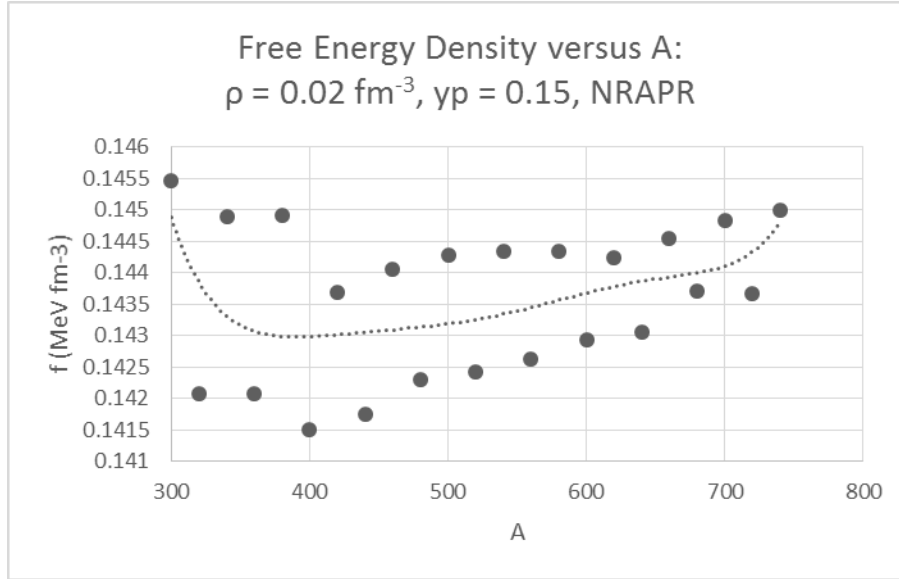


Figure 17: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.02$ (top), 0.03 (bottom) fm^{-3}).

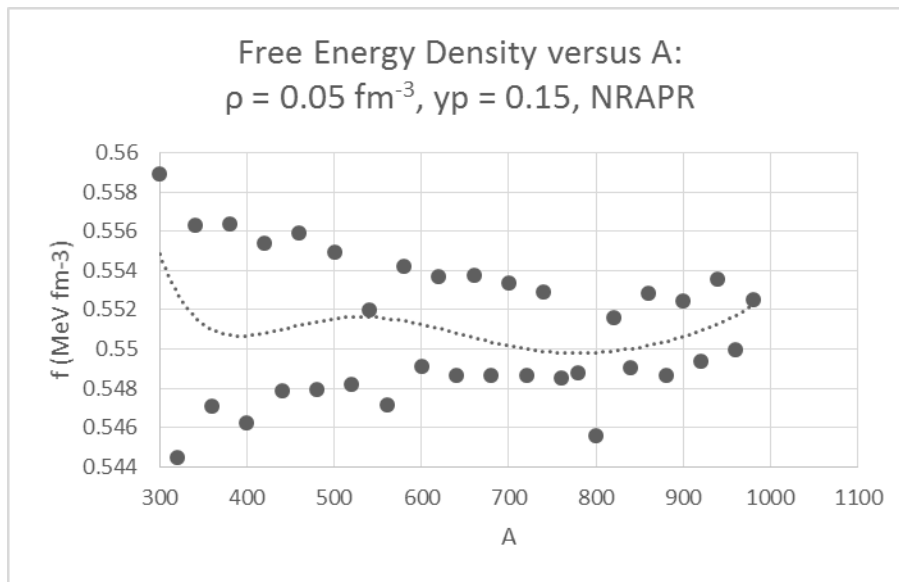
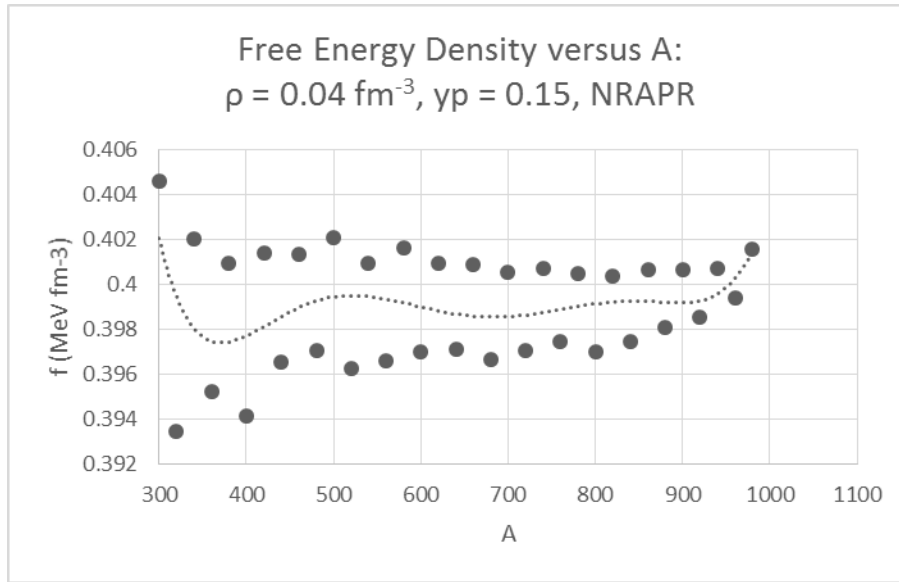


Figure 18: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.04$ (top), 0.05 (bottom) fm^{-3}).

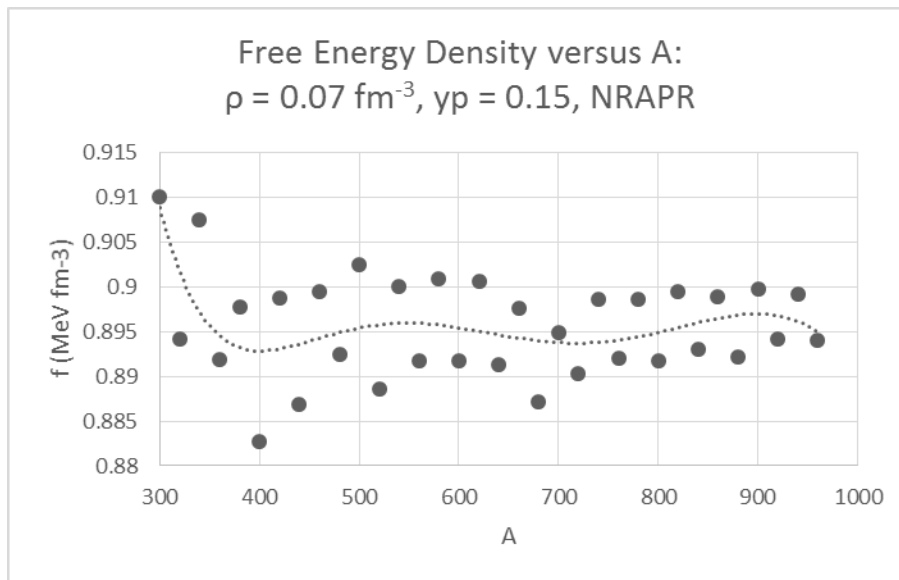
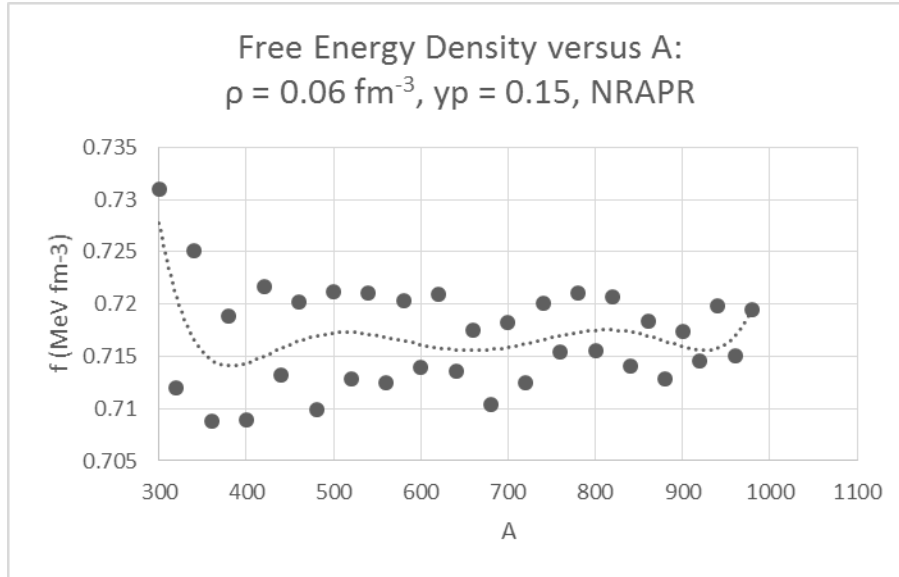


Figure 19: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.06$ (top), 0.07 (bottom) fm^{-3}).

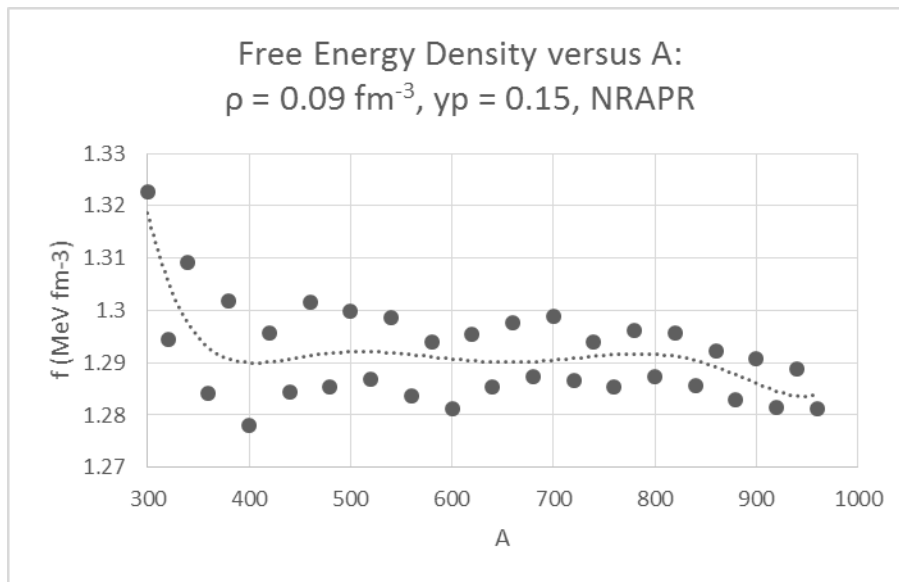
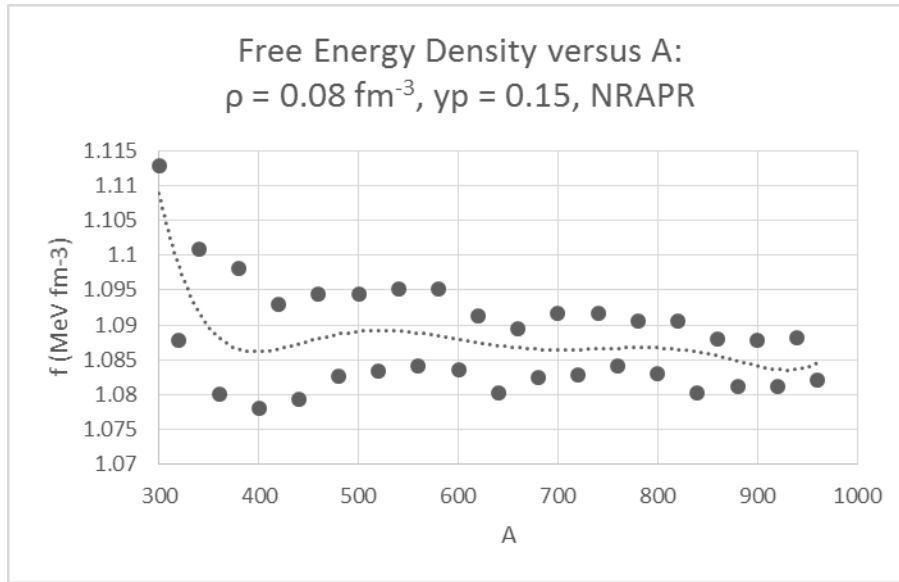


Figure 20: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.08$ (top), 0.09 (bottom) fm^{-3}).

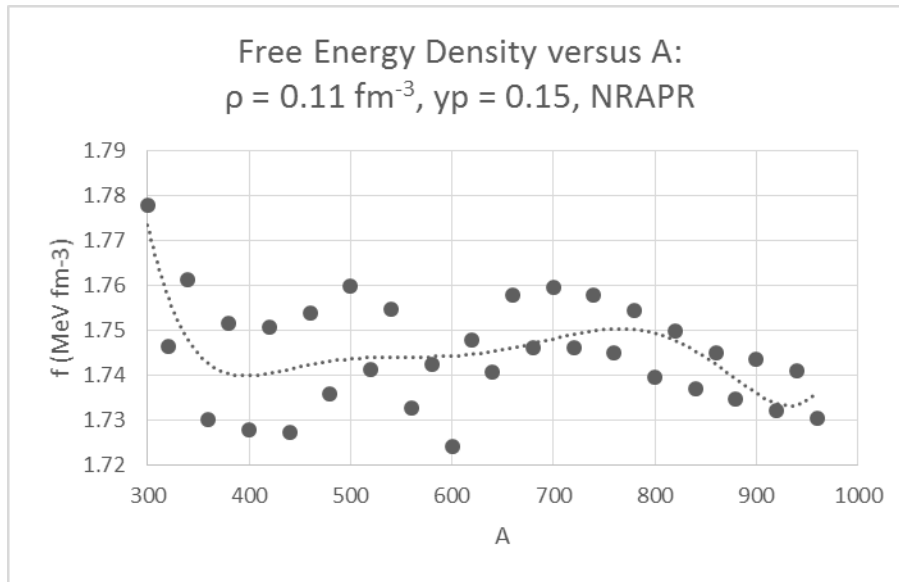
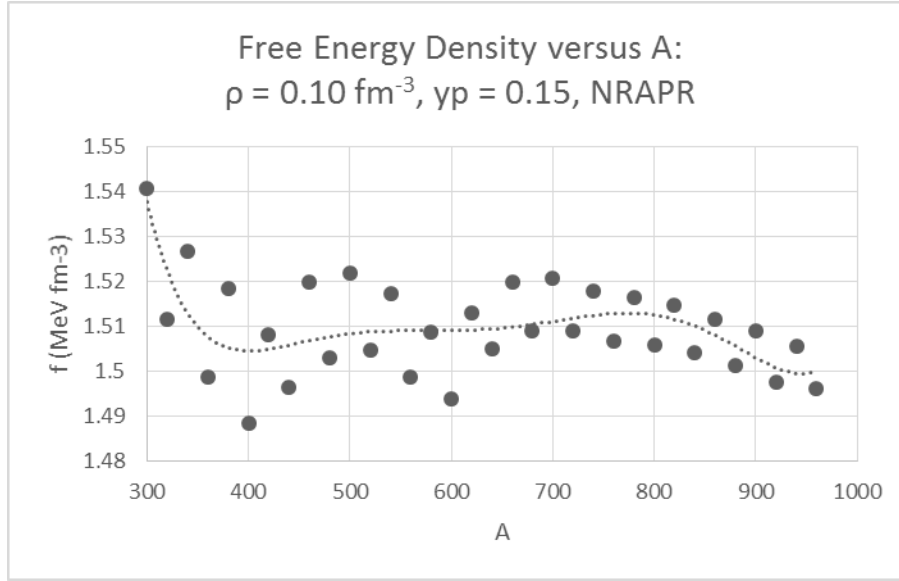


Figure 21: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.10$ (top), 0.11 (bottom) fm^{-3}).

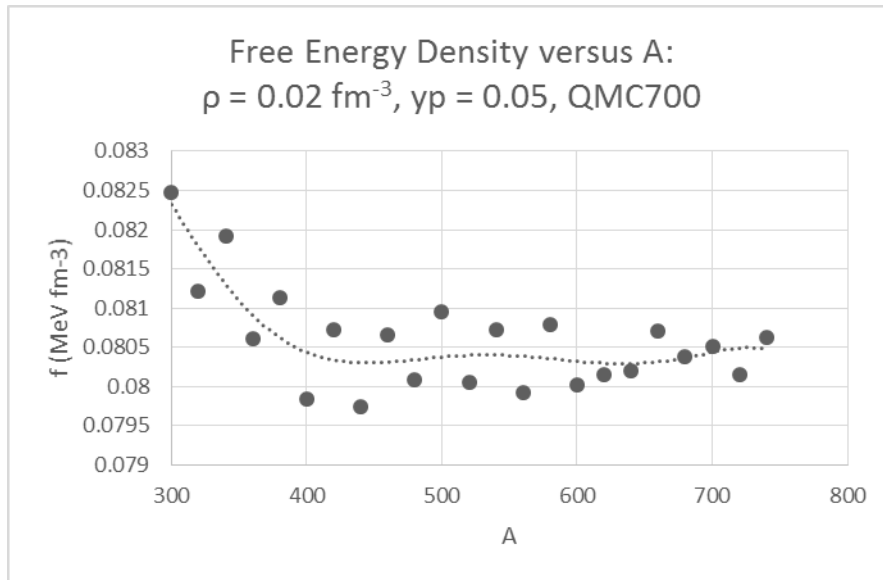
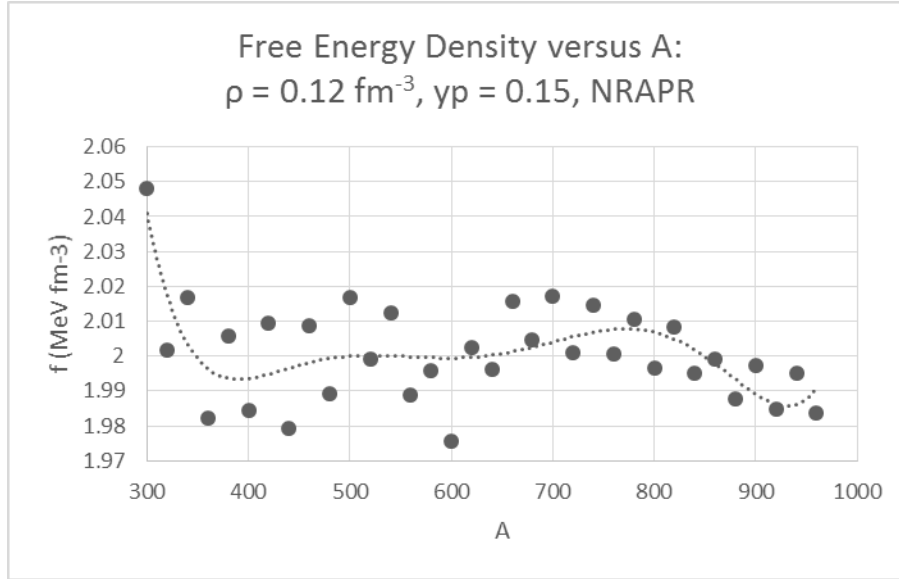


Figure 22: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction NRAPR (top) and QMC700 (bottom), $T = 0 \text{ MeV}$, $y_p = 0.15$ (top), 0.05 (bottom), $\rho = 0.12$ (top), 0.02 (bottom) fm^{-3}).

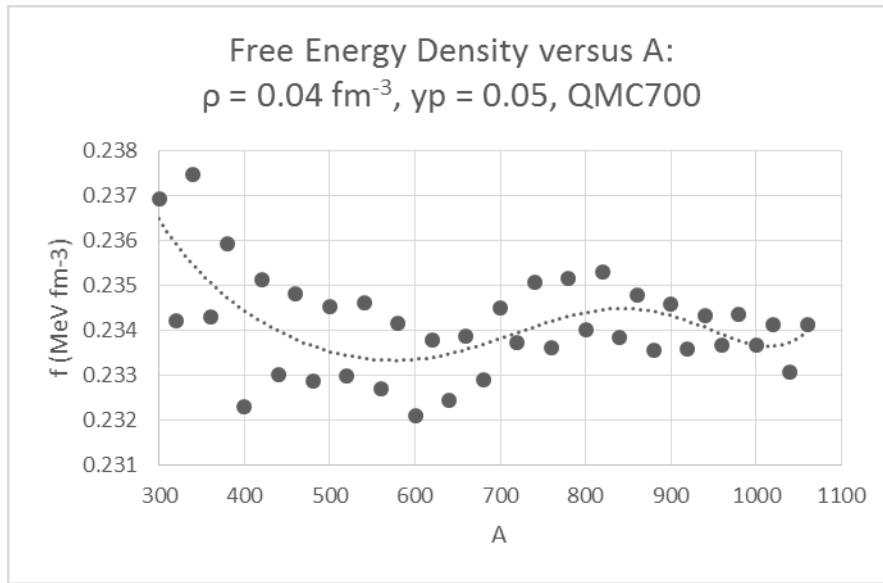
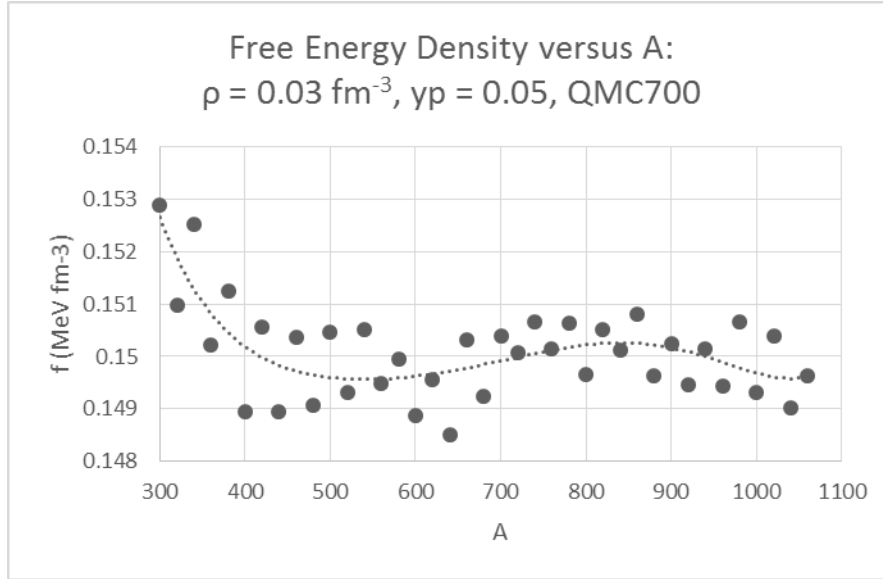


Figure 23: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.03$ (top), 0.04 (bottom) fm^{-3}).

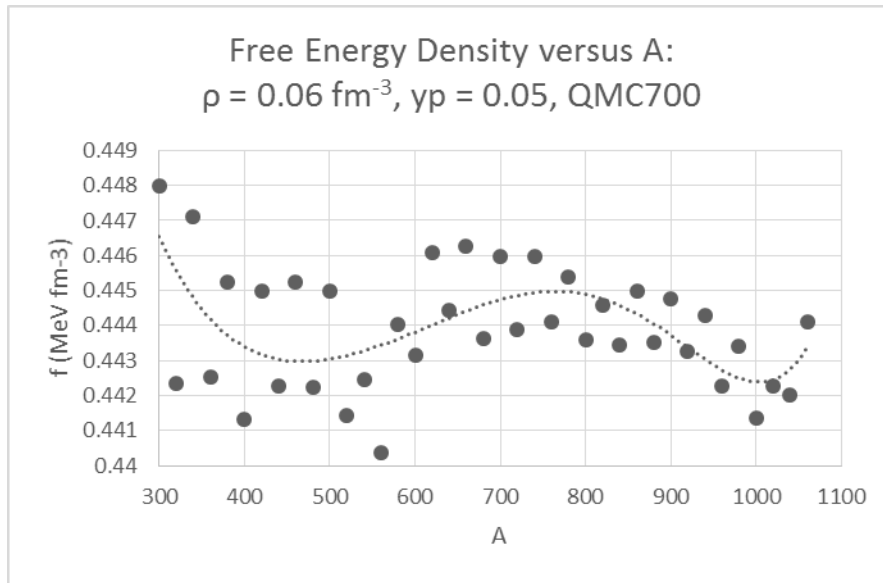
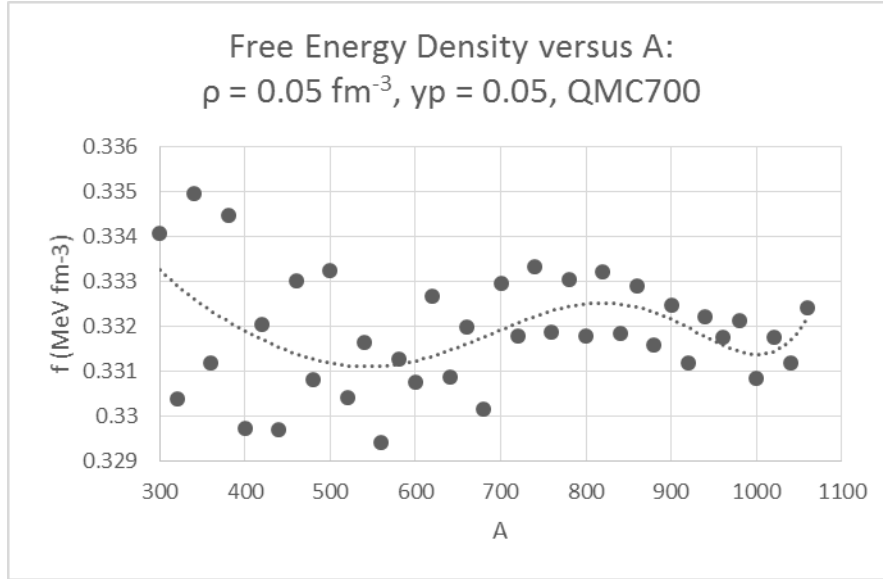


Figure 24: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.05$ (top), 0.06 (bottom) fm^{-3}).

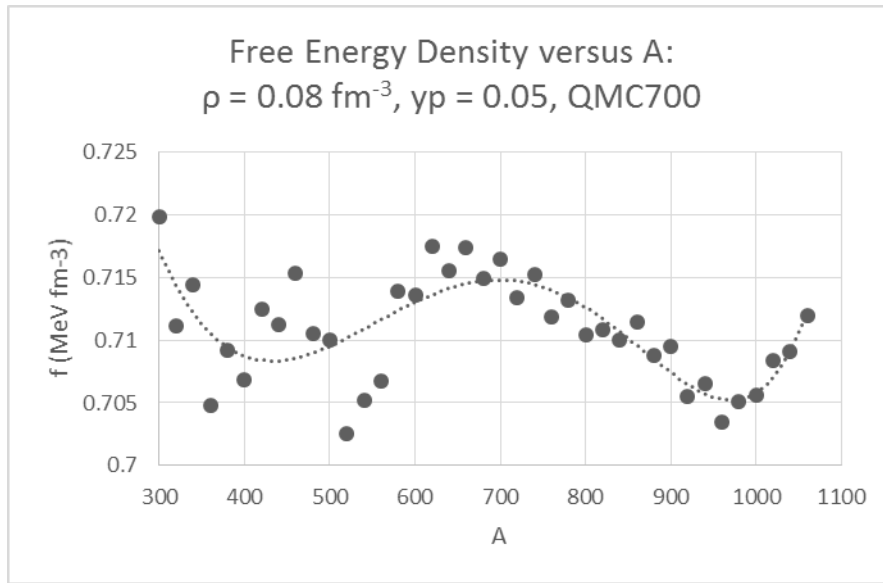
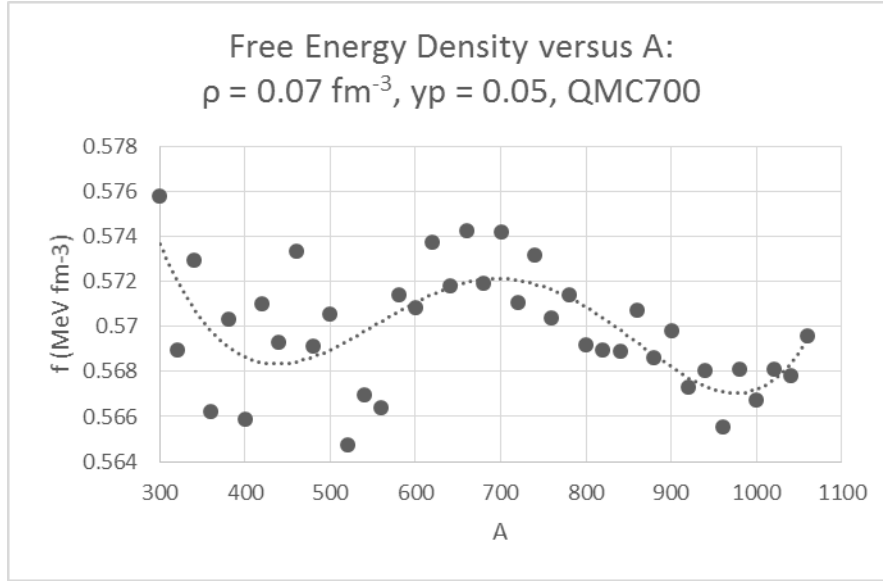


Figure 25: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.07$ (top), 0.08 (bottom) fm^{-3}).

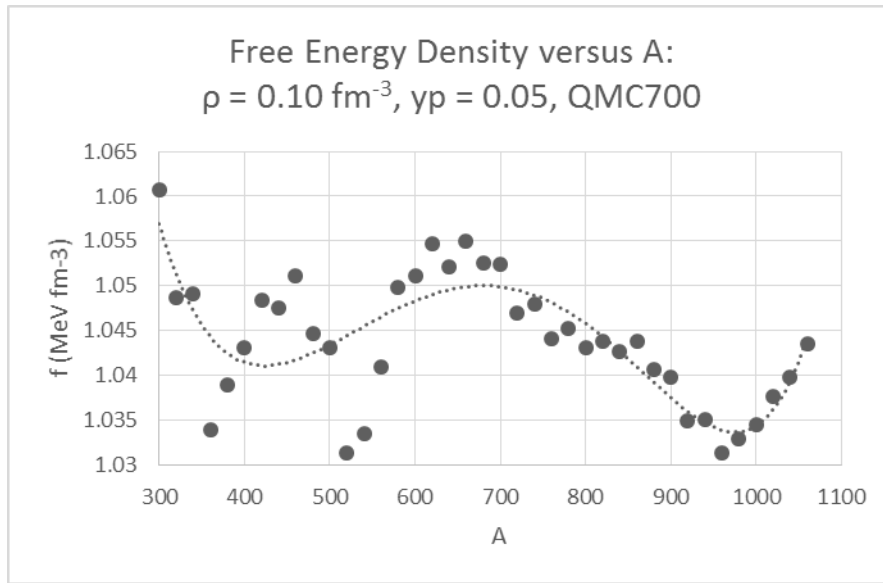
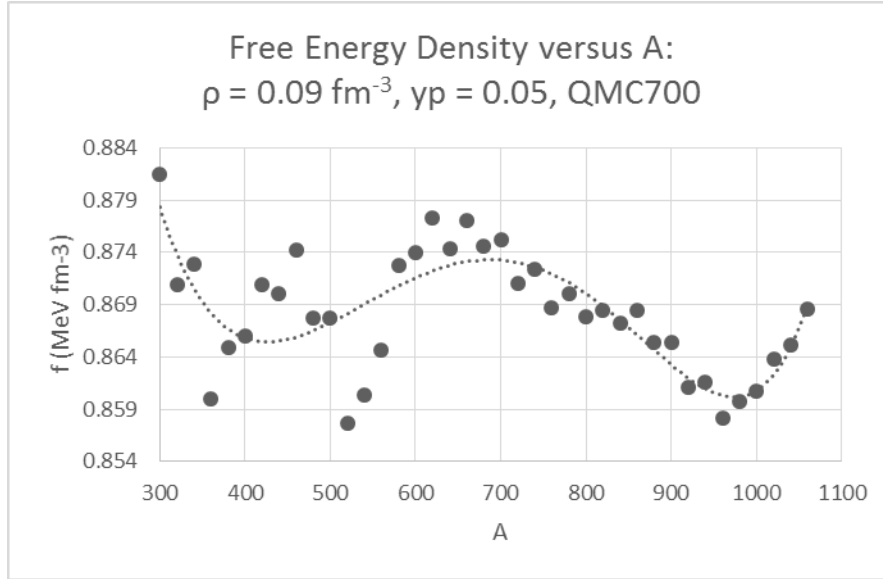


Figure 26: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.09$ (top), 0.10 (bottom) fm^{-3}).

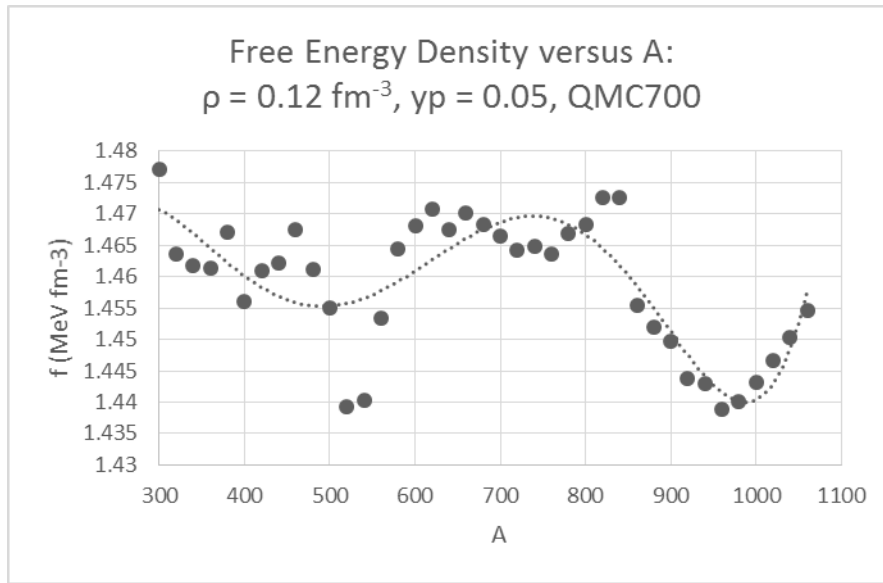
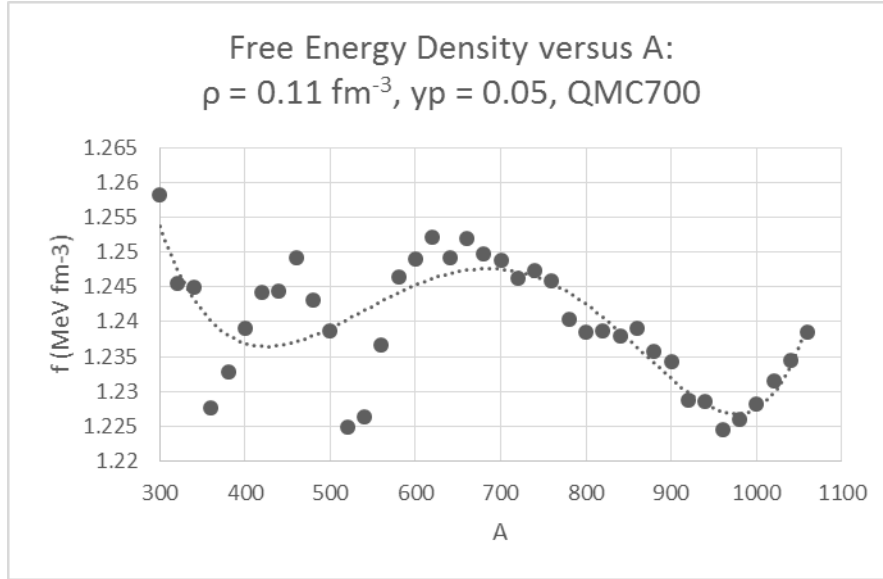


Figure 27: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.11$ (top), 0.12 (bottom) fm^{-3}).

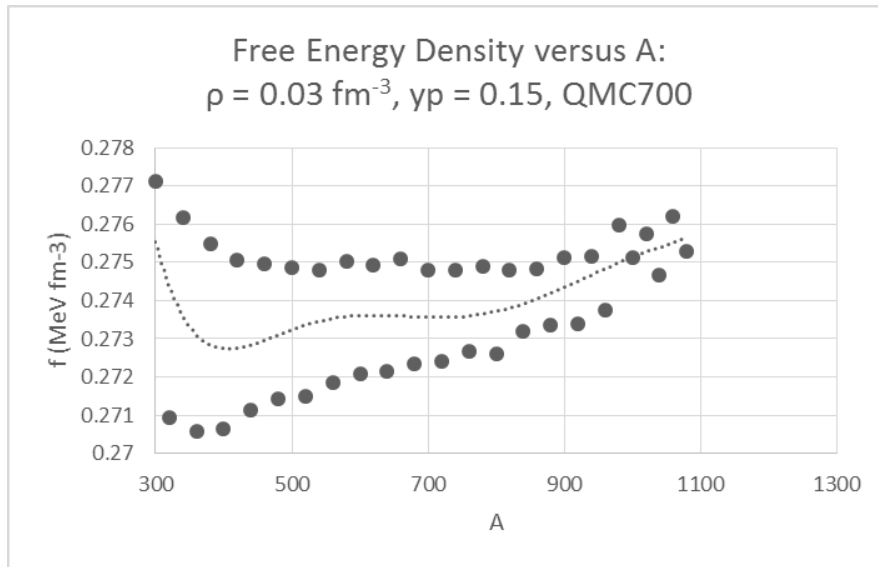
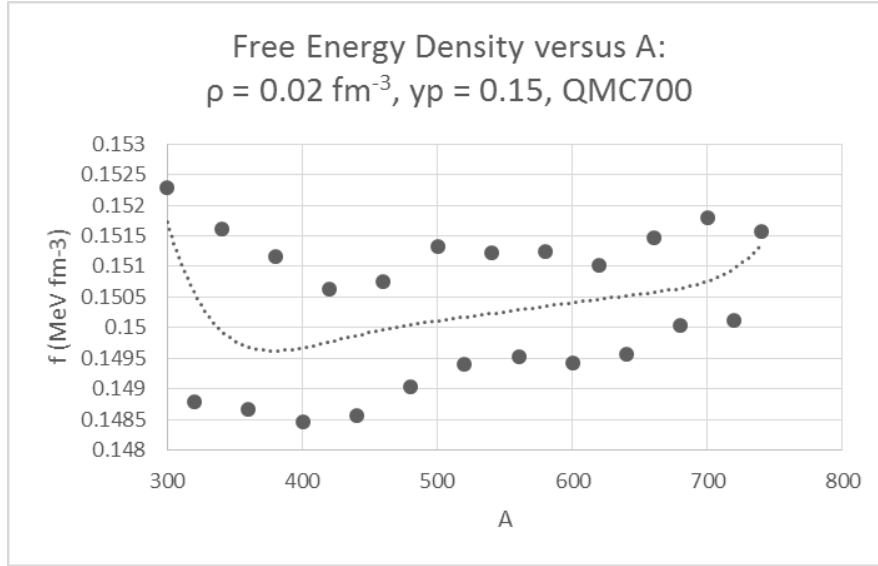


Figure 28: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.02$ (top), 0.03 (bottom) fm^{-3}).

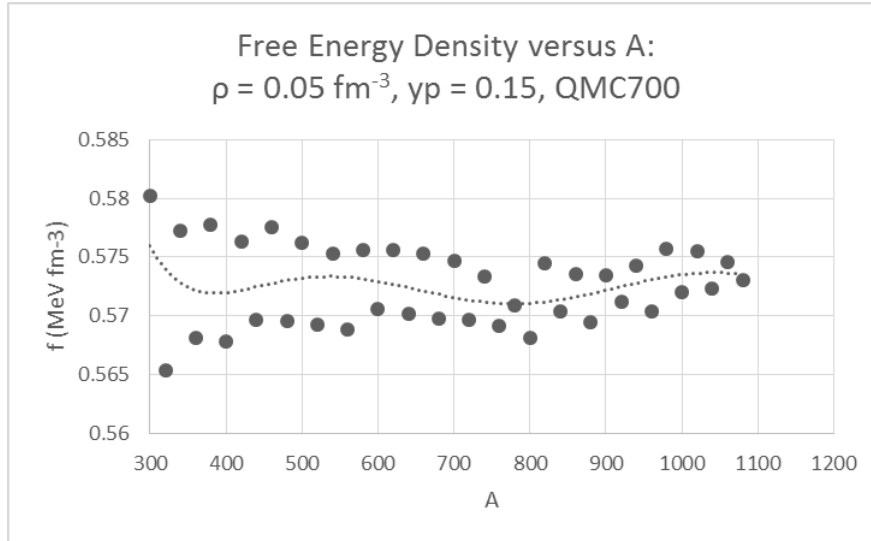
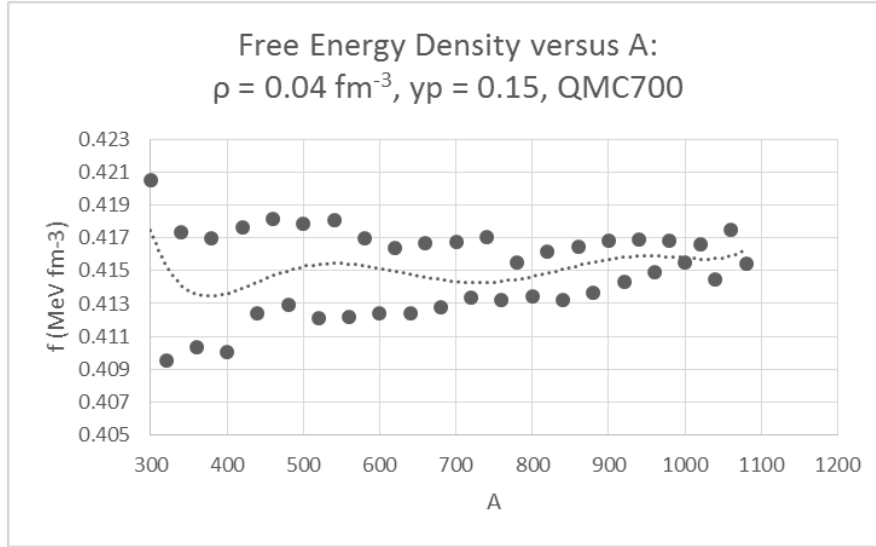


Figure 29: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.04$ (top), 0.05 (bottom) fm^{-3}).

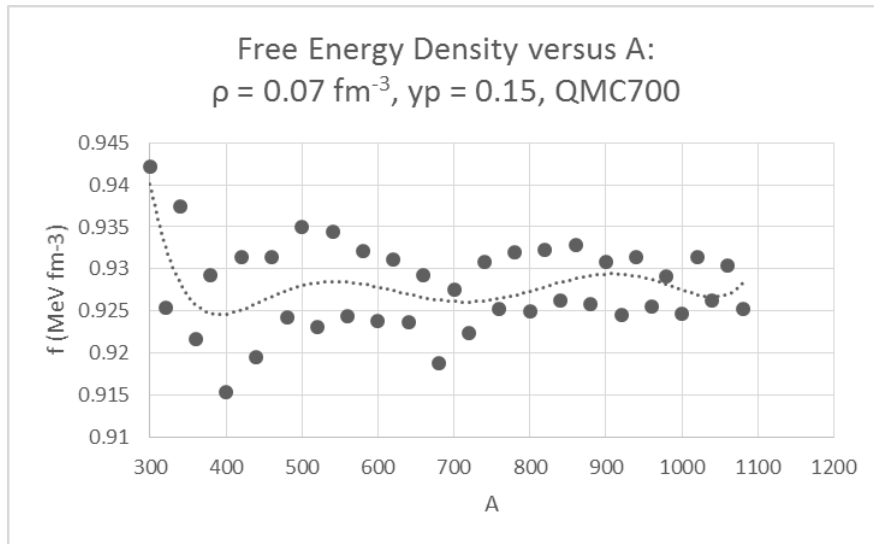
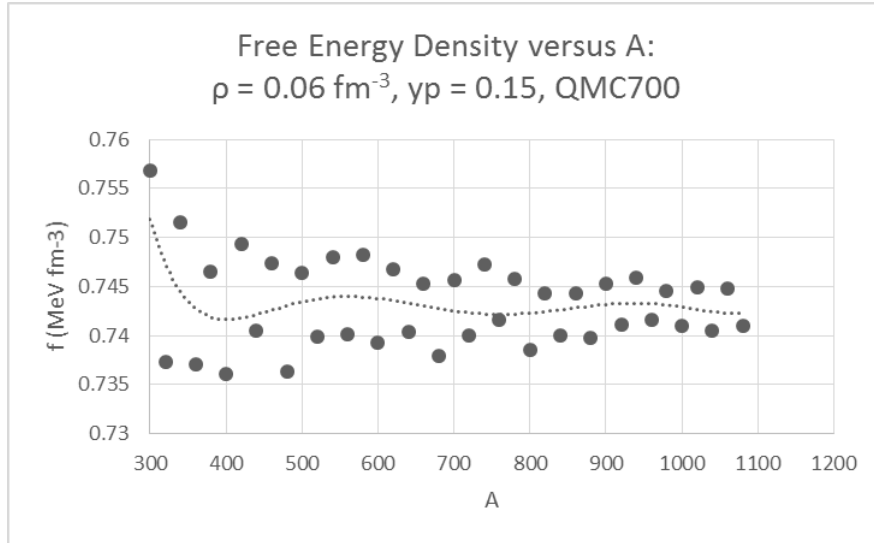


Figure 30: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.06$ (top), 0.07 (bottom) fm^{-3}).

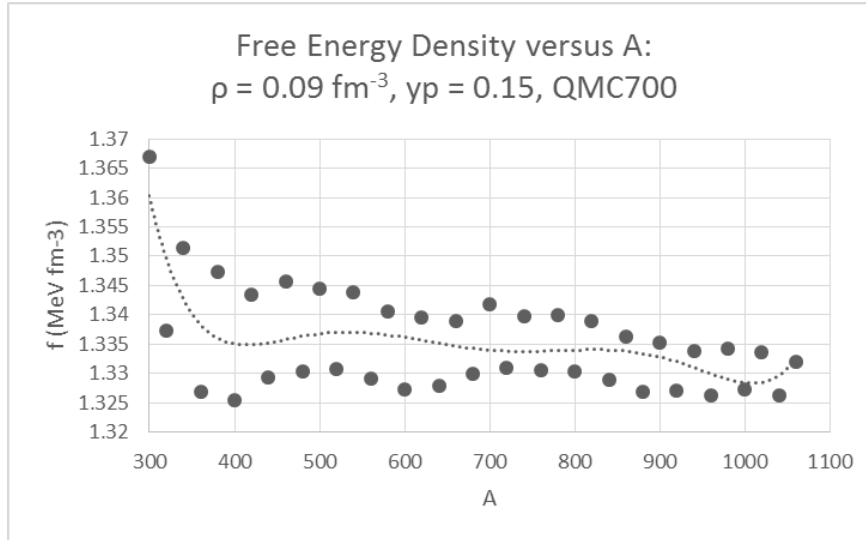
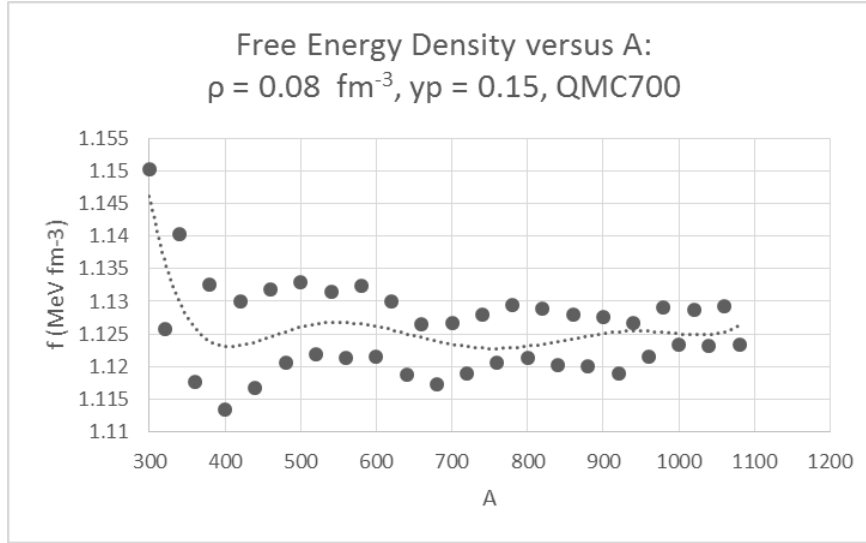


Figure 31: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.08$ (top), 0.09 (bottom) fm^{-3}).

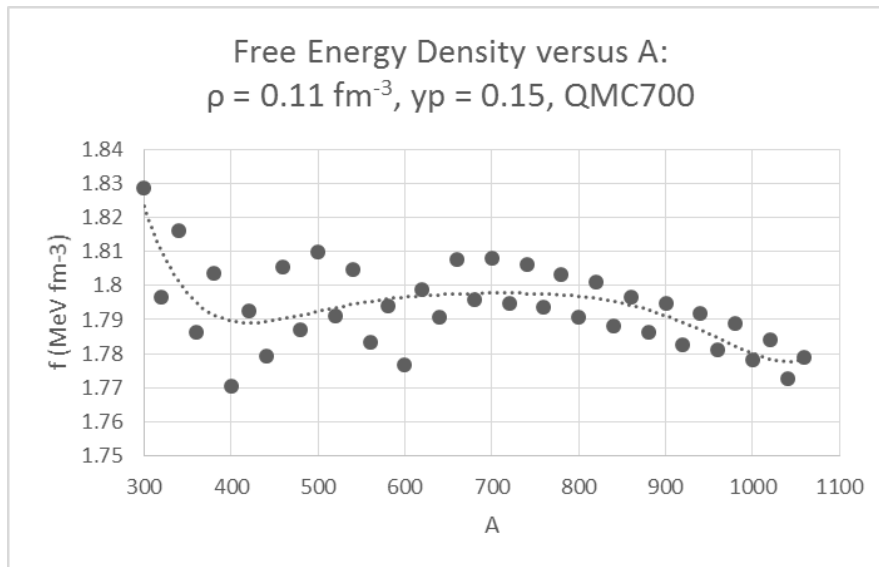
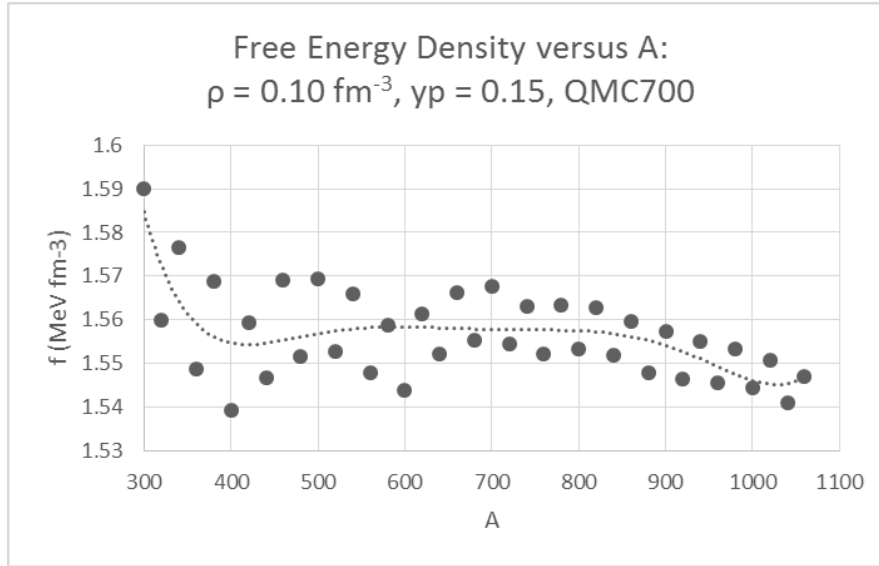


Figure 32: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.10$ (top), 0.11 (bottom) fm^{-3}).

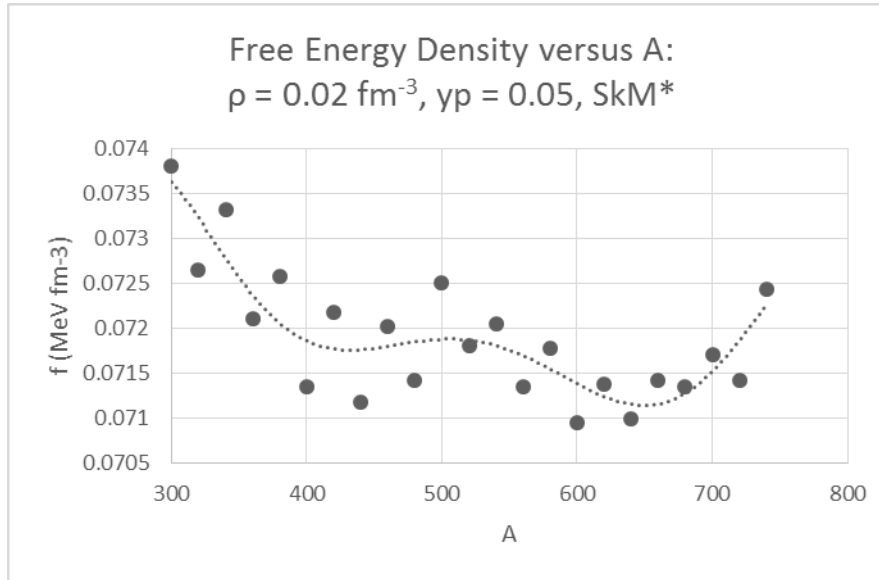
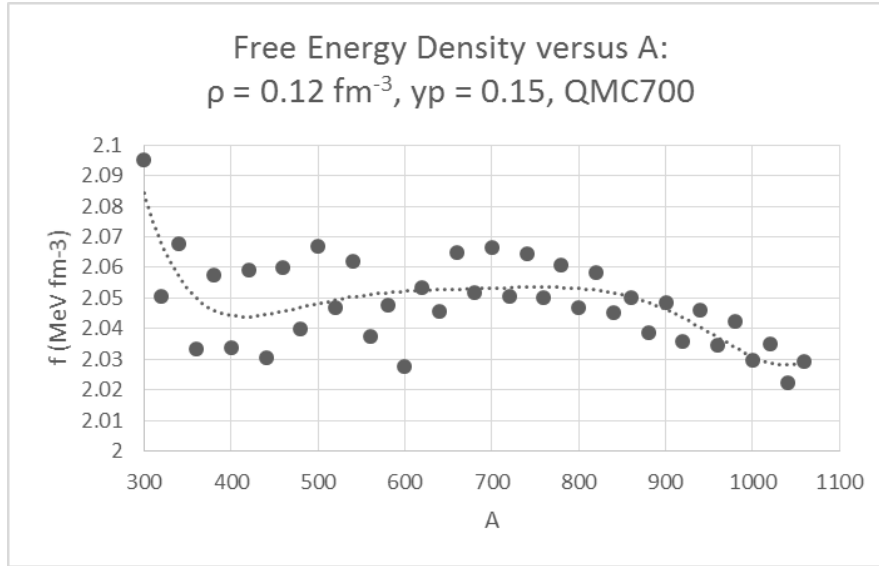


Figure 33: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction QMC700 (top) and SkM* (bottom), $T = 0 \text{ MeV}$, $y_p = 0.15$ (top), 0.05 (bottom), $\rho = 0.12$ (top), 0.02 (bottom) fm^{-3}).

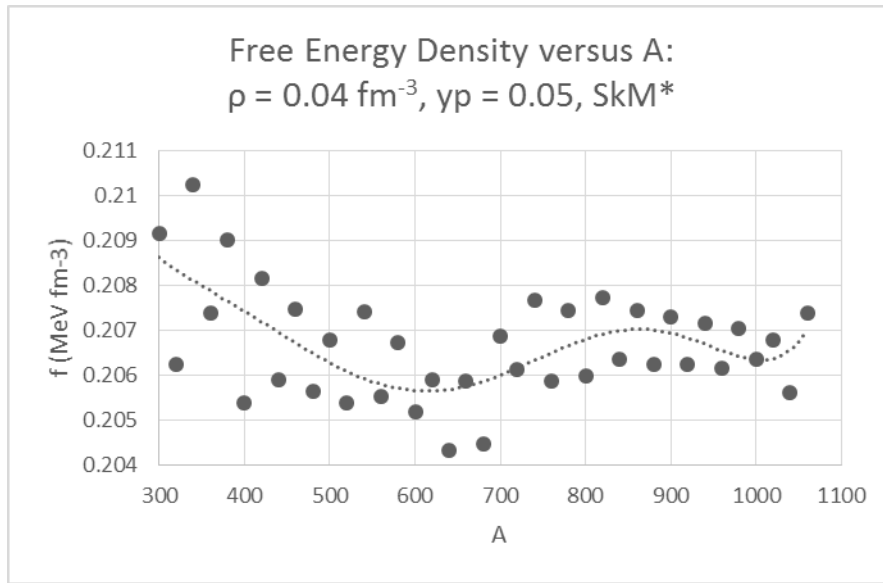
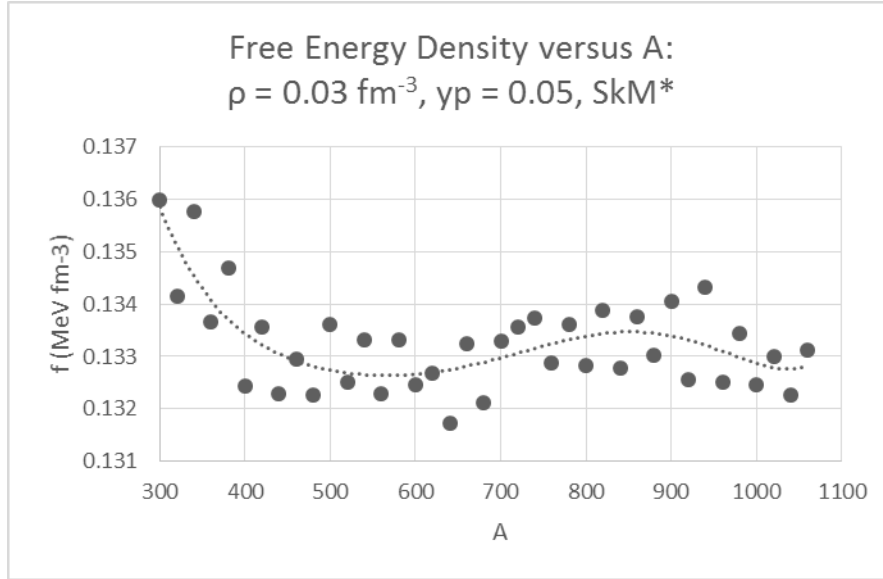


Figure 34: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.03$ (top), 0.04 (bottom) fm^{-3}).

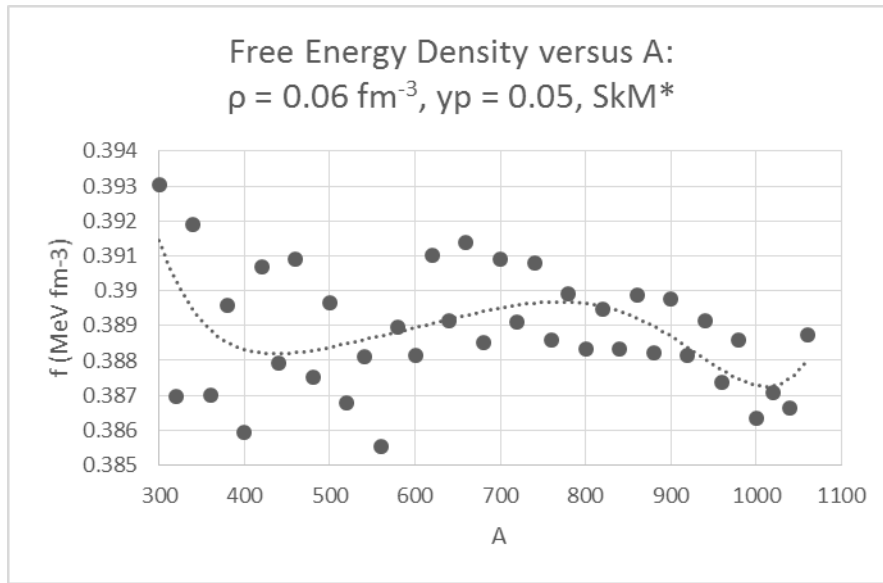
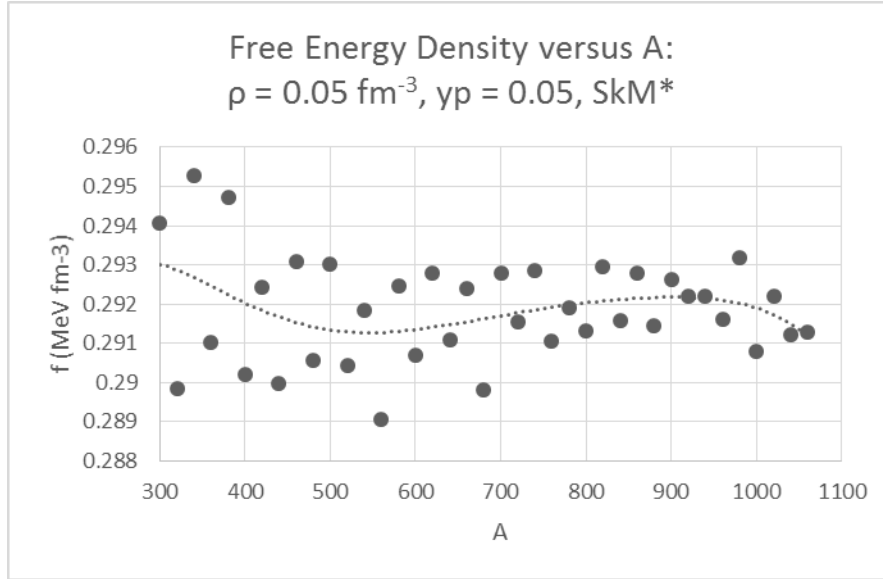


Figure 35: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.05$ (top), 0.06 (bottom) fm^{-3}).

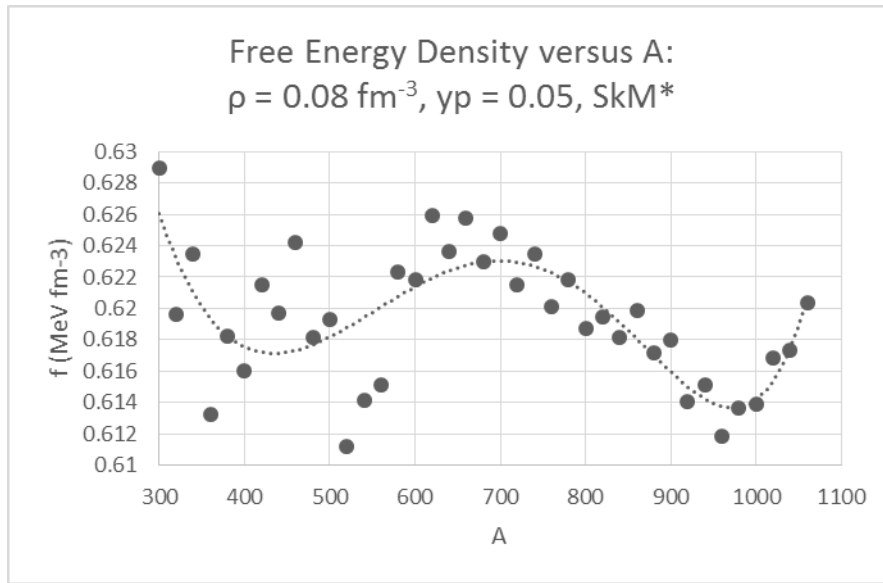
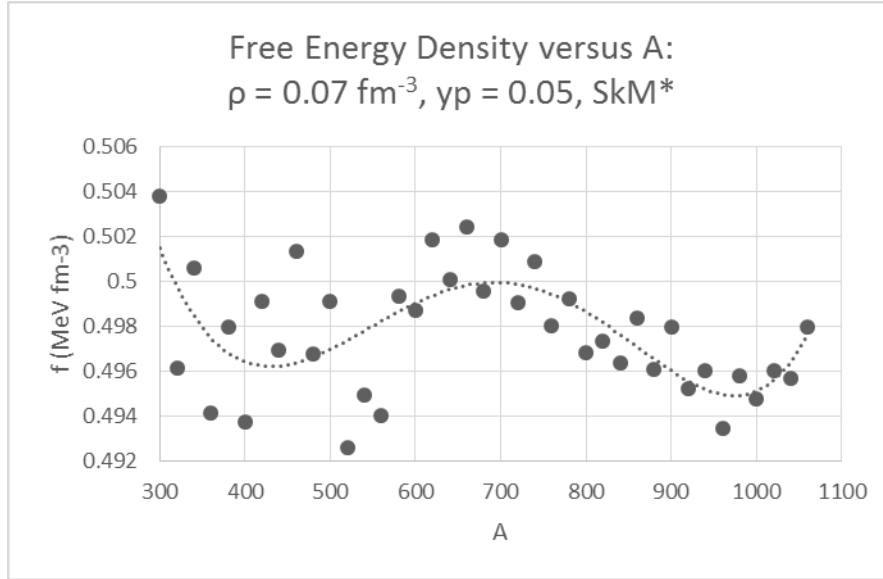


Figure 36: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.07$ (top), 0.08 (bottom) fm^{-3}).

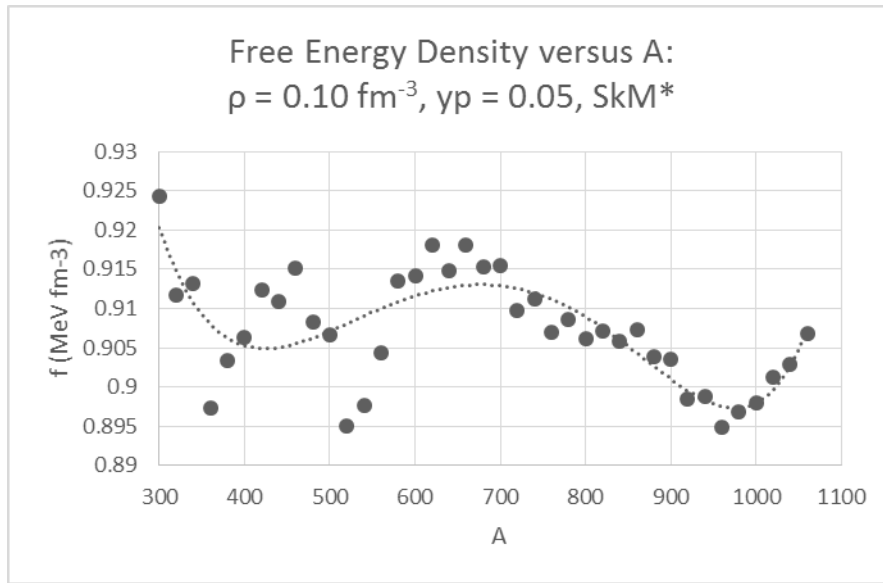
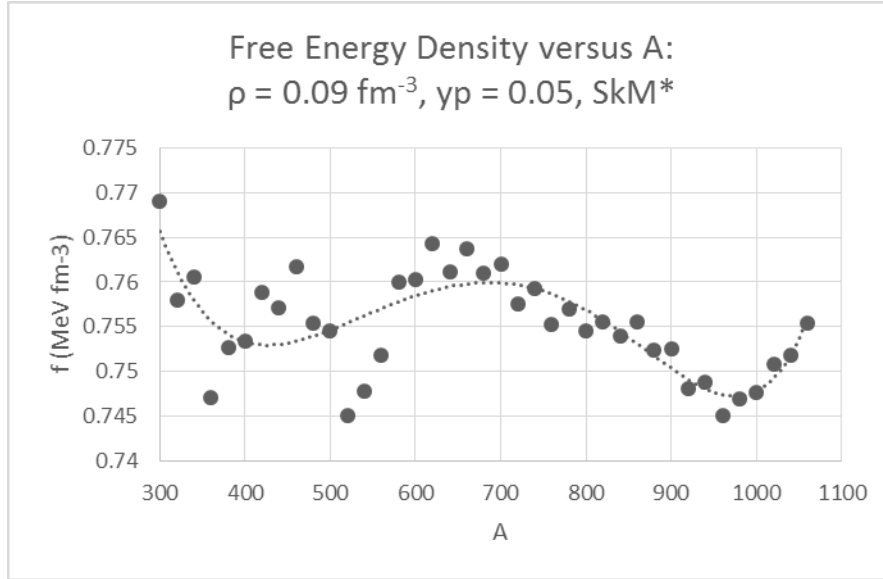


Figure 37: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.09$ (top), 0.10 (bottom) fm^{-3}).

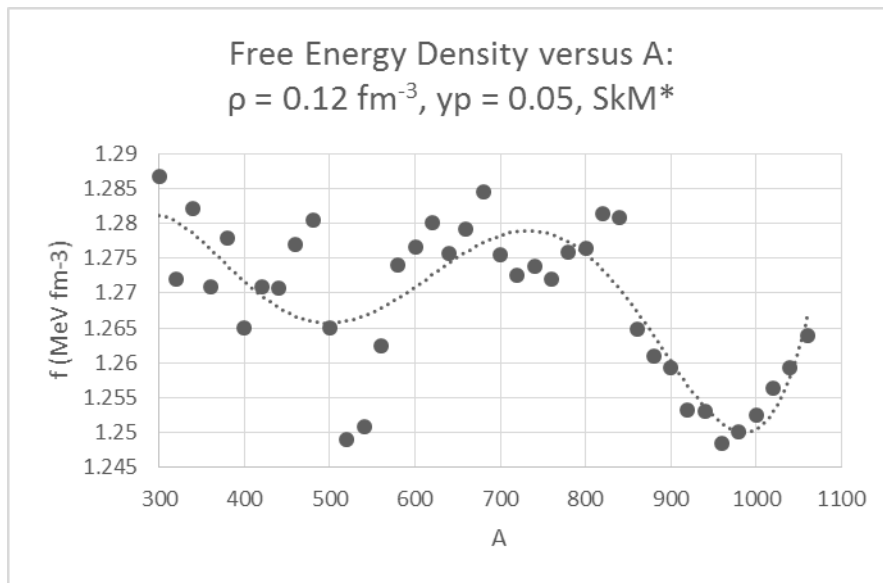
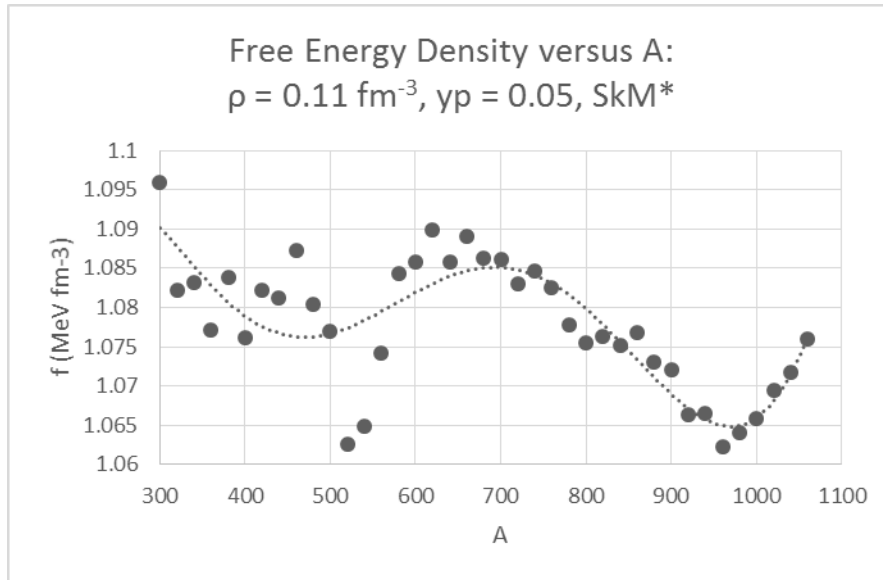


Figure 38: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.05$, $\rho = 0.11$ (top), 0.12 (bottom) fm^{-3}).

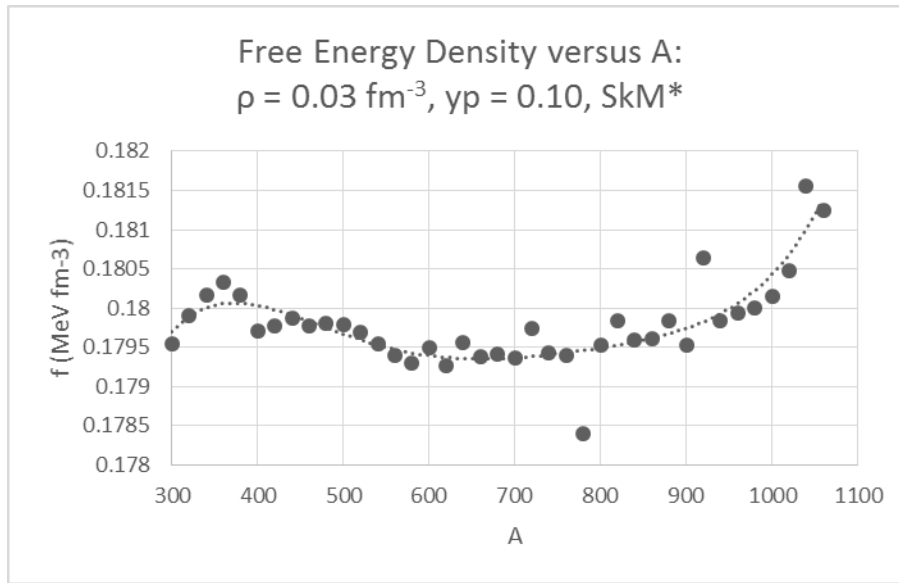
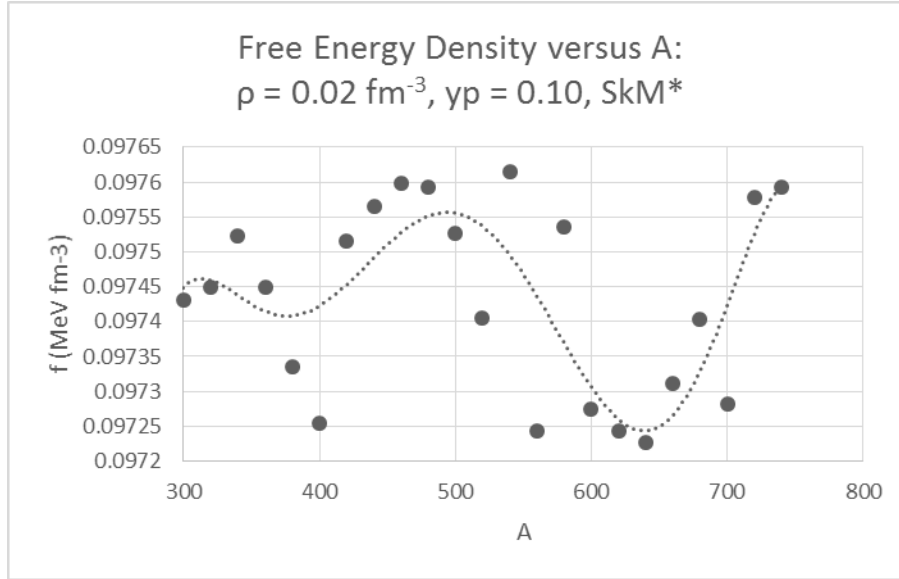


Figure 39: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.02$ (top), 0.03 (bottom) fm^{-3}).

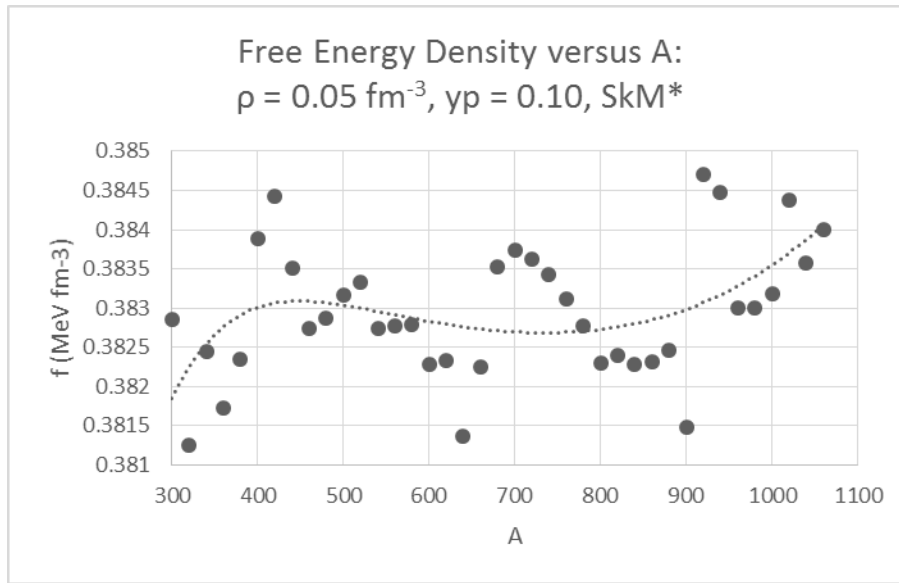
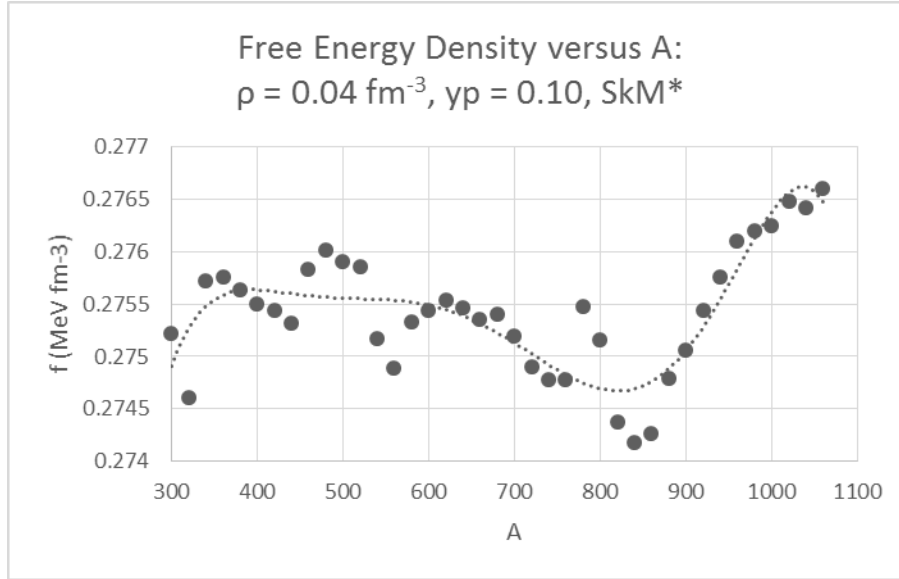


Figure 40: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.04$ (top), 0.05 (bottom) fm^{-3}).

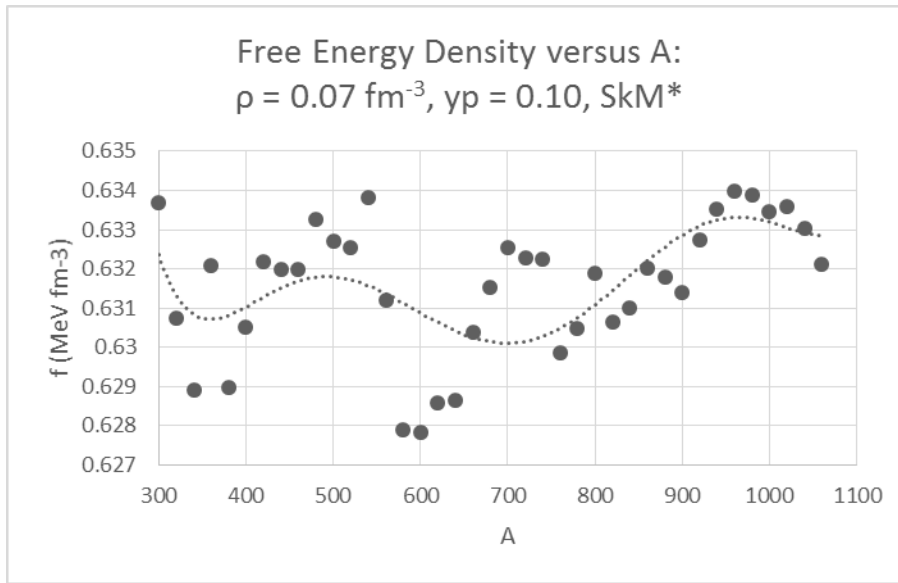
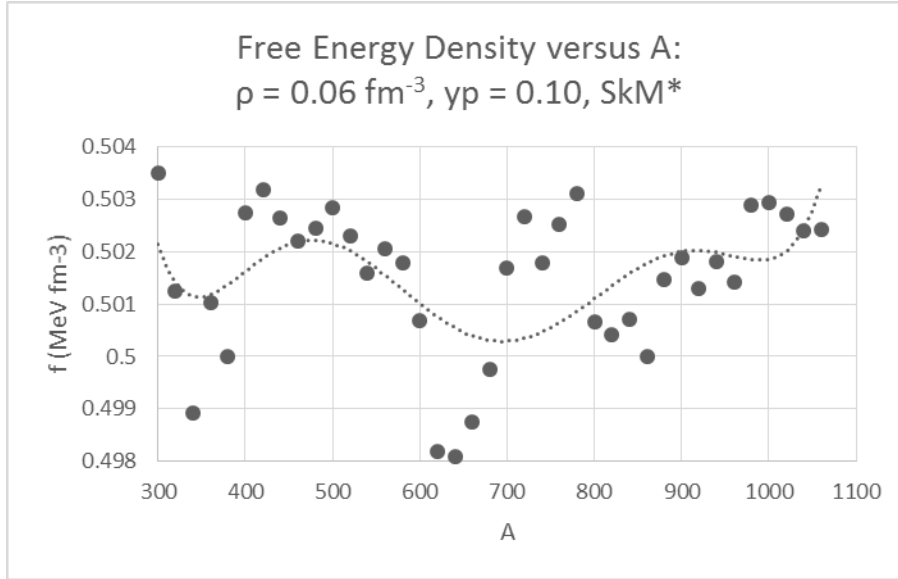


Figure 41: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.06$ (top), 0.07 (bottom) fm^{-3}).

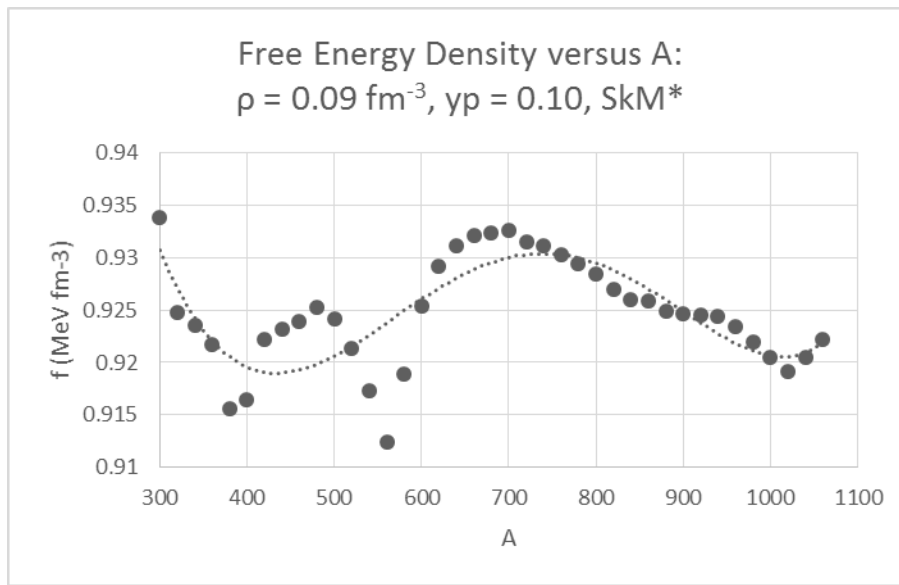
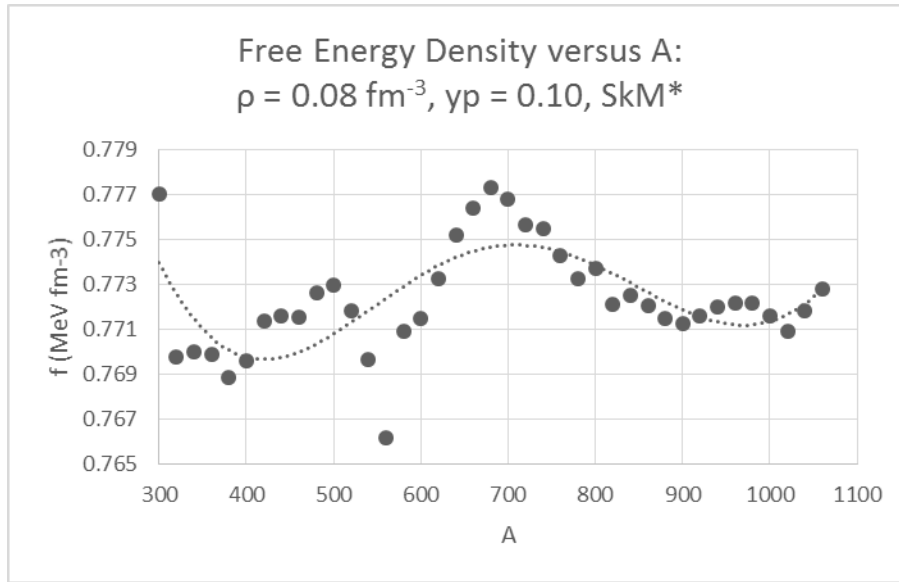


Figure 42: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.08$ (top), 0.09 (bottom) fm^{-3}).

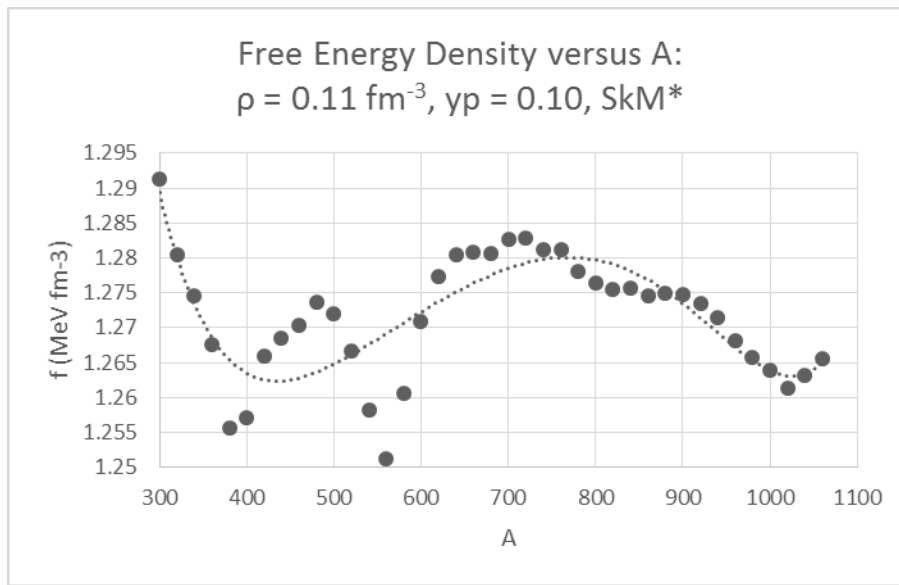
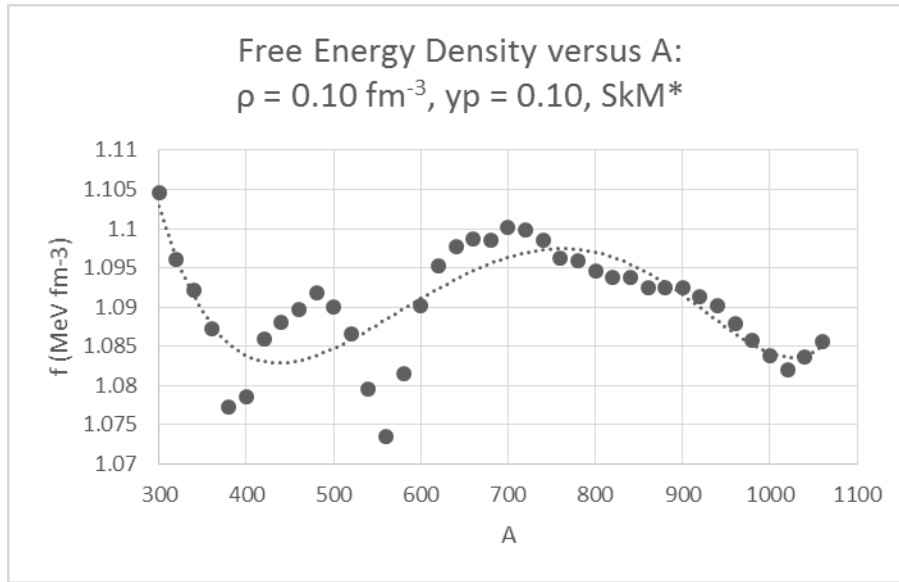


Figure 43: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.10$ (top), 0.11 (bottom) fm^{-3}).

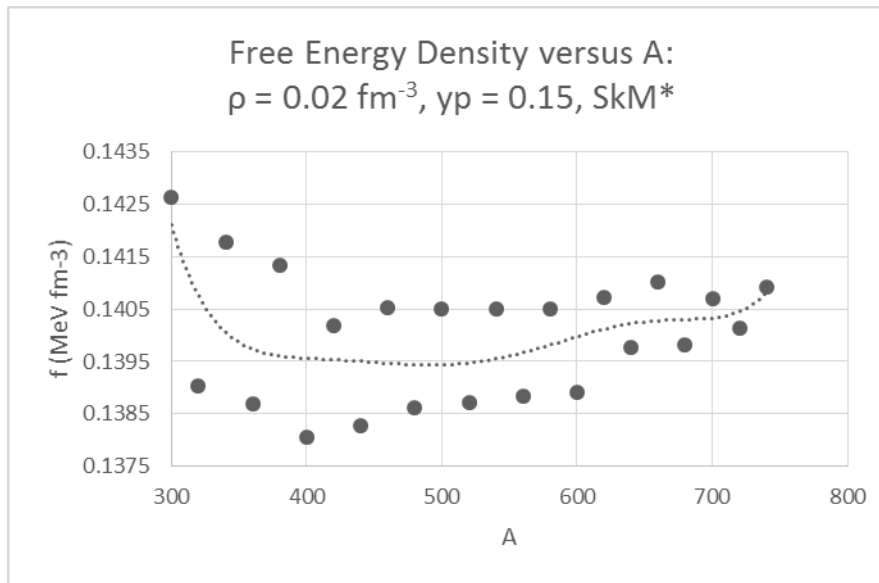
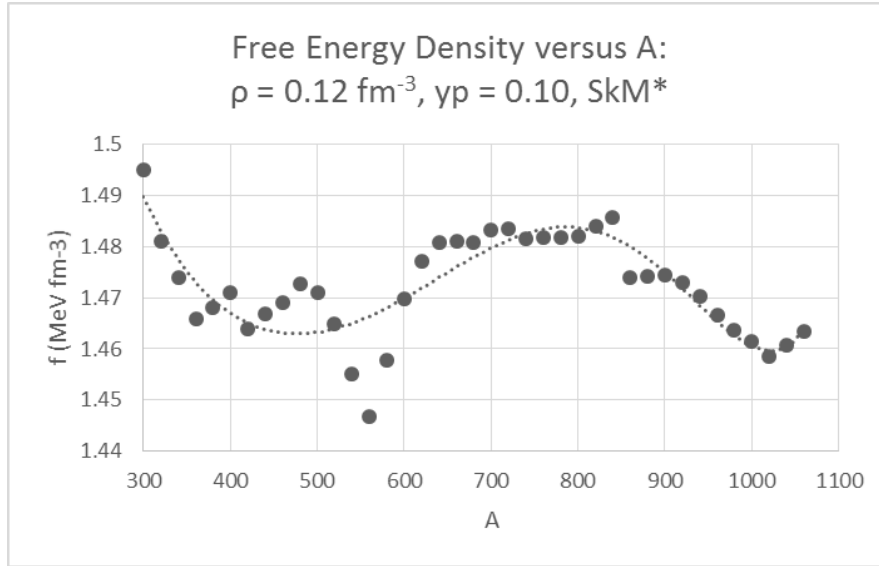


Figure 44: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.10$ (top), 0.15 (bottom), $\rho = 0.12$ (top), 0.02 (bottom) fm^{-3}).

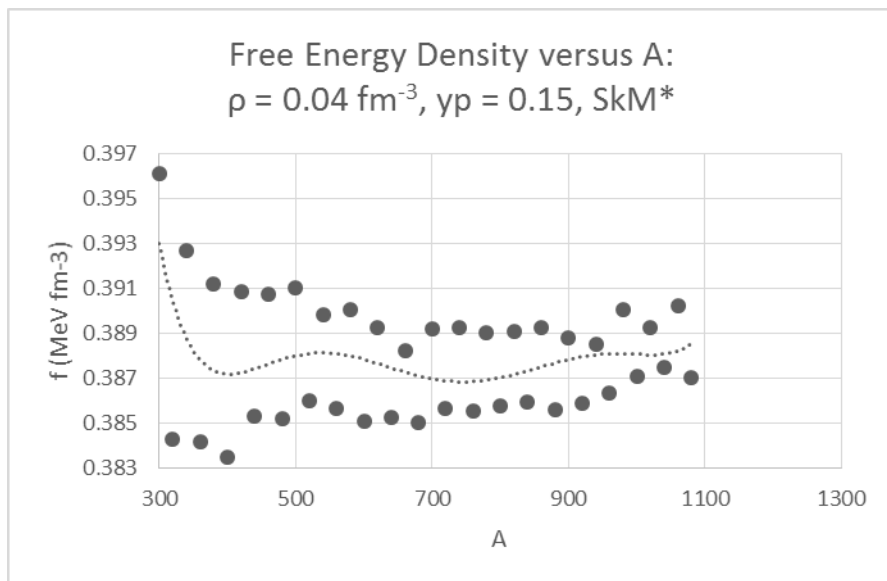
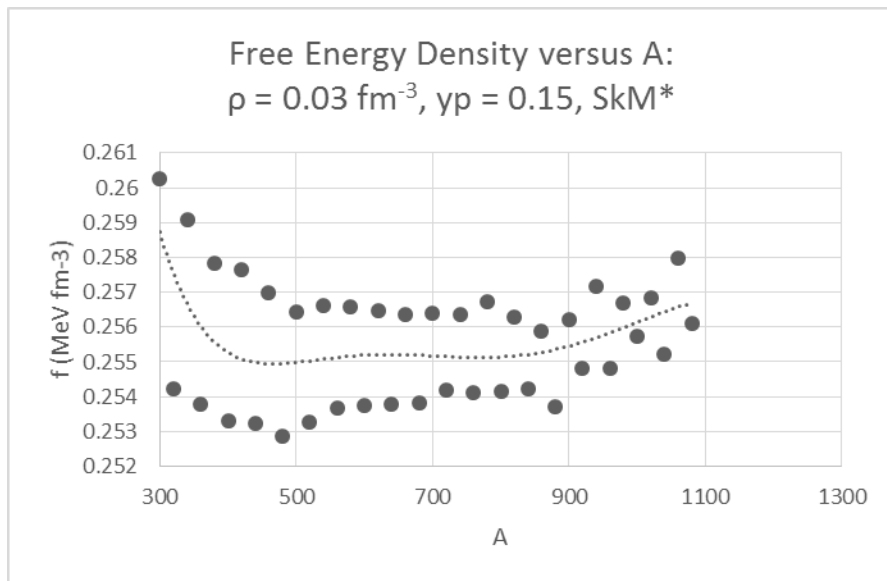


Figure 45: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.03$ (top), 0.04 (bottom) fm^{-3}).

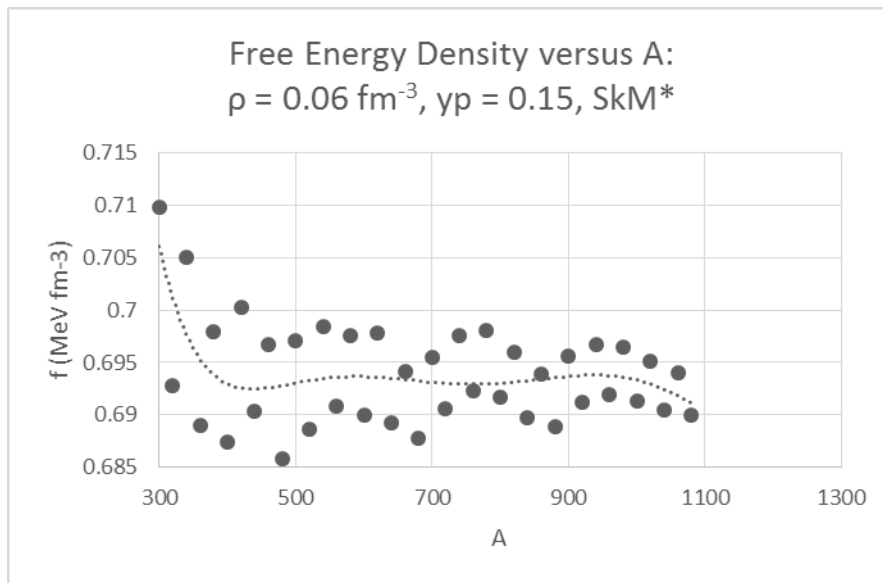
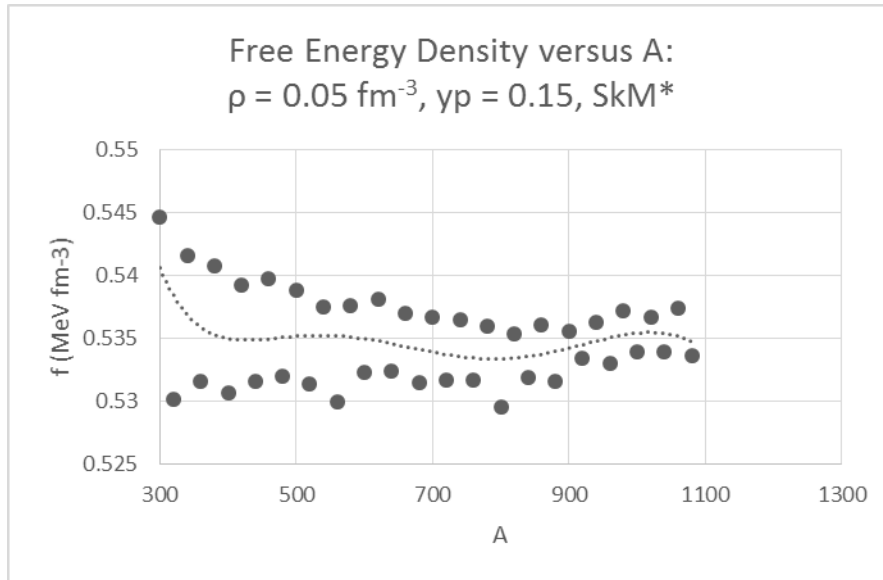


Figure 46: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.05$ (top), 0.06 (bottom) fm^{-3}).

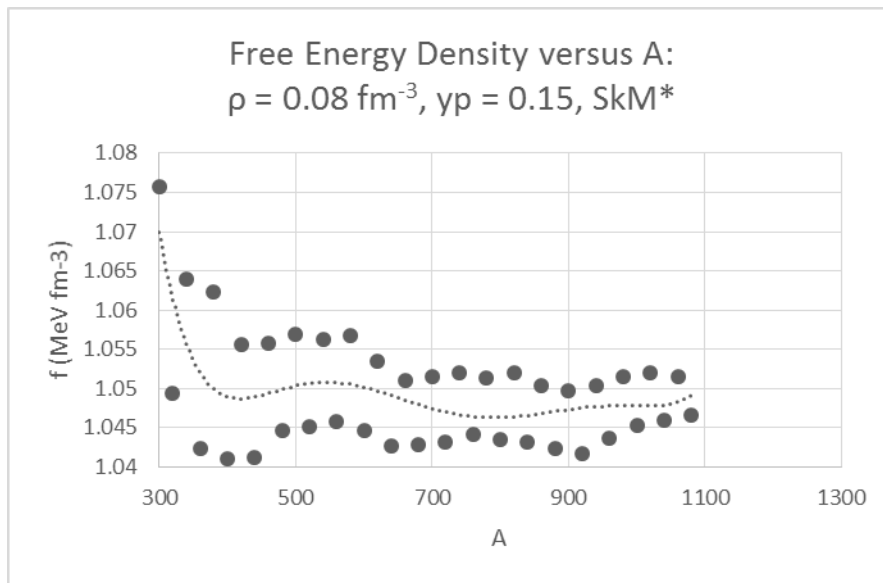
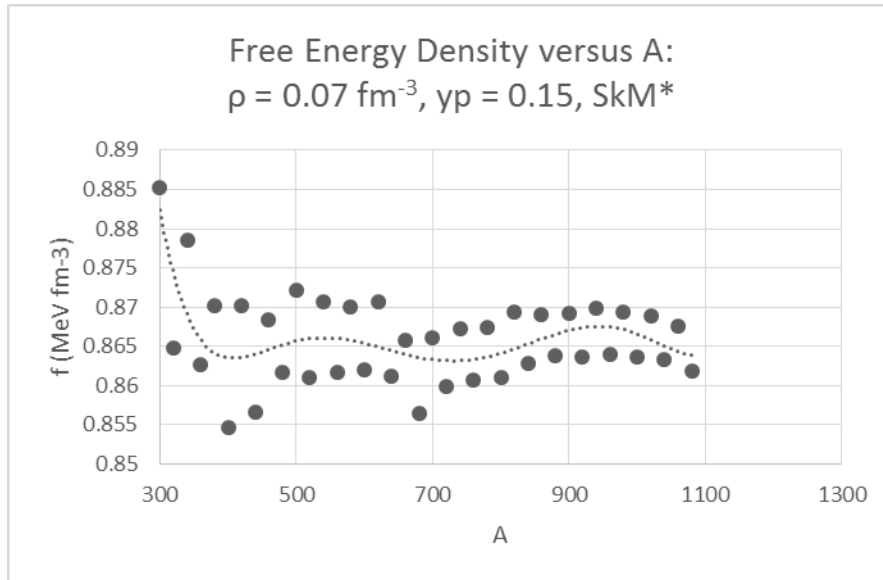


Figure 47: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.07$ (top), 0.08 (bottom) fm^{-3}).

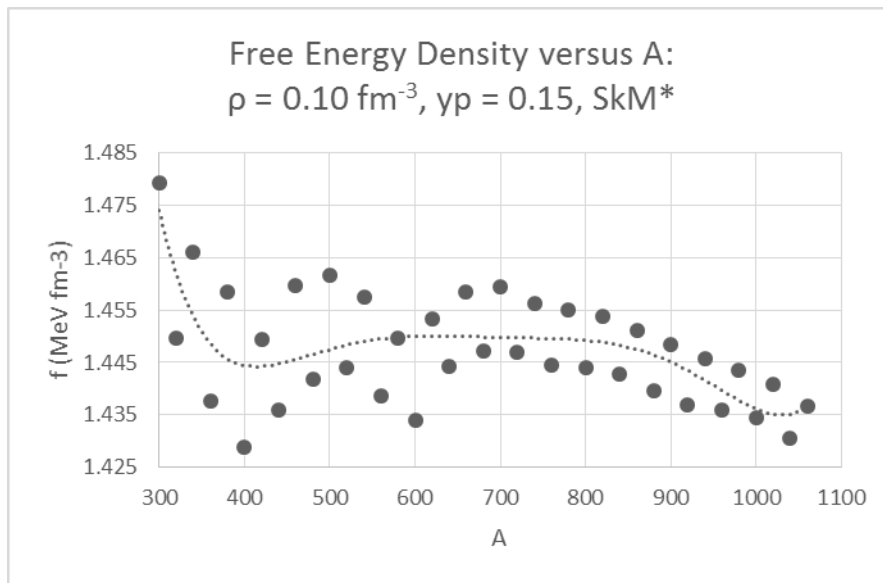
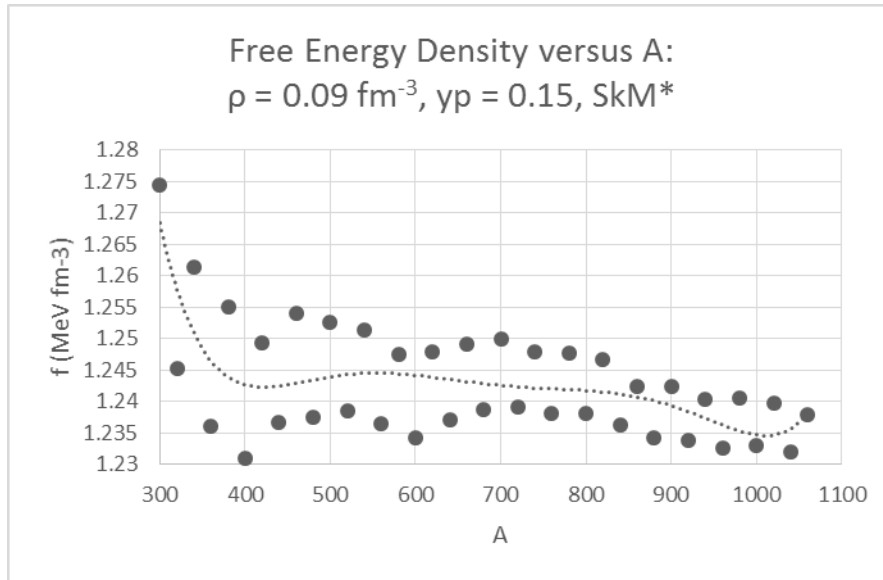


Figure 48: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.09$ (top), 0.10 (bottom) fm^{-3}).

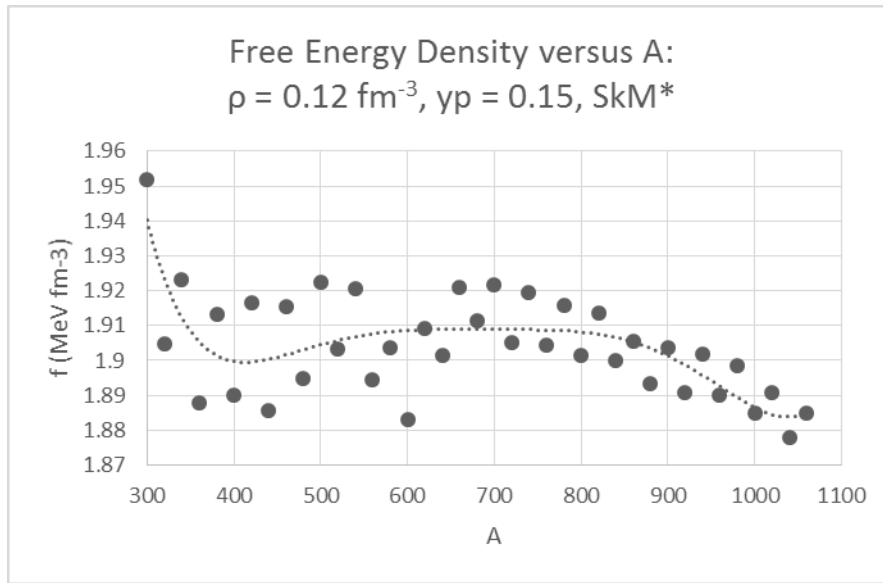
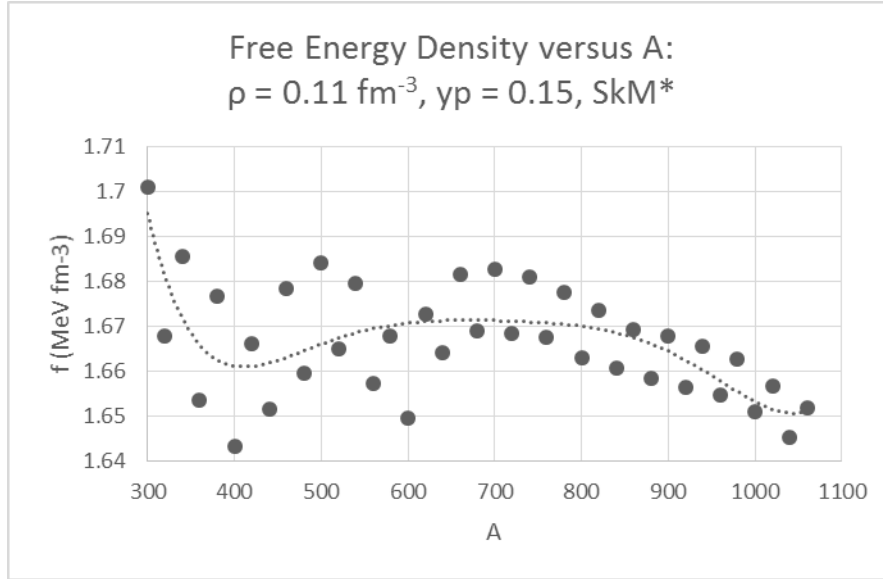


Figure 49: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SkM*, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.11$ (top), 0.12 (bottom) fm^{-3}).

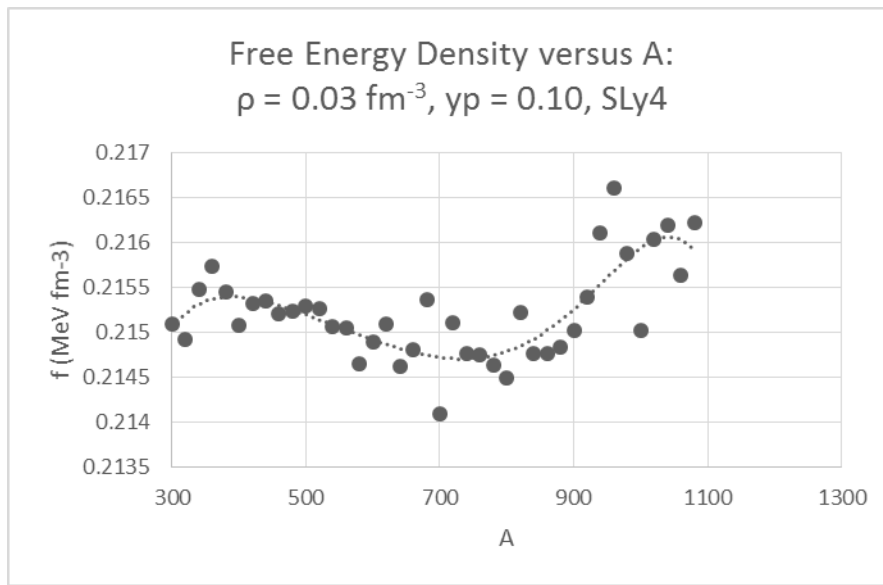
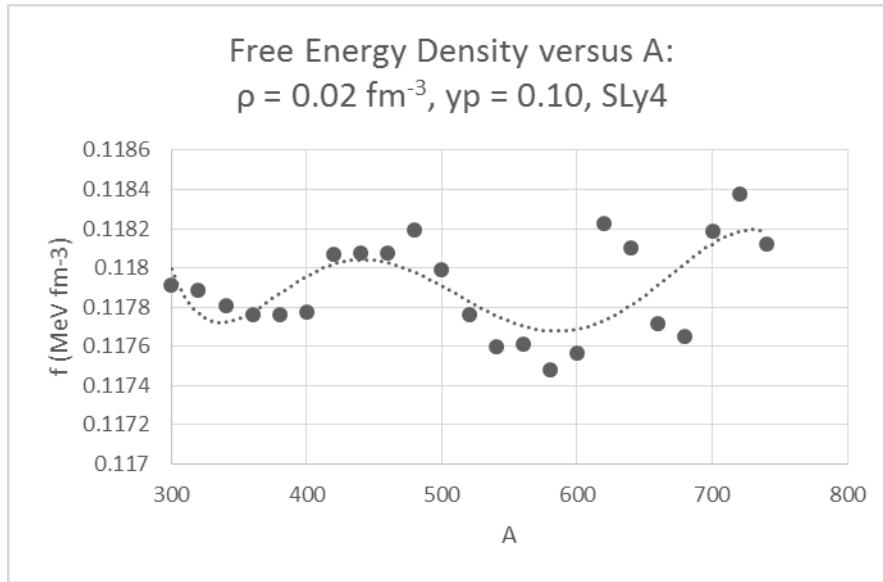


Figure 50: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.02$ (top), 0.03 (bottom) fm^{-3}).

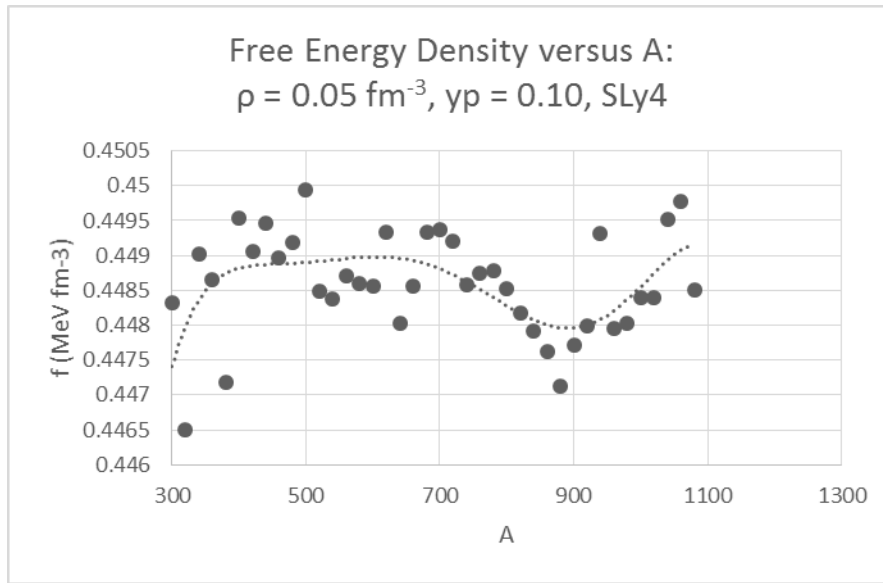
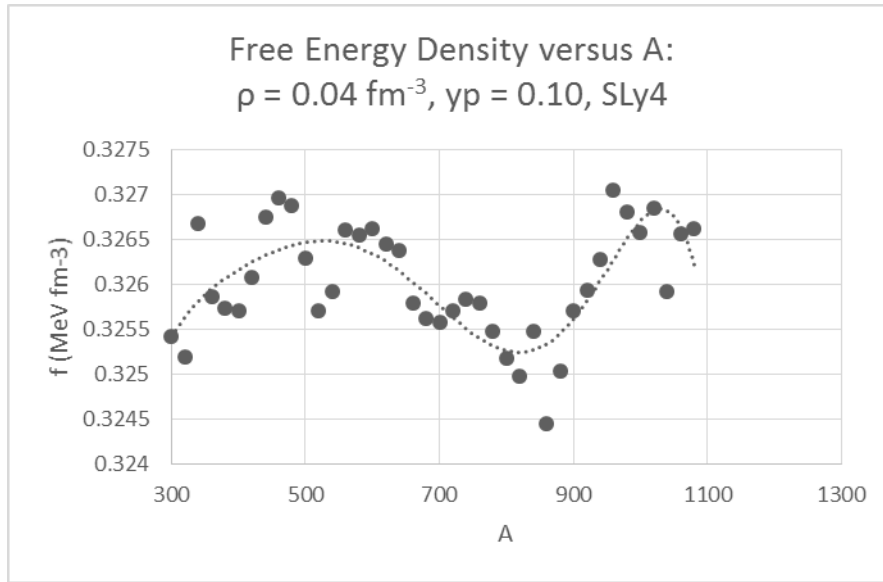


Figure 51: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.04$ (top), 0.05 (bottom) fm^{-3}).

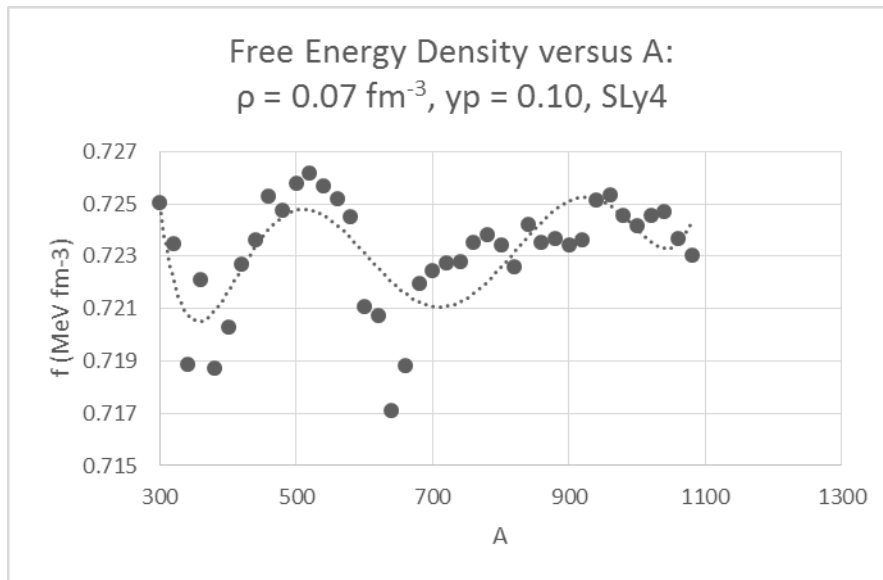
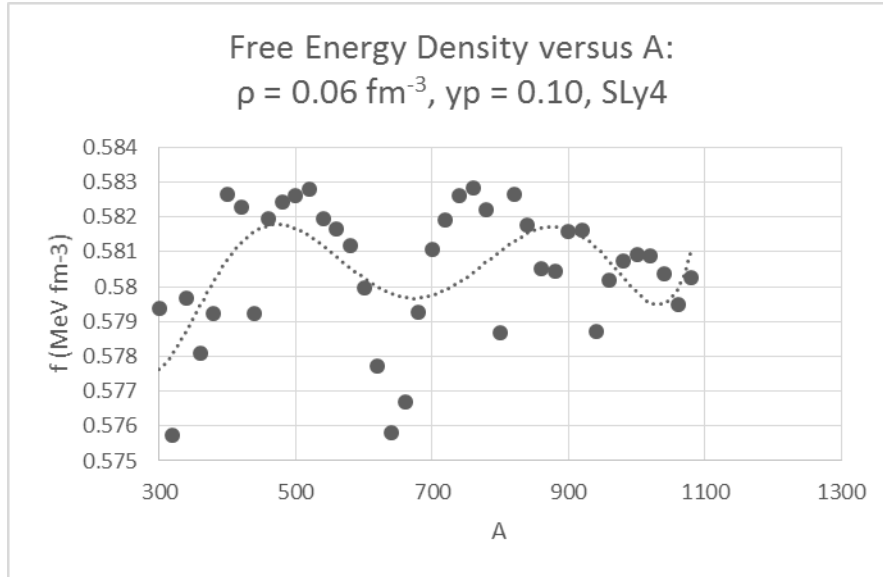


Figure 52: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.06$ (top), 0.07 (bottom) fm^{-3}).

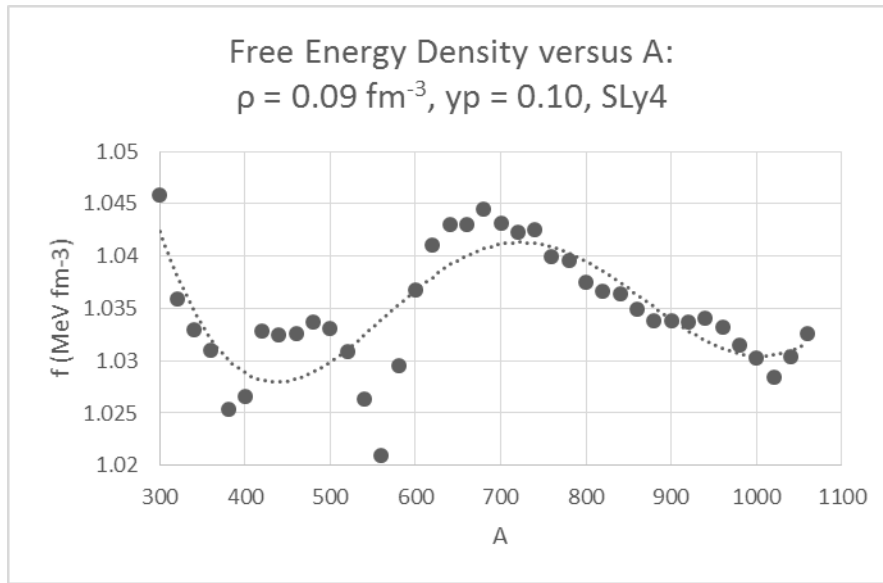
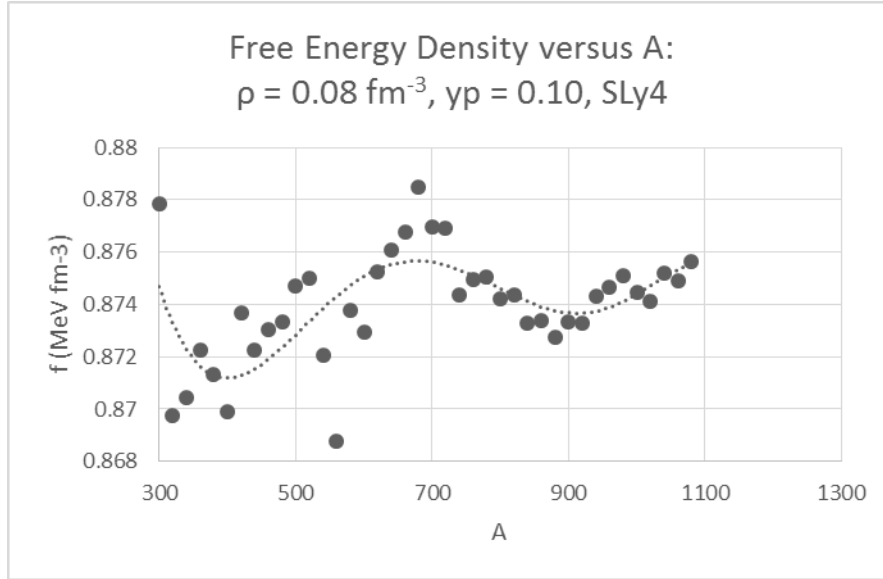


Figure 53: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.08$ (top), 0.09 (bottom) fm^{-3}).

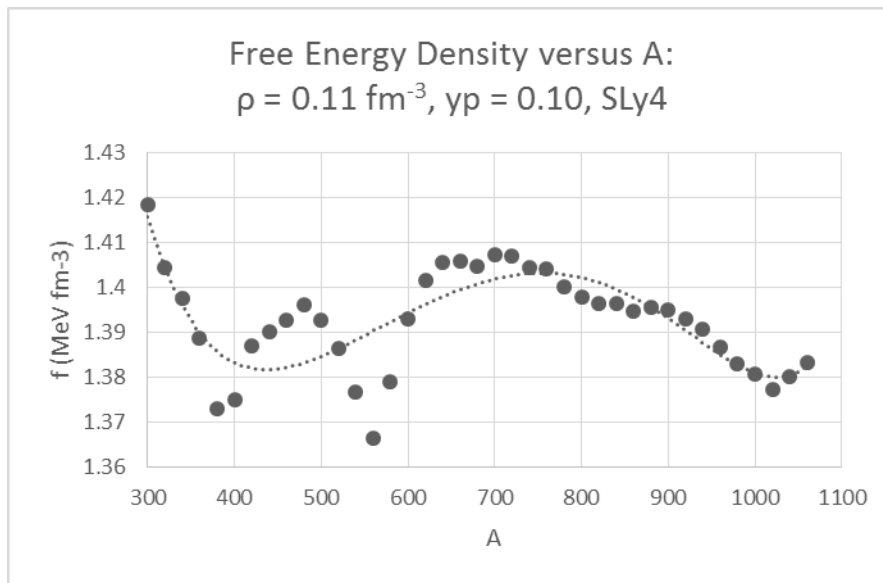
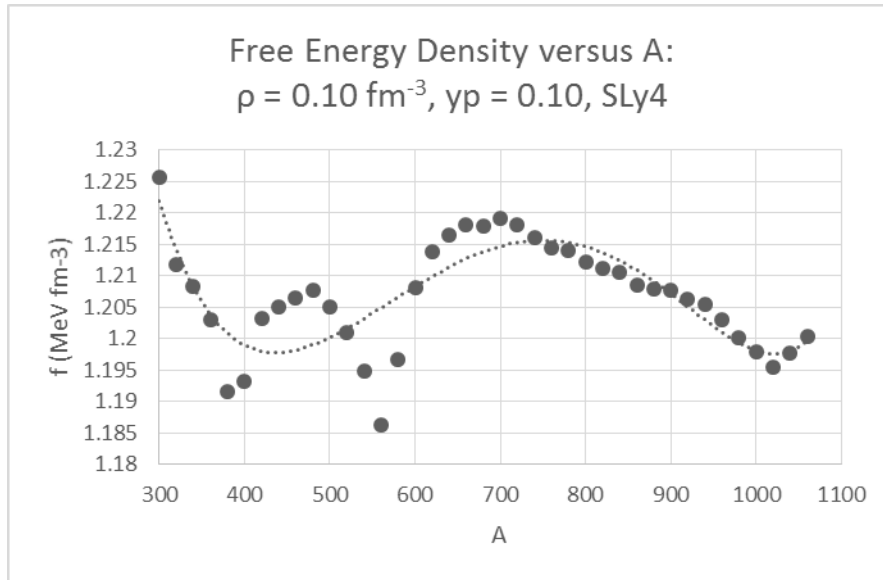


Figure 54: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.10$, $\rho = 0.10$ (top), 0.11 (bottom) fm^{-3}).

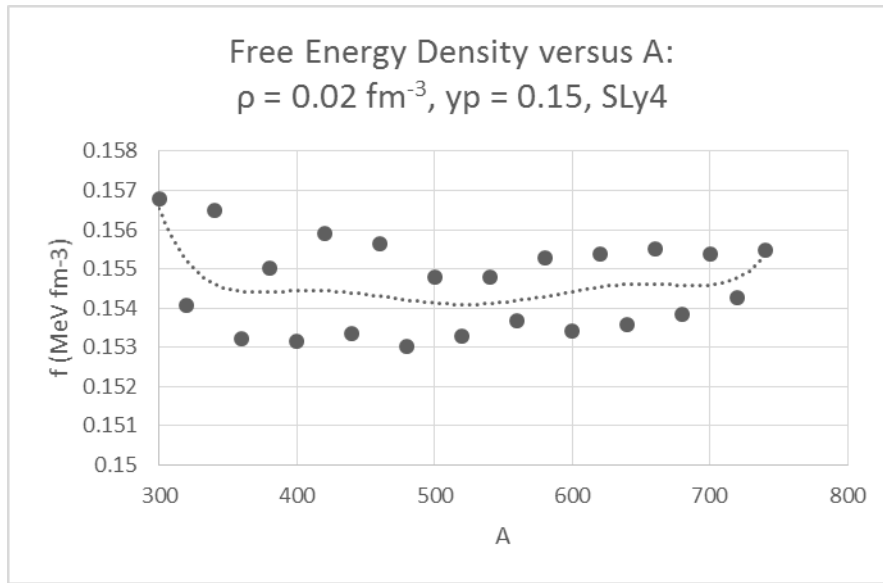
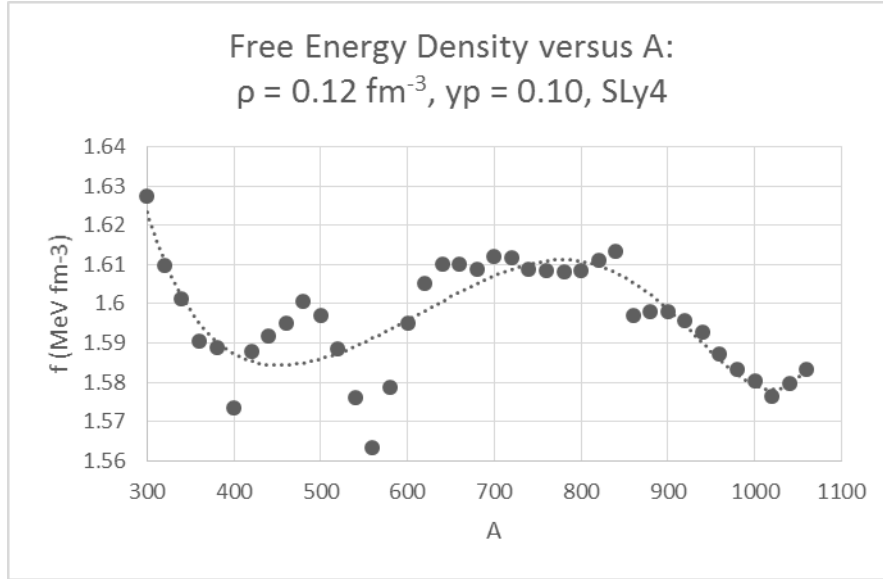


Figure 55: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.10$ (top), 0.15 (bottom), $\rho = 0.12$ (top), 0.02 (bottom) fm^{-3}).

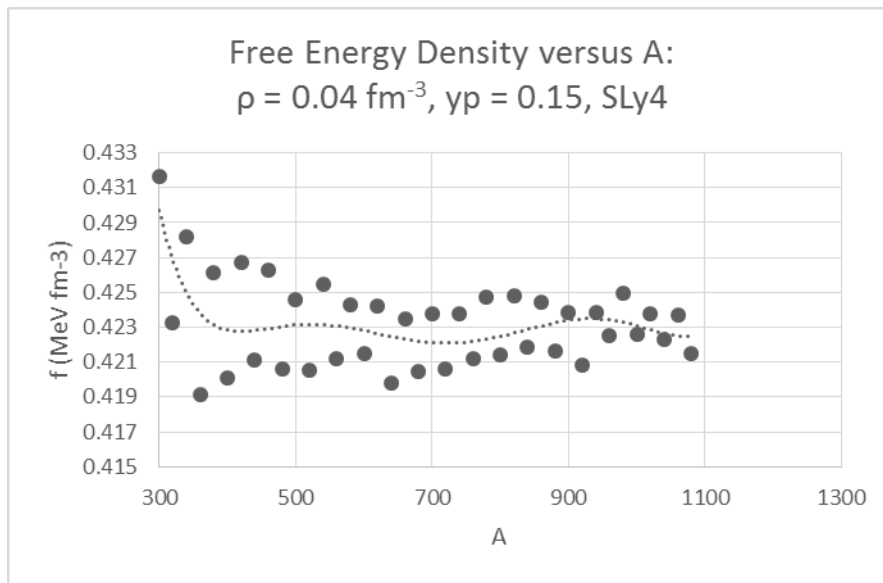
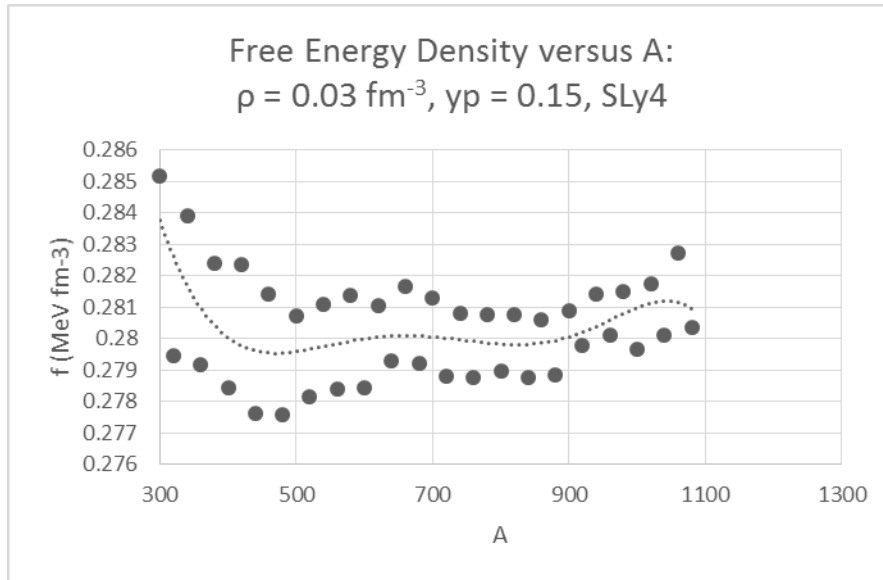


Figure 56: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.03$ (top), 0.04 (bottom) fm^{-3}).

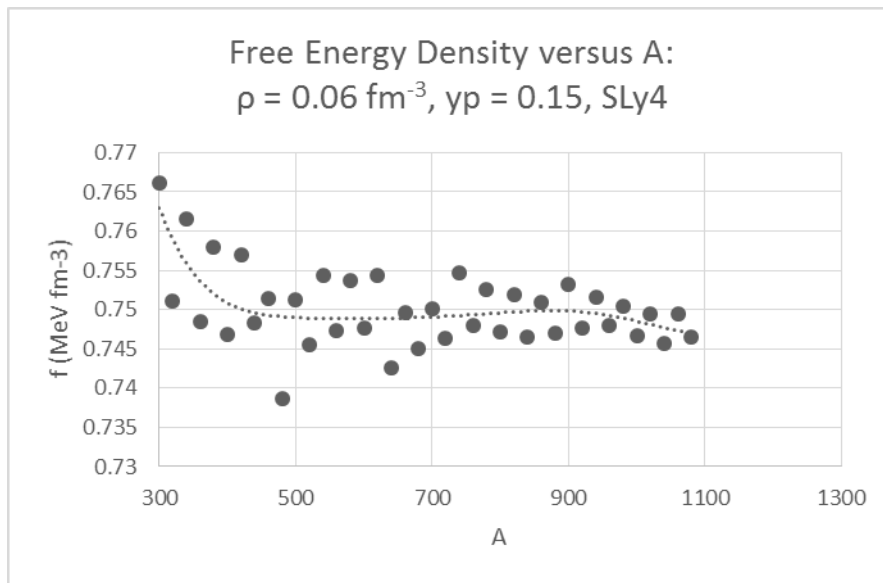
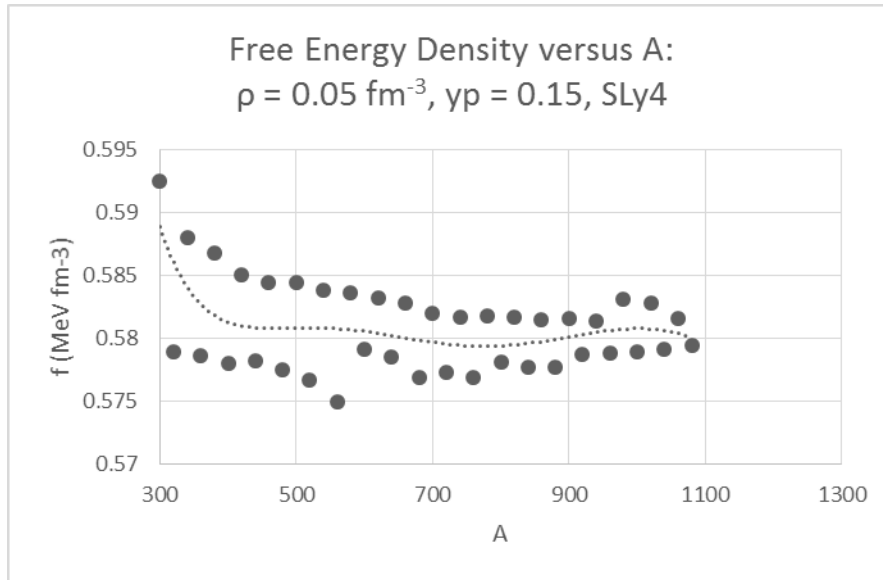


Figure 57: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.05$ (top), 0.06 (bottom) fm^{-3}).

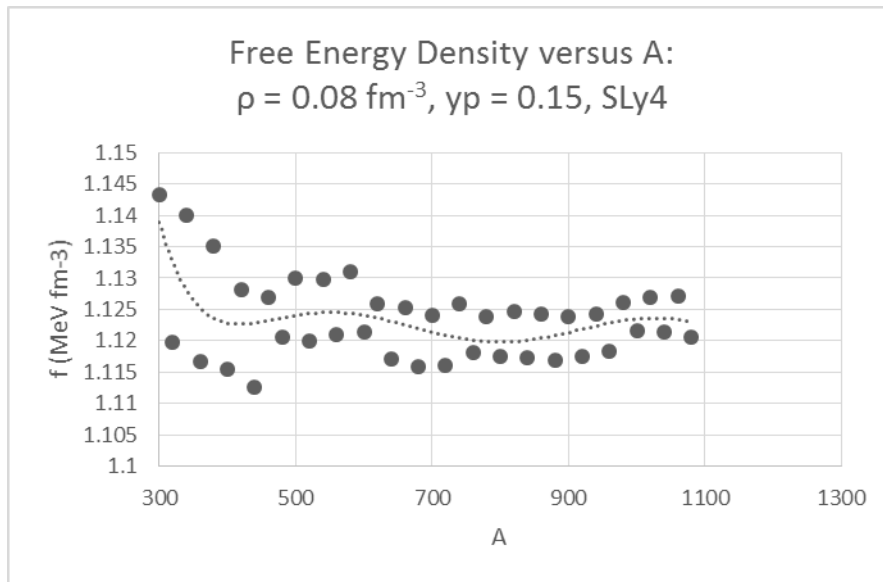
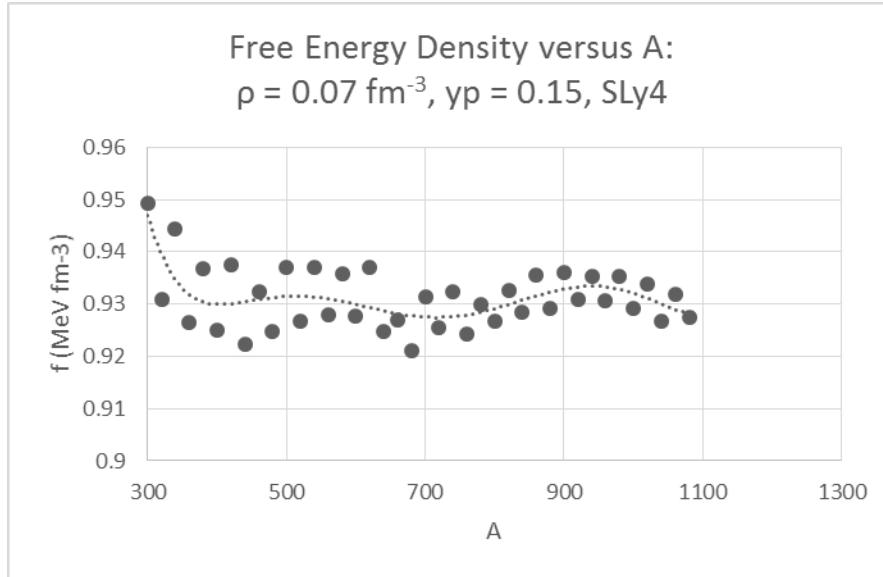


Figure 58: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.07$ (top), 0.08 (bottom) fm^{-3}).

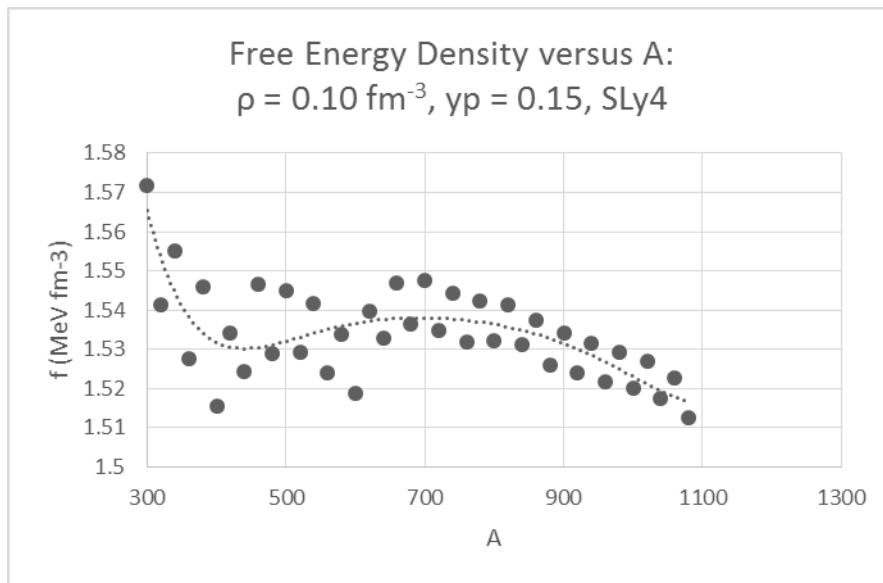
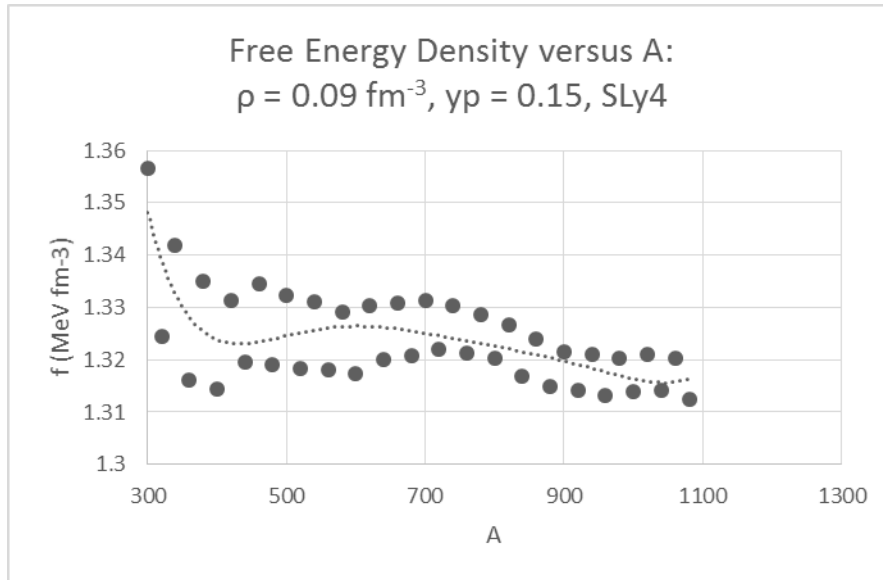


Figure 59: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.09$ (top), 0.10 (bottom) fm^{-3}).

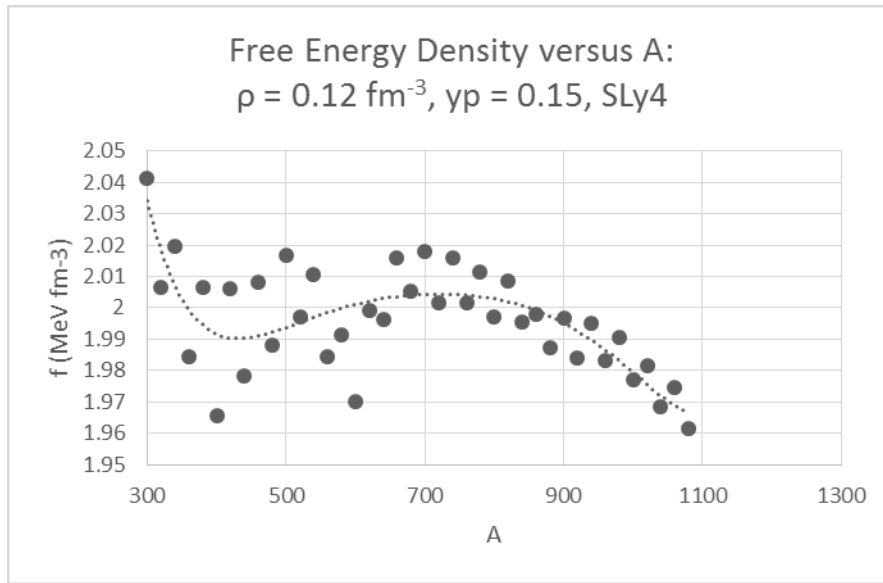
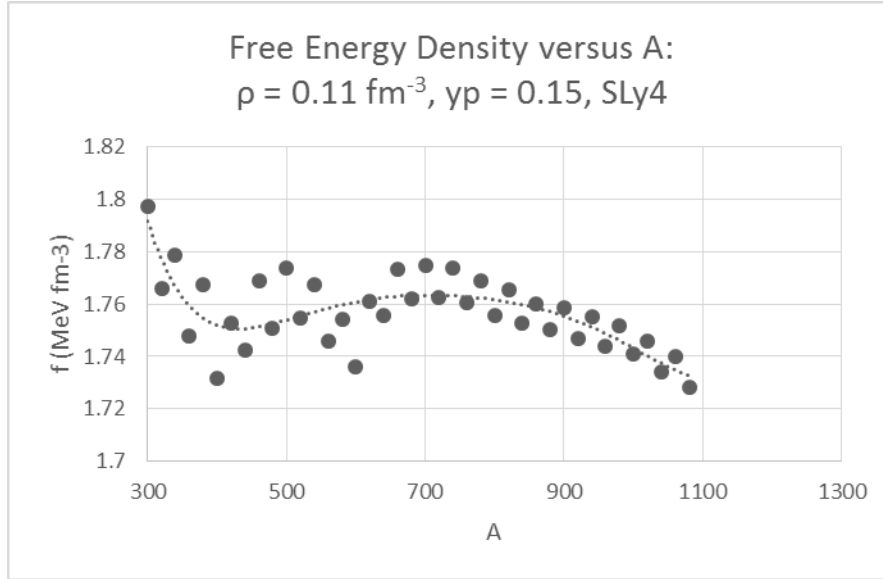


Figure 60: Free energy density f , in MeV fm^{-3} , as a function of nucleon number A for nuclear matter configuration (Skyrme interaction SLy4, $T = 0 \text{ MeV}$, $y_p = 0.15$, $\rho = 0.11$ (top), 0.12 (bottom) fm^{-3}).

$$Y_p = 0.05$$

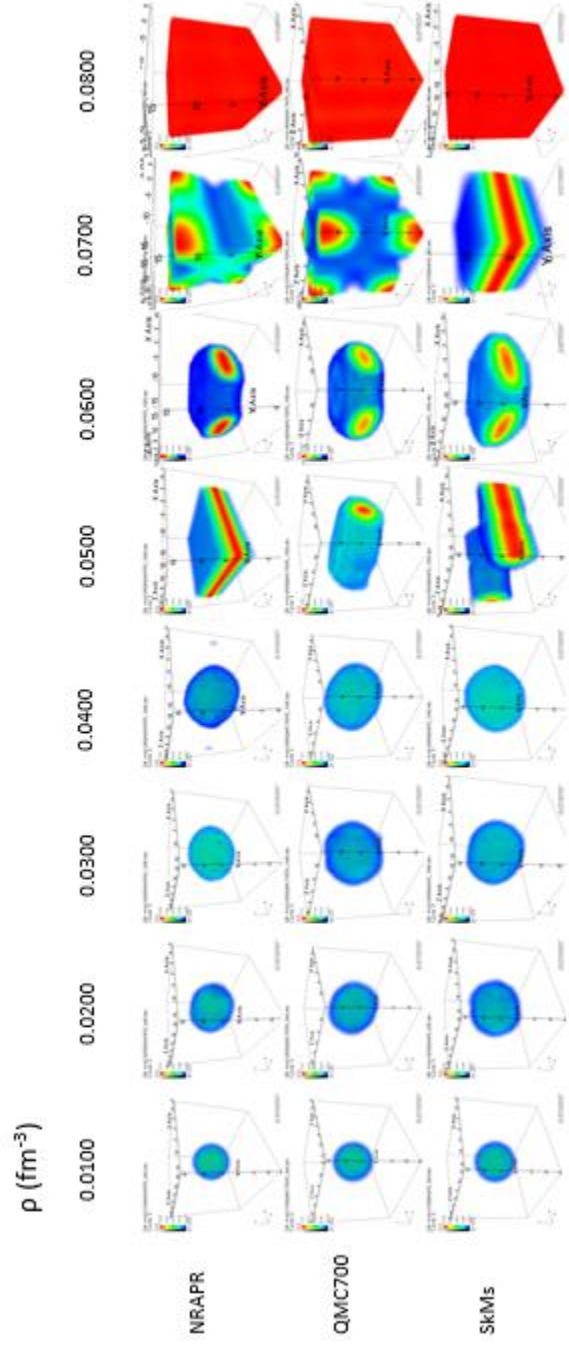


Figure 61: Evolution of the neutron density distribution for $Y_p = 0.05$ and $T = 0$ MeV. Blue indicates low density and the red represents high densities.

$$Y_p = 0.10$$

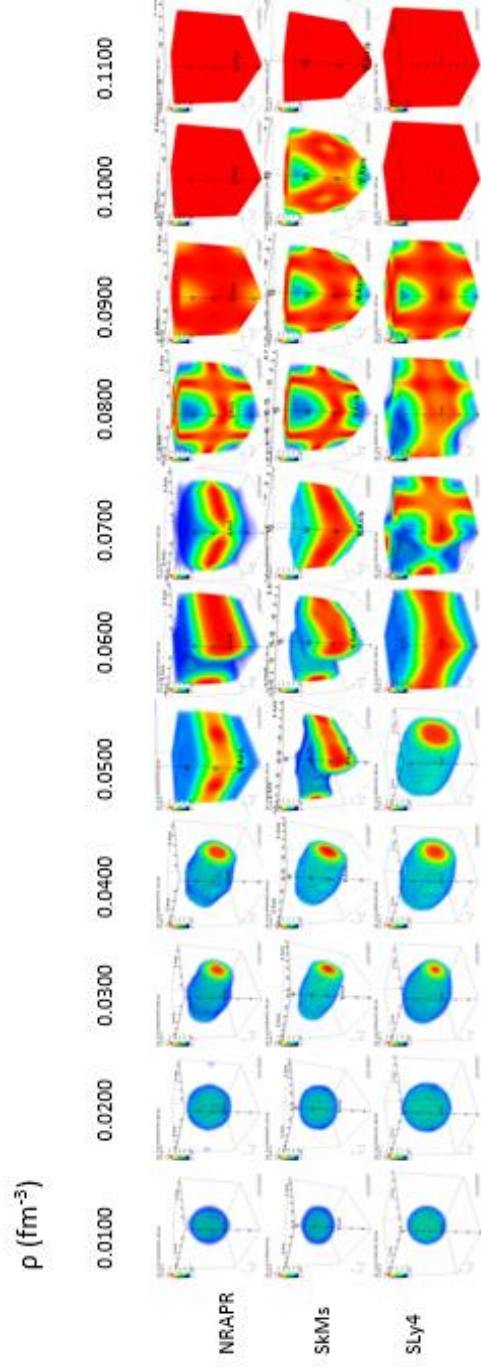


Figure62 : Evolution of the neutron density distribution for $Y_p = 0.10$ and $T = 0$ MeV. Blue indicates low density and the red represents high densities.

$$Y_p = 0.15$$

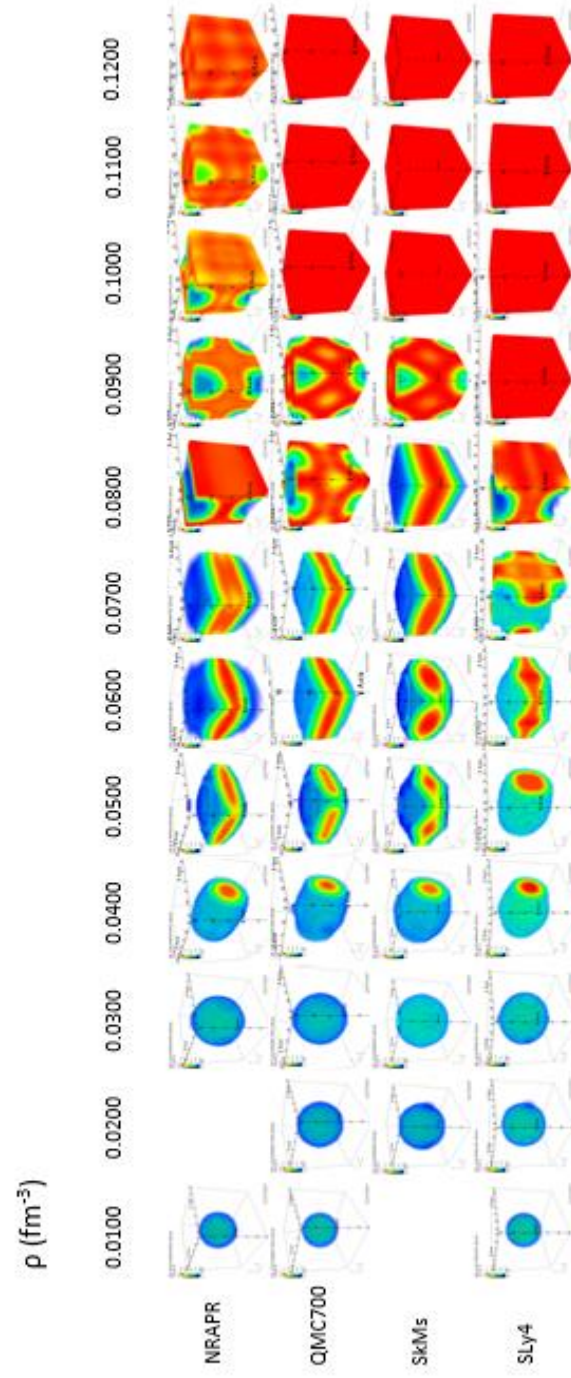


Figure 63: Evolution of the neutron density distribution for $Y_p = 0.15$ and $T = 0$ MeV. Blue indicates low density and the red represents high densities.

Conclusions:

The 3DHF model was used for the first time to calculate fully self-consistent development of the pasta phase as a function of particle number density and the proton/neutron ratio at zero temperature. As the physical size of the unit cell is unknown, we must first calculate the variation of free energy density with nucleon number A for each ρ and select the value of A corresponding to the minimum free energy density. This will identify the volume of the cell $V = A/\rho$.

As shown in this work, this procedure is made difficult by the appearance of serious shell effects, both physical and spurious. The latter effects, originating from constraints due to the finite size of the boxes that are calculated, produce an oscillation in the curves, which may obscure the true minimum of the free energy density with A . Our preliminary procedure to overcome this difficulty seems to be efficient and we find that the typical true minimum in all cases is around $A = 1000$. From these minima, the 3D images were produced (see figures 61, 62, and 63).

We have confirmed that the pasta configurations arise naturally from the self-consistent model not only at a specific case of $T=2$ and $y_p=0.3$, but also for other values of T and y_p [14], [18]. We were able to produce the expected pasta formations as a function of increasing neutron particle number density, although in a somewhat concise form.

A key difference from earlier works we observed is that in the pasta phase is the spread of densities that the pasta appears, transitions, and then disappears into homogeneous matter. In our work, we have seen that the neutron density window for

existence of pasta is smaller at lower y_p . The most extreme example of this is in matter with the lowest ratio of protons. Pais and Stone observed for $y_p = 0.3$ (at $T = 2$ MeV) that typical density threshold for appearance of pasta is at 0.032 fm^{-3} and its dissolution is at 0.114 fm^{-3} [18]. For $y_p = 0.05$, the window for pasta spans from 0.04 fm^{-3} until 0.08 fm^{-3} . As y_p increases to $y_p = 0.15$, the dissolution of pasta appears to be at 0.10 fm^{-3} .

We used four different Skyrme interactions in our work. Although we see some differences in the results, more work would have to be done to make a final conclusion in this area.

Future Work:

Results presented in this thesis represent grounds for further steps towards the investigation of pasta in beta-equilibrium matter in cold neutron stars. More calculations will have to be performed to obtain a fine enough mesh of values of y_p and ρ to find the y_p corresponding to beta-equilibrium matter. Then the pasta formations will be calculated under these conditions for the first time. These results should shed light on the properties of neutron star crusts and their transition to the neutron star core.

Bibliography

- [1] Baade, W. and Zwicky, F. (1934), *Phys. Rev.* 45, 138.
- [2] Bartel, J., Quentin, P., Brack, M., Guet, C., and Håkansson, H.-B. (1982), *Nucl. Phys. A*, 386:79.
- [3] Chabanat, E., Bonche, P., Haensel, P., Meyer, J., and Schaeffer, R. (1997), *Nucl. Phys. A*, 627:710.
- [4] Chabanat, E., Bonche, P., Haensel, P., Meyer, J., and Schaeffer, R. (1998), *Nucl. Phys. A*, 635:231.
- [5] Dutra, M., Lourenço, O., Sá. Martins, J. S., Delfino, A., Stone, J. R., and Stevenson, P. D. (2012), *Phys. Rec. C* 85, 034201.
- [6] Glendenning, Norman K. *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity*. New York: Springer, 2000. Print.
- [7] Greiner, W. and Maruhn, J.A. *Nuclear Models*. New York: Springer. 1996.
- [8] Guichon, P. A. M., Matevosyan, H. H., Sandulescu, N., and Thomas, A. W. (2006), *Nucl. Phys. A*, 772:1.
- [9] Hewish, A., Bell, S.J., Pilkington, D.H., Scott, P.F. and Collins, R.A. (1968), *Nature* 217, Page 709.
- [10] Landau, L. (1932), *Phys. Zeit. der Sowjetunion*, Vol 1, No. 2, Page 285.
- [11] Magierski, P. and Heenen, P.-H. (2002), *Phys. Rev. C* 65, 045804.

- [12] Miller, M. C. "Introduction to Neutron Stars." *Neutron Stars*. University of Maryland, n.d. Web. 21 Apr. 2014.
- [13] Newton, W.G. "The Phase Transition to Uniform Nuclear Matter in Supernovae and Neutron Stars." DPhil Thesis, University of Oxford, 2007.
- [14] Newton, W., and J. Stone. (2009), *Phys. Rev. C* 79, 055801.
- [15] Oppenheimer, J.R. and Volkoff, G.M. (1939), *Phys. Rev.* 55, Page 374.
- [16] Oyamatsu, K. (1993), *Nucl. Phys. A* 561 Page 431.
- [17] Pacini, F. (1967), *Nature* 216, Page 567.
- [18] Pais, Helena, and Stone, J. R. (2012), *PRL* 109, 151101.
- [19] Skyrme, T.H.R. (1956), *Phil. Mag.* 1, Page 1043.
- [20] Sonoda, H., Watanabe, G., Sato, K., Takiwaki, T., Yasuoka, K., and Ebisuzaki, T. (2007), *Phys. Rev. C*, 75:042801(R).
- [21] Steiner, A. W., Lattimer, J. M., and Brown, E. F. (2013), *Astrophys. J.*, 765 L5
- [22] Steiner, A. W., Prakash, M., Lattimer, J. M., and Ellis, P. J. (2005), *Phys. Rep.*, 411:325.
- [23] Stone, J. R., and Reinhard, P. G.(2007), *Prog. Part. Nucl. Phys.* 58, 587.
- [24] Tolman, R. C. (1939), *Phys. Rev.* 55.364.
- [25] Vautherin, D. and Brink, D.M. (1971), *Phys. Rev. C* 5, 3, Page 626.

[26] de Castro Felga Ramos Pais, Helena Sofia, "Three Dimensional Equation of State for Core-Collapse Supernova Matter. " PhD diss., University of Tennessee, 2013.