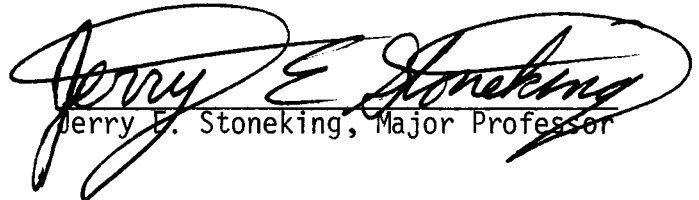
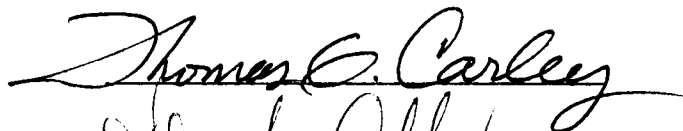


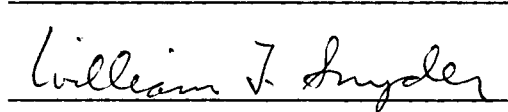
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I am submitting herewith a dissertation written by Danny Lewis Levine entitled "A Finite-Element Based Parametric Study of an Orthopaedic Bone-Plate *In Situ*." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Engineering Science.


Jerry E. Stoneking, Major Professor

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A FINITE-ELEMENT BASED PARAMETRIC
STUDY OF AN ORTHOPAEDIC BONE-
PLATE *IN SITU*

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Danny Lewis Levine
August 1979

1398063

DEDICATION

This dissertation is dedicated to my parents, Ervine and Shirley Levine, whose constant and unselfish support have contributed so much to this academic achievement. It is with the deepest love and devotion that this expression of gratitude is offered.

ACKNOWLEDGEMENTS

The author is indebted to his advisor, Dr. Jerry E. Stoneking, for his guidance and support. Thanks are also extended to the other members of his dissertation committee for their advice and criticism which have contributed significantly to this work.

For its financial support, recognition is also given to the University of Tennessee's Engineering Science and Mechanics Department, and to the National Institutes of Health for grant #BRSg-STONEKING-76. Also, for its invaluable assistance and constant patience the author is grateful for the services provided by the University of Tennessee Computing Center.

Recognition must also be given to Dr. Harry Wilcox of The University of Tennessee School of Dentistry for supplying much needed experimental materials and to Mr. Dale Fitting and Dr. Archie Mathews for their assistance in the execution of portions of the experimental work.

Finally, special thanks must go to the secretaries and machine shop personnel of The University of Tennessee Engineering Science and Mechanics Department, without whom much of this work could not have been accomplished.

ABSTRACT

This study is an examination of some of the factors which affect the load carrying capacity of a plated bone and indicates how the results generated might lead to improved implant design and utilization. The study is confined to a consideration of gross mechanical behaviour of the metal and the cortical bone in the mid-shaft region of a human femur.

The primary analytical tool used was the finite element method, implemented through use of the SAPV computer code. The three-dimensional finite element model developed includes fully anisotropic elastic properties for bone and also includes the presence of a bone fracture realistically simulated by no-tension "gap" elements.

The mechanical response of the bone-plate-screw structure is influenced by many variables. The factors considered in this study include patient weight, bone inside and outside diameters, plate length, distances between screws and angular orientation of the fracture plane. A series of computer "experiments" was designed and implemented to permit assessment of the importance of the parameters studied.

The finite element model was verified experimentally using a strain gauge instrumented bone plate attached to a segment of embalmed human femur. The model gave stress results of the same order of magnitude as the experiment.

The parameter sensitivity study done using the model, indicated that stresses in the structure are most sensitive to changes in

plate length, bone diameters and angulation of the fracture. Closure of the gap between fracture surfaces is also strongly influenced by plate length and is also affected by compression plating. Overall, one can conclude from the observed levels of stress, that static loading alone will probably not cause fracture failures of metal or bone, but under the influence of cyclic loading and the corrosive action of body fluids, such failures are highly probable.

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I. INTRODUCTION

In surgical treatment of certain long-bone fractures, under circumstances which render the use of external support (casts, splints etc.) ineffective, a metal strap known as a bone plate may be implanted so as to provide a rigid fixation. Such a device, when attached to the shaft of the broken bone by screws, promotes healing by assisting in the maintenance of contact between the fracture surfaces and thus expedites the restoration of the bone's original strength and shape.

Inside the body, the metal is subjected to forces of varying magnitude, and is constantly bathed by corrosive saline tissue fluids. These factors may cause failure of metal and/or bone by mechanical action, by corrosion or by some combined mechanism such as corrosion fatigue.

In the time since the first use of the bone plate, orthopaedic surgeons have become aware of the causes of implant failure, and the knowledge borne of their experience has led to improvements in operative techniques and implant design. Although such improvements have made possible a reduction in the percentage of failures, the incidence of mechanical failures is by no means insignificant.

Under the very broad heading of implant related failures, we must include biological mechanisms (inflammation, sepsis, pain or bone non-union) in addition to corrosion related and/or mechanical failures. Within this wide context, the proportion of problems which are purely mechanical is rather small. It is not always easy, however, to isolate any individual causative factor since several may be intimately related.

In published accounts^{1,2,3} of analyses done on removed metal implants, metallurgical factors are discussed at length, and while many fracture or corrosion failures can be traced to material imperfections these cannot account for all observed problems.

Mathematical analyses of the mechanical behaviour of implants *in situ*, by the methods of strength of materials and elasticity require so many simplifications that they can hardly be expected to produce anything more than very crude results. The complex geometrical configurations combined with bone inhomogeneity and anisotropy do not lend themselves to closed form analytical methods. Computer-based numerical methods, on the other hand, provide a means by which these obstacles may be overcome. The finite element method,^{4,5} in particular, permits the development of accurate models of very intricate structures.

For the purposes of the present study, the finite element method was utilized for modelling the mechanical response, *in situ*, of a fractured human femur, fixed by means of a metal bone plate and screws. A linear, static analysis was done in which fully orthotropic bone elastic constants were incorporated and in which fracture surfaces were assumed to transfer compressive but not tensile forces. In addition, a computer code SCALE was developed to facilitate the modification of several dimensions of bone and plate, such as bone inside and outside diameters, plate length, distances between screws etc. Such changes, in general, result in changes in the complex pattern of stresses in bone and metal, and it was the relationship between the

manipulated variables and the predicted stresses that was investigated in this study.

In their experimental work on equine metacarpal bones, Bynum et al.⁶ and Ray et al.⁷ examined the effect of certain plate dimensional changes on the load carrying capacity of a plated bone and they referred to this work as a parametric study. (A somewhat more descriptive phrase is "parameter sensitivity study.") The simple beam mathematical analysis which accompanied these studies could shed no light on the complex mechanical behaviour of the bone-plate-screw structure and hence could do little to explain how the geometrical changes caused the observed variations in load carrying capacity.

With the finite element method, however, questions of this nature can be answered. So, a new parametric study, in which "experiments" were simulated on the computer, was performed. The ability of the developed model to accurately predict stresses was verified by means of a laboratory experiment, which was done prior to the commencement of the parameter sensitivity study.

The body of this dissertation consists of four main parts. First, the review of literature provides, for the non-medical reader, some background information on bone and fracture healing, and in addition gives a historical review of previous experimental and analytical work on bone plates. Next, the development of the present model is discussed with special attention given to assumptions made and to a description of the computer software created for this analysis. Next, the experimental verification of the model is presented, and finally,

the results of, and conclusions based upon the parameter sensitivity study are delineated, from which recommendations about optimum combinations of the design variables were made.

II. LITERATURE REVIEW

Human Cortical Bone: The Tissue

Bone tissue is unique among tissues of the body in that it is the only connective tissue which is hard. This hardness results from the presence of a crystalline mineral imbedded within a matrix of living organic material, and it is the rigidity of bone which enables it to serve as the structural framework of the body.

Within any given bone, such as the femur, there exist two significantly different (differentiated according to microstructure) types of bone tissue.⁸ Compact (or cortical) bone is, as its name implies, quite dense, with its constitutive subunits, called osteones, oriented nearly parallel to the longitudinal axis of the bone. The shafts of long bones such as the femur, tibia and humerus are made of compact bone. At the ends of these bones, however, the mechanical requirements are quite different. We find that the osteones tend to fan out, forming thin plates which are oriented in directions of principal stress.⁹ These thin plates, known as trabeculae, form what is called trabecular bone (also referred to as cancellous or spongy bone). Figures 1 and 2 (adapted from Ref. 10) show, respectively, the gross morphology of a long bone and the microscopic arrangement of the constituents in single osteones.

The osteones (also called haversian systems) are seen to be highly organized, consisting mainly of bone cells (osteocytes), collagen (a fibrous protein) and a crystalline mineral, hydroxyapatite. The haversian system is an irregular cylindrical structure with thick

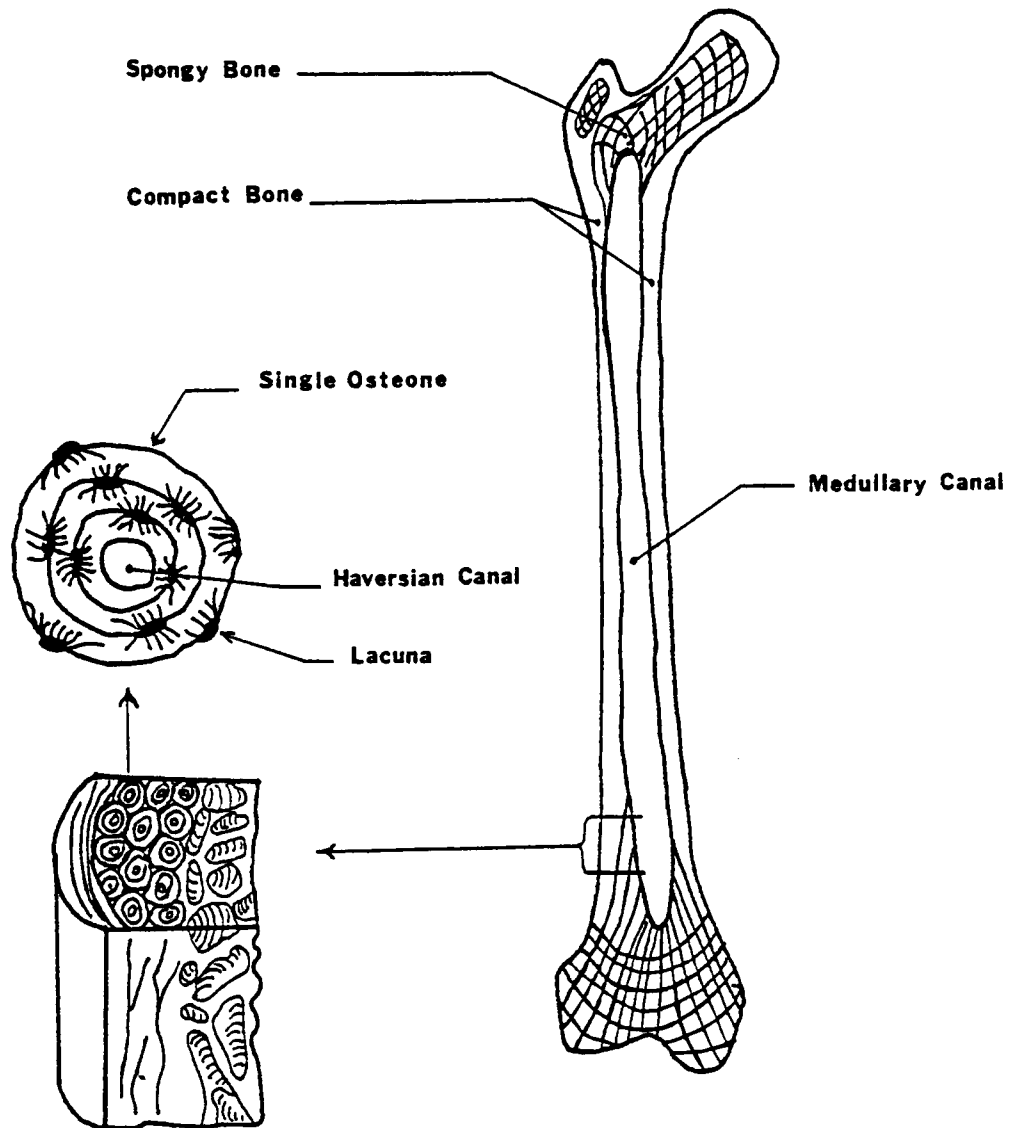


Figure 1. Gross Morphology of a Human Femur

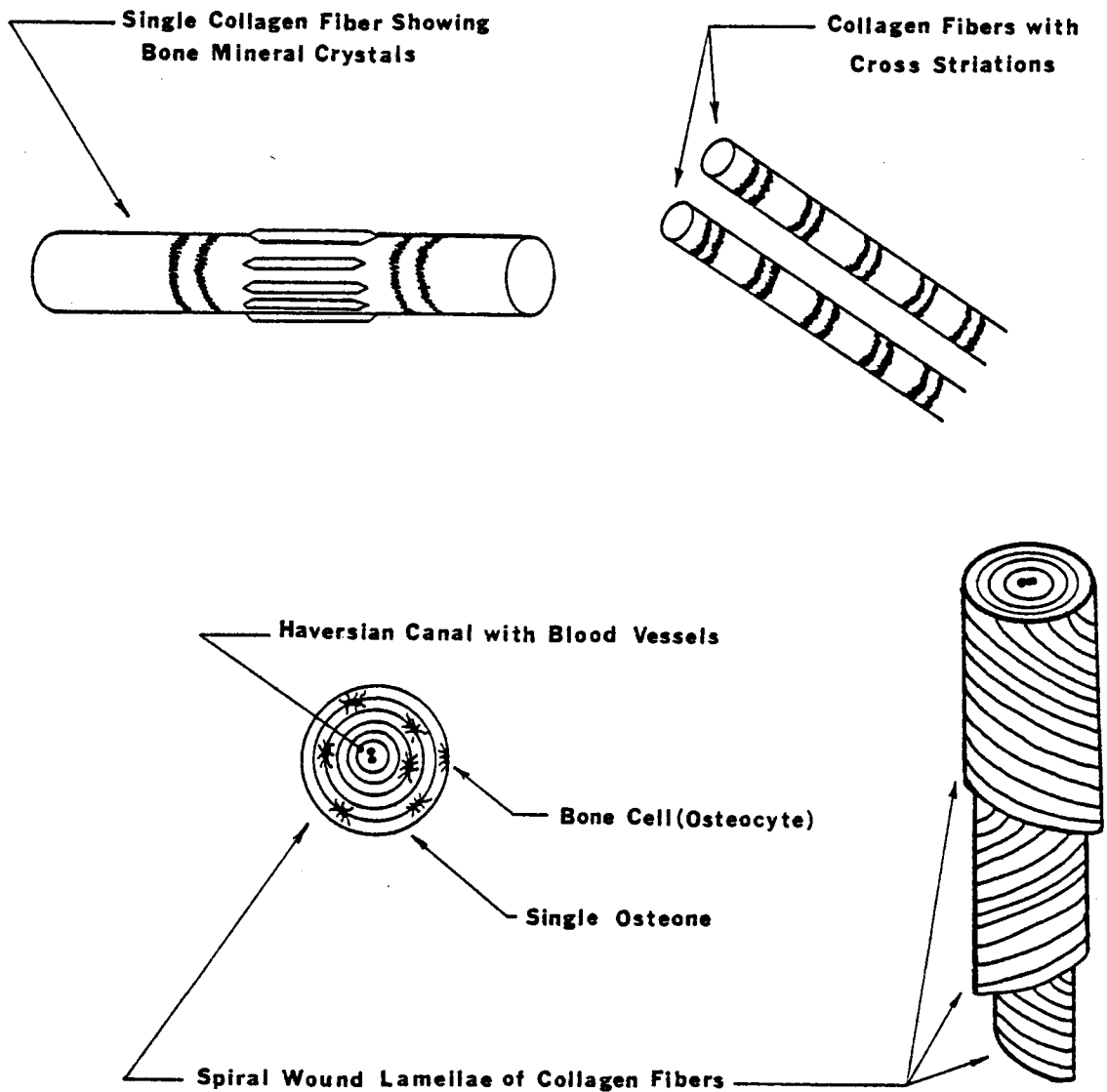


Figure 2. Detail of a Single Osteone

walls and a narrow lumen called the haversian canal, which carries one or more tiny blood vessels. The walls have a lamellar structure (concentric layers) with the fibers of each lamella oriented in a spiral about the axis of the osteone. The direction of the spirals may alternate in successive layers. Between the layers are small spaces known as lacunae, which house the osteocytes, and which are connected to one another and to the lumen of the canal by branched canaliculi (also referred to as canals of Volkmann). Closer examination of the lamellae reveals crystals of hydroxyapatite ($\text{Ca}_{10}(\text{PO}_4)_6(\text{OH})_2$) embedded in an organic matrix. The most prominent organic substance is the fibrous collagen. The bone mineral crystals are aligned parallel to the collagen fibers, which are themselves held together by a group of organic compounds collectively referred to as the "ground substance." Bone mineral makes up about 70% of the weight of dry bone, and of the remaining 30% almost all (90-96%) is collagen.

The osteocytes are the only living parts of bones, but through their action the structure and function of the entire skeletal system is controlled. Under the influence of certain stimuli the osteocytes can be made to change into other forms which have specialized jobs. Osteoblasts (bone forming cells) are responsible for the formation and calcification of the intercellular substance in bone tissue,¹¹ and alternatively, osteoclasts (bone destroying cells) are responsible for the resorption of the intercellular substance. Throughout the body's growth period these cells act in concert to alter the size, shape and internal structure of the bones in order that they might

be adapted to the needs of a developing individual. When maturity is reached, rapid changes in size and shape cease but the action of bone cells can still alter internal structure, and is necessary in the healing of fractures.

During the growth phase of life, the long bones increase in length through the action of growth centers near the bone ends. Through the action of osteoblasts, at these centers (known as epiphyses), the bone shaft (diaphysis) lengthens as new bone material is laid down. The haversian systems thus formed are referred to as primary osteones and they represent the initial state of compact bone. Bones change in size and shape throughout the growth period; however, internal remodelling of bone will continue for the remainder of one's life. This occurs in response to the body's changing requirements for calcium and therefore plays a significant role in the maintenance of life. Such internal remodelling begins with the formation of roughly cylindrical absorption cavities, which are subsequently filled in with new layers of bone substance, forming what are referred to as secondary osteones.

Bone Fracture Healing

In terms of etiology, one can define two main types of fractures, traumatic and pathologic. A traumatic fracture occurs when normal bone is broken as the result of the action of an externally applied force. Should the bone's load carrying capacity be reduced, due to disease or aging, even stresses arising from normal muscular activity may produce a pathologic fracture. It is not uncommon to hear of an

elderly person suffering a "broken hip" which is usually associated with a fall. Cause and effect are not always clearly defined in these instances. For example, the fracture may result from stresses generated in walking or climbing stairs, and the associated pain could cause the fall.

Fractures may also be classified according to the severity of the damage incurred.¹² A complete fracture, for example, is one in which the break has progressed all the way through the bone creating a complete discontinuity. A break in which several fragments are created is termed comminuted. Should the skin be pierced by one or more ends of the broken bone, so that a wound is created, the fracture is said to be compound or open.

The rate at which a fracture heals can be affected by a number of factors, such as the nature and severity of the break, the health of the individual and even on which of the bones is involved. For an average person, not suffering from any bone disease, a fracture in the shaft of the humerus requires 10-12 weeks for union of bone while a femoral shaft fracture may take from 10-12 months for full union.¹³ The construction of various bones differs significantly, which may in part be responsible for the range of healing times, however, the sequence of events in bone union follows a similar pattern in all cases.

As the trauma occurs, numerous blood vessels in the bone, periosteum (membrane which surrounds bone) and in adjacent soft tissues rupture, which gives rise to the development of a large pool of blood called a hematoma in the vicinity of the fracture site. The blood

clots, and within twenty-four hours changes begin to occur in the clot. Somehow, the occurrence of the fracture stimulates the proliferation of osteogenic (bone forming) cells which encircle the bone on each side of the fracture line, and which gradually produce an external "cuff," as shown in Figure 3. The material of which the cuff is made is called callus. The callus experiences a series of morphologic changes which eventually give it the form of normal bone. Then, over a period of months or even years the callus is remodelled so that the original contours of the bone are restored.

Clearly, a complete understanding of the mechanisms by which these events are controlled is desirable in that it can lead to improved treatment methods. As long ago as 1892 Wolff¹⁴ observed a relationship between bone form, and applied stress. He presented a postulate, which has since become known as Wolff's law and which simply states that bone is formed wherever it is needed to withstand stress. More recently, it has been observed that fracture healing, as well as bone remodelling can be affected by applied stress.

Within the last twenty-five years various researchers have studied the phenomenon observed by Wolff, seeking to determine its underlying principle. Of the various mechanisms proposed, the most popular one describes the bone response to stress in terms of an electrical feedback mechanism in which the piezoelectric property of bone¹⁵⁻²² plays an integral part. Externally applied currents have been shown to enhance the healing of fractures²³⁻³¹, and experiments have shown that electrical currents arise in bone which is under

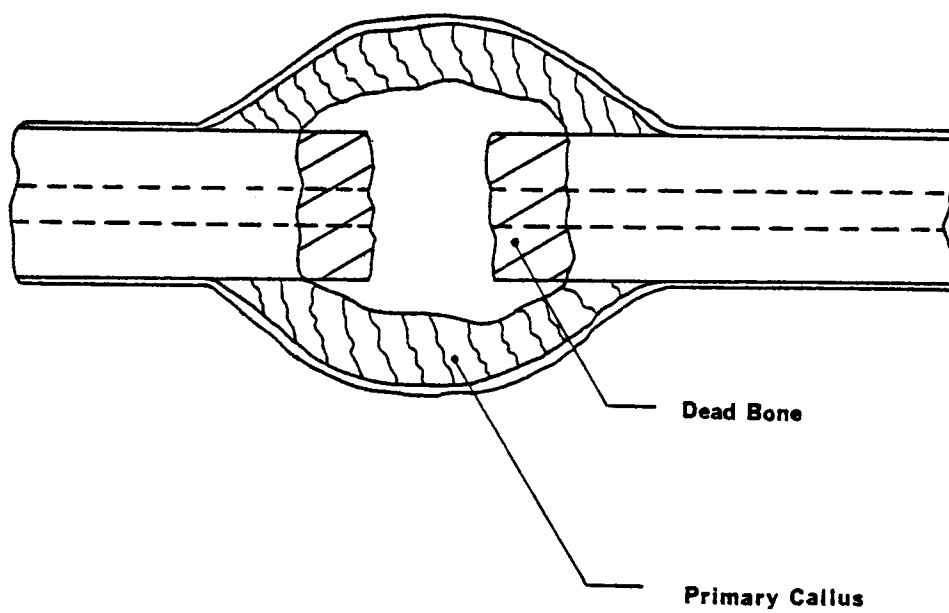


Figure 3. "Cuff" Formed by Callus at a Bone Fracture Site

stress. It is not unreasonable to conclude that there is a connection between these two phenomena. Even so, this mechanism is not universally accepted.

Insofar as the treatment of fractures is concerned, physicians have found that rigid fixation and/or compression at the fracture site increases the chances for restoration of the original geometry. In addition, fracture surfaces maintained in close contact by internal fixation will knit together with a minimum of external callus formation. It seems that the rigid fixation somehow obviates the need for the callus which would otherwise have served to provide some structural support at the fracture site during the healing period. Under conditions of internal fixation with compression, a fracture line may rapidly disappear from x-rays, but this does not mean that the bone at the fracture site has regained its original strength. Most internal fixation devices are not designed for full weight-bearing applications, so a "normal" x-ray does not imply that one may permit full weight load on the affected limb. To do so prematurely, can and does lead to failure of implant, bone or both.

Bone Strength

Among materials of construction, bone is unique, in that its strength can change in response to changing requirements. Consequently, the assessment of maximum load carrying capacity for a bone in any particular situation is not a straightforward task. Each individual configuration of loads and geometry must be examined independently. Clearly the maximum bending stress at a femoral midshaft

cannot be compared with the bending fracture stress of the humerus midshaft, for example, since the material properties may differ greatly between those two locations. The ability to use average data, based on many experiments, as a tool in the evaluation of the capacity of some system is desirable, but it is of no less importance to know a correction factor by which deviations from the norm may be incorporated into one's analysis.

Much experimental measurement of bone strength has been done on specimens ranging from whole bones down to single osteones. Bone is living tissue, and when tested *in vitro* (outside the body) may exhibit significantly different properties from those it possesses *in vivo*. Consequently, the affects of specimen preparation have been given much attention in the literature. Compact cortical bone has been shown to be an orthotropic (hexagonal symmetry) material, exhibiting essentially linear elastic behaviour in quasi-static tests, but with considerable deviations from linear action at high strain rates. Table 1 contains a sample of ultimate strength values obtained experimentally. Comments on these and other data are contained in the following paragraphs.

Aside from physiological factors which may affect the measured mechanical properties of cortical bone, some influential factors for which published data exist are:

1. Specimen preparation techniques
2. Location of specimen in individual bone

Table 1. Fracture Strength of Human Cortical Bone Tested Under Various Conditions

REFERENCE SOURCE	SPECIES	BONE	σ_u (psi)	τ_u (psi)	STRAIN RATE	WET OR DRY	DIRECTION	COMMENTS
Dempster and Liddicoat ¹²	Human	Femur	25680 ± 4740 (C) 15775 ± 388 (C) 17090 ± 3940 (T) 11428 ± 1540 (T) 19201 ± 3139 (C) 18651 ± 1064 (C) 16988 ± 4600 (C) 15366 ± 2800 (C)	-	Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static	Dry Wet Dry Wet Dry Dry Wet Wet	Longitudinal Longitudinal Longitudinal Longitudinal Radial Tangential Radial Tangential	
Dempster and Coleman ¹³	Human	Tibia	13918 ± 3910 (T) 1432 ± 120 (T) 27224 ± 1690 (B) 4639 ± 940 (E) 19939 ± 3120 (T) 1648 ± 372 (T)		Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static	Wet Wet Wet Wet Dry Dry	Longitudinal Transverse Longitudinal Transverse Longitudinal Transverse	
Sedlin and Hirsch ¹⁴	Human	Femur	24100 - 31292 (B)		Quasi-static Quasi-static	Wet	Longitudinal	Bones from patients who who died of various causes.
Reilly and Burstein ¹⁵	Human	Femur	19290 (T) 27992 (C) 7397 (T) 19290 (C)	9863	Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static	Not Clear	Longitudinal Longitudinal Transverse Transverse Torsion	For torsion test, bone longitudinal axis coincides with that of specimen.
Kraus ¹⁶	Human	Femur	11840 (T) 15330 (T)	9800 8000	Quasi-static Quasi-static	Wet Dry	Longitudinal Longitudinal	Specimens from various places on femur. Avg. values given here.
Evans and Bang ¹⁷	Human	Femur Tibia Fibula	11800 (T) 14000 (T) 13500 (T)	9900 11500 11400	Quasi-static Quasi-static Quasi-static			Mech. properties related to micro-structure
Burstein, Currey, Frankel and Reilly ¹⁷	Bovine	Femur	24890 ± 1072 (T) 38687 ± 5959 (B)		Not Clear High Speed Test	Wet Wet	Longitudinal Longitudinal	Significant plastic action at these strain rates.
Saha ¹⁸	Human	Femur		7310 ± 2042		Wet	Longitudinal	Bone embalmed. Test machine crosshead speed = 2.1×10^{-6} m/sec.
Saha and Hayes ¹⁹	Human	Femur	13720 ± 1470 (T) 18320 ± 4800 (T) 14300 ± 3700 (T)		Quasi-static Impact Impact	Wet Wet Wet	Longitudinal Longitudinal Longitudinal	For impact $\dot{\epsilon} = 133$ sec ⁻¹ . Fresh bone in first 2 tests. Embalmed bone for last
McElhanev ²¹	Human	Femur	21800 (C) 26000 (C) 29000 (C) 32000 (C) 40500 (C) 46000 (C)		0.001 sec ⁻¹ 0.01 sec ⁻¹ 0.1 sec ⁻¹ 1 sec ⁻¹ 300 sec ⁻¹ 1500 sec ⁻¹	Wet Wet Wet Wet Wet Wet	Longitudinal Longitudinal Longitudinal Longitudinal Longitudinal Longitudinal	Bone embalmed.

3. Orientation of specimen relative to osteones
4. Moisture content
5. Microstructure
6. Test strain rate
7. Combined loading
8. Repetitive loading

Insofar as specimen preparation is concerned, a number of aspects of this problem have been discussed by Swanson,⁴⁰ Kraus,¹⁰ Sedlin and Hirsh,³⁴ Hermann and Liebowitz⁴¹ and Sedlin.⁴² Drying of bone will increase tensile strength and reduce shear strength as shown in Table 1, but this process can also increase elastic modulus and reduce ductility. Also the ratio of properties in the various directions can be altered by drying. Sedlin and Hirsh³⁴ indicated that such changes begin to occur almost immediately upon removal of the bone from a wet environment. Tests on the affects of embalming show conflicting results but the general conclusion from most studies was that a reduction of fracture strength could be attributed to embalming while the elastic moduli did not show significant changes. Freezing and subsequent heating to human body temperature was shown to induce no significant changes. Exposure to high temperatures, (greater than 100°C) on the other hand can reduce ultimate strength, while increasing elastic modulus. Machining operations were considered safe as long as sufficient lubricant is used to prevent heat buildup.

The influence of specimen site has been examined by Sedlin,⁴² Amtmann⁴³ and by Pope and Outwater.⁴⁴ Longitudinal fracture stress

increases as we scan from the bone ends to the shaft midsection, however radial and circumferential fracture strengths do just the opposite. With respect to quadrant of origin, we find somewhat higher strength (longitudinal) in Lateral and Anterior quadrants in comparison to the specimens obtained from the Medial and Posterior quadrants, however, these variations are not nearly so great as those observed in scanning along the length of the bones.

Somehow, the composite material made from a mineral, hydroxyapatite, and an organic matrix exhibits significantly greater strength than either constituent. As yet nobody has been successful in defining the mathematical relationship between strength and percentage of the various constituents. Two phase material expressions, which are perfectly adequate in many instances, break down here, most probably because of the lack of understanding about the nature of the bonding between apatite and collagen. Amtmann⁴³ examined, and mapped out variations in density of compact bone, and although the pattern is similar to that of tensile strength, an assumption of a direct correlation is not justified.

An examination of microstructural variations brings up more clearly defined correlations. Increasing porosity, for example is associated with a decrease in tensile strength. Also, osteones whose collagen fibrils are predominantly longitudinal in orientation are stronger. Since the appearance of osteones on a cross-sectional micrograph can depend on collagen fiber orientation (those with longitudinal fibers being lighter in colour) one can quantitatively relate

microstructure appearance to tensile and compressive ultimate strengths.^{36,45} Although one can reasonably assume an increase in strength, coinciding with an increased percentage of cross sectional area made up by osteones, one must take care to note whether this increase in percentage of area is due to a greater number of osteones, or rather due to an increase in size of a few osteones. More haversian systems, means more cementing substance between them, and since this cement is weak, more osteones can mean reduced strength. One last important microstructural variable relates to reconstruction, that is the replacement of primary osteones with secondary osteones. Currey⁴⁶ has found a decrease in breaking strength along with increased reconstruction, which is attributed to a decrease in mineral content, even though several other microstructural changes occur as secondary haversian systems are formed.

In terms of the measured mechanical properties, the nature of the testing methods can be as important as the composition and structure of the material being tested. Loading configuration, (e.g. tension, bending, torsion etc.) and rate of applied stress, are, as Table 1 indicates, quite influential. A great deal of attention has been given, in particular, to the affect of strain rate, with several researchers^{10,35,42,46,47} observing viscoelastic and plastic behaviour, and for which numerous mathematical models have been developed. In the upper limit of strain rate (i.e. impact) we note that tensile impact strength may increase by as much as 64% over the quasi-static value³⁹ and compressive impact strength may be more than 100% greater⁴⁸ than

its quasi-static counterpart. It would be naive to expect the *in-situ* loading to which a bone is subjected to be of a single type. Rather, we expect some combination of bending, torsion, compression or tension. Laboratory tests in which only one loading mode is used, provide little information about failure due to combined stresses, and the familiar failure theories such as maximum shear stress theory or maximum distortion energy theory are of no use since they are defined for isotropic materials only. Based on the work of Ashkenazi,⁴⁹ considerable progress toward the definition of a failure theory for bone has been made by Pope and Outwater.⁴⁴ In their impact studies, Lewis and Goldsmith⁴⁸ performed some experiments in which torsion and axial compression were combined. These tests showed that the presence of a compressive stress can bring about a reduction in the required torsional fracture stress. A much wider range of combined impact loading studies would have to be done, however, in order that any general conclusions about high strain rate failure could be made.

During normal activities, one's bones are subjected to repeated (cyclic) loading, but the stresses produced are, in general, much lower than the ultimate strength of bone. While such loading may produce damage on a microscopic scale, the normal physiological activity of osteocytes can usually effect repairs rapidly enough to maintain the integrity of the bone material. If the damage accumulates faster than it can be repaired, the results will be a fatigue failure. Such fractures can and do occur in heavy prolonged exertion such as marching or long-distance running. Experience with many materials has

shown that fatigue failures may occur at stress levels far lower than the ultimate strength. Some materials, such as steel tested in air demonstrate the existence of an endurance limit, which is a stress below which fatigue failure does not occur even at an extremely high number of load cycles. Although something akin to an endurance limit was found by Chamay⁵⁰ in tests on dog ulnas, there was no indication of one given by the tests of Carter and Hayes^{51,52} on bovine femoral specimens (at least not in the range of stresses examined). In the latter works, the influence of several factors on the number of reversals to failure was considered, and the following empirical relationship was derived:

$$\log (2N) = H \log \sigma + JT + K\rho + M$$

where: $2N$ = reversals to failure
 σ = stress amplitude (MN/m^2)
 T = temperature ($^{\circ}\text{C}$)
 ρ = density (gm/cm^3)

The values of the empirical coefficients H , J , K and M are:

$$\begin{aligned} H &= -7.837 \\ J &= -.0213 \\ K &= 4,464 \\ M &= 12.043 \end{aligned}$$

Although one would expect little temperature variation in a deep bone such as the femur, density variations may be great, and the large value of the constant K signifies considerable sensitivity of fatigue life to density changes. It was further shown⁵² that the number of reversals to failure is inversely proportional to the percentage of secondary

osteones. Secondary haversian systems are less dense than primary ones, however the difference in fatigue resistance is much greater than what would be predicted based on a density difference alone. The conclusion reached was that secondary bone is inherently weaker than primary bone and since the fraction of secondary osteones increases with age one could further conclude that fatigue failures increase in likelihood as one ages.

If one accepts the proposition that a fracture has already occurred, and an internal fixation has been performed, then the problem of defining a system failure criterion takes on a new complexity. Clearly a fracture of metal or adjacent bone (possibly weakened by the presence of screw holes) constitutes a failure, however any stress which delays or prevents bone union would produce a clinical failure of no less importance. The healing process can require a great deal of time, and during this time the strength of the callus is much less than that of its surrounding bone matrix. As stated previously, Wolff's Law indicates that bone remodelling, and healing can be augmented by stress, and in fact we find that a certain amount of compressive stress at the fracture site is desirable, while tensile stress is probably harmful in that it may cause distraction of the fracture surfaces. Too much compressive stress, however can be just as bad as too much tension since it may cause pressure necrosis⁵³ (tissue death). Thus, there is a narrow range of stresses within which the system can operate safely and within which healing is not impeded. Quantitative data relative to this question is rather limited, however in their work with sheep,

Perren et al.⁵⁴ found no incidence of pressure necrosis at stress levels as high as 5800 psi. There do not, unfortunately, seem to be any data on the tensile strength of healing human bone, for obvious reasons, and extrapolation of results from animal research may be of only limited value. It is conceivable however that noninvasive techniques such as ultrasonics may one day provide quantitative information on bone strength at a fracture site during various stages of healing.

Bone Elastic Constants

The measured values of bone elastic constants are susceptible to the same factors which affect the strength, however the trends may not be the same for the two properties. A change in some preparation technique or loading parameter could cause an increase in strength and yet may do exactly the opposite to elastic modulus. Hence, these factors must be examined once again, with regard to their influence on elastic moduli.

Table 2 presents several values of experimentally determined bone elastic constants, both from quasi-static and from dynamic tests. The affects of numerous variables are tabulated there, however the information is supplemented in the paragraphs which follow.

As has been mentioned in the previous section, drying in air at room temperature or high temperature can increase elastic modulus. It must also be mentioned however, that drying can markedly alter the shape of the stress-strain curve. Bonfield and Datta⁵⁶ note that wet bone exhibits considerable plastic deformation, which is reduced greatly by air drying. Reilly, Burstein and Franke⁵⁸ have similar

Table 2. Elastic Constants of Human Cortical Bone Obtained by Static, Dynamic and Ultrasonic Testing

REFERENCE SOURCE	SPECIES	BONE	MODULUS OF ELASTICITY (psi x 10 ⁶)	STRAIN RATE	WET OR DRY	DIRECTION	COMMENTS
Kraus ¹⁰	Human	Femur	2.27(T) 2.67(T)	Quasi-static Quasi-static	Wet Dry	Longitudinal Longitudinal	Average of values from several location on femur.
McElhaney ¹¹	Human	Femur	2.2(C) 2.5(C) 2.6(C) 3.2(C) 4.3(C) 5.9(C)	0.001 sec ⁻¹ 0.01 sec ⁻¹ 0.1 sec ⁻¹ 1 sec ⁻¹ 300 sec ⁻¹ 1500 sec ⁻¹	Wet Wet Wet Wet Wet Wet	Longitudinal Longitudinal Longitudinal Longitudinal Longitudinal Longitudinal	Compression on small cubes
Dempster and Liddicoat ¹²	Human	Femur	2.6 ± .167 (C) 2.06 ± .117 (C) 2.69 ± .42 (T) 1.73 ± .128 (T) 1.77 ± .2 (C) 1.04 ± .34 (C) 1.09 ± .16 (C) 1.27 (C) .62 (C) .73 (C)	Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static Quasi-static	Dry Wet Dry Wet Dry Dry Dry Wet Wet Wet	Longitudinal Longitudinal Longitudinal Longitudinal Longitudinal Radial Tangential Longitudinal Radial Tangential	Cylindrical specimens used for longitudinal modulus determination. Cubical specimens used for direction dependency measurements.
Reilly and Burstein ¹³	Human	Femur	1.46 ± .34 (T) 2.60 ± .57 (T) 2.04 ± .48 (T) 2.65 ± .66 (T) 1.70 ± .14 (C) 2.64 ± .12 (C)	0.02-0.05 sec ⁻¹	Wet Wet Wet Wet Wet Wet	Transverse Longitudinal Transverse Longitudinal Transverse Longitudinal	Composite of bones of people aged 41 female, 49, 61, 71 Male. Last 2 tension tests from 21 year old male. Significant plastic action for wet bone demonstrated.
Bonfield and Datta ¹⁴	Bovine	Tibia	3.96(T)	3x10 ⁻⁶ sec ⁻¹	Wet	Longitudinal	Significant an elastic strain in wet bone, recoverable at stress levels as high as 10,000 psi.
Evans and Bang ¹⁵	Human	Femur Tibia Fibula	2.05(T) 2.2(T) 2.4(T)	Quasi-static Quasi-static Quasi-static	Wet Wet Wet	Longitudinal Longitudinal Longitudinal	Embalmed bone from individuals who died of various causes (no bone disease).
Berne, Mengi and Inger ¹⁷	Bovine	Femur	3.07 1.91	Quasi-static Quasi-static	Wet Wet	Longitudinal Transverse	Data from tension, torsion and hydrostatic compression of strain gauge instrumented specimens.
Saha and Hayes ¹⁸	Human	Femur	2.11 ± .5 (tangent modulus) 1.8 ± .42 (secant modulus)	Quasi-static Quasi-static Quasi-static	Wet Wet Wet	Longitudinal Longitudinal Longitudinal	
Evans and Vincentelli ¹⁹	Human	Tibia	2.799 ± .805 (C) (average) 3.387 ± 1.061 (C) (26 yr. old male) 2.675 ± .806 (C) (75 yr. old male)	Quasi-static Quasi-static Quasi-static	Wet Wet Wet	Longitudinal Longitudinal Longitudinal	Bone not embalmed. First value is avg. from 6 males. Other 2 values are avg. from several places in bone. Note E decreases as age increases.
Reilly, Burstein and Frankel ²⁰	Human	Femur	2.48 ± .457 (T)	0.05 sec ⁻¹	Wet	Longitudinal	Av. value from 196 tests. Bilinear σ-ε curve for tension. Elastoplastic curve for compression.

Table 2. (Continued)

REFERENCE SOURCE	SPECIES	BONE	MODULUS OF ELASTICITY (psi x 10 ⁹)	STRAIN RATE	WET OR DRY	DIRECTION	COMMENTS
Currey ⁴⁶	Bovine	Femur	3.29(T)	0.00013 to 0.16 sec ⁻¹	Wet	Longitudinal	Avg. value given. Correlation between E and ϵ discussed.
Lewis and Goldsmith ⁴⁷	Bovine	Tibia	3.3(C)	Impact load duration 10 - 150 μ sec	Semi-dry	Longitudinal	Split Hopkinson bar impact test used.
Bargren, Bassett and Gjelsvik ⁴⁸	Human	Femur	2.29 1.62 1.81	Cyclic loading 7.5 Hz to 0.66 Hz	Wet Wet Wet	Longitudinal 30° from long. 45° from long. axis	Sinusoidal loading (alternating tension and compression).
Sedlin ⁴⁹	Human	Femur	2.0(B)	Quasi-static	Wet	Longitudinal	Fresh bone. Avg. of 43 samples given.
Abendschein and Hyatt ⁴⁰	Human	Tibia	2.86 ± .19(B) (normal bone) 2.36 ± .36(B) (diabetic female) 1.58 ± .39(B) (disuse osteoporosis) 3.53 ± .18 (normal bone) 2.67 ± .17 (diabetic female) 1.98 ± .30 (disuse osteoporosis)	Quasi-static Quasi-static Quasi-static Ultrasonic Ultrasonic Ultrasonic	Wet Wet Wet Wet Wet Wet	Longitudinal Longitudinal Longitudinal Longitudinal Longitudinal Longitudinal	Linear relationship between static and ultrasonic E values.

conclusions and show, for fresh bone a bilinear load-deflection curve in tension and an elastoplastic type curve for compression. Embalming, or freezing with subsequent thawing, however, show no significant effects.

Sampling site has been looked at by Sedlin⁴² and by Pope and Outwater.⁴⁴ In looking at specimens from human mid-femoral diaphyses, Sedlin observed nearly equal values of longitudinal Young's modulus in the lateral, anterior and medial quadrants, but significantly lower E values for the posterior quadrant. In scanning from proximal to distal epiphyses in bovine tibial, Pope and Outwater found that longitudinal modulus increased in moving from the ends to the middle of the shaft while radial and circumferential moduli do the opposite.

Composite material theorems have been mentioned in the previous chapter with regard to their use in predicting bone strength, however, the theories have primarily been aimed at prediction of elastic moduli.^{61,62} Katz has compared three such models, attributed to Voight, Reuss, and Hashin and Shtrikman. Comparison plots based on the three formulae show that these models provide upper and lower bounds on modulus values for the composite. The models assume two material phases, collagen and hydroxyapatite, but fail to consider the amorphous ground substance and the amorphous mineral constituent (part of the hydroxyapatite may be amorphous). The three formulae are thus approximate at best. Unfortunately, very few of the papers in which experimental E values for bone are presented, provide bone composition. Such data would be needed in order to evaluate any composite theory and hence

there is as yet no way to know which model is best. In addition, the spread of the three predictions, increases greatly as the ratio of moduli for the constituents increases. For a collagen-hydroxyapatite composite the spread may be close to an order of magnitude. Until such time as the other bone constituents are accounted for, it would be unwise to rely on such predictions as these for data to be used in mechanical behaviour studies.

Variations in the microstructure of cortical bone show definite correlations with strength as has been previously discussed. Elastic moduli too can be affected by alterations in size and number of microstructural elements. Evans and Vincentelli,⁴⁵ who examined appearance of osteones (light vs. dark), which in turn reflects the collagen fiber orientation within each haversian system, found relatively little correlation between measured E values and percentage of various kinds of osteones. Evans and Bang,³⁶ on the other hand, examined other features of the microstructure and found a strong negative correlation between volume percentage of porosity and elastic modulus. In addition they observed a negative correlation between elastic modulus and the number of osteones, which stems from a corresponding increase in the amount of cement between the osteones. The value of E (longitudinal) is also sensitive to the amount of reconstruction⁴⁶ (percentage of secondary osteones) which is attributed in part to the decrease in mineral content that comes with increased reconstruction.

The mechanical behaviour of bone is influenced by strain rate variations as has been shown by McElhaney⁵⁵ (see Table 2), who observed

an increase in E (longitudinal) as strain rate increased. The magnitude of the E variation is not very great, as discussed by Curry⁴⁶ who observed that a 1000-fold increase in strain rate yielded only a 10% rise in elastic modulus. One of Curry's empirical formulas is:

$$E = 13.5 + 1.37A + .75\dot{\epsilon}$$

where: E = elastic modulus(longitudinal) in GN/m²
A = ash weight percent (percent of bone mineral)
 $\dot{\epsilon}$ = strain rate

At very high strain rates, one interesting additional effect is seen. The maximum strain at failure decreases. This was seen by McElhaney⁵⁵ in tests where strain rate was 300/sec. and 500/sec. Saha and Hayes³⁹ corroborate this last effect with their impact tests.

Observations of non-linearity, and strain-rate sensitivity in the stress-strain behaviour of bone have prompted numerous investigations, both experimental and theoretical. That bone, under certain conditions, behaves as a viscoelastic material is well accepted, although there is, as yet, disagreement as to the best mathematical representation of the viscoelasticity. For the most part, linear viscoelasticity is assumed,^{42,47,59,63} however Lakes and Katz⁶⁴ consider this to be an oversimplification. Unfortunately, without experimental data pertaining to the viscoelastic nature of all the constituents of cortical bone, the development of a model which serves under all loading conditions remains an unreachable goal.

When dynamic loading is applied in a repetitive cyclic fashion, one possible consequence is a fatigue failure. Such loading, however, may

also be applied as a means to gain information about the elastic behaviour (or inelastic behaviour) of a material. The measured E values may differ greatly, depending upon frequency of cycling and on magnitude of applied loads. In general, an increase in E values with frequency has been observed, which, as noted by Black and Korostoff,⁶⁵ is a characteristic of viscoelastic materials.

One cyclic loading technique, involving ultrasonic frequencies yields a relatively large amount of information, for a given number of experiments, simply because of the ease with which the several elastic constants of an anisotropic material can be determined. Sound wave propagation velocity within an elastic continuum is related to material density and elastic modulus as follows:

$$\text{velocity} = \sqrt{\frac{\text{Modulus of Elasticity}}{\text{Density}}}$$

Measurement of density, plus propagation velocities in various directions (relative to the longitudinal axis) using both longitudinal and transverse waves, provides sufficient data for a determination of the complete constitutive matrix for a given material.

Using a driving frequency of 100 kHz, Abendschein and Hyatt⁶⁰ measured the longitudinal modulus in midshaft cortical bone specimens from human femora and tibiae. The ultrasonic values were consistently higher than static E values (obtained in bending tests) and the relationship between static and ultrasonic moduli was found to be a linear one.

In his study of bovine bone, Lang^{66,67} assumed hexagonal symmetry (also referred to as transverse isotropy) based on an examination of

the microstructural arrangement of cortical bone constituents. Although such a material has only 5 independent stiffness constants C_{ij} (as shown below), Lang performed 8 wave propagation velocity measurements, and used the redundant measurements for accuracy checks.

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11}-C_{12}) \end{bmatrix}$$

Further measurements of this kind have been made by Yoon and Katz^{68,69,70} who support Lang's assumption of hexagonal symmetry, based on ultrasonic measurements as well as on the observed microstructure. A comparison of elastic constant values from Lang and from Yoon and Katz is given in Table 3. Note that in both cases the 3 axis was used as the axis of symmetry, and that this axis is parallel to the bone's longitudinal axis. Elastic constant values are specified in psi. In order to compare these data effectively with those obtained by other means, it is necessary to compute the more familiar engineering material constants. Using formulas presented by Tsai⁷¹ the following constants are obtained:

Table 3. Ultrasonically Obtained Elastic Stiffness Coefficients (Units of psi)

Source	Bone	Wet or Dry	C ₁₁	C ₁₂	C ₁₃	C ₃₃	C ₄₄
Lang ^{66,67}	Bovine Femur	Dry	3.45+ ₋ .2x10 ⁶	1.48+ ₋ .08x10 ⁶	1.62+ ₋ .3x10 ⁶	4.84+ ₋ .17x10 ⁶	1.18+ ₋ .03x10 ⁶
	Bovine Phalanx	Dry	3.07+ ₋ 1x10 ⁶	1.38+ ₋ .04x10 ⁶	1.48+ ₋ .2x10 ⁶	5.42+ ₋ .23x10 ⁶	1.09+ ₋ .03x10 ⁶
	Bovine Phalanx	Wet	2.86+ ₋ .07x10 ⁶	1.75+ ₋ .06x10 ⁶	1.83+ ₋ .17x10 ⁶	4.64+ ₋ .16x10 ⁶	.78+ ₋ .01x10 ⁶
Yoon and Katz ⁶⁹	Human Femur	Dry	3.39+ ₋ .04x10 ⁶	1.31+ ₋ .05x10 ⁶	1.32+ ₋ .08x10 ⁶	4.71+ ₋ .06x10 ⁶	1.26+ ₋ .01x10 ⁶

$$E_{\text{longitudinal}} = 3.974 \times 10^6 \text{ psi}$$

$$E_{\text{transverse}} = 2.732 \times 10^6 \text{ psi}$$

(Ref: Yoon and Katz⁶⁹)

We have seen that bone, a viscoelastic composite material, exhibits a wide range of elastic constant values, depending on preparation of specimens and on loading technique. In selecting appropriate data for use in modelling an *in vivo* bone, one must look for experiments whose conditions most nearly approximate the *in situ* case. It would seem that the ultrasonic methods are rendered invalid by this argument, however, there is one important similarity between the low strain rates associated with normal *in vivo* activity and the very high strain rates and low amplitudes of ultrasonics. In both cases, the viscous component of the material's mechanical response is not seen (or at least not to the extent which is measured at strain rates on the order of 1/sec. to 500/sec.). Thus, one may conclude that data such as those obtained by Yoon and Katz⁶⁹ may be effectively used in continuum mechanical models of in-situ bone. With the full set of elastic constants, so obtained, one can develop a fully anisotropic linearly elastic model. The work of Bargren et al⁵⁹ provides further support for the use of a linear stress-strain relationship. Bone specimens, loaded axially via a sinusoidally varying driving force, were instrumented with electrical resistance strain gages. The associated electrical circuitry was calibrated so as to provide two outputs, one proportional to axial stress and the other proportional to axial strain. These outputs were used to drive the horizontal and

vertical deflections on an oscilloscope and the resulting patterns provide a measure of the degree of influence of the viscous effects. At low strain rates (in the range of normal physical activity) the ellipse on the screen narrows, approaching a straight line which is the pattern for linearly elastic behaviour. At higher strain rates (period of cycle = .135 sec.) the ellipse widens indicating an increased energy loss due to viscous effects. So at the lowest strain rate (period of cycle = 1.5 sec.) the viscous component, although not zero, is very small and may be neglected.

If we examine the clinical situation of a fractured bone, supported by some surgically applied internal fixation, the additional question as to the mechanical properties of healing bone must be posed. In the discussion of the healing process, it was noted that a sequence of morphological changes occur with time at the fracture site. As yet, however, no quantitative data exist to show the corresponding variations in mechanical properties. There is an obvious need for a clinical test which would provide reproducible quantitative assessment of callus strength and elastic response, since it could permit accurate definition of time for cast or implant removal or it could allow early prediction of non-union. At present, two main techniques are being tried, one involving the mechanical response of *in-vivo* bone to an impact,⁷² and the second incorporating the observation of response to ultrasonic pulses.⁷³ In both instances, a characteristic wave-shape indicates the presence of a fracture, but as yet, no satisfactory numerical relationship between the characteristic wave and the

RF 1.47

mechanical properties of the callus exists. Thus we must continue to rely on the visual inspection of X-rays, which while appropriate in many cases, may in other instances lead to erroneous conclusions.

Stress Analysis of Intact Bones and Plated Bones

The degree of success or failure of any analysis of a physical system will depend upon the validity of the simplifying assumptions which are invariably made or on how the results are interpreted in light of those assumptions.

One of the earliest mathematical models, developed for a study of stress in bones was that of Koch⁷⁴ who utilized the methods of statics and strength of materials (predominantly simple beam theory) to determine the pattern of stresses in a human femur. In this analysis, Koch neglected muscle force loading, and utilized a body weight load of 100 lb., which, while obviously low for adults, nevertheless allowed a calculation of relative stress values. Through a laborious set of hand calculations, Koch computed bending, shear and axial stresses at various cross sections along the length of the Femur. One of Koch's significant conclusions was that the pattern of trabeculae (plates of spongy bone) at the bone ends coincides with stress trajectories (lines of constant principal stress) at those sites.

In more recent times, other investigators have followed Koch's lead, utilizing not only simple beam models, but also applying methods from elasticity, experimental mechanics and numerical approximations in the form of finite difference or finite element computer programs.

Toridis⁷⁵ refined the mechanics of materials approach by using vector methods to compute stress resultants at various cross sections along a human femur. Knowing the geometrical properties at these sections (i.e. area, second moment of area, first moment of area) it was possible to compute components of the stress tensor in local coordinates defined at each section. Subsequent computation of principal stresses, showed, as in Koch's work, that stress trajectories parallel the pattern of trabeculae.

The finite difference computer program STRESS, written by Piotrowski and Wilcox⁷⁶ provided an improved method for examining torsional and bending stresses, but was limited in that hollow cross sections could not be considered.

Finite element based models of bones, with and without attached implants have proliferated within the last few years, and seem to be very promising in terms of their ability to accurately predict stress distributions in bone. Rybicki, Simonen and Weis⁷⁷ have compared stress calculations from simple beam theory with those from a two-dimensional plane stress finite element model. The results, although of a similar magnitude in the femoral midsection, differed greatly at the bone ends. For applied loading, these authors included the effect of muscle forces in the hip abductor and the tensor fasciae latae as well as an axial load due to body weight. Both analytical techniques showed that the magnitudes of stresses in the bone cortex may be greatly altered by variations in these muscle loads. These authors also examined the question of stress trajectories and bone trabecular

patterns but declined to attribute any causality to this relationship since the pattern of stress trajectories seemed to depend greatly on the analytical method used.

In a similar study, Wood, Valliapan and Svensson⁷⁸ considered a similar pattern of loading in their two-dimensional finite element model of a human femur. The loading, due to body weight, hip abductor muscles and tensor fasciae latae corresponds to a one-legged stance, which, according to these authors is quite similar to the stance phase in slow walking. As in the previous study, the bone was assumed to behave as a linearly elastic isotropic material, however, by varying elastic moduli in the different elements, these authors accounted for the inhomogeneity. In this study too, the importance of including muscle forces in the analysis was demonstrated, since the bending stresses in the femur can be altered greatly by changing force in the tensor fasciae latae.

In another two-dimensional analysis, Viano and Khalil⁷⁹ examined a femur midsection, under an assumption of plane strain. Although these investigators did not include muscle loading, they did consider dynamically applied joint loading at various loading rates (load duration from 3 to 75 milliseconds). Peak stress responses depend upon load duration with "overshoots" of static stress levels for pulse durations of 15-60 ms and "undershoots" for 3 to 15 ms. They further suggested that peak impact loads necessary to induce fracture could vary on the order of $\pm 30\%$ depending upon pulse duration.

In their study of stresses in a human tibia, Piziali, Hight and Nagel⁸⁰ utilized a pair of programs to perform a 3-D (beam element) finite element analysis. The first program provided values of the St. Venant warping function for various irregular cross sections of the tibia, which can be used in calculating the torsional response at these sections. The geometrical properties of each cross section were utilized as part of the input data for a finite element code which contained 3-D beam elements. With this second code, calculations of deflections and stresses at several locations along the tibia were performed. The authors suggest that there is a reasonable correlation between computed stresses and the experimental results of others. In addition, it is suggested that their results are an improvement over those from two dimensional analyses since they have accounted for the irregular cross section geometry of the tibia and also because their beam analysis does consider the axis of the tibia to be a curve in space rather than just lying in a plane. This study also draws attention to the importance of using accurate values for elastic constants.

The study of stress distribution within intact bones provides important information about bone mechanical response and will lead to refined definitions of failure criteria for bone. If, however, one considers the situation in which a fracture has already occurred, and for which some supportive device is required, the analytical model serves a somewhat different purpose. In this latter instance, the mathematical analysis of stresses will aid in the evaluation of the

particular device, and will provide information necessary for improved design of implants.

Mathematical analysis and experiments have been done in an effort to define the mechanical response of a plated bone. The experimental work of Martz⁸¹ provides some of the earliest data relative to the strength of a plated bone. Although Martz did not discuss stress distribution, his work provides a relative measure of bending strength of various long bones and various plates and pins. When plated bones were loaded in bending such that the plates were on the tension side, the resulting strengths were in general much lower than those for intact bones. The plated femur, for example, carried only about half of the load that the intact femur bore prior to breaking.

In another experimental study, Lindahl⁸² plotted deflection vs. applied load in bending and torsion for a number of human femora with "fractures" simulated by transverse sawcuts and plated with a wide variety of commercially available plates. While such empirical data may be of value to the clinical practitioner, they do not provide the kind of specific information necessary for design improvement. The experimental study of Laurence, Freeman and Swanson,⁸³ although similar, did provide some new information. Firstly, these authors included other types of metal implants, such as intramedullary nails, but none of their preparations had the stiffness of an intact bone (in this case human tibiae were used). Secondly, information about the forces in screws used to attach onlay plates, was generated through the use of specially designed strain gage load cells. Even in situations

where bending loads produced large deformations of the plated bones, no screws were subjected to tensile forces sufficient to cause pullout from bone. Furthermore, they suggested that any more than four screws, used in plating a tibia, would be redundant. On the other hand, they observed that for situations in which a metal device was required to support all of the forces imposed by weight bearing, few of the available devices would do so without breaking.

One of the most extensive experimental studies of metal bone plates was the "parametric" study of Ray et al.^{6,7,84} In a planned series of tests, collectively referred to as a "factorial experiment" a number of artificially fractured (via saw cut) equine third metacarpal bones were repaired by different plates. By manufacturing their own plates these authors were able to look at the influence of various dimensional factors, such as plate length, plate width and screw diameter. Plated bones were loaded in bending until the composite ruptured and the failure producing load was compared to that required to fracture an intact specimen from the same bone. By this means, they found plate length to exert a significant influence on the capacity of the structure, and also developed an empirical failure parameter which allows classification of a device as either "safe" or "unsafe." It should be noted at this point that the requirements for implant usage in horses are much more stringent than those for humans since, firstly, the loads are greater with the horse and, secondly, an animal such as the horse cannot remain lying down for several weeks while bone heals. (A horse may develop skin ulcerations at points of contact with the ground if forced to remain lying down.)

In an effort to delineate the stress pattern in a plated bone, Askew, Mow, Wirth and Campbell⁸⁵ utilized a two dimensional photo-elastic model to study the stress field produced by compression plating. The technique of compression plating consists of applying a tensioned steel plate across a fracture so as to force the fracture surfaces together. Unfortunately, from their experiments and from a theoretical analysis based on simple beam theory, Askew et. al. concluded that only a fraction of the fracture surfaces, directly beneath the plate, would be in contact, while on the opposite cortex the bone surfaces would be distracted. This distraction occurs because the plate tension creates a bending moment in addition to compression. Since 80 percent or more of the fracture surface area may not be in contact, the observed clinical success of compression plates must come from increased stability and immobilization afforded by the compression technique. This study did not include the effects of weight and muscle loading, however, based on the work of others, Askew et al. suggest that even in the presence of these loads the fracture gap may not close completely.

In another comparison of experimental vs. theoretical results, Rybicki et al.⁸⁶ compared strains from gages on a plated equine third metacarpal to those predicted by a simple beam theory analysis. Although the theory gave fairly accurate results, assumptions upon which the analysis was based were not satisfied by all of the experimental conditions, so a refined analysis was done by means of a two-dimensional finite element model. Although the results from this computer

model were only preliminary, these authors did show how important it is to have the metal plate on the tension side of the bone. This causes the bone to carry a large fraction of applied loads and reduces stresses and strains in the plate dramatically when compared with the situation in which a plate is on the compression side. In their concluding remarks Rybicki et al.⁸⁶ suggest that the finite element method could serve as the basis for a theoretical parametric study to parallel the experimental one of Ray et al.⁷

Compression plating has also been modelled by Plant and Bartel⁸⁷ using a two dimensional finite element model. This model provides further evidence of the fracture surface distraction effect, as reported by Askew et al.⁸⁵ By means of this technique, Plant and Bartel developed an empirical formula which allows prediction of the linear combination of applied compressive load and bending moment necessary to just reunite the distracted surfaces. They also examined the effect of misalignment of fracture surfaces or plate, and found large changes in the stress distribution resulting from these errors in alignment.

In another series of finite element analyses of plated bones, Simon et al.^{88,89} compared one-, two- and three-dimensional finite element models to each other and to an experiment in which a plated (i.e. with bone plate) aluminum tube represented a bone. The one- and two-dimensional models provided reasonable results in locations which were far removed from geometrical irregularities such as the regions near the bone screws. Only the three-dimensional model gave accurate

results in these areas. One factor which was not considered in these analyses was the effect of a fracture. In the experiment, and in all the finite-element meshes, the "bone" beneath the plate was intact. Nevertheless, the good correlation between the experimental and theoretical results strongly supports the use of the finite element method in orthopaedic implant studies. After thus verifying the efficacy of the three-dimensional finite element model, Simon et. al. went on to examine in detail the theoretical prediction of stress distribution. All computed stresses were found to be in the elastic range, however the authors note that under dynamic loading conditions, that might not be the case. Insofar as the metal implant was concerned, the largest stresses were found to be in the outermost screws, which makes these screws the most likely candidates for fracture failure. An examination of shear stresses between the screws and the bone cortices, gives rise to the suggestion that the outermost screws tend to be pushed out on the nonplated side while the inner screws tend to be pulled out on the plated side. When stresses in the plated "bone" were compared with theoretical (simple beam theory) calculations of bending stresses in an intact bone, it was found that the plate has the effect of reducing the stress in the bone. Since, according to Wolff's law, bone is deposited or resorbed in response to changing stress levels, these authors conclude that there is a possibility of a reduction in bone mass beneath a rigid plate.

If one considers a bone-plate system to comprise a crude two-component composite material, it can readily be shown that a steel plate,

having a much greater elastic modulus than the bone beneath, would carry a proportionately greater fraction of the total applied load. For this simple example we have:⁶¹

$$W_{st} = P \left\{ \frac{V_{st} E_{st}}{V_{st} E_{st} + V_b E_b} \right\}$$

where: W_{st} = the portion of total load carried by steel

P = total load

V_{st} = volume fraction of steel

E_{st} = elastic modulus of steel

V_b = volume fraction of bone

E_b = elastic modulus of bone.

The use of a plate material whose elastic modulus is closer to that of bone has been suggested, however, as shown by Plant and Bartel⁸⁷ this could lead to an increased distraction of fracture surfaces. A trade-off of sorts is required, since the plate must be sufficiently rigid to maintain adequate reduction of the fracture and yet must not reduce the load borne by cortical bone so much as to cause excessive resorption.

The problem of reduced stresses beneath a rigid plate was explored further by Woo et al.⁹⁰ Predictions, based on a one-dimensional finite element model were compared with results from experiments involving dogs. Two plate materials were used, one being Vitallium (A Cobalt-Chromium alloy) and the other being a graphite fiber reinforced methyl methacrylate matrix composite having roughly one tenth the stiffness of the Vitallium. By using the theoretical model, Woo et al. predicted

that the stresses in a plated bone would be lower than those in an intact bone, and that this reduction would be more pronounced in case of the more rigid Vitallium plate. This hypothesis was verified, at least for the Vitallium plate, by the experiments. After 9 and 12 month periods, dogs were sacrificed and the femora removed. Bone from the vicinity of the plates was subjected to mechanical and histological studies, and a marked thinning of the cortex beneath the Vitallium plated bones was observed. There seemed to be no difference in the structure of the bone, only in the amount. Similar experimental results were obtained by Tonino et al.⁹² who plated dog femora with implants of stainless steel and a flouorocarbon plastic.

In addition to plate mechanical properties, a number of other factors related to geometry, and manner of plate application may greatly influence the *in-situ* behaviour of the device. Using a two-dimensional finite element model, Rybicki and Simonen⁹² loaded a plated bone having a 45° oblique fracture (i.e. the normal to the fracture plane was set at 45° from the bone's longitudinal axis). In this study of compression plating, the effects of plate tension, weight-bearing loads and bone screw tightening and screw orientation were considered. With plate tensioning but no weight bearing load, the percent of the fracture surfaces in contact was very small. Tightening the screws was found to increase this area of contact, and applying weight bearing loads increased it still further. Although compressive stresses on the fracture surfaces have been predicted, it is not known whether

stresses of such magnitude are conducive to augmented bone growth. Studies of this kind do provide insight into the mechanical response of this complex system, but as Rybicki and Simonen point out, only through comparison with clinical results will a full understanding of bone plating be made possible.

Closure

The many-faceted problem of examining a bone-plate-screw structural system has clearly generated considerable interest among biomechanics researchers. There is an apparent need for an investigation of the influence of several system parameters on the mechanical behaviour of this system, rather than just one or two parameters as in previously published accounts. Such a parameter sensitivity study, implemented by means of a three-dimensional finite element model, is described in the sections that follow.

III. A THEORETICAL MODEL

Development and Implementation of a Three-Dimensional, Finite Element Based, Model of Plated Human Femur

The viability of the finite element method as a tool for use in biomechanical analysis has been demonstrated by other researchers as stated. The present study expands the base of knowledge about the bone-plate-screw system in two ways. First, the three-dimensional finite element model used incorporates a much larger number of system variables than has heretofore been considered in any single study. Second, establishment of the degree of significance of those system parameters over which the surgeon may exercise control, also defines preliminary guidelines for the selection of an optimum implant to fit any given situation.

For the purposes of this study, a three-dimensional finite element model was developed using the SAPV⁹³ proprietary computer code. The majority of the elements in the mesh are of the 21-node isoparametric type, however a special no-tension element was developed by this author to accurately simulate the mechanical action at mating bone fracture surfaces. The entire package was run repeatedly (using several sets of input data) on an IBM model 370 computer.

Pertinent details about the mesh used, the no-tension element, the applied loading and the material constitutive matrices are presented in the section which follow.

The Mesh and Mesh Scaling

The largest part of the input data for most finite element computer programs is that data which defines the mesh, which represents the discretization of the continuum to be modelled. Two meshes defining plated bones have been prepared, for 6-hole and 4-hole bone plates. Figures 4 and 5 are computer plots of these meshes. The analysis (and these plots) has been done by means of the SAPV⁹³ computer program, which allows modelling of fully orthotropic linearly elastic behaviour in three dimensions. From the mesh plots, it is seen that a plane of symmetry has been assumed, so that only half of the structure to be considered need be discretized. The implicit assumption of symmetry in loading as well as geometry is thus made (see section on applied loading). Another feature of the two meshes is the planar "gap" in the middle which represents a fracture of the bone. All elements, except for the special no-tension elements in the "gap" are of SAPV's variable-number-nodes three-dimensional isoparametric element⁹⁴ type.

The input data for SAPV is stored on a DEC-10 computer, which allows one to interactively modify such stored data. In order to selectively change several dimensional features of each model, a special program SCALE was written for the DEC-10 computer. The SCALE program accepts, as input, a complete SAPV data input file. In response to user commands, typed in on a terminal, SCALE selectively changes the coordinates of the appropriate nodes so as to effect the specified modifications. Figure 6 and 7 show, for the four-hole plate,

MODEL OF HUMAN FEMUR WITH BONE PLATE IN SITU
UNDEFORMED SHAPE
JUNE 04. 1979 03:50:09
IAXIS= 2 ALPHA= 20.00 BETA= -45.00

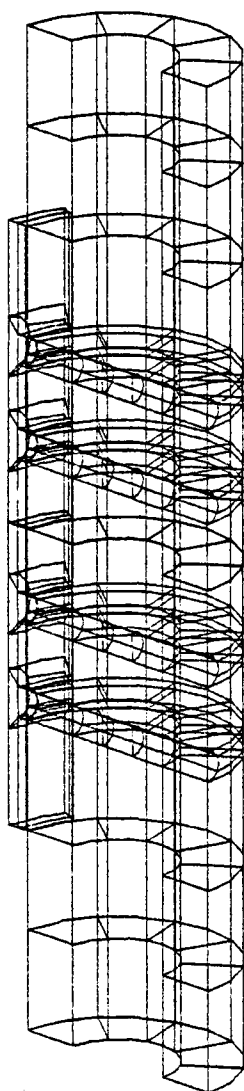


Figure 4. Computer Plot of Bone, Plated with 4-hole Plate

HUMAN FEMUR WITH 6-HOLE BONE PLATE IN SITU
UNDEFORMED SHAPE
JUNE 29, 1979 00:42:29
TAXIS= 2 ALPHA= 20.00 BETA= -45.00

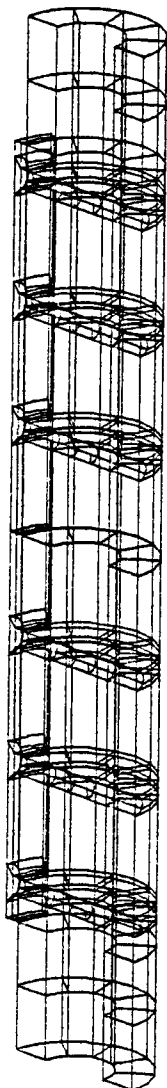
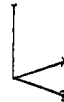


Figure 5. Computer Plot of Bone, Plated with 6-hole Plate

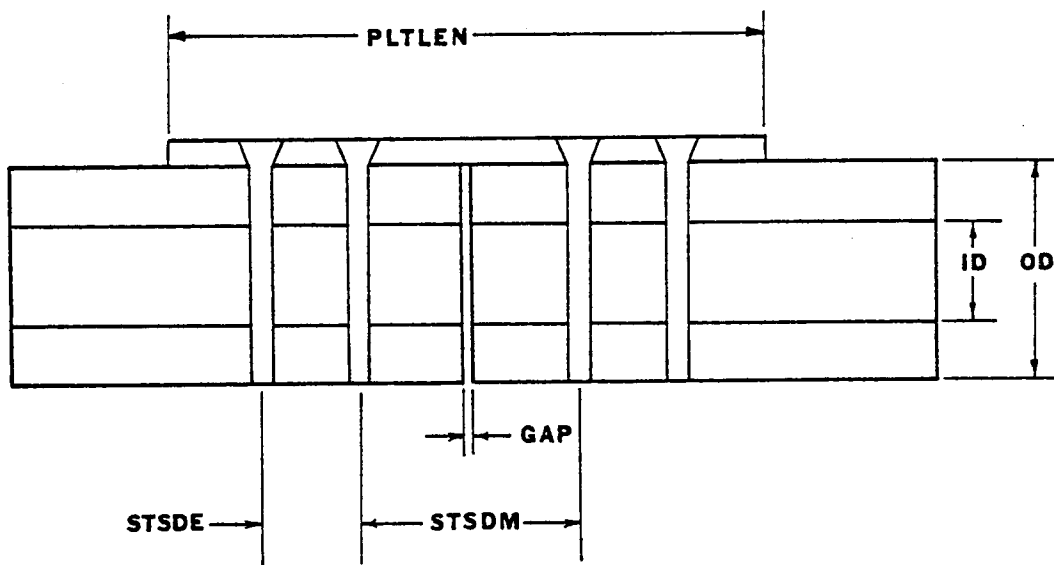


Figure 6. Plated Bone Showing Dimensions which can be Modified by Program SCALE

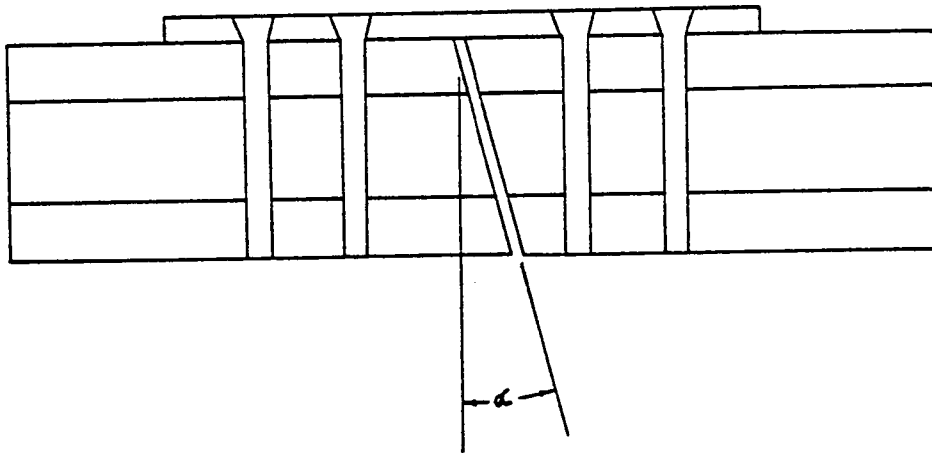


Figure 7. Plated Bone Showing Angle α which can be Modified by Program SCALE

those dimensions (and one angle) which the scaling program may modify. SCALE is not a general purpose program but has been written specifically for this application. There are two versions of SCALE, one each for the four- and six-hole plates. Examples of meshes which have been modified by SCALE are shown in Figures 8 and 9.

A No-Tension Element

Immediately following the surgical reduction of a fracture, there is no permanent bond between the fracture surfaces. They are held together by surrounding tissues and possibly by some implant. As has been stated in the section on fracture healing, as much time as 12 months may be necessary for full restoration of normal bone strength. Thus, surgeons must caution their patients against premature weight bearing on the affected limb.

In terms of the stresses in an implant, such as a bone plate, the "worst case" while might be anticipated, would result from weight bearing immediately after surgery (i.e. within a few days). Most implants are not designed to withstand full weight bearing loads by themselves and would almost certainly fail by fracture if required to do so.

As a means of evaluating the relative capacities of numerous implant designs we have chosen to consider this "worst case" in which no tension is permitted at the fracture. Since material behaviour is assumed to be different in tension and compression, the linearly elastic formulations of the elements in SAPV's element library are

HUMAN FEMUR WITH 6-HOLE BONE PLATE IN SITU
UNDEFORMED SHAPE
MAY 05 1973 09:22:45
IAXIS= 2 ALPHA= 20.00 BETA= -45.00

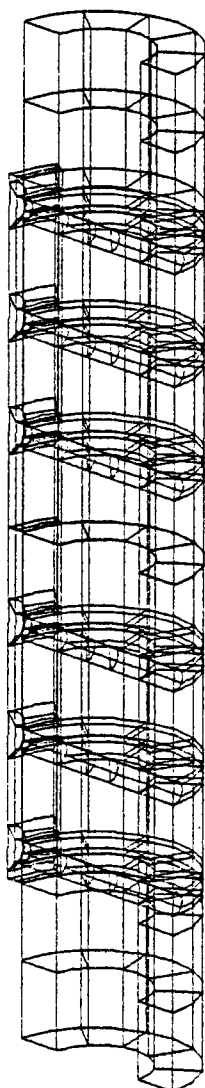


Figure 8. Computer Plot Showing Modified Bone Cortex Diameters

HUMAN FEMUR WITH 5-HOLE BONE PLATE IN SITU
UNDEFORMED SHAPE
NOVEMBER 01 1979 05:24:00
IAXIS= 2 ALPHA= 20.00 BETA= -45.00

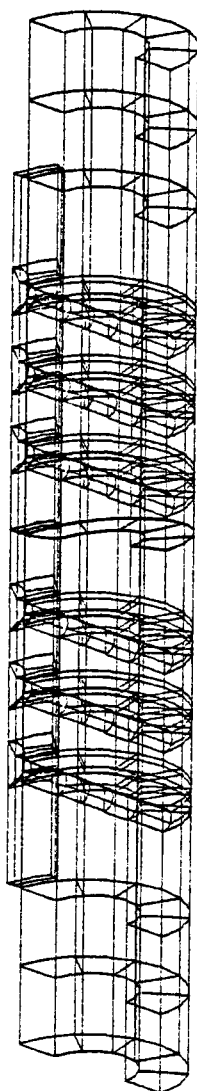
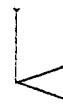


Figure 9. Computer Plot Showing Modified Angle α

not appropriate. Thus it has been necessary to develop a means by which SAPV could perform this particular non-linear analysis.

The new element has been based on SAPV's three-dimensional truss element, and it incorporates many of the features of that element. A direct iterative solution for non-linear materials was adapted from Cook,⁹⁵ and utilizes a secant stiffness approach. Figure 10 (after C. S. Lin⁹⁶) shows load vs. displacement and stress vs. strain as they vary during successive iterations of a secant stiffness analysis of a non-linear material. It is possible to effect a solution without the need for reformulation of the stiffness matrix for stresses beyond the yield stress. Cook⁹⁵ incorporates the von Mises yield criterion into his analysis, defining the effective stress for yielding as:

$$\bar{\sigma} = \frac{1}{2} \left\{ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \right\}^{\frac{1}{2}}$$

The effective strain is:

$$\epsilon^* = \frac{\sqrt{2}}{2(1+\nu)} \left\{ (\epsilon_x - \epsilon_y)^2 + (\epsilon_y - \epsilon_z)^2 + (\epsilon_z - \epsilon_x)^2 + \frac{3}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2) \right\}^{\frac{1}{2}}$$

Line 0-1 in Figure 10 shows a first approximation based on an assumption of linearly elastic behaviour. To find the slope of the curve (E_s = secant modulus) for the next approximation we compute ϵ^* and $\bar{\sigma}$ at pt. 2 (directly below pt 1. but on the non-linear curve) and:

$$E_s = \left(\frac{\bar{\sigma}}{\epsilon^*} \right) \text{ at pt. 2}$$

For any affected element, a reduced stiffness matrix is:

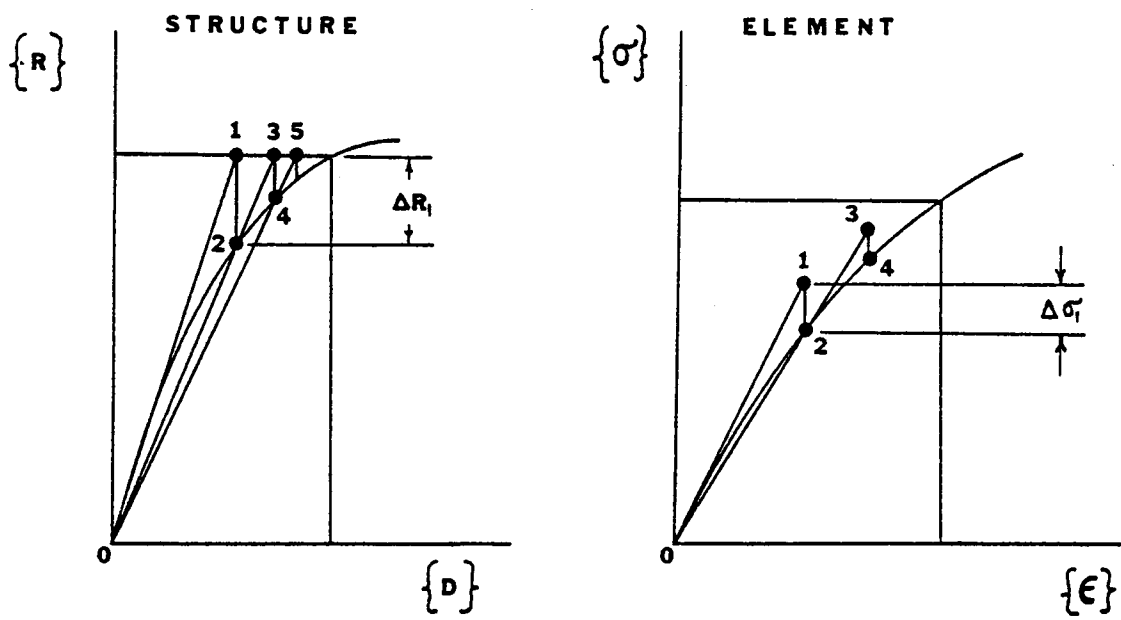


Figure 10. Secant Stiffness Method

$$\begin{bmatrix} k_r \end{bmatrix} = \frac{E_s}{E} \begin{bmatrix} k \end{bmatrix} = \left(1 + \frac{E_s - E}{E}\right) \begin{bmatrix} k \end{bmatrix} = \begin{bmatrix} k \end{bmatrix} + \begin{bmatrix} k_m \end{bmatrix}$$

And:

$$\begin{bmatrix} k_m \end{bmatrix} = \begin{bmatrix} k \end{bmatrix} \frac{(E_s - E)}{E}$$

For a static problem, the direct stiffness method for finite elements is written in matrix notation as:

$$\begin{bmatrix} K \end{bmatrix} \{D\} = \{R\}$$

The modified stiffness can be incorporated into the load vector by defining a new load vector $\{R\}'$ where:

$$\{R\}' = \{R\} - \sum \begin{bmatrix} k_m \end{bmatrix} \{d\}$$

The summation includes all elements in which "yielding" occurs. After $\{R\}'$ is computed we solve:

$$\begin{bmatrix} k \end{bmatrix} \{D\} = \{R\}'$$

The sequence is repeated until the difference between successive $\{R\}'$ vectors is less than some specified convergence criterion.]

Note that, in terms of computational effort, a method such as this is must less costly than one for which reformulation of the system stiffness matrix is required on each iteration, but the method is based on a highly simplified plasticity theory, which does not allow for time dependent effects like creep and stress relaxation.

Figure 11 shows the assumed stress strain curve for the no-tension material. At solution time, each "gap" element is tested for tension, and when tensile stress is detected E_s is set to zero for any elements which are affected. The load vector $\{R\}$ is formed and the solution of equations is repeated. Effectively, all elements found to be in tension are "removed."

This simple element, being based on a truss type element is not designed to account for any frictional affects between contacting areas, but only accounts for force distribution normal to the mating surfaces.

Applied Loading

The activity of "normal walking" involves the precisely phased activation of several muscles (Seireg and Arvikar⁹⁷ list 31 important ones on each side) to produce a smooth coordinated motion. From heel-strike to toe-off (stance phase) to heel strike again (swing phase) the magnitudes and directions of forces and moments within a leg vary through a wide range. These loads are applied to the bones through the joints, muscles, tendons and ligaments thereby producing complex patterns of stress within each bone. The stresses thus produced are dependent upon the size and weight of the individual as well as on factors relating to activity level such as walking speed or stride length.

Insofar as studies of the loading on bones is concerned, one can discern two groups. There are those studies in which joint forces,

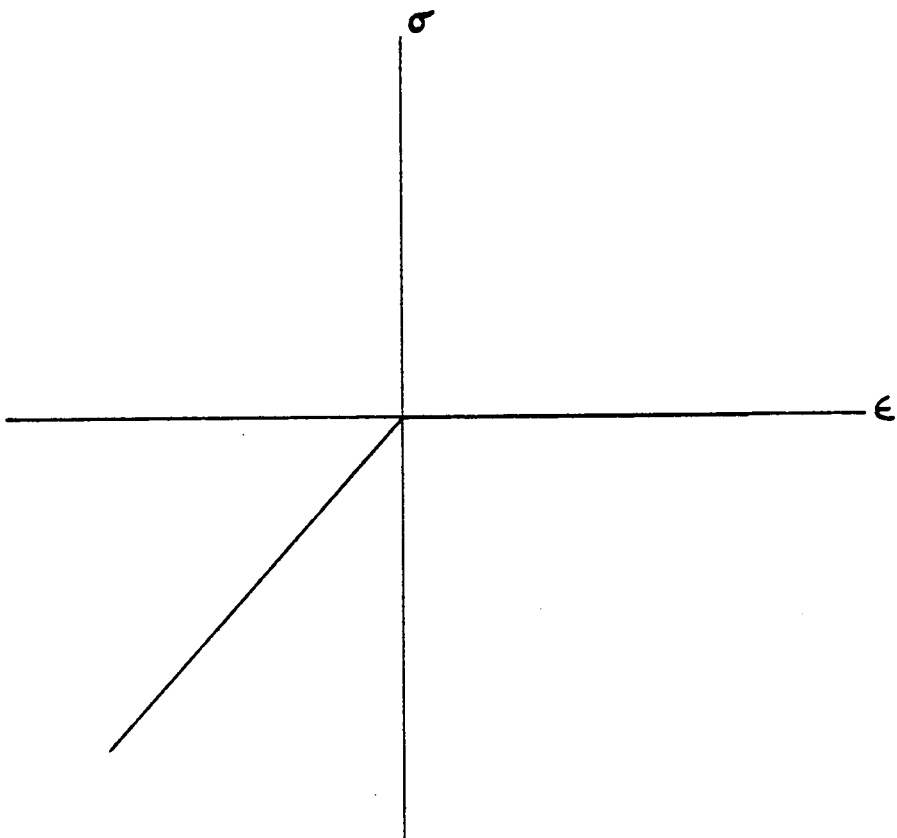


Figure 11. Stress vs. Strain for No-Tension Material

primarily are defined⁹⁸⁻¹⁰² and those in which both muscle and joint forces^{97,103-107} have been computed. Of this latter group, two of the presentations have been concerned with static situations^{103,104} while the remainder have been concerned with forces associated with normal walking.

In the walking studies, careful cinematography of a walking subject (with marks or small LED's at key locations on the leg facing the camera) provide information about the relative position and orientation of the leg segments. In addition, a piezoelectric force-plate mounted in the walkway provides a measure of force and moment reactions between ground and foot. In all cases the bones are considered to form a system of rigid links connected at the joints, with forces transmitted through the joints and with forces and moments transmitted between segments by muscles and ligaments. In these analyses muscles are defined by their attachment points and by lines of action. This is not so easy, since, first of all the attachments are areas, not points and secondly since muscles vary in shape and size from end to end, and since they may wrap around other muscles, the definition of a line of action is complicated. By careful study of anatomical drawings from texts such as Gray's Anatomy¹⁰⁸ one may optimize the selections of attachment points and lines of action. Seireg and Arvikar^{97,104} have even included bearing forces between muscles which wrap around one another.

One common problem presents itself in all these analyses. The force systems defined are indeterminate and hence some approximation

technique is required for a solution. The "reduction" method of Morrison¹⁰⁶ and Paul^{98,105} yields a bounded solution to the distribution of muscular forces. The reduction technique involves grouping functionally similar muscles and their attachment points so as to reduce the system of equations to a determinate one. The linear programming "optimization" method chosen by Seireg and Arivikar^{97,104} and by Crowninshield et al.¹⁰² assumes that the distribution of muscle forces will occur in such a way as to extremize some objective function (such as net force in all muscles or total work done by muscles). There is no way to know, beforehand which objective function will be most appropriate, so some additional verification is needed. Clearly such experimental verification is needed for any analysis, to show whether the magnitudes (or at least the variational trends) of the calculated muscle forces are reasonable.

Sensitive, skin surface electrodes (such as those used in electrocardiography) placed in the vicinity of a muscle can detect the electrical activity associated with a muscle contraction. The recording of such electrical signals, known as electromyography (EMG) gives an indication of the phasic activity of each muscle for which a measurement is made, and in addition the relative magnitude of the muscle force is shown as a function of time on the EMG tracing. There is reason to believe¹⁰² that the actual muscle force magnitude may one day be obtainable from EMG data, but as this is not yet the case, one may only assess the relative merit of any analytical technique in predicting trends of muscle force variation. Still, by using EMG data,

Seireg and Arvikar^{97,104} have been able to rule out some of their suggested objective functions, and to select a function based on minimizing the sum of muscle forces plus some weighted sum of joint moments.

Clearly, during a normal walk cycle, the forces and moments applied to the bones exist in several planes and for a thorough dynamic analysis of such a system, three-dimensionality of both geometry and loads must be considered. For the static finite element analysis at hand, a number of limiting considerations have created the need for some simplification. Due to limitations in available computer facilities, the choice was made to assume symmetry relative to a frontal plane, and to consider only the midsection (shaft) of the femur rather than the entire bone. Some losses in accuracy occur here since forces and moments which are out of the frontal plane cannot be accounted for.

By means of a simple free-body-diagram analysis (refer to Figure 12), forces and moments applied outside our region of interest may be transferred to the chosen shaft segment. By utilizing the largest applied forces which occur during a walk cycle, we expose the plated bone to a "worst case" loading situation. If selected failure criteria under such imposed loading are not exceeded then we may infer something about the viability of a particular device in a given situation. While there is no universally accepted value for the "critical" time in a walk cycle (i.e. the time at which joint and muscle forces are maximized), published results indicate that the loads are maximized during the single support (all loads borne by one foot) portion of the

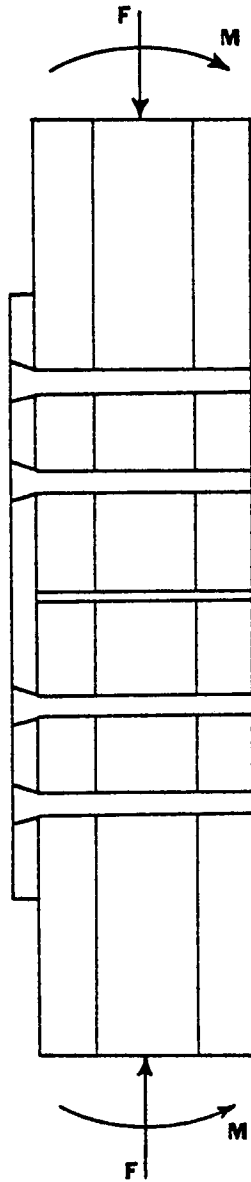


Figure 12. Free Body Diagram Used for Applied Loading

stance phase. Shortly after heel strike, hip joint reaction forces may be as high as 6 times the body weight while knee joint forces may reach values of 3 times the body weight. Another peak in joint forces is seen shortly before toe-off however this peak is generally lower in magnitude, as are the muscle forces^{97,99,102} corresponding to this second peak. For the purpose of this model, the input loads which define the failure criteria for a given implant will correspond to the first force peak (at 50% of the walk cycle).

Bone Anisotropy

The published accounts of other finite-element models of bone-plate systems, mentioned in the previous chapter, have all included an assumption of isotropy for the elastic properties of bone. Such a simplifying assumption must surely cast doubt on the accuracy of predicted stresses, especially near stress concentrations. Studies on the mechanical properties of bone, however, indicate that this material possesses cylindrical orthotropy such that elastic modulus parallel to the bone longitudinal axis is approximately twice the modulus transverse to that axis.

The SAPV isoparametric three-dimensional element permits input of orthotropic material constants, and thus we may use experimentally obtained data defining bone anisotropy to improve the accuracy of stress computations.

In the literature review chapter, the ultrasonic technique for elastic constant measurement was described, and the computation of

the familiar engineering constants (E , G and ν) was discussed. Such constants are required by the SAPV program.

For the purpose of the parameter sensitivity study described in section V, the experimental elastic constant data of Yoon and Katz⁶⁹ has been used, after first computing the engineering constants using the equations from Tsai.⁷¹ Taking the 3-direction to represent the bone's longitudinal axis direction, and letting the 1 and 2 directions be transverse to this, (note that the 1-2 plane is a plane of symmetry with respect to elastic constants) the engineering constants used are:

$$E_{11} = 2,370,000.\text{psi}$$

$$E_{22} = E_{11}$$

$$E_{33} = 3,970,000.\text{psi}$$

$$G_{12} = 1,040,000.\text{psi}$$

$$G_{13} = 1,260,000.\text{psi}$$

$$G_{23} = G_{13}$$

$$\nu_{12} = .312$$

$$\nu_{13} = .279$$

$$\nu_{23} = \nu_{13}$$

These data, which are average values from many experiments are felt to be representative of the adult human population.

IV. EXPERIMENTAL VERIFICATION

In order to assess the relative merit of the finite element model, and the assumptions made, a series of mechanical tests on plated human bones was undertaken.

Embalmed whole human femora were obtained[§] and were stored frozen prior to preparation time. Upon removal of a femur from the freezer, the remaining soft tissue was stripped from the shaft. By means of a silicon carbide cutoff wheel, a shaft segment of approximately 8" length was cut from the midshaft region, and an idealized planar "fracture" transverse to the bone's longitudinal axis was created. The cut pieces were dried in a forced air drying oven at 100°F for several (7-10) days and then stored in a dry place prior to plate attachment. Care was taken to mark the bone so that lateral, medial, anterior and posterior quadrants might be distinguished. On each specimen, a straight, 6-hole bone plate (Wright Mfg. Co. #227-2642) was attached to the bone via screws so that the middle of the plate lines up with the artificial fracture. The plate was located on the lateral side of the bone shaft.

Two of the plated bones, having the least amount of curvature, were selected for instrumentation, and the plate surfaces were abraded lightly (#320 Si-C paper) to facilitate bonding of the strain gages. Onto each plate, were attached two delta rosettes (Micro-Measurements

[§]Bones come from the University of Tennessee School of Dentistry, Dept. of Dental Anatomy, Memphis, TN.

#EA-06-030YB-120) via an acrylic based strain gage cement (M-BOND 200). After all leads were soldered, each rosette was coated with an RTV silicone compound (DOW Corning 3140 RTV Coating) so as to keep out moisture, and the entire assembly was stored in a Drierite filled dessicator.

For the purpose of the mechanical testing, a special clamping device was designed to secure the specimen in an Instron machine. The ends of the bone were inserted in cylindrical wells filled with molten Wood's metal (melts at 71°C). Upon cooling, the Wood's metal firmly holds the bone without crushing or producing severe stress concentrations at the site where the specimen enters the grips. The grips, in addition, were designed so as to permit purely compressive loading or combined compression and bending. The photograph of Figure 13 shows the completed specimen within the special grips.

After each mechanical test, a run of the SAPV computer model was submitted, using dimensional and material property data corresponding to the specimen used in that test.

The full set of elastic constants used was obtained via ultrasonic methods,[§] following the procedure of Yoon and Katz.⁶⁹ A piece from each bone was removed via a cut-off saw. From each piece, small parallelopipeds of bone were cut out on a milling machine. Distilled water was used as a coolant during all machining operations. The

[§]These measurements were made in the laboratories of the University of Tennessee, Dept. of Physics.

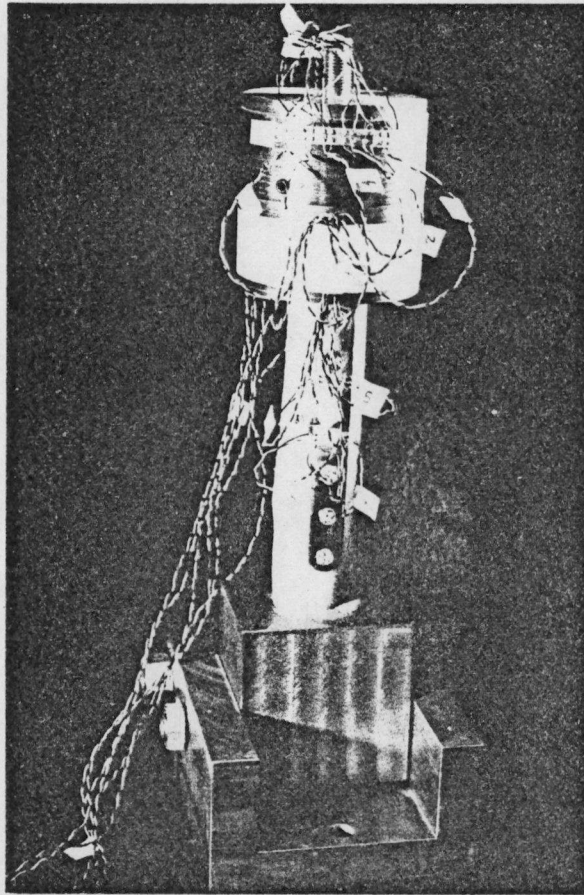


Figure 13. Photograph Showing Plated Human Bone Specimen in Loading Fixture

orientations of the faces of these small blocks, relative to the bone axes, are as specified by Yoon and Katz.⁶⁹ In all, eight ultrasonic velocity measurements are made for each bone specimen and these velocities are in turn related to the elastic constants for the bone (C_{ij}).

The small bone specimens were weighed and measured so that density could be computed. Small piezoelectric crystal transducers were then used to excite the specimens with pulses of high frequency ultrasound. The source of the pulses was a Panametrics Model 5052 P/R (pulser/receiver). By means of an oscilloscope, the transmission time for the ultrasound pulses was measured for both through transmission (2 transducers required) and pulse echo (only 1 transducer required) modes. For each velocity measurement, the mode was chosen which provided the best visual resolution of individual pulses from the oscilloscope screen.

Using averaged velocity values from several transmission time measurements, the elastic constants, C_{ij} , were computed by means of a small FORTRAN computer program, written by this author especially for this purpose. In addition, the C_{ij} matrix was inverted to obtain the compliance matrix (S_{ij}) which in turn was used to compute the familiar engineering constants for input to the SAPV program model of the plated bone. The elastic constants thus obtained are used only for the experimental verification. The parameter sensitivity study employs the data from Yoon and Katz,⁶⁹ listed in section III.

V. DISCUSSION OF RESULTS

Computational Accuracy

In any computational idealization of a continuum mechanical analysis, there will be some solution errors. In order to determine the effects of variations in some geometrical parameters on the accuracy of computed displacements and stresses, a series of preliminary studies was performed.

Experience with the simpler planar elements has suggested that an aspect ratio (ratio of length to width) of about 6 to 1 is the limit, beyond which, computed stress values begin to diverge significantly from closed form solutions. For the purpose of examining aspect ratio sensitivity in SAPV's 21-node isoparametric element, a cantilever beam model was prepared, and its' results were compared to those from a closed form solution presented by Timoshenko and Goodier.¹⁰⁹ By varying the SAPV input data (specifically, some node coordinates were changed), it was possible to alter the aspect ratio of one or more elements along the beam. Figure 14 illustrates the finite element mesh used to model the beam. The end at $x = 6$ in. has been "fixed" according to the various boundary conditions described in reference 109. A single transverse point load applied at the centroid of the free end ($x = 0$ in.) in the negative y direction. In a series of SAPV runs, the distance ξ has been given the values 1 in., 0.2 in., 0.1 in., 0.01 in. and 0.001 in. For the elements having ξ as one of their dimensions, this corresponds to aspect ratios of 1, 5, 10, 100 and 1000 respectively.

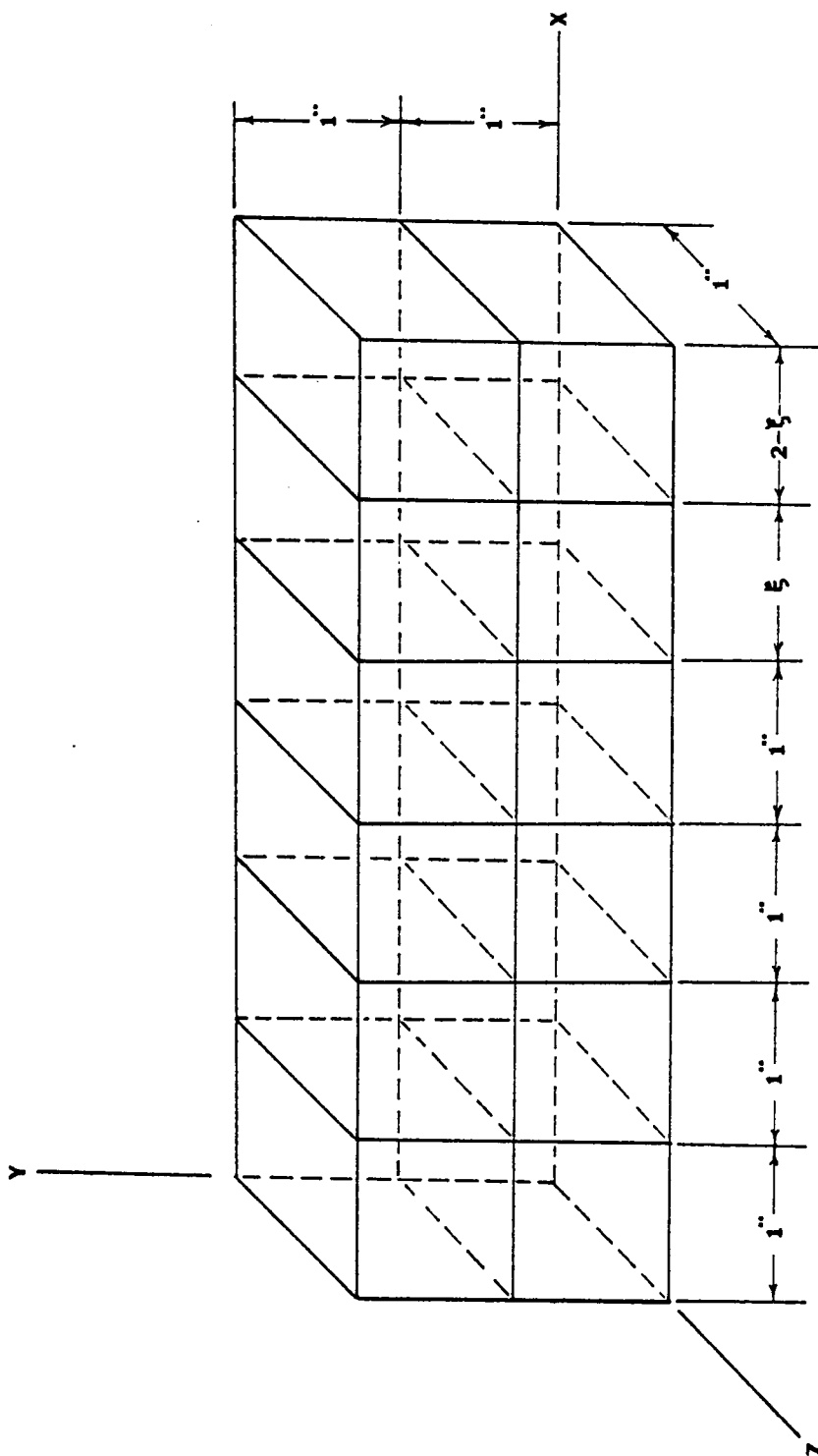


Figure 14. Simple Beam Model - 21 Node Isoparametric Elements

Relative to the fixed end boundary conditions, it was found that constraining all degrees of freedom at $x = 6$ in. gave results comparable to the closed form analysis when the closed form boundary condition $\left(\frac{\partial u}{\partial y}\right)_{\substack{x = \ell \\ y = 0}} = 0$ is used. The variation in aspect ratio did not radically

alter the displacements as shown in Table 4, however the stress values in critical elements (those having x-dimension equal to ξ) diverged radically from the closed form solution at aspect ratios higher than 5. For all analyses, properties of isotropic steel having $E = 30 \times 10^6$ psi, $\nu = 1/3$ and $G = 11.25 \times 10^6$ psi were used.

In its formulation of element stiffness matrices, the SAPV program employs the Gaussian Quadrature method to integrate appropriate functions over the element volume. The number of quadrature points, which is related to solution accuracy, is user defined (it may be 2, 3 or 4) and initially the value 2 was selected. The results in Table 4 corresponds to the runs done with 2 and 4 Gauss points. In elements adjacent to the critical ones, stresses do not depart significantly from theory. When 2 Gauss points are used, the stresses in the critical elements are very sensitive to aspect ratio. For example, when aspect ratio is 10 the outer fiber stresses (using simple beam theory terminology) show 42% error and when the aspect ratio is raised to 100 the algebraic signs of the stresses actually reversed, giving approximately 171% error in the outer fiber stresses. When the number of Gauss points was increased to 4, however, the error in outer fiber stress at an aspect ratio as high as 100 was less than 4% for this geometry.

Table 4. Vertical Displacements of Points on Cantilever Beam Midline for Various X-Coordinate Values (Displacements are in inches)

Source \ x	0	1"	2"	3"	4"	4.2"	4.1"	4.01"	4.001"
Theory	7.2×10^{-4}	5.417×10^{-4}	3.733×10^{-4}	2.25×10^{-4}	1.067×10^{-4}	8.748×10^{-5}	9.687×10^{-5}	1.056×10^{-4}	1.066×10^{-4}
SAPV $\xi = 1"$	7.71×10^{-4} 7.56×10^{-4}	5.748×10^{-4} 5.72×10^{-4}	4.004×10^{-4} 3.966×10^{-4}	2.444×10^{-4} 2.420×10^{-4}	1.187×10^{-4} 1.169×10^{-4}				
SAPV $\xi = .2"$	7.69×10^{-4} ---	5.724×10^{-4} ---	3.98×10^{-4} ---	2.428×10^{-4} ---	1.177×10^{-4} ---	9.954×10^{-5} ---			
SAPV $\xi = .1"$	7.69×10^{-4} 7.39×10^{-4}	5.727×10^{-4} 5.584×10^{-4}	3.987×10^{-4} 2.334×10^{-4}	2.431×10^{-4} 1.115×10^{-4}	1.18×10^{-4} 1.115×10^{-4}		1.098×10^{-4} 1.025×10^{-4}		
SAPV $\xi = .01"$	7.69×10^{-4} 7.397×10^{-4}	5.732×10^{-4} 5.589×10^{-4}	3.99×10^{-4} 3.859×10^{-4}	2.435×10^{-4} 2.336×10^{-4}	1.19×10^{-4} 1.12×10^{-4}			1.195×10^{-4} 1.116×10^{-4}	
SAPV $\xi = .001"$	7.696×10^{-4} ---	5.732×10^{-4} ---	3.99×10^{-4} ---	2.435×10^{-4} ---	1.19×10^{-4} ---				1.204×10^{-4} ---

While it is not valid to extrapolate from these results to other geometrical configurations, one can certainly conclude that the use of 4 Gauss points, for aspect ratios higher than 5, leads to a considerable improvement in accuracy of computed stresses.

One of the features of the 21-node element is the ability to define material property axes which are oriented differently from the system geometrical axes. In order to test this feature a second simple mesh was prepared. In this second case, the mesh represented a straight cylindrical section from the midshaft of a long bone such as the femur (see Figure 15).

Two loading cases were run, one for pure axial compression, and one for pure bending. In both cases, bone material properties utilized in SAPV were derived from the ultrasonically obtained data from Yoon and Katz.⁶⁹

For the case of axial compression, the finite element results were compared to hand calculations based on a simple mechanics of materials approach, and the comparison is as follows:

	<u>Change in length</u>	<u>Axial stress</u>
SAPV	.00247 in.	1414.23 psi
THEORY ¹¹⁰	.00232 in.	1414.70 psi

For this test, the end load was 1200 lb. (uniformly distributed). The specimen length was 6.5 in. and its outside and inside diameters were 1.2 in. and .6 in. respectively. The same mesh was used for a pure bending moment of 720 in lb. applied at the bone ends. A closed form

INTACT HUMAN BONE - COMBINED BENDING AND COMPRESSION.
UNDEFORMED SHAPE
SEPTEMBER 24, 1979 17:35:14
IAXIS= 2 ALPHA= 20.00 BETA= -45.00

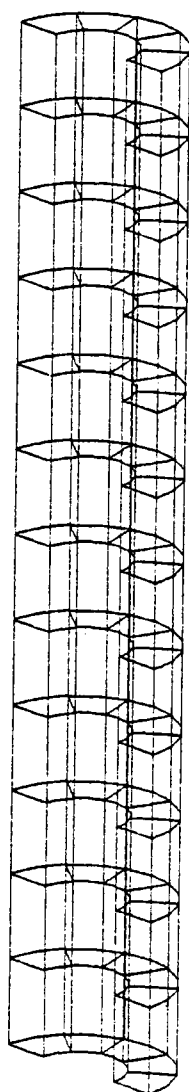
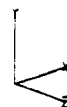


Figure 15. Computer Plot of Intact Cortical Bone

solution, from Lekhnitskii¹¹⁰ served as a basis for comparison. Figure 16 shows a plot (superimposed on a bone cross-section) of bending stresses in the plane of the applied moment both from SAPV and from Lekhnitskii.

The results from these two tests provide a significant support for the 21-node element's ability to accurately handle situations involving anisotropic materials, even with a relatively coarse mesh.

Experimental Results

(i) Ultrasonic measurement of bone elastic constants

As has been stated in section IV, transmission velocities of ultrasound through bone can be related to the elastic constants C_{ij} . It has been experimentally shown^{66,67,68} that bone exhibits hexagonal symmetry (also referred to as transverse isotropy) which means that there are 5 independent elastic constants.

For the purpose of naming the elastic constants, a coordinate system has been defined such that the 3-axis coincides with the bone longitudinal axis, while the 1-2 plane (plane of symmetry) is orthogonal to the bone long axis. Following the convention of Yoon and Katz⁶⁸ the 3-direction is referred to as α and the 1 and 2 directions are labelled γ . In addition to velocity measurements in the α and γ directions, some velocities were measured between planes whose normal was at an angle of 45° with the 1-axis. These coordinates are shown in Figure 17 (adapted from reference 69). For the α , γ and 45° direction, both longitudinal and shear waves were passed through the specimens. For the shear (or transverse) waves, the direction of

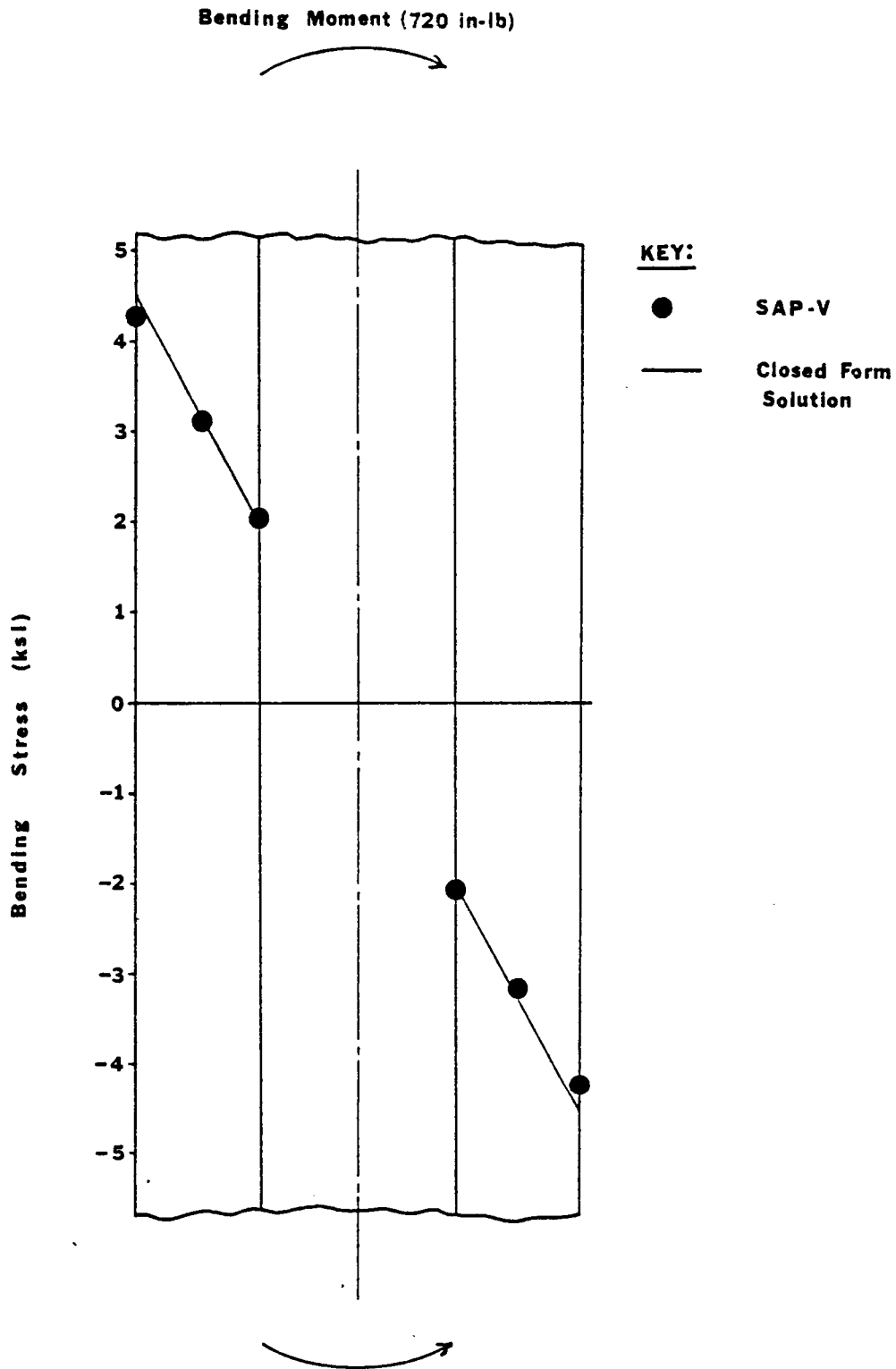


Figure 16. Comparison of Finite Element and Closed Form Solutions to Bending Stress in a Cortical Bone Segment Loaded in Pure Bending

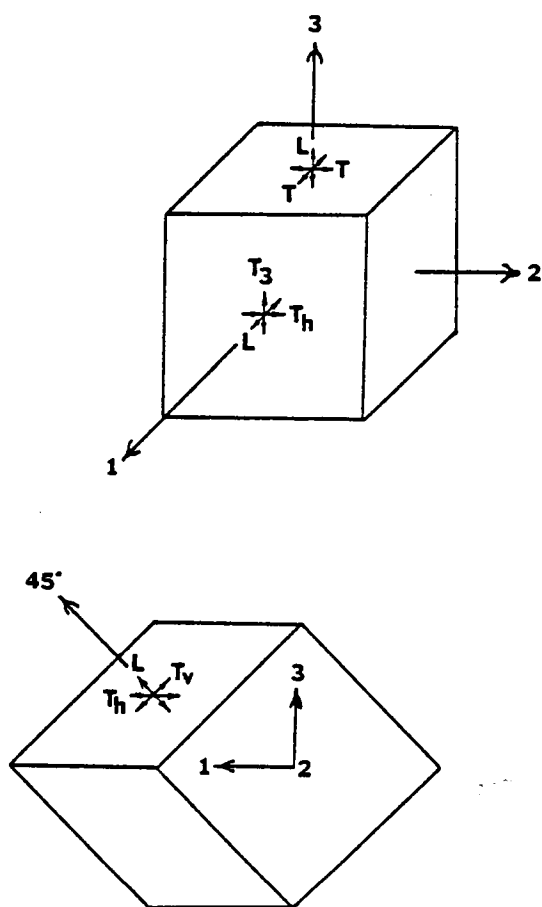


Figure 17. Compact Bone Specimens Showing Transmission Directions for Ultrasonic Testing

propagation and direction of oscillation are important so these directions have both been recorded during the experiments, and the names assigned to the various measurements indicate both. Table 5 lists the average values of all eight measured velocities for the two bones examined, and also gives average density values. The formulas for C_{ij} ⁶⁸ and for the engineering constants⁷¹ have been used, via a FORTRAN computer program (see previous section) and the final values for the engineering constants corresponding to this set of velocity measurements are presented in Table 6.

(ii) Compression and bending tests on plated bones.

The two plated bone specimens, described in section IV, were loaded in compression and then in combined compression with bending by means of an Instron machine and specially designed grips. For both load cases, loads, (as indicated by the Instron machine) of 0 to 1000 lb. (in increments of 100 lb.) were applied.

Strain gauges, mounted on the bone plate, were wired into a Budd: Model P-350 Portable Strain Indicator through a Budd: Model SB-1 Switch and Balance Unit. The latter device permitted rapid selection of an individual gauge, after which one could balance the bridge circuit and thus measure the strain indicated by that particular gauge. Temperature compensation was provided by means of an identically instrumented, plated bone wired into the circuit (a half bridge circuit configuration was used).

As the loading tests were done, significant relaxation was observed. The Instron machine's load indicator showed as much as 10% decrease in

Table 5. Transmission Velocities for Ultrasound Through Human Cortical Bone Measured in Various Transmission Modes.

SPECIMEN	1	2
DENSITY	1.917 (GM/CM ³)	1.66 (GM/CM ³)
MODE	VELOCITY (KM/SEC)	VELOCITY (KM/SEC)
αL	4.30	3.886
αT	1.938	1.968
γL	4.215	3.571
γT_h	2.056	2.051
γT_3	1.840	2.010
45L	4.18	3.571
45T _h	1.950	1.785
45T _v	1.939	1.927

Table 6. Computed Engineering Elastic Constants Based on Measured Ultrasound Velocities.

	SPECIMEN 1	SPECIMEN 2
E_1 (PSI)	3.18×10^6	2.61×10^6
E_2 (PSI)	3.18×10^6	2.61×10^6
E_3 (PSI)	3.36×10^6	3.24×10^6
G_{12} (PSI)	1.18×10^6	1.01×10^6
G_{13} (PSI)	1.04×10^6	9.32×10^5
G_{23} (PSI)	1.04×10^6	9.32×10^5
ν_{12}	.354	.289
ν_{21}	.354	.289
ν_{23}	.325	.176
ν_{32}	.343	.219
ν_{31}	.343	.219
ν_{13}	.325	.176

load in two minutes. So, prior to the recording of strain gauge data, a time of two to five minutes for load stabilization was permitted. While continued relaxation could certainly have occurred beyond the allowed time, it was noted that most of the relaxation that took place occurred during the first minute after motion of the Instron's cross-head was stopped. Rapid recording of strains, which was facilitated by the switch unit described above, ensured that the load changed very little while data were being taken. Figures 18 and 19 show typical results of measured strains vs. applied load for two tests. In the first case, compressive loading only was applied, while in the second a bending moment was superimposed on the compressive load by applying one of the 1000 lb. loads parallel to the bone's longitudinal axis but at a perpendicular distance of 1" from that axis. This is similar to the *in-vivo* situation wherein a load is applied to the femur through the hip. Superimposed on the plot of Figure 18 is a sketch indicating the location and orientation of the strain gauges corresponding to the plot. Each type of test was run three times, and the results indicated a high degree of repeatability in measured strains.

Comparison of Theory and Experiment

The next phase of the study involves the assessment of the capability of the finite element model to predict the stresses in an actual plated bone. [By using dimensions and material properties corresponding to the experiments discussed in the previous sub-section, within the SAPV program input data, we can generate a set of stress predictions. Clearly, the accuracy of such modelling depends upon the numerical

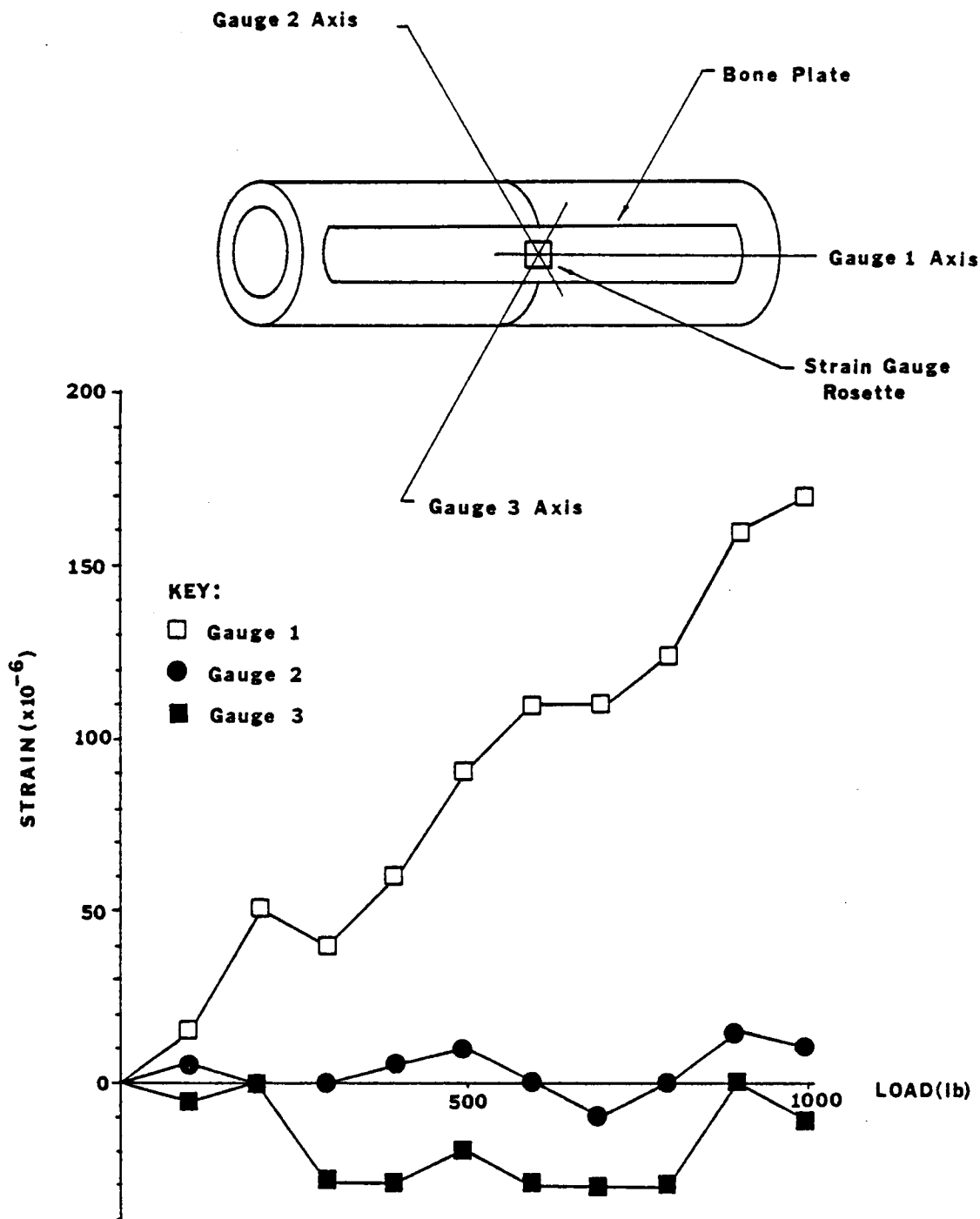


Figure 18. Strain Gauge Readings from the Middle of a Plated Bone Loaded in Pure Compression

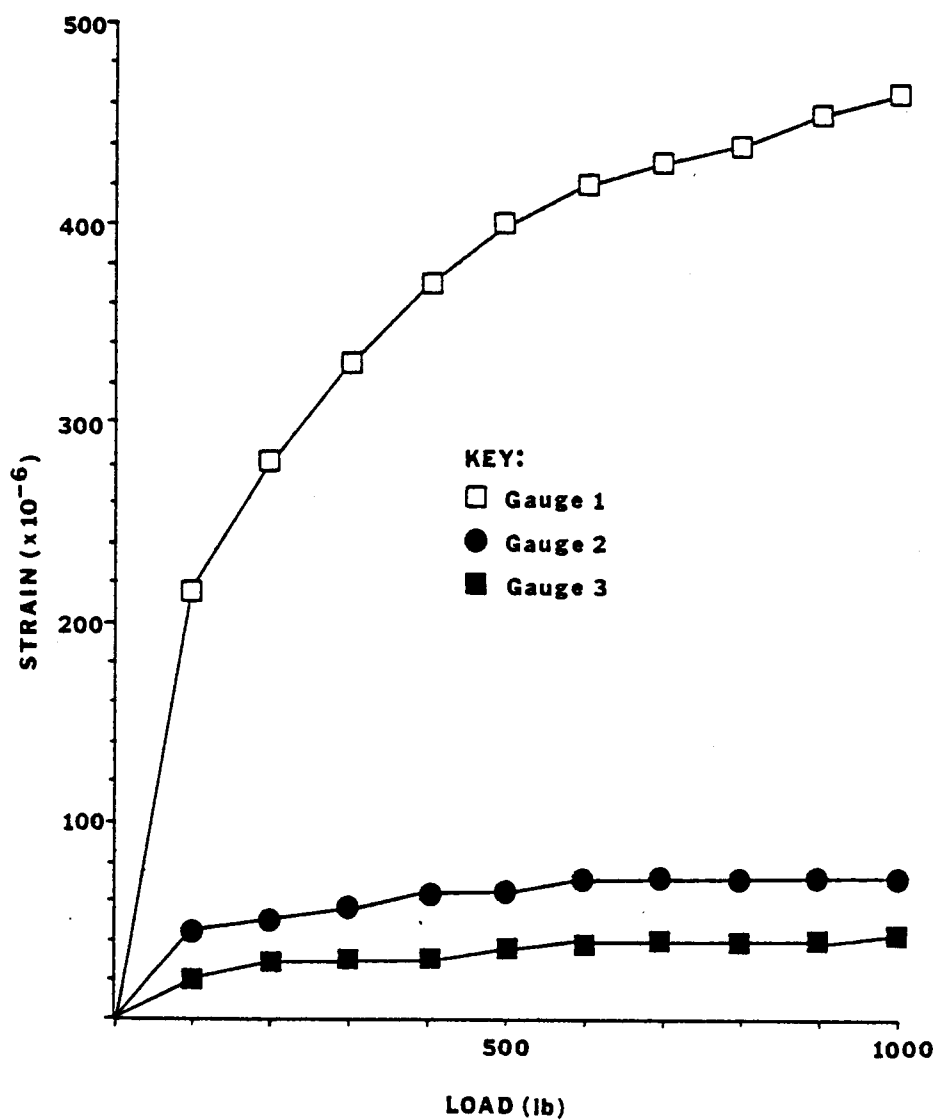


Figure 19. Strain Gauge Readings from the Middle of a Plated Bone Loaded in Compression with Bending

accuracy of the computer analysis scheme as much as it depends upon the accuracy of input data.

Some of the possible error sources, which the present model does not take into consideration are:

1. Bone curvature.
2. Non-homogeneity of bone.
3. Possible misalignment of screws or plate.
4. The screw threads which represent a serious stress concentration.
5. Variation in bone inside and outside diameters along the shaft.

Computation of stresses corresponding to the strains measured in the experiments described, permitted a direct comparison with SAPV program results. The strain gauge rosettes were of the "delta" type (3 gauges at angles of 120° relative to one another). It was necessary to perform a coordinate transformation in order to relate the measured strains to those in some specified cartesian coordinate system defined in a plane tangent to the bone plate's surface. The equations applied to this rotation of coordinates may be found in several basic mechanics of materials texts, such as the one by Popov.¹¹¹ Subsequent stress computations were done using plane stress constitutive equations for an isotropic material with a Young's modulus of 28×10^6 psi and a Poisson's ratio of $1/3$ for the plate's AISI type 316L stainless steel.

The comparison of measured and predicted results for the two experiments depicted in Figures 18 and 19 is presented in Table 7.

(All Stresses shown correspond to an applied load of 1000 lb.)

Table 7. Surface Stresses at Bone Plate Midsection; Comparison of Experimentally Measured and Predicted Stress Values.

	1000 lb. Compressive Load Only	1000 lb. Compressive Load with 1000 in.-lb. Bending Moment
Experiment	$\sigma_{\ell} = 7100 \text{ psi}$ $\sigma_{\tau} = 1900 \text{ psi}$	$\sigma_{\ell} = 13,900 \text{ psi}$ $\sigma_{\tau} = 2,400 \text{ psi}$
SAPV	$\sigma_{\ell} = -400 \text{ psi}$ $\sigma_{\tau} = 800 \text{ psi}$	$\sigma_{\ell} = 19,000 \text{ psi}$ $\sigma_{\tau} = 4,200 \text{ psi}$

σ_{ℓ} = longitudinal stress at plate midsection

σ_{τ} = transverse stress at plate midsection

While the results for the compression + bending test are most encouraging, the comparison for the compression only load case is less fortuitous. Since the experimental results given all correspond to a single specimen, loaded in two different ways, it is interesting to speculate as to the reasons why the predicted values are so much superior in one case. The fact that the algebraic signs for the longitudinal stresses in the compression only case are opposite (i.e. between experiment and theory) provides a possible clue.

Clearly the only way in which a tensile stress (as measured experimentally) could arise is through the addition of a bending moment, however small to the compressive load. Visual inspection of this particular bone shows a very slight curvature which places the plate on the convex side and thus subject to some bending. The question of whether such a slight curvature could indeed be the cause of our observed discrepancy, stimulated further investigation.

A new, modified finite element mesh of the plated bone was prepared, into which was incorporated a slight curvature (radius of curvature of 100"). The bone plate remained straight and was situated on the convex side of the bone. The bone screws remained straight also. Once again the compression only load case was simulated using the newly defined mesh. The longitudinal stress at the middle of the bone plate was now predicted to be + 576 psi as opposed to the -442 psi from the SAPV run on the mesh with no curvature. While this should by no means lead to the conclusion that curvature alone can account for the observed discrepancies, we nevertheless illustrate dramatically the

sensitivity of the SAPV model to very small geometrical changes and incidentally present a very plausible resolution of those discrepancies.

We can conclude that the SAPV based simulation will give acceptable results for the combined compression with bending load, but will probably not do so for the pure compression loading case. The presence of bone curvature has been shown to exert considerable influence on the latter situation, however its effects are somewhat overshadowed in the former case by the large bending moment. In a comparison of two combined load SAPV runs, one with no curvature and one with curvature (radius of curvature = 100 in.) the latter showed approximately 7% higher stress at the plate midsection.

Clearly then, with sufficient careful manipulation of the SAPV model a closer alignment with the experiment can be achieved. Such a model, however, would not necessarily be representative of the general population. Also, that is not the primary objective in this research. That objective, rather, is the assessment of sensitivity to variations in some selected system variables as described in the following section.

The Parameter Sensitivity Study

In their experimental study of plated equine metacarpal bones, Ray et al.⁷ examined the influence of changes in some plate geometrical variables on the capacity (failure producing load) of the system. The independent variables chosen by Ray et al. were plate length, plate width and screw diameter and to each of these factors, a set of 3

values or "levels" was assigned. The total number of possible combinations is therefore: $(\text{levels})^{\text{factors}} = (3)^3 = 27$ and in this instance all 27 experiments were performed. A study of this sort is referred to by statisticians as a full factorial design. The results of the experiments indicated a positive correlation between load capacity and plate length but no clear indication of a correlation relative to the other two independent variables.

Needless to say, even small increases in the numbers of factors and/or levels can rapidly make the total number of combinations grow beyond limits of practicability. Thus the size of studies may have to be limited.

The present, finite element based study involves seven independent variables at a minimum of three levels each. The total possible number of combinations would be approximately $3^7 = 2187$, however only 41 "experiments" were used in the basic parameter sensitivity study. To permit individual assessment of the sensitivity to parameters, the set of runs was designed such that groups existed wherein all but one of the factors were constant while the factor being examined was permitted to assume its different values or levels. Numerical values of the independent variables coincided with or were close to values from commercially available bone plates and typical femurs from adults.

The independent variables chosen for this study are as follows:

1. PLTLEN = length of bone plate (in.)
2. % STSDM = screw to screw distance at plate midsection
expressed as a percentage of plate length

3. % STSDE = screw to screw distance at plate ends expressed as a percentage of plate length
4. BID = bone inside diameter (in.)
5. BOD = bone outside diameter (in.)
6. GAP = gap between fracture surfaces (in.)
7. ALPHA = angle between normal to fracture plane and bone longitudinal axis (degrees)

For most of the "experiments" a constant axial compressive force of 1500 lb. and a bending moment of 1500 in.-lb. (with the metal plate on the tension side) were applied. In addition, all cases were run using the bone elastic property data specified at the end of Chapter III.

The values for the 7 factors used in the computer simulation runs are given for all cases in Table 8.

As computer results were generated a number of stresses were recorded for comparison purposes. Among the values noted were:

1. Maximum tensile stress in the plate
2. Maximum tensile and compressive stresses in the bone
3. Axial stresses along the screw axes
4. Shear stresses between screws and bone
5. Contact stresses on fracture surfaces

A cursory examination of the recorded results indicates that stresses in both metal and bone show the greatest sensitivity to plate length, bone inside and outside diameters, and fracture surface angle.

Table 8. Program of Simulated Experiments

TEST CODE No.	PLTLEN	% STSDM	% STSDE	BID	BOD	GAP	ALPHA
1 - 6S	6"	27	16	.6	1.2	.001	0 ⁰
2 - 6S	6	30	15	.6	1.2	.001	0 ⁰
3 - 6S	6	20	15	.6	1.2	.001	0 ⁰
4 - 6S	6	26	17	.6	1.2	.001	0 ⁰
5 - 6S	6	27	16	.7	1.2	.001	0 ⁰
6 - 6S	6	27	16	.6	1.1	.001	0 ⁰
7 - 6S	6	27	16	.7	1.1	.001	0 ⁰
8 - 6S	6	27	16	.7	1.3	.001	0 ⁰
9 - 6S	6	27	16	.5	1.1	.001	0 ⁰
10 - 6S	6	27	16	.6	1.2	.001	10 ⁰
11 - 6S	6	27	16	.6	1.2	.001	20 ⁰
12 - 6S	6	27	16	.6	1.2	.005	0 ⁰
13 - 6S	6	27	16	.6	1.2	.010	0 ⁰
14 - 6S	4.5	27	16	.6	1.2	.001	0 ⁰
15 - 6S	4.5	30	15	.6	1.2	.001	0 ⁰
16 - 6S	4.5	20	15	.6	1.2	.001	0 ⁰
17 - 6S	4.5	26	17	.6	1.2	.001	0 ⁰
18 - 6S	4.5	27	16	.7	1.2	.001	0 ⁰
19 - 6S	4.5	27	16	.6	1.1	.001	0 ⁰
20 - 6S	4.5	27	16	.7	1.1	.001	0 ⁰
21 - 6S	4.5	27	16	.7	1.3	.001	0 ⁰
22 - 6S	4.5	27	16	.5	1.1	.001	0 ⁰
23 - 6S	4.5	27	16	.6	1.2	.001	10 ⁰
24 - 6S	4.5	27	16	.6	1.2	.001	20 ⁰
25 - 6S	4.5	27	16	.6	1.2	.005	0 ⁰
26 - 6S	4.5	27	16	.6	1.2	.010	0 ⁰
27 - 6S	3	27	16	.6	1.2	.001	0 ⁰
28 - 6S	3	30	15	.6	1.2	.001	0 ⁰
29 - 6S	3	20	15	.6	1.2	.001	0 ⁰
30 - 6S	3	26	17	.6	1.2	.001	0 ⁰
31 - 6S	3	27	16	.7	1.2	.001	0 ⁰
32 - 6S	3	27	16	.6	1.1	.001	0 ⁰
33 - 6S	3	27	16	.7	1.1	.001	0 ⁰
34 - 6S	3	27	16	.7	1.3	.001	0 ⁰
35 - 6S	3	27	16	.5	1.1	.001	0 ⁰

Table 8. (Continued)

TEST CODE No.	PLTLEN	% STSDM	% STSDE	BID	BOD	GAP	ALPHA
36 - 6S	3	27	16	.6	1.2	.001	10 ⁰
37 - 6S	3	27	16	.6	1.2	.001	20 ⁰
38 - 6S	3	27	16	.6	1.2	.005	0 ⁰
39 - 6S	3	27	16	.6	1.2	.010	0 ⁰
40 - 6S	6	13.5	8	.6	1.2	.001	0 ⁰
41 - 6S	6	$\Delta T = -10^0$ APPLIED TO SIMULATE PLATE PRETENSIONS					
42 - 6S	6	26	15	.6	1.2	.001	0 ⁰
43 - 6S	6	26	16	.6	1.2	.001	0 ⁰
44 - 6S	REPEAT CASE 1 - 6S WITH 1200 lb. & 1200 in.lb. LOAD REPEAT CASE 1 - 6S WITH 900 lb. & 900 in.lb. LOAD						
45 - 6S							
01 - 4S	6	27	16	.6	1.2	.001	0 ⁰
02 - 4S	4.5	27	16	.6	1.2	.001	0 ⁰
03 - 4S	3	27	16	.6	1.2	.001	0 ⁰

The importance of plate length was emphasized by Ray et al.⁷ in their experimental parametric study. The finite element model shows that stresses on both plate and bone are strongly dependent upon plate length. Figure 20 shows the maximum tensile stress in each plate plotted for runs 1 - 6S through 43 - 6S (excluding 40 and 41 which are special cases). By clustering the results in groups of equal plate length we immediately see two features. Plate stresses are consistently higher in the longer plates and the spread of results is greater for the longer plates.

Figure 21, in contrast, shows the maximum tensile and compressive stresses in the bone for all runs. There seems to be no definite correlation to plate length. It should be noted however, that these stresses are in fact found in the bone outside the span of the plate, and since the applied loads are the same for all plotted cases, it is perhaps not unreasonable that plate length does not significantly affect bone stresses at such locations. When computer outputs are scanned for stresses in bone beneath the plates, however, we see that increases in plate length, which make the plate stress go up, cause the bone stress to go down. The stresses in the bone immediately beneath a 6" plate are lower than stresses at corresponding locations for a 4.5" and for a 3" plate. If, for example, we look at the maximum tensile stress in elements which represent bone immediately beneath the plate and between the outer two screws (on a six hole plate) we observe values of 1817 psi, 2132 psi and 2555 psi for the 6", 4.5" and 3" plates respectively. (These results correspond to case codes 1 - 6S,

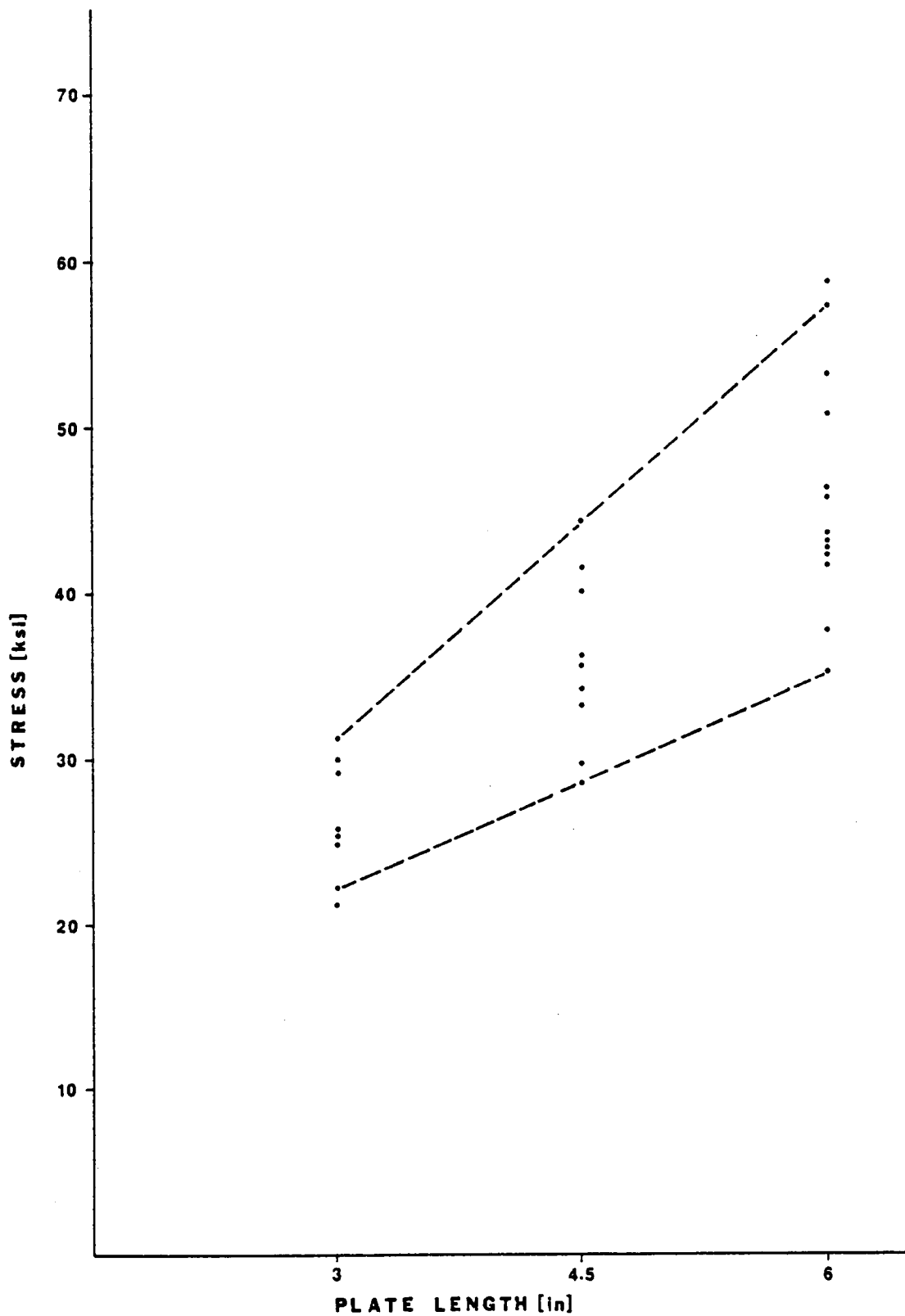


Figure 20. Longitudinal Stress at Middle of Bone Plate Computed in Parametric Study

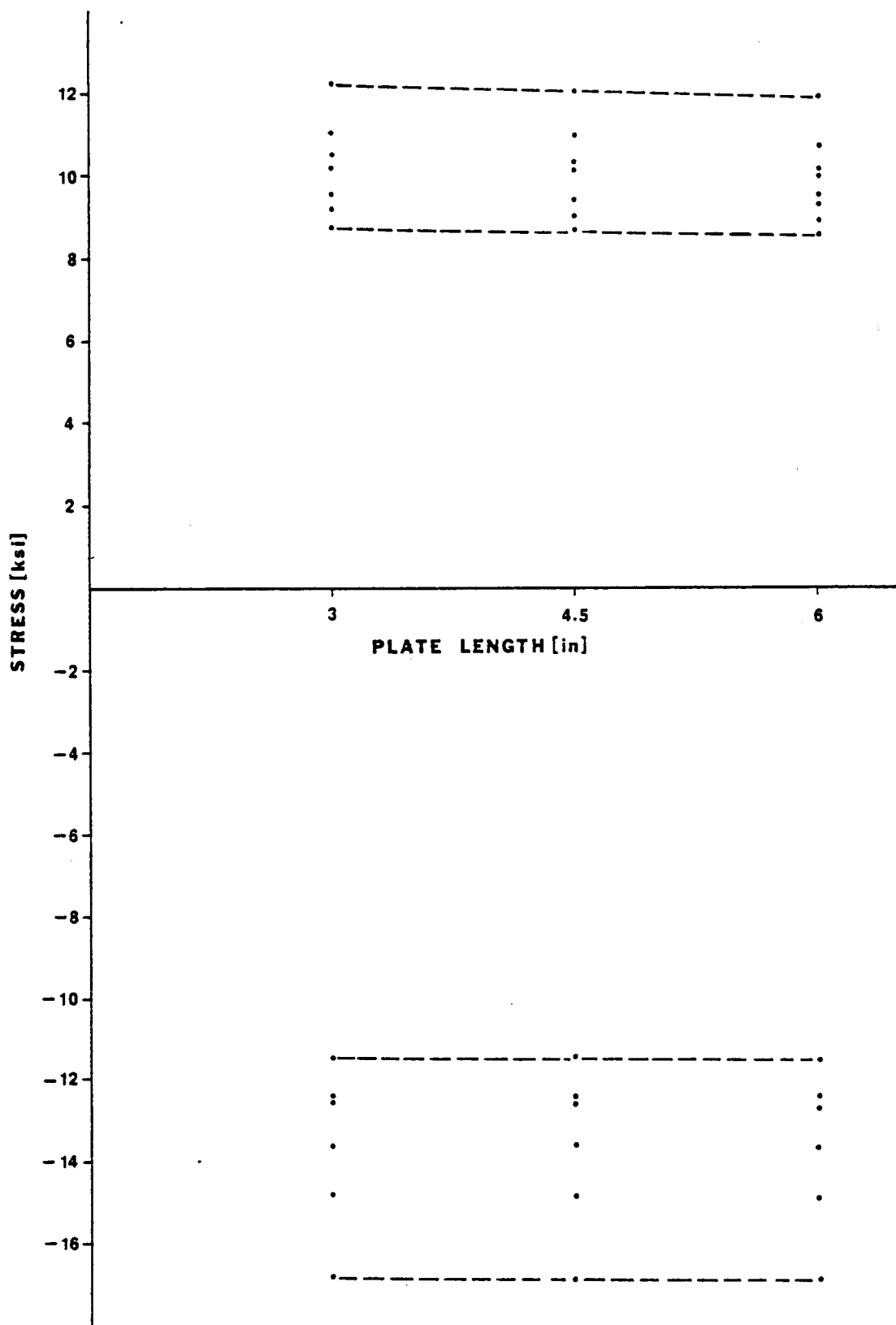


Figure 21. Maximum Tensile and Compressive Stress in Bone Computed in Parametric Study

14 - 6S, and 27 - 6S as listed in Table 8). One possible explanation for these trends is that as the plates get longer they carry an increased fraction of the total applied load and this manifests itself as an increase in plate stress accompanied by a decrease in stress in bone beneath the plate.

The state of stress at every point in the structure changes as the selected input data are modified, however such changes are clearly not random nor independent, as indicated above. So, for this discretization, examination of a single dependent variable should permit the extrapolation necessary to allow conclusions about changes in other dependent variables (in this case other stresses). Ideally the best choice of an output stress is the most sensitive one and this clearly is the tensile stress at the middle of the metal plate. Figures 22 through 28 show how this "test" stress varies as each of the independent variables is operated on. For each each graph, only the factor indicated is being modified. All other factors are held constant.

Figure 22 shows the effects of varying screw-to-screw distances in plate middle and plate ends. If STSDE is held constant, stress rises approximately 1101 psi for every 1% increase in STSDM. Conversely, if STSDM is held constant the test stress rises approximately 462 psi for every 1% increase in STSDE.

Figure 23 illustrates the dependency on fracture angle alpha for all three plate lengths. The anomalous behaviour of the 6" plate bears further consideration.

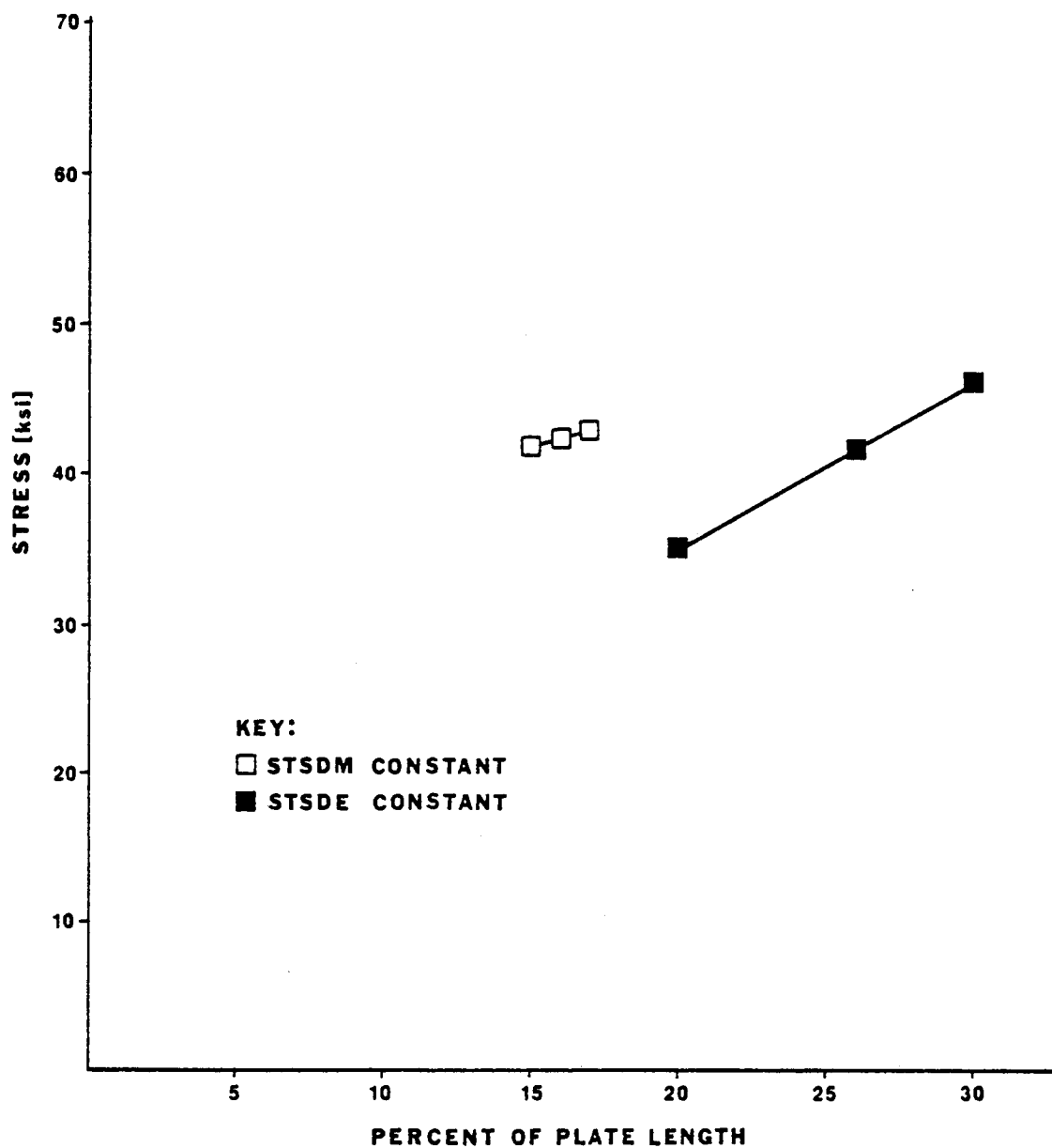


Figure 22. Plate Maximum Tensile Stress Plotted as a Function of % STSDM and % STSDE

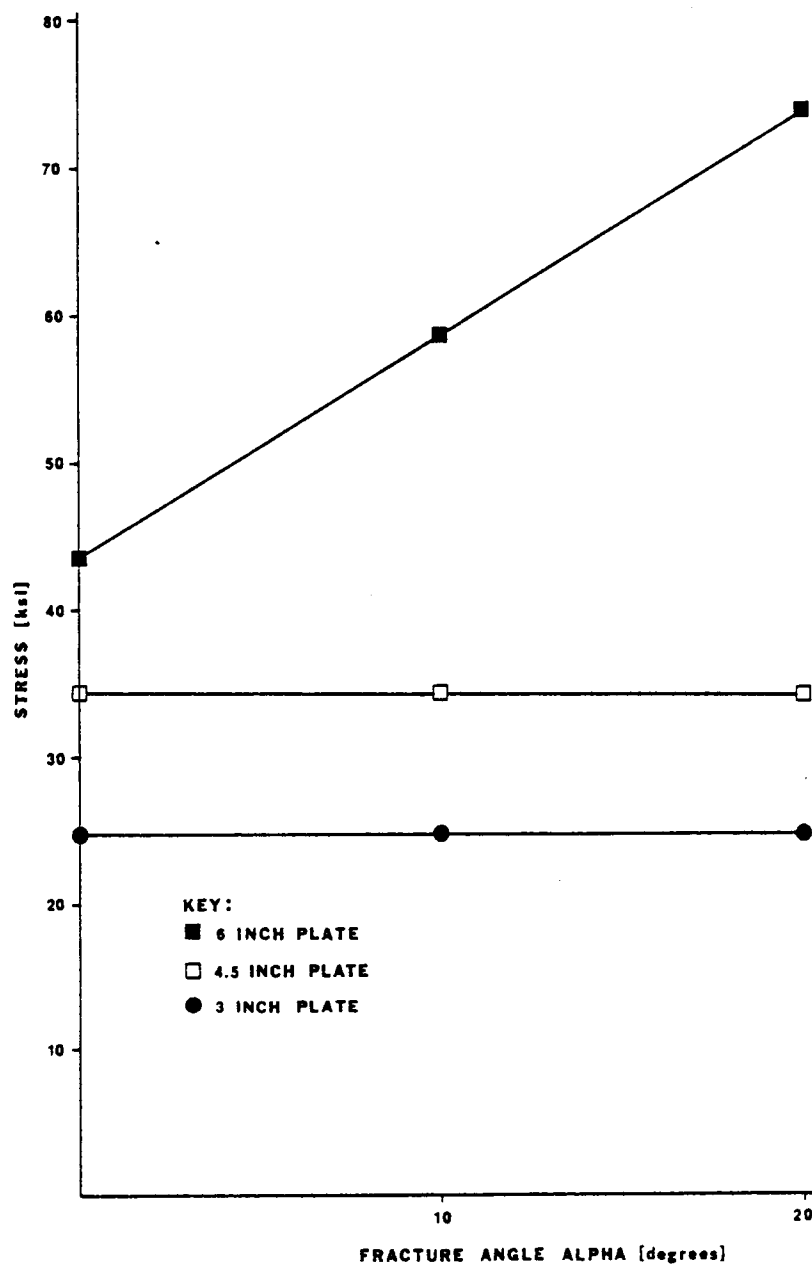


Figure 23. Plate Maximum Tensile Stress Plotted as a Function of Fracture Surface Angle ALPHA

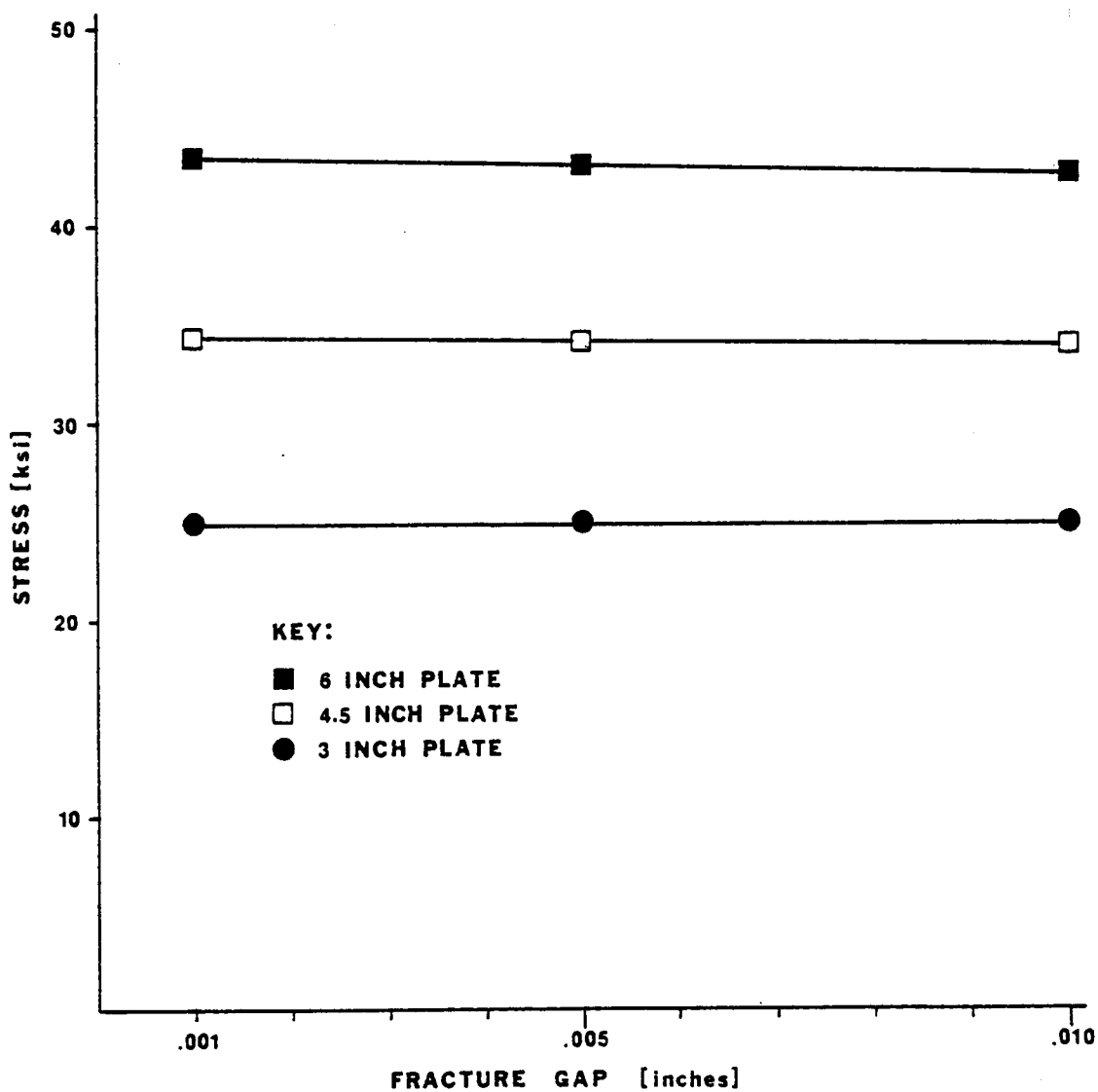


Figure 24. Plate Maximum Tensile Stress Plotted as a Function of Distance GAP between Fracture Surfaces

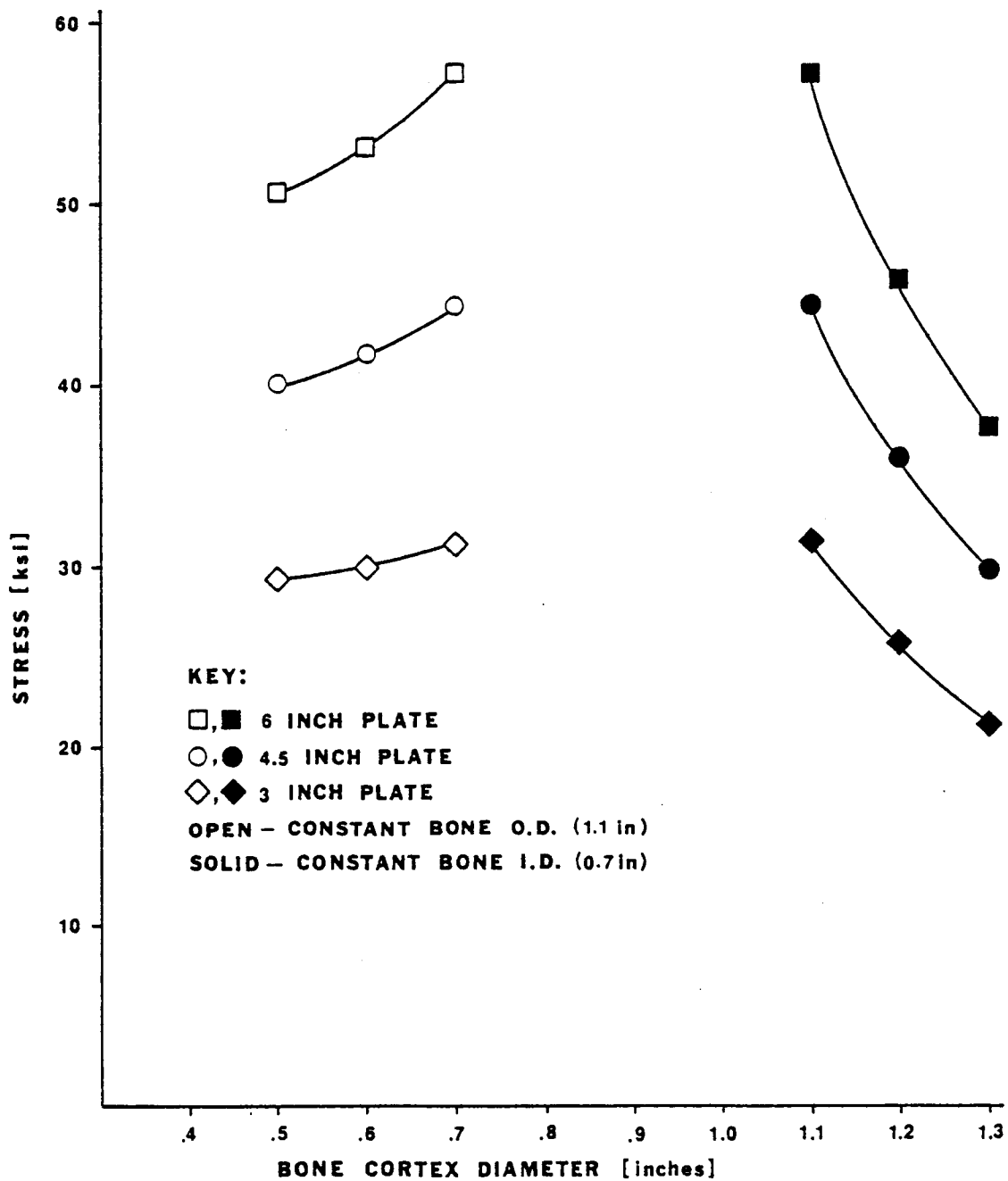


Figure 25. Maximum Tensile Stress in Plate, Plotted as a Function of Bone Cortex Inside and Outside Diameters

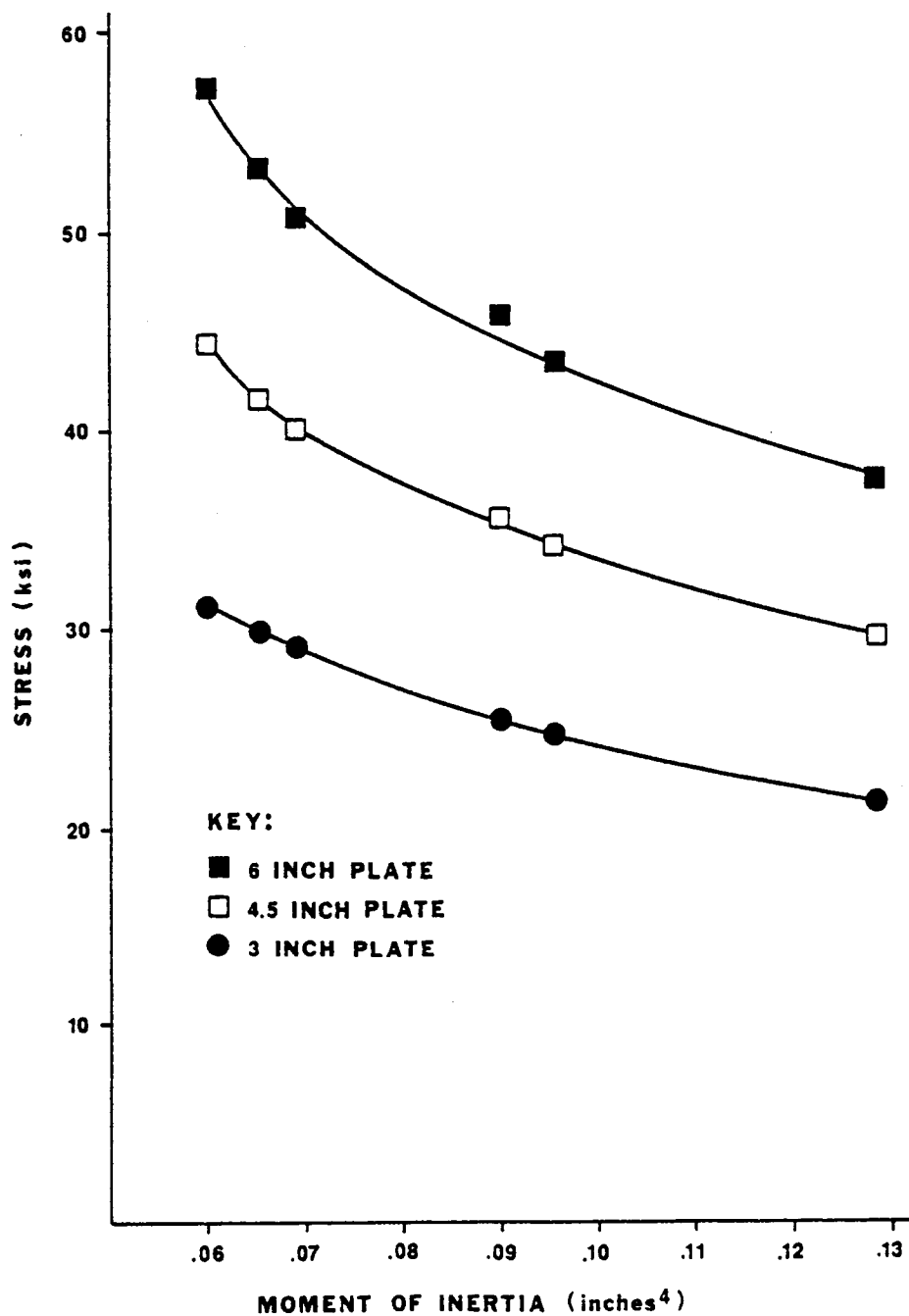


Figure 26. Maximum Tensile Stress in Plate, Plotted as a Function of Bone Cross Section Moment of Inertia

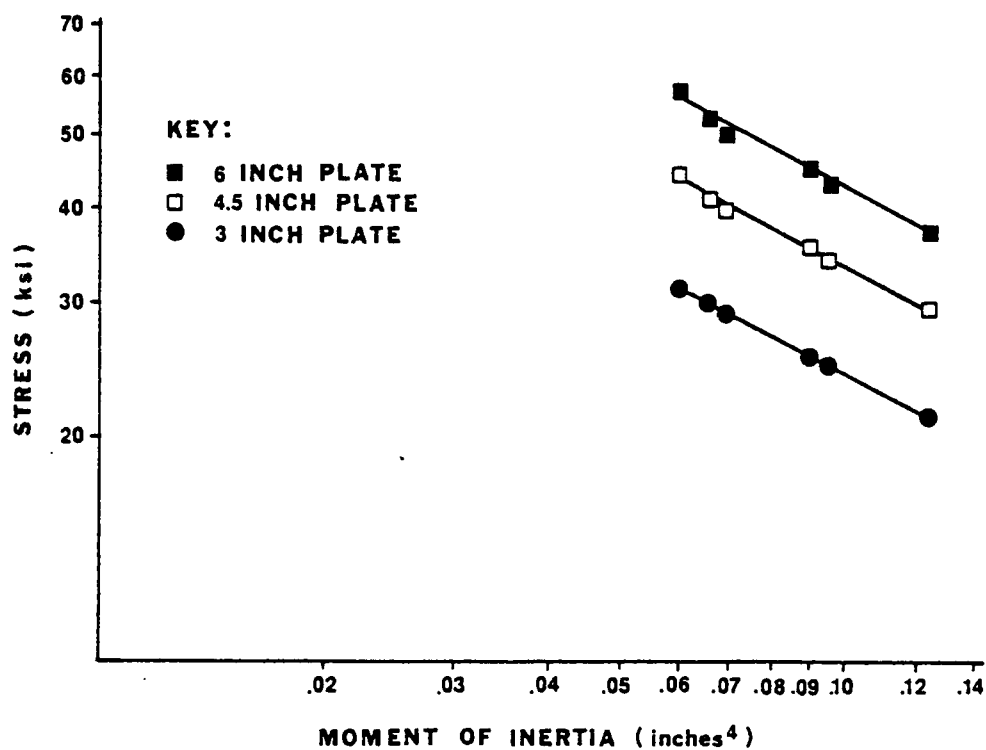


Figure 27. Log-Log Plot of Maximum Tensile Stress in Plate vs. Bone Cross Section Moment of Inertia

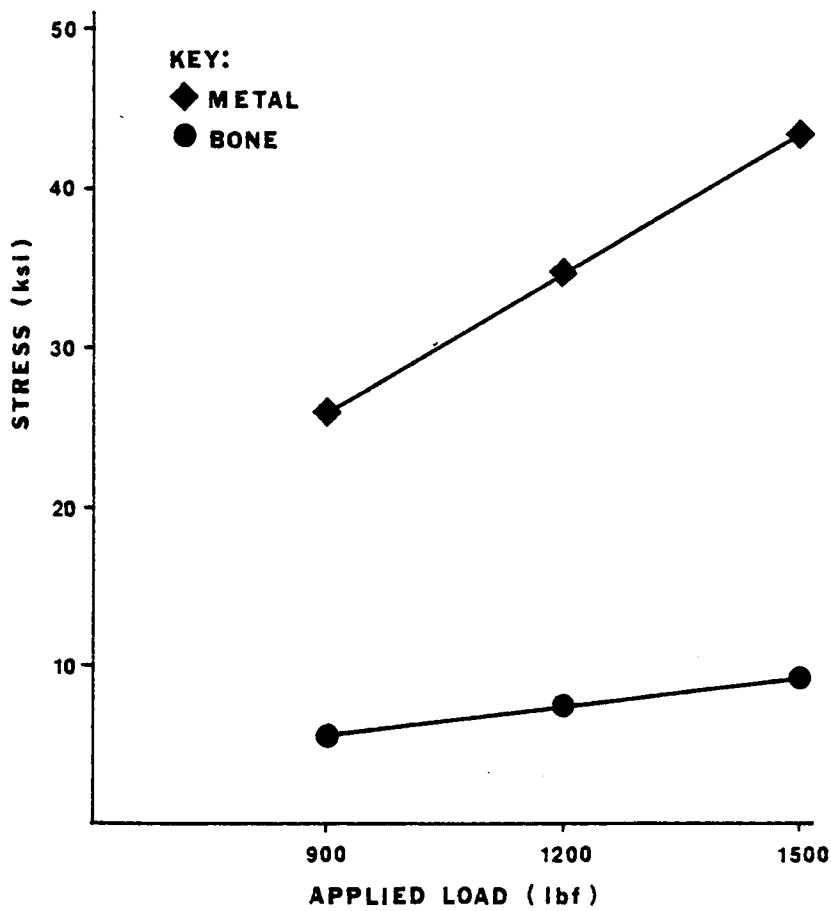


Figure 28. Plate and Bone Maximum Tensile Stress, Plotted as a Function of Applied Load

Although screw to screw distances across the middle of the plate (STSDM) were the same percentage of total plate length for the plotted cases, the absolute values of these distances obviously are greater for the longer plates. This increase in inter-screw distance may, in part, explain the increased sensitivity to angle ALPHA in the 6" plate. Regardless of the cause the distance PLTLEN and the angle ALPHA are in some sense related.

The gap between fracture surfaces is shown to exert relatively little influence as shown by Figure 24.

Bone cortex diameters, however are quite important as indicated in Figures 25 to 27. Figure 25 illustrates the stress variation with inside and outside diameters separately and Figure 26 presents stress vs. a new variable which combines the diameters (moment of inertia). Figure 27 shows the same variables as Figure 26 but on a log-log scale.

An extra variable, applied load, which is not part of the planned parametric study is plotted against plate max. tensile stress in Figure 28. The free body diagram of Figure 13 shows the applied load and the applied bending moment whose magnitude is equal to that of the load. Peak forces transmitted to the femur through the joints may be as high as six times^{97,102} the body weight, thus the loadings indicated in Figure 28 correspond to six times body weights of 150, 200 and 250 lb. persons. The majority of SAPV "experiments" include loading corresponding to the 250 lb. individual. Such loading represents an upper limit on in-service forces, and hence tends to make predictions of failures in bone or plate more likely. This in turn, facilitates

the ranking of certain combinations of independent variables as being safe or unsafe.

The examination of influence exerted by individual factors immediately demonstrates that some are more important than others. Such one-by-one analyses, however, provide little information about possible interactions between factors. Figure 23, for instance, suggests the possibility of an interaction between plate length (PLTLEN) and fracture angle (ALPHA). For further exploration of the correlation between the variables considered, a regression technique known as principal components analysis¹¹¹ was used. The implementation of this analysis was accomplished through the use of the BMDP Biomedical Computer Programs¹¹² package. The dependent variable, it is assumed, may be expressed as a linear combination of the independent variables, the coefficients for which are computed by the principal components program. The user is not restricted to the variables themselves but may, rather, use functions of one or more variables. For example, a decrease in error in the regression equation may be effected through use of $\log I$ rather than I (I = bone moment of inertia). For the first regression computation the plate stress (hereafter called STRESS) is the dependent variable while the geometrical factors PLTLEN, % STSDM, % STSDE, I , GAP and ALPHA are independent. For all cases considered the load was 1500 lb. and 1500 in. lb. The regression equation defined is as follows.

$$\begin{aligned}\text{STRESS} = & -4254.18 + 7187.07(\text{PLTLEN}) + 769.35(\% \text{ STSDM}) \\ & + 443.52(\% \text{ STSDE}) + 639.25(\text{ALPHA}) \\ & - 94155.0(\text{GAP}) - 223818.7(\text{I})\end{aligned}$$

WHERE: PLTLEN = Plate length (inches)

% STSDM = Screw-to-screw distance at plate middle expressed as a percentage of PLTLEN

% STSDE = Screw-to-screw distance at plate ends expressed as a percentage of PLTLEN

ALPHA = Fracture angle (degrees)

GAP = Gap between fracture surfaces (inches)

I = Bone cross section moment of inertia (inches⁴)

Figure 29 is a computer print plot of observed vs. predicted STRESS for this analysis. On the graph P denotes a predicted value, O denotes and observed value and * appears when the two coincide.

While many of the predicted values closely approximate the observed ones, others show rather large error. The largest error coincides with the test case where ALPHA is 20⁰ and PLTLEN is 6 in. Previous observations suggest a correlation of some kind between these two variables and in an effort to improve predictions a number of trials were performed using various combined functions of PLTLEN and ALPHA. The most dramatic reduction in error was found when a new variable APEXP was used in place of ALPHA. This new variable is defined as follows:

$$\text{APEXP} = \text{ALPHA} * \text{EXP}(\text{PLTLEN})$$

WHERE: ALPHA = fracture angle (degrees)

PLTLEN= plate length (inches)

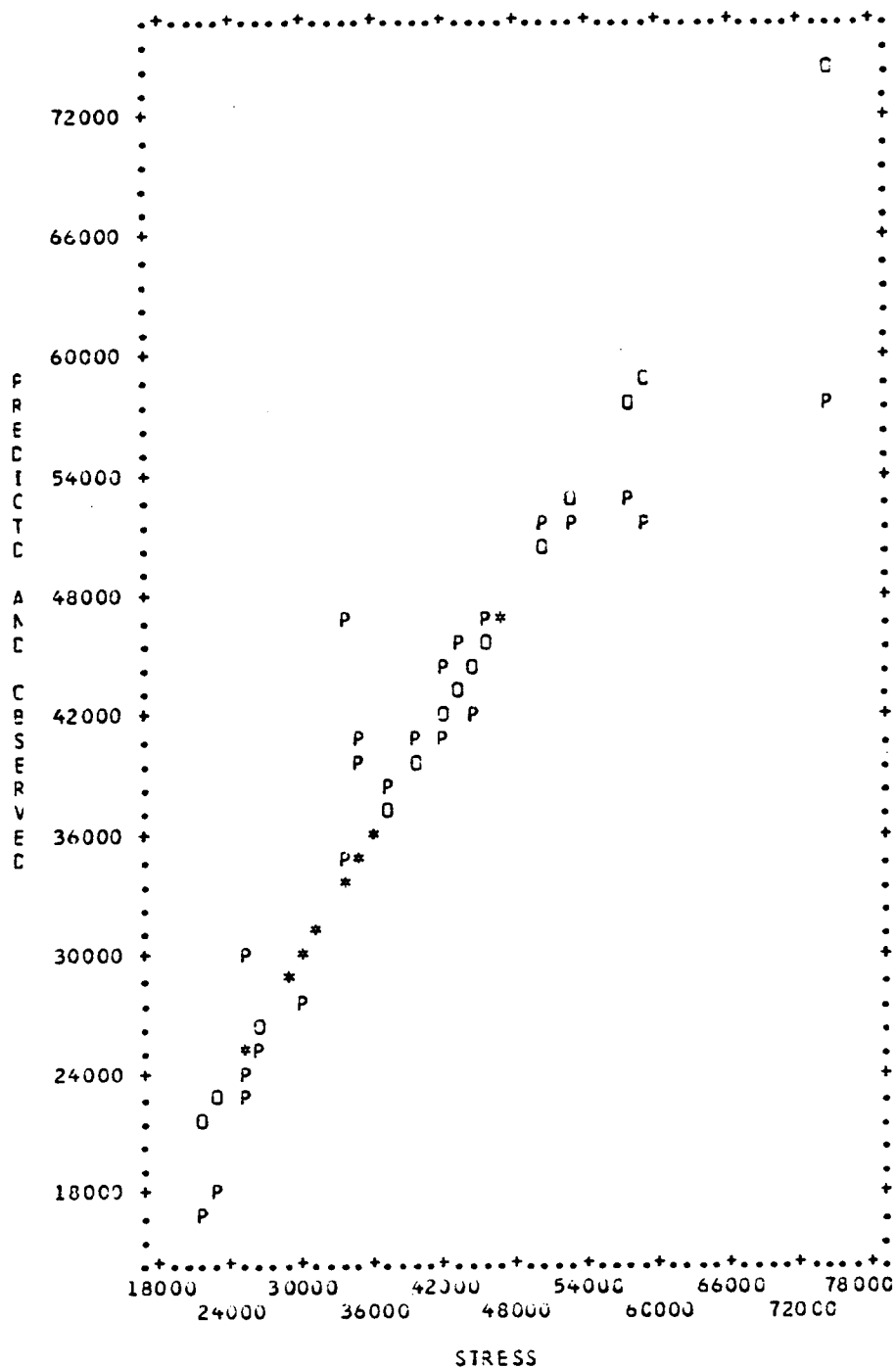


Figure 29. Comparison of Observed Maximum Plate Stress to Predicted Stress from Principal Components Regression. Angle ALPHA Used

The new regression equation is:

$$\begin{aligned}\text{STRESS} = & 649.52 + 6469.24(\text{PLTLEN}) + 773.96(\% \text{ STSDM}) \\ & + 330.03(\% \text{ STSDE}) - 85761.0(\text{GAP}) \\ & - 224817.3(\text{I}) + 3.48(\text{APEXP})\end{aligned}$$

The graph of Figure 30 shows an improvement in predicted vs. observed values corresponding to the new equation. It must be borne in mind that these are purely empirical results. Also, because of the widely varying scales and orders of magnitude of the independent variables, one should not rely only on the relative magnitudes of the regression equation coefficients as an indicator of the sensitivity of the dependent variable to any given factor. The previous discussion of Figures 20 through 27 has already shown that the most prominent factors insofar as plate tensile stress is concerned are plate length and bone cross sectional dimensions. These observations have been further supported by computations of correlation coefficients done by the principal components program. In the case of the second STRESS equation above, the coefficients for PLTLEN, APEXP and I are .8024, .5972 and -.2867 respectively. (If the coefficient, r , is equal to + or - 1, the correlation is perfect.) Coefficients for other variables were all less than .2 indicating a lesser degree of correlation.

When the regression analysis is repeated using first max. bone tensile stress (BONTEN) and then max. bone compressive stress (BONCOM) as the dependent variable the following equations are obtained.

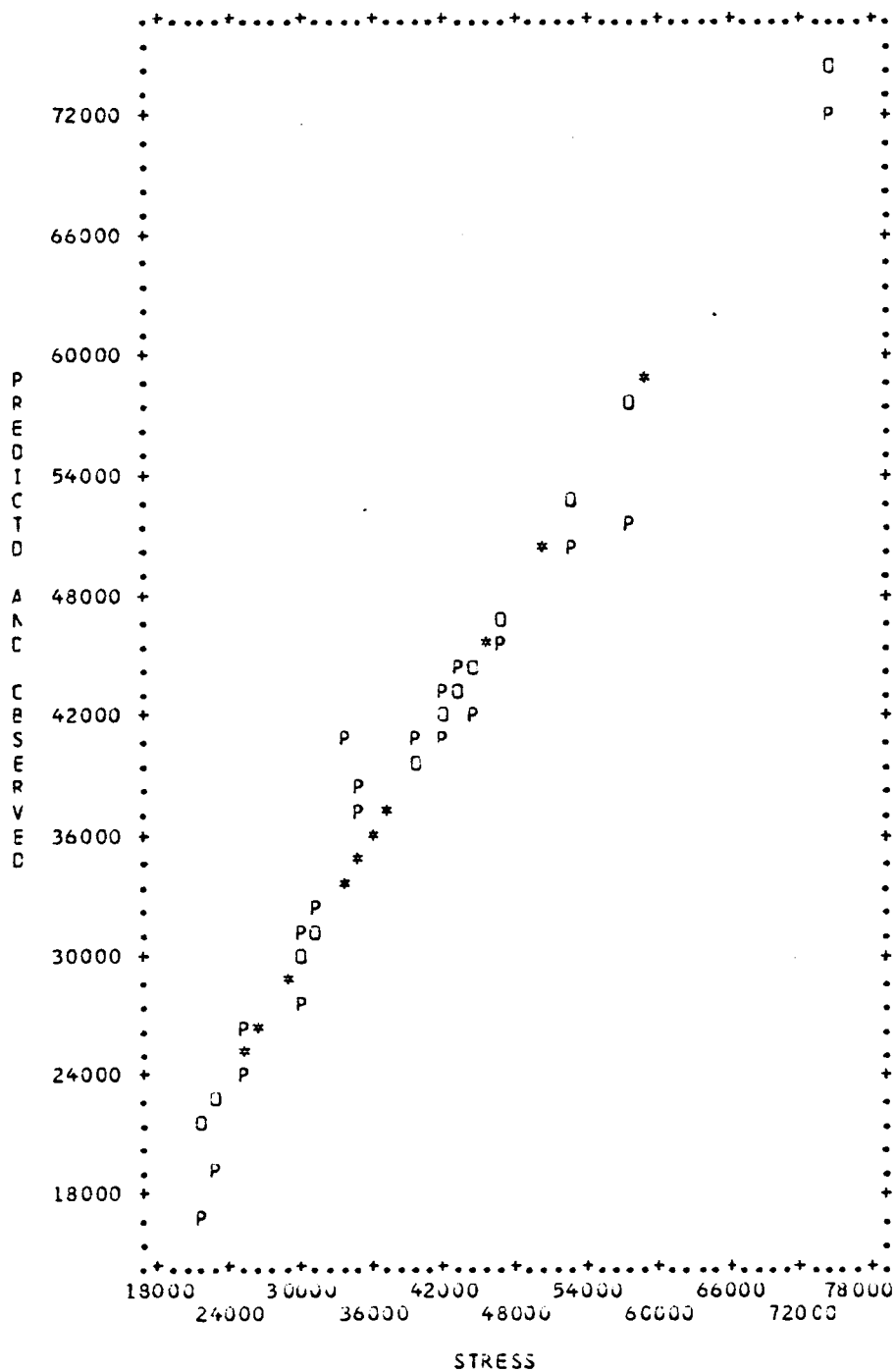


Figure 30. Comparison of Observed Maximum Plate Stress to Predicted Stress from Principal Components Regression. New Variable APEX Used.

$$\begin{aligned} \text{BONTEN} = & 8939.73 - 113.49(\text{PLTLEN}) + 47.77(\% \text{ STSDM}) \\ & + 263.64(\% \text{ STSDE}) - 16.02(\text{ALPHA}) \\ & - 34,484.6(\text{GAP}) - 44,625.5(\text{I}) \end{aligned}$$

$$\begin{aligned} \text{BONCOM} = & -16,366.87 - 24.59(\text{PLTLEN}) - 6.65(\% \text{ STSDM}) \\ & - 178.32(\% \text{ STSDE}) + 28.73(\text{ALPHA}) \\ & + 63,424.5(\text{GAP}) + 68,286.4(\text{I}) \end{aligned}$$

In the BONTEN equation solution, the strongest correlation is to I with a coefficient of $-.8794$ and all other coefficients are much lower. Similarly, for BONCOM the coefficient of correlation between the bone compressive stress and I is 0.8726 while all other coefficients are much lower (magnitudes less than 0.2). Plots of these two regression equations are given in Figure 31 and 32. While the increase in magnitude of bone stress with decreasing moment of inertia is not an unexpected result, it does however suggest that any disease states which operate to reduce the amount of cortical bone present a serious impediment to the success of any plating operation.

A knowledge of stresses in metal and bone provides a means of predicting and preventing fracture failures. Another clinically important stress related problem does not relate to breakage of plate or of bone, but rather relates to healing of the fracture. The primary function of the bone plate is maintenance of contact between the fracture surfaces and avoidance of relative motion between those

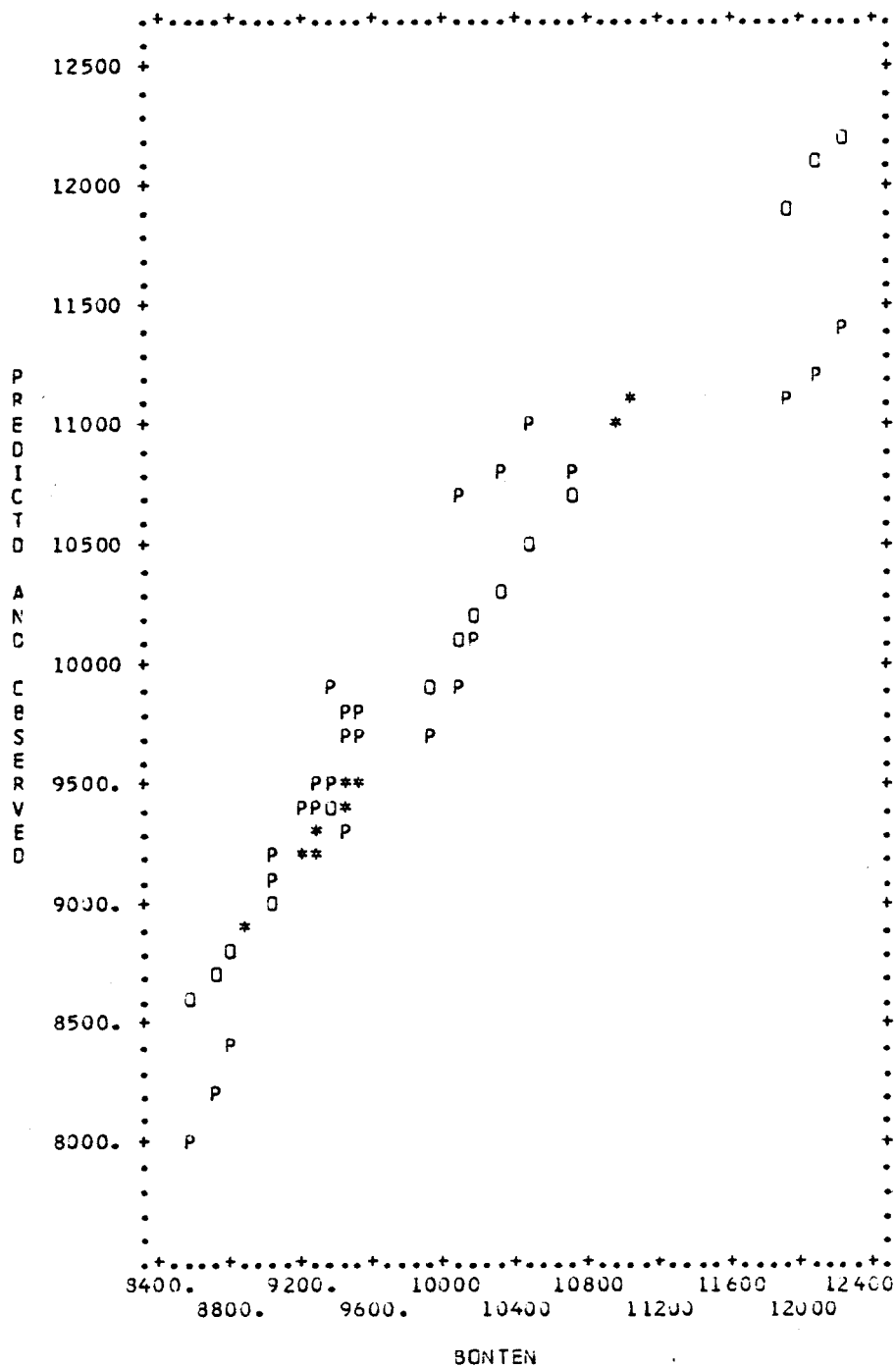


Figure 31. Comparison of Observed Maximum Bone Tensile Stress to Predicted Stress from Principal Components Regression

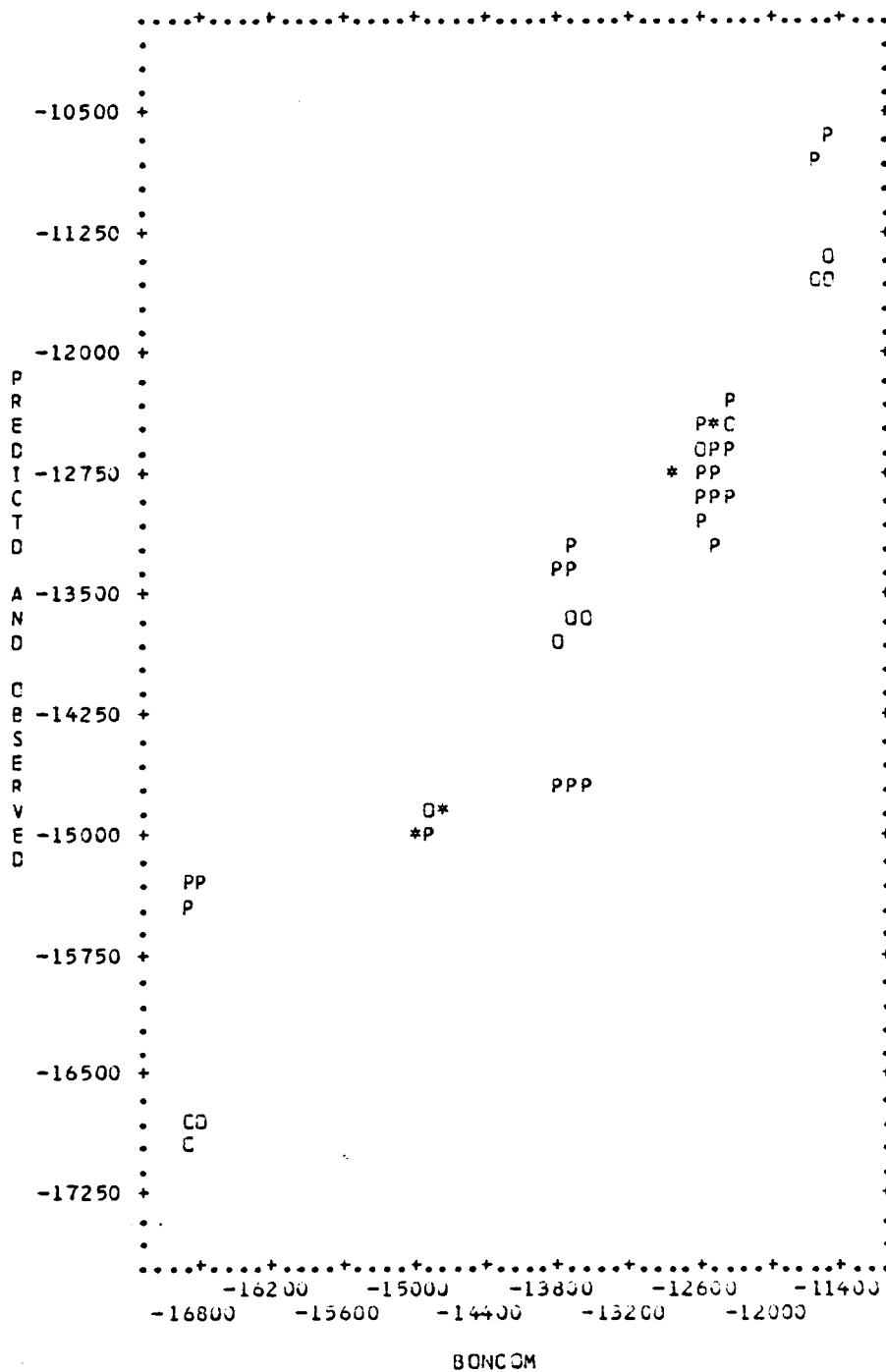


Figure 32. Comparison of Observed Maximum Bone Compressive Stress to Predicted Stress from Principal Components Regression

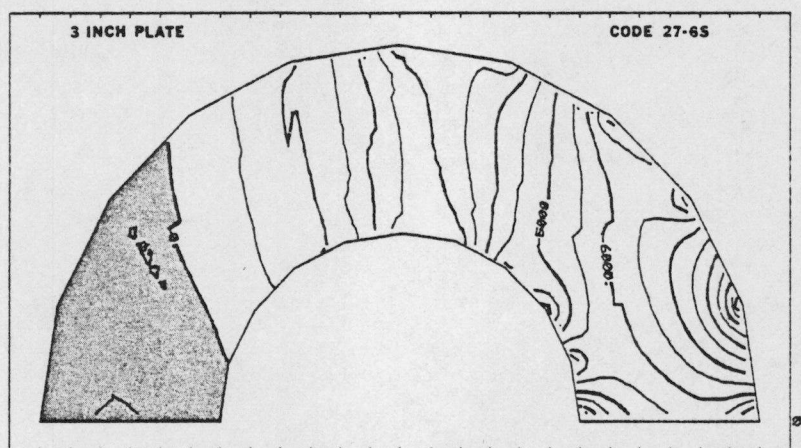
surfaces. The current technique of "compression plating,"[†] for example, is the result of efforts to maintain contact, and, by application of compressive force to expedite the healing process. The modelling of the fracture site, as done in the present finite element analysis, provides the information necessary for an examination of variations in pressure distribution on the fracture surfaces which occur as the selected independent variables are modified.

Values of contact stress at several locations are found in that portion of the SAPV program output corresponding to the no-tension elements described in section III. A graphical representation of the resulting pressure distributions takes the form of contour plots, accomplished through the use of the SURFACE II GRAPHICS SYSTEM.¹¹³ Comparison plots showing the effect of plate length, whose importance has already been demonstrated are given in Figure 33. In addition dependent variables of applied load, applied plate pretensioning load (as in compression plating) and number of bone screws have been selected and their effects on fracture surface stresses are given by Figures 34 - 36 respectively.

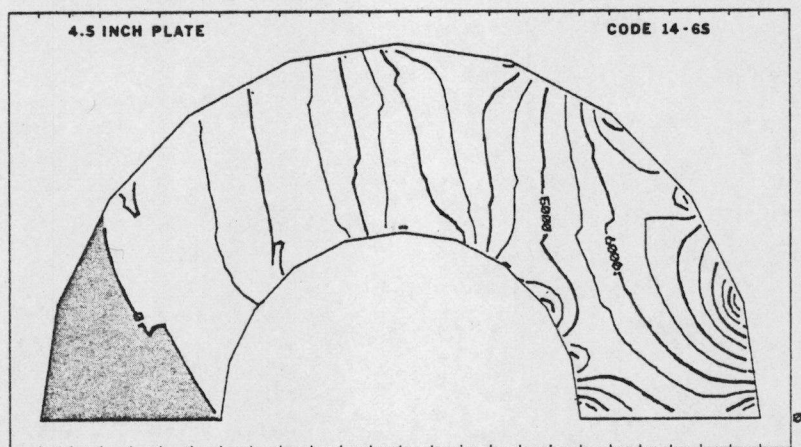
For all the contour plots, the heavier iso-stress lines are drawn at intervals of 1000 psi. The majority of the plots, except for two of those in Figure 34, correspond to an applied loading of 1500 lb. and 1500 in. lb.

[†]Putting the plate into tension, by stretching it and then attaching it to the bone, has the effect of putting the bone immediately adjacent to the plate in compression and forcing a portion of the fracture surfaces together.

* COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
 PLOT NO. 1 DATE 13-JUL-79 TIME 18:49



* COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
 PLOT NO. 1 DATE 13-JUL-79 TIME 18:28



* COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
 PLOT NO. 1 DATE 13-JUL-79 TIME 14:31

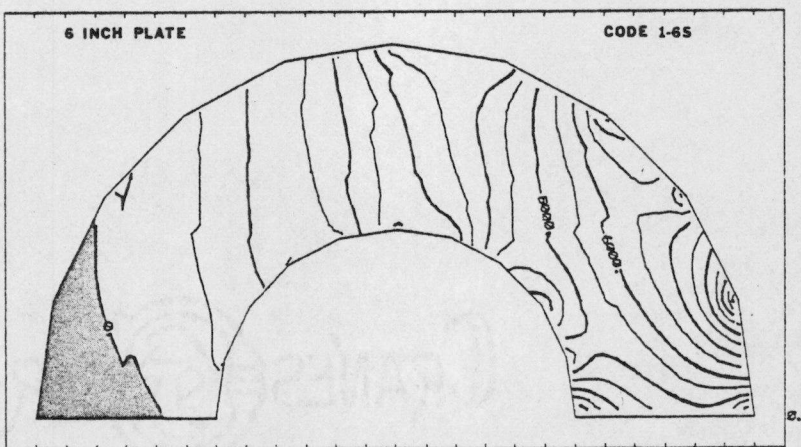


Figure 33. Stress Contours on Bone Fracture Surfaces for
 6, 4.5 and 3" Plates
 113

COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
PLOT NO. 1 DATE 13-JUL-79 TIME 16:28



COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
PLOT NO. 1 DATE 13-JUL-79 TIME 16:12



COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
PLOT NO. 1 DATE 13-JUL-79 TIME 14:31

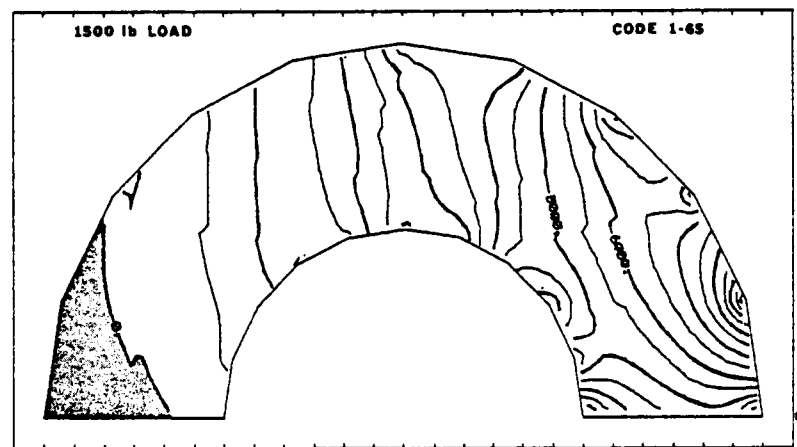
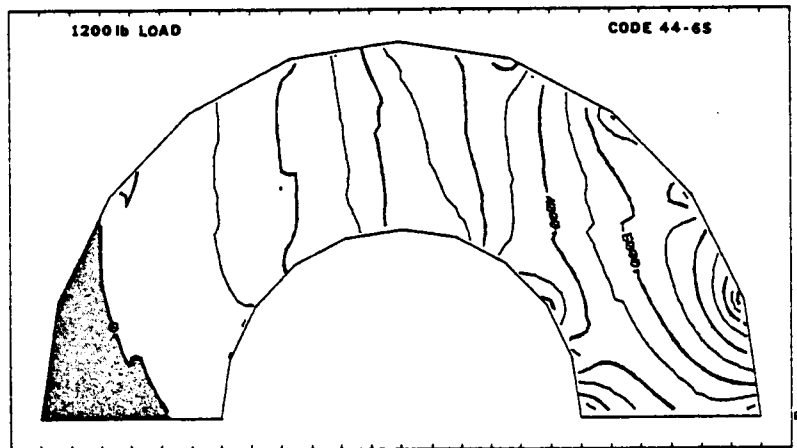
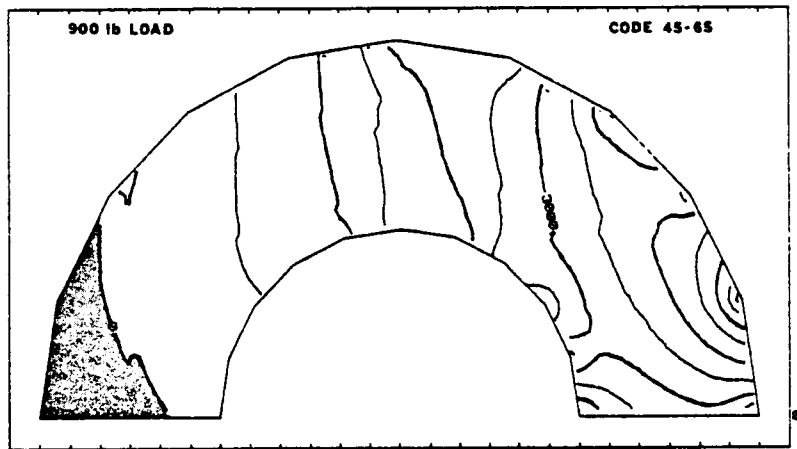


Figure 34. Stress Contours on Bone Fracture Surfaces for 1500 lb., 1200 lb. and 900 lb. loading

COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
 DATE 13-Jul-79 TIME 14:31
 PLOT NO. 1



COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
 DATE 13-Jul-79 TIME 16:39
 PLOT NO. 1

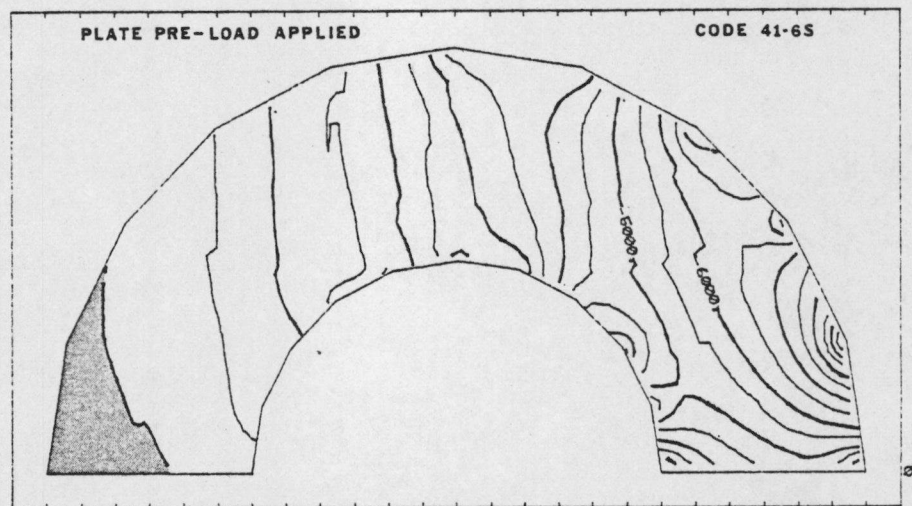
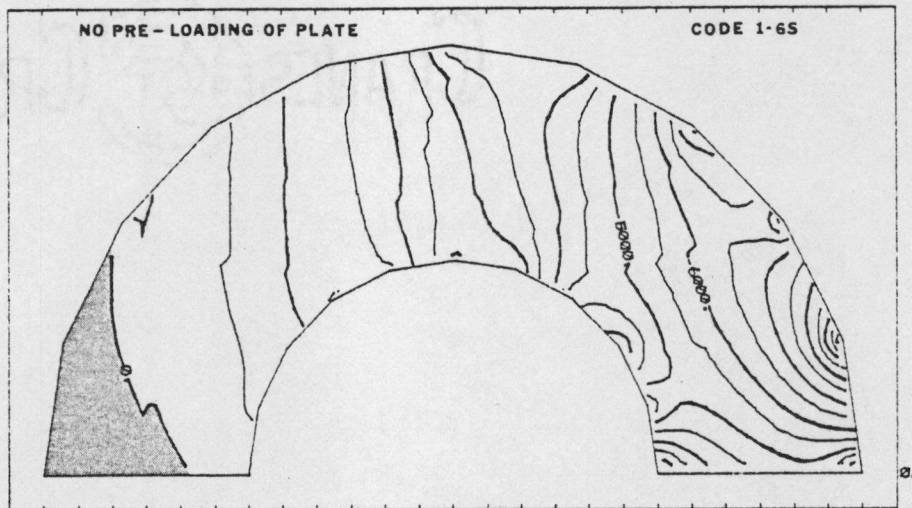


Figure 35. Stress Contours on Bone Fracture Surfaces Without and with Pretensioning Force in Plate

COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
PLOT NO. 1 DATE 13-Jul-79 TIME 16163



COMPRESSIVE STRESS AT LONG BONE FRACTURE SITE
PLOT NO. 1 DATE 13-Jul-79 TIME 14131

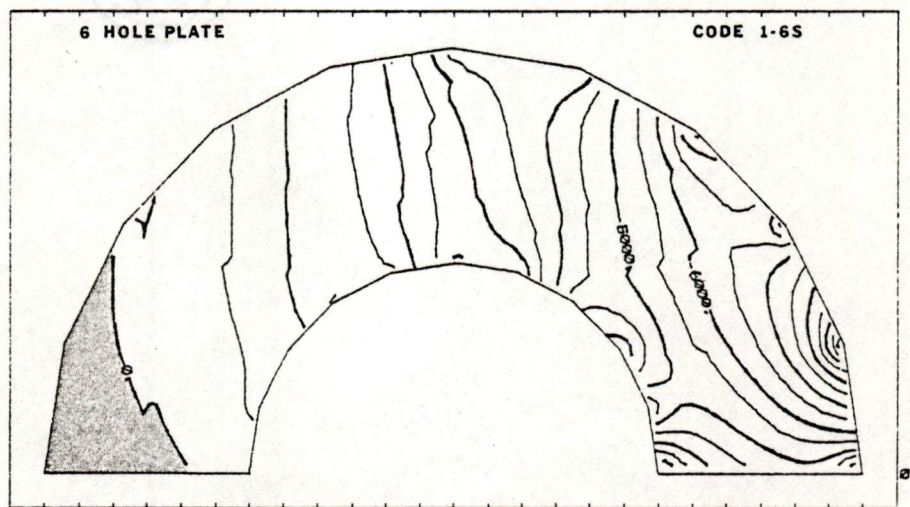
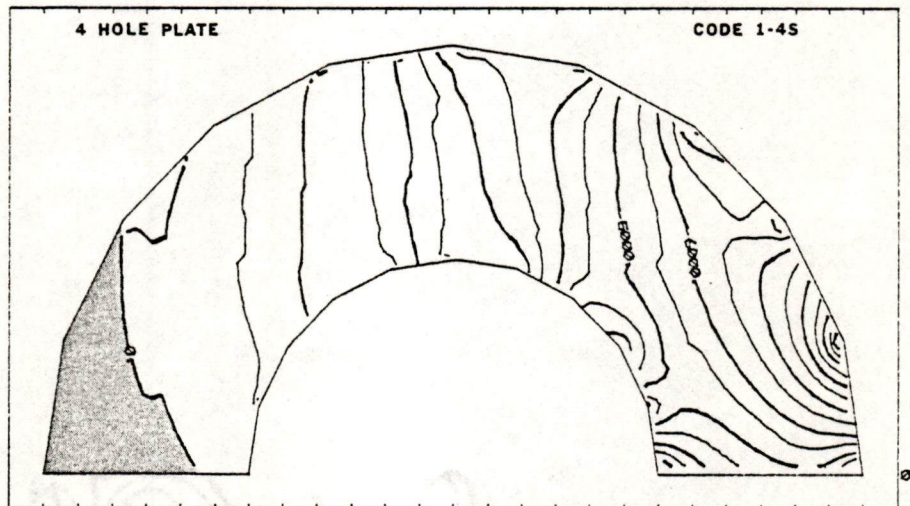


Figure 36. Stress Contours on Bone Fracture for 4-Hole and 6-Hole Plates

The study of the importance of plate length, shown in Figure 33 represents data from "experiments" whose codes (refer to Table 8) are 1 - 6S, 14 - 6S and 27 - 6S. For the applied load sensitivity study of Figure 34 data from tests 1 - 6S, 44 - 6S and 45 - 6S are compared. In order to simulate the application of a compression plate the pre-tensioning force of 195 lb. was applied by imposing a negative temperature difference ($\Delta T = -10^{\circ}\text{F}$) to all nodes in the finite element discretization which designated the plate. A comparison of this results (test code 41 - 6S) with an ordinary plate (test code 1 - 6S) is presented in Figure 35. Finally the comparison of a 4-hole plate (test 1 - 6S) with a 6 - hole plate (test 1 - 6S) is given by Figure 36.

Insofar as the fraction of area in contact is concerned, plate length, of the variables considered, has the most influence. The longer plates hold more of the surfaces together and yet change the magnitude and pattern of contact stresses very little. The compression plating technique also improves the situation, however even a force as high as 195 lb. cannot completely close the gap at all times. Under peak loading associated with a normal walking gait, there can be some distraction of the fracture surfaces. The contact area is little changed by a reduction in applied load, however this reduction does radically change the magnitudes of the contact stresses which may be equally important. Not only is it desirable to have most of the area in contact, but it is also imperative that contact pressures be neither too low nor too high. Too little force, means that the beneficial effect of compression (improved and more rapid healing) will

not be realized, while too much force causes destruction or "pressure necrosis" of tissue. While data regarding the optimum range of loading exists for sheep⁵³ and for rabbit skull bone¹¹⁴ there is as yet no data about human bone available. Since the high compressive stresses of 6000 or 7000 psi, shown on Figures 34 - 37 correspond to peak walking loads and would thus be intermittent, the possibility of pressure necrosis is uncertain but probably not negligible. A change in the number of bone screws results in very little change either in contact area or in pressure distribution. A further examination of peak stresses in plate and bone shows very little difference between plating with 4 and 6 fasteners. Based on their experimental work on plated human tibiae, Laurence et al.⁸³ suggested that at the time of implantation, any more than 4 screws may be redundant, which is consistent with the aforementioned result. These authors noted, however, that such "extra" screws may provide insurance against subsequent structural changes in bone which might act to weaken the system.

The bone screws, represent an extremely important and complex part of the structure. Under the applied loading they are subject to fracture, or to being pulled or pushed out of the bone. The modelling of the fasteners as simple cylinders, means that the importance of stress concentrations at the threads is ignored, yet some general observations on screw mechanical behaviour can be made. A cursory examination of data recorded during the performance of the parametric study indicate relatively little change in magnitude and pattern of stresses in the screws from one "experiment" to the next. For the six-hole plates, if

we look at normal stress on the axis of each screw as it traverses the bone's intramedullary canal, a consistent pattern of tension, compression and tension again is seen as one progresses away from the fracture, thus looking at the innermost, the middle and the outermost screws respectively. The greatest tensile stress observed in any screw is approximately 6500 psi which is much less than the yield stress for cold worked 316L stainless steel (σ yield $\approx$ 100,000 psi.). The greatest tensile stresses are seen in general in the outermost screws. Only in the case of the 6" plate with fracture angle ALPHA different from zero is this pattern disturbed. In such instances the symmetry of observed stresses with respect to the middle of the plate is destroyed, as one should surmise upon examination of Figure 7, p. 50. The greatest stresses for such cases occur on the innermost screws and for experiment 11 - 6S (refer to Table 8, p. 90) a maximum tensile stress of 11,000 psi is observed. The fasteners could also be expected to fail in shear where they pass from bone to plate. The greatest shear stresses of that sort occur (in the plane of the applied load and bending moment) in the outermost screws, at predicted values of 10,000 to 12,000 psi. The possibility of screw pullout or pushout from bone may be assessed through an inspection of bone/screw interface shear stresses where screws pass through the bone cortices. The directions of the computed shear stresses between bone and screws suggest that screws will tend to be pushed out on the non plated side rather than being pulled out by the plate. While these results are not in complete agreement with those of Simon et al.,⁸⁹ they do support the idea that the initially round bone cross section will tend to flatten to an oval

shape because of the bending moment, thus forcing the screws to be pushed out on the non-plated side. For the four-hole plate, the sign and magnitude of stresses on the axes of inner and outer screws corresponds quite closely to values from the middle and outermost screws of comparable six-hole plates. Observed shear stresses in the screws and between screws and bone were also very similar. If anything is to be inferred about benefits associated with the additional two screws, it can certainly not be based on such static analyses as these. One may, in fact, make the general statement that insofar as the screw fasteners are concerned, predicted stresses in metal and adjacent bone do not suggest the possibility of failure from static loading alone.

From the large amount of information produced by the finite element program we have seen a few of the stresses at critical locations, and have observed the response of such stresses to geometrical changes in implant and in bone. In order to predict success or failure of any plating, one must have appropriate stress failure criteria for metal and bone. While familiar combined theories exist for isotropic materials like steel, counterparts for anisotropic bone are neither well developed nor experimentally verified. Furthermore, limitations to stress arise from biological factors as well as from the purely mechanical considerations.

When the maximum distortion energy theory is applied to observed stresses in the steel plate at locations of maximum longitudinal tensile stress, (transverse stresses are also high at such locations)

plastic deformation is not indicated. The much lower stress levels in the screws indicate that yielding is unlikely in them as well. In all cases a yield stress of 100,000 psi¹¹⁵ for cold-worked 316L stainless steel was used in calculations.

As indicated by Table 1, page 15, there is a wide variation in bone ultimate strength values as obtained from the literature. There is however relatively little attention given to yield strengths, but examination of available stress strain curves done on wet bone indicate essentially elastoplastic action in compression (yield and ultimate stresses are the same) and a bi-linear curve in tension for which σ_{ult} is not much greater (perhaps 15 - 20%) than σ_{yield} . Based on the data in Table 1 and on an arbitrarily assumed safety factor of 2, chosen as an allowance for the possibility of fatigue failure, the following uniaxial failure limits for human femur are suggested.

Compression:

Longitudinal	σ allowed $\approx$ 10,000 psi
Transverse	σ allowed $\approx$ 8,000 psi

Tension:

Longitudinal	σ allowed $\approx$ 10,000 psi
Transverse	σ allowed $\approx$ 3,700 psi

Shear:

τ allowed $\approx$ 5,000 psi

The use of such failure criteria, under conditions of multiaxial loading, implies that one ignores the possibility of stress interactions.

That such interactions may occur, has been demonstrated by Lewis and Goldsmith⁴⁸ in their impact studies, but until a more thorough study of the behaviour of bone under multiaxial stress has been made, the uniaxial stress failure criteria represent the best available.

Longitudinal stresses in bone have already been discussed, and for the 1500 lb., 1500 in. lb. loading we note that in several instances the tensile failure limit is exceeded and in all cases the compressive limit is exceeded (refer to Figure 21, page 94) at some location in the bone. Although only a couple of "experiments" were run at lower load levels of 1200 lb. and 900 lb. the observed reduction in bone longitudinal stresses brings them into the safe range. Transverse tensile and compressive stresses, predicted by the SAPV model do not in any case exceed the indicated limits. Shear stresses in bone also do not exceed the limit specified.

Although regression equations of the type defined previously in this section may aid in the assessment of viability of a given bone-plate-screw system, it should be noted that such equations are of limited applicability. The addition of new variables to the parametric study could radically alter the form of any derived regression equations, and even the use of new variables based on the existing ones (for example using $\log I$ instead of I) can exert considerable influence on the end result. The reader may suggest other sets of independent variables which would yield a more intuitively satisfying measure of the physical behaviour of the system.

Overall, one can observe that under the imposed equivalent static loading, failure of the bone-plate-screw structure appears to be relatively unlikely. However, because such loading is applied in a repetitive, cyclic manner in service, the probability of failure for some system geometries can be very high.

VI. SUMMARY AND CONCLUSIONS

Highlights of the present study, along with conclusions and suggestions for future research are herein presented.

1. A three-dimensional finite element model of a fractured human femur with a metal bone plate attached to it by screws, was prepared for the purpose of examining the mechanical response of such a structure to applied static loading.
2. Five anisotropic elastic constants for dry human femoral bone were utilized in the model. As a means of accurately representing the behaviour on the fracture surfaces a no-tension "gap" element was developed and implemented in the SAPV computer code. Symmetry of geometry and loading relative to a frontal plane was assumed.
3. The viability of the model was demonstrated by means of a laboratory experiment wherein a strain-gauge instrumented bone plate was fixed onto a dry, artificially fractured, human femoral shaft. The resulting structure was subjected to a combined compressive and bending load by means of an INSTRON Universal Testing Machine.
4. A series of simulated "experiments" employing the finite-element model was planned and executed on an IBM 370 computer. The parametric or parameter sensitivity study consisted of 48 runs of the finite element code. The geometrical and load parameters studies were manipulated by means of a FORTRAN computer code called SCALE which selectively altered a chosen parameter in response to interactively entered user commands.

5. Various stresses in metal and bone were examined and the response of those stresses to a given parameter change was discussed.
6. The greatest tensile stress in the metal occurred consistently at the middle of the bone plate in a longitudinal direction. The maximum stress in these sites was generally less than half of the yield strength for the chosen alloy.
7. The greatest observed tensile stresses in bone screws were less than 10% of the metal's yield stress, however the stress concentrations associated with screw threads were not considered.
8. The maximum stresses in metal, primarily in the plate, were most sensitive to changes in plate length, bone cross section dimensions and angulation of the fracture.
9. Shear stresses between bone screws and bone cortex were in general less than 1000 psi and their direction indicated the likelihood that screws would be pushed out of bone on the non-plated side.
10. At the highest applied load value of 1500 lb. combined with a 1500 in. lb. moment, maximum tensile stresses in bone ranged from 8400 to 12,200 psi and compressive stresses ranged from -11,500 to -16,800 psi. The peak stresses in bone were most sensitive to changes in bone cross sectional dimensions.
11. An examination of pressure distribution on the fracture surfaces indicate that an increase in plate length can greatly increase the fraction of the surfaces in contact. Compression plating can also bring more of the surface area into contact. Changes in applied load do little to change contact area but can dramatically alter the magnitudes of observed contact stresses.

12. In consideration of the various stresses analyzed there is very little difference between the response of a 6-hole plate and that of a 4-hole plate to static loading.
13. As a general observation, one may state that under the imposed loading and in the circumstances herein described, fracture failure of either bone or metal is unlikely to be caused by static loading alone. Nevertheless, a parametric study of this sort allows one to locate points of greatest stress where repetitive loading, combined with localized corrosion mechanism may cause deterioration and ultimate collapse of either material.
14. Further study of the implant-bone system should concentrate in the following areas:
 - (a) A theoretically based, and experimentally verified failure theory for combined loading of cortical bone should be delineated.
 - (b) A dynamic analysis should be done on a similar finite element model. Inertial loads and non-linear elastic properties of bone should be included.
 - (c) A survey of retrieved bone plates, removed because of in-service fracture failures, could be useful if the mode and location of failures could be categorized according to the values of various system parameters such as those considered in this study. Only through long term study of the behaviour of such devices in practice will the true worth of any modelling technique be determined.

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APPENDICES

APPENDIX A

A NO-TENSION ELEMENT FOR SAPV

This element was added to the SAPV element library so as to permit accurate modelling of the behaviour at the site of a bone fracture. Within this appendix are presented a flow chart (refer to Figure A-1, page 139) of the modified SAPV program, input data instructions and a computer listing of all new and modified subprograms.

The identifying number for this new element is 11(ELTYPE = 11) and input data should be inserted in their proper numerical sequence. All other data for SAPV are as indicated in available programmer's manuals.⁹³

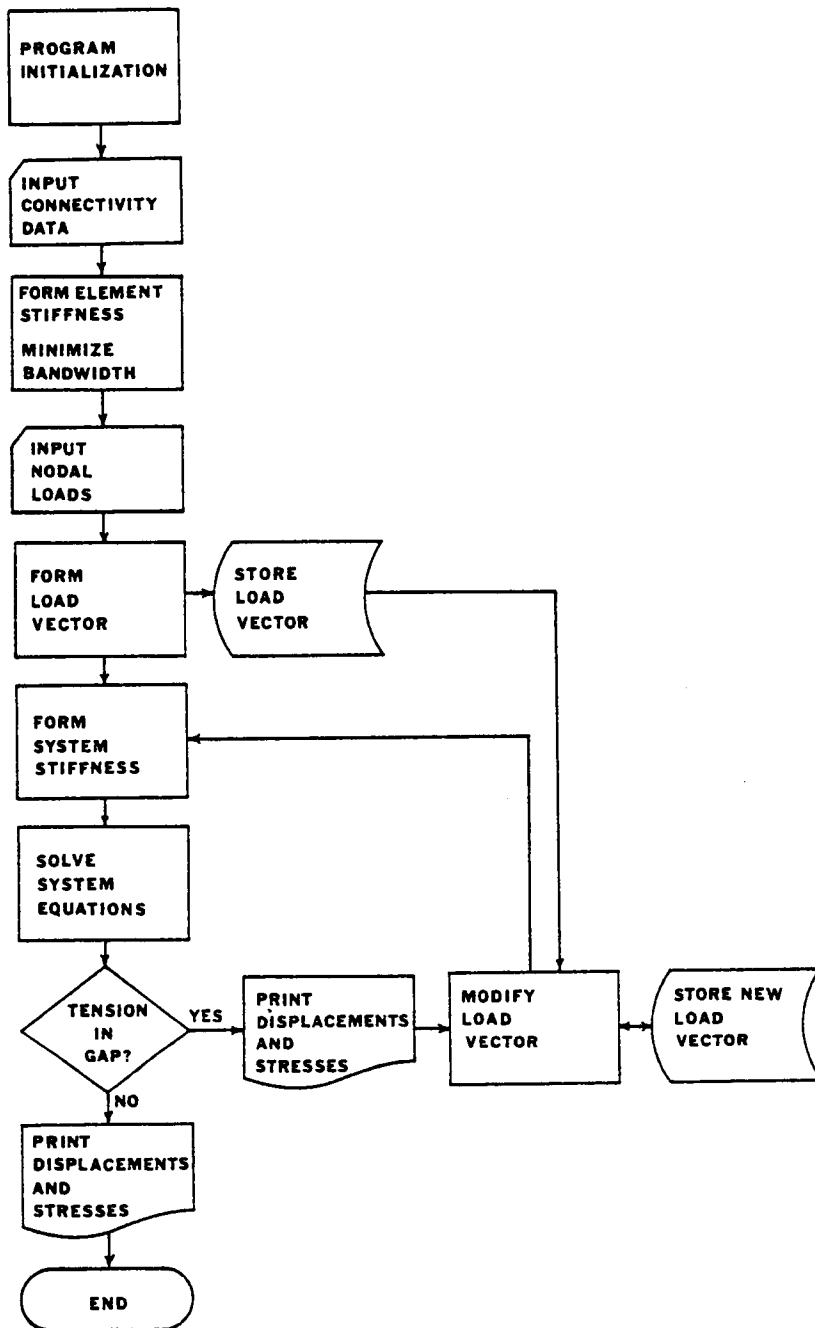


Figure A-1. SAPV Operation with No-Tension Element

ELEMENT DATA

TYPE 11 - NO-TENSION TRUSS TYPE ELEMENT

The no-tension element is identified by the number 11. Axial forces and stresses are calculated for each element.

A. Control Card (315)

Columns 1 - 5 The number 11
6 - 10 Total number of gap elements
11 - 15 Number of material property cards

B. Material Property Cards (I5, 5F10.0)

There need be as many of the following cards as are necessary to define the properties listed below for each element in the structure.

Columns 1 - 5 Material identification number
6 - 15 Modulus of elasticity
16 - 25 Coefficient of thermal expansion
36 - 45 Cross-sectional area
46 - 55 Weight density (used to calculate gravity loads and mass matrix using the specified gravitational constant)

C. Convergence Criterion Card (F10.5)

This card defines the maximum permissible tensile force in any element.

Columns 1 - 10 Gap element convergence force.

D. Element Load Factors (4F10.0) Four cards

Three cards specifying the fraction of gravity (in each of the three global coordinate directions) to be added to each element load case.

Card 1: Multiplier of gravity load in the +X direction

Columns 1 - 10 Element load case A
11 - 20 Element load case B
21 - 30 Element load case C
31 - 40 Element load case D

Card 2: As above for gravity in the +Y direction

Card 3: As above for gravity in the +Z direction

Card 4: This indicates the fraction of the thermal load to be added to each of the element load cases.

E. Element Data Cards (4I5,F10.0.15)

One card per element in increasing numerical order starting with one.

Columns	1 - 5	Element number
	6 - 10	Node number I
	11 - 15	Node number J
	16 - 20	Material property number
	21 - 30	Reference temperature for zero stress
	31 - 35	Optional parameter k used for automatic generation of element (first card of a series)

OPTIONS - NAME= MAIN,CPT=00,LINECNT=59,SIZE=0000K,
SOURCE,EBCDIC,NOLIST,NODECK,LCAD,MAP,NOEDIT,ID,NOXREF

142

C	WRITE (13) HED	00000650
C	RE=START FOR MORE PLCTS	00000660
C		00000670
	IF (IPLT .GE. 0) GO TO 40	00000680
	DO 20 I=1,7	00000690
20	T(I)=T(1)	00000700
C	S(I)=S(1)	00000710
	READ (3) LL,NF,NDYN,NECB,NEQ,NBLOCK	00000720
	CALL SAPLOT(LL,NF,NDYN,NECB,NEQ,NBLOCK)	00000730
	CALL CPUTIM(T(8))	00000740
C	CALL SUPTIM(S(8))	00000750
	DO 30 I=9,12	00000760
30	T(I)=T(8)	00000770
C	S(I)=S(8)	00000780
	GO TO 400	00000790
C		00000800
C	RE=START MODE ACTIVATED IF NDYN.EQ.-2 OR NDYN.EQ.-3	00000810
C		00000820
C	40 IF(NDYN.LT.0) GO TO 160	00000830
C		00000840
C	INPUT JOINT DATA	00000850
C		00000860
	N2=N1+6*NUMNP	00000870
	N3=N2+NUMNP	00000880
	N4=N3+NUMNP	00000890
	N5=N4+NUMNP	00000900
	N6=N5+NUMNP	00000910
	CALL ERROR(N6)	00000920
C		00000930
	CALL INPUTJ(A(N1),A(N2),A(N3),A(N4),A(N5),NUMNP,NEQ)	00000940
C		00000950
C	STORE THE NODAL COORDINATES FOR LATER USE	00000960
C		00000970
C	WRITE (13) (A(I),I=N2,N5)	00000980
C		00000990
C	FORM ELEMENT STIFFNESSES	00001000
C		00001010
	CALL CPUTIM(T(2))	00001020
C	CALL SUPTIM(S(2))	00001030
C		00001040
	MBAND=0	00001050
	NUMEL=0	00001060
	REWIND 1	00001070
	REWIND 2	00001080
C	REWIND 18	00001090
		00001100
	DO 50 M=1,NELTYP	00001110
	READ (5,1003) NPAR	00001120
C***	DATA PORTHOLE SAVE	00001130
	IF(MODEX.EQ.1) WRITE (NT6) NPAR	00001140
	NT1=1	00001150
	IF (MINBND .GE. 1 .AND. MODEX .EQ. 0) NT1=4	00001160
	IF (MINBND .GE. 1 .AND. MODEX .EQ. 1) NT1=7	00001170
	WRITE (NT1) NPAR	00001180
	WRITE(18) NPAR	00001185
	NUMEL=NUMEL+NPAR(2)	00001190

C	MTYPE=NPAK(1)	00001200
C	CALL ELTYPE(MTYPE)	00001210
C		00001220
C	50 CONTINUE	00001230
C		00001240
C	ADD AN END-OF-FILE FLAG ON FILE 13	00001250
C		00001260
C	N=-1	00001270
C	IF (IPLT .GT. 0)WRITE(13)M,M	00001280
C		00001290
C	MINIMIZE THE BANDWIDTH	00001300
C		00001310
C	IF (MINBND .GT. 0) CALL SAPMIN	00001320
C		00001330
C	D E T E R M I N E B L O C K S I Z E	00001340
C		00001350
C	ADOSTF	00001360
C		00001370
C	NEQB=(MTOT - 4*LL)/(MBAND + LL + 1)/2	00001380
C		00001390
C	OVER-RIDE THE SYSTEM MATRIX BLOCKSIZE WITH THE INPUT (NON-ZERO)	00001400
C	VALUE, KEQB.	00001410
C	THIS OVER-RIDE ENTRY IS TO ALLOW PROGRAM CHECKING OF MULTI-	00001420
C	BLOCK ALGORITHMS WITH WHAT WOULD NORMALLY BE ONE BLOCK DATA.	00001430
C		00001440
C	IF(KEQB.LT.NEQB) NEQB = KEQB	00001450
C		00001460
C	GO TO (60,80,60,80,120,80), KQYN	00001470
C		00001480
C	STATIC SOLUTION	00001490
C		00001500
C		00001510
C	60 CONTINUE	00001520
C	NEQB1=(MTOT - MBAND)/(2*(MBAND+LL) + 1)	00001530
C	NEQB2=(MTOT - MBAND - LL*(MBAND-2))/(3*LL + MBAND + 1)	00001540
C	IF (NEQB1.LT.NEQB) NEQB=NEQB1	00001550
C	IF (NEQB2.LT.NEQB) NEQB=NEQB2	00001560
C	NBLOCK = (NEQB-1)/NECB + 1	00001570
C	IF(NEQB.GT.NEQ) NEQB=NEQ	00001580
C	IF(IPLT.EQ.0)GO TO 70	00001590
C	NTOT=20*NUMNP+NEQB*NBLOCK*LL	00001600
C	IF(MTOT.GT.NTOT)GO TO 130	00001610
C	WRITE(6,1006)NTOT	00001620
C	MODEX=1	00001630
C	70 NTOT=6*NUMNP+NEQB*NBLOCK*LL+1	00001640
C	IF(MTOT.GT.NTOT)GO TO 130	00001650
C	WRITE(6,1007)NTOT	00001660
C	MODEX=1	00001670
C	GO TO 130	00001680
C		00001690
C	EIGENSOLUTION	00001700
C		00001710
C	1. DETERMINANT SEARCH ALGORITHM	00001720
C		00001730
C	80 IF (NEQB.LT.NEQ) GO TO 90	00001740
C	NIM=3	00001750
C	NO=NF + NIM	00001760

	NVM=6	00001770
	NCA=NEQ*MAX0(MBAND,NC)	00001780
	NTOT=NCA + 4*NEQ + 2*NVM*NEQ + 5*NC	00001790
	NEIG=0	00001800
	IF (NTOT.LE.MTOT) GO TO 100	00001810
C		00001820
C	2. SUBSPACE ITERATION ALGORITHM	00001830
C		00001840
	90 NV=MIN0(2*NF,NF+8)	00001850
	IF (NAD.NE.0) NV=NAD	00001860
	NEQB1=(MTOT - MBAND)/(2*MBAND + 1)	00001870
	NEQB2=(MTOT - MBAND - 2*NV - NV*(MBAND-2))/(3*NV + MBAND + 1)	00001880
	NEQB3=(MTOT - 3*NV*NV - 3*NV)/(2*NV + 1)	00001890
	NEQB4=(MTOT - 6*NV)/(1 + MBAND)	00001900
	IF (NEQB1.LT.NEQB) NEQB=NEQB1	00001910
	IF (NEQB2.LT.NEQB) NEQB=NEQB2	00001920
	IF (NEQB3.LT.NEQB) NEQB=NEQB3	00001930
	IF (NEQB4.LT.NEQB) NEQB=NEQB4	00001940
	NEIG=1	00001950
C		00001960
	100 CONTINUE	00001970
	NBLOCK = (NEQ-1)/NEQB +1	00001980
	IF (NEQB.GE.NEQ) NEQB=NEQ	00001990
C		00002000
C	HISTORY OR SPECTRUM ANALYSIS	00002010
C		00002020
	KREM = 1000	00002030
	NTOT = NBLOCK*NEQB*NF + KREM	00002040
	IF (MTOT.LT.NTOT) WRITE(6,1002) NTOT	00002050
	IF (IPLT.EQ.0) GO TO 110	00002060
	NTOT=20*NUMNP+NEQB*NBLOCK*NF+2*NF	00002070
	IF (MTOT.GE.NTOT) GO TO 130	00002080
	WRITE(6,1006)NTOT	00002090
	MODEX=1	00002100
	110 NTOT=6*NUMNP+NEQB*NBLOCK*NF+NF	00002110
	IF (MTOT.GE.NTOT) GO TO 130	00002120
	WRITE(6,1007)NTOT	00002130
	MODEX=1	00002140
	GO TO 130	00002150
C		00002160
C	STEP-BY-STEP DIRECT INTEGRATION	00002170
C		00002180
	120 CONTINUE	00002190
C	DISPLACEMENT COMPONENTS FOR DIRECT OUTPUT (*NSD*)	00002200
	NN2 = NEQ	00002210
C	DISPLACEMENT COMPONENTS REQUIRED FOR RECOVERY OF ALL OF THE	00002220
C	REQUESTED ELEMENT STRESS COMPONENTS (*NSS*)	00002230
	NN3 = NEQ	00002240
C		00002250
C	1. DECOMPOSITION	00002260
C		00002270
	NEQB1 = (MTOT-NN2-NN3-NEQ-MBAND)/(2*MBAND+1)	00002280
C		00002290
C	2. TIME INTEGRATION PHASE	00002300
C		00002310
	NEQB2 = (MTOT-MBAND-2*(NN2+NN3)-5*NEQ)/(MBAND+1)	00002320
C		00002330

	IF(NEQB1.LT.NEQB) NEQB = NEQB1	00002340
	IF(NEQB2.LT.NEQB) NEQB = NEQB2	00002350
	IF(NEQB.GT.NEQ) NEQB = NEQ	00002360
	NBLOCK = (NEQ-1)/NEQB + 1	00002370
C		00002380
C	3. INPUT PHASE	00002390
C		00002400
C	NUMBER OF TIME FUNCTIONS (*NFN*)	00002410
	NN2 = 10	00002420
C	MAXIMUM NUMBER OF FUNCTION DEFINITION POINTS (*MXLP*)	00002430
	NN3 = 40	00002440
C		00002450
	NN4 = 6*NUMNP + 2*NN2*NEQ	00002460
	IF(NN4.GT.MTOT) WRITE(6,1002) NN4	00002470
	NN4 = NEQ*2*(NN2+1) + NN2*(1+2*NN3)	00002480
	IF(NN4.GT.MTOT) WRITE(6,1002) NN4	00002490
C		00002500
	130 IF (NEQB .GE. 2) GO TO 140	00002510
	IF (NEQ .LT. 3) GO TO 140	00002520
	WRITE (6,1005) NEQB	00002530
	NEQB=2	00002540
	NBLOCK=(NEQ-1)/NEQB + 1	00002550
	MODEX=1	00002560
	140 CONTINUE	00002570
C		00002580
C	INPUT NODAL LOADS	00002590
C		00002600
	NS3=NEQB*NBLOCK	00002610
	N2=N1+6*NUMNP	00002620
	N3=N2+LL*NS3	00002630
	N4=N3+NS3	00002640
	CALL ERRCR(N4)	00002650
	WRITE (6,1000) NEQ,MBAND,NEQB,NBLOCK	00002660
C		00002670
	CALL CPUTIM(T(3))	00002680
C	CALL SUPTIM(S(3))	00002690
C		00002700
	CALL INL(A(N1),A(N2),A(N3),NUMNP,NEQB,NS3,LL,NBLOCK)	00002710
C		00002720
	CALL CPUTIM(T(4))	00002730
C	CALL SUPTIM(S(4))	00002740
C		00002750
C	CHECK FOR UNRESTRAINED DEGREES OF FREEDOM	00002760
C		00002770
	N3=N2+NEQ	00002780
	CALL CHECK(A(N1),A(N2),NUMNP,NEQ,NUMEL,NELTYP)	00002790
C		00002800
C	FORM TOTAL STIFFNES S	00002810
C		00002820
	141 NE2B=2*NEQB	00002830
	N2=N1+NEQB*MBAND	00002840
	N3=N2+NEQB*LL	00002850
	N4=N3+4*LL	00002860
	NN2=N1+NE2B*MBAND	00002870
	NN3=NN2+NE2B*LL	00002880
	NN4=NN3+4*LL	00002890
C		00002900

	CALL ADDSTF (A(N1),A(NN2),A(NN3),A(NN4),NUMEL,NBLOCK,NE2B,LL,MBAN	00002910
	XD,ANORM,NVV)	00002920
	IF(ITEST.NE.0)GO TO 190	00002921
C		00002930
C	W I A N D C G C A L C U L A T I O N S	00002940
C		00002950
	IF (IWTCG .EQ. 0) GO TO 150	00002960
	N2=N1+NUMNP*6	00002970
	N3=N2+NUMNP	00002980
	N4=N3+NUMNP	00002990
	N5=N4+NUMNP	00003000
	N6=N5+NSB	00003010
	CALL ERROR(N6)	00003020
	CALL WTCG(A(N1),A(N2),A(N3),A(N4),A(N5),NUMNP,NEQB,NEQ,NBLOCK,NSB	00003030
	X)	00003040
150	CONTINUE	00003050
	CALL CPUTIM(T(5))	00003060
C	CALL SUPTIM(S(5))	00003070
C		00003080
C	S O L U T I O N P H A S E	00003090
C		00003100
	IF (GECST) GO TO 170	00003110
160	GO TO (170,210,220,270,330,350), KDCN	00003120
C		00003130
C	STATIC SOLUTION	00003140
C		00003150
170	IF(MODEX.EQ.0) GO TO 190	00003160
	T(6)=T(5)	00003170
C	S(6)=S(5)	00003180
	T(7)=T(5)	00003190
C	S(7)=S(5)	00003200
	IF (IPLT .GT. 0.AND..NOT.GECST)	00003210
	CALL SAPLOT(LL,NF,KDCN,NEQB,NEQ,NBLOCK)	00003220
	CALL CPUTIM(T(8))	00003230
C	CALL SUPTIM(S(8))	00003240
	DO 180 I=9,12	00003250
180	T(I) = T(8)	00003260
C	S(I)=S(8)	00003270
	GO TO 400	00003280
C		00003290
190	REWIND 12	00003300
	CALL SGLEQ	00003310
	IF(ITEST.NE.0)GO TO 141	00003311
	CALL CPUTIM(T(6))	00003320
C	CALL SUPTIM(S(6))	00003330
	T(7)=T(6)	00003340
C	S(7)=S(6)	00003350
	IF (IPLT .GT. 0) CALL SAPLOT(LL,NF,KDCN,NEQB,NEQ,NBLOCK)	00003360
	CALL CPUTIM(T(8))	00003370
C	CALL SUPTIM(S(8))	00003380
	DO 200 I=9,12	00003390
200	T(I) = T(8)	00003400
C	S(I)=S(8)	00003410
	IF (.NOT. GECST) GO TO 400	00003420
	WRITE (6,1004) NUMEL,NE2B	00003430
	CALL ADDEDG (A(N1),A(NN2),NE2B,MBAN,NUMEL,NBLOCK,LL,ANORM)	00003440
	GO TO 160	00003450

C		00003460
C	EIGENVALUE EXTRACTION	00003470
C		00003480
210	T(6) = T(5)	00003490
C	S(6)=S(5)	00003500
	CALL SOLEIG	00003510
	CALL CPUTIM(T(7))	00003520
C	CALL SUPTIM(S(7))	00003530
	IF (IPLT .GT. 0) CALL SAPLCT(LL,NF,KDYN,NEQB,NEQ,NBLOCK)	00003540
	CALL CPUTIM(T(8))	00003550
C	CALL SUPTIM(S(8))	00003560
	T(9) = T(8)	00003570
	T(10)= T(8)	00003580
	T(11)=T(8)	00003590
	T(12)=T(8)	00003600
C	S(9) = S(8)	00003610
C	S(10)= S(8)	00003620
C	S(11)= S(8)	00003630
C	S(12)= S(8)	00003640
	GO TO 400	00003650
C		00003660
C	FORCED DYNAMIC RESPONSE ANALYSIS	00003670
C		00003680
220	T(6) = T(5)	00003690
C	S(6)=S(5)	00003700
	IF(NDYN.LT.0) GO TO 250	00003710
	CALL SOLEIG	00003720
	CALL CPUTIM(T(7))	00003730
C	CALL SUPTIM(S(7))	00003740
	IF (IPLT .GT. 0) CALL SAPLCT(LL,NF,KDYN,NEQB,NEQ,NBLOCK)	00003750
	CALL CPUTIM(T(8))	00003760
C	CALL SUPTIM(S(8))	00003770
	GO TO 260	00003780
230	DO 240 I=1,7	00003790
240	T(I+1)=T(I)	00003800
C	S(I+1)=S(I)	00003810
	REWIND 2	00003820
	READ (2) NEQ,NBLOCK,NEQB,MBAND,N1,NF, (CQ(I), I=1,NF)	00003830
	REWIND 7	00003840
	IMAX=NEQB*NF	00003850
	READ (7) (A(I),I=1,NF)	00003860
	DO 250 L=1,NBLOCK	00003870
250	READ (7) (A(I),I=1,IMAX)	00003880
260	CALL HISTRY	00003890
	CALL CPUTIM(T(9))	00003900
C	CALL SUPTIM(S(9))	00003910
	T(10)= T(9)	00003920
	T(11)= T(9)	00003930
	T(12)= T(9)	00003940
C	S(10)= S(9)	00003950
C	S(11)= S(9)	00003960
C	S(12)= S(9)	00003970
	GO TO 400	00003980
C		00003990
C	RESPONSE SPECTRUM ANALYSIS	00004000
C		00004010
270	T(6) = T(5)	00004020

C	S(6) = S(5)	00004030
	IF (NDYN.LT.0) GO TO 280	00004040
	CALL SQLEIG	00004050
	CALL CPUTIM(T(7))	00004060
C	CALL SUPTIM(S(7))	00004070
	IF (IPLT.GT.0) CALL SAPLOT(LL,NF,KDYN,NEQB,NEQ,NBLOCK)	00004080
	CALL CPUTIM(T(8))	00004090
C	CALL SUPTIM(S(8))	00004100
	T(9)=T(8)	00004110
C	S(9) = S(8)	00004120
	GO TO 310	00004130
280	DO 290 I=1,8	00004140
290	T(I+1)=T(I)	00004150
C	S(I+1)=S(I)	00004160
	REWIND 2	00004170
	READ (2) NEQ,NBLOCK,NEQB,MBAND,N1,NF	00004180
	REWIND 7	00004190
	IMAX=NEQB*NF	00004200
	READ (7) (A(I),I=1,NF)	00004210
	DO 300 L=1,NBLOCK	00004220
300	READ (7) (A(I),I=1,IMAX)	00004230
310	IF (NRSC.LE.0) NRSC=1	00004240
	DO 320 KAPG=1,NRSC	00004250
320	CALL RESPEC	00004260
	CALL CPUTIM(T(10))	00004270
C	CALL SUPTIM(S(10))	00004280
	T(11)=T(10)	00004290
	T(12)=T(10)	00004300
C	S(11)= S(10)	00004310
C	S(12)= S(10)	00004320
	GO TO 400	00004330
		00004340
C	STEP-BY-STEP (DIRECT INTEGRATION) ANALYSIS	00004350
C		00004360
330	DO 340 I=6,10	00004370
340	T(I) = T(5)	00004380
C	S(I) = S(5)	00004390
	CALL STEP	00004400
	CALL CPUTIM(T(11))	00004410
C	CALL SUPTIM(S(11))	00004420
	T(12)=T(10)	00004430
C	S(12)=S(10)	00004440
	GO TO 400	00004450
C		00004460
C	FREQUENCY RESPONSE ANALYSIS	00004470
C		00004480
350	T(6)=T(5)	00004490
C	S(6)=S(5)	00004500
	IF (NDYN.LT.0) GO TO 360	00004510
	CALL SQLEIG	00004520
	CALL CPUTIM(T(7))	00004530
C	CALL SUPTIM(S(7))	00004540
	IF (IPLT.GT.0) CALL SAPLOT(LL,NF,KDYN,NEQB,NEQ,NBLOCK)	00004550
	CALL CPUTIM(T(8))	00004560
C	CALL SUPTIM(S(8))	00004570
	T(9)=T(8)	00004580
	T(10)=T(8)	00004590

	T(11)=T(8)	00004600
C	S(9)=S(8)	00004610
C	S(10)=S(8)	00004620
C	S(11)=S(8)	00004630
	GC 10 390	00004640
	380 DC 370 I=1,10	00004650
	370 T(I+1)=T(I)	00004660
C	S(I+1)=S(I)	00004670
	REWIND 2	00004680
	READ (2) NEQ,NBLOCK,NEQB,MBAND,N1,NF	00004690
	REWIND 7	00004700
	IMAX=NEQB*NF	00004710
	READ (7) (A(I),I=1,NF)	00004720
	DD 380 L=1,NBLOCK	00004730
	380 READ (7) (A(I),I=1,IMAX)	00004740
	390 CALL FREQ	00004750
	CALL CPUTIM(T(12))	00004760
	CALL SUPTIM(S(12))	00004770
C		00004780
C	COMPUTE AND PRINT OVERALL TIME LOG	00004790
C		00004800
	400 TT = 0.0	00004810
C	ST = 0.000	00004820
	DC 410 I=1,11	00004830
	T(I) = T(I+1)-T(I)	00004840
	TT = TT + T(I)	00004850
	410 CONTINUE	00004860
C	S(I) = S(I+1)-S(I)	00004870
C	ST = ST + S(I)	00004880
C		00004890
	WRITE (6,203) (T(K),S(K),K=1,11),TT,ST	00004900
	WRITE (6,1001) (T(K),K=1,11),TT	00004910
C		00004920
	GC 10 10	00004930
C		00004940
	1000 FORMAT (38H1E Q U A T I O N P A R A M E T E R S, //	00004950
	X 34H TOTAL NUMBER OF EQUATIONS =,I5,	00004960
	X /34H BANDWIDTH =,I5,	00004970
	X /34H NUMBER OF EQUATIONS IN A BLOCK =,I5,	00004980
	X /34H NUMBER OF BLOCKS =,I5)	00004990
	1001 FORMAT (11H,31H V E R A L L T I M E L O G, //	00005000
	X 5X,30H NODAL POINT INPUT =, F8.2 /	00005010
	X 5X,30H ELEMENT STIFFNESS FORMATION =, F8.2 /	00005020
	X 5X,30H NODAL LOAD INPUT =, F8.2 /	00005030
	X 5X,30H TOTAL STIFFNESS FORMATION =, F8.2 /	00005040
	X 5X,30H STATIC ANALYSIS =, F8.2 /	00005050
	X 5X,30H EIGENVALUE EXTRACTION =, F8.2 /	00005060
	X 5X,30H STRUCTURE PLOTTING =, F8.2 /	00005070
	X 5X,30H FORCED RESPONSE ANALYSIS =, F8.2 /	00005080
	X 5X,30H RESPONSE SPECTRUM ANALYSIS =, F8.2 /	00005090
	X 5X,30H STEP-BY-STEP INTEGRATION =, F8.2 /	00005100
	X 5X,30H FREQUENCY RESPONSE ANALYSIS =, F8.2 //	00005110
	X 5X,30H TOTAL SOLUTION TIME =, F8.2 /)	00005120
C		00005130
	1002 FORMAT (/ / 47H ** WARNING. ESTIMATE OF STORAGE FOR A DYNAMIC,	00005140
	X 32H ANALYSIS EXCEEDS AVAILABLE CORE,	00005150
	X /,' REQUIRED MTC=' ,I10)	00005160

C		00005170
C		00005180
	1003 FORMAT (14I5)	00005190
	1004 FORMAT ('TOTAL NUMBER OF ELEMENTS',I5,/,	00005200
	X 'NUMBER OF ELEMENTS WITH GEOMETRIC STIFFNESS=',I5)	00005210
	1005 FORMAT ('*** WARNING *** THE AVAILABLE BLANK COMMONT (MTOT) IS',	00005220
	X' TOO SMALL',/, 'COMPUTED NO. OF EQUATIONS PER BLOCK=',I10,/,	00005230
	X' PROCEED IN THE DATA CHECK MODE WITH NEGB=2')	00005240
	1006 FORMAT('*** FATAL ERROR *** AVAILABLE CORE IS TOO SMALL'	00005250
	1, 'FOR PLOTTING, REQUIRED MTOT=',I10/	00005260
	2' CONTINUE IN THE DATA CHECK MODE')	00005270
	1007 FORMAT('*** FATAL ERROR *** AVAILABLE CORE IS INADQUATE'	00005280
	1, 'REQUIRED MTOT=',I10/'CONTINUE IN THE DATA CHECK MODE')	00005290
	END	00005300

CS/360 FORTRAN H

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OPTIONS - NAME= MAIN,CPT=00,LINECNT=59,SIZE=0000K,
          SOURCE,EBCDIC,NCLIST,NODECK,LCAD,MAP,NOEDIT,ID,NOXREF
SUBROUTINE ADDSTF (A,B,STR,TMASS,NUMEL,NBLOCK,NE2B,LL,MBAND,ANGRM,ADUS0100
INVV)
C
C      IMPLICIT REAL*8(A-H,O-Z)
C      LOGICAL ELPRT,ELPCH,GENPRT,GENPCH,DEFPCH
C      REAL PCT
C
C      CALLED BY MAIN
C
C      FORMS GLOBAL EQUILIBRIUM EQUATIONS IN BLOCKS
C
C      DIMENSION A(NE2B,MBAND),B(NE2B,LL),STR(4,LL),TMASS(NE2B)
C
C      COMMON /DYN/ NT,NCT,ALFA,LT,BETA,NFN,NGM,NAT,NDYN
C      COMMON /EM/ LRU,NO,LM(63),IPAC,SS(231)
C      COMMON /EXTRA/ MDEX,NT8
C      COMMON /CTL/ GRAV,ELPRT,ELPCH,GENPRT,GENPCH,DEFPCH
C      COMMON /GAP/ STRS(100,4),ITR8,ITEST
C
C      NEQB=NE2B/2
C      K=NEQB+1
C      X=NBLOCK
C      MB=USQRT(X)
C      Mo=MB/2+1
C      NEBD=MB*NE2B
C      NM=1
C      NDEG=0
C      INV=0
C      ANGRM=0.00
C      AMIN=1.0030
C      AMAX=-AMIN
C      NSHIFT=0
C      NZTRM=0
C      REWIND 3
C      REWIND 4
C      REWIND 9
C      REWIND 15
C      REWIND 17
C      IF(ITR8.GE.1)GO TO 31
C
C      READ ELEMENT LOAD MULTIPLIERS
C
C      2 WRITE (6,1003)
C      DO 10 L=1,LL
C      READ (5,1000) (STR(I,L),I=1,4)
C      10 WRITE (6,1004) L,(STR(I,L),I=1,4)
C      WRITE (17) STR
C      IF(MDEX.EQ.0) WRITE (8) STR
C
C      FOR A STEP-BY-STEP ANALYSIS (NDYN.EQ.4) READ THE SOLUTION
C      CONTROL CARD. THE TIME STEP (DT) AND THE DAMPING COEFFICIENTS
C      (ALFA/BETA) ARE REQUIRED FOR THE ASSEMBLY OF THE EFFECTIVE
C      SYSTEM STIFFNESS MATRIX IN THIS ROUTINE.
C
C      IF(NDYN.NE.4) GO TO 30

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C	READ (5,1001) NFN,NGM,NAT,NI,NCT,CT,ALFA,BETA	ADD S0600
	WRITE (6,1005) NFN,NGM,NAT,NI,NCT,CT,ALFA,BETA	ADD S0610
	IF(NAT.EQ.0) NAT = 1	ADD S0620
	IF(NOT.EQ.0) NCT = 1	ADD S0630
	IF(CT.GT.1.00-12) GO TO 20	ADD S0640
	WRITE (6,1013)	ADD S0650
	STOP	ADD S0660
		ADD S0670
C	COMPUTE INTEGRATION COEFFICIENTS FOR ASSEMBLY OF EFFECTIVE	ADD S0680
C	SYSTEM STIFFNESS (STEP-BY-STEP ANALYSIS ONLY)	ADD S0690
C		ADD S0700
	20 TETA = 1.400	ADD S0710
	DT1 = TETA*DT	ADD S0720
	DT2 = DT1**2	ADD S0730
	AO = (6.00+3.00*ALFA*DT1)/(DT2+3.00*BETA*DT1)	ADD S0740
C		ADD S0750
	30 IF(MODEX.EQ.1) RETURN	ADD S0760
C		ADD S0770
C	FORM EQUATIONS IN BLOCKS (2 BLOCKS AT A TIME)	ADD S0780
C		ADD S0790
	31 IF(ITR8.EQ.0) GO TO 32	ADD S0800
	READ (17) STR	ADD S0801
	32 DO 310 M=1,NBLOCK ,2	ADD S0802
	DO 40 I=1,NE2B	ADD S0810
	DO 40 J=1,MBAND	ADD S0820
	40 A(I,J)=0.00	ADD S0830
	READ (3) ((B(I,L),I=1,NEQB),L=1,LL),(TMASS(I),I=1,NEQB)	ADD S0840
	IF(ITR8.GE.1) READ(15)((B(I,L),I=1,NEQB),L=1,LL)	ADD S0850
	41 IF (M.EQ.NBLOCK) GO TO 50	ADD S0851
	READ (3) ((B(I,L),I=K,NE2B),L=1,LL),(TMASS(I),I=K,NE2B)	ADD S0860
	IF(ITR8.GE.1) READ(15) ((B(I,L),I=K,NE2B),L=1,LL)	ADD S0870
	50 CONTINUE	ADD S0871
C		ADD S0880
	REWIND 7	ADD S0890
	REWIND 2	ADD S0900
	NA=7	ADD S0910
	NUME=NUM7	ADD S0920
	IF (MX.NE.1) GO TO 60	ADD S0930
	NA=2	ADD S0940
	NUME=NUMEL	ADD S0950
	NUM7 =0	ADD S0960
C		ADD S0970
	60 DO 160 N=1,NUME	ADD S0980
	IF(ITR8.EQ.0) GO TO 61	ADD S0990
	READ(17) LRD,ND,(LM(I),I=1,ND),(SS(I),I=1,LRD)	ADD S0991
	GO TO 62	ADD S0992
	61 READ (NA) LRD,ND,(LM(I),I=1,ND),(SS(I),I=1,LRD)	ADD S0993
	WRITE(17) LRD,ND,(LM(I),I=1,ND),(SS(I),I=1,LRD)	ADD S1000
	62 MSHFT=ND*(ND+1)/2+4*ND	ADD S1001
	DO 130 I=1,ND	ADD S1010
	LMN=1-LM(I)	ADD S1020
	II=LM(I)-NSHIFT	ADD S1030
	IF (II.LE.0.OR.II.GT.NE2B) GO TO 130	ADD S1040
	IMS=1+MSHFT	ADD S1050
	TMASS(II)=TMASS(II)+ SS(IMS)	ADD S1060
	DO 70 L=1,LL	ADD S1070
		ADD S1080

DO 70 J=1,4	ADDS1090
KK=ND*(ND+1)/2+ND*(J-1)	ADDS1100
70 B(II,L)=B(II,L)+SS(I+KK)*STR(J,L)	ADDS1110
DO 120 J=1,ND	ADDS1120
JJ=LM(J)+LMN	ADDS1130
IF(JJ) 120,120,80	ADDS1140
80 IF (J-I) 100,90,90	ADDS1150
90 KK=(I-1)*ND-1*(I-1)/2+J	ADDS1160
GO TO 110	ADDS1170
100 KK=(J-1)*ND-J*(J-1)/2+1	ADDS1180
110 A(II,JJ)=A(II,JJ) + SS(KK)	ADDS1190
120 CONTINUE	ADDS1200
130 CONTINUE	ADDS1210
C	ADDS1220
C DETERMINE IF STIFFNESS IS TO BE PLACED ON TAPE 7	ADDS1230
C	ADDS1240
IF (MM.GT.1) GO TO 160	ADDS1250
DO 140 I=1,ND	ADDS1260
II=LM(I) -NSHIFT	ADDS1270
IF(II.GT.NE2B.AND.II.LE.NE2B) GO TO 150	ADDS1280
140 CONTINUE	ADDS1290
GO TO 160	ADDS1300
150 WRITE (7) LRD,ND,(LM(I),I=1,ND),(SS(I),I=1,LRD)	ADDS1310
NUM7=NUM7+1	ADDS1320
C	ADDS1330
160 CONTINUE	ADDS1340
DO 170 L=1,NEQB	ADDS1350
ANORM=ANORM + A(L,1)	ADDS1360
IF (A(L,1) .NE. 0.000 .AND. A(L,1) .LT. AMIN) AMIN=A(L,1)	ADDS1370
IF (A(L,1) .GT. AMAX) AMAX=A(L,1)	ADDS1380
IF (A(L,1) .NE. 0.00) NNEG=NNEG + 1	ADDS1390
IF (A(L,1) .EQ. 0.00) A(L,1)=1.0+20	ADDS1400
IF (TMASS(L) .NE. 0.00) NVV=NVV + 1	ADDS1410
DO 170 KAPG=1,MBAND	ADDS1420
IF (A(L,KAPG) .NE. 0.000) NNZTRM=NNZTRM+1	ADDS1430
170 CONTINUE	ADDS1440
C	ADDS1450
C FOR STEP-BY-STEP ANALYSIS ADD THE MASS CONTRIBUTIONS TO	ADDS1460
C THE EQUATION DIAGONAL COEFFICIENTS	ADDS1470
C	ADDS1480
IF (NDYN.NE.4) GO TO 190	ADDS1490
DO 180 I=1,NEQB	ADDS1500
180 A(I,1) = A(I,1) + AC* TMASS(I)	ADDS1510
WRITE (4) ((A(I,J),I=1,NEQB),J=1,MBAND)	ADDS1520
GO TO 200	ADDS1530
190 WRITE (4) ((A(I,J),I=1,NEQB),J=1,MBAND),((B(I,L),I=1,NEQB),L=1,LL)	ADDS1540
200 WRITE (9) (TMASS(I),I=1,NEQB)	ADDS1550
C	ADDS1560
IF (.NOT. GENPKI) GO TO 230	ADDS1570
WRITE (6,1007) M	ADDS1580
DO 210 I=1,NEQB	ADDS1590
IF (GENPCH) WRITE (11,1003)(A(I,J),J=1,MBAND)	ADDS1600
210 WRITE (6,1009)(A(I,J),J=1,MBAND)	ADDS1610
WRITE (6,1010)	ADDS1620
DO 220 I=1,NEQB	ADDS1630
IF (GENPCH) WRITE (11,1008)(B(I,J),J=1,LL)	ADDS1640
220 WRITE (6,1009)(B(I,J),J=1,LL)	ADDS1650

WRITE (6,1011)	ADD S1660
IF (GENPCH) WRITE (11,1008)(TMASS(I),I=1,NEQB)	ADD S1670
WRITE (6,1009)(TMASS(I),I=1,NEGB)	ADD S1680
230 IF(M.EQ.NBLOCK) GO TO 310	ADD S1690
DO 240 L=K,NE2B	ADD S1700
ANORM=ANORM + A(L,1)	ADD S1710
IF (A(L,1) .NE. 0.000 .AND. A(L,1) .LT. AMIN) AMIN=A(L,1)	ADD S1720
IF (A(L,1) .GT. AMAX) AMAX=A(L,1)	ADD S1730
IF (A(L,1) .NE. 0.00) NDEG=NDEG + 1	ADD S1740
IF (A(L,1) .EQ. 0.00) A(L,1)=1.0+20	ADD S1750
IF (TMASS(L) .NE. 0.00) NVV=NVV + 1	ADD S1760
DO 240 KK=1,MBAND	ADD S1770
IF (A(L,KK) .NE. 0.000) NNZTRM=NNZTRM+1	ADD S1780
240 CONTINUE	ADD S1790
C	ADD S1800
IF(NDYN.NE.4) GO TO 260	ADD S1810
DO 250 I=K,NE2B	ADD S1820
250 A(I,1) = A(I,1) + AC* TMASS(I)	ADD S1830
WRITE (4) ((A(I,J),I=K,NE2B),J=1,MBAND)	ADD S1840
GO TO 270	ADD S1850
260 WRITE (4) ((A(I,J),I=K,NE2B),J=1,MBAND),((B(I,L),I=K,NE2B),L=1,LL)	ADD S1860
270 WRITE (9) (TMASS(I),I=K,NE2B)	ADD S1870
IF (.NOT. GENPRT) GO TO 300	ADD S1880
MPI=M+1	ADD S1890
WRITE (6,1007) MPI	ADD S1900
DO 280 I=K,NE2B	ADD S1910
IF (GENPCH) WRITE (11,1008)(A(I,J),J=1,MBAND)	ADD S1920
280 WRITE (6,1009)(A(I,J),J=1,MBAND)	ADD S1930
WRITE (6,1010)	ADD S1940
DO 290 I=K,NE2B	ADD S1950
IF (GENPCH) WRITE (11,1008)(B(I,J),J=1,LL)	ADD S1960
290 WRITE (6,1009)(B(I,J),J=1,LL)	ADD S1970
WRITE (6,1011)	ADD S1980
IF (GENPCH) WRITE (11,1008)(TMASS(I),I=K,NE2B)	ADD S1990
WRITE (6,1009)(TMASS(I),I=K,NE2B)	ADD S2000
C	ADD S2010
300 IF (MM.EQ.MB) MM=0	ADD S2020
MM=MM+1	ADD S2030
310 NSHIFT=NSHIFT+NE2B	ADD S2040
IF (NDEG.GT.0) GO TO 320	ADD S2050
WRITE (6,1002)	ADD S2060
STOP	ADD S2070
320 ANORM=(ANORM/NDEG)*1.0-8	ADD S2080
NTERM=NEQB*NBLOCK*MBAND	ADD S2090
PCT=100.000*DFLOAT(NNZTRM)/DFLOAT(NTERM)	ADD S2100
RATIO=1.0070	ADD S2110
IF (AMIN .NE. 0.000) RATIO=AMAX/AMIN	ADD S2120
AAVG=ANORM*1.006	ADD S2130
IF(ITER8.GE.1) GO TO 330	ADD S2135
WRITE (6,1012) AMIN,AMAX,RATIO,AAVG,PCT	ADD S2140
C	ADD S2150
330 RETURN	ADD S2160
1000 FORMAT (4F10.0)	ADD S2170
1001 FORMAT (5I5,3F10.0)	ADD S2180
1002 FORMAT (51H0STRUCTURE WITH NC DEGREES OF FREEDOM CHECK DATA)	ADD S2190
1003 FORMAT (/// 10H STRUCTURE,13X,7HELEMENT,4X,4HLOAD,4X,	ADD S2200
X 11HMULTIPLIERS, 10H LOAD CASE,12X,1HA,9X,1HB,9X,1HC,9X,1HD, 1X)	ADD S2210

1004 FORMAT (16,7X,4F10.3)	ADDS2220
1005 FORMAT (45HIS T E P - B Y - S T E P S O L U T I O N ,	ADDS2230
X 37HC O N T R O L I N F O R M A T I O N, ///	ADDS2240
X 5X, 35HNUMBER OF TIME VARYING FUNCTIONS =, 15 //	ADDS2250
X 5X, 35HGROUND MOTION INDICATOR =, 15 /	ADDS2260
X 8X, 10HEQ.0, NONE, /	ADDS2270
X 8X, 29HGT.0, READ ACCELERATION INPUT, //	ADDS2280
X 5X, 35HNUMBER OF ARRIVAL TIMES =, 15 /	ADDS2290
X 8X, 26HEQ.0, ALL FUNCTIONS ARRIVE, /	ADDS2300
X 8X, 18H AT TIME ZERO, //	ADDS2310
X 5X, 35HNUMBER OF SOLUTION TIME STEPS =, 15 //	ADDS2320
X 5X, 35HOUTPUT (PKINT) INTERVAL =, 15 //	ADDS2330
X 5X, 35HSOLUTION TIME INCREMENT =, E14.4 //	ADDS2340
X 5X, 30HMASS- PROPORTIONAL DAMPING, /	ADDS2350
X 5X, 35HCOEFFICIENT (ALPHA) =, E14.4 //	ADDS2360
X 5X, 30HSTIFFNESS-PROPORTIONAL DAMPING, /	ADDS2370
X 5X, 35HCOEFFICIENT (BETA) =, E14.4 /// 1X)	ADDS2380
1006 FORMAT (16,7X,4F10.3)	ADDS2390
1007 FORMAT ('10OVERALL MATRICES, ELOCK',13,/,	ADDS2400
X ' STIFFNESS MATRIX ')	ADDS2410
1008 FORMAT ((1P8E10.3))	ADDS2420
1009 FORMAT (/, (1H ,1P10E13.4))	ADDS2430
1010 FORMAT (///, ' LOAD MATRIX ')	ADDS2440
1011 FORMAT (///, ' MASS MATRIX ')	ADDS2450
1012 FORMAT ('0STIFFNESS MATRIX PARAMETERS',/,	ADDS2460
X 10X, 'MINIMUM NON-ZERO DIAGONAL ELEMENT=' ,1PD10.3,/,	ADDS2470
X 10X, 'MAXIMUM DIAGONAL ELEMENT =', D10.3,/,	ADDS2480
X 10X, 'MAXIMUM/MINIMUM =', D10.3,/,	ADDS2490
X 10X, 'AVERAGE DIAGONAL ELEMENT =', D10.3,/,	ADDS2500
X 10X, 'DENSITY OF THE MATRIX =',DPF5.1, ' PCT.')	ADDS2510
1013 FORMAT (27H0*** ERROR ZERO TIME STEP, / 1X)	ADDS2520
END	ADDS2530

CS/360 FORTRAN H

OPTIONS - NAME= MAIN,OPT=00,LINECNT=59,SIZE=0000K,
SOURCE,EBCDIC,NCLIST,NODECK,LCAL,MAP,NOEDIT,10,NOXREF
SUBROUTINE ELTYPE(MTYPE)
IMPLICIT REAL*8(A-H,O-Z)

C		ELTY0100
C		ELTY0110
C		ELTY0120
C	CALL BY MAIN,STRESS	ELTY0130
C		ELTY0140
C	GO TO (10,20,30,40,50,60,70,80,90,100,110,120),MTYPE	ELTY0150
C		ELTY0160
C	THREE DIMENSIONAL TRUSS ELEMENTS	ELTY0170
C		ELTY0180
C	10 CALL TRUSS	ELTY0190
C	GO TO 130	ELTY0200
C		ELTY0210
C	THREE DIMENSIONAL BEAM ELEMENTS	ELTY0220
C		ELTY0230
C	20 CALL BEAM	ELTY0240
C	GO TO 130	ELTY0250
C		ELTY0260
C	PLANE STRESS ELEMENTS	ELTY0270
C		ELTY0280
C	30 CALL PLANE	ELTY0290
C	GO TO 130	ELTY0300
C		ELTY0310
C	AXISYMMETRIC SOLID ELEMENTS	ELTY0320
C		ELTY0330
C	40 CALL PLANE	ELTY0340
C	GO TO 130	ELTY0350
C		ELTY0360
C	THREE DIMENSIONAL SOLID ELEMENTS	ELTY0370
C		ELTY0380
C	50 CALL THREED	ELTY0390
C	GO TO 130	ELTY0400
C		ELTY0410
C	PLATE BENDING ELEMENTS	ELTY0420
C		ELTY0430
C	60 CALL SHELL	ELTY0440
C	GO TO 130	ELTY0450
C		ELTY0460
C		ELTY0470
C	70 CALL BOUND	ELTY0480
C	GO TO 130	ELTY0490
C		ELTY0500
C	THICK SHELL ELEMENTS	ELTY0510
C		ELTY0520
C	80 CALL SOL21	ELTY0530
C	GO TO 130	ELTY0540
C		ELTY0550
C	90 CALL PSEUD1	ELTY0560
C	GO TO 130	ELTY0570
C		ELTY0580
C	100 CALL ELEM10	ELTY0590
C	GO TO 130	ELTY0600
C		ELTY0610
C	GAP ELEMENTS	ELTY0620
C		ELTY0630
C		ELTY0640

```

110 CALL GAPEL
    GC TO 130
C
C    STRAIGHT OR CURVED PIPE ELEMENTS
C
C 120 CALL PIPE
C
C 130 RETURN
C
    END

```

```

ELTY0650
ELTY0660
ELTY0670
ELTY0680
ELTY0690
ELTY0700
ELTY0710
ELTY0720
ELTY0730
ELTY0740

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CS/360 FORTAN H

OPTIONS - NAME= MAIN,OPT=00,LINECT=59,SIZE=0000K,
SOURCE,EBODIC,NCLIST,NOCHECK,LC=0,MAP,NOEDIT,IO,NOXREF
SUBROUTINE FM00(ID,DISP,FORCE,NSB,NBLOCK,NEQB,LL,NUMNP,NPAR,
ANEZB,M6AND)
IMPLICIT REAL*8 (A-F,G-Z)

C CALLED BY GAPEL
C MODIFIES LOAD VECTOR AS REQUIRED BY GAP ELEMENT SCHEME

COMMON/GAP/STRS(100,4),IIR8,ITEST
COMMON/CONV/GCON
DIMENSION FORCE(NSB,LL),DISP(NSB,LL)
DIMENSION ID(NUMNP,6),LM(6),S(6,6),D(6),R(6),NPAR(14)

READ DISPLACEMENTS INTO GCRE

REWIND 2
NE=NSB
NS=NE+1-NEQB
DO 10 NN=1,NBLOCK
READ(2)((DISP(J,K),J=NS,NE),K=1,LL)
NS=NS-NEQB
10 NE=NE-NEQB

READ LOAD VECTOR

REWIND 15
DO 20 NB=1,NBLOCK
N1=NEQB*(NB-1)+1
N2=NEQB*NB
20 READ(15)((FORCE(N,L),N=N1,N2),L=1,LL)

READ THE ID ARRAY

REWIND 8
READ (8) ID

CHECK ALL GAP ELEMENTS FOR TENSION
MODIFY LOAD VECTOR AS REQUIRED

NUMEL=NPAR(2)
DO 70 L=1,LL
REWIND 16
DO 60 NUM=1,NUMEL
READ(16)N,I,J,((S(NN,K),NN=1,6),K=1,6)
IF(STRS(NUM,L).LE.GCON) GO TO 60
DO 30 K=1,3
D(K)=0.000
D(K+3)=0.000
II=ID(I,K)
JJ=ID(J,K)
IF(II.EQ.0) GO TO 29
D(K)=DISP(II,L)
29 IF(JJ.EQ.0) GO TO 30
D(K+3)=DISP(JJ,L)
30 CONTINUE

FM000100
FM000101
FM000110
FM000120
FM000130
FM000140
FM000150
FM000160

FM000190
FM000200
FM000210
FM000220
FM000230
FM000240
FM000250
FM000260
FM000270
FM000280
FM000290
FM000300
FM000310
FM000320
FM000330
FM000340
FM000350
FM000360
FM000370
FM000380
FM000390
FM000400
FM000410
FM000420
FM000430
FM000440
FM000450
FM000460
FM000470
FM000480
FM000490
FM000500
FM000510
FM000520
FM000530
FM000540
FM000550
FM000560
FM000561
FM000570
FM000580
FM000585
FM000590
FM000595
FM000600
FM000610

DO 40 K=1,6	FMQD0620
R(K)=0.000	FMQD0630
DO 40 M=1,6	FMQD0640
K(K)=R(K)+(S(K,M)*D(M))	FMQD0650
40 CONTINUE	FMQD0660
DO 50 K=1,3	FMQD0670
K3=K+3	FMQD0680
I1=ID(I,K)	FMQD0690
I2=ID(J,K)	FMQD0700
IF(I1.EQ.0) GO TO 49	FMQD0705
FORCE(I1,L)=FORCE(I1,L)-R(K)	FMQD0710
49 IF(I2.EQ.0) GO TO 50	FMQD0715
FORCE(I2,L)=FORCE(I2,L)-R(K3)	FMQD0720
50 CONTINUE	FMQD0730
60 CONTINUE	FMQD0740
70 CONTINUE	FMQD0750
REWIND 15	FMQD0751
DO 80 N6=1,NBLOCK	FMQD0752
N1=NEQB*(N6-1)+1	FMQD0753
N2=NEQB*N6	FMQD0754
80 WRITE(15) ((FORCE(N,L),N=N1,N2),L=1,LL)	FMQD0755
RETURN	FMQD0760
END	FMQD0770

CS/360 FORTRAN H

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OPTIONS - NAME= MAIN,OPT=00,LINECNT=59,SIZE=0000K,
          SOURCE,EBCCIC,NGLIST,NBCECK,LCAD,MAP,NOEDIT,IO,NOXREF
SUBROUTINE GAPEL
IMPLICIT REAL*8(A-H,O-Z)
C
C CALLS RUSS,STRSC
C CALLED BY ELTYPE
C
COMMON A(1)
COMMON/SCL/NBLOCK,NEQB,LL,NF,IFILL(7)
COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQ
COMMON /JUNK/ LT,LH,L,IPAC,SIG(20),N6,N7,N8,N9,N10
COMMON /EXTRA/ MODCX,N18,N10SV,NT10
COMMON/GAP/STRS(100,4),ITR8,ITEST
COMMON/CONV/GCON
C
C IF(NPAR(1).EQ.0) GO TO 10
N6=N5+NUMNP
N7=N6+NPAR(3)
N8=N7+NPAR(3)
N9=N8+NPAR(3)
N10=N9+NPAR(3)
NM=N10+NPAR(3)
CALL ERRCLR(NM)
C
CALL RUSS(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
A(N10),NUMNP)
C
RETURN
C
10 IF(ITEST.EQ.0.AND.ITR8.GT.0)GO TO 40
NUME=NPAR(2)
DO 20 II=1,NUME
DO 20 JJ=1,4
STRS(II,JJ)=0.000
20 CONTINUE
ITR8=ITR8+1
ITEST=0
WRITE (6,1001)
DO 30 NN=1,NUME
CALL STRSC(A(N1),A(N3),NEQ,C)
DO 30 L=LT,LH
CALL STRSC(A(N1),A(N3),NEQ,1)
STRS(NN,L)=SIG(2)
WRITE(6,1999) NN,SIG(1),STRS(NN,L)
1999 FORMAT(' ELEMENT:',13,' STRESS=',2PD15.4,
1' FORCE=',2PD15.4/)
IF(STRS(NN,L).LE.GCON) GO TO 30
ITEST=1
30 CONTINUE
IF(ITEST.EQ.0)RETURN
NE2B=2*NEQB
NSB=NEQB*NBLOCK
N100=N1+(NUMNP*6)
N200=N100+(NSB*LL)
WRITE(6,1003) ITR8
CALL FXCD(A(N1),A(N100),A(N200),NSB,NBLOCK,NEQB,LL,NUMNP,

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GAPE0100
 GAPE0110
 GAPE0120
 GAPE0130
 GAPE0140
 GAPE0150
 GAPE0160
 GAPE0161
 GAPE0170
 GAPE0180
 GAPE0190
 GAPE0200
 GAPE0210
 GAPE0220
 GAPE0230
 GAPE0240
 GAPE0250
 GAPE0260
 GAPE0270
 GAPE0280
 GAPE0290
 GAPE0300
 GAPE0310
 GAPE0320
 GAPE0330
 GAPE0340
 GAPE0350
 GAPE0360
 GAPE0370
 GAPE0380
 GAPE0390
 GAPE0400
 GAPE0410
 GAPE0420
 GAPE0430
 GAPE0440
 GAPE0450
 GAPE0460
 GAPE0470
 GAPE0480
 GAPE0490
 GAPE0500
 GAPE0510
 GAPE0520
 GAPE0525
 GAPE0526
 GAPE0527
 GAPE0528
 GAPE0530
 GAPE0531

ANPAR,NE2B,MBAND)	GAP E0532
RETURN	GAP E0540
C	GAP E0550
40 WRITE (6,1001)	GAP E0560
NUME=NPAP(2)	GAP E0570
DO 50 MM=1,NUME	GAP E0580
CALL STRSC (A(N1),A(N3),NEQ,0)	GAP E0590
WRITE (6,1000)	GAP E0600
DO 50 L=LT,LH	GAP E0610
CALL STRSC (A(N1),A(N3),NEQ,1)	GAP E0620
WRITE (6,1999) MM,SIG(1),SIG(2)	GAP E0630
C** STRESS PORTHOLE	GAP E0640
IF(NIOSV.EQ.1)	GAP E0650
XWRITE (NT10) MM,L,SIG(1),SIG(2)	GAP E0660
50 CONTINUE	GAP E0670
RETURN	GAP E0680
C	GAP E0690
1000 FORMAT (/)	GAP E0700
1001 FORMAT ('IGAP ELEMENT STRESSES AND',	GAP E0710
A ' FORCES:',//////)	GAP E0720
1005 FORMAT(' ITERATION NO. ',I2,' ; TENSION IN GAP ELEMENTS'/)	GAP E0760
END	GAP E0750

CS/360 FORTRAN H

OPTIONS - NAME= MAIN,OPT=00,LINECNT=59,SIZE=0000K,	
SOURCE,BBCDIC,NCLIST,ACDCECK,LCAD,MAP,NOEDIT,ID,NOXREF	
SUBROUTINE INL(ID,B,TMASS,NUMNP,NEQB,NSB,LL,NBLOCK)	INL 0100
IMPLICIT REAL*8(A-H,O-Z)	INL 0110
C	INL 0120
C CALLED BY MAIN	INL 0130
C	INL 0140
C INPUT NODAL LOADS (STATIC SOLUTION) OR NODAL MASSES (DYNAMIC)	INL 0150
C	INL 0160
DIMENSION ID(NUMNP,6),E(NSB,LL),TMASS(NSB)	INL 0170
COMMON /JUNK/ R(6)	INL 0180
COMMON /EXTRA/ MDEX	INL 0190
COMMON/CTL/GRAV	INL 0200
C	INL 0210
C READ THE ID ARRAY FROM FILE 8	INL 0220
C	INL 0230
4 IF (MDEX .EQ. 1) GO TO 20	INL 0240
REWIND 8	INL 0250
READ (8) ID	INL 0260
C	INL 0270
C ZERO THE MATRICES	INL 0280
C	INL 0290
DO 10 N=1,NSB	INL 0300
TMASS(N)=0.000	INL 0310
DO 10 K=1,LL	INL 0320
10 B(N,K)=0.000	INL 0330
C	INL 0340
C READ THE CARDS (TERMINATE WHEN THE NODE NUMBER=0)	INL 0350
C	INL 0360
20 WRITE (6,1001) N,L,R	INL 0370
30 READ (5,1000) N,L,R	INL 0380
IF (N .LE. 0) GO TO 80	INL 0390
IF (L .GT. 0) GO TO 50	INL 0400
DO 40 I=1,6	INL 0410
40 IF (R(I) .LT. 0.000) R(I) = -R(I)/GRAV	INL 0420
IF (R(2) .EQ. 0.000) R(2) = R(1)	INL 0430
IF (R(3) .EQ. 0.000) R(3) = R(1)	INL 0440
50 CONTINUE	INL 0450
WRITE (6,1002) N,L,R	INL 0460
IF (L .GT. LL) GO TO 100	INL 0470
IF (N .GT. NUMNP) GO TO 110	INL 0480
IF (MDEX .EQ. 1) GO TO 30	INL 0490
DO 70 I=1,6	INL 0500
IEQ=ID(N,I)	INL 0510
IF (IEQ .LE. 0) GO TO 70	INL 0520
IF (L .LE. 0) GO TO 60	INL 0530
B(IEQ,L)=R(I)	INL 0540
GO TO 70	INL 0550
60 TMASS(IEQ)=R(1)	INL 0560
70 CONTINUE	INL 0570
GO TO 30	INL 0580
C	INL 0590
C WRITE THE LOAD AND MASS MATRICES ON FILE 3	INL 0600
C	INL 0610
80 IF (MDEX .EQ. 1) RETURN	INL 0620
REWIND 3	INL 0630
REWIND 15	INL 0640

DO 90 NB=1,NBLOCK	INL 0650
N1=NEQB*(NB-1)+1	INL 0660
N2=NEQB*NB	INL 0670
WRITE (3)((B(N,L),N=N1,N2),L=1,LL),(TMASS(N),N=N1,N2)	INL 0680
WRITE(15)((B(N,L),N=N1,N2),L=1,LL)	INL 0690
90 CONTINUE	INL 0700
RETURN	INL 0710
C	INL 0720
C ERROR HANDLING	INL 0730
C	INL 0740
100 WRITE (6,1003) L,LL	INL 0750
MODEX=1	INL 0760
GO TO 30	INL 0770
110 WRITE (6,1004) N,NUMNP	INL 0780
MODEX=1	INL 0790
GO TO 30	INL 0800
C	INL 0810
1000 FORMAT (2I5,7F10.4)	INL 0820
1001 FORMAT ('IN C D A L L O A D S (S T A T I C) C R ',	INL 0830
X'M A S S E S (D Y N A M I C)',///,	INL 0840
A3X,'NODE',3X,'LOAD',	INL 0850
A2(9X,'X-AXIS',9X,'Y-AXIS',9X,'Z-AXIS'),/, ' NUMBER',3X,'CASE',	INL 0860
A3(10X,'FORCE'),3(9X,'MOMENT'),/,1X)	INL 0870
1002 FORMAT (2(3X,14),1P0E15.5)	INL 0880
1003 FORMAT ('0***FATAL ERKCR*** LOAD NUMBER',I10,' IS GREATER',	INL 0890
X'THAN THE MAXIMUM,',I10,/, ' CONTINUE IN DATA CHECK MODE')	INL 0900
1004 FORMAT ('0***FATAL ERKCR*** NODE NUMBER',I10,' IS GREATER',	INL 0910
X'THAN THE MAXIMUM NUMBER OF NODES,',I10, ' CONTINUE IN DATA',	INL 0920
X' CHECK MODE')	INL 0930
END	INL 0940

CS/360 FORTRAN H

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OPTIONS - NAME= MAIN,CPT=00,LINECNT=59,SIZE=0000K,
          SOURCE,EBLDCIC,NCLIST,NCDECK,LCAD,MAP,NOEDIT,ID,NOXREF
SUBROUTINE RUSS (IC,X,Y,Z,T,E,THERM,DEN,AREA,WT,NUMNP)
IMPLICIT REAL*8(A-H,C-Z)
LOGICAL ELPRT,ELPCH,GENPRT,GENPCH,DEFPCH

C
C CALLS CALBAN
C CALLED BY TRUSS
C
C
C DIMENSION X(1),Y(1),Z(1),IC(NUMNP,1),E(1),THERM(1),DEN(1),AREA(1)
C ,T(1),WT(1)
C COMMON /ELPAR/ NPAK(14),NNNN,MBAND,RELTP,N1,N2,N3,N4,N5,MTOT,NEQ
C COMMON /EM/LM(2),ND,N3,S(24,24),P(24,4),XM(24),ST(12,24),TT(12,4)
C COMMON /JUNK/ EMUL(4,4),I,J,K,L,M,N,I1,JJ,KK,MTYPE,TEMP,DX,DY,DZ,
1 XL2,XL,XX,YY,F,FT,FX,FY,FZ,MIN,MAX,NCIF,KKK,TEM,MTYP
C COMMON /EXTRA/ MCODEX,NT8
C COMMON /CTL/ GRAV,ELPRT,ELPCH,GENPRT,GENPCH,DEFPCH,
1 INTCG,MIN3ND,IPL1
C COMMON/CONV/GCCN

C
C CONTROL INFORMATION AND MEMBER PROPERTIES
C
5 KENIND 16
NUME=NPAK(2)
NUMMAT=NPAK(3)
IF(NPAK(1).EQ.1) WRITE (6,1003) NUME,NUMMAT
IF(NPAK(1).EQ.11) WRITE (6,1014) NUME,NUMMAT
WRITE (6,1004)
DO 10 I=1,NUMMAT
READ (5,1000) N,E(N),THERM(N),DEN(N),AREA(N),WT(N)
IF(DEN(N).LE.0.000)
*DEN(N)=WT(N)/GRAV
10 WRITE (6,1005) N,E(N),THERM(N),DEN(N),AREA(N),WT(N)
C*** DATA PORTHOLE SAVE
IF(MCODEX.EQ.1)
*WRITE (NT8) (E(N),THERM(N),DEN(N),AREA(N),WT(N),N=1,NUMMAT)

C
C READ IN GAP ELEMENT CONVERGENCE FORCE
C
IF(NPAK(1).EQ.11) READ(5,1015) GCCN
IF(NPAK(1).EQ.11) WRITE(6,1016) GCCN

C
C ELEMENT LOAD MULTIPLIERS
C
READ (5,1001) EMUL
WRITE (6,1006) EMUL
C*** DATA PORTHOLE SAVE
IF(MCODEX.EQ.1)
*WRITE (NT8) EMUL

C
C ELEMENT INFORMATION
C WRITE (6,1008)

C
N=1
20 READ (5,1002) M,I1,JJ,MTYP,TEM,KK
IF(KK.EQ.0) KK=1

```

RUSS0100
 RUSS0110
 RUSS0120
 RUSS0130
 RUSS0140
 RUSS0150
 RUSS0160
 RUSS0170
 RUSS0180
 RUSS0190
 RUSS0200
 RUSS0210
 RUSS0220
 RUSS0230
 RUSS0240
 RUSS0250
 RUSS0260
 RUSS0270
 RUSS0280
 RUSS0290
 RUSS0300
 RUSS0310
 RUSS0320
 RUSS0330
 RUSS0331
 RUSS0340
 RUSS0350
 RUSS0360
 RUSS0370
 RUSS0380
 RUSS0390
 RUSS0400
 RUSS0410
 RUSS0420
 RUSS0430
 RUSS0440
 RUSS0450
 RUSS0460
 RUSS0470
 RUSS0480
 RUSS0490
 RUSS0500
 RUSS0510
 RUSS0520
 RUSS0530
 RUSS0540
 RUSS0550
 RUSS0560
 RUSS0570

30	IF(M.NE.N) GO TO 40	RUS S0580
	I=II	RUS S0590
	J=JJ	RUS S0600
	MTYPE=MTYP	RUS S0610
	KEFT=TcM	RUS S0620
	KKK=KK	RUS S0630
C		RUS S0640
C	1. FORM ELEMENT STIFFNESS AND STRESS MATRICES	RUS S0650
C		RUS S0660
40	CONTINUE	RUS S0670
	IF(MODEX.EQ.1) GO TO 90	RUS S0680
	DX=X(I)-X(J)	RUS S0690
	DY=Y(I)-Y(J)	RUS S0700
	DZ=Z(I)-Z(J)	RUS S0710
	XL2=DX*DX+DY*DY+DZ*DZ	RUS S0720
	XL=DSQRT(XL2)	RUS S0730
	AX=E(MTYPE)*AREA(MTYPE)*XL	RUS S0740
	ST(1,1)=DX/XL2	RUS S0750
	SI(1,2)=DY/XL2	RUS S0760
	ST(1,3)=DZ/XL2	RUS S0770
	SI(1,4)=-ST(1,1)	RUS S0780
	ST(1,5)=-ST(1,2)	RUS S0790
	ST(1,6)=-ST(1,3)	RUS S0800
C		RUS S0810
	DO 60 L=1,6	RUS S0820
	YY=ST(1,L)*XX	RUS S0830
	DO 50 K=L,6	RUS S0840
	S(K,L)=ST(1,K)*YY	RUS S0850
50	S(L,K)=S(K,L)	RUS S0860
	ST(1,L)=E(MTYPE)*ST(1,L)	RUS S0870
60	ST(2,L)=AREA(MTYPE)*ST(1,L)	RUS S0880
C		RUS S0890
C	2. INERTIA AND THERMAL LOADS	RUS S0900
C		RUS S0910
	F=WT(MTYPE)*AREA(MTYPE)*XL/2.00	RUS S0920
	TEMP=(T(I)+T(J))*0.5 DC- KEFT	RUS S0930
	FI=TEMP*THERM(MTYPE)*C(MTYPE)*AREA(MTYPE)	RUS S0940
	FT = -FI	RUS S0950
	FX=DX*FT/XL	RUS S0960
	FY=DY*FT/XL	RUS S0970
	FZ=DZ*FT/XL	RUS S0980
C		RUS S0990
	DO 70 L=1,4	RUS S1000
	FI(2,L)=EMUL(L,4)*FI	RUS S1010
	TI(1,L)=TI(2,L)/AREA(MTYPE)	RUS S1020
	P(1,L)=EMUL(L,1)*F-EMUL(L,4)*FX	RUS S1030
	P(2,L)=EMUL(L,2)*F-EMUL(L,4)*FY	RUS S1040
	P(3,L)=EMUL(L,3)*F-EMUL(L,4)*FZ	RUS S1050
	P(4,L)=EMUL(L,1)*F+EMUL(L,4)*FX	RUS S1060
	P(5,L)=EMUL(L,2)*F+EMUL(L,4)*FY	RUS S1070
70	P(6,L)=EMUL(L,3)*F+EMUL(L,4)*FZ	RUS S1080
	F=JEN(MTYPE)*AREA(MTYPE)*XL/2.00	RUS S1090
	DO 80 L=1,6	RUS S1100
80	AM(L)=F	RUS S1110
90	CONTINUE	RUS S1120
C		RUS S1130
C	3. FORM LOCATION MATRIX AND COMPUTE BAND WIDTH	RUS S1140

C	DO 100 L=1,3	RUSS1150
	LM(L)=ID(I,L)	RUSS1160
100	LM(L+3)=ID(J,L)	RUSS1170
C		RUSS1180
	ND=6	RUSS1190
	NS=2	RUSS1200
	NDM=24	RUSS1210
	IF (.NOT. ELPRT) GO TO 120	RUSS1220
	WRITE (6,1009) ND,NS	RUSS1230
	IF (ELPCH) WRITE (11,1010) N,ND,NS	RUSS1240
	DO 110 L=1,NS	RUSS1250
	IF (ELPCH) WRITE (11,1011) (ST(L,K),K=1,ND)	RUSS1260
110	WRITE (6,1012) (ST(L,K),K=1,ND)	RUSS1270
	IF (ELPCH) WRITE (11,1011) ((TT(L,K),K=1,4),L=1,NS)	RUSS1280
	WRITE (6,1013) ((TT(L,K),K=1,4),L=1,NS)	RUSS1290
120	CALL CALBAN (MBAND,NDIF,LM,XM,S,P,ND,NDM,NS)	RUSS1300
	NT1=1	RUSS1310
	IF (MINBND .EQ. 0) GO TO 130	RUSS1320
	NT1=4	RUSS1330
	LMI=LM(1)+LM(2)+LM(3)	RUSS1340
	LMJ=LM(4)+LM(5)+LM(6)	RUSS1350
	IF ((LMI*LMJ) .NE. 0) WRITE (3) I,J	RUSS1360
130	CONTINUE	RUSS1370
	IF (IPLT .GT. 0) WRITE (13) I,J	RUSS1380
	IF (MODEX.EQ.0) GO TO 140	RUSS1390
C**	DATA PORTHOLE SAVE	RUSS1400
	WRITE (NT8) N,I,J,MTYPE,REFT	RUSS1410
	GO TO 150	RUSS1420
140	CONTINUE	RUSS1430
	WRITE (NT1) ND,NS,(LM(L),L=1,ND),((ST(L,K),L=1,NS),K=1,ND),	RUSS1440
	X ((TT(L,K),L=1,NS),K=1,4)	RUSS1450
C		RUSS1460
C	4. CHECK FOR MORE ELEMENTS	RUSS1470
C		RUSS1480
	150 CONTINUE	RUSS1490
	WRITE (6,1007) N,I,J,MTYPE,REFT,NDIF	RUSS1500
	WRITE (16) N,I,J,((SIK,L),K=1,6),L=1,6)	RUSS1510
	IF (N.EQ.NUME) RETURN	RUSS1520
	N=N+1	RUSS1530
	I=I+KKK	RUSS1540
	J=J+KKK	RUSS1550
	IF (N.GT.M) GO TO 20	RUSS1560
	GO TO 30	RUSS1570
C		RUSS1580
	1000 FORMAT (15,5F10.0)	RUSS1590
	1001 FORMAT (4F10.0)	RUSS1600
	1002 FORMAT (+15,1F10.0,I5)	RUSS1610
	1003 FORMAT ('1 T R U S S E L E M E N T S',///,	RUSS1620
	X ' NUMBER OF TRUSS MEMBERS=',I5,/,	RUSS1630
	X ' NUMBER OF DIFF. MEMBERS=',I5)	RUSS1640
1004	FORMAT (///1X,4HTYPE,14X,1HE,10X,5HALPHA,12X,3HDEN,11X,4HAREA	RUSS1650
	X 11X,4H WT)	RUSS1660
1005	FORMAT (15,5E15.7)	RUSS1670
1006	FORMAT(///25H ELEMENT LOAD MULTIPLIERS / 20X,1HA,14X,1HB,14X,1HC,	RUSS1680
	X 14X,1HD,/6H X-DIR4E15.0/ 6H Y-DIR4E15.0/ 6H Z-DIR4E15.6/	RUSS1690
	X 6H TEMP4E15.0)	RUSS1700
		RUSS1710

1007 FORMAT (I6,F10.2,I7)	RUSS1720
1008 FORMAT (///42H1 N I J TYPE TEMP BAND)	RUSS1730
1009 FORMAT ('NUMBER OF DISPLACEMENTS PER ELEMENT (ND) =',I3,/,	RUSS1740
X 'NUMBER OF STRESSES PER ELEMENT (NS) =',I3,/,	RUSS1750
X 'ELEMENT STRESS-DISPL MATRIX ')	RUSS1760
1010 FORMAT ('ELEMENT',I3,' NC=',I3,' NS=',I3)	RUSS1770
1011 FORMAT ((1P3E10.3))	RUSS1780
1012 FORMAT (/, (1H ,1P10E13.4))	RUSS1790
1013 FORMAT ('ELEMENT FIXED-NODE STRESSES ',/, (1H ,1P4E13.4))	RUSS1800
1014 FORMAT ('IG A P E L E M E N T S',///,	RUSS1801
X ' NUMBER OF GAP ELEMENTS= ',I5,/,	RUSS1802
X ' NUMBER OF DIFF. MEMBERS=',I5)	RUSS1803
1015 FORMAT(F10.5)	
1016 FORMAT(///' GAP ELEMENT CONVERGENCE FORCE = ',1PD10.4///)	
END	RUSS1810

CS/360 FORTRAN H

```
OPTIONS - NAME= MAIN,CPT=00,LINECNT=59,SIZE=0000K,  
SOURCE,EBCDIC,NOLIST,NODECK,LCAD,MAP,NOEDIT,LD,NOXREF
```

```

SUBROUTINE SOLEQ
IMPLICIT REAL*8(A-H,C-Z)

```

CALLS SESOL,PRINTC,STRESS
CALLED BY MAIN

STATIC SOLUTION PHASE

```
COMMON A(1)
COMMON /ELPAR/ NP(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTCT,NEQ
COMMON /SOL / NOLOCK,NEQB,LL,NF,IFILL(7)
COMMON/GAP/STRS(100,4),ITR8,ITEST
```

REAL TT(4)

SOLVE FOR THE DISPLACEMENT VECTORS

```

10 CALL CPJTIM(TI(1))
   NSB=(MBAND+LL)*NEQB
   NSBB=NEQB*LL*(2+(MBAND-2)/NEQB)
   IF(NSBB.LT.NSB) NSBB=NSB
   N3=N1+NSB
   N4=N3+NSBB
   MI = MBAND + NEQB - 1
   CALL SESOL (A(N1),A(N3),A(N4),NEQ,MBAND,LL,NBLOCK,NEQB,NSB,MI,
X      4,3,2,7)
   CALL CPJTIM(TI(2))

```

PRINT DISPLACEMENTS

```

N2=N1+NUMNP*6
NSB=NEQB+NBLOCK
N4=N2+NSB*LL
N5=N4+1
CALL ERRCK(N5)
CALL PRINTD(A(N1),A(N2),A(N4),1,NEQB,NUMNP,LL,NBLOCK,
            NEQ,NSB,2,1)
CALL CPUTIM(IT(3))

```

COMPUTE AND PRINT ELEMENT STRESSES

```

N2=N1+4*LL                                SOLE0400
N3=N2+NEQB*LL                              SOLE0410
LB=(MTOT-N3)/(NEQ +12)                    SOLE0420
N4=N3+NEQ*LB                              SOLE0425
CALL STRESS(A(N1),A(N2),A(N3),A(N4),NOMNP,NEQB,LB,LL,NEQ,NBLOCK, SOLE0430
1,NELTYP)

```

```
IF(ITEST.NE.0)GO TO 30
CALL CPUTIM(TT(4))
```

COMPUTE TIME LOG FOR THE STATIC SOLUTION PHASE

```

      DO 20 K=1,3
20  TT(K) = TT(K+1)-TT(K)
      WRITE (6,1000) (TT(L),L=1,3)

```

C			SOLE0630
	1000	FORMAT (//// 48H S T A T I C S O L U T I O N T I M E L O G,	SOLE0640
	X	//5X,21HEQUATION SOLUTICN =, F8.2 /	SOLE0650
	X	5X,21HDISPLACEMENT OUTPUT =, F8.2 /	SOLE0660
	X	5X,21HSTRESS RECOVERY =, F8.2 /)	SOLE0670
C			SOLE0680
	30	RETURN	SOLE0690
		END	SOLE0700

CS/360 FORTRAN H

```

OPTIONS - NAME= MAIN,CPT=00,LINECNT=59,SIZE=0000K,
          SOURCE,EDC010,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,NOXREF
SUBROUTINE STRESS(STR,B,J,ID,NUMNP,NEQB,LB,LL,NEQ,NBLOCK,
1 NELTYP)
IMPLICIT REAL*8(A-H,C-Z)
C
C CALLS ELTYPE
C CALLED BY SOLEQ
C
C DIMENSION D(NEQ,LB),B(NEQB,LL),STR(4,LL),ID(NUMNP,6)
COMMON /ELPAR/ NPAR(14)
COMMON /JUNK/ LT,LH
COMMON /EXTRA/ MODEX,NT8,NIOSV,NT10
COMMON/GAP/SIKS(100,4),ITR8,ITEST
C
7 REWIND 8
READ (8) ID
READ (8) STR
NT=(LL-1)/LB +1
LH=0
C*** STRESS PORTHOLE
IF(NIOSV.EQ.1)
XWRITE (NT10) NELTYP,NT
C
C DO 40 II=1,NT
C
C LT=LH+1
C LLT=1-LT
C LH=LT+LB-1
C IF(LH.GT.LL) LH=LL
C
C MOVE DISPLACEMENTS INTO CCKE FOR LB LOAD CONDITIONS
C
C REWIND 2
C*** STRESS PORTHOLE
IF(NIOSV.EQ.1)
XWRITE (NT10) LT,LH
NQ=NEQB*NBLOCK
DO 10 NN=1,NBLOCK
READ (2) B
N=NEQB
IF (NN.EQ.1) N=NEQ-NQ+NEQB
N=NQ-NEQB
DO 10 J=1,N
I=NQ+J
DO 10 L=LT,LH
K=L+LLT
10 D(I,K)=B(J,L)
LK=LH-LT+1
C
C CHECK GAP ELEMENTS FOR TENSION
C
C REWIND 1
C REWIND 18
C DO 20 M=1,NELTYP
READ(1)NPAK
READ(18)NPAK

```

STRE0100
 STRE0110
 STRE0120
 STRE0130
 STRE0140
 STRE0150
 STRE0160
 STRE0170
 STRE0180
 STRE0190
 STRE0195
 STRE0200
 STRE0205
 STRE0210
 STRE0215
 STRE0220
 STRE0230
 STRE0240
 STRE0250
 STRE0260
 STRE0270
 STRE0280
 STRE0290
 STRE0300
 STRE0310
 STRE0320
 STRE0330
 STRE0340
 STRE0350
 STRE0360
 STRE0370
 STRE0380
 STRE0390
 STRE0400
 STRE0410
 STRE0420
 STRE0430
 STRE0440
 STRE0450
 STRE0460
 STRE0470
 STRE0480
 STRE0490
 STRE0500
 STRE0510
 STRE0520
 STRE0530
 STRE0540
 STRE0550
 STRE0555
 STRE0560
 STRE0570
 STRE0575
 STRE0580

MTYPE=NPAP(1)	STRE0590
NPAP(1)=0	STRE0600
CALL ELTYPE(MTYPE)	STRE0620
20 CONTINUE	STRE0630
RETURN	STRE0650
C	STRE0660
C CALCULATE STRESSES FOR ALL ELEMENTS FOR LB LOAD CONDITIONS	STRE0670
C	STRE0680
30 REWIND 1	STRE0690
REWIND 18	STRE0695
DO 40 M=1,NELTY	STRE0700
READ (1) NPAP	STRE0705
READ (18) NPAP	STRE0710
C*** STRESS PORTHOLE	STRE0720
IF(NIOSV.EQ.1)	STRE0730
WRITE (NT10) NPAP	STRE0740
MTYPE=NPAP(1)	STRE0750
NPAP(1)=0	STRE0760
CALL ELTYPE(MTYPE)	STRE0770
40 CONTINUE	STRE0780
C	STRE0790
RETURN	STRE0800
END	STRE0810

CS/360 FORTRAN H

OPTIONS - NAME= MAIN,CPT=00,LINECNT=59,SIZE=0000K,
 SOURCE,EBCLIC,NULIST,NODECK,LOAD,MAP,NOEDIT,ID,NOXREF
 SUBROUTINE STRSC1STR,D,NEQ,NTAG)
 IMPLICIT REAL*8(A-H,C-Z)

C
C
C

CALLED BY TROSS,BEAM,PLANE,THREED,SHELL,BOUND,PIPE,GAPEL

DIMENSION STR(+,1),E(NEQ,1)
 COMMON /JUNK/ LT,LH,L,IPAD,SG(20),SIG(7)
 COMMON /EM/ NS,ND,B(42,63),TI(42,4),LM(63)

C

IF (NTAG.EQ.0) GO TO 60

LL=L-LT+1

DO 30 I=1,NS

SG(I)=0.0D0

DO 30 J=1,4

30 SG(I)=SG(I)+TI(I,J)*STR(J,L)

DO 50 J=1,ND

JJ=LM(J)

IF(JJ.EQ.0) GO TO 50

DO 40 I=1,NS

40 SG(I)=SG(I)+B(I,J)*D(JJ,LL)

C

50 CONTINUE

GO TO 70

60 READ (1) ND,NS,(LM(I),I=1,ND),((B(I,J),I=1,NS),J=1,ND),

1 ((TI(I,J),I=1,NS),J=1,4)

70 RETURN

END

STRS0120
 STRS0130
 STRS0140
 STRS0150
 STRS0160
 STRS0170
 STRS0180
 STRS0190
 STRS0200
 STRS0210
 STRS0220
 STRS0230
 STRS0240
 STRS0250
 STRS0260
 STRS0270
 STRS0280
 STRS0290
 STRS0300
 STRS0310
 STRS0320
 STRS0330
 STRS0340
 STRS0350
 STRS0360
 STRS0370
 STRS0380

APPENDIX B

THE FORTRAN PROGRAM SCALE

This program, as described in section III selectively alters chosen dimensions on the bone plate discretization in response to interactively entered user commands. The program consists of a main program and five subroutines each of which is dedicated to a specific task. If the user wishes to use more than one program option (for example adjust plate dimensions and then adjust cortex diameters) the program must be re-run for the second alteration to the mesh and again for any subsequent operation.

```

C * * * * *
C
C      PROGRAM SCALE:
C      THIS PROGRAM IS DESIGNED TO ALTER THE SCALING OF A
C      FINITE ELEMENT MESH OR PORTIONS OF IT. THE MESH
C      REPRESENTS A MODEL OF A BONE FIXATION PLATE IN SITU
C      ON A PORTION OF A FRACTURED HUMAN FEMUR.
C      THIS CODE ALLOWS VARIATION OF PLATE DIMENSIONS, BONE
C      DIMENSIONS AND ANGULAR ORIENTATION OF THE FRACTURE
C      PLANE. THESE OPERATIONS ARE PERFORMED IN ROUTINES
C      PLTJIM,DIAM AND ANGLE RESPECTIVELY.
C
C * * * * *
C
COMMON ST1(15,20),ST2(1000,20),IX(1220,6),COORD(1220,3),
1 CT(1220),N(1220),KN(1220),T(1220),ICOUNT
DOUBLE PRECISION FILIN,FILOUT
INTEGER CPT
TYPE 100
100 FORMAT(' PROGRAM SCALE: A FINITE ELEMENT MESH SCALING'/
1 ' PROGRAM FOR A THREE DIMENSIONAL MODEL OF A BONE PLATE'//
2 ' IN SITU.'//
3 ' WRITTEN BY: D.L. LEVINE'//
4 ' THE UNIVERSITY OF TENNESSEE'//
5 ' DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS'//
6 ' 11 MAY, 1977'//)
C
C      OPEN THE INPUT AND OUTPUT FILES AND READ ALL DATA FROM THE INPUT
C      FILE.
C
TYPE 104
104 FORMAT(' TYPE THE NAMES OF THE INPUT AND OUTPUT FILES:')
ACCEPT 105,FILIN,FILOUT
105 FORMAT(A10,IX,A10)
OPEN(UNIT=20,DEVICE='DSK',ACCESS='SEQIN',MODE='ASCII',
1 DISPOSE='SAVE',FILE=FILIN)
OPEN(UNIT=21,DEVICE='DSKB',ACCESS='SEQUOT',MODE='ASCII',
1 DISPOSE='SAVE',FILE=FILOUT)
DO 1000 I=1,8
READ(20,201)(ST1(I,J),J=1,20)
201 FORMAT(20A4)
1000 CONTINUE
DO 1001 K=1,1220
READ(20,202) CT(K),N(K),(IX(K,L),L=1,6),(COORD(K,M),M=1,3),
1 KN(K),T(K)
202 FORMAT(A1,14,6I5,3F10.0,15,F10.0)
1001 CONTINUE
KK=1
1002 READ(20,201,END=1003)(ST2(KK,II),II=1,20)
KK=KK+1
GO TO 1002
1003 ICOUNT=KK
TYPE 101
101 FORMAT(' SCALE HAS FIVE OPTIONS AS FOLLOWS:'//
1 ' OPT=1 : ADJUSTS THE ANGLE BETWEEN THE NORMAL TO THE'//
2 ' FRACTURE PLANE AND THE BONE LONGITUDINAL AXIS.'//
3 ' OPT=2 : ADJUSTS VARIOUS DIMENSIONS OF THE BONE PLATE.'//
4 ' OPT=3 : ADJUSTS INSIDE AND OUTSIDE DIAMETERS OF '//
5 ' THE BONE CORTEX.'//
6 ' OPT=4 : CREATES CURVATURE IN FRONTAL PLANE.'//
7 ' OPT=5 : PLATE PRETENSING.'//)

```

```

      3  ' SPECIFY OPT: ' )
      ACCEPT 102, CPT
102  FORMAT(I1)
      IF(OPT.EQ.1)CALL ANGLE
      IF(OPT.EQ.2)CALL FLTDIM
      IF(OPT.EQ.3)CALL CIAM
      IF(OPT.EQ.4)CALL CURVE
      IF(OPT.EQ.5)CALL TEMP
C
C      NOW WRITE OUT THE NEW MESH CONNECTIVITY DATA.
C
      DO 1006 I=1,8
      WRITE(21,201)(ST1(I,J),J=1,20)
1006  CONTINUE
      DO 1007 I=1,1220
      WRITE(21,203)CT(I),N(I),(IX(I,L),L=1,6),(COGRD(I,M),M=1,3),
      1  KN(I),T(I)
      203  FORMAT(A1,I4,6I5,3F10.5,I5,F10.0)
1007  CONTINUE
      DO 1008 I=1,ICCNT
      WRITE(21,201)(ST2(I,J),J=1,20)
1008  CONTINUE
      TYPE 103
      103  FORMAT(' NORMAL TERMINATION OF PROGRAM SCALE.'/)
      STOP
      END

```

```

      SUBROUTINE ANGLE
C * * * * *
C
C      THIS ROUTINE ALTERS THE ANGLE BETWEEN THE NORMAL TO THE
C      BONE FRACTURE SURFACE, AND THE BONE LONGITUDINAL AXIS.
C
C * * * * *
C
      COMMON ST1(15,20),ST2(1000,20),IX(1220,6),COORD(1220,3),
1      CT(1220),N(1220),KN(1220),T(1220),ICOUNT
      TYPE 104
104 FORMAT(' SPECIFY ANGLE ALPHA(MAGNITUDE.LT.25 DEGREES):')
      ACCEPT *,ALPHA
C
C      TYPE THE VALUES FROM THE INPUT MESH FILE WHICH GIVE
C      THE LOCATION OF THE FRACTURE AND THE BONE DIAMETERS.
C
      TYPE 108
108 FORMAT(' ON A SINGLE LINE, USING FREE FORMAT,')
1      ' TYPE THE FOLLOWING:/'
2      ' YOLD16 = THE Y-COORDINATE OF THE FRACTURE GAP BOTTOM'/
3      ' FROM THE INPUT FILE'/
4      ' OLDGAP = THE GAP HEIGHT FROM THE INPUT FILE'/
5      ' DC = BONE OUTSIDE DIAMETER FROM INPUT FILE'/
6      ' DI = BONE INSIDE DIAMETER FROM INPUT FILE'/
      ACCEPT *,YOLD16,OLDGAP,DC,DI
C
C      SCAN THE INPUT DATA AND MODIFY THE NODAL COORDINATES
C      AT THE GAP ACCORDING TO THE MAGNITUDE OF THE ANGLE
C      ALPHA.
C
      RC=DC/2.
      RI=DI/2.
      Y1=YOLD16
      Y2=YOLD16+OLDGAP
      DO 5000 I=1,1220
      X=COORD(I,1)
      Y=COORD(I,2)
      Z=COORD(I,3)
      IF(Y.LT.Y1.OR.Y.GT.Y2)GO TO 5000
      R=SQRT(X**2+Z**2)
      YPLUS=(RC+Z)*(SIND(ALPHA)/CCSD(ALPHA))
      THETA=(ATAN(X/Z))*180./3.141592654
      IF(R.GT.RC.AND.(R-RC).GT.0.004) GO TO 5000
      COORD(I,2)=Y+YPLUS
      SC=SIND(ALPHA)*CCSD(ALPHA)
      IF(ABS(Y-Y2).LE.0.001) COORD(I,3)=Z-(OLDGAP*SC)
5000  CONTINUE
      RETURN
      END

```

```

      SUBROUTINE PLTDIM
C ***
C
C      THIS ROUTINE MODIFIES VARIOUS AXIAL DIMENSIONS IN
C      A BONE PLATE FINITE ELEMENT MESH, IN RESPONSE TO
C      TELETYPE INPUT DATA.
C ***
C
      DIMENSION YOLD(44),YNEW(44),A(3),B(6),IC(6),ID(6)
      COMMON ST1(15,20),ST2(1000,20),IX(1220,6),CCORD(1220,3),
1      CT(1220),N(1220),KN(1220),T(1220),ICOUNT
      SCRDIA=.15
      HEDDIA=.30
C
C      INPUT FROM THE TELETYPE THE VALUES OF THE PREVIOUS
C      SET OF PLATE DIMENSIONS AND COMPUTE CERTAIN Y-COORDINATE
C      VALUES NECESSARY FOR THE SCALING PROCEDURE.
C
      TYPE 300
300  FORMAT(' ON ONE LINE, USING FREE FORMAT,')
1    ' INPUT THE FOLLOWING VALUES FROM THE INPUT FILE:/'
2    ' OLDPLT = OVERALL PLATE LENGTH/'
3    ' OLDSDE = SCREW TO SCREW DISTANCE AT ENDS/'
4    ' OLDSDM = SCREW TO SCREW DISTANCE ACROSS PLATE MIDSECTION/'
5    ' OLDGAP = WIDTH OF GAP BETWEEN BONE SEGMENTS/'
6    ' BOD = BONE OUTSIDE DIAMETER/'
      ACCEPT *, OLDPLT,OLDSDE,OLDSDM,OLDGAP,BOD
      YOLD(1)=0.0
      YOLD(2)=0.5
      YOLD(3)=1.0
      E1=(OLDPLT-OLDSDM-(4.*OLDSDE)-SCRDIA)/2.
      E2=OLDSDE-SCRDIA
      E3=(OLDSDM-OLDGAP-SCRDIA)/2.
      YOLD(4)=YOLD(3)+E1
      YOLD(5)=YOLD(4)+.0375
      YOLD(6)=YOLD(4)+.1125
      YOLD(7)=YOLD(4)+.15
      YOLD(8)=YOLD(7)+E2
      YOLD(9)=YOLD(8)+.0375
      YOLD(10)=YOLD(8)+.1125
      YOLD(11)=YOLD(8)+.15
      YOLD(12)=YOLD(11)+E2
      YOLD(13)=YOLD(12)+.0375
      YOLD(14)=YOLD(12)+.1125
      YOLD(15)=YOLD(12)+.15
      YOLD(16)=YOLD(15)+E3
      YOLD(17)=YOLD(16)+OLDGAP
      YOLD(18)=YOLD(17)+E3
      YOLD(19)=YOLD(18)+.0375
      YOLD(20)=YOLD(18)+.1125
      YOLD(21)=YOLD(18)+.15
      YOLD(22)=YOLD(21)+E2
      YOLD(23)=YOLD(22)+.0375
      YOLD(24)=YOLD(22)+.1125
      YOLD(25)=YOLD(22)+.15
      YOLD(26)=YOLD(25)+E2
      YOLD(27)=YOLD(26)+.0375
      YOLD(28)=YOLD(26)+.1125
      YOLD(29)=YOLD(26)+.15
      YOLD(30)=YOLD(29)+E1

```

```

YOLD(31)=YOLD(30)+.5
YOLD(32)=YOLD(31)+.5
YOLD(33)=YOLD(4)-.075
YOLD(34)=YOLD(7)+.075
YOLD(35)=YOLD(8)-.075
YOLD(36)=YOLD(11)+.075
YOLD(37)=YOLD(12)-.075
YOLD(38)=YOLD(15)+.075
YOLD(39)=YOLD(18)-.075
YOLD(40)=YOLD(21)+.075
YOLD(41)=YOLD(22)-.075
YOLD(42)=YOLD(25)+.075
YOLD(43)=YOLD(26)-.075
YOLD(44)=YOLD(29)+.075

```

C
C
C

TYPE IN THE VALUES OF PLATE DIMENSIONS FOR THE OUTPUT FILE.

TYPE 302

```

302  FORMAT(' ON ONE LINE, USING FREE FORMAT, TYPE IN'/
1    ' THE FOLLOWING VALUES FOR THE OUTPUT FILE: '/
2    ' PLTLEN = OVERALL PLATE LENGTH'/
3    ' STSDE = SCREW TO SCREW DISTANCE AT PLATE ENDS'/
4    ' STSDM = SCREW TO SCREW DISTANCE AT PLATE MIDSECTION'/
5    ' GAP = FRACTURE GAP THICKNESS'/)
ACCEPT *, PLTLEN,STSDE,STSDM,GAP

```

C
C
C
C

BASED ON THE LAST INPUT, COMPUTE THE NEW Y-COORDINATE VALUES FOR THE VARIOUS LAYERS OF THE MESH.

```

YNEW(1)=0.0
YNEW(2)=0.5
YNEW(3)=1.0
D1=(PLTLEN-STSDM-(4.*STSDE)-SCRDIA)/2.
TYPE 310,D1
310  FORMAT(' ',D1=' ',F15.5)
J2=STSDE-SCRDIA
D3=(STSDM-GAP-SCRDIA)/2.
YNEW(4)=YNEW(3)+D1
YNEW(5)=YNEW(4)+.0375
YNEW(6)=YNEW(4)+.1125
YNEW(7)=YNEW(4)+.15
YNEW(8)=YNEW(7)+D2
YNEW(9)=YNEW(8)+.0375
YNEW(10)=YNEW(8)+.1125
YNEW(11)=YNEW(8)+.15
YNEW(12)=YNEW(11)+D2
YNEW(13)=YNEW(12)+.0375
YNEW(14)=YNEW(12)+.1125
YNEW(15)=YNEW(12)+.15
YNEW(16)=YNEW(15)+D3
YNEW(17)=YNEW(16)+GAP
YNEW(18)=YNEW(17)+D3
YNEW(19)=YNEW(18)+.0375
YNEW(20)=YNEW(18)+.1125
YNEW(21)=YNEW(18)+.15
YNEW(22)=YNEW(21)+D2
YNEW(23)=YNEW(22)+.0375
YNEW(24)=YNEW(22)+.1125
YNEW(25)=YNEW(22)+.15
YNEW(26)=YNEW(25)+D2
YNEW(27)=YNEW(26)+.0375

```

```

YNEW(28)=YNEW(26)+.1125
YNEW(29)=YNEW(26)+.15
YNEW(30)=YNEW(29)+.01
YNEW(31)=YNEW(30)+.5
YNEW(32)=YNEW(31)+.5
YNEW(33)=YNEW(4)-.075
IF(YNEW(33).LE.YNEW(3)) GO TO 8888
YNEW(34)=YNEW(7)+.075
YNEW(35)=YNEW(8)-.075
YNEW(36)=YNEW(11)+.075
YNEW(37)=YNEW(12)-.075
YNEW(38)=YNEW(15)+.075
YNEW(39)=YNEW(18)-.075
YNEW(40)=YNEW(21)+.075
YNEW(41)=YNEW(22)-.075
YNEW(42)=YNEW(25)+.075
YNEW(43)=YNEW(26)-.075
YNEW(44)=YNEW(29)+.075

C
C      SCAN THE Y-COORDINATE VALUES FROM THE INPUT FILE
C      AND REPLACE THEM WITH THE COMPUTED NEW VALUES.
C

DO 1004 I=1,1220
X=COORD(I,1)
Y=COORD(I,2)
Z=COORD(I,3)
R=SQRT(X**2+Z**2)
RCUT=(BCL/2.)
DO 1005 J=1,44
YA=ABS(Y-YOLD(J))
IF(YA.GT.1.E-5) GO TO 1005
COORD(I,2)=YNEW(J)
GO TO 1004
1005 CONTINUE
A(1)=.01
A(2)=.075
A(3)=.14
IC(1)=4
IC(2)=6
IC(3)=12
IC(4)=18
IC(5)=22
IC(6)=26
DO 1111 K=1,6
DO 1111 L=1,3
II=IC(K)
AA=A(L)
YTEST=YOLD(II)+AA
YB=ABS(Y-YTEST)
IF(YB.GT.1.E-5) GO TO 1111
IF(R.LE.RCUT.OR.(R-RCUT).LT.0.1) COORD(I,2)=YNEW(II)+AA
IF(R.GT.RCUT.AND.(R-RCUT).GT.0.1) GO TO 1010
GO TO 1004
1111 CONTINUE
1010 B(1)=.02
B(2)=.075
B(3)=.15
B(4)=.225
B(5)=.28
B(6)=.3
ID(1)=53

```

```

      ID(2)=35
      ID(3)=37
      ID(4)=39
      ID(5)=41
      ID(6)=43
      DO 1112 LL=1,6
      DO 1112 KK=1,6
      JJ=ID(LL)
      BB=D(KK)
      TTT=YOLD(JJ)+BB
      YC=ABS(Y-TTT)
      IF(YC.GT.1.E-5)GO TO 1112
      IF(R.GT.RCUT.AND.(R-RCUT).GT.0.1)COORD(1,2)=YNEW(JJ)+BB
      GO TO 1004
1112  CONTINUE
1004  CONTINUE
      GO TO 9999
8388  TYPE 303
303   FORMAT(' ***** ERROR! *****',/,
1      ' INPUT DIMENSIONS HAVE CREATED AN INCONSISTENCY.',/,
2      ' CHECK DATA AND RE-SUBMIT.',//)
      STOP
9999  RETURN
      END

```

```

      SUBROUTINE DIAM
C ***
C
C      THIS ROUTINE CHANGES THE BONE INSIDE AND OUTSIDE
C      DIAMETERS IN A FINITE ELEMENT MESH OF A BONE-PLATE
C      SYSTEM.
C ***
C
      COMMON ST1(15,20),ST2(1000,20),IX(1220,6),COORD(1220,3),
1      CT(1220),N(1220),KN(1220),T(1220),KCOUNT
C
C      TYPE IN BONE INSIDE AND OUTSIDE DIAMETERS FROM THE
C      INPUT FILE, AND DESIRED DIAMETERS FOR THE OUTPUT FILE.
C
      TYPE 205
205      FORMAT(' ON A SINGLE LINE IN FREE FORMAT TYPE:/'
1      ' DIOLD = INSIDE BONE DIAMETER FROM INPUT FILE'/
2      ' DOOLD = OUTSIDE BONE DIAMETER FROM INPUT FILE'/
3      ' DINEW = DESIRED INSIDE DIAMETER FOR OUTPUT FILE'/
4      ' DONEW = DESIRED OUTSIDE DIAMETER FOR OUTPUT FILE'/)
      ACCEPT *,DIOLD,DOOLD,DINEW,DONEW
      RIC=DIOLD/2.
      ROC=DOOLD/2.
      RIN=DINEW/2.
      RON=DONEW/2.
C
C      SCAN INPUT DATA AND MODIFY BONE DIAMETERS TO CONFORM
C      WITH DESIRED VALUES.
C
      DO 1050 I=1,1220
      X=COORD(I,1)
      Z=COORD(I,3)
      R=SQRT(X**2+Z**2)
      TH=ACOS(Z/R)
      RITEST=ABS(R-RIC)
      ROTEST=ABS(R-ROC)
      IF(R.LT.RIC.AND.RITEST.GT.0.005)GO TO 1050
      IF(RITEST.LT.0.005)GO TO 1041
      IF(R.GT.RIC.AND.R.LT.ROC.AND.RITEST.GT.0.005.AND.ROTEST.GT.0.004)
1      GO TO 1042
      IF(ROTEST.LT.0.004)GO TO 1043
      IF(R.GT.ROC.AND.ROTEST.GT.0.004)GO TO 1044
      GO TO 1045
1041      R=RIN
      GO TO 1045
1042      R=RIN+(R-RIC)
      GO TO 1045
1043      R=RON
      GO TO 1045
1044      R=RON+(R-ROC)
1045      COORD(1,1)=R*SIN(TH)
      COORD(1,3)=R*COS(TH)
1050      CONTINUE
      RETURN
      END

```

```

      SUBROUTINE CURVE
C*****
C
C      THIS ROUTINE INTRODUCES A SLIGHT CURVATURE TO THE
C      BONE IN THE FRONTAL PLANE, WITHOUT DISTURBING THE
C      BONE PLATE.
C*****
C
C      COMMON ST1(15,20),ST2(1000,20),IX(1220,6),CCORD(1220,3),
1      CT(1220),N(1220),KN(1220),T(1220),ICOUNT
C
C      ACCEPT FROM ITY THE DATA WHICH DEFINE INPUT MESH
C      DIMENSIONS AND THE AMOUNT OF CURVATURE (DEFINED
C      BY THE Z-COORDINATE OF THE CENTER OF CURVATURE).
C
C      TYPE 401
401      FORMAT(' ON A SINGLE LINE, USING FREE FORMAT, '/
1      ' TYPE THE FOLLOWING: '/
2      ' DI = BONE OUTSIDE DIAMETER FROM THE INPUT FILE. '//
3      ' DI = BONE INSIDE DIAMETER FROM THE INPUT FILE. '//
4      ' YGAP= Y-COORDINATE OF FRACTURE GAP BOTTOM. '//
5      ' ZCEN= Z-COORDINATE OF CENTER OF CURVATURE. '//
      ACCEPT *,DI,DI,YGAP,ZCEN
C
C      SCAN THE INPUT DATA AND INTRODUCE THE SPECIFIED CURVATURE.
C
C      RD=DI/2.
      RI=DI/2.
C
      DO 1010 I=1,1220
      X=CCORD(I,1)
      Y=CCORD(I,2)
      Z=CCORD(I,3)
      R=SQRT(X**2.+Z**2.)
      IF(R.GT.RC.AND.R-RC.GT.0.004) GO TO 1010
      RHC=ZCEN-Z
      YSTAR=Y-YGAP
      RHOC=SQRT(RHC**2.-YSTAR**2.)
      ZSTAR=RHC-RHOC
      CCORD(I,3)=Z+ZSTAR
1010      CONTINUE
      RETURN
      END

```

```

SUBROUTINE TEMP
COMMON ST1(15,20),ST2(1000,20),IX(1220,6),COORD(1220,3),
1 CT(1220),N(1220),KN(1220),T(1220),ICOUNT
C
C THIS ROUTINE EFFECTS A PRETENSIONING OF THE BONE PLATE
C BY APPLYING A TEMPERATURE DIFFERENCE TO ALL THE NODES
C WHICH SPECIFY THE PLATE.
C
TYPE 500
500 FORMAT(' ON A SINGLE LINE IN FREE FORMAT, '/
1 ' TYPE THE FOLLOWING: '/
2 ' DU = BONE OUTSIDE DIAMETER. '/
3 ' TEMP = TEMPERATURE DIFFERENCE APPLIED TO PLATE. '//).
C
ACCEPT *,DU,TEMPF
C
RO=DU/2.
C
DO 1090 I=1,1220
X=COORD(I,1)
Y=COORD(I,2)
Z=COORD(I,3)
R=SQRT(X**2.+Z**2.)
IF((R-RO).GT.0.004)T(I)=TEMPF
1090 CONTINUE
RETURN
END

```

APPENDIX C

THE FORTRAN PROGRAM SONIC

This program reads from a DEC-10 storage file named FOR08.DAT the values of bone density and 8 measured ultrasound propagation velocities, and from these data computes the stiffness and compliance matrices along with a full set of engineering elastic constants for the bone.

```

C      DIMENSION C(6,6),S(6,6),WORK(6,6)

C      READ(8,*)RHC
C      READ(8,*)AL,AT,GL,GTH,GT3,F5L,F5H,F5V

C      A=1.0E3
C      RHO=RHO*A
C      AL=AL*A
C      AT=AT*A
C      GL=GL*A
C      GTH=GTH*A
C      GT3=GT3*A
C      F5L=F5L*A
C      F5H=F5H*A
C      F5V=F5V*A

C      C(3,3)=RHC*(AL**2.00)
C      C(4,4)=RHC*(AT**2.00)
C      C(5,5)=C(4,4)
C      C(1,1)=RHC*(GL**2.00)
C      C(2,2)=C(1,1)
C      C(6,6)=RHC*(GTH**2.00)
C      C(1,3)=(((C(1,1)+C(4,4)-2.00*RHO*(F5L**2.00))*
1          (C(3,3)+C(4,4)-2.00*RHO*(F5L**2.00)))
2          **0.50)-C(4,4)
C      C(2,3)=C(1,3)
C      C(1,2)=C(1,1)-2.00*C(6,6)

C      TEST1=RHC*(GT3**2.00)
C      TEST2=(C(4,4)+C(6,6))*0.50
C      TEST3=(((C(1,1)+C(4,4)-2.00*RHO*(F5V**2.00))*
1          (C(3,3)+C(4,4)-2.00*RHO*(F5V**2.00)))
2          **0.50)-C(4,4)

C      DIFF1=ABS(TEST1-C(4,4))
C      DIFF2=ABS(TEST2-(RHC*(F5H**2.00)))
C      DIFF3=ABS(TEST3-C(1,3))

C      DO 1000 I=1,6
C      DO 1000 J=1,6
C      C(J,I)=C(I,J)
1000  CONTINUE

C      TYPE 120
120  FORMAT(' CONSTITUTIVE MATRIX : ',/)

C      DO 2000 I=1,6
C      TYPE 130,(C(I,J),J=1,6)
2000  CONTINUE
130  FORMAT(' ',6(1PE10.2,2X))

C      TYPE 135,TEST1,TEST2,TEST3
135  FORMAT(// ' TEST1= ',1PE10.2, ' TEST2= ',1PE10.2, ' TEST3= ',
1      1PE10.2)
C      TYPE 140,DIFF1,DIFF2,DIFF3
140  FORMAT(// ' DIFF1= ',1PE10.2, ' DIFF2= ',1PE10.2, ' DIFF3= ',
1      1PE10.2)

C      CALL LINVFIC,6,6,S,C,WORK,IER)

C      TYPE 150

```

```

150  FORMAT(' COMPLIANCE MATRIX : ',/)
C
DO 3000 I=1,6
  TYPE 130,(S(I,J),J=1,6)
3000 CONTINUE
C
  E11=(1.00/S(1,1))
  E22=(1.00/S(2,2))
  E33=(1.00/S(3,3))
  GNU12=-(E11*S(1,2))
  GNU21=-(E22*S(1,2))
  GNU23=-(E22*S(2,3))
  GNU32=-(E33*S(2,3))
  GNU31=-(E33*S(3,1))
  GNU13=-(E11*S(3,1))
  G12=(1.00/S(6,6))
  G13=(1.00/S(5,5))
  G23=(1.00/S(4,4))
  CCNV=1.0/6.894757E3
  E1=E11*CCNV
  E2=E22*CCNV
  E3=E33*CCNV
  GA=G12*CCNV
  GB=G13*CCNV
  GC=G23*CCNV
C
  TYPE 160,E1,E2,E3,GNU12,GNU21,GNU23,GNU32,GNU31,GNU13,
1    GA,GB,GC
160  FORMAT(//' ENGINEERING CONSTANTS : ',/
1    ' E11= ',1PE10.2,' PSI',/
2    ' E22= ',1PE10.2,' PSI',/
3    ' E33= ',1PE10.2,' PSI',/
4    ' NU12= ',1PE10.2,/
5    ' NU21= ',1PE10.2,/
6    ' NU23= ',1PE10.2,/
7    ' NU32= ',1PE10.2,/
8    ' NU31= ',1PE10.2,/
9    ' NU13= ',1PE10.2,/
A    ' G12= ',1PE10.2,' PSI',/
Y    ' G13= ',1PE10.2,' PSI',/
Z    ' G23= ',1PE10.2,' PSI',//)
C
  STOP
  END

```

VITA

Danny Lewis Levine was born in Winnipeg, Manitoba, Canada on January 3, 1950. He attended elementary, junior high and senior high schools in that city and graduated from Grant Park High School in June of 1967. In September of 1967 he entered The University of Manitoba and in June 1971 was awarded the degree Bachelor of Science in Mechanical Engineering. In order to pursue an academic program in the field of Biomedical Engineering he accepted an offer of a research assistantship from the Ohio State University's Department of Metallurgical Engineering, and received the Master of Science degree from that school in December 1973. In January of 1974 he began work toward the Ph.D. degree in The University of Tennessee's Engineering Science and Mechanics Department for which he also became a teaching assistant. He received the Doctor of Philosophy degree in August of 1979.

The author is a member of the Tau Beta Pi and Sigma Alpha Mu honor societies and a member of The Canadian Medical and Biological Engineering Society. He will be employed with the firm Cardiac Pacemakers, Incorporated of St. Paul, Minnesota after graduation.