

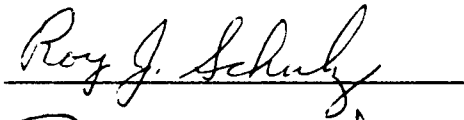
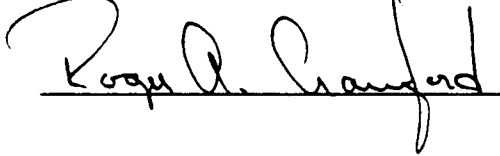
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TRANSIONOSPHERIC PROPAGATION AND DETECTION  
OF BROADBAND ELECTROMAGNETIC WAVEFORMS

A Thesis  
Presented for the  
Master of Science  
Degree  
The University of Tennessee, Knoxville

Joseph A. Miller  
June 1988

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## ABSTRACT

The ability to observe RF emissions through the ionosphere and the degree to which useful information about the source may be gleaned from such measurements is highly dependant upon the type of source, frequency ranges of interest, the state of the ionospheric medium, detector sensitivity and reliability of recovery algorithms used.

This paper analyzes the propagation, detection and discrimination aspects of this observables problem as modeled by the Trans-ionospheric Propagation Code (TIPC). Covered first are several typical theoretical broadband source waveforms. Second, the physics of deterministic dispersion is discussed along with the ionospheric transfer function. Next, results from the TIPC for deterministic and disturbed propagation are presented as well as white noise effects. Results are then presented from a simple detection algorithm utilizing a Gaussian filter function, and finally, a discussion on two methods of source discrimination: spectrum reconstruction and quadrature filtering.

Generally, the TIPC is very useful as a baseline propagation code. The ionospheric transfer function has been validated analytically and provided an accurate representation of the source is used, the higher order corruptive effects of noise and both deterministic and non-deterministic dispersion may be modeled to gain insight to the detectability and usefulness of the transionospheric pulse.

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## INTRODUCTION

Of interest to the observables community is the capability to detect transionospheric Radio Frequency (RF) electromagnetic pulse emissions from a variety of strategic sources below the ionosphere, discriminate among various characteristic signals to positively identify the source, and to precisely determine source location. This capability relies on the modeling of the physics of transionospheric radio wave propagation to support detector development.

Although strategic sources such as Charged Particle Beams (CPB) and nuclear detonations generate electromagnetic pulses (EMP) containing frequencies in all portions of the electromagnetic spectrum, of primary interest are those frequencies in the VHF RF band from 30 MHz through 300 MHz. This frequency range is important because it contains most of the information characterizing broadband pulses which if recovered after propagation may be used in discrimination. Frequencies above 30 MHz have the important characteristic of being able to pass through the ionosphere losing little or no energy and making detection somewhat easier in terms of detectable power levels. However, those above 300 MHz fall in the spectral range where signal strengths are substantially reduced for the majority of sources of interest. In either case, detection is hampered by background interference such as manmade and natural noise as well as broadband transients such as lightning and further

by the signal disruptions caused by RF wave passage through a turbulent ionosphere.

Of critical concern in this detection-location-discrimination process is the feasibility of reconstructing the original time signature of the VHF pulse from its waveform as modified by passage through the ionosphere. This paper discusses a second generation study of this modeling process and how the results affect basic detector design. Presented is the theory behind a transionospheric propagation computer code (TIPC) used to model the propagation and detection of several characteristic waveforms of interest. The TIPC contains a simplified representation of ionospheric propagation effects which includes only the prominent effects of dispersion and background. Such a simplified model is a first step in the development of a far more complex and complete model. As such, it allows definition and fine tuning of baseline transionospheric signal (TIS) reconstruction algorithms based on higher order effects of dispersion. Other effects including reflection, refraction, absorption, and focusing are to be incorporated as the project progresses.

The purpose of this paper is to outline the source, propagation, detection and signal recovery aspects of the VHF observables problem modeled by the TIPC. In presenting the mechanics of the subject code, this paper first discusses source waveforms and the physics of ionospheric dispersion. Second, through treatment of the ionosphere and satellite receiver as a series of linear systems with corresponding transfer functions, modeling of propagation and

detection processes is achieved by convolving the transfer functions against signal inputs. Finally, progress on frequency time-tagging and signal reconstruction algorithms is presented. A graphical representation of the problem and a generalized TIPC algorithm flow chart is presented in Figure 1.

Specifically, the time-tagging algorithm determines time of arrival (TOA) information of individual TIS frequency components. It then uses TOA information to determine range to the source and ionospheric conditions at the time of propagation. Mechanics of the discrimination or signal reconstruction aspect of the problem is presented using two algorithms. The first is an attempt to reconstruct a portion of the frequency spectrum of the source signal using several narrow-band receiver channels over the EM spectrum region of interest. The other uses a quadrature filtering technique to recover post-propagation phase and amplitude information as a function of frequency. It then subtracts the estimated ionospheric conditions at the time of propagation (determined using the TOA algorithm) and inverse transforms the frequency domain signal to obtain the original time signature.

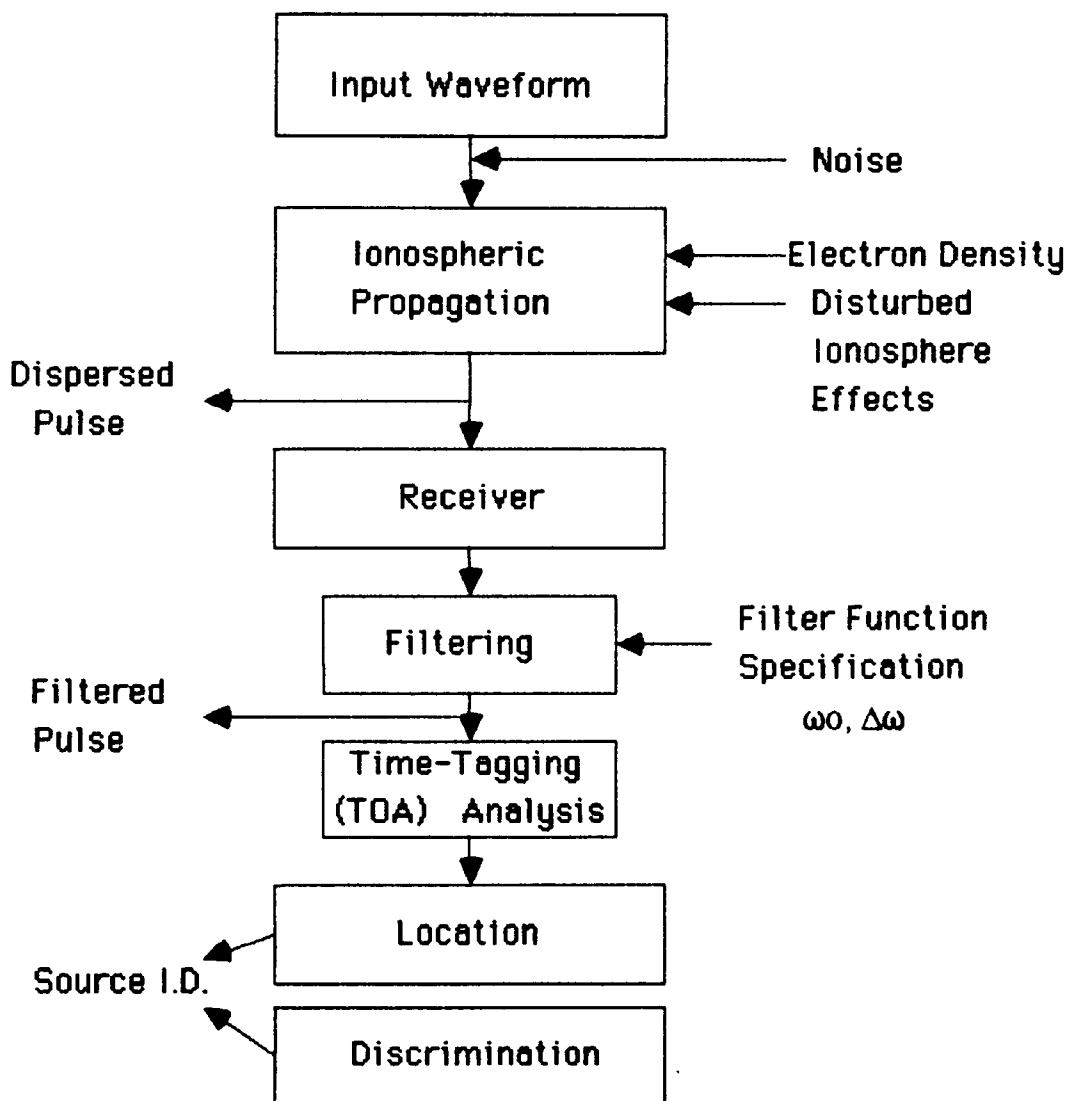
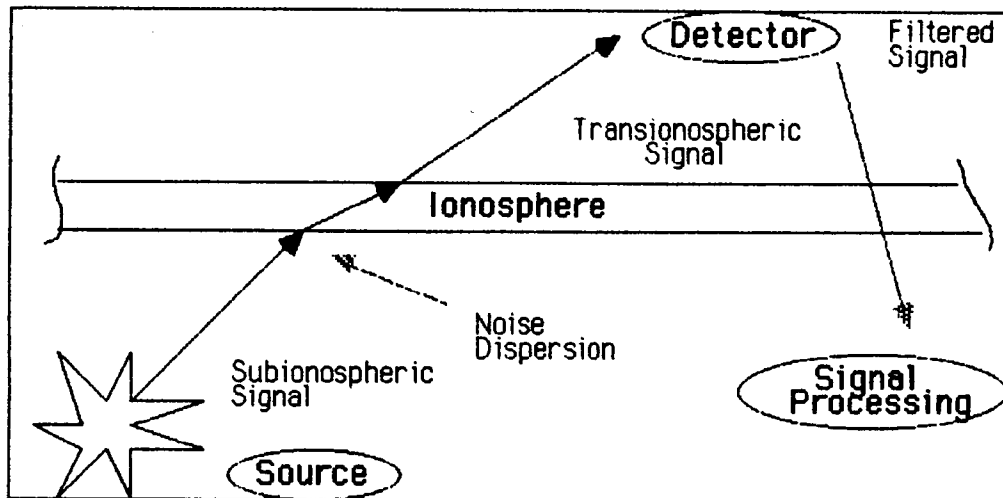


Figure 1. Graphical Presentation of Problem and TIPC Algorithm.

## I. SOURCE WAVEFORMS

A variety of five waveforms have been used in this study as representative signatures from a variety of source generators. They include a simple delta function (impulse) for use in testing the reliability of the code and understanding basic concepts, a theoretical EMP waveform simulating the signature of a nuclear burst, two theoretical charged particle beam (CPB) waveforms for simulating the RF time signatures of CPB's, and finally, one example of actual test ground data recorded at a CPB test. Included below for each are the analytical formula of the pulse, a plot of its time signature, and plots of the Fourier spectrum (FFT of input pulse) on linear and logarithmic scales. The current version of TIPC uses a time resolution of 1.0 nanosecond, a frequency step of 0.1 MHz with 100,000 point arrays. Thus the maximum time window is 100 microseconds and the maximum frequency attainable is 500 MHz. As evident from the figures, all of these pulses are broadband with time durations less than 50 nanoseconds and with a variety of power levels in the frequency domain.

### Delta Function.

The delta or impulse function is a theoretical pulse having finite energy content and instant duration with no pulse width. The spectrum of the impulse function is flat over the entire frequency domain with amplitude one:

$$\int_{-\infty}^{\infty} \delta(t) e^{i\omega t} dt = 1$$

Numerically, the theoretical delta function is here modeled by assigning a single point an arbitrary amplitude. Figure 1-1(a) shows the time signature of a delta function delayed 4.0  $\mu$ sec in a 10.0  $\mu$ sec sampling window. In order to maintain unit amplitude in the frequency domain, the delta function is normalized to amplitude 1000 or one over the nanosecond time resolution in the time domain. Figure 1-1(b) shows its frequency spectrum on a log scale. Note the energy of each frequency component is the same.

#### Electromagnetic Pulse (EMP).

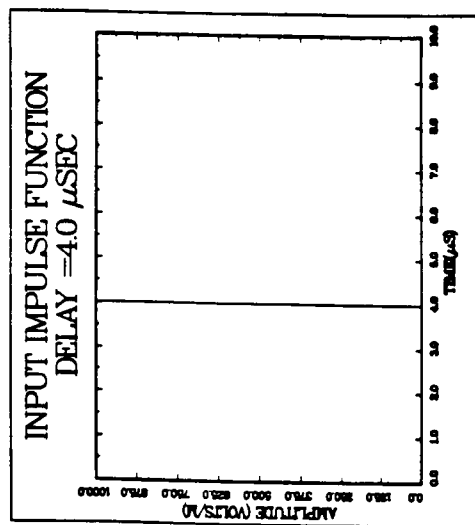
This theoretical EMP is a double exponential of the form:

$$E(t) = E_{\max} [e^{-a(t-t_0)} - e^{-b(t-t_0)}]$$

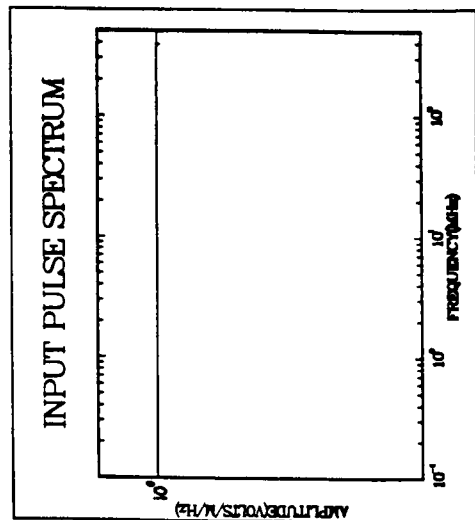
where  $E_{\max}$  is the peak E-field strength,  $a$  and  $b$  are the rise time and fall time of the pulse respectively and  $t_0$  is the peak arrival time defined as

$$t_0 = \ln(a/b)/(a-b).$$

Figure 1-2(a) shows the EMP time signature. Figures 1-2(b) and (c) show the frequency spectrum on linear and log scales respectively.

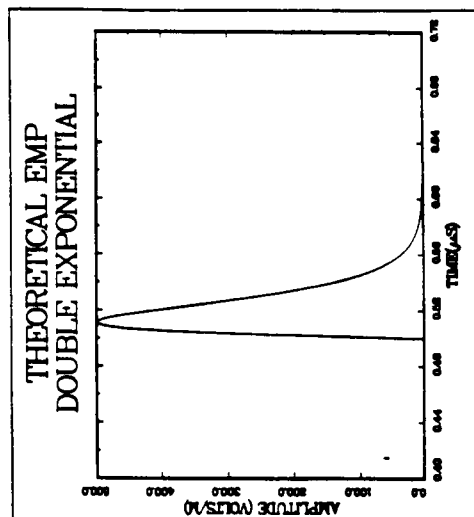


(a)

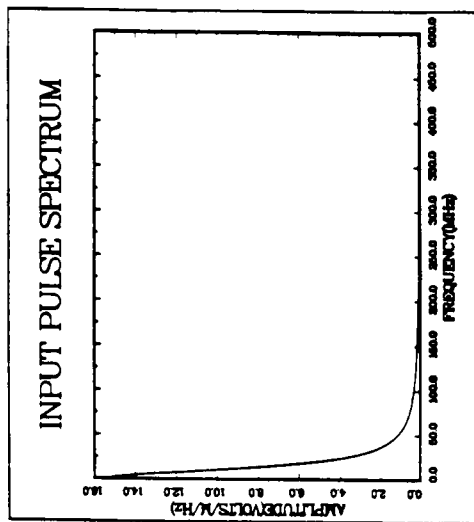


(b)

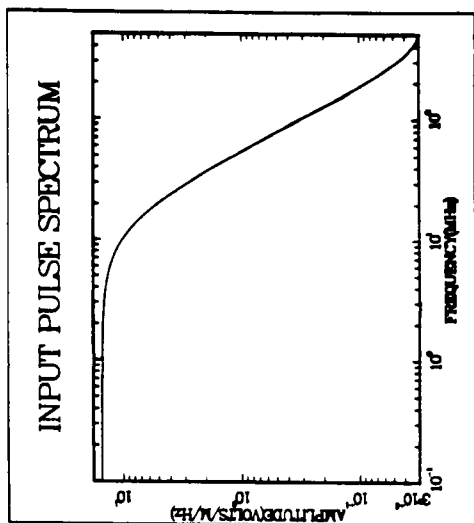
Figure 1-1. Time signature and spectrum for delta function.



(a)



(b)



(c)

Figure 1-2. Time signature and spectra for theoretical EMP.

### Supergaussian CPB.

This CPB waveform is represented here as a Supergaussian function of the form:

$$\begin{aligned} E(t) &= \sum_r A_r e^{-\left(\frac{t-(t_0-t_r)}{\Delta t_r}\right)^n} \\ &= A e^{-\left(\frac{t-t_0}{\Delta t}\right)^n} + \sum_r A_r e^{-\left(\frac{t-t_r}{\Delta t_r}\right)^n} \end{aligned}$$

where A is peak amplitude,  $t_0$  is arrival time of main pulse,  $\Delta t$  is the pulse width, n is an exponent which must be a multiple of two, and the addition sum over r represents the contribution of reflections. Figures 1-3 and 1-4 show the time signature and frequency spectra for an input Supergaussian CPB without and with two reflections respectively. Note the addition of peaks in the spectrum when reflections are added. These reflections are typical of ground and building reflections when the beam is fired.

### Modified Gaussian CPB.

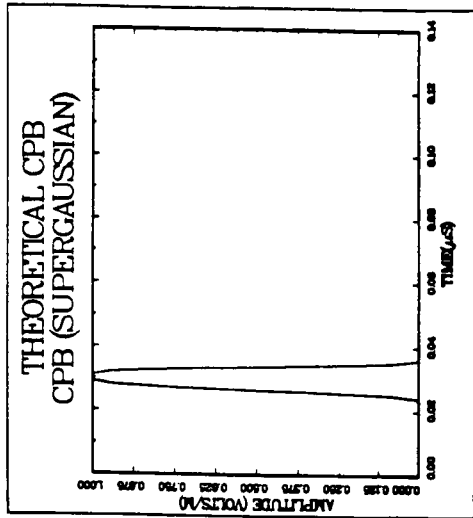
This theoretical CPB waveform is of the form:<sup>7</sup>

$$E(t) = \frac{(R_0 \beta I_n \sin(\theta))}{(1 - \beta \cos(\theta) R)}$$

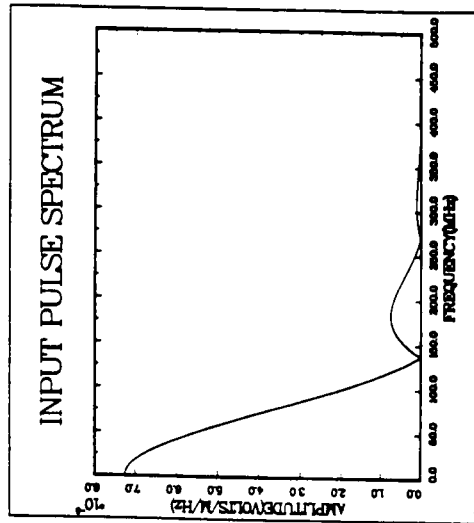
with

$$I_n = I_c \left[ \left( \frac{1}{1 + e^{-A \left( \frac{t-t_w}{t_r} \right)}} \right) - \left( \frac{1}{1 + e^{-\frac{A t}{t_r}}} \right) \right]$$

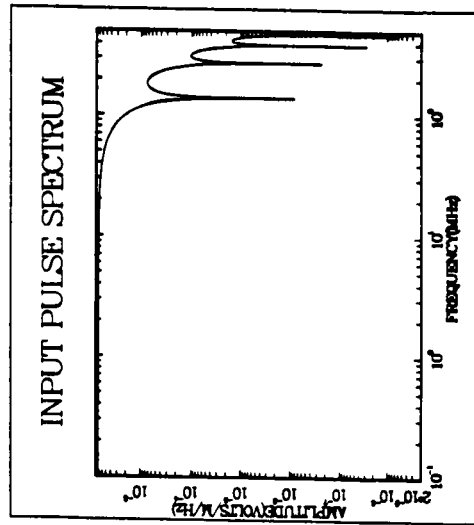
where  $I_n$  = net current



(a)

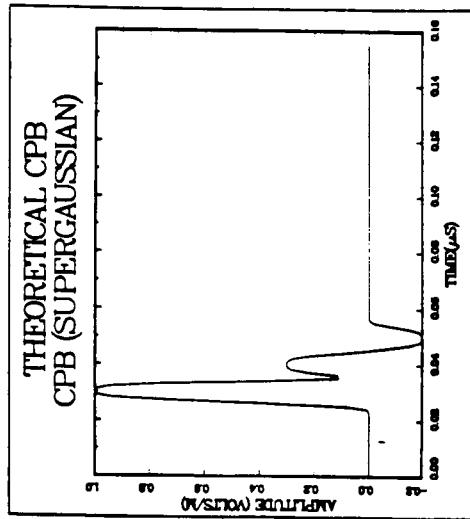


(b)

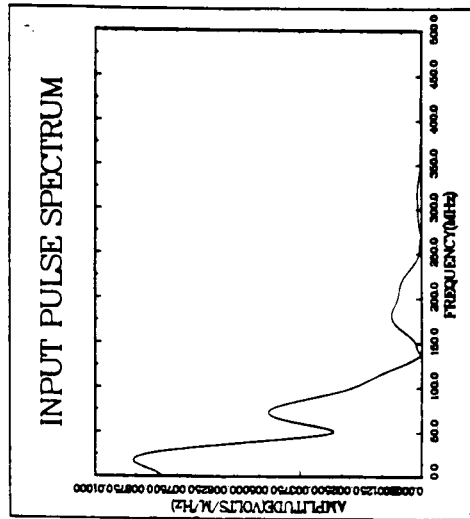


(c)

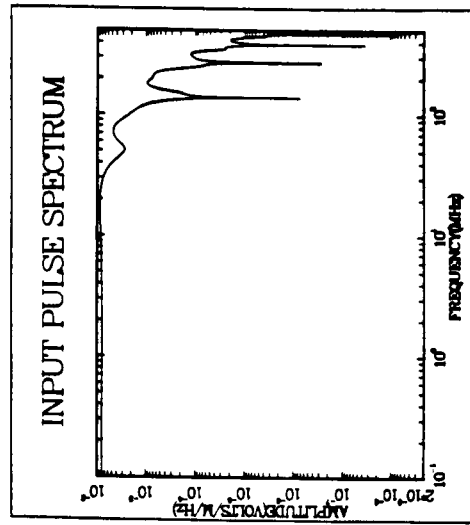
Figure 1-3. Time signature and spectra for Supergaussian CPB.



(a)



(b)



(c)

Figure 1-4. Time signature and spectra for Supergaussian CPB with reflections.

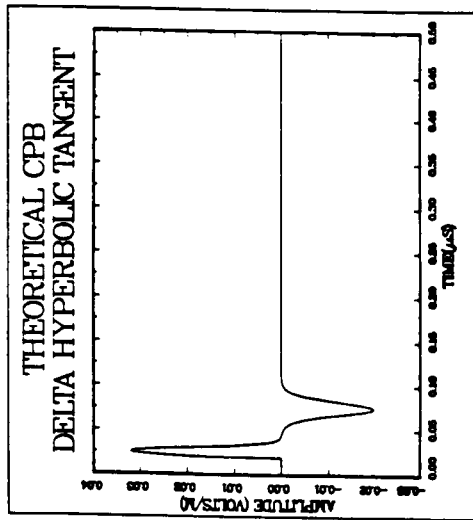
- $\beta$  = beam velocity ( $v/c$ )
- $\theta$  = angle from beam origin off beam axis to observer
- $R_0$  = size of radiating region
- $R$  = distance from origin to observer
- $I_c$  = initial current at beam exit
- $t_r$  = 10% rise time
- $t_f$  = 90% fall time
- $t_w$  = pulse duration
- $A$  = constant = 4.4

Figure I-5 provides the same information as before. This particular pulse has a reflection which overlaps the main pulse causing the evident interference. The main pulse width is about 10 nanoseconds. Note the distinct shape of the spectrum and that most of the energy of the pulse is contained roughly in the 5-80 MHz band.

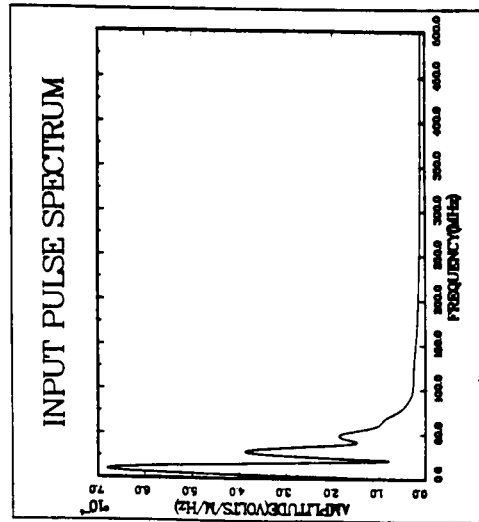
#### Digitized Pulse.

The pulse shown in Figure I-6 is a theoretical representation of an actual CPB device.<sup>9</sup> The similarities between this and the Gaussian type models are evident, especially in the spectrum information.

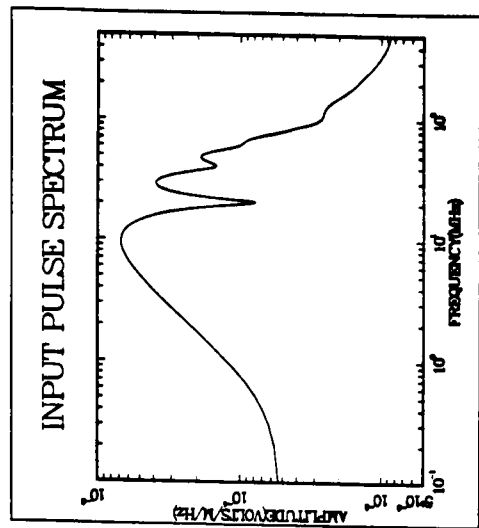
The rest of this paper will demonstrate transionospheric propagation and satellite receiver filtering effects using two of the above waveforms: the impulse function and the Supergaussian CPB with two reflections (Figures I-1 and I-4).



(a)

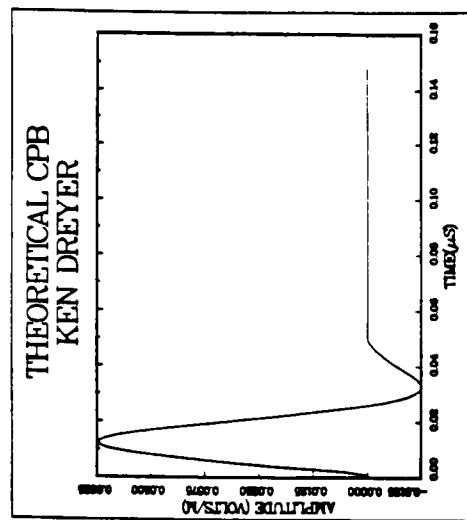


(b)

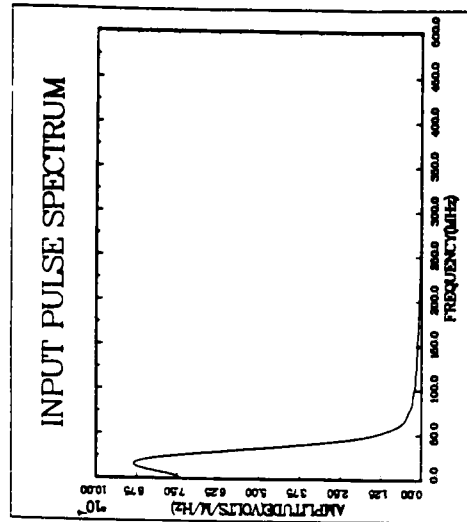


(c)

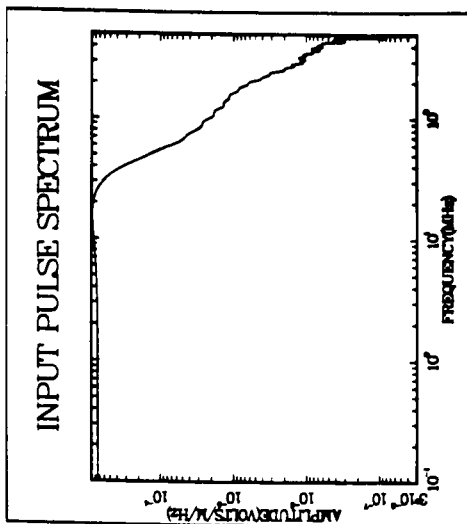
Figure 1-5. Time signature and spectra for theoretical CPB: Delta Hyperbolic Tangent.



(a)



(b)



(c)

Figure 1-6. Time signature and spectra for theoretical CPB: Dreyer.

## II. TRANSIONOSPHERIC PROPAGATION

The following equation governs wave propagation in the ionosphere based on simplifying assumptions and the derivation presented in the Appendix.

$$\partial^2 E_y / \partial^2 z = \mu_0 \epsilon_0 (1 - N_e q_e^2 / \omega^2 \epsilon m_e) \partial^2 E_y / \partial t^2. \quad (II-1)$$

This equation states the time-space variation relationship for the electric field component,  $E_y$ , of an electromagnetic field propagating in the  $z$  direction.

### Phase Function Parameters

Note that (II-1) is of the form of the general equation governing the propagation of any physical quantity,  $\Delta$ , as a wave

$$\partial^2 \Delta / \partial^2 z = 1/v_\phi^2 (\partial^2 \Delta / \partial t^2) \quad (II-2)$$

which, if the propagating wave is sinusoidal and traveling in the positive  $z$  direction, has the general solution of the form

$$\Delta = \Delta_0 \cos \omega(t - z/v_\phi) = \Delta_0 e^{j(\omega t - kz)} \quad (II-2a)$$

where

$v_\phi$  = phase velocity,

$\Delta_0$  = initial amplitude,

$t$  = time,

$k$  = wave number =  $2\pi/\lambda = \omega/v_\phi = 2\pi f/v_\phi$ , and

$z$  = distance of phase propagation from some initial location  $z_0$ .

Also,  $v_\theta = (\mu\epsilon)^{-1/2}$  is the phase velocity in a medium characterized by its permittivity,  $\mu$ , and permeability,  $\epsilon$ . The phase velocity of EMR in free space is the familiar  $c = (\mu_0\epsilon_0)^{-1/2} \approx 3 \times 10^8$  m/s. Further, the ratio  $\eta = c/v_\theta$  is the index of refraction which is medium and frequency dependant for EMR. Rewriting gives

$$v_\theta = c/\eta = \omega/k_0\eta = \omega/k; \quad k = k_0\eta, \quad \eta \leq 1 \quad (11-2b)$$

Hence, the wave number in a plasma medium is some fraction of its free space value,  $k_0$ , as determined by  $\eta$ .

Additionally, from (11-1), the quantity

$$\omega_p = (N_e q_e^2 / \epsilon m_e)^{1/2} \quad (11-3)$$

is the plasma frequency and may be viewed as the natural rate of oscillation of electrons about their neutral positions due to random electron motion in a collisionless medium in the absence of any external electromagnetic field. Reduced,  $\omega_p$  is solely a function of plasma electron density,  $N_e$ ,

$$\omega_p \approx 2\pi 8.98(N_e)^{1/2} \text{ Hz.} \quad (11-3a)$$

This expression is obtained by deriving the current density due to motion of free electrons.<sup>15,21</sup>

Using the expression for  $\omega_p$ , letting  $\Delta = E_y$ , and equating the right-hand-side coefficients of (11-1) and (11-2) gives the phase velocity of a sinusoidal frequency component propagating in the ionosphere as a

function of  $\eta$  which is itself a function of  $\omega_p$ .

$$v_\theta = (\mu_0 \epsilon_0)^{-1/2} (1 - \omega_p^2/\omega^2)^{-1/2} = c/\eta, \quad (11-4)$$

where  $\eta = (1 - \omega_p^2/\omega^2)^{1/2}$ ,

and  $k = k_0 \eta = \omega/c [1 - (\omega_p/\omega)^2]^{1/2} = \omega/v_\theta$  (11-5)

is the phase propagation factor for a wave in an ionized medium as shown in (11-2a). The constant  $k$  may be more formally obtained by substitution of the sinusoidal solution (11-2a) into the wave equation and solving for  $k$  or directly from the Appleton-Hartree formula using the same assumptions.<sup>6,17,21</sup>

### Dispersion.

When the phase velocity and hence wave number  $k$  of electromagnetic radiation propagating in a medium of  $(\mu, \epsilon)$  are dependant upon frequency, that medium is said to be dispersive. If the medium is stable and homogeneous to the extent that its ion/electron density does not vary significantly from location to location, the effects of the plasma on a passing waveform are deterministic; the effects are predictable in the sense that individual frequency components are altered in phase relative to each other in a way defined by the plasma dispersion relation (11-4). If the medium is disturbed having greatly varying electron density, the phases of component frequencies within selective regions of the waves spectrum are altered in a completely random, unpredictable manner

with respect to the normal phase alteration of frequency components outside the selective regions. The resulting time signatures after propagation through the plasma for the benign and disturbed cases may or may not look similar depending completely upon the density structure of the plasma at the time of propagation.

Plasma Frequency Dependence. In an ionized medium the propagation constant,  $k$ , is frequency dependant. Mathematically,  $k$  is generally complex,  $k = k_r + k_i$ . It may be real,  $k_r$ , imaginary,  $k_i$ , or complex dependant upon the relative values of the plasma frequency,  $\omega_p$ , and the component frequency,  $\omega$ , as seen from (II-5). Note from (II-3) that for a given electron density,  $\omega_p$  is constant and in (II-5)  $k$  depends upon  $\omega$ . For all frequencies greater than  $\omega_p$ ,  $k$  is positive and real and a wave propagating in the positive direction is described by

$$E_y = E_0 e^{j(\omega t - kz)}. \quad (II-6)$$

This wave propagates unattenuated in energy with phase velocity unique to each frequency as defined by (II-4). Hence for these frequencies,  $E_y$  will undergo dispersion. Note that for all  $\omega > \omega_p$  the phase velocity of each frequency component is greater than that for free space and as  $\omega$  decreases and approaches  $\omega_p$ , the phase velocity approaches  $\infty$  at  $\omega = \omega_p$ . This apparent divergence must exist mathematically in order for the phase of any element of a component frequency in the plasma to remain oscillating at the frequency of that

component throughout the plasma. More specifically, since  $k$  in a plasma is smaller than  $k_0$  of free space by the factor of  $\eta$ , and since  $k=2\pi/\lambda$ , the wavelength of any component increases in the plasma. Then since  $v_g = \omega/k$ , for  $\omega$  to remain constant as  $k$  decreases,  $v_g$  must increase.

Since physical detection of a single frequency is impossible, it is here appropriate to introduce the concept of group velocity. That is, the velocity of energy transport within a relatively narrow bandwidth of frequency components (wave packet or group) is defined as:

$$v_g = d\omega/dk,$$

where, since from our phase velocity expression  $\omega = (\omega_p^2 + c^2k^2)^{1/2}$ ,

$$v_g = c(1 - \omega_p^2/\omega^2)^{1/2}.$$

Now, for  $\omega > \omega_p$  we see that the group propagates at some  $v_g < c$ . At  $\omega = \omega_p$  propagation ceases since  $v_g = 0$ , and as expected, at  $\omega < \omega_p$ ,  $v_g$  is imaginary. Additionally, since any group of interest propagates at  $v_g$ , the time of arrival of the central frequency of that group may be obtained by integrating  $v_g$  over the path length traveled:

$$\text{TOAg} = \int (1/v_g) dz = \int 1/c (1 + \omega_p^2/2\omega^2) dz = \Delta z + \alpha/\omega^2.$$

Group velocity is useful for determining propagation velocity, energy transport and TOA information for narrowband signals and narrowband measurements of broadband signals.

Continuing, for all  $\omega < \omega_p$ ,  $k$  is imaginary;  $k = -jk_i$ . Hence, since  $j = (-1)^{1/2}$ ,

$$E_y = E_0 e^{j[\omega t - (-jk_i)z]} = E_0 e^{j\omega t - k_i z}. \quad (11-7)$$

The phase  $\omega t$  is now independent of the propagation distance  $z$ , the wave ceases to propagate as its amplitude decreases exponentially with time. If a hypothetical source disturbance consisting only of these frequencies occurs in the plasma, no wave will be generated, only a locally pulsing electromagnetic field which vanishes exponentially with distance from the source.<sup>15</sup> These frequencies cannot be observed through the plasma. Thus, for any electromagnetic disturbance producing these frequencies, its post-propagation signature will lack this information; all  $\omega < \omega_p$  will have been reflected and, if the plasma is collisional, possibly absorbed.

Physically this phenomena works as follows. In a collisionless ionosphere, electrons oscillate at  $\omega_p$  which is a function of the electron density,  $N_e$ . As a wave group passes, its E-field component exerts a force,  $F$ , on the electrons of charge  $q$ , in accordance with Coulomb's law,

$$F = qE,$$

where the resultant instantaneous velocity and acceleration of the electrons is dependant upon the strength of  $E$ .

With the electrons oscillating at some constant value of  $\omega_p$ , for all  $\omega$  less than  $\omega_p$  the passing E-field reverses direction slowly enough to

modify this rate of oscillation, producing electron currents which re-radiate the passing field's energy in such a way as to bend or reflect these wave components away from increasing electron density. For all  $\omega \geq \omega_p$ , the inertia of the oscillating electrons prevents them from reacting to the rapidly changing E-field of these frequencies which propagate essentially unimpeded by the plasma save for refractive path length effects (dispersion) caused by the index of refraction for those  $\omega$  close to  $\omega_p$ . For  $\omega \geq \omega_p$ , the electron's velocity and acceleration are out of phase with the driving force due to their inertia and no energy is transferred in altering the motion of the free electrons.

Total Electron Content. Apparently, from (II-3), if  $N_e$  is known for any given region of the ionosphere then  $\omega_p$  may be calculated for that region. However, the ionosphere has large scale variations in its electron distribution which vary with altitude, time-of-day, geographical and seasonal changes.<sup>12,17</sup> With respect to amplitude variation, the  $N_e$  may range from  $10^8$  to  $10^{12}$  electrons/m<sup>3</sup> with peak values occurring at altitudes close to 250 km or so. The plasma frequency therefore varies from a minimum of 9 kHz in the lower ionosphere to around 3 to 10 MHz at altitudes of peak density in the ionospheric F-region. From (II-4), due to the dependance of the index of refraction on  $\omega_p$  and therefore  $N_e$ , the phase velocity,  $c/\eta$ , of passing wave components may vary with altitude along the

propagation path. Dispersion modeling with this altitude variance is achieved by representing  $N_e$  variation as the line or path integral of  $N_e$  along the propagating path of the frequency components. This integral is naturally derived as a result of integrating the group velocity over the path of propagation to obtain the group time of arrival. This integral is typically on the order of  $10^{13}$  electrons/cm<sup>2</sup> and represents an effective electron density cross section or total electron content (TEC). It is in essence the effective electron density a passing wave would experience and be quantitatively modified by if the electrons over some altitude range were condensed to a thin plane layer.

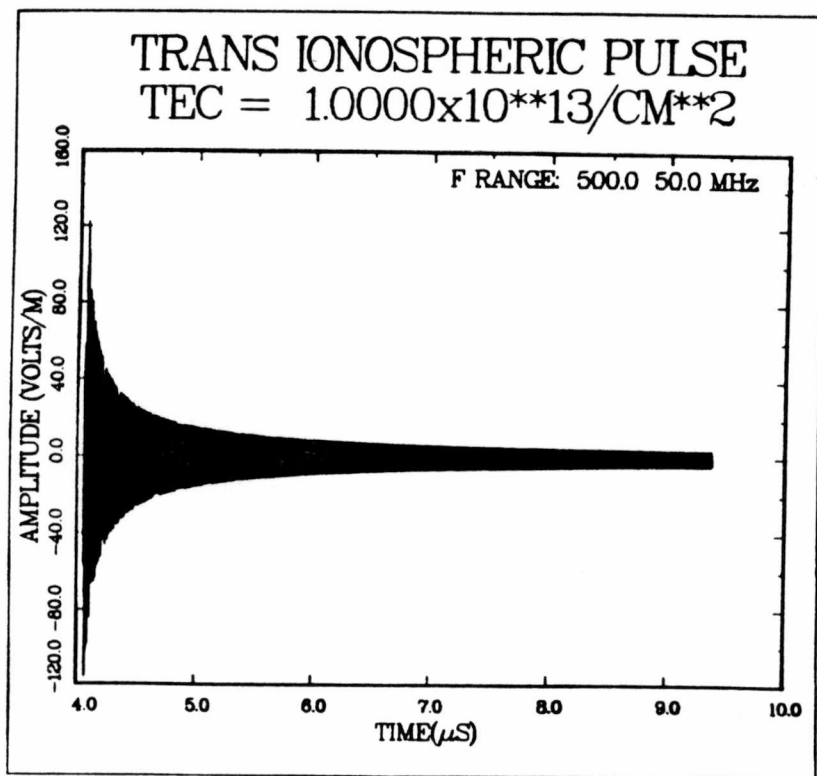
Deterministic Dispersion. In a deterministic medium, the frequency spectrum undergoes dispersion where each component frequency of the spectrum is modified in phase and propagates at a unique phase velocity inversely proportional to that frequency; the higher frequencies propagate at lower phase velocities than the lower frequencies. This effect is evident from (II-4). If all frequency components propagate in the same direction but with minutely varying path lengths the relative effects of phase change in each component causes the destructive or constructive interference of all frequencies over time to produce a pulse extended and distorted from its pre-dispersed form. Hence, an original signal of finite short duration in time would appear greatly expanded in time after passing through the dispersive medium, its distortion in time being a function

of the relative deterministic phase change of each frequency component. The envelope of this dispersed pulse is that of the group or wave train produced by the summation of all frequencies and travels at some average group velocity less than the free space speed of light.

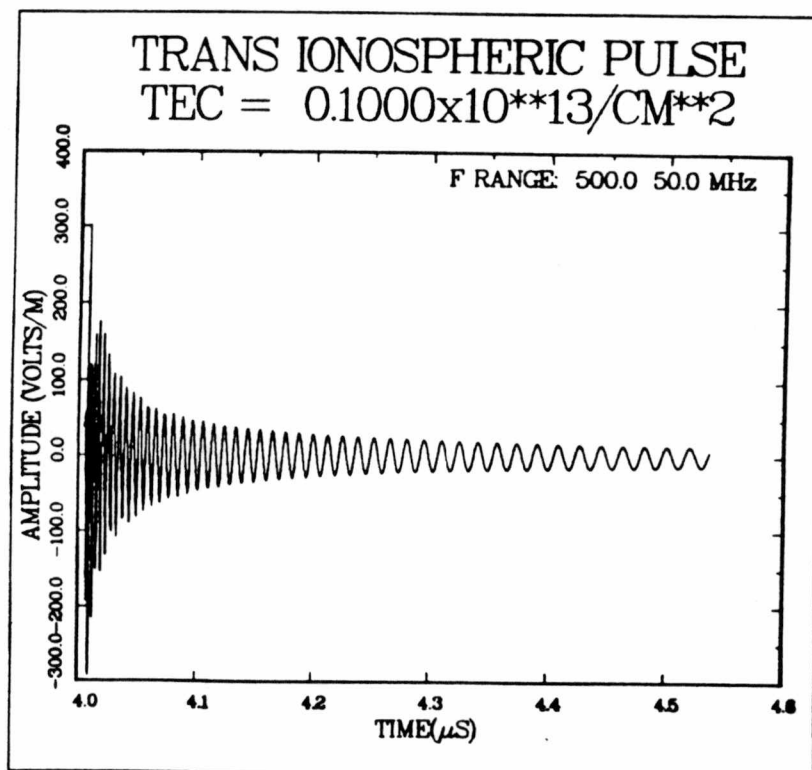
Graphically, dispersion effects may best be observed by analyzing the transionospheric pulse of the delta function. Figure II-1(a) and (b) show examples of a dispersed impulse (delta function) input pulse delayed in time an arbitrary 4  $\mu\text{sec}$  for deterministic ionospheric TEC's of  $1.0 \times 10^{13}$  and  $0.1 \times 10^{13}$  electrons/cm<sup>2</sup> respectively. Shown for both is the 500-50 MHz region of the E field time behavior which is essentially a snapshot of an otherwise infinitely long post-propagation impulse time signature.

In Figure II-1(a) the 500 MHz component arrives at  $t = 4.054 \mu\text{sec}$ , a dispersion delay of .054  $\mu\text{sec}$ , while the 50 MHz component arrives much later in time at 9.37  $\mu\text{sec}$ . As expected, the lower frequencies are delayed far greater in time than the higher frequencies. Since an impulse' spectrum consists of all frequencies, the extremely high frequencies (gamma, X-ray, etc.) arrive virtually undelayed by the plasma at near  $t_0 = 4 \mu\text{sec}$ , the time of the disturbance. The extremely low frequencies arrive far off the shown time scale at near  $t = \infty$ .

Similarly, Figure II-1(b) represents the same 500-50 MHz frequency window but for a TEC of 0.1. As expected, for a lower TEC the plasma effect is less pronounced with both frequencies and all



(a)



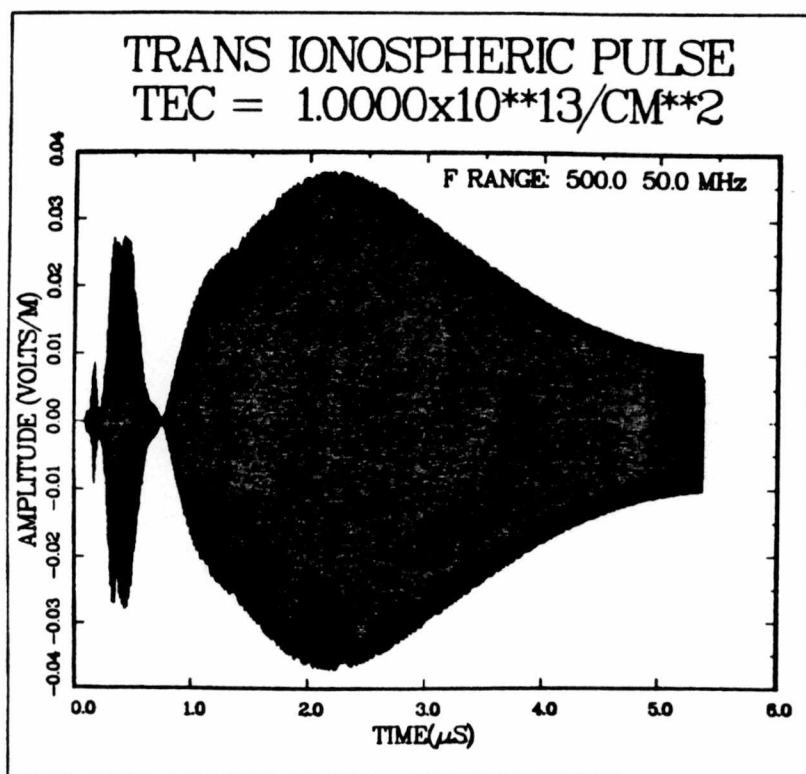
(b)

Figure II-1. 500-50 MHz window of dispersed impulse function for TEC of  
(a)  $1.0 \times 10^{13} / \text{cm}^2$  and (b)  $0.1 \times 10^{13} / \text{cm}^2$ .

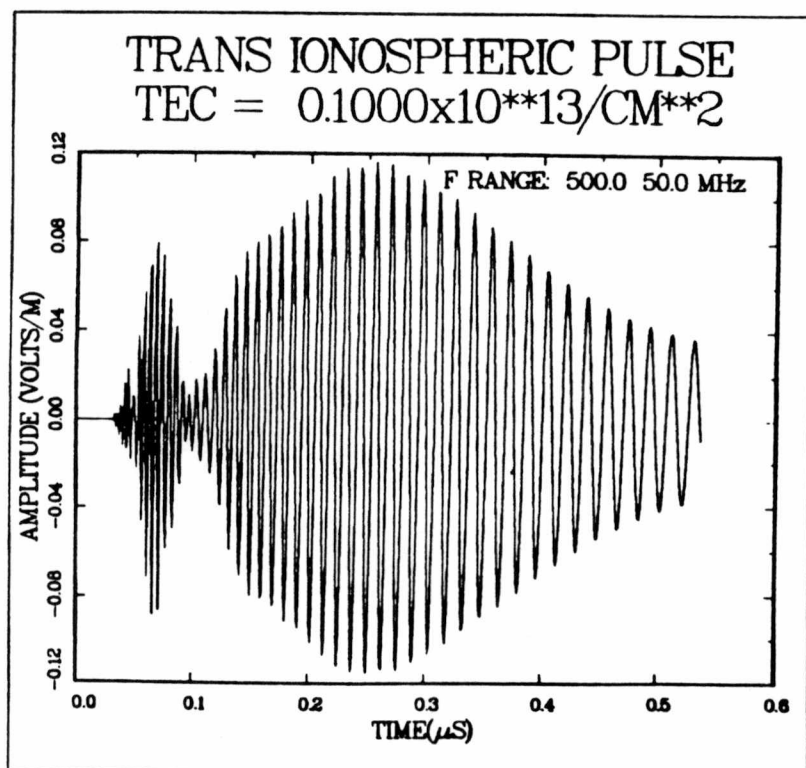
those in between arriving closer in time to the original input  $t_0$ . In this case, the 500 MHz component arrives at 4.0054  $\mu\text{sec}$  or  $t_0 + 5.4$  *nanoseconds* while the 50 MHz component arrives at 4.537 *microseconds*, a delay of 0.537 *microseconds*

Several additional features need to be emphasized. In both (a) and (b), the higher frequencies arrive first, the lower last. Note that although the frequency spectrum shown in Figure I-1(b) shows that all components have the same energy and hence same amplitude at  $t_0$ , dispersion has delayed in time and reduced in amplitude the lower frequency components; the energy of these lower frequencies arriving over a much longer time period. Additionally, comparing the 50 MHz component in each graph, one sees that the amplitude reduction for a TEC of 0.1 is less severe than that for a TEC of 1.0. Thus TEC has a profound effect on the ability of a given detection system to measure the post-propagation power signature at a given frequency. Finally, free space time of propagation is not accounted for in this modeling process, only group delays due to ionospheric dispersion. Also, ionospheric cutoff of low frequency information (reflection of frequencies below  $\approx 20$  MHz) is not modeled. Hence it is unrealistic to window the TIS within these frequency ranges.

Figure II-2(a) and (b) show the transionospheric time signature for the generic electromagnetic pulse (EMP) generated by a charged particle beam (CPB) shown in Figures I-4(a) for the same TEC's of 1.0 and 0.1 respectively. The main pulse is centered at  $\approx t_0 = .03 \mu\text{sec}$



(a)



(b)

Figure II-2. 500-50 MHz window of dispersed Supergaussian CPB for TEC of (a)  $1.0 \times 10^{13} / \text{cm}^2$  and (b)  $0.1 \times 10^{13} / \text{cm}^2$ .

followed by its composite building and ground reflections. Its spectrum is shown in Figure I-4(b) and (c). Similar features may be noted as those for the delta function in that dispersion time delays for all frequencies are identical and amplitude variations occur. However, these variations are more complex due to the nature of the pulse frequency spectrum.

Pulse Amplitude/Energy. To consider a pulse's dispersed amplitude over time, it is instructive, at least intuitively, to consider that the Fourier representation of a pulse waveform consists of an infinite number of frequency components combining constructively to produce that pulse. Mathematically, each frequency sinusoid exists for all time and the pulse is produced at a particular time because of the superposition of the set of all frequencies as defined by their unique phase and amplitude. Dispersion of the pulse changes the relative phase and hence phase velocity of all component frequencies such that the lower frequencies arrive at a detector much later in time than the higher. Now, it was shown above that in a collisionless plasma, little energy loss occurs for  $\omega \gg \omega_p \gg \nu_c$  (where  $\nu_c$  is the electron collision frequency with ions and neutrals). Thus, the energy of a group of these particular components must be the same after propagation as before in accordance with conservation of energy. For these frequencies, then, phase changes but total energy does not during propagation. Effectively, the energy within lower frequency groups is dispersed or spread over a longer time frame than

the higher so that these groups are reduced in amplitude to a greater degree relative to the higher frequency groups. It is important to understand that individual frequencies do not change in amplitude, only groups of frequencies due to phase changes of individual components. Thus, when a particular group of frequency components arrives in time depends upon the relative phase alteration of its components with respect to those of other groups and upon the constructive interference of all individually phased components to produce that group's signature. The change in amplitude of the same group depends primarily upon the severity of dispersion (TEC) and, with respect to measurement, additionally on the relative amplitudes of the neighboring frequencies over some narrow bandwidth about the frequency of interest. Because each frequency component is altered differently, superposition in time results in amplitude loss or gain depending upon how the superposed components interfere within a narrow bandwidth: if a very wide or very narrow bandwidth is used for observation, the more uncertain the amplitude measurement is, this uncertainty being especially dependant upon background noise level. It is important to realize that both time delay and amplitude loss ultimately are results of phase alteration. Observation (filtering) problems associated with power detection and the effects of optimal filtering are discussed in Section IV.

Random Medium (scintillations). When the plasma medium exhibits a random irregularly structured electron density distribution, that

medium is said to be turbulent or disturbed.<sup>14</sup> Whether this irregular structure is natural or manmade, the effect on a passing waveform is to introduce random phase fluctuations to its contributing sinusoids resulting in random amplitude fluctuations in the composed post-propagation time signature. Comparing propagation through a benign or quiet medium, the time signatures of both cases may or may not look similar depending upon the degree of structuring in the medium. These amplitude fluctuations are termed scintillations.

Diffraction effects on propagating waveforms through the ionosphere are probably most pronounced when the ionosphere is disturbed by the EMP from high-altitude nuclear bursts. Propagation of the EMP disrupts the benign electron density distribution creating plasma striations of relatively high and low electron densities aligned along geomagnetic field lines. This is represented in the cartoon form of Figure II-3. This striation approach to modeling a disturbed ionosphere is a good approximation since all such irregular electron density pockets tend to align and elongate themselves along the geomagnetic field lines. Such striations have been observed to develop following barium releases in the ionosphere.<sup>14</sup> Theoretical effects of such a cloud are shown in Figure II-4. Figure II-4(a) shows the relative effects of a deterministic Gaussian-shaped cloud. Figure II-4(b) shows those of a striated cloud. Both effects are shown on a propagating 50 km plane wave front with a carrier frequency of approximately 110-140 MHz and envelope as shown for the undisturbed portions of the deterministic case. In both figures, the

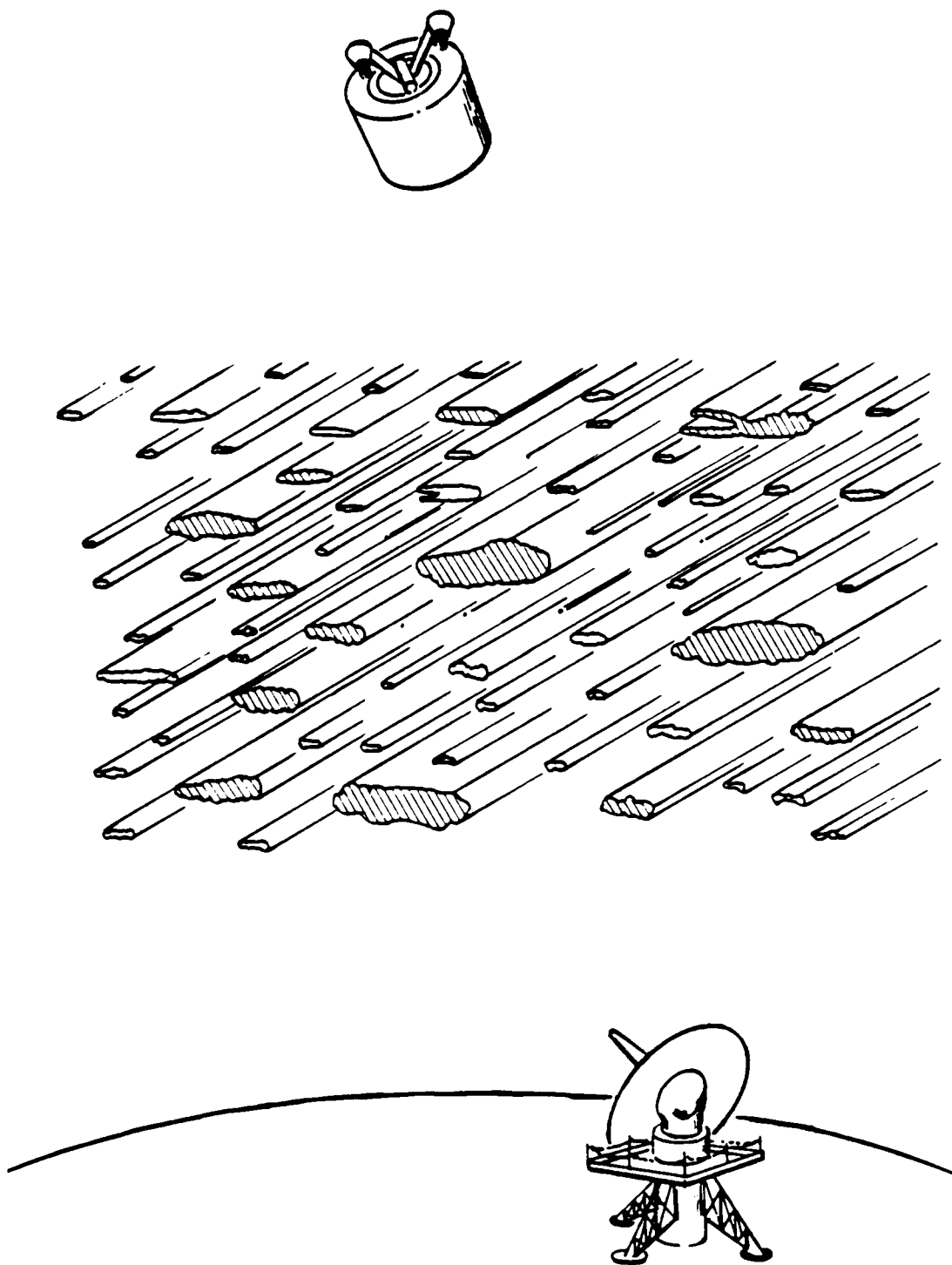
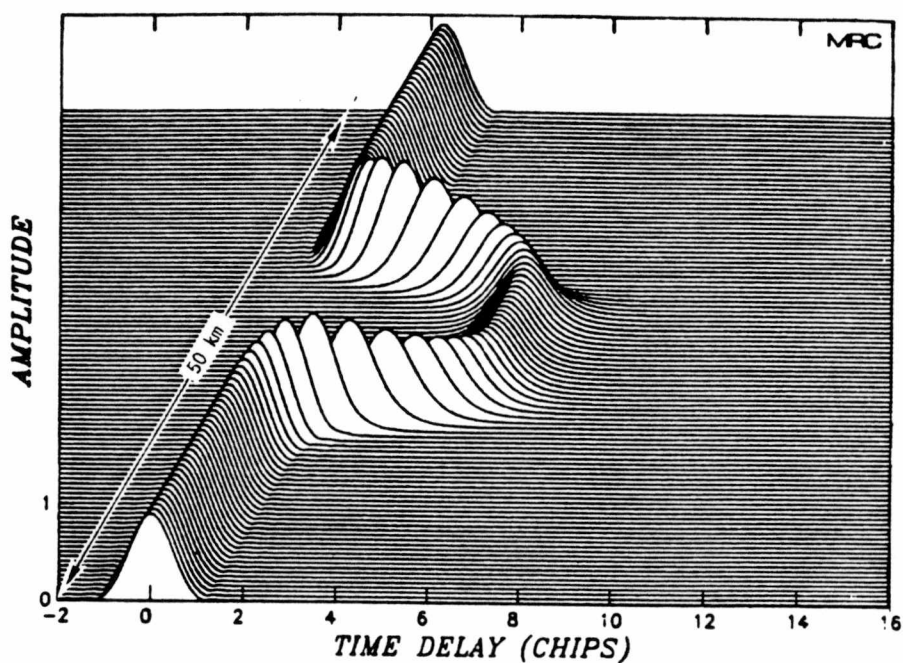
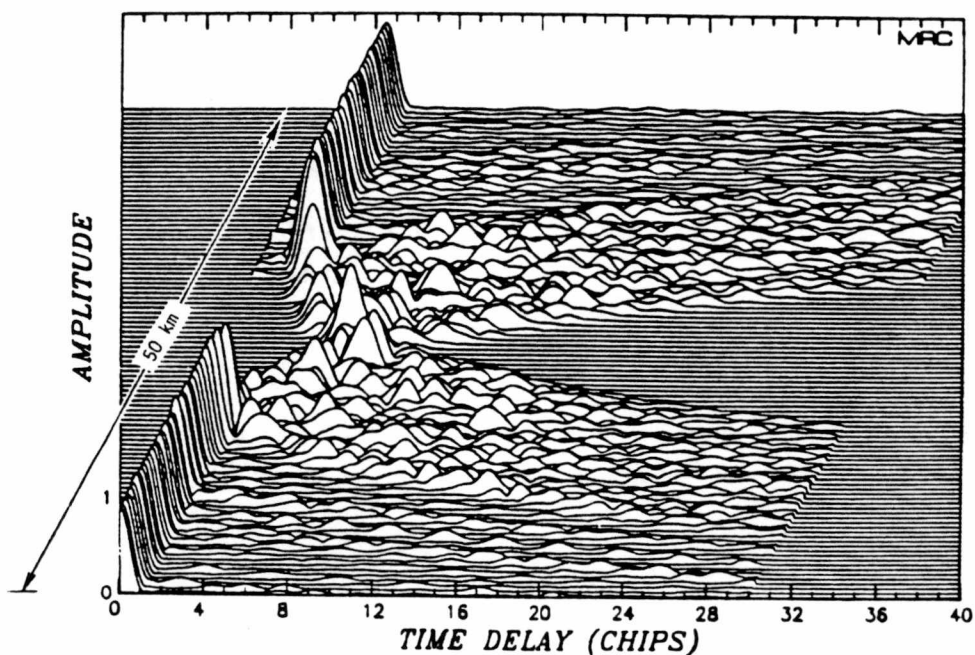


Figure II-3. Propagation of signals through a disturbed transionospheric communications channel.



(a). Envelope of time-domain signal after propagation through deterministic barium cloud.



(b). Envelope of received time-domain signal after propagation through an ionized, striated barium cloud.

Figure II-4. Envelopes for deterministic and striated barium clouds.

wave travels to the left and intercepts the cloud at its center.

Figure II-4(a) exhibits the predictable time delay associated with deterministic dispersive phase alteration. The time scale is in chips where one chip  $\approx$  97 nanoseconds. Portions of the wave intercepting the higher electron density at the center of the cloud show the greatest dispersive time delay. Note that the wave envelopes do not change drastically; the basic form of the pulse is still recognizable. Some effect is noted in amplitude gain for the wavelets intercepting the outer portion of the cloud due to focusing of some frequencies of the inner sections to the outer edge. Beyond this example, however, focusing is not addressed in this paper. Also, significant dispersive spreading of the original pulse of amplitude one occurs for the center section where the highest electron density is encountered.

Figure II-4(b) shows the effects of a disturbed ionosphere for the same propagating plane wavefront. This dispersion pattern is characteristic of the effects of the same broad band pulse used in Figure II-4(a) but propagated through an established barium cloud in which the electron density is now striated. Because of the randomness in the number, density, and dimensions of each striation encountered by each wavelet along its propagation path, the components of each wavelet are modified in phase differently than for any other wavelet along the wavefront. Hence, the composed time signature after propagation for each pulse will differ from any other; each time signature is delayed in time and modified in amplitude differently from any other due to the unique phase delay experienced

by each of its component frequencies. Each in essence is a different "realization" of scintillations effects based on the unique structure encountered by each wavelet. The purpose of Figure II-4(b) is to show the infinite variety of random time signatures possible after transionospheric propagation. In the modeling process, these different realizations are quantified by specifying the frequency coherent bandwidth, or  $f_{coh}$ .

Frequency Coherent Bandwidth. The quiet and disturbed ionosphere may be discussed in terms of being frequency coherent and frequency non-coherent respectively. Frequency coherence refers to the extent that groups of component frequencies propagate unchanged with the same relative phase. Highly coherent or frequency non-selective media allow deterministic phase alteration and coherent propagation of all component frequencies; highly incoherent or frequency selective media cause random phase alterations throughout the medium along a particular propagation path and only allow certain bandwidths of information to propagate through in phase. The so-called coherent bandwidth  $f_{coh}$  is uniquely defined based on the particular structure of the medium along any given path. For the benign ionosphere,  $f_{coh}$  is effectively infinity. For the disturbed ionosphere, each propagation path may be characterized by a unique  $f_{coh}$ , as is apparent in Figure II-4(b). If any two paths are characterized by the same  $f_{coh}$ , portions of the wave propagating along

these two paths may differ in their time signature appearances for two different realizations of path structure for that  $f_{coh}$ .

The following are results from incorporating a scintillations code named CIRF into TIPC.<sup>22</sup> This model introduces phase shifts to frequency components over narrow bandwidths of a propagating signal as parameterized by  $f_{coh}$ . Figure II-5 shows the effects of a disturbed ionosphere on the propagated delta function for a frequency range of 500-70 MHz. Figure II-5(a) is that for a deterministic ionosphere of TEC of 1.0. Figures (b) through (d) are for disturbed ionospheres of TEC = 1.0 additionally characterized by  $f_{coh}$  of 20.0, 10.0 and 0.1 MHz respectively. Amplitudes of the scintillated pulses have not been normalized so those for (b), (c) and (d) are relative and as shown have no relationship to the amplitudes in (a). Note that the time range of the pulse in all cases appears to be identical. Actually this is an effect of numerically windowing the entire pulse based on deterministic time of arrivals for the 500 and 70 MHz limits. Additional dispersion occurs in the disturbed cases which is approximately equal to  $1/f_{coh}$ .<sup>14</sup> Thus, the 20 MHz case has an additional time delay of  $\approx 50$  nanoseconds beyond the end time shown for 70 MHz while the 0.1 MHz case has an additional delay of  $10 \mu sec$  which is not accounted for in the windowing process. In other words, portions of the wave arrive some  $10 \mu sec$  after the 70 MHz time of arrival shown in Figure II-5(d). The frequency content of this extended information is unknown and indeterminable. Note also the

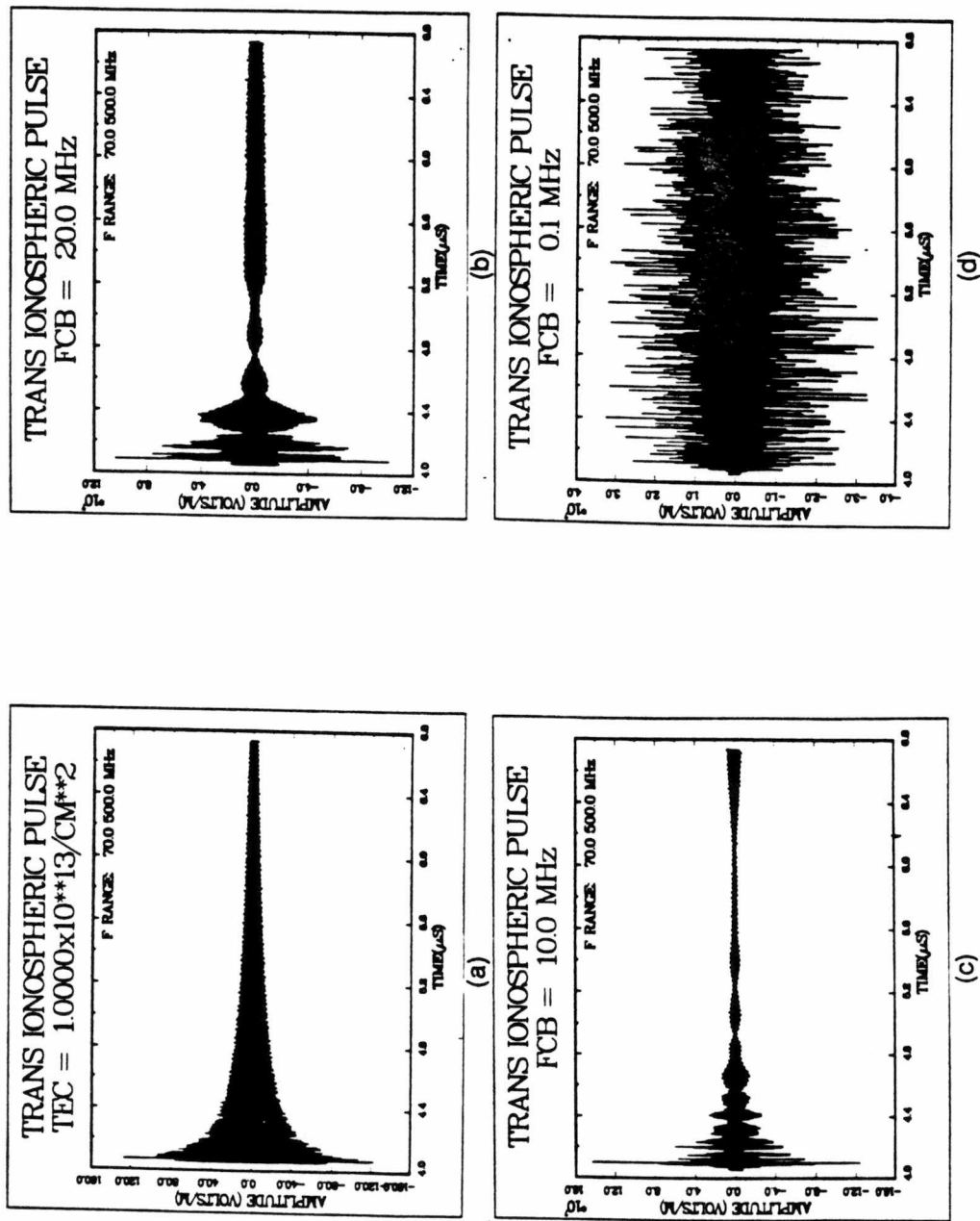
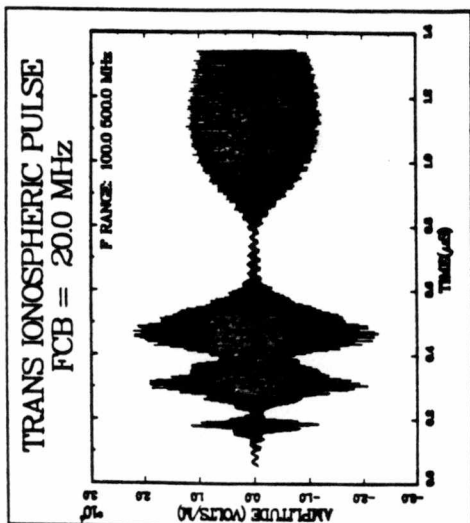


Figure 11-5. 500-70 MHz window of impulse function propagated through disturbed ionospheres of  $\text{TEC} = 1.0 \times 10^{13} \text{ cm}^{-2}$ .

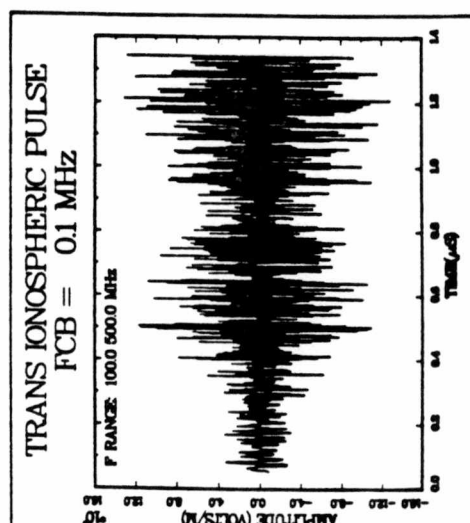
lobed structure of the envelope. Each lobe is roughly equivalent to  $f_{coh}$  in time-width. That is, the lobes are those  $f_{coh}$  frequency ranges in which the components arrive in much the same relative phase as they do in the benign case (a). Regions between these lobes are those where random phase fluctuations have caused severe interference. Even with the distortion the frequency coherent regions roughly maintain the same envelope as the same regions of the benign case as is evident when overlapping the envelope of (a) with (b) or (c). The typical severe interference evident in (d) for very low  $f_{coh}$  prevents envelope comparison. Figure II-6 shows the similar effects for the CPB pulse for a frequency window of 500-100 MHz. Satellite receiver filtering is greatly complicated by such random structure and is discussed in Section IV.

#### Background.

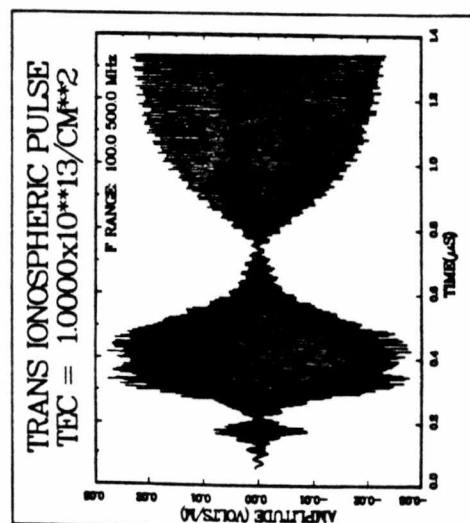
Two types of background should be considered in the propagation and detection of VHF waveforms. Natural background is that due primarily to galactic noise and broadband transients such as lightning. Manmade sources include narrow band interference such as TV, radio, etc. To underscore the significant effect of lightning strokes consider Figure II-7.<sup>19</sup> Without going into details presented in the source for this figure, this entire flash is  $\approx 936$  milliseconds in length with individual short duration (broad bandwidth) return strokes on the order of a few nanoseconds. Note that this is roughly



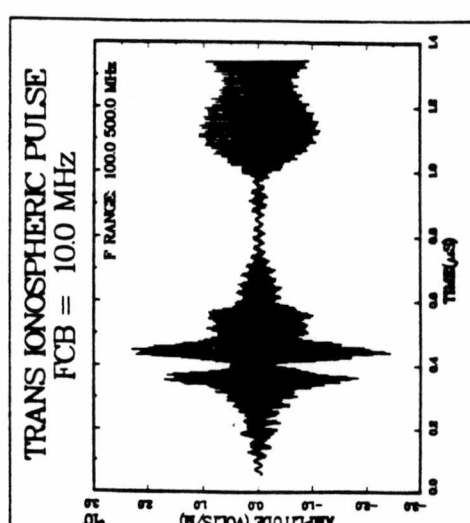
(a)



(b)

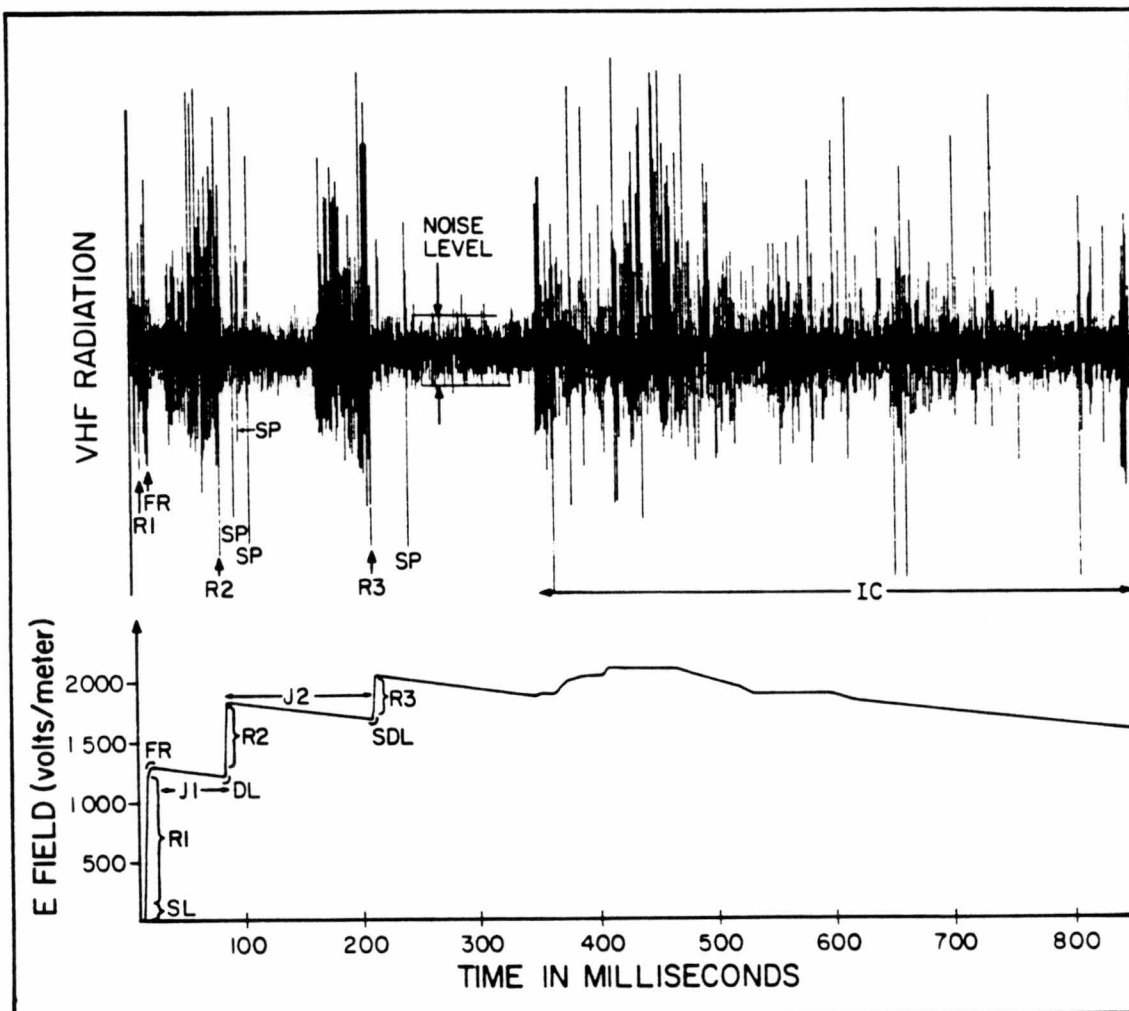


(c)



(d)

Figure II-6. 500-100 MHz window of Supergaussian CPB propagated through disturbed ionospheres of  $TEC = 1.0 \times 10^{13} / \text{cm}^2$ .



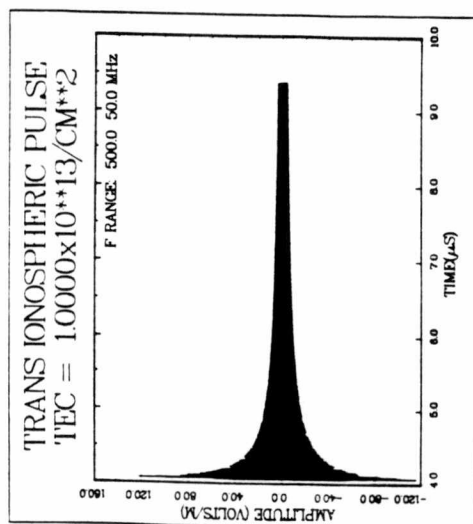
Simultaneous records of the amplitude of the VHF envelope and the electric field intensity at 9 and 11 km, respectively. The total flash lasted 936 ms, of which we show the first 837 ms. R1, R2, and R3 represent the first, second, and third return strokes, respectively; SL is the stepped leader before R1, DL is the dart leader before R2, and SDL is the stepped dart leader before R3. J1 and J2 are the interstroke processes. FR is the activity following the first return stroke; SP's are the solitary pulses. The electric field decays exponentially with a 5 s time constant owing to the antenna system used. Because the VHF recorder had a 400 Hz lower cutoff frequency, the VHF radiation out of the recorder has no frequency components below about 400 Hz and hence is roughly symmetrical about a centerline through the radiation.

Figure II-7. Example of transient broadband background. Lightning stroke recorded at Kennedy Space Center. (Ruston, et al.)

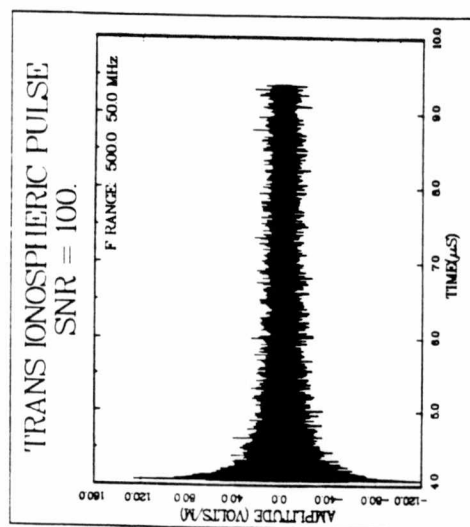
the same duration of the pulses modeled in this study. "IC" in the figure refers to intercloud discharge.

White Noise. Although the existing code does not yet include the capability to add transient background to a propagating signal, it does have the capability to model the background noise similar to that in Figure II-7. White noise may be added to the signal before propagation to study the effects on the propagated waveform as well as on numerical modeling. White noise signal to noise ratio (SNR) is numerically modeled at the ground as the ratio of input signal amplitude at a given frequency over a given bandwidth to white noise amplitude at the same frequency and bandwidth.

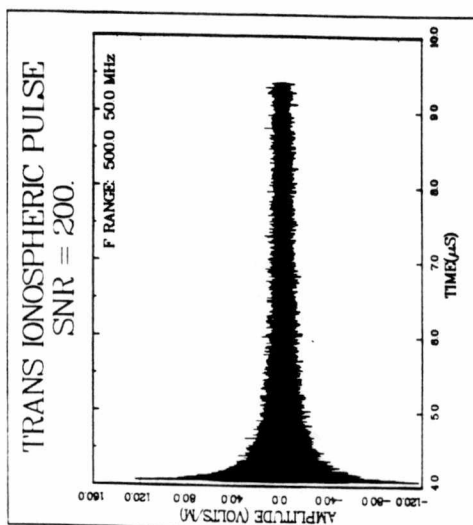
Figure II-8 shows the effects of a noisy delta function propagated through a deterministic ionosphere of  $TEC = 1.0$ . Figure II-8(a) is the input signal with an SNR of 100. Figures (b), (c) and (d) are the 500-50 MHz portion of the transionospheric signal for input SNR's of 200, 100, and 20 respectively. Note that in (b) and (c), the high frequency end is relatively less affected by the noise than the low frequency end. Due to the nature of white noise, dispersion does not affect the noise amplitude from its input value. Noise is produced by adding to each point of the signal array a spike of random amplitude and sign. Each spike is essentially a delta function which when dispersed and summed with all other dispersed spikes produces no net change in the rms noise value. The pulse however, being a single occurrence, reduces in amplitude at the low frequency end. Thus lower



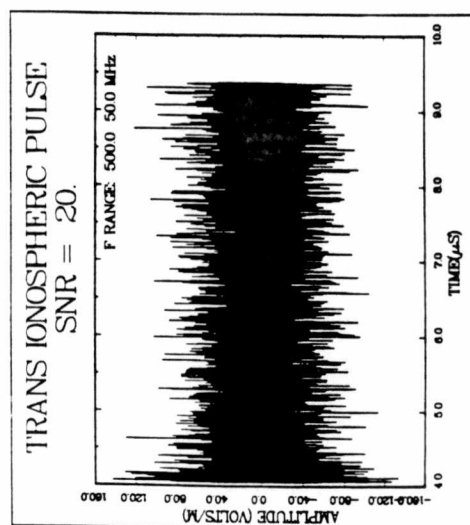
(a)



(c)



(b)

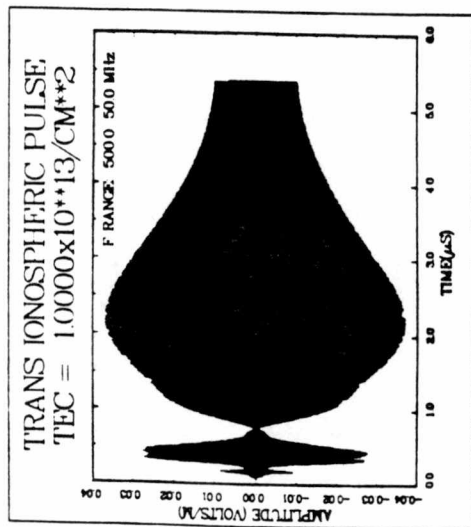


(d)

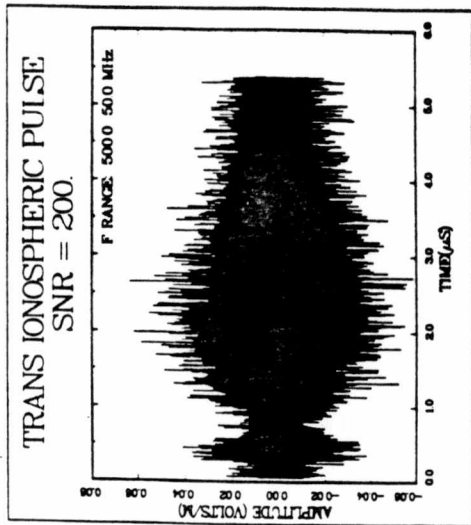
Figure II-8. Noise effects on propagation of delta function for TEC of  $1.0 \times 10^{13} \text{ cm}^{-2}$ .

frequency information is masked by the noise to a greater degree than the higher. Note however, that for the SNR of 20 case in (d) that the noise level is high enough to destroy the pulse regardless of frequency.

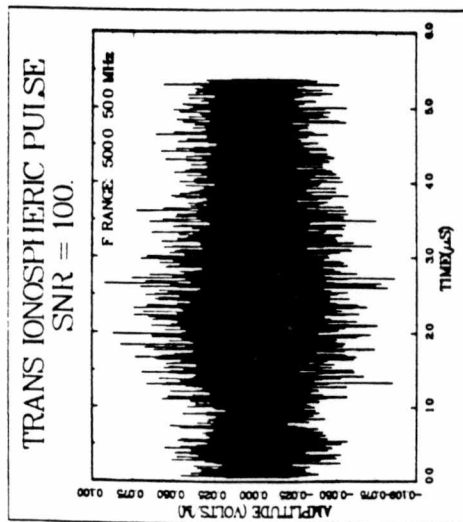
Figure 11-9 shows noise effects using the same SNR for the CPB. Due to the nature of the CPB pulse, a SNR of only 50 is needed to completely obliterate characteristics of the noiseless TIS.



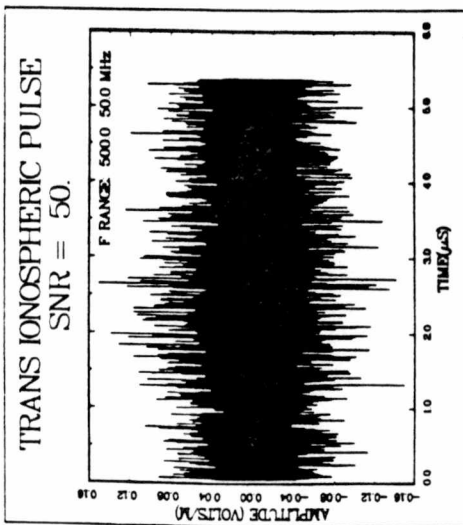
(a)



(b)



(c)



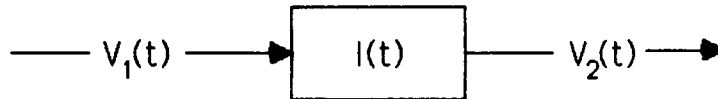
(d)

Figure II-9. Noise effects on propagation of theoretical CPB for TEC of  $1.0 \times 10^{13} \text{ cm}^{-2}$ .

### III. PROPAGATION MODELING

#### Ionosphere as Linear System.

The ionosphere, under the simplifying assumptions presented in the Appendix, may be mathematically modeled as a linear, causal, time-invariant system. Schematically, see below.



Here the input  $V_1(t)$  is operated on by the system impulse response function,  $I(t)$ , resulting in the output,  $V_2(t)$ . System linearity dictates that the output be a linear variation of the input. System time invariance dictates that a time delay in the input produces an equal time delay in the output. Causality dictates that the system produces no response prior to the time of the input.

#### Fourier Analysis.

The reciprocal relationship between frequency and time allows transformation of a waveform as a function of time to one in the frequency domain,

$$S(\omega) = \int V(t)e^{-j\omega t}dt, \quad (\text{III-1})$$

and conversely, from the frequency domain to the time domain,

$$V(t) = 1/2\pi \int S(\omega)e^{j\omega t}d\omega, \quad (\text{III-2})$$

where  $S(\omega)$  is the waveform frequency spectrum.

If a sinusoidal waveform is applied as an input,

$$V_1(t) = \Delta(\cos\omega t + j\sin\omega t) = \Delta e^{j\omega t}, \quad (\text{III-3})$$

then the output is also sinusoidal, of the same frequency, but possibly differing in amplitude and phase as determined completely by the characteristics of the system;

$$V_2(t) = \beta[\cos(\omega t + \theta) + j\sin(\omega t + \theta)] = \beta e^{j(\omega t + \theta)}. \quad (\text{III-4})$$

Mathematically, these characteristics are expressed as the system transfer function as a function of frequency,  $T(\omega)$ , or as its impulse response function as a function of time,  $I(t)$ , both of which are transforms of the other,

$$T(\omega) = \int I(t) e^{-j\omega t} dt \quad (\text{III-5})$$

#### Ionospheric Transfer Function.

From Section II, a propagating waveform was shown to be modified (dispersed) due to the variation of the wave number,  $k$ , as a function of plasma frequency and therefore of ionospheric electron density. The component frequencies of the propagating waveform are modified in phase as a function of frequency as determined by  $k$ . Due to this frequency dependence, modeling of the dispersion process may be conveniently done in the frequency domain. If the transfer function is known, the input  $V_1(t)$  may be decomposed into the spectrum  $S_1(\omega)$  of its component frequencies by applying (III-1). Each frequency may then be operated on by  $T(\omega)$  and modified in phase and/or amplitude as a function of the same frequency. The summation of the results of this operation being performed for all frequencies is the output

waveform as a function of frequency,  $S_2(\omega)$ . This may then be transformed into its time signature by application of (III-2). Mathematically, the frequency domain operation with  $T(\omega)$  is performed by multiplication of the input spectrum by  $T(\omega)$  to obtain the output spectrum,

$$S_2(\omega) = S_1(\omega)T(\omega), \quad (\text{III-6})$$

where, from comparing (III-3) and (III-4),  $T(\omega)$  is of the form

$$T(\omega) = \beta/\Delta e^{j\theta(\omega)}. \quad (\text{III-7})$$

Considering only  $\omega > \omega_p$ , the waveform frequencies are essentially unaffected in total energy by the plasma during propagation,  $\beta = \Delta$  and the transfer function for these frequencies is solely a function of the phase change variable,  $\theta(\omega)$ .

$$T(\omega) = e^{j\theta(\omega)}, \quad (\text{III-8})$$

where  $\theta(\omega)$  is the path integral of the correction term of (II-5),

$$k = k_0 \eta = \omega/c [1 - (\omega_p/\omega)^2]^{1/2}. \quad (\text{II-5})$$

That is, if expressed as the high order terms of a Taylor expansion  $k$  may be written as

$$k \approx \omega/c [1 - 1/2(\omega_p^2/\omega^2)] = \omega/c - \omega_p^2/2\omega c = k_0 + \theta'(\omega),$$

$$\text{where} \quad \theta(\omega) \approx \int \theta'(\omega) dz. \quad (\text{III-9})$$

### Signal Phase Modification.

From the Appendix, the source waveform at some initial position

$z_0$  below the ionosphere is

$$E_y(z_0, t) = E_0 e^{-hz_0} e^{j(\omega t - kz_0)} = E_0 e^{-hz_0} e^{j\omega(t - t_0)}. \quad (\text{III-10})$$

By definition, its Fourier transform is

$$E_y(z_0, \omega) = \int E_y(z_0, t) e^{-j\omega t} dt. \quad (\text{III-11})$$

Using (III-6), for some altitude  $z_1$  above the ionosphere, where

$$\Delta Z = z_1 - z_0,$$

$$E_y(z_1, \omega) = E_y(z_0, \omega) T(\omega), \quad (\text{III-12})$$

$$= T(\omega) \int E_y(z_0, t) e^{-j\omega t} dt, \quad (12a)$$

$$= e^{j\theta(\omega)} \int E_0 e^{-h\Delta Z} e^{j(\omega t - \int k dz)} e^{-j\omega t} dt, \quad (12b)$$

$$= A \int e^{j\omega t} e^{-jk\Delta Z} e^{-j\omega t} e^{j\theta(\omega)} dt, \quad (12c)$$

$$= A \int e^{-jk\Delta Z} e^{j\theta(\omega)} dt, \quad (12d)$$

$$= A \int e^{-j[k\Delta Z + \theta(\omega)]} dt. \quad (12e)$$

Now, transformation back into the time domain gives

$$E_y(z_1, t) = 1/2\pi \int e^{j\omega t} d\omega \int A e^{-j[k\Delta Z + \theta(\omega)]} dt. \quad (\text{III-13})$$

where  $A = E_0 e^{-h\Delta Z}$  and  $h\Delta Z$  is an amplitude decay factor defined in the Appendix. From (III-9) the phase correction term of  $k_0$  is  $\omega_p$  and  $N_e$  dependant and when integrated over some finite altitude range introduces the TEC concept discussed in Section II:

$$\int_{z_0}^{z_1} \phi'(\omega) dz = -(q_0^2 / 2\omega \epsilon_0 m_e c) \int_{z_0}^{z_1} N_0 dz = -1/\omega Q(\text{TEC}) \quad (\text{III-14})$$

where  $Q$  is constant. Using this result, (III-13) may be rewritten as

$$\begin{aligned} E_y(z_1, t) &= 1/2\pi \int d\omega \int A e^{-j[\omega t - \{k\Delta z + \phi(\omega)\}]} dt, \quad (\text{III-15}) \\ &= 1/2\pi \int d\omega \int A e^{-j\omega[t - \{(k/\omega)\Delta z - 1/\omega^2(Q \int N_0 dz)\}]} dt, \\ &= 1/2\pi \int d\omega \int A e^{-j\omega[t - \{(k/\omega)\Delta z - Q(\text{TEC})/\omega^2\}]} dt, \end{aligned}$$

where the quantity within [ ] is the time of arrival (TOA) of each individual frequency component as dispersed in time by the ionosphere. It is this TOA information which must be recovered for each frequency in order to reconstruct the input waveform after propagation through the ionosphere. Methods and problems dealing with this recovery are treated in Section IV.

#### IV. MODELING OF SATELLITE DETECTION

The previous sections dealt with the physics and modeling of propagating a wideband pulse through the ionosphere. This section deals with the detection-location-discrimination problem associated with a space borne receiver. More specifically, detection is here crudely defined to be a simple receiver filtering process which isolates information about a frequency of interest. Location of a VHF source from space is necessarily a ranging problem possibly involving a constellation of many receivers using a triangulation for precision. Finally, discrimination involves the capability to recognize, either by analysis of telemetry on the ground or aboard the platform, VHF portions of characteristic source waveforms or other peculiarities thereof.

This section first discusses the receiver filtering function. It then proceeds with a time of arrival (TOA) analysis useful in the location problem and finally deals with discrimination using TOA information and the technique of quadrature filtering.

##### Receiver Filtering.

Filtering at the receiver allows determination of the time of arrival (TOA), power at the receiver and other useful parameters defining the filtered information of chosen discrete VHF frequency components of the transionospheric waveform. The TOA information may be processed for range determination and in a limited way, signal

reconstruction as discussed in the following sub-section.

As similarly applied to ionospheric propagation, the concept of a causal, time-invariant, linear system may be used to model the receiver filtering process as shown Figure IV-1.

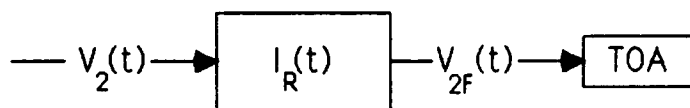


**Figure IV-1. Linear Model of Satellite Receiver.**

Here,  $V_2(t)$  is the transionspheric signal (TIS), the output of Section II propagation modeling,  $I_R(t)$  is the impulse response function or characteristic waveform of the receiver filter, and  $V_{2F}(t)$  is the filtered TIS. In the overall context of signal propagation and detection, and using the basic Fourier analysis concepts of Section III, this series of linear systems appears as in Figure IV-2.

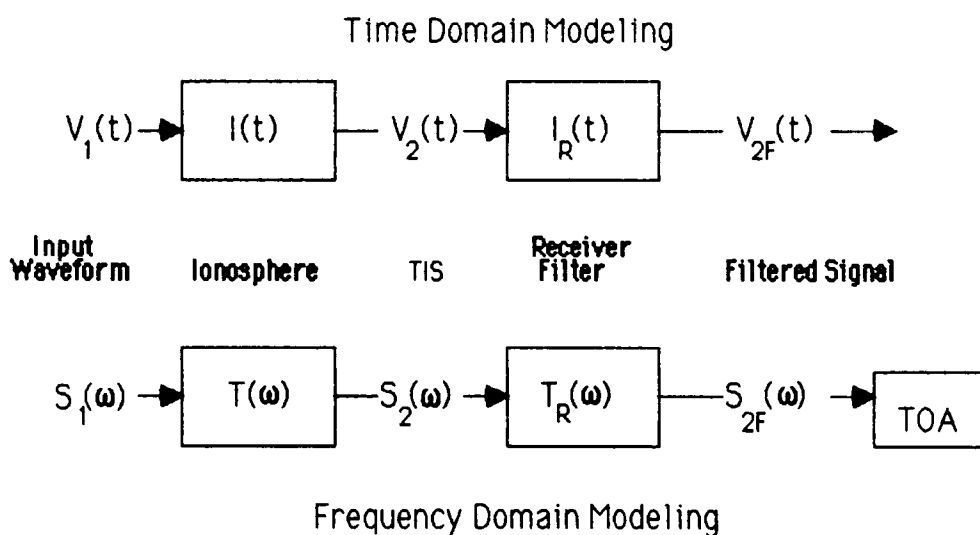


**Figure IV-2. Overall Propagation/Detection System Schematic.**

For this study,  $T_R(\omega)$  is the filter transfer function based on a Gaussian filter function of the form

$$G(t) = Ae^{-[(t-t_0)/\Delta t]^2} \sin \omega_0 t, \quad (IV-1)$$

which is chosen to be of some arbitrary width (FWHM) and centered at some arbitrary central frequency,  $\omega_0$ , and where

$$T_R(\omega) = \int I_R(t) e^{-j\omega t} dt, \quad (IV-2)$$

and,

$$I_R(t) = G(t). \quad (IV-3)$$

The output,  $S_{2F}(\omega)$  is the filtered TIS frequency spectrum.

As in Section III, the filtering may be performed in the time domain as the time convolution of the filter impulse response function and the time domain TIS,

$$V_{2F}(t) = I_R(t) * V_2(t). \quad (IV-4)$$

Heuristically speaking, this convolution process breaks up  $V_2(t)$  into a sequence of impulses and expresses  $V_{2F}(t)$  as the sum of the responses of each component impulse to the filter function. However, since TOA analysis is based on isolation and time tagging of individual frequency components, the filtering may be performed in the frequency domain by using

$$S_{2F}(\omega_0) = T_R(\omega_0) S_2(\omega). \quad (IV-5)$$

Here the component frequencies are isolated by specification of  $G(t)$

and therefore  $T_R(\omega)$  to allow filtering out of all frequencies except those within some narrow bandwidth centered at the frequency,  $\omega_0$ , of interest. As such,  $S_{2F}(\omega)$  contains amplitude and phase information only for the frequencies within the chosen frequency bandwidth centered at  $\omega_0$ . For purposes of example, this paper makes use of Gaussian filter functions centered at  $\omega_0 = 150$  MHz and 50 MHz both with a width of 2 MHz, shown in Figure IV-3(a) and Figure IV-4(a) respectively.

When the 150 MHz filter  $T_R(\omega)$  is applied to the TIS,  $S_2(\omega)$ , in essence all component frequencies of the waveform are filtered out except for 149 MHz through 151 MHz and similarly for the 50 MHz filter. If the impulse TIS shown in Figure II-1(a) is processed with the 150 MHz filter in Figure IV-3(a), the results are the filtered TIS spectrum, filtered time signature and filtered power signature shown in Figures IV-3(b), (c) and (d) respectively. The power signature is obtained by squaring the amplitude signal and filtering out the high frequency oscillations with a 10 MHz low pass filter. The same results for application of the 50 MHz filter are shown in Figure IV-4(b), (c) and (d).

Since only the phase of the TIS is altered and the frequency spectrum remains the same, the information shown for the frequency spectrum in Figures IV-3(b) and IV-4(b) is identical to that contained in the appropriate 2 MHz bandwidths of Figure I-1(b). Additionally,

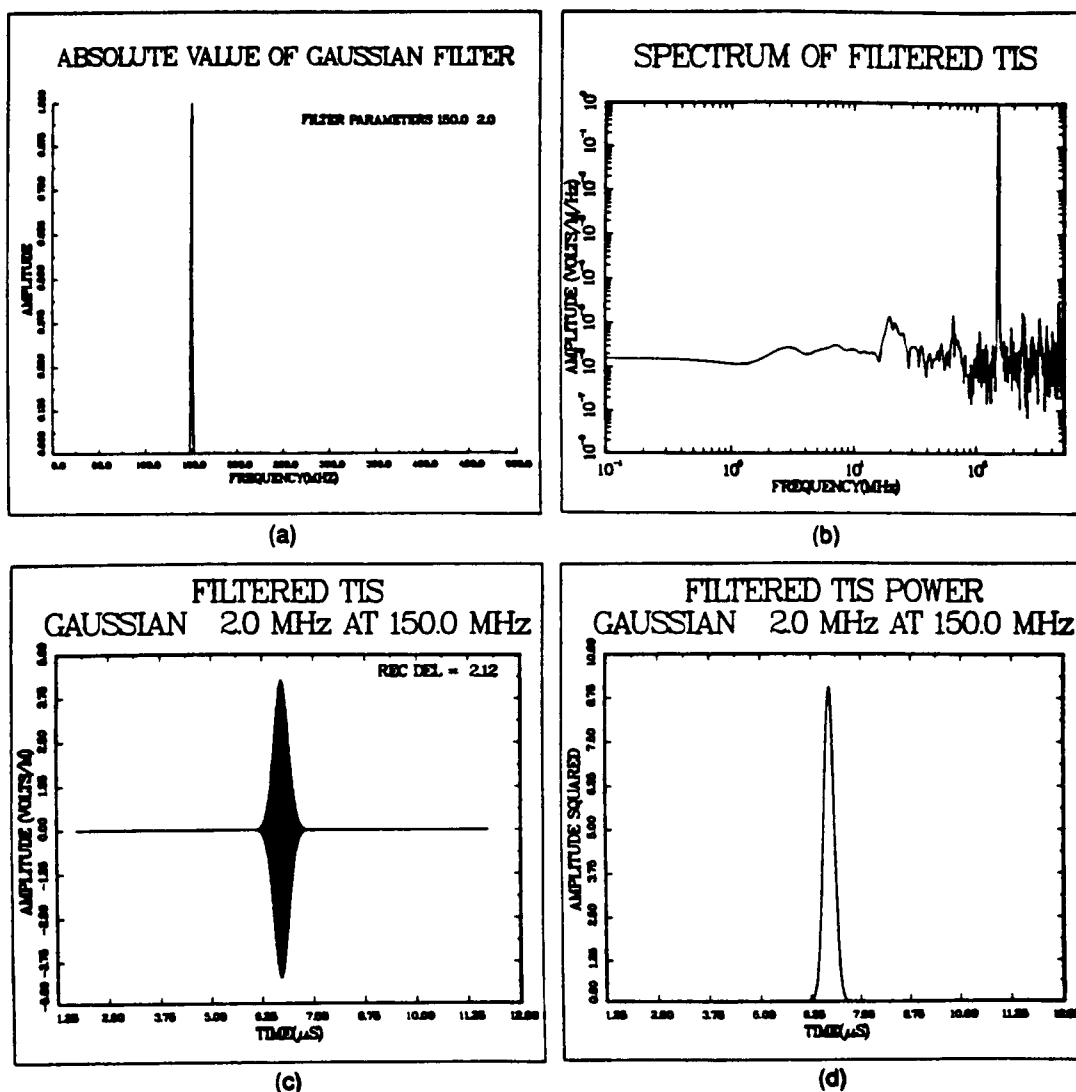


Figure IV-3. Filtered Transionospheric Delta Function (TEC= 1.0, 150 MHz).  
 (a) Supergaussian filter function centered 2 MHz bandwidth about 150 MHz. (b) Delta function filtered power spectrum; oscillations are due to numerical noise. (c) TIS filtered with 150 MHz filter. REC DEL is receiver delay which is a function of central frequency. (d) TIS power produced by squaring signal in (c) and applying a 10 MHz low pass filter.

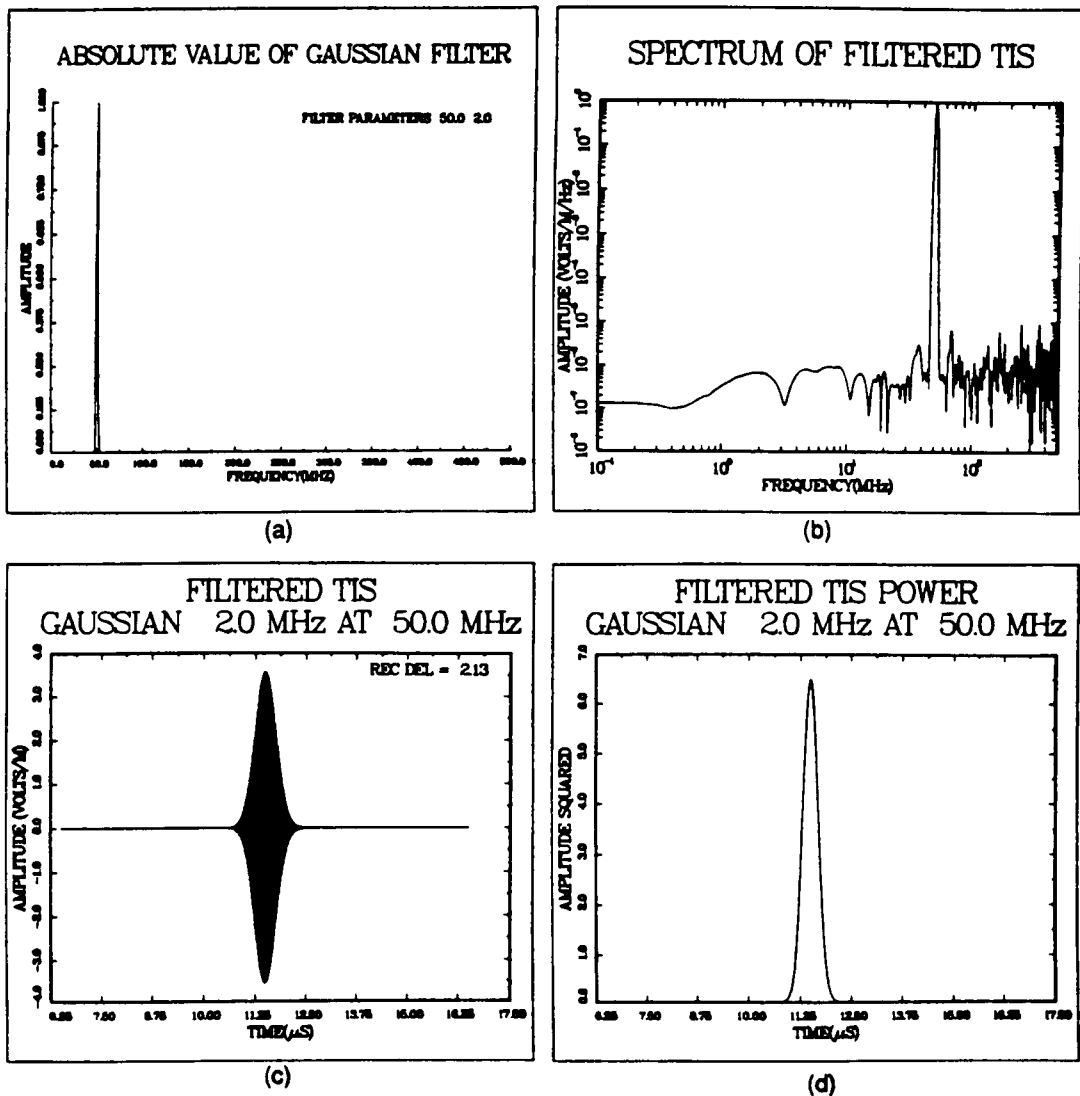


Figure IV-4. Filtered Transionospheric Delta Function (TEC = 1.0, 50 MHz).  
 (a) Supergaussian filter function centered 2 MHz bandwidth about 50 MHz. (b) Delta function filtered power spectrum; oscillations are due to numerical noise. (c) TIS filtered with 50 MHz filter. REC DEL is receiver delay which is a function of central frequency. (d) TIS power produced by squaring signal in (c) and applying a 10 MHz low pass filter.

since the delta function frequency spectrum is unity for all frequencies and the absolute value of the gaussian filters is one, the convolution of the TIS and the gaussian filter produces the filter function itself centered about the  $\omega_0$  of interest with amplitude defined as a function of the bandwidth used and the ionospheric dispersion affects on group energy spread.

The time signatures show the effects of dispersion and time delays associated with the filtering process. Each filtered pulse contains information between  $\omega_0 \pm 1$  MHz. In the 150 MHz case, the actual TOA is 6.71  $\mu\text{sec}$  which is the sum of the initial 4  $\mu\text{sec}$  delay time, the 0.59  $\mu\text{sec}$  ionospheric delay and the 2.12  $\mu\text{sec}$  delay time of the filter. For the 50 MHz signal, the actual TOA is 11.49  $\mu\text{sec}$ , the sum of the same initial delay and filter delay but with a greater dispersion delay of 5.37  $\mu\text{sec}$ .

Note also the peak amplitudes and signature widths. While each component initially propagated with an amplitude of 1000 Volts/m, the reduction in amplitude and hence power over 2 MHz of interest is evident. For the 150 MHz component the signature width is  $\approx 15.7$  nanoseconds, with amplitude of 4.24, reduced by a factor of 236 over the input signature value. Its power is reduced to  $\approx 8.98 \text{ W/m}^2$  from the input  $5(10^5)$ . For the 50 MHz component the signature width is  $\approx 429.6 \text{ nsec}$  with an amplitude of  $\approx 3.6$ , a factor of 278 reduction from the input and a reduction of power to  $6.5 \text{ W/m}^2$ . Note the greater the ionospheric dispersion time, the greater signature spread and

reduction in amplitude and power for the lower frequency groups.

Similar results for the CPB EMP TIS presented in Figure II-2(a) and power spectrum of Figure I-4(c) are shown in Figures IV-5 and IV-6. The TOA information for the same central frequencies are not as predictable since, unlike the delta function, nothing is known about the initial start,  $t_0$ , of each component, except that all of its spectral components superpose within the 20-30 nanosecond width of the pulse as shown in Figure I-4(a). In this respect the delta function is very useful in testing and gaining confidence in a modeling algorithm.

Noise Effects. Figure IV-7 shows the TIS (a), filtered TIS (b) and filtered TIS power (c) for a noisy input impulse function with amplitude signal to noise ratio at the ground (pre-propagation level) of 200. Figure IV-7(d), (e) and (f) are the same for a ground amplitude SNR of 20. The SNR is normally defined to be the ratio between signal power within some narrow bandwidth about some central frequency over the noise power within the same bandwidth:

$$\text{SNR} = \frac{\int_{\omega_0 - \Delta\omega/2}^{\omega_0 + \Delta\omega/2} S^2(\omega) d\omega}{\int_{\omega_0 - \Delta\omega/2}^{\omega_0 + \Delta\omega/2} N^2(\omega) d\omega}$$

Here,  $\omega_0$  is the central frequency of interest and  $\Delta\omega$  is the bandwidth.

For a delta function input, this definition reduces to  $A/N$  where  $A$  is

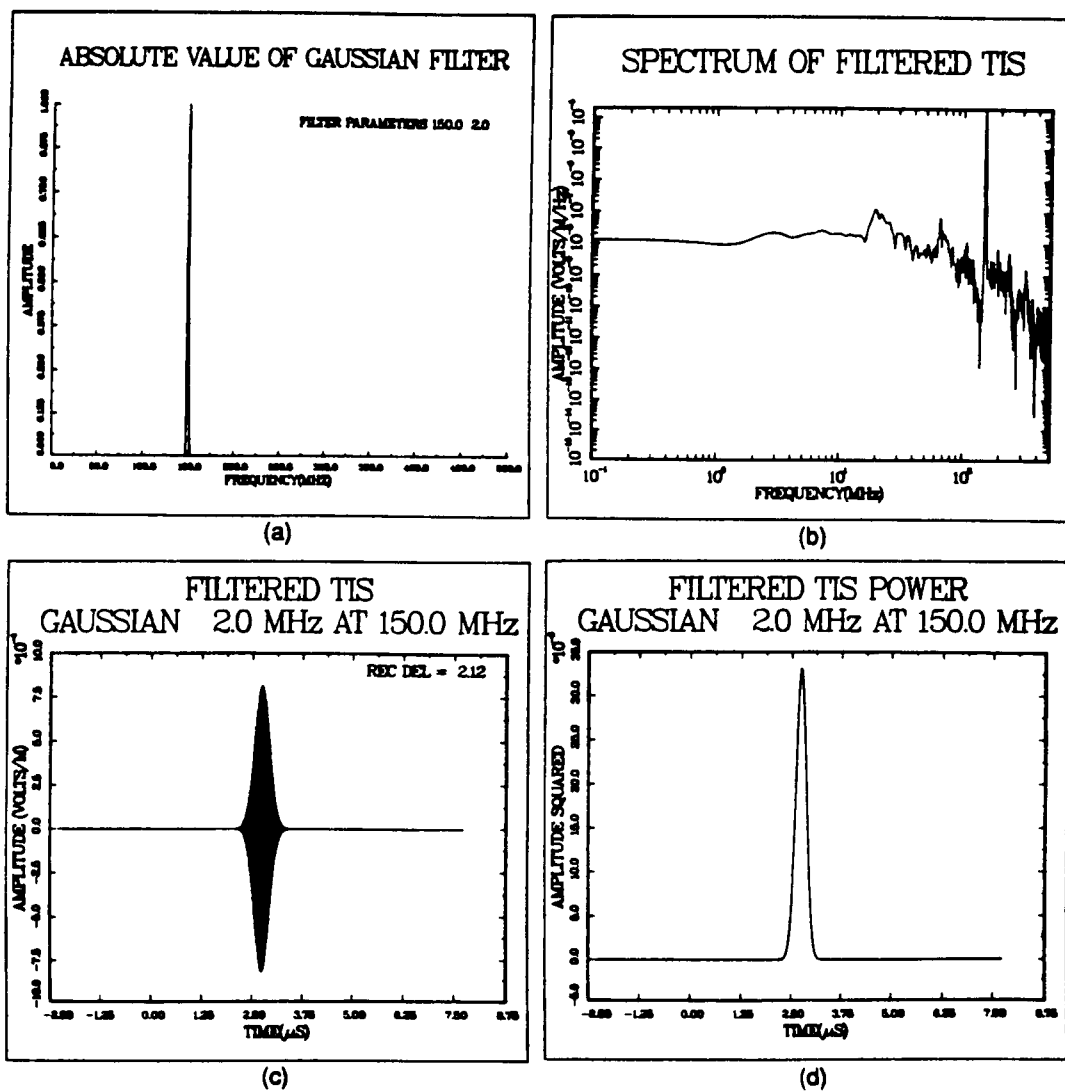


Figure IV-5. Filtered Transionospheric CPB (TEC= 1.0, 150 MHz).  
 (a) Supergaussian filter function centered 2 MHz bandwidth about 150 MHz. (b) CPB filtered power spectrum; oscillations are due to numerical noise. (c) TIS filtered with 150 MHz filter. REC DEL is receiver delay which is a function of central frequency. (d) TIS power produced by squaring signal in (c) and applying a 10 MHz low pass filter.

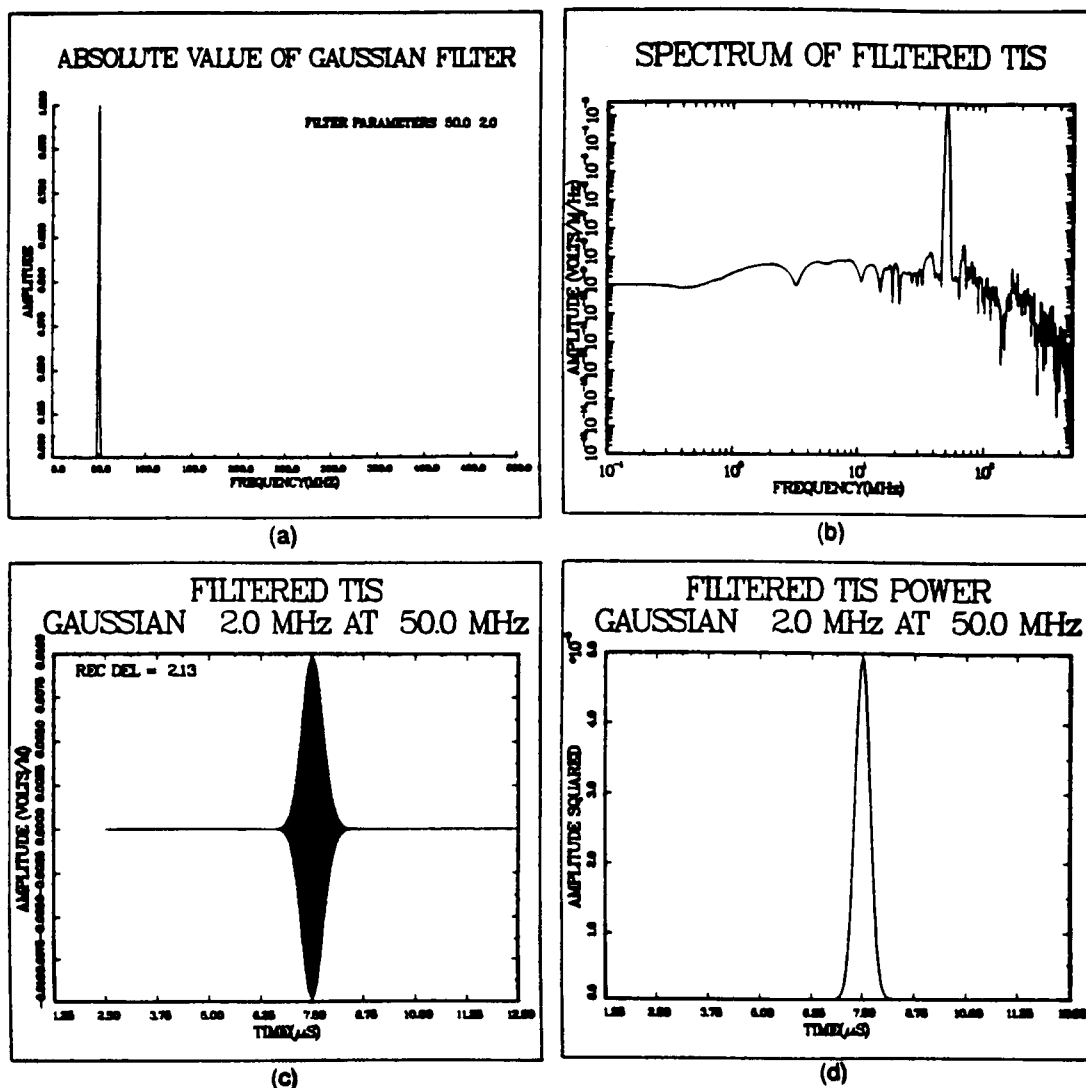


Figure IV-6. Filtered Transitionospheric CPB (TEC= 1.0, 50 MHz).  
 (a) Supergaussian filter function centered 2 MHz bandwidth about 50 MHz. (b) CPB filtered power spectrum; oscillations are due to numerical noise. (c) TIS filtered with 50 MHz filter. REC DEL is receiver delay which is a function of central frequency. (d) TIS power produced by squaring signal in (c) and applying a 10 MHz low pass filter.

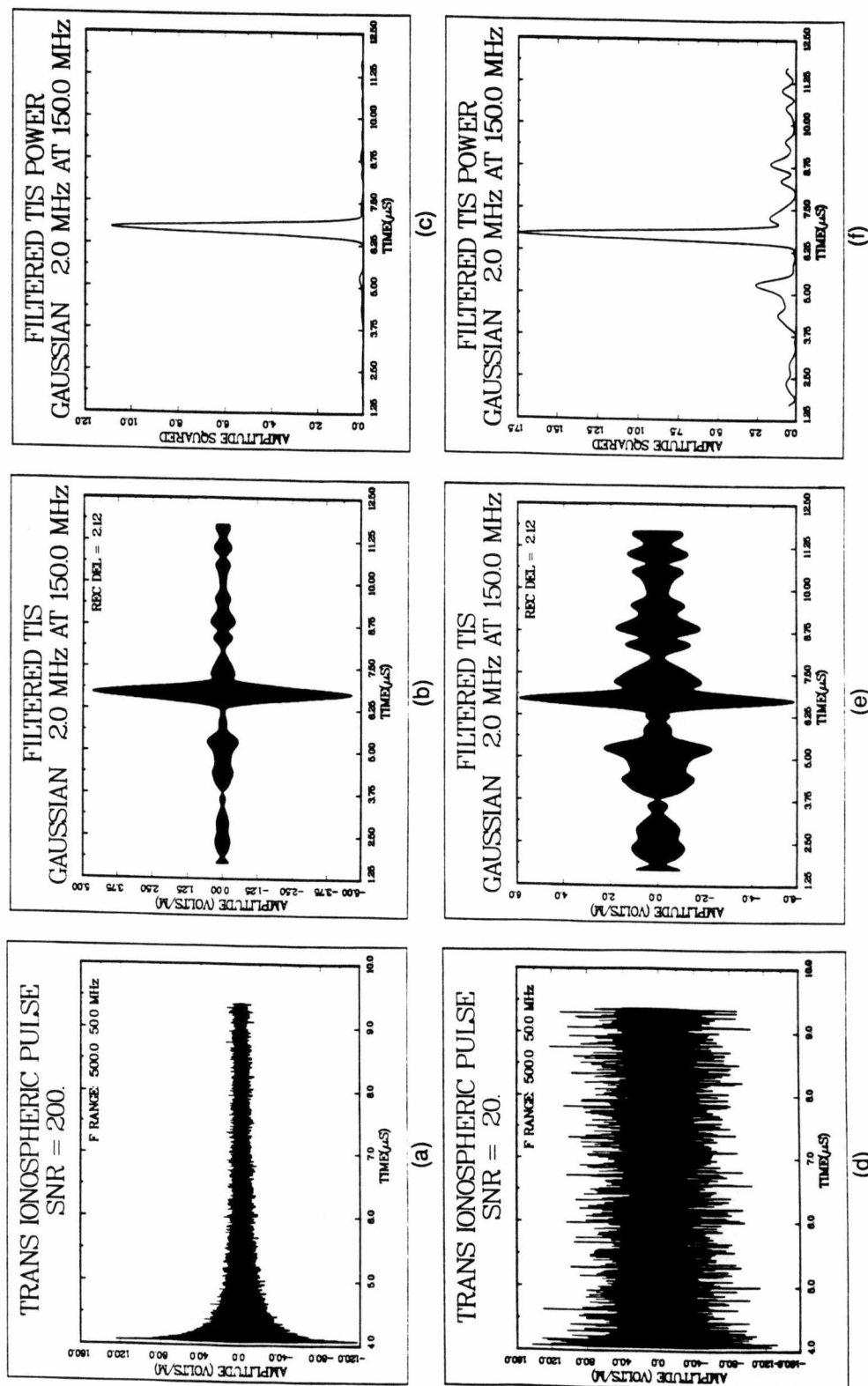


Figure IV-7. Relative noise effects on propagated impulse function filtered with 150 MHz gaussian filter. Figures (a), (b), and (c) show the noisy TIS, filtered TIS, and filtered TIS power signatures for a ground amplitude SNR of 200. Figures (d), (e), and (f) are the same information for a SNR of 20.

the amplitude of the input signal and  $N$  is the rms noise level. Both are filtered with the 150 MHz Gaussian filter used above. Figure IV-8 shows the same information for the CPB EMP. Note the additional 150 MHz noise information scattered randomly about the primary peak in all cases. Also note that for the delta function in Figure IV-7, although the noise power is evident, the peak of interest is very prominent and, depending upon the detection system used, possibly very detectable in each case. In addition, note the amplitude increases for the lower SNR case in all three plots.

For the CPB of Figure IV-8, the effects are much worse due to its unique energy content as shown in its frequency spectrum. In both SNR cases, the 150 MHz information of interest is visible in these time signatures centered at approximately 2.70  $\mu\text{sec}$ . Because the pulse has been centered in the window, noise shown between -2.50 and 0.0  $\mu\text{sec}$  is actually wrapped around from the right sides of Figures IV-8(b), (c), (e) and (f). For SNR=200, the peak power is clearly separate but potential exists for confusion in measurement due to the close proximity of random noise peaks of comparable magnitudes. In the SNR=50 case, however, the 150 MHz information of interest is completely overshadowed by the adjacent noise power peaks. Structure of the noise lobes in each case are similar because the same statistical realization of noise was used in each.

As shown below, such noise levels have the potential of upsetting any time-tagging algorithm, inducing levels of uncertainty dependant upon the degree of noise.

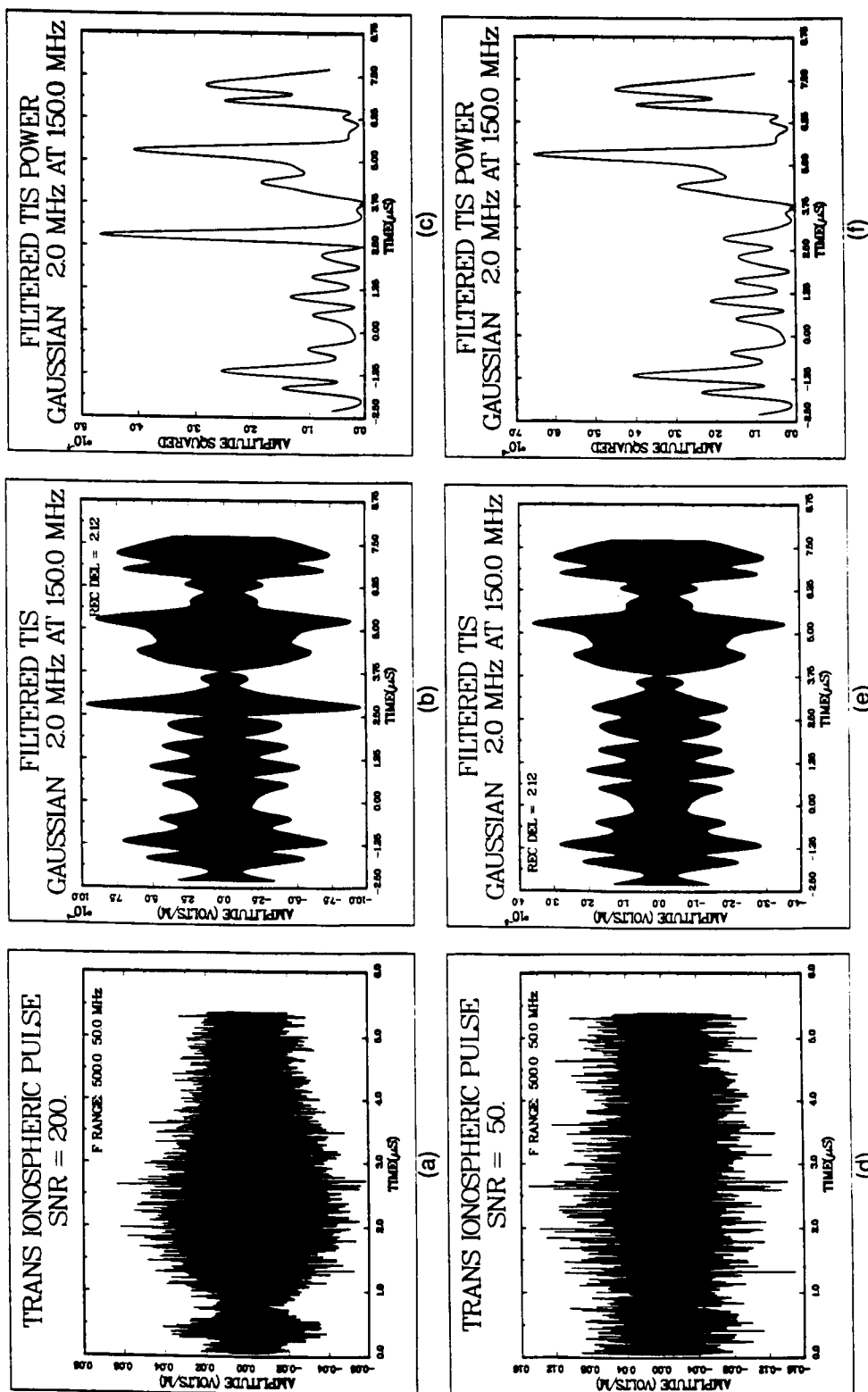


Figure IV-8. Relative noise effects on propagated CPB filtered with 150 MHz gaussian filter. Figures (a), (b), and (c) show the noisy TIS, filtered TIS, and filtered TIS power signatures for a ground amplitude SNR of 200. Figures (d), (e), and (f) are the same information for a SNR of 50.

Scintillations Effects. Figure IV-9 shows the TIS (a), filtered TIS (b) and filtered TIS power (c) for the transionospheric delta function after propagation through a disturbed ionosphere characterized by an  $f_{coh}$  of 20 MHz and TEC of 1.0. Note that the coherence bandwidth is held constant over the entire frequency range shown. That shown in (d), (e) and (f) is for an  $f_{coh}$  of 0.1 MHz. Both cases were filtered using the same 150 MHz gaussian filter with a 2 MHz bandwidth. Similar information for the CPB EMP is shown in Figure IV-10. Note the loss of the signal for the two cases for  $f_{coh}$  of 0.1 MHz in Figures IV-9 and IV-10 (e) and (f). In these cases the correlation time for coherent phase propagation is much less than that for the system operating at a bandwidth of 2 MHz. The potential disruption here may be greater than for white noise depending upon how well noise background is understood and the efficiency in which both effects may be backed out in the location and/or discrimination algorithms. Incidentally, TIPC has the capability to combine noise and scintillations effects simultaneously.

#### Source Location.

TOA Analysis. Due to ionospheric dispersion, the propagation path lengths,  $ct_i$ , of any two discrete frequencies differ, where

$$ct_i = R_i + KN_e/f^2, \quad (IV-6)$$

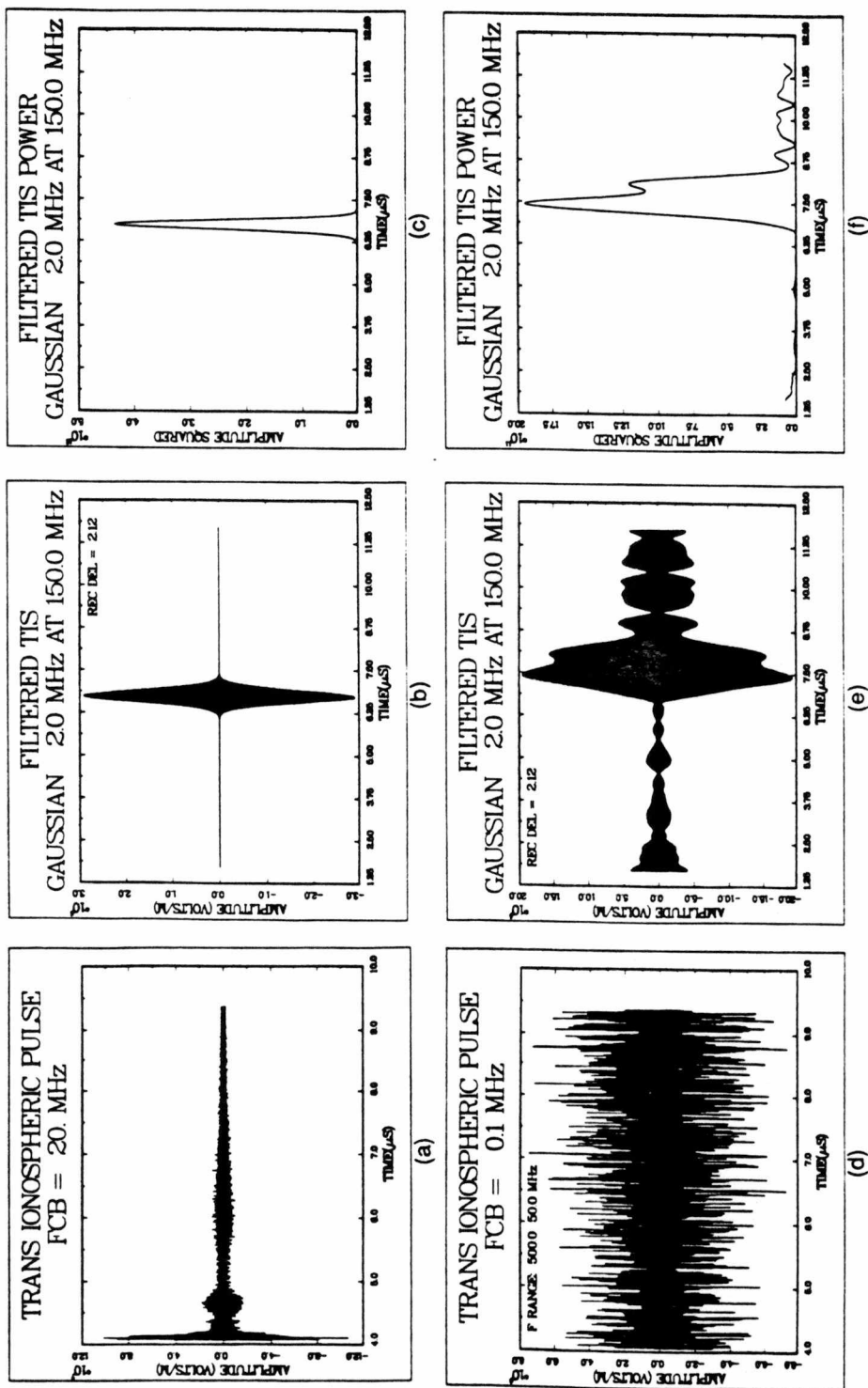


Figure IV-9. Relative scintillations effects on propagated delta function filtered with 150 MHz gaussian filter. Figures (a), (b) and (c) show the TIS, filtered TIS and filtered TIS power signatures for frequency coherent bandwidth of 20 MHz. Figures (d), (e) and (f) are for a coherent bandwidth of 0.1 MHz. Amplitudes are relative. Note loss of signal at  $t = 6.71$  for 0.1 MHz due to system 2 MHz bandwidth being greater than the coherent bandwidth.

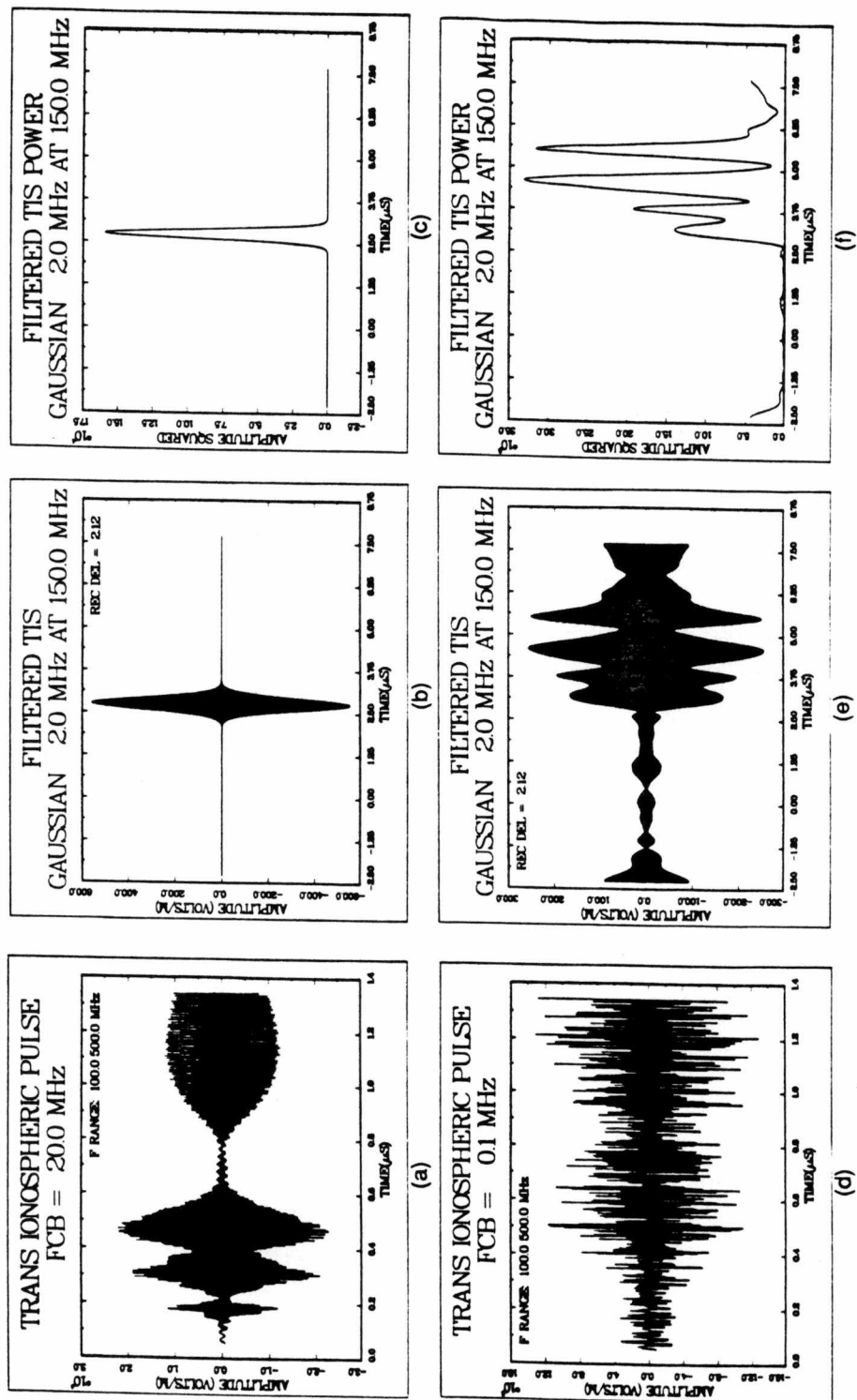


Figure IV-10. Relative scintillations effects on propagated CPB filtered with 150 MHz Gaussian filter. Figures (a), (b) and (c) show the TIS, filtered TIS and filtered TIS power signatures for frequency coherent bandwidth of 20 MHz. Figures (d), (e) and (f) are for a coherent bandwidth of 0.1 MHz. Amplitudes are relative. Note loss of signal at  $t = 2.7$  for 0.1 MHz due to system 2 MHz bandwidth being greater than the coherent bandwidth.

$ct_1$  = actual propagation path length from source to receiver,

$R_1$  = straight line distance from source to receiver,

$K$  = ionospheric transfer function constant,

$N_e$  = electron density which is proportional to IEC,

$c$  = speed of light in a vacuum, and

$t_1$  = TOA of a discrete frequency,  $f_1$ .

Thus for two discrete frequencies,

$$ct_1 = R_1 + KN_e/f_1^2 \quad (IV-6a)$$

and,  $ct_2 = R_2 + KN_e/f_2^2$ .

Now, if  $f_1 \approx f_2$  such that  $|f_1 - f_2| \ll 1$  then  $R_1 \approx R_2$  and both  $R_1$  and  $R_2$  may be replaced by  $R$ , resulting in two equations with two unknowns  $R$ , and  $N_e$ . Solving simultaneously for  $ct_1$  and eliminating  $N_e$  gives

$$ct_1 = R + c(t_2 - t_1) [f_2^2 / (f_2^2 - f_1^2)]. \quad (IV-7)$$

Solving for  $R$  gives

$$R = ct_1 - c(t_2 - t_1) [f_2^2 / (f_2^2 - f_1^2)], \quad (IV-8)$$

where  $t_2 - t_1$  is the delta-TOA or DTOA between the two frequencies.

Now the error in  $R$ , measured as a function of  $t_1$  and  $t_2$  is given as

$$\Delta R = [(\partial R / \partial t_1)^2 \Delta t_1^2 + (\partial R / \partial t_2)^2 \Delta t_2^2]^{1/2} \quad (IV-9)$$

where  $\Delta t_1$  and  $\Delta t_2$  are the uncertainties in the TOA measurements performed by the system in use.<sup>16</sup> More specifically, as applied to  $R$  above,

$$\Delta R = \{c[\Delta t_1^2 - (f_2^2/(f_2^2 - f_1^2))\Delta(\Delta t)]\}^{1/2}, \quad (IV-10)$$

where  $\Delta(\Delta t)$  is the DTOA uncertainty,  $\Delta t$  being  $t_2 - t_1$ . Thus the distance and the uncertainty of this distance from the source to the receiver may be calculated if TOA and TOA uncertainty information is determined for two very close frequencies. If several measurements are simultaneously obtained from differently located receivers, triangulation techniques may be used to precisely locate the source.

TOA and DTOA errors are induced both by receiver limitations and by the degree to which background and ionospheric dispersion disrupt accurate TOA measurements for a given detection system. Therefore, efforts to understand these effects are important. The following subsection discusses one of the modeling methods used in the TIPC for determining TOA and DTOA information and uncertainties for both.

Leading Edge TOA Analysis. This analysis makes a linear fit to the leading edge of the filtered TIS power signature. It finds an estimate for the TOA of the central frequency by using the higher frequency portion of the filtered signal arriving first at the leading edge.

Specifically, this algorithm determines the peak power and locates the 1/3 peak and 2/3 peak points above a chosen noise threshold. A linear fit is made through these two points and the TOA for the central frequency is computed using

$$TOA_{\omega_0} = 2t_{1/3} - t_{2/3}, \quad (IV-11)$$

which is derived from the equation of the straight line fitted. Here,  $t_{1/3}$  and  $t_{2/3}$  are the arrival times of the 1/3 and 2/3 peak power points. Figure IV-11 provides an example of this method using the clean delta function power signature of Figure IV-3(d). As might be

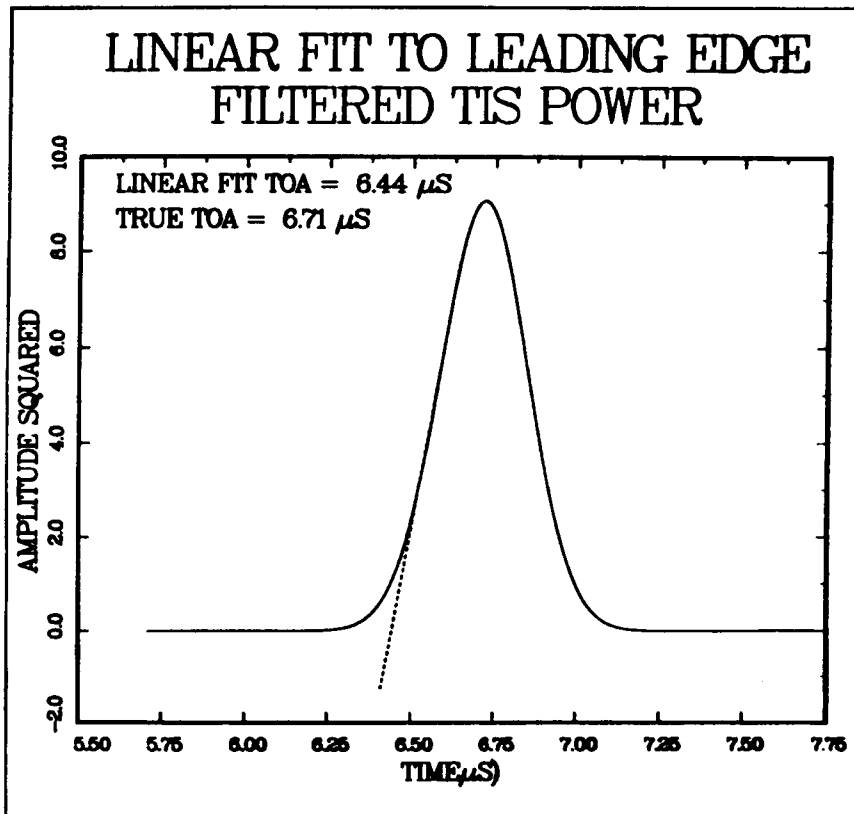


Figure IV-11. Simple example of leading edge fit to 150 MHz delta function power spectrum after propagation through ionosphere of TEC = 1.0.

expected, the TOA information provided is consistently less than the true TOA. Here, the computed TOA from (IV-11) is 6.44  $\mu$ sec and the true TOA is that of the peak at 6.71  $\mu$ sec. This leading edge error,  $\Delta t_{LE}$  must be accounted for in the TOA error,  $\Delta t$ , along with any other

measurement error,  $\Delta t_m$ , in the computation of the range error,  $\Delta R$ , in (IV-10) above:

$$\Delta t_i = (\Delta t_{LE} + \Delta t_m)_i. \quad (IV-12)$$

Although the computed TOA is in error of a few tens of nanoseconds, the TOA leading edge error is consistent between two close frequencies and the DTOA computed between any two channels is as accurate as it would be if the true TOAs were computed. However, TOA and DTOA uncertainties become larger when the leading edge algorithm is applied to noisy or scintillated transionospheric pulses or if less than optimum filtering is performed.

Uncertainty studies have concentrated on DTOA uncertainty due to noise effects. Figures IV-12 and IV-13 show initial noise uncertainty analyses for five ground amplitude signal-to-noise ratios (SNR) of 50 through 250 in increments of 50. All cases presented are for an ionospheric TEC of 1.0. For Figure IV-12, corresponding power SNRs in decibels at the receiver after propagation through a deterministic ionosphere range from 12.9 to 26.7. This study used two gaussian filters centered at 150 MHz and 140 MHz, both with 2 MHz bandwidth. Five statistical realizations of noise were used for each SNR for each central frequency, resulting in 25 realizations over which to average. The dashed curve is the DTOA noise uncertainty mean deviation, computed using the average DTOA over all realizations

$$DTOA_{avg} = [\sum (TOA_{140} - TOA_{150})]/25, \quad (IV-13)$$

with the DTOA mean deviation then computed to be

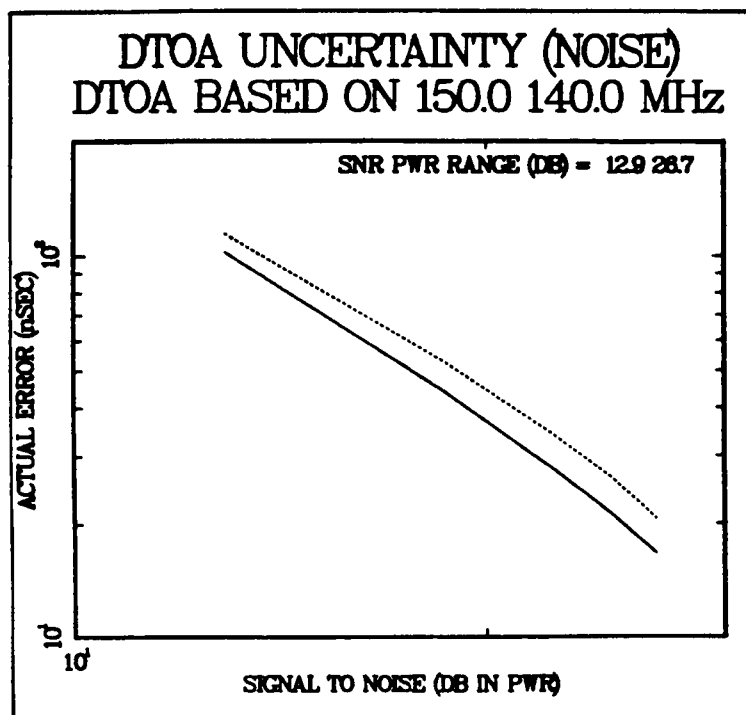


Figure IV-12. Noise DTOA uncertainty study for noisy delta function between 150 MHz and 140 MHz gaussian filter central frequencies with 2 MHz FWHM filter width using five realizations.

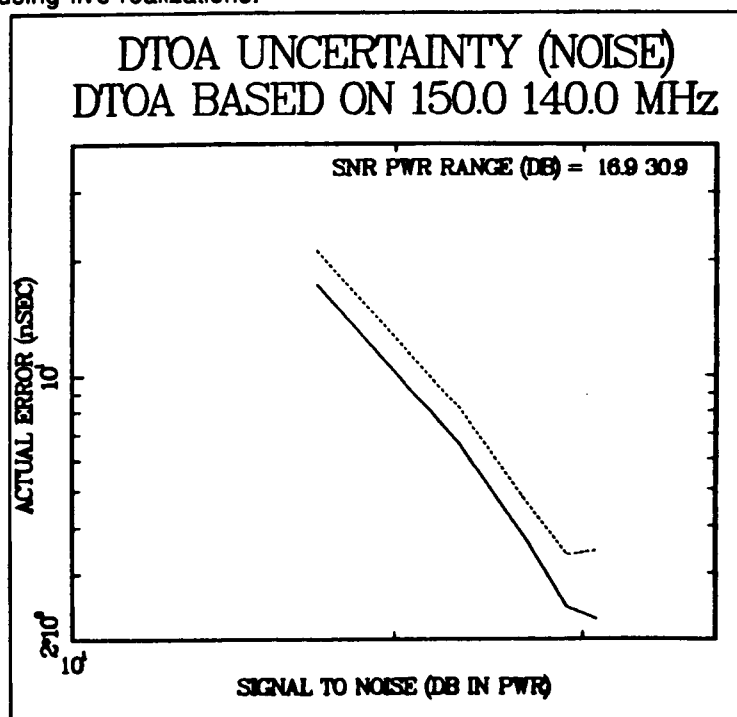


Figure IV-13. Noise DTOA uncertainty study for noisy delta function between 150 MHz and 140 MHz gaussian filter central frequencies with 10 MHz FWHM filter width using five realizations.

$$\text{DTOA}_{\text{md}} = [\sum (\text{DTOA}_i - \text{DTOA}_{\text{avg}})^2]^{1/2} / 25. \quad (\text{IV-14})$$

The solid curve is the actual error in DTOA relative to the known DTOA for a deterministic ionospheric TEC of 1.0. It is computed as the rms average over all 25 DTOA. Both are plotted in nanoseconds.

Figure IV-13 is the same information for the above ground amplitude SNR range, but for a 10 MHz wide filter instead of 2 MHz. Note the reduction in error and increase in the power SNR at the receiver. This effect results from the fact that the ionospheric dispersion time spread,  $\Delta t_d$ , about the central frequency is proportional to the filter width while the time width of the filter,  $\Delta t_f$ , is inversely proportional to the filter frequency width:

$$\Delta t_d = 2 (\Delta f t_d) / f^2, \quad (\text{IV-15})$$

where

$$t_d = \text{TOA}_{\omega_0} \text{ due to dispersion,}$$

and

$$\Delta t_f = 1 / \pi \Delta f_{1/e} \quad (\text{IV-16})$$

respectively. Additionally, the power SNR at the satellite may be expressed as a function of the ground amplitude SNR before propagation, with  $\Delta t_d$  and  $\Delta t_f$ :

$$\text{SNR}_s \propto \text{SNR}_g [\Delta t_d^2 + \Delta t_f^2]^{-1/4}. \quad (\text{IV-17})$$

The filter width governs the amount of signal and noise energy allowed through the system. Up to a certain point widening the filter

allows more signal energy through than the corresponding noise energy.  $SNR_s$  is maximized when  $\Delta t_d$  and  $\Delta t_f$  are of the same magnitude, this value being dependant upon the central frequency used. Hence the filter width for each channel must be unique for achieving optimum  $SNR_s$  and minimum DTOA uncertainty.

Figure IV-14 is the same case as that in Figure IV-12 but for ten realizations of noise rather than five. This shows the better agreement between the two curves as expected when averaging over a greater number of statistical samples.

Finally, a very good estimate of the ionospheric TEC is possible. From (IV-6a) above,  $N_e$  may be replaced  $\int N_e dz$ , or TEC to give

$$ct_1 = R_1 + K \text{ TEC} / f_1^2$$

and, 
$$ct_2 = R_2 + K \text{ TEC} / f_2^2,$$

where  $f_2 > f_1$ . Dividing by  $c$ , assuming  $R_1 \approx R_2$  and subtracting  $t_1$  from  $t_2$  gives

$$\text{DTOA} = t_2 - t_1 = K \text{ TEC} (1/f_2^2 - 1/f_1^2),$$

and, solving for TEC

$$\text{TEC} = [ (f_1^2 - f_2^2) / f_1^2 f_2^2 ] (\text{DTOA} / K). \quad (\text{IV-18})$$

DTOA is measured,  $K$ ,  $f_1$  and  $f_2$  are known. If the TEC is calculated, the dispersive effects of the ionosphere may be subtracted out from phase and amplitude information recorded at the receiver for signal reconstruction as discussed below.

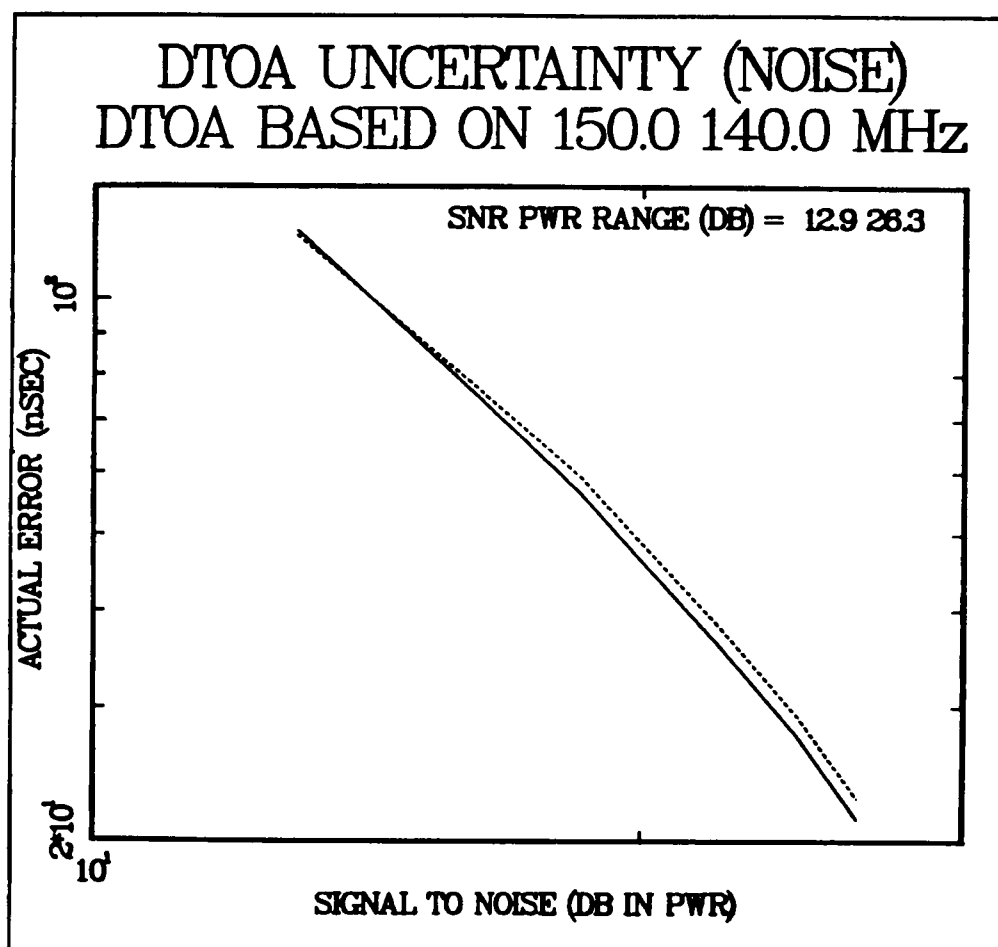


Figure IV-14. DTOA noise uncertainty study of same case as in Figure IV-12 but using ten realizations versus five.

### Source Discrimination.

Discrimination implies being able to distinguish between the EMP signature of one source over another. Since the ionospheric conditions which effect the propagating signal will certainly differ from separate source detections, whether these detections be separated temporally or spacially, these EMP signatures cannot possibly be distinguishable based on transionospheric time signatures; something must be done to remove the alterations in phase and/or amplitude caused by ionospheric propagation in order to recover, at least enough to allow distinction, the original time signature of the device.

Frequency Spectrum Reconstruction. One relatively straight forward method of discrimination is to reconstruct the frequency power spectrum of the source pulse. Note from the spectra plots of Section I that each is unique. If the filter used in the detection process is optimized, the area under the curve of the filtered energy signature is a close estimate of the corresponding value on the source spectrum for the central frequency of interest. If many of such post-propagation power measurements can be made through use of a spectrum analyzer, or by several detectors with judiciously selected central frequencies, portions of the source spectrum may be crudely reconstructed to notice distinguishing characteristics of a particular source type. Since the spectrum undergoes no change for VHF waveforms, then any tell-tale spectrum characteristics may be used

to determine pulse shape characteristics such as width, rise time, fall time, etc., of the source time signature.<sup>7</sup>

However, this method, under the best of circumstances of low noise and deterministic dispersion, allows only an educated guess at the actual shape of the source time signature. What is actually needed is amplitude versus time and phase versus time information over a broad frequency range. Then, if the ionospheric conditions at the time of propagation are known, they may be removed from this information, and the frequency domain TIS may be transformed back to the time domain to recover the source time signature. Such is the approach used in the quadrature filtering technique.<sup>15</sup>

Quadrature Filtering. Using the notation from above, if  $f(t)$  is the filtered TIS, examples being that of Figures IV-4, 5, 6, and 7(b), then the real and imaginary portions of the filtered signal may be separated by the following expressions:

$$\text{Re}\{f(t)\} = |f(t)| \cos \omega_0 t \quad (\text{IV-19})$$

and 
$$\text{Im}\{f(t)\} = |f(t)| \sin \omega_0 t. \quad (\text{IV-20})$$

Here, beating of a cosinusoid of central frequency  $\omega_0$  against the TIS filtered about  $\omega_0$  produces the real component of the filtered TIS. Similarly, beating with a sinusoid of  $\omega_0$  produces the imaginary component. From these the amplitude and phase as functions of  $\omega$  may be obtained by using absolute value and inverse tangent relationships:

$$A(\omega_0) = [ \text{Re}\{f(t)\}^2 + \text{Im}\{f(t)\}^2 ]^{1/2} \quad (\text{IV-21})$$

$$\text{and,} \quad \omega_0 t = \tan^{-1}(\text{Re}/\text{Im}). \quad (\text{IV-22})$$

Several such applications over a large bandwidth provide the numerous data points required to reconstruct the signal. Then, if the ionospheric transfer function is determined from TOA information as in (IV-18), the ionospheric dispersion effects may be backed out and the inverse transform taken to produce the original time signature:

$$f(t) = 1/2\pi \int A(\omega_1) e^{j\omega_1 t} e^{-j\theta(\omega_1)} e^{-j\omega t} d\omega \quad (\text{IV-23})$$

where the integration is carried out over the range of receiver channel central frequencies used, and

$$e^{-j\theta(\omega_1)} = T(-\omega),$$

is the negative of the ionospheric transfer function from (III-8).

Again, the degree to which the TIS has been disturbed by background and dispersion determines the capability of reproducing a source signature reliable enough to be source traceable based on its time characteristics.

## CONCLUSION

This paper has presented the theory behind a relatively large computer code which propagates a broadband pulse through a benign or disturbed ionosphere with or without noise. Ionospheric affects are determined through parameterization of ionospheric TEC, frequency coherent bandwidth and signal to noise ratio. After propagation, the code detects the arriving information of interest based on a linear fit to the leading edge of a group of frequencies centered about a chosen central frequency. Error analysis on time of arrival determination can be performed to study noise and scintillations affects and the effects of less than optimum receiver bandwidth, though this is an initial capability and can be improved. Finally, two methods were presented for recovering information about the type of source, but at the time of this writing, both algorithms were only partially encoded.

The TIPC is a very useful tool for studying the first order effects of deterministic and non-deterministic dispersion and the effects of noise, as well as the numerical modeling of specific receiver filter functions. These results have been analytically validated.<sup>18</sup> More however can be done to improve the versatility and accuracy of the code.

Specifically, a more realistic background model needs to be incorporated to include other noise sources as well as other broadband transients such as lightning. Further, the DTOA uncertainty

algorithms need fine tuning and a comprehensive study performed to analyze the combined effects of noise, scintillations and bandwidth limitations. The code will also eventually include the refraction, focusing and reflection phenomena affecting those frequencies close to the plasma frequency and below as well as any other attenuation or absorption phenomena resulting from more a realistic collisional medium. Finally, signal reconstruction algorithms need to be incorporated based on the most promising of techniques and a comprehensive error analysis needs to be performed.

## LIST OF REFERENCES

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1. Blinchikoff, Herman J., and Anatol I. Zverev, Filtering in the Time and Frequency Domains, Westinghouse Electric Corp., John Wiley & Sons, Inc., 1976.
2. Bracewell, Ron M., The Fourier Transform and its Applications, McGraw-Hill Book Company, Inc., 1965, pp. 24-30, 108 and 177-188.
3. Brandstatter, J. J., An Introduction to Waves, Rays, and Radiation in Plasma Medium, McGraw-Hill Book Company, Inc., 1963.
4. Brigham, E. Oran, The Fast Fourier Transform, Prentice-Hall, Inc., 1974.
5. Brillouin, L., Wave Propagation and Group Velocity, Academic Press, Inc., 1964.
6. Budden, K. G., The Propagation of Radio Waves, Cambridge University Press, 1985.
7. Carron, N. J., "The Exterior Fields of a Charged Particle Beam," MRC-R-918, Mission Research Corporation, 14 May 1985.
8. Davies, Kenneth, Ionospheric Radio Propagation, NBS Monograph 80, U. S. Dept. of Commerce National Bureau of Standards, 1 April 1965.
9. Classified Document.
10. Elliot, Glenn, Signal Processing Considerations for the New Block II-R W/RF Sensor, Sandia National Laboratory, Private Communication, 13 January 1987.

11. Jackson, J. D., Classical Electrodynamics, John Wiley & Sons, Inc., New York, 2nd Edition, 1975, Ch 7.
12. Jasperse, J.R. et. al, "Determination of Ionospheric Electron Density Profiles from Satellite UV Emission Measurements, FY84", AFGL-TR-85-0099, Environmental Research Paper No. 913, Air Force Geophysics Laboratory, Hanscom AFB, MA, 26 April 1985.
13. Jordan, Edward C., and Keith G. Balmain, Electromagnetic Waves and Radiating Systems, Prentice-Hall, Inc., New Jersey, 2nd Edition, 1968, Ch 9 and 17.
14. Knepp, Dennis, Propagation of Wide Bandwidth Signals Through Strongly Turbulent Ionized Media, DNA-TR-81-78, Mission Research Corporation, 15 March 1982.
15. Lorrain, Paul, and Dale R. Corson, Electromagnetic Fields and Waves, 2nd Edition, W.H. Freeman and Co., 1970.
16. Massey, Robert S., Earth and Space Sciences Division, Los Alamos National Laboratory. Private Communication, Jun-Aug 1986 and May-Dec 1987.
17. Rishbeth, Henry and Owen K. Garriot, Introduction to Ionospheric Physics, Academic Press, New York, 1969, Ch II.
18. Roussel-Dupre, Robert, Earth and Space Sciences Division, Los Alamos National Laboratory. Private Communication, Jun-Aug 1986 and May-Dec 1987.
19. Rustan, P. L., et al. Lightning Source Locations From VHF Radiation Data for a Flash at Kennedy Space Center, J. Geophysical Res., Vol 85, No. C9, pp. 4893-4903, Sept. 20, 1980.
20. Classified Document.

21. Skilling, Hugh Hildreth, Fundamentals of Electric Waves, Stanford University, January 1964.
22. Witwer, Leon, DNA, Author of CIRF code used in modeling RF propagation through a disturbed ionosphere, obtained via Roussel-Dupre', Los Alamos National Laboratory.

## APPENDIX

### DERIVATION OF WAVE EQUATION GOVERNING RF TRANSIONOSPHERIC PROPAGATION

## Derivation of Wave Equation Governing RF Transionospheric Propagation

### General.

The wave equations governing electromagnetic energy propagation are derivable from Maxwell's equations relating the electric and corresponding magnetic fields of a propagating electromagnetic disturbance. One is Faraday's law of magnetic induction,

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\partial/\partial t \oint \mathbf{B} \cdot d\mathbf{s}$$

which by Stokes theorem gives

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t. \quad (1)$$

Another is the Ampere-Maxwell relation for magnetism and current,

$$\begin{aligned} \oint \mathbf{B} \cdot d\mathbf{l} &= \mu_0 \oint (\mathbf{H} + \mathbf{M}) \cdot d\mathbf{l} = \mu_0 \oint \mathbf{J}_t \cdot d\mathbf{s} \\ &= \mu_0 \oint (\mathbf{J}_f + \partial \mathbf{D} / \partial t + \mathbf{M} \cdot d\mathbf{l}) \cdot d\mathbf{s} \end{aligned}$$

which, by using the relations  $\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}$  and  $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$  and again invoking Stoke's theorem for a line integral gives,

$$\begin{aligned} 1/\mu_0 (\nabla \times \mathbf{B}) &= \mathbf{J}_f + \partial \mathbf{D} / \partial t + \nabla \times \mathbf{M} \\ &= \mathbf{J}_f + \epsilon_0 \partial \mathbf{E} / \partial t + \partial \mathbf{P} / \partial t + \nabla \times \mathbf{M} = \mathbf{J}_m. \end{aligned} \quad (2)$$

For the above,

$\mathbf{E}$  = total electric field due to bound and free charge,

$B$  = total magnetic induction due to bound and free charge time rates of change (total current),

$P$  = dipole moment per unit volume resultant from polarization of dielectric atoms by an external  $E$  field,

$M$  = magnetization vector representing the number of magnetic dipoles per unit volume of magnetized material;  $M$  is analogous to  $P$ .

$D = \epsilon_0 E + P$ , the total electric field due to all charge except that due to polarization effects. Outside of a dielectric,  $D$  has the same value as  $E$  up to the value allowed by the permittivity,  $\epsilon$ . In free space,  $\epsilon = \epsilon_0$ .

$H = B/\mu_0 + M$ , the magnetic intensity or simply the magnetic field due to all current except that induced by magnetization of a material,  $M$ . Outside of a magnetized material,  $H = B/\mu_0$  only up to the permeability of the medium in case. In free space,  $\mu = \mu_0$ .

$J_t$  = current density due to motion of all bound and free charge and that induced by any magnetic induction field,  $B$ .

$J_f$  = current density of free charge.

$J_m$  = total current density due to motion of all charge within matter, accounting for polarization and magnetization effects,  $P$  and  $M$ .

These relations may be simplified by assuming that the ionosphere is a plasma and void of all magnetization and polarization effects ( $P=M=0$ ). Equations (1) and (2) are then

$$\nabla \times E = -\partial B / \partial t \quad (3)$$

and

$$\nabla \times B = \mu(J_f + \epsilon \partial E / \partial t). \quad (4)$$

Performing the standard simultaneous solution of (3) and (4) (utilizing the curl and substitutions of E and B as appropriate) gives the non-homogeneous wave equations for E and B as

$$\nabla^2 E - \mu \epsilon (\partial^2 E / \partial t^2) = \nabla(\rho_f / \epsilon) + \mu(\partial J_f / \partial t) \quad (5)$$

and

$$\nabla^2 B - \mu \epsilon (\partial^2 B / \partial t^2) = -\mu(\nabla \times J_f).^{15} \quad (6)$$

Assumptions. In this effort the ionosphere is further treated as a homogeneous, isotropic, stationary and linear medium. It is an ionized gas of low density ( $\approx 10^6$  electrons/cm<sup>3</sup>) in which, due to greater mobility, electrons are considered the only constituent of importance. The ionosphere is also considered collisionless, where energy loss due to collisions between electrons, ions, and neutrals are negligible. Additionally, thermal agitation effects are neglected since in the absence of electromagnetic fields the random motion of electrons produces no net current density. Hence, the ionospheric temperature is considered zero. Finally, the Earth's magnetic field is neglected because it has little affect on VHF waveforms and polarization effects on the propagating E-field are neglected; electromagnetic radiation propagates as in free space and the

propagating B field is neglected since its energy is small compared to the E field.

Plane wave/plate ionosphere. This paper considers only the propagation of plane waves originating at or near the Earth's surface and normally incident to a plate ionosphere at the zenith in the z-direction. Neglecting the B-field components of the wave form, only the E-field oscillating in the y direction and propagating in the positive z direction is modeled. Hence  $E_x$  and  $E_z$  do not exist. Also, the electric field strength is uniform throughout the plane of the wave and does not vary as the x or y coordinates change. Hence,

$$E_x = E_z = 0 \quad \text{and} \quad \partial E / \partial x = \partial E / \partial y = 0. \quad (7)$$

Expanding the left hand side of (5) in Cartesian form gives,

$$\begin{aligned} \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} &= \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} + \mu_0 \frac{\partial J_t}{\partial t} + \nabla \frac{\rho r}{\epsilon} \\ \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} &= \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} + \mu_0 \frac{\partial J_t}{\partial t} + \nabla \frac{\rho r}{\epsilon} \\ \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} &= \mu_0 \epsilon_0 \frac{\partial^2 E_z}{\partial t^2} + \mu_0 \frac{\partial J_t}{\partial t} + \nabla \frac{\rho r}{\epsilon} \end{aligned}$$

which by applying (7) reduces to

$$\frac{\partial^2 E_y}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} = \mu_0 \frac{\partial J_t}{\partial t} + \nabla \frac{\rho r}{\epsilon} \quad (8)$$

Further, the plane wave assumption implies the divergence of E is

zero. Since  $E_z = 0 = \partial/\partial x = \partial/\partial y$ ,

$$\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0,$$

and, from another of Maxwell's equations,  $\nabla \cdot \mathbf{E} = \rho_f/\epsilon$ , and the

charge separation term,  $\nabla(\rho_f/\epsilon)$  may be eliminated from (8).

From (8) the space variation of  $E$  is dependant upon the total current density variation and as a function of the time rate of change of the displacement and convection current densities. That is, since

$$J_t = \epsilon(\partial E_y/\partial t) + J_f,$$

then

$$\partial J_t/\partial t = \epsilon_0(\partial^2 E_y/\partial t^2) + \partial J_f/\partial t$$

and, from (8),

$$\partial^2 E_y/\partial z^2 = \mu_0 \epsilon_0(\partial^2 E_y/\partial t^2) + \mu_0(\partial J_f/\partial t) = \mu_0(\partial J_t/\partial t). \quad (10)$$

Through the linearity assumption,

$$J_f = \gamma E_y,$$

where  $\gamma$  is the conductivity of a neutral ionized gas. The conductivity is derived by either analyzing the force on a free

electron due to a passing E-field, or by deriving  $J_r$ , the convection current density.<sup>13,15,21</sup> In either case the result, assuming the collision frequency is zero, is an imaginary conductivity:

$$\gamma = -j(N_e q_e^2 / \omega m_e). \quad (11)$$

Using the last two expressions and the exponential phasor notation for sinusoids,  $j\omega = d/dt$ , and  $j = (-1)^{1/2}$ , (10) reverts to,

$$\begin{aligned} \partial^2 E_y / \partial z^2 &= \mu_0 \epsilon_0 (\partial^2 E_y / \partial t^2) + \mu_0 (\partial \gamma E_y / \partial t), \\ &= -\mu_0 \epsilon_0 \omega^2 E_y + \mu_0 [j\omega (-j N_e q_e^2 / \omega m_e) E_y], \\ &= -\mu_0 \epsilon_0 \omega^2 E_y - j^2 \omega^2 \mu_0 \epsilon_0 (N_e q_e^2 / \omega^2 m_e) E_y, \end{aligned}$$

and finally,

$$\partial^2 E_y / \partial z^2 = \mu_0 \epsilon_0 (1 - N_e q_e^2 / \omega^2 m_e) \partial^2 E_y / \partial t^2. \quad (12)$$

From (12), the coefficient of the right hand side may be written as

$$\Pi^2 = j\omega \mu_0 (\gamma + j\omega \epsilon_0),$$

$$\Pi = j\omega [\mu_0 \epsilon_0 (1 + \gamma / j\omega \epsilon_0)]^{1/2},$$

where, for (12), the solution for a wave traveling in the positive direction is of the form

$$E_y(z, t) = E_0 e^{j\omega t - \Pi z}. \quad (13)$$

Note that if the conductivity of the medium is non-zero and imaginary as for the ionosphere,

$$\Pi = jk = j[k_0 (1 - \omega_p^2 / \omega^2)^{1/2}] = j\omega / v_g,$$

where

$$k = k_0 \eta,$$

$$\eta = (1 - \omega_p^2 / \omega^2)^{1/2}, \text{ the index of refraction,}$$

and

$$\omega_p = N_e q_e^2 / \epsilon m_e, \text{ the plasma frequency.}$$

Thus (13) becomes

$$E_y(z, t) = E_0 e^{j\omega t} e^{-jkz} = E_0 e^{-j\omega(t - z/v_p)},$$

where the wave propagates unattenuated as permitted by  $\mu$  and  $\epsilon$ .

If the conductivity is non-zero and real, then, using  $\gamma = \pm N_e q_e^2 / \omega m_e$  as an example,

$$\Pi = j[k_0(1 \pm \omega_p^2 / j\omega^2)^{1/2}].$$

If expressed as the first term of a Taylor series expansion,

$$\begin{aligned} \Pi &= jk_0 [1 \pm 1/2 (\omega_p^2 / j\omega^2)], \\ &= jk_0 + \omega_p^2 / 2\omega c = jk_0 + h. \end{aligned}$$

Therefore, (13) becomes,

$$\begin{aligned} E_y(z, t) &= E_0 e^{j\omega t} e^{-\Pi z}, \\ &= E_0 e^{j\omega t} e^{-jk_0 z} e^{-hz}, \\ &= E_0 e^{-hz} e^{j(\omega t - k_0 z)}, \end{aligned}$$

where the wave now propagates with some attenuation dependant upon the value of  $h$ , and with some alteration in phase dependant upon  $k$ .

If the conductivity is real and imaginary,

$$E_y(z,t) = E_0 e^{-hz} e^{j(\omega t - kz)},$$

where  $k = k_0 \eta$  as above.

## VITA

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