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BUBBLE MOTION STUDY:
THE WALL EFFECT ON VELOCITY
OF SINGLE AIR BUBBLES
IN METHANOL

33

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BUBBLE MOTION STUDY
THE WALL EFFECT ON VELOCITY OF SINGLE AIR BUBBLES IN METHANOL

SUMMARY

Four tube sizes (1.98, 2.88, 4.06 and 5.90 inch I.D.) were used to study the motion of single air bubbles rising in a column of methanol to determine the effect of the container wall on the steady state rising bubble velocity. Bubbles ranging from 0.0130 to 0.1311 ft. equivalent spherical bubble radius were investigated.

The wall effect was found to be quite significant as indicated by Velocity-radius plots. An equation was proposed, independent of the liquid's physical properties, relating the limiting steady state velocity to the radius of the tube as follows:

$$U_\infty = 0.480\sqrt{ga}$$

Drag coefficients - Reynold's number plots were made which indicate a possible new region of seemingly transitory nature.

A very close comparison and agreement with a previous investigation, using water, was obtained, leading to a more conclusive proof that the wall effect is predominant in determining the steady state rising bubble velocity.

BUBBLE MOTION STUDY: THE WALL EFFECT ON VELOCITY OF SINGLE AIR BUBBLES IN METHANOL

INTRODUCTION

The study of gas bubble motion in liquids has intrigued many investigators as it is an important factor in many physical and chemical operations. Indeed, an ideal illustration of the air lift method would be the Frasch Process to remove liquid sulphur from great depths in the earth. As pointed out by Peebles and Garber[11], in some mass transfer operations it is the very essence of the operation. This is only an illustration to show the importance which bubble motion study contributes.

It would seem that the investigation of gas bubble motion would present an apparently simple problem, however, there has been much investigation with so many and varied results that no one standard of thought prevails on the subject. Peebles and Garber are now attempting to correlate all known data and studies of investigators who have previously sought to understand the process, since there is much need in knowing the mathematical relationships expressing the velocity of a gas bubble moving in a liquid, as a function of all its variables. This was the primary consideration since all previous information was not reliable due to the great diversity in results.

Preliminary investigations of 22 liquids with widely varying physical properties indicated four definite regions of bubble motion[11], with indications of a fifth region where the bubble velocity is constant and independent of the bubble size. This fifth region is considered in the light that quite possibly region 4 is one of a transitory nature. The independence which the size of the bubble has was felt to be due to the wall effect, the primary purpose for this investigation.

Peebles[10] has experimentally and mathematically derived an expression for the steady state velocity concerned primarily with the physical properties of the liquid, thusly:

$$U_\infty = 1.18 \left[\frac{\sigma g g_c}{\rho_l} \right]^{0.25} \qquad \text{(Equation 1)}$$

Wigner[16] derived a similar expression, but Davies and Taylor[5], experimentally and mathematically, show the steady state velocity to be independent of the liquid's physical properties and should only be expressed as a function of the size of the tube. Their equation states:

$$U_\infty = k_1 \sqrt{g\ a} \qquad \text{(Equation 2)}$$

where the constant k_1 is given a value of 0.464 by theoretical treatment and the ratio $V_\infty / \sqrt{g\ a}$ varies from 0.460 to 0.490, with a mean value of 0.464 recommended by experimental treatment using three different sized tubes.

Hullibarger [6], who measured the velocity of air bubbles in water, methanol, and cottonseed oil in 3 inch and 2 inch tubes over bubble sizes in the range

$$0.003 < R_b < 0.032 \text{ ft.}$$

found a tendency toward both an independence of bubble size and of the liquid's physical properties. This independence was later substantiated by Atkins[2], who measured the bubble velocity in water in four tube sizes of 1.98, 2.88, 4.06 and 5.90 inch I.D. A comparison of their results shows the limiting velocity differing from those obtained by Peebles and Garber, and Wigner, only as a function of the tube size - indicating the wall effect to be dominant. In fact, Atkins found the equation

$$U_{oo} = 0.480\sqrt{g\ a}$$

fit the data, obtained by seven investigators using fourteen tube sizes, to an average error of less then 1%.

This present investigation is concerned with observing any similar bubble characteristics between water and methanol using the apparatus which was utilized by Atkins[2]. This similarity, if any, is to be determined by obtaining velocity-radius curves, Drag coefficient-Reynold's number plots for air bubbles ascending through methanol, with special emphasis placed on the largest size bubbles. For this size bubble a point of contradiction

has arisen between investigators in that the bubble velocity is independent of the physical properties of the liquid, and should only be expressed as a function of tube size. It is desired to obtain a more conclusive idea concerning the wall effect of the container on bubble velocity, using a liquid with greatly different physical properties from water.

EXPERIMENTAL INVESTIGATION

Tests were made using four glass tubes of 6 ft. length, having inside diameters equal to 1.98, 2.88, 4.06 and 5.90 inches. Each tube was filled with a sufficient amount of methanol to allow a measured test interval of five feet to be marked on the tube, the bottom mark being placed about 6 inches above the bubble generator to give the bubble sufficient time to reach its steady state velocity before entering the test interval.

Generation of a single bubble in the 2.88, 4.06, and 5.90 inch tubes was accomplished by allowing a steady stream of air to accumulate in a 4 inch stainless steel hemispherical cup placed on the central shaft projecting externally from the generator chamber. With practice, a single large bubble of any size could be released by rotating the hemispherical cup with very little loss due to breaking up of the bubble. For the 1.98 inch tube a 2 inch copper hemispherical cup

was

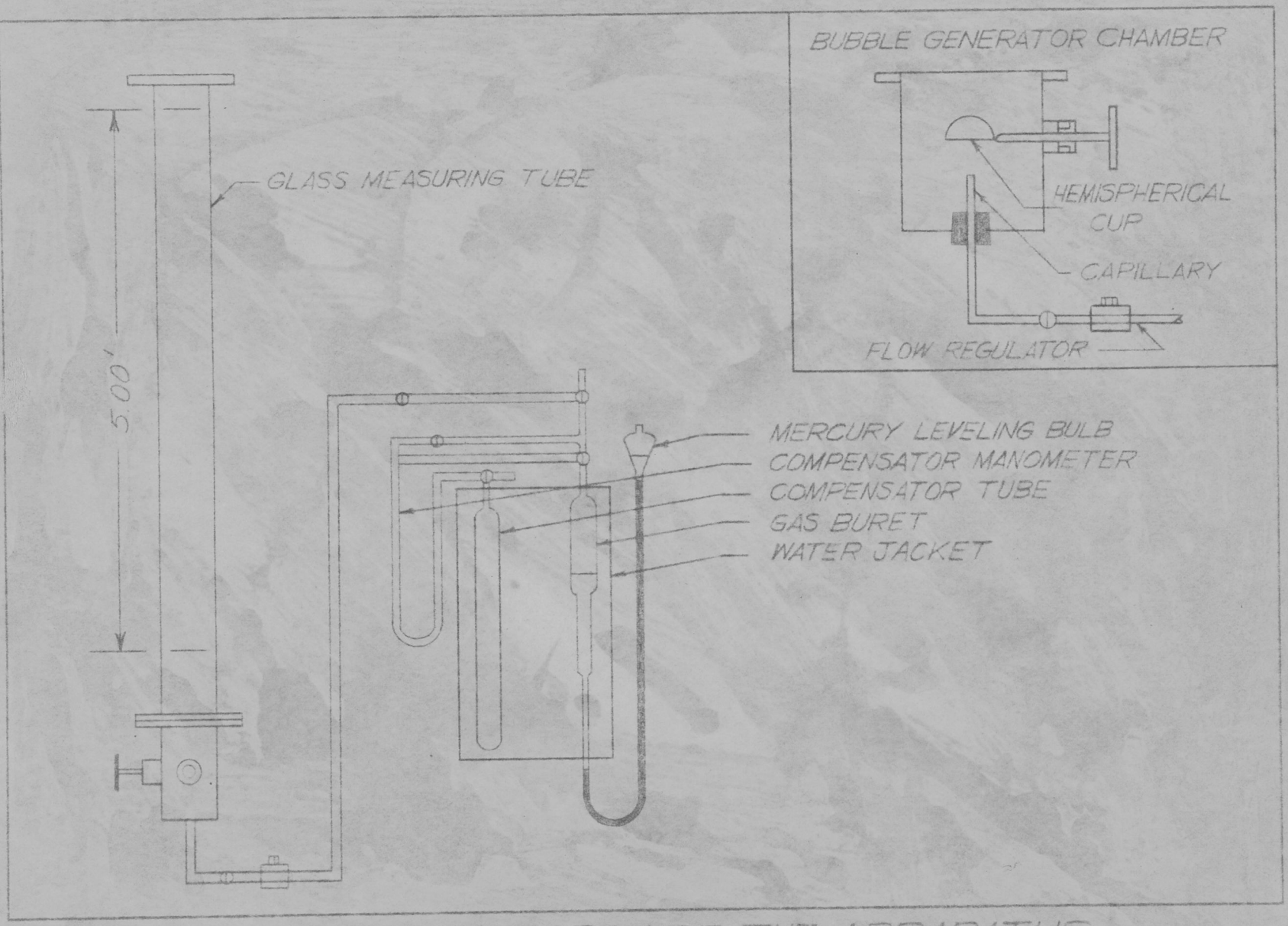
was used in a smaller apparatus of similar design. It was found that in order to produce bubbles of a size capable of filling the tube from wall to wall, a hemispherical cup of diameter equal to the tube diameter was required.

A precision mine-type gas buret equipped with a Hempel-type compensating chamber was used to allow measurement of volumes of each bubble at a known pressure. Temperature fluctuations were reduced by inserting the compensator tube and gas buret in a water jacket. A 100 ml. gas buret graduated so that the lower 6 ml. could be read to 0.05 ml. directly, and estimate to 0.02 ml. A 1/2 inch I.D. piece of copper tubing with a volume in the order of 100 ml. with one end sealed and a 1/8 inch copper tube, silver sorted to the other end, was used as the compensator tube. All valves were Crane stainless steel, packless, bellows-type with 1/4 inch copper tubing soldered in the inlet and outlet ends. The remainder of the air lines were 1/8 inch copper tubing. The flow regulation was accomplished by using a 1/8 inch Imperial brass port valve for off-on regulation. All joints were soldered and painted with Glyptal. The bubble generating apparatus is shown schematically in Figure 1, and a photograph of the apparatus is shown in Figure 2.

The sequence of a bubble run is as follows: (1) start-up: before taking any readings a leak test was made on the

apparatus. This test was made to ascertain that there were no leaks in the system. This test was usually a fifteen minute procedure, whereby pressure was put in the buret by raising the mercury level in the leveling bulb. Having verified that there were no leaks in the system, temperature and barometric pressure recorded, the compensating chamber was shut off from the atmosphere and a series of runs were made.

Each bubble was generated by lowering the mercury leveling bulb and filling the gas buret with air. The valve connecting the gas buret with the atmosphere was then shut off. The pressure in the gas buret was then diverted to the compensator tube where the pressure was equalized in the two chambers as indicated by a manometer. The compensator tube control valve was then closed and the initial buret reading was made. The mercury leveling bulb was then raised, putting pressure in the gas buret, and the Imperial off-on regulator valve was used to control the flow of air into the hemispherical cup. The leveling bulb was next lowered, and the pressure in the gas buret and the compensator chamber were equalized and the final buret reading made. The difference in volume was the volume of air in the bubble at the generating tip. This volume was later adjusted to the volume of the bubble



SCHEMATIC DIAGRAM OF THE APPARATUS

FIGURE 1. SCHEMATIC DIAGRAM OF THE APPARATUS

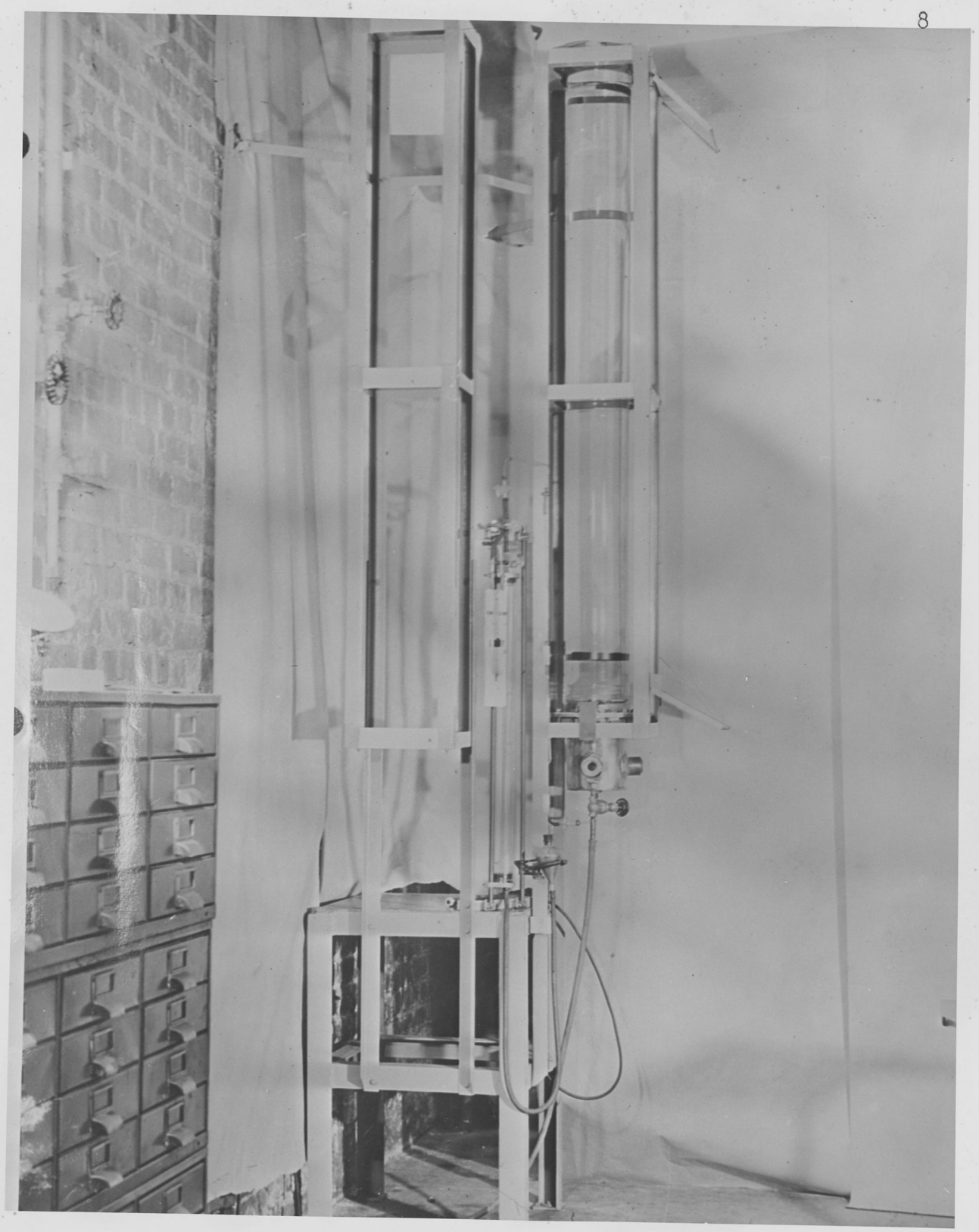


Figure 2. Photograph of Apparatus

under pressure of the liquid head. The air trapped in the hemispherical cup was then released by rotating the central shaft. The time taken for the bubble to pass through the 5 ft. interval was measured with a stop watch.

Procedure 2) was repeated for each bubble.

RESULTS OF THE INVESTIGATION

Notes on The Motion of Bubbles

Observations of bubble motion as related to shape characteristics, confirm to those as reported by Atkins (2) in his observations using water.

The usual classification is as follows:

a. Small bubbles, spherical in shape, moving in straight paths, the very smallest sizes seemingly in accordance to Stoke's law. Experimentation was made primarily on the larger bubbles, therefore few observations were made on the smaller bubbles. Exception was made on a few small sized ones which were detached during the process of generating larger bubbles. The spherical shape and straightness of the path of ascent appear to be accurate unless a larger bubble ascending at the same time does not create any turbulence to disturb the smaller bubble. Any breaking up of these small bubbles did not tend to re-form into a single bubble.

b. Medium sized bubbles usually oblately spheroidal in shape, rose with peculiarly tipped, rocking motion in a roughly spiral path. No notice was made of spiraling in any certain direction. However, it was noted that the bottom of each bubble tended to flare out in a grasping motion toward the wall of the tube.

c. Large bubbles - these bubbles have a tendency toward instability and ascend through the tube with a multitude of small bubbles of varying sizes accompanying in the turbulent wake below the main bubble. These small bubbles, within approximately 6 inches of the main bubble, tended to re-form with it. The remaining bubbles tended to cluster and would not regroup with the main bubble. The general characteristics of these bubbles were that they had spherical tops with ragged lower edges. These bubbles ascended very rapidly with no noticeable rocking or spiraling motion. This group of bubbles was considered up to the point where the edges touched the walls of the tube.

d. Bubbles which filled the tubes - the characteristics of this type of bubble are very easy to observe. These bubbles completely fill the tube and are very well formed, being cylindrical in shape and having rounded tops. These bubbles do not tend to have as noticeable ragged edges as the smaller sized bubbles. In contrast

to bubbles of Type c., they are difficult to break up. All noticeable breaking up of this type bubble was due to shearing effect of the hemispherical cup upon the releasing of the bubble. These small bubbles, however, tended to regroup very rapidly. A bubble of Type b., having been sheared off, tended to ascend and regroup with a velocity much greater than it does when released individually. Bubbles of this type indicate a true constant velocity, whereas bubbles of Types b. and c., seemingly, are in a state of transition.

Bubble Velocity as a Function of Bubble Size

Tables I through IV give a complete summarization of results concerning bubble sizes and velocities.

Figure 3 shows a plot of steady state rising velocity, U, as a function of the equivalent bubble radius, R_b. The bubble sizes range from $0.0130 < R_b < 0.131$ ft. which insure the generation of bubbles of Type d. The characteristic curves agree with those obtained by Atkins[2] in his study using water with a slight decrease in the steady state velocity. It is also noticed that for a given Reynold's number, much higher drag coefficients are realized as the tube size decreases. The characteristic curves tend to point out that the steady state velocity is approaching independence of the characteristics of the liquid. The

TABLE I

SUMMARY OF EXPERIMENTAL DATA FOR
1.98
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
159	19.5	0.685	3.541	3520
154	19.8	0.685	3.600	3604
174	20.1	0.694	3.560	3702
175	22.32	0.714	3.733	4238
151	23.93	0.714	4.012	4530
153	23.93	0.735	3.779	5720
171	24.55	0.725	3.979	4730
160	25.35	0.735	4.012	4946
156	26.2	0.540	4.142	5118
157	26.4	0.735	4.170	5158
144	27.5	0.714	4.600	5484
148	27.75	0.746	4.245	5496
169	28.4	0.735	4.495	5550
147	28.55	0.735	4.513	5562
152	29.6	0.735	4.677	5600
162	29.6	0.735	4.677	5762
150	29.85	0.725	4.846	5758
173	30.65	0.735	4.849	5984
167	30.9	0.735	4.883	6036
155	31.05	0.735	4.910	5800
161	31.2	0.735	4.939	6080
158	31.6	0.746	4.999	6242
149	31.62	0.735	5.010	6180
146	31.8	0.735	5.031	6198
139	32.2	0.735	5.092	6282
145	32.4	0.725	5.251	6240
168	32.6	0.725	5.291	6280
143	32.9	0.758	4.885	6610
142	33.4	0.758	4.951	6720
140	33.6	0.769	4.850	6876
141	34.7	0.758	5.148	6982
172	37.2	0.769	5.365	7600
163	37.45	0.735	5.935	7318

TABLE I
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
1.98
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
164	37.7	0.769	5.442	7682
88	38.7	0.746	5.932	7280
117	39.5	0.769	5.700	7889
87	39.8	0.769	5.745	7740
96	39.8	0.758	5.900	7350
119	40.1	0.758	5.950	7891
86	40.7	0.746	6.240	7650
128	40.7	0.735	6.440	7672
92	40.8	0.769	5.892	7910
93	40.9	0.758	6.558	7830
126	41.2	0.746	6.310	7895
129	41.2	0.758	6.112	8022
133	41.2	0.758	6.112	8022
17	41.8	0.758	6.199	8573
48	41.8	0.758	6.199	8734
67	41.8	0.758	6.199	8670
127	42.4	0.758	6.291	8248
64	42.5	0.769	6.141	8924
90	42.6	0.769	6.148	8260
91	42.7	0.769	6.155	8300
115	43.8	0.758	6.499	8610
132	43.8	0.758	6.499	8520
131	43.9	0.758	6.500	8545
116	44.6	0.758	6.618	8780
89	44.8	0.769	6.460	8700
114	44.8	0.769	6.460	8945
124	45.2	0.758	6.710	8800
18	45.3	0.769	6.540	9445
66	45.8	0.758	6.794	9496
130	45.8	0.746	7.010	8755
61	45.9	0.758	6.805	9500
59	46.3	0.769	6.675	9720

TABLE I
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
1.98
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
94	46.4	0.769	6.690	9010
58	46.5	0.769	6.710	9776
21	46.6	0.758	6.915	9640
25	46.6	0.769	6.730	9800
50	46.7	0.769	6.742	9898
85	47.0	0.769	6.788	9880
23	47.2	0.758	7.000	9780
125	47.2	0.758	7.000	9170
60	47.4	0.746	7.255	9640
166	47.4	0.758	7.035	9524
20	47.6	0.735	7.535	9460
47	47.6	0.769	6.870	10080
63	47.6	0.769	6.870	10000
62	48.1	0.769	6.945	10102
83	48.5	0.769	7.000	10200
44	49.2	0.769	7.100	10430
69	49.2	0.758	7.300	10196
556	49.3	0.806	6.475	10870
81	49.7	0.769	7.165	10442
82	49.8	0.769	7.185	10480
43	49.9	0.769	7.195	10562
113	50.5	0.758	7.498	9935
104	50.7	0.769	7.310	10110
24	51.3	0.758	7.610	10630
109	51.5	0.758	7.645	10120
57	51.7	0.769	7.455	10876
40	51.8	0.758	7.685	10720
42	52.5	0.758	7.795	10884
137	53.0	0.758	7.855	10310
45	53.5	0.769	7.715	11324
107	54.5	0.758	8.092	10710
112	55.0	0.758	8.155	10810

TABLE I
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
1.98
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
78	55.3	0.769	7.985	11624
38	56.4	0.758	8.355	11490
39	56.4	0.758	8.355	11490
105	56.7	0.769	8.190	11300
46	56.8	0.758	8.435	11876
33	56.9	0.769	8.200	11950
30	57.5	0.769	8.299	12090
31	57.5	0.758	8.415	11904
79	58.4	0.746	8.942	11900
53	58.6	0.769	8.455	12302
108	58.9	0.758	8.725	11590
51	59.8	0.769	8.635	12550
27	60.0	0.746	9.190	12220
34	60.4	0.769	8.710	12696
41	60.4	0.758	8.951	12504
55	60.4	0.769	8.710	12696
36	60.5	0.769	8.725	12716
54	60.6	0.769	8.745	12740
106	60.8	0.769	8.760	12120
28	61.0	0.769	8.800	12800
29	61.0	0.758	9.048	12630
32	61.0	0.746	9.340	12450
70	61.0	0.758	9.048	12630
99	61.0	0.735	9.642	11310
35	61.1	0.769	8.815	12820
37	61.1	0.758	9.055	12644
97	61.1	0.758	9.055	11690
76	61.3	0.758	9.095	12698
77	61.9	0.769	8.855	12900
98	61.4	0.758	9.100	11780
100	61.4	0.758	9.100	11780
73	61.5	0.758	9.120	12740

TABLE I
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
1.98
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
101	61.5	0.769	8.873	11920
74	61.6	0.758	9.145	12780
75	61.9	0.769	8.932	13020
121	61.9	0.781	8.625	12400
135	61.9	0.758	9.175	12050
136	62.0	0.746	9.495	11880
72	62.1	0.769	8.955	13040
134	62.1	0.758	9.215	12100
138	62.1	0.758	9.215	12100
102	62.2	0.746	9.525	11700
120	62.5	0.758	9.265	12280
122	63.0	0.758	9.350	12270

TABLE II

SUMMARY OF EXPERIMENTAL DATA FOR
2.88
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
299	14.97	0.625	3.268	2401
294	15.95	0.685	2.903	2810
298	22.0	0.704	3.780	2972
296	23.7	0.794	3.210	4942
295	24.8	0.781	3.468	4968
297	25.6	0.833	3.149	5475
293	25.9	0.820	3.283	5465
300	27.7	0.833	3.404	5929
303	28.78	0.847	3.419	6236
302	33.38	0.877	3.701	7481
304	34.9	0.909	3.601	8100
301	35.2	0.877	3.908	7895
291	36.7	0.877	4.075	8268
292	37.2	0.877	4.139	8388
280	39.9	0.909	4.119	9312
305	40.3	0.893	4.312	9192
307	40.7	0.893	4.355	9275
279	42.9	0.893	4.590	9845
306	43.4	0.909	4.473	10090
290	43.6	0.877	4.848	9830
308	45.6	0.893	4.885	10420
310	48.4	0.926	4.815	11430
311	48.5	0.926	4.828	11450
312	51.1	0.909	5.275	11860
309	52.1	0.926	5.192	12370
315	52.3	0.943	5.025	12590
278	54.6	0.909	5.642	12730
318	55.8	0.943	5.365	13430
317	56.5	0.943	5.435	13600
319	57.9	0.943	5.560	13950
314	58.4	0.943	5.610	14040
316	58.8	0.926	5.850	14150
313	63.6	0.943	6.111	15310

TABLE II
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
2.88
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
320	63.8	0.943	6.138	15380
321	65.3	0.943	6.275	15720
322	65.6	0.909	6.778	15210
323	66.1	0.943	6.350	15390
327	67.5	0.943	6.493	17175
325	70.0	0.943	6.732	16860
328	70.5	0.943	6.772	17960
324	71.3	0.943	6.849	17150
326	72.5	0.943	6.961	18480
329	75.0	0.943	7.202	19080
330	76.1	0.943	7.310	19390
332	77.3	0.943	7.425	19650
331	78.5	0.943	7.548	19990
333	79.9	0.943	7.675	20350
334	80.2	0.943	7.715	20400
336	82.5	0.926	8.210	20650
335	83.6	0.943	8.043	21300
341	83.9	0.943	8.055	21390
342	86.6	0.943	8.330	22050
340	86.7	0.909	8.949	21250
339	90.8	0.943	8.715	23150
337	92.6	0.980	8.245	24530
338	95.2	0.909	9.821	23320
343	97.3	0.926	9.685	24380
345	97.7	0.943	9.398	24900
344	98.3	0.943	9.448	25000
346	99.4	0.926	9.890	24900
347	101.6	0.926	10.100	25400
348	106.7	0.943	10.230	27120
349	108.8	0.943	10.42	27610
350	112.1	0.909	11.59	27580
351	114.1	0.926	11.39	28590

TABLE II
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
2.88
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
352	117.6	0.943	11.29	29900
353	122.2	0.926	12.185	30600
355	125.1	0.943	12.02	31820
354	126.8	0.943	12.18	32220

TABLE III

SUMMARY OF EXPERIMENTAL DATA FOR
4.06
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
364	22.5	0.820	2.858	5215
361	23.8	0.820	3.020	5511
360	25.6	0.877	2.540	6350
362	27.3	0.909	2.819	7010
363	28.32	0.893	3.031	7150
359	28.58	0.909	2.943	7330
357	33.59	0.961	3.100	9135
358	34.0	0.961	3.140	9245
356	37.2	0.961	3.439	10100
365	39.9	0.961	3.685	10840
373	42.5	1.000	3.621	12000
370	47.1	1.000	4.023	13300
366	47.4	1.020	3.892	13650
369	47.8	1.000	4.075	13520
372	52.0	1.064	3.912	15630
367	52.4	1.041	3.730	15420
374	56.4	1.041	4.438	16610
368	57.4	1.064	4.323	17290
371	57.7	1.064	4.345	17350
376	59.4	1.041	4.665	17500
375	60.9	1.111	4.210	19150
377	62.8	1.064	4.733	18900
379	66.9	1.041	5.261	19700
380	68.4	1.041	5.338	20180
378	70.3	1.041	5.530	20750
381	73.7	1.087	5.318	22700
382	74.5	1.064	5.610	22410
413	75.7	1.064	5.699	22200
383	75.9	1.087	5.465	25400
416	77.2	1.041	6.075	22200
384	78.2	1.111	5.412	23900
387	79.7	1.041	6.270	23450
386	79.9	1.087	5.755	24550

TABLE III
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
4.06
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
415	80.1	1.111	5.545	24550
385	81.1	1.087	5.850	24950
396	81.1	1.111	5.610	26400
412	81.1	1.111	5.610	24800
414	81.3	1.111	5.625	24880
390	82.9	1.087	5.968	26410
388	84.9	1.111	5.865	26650
389	85.5	1.111	5.910	27820
391	90.6	1.087	6.540	28900
392	92.4	1.064	6.950	28800
393	94.4	1.111	6.525	30700
394	96.1	1.041	7.560	29340
395	97.9	1.111	6.765	31800
397	100.0	1.064	7.510	30750
398	102.4	1.111	7.085	32820
399	102.8	1.064	7.740	31580
411	102.8	1.087	7.390	31100
400	105.9	1.087	7.632	33210
401	108.1	1.087	7.800	34000
410	111.0	1.087	8.000	33000
402	113.9	1.087	8.210	35750
403	116.7	1.087	8.410	34950
408	117.9	1.087	8.495	35500
406	120.5	1.087	8.685	36050
404	122.7	1.087	8.835	36700
407	123.0	1.087	8.860	36800
405	126.0	1.087	9.080	37700
409	128.0	1.087	9.220	38300

TABLE IV

SUMMARY OF EXPERIMENTAL DATA FOR
5.90
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
226	13.03	0.649	2.643	3405
229	20.10	0.676	3.755	5250
224	20.41	0.725	3.418	5345
225	20.41	0.781	2.860	5345
227	20.41	0.735	3.232	5345
223	26.8	0.893	2.869	7000
217	28.38	0.893	3.038	7405
228	30.0	0.943	2.880	7840
218	30.6	1.000	2.610	7999
220	30.65	0.943	2.945	8010
219	32.79	0.980	2.909	8552
222	33.19	0.961	3.060	8712
221	34.7	1.020	2.848	9065
216	37.0	1.020	3.039	9658
215	40.0	1.020	3.281	10450
213	40.1	0.893	4.285	10488
214	41.5	1.064	3.121	10845
212	46.5	1.087	3.358	12150
211	48.2	1.087	3.478	12590
206	51.9	1.000	4.432	13560
209	52.6	1.111	3.638	13750
210	52.6	1.111	3.638	13750
208	54.1	1.136	3.581	14110
207	55.7	1.111	3.850	14560
205	56.5	1.163	3.560	14780
202	57.4	1.163	3.615	15000
204	60.6	1.163	3.820	15810
197	63.4	1.205	3.720	16560
200	63.6	1.190	3.838	16610
203	67.3	1.205	3.952	17595
201	67.4	1.163	4.249	17600
260	70.2	1.250	3.830	18320
252	70.9	1.190	4.268	18510

TABLE IV
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
5.90
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
199	74.6	1.250	4.075	19500
198	74.8	1.190	4.515	19535
268	75.9	1.190	4.568	19810
251	78.6	1.250	4.292	21667
196	78.7	1.190	4.749	20590
275	79.7	1.250	4.575	20815
194	80.3	1.250	4.382	20990
276	80.8	1.163	5.091	21110
193	83.3	1.190	5.025	21785
190	84.6	1.282	4.391	22130
192	85.6	1.282	4.449	22398
191	86.6	1.25	4.735	22610
195	89.3	1.282	4.641	23330
188	89.4	1.282	4.642	23380
186	90.1	1.282	4.681	23585
189	90.25	1.25	4.925	23599
266	91.1	1.250	4.975	23805
180	92.0	1.282	4.778	24050
185	93.4	1.282	4.849	24400
243	95.0	1.250	5.185	25800
272	95.5	1.282	4.959	25000
273	95.6	1.282	4.965	25050
184	96.1	1.25	5.248	25175
271	96.4	1.282	5.000	25198
242	97.0	1.282	5.040	26375
253	98.3	1.282	5.110	25695
247	98.4	1.351	4.600	27100
182	98.7	1.25	5.391	25800
183	99.7	1.25	5.448	26090
235	100.4	1.250	5.485	26205
234	100.7	1.282	5.225	26295
277	100.9	1.282	5.235	26320

TABLE IV
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
5.90
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
237	102.5	1.282	5.328	27685
244	102.5	1.282	5.328	27615
255	102.9	1.282	5.364	26890
187	105.2	1.282	5.465	27322
265	105.9	1.282	5.495	27610
267	107.0	1.250	5.843	27993
176	107.1	1.316	5.280	28000
264	107.1	1.282	5.562	28000
240	109.0	1.282	5.656	28428
246	110.9	1.282	5.750	30366
261	112.2	1.282	5.838	29335
239	113.0	1.282	5.862	30585
259	113.8	1.282	5.891	29700
256	114.0	1.282	5.918	29798
250	115.1	1.282	5.978	31782
230	116.	1.282	6.020	30310
258	116.0	1.282	6.020	30310
270	116.0	1.282	6.020	30310
254	116.1	1.316	5.625	30395
248	116.6	1.351	5.451	32100
263	118.9	1.282	6.165	31050
232	119.8	1.282	6.215	30255
249	120.0	1.429	5.010	33025
274	120.0	1.282	6.215	31392
241	120.9	1.282	6.271	32800
178	122.4	1.351	5.738	31996
245	123.0	1.282	6.391	33400
177	123.8	1.282	6.425	32310
262	125.1	1.282	6.500	32720
181	125.9	1.282	6.538	32825
236	126.	1.250	6.875	32900
269	126.5	1.282	6.567	33080

TABLE IV
(Continued)

SUMMARY OF EXPERIMENTAL DATA FOR
5.90
INCH TUBE

Run Number	Equivalent Radius R_b ft. x 10^3	Steady State Velocity U ft./sec.	Drag Coefficient C_D	Reynolds Number R_e
257	127.3	1.282	6.615	33250
238	127.5	1.282	6.625	34425
233	127.7	1.250	6.965	33352
231	127.9	1.250	6.972	33400
179	131.1	1.351	6.142	34275

CRANES CREST

steady state velocity for bubbles flowing through water and flowing through methanol tend to approach very close to each other for corresponding tube sizes, though the viscosity and surface tension for the liquids differ considerably. The steady state velocity for bubbles $0.025 \text{ ft.} \lesssim R_b \lesssim 0.060$ ft. rising in a 1.98 inch I.D. tube of water, levels off at 0.782 ft./sec., whereas, for the same size tube filled with methanol, the steady state velocity levels off at 0.769 ft./sec.

A table of comparison is given for the steady state bubble velocity in water and methanol:

TABLE V

Tube Size	Bubble Size R_b	Steady State Velocity U_∞, ft./sec.	
(I.D.) inches	ft. x 10^3	Water	Methanol
1.98	35 - 60	0.782	0.769
2.88	50 -125	0.962	0.943
4.06	70 -125	1.111	1.087
5.90	85 -125	1.316	1.282

Bubbles in each tube exhibited a rapid decrease in velocity from the maximum reported by other investigators (1),(7),(11), at a value of $R_b \sim 0.003$ ft. to a minimum velocity of about 0.7 ft./sec. for the tube sizes and in this work. The minimum velocity, in water, occurs uniformly at a bubble size of $R_b \lesssim 0.0125$ ft., and the minimum velocity in methanol occurs uniformly at a bubble size of $R_b \lesssim 0.015$ ft., although results for tubes smaller

than 1.98 inches would indicate a slightly lower velocity, since apparently the wall effect has become significant for bubbles of a smaller size in these tubes. For bubbles larger than $R_b > \sim 0.015$ ft., the curves gradually become specific for each tube size, signifying the increasing importance of the wall effect which would tend to establish a constant velocity characteristic of the tube size. Table VI gives the values of the ratio of equivalent bubble radius to tube radius where the constant velocity begins.

TABLE VI

VALUES OF $\frac{R_b}{a}$ WHERE BUBBLE VELOCITY BECOMES CONSTANT

Tube Diameter	a	R_b	$\frac{R_b}{a}$
inches	ft.	ft.	
5.90	0.246	0.085	0.346
4.06	0.169	0.075	0.444
2.88	0.120	0.055	0.458
1.98	0.0825	0.040	0.485

These values are approximations, but show the ratio R_b/a to be increasing with decreasing tube sizes with an apparent lessening of the rate of increase for tubes in the 2 inch to 4 inch I.D. range. This, seemingly, would indicate that the wall effect becomes more pronounced in the smaller tube sizes. This is purely speculative, since only one liquid was used in a limited tube size range.

Drag Coefficient As a Function of Reynold's No.

Tables I through IV give a summary of drag coefficients and Reynold's number for each bubble.

It has been the practice of most investigators to represent data on bubble motion studies in terms of the variation of an arbitrarily defined drag coefficient, C_D, with the Reynold's number, R_e, where

$$C_D = \frac{8}{3} g \frac{R_b^2}{U}$$

and

$$R_e = \frac{2 R_b U \rho_1}{\mu}$$

(see p.37 for nomenclature and explanation of symbols).

This was the basis for the classification of bubble motion into the four regions described by Peebles and Garber[11], and followed by Atkins[2]; therefore a similar plot was made to determine if the wall effect was noticeable. Figure 4 is such a plot. The range of variables were:

$$2.5 \sim< C_D <\sim 12.25$$

$$2500 \sim< R_e <\sim 3900$$

This plot also shows the outstanding significance of the wall effect, and indicates the probable existence of a fifth region of bubble motion (using the four regions of Peebles and Garber as a basis). This plot, like the constant velocity portion of the velocity-radius curves

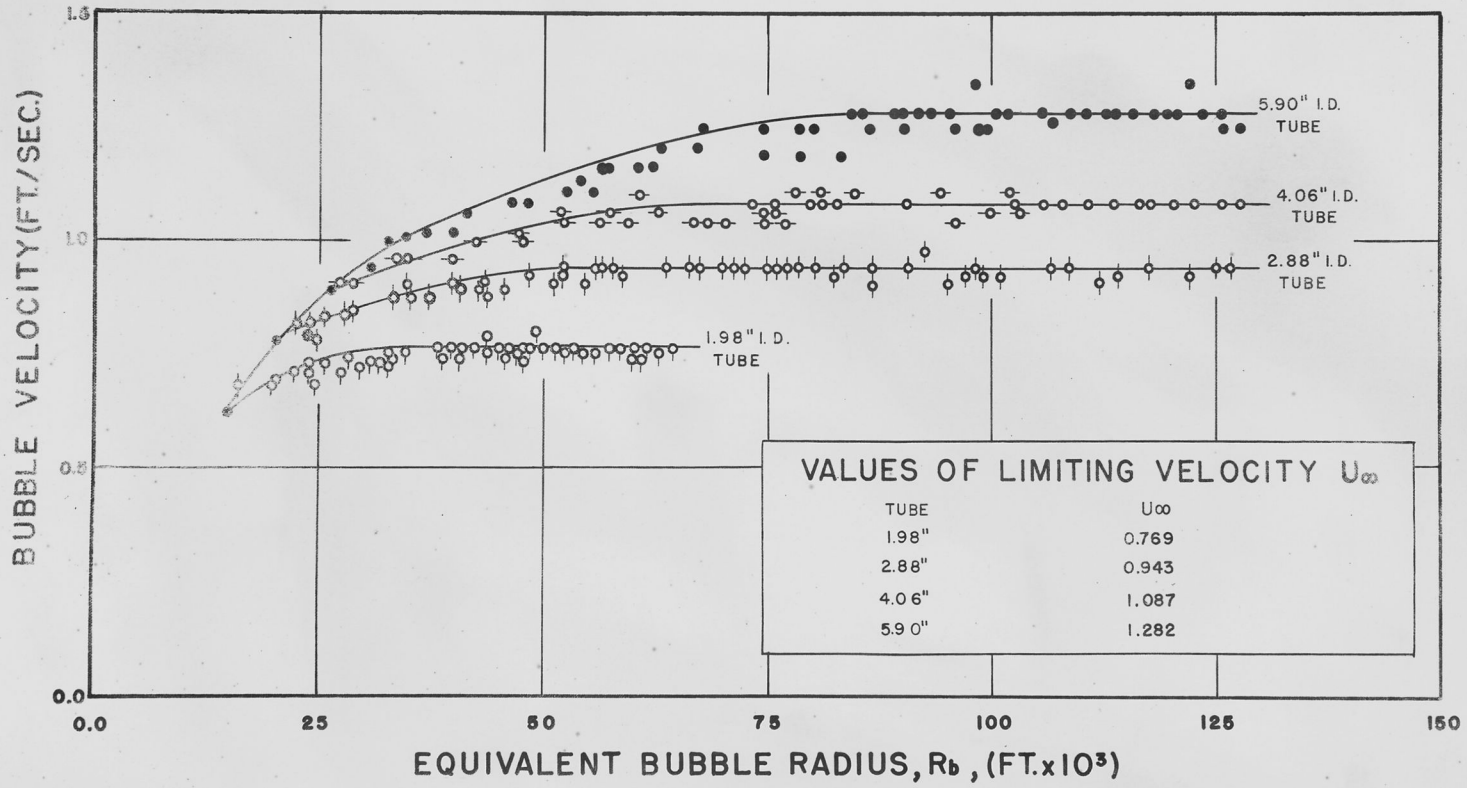


Figure 3. Bubble Velocity vs. Equivalent Radius

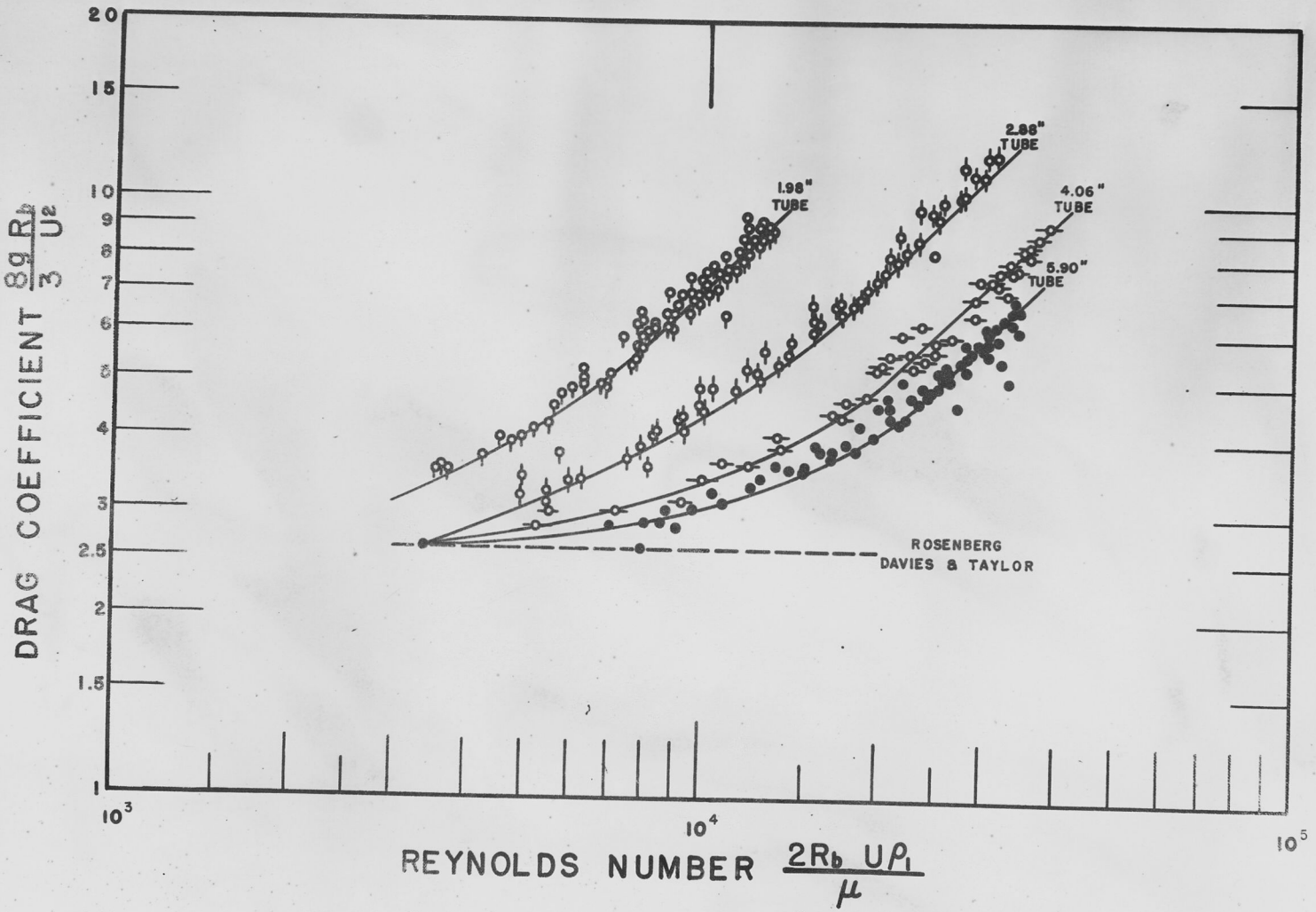


Figure 4. Drag Coefficient vs. Reynolds Number

TABLE VII

MEASURED TERMINAL VELOCITIES AS COMPARED TO CALCULATED VALUES USING EQUATIONS OF THE FORM $U_{\infty} = k_1 \sqrt{ga}$

Reference *	Diameter of Tube (inches)	U_{∞} Measured (ft/sec.)	U_{∞} by Taylor $U_{\infty} = 0.460\sqrt{ga}$	% Variation of Measured U_{∞} From Taylor's Value
Ward & Kessler	0.94	0.52	0.516	+ 0.78
Ward & Kessler	1.0	0.59	0.532	+ 10.89
O'Brien & Gosline	1.18	0.59	0.578	+ 2.08
Miyagi	1.89	0.75	0.732	+ 2.46
Atkins	1.98	0.782	0.749	+ 4.41
Author	1.98	0.769	0.747	+ 2.86
Ward & Kessler	2.0	0.82	0.754	+ 8.89
Owens	2.12	0.81	0.775	+ 4.52
O'Brien & Gosline	2.24	0.78	0.797	- 2.11
Atkins	2.88	0.962	0.902	+ 6.65
Author	2.88	0.943	0.903	+ 4.24
Hoefer	3.07	1.08[b]	0.934	+ 15.64
Atkins	4.06	1.111	1.072	+ 3.64
Author	4.06	1.087	1.082	+ 0.458
Atkins	5.90	1.316	1.292	+ 1.86
Author	5.90	1.282	1.293	- 0.852
Byrn	5.9[a]	1.15[b]	1.292	- 11.00
O'Brien & Gosline	6.0	1.31[b]	1.305	+ 0.38
			Average-	+ 3.875

* All references other than author's are from O'Brien and Gosline (7)

[a] Square sectioned column

[b] Limiting velocity slightly higher

TABLE VII
(Continued)

MEASURED TERMINAL VELOCITIES AS COMPARED TO CALCULATED VALUES USING EQUATIONS OF THE FORM $U_{\infty} = k_1 \sqrt{ga}$

Reference *	Diameter of Tube (inches)	$\frac{U_{\infty} \text{ Measured}}{\sqrt{ga}}$	U_{∞} Calculated by $U_{\infty} = 0.480\sqrt{ga}$	% Variation of Measured U_{∞} from $U_{\infty} = 0.480\sqrt{ga}$
Ward & Kessler	0.94	0.463	0.539	- 3.53
Ward & Kessler	1.0	0.517	0.555	+ 6.31
O'Brien & Gosline	1.18	0.469	0.604	- 0.99
Miyagi	1.89	0.472	0.764	- 1.83
Atkins	1.98	0.480	0.782	- 0.00
Author	1.98	0.473	0.780	- 1.42
Ward & Kessler	2.0	0.501	0.786	+ 4.33
Owens	2.12	0.481	0.808	+ 0.25
O'Brien & Gosline	2.24	0.450	0.831	- 6.14
Atkins	2.88	0.568	0.943	+ 2.02
Author	2.88	0.481	0.941	+ 0.21
Hoefer	3.07	0.532	0.974	+ 9.82
Atkins	4.06	0.477	1.120	- 0.81
Author	4.06	0.462	1.129	- 3.72
Atkins	5.90	0.468	1.350	- 2.52
Author	5.90	0.456	1.349	- 4.96
Byrn	5.9[a]	0.409	1.350	- 14.82
O'Brien & Gosline	6.0	0.462	1.361	- 3.75
	Average	0.479		- 1.20

* All references other than author's are from O'Brien & Gosline (7).

[a] Square sectioned column.

[b] Limiting velocity slightly higher.

by Davies and Taylor[5], for the tube sizes used in this experiment is only - 1.68%. However, the average per cent variation of the measured limiting velocity from the limiting velocity calculated for the fourteen tube sizes, using equation (2) was + 3.88%. A modification of this constant, k_1, to a value $k_2 = 0.480$ fits the data, of all the referred to investigators, to an average error of approximately 1%. This indicates a more realistic value for the limiting velocity would be obtained by using an equation

$$U_{oo} = 0.480 \sqrt{g\ a}\ .$$

This equation was also recommended by Atkins[2]. Such close agreement between two investigators, using two liquids with different physical characteristics in the same size columns, tends to indicate the wall effect is of increasing importance in the value obtained for the limiting velocity.

According to Davies and Taylor[5], and Rosenberg[14], who measured bubble velocities in large tanks, a constant drag coefficient of

$$C_D = 2.6$$

was applicable for Reynold's number from $3000 \sim< R_e <\sim 20000$, (see Figure 4), but the present data, also obtained by Atkins, indicate a lack of consistancy. This inconsistancy indicates the wall effect to be more pronounced than hitherto realized, since the apparent trend for larger tube

sizes is to approach a value of $C_D = 2.6$ as a lower limit, and gradually increase to a value of $R_e > \sim 20000$ where indications point toward a transition region, and the wall effect becomes quite pronounced. In the smaller size tubes the lower limit of C_D is found to be somewhat larger than the minimum C_D for large tanks, and increases quite rapidly, which would indicate the wall effect to be increasing in importance with decreasing tube size.

CONCLUSIONS AND RECOMMENDATIONS

Though the results of this investigation have slight variations (a reflection of the difficulty in measuring with a stopwatch the high velocities obtained by the large bubbles over the short test interval of five feet), sufficient consistancy is obtained to indicate the wall effect to be much more significant than has heretofore been realized. Indications, obtained in this investigation, point toward the wall effect taking place earlier than realized, and eventually being the dominant factor in determining the limiting velocity of the bubble ascending through the liquid. In fact, the wall effect tends to be more significant as tube sizes decrease, and the limiting velocity of the bubble becomes more independent of the characteristics of the liquid. If a general correlation of data from all previous investigations were considered

in this new light, a more concrete conclusion might be obtained concerning bubble motion in liquids in containers of varying sizes and shapes. It must be pointed out that this conclusion is based on the study of liquids independent of great fluctuations in temperature.

According to Peebles and Garber[11], and a point of contradiction, the limiting velocity in region 4 is independent of the wall effect, and defined only in terms of the liquid's physical properties. (See Equation 1, p. 2). Hullibarger[6], who made his investigation using three different liquids, indicates the same velocity for this region regardless of the physical properties of the liquid. If this region 4 is one of a transitory nature (indications seem to verify such phenomenon), connecting region 3 with region 5, the difficulty might be solved and more consistancy between investigators obtained.

It would be most unfair to list recommendations for further study here since a very clear, concise list is given by Atkins[2] on pages 32 - 34 in his report.

However, an important aspect to consider might be to study the effect on bubble motion caused by various materials of column structure.

NOMENCLATURE

a	Tube inside radius, ft.
C_D	Drag coefficient, $\frac{8}{3} g \frac{R_b}{U^2}$, dimensionless
g	Gravitational acceleration, 32.14 ft./sec.^{2}
g_c	Gravitational conversion factor between force and mass, 32.17 lbm. ft./sec. lbf.
h	Head of liquid, in the units indicated
k_1, k_2	Arbitrary constants
P	Total pressure on bubble at generation tip, atm.
P_a	Atmospheric pressure, in the units indicated
P_l	Pressure due to liquid head, atm.
R_b	Equivalent spherical gas bubble radius, ft.
R_e	Reynolds number, $2 R_b U \rho_l / \mu$, dimensionless
t	Time of rise of bubble through test interval, sec.
U	Steady state, rising bubble velocity, ft./sec.
U_∞	Terminal, or limiting, steady state, rising bubble velocity, ft./sec.
V_1	Volume of bubble at pressure in gas buret, ml.
V_2	Volume of bubble at generating tip, in units indicated.
γ	Liquid surface tension, lbf/ft.
μ	Liquid viscosity, lbm/ft. sec., or if accompanied by the figures 0.000672 or 1/1488, the viscosity is given in centipoises, c.p.
ρ_l	Liquid density, lbm/ft.^{3}

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APPENDIX

APPENDIX A

SAMPLE CALCULATIONS

Item No.	Item Name, Symbol, and Location	Value and Units
	Calculation of Equivalent Bubble Radius All Values From Run No. 255 p. 7721, Original Data	
(1)	Atmospheric Pressure, Pa, Original Data, p. 7721	= 738.5 mm. Hg.
(2)	Atmospheric Pressure, Pa, Item (1) x Conversion Factor (738.5 mm. Hg.) x (1.316×10^{-3} atm/mm. Hg.)	= 0.9719 atm.
(3)	Head of Liquid, h, Original Data p. 7721	= 6'-0"
(4)	Temperature, θ , Original Data p. 7721	= 74° F
(5)	Density of Liquid, at Temperature θ, , p.187 Perry(13)	= 49.3 lbm/ft.^{3}
(6)	Pressure Due to Liquid, P_1, Item (3) x Item (5) x Conversion Factor (h) (6.0 ft.) (49.3 lbm/ft.^{3}) (4.7199 ft.^{2} atm/lbm.)	= 0.1396 atm.
(7)	Total Pressure on Bubble at Generating Tip, P, Item (2) + Item (6) (0.9719 atm.) + (0.1396)	= 1.1115 atm.
(8)	Volume of One Bubble at Pressure in Buret, V_1, Original Data, p. 7721	= 143.90 ml.

APPENDIX A (Continued)

SAMPLE CALCULATIONS

Item No.	Item Name, Symbol, and Location	Value and Units

Calculation of Equivalent Bubble Radius
All Values From Run No. 255
p. 7721, Original Data

(9) Volume of Bubble at Generating Tip, V_2,
Item (2) x Item (8) x Conversion Factor
Item (7)

$$\frac{P_a\ (V_1)\ (3.531 \times 10^{-5}\ \text{ft.}^3/\text{ml.})}{P}$$

$$\frac{(0.9719\ \text{atm.})\ (143.90\ \text{ml.})\ (3.531 \times 10^{-5}\ \text{ft.}^3/\text{ml.})}{(1.1115\ \text{atm.})}$$

$$= 44.4 \times 10^{-4}\ \text{ft.}^3$$

(10) Equivalent Radius of Bubble, R_b, (Radius of Sphere With Volume Equal to V_2)
Volume of Sphere = $4/3\ \pi r^3 = V$

$$r = \sqrt[3]{3/4\pi\ V} = \sqrt[3]{0.2387\ V}$$

$$R_b = \sqrt[3]{0.2387\ V_2}$$

$$R_b = (\text{Item (9)} \times 0.2387)^{1/3}$$

$$R_b = (44.4 \times 10^{-4}\ \text{ft.}^3 \times 0.2387)^{1/3}$$

$$= 102.9 \times 10^{-3}\ \text{ft.}$$

(11) Time of 5 foot rise, t, Original Data
p. 7721

$$= 3.9\ \text{sec.}$$

(12) Steady State Bubble Velocity,

$$U_\infty, \frac{5\ \text{ft.}}{\text{Item (11)}} = \frac{5\ \text{ft.}}{3.9\ \text{sec.}} = 1.282\ \text{ft./sec.}$$

Calculation of Experimental Drag Coefficient

$$C_D = 8/3\ g\ \frac{R_b}{U^2}$$

APPENDIX A (Continued)

SAMPLE CALCULATIONS

Item No.	Item Name, Symbol, and Location	Value and Units

Calculation of Equivalent Bubble Radius
All Values From Run No. 225
p. 7721, Original Data

(13) Drag Coefficient, C_D, $\frac{\text{Item (10)}}{\text{Item (12)}^2}$ x conversion Factor x Constant

$$\frac{(102.9 \times 10^{-3} \text{ ft.})}{(1.282 \text{ ft./sec.})^2} (32.14 \text{ ft./sec.}^2) (8/3)$$

$$= 5.364$$

Calculation of Reynold's Number

$$R_e = \frac{2 R_b U \rho_1}{\mu}$$

(14) Liquid Viscosity, , International Critical Tables
At Temperature θ, Item (4) = 0.561 c.p.

(15) Reynold's Number, R_e,

$$\frac{\text{Item (10) x Item (12) x Item (5) x Constant}}{\text{Item (14) x Conversion Factor}}$$

$$\frac{(102.9 \times 10^{-3} \text{ ft.}) (1.282 \text{ ft./sec.}) (49.3 \text{ lbm/ft.}^3) (2)}{(0.561 \text{ c.p.}) \left(\frac{1}{1488} \text{ lbm/ft. sec. c.p.}\right)}$$

$$R_e = 26{,}890$$

APPENDIX B

The following is the method used for computing the straight line portions of the C_D versus R_e plots in figure .

$$C_D = 8/3\ g \frac{R_b}{U_\infty^2}$$

$$R_e = \frac{2\ R_b\ U_\infty \rho_1}{\mu}$$

and

$$\frac{C_D}{R_e} = 4/3\ g \frac{\mu}{\rho_1\ U_\infty^3}$$

or

$$C_D = 4/3\ g \frac{\mu}{\rho_1\ U_\infty^3}\ R_e$$

at the intersection with the $C_D = 2.6$ (from Davies and Taylor and Rosenberg).

$$R_e = \frac{(2.6)\ (3)\ \rho_1\ U_\infty^3}{(4)\ g\ \mu}$$

where $\rho_1 = 49.2$ lbm./ft.^{3}

$\mu = 0.538$ c.p.

$g = 32.14$ ft./sec.^{2}

$$R_e = \frac{(2.6)\ (3)\ (49.2)\ U_\infty^3}{(4)\ (32.14)\ (0.538)\ (0.000672)}$$

$$R_e = 8250\ U_\infty^3$$

and

Tube Dia.	U_∞	U^3_∞	R_e
5.90	1.282	2.06	17000
4.06	1.087	1.28	10550
2.88	0.943	0.84	6940
1.98	0.769	0.45	3710

The slope of the line on a log-log plot, such as in figure , can be seen to be one.

APPENDIX C

NOTES ON INSTALLATION AND FILLING OF TUBES

Each tube was thoroughly washed with a detergent, using a long rod covered with clean cloths, and special care taken not to scratch the wall surface of the tubes. Distilled water was used as a rinse, and the tubes allowed to dry. The bubble tower housing, hemispherical cup, and gaskets were cleaned in the same manner.

Each tube was mounted into the tower structure, aligned vertically, special care being taken not to contaminate the inside surfaces, and rinsed with a small amount of methanol, which was collected and saved for future use where purity is of no consequence. The tube was then filled with methanol and a paper cover taped over the top to prevent dust from entering the tube.

The height of the liquid was measured with little concern for any fluctuation in the height due to vaporization, since a series of runs could be made without any appreciable lowering of the liquid level.

APPENDIX D

LOCATION OF ORIGINAL DATA AND CALCULATIONS

The original data and calculations are on file in the Department of Chemical Engineering, The University of Tennessee.