To the Graduate Council:

I am submitting herewith a dissertation written by James Gerard Gerstle entitled "Mathematical Model Reduction for Process Simulation and Controller Design." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Chemical Engineering.

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ABSTRACT

Techniques of mathematical model reduction are applied to determine reduced models, which are then used for simulation and controller design of the original model. Both single input-single output (SISO) and multiple input-multiple output (MIMO) models are investigated. Numerous existing reduction methods are employed, as well as the development of new methodologies of model reduction. The use of the second order modes emerges as a potential criterion for the selection of the order of the reduced model. The resulting reduced models are evaluated in terms of simulation and controller design of the full model.

For the SISO models, the methods of model reduction are compared with the use of open loop simulations, calculation of the integral of the squared error between the full and reduced models, Bode plots, and PI controller design. From the three examples presented, the Cauer third form of continued fraction expansion is shown to be the best method for SISO model reduction. In the case of MIMO models, the reduction methods are compared using open loop simulations and multivariable PI controller design. Using three examples, the singular perturbation method is suggested for the reduction of MIMO models. For both the SISO and MIMO models the second order modes were found to be useful in selecting the order of the reduced model.

The characteristics of the full model that must be retained by the reduced model are shown to be very different. To circumvent this phenomenon, guidelines for the reduction of SISO and MIMO models are hypothesized. This research illustrates the many problems that may be encountered in model reduction and highlights several promising avenues of research.

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CHAPTER I

INTRODUCTION

Many processes in the chemical industries are described by an extremely high number of first order ordinary differential equations. The simulation and controller design for these processes may pose difficulties, hence an attractive method would be one which approximates the original higher order system with a lower order model. For linear, time-invariant systems the lowering of the order of a model is synonymous with the reduction of dynamic order of the set of equations constituting the model and is known as model reduction.

The incentive for reducing the order of a mathematical model is to enable the engineer to more easily study the dynamic behavior of large systems. With the reduced order model, the simulation of the system is simpler and controller design generally is greatly facilitated.

To date, a vast amount of research has centered on the reduction of single input-single output systems. However, chemical processes are inherently multivariable. At best, many of the single variable techniques have been casually extended to the multivariable case and the reduction of multivariable systems has yet to be fully investigated.

Historically, the applied process control engineer has commonly studied complex processes with simple transfer functions. This transfer function approach corresponds to approximating the system with some low order model by maintaining the characteristics of the system. This method is done empirically and often provides reasonable accuracy. Its application by the practicing engineer attests to its utility. However, when an entire process is studied (e. g., a train of reactors versus a single reactor), the model of this system can rarely be simplified by empirical means with any suitable degree of confidence. What is needed then is a formalized analytical means of reducing the order of the higher order models.

This research investigates the model reduction problem using analytical methods. To address this problem, this work is divided into two major areas of research. The first is the reduction of single variable models. The second phase of the work is the reduction of multivariable models. For both of these cases, various reduction techniques are investigated as well as the application of the reduced models for simulation of, and controller design for, the full model.

Terminology and Notation Conventions

The mathematical models addressed in this dissertation will all be linear, time-invariant, stable, and minimal realizations. Therefore, the definition of model reduction used in the first paragraph of this chapter is applicable. The model that is to be reduced will be refered to as a "full model" and the resulting lower order model as a "reduced model."

As mentioned previously, both single variable and multivariable processes are used in this work. In this dissertation the term multivariable will refer to systems with multiple inputs and/or multiple outputs (MIMO), with a single variable process being a single input-single output process (SISO).

Two types of linear models are typically used in the chemical process industries for dynamic concerns. The first type is the use of transfer functions. For MIMO systems the transfer functions are expressed as a matrix transfer function. The elements of a matrix transfer function are transfer functions which relate each individual input to an individual output. A two input-two output matrix transfer function is,

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \qquad (1.1)$$

where $g_{ij}(s)$ represents the transfer function between the ith output and the jth input.

The second type of model which is often employed is the state space description. In this modeling approach the equations defining the system are all expressed as first order linear differential equations. This set of equations is then expressed in vector-matrix form (state space form),

$$\dot{x}(t) = A\ x(t) + B\ u(t) \qquad (1.2)$$

$$y(t) = C\ x(t) + D\ u(t) \qquad (1.3)$$

where $x(t)$ is the state vector of length n, $u(t)$ is the input vector of length r, $y(t)$ is the output vector of length q, and $\dot{x}(t)$ represents the derivative of $x(t)$ with respect to time. The four matrices of equations (1.2) and (1.3) are defined as: A, a n x n state matrix, (also called the Jacobian matrix), B, a n x r input state matrix, C, a q x n output state matrix, and D, a q x r direction action matrix. Models which represent a chemical process rarely have a D matrix. Throughout this text upper case letters will refer to matrices and lower case letters to vectors or scalars.

Several terms that are used throughout this text are: minimum realization, controllable, and observable. Given the single variable model,

$$C(sI - A)^{-1}B = N(s) \bullet M(s)^{-1} \qquad (1.4)$$

where A, B, and C are defined as in equations (1.2) and (1.3), I is the identity matrix, and $M(s) = \det(sI-A)$, equation (1.4) is a minimum realization if, and only if, the polynomials $M(s)$ and $N(s)$ have no nontrivial common factor [16]. A system is said to be controllable if the input u is capable of moving any state in the state space to any other state in a finite time. And a system is said to be observable if every state may be estimated from the output y.

The Research

The remainder of this text is separated into five chapters. The next chapter reviews the literature pertinent to this research. The first section of this chapter reviews the various methods of model reduction available. Next the application of reduced models for process simulation is discussed. The chapter concludes with a discussion of the application of reduced models for controller design and a review of the MIMO controller known as a multivariable tuning regulator.

Chapter 3 develops methods for comparing the results of different model reduction techniques. Various guidelines for which characteristics of the full model a reduced model should maintain are introduced. These guidelines are discussed with respect to the time domain, frequency domain, and controller design. The chapter concludes with a summary of the criteria by which a reduced model can be termed acceptable.

Chapter 4 deals with the reduction of single variable (SISO) models. Three different SISO models are reduced by the methods described in Chapter 2 and by the methods introduced in this chapter. (The three SISO models presented are commonly used in SISO model reduction examples in the literature.) The resulting reduced models are then compared for simulation and controller design. A PI controller is designed on each of the reduced models and then applied for control of the full model. This controller is employed since it is commonly used in the chemical process industries. A summary of SISO methods of reduction rounds out this chapter.

Chapter 5 describes the reduction of multivariable (MIMO) models. The reduction methods introduced in Chapter 2 are extended to three different MIMO models. (There are no commonly used MIMO models in the reduction literature in part due to the limited amount of literature available in this area. Two of the models have previously been used in MIMO reduction studies [36,38,68]. The third model, a distillation column, was chosen due to its applicability to chemical engineering.) An additional model reduction technique, which can be formulated for the multivariable case, is also introduced and applied to the models. A multivariable tuning regulator is then designed using each of the reduced models. (The multivariable tuning regulator or multivariable PI controller was chosen due to claims of its robustness reported in literature.) The reduced models are compared for simulation and control of the full model. Finally, the results of MIMO model reduction are summarized.

The final chapter, Chapter 6, contains the conclusions of this work. The accomplishments of this work are reviewed and recommendations are given for future work.

CHAPTER II

LITERATURE REVIEW

In this chapter the concept of mathematical model reduction is reviewed. The numerous techniques available for model reduction are discussed with the four techniques (dominant mode, optimal models, continued fraction expansion, and internal balancing) upon which this research is based, discussed in detail. Next is a review of the use of reduced models for simulation and of the methods that determine which reduction techniques give the "best" reduced model. Finally, the use of reduced models for controller design is discussed as well as the multivariable controller adopted in this study.

Review of Model Reduction Techniques

A physical process typically consists of dozens, even hundreds of unit operations. The process model consists of the combination of mathematical expressions of each unit operation which usually results in a very high order model. Hence, the engineer is motivated to determine a method to reduce the order of the system. Such a method is known as a model reduction technique.

Reduction by Physical Reasoning

From a practical point of view, even a very complete and complex mathematical model is only a simplified representation of the physical system. If this model's order is high, the model may be further reduced by physical reasoning. By using experience, certain parts of the model are ignored or are approximated by relatively simple expressions. This technique is known as the pseudo-steady state approximation (or instantaneous steady state or quasi-steady state) method. In this method, one or more terms of the model are assumed to always be at steady state. The basis of this assumption is experience and intuition. Gelinas [32] showed, using a model of air pollution kinetics, that with intuitively sound assumptions the reduced model determined from the pseudo-steady state assumptions disagrees significantly with the complete model. Furthermore, the pseudo-steady state assumption may lead to a change in stability [34]. The difficulties of this technique arise from the prejudice that may result from experience and the unreliability of intuition.

Reduction by the Dominant Mode Methods

Extensions of the pseudo-steady state assumption are known as the dominant mode methods [1,20,21,28]. These methods retain the dominant eigenvalues of the full model and use either matrix diagonalization and diagonal-element discarding [64] or matrix aggregation [1]. These techniques are difficult and contain numerous problems for very high order systems [77]. Moreover, the reduced model may not reproduce the correct steady state value. The fundamental shortcoming of this method is the importance of the fast components (which are eliminated by this method) for controller design.

For the single input-single output case,

$$G(s) = K\,[(s + z_1)(s + z_2)\ \ldots\ (s + z_k)] \bullet [(s + p_1)(s + p_2)\ \ldots\ (s + p_n)]^{-1} \qquad (2.1)$$

where, K is the process gain, p_i are the poles, and z_j are the zeros, with $k < n$, $z_1 \geq z_2 \geq \ldots \geq z_k$, and $p_1 \geq p_2 \geq \ldots \geq p_n$, the reduced model is calculated by retaining the dominant poles of the full model (i. e., neglects the poles that are furthest from the origin). Typically a pole or poles are eliminated if the ratio between two succeeding poles is greater than ten [20]. The zeros of the system are then eliminated in the same manner. For the multivariable problem, this procedure is repeated for each of the elements of the matrix transfer function.

For models expressed in state space form, Davison [20,21,25] suggested a method of constructing a lower order model which has the same dominant eigenvalues and eigenvectors as the original system. Given a full model in the form,

$$\dot{x}(t) = A\ x(t) + B\ u(t) \qquad (2.2)$$

where A is an n x n matrix and B is an n x r matrix, the reduced model is,

$$\dot{x}^*(t) = A^*\ x^*(t) + B^*\ u(t) \qquad (2.3)$$

with A^* an m x m matrix, B^* an m x r matrix, and $m < n$. The vector x^* is a state vector consisting of the m state variables which are retained in the reduced model from the original n state variables. (Note: Davison did not consider the output equation of the state space form; his motivation was to approximate the dominant states of the full model.)

In the reduced model the m variables to be kept will be denoted by, x_a, x_b, x_c, ... The matrix A* is required to have the first m eigenvalues and eigenvectors of the full model. This matrix may be found by analyzing the time solution of the full model and selecting only those terms of the solution containing the first m eigenvalues [20]. The reduced matrix, A*, is then determined such that its time solution is the same as the solution containing the first m eigenvectors. The matrices of the reduced model are [21],

$$A^* = U_1 \Lambda_1 U_1^{-1} \tag{2.4}$$

and

$$B^* = U_1 [U^{-1} B]. \tag{2.5}$$

The matrix U of equation (2.5) is an n x n matrix composed of the eigenvectors of A,

$$U = (u^1, u^2, \ldots u^n) \tag{2.6}$$

where u^i (an eigenvector) is defined by,

$$(A - \lambda_i I)\, u^i = 0, \quad i = 1, 2, \ldots n \tag{2.7}$$

with λ_i the eigenvalues of A, and I is the identity matrix. The product $[U^{-1} B]$ is defined to be an m x r matrix containing the first m rows of $U^{-1}B$.

The U_1 matrix of equation (2.4) and (2.5) is an m x m matrix of the first m dominant eigenvectors corresponding to the retained state variables,

$$(U_1)^T = [u_a \; u_b \; u_c \; \ldots \;]. \tag{2.8}$$

The state variables (x_a, x_b, x_c, ...) retained in the reduced model should be chosen so that the determinant of the U_1 is as large as possible [20].

The remaining matrix, Λ_1, is defined as an m x m diagonal matrix of the m dominant eigenvalues of A,

$$\Lambda_1 = \text{diag}\; [\lambda_1, \lambda_2, \; \ldots, \; \lambda_m]. \tag{2.9}$$

The method described here for calculating the matrices A* and B* is valid both when the eigenvalues of A are real or complex, when the repeated eigenvalues have nondegenerate eigenvectors, and has been extended for the case of degenerate eigenvectors [21].

The reduced model of equation (2.3) approximates the full model of equation (2.2) by retaining the first m dominant modes of the full model. Hence, the overall dynamic response of the full model should be represented by the reduced model. However, there will be an error in the steady state response of the reduced model [21,25,42,44]. The steady state error associated with the reduced model decreases monotically as the order of the reduced model is increased. To eliminate the steady state error, Davison [21] proposed a rather complicated method which has been applied with only marginal success [13,77].

A similar method for reducing a model by retaining the system's dominant eigenvalues was proposed by Elrazaz and Sinha [24,25]. For this method the A matrix of the state space equation (2.2) is diagonalized by the similarity transformation,

$$\Lambda = P^{-1}\ A\ P \tag{2.10}$$

where P is the modal matrix (the columns of P are the eigenvectors of A). Using the linear transformation,

$$x(t) = P\ z(t) \tag{2.11}$$

equation (2.2) may be written as,

$$\dot{z}(t) = \Lambda\ z(t) + P^{-1}B\ u(t) \tag{2.12}$$

which can be rewritten in partitioned form as,

$$\begin{bmatrix} \dot{z}_1(t) \\ \\ \dot{z}_2(t) \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & 0 \\ \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} z_1(t) \\ \\ z_2(t) \end{bmatrix} + \begin{bmatrix} P_1^{-1}\ B \\ \\ P_2^{-1}\ B \end{bmatrix} u(t) \tag{2.13}$$

with the dominant eigenvalues in Λ_{11}. The reduced model is then,

$$\dot{z}_1(t) = \Lambda_{11}z_1(t) + P_1^{-1}\ B\ u(t). \tag{2.14}$$

As before, this reduced model fails to maintain the steady state response of the full model.

A technique called the singular perturbation method [111] closely resembles the method of Elrazaz and Sinha [24,25] except now the steady state gain is maintained by the reduced model. The system is decoupled into a slow and a fast subsystem. The state vector is partitioned and the state space representation is,

$$\begin{bmatrix} \dot{x}_1(t) \\ \mu\dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t) \tag{2.15a}$$

$$y(t) = [C_1 \quad C_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{2.15b}$$

where x_1 contains the slow components and x_2 the fast components. By letting $\mu = 0$, and elimating x_2 from equation (2.15) results in the lower order model,

$$\dot{x}_1(t) = [A_{11} - A_{12}(A_{22})^{-1}A_{21}]x_1(t) + [B_1 - A_{12}(A_{22})^{-1}B_2]\ u(t) \tag{2.16a}$$

$$y(t) = [C_1 - C_2(A_{22})^{-1}A_{21}] - [C_2(A_{22})^{-1}B_2]\ u(t). \tag{2.16b}$$

The steady state gain of the full model is maintained by the feedforward D matrix of equation (2.16b) where,

$$D = [C_2(A_{22})^{-1}B_2]. \tag{2.17}$$

The preceding methods of Davison and Elrazaz and Sinha [24,25] have been shown to be special cases of the aggregation method of Aoki [1,43,58]. This method is based on determining an aggregation matrix which gives the relationship between the state variables of the full and reduced models. Let,

$$z(t) = K\ x(t) \tag{2.18}$$

where K is an m x n constant projection matrix, termed the aggregation matrix.

The reduced model will be of the form,

$$\dot{z}(t) = F\ z(t) + G\ u(t) \tag{2.19}$$

$$y^*(t) = H\ z(t) \tag{2.20}$$

where z, y, and u are vectors of length m, q, and r respectively. The vector y^* of equation (2.20) is to be a close approximation of output vector, y(t), of the full model described by the equations,

$$\dot{x}(t) = A\ x(t) + B\ u(t) \tag{2.21}$$

$$y(t) = C\ x(t) \tag{2.22}$$

where x is a vector of length n.

Differentiating equation (2.18), and substituting for $\dot{x}(t)$ in equation (2.21) gives,

$$\dot{z}(t) = KA\ x(t) + KB\ u(t). \tag{2.23}$$

Hence F and G of equation (2.19) are related to A and B of equation (2.21) by,

$$KA = FK \tag{2.24}$$

and

$$G = KB. \tag{2.25}$$

And H of equation (2.20) is related to C of equation (2.22) by,

$$C \simeq HK. \tag{2.26}$$

Equation (2.26) is considered an approximate equality since in equation (2.20) the output is said to be a "close approximation" of the the full model response.

The matrices F, G, and H may be calculated from equations (2.24) to (2.26) by utilizing the pseudo-inverse of K (since K is rectangular),

$$K^{+} = K^{T}(KK^{T})^{-1}, \tag{2.27}$$

then,

$$F = KAK^{+}, \tag{2.28}$$

$$G = KB, \tag{2.29}$$

and

$$H = CK. \tag{2.30}$$

There are several techniques for calculating the aggregation matrix [1,42,44,59]. The two most commonly accepted methods will be discussed here [43]. The first method [1] uses the controllability matrices of the systems of equations (2.19) and (2.21). From equations (2.24) and (2.25) the controllability matrices of the reduced and full models are related by,

$$KW_A = W_F \tag{2.31}$$

where,

$$W_A = [B,\ AB,\ \ldots,\ A^{n-1}B] \tag{2.32}$$

and

$$W_F = [G,\ FG,\ \ldots,\ F^{n-1}G]. \tag{2.33}$$

Hence,

$$K = W_F W_A^{\ +},$$

where $W_A^{\ +}$ is the pseudo-inverse of W_A as defined in equation (2.27). In this case let $F = \mathrm{diag}\ (\lambda_1,\ \lambda_2,\ \ldots,\ \lambda_m)$, the m dominant eigenvalues of A and choose G so as to make (2.33) completely controllable.

A second method for choosing the aggregation method was proposed by Hickin and Sinha [42]. They have shown that a nontrivial aggregation law exists if and only if all the eigenvalues of F are also the eigenvalues of A. The aggregation matrix is then,

$$K = T[I_m \quad 0]V^{-1} \tag{2.34}$$

where T is any nonsingular m x m matrix and V is the modal matrix of A (the columns of V are either the eigenvectors or generalized eigenvectors of A). By letting T of equation (2.34) be the identity matrix the resulting F matrix of equation (2.24) is diagonal.

The aggregation matrix is to be chosen in such a way that the error in modeling the system by equations (2.19) and (2.20) is minimized. The problem then is determining the proper aggregation matrix. The two methods discussed here have been the most successful in minimizing the modeling error [43]. Unfortunately, the resulting modeling error may still be large when employing either of these methods [111].

A basic assumption that has been made in the foregoing discussion is that the full model contains some dominant modes. Many physical processes do not contain dominant modes [12] and reduction by any of the dominant mode methods is, of course, not possible on such models [77]. Moreover, even when the full model does contain dominant modes, the method suffers from computational problems (e. g., analysis and numerical experimentation) [77]. The main difference between the methods of model reduction by the dominant modes is the technique used for the calculation of the H matrix in equation (2.20). The motivation for each of these methods was to produce a reduced model which minimizes the output error.

Reduction by Curve Fitting

Several methods are available for the determination of reduced models based on fitting either the time-domain response or the frequency-domain plot of the full model with a reduced model [4,23,39,86,88,108]. The methods are essentially identification techniques which numerically fit the full model response with a lower order model. Reduced models calculated by these methods are often referred to as optimum models.

When a step or impulse time-domain response from the full model is simple, the model may be approximated by either a first or second order model with dead time [67,75]. Other process identification techniques are also available for model reduction [111]. These techniques may be used to identify the full model by some lower order model.

For those cases in which the frequency response of the full model is available, Levy [63] has proposed a method by which a reduced model may be calculated. The frequency response curves of the full model are fit numerically with a lower order model in the same manner as in time-domain fitting. The drawback in this method is that the low frequency region fitting is very poor [4]. Moreover, the reduced model obtained from this method offers little or no improvement over the time-domain fit [23]. Since the time-domain response is typically easier to generate, frequency-domain fits are rarely employed.

For calculating an optimum reduced model in the time domain, let the full model be of the form,

$$\dot{x}(t) = A\ x(t) + B\ u(t) \tag{2.35}$$

$$y(t) = C\ x(t). \tag{2.36}$$

Now let the response of this system for a specified input, u(t), be described by a discrete set of values, y(t), taken over an interval of time which contains the full response,

$$Y = [y_0,\ y_1,\ y_2,\ \ldots,\ y_N] \tag{2.37}$$

where,

$$y_i = y_i(t),\ \text{for}\ i = 0,\ 1,\ 2,\ \ldots,\ N. \tag{2.38}$$

The output vector y(t) must be sampled with a frequency that will ensure no significant information is lost. If the output is not sampled often enough, the data used will not contain all of the dynamics of the full model. The resulting reduced model will approximate the erroneous data rather than the process in question.

The reduced model,

$$\dot{x}^*(t) = A^*\ x^*(t) + B^*\ u(t) \tag{2.39}$$

and

$$y^*(t) = C^*\ x^*(t), \tag{2.40}$$

is calculated in such a way that for a given input, the error function,

$$J = f[w_i\ (y_i - y*_i)] \tag{2.41}$$

is minimized, where w_i is a weighting factor and $y*_i$ is the output of the reduced model at each sample time (as in equation (2.38)).

The parameters of the reduced model are the direct result of the error criterion of equation (2.41). The error criterion represents the extent to which the response of the reduced model deviates from the full model. Usually the error criterion is selected as the output error norm raised to some power with a weighting sequence as a multiplying factor. The two common error criteria used are: the absolute error;

$$J = \Sigma\ w_i|y_i - y*_i|, \tag{2.42}$$

and the squared error;

$$J = \Sigma\ w_i(y_i - y*_i)^2. \tag{2.43}$$

Sinha and Bereznai [108] have suggested that the best error criterion would be to minimize the distance between the full and reduced model responses at each sampling instance, (i. e., to minimize the perpendicular line drawn from the full model response at each sample to a point on the reduced model response).

The error function is then minimized by varying the parameters of the reduced models. The parameters are varied using a pattern search method [74,75,108]. For the single input-single output case, the reduced model of equations (2.39) and (2.40) are written in the phase-variable form,

$$\dot{x}^*(t) = A^* x^*(t) + B^* u(t) \tag{2.44a}$$

$$y^*(t) = C^* x^*(t) \tag{2.44b}$$

$$A^* = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 & \cdots & -a_{m-1} \end{bmatrix}, \tag{2.45a}$$

$$B^* = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}, \tag{2.45b}$$

and

$$C^* = [b_0 \quad b_1 \quad b_2 \quad b_3 \quad \cdots \quad b_m]. \tag{2.45c}$$

The advantage of the phase-variable is that at most, 2m parameters, (where m is the order of the reduced model), need to be varied versus $m^2 + 2m$ parameters of the general state space model.

For the multivariable case, the number of parameters which must be varied becomes inhibitively great from the computational standpoint. Gupta, et al. [39] have proposed a method to deal with the multivariable problem. In this method, the reduced model is determined as follows,

$$A^* = \text{diag } [a_1, a_2, \ldots, a_m], \tag{2.46}$$

$$B^* = A^*(C^*)^{-1}C(A)^{-1}B, \tag{2.47}$$

and

$$C^* = CC^T(RC^T)^{-1}, \tag{2.48}$$

where R is a weighting matrix and the superscript T represents the matrix transpose.

With Gupta et al.'s method, only the m parameters of the A^* matrix need to be varied. The matrix R is selected so that the dominant states of the full model are maintained [39]. The success of this model reduction is limited to cases where the eigenvalues of A are distinct [39]. And the eigenvalues of the reduced model are limited to being negative real eigenvalues.

The model reduction method of equation fitting involves several intuitive judgements by the modeler, they being: order of the reduced model, error criteria, and weighting factors. From the techniques discussed here, there are no guidelines for the order of the reduced model. Typically, each input-output relationship is assumed to be second order [13]; otherwise, an order is simply assumed, or the order is determined through a process identification technique [111].

Selecting the error criteria and weighting factors results from the modeler making a subjective judgement concerning the relative importance of the various intervals of the response [13]. Because the error criteria is a weighting factor, the prejudice of a given error criterion may be changed through the use of separate weighting factors.

Reducing models by equation fitting is intuitively apparent; however, there are several pitfalls to this method. One obvious pitfall is that the modeler is making subjective judgements. Additionally, the computer time involved in calculating the reduced model could become excessive. Finally, the optimum model is a function of the input [88].

Reduction by Moment Matching

A number of model reduction techniques are based on matching some properties of the original model. Two parameters that are often matched are the time moments and the Markov parameters of the system.

The matching of time moments for model reduction was first proposed by Paynter and Takahashi [79]. The time moments of the model are directly related to the coefficients of a power series expansion. Let,

$$G(s) = [b_0 + b_1 s + b_2 s^2 + \ldots + b_{n-1} s^{n-1}] \bullet [a_0 + a_1 s + a_2 s^2 + \ldots + a_n s^n]^{-1}. \tag{2.49}$$

The Taylor power series expansion about s = 0 is of the form,

$$G(s) = c_0 + c_1 s + c_2 s^2 + \ldots . \tag{2.50}$$

The terms c_i, the modified time moments, are porportional to the time moments of the impulse response of the model; assuming the system is asympototically stable. Obtaining the modified time moments is complicated and the procedure used for determining modified time moments is unique to each model [7].

For models represented in state space, the coefficients of the power series expansion are calculated as follows, given,

$$\dot{x}(t) = A\ x(t) + B\ u(t) \qquad (2.51)$$

$$y(t) = C\ x(t) + D\ u(t) \qquad (2.52)$$

then,

$$G(s) = C(sI - A)^{-1}B + D, \qquad (2.53)$$

where,

$$c_0 = (CA^{-1}B + D) \qquad (2.54)$$

$$c_i = CA^{-(i+1)}B, \quad \text{for all } i > 0. \qquad (2.55)$$

For model reduction, the term time moment is often used for either the power series expansion coefficients or the actual impulse response time moments [5,8,93,94,111]. In this work, the term impulse response time moment and the term modified time moment will refer to the power series expansion coefficients.

To use time moments for reduction, one retains the first m modified time moments,

$$G^*(s) = c_0 + c_1 s + c_2 s^2 + \ldots + c_m s^m \qquad (2.56)$$

and inverts this expression to a transfer function. By using the modified time moments of a model for model reduction, the steady state or slow modes of the original model are maintained. The steady state response is guaranteed since,

$$G^*(0) = c_0 \qquad (2.57)$$

and

$$G(0) = c_0. \qquad (2.58)$$

With each successive time moment that is maintained in the reduced model, the integral of the squared error is decreased [94]. Summarizing, the initial modified time moments of a system maintain the slow response, but do not maintain the transient or fast response.

The matching of the Markov parameters was introduced to model reduction to retain the transient components of the original model in the reduced order model. Ho and Kalman [46] have shown that the Markov parameters, which in their case were derived from input/output data, may be used to derive a minimal realization of a system.

The Markov parameters are defined as the coefficients of the power series expansion of G(s) in negative powers of s,

$$G(s) = y_1 s^{-1} + y_2 s^{-2} + y_3 s^{-3} + \dots \quad (2.59)$$

There is no simple algorithm for the calculation of the Markov parameters from a transfer function, but one technique (which is shown later) is through continued fraction expansion. For models represented in state space, the Markov parameters may be calculated as follows (given equations (2.51) and (2.52)),

$$G(s) = C(sI - A)^{-1}B + D \quad (2.60a)$$

$$= CABs^{-1} + CA^2Bs^{-2} + CA^3s^{-3} + \dots \quad (2.60b)$$

$$= d_1 s^{-1} + d_2 s^{-2} + d_3 s^{-3} + \dots \quad (2.60c)$$

where,

$$d_i = CA^iB, \quad (2.61)$$

for: $i = 1, 2, \dots, n$.

A reduced model which maintains the fast response alone (i. e., with no interest in the steady state mode) is of little practical interest. Hence, the Markov parameters are usually used in combination with the modified time moments of a system [117]. The fast response of the reduced order model may be improved by choosing the numerator coefficients of the reduced order model to fit both the initial modified time moments and the initial Markov parameters of the original system.

Reduction by Continued Fraction Expansion

A number of model reduction techniques are based on the use of continued fraction expansions. The continued fraction expansion technique is a Pade approximation [5,47,94,95,96,102,124] and is directly related to the matching of moments [5].

The basic theory of continued fractions was proposed in the 1940's by Wall [119] and Frank [29]. The application of the Cauer second form of continued fraction expansion for mathematical model reduction was proposed by Chen and Shieh [9]. Since their work was reported, numerous other authors [33] have investigated the use of continued fractions for reducing mathematical models. The main advantages of using continued fractions reported by these authors include that they: 1) converge faster than other series expansions [4]; 2) contain most of the important characteristics of the system in the first two terms [4]; 3) are well suited to automatic computation, and; 4) are simple to use [5].

In contrast, one main drawback to using this method is that the stability of the reduced model is not guaranteed to be the same as that of the original model [91,124]. Another disadvantage is that the continued fraction expansion may yield complex roots in the reduced model while the original model contains only real roots, or vice versa [91].

Chen and Shieh proposed using the Cauer second form for model reduction [10]. An "improved" continued fraction expansion, the Cauer third or mixed form, was suggested by Shieh and Goldman [100,101]. The Cauer third form is a combination of the Cauer first and second forms; hence, it is often referred to as the mixed form.

Cauer first form. The Cauer first form has not been used directly for model reduction. This form is introduced here for reference when the Cauer third form is discussed. Given the transfer function,

$$\begin{aligned} Y(s) \bullet [U(s)]^{-1} = {} & [A_{2,n}s^{n-1} + A_{2,n-1}s^{n-2} + A_{2,n-2}s^{n-3} \\ & + \ldots + A_{2,2}s + A_{2,1}] \\ & \bullet [A_{1,n+1}s^{n} + A_{1,n}s^{n-1} + A_{1,n-1}s^{n-2} \\ & + \ldots + A_{1,2}s + A_{1,1}]^{-1} \end{aligned} \tag{2.62}$$

the continued fraction expansion of the Cauer first form is,

$$Y(s) \bullet [U(s)]^{-1} = [K_1 s + [K_2 + [K_3 s + [K_4 + [\ \ldots\ [K_{2n}]^{-1}\ \ldots\ ]^{-1}]^{-1}]^{-1}]^{-1}]^{-1}. \qquad (2.63)$$

All of the components of equations (2.62) and (2.63) may be either scalars (SISO) or matrices (MIMO).

The K_i variables of equation (2.63) are the partial fraction coefficients of the Cauer first form. These coefficients may be calculated from a modified Routh array [38]. Employing the transfer function of equation (2.62), the elements of the modified Routh array and the partial fraction coefficients of the Cauer first form are calculated by,

$$K_p = A_{p,n+2-p}[A_{p+1,n+1-p}]^{-1} \qquad (2.64a)$$

$$A_{j,e} = A_{j-2,e+1} - K_{j-2}A_{j-1,e}, \qquad (2.64b)$$

$$A_{p+1,n+1-p} \neq 0, \qquad (2.64c)$$

for: $p = 1, 2, 3, \ldots, 2n;$

$j = 3, 4, 5, \ldots, 2n+1;$

$e = n+1, n-1, \ldots, 1.$

The method described in equation (2.64) may be used for both single input-single output models and multiple input-multiple output models [38]. For multivariable cases, the method is known as a matrix continued fraction [99] and requires the number of inputs to equal the number of outputs, since matrix inversion is required in calculating the partial fraction coefficients.

In model reduction, not all of the partial fraction coefficients, K_i, of equation (2.63) need to be calculated. For the single variable case, the first 2m coefficients must be calculated, where m is the order of the reduced model. In multivariable model case, the order of the reduced model, m, is an integer multiple of r, the number of inputs or outputs. For a given order, the first 2m/r coefficients must be calculated.

Once the required coefficients have been calculated, the resulting continued fraction is inverted. For the Cauer first form, Shieh and Gandiano [99] have proposed the following method of continued fraction inversion. Given the following transfer function,

$$Y(s) \bullet [U(s)]^{-1} = [B_{22}s + B_{21}] \bullet [B_{13}s^2 + B_{12}s + B_{11}]^{-1} \tag{2.65}$$

the expansion and inversion of the Cauer first form is,

$$Y(s) \bullet [U(s)]^{-1} = [K_1 s + [K_2 + [K_3 s + [K_4]^{-1}]^{-1}]^{-1}]^{-1} \tag{2.66}$$

and

$$Y(s) \bullet [U(s)]^{-1} = [(K_2K_3K_4)s\ (K_2 + K_4)] \bullet [(K_1K_2K_3K_4s^2 + (K_1K_2 + K_1K_4 + K_3K_4)s + I]^{-1} \tag{2.67}$$

where I is the identity matrix. In general, the inversion of the Cauer first form is,

$$B_{2n+1,1} = [I] \tag{2.68a}$$

$$B_{p,1} = K_p B_{p+1,1} \tag{2.68b}$$

$$B_{j-2,i+1} = B_{j,i} + K_{j-1}B_{j-1,i+1} \tag{2.68c}$$

for: $p = 2n, 2n-1, \ldots, 2, 1;$

$j = 2n+1, 2n, \ldots, 3;$

$i = 1, 2, \ldots, n.$

The characteristics of the full model retained by the reduced model may be illustrated through equation (2.67). Applying the final value theorem to equation (2.67) with a step input yields,

$$\lim_{s \to 0} Y(s) = (K_2 + K_4) \tag{2.69}$$

while the initial value theorem for a step input gives,

$$\lim_{s \to \infty} Y(s) = K_1^{-1}. \tag{2.70}$$

From equation (2.63) and the initial value theorem, the reduced model of the Cauer first form is seen to maintain the fast components of the system. Similarly, the final value theorem shows that the steady state components are not maintained.

Hence, using the Cauer first form for model reduction will maintain the fast components of the original model. Bosley, et al. [5] have shown that the Cauer first form expansion is equivalent to a Taylor series expansion about $s = \infty$ [5,118].

Cauer second form. The Cauer second form is the continued fraction expansion type originally proposed by Chen and Sinha [9] for model reduction. This method has been successful in approximating many high order models for simulation purposes. Model reduction with the Cauer second form produces a reduced model in which the steady state values dominate the behavior of the system [38].

The Cauer second form continued fraction expansion of a transfer function, equation (2.62), is,

$$Y(s) \bullet [U(s)]^{-1} = [H_1 + [H_2 s^{-1} + [H_3 + [H_4 s^{-1} + [\ \dots\ [H_{2n} s^{-1}]^{-1} \dots\ ]^{-1}]^{-1}]^{-1}]^{-1}]^{-1}. \quad (2.71)$$

As in the case of the Cauer first form, the variables H_i are the partial fraction coefficients of the second form and are calculated from the modified Routh array [11],

$$H_p = A_{p,1}[A_{p+1,1}]^{-1}, \quad (2.72a)$$

$$A_{j,e} = A_{j-2,e+1} - H_{j-2,e+1}, \quad (2.72b)$$

$$A_{p+1,1} \neq 0, \quad (2.72c)$$

for: $p = 1, 2, 3, \dots, 2n;$

$j = 3, 4, \dots, 2n+1;$

$e = 1, 2, 3, \dots, n.$

Again, as in the Cauer first form, the continued fraction expansion described by equations (2.71) and (2.72) may be used for both single input-single output (SISO) and multiple input-multiple output (MIMO) models [13]. A continued fraction expansion of a multivariable model is known as matrix continued fraction expansion [13]. In the multivariable case the number of inputs must equal the number of outputs for the matrix inversion step required in equation (2.72).

For the calculation of an mth order reduced SISO model, a continued fraction containing the first 2m partial fraction coefficients, H_i, are calculated. For MIMO models the order, m, of the reduced models that may be calculated are an integer multiple of r, where r is the number of inputs and outputs, and the first 2m/r partial fraction coefficients must be calculated for model reduction.

From the partial fraction coefficients, the continued fraction may be inverted to a transfer function. For inversion to a transfer function, Chen [15] proposed a simple algorithm which was later modified by Shieh and Gandiano [99] for the multivariable case. Inversion to a transfer function is easily demonstrated through an example. Given,

$$Y(s) \bullet [U(s)]^{-1} = [B_{22}s + B_{21}] \bullet [B_{13}s^2 + B_{12}s + B_{11}]^{-1} \tag{2.73}$$

the Cauer second form expansion is,

$$Y(s) \bullet [U(s)]^{-1} = [H_1 + [H_2 s^{-1} + [H_3 + [H_4 s^{-1}]^{-1}]^{-1}]^{-1}]^{-1} \tag{2.74}$$

and inversion of equation (2.74) is,

$$Y(s) \bullet [U(s)]^{-1} = [(H_2 + H_4)s + H_2H_3H_4 \bullet [Is^2 + (H_1H_2 + H_1H_4 + H_3H_4)s + H_1H_2H_3H_4]^{-1}. \tag{2.75}$$

A general method of inversion of the Cauer second form is,

$$A_{2n+1,1} = I, \tag{2.76a}$$

$$A_{p,1} = H_p A_{p+1,1}, \tag{2.76b}$$

$$A_{j-2,i+1} = A_{j,i} + H_{j-2}A_{j-1,i+1} \tag{2.76c}$$

for: $p = 2n, 2n-1, \ldots, 1;$

$j = 2n+1, 2n, \ldots, 3;$

$i = 1, 2, \ldots, n.$

The reduced model obtained by reduction with the Cauer second form maintains the steady state components of the full model, which may shown with application of the final value theorem to equation (2.75),

$$\lim_{s \to 0} Y(s) = H_1^{-1}, \tag{2.77}$$

and the initial value theorem,

$$\lim_{s \to \infty} Y(s) = (H_2 + H_4). \tag{2.78}$$

The partial fraction coefficient H_1 contains the steady state gains of the full model. The partial fraction coefficients in the continued fraction expansion are in the order of decreasing importance of their influence on the response as steady state is approached [100,107].

The dominant poles of the original model are not retained exactly in the reduced model. The poles of the reduced model are selected to approximate the response of the full model [123]. The response of the reduced model improves monotically with respect to the full model as the order is increased [107].

The motivation for using the Cauer second form is related to the feedback system of Figure 2.1 [12]. This system has a closed loop transfer function of,

$$Y(s) \bullet [U(s)]^{-1} = G(s) \bullet [1 + (G(s)H(s))]^{-1}. \tag{2.79}$$

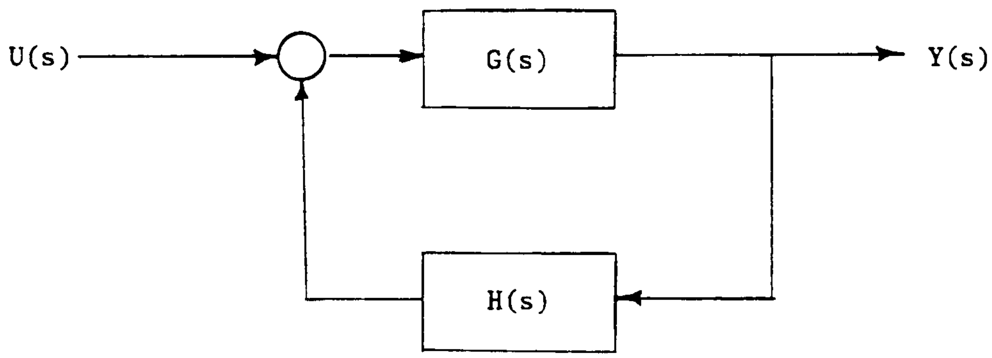


Figure 2.1. Feedback System of the Cauer Second Form.

Dividing the numerator and the denominator of equation (2.79) by G(s),

$$Y(s) \bullet [U(s)]^{-1} = [H(s) + [G(s)]^{-1}]^{-1} \tag{2.80}$$

results in a simple continued fraction. The significance of this continued fraction is that if G(s) is large, then the overall gain of the transfer function may be approximated by H^{-1}; hence, H dominates the behavior of the system.

Bosley et al. [5] have shown that the Cauer second form method of reduction is equivalent to matching moments with a Taylor series expansion about s = 0. Several authors [49,65,77,105,106,110,120] have proposed methods which ensure the stability of the reduced model by matching moments. The resulting reduced model, when stable, has been shown not to retain the characteristics of the full model [95] and in some cases the "reduced" model will be of higher order than the original model [47,97].

Chen and Tsay [14] proposed a squared magnitude method which also guaranteed the stability of the reduced model. Unfortunately, this method may produce a reduced model which is not physically realizable [96], (i. e., the order of the numerator is greater than that of the denominator).

Reducing models with the Cauer second form of continued fraction expansion is a very powerful model reduction method [93]. When the reduced model is stable, the steady state characteristics of the full model are maintained [12,93,107,123].

Cauer third form. Shieh and Goldman [100,101] introduced the Cauer third or mixed form which is a combination of the Cauer first and second forms. The motivation for this method is that the resulting reduced model maintains both the fast and steady state characteristics of the full model. Consider again, the transfer function of equation (2.62). The continued fraction expansion in the Cauer third form is,

$$Y(s) \bullet [U(s)]^{-1} = [K_1 s + H_1 + [K_2 + H_2 s^{-1} + [K_3 s + H_3 + [K_4 + H_4 s^{-1} + [\ldots]^{-1}]^{-1}]^{-1}]^{-1}]^{-1}. \qquad (2.81)$$

The partial fraction coefficients, H_i and K_i, of the Cauer third form are calculated from a modified Routh array [38,100]. Using the notation of the transfer function equation (2.62), the modified Routh array is,

$$H_p = A_{p,1}[A_{p+1,1}]^{-1}, \qquad (2.82a)$$

$$K_p = A_{p,n+2-p}[A_{p+1,n+1-p}]^{-1}, \qquad (2.82b)$$

$$A_{j,e} = A_{j-2,e+1} - H_{j-2}A_{j-1,e+1} - K_{j-2}A_{j-2,e}, \qquad (2.82c)$$

for: $p = 1, 2, 3, \ldots, n;$

$j = 3, 4, \ldots, n+1;$

$e = 1, 2, 3, \ldots n.$

As with the previously discussed cases using the Cauer first and second forms, the method described in equations (2.81) and (2.82) is applicable to both SISO and MIMO systems [38,104]. In the MIMO case, the number of inputs must equal the number of outputs for the matrix inversion step required in equation (2.82).

An mth order reduced model is calculated from the first m partial fraction coefficients, H_i and K_i, for the SISO case. And in the case of MIMO systems, the order of the reduced model that may be calculated are integer multiples of r, the number of inputs or outputs. For an mth order model, the first m/r partial fraction coefficients are calculated.

With the required partial fraction coefficients, the continued fraction of equation (2.81) is inverted to a transfer function. Shieh and Gaudiano [99] demonstrated an algorithm for the inversion of the Cauer third form continued fraction to a transfer function. Consider the second order transfer function,

$$Y(s) \bullet [U(s)]^{-1} = [B_{22}s + B_{21}] \bullet [B_{13}s^2 + B_{12}s + B_{11}]^{-1} \qquad (2.83)$$

which may be expanded to,

$$Y(s) \bullet [U(s)]^{-1} = [K_1s + H_1 + [K_2 + H_1s^{-1}]^{-1}]^{-1} \qquad (2.84)$$

and inverted to,

$$Y(s) \bullet [U(s)]^{-1} = [K_2s + H_2] \bullet [K_1K_2s^2 + (I + H_1K_2 + K_1H_2)s + H_1H_2]^{-1}. \qquad (2.85)$$

Their generalized algorithm for inversion is,

$$A_{n+1,1} = I, \qquad (2.86a)$$

$$A_{p,1} = H_pA_{p+1,1}, \qquad (2.86b)$$

$$A_{p,n+2-p} = K_pA_{p+1,n-1-p}, \qquad (2.86c)$$

$$A_{j-2,i+1} = A_{j,i} + H_{j-2}A_{j-1,i+1} + K_{j-2}A_{j-1,i}, \tag{2.86d}$$

for: $p = n, n-1, \ldots, 1;$

$i = 1, 2, \ldots, (n+2-j);$

$j = n+1, n, \ldots, 3.$

The reduced model obtained by the Cauer third form maintains the fast and steady state components of the full model. Goldman, et al. showed this by applying the initial and final value theorems to equation (2.85),

$$\lim_{s \to \infty} Y(s) = (H_1)^{-1} \tag{2.87}$$

and,

$$\lim_{s \to 0} Y(s) = (K_1)^{-1} \tag{2.88}$$

respectively. Since H_1 contains the steady state gains of the full model and K_1 contains the fast response (i. e., the first Markov parameters) of the reduced model, these characteristics will be maintained in the reduced model.

Hwang [48] showed that the Cauer third form is equivalent to model reduction by matching the first m modified time moments and the first m Markov parameters of the impulse response, which is equivalent to a Taylor series expansion of the transfer function about $s = 0$ and $s = \infty$.

The method of continued fraction expansion by the Cauer third form is considered superior for simulation [38] to the other two Cauer forms, since both the transient and the steady state characteristics of the full model are maintained. As in the other two Cauer forms, the stability of the reduced model is not guaranteed; however, Goldman, et al. [38] assert that of the continued fraction methods this method has the highest probability of being stable.

Summary of continued fraction expansion. Of the three methods discussed here, the Cauer second form is the mostly widely used. Despite the relatively complicated equations, all of the methods are computationally simple to use but may produce unstable reduced models. In general, the methods of continued fraction expansion are considered to be very powerful and efficient for model reduction.

Reduction by Internal Balancing

The method of internal balancing for model reduction has recently been proposed by Moore [71,72,73]. The basis for this method comes from minimal realization theory [51]. According to this theory, there is an identical lower order model for a given model if and only if the original model can be organized in some coordinate system as,

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_R & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_R \\ B_2 \end{bmatrix} u(t) \qquad (2.89)$$

and

$$y(t) = [C_R \quad C_2] \begin{bmatrix} x_1(t) \\ \\ x_2(t) \end{bmatrix} \tag{2.90}$$

where the subsystem (A_R, B_R, C_R) has the same impulse response matrix as the original model.

The motivation of model reduction using internal balancing (and most other methods for that matter) is to eliminate any subsystem that contributes little to the impulse response matrix. For internal balancing the technique is known as model reduction by weak subsystem elimination. In this method of model reduction the full model is reorganized so that a "dominant" subsystem may be identified.

The dominant subsystem of a model is equivalent to the most controllable and observable part of the model [50,86]. The controllability grammian,

$$W_c = \int [\exp(At)\ BB^T \exp(A^T t)]dt \tag{2.91}$$

and the observability grammian,

$$W_o = \int [\exp(A^T t)\ C^T C \exp(At)]dt \tag{2.92}$$

are used to define measures of controllability and observability in certain directions of the state space. The definitions of these grammians require that the system for which they are calculated be asymptotically stable.

Since the controllability and observability grammians are not invariant under coordinate transformations [71], the state space model may be transformed to identify the dominant subsystem of the model. An internally balanced model is defined as a model in which the controllability and observability grammians are "balanced" (i. e., equal and diagonal). The internally balanced model is then partitioned in order to eliminate the weak subsystem of the full model. The reduced model obtained by subsystem elimination is stable and contains the dominant characteristics of the impulse response matrix of the full model [56,82].

For the transformation to an internally balanced model, the first step is the calculation of the controllability grammian from the Lypanov equation [2],

$$AW_c^2 + W_c^2A^T = -BB^T. \tag{2.93}$$

The singular value decomposition of W_c^2 yields,

$$W_c^2 = V_c\Sigma_c^2V_c^T, \tag{2.94}$$

where V_c and Σ_c are the principal component vectors and magnitudes of exp(At)B, respectively.

The coordinate transformation,

$$P_1 = V_c\Sigma_c, \tag{2.95}$$

is then applied to give,

$$A* = P_1^{-1}AP_1, \tag{2.96a}$$

$$B* = P_1^{-1}B, \tag{2.96b}$$

$$C* = CP_1. \tag{2.96c}$$

Now the observability grammian is calculated from the Lypanov equation,

$$(A*)^TW_o^2 + W_o^2A* = -(C*)^TC* \tag{2.97}$$

for the system described by equation (2.96). The component vectors and magnitudes, V^*_o and Σ^*_o, of $\exp(A^{*T})C^{*T}$ are then calculated by the singular value decomposition,

$$W_o^{\,2} = V^*_o \Sigma^*_o{}^2 V^*_o{}^T. \qquad (2.98)$$

The coordinate transformation,

$$P_2 = V^*_o (\Sigma^*_o)^{-0.5} \qquad (2.99)$$

is now applied to A, B, and C to yield the internally balanced model,

$$A'' = P_2^{-1} A^* P_2, \qquad (2.100a)$$

$$B'' = P_2^{-1} B^*, \qquad (2.100b)$$

and

$$C'' = C^* P_2. \qquad (2.100c)$$

Once the model has been expressed in internally balanced form, the model is partitioned for weak subsystem elimination [83]. Let the system described by the matrices A, B, and C be internally balanced and $\sigma_1, \sigma_2, \ldots, \sigma_n$ be the singular values of the controllability and observability grammians, which are defined as the second order modes, with $\sigma_{i+1} \leq \sigma_i$ for i = 1, 2, . . . , n-1. The balanced system may be partitioned as,

$$\begin{bmatrix} \dot{x}_1(t) \\ \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ \\ B_2 \end{bmatrix} u(t) \qquad (2.101a)$$

$$y(t) = [C_1 \quad C_2] \begin{bmatrix} x_1(t) \\ \\ x_2(t) \end{bmatrix} \qquad (2.101b)$$

where A_{11} is m x m. Defining u_1 and u_2 as the minimum norm functions that drive the state from the origin to $(x_1(\tau), 0)^T$ and $(0, x_2(\tau))^T$, respectively, in the time interval $[0,\tau]$, Moore [73] has shown that,

$$[\int \|u_2\|^2 dt] \cdot [\int \|u_1\|^2 dt]^{-1}$$
$$> [\|x_2(\tau)\|^2] \cdot [\|x_1(\tau)\|^2]^{-1}. \qquad (2.102)$$

If $\sigma_m >>> \sigma_{m+1}$ and u_1 and u_2 have the same norms, it follows that,

$$\|x_2(\tau)\| <<< \|x_1(\tau)\|. \qquad (2.103)$$

Hence, the x_2 state variables are only slightly affected by the input when compared with the x_1 state variables.

Similarly, let y_1 and y_2 be the zero input responses (the solution of $x(t) = Ax(t)$ at time t due to the initial condition $x(0)$) from $(x_1(0), 0)^T$ and $(0, x_2(0))^T$, respectively. Then,

$$\int \|y_2(t)\|^2 dt <<< \int \|y_1(t)\|^2 dt \qquad (2.104)$$

if

$$\sigma_m >>> \sigma_{m+1} \qquad (2.105)$$

and

$$\|x_1(0)\| = \|x_2(0)\|. \qquad (2.106)$$

The impulse response implies that the x_2 state variables affect the output only slightly in comparison to the x_1 state variables.

If $\sigma_{m+1} >>> \sigma_m$, a reasonable assumption may be that the x_2 state variables do not affect the input-output of the system significantly. Hence the system (A_{11}, B_1, C_1) is a reasonable low order approximation of the original system (A, B, C).

Summary of Model Reduction Techniques

The preceding description of model reduction methods is not meant to be all-inclusive of the techniques available, but rather to set the stage for the rest of this research. Other methods that were investigated during this research include, reduction by differentiation [41,62], the zeros method [92], partial realization [42], collocation [17,18], and a modal method [122]. These methods were eliminated from this research due to the various limitations, as presented in the literature, that each reduction method has.

As previously mentioned, the methods of reduction discussed here are for linear stable ordinary differential equations. There are a limited number of techniques for nonlinear models [18], for unstable systems [120], and for distributed parameter systems [17]. These techniques were not investigated either, since the success or failure of model reduction with relatively simple models (e. g., linear, lumped parameter, and stable) must first be shown.

Application of Reduced Models to Process Simulation

All of the methods discussed thus far have been used for simulation of full models [4,111]. The question, however, is -- when is the reduced model adequate for simulation of the full model?

The common technique employed for analysis of the reduced model is to compare the open loop response to a step input of the reduced model with the full model. This comparison is usually done graphically by checking critical points such as peak overshoot and steady state. Other authors [4,91,100,111] have opted for a more analytic method of comparing the two responses, namely, using a cost function (e. g., absolute error, squared error).

The drawback to using the above mentioned methods [91] is that a step response is not a high frequency disturbance. In order to investigate a range of frequencies, both Nyquist plots [68,120] and an impulse input [19] have been suggested. In the final analysis, though, it rests with the engineer to determine the degree of precision required for each particular application of the reduced model for simulation of the full model.

Application of Reduced Models to Controller Design

According to Aoki [1], the main objective of model reduction is the study of the feasibility of controlling a system via control laws derived from a reduced model. Reducing the order of the model makes controller design easier [107], and is a necessary step for some control design methods [5]. Historically, countless control system designs have been based on second order approximations with a fair amount of success [47]. Hence, model reduction for controller design would appear to be promising [47,67].

For the control of SISO systems, a feedback loop is typically used. The controller is usually a low pass filter (i. e., PI controller) and will usually only affect the slower modes of the process. In that case, the fast components of the system may be ignored. But Gupta, et al. [39] reported several cases in which the fast components may not be neglected. For these cases, Schwartz and Edgar [91] reported that the method of continued fraction expansion with the Cauer second form is superior to other methods of reduction for controller design. Other authors [12,13,17,40,109] reported similar success in controller design from reduced models while others [18,36,55,67] have reported cases in which the control is not acceptable or is, worse yet, unstable.

The use of reduced models for controller design of MIMO systems has received little attention in literature. Davison [20] and Nicholson [76] successfully designed a suboptiminal controller for a fossil plant using reduced models. However, Marshall [68] demonstrated a case in which a control scheme designed using the inverse Nyquist array and a reduced model, failed to control the full model. Later, Gerstle, et al., [36] investigated the same MIMO model that Marshall [68] had used with the multivariable single input-single output and the singular value decomposition control structures. Again, the controller designed from the reduced models failed to control the full models.

Multivariable PI Controller

The use of reduced models for controller design is not only a test of the success of the model reduction technique, but is also a test of the robustness of the control structure. Davison [22] recently proposed a multivariable tuning regulator structure which is said to be very robust [22,80,81]. This controller is a multivariable PI controller [81].

The multivariable PI controller consists of two parts: the proportional part and the integral part. These two parts are based on the dynamic and steady state interactions of the process. To decouple the dynamic response of the system, the proportional part, K_p, of the controller is set to,

$$K_p = P \text{ diag } [p(1), p(2), \ldots, p(q)] \tag{2.107}$$

where P is a constant r x q matrix and the scalars p(i) are the tuning parameters of the controller. The P matrix is set to [80,81],

$$P = (CB)^+ \tag{2.108}$$

where C and B are the state space matrices and the superscript, $^+$, signifies the matrix pseudo-inverse. The P matrix may also be calculated using the slopes of the initial open loop responses [81]. To eliminate steady state error and to decouple the steady state response the integral part, K_I, is set using the steady state gain matrix K to,

$$K_I = eK^+ \tag{2.109}$$

where e is a scalar tuning parameter.

The multivariable PI controller has been shown to be very effective [80] for systems with or without strong interactions. The controller has been designed for unknown processes with the assumptions that the process is linear, time-invariant, and stable [22]. The structure of this controller is a generalization of the classic single input-single output tuning regulator to the multivariable case.

CHAPTER III

COMPARISON OF REDUCTION TECHNIQUES

The calculation of a reduced model is a simple task once a reduction technique is selected. The question is whether this reduced model is of any use to the engineer for simulation and design purposes. The precision required in the reduced model will vary somewhat depending on the application, but some level of precision should be established.

Obviously if the full model is stable, the reduced model should also be stable. Although the point of stability may seem trivial as a criterion for acceptance, some model reduction techniques (e. g., continued fraction expansion and equation fitting) may fail to preserve the stability of the full model. Since stability is relatively easy to determine by eigenvalue analysis, the modeler should check it before proceeding with the use of the model. Other performance factors are not as easily defined.

The ultimate purpose of the reduced model is important in establishing acceptance criteria for reduced models. If a model is to be used for simulation, such key points as rise time, overshoot, and steady state should be closely approximated. When the reduced model is used for controller design, the said controller should be able to adequately control the full model.

Intuitively, the better that a reduced model can simulate the full model, the more likely that controller design will be successful. The remainder of this chapter deals with those problems in both the single input-single output (SISO) and multiple input-multiple output (MIMO) cases which could deter successful simulation.

Comparison Criteria in the Time Domain

The main method of comparison criteria used in the model reduction literature is the comparison of the open loop responses of the full and the reduced model to a step input. In this way, any key characteristics of the full model (e. g., rise time, overshoot, steady state) can be checked in the reduced model.

Since the nature of a system is different for each system considered, the characteristics which must be maintained vary slightly from system to system. Typically, the fast components of a system are ignored both in reduction methods and in comparison criteria; however, these components may have a devastating effect when control designed for the reduced model is applied to the full model.

One component that might be ignored is what is commonly termed a sag. This term refers to an initial response that is in the opposite direction of the final response, as in a nonminimum phase system [114]. If this response is ignored by the reduced model, the controller designed will have positive feedback, during the sag period, when applied to control of the full model.

Another section of the open loop response is the transient or oscillatory part of the response. If the full model is underdamped, then the reduced model should be underdamped as well. It is highly unlikely that the controller designed using this reduced model will will succeed for the underdamped full model, if the reduced model is overdamped.

The characteristics of the full model and reduced models can be compared in different ways. As mentioned in the previous chapter, this comparison has typically been done graphically -- an open loop response to a step input. Many of the characteristics may also be checked by comparing the poles and zeros of the full and reduced models. Of the methods discussed in Chapter 2, only the dominant mode methods maintain some of the identical poles and zeros of the full model. All of the other methods change the poles and zeros, hence the potential for stability changes. By checking the movement of the poles and zeros, the nature of the model (i. e., whether it is overdamped or underdamped, is a minimum or a nonminimum phase system) is often easier to verify than with the open loop plots.

The motivation for using the pole/zero comparison is, that by comparing only the open loop responses of a step input, the high frequency response of the system is essentially ignored. The high frequency response of the system could be checked by varying the input. Sinusoidal inputs of various frequencies could be used as could a pulse/impulse input. The inspection of the outputs associated with various sinusoidal inputs would be time consuming, whereas an impulse input, (i. e., a theoretical input which contains all frequencies), may be approximated by a pulse function.

Unfortunately, when comparing output responses of the reduced and full models, one is left making an intuitive judgement; mainly, does the reduced model adequately fit the full model? One means of determining the fit would be to calculate some cost function between the reduced and full models. Two commonly used cost functions are, the sum of the absolute error, and the sum of the squared error. The sum of the absolute error, AE, is

$$AE = \Sigma \, |y^*_i - y_i| \qquad (3.1)$$

and the sum of the squared error, SE, is

$$SE = \Sigma \, (y^*_i - y_i)^2 \qquad (3.2)$$

where y^*_i is the output of the reduced model and y_i is the output of the full model.

While these error criteria do supply a method of comparing several reduced models for a given full model, there is no way to define a point at which the reduced model is acceptable when the cost is less than this number. For the cases in which only one reduced model is to be generated, the calculation of a cost gives no more information than simply comparing the open loop responses graphically.

A fundamental drawback to using cost functions is the method in which the errors are weighted. Without weighting factors, a sag or overshoot may not affect the cost substantially if it is sharp. Obviously, steady state must be matched, since otherwise the cost will be infinity; however, adequate simulation and controller design may still be achieved even when the steady state value of the reduced model is in error. Although the cost functions do give a feel for how close the fit of the reduced model is to the full model, it by no means delivers an absolute verdict in determining the adequacy of the reduced model.

Comparison Criteria in the Frequency Domain

Through the use of one of the frequency domain analysis methods (root locus, Bode plot, and Nyquist plot) a model may be analyzed at all frequencies. Marshall [68] has used the Nyquist plot for comparing reduced models with the full model. Since chemical engineers are more familiar with the Bode plot and the Bode plot can handle complex transfer functions much more easily, this tool will be used for analysis of reduced models in the frequency domain.

The Bode plot consists of two plots which compare the magnitude ratio and the phase angle varying with frequency. With the Bode plot, the fast components of the model may be compared in the high frequency region of the plot. Similarly, any dominant characteristics, such as complex poles/zeros, may easily be seen and checked for on the Bode plot.

As in the time domain, the Bode plot reflects which characteristics of the full model are maintained by the reduced model. Similarly, costs functions for both the magnitude ratio and phase angle may be set, as in the case of the time domain. This criterion suffers from the same problems as the time domain. An additional problem, though, is that the error will be infinity due to an offset in the high frequency range as well as in the low frequency range. Since the fast components which affect the high frequency range are eliminated or are substantially adjusted, the high frequency may not be matched; and yet, the reduced model may still be useful for process simulation and controller design.

In the case of model reduction by continued fraction expansion with the Cauer third form, both the low and high frequency regions of the Bode plot are matched. Even in this case, the reduced model may not maintain the dominant characteristics of the full model. If the full model contains complex poles/zeros, the reduced model may not maintain these characteristics and yet match the high and low frequency terms. In this case, the reduced model may not be useful for either simulation or controller design.

Controller Design as an Acceptance Criterion

In a theoretical setting, the acceptance of a reduced model based on whether the controller design succeeds or fails for control of the full model, is possible. But from the practical standpoint, it is a question that should not even be raised -- the control should work. What the two succeeding chapters show, are the reduction techniques which hold the most promise for the design of controllers.

Special Problems of Reducing MIMO Models

Most of the model reduction research published thus far has neglected the reduction of MIMO models. The authors of these papers maintain, however, that the particular method about which they are writing, is applicable to the reduction of multivariable systems.

The reduction of MIMO models is not a simple extension of the methods already developed for SISO models. Not only do they involve more parameters, they also involve process interactions. A fundamental concern of the control engineer is the steady state and dynamic interactions that occur between the input and output variables. In other words, a change in one variable may and, indeed, generally does have an effect on more than one output. The interactions present in the full model must be maintained in the reduced model for acceptable simulation and controller design.

Most multivariable systems do have significant coupling between the inputs and the outputs [30,57,87,114]. Numerous methods have been suggested as measures of interaction in multivariable processes. One of the most widely used of these methods is the Bristol array [6]. The Bristol array measures the amount of steady state interaction. There is no generally accepted measure of dynamic interactions, which is unfortunate since sometimes there are significant dynamic interactions not accounted for in the steady state interaction measurement.

For this research, the control scheme used for the multivariable processes is the multivariable tuning regulator. As previously discussed, this controller contains two decoupling matrices: one for the fast components and one for the steady state components. This controller was selected in order to use the fast component decoupler as a test of any change in dynamic interactions. It was also chosen because it is touted as being very robust [22]. If the controller is as robust as is claimed, then the control scheme will be able to handle small changes in the model while giving information regarding the maintenance of the dynamic and steady state interactions of the full model as witnessed in the reduced model.

Summary of Comparison Techniques

In the preceding four sections of this chapter, numerous methods have been proposed for determining whether a reduced model is acceptable. Unfortunately, each of these methods has its drawbacks. What remains, though, is a set of tools available for making an "engineering judgement" regarding the use of the reduced models. This judgement depends on the complexity of the full model and on how the reduced model is applied. This approach to model reduction enables one to establish the feasibility of model reduction.

CHAPTER IV

ANALYSIS OF THE SINGLE VARIABLE MODEL REDUCTION PROBLEM

This chapter describes the reduction of three different single input-single output (SISO) models. The reduction techniques will be compared in light of simulation and controller design for the full models. The chapter closes with suggestions on how to deal with the single variable problem.

Each of the three models will be reduced by eight different methods:

1) Continued fraction expansion with the Cauer first form,
2) Continued fraction expansion with the Cauer second form,
3) Continued fraction expansion with the Cauer third form,
4) Dominant mode with steady state compensation,
5) Equation fitting with the absolute error as the cost function,
6) Equation fitting with the squared error as the cost function,
7) Internal balancing and subsystem elimination and,
8) Internal balancing and subsystem elimination with steady state compensation.

The basis of all eight of these methods has been introduced in Chapter 2. The dominant mode and internal balancing with steady state compensation methods are extensions of the methods presented in Chapter 2 and are discussed in detail in a subsequent section.

Reductions of two different orders will be performed with every method on each full model. The full models will be reduced to a second order model and another order selected depending on the particular model in question. A second order reduced model will be calculated since a common assumption is that any process may be represented by either a first or second order model. The order of the other reduced model is selected using the criteria established in the internal balancing of the full model.

The sixteen reduced models (two reduced models for each of eight reduction methods) are then compared using the criteria established in Chapter 3. The reduced models are first checked for stability. Then the open loop responses of the remaining stable reduced models are generated. The simulation is performed using a discrete-time model formulated by a diagonal Pade approximation of the matrix exponential [113]. The numerical capabilities of this simulation technique is well established. With the open loop responses, the error criteria integral of the squared error (ISE) is calculated. The Bode plots of the reduced models are then compared visually with the full model and with each other. Finally, a PI controller is designed for each of the reduced models and applied to control of the full model.

Modifications on SISO Methods of Reduction

As mentioned previously, two of the reduction methods for single input-single output models have been extended. The changes were made so that the reduced model will maintain the steady state response of the full model.

Recall that the dominant mode method simply retains the dominant poles and zeros of the full model. This method, developed by Davison [20], is for a general model and thus is quite involved; however, it does not maintain the steady state of the full model. Various schemes have been proposed in the literature to modify the method such that the correct steady state is maintained [1,21]. Instead of using this involved method for the simple case of the SISO model, a different technique is proposed. As before, the dominant poles and zeros are maintained. Then the gain of the reduced model is calculated using the final value theorem to match the steady state gain of the full model.

A similar procedure is performed to maintain the steady state gain of the internally balanced reduced model. The reduced model is calculated by internal balancing and subsystem elimination. Then the resulting state space model is transformed to transfer function form using Leverrier's algorithm [114] Again the final value theorem is applied and the gain calculated to maintain the steady state gain of the full model.

The following sections present three different SISO models for the model reduction techniques to be compared on. For the reduction of these models, several computer program packages were employed

[2,35]. Each model will be introduced, reduced, and analyzed separately. Then all three models will be discussed in the final section which summarizes the SISO model reduction problem and proposes methods for dealing with SISO model reductions problems.

SISO Model Reduction: Case I

The first single variable model to be considered is a sixth order model. This model is taken from a multivariable model that was posed by Goldman, et al [38]. The transfer function for this model is,

$$G(s) = 0.4\ [50 + 125s + 49.5s^2 + 37.0s^3 + 8.5s^4 + s^5] \cdot [510 + 2120s + 2700s^2 + 1120s^3 + 261s^4 + 25.5s^5 + s^6]^{-1}. \qquad (4.1)$$

As mentioned in the introduction to this chapter, this model is to be reduced to a second order model and another order which will depend on the particular model in question. To determine the order of the second reduction, the spread of the second order modes of the full model will be used. For this model, the second order modes of the full model are: {0.020, 0.011, 0.010, 0.00071, 0.000303, and 0.000081}. From the second order modes, an obvious break occurs between the third and fourth singular values which have a ratio of 14. Hence using the criterion of the spread of the second order modes, a third order model is suggested.

The eight reduction techniques, as listed in the introductory comments to this chapter, yield the second order models listed in Table 4.1 and the third order models listed in Table 4.2. The models, except for the two internally balanced models, are listed in phase-variable form (equation (2.45)) This form was used to facilitate changing from the state space representation to a transfer function and back. The two reduced models generated by the internal balancing method are listed as the remaining subsystem of the internally balanced model. In transfer function form, these models are,

$$G(s) = [0.0236s + 2.82E\ {-5}] \bullet [s^2 + 0.607s + 0.00104]^{-1}, \tag{4.2}$$

and

$$G(s) = [0.405s^2 + 0.0498s + 1.47] \bullet [s^3 + 18.8s^2 + 61.6s + 37.8]^{-1}. \tag{4.3}$$

Initial analysis of the 16 reduced models yields evidence that the third order model obtained by the Cauer second form of continued fraction expansion is unstable. The remaining 15 reduced models will be compared graphically, with open loop step responses and Bode plots, by calculating cost criteria, and for PI controller design.

The 15 open loop step responses are shown in Figures 4.1 through 4.4. The input is a unit step at time equals one second. Each figure consists of four plots; a single plot for each of the methods applied and one of the two orders investigated. Following the open

TABLE 4.1. Second Order Reduced Models for SISO Case I

Reduction Methods	State Space Matrices			
	A		B	C^T
Cauer First Form	0 -120.	1.0 -20.7	0 1	1.09 .400
Cauer Second Form	0 -.171	1.0 -.87	0 1	.00671 .02310
Cauer Third Form	0 -5.03	1.0 -18.5	0 1	.197 .400
Dominant Mode	0 -.303	1.0 -1.1	0 1	.0119 .0261
Absolute Error	0 -1.0	1.0 -2.06	0 1	.0392 .0120
Squared Error	0 -.999	1.0 -2.16	0 1	.0392 .0160
Internal Balancing	-.606 -.023	.0231 -8.4E -4	.15400 .00133	.15400 -.00133
Steady State Internal Balancing	0 -.00104	1.0 -.607	0 1	4.08E -5 .0342

Table 4.2. Third Order Reduced Models for SISO Case I

Reduction Method	State Space Matrixes				
	A			B	C^T
Cauer First Form	0	1	0	0	-.272
	0	0	1	0	1.36
	-34.7	-131.	-21.4	1	.400
Cauer Second Form	0	1	0	0	-.0150
	0	0	1	0	-.00613
	-.381	-.785	.157	1	.0189
Cauer Third Form	0	1	0	0	1.79
	0	0	1	0	-.00762
	-45.8	-75.2	-18	1	.400
Dominant Mode	0	1	0	0	.276
	0	0	1	0	.00947
	-7.04	-16.3	-5.25	1	.0715
Absolute Error	0	1	0	0	1.80
	0	0	1	0	.103
	-45.9	-78.4	-22.1	1	.494
Squared Error	0	1	0	0	1.80
	0	0	1	0	.0927
	-45.9	-78	-22.5	1	.490
Internal Balancing	-.606	.0231	3.16	.154	.154
	-.0231	-8.4E -3	-7.78	.00133	-.00133
	3.16	7.78	-18.2	-.618	-.618
Steady State Internal Balancing	0	1	0	0	1.48
	0	0	1	0	.0504
	-37.8	-61.6	-18.8	1	.410

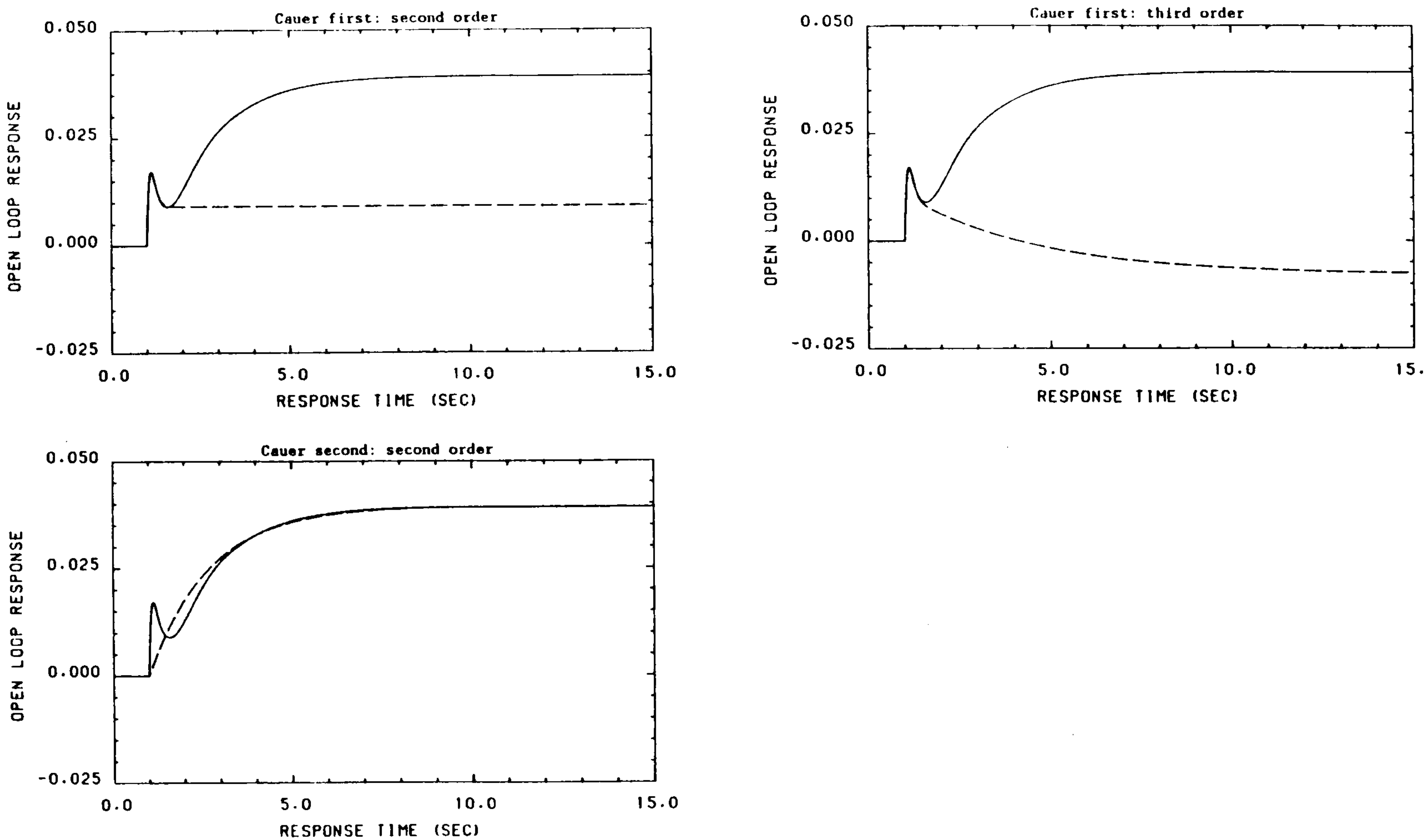


Figure 4.1. Open Loop Responses of the Cauer First and Second Forms for SISO Case I.

Full model: ——— Reduced model: — — — —

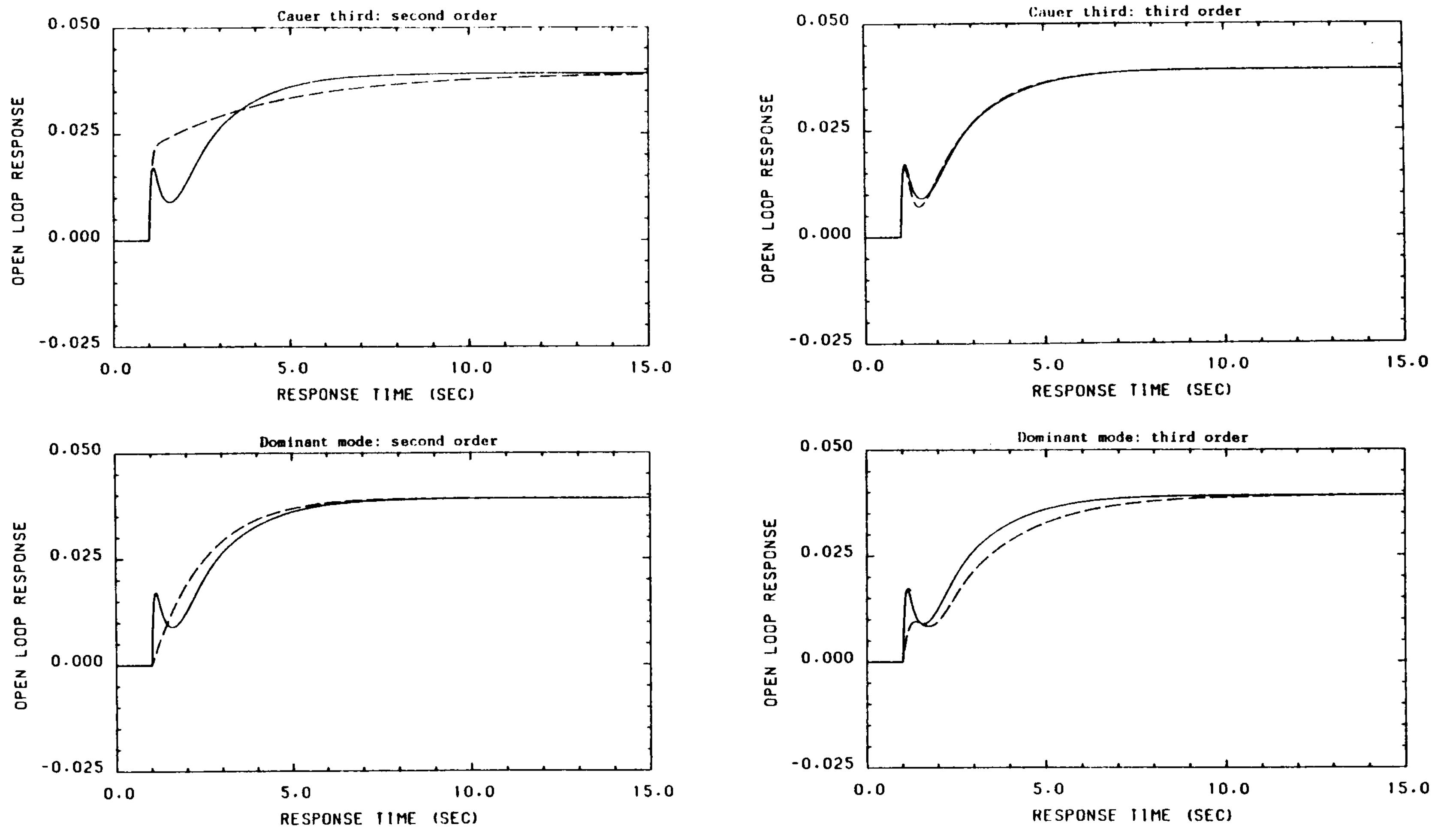


Figure 4.2. Open Loop Responses of the Cauer Third Form and Dominant Mode Method for SISO Case I.

Full model: ——— Reduced model: - - - -

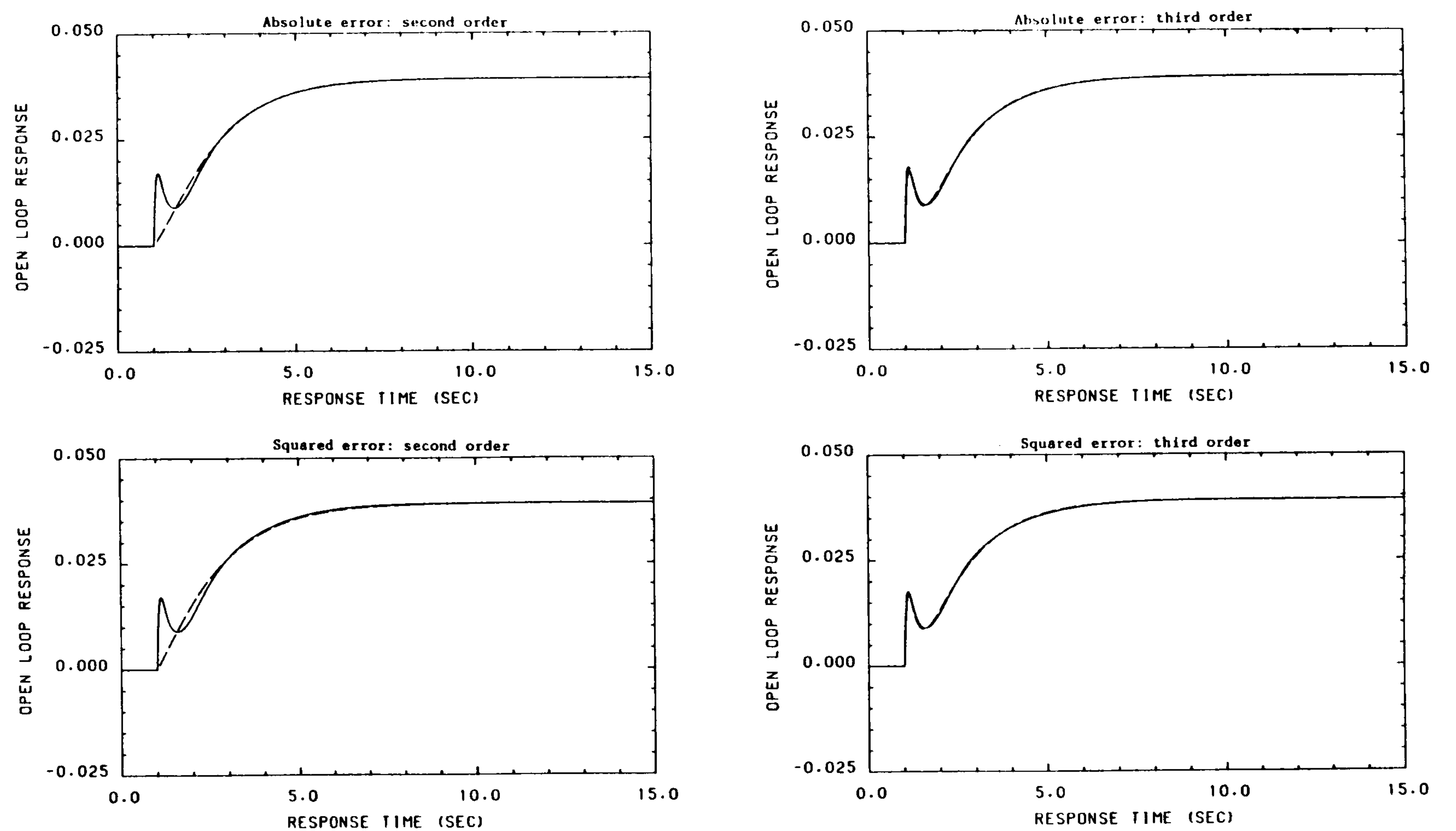


Figure 4.3. Open Loop Responses of the Absolute and Squared Error Methods for SISO Case I.

Full model: ———— Reduced model: – – – –

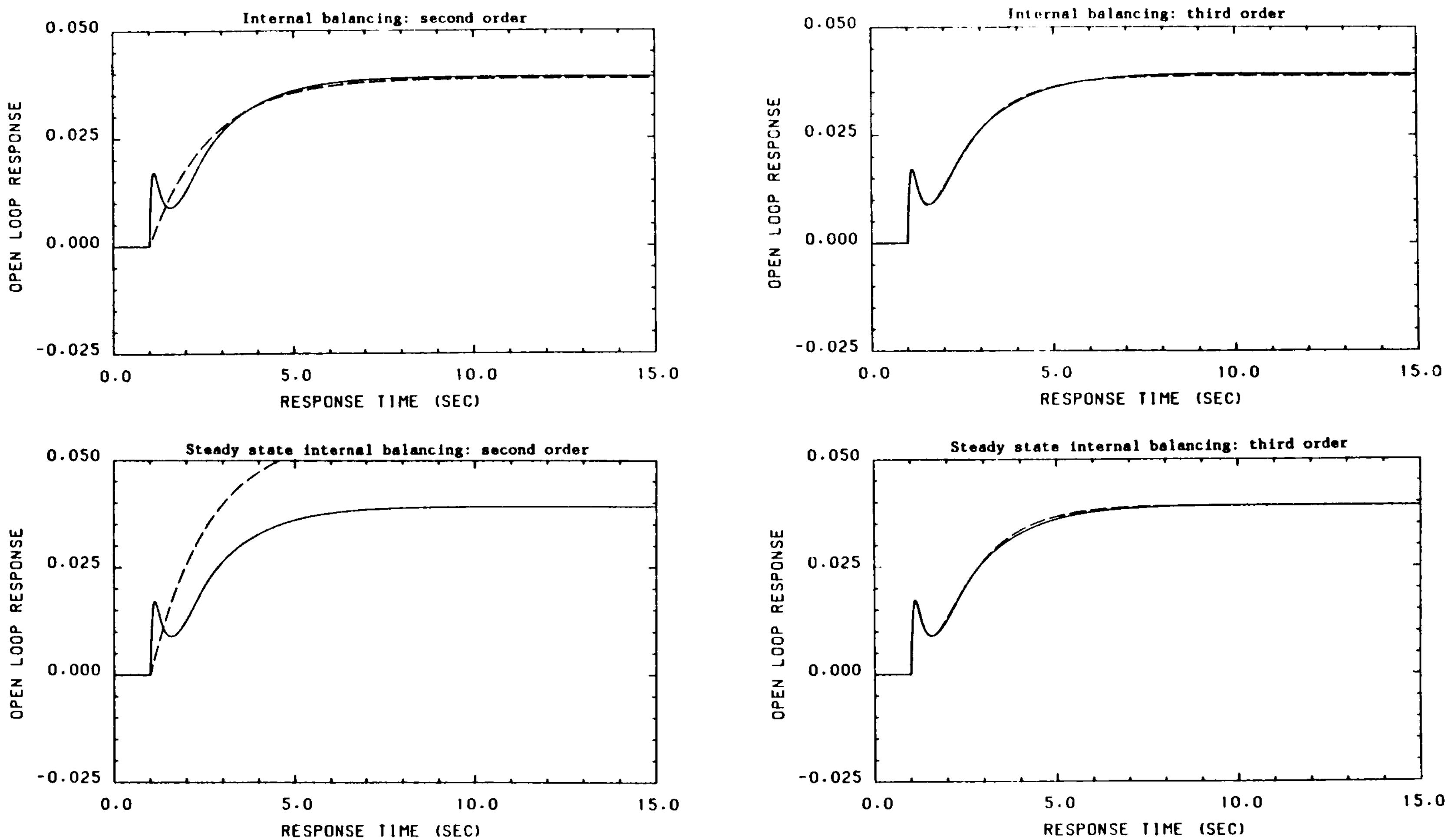


Figure 4.4. Open Loop Responses of the Internal Balancing and Steady State Internal Balancing Methods for SISO Case I.

Full model: ——— Reduced model: — — —

loop responses, is Table 4.3, which contains the integral of the squared error ISE values for the open loop responses. The next set of figures, Figures 4.5 through 4.8, contain the Bode plots of the . reduced and full models. For the Bode plots, the two reductions (if both are stable) are plotted on the same graph with the full model.

The reduced models are now used for controller design. For each of the fifteen stable reduced order models, a PI controller will be designed. The two control parameters are set using the integral of the absolute error (IAE) criterion [66,114] and a pattern search routine [74,75]. The resulting control parameters, proportional band, and integral time are listed in Table 4.4. The fifteen PI controllers are then applied to control of the full model. Of the eight PI controllers designed using a second order model, only two succeeded in control of the full model, the Cauer first and second forms. Whereas with the third order models, six of the seven controllers succeeded with only the dominant mode method failing.

The resulting IAE of each controller when applied on the full model is listed in Table 4.4. The closed loop responses to a unit set point change are shown in Figures 4.9 and 4.10. The responses of the controllers which failed to control the full model are not included in these plots due to the fact the the output increased without bound after only a few iteration steps. The results of the controller design and simulation using these reduced models will be

Table 4.3. ISE Values of the Reduced Models of SISO Case I

Reduction Method	Reduced Order Model	
	Second	Third
Cauer First Form	∞	∞
Cauer Second Form	5.6E -5	unstable
Cauer Third Form	.00026	2.2E -6
Dominant Mode	7.8E -5	9.3E -5
Absolute Error	5.4E -5	5.0E -6
Squared Error	5.2E -5	4.4E -7
Internal Balancing	∞	∞
Steady State Internal Balancing	.0037	1.0E -6

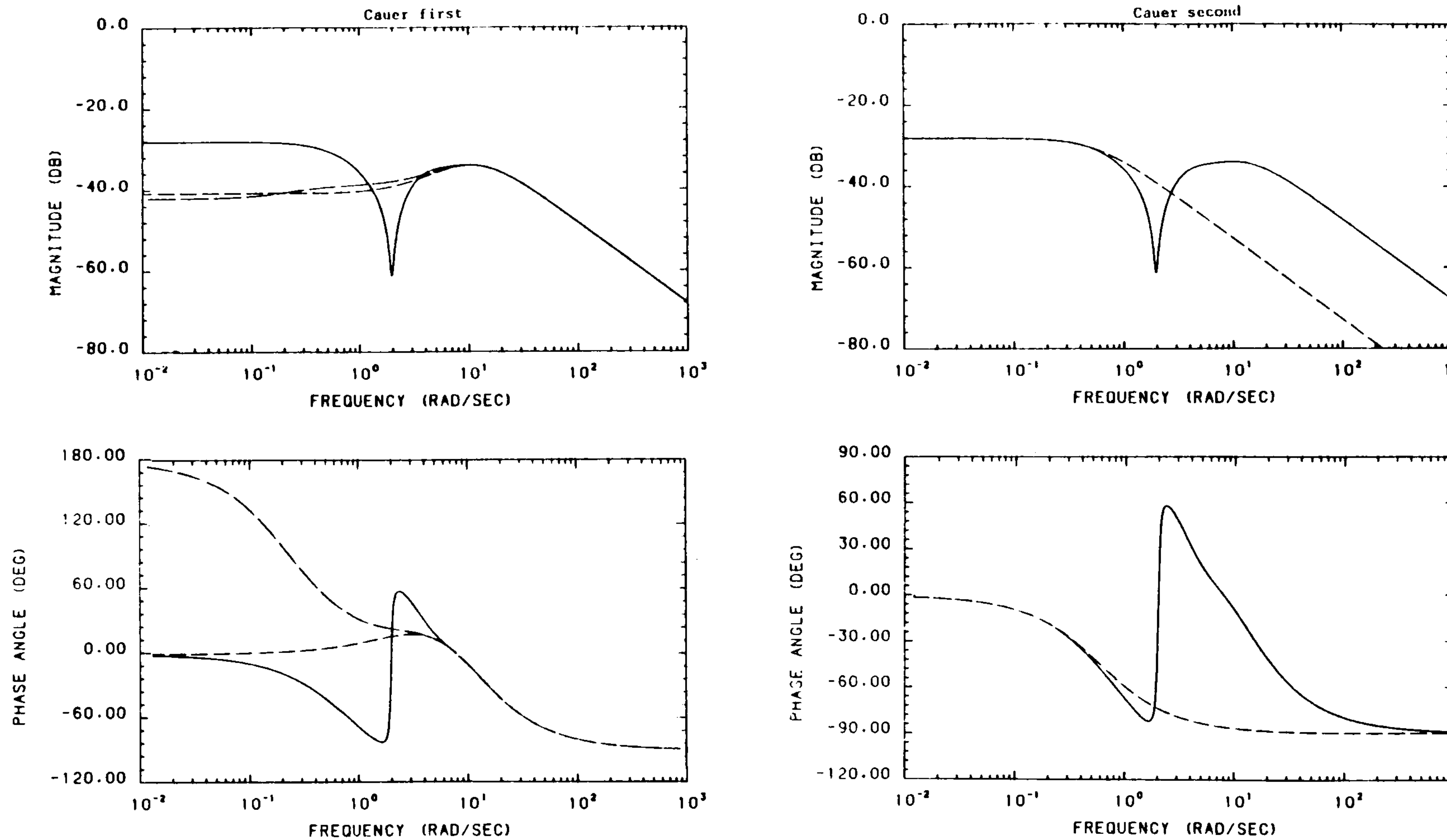


Figure 4.5. Bode Plots of the Cauer First and Second Forms for SISO Case I.

Full model: ———— Second order model: — — — — Third order model: —— —— ——

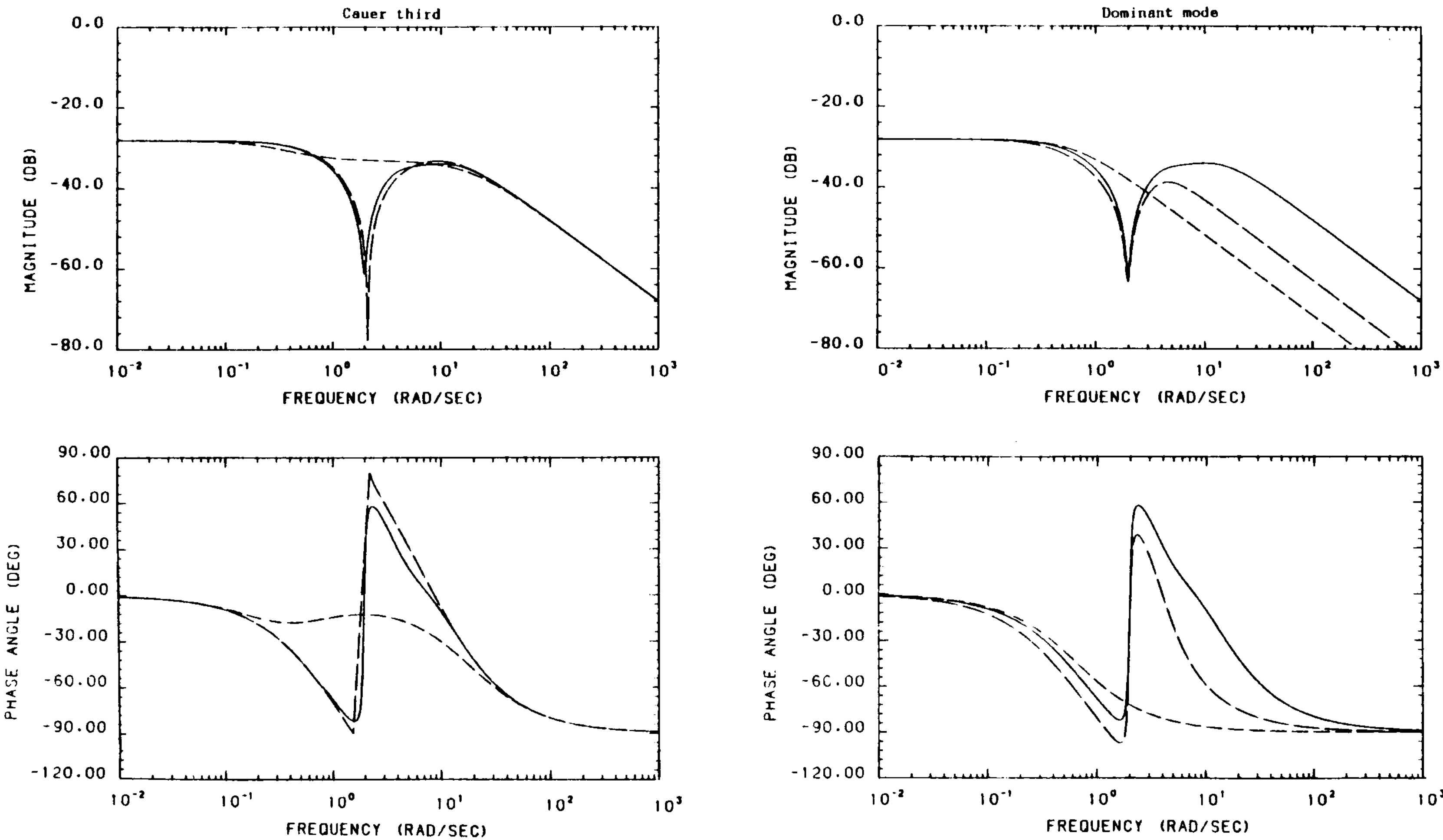


Figure 4.6. Bode Plots of the Cauer Third Form and Dominant Mode Method for SISO Case I.

Full model: ——— Second order model: — — — Third order model: —— —— ——

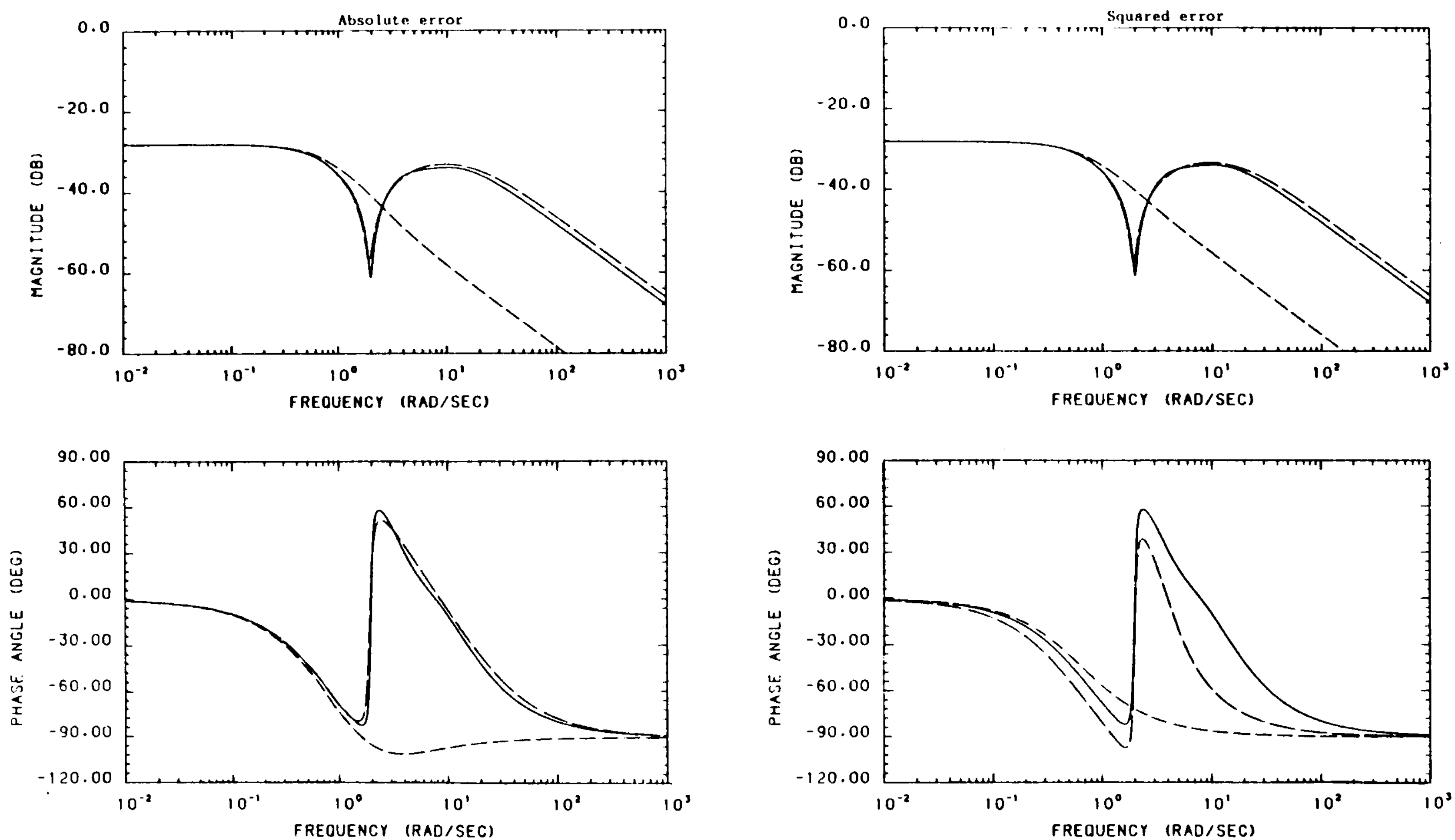


Figure 4.7. Bode Plots of the Absolute and Squared Error Methods for SISO Case I.

Full model: ———— Second order model: — — — Third order model: —— —— ——

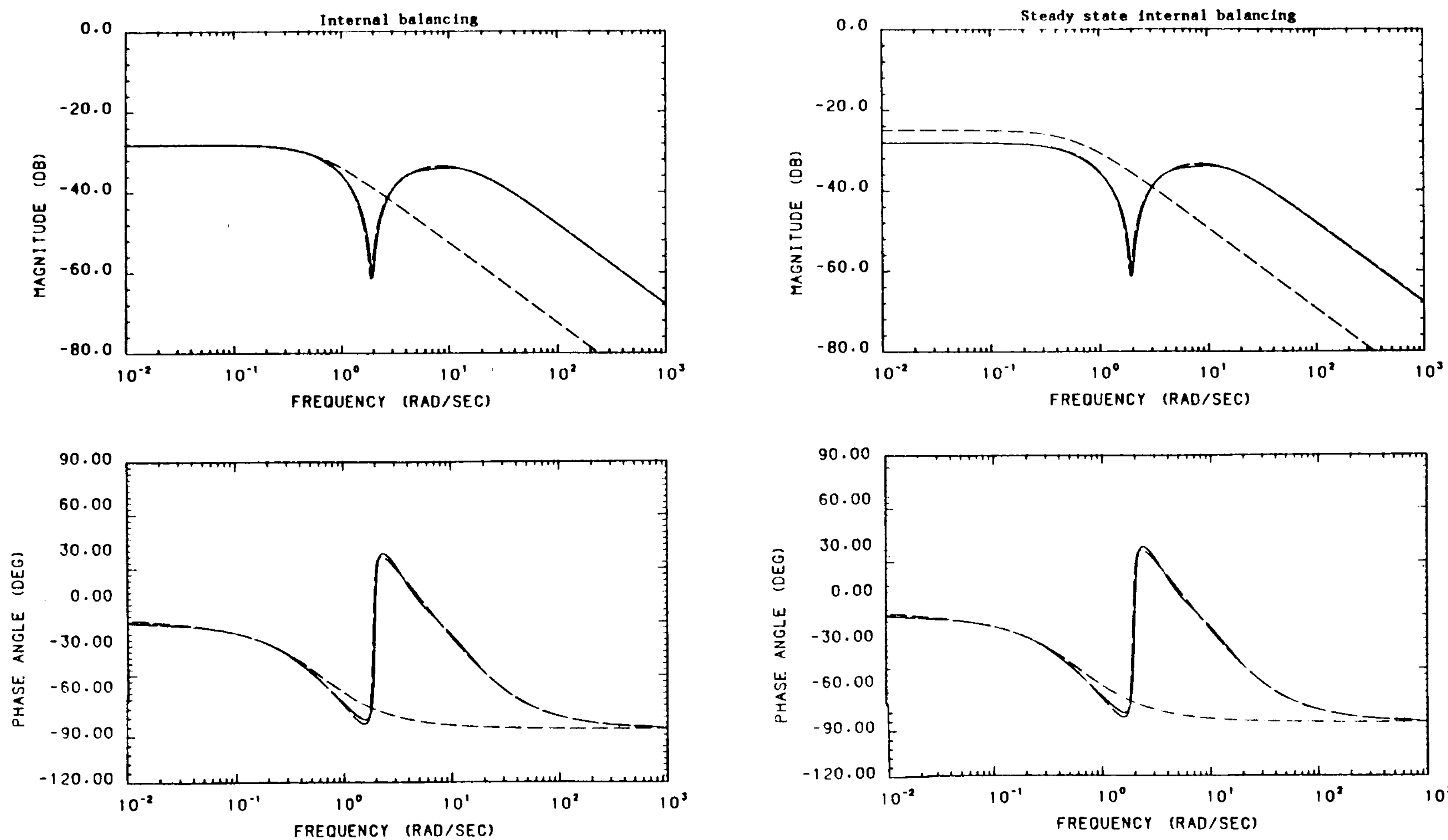


Figure 4.8. Bode Plots of Internal Balancing and Steady State Internal Balancing for SISO Case I.

Full model: ———— Second order model: — - — - Third order model: — — —

Table 4.4. Controller Tuning Parameters for SISO Case I

Reduction Method	Reduced Model Order: Second			Third		
	Gain	Reset time	IAE with full model	Gain	Reset time	IAE with full model
Cauer First Form	56	.02	.38	19	.0016	.20
Cauer Second Form	1700	1.7	∞		unstable	
Cauer Third Form	80	.045	.56	160	2.6	.70
Dominant Mode	1500	1.5	∞	910	4.8	∞
Absolute Error	3400	1.8	∞	140	2.4	.71
Squared Error	2500	1.9	∞	150	2.4	.71
Internal Balancing	1700	1.6	∞	170	2.6	.70
Steady State Internal Balancing	1200	1.6	∞	170	2.6	.70

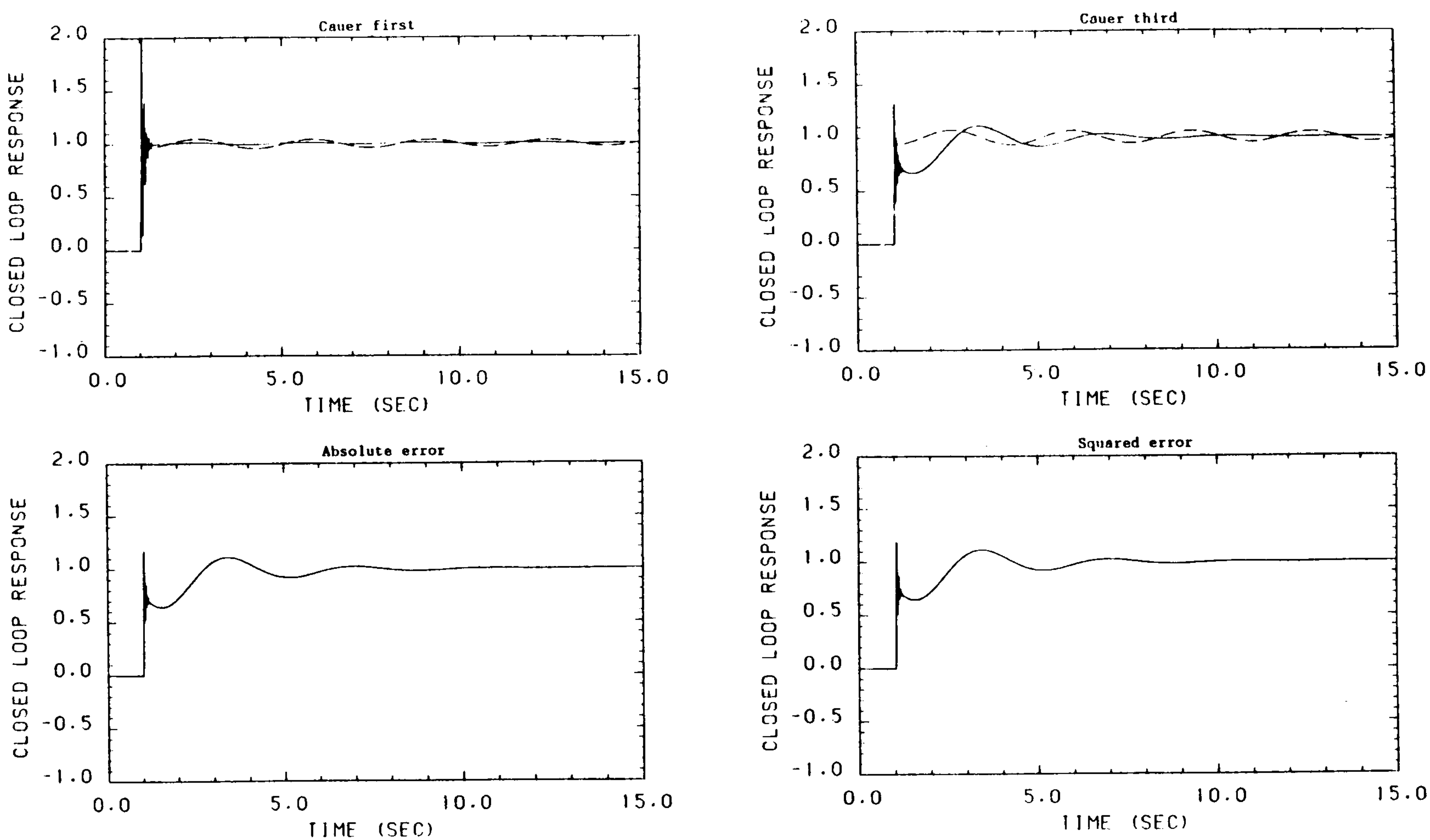


Figure 4.9. PI Control of the Full Model Using Parameters Set with the Reduced Models of Methods One, Three, Five, and Six for SISO Case I.

Using second order model: — — — — Using third order model: ————

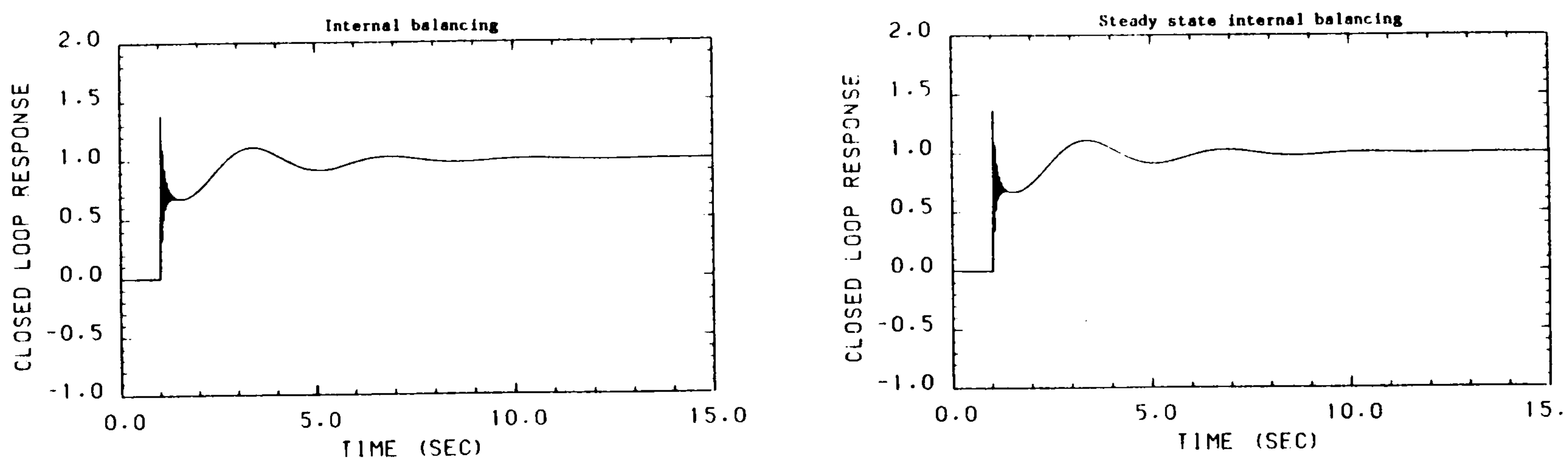


Figure 4.10. PI Control of the Full Model Using Parameters Set with the Reduced Models of Methods Seven and Eight for SISO Case I.

discussed along with the next two case studies in the final section of this chapter.

SISO Model Reduction: Case II

For this case, the SISO model in question will use the same poles as the preceding model with only zeros of the model being changed. In transfer function form, the model is,

$$G(s) = -4\ [-49.8 - 105s - 39.0s^2 - 7.3s^3 - 0.05s^4 + s^5] \bullet [510 + 2120s + 2700s^2 + 1120s^3 + 261s^4 + 26.5s^5 + s^6]^{-1}. \tag{4.4}$$

This model was chosen to demonstrate model reduction of a nonminimum phase system and the effect of the zeros on the second order modes.

This model is to be reduced to two different lower order models with one of the reduced models being second order and the other specified by the model in question. For this model, the second order modes are: {0.22, 0.11, 0.11, 0.036, 0.0050, 0.00007}. From analysis of these second order modes, the only substantial separation occurs between the fifth and sixth modes, which suggests that a fifth order model should adequately represent the system. Assuming that a more substantial reduction is desired, and to investigate the concept of how large the separation must be, a third order reduced model will again be selected. Note that the ratio between the third and fourth modes is approximately three.

The eight reduction techniques mentioned in the introduction of this chapter are applied to this model. The resulting second and third order models are listed in Table 4.5 and Table 4.6 respectively. The reduced models obtained by the internal balancing method are listed in the remaining subsystem with the rest of the models listed in phases-variable form. For completeness, the transfer functions of the two internally balanced reduced models are,

$$G(s) = 0.18\ [s + 0.0011] \cdot [s^2 + 0.42s + 0.00031]^{-1} \tag{4.5}$$

and

$$G(s) = -3.7\ [s^2 - 4.1s - 4.6] \cdot [s^3 + 20s^2 + 166s + 38]^{-1}. \tag{4.6}$$

Stability analysis of the reduced models shows that both the second and third order models obtained by the Cauer second form are unstable. The remaining fourteen reduced models will be used for simulation and controller design.

The reduced models are compared to the full model for simulation purposes in Figures 4.11 through 4.14. The open loop responses shown in these figures are to a unit step at time equals one. Table 4.7 contains the ISE values for each of the open loop responses and the Bode plots of the reduced and full models shown in Figures 4.15 through 4.18.

Again a PI controller was designed for each of the stable reduced models. Using the IAE criterion, the resulting proportional

Table 4.5. Second Order Reduced Models for SISO Case II

Reduction Method	State Space Matrices A		B	C^T
Cauer First Form	0	1	0	20.3
	-135	-21.5	1	-4
Cauer Second Form	0	1	0	3.11
	7.96	15.1	1	.490
Cauer Third Form	0	1	0	6.67
	-17.1	-24.9	1	-4
Dominant Mode	0	1	0	.118
	-.303	-1.1	1	-.0256
Absolute Error	0	1	0	.391
	-1	-2.47	1	.171
Squared Error	0	1	0	.391
	-1	-2.48	1	.170
Internal Balancing	-.408	.0485	-.424	-.4240
	.0485	-.00652	.0375	.0375
Steady State Internal Balancing	0	1	0	.000122
	-.000312	-.415	1	.108

Table 4.6. Third Order Reduced Models for SISO Case II

Reduction Method	State Space Matrices A			B	C^T
Cauer	0	1	0	0	10.5
First	0	0	1	0	15.1
Form	-285	-167	-22.8	1	-4.00
Cauer	0	1	0	0	-2.33
Second	0	0	1	0	-5.97
Form	5.96	27.5	30.6	1	.802
Cauer	0	1	0	0	47.6
Third	0	0	1	0	2.72
Form	-122	-257	-27.6	1	-4.26
	0	1	0	0	2.75
Dominant	0	0	1	0	4.03
Mode	-7.04	-16.3	-5.25	1	-1.00
	0	1	0	0	47.7
Absolute	0	0	1	0	-1.86
Error	-122	-247	-11.9	1	-2.20
	0	1	0	0	48
Squared	0	0	1	0	2.7
Error	-123	-262	-17	1	-3.3
	-.408	.0485	7	-.424	-.424
Internal	.0485	-.00652	-10.4	.0375	.0374
Balancing	-7	10.4	-19.2	-1.97	1.97
Steady State	0	1	0	0	14.8
Internal	0	0	1	0	13.0
Balancing	-37.8	-166	-19.6	1	-3.18

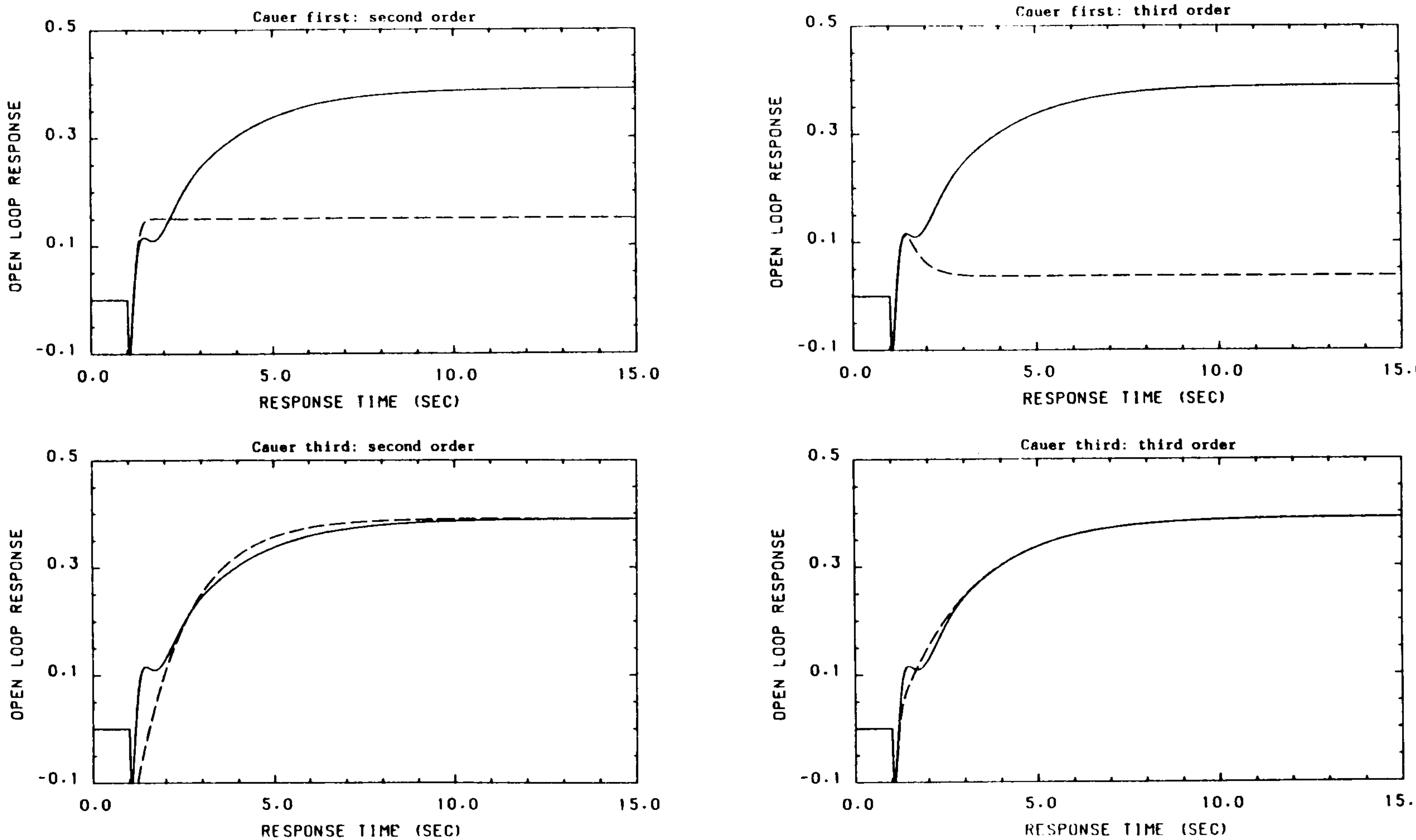


Figure 4.11. Open Loop Responses of the Cauer First and Third Forms for SISO Case II.

Full model: ———— Reduced model: — — —

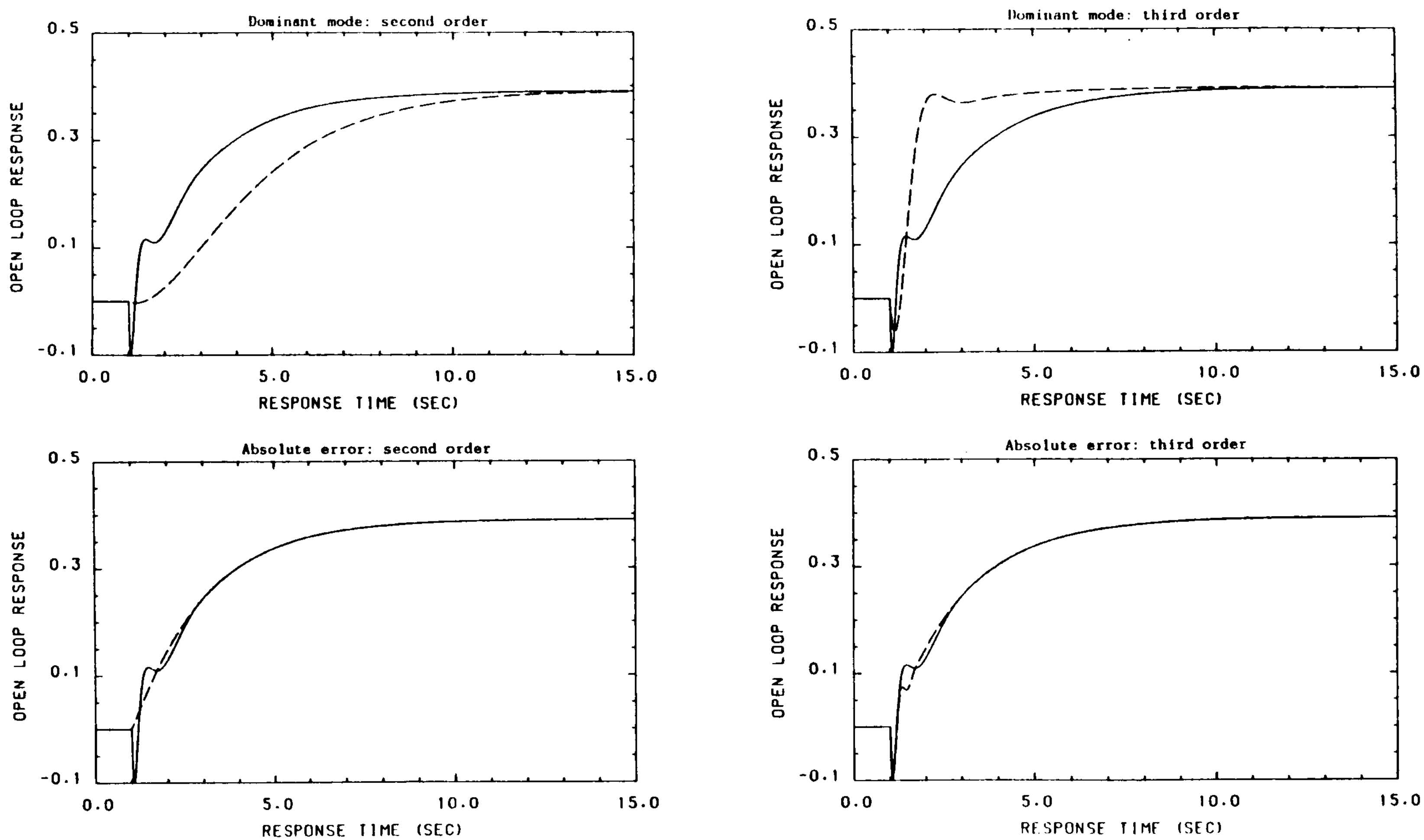


Figure 4.12. Open Loop Responses of the Dominant Mode and Absolute Error Methods for SISO Case II.

Full model: ———— Reduced model: — — — —

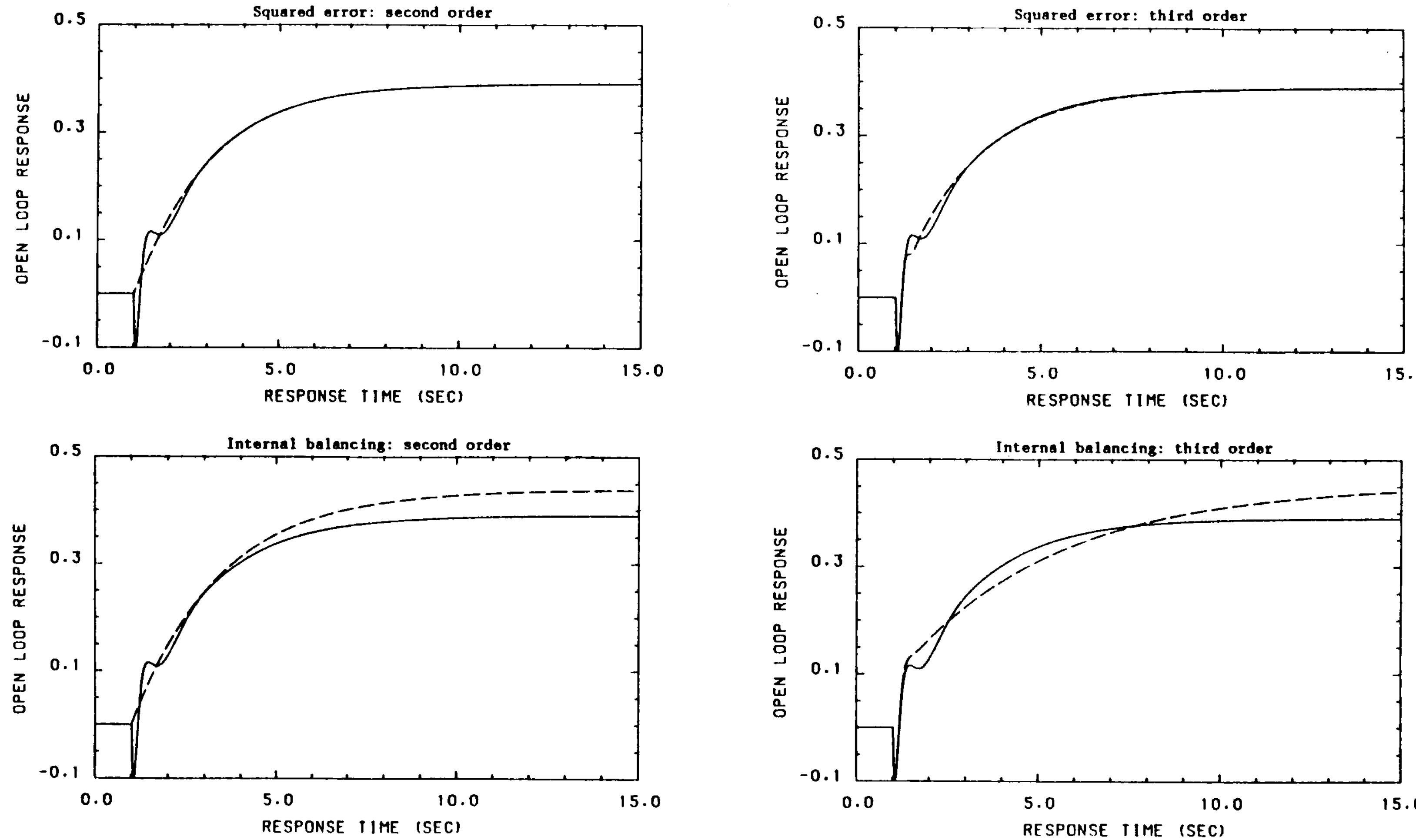


Figure 4.13. Open Loop Responses of the Squared Error and Internal Balancing Methods for SISO Case II.

Full model: ———— Reduced model: — — — —

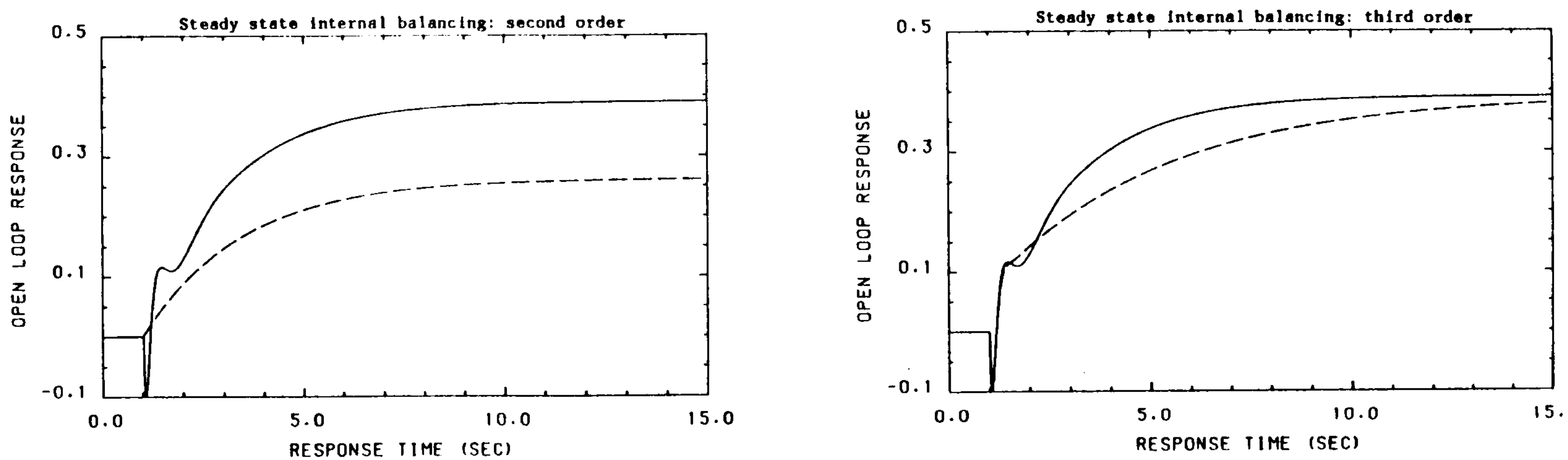


Figure 4.14. Open Loop Responses of the Steady State Internal Balancing Method for SISO Case II.

Full model: ——— Reduced model: — — —

Table 4.7. ISE Values of the Reduced Models for SISO Case II

Reduction Method	Reduced Model Order	
	Second	Third
Cauer First Form	∞	∞
Cauer Second Form	unstable	unstable
Cauer Third Form	.011	.00084
Dominant Mode	.071	.063
Absolute Error	.0019	.00068
Squared Error	.0019	.00056
Internal Balancing	∞	∞
Steady State Internal Balancing	.22	.027

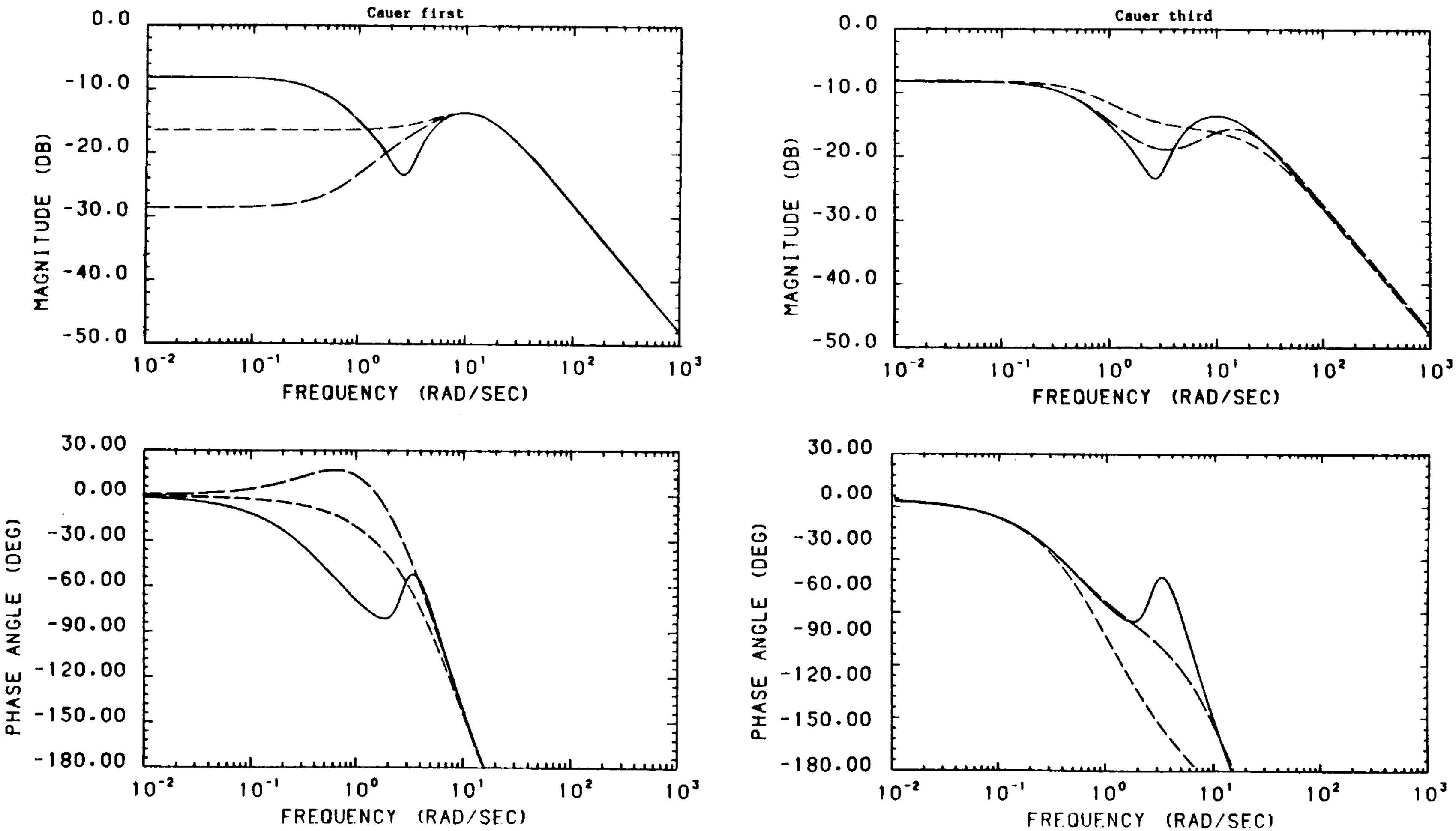


Figure 4.15. Bode Plots of the Cauer First and Third Forms for SISO Case II.

Full model: ——— Second order model: — — — Third order model: —— —— ——

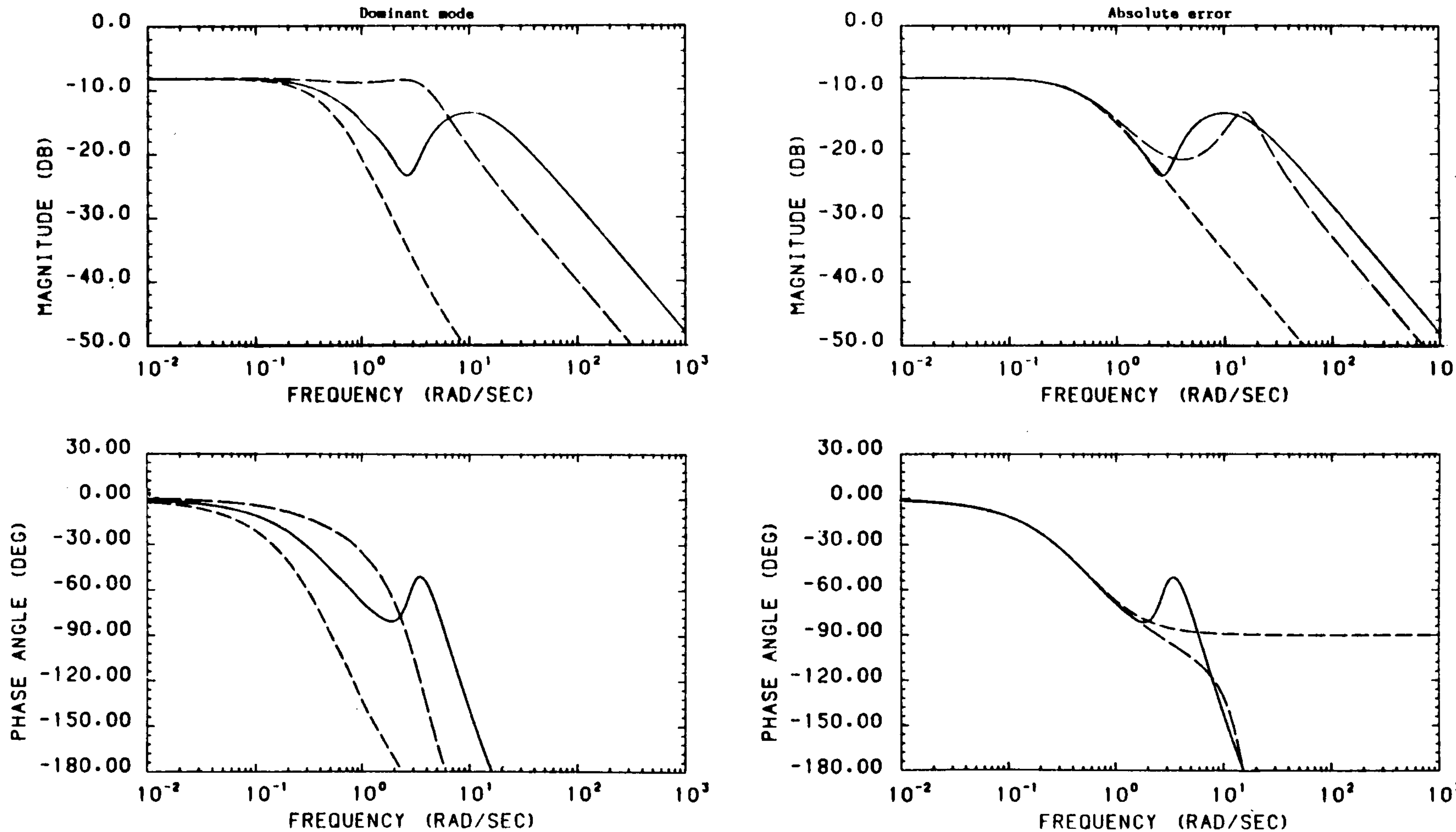


Figure 4.16. Bode Plots of the Dominant Mode and Absolute Error Methods for SISO Case II.

Full model: ———— Second order model: – – – – Third order model: —— —— ——

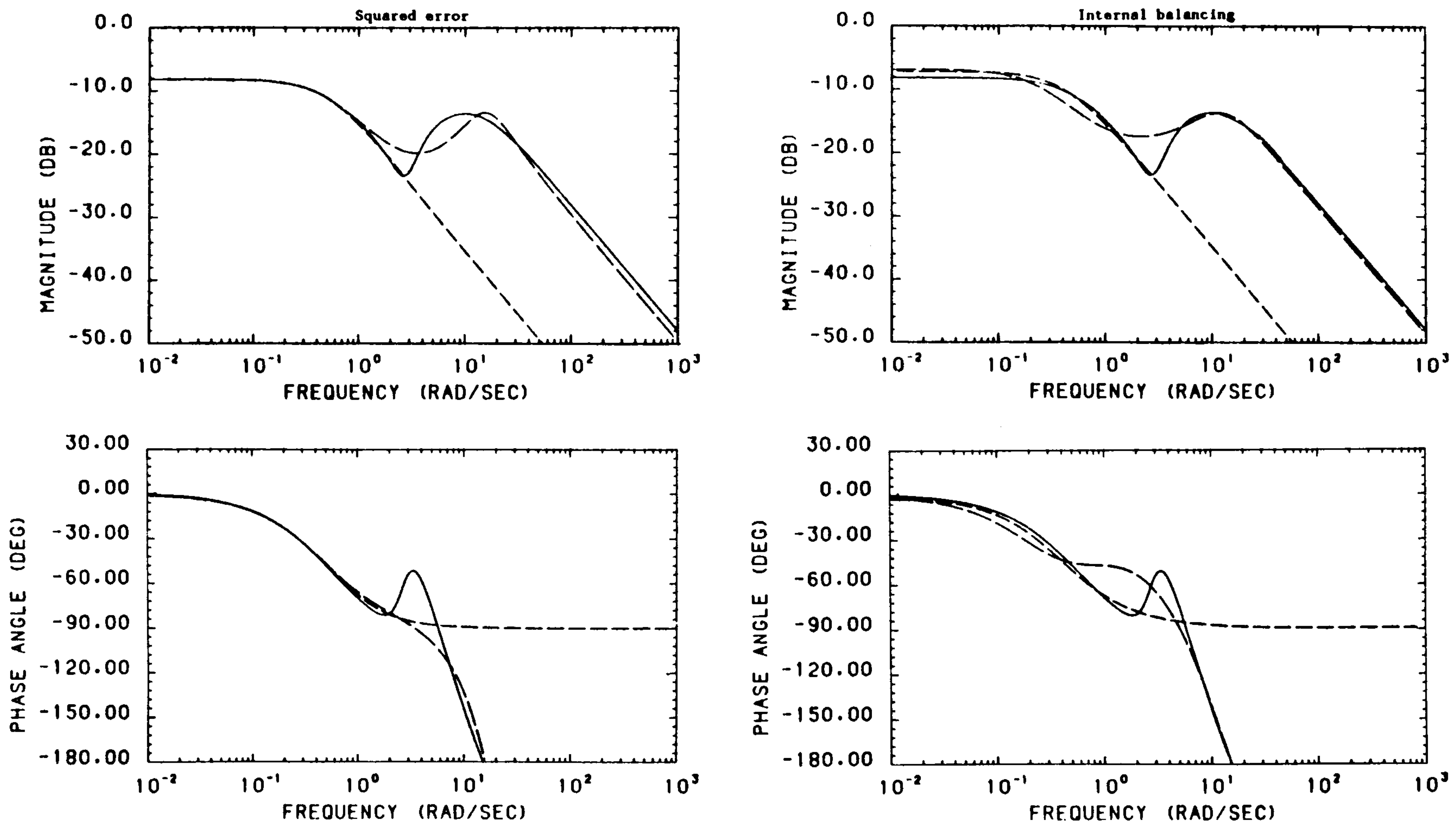


Figure 4.17. Bode Plots of the Squared Error and Internal Balancing Methods for SISO Case II.

Full model: ——— Second order model: — — — Third order model: —— —— ——

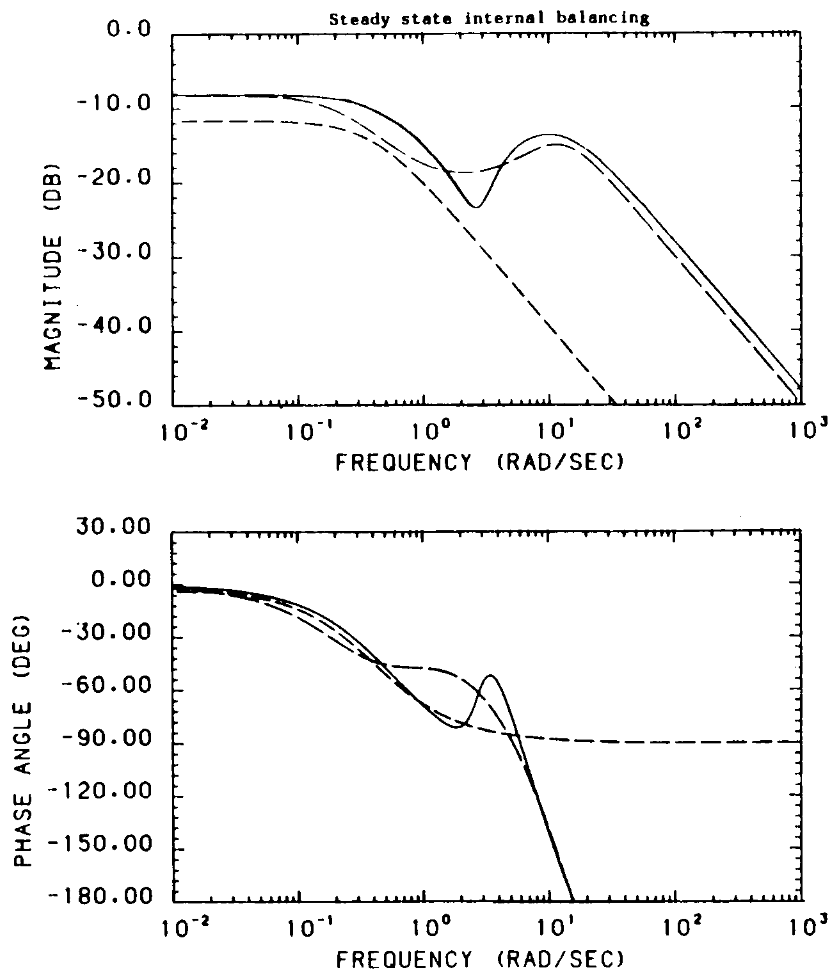


Figure 4.18. Bode Plot of the Steady State Internal Balancing Method for SISO Case II.

Full model: ——— Second order model: – – – Third order model: — — —

bands and integral times are listed in Table 4.8. As in Case I, when using the second order models, only the PI controllers designed with the Cauer first and third forms are successful in control of the full model. However, in this case, all of the controllers designed with the third order models did succeed in controlling the full model.

For comparison of the control of the full models with the PI controllers designed from the reduced models, the IAE is listed in Table 4.8 for each controller when applied to the full model. The closed loop responses of the control of the full model with the various PI controllers are illustrated in Figures 4.19 and 4.20. Again the response of the failed controllers is not included since the control was unstable after a few time steps.

SISO Model Reduction: Case III

The final SISO model to be considered is an eighth order model that has been investigated by Shieh [80] and later by Marshall [74] with the Cauer second form method for simulation purposes. The transfer function of this model is,

$$\begin{aligned} G(s) = 19.8\ [&s^7 + 22s^6 + 244s^5 + 2300s^4 + 12200s^3 \\ &+ 45700s^2 + 95400s + 42500] \\ &\cdot [s^8 + 30s^7 + 358s^6 + 2910s^5 + 18100s^4 \\ &+ 67600s^3 + 173000s^2 \\ &+ 149000s + 37800]^{-1}. \end{aligned} \tag{4.7}$$

Table 4.8. Controller Tuning Parameters for SISO Case II

Reduction Method	Reduced Model Order					
	Second			Third		
	Gain	Reset time	IAE with full model	Gain	Reset time	IAE with full model
Cauer First Form	2.4	.16	1.7	2.5	.11	1.7
Cauer Second Form		unstable			unstable	
Cauer Third Form	3.8	1.4	1.1	4.9	.90	5.9
Dominant Mode	8	5	∞	1.3	.38	2
Absolute Error	230	2.1	∞	4	.93	1.1
Squared Error	230	2.1	∞	3.8	.25	1.4
Internal Balancing	220	2.4	∞	3	.20	1.5
Steady State Internal Balancing	370	2.4	∞	2.9	.14	1.6

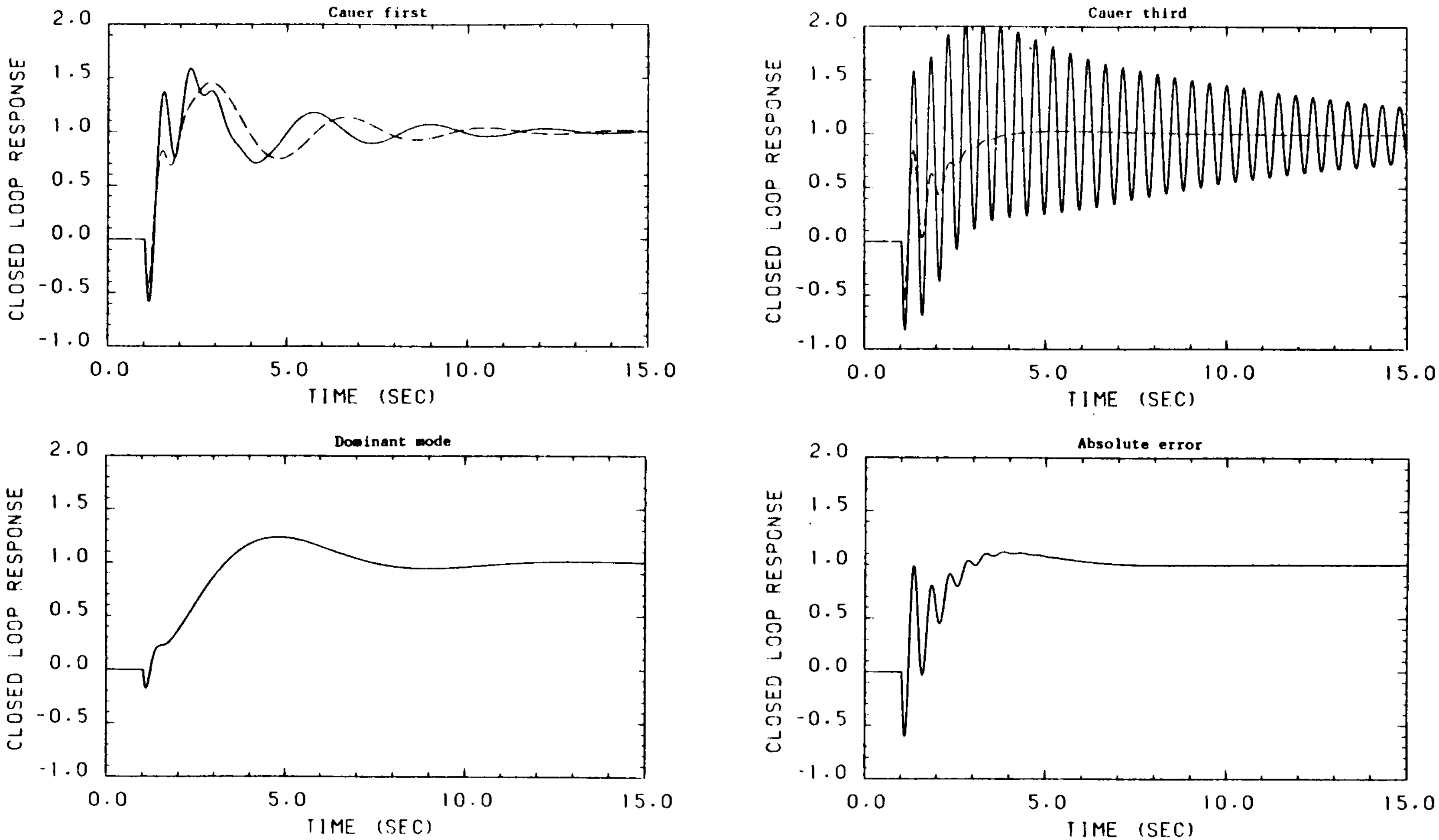


Figure 4.19. PI Control of the Full Model Using Parameters Set with the Reduced Models of Methods One, Three, Four, and Five for SISO Case II.

Using second order model: — — — — Using third order model: ————

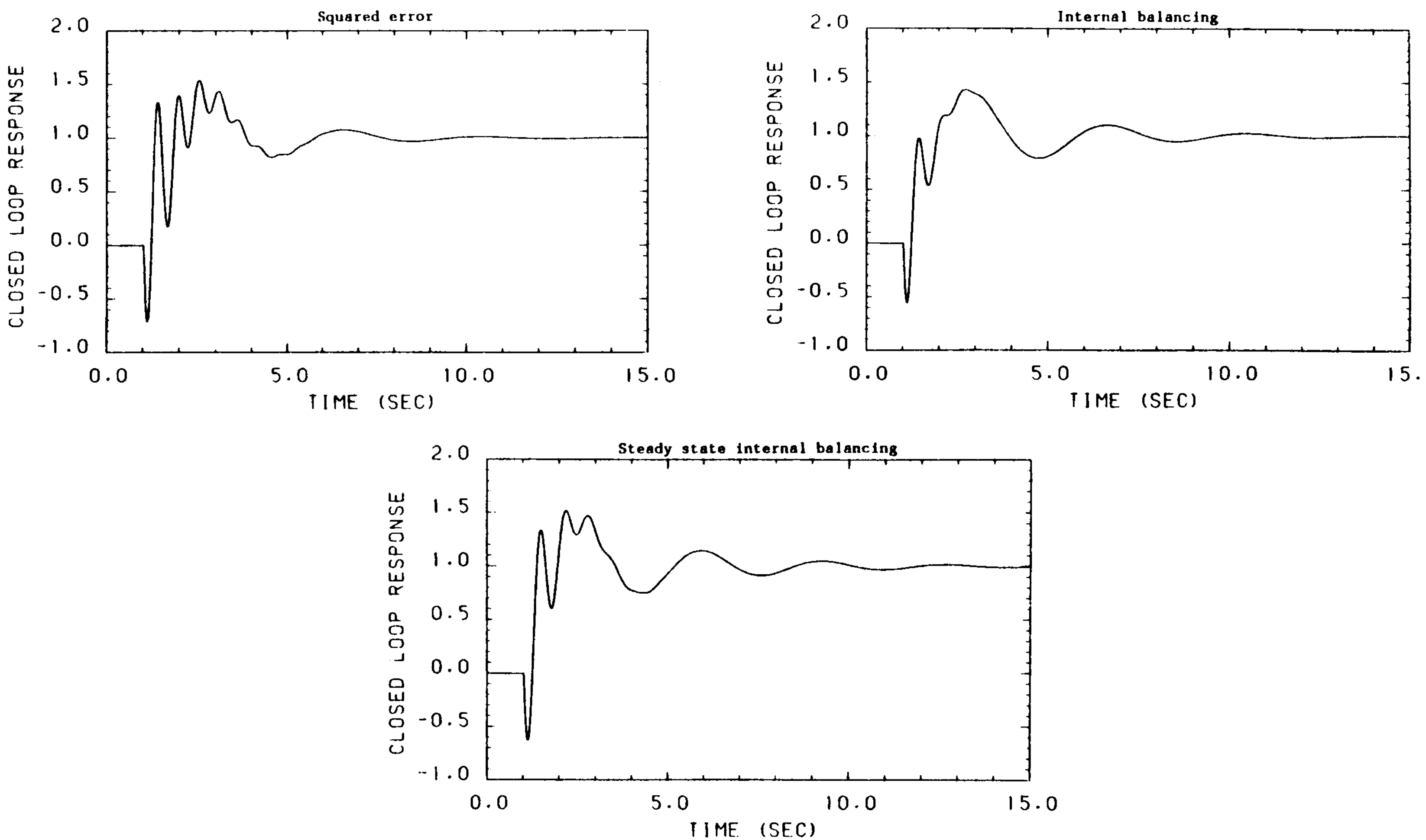


Figure 4.20. PI Control of the Full Model Using Parameters Set with the Reduced Models of Methods Six, Seven, and Eight for SISO Case II.

Using second order model: – – – – Using third order model: ————

As in the previous two cases, this model is to be reduced to a second order model and another order determined by analysis of the second order modes of the full model. The second order modes of the full model are: {14, 3.2, 3.1, 2.3, 1.1, 0.66, 0.077, 0.0011}. To further test the idea of how large the separation of second order modes must be, (as introduced in Case II), a fourth order reduced model will be calculated. The ratio of the fourth and fifth modes is approximately two.

Again the eight reduction techniques introduced in the beginning of this section are used to reduce this model to second and fourth order models. The resulting second order models are listed in Table 4.9 with the fourth order models listed in Table 4.10. The models listed in these tables are in phase-variable form except for the two models obtained by internal balancing and subsystem elimination. In transfer function form, these two models are,

$$G(s) = [17.8s + 3.12] \bullet [s^2 + 0.742s + 0.144]^{-1}, \quad (4.8)$$

and

$$G(s) = [11.5s^3 + 67s^2 + 471s + 1930]$$
$$\bullet\ [s^4 + 2.53s^3 + 46s^2 + 90s + 83.8]^{-1}. \quad (4.9)$$

From stability analysis of the resulting sixteen reduced models, two of the models are unstable. The second order models obtained by the Cauer first and third continued fraction methods are both unstable. Hence, there are fourteen models for further investigation.

Table 4.9. Second Order Models for SISO Case III

Reduction Method	State Space Matrices A		B	C^T
Cauer First Form	0	1	0	8.96
	58.3	-8.43	1	19.8
Cauer Second Form	0	1	0	6.79
	-.304	-1.04	1	20.3
Cauer Third Form	0	1	0	231
	10.4	3.69	1	19.8
Dominant Mode	0	1	0	7.7
	-.345	-1.21	1	13.1
Absolute Error	0	1	0	6.77
	-.303	-1.05	1	22.1
Squared Error	0	1	0	9.94
	-.446	-1.08	1	19.1
Internal Balancing	-.656	.296	4.29	4.29
	-.296	-.0863	.746	-.746
Steady State Internal Balancing	0	1	0	3.23
	-.144	-.742	1	18.4

Table 4.10. Fourth Order Reduced Models for SISO Case III

Reduction Method	State Space Matrices A				B	C^T
Cauer	0	1	0	0	0	15300
First	0	0	1	0	0	2330
Form	0	0	0	1	0	287
	-1340	-845	-171	-22.5	1	19.8
Cauer	0	1	0	0	0	102
Second	0	0	1	0	0	339
Form	0	0	0	1	0	43.2
	-4.58	-17.2	-17.7	-4.55	1	16.1
Cauer	0	1	0	0	0	379
Third	0	0	1	0	0	1260
Form	0	0	0	1	0	158
	-17.0	-64.2	-65.5	-15.9	1	19.8
Dominant	0	1	0	0	0	358
Mode	0	0	1	0	0	618
	0	0	0	1	0	19.8
	-16	-56.6	-47.9	-2.04	1	8.38
Absolute	0	1	0	0	0	102
Error	0	0	1	0	0	338
	0	0	0	1	0	41.8
	-4.58	-17.2	-17.6	-4.5	1	16.3
Squared	0	1	0	0	0	102
Error	0	0	1	0	0	340
	0	0	0	1	0	32.4
	-4.58	-17.3	-17.3	-4.05	1	17
Internal	-.656	.296	-.265	-1	4.29	4.29
Balancing	-.296	-.0863	5.36	.367	.746	-.746
	-.265	-5.36	-.181	-3.84	1.06	1.06
	1	.367	3.84	-1.6	-2.73	2.73
Steady	0	1	0	0	0	1870
State	0	0	1	0	0	455
Internal	0	0	0	1	0	64.8
Balancing	-83.8	-90.1	-46	-2.53	1	11.1

The first step in the investigation of the reduced models is the comparison of the open loop models. Figures 4.21 through 4.24 contain the open loop of response of each of the full models for a step unit at time equals one second. In Table 4.11, the ISE values for each of the open loop responses is listed. And rounding out this phase of investigation, the Bode plots of the models are illustrated in Figures 4.25 through 4.28.

A PI controller is once again designed for each of the reduced models to be applied for control of the full model. Minimization of the IAE is again used for setting the proportional gain and integral times of each of the fourteen PI controllers. The resulting controller parameters are listed in Table 4.12. The application of these controllers to the full model produced some surprising results. All six of the controllers designed with second order models were able to control the full model. But only five of the eight controllers designed with the fourth order models supplied stable control of the full model.

The resulting IAE values of the full model when controlled by fourteen different PI controllers are listed in Table 4.12. The closed loop responses of the full model are illustrated in Figures 4.29 and 4.30 for the eleven successful PI controllers.

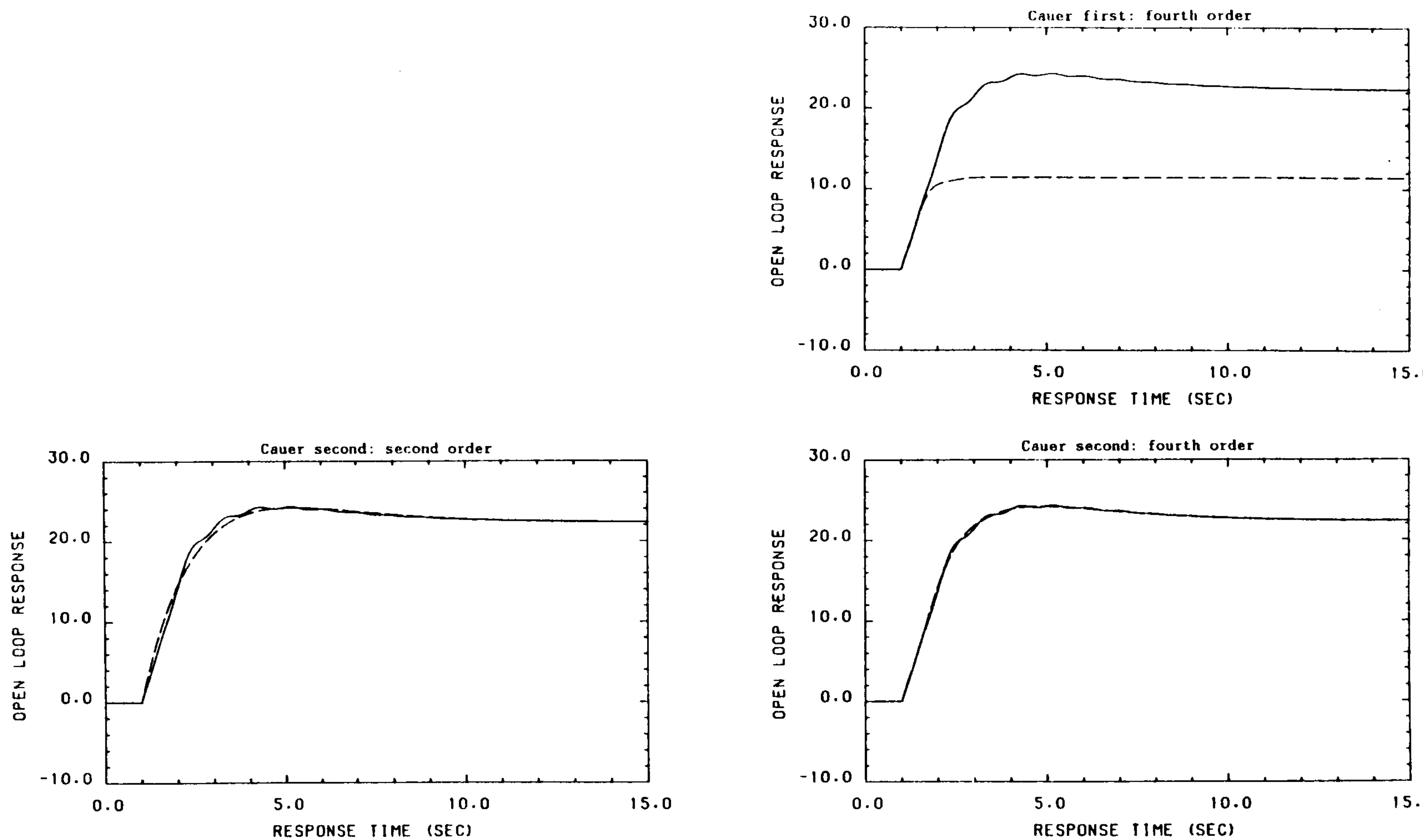


Figure 4.21. Open Loop Responses of the Cauer First and Second Forms for SISO Case III.

Full model: ———— Reduced model: — — —

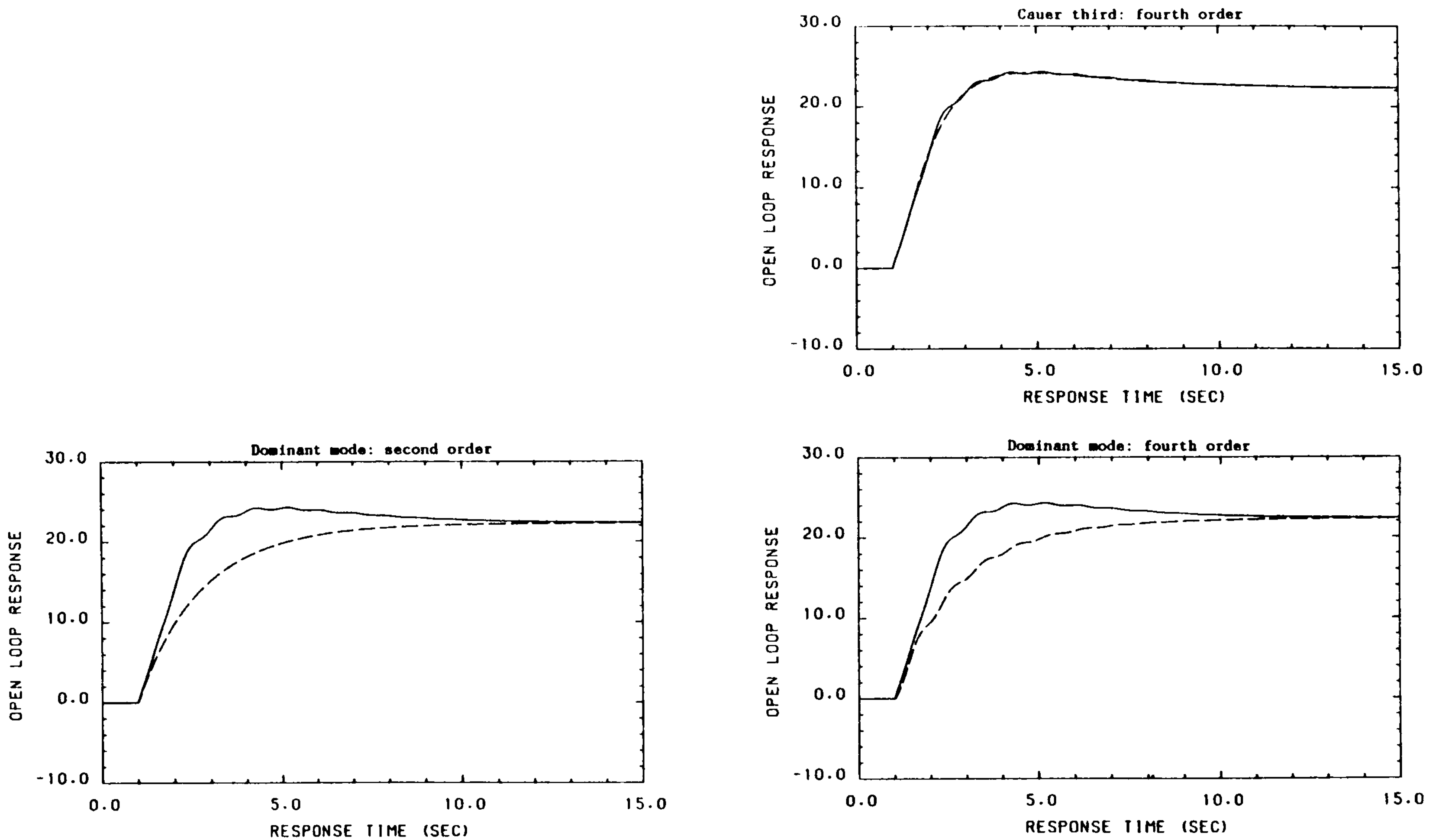


Figure 4.22. Open Loop Responses of the Cauer Third Form and Dominant Mode Method for SISO Case III.

Full model: ———— Reduced model: — — — —

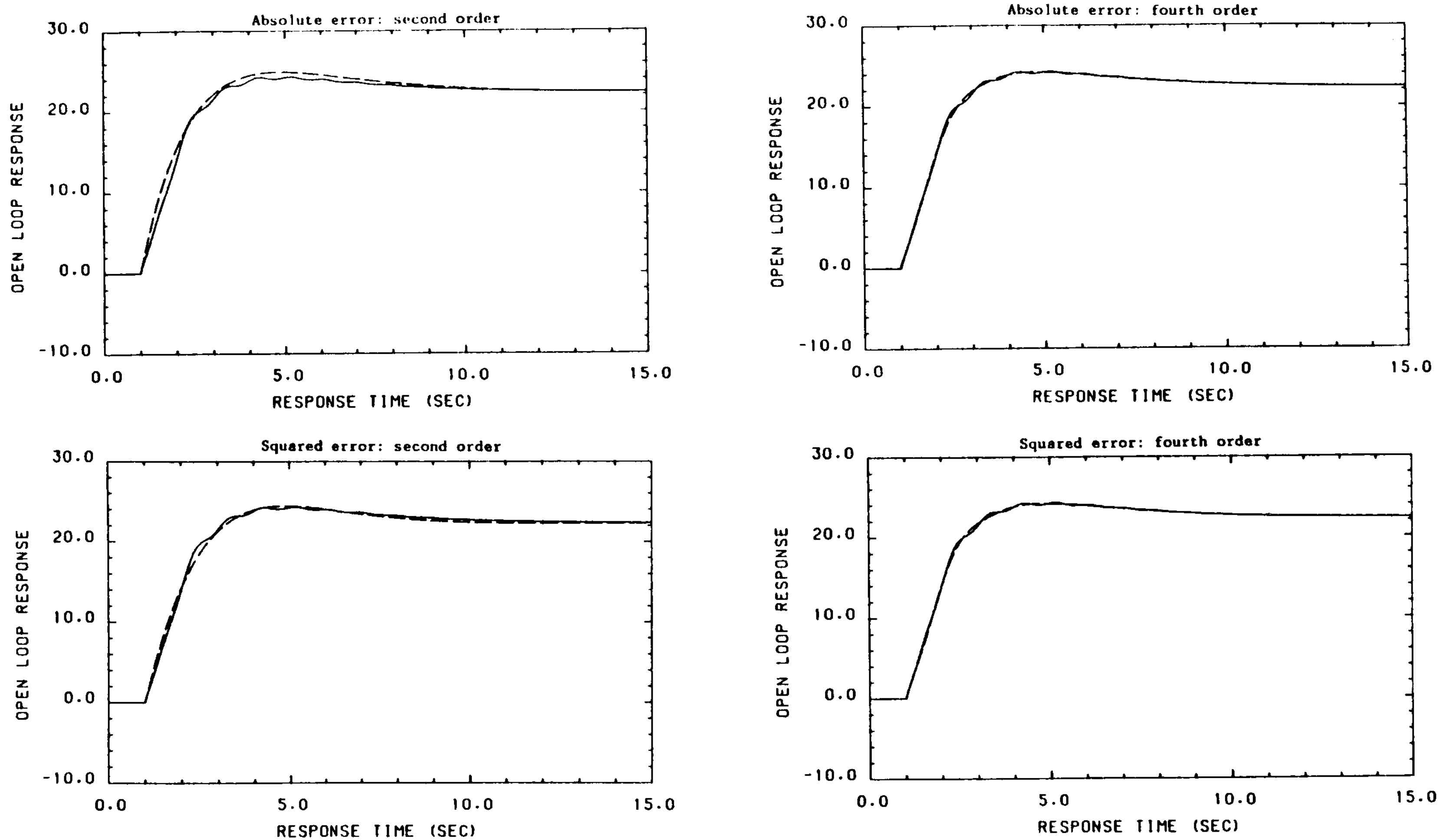


Figure 4.23. Open Loop Responses of the Absolute and Squared Error Methods for SISO Case III.

Full model: ——— Reduced model: — — —

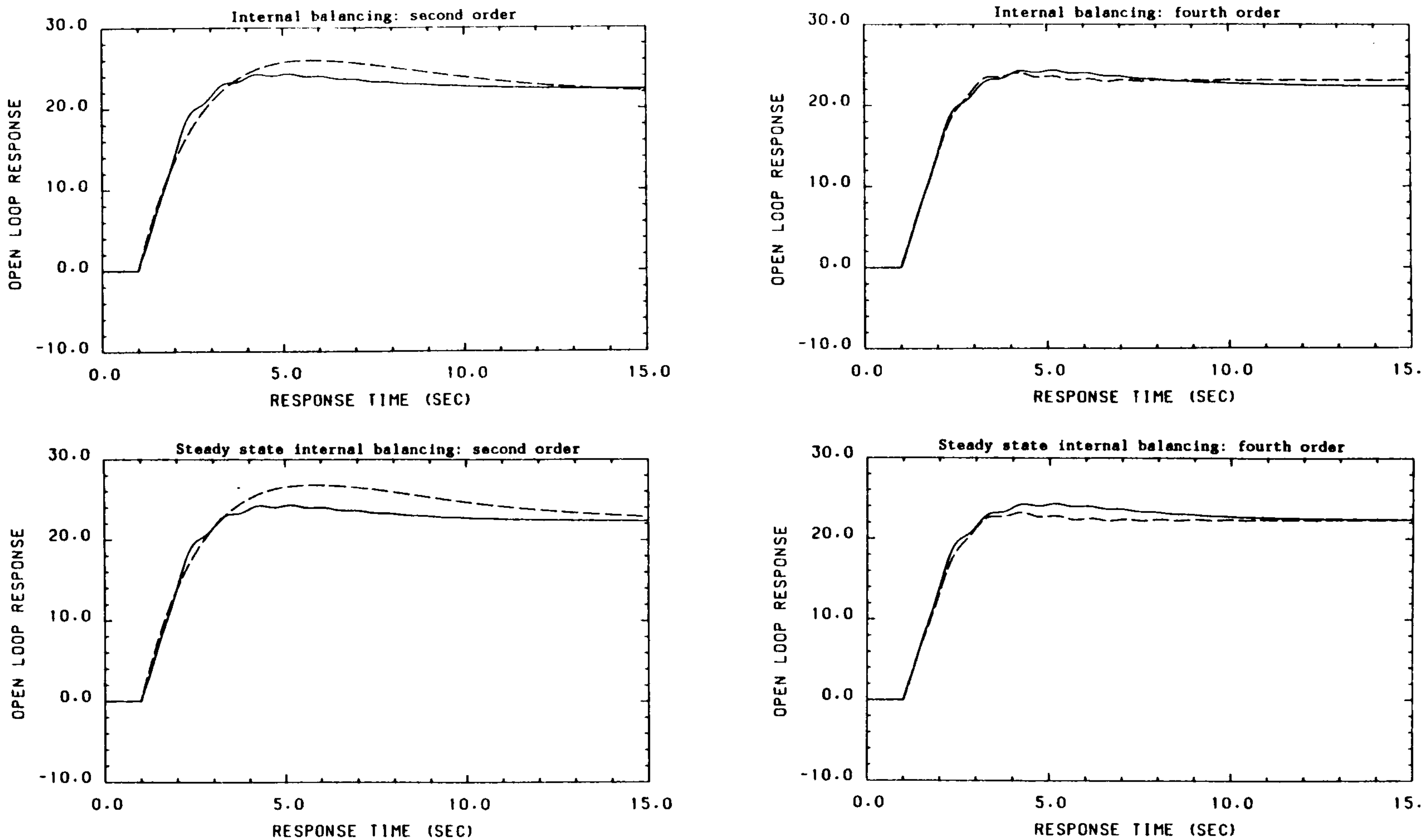


Figure 4.24. Open Loop Responses of the Internal Balancing and Steady State Internal Balancing Methods for SISO Case III.

Full model: ——— Reduced model: — — —

Table 4.11. ISE Values of the Reduced Models for SISO Case III

Reduction Method	Reduced Model Order	
	Second	Fourth
Cauer First Form	unstable	∞
Cauer Second Form	2.7	.23
Cauer Third Form	unstable	.48
Dominant Mode	130	130
Absolute Error	5.4	.22
Squared Error	2.1	.21
Internal Balancing	∞	∞
Steady State Internal Balancing	49	8.9

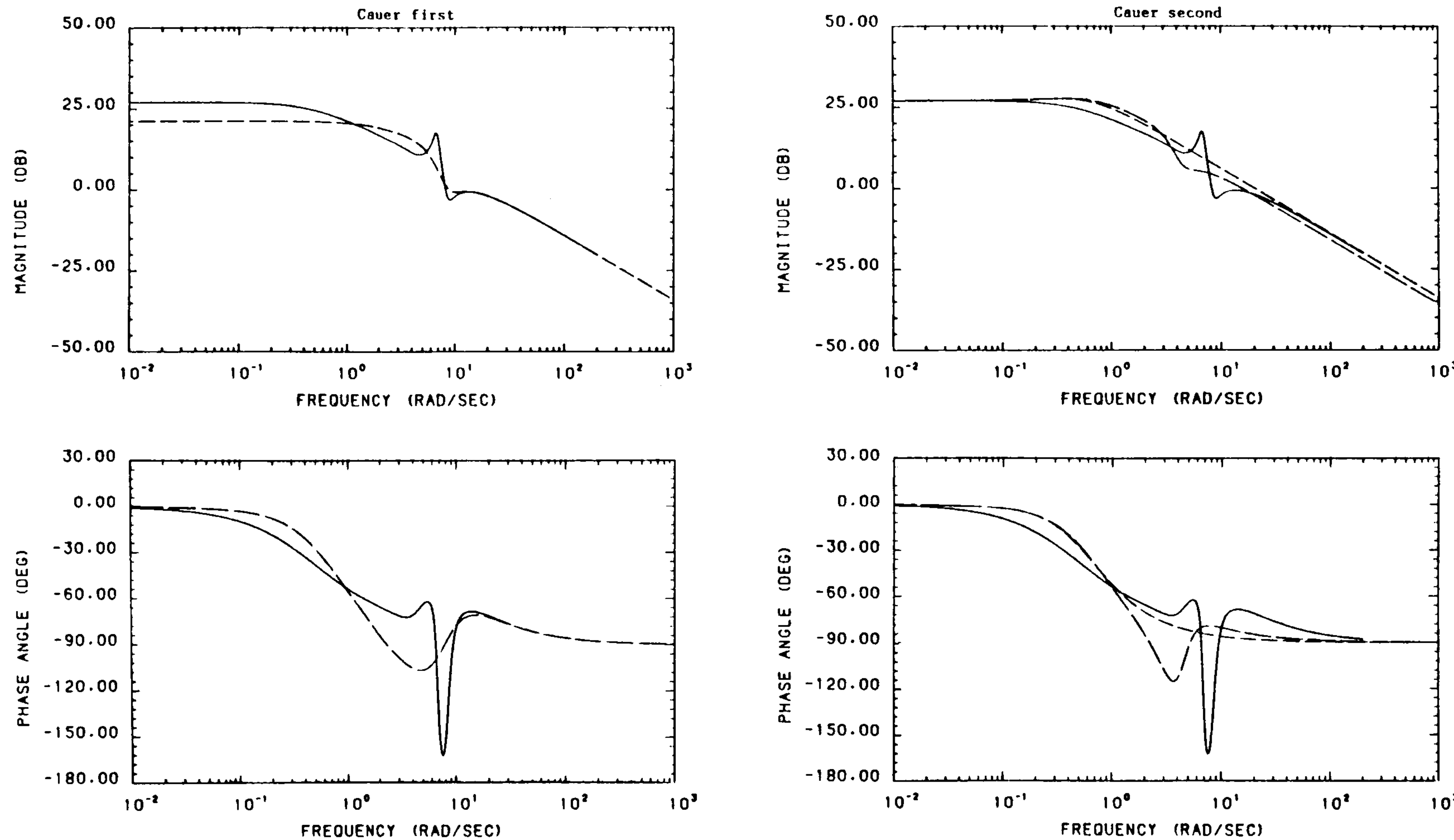


Figure 4.25. Bode Plots of the Cauer First and Second Forms for SISO Case III.

Full model: ———— Second order model: — — — — Fourth order model: —— —— ——

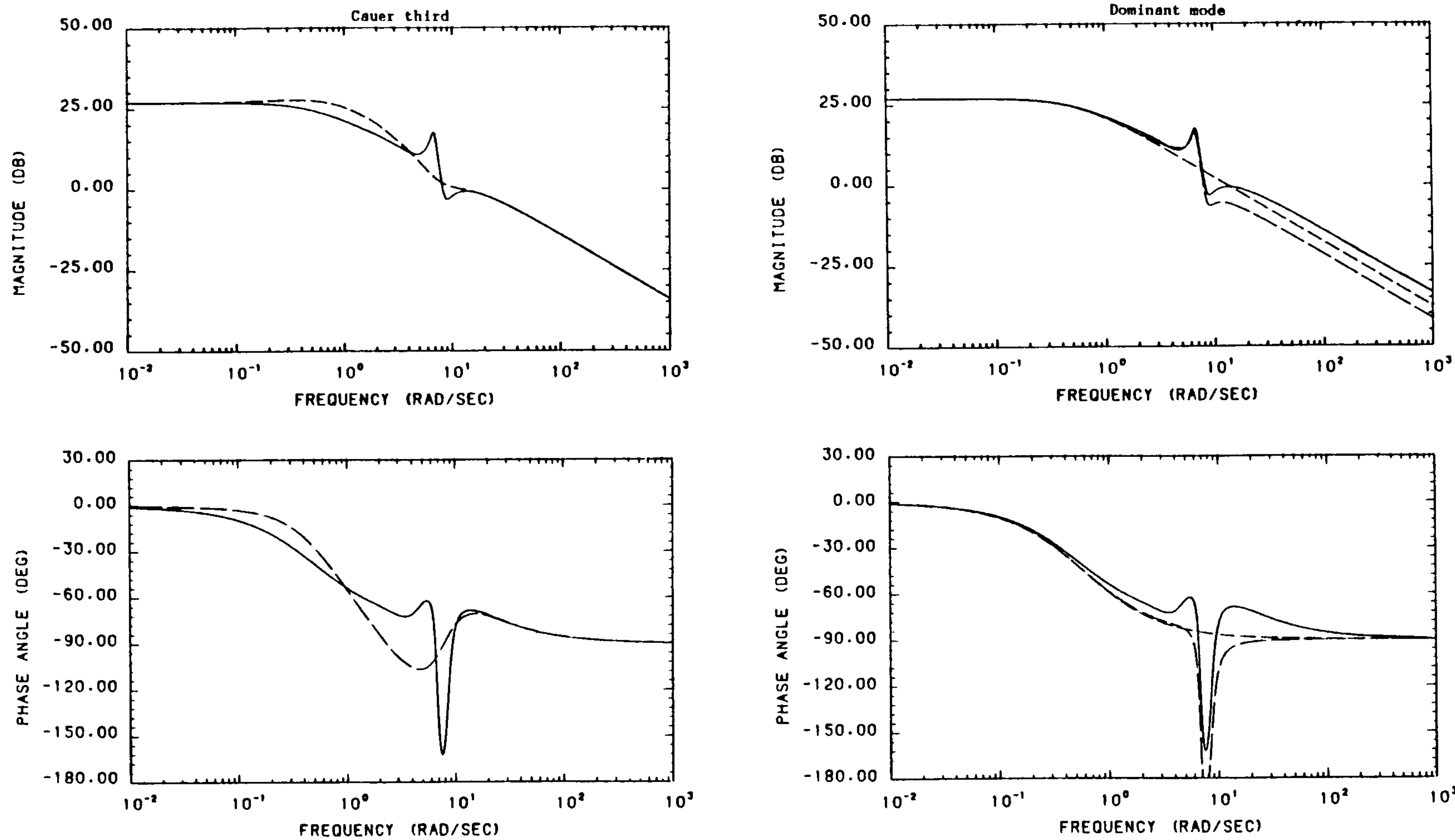


Figure 4.26. Bode Plots of the Cauer Third Form and Dominant Mode Method for SISO Case III.

Full model: ———— Second order model: — — — Fourth order model: — — —

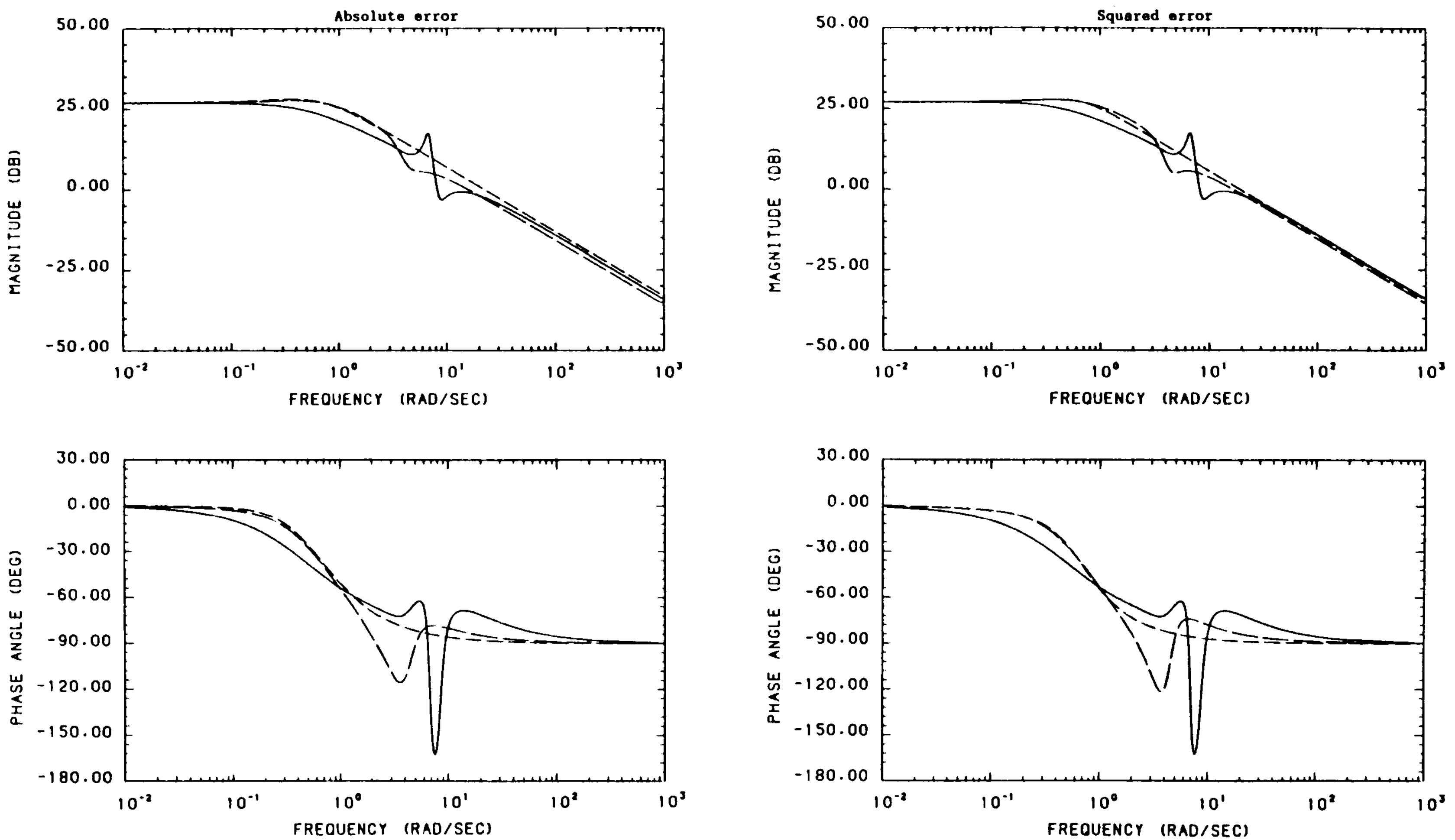
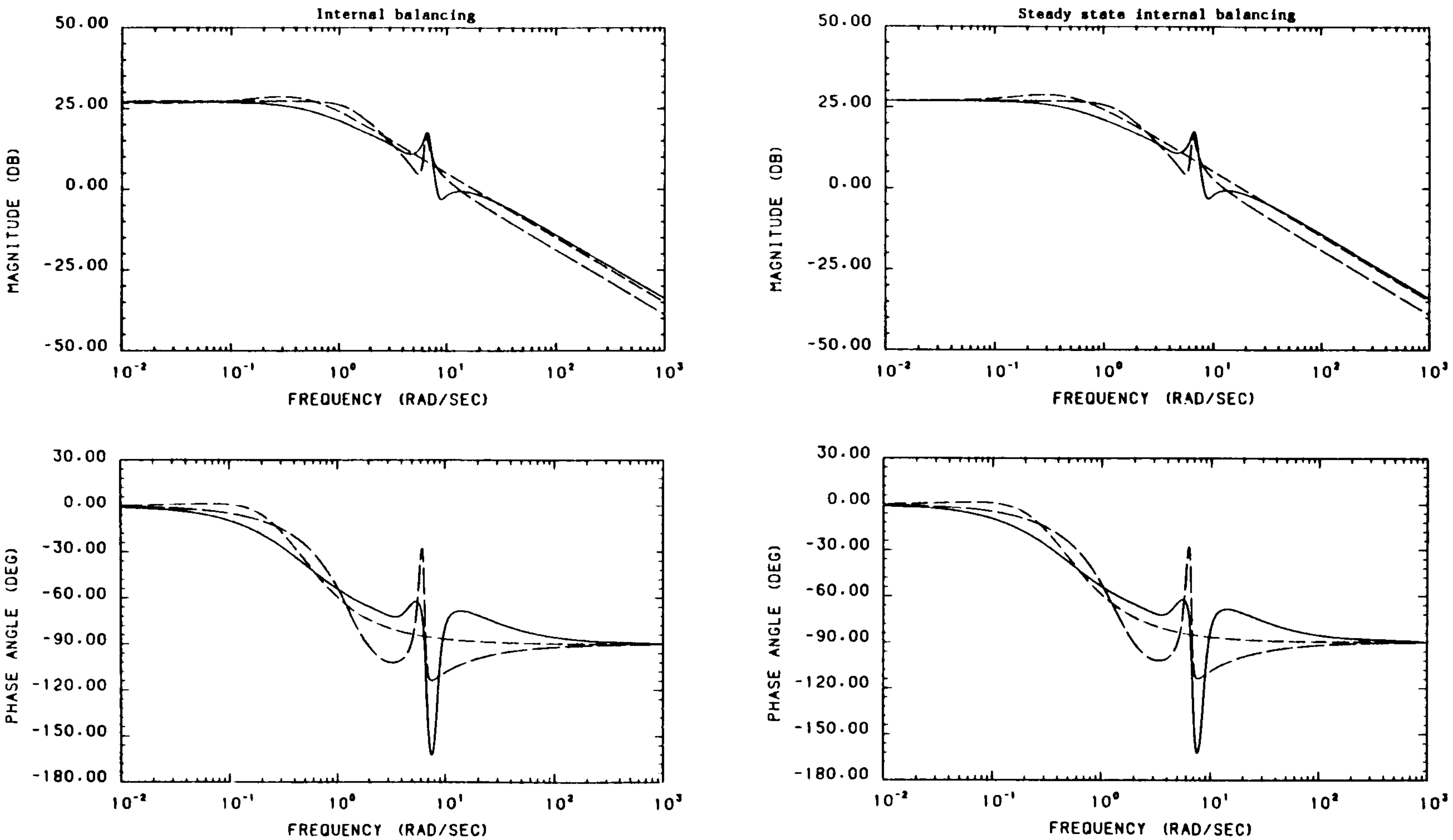


Figure 4.27. Bode Plots of the Absolute and Squared Error Methods for SISO Case III.

Full model: ———— Second order model: — — — — Fourth order model: —— —— ——



Figures 4.28. Bode Plots of the Internal Balancing and Steady State Internal Balancing Methods for SISO Case III.

Full model: ——— Second order model: — — — Fourth order model: —— —— ——

Table 4.12. Controller Tuning Parameters for SISO Case III

Reduction Method	Reduced Model Order					
	Second			Fourth		
	Gain	Reset time	IAE with full model	Gain	Reset time	IAE with full model
Cauer First Form		unstable		2.6	.79	.14
Cauer Second Form	2	1.2	.16	2.6	1.2	.14
Cauer Third Form		unstable		2.3	.93	.14
Dominant Mode	3	1.7	.13	6.6	2.5	∞
Absolute Error	1.8	1.2	.18	2.6	1.2	.133
Squared Error	2.1	1.3	.15	2.8	1.4	.129
Internal Balancing	2.2	1.5	.14	4.4	1.6	∞
Steady State Internal Balancing	2.2	1.5	.14	4.6	1.6	∞

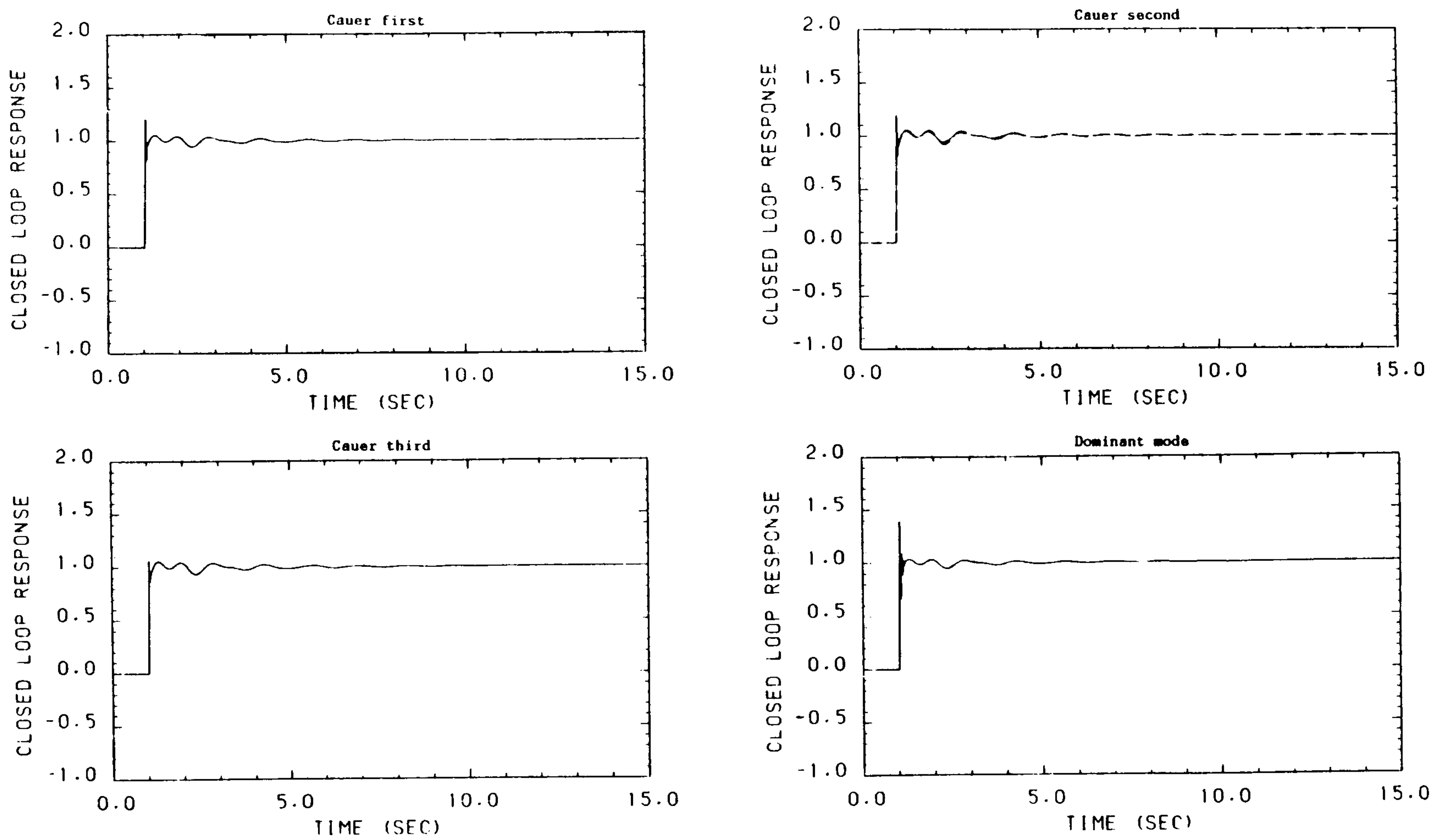


Figure 4.29. PI Control of the Full Model Using Parameters Set with the Reduced Models of Methods One, Two, Three, and Four for SISO Case III.

Using second order model: — — — — Using fourth order model: ————

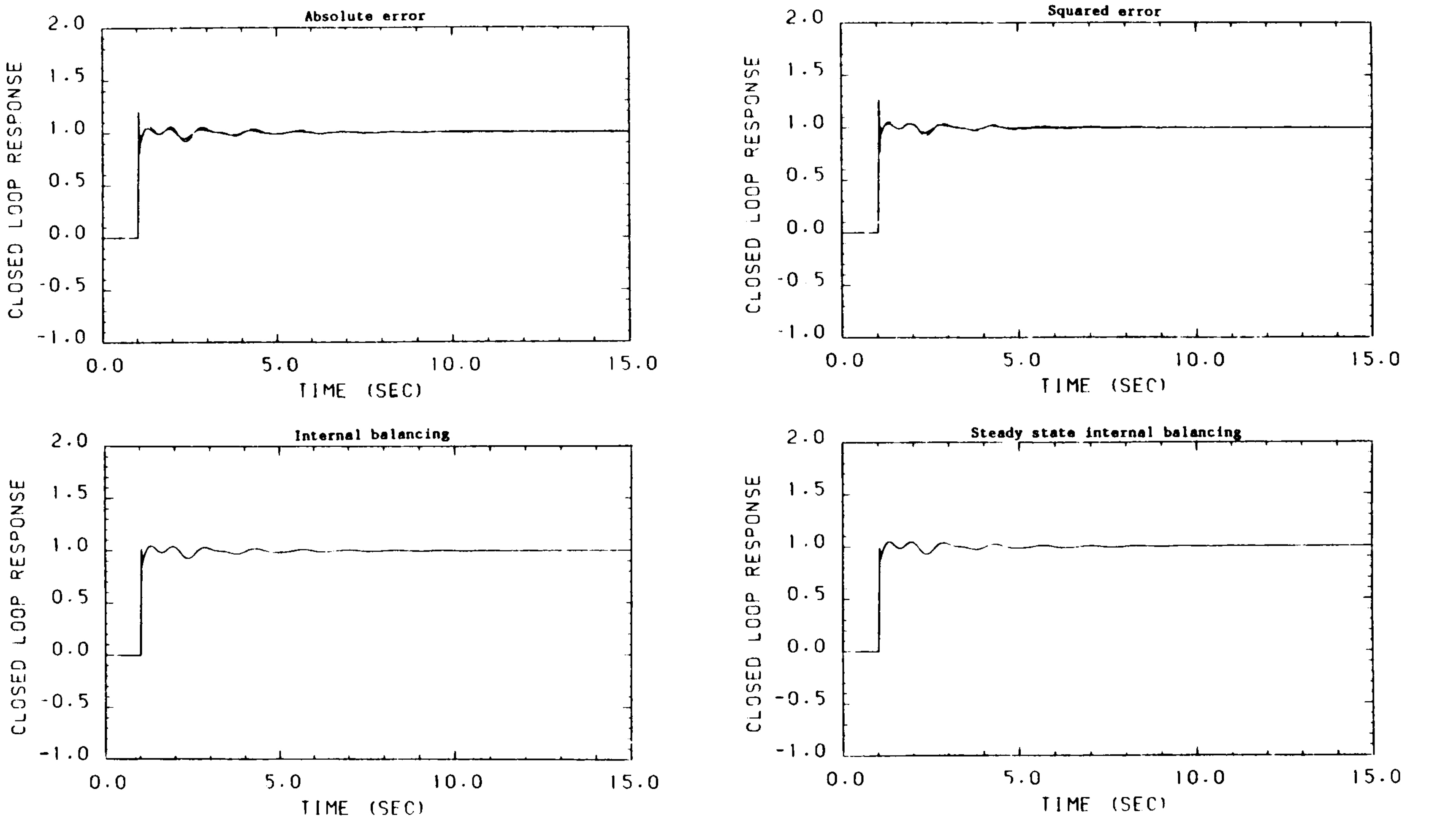


Figure 4.30. PI Control of the Full Model Using Parameters Set with the Reduced Models of Methods Five, Six, Seven, and Eight for SISO Case III.

Using second order model: ----- Using fourth order model: ———

Summary of the Single Variable Model Reduction Problem

The final section of this chapter reviews methods of model reduction for single variable models in light of the three preceding case studies. This section is divided into four parts: simulation, controller design, use of comparison criteria, and concluding remarks. The comments made in this section pertain not only to the three models presented in this chapter, but are based on the results from the numerous other models investigated during the course of this work.

Simulation. Using the open loop responses to unit step inputs, the ability or inability of a reduced model to simulate the full model may be checked graphically. As mentioned in Chapter 3, the idea of a reduced model adequately representing a full model is rather nebulous. At this point, the idea of adequately representing the system is treated in a very general manner.

The parameters to be considered for comparing the models are the steady state response and any dominant transient components. By simply considering the steady state criteria, two of the methods immediately come into question, namely, the Cauer first form and internal balancing methods. This point is illustrated by the open loop responses (Figures 4.1 and 4.4) and the Bode plots (Figures 4.15 and 4.17) of the reduced and full models.

Two of the methods, the Cauer first and third forms, consistently produce reduced models that maintain the fast components of the model. The two continued fraction methods reproduce the initial response quite well, as is evident in the Bode plots (Figures 4.25 and 4.26). While the internal balancing methods do not maintain the fast components, they do maintain any oscillations or other characteristics that occur as the response develops. The internal balancing methods do suffer from containing slower time constants than the other reduction techniques. In fact, the time constants of the second order models have been made so slow, that their effect does not even show up in the Bode plots (Figures 4.8, 4.17, and 4.28). The frequency range could have been changed to illustrate the effect of the time constants, but for consistency, this was not done.

By containing relatively slow time constants, the adjustment of the gain of the reduced model resulting from the internal balancing method may yield disastrous results (Figure 4.4). Since these reduced models do not reach steady state as quickly as the full model, the amplification of the offset from the full model during the transient period may be substantial.

The models obtained from the absolute and squared error equation fitting methods suffer from ignoring any dominant transient components, as in the sag of Case II. The transient could be matched by using different weighting factors, but for consistency, no weighting factors were applied. These models, however, do match the open loop response reasonably well throughout.

The Cauer second form continued fraction method produced models that are almost identical to those obtained with equation fitting (Tables 4.9 and 4.10). With this method, the steady response is always matched at the expense of the fast components.

The success of the dominant mode method in calculating adequate reduced models for simulation purposes, depends on the existence of dominant modes. For the models investigated here, there are no clearly dominant modes. The method was applied so as to give a specified reduced order model rather than to retain all of the dominant modes.

For all of these reduction methods, as the order of the reduced model is increased, the reduced models generated will improve monotonically. Hence, any of the open loop responses from the reduced models may be "improved" by increasing the order of the reduced model.

Controller Design. For the design of PI controllers from reduced order models, two methods, the Cauer first and third forms, hold the most promise for adequate controller design. The method of dominant modes was by far the worst for controller design. The remaining six methods had only marginal success in controller design.

The two continued fraction methods, the Cauer first and third forms, that are reliable for controller design, retain the fast components of the full model. All of the models reported in this study do have dominant characteristics in the transient region of the response. Hence, the study is somewhat biased in favor of methods which do retain the fast components of the full model. In cases in which the open loop response of the full model has no dominant fast components, other reduction methods are applicable for controller design. In these cases the Cauer first form was not as successful as the others while the Cauer third form was equally as successful as they.

In contrast to the simulation results, when the order of the reduced model is increased, the controller designed with the reduced models does not necessarily improve in control of the full model. In fact, for Case III, some of the controllers designed with a fourth order model failed, yet the controllers designed with the second order models, obtained with the same method, succeeded.

Use of comparison criteria. In Chapter 3, several methods were proposed for the comparison of reduced models in order to determine when a reduced model would adequately represent the full model. The results of these criteria, while they may be of theoretical interest, are disappointing.

The ISE values show that the open loop response of the reduced models will improve as the order of the reduced model is increased. However, the ISE values may not reflect the loss of accuracy in a reduced model when a transient component is neglected. A lower ISE open loop value does not imply that the controller designed with the reduced model will perform any better when applied to the full model.

The use of the Bode plot does yield substantially more information about the relationship between the full and reduced models than just an open loop step response. From the Bode plots, all frequencies may be compared. With this information, the Bode plot may give insight into the use of the reduced model for simulation and controller design purposes.

Concluding remarks. The purpose of this last section is to enforce the remarks made in the preceding review and to comment generally on the single variable model reduction problem. A point which has been neglected so far is the use of the second order modes for determining the order of the reduced model.

From the three models discussed in this chapter, the use of the second order modes for determining the order of the reduced model may seem to be of little value, since for controller design, the second order models performed as well as the higher order models. However, the analysis of the three models presented here and seen in other models indicates that not only is the magnitude of the ratio of two succeeding second order modes important, but also that the ratio must be substantially greater than the preceding ratio. Consider again the second order modes of Case III. The ratio between any two adjacent second order modes is essentially constant for the first six modes. Hence, a fourth order model offers little or no improvement over a second order model; moreover, according to the second order mode analysis, the order of the reduced model should have been six. (The order of four was selected to illustrate this point and the success of the sixth order model has been shown by the author elsewhere [35].) The selected orders for the reduced models in the other two cases (and in other cases investigated by the author) support this conclusion. The orders of the reduced models used in this chapter were selected in order to test the hypothesis of the second order modes; hence, the criteria was not followed exactly for each case.

Another point which should be made concerns the equation fitting methods of model reduction. The two cost criteria used in this chapter, absolute error and squared error, yield very similar reduced models. A cost criterion that was introduced in Chapter 2, the perpendicular error, gave parameters identical to those of the absolute error criterion for every case tested. Between the two cost criteria used for the three cases studied here, the resulting reduced models are only slightly different and neither was better for simulation of, or controller design for, the full model.

From the results presented in this chapter and encountered during this research, the methods of continued fraction expansion are extremely powerful for both simulation and controller design purposes. Of the three continued fraction methods studied, the Cauer third form is more successful both in reproducing the open loop response of the full model and for PI controller design. Coupled with the superb results and the ease by which the reduced models may be calculated, the method of the Cauer third form of continued fraction expansion is recommended for reducing single variable models. And, for selecting the order of the reduced model, the use of second order modes is recommended.

CHAPTER V

ANALYSIS OF THE MULIVARIABLE PROBLEM

The reduction of multiple input-multiple output (MIMO) models are discussed in this chapter. As in the previous chapter, the reduction methods will be illustrated through three case studies. The reduced models will be used for open loop simulations and for multivariable PI controller design. The final section of this chapter summarizes the multivariable model reduction problem.

The three models to be investigated will each be reduced by the following seven methods:

1) Matrix continued fraction expansion with the Cauer first form,
2) Matrix continued fraction expansion with the Cauer second form,
3) Matrix continued fraction expansion with the Cauer third form,
4) Davison's dominant mode method,
5) Equation fitting using the state matrix A with the absolute error as the cost function,
6) Internal balancing with subsystem elimination, and
7) The singular perturbation method.

Each of these methods has been discussed in detail in Chapter 2. For method number five, equation fitting, an improvement of the technique outlined in Chapter 2 will be used. The modifications to method five are discussed in detail in the next section of the chapter.

In selecting the three MIMO models for this chapter, the stipulation imposed by the matrix continued fraction methods, that the number of inputs equals the number of outputs, was followed. With this stipulation, all of the methods could be compared for each model. The other four methods are more general since they are applicable to systems with an unequal number of inputs and outputs.

For the three MIMO models considered, a single reduced model will be calculated with each of the seven reduction methods. The order of this reduced model will be selected using the second order modes associated with the internally balanced full model. However, the three Cauer forms have a constraint that the order of the reduced model must be a multiple of the number of inputs. Thus, the order selected will be based on the second order modes so that it is a multiple of the number of inputs.

In the preceding chapter, several comparison criteria were used to determine the effectiveness of the reduction methods. For the MIMO models, the comparison will be made with open loop responses and controller design. The Bode plot comparison that was employed in Chapter 4 is not used here since the Bode plots do not show one of the main problems of MIMO model reduction, which is interaction. The control structure, multivariable PI, was chosen to test whether the reduced model has the same interaction characteristics as the full model. As in Chapter 4, the open loop simulations will be performed using a discrete time model formulated by a diagonal Pade approximation of the matrix exponential [113]. And the controller to be designed is a multivariable PI controller.

The multivariable controller structure selected for this study, the multivariable PI controller, was chosen due to its reputation for being robust and because of its successful applications reported in literature [86]. For controller design with reduced models, the proportional matrix (equation (2.107)) and the integral matrix (equation (2.109)) were both set using the full model. The remaining tuning parameters were obtained using the reduced model and the integral of the absolute error (IAE) tuning criteria. The control matrices are established with the full model since both matrices are easily obtained from the open loop step responses of the full model [81].

A New Formulation of MIMO Model Reduction by Equation Fitting

In Chapter 2, a method of model reduction for multivariable system reduction which was proposed by Gupta, et al. [39] was discussed. Their method concerns finding a reduced model for which a quadratic function between the original and reduced model can be minimized for either a step or impulse input. The drawback to the technique they have proposed is in the selection of a weighting matrix (the matrix R of equation (2.48)). To solve this problem, the following method is proposed and will be used in the subsequent case studies.

In the reduction of models expressed in state space form, the number of state variables is reduced in such a way that the input-output relationship of the full model is maintained. One way to maintain this relationship would be for the state variables of the reduced model to be approximations of the important state variables of the full model. Hence, what is needed is a technique to establish the important state variables of the model.

The internally balanced representation of the full model provides an ordering for the relative importance of its state variables. With the internally balanced model then, the strong and weak state variables may be indentified. The strong state variables of the system may be identified with the second order modes. The important variables established with the internally balanced model are then approximated in the reduced model.

The desired state variables to be approximated in the reduced model are,

$$x''(t) = R\ x(t), \tag{5.1}$$

where,

$$R = [I_m \quad 0] \tag{5.2}$$

is an m x n matrix, x(t) is the full state vector of length n, and I_m is an m x m identity matrix. The internally balanced state matrices are used for calculating the reduced state matrices B* and C*, (equations (2.47) and (2.48)) as specified by Gupta, et al. Finally, the components of the reduced state transition matrix, A*, are varied so as to minimize the absolute error between the output of the reduced and full models using a pattern search routine [74,75].

With the use of the above proposed method, the intuitive decisions as to which components of the full model are the most important, are eliminated. This new proposal will be used, as method five, along with the other six methods mentioned in the introduction to this chapter for the reduction of three selected MIMO models.

MIMO Model Reduction: Case I

The first MIMO system to be considered is a fourth order two input-two output model. This model was originally proposed by Goldman, et al. [38] for simulation studies using the three forms of matrix continued fraction expansion. The model was also used by Marshall [68] for designing a control structure with the inverse Nyquist array and a second order reduced model generated by the Cauer third form for control of the full model. Gerstle, et al. [36] later used the second order reduced models obtained from the three forms of matrix continued fraction expansion and internal balancing for designing a singular value decomposition controller. Both Marshall and Gerstle, et al reported that the controllers designed with these reduced models failed for control of the full model. In fact, Gerstle, et al. [36], reported that the results of the simulation reported in Goldman, et al.'s [38] original paper are in error. This model will again be studied with additional reduction methods and with the multivariable PI controller design.

In state space form the model is,

$$\dot{x} = \begin{bmatrix} 0 & 0 & 3.25 & -1.125 \\ 0 & 0 & 2.25 & -0.125 \\ -1.0 & 0.5 & -1.50 & -0.500 \\ -1.5 & -0.5 & -1.00 & -4.000 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 1.0 & 0.5 \end{bmatrix} u, \qquad (5.3a)$$

$$y = \begin{bmatrix} 1.0 & -1.0 & 1.0 & 1.0 \\ 0 & 1.0 & -1.0 & 1.0 \end{bmatrix} x. \qquad (5.3b)$$

From the internally balanced representation of equation (5.3), the second order modes are found to be: {1.54, 1.42, 0.284, 0.156}. While the ratios of any two of the adjacent modes are relatively low, the ratio between the second and third modes of five is higher than the other ratios. This complies with the matrix continued fraction expansion restriction on selecting the order of the reduced model. Since this is a two input system, the only reductions possible by the matrix continued fraction methods are multiples of two. Hence, a second order model will be calculated for each of the seven reduction techniques.

The A, B, C, and D matrices of the state space model for each of the reductions are listed in Table 5.1. Only the method of singular perturbation has a D matrix. From stability analysis, all seven models are found to be stable.

The open loop responses to a unit step input for each reduced model and the full model are illustrated in Figures 5.1 through 5.7.

Table 5.1. Second Order Reduced Models for MIMO Case I

Reduction Method	State Space Matrices							
	A		B		C		D	
Cauer	-4	-1.5	4	1.5	.385	-.0769		
First Form	-.5	-1.0	.5	1	.0769	.385		
Cauer	-2.59	-.630	1	0	.50	0		
Second Form	-2.68	-1.26	0	1	1.59	.631		
Cauer	-.775	0	1	5.45	-.0665	1.57		
Third Form	0	-3.22	1	.551	.0458	.454		
Dominant	-.0814	-.7720	-.0299	-.141	.979	-.162		
Mode	.7720	-.0814	-.3340	.104	-.734	.502		
Equation	-1.53	-1.30	.0296	.202	35.4	59.6		
Fitting	-.150	-.912	-.0047	-.110	15.2	22.6		
Internal	-.0714	.780	.453	-.125	-.216	.156		
Balancing	-.7530	-.0295	.189	.219	.418	-.244		
Singular	-.0814	.772	.611	-.256	-.372	.174	.666	-.171
Perturbation	-.7720	-.0814	.204	.218	.434	-.323	-.193	.409

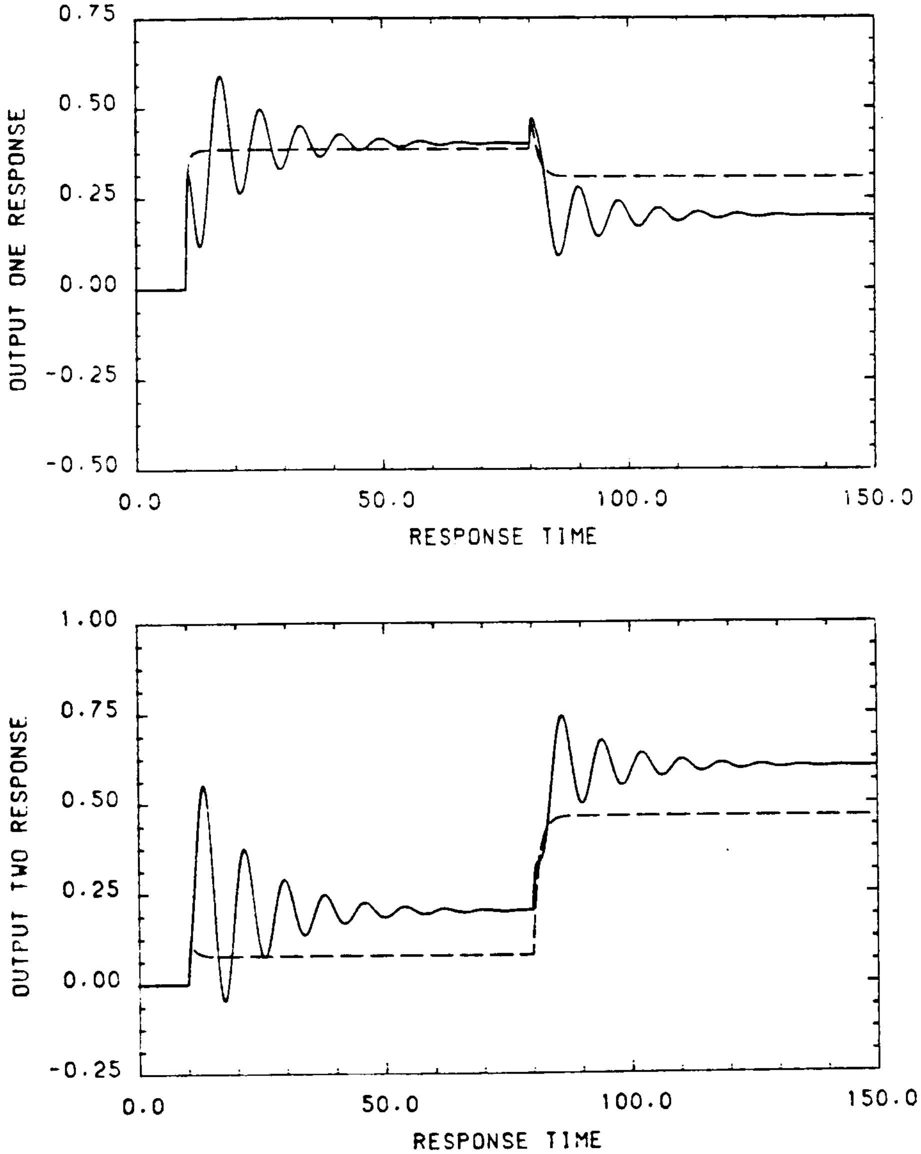


Figure 5.1. Open Loop Responses of the Cauer First Form for MIMO Case I.

Full model: ——— Reduced model: — — —

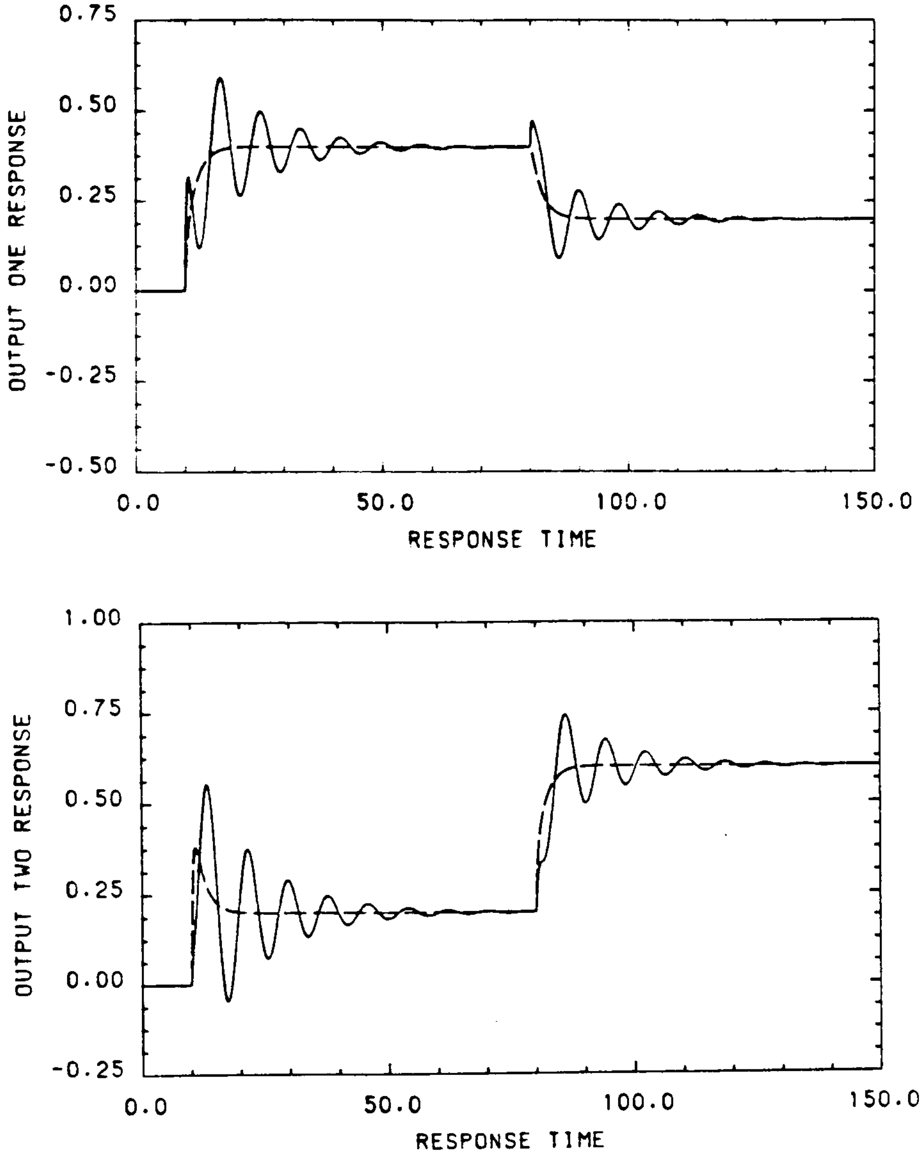


Figure 5.2. Open Loop Responses of the Cauer Second Form for MIMO Case I.

Full model: ———— Reduced model: — — — —

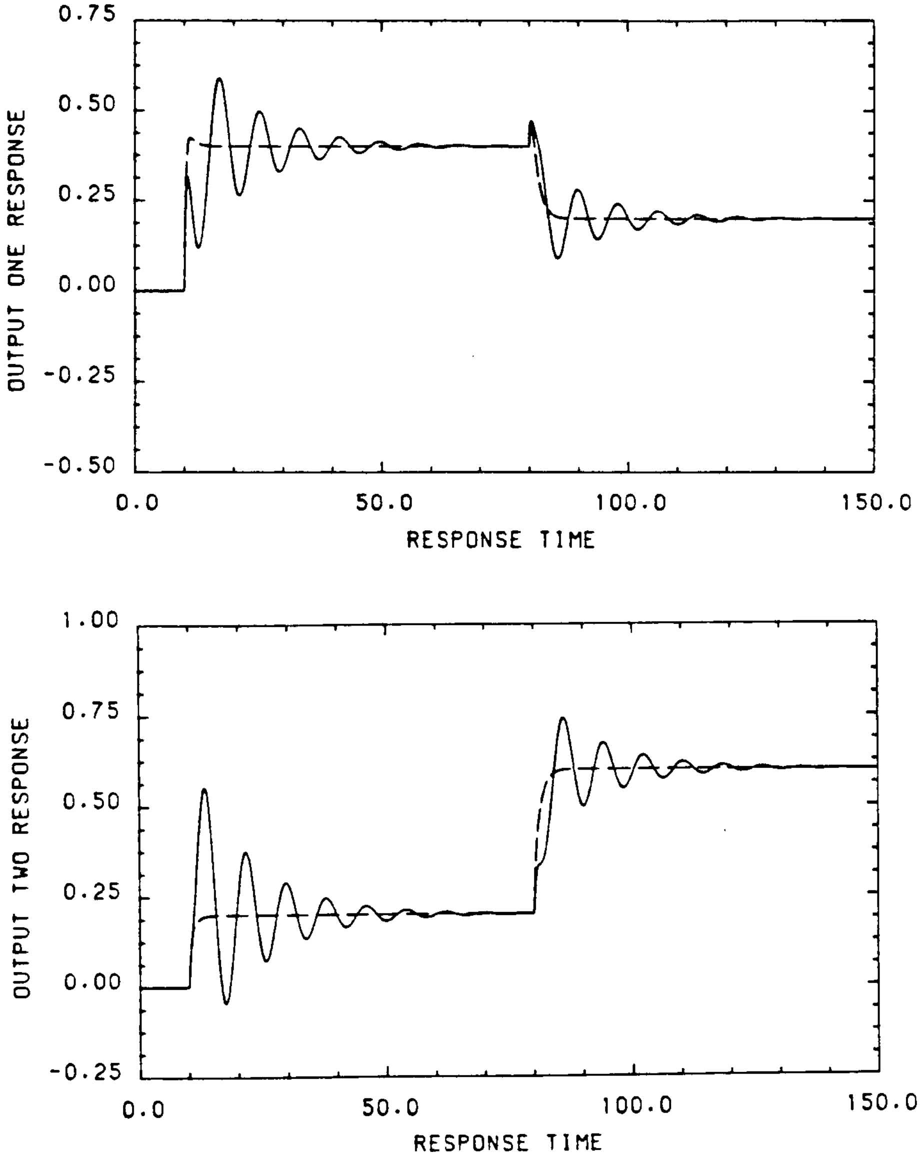


Figure 5.3. Open Loop Responses of the Cauer Third Form for MIMO Case I.

Full model: ———— Reduced model: — — — —

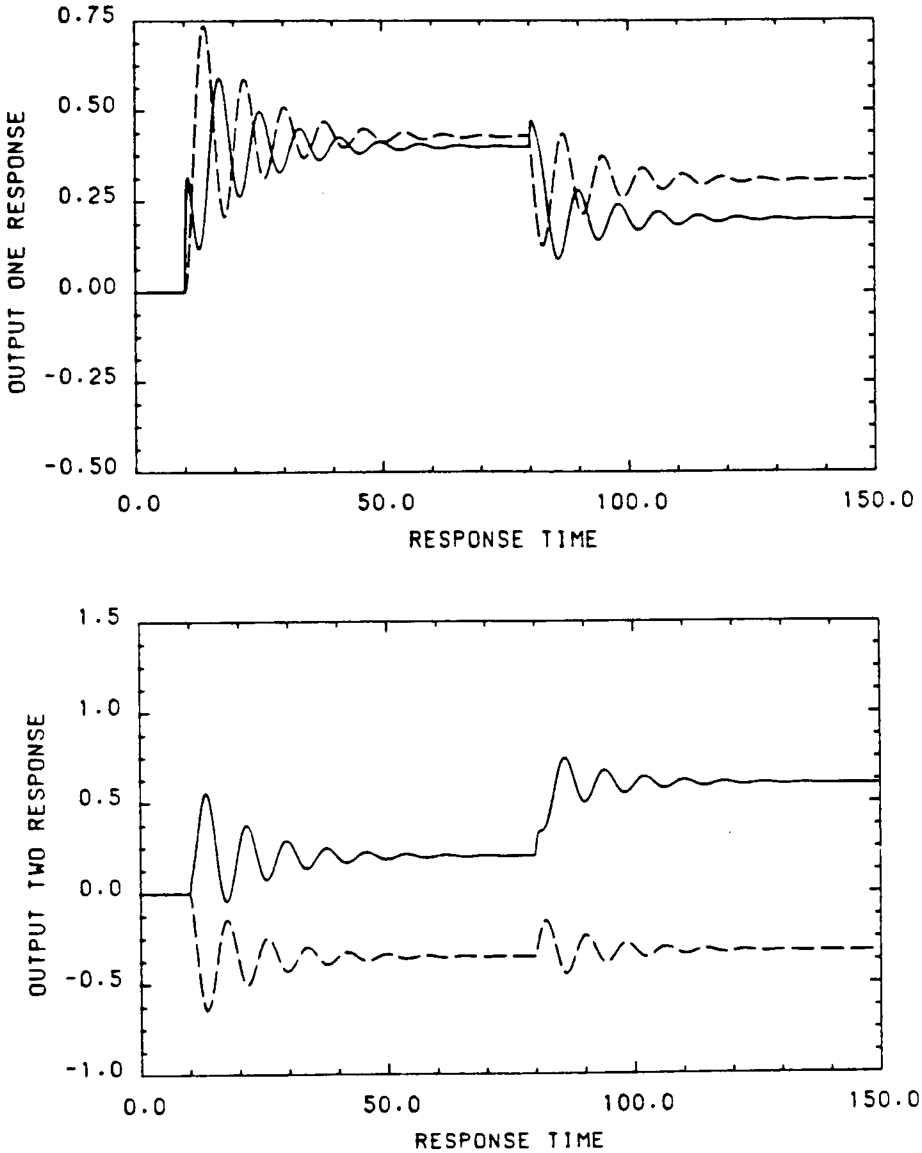


Figure 5.4. Open Loop Responses of the Dominant Mode Method for MIMO Case I.
Full model: ——— Reduced model: — — —

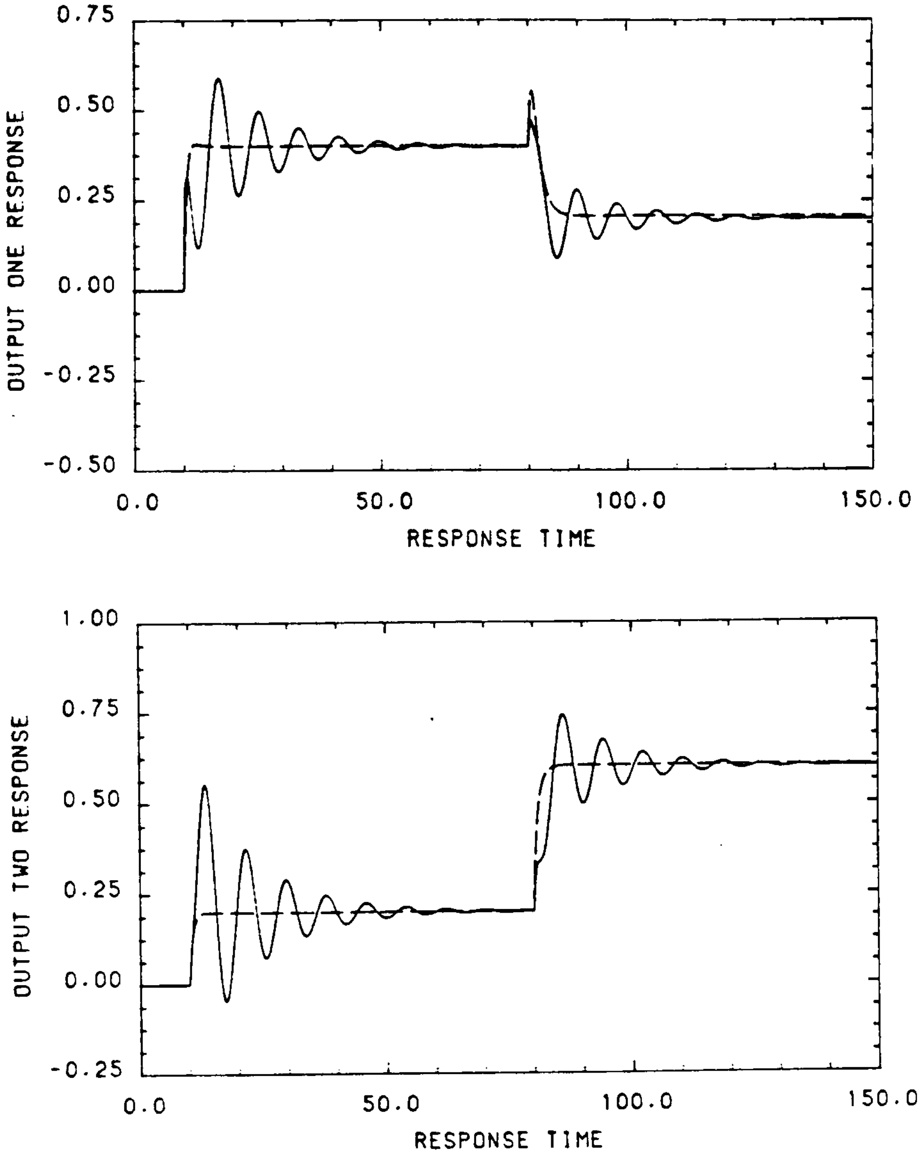


Figure 5.5. Open Loop Responses of the Equation Fitting Method for MIMO Case I.
Full model: ———— Reduced model: — — — —

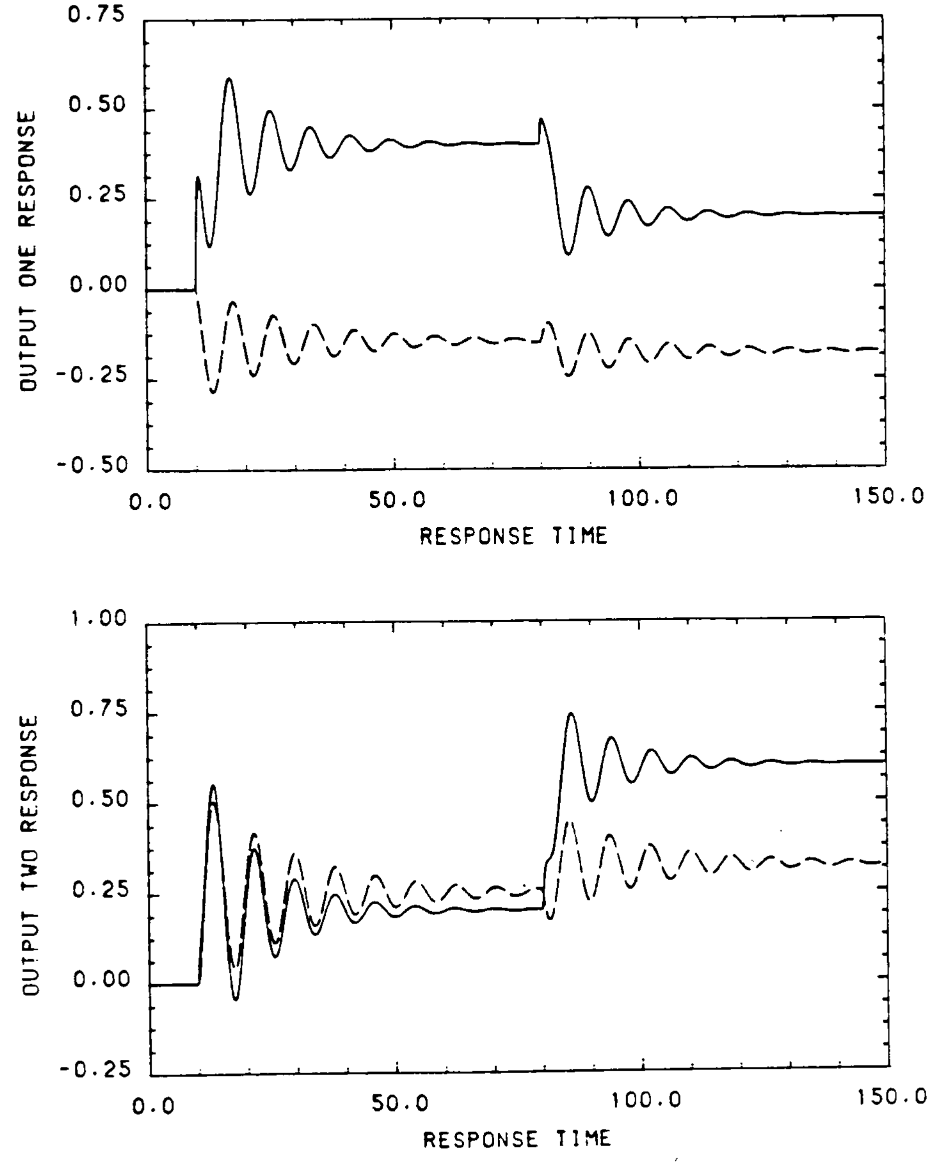


Figure 5.6. Open Loop Responses of the Internal Balancing Method for MIMO Case I.

Full model: ———— Reduced model: – – – –

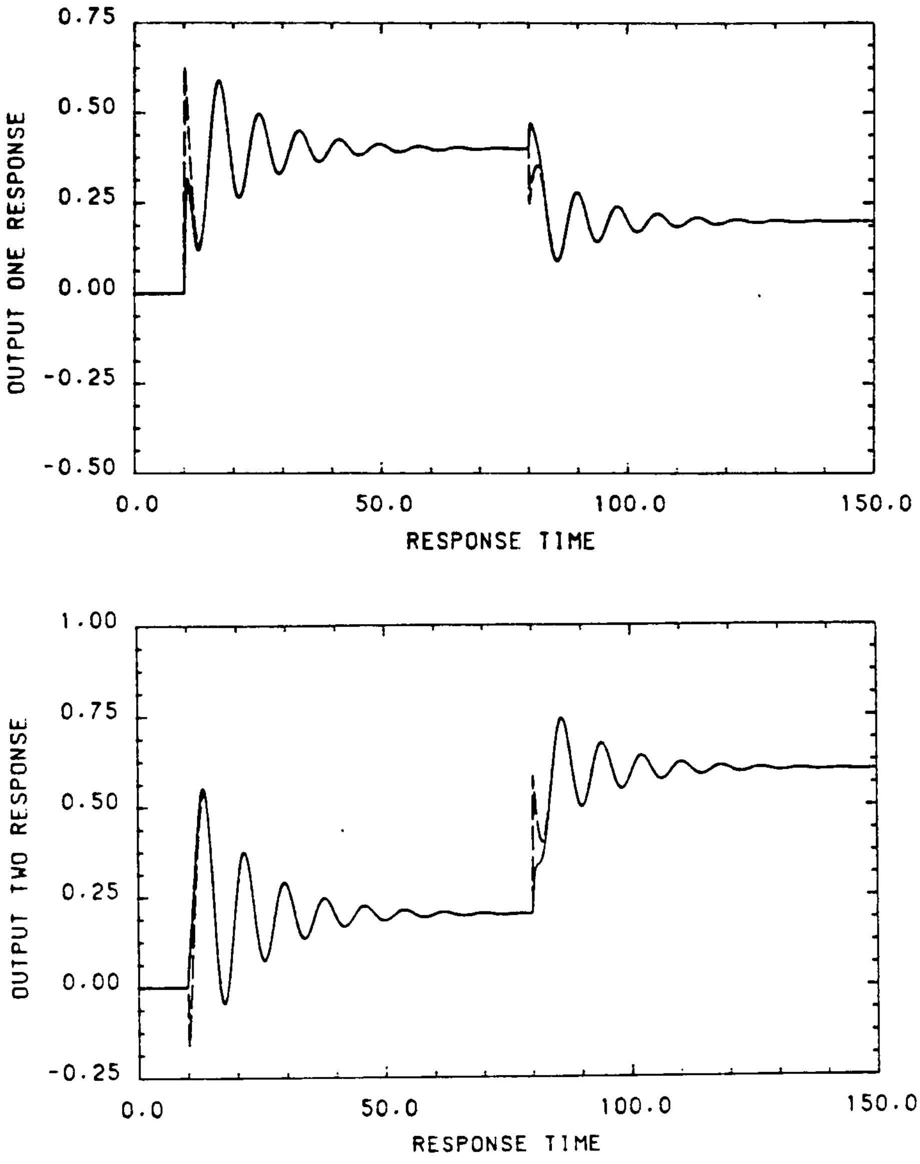


Figure 5.7 Open Loop Responses of the Singular Perturbation Method for MIMO Case I.

Full model: ———— Reduced model: — — — —

The responses shown in these figures are to a unit step in input one at $t = 10$, and unit step in input two, at $t = 80$. Each of the figures contains two plots, one for output one and the other for output two.

From the open loop simulations, it is obvious that none of the Cauer forms contain the dominant characteristics of the full model. The response of the reduced model obtained by equation fitting is very similar to the response generated by the Cauer second form; neither of which match the transients of the full model. The response of the internal balancing reduced model does contain the oscillatory response exhibited by the full model, but fails to match the steady state values of the full model. The final method considered, the singular perturbation method, matched the full model well and is off only in the initial response.

The parameters of the multivariable PI controller were determined by IAE tuning for set point changes using each reduced model. As mentioned in the chapter introduction, the proportional and integral matrices were set with the full model. The matrices for the full model are,

$$P = \begin{bmatrix} 1.0 & -1.0 \\ -1.0 & 3.0 \end{bmatrix}, \tag{5.4}$$

and

$$K^{-1} = \begin{bmatrix} 2.0 & 1.0 \\ -1.0 & 2.0 \end{bmatrix}. \tag{5.5}$$

Using these two matrices, the tuning parameters of the seven reduced model controllers were calculated. The resulting parameters are listed in Table 5.2.

When these controllers are applied to control of the full model, the response to the same set point changes (unit step in set point one at t=10 and a step of 0.5 in set point two at t=80) was unstable for six of the seven control schemes. The only successful control scheme was designed using the singular perturbation reduced model. The closed loop response of this system is shown in Figure 5.8, and the IAE using this controller is 13.2.

Four of the reduced models failed for controller design in part due to the full model being underdamped and the reduced models being overdamped. The remaining three reduced models are underdamped; however, two of the methods, dominant mode and internal balancing, fail to match the steady state gain of the full model. The other model, reduced using the singular perturbation method, is underdamped, matches the steady state of the full model, and is reliable for controller design for this model.

MIMO Model Reduction: Case II

This example, also taken from Goldman, et al. [38], is a sixth order two input-two output system. Because of the high order of the full model, the matrices are easily listed in table form (Table 5.3). This model was also used by Gerstle, et al. [36] for designing a single

Table 5.2. Controller Tuning Parameters for MIMO Case I

Reduction Method	Tuning Parameters			
	P(1)	P(2)	e	IAE with full model
Cauer First Form	39	42	50	∞
Cauer Second Form	130	13	140	∞
Cauer Third Form	39	39	40	∞
Dominant Mode	700	-67	-13	∞
Equation Fitting	65	47	39	∞
Internal Balancing	-1970	360	18	∞
Singular Perturbation	.045	-.18	.22	13.2

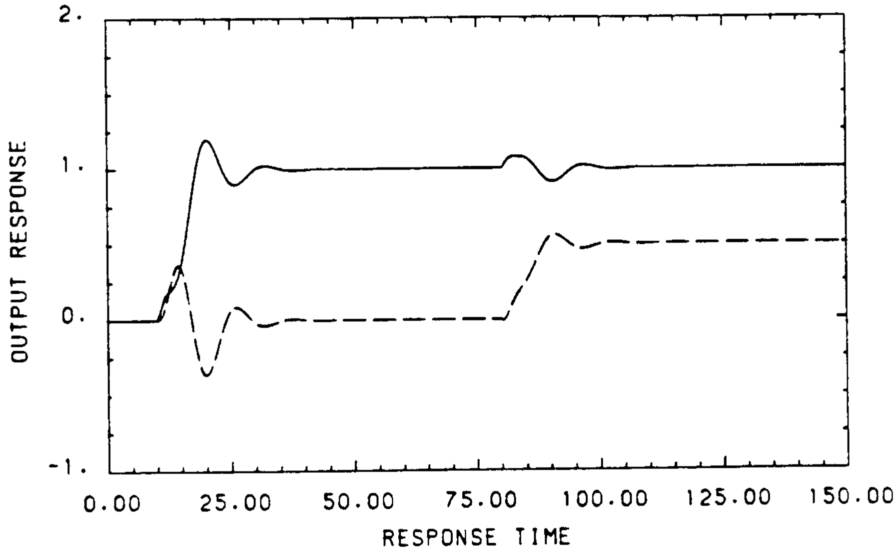


Figure 5.8. Multivariable PI Control of the Full Model Using Parameters Set by the Singular Perturbation Method for MIMO Case I.

Full model: ——— Reduced model: - - - -

Table 5.3. State Space Matrices of Case II

State Space Matrices

A						B		C^T	
0	0	1	0	0	0	0	0	10	1
0	0	0	1	0	0	0	0	.4	4
0	0	0	0	1	0	0	0	1	2
0	0	0	0	0	1	0	0	.2	1
-50	9.96	-108.5	19.6	-16.5	4	2	-1	0	1
49	-19.96	107	-39.6	13	-10	-2	2	.2	0

loop PI controller scheme and a singular value decomposition controller with second order reduced models. They used the three continued fraction methods and the internal balancing method in their study. All of these controllers failed when applied to the full model. This model is now investigated using three additional methods of reduction and for multivariable PI controller design.

To determine the order of the reduced model to be calculated, the second order modes of the internally balanced full model are used. The second order modes for this model are: {0.22, 0.20, 0.059, 0.031, 0.0077, 0.0041}. From analysis of these modes, one break is seen between the fourth and fifth modes and another between the second and third modes. Hence, either a second or fourth order model is suggested. Since both of the breaks are about the same, the lower order model will be calculated.

One of the seven methods of reduction, the Cauer third form, produced an unstable second order model. The state space matrices of the remaining six methods are listed in Table 5.4. Again, the singular perturbation method will be the only model with a D matrix.

Simulation studies used discrete-time models as in the other examples. The responses to a unit step in input one at t=1, and a unit step in input two at t=10, are shown in Figures 5.9 through 5.14. The dynamics of the full model are somewhat simpler than those of the first case considered in this chapter. This results in

Table 5.4. Second Order Reduced Models for MIMO Case II

Reduction Method	State Space Matrices A		B		C		D	
Cauer First Form	-12.1 15.2	6.56 -11.6	12.1 -15.2	-6.56 11.6	.035 .197	.054 .025		
Cauer Second Form	-.482 -.0553	-.00286 -.571	1 0	0 1	.190 .0335	.0235 .228		
Dominant Mode	-.506 0	0 -.599	.0199 .0206	-.00171 .07350	9.39 -2.08	.434 3.29		
Equation Fitting	-.686 -.149	.130 -.395	-.231 -.296	-.136 -.116	1.22 -4.87	-1.61 3.71		
Internal Balancing	-.517 .0178	-.150 -.371	-.202 -.374	-.430 .113	-.157 -.449	-.384 .0695		
Singular Perturbation	-.506 0	0 -.599	-.0364 -.133	.00313 -.474	-5.38 1.46	-.104 -.508	-.0193 .0158	-.0103 -.0107

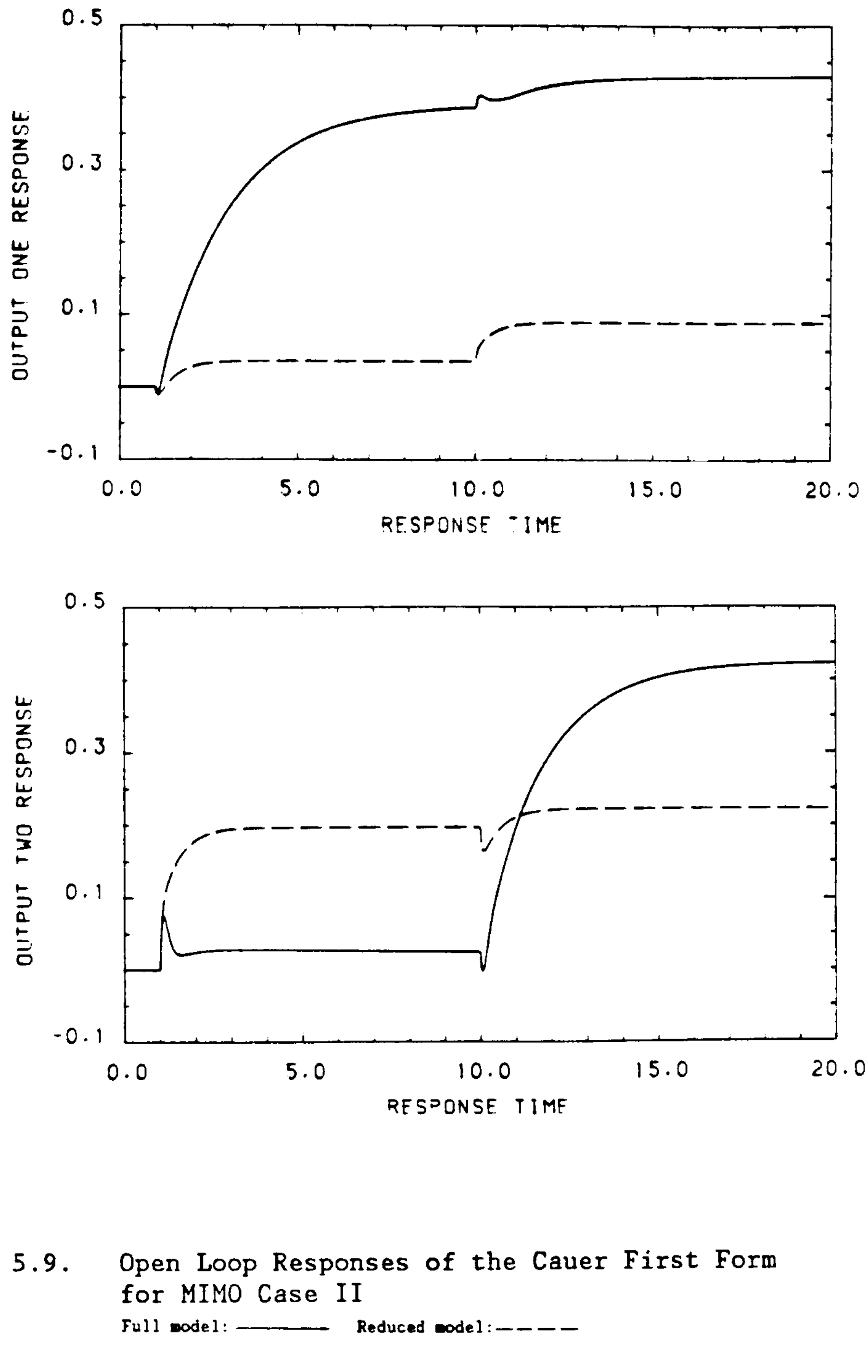
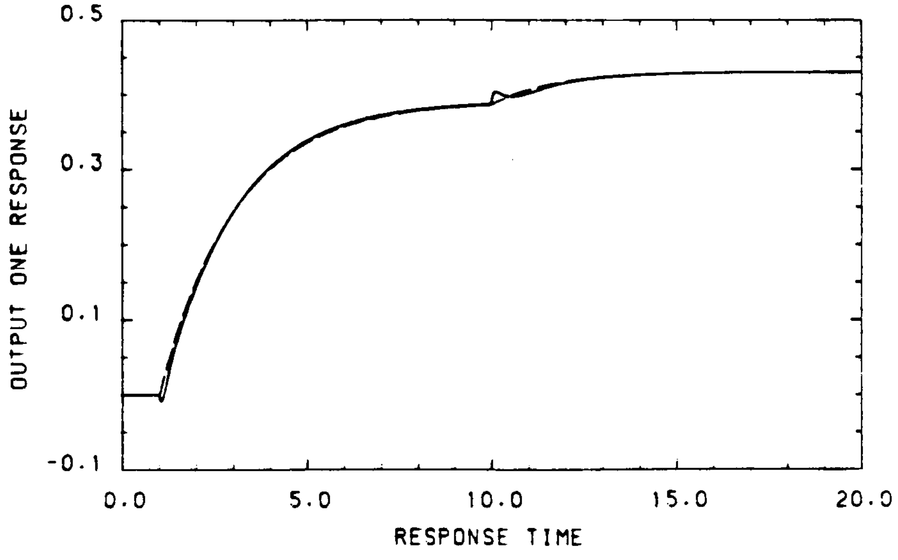


Figure 5.9. Open Loop Responses of the Cauer First Form for MIMO Case II

Full model: ———— Reduced model: — — — —



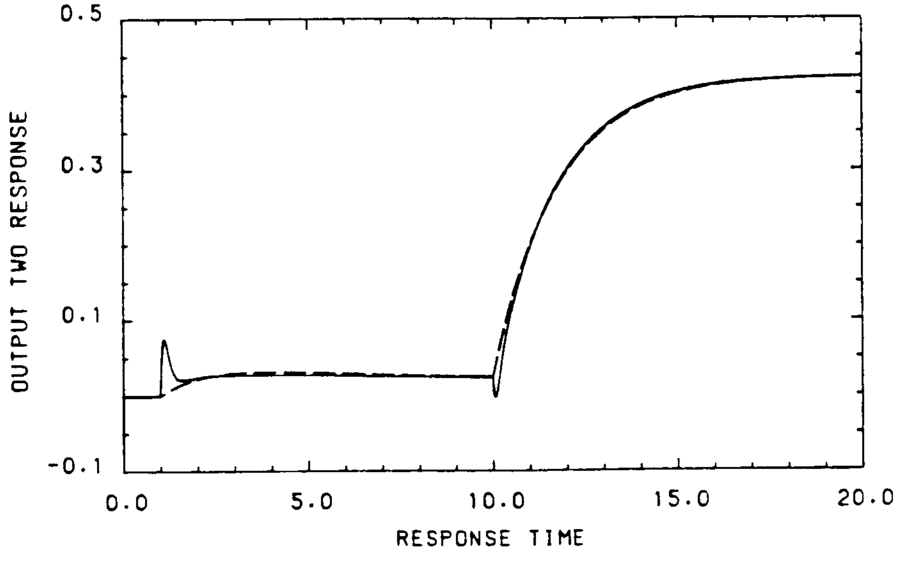


Figure 5.10. Open Loop Responses of the Cauer Second Form for MIMO Case II.

Full model: ———— Reduced model: – – – –

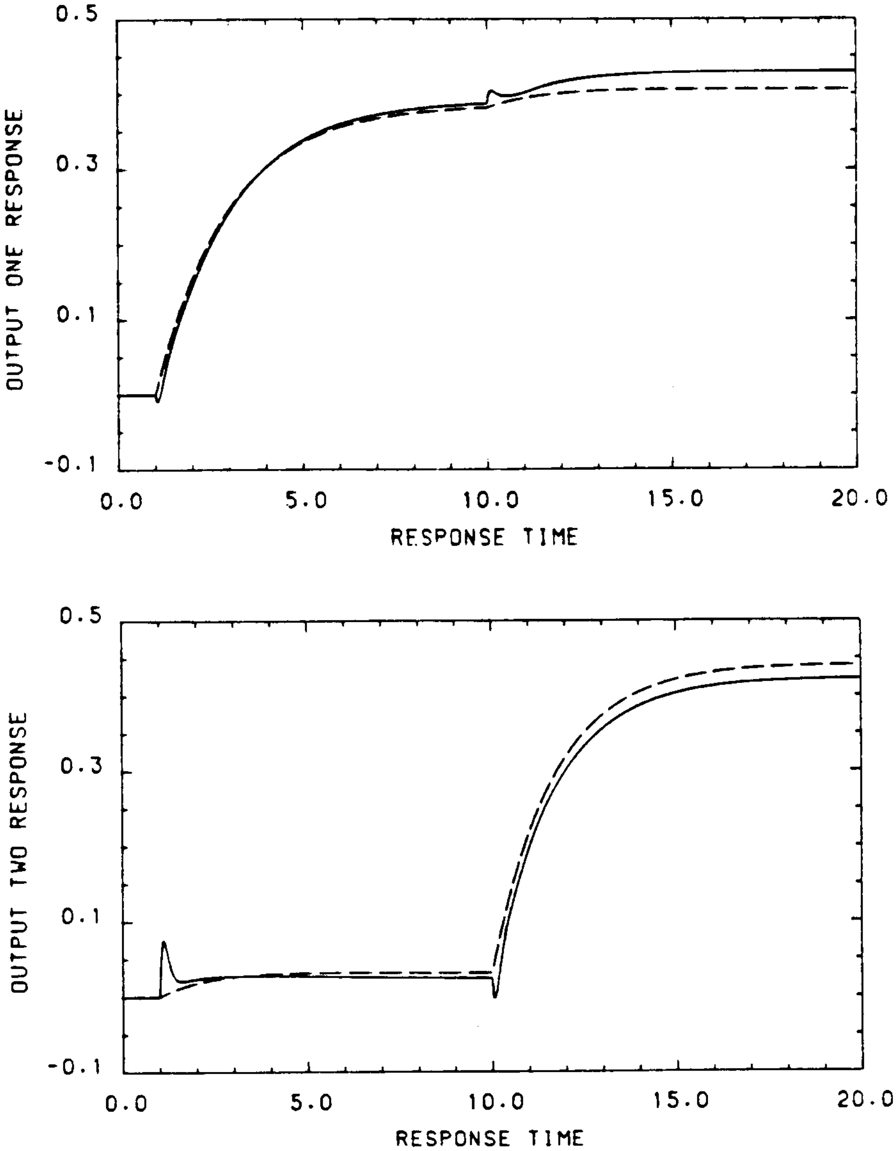


Figure 5.11. Open Loop Responses of the Dominant Mode Method for MIMO Case II.
Full model: ———— Reduced model: – – – –

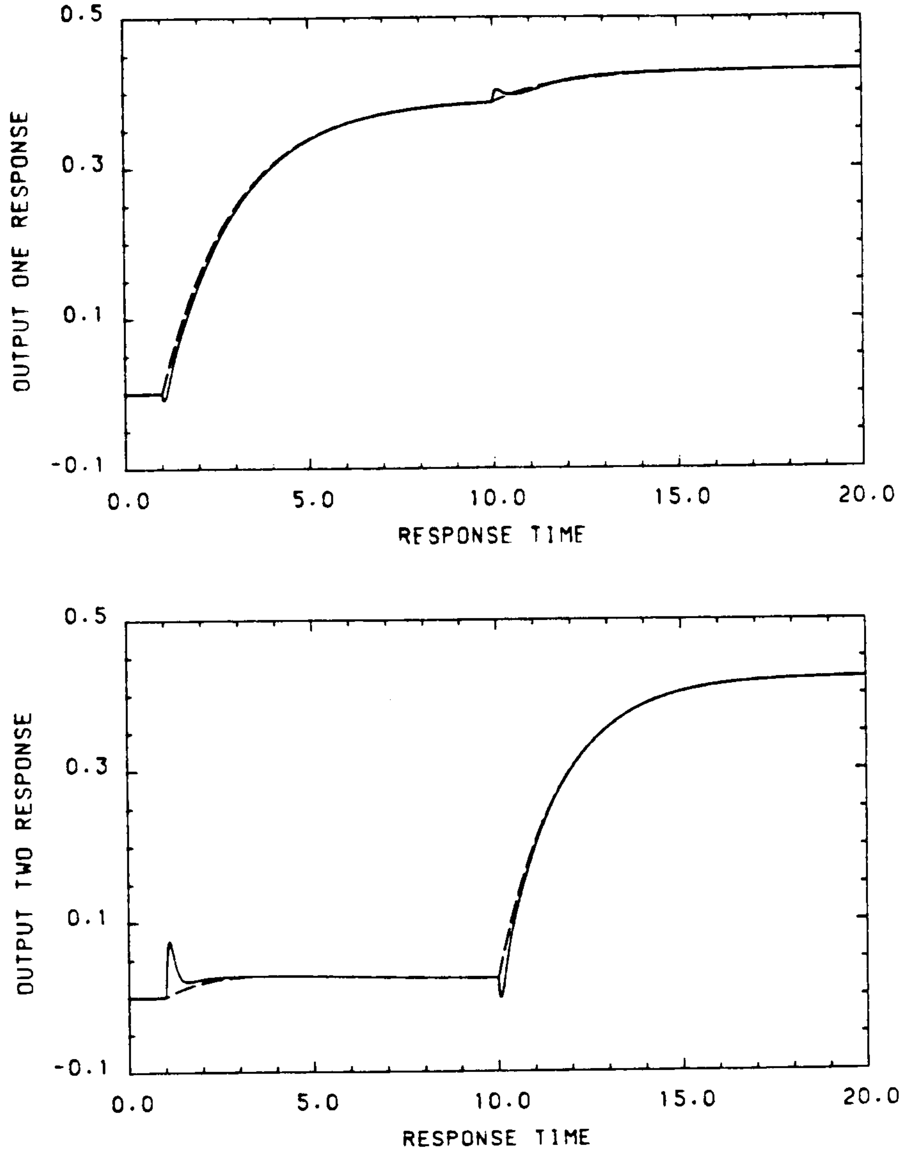


Figure 5.12. Open Loop Responses of the Equation Fitting Method for MIMO Case II.
Full model: ——— Reduced model: – – – –

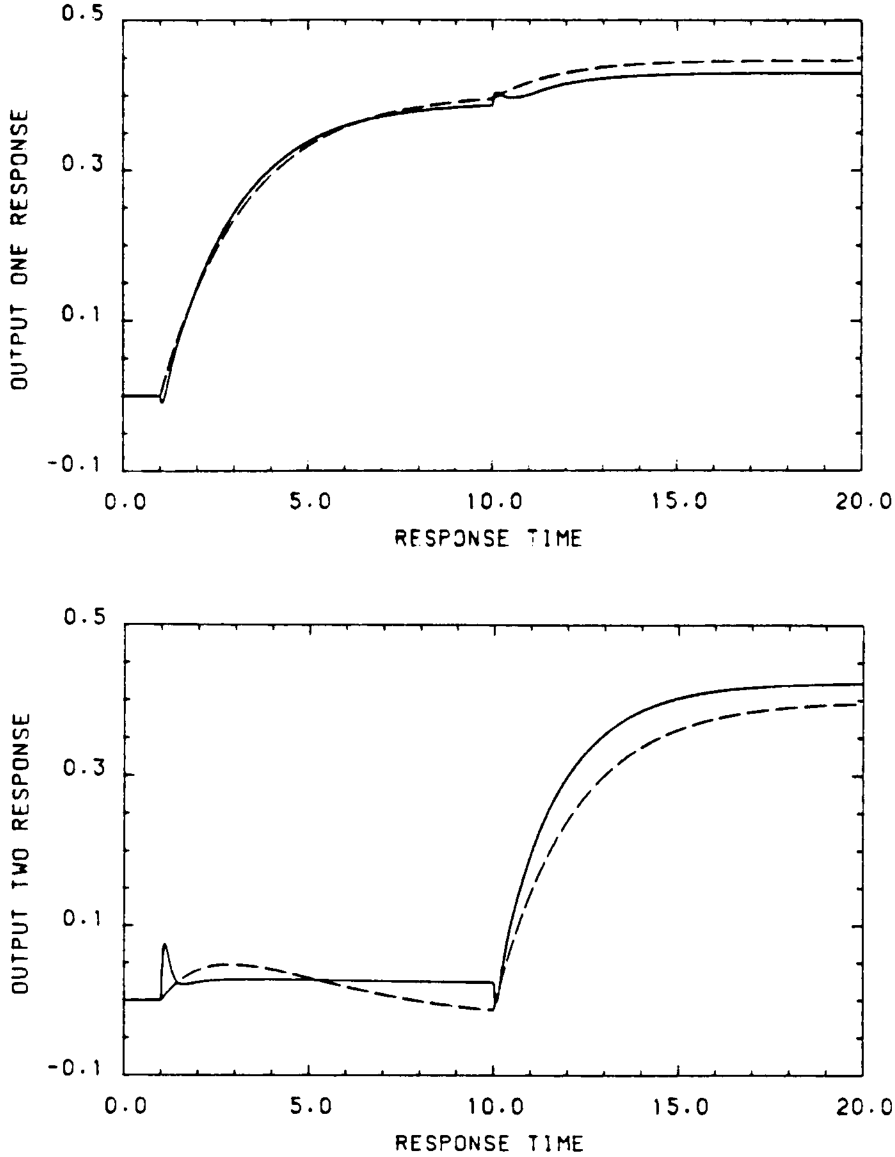


Figure 5.13. Open Loop Responses of the Internal Balancing Method for MIMO Case II.

Full model: ——— Reduced model: – – – –

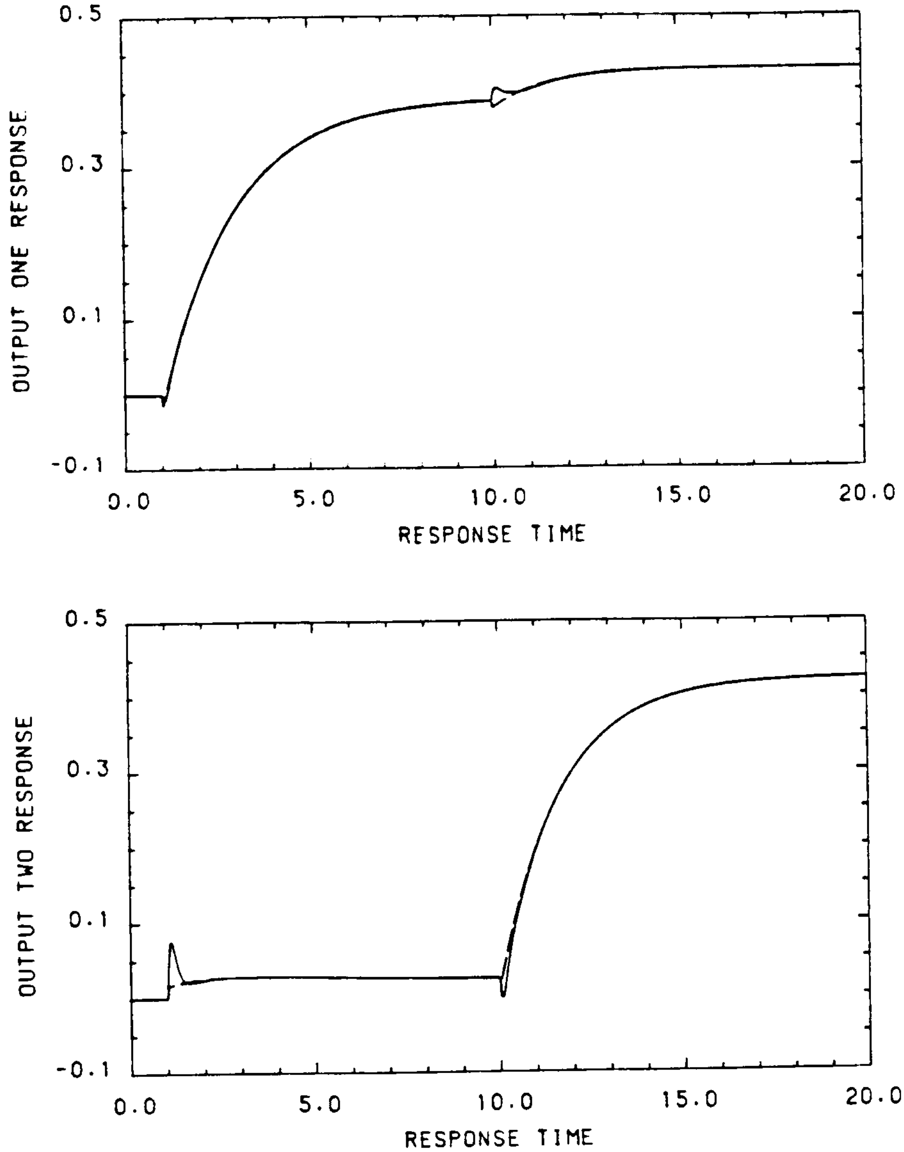


Figure 5.14. Open Loop Responses of the Singular Perturbation Method for MIMO Case II.

Full model: ——— Reduced model: – – – –

reduced models which in general appear to come closer to matching the characteristics of the full model.

As expected, three of the methods, the Cauer second form, equation fitting, and singular perturbation, do match the steady state gain of the full model, while the other three do not. In the transient region, four of the models, the Cauer second form, equation fitting, dominant mode, and singular perturbation, maintain the dominant characteristics of the full model. The reduced models obtained with the Cauer first form and with internal balancing maintain only a few of the characteristics of the full model.

A multivariable PI controller was designed with each of the six reduced models. Using the full model, the proportional and integral matrices were set to,

$$P = \begin{bmatrix} 2.5 & 1.0 \\ 5.0 & 1.0 \end{bmatrix}, \tag{5.6}$$

and

$$K^{-1} = \begin{bmatrix} 2.570 & .251 \\ -.0588 & 2.50 \end{bmatrix}. \tag{5.7}$$

With these two matrices, the control parameters of the multivariable PI controller were calculated. The control parameters determined for the six reduced models are listed in Table 5.5.

Four of the six control structures set with the reduced models proved to be successful for control of the full model. These four, the Cauer second form, dominant mode, equation fitting, and singular

Table 5.5. Controller Tuning Parameters for MIMO Case II

Reduction Methods	Tuning Parameters			
	P(1)	P(2)	e	IAE with full model
Cauer First Form	74	74	-7.1	∞
Cauer Second Form	-.02	3.4	1.1	4.6
Dominant Mode	-.0079	3.4	1.1	4.6
Equation Fitting	-.01	3.1	1.1	4.5
Internal Balancing	-41	370	280	∞
Singular Perturbation	-.022	2.2	.93	4.3

perturbation, all maintained the dominant transient components of the full model. The IAE associated with each controller on the full model is listed in Table 5.5 and the closed loop responses to a unit step in set point one at t=1 and a step of 0.5 in set point two at t=10 are shown in Figures 5.15 and 5.16.

MIMO Model Reduction: Case III

The final multivariable model to be considered is a twelth order three input-three output model of an extractive distillation column. This model was presented by Gupta, Donnelly, and Andre [39] for the simulation of the separation of n-butyl acetate from n-butanol with water as the extractive agent.

For the state space equation,

$$\dot{x}(t) = A\ x(t) + B\ u(t) \tag{5.8a}$$

$$y(t) = C\ u(t) \tag{5.8b}$$

the A matrix is in modified canonical form [114], (i. e., it contains the eigenvalues of the system along its diagonal). The eigenvalues for this system are, in order of their appearance in the A matrix: {-0.009, -0.009, -0.09, -0.04, -0.013, -0.0065, -0.005, -0.005, -0.0063 - j0.005, -0.0063 + j0.005, -0.0042 - j0.0032, -0.0042 + j0.0032}. The B and C matrices of the full model are listed in Table 5.6.

The three input variables for the column are:

u_1 = change in steam rate, kg/hr,

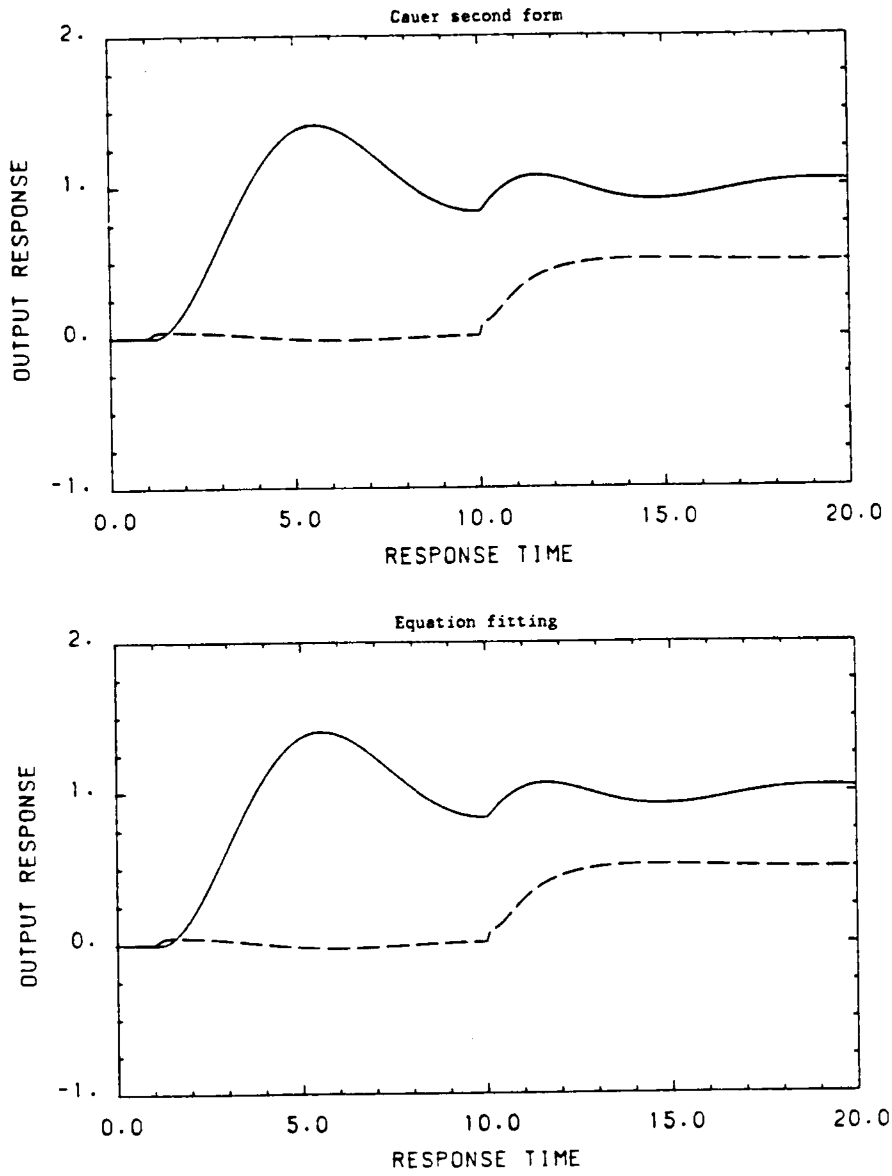


Figure 5.15. Multivariable PI Control of the Full Model Using Parameters Set by the Cauer Second Form and Equation Fitting Method for MIMO Case II.
Output one: ——— Output two: – – – –

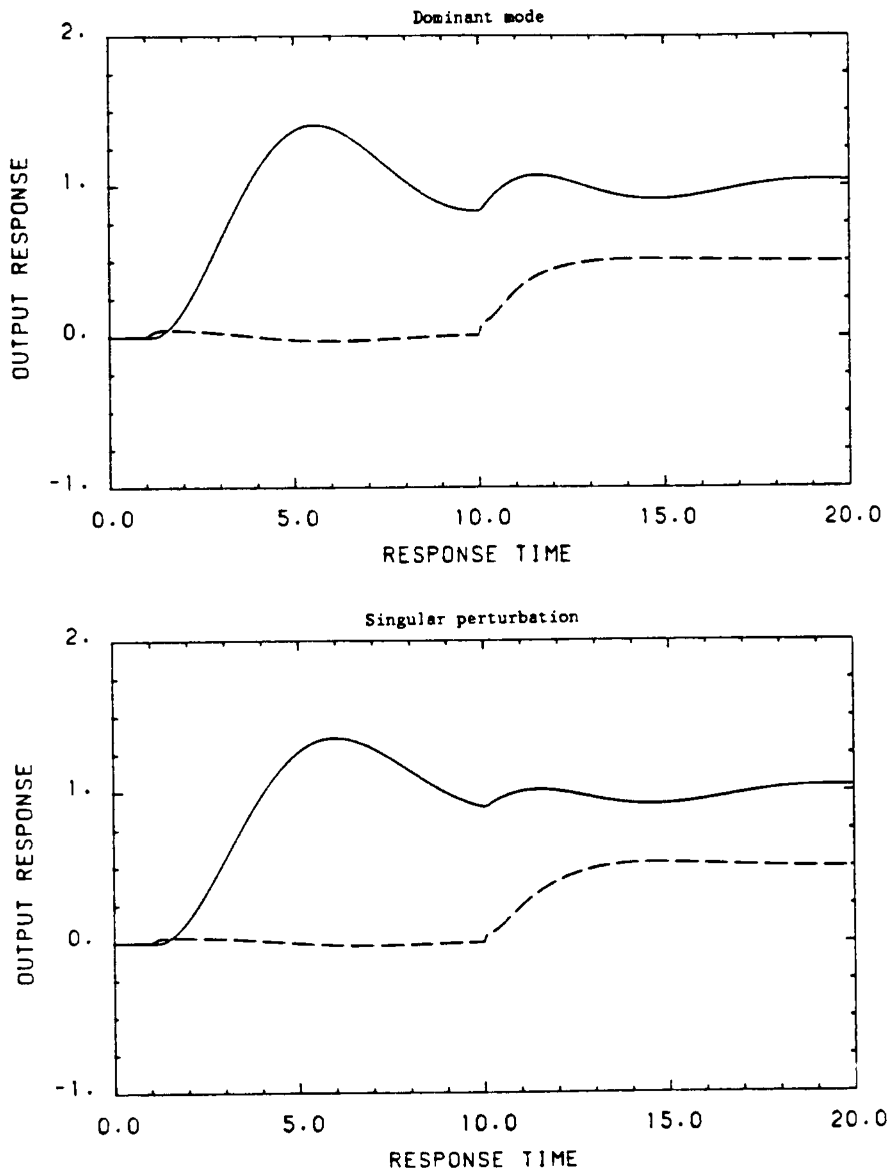


Figure 5.16. Multivariable PI Control of the Full Model Using Parameters Set by the Dominant Mode and Singular Perturbation Methods for MIMO Case II.
Output one: ——— Output two: – – – –

Table 5.6. State Space Matrices of Case III

State Space Matrices					
	B			C^T	
1	0	0	.297	0	-.0198
0	0	1	-.720	-.526	0
0	0	0	1	0	0
0	-.208	.720	1	0	0
-.117	0	.526	0	1	0
0	.0715	0	0	1	0
0	0	0	0	1	0
0	0	0	0	0	1
0	.00485	0	0	0	2
0	-.0133	.0647	0	0	0
0	0	0	0	0	2
0	0	0	0	0	0

u_2 = change in top product reflux rate, kg/hr,

u_3 = change in water reflux rate, kg/hr.

And the three output variables are,

y_1 = change in temperature of liquid at the 19th plate, C,

y_2 = change in top product composition, wt. % butyl acetate,

y_3 = change in composition of liquid at 19th plate, wt. % butyl acetate.

A schematic of the 20 plate extractive distillation column is shown in Figure 5.17.

Analysis of the internally balanced full model yields the second order modes: {40., 11.1, 5.0, 1.85, 0.995, 0.518, 0.335, 0.166, 0.128, 0.0331, 0.0114, 0.0042}, from which the order of the reduced model may be set. There is not a large separation between any two adjacent second order modes. Hence, by second order mode analysis, reducing this model is not suggested.

When Gupta, et al. presented this model, they also reduced it using an equation fitting routine to a third order model. They concluded that the third order model adequately represented the full model for simulation purposes. As a test of both their claims and of the use of the second order modes for order selection, a third order model will be determined with each of the seven MIMO model reduction methods.

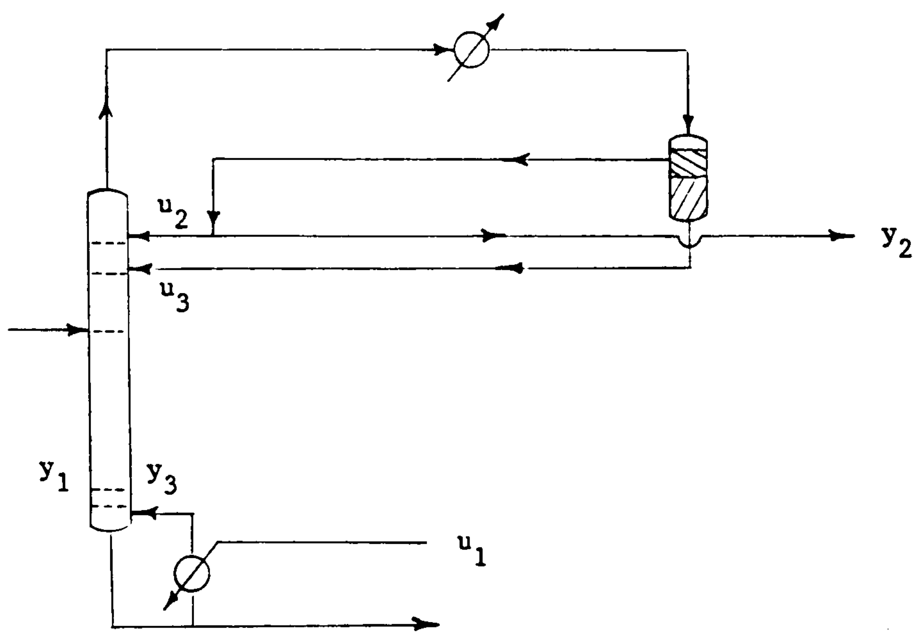


Figure 5.17. Distillation Column for MIMO Case III.

The calculation of a reduced order model from either the Cauer first or third form for this model is impossible. The methods fail due to a required inversion of a singular matrix. The matrix to be inverted in both cases is the zero matrix. The reduced models obtained with the remaining five methods are listed in Table 5.7.

The equation fitting method suggested earlier in this chapter gave essentially the same third order model that was suggested by Gupta, et al. Hence, the intuitive decision made by Gupta, et al. are supported. Moreover, a method by which these judgements are made is now done analytically.

The open loop responses of the five reduced models are illustrated in Figures 5.18 through 5.22. The simulations represent the response to three step inputs; a step of 0.5 in u_1 at $t = 10$, a step of -0.4 in u_2 at $t = 600$, and a step of -0.2 in u_3 at $t = 1200$. From the open loop responses, the reduced models of the Cauer second form and equation fitting are seen to match the original model quite well.

As expected, the open loop plots of the reduced models obtained by internal balancing and by dominant mode models do not maintain the steady state response of the full model. Due to the extremely small values in the C matrix of the dominant mode reduced model, there is hardly any response. The C matrix of the singular perturbation model is very similar to that of the dominant mode method. This method generates a D matrix whose effect on the response is to maintain the

Table 5.7. Third Order Reduced Models for MIMO Case III

Reduction Method	State Space Matrices											
	A			B			C			D		
Cauer Second Form	-.00757	.00140	.00482	1	0	0	.3020	-.0711	-.4760			
	.00336	-.00577	.00203	0	1	0	-.0948	.0774	-.0742			
	.00057	0.0	-.00528	0	0	1	-.0210	.0212	-.0484			
Dominant Mode	-.009	0	0	.312	0	-.796	.5820	-.1640	0			
	0	-.009	0	.110	0	-.879	-.0787	.0933	0			
	0	0	-.0065	.00215	240	-.110	.00588	.00666	0			
Equation Fitting	-.024	0	0	.324	-.0616	-.570	2.61	.379	-.0029			
	0	-.0091	0	-.0388	.0331	-.025	.379	3.28	0			
	0	0	-.0046	-.0022	.0030	-.103	-.0059	0	4.5			
Internal Balancing	-.00550	-.00224	.01170	.315	-.0566	-.580	.6580	.216	-.6190			
	-.00186	-.00882	.00463	.364	-.2410	.0528	.0719	-.368	-.1080			
	-.00935	.00627	-.0404	-.0216	.1920	-.584	.0459	.120	-.0923			
Singular Perturbation	-.00650	0	0	0	.0715	0	0	.297	-.720	0	-5.2	18
	0	-.009	0	1	0	0	1	0	-.526	-9	0	40.5
	0	0	-.009	0	0	1	0	-.0198	0	0	2.999	-10

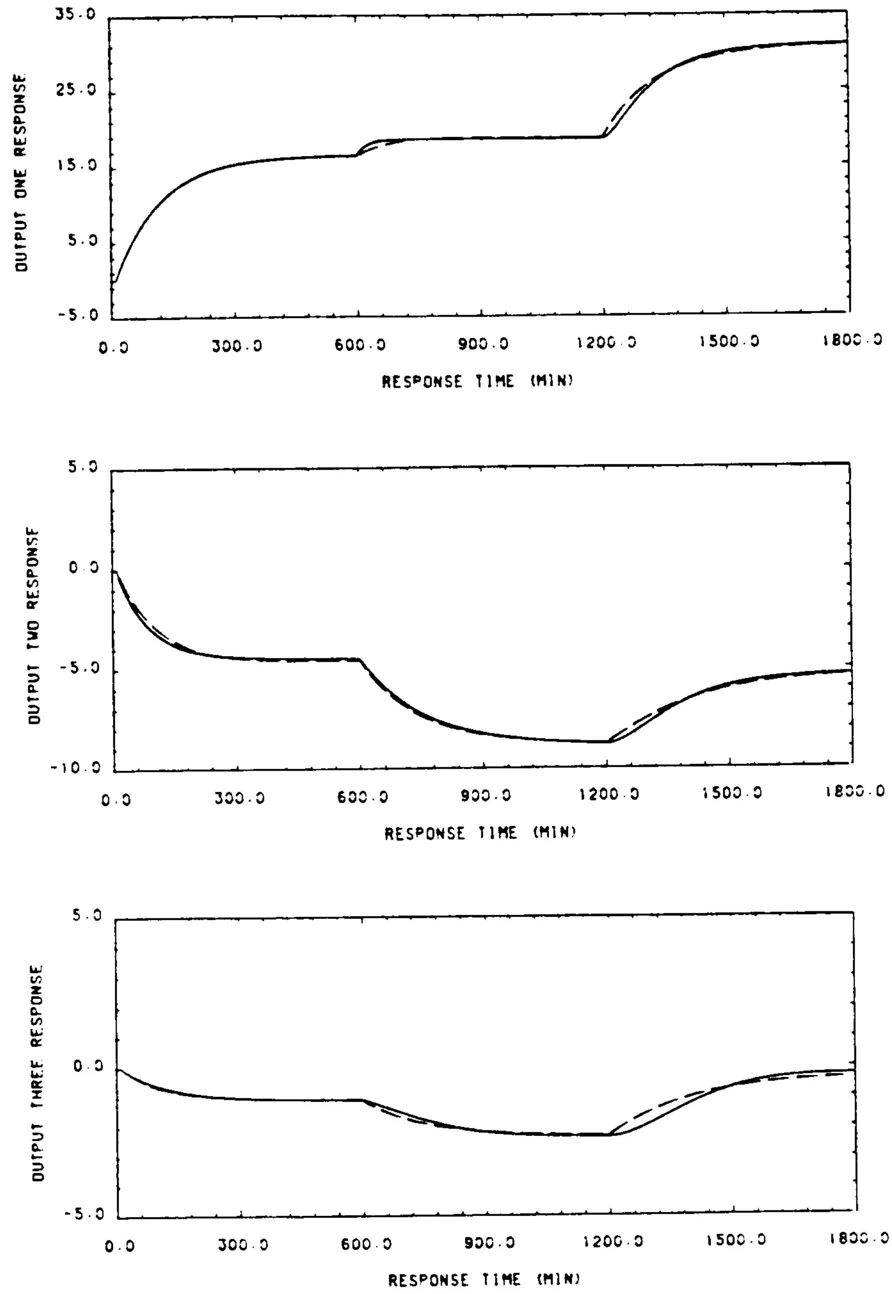


Figure 5.18. Open Loop Responses of the Cauer Second Form for MIMO Case III.

Full model: ———— Reduced model: – – – –

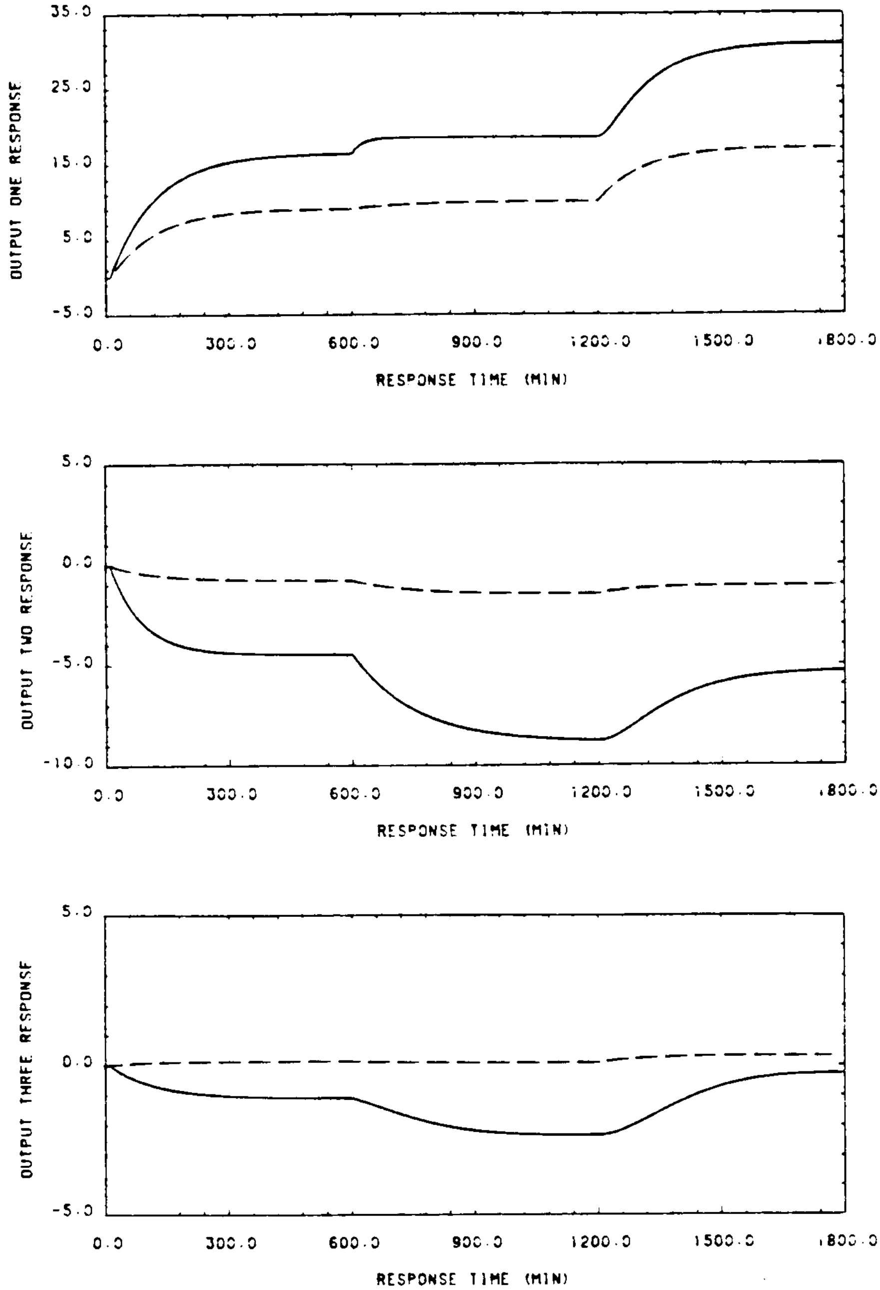


Figure 5.19. Open Loop Responses of the Dominant Mode Method for MIMO Case III.

Full model: ———— Reduced model: — — — —

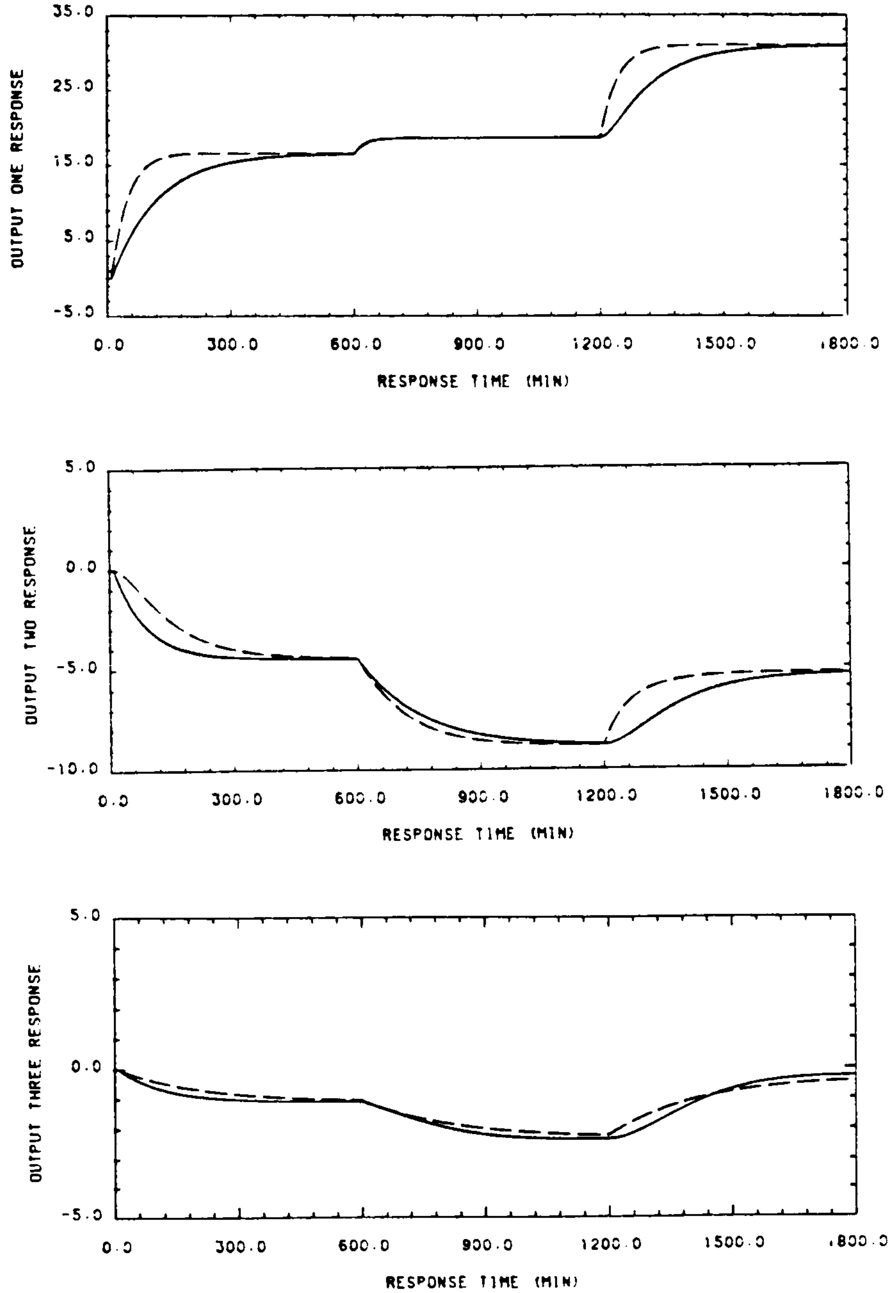


Figure 5.20. Open Loop Responses of the Equation Fitting Method for MIMO Case III.

Full model: ———— Reduced model: — — — —

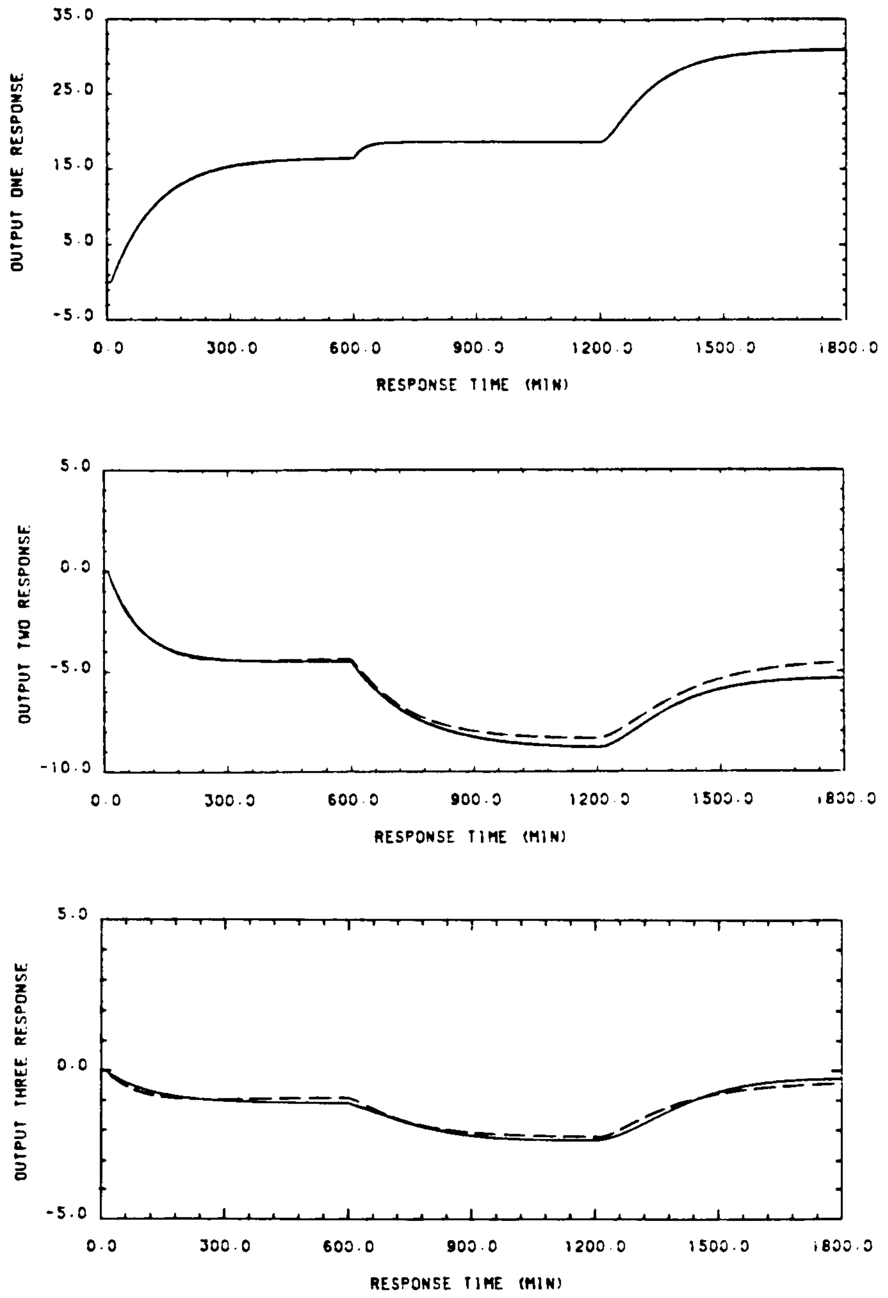


Figure 5.21. Open Loop Responses of the Internal Balancing Method for MIMO Case III.

Full model: ———— Reduced model: — — — —

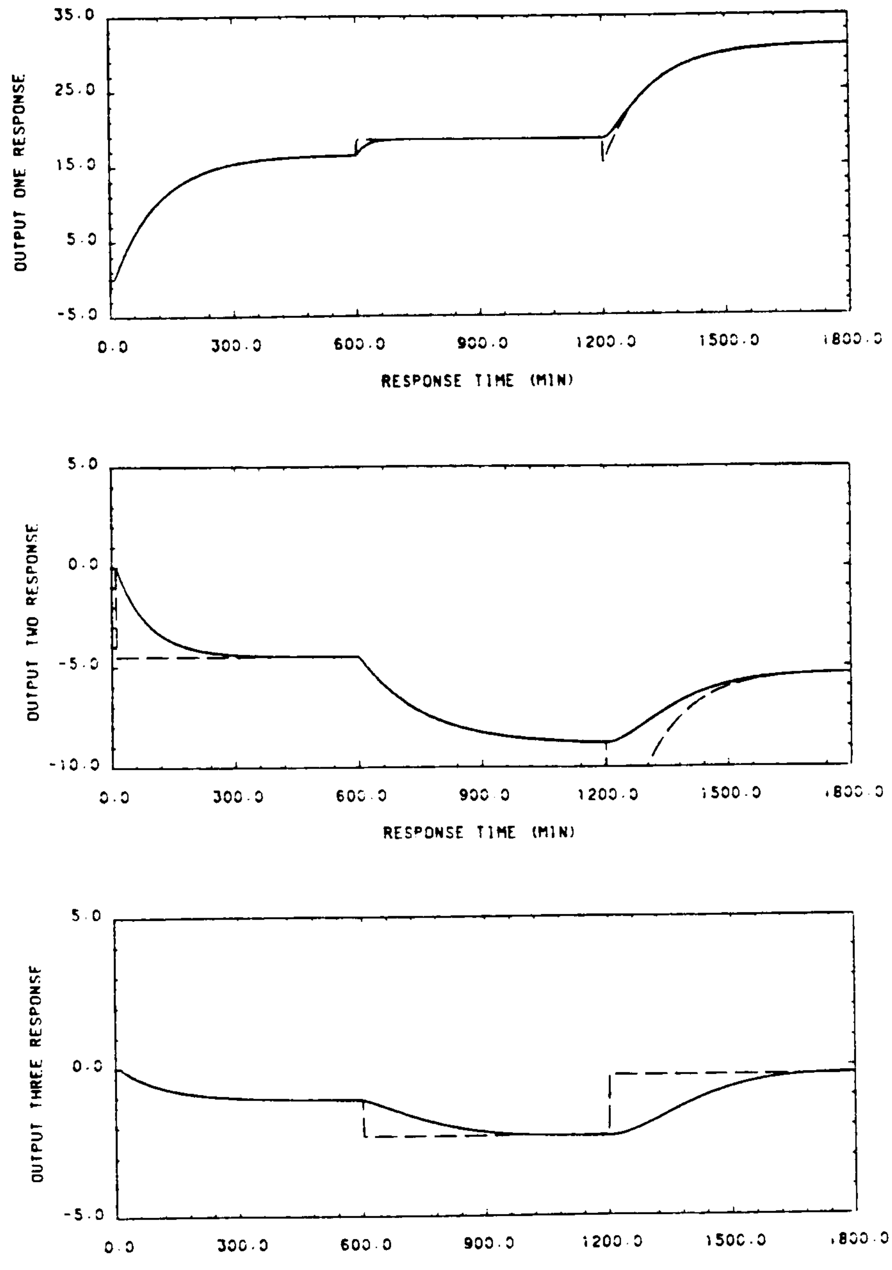


Figure 5.22. Open Loop Responses of the Singular Perturbation Method for MIMO Case III.

Full model: ———— Reduced model: — — — —

original steady state. Any change in the inputs is immediately passed by the D matrix to the outputs. Combining the small values in the C matrix with the relatively large steady state that the D matrix will maintain, produces the "steps" in the responses of Figure 5.22.

The five reduced models were also used to design a multivariable PI controller. As in the previous two case studies, the proportional and integral matrices are set with full model and for this case are,

$$P = \begin{bmatrix} 3.8 & 52 & -300 \\ 5.0 & 94 & -480 \\ -40. & -160 & 320 \end{bmatrix}, \qquad (5.9)$$

and

$$K^{-1} = \begin{bmatrix} .040 & .170 & -.55 \\ .036 & .330 & -.82 \\ .002 & .062 & -.22 \end{bmatrix}. \qquad (5.10)$$

Unfortunately, with these two control matrices, none of the reduced models could be tuned. When the proportional and integral matrices were set by the reduced model, the controllers could be tuned. These controllers did, however, fail in all cases to control the full model.

From the standpoint of controller design, all the reduced models failed. Recall that the second order modes of the full model implied that the full model should not be reduced. Higher order reduced models (orders of six and nine) for this model also failed for proper controller design of the full model. The simulations done with the higher order models do offer improvement, varying from a slight to a substantial degree, on the models presented here.

While investigating this model, the relative gain array [6] was calculated. The relative gain array analysis is useful in determining steady state interactions. The results of the investigation, while somewhat out of line with the theme of this research, are startling.

The first step in relative gain analysis is the calculation of the gain matrix. For this system, the gain matrix is,

$$G(0) = \begin{bmatrix} 33. & -5.2 & -62. \\ -9.0 & 11. & -18. \\ -2.2 & 2.999 & -10. \end{bmatrix} , \tag{5.11}$$

which results in a relative gain array of,

$$\Lambda_1 = \begin{bmatrix} 1.3 & -.18 & -.12 \\ -1.5 & 3.6 & -1.1 \\ 1.2 & -2.4 & 0.2 \end{bmatrix} . \tag{5.12}$$

If the (3,2) element, 2.999, of the gain matrix is rounded to 3.0, the resulting relative gain array is,

$$\Lambda_2 = \begin{bmatrix} .78 & -2.4 & 2.6 \\ -.91 & 2.2 & -.27 \\ 1.1 & 1.2 & -1.3 \end{bmatrix} . \tag{5.13}$$

The two relative gain arrays imply entirely different steady state interactions. In performing the relative gain analysis, inversion of the gain matrix is required. Since matrix inversion is an ill-conditioned operation there is a potential for error with relative gain analysis.

The relative gain array is often used for control system design. In most plant situations, four place accuracy would rarely be assumed for any data. Hence, the control engineer would have assumed that the (3,2) element of the gain matrix was 3.0. In this case, such an assumption lead to incorrect controller pairing.

Summary of the Multivariable Model Reduction Problem

With the three case studies presented and with other models tested during the course of this work, the practicality of multivariable model reduction is discussed. The discussion is divided into three parts: simulation, controller design, and concluding remarks.

Simulation. The open loop responses of the reduced models offer a simple means of checking the effectiveness of the reduction method. Again, the question of what constitutes an adequate representation of the full model is raised. The answer to this question is left to the modeler since he or she would know the stipulations to be put on the reduced model. The remaining discussion centers on what characteristics of the full model are maintained by the reduced models. Of the seven methods used, four of them, the Cauer second and third forms, equation fitting, and singular perturbation, maintain the steady state response of the full model. The Cauer second form and equation fitting maintain the steady state at the expense of the fast components, (i. e., the fast components are ignored). With the Cauer third form, both the fast components and the steady state components of the full model are maintained.

The singular perturbation method maintains the steady state of the full model with a D matrix in the state space equations. The D matrix is a positive feedforward loop from the input to the output. A step input is immediately seen in the output, as a step scaled by the components of the D matrix. If the components of the D matrix are small in comparison to the steady state gains, the step in the output may be inconsequential. Hence, the D matrix not only maintains the steady state, but also approximates the fast components of the full model. However, if the components of the D matrix are large in comparison to the steady gains, large steps may appear in the output (e. g., Figure 5.22).

The singular perturbation method and the dominant mode method both maintain the transient components of the full model. The main difference between them is that the dominant mode method selected does not maintain steady state. As mentioned in Chapter 2, the success of these methods rests on the full model having dominant eigenvalues and the models investigated here do not have dominant eigenvalues.

The remaining two methods employed for MIMO model reduction, Cauer first form and internal balancing, both fail to maintain the steady state of the full model. The internal balancing method does maintain the transient components, however.

Controller Design. Multivariable model reduction for controller design is not only a test of the reduction techniques, but is also a test of the controller's robustness. In this research, the multivariable PI controller was used. Other studies have used the inverse Nyquist array [68], multivariable single input-single output controllers [36], and singular value decomposition controllers [36], all of which report failure of controller design with reduced models.

Of the methods studied in this work the singular perturbation method holds the most promise for multivariable PI controller design. This method is useful only when the components of the D matrix are small in comparison to the steady state of the full model, so that the output does not have significant "steps" in it. The case for singular perturbation method is perhaps the strongest since it is the only method which considers the fast, transient, and steady state components of the full model.

The claim that the multivariable PI controller is robust may fail for some steady state gain matrices. For calculation of the proportional and integral matrices of the multivariable PI controller, a matrix inversion is required (equations (2.108) and (2.109)). Since matrix inversion is an ill-conditioned operation, some steady state gain matrices which are sensitive to inversion may cause the calculation of the wrong control matrices (as in the relative gain analysis for Case III). Since these control matrices are dynamic and steady state decouplers, the tuning of the controller would be difficult, if not impossible.

Concluding remarks. The success of multivariable model reduction techniques has been shown to be limited. For simulation purposes, the Cauer second and third forms appear to be the most promising, whereas for multivariable PI controller design, the singular perturbation method is the optimum choice.

As in the single variable case, the use of the second order modes are very helpful in determining an appropriate order of the reduced model. Case III illustrated a model for which the second order modes implied that the model should not reduced, and from the standpoint of multivariable PI controller design, this conclusion is correct.

A new scheme of model reduction using equation fitting with the internally balanced representation of the full model has been proposed. This method is superior to other methods of MIMO equation fitting, since the decision on which states to approximate, may be made analytically.

The examples investigated demonstrate the potential failures which may arise when using reduced models for multivariable process studies. The reduction of MIMO systems encompass all of the problems encountered with SISO systems plus the additional problem of interactions. Thus, simulation and controller design of multivariable systems using reduced models should be done cautiously and with close consideration of these problems.

CHAPTER VI

CONCLUSION

The impetus of this research is simulation and controller design using reduced order models. This work has shown both the utility and futility of using mathematical model reduction. In this final chapter, the results of this research are summarized and suggestions are made for future work.

Results of this Research

Several avenues of model reduction have been investigated during the course of this work. The major findings of the research are now summarized. First, in the area of single input-single output model reduction, the Cauer third form of continued fraction expansion is found to be superior to the other methods of model reduction for both simulation and controller design. Model reduction by all of the continued fraction methods is straightforward. Various techniques were introduced to compare the reduced model with the full model. Of these, a comparison of the Bode plots of the reduced and full models provides the most information.

For the reduction of multiple input-multiple output models, no single reduction method is superior. However, for certain purposes, some methods operate better than others. For example, when the reduced model is to be used for simulation, both the Cauer second and third forms are the preferred methods. For multivariable PI controller design, the singular perturbation method is the optimum choice. Using the internally balanced representation of the full model to identify the dominant state variables is suggested for multivariable models generated by equation fitting.

The multivariable PI controller, which is both a dynamic and a steady state decoupler, is easily designed. Hence, using the reduced models for multivariable PI controller design provides a simple test of whether this reduced model maintains the interactions present in the full model. The singular perturbation method usually maintains these interactions and is more consistent in maintaining them than the other methods.

The reduction of multivariable systems is not a simple extension of single variable model reduction techniques. The interactions of the full model must be accounted for somewhere within the reduced model. Currently, no method exists which always maintains the interactions of the full model. Hence, any simulation and controller design of multivariable systems using reduced models should be executed with prudence.

Determination of the order of reduced models should be made with the second order modes of the full model. The second order modes may be used with either single variable or multivariable models.

Recommendations for Future Work

Continued research into the use of reduced models for simulation and controller design purposes could lead in several directions. Among these are: refinement of reduction methods, the reduction of other types of models, controller design with reduced models, and further exploitation of the internally balanced state space model representation.

The internal balancing method should be refined so that the reduced model maintains the steady state of the full model. One way to maintain the steady state may be to reduce the internally balanced model in the same way the singular perturbation method is reduced. Since the internal balancing method already maintains the transient components of the system, the addition of steady state matching should make this method extremely powerful.

The single variable methods (and by further extension, the multivariable methods) could be refined by the inclusion of a compensator in the reduced model. A reduced model would first be found using one of the conventional reduction methods. Then, a compensator would be designed with the Bode plot. The compensator is designed so that any characteristics that the first reduced model does not account for, will be taken care of in the "compensated" reduced model. The order of the first reduced model would be increased; for example, if a lead-lag compensator were used the model order would be increased by one.

The research in mathematical model reduction has been centered on the reduction of stable models, but many processes are open loop unstable systems. Hence, model reduction methods for unstable systems would be quite useful. The model reduction could be carried out by retaining the right half plane poles of the full model in the reduced model. The remaining model made up of the left half plane poles would be reduced using one of the conventional techniques.

The models heretofore considered have also all been linear-lumped parameter systems, but chemical processes are inherently nonlinear and many also are distributed parameter systems. Reduction techniques that allow for nonlinear and/or distributed parameter systems are needed. Cho and Joseph [17,18] have begun to investigate this area using collocation methods that have met with some success.

The application of reduced models has been very limited. The literature associated with model reduction has often been concerned with extolling the virtues of a given reduction technique with no regard to either the applications of the reduced model nor to the benefits of the given reduction technique to other methods of reduction. This work has concentrated on filling this void, but there are still questions in this area that remain unanswered.

Only a limited number of control structures have been applied with reduced models. A thorough investigation of other control structures (e. g., singular value decomposition and inverse Nyquist array) is warranted, as well as a comparative study of the design of the different controllers designed with reduced models.

Reduced models would facilitate the design of optimal controllers. They have been designed with reduced models [76,91,109], but no guidelines have been established in this area.

Several avenues of potential research which are not related to model reduction are also suggested as possible extensions to this work. Exploitation of internal balancing systems is one such area; an investigation into the feasibility of using the internally balanced model to answer the scaling problems associated with singular value decomposition controller design is needed [112].

Another area of research would be into the use of the continued fraction expansion for process identification. When Chen, Knox, and Shieh [10] originally presented the continued fraction expansion method for model reduction, they also suggested that the method may lend itself to the identifiction of physical systems. To date no research has been published in this area.

Guidelines have now been proposed for the use and calculation of reduced models. The second order modes of the full model establish a method by which the order of the reduced models are selected. By comparing and contrasting the different reduction techniques with example models, the successes and failures of control design using reduction methods have been illustrated. This knowledge is very useful for understanding the model reduction problem.

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