

**Evaluating the Effects and Social Validity of
Detect, Practice, and Repair for Three
Students with Emotional and Behavioral Disorders**

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DEDICATION

To my wife

Rachel Windler Gordon,

Your ongoing love and support made all of this a possibility for us.

My three sons

Benjamin Martin Gordon,

Noah Henry Gordon,

Joseph Robert Gordon,

You are more than my inspiration; my hope is that you learn as much from me as I have from you.

My parents

Robert L. and Sherry Gordon,

I am completing this PhD. because you first believed in me and supported my education all those years ago.

and family

Dr. Sam Spector and Virginia Spector

Thank you, Sam and Virginia, for showing us all how to be scholarly and most importantly that family comes first.

“What a nice family.” -Sam

Frank Windler and Julie Windler,

Thank you for your ongoing love and support throughout our lives and professional careers.

We appreciate everything you have done for us.

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ABSTRACT

Many students lack basic computational fluency skills that form the foundation for learning algebra in high school. This is particularly true for students with disabilities. Students with emotional and behavioral disorders (EBD) have experienced slower achievement growth rates throughout the pivotal middle grades when compared to other students with and without disabilities. Given the academic challenges for students with EBD, specifically in the area of mathematics, there is a need for researchers to identify and for practitioners to apply research-based best practice in academic intervention and remediation. The purpose of the current study is to examine the effects of a mathematics fluency intervention package, comprised four research-based subcomponents (i.e., taped problems, constant time delay, cover-copy-compare, and explicit timing), on multiplication fluency of middle school-aged student-participants with EBD. The goal of this research is to evaluate the effects and social significance of a modified version of the Detect, Practice, and Repair (DPR; mDPR) intervention in terms of providing students and teachers an acceptable means with which to remediate multiplication fluency deficits in middle school and high school settings. The proposed study integrates systematic instruction with technology and self-monitoring to create a personalized and independent learning activity for students. The proposed modified version of Detect, Practice, and Repair intervention is designed to be efficient for students and practical for teachers.

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CHAPTER ONE

THE SIGNIFICANCE OF THE PROBLEM

Mathematics achievement trends for students from the U.S. are of concern to policy-makers; only 40% of fourth-grade and 34% eighth-grade students perform at or above proficiency on the National Assessment of Educational Progress (NAEP) assessments of mathematics achievement (National Center for Education Statistics, 2017). These data suggest the majority of students in the U.S. have a below basic to basic understanding of mathematics. Just 47% of fourth-grade and 37% of eighth-grade students exceeded intermediate status on the Trends in International Mathematics and Science Study (TIMSS) international benchmark, meaning students from the U.S. lack basic facts and applied problem-solving skills (Provasnik et al., 2016). In the US, math achievement trends for students with disabilities are even more alarming. In 2015, NAEP math proficiency was attained by only 16% of fourth-grade and 9% of eighth-grade students with disabilities. Similar trends were noted for reading achievement (National Center for Education Statistics, 2017). Eckes and Swanado (2009) found that of the schools that failed to meet Annual Yearly Progress (AYP), the failure was most often attributed to the overall poor achievement scores of the students with disabilities subgroup.

Educational outcomes for students with emotional and behavioral disorders (EBD) are perhaps the most alarming. Compared to other disability subgroups, students with EBD are more than twice as likely to be suspended out of school, removed from the school, and/or placed in an alternative educational setting (U.S. Department of Education, 2015). Students with EBD are also more likely to drop out of school and become involved with the criminal justice system (U.S. Department of Health and Human Services, 1999; U.S. Department of Education, 2015; Wagner, Newman, Cameto, & Levine, 2005).

Within a traditional classroom environment it is not uncommon for students with EBD to engage in off-task behaviors that can be maintained by the avoid/escape function (Sprague, Sugai, & Walker, 1988). For students with EBD the interaction between social/behavioral problems and academic learning is complex and teacher behavior, as either antecedent or consequence, can significantly increase or decrease rates of student off-task behavior (Sutherland, Lewis-Palmer, Stichter, & Morgan, 2008). Carnine (1976) concluded that faster teacher presentation rates reduced student off-task behaviors. Teachers can also increase the probability of on-task behaviors by increasing rates of reinforcement for academically engaged behaviors (Myerson & Hale, 1984) and providing students with increased opportunities to respond (Menzies, Lane, Oakes, & Ennis, 2016).

Due to the unique academic and behavioral support needs associated with students with EBD, educational services are often delivered in small-group special education settings (Maggin, Wehby, Moore-Partin, Robertson, & Oliver, 2011). Students with EBD are more likely to receive educational services in self-contained classrooms, self-contained schools, and are less likely to respond to academic interventions (Lane, Wehby, Little, & Cooley, 2005). Wagner and Davis (2006) reported that approximately 33% of elementary and middle school students with EBD receive educational services in a special education setting, excluded from the least restrictive general education classrooms. By contrast, only 14% of elementary school students and 18% of middle school students with other disabilities, such as specific learning disabilities (SLD), are placed in special education settings (Wagner & Davis, 2006).

Students with EBD have cognitive similarities to students with SLD, yet students with EBD typically make less progress in literacy and mathematics achievement, are more likely to qualify for free/reduced lunch (i.e., low socio-economic status), and experience poor attendance

(Anderson, Kutash, & Duchnowski, 2001). Compared to other high incidence disability subgroups, students with EBD are more likely to drop out and demonstrate lower achievement in all academic areas, and this is especially true for mathematics (Reid, Gonzalez, Nordness, Trout, & Epstein, 2004). Although students with EBD typically have higher full-scale intelligence quotient scores compared to students with SLD, performance on tests of achievement are generally lower, and students with EBD typically make less achievement gains over time (Anderson et al., 2001; Mattison & Blader, 2013). Excluding students with intellectual disabilities, research has shown that students with emotional disturbance (ED) eligibility on average demonstrate the lowest scores in literacy (Schulte, Stevens, Elliott, Tindal, & Nese, 2016) and numeracy (Stevens, Schulte, Elliott, Nese, & Tindal, 2015) achievement when compared to other IDEA eligibility subgroups. Also, mathematics and reading achievement gaps between general education students and students with EBD emerge as early as kindergarten, widen through the seventh grade, and remain persistent throughout high school (Stevens et al., 2015).

Research in Math Trajectories for Students with Disabilities

The available mathematics achievement trajectories research for students with disabilities and/or at risk of developing a disability is sparse. One study (Kohli, Sullivan, Sadeh, & Zopluoglu, 2015) included data for students with a specific learning disability (SLD) in a comparison to students without disabilities. The researchers evaluated linear, quadratic, and piecewise linear mixed-effects models to examine mathematics achievement trajectories from kindergarten through eighth-grade for students with SLD and students without disabilities (Kohli et al., 2015). The sample ($N = 2150$) was drawn from Early Childhood Educational Study (ECLS-K) database. Scores from the ECLS-K Mathematics test, a computer-assisted adapted test

based on the NAEP 1996 framework, served as the math achievement measure (Reese, 1997). The researchers converted raw scores to Item Response Theory (IRT) scale scores and constructed linear, quadratic, and piecewise linear mixed-effects models to account for time in terms of developmental stages (i.e., time as an ordinal scale; e.g., grade levels: k-1, 2-4, 5-8 represent three stages of development) as opposed to annual years (i.e., time as an interval scale) throughout the longitudinal study.

Using the Akaike Information Criterion (AIC) model fit index (Akaike, 1987), Kohli and colleagues (2015) determined that the 2-level (i.e., time at level-1 and student SLD eligibility at level-2) piecewise linear mixed-effects model with unknown random knot locations to be the best fitting model compared to linear models and quadratic models (Kohli et al., 2015).

Authors used the piecewise linear mixed-effects model to identify the location of the knot (i.e., transition point between linear and curvilinear growth, and indicative of students' transitions between developmental stages) to occur in early fourth grade on average for students without disabilities, and after the fourth grade academic year on average for students with SLD. The knot represents a transitional point where the rate of growth changed from a linear functional form to a curvilinear, or quadratic functional form, as decelerated growth occurred throughout the middle grades. The piecewise model knot, the joining of linear and quadratic (e.g., logarithmic) functional forms, identifies a change brought on by students' transition through developmental stages of learning where students begin to apply basic math facts and develop more advanced problem-solving skills.

Findings from Kohli and colleagues (2015) also include the identification of a math achievement gap for students with SLD, compared to students without disabilities, and statistically significant differences in math trajectories between students with SLD and students

without disabilities. IRT scale scores trajectories demonstrated a similar shape; however, students with SLD performed below students without disabilities at kindergarten and the achievement gap widened through early fourth grade. On average students with SLD demonstrated a slower rate of growth from kindergarten through third grade and a late growth surge after the completion of fourth grade, when compared to students without disabilities, which indicates a later than typical transition to the next ordinal developmental stage. The achievement gap expanded during the elementary grades to remain relatively stable throughout the middle grades. The gap did not widen or narrow, yet it remained stable through the middle grades.

Given this math achievement gap, Kohli and colleagues (2015) urged policymakers to consider adjusting expectations for students identified with SLD to account for these differences in mathematics achievement. The authors report that students with SLD should not be expected to demonstrate the same growth rates as students without SLD. According to the authors, curricula for students with SLD should be match developmental readiness; curricula for students with SLD should focus on basic skill mastery prior to developing more advanced problem-solving applications. Findings from Kohli and colleagues (2015) are consistent with previous research by Shin, Davison, Long, Chan, and Heistad (2003): students who are performing lower at kindergarten, including students with SLD, tend to demonstrate slower growth patterns through elementary and middle school resulting in a widening of the achievement gap as higher-performing students demonstrate more accelerated trajectory during primary grades.

Judge and Watson (2011) modeled math achievement trajectories to evaluate achievement gaps from subgroups of students identified as having learning disability (SLD) during the elementary school years. They used a full maximum likelihood estimation 2-level hierarchical linear model (Bryk & Raudenbush, 1992) with time (level-1) nested within the

student (level-2). The level-2 variables included sociodemographic variables as well as disability eligibility membership. Subgroups were defined by identification of SLD during the early primary grades (i.e., early emerging SLD, kindergarten and first grade), the later primary grades (i.e., emerging SLD, second and third grades), the upper elementary grades (i.e., late emerging SLD, fourth and fifth grades), or children without a specific learning disability. The authors used the ECLS-K database to evaluate trajectory models. Achievement scores derived from cognitive tests in mathematics, taken during kindergarten, first, third, and fifth grades, were converted to IRT scale scores. Students identified in the early emerging subgroup demonstrated math achievement scores approximately seven points lower than other students that were identified for SLD in later grades. Emerging SLD students' mathematics scores were about six points lower. Late-emerging SLD students' mathematics scores were approximately five points lower. On average students from ethnic minority groups started kindergarten with achievement scores approximately two points lower than non-minority students. Results from Tukey post hoc comparisons indicated statistically significant group differences; all three SLD subgroups had statistically significantly lower SES scores compared to a non-SLD group.

Judge and Watson's (2011) analysis of linear growth models resulted in additional significant findings, including differences between subgroups of students with SLD. Although all students made math achievement gains, differences in growth patterns were apparent in the slope coefficient for students with early-emerging SLD (i.e., SLD emerging during kindergarten and first grades), for students with emerging SLD (i.e., SLD emerging during second and third grades), and for students with late-emerging SLD (i.e., SLD emerging during fourth and fifth grades). These data showed that all three SLD subgroups performed lower on math achievement measures and demonstrated a slower growth rate over time, compared to students without SLD.

The Math Gap and Students with EBD

In most published research for students with EBD, researchers have targeted social skills outcomes over academic skills, and most of the academic research has focused on reading outcomes (Wehby, Lane, & Falk, 2003). Mulcahy, Krezmien, and Travers (2016) reviewed mathematics literature, in the context of methodological standards (i.e., Council for Exceptional Children; CEC, 2014; Horner et al., 2005; Kratochwill et al., 2010), on math interventions for secondary students with EBD. Mulcahy and colleagues (2016) found that only 8 of 19 studies included in their review evaluated an academic outcome; the remaining studies focused on behavioral interventions during mathematics instructional time. In light of these findings, Mulcahy and colleagues (2016) argued for increased academic research for students with EBD.

Using a large statewide sample of ($N = 92,045$), Stevens and colleagues (2015) examined math achievement growth trajectories of students grouped into nine subgroups including students categorized into seven subgroups based upon IDEA eligibility (i.e., autism, emotional disturbance, hearing impairment, mild intellectual disability, health impairment, SLD, speech and language impairment) and two subgroups of students without disabilities (i.e., general education, academically/intellectually gifted) across the 5-year longitudinal study (i.e., third grade through seventh grade). Stevens and colleagues (2015) used achievement data from students' scores on state-mandated tests of achievement, the North Carolina End of Grade (EOG) Math Test. The authors modeled student growth using multilevel longitudinal analyses (Raudenbush & Bryk, 2002), full maximum likelihood estimation, and specification of model parameters as random effects.

To account for acceleration or deceleration in growth trajectories, Stevens and colleagues (2015) constructed models from both linear and quadratic functional forms (i.e., time and time

squared). The researchers first compared two fully unconditional (i.e., random effects) models for the full sample: linear and quadratic longitudinal level-1 model. The quadratic unconditional model proved to be a statistically better fit compared to the linear unconditional model confirming the curvilinear functional form of growth data. Comparisons were then drawn from two conditional 2-level models that added predictors to the previous unconditional models. The first conditional model included disability status, and the second conditional model (i.e., full model) included demographic variables such as ethnicity, SES, limited English, and parental education.

Results reported by Stevens and colleagues (2015) indicated that all nine subgroups of students demonstrated progress over time although year-to-year growth decelerated for all subgroups. Also, all seven IDEA eligibility subgroups, and academically/intellectually gifted subgroup, were statistically significantly different from the general education reference group at time third grade (i.e., intercept). In terms of growth (i.e., slope), students with ED eligibility were the greatest contrast from the general education subgroup; curvature in the growth function was -0.77 developmental scale score points for students with ED, compared to -0.56 developmental scale points from the general education student subgroup, which was statistically significant (Stevens et al., 2015).

Stevens and colleagues (2015) also examined growth in terms of effect sizes at transitions between grade levels by student subgroups. Students with ED demonstrated the greatest drop in terms of magnitude of year-to-year growth effect sizes. The year-to-year growth effect sized for students with ED decreased across elementary and middle school, from $d = 0.9$ during the third-to-fourth-grade transition to $d = 0.2$ during sixth-to-seventh-grade transition, compared to general education students, $d = 1.2$ during the third-to-fourth grade transition to $d = 0.5$, and

compared to students with SLD, $d = .8$ during the third-to-fourth grade transition to $d = 0.3$ during sixth-to-seventh-grade transition. This data confirms that students with ED demonstrated much slower growth rates throughout the middle grades when compared to other students without disabilities and other disability subgroups (e.g., SLD). Growth rate trajectories were curvilinear, with decelerated growth across grade levels over time. This decelerated pattern was most apparent among the students with ED subgroup and illustrated a widening achievement-growth gap through the middle grades for this population. Compared to all other subgroup of students with and without disabilities, students with ED were the slowest growing group from fifth grade through seventh grade. Schulte and Stevens (2016) have modeled similar growth trajectories with reading achievement scores among students with ED.

Given the academic challenges for student with EBD, specifically in the area of mathematics, there is a need for researchers to identify and practitioners to apply, research-based practices for academic interventions. The purpose of the current study is to examine the effects of a mathematics fluency intervention package, that consists of research-based subcomponents, on multiplication fluency of middle school-aged student-participants with EBD. The goal of this research is to evaluate the effects and acceptability of the Detect, Practice, and Repair (DPR) intervention, in terms of providing students and teachers a socially-valid means with which to remediate mathematics fluency deficits prior to high school. Access to twenty-first century jobs in a competitive and growing global economy depend on technology and mathematics skills (Zhao, 2009). The intervention used in the current study has been designed to help correct math fluency skill deficits before they negatively impact educational and economic opportunities for students with EBD at the secondary education level before transition to adulthood.

CHAPTER TWO

MATH FLUENCY INTERVENTIONS: A REVIEW OF LITERATURE

According to the National Mathematics Advisory Panel (2008), students lack basic computational fluency skills which form the foundation for learning algebra in high school. The term fluency refers to the speed and accuracy in which students respond to academic skills prompts (Shapiro, 2011), and it forms the foundation of learning-hierarchies (Haring & Eaton, 1978). For math facts (for addition, subtraction, multiplication, and division), digits correct per min (DC/M) scoring is typically used to measure fluency (Shinn, 1989). Identifying appropriate and effective mathematics interventions is a top priority for elementary and middle school educators and administrators (Gersten et al., 2009). Many interventions are implemented within the Response to Intervention (RTI) framework for students with learning disabilities or students at risk of developing a learning disability (Coddington, 2011). Typically, math fluency interventions are conducted in elementary school settings, ranging from 15-40 min per intervention session, although it is recommended that interventions at all grade levels include approximately 10 min each session to single-digit math fact drills (Gersten et al., 2009).

Mathematics is an area in which speed and accuracy are pivotal for building the foundation for future success in school (Shapiro, 2011, p. 237). Children who exhibit poor fluency often struggle with more challenging tasks in the hierarchy as too much attention, time, and cognitive efforts are used on retrieval of basic facts, which overloads working memory (Kane & Engle, 2000). When students respond with automaticity, cognitive effort is conserved, which can then be applied to more challenging tasks in the hierarchy. Math fluency is the cornerstone of math education and forms the foundation from which students build, develop, and apply to more advance math tasks (Skinner, Fletcher, & Hennington, 1996). However, students

with poor math fluency tend to avoid math tasks likely due to the perception of task difficulty and low rates of reinforcement (e.g., lacking immediate positive academic feedback). In addition, poor performance on math tasks can contribute to math anxiety (Cates & Rhymer, 2003) or even have a punitive effect for students with low fluency skills (Skinner, Pappas, & Davis, 2005). The National Council of Teachers of Math (NCTM, 2009) has underscored the importance of teaching fluency, although there is much debate in terms of identifying successful instructional interventions with which to facilitate this standard.

Evaluating Interventions by Learning Rate

It is difficult to evaluate fluency-instruction research without first noting the pedagogical differences between leading theories of education, publications, and authors. Constructivism and behaviorism represent counter educational philosophies (Poncy, McCallum, & Schmitt, 2010). The constructivist approach to learning (e.g. problem-based or discovery learning) is defined through individual learner's interpretation of experiences within a "subjective reality" (Von Glasserfeld, 1998). Namely, learning is viewed as a collection of covert cognitions, unique interpretations of experiences, and application of previous knowledge within a social environment. In this sense, learning is defined as uniquely constructed interpretations of social experiences within a subjective reality (Kirschner, Sweller, & Clark, 2006). The constructivist approach is student-centered, attributing teachers as guiders of discovery-based learning experiences (Jensen, 2006).

From a behaviorist perspective, learning is defined as a long-term change in behavior, the expansion of one's own behavioral repertoire (Williams, 1999). According to behavioral learning theory, students' learning can be defined in overt terms, observed, and objectively measured; however, not all behavioral change is learning, and learning can occur without an overt change in

behavior. A behaviorist teaching model is typically teacher-centered and facilitated through direct and explicit instruction that defines and targets specific skills which are measured, monitored, and taught toward a defined criterion outcome. Teaching methods are facilitated through modeling (Bandura, 1965, 1974) and feedback (i.e., reinforcement, arranging contingencies that bring about behavioral change; Skinner, 1953, 1963, 1974; Thorndike, 1927; Watson, 1930). Math fluency interventions rooted in constructivist learning theory (e.g. strategy-based instruction) have used more instructional time and led to slower learning rates (Poncy, Solomon, Duhon, Skinner, Moore, & Simons, 2015). The current review focuses on peer-reviewed, published research studies examining effects of fluency-based instructional interventions primarily aligned with behavioral approach to instruction.

Cumulative Instructional Time and Social Validity

Despite pedagogical debate, it is important to note the overall social significance and context validity of an intervention. Social validity is often considered the “acceptability, or consumer satisfaction” of an intervention (Gordon, Franklin, Scott, Thompson, & Moore, 2016; Kazdin, 1977; Wolf, 1978). In educational research, “consumers” would be defined as students (i.e., participants) teachers, school administrators, and perhaps parents and/or community members— anyone with direct contact to the intervention. For an intervention to be deemed “socially valid” “consumers” report that the intervention was beneficial and worth the time and resources spent during planning, development, and implementation. Several research-based social validity scales are available to assess behavioral interventions (Carter, 2007; Elliott & Treuting, 1991; Finn & Sladeczek, 2001; Lane, Robertson, & Wehby, 2002; Lane, Kalberg, Bruhn, Driscoll, Wehby, & Elliott, 2009; Martens & Witt 1982; Reimers & Wacker, 1988;

Reimers, Wacker, & Koepl, 1987; Witt & Elliott, 1985; Witt & Martens, 1983; Witt & Martens, 1984).

It is important to note the perspective from which interventions are evaluated. In the current review, learning rates are evaluated and discussed in terms of cumulative instructional time (CIT), not strictly number of intervention sessions, as session length varies widely across studies. According to Poncy and colleagues (2015), the social significance of academic interventions should consider and account for learning rates in terms of cumulative instructional time—the summation of instructional time participants spend during the intervention (Poncy, et al., 2015; Skinner, 2008). By defining learning as a function of instructional time, researchers can evaluate instructional interventions in meaningful and socially-valid ways, and practitioners can select intervention strategies that have a large social and academic impact in terms of students' and teachers' invested time. This is particularly important for older students (i.e., students in middle grades and above) whose math fluency is far behind that of students without disabilities. Spending much effort and time on interventions that produce little effect is counterintuitive to education in general, and teachers need to identify interventions that can be applied to immediately and efficiently address skill deficits (Skinner et al., 1996).

When comparing multiple academic interventions that produce a similar effect, the intervention that requires less time should be given preference (Cates, Skinner, Watson, Meadows, Weaver, & Jackson, 2003). Interventions that produce large effects in a relatively brief time (i.e., a continuous account of cumulative instructional time as opposed to a discrete number of sessions that vary according to time per session) would then be viewed as more socially valid. Poncy and colleagues (2015) recently conducted a meta-analysis of the math intervention literature and have called for researchers to report cumulative instructional time and

for practitioners to implement interventions that have demonstrated rapid learning rates in terms of cumulative instructional time.

Additional factors in determining the effectiveness of math fluency instruction include an account of curricular scope (e.g., number of math facts; Poncy et al., 2015). For example, effect sizes derived from a disproportionate number of items in the dependent variable over disproportionate cumulative instructional time will result in inoperative comparisons across intervention studies. Meaning, a meta-analysis comprised studies with limited curricular scope (e.g., a small number of math facts) and a small number of, but lengthy, sessions will result in an inflated effect size. In order to standardize comparisons across studies, researchers should report both curricular scope (e.g., number of multiplication fact problems completed: 6 versus 25 problems) and cumulative instructional time (i.e. a cumulative running record of total instructional minutes) to account for learning rate increases, such as digits correct per instructional min (DC/IM) (Poncy et al., 2015).

Upon review of the available studies that have included data for curricular scope and cumulative instructional time ($N = 24$), Poncy and colleagues (2015) identified four intervention strategies that have generated larger effect sizes when accounting for curricular scope and cumulative instructional time: cover-copy-compare (CCC); taped problems; explicit timing; and Detect, Practice and Repair (DPR). However, Detect, practice, and Repair, as well as explicit timing, were represented by only one study each. Incremental rehearsal, drill and practice, and comprehensive instruction were additional interventions included in Poncy et al.'s (2015) review, and although these intervention studies led to positive student outcomes, they were determined to be much less efficient when accounting for cumulative instructional time and curricular scope.

Effective Research-based Math Fluency Interventions

The development of best practice in education is informed through review of the published scientific literature (Stanovich & Stanovich, 2003). A search of peer-reviewed empirical studies published in academic scholarly journals revealed three commonly researched math fluency interventions: CCC (20 studies), taped problems (10 studies, 2 of which were comparisons with CCC), and explicit timing (9 studies) as well as DPR (an intervention that combines elements of taped problems, CCC, and explicit timing; 6 studies). Although few published studies report cumulative instructional time, CCC, taped problems, explicit timing, and DPR interventions all led to increased math fluency for study participants across the literature.

In addition, eight literature reviews that summarized and evaluated math fluency instructional interventions across the body of available literature were included as sources for the current review. Four reviews focused on outcomes for students with and without disabilities (Burns, Coddling, Boice, & Lukito, 2010; Coddling, Burns, & Lukito, 2011; Joseph, Konrad, Cates, Vajner, Eveleigh, & Fishley, 2012; Poncy et al., 2015), and four reviews focused on outcomes for students with disabilities (Hodge, Riccomini, Buford, & Herbst, 2006; Mulcahy, Krezmien, & Travers, 2016; Stocker & Kubina, 2016; Templeton, Neel, & Blood, 2008), three of these reviews were specific to students with EBD (Hodge et al., 2006; Mulcahy et al., 2016; Templeton et al., 2008). Two reviews were devoted specifically for CCC interventions (Joseph et al., 2012; Stocker & Kubina, 2016).

According to Poncy and colleagues' meta-analysis (2015), when accounting for curricular scope and cumulative instructional time, cover-copy-compare (CCC), taped problems, explicit timing, and Detect, Practice and Repair (DPR) have generated large effect sizes across the literature base. CCC, taped problems, explicit timing, and DPR are interventions designed to

occasion high rates of academic responding and limit wait time (i.e., down time) between antecedent and behavior. Previous researchers have suggested that faster teacher presentation rates reduce student off-task behavior (Carnine, 1976).

Cover-Copy-Compare

CCC is a self-management strategy that integrates repetition and practice with self-monitoring, leading to near errorless learning when implemented with high fidelity (Skinner et al., 1996; Skinner, McLaughlin, & Logan, 1997). CCC has been widely evaluated and regarded as a highly effective intervention for teaching spelling and mathematics skills (Joseph et al., 2012; Poncy et al., 2015). CCC is a low-tech, low-cost intervention designed to increase students' active and accurate academic responding which can directly contribute to increased learning rates and increased skill fluency (Berliner, 1984; Greenwood, Delquadri, & Hall, 1984; Skinner et al., 1997; Skinner et al., 2005).

CCC interventions are based on a set of rules and procedures defined by student responses to the presentation of a series of stimuli (i.e., models). In terms of mathematics instruction, the student is presented with a stimulus, a visual model of a math fact (e.g., $9 \times 5 = 45$). The student briefly examines the visual model, then covers it from view with his hand, or blank notecard. While the model is covered, the student quickly recreates it from memory, typically in written form. The student then uncovers the model to compare with the recreated version, this allows for immediate feedback on whether or not the student's written response was correct. Although there are several iterations of CCC procedures available in the literature, typically, when a student writes a correct response, he or she then progresses to the next prompt in the series (e.g., a list of math facts). When a student responds incorrectly, a positive practice error correction procedure is initiated where the student rehearses the correct model consecutive

times (i.e., error correction/positive practice, typically three to five times consecutively) to improve accuracy and to penalize incorrect responding (Skinner et al., 1997). The CCC method is then repeated for the duration of the activity (typically a math fact set of approximately 12 facts).

In most studies, student responses were in written behavioral topography; however, student responses have also included verbal, and sub-vocal (i.e., the student responded cognitively, silently). Researchers have also combined verbal and written responses (Bolich, Kavon, McLaughlin, Williams, & Urlacher, 1995). Both verbal responses (Skinner, Belfiore, Mace, Williams, & Johns, 1997; Skinner, Ford, & Yunker, 1991) and sub-vocal responses (i.e., cognitive CCC) (Skinner, Bamberg, Smith, & Powell, 1993) have been the most efficient, and arguably the most effective, iterations of the CCC when compared to written response. Verbal and sub-vocal topographies have an efficiency advantage over written responding as they require less effort and time (Zipf, 1949). Therefore, verbal and sub-vocal iterations of CCC allow for increased opportunities for students to respond (Stocker et al., 2016) and an efficient use of instructional time (Poncy et al., 2015).

CCC interventions have been used to increase accurate responding and/or math fluency (i.e., addition, subtraction, multiplication, division, and multi-skill or mixed probes) in the 21 empirical studies included in this review and across $N = 399$ students. Eighteen studies were conducted using single-subject design intervention. In six studies, researchers compared CCC with other math facts interventions including: explicit timing (Coddington et al., 2007), taped problems (Poncy, Skinner, & Jaspers, 2007; Poncy, Skinner, & McCallum, 2012), constant time delay (Reisener, Dufrene, Clark, Olmi, & Tingstrom, 2015), and constructivist methods (Mong

& Mong 2010; Poncy et al., 2010). To date, 12 published studies have included students with disabilities and/or behavioral support needs.

Of the intervention research included in this review (i.e., CCC, taped problems, and explicit timing), students with disabilities have been included more in CCC intervention research (Becker, McLaughlin, Weber, & Gower, 2009; Bolich, 1995; Ozaki, Williams, & McLaughlin, 1996; Cieslar, McLaughlin, & Derby, 2008; Poncy, Skinner, & Jaspers, 2007; Reisener, Dufrene, Clark, Olmi, & Tingstrom, 2015; Schrauben, & Dean, 2018; Skinner et al., 1993; Skinner, Belfiore et al., 1997; Skinner et al., 1991; Skinner, Turco, Beatty, & Rasavage, 1989; Stading, Williams, & McLaughlin, 1996). The remaining nine studies have either not reported or excluded students with disabilities and/or behavioral support needs (Coddling, Chan-Iannetts, Paul, & Lukito, 2009; Coddling, Eckert, Fanning, Shiyko, Solomon, & 2006; Coddling, Shiyko, Russo, Birch, Fanning, & Jaspen, 2007; Grafman & Cates, 2010; Lee & Tingstrom, 1994; Mong & Mong, 2010; Poncy et al., 2010; Poncy, Skinner, & McCallum, 2012; Skinner, Shapiro, Turco, Cole, & Brown, 1992;).

Across studies, student participation in this research includes students with EBD ($n = 15$ students), specific learning disability ($n = 4$ students), attention deficit hyper-activity disorder ($n = 2$ students), and intellectual disability or developmental delay ($n = 1$ student). Students described in research as EBD or behavior disorder (BD) have not necessarily met eligibility criteria to receive special education services for ED (Merrell & Walker, 2004). Students identified as having EBD often have coexisting conditions such as attention deficit hyperactivity disorder (ADHD) or conduct disorder (Cullinan & Epstein, 2001). Six published studies have evaluated CCC on math outcome variables for students with emotional and/or behavioral difficulties. Participants' ages have ranged from 9 to 16 years, and representative of the middle

grades, a notoriously challenging population in terms of finding scientifically-valid yet practical interventions for meeting academic and behavioral needs (Lane, 2007). Middle school-aged students are also known to experience low levels of motivation and engage in increased rates of problem behavior (Anderman & Maehr, 1994) notwithstanding increased academic rigors while preparing for high school algebra (Cavanagh, 2008). Researchers have addressed student motivation through integrating behavioral supports (e.g., token economy) with CCC math intervention for middle school students with ADHD (Bolich et al., 1995) and found to have little impact on academic performance, although the behavioral supports might have influenced task engagement.

Skinner and colleagues (1989) were the first to apply CCC to a math outcome (i.e., single-digit multiplication fact accuracy) for students with EBD. The authors used a multiple baseline across participants design (i.e., two fourth grade and two 10th grade students), all males with ED receiving educational services in an alternative setting at the time of the study. The intervention improved multiplication accuracy to 100% with very little instructional time, just 3 min of instruction per day for seven or less days (i.e., three to seven days, or 9-21 min of cumulative instruction; Poncy et al., 2015). All students met or exceeded mastery criteria (i.e., 40 DC/M at 90% accuracy). Instructional time was applied efficiently because students responded to each stimulus until just one accurate response was written, meaning CCC was implemented without an overcorrection procedure, which in turn contributed to the efficient use of instructional time. It is important to note curricular scope (Poncy et al., 2015). Here, only 10 items were included in each student's multiplication fact set. The application of CCC in this sense enhance learning rates to normative levels while reducing instructional time to less than 21 cumulative minutes.

In a subsequent study, Skinner and colleagues (1991) applied an adapted alternating treatments design to compare two variations of CCC across repeated math fluency measures (i.e. DC/M) for two males with EBD, 9 and 11 years of age. The authors compared a verbal-response version of CCC (VCCC), a written-response version (WCCC), and a no treatment condition on students' written math fluency performance (i.e., the assessment topography). The authors controlled for time across interventions, meaning student participated in verbal and written conditions in equal amounts of time. Due to the more efficient nature of verbal communication, students had twice as many learning trials during the verbal condition, which contributed to higher learning rates compared to WCCC (Skinner et., al. 1991). Findings demonstrated that VCCC was a more efficient intervention; both students participants generalized math fluency skills to written behavioral topography. The VCCC intervention was shown to improve written multiplication fluency; specifically, both students generalized multiplication fluency in verbal form to a more laborious topography (i.e., written expression).

In a follow-up study, Skinner, Belfiore, and colleagues (1997) used a multiphase alternating treatments design to examine the effects of VCCC and WCCC on multiplication fluency and accuracy for two students, 10- and 11-year-old males, receiving psychological and educational services in an alternative setting. The researchers compared students' written multiplication fluency performance across baseline, trials held constant, and time held constant phases. Findings suggest that even when assessment requires a written response, the efficiency of the verbal response topography led to increased opportunities to respond and a more efficient use of instructional time as VCCC contributed to higher learning rates as measured by both fluency and accuracy outcomes.

There have been several iterations of CCC interventions tested in the mathematics literature (Bolich et al, 1995; Ozaki et al., 1996) including iterations based on sub-vocal topography, or cognitive responding (Skinner et al., 1993). Related, Cieslar and colleagues (2008) used copy-first iteration of CCC to address math accuracy deficits for a 16-year-old male with EBD in a high school setting. The participating student also received special education services for SLD in reading, written language, and mathematics. The authors used a multiple baseline across behaviors (i.e., three sets of math facts comprised division facts and sums of fractions with like denominators. Although the student improved to 100% accuracy and $PND = 100\%$ across all sets, the curricular scope was relatively low (i.e., 6 problems per set). In this study, the student's computational fluency was not reported.

The success of CCC has largely been attributed to improved learning rates through increased opportunities to respond (Skinner, McLaughlin, & Logan, 1997; Skinner et al., 2005). During CCC interventions, students are actively engaged in ongoing practice while being provided with consistent and immediate feedback. Although overcorrection/positive practice procedures have been used to penalize inaccurate responding, the most efficient CCC interventions have omitted the overcorrection/positive practice and applied verbal response topography (Skinner et al., 1991).

Taped Problems

In comparison studies, researchers used adapted alternating treatments designs to evaluate the relative effectiveness of CCC and taped problems interventions on addition and subtraction fluency (Poncy et al., 2007; Poncy, Skinner, & McCallum, 2012). Results confirmed that both interventions were effective; however, the taped problems interventions have required even less instructional time which contributed to slightly higher learning rates. Like CCC, taped

problems interventions support students thorough providing immediate access to corrective feedback.

Taped problems interventions have been well represented across the literature base and regarded as highly effective interventions for increasing math (Poncy et al., 2015) and reading fluency outcomes (i.e., taped words intervention; Freeman & McLaughlin, 1984; Skinner & Shapiro, 1989). In terms of math instruction, taped problems interventions are implemented using prerecorded audio clips of math facts arranged in the same sequence as student worksheets (i.e., follow-along worksheets), where students attempt to “race against the tape” by accurately responding to the audio prompt prior to reveal (i.e., the tape-recorded voice tells them the answer). The audio tapes are constructed to provide an audible prompt set to a time-delay (e.g., 3-s delay). Essentially, students complete the worksheet while listening alongside the prerecorded audio. Students are instructed to work alongside the taped audio; however, in previous studies, researchers have typically not addressed procedural integrity in terms of student-participants working ahead of, or behind the tape. Despite potential procedural integrity concerns, taped problems interventions have been a highly effective instructional method (Poncy et al., 2015).

Taped problems are based on a response prompting strategy and can be applied with or without constant or variable time delay (i.e., constant time delay, progressive time delay, or simultaneous prompting). Constant time delay is a response prompting strategy that involves discrete trials based on a fixed response duration interval. On the initial trials the presentation of the target stimulus is immediately followed with the immediate delivery of a controlling prompt. On subsequent trials the presentation of the target stimulus is followed by a response duration set on a fixed time interval (e.g., 3-s delay) where students have opportunity to respond

independently prior to the presentation of the controlling prompt (Wolery et al., 1992). Students are instructed to respond only if they are confident they know the answer. Typically, the teacher scores student responses as: “unprompted anticipations,” “prompted correct,” or “prompted error.” With progressive time delay the interval is progressively increased over time as students build independence and skill mastery. Simultaneous prompting is a zero time delay, where both the target stimulus and controlling prompt are presented to the student simultaneously.

Taped problems interventions are designed to occasion high rates of accurate responding linked to immediate feedback (i.e., reinforcement). Most taped problems interventions have been facilitated through progressive or varying time delay (Aspiranti, Skinner, McCleary, & Cihak, 2011; Bliss, Skinner, McCallum, Saeker, Bryant, & Brown, 2010; McCallum, Skinner, & Hutchins, 2004; McCallum et al., 2006; Miller, Skinner, Gibby, Gaylon, & Meddows-Allen, 2011; Poncy, Skinner, & McCallum, 2012; Windingstad, Skinner, Rowland, Cardin, & Fearington, 2009). In addition, 2-s constant time delay (McCallum et al., 2010; McCallum, & Schmitt, 2011) and 4-s constant time delay (Poncy et al., 2007) procedures have been successfully used in research. Note, two of the three taped problems interventions that used constant time delay led to increased learning rates for student with moderate intellectual disability (McCallum, & Schmitt, 2011; Poncy et al., 2007).

Progressive time delays in taped problems research have typically included three phases, a brief time delay to promote accuracy, an extended time delay for practice, then a return to brief time delay (i.e., fluency training). Aspiranti and colleagues (2011) used group contingency with taped problems on a variable time delay (i.e., from 0-s to 2-s to 1-s delay) in a classroom of 20 first grade general education students to examine effects on addition fluency. Alternatively, Poncy, Jaspers, Hansmann, Bui, and Matthew (2015) discovered that a 0-s (i.e., simultaneous

prompting) procedure was more effective for increasing accurate responding, improving learning rates, and saving about 33% more instructional time when compared to a 2-s time-delay. Poncy, Jaspers and colleagues (2015) reported that a no time delay (i.e., simultaneous prompting) condition contributed to higher addition fluency learning rates for second grade students in their study.

Taped problems interventions have been successful, in part, due to the flexibility of time delay procedures to differentiate tasks to meet individual student needs (McCallum, Skinner, Turner, & Saecker, 2006; Poncy et al., 2007; Poncy, Skinner, & McCallum, 2012) which have contributed to efficient use of instructional time, averaging 49 min of cumulative instructional time per study according to Poncy et al's., meta-analysis (2015). Carnine (1976) argued slow instructional pacing (i.e., long wait times and slow teacher presentation rates) contributed to off-task behaviors which negatively impact learning rates.

In a recent meta-analysis Kleinert, Coddington, Minami, and Gould (2017) found that on average taped problems interventions used approximately 9.32 min of instructional time at 4.8 sessions per week for 16 sessions, accounting for two and one-half hours of cumulative instructional time across less than four weeks of instruction. According to Kleinert et al. (2017), across the literature base, taped problems interventions have led to large effect sizes as defined by both parametric and nonparametric statistics.

Eleven published taped problems intervention studies were included in the current review; two studies, as noted above, were comparisons with CCC. Six studies were implemented in a whole-class as single-subject design (Aspiranti et al., 2011; Miller et al., 2011; McCallum, Skinner, Turner, & Saecker, 2006; Poncy, Jaspers et al., 2015; Poncy et al., 2012; Windingstad et al., 2009). Four studies examined effects on individual students (Bliss et al., 2010; McCallum, &

Schmitt, 2011; McCallum, Skinner, & Hutchins, 2004; Poncy, Skinner, & Jaspers, 2007), and one study used between-subjects group design to examine effects across classes (McCallum, Schmitt, Schneider, Rezzetano, & Skinner, 2010). Taped problem interventions have shown to be effective across several math fluency measures, including addition, subtraction, multiplication, division and mixed addition and subtraction problems. Most of the research in this area has been for students without disabilities. As noted by Poncy et al., (2007), additional studies that include students with high incidence disabilities would be a contribution to the literature. Although McCallum and colleagues (2010) examined the impact of a group contingency on a taped problems intervention, there was no statistically significant difference between groups, taped problems, and taped problems with the addition of a group contingency.

Explicit Timing

Explicit timing is an antecedent-based math fluency building strategy that has resulted in increased rates of responding by providing repeated practice opportunities within a series of timed drills simply by alerting students to a time limit. During explicit timing interventions students are given a math facts probe, prompted to work independently, and timed for the duration of the intervention session. Explicit timing procedures have typically been based upon a series of timed 60-s simple computation drills (i.e., 1-step math fluency exercises such as single digit addition, subtraction, multiplication, or division).

Van Houten and Thompson (1976) were of the first to report results involving explicit timing as an independent variable. By timing second grade students across 30 consecutive 60-s math work sessions, the researchers found that participants completed more single digit addition and subtraction problems during the timed condition while not compromising accuracy. By simply modifying the learning environment to include a timer, participants more than doubled

their rate of responding while maintaining high levels of accuracy. The intervention was implemented with the whole class using a withdrawal-reversal design. The class more than doubled the number of problems completed during the timed condition, and levels immediately dropped when the timed condition was removed.

Explicit timing procedures have been tested in multiple research designs, including two single-subject design studies: an alternating treatments design (Rhymer, Dittmer, Skinner, & Jackson, 2000) and a multiple baseline design (Poncy, Duhon, Lee, & Key, 2010). Six additional studies used within-subjects group designs (Rhymer & Morgan, 2005; Rhymer, Skinner, Jackson, McNeill, Smith, & Jackson, 2002), between subjects designs (Gross, Duhon, Hansen, Rowland, Schutte, & Williams, 2014; Woodward, 2006), and mixed model designs (Duhon, House, Hastings, Poncy, & Solomon, 2015). Rhymer, Skinner, Jackson, McNeill, Smith, and Jackson (2002) used a 2 by 3 factorial ANOVA to examine two timing conditions (i.e., explicit timing and no timing condition) across forty-four sixth grade students' performance on three sets of math problems grouped by level of difficulty (i.e., one-step addition, three-step subtraction, and a complex multiplication task). The researchers found that explicit timing procedures were effective at increasing the number of problems completed per min but had little to no effect on accuracy (i.e., percent of completed problems correct).

Explicit timing has been identified as an effective intervention, leading to increased learning rates with relatively little cumulative instructional time invested (i.e., approximately 40 minutes) according to Poncy's (2015) meta-analysis; however, only one explicit timing study reported adequate details for which to measure cumulative instructional time (Poncy, Duhon et al., 2010). Thirty-one students with disabilities have participated in explicit timing research across the studies included in the current review. Most studies described student eligibility in

terms of SLD or special education service eligibility per individualized education program (IEP).

Students with high incidence disabilities have been included in this line of research, 15 students with SLD eligibility in math (Woodward, 2006), and 6 students with IEPs (Gross et al., 2014), and 10 “students with IEPs” (Gross, Duhon, Shutte, & Rowland, 2016). A limitation to this research is the lack of detailed reporting in terms of identifying eligibility criteria for special education services beyond “students with an IEP”. In the Woodward (2006) study, participating students met eligibility criteria for a specific learning disability in mathematics. However, despite the 4-week long intervention, participants in general were unable to meet mastery criteria. Study results indicated very little practical difference between group performance based on participation with strategy intervention combined with explicit timing or explicit timing alone. Other researchers have described student eligibility in more general terms. Gross et al. (2016) found explicit timing procedures that integrated independent group contingencies were more effective in increasing the addition fluency of first grade participants with IEPs compared to explicit timing alone or explicit timing with dependent group contingencies.

Explicit timing has been successful when applied as subcomponent as part of a larger intervention package, such as DPR (Poncy, Skinner, & O’ Mara, 2006). Duhon and colleagues (2015) found that the addition of goal setting and immediate feedback enhanced the effects of explicit timing on subtraction fluency of second graders without disabilities by gains of more than two standard deviations. Daily 120-s timed drills were implemented through web-based program. The program provided students with auditory immediate performance feedback (i.e., ding when correct, buzz when incorrect) which likely made a positive impact on learning (Duhon et al., 2015). In five of nine studies reviewed, explicit timing was included as a subcomponent in larger intervention packages. Rhymer and colleagues (2000) used chess clocks to combine a

timed flashcard drill with peer tutoring, corrective feedback, and overcorrection procedures (i.e., positive practice overcorrection- writing the problem and correct answer three times immediately following each incorrect response). Explicit timing procedures have been included with other methods, such as strategy instruction (Poncy, Duhon et al., 2010; Woodward, 2006), goal setting (Duhon et al., 2015; Gross et al., 2014), and group contingencies (Gross et al., 2016) as well as other multi-component interventions such as Detect, Practice, and Repair.

Detect Practice and Repair Intervention

DPR, developed by Poncy, Skinner, and O'Mara (2006), is a class-wide multicomponent intervention that has consistently resulted in increased math fluency for students ranging from third to eighth grade, 8 to 15 years of age. DPR has been evaluated in published research exclusively for testing effects on mathematics fluency, specifically subtraction (Poncy, Skinner, & O'Mara, 2006), multiplication (Musti-Rao & Plati, 2015; Parkhurst, Skinner, Yaw, Poncy, Adcock, & Luna, 2010), and division (Axtell, McCallum, Bell, & Poncy, 2009; Poncy, Skinner, & Axtell, 2010). DPR consists of four components: pre-assessment (Detect), instruction (Practice), post-assessment (Repair), and self-monitoring (student self-graphing). Although DPR does include a personalized assessment component that has addressed the idiosyncratic nature of learning basic mathematics skills, the intervention was designed for use in whole-class/large-group settings. DPR has led to increased levels of active and accurate, academic responding, which has been shown to increase learning rates (Skinner et al., 2005). Currently, there have been six published DPR studies.

Detect condition. During the Detect procedure students participate in a brief formative assessment designed to identify math facts on which they are fluent and math facts with which they are not fluent. Students are instructed to respond accurately and immediately (e.g., if known

respond accurately, if unknown immediately cross-out and skip) to each math fact prompts within a given time delay. Inaccurate responses and skipped facts are identified and practiced during the instructional condition (i.e., practice). Originally, and in most studies, the presentation of single-digit math facts was set to a 1.5s time delay. Essentially, students had slightly less than 1.5 seconds to respond to each presented fact (Poncy, 2006). However, authors of two studies have integrated technology by presenting facts through a *Microsoft* PowerPoint slideshow set to a 3-s time delay (Parkhurst et al., 2010; Musti-Rao & Plati, 2015). This adaptation likely improved treatment fidelity in terms of controlling for students working beyond the time delay. In other DPR research studies it was possible for students to work ahead and beyond the parameters of the 1.5s time delay, skip problems or perhaps resort to finger-counting strategies in response to non-fluent facts.

Learning mathematics is an idiosyncratic process. The Detect procedure was designed to, and intended for, students to identify only the math facts in need of remediation (i.e., non-fluent facts). The identified non-fluent facts were used during the practice procedure to generate a unique curriculum for each student. In most published studies the Detect procedure has taken less than 2 min per session while efficiently yielding much needed information through the identification of a personalized set of targeted non-fluent math facts for participants, thus eliminating the use of instructional time spent on practicing already fluent facts.

Practice condition. After students have completed the Detect procedure a direct instructional procedure (i.e., practice) is initiated. The practice component of DPR applies direct instruction through a Cover-Copy-Compare (CCC) method (Hansen, 1978). Using the first five skipped or inaccurate responses identified from the Detect procedure students first view the fact from a given model (i.e., answer key), cover the fact, with a card or by hand, and reproduce (i.e.,

copy) it from memory, then uncover to compare for accuracy. The method is repeated until each identified non-fluent fact (i.e., the first 5 facts from Detect) is practiced five consecutive times. The practice procedure (CCC) was designed to allow students to work quickly and independently as it occasions high rates of active and accurate academic responding, repetition, and immediate feedback. CCC was developed by Hansen (1978) for spelling instruction; however, the intervention procedures also have been applied to mathematics. Skinner, Turco, and colleagues (1989) were the first to successfully apply CCC to improve multiplication accuracy and increase learning rates for fourth grade and 10th grade students with EBD.

In their review of mathematics interventions for students with EBD, Hodge and colleagues (2006) noted that CCC has been a very effective intervention for increasing and maintaining high rates of accurate math fact responding (Skinner et al., 1993; Skinner et al., 1989) especially when a vocal response is included (Skinner et al., 1991). It is important to note that Skinner and colleagues (1991) implemented the CCC intervention over brief time (i.e., 21 min of cumulative instructional time across 7 sessions). Poncy et al. (2015) found CCC to be a highly effective intervention contributing to some of the highest learning rates in children in terms of cumulative instructional time.

Repair condition. The third component of DPR is the Repair procedure. The Repair procedure is a timed assessment that has varied across published studies from 30-s (Parkhurst et al., 2010), 60-s (Poncy et al., 2010; 2013; Musti-Rao, & Plati, 2016), or 120-s (Axtell et al., 2009; Poncy et al., 2006) “math sprints”. The “math sprint” has been based on explicit timing procedures that assess math fluency skill mastery across repeated measures of DC/M.

Upon completion of the Repair procedure, students are instructed to self-monitor progress with the intervention by self-tracking DC/M across baseline and intervention phases. In most

studies students were given a folder with a bar graph template stapled to the inside cover.

Students track progress throughout the intervention by recording DC/M and creating a bar graph to help them self-monitor their progress over number of sessions. In academic research for students with high-incidence disabilities, including students with EBD (Carr & Punzo, 1993), self-monitoring academic progress has been an effective way to improve academic accuracy and productivity as well as increase on-task behaviors (Mooney, Ryan, Uhing, Reid, & Epstein, 2005; Shimabukuro, Prater, Jenkins, & Edelen-Smith, 1999).

DPR Research

Researchers have examined the effects of DPR on mathematics fluency in large group or whole-class settings. Across studies, dependent variables have exclusively been basic fluency, including subtraction facts (Poncy et al., 2006), multiplication facts (Musti-Rao, & Plati, 2015; Parkhurst et al., 2010), division facts (Axtell et al., 2009; Poncy et al., 2010), or idiosyncratic mixed facts (Poncy et al., 2013). Students have ranged from 8 to 15 years of age. In most studies, student-participants have been elementary-aged students: third graders (Musti-Rao, & Plati, 2015; Poncy, Skinner, & Axtell, 2010; Poncy et al., 2006), fourth graders (Poncy, Fontenelle IV, & Skinner, 2013) and fifth graders (Parkhurst et al., 2010); another study included middle school-aged students (Axtell, McCallum, Bell, & Poncy, 2009). A total of 90 students have participated with DPR intervention research, 17 of whom have been students with a documented disability at the time of the study. These include: SLD ($n = 3$ students; Poncy et al., 2006), ($n = 11$ students; Axtell et al., 2009), ($n = 3$ students; Poncy et al., 2013). Other studies have included students described as “low achievers in math” ($n = 10$; Parkhurst et al., 2010 and $n = 12$; Musti-Rao & Plati, 2016).

Limitations and Gaps in DPR Research Literature

Students with EBD or students from low incidence disability subgroups have yet to participate in a published DPR research study. Most students have responded positively to DPR; however, according to the current review lower achieving students, students with limited skills, and students with disabilities have not had as much success with the intervention. Poncy and colleagues (2013) hypothesized that the response failure for students with learning disabilities can likely be attributed to procedural or curricular issues. For example, the time delay during Detect (i.e., 1.5-s) might be too rapid and result in inaccuracies in terms of students identifying and practicing only non-fluent facts (i.e., procedural) as intended; or perhaps the set size represents too large a ratio of non-fluent to fluent facts (i.e., curricular). Another possible explanation is that previous DPR research has been applied in group and whole-class settings where phase change decisions were made based upon group means, as individual students with disabilities may have needed more time to respond to the intervention. Further, DPR has been conducted only in group settings; individual student graphs and subsequent visual analysis of individual student data have not been reported by researchers.

By design, the purpose of previous DPR intervention research was for whole-class applications. DPR has typically been evaluated through multiple baseline or multiple probe across fact sets in a whole-class as a single-subject format, although research designs have varied to include AB empirical case study (Poncy, 2006), group design via random assignment (Axtell et al., 2009), and adapted alternating treatment design (Musti-Rao & Plati, 2015). The original DPR study was conducted through AB empirical case study, a non-experimental design that does not control for threats to internal validity such as history, maturation, and testing (Gast, 2009). By reporting intervention effects as whole-class, many of the nuances of single-subject designs

are lost; this would include visual analysis of individual participant data and phase change decision criteria for individual students.

A limitation to previous DPR research has been oversight in reporting comprehensive procedural fidelity data. Generally, authors have collected procedural fidelity on researcher protocol but have been unable to evaluate the procedural fidelity of individual student participants. This oversight can also be attributed to the whole-class design of DPR. For example, students have worked independently during Detect and practice conditions, yet researchers have been unable to collect procedural fidelity on the implementation procedures of individual students. Questions concerning procedural fidelity include: Have students responded accurately to timing procedures during Detect (i.e., tap-a-problem procedure)? Have students practiced identified problems correctly during CCC/practice procedure? Despite the use of a metronome to prompt response timing during Detect, there have been no guarantees that all student-participants were using a true 1.5-s response time. Except for Parkhurst and colleagues (2010) and Musti-Rao and Plati (2015), researchers have not controlled for students working at their own pace during DPR-Detect. Again, due to the group implementation of DPR, procedural fidelity has likely been a challenge in terms of teachers and researchers monitoring individual students in a whole-class setting.

Another limitation with previous literature includes a lack of a social validity reporting from students and teachers. In five of six studies on DPR, intervention procedures implemented by researchers not teachers. DPR implementation might be a laborious for teachers (Poncy, Fontenelle IV, & Skinner, 2013), there is little information available to evaluate this aspect of social validity. Also, little is known about treatment acceptability. Parkhurst and colleagues (2010) noted that students appeared to lose interest as the DPR intervention progressed. The

authors added a reinforcement procedure across the third and final set of facts during intervention, and they reported the reinforcement system was effective and improved student performance, participation, and engagement. DPR was implemented by a classroom teacher in only one study (Musti-Rao & Plati, 2015). Musti-Rao and Plati (2015) used the *Teacher Post-Intervention Acceptability and Importance of Effects Survey* (Lane & Beebe-Frankenberger, 2004) to assess treatment acceptability and inform social validity determination. This study tested the effects of two interventions, DPR against the Math Drills iPad application, on single digit multiplication skills of third-grade students. Musti-Rao and Plati (2015) reported that students, parents, and teachers preferred the Math Drills intervention, citing Math Drills to be more conducive with classroom culture, more motivating for students, and easier to use. Both interventions lead to mastery fluency levels; however, Math Drills was somewhat more effective and led to higher learning rates in this study.

Additional limitations to previous DPR research have been the low number of participants with disabilities. Although DPR studies have targeted students with basic mathematics skills deficits, students with disabilities have been included in three studies (Axtell et al., 2009; Poncy et al., 2013; Poncy et al., 2006). Just 17 of the 90 students included in this line of research had a documented disability at the time of the study. Musti-Rao and Plati (2015) conducted their study in a co-taught classroom, implemented as a collaboration between the general and special education teachers; however, none of the participating students had a disability at the time of the study. Also, only two studies included a maintenance phase (Poncy et al., 2013; Poncy, Skinner, & Axtell, 2010), and little is known about the long-term effects of the intervention for students in general, especially students with disabilities. To address these gaps in

the literature, the proposed study focuses on the effects of a modified DPR on multiplication fluency and skill maintenance for middle school-aged students with EBD.

Purpose of the Study

Students with behavioral, learning, and/or attention problems who might have had punitive educational experiences with mathematics in the past, can experience high levels of reinforcement in a relatively short time span through interventions such as CCC and taped problems (Skinner et al., 2005). Students with learning and behavioral difficulties often require additional opportunities to respond and teachers of students with EBD infrequently provide those opportunities (Sutherland & Wehby, 2001). For students with disabilities, increasing rates of responding has had the biggest impact on math fluency, this is evident throughout the intervention research summarized in the current review and especially true for CCC research (Stocker & Kubina, 2016). In studies discussed here, interventions can be modified to increase opportunities to respond; efficiency can also be increased by using more efficient behavioral topographies (i.e., sub-vocal and verbal topographies), even when assessments are in written form (Bolich, Kavon, McLaughlin, Williams, & Urlacher, 1995; Skinner et al., 1993; Skinner, Belfiore et al, 1997; Skinner et al, 1991). The interventions discussed in the current review hold promise, individually as well as collectively, as part of a multicomponent intervention such as DPR.

Due to the lack of evidence in the area of mathematics intervention research for middle and high school aged students with EBD (Mulcahy, Krezmien, & Travers, 2016), there is a need to address, with rigor, replication studies in order to build an informative literature base (Cook, 2014). Current research should address ways to integrate combinations of CCC, taped problems, and explicit timing procedures into multicomponent interventions designed to maximize the use

of instructional time and remediate skill deficits in time-efficient and socially valid ways. Interventions can be structured to promote time efficiency (i.e., maximizing the use of cumulative instructional time) and modified accordingly (i.e., verbal response topography) to provide students with adequate opportunities to respond actively and accurately to support increased learning rates. Interventions that include errorless or near errorless learning, such as CCC and taped problems iterations with 0-s delay (i.e., simultaneous prompting), contribute to improved learning environments as punitive consequences are removed from the learning experience (Skinner et al., 2005). Students with low fluency also benefit from interventions like CCC that include modeling (Coddington et al., 2007).

Benner, Kutash, Nelson, and Fischer (2013) have supported multi-tiered systems of academic support for students with EBD. The authors' recommendations for practice include the use of formative assessment, explicit instruction, guided practice, independent practice, and structuring lessons to give students a sense of quick instructional pacing with frequent transitions between activities. The modified version of the DPR intervention proposed in the following chapter, and inspired by Poncy and colleagues' research (2006), is well aligned with the instructional model proposed by Benner, Kutash, Nelson, and Fischer (2013), and integrates components of taped problems, CCC, and explicit timing procedures.

The purpose of the current study is to evaluate the effects of a DPR intervention on the multiplication fluency skills of secondary-aged (i.e., middle school and high school) students with EBD participating in academic interventions within a small-group setting.

Research Questions

- 1) Is a modified version of DPR (mDPR) an effective intervention for teaching or remediating multiplication fluency skills for students, secondary-aged, with disabilities currently placed in alternative school programs for students with EBD?
- 2) While working independently, can student participants follow the mDPR steps with a high degree of procedural fidelity throughout the entire intervention?
- 3) According to students is mDPR a socially-valid intervention?
- 4) Will students maintain fluency and accuracy skills approximately six weeks after the intervention?

CHAPTER THREE

METHOD

For the current study, a modified version of the DPR intervention was implemented through a multiple probe design across three participants. Modifications to DPR include the use of computer aided instruction to embed a constant time delay within the DPR Detect condition. An additional modification includes the use of a sub-vocal form of CCC during DPR practice condition.

Participants and Setting

Three students (13-17 years of age) participated in this research. At the time of the study, all three students were receiving special education services in an alternative program designed to support students with EBD. All three students had previously been placed in the alternative school setting by their respective IEP teams due to both behavioral and academic support needs. The IEP teams had made additional recommendations for all three students to attend summer school- extended school year (ESY). According to educational records, all three participants had a history of problem behaviors that interfere with typical instructional practice. The current study was conducted in a semi-private setting (i.e., school library) within an alternative summer school program located in a large urban school district in the Southeastern U.S. The school serves students in kindergarten through twelfth grade.

Three students from two different classrooms participated in the study. All three students received special education services in an alternative setting the academic year immediately preceding the current study. One student, Larry, received educational services in a self-contained special education middle school classroom. Two students, Eddy and Paul, received educational services in a self-contained special education high school classroom. Both classrooms were

designed to support academic, emotional, and behavioral support needs of students with disabilities. Students are placed in the ESY special education program as part of an IEP team decision to provide additional, and more comprehensive, support throughout the summer. All students were identified by their ESY classroom teachers for multiplication fluency deficits which inhibited problem-solving skills and overall performance with the middle school and high school mathematics curriculum (i.e., pre-algebra and algebra mathematics curriculum). The students were referred to the researcher by their classroom teachers for participation in the current study.

Eddy

At the time of the study Eddy was a 15-year-old male student receiving special education services for Autism Spectrum Disorder (ASD) comorbid with EBD. According to his special education case manager, Eddy had average communication skills but struggled mostly in mathematics. Eddy received special education services in all academic areas, 100% of the time (i.e., 35 hours per week), in a self-contained program with access to comprehensive therapeutic services including behavior modification, counseling, and specialized academic instruction in small groups. A Functional Behavior Assessment (FBA) and Behavior Intervention Plan (BIP) to help reduce escape-maintained behaviors and improve social skills and academic engagement, were an active part of his current IEP and ESY educational programming. Eddy's special education case manager reported that his adaptive behaviors, according to *Adaptive Behavior Assessment System*, Second Edition (Harrison & Oakland, 2006), were in low average range (i.e., conceptual 84, social 75, practical 75, composite 72). *Key Math-3 Diagnostic Assessment* (Connolly, 2007) placed Eddy at the 5.6 grade level for mathematics calculation. The case manager and school administrator were unsure if calculator accommodations, per IEP, were used

on the math calculation assessment. According to the *Wechsler's Intelligence Scale for Children, Fourth Edition* (WISC-IV; Wechsler, 2003), Eddy's scores were in the low average to average range (i.e., verbal 104, perceptual reasoning 84, working memory 99, processing speed 83, Full Scale Intelligence Quotient 91). Eddy completed ninth grade in spring prior to the study.

Larry

At the time of the study, Larry was a 13-year-old male receiving special education services for emotional disturbance (ED; primary) and Other Health Impairment (OHI) due to ADHD (secondary). Larry's special education case manager reported that he had strong communication and vocabulary skills but struggled with simple math tasks. Adaptive behavioral areas of difficulty, as noted by the special education case manager, included impulse control, distractibility, and intolerance of routines. *Key Math-3 Diagnostic Assessment* (Connolly, 2007) placed Larry at the 1.9 grade level for mathematics calculation. Larry received special education services in all academic areas 100% of the school day (i.e., 35 hours per week) in a self-contained program with access to comprehensive therapeutic services including behavior modification, counseling, and specialized academic instruction in small groups. At the time of the study, Larry's official FBA/BIP had lapsed. However, his classroom teacher continued to employ the use of a consequence-based behavior modification that gave Larry access to 10 minutes of game time contingent upon academic work completion and the use of appropriate social skills. Larry completed seventh grade in spring prior to the study.

Paul

At the time of the study, Paul was a 17-year-old male receiving special education services for ASD (primary) and OHI due to ADHD (secondary). Paul's special education case manager reported that he had near grade-level reading skills, was kind, but did not attend school regularly.

Key Math-3 Diagnostic Assessment (Connolly, 2007) placed Paul at the second-grade level for mathematics calculation. Paul received special education services in all academic areas 100% of the school day (i.e., 35 hours per week) in a self-contained program with access to comprehensive therapeutic services including behavior modification, counseling, and specialized academic instruction in small groups. Paul completed tenth grade in spring prior to the study.

Materials

A variety of low-tech and high-tech materials were used for mDPR intervention. Low-tech materials included: pencil, math fluency worksheets, procedural fidelity checklist, self-monitoring form (i.e., “myDPR” form for students to use for self-monitoring), and multiplication fact sheets (made from printed *Microsoft* PowerPoint slides). See Figure 1 and Figure 2 in Appendix B. High-tech materials needed for this intervention were: laptop computer with PowerPoint software, mobile phone with stopwatch application, and PowerPoint slideshows. The *Microsoft* PowerPoint slideshows were designed by the primary researcher to include multiplication facts based upon level of difficulty research (van der Ven, Straatemeier, Jansen, Kinkenbrg, & van der Mass, 2015). See Table 1 and Table 2 in Appendix A for a summary of multiplication facts level of difficulty based upon van der Ven and colleagues’ research (2015).

Given the nature of individual student participant skill level, and per classroom teacher recommendation, two slideshow presentations were used in the current study (i.e., one slide show for each fact set used; factors 2-6 and factors 6-9). The factors of six through nine slideshow presentation included the multiplication facts identified by van der Ven and colleagues’ (2015) research to be the most difficult. The factors of two through six slideshow presentation included the multiplication facts identified by van der Ven and colleagues’ (2015) research to be the least difficult. Per teacher recommendation, Eddy participated using the more

difficult fact set (i.e., factors 6-9); Larry and Paul participated using the less difficult fact set (i.e., factors 2-6). Both slideshows were set to a 3-s time delay, with 4-s buffers between slides so students have time to self-monitor performance using the “myDPR Math” form during the Detect condition.

According to Butterworth, Marchesini, and Girelli (2003), the larger factor is likely easier to recall from memory, therefore in effort to reduce problem sets to a manageable number of math prompts per set, the researcher accounted for the commutative property of multiplication during the Detect condition by consistently arranging math fact presentation by the larger factor first (e.g., 6×3 , not 3×6) as opposed to alternating presentation of the larger factor (Baroody, 1999). To account for skill generalization, prompts on the 60-s math fluency measures were randomly mixed between smaller and larger operands.

Design

Although reversal designs are known for robust experimental control and strong internal validity they are not appropriate for measuring functionally-irreversible behaviors, such as academic behaviors (Baer, Wolf, & Risley, 1968; Gast, 2010; Kratochwill et al., 2010). Multiple probe designs and multiple baseline designs are similar in that a functional relation can be established without withdrawing the intervention (Alberto & Troutman, 2009). Multiple baseline and multiple probe designs can be applied across behaviors or participants (Horner & Baer, 1978). Multiple probe designs allow for replication across participants while also controlling for facilitative practice effects when measuring functionally irreversible behaviors such as academic skills (Gast, 2010; Gall, Gall, & Borg, 2007). Multiple baseline and multiple probe designs differ only in the frequency with which pre-intervention data are collected. With multiple baseline designs, data are collected continuously; however, multiple probe designs accommodate

researchers in the collection of pre-intervention data intermittently (Gast, 2010). Multiple-probe designs are preferred over multiple-baseline designs for measuring academic behaviors as they offer an appropriate way to assess learner performance, and account for threats to internal validity while reducing time performing baseline tasks (Cuvo, 1979).

Once a trend of three consecutive fluency score data points at the instructional level fluency norms for students in fourth grade and above, and with accuracy at or above 90%, the intervention was ceased (Burns, Van Der Heyden, & Jiban, 2006; Deno & Mirkin, 1977). According to Deno and Mirkin (1977) an appropriate instructional criterion for students in fourth grade and above is 29 DC/M, whereas skill mastery criteria is 40 DC/M.

For multiple probe across participants designs, Gast (2010) recommended for researchers to set a priori criterion for introducing the independent variable to subsequent participants given that prior data has met the 80/20 stability envelope criterion. As defined by the lead researcher for the current study, intervention phase change decisions were determined through visual analysis, once a series of at least four of five data points of 100% *PEM* (Percent of data Exceeding the Median) was established. For example, when a 100% *PEM* across at least four of five fluency data points was established for the first participant, then the intervention was introduced to the second participant. Once a 100% *PEM* across at least four of five data points was established for the second participant, then the intervention was introduced to the third participant. The minimum standard in single-case design research for a multiple probe design is to include at least three data points in each phase for each participant (Kratochwill, 2012). The current design meets the minimum standard.

Dependent Variables

Two academic dependent variables were measured throughout this study: multiplication fluency (i.e., the speed in which multiplication facts are communicated correctly through written responses, *digits correct per minute; D/CM*) and multiplication fact accuracy (i.e., *percentage of correct responses*, the number of correct responses divided by total number of correct and incorrect responses). Multiplication fluency served as the primary dependent variable and multiplication accuracy served as the secondary dependent variable. Both dependent variables were measured during 60-s math fluency probes throughout baseline, intervention and maintenance phases.

Multiplication Fluency

Student responses were scored as DC/M in accordance with Deno and Mirkin (1977): number of the correct digits recorded/written in the correct place value. For example, $7 \times 4 = \underline{\quad}$; a response of 28 is scored as 2 digits correct whereas responses of 21 or 18 are scored as only 1 digit correct. The total number of correct digits was then divided by time in minutes. If the student completed the 35-item probe before the 60-s time limit, the time spent to complete the probe was used in the calculation of DC/M. For example, if the student completed the probe in 57 seconds (i.e., $\frac{57}{60}$ is .95 minute), then .95 was applied to the denominator of the DC/M formula. As a measure of fluency DC/M has been used consistently throughout the educational literature, and across operations and domains, to determine appropriate instructional leveling for students

with learning and behavior problems (Shapiro, 2011; Deno & Mirkin, 1977). Responses recorded after the prompt to stop were not included in the DC/M calculation.

Multiplication Accuracy

Percent correct was calculated by dividing the number of correct items by the number of correct and incorrect items from student performance on the multiplication probe during the 60-s multiplication fluency measures. Skipped items were counted as incorrect as students were instructed to work from left to right and respond to each problem as quickly as possible. Responses recorded after the prompt to stop were excluded from the accuracy calculation.

Baseline Procedures

The assessment procedures used during baseline, intervention, and maintenance were identical for all students. Refer to Figure 3 in Appendix B. During baseline, students participated in multiplication fluency probes (i.e., 60-s multiplication fluency measures) across at least three repeated measures (i.e., sessions). The multiplication fluency probes included six random variations of the single-digit multiplication facts from the multiplication sets noted by van der Ven and colleagues' (2015) research. See Table 1 and Table 2 in Appendix A. The lead researcher worked with students individually in a semi-private classroom setting. Using a stopwatch, the lead researcher timed students during the 60-s multiplication fluency measures. Each student was instructed to immediately stop working when prompted by the lead researcher to "stop" at the end of the 60-s time period. Digits correct per minute (DC/M) were then calculated for each student based on the method used by Deno and Mirkin (1977).

The math probes were created by the lead researcher through resources found on the Worksheet Genius website, (Worksheet Genius, 2019). The website allowed for specific facts to be included in the probes. See Figure 4 in Appendix B. Students received at random one of six

multiplication fluency probes each time the dependent variable was measured. Probes were numbered from 1 to 6. The lead researcher used a six-sided dice and controlled so the same probe was not used twice consecutively. Each multiplication fluency probe contained a unique random variation of 35 problems (i.e., five columns across seven rows) from the single-digit multiplication facts identified by van der Ven and colleagues' (2015) research as the most difficult (Eddy; factors 6-9) and less difficult (Larry and Paul; factors 2-5). For Eddy, the fluency measure included random arrangements of the multiplication facts identified by van der Ven and colleagues (2015) as the most difficult (i.e., 6, 7, 8, and 9) and excluded the factors 2, 3, 4, and 5 (i.e., a total of 18 facts in the set in terms of curricular scope). See Table 1 in Appendix A. Since Larry and Paul were not yet fluent with the more basic multiplication facts, per teacher recommendation their fact set included factors 2, 3, 4, 5, and 6 and excluded 7, 8, 9 (i.e., a total of 15 facts in the set; curricular scope). See Table 1 and Table 2 in Appendix A.

As recommended by Gast (2010), baseline data were collected on all participants prior to the introduction of the intervention to the first participant. When a stable baseline was established, defined by a 80/20 stability envelope criterion (i.e., 80% of the data points in the baseline series fall on or within $\pm 20\%$ range of the median) for a condition length of at least three consecutive repeated measures (Gast, 2010), the mDPR intervention was started for the first participant. Because means are influenced by extreme values, Gast recommends using the median for visual analysis and determining baseline stability. Once the intervention was introduced to the first participant, intermittent measurements (i.e., probe trials) were continued for subsequent participants. Probe trial data were collected at least weekly, but no more than twice in the same week, for subsequent participants.

Independent Variable

For the current study, the independent variable was a modified version of the DPR (mDPR) intervention described by Poncy et al. (2006). Similar to Parkhurst and colleagues (2010), the DPR intervention used in the current study was modified to include technology, via PowerPoint and a personal computer. However, the current study used the slideshow to integrate an embedded constant time delay procedure during the DPR Detect condition. Another modification to the current study included the use of sub-vocal responses (i.e., cognitive CCC; Skinner, Bamberg, Smith, & Powell, 1993) during the mDPR Practice procedure.

Modified Detect, Practice, and Repair

Research in mathematics instruction has emphasized the importance of providing clear and immediate feedback (Baker, Gersten, & Dae-Sik, 2002). However, all previous DPR research studies have withheld reinforcement (i.e., academic feedback) during the Detect condition; students were only given access to the answer key at the end of the Detect condition. In previous research the Detect condition was designed for assessment only and was not directly intended for instruction. Therefore, modifications to DPR for the current study include an embedded constant time delay during the DPR-Detect condition.

The current study was designed to make a more efficient use of the students' time by embedding an instructional component (i.e., 3-s constant time delay) during DPR-Detect. McCallum and colleagues (2004) used a progressive time delay procedure to facilitate a taped problems intervention which improved division fluency for a 10-year old male with a suspected learning disability. Parkhurst and colleagues (2010) and Musti-Rao and Plati (2015) applied a 3-s delay to DPR-Detect although the intent was not instructional, but strictly for formative assessment. In both studies the authors withheld reinforcement (i.e., the answer key) until

presentation of the final slide. The addition of an embedded constant time delay integrated an instructional component, like taped problems, to the mDPR-Detect condition which can lead to more efficient use of instructional time and contribute to higher learning rates.

For the current study, a 3-s constant time delay was embedded within the Detect condition and implemented similarly to taped problems interventions (McCallum, Skinner, & Hutchins, 2004) where students were instructed to respond to the prompt before the answer was provided (i.e., within 3-s delay). The advantage here is that students continue to engage with instruction even during mDPR Detect (i.e., assessment) condition so that instructional time is used more efficiently. Ongoing formative assessment and instruction can occur simultaneously with the current modified version of DPR. Because of the embedded 3-s constant time delay, the Detect condition simultaneously serves two purposes: systematic instruction and ongoing formative assessment which enables the learner to quickly discriminate between fluent and non-fluent facts, while fluent facts are immediately reinforced.

The mDPR intervention was implemented twice daily on Tuesdays, Wednesdays, and Thursdays (i.e., 6 times each school week across 5 weeks of ESY). The procedures supplemented each student's ongoing regular math instruction during ESY. Since the mDPR intervention is designed to be a unique and personalized learning program for each student, it was implemented at the individual student level and in an individualized semi-private setting in the school library. Immediately following each intervention session, students completed the 60-s multiplication fluency measure. The 60-s multiplication fluency measurement procedures were identical to the procedures used during baseline.

mDPR Detect. After completing the last 60-s math fluency measure during baseline for the first participant, or probe for subsequent participants, students immediately began the mDPR

intervention Detect condition. The mDPR-Detect condition consisted of a *Microsoft* Powerpoint slideshow presentation with an embedded 3-s constant time delay. The slideshow presentation was stratified according to problem difficulty and to avoid the presentation of naturally and consecutively occurring facts.

The slideshow functioned similarly to a timed flashcards drill. First the student was presented with a prompt (e.g., 9×8), followed by an automatic 3-s delay that was built into the slideshow animations and transitions. At the end of the 3-s delay, the student was presented automatically with academic feedback (i.e., the correct answer was automatically presented as part of the animated slide show advanced features; e.g., $9 \times 8 = 72$). The academic feedback was presented to the students in a visual format through advanced animation features. Students were instructed to avoid guessing and respond verbally to the prompt only when they knew the correct answer (i.e., errorless learning).

Similar to the taped problems intervention, the slideshow prompts and answers were controlled with a 3-s time delay before the correct answer was presented via the computer slideshow. In this way, the answer key (i.e., academic feedback; visual feedback) was naturally built into the presentation of each slide. The immediate academic feedback (i.e., reinforcement of an accurate response, or corrective feedback for an inaccurate response) adds an instructional component to traditional DPR-Detect. For each correct response, the student circled the green thumbs up icon on the *My DPR* form. See Figure 1 in Appendix B. If the fact was known (i.e., fluent), students were instructed to call out, verbalize their answer. If the fact was unknown (i.e., nonfluent), students were instructed to wait on the answer then circle the red thumbs down icon on their *my DPR* form. If the student responded incorrectly, he was instructed to circle the corresponding red thumbs down icon on the *my DPR* form.

Once the student had identified five incorrect (i.e., non-fluent) facts then he stopped the slideshow and immediately began the next phase, DPR-Practice (i.e., cognitive Cover-Copy-Compare; Sub-vocal CCC). The lead researcher also collected inter observer agreement (IOA) data on the accuracy of student self-monitoring (i.e., *My DPR* form). Student-to-researcher IOA on student self-monitoring was calculated as number of agreements (i.e., number of items scored correctly) divided by the number of agreements and number of disagreements. The student self-monitoring IOA data were used to evaluate procedural fidelity.

mDPR Practice. During the next condition (DPR-Practice) students practiced only the first 5 non-fluent facts identified during the Detect condition. As students increased fluency skills their respective 5-fact practice sets continually evolved to include an ongoing personalized set of non-fluent facts. Students used the math fact sheets (i.e., Figure 2 in Appendix B) during mDPR-Practice (i.e., cognitive Cover-Copy-Compare). The math facts sheets were printed copies of the individual *Microsoft* Powerpoint slides (i.e., math facts with correct products) for each of the multiplication facts included in this study. The math facts sheets were printed on cards that were approximately 3” in height by 4” at base. The student read the identified nonfluent fact subvocally, then covered the fact with his hand, then stated the complete fact subvocally (i.e., cognitive) to self while recalling from memory (e.g., “nine times seven is sixty-three”), then uncovered to check for accuracy with the correct model (i.e., math fact sheet). This process was repeated five consecutive times for each non-fluent fact ($n = 5$) identified during the DPR Detect condition. Refer to Figure 4 in Appendix B. At the end of the DPR Practice condition, the researcher used random verbal praise statements (e.g., “Good work. These are really tough problems.”). Refer to Figure 4 in Appendix B.

mDPR Repair. Immediately following the completion of mDPR Practice, students responded to math facts during a 60-s multiplication fluency measure. A digital timer was used and each student was instructed to immediately stop working when he was prompted to stop at the end of the 60-s time period. Responses recorded after the prompt to stop were not included in the DC/M or accuracy calculations. The mDPR Repair procedures were the same procedures used during baseline as the lead researcher followed a teaching script throughout baseline and intervention phases. See Figure 4 in Appendix B.

Maintenance

The maintenance phase procedures were the same as those used during baseline for each participant and consisted of a randomly drawn variation of the same set of six probes used during baseline, and mDPR intervention. Approximately six weeks after the mDPR intervention was stopped, the lead researcher returned to the alternative school site and collected multiplication probe data from Eddy and Paul. Due to absences, approximately seven weeks after the mDPR intervention was stopped, the lead researchers returned to the alternative school setting and collected multiplication probe data from Larry.

Procedural Fidelity

Training Session

Prior to the intervention, there was a brief individualized training session to teach each student how to use the slideshow, identify non-fluent multiplication facts (i.e., mDPR Detect), and practice non-fluent facts (i.e., mDPR Practice). During the training session, the procedural fidelity checklist (see Figure 5 in Appendix B) was used to monitor fidelity. Students participated in mDPR with a set of addition facts embedded into the slide show. To limit potentially confounding variables which might impact measurement of the dependent variable,

only the addition facts were included during the training session. The training sessions facilitated modeling, guided practice, and independent practice to teach intervention procedures to all student participants, individually. The intervention procedures were rehearsed until each student independently established a 100% fidelity score via the procedural fidelity checklist. In addition, the research assistant attended a training session and demonstrated understanding of the procedures by scoring a 100% fidelity score on the procedural fidelity checklist. A 100% score on the procedural fidelity checklist is contingent upon all procedural steps being conducted independently by the participant and in the correct sequence.

Procedural Fidelity of Student Self-Monitoring

All participants' self-monitoring behaviors were monitored throughout the intervention to observe the degree to which students could follow the intervention steps. Since DPR was designed to be an academic self-monitoring intervention, procedural fidelity of student self-monitoring was evaluated by the lead researcher on 100% of mDPR sessions. See Table 3 in Appendix A. When student self-monitoring fidelity was low, the lead researcher used the teaching script to re-teach the mDPR procedures. Refer to Figure 4 in Appendix B. Using the student self-monitoring procedural fidelity checklist, the graduate research assistant gathered IOA data with the lead researcher on observations of students' self-monitoring across eight of 34 mDPR sessions (i.e., 24%) across all participants. Researcher IOA on students self-monitoring observations (i.e., number of agreements/number of agreement and disagreements) ranged from 90% - 100%, with a mean of 94%, across the eight sessions.

Students' fidelity of self-monitoring scores are presented in Table 3 in Appendix A as a percentage of steps completed, means, standard deviations, and range for mDPR Detect, mDPR Practice, grand means across all sessions. Overall, students appeared to be accurate self-

monitors; however, the lead researcher noted concerns with the validity and reliability of measuring fidelity during mDPR Practice (i.e., subvocal/cognitive-CCC). Eddy's self-monitoring scores were consistently lower than Larry and Paul, as evident by grand means and standard deviations across sessions. Larry ($M = 99\%$, $SD = 3.7$) and Paul demonstrated higher and less variable fidelity scores ($M = 98\%$, $SD = 4.5$) than Eddy ($M = 79\%$, $SD = 16$). Larry and Paul scored consistently high across all sessions of the mDPR Detect and mDPR Practice conditions as well; however given the nature of overt behavior, researchers were unable to gather definitive evidence that all students were implementing mDPR Practice procedures (i.e., subvocal/cognitive-CCC) with fidelity.

Eddy demonstrated lower fidelity score means across all sessions and conditions of the intervention. During mDPR Detect, Eddy frequently scored his response as accurate (i.e., within the 3-s delay) on the *my DPR* self-monitoring form despite responding on or after the delay. During mDPR Detect Eddy stated several times that he was trying to “get to the end of the game (i.e., 3-s delay slideshow) with a 100% on the *my DPR* form used by the students for self-monitoring the mDPR Detect condition. Eddy's score during mDPR Detect condition was $M = 74\%$ fidelity, $SD = 19$, across all sessions. Due to this low fidelity of implementation, the mDPR intervention was retaught to Eddy five times during the study.

Inter-Scorer Agreement for Fluency and Accuracy

Multiplication fluency was scored using Deno and Mirkin's (1977) method for measuring DC/M. The lead researcher served as the primary scorer. A doctoral student and graduate research assistant served as the secondary scorer and assessed inter-scorer agreement on a random sample of 22% of all probes across all participants and phases of the experiment (i.e., baseline, intervention and maintenance). Performance on the 60-s multiplication fluency measure

was a permanent product with which to measure the primary and secondary dependent variables. Throughout the intervention the lead researcher scored all of the multiplication probes. At the close of the experiment, after maintenance data had been gathered, probes were drawn at random, across all participants and phases of the experiment, and scored by the graduate research assistant. Inter-scorer agreement was 100% on fluency and 100% accuracy measures.

Social Validity

Finn and Sladeczek (2001) claimed that best practice for social validity measurement should include multiple methods of investigating treatment acceptability. These methods include, but are not limited to, the use of questionnaires, semi structured interviews, and treatment acceptability evaluations. According to Finn and Sladeczek, students, parents, and facilitators (i.e., school personnel) should be involved in the determination of treatment acceptability. To evaluate treatment acceptability for the current study, researchers conducted a student survey based on an adapted version of the *Post-Intervention Acceptability & Outcome Survey*, a paper-based survey previously adapted from Lane, and Beebe-Frankenberger (2004) by the University of Nebraska, Lincoln, College of Education and Human Sciences, Center for Research on Children, Youth, and Families (CYFS, 2016). Upon completion of the study, all students individually participated in a brief social validity survey which included 14 items based on a 5-point Lickert-type scale and three open response items. See Figure 6 in Appendix B. Students completed a hard copy of the survey in the school library, the same place the intervention occurred and a semi-private setting at the school. To accommodate student participants, the lead researcher read aloud and explained the instructions and each item on the survey. The lead researcher recorded participant's verbal responses to all Lickert-type survey items and open

responses. Each participant's responses were recorded onto separate social validity student rater scale forms.

Cumulative Instructional Time

For interventions to be socially valid they must work efficiently (i.e., lead to increased learning rates within a minimal time frame). In addition to the student survey the primary researcher assessed the efficiency of the mDPR intervention through visual and statistical analysis of student performance (i.e., fluency and accuracy scores) during the mDPR intervention over time (CIT). In addition to DC/M fluency scores, learning rates were also converted to digits correct per instructional min (DC/IM) to account for cumulative instructional time (CIT; Poncy et al., 2015) and contribute to the overall assessment of the social significance of mDPR (i.e., social validity) in terms of maximizing the efficiency of instructional time for both teachers and students. Only time spent in the intervention factored into CIT. Time spent in baseline and training were omitted from the CIT calculation.

CHAPTER FOUR

RESULTS

For the current study the effects of mDPR were measured on two dependent variables via multiple probe design across three student participants. Multiplication fluency (i.e., DC/M) served as the primary dependent variable, and multiplication accuracy (i.e., percent correct) served as the secondary dependent variable. Multiplication fluency scores across phases for all three student participants are presented in Figure 7 in Appendix B. Multiplication accuracy scores across phases for all three student participants are presented in Figure 8 in Appendix B.

Dependent Variables

Multiplication Fluency

Throughout baseline, fluency scores were generally stable across participants, per Gast's (2010) stability envelope criterion; 80% of baseline data should fall within the stability envelope of +/- 20% of the median.

Eddy. Eddy's baseline fluency data demonstrated an ascending trend as evident by visual analysis and least squares linear regression equation ($y = a + bx$) fit line ($y = 12.33 + 1.5x$). Eddy's median fluency score during baseline was 15 DC/M; his scores ranged from 12 – 19 DC/M while the stability envelope ranged from 12 – 18 DC/M. Although only two of three data points (i.e., 67%) were within the stability envelope, the outlier was only 1 point (i.e., 19) from meeting the predetermined criterion (i.e., 80/20 stability envelope). Based upon recommendations from his special education case manager and ESY teachers, and concerns regarding inconsistencies for the other two participants' attendance, the lead researcher decided to first introduce the intervention to Eddy.

Immediately after the presentation of the independent variable, Eddy's multiplication fluency scores increased; however, during intervention Eddy's fluency scores varied widely ranging from 12 – 36 DC/M, with $Mdn = 28.5$ DC/M. During session 7 (i.e., the 4th mDPR session) Eddy scored 12 DC/M which was his lowest score during mDPR and does overlap with the baseline range. On this day his teachers reported that he had a “rough morning” and was emotionally dealing with personal matters stemming from home. Despite high variability in Eddy's fluency data during mDPR, this data point was the only overlapping fluency data point during mDPR. Throughout the mDPR intervention Eddy's fluency data demonstrated increased intercept, compared to baseline, and ascending trend as evident by visual analysis and least squares regression equation fit line ($y = 23.55 + .46x$). According to Deno and Mirkin (1977) the minimum instructional level for students at fourth grade and above is 29 DC/M. During the mDPR intervention Eddy met this criterion on 8 of 16 occasions. The predetermined criterion set to close the mDPR intervention was three consecutive data points at the instructional level (i.e., 29 DC/M or above) with 90% or above accuracy. Eddy met this criterion on session 19 (i.e., mDPR session 15). Eddy maintained this level of fluency scoring 28 DC/M six weeks after the completing of the intervention.

Ma (2006) claimed that Percent Exceeding the Median (*PEM*) effect size calculation is more closely related to authors' judgement and visual analysis and perhaps a more accurate measure of magnitude as it is less prone to outliers in the data. Although limitations with interpretations of *PEM* are apparent (i.e., *PEM* does not account for trend, and 100% *PEM* can be achieved despite there being little difference between baseline and intervention data), researchers have suggested the use of *PEM* for measuring the magnitude of effect for single-case designs (Ma, 2006). Fluency score effect sizes in terms of *PEM* might be a more appropriate

index for determining magnitude of effect for academic interventions. In terms of fluency effect size calculation for Eddy, 15 of 16 data points (i.e., 94%) exceeded the baseline median of 15 DC/M ($PEM = 94\%$). Ninety-four percent PEM is considered to be a large magnitude of effect. Eddy demonstrated skill maintenance by scoring 28 DC/M six weeks after the close of the intervention.

Larry. Larry's median fluency score during baseline was 17 DC/M; his scores ranged from 13 – 20 DC/M, while the stability envelope ranged from 13.6 – 20.4 DC/M. All five data points (i.e., 100%) were within the stability envelope. Baseline trend was decreasing as evident by visual analysis and least square linear regression equation fit line ($y = 20.7 - 1.5x$). There was overlap across the first three fluency scores during mDPR. Both the lead researcher and graduate research assistant observed limitations in Larry's level of engagement (i.e., lack of enthusiasm with the mDPR intervention procedures). Larry stated: "This is boring," and "I expected this to be more like a game" (i.e., video game). Although Larry's fidelity scores were consistently high throughout the mDPR intervention, his initial data did not appear to illustrate increased learning rates.

After a brief meeting with Larry's ESY teacher it was decided that his participation with mDPR would be reinforced via a continuation of his ESY behavioral programming which allowed Larry to earn video game time upon compliance with school expectations and participation with academic instruction. Just prior to session 12 (i.e., mDPR session 4) the ESY teacher and lead researcher met with Larry to explain the reinforcement procedures and offer this contingency. Larry stated that he liked working for video game time and he was willing to participate in the "computer math" research given the opportunity to earn game time as provided by his current ESY classroom programming.

Larry was presented with a behavioral contingency which allowed him to earn 10 minutes of game time on his portable gaming system upon completion of the mDPR session and he verbally accepted the contingency. It is important to note, Larry's current educational programming provided an escape-maintained behavioral contingency allowing him to work for brief periods of time in exchange for game time. This was the same contingency in place for Larry in his ESY classroom; however, in the classroom he typically had to complete a worksheet and/or read an excerpt from a textbook which took significantly longer than mDPR. His teacher reported that he typically would work for 20 – 30 minutes to earn 10 minutes of game time during ESY. This contingency might have offered too much pay off to Larry. Researchers and teachers should have considered a random variable ratio schedule, or a performance-based contingency. Although, augmenting mDPR to include a performance-based contingency would have compromised the internal validity of the current design by facilitating the influence of potentially confounding variables on the dependent variables. There is a noticeable surge in Larry's fluency scores immediately after this contingency was initiated; however, this surge only lasted for three sessions before a week-long Independence Day holiday break.

Larry's median fluency score during intervention was 18 DC/M; his scores ranged from 10 – 27 DC/M, with the last two scores being the lowest (i.e., 12 DC/M and 10 DC/M). Larry's fluency scores were highly variable during mDPR. Larry's fluency data demonstrated descending trend line ($y = 20.81 - .34x$). Larry's fluency scores never met the predetermined criterion to introduce mDPR to the final participant (i.e., when a 100% *PEM* across at least four of five stable data points was established); however, the intervention was introduced to the final participant (i.e., Paul) in effort to be ethically responsible with this research project. Also, during the intervention, the lead researcher and graduate research assistant shared concerns about the

use of cognitive-CCC and whether or not Larry was actually participating in cognitive-CCC or if he was merely “going through the motions” to obtain the game time. Cognitive-CCC might not have been an appropriate iteration of CCC for Larry. Larry might have needed to practice multiplication facts with an overt behavioral topography such as traditional CCC, or verbal CCC. There appears to be very little difference in Larry’s baseline and mDPR fluency scores. For Larry, seven of 13 fluency measure data points exceeded the baseline median of 17 DC/M (*PEM* = 54%) and considered a low magnitude of effect. Larry did not reach the predetermined criterion. Larry’s maintenance fluency data do not appear to be different from measures obtained during baseline. Although Larry did not meet the predetermined criterion for mDPR, approximately seven weeks after the close of the intervention, he scored 17 DC/M on the 60-s multiplication fluency probe. This score was one point lower than baseline median.

Paul. Paul’s median fluency score during baseline was 14.5 DC/M; his scores ranged from 12 – 19 DC/M, while the stability envelope ranged from 11.6 – 17.4 DC/M. Five of six data points (i.e., 83%) were within the stability envelope. Paul’s baseline trend was also slightly decreasing as evident by visual analysis and least square regression equation fit line ($y = 17.07 - .69x$). Considering the slight overlap during the first mDPR session, there is an increase in Paul’s fluency scores immediately after the presentation of the independent variable, across session 19 and 20 (i.e., mDPR session 1 and 2) which occurred on a Thursday; ESY sessions were Tuesday – Thursday each week. Paul’s median fluency score during mDPR was 18 DC/M; his scores ranged from 16 – 26 DC/M. Paul’s attendance was inconsistent throughout baseline/probe, and intervention. The mDPR intervention was introduced to Paul during the last two weeks of ESY, in which there was a long break between sessions 20 and 21 (i.e., mDPR sessions 3, 4, and 5 occurred after a 5-day weekend break). There is a decrease in Paul’s fluency scores during that

time. Paul's fluency data during mDPR demonstrated a descending trend line ($y = 22.4 - x$), although intercept increased compared to baseline. Paul participated in five mDPR sessions; additional session might have improved this trend line. For Paul five of five data fluency measure points during intervention ($PEM = 100\%$) exceeded the baseline median of 14.5 DC/M. Paul did not reach the predetermined criterion, and per visual analysis, Paul's maintenance phase fluency data point does not appear to be different from measures obtained during baseline. Paul scored 15 DC/M approximately 6 weeks after the close of the mDPR intervention while his baseline median score was 14.5 DC/M.

Multiplication Accuracy

On the measures of the secondary dependent variable (i.e., multiplication accuracy), baseline stability was achieved for two of three participants (Larry and Paul). Although phase change decisions were based on the primary dependent variable (i.e, multiplication fluency), multiplication accuracy was a significant part of the research questions asked via the current study. If fluency increases but results in lower accuracy, then the validity of the intervention is called into question. For the current study, two students demonstrated improvements in fluency and two students demonstrated improvement in accuracy. Eddy showed improvement in both fluency and accuracy.

Eddy. Eddy's median accuracy score during baseline was 60% correct. His scores ranged from 43% – 86% during baseline while the stability envelope ranged from 48% – 72% correct. Only one data point fell within the stability envelope. During baseline Eddy's accuracy data demonstrated a descending trend line ($y = 89 - 13x$) while his mDPR accuracy data demonstrated an ascending trend ($y = 69 + 1.09x$) which suggests the mDPR intervention contributed to Eddy's learning. However, Eddy's accuracy data is highly variable. For Eddy, 14 of 16 ($PEM = 88\%$)

accuracy measure data points exceeded baseline median of 60% correct and is considered to be a large effect size.

During mDPR Eddy's median accuracy score was $Mdn = 82.5\%$ correct as scores ranged from 33 -100% correct. The median difference between phases (i.e., median value of baseline data subtracted from the median value of adjacent intervention data) was $Mdn\ Diff = 22.5\%$ correct which suggests that the mDPR intervention contributed to increased multiplication accuracy learning rates in multiplication accuracy for Eddy. During sessions 11 and 12 (i.e., mDPR sessions 8 and 9, the lead researcher noticed that Eddy appeared hurried and was working through the 60-s multiplication fluency measure much faster, but also making more errors than usual. When asked, Eddy stated that he was trying to "finish the worksheet before the timer ended," which might have contributed to errors. In Addition to the lead researcher routinely reading the teaching script immediately before all 60-s multiplication probes throughout the study (baseline, mDPR, and maintenance), after session 12 and again just prior sessions 13 and 14, the lead researcher reread the 60-s math fluency measure instructions from the teaching script to Eddy for clarification. Additionally, the lead researcher retaught mDPR procedures (i.e., modeling and practicing) with Eddy five times throughout the intervention.

Approximately 6 weeks after the close of mDPR intervention, Eddy's fluency and accuracy data illustrate maintenance (i.e., 28 DC/M at 93% correct) one point from the instructional level defined by Deno and Mirkin (1977).

Larry. Larry's accuracy baseline data demonstrated an ascending trend ($y = 61.7 + x$). Median accuracy score during baseline was $Mdn = 67\%$ correct; his scores ranged from 41% – 69% correct during baseline while the stability envelope ranged from 53.6% – 80.4% correct. Four of five data points (i.e., 80%) were within the stability envelope. During mDPR Larry's

median accuracy score was $Mdn = 74.5\%$ correct while scores ranged from 56- 94% correct. Median difference between phases was $Mdn\ Diff = 7.5\%$ correct as the mDPR intervention appeared to contribute to slightly higher learning rates in multiplication accuracy for Larry. Larry's mDPR accuracy data demonstrates a slight descending trend ($y = 75.42 - .15x$). For Larry, 9 of 13 ($PEM = 69\%$) accuracy measure data points exceeded the baseline median of 67% correct and considered to be a moderate magnitude of effect. Larry's data did not reach the predetermined criterion for the current study, and per visual analysis accuracy data gathered during mDPR appear to be slightly higher than measures obtained during baseline.

Approximately seven weeks after the close of the mDPR intervention, Larry scored 77% correct on the maintenance probe. This score was 10 points higher than the median baseline score of 67% correct and does demonstrate a slight increase in multiplication accuracy learning rate as well as skills maintenance.

Paul. Paul's median score was $Mdn = 71\%$ correct during baseline; his scores ranged from 38% - 83% correct while the stability envelope ranged from 56.8% - 85.2% correct. Five of six data points (i.e., 83%) were within the stability envelope. Paul's baseline accuracy data demonstrated a slight descending trend ($y = 70.3 - 1.11x$). During mDPR Paul's accuracy scores ranged from 67% - 83% correct, with $Mdn = 71$. Paul's mDPR accuracy data demonstrate an ascending trend ($y = 70.3 + 1.1x$). Note, comparing baseline to mDPR, there was no difference between intercept and slope for Paul's data. Since fluency was the primary dependent variable, phase change decisions were based on the fluency baseline stability. For Paul, two of five ($PEM = 40\%$) accuracy measure data points exceeded the median of 71% correct. Paul did not reach the predetermined criterion, and per visual analysis, accuracy data do not appear to be different from measures obtained during baseline. However, approximately 6 weeks after the close of the

mDPR intervention Paul scored 75 percent correct during the maintenance probe. This score was 4 points higher than the median baseline score of 71 percent correct.

Social Validity

Social Validity Student Rater Scale and Survey

Results of the social validity student rater scale are presented in Table 4, Appendix A. Participant responses to the Lickert-scale items indicate that Paul and Eddy reported generally more approval of the intervention procedures compared to Larry. Participant mean scores were higher for Paul ($M = 4.29$, $SD = .83$) and Eddy ($M = 3.93$, $SD = 1.64$) compared to Larry ($M = 2.96$, $SD = 1.45$), and Paul's responses were less variable compared to his peers.

Across participants, the *most* favorable responses (i.e., item mean scores from 4 - 4.67 across all participants) reported were for items: "I like learning math this way," "My classmates would like learning math this way," "The computer would help me learn other academic skills," "Computer math and practice was fun," "The steps were easy," "I like trying to be faster than the computer," and "I liked practicing math facts by using the cover, copy, and compare technique". The *least* favorable responses (i.e., item means scores less than 3 across all participants) included items: "I would do computer math and practice on my own," and "Computer math and practice would help me learn other facts"- only Paul rated these items "Agree" or higher.

Larry rated the intervention *least* favorably compared to his peers. Larry's disagreements with specific survey items included: "Computer math and practice would help my friends learn math," "I like trying to be faster than the computer,". Specifically, Larry's strong disagreements were: "I liked the 1-munite math worksheets," "I would do computer math and practice on my own," and "Computer math and practice would help me learn other math facts". Larry reported agreement with "The computer would help me learn other academic skills," "Computer math and

practice was fun,”. Larry reported strong agreement with “The steps were easy,” and “I would do computer math and practice again”. However, both the lead researcher and the graduate assistant had concerns about Larry’s engagement with mDPR Practice procedures and that Larry might have gone through the motions of cognitive-CCC but had not actually attended to task. As previously discussed, the fidelity measure used during mDPR Practice might not have been a valid for Larry during this study.

Eddy strongly disagreed with: “I would do computer math and practice on my own,” “Computer math and practice would help me learn other facts” and “I would do computer math and practice again”. Paul was neutral about: “Computer math and practice helped me learn” and “My classmates would like learning math this way”. Paul agreed or strongly agreed with all other social validity items. Paul participated with mDPR for only five sessions.

Open response items included three questions. To the first question, “What was the most helpful part about computer math and practice?” Larry stated, “getting faster on ones I already knew,” and “playing my games” (i.e., video games as a participation incentive currently implemented in his ESY classroom and extended to mDPR during session 12). Paul stated, “I like the practice and computer race- it helps with reflexes and playing games”. When asked by the lead researcher, “Do you mean video games?” Paul nodded his head then stated, “yes”.

For the second question, “What was the least helpful part of computer math and practice?” Eddy stated, “practicing the problems I got red”. Larry stated, “math practice and 1-minute worksheets”. Paul stated, “the speed and wrong answers”. For the third and final; question, “What would you change about computer math and practice, and why?” Larry stated, “more gaming features and free-play options”. Paul stated, “more than three seconds” (i.e., 3-s delay between prompt and reveal during mDPR Detect).

Cumulative Instructional Time

CIT, and curricular scope data for each participant are presented in Table 5, Appendix A. Accuracy and fluency learning rate data in terms of CIT are presented in Figure 9 (Eddy), 10 (Larry), and 11 (Paul) in Appendix B. Additionally, fluency learning rate data are presented in terms of median difference between phases over CIT and standardized to digits correct per instructional minute (DCIM) as a measure of learning rate for each participant. See Table 5 in Appendix A.

Eddy met the predetermined criterion (i.e., three consecutive data points at 29 DC/M or above and with 90% accuracy or above) to stop the intervention at 90.68 minutes CIT. However, Eddy did participate in one more mDPR session on the following morning. This additional session was an oversight by the lead researcher. With the addition of this last mDPR session, Eddy spent 95.37 minutes of instruction across 16 mDPR sessions. Note the shape of the multiplication accuracy and fluency line graphs which visually illustrate the covariance between Eddy's fluency and accuracy scores during the mDPR intervention. See Figure 9 in Appendix B. For Eddy, mDPR averaged 5.96 minutes per session.

Larry continued to participate in the mDPR intervention until the last week of ESY. Larry participated in mDPR for 55.6 minutes CIT across 13 mDPR sessions. Despite little change in fluency scores, Larry's accuracy appeared to improve slightly during mDPR. The mDPR intervention was introduced to Paul last, as he had access to five mDPR sessions. Note the shape of the multiplication accuracy and fluency line graphs and the apparent covariance between Larry's fluency and accuracy scores during the mDPR intervention. See figure 11 in Appendix B.

Paul spent 19.48 minutes of CIT across 5 mDPR sessions. For Larry, mDPR averaged 4.27 minutes per session. Although Paul's accuracy data suggests the mDPR intervention had no effect on multiplication accuracy, fluency scores increased slightly across five mDPR sessions. For Paul, mDPR averaged about 3.9 minutes per session. See Table 5, Appendix A. Similar to Eddy and Larry, the shape of the multiplication accuracy and fluency line graphs demonstrate an apparent covariance between Paul's fluency and accuracy scores during the mDPR intervention. See figure 12 in Appendix B.

CHAPTER FIVE

DISCUSSION

Researchers have examined the effects of DPR on math fluency skills across a whole-class setting (Axtell et al., 2009; Musti-Rao & Plati, 2016, Parkhurst et al., 2010; Poncy et al., 2013; Poncy, Skinner, & Axtell, 2010; Poncy Skinner, & Omara, 2006). Researchers in two studies have integrated the use of technology (Musti-Rao & Plati, 2016; Parkhurst et al., 2010). The effects of DPR on math fluency has been evaluated in six published studies across 90 students spanning from 8 to 15 years of age and from first to eighth grade. Of the 90 students who have previously participated in DPR, 17 have been students with a documented disability at the time of the study. Previous studies have included students with SLD ($n = 3$ students; Poncy et al., 2006; $n = 11$ students; Axtell et al., 2009; $n = 3$ students; Poncy et al., 2013). Other studies have included students described as “low achievers in math” ($n = 10$; Parkhurst et al., 2010 and $n = 12$; Must-Rao & Plati, 2016).

In previous literature DPR has contributed positively to improving math fluency learning rates, although students with lower achievement and students with learning disabilities have demonstrated less success than students with average achievement and students without disabilities. Additional limitations to previous DPR research have included whole-class implementation during the Detect phase. Although the DPR intervention was intentionally designed for whole-class implementation, the current study attempted to examine effects on individual students, and in doing so, evaluate fidelity of implementation at the student self-monitoring level. Except for Parkhurst and colleagues (2010) and Musti-Rao and Plati (2015), previous research has been unable to integrate technology and control for students working at their own pace during DPR Detect (i.e., tap-a-problem) and inevitably connecting the correctly

identified non-fluent facts to DPR Practice (i.e., CCC). Additionally, fidelity of students' self-monitoring behaviors during DPR Practice had not previously been measured by researchers.

The current research examined the effects of a modified version of DPR on multiplication fluency and accuracy across three secondary-aged students receiving special education services in an alternative setting designed to comprehensively support students with EBD. The current iteration of DPR integrated the use of computer aided instruction to embed a constant time delay procedure within the DPR Detect condition. Additional modifications to mDPR included a cognitive (i.e., subvocal/cognitive) iteration of the CCC, first employed by Skinner and colleagues (1993).

Findings

In the current study mDPR appeared to contribute to increased learning rates for two of three participants, but these effects were unable to be replicated with a third student. In order to establish a functional relation between the independent and dependent variables, data must demonstrate experimental control via the systematic presentation of the independent variable that leads to a predicted change in the dependent variable, verification of that change once enough data are gathered, and replication across three participants, behaviors, or settings. The current intervention appeared to contribute to increased multiplication fluency learning rates for Eddy and Paul but not for Larry. The current intervention appeared to contribute to increased multiplication accuracy learning rates for Eddy and Larry but not for Paul. Findings do not demonstrate functional relations between mDPR and multiplication fluency or multiplication accuracy.

Limitations

In the current study a functional relation between independent and dependent variables was unable to be established. Any conclusions drawn from the current research should be weighed with caution. Limitations to the current research include design and setting. As with any single-case design research results are not generalizable to populations as a whole. Single-case designs are typically robust in terms of internal validity; however, the nature of the single-case design research (i.e., small “*n* size”) lacks external validity. For the current research, the alternative school setting would also be a limitation. Student participants have been placed in the alternative school setting as part of their respective IEP team decisions. The alternative school setting was a placement option for all three participants due to academic and behavioral needs that were not previously being met at their respective base schools. Also, according to school and district administration, the ESY program is designed to assist students in terms of skill maintenance during the summer months. The ESY program is *not* designed for students to learn new skills, but to practice and maintain present levels of performance across academic and social skills.

Additional limitations include the methods used to measure student self-monitoring procedural fidelity. Using the student self-monitoring procedural fidelity checklist to guide observations of student behavior during mDPR Practice might not have led to a valid or reliable measure of fidelity. The cognitive-CCC procedure used during mDPR Practice was based on a covert (i.e., internal) behavior, yet the research team could only observe the overt behaviors of the students’ eyes being oriented to the practice card, and the motion of their hands to cover and uncover the card. The research team cannot claim that students were actually attending to the task. It is possible that students were simply going through the motions of mDPR Practice. The

lead researcher and graduate research assistant were concerned with Larry, specifically as cognitive CCC might not have been a good fit for Larry's learning style. Given the nature of Larry's current behavioral programming (i.e., earning video game time for compliance with school expectations and participation with academics), the use of cognitive CCC might have contributed to Larry's ongoing escape-maintained behaviors. For this reason, Larry might have needed to practice multiplication facts with a more overt behavioral topography such as traditional CCC or verbal CCC.

Implications for Practice

In the current study, a functional relation between the mDPR intervention and multiplication fluency or accuracy was not established; however, information can still be gleaned from the scientific process and applied to inform future applications of DPR, mDPR, or similar interventions designed to support struggling learners and/or students with disabilities. Of particular interest in the current study might be an examination of social validity and student self-monitoring.

There are limitations in the way student self-monitoring fidelity was measured which call into question the validity and reliability of the student self-monitoring fidelity procedural checklist and research team's process used to organize observations of students' procedural fidelity throughout mDPR. Two of three participants were able to demonstrate consistently high fidelity of implementation scores during mDPR Detect and mDPR Practice. Eddy had generally lower and highly variable fidelity scores, yet the mDPR intervention appeared to contribute to increased multiplication fluency and accuracy rates. Larry had consistently high fidelity scores, yet the mDPR intervention appeared to have little to no impact on multiplication fluency. Paul only spent five sessions in mDPR and appeared to demonstrate consistently high fidelity of

implementation. Although the mDPR intervention appeared to contribute to slightly increased learning rates in multiplication fluency, there was virtually no change in multiplication accuracy for Paul.

Although the self-monitoring procedural fidelity scores might suggest otherwise, the current iteration of mDPR might have been too complicated for the current student participants in the ESY setting to follow efficiently in terms of sustained attention and engagement over time. The ESY program was designed to first support social and behavioral skills. Maintaining, but not learning new, academic skills is an additional goal of ESY.

Two of three students demonstrated high self-monitoring fidelity scores; however, there were noted inconsistencies. Namely, Eddy was frequently inaccurate with mDPR Detect procedures (i.e., 74% fidelity with a range of 48-100; see Table 3, Appendix A) and researchers were unable to observe sub-vocal (i.e., cognitive; covert) behavioral responses during the mDPR Practice condition. Specifically, the research team was concerned with Larry's fidelity of implementation during mDPR Practice. Although his eyes remained oriented to the practice cards and he consistently covered/uncovered the cards, Larry might not have actually been practicing. It is possible that Larry was simply moving his hand to cover/uncover the practice cards but not rehearsing the facts at the sub-vocal/cognitive level. Cognitive-CCC might not have been an appropriate way for Larry to practice multiplication facts.

Interventions need to be designed with both the students and practitioners in mind. In terms of the student perspective, the current iteration of DPR comprised several procedures and steps that might have been disengaging for the student participants. The intervention process, steps, behaviors should be simple, efficient for student participants, and easy to perform. Parkhurst and colleagues (2010) noted that as the DPR intervention sessions progressed students

appeared to lose interest and become disengaged over time. The researchers added a reinforcement component to DPR during the third and final set of facts and reported that the reinforcement system was effective and improved students' engagement and performance.

Musti-Rao and Plati (2016) concluded that *Math Drills*, an apps-based software available on iPad, provided students with increased OTR rates and led to slightly higher math fluency learning rates for student participants as compared to DPR. In addition, teachers and students reported a preference for the *Math Drill* iPad application. Researchers and practitioners should continue to evaluate and select interventions that are socially valid for consumers (i.e., students). Interventions should be research-based, simple for teachers to implement and students to use, designed to engage students, support sustained attention, and lead to increased learning rates.

In selecting interventions used for RTI or remediation, practitioners also should consider CIT in the context of social validity for all students— especially for students with disabilities in the secondary grades when transition to the adult world is imminent. Academic interventions selected by practitioners should be designed and implemented in ways to make efficient use of CIT in terms of leading to higher learning rates and remediating skill deficits as quickly as possible. The behavioral approach to learning lends itself to quantifying behavioral change (i.e., learning) over time. As opposed to just recording the number of sessions, future research should consistently record and report the duration of each session in seconds then convert to CIT in minutes. Interventions can then consistently be evaluated in terms of socially valid practice and time efficiency. Also, by consistently reporting CIT researchers can make more meaningful interpretations through meta-analysis once a minimum number of studies have been published.

The most effective interventions should be rated high in consumer satisfaction, practitioner feasibility, and learning rate in terms of CIT. Regarding interventions, future

research should address the following questions: Are the intervention procedures easy to implement? Do participants approve of the intervention procedures? Does the intervention lead to increased learning rates?

Future Research

Future researchers might examine iterations of mDPR integrated with computer-aided instructional technology and reinforcement contingent on academic progress as opposed to just participation. It might also be beneficial to sequence the mDPR subcomponents into smaller steps throughout the day- providing transitions to other academic activities and routines between mDPR conditions (i.e., Detect, Practice, Repair conditions). This modification would have little to no impact on CIT. Additionally, alternating treatments designs comparing subcomponents of mDPR as individual interventions (e.g., constant time delay with slideshows vs CCC) might also produce interesting findings for researchers and practitioners.

In applying the student social validity survey data gathered from the current study to future research, researchers should consider examining ways to integrate gaming with computer-aided mathematics instruction. Perhaps other forms of technology can be utilized (e.g., augmented reality, or virtual reality) to create a more game-like experience for students to practice basic math facts and applied problem solving in settings conducive for supporting engagement, and sustained attention.

Still yet, there is little known about the long-term effects of DPR. Only two prior research studies (Poncy et al., 2013; Poncy, Skinner, & Axtell, 2010) have included a maintenance phase. Although a functional relation between mDPR and multiplication fluency was not established in the current study, Eddy demonstrated skill maintenance up to six weeks after the intervention

was completed. Future research should examine the long-term effects of DPR on fluency and accuracy skills.

By reanalyzing data from an alternating treatments design that compared a 1-s intervention to a 5-s intervention, Skinner, Belfiore, and Watson, (2002) demonstrated how conclusions drawn from intervention data can be influenced by how that data is presented—calling for a more accurate measure of learning rate. Namely, when data are presented in terms of cumulative instructional time, as opposed to number of sessions, then learning rates can be examined with more precision. As part of Poncy and colleagues’ meta-analysis (2015), CIT from available mathematics intervention research, including DPR, CCC, TP, and comprehensive instruction, were evaluated and summarized. From the data available to Poncy and colleagues (2015), on average mathematics interventions take about 93minutes (mean CIT across studies) of CIT, (*Mdn* = 67.5) ranging from 22 – 220 with curricular scope about 18 facts per study, (*Mdn* = 12) ranging from 4 – 64 math facts. Future research should continue to target learning rates in terms of CIT, while also evaluating the long-term effects of interventions designed to increase learning and maximize time efficiency. Two of three student in the current study reported to strongly disagree with implementing mDPR on their own. Future research should continue to examine ways in which students can engage in meaningful ways with personalized computer-aided instruction to practice skills and potentially build fluency while self-monitoring.

Researchers should consider computer-aided instructional technologies, as Parkhurst and colleagues (2010), Musti-Rao and Plati (2016), and the current study have done; although the inclusion of the practitioner’s perspective in terms of social validity would contribute to the literature base. Except for Must-Rao and Plati, teachers have not been directly involved in DPR intervention research. The preparatory work involved in using computer-aided instructional

technology with DPR might be viewed as time-consuming and therefore could be perceived as labor intensive by practitioners. Would DPR assist teachers with differentiated instruction facilitation, or be viewed by teachers as a labor-intensive practice?

Conclusion

Although a functional relation between mDPR and multiplication fluency or multiplication accuracy was not established in the current study, results can be used to inform future studies. Two of three participants demonstrated a successful response to mDPR. The current iteration likely did not meet the needs of Larry's current education programming. The current version of mDPR could have been modified to more closely and directly support Larry's education needs. Such adaptations might include a more overt behavioral topography (e.g., traditional written form or verbal iteration of CCC) during mDPR Practice, and/or the addition of reinforcement contingent on academic progress.

With the widespread use of digital technologies in twenty-first century classrooms, as instructional technology advances, the use of computer-aided instructional applications of traditional research-based practice could prove to be an interesting and meaningful endeavor for future researchers, practitioners, and students. As technology becomes an increasing part of our daily lives, it will inevitably become an ever-evolving part of our classrooms as teachers prepare students for a demanding and competitive world and ultimately to become lifelong learners. For students with disabilities, skill gaps often emerge during elementary school and widen throughout secondary education (Kohli, 2015; Stevens et al., 2015). Researchers should continue to examine innovative and socially valid ways to evaluate instructional interventions based upon learning rates and CIT.

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APPENDICES

Appendix A Tables

Table 1

Multiplication Facts Identified by Van der Ven et al., (2015) as The Most Difficult

6×3	6×4	6×6			
7×3	7×4	7×6	7×7		
8×3	8×4	8×6	8×7	8×8	
9×3	9×4	9×6	9×7	9×8	9×9

Table 2

Multiplication Facts Identified by Van der Ven et al., (2015) as The Least Difficult

2×2				
2×3	3×3			
2×4	3×4	4×4		
2×5	3×5	4×5	5×5	
2×6	3×6	4×6	5×6	6×6

Table 3

Student Fidelity of Self-Monitoring as a Percentage of Implementation for each mDPR Condition

Student	mDPR Detect Condition Fidelity ^a			mDPR Practice Condition Fidelity ^b			Fidelity Score Session Grand Means ^c			Fidelity Pre-Corrects ^d
	<i>mean</i>	<i>sd</i>	<i>range</i>	<i>mean</i>	<i>sd</i>	<i>range</i>	<i>mean</i>	<i>sd</i>	<i>range</i>	
Eddy	74	19	48-100	84	18	45-100	79	16	47-100	5 of 16
Larry	97	7.5	75-100	100	0	N/A	99	3.7	89-100	2 of 13
Paul	100	0	N/A	96	8.9	80-100	98	4.5	91-100	0 of 5

Note. Scores were calculated as percentage of correctly completed steps from the procedural fidelity checklist items. Means, standard deviations and ranges were calculated by mDPR condition (Detect, Practice) and across all sessions (grand mean); ^amean, sd and range of fidelity scores from the mDPR Detect condition; ^bmean sd and range of fidelity scores from the mDPR Practice condition; ^cgrand mean, sd, and range of scores by session; ^dnumber of times the lead researcher retaught the intervention procedures to the participant due to decreasing fidelity trend. All students scored 100% fidelity during mDPR Repair condition (i.e., 60-s multiplication fluency measure).

Table 4

Social Validity

Lickert-type scale: 1= Strongly Disagree, 2 = Disagree, 3= Neutral, 4= Agree, 5= Strongly Agree	Eddy	Larry	Paul	Item means	Item sd
Computer math and practice helped me learn.	4	3.5	3	3.50	0.50
I quickly improved my math skills.	4	3	4	3.67	0.58
Computer math and practice would help my friends learn math.	5	2	4	3.67	1.53
I like learning math this way.	5	3	5	4.33	1.16
My classmates would like learning math this way.	5	3	3	3.67	1.16
The computer would help me learn other academic skills.	5	4	5	4.67	0.58
Computer math and practice was fun.	5	5	4	4.67	0.58
The steps were easy.	5	5	3	4.33	1.16
I like trying to be faster than the computer.	5	2	5	4.00	1.73
I liked practicing math facts by using the cover, copy, and compare technique.	5	3	5	4.33	1.16
I liked the 1-munite math worksheets.	4	1	5	3.33	2.08
I would do computer math and practice on my own.	1	1	4	2.00	1.73
Computer math and practice would help me learn other facts.	1	1	5	2.33	2.31
I would do computer math and practice again.	1	5	5	3.67	2.31
means by participant	3.93	2.96	4.29		
Sd by participant	1.64	1.45	0.83		

Note. Participants' responses, means and standard deviation to the SV measure on a Lickert-type scale: 1= Strongly Disagree, 2 = Disagree, 3= Neutral, 4= Agree, 5= Strongly Agree; Larry circled both "Neutral" and "Agree" on the first item. When asked he stated that he felt "half and half" about it.

Table 5

Curricular Scope, Median Difference Between Phases, Average Session Instructional Time, Cumulative Instructional Time, and Number of mDPR Sessions.

Participant	CS ^a	mDPR ^b <i>Mdn</i>	Baseline <i>Mdn</i>	<i>Mdn Diff</i>	DCPIM ^c	Average Session (min)	CIT ^d (min)	Intervention Sessions
Eddy	18	28.5	15	13.5	.142	5.96	95.37	16
Larry	15	18	17	1	.018	4.27	55.6	13
Paul	15	18	14.5	3.5	.179	3.90	19.48	5

Note. ^aCS = Curricular Scope represents the fact set size; ^bmDPR *Mdn* = median fluency score; ^cDCPIM = Digits Correct per Instructional Minute is calculated by dividing mean difference between phases by CIT; ^dCIT = cumulative instructional time the student spent in the mDPR intervention across the study.

Appendix B Figures

My DPR form

Name: _____

Date: _____

If correct, circle Thumbs up

If incorrect, circle thumbs down

A		
B		
C		
D		
E		
F		
G		
H		
I		
J		
K		
L		
M		
N		
O		
P		
Q		
R		

Figure 1. My DPR Form. Students used this form to record the first 5 unknown facts during the Detect procedure. The presentation of facts were stratified by level of difficulty.

A

$$\begin{array}{r} 9 \\ \times 3 \\ \hline 27 \end{array}$$

B

$$\begin{array}{r} 8 \\ \times 4 \\ \hline 32 \end{array}$$

Figure 2. Example Math Fact Sheets used by students during DPR-Practice (Cognitive Cover-Copy-Compare). Note, the student used the My DPR form to identify fluent and non-fluent facts from the slideshow.

Baseline instructions:

- 1) Place timer out on the table in plain sight.
- 2) "You are going to work on some math problems and I am going to time you. When I say start, immediately begin working. Do them in order from left to right as fast as you can. Don't Skip. When you come to a problem you don't know, just do your best and move on. When I say stop, place your pencil down immediately. Do not continue working.
- 3) "Do you have any questions?"
- 4) Answer student's relevant questions
- 5) "Turn over your worksheet."
- 6) "Get ready, go!"
- 7) Begin timer.
- 8) At 1 minute state "Stop."
- 9) If student has complied, state: "Thank you for putting your pencil down immediately."
- 10) "We are all done for right now. Thank you."

mDPR Detect Instructions:

- 1) "Work through the math problems on the computer slideshow."
- 2) "We are just going to do the hard ones."
- 3) "If you know the correct answer, call it out."
- 4) "If you are correct, circle green thumbs up."
- 5) "If you are incorrect, circle red thumbs down."
- 6) "If you don't know the answer, don't guess. Just wait for the answer then circle red thumbs down."
- 7) "When you get to 5 thumbs down, stop."
- 8) "Are you ready?"
- 9) Start slideshow.

mDPR Practice Instructions

- 1) "Now you are ready to practice"
- 2) "Find the practice card for your first problem."
- 3) Student will find the practice cards that correspond to their 5 incorrect responses (red thumbs down).
- 4) Cover-Copy Compare:
 - a. Cog-CCC: "Look at the card, then cover it with your hand and recall math fact from memory, or with an inside voice".
- 5) "Then repeat five times and continue for all five of your math facts."

mDPR Repair Instruction (60-s math fluency measure):

- 1) Place timer out on the table in plain sight.
- 2) "You are going to work on some math problems and I am going to time you. When I say start, immediately begin working. Do them in order from left to right as fast as you can. Don't skip. When you come to a problem you don't know, just do your best and move on. When I say stop, place your pencil down immediately. Do not continue working.
- 3) "Do you have any questions?"
- 4) Answer student's relevant questions
- 5) "Turn over your worksheet."
- 6) "Get ready, go!"
- 7) Begin timer.
- 8) At 1 minute state "Stop."
- 9) If student has complied, state: "Thank you for putting your pencil down immediately."
- 10) "We are all done for right now. Thank you."

Figure 3. Teaching scripts used during training, baseline, intervention, and maintenance phases

Name _____
Date _____

Multiplication UTK1

$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$
$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$
$\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$

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Name _____
Date _____

Multiplication* UTK1

$\begin{array}{r} 2 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times 3 \\ \hline \end{array}$
$\begin{array}{r} 3 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 4 \\ \hline \end{array}$
$\begin{array}{r} 2 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$
$\begin{array}{r} 2 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 3 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ \times 6 \\ \hline \end{array}$

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Figure 4. Sample Multiplication Facts Probes for each Factor Set used during 60-s Multiplication fluency Measures.

Procedural Fidelity Checklist for modified Detect Practice and Repair

Observer's Name: _____

Student's Name: _____

Date: _____ Time: _____ 20% Researcher Fidelity IOA Check: Y/N

DETECT (Slideshow, Taped Problems, Constant Time Delay)		Start Timer
1	The student was engaged (eyes oriented to the computer screen, waited, or responded verbally to each PPT slide within the 3-second delay), until five nonfluent facts were identified.	Y/N/p
2	The student used the "MyDPR" form to record correct and incorrect responses.	Y/N/p
3	Use the adjacent cell to record the student's "MyDPR" form accuracy. This # is the rater's IOA with the student (#agree/#agree + disagree).	
4	The student identified the first five nonfluent facts, then stopped the "detect condition".	
PRACTICE (Verbal Cover Copy Compare)		
5	The student used the appropriate math fact sheet to practice the five identified nonfluent facts (5x each).	
	Fact 1: Y/N/p	Fact 2: Y/N/p
	Fact 3: Y/N/p	Fact 4: Y/N/p
	Fact 5: Y/N/p	
6	The student appeared to read the math fact silently (oriented his/her eyes to the math fact) (5x each).	
	Fact 1: Y/N/p	Fact 2: Y/N/p
	Fact 3: Y/N/p	Fact 4: Y/N/p
	Fact 5: Y/N/p	
7	The student covered the math fact sheet with his/her hand (5x each).	
	Fact 1: Y/N/p	Fact 2: Y/N/p
	Fact 3: Y/N/p	Fact 4: Y/N/p
	Fact 5: Y/N/p	
8	The student verbally stated the fact correctly (5x each). <u>N/A if CogCCC (subvocal)</u>	
	Fact 1: Y/N/p	Fact 2: Y/N/p
	Fact 3: Y/N/p	Fact 4: Y/N/p
	Fact 5: Y/N/p	
9	The student uncovered the math fact sheet and appeared to make a comparison (5x each).	
	Fact 1: Y/N/p	Fact 2: Y/N/p
	Fact 3: Y/N/p	Fact 4: Y/N/p
	Fact 5: Y/N/p	
REPAIR (Explicit Timing, 60-s fluency measure)		
10	The student completed the multiplication fluency measure as instructed.	
		Y/N/p

Score:

Y "Yes"

N "No"

P "Prompt provided"

Cover-Copy-Compare checklist															
	Fact 1			Fact 2			Fact 3			Fact 4			Fact 5		
1x	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm
2x	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm
3x	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm
4x	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm
5x	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm	Cv	Cp	Cm

Use a checkmark to document the following behaviors (5x for each fact):

Cv- Cover

Cp- Copy

Cm- Compare

Record Cumulative Instructional Time for this Session:

CIT:	_____ Sec
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Figure 5. Student Self-monitoring procedural fidelity checklist used by researchers to measure procedural fidelity of student self-monitoring during mDPR.

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Name: _____ Date: _____

For each item, please circle the number that best tells what you think about the computer math and practice intervention that you just finished.

Computer math and practice	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1. Computer math and practice helped me learn.	1	2	3	4	5
2. I quickly improved my math skills.	1	2	3	4	5
3. Computer math and practice would help my friends learn math.	1	2	3	4	5
4. I like learning math this way.	1	2	3	4	5
5. My classmates would like learning math this way.	1	2	3	4	5
6. The computer would help me learn other academic skills.	1	2	3	4	5
7. Computer math and practice was fun.	1	2	3	4	5
8. The steps were easy.	1	2	3	4	5
9. I like trying to be faster than the computer.	1	2	3	4	5
10. I liked practicing math facts by using the cover, copy, and compare technique.	1	2	3	4	5
11. I liked the 1-minute math worksheets.	1	2	3	4	5
12. I would do computer math and practice on my own.	1	2	3	4	5
13. Computer math and practice would help me learn other facts.	1	2	3	4	5
14. I would do computer math and practice again.	1	2	3	4	5

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Open Responses:

15. What was the most helpful part about computer math and practice?

16. What was the least helpful part of computer math and practice?

17. What would you change about computer math and practice, and why?

Adapted from the *Student Post-Intervention Acceptability & Outcome Survey*, a paper-based survey previously adapted from Lane, and Beebe-Frankenberg (2004) by the University of Nebraska, Lincoln, College of Education and Human Sciences, Center for Research on Children, Youth, and Families (CYFS, 2016).

CYFS. (2016) The Nebraska center for research on children, youth, families and schools. Lincoln, NE. Retrieved from: <http://cyfs.unl.edu/>

Lane, K. L., & Beebe-Frankenberg, M. E. (2004). *School-based interventions: The tools you need to succeed*. Boston, MA: Allyn & Bacon.

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Figure 6. Social validity student rater scale based on an adapted version of the *Post-Intervention Acceptability & Outcome Survey*, a paper-based survey previously adapted from Lane, and Beebe-Frankenberg (2004) by the University of Nebraska, Lincoln, College of Education and Human Sciences, Center for Research on Children, Youth, and Families (CYFS, 2016) .

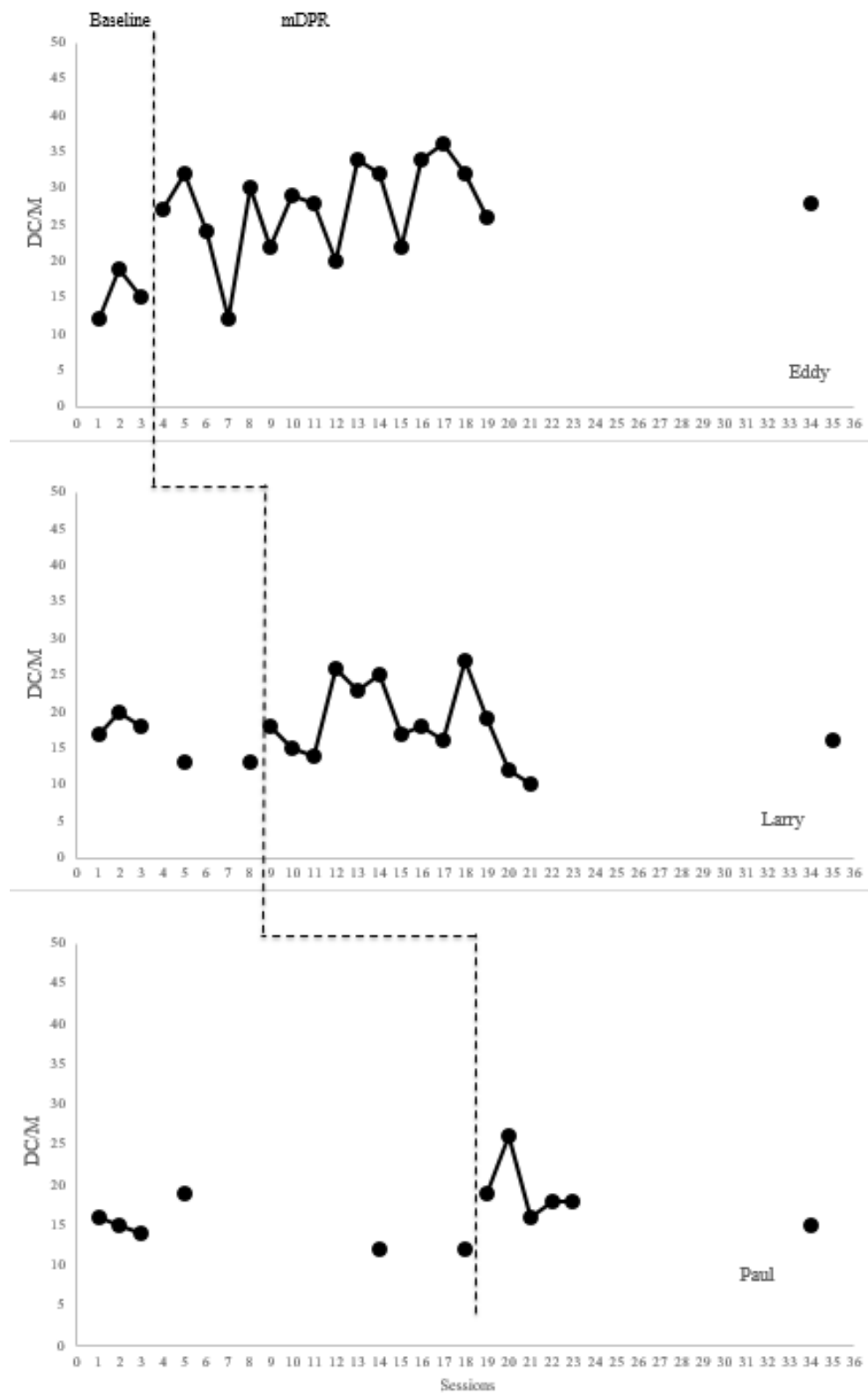


Figure 7. Digits Correct per min (DC/M) over number of sessions.

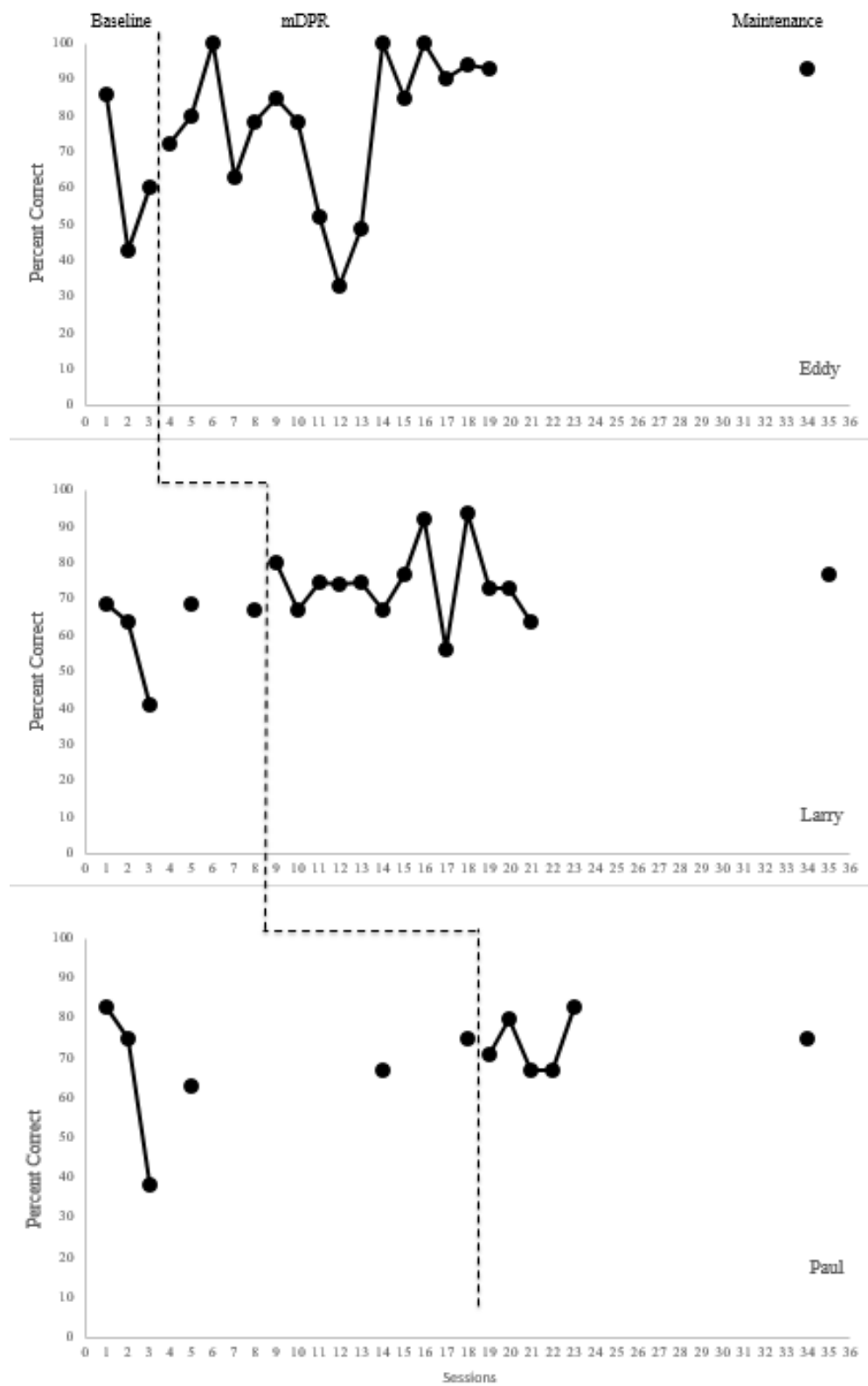


Figure 8. Percent Correct over number of sessions.

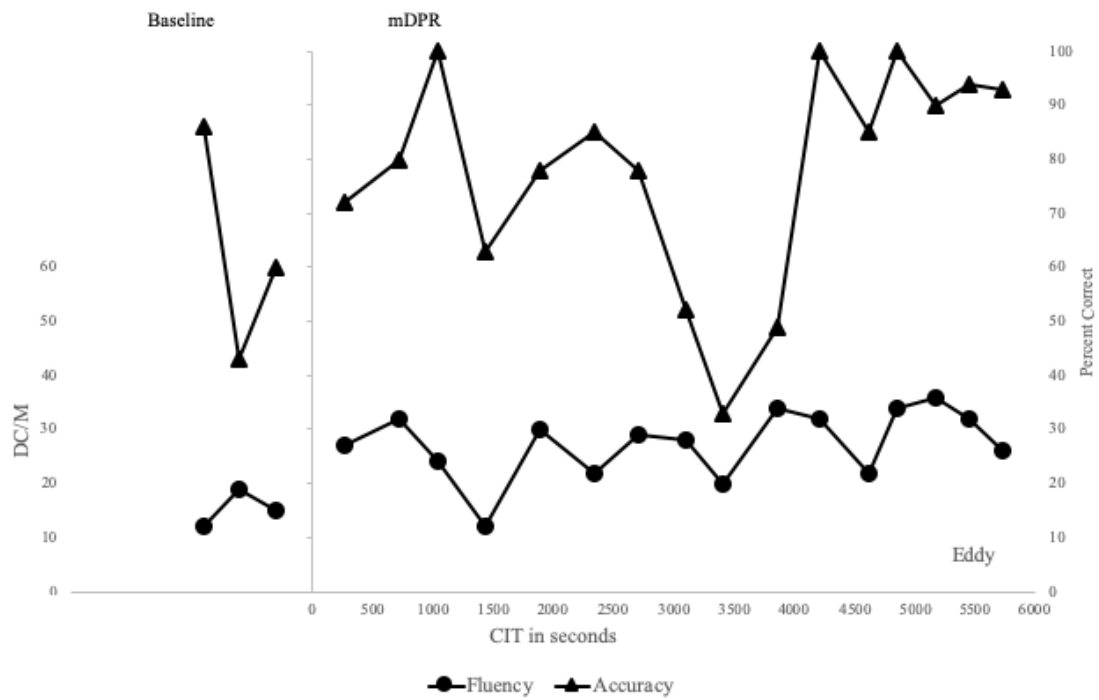


Figure 9. Eddy's Accuracy and Fluency Scores over Cumulative Instructional Time.

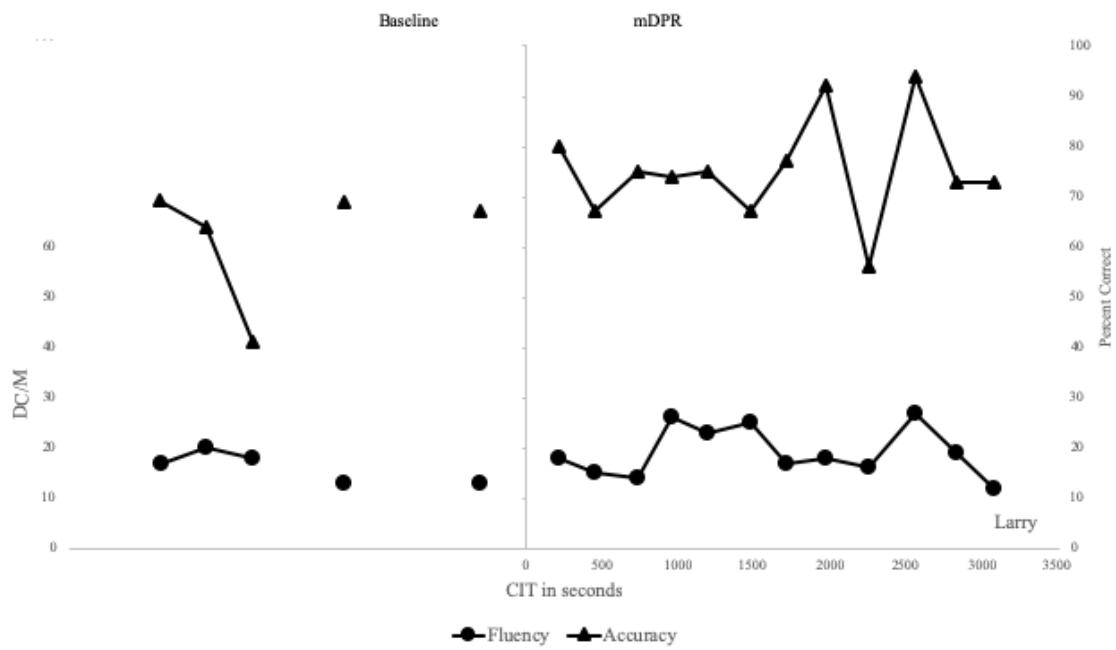


Figure 10. Larry's Accuracy and Fluency Scores over Cumulative Instructional Time.

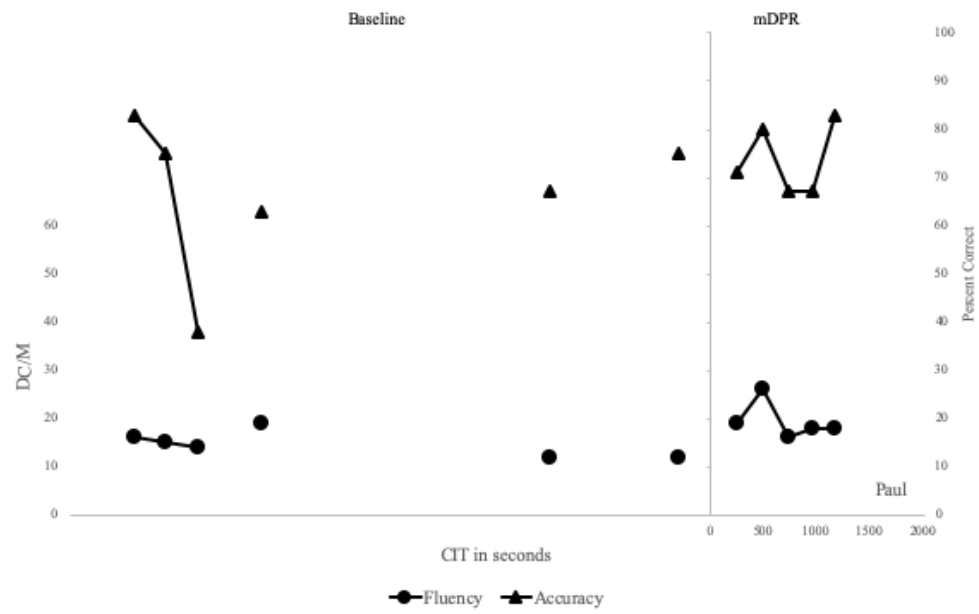


Figure 11. Paul's Accuracy and Fluency Scores over Cumulative Instructional Time.

VITA

Jason R. Gordon is originally from Ravenswood, West Virginia, where he graduated from Ravenswood High School in 1994. He graduated from Glenville State College in 1998 with a Bachelor of Arts degree in History while minoring in Social Science, Psychology, and Political Science. Prior to starting the doctoral program at the University of Tennessee, Knoxville (UTK), Jason completed a Master's of Science in Education degree (UTK) in 2005, and an Educational Specialist degree from Lincoln Memorial University in 2014. Throughout his career, Jason has served the field of education as a counselor, an educator of students with special needs, building-level mentor teacher, and special education program supervisor.

Currently, Jason is an educational consultant for the Tennessee Behavior Supports Project (TBSP) at UTK. In his current role, Jason consults with teachers, principals and district-level administrators, provides professional development and technical assistance to school and district personnel, designs data summary tools used to support school leadership teams with RTI²-B implementation, and manages a large database for TBSP. Jason has also taught courses in education and applied behavior analysis for the College of Education, Health and Human Sciences at the University of Tennessee, Knoxville. As part of his PhD program, Jason has completed approved coursework, and is currently fulfilling supervised fieldwork, for Board Certified Behavior Analyst certification and has earned a minor in Statistics. Jason plans to continue making contributions to publications across multiple lines of research including behavioral interventions, academic interventions, and quasi-experimental research that lead to improved outcomes for students, teachers, and people with disabilities. Jason received his PhD in December 2019.