Journal of Institutional and Theoretical Economics (JITE) 147 (1991), 499–516
Zeitschrift für die gesamte Staatswissenschaft

Stochastic Demand, Inventory Management, and Chamberlinian Excess Capacity

by

HANS HALLER and DANIEL ORR *

1. Introduction and Objectives

A theorem from the nineteen thirties which has become a minor textbook canon in microeconomics states that in long run equilibrium, monopolistic competition implies excess capacity in all individual firms.[1] In a geometric analysis too familiar to replicate, EDWARD H. CHAMBERLIN [1933] asserts that in markets with (1) numerous single plant, single product firms, (2) a market demand structure in which outputs of different firms are 'close but not perfect' substitutes (so that each firm faces a demand structure that is less than infinitely elastic in own price), and (3) free entry, the long run equilibrium of any supplier will be at a point of tangency between its demand curve and its long (and short) run average cost curve. It necessarily follows from the negative slope of the demand curve that (a) the plant of choice is underutilized, and (b) the plant of choice is smaller than the competitively optimal one. Chamberlin further speculated that this "excess capacity theorem" need not imply resource misallocation; product variety was seen as unobtainable without it.[2]

To many economists, Chamberlin's large-group equilibrium is synonymous with monopolistic competition; and excess capacity is the most important economic implication of that equilibrium, in which (1) every firm perceives the finite-elasticity response of demand to own-price changes; (2) the firm's demand

* We are indebted to Richard Ashley, James Friedman, Roderick Reasor, Nicolaus Tideman, and two referees for advice and beneficial critique.

[1] See recent texts as diverse as VARIAN [1984] and WONNACOTT-WONNACOTT [1987] for the importance now assigned to the excess capacity postulate, and CASSELS [1937], CHAMBERLIN [1937], [1938], KAHN [1935], KALDOR [1935] for a tour of origins.

[2] That particular insight has been made more precise in work by DIXIT-STIGLITZ [1977] and LANCASTER [1979], [1980]. Dixit and Stiglitz express desirability of variety via convexity of utility functions. They present cases where (constrained) optimality of an allocation requires excess capacity and where equilibrium outputs are at or beyond the constrained optimal levels. Lancaster emphasizes diversity of tastes across consumers and arrives at similar conclusions: that optimality requires excess capacity; that sometimes monopolistic competition results in a sub-optimal configuration, with less excess capacity than at the social optimum.

curve shifts in response to price changes of other firms, or entry by other firms; (3) every firm in the 'group' (a set of firms producing closely substitutable output under very similar technological and cost conditions) maximizes profit with respect to own price and scale of plant; and (4) there are no profits anywhere in the group that could be dissipated by further entry (i.e., all firms earn zero profit net of imputable rents).

This may be formalized: the vector of prices of all firms in a Chamberlinian group is $\mathbb{P}$, a representative firm's demand function is $\bar{q}(\mathbb{P})$, its cost function is $C[\bar{q}(\mathbb{P})]$, and its own price is p. Chamberlinian equilibrium holds in the long run when

$$\max_p \pi = p \cdot \bar{q}(\mathbb{P}) - C[\bar{q}(\mathbb{P})] \tag{1}$$

is solved such that

$$\pi = 0\,, \tag{2}$$

and when the cost function C is associated with an optimal scale of plant. The conditions (1) and (2) hold for all firms of the group. Excess capacity follows immediately from the $MC = MR$ condition along with zero profit ($AC = AR$) so long as $q' \equiv \partial\bar{q}/\partial p < 0$, as is usual.

A literature has grown sporadically since the nineteen fifties, which has tried to unseat the excess capacity theorem, through attempts to show that excess capacity ceases to hold as a condition of equilibrium if explicit analytical allowance is made for other cost-related features of monopolistic competition, such as advertising or differentiated product quality among firms. The excess capacity theorem has, however, survived those challenges.[3]

Like those efforts, this paper seeks to counter-exemplify excess capacity by enriching the firm's environment, in ways consistent with Chamberlin's vision. With U-shaped average cost curves and downward sloping demand curves, as we shall assume, an additional cost element not inherent in the production technology is needed to make the excess capacity theorem fail. Brand variety with spill-over between brands may do.[4] So do demand shocks – or demand shocks buffered by inventory management. We will portray the firm as an expected-returns maximizer over time, facing a stochastic demand process (modelled in discrete time by a relation in which the firm's own price and mean quantity demanded per period of time are inversely related; and there is an additive random zero-mean quantity shock in each period).

The modification introduced here, stochastic demand, is a natural extension of Chamberlin's own analysis, which sought more realistic portrayals of the

[3] See SCHMALENSEE [1972] and MARGOLIS [1985] for analysis and references.

[4] MARGOLIS [1989] successfully follows this route which is a departure from the single product firm.

market process than monopoly or competition alone afforded. Most importantly, Chamberlin's tangency solution is a product of VINER's [1931] famous envelope of U-shaped short run average cost curves; and (as STIGLER [1939] argued in another famous old paper) it makes sense to adopt a production technology with U-shaped short run average cost only when the firm expects to vary output in the short run. Stigler identifies "capital flexibility" as a costly attribute which enables the firm to vary output without incurring severe cost penalties. When sales fluctuate stochastically and inventories are absent or costly to maintain at extremely high or low (negative) levels, output variation is necessary, and capital flexibility is desirable. Heretofore there has been no explicit rationale for U-shaped short run average cost in the analysis of Chamberlinian firms, but our introduction of stochastic demand does provide an economic justification.

The prevalent response to stochastic demand fluctuations, both in theory and in practice, is to introduce inventory: the firm's output has to be storable; the firm buffers against stochastic quantity shocks with finished goods inventory; the output rate adjusts to inventory movement through time as prescribed by 'inventory theory'. The quantity-buffering inventory management considered below is a variant of normative production control analyses widely published in the management science literature for use by firms in the same kinds of branded goods markets which motivate Chamberlin's work. Our specific analysis is abetted by a stylized and simple framework, drawn from yet more old work which in the last decade has enjoyed a vigorous revival, the quadratic criteria analysis of production and inventory planning, exemplified in the classic of HOLT, MODIGLIANI, MUTH and SIMON [1960].

One of the referees pointed out that our main conclusions might also be obtained with a zero inventory policy. The crucial fact in both cases is that the variance of demand enters the firm's expected operating cost linearly.[5] Since the zero inventory regime makes the analysis somewhat simpler and more transparent, we deal with this regime first, in Section 7, before we turn to an inventory management regime in Section 8. Traditional inventory management has been and very likely will be the preferred response of most manufacturers to demand shocks. However, following the Japanese lead, an increasing number of US manufacturers opt for zero or close to zero inventory; the new technical term is 'just-in-time' (JIT).[6] Therefore, apart from being analytically convenient, the assumption of zero inventory is becoming empirically relevant.

---

[5] See equation (5).

[6] See GILBERT [1990]. The traditional procedure basically applies a batch-push system where parts are made and assembled in large batches and pushed to the next operation on a fixed schedule. Finished goods are stockpiled as inventory. Just-in-time production attempts to produce only the necessary products in the necessary quantities at the necessary time. It is a pull system which means that sub-assemblies, components, parts, and raw materials are pulled forward as they are needed in the next step of operation. In the ideal scenario, no inventories are accumulated (and no backlogs occur).

The following investigation of how a firm faced with stochastic demand would act in equilibrium, answers two questions related to excess capacity: will the firm, on average, necessarily operate at a production rate less than the average-cost minimizing rate in the plant of its choice; and will it necessarily choose a plant smaller in scale than one which yields the lowest possible cost per unit of output? Sections 2–5 introduce the model and provide general characterization results. Section 6, more specifically, defines and discusses a symmetric long-run equilibrium in a model with identical firms. Sections 7 and 8 explore the more specific structure and counter-exemplify the excess capacity condition, for a JIT production schedule and for an intertemporal inventory management setting, respectively. Section 9 offers concluding remarks.

When a firm faces stochastic demand fluctuations and maximizes net expected revenue, the excess capacity condition, an average rate of production that lies to the left of the output level at which average production costs are minimized, can fail. The reason is that demand variance enters as a term in the expected operating cost formula (5). That term cannot be simply derived from the technological specification of the plant, but is a consequence of stochastic demand. In the standard Chamberlinian analysis, the maximization of net return necessitates that a plant is operated at a rate at which unit costs of manufacture are falling. We will show here that when a stochastic element of demand imposes additional costs on the firm, none of which are direct costs of manufacture, expected returns maximization no longer implies that "excess capacity" condition. At the very least, this demonstrates that a clean separation between cost-related and demand-related components in the firm's objective function cannot be relied on; cost no longer is a matter only of technological plant specification and input prices. The position of the average rate of production, relative to the technologically defined output level of minimum unit cost of output, can no longer be predicted for an equilibrated firm, unless its exact demand conditions are known.[7, 8]

The efficiency or social welfare consequences of this finding are obscure. Nonetheless, the finding prompts a new view of branded goods markets which promises to improve our understanding of those markets in a significant way, as follows. With the costs of meeting demand variation incorporated into the analysis, it is no longer true that firms can be charged with producing too little output in too small a plant at too high a cost, and selling it at too high a price, even at the level at which that charge is true in the standard Chamberlinian

[7] These observations could also be relevant for empirical studies of capacity choice and capacity utilization.

[8] DE VANY-SAVING'S [1983] model remotely resembles ours. Although they consolidate technological production costs and demand-related costs, they counter the excess capacity condition. The reason is that in their model, capacity choice influences the expected waiting time for customers; consumers consider waiting time as a quality parameter and adjust their demand (decision to join a particular queue) accordingly. Hence a novel link between capacity choice and demand is postulated.

analysis. One of the "costs of variety" in branded goods markets is the maintenance of stock levels or the fulfillment of delivery times to avoid customer disaffection. Accompanying the sale of branded goods is demand fluctuation, represented (albeit imperfectly no doubt) herein by an additive i. i. d. error term. It may be unnecessary to search for a reasonable tradeoff between variety and excess capacity; survival in a branded goods market may depend more on logistics (along lines suggested in our analysis below) than on capacity. Logistics, like capacity, is a use of resources; but those resources are probably less brand specific than capacity, and more readily transferred to other uses in the event of brand failure.

2. Demand Uncertainty

Analysis of stochastic demand begins with one of the firms in a Chamberlinian group that is equilibrated in the long run. That is, prices, inventory maintenance decisions, physical plant investment, advertising, product quality and other choices affecting competition or cost within the group are assumed to be in long run equilibrium. We will explicitly consider price, output, inventory maintenance (when applicable) and plant scale decisions, but none of the other competitive dimensions. Let x^* denote the scale of plant and p denote the price of the firm under consideration. The demand process faced by a representative firm is of the form

$$q_t = \bar{q}(\mathbb{P}) + \varepsilon_t, \quad t = 1, 2, \ldots, \tag{3}$$

where $\mathbb{P}$ is the vector of group equilibrium prices (which includes the firm's own price), and $\{\varepsilon_t\}$ is a sequence of independent, identically distributed random variables with zero mean and finite variance $V > 0$.

The firm seeks production rate and inventory level sequences which minimize expected cost of operation. The period t cost of producing depends on the output rate x_t in that period and is

$$a x_t + b(x_t - x^*)^2 , \tag{4}$$

where a and b ($b > 0$) are parameter values when the firm has competitive market access to all variable inputs, and x^* is the plant scale parameter (identical with the short run minimum-average-cost output rate). The plant scale x^* is a choice variable considered explicitly in Section 5. For now x^* ist taken as exogenous and assumed to be survivable, i. e. capable of yielding normal returns in long run equilibrium. The period t operating cost of the firm is denoted γ_t: it is given by (4) plus eventual inventory costs. We proceed under the assumption that the expected period t operating cost takes the general form

$$E\gamma_t = a\bar{q} + \phi(\bar{q} - x^*)^2 + \theta V \tag{5}$$

with $\phi > 0$ and $\theta > 0$. In the sequel, we determine the coefficients ϕ and θ for two prominent operating regimes: a 'just-in-time' production schedule in Section 7 and an 'optimal partial adjustment rule' in Section 8.

3. *Properties of Equilibrium Price*

Section 2 presumes a specific plant such that the firm's actual cost of production at the rate x_t is (4), with a, b, and x^* treated as given parameters. In contrast, the firm faces the expected operating cost (5) which is obtained after an 'optimal' inventory-output response to stochastic demand, with plant scales and equilibrium prices fixed throughout the group. The functional form (5) explicitly reflects the effects of the firm's price choice. The firm's chosen price, in the stochastic version of Chamberlinian equilibrium, will maximize its expected net revenue; but because of the entry and equilibrium price decisions of all firms, expected net revenue will be zero. The stochastic market equivalent of (1) and (2) then is, with D the firm's discount factor, $0 < D < 1$,

$$\text{(6)} \qquad \operatorname*{Max}_{p} E\pi = E \sum_{t=1}^{\infty} D^{t-1} [q_t(\mathbb{P}) \cdot p - \gamma_t] = 0 \,.$$

Using (5) and suppressing the discount factor D, the maximand (6) becomes

$$\text{(7)} \qquad \operatorname*{Max}_{p} E\pi = p \cdot \bar{q}(\mathbb{P}) - a\bar{q}(\mathbb{P}) - \phi \cdot (\bar{q}(\mathbb{P}) - x^*)^2 - \theta V.$$

The firm and group are in equilibrium when every firm has chosen its component of $\mathbb{P}$ to satisfy (7) and when every firm's choice of plant is consistent with meeting demand at minimum cost. In a representative firm, those decentralized choices result in zero maximum expected profit. Hence the equilibrium conditions are, with $q' = \partial\bar{q}/\partial p$:

$$\text{(8)} \qquad \partial E\pi/\partial p = \bar{q} + (p - a)\, q' - 2q'\phi(\bar{q} - x^*) = 0\,;$$

$$\text{(9)} \qquad E\pi = (p - a)\,\bar{q} - \phi\,(\bar{q} - x^*)^2 - \theta V = 0\,.$$

4. *Utilization of the Chosen Plant*

Let p and $\bar{q}$ denote the values which solve (8) and (9). (This may be thought of as price being the choice of the firm, and the associated mean quantity demanded being determined by the process of profit-dissipating entry.) Let again $q' = \partial\bar{q}/\partial p < 0$ be the slope of the firm's demand with respect to its own price

p, evaluated at the equilibrium values. From (8) and (9), we have that

$$\bar{q} = [-q'(\phi x^{*2} + \theta V)/(1 - \phi q')]^{1/2} . \tag{10}$$

In order for the excess capacity theorem to hold, it is necessary that $\bar{q} < x^*$. Obviously no such necessity holds here. Indeed, if $\bar{q}^2 < -q'\theta V$, then also $\bar{q} > x^*$. This result is a

Theorem. In a plant whose scale x^* is consistent with long run equilibrium, the likelihood that $\bar{q} > x^*$ increases with $-q'$, the absolute value of the partial slope of the (mean) demand function $\bar{q}(\mathbb{P})$, and with V, the variance of the demand process.

The proof of the theorem is immediate from (10). The intuitive basis of that result is clear. In the deterministic Chamberlin world, the steeper the demand curve (whose slope is $1/q'$), the farther to the left on the AC curve the point of tangency occurs, and the greater the degree of excess capacity.[9] That effect is preserved her. Variance is seen in (5) to contribute to expected operating cost. In the process of group equilibration, the firms's demand curve is shifted downward by entry so long as there are positive expected excess returns. The greater the costs imposed by variance, the smaller that downward shift, and the greater the likelihood of $\bar{q} > x^*$.

So, our first finding is that mean output can exceed the minimum-average-cost output rate within a survivable plant whose technical cost specification is given by (4). But will equilibrium nevertheless always occur in a plant that is 'too small', so that excess capacity remains in the long run with reference to a competitively scaled plant? That question is taken up next.

5. *Choice of a "Too Small" Plant*

In Chamberlin's depiction of the process of equilibration, it is strongly implied that entry or exit in the group is accompanied by moves to smaller sized plants in the ongoing firms. He views those adjustments toward small plants as the more important component of excess capacity from the standpoint of resource use; under-use of the chosen plant, the problem studied in the preceding section, is a secondary component of excess capacity.

Exploration continues, then, with the introduction of an explicit quadratic long run cost envelope curve. The fixed cost coefficient is positive; otherwise there is no minimum average cost output to measure capacity by. So despite the

[9] When drawing a demand curve, we follow the convention of partial equilibrium analysis. That is the firm's price is depicted on the vertical axis, whereas quantity is depicted on the horizontal axis.

maxim that there are no fixed costs in the long run, we introduce a cost component C which can be taken as start-up costs associated with any finite production rate, however small, and which is avoidable by shutdown. In

$$LRTC = Ax^2 + Bx + C, \tag{11}$$

the coefficient sign restrictions $A, B, C > 0$ assure that marginal cost is positive and rising for all x; and $B > 0$ is further necessary in order that average cost be positive at its minimum value.

Next, define a parametrized family of short run total cost curves

$$SRTC_k = a_k x + b(x - x_k^*)^2 \tag{12}$$

whose envelope is (11). The minimum point for average cost on each short run curve is x_k^*. The index k is conveniently chosen to be the slope of $LRAC$ at x_k, the point of tangency with $SRAC_k$. Thus if x_0 is the minimum point of $LRAC$, then x_0 is both the point of tangency of $LRAC$ with $SRAC_0$ and the minimum point of $SRAC_0$: $x_0 = x_0^*$.

Examine that minimum point x_0^*. We have $LRAC' = SRAC' = 0$, i.e.

$$A - C/x^2 = b(x^2 - x_0^{*2})/x^2 = 0$$

at $x = x_0$, so

$$x_0 = x_0^* = (C/A)^{1/2}. \tag{13}$$

Moreover, $SRAC$ is more convex than $LRAC$, i.e.

$$SRAC'' > LRAC'' > 0;$$

so $bx_0^{*2} > C$, or $b > A$.

Next consider the $SRAC_k$ curve. Its minimum point is x_k^*. We have that

$$Ax_k^2 + Bx_k + C = a_k x_k + b(x_k - x_k^*)^2 \tag{14}$$

and, from $LRAC' = SRAC' = k$,

$$A - C/x_k^2 = b(x_k^2 - x_k^{*2})/x_k^2 = k,$$

which implies $k < A$, since $C > 0$; and further $C = (A - k)/x_k^2$ and

$$x_k^2 = C/(A - k) = bx_k^{*2}/(b - k)$$

which in turn gives

$$x_k^* = [(b - k)C/(A - k)b]^{1/2}. \tag{15}$$

Note that x_k increases and x_k^*/x_k decreases as k increases. From (14) and $LRMC = SRMC$ at x_k follows

$$2Ax_k + B = a_k + 2b(x_k - x_k^*) \quad \text{or}$$

$$a_k = B - 2(b-A)\left(\frac{C}{A-k}\right)^{1/2} + 2b\left(\frac{(b-k)C}{(A-k)b}\right)^{1/2}. \tag{16}$$

Next, to simplify further, we linearize the firm's partial demand relation:

$$\bar{q} = F - fp.$$

The maximand (7), with the scale variable k introduced as a decision variable, then is

$$\underset{p,k}{\text{Max}}\, E\pi = (p - a_k)\cdot(F - fp) - \phi(F - fp - x_k^*)^2 - \theta V.$$

The three conditions which now characterize long run equilibrium are

$$E\pi = 0, \tag{17}$$

$$\partial E\pi/\partial p = F - 2fp + fa_k + 2\phi f(F - fp - x_k^*) = 0, \tag{18}$$

$$\partial E\pi/\partial k = 2\phi(F - fp - x_k^*)\, dx_k^*/dk - (F - fp)\, da_k/dk = 0. \tag{19}$$

The three variables that adjust to meet these conditions are k, p, and F, the demand curve intercept (which is shifted by entry).

The two matters to investigate are: (a) Can the three long run equilibrium conditions hold when $k \geq 0$? (b) If so, can the result $\bar{q} > x_k^*$ be observed with $k \geq 0$? That investigation proceeds by substituting $\bar{q} = F - fp$ back into (17)–(19) and eliminating the terms $(p - a_k)$ from (17) and (18). The resulting expressions, using

$$dx_k^*/dk = (b-A)\cdot C^{1/2}/2(A-k)^{3/2}[b\,(b-k)]^{1/2} \quad \text{and}$$

$$da_k/dk = (b-A)\cdot C^{1/2}[b^{1/2} - (b-k)^{1/2}]/(A-k)^{3/2}(b-k)^{1/2}$$

are

$$[\phi(\bar{q} - x_k^*)^2 + \theta V]/\bar{q} = \bar{q}/f + 2\phi(\bar{q} - x_k^*) \tag{20}$$

and

$$\phi(\bar{q} - x_k^*) = \bar{q}[b - b^{1/2}(b-k)^{1/2}]. \tag{21}$$

From (21), $\bar{q} > x_k^*$ iff $k > 0$; and if $k = 0$, $\bar{q} = x_0^*$. (20) is a quadratic in $\Delta = \bar{q} - x_k^*$, with roots

$$\Delta = \bar{q} \mp \bar{q}[4\phi^2 - 4\phi(\theta V/\bar{q} - \bar{q}/f)/\bar{q}]^{1/2}/2\phi.$$

If the positive sign is used, Δ is of the order of $2\bar{q}$, which is unlikely. Using the negative sign, we have that

$$\Delta > 0 \text{ iff } \bar{q}^2 < f\theta V.$$

Since here $-q' = -\partial\bar{q}/\partial p = f$, the finding of Section 4 is confirmed. The likelihood of $\bar{q} > x^*$ increases with the absolute value of the partial slope of the (mean) demand function and with the variance of the demand process.

6. *Structure of Equilibrium*

To illustrate and substantiate our previous analysis, we are going to show, for particular parameter values, the existence of a long run equilibrium with $\bar{q} > x_k^*$. To this end, we simplify a step further by assuming all firms identical and each one exposed to a residual demand of the form

$$F = g\bar{p}$$

with g a constant and $\bar{p}$ the 'average price'. If individual firms are negligibly small, $\bar{p}$ means the group's average price; otherwise $\bar{p}$ stands for the average price of the group, excluding the representative firm. The conditions defining *symmetric long run equilibrium* are:

$$(p, k) \in \operatorname{argmax} E\pi, E\pi = 0, \text{ and } p = \bar{p}. \tag{22}$$

In such an equilibrium, all firms face the same partial demand schedule $q(p) = F - fp = g\bar{p} - fp$; each firm chooses the profit maximizer (that is expected revenue maximizer) (p, k) and the average price $\bar{p}$ happens to be equal to p. Thus the firms operating in the market play a symmetric Nash equilibrium point of the strategic game with (p, k)-strategy space(s) and payoff function(s) $E\pi$. Moreover, each operating firm makes a normal profit equal to zero: $E\pi = 0$. A potential entrant, if it is an identical clone of the firms already in the market, would also face the partial demand schedule $q(p) = g\bar{p} - fp$ and at best make zero profit. In this sense, the equilibrium precludes profitable entry.

7. *Existence of Equilibrium*: *'Just-In-Time' Production Schedule*

In this section, we assume that the representative firm produces to order or 'just-in-time'. We further assume, as in Section 6, that all firms are identical clones and face a linear demand curve of the form $q(p) = F - fp = g\bar{p} - fp$. We want to show that for specific parameter values, there exists a symmetric long run equilibrium with $\bar{q} > x_k^*$.

First of all, our foregoing analysis applies, when the firm follows a 'just-in-time' production schedule. For then a short run production cost function (4) leads to an expected operating cost function (5) with $\phi = \theta = b$. Now we are ready to state a

Theorem. There are values of the parameters A, B, C, b, f, g, and V such that a symmetric long run equilibrium with $\bar{q} > x_k^*$ exists.

Proof. Put $A = b/2$, $B = 6$, $C = b = 1.25$, $f = 0.2$. With these parametric choices, $\phi = \theta = 1.25$ and $2f\phi = 0.5$. Further, the condition $k < A$ becomes $k < b/2$ and (15) becomes

$$x_k^{*2} = \frac{b-k}{b/2-k}. \tag{23}$$

x_k^* is increasing in k with $x_k^* \to 1$ as $k \to -\infty$ and $x_k^* \to \infty$ as $k \to b/2$. From (23),

$$k = \frac{b}{2} \cdot \frac{x_k^{*2} - 2}{x_k^{*2} - 1} \quad \text{and} \quad \frac{b}{2} - k = \frac{b}{2} \cdot \frac{1}{x_k^{*2} - 1}.$$

Therefore (16) yields

$$a_k = B - b\sqrt{2} \cdot (x_k^{*2} - 1)^{1/2} + 2bx_k^*. \tag{24}$$

Now we are prepared for an explicit solution of the problem Max $E\pi$. Rearranging terms yields

$$E\pi = -p^2 r + ps_k - d_k - \theta V \tag{25}$$

with

$$r = f + \phi f^2 = 1/4,$$

$$s_k = F + a_k f + 2f\phi(F - x_k^*),$$

$$d_k = a_k F + \phi(F - x_k^*)^2.$$

Let us first maximize $E\pi$ with respect to p, given k. The solution is given by the first order condition

$$2pr = s_k \text{ or } p = s_k/2r$$

and therefore

$$E\pi(k) = \frac{s_k^2}{4r} - d_k - \theta V, \tag{26}$$

provided $s_k > 0$. (Otherwise $p = 0$ is optimal. This possibility will be excluded later.) Hence, with $f = 0.2$, $\phi = 1.25$, $2f\phi = 0.5$, and $r = 1/4$, one obtains

$$(27) \qquad E\pi(k) = s_k^2 - d_k - \theta V$$
$$= (1.5 \cdot F + 0.2 \cdot a_k - 0.5 \cdot x_k^*)^2 - a_k F - 1.25 \cdot (F - x_k^*)^2 - \theta V.$$

Using (24), (27) can be expressed in terms of x_k^* only:

$$(28) \quad E\pi(x_k^*) = (1.5 \cdot F + \tfrac{1}{5} B - \tfrac{1}{4}\sqrt{2} \cdot (x_k^{*2} - 1)^{1/2})^2$$
$$- (B - b\sqrt{2} \cdot (x_k^{*2} - 1)^{1/2} + 2bx_k^*) F - 1.25 \cdot (F - x_k^*)^2 - \theta V.$$

Let ℓ solve $s_\ell = 0$.

Lemma 1. For $F = 16$, $E\pi(x_k^*)$ attains a maximum and all its maximizers x_m^* satisfy $2 < x_m^* < x_\ell^*$.

Proof. Postponed to the appendix. Q.E.D.

Next fix $F = 16$ and choose a corresponding maximizer x_m^* of $E\pi(x_k^*)$. For $x_k^* = 3$ or $k = \frac{7}{16} b$, $s_k^2 - d_k = [25.2 - 1]^2 - 16 \cdot [6 + 2b] - 1.25 \cdot 13^2 > 0$.
Hence $s_m^2 - d_m \geq s_k^2 - d_k \geq 0$. Choose $V = \theta^{-1} \cdot [s_m^2 - d_m]$. Then

$$(29) \qquad E\pi(x_m^*) = 0\,.$$

Since $x_m^* < x_\ell^*$, $s_m > 0$. Put $p = \bar{p} = 2s_m$ and $g = 8/s_m$. Then $F = g\bar{p} = 16$ and

$$(30) \qquad p = \bar{p}\,.$$

It remains to be shown that, indeed,

$$(31) \qquad (\bar{p}, x_m^*) \in \operatorname{argmax} E\pi, \text{ if } F = 16\,.$$

It suffices to exclude the possibility $p = 0$ when maximizing (25). By (24) and (25), $E\pi(0, x_k^*) = -d_k - \theta V < 0$, whatever x_k^*. Since $E\pi = 0$ can be achieved, choosing $p = 0$ is sub-optimal.
Finally, $x_m^* > \sqrt{2} = x_0^*$ implies $m > 0$. Hence, by (21),

$$(32) \qquad \bar{q} > x_k^* \quad \text{for} \quad k = m\,.$$

As asserted by the theorem, we have found parameters and $p = \bar{p} = 2s_m$, $k = m$ such that the long run equilibrium conditions (22) are satisfied with capacity x_k^* less than permanent demand $\bar{q} = F - fp = g\bar{p} - fp$. Q.E.D.

8. *Existence of Equilibrium: Intertemporal Inventory Management*

In this section, we assume that the representative firm engages in intertemporal inventory management to buffer demand shocks. Costs of maintaining (or running out of) inventory in period t are given by

$$(33) \qquad h \cdot (I_t - I^*)^2\,,$$

$h > 0$ and I^* parameters. The inventory process is subject to the material balance identity

$$I_t = I_{t-1} + x_t - q_t, \tag{34}$$

where q_t is realized period t demand, generated by the process (3). There are no nonnegativity constraints on inventory. (But normally large back-order accumulations will be very costly.)

From (4) and (33), the period t operating cost of the firm amounts to

$$\gamma_t = a x_t + b(x_t - x^*)^2 + h(I_t - I^*)^2, \tag{35a}$$

and the total expected operating cost over an infinite horizon is given by

$$E\Gamma = E \sum_{t=1}^{\infty} D^{t-1} \gamma_t, \tag{35b}$$

where D is the firm's discount factor, $0 < D < 1$. We assume that the firm determines its production schedule by the 'partial adjustment rule'

$$x_t = \bar{q} - \alpha(I_{t-1} - I^*)^2, t = 1, 2, \ldots, \tag{36a}$$

where α is determined by the coefficients of the objective function:

$$\alpha = \{(h/b)^{1/2} [4 + (h/b)]^{1/2} - (h/b)\}/2. \tag{36b}$$

Note that $0 < \alpha < 1$ for all $b > 0$ and $h > 0$. In an ealier version of this paper, we resort to old-established inventory theory as in HOLT et al. [1960] and show that (36) is the limit of the optimal adjustment rule as $D \to 1$. It can also be shown to be optimal over an undiscounted finite horizon. Use of (36) can be justified on the ground that the scheduling interval will be a short period of time, e.g. a week; and hence the discount factor will differ only very slightly from 1.

A scheduling rule of the form (36), when demand is a stationary process, implies a stochastic steady state, in which inventory levels and output rates can be predicted up to probability distributions, which depend on $\bar{q}$ and V, the moments of the demand process. Those relations are made explicit in

Lemma 2. When a production scheduling rule of the form (36) is applied, the first two moments of the output and inventory processes $\{x_t\}$ and $\{I_t\}$ are, respectively,

$$Ex = \bar{q}, \qquad V_x = \alpha^2 V/[1 - (1 - \alpha)^2]; \tag{37a}$$

$$EI = I^* - \frac{1}{\alpha}(\bar{q} - x^*), \qquad V_I = V/[1 - (1 - \alpha)^2], \tag{37b}$$

where $\bar{q}$ and V are the first two moments of the demand process (3).

Proof. Published by HARLAN D. MILLS [1967]. Q.E.D.

Now for the firm with plant scale x^*, production cost function (4) and inventory management cost function (33), the expected operating cost in period t is

$$E\gamma_t = E[ax_t + b(x_t - x^*)^2 + h(I_t - I^*)^2],$$

and when the decision rule (36) is used, we can substitute from (37) to get the functional form (5) with

$$\phi = b + h/\alpha^2 > 0, \theta = (b\alpha^2 + h)/[1 - (1 - \alpha)^2] > 0. \tag{38}$$

Again, we obtain a

Theorem. There are values of the parameters A, B, C, b, f, g, h, and V such that a symmetric long run equilibrium with $\bar{q} > x_k^*$ exists.

Sketch of Proof. Put $A = b/2$, $B = 6$, $C = b$, $f = 1/6$, $\phi = 3$. With $\beta^2 = h/b$, one gets $\alpha = \frac{1}{2}\{\beta \cdot (4 + \beta^2)^{1/2} - \beta^2\}$, $\phi = b \cdot (1 + \beta^2/\alpha^2)$. Choosing $\beta = \frac{1}{4}$, one gets $\alpha \approx 0.221$, $1 + \beta^2/\alpha^2 \approx 2.283$. Hence $\phi = 3$ requires $b = \dfrac{3}{1 + \beta^2/\alpha^2} \approx 1.314$ and $h = \beta^2 b \approx 0.082$. Choose these values for b and h. After these parameter choices, the remainder of the proof parallels the one in the previous section. Details are provided in an earlier draft of the paper. Q.E.D.

9. *Concluding Remarks and Qualifications*

This is not the first inquiry into excess capacity in a stochastic demand setting, or in an inventory-holding firm. But it is the first fully dynamic model which considers excess capacity, inventories, and stochastic demand. GOULD [1978], in a model with inventories and stochastic demand, finds, in passing, that the Chamberlin theorem holds up. His model, though multi-period, is not intertemporal: it is a variant of the 'newsboy problem' of LADERMAN, LITTAUER and WEISS [1953]; goods put into stock for sale on day t cannot be sold on day $t + 1$. Nor is there any price-searching behavior, or group price equilibration in Gould's model.

DRÈZE and GABSZEWICZ [1967] also find in favor of necessary excess capacity. Their cubic cost firm, though facing stochastic demand, shares its market with producers of identical goods, and cannot store output for future sales. Their model omits the vague Chamberlinian interplay of prices among differentiated-product suppliers that has been modelled in this paper by negatively sloped demand and zero long run profit; the products within their group are perfect substitutes.

In contrast, the crucial elements of this model are producer response to stochastic demand (under the JIT regime or under an inventory management regime) and the Chamberlin large-group hypothesis of competition with some vestigial price inelasticity to exploit, but less than oligopolistic dependency. The quadratic example shows that 'excess capacity' can no longer be concluded. The result suggests that the equilibria of large-group monopolistically competitive and perfectly competitive firms may be less different in terms of allocative implications than had previously been believed, even by those most ready to question the importance of those differences.

Summary

A model of the firm is presented which preserves the three crucial features of Chamberlinian large-group equilibrium (numerous firms, free entry, finite demand elasticity), but which also stipulates additive i.i.d. stochastic demand shocks in discrete time. Storable, but not necessarily stored output is considered. Production costs are assumed to be quadratic and non-stochastic. In such a regime it is found that the excess capacity theorem need not hold: mean output rates can exceed the minimum average cost level of output in both the short run and the long run.

Zusammenfassung

Es wird ein Modell der Unternehmung mit den drei entscheidenden Eigenschaften des Gleichgewichts großer Gruppen im Sinne von Chamberlin (zahlreiche Firmen, freier Marktzutritt, endliche Nachfrageelastizität) vorgestellt. Zusätzlich werden additive, identisch und unabhängig verteilte stochastische Nachfrageschocks in diskreter Zeit vorausgesetzt. Der Output is lagerbar, wobei nicht unbedingt Lagerhaltung betrieben wird. Die Produktionskostenfunktion ist quadratisch und nicht stochastisch. Unter diesen Voraussetzungen braucht das „Excess Capacity Theorem" nicht zu gelten: Kurzfristig und langfristig kann die mittlere Outputrate größer sein als das Produktionsniveau, welches die Stückkosten minimiert.

Appendix: Proof of Lemma 1

Let $F = 16$. $E\pi(x_k^*)$ as given by (28) is differentiable and

$$\frac{dE\pi}{dx_k^*} = -2s_k \cdot c_1 - bF \cdot c_2 + 2.5 \cdot (F - x_k^*), \text{ with}$$

$$s_k = 1.5 \cdot F + \tfrac{1}{5}B - \tfrac{1}{4}\sqrt{2}(x_k^{*2} - 1)^{1/2},$$

$$c_1 = \tfrac{1}{4}\sqrt{2} \cdot \frac{x_k^*}{(x_k^{*2} - 1)^{1/2}},$$

$$c_2 = 2 - \sqrt{2} \cdot \frac{x_k^*}{(x_k^{*2} - 1)^{1/2}}.$$

It suffices to show that $\frac{dE\pi}{dx_k^*} > 0$ for $x_k^* \leq 2$ and $\frac{dE\pi}{dx_k^*} < 0$ for $x_k^* \geq x_\ell^*$. Recall $B = 6$, $b = 1.25$.

Case 1. $x_k^* \leq 2$.

Then $0 \leq 2s_k \leq 56$ and $c_1 = \tfrac{1}{4}\sqrt{2} \cdot \frac{x_k^*}{(x_k^{*2} - 1)^{1/2}}$.

Collecting negative terms (neg) of $\frac{dE\pi}{dx_k^*}$ yields

$$\Sigma|\text{neg}| = 2s_k c_1 + 2bF \leq 40 + 14 \cdot \sqrt{2} \cdot \frac{x_k^*}{(x_k^{*2} - 1)^{1/2}}.$$

Collecting positive terms (pos) of $\frac{dE\pi}{dx_k^*}$ yields

$$\begin{aligned}\Sigma\,\text{pos} &= b\sqrt{2} \cdot F \cdot \frac{x_k^*}{(x_k^{*2} - 1)^{1/2}} + 2.5 \cdot (F - x_k^*) \\ &\geq 35 + 20\sqrt{2} \cdot \frac{x_k^*}{(x_k^{*2} - 1)^{1/2}} > 41 + 17\sqrt{2} \cdot \frac{x_k^*}{(x_k^{*2} - 1)^{1/2}},\end{aligned}$$

since $1 < x_k^* \leq 2$ implies $\frac{x_k^*}{(x_k^{*2} - 1)^{1/2}} \geq 2$.

Hence $\Sigma\,\text{pos} > \Sigma|\text{neg}|$ and $\frac{dE\pi}{dx_k^*} > 0$.

Case 2. $x_k^* \geq x_\ell^*$.

Then $\tfrac{1}{4}\sqrt{2} \cdot x_k^* \geq \tfrac{1}{4}\sqrt{2} \cdot (x_k^{*2} - 1)^{1/2}$

$$\geq 25.2,\ x_k^* \geq 71, \frac{x_k^*}{(x_k^{*2} - 1)^{1/2}} \leq \frac{71}{(71^2 - 1)^{1/2}} < 1.001,$$

$s_k \leq 0$, $|s_k| < \tfrac{1}{4}\sqrt{2} \cdot x_k^*$, $c_1 \leq \frac{\sqrt{2}}{4} \cdot 1.001$.

Collecting positive terms of $\frac{dE\pi}{dx_k^*}$ yields

$$\Sigma \text{ pos} = 2\,|s_k|\,c_1 + \sqrt{2}\cdot\frac{x_k^*}{(x_k^{*2}-1)^{1/2}}\,bF + 2.5F < \frac{1.001}{8}\cdot x_k^* + 68.32 < 0.126\,x_k^* + 68.32.$$

Collecting negative terms yields

$$\Sigma|\text{neg}| = 2bF + 2.5\,x_k^* = 40 + 2.5\,x_k^* \geq 1.5\,x_k^* + 111.$$

Thus $\Sigma|\text{neg}| > \Sigma$ pos and $\frac{dE\pi}{dx_k^*} < 0$. Q.E.D.

References

Cassels, J. [1937], "Excess Capacity and Monopolistic Competition," *Quarterly Journal of Economics*, 51, 426–443.

Chamberlin, E. [1933], *The Theory of Monopolistic Competition,* 6th edition 1950, Cambridge, MA.

-- [1937], "Monopolistic or Imperfect Competition?," *Quarterly Journal of Economics*, 51, 557–580.

-- [1938], "Reply to Kaldor," *Quarterly Journal of Economics*, 52, 530–538.

De Vany, A. S., and Saving, T. R. [1983], "The Economics of Quality," *Journal of Political Economy*, 91, 979–1000.

Dixit, A. K., and Stiglitz, J. E. [1977], "Monopolistic Competition and Optimal Product Diversity," *American Economic Review*, 67 (3), 297–308.

Drèze, J., and Gabszewicz, J. J. [1967], "Demand Fluctuations, Capacity Utilization and Prices," *Operations Research Verfahren* III, 119–141.

Gilbert, J. P. [1990], "The State of JIT Implementation and Development in the USA," *International Journal of Production Research*, 28, 1099–1109.

Gould, J. [1978], "Inventories and Stochastic Demand: Equilibrium Models of the Firm and Industry," *Journal of Business*, 51, 1–42.

Holt, C., Modigliani, F., Muth, J., and Simon, H. [1960], *Planning Production, Inventories and Workforce*, Eglewood Cliffs, NJ.

Kahn, R. F. [1935], "Notes on Ideal Output," *Economic Journal,* 45, 1–35.

Kaldor, N. [1935], "Market Imperfection and Excess Capacity," *Economica*, 2, 33–50.

Laderman, J., Littauer, S., and Weiss, L. [1953], "The Inventory Problem," *Journal of the American Statistical Association*, 48, 717–732.

Lancaster, K. [1979], *Variety, Equity and Efficiency*, New York.

-- [1980], "Competition and Product Variety," *Journal of Business*, 53, S79–S103.

Margolis, S. E. [1985], "The Excess Capacity Controversy: A Critique of Recent Criticism," *Economic Inquiry*, 24, 265–275.

-- [1989], "Monopolistic Competition and Multiproduct Brand Names," *Journal of Business*, 62, 199–209.

Mills, H. D. [1967], "Smoothing in Inventory Processes," pp. 131–148 in: M. Shubik (ed.), *Essays in Mathematical Economics in Honor of Oskar Morgenstern*, Princeton, NJ.

SCHMALENSEE, R. [1972], "A Note on Excess Capacity and Monopolistic Competition," *Journal of Political Economy*, 80, 586–591.

STIGLER, G. [1939], "Production and Distribution in the Short Run," *Journal of Political Economy*, 47, 305–327.

VARIAN, H. R. [1984], *Microeconomic Analysis*, 2nd edition, New York.

VINER, J. [1931], "Cost Curves and Supply Curves," *Zeitschrift für Nationalökonomie*, 3, 23–46.

WONNACOTT, P., and WONNACOTT, R. W. [1987], *Economics*, New York.

Hans Haller
Department of Economics
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061
U.S.A.

Daniel Orr
Department of Economics
University of Illinois
Champaign, IL 61820
U.S.A.