To the Graduate Council:

I am submitting herewith a dissertation written by Ida McCalip Dell'Isola entitled "An investigation of the use of algebra tiles to introduce the basic algebraic concepts of integers, equations, and polynomials in community college developmental algebra classes." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Education.

John R. Ray, Major Professor

We have read this dissertation
and recommend its acceptance:

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Associate Vice Chancellor and
Dean of The Graduate School

An investigation of the use of algebra tiles to introduce the basic algebraic concepts of integers, equations, and polynomials in community college developmental algebra classes

A Dissertation

Presented for the

Doctor of Philosophy

Degree

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Ida McCalip Dell'Isola

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Dedication

I dedicate my dissertation to the memory of my parents, Obed and Willie McCalip. The memory of their love, sacrifice, and devotion continues to inspire me.

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ABSTRACT

The purpose of this study was to examine the effectiveness of using activities involving algebra tiles to introduce algebra concepts. The study was conducted with 119 students enrolled in six elementary algebra classes at Walters State Community College. Three instructors each taught a control and a manipulative-based class. The study was designed to compare the mathematical achievement, retention, anxiety, and attitude of students based on treatment groups and past algebra experience groups.

In this quasi-experimental study, equivalency of treatment groups was determined by independent samples *t*–tests on pre-test scores. Analysis of variance on pre-and post-tests with treatment, instructor, and past algebra experience as factors and pre-test and post-test as repeated factors was used to analyze differences in achievement, attitude, and anxiety. To measure retention, the post-test was used as a covariate in an analysis of variance on the final exam.

The most significant finding ($p = .045$) was that students with no recent experience in the manipulative-based group experienced greater retention than did students with no recent experience in the control group. The mean achievement gain of all no-recent-experience students was significantly higher than the mean gain of recent-experience students ($t = -4.884$, $df = 117$, and $p < .001$). Additionally, the mean attitude of all no-recent-experience students was better than that of recent-experience students ($p < .001$). There was a significant time by treatment group interaction for attitude ($p = .034$). At pre-test the control group attitude score appeared to be higher than the manipulative group score. Over time a trend appeared to be developing with the

manipulative group's attitude improving so that the gap was narrowing. There was no significant difference in the anxiety of students in the treatment and control groups. Both groups experienced a significant lessening of anxiety.

The researcher believes it is significant that the previous experience group with the better attitude experienced a significant difference in retention when instructed with manipulatives. With the attitude of the manipulative group showing a trend toward improvement, there is reason to believe that the treatment might have produced greater differences in a longer study.

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Introduction

Thirty-four percent of all entering first-time freshmen at public two-year colleges must enroll in a remedial or developmental mathematics course, according to the U. S. Department of Education, National Center for Education Statistics. A profile of these students reveals that they fall into two categories: traditional college age students and non-traditional returning adults.

Traditional age students are in the developmental classes because of low scores on the ACT or SAT college entrance exam. Although most of them completed the required algebra courses in high school, they failed to retain algebra skills and lacked the ability to transfer skills and concepts studied in mathematics classes to unfamiliar situations.

Non-traditional, returning adult students have been away from school for several years and usually need to be reintroduced to each concept. Many students in this category do not have high school diplomas and have entered college by taking the GED. They may have never had an algebra course and are often very anxious about studying mathematics.

Finding new teaching methods that will result in successful completion of developmental mathematics for both of these groups of students is the focus of this study. Davis (1986) said that when students are given step-by-step instructions to perform mathematical operations and are not provided with conceptual tools to allow them to think about the mathematics being presented, they will only be able to carry out the procedure when tasks are presented to them in some standard fashion. This may be the reason for so many students failing to make a satisfactory score on the college entrance exams. Since most topics studied in developmental classes repeat those previously studied in high school, student apathy and even resentment are commonplace. Many of these students have successfully completed at least two algebra courses in high school, but low ACT or SAT scores have labeled them as academically deficient. If college developmental mathematics courses merely repeat unsuccessful traditional methods there is reason to believe that results will be the same.

The National Council of Teachers of Mathematics (NCTM) advocates "changes in instructional patterns and in the roles of both teachers and students" (NCTM, 1989, p. 125). Instructors must encourage students to become active learners. In the algebra classroom, students should actively investigate mathematical concepts by using manipulative materials and technology and by engaging in dialog with other students. The goal is to move away from encouraging students to memorize procedures and toward promoting mathematical reasoning in all students. In addition, research indicates that a change is needed in the way mathematics is organized and taught, and in teacher beliefs about what it means to know and do mathematics. "The goal should be... to learn ways of making sense of mathematics, inventing procedures to solve new problems, and building models to understand mathematical situations" (Schram, 1988, p. 3).

In the traditional classroom, algebra is presented as sets of isolated algorithms and abstract concepts which students practice as decomposed skills. Campione (1990) said that these methods of instruction most often lead to what may be referred to as "inert knowledge"--a set of skills and facts that are only used when prodding occurs from the instructor. Mathematics curricula assume that, armed with the necessary skills, students will know when and how to use them. However, many students see no relation between these skills and the situations in which the skills are useful. This may be because most activities in the algebra classroom begin and end in the abstract. The philosophy is that knowing mathematics is a totally rational experience. Students are not given the opportunity to experience the mathematics through their senses and therefore have no ownership of it.

Consequently, many of these students fail to make satisfactory grades on the ACT or SAT exam and are placed in developmental mathematics classes in college. These courses are intended to remedy deficiencies from lack of mathematical preparation or to update skills for those students who have not taken a mathematics course in a long period of time. When these courses merely repeat traditional practices, students become apathetic, resentful, or at best owners of "inert" discrete skills. Experiencing mathematics through activities involving manipulative materials may give these students opportunities for insights into mathematical concepts that they have never had before. It is a chance for both teachers and students to approach learning mathematics from a new standpoint, which may yield better results.

The Problem

Dyer (1997) found that although research has shown that instructional methods using manipulative materials have been successful in teaching mathematical concepts to elementary and secondary students, little research exists addressing the use of such strategies with college level students. College professors assume that college students think rationally and can follow lectures on the logical development of algebraic concepts. But Renner (1976) found that approximately half of college freshmen were still operating on the concrete-operational level as defined by Piaget. These students resort to trying to memorize steps for solving each type of problem that they encounter in mathematics. When the impossibility of this undertaking hits them, they become anxious and develop negative attitudes toward mathematics. Many of these students may be able to bridge the gap from concrete-operational thought to formal-operational thought by investigating mathematical concepts through activities with manipulative materials. According to Piaget, a student's logical thought processes change significantly when he/she is permitted to interact with objects, events, and situations in his environment (Renner, 1976).

Kinard (1996) recommended research on the best way to provide college developmental mathematics students with the means of interacting with their environment and experiencing mathematics. This study will address the problem of finding whether or not manipulative activities are appropriate for helping these students develop mathematical concepts, apply these concepts when appropriate, and retain mathematical skills and knowledge that they have acquired.

The Purpose

The purpose of this study was to investigate relationships between manipulative-based instruction and achievement, retention, anxiety, attitude, and past algebra experience of community college elementary algebra students. The experimental classes participated in activities designed to allow students to demonstrate a concept concretely, draw diagrams representing the concept, and translate the concept to symbols. All of the activities were planned to allow students to work in small groups to discuss and share ideas while being guided by the instructor. The study was intended to compare mathematical achievement, retention, anxiety, and attitude of students based on treatment groups and past algebra experience groups. The following questions were investigated:

1. How do instructional activities involving manipulative materials and/or past algebra experience effect the mathematical achievement of students in developmental college algebra courses?
2. How do instructional activities involving manipulative materials and/or past algebra experience effect the concept retention of students in developmental college algebra courses?
3. How do instructional activities involving manipulative materials and/or past algebra experience effect the anxiety of students in developmental college algebra courses?

4. How do instructional activities involving manipulative materials and/or past algebra experience effect the attitude of students in developmental college algebra courses?

Need for the Study

As a result of the reform movements, teaching strategies are being scrutinized at all levels of mathematics education. It is especially important that the best methods for teaching college developmental algebra students be found. It seems obvious that traditional methods have failed for most of these students. Therefore, investigating the effects of using manipulative materials to introduce algebra concepts is a very relevant and useful study.

In 1996 Kinard studied the use of manipulatives in developmental algebra classes. She found no significant differences in achievement of control and experimental groups, but recommended replicating the study with a larger sample size, better-trained instructors, and homemade algebra tiles given to students to use whenever they needed. She also suggested that more time be spent familiarizing students with the tiles and developing the concept of negative numbers. Additionally, Kinard questioned whether age played a role in students' resistance to the use of manipulatives. The design of this study incorporated Kinard's suggestions and questions.

Dyer (1996) found a significant increase in the content learning of polynomial multiplication by college algebra students when receiving manipulative instruction. She suggested that using algebra tiles assisted students in becoming mature in their mathematical thinking and improved their attitudes. As a result, student performance was also improved. Dyer recommended that college instructors experiment to find manipulative instructional methods that best fit their students' learning needs.

Instructional methods involving manipulative materials need to be incorporated into the college algebra classroom. Drapac (1980) suggested that a study should be done with adults with no previous background in algebra to determine the usefulness of algebra tiles without contamination of previously learned habits or attitudes. Kinard (1988) said that a study of using algebra tiles in developmental classes by more than one teacher should be done. Kanai (1994) said that experimental studies are needed to explore which topics in particular would benefit from the use of manipulatives. And Goldsby (1994) indicated that research on manipulatives is needed, especially for longer periods of usage.

In summary, the purpose of this study was to analyze the relative effectiveness of traditional and manipulative-based instruction in community college elementary algebra classes and give a better perception of whether student performance and retention can be enhanced by using manipulatives. Additionally, the study investigated any interactions between previous algebra experience and the effectiveness of manipulative instruction. And finally, since college developmental algebra students' high levels of mathematics anxiety often prevent them from succeeding in algebra courses, this study was designed to give insight into the usefulness of manipulatives in reducing mathematics anxiety and improving students' attitudes toward math.

Assumptions

The following assumptions were made for this study:

1. Students responded honestly to the attitude, anxiety, student information, and exit surveys administered as part of the study.
2. Students' mathematical anxieties and mathematical attitudes can be measured. The instruments selected to measure these variables in this study are valid and reliable. Data for the Fennema-Sherman Attitude Scales are available (O'Neal, 1988 and Thompson, 1993).
3. The test developed to measure achievement was appropriate and covered the course curriculum objectives. The test was developed by the researcher and reviewed by faculty from Walters State Community College and Pellissippi State Technical Community College for face validity.
4. The researcher and other instructors participating in the study introduced no bias into the study.

Limitations

The study was limited to six intact elementary algebra classes at WSCC. Students were not randomly assigned to classes. Therefore, the results of this study are limited in generalizability.

Student responses to the mathematics anxiety and attitude surveys only relate what students perceive and were willing to relate about their actual attitudes and anxieties.

The study was limited to one unit of instruction lasting six weeks and covering three algebraic concepts. Time for manipulative activities was limited by the requirements of an extremely tight course syllabus.

The manipulatives used in this study were limited to algebra tiles.

Delimitations

Students involved in the study were elementary algebra students of three instructors at WSCC in the Fall Semester of 1998 and may not be representative of the general population.

The research analysis was limited to students who completed the unit of study.

Definition of Terms

The following definitions were used in this study:

Manipulative materials: Concrete materials appealing to several senses that can be handled, touched, and moved about by the students to represent mathematical concepts. In this study the manipulatives used were algebra tiles.

Mathematics anxiety: Feelings of tension which interfere with the manipulation of numbers and the solving of mathematical problems either in ordinary life situations or in academic setting as measured by the Fennema-Sherman Mathematics Attitude Scale.

Attitude toward mathematics: A learned favorable or unfavorable response to mathematics as measured by the Fennema-Sherman Mathematics Attitude Scale.

Polynomial: An algebraic expression in which the terms are limited to numbers and/or variables with positive integral exponents.

Integers: Positive or negative whole numbers and zero.

Equations: Equations to be solved in this study involved polynomials with integral exponents no higher than one.

Previous algebra experience: Students who have taken algebra courses within the past three years versus students whose algebra courses were more than three years ago or never.

Concept retention: The ability to recall and use algebra skills and concepts a period of time after first studying them. In this study the period of time was seven weeks.

Hypotheses

The following null hypotheses were tested in this study:

H_1: There will be no significant differences in mathematics achievement between treatment groups and/or experience groups.

H_2: There will be no significant differences in concept retention between treatment groups and/or experience groups.

H_3: There will be no significant differences in mathematics anxiety between treatment groups and/or experience groups.

H_4: There will be no significant differences in mathematics attitude between treatment groups and/or experience groups.

Organization of the Study

This study is composed of five major chapters. Chapter I includes the introduction to the study, a statement of the problem and purpose, need for the study, assumptions, delimitations, definitions of terms, hypotheses, and organization of the study. A review of the literature related to learning theories, philosophies, and manipulative usage effecting mathematics achievement, attitude, and anxiety is covered in Chapter II. An explanation of the methodology and procedures employed in the study is presented in Chapter III. Chapter IV reports on the results of statistical analysis of the data, and Chapter V summarizes the results of the study and outlines conclusions and recommendations.

Chapter II

Review of the Literature

This study will investigate the effects of using algebra tiles to teach the basic algebra concepts of integers, equations, and polynomials. This chapter summarizes findings from a review of the literature addressing four areas relating to the study. The areas are: a) historical and philosophical background, b) supportive learning theories, c) rationale for change, and d) research on the impact of manipulatives on mathematical achievement, attitude, and anxiety.

Historical and philosophical background

"I hear and I forget,

I see and I remember,

I do and I understand."

Chinese Proverb

(Welchman-Tischler, 1992, p.1)

The debate between two main theories of the basis of knowledge is being continued in mathematics education today. Is the basis of mathematical knowledge empiricism or rationalism? Traditional mathematics education, especially mathematics of algebra and beyond, adheres to the philosophy of rationalism. But recent reform movements advocate an empirical basis for mathematical knowledge.

Following Descartes' philosophy, mathematicians have historically rejected sense experience as being necessary to obtain mathematical knowledge. Knowledge of mathematics had to be acquired through reason, independently from sense experience. All mathematical knowledge was held to come from intuition and deduction. According to Descartes, knowledge from intuition is grasped immediately by direct awareness. He believed that this innate awareness was the basis for deduction of other truths.

This "certainty of demonstrations and evidence of reasoning" in mathematics appealed to philosophers such as Descartes and to other mathematicians, and has been used traditionally to teach mathematics (Miller, 1987, p. 211). Teachers assume that students have intuitive knowledge of mathematical concepts such as variables, equations, derivatives, etc. Therefore, students are expected to understand the concepts that are developed logically and efficiently by the teacher. However, when asked to explain the reasoning behind the operations or to solve problems requiring the skills, students are exposed as only having memorized the algorithms.

In defining experience as the source of ideas, Locke named two kinds of experience--sensation and reflection (Miller, 1987). The basis for reform in mathematics education corresponds with Locke's description of how knowledge is obtained. External experiences are provided for students through the use of manipulatives, technology, and dialog with other students. Then students are given opportunities to internalize experiences provided in the mathematics class by reflection -- thinking, believing, doubting, affirming, denying, and comparing.

Most algebra lectures are not helpful to a large number of students. What the teacher says is not nearly as important as what the students think. Their innate ideas, or intuitions, as described by Descartes, are not as well formed as teachers have assumed them to be. Kant said that all human cognition begins with intuitions, proceeds to conceptions, and ends with ideas. But Polya (1963) argued that learning begins with action and perception, proceeds to words and concepts, and should end in desirable mental habits.

Action and perception suggest manipulating and seeing concrete things such as algebra tiles, pattern blocks, graphing calculators, etc. Polya maintained that for efficient learning to take place students should proceed through phases of exploration, formalization, and assimilation. Eventually the material should become part of the mental attitude of the learner.

Traditional teaching emphasis on practice in symbol manipulation and algorithms before solving problems ignores the fact that knowledge often emerges from the problems. Aristotle stressed the need for the right preliminary questions. Promoting active learning by engaging students in the solution of a carefully selected problem followed by appropriate questioning from the instructor underlies the idea of the "Socratic method." Polya (1963) said that the fundamental art form of teaching is the Socratic dialogue. By providing appropriate experiences and asking appropriate questions the teacher, imitating Socrates, extracts the desired concepts from the students.

Spando and Zeidler (1996) characterized the difference between algorithmic and conceptual learning as the difference between *faith* and *deed.* In traditional mathematical classes, learning outcomes are predetermined and very limited for most students. Knowledge is delivered to the students through instructional techniques of the teacher. The student has faith in what the teacher knows and believes what the teacher teaches. However, when teachers expand the parameters and provide concrete experiences for students to pursue mathematical investigations, they are able to inspire students to deeper understanding by Socratic inquiry. The student then develops an ownership of understanding by deed and believes what experience teaches. Instead of viewing the teacher as the sole possessor and authority of knowledge, students begin to decide for themselves what is true.

Supportive learning theories

Learning theorists have suggested for some time that true knowledge evolves through direct interaction with the environment. Jean Piaget (1972) suggested that increasingly sophisticated cognitive structures are responses to physical experiences within the environment. He called his general theoretical framework genetic epistemology because he was interested in how human knowledge is developed. Jerome Bruner (1966) and other constructivists theorized that learning is an active process in which learners construct new concepts based upon current and past knowledge. A newer theory, called situated learning, asserts that knowledge needs to be presented and learned in a situation that would normally involve that knowledge. Each of these theories places great emphasis on the process dimension of learning, and is focused on how, rather than what, students learn. Highest priority is given to understanding which "can only follow the individual's personalized perception, synthesis and assimilation of relationships as these are encountered in real situations" (Post, 1980, p.113).

Piaget identified four, now famous, development stages: sensorimotor, pre-operational, concrete-operational, and formal-operational. Many of his experiments were focused on the development of logical and mathematical concepts. Piaget's research indicated that children learn best from concrete activities involving materials appropriate for their current developmental stage. Knowledge derives from the adaptation of the individual to his or her environment. The process of learning is assimilation of new situations and new objects to former structures and modifying these structures to the new characteristics of those objects. Piaget described this cognitive structure as a person's internal "mental map" of concepts for understanding and responding to experiences in the environment.

Post (1981) noted that Piaget emphasized the important role of social interaction in the rate and quality of intellectual development. Exchanging and discussing ideas, and reflecting upon one's own ideas and the ideas of others, leads to a more critical and realistic view of one's self and others.

Much of the constructivism theory is linked to research originating with Piaget. Bruner, like Piaget, described learning as an active process in which learners construct new ideas or concepts based upon their current and past knowledge. He said that the key to readiness for learning was intellectual development, or how a student views the world. Referring to Piaget, he suggested that the teacher's role was to help students progress from concrete to more conceptual ways of thinking.

The key ideas of constructivism are that learning is active rather than passive and that learners construct new understandings using what they already know. When confronted with new experiences inconsistent with current understanding, learners make judgments and modify knowledge. Piaget referred to this as assimilation/accommodation.

Information processing theories deal only with assimilation and do not incorporate accommodation, which is where knowledge is actively constructed. Understanding cannot be achieved through sets of rules or other symbolic structures. Kloosterman (1993) suggested that people are better at remembering information that they create for themselves than information that they receive passively. And in emphasizing the social part of learning Lev Vygotsky, a Russian psychologist, stressed that accommodating new knowledge involves discussing and defending points of view.

In addition to contributing to the constructivism theory of learning, Vygotsky provided a foundation for the theory of situated learning. By framing the act of learning in both social and cultural contexts, Vygotsky did not focus solely on the cognitive activity of the individual, but also placed attention on the social setting of learning activities. Learning is determined by and connected to the activity and environment in which it is developed. Wilson (1993) said that a key component of situated learning is that activities be authentic or meaningful and purposeful.

Jean Lave (1982) is a leading proponent of situated learning. She asserts that the activities of a domain are framed by its culture. As an example in her research she describes a group of California housewives who did very well at figuring best buys at a supermarket, but who failed to solve school-like pencil and paper mathematics problems with similar solutions. Many of the activities students perform in school are not recognized by members of the culture that they are intended to represent. They are not authentic activities.

Traditional textbook word problems are typically out-of-context problems that deal with contrived situations. Mathematics knowledge tends to be separated from its uses in real life. Wilson (1993) recalled that, as early as 1929, A. N. Whitehead coined the term "inert knowledge" to describe knowledge that can be recalled by students when explicitly asked to do so, but is not used spontaneously when it is relevant in problem-solving. Symbolic reasoning and generalities used to solve problems in school are in direct contrast to "situation-specific competencies and context-based reasoning required to deal with society's economic realities" (Wilson, p.10, 1993).

In the real world single individuals rarely accomplish tasks alone. Therefore, school settings designed around the individual fail to provide activity relevant to successful performance outside the classroom. The theory of situated learning says that learning both inside and outside school advances through social interaction and the social construction of knowledge.

All three of these learning theories imply that teaching cannot be viewed as the transmission of knowledge from the enlightened to the unenlightened. Teachers are no longer in the role of the "sage on stage", but rather act as "guides on the side" to provide learning experiences appropriate for their students. The teacher must pose interesting problems, and then guide, question, and inspire the student on an exploration to make sense of mathematics. This sometimes involves telling students things about mathematics. But the telling should occur only as the need arises after students have had time to explore problems individually and with their classmates. To encourage students to discover principles by themselves, teachers should engage in active dialog (Socratic questioning) with the students.

Providing experiences involving active problem solving with manipulative materials, technology such as graphing calculators, and cooperative learning may be the means of bridging the gap from concrete-operational to formal-operational thought. Meyers (1988) asserted that, according to Piaget, if the student had the prerequisite psychological maturity, his logical thought processes change significantly when he is permitted to interact with objects, events, and situations in his environment.

Rationale for change

Research indicates that a change is needed in the way mathematics is organized and taught, and in teacher beliefs about what it means to know and do mathematics. "The goal should not be the accumulation of large numbers of problems with appropriate algorithmic solutions, but to learn ways of making sense of mathematics, inventing procedures to solve new problems, and building models to understand mathematical situations" (Schram, 1988, p.3).

In its three standards documents, the National Council of Teachers of Mathematics advocates "changes in instructional patterns and in the roles of both teachers and students" (NCTM, 1989, p.125). Instructors must encourage students to become active learners. In the algebra classroom, students should actively investigate mathematical concepts by using manipulative materials and technology, and by engaging in dialog with other students. The goal is to move away from encouraging students to memorize procedures and toward promoting mathematical reasoning in all students.

The curriculum must be developmentally appropriate for students' logical and conceptual development. The principle role of the teacher is to present material to be learned to match the learner's current level of understanding. Curriculum should be organized in a spiral manner so that students continually build upon what they have already learned. An innovative curriculum that provides opportunities for students to explore, collaborate, discover, and reflect will promote active learning. Real inquiry crosses disciplines, reaches into the real world, and involves open-ended problems. If new knowledge is to be actively built, time is needed to build it. Students must have time to reflect on new experiences and reshape previous knowledge structures to accommodate the new experiences.

The NCTM standards give a vision of a curriculum that focuses on students developing mathematical thinking and reasoning through opportunities to solve problems and understand concepts. Content emerges from in-depth experiences with mathematical ideas. Seeley (1995) described rules, facts, and algorithms as necessary knowledge to solve problems which should be the starting point rather than the result of a student's work.

A “buzzword” in mathematics education today is “algebra for all.” Basic algebra serves as the gatekeeper for entry into the successful study of higher mathematics. Instructional practices in the past have led to a narrowing in the number of students who are successful or even enrolled in mathematics courses after the eight grade. When algebra teachers introduce concepts by using abstract logic to connect new concepts to previously learned ideas, only those students functioning at the formal-operational level of intellectual development as described by Piaget can follow the reasoning. The age at which this level is attained is not known, but Meyer (1988) said that it could be anywhere from early adolescence to adult. When confronted with these situations, concrete-operational students resort to memorizing steps. They have no means of incorporating new ideas into a conceptual framework. This inability to think algebraically keeps the door closed to higher mathematics.

Many national study groups have called for a balanced instructional approach, which incorporates problem solving, understanding, and real-life applications into the mathematics curriculum. In 1980 the NCTM issued its *Agenda for Action* which recommended a change in the focus of mathematics education from numbers and computations to problem-solving. This shift in focus is a reflection of cognitive science, which indicates that understanding how students learn is as important as what they learn. In the document, *Everybody Counts*, the National Research Council maintained, "Curricula and instruction in our schools are years behind the times. They reflect neither the increased demand for higher order-thinking skills, nor the greatly expanded uses of the mathematical sciences, nor what we know about the best ways for students to learn mathematics" (p. 74).

In 1995 the American Mathematical Association of Two-Year Colleges (AMAYTC) released *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*. This document addresses the need to change and adapt the traditional mathematics curriculum so students will be involved as active learners participating in worthwhile mathematical tasks. These problem-solving activities include manipulating concrete objects to explore new ideas. The standards published by AMATYC propose work with manipulatives as one way to bridge the gap between the reform calculus movement and introductory college mathematics courses. Jacobson (1993) suggests that most educators feel that mathematics topics from arithmetic to the foundations of calculus can be more easily understood through the instruction involving more hands-on collaborative inquiry.

Students of mathematics have been viewed historically as passive receptacles into which knowledge is poured. Schram (1988) described mathematics content as being broken down into small pieces, "today's lesson," which students are rarely given opportunities to fit together in the larger field of mathematics. The basic ideas elude those students who have not yet developed the ability to reason abstractly. In addition, those functioning on a higher level of reasoning ability are denied opportunities to explore the meaning of the algorithms more thoroughly. Regardless of ability level, active learning with concrete examples is a key part of promoting the transition to more abstract thought processes.

Students' opportunities to learn mathematics are greatly enhanced when the ideas are drawn from their own experiences. Davis (1986) said that if students' perceptions of algebraic concepts are limited to viewing meaningless symbol manipulations on a chalkboard, very little concept building takes place. But if students are engaged in solving a problem in which concrete examples are touched and manipulated, then valuable mental representations are virtually guaranteed.

Schram (1988) recommended a change in the way mathematics is organized and taught, and in teacher beliefs about what it means to know and do mathematics. Traditional "show and tell" teaching of algorithms promotes memorizing procedures without developing reasoning ability. Students can only solve problems presented to them in a familiar sitting. Providing students with hands-on activities that allow them to develop their own rules and procedures may lead students to develop a conceptual framework for handling new situations and arriving at solutions.

Research on the impact of manipulatives on mathematical achievement, attitude, and anxiety

Manipulative mathematics materials are concrete examples that can be used to model abstract mathematical concepts. They appeal to several senses and can be touched and moved around by students. Such materials range from those known almost synonymously with brand names, like Cuisenaire rods, to ordinary objects such as Popsicle sticks, paper clips, or even gummy bears. Suydam (1985) found that manipulative materials are widely used in the primary grades to introduce counting concepts, but their use decreases throughout the elementary grades. Therefore, by the time a student reaches algebra classes, concrete representations of algebraic concepts are rarely used.

Varying levels of cognitive development exist at all grade levels, including high school and college. Many college students still need to see concrete examples to develop mathematical ideas. If the material fits the mathematical idea and fits the learner's needs manipulative material is useful right through high school and beyond. Suydam suggested that the proper use of manipulative materials has a high probability of increasing mathematical achievement for most students. Her surveys show this to be true across a variety of mathematical topics, at every grade level, at every achievement level, and at every ability level. She concludes that a major reason for patterns of errors in using paper-and-pencil procedures is that manipulative materials are abandoned too early. Stodolsky (1985) believes this move away from concrete experiences correlates to the decline in attitude toward mathematics from early elementary years to high school when mathematics is described as the least-liked subject. He concluded that studies of learning under very different conditions are needed to confirm the relations suggested between instructional forms, attitudes, and conceptions of learning.

Sara (1990) also found this to be an important question. She noted that the number of students taking mathematics classes decreases by half each year after mathematics becomes an elective class. To help answer the question, she suggested research to find how attitude is effected by instruction beginning with manipulative mathematics materials and progressing to abstractions.

In studying the effect of instruction with manipulatives with gifted elementary students, Sara found that regardless of ability level, manipulatives are a key part of promoting the transition to more abstract thought processes. Stallings-Roberts established that using a manipulative device gave lower ability high school students the opportunity to move in a thoughtful manner from concrete examples to the abstract complexities of absolute value. After working with manipulative devices to find surface area, college students expressed confidence in their ownership of the concept of area in a study by Schram (1988). Martelly (1998) found that manipulatives and concrete experiences are beneficial to adult learners' mathematical achievement and attitude. She indicated that manipulative activities should be considered in designing learning experiences for the adult learner. Martelly also observed that there seemed to be a correlation between previous algebra experience and the effectiveness of manipulative instruction. She suggested that a comparison be made by using manipulative materials to teach algebra to groups with and without previous algebra experience.

LeNoirn's (1989) meta-analysis of five studies suggested that manipulatives may be beneficial in teaching mathematical concepts to non-remedial college students as well as high school students. Archer (1972) found significantly higher achievement and retention scores among college level mathematics students when instructed with manipulative materials. Schram (1990) found that when instruction involved manipulatives, prospective elementary teachers said that for the first time they understood why rules and procedures they had memorized years before really worked. However, at least half of these future teachers still saw themselves teaching the rules of arithmetic in the traditional manner when they were given their own classes. Their limited knowledge of the mathematics curriculum and their expectations of young learners were not significantly changed or altered. Yet the results of this pilot study did seem to show important changes in students' thinking about teaching and learning mathematics for themselves. Curiously, they saw the benefits of understanding mathematics at a conceptual level for themselves, but that did not seem to carry over to how they thought about mathematics for children. Wiesner (1989) found that in-service elementary and secondary mathematics teachers could significantly improve their understanding of probability and statistics by using manipulatives.

While a meta-analysis by Sowell (1989) found no significant differences in achievement using manipulatives in college mathematics classes, significant improvement in attitude was indicated in the study. Harris (1989) conducted a study using manipulatives to teach ratio and proportion to remedial college students. The immediate achievement of the manipulative-based group was not significantly different from the control group, but retention of the learned concepts was significantly different for the manipulatives group. In a study of college mathematics students, Smith (1974) found that using manipulatives significantly improved achievement and also improved attitude toward the required mathematics course. However, Smith suggested that manipulative activities must be carefully selected and not overused.

Drapac (1980) used Mathtiles to teach the concepts of integers and polynomials to developmental college algebra students. She found significant improvement in achievement, attitude toward mathematics, and mathematics anxiety from pre-tests to post-tests. To test for retention Drapac compared end of term tests of experimental and control groups using pre-test scores as a covariate. Significantly higher achievement scores were obtained by the manipulative-based group. Archer (1972) also found significantly higher retention scores among students using manipulatives in college-level mathematics courses.

In a study of factors related to academic achievement of developmental algebra students, Barker (1994) found the use of manipulatives to be a significant factor to improve achievement. He concluded that students benefit from methods that diverge from the traditional lecture and discussion format.

After conducting a study using manipulatives with developmental algebra students, Dyer (1996) suggests that college instructors should acknowledge the role that students' attitudes have on performance. She suggests that using alternative techniques such as manipulatives can improve attitudes and performance. Dyer found a significant increase in the content learning of polynomial multiplication by college algebra students receiving manipulative instruction. The use of algebra tiles seemed to help them mature in their mathematical thinking and reasoning.

In her study of the relationship between using a graphing calculator and attitude and anxiety levels of intermediate algebra students, Gardner (1996) found a relationship between age of students and anxiety and attitude. While older students appeared to be more mathematics anxious, they also held more positive attitudes toward mathematics. However, Cook (1997) found that age was not significantly correlated to mathematics anxiety levels.

Summary

The review of the literature for this study researched effects of manipulatives on student achievement, attitude and anxiety. The use of manipulatives was considered from the perspective of philosophy, learning theory, need for change, and past research. Adhering to the constructivist theory, both NCTM and AMAYTC recommend providing opportunities for students to actively investigate mathematical concepts utilizing manipulative materials. Stodolsky and Sara suggested a relationship between the decrease in the manipulative instruction and the decline in attitude toward mathematics after early elementary grades. Martelly observed a correlation between previous algebra experience and the effectiveness of manipulative instruction. Several studies found manipulative instruction to improve achievement and retention. Dyer observed that algebra tiles seemed to help students mature in their mathematical thinking and reasoning. Results of studies linking anxiety and older students were mixed.

The literature review developed the context for this study to examine interactions between achievement, retention, attitude, and anxiety of groups based on manipulative treatment and previous algebra experience.

Chapter III

Methods and Procedures

Descriptions of the sample selection, instructional organization, data collection procedures, and statistical analysis are presented in this chapter. The focus of the investigation was to study the effectiveness of activities involving manipulative materials on academic achievement and retention. Other variables examined were attitude, anxiety, and previous algebra experience.

Setting

Walters State Community College is a two-year college located in Morristown, Tennessee. As a member of the Tennessee Board of Regents, WSCC has an open door admissions policy. Any student with a high school diploma or equivalent may be admitted into the college. Many of these students are not adequately prepared for college mathematics. The developmental mathematics courses at WSCC are designed for such students. During the Fall, 1998 Semester, approximately 50% of all mathematics students at WSCC were enrolled in a developmental mathematics course.

Subjects

Data for the study was collected from students enrolled in six developmental elementary algebra classes at Walters State Community College, Fall Semester, 1998. Elementary algebra students at WSCC are students who are inadequately prepared for college level mathematics courses as evidenced by ACT scores and/or Academic Assessment and Placement Program (AAPP) testing. This is a placement test that Tennessee community colleges administer prior to a student enrolling in any developmental course in order to determine levels of ability. Students in the elementary algebra classes have either completed a pre-algebra course at WSCC or passed the arithmetic portion of the AAPP.

Within the first week of classes students were told that the classes would a part of an educational study and that participation was voluntary and limited to those students eighteen years of age or older. Students were switched to other classes if they wished or were under age. After the class roster was permanent for the semester, human consent forms were signed by participating students (see Appendix A).

A total of 146 subjects began the study. However, only the 119 who completed the unit of instruction are included in the data analysis.

Procedures

Two instructors volunteered to participate in the study with the researcher. Two classes of each of these instructors along with two classes of the researcher comprised the six classes in the study. All three teachers are experienced in teaching elementary algebra and are familiar with manipulative materials. One of each instructor's classes was randomly chosen as an experimental class. Thus, each instructor taught both a treatment and a control class. Analyses of the results include the teacher factor, so that any effects due to experimenter bias were identified. The study was carried out over a period of six weeks.

To begin the study, students completed pre-tests for achievement, mathematics anxiety, and attitude toward mathematics. In addition, they completed questionnaires on age, gender, and previous algebra experience. After the unit of instruction, students completed post-tests for achievement, mathematics anxiety, and attitude toward mathematics. Students in the treatment classes also completed exit questionnaires about their opinions of using manipulatives in algebra class. At the end of the semester, seven weeks later, students took a final exam which included certain questions deemed directly related to concepts taught in the unit of study. Results of these questions were used as a second post test to measure retention.

All classes used the same textbook, *Introductory Algebra*, by Alice Kaseberg. Algebra topics included in the unit of study were integers, equations, and an introduction to polynomials. Teaching methodology in the control classes was considered traditional. No manipulatives or other visual aids, except overhead projectors and graphing calculators, were used. Students were introduced to the concepts using algebraic reasoning, symbol manipulation, and the graphing calculator as suggested in the textbook. While using the graphing calculator may not be considered traditional, since it was part of the textbook methodology and was used in both classes, it was not considered as a factor in the study.

The manipulative groups were introduced to each concept with algebra tile activities focusing on the meaning of each concept. Each student was given a set of tiles to use in class and to use with homework if desired. Each activity was designed to allow the students to demonstrate a concept concretely, draw diagrams representing the concept, and translate the concept to symbols. During the study the manipulative-based classes spent about one-third of class time participating in activities involving manipulative materials.

Description of the Treatment

The treatment consisted of eight activities using algebra tiles to develop concepts of integers, equations, and polynomials (see Appendix B). Since algebra tile activities conceptualize polynomials as areas and perimeters, the first activity utilized a problem solving approach to strengthen students' knowledge of area and perimeter. The following three activities developed rules for integer operations as well as the concepts of opposites, absolute value, and the commutative and associative properties of addition. The focus of the fifth activity developed the concepts of variables and exponents. Algebraic expressions and solving equations were explored in the sixth activity. The final two activities incorporated simplifying expressions, polynomials, the distributive property, and factoring. All of the activities were designed to allow students to work in small groups to discuss and share ideas while being guided by the instructor.

Instruments Used in the Study

A unit test prepared by the researcher and reviewed by other members of the WSCC faculty was used as a pre-test and post-test to obtain a measure of achievement. A Pellissippi State Technical Community College faculty member, familiar with the Kaseberg textbook, also reviewed the test. These reviewers agreed that the content was appropriate and clear. Selected questions from the departmental final examination given to all elementary algebra students at WSCC were used as a second post-test to measure retention. The questions used from the final exam were those that required the basic algebra concepts involved in the study. Scores on the first post-test were used as a covariate in the analysis of the final examination.

Achievement pre-tests and post-tests were graded collaboratively by the three instructors to avoid bias and provide uniformity of grading. The attitude and anxiety tests have fixed scoring and therefore, collaborative grading was not necessary.

Two scales of the Fennema-Sherman Mathematics Attitude Scales were used to measure student attitude and anxiety at the beginning and end of the study. The instrument consists of nine Likert-type scales, which measure attitudes related to mathematics learning.

The following is a description of the scales provided in the document manual and descriptions of the two scales used in the study:

> Each scale consists of six positively stated and six negatively stated items with five response alternatives: strongly agree, agree, undecided, disagree and strongly disagree. Each response is given a score from 1-5 and on each scale, except the Male Domain Scale, the weight of 5 is given to the response that is hypothesized to have a positive effect on the learning of mathematics. The person's total score on each of these scales is their cumulative total and the higher the score, the more positive their attitude. (Fennema & Sherman, 1986, p. 7)

Mathematics Anxiety Scale

> The Mathematics Anxiety Scale (A) is intended to measure feelings of anxiety, dread, nervousness and associated bodily symptoms related to doing mathematics. The dimension ranges from feeling at ease to those of distinct anxiety. The scale is not intended to measure confidence in or enjoyment of mathematics. (Fennema & Sherman, 1986, p. 4)

Effectance Motivation Scale in Mathematics

> The Effectance Motivation Scale in Mathematics (E) is intended to measure effectance as applied to mathematics. The dimension ranges from lack of involvement in mathematics to active enjoyment and seeking of challenge. The scale is not intended to measure interest or enjoyment of mathematics. (Fennema & Sherman, 1986, p. 5)

Fennema and Sherman obtained individual item and scale statistics by testing two high school age populations (N=1600). Split-half reliabilities showed the following values for the scales being utilized in this study: Mathematics Anxiety, .89 and Effectance Motivation in Mathematics, .87.

Fennema and Sherman established construct validity of the mathematics attitude scales by a principal components factor analysis. Although a correlation study between the scales showed some interrelation, each scale measured the construct it was designed to measure. The test may be given in any combination of scales to measure particular constructs. The questions from the Mathematics Anxiety Scale and the Effectance Motivation Scale in Mathematics were placed together in a random order and used to measure anxiety and attitude for participants in the study. The instrument appears in Appendix C. For data analysis, the two scales were separated again and each question was encoded with the correct 1 to 5 response depending on whether it was a positive or negative question. Data were organized in an Excel spreadsheet program to obtain pre-test and post-test anxiety and attitude scores for each student to use for statistical analysis.

Information about past algebra experience of subjects was obtained from a questionnaire developed by the researcher (see Appendix D). Students were asked gender, age, and the year of any previous algebra class. In analysis subjects were divided into two groups: 1) those students whose algebra experience occurred three years ago or less; and 2) those students whose algebra experience occurred more than three years ago or never. For any analysis involving age subjects were divided into three groups: 1) those 18-20 years old; 2) those 21-30 years old; and 3) those over 30 years old. However, since one instructor had only one student in one of these groups, no analysis used age as a factor. The raw data from pre-and post-achievement tests, final examinations, Fennema Mathematics Attitude Scales, and student information questionnaires are included in Appendix E.

At the end of the study students in the experimental group were asked to respond to a questionnaire to give opinions about instruction with algebra tiles (see Appendix D). These responses were used in discussion of the study.

Data Analysis

Hypothesis 1 was designed to answer the first research question examining the effect of manipulative instruction and/or past algebra experience on achievement. The following null hypothesis was tested in the manner described:

H_1: There will be no significant differences in mathematics achievement between treatment groups and/or experience groups.

An independent samples *t*-test was used on the achievement pre-test to examine the initial equivalence of the control and treatment groups with respect to achievement. A MANOVA with treatment, instructor, and past algebra experience as factors, and pre-test and post-test as repeated factors, was used to analyze differences in achievement. If significant factors were found, appropriate post-hoc analysis was performed using multiple *t*-tests.

Hypothesis 2 was designed to answer the second research question examining the effect of manipulative instruction and/or past algebra experience on retention. The following null hypothesis was tested in the manner described:

H_2: There will be no significant differences in concept retention between treatment groups and/or experience groups.

Retention was analyzed with an ANOVA on post-test 2 using treatment, instructor, and experience as factors and the post-test 1 score as a covariate. If significant factors were found, appropriate post-hoc analysis was performed using confidence intervals.

Hypothesis 3 was designed to answer the third research question examining the effect of manipulative instruction and/or past algebra experience on anxiety. The following null hypothesis was tested in the manner described:

H_3: There will be no significant differences in mathematics anxiety between treatment groups and/or experience groups.

An independent samples *t*-test was used on the anxiety pre-test to examine the initial equivalence of the control and treatment groups with respect to anxiety. A MANOVA with treatment, instructor, and past algebra experience as factors, and pre-test and post-test as repeated factors, was used to analyze differences in anxiety. If significant factors were found, appropriate post-hoc analysis was performed using multiple *t*-tests.

Hypothesis 4 was designed to answer the fourth research question examining the effect of manipulative instruction and/or past algebra experience on attitude. The following null hypothesis was tested in the manner described:

H_4: There will be no significant differences in mathematics attitude between treatment groups and/or experience groups.

An independent samples *t*-test was used on the attitude pre-test to examine the initial equivalence of the control and treatment groups with respect to attitude. A MANOVA with treatment, instructor, and past algebra experience as factors, and pre-test and post-test as repeated factors, was used to analyze differences in attitude. If significant factors were found, appropriate post-hoc analysis was performed using multiple *t*-tests.

The results of the data analyses are given in Chapter Four.

Chapter IV

Findings

This chapter presents the analysis of data needed to test the stated hypotheses. Results of data analysis are given for each hypothesis individually. All hypotheses were tested at the .05 level of significance using the Statistical Products and Service Solutions data analysis program. A final summary of results will conclude this chapter.

Hypothesis 1

H_1: There will be no significant differences in mathematics achievement between treatment groups and/or experience groups.

Since intact classes were used in this study, the research design was quasi-experimental. To determine the equivalency of the two treatment groups, an independent samples *t*-test was utilized on the pre-test achievement scores of students in the control and manipulative groups. The test found no significant differences in pre-test scores for academic achievement of students in the two treatment groups ($t = -.70$, $df = 117$, $p = .488$). This information supports the assumption that although the treatment groups were not randomly assigned, they were not different in achievement at the beginning of the study.

Statistical analysis of achievement was accomplished using the general linear model procedure within the SPSS software system with the repeated measures option for the time variable. Hypothesis 1 was tested by this procedure for performing a repeated measure analysis of variance using a multivariate approach. The raw data are presented in Appendix E and the results of multivariate tests of achievement are presented in Appendix F.

There was a significant time by experience interaction, $p < .001$. To locate the source of these differences, pre-test scores of all students whose experience in algebra occurred more than three years ago or never (no-recent-experience group) were compared with scores of all students whose algebra experience was more recent (recent-experience group). An independent samples *t*-test showed that the pre-test scores of the two groups were significantly different, ($t = 4.73$, $df = 117$, $p < .001$). The mean pre-test score of the no-recent-experience group was 27.66 and the pre-test mean for the recent-experience group was 40.66.

Next, the post-test scores of the two experience groups were examined. These post-test scores showed no differences. The mean post-test score for all students with no recent experience was 67.32 and the mean post-test score for all recent-experience students was 66.80. Figure 4.1 presents a graph of pre-test and post-test results.

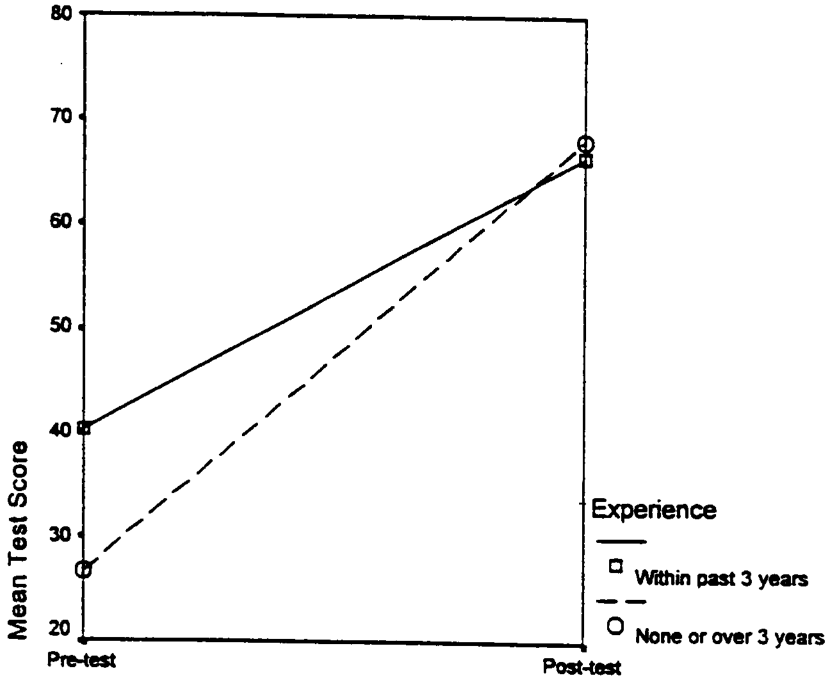

Figure 4.1: Achievement Scores of Experience Groups

Looking within the pre-test and post-test scores of the two experience groups, a paired samples *t*-test showed significant differences for the no-recent-experience group ($t = 18.20$, $df = 43$, and $p < .001$) and also for the recent-experience group ($t = 15.55$, $df = 74$, and $p < .001$). From pre-test to post-test, the difference in the means of the recent-experience group was 26.17 and the no-recent-experience group difference was 39.66. Although no differences existed in gain scores of experience groups based on treatment, an independent samples *t*- test determined that the achievement gain of the whole no-recent experience group was significantly higher than the gain of the recent-experience group ($t = -4.884$, $df = 117$, and $p < .001$).

No significant differences were found in achievement based on treatment group or experience groups within treatment groups. Achievement scores of students in the manipulative-based group were not significantly different from achievement scores of students in the control group.

The manipulative treatment did not cause any significant differences in achievement scores, but data analysis showed significantly higher gains in achievement based on experience groups. Students with no recent experience in both control and manipulative groups gained significantly more in achievement than did students with recent algebra experience in control and manipulative groups.

Hypothesis 1 was rejected.

Hypothesis 2:

H_2: There will be no significant differences in concept retention between treatment groups and/or experience groups.

Since the final examination, which was used to measure retention, was different from the pre-test and the first post-test, the first post-test was used as a covariate in an analysis of variance for retention. The ANOVA on the final examination used treatment, instructor, and experience as factors and the post-test 1 score as a covariate. The raw data are presented in Appendix E and results of the univariate analysis of variance for retention are presented in Appendix F.

There was a significant relationship in scores from post-test 1 to final exam ($p < .001$). The correlation of the two tests ($r = .70$) indicates a significant relationship. Those students scoring higher on post-test 1 tended to score higher on the final examination. In addition, there was a significant interaction in retention between experience and treatment groups ($p = .045$). Since retention was measured by the final exam which was different form the first post-test, *t*-tests could not be used to verify these findings. Therefore, confidence intervals on the estimated marginal means were used to examine the interaction in retention between experience and treatment groups. Post hoc analysis did not control for type I error. This must be considered when using the results. Table 4.1 displays these results.

Table 4.1: Confidence Intervals for Retention

				95% Confidence Interval	
Experience	Treatment	Mean	Std. Error	Lower Bound	Upper Bound
Recent Experience	Control	74.987[a]	2.019	70.980	78.994
	Experimental	72.440[a]	1.953	68.563	76.317
No Recent Experience	Control	73.230[a]	3.055	67.167	79.293
	Experimental	80.717[a]	2.646	75.465	85.969

a. Evaluated at covariates appeared in the model: Post-test 1 = 67.9450.

Looking within the recent experience group shows no difference between the control and manipulative-based groups. This is evidenced by the fact that the mean of the control group falls within the confidence interval of the manipulative group. However, looking within the no-recent-experience group, differences can be observed. The mean of the no-recent-experience students in the manipulative-based group falls outside the confidence interval of the no-recent-experience students in the control group. The mean of no-recent-experience students in the manipulative-based group is higher than the mean of the no-recent-experience students in the control group. The mean of the no-recent-experience students in the manipulative group also falls outside the confidence intervals of both treatment groups within the group of recent-experience students.

No-recent experience students in the manipulative group scored significantly higher in retention than no-recent experience students in the control group. These results indicate that the work with manipulatives was beneficial in the retention of mathematics concepts for students who had no recent experience with algebra.

Hypothesis 2 was rejected.

Hypothesis 3

H_3: There will be no significant differences in mathematics anxiety between treatment groups and/or experience groups.

To determine the equivalency of the two treatment groups, an independent samples *t*-test was utilized on the pre-test anxiety scores of students in the control and manipulative-based groups. The test found no significant differences in pre-test scores for anxiety of students in the two groups ($t = -.22$, $df = 115$, $p = .826$).

Statistical analysis of anxiety was accomplished using the general linear model procedure within the SPSS software system with the repeated measures option for the time variable. Hypothesis 3 was tested by this procedure for performing a repeated measures analysis of variance using a multivariate approach. A MANOVA with treatment, instructor, and past algebra experience as factors, and pre-test and post-test as repeated factors, was used to analyze differences in anxiety. The raw data are presented in Appendix E and the results of these analyses are presented in Appendix F.

Time was the only significant factor. A significant difference was found in the means of the pre-test and post-test scores of all students in the study for anxiety with $p < .001$. A paired-samples t-test showed significant differences in pre-test and post-test means for participants in the study ($t = 4.83$, $df = 114$, and $p < .001$).

Figure 4.2 displays a graph of the mean anxiety scores. The pre-test mean for all students was 31.45 and the post-test mean was 34.65. (A higher score shows less anxiety than a lower score). No significant differences were found in the anxiety levels of students in the manipulative-based group and the control group. No significant differences were found in the anxiety levels of students in the recent-experience group and students in the no-recent experience group. No significant differences were found in the anxiety levels of experience groups within treatment groups. Over time the mean anxiety level decreased for all students.

Hypothesis 3 was not rejected.

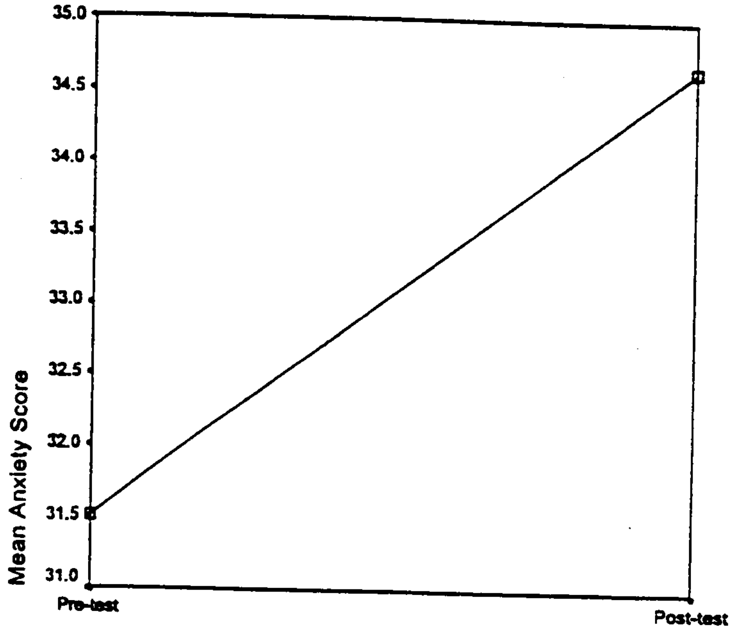

Figure 4.2: Mean Anxiety Scores of All Participants

Hypothesis 4

H_4: There will be no significant differences in mathematics attitude between treatment groups and/or experience groups.

To determine the equivalency of the two treatment groups, an independent samples *t*-test was utilized on the pre-test attitude scores of students in the control and treatment groups. The test found no significant differences in pre-test scores for attitude of students in the two groups ($t = 1.81$, $df = 115$, and $p = .074$). This information supports the assumption that although the groups were not randomly assigned, they were not different in attitude at the beginning of the treatment.

Statistical analysis of attitude was accomplished using the general linear model procedure within the SPSS software system with the repeated measures option for the time variable. Hypothesis 4 was tested by this procedure for performing a repeated measure analysis of variance using a multivariate approach. The raw data are presented in Appendix E and the results of multivariate tests of achievement are presented in Appendix F.

There was a significant time by treatment group interaction, (p = .034). To locate the source of these differences, pre-test attitude scores of students in the control group were compared with pre-test scores of students in the manipulative group. The independent samples t –test showed that the pre-test scores of the two groups were not significantly different (t = 1.81, df = 115, and p = .07). The mean pre-test score of the control group was 37.54 and the mean pre-test score of the manipulative group was 34.36.

Next, the post-test scores of the two groups were examined. These post-test scores also showed no significant differences (t = .54, df = 115, and p = .59). The mean post-test score for the control group was 36.11, and the mean post-test score for the manipulative group was 35.10. The difference between the two treatment groups was larger at pre-test than at post-test. This indicates that although, the post-hoc analysis showed no significant differences, a trend seemed to be developing. It appears that the differences in attitude scores of the two groups are narrowing over time. This can be seen in the graph of mean attitude scores displayed in Figure 4.3.

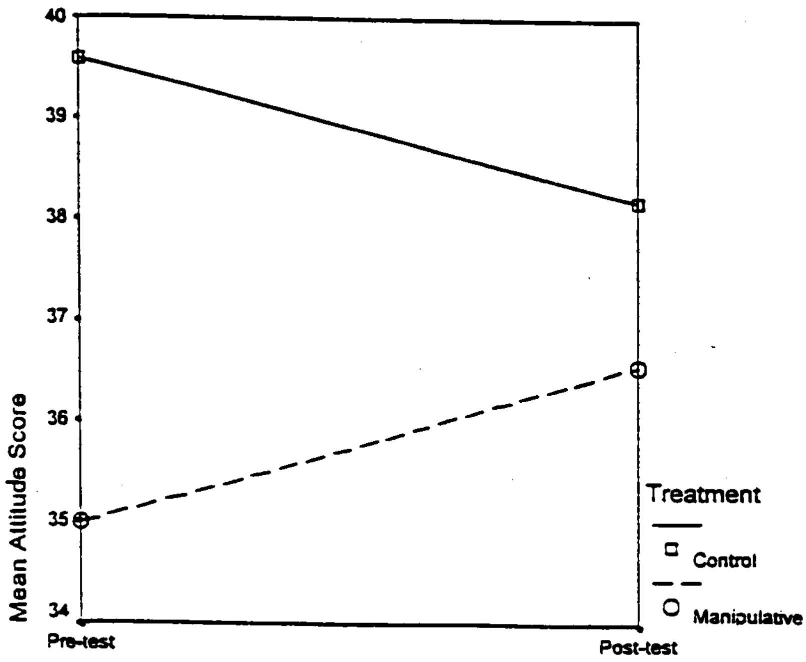

Figure 4.3: Attitude Scores of Treatment Groups

Past algebra experience was a significant factor for attitude ($p < .001$). The marginal means of attitude for the no-recent-experience group was 41.17 and the marginal means of attitude for the recent-experience group was 33.49. The attitude of students whose experience occurred more than three years ago or never was better than the attitude of students whose mathematics experience occurred three years ago or less. This is displayed in the graph in Figure 4.4.

No significant differences in attitude were found between experience groups based on treatment groups. Experience groups in control groups had attitudes not significantly different from experience groups in manipulative groups.

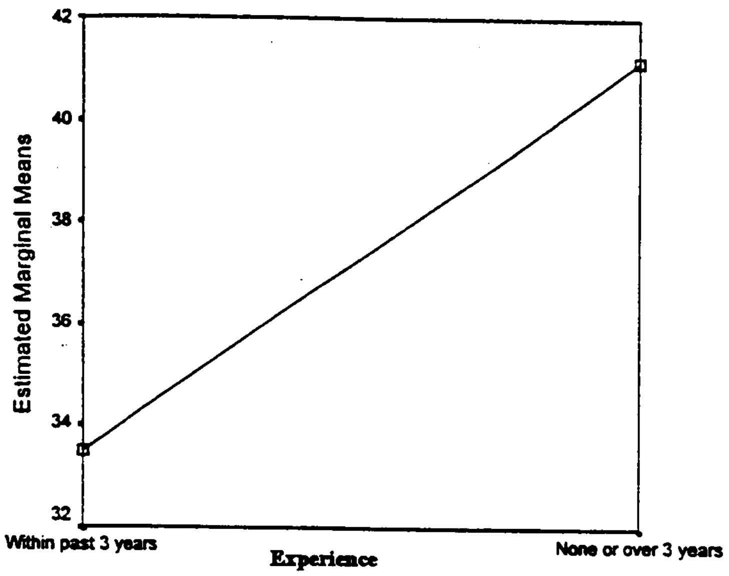

Figure 4.4: Attitude Scores of Experience Groups

The analysis also indicated a significant treatment group by teacher interaction ($p = .047$). There appears to be no difference in the mean attitude scores of manipulative and control groups for two of the instructors, but the third instructor's manipulative and control groups appear to have different attitudes. With a p-value so close to .05, the difference may not be strongly significant. The instructor whose groups seemed different was the researcher. The reason for any difference could be that she was more involved in the investigation. Another possibility is that the researcher's efforts to avoid bias toward the manipulative group led to the more positive attitude score of the control group. Since attitudes caused by teacher differences were not the focus of this study, these differences were not analyzed further. Figure 4.5 shows a graph of attitude means of instructor groups.

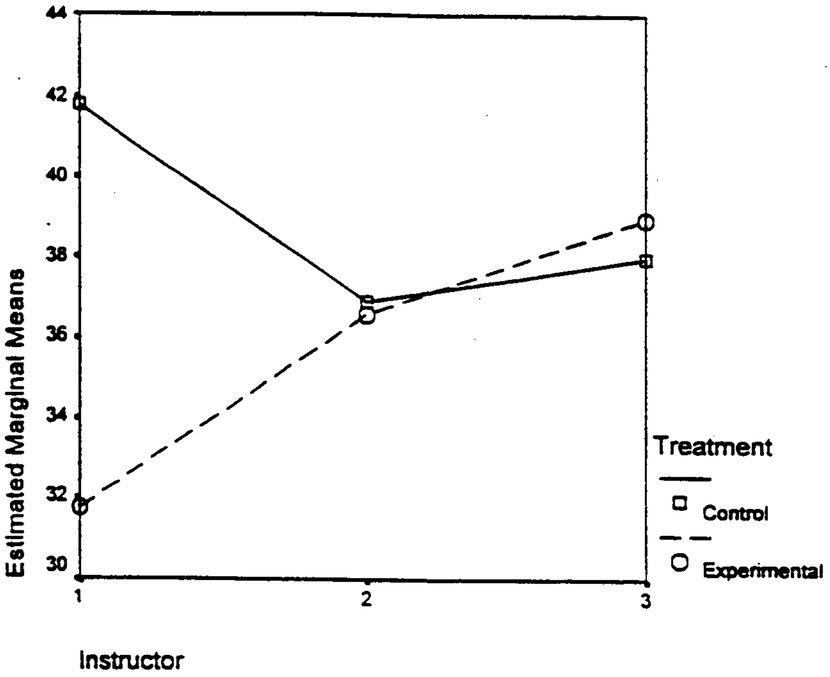

Figure 4.5: Attitude Scores of Instructor Groups

The attitude of students in the manipulative group was not significantly different from the attitude of students in the control group. However, a trend appeared to be developing in the attitudes of these two groups, with the gap between the mean scores narrowing. The attitude of all students in the no-recent experience group was significantly more positive than the attitude of all students in the recent-experience group.

Hypothesis 4 was rejected.

Summary of the Results

In summarizing the analysis of data, the following results were found:

H_1 (rejected): No significant differences exist in achievement scores for students in the control and the manipulative based groups and no significant differences exist in the achievement scores of experience groups based on treatment groups. However, significant differences exist in achievement gains of experience groups. The no-recent experience students from both control and manipulative groups gained significantly more than the recent-experience students from both treatment groups.

H_2 (rejected): Significant differences exist between the retention scores of no-recent experience students in the control group and no-recent-experience students in the manipulative group. Retention scores of no-recent experience students in the manipulative-based group were significantly higher than retention scores of no-recent-experience students in the control group.

H_3 (not rejected): No significant differences exist in anxiety levels of the treatment groups and no significant differences exist in the anxiety levels of the experience groups. No significant differences exist in anxiety levels of experience groups based on treatment groups.

H_4 (rejected): No significant differences exist in attitudes of students in the control and the manipulative based groups and no significant differences exist in the attitudes of experience groups based on treatment groups. However, significant differences in attitude do exist in the experience groups. Students with no-recent experience in both treatment groups had attitudes significantly higher than did students with recent-experience in both treatment groups.

Three of the null hypotheses were rejected as a result of the statistical analysis. A discussion of the findings is presented in Chapter Five.

Chapter V

Summary, Conclusions, Discussion, and Recommendations

This chapter summarizes the purpose of the study and its findings. A discussion based upon instructor and student feedback provides further information relating to the effects of the manipulative instruction. The chapter concludes with implications of this research and recommendations for further study.

Summary

This research was a study designed to investigate the effects of using algebra tiles to teach the basic algebraic concepts of integers, equations, and polynomials. The research design was quasi-experimental because subjects could not be randomly assigned to groups. A review of the literature revealed that research on manipulative use in college classrooms is only in the beginning stage. Past studies suggested that the effects of using manipulatives on attitude and anxiety of developmental education students should be studied. One such study also suggested studying the effect of past algebra experience on the effectiveness of using manipulatives. The researcher designed a study to examine these factors.

Three instructors each taught a control and a manipulative-based group. Students completed pre-tests and post-tests for academic achievement, attitude, and anxiety. They also completed information forms on previous algebra experience. Experimental groups participated in activities involving algebra tiles during the unit of instruction. The activities were designed to allow students to model the algebraic concepts of integers, equations, and polynomials concretely, represent the concepts with diagrams, and translate the concept to symbols. The 119 students who completed the unit of study were included in the statistics.

Analysis of the data showed significant differences in concept retention of no-recent-experience students receiving instruction with the manipulatives and no-recent-experience students who did not receive instruction with manipulatives. Significant differences were also found in achievement gains and attitudes of recent-algebra experience groups and no-recent algebra experience groups. The results and analyses were presented in table, graph, and narrative forms in Chapter Four.

Conclusions

Hypothesis 1 relates to the effects of manipulative-based instruction and previous algebra experience on algebraic achievement. This hypothesis was rejected. This answered the first question posed under the problem section of this study (p. 5):

Question: How do instructional activities involving manipulative materials and/or past algebra experience effect the mathematical achievement of students in developmental college algebra courses?

Answer: In this study, instructional activities involving manipulative materials did not effect the achievement of students in developmental college algebra courses. Students who received instruction in mathematics concepts through manipulative based activities experienced achievement not significantly different from students who were taught by more traditional methods. Both treatment groups achieved significant increases in content knowledge from the pre-test to the post-test.

However, past algebra experience does appear to effect the mathematical achievement of students. Analysis of data showed a significant interaction between previous experience groups. The achievement gain of students with no recent algebra experience in both treatment groups was significantly higher that the achievement gain of students with recent experience in both treatment groups.

Hypothesis 2 relates to the effects of manipulative-based instruction and previous algebra experience on concept retention. This hypothesis was rejected. This answered the second question posed under the problem section of this study (p. 5):

Question: How do instructional activities involving manipulative materials and/or past algebra experience effect the concept retention of students in developmental college algebra courses?

Answer: There was a significant difference in concept retention within the group of students with no recent algebra experience. Supporting the findings of Archer (1972), Drapac (1980), and Harris (1989), students with no recent experience who were in the manipulative-based group had significantly higher retention than did students with no recent experience from the control group. Manipulatives appear to be a significant factor in the retention of algebraic concepts for students with no recent algebra experience.

Hypothesis 3 relates to the effects of manipulative-based instruction and previous algebra experience on anxiety. This hypothesis was not rejected. This answered the third question posed under the problem section of this study (p. 5):

Question: How do instructional activities involving manipulative materials and/or past algebra experience effect the anxiety of students in developmental college algebra courses?

Answer: No significant differences were found in anxiety levels of students in the manipulative-based group and students in the control group. Also, no differences were found in the anxiety levels of students in the recent-experience group and students in the no-recent experience group. No differences were found in the anxiety levels of students in either the control or manipulative groups based on previous experience. Over time the mean anxiety levels of all participants decreased. The use of manipulatives was not detrimental to the anxiety level of the students, nor did it have a positive effect.

Hypothesis 4 relates to the effects of manipulative-based instruction and previous algebra experience on attitude. This hypothesis was rejected. This answered the fourth question posed under the problem section of this study (p. 6):

Question: How do instructional activities involving manipulative materials and/or past algebra experience effect the attitude of students in developmental college algebra courses?

Answer: The findings of this study suggest that previous algebra experience has an effect on attitude. It appears that students with no recent algebra experience have better attitudes toward mathematics than do students with recent algebra experience.

The effect of manipulative instruction on attitude is not as clear. No significant differences were found in the attitudes of the manipulative-based and the control groups nor in the attitudes of experience groups based on treatment groups. However, the significant interaction in attitude of treatment groups from pre-test to post-test suggests a positive trend in the attitude of students taught with manipulatives and a negative trend in attitude of the control group.

Analysis of the data indicated a possible effect on attitude by one of the three instructors involved in the study. Since attitudes caused by teacher differences were not the focus of the study, these differences were not analyzed further. However, it must be acknowledged that this factor could have been an influence in the attitude scores.

Discussion

The power of the design was weakened by the time limit imposed by the course syllabus. The literature indicates that in order for manipulative activities to make an impact on students' learning, adequate time for reflection and discussion is necessary.

Perhaps, when students were not given sufficient time to study and reflect upon the activities, they became confused and discouraged. However, both treatment groups had a significant gain in achievement. Additionally, students with no recent algebra experience in the manipulative group retained concepts significantly better than did students with no recent experience in the control group.

Manipulative activities can only become beneficial to students when they are convinced of the genuineness of such activities. The prevalent attitude toward mathematics is that it is just a set of steps to memorize in order to complete the course. Many students resisted trying to develop an understanding of the concepts. Just as Kinard (1996) found in her study, students in this study exhibited much resistance to using the tiles. This is evidenced by comments such as these on the exit questionnaire:

> "The algebra tiles took something that was simple to learn and made it hard."
>
> "I made all A's last semester in math class. I believe these tiles are the reason why I am receiving a C in this class now. Those tiles make everything twice as hard and twice as confusing."

It seems that attitudes toward what it means to do mathematics and to know mathematics must be changed before most students will see the benefit of these kinds of activities.

Manipulative instruction may be more effective when used to introduce students to algebraic concepts. As suggested by Drapac (1980), it appears that if they had already developed some methods for working with these problems, students in this study were very resistant to looking at them in a different way. The findings in this study may indicate the need to use manipulative materials in pre-algebra and first year algebra classes in middle and high schools.

The treatment with manipulative activities had a significant effect on the retention of students who had no recent algebra experience. These students scored significantly higher on the second post-test than students with recent algebra experience in either treatment group and also higher than other no-recent-experience students in the control group. Manipulative-based methods may work better when students do not have pre-conceived notions of what it means to do algebra. Some students made similar observations in their comments:

> "I believe if I had been using Algebra Tiles since the beginning of my algebra courses they would have been helpful, but changing the way to solve something I already knew how to do confused me and wasted my time on many problems."

> "I think algebra tiles could be helpful to a younger age group who haven't already learned to do math in a specific way."

> "I disagree very strongly with using the algebra tiles. I might get something out of them if I started using them in the 8th grade, but I think they are very confusing."

Results of the exit survey showed that most of the students (64%) said that they never used the tiles on a test (see Appendix G). However, one student who said that she never used the tiles on a test also commented:

> "Even though I didn't use the real tiles, they were very helpful during test time because you could draw them."

The fact that 56% of the students felt that the manipulative activities were more meaningful when working in a small group may indicate the importance of incorporating cooperative learning into classroom instruction. This gives students an opportunity to question, state, and test their beliefs about mathematics concepts in a non-threatening situation and much learning can take place.

Student feedback indicates that only 20% of the students felt that the algebra tiles were helpful to them. However, these exit questionnaire comments indicated that, although students felt that the tiles were not beneficial to them personally, they could see how others found them helpful:

> "I feel the algebra tiles confused me more than (they) helped me. Probably because I was used to the old fashioned way. However, I did see some students benefit from using tiles."

> "I feel that manipulatives are a great asset in mastering a skill. Please do not take my negative answers the wrong way. They were just unnecessary in my case."

The data analysis found no significant differences in mathematics attitudes of students in control and manipulative-based groups. But, a graph of pre-test and post-test scores of the two treatment groups reveals the possibility of a developing trend. If this trend could be shown to hold for a longer period of time, there is reason to suspect that the manipulative group would experience a significant positive change in attitude.

The group of students with no recent experience was, on the average, older than the group with recent experience. The study found no difference in the mathematics anxiety levels of these two groups. This supports Cook's (1997) finding that age was not significantly correlated to mathematics anxiety levels.

For students without pre-conceived notions about how algebra should be done, manipulatives may be helpful at all ages. And when introduced to a concept with a concrete representation, some students are able to hold on to that concept better than when the only instruction is abstract. The finding that no-recent-students experienced significantly better retention when instructed with manipulatives may also respond to Kinard's question about the role of age in the resistance to the use of manipulatives. It seems possible that the group did better with manipulative instruction because they were less resistant to their use. This also supports Martelly's (1998) observation that there is a correlation between previous algebra experience and the effectiveness of manipulative instruction and Archer's (1972) finding that using manipulatives can increase retention among college mathematics students.

Recommendations for further study

Research is needed to explore ways to encourage students to understand mathematical concepts and learn to think mathematically. Many students in this study were resistant to working with algebra tiles because they believe that knowing and doing mathematics begins and ends with paper-and-pencil manipulative skill work driven by teacher exposition. Research needs to be done on developing ways to involve students in constructing and applying mathematical ideas as recommended by the National Council of

Teachers of Mathematics. Such research needs to incorporate experiential learning and the use of manipulatives. The following recommendations are made for further research:

1. Replicate this study with a longer treatment period or a time in which total attention can be given to the concepts taught in the unit. For maximum effectiveness, it appears that students need more time to think, discuss, question, and reflect on the mathematics.

2. Replicate this study with beginning high school or junior high school algebra students. If algebra concepts can first be presented in a concrete manner using activities with manipulatives to develop an understanding of the concepts, they may not need to take developmental mathematics in college. Work with manipulatives in this study was impeded by the attitude that they are unnecessary because students have not worked with them since elementary school, if at all.

3. Replicate this study using the same post-test for achievement and retention to confirm the higher retention rate of students with no recent algebra experience. Since the test for retention used in this study was not the same test used to measure achievement, confidence intervals on the estimated means were used to examine the interaction in retention between experience and treatment groups. If the same test had been used a t-test could be used to verify findings. It would also be better if the time between the treatment and the test for retention could be longer than the seven weeks used in this study.

4. Replicate this study over a longer period to examine the trend in attitude of

treatment groups. Although no significant differences in attitude between the manipulative and control groups were found in this study, it appears that the attitude scores of the manipulative group were improving over time. A longer study could examine whether this trend develops into a significant difference.

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Appendices

Appendix A

Informed Consent Statement

INFORMED CONSENT STATEMENT

"An investigation of the use of algebra tiles to introduce the algebra concepts of integers, equations, and polynomials in community college developmental algebra classes"

You are invited to participate in a research study. You must be 18 years of age or older to participate. The purpose of this study is to investigate the use of manipulative materials (algebra tiles) in community college elementary algebra classes.

INFORMATION ABOUT PARTICIPANTS' INVOLVEMENT IN THE STUDY

The following procedures will be followed in the study:

> Students will complete a pre-test and post-tests to measure achievement and retention of concepts taught during the study. These tests will be the regular unit test given by the instructor for this unit of study and parts of the departmental final exam. Each one will require no more than one class period.
>
> Students will complete the Fennema-Sherman rating scale of math anxiety and attitude as a pre-test and a post-test. Each of these tests will require approximately 10 minutes.
>
> Students will complete a questionnaire concerning previous algebra experience. And at the end of the semester students who use algebra tiles will be asked to respond to some opinion questionnaires. These questionnaires will require approximately 10 minutes.
>
> Some classes in the study will participate in learning activities involving algebra tiles. These activities will be limited to 30 to 45 minutes in approximately six class periods and for shorter periods of time in a few more class periods.
>
> The length of the study will be approximately six weeks.

Approximately 140 students will participate in this study.

If you feel that you can not participate in this study you may see your instructor about transferring to another elementary algebra class. Credit given for the course will be the same as for all other Math 0820 classes.

RISKS

There are no foreseeable risks in any of the procedures to be used in the study.

______ Participant's initials

BENEFITS

There will be no direct benefits to students participating in this study. However, investigating the effects of using manipulative materials to introduce algebra concepts is a very relevant and useful study. Analyzing the relative effectiveness of traditional and manipulative-based instruction will give a better perception of student performance and its relation to the success of these methods. Also developmental college algebra students have high levels of math anxiety which often prevent them from succeeding in algebra courses and this study will give insight into the usefulness of manipulatives to reduce math anxiety and improve students' attitudes toward math.

CONFIDENTIALITY

The information in the study records will be kept confidential by your instructor and the researcher. Data will be stored securely and will be made available only to persons conducting the study unless participants specifically give permission in writing to do otherwise. No reference will be made in oral or written reports which could link participants to the study. Data will be stored for three years following completion of the study.

CONTACT INFORMATION

If you have questions at any time about the study or the procedures, (or you experience adverse effects as a result of participating in this study,) you may contact the researcher, Lee Dell'Isola, at LSCI 118, and 585-6871. If you have questions about your rights as a participant, contact the Compliance Section of the Office of Research at (423) 974-3466.

PARTICIPATION

Your participation in this study is voluntary; you may withdraw from the study at anytime without penalty and without loss of benefits to which you are otherwise entitled. If you withdraw from the study before data is completed your data will be returned to you or destroyed.

CONSENT

I have read the above information. I have received a copy of this form. I agree to participate in this study.

Participant's signature ______________________________ Date ______________

Investigator's signature ______________________________ Date ______________

Appendix B

Instructional Materials

POLYOMINOES

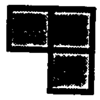
(Area, Perimeter, and Variables)

Name ____________________ SS No. ____________________

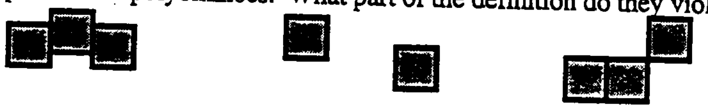
You will need: congruent squares and graph paper

Definition: **Polyominoes** are shapes that are made by joining squares so that they touch along entire sides, but do not overlap. The best known polyomino is the domino, made with two squares.

Using three squares, you can find two different polyominoes, the straight one and the bent one:

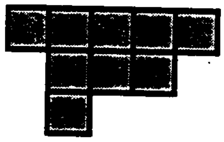


The following shapes are not polyominoes. What part of the definition do they violate? ____________



Definition: The **area** of a geometric figure is the number of unit squares it would take to cover it. The **perimeter** of a figure is the distance around it. For **example,** the area of the domino (at the top of the page) is 2 and the perimeter is 6.

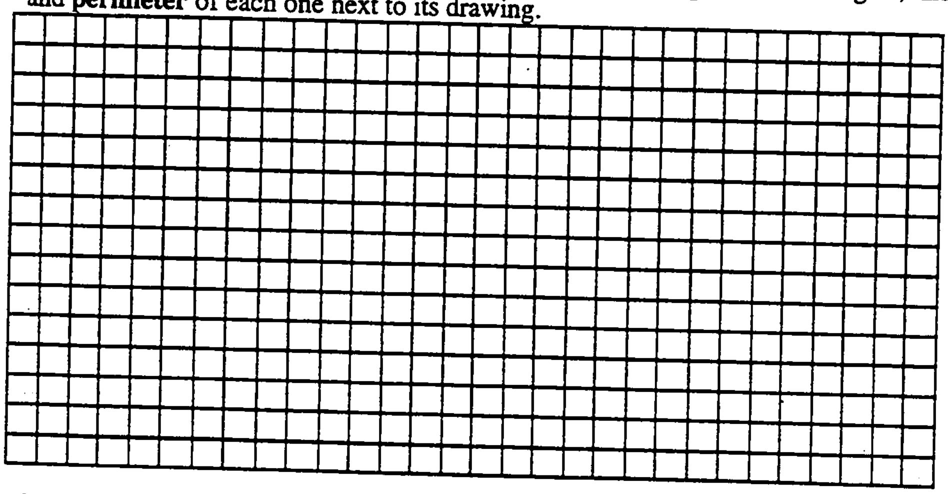
1. Here is a 9-omino. What is its area? What is its perimeter?



area = __________

perimeter = __________

2. **Make five** more different 9-ominoes. **Draw** each arrangement on the grid, and **indicate the area** and **perimeter** of each one next to its drawing.



3. What are the **largest** and **smallest perimeters** that you found? ____________________

What are the **largest** and **smallest areas** that you found? ____________________

Did any of the 9-ominoes have an **odd** number for the **perimeter?** ________________

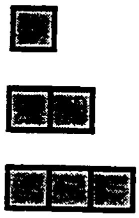
If your answer is yes, check your work. If your answer is no, explain why.__________

____

4. Complete this table of the perimeters of single row polyominoes (as shown at right) made with the given number of squares. Look for a pattern that relates the number of squares (input) to the area and the perimeter (outputs). In the last row write an algebraic expression for the perimeter of a single row polyomino with n squares.

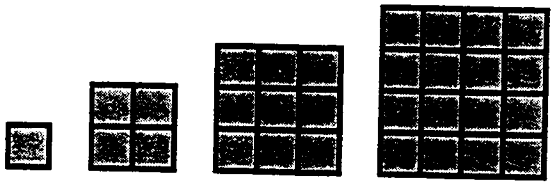
no.of squares	area	perimeter
1		
2		
3		
4		
10		
25		
n		

1
2
3



5. Form different square polyominoes. Build the first few polyominoes using the pattern shown. Look for a pattern that relates the number of the figure or input. to the area and the perimeter (outputs). Find an algebraic expression for both the area and the perimeter of the nth polyomino.

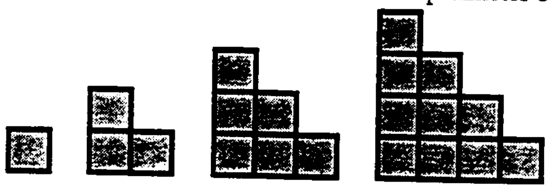
Polyomino



Polyomino number	1	2	3	4	5	n
Area	1	____	____	____	____	____
Perimeter	4	____	____	____	____	____

6. Build polyominoes using the pattern shown below. What are the area and the perimeter? Find the pattern and use algebraic expressions for the area and perimeter of the nth polyomino.

Polyomino

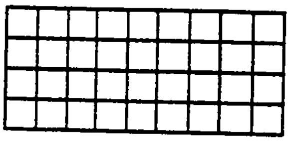


Polyomino number	1	2	3	4	5	n
Area	1	____	____	____	____	____
Perimeter	4	____	____	____	____	____

7. Area and perimeter are related. It is not a simple relationship. For a given area there may be more than one perimeter possibility. For a given perimeter, there may be more than one area. Let's look at some rectangles. Find all possible **rectangles** that have an **area** of 36 that can be made with your squares.

a) **Sketch** the rectangles on graph paper and **find the perimeter** of each.

Example:



area = 36

perimeter = 26

b) What is the **smallest perimeter**? ______________

c) What is the **largest perimeter**? ______________

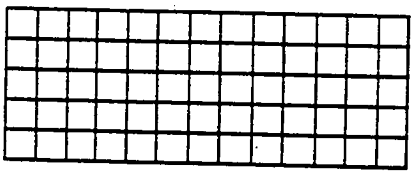
d) **Describe** how you would enclose a rectangular garden and **use the least amount** of fencing.

____

8. Find 5 different rectangles that have a **perimeter** of 36.

a) **Sketch** the rectangles on graph paper and **find the area** of each.

Example:



area = 65

perimeter = 36

b) What is the **smallest** area that you found? ______________

c) What is the **largest** area that you found? ______________

d) Look for a **pattern**. What do you think is the **smallest** possible area for a **perimeter of 36** (using whole number lengths for the sides)? ______________

e) What do you think is the **largest** possible area for a **perimeter of 36**? ______________

f) How would enclose a garden using **100 feet of fencing** to have the **largest possible** garden?

____

Signed Numbers

Name ____________________ SSNo. ____________________

You will need small square algebra tiles for this activity.

The **counting numbers** {1,2,3..} and **their opposites** {-1,-2,-3...} and **zero** {0} make up the set of numbers called **integers** and can be shown on a number line. A number line illustrates that there is no largest or smallest number. The line extends indefinitely.

1. **Finish numbering** the number line below.

___ ___ ___ -3 -2 -1 0 1 2 3 ___ ___ ___

Science uses positive and negative particles to show how **opposite particles** that are **combined** will **neutralize and cancel out.** This idea will be used to help us understand the operations with positive and negative numbers and later to solve equations.

Let the small black algebra tile squares represent positive integers and the small white squares represent negative integers. The symbols used in this handout are:

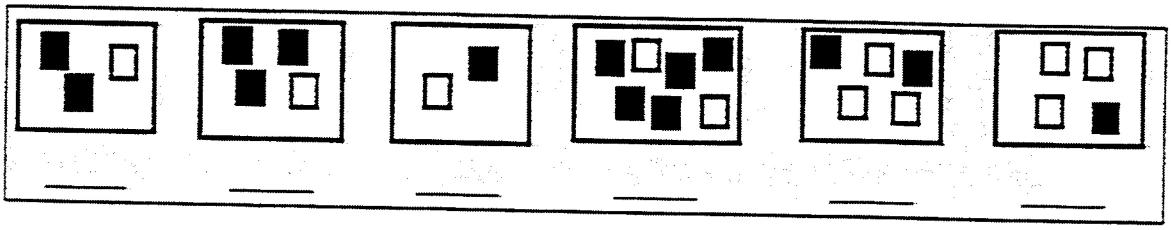
■ = positive 1 and □ = negative 1

If we **combine positive 1 with negative 1,** they will cancel out and **equal zero.**

■ + □ = 0

1 + -1 = 0

2. **Combine** positive and negative particles to make "zeros" and then write the value of each set below the picture.



3. Use your black or white squares to represent +3 and -3 in different ways. **Draw diagrams** for these integers below.

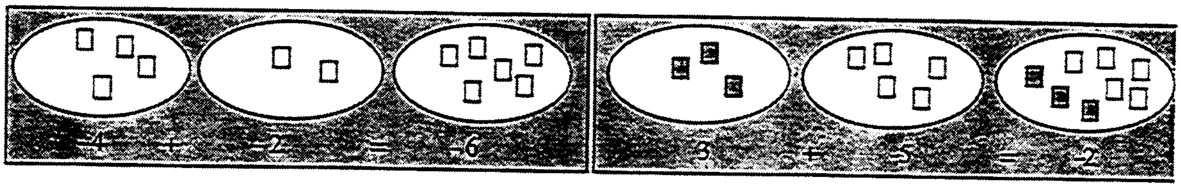
+3	+3	-3	-3

Can you show +3 and -3 using 5 tiles? ____ 9 tiles? ____ 10 tiles? _____ **Explain** your answer.

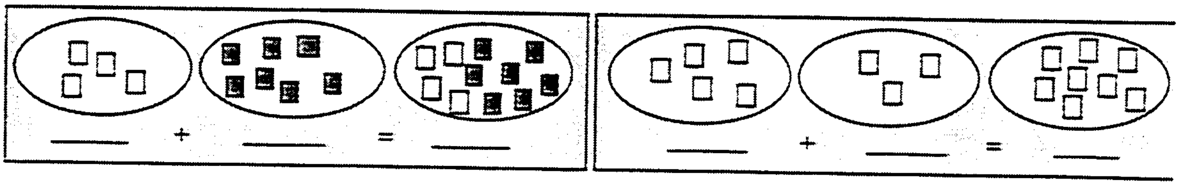
Adding Integers

Use your small black or white squares to add the following sets of integers. (Remember that black is positive and white is negative).

Examples:



4. Complete:



5. Try these:

a) $-6 + 5 =$ ____ b) $7 + (-3) =$ ____ c) $-6 + (-5) =$ ____ d) $8 + (-5) =$ ____

e) $-9 + 4 =$ ____ f) $8 + (-9) =$ ____ g) $-6 + (-7) =$ ____ h) $5 + 6 =$ ____

6. Describe any discoveries you made about adding integers. ________________________________

____

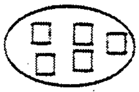
If you wanted to explain to your friend how to add integers of the same sign, what would you say? ____

____

If you wanted to explain to your friend how to add integers of different signs, what would you say? ____

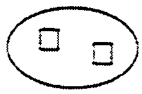
____

7. The distance a number is from zero on the number line is its **absolute value**. **Read** about absolute value on **page 44** of your textbook. Give the **absolute value** of each modeled number:



$|-5| =$ ____

$|3| =$ ____

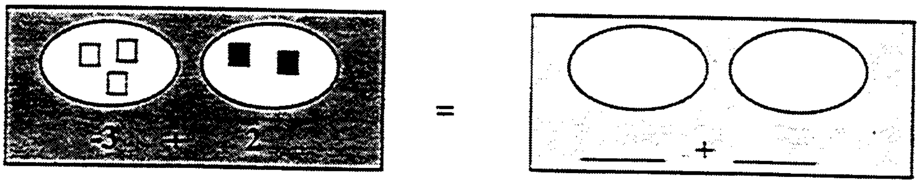


$|-2| =$ ____

When you model a number with your algebra tiles, its **absolute value** is shown by ______________.

In problems like $-9 + 7$, the sign of the answer is the same sign as the -9, because it has the larger ______________________.

8. The **commutative property of addition** says that for any numbers a and b, $a + b = b + a$.
   a) **Draw a diagram** with your tiles demonstrating this property.

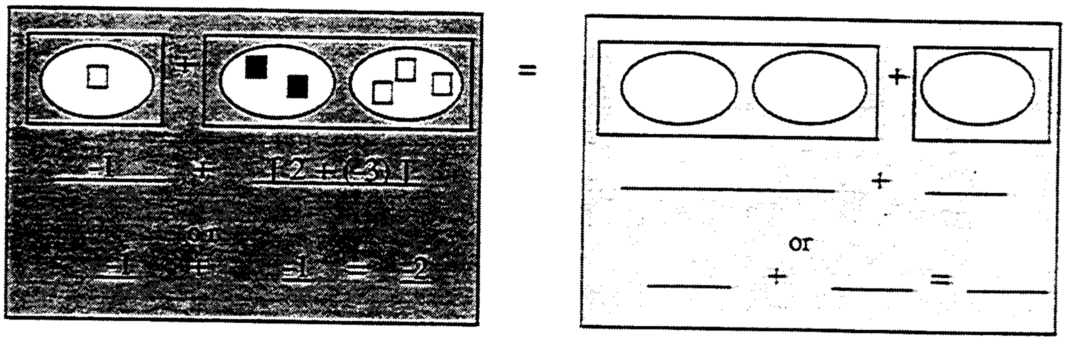


   b) What does this property say in your own words? ______________________________.

9. The **associative property of addition** says that for numbers a, b, and c,

$$(a + b) + c = a + (b + c).$$

Complete the following diagrams to demonstrate this property with your tiles, using the problem **-1 + 2 + -3 = _______.**

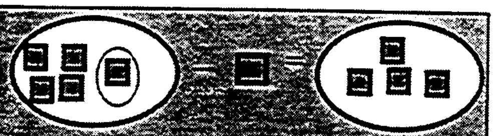


Subtracting Integers

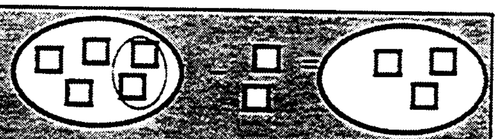
Materials needed: Algebra Tile Small Squares
Name_____________________ SS No.____________________

To understand subtraction of signed numbers, we will use the definition that **subtract** means **"take away"**. A **circle** is drawn around the number that is **"taken away"**.

Examples:



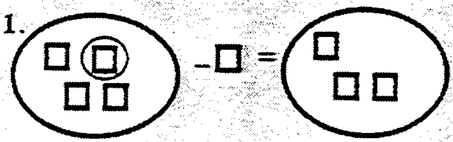
5 - 1 = 4



-5 - (-2) = -3

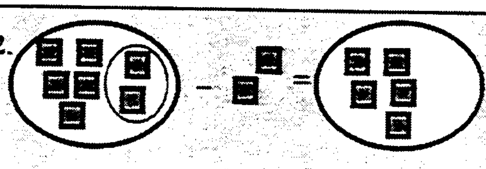
Use your algebra tiles to count out the number of black or white squares that are shown in the first picture of each set below. "Take away" the circled number of disks. **Write** the **equation** for each set of pictures.

1.



____ - ____ = ____

2.



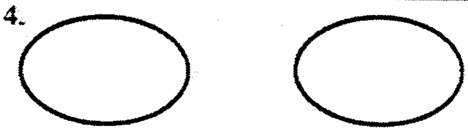
____ - ____ = ____

Use your tiles to model the following subtractions. Sketch your models below.

3.

-6 - (-2) = ________

4.



5 - 3 = _____

You may always add zero and not change the value of an expression. Sometimes it is necessary to **add** sets of positive and negative tiles to make **zeros** before you can do a subtraction problem.

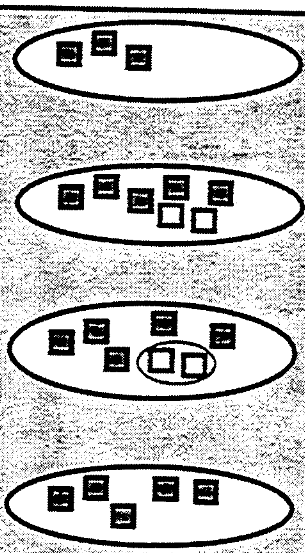
Example: 3 - (-2) = ____

Step 1: Count out three black (positive) tiles.

Step 2: Add two sets of positive and negative tiles.

Step 3: Circle and take away two white (negative) tiles.

Step 4: Write the numeral for the number that remains.



5

Follow the steps above to solve these problems. Sketch each step.

5. 4 - 5 = ____

Step 1	Step 2	Step 3	Step 4

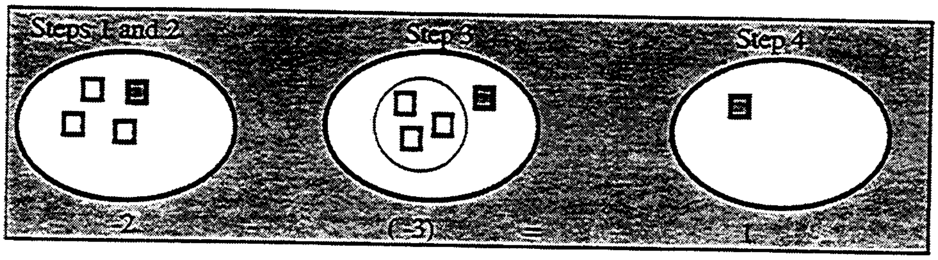
6. -2 - 3 = ____

Step 1	Step 2	Step 3	Step 4

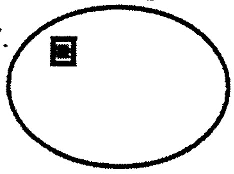
For the problems below complete the following steps:

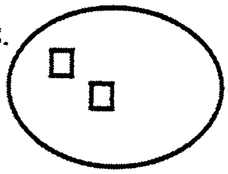
a) Count out the number of black or white tiles to show the beginning number.
b) Put in zeros if necessary for subtraction.
c) Draw a circle around the tiles to be subtracted.
d) Take away the tiles that are circled.
e) Write the numeral for the number of tiles that remain.

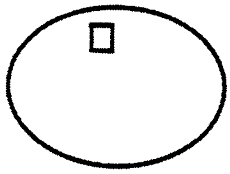
Example: -2 - (-3) = 1

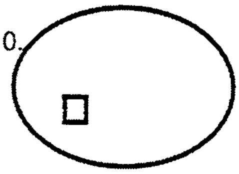


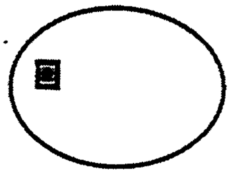
Follow the steps above to solve these problems.

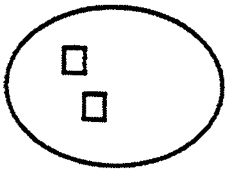
7.

1 - (-3) = _____

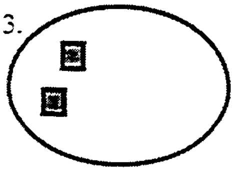
8.

-2 - (-4) = _____

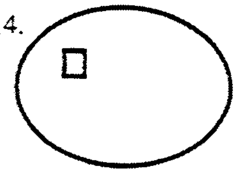
9.

-1 - 2 = _____

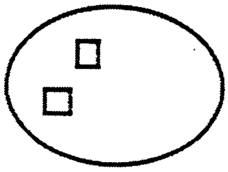
10.

-1 - (-3) = _____

11.

1 - (3) = _____

12.

-2 - (1) = _____

13.

2 - (4) = _____

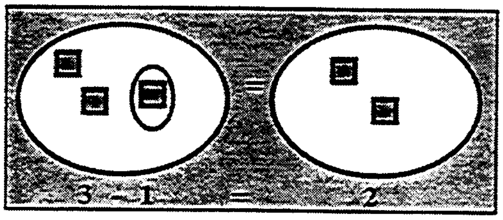
14.

-1 - (-2) = _____

15.

-2 - (2) = _____

Addition and subtraction are **opposite** operations.

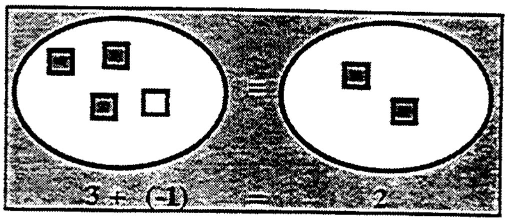
Example: 3 - 1 = 2

Subtraction



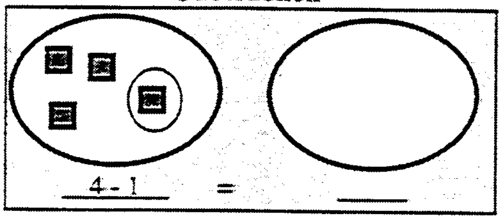
and

Addition



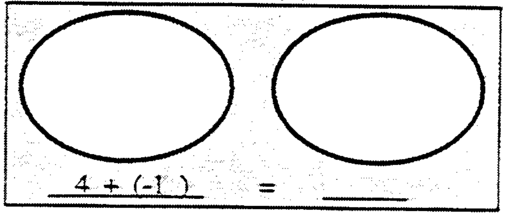
Count out tiles and **draw diagrams** to show that **addition and subtraction** are **opposite** operations.

16. **Subtraction**

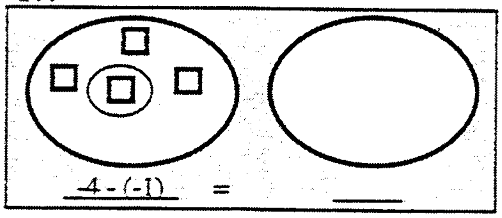


and

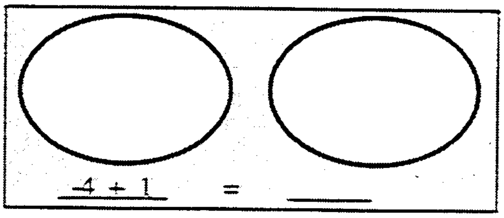
Addition



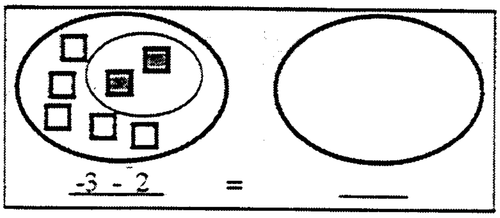
17.



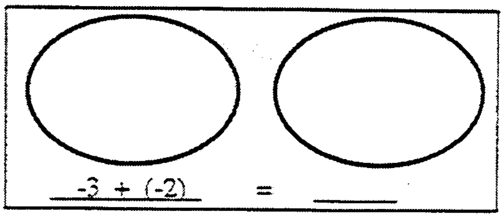
and



18.



and



19. **Subtraction** problems can be solved by **adding** ________________________.

20. Write a related addition problem for the following subtraction problem and add.

a) 5 - (-3) =　　　　b) -4 - 3 =　　　　c) -2 - (-7) =

Multiplying Integers

Name ____________________ SS No.________________

Materials: Small Square Algebra Tiles

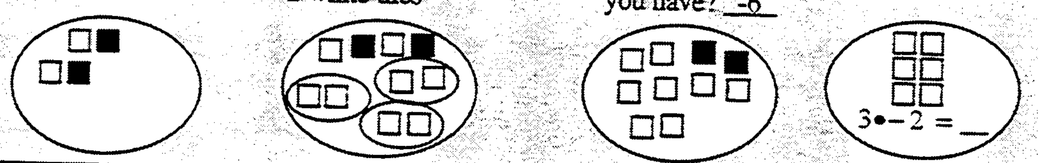
Multiplying by a **positive** integer can be modeled by **putting tiles into** a set. The answer can be shown as a **rectangle.**

Example: $3 \bullet 2 =$ ____

1. Start with zero:
2. Put in three sets of 2 black tiles
3. How many do you have? _6_
4. Make a rectangle:

Example: $3 \bullet -2 =$ ____

1. Start with zero
2. Put in three sets of 2 white tiles
3. How many do you have? _-6_
4. Make a rectangle:



1. **Model** these multiplications and **draw diagrams** of your models:

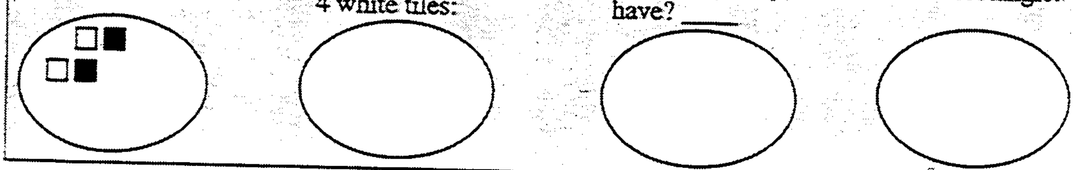
a) $2 \bullet -4 = -8$

Start with zero

Put in 2 sets of 4 white tiles:

How many do you have? ____

Make a rectangle:



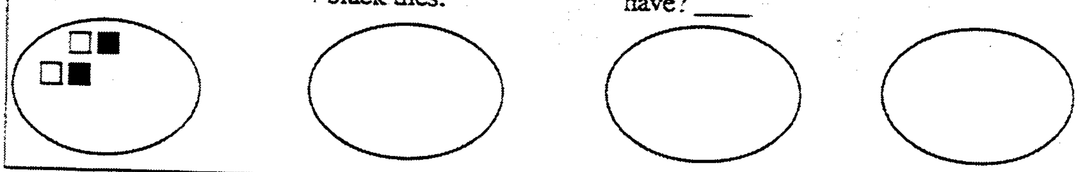
b) $2 \bullet 4 = 8$

Start with zero:

Put in two sets of 4 black tiles:

How many do you have? ____

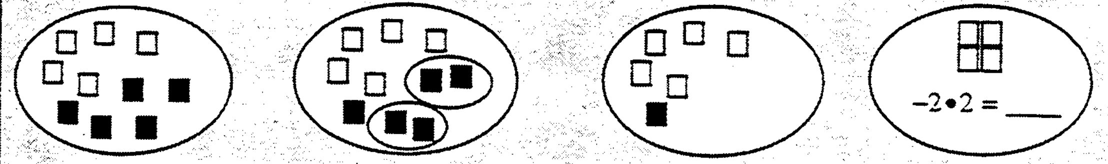
Make a rectangle:



Multiplying by a **negative** number is modeled by **taking tiles out** of a set.

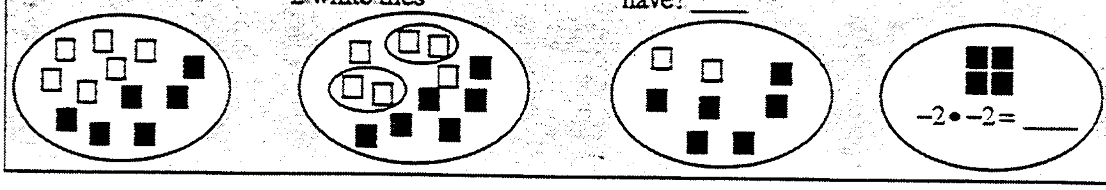
Example: $-2 \bullet 2 =$ ____

1. Start with zero.
2. Take out 2 sets of 2 black tiles
3. How many do you have? ____
4. Make a rectangle



Example: $-2 \bullet -2 =$ ____

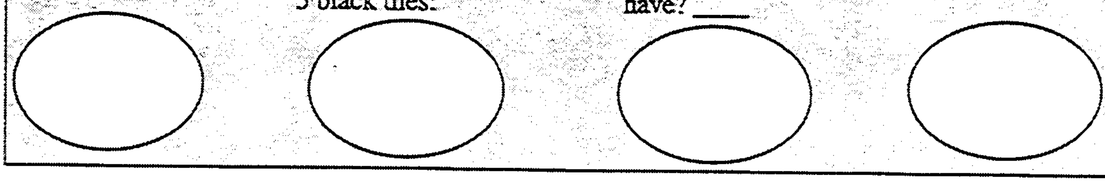
1. Start with zero
2. Take out 2 sets of 2 white tiles
3. How many do you have? ____
4. Make a rectangle



2. **Model** these multiplications and draw diagrams of your models:

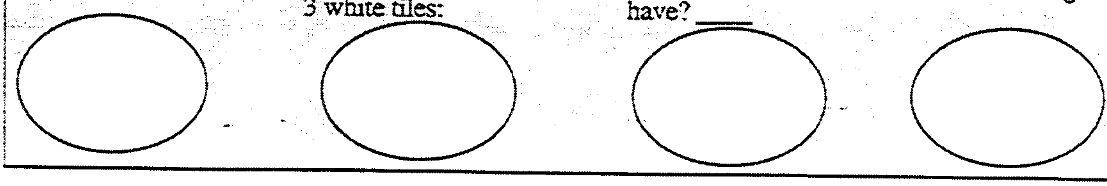
a) $-1 \bullet 5 = -5$

Start with zero: | Take out 1 set of 5 black tiles: | How many do you have? ____ | Make a rectangle:



b) $-2 \bullet (-3) = 6$

Start with zero: | Take out 2 sets of 3 white tiles: | How many do you have? ____ | Make a rectangle:



3. Use the preceding problem as an example to model these multiplication problems. Notice that the **order** of the numbers has been **reversed**.
   a) $5 \bullet -1 =$ ____
   b) $-3 \bullet (-2) =$ ____
   c) Tell how changing the order effects the answer.

4. Use your algebra tiles to model these multiplications. For *a - f*, sketch the process in two or three steps. (put on / make a rectangle; or add zero / take off / make a rectangle.)

a) 3 (-4) = _____

c) -4 (-5) = _____

e) -10 (2) = _____

b) -3 (2) = _____

d) -6 (-2) = _____

f) 7 (-3) = _____

Compute:

g) -9 + 6 = _____

i) -3 - 2 = _____

h) 2 - (-4) = _____

j) -3 (-2) = _____

k) **Complete**: Problem *i* above is a subtraction, while problem *j* is a _________________.

5. Use your tiles, if needed, to do these multiplications:

a) -1 (-2) = _____

c) 4 (-1) = _____

b) -1 (3) = _____

d) 5 (-1) = _____

c) **Explain** what happens when a number is multiplied by -1. _________________________

d) What happens to the tiles when you multiply by -1? Sketch the tiles that represent the number before and after multiplying by -1. What happens in each case? _________________________

Before multiplying by -1

After multiplying by -1

Write each of the following problems **under the correct heading** below and then **solve** the problems. Use your black and white tiles to check your answers.

2 *x* 3	-2 *x* (-7)	-8 *x* 9	-8 *x* (-6)
-2 *x* 3	-2 *x* 7	5 *x* (-8)	5 *x* 8
-9 *x* (-5)	-2 *x* (-3)	5 *x* (-2)	4 *x* (-9)
4 *x* 6	-8 *x* 6	-3 *x* (-3)	2 *x* (-7)
4 *x* 9	-9 *x* 5	2 *x* (-3)	4 *x* (-4)

POSITIVE *x* POSITIVE = POSITIVE				
2	*x*	3	=	6
______	*x*	______	=	______
______	*x*	______	=	______
______	*x*	______	=	______
______	*x*	______	=	______

NEGATIVE *x* NEGATIVE = POSITIVE				
-2	*x*	(-3)	=	6
______	*x*	______	=	______
______	*x*	______	=	______
______	*x*	______	=	______
______	*x*	______	=	______

POSITIVE *x* NEGATIVE = NEGATIVE				
2	*x*	(-3)	=	-6
______	*x*	______	=	______
______	*x*	______	=	______
______	*x*	______	=	______
______	*x*	______	=	______

NEGATIVE *x* POSITIVE = NEGATIVE				
-2	*x*	3	=	-6
______	*x*	______	=	______
______	*x*	______	=	______
______	*x*	______	=	______
______	*x*	______	=	______

Write a **rule** for multiplying **two positive** integers. ______________________________

____

Write a **rule** for multiplying **two negative** integers. ______________________________

____

Write a **rule** for multiplying a **positive integer by a negative integer.** ______________________

____

Solving Linear Equations Physically

Materials Needed: algebra tiles and diagram of balance scale

Name____________________
SS No. __________________

For this activity, assume that the weight of
a) a small black square is (+1) unit
b) a small white square is (-1) unit
c) a black rectangle is a variable quantity we will name S

1a) The expression **2S + 6** can be represented physically by **two black rectangles** and **six black squares.** Place these eight pieces on your desk.

b) Rearrange the pieces to **form a rectangle. Draw a diagram** of your rectangle (showing individual the pieces).

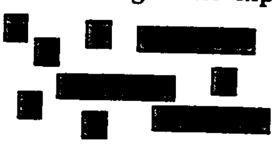
c) **Explain** why the weight of these eight pieces can also be expressed as **2(S+3)**

2. Represent each algebraic expression with rectangles and squares. **Draw diagrams** of your solutions.

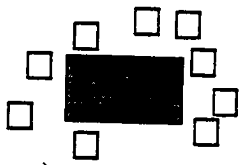
a) $2S + 4$ b) $2S + (-4)$ c) $2(S+4)$

d) $3S + (-1)$ e) $3[S + (-4)]$ f) $3(2S + 1)$

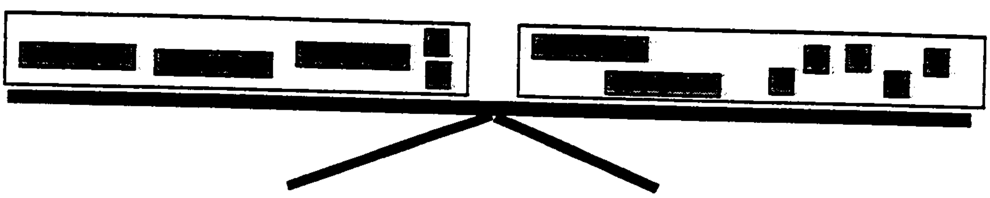
3. Write an **algebraic expression** for each diagram.


a) ______________


b) ______________


c) ______________

4. To represent the linear equation **3S + 2 = 2S + 5** physically, place your tiles on your balance scale as shown below:



a) Assume that **S = 1** and replace each rectangle with one blue square. Would your scale remain balanced? __________
b) Restore the original pieces to the scale.
c) Assume that **S = -2** and replace each rectangle with two white squares.
d) Simplify each side. (Take out zeros: $(-1) + (+1) = 0$)
e) **Describe** the contents of the left side of the scale: ______________________________
The right side: ______________________________________ Does the scale balance? ______

5. To solve the linear equation **3S + 2 = 2S + 5** physically, we must determine the number of squares that can be used to replace each rectangle and keep the scale balanced. (In other words we need to know the weight of 1 rectangle.) This can be accomplished by removing **equally weighted** pieces from each side of the scale until you have only one rectangle remaining on the scale.

a) **Represent** the equation **3S + 2 = 2S + 5** on your balance scale.
b) **Remove** two squares from **each side.**
c) Now **remove** two rectangles from **each side.**
d) The weight of **one rectangle** equals the weight of _______ **black squares**. Thus, the **solution** of the equations is S = ___.

6. Use the **method in exercise 5** to **solve** each of the following linear equations. Write your solutions in the spaces provided.

a) $2S + 4 = S + 5$ S =____________ b) $4S = 3S + 7$ S =____________

c) $5S + 3 = 4S + 8$ S =____________ d) $3S = 6$ S =____________

e) $4S + 1 = 2S + 7$ S =____________

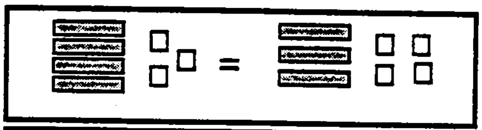
7. Try solving each of the following equations **without using your tiles.** Physically check your solutions by using the method in exercise 5.

a) $S + 5 = 9$ S =____________ b) $2S + 6 = S + 13$ S =____________

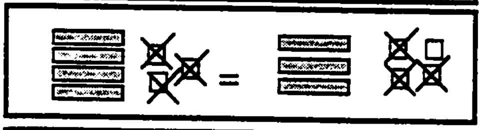
c) $5S + 2 = 3S + 6$ S =____________

8. Since we do not usually think of putting a weight of -1 on a scale, it is helpful also to look at **pictorial methods** for methods for solving linear equations. For example, to solve the equation **4S + (-3) = 3S + (-4)** pictorially, we again use the process **of removing equally valued pieces from (or adding equally valued pieces to) each side** of the configuration.

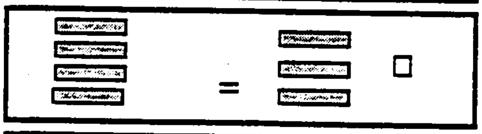
Step 1



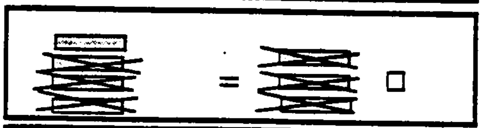
Step 2



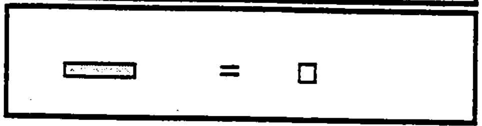
Step 3



Step 4



Solution:



The **solution is:** S = -1.

Note that to remove (or **subtract**) a square or a rectangle, we cross it out with an "X". In Step 2 we could have **added three black squares** to each side **instead of removing three white ones. Explain why.** ____

8. **Use the method described above** to **solve** each of the following linear equations.

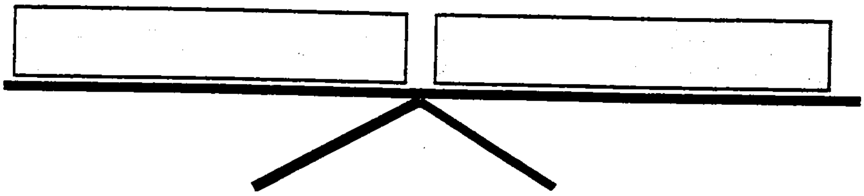
a) $2S + (-5) = -9 + S$ S =______ b) $3S + (-6) = S + (-8)$ S =______

c) $5S + 3 + 2[S + (-3)]$ S =______

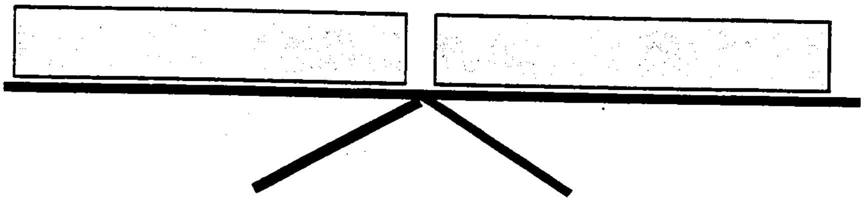
9. Try to **solve** each of the following equations **without using pictorial representations.** Check your solutions by using the method shown at the top of this sheet.

a) $4S + (-2) = 2S + (-10)$ S =______ b) $5S + (-6) = 4S + 8$ S =______

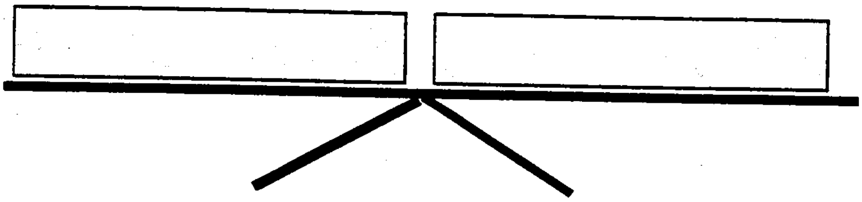
10. a) Illustrate on the scale below how you would represent the linear equation **$3S + (-4) = 2$**



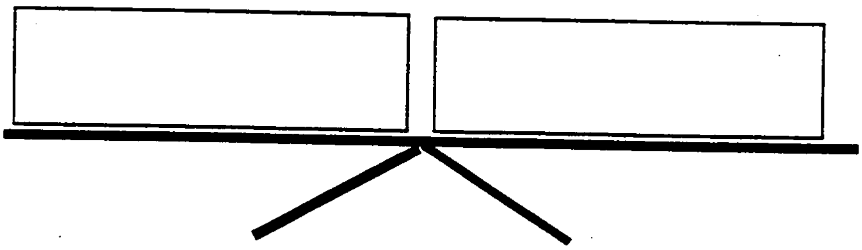
b) Since we can not take 4 negative tiles away from both sides (the right side only has 2 positive tiles), we can **add 4 positive tiles** to each side so that the (-4) is changed to zero. **Show adding +4 to each side** of the equation on the scale below:



c) **Remove zeros to simplify** the equation:



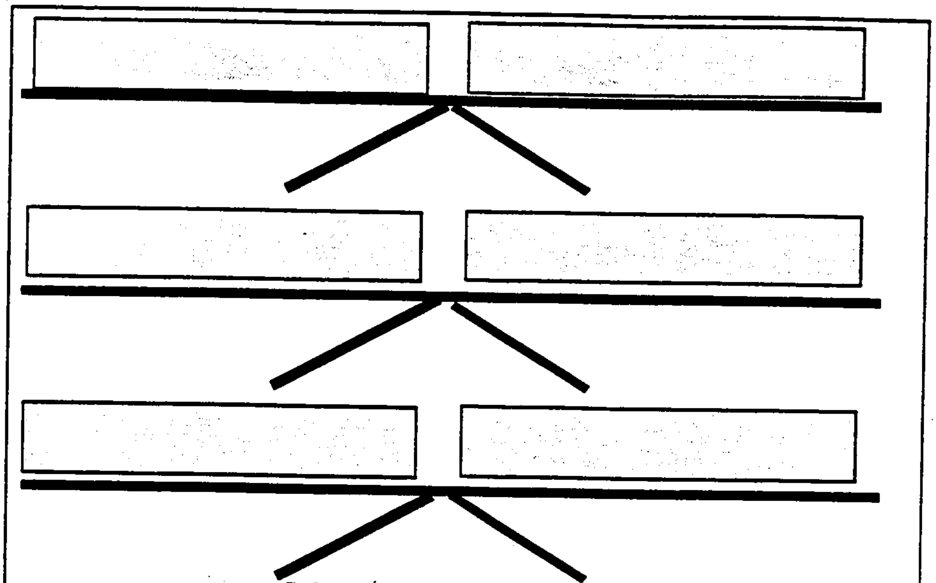
c) **Divide both sides by 3** to find the value of one "S."



Solution: S = ___

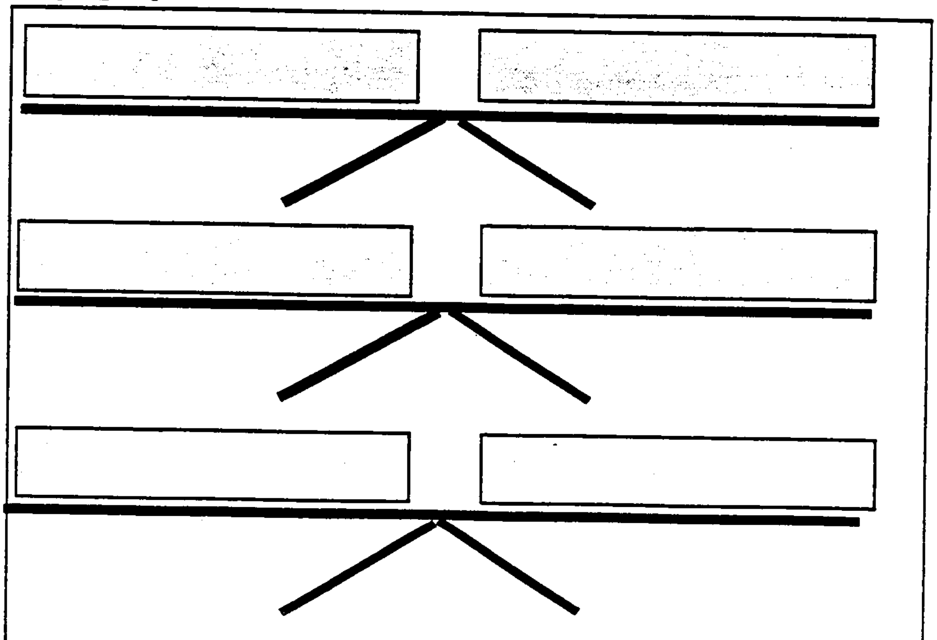
Solve these equations by **adding positive or negative tiles** to **each side** of the equation and then **dividing if necessary**. Use your tiles to help you and draw diagrams.

11. a) **2S + 3 = -1**



Solution: S = ____

b) **4S - 2 = 6**



Solution: S = ____

c) 4S - 3 = S + 3 S =__________

d) 5(S-1) = 2S + 7 S =__________

e) -3 + 2S = 3(S - 2) S =__________

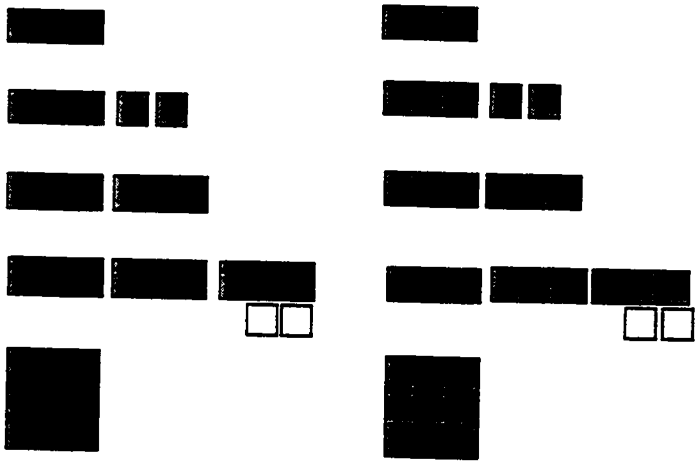
Writing and Evaluating Expressions

Materials: Algebra tiles
Name ____________________ SS No. ______________

Variables are one of the most important concepts of algebra. **A variable can stand for different numbers at different times.** For example, x could be a positive or a negative number or 0. Because they do not change numbers are called **constants**. Replacing a variable with a constant amount is called **substitution.**

1. The following figures show how the algebra tiles can be used to show the substitution $x = 3$ for the expressions x, $x+2$, $2x$, $3x + (-2)$, and x^2. **Complete the values column.**

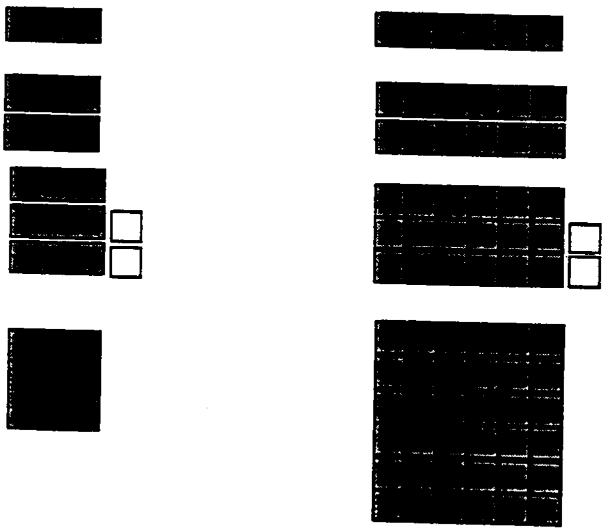
Model **Model if $x = 3$** **value, if x = 3**



	value, if x = 3
x	3
$x + 2$	____
$2x$	____
$3x + (-2)$	____
x^2	____

2. The following figures show how the algebra tiles can be used to show the substitution $x = 6$ for the expressions x, $2x$, $3x + (-2)$, and x^2.

Model **Model if $x = 6$** **value, if x = 6**



	value, if x = 6
x	6
$2x$	____
$3x + (-2)$	____
x^2	____

3. **Build** $3x + 2$ with the algebra tiles and **sketch** your model below. Then **sketch** what it looks like if the following **substitutions** are done. Then **give the value** of the expression in each case.

Sketch $3x + 2$:

a. if $x = 5$, sketch and give value of $3x + 2$

b. if $x = -1$, sketch and give value of $3x + 2$

4. **Build** $x^2 - 2$ with the algebra tiles and **sketch** your model below. Then **sketch** what it looks like if the following **substitutions** are done.

Sketch $x^2 - 2$:

a. if $x = 5$, sketch and give value of $x^2 - 2$

b. if $x = 1$, sketch and give value of $x^2 - 2$

5. **Show** by modeling with algebra tiles the **difference** between $3 + x$ and $3x$.

6. Find the **value** of $3 + x$ when:
   a) $x = 0$ **b)** $x = 5$ **c)** $x = 0.5$

7. Find the **value** of $3x$ when:
   a) $x = 0$ **b)** $x = 5$ **c)** $x = 0.5$

8. For most values of x, $3x$ does not equal $3 + x$. Find the **only number** you can substitute for x that will make $\mathbf{3 + x}$ **equal to** $\mathbf{3x}$. Draw a diagram. (You may need to show a fractional part of a tile.)

9. a) Model $\mathbf{3x^2}$ with your tiles and draw a diagram of your model. What is the **value** of this area if $\mathbf{x = 2}$?

b) Model $\mathbf{(3x)^2}$ and draw a diagram of your model. What is the **value** of this area if $\mathbf{x = 2}$?

10. a) Our algebra tiles only model x*'s* and x^2*'s*. Use your x^2 tile as a pattern and make a model of x^3. Draw a sketch of your model:

b) For a square, x^2 represents the **area** of the square with side of length x. What does x^3 represent for a cube with each edge of length x ? ______________________________

c) Find the **value** of x^3 if $x = 2$.

10. a) Remember that the distance a number is from zero is its **absolute value.** For numbers other than zero, this is always a ______________ (positive or negative) value.

The **absolute value** of an **expression** is the distance that the **value of the expression** is from zero. But first we must **substitute a value** for the variable. Use your algebra tiles to help you with the substitutions if necessary.

b) Complete the following examples:

if $x = 1$, $|x+2| = |1+2| = |3| =$ _____

if $x = -5$, $|x+2| = |-5+2| = |-3| =$ _____

c) Try these if $\mathbf{x = -2}$:

$|x+6| =$ _______ $|x-1| =$ _______ $|4+x| =$ _______

Algebraic Expressions

Name ________________ SS NO. ____________

Materials needed: Algebra Tiles

Meanings of minus:

The minus sign can mean three different things, depending on context:

- It can mean **negative**. In front of a positive number, it can mean negative. For example, -5 means negative 5.
- It can mean **opposite**. For example, -5 can mean the opposite of 5, which is negative 5. And - (-5) means the opposite of -5, which is 5. Likewise, -x means the opposite of x and -(x + 6) means the opposite of (x+6).
- It can mean **subtract**. Between two expressions it can mean to subtract the second expression from the first one. For example, x - 3 means subtract 3 from x (from our work in subtraction we know that x + (-3) gives the same answer).

1. For each of the following, write an **explanation** of what the **minus sign means.**
   a) y - 5 ____________________

   b) -(5x + 1) ____________________

   c) -2 ____________________

   d) -y ____________________

2. If $x \neq 0$, tell whether each of the following is **true or false**: (if false, give an example to show why)
   a) -x is always negative ____________________

   b) -x can be positive ____________________

   c) x^2 is always positive ____________________

   d) $-x^2$ is always negative ____________________

3. In your algebra tiles, the white rectangles represent *-x*. From your work above, discuss the value of these *-x* tiles. ____________________

   ____

4. Does x + -(x) always equal zero? How would you convince a friend of this? ____________

   ____

5. Does $x^2 + (-x^2)$ always equal zero? How would you convince a friend of this? ____________

   ____

6. Name each of the white algebra tile pieces. Decide if you think the best term should include the word negative or the word opposite.

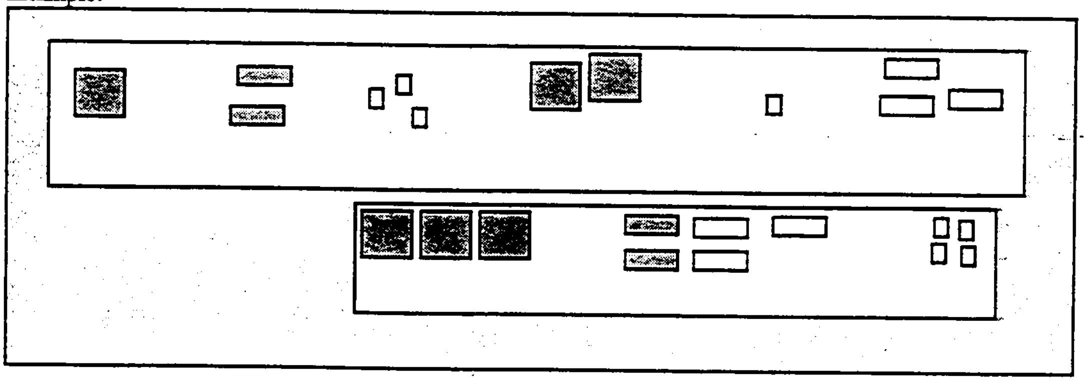
$\underline{x^2}$ $\quad$ $\underline{x}$ $\quad$ $\underline{1}$ $\quad$ ______ $\quad$ ______ $\quad$ ____

In the expression $3x^2 + x - 4$, three quantities are added or subtracted, so we say that there are three **terms**: $3x^2$; x ; and -4. Note that a **term** is the **product of numbers and variables**. The 3 in the term $3x^2$ is called the **coefficient**. A term like x has an invisible coefficient of 1, since $1x$ is usually written as x. **Like terms** have **identical variables with identical exponents.**

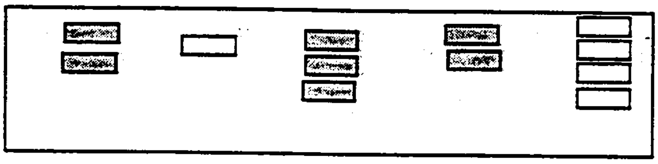
7. How can you recognize like terms with your algebra tiles? ________________________

8. For each set of tiles below, write an **expression** for each **term**, **combine like terms**, and write the **simplified expression**.

Example:

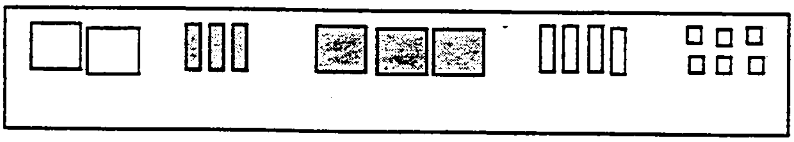


a)

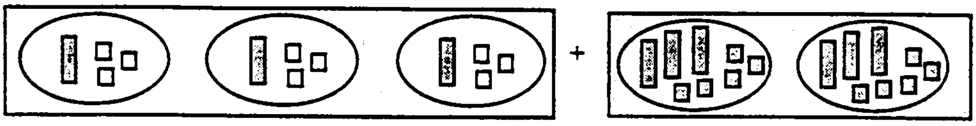


= ________________

b)



= ____________

c) 

Complete the expression for these tiles, then **simplify** the expression

3(________) + 2(________) = ____________________

9. Use your tiles to **draw diagrams** of these problems and then **combine like terms** and write the **simplified** expression.

a) $x^2 + 6 + 4x^2 + 8$ = ____________

b) $5x - 4 - 2x + 1$ = ____________

c) $2(x^2 - 2) + 3(x^2 + 2)$ = ____________

10. **Draw a diagram** that would help you use your algebra tiles to explain to a friend why

$$x^2 - 3x + 1 + 2(x^2 + x - 5) = 3x^2 - x - 9$$

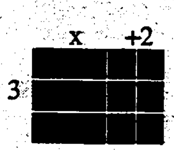
Distributive Property and Factoring

Name ______________________ SSNo. ____________

> Remember that the **area** of a rectangle is found by **multiplying** the ____________ by the ____________.

Example:

Make a rectangle with **width** 3 and **length** x + 2 with your algebra tiles. Draw the rectangle below:



What is the **area** of the rectangle? It takes 3 **x's** and 6 **units** to cover the rectangle, therefore the area is 3x + 6

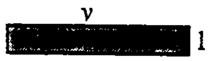
Write an equation for the area of the rectangle: 3(x + 2) = 3x + 6

1. a) Make a rectangle with **width** 2x and **length** x + 1 with your algebra tiles. Draw the rectangle below:

b) What is the **area** of the rectangle? ________________

c) Write an equation for the area of the rectangle. _____ x (______) = ___________

2. Suppose that another tile has **length** y. It looks like this:



a) Sketch a rectangle that has **width** 2 and **length** 3x + y + 5:

b) What is the area of the rectangle? ____________________

c) Write an equation for the area of the rectangle. ____ x _________ = ___________

3. **a)** Using the same y as in #2 sketch a rectangle that has **width** $\underline{2x}$ **and length** $\underline{y + 3}$.

b) What is the area of the rectangle? ____________________

c) Write an equation for the area of the rectangle. ____ x __________ = ___________

4. Use your equations above as examples to multiply:

a) $12(3x + 5)$ **b)** $6a(2a + 3b + 4)$

5. The **distributive property** says: $a(b + c) = ab + ac$. Were you using the distributive property in your work for number 4? _____ In your own words what does the distributive property tell you to do? ____

Opposites

The opposite of an algebraic expression can be illustrated with your algebra tiles.

$\underline{-(2x + 1)} = -($  $)$ =  or $\underline{-2x \; -1}$

$\underline{-(x - 3)} = -($  $)$ =  or $\underline{-x + 3}$

6. **Illustrate** and **name** the following opposites:

a) $-(3x - 2)$

b) $-(-2x + 1)$

7. How does the **distributive property** relate to **opposites**? ____________________

Factoring

Example:
Make a rectangle of **area** $4x + 6$ with your algebra tiles. What are the length and the width of the rectangle?

the width is ______ and the length is ______

8. a) Make a rectangle of **area** $2x^2 + 6x$. **Sketch** the **rectangle** below.

b) The width is __________ and the length is __________.

9. a) Make a rectangle of area $4x^2 + 2x + 8$. **Sketch** the **rectangle** below.

b) The width is __________ and the length is __________.

> Given the rectangle and looking for the length of a rectangle is like reversing the distributive property. It is called **factoring.**

10. Factor the following expressions. You may use your tiles to help you if needed.

a) $5x + 10$ **b)** $x^2 + 4x$ **c)** $3x^2 + 3x$

11. Factor these using your sign rules.

a) $3x - 3$ **b)** $2x^2 - 6x + 4$

c) $x^3 - 6x^2 + 5x$ **d)** $6a^2 - 9ab + 12ac$

Appendix C

Fennema-Sherman Mathematics Attitude Scales

FENNEMA-SHERMAN MATHEMATICS ATTITUDE SCALE

Elizabeth Fennema - Julia A. Sherman
University of Wisconsin-Madison

Name ____________________________ Social Security No.____________________

Please respond to the following statements. There are no correct answers for these statements. They have been set up in a way which permits you to indicate the extent to which you agree or disagree with the ideas expressed. The only correct responses are those that are true for you.

Please circle A is you strongly agree with the statement, B if you somewhat agree, C if you feel neutral, D if you somewhat disagree, and E if you strongly disagree.

A B C D E 1. Math doesn't scare me at all.

A B C D E 2. I don't understand how some people can spend so much time on math and seem to enjoy it.

A B C D E 3. I usually have been at ease in math classes.

A B C D E 4. Mathematics is enjoyable and stimulating to me.

A B C D E 5. Math puzzles are boring.

A B C D E 6. Mathematics makes me feel uneasy and confused.

A B C D E 7. When a question is left unanswered in math class, I continue to think about it afterward.

A B C D E 8. Mathematics makes me feel uncomfortable, restless, irritable, and impatient.

A B C D E 9. A math test would scare me.

A B C D E 10. I am challenged by math problems I can't understand immediately.

A B C D E 11. I like math puzzles.

A B C D E 12. When a math problem arises that I can't immediately solve, I stick with it until I have the solution.

A B C D E 13. It wouldn't bother me at all to take more math courses.

A B C D E 14. Figuring out mathematical problems does not appeal to me.

A B C D E 15. I would rather have someone give me the solution to a difficult math problem than to have to work it out for myself.

A B C D E 16. The challenge of math problems does not appeal to me.

A B C D E 17. I get a sinkng feeling when I think of trying hard math problems.

A B C D E 18. I haven't usually worried about being able to solve math problems.

A B C D E 19. I usually have been at ease during math tests.

A B C D E 20. I almost never have gotten shook up during a math test.

A B C D E 21. My mind goes blank and I am unable to think clearly when working mathematics.

A B C D E 22. I do as little work in math as possible.

A B C D E 23. Mathematics usually makes me feel uncomfortable and nervous.

A B C D E 24. Once I start trying to work on a math puzzle, I find it hard to stop.

Appendix D

Student Questionnaires

STUDENT INFORMATION

Name ______________________ Social Security No. _________________

Gender: Age:

Male _____ 18-20 _____
Female _____ 21-30 _____
over 30 _____

Mathematics Background:

___ No previous algebra experience

High School Experience:

___ Algebra I Year: ____
___ Algebra II Year: ____
___ Geometry Year: ____
___ Advanced Math Year: ____
___ Other Year: ____

WSCC Experience:

___ Math 0710, Basic Math Year: ____
___ Math 0820, Elementary Algebra Year: ____

Algebra Tiles

Student Exit Questionnaire

Name ________________________ Social Security No. ____________________

Circle the letter that most nearly represents your feelings toward working with algebra tiles.

1. I found using algebra tiles to be
   a) a help to me
   b) a waste of my time
   c) a confusing activity

2. I find algebra tiles to be most beneficial when working
   a) in small groups
   b) by myself
   c) as a whole class

3. Algebra tiles were most helpful to me when studying
   a) signed numbers
   b) equations
   c) polynomials
   d) other: ____________

4. How do you think algebra tiles may have influenced your performance in this course?
   a) raised my grade
   b) no effect on my grade
   c) lowered my grade

5. Do you feel that working with algebra tiles helped you to understand algebra better?
   a) yes
   b) no
   c) uncertain

6. How often did you use algebra tiles outside of class?
   a) often
   b) seldom
   c) never

7. How often did you use algebra tiles on tests?
   a) often
   b) seldom
   c) never

Appendix E

Raw Data

Raw Data

Student #	Achievement			Attitude		Anxiety		Age	Experience	Teacher	Treatment Group
	Pre	Post #1	Post #2	Pre	Post	Pre	Post	1= 18-20 2= 21-30 3= >30	1= Recent 2= No Recent		1= Control 2= Experimental
1	12	62	24	50	49	32	32	3	2	1	1
2	55	62	82	29	36	40	40	1	1	1	1
3	32	59	61	24	30	15	21	1	1	1	1
4	15	29	31	27	32	20	21	1	1	1	1
5	32	84	100	45	47	40	50	1	1	1	1
6	29	86	97	47	54	16	24	3	2	1	1
7	63	92	97	47	41	43	43	1	1	1	1
8	53	57	75	29	20	29	30	1	1	1	1
9	8	42	78	50	47	21	24	3	2	1	1
10	35	76	76	34	23	22	36	1	1	1	1
11	29	99	97	43	38	23	29	1	1	1	1
12	23	45	48	15	12	20	20	1	2	1	2
13	24	74	83	38	33	32	28	1	1	1	2
14	30	44	63	23	28	15	12	1	1	1	2
15	62	75	90	26	25	21	24	1	1	1	2
16	38	76	87	27	37	14	29	3	2	1	2
17	47	80	97	38	43	41	55	1	1	1	2
18	37	53	51	23	19	29	28	1	1	1	2
19	15	50	76	48	42	36	33	3	2	1	2
20	61	70	63	25	21	33	42	1	1	1	2
21	43	73	70	29	21	24	23	1	1	1	2
22	44	90	99	43	25	38	50	1	1	1	2
23	22	51	60	27	18	23	14	1	1	1	2
24	18	76	69	16	36	12	37	1	2	1	2
25	27	66	76	42	43	26	24	3	2	1	2

Student #	Achievement			Attitude		Anxiety		Age	Experience	Teacher	Treatment Group
								1= 18-20 2= 21-30 3= >30	1= Recent 2= No Recent		1= Control 2= Experimental
	Pre	Post #1	Post #2	Pre	Post	Pre	Post				
26	52	79	78	25	35	33	46	1	1	1	2
27	43	63	65	24	31	23	33	1	1	1	2
28	21	64	59	33	32	38	41	1	1	1	2
29	28	63	74	20	26	30	31	1	1	1	2
30	22	73	92	45	48	52	50	3	2	1	2
31	42	87	84	32	33	25	23	1	2	1	2
32	51	89	88	26	29	30	42	1	1	1	2
33	17	58	59	37	33	28	26	1	1	2	1
34	8	75	96	41	39	50	59	3	2	2	1
35	9	50	49	58	60	49	57	2	2	2	1
36	64	90	87	43	46	41	53	1	1	2	1
37	23	51	71	41	39	28	36	3	2	2	1
38	28	44	51	43	38	24	31	1	1	2	1
39	58	81	76	30	30	22	27	1	1	2	1
40	22	67	74	42	43	21	23	3	2	2	1
41	47	62	70	28	36	27	40	1	1	2	1
42	25	69	81	29	30	31	24	1	1	2	1
43	55	67	70	40	35	37	42	1	1	2	1
44	36	58	59	37	36	13	32	3	2	2	1
45	28	87	86	37	45	30	42	2	2	2	1
46	15	52	68	30	37	25	21	3	2	2	1
47	30	62	70	50	32	43	32	1	1	2	1
48	25	72	73	28	21	17	14	2	2	2	1
49	48	60	67	22	23	25	42	1	1	2	1
50	31	67	82	43	43	31	36	1	1	2	1

Student #	Achievement			Attitude		Anxiety		Age	Experience	Teacher	Treatment Group
								1= 18-20 2= 21-30 3= >30	1= Recent 2= No Recent		1= Control 2= Experimental
	Pre	Post #1	Post #2	Pre	Post	Pre	Post				
51	27	75	75	45	44	25	35	2	2	2	2
52	36	48	48	35	31	37	29	1	1	2	2
53	43	81	63	27	39	29	35	1	1	2	2
54	3	56	90	43	29	30	27	3	2	2	2
55	28	60	87	53	55	38	38	2	2	2	2
56	47	64	77	39	37	31	38	1	1	2	2
57	22	36	48	21	19	14	14	1	1	2	2
58	26	67	79	47	59	54	60	2	2	2	2
59	38	71	67	20	26	23	26	1	1	2	2
60	56	88	93	40	47	44	59	2	2	2	2
61	34	69	63	36	37	19	23	2	2	2	2
62	39	67	70	33	29	33	36	1	1	2	2
63	32	58	78	21	24	40	39	1	2	2	2
64	52	91	96	51	60	58	60	2	2	2	2
65	22	43	52	35	39	44	46	1	1	2	2
66	59	67	67	36	29	21	24	2	2	2	2
67	19	76	91	29	32	26	21	3	2	2	2
68	62	69	79	39	41	46	44	1	1	2	2
69	44	64	71	26	26	32	38	1	1	3	1
70	56	74	81	32	35	26	31	1	1	3	1
71	30	55	61	53	36	48	41	3	2	3	1
72	27	55	36	25	32	45	42	1	1	3	1
73	52	79	66	37	34	46	45	1	1	3	1
74	27	66	83	23	24	18	26	1	1	3	1
75	43	68	83	23	19	12	29	1	1	3	1

Student #	Achievement			Attitude		Anxiety		Age	Experience	Teacher	Treatment Group
								1= 18-20 2= 21-30 3= >30	1= Recent 2= No Recent		1= Control 2= Experimental
	Pre	Post #1	Post #2	Pre	Post	Pre	Post				
76	31	56	55	39	33	26	28	1	1	3	1
77	25	70	70	55	53	46	46	2	2	3	1
78	34	55	77	46	53	34	37	2	1	3	1
79	16	55	71	43	42	28	31	3	1	3	1
80	19	73	95	47	49	43	52	3	2	3	1
81	30	56	75	36	31	27	28	2	2	3	1
82	28	53	67	28	26	17	13	1	1	3	1
83	52	96	79	31	31	48	56	2	2	3	1
84	72	94	99	46	45	51	48	1	1	3	1
85	54	69	72	40	40	40	45	1	1	3	1
86	27	81	97	38	36	37	36	1	1	3	1
87	35	56	84	36	35	38	39	1	1	3	2
88	45	82	100	28	43	17	29	2	2	3	2
89	61	89	81	43	44	42	53	1	1	3	2
90	54	73	82	44	38	38	39	1	1	3	2
91	28	64	70	35	46	33	34	1	1	3	2
92	53	65	87	34	34	22	25	1	1	3	2
93	34	72	70	32	37	21	37	1	1	3	2
94	20	55	39	23	26	16	16	1	1	3	2
95	56	63	61	36	30	39	30	1	1	3	2
96	54	59	66	36	37	39	44	1	1	3	2
97	39	71	85	45	48	34	43	1	1	3	2
98	51	74	69	38	26	35	25	1	1	3	2
99	6	55	85	41	42	26	13	3	2	3	2
100	48	73	90	34	38	32	38	1	1	3	2

Student #	Achievement			Attitude		Anxiety		Age	Experience	Teacher	Treatment Group
								1= 18-20 2= 21-30 3= >30	1= Recent 2= No Recent		1= Control 2= Experimental
	Pre	Post #1	Post #2	Pre	Post	Pre	Post				
101	37	100	99	54	58	60	60	3	2	3	2
102	61	74	88	36	35	49	44	1	1	3	2
103	33	73	78	27	22	14	14	1	1	3	2
104	53	83	93	32	36	26	20	1	1	3	2
105	36	81	90	37	37	44	47	2	2	3	2
106	19	89		49	50	37	44	2	1	1	2
107	34	76		50	45	47	40	2	2	2	1
108	46	50		29		35		1	1	2	1
109	21	34		18	16	25	15	3	2	2	1
110	20	64		46	42	54	50	1	1	2	2
111	32	46		35	27	13	16	3	2	2	2
112	65	58		26	28	13	22	1	1	3	1
113	19	60		55	51	52	54	3	2	3	1
114	29	66		37	34	36	48	2	2	3	1
115	12	23		34	16	17	12	1	1	3	1
116	51	72	87	37		35		1	1	3	1
117	11	69	85		31		26	3	2	1	2
118	52	76	91		33		43	1	1	1	1
119	55	61	75	44	35	24	36	1	2	2	1

Appendix F

Analysis of Variance Tables

Table E.1: Multivariate Tests for Achievement

Effect	Wilks' Lambda	F	Hypothesis df	Error df	Sig.
Time	.177	497.981[a]	1.000	107.000	<.001 *
Time * Experience	.806	25.818[a]	1.000	107.000	<.001 **
Time * Treatment	.999	.109[a]	1.000	107.000	.741
Time * Teacher	.957	2.388[a]	2.000	107.000	.097
Time * Experience * Treatment	1.000	.009[a]	1.000	107.000	.923
Time * Experience * Teacher	.990	.559[a]	2.000	107.000	.573
Time * Treatment * Teacher	.992	.435[a]	2.000	107.000	.649
Time * Experience * Treatment * Teacher	.990	.516[a]	2.000	107.000	.598

a. Exact statistic

Between for Achievement

Measure: MEASURE_1

Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	474874.301	1	474874.301	1461.911	<.001
Experience	1721.314	1	1721.314	5.299	.023 ***
Treatment	561.225	1	561.225	1.728	.192
Teacher	788.412	2	394.206	1.214	.301
Experience * Treatment	511.907	1	511.907	1.576	.212
Experience * Teacher	771.062	2	385.531	1.187	.309
Treatment * Teacher	188.958	2	94.479	.291	.748
Experience * Treatment * Teacher	242.621	2	121.310	.373	.689
Error	34756.932	107	324.831		

Treatment Groups: Control Group
Manipulative Group

Experience Groups: Recent-Experience Group
No-Recent-Experience Group

Teacher: 1
2
3

* Significant difference in pre-test and post-test achievement scores.

** Significant time by experience group interaction.

*** Significant interaction between experience groups.

Table E.2: ANOVA Tests for Retention

Dependent Variable: POST2

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	14103.704[a]	12	1175.309	9.053	<.001
Intercept	2575.391	1	2575.391	19.838	<.001
Post1	10476.276	1	10476.276	80.699	<.001 *
Experience	228.641	1	228.641	1.761	.188
Treatment	130.883	1	130.883	1.008	.318
Teacher	178.398	2	89.199	.687	.505
Experience * Treatment	533.077	1	533.077	4.106	.045 **
Experience * Teacher	276.612	2	138.306	1.065	.349
Treatment * Teacher	92.510	2	46.255	.356	.701
Experience * Treatment * Teacher	10.311	2	5.155	.040	.961
Error	12462.626	96	129.819		
Total	642695.000	109			
Corrected Total	26566.330	108			

a. R Squared = .531 (Adjusted R Squared = .472)

Treatment Groups: Control Group
Manipulative Group

Experience Groups: Recent-Experience Group
No-Recent-Experience Group

Teacher: 1
2
3

* Significant relationship between scores on post-test 1 and final exam (post-test 2)
** Significant interaction between experience and treatment groups.

Table E.3: Multivariate Tests for Anxiety

Effect	Wilks' Lambda	F	Hypothesis df	Error df	Sig.
Time	.859	16.937[a]	1.000	103.000	<.001 *
Time * Experience	1.000	.008[a]	1.000	103.000	.927
Time * Treatment	.992	.860[a]	1.000	103.000	.356
Time * Teacher	.988	.618[a]	2.000	103.000	.541
Time * Experience * Treatment	1.000	.037[a]	1.000	103.000	.848
Time* Experience * Teacher	.998	.102[a]	2.000	103.000	.903
Time * Treatment * Teacher	.995	.256[a]	2.000	103.000	.775
Time *Experience * Treatment * Teacher	.996	.198[a]	2.000	103.000	.821

a. Exact statistic

Between for anxiety

Measure: MEASURE_1
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	198633.131	1	198633.131	770.653	<.001
Experience	65.931	1	65.931	.256	.614
Treatment	14.880	1	14.880	.058	.811
Teacher	1354.116	2	677.058	2.627	.077
Experience * Treatment	32.815	1	32.815	.127	.722
Experience * Teacher	1456.723	2	728.361	2.826	.064
Treatment * Teacher	287.286	2	143.643	.557	.574
Experience *Treatment * Teacher	340.818	2	170.409	.661	.518
Error	26547.890	103	257.747		

Treatment Groups: Control Group
Manipulative Group

Experience Groups: Recent-Experience Group
No-Recent-Experience Group

Teacher: 1
2
3

* Significant difference in pre-test and post-test scores.

Table E.4: Multivariate Tests for Attitude

Effect	Wilks' Lambda	F	Hypothesis df	Error df	Sig.
Time	1.000	.014[a]	1.000	103.000	.907
Time * Experience	.987	1.336[a]	1.000	103.000	.250
Time * Treatment	.957	4.628[a]	1.000	103.000	.034 *
Time * Teacher	.997	.154[a]	2.000	103.000	.858
Time * Experience * Treatment	.989	1.145[a]	1.000	103.000	.287
Time * Experience * Teacher	.992	.436[a]	2.000	103.000	.648
Time * Treatment * Teacher	.988	.624[a]	2.000	103.000	.538
Time * Experience * Treatment * Teacher	.980	1.055[a]	2.000	103.000	.352

a. Exact statistic

Between Subjects Tests for Attitude

Measure: MEASURE_1

Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	252879.735	1	252879.735	1656.361	<.001
Experience	2675.787	1	2675.787	17.526	<.001 * *
Treatment	441.997	1	441.997	2.895	.092
Teacher	121.778	2	60.889	.399	.672
Experience * Treatment	113.267	1	113.267	.742	.391
Experience * Teacher	236.147	2	118.074	.773	.464
Treatment * Teacher	960.066	2	480.033	3.144	.047 * * *
Experience * Teacher * Treatment	423.945	2	211.972	1.388	.254
Error	15725.207	103	152.672		

Treatment Groups: Control Group
Manipulative Group

Experience Groups: Recent-Experience Group
No-Recent-Experience Group

Teacher: 1
2
3

* Significant time by experience group interaction.
** Significant interaction between treatment groups.
*** Significant interaction between teacher groups.

Appendix G

Exit Survey Results

Results of Student Exit Survey

1. I found using algebra tiles to be
   - 20% a) a help to me
   - 17% b) a waste of my time
   - 63% c) a confusing activity

2. I find algebra tiles to be most beneficial when working
   - 56% a) in small groups
   - 11% b) by myself
   - 32% c) as a whole class

3. Algebra tiles were most helpful to me when studying
   - 32% a) signed numbers
   - 21% b) equations
   - 27% c) polynomials
   - 19% d) other: ___________

4. How do you think algebra tiles may have influenced your performance in this course?
   - 17% a) raised my grade
   - 44% b) no effect on my grade
   - 39% c) lowered my grade

5. Do you feel that working with algebra tiles helped you to understand algebra better?
   - 17% a) yes
   - 48% b) no
   - 34% c) uncertain

6. How often did you use algebra tiles outside of class?
   - 8% a) often
   - 36% b) seldom
   - 56% c) never

7. How often did you use algebra tiles on tests?
   - 5% a) often
   - 33% b) seldom
   - 62% c) never

Vita

Ida (Lee) McCalip Dell'Isola was born in McComb, Mississippi, on September 16, 1944. She is married to Phil Dell'Isola and is the mother on one son, Philip.

Mrs. Dell'Isola received the Bachelor of Science degree in Mathematics in 1966 and the Master of Education degree in Curriculum and Instruction in 1986, both from the University of Southern Mississippi in Hattiesburg, Mississippi. She received the Doctor of Philosophy degree in Education with a concentration in community college mathematics instructional theory and practice from the University of Tennessee, Knoxville, in May 1999.

Mrs. Dell'Isola has taught middle school mathematics in the public school systems of Huntsville, Alabama, North Pike County, Mississippi, and Nashville, Tennessee. She currently teaches developmental and college-level mathematics at Walters State Community College in Morristown, Tennessee.

Improving classroom instruction in mathematics is a major interest of Mrs. Dell'Isola. She enjoys working with public school teachers to help implement the teaching standards of the National Council of Teachers of Mathematics.