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DEVELOPMENT OF A CIRCUIT THAT LINEARIZES THE RESPONSE OF A
PLATINUM RESISTANCE TEMPERATURE DETECTOR

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Gus C. Pabon

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DEDICATION

I wish to dedicate this thesis to my family, my mother, Carmen, whose never-ending love and unselfishness makes her a wonderful person, my father, Gus, who inspired me by completing his Master's Degree, and my sister Jackie, my brothers Bernie and Chris, who have shared some of the best times of my life. Though great distances have seperated us for some time, we are together in our thoughts and in our hearts.

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ABSTRACT

This thesis presents the development and analysis of a signal processor that linearizes the response of a Platinum Resistance Temperature Detector, commonly referred to as RTD.

The function of the RTD Processor is to accurately measure temperature in any resistance thermometry application which uses the RTD as a temperature transducer. The characteristic change of the RTD's electrical resistance with temperature makes possible this measurement; however, a linearizing circuit is needed because the resistance change with temperature is not linear.

Calculated results show that the circuit is capable of producing a temperature inaccuracy of 0.005% of full scale temperature for 0 to 200 °C and 0.05% inaccuracy of full scale temperature for the range of 0 to 600 °C. Experimental results confirm the linearity of the output response of the circuit and are correlated with predicted results given in this thesis. The maximum error encountered in experimentation was 0.02% of full scale temperature for the range of 0 to 200 °C.

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CHAPTER 1

INTRODUCTION

A circuit to linearize the response of a platinum resistance temperature detector was developed by this author for application in the Nuclear Power industry while he was employed at the Technology for Energy Corporation, hereafter referred to as TEC. The work at TEC required the development of a complex safety-monitoring system for Public Service Electric and Gas' Hope Creek Nuclear Power Plant. This plant has a boiling water reactor and has, as an integral part, a Suppression Pool System. This Suppression Pool is part of the Reactor Core Isolation Cooling System^{1,2} and is used in the event that the reactor core is isolated from its normal cooling system. The pressurized steam is vented to the suppression pool for cooling in an emergency situation; therefore, temperature measurement of the pool water is very important. The temperature is measured using 100 ohm platinum RTD transducers as specified in the contract. TEC supplied the RTD Processors as part of the monitoring instrumentation.

The linearizing electronics, TEC Model 982-1 RTD Processor, is designed to be housed in an instrumentation rack of the TEC Model 185 Control Room Cabinet. Field-mounted, platinum RTD's connect to the Processor and allow temperature measurements to be made at remote locations in the plant.

Resistance Thermometry had applications long before technology levels we are now accustomed to ever existed so a review of its history and background is of interest.

1.1 History of Resistance Thermometry

Resistance thermometry from its birth to the present has a long rich history in science dating back to 1821. It was in this year that while performing some experiments using platinum wire, Humphry Davy³ at the Royal Institution, discovered that the electrical resistance of a metal varies with temperature. As the interest in this phenomenon grew, Carl Wilhelm Siemens, naturalized as Charles William Siemens, recognized this phenomenon's first application in thermometry.⁴ In 1850, he used copper wire to sense the spontaneous generation of heat inside spools of telegraph cable which were being shipped to the Far East. These spools were part of the investment in his business and Siemen's idea prevented the deterioration of this investment. His work continued with platinum and is well documented.

Experimentation and prediction of results in resistance thermometry were very difficult without first establishing an accurate relationship between temperature and resistance change of a metal. In 1885 Hugh Longbourne Callendar⁵, while doing research under J.J Thompson, developed an equation defining the change in platinum's resistance to temperature. This equation was modified in 1924 by Milton S. Van Dusen⁶ as a result of work performed for the National Bureau of Standards. This equation is popularly known as the

Callendar-Van Dusen Equation and is presently used to establish the International Practical Temperature Scale^{7,8} in the range of -190°C to 660°C using a Standard Platinum Resistance Thermometer.

1.2 Scope of Work

This thesis presents theory and analysis on the subject of linearizing a Platinum RTD. Much literature was researched in preparation of this text and this work attempts to incorporate the ideas of the various authors. The approach is to analyze the 982-1 circuit from which a transfer function relating output voltage to sensed temperature is obtained. From this transfer function, the series representation of the transfer function is developed and the linearizing criteria will be derived.

Chapter 2 reviews resistance thermometers available in industry and discusses their properties. Previous work on this subject is also discussed with emphasis on the work that most closely relates to the approach taken in this thesis.

Chapter 3 discusses the concept of using a two-resistor network to linearize the response of a resistance thermometer. The Callendar-Van Dusen Equation which defines the resistance to temperature relationship is introduced here.

Chapter 4 includes the derivation of the transfer function and introduces the TEC 982-1 circuit. The positive feedback implemented to linearize the platinum RTD is reviewed. This chapter also analyzes the Howland current pump, which provides the linearizing feedback current. The output resistance of the Howland current pump is derived

as well as the loop transmission expression for frequency stability analysis.

Chapter 5 presents the development of the power series representation of the transfer function. A linearization approach is given as part of the analysis.

Chapter 6 discusses a more practical approach to optimizing the linearizing property of the circuit. The experimental results are presented as well as the calculated results of linearization.

Chapter 7 has suggestions for improvement of the design and summarizing discussion.

CHAPTER 2

TRANSDUCER TYPES AND PREVIOUS WORK

There are many temperature transducers available in industry today and though some of them are briefly discussed in this text, it is recommended that the reader refer to the references given for further information.

Resistance thermometers can be divided into three groups: Metallic Conductors, Semiconductors, and Insulators. Their application is usually determined by the temperature range to be measured, with insulators considered for extremely high temperatures and semiconductors for low temperatures. Metallic conductors have the widest range of application.

2.1 Metallic Conductors

The most commonly used metals for resistance thermometers include platinum, copper and nickel. Among these choices, platinum is the most popular. Platinum exhibits a high melting point and is available in pure form. It is very stable chemically and resists oxidation at high temperatures. Platinum RTD's⁹ are available in a laboratory class with stringent specifications and in an industrial class where the emphasis is on durability but both applications benefit from the superior properties of platinum.

Nickel resistance thermometers are less expensive than platinum resistance thermometers and have a higher coefficient of resistivity which gives them the advantage of higher sensitivity in electronic

measurements. This metal is, however, less stable than platinum in that its resistance characteristics become unpredictable at temperatures much above 315 °C.¹⁰

Copper resistance thermometers have the unique property of being linear but only within the narrow limits of -50 to +200 °C.¹⁰ Copper's cost is less than nickel and platinum but the metal tends to oxidize at temperatures much above 200 °C.

2.2 Semiconductors

Similar to the application of metals as resistance thermometers, some semiconductors may be exploited with advantages for the purpose of temperature measurement. Thermistors and germanium thermometers are semiconductors frequently used as temperature transducers.

A property of semiconductor resistance thermometers is the negative slope of the resistance vs. temperature response of these elements. At low temperatures the sensitivity of resistance to temperature change for both silicon and germanium is very high allowing for accurate measurements to be made over narrow spans of temperature. The function defining the resistance to temperature change for a thermistor is approximately exponential¹¹ but for a germanium thermometer this function cannot be as clearly defined. Small changes in the amount of impurity in a germanium element make the reproducibility of critical parameters difficult from batch to batch. Purely empirical polynomials must be used to define the resistance to temperature relation and each thermometer must be calibrated at a number of different points.^{12,13}

2.3 Insulators

For extremely high temperature ranges which exceed the melting point of metals, insulator resistance thermometers¹⁴ are the only materials capable of withstanding the heat. Ceramic thermistors have been extensively studied, especially with the advent of the jet engine. These resistance thermometers resist oxidation effects and have the property of decreasing resistance with increasing temperatures. The attachment of leads in the process of experimentation is one of many problems that have been encountered in the study of the elements. Research, however, continues with the hope of using these elements as temperature standards for interpolating between fixed points in the regime above 2300 °R.

2.4 Previous Work

Much work has been done on techniques to linearize the response of resistance thermometers. Diamond¹⁵, in 1969, presented a paper which deals with the linearization of resistance thermometers. In this paper a very generalized approach is outlined in which some conditions for linearization arise from his analysis. Diamond uses a Thevenin equivalent of a general circuit to develop a two series resistor circuit which is the basis for his analysis. The approach of using this simplified circuit proves to be very important since most circuits can be simplified using their Thevenin equivalent.

Other papers of particular interest deal with thermistor linearization. Beakley¹⁶ introduces an approach to analyzing the problem using the power series representation of the circuit transfer

function. He eliminates the second order term in the series and shows that the higher order terms are negligible which results in an approximately linear function. Beakley's approach is very analytical and is very similar to the approach taken in this thesis. Though Beakley's circuit deals with the thermistor circuit, it is applicable to a platinum resistance linearizing circuit and the details of the derivation are presented in this thesis.

Hodge¹⁷ follows up on Beakley's paper and compares eight different circuits and their thermistor linearization accuracy. He concludes that, according to his work, all the circuits are equally accurate in linearizing the thermistor's response.

Platinum resistance thermometers are addressed in Bolk's work presented in 1979.¹⁸ Bolk's analysis deals with both thermistor¹⁹ and platinum thermometers. In both his papers he takes a different approach in doing the analysis of his circuit response, $V(t)$, by working with three points and setting the conditions such that the response of the circuit matches the ideal linear response at these three points. He specifies that the three points on the response plot lie at the beginning, $V(l)$, the end, $V(h)$, and the midpoint, $V(m)$, of the temperature range from which he introduces the condition

$$2V(m) = V(l) + V(h), \text{ where } m = l + h \quad . \quad (2.1)$$

The parameter he calls the response in his circuit is the equivalent resistance of the transducer in shunt with a fixed resistance whose value is chosen to fit the response at the three desired points.

The equation of the response is the equivalent resistance expression of two resistors in parallel. Substituting the expression for the equivalent resistance at each of the three points into Eq. (2.1), he solves three simultaneous equations to obtain an expression giving the value of fixed resistance needed to linearize the response. Much of these details he leaves out of his presentation but the importance is in how the basic approach follows that of Diamond.

Nobbs and Carius have documented work of particular interest. Nobbs'²⁰ paper on platinum resistance thermometers deals with using least-squares fitting to obtain linearizing parameters, but Bolk²¹ comments on Nobbs' work and offers simplification using his own method of analysis. Carius²², on the other hand, follows the approach outlined by Beakley of using the series representation of the transfer function to design a linear-responding circuit.

CHAPTER 3

LINEARIZATION OVERVIEW

3.1 First Circuit

Let us consider a simple circuit which consists of two resistors connected in series and biased by a constant voltage source. This circuit seen in Figure 3.1 has a fixed-value resistor, R_1 , a temperature-varying resistor, $R(T)$, and a constant voltage source, E_r . We are interested in the response of voltage developed across $R(T)$ with temperature change and the criteria for linearizing this response.

The expression for the voltage across $R(T)$ will be called $V_o(T)$ and is given by

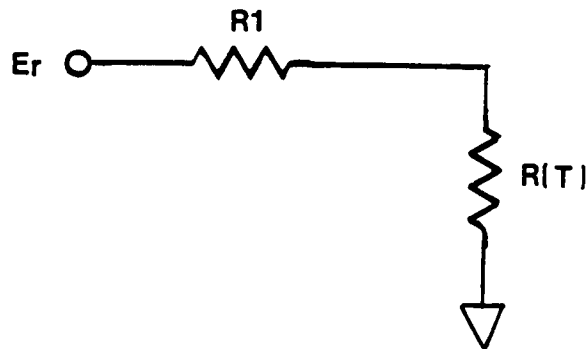


Figure 3.1 A Two Series Resistor Network.

$$V_o(T) = E_r \frac{R(T)}{(R_1 + R(T))} , \quad (3.1)$$

which is simply the voltage division of the two resistors. If we differentiate (3.1) with respect to temperature we get

$$V_o(T)' = E_r \frac{R_1 R(T)'}{(R_1 + R(T))^2} , \quad (3.2)$$

where $V_o(T)'$ and $R(T)'$ are the first derivatives with respect to temperature. Unless otherwise specified, derivatives and linearity will, hereafter, be referred with respect to temperature. For simplification of notation we make the following definitions

$$R = R(T), R' = R(T)', R'' = R(T)'', \quad (3.3)$$

from which Eq. (3.2) becomes

$$V_o(T)' = E_r \frac{R_1 R'}{(R_1 + R)^2} , \quad (3.4)$$

The linearity of Eq. (3.1) depends on the constancy¹⁵ of its first derivative, Eq. (3.4). If R is an increasing function of temperature, the constancy of its first derivative is, at least, dependent on R'

being also an increasing function of temperature. Another condition must also be met before this response can be linearized.

Consider the second derivative which is given by

$$V_o(T)'' = E_r \frac{R_1}{(R_1 + R)^3} [(R_1 + R)R'' - 2(R')^2] \quad (3.5)$$

If the response is to be linear, then the second derivative must go to zero. For Eq. (3.5) to go to zero

$$R'' = \frac{2(R')^2}{(R_1 + R)} \quad , \quad (3.6)$$

which implies that R'' must be non-negative. Since R_1 is a fixed-value, non-negative, linear resistor

$$R'' \geq \frac{2(R')^2}{R} \quad , \quad (3.7)$$

from which we can obtain the requirements for linearization of this circuit

$$\frac{2(R')^2}{R} \geq R'' \geq 0 \quad . \quad (3.8)$$

At this point it would be beneficial to review the characteristics of a platinum RTD. Its resistance to temperature relationship is defined by the Callendar-Van Dusen Equation:^{6,9,23}

$$R(T) = R_0 \{ 1 + AT - BT^2 - C(T - 100)T^3 \} , \quad (3.9)$$

with

$$A = \frac{\alpha (100 + \delta)}{100 \text{ } ^\circ\text{C}} , \quad (3.10)$$

$$B = \frac{(\delta * \alpha)^2}{10^3 \text{ } ^\circ\text{C}} , \quad (3.11)$$

$$C = \frac{\beta * \alpha}{10^8 \text{ } ^\circ\text{C}} , \text{ with } \beta = 0 \text{ for } T > 0 \text{ } ^\circ\text{C} , \quad (3.12)$$

where the temperature coefficient, α , is determined at the steam point ($100 \text{ } ^\circ\text{C}$) and is usually a good indication of purity of platinum. This value of this temperature coefficient must meet stringent RTD manufacturing standards that require the value of the coefficient be $0.00385 \text{ (} ^\circ\text{C)}^{-1}$ or $0.00392 \text{ (} ^\circ\text{C)}^{-1}$. δ is determined at the boiling point of sulfur ($444.6 \text{ } ^\circ\text{C}$) and β is determined at the boiling point of oxygen ($-182.97 \text{ } ^\circ\text{C}$). The value of δ is not as carefully controlled as this coefficients value is usually determined in calibration each transducer. A typical value of δ is $1.526 \text{ (} ^\circ\text{C)}^{-2}$. One can see that the first derivative increases with temperature and that the second derivative is negative. From this observation, it becomes apparent that a platinum RTD cannot be linearized by the method outlined above, however, with certain modifications, linearization of

transducers with concave-down resistance to temperature characteristics is possible. A modification of the two-resistor network yields some interesting results.

3.2 Second Circuit

We now consider another circuit which is formed by replacing $R(T)$ of Figure 3.1 with a negative, temperature-varying resistor, $-R(T)$. The $-R(T)$ in Figure 3.1 is not meant to imply that there exists a negative, temperature-varying resistance but rather that a positive, temperature-varying resistance can, with the use of active components, be made to appear negative. Except for changing $R(T)$, this second circuit is identical to that of Figure 3.1 and is shown below in Figure 3.2.

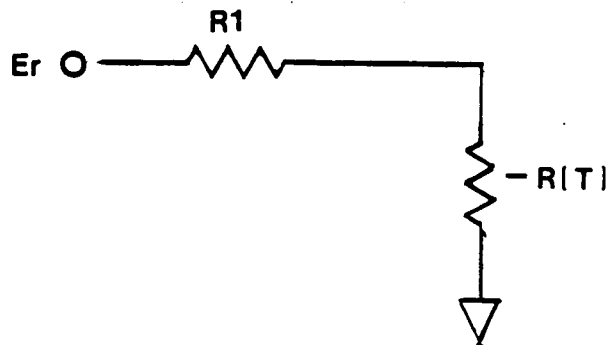


Figure 3.2 A Two Resistor Network With One Resistance Being Negative.

This change results in Eq. (3.1) becoming

$$V_o(T) = E_r \frac{-R(T)}{(R_1 - R(T))} \quad . \quad (3.13)$$

With the same conditions given in Eq. (3.3), we follow the same approach of that in Section 3.1, giving us the modified first derivative expression

$$V_o'(T) = E_r \frac{-R_1 R'}{(R_1 - R)^2} \quad , \quad (3.14)$$

to which we apply the constancy test discussed in Section 3.1. The magnitude of the denominator will decrease with temperature if R increases with temperature for the condition $R_1 > R$ in the temperature range of interest. If the first derivative of R normally decreases with temperature, the numerator's magnitude response also decreases in magnitude. It therefore seems possible to abide by the constancy condition and linearize this type of transducer but before we can make this statement the second condition must be checked.

The second derivative expression becomes

$$V_o''(T) = E_r \frac{-R_1}{(R_1 - R)^3} [(R_1 - R)R'' + 2(R')^2] \quad . \quad (3.15)$$

which once again must go to zero if the response is to be linear. For Eq. (3.15) to go to zero the following must be true:

$$R'' = \frac{-2(R')^2}{(R_1 - R)} , \quad (3.16)$$

from which one can see that R'' can be negative for the range where, $R_1 > R$ since the denominator will always be positive. Since R_1 is a fixed-value, non-negative, linear resistor

$$R'' \leq \frac{2(R')^2}{R} , \quad (3.17)$$

It has been proven qualitatively that a platinum RTD can be linearized but the scheme requires negative resistance incorporated into the circuit. A negative resistance is, of course, not physically realizeable but a negative resistance can be simulated using active components. The Howland current pump^{24,25,26} will be shown to exhibit a negative output impedance.

CHAPTER 4

THE TEC MODEL 982-1 RTD PROCESSOR

4.1 Positive Feedback in the Circuit

The TEC Model 982-1 RTD Processor uses positive feedback control of the biasing current to the platinum RTD to make this current vary non-linearly with temperature to compensate for the non-linear resistance change with temperature of the transducer. The final circuit schematic is seen in Plate 4.1. The feedback loop of the circuit is seen in Figure 4.1 and consists of the RTD transducer, $R(T)$, an instrumentation amplifier, U8, a voltage offsetting amplifier, U3, a voltage reference, U2, and a current source, U4.

The operation of the feedback is such that when the resistance of the RTD changes due to a temperature change, the voltage across the RTD reflects this change which appears as a differential input voltage to the instrumentation amplifier. This signal is then inverted, amplified, summed with an offsetting voltage and fed to the current source as a voltage, V_o , through resistor R_{10} .

The current source is known in literature as a Howland current pump^{24,25,26} and is discussed more thoroughly later in this chapter but for the present we accept that the output current can be defined by

$$I = \frac{E_R}{R_8} - \frac{V_o}{R_{10}}, \quad (4.1)$$

where E_R is the reference voltage which determines the quiescent current and voltage V_O determines the feedback current. It is important to note that R_{10} independently controls the feedback current and therefore must play an important part in the circuit linearization.

The feedback loop is then completed by the voltage V_O changing the biasing current to the RTD. If one follows the operation described, it is clear that an increase in RTD resistance yields an increase in its biasing current (or vice-versa), which characterizes the feedback loop as being positive.

4.2 The Circuit Transfer Function

The transfer function describing the output voltage, V_O , as a function of the RTD resistance, $R(T)$, is now derived directly from Figure 4.1.

$$V_O = (-G)IR(T) + V_r , \quad (4.2)$$

where

$$V_r = E_R \frac{R_5}{(R_5 + R_a)} \frac{(R_3 + R_2)}{R_3} , \quad (4.3)$$

with R_a = a potentiometer. The gain of the instrumentation amplifier is, G , and the current, I , was defined earlier in Eq. (4.1). Upon substituting these expressions into Eq. (4.2) and solving for V_O we get

$$V_O = -E_R \frac{\frac{GR(T)}{R_8} - \frac{R_5}{(R_5 + R_a)} * \frac{(R_3 + R_2)}{R_3}}{1 - \frac{(GR(T))}{R_{10}}}, \quad (4.4)$$

The temperature dependency of Eq. (4.4) comes from the term, $R(T)$, which is, the resistance of the RTD. We will linearize the voltage with respect to temperature. The actual output of the circuit is this voltage processed through another circuit stage. Note that in the final circuit schematic of Plate 4.1, R_{10} is actually the combination of $R_9 + R_{10}$.

4.3 Introduction and Background of The Howland Current Pump

The Howland current pump is a very important component of the circuit since it supplies not only the bias current to the RTD but also the feedback current which makes possible the linearization of the voltage developed across the RTD. Its importance warrants a closer look at this ingenious and effective current source. This author, as part of a very concentrated research effort for this thesis, was not able to locate any in-depth frequency stability analysis documented on the Howland current pump. It was only through an article in an electronics design magazine on the Howland current pump^{24,25,26} and a telephone conversation with the author of this article that some insight into the lack of documentation was explained. Robert A. Pease, the author of the current pump article, revealed to me that, while at

MIT Research Labs in 1959, Brad Howland invented this circuit but for unknown reasons did not document his invention. It was only due to George Philbrick, a noted scientist and co-worker of Howland, that this invention was presented to the outside sources. Mr. Pease claims to not have analyzed this circuit but only to have experimentally tested it. Further insight into this circuit will be acquired from an analysis to follow. The basic configuration is first investigated and then the analysis will be extended to a modified version of the Howland current pump used in the 982-1. The closed-loop stability of the current pump will be analyzed with the aid of macro-modeling of the op-amp using the circuit analysis computer program, SPICE²⁷.

4.4 Circuit Operation

The basic current pump circuit can be seen in Figure 4.2 and is designed to drive a ground-referenced load. Upon inspection one can see that the circuit contains both positive and negative feedback control and has fully differential inputs. Consider the operation of the circuit with its load connected at the non-inverting terminal and a voltage reference, E_R , as an input. As the load resistance increases, the voltage level at the non-inverting terminal increases which, by virtue of feedback, forces the voltage at the inverting terminal to rise to the same potential. The output of the op-amp rises above the inverting terminal potential and supplies the additional current to the load that is no longer supplied by E_R . As the load resistance increases further, less and less current is taken from E_R and the op-amp supplies the extra current needed thereby regulating

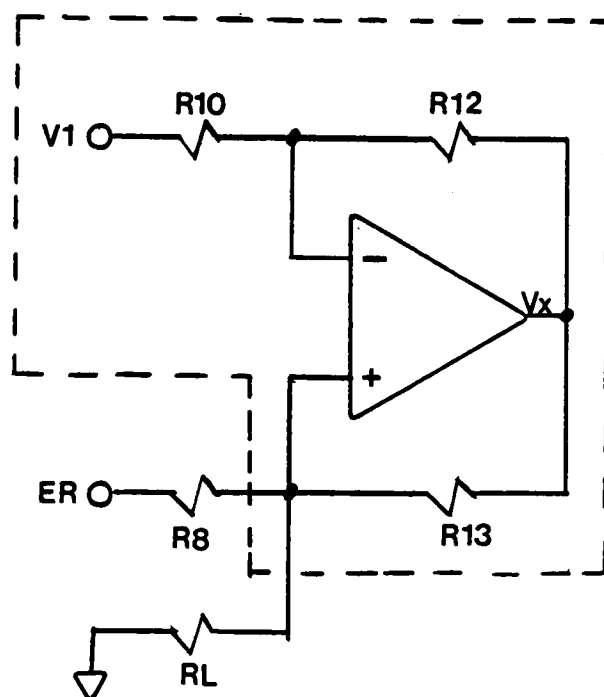


Figure 4.2 The Basic Howland Current Pump Configuration.

the current and using its positive feedback to make the circuit's output impedance, as seen looking into the non-inverting terminal, appear infinite.

4.5 Derivation of Output Resistance and Current

To maintain the high output impedance and current regulation of the current pump requires that the Figure 4.2 circuit resistances maintain the relationship:

$$\frac{R_{12}}{R_{13}} = \frac{R_{10}}{R_8} = n , \quad (4.5)$$

where n is a positive rational number. The importance of this relationship becomes more obvious in studying the output impedance and output current derivation.

The analysis of the circuit is greatly facilitated by using the Thevenin and Norton theorems of circuit analysis. We will take the approach of deriving the output impedance seen at the non-inverting terminal of the circuit. As Thevenin's Theorem dictates, the output impedance analysis is performed with V_1 shorted to ground. Next, the analysis continues by deriving the open-circuit voltage from the non-inverting terminal to ground with V_1 as an input. This exercise allows the circuit of Figure 4.2 to be represented by a simple circuit consisting of its output impedance, as seen looking into the non-inverting terminal, in series with a voltage source equal to the open circuit voltage also seen looking into the non-inverting terminal. This simplified circuit is then connected to the voltage reference, E_R and the biasing resistor R_8 from which the expression of the current to the load is derived.

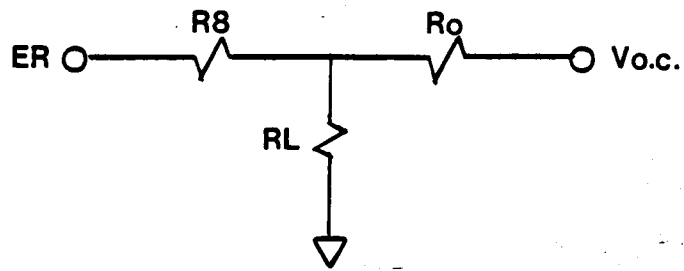
The output impedance is derived by applying a test voltage, V_t , from the non-inverting terminal to ground from which a test current, I_t , is obtained. The ratio of V_t to I_t is the output resistance, R_o . This output impedance equation results as

$$R_o = - R_{13} \frac{R_{10}}{R_{12}} , \quad (4.6)$$

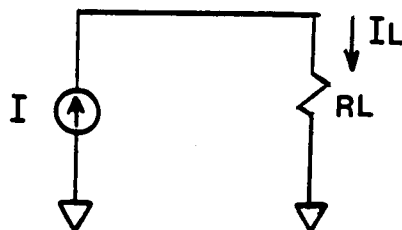
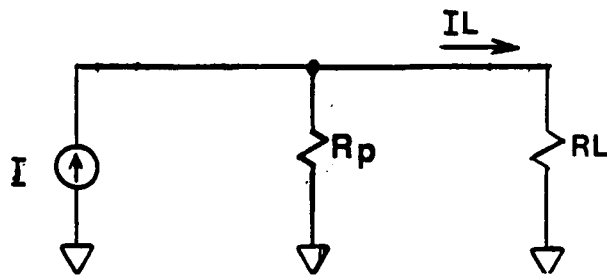
which indicates the circuit as being a negative impedance converter. To complete the derivation, the open-circuit voltage appearing from the non-inverting terminal to ground is needed. Since the circuit's two feedback loops remain intact in this analysis and the circuit is stable, it is apparent that the voltage at the non-inverting terminal must equal the voltage at the inverting terminal, assuming an ideal op-amp. Let I_{13} be the current flowing through R_{13} and I_{12} be the current flowing through R_{12} (currents not shown in Figure 4.2). Owing to the very high input impedance of the op-amp's terminals, I_{13} can be approximated at zero, forcing $V_x=0$ and I_{12} to be also zero resulting in

$$V_{o.c.} = V_1 . \quad (4.7)$$

The circuit in the dashed box of Figure 4.2 is now replaced with its Thevenin equivalent and Figure 4.2 becomes Figure 4.3(A). Using the Norton transformation, the voltage sources become current sources with their associated resistances in parallel as seen in Figure 4.3(B). The load is left intact with its current, I_L . Combining the parallel sources and their resistances with the infinite resistance, R_p , removed, the lower circuit of Figure 4.3(B) is obtained. The parallel combination of R_8 and $-R_{13}*(R_{10}/R_{12})$ appears as R_p in Figure 4.3(B) and is simplified to



(A) The Thevenin Equivalent Circuit.



(B) Using Norton Transformations to Further Simplify the Circuit.

Figure 4.3 The Thevenin and Norton Equivalent Circuits Used to Derive the Load Current of the Howland Current Pump.

$$R_p = - \frac{R_8(R_{13})R_{10}}{R_8R_{12} - R_{13}R_{10}} \quad (4.8)$$

The sources output current is given by

$$I = \frac{E_R}{R_8} - \frac{R_{12}}{R_{13}R_{10}} V_1 \quad (4.9)$$

The resistor values are now chosen such that the important condition

$$R_{13} = nR_{12}, R_8 = nR_{10}, \quad (4.10)$$

is met. Making the substitutions of Eq. (4.10) into Eq. (4.8) and (4.9) yields

$$R_p = \text{infinity} \quad , \quad (4.11)$$

$$I = \frac{E_R - V_1}{R_8} \quad (4.12)$$

From Eq. (4.12) one can see that the current supplied to the load is proportional to the difference in input voltages and the biasing resistor, R_8 . In the ideal case, the current pump exhibits infinite output resistance as seen by the load and as would be expected of an ideal current source; however, in reality the tolerances of the circuit resistors do not allow perfect matching and therefore prevent an infinite output resistance. For 10K, 0.1% resistors, the worst case tolerance matching yields a 5 megohm output resistance as calculated

using Eq. (4.8). An analysis of the close-loop stability of the Howland current pump is outlined in Section 4.8.

4.6 The Modified Howland Current Pump

The basic Howland current pump configuration, previously presented in Figure 4.2, was analyzed and its output current to the load found to be given by Eq. (4.12). For the 982-1, the preference is to control the current with two input voltages whose biasing resistors are independent of each other. In this manner, E_R and R_g would control and maintain a fixed biasing current to the load and V_O with its associated resistance, which will be called R_z , would provide a varying component of load current. Since V_O in the 982-1 is the voltage across the RTD which varies with temperature, this voltage would determine the varying component of load current which, in effect, is the linearizing feedback current. It is then clear that this configuration is desirable since the modification would allow the quiescent current to be set by R_g and the linearizing feedback current by R_z . The basic Howland current pump has the single resistor R_g as the only means of control for both the quiescent and feedback current.

The modified Howland current pump is seen in Figure 4.4 with R_z being the parallel combination of R_{10} and R_{11} . This new configuration lends itself to the identical method of analysis as the basic current pump with but a few subtle differences. The similarities are enough that it allows the same Thevenin and Norton equivalent circuits of Figures 4.3 and 4.4 to be used in the analysis of the modified current pump. In addition to the analysis presented for the basic current

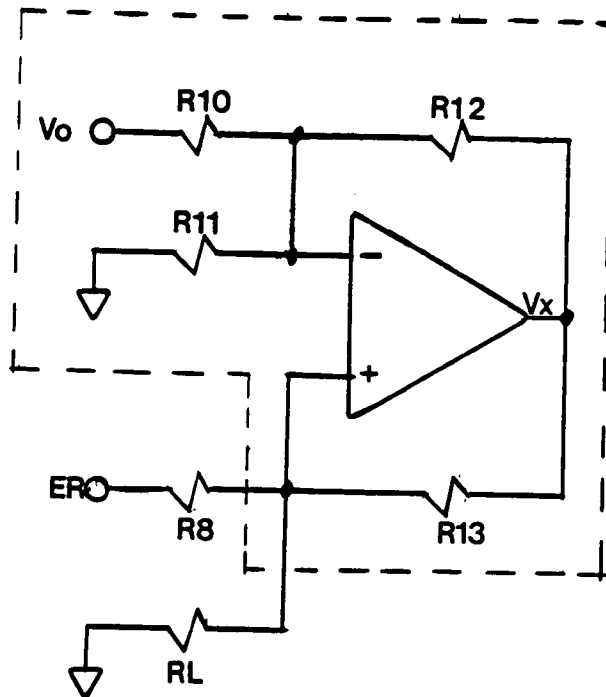


Figure 4.4 The Modified Howland Current Pump.

pump, a SPICE macro-model for the modified Howland current pump was developed and used to study the closed-loop stability of the circuit.

4.7 Modified Circuit Output Current and Output Resistance

Once again, we employ the Thevenin and Norton Equivalents of the modified current pump to facilitate the analysis. Following the same procedure outlined in Section 4.5, the circuit in Figure 4.4 is analyzed. The output resistance, seen looking into the non-inverting terminal and as described in Section 4.5, is the ratio of the applied test voltage, V_t , to the return test current, I_t , and results as

$$R_o = -R_{13} \frac{R_z}{R_{12}} , \quad (4.13)$$

where R_z is the parallel combination of R_{10} and R_{11} .

The open circuit voltage is calculated with no currents flowing through resistors R_{12} and R_{13} as explained in Section 5.3 leaving $V_{o.c.}$ equal to the voltage division across R_{10} and R_{11} or

$$V_{o.c.} = \frac{R_{11}}{R_{11} + R_{10}} V_o . \quad (4.14)$$

The resulting Thevenin equivalent circuit replaces the dashed-box portion of Figure 4.4 which becomes Figure 4.3 and upon making the Norton transformation becomes Figures 4.4 (A) and (B). The resulting Norton source has an output current of

$$I = \frac{E_R}{R_8} - \frac{R_{12}R_{11}}{R_{13}R_z(R_{11} + R_{10})} V_o , \quad (4.15)$$

and an equivalent parallel combination results

$$R_p = - \frac{R_8(R_{13})R_z}{R_8R_{12} - R_{13}R_z} . \quad (4.16)$$

The condition for resistance matching in the modified Howland Current Pump is

$$\frac{R_{12}}{R_{13}} = \frac{R_z}{R_8} = n , \quad (4.17)$$

where R_z is the parallel combination of R_{10} and R_{11} . Note that n will be a positive rational number. Substituting the appropriate resistance ratios of Eq. (4.17) into (4.15) and (4.16) yields

$$R_p = - R_g R_{13} \frac{R_z}{R_g n R_{13} - R_{13} n R_g} = \text{infinity} , \quad (4.18)$$

with R_p being the resistance seen by the load; the current expression is simplified to

$$I = \frac{E_R}{R_g} - \frac{n V_o}{R_{10}} , \quad (4.19)$$

with n being a positive rational number. From Eq. (4.19), one can see that the amount of feedback current, and therefore the linearity adjust, can be controlled by choosing the value of R_{10} while the quiescent current value can be adjusted by R_g . In the final design of the 982-1, the value of n is set to unity as all resistors were chosen to be 10K, 0.1% tolerance. Note that in the complete circuit schematic, Plate 4.1, the parallel combination of R_{11} and R_{10} equals 10K.

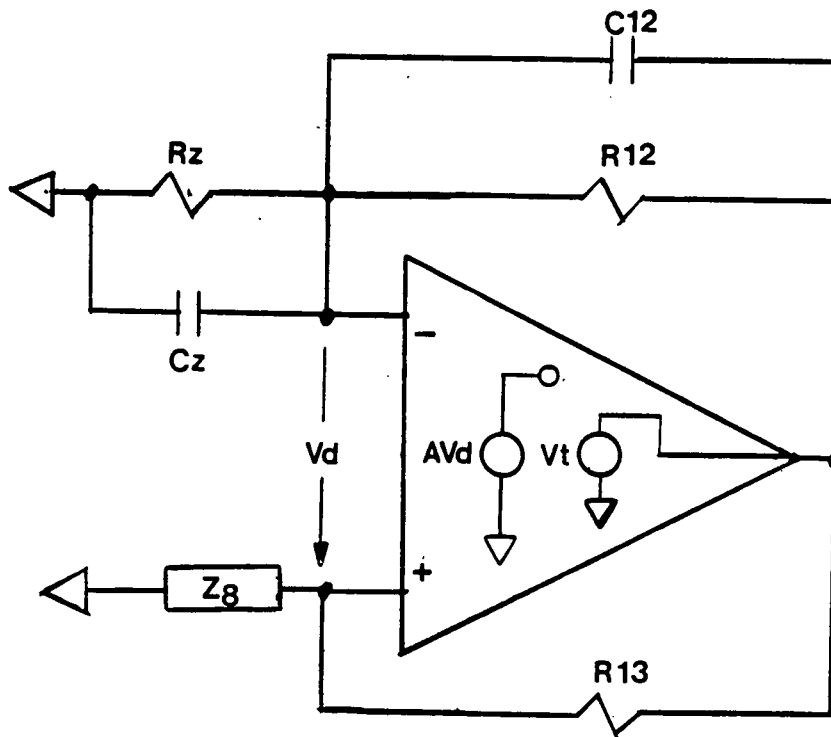
4.8 Closed-loop Analysis of the Modified Howland Current Pump

The closed-loop stability analysis for the Howland current pump requires inspection because the circuit employs both a positive and a negative feedback loop and thus the possibility for instability

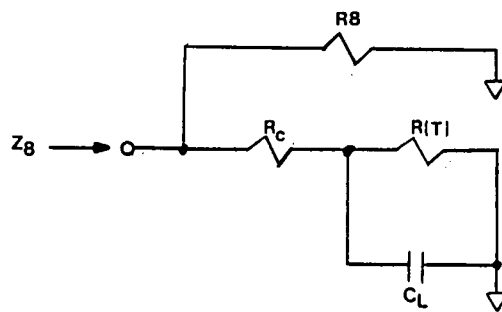
exists. The analysis proves the Howland current pump to be a stable feedback system in the frequency domain for most cases; however, in the extreme cases, as with any multiple-feedback system, there exist certain conditions where the stability can be compromised, particularly where parasitic capacitances introduce unwanted phase shift in the feedback loop. The analysis, though not a trivial matter, generated results which were verified by SPICE proving the derived loop transmission transfer function to be correct. The program listing for this SPICE analysis includes the macro-model of the circuit's op-amp and can be found in Appendix E.

The system's loop transmission derivation characterizes the system as having four poles and two zeroes in the loop transmission transfer function. The origin of these poles and zero will now be discussed. The frequency stability requires, of course, knowledge of the loop transfer function of the current pump. The circuit model used for the derivation is seen in Figure 4.5, where A is the op-amp's open loop gain which is a single pole function and V_d is the differential input voltage.

The most logical and simplest approach is to break the loop in the forward path of the op-amp, insert a test a.c. voltage source at this point and monitor the returning signals. The ratio of the return signal to the test signal is the loop transmission transfer function, T . In Figure 4.5(A) one can see the model used where V_t is the test voltage applied and the return voltage sensed across voltage source AV_d . This voltage source represents the op-amp's open loop gain



(A) The Operational Amplifier Model.



(B) The Network for the Impedance, Z_8 .

Figure 4.5 The Circuit Used to Derive the Loop Transmission Equation.

multiplying the differential input signal. The transfer simplified function results as

$$T = \frac{AV_d}{V_t} = \frac{A_o}{[1 + j(f/f_1)][1 + j(f/f_2)]} * \left[\frac{Z_8}{Z_8 + R_{13}} - \frac{Z_z}{Z_{12} + Z_z} \right] . \quad (4.20)$$

where Z_8 represents the impedance formed by the network seen in Figure 4.5 (B). This network consists of: R_C , the series combination of the calibration resistors that appear in series with the RTD in the final design, $R(T)$, the RTD load resistance at its maximum value, C_L , the lead capacitance of the RTD and R_8 , the biasing resistor. The lead capacitance of the RTD can be of significant value since the length of the RTD leads can exceed 200 ft. and therefore must be considered. Note that R_z , seen in Figure 4.5(A), is still defined as the parallel combination of R_{10} and R_{11} while, in Eq. (4.20), Z_z and Z_{12} represent the parallel combinations of C_z , R_z and C_{12} , R_{12} respectively.

To be complete in the stability analysis, one must include possible parasitic capacitances where they will adversely affect the loop transmission stability and where they are most likely to be found in an actual circuit assembly. The crucial parasitic capacitance appears from the negative input to ground, while the lead capacitance would appear as part of the network in Figure 4.5 (B), from the positive terminal to ground. A compensation capacitor is also included in the negative feedback loop. These capacitances can be seen in Figure 4.5.

The resulting loop transmission transfer function, T, consists of four poles and two zeroes. The op-amp's pole associated with the critical frequency f_1 , is the first pole and the second pole critical frequency, f_2 , is also contributed by the op-amp. These first two poles, f_1 and f_2 , were determined from the manufacturers data sheets for the op-amp and are located at 1.0 Hz and 1.0 MHz respectively. The third pole critical frequency, f_3 , and zero critical frequency, f_5 , are determined by combination of R_{13} and Z_8 . The fourth pole f_4 , is given by the sum of C_z and C_{12} combined with resistors R_{12} and R_z in parallel. The phase shift associated with this pole is critical and can destabilize the current pump. The zero critical frequency of the system f_6 , is not as easily defined but results from the interaction of the third and the fourth poles coupled with C_{12} and R_{12} . The function in Bode form is

$$T = \frac{A_o[(K_1 - K_2)f_3f_5f_6]}{f_3f_5f_6} * \left[\frac{[1+j(f/f_5)][1+j(f/f_6)]}{[1+j(f/f_1)][1+j(f/f_2)][1+j(f/f_3)][1+j(f/f_4)]} \right] .$$

(4.21)

The critical frequency, f_1 , of the op-amp dominates the initial roll-off of the Bode plot. The frequency location of f_4 is the major concern from the stability viewpoint. Without the use of C_{12} , this pole, f_4 , introduces sufficient phase shift and magnitude roll-off to destabilize the circuit from parasitic capacitance of C_z . The matter

is not of great concern, however, since SPICE simulation, Figure 4.6, shows that, for C_Z 's capacitance value up to 50pF and no compensation, the circuit will exhibit about a 44° phase margin. Note that the phase plot is converted to $+180^\circ$ at -180° , which has the same phase angle-shift significance. This phase angle change in the plot appears as a sudden jump in the curve. The resistance values used to obtain this phase margin were the actual values on the 982-1 device. Resistances R_Z, R_{12}, R_{13} and R_g were 10K. The value of R_C was 276 ohms, and $R(T)$ was 176 ohms. Compensation of this critical frequency, f_4 , is possible through using C_{12} connected in shunt with the negative feedback resistor R_{12} . It is also noted that the parasitic capacitance, seen as part of the network from the positive input to ground, should be considered since the lead capacitance of the RTD can be significant. Analysis shows that the critical frequencies associated with this pole and zero lie virtually in the same location, even for worse case conditions of large lead capacitances, and therefore their effects cancel each other. This cancellation is very important; since it makes the Howland current pump immune to destabilization from RTD lead capacitance.

A Bode plot of the circuit's loop transmission is given in Figure 4.7 for typical component values of: $C_{12}=20\text{pf}$, $C_L=0.01\mu\text{F}$, $C_Z=20\text{pF}$, $R_{12}=R_{13}=R_Z=10\text{K}$, $R_C=276$ and $R(T)=176$ ohms. The plot found in Figure 4.7 contains the worst case lead capacitance, the worst case capacitance C_Z , from the negative terminal to ground, and the compensation capacitor needed to assure a desirable phase margin. One can see that the system is very stable with a phase margin of 69° at the

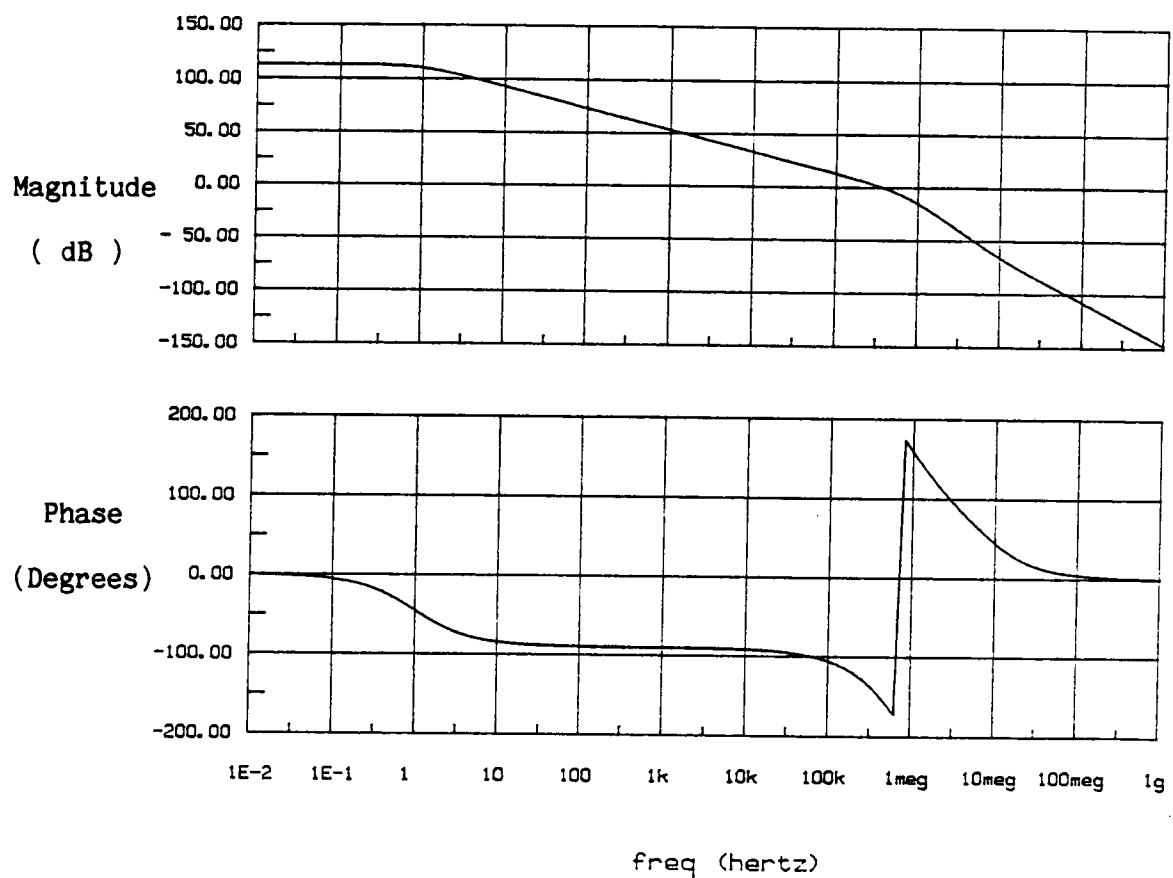


Figure 4.6 Bode Plot of the Howland Current Pump's Loop Transmission Response for $C_Z = 50$ pF.

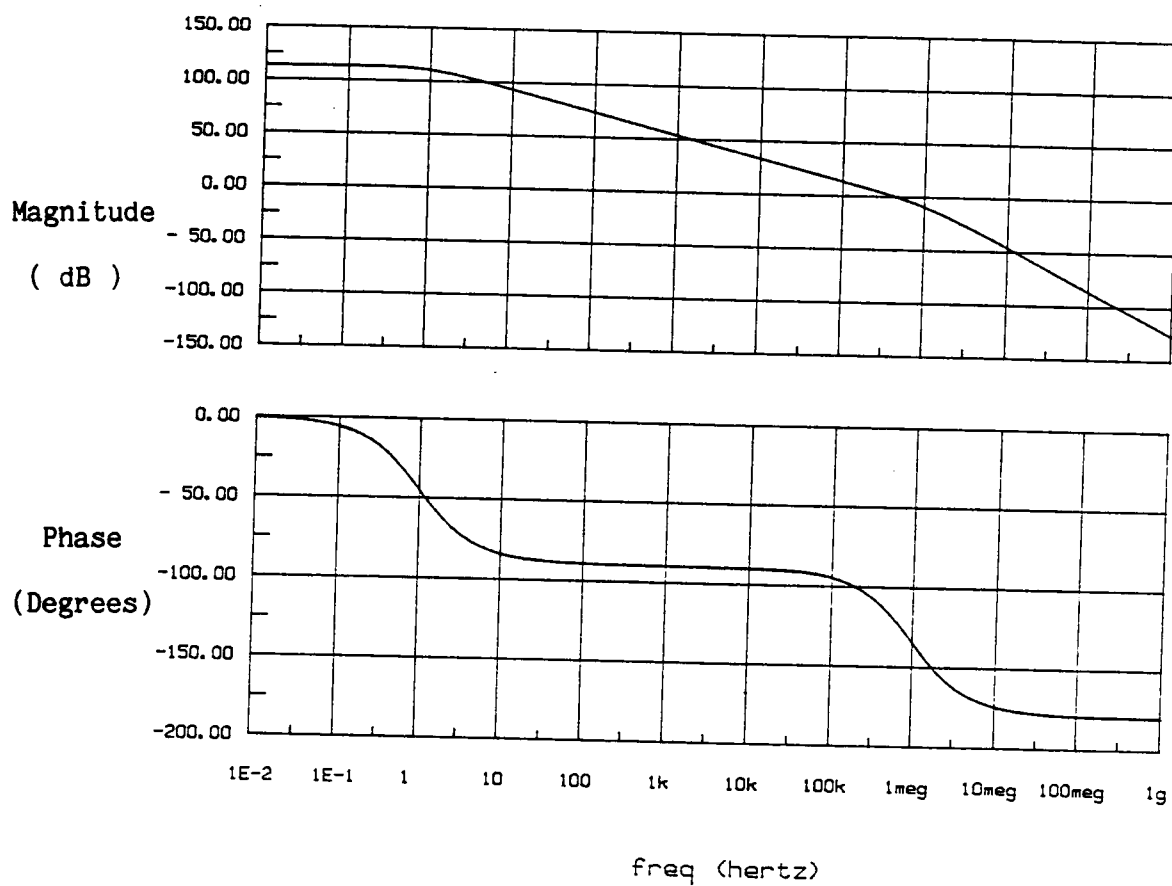


Figure 4.7 Bode Plot of the Howland Current Pump With Typical Circuit Values of $C_Z=20\text{pF}$, $C_{12}=20\text{pF}$, $C_L=0.01\mu\text{F}$, $R(T)=176\text{ ohms}$, $R_{12}=10\text{K ohms}$, $R_C=276\text{ ohms}$, $R_{13}=R_Z=R_8=10\text{K ohms}$.

unity cross-over frequency. The final 982-1 design had its printed circuit board designed to minimize any stray capacitance and therefore did not need any compensation.

In summary, it should be said that Brad Howland's invention has proven to be an effective current source. The analysis indicates a high margin of stability against oscillations even under some worse case conditions.

CHAPTER 5

LINEARIZATION CRITERIA

The approach taken in the development of the linearization criteria is to derive the power series representation of the circuit transfer function with temperature as the independent variable and expand around a temperature point in the range of interest by using Taylor's Theorem.²⁸ The intent is to analytically explore the resulting series representation, truncated to four terms and attempt to manipulate the coefficients of the series terms in such a way as to maximize the linearity. The procedure requires expanding the function, $V_O(T)$, using the derivatives with respect to temperature evaluated at a point. The independent variable in the series is temperature, T , and the dependent variable is the voltage, V_O . The chain rule of calculus is utilized in obtaining these derivatives by first taking the derivative of $V_O(T)$ with respect to $R(T)$, and multiplying it with the derivative of $R(T)$ with respect to temperature which results in the derivative of $V_O(T)$ with respect to temperature. With this power series representation, the coefficients of the series equation can be chosen to minimize the deviation from perfect linearity. The resulting function is then linear within a specified accuracy for a specified temperature range. The response chosen for the 982-1 required measurement over a temperature range of 0 to 200 °C and the error was minimized to less than 0.005% of measured value at worst case as outlined in Chapter 6. The error is defined as 100% times the difference in the measured value and the expected value, divided by the full-scale

value. Analysis for a wider temperature range of measurement of 0 to 600 °C yielding about 0.05% error is also presented in Chapter 6. The error figure presented in the tables of Chapter 6 is defined as 100% times the difference in the measured value and the expected value, divided by the expected value. This error presented was used to calculate the temperature deviation of measured to expected temperatures at each point and is explained in Section 6.2.3.

5.1 The Derivation of the Power Series

We begin with the transfer equation for $V_O(T)$ as a function of the RTD resistance, $R(T)$,

$$V_O(T) = -E_R \frac{\frac{GR(T)}{R_8} - \frac{R_5}{(R_5 + R_a)} * \frac{(R_3 + R_2)}{R_3}}{1 - \frac{(GR(T))}{(R_9 + R_{10})}}, \quad (5.1)$$

and rewrite it as

$$V_O(T) = E_R \frac{\frac{R_5}{(R_5 + R_a)} * \frac{(R_3 + R_2)}{R_3} - \frac{GR(T)}{R_8}}{1 - \frac{(GR(T))}{(R_9 + R_{10})}}, \quad (5.2)$$

to which some substitutions are made in order to simplify the expression. Let the constants C, F, and D be defined as follows:

$$C = \frac{R_5}{(R_5 + R_a)} * \frac{(R_3 + R_2)}{R_3} , \quad (5.3)$$

$$F = - \frac{GR(T)}{R_8} , \quad (5.4)$$

$$D = - \frac{G}{(R_9 + R_{10})} . \quad (5.5)$$

The resulting expression upon substituting into Eq. (5.2) becomes

$$V_O(T) = E_R \frac{C + FR(T)}{1 + DR(T)} . \quad (5.6)$$

For temperatures above 0 °C, the Callendar-Van Dusen equation defines the resistance change to temperature as was discussed in Section 3.1 and is rewritten

$$R(T) = R_O (1 + AT - BT^2) . \quad (5.7)$$

This equation can be written as

$$R(T) = R_O [1 + A(T - T_O) - B(T^2 - T_O^2)] , \quad (5.8)$$

where T_0 can be any temperature in the desired range. The physical implementation of this adjustment to the equation would be to electronically subtract two RTD's having the same coefficients in which one RTD has double the characteristic resistance, R_0 , of the other and is held at a constant temperature, T_0 . It is interesting to note that a fixed-value resistor could simulate the RTD held at constant temperature. The resulting expression would be in terms of voltage instead of resistance but the similarity to Eq. (5.8) still exists.

Taylor's theorem²⁸ states: Let $f(z)$ be analytic in a domain D and let $z=z_0$ be any point in D . Then there exists precisely one power series with center at z_0 which represents $f(z)$; this series is of the form

$$f(z) = \sum_{n=0}^{\infty} b_n(z - z_0)^n, \quad (5.9)$$

where

$$b_n = \frac{f^{(n)}(a)}{n!}, \quad n = 0, 1, \dots \quad (5.10)$$

In applying Taylor's Theorem to Eq. (5.6), the derivatives with respect to temperature will have to be calculated. As previously discussed in the beginning of this chapter, it is much easier to use the chain rule of calculus to determine these derivatives. The first four terms in the series are calculated and sufficiently represent the equation.

In deriving each coefficient of the power series, its chain rule equation will be listed followed by the coefficient equation obtained by substituting the derivatives of the transfer function of Eq. (5.6) with respect to $R(T)$ and the derivatives of $R(T)$ of Eq. (5.8) with respect to temperature, T , into the chain rule expression and simplifying. In order to use Taylor's Formula, the derivatives will be presented evaluated at temperature, T_0 , as part of the simplification.

We begin the derivation considering the circuit whose transfer function is represented by Eq. (5.6). The 982-1 circuit has component values selected and is calibrated such that at temperature, T_0

$$\frac{R_5}{(R_5 + R_a)} * \frac{(R_3 + R_2)}{R_3} - \frac{GR_0}{R_8} = 0, \quad (5.11)$$

which, upon substitution of the constants, translates to

$$C + FR_0 = 0. \quad (5.12)$$

The chain rule expression is written in its customary form but its associated derivative of the transfer function will be written with a different notation. As an example, $V_0''(T_0)$, will represent the second derivative of V_0 with respect to temperature, T , evaluated at $T=T_0$. Upon inspecting Eq. (5.8) one should note that all derivatives of $R(T)$ with respect to temperature, beyond the second derivative, are equal to zero. The first derivative of Eq. (5.6) is

$$\frac{dV_O}{dT} = \frac{dV_O}{dR(T)} * \frac{dR(T)}{dT} , \quad (5.13)$$

$$V_O'(T) = E_R (E + DR(T))^{-1} [F - (C + FR(T)) (E + DR(T))^{-1} D] \\ * (B + 2CT_O) . \quad (5.14)$$

T_O be consistent with Taylor's Formula we evaluate Eq. (5.14) at a temperature $T = T_O$ and apply the condition given in Eq. (5.12) which results in

$$V_O'(T_O) = E_R * (E + DR_O)^{-1} [F] * (B + 2CT_O) . \quad (5.15)$$

The remaining derivatives are formed in a similar manner: for the second derivative

$$\frac{d^2V_O}{dT^2} = \frac{dV_O}{dR(T)} * \frac{d^2R(T)}{dT^2} + \left[\frac{dR(T)}{dT} \right]^2 * \frac{d^2V_O}{dR(T)^2} , \quad (5.16)$$

$$V_O''(T_O) = E_R(2)(E + DR_O)^{-1} [F] \\ * \{ C - (B + 2CT_O)^2 (E + DR_O)^{-1} D \} , \quad (5.17)$$

For the third derivative

$$\begin{aligned} \frac{d^3 V_O}{dT^3} = & \frac{dV_O}{dR(T)} * \frac{d^3 R(T)}{dT^3} + 3 \frac{d^2 R(T)}{dT^2} * \frac{dR(T)}{dT} * \frac{d^2 V_O}{dR(T)^2} + \\ & \left[\frac{dR(T)}{dT} \right]^3 * \frac{d^3 V_O}{dR(T)^3} , \end{aligned} \quad (5.18)$$

$$\begin{aligned} V_O'''(T_O) = & E_R(-6)(E + DR_O)^{-2} (D)(B + 2CT_O)[F] \\ & * \{ 2C - (B + 2CT_O)^2(E + DR_O)^{-1} D \} , \end{aligned} \quad (5.19)$$

For the fourth and final derivative

$$\begin{aligned} \frac{d^4 V_O}{dT^4} = & \frac{dV_O}{dR(T)} * \frac{d^4 R(T)}{dT^4} + 4 \frac{d^3 R(T)}{dT^3} * \frac{dR(T)}{dT} * \frac{d^2 V_O}{dR(T)^2} + 6 \frac{d^3 V_O}{dR(T)^3} * \left[\frac{dR(T)}{dT} \right]^2 \\ & * \frac{d^2 R(T)}{dT^2} + 3 \frac{d^3 V_O}{dR(T)^3} * \left[\frac{d^2 R(T)}{dT^2} \right]^2 + \left[\frac{dR(T)}{dT} \right]^4 * \frac{d^4 V_O}{dR(T)^4} , \end{aligned} \quad (5.20)$$

$$\begin{aligned} V_O''''(T_O) = & E_R(24)(E + DR_O)^{-2} D[F] * \\ & \{ [3C - (B + 2CT_O)^2(E + DR_O)^{-1} D] * (B + 2CT_O)^2(E + DR_O)^{-1} D - C^2 \} \end{aligned} \quad (5.21)$$

The derivatives calculated to this point are used in conjunction with Taylor's Formula to develop the series representation of Eq. (5.6). The series as dictated by Eq.'s (5.9) and (5.10),

considering the first four terms, is given in general form as

$$V_0(T_0) = \frac{V_0'(T_0)}{1!} * (T - T_0) + \frac{V_0''(T_0)}{2!} * (T - T_0)^2 + \frac{V_0'''(T_0)}{3!} * (T - T_0)^3 + \frac{V_0''''(T_0)}{4!} * (T - T_0)^4 + \dots \quad (5.22)$$

The initial strategy is to cancel the second derivative of this truncated series by appropriately selecting the coefficients required to achieve this cancellation. An inspection of Eq. (5.17) shows that the second derivative can be canceled by the condition that

$$[C - (B + 2CT_0)^2(E + DR_0)^{-1}D] = 0 \quad . \quad (5.23)$$

The method by which this manipulation is performed will be discussed shortly but for now we concentrate on the effect of Eq. (5.23) with respect to the remaining derivatives. Substituting (5.23) into Eq.'s (5.19), and (5.21) introduces a great deal of simplification by reducing the equations to the form

$$V_0'(T_0) = E_R (E + DR(T))^{-1} [F] * (B + 2CT_0) \quad , \quad (5.24)$$

$$V_0'''(T_0) = V_0'(T_0) * (-6)C^2(B + 2CT_0)^{-2} \quad , \quad (5.25)$$

$$V_0''''(T_0) = V_0'(T_0) * (24)C^3(B + 2CT_0)^{-3} \quad , \quad (5.26)$$

The simplified derivatives are shown as multiples of the first derivative and upon substituting Eq.'s (5.24), (5.25), and (5.26) into (5.22) results in the series representation of the circuit transfer function and is given by

$$V_O(T) = V_O(T_O)'(T - T_O) * \{ 1 - \text{ERROR} \} , \quad (5.27)$$

where

$$\text{ERROR} = [C^2(B + 2CT_O)^{-2}](T - T_O)^2 - [C^3*(B + 2CT_O)^{-3}](T - T_O)^3 , \quad (5.28)$$

Eq. (5.27) is a linear expression for the voltage V_O as a function of temperature multiplied by an error term. It is very interesting to notice that the error term is completely a function of the RTD characteristics and the sensed temperature with the magnitude of the error increasing as the range of measured temperature increases. The slope of the equation, or the sensitivity of the response, is determined by the circuit parameters, as one would expect. We now look at the cancellation of the second derivative to see how this is achieved.

The constants appearing in Eq. (5.22) consist of components used in the 982-1 circuitry as well as the characteristic constants of the RTD. The term D , seen in Eq. (5.5) consists of the instrumentation amplifier's gain, G , and the feedback resistors, R_9 and R_{10} . These parameters control the amount of bias-current variation that the RTD

will experience, therefore, the physical interpretation of the manipulation in setting the condition of Eq. (5.22) is that these components need to be chosen to optimize the amount of feedback required to linearize the voltage developed across the RTD. The gain term, G , influences not only the linearity but also the sensitivity of the response whereas the series combination of resistors, R_9 and R_{10} , influences only the linearity of the response and, therefore, it is the logical choice to have the value of $R_9 + R_{10}$ determine the linearization criteria.

We take Eq. (5.23)

$$[C - (B + 2CT_0)^2 (E + DR_0)^{-1} D] = 0 , \quad (5.29)$$

and substitute into it the resistor expressions in Eq. (5.5) for D which results in

$$C + \frac{(B + 2CT_0)^{-2} G}{(R_9 + R_{10}) - GR_0} = 0 . \quad (5.30)$$

Solving Eq. (5.28) for $R_9 + R_{10}$ and evaluating at the temperature $T_0 = 0^\circ\text{C}$ we get that the linearizing criteria set by the resistors must be

$$R_9 + R_{10} = GR_0 \frac{\alpha}{\delta} [1 + (100 + \delta)^2] . \quad (5.31)$$

As an example, the characteristics of one of the RTD's used with the 982-1 are : $\alpha = 0.00385$, $\delta = 1.526$, and $R_0 = 100$ ohms which are typical values for these transducers and upon using these values in Eq. (5.31) the amount of feedback resistance needed is found to be

$$R_9 + R_{10} = 270.050 \text{ K ohms} \quad . \quad (5.32)$$

This resistance value cancels the second order term of the series expansion but does not achieve the optimum accuracy as is discussed in Chapter 6.

CHAPTER 6

982-1 CIRCUIT RESPONSE AND ERROR ANALYSIS

To be thorough in the investigation of the response of the 982-1, the analytical calculations were checked experimentally. Analysis showed that the truncated series representation accurately describes the transfer function of the 982-1. Analysis also shows, for the range of 0 to 200 °C, a maximum error of 0.1% of full scale temperature and a maximum deviation of actual to measured temperature of 0.1 °C using the resistance value of $R_9 + R_{10}$ equal to 270 Kohms as given by Eq. (5.32). This error, however is not the minimum obtainable error as a resistance value of $R_9 + R_{10}$ equal to 258.5 Kohms yields a maximum error of 0.02%. For ease of description, the linearizing feedback resistor, $R_9 + R_{10}$, will be referred to as R_x . We again define the error figure appearing in tables of this chapter as 100% times the difference in the measured value and the expected value, divided by the expected value. The 0.02% error is defined as 100% times the difference in the measured value and the expected value, divided by the full-scale value at the maximum deviation.

6.1 Referencing the Transfer Function to the Output of the 982-1

The series representation of the 982-1 transfer function was discussed in Chapter 4 and is given by Eq. (5.22) where the derivatives of $V_O(T_O)$ are given by Eq.s (5.15), (5.17), (5.19) and (5.21). Chapter 4 derives the transfer function from which the truncated series is developed and is concerned with only the positive feedback

loop. It should be noted that the equation for $V_O(T)$ is the voltage at the output of U8 and now we are concerned with output voltage of the circuit which is $V_O(T)$ processed through a gain stage of U5 as seen in the schematic of Plate 4.1. We make a minor modification to the transfer function of given in Eq. (5.1) by introducing an inverting gain term of U5 which will be called, K_1 . The resulting equation is

$$V_{out}(T) = (-K_1)*(-E_R) \frac{\frac{GR(T)}{R_8} - \frac{R_5}{(R_5 + R_a)} * \frac{(R_3 + R_2)}{R_3}}{1 - \frac{GR(T)}{(R_9 + R_{10})}} \quad (6.01)$$

The modification of adding the gain term, K_1 , to the transfer function affects Eq.'s (5.15), (5.17), (5.19) and (5.21) only by replacing E_R with $-K_1*E_R$. Eq. (5.22) remains in its same form. The K_1 gain term allows the circuit output to equal to +5.0 Volts at the maximum temperature.

6.2 Calculation of Data Using an R_x Value of 270 Kohms

The series representation, Eq. (5.22), in general form is cumbersome to work with, so it is now simplified by substituting the circuit component values used in the 982-1 with the exception of R_x . Since the value of R_x will determine the linearity of the circuit, its resistance value is left as a variable to be manipulated so as to

minimize the error. The remaining component values in the 982-1 design can be seen in the schematic of Plate 4.1 to be

$R_2=10K$, $R_3=10K$, $R_5=10K$, $R_8=10K$, $R_6=9.09K$,

R_7 = adjustable potentiometer,

$R(T)$ = RTD with characteristics:

$\alpha=0.00385$, $\delta=1.526$, $R_0=100$ ohms .

Upon substituting these values into Eq.'s (5.15), (5.17), (5.19), (5.21) and (5.22) the simplified equation results as:

$$\begin{aligned}
 V_{out}(T) = K_1 * \frac{(-0.0196)}{(R_x - 10K)} & \left[T - \left(1.503 * 10^{-4} - \frac{5.875 * 10^{-3}}{(R_x - 10K)} \right) T^2 \right. \\
 & - \left(\frac{0.0118}{(R_x - 10K)} - \frac{3.452 * 10^{-5}}{(R_x - 10K)^2} \right) T^3 + \\
 & \left. + \left(\frac{8.83 * 10^{-7}}{(R_x - 10K)} - \frac{0.689}{(R_x - 10K)^2} - \frac{5.973 * 10^{-4}}{(R_x - 10K)^3} \right) T^4 \right] . \quad (6.02)
 \end{aligned}$$

Eq. (6.02) is a fourth order equation whose value is zero at 0 °C and the value of the equation at the maximum temperature is controlled by the gain constant, K_1 . The 982-1 instrument is calibrated for 0 to +5.0V output corresponding to a 0 °C to 200 °C temperature range. The ideal linear response, a straight line through the origin and +5.0V at its maximum point, will serve as a reference for the accuracy of the actual response. The coefficients of the equation are dependent on

R_X . This dependence on R_X is appropriate since the resistance value of R_X will determine the value of the coefficients and consequently the behavior of the equation over the temperature range desired. Since the sign of the coefficients are different, one would suspect that certain terms will dominate in given temperature ranges causing the slope of the curve to change over the temperature range. Our interest is focused on the amount of deviation from the straight line. This deviation will need to be minimized in order to obtain the maximum accuracy. For typical values of R_X around 200K, the plot of the actual data will exhibit a deviation which will occur in three different modes.

The first deviation is a bow shaped pattern concave-down and above the ideal linear plot and the second deviation pattern is also a bow-shaped pattern but concave up and below the ideal linear plot. The third deviation pattern contains an inflection point forcing the actual and ideal plots being exactly equal at this inflection point in addition to the two endpoints of the plots. The third deviation pattern of the actual response yields the minimum deviation and is preferred. The most accurate results were obtained with the inflection point location occurring at approximately one-half of the temperature range.

6.2.1 Choosing an Optimum R_X Value. Though it would be desirable to derive an analytical expression for choosing the optimum value of R_X , this approach is not practical as will become obvious in the explanation that follows.

The point of interest, in the performance of the circuit, is in studying the deviation point per point between the actual response and the ideal response which, from a mathematical point of view, involves taking the difference of the expression of the actual response and the ideal response. Eq. (6.02) can be expressed in general terms as

$$V_{out}(T) = K*(T + K_2T^2 + K_3T^3 + K_4T^4) , \quad (6.03)$$

and the ideal response can be expressed as

$$V_{out}(T)^* = K_5T , \quad (6.04)$$

where K, K_2, K_3, K_4 are real positive or negative constants. The difference is then

$$V_{out} - V_{out}^* = [K-K_5]T + K_2T^2 + K_3T^3 + K_4T^4 . \quad (6.05)$$

The greatest deviations will occur at the minimum and maximum points of Eq. (6.05). The temperatures at which these min. and max. values will occur are found by taking the first derivatives of the equation, setting the resulting expression equal to zero and solving for the roots. With knowledge of the roots, their temperature values can be substituted into Eq. (6.05) to find the magnitude of the maximum deviation.

It is obvious that the first derivative of Eq. (6.05) will be a cubic equation. The roots of a cubic equation²⁹ can be found using

techniques developed in many text books without much difficulty if the coefficients of the equation of interest are constant values. For this application, the coefficients of the equation of interest are not constant but are a function of R_x . Following the procedure for solving the roots of a cubic equation with variable coefficients is not as practical as other approaches available. The approach outlined above, however, serves as an excellent approach for verifying results once an optimum value of R_x has been chosen.

6.2.2 Using a Computer for Choosing the Optimum Value of R_x . A more practical approach to finding the optimum value of R_x is to employ the aid of a computer to calculate the data points of the series representation of the transfer function. This approach is unlike using a least-squares fitting algorithm for several reasons. A least-squares fit will yield a straight line that does not necessarily cross at the desired output levels needed for proper calibration of the instrument. The least squares fit is more of a trial and error approach than programming into a computer the calculated series representation of the transfer function. With the series representation, the relative influence of each factor multiplying the higher order terms in the equation can be observed. An initial value of R_x can be calculated by simply cancelling the second order term of Eq. (6.02) as discussed in Chapter 5.

A Fortran program was written to aid in determining the optimum value of R_x and the source listing can be found in Appendix A. This program uses the derivative expressions of Eq.'s (5.15), (5.17),

(5.19) and (5.21) which are substituted into Eq. (5.22) along with the output gain constant $-K_1$ to obtain the series representation of the transfer function as in Eq. (6.02). The program lists the coefficients of each temperature term with CONSTL, CONSTM, CONSTN being the factors of T^2, T^3, T^4 respectively for a particular R_x value. The factor outside the brackets in Eq. (6.02) is given by CONSTS and the output inverting gain given by K_1 .

6.2.3 The Optimum Value for R_x in the Range of 0 to 200 °C. A temperature range of 0 to 200 °C, a value of 258.5K yielded the most accurate results with a calculated maximum temperature deviation of 0.0096 °C from the actual temperature. This deviation error translates to a linearity error of only 0.005% max. referred to full-scale temperature over the total temperature range of measurement. The data is more meaningful when it is expressed in terms of calculated deviation from the nominal temperature reading at every point. The data is presented in this format with Table 6.1 showing the actual temperature value, the calculated (indicated) temperature value, and the difference defined as calculated minus nominal. The percent error shown in the tables are defined as 100% multiplying the difference in measured (indicated) value and expected (ideal) value divided by the expected value. The calculated deviation is obtained by multiplying the percent error shown in the tables with the expected (ideal) temperature value at each point. The percent error is the ratio of the temperature deviation to the actual temperature multiplied by 100%. A plot of the calculated deviation versus the actual

Table 6.1 A Comparison of the Ideal Response and the Calculated Linearized Response in Terms of Temperature for the 982-1 RTD Processor.

ACTUAL TEMP. (°C)	INDICATED TEMP. (°C)	TEMP. DIFFERENCE (°C)	ERROR (%)
0.000000	0.000000	0.000000	0.000000
4.000000	3.998247	-0.001753	-0.043830
8.000000	7.996708	-0.003292	-0.041144
12.000000	11.995376	-0.004624	-0.038531
16.000000	15.994241	-0.005759	-0.035991
20.000000	19.993295	-0.006705	-0.033523
24.000000	23.992529	-0.007471	-0.031128
28.000000	27.991934	-0.008066	-0.028806
32.000000	31.991502	-0.008498	-0.026557
36.000000	35.991223	-0.008777	-0.024382
40.000000	39.991088	-0.008912	-0.022279
44.000000	43.991090	-0.008910	-0.020251
48.000000	47.991218	-0.008782	-0.018296
52.000000	51.991465	-0.008535	-0.016414
56.000000	55.991820	-0.008180	-0.014607
60.000000	59.992276	-0.007724	-0.012874
64.000000	63.992822	-0.007178	-0.011215
68.000000	67.993451	-0.006549	-0.009630
72.000000	71.994153	-0.005847	-0.008120
76.000000	75.994919	-0.005081	-0.006685
80.000000	79.995740	-0.004260	-0.005324
84.000000	83.996607	-0.003393	-0.004039
88.000000	87.997511	-0.002489	-0.002828
92.000000	91.998442	-0.001558	-0.001693
96.000000	95.999392	-0.000608	-0.000633
100.000000	100.000351	0.000351	0.000351
104.000000	104.001310	0.001310	0.001260
108.000000	108.002260	0.002260	0.002093
112.000000	112.003192	0.003192	0.002850
116.000000	116.004095	0.004095	0.003531
120.000000	120.004962	0.004962	0.004135
124.000000	124.005783	0.005783	0.004663
128.000000	128.006547	0.006547	0.005115
132.000000	132.007247	0.007247	0.005490
136.000000	136.007872	0.007872	0.005788
140.000000	140.008413	0.008413	0.006010
144.000000	144.008862	0.008862	0.006154
148.000000	148.009207	0.009207	0.006221

Table 6.1 (continued)

ACTUAL TEMP. (°C)	INDICATED TEMP. (°C)	TEMP. DIFFERENCE (°C)	ERROR (%)
152.000000	152.009440	0.009440	0.006210
156.000000	156.009551	0.009551	0.006122
160.000000	160.009531	0.009531	0.005957
164.000000	164.009370	0.009370	0.005714
168.000000	168.009059	0.009059	0.005392
172.000000	172.008588	0.008588	0.004993
176.000000	176.007947	0.007947	0.004515
180.000000	180.007126	0.007126	0.003959
184.000000	184.006117	0.006117	0.003325
188.000000	188.004909	0.004909	0.002611
192.000000	192.003493	0.003493	0.001819
196.000000	196.001859	0.001859	0.000949
200.000000	200.000000	0.000000	0.000000

temperature value shows the 's' shaped behavior of the response with its inflection point. Additionally, two other plots are also provided with different R_X values to show that the error is greater for resistance values other than 258.5K. Note that one of the plots corresponds to $R_X=270$ Kohms, the value needed to cancel the second order term in the series representation as discussed in Chapter 5. The plot can be seen in Figure 6.1.

The data obtained using the truncated series representation very accurately represents the data that would be compiled if the ideal infinite series representation of the transfer function were used. If the transfer function of Eq. (5.1) is used to directly calculate data points for different values of $R(T)$, RTD resistances, then these data points would represent the infinite series representation. These data points could then be compared with the data points of the truncated series representation and the validity of the truncated series would be obvious. This comparison is made in Table 6.2 which shows the truncated series data points previously presented in Table 6.1 and the data points calculated directly from the transfer function. The Fortran program listing used to calculate the transfer function data points is found in Appendix B. In the table, the data points corresponding to the transfer function are abbreviated T.F. and the data corresponding to the series representation are labeled, SERIES. The comparison is made between the temperature difference between actual and indicated temperatures and the corresponding errors for both cases at each point.

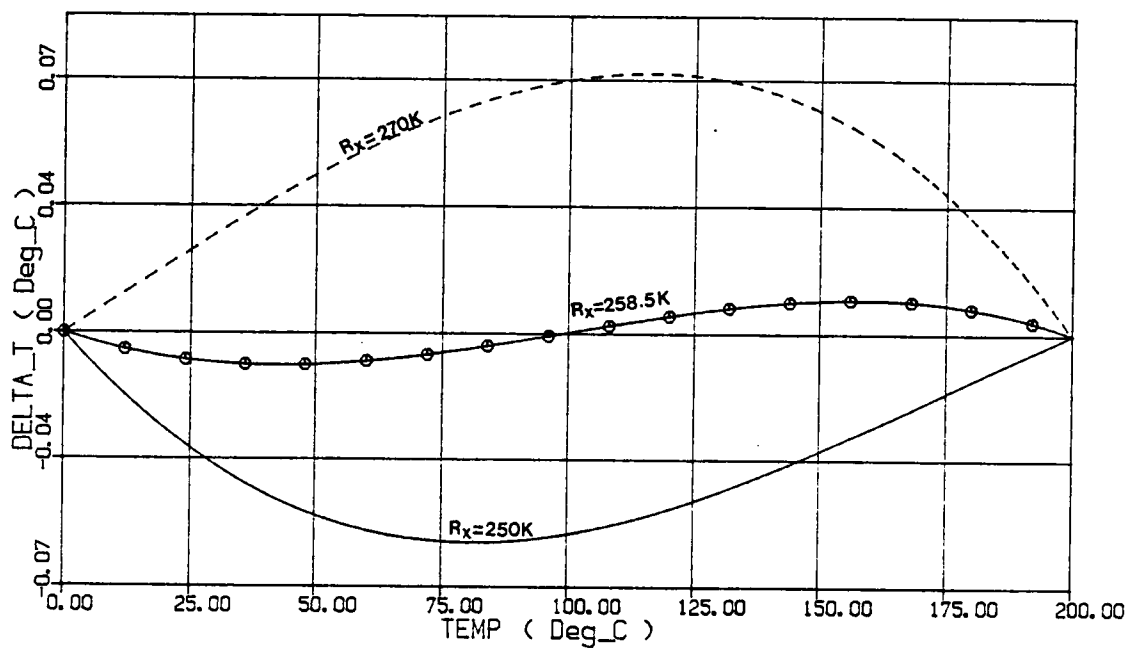


Figure 6.1 A Plot of the Temperature Deviation Between Calculated Linearized Temperatures and Ideal Temperatures Versus Ideal Temperature.

Table 6.2 A Comparison of Data Derived From the Actual Transfer Function and Data Derived From the Series Equation.

TEMP (°C)	T.F. TEMP DIFFERENCE (°C)	T.F. ERROR (%)	SERIES TEMP DIFFERENCE (°C)	SERIES ERROR (%)
0.000000	0.000000	0.000000	0.000000	0.000000
4.000000	-0.001753	-0.043824	-0.001753	-0.043830
8.000000	-0.003291	-0.041139	-0.003292	-0.041144
12.000000	-0.004623	-0.038526	-0.004624	-0.038531
16.000000	-0.005758	-0.035986	-0.005759	-0.035991
20.000000	-0.006704	-0.033518	-0.006705	-0.033523
24.000000	-0.007469	-0.031123	-0.007471	-0.031128
28.000000	-0.008064	-0.028801	-0.008066	-0.028806
32.000000	-0.008497	-0.026552	-0.008498	-0.026557
36.000000	-0.008775	-0.024376	-0.008777	-0.024382
40.000000	-0.008910	-0.022274	-0.008912	-0.022279
44.000000	-0.008908	-0.020245	-0.008910	-0.020251
48.000000	-0.008779	-0.018290	-0.008782	-0.018296
52.000000	-0.008533	-0.016409	-0.008535	-0.016414
56.000000	-0.008177	-0.014602	-0.008180	-0.014607
60.000000	-0.007721	-0.012869	-0.007724	-0.012874
64.000000	-0.007174	-0.011210	-0.007178	-0.011215
68.000000	-0.006545	-0.009625	-0.006549	-0.009630
72.000000	-0.005843	-0.008115	-0.005847	-0.008120
76.000000	-0.005077	-0.006680	-0.005081	-0.006685
80.000000	-0.004255	-0.005319	-0.004260	-0.005324
84.000000	-0.003388	-0.004034	-0.003393	-0.004039
88.000000	-0.002484	-0.002823	-0.002489	-0.002828
92.000000	-0.001553	-0.001688	-0.001558	-0.001693
96.000000	-0.000603	-0.000628	-0.000608	-0.000633
100.000000	0.000356	0.000356	0.000351	0.000351

Table 6.2 (continued)

TEMP (°C)	T.F. TEMP DIFFERENCE (°C)	T.F. ERROR (%)	SERIES TEMP DIFFERENCE (°C)	SERIES ERROR (%)
104.000000	0.001315	0.001265	0.001310	0.001260
108.000000	0.002265	0.002098	0.002260	0.002093
112.000000	0.003197	0.002854	0.003192	0.002850
116.000000	0.004101	0.003535	0.004095	0.003531
120.000000	0.004968	0.004140	0.004962	0.004135
124.000000	0.005788	0.004668	0.005783	0.004663
128.000000	0.006553	0.005119	0.006547	0.005115
132.000000	0.007252	0.005494	0.007247	0.005490
136.000000	0.007878	0.005792	0.007872	0.005788
140.000000	0.008419	0.006014	0.008413	0.006010
144.000000	0.008867	0.006158	0.008862	0.006154
148.000000	0.009212	0.006224	0.009207	0.006221
152.000000	0.009445	0.006214	0.009440	0.006210
156.000000	0.009556	0.006126	0.009551	0.006122
160.000000	0.009536	0.005960	0.009531	0.005957
164.000000	0.009375	0.005716	0.009370	0.005714
168.000000	0.009063	0.005395	0.009059	0.005392
172.000000	0.008591	0.004995	0.008588	0.004993
176.000000	0.007950	0.004517	0.007947	0.004515
180.000000	0.007129	0.003961	0.007126	0.003959
184.000000	0.006120	0.003326	0.006117	0.003325
188.000000	0.004911	0.002612	0.004909	0.002611
192.000000	0.003495	0.001820	0.003493	0.001819
196.000000	0.001860	0.000949	0.001859	0.000949
200.000000	0.000000	0.000000	0.000000	0.000000

As promised, the analysis was extended to a wider temperature range of 0 to 600 °C. The value of resistance that optimized the linearity of the response was found to be 236 Kohms. The maximum temperature deviation resulted as 0.29 °C at 480 °C which translates to a 0.05% maximum linearity error when referred to full-scale temperature over the total temperature range. This extended temperature range could be supported by the current 982-1 design with only minor modifications. These data are plotted in Figure 6.2. and are found in Table 6.3.

6.2.4 Experimental Results. The experimentation process of the 982-1 had to be carefully planned in order not to introduce any error in the measurement process that would not be attributed to the actual performance of the circuit. The experimental process involved simulating the RTD transducer by applying known resistance inputs to the circuit, calculating the equivalent temperature they correspond to, as dictated by the Callender-Van Dusen equation discussed in Sections 3.1 and 5.1, and measuring the output voltage. The circuit is calibrated to output 0 Volts for a 100 ohm input and +5.0 Volts for a 175.83 ohm input. The resistance values correspond to 0 and 200 degrees Centigrade temperatures respectively. The RTD coefficients used in the Callender-Van Dusen equation correspond to a typical industrial RTD which would be used in the field application of the 982-1. These constants were given in the text following Eq. (5.31) and are repeated again

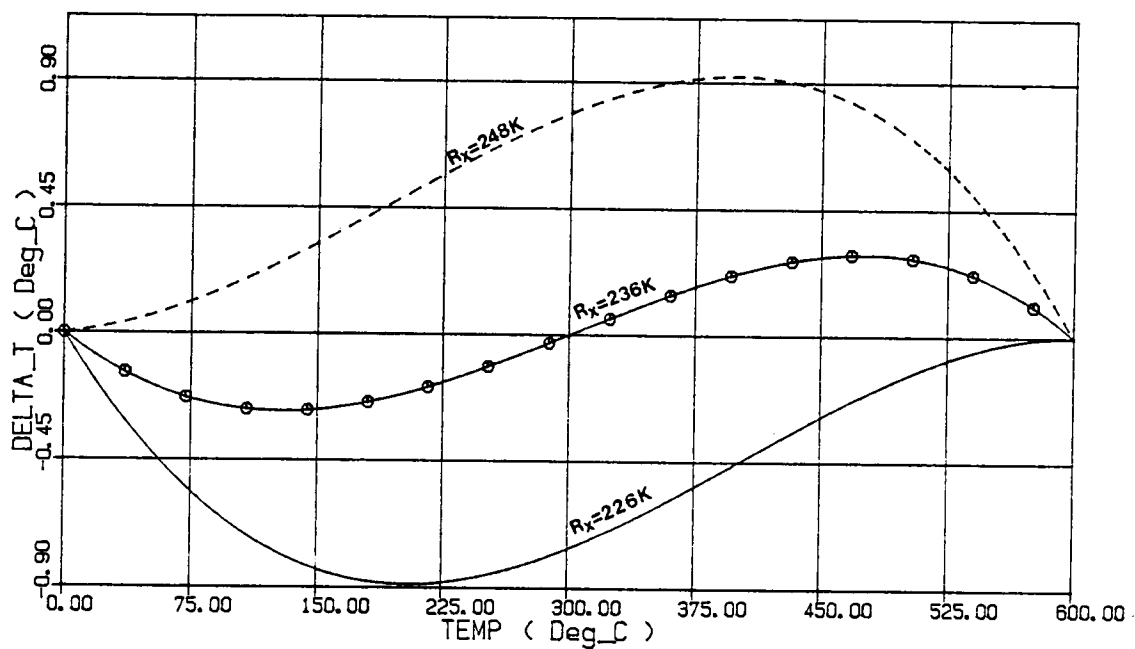


Figure 6.2 A Plot of the Temperature Deviation Between Calculated Linearized Temperatures and Ideal Temperatures Versus Ideal Temperature for an Extended Temperature Range of 0 to 600 $^{\circ}\text{C}$.

Table 6.3 A Comparison of the Ideal Response and the Calculated Linearized Response in Terms of Temperature for the 982-1 RTD Processor for an Extended Temperature Range.

ACTUAL TEMP. (°C)	INDICATED TEMP. (°C)	TEMP. DIFFERENCE (°C)	ERROR (%)
0.000000	0.000000	0.000000	0.000000
12.000000	11.947215	-0.052785	-0.439872
24.000000	23.900694	-0.099306	-0.413777
36.000000	35.860204	-0.139796	-0.388323
48.000000	47.825512	-0.174488	-0.363517
60.000000	59.796383	-0.203617	-0.339361
72.000000	71.772580	-0.227420	-0.315861
84.000000	83.753862	-0.246138	-0.293022
96.000000	95.739987	-0.260013	-0.270847
108.000000	107.730712	-0.269288	-0.249341
120.000000	119.725789	-0.274211	-0.228509
132.000000	131.724970	-0.275030	-0.208356
144.000000	143.728005	-0.271995	-0.188885
156.000000	155.734640	-0.265360	-0.170103
168.000000	167.744620	-0.255380	-0.152012
180.000000	179.757689	-0.242311	-0.134617
192.000000	191.773586	-0.226414	-0.117924
204.000000	203.792049	-0.207951	-0.101937
216.000000	215.812816	-0.187184	-0.086659
228.000000	227.835619	-0.164381	-0.072097
240.000000	239.860191	-0.139809	-0.058254
252.000000	251.886262	-0.113738	-0.045134
264.000000	263.913557	-0.086443	-0.032743
276.000000	275.941804	-0.058196	-0.021086
288.000000	287.970724	-0.029276	-0.010165
300.000000	300.000040	0.000040	0.000013

Table 6.3 (continued)

ACTUAL TEMP. (°C)	INDICATED TEMP. (°C)	TEMP. DIFFERENCE (°C)	ERROR (%)
312.000000	312.029468	0.029468	0.009445
324.000000	324.058726	0.058726	0.018125
336.000000	336.087529	0.087529	0.026050
348.000000	348.115587	0.115587	0.033215
360.000000	360.142612	0.142612	0.039615
372.000000	372.168311	0.168311	0.045245
384.000000	384.192389	0.192389	0.050101
396.000000	396.214550	0.214550	0.054179
408.000000	408.234496	0.234496	0.057474
420.000000	420.251924	0.251924	0.059982
432.000000	432.266532	0.266532	0.061697
444.000000	444.278015	0.278015	0.062616
456.000000	456.286065	0.286065	0.062733
468.000000	468.290372	0.290372	0.062045
480.000000	480.290624	0.290624	0.060547
492.000000	492.286508	0.286508	0.058233
504.000000	504.277707	0.277707	0.055101
516.000000	516.263903	0.263903	0.051144
528.000000	528.244774	0.244774	0.046359
540.000000	540.219999	0.219999	0.040741
552.000000	552.189252	0.189252	0.034285
564.000000	564.152206	0.152206	0.026987
576.000000	576.108531	0.108531	0.018842
588.000000	588.057897	0.057897	0.009846
600.000000	600.000000	0.000000	0.000000

RTD with characteristics:

$\alpha=0.00385$, $\delta=1.526$, $R_0=100$ ohms .

It was very important to use stable resistances as the inputs in order to avoid their resistance value changing with temperature in the process of experimentation. A logical choice was to use a decade-resistance box which provide stable resistance and repeatable values. The tolerance of the decade-resistance box was specified at 0.1% but each resistance value used in the experiment was measured to determine the exact value. The output voltage was measured accurately with a six digit voltmeter and proper shielding was used to avoid any interference pick-up from the surrounding environment. All readings were taken only after ample time was allowed between resistance changes to assure steady state performance. The test equipment listed below and Figure 6.3 is a photograph of the test set-up.

Decade Box: General Radio Company, 0.1%, Type 1432-N

Digital Voltmeter: Hewlett Packard, Model 3468A

Power Supply: Power Designs, Tri-volt Model TP325

Electronics Module: RTD Processor, TEC Model 982-1

The experimental results were very successful and are tabulated in Table 6.4. The data was collected with the aid of a Fortran program that was used during the process of experimentation. This program calculates the equivalent temperatures for each corresponding input resistances and prompts the operator for the measured voltage

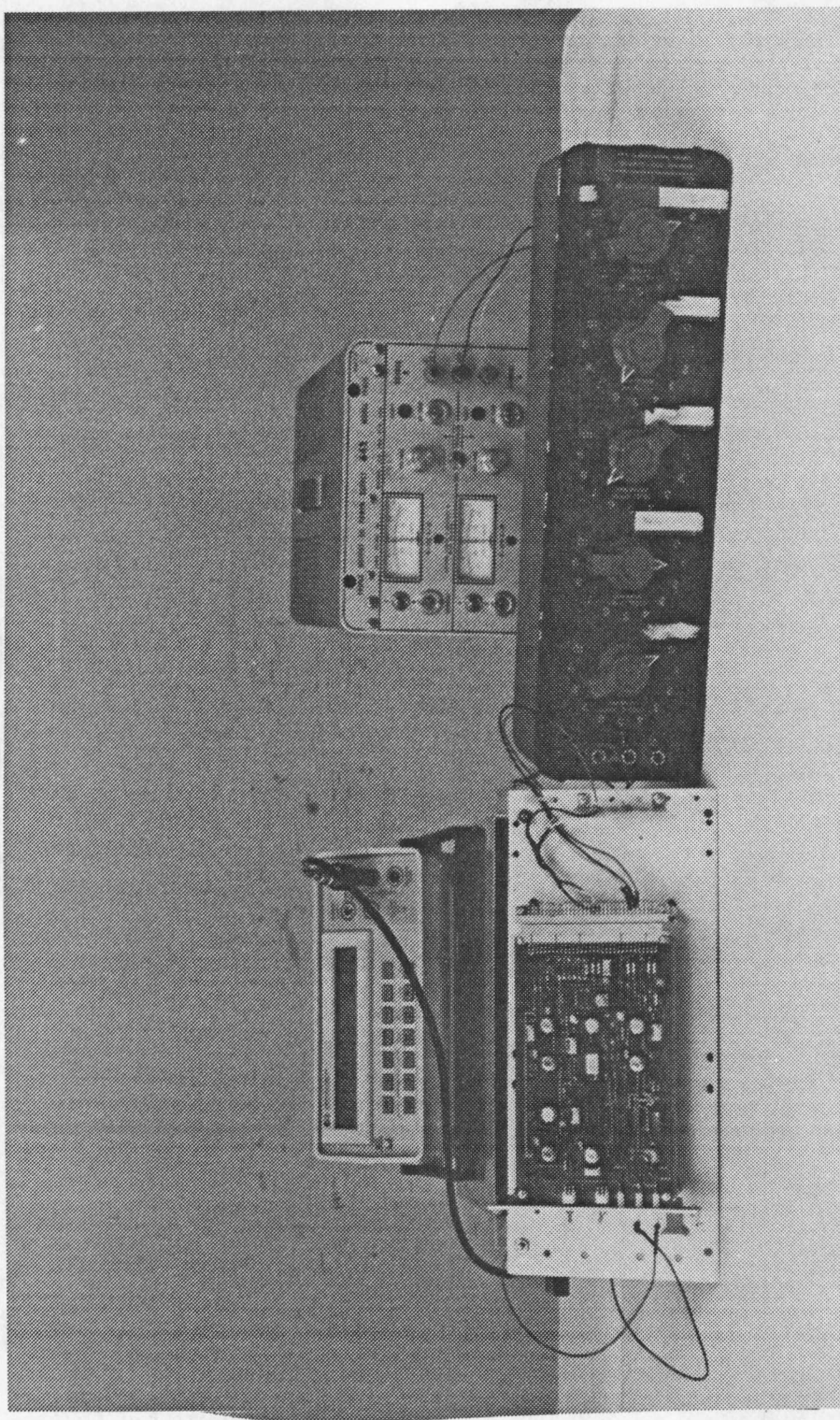


Figure 6.3 A Photograph of the Experimental Set-up.

Table 6.4 Experimental Data for the 982-1 for the Specified Temperature Range of 0 to 200 °C.

IDEAL TEMP (°C)	RTD VALUE (Ohms)	MEASURED VOLTAGE (Volts)	IDEAL VOLTAGE (Volts)	ERROR ERROR (%)	TEMP DIFFERENCE (°C)
0.145828	100.057	0.000100	0.000010	0.000000	0.000000
10.395516	104.057	0.255780	0.255787	-0.002732	-0.000284
23.262650	109.061	0.576610	0.576769	-0.027562	-0.006412
46.529426	118.060	1.157120	1.157180	-0.005186	-0.002413
59.502421	123.050	1.480720	1.480800	-0.005400	-0.003213
83.026524	132.048	2.067400	2.067600	-0.009674	-0.008032
93.552096	136.053	2.330100	2.330200	-0.004289	-0.004012
106.72511	141.047	2.679040	2.678810	0.008584	0.009162
119.94426	146.038	2.989300	2.988600	0.023423	0.028094
130.57281	150.036	3.254500	3.253700	0.024585	0.032101
143.93916	155.045	3.587600	3.587200	0.011151	0.016050
154.64004	159.040	3.854800	3.854100	0.018162	0.028086
168.07860	164.038	4.190000	4.189300	0.016711	0.028087
178.89673	168.046	4.459500	4.459200	0.006724	0.012029
189.70602	172.037	4.729100	4.728800	0.006349	0.012044
200.57989	176.038	5.000100	5.000100	0.000000	0.000000

at which point the temperature deviation and error is given for the data reading. The listing is found in Appendix D.

A resulting maximum temperature deviation of 0.032°C maximum over the total temperature range translates to 0.02% error of full scale temperature was measured establishing the instrument as a very accurate temperature measuring device. The final feedback linearizing resistor value used in experimentation was 257.99K ohm. A photograph of the actual circuit is seen in Figure 6.4.

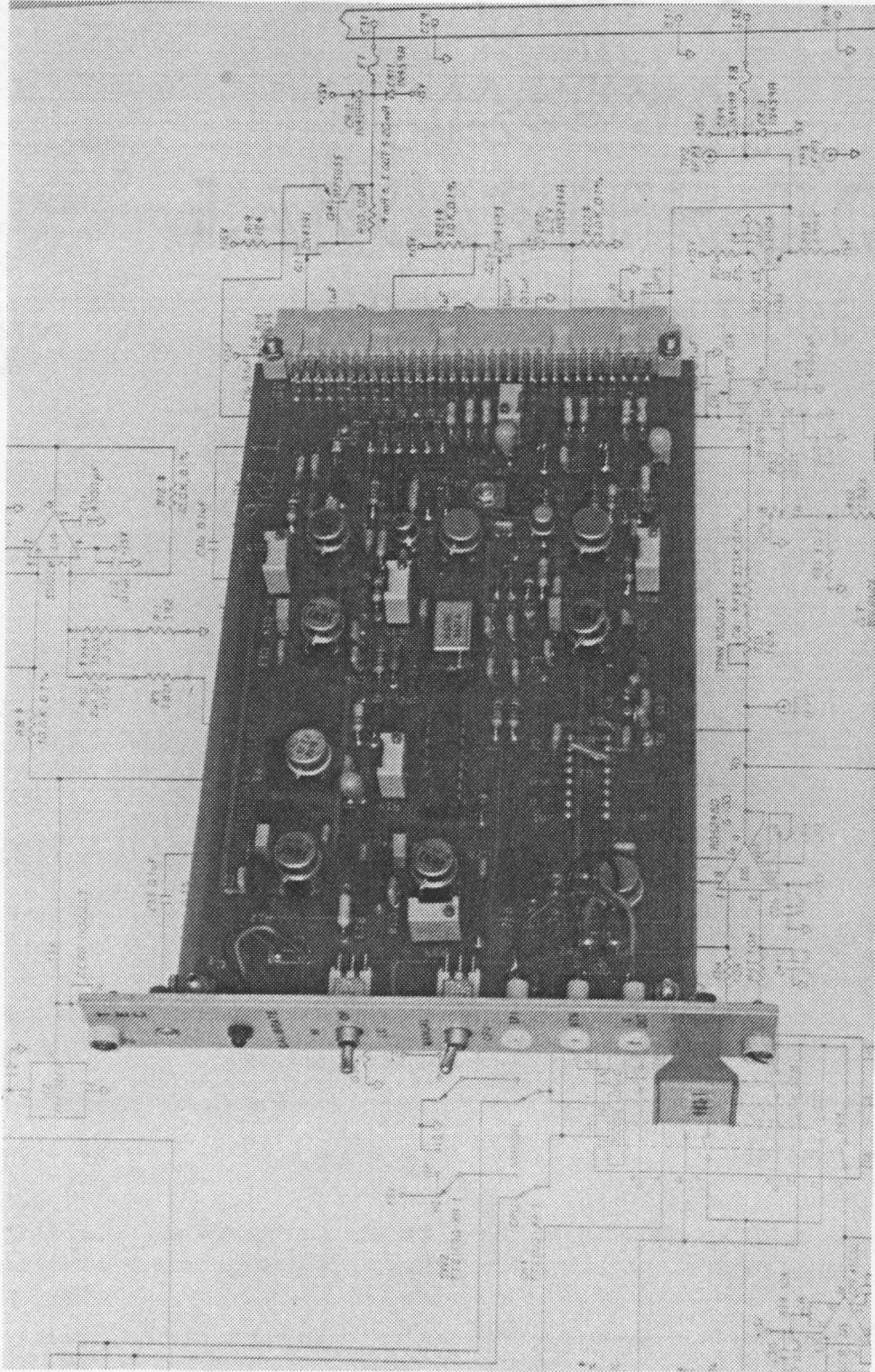


Figure 6.4 A Photograph of the 982-1 RTD Processor.

CHAPTER 7

CONCLUSIONS

7.1 Summarizing Discussion

The analysis presented in this text demonstrated that a non-linear temperature transducer can be very accurately linearized by using feedback linearization. It has been shown that the inaccuracy of the response can be minimized by appropriately choosing a feedback resistor, R_x and that the optimum response contains an inflection point located at approximately the midpoint of the temperature range. Results from experimental work show that the linearized response, for the range of 0 to 200 °C, contained an inaccuracy of only 0.02% of full-scale temperature.

Based on the derivations presented, a truncated-series representation of the transfer function was analyzed to understand how the linearization is made possible. The development of the series representation provides much detail derivation which is presented in general form, beginning with Eq. (5.6) and followed by chain rule expansion to the fourth order shown in Eq. (5.16), (5.18), and (5.20). Since the derivations are presented in general form, they are applicable to analyze any linearizing circuit whose transfer function can take the form of Eq. (5.6). The information provided in these derivations is important because it is difficult to find in the existing literature.

The Howland current pump frequency stability analysis is documented in this text along with the SPICE macro-model shown in Appendix E and the listing shown in Appendix E. Most text books that discuss

this current pump do not provide information dealing with the frequency stability, but this text discusses the effects of parasitic capacitances which can destabilize the current pump circuit.

7.2 Suggested Improvements for Future Designs

When dealing with extreme accuracy requirements in a design of a circuit, noise can degrade the performance. Though the noise interference in the design presented in this text was insignificant, more demanding accuracy specifications would require a closer inspection of these noise problems. The second generation RTD Processor could be improved by designing a discrete, three-op-amp, instrumentation amplifier using low noise components. The major part of the circuit gain should be contained in this instrumentation amplifier stage.

The response of the future designs could be simulated using a SPICE model for the RTD. A model for the RTD in SPICE, as a temperature-varying resistor, is not available; however, it is possible to indirectly model the RTD and simulate the transducer's signal variation with temperature and feedback excitation. SPICE provides non-linear, current-controlled, voltage sources of two variables. One of the two variables can be the feedback current that flows through the RTD. The second variable can be a current that simulated temperature ramping. Specifying a piece-wise linear current source, whose current is the second controlling variable in the non-linear source, allows the circuit to respond in the same manner that the circuit would respond to changes in the RTD resistance due to changes in the sensed

temperature. This model, though not included in this text, was successfully developed.

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LIST OF REFERENCES

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APPENDIXES

APPENDIX A

SOURCE LISTING FOR A PROGRAM THAT CALCULATES THE DATA
POINTS USING THE SERIES REPRESENTATION

```

C*****
C
C PROGRAM'S NAME : SERIES.FOR
C
C DATE CREATED : 6/5/85
C PROGRAMMER : GUS C. PABON
C
C PROGRAM DESCRIPTION:
C
C THIS PROGRAM CALCULATES THE DATA POINTS USING
C THE SERIES REPRESENTATION OF THE 982-1 TRANSFER
C FUNCTION FOR DIFFERENT VALUES OF FEEDBACK RESIS-
C TANCE RX. THIS METHOD ALLOWS US TO SEE THE
C " MOST LINEAR " DATA POINTS FOR A GIVEN RX SINCE
C THE VALUE OF RX IS ENTERED AFTER A PROGRAM PROMPT
C
C*****
C VARIABLES DEFINED
C*****
C
C RX, THE VALUE OF THE LINEARIZING FEEDBACK RESISTOR
C IRANGE, THE RANGE OF TEMPERATURE TO BE USED IN THE CALCU-
C LATIONS.
C INTEMP, THE TEMPERATURE INTERVAL USED TO INCREMENT EACH TEM-
C PERATURE VALUE IN THE PROGRAM LOOP.
C CONSTC,F,D,E, THE CONSTANTS FOUND IN THE SIMPLIFIED TRANSFER
C FUNCTION, EQUATION (6.6), IN THE thesis TEXT.
C CONSTA,B THE RTD CONSTANTS DEFINED IN EQUAT. (3.10)&(3.11)
C OF THE thesis TEXT.
C CONSTW,X,Y,Z,S,L,M,N, THE CONSTANT EXPRESSIONS FOR EACH
C COEFFICIENT OF EACH TERM OF THE
C SERIES EXPANSION. THEY ARE FOUND IN
C THE THESIS TEXT IN EQUATIONS:(6.15),
C (6.17),(6.19),(6.21).
C K1, THE SCALING FACTOR TO FORCE VOUT TO EQUAL +5.0V @ TMAX.
C NOTE THAT THIS IS THE OUPUT STAGE GAIN OF THE CIRCUIT.
C VO(I), AN ARRAY CONTAINING THE DATA FOR THE OUTPUT OF FEED-
C BACK LOOP IN THE CIRCUIT.
C VOUT(I), AN ARRAY CONTAINING THE LINEARIZED OUTPUT VOLTAGE
C DATA OF THE CIRCUIT. NOTE VOUT EQUALS VO MUL-
C TIPLIED BY THE OUTPUT STAGE GAIN, K1, OF THE CKT.
C IDLVOL(I), AN ARRAY CONTAINING DATA OF THE IDEAL OUTPUT
C VOLTAGE AT EACH TEMPERATURE POINT.
C PCTERR(I), AN ARRAY CONTAINING THE PERCENT ERROR BETWEEN
C VOUT AND IDLVOL AT EACH POINT.
C TEMDIF(I), AN ARRAY CONTAINING THE DIFFERENCE OF THE IDEAL
C TEMPERATURE AND THE CALCULATED LINEARIZED TEMP'S.
C TEMP2(I), AN ARRAY CONTAINING THE CALCULATED LINEARIZED
C TEMPERATURE DATA POINTS.
C

```

```

C*****
C*****
C
C   VARIABLE TYPE DECLARATION
C
REAL   INTEMP,R2,R3,R5,R7,GAIN,R8,RX,ER,RO,alpha,delta
REAL*8 K1,CONSTA,CONSTF,CONSTD,CONSTE,CONSTB,CONSTC,CONSTW
REAL*8 CONSTX,CONSTY,CONSTZ,CONSTS,CONSTL,CONSTM,CONSTN
REAL*8 TEMP(200),TEMP2(200),IDLVOL(200),VO(200),VOUT(200)
REAL*8 PCTERR(200),TEMDIF(200)
INTEGER I,UPTEMP,LOTEMP,NPOINT,IRANGE
C
C   CHARACTER ANS
C
C   DATA DECLARATION USING CIRCUIT VALUES AND RTD CHARACTERISTICS
C
DATA R2/10.0E3/,R3/10.0E3/,R5/10.0E3/,R7/10.0E3/
DATA GAIN/100.0/,R8/10.0E3/,ER/5.0/
DATA RO/100.0/,alpha/0.00385/,delta/1.526/
C
OPEN(UNIT=1,NAME='NEWTOT.DAT',TYPE='NEW')
OPEN(UNIT=2,NAME='DAT.DAT',TYPE='NEW')
OPEN(UNIT=3,NAME='FINSER.DAT',TYPE='NEW')
OPEN(UNIT=4,NAME='COMPARE2.DAT',TYPE='NEW')
C
C
TYPE 50 ,R2,R3,R5,R7,GAIN,RO,alpha,delta
WRITE(1,50) R2,R3,R5,R7,GAIN,RO,alpha,delta
C
50   FORMAT(/,5X,'THE PRESET PARAMETER VALUES ARE : R2= ',1PG7.1,/,
1      5X,' R3= ',1PG7.1,' R5= ',1PG7.1,' R7= '
1      ',1PG7.1,/,
1      5X,' GAIN= ',1PG7.1,' RO= ',1PG7.1,/,
1      5X,' alpha= ',1F12.6,' delta= ',1F12.6,/)
C
C   Operator program prompts
C-----
TYPE 60
60   FORMAT(5X,'ENTER THE UPPER TEMPERATURE VALUE (INTEGER #): ',,$)
ACCEPT 61,UPTEMP
61   FORMAT(I6)
C
TYPE 70
70   FORMAT(5X,'ENTER THE LOWER TEMPERATURE VALUE (INTEGER #): ',,$)
ACCEPT 61,LOTEMP
C
TYPE 80
80   FORMAT(5X,'ENTER THE TOTAL NUMBER OF TEMPERATURE POINTS
1      '(MAX=200) ',,$)
ACCEPT 61,NPOINT
C

```

```

85     TYPE 90
C
C     NOTE THAT RX IS DEFINED AS R9+R10 IN THE thesis.
C
90     FORMAT(5X,'ENTER THE VALUE FOR RX DESIRED (i.e. 200.0K): ', $)
      ACCEPT 91,RX
91     FORMAT(1PG14.7)
C
C-----
C
      TYPE 95,RX
      WRITE(1,95) RX
95     FORMAT(//,5X,'THE VALUE OF RX FOR THE DATA BELOW IS  RX= '
1       ,1PG14.7,///)
C
C
      TYPE 100
      WRITE(1,100)
100    FORMAT(/,5X,'#      TEMPS      IDEAL VOLTAGE      ACTUAL VOLTAGE'
1       '      Vo VOLTAGE      TEMP DIFFERENCE      %ERROR',/)
C
C     Calculations made to properly increment the temperatures.
C
      IRANGE = UPTEMP - LOTEMP
      INTEMP = FLOAT(IRANGE)/(FLOAT(NPOINT)-1.0)
      TEMP(1) = FLOAT(LOTEMP)
      TMAX = UPTEMP
C
C     Constants being calculated as dictated by the derived series.
C
      CONSTA = ((R2+R3)*R5)/((R7+R5)*R2)
      CONSTF = -GAIN/R8
      CONSTD = -GAIN/RX
      CONSTE = 1.0

      CONSTB = RO*alpha*(1+(delta/100.0))
      CONSTC = (-RO*alpha*delta)/100.0**2

      CONSTW = 1/(CONSTE + CONSTD*RO)
      CONSTX = CONSTD*CONSTW
      CONSTY = (CONSTB**2)*CONSTX
      CONSTZ = CONSTC - CONSTY
      CONSTS = ER*CONSTW*CONSTF*CONSTB

      CONSTL = CONSTZ/CONSTB
      CONSTM = CONSTX*(CONSTC+CONSTZ)
      CONSTN =(CONSTX/CONSTB)*((2*CONSTC+CONSTZ)*CONSTY-(CONSTC**2))
C

```

```

C      The scaling factor to force VOUT=+5.0V @ TMAX.
C
C      K1 = 5.0/ (CONSTS*( TMAX + CONSTL*(TMAX**2)
1          - CONSTM*(TMAX**3) + CONSTN*(TMAX**4) ))
C
C
C      Loop calculates all the data points for the variables.
C
C      DO 300 I = 1,NPOINT
C
C          VO(I) = CONSTS*( TEMP(I) + CONSTL*(TEMP(I)**2)
1          - CONSTM*(TEMP(I)**3) + CONSTN*(TEMP(I)**4) )
C
C          VOUT(I) = K1*VO(I)
C
C
C          IDLVOL(I) = (5.0/FLOAT(IRANGE))*TEMP(I)
C
C
C          PCTERR(1)= 0.0
C          IF (IDLVOL(I) .EQ. 0.0 )GO TO 150
C
C          PCTERR(I) = ( ( VOUT(I) - IDLVOL(I) )/IDLVOL(I) )*100.0
C
C          TEMDIF(I) = (PCTERR(I)*TEMP(I))/100.0
C
C          TEMP2(I) = TEMP(I)+TEMDIF(I)
C
C      150      TYPE 200, I,TEMP(I),IDLVOL(I),VO(I),VOUT(I),
1          TEMDIF(I),PCTERR(I)
C
C      All the data is transfered to filename 'NEWTOT.DAT'.
C
C          WRITE(1,200) I,TEMP(I),IDLVOL(I),VO(I),VOUT(I),
1          TEMDIF(I),PCTERR(I)
C
C
C      200      FORMAT(3X,I3,3X,1F12.6,3X,1F12.6,3X,1F12.6,3X,
1          1F12.6,3X,1F12.6,3X,1F12.6)
C
C      Data is transfered to filename 'FINSER.DAT' to create a table
C      in the thesis text that compares ideal and calculated temp's.
C
C          WRITE(3,210) TEMP(I),TEMP2(I),TEMDIF(I),PCTERR(I)
210      FORMAT (5X,1F12.6,1X,1F12.6,1X,1F12.6,1F12.6)
C
C      Data is transfered to filename 'PLOT.DAT' to create a plotting
C      file of the data.
C

```



```

                WRITE(2,250) TEMP(I),TEMDF(I)
250             FORMAT(2F12.6)
C
C      Data is transfered to filename 'COMPARE2.DAT' to create a table
C      in the thesis text that compares this data with that of data
C      of the actual transf. funct. received through prog. 9821TF.FOR.
C
                WRITE(4,270) TEMDF(I),PCTERR(I)
270             FORMAT(2F12.6)
C
                TEMP(I+1) = TEMP(I) + INTEMP
C
300      CONTINUE
C
C
                TYPE 301
                WRITE(1,301)
301      FORMAT(//,5X,'THE DATA ABOVE IS CALCULATED USING',/,
1          5X,'THE FOLLOWING CONSTANTS:',/)
C
                WRITE(1,302)CONSTB,CONSTC,CONSTW,CONSTX,CONSTY,CONSTZ,CONSTS
                TYPE 302, CONSTB,CONSTC,CONSTW,CONSTX,CONSTY,CONSTZ,CONSTS
302      FORMAT(/,5X,'B= ',1PG12.4,' C= ',1PG12.4,' W= ',1PG12.4,
1          ' X= ',1PG12.4,' Y= ',1PG12.4,' Z= ',1PG12.4,' S= '
1          ',1PG12.4,/)
C
                TYPE 303,CONSTL,CONSTM,CONSTN,K1
                WRITE(1,303) CONSTL,CONSTM,CONSTN,K1
303      FORMAT(5X,' L= ',1PG12.4,' M= ',1PG12.4,' N= ',1PG12.4,
1          ' K1= ',1PG12.4,/)
                TYPE 310
C
310      FORMAT(//,5X,'DO WISH TO CONTINUE ? (Y OR N): ',$,)
                ACCEPT 311,ANS
311      FORMAT(A)
C
                IF (ANS .EQ. 'Y' )GO TO 85
C
C
                CLOSE (UNIT=1)
                CLOSE (UNIT=2)
                CLOSE (UNIT=3)
                CLOSE (UNIT=4)
C
                STOP 'NORMAL END'
                END

```

APPENDIX B

SOURCE LISTING FOR A PROGRAM THAT CALCULATES THE DATA
POINTS USING THE TRANSFER FUNCTION IN GENERAL FORM

THE TRANSFER FUNCTION OF THE 982-1 WITHOUT SIMPLIFICATION

The equation below is the transfer function for the 982-1 without the simplification of perfect matching Howland current pump resistances. If the conditions of Eq. (4.17) is substituted into this equation shown below will simplify to the form presented in Eq. (6.1). This equation is programmed in the listing seen of program 9821TF.FOR.

$$V_{OUT} = -K_1 *$$

$$\frac{ER(R_4+R_6)R_7}{(R_7+R_8)R_4} * \left[1 + \left(\frac{1}{R_8} - \frac{R_{12}}{R_{13}R_{10}} - \frac{R_{12}}{R_{13}R_{11}} \right) * RTD2 - \frac{RTD1(ER)G}{R_8} \right]$$

$$\left[1 + \left(\frac{1}{R_8} - \frac{R_{12}}{R_{13}R_{10}} - \frac{R_{12}}{R_{13}R_{11}} \right) RTD2 \right] - \frac{RTD1(ER)G}{R_{13}R_{10}}$$

(B-1)


```

C*****
C*****
C
C      Variable Declaration
C
      REAL R12,R13,R10,R11,R4,R6,R7,R8,GAIN,ER
      REAL CHARES,ALPHA,DELTA,RSER,INTEMP
      REAL*8 CONSTA,CONSTB,CONSTC,CONSTD,CONSTE,CONSTF,K1,R8
      REAL*8 RT1MAX,RT2MAX,CONSTG,CONSTH,CONSTJ,CONSTL,VOMAX
      REAL*8 RTD1(200),RTD2(200),VO(200),VOUT(200),TEMP(200)
      REAL*8 IDLVOL(200),DIFF(200),TEMDF(200),PCTERR(200)
      INTEGER I,UPTEMP,LOTEMP,NPOINT,IRANGE
C
C
C      Data declaration using circuit values and rtd characteristics
C
      DATA R12/10.0E3/,R13/10.0E3/,R10/258.5E3/,R11/10.402414E3/
      DATA R4/10.0E3/,R6/10.0E3/,R7/10.0E3/,R8/10.0E3/
      DATA GAIN/100.0/,ER/5.0/,RSER/243.0/,CHARS/100.0/
      DATA ALPHA/0.00385/,DELTA/1.526/
C
      OPEN(UNIT=1,NAME='TRANF.DAT',TYPE='NEW')
      OPEN(UNIT=2,NAME='COMPARE1.DAT',TYPE='NEW')
C
C      Operator program prompts
C-----
      TYPE 60
60      FORMAT(5X,'ENTER THE UPPER TEMPERATURE VALUE (INTEGER #): ',%)
      ACCEPT 61,UPTEMP
61      FORMAT(I6)
C
      TYPE 70
70      FORMAT(5X,'ENTER THE LOWER TEMPERATURE VALUE (INTEGER #): ',%)
      ACCEPT 61,LOTEMP
C
      TYPE 80
80      FORMAT(5X,'ENTER THE TOTAL NUMBER OF TEMPERATURE POINTS
1          '(MAX=200) ',%)
      ACCEPT 61,NPOINT
C-----
C
C
      TYPE 90 ,R12,R13,R10,R11,R4,R6,R7,GAIN,ER,CHARS,ALPHA,DELTA
      WRITE(1,90) R12,R13,R10,R11,R4,R6,R7,GAIN,ER,
1          CHARES,ALPHA,DELTA
C
90      FORMAT(/,5X,'THE PRESET PARAMETER VALUES ARE : R12= ',1PG7.1,/,
1          5X,' R13= ',1PG7.1,' R10= ',1PG7.1,' R11= ',1PG16.10,/,
1          5X,' R4= ',1PG7.1,' R6= ',1PG7.1,' R7= ',1PG7.1,/,
1          5X,' GAIN= ',1PG7.1,' ER= ',1PG7.1,/,

```

```

1      5X,' Ro= ',1PG7.1,' alpha= ',1F12.6,
1      ' delta= ',1F12.6,/)
C
C
C      TYPE 100
C      WRITE(1,100)
100     FORMAT(/,5X,'#      TEMPS      RTD VALUE      Vo VOLTAGE'
1      ' Vout VOLTS  TEMP. DIFFERENCE  %ERROR',/)
C
C      Calculations made to properly increment the temperatures.
C
C      IRANGE = UPTEMP - LOTEMP
C      INTEMP = FLOAT(IRANGE)/(FLOAT(NPOINT)-1.0)
C      TEMP(1) = FLOAT(LOTEMP)
C      TMAX = FLOAT(UPTEMP)
C
C
C      Program loop to calculate the data points for each
C      temperature.
C
C      DO 170 I = 1 , NPOINT
C
C          CONSTA = CHARES*ALPHA*(1+(DELTA/100))
C          CONSTB = -(CHARS*ALPHA*DELTA)/(100**2)
C
C          RTD1(I) = CHARES + CONSTA*TEMP(I) + CONSTB*(TEMP(I)**2)
C
C          RTD2(I) = RTD1(I) + RSER
C
C          CONSTC = 1 + ( (1/R8) - ( R12/(R13*R10) ) -
1          ( R12/(R13*R11) ) ) *RTD2(I)
C
C          CONSTE = ( GAIN*RTD1(I)*ER )/R8
C
C          IF ( I .GT. 1 ) GO TO 110
C
C-----
C      The value of 'potentiometer' R8 is calculated so that VOUT(1)
C      equals zero at TEMP=0.0 for different tolerances and values of
C      the associated resistors.
C
C          CONSTM =( ER*(R4+R6)*R7 )/R4
C
C          R8 = CONSTM*( CONSTC/CONSTE) - R7
C
C-----
C
110     CONSTD = ( ER*(R4+R6)*R7 )/( (R7+R8)*R4 )
C
C          CONSTF = ( GAIN*RTD1(I)*R12 )/( R13*R10 )

```

```

C
C      Allows the maximum RTD value to be calculated only on the
C      first pass of the program loop.
C
C      IF ( I .GT. 1 )GO TO 120
C
C      RT1MAX = CHARES + CONSTA*TMAX +
1          CONSTB*(TMAX**2)
C
C      RT2MAX = RT1MAX + RSER
C
C      CONSTG = 1 + ( (1/R8) - ( R12/(R13*R10) ) -
1          ( R12/(R13*R11) ) ) *RT2MAX
C
C      CONSTH = ( GAIN*RT1MAX*ER )/R8
C
C      CONSTJ = ( ER*(R4+R6)*R7 )/( (R7+R8)*R4 )
C
C      CONSTL = ( GAIN*RT1MAX*R12 )/( R13*R10 )
C
C      VOMAX = ( (CONSTD*CONSTG) - CONSTH )/( CONSTG - CONSTL )
C
C      The scaling factor to force VOUT = +5.0V @TMAX.
C
C      K1 = 5.0 / VOMAX
C
C      120  VO(I) = ( (CONSTD*CONSTC) - CONSTE )/( CONSTC - CONSTF )
C          VOUT(I) = K1*VO(I)
C
C      IDLVOL(I) = (5.0/FLOAT(IRANGE))*TEMP(I)
C
C
C      PCTERR(1)= 0.0
C      IF (IDLVOL(I) .EQ. 0.0 )GO TO 130
C
C      PCTERR(I) = ( ( VOUT(I) - IDLVOL(I) )/IDLVOL(I) ) *100.0
C
C      TEMDIF(I) = (PCTERR(I)*TEMP(I))/100.0
C
C      All the data is transfered to filename 'TRANF.DAT' .
C
C      130  TYPE 150, I ,TEMP(I),RTD1(I),VO(I),VOUT(I),
1          TEMDIF(I),PCTERR(I)
C      WRITE(1,150) I ,TEMP(I),RTD1(I),VO(I),VOUT(I),
1          TEMDIF(I),PCTERR(I)
C

```

```

150      FORMAT(3X,I3,3X,1F12.6,3X,1F12.6,3X,1F12.6,3X,1F12.6,
1          3X,1F12.6,3X,1F12.6)
C
C      Data is transfered to filename 'COMPARE1.DAT' to create a
C      table in the thesis text that compares this data with that of
C      data of the actual transf. funct. received through program
C      SERIES.FOR
C
C      WRITE(2,160) TEMP(I),TEMDF(I),PCTERR(I)
160      FORMAT(3F12.6)
C
C      TEMP(I+1) = TEMP(I) + INTEMP
C
C      170      CONTINUE
C
C      TYPE 180
180      FORMAT(//,3X,'THE PRECEDING DATA POINTS WILL BE STORED IN',/,
1          ' FILENAME: TRANF.DAT')
C
C
C      CLOSE(UNIT=1)
C      CLOSE(UNIT=2)
C
C      STOP 'NORMAL END'
C      END

```


APPENDIX C

SOURCE LISTING FOR A PROGRAM USED TO MERGE DATA
FROM OTHER FILES TO ENABLE PLOTTING

```

C*****
C  PROGRAM'S NAME : PLOT.FOR
C  DATE CREATED   : 6/20/85
C  PROGRAMMER     : GUS C. PABON
C
C
C  PROGRAM DESCRIPTION:
C      THIS PROGRAM READS THE SELECTED COLUMNS
C      OF DATA FROM DIFFERENT DATA FILES AND CREATES
C      A NEW DATA FILE CALLED PLOT.DAT WHICH CONTAINS
C      THE SELECTED DATA COLUMNS FROM THE DIFFERENT
C      FILES. THIS NEW DATA FILE IS USED FOR PLOTTING
C      THE COMPOSITE DATA POINTS ON THE SAME PLOT.
C
C*****
C      VARIABLES DEFINED
C*****
C      TOTCOL,    THE NUMBER OF COLUMNS THAT THE FINAL PLOTTING
C                  PROGRAM WILL CONTAIN. DEPENDS ON THE NUMBER OF
C                  TIMES THE OPERATOR ANSWERS 'Y' TO LINE 110.
C
C      INPUT(I,J), AN ARRAY THAT TEMPORARILY STORES ALL THE DATA OF
C                  THE FILE BEING READ IN. NOTE THAT IT IS SET UP FOR
C                  ONLY TWO OR THREE COLUMNS OF DATA AS DICTATED BY
C                  THE FORMAT STATEMENT AND DEPENDING ON THE STATUS
C                  OF COLINP.
C
C      DATA(I,TOTCOL), AN ARRAY THAT STORES ONLY SELECTED COLUMNS OF
C                  ARRAY INPUT(I,J)
C
C*****
C*****
C
C      REAL*8 DATA(200,200), INPUT(200,10)
C      INTEGER COLINP,COLNUM,NPOINT,TOTCOL, I ,J
C      CHARACTER*32 ANS,FILNAM
C
C
C      OPEN(UNIT=2,NAME='PLOT.DAT',TYPE='NEW')
C
C      TOTCOL=0
C
C      The number of columns that the composite data file will
C      contain is incremented.
C
10  TOTCOL= TOTCOL + 1
C
C      The filename containing the data to be read in is entered.
C

```

```

20      TYPE 20
      FORMAT( '      ENTER DATA FILENAME TO BE COPIED: ', $)
      ACCEPT 21, FILNAM
21      FORMAT(A)
C
      OPEN(UNIT=1, NAME=FILNAM, TYPE='OLD')

C
      TYPE 25
25      FORMAT( '      ENTER THE NUMBER OF COLUMNS IN THIS FILE: ', $)
      ACCEPT 26, COLINP
26      FORMAT(I3)
C
C
C      The column to be copied into DATA(I,COLUM) is entered.
C
      TYPE 30
30      FORMAT( '      ENTER THE COLUMN NUMBER TO BE COPIED: ', $)
      ACCEPT 31, COLNUM
31      FORMAT(I3)
C
C
      TYPE 40
40      FORMAT(5X, 'ENTER THE TOTAL NUMBER OF DATA  POINTS (MAX=200)', $)
      ACCEPT 41, NPOINT
41      FORMAT(I3)

C
C      This if statement allows the correct format statement
C      to be accessed for the correct number of columns of F12.6
C      data contained.
C
      IF( COLINP .EQ. 3 )GO TO 60

      READ(1,51) ( (INPUT(I,J), J= 1,COLINP), I= 1,NPOINT )
51      FORMAT(2F12.6)
C
      GO TO 65

C
60      READ(1,61) ( (INPUT(I,J), J= 1,COLINP), I= 1,NPOINT )
61      FORMAT(3F12.6)
C
65      DO 100 I= 1,NPOINT
C
C          The desired column # to be added to the ones already
C          existing (after the first pass) is stored in DATA(I,COLUM).
C
          DATA(I,TOTCOL)=INPUT(I,COLNUM)
C
100      CONTINUE

```

```

C
C
      CLOSE(UNIT=1)
C
C
      TYPE 110
110    FORMAT(' DO YOU WISH TO CONTINUE ? :', $)
      ACCEPT 120 ,ANS
120    FORMAT(A)
C
      IF ( ANS .EQ. 'Y' ) GO TO 10
C
C      The composite data array DATA(I,J) is written to
C      a data file, PLOT.DAT to allow plotting of the points.
C      Once again the proper format statement must be chosen
C      for the amount of data columns in the composite data file.
C
      WRITE(2,200) ((DATA(I,J), J= 1,TOTCOL), I= 1,NPOINT)
200    FORMAT(4F12.6)
C200  FORMAT(5F12.6)
C
      TYPE 210, ((DATA(I,J), J=1,TOTCOL), I= 1,NPOINT)
210    FORMAT(4F12.6)
C210  FORMAT(5F12.6)
C
      CLOSE(UNIT=2)
C
      STOP 'NORMAL END'
      END

```

APPENDIX D

SOURCE LISTING FOR A PROGRAM USED TO COLLECT
DATA IN THE PROCESS OF EXPERIMENTATION

```

C*****
C  PROGRAM'S NAME : 982EXP.FOR
C  DATE CREATED   : 6/23/85
C  PROGRAMMER     : GUS C. PABON
C
C  PROGRAM DESCRIPTION:
C
C      THIS PROGRAM IS USED DURING EXPERIMENTAL TESTING OF
C      THE 982-1. THE PROGRAM RECEIVES EXPERIMENTAL RESULTS
C      AS THEIR BEING COLLECTED AND CALCULATES THE ERRORS OF
C      THE MEASURED DATA. ALL DATA ARE STORED IN FILENAME
C      982EXP.DAT.
C*****
C      VARIABLES DEFINED
C*****
C
C      MAXVOL, THE VALUE OF THE MAXIMUM VOLTAGE TO BE MEASURED
C      MINVOL, THE VALUE OF THE MINIMUM VOLTAGE TO BE MEASURED
C      RTDMAX, THE VALUE OF THE MAXIMUM RTD TO BE USED (NEEDED TO
C              CALCULATE THE MAXIMUM TEMPERATURE ).
C      RTDMIN, THE VALUE OF THE MINIMUM RTD TO BE USED (NEEDED TO
C              CALCULATE THE MINIMUM TEMPERATURE ).
C      TMAX,   THE MAXIMUM TEMPERATURE VALUE IN THE RANGE DESIRED.
C      TMIN,   THE MINIMUM TEMPERATURE VALUE IN THE RANGE DESIRED.
C      RTD ,   THE RESISTANCE VALUE USED FOR EACH DATA POINT IN THE
C              EXPERIMENT.(THE EQUIVALENT TEMPERATURE IS OBTAINED)
C      VOMEAS, THE EXPERIMENTALLY-MEASURED VOLTAGE FOR EACH DATA
C              POINT OF THE EXPERIMENT.
C      CONSTS, THE SLOPE OF THE IDEAL LINE.
C      CONSTA,B, THE COEFICIENTS OF THE CALLENDAR VAN-DUSEN EQUATION.
C      CONSTC, THE QUADRATIC EQUATION VALUE USED TO DETERMINE THE
C              EQUIVALENT TEMPERATURE VALUE FOR EACH RESISTANCE
C              ,RTD, INPUT.
C      VOIDL,  THE VALUE OF THE IDEAL VOLTAGE AT EACH DATA POINT.
C      PCTERR, THE % ERROR DEFINED AS THE DIFFERENCE IN THE
C              MEASURED AND THE IDEAL DIVIDED BY THE IDEAL AT EACH
C              DATA POINT.
C      TEMDIF, THE DIFFERENCE IN THE MEASURED AND THE IDEAL
C              TEMPERATURES (CALCULATED FROM THE PCTERR ).
C      TEMP,   THE EQUIVALENT TEMPERATURE CALCULATED FOR EACH
C              RTD RESISTANCE INPUT.
C      YINT,   THE Y-INTERCEPT FOR THE IDEAL VOLTAGE EQUATION.
C*****
C

```

```

C      VARIABLE TYPE DECLARATION
      REAL MAXVOL,MINVOL,RTD,RTDMAX,TMAX,TMIN,RTDMIN,VOMEAS
      REAL*8 CONSTS,VOIDL,CONSTA,CONSTB,CONSTC
      REAL*8 PCTERR,TEMDIF,TEMP,YINT
      INTEGER X
      CHARACTER ANS

C
C      Data declaration using typical RTD charecteristic values.
C
      DATA CHARES/100.0/,alpha/0.00385/,delta/1.526/

C
      OPEN(UNIT=1,NAME='982EXP.DAT',TYPE='NEW')

C
C      Operator program prompts.
C
      TYPE 5
5      FORMAT (3X,'ENTER THE MAXIMUM OUTPUT VOLTAGE READING: ', $)
      ACCEPT 6,MAXVOL
6      FORMAT(1F12.6)
C
      TYPE 7
7      FORMAT (3X,'ENTER THE MINIMUM OUTPUT VOLTAGE READING: ', $)
      ACCEPT 8,MINVOL
8      FORMAT(1F12.6)
C
      TYPE 10
10     FORMAT (/ ,3X,'ENTER THE MAXIMUM RTD VALUE USED: ', $)
      ACCEPT 11,RTD
11     FORMAT (1F12.3)
C
      RTDMAX = RTD
      X=0

C
C      Program branches to calculate the maximum temperature limit.
C
      GO TO 70
20     TMAX = TEMP
C
      TYPE 30
30     FORMAT(/ ,3X,'ENTER THE MINIMUM RTD VALUE USED: ', $)
      ACCEPT 31,RTD
31     FORMAT(1F12.3)
C
      RTDMIN = RTD
      X=1

C
C      Program branches to calculate the minimum temperature limit.
C
      GO TO 100
40     TMIN = TEMP
C

```

```

C      The operating limits are exhibited.
C
      WRITE (1,41) RTDMAX,TMAX,MAXVOL,RTDMIN,TMIN,MINVOL
      TYPE 41, RTDMAX,TMAX,MAXVOL,RTDMIN,TMIN,MINVOL
41     FORMAT(/,3X,'RTDMAX= ',1F12.3,' TMAX= ',1F12.6,
1      ' MAXVOL= ',1F12.6,/,3X,'RTDMIN= ',1F12.3,' TMIN= ',1F12.6,
1      ' MINVOL= ',1F12.6)
C
      WRITE (1,42)
42     FORMAT(3X,'TEMPERATURE RTD VALUE MEAS. VOLTAGE'
1      ' IDEAL VOLTAGE %ERROR TEMP. DIFFERENCE',/)
C
C      Branch test variable set to avoid branching to line 20 or 40.
C
      X=100
C
      CONSTS = (MAXVOL - MINVOL)/(TMAX - TMIN)
C
45     TYPE 50
50     FORMAT(/,3X,'ENTER THE RTD VALUE: ',$,)
      ACCEPT 51,RTD
51     FORMAT(1F12.3)
C
70     CONSTA = CHARES*ALPHA*(1+(DELTA/100))
      CONSTB = -(CHARS*ALPHA*DELTA)/(100**2)
C
C      The quadratic formula calculates the equivalent temp. for a
C      resistance , RTD , input .
C
100    CONSTC = SQRT( (CONSTA**2) - ( 4*CONSTB*(CHARS - RTD) ))
C
      TEMP = (-CONSTA + CONSTC)/(2*CONSTB)
C
C      Branch to redefine the temperature variable value for
C      Tmin and tmax conditions.
C
      IF( X .EQ. 0 ) GO TO 20
      IF( X .EQ. 1 ) GO TO 40
C
      YINT = MINVOL - (CONSTS*TMIN)
C
C      The ideal voltage calculated using the form y=mx+b .
C
      VOIDL = CONSTS*TEMP + YINT
C
      TYPE 55, VOIDL
55     FORMAT (/,3X,'THE IDEAL VOLTAGE IS: ',1F12.6)
C

```



```

TYPE 56
56      FORMAT (/,3X,'ENTER THE ROUND-OFF VALUE OF THE IDEAL',/,3X,
1      'VOLTAGE DESIRED: ', $)
      ACCEPT 57 ,VOIDL
57      FORMAT (1F12.6)
C
C      Program prompts to input experinently measured values.
      TYPE 60
60      FORMAT (/,3X,'ENTER THE MEASURED VOLTAGE: ', $)
      ACCEPT 61,VOMEAS
61      FORMAT (1F12.6)
      PCTERR = ( (VOMEAS - VOIDL)/ VOIDL)*100.0
C
      TEMDIF = (PCTERR*TEMP)/100.0
C
      TYPE 200
200     FORMAT(/,3X,'TEMPERATURE  RTD VALUE  MEAS. VOLTAGE'
1     ' IDEAL VOLTAGE  %ERROR  TEMP. DIFFERENCE',/)
C
      TYPE 210,TEMP,RTD,VOMEAS,VOIDL,PCTERR,TEMDIF
      WRITE(1,210) TEMP,RTD,VOMEAS,VOIDL,PCTERR,TEMDIF
210     FORMAT(1F12.6,1X,1F12.3,1X,1F12.6,1X,1F12.6,1X,1F12.6,
1         1X,1F12.6,/)
C
C      Answer 'Y' for yes to the following prompt if more
C      experimental data points are desired.
C
      TYPE 310
310     FORMAT(/,5X,'DO WISH TO CONTINUE ? (Y OR N): ', $)
      ACCEPT 311,ANS
311     FORMAT(A)
C
      IF (ANS .EQ. 'Y' )GO TO 45
C
C
C      CLOSE (UNIT=1)
C
      END

```

APPENDIX E

SPICE MACRO-MODELS AND PROGRAM SOURCE LISTING FOR THE LOOP TRANSMISSION ANALYSIS

SPICE MODEL OF THE HOWLAND CURRENT PUMP USING THE BURR BROWN 3510CM OPERATIONAL AMPLIFIER

Macro-modeling is a very powerful tool that allows one to take advantage of the SPICE analysis program by modeling an operational amplifier without having to construct the discrete, multi-transistor circuit. Depending on the analysis desired, the macro-model may be made complex or simple. The model used to determine the frequency response of the Howland current pump did not require great complexity but was more than adequate to predict the response of the current pump circuit. The operational amplifier modeled by this analysis is the Burr Brown 3510CM and is used extensively in the 982-1 instrument.

The model, seen in Figure E.1, contains the input stage, with the input resistance of the op-amp. The second stage contains a dependent voltage source which is dependent on the input, differential voltage and provides the open-loop gain of the amplifier. The first pole of the amplifier is realized in this stage with the network, R_1-C_1 , determining the critical frequency f_1 , of Eq. (4.21). The second stage realizes the second pole of the op-amp with the network, R_2-C_2 , determining the critical frequency, f_2 , also seen in Eq. (4.21). The pole locations of the op-amp were obtained from the manufacturer's data sheets and were found to be 1.0 Hz and 1.0 MHz for f_1 , and f_2 , respectively.

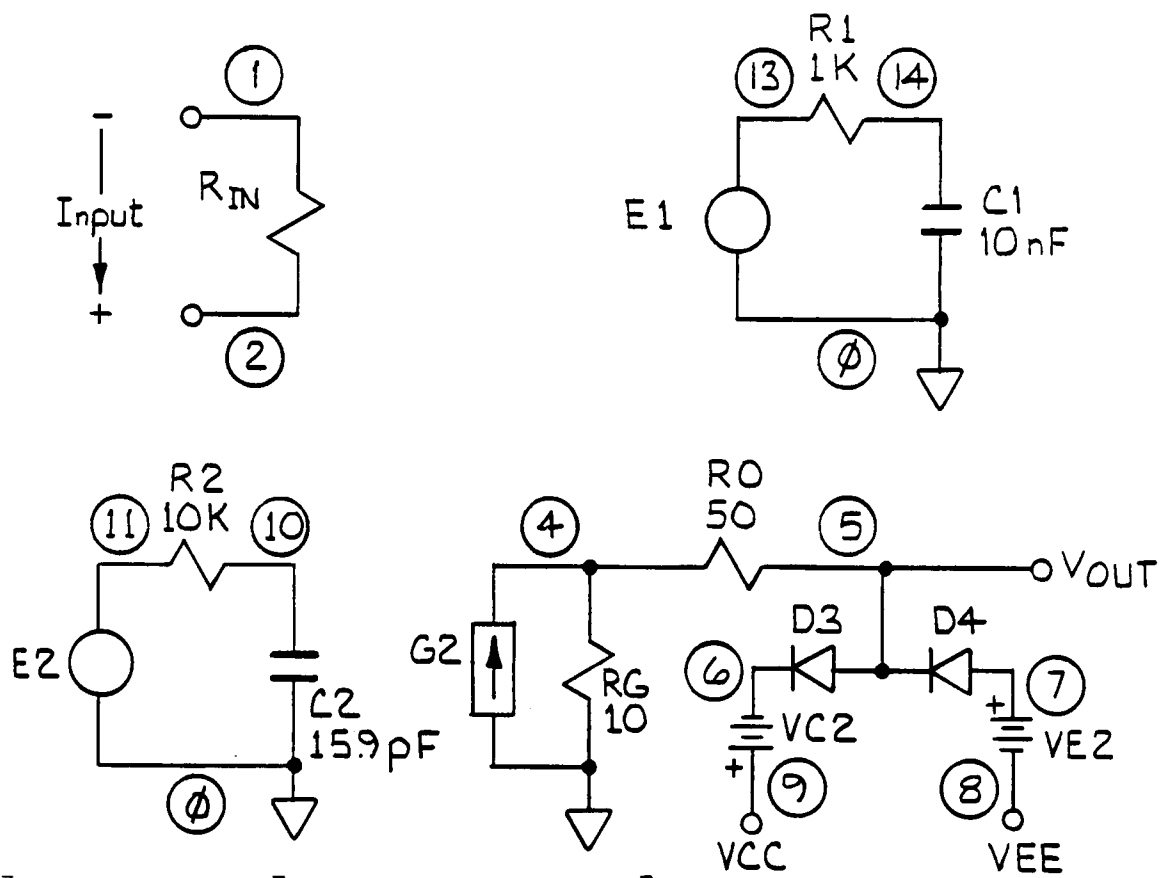


Figure E.1 Op-amp Macro-model for SPICE Analysis.

The output stage contains the amplifiers output impedance and circuitry which limits the dynamic range of the output voltage swing. As previously mentioned, the circuit can be made more complex if required by adding voltage sources and current sources that drift with temperature for drift analysis or sensitivity analysis. Additionally, the input stage can be modeled as an actual differential-pair, transistor network which would very accurately model non-linear effects and slewing as would occur in an actual circuit. The source listing for the SPICE program follows.

HOWLAND CURRENT PUMP

SOURCE LISTING FOR THE SPICE ANALYSIS OF THE HOWLAND CURRENT PUMP.

VCC 7 0 +15

VEE 8 0 -15

VIN 4 0 AC 1.0

RL 4 0 176.0

RCAL 3 6 273.0

CL 4 0 0.01UF

R10 1 0 10K

R12 1 4 10K

R8 3 0 10K

R13 3 4 10K

C10 1 0 90PF

C12 1 4 10PF

* BURR BROWN OP-AMP SUBCKT CALL

* - + +PS VOUT -PS MODEL

X4 1 3 7 2 8 BB3510

THE BURR BROWN OP AMP CIRCUIT MACRO-MODEL

* BURR BROWN CKT

* - + +PS VOUT -PS

.SUBCKT BB3510 1 2 9 5 8

E1 13 0 12 2 1E+6

R1 13 14 1.592MEG

C1 14 0 0.1UF

E2 11 0 14 0 1.0

R2 11 10 10K

C2 10 0 15.9PF

G2 0 4 10 0 0.10

RG 4 0 10.0

VC2 9 6 4.2

VE2 7 8 4.2

RIN 12 2 1MEG
RO 5 4 50

D3 5 6 DIODE2
D4 7 5 DIODE2

.MODEL DIODE2 IS=8E-16

.ENDS BB3510

.OPTIONS NOMOD
.AC DEC 10 0.01 1GHZ
.PRINT AC VDB(2) VP(2)
.PLOT AC VDB(2) VP(2)

.END

VITA

Gus Charles Pabon was born in La Paz, Bolivia, South America on March 23, 1958 and has lived most of his life in Knoxville, Tennessee. He graduated from West High School in Knoxville and then attended the University of Tennessee in Knoxville. At the University of Tennessee, he received a Bachelor of Science Degree and a Master of Science Degree in Electrical Engineering.

He was employed at Technology for Energy Corporation in Knoxville as an electronic design engineer during the developement of his thesis. He is currently employed by ITT Power Systems Division in the High-frequency, Advanced-development Group in Tucson, Arizona.

Mr. Pabon, in addition to electronic design, has a great interest in the outdoors, sports and athletics. He was recently awarded a Black-Belt Degree in Taekwondo, a Korean-style Karate and also enjoys weightlifting. His leisure time is spent on the snow-ski slopes during the Winter and on the lake, water-skiing, in the Summer. He would like to thank his thesis for having forced him to finally learn to type.