A STUDY OF TRANSMISSION LINES

by

David C. Merrill

A THESIS SUBMITTED FOR THE DEGREE OF

BACHELOR OF SCIENCE

ELECTRICAL ENGINEERING COURSE

The University of Tennessee

1930

TABLE OF CONTENTS

TABLE OF CONTENTS (CONTINUED)

PREFACE

This work deals with the theory of the transmission line. A discussion of the losses involved and the formulas used when considering such losses are shown. A discussion of the effect of charging current and capacitance is given. Different means, whereby the voltage of the line is kept constant at the receiver end, are is shown, together with the problems which arise.

The methods for calculating losses and the derivation of equations for conditions required at supply end, when receiver conditions are known, are divided into two classes; namely , Short Line Calculations and Long Line Calculations. In the Short Line Calculations, different assumptions are made as to the location of the capacitance of the line, which gives varying degrees of accuracy. The Long Line Calculations are exact solutions to the problems arising in power transmission over long lines. In connection with this subject is given an explanation of the hyperbolic functions which are used in the formulas.

Considerable space is given to the derivation of equations for the circle diagrams, because this method affords a practical means of solving problems of a series, or network, of lines. For any given conditions of receiver- or load-end a circle diagram may be constructed which gives the complete performance of the line

In a system of lines, or a network of variaus combinations of series and parallel groupings of their constituent elements, it is possible to combine them into an eqivalent network. Then, for this equivalent network, it is possible to get the equations for voltage and current at the supply end in terms of the receiver voltage and current in the form of the general equations. These steps are shown under Equivalent Networks.

Then, under Calculations, an assumed line is taken and the calculations made,whereby it is possible to construct a circle diagram and plot curves showing the performance of the line.

In preparing this work due acknowledgment is made to Professor Tarboux for the assistance given. Also, much assistance was derived from the book, "Electrical Power Transmission", by E. A. Loew.

GENERAL SURVEY OF THE FIELD

Trans mission of electrical energy has played an important part in the development of the country's natural resources. Lines, bringing power over long distances, has had to compete with local power plants. This has lead to a minute study of the electrical properties of the line in order to accertain the losses within the line itself. Also, as the distance over which power was transmitted increased, it became necessary to accertain just what takes place in the line in order to fix the value of supply voltage so that a given receiver voltage and current could be sustained at the receiver end. This has lead to the development of formulas which give the values required at the supply end. In short lines these formulas need only be approximate, but in lines over 50 or 60 miles in length the error becomes appreciable and a more exact method is needed. Exact equations are obtainable which give values, the accuracy of which depends on the number of terms used in the infinite series accuring in the formulas.

In general practice both the supply- end and receiver-end voltages arekept constant, synchronous condensers being put in the load-end of the line to control the power factor.

This work tends to show the derivation of the formulas and their application to the solution of a given problem.

CHAPTER 1

THEORY OF THE TRANSMISSION LINE

Losses Found in Transmission Lines.

Three fairly distinct losses are recognized as taking place in a transmission line in addition to the resistance losses. They are, namely, (a) normal conduction loss, which is due to the imperfectly insulating materials which support and surround the conductors; (b) dielectric hysteresis loss and, (c) the corona loss.

The Normal Conduction Loss. - Because of imperfect line insulation, the voltage between the line wires causes a leakage current to flow from wire to wire, which represents power loss. The current is not proportional to the voltage. No insulating substance conforms to Ohm's Law. If the voltage is doubled across the plate of a condenser the leakage current is not doubled. In a cable the conduction loss increases with the temperature.

The Dielectric Hysteresis Loss. - If a condenser is charged a certain amount of work is done. Now if the condenser is discharged all the energy is not given back. Some of the energy loss will be due to conduction loss as explained above. However all the energy loss will not be due to the normal conductive loss. This extra loss is called dielectric hysteresis loss.

The Corona Loss. - This loss exists on an air line when the voltage gradient about the conductors rises above a certain critical value, which is about 30 kv. per cm. As this critical point is reached the air about the conductors becomes conducting. If viewed in the dark a violet glow becomes visible, a hissing sound is heard, and, if conditions are favorable, ozone may be detected by its odor.

If the voltage be raised to still higher values, the violet glow will reach out farther and farther until finally, the insulation afforded by the air is broken down completely and an arc passes between the conductors. this voltage, which is higher than the visible corona voltage, is called the spark-over voltage. If the voltage is held at this value the arc will be intermittent, corona being visible between successive discharges.

In transmission lines, the diameter of the conductor is made large enough so that the voltage gradient at the surface of the wire is below the critical value.

Charging Current. - In the case of a transmission line carrying no load there is still a current flowing in the wire. At the receiver end the current is zero and increases towards the generator end where its value is maximum. Due to the current flowing in the line there is a drop of voltage. At the receiver end the voltage is maximum and decreases towards the supply end. In a long line the former may be considerably larger than the latter. This current, which is lost in the line due to impedence capacitance and hysteresis losses of the circuit, is called charging current.

Effect of capacitance. - A very important factor in the operation of very long lines, and one that must be at least approximated for all lines except very short ones, is that of line capacitance.

Capacitance, like resistance and inductance is distributed uniformly along the line. Each unit length of line has a definite current to charge the capacitance associated with it. This charging current leads the potential difference between line conductors at the particular point on the line by 90°.

Leading current flowing in the inductive reactance of the line causes the voltage to rise towards the end of the line. Therefore, the line capacitance has the effect of decreasing the line drop for a given load current.

H- Voltage Control of Transmission Lines .

Generally, alternating current transmission systems maintain a constant voltage at the receiver end. This is necessary, because approximately constant voltage is required at the distribution center at all loads.

(c) Voltage Control by Generator Exitation

Within practical operating limits the voltage may be kept constant at the receiver end by varying the exitation of the generator field. This is only practical on short lines, because the voltage difference required between full-load and no load on long lines becomes excessive.

(c) Line Regulation

The regulation of a transmission line is the change in voltage at the receiver end of the line between rated, non-inductive load and no load, with constant voltage impressed upon the supply end. The regulation is usually expressed as a percentage of the terminal rated voltage at the load, and, when so expressed, it is the percentage change in the receiver voltage with respect to its normal rated value.

(c) Constant -Voltage, Variable Power-factor Control,

This method of voltage control, commonly referred to as phase control, is commonly used in long lines, as well as on more important short lines.

The supply-end voltage and receiver-end voltage are both kept constant for all loads, but both receiving-end and supply-end power factors vary, as demanded by the load requirements. This is made possible by the use of synchronous reactors connected to the receiver end of the line in parallel with the load and the excitation being automatically controlled by the usual type of voltage regulators.

This method of control has the advantage of maintaining constant voltage at both supply-end and receiver end independent of the transformer characteristics and the variable drop in the line due to change of load.

This method has some disadvantages, When the synchronous reactors are used at the receiver end only, the voltage at any point on the line other than its ends varies with the load. This makes it impossible to have any constant voltage taps at any intermediate point on the line.

In long lines, if, during light loads, for any reason the synchronous reactors should get out of step and be disconnected from line, the voltage at the supply end would rise to dangerously high values. For this reason, long lines should be built with considerably higher factors of safety than for short lines.

For long straigth transmission lines it would be advantageous to connect one or more synchronous reactors at different points along the line. In this way points of constant voltage, at intervals along the line would avoid the danger of excessive voltages.

<u>Methods of Calculating Line Losses</u>

In a transmission line the construction is such that the electrical properties per unit length are practically constant. The properties are represented by $\underline{r}$, $\underline{g}$, $\underline{L}$ and $\underline{C}$, and are called the constants of the line.

Because these constants for a given transmission do not vary, the energy transfer follows laws which may be expressed in mathematical form.

The exact equations involving the correct assumption of uniform distributed line capacitance, although simple in form, involve considerable calculation. In order to simplify the calculations one of two expedients may be resorted to.

1. The use of only one or two terms of the infinite series used in the exact equations.

2. The substitution of hypothetical circuits having lumped capacitance which perform very near like the actual line.

The method under(1) will be shown later under the exact equations which one used in long line calculations. Under (2) a number of different approximations are used, depending principally in the method employed to approximate the line capacitance.

CHAPTER 11

SHORT LINE CALCULATIONS

Simple Impedence Circuits.

In this the line is assumed to have no capacitance. The circuit is then only a series circuit containing resistance and reactance. This hypothetical circuit will usually have a greater line drop, poorer power facter and larger generatorcurrent than the actual line. For lines up to 30 miles in length the error is negligable.

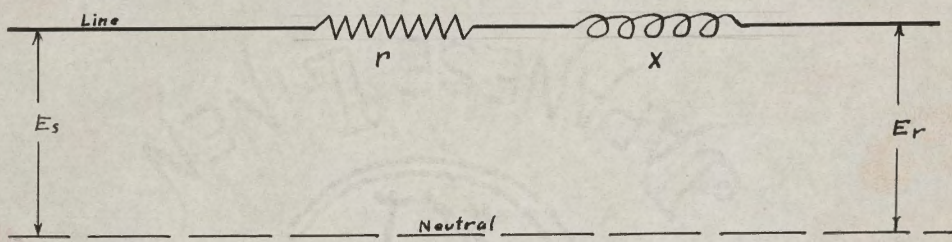


Fig. 1. Diagram of simple impedence circuit.

In figure 1 is represented one leg of a three phase line, over which one third of the power is to be transmitted. Assumptions are made that the receiver load, power factor, voltage and current are known. The supply end voltage is varied to keep the receiver voltage constant. The receiver voltage is used as vector of reference.

Assuming, since the conductance of the leakage path and the capacitance of the line are both negligably small, that the supply voltage and receiver voltage are equal.

Thus, assuming either leading or lagging receiver current, the line current is

$$I = I_1 \pm jI_2 \text{ vector amperes.*}$$

*(When double sign occurs upper is leading,- loweris lagging-, power factor.)

and the receiver power factor is

$$\theta_r = \pm \tan^{-1} \frac{I_2}{I_1}$$

The line impedence is

$$Z = r + jx \text{ vector ohms.}$$

Where r = total resistance of one conductor

x = total inductive reactance of one conductor

Impedence drop in line is

$$ZI = (r + jx)(I_1 \pm jI_2)$$

$$= I_1(r = jx) + I_2(\mp x \pm jr) \text{ vector volts.} \qquad (1.)$$

Supply-end voltage is equal to the receiver voltage plus the impedence drop.

$$E_s = E_r + ZI$$

$$= E_r + rI_1 + xI_2 + j(xI_1 \pm rI_2)$$

$$= {}_sE_1 + j\,{}_sE_2 \qquad (2.)$$

Where ${}_sE_1$ and ${}_sE_2$ are the real and quadrature components of the supply voltage, respectively.

The angle of supply voltage referred to receiver voltage is

$${}_e\theta_s = \tan^{-1} \frac{{}_sE_2}{{}_sE_1} \qquad (3.)$$

Supply power factor angle is

$$\theta_s = {}_e\theta_s - (\pm \theta_r)$$

$$= {}_e\theta_s \mp \theta_r \qquad (4.)$$

In terms of absolute values

$$E_s = \sqrt{{}_sE_1^2 + {}_sE_2^2}$$

$$= \sqrt{(E_r + rI_1 + xI_2)^2 + (xI_1 \pm rI_2)^2} \qquad (5.)$$

Percent line regulation

$$= \frac{100(E_s - E_r)}{E_r} \qquad (6.)$$

Supply output is

$$= E_s I \cos\theta_s \qquad (7.)$$

The receiver input

$$= E_r I \cos\theta_r \qquad (8.)$$

The line loss is

$$= E_s I \cos\theta_s - E_r I \cos\theta_r \qquad (9.)$$

or

$$\text{Loss} = \frac{I^2 r}{1000} \text{ KW.} \qquad (10.)$$

Nominal T Line.

In this assumption the line capacitance is assumed at the mid-point with one-half the impedence of the line on each side of the condenser as shown in Fig. 2.

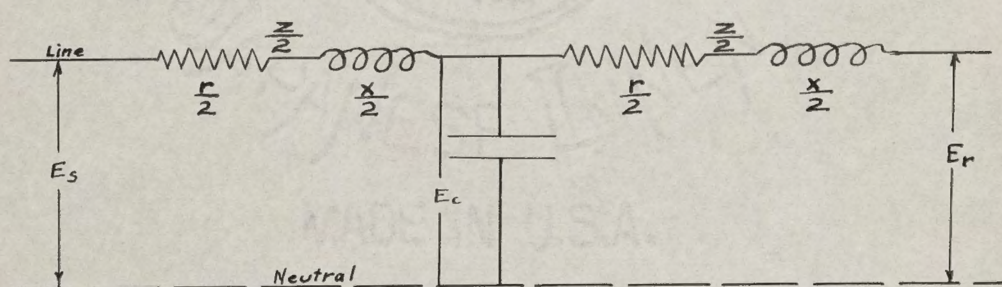


Fig. 2. Diagram of Nominal T Line.

The charging current required for the capacitance, flowing through one-half the impedence of the line, will cause a drop approximately equal to the actual line drop. Also, E_c, the voltage across the condenser is nearly equal to the average value for the voltage.

$$E_c = E_r + I_r \frac{Z}{2} \qquad (11.)$$

$$E_c = E_r + \tfrac{1}{2}(I_1 - jI_2)(r + jx) \qquad (12.)$$

Where "r" and "x" are the resistance and reactance of one conductor and ${}_cE_1$ and ${}_cE_2$ are the real and quadrature components of the potential difference E_c, respectively.

The size of the vector E_c is

$$E_c = \sqrt{{}_cE_1^2 + {}_cE_2^2} \quad \text{volts} \qquad (13.)$$

Now

$$E_s = E_c + I_s \frac{Z}{2} \qquad (14.)$$

From equation (11.)

$$E_s = (E_r + I_r \frac{Z}{2}) + I_s \frac{Z}{2} \qquad (15.)$$

$$= E_r + \frac{Z}{2}(I_r + I_s) \qquad (16.)$$

But

$$I_s = I_c + I_r \qquad (17.)$$

Also

$$I_r = E_r Y = YE_r + \frac{YZ}{2} I_r \qquad (18.)$$

Then

$$I_s = I_r + YE_r + \frac{YZ}{2} I_r \qquad (19.)$$

Now from equation (15.)

$$E_s = E_r + I_r \frac{Z}{2} + \frac{Z}{2} I_r + \frac{Z}{2} YE_r + \frac{Z}{2} Y \frac{Z}{2} I_r \qquad (20.)$$

$$= (1 + \frac{Z}{2} Y)E_r + (Z + \frac{Z^2}{4} Y) I_r$$

$$= YE_r + (1 + \frac{YZ}{2}) I_r \qquad (21.)$$

Nominal π Line.

In this assumption only one-half the line capacitance is put in parallel with the load, the other half being put from line to neutral at the generator end.

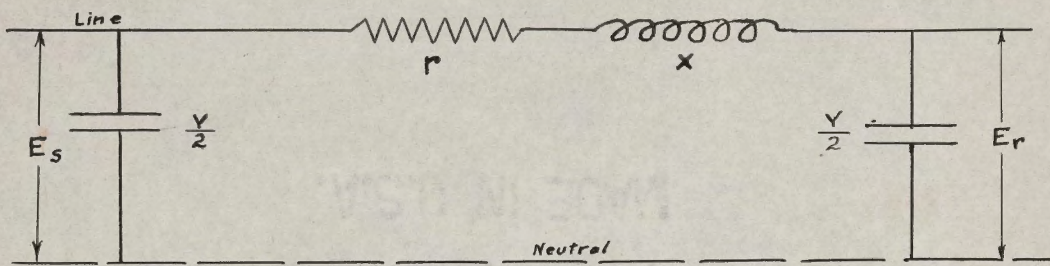


Fig. 3. Diagram of Nominal π Line.

From Fig. 3.

$$I_m = I_r + E_r \frac{Y}{2} \qquad (22.)$$

$$I_s = I_m + E_s \frac{Y}{2} \qquad (23.)$$

$$= I_r + E_r \frac{Y}{2} + E_s \frac{Y}{2}$$

$$= E_r \left(Y + Z \frac{Y^2}{4}\right) + I_r \left(1 + \frac{ZY}{2}\right) \qquad (24.)$$

$$E_s = E_r + ZI_m \qquad (25.)$$

$$= E_r + ZI_r + E_r \frac{ZY}{2}$$

$$= E_r \left(1 + \frac{ZY}{2}\right) + ZI_r \qquad (26.)$$

From the foregoing it may be seen that there is a general form for E_s and I_s as

$$E_s = A\,E_r + B\,I_r$$

and $$I_s = C\,E_r + D\,I_r$$

Also, it may be seen that

For D. C.	$A = 1$	$B = r$	$C = 0$	$D = 1$
For Impedence	$A = 1$	$B = Z$	$C = 0$	$D = 1$
For Nominal T	$A = 1 + \frac{ZY}{2}$	$B = Z + \frac{Z^2Y}{4}$	$C = Y$	$D = 1 + \frac{ZY}{2}$
For Nominal π	$A = 1 + \frac{ZY}{2}$	$B = Z$	$C = Y + \frac{ZY^2}{4}$	$D = 1 + \frac{ZY}{2}$

CHAPTER 111

HYPERBOLIC SINE AND COSINE

It may be shown that the functions, sine and cosine, are the sums of an infinite series of powers of "x". This series, called Maclaurin's series is as follows:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots\ldots\ldots\ldots \qquad (27.)$$

and

$$\text{cosine } x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots\ldots\ldots\ldots \qquad (28.)$$

Also it may be shown that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \ldots\ldots\ldots\ldots \qquad (29.)$$

$$e^{jx} = e^{\sqrt{-1}\,x} = 1 + jx - \frac{x^2}{2!} \ldots\ldots\ldots \qquad (30.)$$

$$= j\frac{x^3}{3!} + \frac{x^4}{4!} + j\frac{x^5}{5!} \ldots \qquad (30.)$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots\ldots\ldots\ldots \qquad (31.)$$

$$e^{-jx} = 1 - jx - \frac{x^2}{2!} + j\frac{x^3}{3!} + \frac{x^4}{4!} - j\frac{x^5}{5!} \ldots\ldots\ldots \qquad (32.)$$

Adding (29) and (31) and taking half their sum the equation for the hyperbolic cosine (cosh) is obtained.

$$\cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} \ldots\ldots\ldots \qquad (33.)$$

And by taking the difference between (29) and (31) the equation for the hyperbolic sine (sinh) is obtained.

$$\sinh x = \frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \ldots\ldots\ldots \qquad (34.)$$

Also

$$\cos x = \frac{e^{jx} - e^{-jx}}{2} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots\ldots\ldots\ldots \qquad (35.)$$

And

$$\sin x = \frac{e^{jx} - e^{-jx}}{2} = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots\cdots \qquad (36.)$$

This gives an expression for sin x , cos x , coshx, and sinh x in terms of " e ", the base for natural logarithms.

Also it may be shown that

$$\cosh(u + jv) = \tfrac{1}{2}\left(e^{u + jv} + e^{-u - jv}\right) \qquad (37.)$$

$$= \left(\frac{e^{u} + e^{-u}}{2} \cdot \frac{e^{jv} + e^{-jv}}{2}\right) + j\left(\frac{e^{u} - e^{-u}}{2} \cdot \frac{e^{jv} - e^{-jv}}{2j}\right)$$

Then

$$\cosh(u + jv) = \cosh u \cos v + j \sinh u \sin v \qquad (38.)$$

Also

$$\sinh(u + jv) = \frac{e^{u + jv} - e^{-u - jv}}{2} \qquad (39.)$$

$$= \left[\frac{e^{u} - e^{-u}}{2} \cdot \frac{e^{jv} + e^{-jv}}{2}\right] + j\left[\frac{e^{u} + e^{-u}}{2} \cdot \frac{e^{jv} - e^{-jv}}{2}\right] \qquad (40.)$$

Then

$$\sinh(u + jv) = \sinh u \cos v + \cosh u \sin v \qquad (41.)$$

CHAPTER IV

LONG LINE CALCULATIONS

Fundamental Differential Equations.

In these equations only one leg of transmission circuit is considered. All voltages are referred to neutral and line length is considered positive in direction from receiver to generator end.

At any point "p" at a distance "l" from receiver end let

I = vector current in line conductor.

E = vector potential difference from line to neutral.

Let the constants per mile of one line conductor be as follows:

r_1 = resistance of one conductor

x_1 = inductive reactance of one conductor

$z_1 = \sqrt{r_1^2 + x_1^2}$ = numerical impedence of the series circuit

$Z_1 = r_1 + jx_1$ = complex " " " " "

g_1 = conductance of shunt leakage path per conductor

b_1 = susceptance of shunt leakage

$y_1 = \sqrt{g_1^2 + b_1^2}$ = numerical admittance of the shunt leakage path per conductor

$Y_1 = g_1 + jb_1$ = complex admittance per conductor to neutral

Let vector voltage from line to neutral at point "l" from receiver with receiver voltage as vector of reference be

$$E = E_1 + jE_2 \text{ vector volts} \tag{42.}$$

If vector current for same point is

$$I = I_1 - jI_2 \text{ vector amperes}$$

then the drop dE due to the current flowing in an elemental length, dl, of line is

$$dE = (I_1 - jI_2)(r_1 + jx_1)dL$$

$$= (I_1r_1 + I_2x_1)dl + j(I_1x_2 - I_2r_1)dl \tag{43.}$$

and the potential difference from line to neutral at point, $l + dl$, miles from receiver is

$$E + dE = E_1 + (I_1r_1 + I_2x_1)dl + j\left[E_2 + (I_1x_1 + I_2r_1)dl\right] \text{ vector volts} \quad (44.)$$

The current in the leakage path of the element dl is

$$dI = (E_1 + jE_2)(g_1 + jb_1)dl \quad (45.)$$

$$= (E_1g_1 - E_2b_1)dl + j(E_1b_1 + E_2g_1)dl \text{ vector amps.}$$

Current in the line at a distance, $l + dl$, miles from the receiver is

$$I + dI = I_1 + (E_1g_1 - E_2b_1)dl - j\left[I_2 - (E_1b_1 + E_2g_1)dl\right]$$

Using the more general form of notation, the increments, of vector-voltage rise in the element in the series circuit, is

$$dE = Z_1Idl$$

$$\frac{dE}{dl} = Z_1I \quad (46.)$$

and the current, leaking to neutral in dl of line, is

$$dI = EY_1\, dl$$

$$\frac{dI}{dl} = EY_1 \quad (47.)$$

Equations (46) and (47) are differential equations of the circuit under consideration. Their solution will give the value of E or I for any point on the line.

Solution of Equations.

Differentiating (46) with respect to l

$$\frac{d^2E}{dl^2} = Z_1\frac{dI}{dl} \quad (48.)$$

Differentiating (47) with respect to l

$$\frac{d^2I}{dl^2} = Y_1\frac{dE}{dl} \quad (49.)$$

Substituting (47) in (48)

$$\frac{d^2E}{dl^2} = Z_1Y_1E \quad (50.)$$

Substituing (46) in(49)

$$\frac{d^2I}{dl^2} = Y_1Z_1I \qquad (51.)$$

It will be seen that (50) and (51) are alike in form and must therefore have the same general solution.

What ever differences appear in the values E and I will result from constants of intergration, i. e., from assumed terminal conditions

Multiplying both sides of equation (49) by $\frac{dI}{dl}$,

$$\frac{dI}{dl} \cdot \frac{d^2I}{dl^2} = Y_1Z_1I\,\frac{dI}{dl} \qquad (52.)$$

Intergrating

$$\frac{1}{2}\left(\frac{dI}{dl}\right)^2 = \frac{\overline{Y_1Z_1I}^{\,2}}{2} + \frac{C^2}{2} \qquad (53.)$$

$\frac{C^2}{2}$ being the constant of intergration.

Let $C = Y_1Z_1C_1^{\,2}$

Then from (53)

$$\frac{dI}{dl} = Y_1Z_1 \cdot \sqrt{I^2 + C_1^{\,2}} \qquad (54)$$

Or

$$\frac{dI}{\sqrt{I^2 + C_1^2}} = \sqrt{Y_1Z_1} \cdot dl \qquad (55.)$$

Equation (55) may be intergrated by making the following substituions:

Let
$I = C_1 \sinh u$

Then
$dI = C_1 \sqrt{\sinh^2 u + 1} = C_1 \cosh u$

Making these substitutions in (55) and solving for "u",

$$\int du = \sqrt{Y_1Z_1}\int dl + C_2$$

Or

$$u = Y_1Z_1\, l + C_2 \qquad (56)$$

And

$$I = C_1 \sinh\left(\sqrt{Y_1 Z_1}\cdot l + C_2\right) \tag{57.}$$

Differentiating (57)

$$\frac{dI}{dl} = \sqrt{Y_1 Z_1}\, C_1 \cosh\left(\sqrt{Y_1 Z_1}\; l + C_2\right) \tag{58.}$$

Substituting equation (47) in equation (58) and solving for E

$$E = \frac{Z_1}{Y_1}\, C_1 \cosh\left(\sqrt{Y_1 Z_1}\; l + C_2\right) \tag{59}$$

Equations (58) and (59) are the derired equations for E and I in which it remains to find the constants C_1 and C_2 from known terminal conditions.

In transmissions problems the current and voltage at the receiver end are known, or can readily be estimated. If "l" be taken as positive in direction from receiver to generator, the assumed known terminal conditions (for l = 0) are

$E = E_r$, the receiver vector voltage

$I = I_r$, the receiver vector current

Expanding (57)

$$I = C_1\left(\sinh\sqrt{Y_1 Z_1}\; l \cosh C_2 + \cosh\sqrt{Y_1 Z_1}\; l \sinh C_2\right) \tag{60}$$

Or, since

$I = I_r$ for $l = 0$

$I_r = C_1 \sinh C_2$

And

$$\sinh C_2 = \frac{I_r}{C_1} \tag{61}$$

Expanding (59)

$$E = C_1 \frac{Z_1}{Y_1}\left(\cosh\sqrt{Y_1 Z_1}\; l \cosh C_2 + \sinh\sqrt{Y_1 Z_1}\; l \sinh C_2\right) \tag{62}$$

Or, since

$E = E_r$ when $l = 0$

$$E_r = C_1\sqrt{\frac{Y_1}{Z_1}}$$

Or

$$\cosh C_2 = \frac{E_r}{\sqrt{\frac{Y_1}{Z_1}}} \tag{63}$$

Substituting equations (61) and (63) in equations (60) and (62)

$$E = E_r \cosh\sqrt{Y_1 Z_1}\cdot l + I_r \frac{Z_1}{Y_1} \sinh\sqrt{Y_1 Z_1}\cdot l \tag{64}$$

$$I = I_r \cosh\sqrt{Y_1 Z_1}\, l + E_r\sqrt{\frac{Y_1}{Z_1}} \sinh\sqrt{Y_1 Z_1}\, l \tag{65}$$

Since E_r and I_r, respectively, the voltage to neutral and the current in a line conductor at the receiver or load end of line, E and I of equations (64) and (65) are the corresponding quantities at any point on the line at a distance, l, from the receiver end.

If the impedence and admittance of the entire line be used instead of that for unit length, the substitutions below are required.

$Z = Z_1\, l$ = total impedence on one conductor.

$Y = Y_1\, l$ = total shunted admittance of one conductor.

Substituting the above in equations (64) and (65a)

$$E_s = E_r \cosh\sqrt{YZ} + I_r\sqrt{\frac{Z}{Y}} \sinh\sqrt{YZ} \tag{66}$$

$$I_s = I_r \cosh\sqrt{YZ} + E_r\sqrt{\frac{Y}{Z}} \sinh\sqrt{YZ} \tag{67}$$

Equations (66) and (67) are the general equations for the entire line in terms of the assumed known values of E_r and I_r.

Equations (66) and (67) may be somewhat simplified by writing

$$E_s = A\, E_r + B\, I_r$$

$$I_s = A\, I_r + C\, E_r$$

Where the constants A, B, and C are derived or auxillary line constants and are, from equations (66) and (67), equal to

$$A = \cosh\sqrt{YZ} \tag{68}$$

$$B = \sqrt{\frac{Z}{Y}} \sinh\sqrt{YZ} \tag{69}$$

$$C = \sqrt{\frac{Y}{Z}} \sinh \sqrt{YZ} \tag{70}$$

Or

$$A = a_1 + ja_2 \text{ , a complex number} \tag{71}$$

$$B = Z_o \sinh \sqrt{YZ}$$

$$= b_1 + jb_2\text{, a complex number} \tag{72}$$

$$C = Y_o \sinh \sqrt{YZ}$$

$$= c_1 + jc_2\text{, a complex number} \tag{73}$$

It may also be noted that

$$Z_o = \sqrt{\frac{Z}{Y}} \text{ a complex number, called the surge impedence of the line.} \tag{74}$$

$$Y_o = \sqrt{\frac{Y}{Z}} = \frac{1}{Z_o}\text{, a complex number , called the surge admittance of the line.} \tag{75}$$

And

$$\sqrt{YZ} = \theta$$

$$= \alpha + j\beta\text{, a complex number, called the complex angle of the line.} \tag{76}$$

But under Hyperbolic functions it was shown that , in equations (38) and (41) that

$$\cosh (\alpha + j\beta) = \cosh\alpha \cos\beta + j \sinh\alpha \sin\beta \tag{77}$$

and

$$\sinh (\alpha + j\beta) = \sinh\alpha \cos\beta + j \cosh\alpha \sin\beta \tag{78}$$

Then

$$A = \cosh\alpha \cos\beta + j \sinh\alpha \sin\beta \tag{79}$$

$$B = Z_o(\sinh\alpha \cos\beta + j \cosh\alpha \sin\beta) \tag{80}$$

$$C = Y_o(\sinh\alpha \cos\beta + j \cosh\alpha \sin\beta) \tag{81}$$

CHAPTER V

EQUIVALENT NETWORKS

When networks consist of various combinations of series and parallel groupings of their constituent elements, it is sometimes desireable to combine the elements into an equivalent network having only the usual generalized constants.

When a single line is considered the general equations are:

$$E_s = A\,E_r + B\,I_r$$

$$I_s = C\,E_r + D\,I_r$$

But, in the case of transformers in the line, or when line is connected to a second line with different characteristics, the constants involved rapidly become very complex and hard to handle. In general, equivalent constants, A_o, B_o, C_o, and D_o can always be found, such that the supply voltage and supply current are given in terms of the receiver voltage and receiver current and the equivalent constants, by the equations,

$$E_s = A_oE_r + B_oI_r$$

$$I_s = C_oE_r + D_oI_r$$

Taking , for instance, a network with transformers at receiver and supply end .

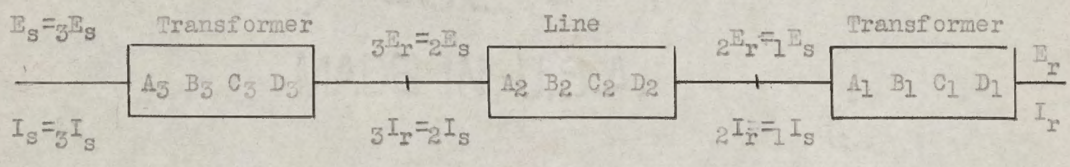


Fig. 4.- Diagram showing constants for a line with step-up and step-down transformers.

From figure 4 it is seen that

$${}_2E_r = {}_1E_s \;;\quad {}_2I_r = {}_1I_s \;;\quad {}_3E_r = {}_2E_s \;;\quad {}_3I_r = {}_2I_s \qquad (1.)$$

Then

$$ {}_1E_s = A_1 \; E_r + B_1 \; I_r $$
$$ {}_1I_s = C_1 \; E_r + D_1 \; I_r \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (2.) $$

And

$$ {}_2E_s = A_2 \; {}_2E_r + B_2 \; {}_2I_r $$

But from equations (1) and (2)

$$ {}_2E_s = A_2(A_1E_r + B_1I_r) + B_2(C_1E_r + D_1I_r) \qquad (3.) $$

$$ {}_2I_s = C_2 \; {}_2E_r + B_2 \; {}_2I_r $$
$$ = C_2(A_1E_r + B_1I_r) + B_2(C_1E_r + D_1I_r) \qquad (4.) $$

Also

$$ {}_3E_s = A_3 \; {}_3E_r + B_3 \; {}_3I_r $$
$$ = A_3\Big[A_2(A_1E_r + B_1I_r) + B_2(C_1E_r + D_1I_r)\Big] + $$
$$ B_3\Big[C_2(A_1E_r + B_1I_r) + D_2(C_1E_r + D_1I_r)\Big] \ldots\ldots \qquad (5.) $$

$$ {}_3I_s = C_3 \; {}_3E_r + D_3 \; {}_3I_r $$
$$ = C_3\Big[A_2(A_1E_r + B_1I_r) + D_2(C_1E_r + D_1I_r)\Big] + $$
$$ D_3\Big[C_2(A_1E_r + B_1I_r) + D_2(C_1E_r + D_1I_r)\Big] \ldots\ldots \qquad (6.) $$

But

$$ {}_3E_s = E_s \qquad \text{and} $$
$$ {}_3I_s = I_s $$

Then

$$ E_s = A_3A_2A_1E_r + A_3A_2B_1I_r + A_3B_2C_1E_r + A_3B_2D_1I_r + B_3C_2A_1E_r + $$
$$ B_3C_2B_1I_r + B_3D_2C_1E_r + B_3D_2D_1I_r \qquad (7.) $$

$$ = (A_3A_2A_1 + A_3B_2C_1 + B_3C_2A_1 + B_3D_2C_1)E_r + $$
$$ (A_3A_2B_1 + A_3B_2D_1 + B_3C_2B_1 + B_3D_2D_1)\; I_r \qquad (8.) $$

Or

$$E_s = A_oE_r + B_oI_r$$

$$I_s = C_3A_2A_1E_r + C_3A_2B_1I_r + C_3B_2C_1E_r + C_3B_2D_1I_r + D_3C_2A_1E_r +$$

$$D_3C_2B_1I_r + D_3D_2C_1E_r + D_3D_2D_1I_r$$

$$= (C_3A_2A_1 + C_3B_2C_1 + D_3C_2A_1 + D_3D_2C_1)E_r +$$

$$(C_3A_2B_1 + C_3B_2D_1 + D_3C_2B_1 + D_3D_2D_1)I_r \qquad (9.)$$

$$= C_oE_r + D_oI_r$$

From the above it is seen that, for any network, it is possible to get the equations for supply voltage and supply current in terms of receiver voltage and receiver current times some general constants, as in equations (8) and (9).

Now take the case of a line with step-up and step-down transformers, in which the magnetizing current and losses in the transformers are neglected.

In this case the equivalent transformer circuit becomes a simple impedence circuit , as shown in figire 5.

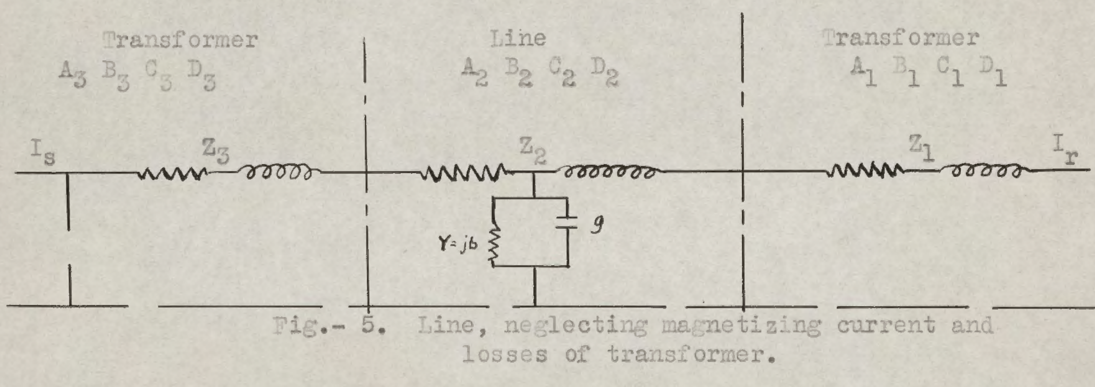


Fig.- 5. Line, neglecting magnetizing current and losses of transformer.

In simple impednce circuit

$$E_s = A_oE_r + B_oI_r = E_r + ZI_r$$

$$I_s = C_oE_r + D_oI_r = I_r + 0$$

Then

$$A_o = 1 \;; \quad B_o = Z \;; C_o = 0 \;; \quad D_o = 1 . \tag{10}$$

The total equivalent circuit consists of the transmissionn line itself, plus the series impedence, Z_r, at the receiver end , plus the series impedence, Z_s, at the supply end, where Z_r and Z_s are the impedences of the step-up and step-down transformers, respectively.

Following the proceedure as given at the first of this chapter:

For first section

$$\begin{aligned} E_2 &= A_1E_r + B_1I_r \\ I_2 &= C_1E_r + D_1I_r \end{aligned} \tag{11.}$$

For second section

$$\begin{aligned} E_3 &= A_2E_2 + B_2I_2 \\ I_3 &= C_2E_2 + D_2I_2 \end{aligned} \tag{12.}$$

For third section

$$\begin{aligned} E_S &= A_3E_3 + B_3I_3 \\ I_s &= C_3E_3 + D_3I_3 \end{aligned} \tag{13.}$$

From equations (8) and (9)

$$\begin{aligned} A_0 &= A_3(A_1A_2 + C_1B_2) + B_3(A_1C_2 + C_1D_2) \\ B_o &= A_3(B_1A_2 + D_1B_2) + B_3(B_1C_2 + D_1D_2) \\ C_o &= C_3(A_1A_2 + C_1B_2) + D_3(A_1C_2 + C_1D_2) \\ D_o &= C_3(B_1A_2 + D_1B_2) + D_3(B_1C_2 + D_1D_2) \end{aligned} \tag{14.}$$

From the preceeding results in equation (10) with subscipts(1) and (3) denoting transformers:

$$\begin{aligned} A_1 &= A_3 = 1 \\ B_1 &= Z_r \\ B_3 &= Z_s \\ C_1 &= C_3 = 0 \\ D_1 &= D_3 = 1 \end{aligned} \tag{15}$$

The constants carrying subscript (2) are of the line itself. Substituting equation (6) in (5)

$$\begin{aligned} A_o &= A + C\,Z_s \\ B_o &= B + A(Z_r + Z_a) + C\,Z_r Z_s \\ C_o &= C \\ D_o &= A + CZ_r \end{aligned} \tag{16}$$

CHAPTER VI

CIRCLE DIAGRAMS

Power Circles - Receiver.

Fundamental equations. *

$$E_s = A E_r + B I_r \tag{1}$$

$$E_r = D E_r - B I_r \tag{2}$$

Conjugate functions. *

$$\overline{E}_s = \overline{A}\,\overline{E}_r + \overline{B}\,\overline{I}_r \tag{3}$$

$$\overline{E}_r = \overline{D}\,\overline{E}_r - \overline{B}\,\overline{I}_r \tag{4}$$

Equations (1) to (4) are for single phase lines, or line to neutral conditions of polyphase lines. All voltages are in volts and current in amperes.

Multiplying (1) by (3)

$$E_s\overline{E}_s = (A E_r + B I_r)(\overline{A}\,\overline{E}_r + \overline{B}\,\overline{I}_r) \tag{5}$$

$$= A\overline{A} E_r\overline{E}_r + \overline{A} B \overline{E}_r I_r + A\overline{B} E_r\overline{I}_r + \overline{B} B I_r\overline{I}_r$$

Multiplying through by $E_r\overline{E}_r$

$$E_s\overline{E}_s E_r\overline{E}_r = A\overline{A}(E_r\overline{E}_r)^2 + (\overline{A} B \overline{E}_r I_r + A\overline{B} E_r\overline{E}_r)E_r\overline{E}_r + B\overline{B} E_r\overline{I}_r E_r I_r$$

But from the relation of conjugate functions this equation becomes:

$$E_s^2E_r^2 = A^2E_r^4 + \left[\overline{A} B (P_r + jQ_r) + A\overline{B}(P_r - jQ_r)\right]E_r^2 + B^2(P_r + jQ_r)(P_r - jQ_r)$$

$$= A^2E_r^4 + (\overline{A}B + A\overline{B})P_rE_r^2 - j(A\overline{B} - \overline{A}B)Q_rE_r^2 + B^2(P_r^2 + Q_r^2)$$

Dividing through by B^2 and rearranging:

$$P_R^2 + \frac{\overline{A}B + A\overline{B}}{B^2} P_rE_r^2 + Q_r - j\frac{A\overline{B} - \overline{A}B}{B^2} Q\,E = \frac{E_s^2E_r^2}{B^2} - \frac{A^2}{B^2}E_r^4 \tag{6}$$

The above equation may be made a complete quadratic by the addition on both sides of the following quantities:

$$\left(\frac{\underset{\cdot}{A}\,\underset{\cdot}{\overline{B}} + \underset{\cdot}{\overline{A}}\,\underset{\cdot}{B}}{2B^2}\,E_r^2\right)^2 + \left(j\frac{A\,B - A\,B}{2B^2}\right)^2 =$$

$$= 4\,\underset{\cdot}{A}\,\underset{\cdot}{\overline{B}}\,\underset{\cdot}{\overline{A}}\,\underset{\cdot}{B}\,\frac{E_r^2}{4B^4}$$

$$= \frac{A^2B^2E_r^4}{B^4} = \frac{A^2}{B^2}E_r^4 \qquad (7)$$

From which equation (6) becomes:

$$\left(P + \frac{\underset{\cdot}{A}\,\underset{\cdot}{\overline{B}} + \underset{\cdot}{\overline{A}}\,\underset{\cdot}{B}}{2B^2}\,E_r^2\right)^2 + \left(Q_r - j\,\frac{\underset{\cdot}{A}\,\underset{\cdot}{\overline{B}} - \underset{\cdot}{\overline{A}}\,\underset{\cdot}{B}}{2B^2}\right)^2 = \frac{E_s^2E_r^2}{B^2} \qquad (8)$$

Equation (8) may be written thus:

$$\left(P_r + l\,E_r^2\right)^2 + \left(Q_r - mE_r^2\right)^2 = n\,E_s^2E_r^2 \qquad (9)$$

Where

$$l = \frac{\underset{\cdot}{A}\,\underset{\cdot}{\overline{B}} + \underset{\cdot}{\overline{A}}\,\underset{\cdot}{B}}{2\,B^2}$$

$$m = j\,\frac{\underset{\cdot}{A}\,\underset{\cdot}{\overline{B}} - \underset{\cdot}{\overline{A}}\,\underset{\cdot}{B}}{2\,B^2} \qquad (10)$$

$$n = \frac{1}{B}$$

In the above P_r and Q_r are per phase, P_r being in watts and Q_r being in volt- amperes.

Equation (9) is that of a circle with center at co-ordinates, $-l\,E_r^2$ and $+m\,E_r^2$, and of radius $n\,E_sE_r$.

To evaluate l, m, n:

$$\underset{\cdot}{A} = a_1 + ja_2 \quad ; \quad \underset{\cdot}{\overline{A}} = a_1 - j\,a_2$$

$$\underset{\cdot}{\overline{B}} = b_1 - jb_2 \quad ; \quad \underset{\cdot}{B} = b_1 + jb_2 \tag{11}$$

$$\underset{\cdot}{A}\,\underset{\cdot}{\overline{B}} + \underset{\cdot}{\overline{A}}\,\underset{\cdot}{B} = 2(a_1b_1 + a_2b_2) \tag{12}$$

$$B^2 = b_1^2 + b_2^2 \tag{13}$$

Therefore

$$l = \frac{a_1b_1 + a_2b_2}{b_1^2 + b_2^2} \tag{14}$$

It may also be shown that

$$j(\underset{\cdot}{A}\,\underset{\cdot}{\overline{B}} - \underset{\cdot}{\overline{A}}\,\underset{\cdot}{B}) = -2(a_2b_1 - a_1b_2)$$

Therefore

$$m = \frac{a_1b_2 - a_2b_1}{b_1^2 + b_2^2} \tag{15}$$

And

$$n = \frac{1}{B} = \frac{1}{\sqrt{b_1^2 + b_2^2}} \tag{16}$$

Power Circles -- Sending.

Multiplying (2) by (4)

$$E_r\bar{E}_r = (DE_s - BI_s)\,(\bar{D}\bar{E}_s - \bar{B}\bar{I}_s) \tag{17}$$

$$= D\,\bar{D}\,E_s\,\bar{E}_s - BDE_sI_s - B\bar{D}\bar{E}_sI_s + B\bar{B}I_s\bar{I}_s$$

Multiplying through by $E_s\bar{E}_s$

$$E_r^2E_s^2 = D^2E_s^4 - \left[\bar{B}D(P_s - jQ_s)\right]E_s^2 - \left[B\bar{D}(P_s + jQ_s)\right]E_s^2 +$$

$$B^2(P_s + jQ_s)\,(P_s - jQ_s)$$

Dividing through by B^2 and rearranging:

$$P_s^2 - \frac{\bar{B}D - B\bar{D}}{B^2}P_sE_s^2 + Q_s^2 + j\,\frac{\bar{B}D \ast B\bar{D}}{B^2}Q_sE_s^2 = \frac{E_r^2\,E_s^2}{B^2} - \frac{D^2E_s^4}{B^2} \tag{18}$$

To complete the squares add the following to both sides of (18)

$$\left(\frac{\bar{B}D + B\bar{D}}{2B^2}E_s^2\right)^2 + \left(j\frac{\bar{B}D \ast B\bar{D}}{2B^2}E_s^2\right)^2 = \frac{D^2}{B^2}E_s^4$$

Therefore (18) becomes:

$$\left(P_s - \frac{BD + BD}{2B^2}E_s^2\right)^2 + \left(Q_s + j\,\frac{BD \ast BD}{2B^2}E_s^2\right)^2 = \frac{E_s^2\,E_r^2}{B^2} \tag{19}$$

Equation (19) may be written:

$$(P_s - l_1E_s^2)^2 + (Q_s + m_1E_s^2)^2 = n^2E_s^2E_r^2 \tag{20}$$

Where

$$l_1 = \frac{\bar{B}D + B\bar{D}}{2B^2} = \frac{d_1b_1 + d_2b_2}{b_1^2 + b_2^2} \tag{21}$$

and

$$m_1 = \frac{\bar{B}D - B\bar{D}}{2B^2} = \frac{d_1b_1 - d_2b_2}{b_1^2 + b_2^2} \tag{22}$$

Note:

Equations (21) and (22) are similar to equations (14) and (15) the constant A being replaced by D. Also, $n = n_1$

CHAPTER VII

CALCULATIONS

A transmission line was assumed with the following characteristics;

Length of line - - - - - - - 400 miles

Voltage - - - - - - - - - - 220,000 volts

Power factor of load - - - - 80 %

Transformers at both ends -- 220,000/11,000

For transformers

125,000 KW. or 156,000 Kva. at 80 % p.f.

which requires 52,000 Kva. transformers.

Spacing - The clearances which must be provided between the conductor of a line are determined largely by electrical safety. Sufficient space to allow a reasonable factor of safety againt spark-over between conductors should be provided.

$$d = 20 - \frac{Kv}{1.4} - \frac{(l)^2}{(120)^2} \quad *$$

Assuming l (length of spaces) as an average of 1000 ft.

$$d \text{ (distance in inches between conductors)} = 20 + \frac{220}{1.4}\left(\frac{1000}{120}\right)^2$$

$$= 246.5'' \text{ or } 20.5 \text{ feet.}$$

* Still - - "Electrical Power Transmission," pp. -232.

Size of Conductors - The conductor size is usually fixed, in high voltage transmission lines, by the voltage at which corona appears, rather than by I^2R losses.

In a conductor the critical disruptive voltage is:

$$E_0 = 123\, m\theta\, r \log \frac{D}{r}\left(1 - \frac{.189}{\sqrt{\theta r}}\right)^{*} \text{ to neutral}$$

Where m = .87 (roughness factor for stranded cable)

θ = Air density factor (taken as 1 at 77°F. - 29.9"Hg.)

r = Radius of conductor in inches

D = Spacing of conductors in inches

The, taking r = .5"

$$E = 123 \times 0.87 \times 0.5 \log \frac{246.5}{0.5}\left(1 - \frac{0.189}{\sqrt{0.5}}\right)$$

$$= 183 \text{ Kv.}$$

$$\frac{220{,}000}{1.73} = 127, \text{ Kv. per phase}$$

Then a one inch conductor is large enough to prevent corona at 220,000 volts on the line.

Characteristics of conductor used ***

Circular Mils	No. Strands Steel	: Al.	Resistance	Reactance
518,000	19	42	.180 ohms / mile	.755 ohms/miles

*"Electrical Power Transmission," by E. A. Loew, pp.95,

*** "Electrical Characteristics of Transmission Circuits" by Westinghouse Engineers.

From "Electrical Characteristics of Transmission Circuits"

Susceptance to neutral ----b = 9.06×10^{-6} mhos

In transformers

Resistance drop in percent ---------- 0.3 %

Reactance drop in percent ----------- 7.0 %

Resistance of transformers

$$\frac{.003 \times 127{,}000}{412.5} = .925 \text{ ohms resistance}$$

Reactance of transformers

$$\frac{.07 \times 127{,}000}{412.5} = 21.55 \text{ ohms reactance}$$

Referring to Chapter lV and using notations as given there:

$r_1 = .180$ ohms per miles $\qquad g_1 = 0$

$x_1 = .755$ ohms per mile $\qquad b_1 = 9.06 \times 10^{-6}$ mhos per mile

Unit Angle.

$$\theta = \sqrt{Y_1 Z_1}$$

$$Z_1 = .180 + j.755 \text{ vector ohms}$$

polar

$$Z_1 = z_1 \underline{/\tan^{-1} \frac{x_1}{r_1}}$$

$$= .783 \underline{/76^\circ\ 34.6'}$$

Similarly

$$Y_1 = 9.06 \times 10^{-6} \underline{/90^\circ\ 00.0'}$$

then

$$Y_1 Z_1 = 9.06 \times 10^{-6} \times .783 \underline{/76^\circ\ 34.6' + 90^\circ}$$

and

$$\alpha_1 + j\beta_1 = \sqrt{Y_1 Z_1}$$

$$= .002661 \underline{/83^\circ\ 17.3'}$$

In rectangular co-ordinates:

$$\alpha_1 + j\beta_1 = .002661 \left[\cos(83^\circ\ 17.3') + j\sin(83^\circ 17.3')\right]$$

$$= .0003108 + j\ .002641$$

or

$$\alpha_1 = .0003108$$

$$\beta_1 = .002641$$

then

$$\alpha = \alpha_1 l = .12432 \quad \text{hyperbolic radians}$$

$$\beta = \beta_1 l = 1.0564 \quad \text{circular radians}$$

Surge Impedence

$$\frac{Z_1}{Y_1} = \frac{.783\ \angle 76^\circ\ 34.6'}{9.06 \times 10^{-6}\ \angle 90^\circ\ 00.0'}$$

$$= 81,430 \quad -13^\circ\ 25.4'$$

$$Z_o = \sqrt{\frac{Z_1}{Y_1}} = \sqrt{81,430}\ \angle -\tfrac{1}{2}\ (13^\circ\ 25.4)$$

$$= 285.3 \quad -6^\circ\ 42.7'$$

in rectangular co-ordinates:

$$Z_o = 285.3 \left[\cos(6^\circ\ 42.7') - j\sin(6^\circ\ 42.7')\right]$$

$$= 283.1 - j\ 33.29$$

The surge admittance is:

$$Y_o = \frac{1}{Z_o} = \frac{1}{285.3}\ \angle 6^\circ\ 42.7'$$

$$= .0035\ \angle 6^\circ\ 42.7'$$

in rectangular co-ordinates:

$$Y_o = .0035 \left[\cos(6^\circ\ 42.7') + j\ \sin(6^\circ\ 42.7')\right]$$

$$= .00348 + j\ .000409$$

Constants A, B, C.

From equations (79), (80) and(81) Chapter lV.

$A = \cosh\alpha \cos\beta + j \sinh\alpha \sin\beta$

$B = Z_o(\sinh\alpha \cos\beta + j \cosh\alpha \sin\beta)$

$C = Y_o(\sinh\alpha \cos\beta + j \cosh\alpha \sin\beta)$

$\cosh\alpha = \cosh .12432 = 1.0076$

$\sinh\alpha = \sinh .12432 = .1245$

$\cos\beta = \cos(1.0564 \times 57.2958)^\circ$

$= \cos 60.55^\circ = .4916$

$\sin\beta = .8708$

Then

$A = 1.0076 \times .4916 + j\ .1245 \times .8708$

$= .4948 + j\ .1084$

$a_1 = .4948$

$a_2 = .1084$

$B = (283.1 - j\ 33.29)(.1245 \times .4916 + j\ 1.0076 \times .8708)$

$= 45.85 + j\ 246.435$

$b_1 = 46.53$

$b_2 = 246.435$

$C = (.00349 + j\ .000409)(.1245 \times .4916 + j\ 1.0076 \times .8708)$

$= -.0001536 + j\ .003076$

$c_1 = -.0001536$

$c_2 = .003076$

Referring to equations (16) and keeping in mind that, with similar transformers at receiver and supply end,

$$Z_r = Z_s \qquad Z_r + Z_s = 2Z_s \qquad Z_r Z_s = Z_s^2$$

and $\quad A_o = D_o$

it is seen that

$A_o = .46831 + j\ .1089$

$a_1 = .46831$

$a_2 = .1089$

$B_o = 41.909 + j\ 265.033$

$b_1 = 41.909$

$b_2 = 265.033$

$C_o = -.0001536 + j\ .003076$

$c_1 = -.0001536$

$c_2 = .003076$

$D_o = .46831 + j.1089$

$d_1 = .46831$

$d_2 = .1089$

Referring to Chapter VI, equations (14), (15) and (16)

$$l = \frac{a_1 b_1 \quad a_2 b_2}{b_1^2 \quad b_2^2}$$

Taking values from above:

$l = .00064$

Similarly

$m = .00156$

$n = .00371$

It will be seen that, with equal impedense at both ends of the line, which is the case when similar transformers are used, the constants l_1 , m_1, n_1 for the sendind power circle will be equal to the constants l, m, n, for the receiver power circles.

These constants, as derived, were used to construct the power circles shown in the appendix.

CHAPTER VIII

CONCLUSIONS

The question of power limits of a transmission line is a very complex one and will not be gone into here. It might be defined, however. "The power limit of a transmission line is not fixed by the line alone, but depends to some extent upon the nature and characteristics of every piece of apparatus connected to it. It is reached at the point where, for what'ever reason, the synchronous apparatus at the two ends of the link break out of step. This may happen for either one of two reasons, namely, (a) because, due to gradually applied loads, the angle of displacement between the excitation voltages at the two ends of the line has reached its maximum possible value, or, (b) because the same angle has been reached due to changes in load resulting from a short circuit, the switching in of load, or other cause." **

In a line the length of the one which has been dealt with in this work, it will be seen that the power limit is not one which is determined from either of the above, but will be determined by the economical limit. It will be seen fromm the circle diagram in the appendix of this thesis that, with any increase in load, the synchronous Kv-a. capacity is rapidly increased. For a load greater than 40,000 KW. per phase the required synchronous reactive Kv-a. would more than offset the saving due to increase in efficiency.

** "Electrical Power Transmission" -- E. A. Loew.

APPENDIX

CIRCLE DIAGRAM OF 220 Kv. LINE

Length of line - - - - - - - - - - - - - -	400 miles
Line Voltage at Receiver End - - - - - -	220,000
Transformers Used	
Rated at - - - - - - - - - - - -	52,000 Kva.
Resistance drop - - - - - - - - -	0.3 %
Reactance drop - - -- - - - - - -	7.0%
Conductors	
Three - - - - - - - - - - - - - - -	518,000 C. M.
Steel-reinforced aluminum	
Diameter - - - - - - - - - - - -	1 inch
Spacing - - - - - - - - - - - - -	20.5 feet eqio;atera;

From the data obtained under "Calculations", a circle diagram was drawn with centers at $-\ lE_r^2$ and $-\ mE_r^2$ for the receiving circles, and at $l_1E_r^2$ and $-m_1E_r^2$ for sending circles.

With the receiver voltage constant at 220,000volts, circles were drawn with E_s equal to 240,000 volts, 220,000 volts, and 200,000 volts.

For a given power out-put at receiver end a line is drawn through the center of the receiver circle.

The angles, 0, that this line makes with the line laid off in the direction of E_r is transferred to the sending circle.

Where the line OC cuts the sending circle locates the ordinate giving sending power.

The angles , and give the sending and receiving power factors, respectively.

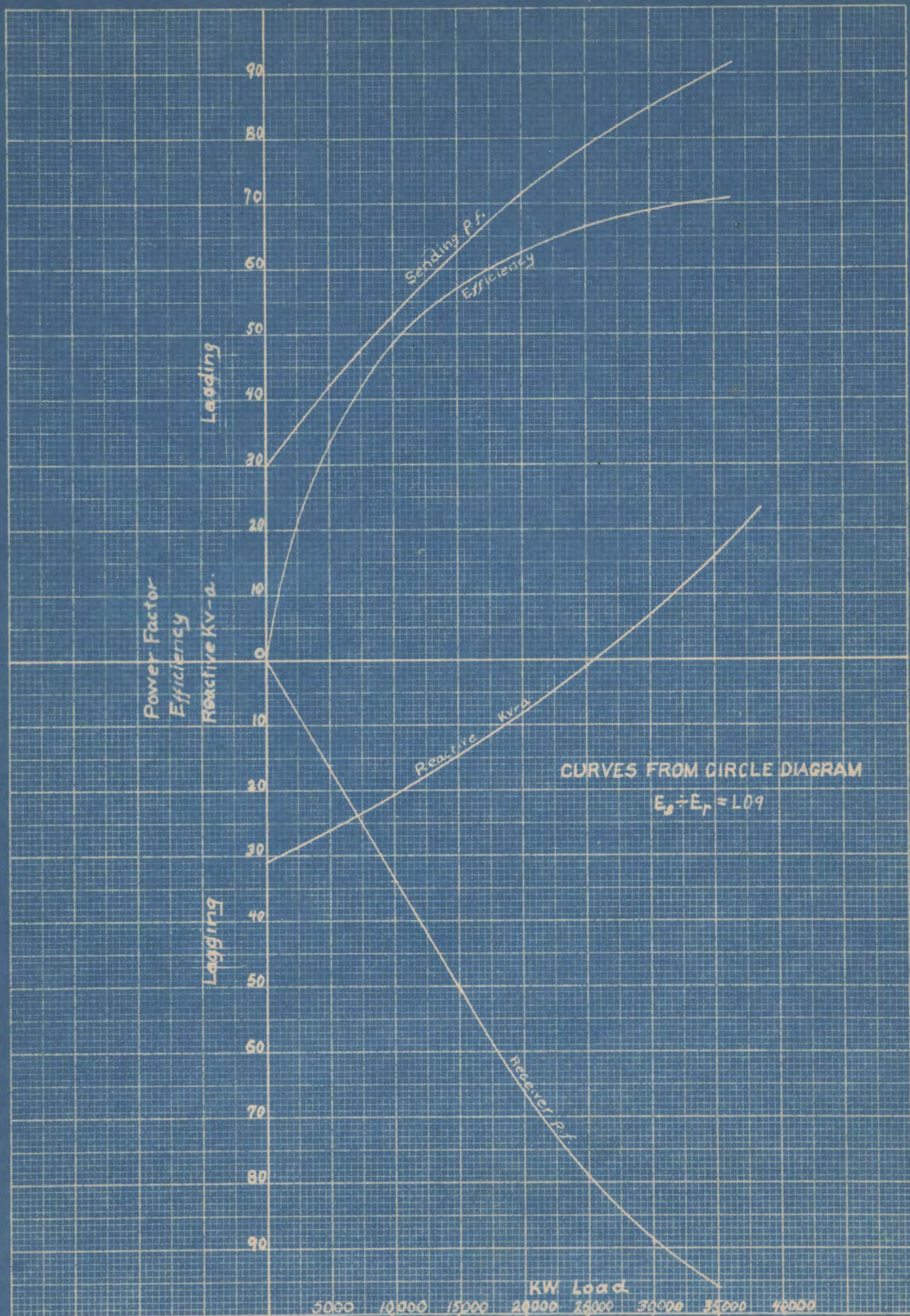
Kva. required at the receiver end in the form of a synchronous condenser.

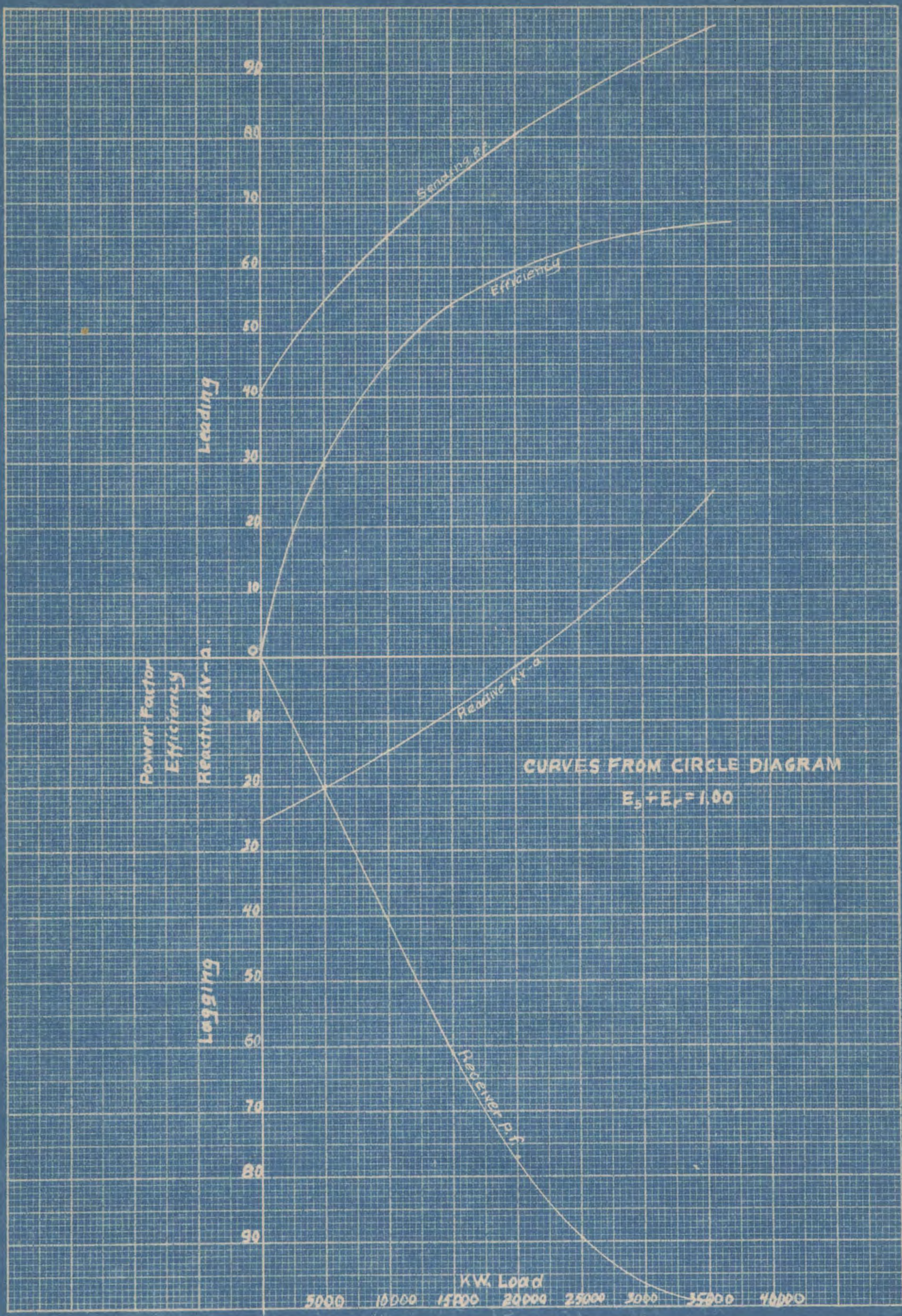
The efficiency of the line is given by the ratio of receiver power to sending power.

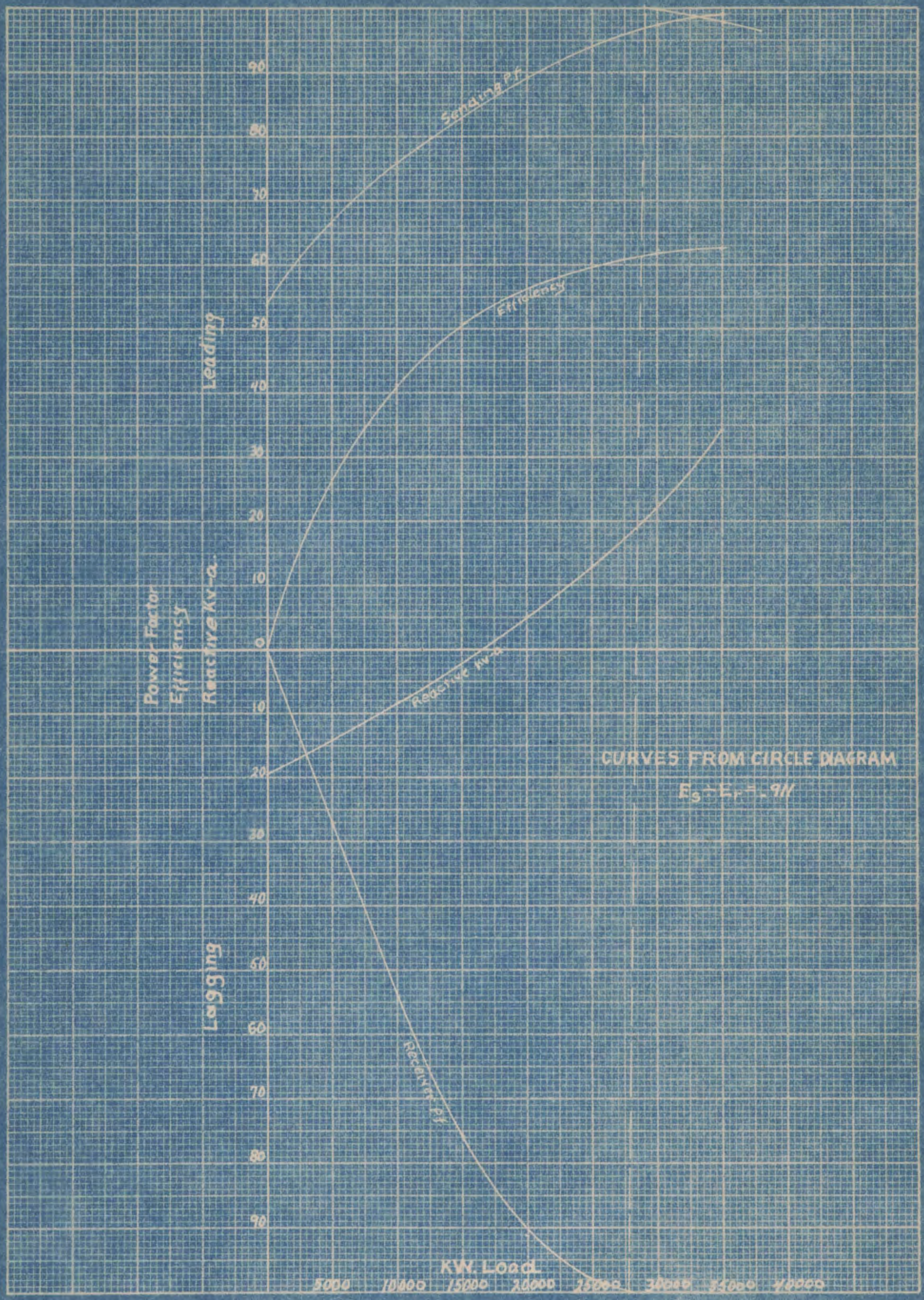
With receiver power as abscissa, curves were plotted with required synchronous condenser, receiver power factor, sending power factor and efficiency as ordinates.

POWER CIRCLES

CURVES TAKEN FROM POWER CIRCLES


CURVES FROM CIRCLE DIAGRAM
$E_s \div E_r = 1.09$
Sending P.F.
Efficiency
Reactive Kv-a
Receiver P.F.
Power Factor
Efficiency
Reactive Kv-a
Leading
Lagging
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
KW Load
5000
10,000
15000
20000
25000
30000
35000
40000


CURVES FROM CIRCLE DIAGRAM
$E_s \div E_r = 1.00$
Sending P.F.
Efficiency
Reactive Kv-a
Receiver P.F.
Power Factor
Efficiency
Reactive Kv-a.
Leading
Lagging
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
KW. Load
5000
10000
15000
20000
25000
3000
35000
40000


CURVES FROM CIRCLE DIAGRAM
$E_s \div E_r = .911$
Sending P.F.
Efficiency
Reactive Kv-a.
Receiving P.F.
Leading
Lagging
Power Factor
Efficiency
Reactive Kv-a.
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
KW. Load
5000
10000
15000
20000
25000
30000
35000
40000

BIBLIOGRAPHY

Transmission Line Formulas
by
Herbert B. Dwight

Transmission Line Theory
by
William S. Franklin
and
Frederick E. Terman

Electrical Characteristics of Transmission Circuits
by
Westinghouse Engineers

Electrical Power Transmission
by
E. A. Loew

Circle Diagrams
from
Electric Journal Dec. 1921, pp.- 530

M. W. Smith - Water Wheel Generators and Synchronous Condensers for Long Transmission Lines.
from
Transactions A. I. E. E. Vol. XLII, '23

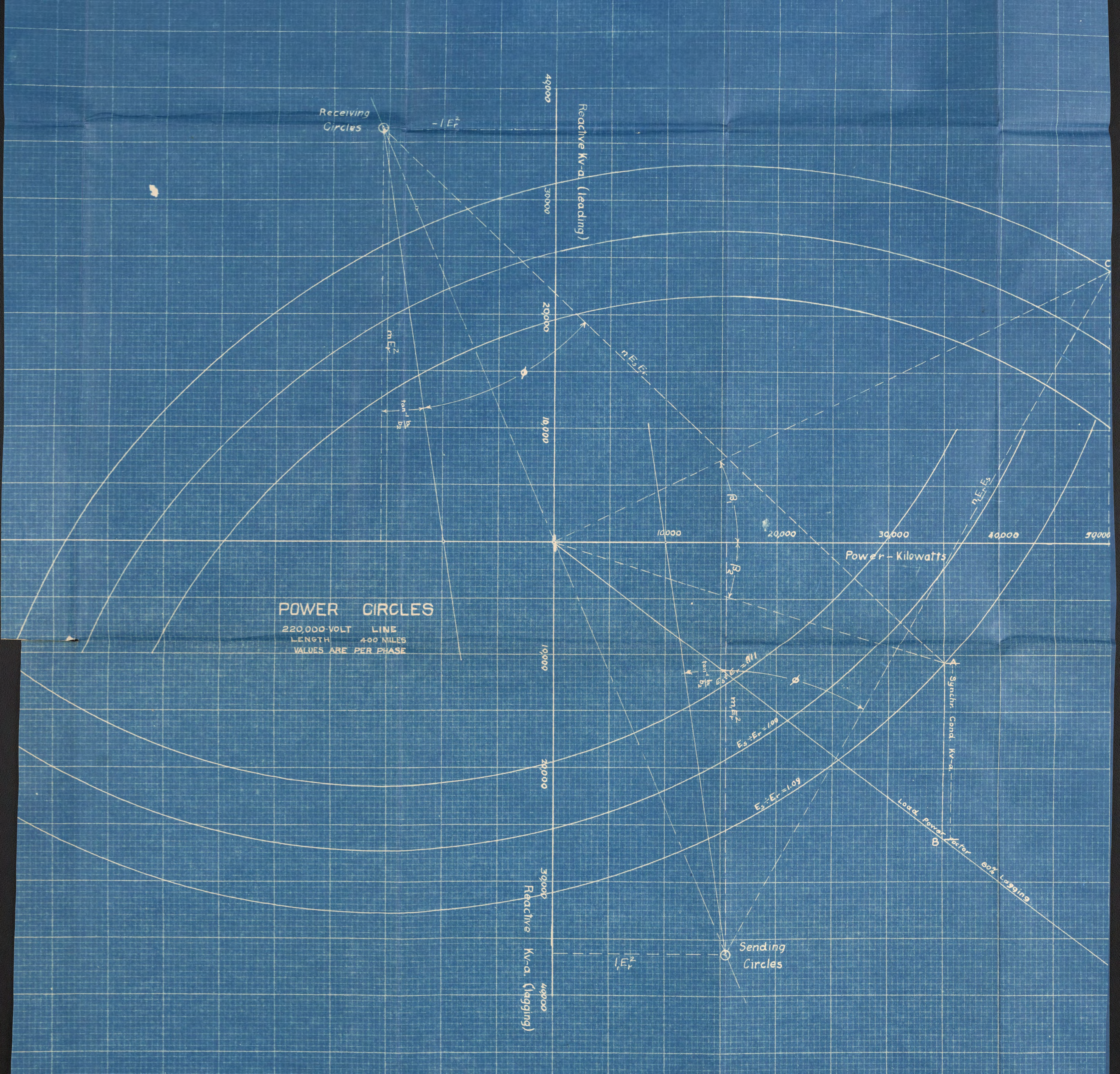
C. L. Fortescue and C. F. Wagner - Some Theoretical Considerations of Power Transmission Systems.
from
Transactions A. I. E. E. Vol. XLIII, '24

APPROVAL

The foregoing thesis is hereby approved as a creditable study of an engineering subject, carried out and presented in a manner sufficiently satisfactory to warrant its acceptance as a prerequisite to the degree for which it has been submitted. It is to be understood that by this approval the undersigned does not necessarily endorse or approve any statement made, opinions expressed, or conclusions drawn therein, but approves the thesis only for the purpose for which it is submitted.

Name J. S. Triboux

Title Professor of Elec Eng.



Receiving Circles
-lEr²
Reactive Kv-a (leading)
40000
30000
20000
10000
mEr²
nEsEr
Power - Kilowatts
10,000
20,000
30,000
40,000
50,000
POWER CIRCLES
220,000-VOLT LINE
LENGTH 400 MILES
VALUES ARE PER PHASE
Es÷Er=1.11
Es÷Er=1.00
Es÷Er=1.09
Synchr. Cond. Kv-a.
A
B
Load Power Factor 80% Lagging
Sending Circles
lEr²
Reactive Kv-a (lagging)