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I am submitting herewith a thesis written by Mark Hastings Neville entitled "A Sensitivity Analysis of a Nulling Network Used on an Inductance Type Lubricating Film Thickness Probe." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

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A SENSITIVITY ANALYSIS OF A NULLING NETWORK USED ON AN INDUCTANCE TYPE LUBRICATING FILM THICKNESS PROBE

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Mark Hastings Neville

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ABSTRACT

This thesis investigates the application of classical sensitivity to the drift problems encountered in inductive film thickness probes used on dynamic seals. Of primary interest were the sensitivities of the measurement system to component values of the nulling network.

The sensitivities of the circuit model were found in two different ways. The first method was to derive the sensitivity functions from the transfer function by taking derivatives. The second method was to use a numerical technique to compute the values of the sensitivities. The sensitivity functions were evaluated and compared to the ones found by numerical techniques. The sensitivities of the null network components was measured by making small changes to these parameters and observing the output change. Techniques for finding probe model parameters by fitting techniques were developed. Computer programs were written to support model parameter estimation and sensitivity evaluation.

The derived sensitivity functions were found to yield values consistant with those found by numerical techniques. Measured values of sensitivity were significantly different from the calculated values. The fitting techniques used to find model component values give reasonable results.

Derivation of sensitivity functions for circuits of this complexity was found not to be useful enough to justify the

effort required to arrive at them. Numerical techniques can yield as much information and require much less effort to arrive at the values. Numerical techniques are significantly less prone to error. Although the sensitivity functions work well with the model used, evidence indicates that a more complex model must be used to account for the deviations from the measured values. The definition of classical sensitivity does not lend itself well to circuits which can have zero output. A simple modification to this definition will yield a significant improvement in the insight derived from the resulting sensitivity values.

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LIST OF SYMBOLS

SYMBOL	DEFINITION
C_1	The equivalent capacitance shunting the drive coil number 1.
C_2	The equivalent capacitance shunting the drive coil number 2.
C_9	Coil shunt capacitance
H	L^2
i	Current through the inductor
L	Coil leakage inductance.
L_1	The inductance of drive coil number one.
L_2	The inductance of drive coil number two.
L_3	The inductance of drive coil number three.
L_4	The inductance of drive coil number four.
M_{12}	The mutual inductance between coils one and two.
M_{13}	The mutual inductance between coils one and three.
M_{14}	The mutual inductance between coils one and four.
M_{23}	The mutual inductance between coils two and three.
M_{24}	The mutual inductance between coils two and four.
M_{34}	The mutual inductance between coils three and four.
R_1	The equivalent shunting resistance of the drive coil number one.
R_1	The reference resistor
R_2	The equivalent shunting resistance of the drive coil number two.

SYMBOL	DEFINITION
R_2	The ohmic resistance of the coil wire
R_3	The sum of the reference resistor R_1 and the ohmic resistance of the coil wire R_2
r_1	The equivalent series resistance of the drive coil number one.
r_2	The equivalent series resistance of the drive coil number two
R_s	The generator output resistance.
V_a	The voltage measured across the unknown impedance.
V_o	The output voltage measured at the pickup coils.
V_{ref}	The voltage of at the input side of the reference resistor.
V_4	The voltage between the pickup coils with respect to ground.
DET	The determinant of a matrix.
S_x^N	The sensitivity of function N with respect to parameter x.
$\frac{dY}{dx}$	The derivative of function Y with respect to parameter X.
$\frac{\partial Y}{\partial x}$	The partial derivative of function Y with respect to x.
$\|X\|$	The magnitude of complex number X.
ang X	The phase angle of complex number X.
S	Complex frequency, $S = \sigma + j\omega$.

SYMBOL	DEFINITION
ω	The radian frequency.
j	The square root of negative one.
ΔX	An incremental change in X.

INTRODUCTION

Background

In machinery it is quite often necessary to pass a shaft through a housing or barrier, and dynamic seals are often employed at these points. Even though their occurance is commonplace, their theory is poorly understood.

To the end of acquiring experimental data to provide supporting evidence for one of the theories, the Mechanical and Aerospace Engineering Department at the University of Tennessee, under contract to the United States Department of the Navy, undertook the investigation of dynamic seals.

One challenging aspect of this research was the need to study the behavior of lubricating films between seal faces under dynamic conditions. The accuracy of the measurement of lubricating film thickness was of great importance. The further constraint that the measurement not disturb the fluid between the seal faces complicated the problem considerably. To perform this measurement of film thickness, a nonpenetrating inductance probe was developed by the Electrical Engineering Department at the University of Tennessee.

To obtain a basic understanding of this thesis, it is helpful to understand the nonpenetrating inductance probe. This is true because the parameters of the probe are inseparable from those of the nulling network. The following is a description of the probe developed by W. R. Duncan [4].

The Figure 0.1 on page 3 shows a cutaway diagram of the probe used in this thesis. One can see that it is composed of four coils sharing a common axis. The outer coils are the drive coils; the inner two are the pickup coils. Separation is maintained between the pickup coils by means of a nonmetallic spacer. The probe is mounted in a hole bored through a stainless steel seal face. This is done by first boring a hole, then covering the face of the seal with a thin sheet of stainless steel. The probe is then affixed to the back of the membrane with epoxy glue, and the rest of the probe cavity is filled with the epoxy glue to keep the membrane rigid.

To understand the functioning of the probe, envision this probe fabricated perfectly with a stainless steel membrane at each end. If the drive coils have equal currents in them such that their flux is additive, then the voltages induced in each of the pickup coils would be equal. If the pickup coils are then connected differentially, the voltages would cancel. However, if there was another seal face brought in close proximity to the lower membrane, and therefore the lower pickup coil, eddy currents would cause a change in the electromagnetic vector potential in the region of the probe. The change would be most radical in the region of the lower coil and therefore, its output voltage would change the most. The pickup voltages would no longer be equal, and the output of the pickup coils would not be zero. Duncan [4] found that

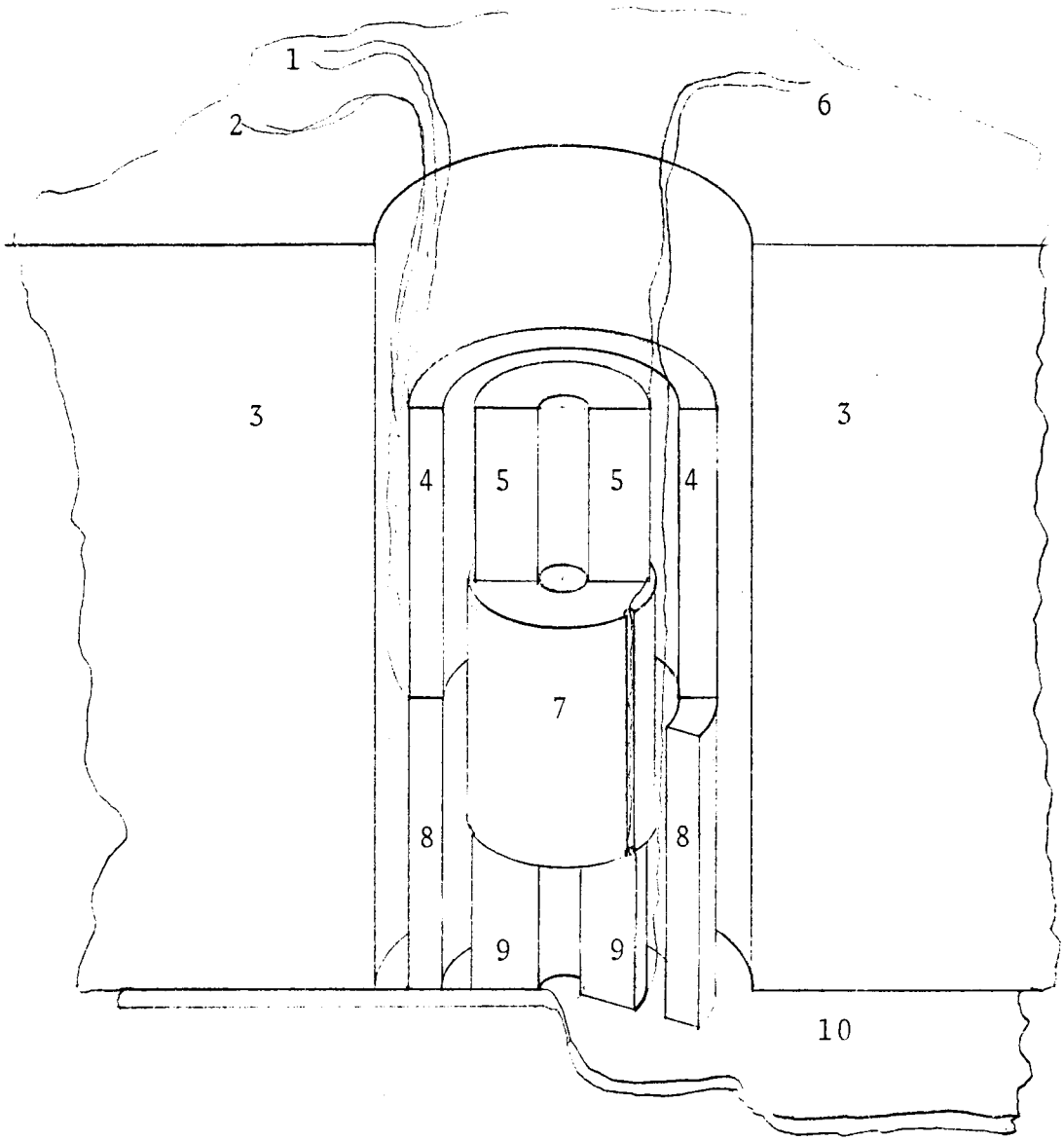


Figure 0.1. A cutaway diagram of the nonpenetrating inductance probe. The probe is mounted in a hole bored in the seal face (3) and the face is covered with a thin sheet of stainless steel (10). The pickup leads (6) are brought up from the lower pickup coil (9) through a channel cut into the spacer (7) and connected to the upper pickup coil (5). These leads are then brought out the back side of the seal. The leads (2) from the lower drive coil (8) and the leads (1) to the upper drive coil (4) are also brought out the back.

the change in output voltage of the probe was a reasonably linear function of the change in the oil film thickness. He found that if the seals are in close proximity, changes of a few microinches could be detected. The advantage of this technique is that it provides a high resolution measurement without perturbing the oil film between the seal faces.

One of the difficulties in using this system is the relatively high level signal that is needed on the drive side of the probe which must be canceled by balancing the probe. Small changes in the parameters of the probe and the nulling network used to balance it may cause output variations that are indistinguishable from those caused by changes is oil film thickness. Galyon [7] found the probe output to be very sensitive to temperature effects on probe dimensions and seal permeability. Temperature changes also affect the nulling network components.

In practice, perfect probes are hard to realize; and even if they could be made, they could not be ideally mounted, and slight imbalances would occur. To overcome the imbalance problem, many methods were attempted by Duncan [4] and one was selected. By splitting the drive coil into two sections, as in Figure 0.1, the probe output could be balanced by adjusting the phase and magnitude of the currents in the two coils. This is achieved by paralleling a resistor and a capacitor with each of the drive coils, and driving the coils in series. By choosing nulling network values for one coil

and making the values of the other coil variable, Duncan [4] found that the probe may be nulled at any particular frequency and seal to seal spacing.

Purpose

One limiting factor in the measurement system was that the passive components have certain temperature coefficients, and these may cause significant drifts in the system. The purpose of this thesis is to investigate the possibility of applying classical sensitivity to the circuit model used by Duncan [4] with the intent that once these sensitivity functions are derived, one would be able to gain insight into how to choose null network components such that the sensitivity of the probe output to parameters which are highly variable is minimized.

Scope of the Thesis

This thesis is devoted to a study of the sensitivity of the probe output to nulling network and probe parameter changes. Chapter I contains a discussion of the circuit model, derivation of the transfer function and a discussion of sensitivity and the sensitivity functions of the probe. Chapter II contains a discussion of the experimental and mathematical techniques used to arrive at model parameters. Chapter II also contains a discussion of the measurement techniques used to find the sensitivities of the probe output to changes in nulling network parameters. Chapter III

contains a discussion of the results, the conclusions and recommendations for further work. Appendix A contains the derivation of the sensitivity functions which describe the effect of changes in nulling network parameters and probe model parameters on the probe output. Appendix B contains the descriptions and listings of each of the computer programs used in this thesis.

CHAPTER I

THE CIRCUIT MODEL AND CLASSICAL SENSITIVITY ANALYSIS

The Circuit Model

Examination of the model depicted in Figure 1.1 reveals that several approximations have been made. The most obvious is that the output of the pickup coils is terminated in an open circuit. This approximation is made because the frequency of operation is only 98 K Hertz and this output is connected to an amplifier with a very high input impedance. The fact that the frequency is relatively low makes the effect of the capacitance of the amplifier and the connecting cable on the pickup coils small. The high input impedance of the amplifier allows one to make the assumption that the current flowing in the pickup coils is not significant, and therefore the drive side of the probe can be analyzed independently from the output side.

This may be easily seen from examination of Equation 1.001 found in Hayt and Kemmerly [9] which describes the input impedance of an air core transformer.

$$Z_{in} = r_1 + SL_1 - \frac{S^2M^2}{Z_{22}} \qquad (1.001)$$

where r_1 is the series primary resistance, L_1 is the primary leakage inductance, M is the mutual inductance and Z_{22} is the series secondary impedance including load.

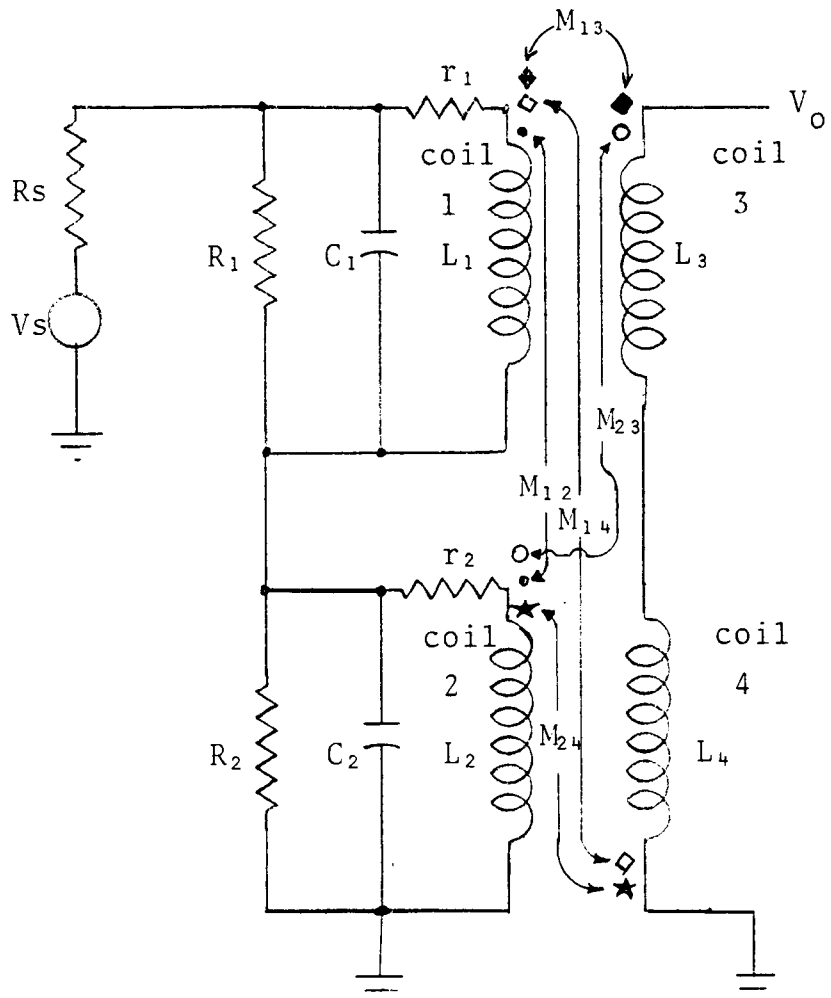


Figure 1.1. The circuit model used for the probe and nulling network.

As Z_{22} becomes large the effect of the secondary on the primary is reduced.

The voltage source shown in Figure 1.1 is the signal generator and the resistance R_s shown is the generator output impedance. The capacitance of the connecting cable has been neglected because R_s is small (typically 50 ohms). Components affecting both drive coils will have little effect upon the output due to the differental nature of the probe. The capacitances C_1 and C_2 are lumped equivalent values which include both the physical capacitor in the nulling network and the equivalent capacitance across the respective drive coils. The resistors r_1 and r_2 represent the resistance of the wire in the drive coils. The resistors R_1 and R_2 represent the resistance of the nulling network resistor and the equivalent loss resistor shunting the respective drive coils. Inductances L_1 and L_2 represent the leakage inductances of the drive coils. Similarly L_3 and L_4 represent the leakage inductances of the pickup coils. The mutual inductances M_{13} and M_{14} represent mutuals linking coils 1 and 3 and linking coils 1 and 4 respectively. Similarly M_{23} and M_{24} represent the mutual inductances between coils 2 and 3 and between coils 2 and 4 respectively. Using similar notation M_{12} and M_{21} will be represented by only M_{12} since they are equivalent due to reciprocity. The mutuals M_{34} and M_{43} have been neglected since the current in the pickup coils is small.

The drive side of the probe may be analyzed separately

because of negligible influence by the pickup coils. The equivalent circuit used in the analysis is illustrated in Figure 1.2. The basic drive circuit may be reduced to simplify analysis by substituting Z_1 for the parallel combination of the impedances of elements R_1 and C_1. Similarly, Z_2 is the equivalent impedance of the parallel combination of the elements R_2 and C_2. This reduces the circuit to a more easily analyzed three loop problem. Summing the voltages about the loops in Figure 1.3 yields the following equations:

$$V_o = I_1(-Z_1) + I_2(-Z_1) + I_3(R_s + Z_1 + Z_2), \tag{1.002}$$

$$0 = I_1(Z_1 + r_1 + SL_1) + I_2(SM_{12}) + I_3(-Z_1), \text{ and} \tag{1.003}$$

$$0 = I_1(SM_{12}) + I_2(Z_2 + r_2 + SL_2) + I_3(-Z_2). \tag{1.004}$$

To simplify the handling of the symbols, some defining equations are used along with matrix notation to express the above equations in compact form. The symbol definitions may be found in Table 1.1.

$$\begin{pmatrix} V_s \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -Z_1 & -Z_2 & Z_3 \\ Z_4 & SM_{12} & -Z_1 \\ SM_{12} & Z_5 & -Z_2 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} \tag{1,005}$$

Substituting the vector on the left of Equation 1.005 for each of the columns and finding the determinants of the resulting matrix yields Equations 1.006, 1.007, and 1.008 found on page 14.

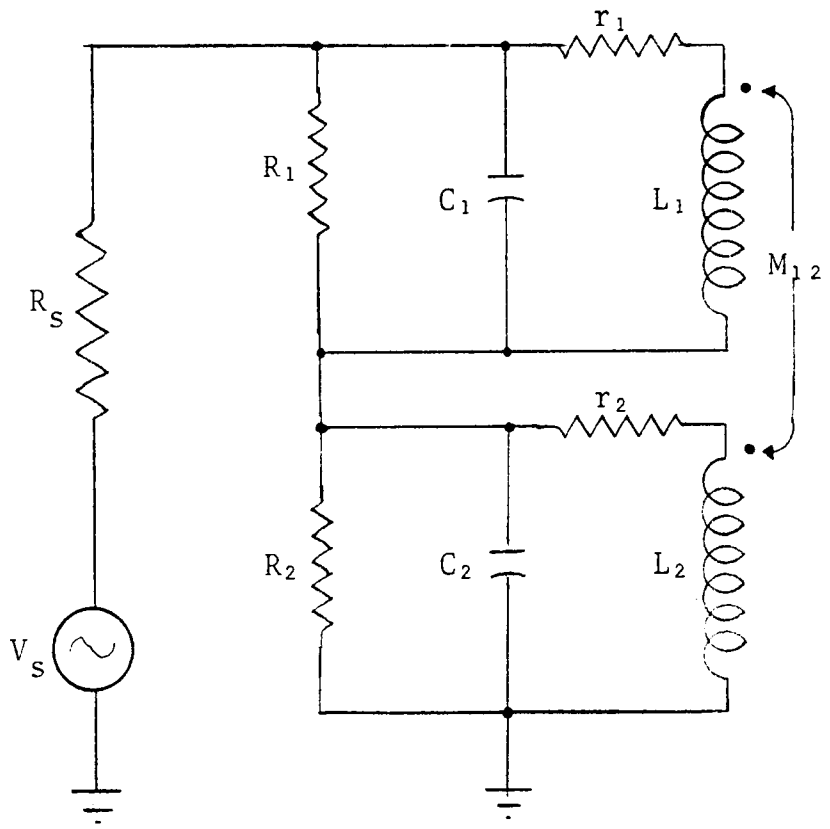


Figure 1.2. The circuit model showing only the required primary side.

Table 1.1

Simplifying Variable Definitions

$$Z_1 = R_1/(SR_1C_1 + 1)$$

$$Z_2 = R_2/(SR_2C_2 + 1)$$

$$Z_3 = R_s + Z_1 + Z_2$$

$$Z_4 = SL_1 + r_1 + Z_1$$

$$Z_5 = SL_2 + r_2 + Z_2$$

$$Z_6 = Z_2Z_4 - SM_{12}Z_1$$

$$Z_7 = Z_1Z_5 - SM_{12}Z_2$$

$$Z_8 = -Z_1^2Z_5$$

$$Z_9 = -Z_2^2Z_4$$

$$Z_{10} = Z_3(-S^2M_{12}^2 + Z_4Z_5)$$

$$Z_{11} = 2SM_{12}Z_1Z_2$$

$$P = Z_6 + Z_9 + Z_{10} + Z_{11}$$

$$Q = S[(M_{13} - M_{14})Z_7 + (M_{23} - M_{24})Z_6)]$$

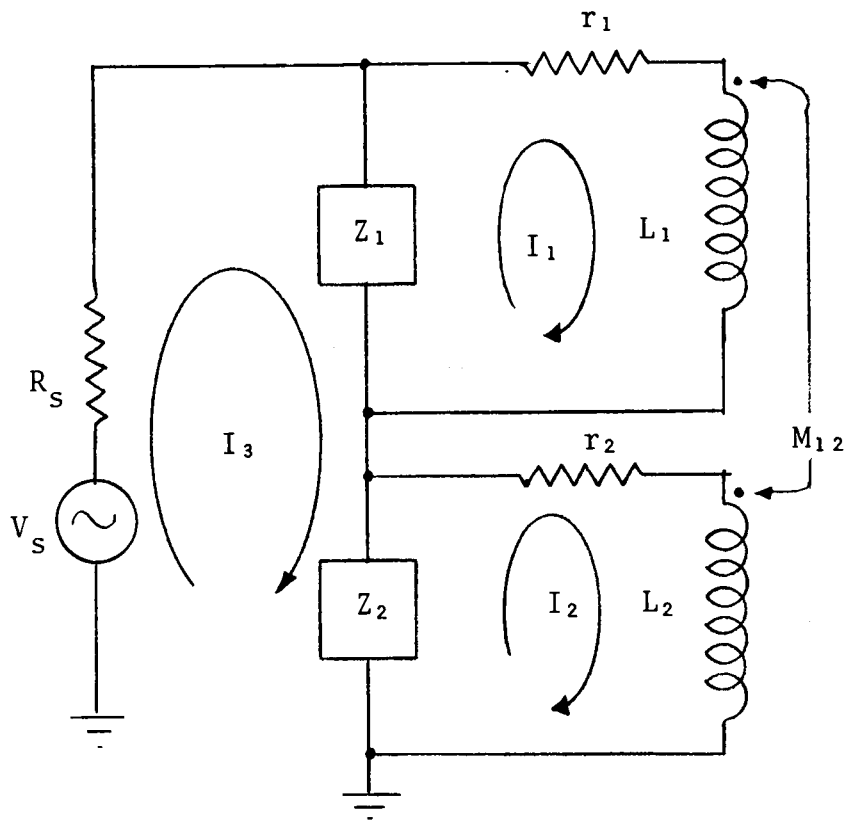


Figure 1.3. Circuit diagram of the primary side of the probe and nulling network which shows the substitution of the simplifying impedances Z_1 and Z_2 and the definition of currents I_1, I_2 and I_3.

$$DET_{I_1} = V_S(-SM_{12}Z_2+Z_1Z_5), \quad (1.006)$$

$$DET_{I_2} = V_S(-SM_{12}Z_1+Z_2Z_4), \text{ and} \quad (1.007)$$

$$DET_{I_3} = V_S(-S^2M_{12}^2+Z_4Z_5). \quad (1.008)$$

The determinant of the basic matrix is found to be

$$DET_O = Z_3(Z_4Z_5-S^2M_{12}^2)-Z_2^2Z_4-Z_1^2Z_5+2Z_1Z_2SM_{12}. \quad (1.009)$$

From these determinants, the currents may be found

$$I_1 = \frac{DET_{I_1}}{DET_O} = \frac{V_S(-SM_{12}Z_2+Z_1Z_5)}{DET_O}, \quad (1.010)$$

$$I_2 = \frac{DET_{I_2}}{DET_O} = \frac{V_S(-SM_{12}Z_2+Z_2Z_4)}{DET_O}, \text{ and} \quad (1.011)$$

$$I_3 = \frac{DET_{I_3}}{DET_O} \quad \frac{V_S(-S^2M_{12}^2+Z_4Z_5)}{DET_O}. \quad (1.012)$$

Now that the primary currents are known, it is a simple process to find the output voltage. The Equations 1.013 and 1.014 are the result of examining the partial circuit shown in Figure 1.4 on page 15.

$$V_4 = -SM_{24}I_2-SM_{14}I_1, \quad (1.013)$$

$$V_O-V_4 = SM_{13}I_1+SM_{23}I_2. \quad (1.014)$$

Solving for V_O yields

$$V_O = I_1(SM_{13}-SM_{14})+I_2(SM_{23}-SM_{24}). \quad (1.015)$$

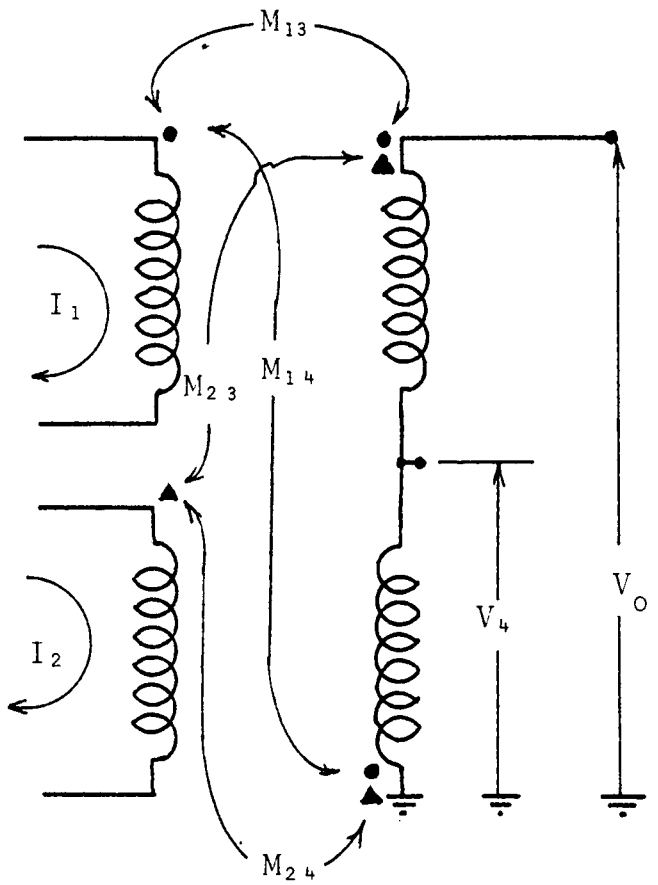


Figure 1.4. A circuit diagram of the probe which shows only the coupling of the primary currents into the pickup coils.

The substitution of Equations 1.010 and 1.011 into Equation 1.015 results in the transfer function from the generator to the output of the pickup coils

$$\frac{V_o}{V_s} = \frac{(Z_1Z_5 - SM_{12}Z_2)[S(M_{13} - M_{14})]}{DET_o} + \frac{S(M_{23} - M_{24})(Z_2Z_4 - SM_{12}Z_1)}{DET_o} . \quad (1.016)$$

This function may be resolved into a numerator which is a fifth degree polynomial in S and a denominator which is a sixth degree polynomial in S.

Classical Sensitivity

The definition of classical sensitivity used in this thesis was found in Huelsman [11]:

$$S_x^N(S,x) = \frac{\frac{dN}{N}}{\frac{dx}{x}}, \quad (1.017)$$

where N is the network function being changed as a response to a change in parameter x. S_x^N is the function which relates the differential change in parameter x to a differential change in response N. Huelsman further states that sensitivity can be found by using the equation:

$$S_x^N(S,x) = x\left(\frac{Q'}{Q} - \frac{P'}{P}\right), \quad (1.018)$$

where $Q' = \frac{\partial Q}{\partial x}$, (1.019)

and $P' = \frac{\partial P}{\partial x}$. (1.020)

The quantity Q is the numerator of the network function N and P is the denominator of N.

Another useful form of the sensitivity equation found in Huelsman [11] is

$$S_x^N(j\omega,x) = \frac{\frac{d|N(j\omega,x)|}{|N(j\omega,x)|}}{\frac{dx}{x}} + j\,\frac{d\ \text{ang}\ N(j\omega,x)}{\frac{dx}{x}}. \quad (1.021)$$

From Equation 1.021 it may be seen that the real part of the sensitivity gives information about the change in the magnitude of the transfer function, and the imaginary part gives information about the change in phase. Since the primary interest in this analysis is the change in magnitude, it is Equation 1.021 that stimulated interest in using classical sensitivity.

In reading the literature about different types of sensitivity, the subject of bilinearity was mentioned repeatedly. The assumption of bilinearity was made time after time in the development of various forms of and short-cuts to making sensitivity calculations. A general network function is considered a bilinear function of parameter x if it can be

expressed as

$$N(S,x) = \frac{C(S) + xD(s)}{A(S) + xB(S)} , \qquad (1.022)$$

where A(S),B(S),C(S) and D(S) are polynomials in S and not functions of x [11]. Huelsman [11] did not mention this requirement in his development of classical sensitivity, but the fear that it was assumed in a roundabout manner motivated a search for the origin of this requirement. Huelsman discussed bilinearity at some length in sections of his book following the one on classical sensitivity. In particular he mentioned that although most situations may be treated as bilinear, the exception that he noted was the "dependence upon the coefficient of coupling of a pair of coupled coils is not bilinear." He did not elaborate further and a search of references resulted in the discovery that Bode [1] defined sensitivity to be the reciprocal of Equation 1.017 on page 16 and appears to be the originator of this type of sensitivity. However, the requirement for bilinearity was not apparent in Bode's discussions relating sensitivity and the return difference. I. M. Horowitz [10] states that coupled coil parameters are not bilinear, but the necessity of bilinearity to use classical sensitivity was still not resolved. The source of the requirement comes from a discussion between L. Tasny-Tschiassy [2] and I. Cederbaum and A Fuchs on the necessity that a network function be bilinear with respect to

a parameter in order that sensitivity could be calculated by "Bode's Theorem": "The sensitivity and the return difference are equal for any element whose vanishing leads to zero transmission through the circuit as a whole." The specific discussion was upon the subject of "Bode's Theorem" not applying for mutual inductances. As an argument L. Tasny-Tschiassny shows that by using the more basic definition, which is in terms of derivatives as is Equation 1.017, one arrives at a different solution than the one found by using the return difference method of derivation. He implies that the derivative method is the correct one since he uses it as a standard.

Considering that any of the classical sensitivity formulas given in Huelsman [11] may be derived from the basic definition of classical sensitivity (Equation 1.017 on page 16) by applying the rules of derivatives of complex numbers, then the only requirements are that the function be analytic in the region of interest (see Churchill [3]). It is for this reason that the techniques used for the derivation of the sensitivity functions for mutual inductances were those described in Huelsman.

If Q equals the numerator of the transfer function from the generator to the output, then, from Equation 1.016, Q may be defined as in Equation 1.023.

$$Q=S(M_{13}-M_{14})(Z_1Z_5-SM_{12}Z_2)+(M_{23}-M_{24})(Z_2Z_4-SM_{12}Z_1). \quad (1.023)$$

The quantity P may be defined as being equal to the determinant DET_0 which may be found in Equation 1.009 on page 14.

For the sake of compactness let us further define the following variables:

$$Z_6 = Z_2Z_4 - SM_{12}Z_1, \text{ and} \tag{1.024}$$

$$Z_7 = Z_1Z_5 - SM_{12}Z_2. \tag{1.025}$$

These definitions may also be found in Table 1.1 on page 12. Applying these definitions to Equation 1.023 on page 19 and Equation 1.009 on page 14 yields

$$Q = S[(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6]. \tag{1.026}$$

To express Equation 1.009 is a reasonable amount of space, let us define the following variables:

$$Z_8 = -Z_1^2Z_5, \tag{1.027}$$

$$Z_9 = -Z_2^2Z_4, \tag{1.028}$$

$$Z_{10} = Z_3(-S^2M_{12}^2+Z_4Z_5), \text{ and} \tag{1.029}$$

$$Z_{11} = 2SM_{12}Z_1Z_2. \tag{1.030}$$

The expression for the denominator P, may now be written as:

$$P = Z_8+Z_9+Z_{10}+Z_{11}. \tag{1.031}$$

The results of applying the classical sensitivity equation (See Equation 1.018 on page 16) to the expressions for P and Q for each of the nulling network and probe parameters appears in Table 1.2 on page 21. The derivation of these

Table 1.2

Classical Sensitivities of V_o/V_s

$$S_{R_1}^{T} = R_1 \frac{\partial Z_1}{\partial R_1} \left\{ \frac{(M_{23}-M_{24})(Z_2-SM_{12})+Z_5(M_{13}-M_{14})}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_5(-2Z_1+Z_4+Z_3)-(Z_2-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right\}$$

$$S_{R_2}^{T} = R_2 \frac{\partial Z_2}{\partial R_2} \left\{ \frac{(M_{13}-M_{14})(Z_1-SM_{12})+Z_4(M_{23}-M_{24})}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_4(-2Z_2+Z_5+Z_3)-(Z_1-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right\}$$

$$S_{C_1}^{T} = C_1 \frac{\partial Z_1}{\partial C_1} \left\{ \frac{(M_{23}-M_{24})(Z_2-SM_{12})+Z_5(M_{13}-M_{14})}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_5(-2Z_1+Z_4+Z_3)-(Z_2-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right\}$$

$$S_{C_2}^{T} = C_2 \frac{\partial Z_2}{\partial C_2} \left\{ \frac{(M_{13}-M_{14})(Z_1-SM_{12})+Z_4(M_{23}-M_{24})}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_4(-2Z_2+Z_5+Z_3)-(Z_1-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right\}$$

$$S_{R_S}^{T} = -R_S \frac{Z_4Z_5-S^2M_{12}^2}{Z_8+Z_9+Z_{10}+Z_{11}}$$

$$S_{L_1}^{T} = SL_1 \left\{ \frac{(M_{23}-M_{24})Z_2}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_3Z_5-Z_2^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right\}$$

Table 1.2 (continued)

$$S_{L_2}{}^{T} = SL_2 \left(\frac{(M_{13}-M_{14})Z_1}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_3Z_4-Z_1{}^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right)$$

$$S_{r_1}{}^{T} = r_1 \left(\frac{(M_{23}-M_{24})Z_2}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_3Z_5-Z_2{}^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right)$$

$$S_{r_2}{}^{T} = r_2 \left(\frac{(M_{13}-M_{14})Z_1}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_3Z_4-Z_1{}^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right)$$

$$S_{M_{12}}{}^{T} = -SM_{12} \left(\frac{Z_2(M_{13}-M_{14})+Z_1(M_{23}-M_{24})}{Z_7(M_{13}-M_{14})+Z_6(M_{23}-M_{24})} + \frac{2(Z_1Z_2-Z_3SM_{12})}{Z_8+Z_9+Z_{10}+Z_{11}} \right)$$

$$S_{M_{13}}{}^{T} = \frac{M_{13}Z_7}{Z_7(M_{13}-M_{14})+Z_6(M_{23}-M_{24})}$$

$$S_{M_{14}}{}^{T} = \frac{-M_{14}Z_7}{Z_7(M_{13}-M_{14})+Z_6(M_{23}-M_{24})}$$

$$S_{M_{23}}{}^{T} = \frac{M_{23}Z_6}{Z_7(M_{13}-M_{14})+Z_6(M_{23}-M_{24})}$$

$$S_{M_{24}}{}^{T} = \frac{-M_{24}Z_6}{Z_7(M_{13}-M_{14})+Z_6(M_{23}-M_{24})}$$

Table 1.2 (continued)

$$S_S^T = S\left\{\frac{(M_{13}-M_{14})\left[Z_7+S\left\{\frac{\partial Z_1}{\partial S}Z_5+\frac{\partial Z_2}{\partial S}(Z_1-SM_{12})+Z_1L_2-Z_2M_{12}\right\}\right]}{S\left[(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6\right]}\right.$$

$$+\frac{(M_{23}-M_{24})\left[Z_6+S\left\{\frac{\partial Z_2}{\partial S}Z_4+\frac{\partial Z_1}{\partial S}(Z_2-SM_{12})+Z_2L_1-Z_1M_{12}\right\}\right]}{S\left[(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6\right]}$$

$$-\frac{\partial Z_1}{\partial S}\left(\frac{Z_5(Z_3+Z_4-2Z_1)-(Z_2-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}}\right)$$

$$-\frac{\partial Z_2}{\partial S}\left(\frac{Z_4(Z_3+Z_5-2Z_2)-(Z_1-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}}\right)$$

$$\left.-\frac{-Z_1^2L_2-Z_2^2L_1+Z_3(Z_5L_1+Z_4L_2-2SM_{12}^2)+2M_{12}Z_1Z_2}{Z_8+Z_9+Z_{10}+Z_{11}}\right)$$

Where:

$$T=\frac{V_o}{V_s}$$

sensitivity functions is found in Appendix A. It may be seen that these results are relatively intractable and for this reason, they were evaluated by a digital computer. The listing of the program called START used in this evaluation is found in Appendix B. The data for the sensitivity values listed in Table 1.3 on page 25 were generated by the program START.

To obtain some level of confidence that these formulas are accurate, another approach was taken to obtain the values. J. K. Fidler [6] uses an incremental technique to estimate the derivative of a network function with respect to parameters of that function. In Table 1.4 on page 26 is a comparison of the results obtained by the two methods. The values found by the derived sensitivity functions were subtracted from those found by Fidler's method and then normalized by the values computed by the sensitivity functions. The values of the sensitivities matched very well.

The technique used by J. K. Fidler makes the assumption that the function is bilinear in the parameter of interest. It is fairly obvious that the mutuals between the drive and pickup coils satisfy bilinearity definition, Equation 1.022 on page 18. This is clear since the mutuals M_{13}, M_{14},M_{23} and M_{24} are only in the numerator of the transfer function and are of first order. The mutual M_{12} which couples the drive coils may not be bilinear as it is not easy to put it in the form of Equation 1.022. Since the values computed by

Table 1.3

Classical Sensitivity Values

Parameter (Units)		Nominal Values Real Part	Nominal Values Imaginary Part	Sensitivity Real Part	Sensitivity Imaginary Part
S	(rad/s)	0.0	6.158×10^{5}	-6.221×10^{2}	8.574×10^{2}
R_1	(Ω)	1.357×10^{4}	0.0	7.093×10^{2}	4.857×10^{3}
R_2	(Ω)	1.248×10^{4}	0.0	-8.630×10^{2}	-5.446×10^{3}
C_1	(F)	1.429×10^{-10}	0.0	5.796×10^{3}	-8.465×10^{2}
C_2	(F)	1.474×10^{-10}	0.0	-6.170×10^{3}	9.779×10^{2}
L_1	(H)	2.250×10^{-3}	0.0	5.086×10^{3}	5.532×10^{3}
L_2	(H)	2.315×10^{-3}	0.0	-5.313×10^{3}	6.263×10^{3}
M_{13}	(H)	3.565×10^{-4}	-5.137×10^{-6}	4.262×10^{4}	6.176×10^{-9}
M_{24}	(H)	3.542×10^{-4}	0.0	-4.262×10^{4}	-6.176×10^{-9}
r_1	(Ω)	1.027×10^{2}	0.0	-4.102×10^{2}	-3.771×10^{2}
r_2	(Ω)	9.958×10^{1}	0.0	4.375×10^{2}	-3.711×10^{2}
M_{12}	(H)	3.571×10^{-5}	1.318×10^{-4}	-2.209×10^{1}	-5.085
R_S	(Ω)	5.000×10^{1}	0.0	-2.399×10^{-3}	1.520×10^{-2}

Table 1.4

Normalized Difference Between Sensitivities
Calculated by Huelsman's Method and Fidler's Method*

Parameter	Error	
	Real Part	Imaginary Part
S	-6.378×10^{-10}	-1.993×10^{-10}
R_1	-1.292×10^{-11}	-1.198×10^{-12}
R_2	-2.157×10^{-11}	-2.794×10^{-13}
C_1	2.189×10^{-13}	-3.166×10^{-12}
C_2	-1.502×10^{-12}	-6.794×10^{-13}
L_1	6.965×10^{-13}	-1.582×10^{-12}
L_2	1.740×10^{-12}	1.239×10^{-12}
M_{13}	6.277×10^{-13}	7.208×10^{-2} **
M_{24}	-8.508×10^{-13}	-2.889×10^{-2} **
r_1	9.932×10^{-12}	1.435×10^{-11}
r_2	3.125×10^{-11}	2.438×10^{-11}
M_{12}	3.901×10^{-10}	-1.272×10^{-11}
R_s	-1.176×10^{-10}	-7.461×10^{-10}
Nominal Transfer function = $1.794\times10^{-6} + j\ 3.977\times10^{-8}$		

* Step size used was 1×10^{-4} times nominal value.

** Round off error of computer due to the small size of the nominal value of the imaginary part. See Table 1.3 on page 25.

the two methods are so close, it would appear that M_{12} may be treated as bilenear in this case.

A comparison between the measured magnitude changes and those calculated is shown in Table 1.5 on page 28. The discrepancy between the two columns of sensitivity is readily noticable. The difference in sign is caused by the fact that the measurement was taken close to null. A change in any parameter in the system must cause the output to increase. The only measurements which gave negative values for the magnitude sensitivity were those whose nominal values were reduced to make the sensitivity measurement. The computer calculation of sensitivity is not hampered by the fact that its output is close to zero, since it can handle the small step sizes required to move closer to null. Th proximity to null was a significant cause of problems in making this measurement.

In this chapter, the probe and nulling network model was described, the transfer function was derived, and classical sensitivity functions found for each parameter. Also, the values of the classical sensitivity functions were calculated by two methods. The values compared very favorably. The calculated and measured sensitivity were also compared. The capacitances shunting the probe were found to be the most sensitive components in the nulling network.

In the next chapter, the experimental and analytical techniques for arriving at model parameters are described,

Table 1.5

Measured Classical Sensitivities

Parameter	Measured Sensitivity	Calculated Sensitivity
C_1	1.58×10^4	5.80×10^3
R_1	-1.23×10^4	7.09×10^2
L_1	4.37×10^3 §	5.09×10^3
r_1	3.02×10^2 §	-4.10×10^2
C_2	1.57×10^4	-6.17×10^3
R_2	-1.55×10^4	-8.63×10^2
S	1.07×10^4	-6.22×10^2

§ Measurement technique not valid if drive coils have significant input capacitance and equivalent loss resistance.

and the experimental techniques for measuring sensitivity are described.

CHAPTER II

EXPERIMENTAL TECHNIQUES

In this chapter, the various experimental techniques used are described along with the methods used to find the various parameters that make up the probe model. Computer programs were used extensively to assist in the evaluation of data and are described in Appendix B.

One of the primary difficulties encountered in evaluating the probe sensitivities to changes in the parameters was the determination of nominal parameter values. Use of Duncan's [4] computer models of the probe left much to be desired when it came to working with real probes. As a design tool, they may be relatively effective, but the parameter values that his technique produced were perfectly matched, and excluded capacitive effects. A great deal of time and effort was put into attempting to use these parameter values, but empirical techniques were found to be more productive.

Impedance Measurement Technique

Due to the difficulties in obtaining equipment which could properly measure the probe impedances, a technique was chosen which is based on the definition of impedance. The system is illustrated in Figure 2.1. It consists of measuring the voltage on either side of a resistor which

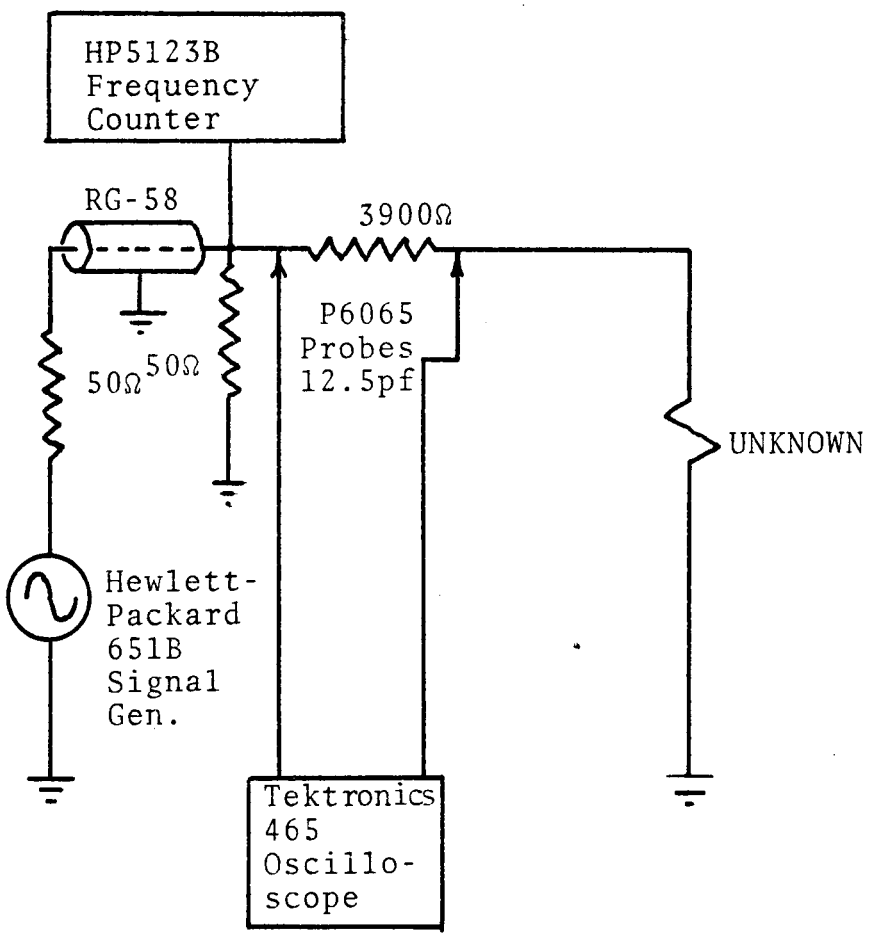


Figure 2.1. Impedance measurement system.

feeds the unknown impedance. Since the resistor is known, and the voltage on either side is known, the current is defined. Knowing the voltage across the unknown, and the current through it, allows the impedance of the unknown to be calculated. A Tektronics 465 Oscilloscope and p6065 probe were used to make the voltage measurement. A Hewlett-Packard 651B signal generator was used to drive the measurement circuit with a sine wave.

The impedance of one of the drive coils could be modeled as in Figure 2.2. Figure 2.2a is the low frequency model in which the coil inductance and capacitance have no significant effect, which leaves only the D. C. resistance of the probe wire. Figure 2.2b is a moderate frequency model in which the inductive nature of the probe is affecting the impedance value, and the capacitive nature of the probe has no significant contribution. In Figure 2.2c the frequency is very high, and the capacitive effect dominates. All three components are combined in Figure 2.2d. The voltage magnitude in each of the three regions can be measured, and each of these model parameters can be computed in all regions. Examination of the data for consistant parameter values identifies the location of these regions, and provides the values of the parameters.

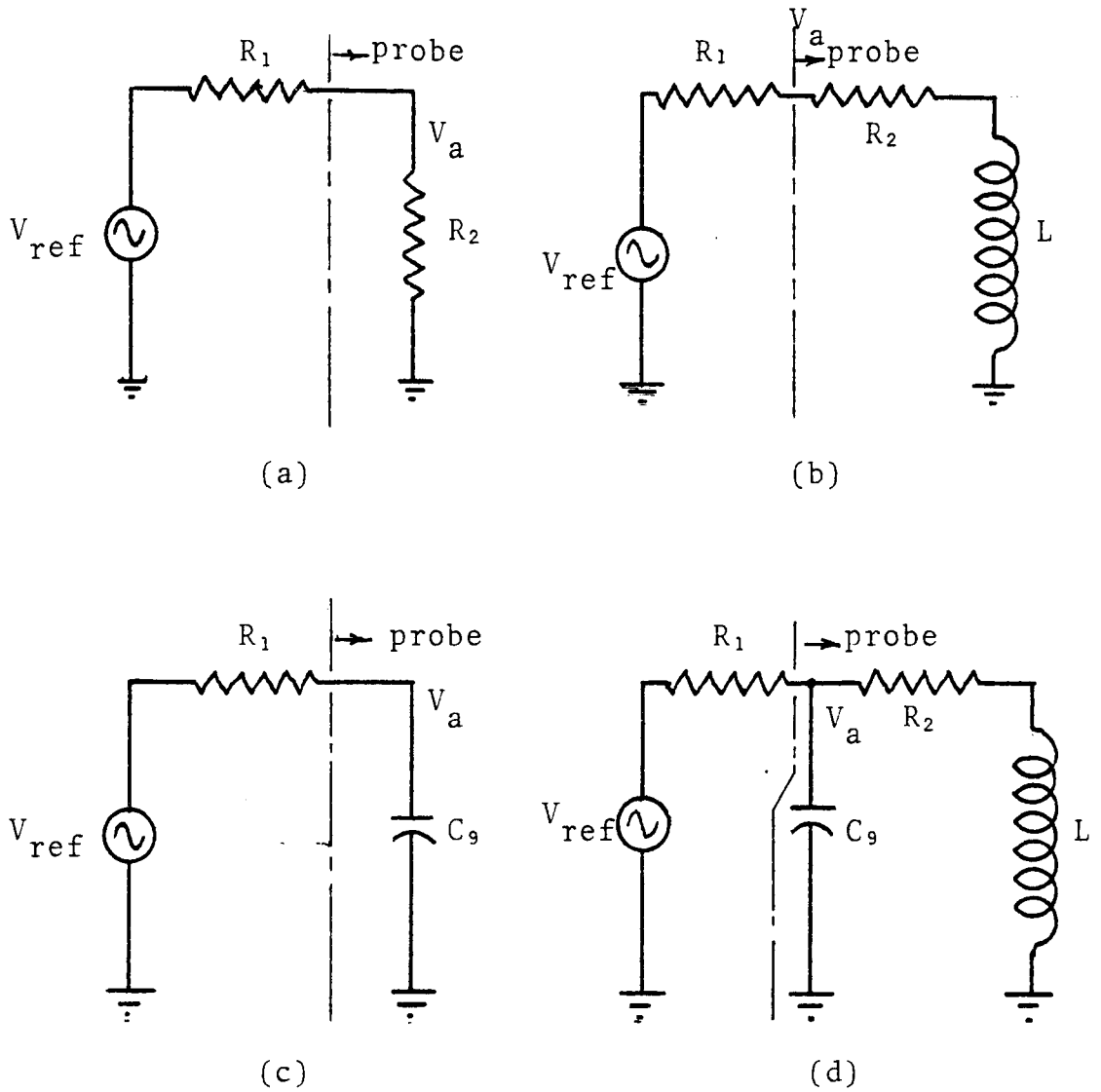


Figure 2.2. Drive coil model for (a) low, (b) moderate, (c) high and (d) all frequency regions.

Three Region Parameter Calculation Technique

The series resistance of the wire may be measured at low frequencies or at D. C.. Solution of the equations for inductance generated by analysis of Figure 2.2b is more difficult if only magnitude of the voltage across the unknown is available. The solution of the problem utilizing only magnitudes generates an extra root, but it is relatively easy to discover which root is erroneous. Only the true root gives consistant results. The equation for inductance is found from writing these equations from Figure 2.2b:

$$i = \frac{V_{ref}}{R_1+R_2+j\omega L} \qquad (2.001)$$

where R_1 is the reference resistor, R_2 is the resistance of the wire, L is the coil inductance, V_{ref} is the voltage of the driving source, and ω is radian frequency. The current through the inductor is i. If R_3 is equivalent to R_1 plus R_2, then the denominator will be found to yield

$$i = \frac{V_{ref}(R_3-j\omega L)}{R_3^2+\omega^2 L^2} \; . \qquad (2.002)$$

From Equation 2.002 the magnitude of the voltage across the probe may be found:

$$|V_a| = \frac{V_{ref}}{R_3^2+\omega^2L^2}\{(R_2R_3+\omega^2L^2)^2+\omega^2L^2R_1^2\}. \qquad (2.003)$$

Rewriting Equation 2.003 as a polynomial in L yields

$$L^4\omega^4(|V_a|-|V_{ref}|)+L^2\omega^2[|V_a|^2(2R_3^2)-|V_{ref}|^2(R_2R_3-R_1^2)] + |V_a|^2R_3^4-|V_{ref}|^2R_2^2R_3^2 = 0 . \qquad (2.004)$$

If L is defined to be equal to the new variable H, and H is substituted into Equation 2.004, the resulting equation will be in quadratic form in terms of H. Let

$$A = \omega^4(|V_a|^2-|V_{ref}|^2),$$
$$B = \omega^2(2R_3^3|V_a|^2-2|V_{ref}|^2R_2R_3-|V_{ref}|^2R_1^2), \text{ and}$$
$$C = R_3^4|V_a|^2-|V_{ref}|^2R_3^2R_2^2.$$

The expression for H is

$$H = \frac{-B\pm(B^2-4AC)^{.5}}{2A} . \qquad (2.005)$$

Keeping only the positive roots of L,

$$L = \sqrt{H} . \qquad (2.006)$$

The computer program CALEL, in Appendix B, page evaluates Equation 2.006.

Writing equations for the high frequency probe model depicted in Figure 2.2c on page 33, and solving for the voltage magnitude across the probe, it can be shown that

$$|V_a| = \frac{|V_{ref}|}{(1+\omega^2 C_9^2 R_1^2)^{.5}} . \qquad (2.007)$$

Solving Equation 2.007 for the capacitance shunting the coil C_9,

$$C_9 = \frac{(|V_{ref}|^2 - |V_a|^2)^{.5}}{|V_a|\omega R_1} . \qquad (2.008)$$

The Equation 2.008 is also evaluated by the program CALEL.

Although Equations 2.006 and 2.008 were verified by using ECAP [5] to generate an ideal set of input data, actual measured data proved to give less satisfactory results. The high frequency model was difficult to evaluate since the magnitude at high frequencies was hard to measure. This made the results in this region inconsistant, and the accuracy of the shunt capacitance was doubtful.

Three Parameter Fitting Technique

A model fitting technique seemed the best approach to the problem of how to arrive at the most representative set of components. The problem that arises is finding a criterion for the quality of the fit. The problem is complicated by the fact that both magnitude and phase, or the real part and the imaginary part, of the impedance must be fitted at the same time. The most logical choice is to attempt to minimize the sum of square errors for

both the imaginary and the real parts of the impedance; but in practice, this type of fit was found unsatisfactory. It is the logarithmic nature of circuits that causes the problem. The drive coil is essentially a resonant circuit, and fitting in this manner tends toward relatively low errors at this high impedance point, and high errors where the impedance is low. For example, one hundred ohms may be an insignificant error at resonance, but at lower or higher frequencies, where the impedance is comparable to that value, one hundred ohms is too much. The logarithm of the real and imaginary parts cannot be used as an error criterion to overcome this problem, since the imaginary part may easily be zero or negative. Summing the absolute value of the difference in the log magnitude and the absolute value of the difference in radian phase error is an error criterion that works fairly well, up to a point. This last criterion seems to be biased toward phase errors, but after working with several other criterion, this one seemed to work best, in terms of arriving at components which apparently generate the same magnitude and phase as the measured data. A better criterion would be desirable.

It seemed that a reasonable approach to take was to model the error as a function of the different parameters of the circuit model, and then change the parameters as necessary to reduce the error. To accomplish this, a design matrix was generated as suggested by Hunter [12],

which contains the points at the corners of a cube in parameter space, points at the origin, and points on axis. The points on the axis are of such a distance as to allow the design to be rotatable. Such matrices are called rotatable central composite designs [12]. A guess was made as to the values of the model components; then the error was computed for this point, (the origin), and the perturbations around that point. Then the best one of these points was selected as indicative of the direction to proceed in order to reduce the error. The parameters were changed in this direction until the error began to increase. The minimum point was selected as the new origin, and the process was repeated to find a new direction. This was continued until the error was minimized. An attempt was made to fit a line to the error and use the resulting equation to determine the best possible direction to move, but, due to lack of time this method was not used. The programs used to reduce the error by an iterative method are: DATA∆INPUT, DZIN, EFIT, ERR, FIT, FLOORVALUE, LG, LOG, MOVE, MOVEIT, SETREZ, and, TRY. These are found in Appendix B along with several useful routines, such as PLOTZ and LPT, which provide graphical output on a printer, comparing the fitted and measured impedance magnitudes versus frequency. There are also several other routines used to check the validity of the fitted component values.

Four Parameter Fitting Technique

Fitting of the three components revealed that there were more losses in the resonant circuit than could be accounted for in the wire resistance alone. The resonant peak on the fitted model was too high by several orders of magnitude. This led to including a shunt resistance in the probe model as shown in Figure 2.3. A similar response surface technique was used to fit the four parameter model using the programs DATAΔINPUT, CHK4, DZN4, EFIT, ER4, FIT, FLOORVALUE, LG, LOG, MOVEIT, MOV4, PHASE, SETREZ, and TRY. These programs and an explanation of their uses may be found in Appendix B.

When the fitting routine reached a certain point, that is, when the resonant point was well fitted, it was necessary to monitor both the phase and the magnitude errors independently to assure that neither one suffered unduly from the improvement in fit to the other. This usually meant that some phase error had to be sacrificed for the good of the magnitude fit. The quality of the fit was judged subjectively by plotting the curves or looking at listing, and comparing the computed and measured values.

Some problems were caused when the model did not resonate at the same frequency as the measured data. The data was not dense enough around the resonant point to allow an accurate fit in magnitude at resonance. The magnitude of the impedance computed using the model values

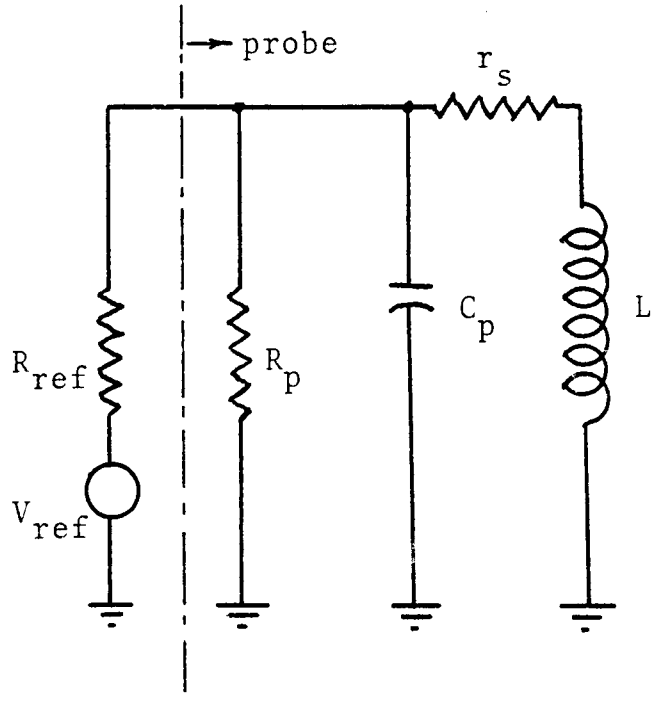


Figure 2.3. Drive coil model expanded to include a loss resistor shunting the coil.

at resonance could be well matched to the data, even though the model resonated at a slightly different frequency, and had a higher peak impedance. To compensate for errors of this type, the resonant point was monitored for assurance that the magnitude error was meaningful. Increasing the number of data points about the resonant point could help prevent this error.

The parameters for the two drives were measured with the terminals of all other coils open. This assumes that the drive coils do not load with each other in this case, and that no current flows in the pickups. The fits for the drive coil models may be seen in Figures 2.4 and 2.5.

Calculation of the Mutual M_{12}

Calculation of the mutual inductance between the two drive coils was accomplished by using a drive signal on one of the drive coils, and measuring the output on the other, as illustrated in Figure 2.6 on page 44. The assumed circuit is that of a standard transformer, as illustrated in Figure 2.7 on page 45. The primary section of that circuit is illustrated in Figure 2.8 on page 46. I_1 in Figure 2.8a is easily found. The resulting equation may be written in terms of the input impedance of the coil as

$$I_1 = \frac{V_s Z_1}{R_s(Z_1+Z_{in})+Z_1 Z_{in}} . \qquad (2.009)$$

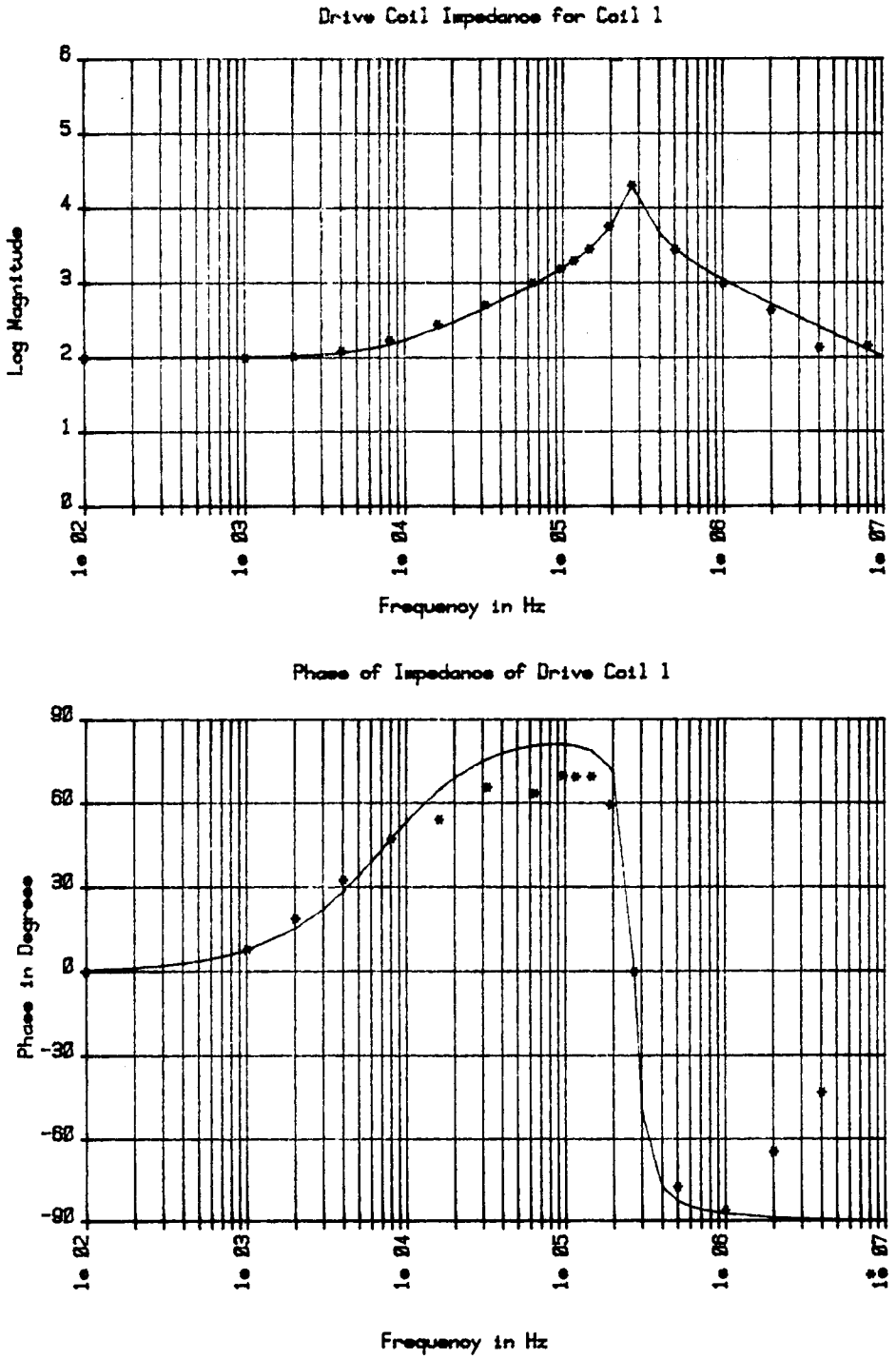


Figure 2.4. The probe drive coil impedance for coil 1 is shown versus frequency. The curves were generated from a model where r_1=102.72Ω, C_1=155.35 pF, R_1=23442Ω, and L_1=2.25 mH.

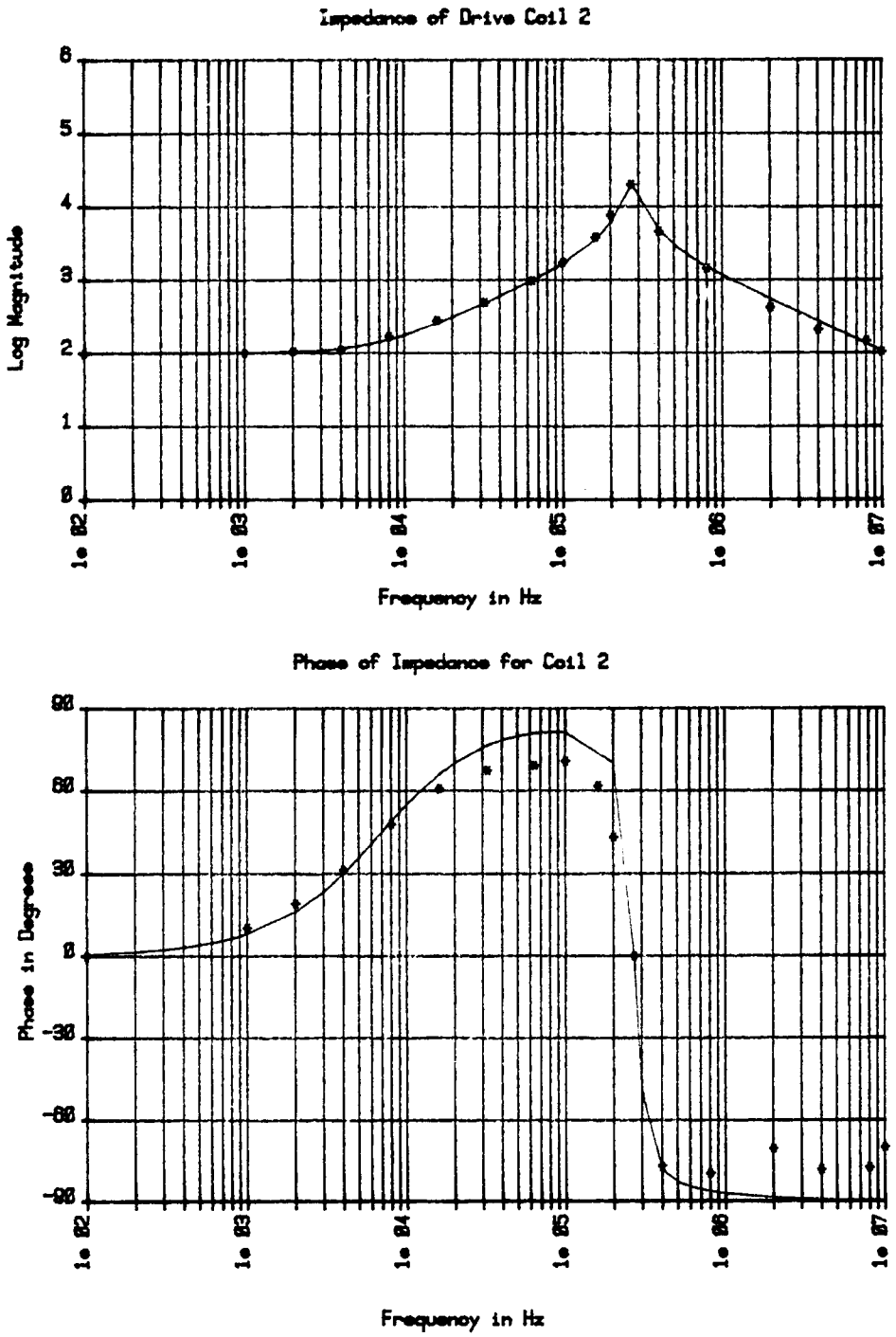


Figure 2.5. The probe drive coil impedance for coil 2 is shown versus frequency. The curves were generated from a model where r_2=99.575Ω, R_2=23137.11Ω, C_2=151.826 pF, and L_2=2.31515 mH.

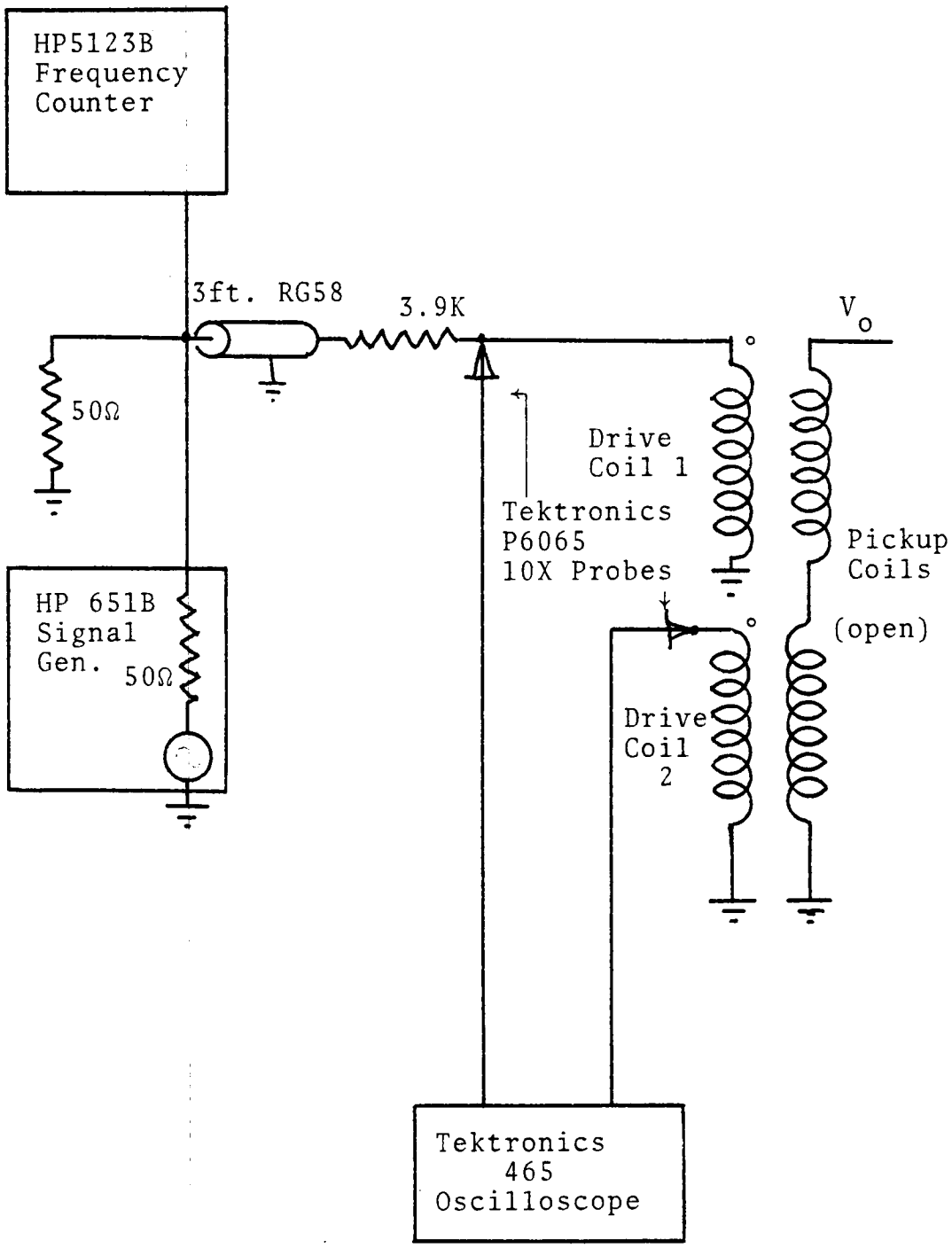
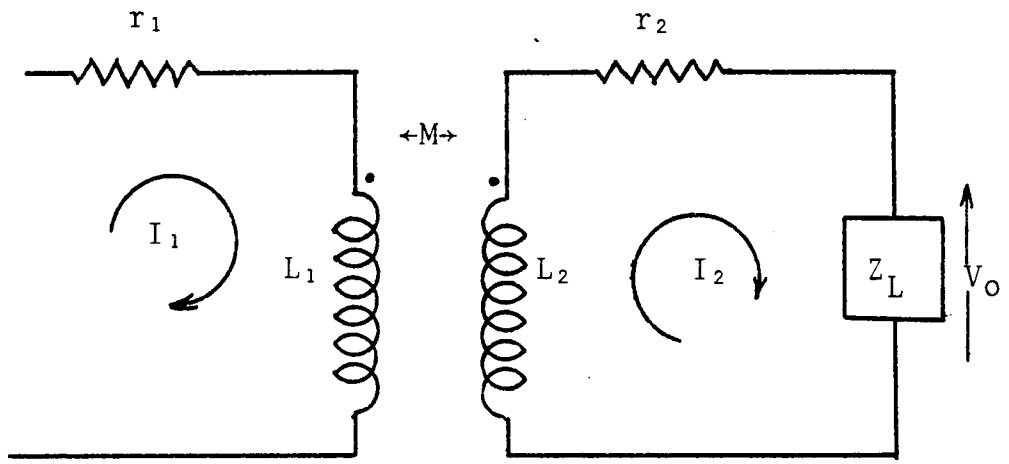
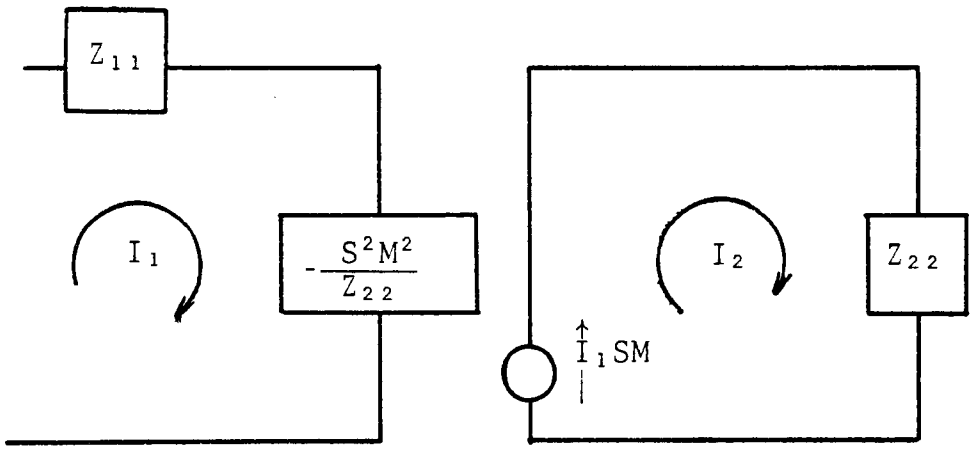


Figure 2.6. Experimental set up used in the measurement of the mutual inductance M_{12}.

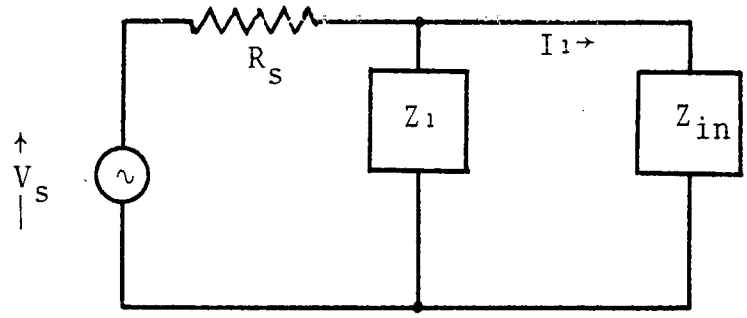


(a)

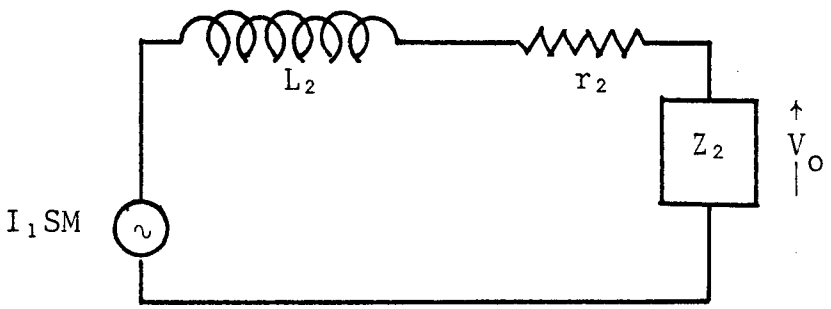


(b)

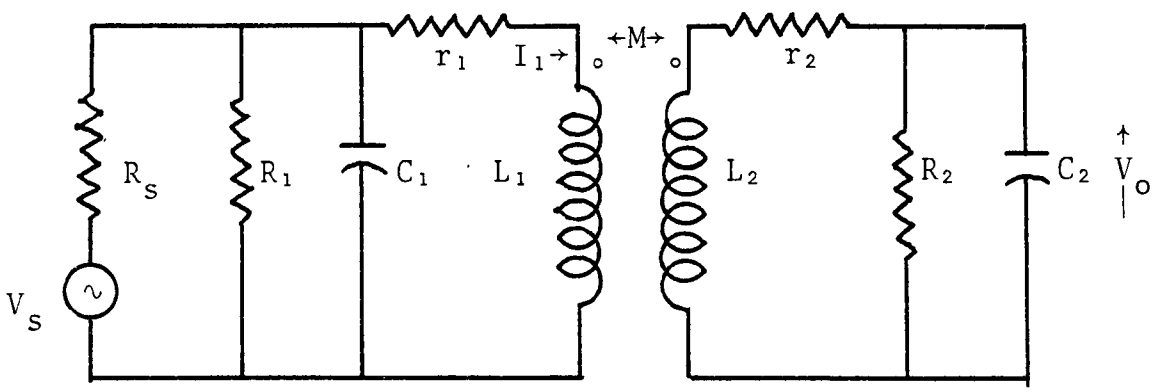
Figure 2.7. (a) Standard transformer model from Hayt and Kemmerly [9] and a modified circuit (b) illustrating the matrix notation used.



(a)



(b)



(c)

Figure 2.8. The drive coil coupling models used: (a) primary model, (b) secondary model, (c) combined primary and secondary circuits in detail.

From Hayt and Kemmerly [9] and Figure 2.7 on page 45, it is known that

$$Z_{in} = Z_{11} - \frac{S^2M^2}{Z_{22}} \quad , \tag{2.010}$$

where, in this case,

$$Z_{11} = r_1 + SL_1 ,$$
$$Z_{22} = r_2 + SL_2 + Z_2 ,$$
$$M = M_{12} .$$

By substituting Equation 2.010 into 2.009, and utilizing the definitions of Z_4 and Z_5 from Chapter I, page 12, it is found that

$$I_1 = \frac{V_S Z_5 Z_1}{R_S(Z_4Z_5 - S^2M_{12}^2) + Z_1[Z_5(r_1+SL_1) - S^2M_{12}^2]} . \tag{2.011}$$

From Figures 2.7b and 2.8b, it is known that

$$V_o = \frac{I_1 Z_2 S M_{12}}{Z_5} . \tag{2.012}$$

Equation 2.011 may be substituted into Equation 2.012, and the resulting equation manipulated into a quadratic form in terms of M_{12}.

$$M_{12}^2[S^2V_o(R_S+Z_1)] + M_{12}(V_oZ_1Z_2S) - V_oZ_5[R_SZ_4 + Z_1(r_1+SL_1)] = 0 . \tag{2.013}$$

Using the quadratic formula to solve Equation 2.013 yields the value of M_{12} as well as an extra root. In practice it is relatively easy to pick out the correct root, due to its value being within the expected range. The program MU12 evaluates Equation 2.013. A conversational routine which asks for the required input, called CONMU12, may also be used to find the value of M_{12}. The routine CQUAD is used to perform the quadratic solution by the standard techniques. These programs are all found in Appendix B.

Calculation of the Mutuals M_{13}-M_{14} and M_{23}-M_{24}

Finding the mutuals between the pickups and the drive coils is a difficult problem, since the center of the two pickup coils is not brought out. Therefore, only the difference in two mutuals may be found. Duncan [4] used only one mutual to represent this difference. This proved convenient for the writing of equations in a compact form. It is also mathematically correct for this simple model, but more complex models would require both mutuals to be computed. Duncan found that the ratio of the mutuals at null were equal to the reciprocal of the drive currents

$$X = \frac{M_{13}-M_{14}}{M_{23}-M_{24}} = \frac{I_2}{I_1} = \frac{Z_2Z_4-SM_{12}Z_1}{Z_1Z_5-SM_{12}Z_2}\ \frac{Z_6}{Z_7} \qquad (2.014)$$

From Equation 2.014 it may be seen that

$$M_{13}-M_{14} = (M_{23}-M_{24})\frac{I_2}{I_1} \ , \qquad (2.015)$$

where I_1 and I_2 are the drive currents at null. It is known that if the drive currents for a point off null can be found, the probe output voltage is

$$V_o' = I_1'S(M_{13}-M_{14})+I_2S(M_{23}-M_{24}) \ , \qquad (2.016)$$

where I_1' and I_2' are the drive currents when the probe is at a point off null, and V_o' is the output voltage at that point. If the difference in the mutuals M_{23} and M_{24} is assumed to be real, then solution for that difference in terms of the magnitude of output voltage V_o' is

$$M_{23}-M_{24} = \frac{V_o'}{|S(I_2'+XI_1')|} \ , \qquad (2.017)$$

where X is defined as in Equation 2.014. The quantity X may be found by knowing the values of the components in the nulling network at null, and calculating the currents as they are defined by Equations 1.010 and 1.011 on page 14. A similar process may be used to find I_1' and I_2' for substitution into Equation 2.017. The circuit used to find V_o' and to compute I_1' and I_2' is shown in Figure 2.9. The nulling network was not used in this experiment, and the currents were computed from the fitted probe parameters alone. The currents I_1 and I_2 for the probe at null were computed from the values of the nulling network components required for null.

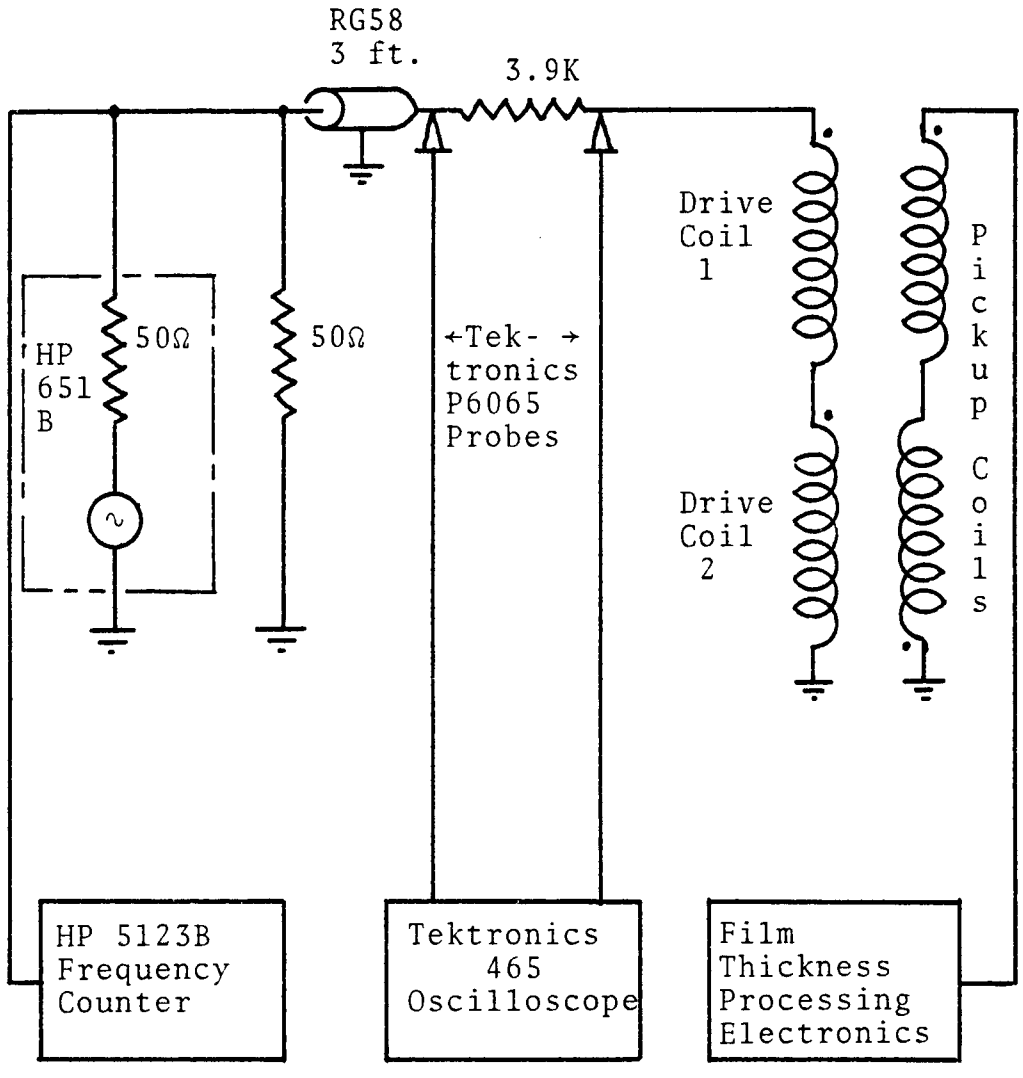


Figure 2.9. Measurement system used for finding the values of the mutuals M_{13}-M_{14} and M_{23}-M_{24}.

The signal processor used in measuring the oil film thickness is shown in Figure 2.9. Details of the processor may be found in Wilkerson [17]. A block diagram of the processor is shown in Figure 2.10. The probe was nulled at a distance from all metallic objects that might cause variations in the probe's output; and after sensitivities were measured, the nulling network was taken off, and the nulling network components were individually measured. The program EMU, found in Appendix B, solves Equation 2.015 conversationally from the previously measured nulling network and parameters. The program then uses Equation 2.015 to find M_{13}-M_{14}. The resulting parameters null the probe so effectively that the mutuals had to be modified slightly, out past the significance of the measurement, to bring the output up to the actual measured nulled value. This was done to avoid the distortion in the sensitivity measurements due to the singularity which exists in classical sensitivity at null.

Measurement of Classical Sensitivity

The Figure 2.11 on page 54 shows the nulling network used in the measurement of the sensitivities. The first thing that is noticed about this set up is that the null network contains six switches. These switches are part of a "dual-in-line" switch of the variety used to set binary data in digital equipment. It was used for its compact size, low on resistance, and low off capacitance.

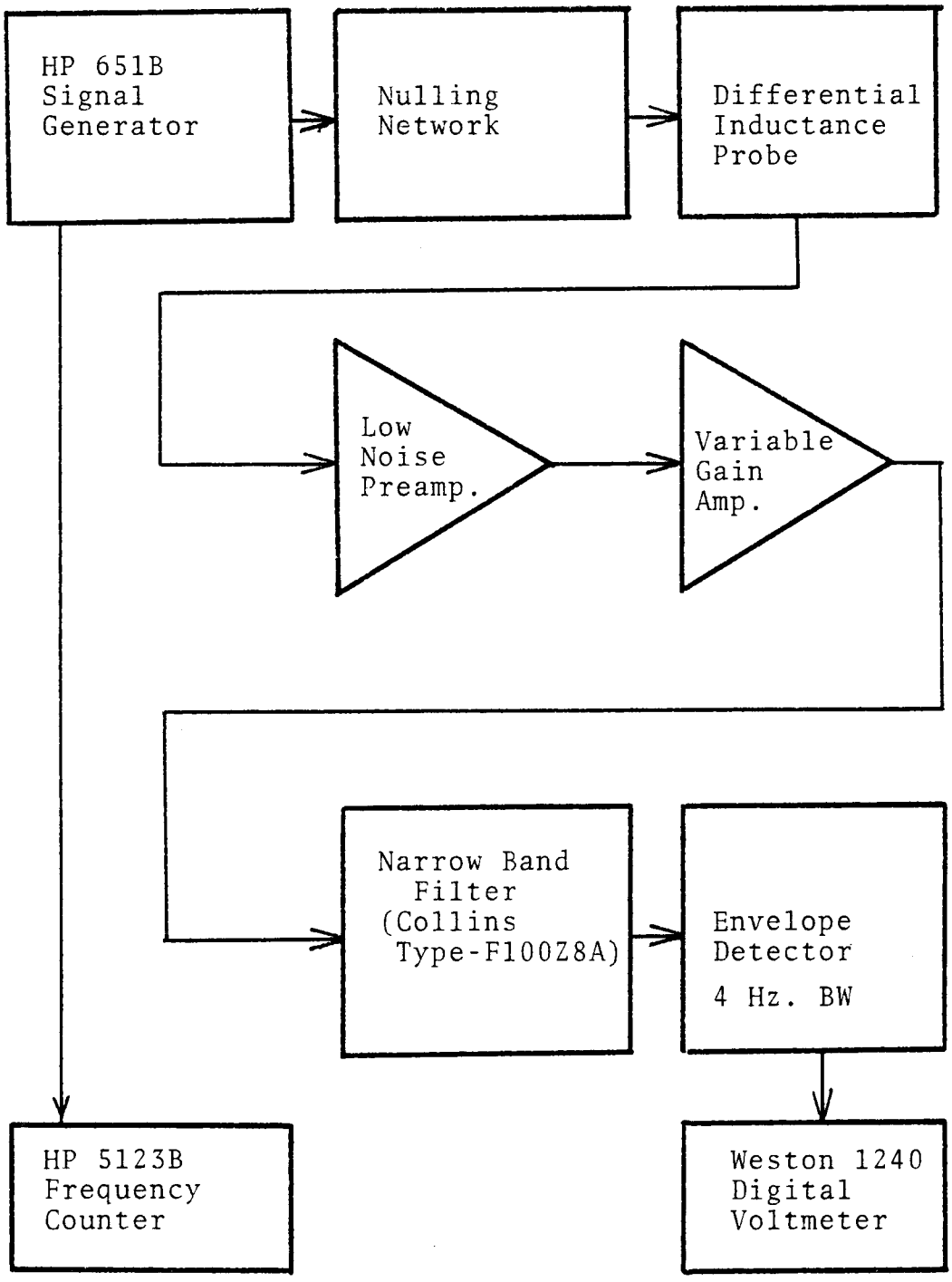


Figure 2.10. Block diagram of measurement system used both in measurement of film thickness and in finding the mutual impedances. This system was also used in determining the sensitivities of the nulling network.

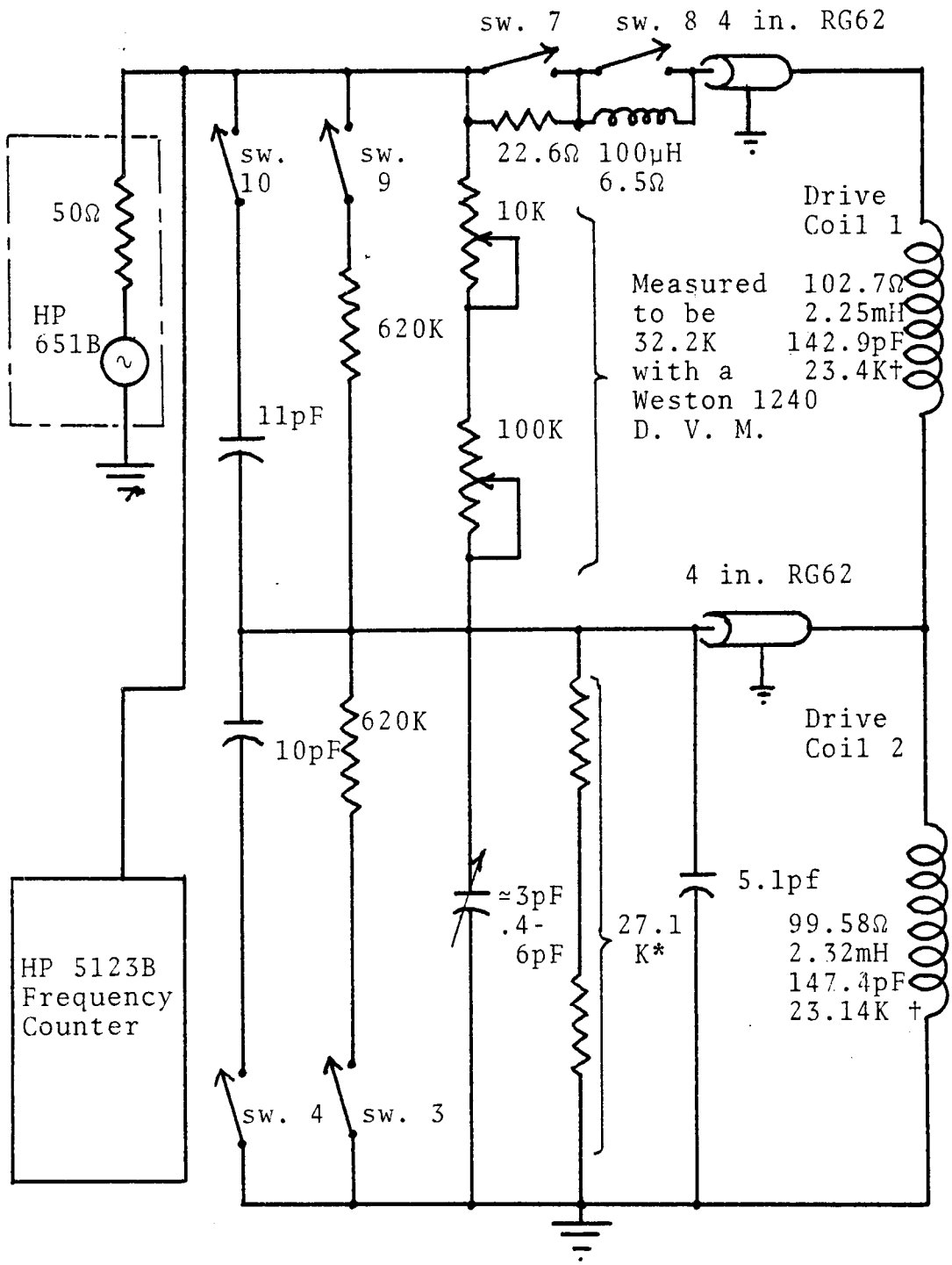


Figure 2.11. Nulling network model used for measurement of sensitivities. Switches are shown in the positions for null except for switches 7 and 8. * Measured.
† Coil parameters given are r_x, L_x, and shunting R and C.

The purpose of these switches is to be able to quickly change the value of a null network parameter several times. The need to change the parameters rapidly was due to the swift system drift. Other means of changing the parameters required so much time that there was much doubt about the value of the output voltage with the parameter unchanged. Techniques such as soldering caused gross output changes due to the altered temperature of the nulling network. The switches allowed each measurement to be made many times, and the change in the output to be quickly noted. The mean changes in the output were computed and used to compute the sensitivities. The probe set up was such that very few of the parameters could be changed. Inductance of drive coil changes were simulated by opening switch 8, which put 100 micro-Henrys in series with drive coil 1. Similarly, switch 7 provided a change in the series resistance of the coil. Switches 10 and 9 provided changes in the shunt capacitance and resistance of drive coil 1. Similarly, switches 3 and 4 changed the shunt capacitance and resistance of drive coil 2. Basically, only the null network parameters could be easily changed. Probe parameters were difficult to modify. Changes in frequency were accomplished by changing the frequency of the signal generated by the Hewlett-Packard 651B signal generator, and observing the change in frequency on a Hewlett-Packard 512B frequency counter. The output of the pickup coils was fed into the

processing electronics normally used for film thickness measurements. A scale factor was measured for the processor and used to convert the D. C. output of the processor to a peak-to-peak voltage measurement. This was required in order to use a set of units consistant with those used in the measurement of the drive level. Although not shown in Figure 2.11 on page 53, a Tektronics 465 Oscilloscope was used to measure the output level of the signal generator.

In Table 1.5 on page 28, the result of these measurements and computations is shown. The parameter changes used in the measurement of sensitivity were made large, so that the error due to the drift in the output of the probe would cause less error in the measurements. The output can easily go in a negative direction, through zero, and then increase the magnitude of the output. Therefore, the sign of the measured output is not very meaningful. The size of the change, however, is quite important, since the change in the output is approximately the same whether the change was an increase or a decrease, and the value of the output was small compared to the change. The calculated sensitivities for null network parameters and frequency were at least an order of magnitude low. The only measurements that were relatively close were those of the inductance and resistance changes; but, if these are accurate, the results are still confusing, since the measurement technique is only valid if the shunt capacitance and

resistance innate to the coils are small. The fitting of the probe parameters and measurements on similar coils used by Galyon [7] tends to discount this possibility.

Summary

In this chapter techniques of obtaining probe parameters from a minimum amount of data were developed. The measurement techniques for finding sensitivity values were described. Some of the problems in making these measurements were illuminated.

In the next chapter, the conclusions reached and the recommendations will be discussed. These will include not only recommendations about the improvements in sensitivity related areas, but general approaches to understanding the functioning of the probe and related circuits.

CHAPTER III

RESULTS, CONCLUSIONS AND RECOMMENDATIONS

Results

In this thesis, the classical sensitivities were derived for the probe and nulling network model parameters. A numerical technique to find the values of the sensitivities was found in the literature and applied to the transfer function, Equation 1.016 on page 16. The values of the sensitivities computed from the numerical technique and the values of the sensitivities found by evaluating the sensitivity functions had excellent correlation. The high correlation between the two calculation techniques gives support for the derivations of the sensitivity functions. The sensitivities of the nulling network parameters were measured and compared with the computed values. The comparison of the measured and the computed sensitivities was not good. Although some of the techniques used to estimate probe parameters were somewhat weak, the errors in these estimates are not considered to be of such a magnitude as to cause this size of an error. The problems encountered with modeling and sensitivity measurement are believed to be from four sources: insufficient experimental data was available to make accurate estimations of probe parameters, the probe model was inadequate to explain observed probe behavior, high level of drift in the probe output and the fact that the sensitivity

was measured with the probe output so close to zero.

Much more attention must be paid to the capacitive effects in both the drive coils and the pickup coils. It is possible that the apparent values of capacitances measured at the input of the drive coils may be reflected from the pickup coils or from the other drive coil. This might cause large enough errors in the estimates of these parameters to bring about differences in the measured and calculated values of sensitivity. The experimental data available was not sufficient to resolve this question.

Although the probe model may require improvements to yield significant increases in the correlation between the calculated and measured sensitivity values, some indication of the relative sensitivity of the probe parameters is available by examining the sensitivity values in Table 1.3 on page 25. The magnitude of the probe output is most sensitive to changes in the four mutual inductances which couple the drive coils to the pickup coils. The next most sensitive group of components consists of the effective capacitances which shunt the drive coils and the leakage inductances of the drive coils. The probe is least sensitive to changes in the source resistance of the signal generator. The output is relatively insensitive to changes in the mutual inductance which couples the two drive coils.

The most important observation to be made by examining the computed values of sensitivity is to note that all of

the sensitivities related to drive coil components have an opposite and nearly equal sensitivity listed in Table 1.3 on page 25. These opposite sensitivities are the sensitivities of corresponding components in each of the drive coils. For example, the sensitivities of the two capacitances C_1 and C_2 are nearly equal. This reinforces the intuitive feeling that a probe which is fabricated such that it has very well matched parameters will tend to be much less prone to drift. This assumes that the well matched parameters will have nearly equivalent temperature coefficients and the temperature changes in each of the components will be nearly equal.

Conclusions

From working with and deriving the classical sensitivity functions, it is apparent that when the circuit model reaches this level of complexity, the derivation of formula for the sensitivities is of little value. Only the values of the sensitivities are very useful in determining the best way to decrease probe output drift. These values may be found by the numerical method used in this thesis, with much less effort than going through the derivation and evaluation of these functions. If a more complex model is used in future work, this numerical technique could be easily applied once the transfer function of the new probe model is derived and the model components found.

Classical sensitivity is not very effective when applied to circuits whose output is the difference of two or more expressions. This is due to the fact that the output of the circuit may have the value zero, and this causes the sensitivity to become infinite. A simple modification of the definition can overcome this problem.

The statistical modeling techniques used in Chapter II have general application in the optimization of any reasonably well behaved function. In Chapter II this function was the error between the impedance versus frequency curve generated by the probe model, and the measured data points. This type of optimization is called response surface methodology, and could have extensive use in estimating model component value. It may also prove useful in the optimization of probe performance.

The model used in this theseis, Duncan's [4] thesis, and Galyon's [7] thesis, was not found to be representative of the real probes. Improvements in the probe design may allow the sensitivity functions derived in this thesis to be applied to reduce probe output drift.

The major contributions of this thesis are: The derivation of the sensitivity functions for the probe model, the finding of a suitable numerical technique which will evaluate probe sensitivity and which is applicable to almost any probe model, and the application of response surface methodology to the fitting of model component values to observed circuit performance.

Recommendations for Further Research

Any additional work should begin with a thorough empirical modeling of the probe. This should be done before any changes are made to any part of the system. The probe needs to be resolved into an equivalent circuit which will be representative of how the probe actually behaves. It seems highly unlikely that improvements can be made to a measurement system until a working knowledge of the starting point is obtained. The fitting techniques described in this thesis should help toward this end. The author suggests that a new error criterion be found. A criterion that seems promising is

$$\text{Error} = \sum_{i=1}^{n} \left(\frac{\text{Re}(\hat{Z}_i) - \text{Re}(Z_i)}{|Z_i|} \right)^2 + \left(\frac{\text{Im}(\hat{Z}_i) - \text{Im}(Z_i)}{|Z_i|} \right)^2, \quad (3.001)$$

where n is the number of data points, Z_i is the i'th measured impedance and $\hat{Z}_i$ is the estimate of the i'th impedance. Equation 3.001 may allow the fitting process to be fully automated. It normalizes the real and imaginary parts of the error by the magnitude of the data point and sums the squares of these errors.

Further work in the area of sensitivity should be done on a thoroughly tested model. Attempts at deriving sensitivity from the classical sensitivity definition should be abandoned. Numerical techniques such as those used in this thesis and described by Fidler [6] are probably more useful

considering the complexity of the sensitivity functions and the time that was required to find them. When using techniques such as Fidler's, care must be taken in knowing that the function is bilinear in the parameter of interest.

An alternate definition of sensitivity which is not normalized to the function would be more rewarding in this application, since it would not have a singularity when the function is equal to zero. Equation 3.002 is such a definition.

$$S_x^N = \frac{dN}{\frac{dx}{x}} , \qquad (3.002)$$

where N is the function, x is the parameter of interest, and S_x^N is the sensitivity of N with respect to x.

Statistical techniques would be helpful in experiments performed on the probe. Properly designed experiments tend to eliminate biased experiments and compensate for changes in the system with time.

Response surface methodology [12] would find application in improving probe performance in a minimum number of trials even if the true nature of the probe is poorly understood. A performance criterion could be established and data points taken to indicate what is the best way to modify the probe to improve its performance. This is an expensive approach if probes have to be fabricated for each data point, but it may be the only technique which would lead to an acceptable

level of probe performance. Response surface methodology could have been applied to Duncan's [4] work when he was optimizing the probe performance on the computer. The one at a time approaches to optimization will not reveal cross terms as readily as statistical approaches.

The subject of stray capacitances and parasitic capacitances is found in both Duncan's and Galyon's [7] theses. It is a topic which deserves much attention since it is the problems in this area that make the model used in this thesis suspect. In the field of transformer design, these matters have been considered in the past. Grossner [8] provides quite a bit of information on designing transformers to reduce these effects. He has a formula for estimation of the effective capacitance, and also discusses several techniques for reduction of these capacitances. One of the techniques is sectored primary and secondary windings. The principle underlying this is that if a part of the coil, such as the primary, is broken into, let us say, two pieces, each will have approximately half the capacitance of the previously single sector coil. Then, if these sectors are connected in series, the coil will have one quarter of the capacitance of the original single sector coil. From this, and other topics discussed in Grossner, the author would like to suggest a probe structured as illustrated in Figure 3.1. This structure would help to reduce and balance capactances. It would seem appropriate to mention that the drive

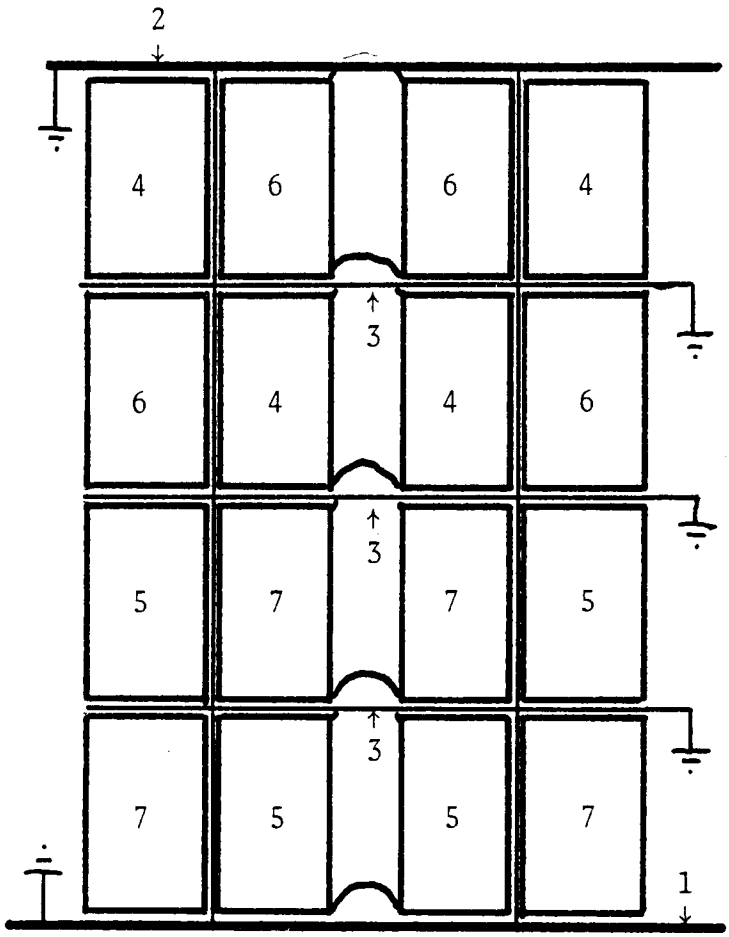


Figure 3.1. A sector wound probe which uses screens to reduce interwinding capacitance. This probe would consist of (1) a stainless steel seal face, (2) a matching stainless steel membrane, (3) screens, (4) sectors of the upper drive coil, (5) sectors of the lower drive coil, (6) sectors of the upper pickup coil, and (7) sectors of the lower pickup coil.

technique used on this probe tends to be inherently imbalanced. Capacitances between the center tap and ground appear to be across the second drive coil, while those between the input of the upper coil and ground appear across both coils.

It would seem wise to investigate the possibility of using screens to reduce the interwinding capacitances instead of separation as was used in Duncan's [4] design. This might allow a smaller probe with lower thermal time constants and a higher sensitivity to be fabricated. Screens also tend to define and make predictable stray capacitances.

Galyon's [7] work with the thermal problems of the probe was very good in that it gave an estimate of the order of magnitude of several of the temperature change and temperature gradient effects on the probe's output. Several temperature effects are not mentioned. One was the effect of the dimensional change in the spacer between the pickup coils. Changes in the pickup coil locations could have significant effect. Also, there was the possibility of the probe becoming asymmetric due to the seal face limiting the expansion in that direction. The point of symmetry for the pickup coils may not move at the same rate as the corresponding point in the drive coils. This is another reason to investigate the feasibility of using a screen instead of a spacer to reduce interwinding capacitances.

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APPENDICES

APPENDIX A

DERIVATION OF CLASSICAL SENSITIVITIES FOR NULLING NETWORK PARAMETERS

The derivation of the equations listed in Table 1.2 on page 21 is contained in this appendix. The approach used was to apply the formula for classical sensitivity, Equation 1.018 on page 16, to the transfer function, Equation 1.016 on page 16 for each of the nulling network and probe parameters. Due to the large number of parameters involved, it was found to be easiest to find the partial derivatives of each of the simplifying terms listed in Table 1.1 on page 12 with respect to the nulling network and probe parameters. Then these partials were used to find the partials of the numerator Q and the denominator P with respect to each of the parameters. From this point Equation 1.018 may be readily applied to find the sensitivity. In this derivation it is assumed that none of these derivatives or sensitivities will be evaluated at a singularity and therefore they exist in the region of interest.

Partial Derivatives of Z_1 and Z_2

The equation defining Z_1 found in Table 1.1 on page 12 is repeated here:

$$Z_1 = \frac{R_1}{1+R_1C_1S} \quad . \tag{A.001}$$

Upon taking the partial derivative of Equation A.001 with respect to R_2, it is found that

$$\frac{\partial Z_1}{\partial R_1} = \frac{(1+R_1C_1S)(1)-(R_1)(C_1S)}{(1+R_1C_1S)^2}, \qquad (A.002)$$

which when reduced yields

$$\frac{\partial Z_1}{\partial R_1} = \frac{1}{(1+R_1C_1S)^2}. \qquad (A.003)$$

A similar process performed upon the definition of Z_2 on page 12 in Table 1.1 gives similar results:

$$\frac{\partial Z_2}{\partial R_2} = \frac{1}{(1+R_2C_2S)^2}. \qquad (A.004)$$

Taking the partial of Equation A.001 with respect to C_1 gives

$$\frac{\partial Z_1}{\partial C_1} = \frac{-R_1^2S}{(1+R_1C_1S)^2}. \qquad (A.005)$$

Following a similar procedure it is found that the partial derivative of Z_2 with respect to C_2 is:

$$\frac{\partial Z_2}{\partial C_2} = \frac{-R_2^2S}{(1+R_2C_2S)^2}. \qquad (A.006)$$

With respect to complex frequency, it is found that the partial derivatives of Z_1 and Z_2 are

$$\frac{\partial Z_1}{\partial S} = \frac{-R_1^2 C_1}{(1+R_1 C_1 S)^2}, \qquad (A.007)$$

and

$$\frac{\partial Z_2}{\partial S} = \frac{-R_2^2 C_2}{(1+R_2 C_2 S)^2}, \qquad (A.008)$$

respectively.

The partial derivatives with respect to any other parameter is equal to zero for both Z_1 and Z_2 since they are not functions of any other parameters.

Partial Derivitives of Z_3

The equation defining Z_3 is repeated here for reference:

$$Z_3 = Z_1+Z_2+R_S . \qquad (A.009)$$

Taking the derivative of Z_3 with respect to R_1 yields

$$\frac{\partial Z_3}{\partial R_1} = \frac{\partial Z_1}{\partial R_1} = \frac{1}{(1+R_1 C_1 S)^2} . \qquad (A.010)$$

Similarly one can take the partials with respect to R_2, C_1 and C_2:

$$\frac{\partial Z_3}{\partial R_2} = \frac{\partial Z_2}{\partial R_2} = \frac{1}{(1+R_2 C_2 S)^2}, \qquad (A.011)$$

$$\frac{\partial Z_3}{\partial C_1} = \frac{\partial Z_1}{\partial C_1} = \frac{-R_1^2 S}{(1+R_1 C_1 S)^2}, \text{ and} \qquad (A.012)$$

$$\frac{\partial Z_3}{\partial C_2} = \frac{\partial Z_2}{\partial C_2} = \frac{-R_2^2 S}{(1+R_2 C_2 S)^2} . \qquad (A.013)$$

Taking the partial derivative of Z_3 with respect to complex frequency yields

$$\frac{\partial Z_3}{\partial S} = \frac{\partial Z_1}{\partial S} + \frac{\partial Z_2}{\partial S} . \qquad (A.014)$$

Substituting Equations A.008 and A.009 into A.014 gives

$$\frac{\partial Z_3}{\partial S} = \frac{-R_1^2 C_1}{(1+R_1 C_1 S)^2} + \frac{-R_2^2 C_2}{(1+R_2 C_2 S)^2} . \qquad (A.015)$$

By inspection of Equation A.009 the derivative with respect to R_s is

$$\frac{\partial Z_3}{\partial R_s} = 1 . \qquad (A.016)$$

All other derivatives with of Z_3 are equal to zero.

Partial Derivatives of Z_4

The definition of Z_4 is repeated here for convenience:

$$Z_4 = Z_1 + r_1 + SL_1 . \qquad (A.017)$$

Finding the derivative of Z_4 with respect to r_1 by inspection:

$$\frac{\partial Z_4}{\partial r_1} = 1 . \qquad (A.018)$$

Since the derivative of Z_4 with respect to Z_1 is also one, the derivatives of R_1 and C_1 are equal for Z_4 and Z_1. Therefore the following equations are true:

$$\frac{\partial Z_4}{\partial R_1} = \frac{\partial Z_1}{\partial R_1} = \frac{1}{(1+R_1C_1S)^2}, \text{ and} \qquad (A.019)$$

$$\frac{\partial Z_4}{\partial C_1} = \frac{\partial Z_1}{\partial C_1} = \frac{-R_1^2S}{(1+R_1C_1S)^2}. \qquad (A.020)$$

The partial of Z_4 with respect to complex frequency is

$$\frac{\partial Z_4}{\partial S} = \frac{\partial Z_1}{\partial S} + L_1 = \frac{-R_1^2C_1}{(1+R_1C_1S)} + L_1 . \qquad (A.021)$$

By inspection of Equation A.017 the derivative with respect to L_1 is found to be

$$\frac{\partial Z_4}{\partial L_1} = S . \qquad (A.022)$$

All other derivatives of Z_4 are equal to zero. This is due to the fact that there are no other parameters in Z_4.

Partial Derivitives of Z_5

As before the definition of Z_5 is repeated here for convenience:

$$Z_5 = Z_2 + r_2 + SL_2 . \qquad (A.023)$$

Since Z_5 is analogous to Z_4 when one considers that one is the sum of the impedances around the loop of drive coil 2 and the other is the sum of impedances around the loop of drive coil 1 excluding reflected impedances, a similar set of derivatives can be generated:

$$\frac{\partial Z_5}{\partial r_2} = 1 , \qquad (A.024)$$

$$\frac{\partial Z_5}{\partial R_2} = \frac{1}{(1+R_2C_2S)^2} , \qquad (A.025)$$

$$\frac{\partial Z_5}{\partial C_2} = \frac{-R_2{}^2S}{(1+R_2C_2S)^2} , \qquad (A.026)$$

$$\frac{\partial Z_5}{\partial S} = \frac{-R_2{}^2C_2}{(1+R_2C_2S)^2} + L_2 , \text{ and} \qquad (A.027)$$

$$\frac{\partial Z_5}{\partial L_2} = S . \qquad (A.028)$$

And as before all other first derivatives are equal to zero since Z_5 is not a function of any other parameters.

Partial Derivatives of Z_6

The partial derivatives of Z_6 are a little more complex that the previous ones due to the fact that it contains several of the previous simplifying expressions within it. The definition of Z_6 is repeated here to illustrate this:

$$Z_6 = Z_2Z_4 - SM_{12}Z_1 . \qquad (A.029)$$

Attacking each of these partials in turn starting with the partial with respect to R_1:

$$\frac{\partial Z_6}{\partial R_1} = Z_2\frac{\partial Z_4}{\partial R_1} + Z_4\frac{\partial Z_2}{\partial R_1} + (-SM_{12})\frac{\partial Z_1}{\partial R_1} . \qquad (A.030)$$

Substituting Equations A.019 from page 74 and A.003 from page 71 into Equation A.030 and knowing that Z_2 is not

a function of R_1 one arrives at the equation,

$$\frac{\partial Z_6}{\partial R_1} = \frac{\partial Z_1}{\partial R_1}(Z_2 - SM_{12}) . \qquad (A.031)$$

When taking the derivative with respect to R_2 one notices that only Z_2 is a function of R_2. This allows the derivative to be taken by inspection.

$$\frac{\partial Z_6}{\partial R_2} = Z_4 \frac{\partial Z_2}{\partial R_2} . \qquad (A.032)$$

If one notices that all of Equation A.031 excluding the derivative of Z_1 with respect to R_1 is equal to the derivative of Z_6 with respect to Z_1, then one may easily see that

$$\frac{\partial Z_6}{\partial C_1} = \frac{\partial Z_1}{\partial C_1}(Z_2 - SM_{12}) . \qquad (A.033)$$

Similar thinking about Equation A.032 results in the equation below:

$$\frac{\partial Z_6}{\partial C_2} = Z_4 \frac{\partial Z_2}{\partial C_2} . \qquad (A.034)$$

The only occurance of L_1 in Z_6 is in Z_4, so the derivative with respect to L_1 may be easily determined by examining Equation A.029 and Equation A.022:

$$\frac{\partial Z_6}{\partial L_1} = SZ_2 . \qquad (A.035)$$

By examining Equation A.029, it may be observed that only Z_4 is a function of r_1 and its derivative with respect to r_1 is Equation A.018 on page 73. From these it follows that

$$\frac{\partial Z_6}{\partial r_1} = Z_2 \ . \qquad \text{(A.036)}$$

Taking the partial derivative of Z_6 with respect to M_{12} directly and knowing its only occurrence within Z_6 is the obvious one in the second term, it is found that

$$\frac{\partial Z_6}{\partial M_{12}} = -SZ_1 \ . \qquad \text{(A.037)}$$

The derivative of complex frequency is always more difficult due to the fact that it may be found in every term of the defining Equation A.029. It follows from this that

$$\frac{\partial Z_6}{\partial S} = Z_2\frac{\partial Z_4}{\partial S} + Z_4\frac{\partial Z_2}{\partial S} - SM_{12}\frac{\partial Z_1}{\partial S} - Z_1\frac{\partial SM_{12}}{\partial S} . \qquad \text{(A.038)}$$

Using the Equation A.022 on page 72, carrying the partial derivatives of Z_1 and Z_2 with respect to S, performing the required operation on the last term and reducing yields

$$\frac{\partial Z_6}{\partial S} = \frac{\partial Z_1}{\partial S}(Z_2 - SM_{12}) + \frac{\partial Z_2}{\partial S}(Z_4) - M_{12}Z_1 + Z_2L_1 \ . \qquad \text{(A.039)}$$

The motive for carrying the derivatives of Z_1 and Z_2 becomes obvious as the process continues. The sheer size of the expressions becomes quite a problem.

Partial Derivatives of Z_7

As in the past, the defining equation for Z_7 is repeated here for convenience:

$$Z_7 = Z_1 Z_5 - SM_{12} Z_2 \ . \qquad (A.040)$$

The process of arriving at the derivatives with respect to Z_7 is so similar to arriving at those for Z_6 that they are just listed below:

$$\frac{\partial Z_7}{\partial R_2} = \frac{\partial Z_7}{\partial R_2} (Z_1 - SM_{12}), \qquad (A.041)$$

$$\frac{\partial Z_7}{\partial R_1} = Z_5 \frac{\partial Z_1}{\partial R_1} \ , \qquad (A.042)$$

$$\frac{\partial Z_7}{\partial C_2} = \frac{\partial Z_2}{\partial C_2} (Z_1 - SM_{12}) \ , \qquad (A.043)$$

$$\frac{\partial Z_7}{\partial C_1} = Z_5 \frac{\partial Z_1}{\partial C_1} \ , \qquad (A.044)$$

$$\frac{\partial Z_7}{\partial L_2} = SZ_1 \ , \qquad (A.045)$$

$$\frac{\partial Z_7}{\partial r_2} = Z_1 \ , \qquad (A.046)$$

$$\frac{\partial Z_7}{\partial M_{12}} = -SZ_2 \ , \text{ and} \qquad (A.047)$$

$$\frac{\partial Z_7}{\partial S} = \frac{\partial Z_1}{\partial S}(Z_5) + \frac{\partial Z_2}{\partial S}(Z_1 - SM_{12}) + Z_1 L_2 - Z_2 M_{12} . \qquad (A.048)$$

Partial Derivatives of Z_8

Below is the the defining relation for Z_8 repeated here for convenience:

$$Z_8 = -Z_1^2 Z_5 \ . \tag{A.049}$$

The partial derivative may be found by utilizing the fact that Z_5 is not a function of R_1, and Z_1 is

$$\frac{\partial Z_8}{\partial R_1} = -2Z_1 Z_5 \frac{\partial Z_1}{\partial R_1} \ . \tag{A.050}$$

A similar process used for the same reason when finding the derivative with respect to C_1 yields

$$\frac{\partial Z_8}{\partial C_1} = -2Z_1 Z_5 \frac{\partial Z_1}{\partial C_1} \ . \tag{A.051}$$

When finding the partial with respect to R_2, it is helpful to note that only Z_5 is a function of R_2 and therefore if one uses Equations A.025 on page 75 and A.004 on page 71, it is found that

$$\frac{\partial Z_8}{\partial R_2} = -Z_1^2 \frac{\partial Z_2}{\partial R_2} \ . \tag{A.052}$$

Similarly employing equations A.006 on page 71 and A.026 on page 75 yields

$$\frac{\partial Z_8}{\partial C_2} = -Z_1^2 \frac{\partial Z_2}{\partial C_2} \ . \tag{A.053}$$

Since L_2 is only found in Z_5, one can arrive at Equation A.054 by using the Equations A.049 and A.028 on page 75. The resulting partial derivative is

$$\frac{\partial Z_8}{\partial L_2} = -Z_1^2 S \ . \qquad \text{(A.054)}$$

Considering the derivative of Z_8 with respect to r_2, one finds that again only Z_5 is a function of r_2. Applying the normal derivative rules and using Equation A.024 on page 75 to find the derivative of Equation A.049 results in

$$\frac{\partial Z_8}{\partial r_2} = -Z_1^{\ 2} \ . \qquad \text{(A.055)}$$

Since both Z_1 and Z_5 are functions of S, the partial derivative with respect to Z_8 is more complex. From Equations A.049 and A.028 on page 75, one finds that

$$\frac{\partial Z_8}{\partial S} = -2Z_1 Z_5 \frac{\partial Z_1}{\partial S} - Z_1^{\ 2} \frac{\partial Z_2}{\partial S} - Z_1^{\ 2} L_2 \ . \qquad \text{(A.056)}$$

Partial Derivatives of Z_9

The definition of Z_9 is

$$Z_9 = -Z_2^{\ 2} Z_4 \ . \qquad \text{(A.057)}$$

Notice the similarity in form between Equations A.057 and A.049 on page 78. The derivatives will be of analagous forms and are

$$\frac{\partial Z_9}{\partial R_1} = -Z_2{}^2 \frac{\partial Z_1}{\partial R_1} , \qquad (A.058)$$

$$\frac{\partial Z_9}{\partial R_2} = -2Z_2 Z_4 \frac{\partial Z_2}{\partial R_2} , \qquad (A.058)$$

$$\frac{\partial Z_9}{\partial C_1} = -Z_2{}^2 \frac{\partial Z_1}{\partial C_1} , \qquad (A.060)$$

$$\frac{\partial Z_9}{\partial C_2} = -2Z_2 Z_4 \frac{\partial Z_2}{\partial C_2} , \qquad (A.061)$$

$$\frac{\partial Z_9}{\partial L_1} = -Z_2{}^2 S , \qquad (A.062)$$

$$\frac{\partial Z_9}{\partial r_1} = -Z_2{}^2 , \text{ and} \qquad (A.063)$$

$$\frac{\partial Z_9}{\partial S} = -2Z_2 Z_4 \frac{\partial Z_2}{\partial S} - Z_2{}^2 \frac{\partial Z_1}{\partial S} - Z_2{}^2 L_1 . \qquad (A.064)$$

Partial Derivatives with Respect to Z_{10}

The definition of Z_{10} is repeated in Equation A.065:

$$Z_{10} = Z_3(-S^2 M_{12}{}^2 + Z_4 Z_5) . \qquad (A.065)$$

Starting with R_1,

$$\frac{\partial Z_{10}}{\partial R_1} = \frac{\partial Z_3}{\partial R_1}(-S^2 M_{12}{}^2 + Z_4 Z_5) + Z_3\left(Z_5 \frac{\partial Z_4}{\partial R_1}\right) . \qquad (A.066)$$

Drawing upon Equations A.065, A.010 on page 72 and A.019 on page 74 to give the equation,

$$\frac{\partial Z_{10}}{\partial R_1} = \frac{\partial Z_1}{\partial R_1} (-S^2M_{12}^2+Z_4Z_5+Z_3Z_5) \cdot \qquad (A.067)$$

Utilizing the similar equations relating to R_2 yields corresponding results. From Equations A.065, A.011 on page 72 and A.025 on page 75, it follows that

$$\frac{\partial Z_{10}}{\partial R_2} = \frac{\partial Z_2}{\partial R_2} (-S^2M_{12}^2+Z_4Z_5+Z_3Z_4) \ . \qquad (A.068)$$

Knowing that the expressions in parenthesis in Equations A.066 and A.067 are partial derivatives of Z_{10} with respect to Z_1 and Z_2 respectively it follows that the derivatives with respect to C_1 and C_2 are

$$\frac{\partial Z_{10}}{\partial C_1} = \frac{\partial Z_1}{\partial C_1} (-S^2M_{12}^2+Z_4Z_5+Z_3Z_5), \text{ and} \qquad (A.069)$$

$$\frac{\partial Z_{10}}{\partial C_2} = \frac{\partial Z_2}{\partial C_2} (-S^2M_{12}^2+Z_4Z_5+Z_3Z_4) \ . \qquad (A.070)$$

From Equations A.018 on page 73 and A.024 on page 75, it may be seen that Z_4 has a non-zero derivative with respect to r_1 and Z_5 has a non-zero derivative with respect to r_2. All other terms in Z_{10} have zero as the derivative with respect to these impedances. Therefore it follows that

$$\frac{\partial Z_{10}}{\partial r_1} = Z_3Z_5 \ , \qquad (A.071)$$

and

$$\frac{\partial Z_{10}}{\partial r_2} = Z_3 Z_4 \ . \qquad (A.072)$$

Since only Z_3 is a function of R_s and from Equation A.016 on page 73 it is known that the derivative is unity, it becomes obvious from Equation A.065 on page 81 that

$$\frac{\partial Z_{10}}{\partial R_s} = -S^2 M_{12}^2 + Z_4 Z_5 \ . \qquad (A.073)$$

It has been shown that a non-zero partial derivative exists for Z_4 with respect to L_1 and Z_5 with respect to L_2. From Equations A.022 on page 74 and A.028 on page 75 it follows that

$$\frac{\partial Z_{10}}{\partial L_1} = Z_3 Z_5 S \ , \text{ and} \qquad (A.074)$$

$$\frac{\partial Z_{10}}{\partial L_2} = Z_3 Z_4 S \ . \qquad (A.075)$$

The derivative with respect to M_{12} must be taken directly from Equation A.065 on page 81:

$$\frac{\partial Z_{10}}{\partial M_{12}} = -2 Z_3 S^2 M_{12} \ . \qquad (A.076)$$

Taking the derivative of Equation A.065 with respect to complex frequency must be done in several steps since

all terms in the equation are functions of it. From Equations A.014 on page 73, A.021 on page 74, and A.027 on page 75, it follows that

$$\frac{\partial Z_{10}}{\partial S} = \frac{\partial Z_3}{\partial S}(-S^2M_{12}^2+Z_4Z_5)+Z_3\frac{\partial(-S^2M_{12}^2+Z_4Z_5)}{\partial S} . \qquad (A.077)$$

Equation A.076 expands to

$$\frac{\partial Z_{10}}{\partial S} = \frac{\partial Z_1}{\partial S}(-S^2M_{12}^2+Z_4Z_5+Z_3Z_5) + \frac{\partial Z_2}{\partial S}(-S^2M_{12}^2+Z_4Z_5+Z_3Z_4) + Z_3(-2M_{12}^2+Z_5L_1+Z_4L_2) . \qquad (A.078)$$

Partial Derivatives of Z_{11}

For convenience the defining equation for Z_{11} is repeated here:

$$Z_{11} = 2SM_{12}Z_1Z_2 . \qquad (A.079)$$

The derivatives with respect to R_1, R_2, C_1, C_2 and M_{12} may be easily found from Equation A.079:

$$\frac{\partial Z_{11}}{\partial R_1} = 2SM_{12}Z_2\frac{\partial Z_1}{\partial R_1} , \qquad (A.080)$$

$$\frac{\partial Z_{11}}{\partial R_2} = 2SM_{12}Z_1\frac{\partial Z_2}{\partial R_2} , \qquad (A.081)$$

$$\frac{\partial Z_{11}}{\partial C_2} = 2SM_{12}Z_1\frac{\partial Z_2}{\partial C_2} \quad , \tag{A.082}$$

$$\frac{\partial Z_{11}}{\partial C_1} = 2SM_{12}Z_2\frac{\partial Z_1}{\partial C_1} \quad ; \text{ and} \tag{A.083}$$

$$\frac{\partial Z_{11}}{\partial M_{12}} = 2SZ_1Z_2 \quad . \tag{A.084}$$

The derivative with respect to S also may be found directly from Equation A.079:

$$\frac{\partial Z_{11}}{\partial S} = 2M_{12}(Z_1Z_2+SZ_2\frac{\partial Z_1}{\partial S}+SZ_1\frac{\partial Z_2}{\partial S}) \quad . \tag{A.085}$$

The Partial Derivitives of Q

The definition of Q is repeated here for reference:

$$Q = S[(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6] \quad . \tag{A.086}$$

It follows from Equations A.086, A.031 on page 76, and A.042 on page 78 that

$$\frac{\partial Q}{\partial R_1} = \frac{\partial Z_1}{\partial R_1} S[(M_{13}-M_{14})Z_5+(M_{23}-M_{24})(Z_2-SM_{12})] . \tag{A.087}$$

Similarly it follows from Equations A.086, A.042 on page 78, and A.032 on page 76 that

$$\frac{\partial Q}{\partial R_2} = \frac{\partial Z_2}{\partial R_2} S[(M_{23}-M_{24})Z_4+(M_{13}-M_{14})(Z_1-SM_{12})] . \tag{A.088}$$

From Equations A.086, A.044 on page 78, and A.033 on page 76, it follows that

$$\frac{\partial Q}{\partial C_1} = \frac{\partial Z_1}{\partial C_1} S[(M_{13}-M_{14})Z_5+(M_{23}-M_{24})(Z_2-SM_{12})] \quad . \quad (A.089)$$

Similarly it follows from Equations A.086, A.043 on page 78, and A.034 on page on page 76 that

$$\frac{\partial Q}{\partial C_2} = \frac{\partial Z_1}{\partial C_2} S[(M_{23}-M_{24})Z_4+(M_{13}-M_{14})(Z_1-SM_{12})] \quad . \quad (A.090)$$

Since only Z_6 is a function of r_1, and only Z_7 is a function of r_2, then from Equations A.046 on page 78, and A.036 on page 77, the derivatives of Equation A.086 are

$$\frac{\partial Q}{\partial r_1} = S(M_{23}-M_{24})Z_2 \quad , \text{ and} \quad (A.091)$$

$$\frac{\partial Q}{\partial r_2} = S(M_{13}-M_{14})Z_1 \quad . \quad (A.092)$$

Similarly since only Z_6 is a function of L_1, and only Z_7 is a function of L_2, then from Equations A.045 on page 78, and A.035 on page 76, the derivatives are

$$\frac{\partial Q}{\partial L_1} = S^2(M_{23}-M_{24})Z_2 \quad , \text{ and} \quad (A.093)$$

$$\frac{\partial Q}{\partial L_2} = S^2(M_{13}-M_{14})Z_1 \quad . \quad (A.094)$$

Both Z_6 and Z_7 are functions of M_{12} which may be seen from Equations A.037 and A.047. It follows from Equation A.086 that the derivative is

$$\frac{\partial Q}{\partial M_{12}} = -S^2[Z_2(M_{13}-M_{14})+Z_1(M_{23}-M_{24})] \ . \qquad (A.095)$$

The derivative with respect to complex frequency may be found by applying the rules for derivatives to A.086 and expanding them by using the results already found for Z_6 in Equation A.039 on page 77 and for Z_7 in Equation A.048 on page 78:

$$\frac{\partial Q}{\partial S} = (M_{13}-M_{14})(Z_7+S\frac{\partial Z_7}{\partial S})+(M_{23}-M_{24})(Z_6+S\frac{\partial Z_6}{\partial S}) \ , \text{ or } \qquad (A.096)$$

$$\frac{\partial Q}{\partial S} = (M_{13}-M_{14})\{Z_7+S[Z_5\frac{\partial Z_1}{\partial S} + \frac{\partial Z_2}{\partial S}(Z_1-SM_{12})+Z_1L_2-Z_2M_{12}]\}+$$

$$(M_{23}-M_{24})\{Z_6+S[\frac{\partial Z_1}{\partial S}(Z_2-SM_{12})+\frac{\partial Z_2}{\partial S}Z_4+Z_2L_1-Z_1M_{12}]\}. \qquad (A.097)$$

The partial derivatives with respect to the mutuals coupling the drive coils to the pickups are easily obtaine- for Equation A.086 directly:

$$\frac{\partial Q}{\partial M_{13}} = SZ_7 \ , \qquad (A.098)$$

$$\frac{\partial Q}{\partial M_{14}} = -SZ_7 \ , \qquad (A.099)$$

$$\frac{\partial Q}{\partial M_{23}} = SZ_6 \text{ , and} \qquad \text{(A.100)}$$

$$\frac{\partial Q}{\partial M_{24}} = -SZ_6 \text{ .} \qquad \text{(A.101)}$$

Partial Derivatives of P

The quantity P is defined as

$$P = Z_8+Z_9+Z_{10}+Z_{11} \text{ .} \qquad \text{(A.102)}$$

Using Equations A.050 on page 79, A.058 on page 81, A.065 on page 81, and A.080 on page 84, one may find the derivative with respect to R_1:

$$\frac{\partial P}{\partial R_1} = \frac{\partial Z_1}{\partial R_1}(-Z_2^2-2Z_1Z_5+Z_3Z_5-S^2M_{12}^2+Z_4Z_5+2SM_{12}Z_2) \text{.} \qquad \text{(A.103)}$$

From Equations A.052 on page 79, A.058 on page 81, A.068 on page 82 and A.081 on page 83, one can arrive at the derivative with respect to R_2:

$$\frac{\partial P}{\partial R_2} = \frac{\partial Z_2}{\partial R_2}(-Z_1^2-2Z_2Z_4+Z_3Z_4-S^2M_{12}^2+Z_4Z_5+2SM_{12}Z_1) \text{.} \qquad \text{(A.104)}$$

Taking advantage of Equations A.051 on page 79, A.060 on page 81, A.069 on page 82, and A.082 on page 84, one may obtain the derivative with respect to C_1:

$$\frac{\partial P}{\partial C_1} = \frac{\partial Z_1}{\partial C_1}(-Z_2^2-2Z_1Z_5+Z_3Z_5-S^2M_{12}^2+Z_4Z_5+2SM_{12}Z_2) \text{.} \qquad \text{(A.105)}$$

Similarly from Equations A.053 on page 79, A.061 on page 81, A.070 on page 82, and A.082 on page 85, the derivative with respect to C_2 is found to be

$$\frac{\partial P}{\partial C_2} = \frac{\partial Z_2}{\partial C_2}(-Z_1^2-2Z_2Z_4+Z_3Z_4-S^2M_{12}^2+Z_4Z_5+2SM_{12}Z_2). \quad (A.106)$$

Using Equations A.062 on page 81, and A.074 on page 83, one may find the derivative with respect to L_1

$$\frac{\partial P}{\partial L_1} = S(-Z_2^2+Z_3Z_5). \quad (A.107)$$

It follows from Equations A.054 on page 80, and A.075 on page 83 that

$$\frac{\partial P}{\partial L_2} = S(-Z_1^2+Z_3Z_4). \quad (A.108)$$

Drawing upon Equations A.063 on page 81, and A.071 on page 82, it is found that

$$\frac{\partial P}{\partial r_1} = -Z_2^2+Z_3Z_5. \quad (A.109)$$

A similar set of equations, A.055 on page 80, A.072 on page 83, results in

$$\frac{\partial P}{\partial r_2} = -Z_1^2+Z_3Z_4. \quad (A.110)$$

Continuing to sum the derivatives of Z_8, Z_9, Z_{10}, and Z_{11}

results in the rest of the partials with respect to P. From Equations A.076 on page 83, and A.084 on page 85 comes

$$\frac{\partial P}{\partial M_{12}} = 2S(-Z_3SM_{12}+Z_1Z_2). \qquad (A.111)$$

From Equations A.073 on page 83 comes the equation

$$\frac{\partial P}{\partial R_s} = -S^2M_{12}^2+Z_4Z_5 . \qquad (A.112)$$

Finally utilizing Equations A.056 on page 80, A.064 on page 81, A.078 on page 84 and A.084 on page 85, the derivative with respect to complex frequency may be found

$$\frac{\partial P}{\partial S} = \frac{\partial Z_1}{\partial S}[-Z_2^2+Z_5(-2Z_1+Z_4+Z_3)-S^2M_{12}^2+2SM_{12}Z_2]+$$
$$\frac{\partial Z_2}{\partial S}[-Z_1^2+Z_4(-2Z_2+Z_5+Z_3)-S^2M_{12}^2+2SM_{12}Z_1]+$$
$$2M_{12}Z_1Z_2-Z_1^2L_2-Z_2^2L_1+Z_3(Z_5L_1+Z_4L_2-2SM_{12}^2). \qquad (A.113)$$

Sensitivities

Having taken all the partial derivatives with respect to each of the basic circuit parameters, it is now easy to apply the classical sensitivity equation

$$S_x^N = x\left(\frac{\frac{\partial Q}{\partial x}}{\partial Q} - \frac{\frac{\partial P}{\partial x}}{\partial P}\right) \quad \text{where } N=\frac{P}{Q}. \qquad (A.114)$$

Substituting Equations A.102, A.103, A.086 on page 85, and A.087 into A.114 and reducing where possible yields

$$S_{R_1}^{T=R_1\frac{\partial Z_1}{\partial R_1}}\left\{\frac{(M_{23}-M_{24})(Z_2-SM_{12})+Z_5(M_{13}-M_{14})}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6}-\right.$$

$$\left.\frac{Z_5(-2Z_1+Z_4+Z_3)-(Z_2-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}}\right\}, \qquad \text{(A.115)}$$

where $T=\frac{Q}{P}=\frac{V_o}{V_s}$.

Similarly from Equations A.102, A.104, A.086 on page 85, and A.088 on page 85, it may be found that Equation A.114 results in the equation

$$S_{R_2}^{T=R_2\frac{\partial Z_2}{\partial R_2}}\left\{\frac{(M_{13}-M_{14})(Z_1-SM_{12})+Z_4(M_{23}-M_{24})}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6}-\right.$$

$$\left.\frac{Z_4(-2Z_2+Z_3+Z_5)-(Z_1-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}}\right\}. \qquad \text{(A.116)}$$

From Equations A.114, A.103, A.089 on page 86, A.102, and A.086 one can find the sensitivity of the transfer function T with respect to parameter C_1,

$$S_{C_1}^{T=C_1\frac{\partial Z_1}{\partial C_1}}\left\{\frac{(M_{23}-M_{24})(Z_2-SM_{12})+Z_5(M_{13}-M_{14})}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6}-\right.$$

$$\left.\frac{Z_5(-2Z_1+Z_4+Z_3)-(Z_2-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}}\right\}. \qquad \text{(A.117)}$$

Similar action may be taken upon Equation A.114 using

Equations A.086 on page 85, A.090 on page 86, A.102 on page 88, and A.106 on page 89, the sensitivity with respect to C_6 is

$$S_{C_2}^{T} = C_2 \frac{\partial Z}{\partial C} \left[\frac{(M_{13}-M_{14})(Z_1-SM_{12})+Z_4(M_{23}-M_{24})}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_4(-2Z_2+Z_3+Z_5)-(Z_1-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right] . \quad (A.118)$$

With respect to the source resistance, the sensitivity may be found by knowing that Q is not a function of R_s, and applying Equations A.112 and A.102 to Equation A.114

$$S_{R_s}^{T} = -R_s \left[\frac{-S^2M_{12}^2+Z_4Z_5}{Z_8+Z_9+Z_{10}+Z_{11}} \right] . \quad (A.119)$$

Substituting into Equation A.114 the Equations A.093 on page 86, A.086 on page 85, A.102 on page 88 and A.107, the sensitivity with respect to L_1 is found to be

$$S_{L_1}^{T} = SL_1 \left[\frac{(M_{23}-M_{24})Z_2}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_3Z_5-Z_2^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right] . \quad (A.120)$$

Similar utilization of Equation A.114, and the Equations A.086 on page 85, A.094 on page 86, A.102 on page 88 and A.107 on page 89 results in

$$S_{L_1}^{T} = SL_2 \left[\frac{(M_{13}-M_{14})Z_1}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} - \frac{Z_3Z_4-Z_1^2}{Z_8+Z_9+Z_{10}+Z_{11}} \right] . \quad (A.121)$$

The sensitivity with respect to r_1 may be found by substituting Equations A.086 on page 85, A.091 on page 86, A.102 on page 88, and A.109 on page 89 into Equation A.114 on page 90

$$S_{r_1}^{T=r_1}\left[\frac{(M_{23}-M_{24})Z_2}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6}-\frac{Z_3Z_5-Z_2^2}{Z_8+Z_9+Z_{10}+Z_{11}}\right]. \qquad (A.122)$$

A parallel procedure using Equations A.086 on page 85,, A.091 on page 86, A.102 on page 88, and A.110 on page 89 into Equation A.114 on page 90 yields

$$S_{r_2}^{T=r_2}\left[\frac{(M_{13}-M_{14})Z_1}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6}-\frac{Z_3Z_4-Z_1^2}{Z_8+Z_9+Z_{10}+Z_{11}}\right]. \qquad (A.123)$$

The sensitivity of the mutual inductance linking the drive coils may be found by substituting Equations A.111 on page 90, A.102 on page 88, A.095 on page 87, and A.086 on page 85 into A.114 on page 90

$$S_{M_{12}}^{T=-SM_{12}}\left[\frac{Z_2(M_{13}-M_{14})+Z_1(M_{23}-M_{24})}{Z_7(M_{13}-M_{14})+Z\ (M_{23}-M_{24})}+\frac{2(Z_1Z_2-Z_3SM_{12})}{Z_8+Z_9+Z_{10}+Z_{11}}\right]. \qquad (A.124)$$

It may be noted that none of the mutuals which couple the drive coils to the pickup coils are found in the denominator P. Therefore, only Q and its derivatives need be used to find sensitivities with respect to these parameters. Substituting the equations noted and Equation A.086 on page 85, into A.114 on page 90 for each mutual yields

$$S_{M_{13}}^{T} = \frac{M_{13}Z_7}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} \quad \{\text{from A.098 on page 87}\}, \tag{A.125}$$

$$S_{M_{14}}^{T} = \frac{-M_{14}Z_7}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} \quad \{\text{from A.099 on page 87}\}, \tag{A.126}$$

$$S_{M_{23}}^{T} = \frac{M_{23}Z_6}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} \quad \{\text{from A.100 on page 88}\}, \tag{A.127}$$

$$S_{M_{24}}^{T} = \frac{-M_{24}Z_6}{(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6} \quad \{\text{from A.101 on page 88}\}. \tag{A.128}$$

By taking the equations for the derivative of P with respect to S (A.113 on page 90), the derivative of Q with respect to S (A.096 On page 87), the definition of Q (A.086 on page 85) and the definition of P (A.102 on page 87), and substituting these into the definition of sensitivity (A.114 on page 90), the resulting equation may be manipulated to yield the sensitivity with respect to complex frequency.

$$S_S^T = S\left\{\frac{(M_{13}-M_{14})\left[Z_7+S\left\{\frac{\partial Z_1}{\partial S}Z_5+\frac{\partial Z_2}{\partial S}(Z_1-SM_{12})+Z_1L_2-Z_2M_{12}\right\}\right]}{S\left[(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6\right]}\right.$$

$$+\frac{(M_{23}-M_{24})\left[Z_6+S\left\{\frac{\partial Z_2}{\partial S}Z_4+\frac{\partial Z_1}{\partial S}(Z_2-SM_{12})+Z_2L_1-Z_1M_{12}\right\}\right]}{S\left[(M_{13}-M_{14})Z_7+(M_{23}-M_{24})Z_6\right]}$$

$$-\frac{\partial Z_1}{\partial S}\left(\frac{Z_5(Z_3+Z_4-2Z_1)-(Z_2-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}}\right)$$

$$-\frac{\partial Z_2}{S}\left(\frac{Z_4(Z_3+Z_5-2Z_2)-(Z_1-SM_{12})^2}{Z_8+Z_9+Z_{10}+Z_{11}}\right)$$

$$\left.-\frac{-Z_1^2L_2-Z_2^2L_1+Z_3(Z_5L_1+Z_4L_2-2SM_{12}^2)+2M_{12}Z_1Z_2}{Z_8+Z_9+Z_{10}+Z_{11}}\right\}. \quad (A.129)$$

APPENDIX B

COMPUTER PROGRAMS

This appendix is composed of four parts. The first section describes programs used in fitting the component parameters to the drive coils. The second is a listing of the fitting programs in alphabetical order. The third is a description of the programs used in finding mutual inductances of the probe and also the sensitivities of the probe to various component parameters. The fourth is a listing of the programs described in the third part. These programs were written in APL [13], [14] for use on an International Business Machines 5100 portable computer. The language is also applicable to other systems.

Description of Fitting Programs

CALC

CALC is listed on page 112. The purpose of this program is to find the capacitance shunting a series resistance and inductance when the resonant frequency, the resistance and inductance are known. It is executed in this form:

```
100 2.3E¯3 CALC 200E3 ,
```

where the first two numbers are the resistance and inductance and the number following the function name is the frequency in Hertz.

CALEL

The program CALEL is used to find the capacitance and inductance of a series resistance and inductance shunted by a capacitance. A listing of CALEL may be found on page 112. It is assumed that the circuit is being driven by a generator of voltage VREF which is equal to 8 Volts and having a known source impedance of R1 equal to 3900 ohms. Other generator voltages and source impedances may be used by changing lines nine and ten of the program. The program requires the arguments R and Z, where R is the series resistance of the circuit, and Z is a matrix in which the first column is assumed to be a vector of frequencies, the second column is a vector of voltage magnitudes measured across the circuit. The output of the program, L is a matrix having four coulumns. The first column is the input frequencies, the second and third columns are the two roots of the solution for the inductance and the fourth column is the computed capacitance. An example of the form of the input is illustrated below:

```
100 CALEL 3 2ρ100 .2 1000 .4 2000 .6 ,
```

where 100 ohms is the series resistance; 100, 1000, and 2000 are the frequencies; and .2, .4 and .6 are the voltages read across the circuit. The 3 is the number of rows in the input matrix and 2 is the number of columns.

CHECK

This program is used to check the error between the impedance data, DATA and the impedance computed for the same frequencies using the first, third and the fifth elements of the global vector AP. A listing of CHECK may be found on page 112. The first element of AP is the series resistance of the coil, the third element is the shunt capacitance and the fifth is the inductance. This program calls various other routines to compute the various error criterion. The input matrix is generated by the interactive program DATA∆INPUT.

The output of this program is in the form of a printed table or results, the form of which is

```
USING AP[1 2 5]=100 1.5E¯10 2.3E¯3
SUM OF PRODUCT ERROR=X.XXXXXE XX
SUM LOG MAG ERROR X.XXXXXE XX
SUM PHA ERROR= X.XXXXXE XX DEGREES
COMPUTED PHASE AT REZ= X.XXXXXE XX DEGREES
COMPUTED MAGNITUDE ERROR AT REZ=X.XXXXXE XX,
```

where the first line gives the values of interest in the model vector AP, which in this case is a 6 element vector. Only the odd three elements are displayed. The even three elements give the size of a unit perturbation of the parameter and is not used in this program. The sum of product error in the second line is really a misnomer, since the value is equal to the sum of the global variables PE and ME. ME is the sum of the absolute values of

of the differences between the log of the magnitudes measured and the computed log magnitudes at the same frequency. PE is the sum of the absolute value of the measured and computed phases in radians. The sum log magnitude error in the third line is ME. The sum phase error is the value of PE converted from radians to degrees. The purpose of the sum of product error was to give a variable which varied with the fitting of both magnitude and phase. The attempt was not totally sucessful, since these are not comparable quantities. For this reason the log magnitude error and the phase error were also monitored independently. The global variable NREZ must be set to the index of the data point where the circuit resonates. This routine uses this index to compute the errors in magnitude and phase at this data point. These errors are printed in the fifth and sixth lines.

CHECKL

A listing the routine CHECKL may be found on page 113. This program uses the formated data generated by *DATAΔINPUT* and a known series resistance R and computes an inductance for that data point, assuming that there is no shunt capacitance and no other losses. It is executed in the form:

```
100 CHECKL DATA9←DATAΔINPUT
```

where 100 is the series resistance, DATAΔINPUT assigns to

the arbitrary variable DATA9 , the result of its execution. In this case, DATA9 would be a matrix in which the first column is frequencies, the second is the real part of the impedance at the corresponding frequency and the third is the imaginary part at that frequency.

CHK4

This program is similar to CHECK except that it considers a circuit containing four elements. In this case, the vector AP must be an eight element vector whose odd elements are series resistance, parallel resistance, parallel capacitance and series inductance. Its output is also similar. The listing of CHK4 may be found on page 113.

DATAΔINPUT

The purpose of this program is to interactively compute and format impedance data. This data is measured by taking a known resistor and a signal source and driving the unknown through the known resistor. The voltage on both sides of the resistor are measured with an oscilloscope and the time between the zero crossings of the two signals is also recorded. The signal source is used as a phase reference and voltage reference. The program asks one time for these references and then repeatedly for sets of data consisting of first the frequency, then the magnitude of the voltage across the unknown and then the time between

the two zero crossings. To terminate the input, the user must give four elements instead of three. The function is executed in this form:

DATA9←DATAΔINPUT,

where DATA9 is an arbitrary matrix in which the formatted input is to be saved. The matrix DATA9 is described in the description of the function CHECKL and will not be repeated here. The listing of the DATAΔINPUT is on page 113.

DZIN

DZIN which is listed on page 114, generates an experimental design matrix for three variables. It generates the design matrix in three orthogonal blocks BL1, BL2, and BL3. ALL is the combination of all three blocks. These are used to generate perterbations about a selected estimate of component values. The four matrices are global and this function only has to be executed once to create them. The form of the execution of DZIN is

DZIN.

DZN4

DZN4 which is listed on page 114, is similar to DZIN except that it generates a design matrix for four variables. The form of its execution is also similar to DZIN. These matrices are described in Hunter [12].

EFIT

This function is used to estimate the response of the error computed by ERR or ER4 to changes in model parameters. A listing of this function may be found on page 115. Perturbations about the currently selected component values given by the global variable AP's odd elements and of magnitudes scaled by its even elements. The input matrix DZN, which is usually BL1, B12, BL3 or ALL as generated by DZIN or DZN4, determines the relative size of the perturbation. An example of the form required for execution is

*COEF←ALL EFIT DATA*9,

where COEF are the coefficients of a first order fit of the parameters to the error response. ALL is the design matrix. DATA9 is the formatted impedance data from DATA∆INPUT.

ERR

This function is listed on page 115. It computes the various error criterion for the three component model. The input consists of the vector F which contains the model values of series resistance, shunt capacitance and coil inductance. It generates four global variables: ZC, which is a matrix consisting of the real and imaginary parts of the impedance of the model circuit corresponding to each of the measured data points contained in Z. PE is the sum of the absolute values of the radian phase errors. PHREZ is the phase error computed at the data point indexed

by NREZ. ME is the sum of the absolute value of the log magnitude errors. The esplicit result of the function, E is the sum of PE and ME. An example of the form required for execution is

```
ERROR←AP[1 3 5] ERR DATA9,
```

where ERROR is the variable into which the resulting error criterion is stored, AP[1 3 5] is the part of the vector AP which describes the components of the model, and DATA9 is the formatted impedance data generated by DATAΔINPUT.

ER4

A listing of ER4 may be found on page 116. This function is similar to ERR except that it computes errors for the four parameter model. It would be executed in the form:

```
ERROR←AP[1 3 5 7] ER4 DATA9.
```

This uses similar notation to ERR.

FIT

This function does a least square fit on a matrix of stimuli X and a vector of responses Y. It is called by the function EFIT to find the coeficients of the function which approximates the error equation. The listing is found on page 116.

FLOORVALUE

A listing of the program FLOORVALUE may be found on page 117. Its function is to find the minimum error point in the design matrix. It is called by the function TRY and it is always called after the function EFIT has been called. EFIT generates a matrix of errors corresponding to each of the points in the design matrix. This matrix of errors is called Y. Y is a global variable. The design matrix point with the minimum error is the explicit result J. The output of the program which is routed to the printer is in this form:

```
THE MINIMUM ERROR IS   X.XXXXXE XX
THE INPUT USED IS 1 1 ¯1 ¯1
THE MINIMUM PHASE ERROR=  X.XXXXXE XX
THE INPUT USED IS 1 1 1 1 .
```

The first line is the miminum error found and the second line gives design vector which gave that minimum error. The third and fourth lines are erroneous. The intent was that this output be the design vector which gives the minimum phase error. However due to an error in line three, and back in EFIT, this is not the case. Line three was supposed to find the minimum of a matrix of phase errors generated from a series of PE's. This matrix is not generated, therefore it is hard to find its minimum. The minimum of a single point is found instead.

LG

This program is listed on page 117. Its purpose is to take a matrix DATA in which the first column is assumed to be the real part of a complex number and the second column is assumed to be the imaginary part of a complex number. The explicit result is vector equivalent to the log magnitudes of the input matrix. It is executed in this form:

```
LOGMAG←LG DATA,
```

where LOGMAG is the arbitrary vector into which the explicit result of LG is stored. DATA is the input matrix.

LOG

LOG is a convenience program which takes the log magnitude of the second and third columns and the log magnitude of the fourth and fifth columns of the input matrix DATA. Its purpose is to compare the log magnitudes of the fitted impedance and the measured impedance. Column one of the matrix is expected to be the frequency of the data point. Execution of the program listed on page 117 is usually in this form:

```
□←13 ¯5▼ LOG DATA9,ZC.
```

where ZC is the global matrix generated by ERR or ER4 and DATA9 is the explicit result of DATA∆INPUT.

LPT

This program is listed on page 118 and its output is shown in Figure B.1. It is called by the routine PLOTZ. Its function is to plot on the printer the comparison between two magnitudes vs frequency. It is capable of plotting magnitudes in the range for 10 to 100,000 ohms and a frequency range of 100 Hz to 10 mega-Hertz. The input is in the form of a matrix XY which contains columns which correspond to frequency, the first magnitude and the second magnitude. Its form of execution is described in line one of the function PLOTZ.

MOVE

MOVE is listed on page 118. Its function is to change the selected values of the three components which describe the model of the drive coil. It changes the value of the vector AP[1 3 5] by the scale vector AP[2 4 6] times the input vector HOW. The form of execution is

```
MOVE ¯1 1 1 ,
```

where -1 1 1 is the input vector HOW.

MOVEIT

MOVEIT is listed on page 119. Its function is to change the selected values of the components which describe the design matrix by the design vector which describes the

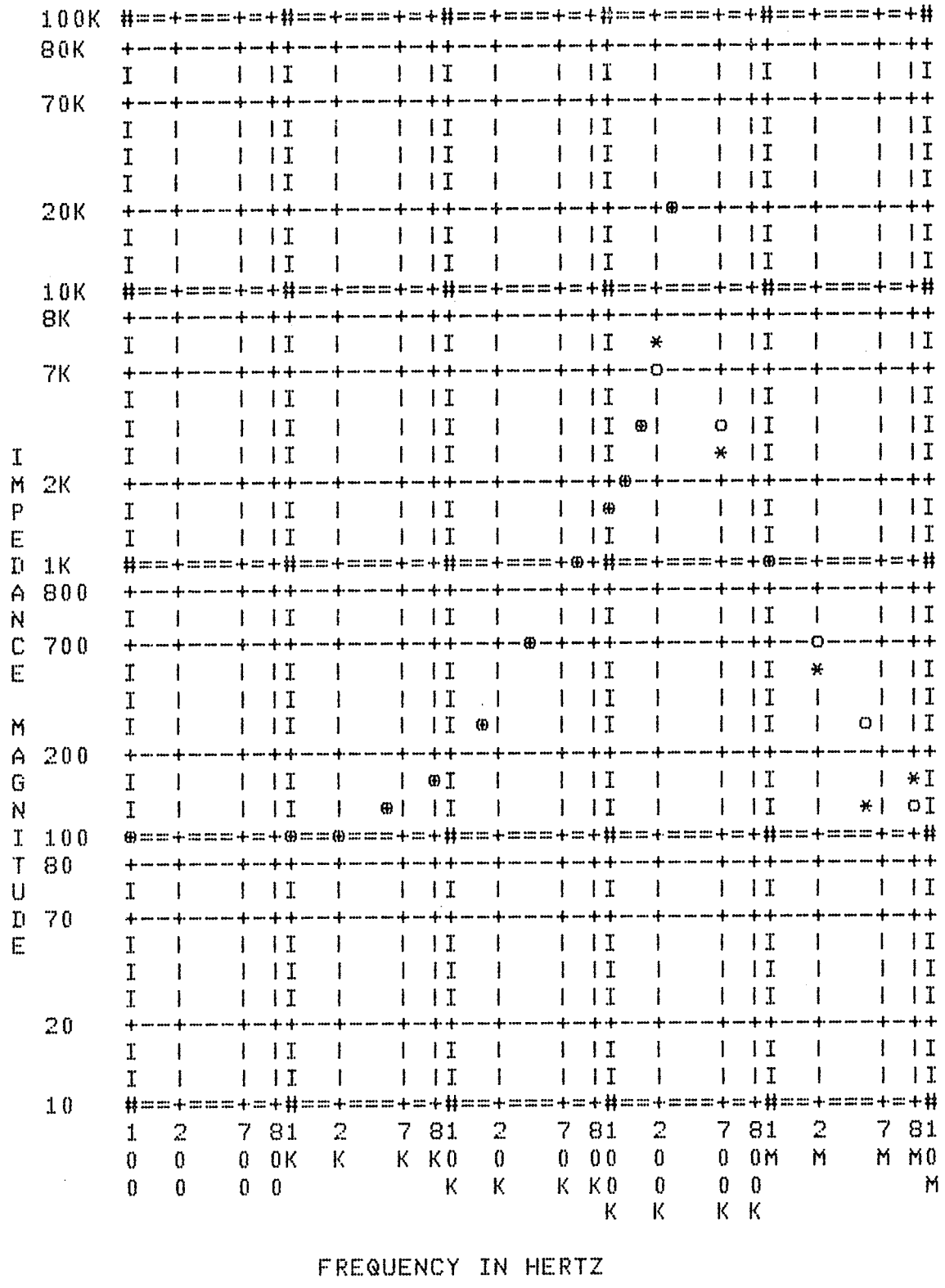


Figure B.1. Printer plot generated by the program LPT.

best direction to move in to improve the error times the scale vector. In symbols it multiplies the design vector found by the routine TRY called MM times the scale vector AP[2 4 6 (8)] and adds that to AP[1 3 5 (7)] which is reasigned to AP. After "moving" the model to a new set of values, it is compared with the input data set in lines three or nine. This allows one to generate a vector describing the best direction to change the model by executing the function TRY and continue in that direction by executing the function MOVEIT until the error starts to increase. It is important to note that as listed the function will compare the model to the data stored in the matrix DATA9. It is executed by just the program name.

MOV4

MOV4 is listed on page 119. It performs the same function for the four parameter model as MOVE provides for the three parameter model. It expects a four element input and an eight element global variable AP to exist.

PHASE

PHASE is listed on page 119. It generates a list of the phases of complex numbers in the input variable DATA vs frequency. The frequency is expected to be in column one of DATA and the complex numbers in cartesian form are expected to be in columns two and three and columns four

and five. The matrix that is output from PHASE contains three columns. The first is the input frequency from column one of DATA, the sceond is the phase of the impedance in columns two and three and the third is the phase of the impedance in columns four and five. The intended use of PHASE is to compare the phases vs frequency of the fitted and measured data. The form of execution in that case is

```
□←13 ¯5▼ PHASE DATA9,ZC,
```

where DATA9 is the measured data and in the format of the output of the program DATAΔINPUT. ZC is the matrix of computed impedances in the form of a column of real parts and a column of imaginary parts.

PLOTZ

This program is listed on page 119. Its function is to plot the fitted data ZC which is generated by the program ERR or ER4 and the measured data DATA. It calls the plotting routine LPT and generates the output as is shown if Figure B.1. It also calls the routine PHASE which generates a tabular listing comparing the measured and fitted phases vs frequency. The form of execution is usually

```
PLOTZ DATA9,
```

where DATA9 is the measured data generated by the routine DATAΔINPUT. It is assumed that either ERR or ER4 has already been executed on the same data set.

SETREZ

This routine is listed on page 119. Its function is to set the variable NREZ to the index of the resonant point in the measured data. This is used to indicate to the program ERR or ER4 the row of the measured data which contains information taken at resonance. This must be executed before the results of FLOORVALUE, CHECK or CHK4 will give the correct error at resonance. The execution of this function and the computer response is of this form:

```
USER:      SETREZ
COMPUTER:  THE RESONANT POINT IS THE DATA POINT NO.
USER:      13,
```

where 13 is the number of the line in the input data which was measured at resonance.

TRY

TRY is listed on page 120. It is a conversational function which finds the best direction in which to "move" the selected model in parameter space to decrease the error criterion. The interaction between the user and the computer is quite lengthy and only the initiation of that is shown below:

```
TRY DATA9,
```

where DATA9 is the formatted measured data output by the program DATAΔINPUT. This program calls EFIT and FLOORVALUE.

Listings of Fitting Programs

This part of the appendix comtains the listings of those functions that are used to fit the models of the drive coils to the measured impedances. There are two listings that are not included in this section. They are also not described in part one. They are CDIV2 and CMUL2. The reason that they are not included in this part of the appendix is that they are not original to this thesis. They may be found in Wiedman [16] and are used to divide and multiply complex arguments. They are much more general than the function DV and M which are described in the third section of this appendix.

```
      ∇C←RL CALC F
[ 1]  ⍝ THIS FUNCTION CALCULATES THE
[ 2]  ⍝ CAPACITANCE OF AN SERIES RL
[ 3]  ⍝ AND PARALLEL C KNOWING R L AND F
[ 4]  ⍝ THE INPUT RL IS A 2 ELEMENT VECTOR
[ 5]  ⍝ CONTAINING R AND L RESPECTIVELY
[ 6]  ⍝ F IS THE RESONANT FREQUENCY
[ 7]   C←1÷(((○2×F)*2)×RL[2])+RL[1]*2
      ∇


      ∇L←R CALEL Z;VREF;R1;R3;W;W2;W4;VA2;VREF2;R32;A;B;C
[  1]  ⍝ THIS PROGRAM ESTIMATES L FOR EACH DATAPOINT
[  2]  ⍝ ASSUMING NO CAPACITANCE
[  3]  ⍝ AT LOWER FREQUENCIES THIS SHOULD BE CLOSE
[  4]  ⍝ Z IS A MATRIX OF FREQUENCIES AND MAGNITUDES
[  5]  ⍝ THE FIRST COLUMN BEING FREQUENCY AND THE
[  6]  ⍝ SECOND BEING MAGNITUDES OF THE VOLTAGE ACROSS
[  7]  ⍝ THE IMPEDANCE
[  8]  ⍝ THE REFERENCE RESISTOR IS ASSUMED TO BE 3900
[  9]  ⍝ THE REFERENCE VOLTAGE IS ASSUMED TO BE 8 VOLTS
[ 10]   VREF←8
[ 11]   R1←3900
[ 12]   W4←(W2←(W←(Z[;1]×○2))*2)*2
[ 13]   VREF2←(1↑⍴Z)⍴VREF*2
[ 14]   VA2←Z[;2]*2
[ 15]   A←W4×VA2-VREF2
[ 16]   R3←R+R1
[ 17]   R32←R3*2
[ 18]   B←W2×(⍴W2)⍴(2×R3×((R3×VA2)-R×VREF2))-(R1*2)×VREF2
[ 19]   C←R32×(R32×VA2)-VREF2×R*2
[ 20]   H1←((⍴H1),1)⍴H1←(-B-TT←((B*2)-4×A×(⍴A)⍴C)*0.5)÷2×A
[ 21]   H2←((⍴H2),1)⍴H2←(-B+TT)÷2×A
[ 22]   L←(⌈H1,H2)*0.5
[ 23]   C9←((⍴C9),1)⍴C9←((VREF2-VA2)*0.5)÷Z[;2]×W×R1
[ 24]   L←Z[;1],L,C9
      ∇


      ∇CHECK DATA
[ 1]  ⎕←'USING AP[1 3 5]='
[ 2]  ⎕←'SUM PRODUCT ERROR=', 13 ¯5 ⍕(⎕←AP[1 3 5]) ERR DATA
[ 3]  ⎕←'SUM LOG MAG ERROR', 13 ¯5 ⍕ME
[ 4]  ⎕←'SUM PHA ERROR=',(13 ¯5 ⍕PE×180÷○1),' DEGREES'
[ 5]  ⎕←'COMPUTED PHASE AT REZ=',(13 ¯5 ⍕PHREZ),' DEGREES'
[ 6]  ⎕←'COMPUTED MAGNITUDE AT ERROR AT REZ=',(13 ¯5 ⍕ER[NR
       EZ])
      ∇
```

```
        ∇L←R CHECKL Z
[ 1]    L←(Z[; 2 3]-(ρZ[; 2 3])ρR,0) CDIV2((1↑ρZ)ρ0),(o2)xZ[;
          1]
        ∇


        ∇CHK4 DATA
[ 1] ⍝ THIS PROGRAM LISTS THE COMPUTED ERRORS GENERATED
[ 2] ⍝ WHEN PROGRAM ER4 IS EXECUTED
[ 3]   ⍞←'USING AP[1 3 5 7] '
[ 4]   ⍞←'SUM PHASE & MAG ERRORS', 13 ¯5 ▼(⍞←AP[1 3 5 7]) ER
         4 DATA
[ 5]   ⍞←'SUM OF LOG MAGNITUDE ERRORS ', 13 ¯5 ▼ME
[ 6]   ⍞←'SUM OF RADIAN PHASE ERRORS ', 13 ¯5 ▼PE
[ 7]   ⍞←'SUM OF DEGREE PHASE ERRORS ',(13 ¯5 ▼PEx180÷o1),'D
         EG.'
[ 8]   ⍞←'COMPUTED PHASE AT RESONANCE ',(13 ¯5 ▼PHREZ),' DEG
         .'
[ 9]   ⍞←'COMPUTED MAGNITUDE ERROR AT RESONANCE', 13 ¯5 ▼ERC
         NREZ]
        ∇


        ∇Z2←DATAΔINPUT;T
[  1] ⍝THIS INPUTS DATA FOR IMPEDANCE CALCULATION
[  2]  Z2←Z← 0 3 ρ0
[  3]  ⍞←'INPUT REFERENCE AMPLITUDE'
[  4]  A←⍞,0
[  5]  ⍞←'REFERENCE RESISTOR='
[  6]  R←⍞
[  7] INP:⍞←'INPUT FREQUENCY,MAGNITUDE B,ΔT='
[  8] ⍝ ΔT IS THE TIME DELAY BETWEEN THE PHASES OF VA AND B
[  9]  T←⍞
[ 10]  →((ρT)≠ 3 4)/YES,DUN
[ 11]  ⍞←'ERROR, 3 ELEMENTS PLEASE'
[ 12]  →INP
[ 13] YES:Z←Z,[1] T
[ 14]  Z2←Z2,[1] T[1],(RxB) CDIV2 A-B←((2oPH),1oPH←x/T[1 3]
         ,o2)xT[2]
[ 15]  →INP
[ 16] DUN:
        ∇
```

```
      ∇DZIN;AP
[  1] ⍝ THIS FUNCTION GENERATES A DESIGN MATRIX
[  2] ⍝ FOR A 3 VARIABLE EXPERIMENT
[  3] ⍝ SEE DESIGN OF EXPERIMENTS COURSE VOL.7
[  4] ⍝ PAGE 127
[  5]  BL1← 6 3 ⍴ ¯1 ¯1 ¯1 1 1 ¯1 1 ¯1 1 ¯1 1 1 ,6⍴0
[  6]  BL2← 6 3 ⍴ 1 1 1 ¯1 ¯1 1 ¯1 1 ¯1 1 ¯1 ¯1 ,6⍴0
[  7]  AP←8*0.25
[  8]  BL3←(-AP), 0 0 ,AP, 0 0 0 ,(-AP), 0 0 ,AP, 0 0 0 ,(-
        AP), 0 0 ,AP,6⍴0
[  9]  BL3← 8 3 ⍴BL3
[ 10]  ALL←BL1,[1] BL2,[1] BL3
      ∇

      ∇DZN4;AP
[  1] ⍝ THIS FUNCTION GENERATES A DESIGN MATRIX
[  2] ⍝ FOR A 4 VARIABLE EXPERIMENT
[  3] ⍝ SEE DESIGN OF EXPERIMENTS COURSE VOL.7
[  4] ⍝PAGE 130
[  5]  BL1← ¯1 ¯1 ¯1 ¯1 1 1 ¯1 ¯1 1 ¯1 1 ¯1
[  6]  BL1←BL1, ¯1 1 1 ¯1 1 ¯1 ¯1 1 ¯1 1 ¯1 1
[  7]  BL1← 10 4 ⍴BL1, ¯1 ¯1 ,(6⍴1),8⍴0
[  8]  BL2← 1 ¯1 ¯1 ¯1 ¯1 1 ¯1 ¯1 ¯1 ¯1 1 ¯1
[  9]  BL2←BL2, 1 1 1 ¯1 ¯1 ¯1 ¯1 1 1 1 ¯1 1 1 ¯1 1 1
[ 10]  BL2← 10 4 ⍴BL2, ¯1 1 1 1 ,8⍴0
[ 11]  BL3← ¯2 0 0 0 2 0 0 0 0 ¯2 0 0 0 2 0 0
[ 12]  BL3←BL3, 0 0 ¯2 0 0 0 2 0 0 0 0 ¯2 0 0 0 2
[ 13]  BL3← 10 4 ⍴BL3,8⍴0
[ 14]  ALL←BL1,[1] BL2,[1] BL3
      ∇
```

```
      ∇COE←DZN EFIT Z
[  1] ⍝ THIS FUNCTION USES THE DESIGN MATRIX
[  2] ⍝ GENERATED BY DZIN TO ESTIMATE THE RESPONSE OF
[  3] ⍝ THE ERROR COMPUTED BY ERR TO CHANGES IN PARAMETERS
[  4] ⍝ AP, WHERE AP=RD,ΔRD,C,ΔC,L,ΔL
[  5] ⍝ OR AP=RD,ΔRD,RE,ΔRE,C;ΔC,L,ΔL
[  6]  EXN←0
[  7]  Y← 0 1 ⍴0
[  8]  X←DZN
[  9]  K←(2×⍳(⍴AP)÷2)
[ 10] NEXT:→((1↑⍴DZN)<EXN←EXN+1)/ENUF
[ 11]  →((⍴K)= 3 4)/N3,N4
[ 12] N3:Y←Y,[1](X[EXN;]←AP[K-1]+DZN[EXN;]×AP[K]) ERR Z
[ 13]  →NEXT
[ 14] N4:Y←Y,[1](X[EXN;]←AP[K-1]+DZN[EXN;]×AP[K]) ER4 Z
[ 15]  →NEXT
[ 16] ENUF:COE←((((1↑⍴X),1)⍴1),X) FIT Y
      ∇


      ∇E←F ERR Z
[  1] ⍝ THIS FUNCTION TAKES THE EMPIRICAL DATA
[  2] ⍝ AND FINDS THE SUM OF THE ABSOLUTE VALUES
[  3] ⍝ OF THE LOG MAGNIUDE ERROR BETWEEN THE DATA
[  4] ⍝ AND THE VALUES THAT ARE
[  5] ⍝ COMPUTED FOR THE MODEL PARAMETERS F
[  6] ⍝ WHERE F←DRIVE RESIST,CAP,INDUCTANCE
[  7]  ZC←(⍴Z)⍴0
[  8]  W2←(W←○2×((1↑⍴Z),1)↑Z)*2
[  9]  RD←1↑F
[ 10]  C← 0 1 0 /F
[ 11]  L←¯1↑F
[ 12]  ZC←(((⍴W)⍴RD),W×L) CDIV2(1-W2×L×C),W×RD×C
[ 13]  ME←+/,|ER←(LG(Z[; 2 3]))-LG(ZC)
[ 14]  PE←+/,|PER←(PHA(Z[; 2 3]))-PHA(ZC)
[ 15]  PHREZ←(PHA(ZC[NREZ;]))×180÷○1
[ 16]  E←PE+ME
      ∇
```

```
        ∇E←F ER4 Z;W2;W;RD;RP;C;L;K
[  1]  ⍝THIS FUNCTION TAKES THE EMPRICAL DATA
[  2]  ⍝ AND FINDS THE SUM OF THE ERRORS BETWEEN
[  3]  ⍝ THE COMPUTED VALUES USING THE MODEL
[  4]  ⍝ PARAMETERS F, WHERE F[1] IS COIL RESISTANCE
[  5]  ⍝ F[2] IS EDDY CURRENT RESISTANCE COEF
[  6]  ⍝ F[3] IS SHUNT CAPACITANCE
[  7]  ⍝ F[4] COIL INDUCTANCE
[  8]  ⍝ Z IS AN ARRAY OF IMPEDANCE VS FREQ
[  9]  ⍝ Z[;1] IS A FREQUENCY VECTOR
[ 10]  ⍝ Z[;2] IS A REAL PART VECTOR
[ 11]  ⍝ Z[;3] IS THE IMAGINARY PART VECTOR
[ 12]   ZC←(⍴Z)⍴0
[ 13]   W2←(W←○2×((K←1↑⍴Z),1)↑Z)*2
[ 14]   RD←1↑F
[ 15]   RE← 0 1 0 0 /F
[ 16]   C← 0 0 1 0 /F
[ 17]   L←¯1↑F
[ 18]   ZC←((K⍴RE×RD),W×L×RE) CDIV2(RD+RE-W2×RE×L×C),W×L+C×R
          E×RD
[ 19]   ME←+/,|ER←(LG ZC)-LG Z[; 2 3]
[ 20]   PE←+/,|PER←(PHA ZC)-PHA Z[; 2 3]
[ 21]   PHREZ←(PHA(ZC[NREZ;]))×180÷○1
[ 22]   E←ME+PE
        ∇


        ∇B←X FIT Y
[  1]  ⍝ THIS PROGRAM DOES A LEAST SQ FIT ON
[  2]  ⍝ A MATRIX OF STIMULI X AND A VECTOR OF
[  3]  ⍝ RESPONSES Y.  SEE DESIGN OF EXPERIMENTS
[  4]  ⍝ COURSE BY DR. J. STUART HUNTER. VOL. 7
[  5]  ⍝ PG. 15 TO  18 , COPYRIGHT 1968 BY WESTINGHOUSE
[  6]  ⍝ ELECTRIC CORP.
[  7]   SLY←(+/[1]|Y)÷(1↑⍴Y)
[  8]   SCLX←(⍴X)⍴(+/[1]|X)÷(1↑⍴X)
[  9]   X←X÷SCLX
[ 10]   Y←Y÷SLY
[ 11]   B←((⌹X)+.×Y)⌸((⌹X)+.×X)
[ 12]   B←B÷(⍴B)⍴SCLX
[ 13]   B←B×SLY
        ∇
```

```
        ∇J←FLOORVALUE
[  1] ⍝ THIS FUNCTION FINDS THE MIN. VALUE OF Y
[  2] ⍝ AND GIVES THE X WHICH BROUGHT IT ABOUT
[  3]  PF←(⌊/,PE)
[  4]  YF←(⌊/,Y)
[  5]  ⎕←'THE MININUM ERROR IS'
[  6]  ⎕← 13 ¯5 ⍕YF
[  7]  ⎕←'THE INPUT USED IS'
[  8]  ⎕←J←1+⌊((YF=,Y)/⍳⍴,Y);⌋
[  9]  ⎕←'THE MINIMUM PHASE ERROR='
[ 10]  ⎕← 13 ¯5 ⍕180×PF÷○1
[ 11]  ⎕←'THE INPUT USED IS '
[ 12]  ⎕←1+⌊((PF=,PE)/⍳⍴,PE);⌋
[ 13]  ⎕←'PHASE AT RESONANCE=',(13 ¯5 ⍕PHREZ),' DEGREES'
        ∇


        ∇OUT←LG DATA
[ 1] ⍝ THS FUNCTION TAKES THE LOG OF THE
[ 2] ⍝ MAGNITUDES OF THE ARRAY DATA
[ 3] ⍝ THE FIRST COLUMN IS ASSUMED TO BE
[ 4] ⍝ THE REAL PART AND THE SECOND IS ASSUMED
[ 5] ⍝ TO BE THE IMAGINARY PART
[ 6]  OUT←10⍟MAG DATA
        ∇


        ∇OUT←LOG DATA
[ 1] ⍝ THIS FUNCTION TAKES THE LOG OF DATA
[ 2] ⍝ MAGNITUDES COMPUTED FROM DATA COLUMNS
[ 3] ⍝ 2 3 AND 4 5
[ 4] ⍝ IT IS USED TO DISPLAY THE INPUT DATA
[ 5] ⍝ NEXT TO THE FITTED DATA ZC
[ 6] ⍝ COLUMN 1 IS EXPECTED TO CONTAIN THE
[ 7] ⍝ THE FREQUENCY
[ 8]  OUT←DATA[;1],(LG(DATA[; 2 3])),[1.1] LG(DATA[; 4 5])
        ∇
```

```
        ∇PLT←LPT XY;TU;TRE;FOE;PC;VLN;LIN;INDEX;INDY1
[  1]   PC←'*o⊕'
[  2]   XY←10⊕XY
[  3]   PLT← 51 60 ⍴' '
[  4]   LIN←(9⍴' '),(50⍴'#--+---+--+'),'#'
[  5]   PLT[,(10×(⍳4)-1)∘.+ 2 4 8 ;]← 12 60 ⍴LIN
[  6]   VLN←'+',(40⍴'+|+|||+||+'),'           '
[  7]   PLT[;(,(10×⍳5)∘.+ 3 7 9)]←⍉ 15 51 ⍴VLN
[  8]   DLN←(9⍴' '),(50⍴'#==+===+=+'),'#'
[  9]   PLT[1 11 21 31 41 ;]← 5 60 ⍴DLN
[ 10]   DVL←'#',(40⍴'+I+III+II#'),'           '
[ 11]   PLT[;10×⍳6]←⍉ 6 51 ⍴DVL
[ 12]   PLT[42;]←((9⍴' '),50⍴'1  2   7 8'),'1'
[ 13]   PLT[;5]←(40⍴'18 7   2  '),'1',10⍴' '
[ 14]   PLT[43;]←(9⍴' '),'0  0   0 0K  K   K K0  0   0 00  0
            0 0M  M   M M0'
[ 15]   PLT[44;]←(9⍴' '),'0  0   0 0           K  K   K K0  0
            0 0           M'
[ 16]   PLT[45;]←(9⍴' '),'                                 K  K
            K K           '
[ 17]   PLT[47;]←(21⍴' '),'FREQUENCY IN HERTZ',21⍴' '
[ 18]   PLT[;6]←'00 0   0  0K K   K  K0 0   0  00 0   0  0 '
         ,9⍴' '
[ 19]   PLT[;7]←'0K K   K  K           0 0   0  0            '
         ,9⍴' '
[ 20]   PLT[;3]←(16⍴' '),'IMPEDANCE MAGNITUDE',16⍴' '
[ 21]   PLT[1;8]←'K'
[ 22]   INDEX←10+⌊10×XY[;1]-1.95
[ 23]   INDY1←41-⌊10×XY[;2]-0.95
[ 24]   INDY2←41-⌊10×XY[;3]-0.95
[ 25]   TRE←⍴INDEX
[ 26]   FOE←0
[ 27] NUTHER:→(TRE<FOE←FOE+1)/NUF
[ 28]   PLT[INDY1[FOE];INDEX[FOE]]←PC[1]
[ 29]   PLT[INDY2[FOE];INDEX[FOE]]←PC[2]
[ 30]   →(INDY1[FOE]≠INDY2[FOE])/NUTHER
[ 31]   PLT[INDY1[FOE];INDEX[FOE]]←PC[3]
[ 32]   →NUTHER
[ 33] NUF:→0
        ∇


        ∇MOVE HOW
[  1]   AP[1 3 5]←AP[1 3 5]+AP[2 4 6]×HOW
        ∇
```

```
      ∇MOVEIT
[ 1]  →((,ρMM)=4)/N5
[ 2]  MOVE,MM
[ 3]  CHECK DATA9
[ 4]  →0
[ 5] N5:MOV4,MM
[ 6] ⍝ INORDER TO USE THIS ROUTINE DATA NAME
[ 7] ⍝ SHOULD FOLLOW CHK4 ON FOLLOWING LINE
[ 8] ⍝ AND CHECK ON LINE [3]
[ 9]  CHK4 DATA9
      ∇


      ∇MOV4 HOW
[ 1]  AP[1 3 5 7]←AP[1 3 5 7]+AP[2 4 6 8]×HOW
[ 2] ⍝THIS FUNCTION ALLOWS ONE TO CHANGE
[ 3] ⍝THE MODEL PARAMETERS AP BY INCREMENTAL
[ 4] ⍝AMOUNTS.
[ 5] ⍝IT IS USED WITH CHK4
      ∇


      ∇OUT←PHASE DATA
[ 1] ⍝ THIS FUNCTION COMPARES THE PHASE OF THE
[ 2] ⍝ DATA VARIABLE.  COLUMN 1 IS EXPECTED TO
[ 3] ⍝ BE FREQUENCY, COLUMN 2 AND 3 THE REAL
[ 4] ⍝ AND IMAGINARY PARTS OF THE INPUT DATA
[ 5] ⍝ COLUMN 4 AND 5 THE REAL AND IMAGINARY
[ 6]  OUT←PHA(DATA[; 4 5])
[ 7]  OUT←DATA[;1],(180÷○1)×(PHA(DATA[; 2 3])),[1.1] PHA(DA
        TA[; 4 5])
[ 8] ENDS:
      ∇


      ∇PLOTZ DATA
[ 1]  ⍞←LPT DATA[;1],(MAG DATA[; 2 3]),[1.5] MAG ZC
[ 2]  ⍞←PHASE DATA,ZC
      ∇


      ∇SETREZ
[ 1]  ⍞←'THE RESONANT POINT IS THE DATA POINT#'
[ 2]  NREZ←⍞
      ∇
```

```
      ∇TRY DATA;COE
[ 1]  ⎕←'INPUT DESIGN MATRIX '
[ 2]  DZN←⎕
[ 3]  ⎕←'INPUT PARAMETERS RD,ΔRD,(RE,ΔRE),C,ΔC,L,ΔL'
[ 4]  ⎕←AP←⎕
[ 5]  ⎕←'COEF EQUAL'
[ 6]  ⎕← 16 ¯6 ⍕COE←DZN EFIT DATA
[ 7]  ⎕←'COEF TIMES NOMINAL VALUES'
[ 8]  ⎕←COE×(⍴COE)⍴1,AP[(2×⍳(⍴AP)÷2)-1]
[ 9]  MM←FLOORVALUE
      ∇
```

Mutual Inductance and Sensitivity Programs

This section of the appendix describes the programs used to calculate the mutual inductances M_{12}, M_{13}-M_{14}, and M_{23}-M_{24}. The programs associated with the calculation of the sensitivity of the transfer function to probe and nulling network parameters are also described in this part of the appendix. The programs are described in alphabetical order.

APPROX

The function APPROX is listed on page 137. It computes the sensitivities for the parameters K by using Fidler's [6] technique. The size of the equal and opposite perturbations used is specified by the input variable SIZ. It is a moderately interactive function which is executed in this form:

```
1E-6 APPROX K,
```

where 1E-6 is the perturbation size and K is a vector of probe parameters. K is usually generated by the function INPUT. This program takes about 15 minutes to run on the IBM 5100 computer, and for this reason it prints numbers one through fifteen as it completes the computation of the sensitivity for each of the fifteen probe parameters. This gives the user some indication of how long the program

has left to run. When the computation is complete the function instructs the user to turn to a fresh page so that the output may be printed on a single sheet and not between pages. The printed output lists the size of the step taken and the run number. Below this it lists five columns. The first is the name of the parameter. The second and third are the real and imaginary parts of the input parameter respectively. The fourth and fifth are the real and imaginary parts of the sensitivity.

CLS

CLS is listed on page 137. It calculates the classical sensitivity formula from the global variables Q, QP, P and PP, where Q is the numerator of the transfer function, QP is the derivative of the numerator of the transfer function, P is the denominator of the transfer function and PP is the derivative of the denominator of the transfer function. The function has an explicit result CSENCE. It is used in SEN. It is not usually called by itself.

CONMU12

This program is listed on page 138. It is a conversational program that calculates the mutual between the drive coils. It requests the known input parameters in the form of a matrix K. If the matrix K is not in existance, the program is capable of calling the routine INPUT to obtain it.

The program expects the user to input dummy values for the mutual impedances. The parameters input should be the equivalent shunting resistances for R1 and R2, the equivalent shunting capacitances for C1 and C2, the series resistances for RD1 and RD2. The inductances should be input for L1 and L2. RS is the source resistance. Inputs asking for RE[X] or IM[X] are asking for the real part of parameter X or the imaginary part of parameter X respectively. These inputs are intended to describe the circuit when drive coil 1 is driven from a source with impedance RS and its other terminal grounded. The second drive coil is to be connected such that the output may be observed on an oscilloscope. The test setup used is described in Figure 2.6 on page 44. The program then requests the source voltage, the output voltage and the output phase. From this is computes the global variable M12.

CONVERT

CONVERT is listed on page 138. Its function is to take the input matrix K and convert it to individual variables with familiar names. K is of a form generated by the program INPUT. CONVERT is usually not called by itself. Its form of execution is

CONVERT K .

CQUAD

This function is listed on page 139. It finds the solution of the quadratic equation:

$$AX^2+BX+C=0 \ , \qquad (B.001)$$

where A=1 and B and C are complex inputs of the function. The resulting solution for X is also complex. It is done by applying the Equation 2.005 on page 35. The form of execution is

```
10 11 CQUAD 5 ¯3,
```

where 10 and 11 are the real and imaginary parts of B respectively and 5 and -3 are the real and imaginary parts of C respectively. The function has an explicit result.

DEG

The function DEG is listed on page 139. It converts the last column of the input matrix Y from radians to degrees. The first column is not changed in the result.

DV

DV is listed on page 139. It is a function whose explicit result is the division of the complex numbers input A and B. It will only handle numbers in which are two element vectors. The first element is the real part and the second is the imaginary part. The executable from is

```
10 11 DV 5 ¯3,
```

where the 10 and 11 is the complex number A and where 5 and -3 is the complex number B. The result will be A divided by B. This function is used in many of the other functions.

EMU

EMU is listed on page 140. It computes the value of the mutuals M_{13}-M_{14} and M_{23}-M_{24}. It does this by evaluating the Equations 2.017 on page 49 and 2.015 on page 48. This is a conversational program which asks for the required inputs. It first asks for the input parameters that were required for null. This is in a form of the matrix K generated by the function INPUT. If the matrix is not already in existence, the program can be made to call it. Input dummy values for the unknown mutuals between the drive coils and the pickups. The program then computes the value of X as defined by equation 2.014 on page 58. It then asks for the off null conditions. It expects here also to recieve an input in the form of generated by the program INPUT. It then asks for the source voltage in the off null experiment and magnitude of the measured output at the pickup coils. A diagram of this experiment may be found in Figure 2.9 on page 50. The outputs of this function are not explicit. They are the global variables M14, M24, M23 and M24 representing the corresponding mutual inductances.

FIGURE

This program is listed on page 140. Its function is to compute the output voltage of the second drive coil when the first drive coil is driven as in Figure 2.6 on page 44. Its purpose is to show that the mutual calculated for the two drive coils would give the correct output voltage. This function is not called by itself. The function CONMU12 or MU12 must have been executed first. This function has no arguments. Its output is in the form of the global variable VO.

IMAG

This function finds the imaginary part of its complex input. The input is in Cartesian form. This function is listed on page 141. It may be executed in the form:

```
IMAG 10 11,
```

where 10 and 11 are the real and imaginary parts of the complex input. The explicit result in this case will be 11.

INCREMENT

This function is used in the calculation of sensitivities. It steps the values of the input matrix K by the step matrix STEP in both increasing and decreasing directions. The resulting outputs are output explicitly in the form of a two by two matrix. The first row is the output when the value is decreased and the second is the value when the

value is increased. The assumed source voltage is unity. This function is listed on page 141. The executable form is

(1E⁻5 *STPGEN* 3) *INCREMENT K*,

where the function STPGEN generates a matrix of steps the same shape as K and haveing all elements zero except the third row which is of the elements equal to 1×10^{-5}. K is of the form of the matrix generated by the function INPUT.

INPUT

This function is listed on page 142. This function generates the matrix describing the probe and null network parameters. It asks for each parameter in turn, and then returns these in the form of a global variable K. It has no arguments and no explicit result.

LISTK

This function is listed on page 142. It is used to check the data that was input into the function INPUT. It lists the values of K along with the labels which describe what each of these values represent. It has no explicit result. It prints a matrix in which the first column is the parameter name, the second is the real part of the parameter and the third is the imaginary part.

M

This function is listed on page 143. It multiplies complex numbers A and B and yields the result explicitly. The form of execution is

```
10 11 M 5 -3,
```

where 10 and 11 is the first complex number and 5 and -3 is the second. The explicit result is the product of the two numbers.

MAG

MAG is listed on page 143. It finds the magnitude of a complex number arguement. The input number is in rectangular form. The form of execution is

```
MAG 3 4,
```

The 3 and 4 are the real and imaginary parts of the complex number respectively. The explicit result would be 5 in this case.

MU12

This function computes the mutual inductance between the two drive coils given that the values of the model already exist and given the arguments VS and VO which are the source voltage and the output voltage respectively. The experiment for which this program was designed, may be found in Figure 2.6 on page 44. In this experiment the first

drive coil is driven from a signal source with amplitude VS and the output is measured to have the value VO. The program solves Equation 2.013 on page 47 with the function CQUAD to find the roots. Its explicit result consists of a two by two matrix in which each row is one of the roots. The input arguments are expected to be complex and in rectangular form. The form of execution is

```
□←8 0 MU12 2 2,
```

where 8 and 0 are the real and imaginary parts of the source voltage in which the imaginary part is always zero. The 2 and 2 are the real and imaginary parts of the output voltage. This function is call by the conversational function CONMU12.

PHA

This function is listed on page 144. It returns the phase of a complex number in radians as an explicit result. The form of execution is

```
□←PHA 4 4,
```

where 4 4 is the complex argument. The explicit result of this particular case would be $\pi/4$.

POL2REC

This function converts polar complex numbers into the rectangular form. Its input is the number to be converted

the alternate form. The explicit result is the converted form. The form of execution is

POL2REC 1 3.1415926,

where 1 is the magnitude and 3.1415926 is the angle in radians. This function is listed on page 144.

RAD

This function is listed on page 144. It converts the angle of a complex number that is in polar form from degrees to radians. The form of execution is

RAD 1 180,

where 1 is the magnitude of the input and 180 is the angle in degrees. The explicit result in this case would be 1 and 3.14159265.

REAL

This function is listed on page 144. It finds the real part of a complex number in rectangular form. Its form of execution is

REAL 3 4,

where 3 and 4 are the real and imaginary parts of the input variable. The explicit result in this case would be 3.

REC2POL

This function is listed on page 144. Its purpose is to convert the complex number argument from rectangular to polar form. The form of execution is

REC2POL 10 10,

where in this case the real and imaginary parts of the input complex number are both 10.

RPARA

This function is used to find the equivalent resistance of two resistors in parallel. This convenience function is listed on page 145. Its use was to quickly find the value of resistance to put in the model for the parallel combination of the coil shunting resistance and the null network shunting resistance. The form of execution is

2 *RPARA* 3,

where 2 and 3 are the two resistance values. The explicit result in this case is the value 1.2.

SEEZ

SEEZ is listed on page 145. Its function is to print out the values of the matrix Z. The matrix Z is a matrix that contains the values of the Z's defined in Table 1.1 on page 12. The first row is the real and imaginary parts of Z_1 and the second is the real and imaginary parts of Z_2 and so

on. There is no explicit result of SEEZ. Its primary purpose was to assist in the varification of the computer programs. However, it may be a useful tool in reducing the sensitivity of the system to a particular parameter. It must not be called until SIMPLE or a routine which calls SIMPLE is called. This is because Z will not exist until then. The form of execution is

SEEZ.

SEN

SEN is listed on page 146. This function takes the parameter matrix generated by the function INPUT and finds the classical sensitivities for each parameter in the matrix. SEN uses the equations in Table 1.2 on page 21 to do this. This program takes about 10 minutes to execute on the IBM 5100 computer. It is somewhat faster than the incremental approach used in the function APPROX. The results of this function are output explicitly in a matrix in which each contains the sensitivity of the corresponding element in the input matrix K. The form of required for execution is

SEN K,

where K is the imput matrix of parameter values generated by INPUT. SEN is usually not called directly, but called by the conversational routine START.

SENSELIST

This function is listed on page 147. It lists the calculated classical sensitivities to 15 places. It is convenient to list these when comparing the results of the program SEN and APPROX. This function assumes that the result of SEN is stored in the global variable SVO. The form required for execution is simply the program name.

SIMPLE

This function is listed on page 147. It calculates the simplifying terms listed in Table 1.1 on page 12. This function assumes that the function CONVERT has been executed and the global variables describing the probe parameters exist. It has no explicit result. It creates the global matrix Z which contains the Z's listed in Table 1.1. Also the denominator of the transfer function, the numerator of the transfer function, and the transfer function itself is calculated and stored in the global variables P,Q and TFVO respectively. The form required for execution is simply the program name.

SIMPLIFY

This function is listed on page 148. It calculates the simplifying terms for the classical sensitivity calculation by calling the function SIMPLE. It then uses those terms to calculate the devatives of several of the basic

simplifying equations. This function requires no arguments and it requires only that the requirements for the execution of SIMPLE be met. It yields no explicit result and is not usually called by itself. It is usually called by the function START. Its output is in terms of several global variables. PZ1R1, PZ2R2, PZ1C1, PZ2C2, PZ1S, and PZ2S which are the derivatives of Z_1 with respect to R_1, Z_2 with respect to R_2, Z_1 with respect to C_1, Z_2 with respect to C_2, Z_1 with respect to S and Z_2 with respect to S respectively. The form that is required for execution is simply the function name.

SQ

This function is listed on page 148. Its explicit result is the square of its complex number argument. The form required for execution is

SQ A ,

where A is a complex number in rectangular form.

START

This function is conversational in nature and is designed to lead the user through the computation of classical sensitivities. The function is listed on page 148. It does require that the variable RUN have a value. RUN is the run number. Once a value for RUN is established, it need not be given a new value unless the user desires it. RUN is incremented each time START or APPROX is executed.

Unless START has been executed, the global variable LABELS will have no value. Therefore many functions which list the values of K will not function. START requires no arguments. Therefore its form of execution for the initial time is

```
RUN←0,
START,
```

where the global variable RUN is assigned the initial value of 0. Additional executions do not require this asignment. The result of START is a table of five columns. The first is a label describing the parameter. The second and third is the value of the parameter. The fourth and fifth is the sensitivity of the parameter.

STPGEN

This function is listed on page 149. Its purpose is generate a matrix of increments for each of the parameters in K. It has two arguments N and SIZ. N is the index of the element of K to be stepped. SIZ is the relative magnitude of the step. The explict result is a matrix with fifteen rows and two columns, all of which are zero except the Nth row which contains elements equal to SIZ. The form required for execution is

```
1E¯6 STPGEN 4,
```

where 1E-6 is the step size argument and 4 indicates that the fourth row will contain elements equil to 1×10^{-6}.

Listings of Mutual Inductance and Sensitivity Programs

The programs in this part are used to find the mutual inductances between the probe coils from experimental data and to find the classical sensitivities of the transfer function to probe and nulling network parameters. The main sensitivity programs listed are START, which calculates the classical sensitivities from the formulas derived in Chapter I and APPROX which calculates classical sensitivies from an incremental technique described in Chapter I. The listings are in alphabetical order.

```
        ∇SIZ APPROX K
[  1] ⍝ THIS FUNCTION APPROXIMATES SENSITIVITIES
[  2] ⍝ BY MAKING SMALL STEPS IN PARAMETERS
[  3] ⍝ AND ESTIMATING THE SENSITIVITY
[  4]  ASEN← 15 2 ⍴0
[  5]  RUN←RUN+1
[  6]  N←0
[  7]  CONVERT K
[  8]  SIMPLE
[  9]  NVO←TFVO
[ 10] ONE:→(15<N←N+1)/TWO
[ 11]  VO←(SIZ STPGEN N) INCREMENT K
[ 12] ⍝ SENSITIVITIES ARE ESTIMATED BY CALCULATING THE
[ 13] ⍝    DERIVITIVE OF VO WITH RESPECT TO PARAMETER K[N]
[ 14] ⍝ BY THE TECHNIQUE DESCRIBED BY J. K. FIDLER
[ 15] ⍝ AND NORMALIZING IT TO OBTAIN CLASSICAL SENSITIVITY
[ 16]  ASEN[N;]←(2 0 M T1 M T2) DV((T1←VO[1;]-NVO)-T2←VO[2;
         ]-NVO) M SIZ×K[N;]
[ 17]  ASEN[N;]←(ASEN[N;] M K[N;]) DV NVO
[ 18]  ⍞← 2 0 ⍕N
[ 19]  →ONE
[ 20] TWO:⍞←'TURN TO TOP OF PAGE FOR OUTPUT'
[ 21]  ANS←⍞
[ 22]  ⍞←60⍴'='
[ 23]  ⍞←'           APPROXIMATED SENSITIVITIES '
[ 24]  ⍞←'                      RUN NO. ', 4 0 ⍕RUN
[ 25]  ⍞←60⍴'='
[ 26]  ⍞←' STEP SIZ USED=', 11 ⍕4 ⍕SIZ
[ 27]  ⍞←'NOMINAL TRANSFER FUNCTION=',(12 ⍕4 ⍕NVO[1]),' + J
         ', 12 ⍕4 ⍕NVO[2]
[ 28]  ⍞←' '
[ 29]  ⍞←'                INPUT VALUES          SENSITIVITY'
[ 30]  ⍞←'            REAL      IMAGINARY    REAL      IMAGINAR
        Y'
[ 31]  ⍞←60⍴'-'
[ 32]  ⍞←LABELS, 12 ⍕4 ⍕K,ASEN
       ∇


       ∇CSENCE←CLS
[  1] ⍝ THIS ROUTINE CALCULATES CLASSICAL SENSITIVITY
[  2]  CSENCE←(QP DV Q)-PP DV P
       ∇
```

```
      ∇CONMU12
[  1] ⍝ THIS IS A CONVERSATIONAL PROGRAM FOR
[  2] ⍝ CALCULATION OF THE MUTUAL BETWEEN THE
[  3] ⍝ DRIVE COILS M12
[  4]  ⍞←'INPUT KNOWN PARAMETERS'
[  5]  ⍞←'USE DUMMYS FOR M12,M13,M14,M23,M24'
[  6]  ⍞←'IS DATA IN MATRIX FORM?'
[  7]  →('N'=⍞)/INY
[  8]  ⍞←'INPUT MATRIX K'
[  9]  K←⍞
[ 10]  →ELSY
[ 11] INY:INPUT
[ 12] ELSY:CONVERT K
[ 13]  ⍞←'SOURCE VOLTAGE='
[ 14]  VS←⍞
[ 15]  ⍞←'OUTPUT VOLTAGE MAGNITUDE'
[ 16]  VO←⍞
[ 17]  ⍞←'OUTPUT PHASE(DEG.)='
[ 18]  VO←VO,⍞×○1÷180
[ 19]  VO←POL2REC VO
[ 20]  M12←VS MU12 VO
      ∇


      ∇CONVERT K
[  1] ⍝THIS ROUTINE CONVERTS K TO FAMILIAR SYMBOLS
[  2]  S←K[1;]
[  3]  R1←K[2;]
[  4]  R2←K[3;]
[  5]  C1←K[4;]
[  6]  C2←K[5;]
[  7]  L1←K[6;]
[  8]  L2←K[7;]
[  9]  M13←K[8;]
[ 10]  M14←K[9;]
[ 11]  M23←K[10;]
[ 12]  M24←K[11;]
[ 13]  RD1←K[12;]
[ 14]  RD2←K[13;]
[ 15]  M12←K[14;]
[ 16]  RS←K[15;]
      ∇
```

```
      ∇X←B CQUAD C;T
[ 1] ⍝THIS FUNCTION PERFORMS A
[ 2] ⍝QUADRATIC SOLUTION UPON
[ 3] ⍝COMPLEX ARGUMENTS
[ 4] ⍝    WHERE:
[ 5] ⍝ X*2 COEF ASSUMED 1
[ 6] ⍝ X COEF=B
[ 7] ⍝ CONTSANT=C
[ 8]  X←((-B)-T←CSQR(SQ B)-4×C)÷2
[ 9]  X←⊖X,[1.5]((-B)+T)÷2
      ∇


      ∇X←DEG Y
[ 1]  X←(1↑Y),(¯1↑Y)×180÷○1
      ∇


      ∇Z←A DV B
[ 1] ⍝ THIS FUNCTION DIVIDES COMPLEX NUMBERS
[ 2]  Z←((A+.×B),-/(⌽A)×B)÷B+.×B
      ∇
```

```
      ∇EMU
[  1] ⍝ THIS FUNCTION ESTIMATES THE MUTUALS BETWEEN
[  2] ⍝ THE DRIVE AND PICKUP COILS OF THE PROBE
[  3]  ⎕←'INPUT CONDITIONS FOR NULL'
[  4]  ⎕←'USE DUMMY NUMBERS FOR M13 M14 M23 M24'
[  5]  ⎕←'IS INPUT IN MATRIX FORM?'
[  6]  →('N'=1↑⍞)/THEN
[  7]  ⎕←'INPUT MATRIX'
[  8]  K←⎕
[  9]  →ENDIF
[ 10] THEN:INPUT
[ 11] ENDIF:CONVERT K
[ 12]  SIMPLE
[ 13]  XX←Z[6;] ⌹ Z[7;]
[ 14]  ⎕←'INPUT OFF NULL CONDITIONS'
[ 15]  ⎕←'USE DUMMY NUMBERS FOR M13 M14 M23 M24'
[ 16]  ⎕←'IS DATA IN MATRIX FORM'
[ 17]  →('N'=1↑⍞)/THN
[ 18]  ⎕←'INPUT MATRIX'
[ 19]  K←⎕
[ 20]  →ENIF
[ 21] THN:INPUT
[ 22] ENIF:CONVERT K
[ 23]  SIMPLE
[ 24]  ⎕←'INPUT SOURCE VOLTAGE'
[ 25]  VSS←⎕,0
[ 26]  ⎕←'INPUT MAGNITUDE VO'
[ 27]  VVO←⎕
[ 28]  I1←VSS M Z[7;] ⌹ P
[ 29]  I2←VSS M Z[6;] ⌹ P
[ 30]  M14← 0 0
[ 31]  M24←(VVO÷MAG S M(I1 M XX)-I2),0
[ 32]  M23←M14
[ 33]  M13←XX M M24
       ∇


      ∇FIGURE
[ 1]  ZIN←RD1+(S M L1)-(SQ S M M12) ⌹ Z5
[ 2]  DET←((RS+Z1) M(Z1+ZIN))-SQ Z1
[ 3]  DET1←Z1 M VS
[ 4]  I1←DET1 ⌹ DET
[ 5]  VO←(Z2 M I1 M S M M12) ⌹ Z5
      ∇
```

```
      ∇X←IMAG A
[ 1]  X←¯1↑A
      ∇


      ∇VO←STEP INCREMENT K
[ 1] ⍝ THIS FUNCTION STEPS THE VALUES OF K
[ 2] ⍝ AND COMPUTES THE OUTPUT VOLTAGE AND CHANGE
[ 3]  VO← 2 2 ⍴0
[ 4]  CONVERT K×1-STEP
[ 5]  SIMPLE
[ 6]  VO[1;]←TFVO
[ 7]  CONVERT K×1+STEP
[ 8]  SIMPLE
[ 9]  VO[2;]←TFVO
      ∇
```

```
        ∇INPUT
[  1] ∩THIS FUNCTION INPUTS VARIABLES CONVERSATIONALLY
[  2]  K← 15 2 ⍴0
[  3]  ⍞←'FREQUENCY IN HZ='
[  4]  K[1;]←0,(○2)×F←⍎⍞
[  5]  ⍞←'R1='
[  6]  K[2;]←⍎⍞,0
[  7]  ⍞←'R2='
[  8]  K[3;]←⍎⍞,0
[  9]  ⍞←'C1='
[ 10]  K[4;]←⍎⍞,0
[ 11]  ⍞←'C2='
[ 12]  K[5;]←⍎⍞,0
[ 13]  ⍞←'RE[ L1 ]='
[ 14]  K[6;1]←⍎⍞
[ 15]  ⍞←'IM[ L1 ]='
[ 16]  K[6;2]←⍎⍞
[ 17]  ⍞←'RE[ L2 ]='
[ 18]  K[7;1]←⍎⍞
[ 19]  ⍞←'IM[ L2 ]='
[ 20]  K[7;2]←⍎⍞
[ 21]  ⍞←'RE[ M13-M14]='
[ 22]  K[8;1]←⍎⍞
[ 23]  ⍞←'IM[ M13-M4 ]='
[ 24]  K[8;2]←⍎⍞
[ 25]  K[9;]← 0 0
[ 26]  ⍞←'RE[ M24-M23 ]='
[ 27]  K[11;1]←⍎⍞
[ 28]  ⍞←'IM[ M24-M23 ]='
[ 29]  K[11;2]←⍎⍞
[ 30]  K[10;]← 0 0
[ 31]  ⍞←'RD1='
[ 32]  K[12;]←⍎⍞,0
[ 33]  ⍞←'RD2='
[ 34]  K[13;]←⍎⍞,0
[ 35]  ⍞←'RE[ M12 ]='
[ 36]  K[14;1]←⍎⍞
[ 37]  ⍞←'IM[ M12 ]='
[ 38]  K[14;2]←⍎⍞
[ 39]  ⍞←'RS='
[ 40]  K[15;]←⍎⍞,0
        ∇


        ∇LISTK
[ 1]   ⍞←LABELS, 24 ¯15 ⍕K
        ∇
```

```
        ∇Z←A M B
[ 1] ⍝ THIS FUNCTION MULTIPLIES COMPLEX NUMBERS
[ 2]  Z←(-/A×B),+/A×⌽B
        ∇


        ∇Z←MAG A
[ 1] ⍝ THIS FUNCTION FINDS THE MAGNITUDE OF A COMPLEX NUMBE
       R
[ 2]  Z←(+/A*2)*0.5
        ∇


        ∇M12←VS MU12 VO
[  1] ⍝ THIS FUNCTION CALCULATES THE MUTUAL INDUCTANCE
[  2] ⍝ BETWEEN TWO COILS
[  3]  Z1←R1 DV 1 0 +S M R1 M C1
[  4]  Z2←R2 DV 1 0 +S M R2 M C2
[  5]  Z5←(S M L2)+RD2+Z2
[  6]  Z4←(S M L1)+RD1+Z1
[  7]  Z11←RD1+S M L1
[  8]  A←VO M(RS+Z1) M SQ S
[  9]  B←VS M Z2 M S M Z1
[ 10]  C←-VO M Z5 M(RS M Z4)+Z1 M Z11
[ 11]  M12←(B DV A) CQUAD C DV A
        ∇
```

```
        ∇B←PHA A
[  1]  ⍝THIS FUNCTION RETURNS THE PHASE
[  2]  ⍝OF A COMPLEX NUMBER IN RADIANS
[  3]   →((⍴⍴A)=2)/DEU
[  4]   →((x1↑A)= ¯1 0)/NEG,ZER
[  5]   B←¯3○÷/ΦA
[  6]   →0
[  7]  NEG:B←(○(x¯1↑A)+((x¯1↑A)=0))+¯3○÷/ΦA
[  8]   →0
[  9]  ZER:B←○0.5x(x¯1↑A)
[ 10]   →0
[ 11]  DEU:TTT←1↑⍴A
[ 12]   QQQ←xA
[ 13]   TT1←(((,(TTT,1)↑QQQ))=TTT⍴1)/⍳TTT
[ 14]   TT2←((,((TTT,1)↑QQQ))=TTT⍴0)/⍳TTT
[ 15]   TT3←((,((TTT,1)↑QQQ))=TTT⍴¯1)/⍳TTT
[ 16]   B←TTT⍴0
[ 17]   B[TT2]←○0.5xQQQ[TT2;2]
[ 18]   B[TT1]←¯3○÷/ΦA[TT1;]
[ 19]   B[TT3]←(○QQQ[TT3;2]+QQQ[TT3;2]=0)+¯3○÷/ΦA[TT3;]
        ∇


        ∇X←POL2REC Y
[ 1]   ⍝THIS FUNCTION CONVERTS POLAR COMPLEX VARIABLES
[ 2]   ⍝TO RECTANGULAR FORM. ANGLES ARE IN RADIANS
[ 3]    X←((1↑Y)x2○(¯1↑Y)),(1↑Y)x1○¯1↑Y
        ∇


        ∇X←RAD Y
[ 1]    X←(1↑Y),(¯1↑Y)x○1÷180
        ∇


        ∇X←REAL A
[ 1]    X←1↑A
        ∇


        ∇X←REC2POL Y
[ 1]    X←(MAG Y),PHA Y
        ∇
```

```
      ∇Z←X RPARA Y
[ 1] ⍝THIS FUNCTION CALCULATES THE PARALLEL COMBINATION
[ 2] ⍝OF TWO RESISTORS
[ 3]  Z←(X×Y)÷X+Y
      ∇


      ∇SEEZ
[ 1] ⍝ THIS PROGRAM LISTS Z
[ 2]  ' Z           RE[Z]                    IM[Z] '
[ 3]  2 0 23 ¯15 23 ¯15 ⍉(⍳11),Z
      ∇
```

```
      ∇SVO←SEN K
[  1] ⍝ THIS ROUTINE CALCULATES CLASSICAL SENSITIVITIES
[  2] ⍝ FOR NULL NETWORK AND PROBE PARAMETERS K
[  3]  CONVERT K
[  4]  SIMPLIFY
[  5]  QP←S M(M1314 M Z[5;])+M2324 M Z[2;]-SM12
[  6]  PP←(Z[5;] M Z[3;]+Z[4;]- 2 0 M Z[1;])-SQ Z[2;]-SM12
[  7]  SVO← 15 2 ⍴0
[  8]  SVO[2;]←R1 M PZ1R1 M T←CLS
[  9]  SVO[4;]←C1 M PZ1C1 M T
[ 10]  QP←S M(M1314 M Z[1;]-SM12)+Z[4;] M M2324
[ 11]  PP←(Z[4;] M Z[3;]+Z[5;]- 2 0 M Z[2;])-SQ Z[1;]-SM12
[ 12]  SVO[3;]←R2 M PZ2R2 M T←CLS
[ 13]  SVO[5;]←C2 M PZ2C2 M T
[ 14]  PP←(Z[4;] M Z[5;])-SQ SM12
[ 15]  SVO[15;]←-RS M PP DV P
[ 16]  QP←S M M2324 M Z[2;]
[ 17]  PP←(Z[3;] M Z[5;])-SQ Z[2;]
[ 18]  SVO[6;]←S M L1 M T1←CLS
[ 19]  QP←S M M1314 M Z[1;]
[ 20]  PP←(Z[3;] M Z[4;])-SQ Z[1;]
[ 21]  SVO[7;]←S M L2 M T2←CLS
[ 22]  SVO[12;]←RD1 M T1
[ 23]  SVO[13;]←RD2 M T2
[ 24]  QP←-S M(Z[2;] M M1314)+Z[1;] M M2324
[ 25]  PP← 2 0 M(Z[1;] M Z[2;])-Z[3;] M SM12
[ 26]  SVO[14;]←SM12 M CLS
[ 27]  →(0=MAG M13)/SKIP1
[ 28]  SVO[8;]←(S M M13 M Z[7;]) DV Q
[ 29] SKIP1:→(0=MAG M14)/SKIP2
[ 30]  SVO[9;]←-(S M M14 M Z[7;]) DV Q
[ 31] SKIP2:→(0=MAG M23)/SKIP3
[ 32]  SVO[10;]←(S M M23 M Z[6;]) DV Q
[ 33] SKIP3:→(0=MAG M24)/SKIP4
[ 34]  SVO[11;]←-(S M M24 M Z[6;]) DV Q
[ 35] SKIP4:A←(Z[1;] M L2)-Z[2;] M M12
[ 36]  A←A+PZ2S M Z[1;]-SM12
[ 37]  A←S M A+PZ1S M Z[5;]
[ 38]  A←M1314 M Z[7;]+A
[ 39]  B←(PZ1S M Z[2;]-SM12)+(Z[2;] M L1)-Z[1;] M M12
[ 40]  B←Z[6;]+S M(PZ2S M Z[4;])+B
[ 41]  QP←A+M2324 M B
[ 42]  A←Z[5;] M Z[3;]+Z[4;]- 2 0 M Z[1;]
[ 43]  A←PZ1S M A-SQ Z[2;]-SM12
[ 44]  B←SQ Z[1;]-SM12
[ 45]  A←A+PZ2S M(Z[4;] M Z[3;]+Z[5;]- 2 0 M Z[2;])-B
[ 46]  B←Z[3;] M((Z[5;] M L1)+Z[4;] M L2)- 2 0 M S M SQ M12
[ 47]  B←(-L1 M SQ Z[2;])+(-L2 M SQ Z[1;])+B
[ 48]  B←B+ 2 0 M M12 M Z[1;] M Z[2;]
[ 49]  PP←A+B
[ 50]  SVO[1;]←S M CLS
```

```
        ∇SENSELIST
[  1]  ⍝ THIS ROUTINE LISTS THE CALCULATED CLASSICAL
[  2]  ⍝       SENSITIVITIES CALCULATED BY SEN
[  3]  ⍝            TO 15 DIGITS
[  4]   ⎕PW←120
[  5]   ⎕←'                                    CLASSICAL SENSITIVITIE
         S'
[  6]   ⎕←'                                         RUN NO. ', 4 0 ⍕R
         UN
[  7]   ⎕←120⍴'='
[  8]   ⎕←(24⍴' '),'INPUTS',(40⍴' '),'SENSITIVITIES'
[  9]   ⎕←120⍴'-'
[ 10]   LABELS, 23 ¯15 ⍕K,SVO
[ 11]   ⎕PW←60
        ∇


        ∇SIMPLE
[  1]  ⍝ THIS ROUTINE CALCULATES SIMPLIFYING TERMS
[  2]   Z← 11 2 ⍴0
[  3]   Z[1;]←R1 DV 1  0 +SMR1C1←S M R1 M C1
[  4]   Z[2;]←R2 DV 1  0 +SMR2C2←S M R2 M C2
[  5]   Z[3;]←+/[1] RS,[1] Z[⍳2;]
[  6]   Z[4;]←(S M L1)+RD1+Z[1;]
[  7]   Z[5;]←(S M L2)+RD2+Z[2;]
[  8]   Z[6;]←(Z[2;] M Z[4;])-Z[1;] M SM12←S M M12
[  9]   Z[7;]←(Z[1;] M Z[5;])-Z[2;] M SM12
[ 10]   Z[8;]←-Z[5;] M SQ Z[1;]
[ 11]   Z[9;]←-Z[4;] M SQ Z[2;]
[ 12]   Z[10;]←Z[3;] M(Z[4;] M Z[5;])-SQ SM12
[ 13]   Z[11;]← 2 0 M SM12 M Z[1;] M Z[2;]
[ 14]   P←+/[1] Z[7+⍳4;]
[ 15]   Q←S M((M1314←M13-M14) M Z[7;])+Z[6;] M M2324←M23-M24
[ 16]   TFVO←Q DV P
        ∇
```

```
      ∇SIMPLIFY
[  1] ⍝ THIS ROUTINE CALCULATES PARTIALS OF Z1 AND Z2
[  2] ⍝         WITH RESPECT TO R1,C1,R2,C2 AND S
[  3] ⍝         AND CALLS SIMPLE
[  4]  SIMPLE
[  5]  PZ1R1← 1 0 DV SQ 1 0 +SMR1C1
[  6]  PZ2R2← 1 0 DV SQ 1 0 +SMR2C2
[  7]  PZ1C1←S M T1←-(SQ R1) M PZ1R1
[  8]  PZ2C2←S M T2←-(SQ R2) M PZ2R2
[  9]  PZ1S←C1 M T1
[ 10]  PZ2S←C2 M T2
      ∇


      ∇Z←SQ A
[ 1] ⍝ THIS FUNCTION SQUARES COMPLEX NUMBERS
[ 2]  Z←(A-.×A),A+.×⌽A
      ∇


      ∇START
[  1] ⍝ THIS ROUTINE CALCULATES SENSITIVITIES CONVERSATIONA
       LLY
[  2]  ⎕←'IS INPUT IN 15 BY 2 MATRIX FORM?'
[  3]  →('N'=1↑⍞)/IN
[  4]  ⎕←'INPUT S,R1,R2,C1,C2,L1,L2,M13,M14,M23,'
[  5]  ⎕←'       M24,RD1,RD2,M12,RS'
[  6]  K←⎕
[  7]  →IN2
[  8] IN:INPUT
[  9] IN2:SVO←SEN K
[ 10]  RUN←RUN+1
[ 11]  LABELS←'S    R1   R2   C1   C2   L1   L2   '
[ 12]  LABELS←LABELS,'M13 M14 M23 M24 RD1 '
[ 13]  LABELS←LABELS,'RD2 M12 RS  '
[ 14]  LABELS← 15 4 ⍴LABELS
[ 15]  ⎕←'       CLASSICAL SENSITIVITY CALCULATIONS'
[ 16]  ⎕←'                        RUN NO.', 4 0 ⍕RUN
[ 17]  ⎕←'          INPUT VALUES              SENSITIVITY'
[ 18]  ⎕←'        REAL      IMAGINARY     REAL      IMAGINARY'
[ 19]  ⎕←'        PART         PART       PART         PART'
[ 20]  ⎕←LABELS, 11 ¯4 ⍕K,SVO
[ 21]  ⎕←'DATA STORED IN K MODIFY AND RERUN''
      ∇
```

```
      ∇STP←SIZ STPGEN N
[ 1] ⍝THIS FUNCTION GENERATES THE STEP MATRIX
[ 2] ⍝VARYING EACH OF THE PARAMETERS BY SIZ
[ 3]  STP← 15 2 ⍴0
[ 4]  STP←(N=(\15),[1.2]\15)×SIZ
      ∇
```

VITA

Mark Hastings Neville was born in Wyandotte, Michigan on September 27, 1948. He attended elementary schools in that city. He moved to Memphis, Tennessee in January of 1961 and attended Kingsbury Junior and Senior High School. He graduated from there in 1967 and entered The University of Tennessee at Martin that fall. He participated in the Co-operative Education program during his sophomore year at Martin and junior year at Knoxville. He Co-oped with the Memphis Light, Gas and Water. He received his Bachelor of Science in Electrical Engineering with honors in August of 1972. He entered graduate school at the University of Tennesse at Knoxville in September of that year. At this time he accepted a research assistantship there also and began pursuing a Master's degree.

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