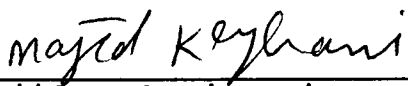
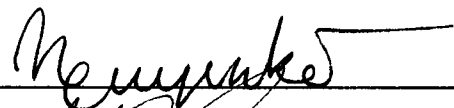
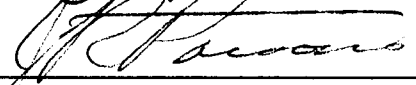
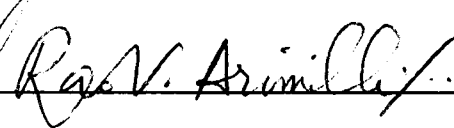


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HOLOGRAPHIC INTERFEROMETRIC STUDY OF
NATURAL CONVECTION ALONG A VERTICAL PLATE

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

William P. Partridge Jr.

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ABSTRACT

The full range of information pertaining to holographic interferometry has been documented. The phenomenon of light interference is explained and subsequently used to develop the equation of ideal interferometry. The non-ideal characteristics of interferometric research are discussed and the corresponding correction terms are presented. Moreover, thorough consideration is given to the configuration, adjustment and operation of a holographic interferometer. The influence of the experimental apparatus on the accuracy of the interferometric results is addressed. The use of holographic interferometry in research is demonstrated via investigation of natural convection heat transfer along a vertical isothermal surface. The procedure for analysis of the resulting interferogram for determination of local surface temperature and Nusselt number is given. The analysis procedure presented in this thesis differs from traditional methods in that both bright and dark fringes are used, and a linear polynomial is used to extrapolate the surface temperature and temperature gradient. Additionally, a statistical method

of determining the number of interferometric data sets used to produce the first order temperature profile is proposed. The analysis presented strives to generalize the procedure in order to create an analysis method which can be utilized in other applications. The interferometric results are compared to thermocouple measured surface temperature and similarity solution to determine their accuracy.

It was found that using both fringe types provides sufficient data density to support accurate first order extrapolation of the results. The average error in the Nusselt numbers thus obtained was 6.81 percent, which is within the expected range for the number of fringes in the evaluated interferogram.

TABLE OF CONTENTS

SECTION	PAGE
1. Introduction.....	1
1.1 Background Information.....	1
1.2 Mach-Zehnder Interferometer.....	6
1.3 Holographic Interferometer.....	9
1.4 Comparison of MZI and HI.....	13
1.4.1 Similarities.....	13
1.4.2 Differences.....	14
1.4.3 Benefits of Holographic Interferometry.....	14
1.5 Objective of the Present Study.....	15
2. Ideal Interferometry.....	17
2.1 Wavefront Distortion by a refractive Index Gradient.....	17
2.2 Light Interference.....	20
2.3 Relation of Fringes to Fringe Shift.....	23
2.4 Ideal Interferometry Equation for Temperature Measurement.....	29
3. Real Interferometry.....	33
3.1 Corrections.....	33
3.1.1 Errors Produced by Refraction.....	33
3.1.1.1 Displacement Error.....	34
3.1.1.2 Refraction Error.....	46

SECTION	PAGE
3.1.2 End Effect Errors.....	49
4. Setup Parameters Options.....	56
4.1 Fringe Field.....	56
4.1.1 Infinite-Fringe Interferogram.....	56
4.1.2 Finite-Fringe Interferogram.....	57
4.1.3 Comparison of Infinite and Finite-Fringe	65
4.2 Data presentation.....	69
4.2.1 Double Exposure Interferometry.....	69
4.2.2 Real Time Interferometry.....	71
4.2.3 Comparison of Double Exposure and Real Time.....	73
4.3 Holographic Recording Medium.....	77
4.3.1 Recording Medium Types.....	78
4.3.1.1 Thermoplastic Film Plate....	78
4.3.1.2 Silver Halide Emulsion Film Plates.....	79
4.3.1.3 Recording Medium Selection Criteria.....	84
4.3.2 Holographic Plate Exposure.....	88
4.3.2.1 Angle of Incidence.....	89
4.3.2.2 Irradiance Ratio.....	92
4.3.2.3 Exposure Time.....	94
4.3.3 Processing.....	98

SECTION	PAGE
4.3.3.1 Amplitude Hologram.....	99
4.3.3.2 Phase Hologram.....	101
5. Holographic Interferometric Laboratory.....	105
5.1 The Interferometer.....	105
5.1.1 Interferometer Components.....	105
5.1.2 Setup Adjustments.....	112
5.1.2.1 Optical table.....	113
5.1.2.2 Pathlength.....	117
5.1.2.3 Variable Attenuator/Beamsplitter.....	118
5.1.2.4 Mirrors.....	119
5.1.2.5 Objective/Pinhole Assembly..	122
5.1.2.6 Collimating lens.....	128
5.1.2.7 Film Plate Holder Positioning.....	150
5.1.2.8 Projection Screen Positioning.....	151
5.1.2.9 Imaging Lens Positioning....	151
5.2 Data Collection.....	152
5.2.1 Temperature Measurement.....	154
5.2.2 Pressure Measurement.....	155
5.2.3 Interferogram Digitization and Storage.....	155
6. Experimental Apparatus.....	157

SECTION	PAGE
6.1 Design Considerations.....	157
6.2 Apparatus.....	161
6.3 Test Section Alignment	165
6.4 Experimental Procedure.....	169
7. Results.....	171
7.1 Holograms.....	171
7.2 Data reduction.....	174
7.2.1 Analysis of the Temperature Field.....	175
7.2.2 Determination of Wall Temperature and Nusselt Number.....	194
7.3 Conclusions.....	210
APPENDIXES.....	215
APPENDIX A. RAY REFRACTION IN A LINEAR STRATIFIED REFRACTIVE INDEX MEDIUM.....	216
APPENDIX B. END EFFECT CORRECTION.....	221
APPENDIX C. UNCERTAINTY ANALYSIS.....	226
APPENDIX D. EXPERIMENTAL RESULTS.....	240
APPENDIX E. DATA REDUCTION PROGRAM.....	251
VITA.....	269

LIST OF FIGURES

FIGURE	PAGE
1.1 Mach-Zehnder Interferometer layout.....	7
1.2 Holographic Interferometer layout.....	10
1.3 Holographic Interferometric process.....	12
2.1 Wavefront interference of two light beams.....	27
3.1 Refraction of a light ray in a linearly varying refractive index field.....	36
3.2 Locating an image plane, I_c , corresponding through an imaging lens to a given focal plane, F_c	38
3.3 Displacement error associated with a given focal plane.....	42
3.4 Pathlength of the ray closest to the heater surface in a homogeneous and nonhomogeneous medium.....	48
3.5 End effect due to boundary layer termination at heater surface ends.....	51
4.1 Irradiance distribution resulting from the interference of two slightly diverging light beams.....	59
4.2 Deflection of and method of assigning fringe shift values to reference fringes.....	62
4.3 Thermoplastic plate operation.....	80
4.4 Modulation transfer function (MTF) curve.....	91
4.5 Density log exposure ($D \log E$) curve.....	96
4.6 Transmittance log exposure ($T \log E$) curve.....	96
5.1 Optical table leveling.....	116

FIGURE	PAGE
5.2	Mirror adjustment.....121
5.3	Objective pinhole assembly (OPA) adjustment...123
5.4	Collimating lens (L1) adjustment.....130
5.5	Focal length adjustment.....134
5.6	Collimation tool.....137
5.7	Effect of lens positioning on spherical aberration.....144
5.8	Spherical aberration measuring tool (SAMT)....147
5.9	Imaging lens(L2) adjustment.....153
6.1	Experimental Apparatus.....163
7.1	Interferogram of natural convection boundary layer along a vertical isothermal surface.....172
7.2	Scale standard to determine scale factor.....173
7.3	Interferometric data reduction technique.....178
7.4	Combined bright and dark fringe interferometric data and results for analysis at $y/h = 0.5$ 182
7.5	Fringe shift variation through the boundary layer.....184
7.6	Boundary layer temperature profiles.....186
7.7	Fringe shift gradient distribution within the boundary layer.....189
7.8	Temperature gradient distribution within the boundary layer at $y/h = 0.33$ 190
7.9	Temperature gradient distribution within the boundary layer at $y/h = 0.50$ 191
7.10	Temperature gradient distribution within the boundary layer at $y/h = 0.68$ 192

FIGURE		PAGE
7.11	Results of interferometric analysis including surface temperature and Nusselt number at $y/h = 0.50$	206
7.12	Comparison of interferometrically determined and exact Nusselt numbers.....	208

LIST OF SYMBOLS

NOMENCLATURE

A	amplitude
BS _i	i'th beamsplitter
b	fringe spacing
c	speed of light
C ₁ , C ₂	coefficients of the linear temperature profile fit
d	relative phase difference
d	length of end effect influence
D	density
e	electrical permittivity
E	electric intensity or exposure
f	focal length
f/	f-number. ratio of focal length to usable diameter
F	focal plane
g	acceleration due to gravity
Gr	Grashof number
h	test section height
HI	holographic interferometer
I	irradiance or image plane or total power density
I _i	i'th iris diaphragm
k	propagation number
K	Gladstone-Dale constant

L	test section length
Li	i'th lens
Mi	i'th mirror
MTF	modulation transfer function
MZI	Mach-Zehnder interferometer
n	refractive index
Nu	nusselt number
OPA	objective pinhole assembly
OPD	optical pathlength difference
P	pressure
PC	column pixels
PR	row pixels
Pr	Prandtl number
PS	projection screen
r	specific refractivity\
R	gas constant
S	source or fringe shift or sensitivity
s	length as measured along a light ray
SAMT	spherical aberration measuring tool
SF	spatial filter
SIDEAL	ideal fringe shift
AS1	fringe shift correction for refraction error
AS2	fringe shift correction for end effect error
ti	time i
T	temperature or transmittance

ΔT	temperature above ambient
u	fluid velocity in x direction
v	fluid velocity in y direction or speed of propagation
VAB	variable attenuator/beamsplitter
w	angular velocity or uncertainty
x	coordinate axis perpendicular to surface
y	coordinate axis parallel to surface
z	coordinate axis parallel to object beam

SUBSCRIPTS

o	values in vacuum
∞	values at ambient
C	correct
D	downstream
u	uncorrected
U	upstream
wall	value at the surface
y	value at local y

SUPERSCRIPTS

$*$	differentiation with respect to x
$'$	differentiation with respect to z

GREEK SYMBOLS

λ	wavelength
θ	impingement angle

ϕ	phase angle
Φ	optical pathlength
$\Delta\Phi$	optical pathlength difference
ρ	density
δ	boundary layer thickness
α	thermal diffusivity
β	volumetric thermal expansion coefficient
ν	kinematic viscosity

1. Introduction

1.1 Background Information:

The result of an interferometric process, as implied by the name, is an interference pattern formed by two or more beams of light. This interference pattern is observed as alternating dark and bright lines which are called fringes. The interference pattern is referred to as an interferogram. Light is an electromagnetic wave and behaves accordingly, and is described just as are more familiar macroscopic waves (i.e. oceanic, trigonometric, etc...). Because of this wave similarity, irrespective of dimension, one may investigate the foundation of interference by applying the linear superposition principle to any wave. The bounding limits of the linear superposition principle occurs when the waves are in phase and 180 degrees out of phase, which results in a doubling and nullifying the amplitude, respectively. This addition and subtraction of the amplitudes is referred to as constructive and destructive interference, respectively. When one observes the interference pattern formed by two light beams, what the eye interprets as bright and dark fringes is actually variations in the irradiance of the resultant beam. It may be shown that the irradiance of the combined beam is directly proportional to the

amplitudes of the participating beams. Therefore, one expects to observe a dark fringe in areas of destructive interference, and a bright fringe in areas of constructive interference. Thus, the fringes represent lines of constant phase shift. Although the above discussion is a simplistic presentation of the interferometric process, it fulfills the purpose of providing an initial means of fathoming the concept of light interference. A more formal and thorough presentation of this phenomenon will be in a forthcoming section.

A broad variety of interferometers have been successfully developed to perform specific measurements. Born and Wolf (1964) provide a thorough summary of the different types of interferometers. Prior to the advent of the Holographic Interferometer (HI), the most commonly employed interferometer in heat transfer research was the Mach-Zehnder Interferometer (MZI). The MZI was conceived independently by Zehnder (1891) and Mach (1892). Development of the hologram (Gabor 1948, 1949, 1951) and laser (Schawlow and Townes 1958), which is a highly coherent and monochromatic light source, provided new components which facilitated the modification of the existing measurement techniques. The use of a hologram in interferometry was first proposed by Horman (1965). The introduction of HI by Horman was met enthusiastically by

the scientific community, and was used extensively in a variety of disciplines.

As discussed earlier, a fringe pattern is the result of a phase shift between two beams of the interferometer. In application of interferometry to thermal sciences, this phase shift is due to density variations within the fluid through which the light beams are propagating.

Interferometry can provide both qualitative and quantitative information about the density field.

Qualitatively, interferometry is used extensively for flow visualization such as thermal boundary layers, plumes, and shock waves. Quantitative information such as spatial temperature or pressure distributions, concentration gradients, and conductive heat transfer coefficients can be obtained from an interferogram. These results can be deduced from relating density field to the parameter of interest by invoking appropriate laws and constitutive equations. It should be noted that in applications of HI to thermal sciences research, the object beam passes through the field of interest. Thus, this class of HI applications are referred to as transmission mode interferometry.

Interferometry may also be applied to the study of solid mechanics. This type of application is referred to as reflective mode interferometry. In solid mechanics

applications, the phase shift between the two beams is introduced via a physical lengthening (+ or -) of the path of one of the beams due to the process under study. In the reflective mode, the object under investigation is incorporated into the optical set-up as a mirror. For instance, if one of the light beams was to be reflected off of a speaker surface at some point along its path, its pathlength and thus relative phase would be dependent on the position of the speaker surface. As this surface oscillated or deformed, the relative phase would change accordingly, producing an interferometric pattern. In this type of application, fringes are formed by pathlength differences between the two light beams corresponding to increments of one half the wavelength of light being used. Since reflective mode investigation is not the focus of this paper, reference to the interferometer's application to solid mechanics will not be discussed further.

The advantages and disadvantages of interferometry are in general the consequence of it's optical measurement basis. One of the most desirable and unique characteristics of the interferometer, is it's non-intrusive measurement capability. Thermocouples and probes disturb the field of interest. Compared to these classical techniques which physically alter the field of measurement, HI imposes only a low intensity optical wave

on the field leaving it virtually unaltered. The form of the output of an interferometer also adds to it's desirability. A single interferogram contains global information of the entire field. Thus, one may directly obtain quantitative information on any point in the field of the interferogram. This is in contrast to measurement techniques which produce point information, and require interpolation to deduce intermediate spacial values. The effect of this interpolation is degraded accuracy. An additional highly desirable characteristic of HI is it's compatibility to transient processes. Because light is practically inertialess, the interferometer induces no time lag or settling time in it's output (i.e. the interferogram). That is, changes in the interferogram occur simultaneously with the changing process being observed.

The primary factors which limit the application of interferometers are cost and portability. Even the most elementary interferometer would cost several hundreds of dollars. Although portable interferometers exist, they are less convenient than other measurement techniques such as the thermocouple or hot wire anemometer. The above factors aside, the most prominent disadvantage of interferometers is their indirect presentation of quantitative results. That is, these results must be

extracted from the interferogram. A detailed comparison of MZI and HI will be presented following a general description of their configuration which appears in the following sections.

1.2 Mach-Zehnder Interferometer:

The configuration of a MZI employing a laser light source is depicted in Fig. 1.1. When discussing optical arrangements, it is convenient to invoke a river/light analogy. That is to define the terms upstream and downstream relative to the propagation of light. Herein, upstream and downstream will be used to indicate locations along the optical axis relative to a reference point in the direction and counter-direction of light propagation, respectively. The laser is expanded and collimated by lenses L1 and L2 as shown in Fig. 1.1. The beam is then divided into two beams by beamsplitter BS1. These two beams are referred to as the object beam and the reference beam. The object and reference beams are reflected by mirrors M1 and M2, respectively, such that they will be incident on beamsplitter BS2. The two beams are recombined by BS2 and are focused onto projection screen PS by lens L3. Notice that a recombined beam will also leave BS2 along the optical axis between M2 and BS2. For viewing the hologram, the use of this recombined beam, in

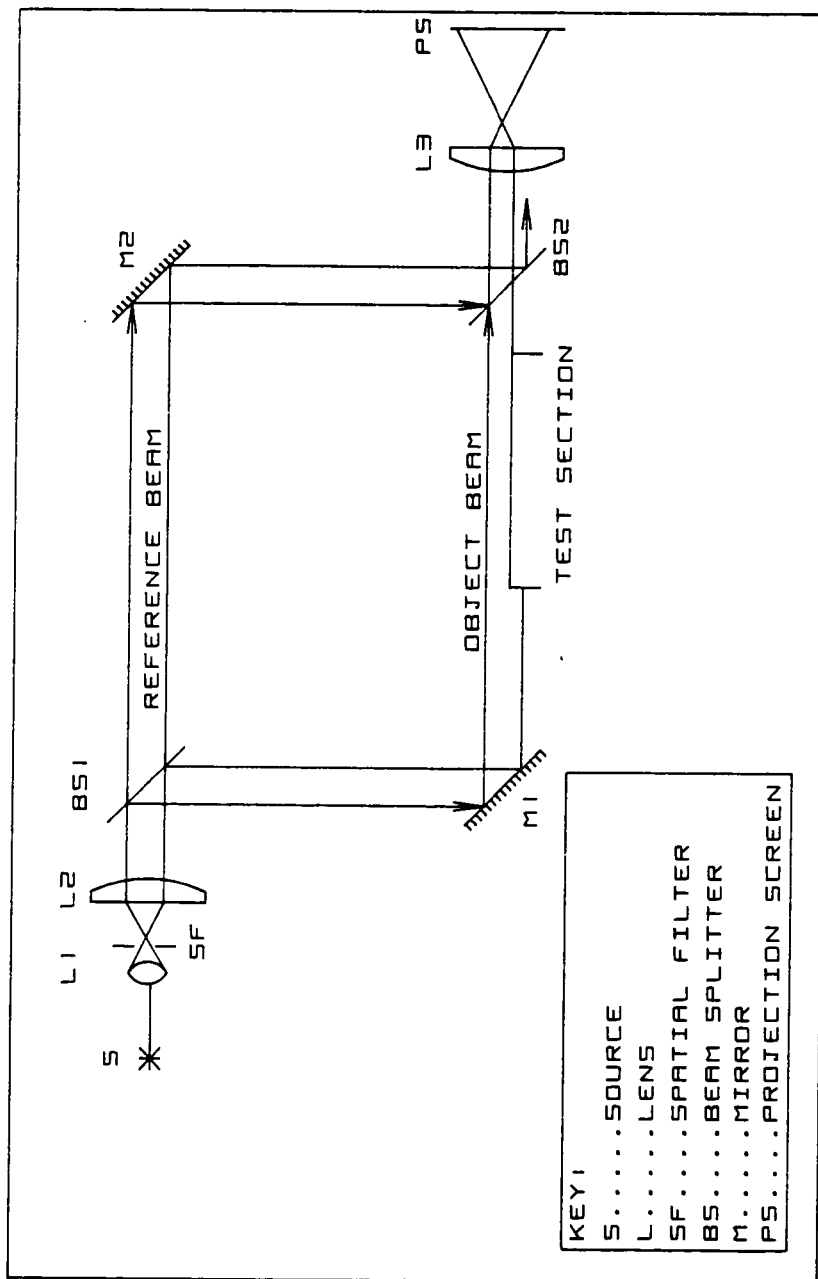


Fig. 1.1 Mach-Zehnder Interferometer layout.

place of the configuration displayed in Fig. 1.1, is equally valid. This general configuration is adjusted such that, in the absence of any pathlength changing phenomenon throughout both beams, the beams recombine in phase to produce a fringe free interferogram. The test section is introduced into the path of the object beam. It is therefore the object beam which experiences the pathlength changing phenomenon resulting from the process under investigation. Thorough discussions of alignment of the MZI are given by Hauf and Grigull (1970), and Jessee (1983).

In order to discuss the basic operation of the MZI, let's assume that all of the optical components are ideal, and that the test section is an isothermal plate at a temperature above ambient. Under these circumstances, the wavefronts, surfaces of constant phase, of the reference beam will remain planar. Additionally, the wavefronts of the object beam will also remain planar upstream of the test section. Because of the temperature gradient normal to the heated plate, the wavefront of the object beam will be distorted from it's planar shape as it translates through the medium adjacent to the plate. Downstream of the test section, there is no refractive index gradient normal to the direction of propagation. Therefore, the distorted wavefront of the object beam will remain

unchanged downstream of the test section. Actually, the preservation of a plane wavefront in the reference beam additionally assumes that this beam is not violated by the effects of the test section which produce the phase shift in the object beam. During initial configuration of the optical layout, the path of the reference beam is positioned an appropriate distance from the test section to validate the above consideration. Downstream of beamsplitter BS2 shown in Fig. 1.1, the relative phase shift between the superimposed waves produces the interference pattern discussed earlier.

1.3 Holographic Interferometer:

A schematic representation of the layout of the HI is presented in Fig. 1.2. This configuration, developed by Leith and Upatnieks (1962, 1963, 1964), is termed off-axis interferometry. Note that the functions of the optical components common to both the MZI and HI are identical. The only new component is the holographic plate. Additionally, the spatial behavior of the wavefronts is identical between common path sections of the MZI and HI. The uniqueness of HI lies in the function performed by the object and reference beams, and the operation of the interferometer. The sole purpose of the reference beam is to recreate the image recorded by the holographic plate.

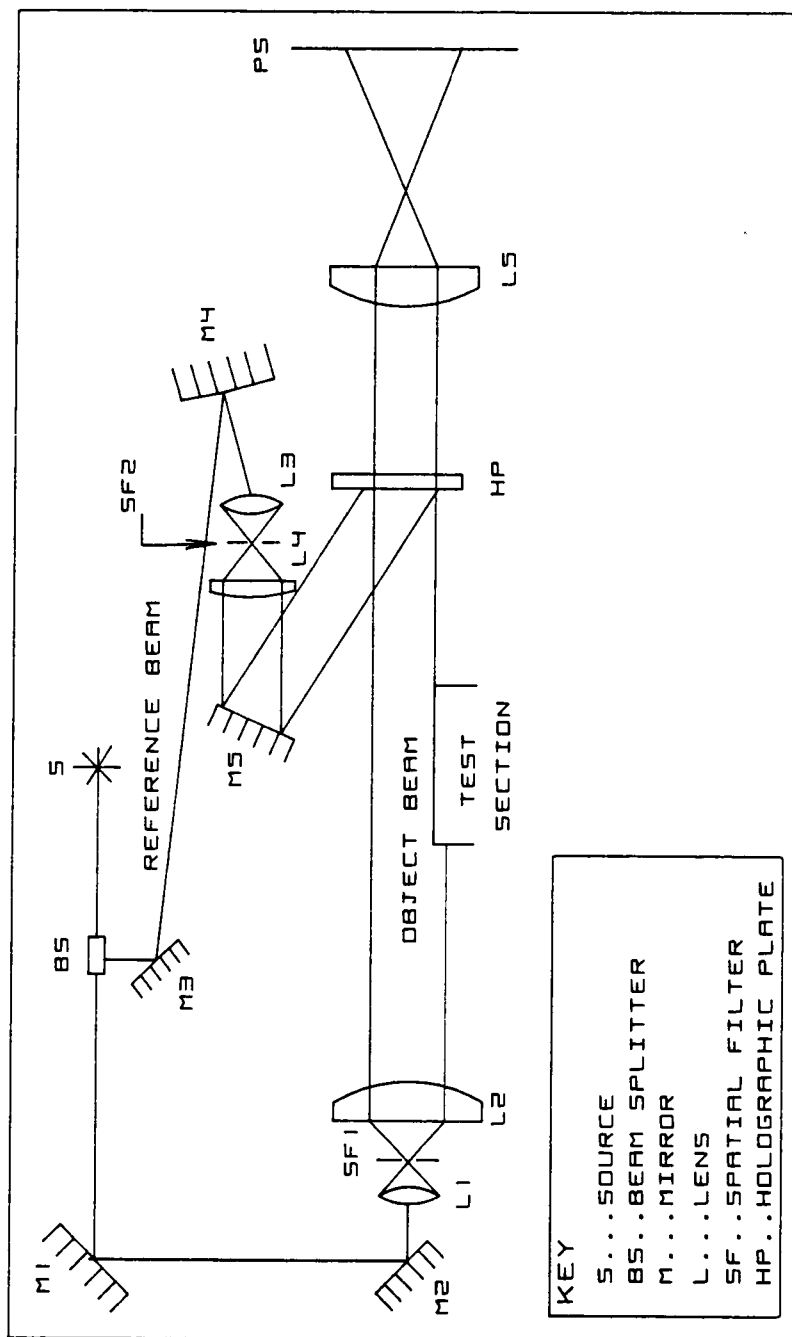


Fig. 1.2 Holographic Interferometer layout.

That is, the reference wave in no way participates directly in the interference process. The interferogram is produced by the interference of two temporally separated object waves. As will become more salient, through the following discussion of the HI process, this remarkable technique of interferometry is made possible via the use of the hologram. The process of HI can best be illustrated via a discussion of Fig. 1.3 which is adapted from a paper presented by Mayinger and Panknin (1974). As depicted in Fig. 1.3a, at time t_1 , prior to the onset of the process to be investigated, the hologram is exposed to both the plane waves of the object and reference beams. Note that the waves of the object beam remain plane during their traverse of the test section at time t_1 , as a refractive index gradient has not been introduced to distort the wavefront. Following development of the hologram, it's subsequent illumination by the reference beam produces a refraction downstream of the hologram along the optical axis of the object beam which is now blocked by a light trap (Fig. 1.3b). Note that this refracted beam is similar to the original object beam in it's direction of propagation, which is referred to as zeroth order refraction. Moreover, the wavefront of the refracted beam is a replica of that of the object beam with which the hologram was exposed (Leith and Upatnieks

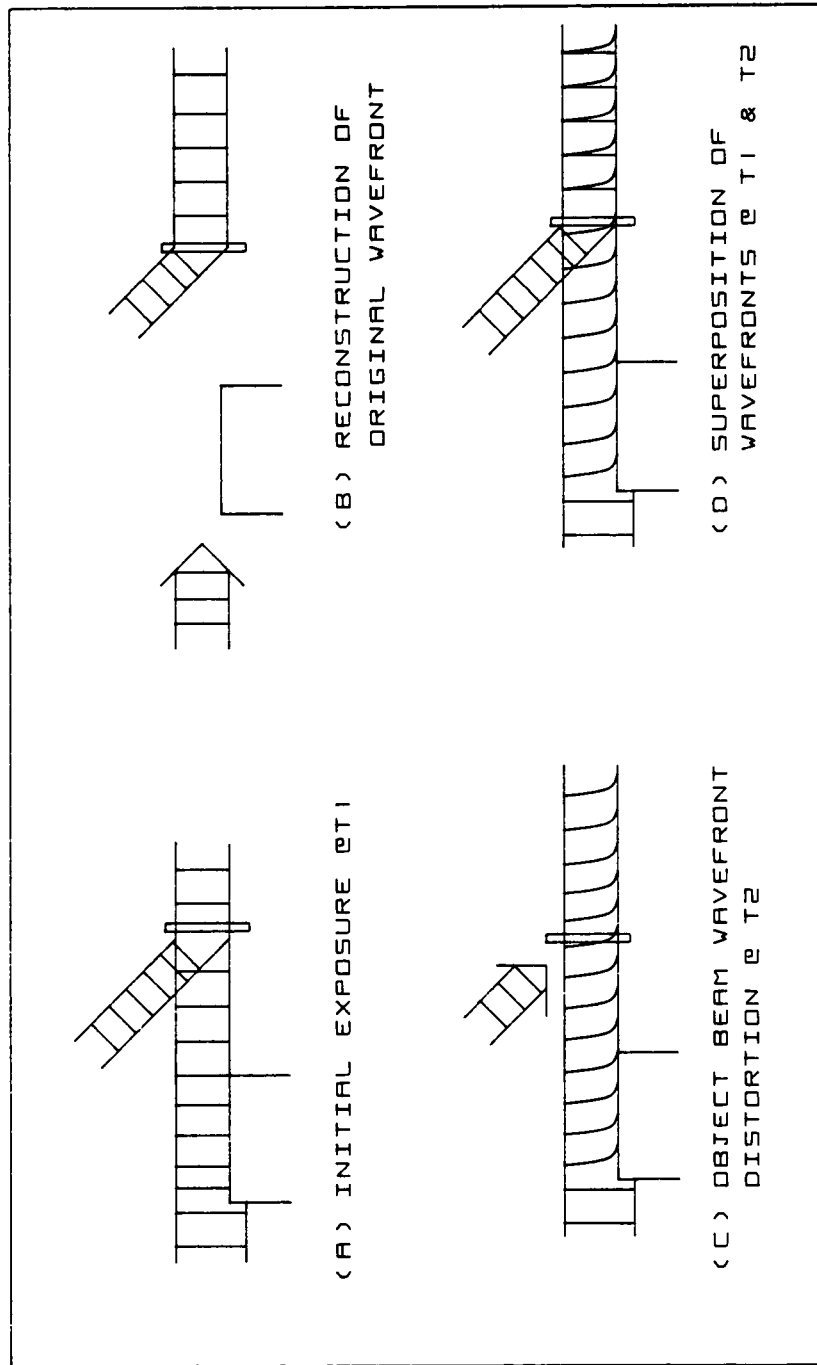


Fig. 1.3 Holographic Interferometric process.

1961). Thus, the hologram provides a means of recreating a facsimile of the wavefront of the object beam, which existed at time t_1 , at any subsequent time. The process to be investigated is started at some time t_2 ($t_2 > t_1$) resulting in distortions of the measuring beam wavefronts as depicted in Fig. 1.3c. The distorted wavefronts produced at t_2 pass through the hologram with some attenuation, but along the optical axis of the zeroth order refracted beam. If the object and the reference beams are allowed to impinge upon the hologram at t_2 , the wavefronts of the object beam at both t_1 and t_2 will simultaneously exist downstream of the hologram. The resulting superposition of these wavefronts produces an interference pattern.

1.4 Comparison of MZI and HI:

1.4.1 Similarities:

The theoretical foundations of fringe formation and data reduction are equivalent in both MZI and HI. In fact, a large portion of the information presented here, was gathered from the literature addressing MZI. This similarity has allowed researchers working with HI to draw on the coffer of knowledge developed earlier for MZI, and to focus on refining the applications and use of HI.

1.4.2 Differences:

The basic difference between MZI and HI lies in the variations in the temporal and spatial descriptions of the two waves that produce the interference pattern. The interferogram of a MZI is produced by two waves which simultaneously travel different paths. In HI, these two waves travel the same path at two distinct times. As will be demonstrated, it is this basic difference in the two interferometers which is responsible for the superiority of HI.

1.4.3 Benefits of Holographic Interferometry:

An important feature of HI is that the object beam, which is the sole contributor to the interferogram, follows the same path in consecutive exposures. Application of the MZI requires equalization of the object and reference beam paths to within a wavelength of the light source used. This is a very tedious and time consuming process. In contrast, as long as the object and reference beams of a HI are equalized to within the coherence length of the source (approximately 20cm for laser), an interferogram may be produced. Thus, the constraints on the optical set-up of a HI are relaxed in comparison to the MZI. An additional benefit of HI arises in considering the imperfections of the optical components

such as digs, impurities, or dust particles on their surface. Such random imperfections produce diffraction of the light waves as they pass through the optical system, thus distorting their wavefronts. In the MZI, this diffraction would contribute to the interferogram, and in some cases, to such an extent which deems the interferogram useless. Thus the MZI requires high quality optics. In HI, since the object beam traverses the same path containing temporally constant imperfections, this secondary diffraction produces no temporal relative differences in the wavefronts producing the interferogram. Accordingly, fringe formation in HI is due exclusively to wavefront distortion in the test section. Moreover, the above discussion applies to cuvettes containing non-ambient fluids and their windows, which must be compensated for in the reference beam of the MZI. Thus, HI further relaxes the requirements on optical quality.

The use of the hologram in HI facilitates the analyses of three-dimensional phenomena. Although this type of application will not be discussed here, Vest (1979) addresses this issue and provides several references.

1.5 Objective Of The Present Study:

The purpose of this thesis is to develop and document a working knowledge of the Mechanical and Aerospace

Engineering's Holographic Interferometric Laboratory. The major topics which will be addressed are: the methodology and techniques for configuration and operation of the HI, development of the interferogram evaluation techniques as applied to heat transfer experimentation, and general considerations pertinent to design of test sections. The above objective will be met via presentation of the experimental procedure, data reduction techniques, and the results of investigation of natural convection heat transfer along an isothermal vertical plate in an infinite medium. The accuracy of the experimental heat transfer coefficients will be discussed through a comparison with the exact solution.

2. Ideal Interferometry

2.1 Wavefront Distortion by a Refractive Index Gradient:

In discussing the theoretical foundation of interferometry, it is convenient to consider a beam of light as an assembly of rays. A ray describes the path of the light as it propagates through a medium, and is defined as a curve which at any time is perpendicular to the wavefront (Vest 1979). With this perspective, the behavior of the individual rays may be determined, and the wavefront of the light beam may then be deduced from the global ray behavior. It may be noted that the distortion of the object beam wavefront ultimately produces the interference pattern.

As a light ray propagates through a medium, it's speed and wavelength are dictated by a property of the medium known as refractive index n

$$n = \frac{c_o}{c} = \frac{\lambda_o}{\lambda} \quad (2.1)$$

where c and λ are the speed and wavelength of the light, respectively. The subscript o denotes values in a vacuum. At this point, let's define a homogeneous and a nonhomogeneous medium as mediums in which the refractive index is constant and variable, respectively.

Equation (2.1) suggests that in an optically nonhomogeneous medium, the speed of the ray is greatest in regions where the refractive index is the lowest. Alternately, the wavelength of a ray increases as the refractive index decreases. Thus, as the refractive index of a medium decreases, the distance between points of equal phase along the ray increases. The result is that an optical nonhomogeneity produces a distortion of the wavefront. Consequently, there will exist a phase shift between the wavefronts of two light beams which are subjected to different nonhomogeneities. Note that this does not necessarily imply that the two light beams propagate through different mediums, since the refractive index distribution of a given medium may change with time.

Two general categories of nonhomogeneities exist, which are referred to as surface and volume nonhomogeneities. A surface nonhomogeneity occurs at the interface of two homogeneous mediums with different refractive indices (e.g. the situation at the surface of a lens). In passing through a surface nonhomogeneity, a ray's direction of propagation is altered according to Snell's Law

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2). \quad (2.2)$$

In Snell's Law, n and θ refer to the refractive index and the angle of incidence relative to the surface normal at the point of incidence in the two mediums bounding the

surface nonhomogeneity, respectively. The general consequence of Snell's Law is that rays are refracted towards and away from the surface normal in mediums with relatively higher and lower refractive indices, respectively. A volume nonhomogeneity refers to a medium in which the refractive index varies continuously in one or more directions within the medium. Rays subjected to such a variation of refractive index follow a continuous curved path which is described by the ray equation

$$\frac{d}{ds} \left[n \frac{d\vec{r}}{ds} \right] = \nabla n \quad (2.3)$$

where $\vec{r}$ is the position vector of points along the ray, s is length of the ray relative to any point along the ray, and ∇ is the del operator. This type of nonhomogeneity is characteristic of the refractive index field adjacent to a heated plate. If the refractive index field varies only in the direction perpendicular to the heated surface, then the ray equation reduces to

$$\frac{d}{ds} \left[n(x) \frac{dx}{ds} \right] = \frac{dn}{dx} \quad (2.4)$$

where x is the direction normal to the heated surface. Consider that the ray adjacent to the heated surface enters the test section parallel to that surface. It can be shown (see Appendix A for details) that for such a ray

exposed to a linearly varying refractive index field in the x-direction, the ray equation further reduces to

$$x - x_0 = \frac{1}{2} \frac{n'}{n_0} z^2 , \quad (2.5)$$

where z is the direction of ray propagation prior to entering the test section, and the prime denotes differentiation with respect to x . Thus, it is apparent that the deflection of the ray adjacent to the heated surface in such a refractive index field is approximately parabolic.

Thus far, it has been established that the variation of refractive index along the ray path produces the relative phase shift between wavefronts. At this point, it is appropriate to introduce the concept of optical pathlength Φ ,

$$\Phi = \int n \, ds. \quad (2.6)$$

The above equation states that the optical pathlength is the path integral of refractive index along the ray. This is also known as Fermat's principle.

2.2 Light Interference:

The following discussion of light interference is based on similar presentations by Born and Wolf (1964), Vest

(1979), and Hecht (1987). Hauf and Grigull (1970) present a very interesting alternate approach that will not be discussed here. It is important to note that when two waves intersect, neither is altered in any way by the other. That is, there is no interference between the two waves in the true sense of the word. Rather, upon intersection, the two waves obey the principle of superposition and their amplitudes add algebraically. Hecht (1987, p.242) provides a very good graphical representation of the superposition principle. Despite the misnomer, the term interference will be used throughout this paper to adhere to the traditional interferometric literature. Consider two linearly polarized light waves given by,

$$E_1 = A_1 \cos(\omega t - k_1 r) \quad (2.7)$$

$$E_2 = A_2 \cos(\omega t - k_2 r + \phi) \quad (2.8)$$

In the above equations, E is the component of the electric intensity vector parallel to the direction of polarization (the other two components are zero), A is its amplitude, ω is its angular frequency, k is its propagation number, and ϕ is its phase. Specific discussion of the terms operated on by the trigonometric function is contained in the

literature. The oscillations of these waves are on the order of 10^{15} Hz. Such variations are too high to be detected by eye, film, or other practical detectors. What these detectors do respond to is the time averaged energy flux, or irradiance. Irradiance is more traditionally referred to as the magnitude of the Poynting vector (Borne and Wolf 1964). Irradiance is denoted by I , and is represented as

$$I = ev\langle E^2 \rangle, \quad (2.9)$$

where e is the electrical permittivity of the medium through which the wave is propagating, v is the speed of propagation, and $\langle E^2 \rangle$ is the time averaged value of the square of the magnitude of the electric intensity vector. In the present discussion, the relative irradiance resulting from the superposition of two waves traveling through the same medium is the quantity of interest. Therefore, the constant prefix, ev , in Eq. (2.9) may be dropped in the following discussion. Adhering to the principle of superposition, the waves of Eq. (2.7) and (2.8) combine as

$$E = E_1 + E_2. \quad (2.10)$$

The resulting irradiance is

$$I = \langle E^2 \rangle = \langle E_1^2 \rangle + \langle E_2^2 \rangle + 2\langle E_1 \cdot E_2 \rangle. \quad (2.11)$$

Carrying out the algebra and noting that $\langle \cos(wt) \rangle = \langle \sin(wt) \rangle = 1/2$, and $\langle \cos(wt) \cdot \sin(wt) \rangle = 0$, one obtains

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(d), \quad (2.12)$$

assuming that $I_1 = \langle E_1^2 \rangle$, and $I_2 = \langle E_2^2 \rangle$ (Hecht 1987). In the above equation, d is the relative phase difference between the two waves and is of the form

$$d = k_2 r - k_1 r - \phi. \quad (2.13)$$

Thus, the total irradiance of the combined beams varies sinusoidally from a maximum when $\cos(d) = +1$, to a minimum value when $\cos(d) = -1$. Maximum irradiance is referred to as constructive interference, and is perceived in the interferogram as a bright fringe. Minimum irradiance is referred to as destructive interference, and corresponds to a dark fringes on the interferogram.

2.3 Relation of Fringes to Fringe Shift:

Recalling the above discussion of optical pathlength, d may alternately be represented as

$$d = \frac{2\pi}{\lambda} [\Phi_1(x,y) - \Phi_2(x,y)] \quad (2.14)$$

or

$$d = \frac{2\pi}{\lambda} \Delta\Phi(x,y) , \quad (2.15)$$

where $\Delta\Phi(x,y)$ is the optical pathlength difference, OPD, between the two beams. The OPD, such that the constraints on d are preserved, is given by

$$\Delta\Phi(x,y) = \int [n_1(x,y,z) - n_2(x,y,z)] dz = S \lambda, \quad (2.16)$$

where S is a scalar variable termed the fringe shift. The fringe shift assumes integer values (ie. 0,1,2,3,4,...) along bright fringes, and integer plus one half values (ie. 0.5,1.5,2.5,3.5,...) along dark fringes. This behavior of the fringe shifts follows directly from Eq.s (2.15) and (2.12).

It should be noted that all points in space with equivalent values of fringe shift will appear equally bright or dark on the interferogram. Moreover, if the refractive index field is continuous, these points of equal fringe shift will form continuous fringes. Thus, each fringe of the interferogram represents a curve of constant fringe shift. With the fringe shifts assigned

via the interferogram analysis, then Eq. (2.16) can be inverted to produce the refractive index difference. The present study is restricted to two-dimensional phase objects. Vest presents analysis of more complex radially symmetric and asymmetric phase objects (1979). Normally, the refractive index field experienced by one of the wavefronts is constant and known (one of the beams used to produce the interferogram propagates through an optically homogeneous medium). For two-dimensional phase objects, the test section and alignment of the interferometer may be configured such that the refractive index is not a function of the propagation direction. Hence, Eq. (2.16) may be integrated over the region of the path where in optical nonhomogeneities are experienced. This region is the length of the test section, and thus, the limits of integration are from 0 to L, where L is the length of the test section. The resulting algebraic equation is

$$n_0 - n(x,y) = S(x,y) \lambda / L \quad (2.17)$$

This equation is known as the equation of ideal interferometry, and provides the basis of interferogram analysis. Equation (2.17) shows that the region of the interferogram where n is preserved corresponds to a fringe shift of zero. Consecutive dark and bright fringes

are assigned fringe shifts of $S = 0.5, 1.0, \dots$ etc. going from the homogeneous field into the nonhomogeneity. The application of interferometry to extract information other than refractive index field is performed by obtaining the appropriate constitutive equation relating refractive index and the variable of interest, and substituting into Eq. (2.17). Such a transformation of Eq. (2.17) to accommodate temperature field evaluation is presented in the following section.

The interference of two light beams is graphically depicted in Fig. 2.1, where the geometrical scales are exaggerated for clarity. Two sets of wavefronts represented by the solid and dashed lines appear in this figure. These solid and dashed lines are surfaces of maximum and minimum values of the electric intensity vector, respectively. This is evident from the relationship between the wavefronts and the sinusoidal characteristic of the electric intensity vector shown in the top of the figure. The set of planar wavefronts have been exposed to a homogeneous field and are therefore undistorted. The constant refractive index, n_0 , of this homogeneous field which is assumed to be known in Eq. (2.17). The beam exposed to the homogeneous field is referred to as the comparison beam. The set of non-planar wavefronts have been exposed to a nonhomogeneous field.

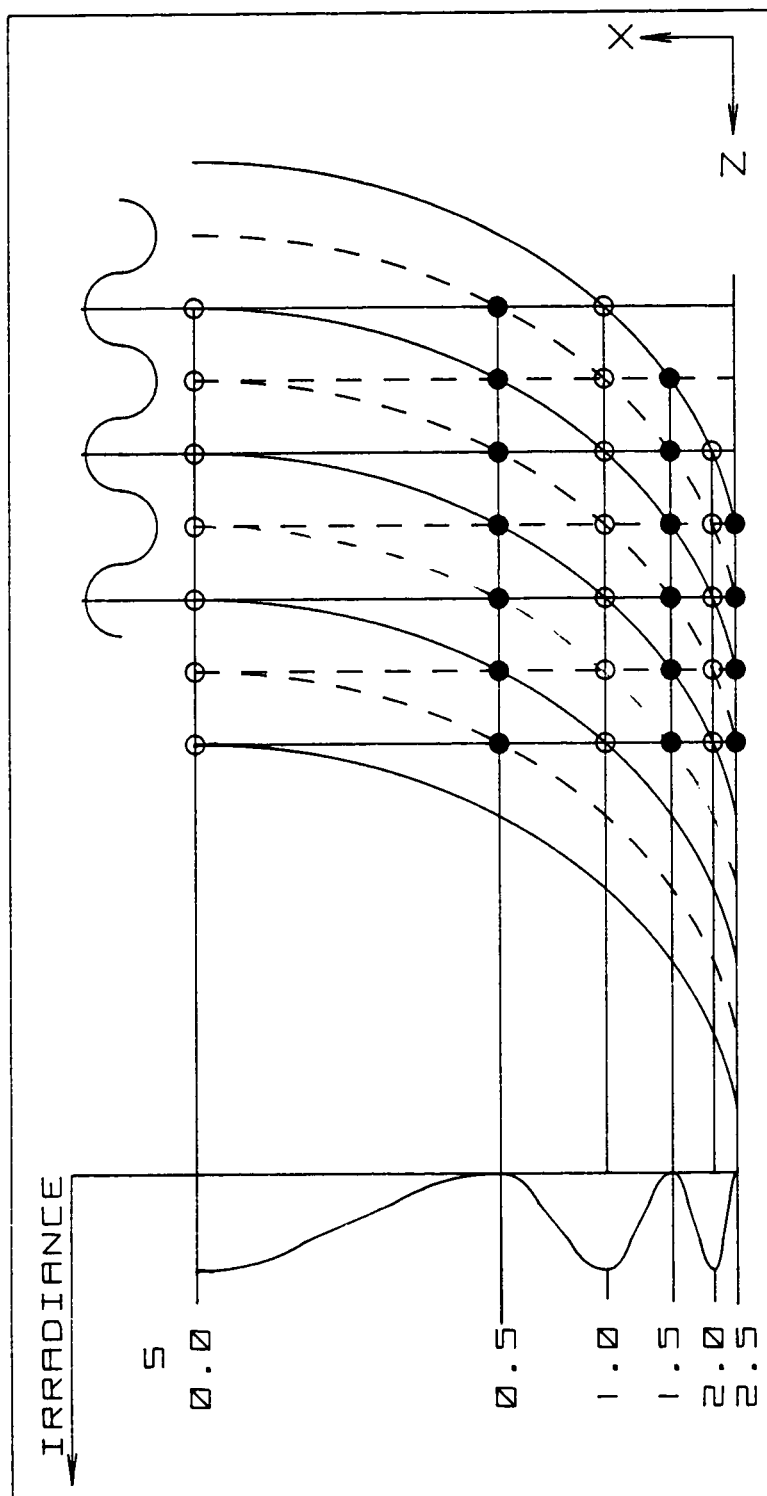


Fig. 2.1 Wavefront interference of two light beams.

The nonhomogeneous field produces a variation in the wavelengths of the individual rays across the beam according to Eq. (2.1), thus distorting the wavefronts. The beam exposed to the nonhomogeneous field is referred to as the measuring beam. The distortion causes a change in the OPD between the comparison and measuring beams whose wavefronts are shown. It should be evident that if this change in OPD is any integer plus one half multiple of the homogeneous fields wavelength, destructive interference occurs. In the figure, this situation corresponds to the intersection of solid and dashed wavefronts, and is marked as solid circles. Conversely, OPD of integer multiples of the homogeneous fields wavelength produce constructive interference. Constructive interference is indicated in the figure as the intersection of wavefronts equal values of the electric intensity vector, and is marked as open circles. Thus, constructive interference occurs at the intersection of two solid or two dashed wavefronts in Fig. 2.1. Notice that a region of preservation of the refractive index of the homogeneous field exists in the top of Fig. 2.1, as required for interferometric analysis. Projecting this interference field, along lines connecting points of equivalent phase shift, onto a projection screen placed in the path of the beams yields an interferogram. The

interferogram is observed as sinusoidal variations of irradiance indicative of the variations of the OPD across the superimposed beams. Thus, as previously discussed, bright and dark fringes are formed corresponding to areas on the interferogram of constructive and destructive interference, respectively. The method of assigning fringe shift values to the individual fringes is indicated in the left hand side of Fig. 2.1. Notice that neither the fringe width or spacing is constant across the interferogram. This in general is indicative of most interferograms, and should be expected. Additionally, it is apparent that the slope of the distorted wavefronts provides an indication of the local refractive index gradient. More specifically, the slope (i.e., dx/dz) decreases as the refractive index gradient increases. From this information and the irradiance distribution depicted in Fig. 2.1, one may conclude that the fringe density observed in the interferogram is greatest in regions of maximum refractive index gradient.

2.4 Ideal Interferometry Equation for Temperature Measurement:

The first transformation of the ideal interferometry equation involves the conversion of refractive index to density. This form may be directly applied to determine

concentration fields. The general relationship between the refractive index n and density ρ is given by the Lorentz-Lorenz equation:

$$\frac{n^2 - 1}{\rho(n^2 + 2)} = r(\lambda) \quad (2.18)$$

where $r(\lambda)$ is called the specific refractivity, which is a function of the medium under analysis and the wavelength of the source. Because gases have refractive indices close to unity, it is sufficient to use a first order approximation to Eq. (2.18) known as the Gladstone-Dale equation:

$$n - 1 = K\rho \quad (2.19)$$

where K is the Gladstone-Dale constant. This constant is a function of the gas, and has a weak dependence upon the wavelength of the source, however, it is practically independent of temperature and pressure (Vest 1979). The Gladstone-Dale equation is often expressed in terms of specific refractivity via the relationship $K=2r/3$. Vest (1979) provides tabulated values of K and r , and presents a method for determining their value in nonstandard gaseous mixtures.

Since natural convection in air is the phenomenon of interest in this thesis, it is appropriate to apply the

Gladstone-Dale equation to Eq. (2.17). Implementing this substitution to solve for the left hand side of Eq.

(2.17), one obtains

$$\begin{aligned} n_0 - n(x,y) &= [K\rho_0+1] - [K\rho(x,y)+1] \\ &= K[\rho_0 - \rho(x,y)], \end{aligned} \quad (2.20)$$

The final transformed form of Eq. (2.17) is then given by,

$$\rho_0 - \rho(x,y) = S\lambda/KL. \quad (2.21)$$

Thus, by assigning fringe shift values to the interferogram, and knowing the remaining quantities on the right hand side of Eq. (2.21), the density field may be determined.

The ideal gas equation may be invoked to obtain either temperature or pressure information.

$$\rho = P/RT \quad (2.22)$$

Note that when using this equation of state, either the temperature or pressure field must be known. The normal experimental technique is to maintain either an isothermal or isobaric test section when investigating pressure or temperature fields, respectively. Note that in the case

of natural convection, the pressure variation compared to the mean value is negligible, and the field is thus approximately isobaric. Accepting that the pressure is constant, Eq. (2.22) may be substituted into (2.21) and reduced to obtain

$$T = \frac{T_{\infty}}{1 - aS} \quad (2.23)$$

where

$$a = \frac{\lambda RT_{\infty}}{KLP_{\infty}} \quad (2.24)$$

The above equation is the equation of ideal interferometry in terms of temperature. Notice that for a given experiment, the term 'a' is a constant, and thus the temperature at any point is only a function of the fringe shift. Recall that fringes, either bright or dark, represent curves of constant fringe shift. Thus, the fringes of interferograms to be analyzed by Eq. (2.23) and (2.24) represent isotherms.

Notice that in applying Eq. (2.23), as evident from its form, the temperature increases in the direction of increasing fringe shift. If the experiment is such that the opposite is true, fringe shifts take negative values.

3. Real Interferometry

3.1 Corrections:

The equation of ideal interferometry provides a means of determining a temperature field from interferometric data in the form of fringe shifts. The development of this equation ignored several non-idealistic phenomena which in actual experimentation are inescapable. Such phenomena either degrade the quality of the interferogram, or pollute the experimental results with error. This chapter explains these non-idealized phenomena, and provides means of correcting for their effects. The basic method of implementing these corrections is to modify the ideal fringe shift recorded on the interferogram. By investigating the effect of the error mechanism on the fringe shift, a change in fringe shift solely due to the error may be derived. These values are then appropriately added or subtracted from the fringe shift to obtain the ideal fringe shift. This ideal fringe shift may then be introduced into Eq. (2.23) to determine the actual temperature field.

3.1.1 Errors Produced by Refraction:

Recall that the equation of ideal interferometry assumed that the rays of the object beam remain parallel

while traversing the test section. In reality, the rays are refracted (i.e., deflected) in the presence of a refractive index gradient. This refraction produces two types of error known as displacement error, and refraction error.

3.1.1.1 Displacement Error:

Displacement error is observed as a displacement of points in the interferogram from their proper position. The specific points that are of interest are those which combine to form a fringe, test section heated surface, and scale probe locations. A variable relative displacement of these points in the interferogram results in the distortion of the image. It should be noted that it is impossible to eliminate displacement error throughout the entire field of the interferogram. Therefore, it is appropriate to choose to eliminate this type of error in the regions of most importance. When the surface temperature and local heat transfer coefficient on a heated plate are of interest, then it is desirable to eliminate displacement error in a region just adjacent to the heated surface.

In order to investigate the method of correcting for displacement error in the immediate vicinity of the heated surface, the behavior of the light ray closest to that

surface should be discussed. Notice that this ray is an indicator of the heated surface position. The heated surface and the ray closest to it are shown in Fig. 3.1. The ray refraction depicted is based on the assumption that the refractive index gradient introduced in the test section behaves as a step function at the upstream and downstream edges of the test section. The light ray is assumed to enter the test section parallel to the heated surface. This is achieved by proper alignment of the interferometer. The path of the ray through a homogeneous test section is depicted by the dashed line. As mentioned earlier, the path of the ray propagating through a homogeneous medium does not deviate from its initial straight line path. Conversely, Snell's Law states that a ray propagating through a nonhomogeneous medium is deflected towards regions of higher refractive index. It is clear that the required conduction/convection balance at the surface dictates that the refractive index gradient adjacent to the heated surface is should be linear.

Recall, from the previous chapter, that a ray propagating through a linear refractive index gradient follows a parabolic path to a first approximation. Thus, the ray propagating through a nonhomogeneous test section follows a parabolic path depicted by the solid line in Fig. 3.1. Downstream of the test section, this ray

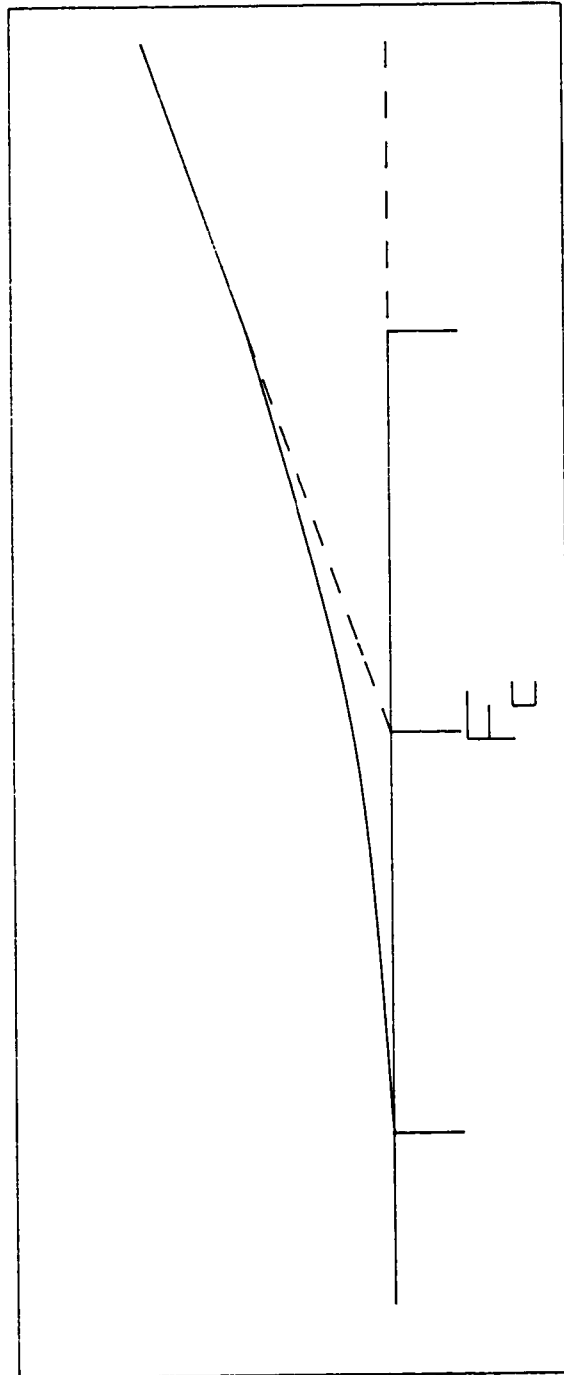


Fig. 3.1 Refraction of a light ray in a linearly varying refractive index field.

experiences the ambient homogeneous fluid and thus follows a straight path. Note that since the ray path is continuous, its path downstream of the test section is tangent to that at the exit of the test section. Extrapolating this linear path back into the test section, a common virtual origin of the two ray paths may be found. Downstream of the test section, both ray paths appear to originate from this virtual origin. If the test section is aligned parallel to the unrefracted ray path, it is evident from Fig. 3.1 that the virtual origin must exist at some location within the test section. Since the two ray paths must intersect in order to form an image (i.e., interference pattern), an imaging lens must be introduced to refract the diverging rays. Figure 3.2 is a reproduction of Fig. 3.1 with the introduction of an imaging lens of focal length f . In Fig. 3.2, F_c designates the location of the focal plane within the test section containing the virtual origin. Introduction of the imaging lens forces all rays emanating from the virtual origin converge to a single point, thus forming a real image of the virtual origin in the image plane I_c .

Inherent with its definition, the formation of a real image is subject to the constraint that the imaging plane be positioned at a distance downstream of the imaging lens greater than the focal length of that lens. Varying

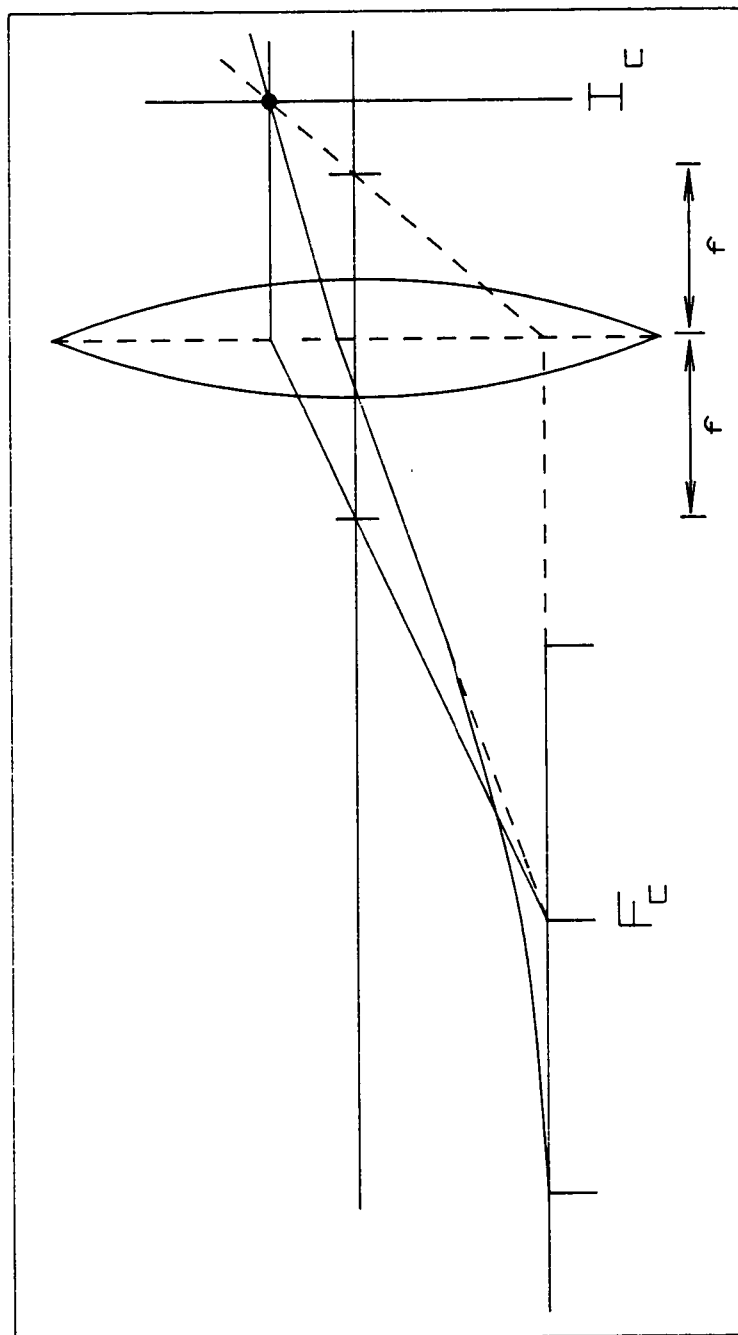


Fig. 3.2 Locating an image plane, I_c , corresponding through an imaging lens to a given focal plane, F_c .

degrees of refractive index gradient within the test section produce proportional degrees of refraction of the rays. However, provided that the refractive index gradient is linear, the resulting ray path is parabolic. Consequently, the virtual origin of the ray remains in the focus plane F_c for any degree of refractive index gradient. Therefore, regardless of the degree of refraction, the refracted ray converges to the real image of the virtual origin in image plane I_c . The position of the real image is found by standard ray tracing procedures. This procedure is implemented by the dashed line and the solid line emanating from the virtual origin and intersecting the optical axis at the focal length of the lens. Thus, the image of the wall and its adjacent fringe is temporally stable, and as the test section experiences the transient process of a developing boundary layer is not displaced from its original position in I_c . This statement implies that the image is stable between any two steady state conditions of the test section. Therefore, the location of the wall and the fringe closest to the wall is imaged correctly in the interferogram provided the projection screen of the interferometer is positioned in the plane I_c . Thus, assuming that the ray adjacent to the heated surface follows a parabolic path through the test section, the displacement error at the

heated surface is eliminated by positioning the projection screen in the image plane I_c .

Notice that rays outside the boundary layer propagate through a homogeneous medium, and thus follow identical linear paths at all points in time. Consequently, the rays outside the boundary layer are imaged correctly in any image plane. The shape of the path followed by the boundary layer rays varies from parabolic at the heated surface to linear at the boundary layer edge.

Accordingly, the focus plane containing the virtual origin of each ray will be unique to that ray. The specific location of focus plane for each ray varies from F_c to infinity in a direction upstream of F_c as rays are considered from the heated surface to those outside the boundary layer, respectively. This fact implies that if I_c is chosen as the image plane, then all rays, excluding the one at the heated surface and those outside the boundary layer contain some amount of displacement error. Therefore, choosing I_c as the image plane imposes null boundary conditions on the displacement error of the boundary layer. This error assumes a maximum value at some point within the boundary layer, and decreases to zero as one moves towards the heated surface or the boundary layer edge. It should be clear from the above discussion that there is no single image plane in which

displacement error is globally eliminated. It is thus appropriate to choose the image plane in which displacement error is minimized in the region of maximum interest.

The effect of image plane location on displacement error is illustrated in Fig. 3.3. In this figure, three focus planes F_u , F_c , and F_d are shown along with their corresponding image planes I_u , I_c , and I_d , respectively. The focus planes F_u and F_d correspond to the upstream and downstream edges of the test section, respectively. Focus plane F_c contains the virtual origin of the ray next to the heated surface which correctly locates the image of the heated surface and its adjacent ray. Consequently, the correct location of the heated surface is indicated by the image point C in the image plane I_c . The image planes are determined by ray tracing as indicated in the figure. Recognizing that the image is inverted by the imaging lens, the result of choosing focus planes other than F_c may be investigated.

Consider the selection of focus plane F_d . In this situation, the virtual origin is located at focus point D in focal plane F_d . As discussed earlier, the interference pattern is the result of the superposition of the two temporally separated rays emanating from the virtual origin. However, in this case these two rays are

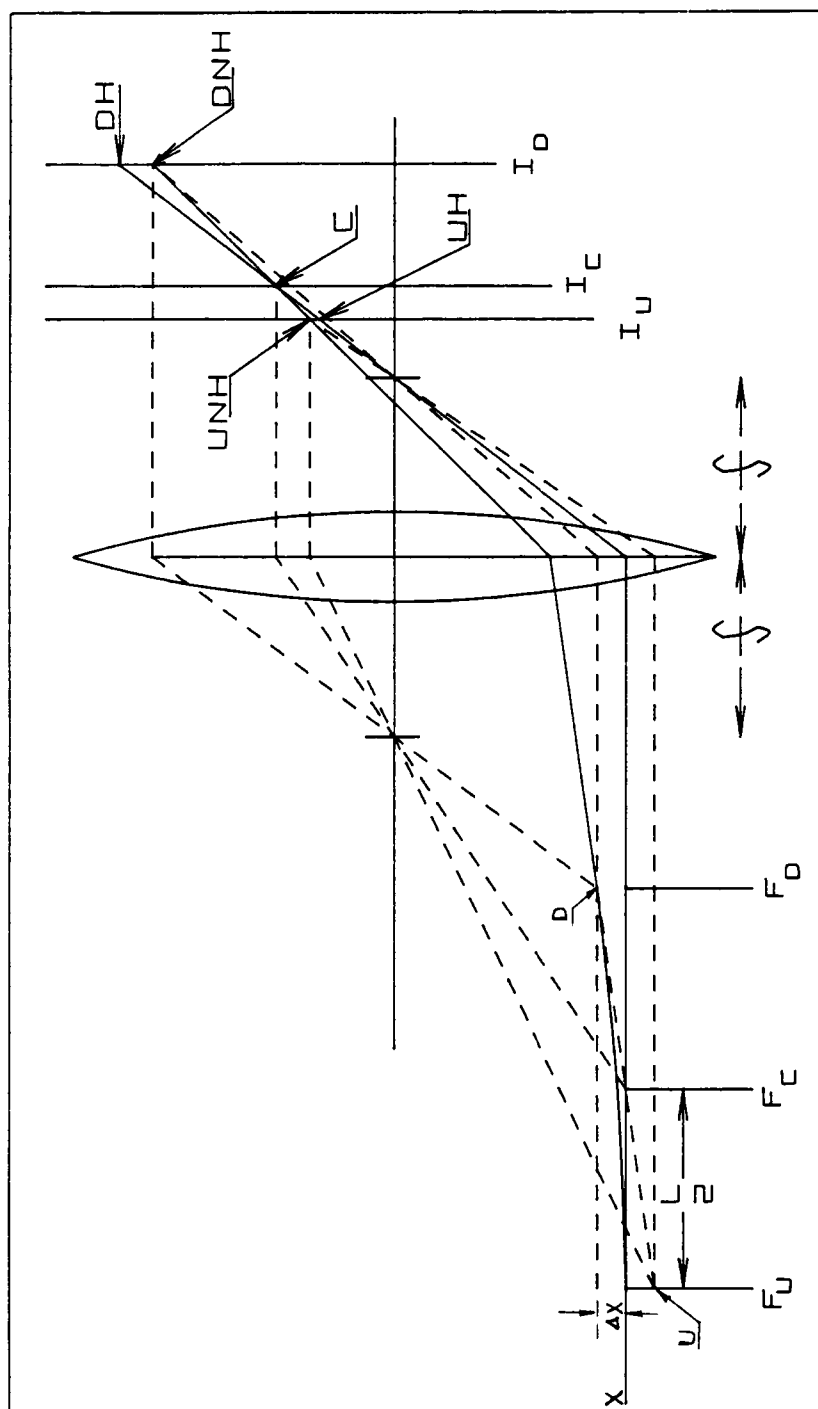


Fig. 3.3 Displacement error associated with a given focal plane. Choosing focal planes F_u and F_d result in displacement errors of UH-UNH and DNH-DH, respectively. Only the focal plane at the test section mid-length, F_c results in zero displacement error at C.

distinctly different, and thus the interference is between the refracted ray initially at a position X_0 , and the ray at $X_0 + \Delta X$. If the test section is truly homogeneous in the initial exposure, the initial wavefront will be planar, and thus the optical pathlength of the ray at X_0 and ΔX will be identical. Therefore, it is of no consequence that the refracted ray originating from X_0 interferes with the ray at $X_0 + \Delta X$ rather than its unrefracted twin. If the test section is nonhomogeneous in the initial exposure, the refractive index along the ray path at $X_0 + \Delta X$ contributes to the image of the ray at X_0 . In that case, the resulting interferogram contains erroneous data. A similar argument may be made for positioning the focal plane at F_0 . However, the heated assembly will block the ray at $X_0 - \Delta X$, and thus no interference occurs in the image plane I_0 .

The image point C in image plane I_c is said to be devoid of displacement error because of its temporal stability of location. Notice that this stability is unique to imaging in the plane I_c . Consider imaging in the plane I_0 , which is the conjugate plane of the focal plane F_0 . When the test section is homogeneous, the ray at X_0 is imaged in the plane I_0 at the image point DH. However, when nonhomogeneities are introduced in the test section, the image of this ray is displaced to the image

point DNH. This temporal displacement of the image in plane I_0 causes there is no direct geometrical relationship between the interferogram and the refractive index field in plane F_0 . This displacement error is observed in the interferogram as a distortion of the heated surface into the field of the test section. This is evident from the direction of the displacement error in the image plane I_0 . Similarly, imaging in the plane I_u , the conjugate to focal plane F_u , introduces displacement error. In this configuration, the image of the ray at X_0 is displaced from UH to UNH when the test section contains a homogeneous and nonhomogeneous refractive index field, respectively. This refraction error is observed in interferograms of image plane I_u as an erosion of the heated surface. It should be clear that displacement error occurs for imaging in any plane other than I_c . The focal planes F_u and F_0 were chosen for their convenience and the fact that they bound the correct focal plane F_c . In general, the behavior of displacement error, due to improper imaging for similar deviation from the ideal image plane I_c , is analogous to that discussed earlier.

The location of the correct focal plane F_c within the test section may be found by assuming that the ray nearest to the wall follows a parabolic path through the test

section. As presented in the previous chapter, the equation describing this parabolic path is

$$x = x_o + \frac{1}{2} \frac{n^*}{n_o} z^2 . \quad (3.1)$$

Therefore, the cumulative refraction at the end of a test section of length L is

$$REF = x(L) - x(0) = \frac{1}{2} \frac{n^*}{n_o} L^2 . \quad (3.2)$$

The slope of the refracted ray downstream of the test section is

$$SLOPE = x'(L) = \frac{n^*}{n_o} L , \quad (3.4)$$

where the prime denotes the derivative with respect to z . Accordingly, the position of the correct focal plane POS relative to the downstream edge of the test section is

$$POS = \frac{REF}{SLOPE} = \frac{L}{2} . \quad (3.4)$$

Thus, a correctly imaged interferogram is produced in I_c by choosing the focal plane F_c to coincide with the center of the test section.

The information presented above was derived from Hauf and Grigull (1970) and Vest (1979). It is appropriate to mention that alternate positions of the correct focal plane have been suggested by Vest (1979). Vest (1979)

plane have been suggested by Vest (1979). Vest (1979) suggests the selection of a focal plane $L/3$ from the downstream edge of the test section. However, this value is derived to eliminate refraction error in the vicinity of the wall. If such a focal plane is chosen, one may expect to observe displacement error in the vicinity of the heated surface. It will be established in the following section that the refraction error may be accounted for mathematically in the data reduction process. Therefore, it is logical to select the focal plane such that the displacement error is eliminated, and correct for refraction error mathematically.

3.1.1.2 Refraction Error:

Refraction error is introduced in the interferogram due to the deviation of the light rays, propagating through a nonhomogeneous medium, from a linear path as assumed in ideal interferometry. The result of this non-ideal ray behavior is a decreasing of the optical pathlength. Recall that in the application of interferometry to phase objects, the phase shift is assumed to be the direct result of the temporal variation in the refractive index field within the test section. This is in contrast to a phase shift due to a geometrical lengthening of the optical pathlength, which is the case in reflective mode

interferometry. By quantifying the relative amount that the optical pathlength is decreased, due to the geometrical distortion, the equation of ideal interferometry may be modified to produce accurate interferometric results.

The varying paths that are taken by the ray closest to the heated surface are shown in Fig. 3.4. The path of this ray through the homogeneous test section is represented by the dashed line ABCD. The actual path of the ray through the nonhomogeneous test section is refracted to the path AED. The virtual path of the refracted ray through the nonhomogeneous test section is ABED. Assuming ideal properties for the imaging lens, which are assumed, the unrefracted and virtual refracted ray paths, ABCD and ABED, are equal. If the refracted ray was to actually follow its virtual path, the optical pathlength would be solely due to the refractive index within the test section. However, this is not the case, and as may be seen, the actual path of the refracted ray AED is shorter than its virtual path ABED. Thus, interference between the temporally separated rays is the net effect of the refractive index field and the geometrical distortion of the ray.

The phase shift due to the geometrical distortion of the ray may be represented in terms of a fringe shift

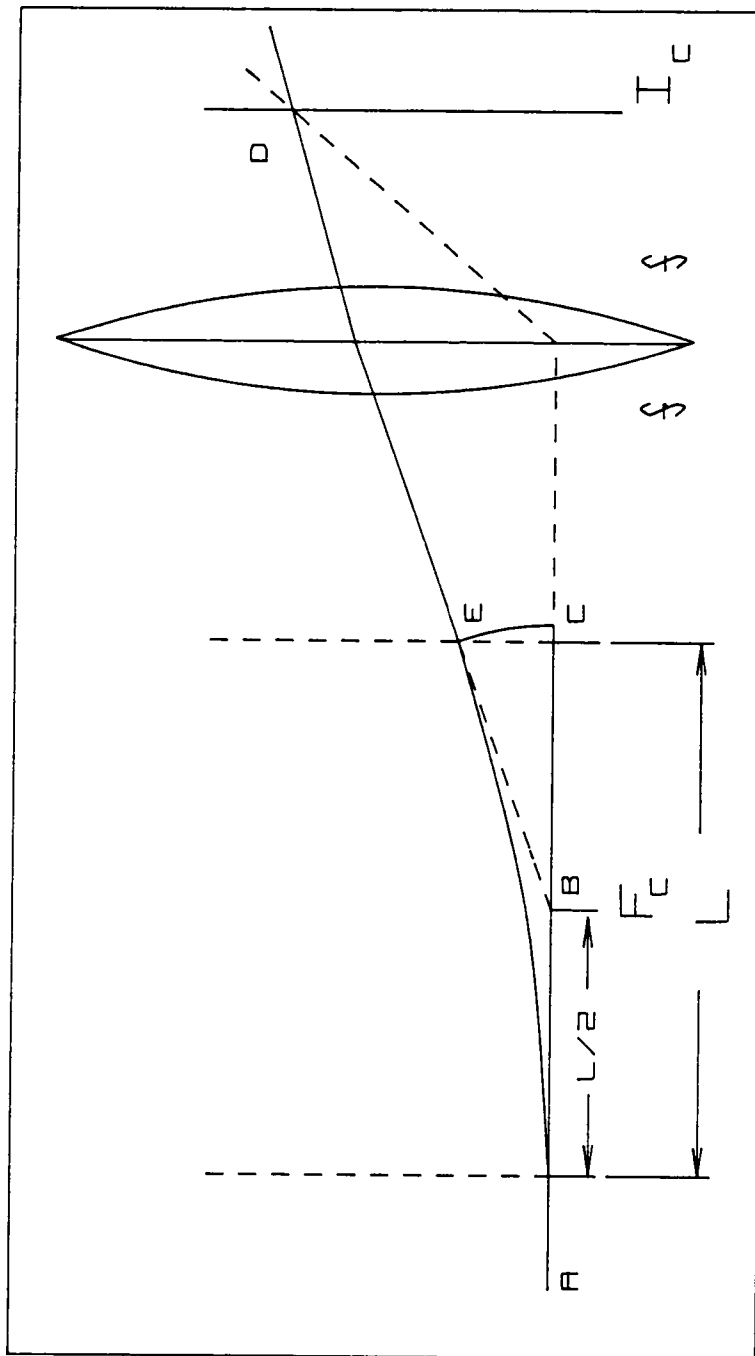


Fig. 3.4 Pathlength of the ray closest to the heater surface in a homogeneous and nonhomogeneous medium.

correction. Hauf and Grigull (1970) and Vest (1979) provide a very thorough presentation of this correction and show it to be

$$\Delta S1 = \frac{n_o \lambda L}{12b^2} , \quad (3.5)$$

where n_o is the refractive index of the homogeneous medium, λ is the wavelength of light source, L is the length of the test section, and b is the fringe spacing. This correction term is added to the fringe shift as indicated by the interferogram to produce an ideal fringe shift.

$$SIDEAL = S + \Delta S1 \quad (3.6)$$

In the above equation, S is the fringe shift depicted on the interferogram. The corrected fringe shift $SIDEAL$ may be introduced into the equation of ideal interferometry (2.23) to produce accurate interferometric results corrected for refraction error.

3.1.2 End Effect Errors:

Up to this point, the nonhomogeneous field within the test section has been assumed to be strictly confined to the test section. That is, that the refractive index field has been assumed to exhibit the behavior of a step

function at the upstream and downstream edges of the test section. Figure 3.5 shows the boundary layer due to natural convection along an isothermal finite-length plate. The lines represent locus of constant refractive index or temperature. The term δ denotes the thickness of the boundary layer. It is clear from Fig. 3.5 that the nonhomogeneities are not confined strictly to the test section. Rather, the nonhomogeneities extend beyond upstream and downstream of the test section, and gradually decay. Note that isotherms intersect adiabatic boundaries perpendicularly. Therefore, one may imagine that the isotherms, extending upstream and downstream of the heated surface, intersect an imaginary boundary extension, of the heated surface along the z-axis, perpendicularly. This behavior has been verified experimentally by holographic interferometry. Thus, the general shape of the end effects is that of a quarter cylinder. A nonhomogeneous field of such shape is termed a radially symmetric phase object.

The end effects impose an additional phase shift upon the rays propagating through the nonhomogeneous test section. Thus, the fringe shift indicated in the interferogram is the net result of that due to activity within the test section, less refraction error, plus end effect error. Therefore, in applying the equation of

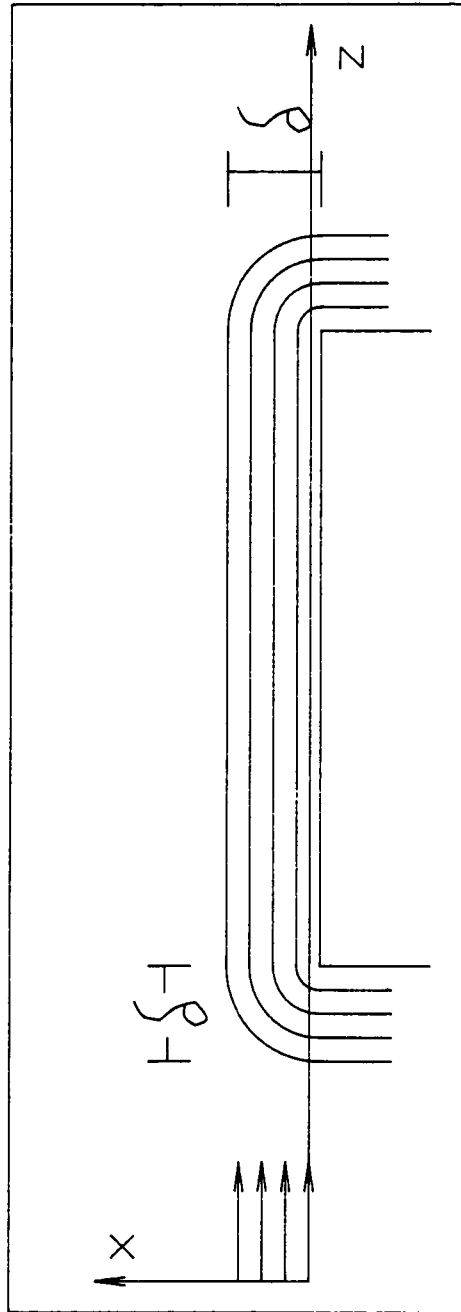


Fig. 3.5 End effect due to boundary layer termination at heater surface ends.

ideal interferometry, if the fringe shift is not corrected to account for the additional fringe shift induced by the end effects, the temperatures indicated by interferometry will be greater than the actual temperature. The standard method of accounting for the end effects is to quantify their affect on the phase shift in terms of an equivalent fringe shift. This term is then subtracted from the fringe shift as indicated by the interferogram. The form of the ideal fringe shift accounting for refraction and end effect errors is then given by

$$SIDEAL = S + \Delta S1 - \Delta S2, \quad (3.7)$$

where $\Delta S2$ is the correction for the end effects. In this form, SIDEAL may be applied to the equation of ideal interferometry (2.23) to obtain interferometric results corrected for refraction and end effect errors.

Methods of determining the fringe shift produced by a radially symmetric phase object have been presented by Vest (1979) and Hauf and Grigull (1970). In the stringent method of analysis described by these authors, the fringe shift uncorrected for end effects is used to determine an initial approximation of temperature distribution within the test section. These values are then used to define

the temperature distribution along the length of each ray through the end effect. Recall that in the development of the equation of ideal interferometry in the previous chapter, the refractive index field was assumed to be two dimensional in a plane perpendicular to the direction of beam propagation. Thus, the refractive index distribution of the nonhomogeneous medium was treated as a constant in the z direction. In the present situation, the refractive index field is a function of the direction of propagation, and thus must remain inside the integral. However, substitution of the Gladstone-Dale and Ideal Gas equations into the unintegrated form of the equation of ideal interferometry remain valid. With the approximate temperature distribution established, such a form of the equation of ideal interferometry may be used to determine the effective fringe shift due to the end effects. This term may then be used to correct the fringe shift and upon application of the original equation of ideal interferometry, obtain an accurate temperature distribution within the test section.

A much more simple method, which is used in the present study, involves assuming a temperature distribution along the end effect length of each ray. Thus, the step of data reduction to determine the initial temperature distribution, as required in the previously presented

method, is eliminated. Such a method of determining the end effect correction has been broadly applied to interferometric analysis to obtain highly accurate results. The specific developments presented here in, were derived from work by Jessee (1983). The assumed temperature distribution is applied bounded by T_i and T_o , where T_i is the temperature of the i 'th ray and T_o is the temperature outside the boundary layer (ie. the temperature of the homogeneous field). Possible temperature distributions between T_i and T_o could be the mean, linear, or parabolic. The development of a parabolic temperature distribution, its introduction into the equation of ideal interferometry, and the subsequent integration to obtain the end effect correction term are given in Appendix B. Jessee (1983) gives the results for mean and linear temperature distributions. The end effect correction, assuming a parabolic temperature distribution is found to be

$$\Delta S2 = A \left[\frac{1}{T_o} - \frac{1}{\sqrt{\Delta T T_o}} \tan^{-1} \frac{\Delta T}{\sqrt{\Delta T T_o}} \right] \quad (3.8)$$

where

$$A = \frac{KP_o}{R} \sqrt{\delta^2 - x^2} \quad (3.9)$$

and

$$\Delta T = T_i(x) - T_{\infty} \quad (3.10)$$

Actual application of the above equation to correct interferometric data will be demonstrated in the results section of this thesis.

4. Setup Parameter Options

4.1 Fringe Field:

Up to this point, all reference to interferograms has assumed that the interferometer is adjusted to what is termed the infinite-fringe mode. In general, there are two forms of interferograms which produce different fringe fields under the influence of the same nonhomogeneity. These two types of interferograms are referred to as the infinite-fringe and finite-fringe interferograms.

4.1.1 Infinite-Fringe Interferogram:

If the interferometer is not altered between exposures, an infinite fringe interferogram is obtained. Assume that the comparison and measuring beams are exposed to identical homogeneous fields within the test section. The resulting interferogram is a field of uniform irradiance, or more formally, a fringe field of infinite fringe spacing. Thus the term infinite-fringe interferogram. If the measuring beam traverses a nonhomogeneity, the resulting fringes are lines of constant phase shift. Thus, the fringes of an infinite-fringe interferogram represent lines of constant density, or may be made to represent isotherms or isobars. The method for assigning

fringe shift values to the fringes of an infinite-fringe interferogram was outlined in section 2.

4.1.2 Finite-Fringe Interferogram:

A finite-fringe interferogram is produced by introducing reference fringes into the interferogram. Reference fringes are introduced by altering the interferometer between the comparison and measuring beam exposures. If the comparison and measuring beams are subjected to identical homogeneous fields, the reference fringes are observed as linear, parallel, equally spaced lines. Additionally, the reference fringes should be aligned parallel to the anticipated direction of heat flux in the nonhomogeneous field. When nonhomogeneities are introduced into the test section and subsequently distort the wavefront of the measuring beam, the reference fringes are distorted from their original linear shape. This distortion is proportional to the degree of phase shift introduced by the nonhomogeneity, and thus may be expected to be a maximum in the vicinity of the nonhomogeneity source (e.g., the test section heated surface). It will become evident that the shape of the reference fringes is indicative of the shape of the wavefronts of the measuring beam.

Vest (1979) presents several methods of introducing reference fringes. Basically, reference fringes are introduced by altering the optical set-up such that the comparison and measuring object beams slightly diverge. One method of creating this divergence is to rotate M5 (see Fig. 1.2) after exposing the hologram to the comparison beam. This causes the replica of the comparison beam, which is produced via the hologram, to diverge from the zeroth-order refraction direction. Despite the alteration of the optical set-up, the measuring beam will propagate along the zeroth-order refraction direction. Thus, the desired divergence is produced. Figure 4.1 depicts such a situation where the comparison and measuring beams are subjected to identical homogeneous fields, and thus their wavefronts remain planar. In this figure, the wavefronts of maximum values of the electric intensity vector for both the comparison and measuring beams are indicated by the solid lines. Note that the spacing of these wavefronts parallel to their direction of propagation is equal to the wavelength of the light source utilized. Lines connecting the points of intersection of these wavefronts of equal values of electric intensity correspond to points of maximum irradiance, or a bright fringe, on a projection screen placed in the path of the beams. As previously discussed,

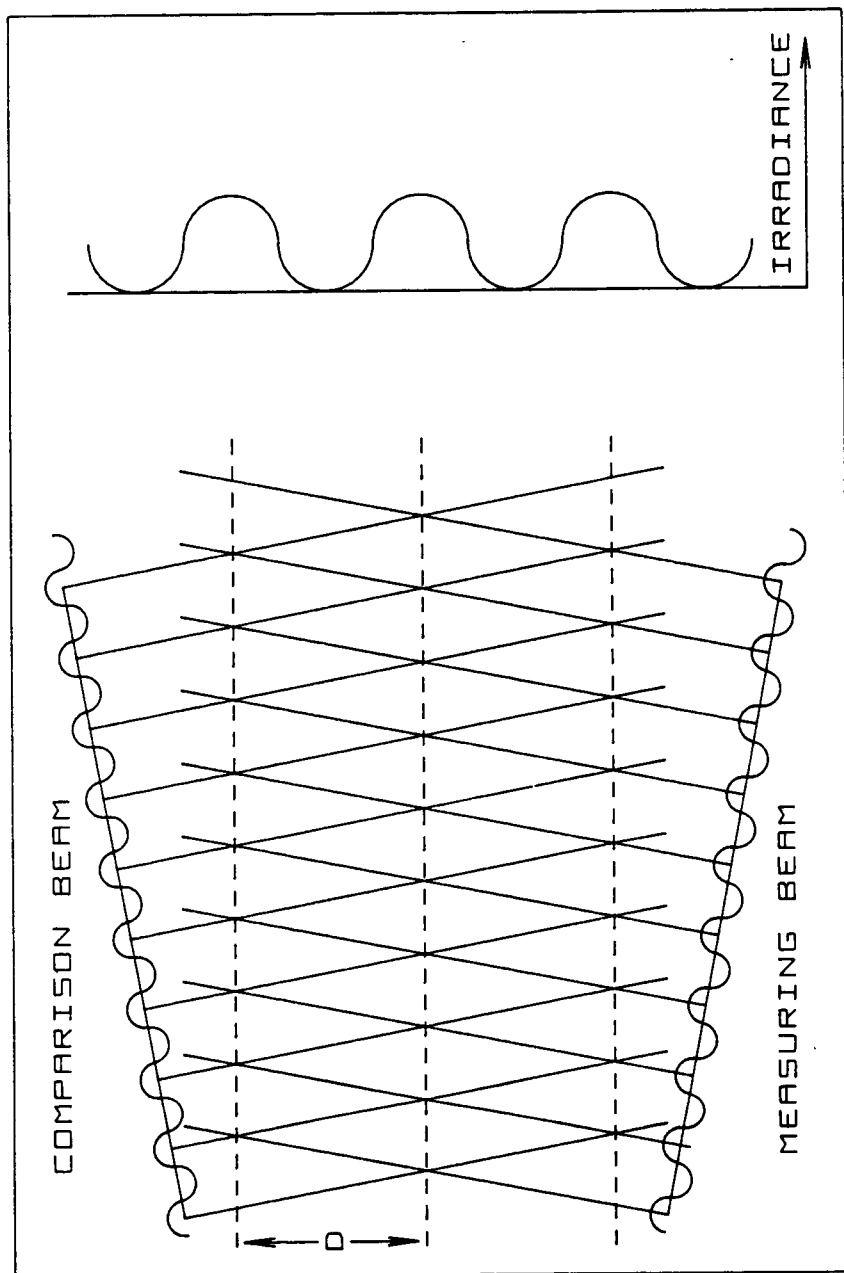


Fig. 4.1 Irradiance distribution resulting from the interference of two slightly diverging light beams.

the irradiance varies sinusoidally between maximum and minimum values, and is indicated as such in Fig. 4.1. The resulting interferogram is a field of equally spaced linear fringes. Thus the spacing of the observed similar fringes, be it bright or dark, represent a unit wavelength phase shift between the comparison and measuring beams. Note that this interferogram is indicative of a linear variation of OPD across the field of view (Eckert and Goldstein 1976). From Fig. 4.1, it can be observed that only along the central bright fringe is the optical pathlength of comparison and measuring beams identical. Below the centerline, the optical pathlength of the comparison beam is greater than that of the measuring beam. Above the centerline, the opposite is true. It should be noted that the OPD, as defined by Eq. (2.16), increases in Fig. 4.1 in a direction from top to bottom.

Visualize the effect of introducing a nonhomogeneity into the path of the measuring object beam, in the form of a thermal boundary layer off of a vertical plate. In such a situation, the reference fringes would initially be aligned horizontally. Additionally, the wavefronts of the comparison beam are as shown in Fig. 4.1. Outside the boundary layer, the original homogeneous field is preserved, and the wavefronts of the measuring beam remain undistorted. Thus, the image in Fig. 4.1 is indicative of

a slice, parallel to the heater surface, of the superimposed comparison and measuring beams in the far field of the interferogram outside the boundary layer. Viewing similar slices of the superimposed beams inside the boundary layer, one would observe the spacing of the wavefronts of the measuring beam to increase as the slices approach the heated wall. At the same time, the spacing of the wavefronts of the comparison beam remains constant in all such slices. In summary, the fringes shown in Fig. 4.1 tend to maintain their constant spacing, but shift downward as the slices approach the wall. Assembling all such slices and viewing them along the axis of the zeroth order refraction, one would see the finite-fringe interferogram shown in Fig. 4.2. The dashed lines in the figure represent the original position of the reference fringes, and the solid lines represent their displaced position under the influence of the nonhomogeneity within the test section. Notice that Fig. 4.2 shows the global behavior of the reference fringes. Meaning that the original reference fringes are preserved outside the boundary layer, and the degree of their shift becomes more severe as the heated wall is approached.

The method of assigning fringe shift values to the fringes of a finite-fringe interferogram is much different from that described for an infinite-fringe interferogram.

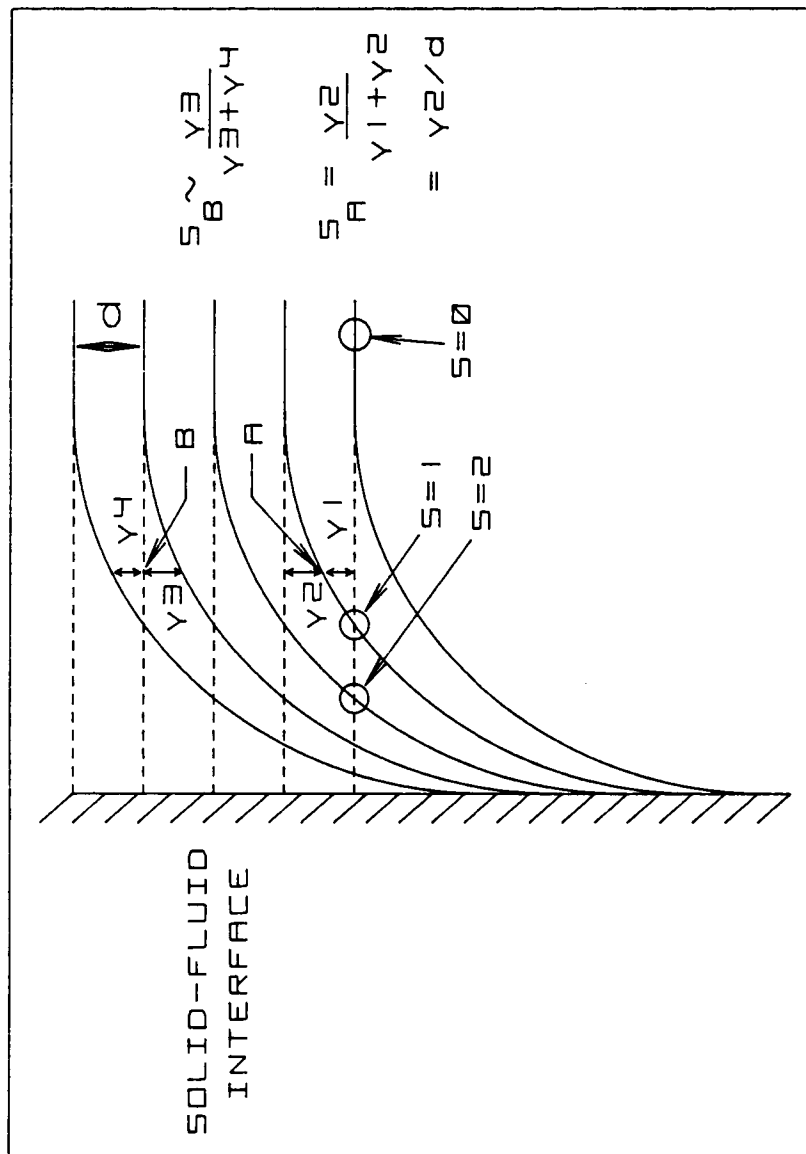


Fig. 4.2 Deflection of and method of assigning fringe shift values to reference fringes.

The only similarity is in assigning the zeroth-order fringe shift. Recall that the zeroth order fringe shift is reserved for the section of the interferogram in which the original homogeneous field is preserved. Thus, in Fig. 4.2, the straight fringes outside the boundary layer are assigned a fringe shift of zero. This may appear contradictory, since as mentioned earlier, the OPD varies across the span of reference fringes. However, interferogram analysis is a stepping process in a direction parallel to the undistorted reference fringes. Thus, as will become apparent, only the relative OPD along such a path from a zeroth order fringe shift is of importance. Recalling the direction of increasing OPD as inferred from Fig. 4.1, and noting the deflection of the reference fringes in Fig. 4.2, it is observed that as the heated surface is approached, the reference fringes shift towards higher fringe shift values. Note that the reference fringes may shift in an opposite direction under other circumstances as will be discussed later. Furthermore, recall the conclusion that the spacing of like fringes represent unit fringe shift values of OPD. This provides a method of assigning unit fringe shift values. Deflection of a given reference fringe to such a degree that it intersects the original position, as indicated by the dashed lines of Fig. 4.2, of its adjacent

fringe corresponds to a unit fringe shift of specified sign. The same applies to further deflection to the next adjacent fringe and so on. Recall that in taking slices of the superimposed beams, the spacing of the reference fringes remained constant in all slices. Thus, adjacent fringes are anticipated to take incremental fringe shifts approximately simultaneously. Therefore, following the path of the undistorted reference fringe, the point of intersection with the deflected adjacent fringe is indicative of the point at which that fringe obtains a unit increment in fringe shift. Assuming that the reference fringes deflect towards higher fringe shifts, as for the case of a thermal boundary layer, the unit fringe shifts are assigned as indicated in Fig. 4.2. The method of assigning fractional fringe shifts is also demonstrated in Fig. 4.2. Normally, the approximate method of determining the additive fractional fringe shift is used in analyzing finite-fringe interferograms. This is due to the fact that the measurements are made from an imaginary line, shown by a dashed line in Fig. 4.2, to existing lines. Thus only the position of the imaginary line being used for the analysis must be approximated. This is in contrast to the exact solution in which the position of two imaginary lines must be approximated, which increases the actual error incorporated in the analysis. A thorough

presentation of the finite-fringe interferometry analysis technique is given by Jessee (1983).

As stated earlier, the reference fringes of a finite-fringe interferogram are introduced via a rotation of M5 in Fig. 1.2, and their virtual image is located in the plane of M5. The highest contrast in the reference fringes are obtained when the imaging screen is positioned in an image plane conjugate to a focal plane in the plane of M5. However, recall that the imaging screen is focused on the midplane of the test section, and therefore its position is fixed. Therefore in order to simultaneously meet the above two requirements the distance from M5 to the hologram must be approximately equal to that of the midplane of the test section to the hologram. This should be anticipated and incorporated into the initial configuration of the optics of the interferometer if finite-fringe interferograms are to be produced.

4.1.3 Comparison of Infinite and Finite-Fringe:

The fringes of an infinite-fringe interferogram represent lines of constant density, and qualitative information is directly apparent from this type of interferogram. With such a representation of the field of interest, isotherms of a heat transfer process, or isobars of a flow field may be directly observed. Although this

information is indirectly inferred by a finite-fringe interferogram, its extraction requires much more imagination and a broader understanding of the interferometric process. It is possible to convert a finite-fringe interferogram to an infinite-fringe interferogram. This is accomplished by subtracting the image of the original undeflected reference fringe field from the interferogram of the deflected reference fringe field. Although this procedure produces an infinite-fringe interferogram, the resulting fringes are not smooth, and take on a moire appearance. Thus, interferometric data intended for purely qualitative analysis is most effectively presented in the infinite-fringe form.

Hauf and Grigull (1970) point out that since the fringe shift values of an infinite-fringe interferogram are restricted to half unit steps, quantitative results obtained from such an interferogram are only valid for fringe fields containing approximately five or more fringes. This places a lower limit on the temperature difference of fields to be analyzed by infinite-fringe interferometry. Note that a similar restriction on the values of the fringe shift of a finite-fringe interferogram does not exist. As indicated in the previous section, any number of fringe shift values may be

interpolated between integer values of fringe shift in a finite-fringe interferogram. However, in fields of small overall temperature gradient, which limit the application of infinite-fringe interferometry, the deflection of reference fringes is slight. If the order of magnitude of this deflection is much less than that of the reference fringe spacing, interpolation error may be significant. Recall, though, that this reference fringe spacing is dictated by the degree of rotation of M5 (Fig. 1.2), and is thus controllable. Therefore, in analyzing the field at hand, this spacing may be adjusted appropriately. It should be noted that, increasing the reference fringe spacing correspondingly magnifies the deflection. The result is increased sensitivity in the finite-fringe interferogram, and thus reduction of the interpolation error.

A serious limitation of infinite-fringe interferometry is its inherent sign ambiguity. As noted earlier, nonhomogeneities that differ only in sign (e.g., heat fluxes of equal magnitude but opposite direction) produce identical infinite-fringe interferograms. Thus, such interferograms do not indicate whether positive or negative fringe shift values should be assigned. This limitation may be overcome by knowledge of the process under investigation. For example, if a wall within the

test section was known to be of lower temperature in the measuring beam relative to the comparison beam, negative fringe shifts would be assigned to the fringes of the infinite-fringe interferogram. This is in contrast to the situation of a thermal boundary layer previously discussed. However, such knowledge may not always be available. The aforementioned limitation does not apply to finite-fringe interferometry. Recall that the direction of increasing OPD may be deduced from the initial reference fringes. Actually, this is dictated by the direction of rotation of M5 (Fig. 1.2) used to create the reference fringes. The sign of the fringe shift values is dictated by the direction that the finite-fringes are displaced relative to the sign of the OPD. Positive fringe shift values are assigned if the reference fringes shift in a direction of increasing OPD. As shown in Fig. 4.2, the fringes are shown shifting downward as the heated plate is approached. This behavior could have been predicted by the conclusions drawn from Fig. 4.1, specifically, the direction of increasing optical pathlength difference. Fringe shifts take negative values if the reference fringes shift in a direction of decreasing OPD. Fringe shift values are assigned in the same manner regardless of their sign. Thus knowledge of

how the reference fringes are introduced eliminates sign ambiguity in the finite-fringe interferogram.

4.2 Data Presentation:

The two general methods of presenting interferometric data are referred to as double exposure and real time interferometry. The two methods utilize identical optical set ups, and their utility is independent of the type of fringe field used. The choice of presentation method is in general dictated by the type of phenomenon under observation. In the following sections, these methods are described and compared.

4.2.1 Double Exposure Interferometry:

In double exposure interferometry the holographic plate is subjected to two exposures prior to its development. These two exposures correspond to the comparison and measuring beams. Assume, for the moment, that the same reference beam is used in both exposures. In one exposure the object beam traverses a homogeneous field within the test section. In the other exposure the object beam encounters a nonhomogeneous field within the test section. The sequence of the exposures is irrelevant, and the resulting hologram will be the same in either situation. Following its development, the hologram is placed in its

original position in the interferometer. Thereafter, blockage of the object beam while the hologram is illuminated with the reference beam reconstructs the comparison and measuring beams downstream of the hologram. The resulting interferogram may be viewed on the projection screen.

Recall that in the above discussion of double exposure interferometry, it was assumed that the same reference beam was used in both exposures. Although the optical set up for this method is simple, it makes it impossible to independently reconstruct the comparison and measuring beams. Alternately, the hologram may be exposed to the comparison and measuring beams using two separate reference beams, one for each exposure. To produce multiple reference beams, a beamsplitter is inserted in the path of the reference beam. This produces two reference beams which are then expanded and directed to the holographic plate. Following exposure and development of the hologram, illumination of the hologram by one of the reference beams reconstructs the corresponding wavefront downstream of the holographic plate. Simultaneous illumination of the hologram by both reference beams reconstructs both wavefronts. Although they do not specify to what degree, Mayinger and Panknin (1974) indicate that there must be sufficient angular

separation of the two reference beams so that independent reconstruction is achieved. Recall that interference requires the superposition of at least two beams. Thus, the reconstruction of only a single wavefront does not produce an interference pattern. The utility of the multiple reference beam double exposure technique is that either an infinite or finite-fringe interferogram may be produced from a single interferogram. Leaving the optical set up unaltered, illuminating the hologram with both reference beams naturally produces an infinite-fringe interferogram. To produce a finite-fringe interferogram, the angle of incidence of the reference beam, which was used to record the comparison beam, upon the holographic plate may be altered. Thus a multiple reference beam interferometer allows the choice of type of fringe field to follow the development process. Such freedom is desirable when the particular phenomenon requires the use of reference fringes for quantitative analysis. However, the interferometer could be subsequently adjusted to display the qualitative characteristics of the phenomenon in the infinite-fringe mode.

4.2.2 Real Time Interferometry:

In real time interferometry, the holographic plate is exposed to the reference beam and either the comparison or

the measuring beam before development. This is the process which was depicted in Fig. 1.3. Regardless of whether the object beam used for this exposure experiences a homogeneous or nonhomogeneous field within the test section, it is necessary for that field to be at a steady state condition. For this reason, the comparison beam is most easily used for exposing the holographic plate. The following discussion assumes that this is the case. Following exposure and development, the hologram is replaced in the interferometer in its original position. Subsequent illumination of the hologram by the reference beam used for its exposure reconstructs the wavefront of the comparison beam. Blocking the reference beam, the measuring beam passes unaffected through the hologram. Simultaneous illumination of the hologram with both the reference and object beams results in the superposition of the comparison and measuring beams downstream of the hologram. Assuming that the hologram is replaced accurately and that the field of the test section is identical in the comparison and measuring beams, a field of uniform irradiance is observed on the projection screen. At this point, if it is desired to use the finite-fringe method, reference fringes may be introduced. Regardless of the type of fringe field used, consider the consequence of initiating the phenomenon to be observed

within the test section. In such a situation, the comparison beam is continuously reconstructed and the wavefronts of the measuring beam continuously distort according to the nonhomogeneities of the phenomenon under consideration. Thus, the temporal development of the resulting interference pattern is continuously observable.

4.2.3 Comparison of Double Exposure and Real Time:

The primary difference in double exposure and real time HI is the temporal extent of information available from each type of resulting interferogram. Double exposure HI provides information concerning the phenomenon of interest at a selected discrete time, where real time HI provides the information continuously as the process is taking place. If only steady state processes are of interest, double exposure and real time HI are equally applicable in terms of information content of their resulting interferograms. Their most prominent difference lies in their application to the study of transient phenomenon. To observe such a phenomenon through double exposure HI requires the step-wise production of individual interferograms at distinct time increments throughout the transient. Clearly, such a process is tedious, and the resulting information is discontinuous through the transient. Moreover, interpolation of information between

time steps introduces error additional to that already present in the experimental results. In contrast, continuous data collection, as required in the study of transients, is natural in the application of real time HI. Applying real time HI, the entire transient process may be video recorded.

It was assumed in the preceding paragraph that film plates can be repositioned exactly following their processing. However, this is not the case. The degree of accuracy of the repositioning required for double exposure and real time HI is a fundamental difference between the two methods. Double exposure HI requires only that the reference beam, used to reconstruct the image stored in the hologram, be of approximately the same angle of incidence as that with which the hologram was exposed. The hologram may be reconstructed by any beam at an angle of incidence within a range centered about that of the reference beam used for its exposure. As indicated earlier, if multiple images are to be stored on a single interferogram, a finite angular separation of the reference beams used to create the individual images is required. The consequence of this relaxation of the requirements for the reconstruction of the reference beam is an equivalent relaxation of the film plate repositioning requirements. Thus, there is a range of

acceptable film plate misalignment which nonetheless, results in reconstruction of the measuring and comparison beams. Confinement of the misalignment to this range is easily met by standard film plate holders. In the presence of such misalignment, the zeroth-order refraction direction deviates from the optical axis of the object beam. However, this is only slight, and as this deviation is equally present in both the measuring and comparison beams it is of no significance. Thus, double exposure HI may be implemented with relative ease.

Real time HI is rather extremely sensitive to film plate misalignments. In real time HI, plate misalignments translate into phase shifts in the comparison beam which are not present in the measuring beam. To quantify the effects of plate misalignment on the interferometric results, consider a misalignment only along the optical axis of the object beam. Under such circumstances, using a helium-neon laser, a misalignment of only 632 nanometers produces a unit fringe shift. Equally stringent criteria are required for angular alignment of the film plate. Moreover, misalignments of this order of magnitude are imposed upon the plate in the form of its deformation by standard film plate mounts. It is unlikely that this deformation will be constant to within 632 nanometers for consecutive placements of the plate in the plate mount.

Even with highly accurate repositioning tooling built into the plate holder, such accurate repositioning is practically impossible. Thus, even if the comparison and measuring beams experience identical homogeneous fields, a fringe pattern inevitably is observed on the imaging screen. Aung and O'Regan (1971) reported that such extraneous misalignment fringes may be eliminated by rotation of the hologram about two perpendicular axes in the plane of the hologram. Alternately, consider accepting such misalignment fringes, and using them as a basis for subsequent interferogram analysis. This extraneous fringe field is considered to represent a reference interferogram of zero OPD in terms of activity within the test section. Beginning the phenomenon to be observed within the test section, distorts the wavefronts of the measuring beam, and subsequently causes the base fringes to displace. By measuring the displacement of the base fringes relative to their original position, fringe shift values may be assigned. This is similar to finite-fringe HI in which reference fringes are used as a basis for subsequent interferogram analysis. Recall that in the consideration of finite-fringe HI, it was noted that an infinite-fringe interferogram may be obtained by subtracting the image of the original reference fringes from the finite-fringe interferogram. This is possible

only because the original reference fringes are static in time, as long as the measuring beam experiences a homogeneous field identical to that of the comparison beam. The extraneous fringes introduced in real time HI, due to plate misalignments, display this same static characteristic. Thus, real time interferograms may be analyzed in a manner similar to that of finite-fringe interferograms by measuring the fringe displacement relative to the original fringe field. Such an analysis method has been used by Behbahani and Stewart (1989). Alternately, a infinite-fringe interferogram may be produced, by subtraction of the original fringe field from the real time fringe field, and analyzed in the typical manner.

4.3 Holographic Recording Medium:

Once the optics of an interferometer are properly configured, it may be applied to experimental analysis indefinitely, assuming that it is not mechanically disturbed. This includes thermal expansion of components resulting in misalignment. Thus actual use of HI ultimately reduces to selection, exposing, and processing of the hologram. The choices and decisions pertinent to each of these steps is considered in the following

sections. The majority of the information presented is derived from a thorough text edited by Caulfield (1979).

4.3.1 Recording Medium Types:

The formation of a hologram coincides to the production of a diffraction grating. This diffraction grating is to such specifications as to refract a portion of the reference beam along the zeroth-order direction and reconstruct a replica of the comparison object beam with which it was formed. This process, in itself, is quite impressive. However, any material which is capable of responding or may be made to respond to the spacial variation of irradiance could possibly be implemented as a hologram. Two recording mediums which display the above response characteristics will be discussed. These are the thermoplastic and silver halide emulsion film plates.

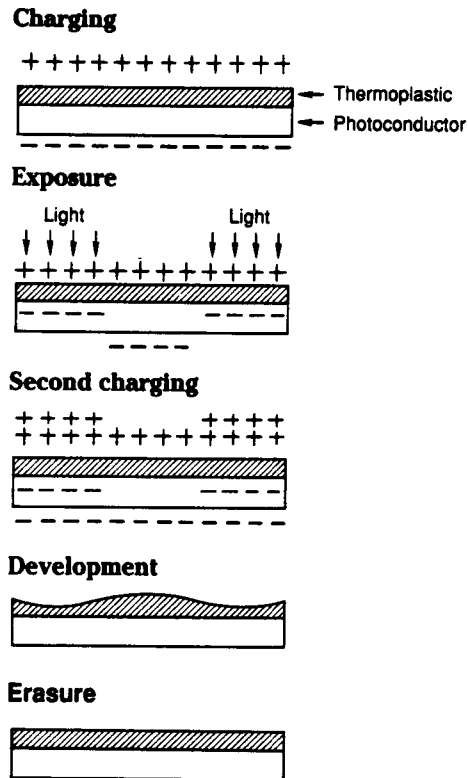
4.3.1.1 Thermoplastic Film Plate:

Thermoplastic film plates are formed in layers consisting of a quartz base, a transparent electrical conductor, and a deformable thermoplastic. It is the deformable thermoplastic which forms the diffraction grating. However, the thermoplastic does not respond directly to the variations in irradiance. The formation of a diffraction grating is a step wise process depicted

in Fig. 4.3 (derived from Newport Corp. literature). Initially, a uniform electrical charge distribution is produced on the surface of the plate by an integral plate holder/processor unit. Exposure of the charged plate causes the uniform charge to redistribute locally relative to the degree of local irradiance. A second charging of the plate intensifies the displaced charge distribution. By heating the thermoplastic via passing current through the electrically conductive layer, a diffraction grating is formed as the thermoplastic layer deforms under the influence of the nonuniform charge distribution. Stopping the current flow and cooling the thermoplastic produces a stable hologram. A most beneficial feature of thermoplastic plates is their recyclability. This is achieved by a second heating which allows the residual stresses within the thermoplastic to evenly redistribute the thermoplastic layer to its original shape.

4.3.1.2 Silver Halide Emulsion Film Plate:

The following discussion of silver halide emulsion film plates applies also to ordinary film of the same emulsion. The term plates is used to specify film in which the emulsion is deposited on a mechanically rigid base such as glass. In contrast, film refers to recording material in which the emulsion is deposited on a nonrigid base. The



Charging: A uniform charge is deposited on the thermoplastic and photoconductor.

Exposure: Exposure to laser fringe pattern redistributes the charges through the photoconductor.

Second Charging: Recharging increases the electric field across the exposed area of the thermoplastic.

Development: Heating of thermoplastic causes permanent deformation. The irregularities will diffract light to recreate the original image.

Erasure: Erasure is done by controlled heating.

Fig. 4.3 Thermoplastic plate operation. Used with permission from Newport Corp.

application of both types of films were investigated in the present study. Short of sandwiching the film between two pieces of glass, it was very difficult to maintain a constant positioning of the film during use. This is due to its nonrigid character. Displacement of the film during operation introduces an additional phase shift which is in no way constant across the field of the interferogram. This nonuniform and time varying additional phase shift deemed the interferograms obtained from film holograms virtually impossible to analyze. Using the sandwich method of mounting the film, resulted in stable interferograms, but were littered with secondary fringe patterns resulting from reflection of the laser back into the emulsion off the sandwiching surfaces of glass. Such secondary fringes are referred to as Fizeau fringes. Note that since film plates utilize a glass base, similar secondary fringe patterns exist, but not to as large an extent. The implications of such secondary fringe patterns are discussed later. In the present study, the beam size is approximately one inch. Due to the fact that the cost of small film plates is comparable to that of film, and the superiority of plates, film plates were used in the experiments. Thus the following discussion is based specifically on film plates.

In general, the emulsion of film plates is a gelatin which supports an array of silver salts. To expose a film plate, it is positioned such that the laser beams are initially incident upon the emulsion. That is, that the emulsion is on the upstream side of the plate. The emulsion side of the plate is tacky and may be detected by touching a corner of the plate with a moisten lip. Thus, the emulsion side can be identified and the film plate may be properly positioned. If the plate is exposed to light of sufficient energy, the silver salts are converted into silver atoms. It is in this manner that the emulsion responds to the spatial variation in irradiance. In processing the exposed plates, steps are taken to convert the silver atoms into metallic silver, and remove the silver salts from the areas of the plate which did not receive sufficient energy during the exposure. After the plate is processed and dried, it may be replaced in the interferometer to reconstruct the object wavefront with which it was exposed.

As discussed earlier, a secondary fringe pattern results in addition to the desired fringe pattern in the production of a film plate hologram. This is the result of reflection of the light rays, off the glass base, back into the emulsion. Areas of the beam with a sufficiently high irradiance to convert the silver salts to silver

atoms produce the desired refracting effects upon the first passage through the emulsion. However, the rays which make up these areas reflect off the glass base and pass back through the emulsion at a displaced location creating secondary sights of silver atoms. Although this secondary fringe pattern is undesirable, it does not in any way effect the primary desired fringe pattern (Jeong 1990). Vest (1979) states that multiple exposures may be stored on a single hologram by utilizing multiple reference beams. Moreover, it is stated that the effects of each of these exposures are independent. This supports Jeong's statement that the primary interference pattern is independent of the secondary fringe pattern.

Although the secondary fringes do not influence the interferometric analysis, they do degrade the overall image quality. Additionally, these fringes detract an observers focus on the fringe field of interest. It is thus desirable to devise some means of eliminating the secondary fringes. This is achieved by backing the glass base on the side opposite the emulsion with what is termed an anti-halation layer. Kodak offers most of its film plates in either the backed or unbacked form. The film plates used in the research being presented were AGFA 8E75HD, which are unbacked as are all AGFA film plates. However, techniques exist for the individual consumer to

apply an anti-halation backing to the plates prior to use (Redjikowski 1990). The method of producing such a backing involves applying a layer of Speedball black water soluble block printing ink. A liberal coating of this ink is applied with a rubber ink roller to the back side of the film plates. The applied ink is allowed to dry in open air for approximately twelve hours. The plates are then placed in the plate storage container and allowed to dry for an additional twenty four hours. Upon removal of the backed plates for use, it may be observed that the ink is still tacky. This method of film plate backing provides very favorable results. Although secondary fringes may be observed in the actual hologram, their corresponding image on the imaging screen is practically unobservable relative to that of the primary fringe pattern.

4.3.1.3 Recording Medium Selection Criteria:

A prominent difference between thermoplastic and film plate holograms is the ease with which each may be applied to interferometry. In storage, thermoplastics may be exposed to normal ambient light. This is a benefit of recording media which are not directly effected by the light intensity. In contrast, care absolutely must be

taken to keep the film plates sealed in a light tight container until a particular plate is intended for use.

Comparing the processing requirements reveals great difference between the two methods. Processing of a film plate involves several chemical development steps. This involves preliminary mixing of chemicals and the stabilization of their temperatures as a processing system. The time of processing in each step is of varying criticality, and is normally determined experimentally. Thus, several preliminary experiments are required to simply determine the correct processing times. Once these are determined, the total processing time may be up to fifteen minutes. This does not include the time required for the film plate to air dry. Accordingly, not only is the processing of film plates time consuming, but it requires a more thorough level of interaction and understanding of the development process by the experimenter. Processing of a thermoplastic plate may be completed in less than one minute, and involves pressing indicated switches on a controller unit at prompted intervals. All decision making, such as the degree of development time required, is performed by the controller and plate holder/processor unit. Hence, the use of thermoplastic recording medium reduces not only the processing time required, but additionally, the required

processing guesswork. Thus, other factors to be discussed aside, the use of thermoplastic recording medium provides the quickest and easiest experimental results. For these reasons, the use of the thermoplastic plate is most useful in initial experimentation. Such initial experimentation may be sufficient to dictate the feasibility of further investigation. If such experimentation indicates that improvements to the model or optical set up are required, time and expenses are conserved relative to that which would have been incurred in the use of film plates.

As stressed above, the use of film plates requires greater time and understanding of the development process relative to that for thermoplastic plates. However, the additional effort invested in using film plates is justified in the resulting interferograms. The fringes of film plate interferograms are crisp and of high contrast and resolution. Contrast relates the relative degree of changes in the shades of gray between adjacent bright and dark fringes. Higher contrast results in more distinct fringes. Although thermoplastic plates have definite utility, their resulting interferograms are of lower contrast. Thus the transition between adjacent fringes is very gradual. This makes it more difficult to accurately specify the location of the fringe centers, at which the fringe shift values are assigned. Thermoplastic plate

produced interferograms are further degraded by surface dust particles. When the thermoplastic plate is charged in the processing, it becomes an accumulator for oppositely charged dust particles. Subsequent cycling of the plate through heating steps further imbeds these particles into the thermoplastic. Since the thermoplastic layer is extremely delicate, it may be somewhat cleaned by a stream of low velocity filtered dry gas. Nonetheless, gradually particles become entrapped in the thermoplastic layer and litter all ensuing interferograms.

Recall the inherent difficulties of hologram realignment associated with real time HI. In the processing of thermoplastic plates, the plate remains stationary in the optical set up, and all processing is brought to the plate via its holder/processor unit. Thus thermoplastic plates are naturally compatible to the application of real time HI. A similar but more awkward approach is available in the processing of film plates. This is through the use of a film plate mount which also serves as a development tank. With such equipment, the developing chemicals are brought to the plate by being cycled individually through the tank. The chemicals enter the top of the tank, and are drained after the appropriate reaction time. It should be added that the interferometer

will not be effected by the end windows of such a development tank.

4.3.2 Holographic Plate Exposure:

Proper exposure of a holographic plate is based upon the setting of three parameters. These are the angle of incidence of the object and reference beams relative to the plate normal, the irradiance ratio of reference beam to the object beam, and the exposure time. The setting of these parameters is based on properties of the recording medium being used. Although these properties are often specified by the manufacturer, there exist much disagreement on their actual values within the experimental community. For example, for the setting of a single parameter for a given film plate, one may find varied opinions in the literature concerning the actual value of the properties upon which the setting is based. It is thus most beneficial to obtain a knowledge of the origin of these properties. These properties are usually reported in the form of characteristic curves describing the particular film to be used. A brief description of such curves and the method of extracting plate properties will be presented. With such knowledge, one may define the value of the individual properties and compare with

the published values. Caulfield (1979) presents a thorough description of these processes.

4.3.2.1 Angle of Incidence:

The angle of incidence of the object and reference beams relative to the plate normal is a function of the wavelength of the light source and the resolution of the film plate. Resolution has units of lines/mm and is an indication of the film plate's ability to resolve spatial variations in irradiance. The relevance of resolution to interferometry is that it sets an upper limit on the linear fringe density, fringes/mm, which may be resolved by a film plate. It is unlikely that this linear fringe density will be even approached in typical experiments. However, this upper value is used in determining the angle of incidence of the beams in order to create the maximum linear fringe density working envelope. The equation relating these terms is

$$\sin \theta_{\text{OBJ}} + \sin \theta_{\text{REF}} = \lambda R, \quad (4.1)$$

where θ is the angle between the object or reference beam (as specified) and the plate normal, λ is the wavelength of the light source used, and R is the resolution of the film plate used. Note that θ_{OBJ} is normally zero, and thus

the first term on the left hand side of Eq. (4.1) drops out. In such a situation, θ_{REF} may be found directly once the film plate resolution is determined.

The resolution of a particular film plate is determined by the behavior of what is called the modulation transfer function, MTF. The MTF is the characteristic curve from which the film plate property of resolution is extracted. The MTF is dependent upon the spatial variation of irradiance, I , and is mathematically represented as

$$MTF = \frac{(I_{min} - I_{max})}{(I_{min} + I_{max})} \quad (4.2)$$

In Eq. (4.2) I_{min} and I_{max} represent irradiance of dark and bright fringes, respectively. Recall that irradiance varies sinusoidally between maximum and minimum values. In a coordinate plane, it may be determined that Eq. (4.2) represents the ratio of the amplitude of the variation to the mean value of the irradiance. The values of MTF range from zero to one. The typical form of presentation of the MTF is a plot of its variation with resolution, as shown in Fig. 4.4. Another name for resolution which is often used in literature is spatial frequency. From Fig. 4.4, it is observed that the MTF decreases with increasing resolution.

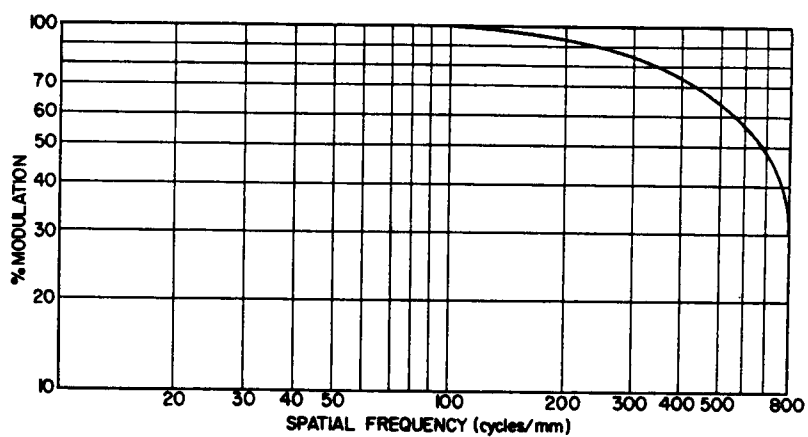


Fig. 4.4 Modulation transfer function (MTF) curve. Used with permission from Academic Press Inc.

Note that the maximum contrast interferogram results when the dark fringes have irradiance values of zero. This is clear from the definition of contrast given as $I_{\max}/I_{\min}$. Striving to achieve such an ideal interferogram provides a means of extracting the resolution of a film plate from the MTF vs. resolution curve. Thus, it is evident from Eq. (4.2) the maximum contrast results when the MTF takes a value of unity. It is now clear that maximum resolution of a given film plate is indicated by the point at which the curve of Fig. 4.4 deviates from unity. Normally, this ideal requirement is somewhat relaxed by accepting a value of MTF less than one. However, beware that manufactures often take this relaxation to extreme limits in order to glamorize their product. Therefore, if manufactures specifications of resolution are used, it is appropriate to actually enter 75% of that value into Eq. (4.1) to determine the reference beams angle of incidence (Newport 1989). This method has been used in experimentation, and proved satisfactory.

4.3.2.2 Irradiance Ratio:

The ratio between the irradiance of the reference and object beams influences the contrast of the interferometric fringes. The setting of this ratio is

simplified by the use of a variable beamsplitter. Note that it is the irradiance ratio at the film plate that is of interest. Thus, the irradiance of the two beams is measured by placing a power detector in the plane which the film plate will occupy during its exposure. By alternately blocking one beam and measuring the power density of the other, the variable beamsplitter may be adjusted to obtain a desired irradiance ratio.

There is vast disagreement in the literature concerning the appropriate value of this ratio. Vest (1979) suggest a ratio of one, Behbahani and Stewart (1989) use a value of one to four, and technical literature from Newport, AGFA, and Kodak suggest ratio values of four, one to two, and five to ten, respectively. From these few sources, the vast disagreement is apparent. Unlike the determination of angle of incidence, no basic theoretical technique exists for the determination of irradiance ratio. Thus, the determination of irradiance ratio is based solely on experimentation. Published values may be used to guide such experimentation. Irradiance ratio values of one and three were used in the final experimentation of this thesis, and there was no noticeable difference in contrast between the resulting interferograms. Thus, it does not appear that this parameter is of significant importance as long as

reasonable values are chosen. Following the conclusion of the experimental stage of this thesis, a paper was discovered which investigates the value of optimum beam ratio. In this paper (Sollid and Swint 1970), irradiance ratios ranging from 0.41 to 4 were used with Kodak 649F plates. The authors conclude that the optimum irradiance ratio is 1.2 ± 0.2 .

4.3.2.3 Exposure Time:

The film plate property which is used to determine the appropriate exposure time is sensitivity (in some literature it is called exposure) which has the units of energy per unit area. The system parameter on which exposure time is dependent is the total power density that the film plate is exposed to prior to development. The relationship between these quantities and exposure time is

$$t = \frac{S}{I} . \quad (4.3)$$

In Eq. (4.3), t is the exposure time, S is the sensitivity of the film plate, and I is the total power density at the plate due to the combined effect of the object and reference beams. The total power density, I , is simply the power meter reading, as discussed above in setting irradiance ratios, with both beams unblocked. Equation (4.3) applies directly when using real time HI. However,

this value must be divided by two when using double exposure HI since the plate is exposed to the object and reference beams twice before its development.

The film plate characteristics which are used to determine its sensitivity are density, D , and/or amplitude transmittance, T . These terms are defined as

$$T = \frac{E_1}{E_2} = \frac{I_1}{I_2} , \quad (4.4)$$

and

$$D = \log \left(\frac{1}{T} \right) \quad (4.5)$$

where E is the amplitude of the electric intensity vector and I is the corresponding irradiance. The subscripts 1 and 2 refer to values of the electric intensity vector or irradiance of the reference beam impinging upon the plate and along the zero-order diffraction direction, respectively. Plotting these parameters verses $\log$ exposure (sensitivity) produces the characteristic curves from which sensitivity is determined. The resulting characteristic curves are referred to as $D\text{-}\log E$ or $T\text{-}\log E$ curves, and sample curves are shown in Figs. 4.5 and 4.6, respectively. Although in these curves, exposure and sensitivity have the same meaning, the term exposure is

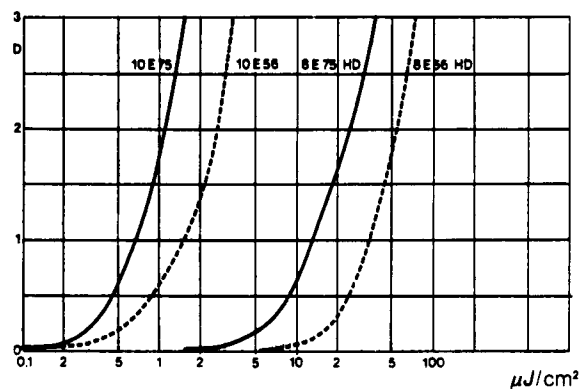


Fig. 4.5 Density log exposure (D logE) curve. Used with permission from AGFA corp.

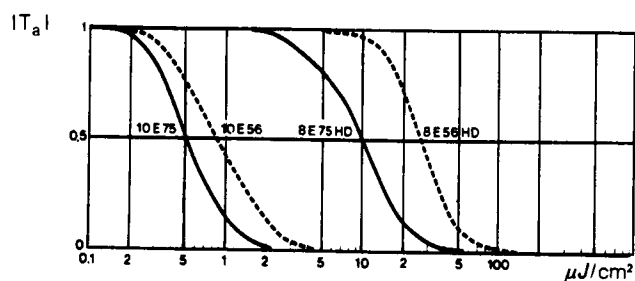


Fig. 4.6 Transmittance log exposure (T logE) curve. Used with permission from AGFA corp.

used to adhere to tradition. These curves are always referred to as being a function of log exposure, even in literature which uses the more accepted terminology of sensitivity. Density and amplitude transmittance are plotted against the log of exposure because the human eye interprets differences in these parameters on approximately a logarithmic basis (Caulfield 1979). It is appropriate to note that these characteristic curves are derived experimentally by the manufacturer for a given processing procedure. The general shape and position of these curves may be altered to some degree by variations in the processing. Consider the case of decreasing the development processing time relative to that used to derive the characteristic curves. In such a case, the D-logE curve would be shifted down, and the T-logE curve would be shifted up. The opposite is true for a relative increasing in the development time.

The correct value of sensitivity, exposure, is selected from the characteristic curves such that a linear hologram will result. Caulfield (1979) develops a relationship concluding that linearity is achieved when gamma, which is the slope of the D-logE curve, takes on a value of two. However, many film plates, most specifically those used in this thesis, never reach this value of gamma. The same exists in the D-logE curves presented in Caulfield's text.

A more accepted method is to obtain the sensitivity of film plates from their T-logE characteristic curve. "To obtain a linear hologram, the average exposure should be selected near the center of the linear portion of the T-logE curve (which does not coincide with the linear portion of the D-logE curve)" (Vest 1979). This corresponds to reading the sensitivity off the T-logE curve at an amplitude transmittance of 0.5. Equation (4.5) indicates that this is equivalent to reading the sensitivity off the D-logE curve at a density of 0.6. For example, from Figs. 4.5 and 4.6, the value of sensitivity of film plate 10E75 is approximately 5 uJ/cm (whether determined from the D-logE or T-logE characteristic curve). This method of determining sensitivity has been used experimentally with Kodak S0253 film and was found to produce very good interferograms.

4.3.3 Processing:

Chemical processing is a required step in the use of silver halide emulsion film plates. This step follows the exposure, and completes the alteration of the plate emulsion via chemical reactions. Chemical processing involves agitating the exposed film plate in various chemical solutions for specified periods of time. The nominal processing time required in each solution is

usually specified by the film manufacturer. These times vary depending upon the exposure and solution temperature. Thus, development of good holograms usually requires some fine tuning of the processing times in the solutions. This fine tuning is performed experimentally. Depending upon the steps included in the chemical processing, the resulting hologram is referred to as a amplitude or phase hologram. These two types of holograms are discussed in the following sections.

4.3.3.1 Amplitude Holograms:

The two basic chemical processing steps necessary to produce an amplitude hologram are referred to as development and fixing. Recall that in the exposure process the silver salts of the emulsion are converted to silver atoms. In the development process, these silver atoms are converted to metallic silver. These areas of the plate are consequently opaque. The fixing step removes the unconverted silver salts from the emulsion. The resulting processed hologram thus consist of opaque and transparent areas. The hologram may thus be thought of as a binary filter where in the resulting reconstructed beam the irradiance distribution is approximately a maximum or a minimum corresponding to the transparent and opaque regions of the hologram, respectively. In reality,

the entire illuminated portion of the plate will contain some finite density of silver atoms which are subsequently converted to metallic silver. Thus, areas of the plate in which destructive interference occurs do not appear totally transparent. As previously discussed, the actual variation between bright and dark fringes is sinusoidal rather than a step function. However, the basic influence of the hologram on the beam used to form the reconstruction is one of amplitude variation. Therefore, this type of hologram is referred to as an amplitude hologram.

Each chemical processing step should be followed by a water rinsing to remove the excess solution from the plate. This water bath must be continually replenished to avoid excess contamination by the development and fixing solutions. Additionally, the processing may include a Kodak photoflo step which eliminates water marks from the resulting hologram. The chemical processing steps used in this thesis to process AGFA 8E75HD film plates were as follows:

- 1) the plates were developed with agitation in Kodak D-19 developer for 2 minutes and 30 seconds,
- 2) they were then rinsed with agitation in a water bath for 30 seconds,
- 3) they were fixed with agitation in Kodak fixer for 6 minutes,

- 4) they were rinsed with agitation in a water bath for 5 minutes,
- 5) they were soaked in Kodak photoflo solution for 3 minutes.

Following chemical processing, the excess solution was removed from the plates using a squeegee, and the plates were hung vertically to air dry.

4.3.3.2 Phase Holograms:

Amplitude holograms are characterized by opaque regions due to the presence of metallic silver, and all regions of the plate contain some degree of silver density. The presence of silver in the areas of the plate where destructive interference occurs reduces the diffraction efficiency of the hologram. Consequently, the resulting hologram is relatively dim. The motivation for producing a phase hologram is to increase the diffraction efficiency of the hologram, and thereby increasing the brightness of the resulting interferogram. This is achieved by subjecting the hologram to a bleaching step. The bleaching step may be applied to a previously processed amplitude hologram or incorporated directly into the development process. In the bleaching process the metallic silver produced in the development process is converted to silver salts or completely removed from the emulsion. The result will be, depending upon the local

degree of metallic silver conversion or removal, there exists a variation in the local refractive index. Thus, rather than varying the amplitude, a phase hologram alters the phase of the beam used to produce the reconstruction. Consequently, the nominal irradiance in the interferogram is increased relative to that of an amplitude hologram.

The chemical processing steps used in this thesis to produce phase holograms from AGFA 8E75HD plates was as follows:

- 1) develop with agitation plates in Kodak D-19 developer for 2 minutes,
- 2) rinse with agitation in a water bath for 3 minutes,
- 3) bleach with agitation in JD-2 solution (obtained from Integraph Lake Forest, IL) for 1 minute and 45 seconds,
- 4) rinse with agitation in water for 3 minutes,
- 5) soak in Kodak photoflo solution for 3 minutes.

It should be noted that the bleaching process goes to completion and then stops. Therefore, the upper time limit on the bleaching step is not particularly critical. The JD-2 bleaching solution used in this thesis was suggested and developed by Dr. Jeong of Integraph corp. The production of phase holograms and bleaching solutions has been thoroughly addressed in the literature (Upatnieks and Leonard, 1969, Urbach and Meier, 1969, Graube, 1974,

Smith, 1968). If phase holography is to be utilized, it is suggested that this literature be referenced.

As mentioned earlier, the processing steps for production of a phase hologram increases the nominal irradiance of the interferogram. Thus, this type of hologram is highly desirable in situations where an amplitude hologram would produce a dim interferogram. However, a shrinkage of the emulsion is associated with the bleaching process. This shrinkage of the emulsion induces extraneous diffraction which will be reflected in the interferogram. Recall the discussion of secondary Fizeau fringes (section 4.3.1.2) in the hologram resulting from reflection of the beams off the film plate base back into the emulsion. Through experimentation, it was found that when bleaching was used, the relative irradiance of the dark fringes resulting from the phenomenon under investigation was of the same order of magnitude as the dark Fizeau fringes. Recall also that application of an anti-halation backing reduces the reflection back into the emulsion. Experimentation proved that the use of the anti-halation backing in conjunction with amplitude hologram processing produces fringes of the maximum contrast, while minimizing the observable effects of the Fizeau fringes. Thus the final experiments performed for

this thesis utilized amplitude holograms containing the anti-halation backing discussed in section 4.3.1.2.

5. Holographic Interferometric Laboratory

5.1 The Interferometer:

The funding for the development of the University of Tennessee's Mechanical and Aerospace Engineering Department's Holographic Interferometric Research Laboratory was provided through a cost sharing equipment grant from the National Science Foundation. The interferometer consist of components which are appropriately assembled to accommodate a particular experiment. The majority of these components were purchased from Newport Corporation.

5.1.1 Interferometer Components:

The interferometer is configured on a vibration isolated optical table. The table top rests on four air spring supports which are supplied by pressurized nitrogen to provide the vibration isolation. The optical table and supports are Newport model RS-48-12 and XL4A-22, respectively. The optical table surface is a ferromagnetic material and contains tapped $1/4$ - 20 holes at regular one inch intervals. These characteristics of the surface provide the means of attaching the individual components to the table surface. The components may either be directly anchored via the threaded holes, or

attached to magnetic bases and locked in place magnetically.

Normally, it is desired to elevate the beams of the interferometer above the table surface. A beam steering device is used to provide the transition between the laser output and working elevations. The beam steering device also allows pitch, and yaw adjustments to be made to the beam at the elevated level. With the working level elevated as such, it is necessary to elevate the optics of the interferometer accordingly. This is achieved by mounting the various components on posts. Several different types of posts of varying size and degree of adjustability are available in the laboratory.

The interferometer uses a Spectra-Physics, model 127, 60 mW, helium-neon laser as a light source. The output beam of helium-neon lasers is in the red regime of the spectrum at a wavelength of 632.8 nm. The laser is cooled by natural convection, which is enhanced by the vent slots on the upper and lower surfaces of the laser housing. These vent slots allow light from the laser tube to escape into the laboratory. Such stray light destroys the total darkness of the laboratory, which is necessary during operation. It is thus desirable to operate the laser only for short periods during the exposure process. The laser contains interlocking capabilities but, these have not

been utilized. Rather, for simplicity, the laser receives power through a switchable outlet, to provide remote operation. Note that the manufacturer suggest a warm-up time of two hours. However, allowing the laser to warm-up and then, once experimentation begins, operating it only during exposures presented no detrimental effects on the interferometric results. Since the laser simply rests on the optical table surface, the entire optical system may become easily misaligned due to slight unintentional bumping of the laser. To reduce the possibility of such misalignment occurring, brackets were manufactured to lock the laser in place.

The single laser beam is split into two beams to form the object and reference beams. This is accomplished via a Newport model 930-63 variable attenuator/beamsplitter (VAB). The output beams of the VAB are at right angles, and one is in line with the input beam. The VAB allows continuous adjustment of the irradiance ratio of the reference and object beams by manipulation of an adjustment knob.

Two inch O.D., first surface, dielectric coated mirrors are used to direct the laser beam in the working plane. The mirror mounts used in the laboratory have integral micrometer head pitch and yaw adjustment screws to accommodate fine adjustment of the laser beam direction.

The mirrors and mirror mounts are Newport model numbers 20D20BD.1 and 630A-2, respectively.

The process of expanding the laser beam incorporates a spatial filtering process. Expansion of the beam is achieved by placing a small focal length lens in the path of the beam. Microscope objectives are used for this process. The beam is filtered by placing a small diameter aperture along the optical axis at a distance downstream of the objective equal to its focal length. Such apertures are referred to as pinholes. Although the mean wavelength of light emitted by the laser is 632.8 nm, there is actually a finite range of wavelengths around this mean value emitted. The distribution of this wavelength spread is naturally gaussian. Spatial filtering reduces the standard deviation of the wavelength distribution. The objective and pinhole are mounted in a component referred to as the objective pinhole assembly (OPA). This component was purchased from Newport as compact spatial filter model 910B. The OPA contains fine adjustment screws to position the pitch and yaw of the assembly. The pinhole is positioned at the focal length by combined gross adjustment of the objective position, and a fine adjustment threaded housing of the pinhole. Additional adjustments available are two dimensional

positioning of the pinhole in a plane perpendicular to the optical axis.

To collimate the expanding laser beam, a plano-convex lens is introduced into the path of the beam. A plano-convex lens is characterized by a planar side and a curved convex side. To hold the lenses of the interferometer, lens holders were designed and subsequently fabricated by the department's machine shop. The design consist of simply clamping the lens in a V-shaped groove in the plane of the lens. To protect the delicate lens edges from damage, rubber bases were cemented to the aluminum frame at the points of lens contact. Clamping the lens holder to a vertical post, via a 90 degree clamp provides sufficiently controlled pitch and yaw adjustment of the lens. The lens elevation is adjusted by elevating the post to which the lens holder is attached.

Numerous components of the interferometer require controlled linear adjustments. Most specifically, positioning of the lenses of the interferometer require such adjustment in a plane parallel to that of the optical table surface, i.e., perpendicular to the plane of the lens. To provide this adjustment, a linear translator design was acquired from Dr. Nguyen of the Mechanical and Aerospace Department, and fabricated by the department's machine shop. The linear translators provide one half

inch of total travel, using commercial micrometer heads and ball bearing slides for precision motion. The design allows perpendicular cascading of the translators to achieve the two dimensional translation adjustment required for lens positioning. The translators are designed to incorporate the use of magnetic bases to provide table top anchoring. The upper surface of the translators contains mounting holes identical to those of the optical table.

Normally, only an axially symmetric fraction of the expanded beam is utilized to produce the collimated beam. Therefore, the expanded beam must be stopped down to an appropriate degree. This is similar to setting the f-stop on a camera. This need to stop down the expanded beam is met by an iris diaphragm. The iris diaphragm provides an approximate circular aperture of adjustable diameter from 2.4 to 41.3 mm. This component was purchased from Newport as model ID-1.5. The aperture diameter is adjusted by translation of an knob along the iris diaphragm perimeter. The iris diaphragm is designed to accommodate mounting via post attachment.

The film plate holder in the laboratory is a Newport model 535. The film plate holder uses three springed pins and three conjugate fixed pins to hold the plate in a specified plane during exposure. The upper pin set is

centered in the assembly. The lower pin sets are centered a distance of 3.25 inches apart. An adjustable corner enhances relocation of the film plate in the film plate holder. The film plate holder may accommodate plates up to ten inches wide. However, due to the spacing of the lower pin sets, the 2.5 by 2.5 inch plates used in this thesis could not directly be positioned with the available film plate holder. To address this shortcoming, aluminum strips were machined to join the like pins of the lower pin set. This arrangement created a continuous surface contact clamping between the lower pin sets, in contrast to the natural linear contact clamping of the film plate holder.

The projection screen is the surface upon which the interferogram is imaged. A piece of ground glass is used as a projection screen in the interferometer. This ground glass is actually a replacement imaging screen for a Graflok camera, and was purchased at a local photography shop. A frame in which to mount the projection screen was fabricated from plexiglass. This frame is designed to be mounted on a post, and contains the projection screen in a recessed groove by plexiglass leaf springs. Elevation and yaw adjustments of the projection screen are accomplished via the post holder. Pitch adjustment requires appropriate shimming of the post holder base.

As previously discussed, the irradiance ratio of the reference and object beams is a critical parameter to the exposure process. The method of determining this ratio involves the use of a power detector and meter as previously defined. The digital power meter is a Newport model 815. There are two power detectors available in the laboratory, which are Newport models 816 and HC-302. The detector surface area of each of these detectors is one square centimeter. Thus, power density readings on the power meter have actual units of $\mu\text{W}/\text{cm}^2$.

An additional parameter critical to the exposure process is the exposure time. The use of Newport model 845 digital shutter system provides controlled exposures of the film plate. The mechanical shutter mechanism is best positioned just downstream of the beam steering device. This reduces unwanted light in the laboratory due to scattering of the laser light by the optical components. The shutter controller provides a range of exposure times from 10 ms to 990 s. Additionally, the controller provides selection of exposure delay, and remote operation.

5.1.2 Setup Adjustments:

In the previous section, the major components of the interferometer were introduced. It is the purpose of this

section to provide the details of the procedure for the configuration of the interferometer. A number of techniques of varying degrees of precision exist for the configuration of the components of a holographic interferometer. The following presentation of component configuration represents a compilation of the most successful techniques discovered over a period of approximately two years of working in the holographic interferometric research laboratory. Thus, it is anticipated that, use of the following techniques will alleviate much of the initial time required to become familiar with component alignment. Beginning experimental work with such a foundation, should allow future experimenters to focus on the investigation of the phenomenon under study, rather than on the basics of interferometer alignment. In the following presentation, only the configuration of the object beam is fully considered. The techniques for configuration of the reference beam are identical to those presented for the object beam.

5.1.2.1 Optical Table:

The optical table is vibration isolated via its nitrogen activated air spring supports. The nitrogen is provided by commercially bottled nitrogen which is

regulated down to approximately 70 psi. Three of the four table supports have leveling valves which are used to adjust the elevation and level of the optical table. These valves are rigidly attached to the supports and have an activating arm which senses the local elevation of the table. The level of each valve arm dictates whether the valve is opened to supply, closed, or opened to bleed. Each leveling valve arm contains an adjustment screw which allows variation of the valve position for a given table elevation. Thus when the table is below its equilibrium position, the leveling valve opens to supply pressure to the corresponding support to return the table to its equilibrium position. In contrast, when the table is above this equilibrium position, the valve opens to bleed off pressure from the corresponding support. Additionally, the valves contain a gage which displays the pressure within the support. The fourth support which does not have a leveling valve is supplied by one of the adjacent valves. This support is referred to as the slave. Thus, the elevation of each corner of the optical table, the equilibrium position, may be adjusted by the adjustment screw on the leveling valve arm.

Although the regulated pressure of the supply was stated to be approximately 70 psi, this value may change depending upon the weight of the components upon the

table. The supply pressure is dictated by the pressure within the supports required to support the table. The supply pressure should be approximately 10 psi higher than this value. If the supply pressure is set below this ideal pressure, but above the support pressure, the table may continually oscillate about, "hunting for", the equilibrium position.

In many applications of the interferometer, it is desired for the optical table to be level. The process of leveling the table is now described. Figure 5.1 shows a sketch of the optical table system. This figure displays the points of contact between the table and supports and their location relative to the table edges. In Fig. 5.1, supports 1,2,and 3 have leveling valves. Support 4 is the slave, specifically of the leveling valve of support 1. The level of the table is indicated by a bubble level placed along cross hairs connecting the points of contact of supports 1 and 3, and supports 2 and 4. The bubble level should be verified as a true indicator of level. The operator should place the bubble level along the cross hair connecting supports 1 and 3, and independently adjust the valve arm adjustment screws such that level is indicated, and the supports are midway between their corresponding stops. Thereafter, the bubble level should

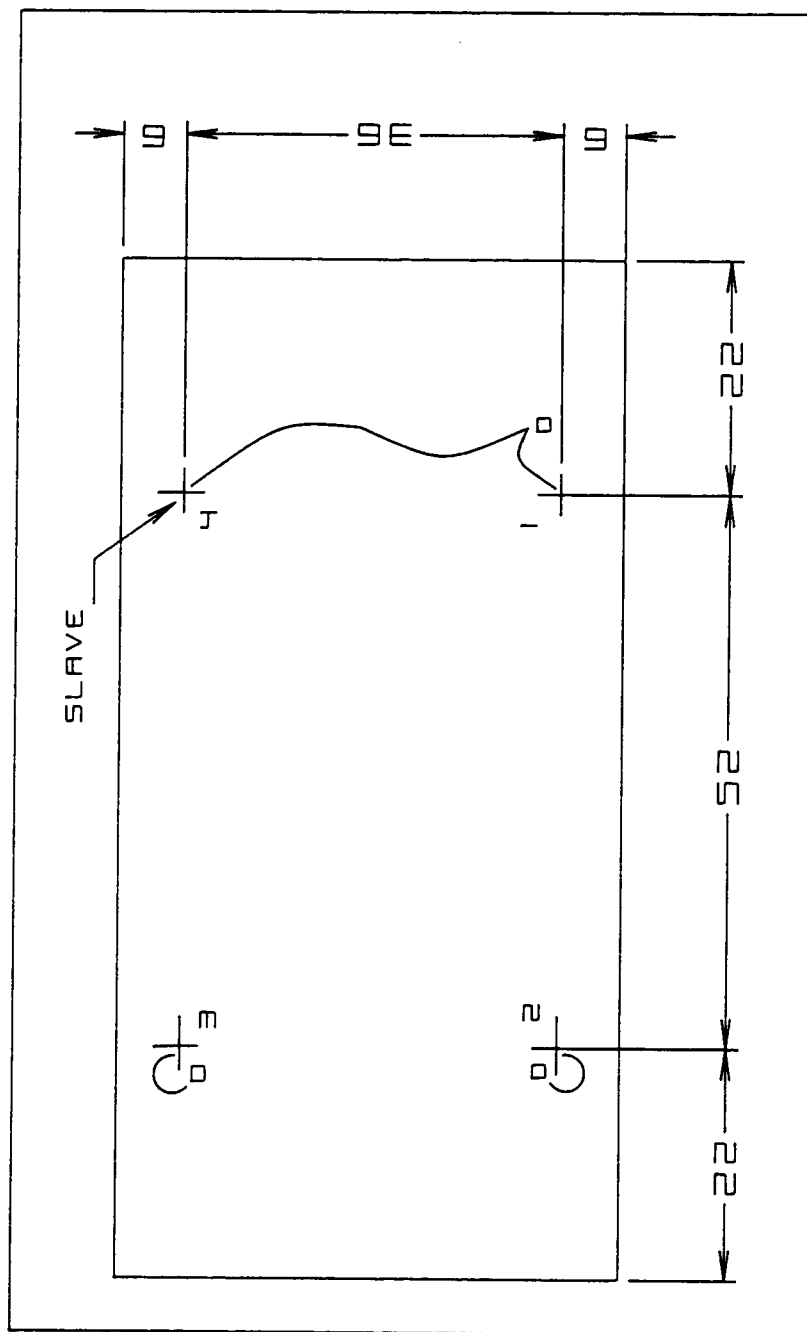


Fig. 5.1 Optical table leveling.

be placed along the cross hair connecting support 2 and 4. Note that adjustment of level along this cross hair does not effect the previous adjustment as the table will pivot about the cross hair connecting supports 1 and 3. Note though, that since support 4 is the slave, its elevation may not be directly adjusted without disturbing the previous adjustment. However, there are two ways to raise a given support. These are, by directly raising that support, or by lowering the diagonally situated support. The first method was used to level supports 1 and 3. The second method is used to level the table along the cross hair connecting supports 2 and 4. Thus, adjusting the adjustment arm of the leveling valve on support 2, and observing the bubble level, the table leveling is finalized. Rotating the bubble level back to the other cross hair, or any position should indicate a level condition. Once the optical table has been leveled, the supports should be verified to be approximately midway between their stops. If the supports are not so situated, the table should be releveled to meet this final criterion.

5.1.2.2 Pathlength:

Recall that the pathlengths of the object and reference beams must be equalized to within the coherence length of

the light source being used. The pathlength of concern is that from the beamsplitter to the recording medium. As stated earlier, typical coherence lengths of lasers are approximately 20 cm. With such a wide range of acceptable pathlength deviation, the pathlengths may be sufficiently equalized by measuring one with a piece of string, and configuring the other to be approximately equal.

Normally, the optics of the object beam are configured first to accommodate the test section at hand. With the object beam configured, the reference beam pathlength may be equalized by adding a compensating leg into its path. A compensating leg is extraneous travel of the reference beam to increase its pathlength. The use of a compensating leg assumes that the pathlength of the object beam is greater than the minimum required pathlength of the reference beam. If this is not so, and these two pathlengths are not equal, the pathlengths may not be equalized. Therefore, this requirement must be foreseen and the object beam pathlength be planned accordingly.

5.1.2.3 Variable Attenuator/Beamsplitter:

The only concerns pertinent to the positioning of the VAB, are that it be mounted on a horizontal plane, and that the entering and exiting beams pass unrestricted through the VAB windows. Proper mounting is easily

achieved by mounting the VAB on a platform attached to a post. Meeting the second requirement involves centering the entering and exiting beams on the VAB windows. Note that the laser beam is visible only when it strikes a surface. Thus the laser beam may be visualized in air only in the presence of dust particles or smoke, both of which are detrimental to the optics of the interferometer. To create a safe means of visualizing the beams position at the VAB windows, transparent tape may be placed over the openings. Using this method, the VAB window perimeter and laser position at the window may be simultaneously visualized. However, such tape acts as a volume diffuser. Therefore, the position of the laser beam at the exiting windows is observable only with the tape at the entrance window removed. Proper alignment of the VAB may be achieved by iteratively observing the laser beam position in the exit and entrance windows.

5.1.2.4 Mirrors:

Mirrors are used in the interferometer to direct the laser light. Once the path of the object beam has been determined, mirrors are positioned to achieve the desired path. The mirror mounts are attached to posts which are in turn attached to a magnetic base. Approximate positioning of the path is accomplished through

translation and rotation of the entire mirror assembly. Fine adjustment is provided by the yaw adjustment on the mirror mount. Adjusting the pitch micrometer on the mirror mount provides a means of aligning the beam in a plane parallel to the optical table surface. The position of the beam above the table is easily measured by using a simple ruler. Note that in legs of the interferometer which contain optical elements, it is convenient to position the laser beam such that its optical axis is directly above a line of mounting holes in the optical table surface. This simplifies the attachment of the components mounted on the posts to the optical table. A method for achieving such alignment is shown in Fig. 5.2. In Fig. 5.2 and subsequent figures, the star represents the laser light source, M_i represents the i 'th mirror, and I_i represents the i 'th iris diaphragm. The path of the laser downstream of M_2 is the final leg of the object beam path. This is the leg of the object beam which is desired to be positioned above a line of mounting holes in the optical table surface. Initially, the constricted apertures of I_1 and I_2 are mounted on posts and adjusted to the desired elevation. This is simplified by appropriately adjusting their elevation at some point in the path. For instance, assuming that the beam downstream of M_1 has been leveled to the desired elevation, the

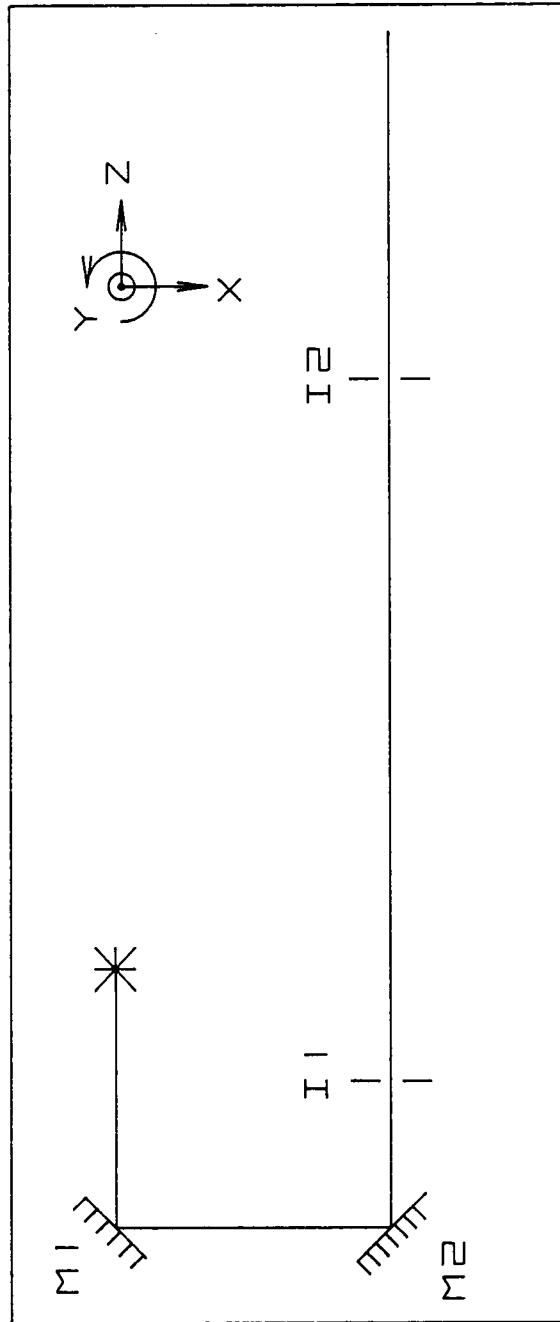


Fig. 5.2 Mirror adjustment. M = mirrors, I = iris diaphragm.

elevation of I1 and I2 may be alternately adjusted in the same table mounting hole just downstream of M1 and then positioned as in Fig. 5.2. Thus, the final leg of the interferometer is positioned as desired if the laser beam downstream of M2 passes through the constricted aperture of both I1 and I2. The positioning of M2 is achieved by simultaneous manual translation and rotation until this criterion is met. For maximum positioning accuracy, I1 and I2 should be positioned as far apart as possible.

Note that the coordinate system and sign convention used throughout this discussion are defined in Fig. 5.2. The coordinate system consists of three mutually perpendicular axes with the z-axis aligned with the optical axis of the object beam, and the y-axis vertical. The sign of pitch, yaw, and roll about the x, y, and z-axes, respectively, is defined by the right hand rule. That is, aligning the thumb of one's right hand in the positive axis direction, the fingers naturally curl in the direction of positive rotation. For clarity, the positive yaw direction is indicated in the coordinate legend.

5.1.2.5 Objective/Pinhole Assembly:

Prior to the installation of the OPA, I3 should be installed approximately 4 cm downstream of the planned location of the OPA, as shown in Fig. 5.3. The ultimate

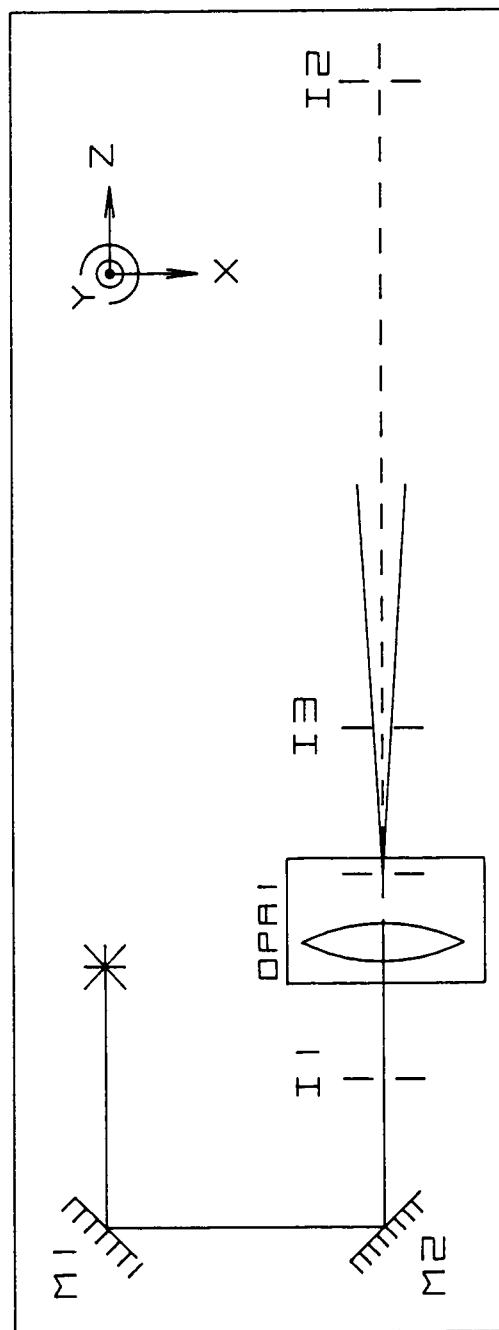


Fig. 5.3 Objective pinhole assembly (OPA) adjustment. M = mirrors, I = iris diaphragm.

use of I3 will be to reduce spherical aberration in the collimated beam as will be discussed in a following section. The OPA should be mounted on a post using a Newport post holder model VPT-4. This type of post holder incorporates the use of a fine adjustment screw which allows controlled adjustment of the elevation of the OPA. This assembly is in turn mounted on a Newport model 150 magnetic base.

The process of aligning the OPA in the position shown in Fig. 5.3 is simplified by initially focusing it at some convenient location in the optical path. Focusing of the OPA involves positioning the aperture of the pinhole at the focal point of the objective. To begin this process, one should adjust the elevation of the assembly such that the constricted iris of the OPA is aligned with the optical axis of the laser beam. Thereafter, the pitch and yaw of the OPA should be adjusted to their mean position. This is the position in which the rotations have equal possible magnitude in both the positive and negative directions. This positioning is not extremely critical, and is judged by visual inspection of the OPA. The operator should also place the x and y-axis pinhole adjustments to their mean position. The OPA can now be aligned with the optical axis by positioning the assembly such that the laser beam passes through the constricted

iris of the objective mount, with the objective removed, and the pinhole. The beam passes through the pinhole when maximum irradiance is observed emanating from the pinhole. Once this alignment is achieved, the magnetic base is locked in place. At this point, the objective should be installed in its mount. When the OPA is correctly focused, uniform illumination of maximum irradiance is observed on a screen placed downstream of the OPA. This screen may simply be a piece of white cardboard. To focus the OPA, one should carefully move the objective mount manually towards the pinhole until some illumination is observed on the screen. When this illumination is observed, then the objective should be locked in place in the OPA. More likely than not, this illumination will not be uniform or symmetric. Appropriate adjustments of the x and y positions of the pinhole should be made until the illumination becomes symmetric. The pattern observed on the screen should be one of alternating concentric bright and dark circular fringes. The fine adjustment screw which houses the pinhole may now be used to bring the OPA closer into focus. As this process is implemented, the irradiance pattern on the screen will probably become non-symmetric once again, and should be brought back into symmetry. Iterative application of these adjustments will focus the OPA. The actual point of focus is best judged

by traveling through the focus position from either side. Deviation from the focus position in either direction will result in the appearance of the circular fringes discussed earlier. Observing the appearance and disappearance of these fringes from both sides of the focus point provides an indication of the actual point of maximum focus.

Once the OPA is focused, it may be located, as shown in Fig. 5.3, approximately 4 cm upstream of I3. The iris diaphragm I1 may now be removed since it has fulfilled its purpose. The remaining adjustments of the OPA are x-axis position, yaw, elevation (y-position), and pitch.

Initially, one should constrict the iris of the OPA and partially open I3. Thereafter, the operator should align the OPA iris with the laser beam. While maintaining this alignment, one should carefully manually position the entire assembly such that an expanding beam exits the pinhole. In this position, the magnetic base to which the OPA is mounted should be partially locked down. Placing the screen downstream of I3 allows the observation of the irradiance pattern formed by the portion of the expanding beam exiting the OPA which passes through I3. The proper position of the OPA is indicated by a radially symmetric irradiance distribution over the illuminated portion of the screen. Note that the sensitivity of adjustments is

increased proportional to the distance of the screen downstream of I3.

Initially consider the x-position and yaw adjustments of the OPA. Observing the illuminated screen, these adjustments are initiated with the end result of producing an irradiance distribution which is symmetric about the y-axis. Note that, making a yaw adjustment with the corresponding OPA adjustment screw causes the OPA iris to become misaligned with the impinging laser beam. These adjustments must therefore be iteratively addressed until the desired irradiance distribution is produced. Consider now, the elevation and pitch adjustments of the OPA. The end result of the execution of these adjustments is to produce an irradiance distribution on the screen symmetric about the x-axis. As before, the overall adjustment procedure is an iterative process. If these steps are properly executed, the illuminated portion of the screen should be of equal irradiance at every point along any given radius from the optical axis, i.e., the irradiance distribution should be radially symmetric about the optical axis. When the OPA is satisfactorily aligned, its magnetic base should be fully locked in place.

5.1.2.6 Collimating Lens:

The collimating lens is held in a lens holder with the holder mounting axis positioned parallel to the x-axis, and the lens below this axis. The lens holder mounting axis is attached via a 90 degree clamp to a vertical post. Rotation of the lens holder axis and post axis provide pitch and yaw adjustment capability, respectively. Note that since these axes are not about the optical center of the collimating lens, pure pitch and yaw adjustments are not possible. Thus, executing these adjustments disturbs the lens position with respect to other adjustments such as x, y, and z-position. However, this is only an inconvenience, and since much more expensive lens holders provide only pure rotation adjustment, the present holders are utilized. A Newport post holder model VPT-4 is used to provide fine adjustment of the elevation of the lens. This post holder is mounted on cascaded linear translators aligned parallel to the x and z-axes to provide the respective adjustment capabilities. The linear translators are in turn mounted to a Newport model 100 magnetic base. To provide crude adjustment along the z-axis, focal axis, the magnetic base is abutted to an aluminum spacer bolted to the optical table and aligned with the z-axis. Since the linear translators provide only one half inch of travel, the proper positioning of

the collimating lens along the z-axis must be accurately approximated.

Prior to positioning of the collimating lens in the optical setup, some supplementary components must be positioned which are necessary for proper alignment of the lens. These are M3 and I4 as shown in Fig. 5.4. The I4 is installed between M1 and M2 with its constricted aperture aligned with the optical axis. To position M3, the objective and pinhole of the OPA must be removed. If this removal is performed with caution, the only readjustment of the OPA upon replacement will be that of adjusting the focal length. With these subcomponents removed, M3 is positioned such that the reflected laser beam retraces its path from I4 to M3, i.e., the laser beam passes back through the constricted aperture of I4. With these supplementary components installed, the positioning of the collimating lens may be initiated.

X-Y Plane, Pitch, and Yaw Positioning. Positioning of the collimation lens begins with the objective and pinhole of the OPA removed and M3 shielded from the incident beam by a screen placed between I2 and M3. Initial adjustment of the collimation lens involves centering the lens on the laser beam and positioning the lens perpendicular to the direction of beam propagation. These two adjustments

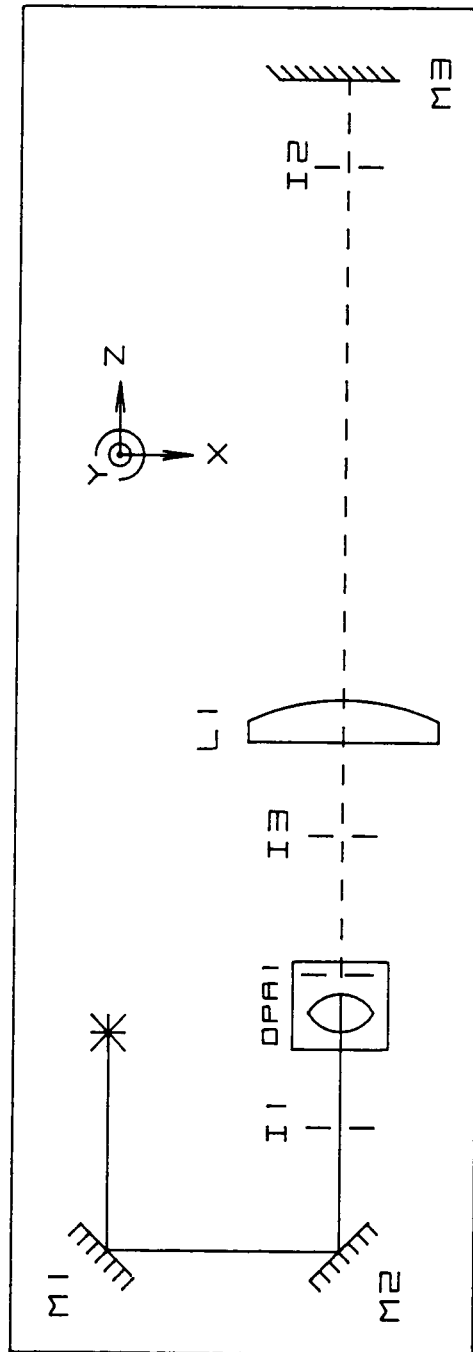


Fig. 5.4 Collimating lens (L1) adjustment.

correspond to adjustment in the x-y plane, and pitch and yaw of the lens position, respectively.

Initially, consider centering the lens on the object beam optical axis which corresponds to the unexpanded laser beam path. Adjustments of the lens in the x-y plane are dictated by the transmitted portion of the laser beam's position relative to the constricted aperture of I2. Note that when the laser beam intercepts the lens there is some degree of reflection and transmission at each surface. The lens should be adjusted in the x-y plane such that the transmitted portion of the laser beam is centered on I2.

Next, consider adjusting the lens such that it is perpendicular to the optical axis. This adjustment is performed by observing the position of the reflected portion of the laser beam relative to constricted aperture of I4. Pitch and yaw of the lens should be appropriately adjusted in order to center the reflected portion of the laser beam on I4.

When the reflected and transmitted portions of the laser beam are simultaneously centered on the constricted apertures of I4 and I2, respectively, the lens is positioned correctly with respect to all adjustments other than that along the z-axis. Note that, even if the pitch and yaw axes passed through the lens's optical center, the

two adjustment sets are not independent. Thus, several iterations of the two adjustment sets may be required to finalize this positioning.

The only adjustment now remaining is positioning L1 along the z-axis one focal length downstream of the pinhole of the OPA. Ideally, the lens assembly is locked down to the optical table within one half inch of this location before the previous adjustments are made. If this is not the case, the lens assembly's magnetic base may be unlocked and moved along the z-axis while remaining abutted to the aluminum spacer block. This manual readjustment should not drastically effect the previous adjustments if it is performed with care. However, if this process does result in misalignment, only minor readjustments to the x-y plane, and pitch and yaw position of L1 will be required and should be performed as described above.

Z-Axis Positioning. To collimate the object beam downstream of L1, the objective and pinhole should be reinstalled in the OPA, and brought into focus. Thereafter, the position of L1 along the z-axis should be adjusted to a distance of one focal length downstream of the pinhole. The focal length of interest is that of L1, and not the objective. When collimation of the object beam is achieved, the individual paths of every light ray

emanating from L1 are mutually parallel. The result of positioning L1 at any position other than that one focal length downstream of the pinhole is either a converging or diverging beam downstream of L1. A converging beam results from the pinhole being outside the focal length of L1, as depicted in Fig. 5.5a. Conversely, positioning the pinhole within the focal length of L1 produces a diverging beam downstream of L1, as shown in Fig. 5.5b.

Several techniques can be used for confirming if the beam is properly collimated. The simplest and most common method involves measuring the beam diameter at different positions along the optical axis. When the measured diameter of the beam is constant along the optical axis, the beam is considered to be collimated. This method is only as reliable as the scale used to measure the beam diameter and the experimenter's ability to accurately measure the diameter along its path. Additionally, this method incorporates no amplification of the beam's deviation from collimation which would result in increased accuracy of the above measured diameter. A second method proposed by Jessee (1983) involves measuring the separation distance of shadows produced by two wires of known separation distance placed just downstream of L1. This method incorporates an amplification of the deviation from collimation by reflecting the "collimated" beam off a

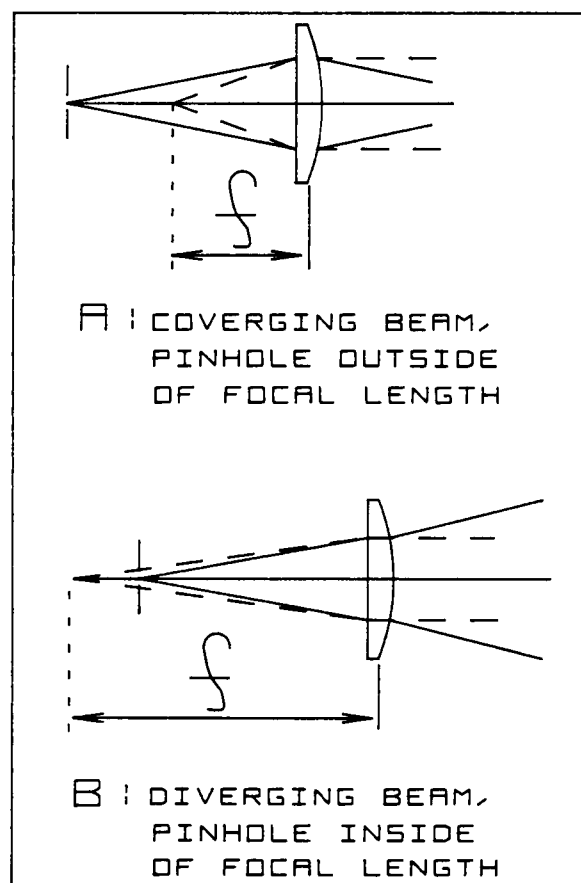


Fig. 5.5 Focal length adjustment. (a) Converging beam, (b) Diverging beam.

mirror, thus increasing the optical pathlength of the beam between the actual separated wires and their shadow. Increasing this distance proportionally increases or decreases the measured diameter for a given divergence or convergence, respectively. This method is complicated due to the existence of diffraction patterns introduced by the presence of the wires introduced into the beam. The actual pattern observed on the screen is that of two dark shadows, representing the wire positions, surrounded by parallel diffraction fringes of decreasing darkness with increasing distance from each wire. The difficulty encountered in using this method is the similarity between the two main shadows and the diffraction fringes, making it difficult to distinguish one from the other. The major drawback of these two verification techniques is that they provide no means of determining the presence of spherical aberration, which is addressed later, in the "collimated" beam.

After a thorough review of the literature and discussions with several researchers in the area of optics, it was concluded that there does not exist sufficient information on an accepted method for the collimation of free lenses. This is not necessarily a reflection on the researchers knowledge or ability since many laboratories use equipment which combines the

function of the OPA and the collimating lens into a single component referred to as a "collimator". Apparently, such collimators are relatively easy to position accurately. However, a method has been developed, which may be applied with relative ease, to accurately collimate a laser beam. Recall that the rays of a collimated beam are mutually parallel. Note that the central ray of the collimated beam defines the optical axis, and is the central ray of the unexpanded laser beam with which M3 was positioned. Therefore, the central ray will retrace its original path following reflection off of M3. Thus, if a beam is truly collimated (i.e. the rays are mutually parallel), all rays of the expanded beam will retrace their original paths following their reflection off of M3. This behavior will be referred to as the "path retracing rule of collimation". Note that this assumes that M3 is a planar mirror. The present collimation method strives to isolate individual rays and subsequently investigate their adherence to the path retracing rule of collimation. This method invokes the use of a collimation tool shown in Fig. 5.6a. The collimation tool contains four small diameter holes (0.05 inches) 90 degrees apart along a circumference of given diameter as shown in Fig. 5.6a. Note that the position of the holes in the collimation tool is insignificant. The arrangement shown was chosen for

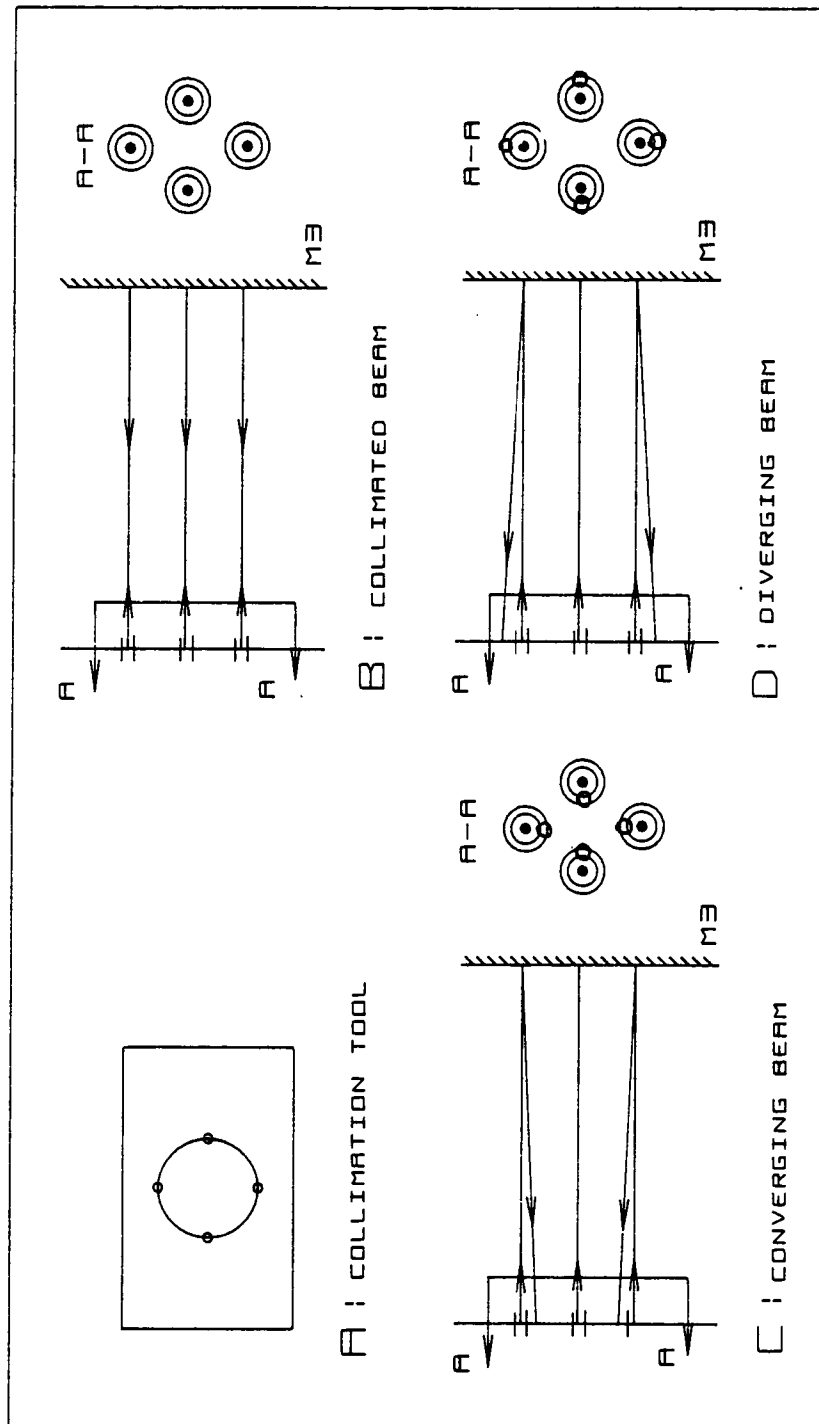


Fig. 5.6 Collimation tool. (a) collimation tool, (b) image observed when beam is collimated, (c) image of converging beam, (d) image of diverging beam.

reasons to be discussed later. The collimation tool is used by placing it in the path of the beam to be collimated, perpendicular to the optical axis, such that the beam is incident upon the holes. The position of the collimation tool along the z-axis should be just downstream of L1. The holes in the collimation tool are an attempt to isolate individual rays by allowing only a single ray to pass through the collimation tool, and blocking the propagation of all others. Although this is an impossibility, the behavior of the ray bundles which passes through the collimation tool holes sufficiently model the behavior of a single ray to provide information on the degree to which the individual rays would obey the path retracing rule of collimation. Thus, it may be expected that the ray bundles, following reflection off of M3, will pass back through the holes from which they emanate. Similar to Jessee's collimation method, introduction of the collimation tool into the path of the object beam produces a diffraction pattern which is reflected off of M3 and observed on the downstream side of the collimation tool. However, because this diffraction pattern is in the form of circular fringes, it tends to enhance the definition of the image of the ray bundle rather than obscure it.

Use of the collimation tool involves observation of the image of the reflected ray bundles and their corresponding diffraction pattern on the downstream side of the collimation tool. The introduction of the collimation tool into a collimated beam is shown in Fig. 5.6b. As discussed earlier, in this situation, the rays retrace their original paths following reflection and pass back through the holes from which they emanate. Section A-A of Fig. 5.6b displays the image observed on the collimation tool when the beam is collimated. Notice that the diffraction patterns form bulls eye target patterns centered on each hole thus contributing to the definition of the central image of the ray bundles. The ray bundle behavior and resulting image pattern for converging and diverging beams are shown in Figs. 5.6c and 5.6d, respectively. It should be noted that by approaching the collimated position from either side, the image of the ray bundles are expected to simultaneously and symmetrically approach the hole positions.

It is most likely that the initial image observed, upon implementation of the collimation tool, will be either that of Section A-A in Figs. 5.6c or 5.6d. To bring the beam into collimation, the z-axis linear translator, upon which the collimating lens is mounted, should be appropriately adjusted until the image in section A-A of

Fig. 5.6b is observed. Such translation along the z-axis should not effect the initial x-y plane, pitch, and yaw adjustments if the magnetic base of the lens assembly is abutted to the aluminum spacer block as specified.

The collimation tool also provides a means for fine adjustments of the initial x-y plane. Consider the situation in which the collimating lens is slightly misaligned in the vertical direction. In such a case, at collimation, the observed image would be similar to that of Fig. 5.6b, but would be either above or below the holes. Specifically, each ray bundle image would be an equal distance above or below its corresponding hole. If the image of the ray bundles is above (below) the holes of the collimation tool, the y-axis position of the collimation lens should be decreased (increased) until vertical alignment with the holes is achieved. The collimation tool may be similarly used for fine adjustment of the x-axis position of the collimating lens.

Spherical Aberration. Spherical aberration is a property of all spherical surfaced lenses and must therefore be considered. Newport's catalog tutorial written by Dr. W. Bridges of the California Institute of Technology gives an adequate explanation of this phenomenon, it's implications, and how to minimize it's effects. In this reference, the rays which constitute the

body of a collimated beam are divided into two major regimes referred to as marginal and paraxial rays. Marginal rays in general are those rays situated at the border or outer limits of the collimated beam. Paraxial rays are those rays in closest proximity to the beam axis.

In the investigation of spherical aberration, it is standard procedure to assume that the process is initiated with the existence of a collimated beam which is then focused to a point by a collimating lens. Due to geometric optics, this process may be reversed. Where necessary, the less salient implications of direction reversal are investigated. With an ideal lens, the rays of a collimated beam are refracted as a function of radius from the optical axis to converge to a single point on the optical axis at the focal length of the lens. Spherical aberration results due to the outer regions of the lens refracting the collimated beam to a higher extent relative to the refraction which occurs in the center portion. The result is that the marginal rays are severely refracted and therefore converge to a focal point different from and shorter than the focal point formed by the paraxial rays. The focal point specified for a given lens is that of the paraxial rays. By making proper decisions in lens selection, system design/orientation, and adjustments, the

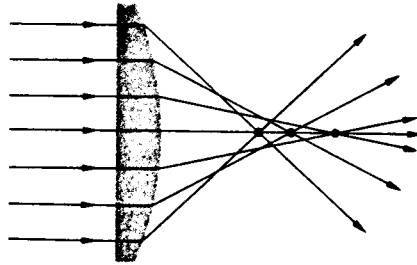
effects of spherical aberration may be reduced to an acceptable degree.

In selecting a lens, one should strive to minimize the refraction of the marginal rays. The achievement of this minimization is referred to as minimum deviation. It can be shown (Hecht 1987) that minimum deviation occurs when the total refraction of the rays is equally divided between the first and second optical surfaces. Image and object distances refer to the distances from the lens to the image and object, respectively. These distances are related by the conjugate ratio, which is the ratio of the image to the object distance. For a collimated beam (i.e., focusing on an object positioned at infinity) minimum deviation is achieved by using a plano-convex lens. In a situation where the image and object distances are equal, a symmetric lens will produce equal refraction at the two optical surfaces and therefore result in minimum deviation. The plano-convex and symmetric lenses represent the bounds of the conjugate ratio, infinity and one, respectively.

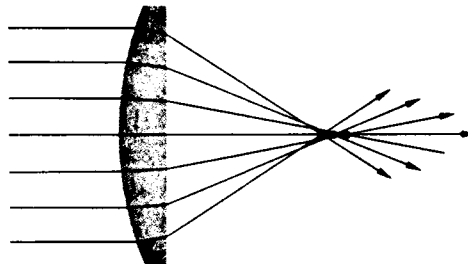
In the optical system design and orientation, spherical aberration may be further reduced by once again considering the condition of minimum deviation. In this step of the process, minimum deviation dictates which side of the plano-convex lens should be positioned facing the

collimated beam. The lens should be positioned such that the refractions of the rays entering and exiting the lens are approximately equal. Referring to Fig. 5.7a, if the flat side of the lens is positioned facing the collimated beam, the rays will intersect the lens perpendicular to the first surface. In this position, no refraction will occur at the first surface of the lens, and all refraction must therefore take place at the second surface. If however, the lens is positioned with the convex side facing the collimated beam as in Fig. 5.7b, the refractions occurring at the two surfaces are approximately equal. Such positioning satisfies the concept of minimum deviation, and consequently minimizes the effects of spherical aberration as shown in the figure. Note that the degree of spherical aberration in these figures has been greatly exaggerated to demonstrate the reasoning behind proper lens orientation.

Figure 5.7b shows that a degree of spherical aberration persists even after proper lens selection and orientation. One final step for minimization of this phenomenon may be executed in the optical adjustment stage of the process. Referring to Fig. 5.7b, in expanding and collimating a beam, the rays propagate from right to left with the pinhole positioned at the focal point on the optical axis. The question which immediately arises is which focal



a



b

Fig. 5.7 Effect of lens positioning on spherical aberration. Used with permission from Addison-Wesley Publishing Co.

point, since there are a range of focal points resulting from spherical aberration. This point of view requires that all rays, marginal and paraxial, have a common focal point at the pinhole. The effect of spherical aberration will now be observed on the 'collimated' side of the lens. Consider an expanding beam covering the entire diameter of the lens in Fig. 5.7b with the pinhole positioned at the focal point of the paraxial rays. All rays are required to have a common focal point, therefore the focal point of the marginal rays should move back to that of the paraxial rays. The result is that although in this configuration the paraxial rays exit the lens collimated, the marginal rays will converge due to the displacement of their focal point. Conversely, with the pinhole positioned at the focal point of the marginal rays, the marginal rays will exit the lens collimated and the paraxial rays will exit diverging. The conclusion is that with the finite amount of spherical aberration present, it is impossible for the paraxial and marginal rays to be simultaneously collimated for a given focal point. This is the justification for the final adjustment to minimize the effects of spherical aberration. The optics should be adjusted such that the paraxial rays exit the lens collimated, and the marginal rays blocked by constricting the aperture of an iris between the pinhole and the lens.

The obvious question which must now be answered is, to what radius from the optical axis can the rays be considered to be paraxial. The effect of spherical aberration is a continuous phenomenon which increases in extent from the optical axis outward. There is therefore no one aperture at which spherical aberration is completely eliminated. Therefore, an aperture must be selected which maximizes the usable diameter of the exiting collimated beam while minimizing the most severe spherical aberration effects. This aperture size is controlled via the iris diaphragm I3 of Fig. 5.4. An experimental method for selection of the aperture size was developed. This method utilizes spherical aberration measuring tool (SAMT) displayed in Fig. 5.8. The theory of operation of the SAMT is similar to that of the collimation tool. The rays of a collimated beam passing through the holes of the SAMT and reflected back along the same path should pass back through the same holes from which they previously exited. If in fact, the paraxial rays are collimated, the only deviation from this idealization will be due to spherical aberration. In order to utilize the SAMT, it should be positioned approximately perpendicular to the optical axis with the common centerline of its holes intersecting the optical axis. With the aperture of an iris between the pinhole

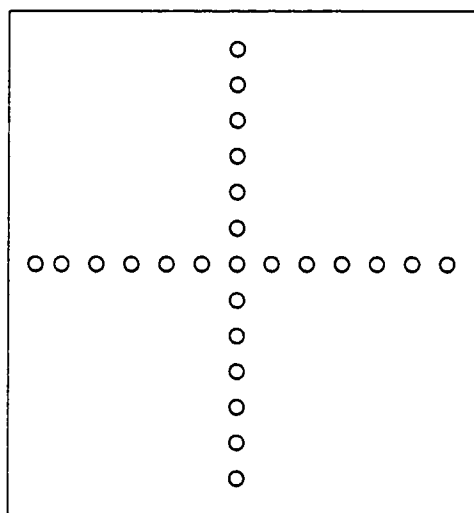


Fig. 5.8 Spherical aberration measuring tool (SAMT).

and lens opened to illuminate the entire lens, the downstream side of the SAMT is observed. It is observed that the paraxial rays indeed pass back through their original holes. If this is not the case, the beam should be recollimated. Additionally, it may be observed that the marginal rays converge towards the optical axis, and therefore do not pass back through their original holes. Although the effect of spherical aberration is a continuous function of radius from the optical axis, it has been observed that there is a radius at which its effect becomes increasingly severe per unit radius change. In observation, this functional dependency appears similar to a ramp function. That is to say, out to a certain radius, the effect of spherical aberration are not noticeable, and thereafter it becomes increasingly severe. It is this radius which defines the extent to which the rays may be considered paraxial. The iris aperture should be adjusted to block all marginal rays outside this radius. Twice this radius, as measured in the collimated beam, defines the usable diameter.

Although this correction is not implemented until the optical adjustment stage of the overall process, it must be anticipated and incorporated into the design stage, as it limits the usable diameter of a lens beyond that specified by the $f/$ number of the lens. However, the

method of determining the extent of this limitation described above is experimental and therefore one can not exactly predict its degree for a particular lens. A suggested rule of thumb is to design for a working diameter of half that specified by the lens f/number (Nguyen 1989).

Note that the effect of spherical aberration must be considered in the collimation process. Also, recall that the above process for determining the usable diameter of a lens assumes that a collimated beam is available. If the holes in the collimation tool are on a diameter larger than that of the usable diameter of the lens at hand, collimation will be based on marginal rays. If this is the case, the erroneous collimation will be discovered when selecting the usable diameter. The effect will be that the paraxial rays will diverge. To correct this error, the diameter of the holes of the collimation tool must be reduced to less than that of the usable diameter of the lens. Notice also, that the effect of spherical aberration further complicate the method of collimation by diameter measurement. This method is inherently based on dictating collimation from the most marginal rays present. If the marginal rays are outside the usable diameter of a lens, and the paraxial rays do not diverge to the diameter of the marginal rays within the measurable range of the

beam, then this method may be in error. Therefore, if the diameter measuring collimation technique is to be used, it is of utmost importance to insure that the exiting collimated beam is within the usable diameter of the lens being used.

5.1.2.7 Film Plate Holder Positioning:

The desired position of the holographic film plate during exposure is such that it is perpendicular to the object beam's direction of propagation. In adjusting the film plate holder to meet this criterion, a beam reflection method similar to that used in lens alignment is used to dictate alignment. To execute this alignment procedure, a piece of thin glass was cut to the appropriate dimensions, and placed in the film plate holder. When the object beam impinges upon this glass, it is partially reflected and partially transmitted. Pitch and yaw adjustments are made to the film plate holder such that the reflected portion of the object beam off the glass retraces its path to the glass. An iris diaphragm placed in the object beam path, upstream of the film plate holder, may be used to enhance this adjustment. Following alignment, the glass alignment plate should be removed from the film plate holder before aligning the subsequent components.

5.1.2.8 Projection Screen Positioning:

To accommodate positioning of the projection screen in the image plane of the test section midplane, it should be mounted on a linear ball bearing slide. Initially, this slide is aligned with the z-axis and bolted in place. This simply involves aligning the slide with a line of mounting holes in the optical table surface. The slide is a ferromagnetic material which allows the projection screen to be locked in place on the slide via a magnetic base. The projection screen should be positioned such that it is perpendicular to the direction of propagation of the object beam. The method for this alignment is identical to that used for the film plate holder. Conveniently, the imaging screen used is ground only on one surface. Therefore, the screen may be positioned in its holder such that its unground side is facing the impinging object beam. Once the projection screen is aligned, it should be carefully reversed so that the ground side is facing the impinging object beam. This is the position that should be used during the operation.

5.1.2.9 Imaging Lens Positioning:

The imaging lens should be placed just downstream of the film plate holder in the path of the object beam. Alignment of the imaging lens follows a procedure similar

to that presented for the collimation lens. However, this alignment is somewhat more difficult due to the presence of an expanded object beam. This presents only a minor obstacle, and an iris diaphragm may be used to block the expanded portion of the object beam and simulate the original unexpanded laser beam. This step requires the introduction of I5 as shown in Fig. 5.9. The x, y-position, pitch, and yaw of the imaging lens are appropriately adjusted such that the reflected and transmitted portion of the stopped down object beam are simultaneously centered on the constricted apertures of I5 and I2, respectively.

5.2 Data Collection:

Since it is the primary intent of this thesis to present a thorough description of the physics and application of the HI method, these topics are stressed herein. The actual tools used to determine the secondary measurements are therefore given less emphasis. However, in order to maintain completeness, these tools are briefly discussed in the following sections. What is important about these secondary tools is their influence on the achievable accuracy of the interferometric results. This influence is dictated by the uncertainty analysis contained in Appendix C. Thus, for a given experiment,

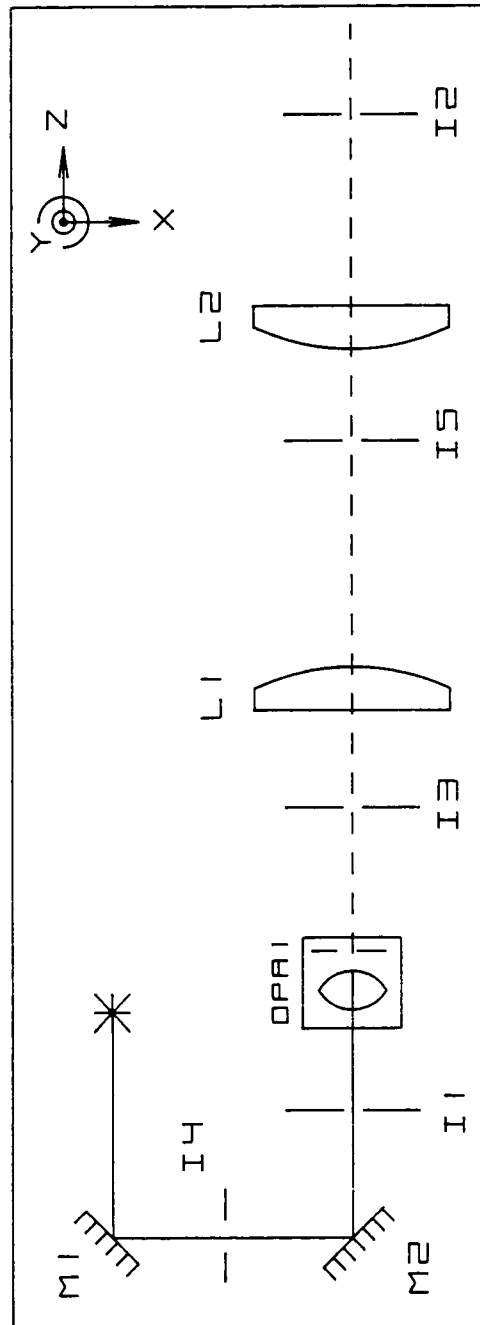


Fig. 5.9 Imaging lens(L2) adjustment.

the uncertainty analysis may be invoked to predict the required accuracy of these secondary tools required.

5.2.1 Temperature Measurement:

Temperatures were measured using copper constantan thermocouples and read using an Omega mode HH-73T thermocouple reader. The temperatures were read in degrees celsius to 0.1 °C. The thermocouples and meter system was calibrated over the temperature range of 25.00 to 54.70 °C, and found to underpredict the temperature by at most 0.1 to 0.7 °C, respectively. Thus the ambient temperature upon which interferometric results are based is known to within approximately 0.1 °C. With this degree of uncertainty in the ambient temperature, this term dominates the uncertainty in the fringe field temperature uncertainty. Specifically, in this thesis, the ambient temperature contribution to the fringe field temperature uncertainty is an order of magnitude greater than that of the a term or the corrected fringe shift. In order to approximately equalize the contribution of the ambient temperature to the fringe field temperature uncertainty, it would have to be measured with an accuracy of approximately 0.04 °C.

5.2.2 Pressure Measurement:

Pressure measurements were taken from a barometer on the same floor of Dougherty Engineering Building as the HI Research Laboratory. The particular barometer used provided resolution to within ± 0.01 inches using a vernier scale. Unfortunately, it was not realized that the reading was not corrected for local gravity until after the fact. This term is four times that of the barometer resolution. The assigned pressure uncertainty accounted for both of these. Thus, using proper correction procedures would reduce this uncertainty by five fold. The pressure ultimately effects the interferometric definition of the temperature field. Despite the unnecessarily high pressure uncertainty, it is revealed via the uncertainty analysis that this term is not the major contributor to the temperature field uncertainty. Thus, this particular barometer or one of similar specifications should be sufficient for use in future interferometric investigations.

5.2.3 Interferogram Digitization and Storage:

The interferograms obtained were recorded using an imaging tool. The foundation of the imaging tool is an image processing software package purchased from Werner Frei Associates. Peripheral equipment upon which this

software is dependent consist of a computer, video camera, and monitor.

The image of an interferogram is digitized from the imaging screen using the video camera and software. The digitized interferograms may then be stored on the computer hard drive for analysis at a later time. Interferogram analysis is performed from the monitor using the imaging software. The most analysis applicable features of the imaging software include contrast enhancement, pixel relative position and irradiance value determination, image algebra, and image rotation capabilities. The software provides image enlargement capability. It was intended to use this feature to provide selectivity of the scale factor between the test cell and the stored interferogram. However, associated with the digitization process is a fixing of the image resolution. Thus, as the image is enlarged, the resolution is equivalently decreased. For this reason, the interferograms in this thesis were analyzed using the image as digitized. In subsequent investigations, the scale factor may be determined by proper selection of the imaging lens.

6. Experimental Apparatus

6.1 Design Considerations:

Design of an experimental apparatus to be used in holographic interferometry research imposes several constraints on the design. Disregarding these constraints may simply limit the range of the analysis or actually make the results useless. Thus, it is imperative to incorporate these considerations into the original design. Unfortunately, several of the design considerations that will be presented were discovered after the design and fabrication of the apparatus used in this thesis. The result of these design flaws was that the original phenomenon which was to be observed had to be abandoned. Although this was only a disappointment in the present work, it could be absolutely devastating in others. Undoubtedly, the following discussion is incomplete. However, it represents the major design considerations that were discovered through personal experience and in the literature.

The anticipated range of the phenomenon to be investigated must be considered in the apparatus design. In the present investigation, this would be the maximum temperature range within the test section. The consideration of interest is the temperature change per

fringe shift. The larger (smaller) the temperature change per fringe shift, the less (greater) the number of fringes that will be present in an interferogram for a given overall temperature difference. Recall that infinite fringe interferograms, from which quantitative information is to be extracted, should possess five or more fringes in order to be accurate. For a given temperature range, the adherence to this rule is governed by the temperature change per fringe shift. If the temperature change per fringe shift is too large, there may not be a sufficient number of fringes present. In contrast, consider having a small temperature change per fringe shift. This results in an increase in the fringe density in the interferogram. The upper limit of acceptable fringe density is imposed either by the resolution of the holographic film used or that of the fringe location measurement system. Thus, the parameters influencing the selection of the temperature change per fringe shift are the range of the phenomenon to be investigated, the type of holographic film plate to be used, and the measurement system available. The relationship governing this variable is derived from Eq. (2.17) and is of the form

$$\frac{\partial T}{\partial S} = \frac{\lambda RT_{\infty}^2}{KLP_{\infty}} \quad (6.1)$$

Thus, it is apparent that for an ideal gas the temperature change per fringe shift is a function of the ambient temperature and pressure, the wavelength of the light source used, and the length of the test section.

Operating the test section at ambient temperature and pressure normally reduces the complexity of the experimental apparatus. Thus these variables may be assumed to be approximately fixed. Additionally, in many situations, the wavelength of the light source used is dictated by availability rather than particular selection. For all practical purposes, the Gladstone Dale and gas constants are in fact constant and may thus not be varied to accommodate a selected temperature change per fringe shift. The relative dependence of the Gladstone Dale constant upon wavelength, temperature, and pressure were discussed in section 2.4. Therefore the only variable which influences the temperature change per fringe shift, and the designer may be able to vary, is the length of the test section. Estimating an average desired temperature change per fringe shift and knowing the values of the other variables upon which it is dependent, the appropriate test section length may be determined.

It was noted earlier that a benefit of holographic interferometry is that it is insensitive to the quality of the end windows used in the test section. However, HI is sensitive to variations in the test section end windows that are not present in both exposures of the hologram. Thus, if the end windows experience thermal expansion between the first and second exposures, the effect of this change on the light rays influences the resulting interferogram. More than likely, such undesirable behavior will destroy the utility of the interferogram. Proper design of the experimental apparatus may eliminate thermal expansion of the end windows. This is normally achieved by positioning the end windows outside the range of the thermal effect of the heated element. The same consideration must be given to mechanical deformations of the end windows due to events such as a pressure change within the test cell between the first and second exposures.

An experimental apparatus to be used in interferometry should be designed such that all refractive index changing parameters throughout the experiment other than that which is to be investigated, are constant. Thus, the temperature field within the test section must be uniform during the initial exposure. Additionally, thermal and hydrodynamic entrance lengths must be considered in the

apparatus design if their effects are not pertinent to the investigation. Panknin et al. (1974) discuss the design of an apparatus in which such considerations are relevant. Incorporating sufficient entrance length in the apparatus allows the stabilization of these effects.

A more elusive task is fabricating an apparatus which accurately models the desired phenomenon. If the interferograms of a phenomena are to be evaluated as two dimensional, the apparatus must be designed to simulate, as closely as possible, a two dimensional behavior. Consider a vertical heated plate without adiabatic extension walls around its perimeter. Without such walls there exists entrainment of the fluid from behind the heated plate into the boundary layer. Incorporating such extension walls eliminates this undesirable entrainment. This is just one example which is applicable to the present investigation. Similar considerations must be given to each specific experiment.

6.2 Apparatus:

The goal in designing the present experimental apparatus was to create an isothermal surface at one end of a rectangular box while maintaining adiabatic boundary conditions on the remaining surfaces. As these

requirements were only desired to be approximate, a relatively simple design was selected.

Prior to the fabrication of the model, it was numerically modeled in order to investigate the adherence to the desired design criterion. A variable finite difference mesh was deployed on the solid heated assembly to account for the conduction heat transfer within all portions of the model. The radiation heat transfer was neglected, and convective heat transfer was accounted for by using estimated heat transfer coefficients for the exterior surfaces of the assembly. This numerical model was programmed in Fortran on the University of Tennessee's Engineering Vax. This investigation predicted that the heated surface will be nearly isothermal.

The schematic of the experimental apparatus is shown in Fig. 6.1. The design consists of a trapezoidal profile copper diffuser plate inserted in the end of a hollow plexiglass box beveled to accommodate the profile. A Minco 0.5 inch thick thermfoil strip heater was cemented to the surface of the diffuser element opposite the exterior isothermal surface. The diffuser element was chosen to have such a trapezoidal profile with the intent of channeling and diffusing the conduction heat transfer to the full width of the exterior surface of the diffuser

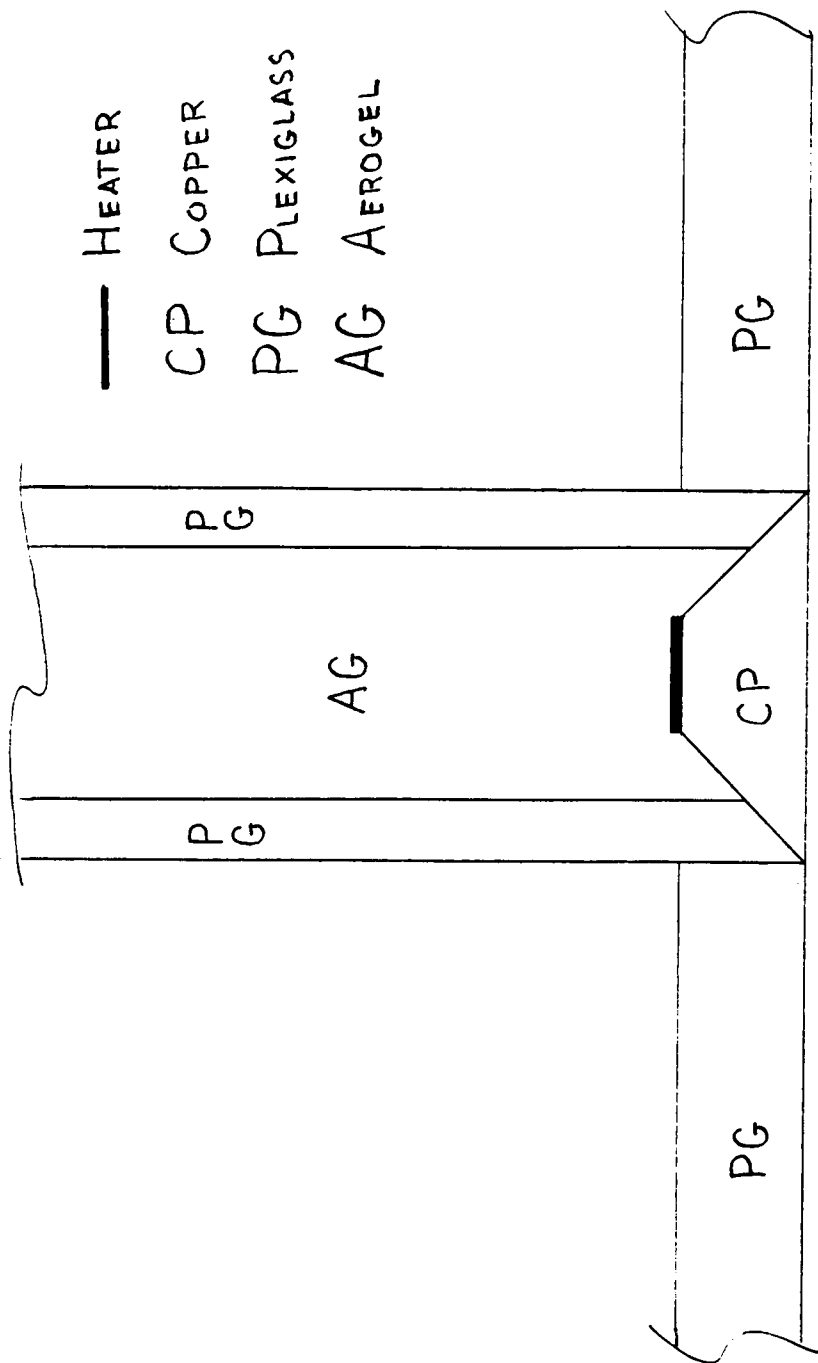


Fig. 6.1 Experimental Apparatus.

plate. This design resulted in a nearly isothermal surface, while minimizing the heat transfer through the plexiglass walls. The exterior surface of the diffuser plate is approximately 7.5 inches in the z direction and 0.73 inches in the y direction. The diffuser plate contains six evenly spaced type T 30 AWG thermocouples, placed 1/16 inches from the exterior surface to measure the surface temperature. The plexiglass housing was designed hollow to minimize heat transfer through the base. To minimize radiation heat transfer between extreme surfaces of the housing, the void was filled with Aerogel with a thermal conductivity lower than that of air. Thus, minimizing the conduction heat transfer through this filler material.

Although the numerical model of the experimental apparatus predicted that it would meet the specified design criteria, its actual performance was investigated interferometrically. To test the performance of the experimental apparatus, double exposure HI was used to produce an interferogram of the apparatus with the isothermal surface horizontal and facing upward. The adherence to the design criterion was then evaluated based on the qualitative fringe behavior. The results showed that the fringes just above the isothermal surface were parallel to that surface. This fringe behavior is

characteristic of an isothermal surface. Additionally, the fringes were observed to wrap around the sides of the experimental apparatus and intercept the plexiglass housing approximately perpendicularly. Recalling that fringes represent isotherms, this fringe behavior is characteristic of an adiabatic surface. Thus, through HI analysis, the performance of the experimental apparatus was verified as approximating a nearly isothermal surface at the diffuser plate, and adiabatic housing walls.

6.3 Test Section Alignment:

Several standard alignment methods are used in interferometric studies and have proven to be highly reliable (Hauf and Grigull, 1970, and Jessee, 1983). The methods used in the present experiments represent an adaptation of these standard methods to the specific apparatus at hand. The proper position of the plate is such that it is parallel to the rays of the collimated object beam. This is achieved by proper pitch and yaw adjustment of the heated assembly. Since the phenomenon under investigation is natural convection along a vertical plate, an additional constraint on the alignment is that the plate be vertically situated. Each of these adjustments is addressed individually, and will be presented as such. Due to the limitations of the mounting

hardware, each adjustment normally has some effect on the others. Therefore, the overall adjustment requires an iterative approach similar to that required for lens positioning.

Consider the adjustment of the pitch of the heated assembly. This adjustment is governed by the behavior of the rays adjacent to the upper and lower surfaces of the heated assembly. However, to observe such behavior in the present system requires opening the spherical aberration eliminating iris diaphragm, I3 of Fig. 5.4 discussed in section 5.1.2.6.3. Thus this maximum spherical aberration free collimated diameter must in some manner be recorded prior to making such an alteration. This is easily achieved by positioning an iris diaphragm downstream of the test section and stopping it down to a diameter equal to that of the spherical aberration free collimated object beam. The spherical aberration eliminating iris diaphragm may now be opened to allow the adjustment to be made. Consider the situation of a positive pitch misalignment. In such a scenario, the rays above the heated assembly would impinge upon its upper surface and reflect. The image of this phenomenon may be observed, expanded through a short focal length lens, on a screen placed downstream of the test section. This observation would reveal the shadow of the heated assembly surrounded by the

illumination of the object beam. More importantly, a bright streak in the illuminated field would be observed parallel to the upper surface of the heated assembly due to the reflection of the rays off that surface. Consider now, the behavior of the bright streak as the heated assembly is brought into alignment. As this procedure is administered, the bright streak would continually merge towards the shadow of the upper surface of the assembly. If the pitch rotation is continued past the aligned position, a similar bright streak will appear off the shadow of the lower surface of the heated assembly. Similarly, this bright streak is the result of reflection of the rays of the lower surface of the heated assembly. Thus, the presence of such bright streaks in the image may be used as an indicator of misalignment. The heated assembly is considered to be aligned with respect to pitch when its image is free of such bright streaks.

To align the heated surface in the vertical orientation, its image was compared to that of a vertical standard. The vertical standard used was a plumb line with the bob suspended in a liquid to dampen out oscillations. To increase the sensitivity of the alignment, the images were expanded through the imaging lens L2 of Fig. 5.9. The expanding images were projected onto white poster board taped to the far wall of the

corridor (outside the lab). This increased the z-axis travel of the beam by approximately 2.59 meters, which increased the image scale by approximately 1.86. The image of the plumb line is that of a central dark fringe surrounded by a diffraction pattern. The roll of the heated assembly was adjusted until the image of the heated surface was parallel to the central dark fringe of the plumb line.

The remaining alignment is that of yaw, i.e., positioning of the heated surface parallel to the rays of the object beam. This alignment procedure involves observing the expanded image of the heated surface and the adjacent illuminated field. In the presence of a positive yaw misalignment, the rays of the object beam impinge upon the surface producing a vertical bright streak in the image of the illuminated field. The opposite bound of the aligned position is indicated by the presence of a diffraction pattern produced by the upstream edge of the heated surface. As the heated surface is rotated into a negative yaw misalignment, this diffraction pattern is observed by the appearance of a broad diffraction fringe departing from the image of the heated surface. As the bright streak approaches the heated surface image its brightness decreases. This problem may be resolved by incorporating a second smaller expansion lens in the path

of the expanding beam. Observing the image of the bright streak in this doubly expanded image and the diffraction pattern in the singly expanded image is most effective. The optics may be configured such that both images are in close proximity to allow simultaneous viewing. By carefully rotating the assembly through the aligned position in both directions, one may alternately observe the development and rate of propagation of the bounding indicators. Through several such oscillations, one may approximate a common origin of the two indicators. Positioning the heated plate such that both indicators are at the common origin produces the desired yaw alignment.

6.4 Experimental Procedure:

In this section, the steps involved in performing an experimental run are described. Most of these have been discussed in depth in the body of this thesis. Therefore, only the general steps and their sequential order are indicated.

- 1) Allow the laser to warm up for one hour.
- 2) Prepare and equalize the temperature of the chemical processing solutions used to develop the hologram.
- 3) Activate pneumatic optical table supports.
- 4) Adjust the reference and object beam irradiance ratio at the hologram via the Variable Attenuator/Beamsplitter.

- 5) Determine and set the exposure time using the digital shutter controller.
- 6) Create a completely dark environment in the laboratory. This includes turning off the laser and covering any LED on the equipment in the laboratory.
- 7) Place holographic plate in the plate holder.
- 8) Take the initial exposure. To do this, the laser is turned on, the plate is exposed and the laser is turned off again.
- 9) Apply power to the experimental apparatus.
- 10) Verify that the test section has reached steady state. This was done by sampling the temperatures of the isothermal surface at 30 minute intervals. When two sequential sample sets indicated the same temperatures, the surface was assumed to have reached steady state.
- 11) Take the second exposure. This step is performed similar to step 8.
- 12) Remove the hologram from the interferometer and process in the chemical solutions as outlined earlier. When processing the hologram, avoid allowing direct light from the luminescent dark room timer from impinging on the hologram.
- 13) After the hologram has dried, replace it in the interferometer in the position in which it was exposed, and illuminate with the reference beam only.
- 14) Record the interferogram from the imaging screen using the image processing tool.

7. Results

7.1 Holograms:

Figures 7.1 and 7.2 display the interferogram of the boundary layer off the vertical isothermal surface and the image of the scale standard, respectively. The interferogram in Fig. 7.1 was produced from an amplitude hologram using AGFA 8E75HD plates. This hologram was produced using the double exposure technique with an irradiance ratio of 3.08 and an exposure time of 0.04 seconds in both exposures. There are several visual conclusions that draw from the interferogram in Fig. 7.1. Holographic has indeed provided a very thorough qualitative description of the boundary layer. The pin in the lower region of the interferogram marks the vertical position of the leading edge of the isothermal surface. Note that the leading edge of the boundary precedes that of the surface due to substrate conduction. Thus deviation of the experimental determination of Nusselt number from the exact solution is expected. This will be discussed in a following section. Finally, notice the behavior of the fringes following the trailing edge of the isothermal surface. Note that these fringes in this area follow an approximate constant radius centered on the surface trailing edge. This observation supports the

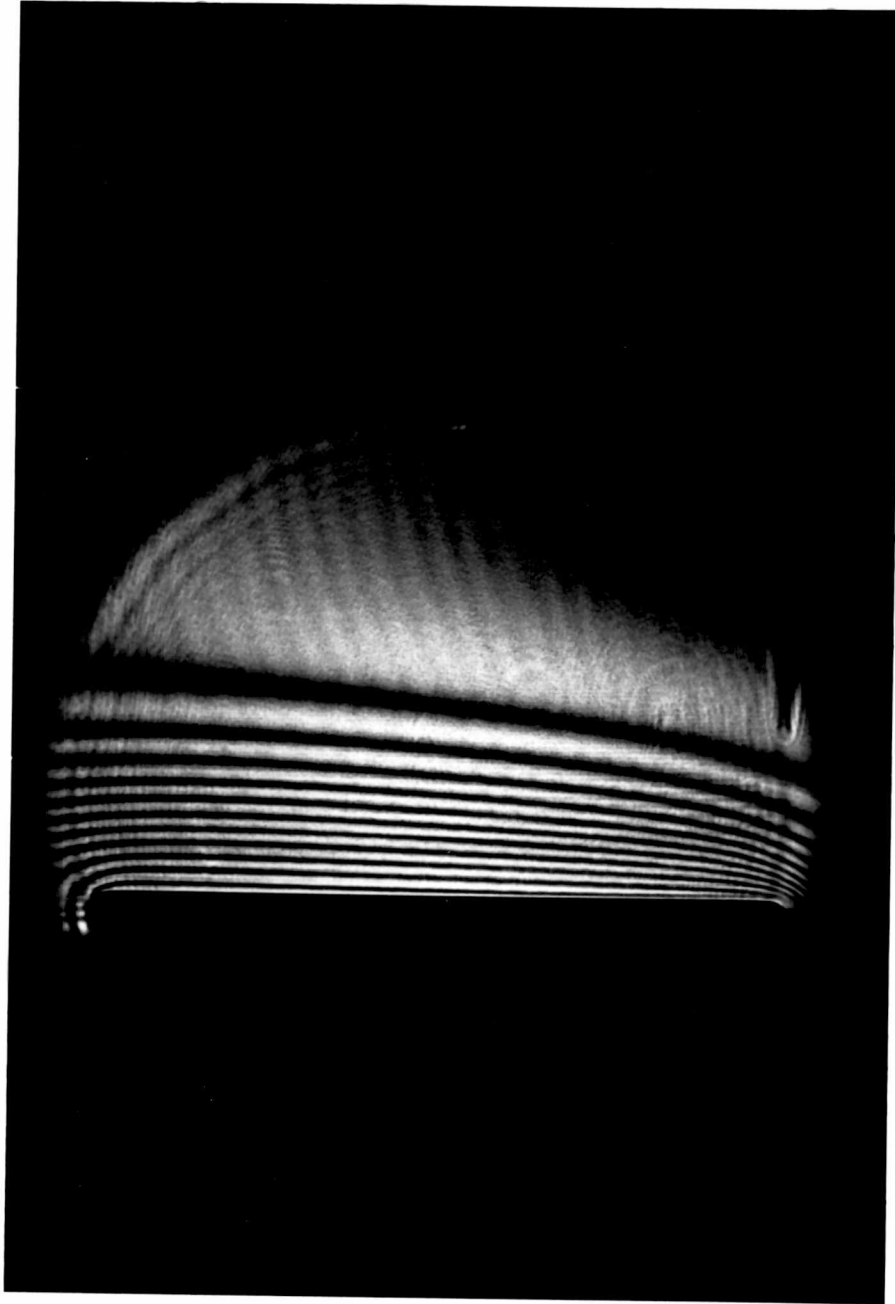


Fig. 7.1 Interferogram of natural convection boundary layer along a vertical isothermal surface.

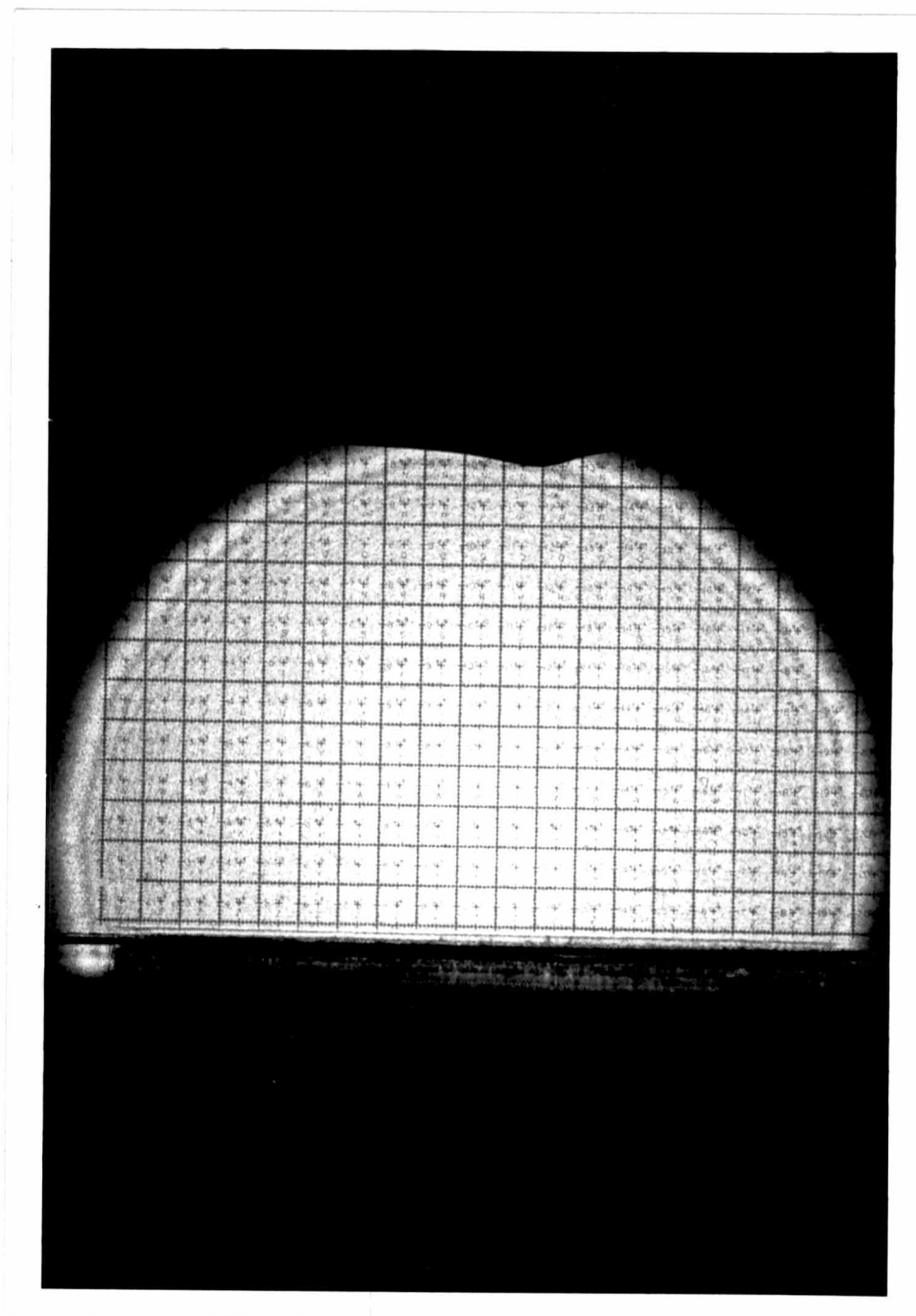


Fig. 7.2 Scale standard to determine scale factor.

assumed shape of the end effects. Although this observation is not directly that of the end effects, the fringe behavior at the end effects is expected to decay similarly.

7.2 Data Reduction:

The initial step in the data reduction process is to determine the temperature field adjacent to the heated surface. The methods used to define this temperature field using interferometric data follows accepted methods which are discussed by Hauf and Grigull (1970), and Jessee (1983). For completeness, these steps will be discussed in a following section. The determination of heated wall temperature and Nusselt number are based on the adjacent temperature field. The standard method of analysis is to fit a curve to the temperature field in close proximity to the heated surface. With this curve fit established, the wall temperature is extrapolated. The determination of the Nusselt number is based on the extrapolated wall temperature and the slope of the temperature curve fit at the wall position. Significant differences exist between the traditionally accepted methods and that used in this thesis for the determination of the wall temperature and Nusselt number. These methods as well as the justification for the method used in this thesis will be

discussed in the following sections. The data reduction and uncertainty analysis procedure was programmed using QuickBASIC 4.5. A listing of this program is contained in Appendix E.

7.2.1 Analysis of the Temperature Field:

The fringe center coordinates are needed in order to interferometrically define the temperature field adjacent to the heated surface. These coordinates are measured from the interferogram. In this thesis, the interferograms were digitized via the imaging hardware and software. Using the imaging tool, the fringe positions were measured from the digitized interferogram from the computer monitor. These measurements have dimensions of column pixels or row pixels. Other methods of making these measurements first involve photographing the interferogram. The measurements may then be taken from the resulting negatives or prints using a microscope and transnational stage. Regardless of the method used to extract these measurements from the interferogram, there exist some scale factor relating the image of the test cell, the interferogram, and the actual test cell.

The normal method of determining the scale factor for a particular experimental setup, consists of placing a scale standard in a selected plane of the test section and

comparing actual lengths of the standard to corresponding lengths in the recorded interferogram. Recall from the discussion of refraction error, that the imaging screen is focused through the imaging lens on the mid-plane of the test section. Therefore, the scale standard is placed in the mid-plane of the test section for accurate determination of the scale factor. The scale standard used in this thesis is a Field Finder made by Teledyne Gurley. The Field Finder is a microscope slide on which a uniform square grid of one millimeter spacing is etched. Each grid of the Field Finder is further divided into quadrants with identifying numbers. The fine resolution of these identifying numbers may be used as an aid in the focusing of the projection screen.

For the determination of the scale factor, the grid lines of the Field Finder is aligned with the x and y-axes. An image of the field finder is digitized and its grid lines were aligned with the pixel columns and rows using the imaging software's rotation option when necessary. A scale factor may then be determined as the number of grid lines in millimeters divided by the corresponding pixel length. Thus, the scale factor has units of millimeters per pixel, column or row. Note that because the pixels are rectangular rather than square, independent scale factors must be determined corresponding

to the column and row pixels. If interferograms are recorded without altering the interferometer or data collection system, the same scale factors apply to the interferogram.

With the scale factors for a given experiment established, the fringe centers may be measured in pixels and subsequently converted to a standard length scale of millimeters.

Recall that by proper experimental apparatus design, an approximate two dimensional phenomenon may be produced. The basic motivation for producing a two dimensional phenomenon is to eliminate the z dependence. Thus, the refractive index and fringe shift fields are only a function of x and y as given in Eq. (2.17). Notice that the y dependence may be additionally eliminated by evaluating the interferogram along a given y elevation. The two dimensional field may then be defined by performing the interferometric analysis at discrete y locations. This is the analysis scheme used in this thesis.

The determination of the temperature field adjacent to the heated surface is demonstrated in Fig. 7.3. Figure 7.3 corresponds to the analysis performed at the mid-height of the heated surface using dark fringes. Information pertaining to a single fringe is read from the

S	PC	x (mm)	b (mm)	b _{AVE.}	$\Delta S1$	$\Delta S2$	SIDEAL	T (°C)
10.50	140.00	8.400		0.345	0.084	0.200	10.384	62.34
			0.330					
9.50	145.50	8.730		0.345	0.084	0.179	9.405	57.72
			0.360					
8.50	151.50	9.090		0.345	0.084	0.158	8.427	53.23
			0.330					
7.50	157.00	9.420		0.345	0.084	0.136	7.448	48.87
			0.360					
6.50	163.00	9.780		0.360	0.078	0.115	6.462	44.58
			0.360					
5.50	169.00	10.140		0.360	0.078	0.094	5.483	40.44
			0.360					
4.50	175.00	10.500		0.405	0.061	0.074	4.487	36.34
			0.450					
3.50	182.50	10.950		0.480	0.044	0.054	3.490	32.33
			0.510					
2.50	191.00	11.460		0.615	0.027	0.035	2.491	28.42
			0.720					
1.50	203.00	12.180		0.930	0.012	0.017	1.495	24.62
			1.140					
0.50	222.00	13.320		0.000	0.000	0.000	0.500	20.92
			0.000					

Fig. 7.3 Interferometric data reduction technique.
Results shown are at $y/h = 0.5$.

corresponding row of Fig. 7.3. Uncorrected fringe shift values are assigned to the fringes as specified in Section 2.3, in a direction from the ambient towards the surface. These values are entered in column S of Fig. 7.3. The fringe with an S value of 0.5 is at the outer edge of the boundary layer, while that with a value of 10.5 is the dark fringe adjacent to the surface. The measured fringe center positions in terms of column pixels are listed in column PC. Note that these values are not relative to a surface position of zero. The fringe center positions are multiplied by the scale factor to be converted to millimeters. The fringe center positions in millimeters is entered in column x of Fig. 7.3. Recall that the correction to the fringe shift to account for refraction error requires the local value of fringe spacing. To obtain the fringe spacing in millimeters, adjacent fringe positions are subtracted and entered in column b of Fig. 7.3. Associated with each measurement of the fringe center is some finite error. This local error subsequently influences the local value of fringe spacing. To dampen out inconsistencies in the measurement error between fringes, the fringe spacing values bounding a given fringe are averaged to determine an average fringe spacing for that fringe. These values correspond to column b_{AVE} of Fig. 7.3. However, the average fringe

spacing for the fringe adjacent to the heated surface may not be determined in this manner. Rather, its value of average fringe spacing is determined as the average of those of the three neighboring fringes. With the average fringe spacing values established, these values are used with to Eq. (3.5) to determine the local correction to the uncorrected fringe shift for refraction error. These values are entered in the ΔS_1 column of Fig. 7.3. The column ΔS_2 of Fig. 7.3 contains the local values of the fringe shift correction for end effect error. Recall that the equation to determine this correction, Eq. (3.8), requires the fringe temperature and the fringe position relative to the wall. The fringe position relative to the wall is found simply by subtracting the wall location in millimeters from the fringe position in column x . The fringe temperatures that are used to determine the end effect corrections are those found by applying the uncorrected fringe shift values to Eq. (2.23). These values are then used with Eq. (3.8) in order to calculate the end effect correction. Once the refraction and end effect correction terms are determined, they are added to and subtracted from, respectively, the uncorrected fringe shift value to determine the ideal fringe shift. The resulting values of ideal fringe shifts are listed in column $SIDEAL$ of Fig. 7.3. These corrected fringe shift

values are in turn inserted in Eq. (2.23) to determine the corrected temperature field adjacent to the plate as listed in column T of Fig. 7.3.

In this thesis, bright as well as dark fringes are used to define the temperature field adjacent to the heated surface. Since information concerning both fringe types are available in any interferogram, the additional effort required to incorporate both types into the analysis is minimal. Moreover, analysis incorporating both bright and dark fringes provides twice the resolution in the definition of the temperature field adjacent to the heated surface than would result from the normal practice of using one fringe type. Since the fringe spacing variable (b) required for the refraction error correction term is the distance between like fringes, it is convenient to analyze bright and dark fringes separately. Interferometric analysis of the temperature field using bright fringes follows the same procedure as outlined in Fig. 7.3 with the exception of assignment of the uncorrected fringe shift values, S.

Once the temperature field with respect to bright and dark fringes is determined, the results are combined as shown in Fig. 7.4. Notice that Fig. 7.4 contains the major data from Fig. 7.3, and that the fringe position (x) has been shifted relative to the heated surface.

S	x (mm)	$\Delta S1$	$\Delta S2$	SIDEAL	T (°C)	w_T (°C)
11.00	0.030	0.149	0.176	10.972	65.17	0.39
10.50	0.180	0.174	0.168	10.507	62.92	0.37
10.00	0.300	0.138	0.159	9.979	60.41	0.35
9.50	0.480	0.138	0.150	9.488	58.11	0.33
9.00	0.570	0.154	0.141	9.013	55.91	0.32
8.50	0.720	0.174	0.132	8.542	53.76	0.30
8.00	0.810	0.138	0.124	8.014	51.38	0.29
7.50	0.960	0.138	0.115	7.523	49.20	0.27
7.00	1.110	0.124	0.106	7.018	46.98	0.26
6.50	1.260	0.112	0.097	6.514	44.81	0.24
6.00	1.380	0.138	0.089	6.049	42.82	0.23
5.50	1.560	0.112	0.080	5.532	40.64	0.22
5.00	1.650	0.092	0.072	5.021	38.52	0.21
4.50	1.860	0.078	0.063	4.515	36.45	0.19
4.00	2.040	0.066	0.054	4.012	34.41	0.18
3.50	2.280	0.057	0.046	3.511	32.42	0.17
3.00	2.430	0.050	0.038	3.012	30.45	0.16
2.50	2.700	0.039	0.030	2.509	28.49	0.15
2.00	2.940	0.024	0.022	2.002	26.55	0.14
1.50	3.300	0.015	0.015	1.501	24.65	0.13
1.00	3.720	0.000	0.000	1.000	22.77	0.12
0.50	4.320	0.000	0.000	0.500	20.92	0.11

Fig. 7.4 Combined bright and dark fringe interferometric data and results for analysis at $y/h = 0.5$.

Moreover, the uncertainties in the interferometrically determined temperature, w_T , are also presented in the right hand column of Fig. 7.4. The method used to obtain the uncertainties is discussed in Appendix C. It is interesting to note from Fig. 7.4 that the general trend in the fringe shift correction terms is as expected. Recall from Eq. (3.5), that for a given experiment, the refraction error correction term is a constant divided by the square of the fringe spacing. Since the fringe spacing decreases as the heated surface is approached, the magnitude of this correction term is expected to increase accordingly. The expected trend in the end effect correction terms is most evident from Fig. 3.5. The end effect correction term is proportional to the length over which the corresponding rays are influenced by the end effect. From Fig. 3.5, it is apparent that this length of influence of the end effect increases as the heated surface is approached. Thus, the end effect correction term is expected to increase as the surface is approached. More importantly, the increased resolution in the temperature field definition associated with the use of both bright and dark fringes is evident when one compares Figs. 7.3 and 7.4.

Figure 7.5 displays the behavior of the ideal fringe shift, $SIDEAL$, with fringe position relative to the heated

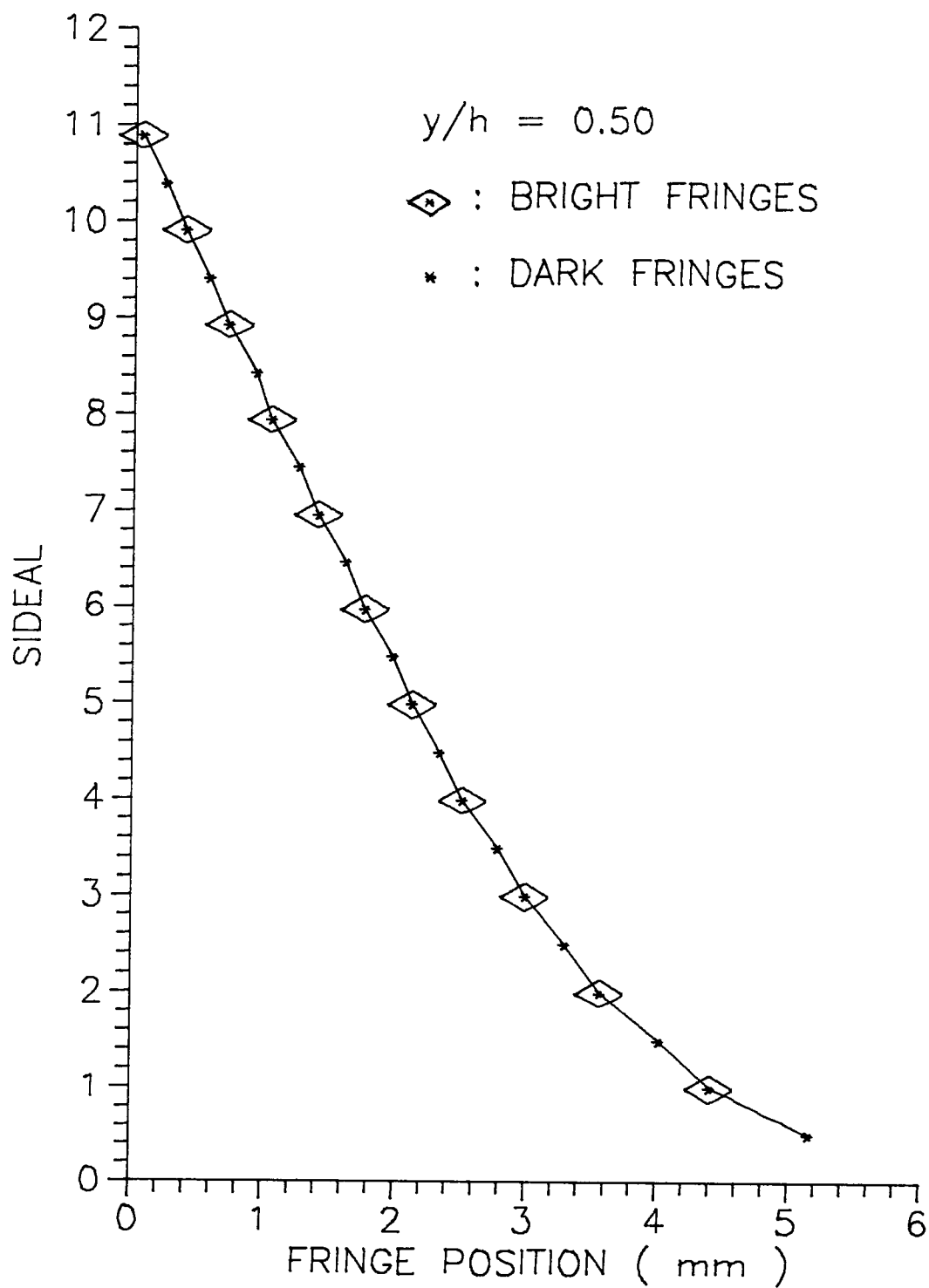


Fig. 7.5 Fringe shift variation through the boundary layer.

surface. The bright and dark fringe contributions to the curve are as indicated in the legend of Fig. 7.5. Notice that the continuity of the curve is not sacrificed by combining the results from bright and dark fringe analysis. Although this preservation of continuity should be expected, it is mentioned to emphasize the compatibility of the results of analysis based on the two fringe types. The general shape of the curve in Fig. 7.5, which is for the heated surface mid-height, is characteristic of all elevations along the surface. The most notable characteristic of the curve in Fig. 7.5 is the behavior of the slope with fringe position. It can be seen that the slope is approximately constant up to $x = 1.2$ mm, and thereafter gradually decays.

Recalling the equation of ideal interferometry in terms of temperature, Eq. (1.23), one would expect the general shape of the curve of Fig. 7.5 to be similar to that of temperature versus fringe position. Figure 7.6, which presents the temperature profile above the ambient value, indeed displays this similarity. Although bright and dark fringe analysis results are not distinguished in Fig. 7.6, the combined results are used. Three temperature profiles for elevations indicated in the legend are shown. The equation of ideal interferometry in terms of temperature, Eq. (2.23), specifies that the ideal fringe shift is only

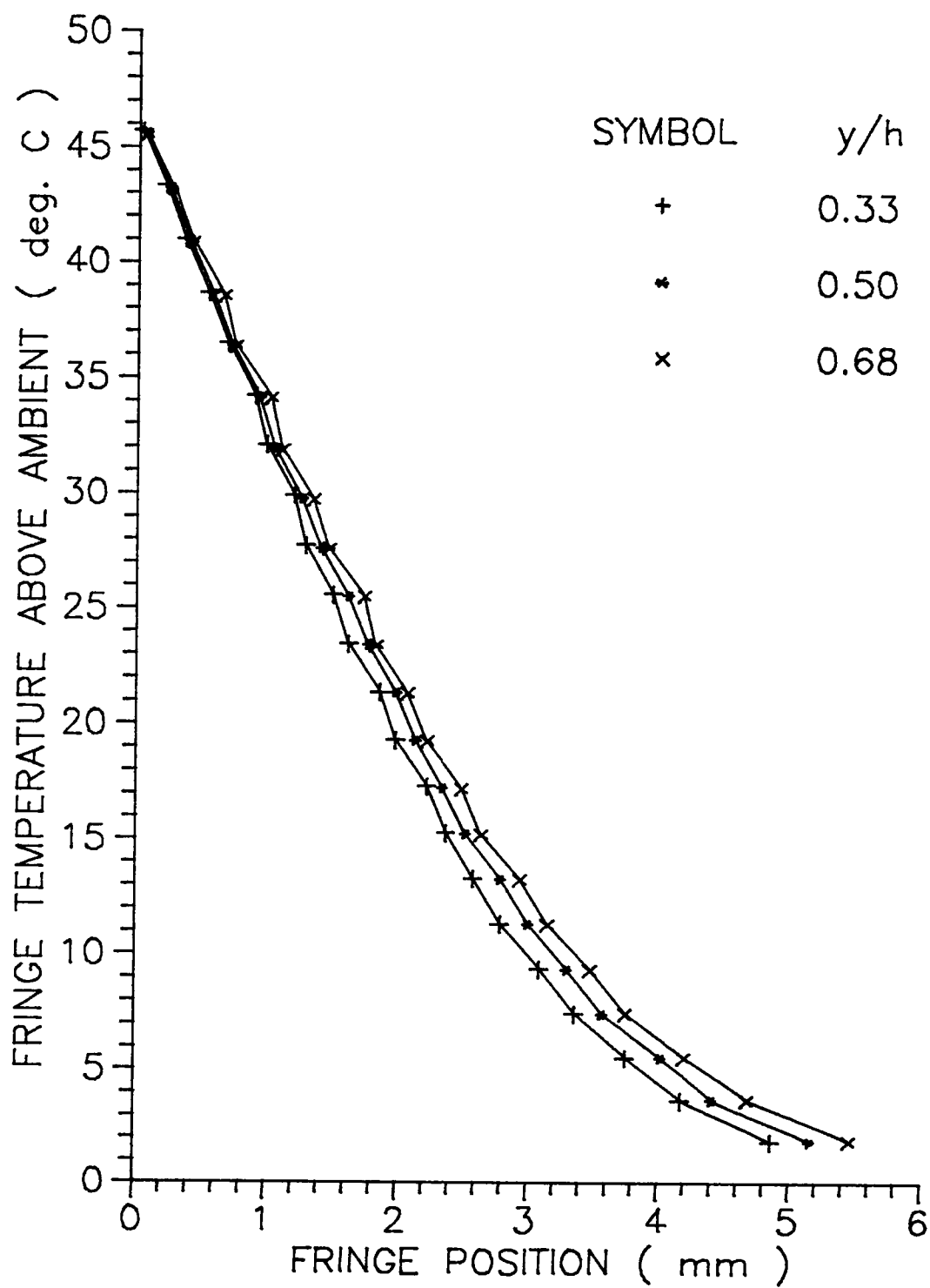


Fig. 7.6 Boundary layer temperature profiles.

one of many variables upon which temperature is dependent. Each of these independent variables has some degree of associated error. Thus, in general, the error associated with the ideal fringe shift is only a fraction of that of the temperature. For this reason, the temperature profile displayed in Fig. 7.6 is not as smooth as the ideal fringe shift profile of Fig. 7.5. Notice that Fig. 7.6 demonstrates the approximate isothermal nature of fringes as discussed in Section 2.4. This is evident from the fact that the three data points corresponding to the same fringe approximately have the same temperature.

The ultimate goal of the present analysis is to determine the surface temperature and Nusselt number. The slopes of the temperature-fringe position curves of Fig. 7.6 has significant influence on the determination of these parameters. It can be seen in Fig. 7.6 that the slope of each curve is approximately constant up to approximately $x=1$ mm, and thereafter decays as the boundary layer is further penetrated. The total heat transfer from the surface is by conduction at $x=0$. Recalling the form of Fourier's Law, the slope of curves in Fig. 7.6 at $x=0$ is an indication of the magnitude of the conduction heat transfer. With this in mind, it is appropriate to conclude that close to the wall where the slope is approximately constant is the region where

convection heat transfer is negligible. Therefore, it appears that conduction is the major contributor to the total heat transfer up to approximately one millimeter from the surface. As the boundary layer is further penetrated, the significance of convection heat transfer increases. This is reflected in the curves of Fig. 7.6 by the gradual decay of the slope as the boundary layer is further penetrated. Although the results at only three elevations are shown, the same general conclusions may be made for other elevations along the surface.

Figure 7.7 shows the behavior of the ideal fringe shift gradient as the boundary layer is penetrated. This gradient was calculated for bright and dark fringes independently and the results were subsequently combined. Figure 7.7 displays the same results concerning the compatibility of dark and bright fringe analysis, and the local ideal fringe shift gradient as discussed in relation to Fig. 7.5. One may initially question the compatibility of the two fringe type results due to the non-continuous nature of the curve in Fig. 7.7. However, recognizing the amplification of error associated with differentiation, the overall trend is acceptable.

Figures 7.8, 7.9, and 7.10 show the variation of the temperature gradient as a function of position within the boundary layer at three elevations. In these figures, the

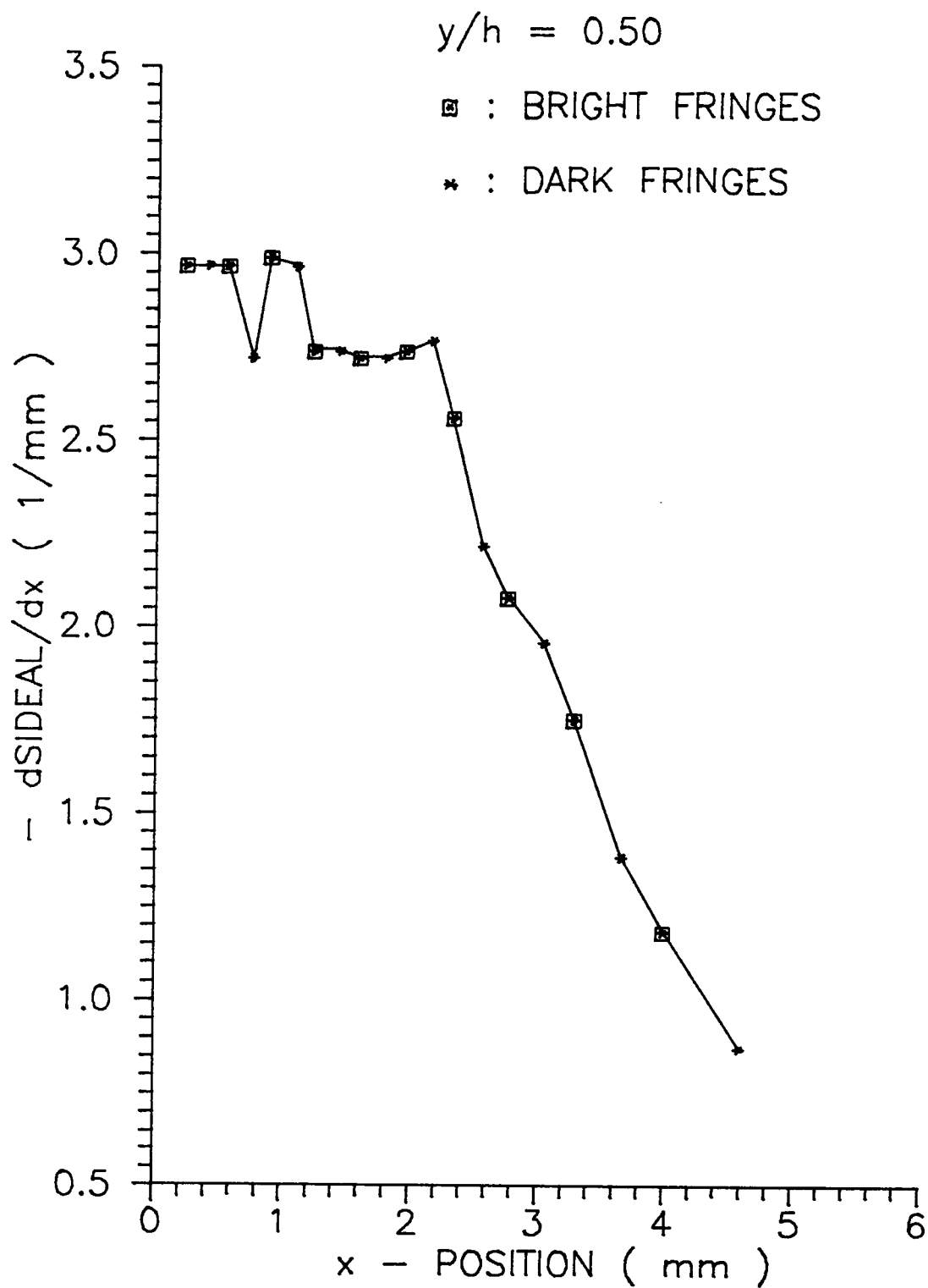


Fig. 7.7 Fringe shift gradient distribution within the boundary layer.

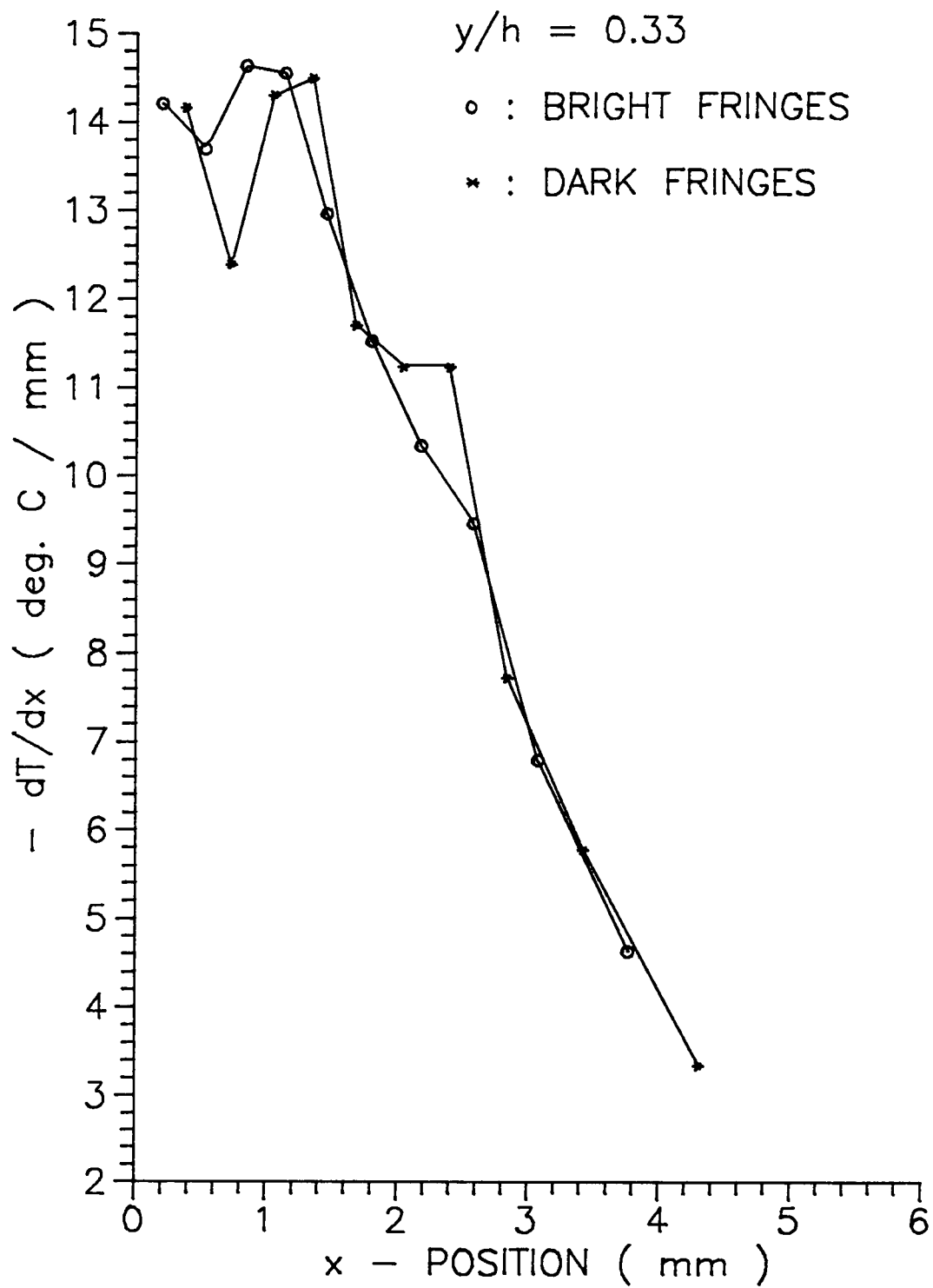


Fig. 7.8 Temperature gradient distribution within the boundary layer at $y/h = 0.33$.

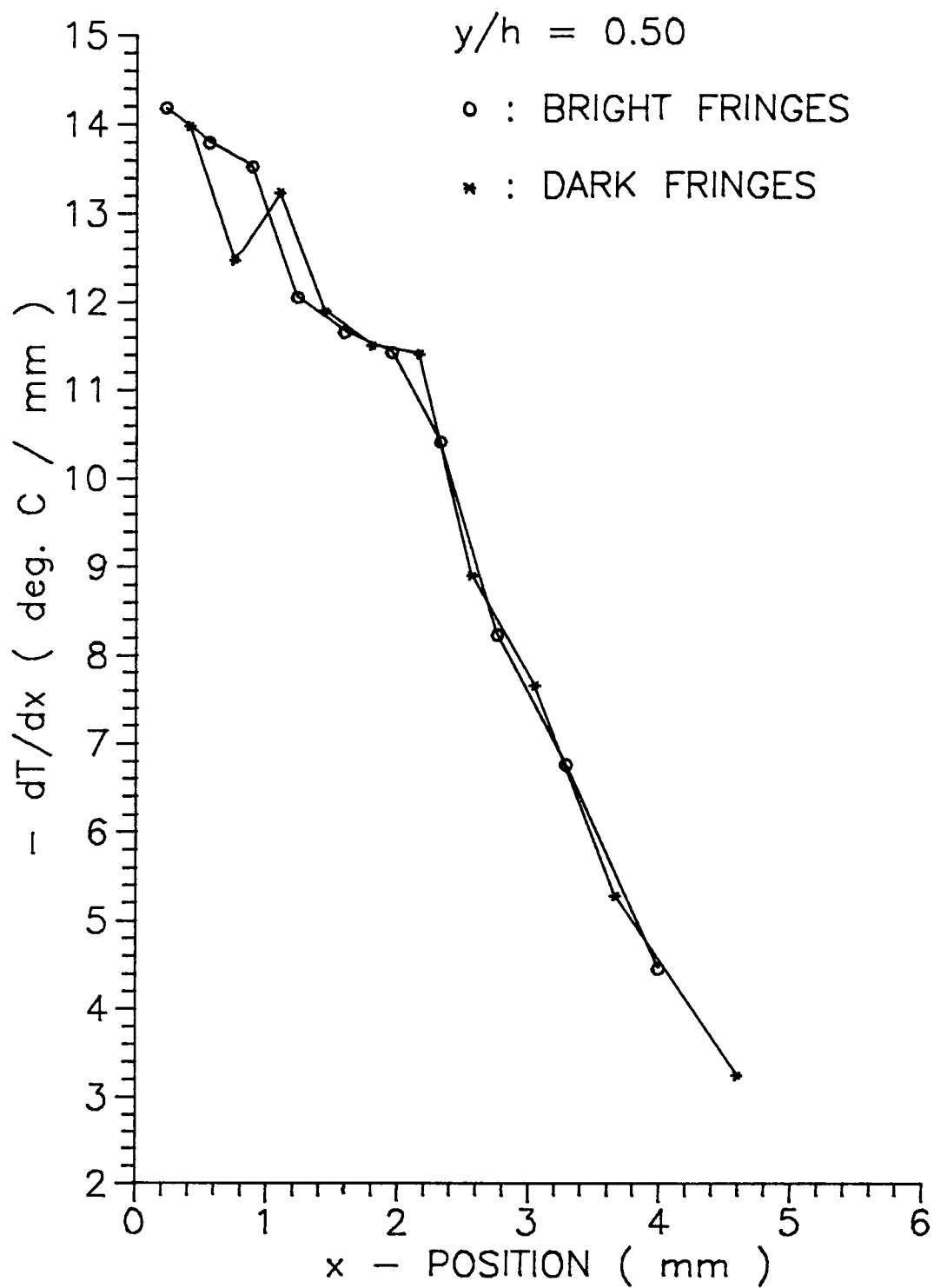


Fig. 7.9 Temperature gradient distribution within the boundary layer at $y/h = 0.50$.

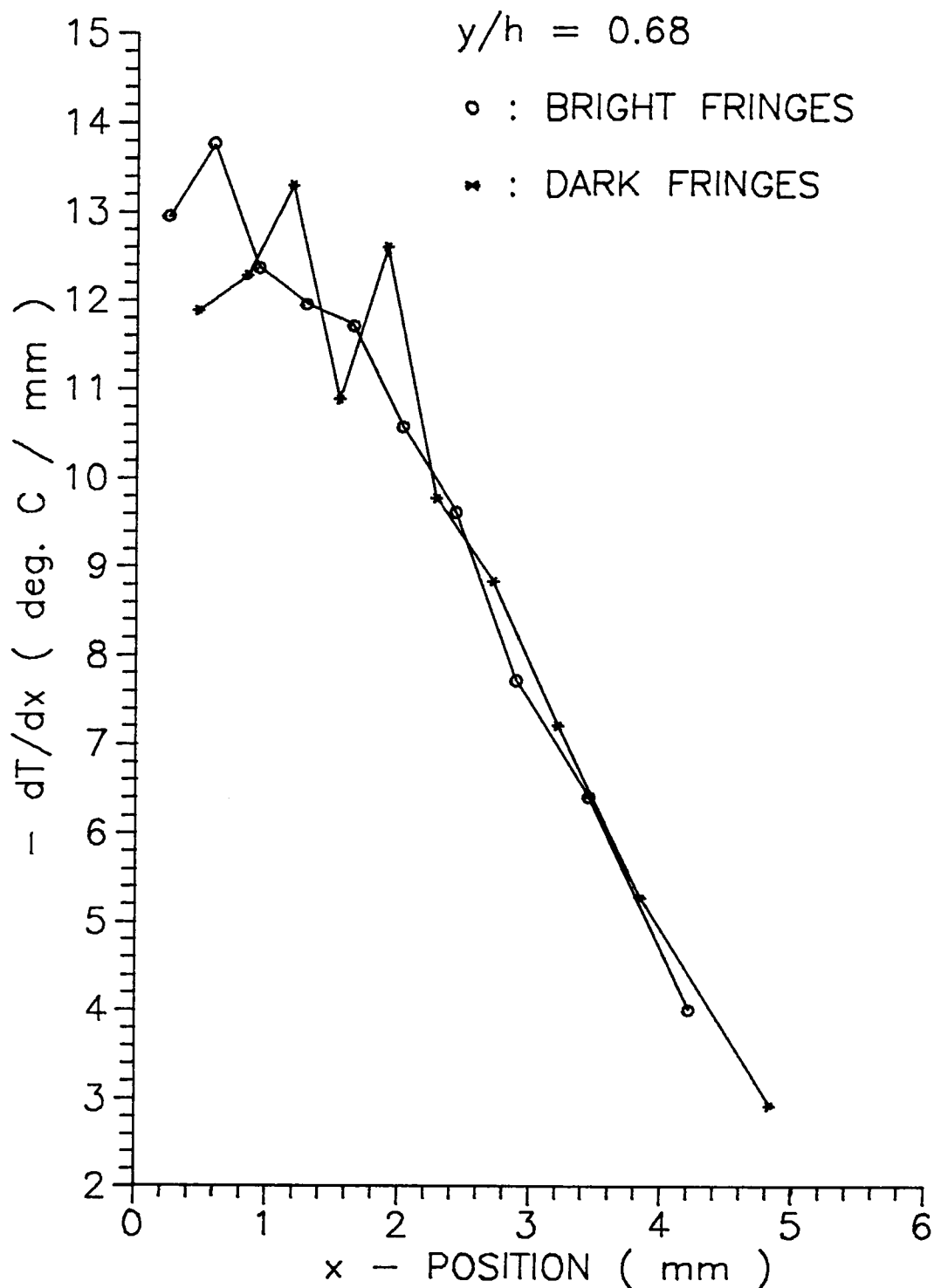


Fig. 7.10 Temperature gradient distribution within the boundary layer at $y/h = 0.68$.

curves corresponding to bright and dark fringe analysis are superimposed. The data points resulting from bright and dark fringe analysis are marked as indicated in the legend. As mentioned earlier, the amplification of the error associated with differentiation is expected and is evident in these figures. Moreover, considering the relative magnitudes of the uncertainties in the temperature and ideal fringe shift, the data scatter of the temperature gradient figures is expected to be greater than that displayed in Fig. 7.7. However, visually averaging this scatter in Figs. 7.8, 7.9, and 7.10 an average temperature gradient profile may be visualized. Visualizing such averaged curves, the conclusions drawn from Fig. 7.6 are supported. The figures of temperature gradient indicate that there is a region in close proximity to the surface over which the temperature gradient is approximately constant. As previously discussed, this constant temperature gradient is indicative of the conduction dominate regime of the boundary layer. The increasing significance of convection heat transfer as the boundary layer is penetrated beyond the conduction dominated regime is indicated by the corresponding temperature gradient decay.

7.2.2 Determination of Wall Temperature and Nusselt Number:

In the beginning of this section, the standard method of interferometrically determining wall temperature and Nusselt number were outlined. Additionally, it was mentioned that there exist differences between the traditionally accepted analysis methods and that used in this thesis. The similarities, differences and the justification for the applied method of analysis will now be discussed.

With the temperature field adjacent to the surface defined, a portion of this data is then used to produce a temperature profile function by applying the method of least squares. With such a function, the wall temperature may be determined by extrapolating to the surface position. From the definition of Nusselt number,

$$Nu_y = \left[\frac{y}{T_{wall} - T_{\infty}} \right] \left[\frac{dT}{dx} \right]_{wall} \quad (7.1)$$

it is evident that the temperature gradient at the surface is needed for prediction of the Nusselt number. This parameter is extracted from the least squares curve fit results by determining the value of its derivative with respect to position at the surface. The temperature difference in the denominator of Eq. (7.1) is that between the surface and ambient. The surface temperature used to

evaluate this difference is either determined interferometrically or measured by thermocouple. This procedure for determination of wall temperature and Nusselt number is generally applied to interferometric analysis including that in this thesis.

In traditional interferometric analysis procedure, researchers use only bright or dark fringes to define the temperature field adjacent to the surface. Although dark fringes are most predominately used, the choice between the two fringe types is totally random. The resulting temperature field would thus be defined as was presented in Fig. 7.3. With this temperature field established, the least squares method is used to generate a second order polynomial for the temperature profile. The notable features of traditional interferometric analysis are the use of a single fringe type and the use of a second order temperature profile. The broad adherence to this procedure can be observed throughout the literature (Jessee, 1983, Chadwick, 1989, and McKinley, 1985).

The information extracted from the curve fit of the temperature field data is at the surface location. In the case of applying holographic interferometry to any general problem, the global behavior of the temperature field profile may not be readily apparent. Thus, it is desirable to generalize the interferometric analysis

procedure such that it may be confidently applied to investigate more complex phenomena. What is known about all thermal phenomenon, is that they must adhere to established physical laws of heat transfer. This assumes that such established laws exist for the process being investigated. Such generality is achieved by basing the analysis model on the physics of the phenomenon under investigation.

The physics of the present investigation is specified by the energy equation,

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (7.2)$$

In Eq. (7.2), u and v are the fluid velocities in the x and y directions, respectively, and α is the thermal diffusivity. Since the fluid behavior at the surface is of interest, it is appropriate to investigate the energy equation at that position. Recognizing that the surface is impermeable, and observing the no slip condition requires that $u(x=0) = v(x=0) = 0$. Thus, the physics of the phenomenon, revealed via the energy equation, requires that the second derivative of temperature with respect to x be zero at the surface. Now, assume a second order temperature profile, and differentiate it with respect to position twice. Combining the results of this differentiation with that provided by the energy equation

requires the coefficient of the second order term to be zero. Thus, in order to adhere to the physical laws governing the process, the temperature profile adjacent to the surface should be modeled as linear.

In the preceding discussion, the consequences of the energy equation were investigated at the surface and used to predict the appropriate temperature profile model. However, the points used to produce the temperature profile function are not at the surface. Rather, these data sets are some distance into the thermal boundary layer. Consequently, the fluid at these data points will have some finite velocity in the x and y direction. Thus, the second derivative of temperature with respect to temperature takes on some finite value. Recalling the implications of combining the results of the energy equation with the twice differentiated second order temperature profile, one realizes that the second order term assumes a finite magnitude as the boundary layer is penetrated. Moreover, for the case of a thermal boundary layer, to a certain extent, the fluid velocities increase as the boundary layer is further penetrated. Accordingly, the temperature profile becomes increasingly second order in nature as the boundary layer is further penetrated. This phenomenon persists up to the point where the velocity in the y direction reaches its maximum, and thereafter

begins to decay to zero. Note that this reflects the relative significance of conduction and convection heat transfer along the boundary layer thickness.

In traditional interferometric analysis procedure valuable data is disregarded due to the use of a single fringe type data. Therefore, the boundary layer is unnecessarily penetrated, more than that needed otherwise, in order to have sufficient data to generate a curve. Due to this excessive penetration, the range of data points used to form the curve display obvious second order characteristics. This is the main reason traditional interferometric analysis procedure invokes a second order temperature profile to extrapolate the temperature and temperature gradient at the surface. The use of a second order curve fit in itself exacerbates the situation by requiring an absolute minimum of three data sets to support such a curve fit. Undoubtedly, further penetration of the boundary layer will be required in order to increase the accuracy of the curve fit.

Although at the point of interest, the temperature profile is linear in nature, traditional interferometric analysis applies a second order model. The basic drive requiring the use of a second order temperature profile is lack of sufficient data density. This reveals the motivation for using both bright and dark fringe analysis

to extract the full amount of data from the interferogram. As previously indicated, interferometric analysis using both fringe types produces a doubling of the data density. Thus, for a given distance into the boundary layer, twice as many data points are produced. Specifically, this corresponds to twice as many data sets in the near vicinity of the surface where convection heat transfer is relatively insignificant.

By using bright and dark fringe analysis, sufficient data is available to produce a good curve fit in a region of the boundary layer in which conduction is the predominant mode of heat transfer. This was visually demonstrated in Fig. 7.6. Thus, a first order temperature profile is appropriate. The use of a first order temperature profile combined with the increased fringe density reduces the required penetration of the boundary layer, making the pure conduction model even more appropriate. For this reason, interferometric analysis in this thesis is based on both bright and dark fringe data and the use of a linear temperature profile.

By investigating the physical laws governing the phenomenon under investigation, a first order temperature profile model was selected. With this established, a range of data sets, for the generation of the curve fit, must be specified. The lower and upper bounds of this

limit are set statistically and by the nature of the phenomenon being investigated, respectively.

Although a linear function may be fit exactly to any two data points, three data points defines the statistical minimum to produce such a fit. Because each data point contains some degree of uncertainty, increasing the number of data points used to define the linear temperature profile model, correspondingly increases the accuracy with which it reflects the global behavior of nominally linear data. Thus, using the statistical minimum number of data sets required to produce a given fit is not expected to result in a good curve fit. For these reasons, four data sets was chosen as the lower bound of the range with which to produce the temperature profile model. It is important to note that with the combined interferometric data this lower limit of four data sets corresponds to two fringes in traditional interferometry.

Since a first order temperature profile model was used in the analysis, it is desirable to sequester the data set range, to a portion of the boundary layer in which the actual temperature profile is nearly linear. The actual limit at which the temperature profile becomes nonlinear is indicated by the interferometrically determined temperature field as displayed in Fig. 7.6. From Fig. 7.6 it is evident that this upper limit of the data range is

at approximately the sixth fringe off the surface. Note that this maximum extent of the data range used to fit the first order curve is equivalent to the minimum of three fringes used in traditional interferometric analysis.

With the allowable data range defined, the only remaining decision to be addressed is the procedure for the selection of the number of data points, within the specified range, which will be used to generate the curve fit. Recall that it is the desire in this thesis to standardize general procedures to form the analysis method. Thus generalizing the analysis method development, allows that method to be tested on phenomena with known solutions and subsequently be applied to more significant unknown phenomena. Tables 7.1 and 7.2 are used to demonstrate the selection criterion used to choose the number of data sets with which to model the temperature profile. Table 7.1 shows the percent excess temperature and correlation coefficient for each number of data sets in the range for ten elevations along the surface. Table 7.2 is similar except that the displayed variables are the percent error in the holographic interferometric determination of Nusselt number and the correlation coefficient. In Table 7.1 and 7.2 the bottom row contains the average deviation or error for the three choices of number of data sets used. Thus one concludes

TABLE 7.1: Percent deviation, Dev.(%), in excess surface temperature, $T_{WALL} - T_{AMB}$, as predicted by holographic interferometry from the average thermocouple plate temperature, and correlation coefficient, R, for curve fits to 4, 5, and 6 data sets. The result with the best correlation coefficient is selected.

Number of data sets used				

	4	5	6	

y/h	Dev., R	Dev., R	Dev., R	Statistical selection

0.15	4.35, 0.99385	4.62, 0.99533	4.58, 0.99738	4.58
0.24	3.81, 0.99997	3.75, 0.99989	3.72, 0.99993	3.81
0.33	3.50, 0.99903	3.54, 0.99954	3.21, 0.99852	3.54
0.42	4.04, 0.98882	4.53, 0.99004	4.42, 0.99450	4.42
0.50	4.26, 0.99933	4.31, 0.99965	3.95, 0.99860	4.31
0.59	4.42, 0.99391	4.91, 0.99289	4.28, 0.99333	4.42
0.68	3.63, 0.99525	4.13, 0.99207	3.68, 0.99388	3.63
0.77	4.19, 0.99255	4.28, 0.99655	4.04, 0.99762	4.04
0.86	4.91, 0.99485	4.73, 0.99738	4.87, 0.99842	4.87
0.94	3.57, 0.98881	3.92, 0.99297	3.75, 0.99590	3.75

Average deviation	4.07	4.27	4.05	4.14

TABLE 7.2: Percent error in Nusselt number, % error, as predicted by holographic interferometry from the exact solution, and correlation coefficient (R) for curve fits to 4, 5, and 6 data sets. The result with the best correlation coefficient is selected.

Number of data sets used				

	4	5	6	

y/h	% error, R	% error, R	% error, R	Statistical selection

0.15	16.01, 0.99385	12.16, 0.99533	12.70, 0.99738	12.70
0.24	7.26, 0.99997	8.25, 0.99989	8.62, 0.99993	7.26
0.33	10.02, 0.99903	9.58, 0.99954	12.87, 0.99852	9.58
0.42	14.54, 0.98882	8.46, 0.99004	9.36, 0.99450	9.36
0.50	1.00, 0.99933	0.21, 0.99965	3.77, 0.99860	0.21
0.59	7.47, 0.99391	1.29, 0.99289	6.72, 0.99333	7.47
0.68	7.96, 0.99525	0.99, 0.99207	5.32, 0.99388	7.96
0.77	1.32, 0.99255	0.34, 0.99655	2.57, 0.99762	2.57
0.86	3.78, 0.99485	1.74, 0.99738	2.96, 0.99842	2.96
0.94	4.81, 0.98881	9.88, 0.99297	8.05, 0.99590	8.05

Average error	7.42	5.29	7.30	6.81

that using six and five data sets produces, on the average, the most accurate determination of wall temperature and Nusselt number, respectively. However, although a particular data set may produce the best results for a specific investigation, the results of that data set may not be universally favorable. Moreover, when investigating phenomena in which an exact solution is not available, there is no way of producing tables such as Tables 7.1 and 7.2, and thus, no means of determining what particular number of data sets produces the most accurate results. Hence, one is motivated to base the selection of the number of data points used to produce the temperature profile on a criterion which is blind to the final results. Such a criterion is available via the use of statistical selection based on the best correlation coefficient. Use of a statistical selection scheme eliminates any bias in the selection of a particular set of results. The right hand column of Tables 7.1 and 7.2 displays the individual results and cumulative average ensuing from the application of statistical selection.

In neither case is the resulting average deviation or error as small as that obtained with the most favorable selection of a single number of data sets. However, in neither case is this average as large as the most undesirable selection of a single number of data sets.

The statistical selection of number of data sets used, based on the best correlation coefficient, was used in this thesis.

Figure 7.11 shows the results of the full fringe analysis using the statistically selected set of data points to produce a linear temperature profile. The data from Fig. 7.4 is repeated along with the temperature profile function results. The derivation of all uncertainties listed in Fig. 7.11 is shown in appendix C. As indicated in Fig. 7.11, C1 is the surface temperature and C2 is the temperature gradient as predicted by interferometry. These values were determined using standard least squares method. The correlation coefficient and standard error for the temperature profile curve fit are also listed in Fig. 7.11. The thermocouple measured averaged local and overall averaged surface temperatures and corresponding uncertainties are listed under the heading Wall Temperature in fig. 7.11. The Grashof number is defined as

$$Gr_y = \frac{g\beta(T_{wall} - T_{\infty})x^3}{\gamma^2} \quad (7.3)$$

The Nusselt number results are displayed in Fig. 7.11 under the heading Nusselt Number. The theoretical Nusselt

S	x (mm)	ΔS_1	ΔS_2	SIDEAL	T (°C)	w _T (°C)
11.00	0.030	0.149	0.176	10.972	65.17	0.39
10.50	0.180	0.174	0.168	10.507	62.92	0.37
10.00	0.300	0.138	0.159	9.979	60.41	0.35
9.50	0.480	0.138	0.150	9.488	58.11	0.33
9.00	0.570	0.154	0.141	9.013	55.91	0.32
8.50	0.720	0.174	0.132	8.542	53.76	0.30
8.00	0.810	0.138	0.124	8.014	51.38	0.29
7.50	0.960	0.138	0.115	7.523	49.20	0.27
7.00	1.110	0.124	0.106	7.018	46.98	0.26
6.50	1.260	0.112	0.097	6.514	44.81	0.24
6.00	1.380	0.138	0.089	6.049	42.82	0.23
5.50	1.560	0.112	0.080	5.532	40.64	0.22
5.00	1.650	0.092	0.072	5.021	38.52	0.21
4.50	1.860	0.078	0.063	4.515	36.45	0.19
4.00	2.040	0.066	0.054	4.012	34.41	0.18
3.50	2.280	0.057	0.046	3.511	32.42	0.17
3.00	2.430	0.050	0.038	3.012	30.45	0.16
2.50	2.700	0.039	0.030	2.509	28.49	0.15
2.00	2.940	0.024	0.022	2.002	26.55	0.14
1.50	3.300	0.015	0.015	1.501	24.65	0.13
1.00	3.720	0.000	0.000	1.000	22.77	0.12
0.50	4.320	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

T(x,y= 2.73) = C1 + C2*x

Number of data sets used = 6

C1 = 65.721 +/- 0.485 °C

C2 = -16.688 +/- 1.062 °C/mm

Correlation coefficient = 0.99738

Standard error = 0.311 °C

Wall Temperature:

Average of the upper wall section is .. 63.77 +/- 0.7 °C

Average of the lower wall section is .. 63.60 +/- 0.7 °C

Average of the wall is 63.68 +/- 0.7 °C

Determination by interferometry is 65.72 +/- 0.70 °C

Grashof Number = 99.7 +/- 2.74 %

Nusselt Number:

Theoretical 1.118

Using interferometry wall temperature .. 0.976 +/- 0.064

Using thermocouple wall temperature 1.018 +/- 0.067

Fig. 7.11 Results of interferometric analysis including surface temperature and Nusselt number at y/h = 0.50.

number is determined using the similarity solution

$$Nu_y = \left(\frac{Gr_y}{4} \right)^{\frac{1}{4}} g(Pr) , \quad (7.4)$$

where

$$g(Pr) = \frac{0.75Pr^{0.5}}{(0.609 + 1.221Pr^{0.5} + 1.238Pr)^{0.25}} . \quad (7.5)$$

The Nusselt numbers listed are calculated based on Eq. 7.1. The temperature gradient used is C2 listed in Fig. 7.11, and the surface temperature used to define the temperature difference is as indicated. The full results, similar to Fig. 7.11, for ten elevations along the surface are given in Appendix D.

The experimental Nusselt numbers, based HI wall temperatures and thermocouple measured wall temperature, are shown in Fig. 7.12 along with the theoretical values. In general, the experimentally determined Nusselt numbers are very close to the exact values, and have an average error of 6.81 percent. Note that this average error is within the 8-10 percent suggested by Hauf and Grigull (1970) for interferograms containing less than 30 fringes. The experimental Nusselt numbers based on the thermocouple wall temperature, on average, provide a more accurate prediction. This is to be expected since this method uses

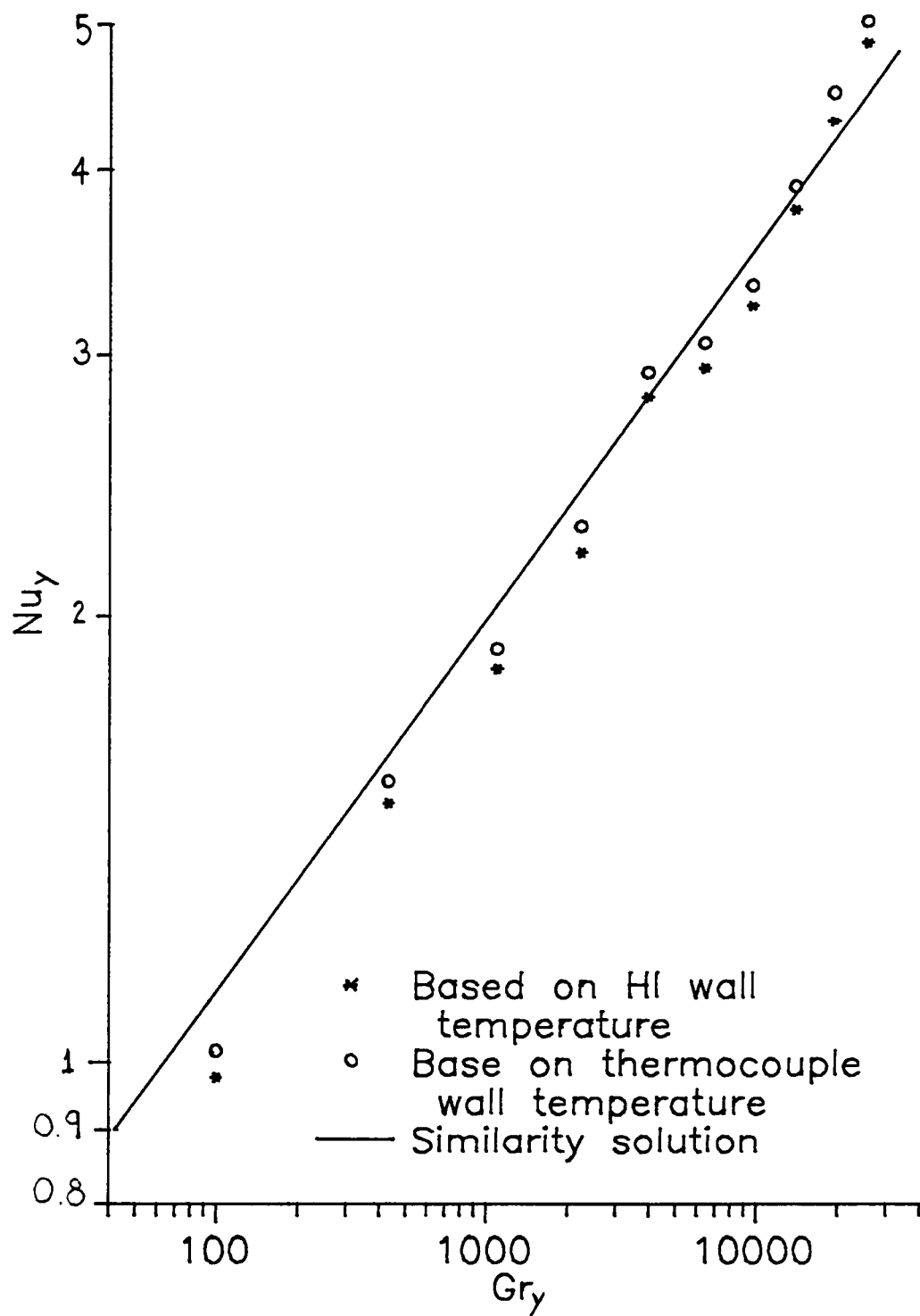


Fig. 7.12 Comparison of interferometrically determined and exact Nusselt numbers.

non-interferometrically defined feed back in the form of a more accurate wall temperature. It should be noted that the experimentally determined Nusselt numbers at six y values are less than the similarity solution. This should be expected due to the substrate conduction into the plexiglass shield around the heater. The result of this substrate conduction is that the actual leading edge of the boundary layer is somewhat below $y=0$ location. This behavior is observed in the interferogram of the boundary layer. Since the heat transfer coefficient decreases along the surface in the direction of buoyant flow, the experimental Nusselt numbers are under estimated by some constant amount. It is worth noting that the over prediction of the Nusselt number at the two elevations furthest downstream of the leading edge should be expected by a visual inspection of the interferogram. The experimental model's surface is not an infinitely tall plate. Specifically, following the trailing edge of the surface is an approximately adiabatic shield. The adiabatic approximation is evident in the interferogram as the isotherms are almost perpendicular to this shield. Thus, near the trailing edge of the surface the fringes begin to curve towards the surface. The effect of this non-ideal behavior of the fringes is an increased fringe density at these elevations. This corresponds to an over

estimation of the local temperature gradient, and subsequently, an over prediction of the Nusselt number as seen in Fig. 7.12. This temperature gradient behavior may be directly observed from the trends in the temperature gradient, C_2 , in the compiled results of Appendix D. The tabulated data shows that the temperature gradient decreases (with one minor exception) as the surface is traversed until the final three fringes.

7.3 Conclusions:

The specific issues of implementing a holographic interferometry research task has been documented. These include, the physics of interference, setup and operation of the interferometer, and data reduction technique. The data reduction technique was demonstrated for the investigation of natural convection along a vertical isothermal surface. The data reduction technique was verified by comparing the experimental results for surface temperature and local Nusselt number to the thermocouple measured surface temperature and exact solution via the similarity solution, respectively. The specific contributions of this thesis to the data reduction procedure are:

Use of both bright and dark fringes to extract the maximum data from the interferogram, thus doubling the information density relative to traditional analysis methods.

Implementation of a first order curve fit to the interferometric results to extrapolate the surface temperature and temperature gradient. This choice is dictated by phenomenon's governing equations rather than the interferometrically determined temperature field results.

Application of a statistical selection procedure to choose the number of data sets to be used to produce the first order curve fit. This generalizes the procedure for obtaining results and prevents bias towards unusually favorable results. The results thus obtained have an average error of 6.81 percent which is within the accuracy bound of 8-10 percent as suggested by Hauf and Grigull (1970) for the present number of fringes.

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APPENDIXES

APPENDIX A

RAY REFRACTION IN A LINEAR STRATIFIED REFRACTIVE INDEX MEDIUM

In this appendix, the shape of the path of a light ray propagating through a linearly stratified nonhomogeneous volume is investigated. For this investigation, assume that the unrefracted rays propagate in the z direction and the refraction index variation is in the x direction. This is the coordinate system which is used for the interferogram presented in this thesis.

Applying Eq. (2.4) to the x and z directions yields

$$\frac{d}{ds} \left[n(x) \frac{dx}{ds} \right] = \frac{dn}{dx}, \quad (\text{A.1})$$

$$\frac{d}{ds} \left[n(x) \frac{dz}{ds} \right] = 0. \quad (\text{A.2})$$

Before attempting to reduce these equations, note the following two relations

$$ds = \sqrt{dz^2 + dx^2} = \left[1 + \left(\frac{dx}{dz} \right)^2 \right]^{\frac{1}{2}} dz, \quad (\text{A.3})$$

$$x'' dx = \frac{d^2 x}{dz^2} dx = \frac{d}{dz} \frac{dx}{dz} dx = \frac{dx}{dz} d \frac{dx}{dz} = x' dx', \quad (\text{A.4})$$

where the prime, ', represents differentiation with respect to z . Combining the above equations results in

$$\frac{x' dx'}{1 + (x')^2} = \frac{dn}{n} = d(\ln n) \quad (\text{A.5})$$

Integrating this relationship with respect to z between upper, 2, and lower, 1, limits and eliminating the natural logs on both sides of the resulting equation provides

$$\frac{1 + (x'_2)^2}{1 + (x'_1)^2} = \left[\frac{n_2}{n_1} \right]^2. \quad (\text{A.6})$$

The length which is bounded by the limits of integration, 1 and 2, is that over which the rays are influenced by the nonhomogeneity. Recall that the coordinate system was positioned such that the rays were parallel to the z -axis prior to their refraction by the nonhomogeneity in the medium. Thus, at the lower limit of integration, 1, the rays are parallel to the z -axis. Therefore, the following boundary condition is applicable.

$$x'(z=1) = x'_1 = 0 \quad (\text{A.7})$$

This boundary condition may be applied to obtain

$$1 + (x'_2)^2 = \left[\frac{n_2}{n_1} \right]^2 \quad (\text{A.8})$$

$$dx_2 = \left\{ \left[\frac{n_2}{n_1} \right]^2 - 1 \right\}^{\frac{1}{2}} dz_2. \quad (\text{A.9})$$

This is the equation describing the path of a ray through a nonhomogeneous medium. Once a particular nonhomogeneity is described, it may be applied to this equation to determine the solution.

As previously stated, the nonhomogeneity of interest is one which is linearly stratified in the x direction. Such a nonhomogeneity is described by

$$n_2 = n_1 + n_2^*(x_2 - x_1) \quad (\text{A.10})$$

$$\left[\frac{n_2}{n_1} \right]^2 = 1 + 2 \frac{n_2^*}{n_1} (x_2 - x_1) + \left[\frac{n_2^* (x_2 - x_1)}{n_1} \right]^2 \quad (\text{A.11})$$

where the superscript, $*$, denotes differentiation with respect to x . Since n^* is normally very small, the last term in Eq. (A.11) may be dropped to obtain an approximation of the refractive index field stratification. If this approximation is substituted into the equation describing the ray path through a nonhomogeneity, and the resulting equation integrated from 1 to 2 one obtains

$$x_2 - x_1 = \frac{1}{2} \left[\frac{n_2^*}{n_1} \right] (z_2 - z_1)^2. \quad (\text{A.12})$$

Thus, the ray path through a linearly stratified nonhomogeneity in the x direction is approximately parabolic. Note that this is an approximation since the higher order term in the refractive index field model was dropped.

APPENDIX B

END EFFECT CORRECTION

Due to the finite length of the test section in the z direction, the boundary layer wraps around these ends. This behavior of the boundary layer at the test section ends is referred to as the end effects. The end effects introduce additional refraction of the light rays beyond that of the ideal phenomenon. In HI analysis, it is desirable to eliminate or account for all phenomena influencing the light rays. Such accounting allows the influence of undesirable effects to be subtracted from the final interferometric results. Thus, it is necessary to develop a method of accounting for the influence of the end effects on the light rays. In this appendix, this influence is quantified in terms of fringe shift. This term, the end effect fringe shift correction term, ΔS_2 , is then subtracted from the uncorrected fringe shift to account for the end effects.

Recall from Fig. 3.5 that the length over which the end effect influences a given ray is dependent upon the x position of that ray relative to the surface. Additionally, it was proposed to model the end effects as circular arcs centered on the surface ends as shown in Fig. 3.5. Thus, the length d over which each ray is influenced by each end effect is

$$d = \sqrt{\delta^2 - x^2} \quad (B.1)$$

In this thesis, the temperature decay along a given ray passing through the ends of the test section is approximated as a second order polynomial. Thus, the temperature along a ray at any position in the boundary layer is

$$T(x, z) = T_u(x) + Az + Bz^2 \quad (\text{B.2})$$

Where T_u is the uncorrected ray temperature predicted by applying the uncorrected fringe shift at that ray to Eq. (2.23). The boundary conditions which may be applied to determine the coefficients A and B are

$$\underline{\text{(B.3)}} \quad T(x, z=d) = T_\infty$$

$$\underline{\text{(B.4)}} \quad \left[\frac{\partial T}{\partial z} \right]_{z=d} = 0.$$

Applying these boundary equations, the temperature decay equation becomes

$$T = T_u + (T_\infty - T_u) \left[\left(\frac{z}{d} \right)^2 - 2 \left(\frac{z}{d} \right) \right]. \quad (\text{B.5})$$

In the above equation, as with ensuing equations, the x and z dependence has been dropped for clarity.

Recall that combining the Gladstone Dale equation and the Ideal Gas equation provides

$$n - 1 = K\rho = \frac{KP_{\infty}}{RT}. \quad (\text{B.6})$$

The variation of the refractive index within the end effect may be found by combining the above two equations to provide

$$n = \frac{KP_{\infty}}{R} \left\{ T_u + (T_{\infty} - T_u) \left[\left(\frac{z}{d} \right)^2 - 2 \left(\frac{z}{d} \right) \right] \right\}^{-1} + 1. \quad (\text{B.7})$$

Recall that the relation between the refractive index distribution and the fringe shift is given by the equation of ideal interferometry, Eq. (2.16).

$$S\lambda = n_{\infty}L - \int_0^L n \, dz \quad (\text{B.8})$$

Substituting Eq. (B.7) in the above equation yields

$$S\lambda = n_{\infty}L - \int_0^L dz - \quad (\text{B.9})$$

$$\frac{KP_{\infty}}{R} \int_0^L \left[\Delta T \left(\frac{z}{d} \right)^2 - 2\Delta T \left(\frac{z}{d} \right) + T_u \right]^{-1} dz.$$

where $\Delta T = T_u - T_{\infty}$. Carrying out the integration, invoking the combined Gladstone Dale and Ideal Gas equations, dividing by λ , and multiplying by two to account for the end effects at each end of the test section, one obtains the

quantitative expression for the end effect fringe shift correction, ΔS_2 ,

$$\Delta S_2 = A \left[\frac{1}{T_\infty} - \frac{1}{\sqrt{\Delta T T_\infty}} \tan^{-1} \frac{\Delta T}{\sqrt{\Delta T T_\infty}} \right] \quad (\text{B.10})$$

where

$$A = \frac{2KP_\infty}{R} \sqrt{\delta^2 - x^2} \quad (\text{B.11})$$

APPENDIX C

UNCERTAINTY ANALYSIS

In this appendix, the uncertainty analysis of the interferometric data is presented. The Kline and McClintock method of uncertainty analysis was used to calculate the reported values. Several variables used in the interferometric analysis were assumed to have negligible uncertainties.

Ideal gas constant:

The ideal gas constant, R, was obtained from tables in the Handbook of Chemistry and Physics (1979). The uncertainty in this variable was assumed to be negligible. The R value used was:

$$R = 2.15297 \text{ Torr mm}^3 / \text{kg K.} \quad (\text{C.1})$$

Gladstone Dale Constant:

The Gladstone Dale constant, K, was determined from the specific refractivity, SR. The value of specific refractivity was taken from a table in Advances in Heat Transfer (Hauf and Grigull 1970). The Gladstone Dale constant was assumed to have negligible uncertainty. The K value used was:

$$K = \frac{3}{2} SR = \frac{3}{2} 1.504 \cdot 10^{-4} \frac{\text{m}^3}{\text{kg}} = 1.128 \cdot 10^{-4} \frac{\text{m}^3}{\text{kg}} \quad (\text{C.2})$$

Thermophysical Properties of Air:

The thermophysical properties of air needed for the determination of the Nusselt and Grashof numbers were calculated using National Bureau of Standards equations. The uncertainties in these values were assumed to be negligible.

Refractive Index of Air:

The refractive index of air was calculated using a relation provided by Radulovic (1977),

$$n - 1 = \frac{0.292015 \cdot 10^{-3}}{1 + T \cdot 0.368184 \cdot 10^{-2}}. \quad (\text{C.3})$$

which applies to the use of a helium neon laser. Using this equation, the value of refractive index over the range of temperatures experienced in the experimentation was found to be

$$n = 1.00025 \pm 0.00005. \quad (\text{C.4})$$

Thus, the nominal value of refractive index was used and its uncertainty was assumed to be negligible compared to that of other variables.

Laser Wavelength:

The wavelength of helium neon laser is 632.8 nm. This value was used in the data reduction, and its uncertainty was assumed to be negligible.

x Direction Length Scale:

The uncertainty in the x direction length scale is a function of the resolution of the pixel scale of the image processing tool. The location of the fringe centers were measured using the image processing tool to within ± 0.5 column pixels, PC. This uncertainty must be multiplied by the appropriate scale factor to account for the imaging lens magnification, which was determined to be 0.060 mm/PC. Thus, the uncertainty in the x-direction length scale was

$$w_x = (0.5 \text{ PC}) * (0.060 \text{ mm/PC}) = 0.03 \text{ mm} \quad (\text{C.5})$$

y Direction Length Scale:

The uncertainty in the y direction length scale is determined similar to that for the x direction. The elevation was measured to within ± 0.5 row pixel, PR, and the imaging scale factor was determined to be 0.0478 mm/PR. Thus, the uncertainty in the y-direction length scale was

$$w_y = (0.5 \text{ PR}) * (0.0478 \text{ mm/PR}) = 0.024 \text{ mm}. \quad (\text{C.6})$$

Test Section Length:

The uncertainty in the test section length was specified in the apparatus design.

$$w_L = 0.01 \text{ in} = 0.0254 \text{ mm} \quad (\text{C.7})$$

$$\epsilon_L = (0.0254 \text{ mm}) / (7.50 \text{ mm}) = 0.0017 \quad (\text{C.8})$$

Thermocouple Measured Temperatures:

The uncertainty in the thermocouple measured temperatures was determined from the results a calibration test. The uncertainty in the ambient temperature was

$$w_{T_o} = 0.1 \text{ }^\circ\text{C} \quad (\text{C.9})$$

$$\epsilon_{T_o} = (0.1 \text{ }^\circ\text{C}) / (19.1 \text{ }^\circ\text{C}) = 0.005 \quad (\text{C.10})$$

Ambient Pressure:

Although the barometer was corrected for temperature, after the data reduction was complete it was realized that the pressure was not corrected for gravity. This correction was thus lumped in with the estimated pressure reading uncertainty, and the resulting uncertainty was

$$w_{p_o} = 1.27 \text{ Torr} \quad (\text{C.11})$$

$$\epsilon_{p_o} = (1.27 \text{ Torr}) / (746.49 \text{ Torr}) = 0.0017 \quad (\text{C.12})$$

Local Field Temperature:

The equation of ideal interferometry relating fringe shift values to temperatures is

$$T(x,y) = \frac{T_{\infty}}{1 - a*S_{IDEAL}} , \quad \text{where} \quad a = \frac{\lambda RT_{\infty}}{LKP_{\infty}} \quad (C.13)$$

Before developing the equation predicting the local uncertainty in field temperature, it is appropriate to obtain expressions for the individual uncertainties upon which it is dependent.

Uncertainty in a:

Thusfar, all of the uncertainties of the variables upon which a is dependent on have been defined. Therefore, the uncertainty in a can be calculated via

$$e_a = \{ e_{T_{\infty}}^2 + e_L^2 + e_{P_{\infty}}^2 \}^{\frac{1}{2}} \quad (C.14)$$

Uncertainty in SIDEAL:

The ideal fringe shift values were calculated by

$$SIDEAL = S + \Delta S1 - \Delta S2. \quad (C.15)$$

The uncertainties in the individual terms upon which SIDEAL is dependent on must be determined before that of SIDEAL can be calculated.

Uncertainty in S:

The basis of the development of the equations of interferometry is that bright and dark fringes take on integer and integer plus one half fringe shift values, respectively. Thus, the uncertainty in S is zero.

Uncertainty in $\Delta S1$:

The equation for calculation of $\Delta S1$ was

$$\Delta S1 = \frac{n_{\infty} \lambda L}{12b^2}. \quad (C.16)$$

In the above equation, the uncertainties in all parameters are known except for that of the fringe spacing, b. The fringe spacing is simply the difference in x position between adjacent fringes. Thus the uncertainty in the fringe spacing becomes the radical of the sum of the squares of the uncertainties in the location of adjacent fringe center locations. However, the uncertainty in location is the same for all fringes. Thus, the uncertainty in the fringe spacing becomes,

$$w_b = \sqrt{2} w_x, \quad (C.17)$$

The uncertainty in $\Delta S1$ may now be evaluated as

$$w_{\Delta S1} = \left\{ \left[\frac{n_{\infty} \lambda}{12b^2} w_L \right]^2 + \left[\frac{-2n_{\infty} L}{12b^3} w_b \right]^2 \right\}^{\frac{1}{2}} . \quad (C.18)$$

It should be noted that the uncertainty in $\Delta S1$ is variable due to the fringe spacing term.

Uncertainty in $\Delta S2$:

Recall that the end effect fringe shift correction, $\Delta S2$, was calculated via

$$\Delta S2 = A \left[\frac{1}{T_{\infty}} - \frac{1}{\sqrt{\Delta T T_{\infty}}} \text{TAN}^{-1} \frac{\Delta T}{\sqrt{\Delta T T_{\infty}}} \right] . \quad (C.19)$$

where

$$A = \frac{KP_{\infty}}{R} \sqrt{\delta^2 - x^2} \quad (C.20)$$

where $\Delta T = T_i(x) - T_{\infty}$. Rather than delving into the task of stringently evaluating the uncertainty in the end effect fringe shift correction term, assume this uncertainty to be of the same order of magnitude as its value. If the correction is a valid model of the end effect, as it has been shown to be, such simplified assumption should more than adequately cover the actual uncertainty range. Thus, the uncertainty in the end effect fringe shift correction term was taken as

$$w_{\Delta S2} = \Delta S2 . \quad (C.21)$$

With these uncertainties evaluated, the uncertainty in the ideal fringe shift becomes

$$W_{SIDEAL} = \left\{ W_{\Delta S1}^2 + W_{\Delta S2}^2 \right\}^{\frac{1}{2}} \quad (C.22)$$

Note that since the uncertainties in the correction terms vary from fringe to fringe, so will the value of uncertainty in the ideal fringe shift.

The percent uncertainty in the field temperature can now be calculated from

$$\epsilon_{T(x,y)} = \left\{ \epsilon_{T_{\infty}}^2 + \left[\frac{S_{\infty} W_a}{1 - a S_{\infty}} \right]^2 + \left[\frac{a W_{S_{\infty}}}{1 - a S_{\infty}} \right]^2 \right\}^{\frac{1}{2}} \quad (C.23)$$

Wall Temperature:

The wall temperature was determined using the first 4, 5, or 6 field temperature data points adjacent to the wall to produce a first order temperature profile model by applying the least squares method. Thus, the temperature profile adjacent to the wall is given by,

$$T(x) = C1 + C2*x, \quad (C.24)$$

where,

$$C1 = \frac{(\sum T_i) (\sum x_i^2) - (\sum x_i T_i) (\sum x_i)}{n \sum x_i^2 - (\sum x_i)^2} , \quad (C.25)$$

$$C2 = \frac{n \sum x_i T_i - (\sum x_i) (\sum T_i)}{n \sum x_i^2 - (\sum x_i)^2} . \quad (C.26)$$

From the temperature profile model, it is evident that the uncertainty in any given temperature predicted via the curve is

$$W_{T(x)} = \left\{ (W_{C1})^2 + (x W_{C2})^2 + (C2 W_x)^2 \right\}^{\frac{1}{2}} . \quad (C.27)$$

The fringe position (x) used to develop the temperature profile model was that relative to the surface. Thus, the wall temperature is the value of the function at x=0. Thus, the uncertainty in the wall temperature as predicted by the present method of analysis is

$$W_{T_{wall}} = W_{T(x=0)} = \left\{ (W_{C1})^2 + (C2 W_x)^2 \right\}^{\frac{1}{2}} \quad (C.28)$$

and

$$e_{T_{wall}} = \frac{w_{T_{wall}}}{C1} . \quad (C.29)$$

Thus, evaluation of the uncertainty in the wall temperature, as predicted by interferometry, requires the evaluation of the uncertainty in the coefficient to the zeroth order term, C1. Applying the Kline and McClintock method to the equation for C1 one obtains

$$w_{C1} = \left\{ \sum \left(\frac{\partial C1}{\partial x_j} w_{x_j} \right)^2 + \sum \left(\frac{\partial C1}{\partial T_j} w_{T_j} \right)^2 \right\}^{\frac{1}{2}} \quad (C.30)$$

where

$$\frac{\partial C1}{\partial x_j} = \frac{2x_j \sum y_i - y_j \sum x_i - \sum x_i y_i}{n \sum x_i^2 - (\sum x_i)^2} - \quad (C.31)$$

$$\frac{2 [(\sum y_i) (\sum x_i^2) - (\sum x_i y_i) (\sum x_i)] [nx_j - \sum x_i]}{[n \sum x_i^2 - (\sum x_i)^2]^2}$$

and

$$\frac{\partial C1}{\partial T_j} = \frac{\sum x_i^2 - x_j \sum x_i}{n \sum x_i^2 - (\sum x_i)^2} . \quad (C.32)$$

Nusselt Number:

The definition of Nusselt number indicates that it is dependent upon the coefficient of the first order term of the temperature profile model, C2.

$$Nu_y = \left[\frac{y}{\Delta T_{wall}} \right] \left[\frac{\partial T}{\partial x} \right]_{wall} = \frac{C2 y}{\Delta T_{wall}} \quad (C.33)$$

From the above equation, the uncertainty in the Nusselt number can be obtained from

$$e_{Nu_y} = \left\{ (e_{C2})^2 + (e_y)^2 + (e_{\Delta T_{wall}})^2 \right\}^{\frac{1}{2}} \quad (C.34)$$

C2:

The uncertainty in the coefficient to the first order term in the temperature profile is

$$e_{C2} = \frac{w_{C2}}{C2} , \quad (C.35)$$

where

$$w_{C2} = \left\{ \left(\frac{\partial C2}{\partial x_j} w_{x_j} \right)^2 + \left(\frac{\partial C2}{\partial T_j} w_{T_j} \right)^2 \right\}^{\frac{1}{2}} , \quad (C.36)$$

where

$$\frac{\partial C2}{\partial x_j} = \frac{nT_j - \sum T_i}{n\sum x_i^2 - (\sum x_i)^2} - \quad (C.37)$$

$$\frac{2 [n\sum x_i T_i - (\sum x_i) (\sum T_i)] [nx_j - \sum x_i]}{[n\sum x_i^2 - (\sum x_i)^2]^2}$$

and

$$\frac{\partial C_2}{\partial T_j} = \frac{nx_j - \sum x_i}{n\sum x_i^2 - (\sum x_i)^2} . \quad (C.38)$$

Temperature Difference:

The uncertainty in the temperature difference was calculated via the following equations.

$$\Delta T_{wall} = T_{wall} - T_{\infty} \quad (C.39)$$

$$e_{\Delta T_{wall}} = \{ e_{T_{wall}}^2 + e_{T_{\infty}}^2 \}^{\frac{1}{2}} \quad (C.40)$$

Grashof Number:

The Grashof number is defined as

$$Gr_y = \frac{g\beta (T_{wall} - T_{\infty}) y^3}{\nu^2} . \quad (C.41)$$

By substituting the inverse of the film temperature for the coefficient of thermal expansion one obtains,

$$Gr_y = \frac{2g}{\nu^2} \frac{(T_{wall} - T_{\infty}) y^3}{(T_{wall} + T_{\infty})} . \quad (C.42)$$

Applying the Kline and McClintock method the uncertainty in the Grashof number becomes

$$\epsilon_{Gr_y} = \left\{ \left[\frac{2T_{\infty} W_{T_{wall}}}{T_{wall}^2 - T_{\infty}^2} \right]^2 + \left[\frac{2T_{wall} W_{T_{\infty}}}{T_{wall}^2 - T_{\infty}^2} \right]^2 + 9\epsilon_y^2 \right\}^{\frac{1}{2}} . \quad (C.43)$$

APPENDIX D

EXPERIMENTAL RESULTS

TABLE D.1: Summary of fringe data and results at $y/h = 0.15$

S	x (mm)	$\Delta S1$	$\Delta S2$	SIDEAL	T (°C)	w_T (°C)
11.00	0.030	0.149	0.176	10.972	65.17	0.39
10.50	0.180	0.174	0.168	10.507	62.92	0.37
10.00	0.300	0.138	0.159	9.979	60.41	0.35
9.50	0.480	0.138	0.150	9.488	58.11	0.33
9.00	0.570	0.154	0.141	9.013	55.91	0.32
8.50	0.720	0.174	0.132	8.542	53.76	0.30
8.00	0.810	0.138	0.124	8.014	51.38	0.29
7.50	0.960	0.138	0.115	7.523	49.20	0.27
7.00	1.110	0.124	0.106	7.018	46.98	0.26
6.50	1.260	0.112	0.097	6.514	44.81	0.24
6.00	1.380	0.138	0.089	6.049	42.82	0.23
5.50	1.560	0.112	0.080	5.532	40.64	0.22
5.00	1.650	0.092	0.072	5.021	38.52	0.21
4.50	1.860	0.078	0.063	4.515	36.45	0.19
4.00	2.040	0.066	0.054	4.012	34.41	0.18
3.50	2.280	0.057	0.046	3.511	32.42	0.17
3.00	2.430	0.050	0.038	3.012	30.45	0.16
2.50	2.700	0.039	0.030	2.509	28.49	0.15
2.00	2.940	0.024	0.022	2.002	26.55	0.14
1.50	3.300	0.015	0.015	1.501	24.65	0.13
1.00	3.720	0.000	0.000	1.000	22.77	0.12
0.50	4.320	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$T(x, y=2.73) = C1 + C2 \cdot x$

Number of data sets used = 6

$C1 = 65.721 \pm 0.485^\circ\text{C}$

$C2 = -16.688 \pm 1.062^\circ\text{C/mm}$

Correlation coefficient = 0.99738

Standard error = 0.311 °C

Wall Temperature:

Average of the upper wall section is .. 63.77 $\pm$ 0.7 °C

Average of the lower wall section is .. 63.60 $\pm$ 0.7 °C

Average of the wall is 63.68 $\pm$ 0.7 °C

Determination by interferometry is 65.72 $\pm$ 0.70 °C

Grashof Number = 99.7 $\pm$ 2.74 %

Nusselt Number:

Theoretical 1.118

Using interferometry wall temperature .. 0.976 $\pm$ 0.064

Using thermocouple wall temperature 1.018 $\pm$ 0.067

TABLE D.2: Summary of fringe data and results at $y/h = 0.24$

S	x (mm)	$\Delta S1$	$\Delta S2$	SIDEAL	T (°C)	w_f (°C)
11.00	0.030	0.112	0.188	10.923	64.93	0.39
10.50	0.180	0.112	0.179	10.432	62.57	0.37
10.00	0.330	0.112	0.170	9.942	60.23	0.35
9.50	0.480	0.112	0.160	9.451	57.94	0.33
9.00	0.630	0.124	0.151	8.973	55.72	0.32
8.50	0.780	0.112	0.142	8.470	53.43	0.30
8.00	0.900	0.124	0.132	7.991	51.28	0.29
7.50	1.080	0.112	0.123	7.489	49.05	0.27
7.00	1.200	0.112	0.113	6.998	46.90	0.26
6.50	1.380	0.101	0.104	6.498	44.73	0.24
6.00	1.500	0.092	0.095	5.998	42.60	0.23
5.50	1.710	0.092	0.085	5.507	40.54	0.22
5.00	1.860	0.084	0.076	5.009	38.47	0.21
4.50	2.040	0.078	0.067	4.511	36.43	0.19
4.00	2.190	0.071	0.058	4.013	34.42	0.18
3.50	2.430	0.053	0.049	3.504	32.39	0.17
3.00	2.610	0.044	0.041	3.003	30.41	0.16
2.50	2.910	0.033	0.032	2.501	28.46	0.15
2.00	3.150	0.021	0.024	1.997	26.53	0.14
1.50	3.540	0.014	0.016	1.498	24.64	0.13
1.00	3.990	0.000	0.000	1.000	22.77	0.12
0.50	4.620	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$T(x, y = 4.45) = C1 + C2 \cdot x$

Number of data sets used = 4

$C1 = 65.383 \pm 0.547^\circ\text{C}$

$C2 = -15.550 \pm 1.760^\circ\text{C/mm}$

Correlation coefficient = 0.99997

Standard error = 0.023°C

Wall Temperature:

Average of the upper wall section is .. $63.77 \pm 0.7^\circ\text{C}$

Average of the lower wall section is .. $63.60 \pm 0.7^\circ\text{C}$

Average of the wall is $63.68 \pm 0.7^\circ\text{C}$

Determination by interferometry is $65.38 \pm 0.72^\circ\text{C}$

Grashof Number = $430.7 \pm 1.80\%$

Nusselt Number:

Theoretical 1.612

Using interferometry wall temperature .. 1.495 ± 0.171

Using thermocouple wall temperature 1.547 ± 0.177

TABLE D.3: Summary of fringe data and results at $y/h = 0.33$

S	x (mm)	ΔS_1	ΔS_2	SIDEAL	T (°C)	w_T (°C)
11.00	0.030	0.092	0.198	10.894	64.79	0.39
10.50	0.210	0.095	0.188	10.407	62.44	0.37
10.00	0.360	0.092	0.179	9.914	60.10	0.35
9.50	0.540	0.084	0.169	9.416	57.77	0.34
9.00	0.690	0.101	0.159	8.943	55.58	0.32
8.50	0.900	0.092	0.148	8.444	53.31	0.30
8.00	0.990	0.112	0.139	7.973	51.19	0.29
7.50	1.200	0.112	0.128	7.483	49.02	0.27
7.00	1.290	0.101	0.119	6.982	46.83	0.26
6.50	1.500	0.092	0.109	6.484	44.67	0.24
6.00	1.620	0.084	0.099	5.985	42.55	0.23
5.50	1.860	0.078	0.089	5.489	40.46	0.22
5.00	1.980	0.071	0.080	4.992	38.40	0.21
4.50	2.220	0.078	0.070	4.508	36.42	0.19
4.00	2.370	0.061	0.061	4.001	34.37	0.18
3.50	2.580	0.053	0.051	3.502	32.38	0.17
3.00	2.790	0.041	0.042	2.999	30.40	0.16
2.50	3.090	0.029	0.033	2.496	28.44	0.15
2.00	3.360	0.021	0.025	1.996	26.52	0.14
1.50	3.750	0.013	0.016	1.496	24.63	0.13
1.00	4.170	0.000	0.000	1.000	22.77	0.12
0.50	4.860	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$T(x, y = 6.08) = C_1 + C_2 \cdot x$

Number of data sets used = 5

$C_1 = 65.256 \pm 0.470^\circ\text{C}$

$C_2 = -13.982 \pm 1.055^\circ\text{C/mm}$

Correlation coefficient = 0.99954

Standard error = 0.111 °C

Wall Temperature:

Average of the upper wall section is .. 63.77 $\pm$ 0.7 °C

Average of the lower wall section is .. 63.60 $\pm$ 0.7 °C

Average of the wall is 63.68 $\pm$ 0.7 °C

Determination by interferometry is 65.26 $\pm$ 0.63 °C

Grashof Number = 1094.9 $\pm$ 1.38 %

Nusselt Number:

Theoretical 2.036

Using interferometry wall temperature .. 1.841 $\pm$ 0.141

Using thermocouple wall temperature 1.899 $\pm$ 0.147

TABLE D.4: Summary of fringe data and results at $y/h = 0.42$

S	x (mm)	ΔS_1	ΔS_2	SIDEAL	T (°C)	w_f (°C)
11.00	0.060	0.092	0.205	10.887	64.76	0.39
10.50	0.270	0.092	0.195	10.397	62.40	0.37
10.00	0.390	0.092	0.185	9.907	60.07	0.36
9.50	0.630	0.092	0.175	9.418	57.78	0.34
9.00	0.720	0.101	0.165	8.937	55.56	0.32
8.50	0.930	0.101	0.154	8.447	53.33	0.30
8.00	1.020	0.101	0.144	7.957	51.12	0.29
7.50	1.260	0.084	0.133	7.451	48.88	0.27
7.00	1.350	0.084	0.123	6.961	46.74	0.26
6.50	1.620	0.078	0.112	6.465	44.60	0.24
6.00	1.710	0.084	0.103	5.982	42.54	0.23
5.50	1.980	0.078	0.092	5.486	40.45	0.22
5.00	2.040	0.078	0.083	4.995	38.41	0.21
4.50	2.340	0.078	0.072	4.506	36.41	0.19
4.00	2.430	0.053	0.063	3.990	34.33	0.18
3.50	2.700	0.050	0.053	3.497	32.36	0.17
3.00	2.910	0.036	0.044	2.993	30.37	0.16
2.50	3.240	0.028	0.034	2.494	28.43	0.15
2.00	3.480	0.019	0.026	1.994	26.51	0.14
1.50	3.900	0.012	0.017	1.496	24.63	0.13
1.00	4.350	0.000	0.000	1.000	22.77	0.12
0.50	5.040	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$T(x, y = 7.70) = C_1 + C_2 \cdot x$

Number of data sets used = 6

$C_1 = 65.651 \pm 0.447^\circ\text{C}$

$C_2 = -13.339 \pm 0.749^\circ\text{C/mm}$

Correlation coefficient = 0.99450

Standard error = 0.448 °C

Wall Temperature:

Average of the upper wall section is .. 63.77 $\pm$ 0.7 °C

Average of the lower wall section is .. 63.60 $\pm$ 0.7 °C

Average of the wall is 63.68 $\pm$ 0.7 °C

Determination by interferometry is 65.65 $\pm$ 0.92 °C

Grashof Number = 2243.4 $\pm$ 1.33 %

Nusselt Number:

Theoretical 2.435

Using interferometry wall temperature .. 2.207 $\pm$ 0.132

Using thermocouple wall temperature 2.297 $\pm$ 0.134

TABLE D.5: Summary of fringe data and results at $y/h = 0.50$

S	x (mm)	$\Delta S1$	$\Delta S2$	SIDEAL	T (°C)	w_T (°C)
11.00	0.060	0.092	0.210	10.882	64.73	0.40
10.50	0.240	0.084	0.200	10.384	62.34	0.38
10.00	0.390	0.092	0.190	9.903	60.05	0.36
9.50	0.570	0.084	0.179	9.405	57.72	0.34
9.00	0.720	0.092	0.169	8.924	55.50	0.32
8.50	0.930	0.084	0.158	8.427	53.23	0.30
8.00	1.050	0.084	0.147	7.937	51.03	0.29
7.50	1.260	0.084	0.136	7.448	48.87	0.27
7.00	1.410	0.078	0.126	6.952	46.69	0.26
6.50	1.620	0.078	0.115	6.462	44.58	0.24
6.00	1.770	0.078	0.105	5.973	42.50	0.23
5.50	1.980	0.078	0.094	5.483	40.44	0.22
5.00	2.130	0.071	0.084	4.987	38.38	0.21
4.50	2.340	0.061	0.074	4.487	36.34	0.19
4.00	2.520	0.053	0.064	3.989	34.32	0.18
3.50	2.790	0.044	0.054	3.490	32.33	0.17
3.00	3.000	0.036	0.045	2.992	30.37	0.16
2.50	3.300	0.027	0.035	2.491	28.42	0.15
2.00	3.570	0.020	0.026	1.994	26.52	0.14
1.50	4.020	0.012	0.017	1.495	24.62	0.13
1.00	4.410	0.000	0.000	1.000	22.77	0.12
0.50	5.160	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$T(x, y = 9.33) = C1 + C2 \cdot x$

Number of data sets used = 5

$C1 = 65.604 \pm 0.500$ °C

$C2 = -13.981 \pm 1.060$ °C/mm

Correlation coefficient = 0.99965

Standard error = 0.097 °C

Wall Temperature:

Average of the upper wall section is .. 63.77 $\pm$ 0.7 °C

Average of the lower wall section is .. 63.60 $\pm$ 0.7 °C

Average of the wall is 63.68 $\pm$ 0.7 °C

Determination by interferometry is 65.60 $\pm$ 0.98 °C

Grashof Number = 3983.2 $\pm$ 1.27 %

Nusselt Number:

Theoretical 2.811

Using interferometry wall temperature .. 2.805 $\pm$ 0.221

Using thermocouple wall temperature 2.916 $\pm$ 0.226

TABLE D.6: Summary of fringe data and results at $y/h = 0.59$

S	x (mm)	$\Delta S1$	$\Delta S2$	SIDEAL	T (°C)	w_T (°C)
11.00	0.090	0.087	0.218	10.869	64.67	0.40
10.50	0.270	0.084	0.207	10.377	62.30	0.38
10.00	0.420	0.092	0.196	9.896	60.02	0.36
9.50	0.660	0.071	0.185	9.386	57.63	0.34
9.00	0.750	0.084	0.174	8.910	55.43	0.32
8.50	1.020	0.084	0.163	8.422	53.21	0.31
8.00	1.110	0.078	0.152	7.925	50.98	0.29
7.50	1.350	0.092	0.141	7.451	48.88	0.27
7.00	1.470	0.084	0.130	6.954	46.71	0.26
6.50	1.680	0.078	0.119	6.458	44.57	0.25
6.00	1.800	0.078	0.109	5.969	42.48	0.23
5.50	2.070	0.066	0.097	5.469	40.38	0.22
5.00	2.190	0.061	0.087	4.974	38.33	0.21
4.50	2.460	0.061	0.076	4.485	36.33	0.19
4.00	2.610	0.050	0.067	3.983	34.30	0.18
3.50	2.880	0.044	0.056	3.488	32.32	0.17
3.00	3.090	0.034	0.046	2.988	30.36	0.16
2.50	3.420	0.025	0.036	2.489	28.42	0.15
2.00	3.690	0.018	0.027	1.991	26.50	0.14
1.50	4.140	0.011	0.018	1.493	24.62	0.13
1.00	4.590	0.000	0.000	1.000	22.77	0.12
0.50	5.340	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$$T(x,y=10.96) = C1 + C2 \cdot x$$

Number of data sets used = 4

$$C1 = 65.646 \pm 0.537 \text{ } ^\circ\text{C}$$

$$C2 = -12.469 \pm 1.258 \text{ } ^\circ\text{C/mm}$$

Correlation coefficient = 0.99391

Standard error = 0.333 °C

Wall Temperature:

Average of the upper wall section is .. 63.77 $\pm$ 0.7 °C

Average of the lower wall section is .. 63.60 $\pm$ 0.7 °C

Average of the wall is 63.68 $\pm$ 0.7 °C

Determination by interferometry is 65.65 $\pm$ 1.24 °C

Grashof Number = 6455.0 $\pm$ 1.41 %

Nusselt Number:

Theoretical 3.172

Using interferometry wall temperature .. 2.935 $\pm$ 0.307

Using thermocouple wall temperature 3.054 $\pm$ 0.312

TABLE D.7: Summary of fringe data and results at $y/h = 0.68$

S	x (mm)	ΔS_1	ΔS_2	SIDEAL	T (°C)	w_T (°C)
11.00	0.060	0.082	0.223	10.859	64.62	0.40
10.50	0.270	0.078	0.212	10.366	62.25	0.38
10.00	0.420	0.084	0.201	9.884	59.96	0.36
9.50	0.660	0.071	0.189	9.382	57.61	0.34
9.00	0.750	0.084	0.178	8.906	55.42	0.32
8.50	1.020	0.084	0.167	8.418	53.19	0.31
8.00	1.110	0.078	0.156	7.922	50.96	0.29
7.50	1.350	0.078	0.144	7.433	48.80	0.27
7.00	1.470	0.078	0.133	6.944	46.66	0.26
6.50	1.740	0.078	0.122	6.456	44.56	0.25
6.00	1.830	0.071	0.111	5.960	42.44	0.23
5.50	2.070	0.071	0.100	5.471	40.39	0.22
5.00	2.220	0.061	0.090	4.972	38.32	0.21
4.50	2.490	0.053	0.078	4.475	36.28	0.19
4.00	2.640	0.046	0.068	3.978	34.28	0.18
3.50	2.940	0.041	0.057	3.484	32.31	0.17
3.00	3.150	0.033	0.048	2.985	30.34	0.16
2.50	3.480	0.025	0.037	2.488	28.41	0.15
2.00	3.750	0.017	0.028	1.989	26.50	0.14
1.50	4.200	0.010	0.018	1.492	24.61	0.13
1.00	4.680	0.000	0.000	1.000	22.77	0.12
0.50	5.460	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$T(x,y=12.58) = C_1 + C_2 \cdot x$

Number of data sets used = 4

$C_1 = 65.298 \pm 0.502$ °C

$C_2 = -11.876 \pm 1.177$ °C/mm

Correlation coefficient = 0.99525

Standard error = 0.293 °C

Wall Temperature:

Average of the upper wall section is .. 63.77 $\pm$ 0.7 °C

Average of the lower wall section is .. 63.60 $\pm$ 0.7 °C

Average of the wall is 63.68 $\pm$ 0.7 °C

Determination by interferometry is 65.30 $\pm$ 0.87 °C

Grashof Number = 9729.4 $\pm$ 1.08 %

Nusselt Number:

Theoretical 3.515

Using interferometry wall temperature .. 3.235 $\pm$ 0.326

Using thermocouple wall temperature 3.341 $\pm$ 0.335

TABLE D.8: Summary of fringe data and results at $y/h = 0.77$

S	x (mm)	$\Delta S1$	$\Delta S2$	SIDEAL	T (°C)	w_T (°C)
11.00	0.090	0.082	0.230	10.852	64.59	0.40
10.50	0.270	0.082	0.219	10.363	62.24	0.38
10.00	0.420	0.078	0.207	9.870	59.90	0.36
9.50	0.660	0.071	0.196	9.376	57.58	0.34
9.00	0.810	0.078	0.184	8.893	55.36	0.33
8.50	1.020	0.084	0.172	8.412	53.17	0.31
8.00	1.140	0.084	0.161	7.923	50.97	0.29
7.50	1.350	0.078	0.149	7.428	48.78	0.28
7.00	1.500	0.078	0.138	6.939	46.64	0.26
6.50	1.740	0.071	0.126	6.445	44.51	0.25
6.00	1.860	0.071	0.115	5.956	42.43	0.23
5.50	2.100	0.066	0.104	5.462	40.35	0.22
5.00	2.250	0.057	0.093	4.964	38.29	0.21
4.50	2.520	0.050	0.081	4.468	36.26	0.19
4.00	2.700	0.044	0.071	3.973	34.26	0.18
3.50	3.000	0.041	0.060	3.481	32.30	0.17
3.00	3.210	0.031	0.049	2.982	30.33	0.16
2.50	3.510	0.025	0.039	2.486	28.40	0.15
2.00	3.840	0.016	0.029	1.987	26.49	0.14
1.50	4.260	0.009	0.020	1.489	24.60	0.13
1.00	4.800	0.000	0.000	1.000	22.77	0.12
0.50	5.640	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$T(x,y=14.21) = C1 + C2*x$

Number of data sets used = 6

$C1 = 65.484 \pm 0.431^\circ\text{C}$

$C2 = -12.254 \pm 0.658^\circ\text{C/mm}$

Correlation coefficient = 0.99762

Standard error = 0.295°C

Wall Temperature:

Average of the upper wall section is .. $63.77 \pm 0.7^\circ\text{C}$

Average of the lower wall section is .. $63.60 \pm 0.7^\circ\text{C}$

Average of the wall is $63.68 \pm 0.7^\circ\text{C}$

Determination by interferometry is $65.48 \pm 1.18^\circ\text{C}$

Grashof Number = $14049.2 \pm 1.30\%$

Nusselt Number:

Theoretical 3.853

Using interferometry wall temperature .. 3.754 ± 0.224

Using thermocouple wall temperature 3.893 ± 0.218

TABLE D.9: Summary of fringe data and results at $y/h = 0.86$

S	x (mm)	ΔS_1	ΔS_2	SIDEAL	T (°C)	w_T (°C)
11.00	0.120	0.082	0.234	10.848	64.57	0.41
10.50	0.270	0.078	0.222	10.355	62.20	0.38
10.00	0.450	0.078	0.211	9.867	59.88	0.36
9.50	0.660	0.078	0.199	9.379	57.60	0.34
9.00	0.840	0.078	0.187	8.891	55.35	0.33
8.50	0.990	0.084	0.175	8.409	53.15	0.31
8.00	1.170	0.084	0.164	7.921	50.96	0.29
7.50	1.350	0.078	0.152	7.426	48.77	0.28
7.00	1.530	0.078	0.140	6.937	46.63	0.26
6.50	1.710	0.078	0.129	6.449	44.53	0.25
6.00	1.890	0.066	0.117	5.949	42.40	0.23
5.50	2.070	0.066	0.106	5.460	40.34	0.22
5.00	2.310	0.057	0.094	4.963	38.28	0.21
4.50	2.490	0.046	0.083	4.463	36.24	0.19
4.00	2.730	0.046	0.072	3.975	34.26	0.18
3.50	3.000	0.039	0.061	3.478	32.28	0.17
3.00	3.240	0.031	0.050	2.981	30.33	0.16
2.50	3.510	0.022	0.040	2.482	28.39	0.15
2.00	3.870	0.015	0.030	1.985	26.48	0.14
1.50	4.350	0.008	0.020	1.488	24.60	0.13
1.00	4.860	0.000	0.000	1.000	22.77	0.12
0.50	5.730	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$T(x, y=15.84) = C_1 + C_2 \cdot x$

Number of data sets used = 6

$C_1 = 65.849 \pm 0.458^\circ\text{C}$

$C_2 = -12.717 \pm 0.699^\circ\text{C/mm}$

Correlation coefficient = 0.99842

Standard error = 0.240 °C

Wall Temperature:

Average of the upper wall section is .. 63.77 $\pm$ 0.7 °C

Average of the lower wall section is .. 63.60 $\pm$ 0.7 °C

Average of the wall is 63.68 $\pm$ 0.7 °C

Determination by interferometry is 65.85 $\pm$ 1.59 °C

Grashof Number = 19549.5 $\pm$ 1.63 %

Nusselt Number:

Theoretical 4.184

Using interferometry wall temperature .. 4.308 $\pm$ 0.279

Using thermocouple wall temperature 4.503 $\pm$ 0.258

TABLE D.10: Summary of fringe data and results at $y/h = 0.94$

S	x (mm)	$\Delta S1$	$\Delta S2$	SIDEAL	T (°C)	w_T (°C)
11.00	0.060	0.082	0.237	10.845	64.55	0.41
10.50	0.270	0.084	0.226	10.359	62.21	0.39
10.00	0.390	0.084	0.214	9.870	59.90	0.37
9.50	0.630	0.084	0.202	9.382	57.62	0.35
9.00	0.750	0.078	0.190	8.887	55.33	0.33
8.50	0.960	0.084	0.178	8.406	53.14	0.31
8.00	1.110	0.078	0.167	7.911	50.92	0.29
7.50	1.320	0.078	0.155	7.423	48.76	0.28
7.00	1.470	0.078	0.143	6.935	46.62	0.26
6.50	1.680	0.071	0.131	6.440	44.49	0.25
6.00	1.830	0.071	0.120	5.952	42.41	0.23
5.50	2.070	0.061	0.108	5.454	40.32	0.22
5.00	2.220	0.053	0.097	4.957	38.26	0.21
4.50	2.490	0.046	0.085	4.462	36.23	0.19
4.00	2.700	0.044	0.074	3.970	34.25	0.18
3.50	3.000	0.039	0.062	3.476	32.28	0.17
3.00	3.180	0.029	0.052	2.977	30.31	0.16
2.50	3.510	0.022	0.041	2.481	28.38	0.15
2.00	3.870	0.014	0.031	1.983	26.47	0.14
1.50	4.350	0.008	0.020	1.487	24.60	0.13
1.00	4.890	0.000	0.000	1.000	22.77	0.12
0.50	5.820	0.000	0.000	0.500	20.92	0.11

Extrapolation using x values to find T_{wall} :

$T(x,y=17.46) = C1 + C2*x$

Number of data sets used = 6

$C1 = 65.352 \pm 0.439^\circ\text{C}$

$C2 = -12.863 \pm 0.714^\circ\text{C/mm}$

Correlation coefficient = 0.99590

Standard error = 0.387°C

Wall Temperature:

Average of the upper wall section is .. $63.77 \pm 0.7^\circ\text{C}$

Average of the lower wall section is .. $63.60 \pm 0.7^\circ\text{C}$

Average of the wall is $63.68 \pm 0.7^\circ\text{C}$

Determination by interferometry is $65.35 \pm 0.89^\circ\text{C}$

Grashof Number = $26027.5 \pm 1.02\%$

Nusselt Number:

Theoretical 4.495

Using interferometry wall temperature .. 4.857 ± 0.286

Using thermocouple wall temperature 5.022 ± 0.290

APPENDIX E

DATA REDUCTION PROGRAM

```

'***** W. P. PARTRIDGE JR.*****
'
'***** PROGRAM NAME = REDUCE6.BAS *****
'
'*****HOLOGRAPHIC INTERFEROMETRIC DATA REDUCTION *****
'
'PROGRAMMED IN QUICK BASIC 4.5
'
'Table VI from Hauf and Grigull---see H&G p.239 for eq.s
'
N = 1.00025 'approximate refractive index of
'air may correct the above approximation by implementing
'Vest's eq.

L = .1905 'test section length in (m)
LMM = L * 1000 'test section length in (mm)
H = 18.5 'test section height in (mm)
LMBDA = 6.328E-07 'wavelength of laser (m)
SR = .0001504 'specific refractivity (m^3 / kg)
R = 2.15297 'universal gas constant
' (Torr-m^3 / kg-K)
K = 3 * SR / 2 'Gladstone-Dale constant
' (m^3 / kg)

INPUT "ENTER EXPERIMENT I.D. NUMBER FOR DARK FRINGES";
STRING1$
DARK$ = STRING1$
PRINT " "
'
'inporting data from previously created file named STRING1$
'
OPEN STRING1$ FOR INPUT AS #1
INPUT #1, LE, YRP, PRTOM, YMM
INPUT #1, TWALL, TINFC, TINFK
INPUT #1, P
INPUT #1, PCTOM
INPUT #1, NS
DIM S(NS): DIM PC(NS + 1)
FOR I = 1 TO NS
    INPUT #1, S(I), PC(I)
NEXT I
CLOSE #1

'
' DIM S(NS) 'fringe number
' DIM PC(NS + 1) 'column pixel location of fringes
DIM PCR(NS + 1) 'column pixel location of fringes
'relative to wall

```

```

DIM X(NS + 1)      'fringe location in millimeters
DIM XREL(NS + 1)   'relative fringe location in millimeters
DIM B(NS)          'fringe spacing
DIM BAV(NS)        'averaged fringe spacing
DIM DELS(NS)       'refraction error correction
DIM DELS2(NS)      'end effect error correction
DIM SIDEAL(NS)     'ideal fringe number SI=S+DELS
DIM DSIDEAL(NS)
DIM TUC(NS)        'uncorrected fringe temp (needed for end
DELS2)
DIM TIK(NS)
DIM TIC(NS)        'temperature of the i'th fringe in
                    'degrees celsius

DIM TRELC(NS)
DIM DELTIC(NS)     'temperature difference per fringe in
                    'degrees celsius
DIM DELTICAV(NS)   'average temp. diff. per fringe in
                    'degrees celsius
'
111 'calculating fringe position in millimeters
'
FOR I = 1 TO NS
X(I) = PC(I) * PCTOM
NEXT I
'
'calculating fringe spacing
'
FOR I = 1 TO NS - 1
J = I + 1
B(I) = X(J) - X(I)
NEXT I
'
'
'calculating average fringe spacing
'
FOR I = 2 TO NS - 1
J = I - 1
BAV(I) = (B(I) + B(J)) / 2
NEXT I
'
'calculating displacement error
'
CONS = N * LMBDA * L * (1000 ^ 2) / 12
FOR I = 2 TO NS - 1
DELS(I) = CONS / (BAV(I) ^ 2)
NEXT I
'
'calculating uncorrected fringe temperatures
'
FOR I = 1 TO NS

```

```

C = LMBDA * R * TINFK / (L * K * P)
TUC(I) = TINFK / (1 - S(I) * C)
NEXT I
'
'calculating relative fringe position
'
FOR I = 1 TO NS
XREL(I) = X(I) - X(1)
NEXT I
'
'calculating end effect error
'
IF STRING1$ = DARK$ THEN D = XREL(NS)      'approx. boundary
layer edge.
FOR I = 2 TO NS - 1
NUMER = (D ^ 2 - XREL(I) ^ 2) ^ .5
DENOM = ((TUC(I) - TINFK) * TINFK) ^ .5
ENDEF = (TUC(I) - TINFK) / DENOM
ENDEF = ATN(ENDEF)
ENDEF = ENDEF / DENOM
ENDEF = (1 / TINFK) - ENDEF
ENDEF = NUMER * ENDEF
ENDEF = .3311 * P * ENDEF
DELS2(I) = ENDEF                        'end effect correction
NEXT I
'
'calculating ideal fringe shift
'
FOR I = 2 TO NS
SIDEAL(I) = S(I) + DELS(I) - DELS2(I)
NEXT I
'
'calculating the fringe temperatures
'
C = LMBDA * R * TINFK / (L * K * P)
FOR I = 2 TO NS
TIK(I) = TINFK / (1 - SIDEAL(I) * C)
TIC(I) = TIK(I) - 273.15
NEXT I
'
' display preliminary output for bav(2) correction
'
PRINT "  S      PC      X      B      BAV      DELS
      DELS2      SIDEAL"
B$ = "###.##  ###.##  ##.#####  ##.#####  ##.#####
###.#####  ##.####"
C$ = "                                ##.#####"
FOR I = 1 TO 7
PRINT USING B$; S(I); PC(I); X(I); BAV(I); DELS(I);
DELS2(I); SIDEAL(I)

```

```

PRINT USING C$; B(I)
NEXT I
'
'correct average fringe spacing next to wall
'
INPUT "ENTER NEW VALUE OF BAV(2)"; BAV(2)
DELS(2) = CONS / (BAV(2) ^ 2)
SIDEAL(2) = S(2) + DELS(2) - DELS2(2)
TIK(2) = TINFK / (1 - SIDEAL(2) * C)
TIC(2) = TIK(2) - 273.15
'
'calculating temperature difference per fringe
'
FOR I = 2 TO NS - 2
J = I - 1
DELTIC(I) = TIC(J) - TIC(I)
NEXT I
'
'calculating average temperature difference per fringe
'
FOR I = 2 TO NS - 2
J = I + 1
DELTICAV(I) = (DELTIC(I) + DELTIC(J)) / 2
NEXT I
'
' finding temperature gradient between the first and second
' fringes off the wall.
'
HFLUX23 = DELTIC(3) / B(2) ' degrees C / mm
'
'finding approximate surface temp. by extrapolating to the
'heater surface using the above underpredicted wall
gradient.
'
TIC(1) = HFLUX23 * B(1) + TIC(2)
PDIFTIC23 = ABS(TWALL - TIC(1)) * 100 / TWALL
'
'finding approx. surface temp. by linear extrapolation of
the
' ideal fringe shift values vs. x-position.
'
MS = (SIDEAL(2) - SIDEAL(3)) / (X(2) - X(3))
BS = SIDEAL(2) - MS * X(2)
SIDEAL(1) = BS + MS * X(1)
TWALLEXPS = (TINFK / (1 - C * SIDEAL(1))) - 273.15
PDIFTWALLEXPS = ABS(TWALL - TWALLEXPS) * 100 / TWALL
'
' calculating sideal difference per fringe
'
FOR I = 2 TO NS

```

```

J = I - 1
DSIDEAL(I) = SIDEAL(J) - SIDEAL(I)
NEXT I
'
'display corrected output
'

TEXT1$ = "CORRECTED OUTPUT"
TEXT2$ = "EXPERIMENT NO. "
TEXT3$ = "Y ="
TEXT4$ = "mm"
TEXT5$ = " deg C"
TEXT6$ = " mm HG"
TEXT7$ = " %"
TEXT8$ = " deg C / mm"

555 'exporting data to string1$.out file to use for plotting
'
'
' output info to data files
'
S2$ = STRING1$ + "A.DAT"
S3$ = STRING1$ + "S.DAT"
S4$ = STRING1$ + "DS.DAT"
S5$ = STRING1$ + "T.DAT"
S6$ = STRING1$ + "DT.DAT"
OPEN S2$ FOR OUTPUT AS #2
OPEN S3$ FOR OUTPUT AS #3
OPEN S5$ FOR OUTPUT AS #5
FOR I = 2 TO NS - 1
    WRITE #2, XREL(I), BAV(I), S(I), DELS(I), DELS2(I),
SIDEAL(I), TIC(I)
NEXT I
WRITE #2, XREL(NS), B(NS - 1), S(NS), DELS(NS), DELS2(NS),
SIDEAL(NS), TIC(NS)
CLOSE #2

FOR I = 2 TO NS
'
' create data file for SIDEAL vs. X plot
'
    WRITE #3, XREL(I), SIDEAL(I)
'
' create data file for T VS. X plot
'
    TRELC(I) = TIC(I) - TINFC
    WRITE #5, XREL(I), TRELC(I)

NEXT I
CLOSE #3

```

CLOSE #5

OPEN S4\$ FOR OUTPUT AS #4

OPEN S6\$ FOR OUTPUT AS #6

FOR I = 2 TO NS - 1

,

' create data file for - DSDX vs. X and - DTDX vs. X plot

,

J = I + 1

XRELMN = (XREL(I) + XREL(J)) / 2

DSDX = (SIDEAL(I) - SIDEAL(J)) / (XREL(J) - XREL(I))

DTDY = (TIC(I) - TIC(J)) / (XREL(J) - XREL(I))

WRITE #4, XRELMN, DSDX

WRITE #6, XRELMN, DTDY

NEXT I

CLOSE #4

CLOSE #6

,

999 ' incorporating bright fringes in the analysis

,

IF FLG1 = 1 THEN GOTO 222

TICDARK23 = TIC(1)

NUYDARK23 = NUYEXP23

PDIFNUYDARK23 = PDIFNUYEXP23

INPUT "DO YOU WANT TO USE BRIGHT FRINGES TOO? { Y=1, N=2 }

", FLG1

IF FLG1 = 2 THEN GOTO 333

INPUT "ENTER EXPERIMENT I.D. NUMBER FOR BRIGHT FRINGES";

STRING1\$

BRIGHT\$ = STRING1\$

,

'importing data from previously created file named STRING1\$

,

OPEN STRING1\$ FOR INPUT AS #1

INPUT #1, LE, YRP, PRTOM, YMM

INPUT #1, TWALL, TINFC, TINFK

INPUT #1, P

INPUT #1, PCTOM

INPUT #1, NS

FOR I = 1 TO NS

INPUT #1, S(I), PC(I)

NEXT I

CLOSE #1

DIM XXREL(2 * NS)

DIM BBAV(2 * NS)

DIM SS(2 * NS)

DIM DELSS(2 * NS)

```

DIM DELSS2(2 * NS)
DIM SSIDEAL(2 * NS)
DIM TTIC(2 * NS)
DIM TTRELC(2 * NS)
DIM TWSFITXXRELC(10)
DIM TWXFITXXRELC(10)

PRINT " "
'
222 'combining data from dark and bright fringes
'
TICBRIGHT23 = TIC(1)
NUYBRIGHT23 = NUYEXP23
PDIFNUYBRIGHT23 = PDIFNUYEXP23
IND$ = DARK$ + "A.DAT"
INB$ = BRIGHT$ + "A.DAT"

OPEN IND$ FOR INPUT AS #7
OPEN INB$ FOR INPUT AS #8
FOR I = 2 TO 2 * (NS - 1) STEP 2
    J = I + 1
    INPUT #8, XXREL(I), BBAV(I), SS(I), DELSS(I),
DELSS2(I), SSIDEAL(I), TTIC(I)
    INPUT #7, XXREL(J), BBAV(J), SS(J), DELSS(J),
DELSS2(J), SSIDEAL(J), TTIC(J)
NEXT I
CLOSE #7
CLOSE #8
'
' output combined info to data files
'
S9$ = DARK$ + "3S.DAT"
S10$ = DARK$ + "3DS.DAT"
S11$ = DARK$ + "3T.DAT"
S12$ = DARK$ + "3DT.DAT"
S13$ = BRIGHT$ + "DS.DAT"
S14$ = DARK$ + "DS.DAT"
OPEN S9$ FOR OUTPUT AS #9
OPEN S11$ FOR OUTPUT AS #11
FOR I = 2 TO 2 * NS - 1
'
'create data file for SIDEAL vs. X plot
'
    WRITE #9, XXREL(I), SSIDEAL(I)
'
' create data file for T VS. X plot
'
    TTRELC(I) = TTIC(I) - TINFC
    WRITE #11, XXREL(I), TTRELC(I)

```

```

NEXT I
CLOSE #9
CLOSE #11
OPEN S10$ FOR OUTPUT AS #10
OPEN S12$ FOR OUTPUT AS #12
OPEN S13$ FOR INPUT AS #13
OPEN S14$ FOR INPUT AS #14
'
' create data file for - DSDX vs. X
'
FOR I = 2 TO NS - 1
    INPUT #13, XRELMN, DSDX
    WRITE #10, XRELMN, DSDX
    INPUT #14, XRELMN, DSDX
    WRITE #10, XRELMN, DSDX
NEXT I
CLOSE #13
CLOSE #14
'
' create data file for - DSDX vs. X and - DTDX vs. X plot
'
FOR I = 2 TO 2 * NS - 2
    J = I + 1
    XRELMN = (XXREL(I) + XXREL(J)) / 2
    DSDX = (SSIDEAL(I) - SSIDEAL(J)) / (XXREL(J) -
XXREL(I))
    DTDX = (TTIC(I) - TTIC(J)) / (XXREL(J) - XXREL(I))
    WRITE #10, XRELMN, DSDX
    WRITE #12, XRELMN, DTDX
NEXT I
CLOSE #10
CLOSE #12

'
' setup least squares fit program to fit six data sets to
' a second order curve.
'

NN = 9
ORD = 1
DIM EX(20)
DIM FX(NN)
DIM FY(NN)
DIM EXY(NN)
DIM A(NN, NN)
DIM BB(NN)
DIM BBB(2, NN, ORD + 1)
DIM SIGMAY(20)
DIM SIGMAYX(20)

```

```

DIM CC(20)
DIM C1(20)
DIM DC1X(9, NN)
DIM DC1Y(9, NN)
DIM WC1(20)
DIM ETAC1(10)
DIM C2(20)
DIM DC2X(9, NN)
DIM DC2Y(9, NN)
DIM WC2(20)
DIM ETAC2(20)
DIM WTWALL(NN)
DIM ETATWALL(NN)
DIM WDTWALL(NN)
DIM ETADTWALL(NN)
DIM ETAY(NN)
DIM ETANUY(NN)
DIM ETAT(2 * NS)
DIM WT(2 * NS)
'
'
' extrapolating to the wall
'
'
' interpolation using XXREL values to find SSIDEALwall
'
'LPRINT "Interpolation using X values to find SIDEALwall :"
'LPRINT USING "SIDEAL(x,y=#.##) = C1 + C2*X + ....."; YMM
DATAINDEX = 1
FOR I = 2 TO NN + 1
J = I - 1
FX(J) = XXREL(I)
FY(J) = SSIDEAL(I)
NEXT I
GOSUB LSQRS
SWFITXXREL = BB(1) + XXREL(1) * BB(2)
TWFITXXRELK = TINFK / (1 - SWFITXXREL * C)
TWSFITXXRELC(NN) = TWFITXXRELK - 273.15
'
'LPRINT "T(x,y) = Tinf/(1 - A*S); A = "; C
'LPRINT " "
'LPRINT USING "Twall = ##.### C"; TWSFITXXRELC(NN)
'
'      L           P           R           I           N           T
'-----"
'LPRINT " "
'
' finding temperature flux dT/dX |wall
'
'

```

```

' interpolation using XXREL values to find dT/dX |wall :
,
DATAINDEX = 2
FOR NN = 4 TO 6
FOR I = 2 TO NN + 1
J = I - 1
FX(J) = XXREL(I)
FY(J) = TTIC(I)
NEXT I
GOSUB LSQRS
PRINT "CC"; NN; " = "; CC(NN)
'LPRINT "CC"; NN; " = "; CC(NN)
NEXT NN
INPUT "How many fringes should be used for the curve fit? ";
NN
FOR I = 2 TO NN + 1
J = I - 1
FX(J) = XXREL(I)
FY(J) = TTIC(I)
NEXT I
GOSUB LSQRS
,
' finding properties based on interferometry plate temp.
,
'AIR STANDARDS TAKEN FROM CIRCULAR SB4 PUBLISHED BY
'NATIONAL BUREAU OF STANDARDS (obtained from M. KEYHANI)
'*****
TK = (BB(1) + 273.15 + TINFK) / 2      'film temperature
TK1 = 6.325E-06 * (TK ^ 1.5)
G = -12 / TK
TK2 = 245.4 * (10 ^ G)
KAIR = 418.3709 * TK1 / (TK + TK2):    'W/M-K
BETA = 1 / TK:                        '1/K COEF. OF THERMAL
EXPANSION
CPAIR = 981.5714 + .085714 * TK:      'J/KG-K
DENSITY = 352.5 / TK:                 'KG/M^3 DRY AIR @
ATM. PRESS.
DYNVIS = 1.458E-06 * (TK ^ 1.5) / (TK + 110.4): 'KG/M-S
KINVIS = DYNVIS / DENSITY:             'M^2/S
ALPHA = KAIR / (DENSITY * CPAIR):      'M^2/S
PR = KINVIS / ALPHA:                   'PRANDTL NUMBER
GNUMER = .75 * (PR) ^ .5
GDENOM = (.609 + 1.221 * (PR) ^ .5 + 1.238 * PR) ^ .25
GPR = GNUMER / GDENOM
GRPROP = 9.810001 * BETA / (KINVIS ^ 2) '1/(K*M^3)
GRPROP = GRPROP / (1000 ^ 3)           'convert to 1/K-mm^3
RAPROP = GRPROP * PR                   '1/(K-mm^3)
,
' Temperature based on interferometric data
,

```

```

TWXFITXXREL(NN) = BB(1)
'
' Nusselt No., and Grahsof No. based on interferometric data
'
' Theoretical values based on interferometry wall temp.
GRY = GRPROP * (BB(1) - TINFC) * YMM ^ 3
RAY = RAPROP * (BB(1) - TINFC) * YMM ^ 3
NUYTHEO = GPR * (GRY / 4) ^ .25
'
' Nusselt number based on interferometry
'
GRADWFITXXREL = BB(2)
NUYFIT = -GRADWFITXXREL * YMM / (TWXFITXXREL(NN) - TINFC)
PDIFNUYFIT = ABS(NUYFIT - NUYTHEO) * 100 / NUYTHEO
OPEN "NUYVSGRY.DAT" FOR APPEND AS #13
WRITE #13, GRY, NUYFIT
CLOSE #13
' Nusselt number based on thermocouple measurement of wall
temp.
NUYCOUPLE = -BB(2) * YMM / (TWALL - TINFC)
PDIFNUYCOUPLE = ABS(NUYCOUPLE - NUYTHEO) * 100 / NUYTHEO
OPEN "NUGR2.DAT" FOR APPEND AS #15
WRITE #15, GRY, NUYCOUPLE
CLOSE #15
'
' Finding the uncertainty in the variables
'
GOSUB UNCERT
'
' Printing the final output
'
LPRINT CHR$(27); "x1";
LPRINT ""
G$ = "TABLE      : Summary of fringe data and results at y/h
= #.## "
LPRINT USING G$; YMM / H
LPRINT " "
LPRINT "  S          x (mm)      S1          S2          SIDEAL      T (C)
w (C)"
LPRINT ""
F$ = "###.##      #.###      #.###      #.###      ##.###      ##.##
#.##"
FOR I = 2 TO 2 * NS - 1
LPRINT USING F$; SS(I); XXREL(I); DELSS(I); DELSS2(I);
SSIDEAL(I); TTIC(I); WT(I)
NEXT I
L      P          R          I          N          T
"-----"
---"
LPRINT " "

```

```

LPRINT "Extrapolation using x values to find Twall :"
```

$$T(x,y=##.##) = C1 + C2 \cdot x$$

```

LPRINT USING "Number of data sets used = #"; NN
LPRINT USING "C1 = ##.### +/- ##.### C"; BB(1); WC1(NN)
LPRINT USING "C2 = ###.### +/- ##.### C/mm"; BB(2); WC2(NN)
LPRINT USING "Correlation coefficient = #.#####"; CC(NN)
LPRINT USING "Standard error = ##.### C"; SIGMAYX(NN)
'LPRINT "SIGMAY = "; SIGMAY(NN)
LPRINT ""
LPRINT "Wall Temperature:"
LPRINT ""
LPRINT "Average of the upper wall section is .. 63.77 +/-
0.7 C"
LPRINT "Average of the lower wall section is .. 63.60 +/-
0.7 C"
LPRINT "Average of the wall is ..... 63.68 +/-
0.7 C"
LPRINT USING "Determination by interferometry is .... ##.##
+/- ##.## C"; TWXFITXXREL(NN); WTWALL(NN)
LPRINT ""
LPRINT ""
LPRINT USING "Grashof Number = #####.## +/- ##.## %"; GRY;
ETAGRY(NN) * 100
LPRINT ""
LPRINT ""
LPRINT "Nusselt Number:"
LPRINT ""
LPRINT USING "Theoretical .....
##.### "; NUYTSEO
LPRINT USING "Using interferometry wall temperature ..
##.### +/- ##.### "; NUYFIT; ABS(WNUY(NN))
LPRINT USING "Using thermocouple wall temperature ....
##.### +/- ##.### "; NUYCOUPLE; ABS(WNUYCOUPLE(NN))
LPRINT " "

```

333 END

LSQRS:

```

'*****
'*** W.P. PARTRIDGE JR. *****
'*** PROGRAM NAME = LSQRS2.BAS *****
'*** PROGRAMMED IN QUICK BASIC *****
'
' to find the temperature of the "wall fringe", a first
order curve
' is fit to the NN fringes and their corresponding
temperatures
' closest to the wall. The "wall fringe" temperature is
then found

```

```

' by extrapolating back to the "wall"
'
' least squares fit of a ORD order polynomial
'
'  $FY = C1 + C2(FX)^1 + C3(FX)^2$ 
'  $FY(I), FX(I)$  where  $I=2,3,4,\dots$ 
'
' initializing variables to zero
FOR I = 1 TO NN
  DC2X(NN, I) = 0
  DC2Y(NN, I) = 0
  WC2(NN) = 0
  YM = 0
  TIC = 0
  SIGMAY = 0
  SIGMAYX = 0
  FOR J = 1 TO NN
    A(I, J) = 0
  NEXT J
  BB(I) = 0: EXY(I) = 0
NEXT I
FOR I = 1 TO 10
  EX(I) = 0
NEXT I
'
EX(0) = NN
'
FOR I = 1 TO NN
  FOR J = 1 TO 2 * ORD
    EX(J) = EX(J) + FX(I) ^ J
  NEXT J
  FOR J = 1 TO ORD + 1
    IF J = 1 THEN EXY(J) = EXY(J) + FY(I)
    IF J <> 1 THEN EXY(J) = EXY(J) + FY(I) * (FX(I) ^ (J - 1))
  NEXT J
NEXT I
'form of least squares matrix
'
'      |  NN      EX1      EX2 |      |  C1  |      |  EXY1  |
'      |  EX      EX2      EX3 |      |  C2  |      |  EXY2  |
'      |  EX2      EX3      EX4 |      |  C3  |      |  EXY3  |
'
'
' Gauss elimination method
'
' I'th row, J'th column
FOR I = 1 TO ORD + 1 'rows
FOR J = 1 TO ORD + 1 'columns

```

```

K = I + J - 2
A(I, J) = EX(K)
NEXT J
BB(I) = EXY(I)
NEXT I
,
FOR IPV = 1 TO ORD + 1
    PIVOT = A(IPV, IPV)
    IF ABS(PIVOT) < .000001 THEN PRINT "NEAR SINGULAR", IPV,
    IPV, PIVOT
    'normalize pivot row by dividing by pivot
    FOR J = 1 TO ORD + 1
        A(IPV, J) = A(IPV, J) / PIVOT
    NEXT J
    BB(IPV) = BB(IPV) / PIVOT
,
' "replace row-irow with row-irow+factor*(pivot-row)
' with factor chosen to obtain zero in pivot column"
,
FOR IROW = 1 TO ORD + 1
    IF IROW = IPV THEN 3260
        FACTOR = A(IROW, IPV)
        FOR ICOL = 1 TO ORD + 1
            A(IROW, ICOL) = A(IROW, ICOL) - FACTOR *
A(IPV, ICOL)
        NEXT ICOL
        BB(IROW) = BB(IROW) - FACTOR * BB(IPV)
3260 NEXT IROW
NEXT IPV
FOR I = 1 TO ORD + 1
    BBB(DATAINDEX, NN, I) = BB(I)
NEXT I
,
' finding the correlation coefficient
' see method in Holman p.87
,
FOR I = 1 TO NN
    YM = YM + FY(I)
NEXT I
YM = YM / NN ' mean value of y
FOR I = 1 TO NN
    SIGMAY = SIGMAY + (FY(I) - YM) ^ 2
    YIC = BB(1)
    FOR J = 2 TO ORD + 1
        YIC = YIC + BB(J) * (FX(I) ^ (J - 1))
    NEXT J
    SIGMAYX = SIGMAYX + (FY(I) - YIC) ^ 2
NEXT I
SIGMAY(NN) = (SIGMAY / (NN - 1)) ^ .5
SIGMAYX(NN) = (SIGMAYX / (NN - 2)) ^ .5

```

```

CC(NN) = (1 - (SIGMAYX(NN) / SIGMAY(NN)) ^ 2) ^ .5
RETURN
'*****

UNCERT:
'*****'
'**** W.P. PARTRIDGE JR. *****'
'*** PROGRAM NAME = UNCERT.BAS *****'
'*** PROGRAMMED IN QUICK BASIC *****'
'
' Subroutine to find the uncertainties in the variables
'
WC1(NN) = 0
WC2(NN) = 0

WX = .03      'uncertainty in x position in (mm)
WY = .024     'uncertainty in y position in (mm)
WTINF = .1    'uncertainty in T-infinity in (deg. C )
ETATINF = WTINF / TINFC
WL = .254     'uncertainty in heater length in (mm)
ETAL = WL / LMM
WP = 1.27     'uncertainty in P-infinity in (TORR)
ETAP = WP / P

'
' Finding the uncertainty in the temperatures
'
FOR I = 2 TO 2 * NS - 1
J = I - 1
' uncertainty in a:
ASMULT = C * SSIDEAL(I) / (1 - C * SSIDEAL(I))
ETAA = ASMULT * ((ETATINF ^ 2 + ETAL ^ 2 + ETAP ^ 2) ^ .5)
' uncertainty in S:
ETAS = (DELSS(I) ^ 2) * (ETAL ^ 2 + 8 * ((WX / BBAV(I)) ^ 2))
ETAS = ETAS + DELSS2(I) ^ 2
ETAS = (ASMULT / SSIDEAL(I)) * (ETAS ^ .5)
' uncertainty in T(x,y):
ETAT(J) = (ETATINF ^ 2 + ETAA ^ 2 + ETAS ^ 2) ^ .5
'LPRINT USING "The uncertainty in fringe # is ###.### %"; J;
ETAT(J) * 100
WT(I) = ETAT(J) * TTIC(I)
NEXT I
'
' finding the slope directly as specified by a = ... in
' Holman p.74
'
TOP = NN * EXY(2) - EX(1) * EXY(1)

```

```

TOP2 = EXY(1) * EX(2) - EXY(2) * EX(1)
BOTTOM = NN * EX(2) - EX(1) ^ 2
C2(NN) = TOP / BOTTOM
'
' Evaluating the derivatives of C2
' Obtained from error propagation in "a" of Holman p.
' dc2/dX = DC2X ; dc2/cY = DC2Y
'
FOR I = 1 TO NN
DC2X(NN, I) = (BOTTOM * (NN * FY(I) - EXY(1)) - 2 * TOP *
(NN * FX(I) - EX(1))) / (BOTTOM ^ 2)
DC2Y(NN, I) = (NN * FX(I) - EX(1)) / BOTTOM
WC2(NN) = WC2(NN) + (DC2X(NN, I) * WX) ^ 2 + (DC2Y(NN, I) *
ETAT(I) * FY(I)) ^ 2
NEXT I
WC2(NN) = (WC2(NN)) ^ .5 'uncertainty in C2
ETAC2(NN) = WC2(NN) / C2(NN)
'
' Evaluating the derivatives of C1
' Obtained from error propagation in "b" of Holman p.
' dc1/dX = DC1X ; dc1/dY = DC1Y
'
C1(NN) = BB(1)
FOR I = 1 TO NN
DC1X(NN, I) = (BOTTOM * (2 * FX(I) * EXY(1) - FY(I) * EX(1)
- EXY(2)) - 2 * TOP2 * (NN * FX(I) - EX(1))) / (BOTTOM ^ 2)
DC1Y(NN, I) = (EX(2) - FX(I) * EX(1)) / BOTTOM
WC1(NN) = WC1(NN) + (DC1X(NN, I) * WX) ^ 2 + (DC1Y(NN, I) *
ETAT(I) * FY(I)) ^ 2
NEXT I
WC1(NN) = (WC1(NN)) ^ .5 'uncertainty in C1
ETAC1(NN) = WC1(NN) / C1(NN)
'
'Evaluating the uncertainty in T-wall = WTWALL
'
WTWALL(NN) = (WC1(NN) ^ 2 + (C2(NN) * XXREL(2)) ^ 2) ^ .5
ETATWALL(NN) = WTWALL(NN) / C1(NN)
'
'Evaluating the uncertainty in delta Tw = (Twall -
Tinfinity)
'
WDTWALL(NN) = (WTWALL(NN) ^ 2 + WTINF ^ 2) ^ .5
ETADTWALL(NN) = WDTWALL(NN) / (TWXFITXXREL(NN) - TINFC)
WDTWALLCOUPLE(NN) = (.7 ^ 2 + WTINF ^ 2) ^ .5
ETADTWALLCOUPLE(NN) = WDTWALLCOUPLE(NN) / (TWALL - TINFC)
'
'Evaluating the uncertainty in the y-location
'
ETAY(NN) = WY / YMM

```

```

|
| ' evaluating the uncertainty in Nu(y) using interferometry
| wall temp.
|
ETANUY(NN) = (ETAC2(NN) ^ 2 + ETAY(NN) ^ 2 + ETADTWALL(NN)
^ 2) ^ .5
WNUY(NN) = ETANUY(NN) * NUYFIT
|
| 'Evaluating the uncertainty in Nu(y) using thermocouple wall
| temp.
|
ETANUYCOUPLE(NN) = (ETAC2(NN) ^ 2 + ETAY(NN) ^ 2 +
ETADTWALLCOUPLE(NN) ^ 2) ^ .5
WNUYCOUPLE(NN) = ETANUYCOUPLE(NN) * NUYCOUPLE
|
| 'Evaluating the uncertainty in Gr(y)
|
ETA2TW = 2 * TINFC * WTWALL(NN) / (BB(1) ^ 2 - TINFC ^ 2)
ETA2TINF = -2 * BB(1) * WTINF / (BB(1) ^ 2 - TINFC ^ 2)
ETAGRY(NN) = (ETA2TW ^ 2 + ETA2TINF ^ 2 + 9 * ETAY(NN) ^ 2)
^ .5

RETURN
| *****

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VITA

William Powell Partridge Jr. was born in Knoxville, Tennessee on January 19, 1964. He attended The Webb School of Knoxville and graduated in the Spring of 1983. The following fall, he began undergraduate study at Auburn University. While at Auburn, Mr. Partridge entered the Co-op program where he worked with Du Pont at the Savannah River Plant. In the fall of 1985 he transferred to The University of Tennessee where he completed the Bachelor of Science in Mechanical Engineering program in the summer of 1988. The following fall of 1988 he accepted a teaching assistantship with the Department of Mechanical and Aerospace Engineering of The University of Tennessee and began working towards his Master's degree. This degree was conferred in August of 1990.

Mr. Partridge is a member of Tau Beta Pi, Pi Tau Sigma, Phi Kappa Phi, and the American Society of Mechanical Engineers. Mr. Partridge will continue work at Purdue University towards a Doctor of Philosophy degree in the fall of 1990.