Multiple Positron Emission Particle Tracking Using the Gap Statistic

Matthew Herald – Nuclear Engineering

Introduction

Multiple Positron Emission Particle Tracking (mPEPT) is the 3D position reconstruction of radio-labeled tracers in a flow or process.

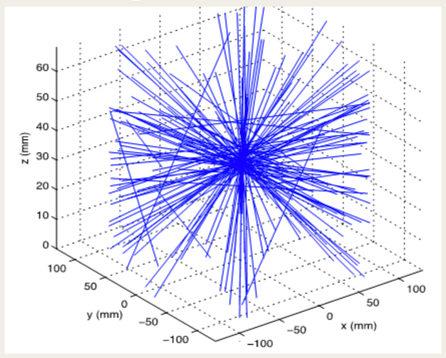


Fig. 1. 511 keV gammas produced by positron annihilation near radio-tracer.

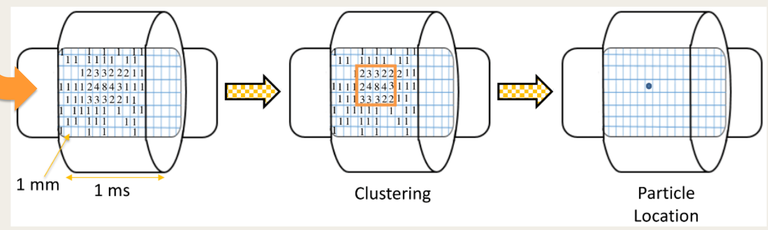
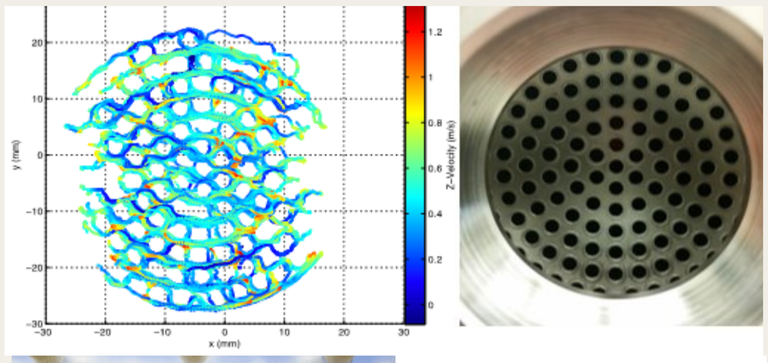


Fig. 2. mPEPT measurement using PET scanner to detect 511 keV gammas.

Highly penetrating gamma radiation passes through pipe walls and opaque media.

mPEPT measurements can be used where optical methods can't.



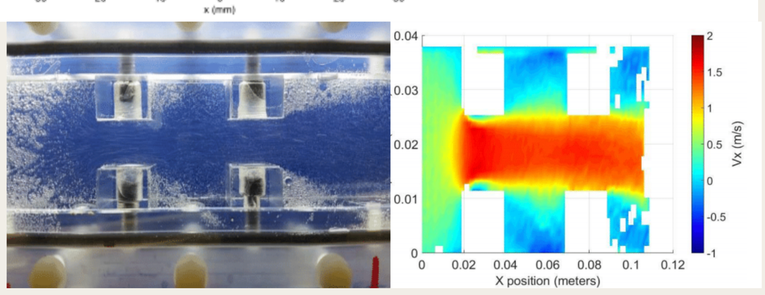


Fig. 3. Multiple trajectories can be used to develop 3D velocity fields.

Opportunity

In pipe flow, tracers are simultaneously entering and exiting the imaging volume altering the number of particles tracked.

Clustering algorithms must be passed a value for the number of clusters as a search parameter.

Determining the number of clusters in a dataset without prior knowledge is difficult; poor values cause clustering inaccuracies.

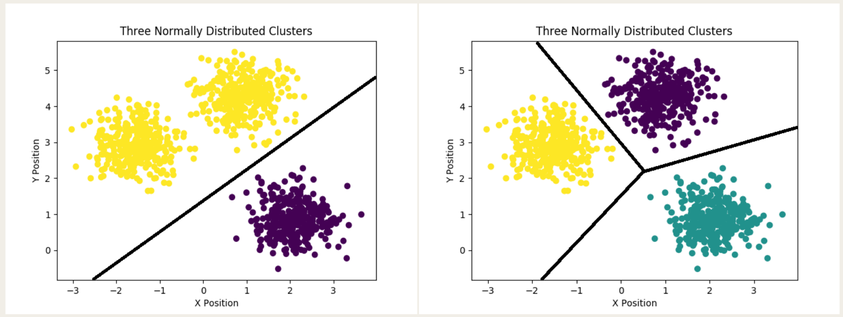


Fig. 4. Same-set clustering using a value of 2 (left) and 3 (right) for the number of clusters.

An <u>autonomous, robust method</u> for determining the number of clusters using measurable attributes of dataset must be developed.

This value could then be passed to clustering algorithm, <u>eliminating user input</u> needed by other methods.

Positions are then linked into tracer trajectories to study attributes of the flow.

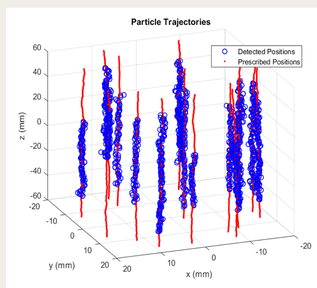


Fig. 5. mPEPT tracer trajectories from a simulated pipe flow experiment using pre-existing mPEPT software.

Solution

The gap statistic is a method used to measure the 'togetherness' of a dataset.

Scoring several clustering runs with incremental values for the number of clusters, the best scoring value is used to calculate tracer positions.

$$Residuals = \sum_{i=1}^{n} \sqrt{(x_i - x_{centroid})^2 + (y_i - y_{centroid})^2}$$

$$Gap = \ln residuals_{k_{null\,reference}} - \ln residuals_k$$

Apply null reference: evenly distributed data will result in the highest residual.

The largest 'gap' reveals the correct number of clusters.

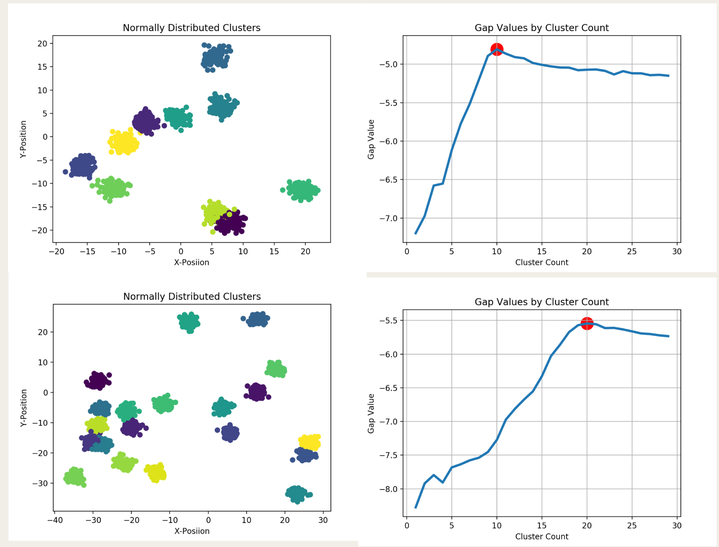


Fig. 6. Demonstration of gap statistic on normally distributed simple test cases.

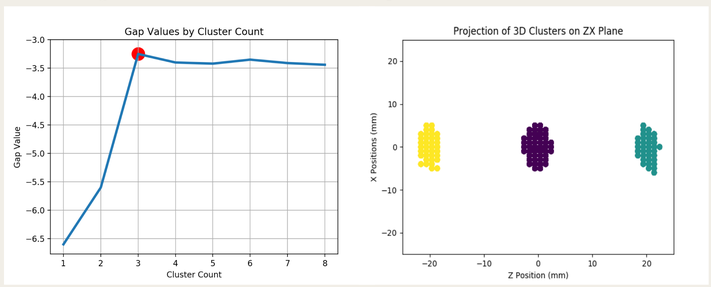


Fig. 7. Three stationary PEPT tracers, simulated under representative* conditions.

TABLE I. Reconstructed Position Errors

Tracer	|Δx| mm	|Δy| mm	|Δz| mm
1	0.0062	0.0615	0.0482
2	0.0716	0.0898	0.0795
3	0.0537	0.0888	0.0021
Average	0.0438	0.0800	0.0433

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- Verification of method is conducted using Geant4 Application for Tomographic Emission (GATE), which simulates the decay physics and electronic chain of a PET scanner. An independently produced model of the Inveon PET scanner is used for the measurement.

Wiggins, C., Santos, R., Ruggles, A., "A novel clustering approach to positron emission particle tracking," Nucl. Instr. Meth. Phys. Res. Sec. A, vol. 811, pp. 18-24, (2016)