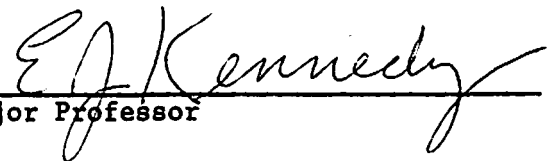


November 19, 1970

To the Graduate Council:

I am submitting herewith a dissertation written by William Pinkston Albritton, Jr., entitled "Application of Positive Feedback Techniques to Charge-Sensitive Preamplifiers." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Electrical Engineering.


Major Professor

We have read this dissertation
and recommend its acceptance:









Accepted for the Council:


Vice Chancellor for
Graduate Studies and Research

**APPLICATION OF POSITIVE FEEDBACK TECHNIQUES
TO CHARGE-SENSITIVE PREAMPLIFIERS**

**A Dissertation
Presented to
the Graduate Council of
The University of Tennessee**

**In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy**

**by
William Pinkston Albritton, Jr.
December 1970**

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W. P. A.

December, 1970

ABSTRACT

The application of positive feedback techniques to charge-sensitive preamplifiers for the purpose of improving their performance characteristics and versatility is considered. Improvements in sensitivity of charge gain to changes in input capacitance, preamplifier output pulse rise-time, and ability to terminate long input cables are discussed. In each case, theoretical developments are carried out in order to determine the optimum positive feedback conditions. A practical charge-sensitive preamplifier design is discussed and the effects of applying positive feedback are delineated.

For the experimental preamplifier, the application of positive feedback resulted in a reduction in charge gain sensitivity to input capacitance changes of almost an order of magnitude for a 100 pf. change in input capacitance. The output pulse rise-time without positive feedback was approximately 90 nsec. with 100 pf. detector capacitance. This was reduced to approximately 15 nsec. by the application of positive feedback. The equivalent noise charge of the experimental preamplifier was approximately 4×10^{-17} rms coulombs with 0 pf. detector capacitance and for 1 μ sec. RC-RC shaping. The use of positive feedback did not affect the noise performance. The experimental preamplifier was not designed with low noise as a prime requisite.

The application of positive feedback was shown to provide tremendous improvement in the ability of the experimental preamplifier

to terminate long input cables. Different cables with characteristic impedances, Z_0 , from 50 ohms to 950 ohms were attached to the input of the preamplifier. Theoretical equations were developed for the positive feedback conditions which would cause the input impedance of the preamplifier to be equal to the Z_0 of the cables. Experimental results provided excellent confirmation of the theory. Output signal waveforms, which formerly were completely useless for normal pulse-shaping networks, were in many cases essentially undistorted after correct use of the cable-termination theory.

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CHAPTER 1

INTRODUCTION

A. Background

In a nuclear detection system the nuclear preamplifier is assigned the task of accepting the output signal of a nuclear radiation detector, usually a current pulse, and operating on that current pulse in such a manner that the preamplifier output signal will yield information concerning nuclear radiation incident on the detector.

The information required may be the amount of energy given up in the detector by the incident radiation, the time at which the incident radiation penetrates the detector, or both.

Timing information is useful, for example, in multiple detector systems where the time fiducials extracted from two different detectors may be used to determine the velocity of an incident particle.

The total energy of an incident particle would be the sum of the energies given up by the particle in each detector, including the detector in which the particle stopped.

Particle identification could then be accomplished by using the energy and velocity to determine particle mass.

As a particle penetrates a detector, it ionizes some of the detector atoms along its path. Since each ionization requires a specific expenditure of particle energy, the exact amount depending on the detector material, and since each ionization produces a specific

charge, one hole-electron pair, the energy given up by the particle is directly proportional to the amount of charge produced.

Because the amount of charge produced in the detector is linearly dependent upon the energy given up by the incident radiation, the area under the detector's output current pulse contains the desired energy information. Extraction of energy information therefore requires integration of the output current pulse from the detector.

The required integration may be accomplished by depositing the charge contained in the current pulse on a capacitor. The resulting voltage on the capacitor is proportional to the deposited charge and, therefore, to the energy of the incident radiation. This process may be carried out in either of three ways. The capacitor which collects the charge may be placed at the preamplifier output, at the preamplifier input, or from input to output of the preamplifier as a feedback element.

The first method, with the charge collection capacitor placed at the preamplifier output, dictates amplification of the output current pulse from the detector by the preamplifier and deposition of the amplified current pulse on the charge collection capacitance. That is, the preamplifier must be a current amplifier. This is generally referred to as a current-sensitive configuration. The current amplifier used in this configuration must have low-noise characteristics and be fast enough to amplify the very short (typically less than 10 nsec.) current pulses from the detector. Present semiconductor devices are

capable of producing the necessary speed of response. High-speed and low-noise are, however, conflicting requirements in current amplifiers.¹ The result of the conflict is that present current-sensitive systems have poorer noise performance than that which can be realized with the other techniques.

It is sometimes necessary to deliver the output current pulse from the detector to the preamplifier via a long cable. A long cable, in this context, means one whose time delay is long compared to the rise-time constant of the preamplifier. This same meaning will also be ascribed to the term "long cable" in subsequent discussions. Since the detector appears to be a current source, the cable will not be terminated at the sending-end. A current amplifier at the receiving-end, however, can provide a good termination² and multiple reflection problems on the input cable would then be eliminated.

The second method, with the charge-collection capacitor placed at the preamplifier input, dictates amplification by the preamplifier of the voltage developed on the charge-collection capacitor. The preamplifier must be a voltage amplifier, which will be hereafter referred to as the voltage-sensitive case.

The output pulse rise-time of the voltage-sensitive preamplifier must be short compared to the time constants (typically in the micro-second range) associated with the pulse shaping networks which follow the preamplifier. Necessary speeds can be fairly easily obtained. Usually, the output pulse rise-time is determined by the dominant time-

constant within the preamplifier, which means that the rise-time will not be altered by changes in the detector's capacitance.³

For best noise performance, the charge-collection capacitance is comprised of the detector capacitance in parallel with the preamplifier input capacitance. Variations in detector capacitance will therefore cause changes in output pulse height.

Further, if the input to the voltage-sensitive preamplifier is connected to a detector via a long cable, the cable will not be terminated at either end and multiple reflections will therefore result.

The third method, with the charge-collection capacitor placed from input to output of the preamplifier as a feedback element, constitutes an operational integrator. The amplifier is, therefore, a voltage amplifier. Since the preamplifier output voltage is proportional to the integral of the detector's output current pulse (charge), this configuration is designated as a charge-sensitive configuration.

The rise-time of the preamplifier output pulse is usually determined by the closed-loop rise-time of the preamplifier. Since the feedback ratio at high frequencies is dependent on the total capacitance at the preamplifier input, and the detector capacitance is part of that total, the output pulse rise-time will change with variations in detector capacitance.

This variation need not be a serious problem, however, because the rise-time for a charge-sensitive configuration can be faster than the rise-time for a voltage-sensitive system by a factor equal to the feedback ratio. As previously stated, the output pulse rise-time must

be short compared to the time constants associated with the shaping networks which follow the preamplifier. Since this constraint can be satisfied with a voltage-sensitive configuration, it can also be satisfied with a charge-sensitive configuration whose rise-time, though varying, may be faster than that for a voltage-sensitive configuration.

Output pulse height is not as sensitive to detector capacitance variations for the charge-sensitive case as for the voltage-sensitive case. This is because the feedback capacitance appears, when reflected to the input, to be multiplied by the gain magnitude of the open-loop amplifier. Thus, the detector capacitance is only a small portion of the total effective capacitance on which the detector's charge is deposited. This results in a greatly reduced output pulse-height sensitivity to detector capacitance variations.

Noise performance for the charge-sensitive and voltage-sensitive cases is not appreciably different.

Long cables feeding the input of the charge-sensitive preamplifier will not be terminated because the dominant component in the preamplifier's effective input impedance is the feedback capacitance multiplied by the open-loop gain magnitude. As with the voltage-sensitive preamplifier, the charge-sensitive preamplifier may have multiple reflection problems when driven from long input cables.

For a particular experiment the most appropriate energy detection technique of the three discussed above will, of course, depend upon the requirements of the experiment itself. A requirement of best noise performance suggests the voltage-sensitive or charge-

sensitive configuration. In general, the charge-sensitive preamplifier's lower sensitivity to input capacitance variations makes it more attractive than the voltage-sensitive preamplifier. Where long cables are unavoidable, the current-sensitive preamplifier might be the proper choice. Conflicting requirements will force compromises in many systems. At any rate, the charge-sensitive preamplifier offers many advantages when extracting the energy information from an appropriate detector.

The subject of this thesis is the improvement of the performance characteristics and versatility of charge-sensitive preamplifiers by application of positive feedback techniques.

B. Scope of the Thesis

Positive feedback has been applied to charge-sensitive preamplifiers previously by Chase et al.,⁴ Blalock,^{5,6} Radeka,^{7,8} and others who designed preamplifiers utilizing "bootstrapped"⁹ dynamic load impedances for the purpose of increasing the open loop gain. The loop-gain of these "bootstrap" positive feedback circuits is constrained to be less than unity. Loop gains equal to or greater than unity are possible in the positive feedback configurations reported by Fairstein,¹⁰ Hahn and Mayer,^{11,12} Goldsworthy,¹³ and Hill and Albritton.¹⁴ Configurations of the latter type will be considered in this thesis. The Fairstein and Goldsworthy preamplifiers actually contained both types of positive feedback.

The positive feedback efforts referenced above were directed toward further reducing the sensitivity of the closed-loop charge-

sensitive preamplifier's charge-to-voltage gain to input capacitance variations by providing increased open-loop gain. Complete theoretical treatments of the sensitivities resulting from the use of positive feedback in those preamplifiers have not been presented. Theoretical analysis of the effect of positive feedback on noise performance is also lacking.

Chapter 2 of this thesis presents theoretical treatments of positive feedback applied to a charge-sensitive preamplifier for the purposes of making the charge gain of the preamplifier insensitive to input capacitance variations and of reducing the preamplifier output pulse rise-time. Past efforts are briefly discussed and equations are developed for finding the correct amount of positive feedback necessary to attain the established goals. Oscillation stability and noise performance are considered.

Chapter 3 considers the very practical case of termination problems which occur when a charge-sensitive preamplifier must be fed from a long input cable. A positive feedback technique is developed which will allow a charge-sensitive preamplifier to provide an effective termination for a long input cable. The output pulse expression for a preamplifier having a long input cable is developed. Effects of long input cables on preamplifier output noise are briefly discussed.

Experimental results are shown in Chapter 4. The design of an experimental preamplifier is discussed. Positive feedback is applied to the experimental preamplifier in accordance with the results of the analyses in Chapters 2 and 3. The theoretical and experimental

performance characteristics of the preamplifier with and without positive feedback are compared.

A brief summary of the work is given in Chapter 5. Also included are suggestions for further study in the application of positive feedback techniques.

CHAPTER 2

IMPROVING THE CHARGE GAIN STABILITY OF CHARGE-SENSITIVE PREAMPLIFIERS BY APPLICATION OF POSITIVE FEEDBACK

In this chapter the variations in charge gain of a charge-sensitive preamplifier due to changes in capacitance at the preamplifier's input are considered. Charge gain is by definition the ratio of peak output voltage to input charge.

Two methods of applying positive feedback for stabilizing the charge gain against variations in input capacitance are considered in Appendix A. Simplified analyses are carried out in order to determine which of the two methods is better. The method thus selected is more thoroughly analyzed in this chapter.

A. Basic Charge-Sensitive Preamplifier Circuit Configuration

Most existing charge-sensitive preamplifier designs are based, with minor variations, on the circuit configuration shown in Figure 2-1.

The input device, an FET, could be replaced by a bipolar transistor or a vacuum tube. Cooled FETs offer the best noise performance, with vacuum tubes next and bipolar transistors a poor third.⁸

In order that the noise contribution of the second stage be small compared to that of the input FET, the input impedance of the second stage should be small compared to the output impedance of the FET.³ The required low input impedance of the second stage leads to its designation as a current amplifier.

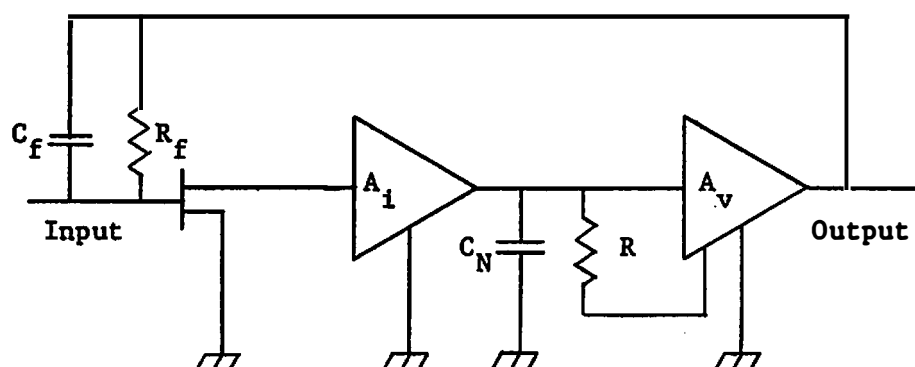


Figure 2-1. Basic charge-sensitive preamplifier circuit configuration.

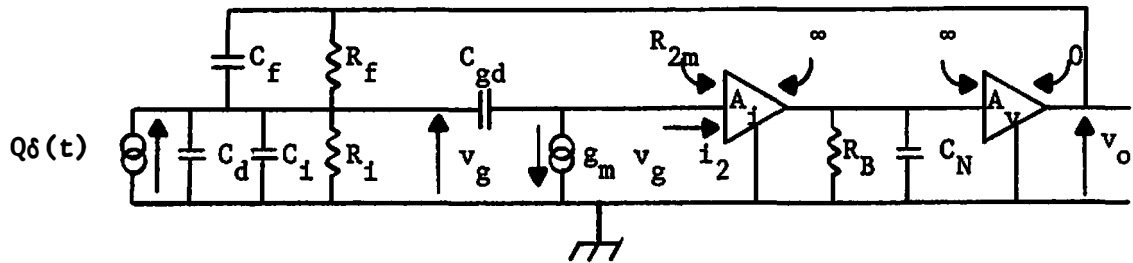
The input FET and its load, the low input impedance current amplifier, comprise a cascode pair. One well known advantage of the cascode connection is the small Miller capacitance seen at the input of the first stage. It will be shown later that a low value of Miller capacitance has a beneficial effect on the preamplifier's output pulse rise-time.

The current amplifier may be, and quite often is, simply a bipolar transistor in the common base configuration.

The load for the second stage current amplifier is its own high output impedance, the large input impedance of the voltage amplifier, capacitance C_N , and load resistor R . The load resistor is usually bootstrapped at some later point in the voltage gain section, A_v , to raise the load impedance level for the current amplifier in order to increase the low frequency open-loop gain of the preamplifier. Capacitance C_N is the sum of all device and stray capacitances at this high impedance point. Thus, it is at this point that the preamplifier's dominant open-loop pole is determined.

The voltage amplifier is usually some type of emitter-follower configuration which provides a low output impedance as well as good high-frequency capability.

An equivalent circuit model for the basic charge-sensitive configuration is shown in Figure 2-2. Note that the device capacitances associated with the input FET have been included and the effective resistance of the bootstrapped load, the current amplifier's



$Q\delta(t)$ = Detector output current pulse.

C_f = Feedback capacitor.

R_f = Feedback resistor.

C_d = Detector capacitance.

C_i = Sum of gate to source capacitance of FET, C_{gs} , and input stray capacitance, C_s .

R_i = Combination of detector leakage resistance, gate leakage resistance of the FET, and other leakage paths to ground at the input.

C_{gd} = Gate-to-drain capacitance of the FET.

$g_m v_g$ = Equivalent drain current generator of the FET.

R_{2m} = Input resistance of the second stage.

A_i = Current gain.

R_B = Combination of current amplifier output resistance, voltage amplifier input resistance, and effective bootstrapped load resistance at the high impedance point.

C_N = Sum of device capacitances and stray capacitances at the high impedance point.

A_v = Voltage gain.

Figure 2-2. Equivalent circuit model for the basic charge-sensitive preamplifier.

output resistance, and the voltage amplifier's input resistance have all been lumped together as one resistance to ground, R_B .

The output pulse expression will be calculated from the equivalent circuit of Figure 2-3. The circuit in Figure 2-3 is the same as that in Figure 2-2 except that the Miller capacitance associated with the FET is shown as a capacitance to ground and all parameters have been replaced by their Laplace transformed equivalents.

The following quantities will be defined in order to simplify the equations.

$$C_T \equiv C_d + C_i + C_m \quad (2-1)$$

$$A_2 \equiv \frac{V_o}{I_2} = \frac{A_i R_B A_v}{1 + R_B C_N s} \equiv \frac{A_{2m}}{1 + \tau_2 s} \quad (2-2)$$

$$A_{2m} \equiv A_i R_B A_v \quad (2-3)$$

$$\tau_2 \equiv R_B C_N \quad (2-4)$$

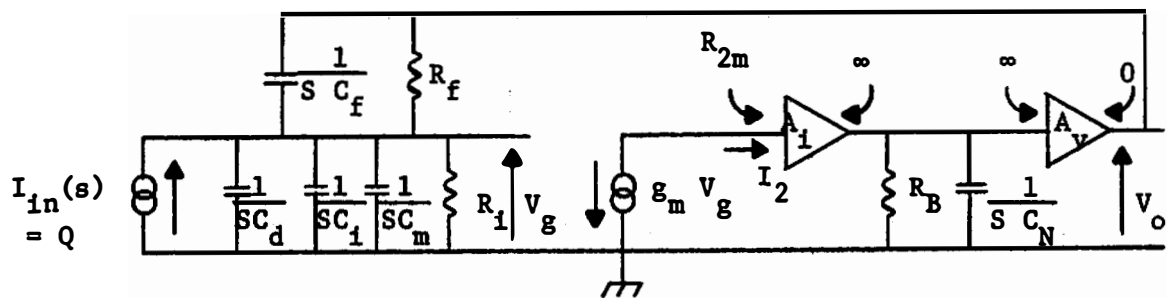
$$R \equiv \frac{R_i R_f}{R_i + R_f} \quad (2-5)$$

The required equations are the Kirchoff current equation at the V_g node,

$$I_{in}(s) = Q = (s C_T + \frac{1}{R_i}) V_g + (s C_f + \frac{1}{R_f})(V_g - V_o) \quad (2-6)$$

and the open-loop gain expression,

$$\frac{V_o}{V_g} = \frac{I_2}{V_g} \frac{V_o}{I_2} = -g_m A_2 \quad (2-7)$$



$$C_m \equiv C_{gd}(1 + g_m R_{2m}) = \text{Miller capacitance.}$$

Figure 2-3. Laplace transformed equivalent circuit model for the basic charge-sensitive preamplifier.

Solution of Equations (2-6) and (2-7) yields the Laplace transformed output pulse expression which, with the aid of Equations (2-1) - (2-5), may be written in the form

$$V_o = \frac{-\frac{g_m A_{2m}}{(C_f + C_T) \tau_2} Q}{s^2 + s \frac{\tau_2 + R C_f g_m A_{2m} + R(C_f + C_T)}{\tau_2 R(C_f + C_T)} + \frac{1 + \frac{R}{R_f} g_m A_{2m}}{\tau_2 R(C_f + C_T)}} \quad (2-8)$$

Defining the quantities

$$a_o \equiv \frac{1 + \frac{R}{R_f} g_m A_{2m}}{\tau_2 R(C_f + C_T)} \quad , \quad (2-9)$$

$$a_1 \equiv \frac{\tau_2 + R C_f g_m A_{2m} + R(C_f + C_T)}{\tau_2 R(C_f + C_T)} \quad , \quad (2-10)$$

and

$$\delta \equiv \frac{4 a_o}{a_1^2} \quad , \quad (2-11)$$

the roots of the characteristic equation,

$$s^2 + a_1 s + a_o = 0 \quad , \quad (2-12)$$

are

$$-\frac{1}{\tau_r} = -\frac{a_1}{2} [1 + (1 - \delta)^{1/2}] \quad (2-13)$$

and

$$-\frac{1}{\tau_d} = -\frac{a_1}{2} [1 - (1 - \delta)^{1/2}] . \quad (2-14)$$

The output pulse expression, Equation (2-8), may now be rewritten as

$$v_o = \frac{\frac{-g_m A_{2m}}{(C_f + C_T) \tau_2} Q}{(S + \frac{1}{\tau_r})(S + \frac{1}{\tau_d})} . \quad (2-15)$$

Taking the inverse Laplace transform¹⁵ of Equation (2-15) yields the time domain output pulse expression,

$$v_o = \frac{-\frac{g_m A_{2m}}{(C_f + C_T) \tau_2} Q}{(\frac{1}{\tau_r} - \frac{1}{\tau_d})} \left[\frac{t}{\tau_d} e^{-\frac{t}{\tau_d}} - \frac{t}{\tau_r} e^{-\frac{t}{\tau_r}} \right] . \quad (2-16)$$

The subscripts on τ in Equations (2-13) and (2-14) were chosen such that τ_r is the rise time constant and τ_d is the decay time constant in Equation (2-16).

Differentiating Equation (2-16) with respect to time and setting the derivative equal to zero yields the time of occurrence of the peak value of v_o ,

$$t_p = \frac{1}{\frac{1}{\tau_r} - \frac{1}{\tau_d}} \ln \frac{\tau_d}{\tau_r} . \quad (2-17)$$

The peak value of the output pulse is, then,

$$v_{op} = v_o(t_p) = \frac{-\frac{g_m A_{2m}}{(C_f + C_T) \tau_2} Q}{\left(\frac{1}{\tau_r} - \frac{1}{\tau_d}\right)} \left[\begin{array}{c} -\frac{\frac{1}{\tau_d}}{\frac{1}{\tau_r} - \frac{1}{\tau_d}} \ln \frac{\tau_d}{\tau_r} - \frac{\frac{1}{\tau_r}}{\frac{1}{\tau_r} - \frac{1}{\tau_d}} \ln \frac{\tau_d}{\tau_r} \\ \epsilon \end{array} \right] \quad (2-18)$$

With the aid of Equations (2-9) - (2-14), the expression in Equation (2-18) may be rewritten in the form

$$A_c \equiv \frac{v_{op}}{Q} = \frac{-g_m A_{2m} R^{1/2}}{[\tau_2 (1 + \frac{R}{R_f} g_m A_{2m}) (C_f + C_T)]^{1/2}} \epsilon^{\frac{1}{(1-\delta)^{1/2}} \ln \frac{\delta^{1/2}}{1+(1-\delta)^{1/2}}} \quad (2-19)$$

where A_c is the charge gain as previously defined.

B. Development of the Expression for the Negative Resistance which Eliminates Changes in Charge Gain Due to Variations in Input Capacitance

The derivative of A_c with respect to C_T , normalized to A_c , is found, again aided by Equations (2-9) - (2-14), to be

$$\frac{1}{A_c} \frac{dA_c}{dC_T} = \frac{[\tau_2 + RC_f g_m A_{2m} - R(C_f + C_T)] \delta}{2(C_f + C_T) [\tau_2 + RC_f g_m A_{2m} + R(C_f + C_T)] (1-\delta)^{3/2}} \quad (2-20)$$

$$\times \left\{ \left[\frac{-\tau_2 - RC_f g_m A_{2m} - R(C_f + C_T)}{2\tau_2 (1 + \frac{R}{R_f} g_m A_{2m})} + 1 \right] \left[\frac{\tau_2 + RC_f g_m A_{2m} + R(C_f + C_T)}{\tau_2 + RC_f g_m A_{2m} - R(C_f + C_T)} (1-\delta)^{\frac{1}{2}} \right] + \ln \frac{\delta^{1/2}}{1+(1-\delta)^{1/2}} \right\}.$$

This derivative is the same as the derivative with respect to C_d since, from Equation (2-1), with C_i and C_m constant,

$$\frac{d C_T}{d C_d} = 1 . \quad (2-21)$$

Approximations will now be made in order to simplify Equation (2-20). Using Equations (2-9) - (2-11) it can be seen that

$$\delta = \frac{4(1 + \frac{R}{R_f} g_m A_{2m}) \tau_2 R(C_f + C_T)}{[\tau_2 + R C_f g_m A_{2m} + R(C_f + C_T)]^2} . \quad (2-22)$$

If

$$\left| \frac{R}{R_f} g_m A_{2m} \right| \gg 1 \quad (2-23)$$

and

$$\left| R C_f g_m A_{2m} \right| \gg \left| \tau_2 + R(C_f + C_T) \right| , \quad (2-24)$$

Equation (2-22) may, with the aid of Equations (2-3) and (2-4), be written in the approximate form

$$\delta \approx \frac{4C_N(C_f + C_T)}{C_f^2 g_m A_i A_v R_f} . \quad (2-25)$$

In addition, if

$$C_f^2 g_m A_i A_v R_f \gg 4C_N(C_f + C_T) , \quad (2-26)$$

it can be seen that

$$\delta < 1. \quad (2-27)$$

Applying the inequalities of Equations (2-23), (2-24), and (2-27) to Equation (2-20) yields the approximate equation

$$\frac{1}{A_c} \frac{dA_c}{dC_T} \approx \frac{C_N}{C_f^2 g_m A_1 A_v R_f} \left\{ -\frac{\tau_f}{\tau_2} + \ln \left[\frac{\epsilon^2 C_N (C_f + C_T)}{C_f^2 g_m A_1 A_v R_f} \right] \right\} \quad (2-28)$$

where

$$\tau_f \equiv R_f C_f \quad (2-29)$$

is the feedback time constant.

Equation (2-28) contains, approximately, the condition which must be satisfied in order that $\frac{1}{A_c} \frac{dA_c}{dC_T}$, Equation (2-20), be equal to zero. The optimum value of R_B , which makes $\frac{1}{A_c} \frac{dA_c}{dC_T}$ approximately zero, is found by setting the right hand side of Equation (2-28) equal to zero and solving for R_B . The result is

$$R_{B(\text{opt.})} = \frac{\frac{R_f C_f}{C_N}}{\ln \left[\frac{\epsilon^2 C_N (C_f + C_T)}{C_f^2 g_m A_1 A_v R_f} \right]} \quad (2-30)$$

It can be seen from Equations (2-25) and (2-27) that the argument of the $\ln$ function in Equation (2-30) is a number much less than one.

This means that $R_{B(\text{opt.})}$ is a negative number. Negative values of R_B may be realized by using negative immittance converter techniques,¹⁶⁻²¹ that is, by applying positive feedback.

The value of $R_{B(\text{opt.})}$, Equation (2-30), is unfortunately a function of C_T . This means that, if R_B is chosen to be equal to $R_{B(\text{opt.})}$ in order to make $\frac{1}{A_c} \frac{dA_c}{dC_T}$ be approximately zero, as soon as C_T changes R_B is no longer the proper value for compensation and $\frac{1}{A_c} \frac{dA_c}{dC_T}$ changes. This problem does not invalidate the technique, however, because with positive feedback applied to make R_B equal to $R_{B(\text{opt.})}$, the value of $\frac{1}{A_c} \frac{dA_c}{dC_T}$ will be much less than it would be with no positive feedback, even for violent changes in C_T .

This may best be demonstrated with an example. Consider the preamplifier of Figure 2-3, page 14, with the following circuit parameter values :

$$A_v = A_1 = 1$$

$$C_N = 15 \text{ pf.}$$

$$R_{2m} = 50 \text{ ohms}$$

$$g_m = 5 \text{ mmhos}$$

$$R_1 = 10^9 \text{ ohms}$$

$$C_1 + C_m = 10 \text{ pf.}$$

$$R_f = 10^8 \text{ ohms}$$

$$C_f = 1 \text{ pf.}$$

(2-31)

In addition, let the detector capacitance, C_d , have a reference (starting) value of

$$C_{do} = 20 \text{ pf.} \quad (2-32)$$

The total input capacitance, C_T , then has a reference value of

$$C_{To} = C_{do} + C_i + C_m = 30 \text{ pf.} \quad (2-33)$$

Without positive feedback R_B will be a positive number. As an example, let

$$R_B = 250 \text{ K ohms.} \quad (2-34)$$

The normalized rate-of-change of charge gain with respect to input capacitance may be calculated from Equation (2-20). For this example it is found to be

$$\left. \frac{1}{A_c} \frac{dA_c}{dC_T} \right|_{\substack{C_T = C_{To} \\ R_B = 250 \text{ K } \Omega}} \approx -9.41 \times 10^{-2} \frac{\%}{\text{pf.}} \quad (2-35)$$

With positive feedback R_B can be made negative and, in particular, may be made equal to $R_{B(\text{opt.})}$. The value of $R_{B(\text{opt.})}$ is found from Equation (2-30) to be

$$R_{B(\text{opt.})} \approx -1.34 \text{ Meg ohms.} \quad (2-36)$$

The normalized rate-of-change of charge gain with respect to input capacitance, for R_B equal to $R_{B(opt.)}$, is found from Equation (2-20) to be

$$\left. \frac{1}{A_c} \frac{dA_c}{dC_T} \right|_{\substack{C_T = C_{To} \\ R_B = R_{B(opt.)}}} \approx - 2.71 \times 10^{-4} \frac{\%}{pF}. \quad (2-37)$$

Note that the value of the normalized rate-of-change for R_B equal to $R_{B(opt.)}$ was not zero. This is because Equation (2-30), from which $R_{B(opt.)}$ was determined, is only an approximate expression. Also note, however, that the value of the normalized rate-of-change for R_B equal to $R_{B(opt.)}$, Equation (2-37), is smaller (i.e., better) by a factor of about 350 than the value found for R_B equal to $250 \text{ K } \Omega$. For this example, therefore, application of positive feedback reduces the sensitivity of output pulse height to input capacitance variations by between two and three orders of magnitude.

As stated previously, the size of the normalized rate-of-change is a function of C_T , even for the case of positive feedback with R_B equal to $R_{B(opt.)}$.

The effects of this fact, for the present example, are shown in Figure 2-4. In Figure 2-4, the values of the normalized rate-of-change for different values of C_T , as determined from Equation (2-20), are plotted against C_T normalized to the reference value, C_{To} . This is done for the case of positive feedback, $R_B = R_{B(opt.)}$, and the case

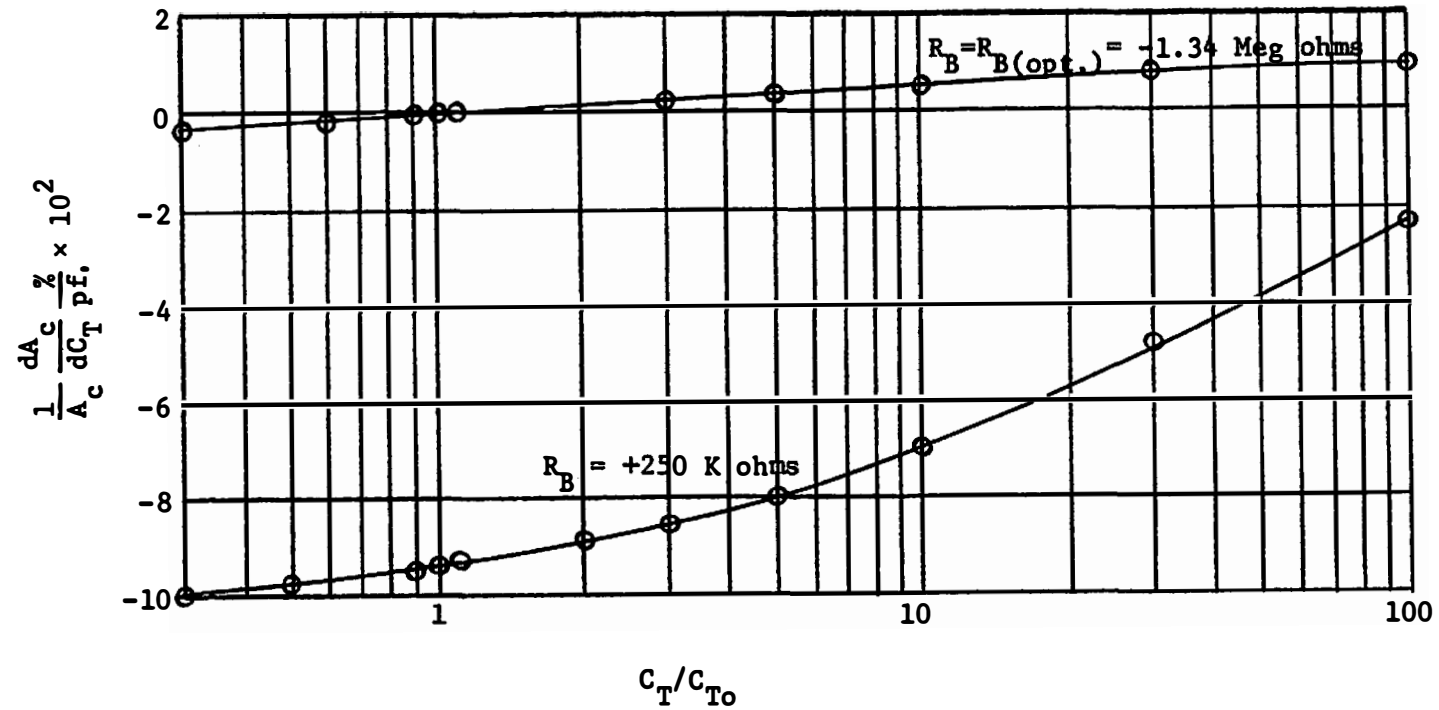


Figure 2-4. Variations in normalized rate-of-change of charge gain with changes in total input capacitance.

of no positive feedback, $R_B = 250 \text{ K ohms}$. Figure 2-4 shows that, for the present example, the normalized rate-of-change of charge gain with respect to C_T is considerably less for the case of positive feedback than for the case of no positive feedback, even for violent changes in C_T .

The expression for $R_{B(\text{opt.})}$ may be put in a more convenient form by applying the approximations of Equations (2-23) - (2-27) to the expressions for the rise- and decay-time constants, Equations (2-13) and (2-14). Carrying this out yields

$$\tau_r \approx \frac{C_N(C_f + C_T)}{g_m A_i A_v C_f} \quad (2-38)$$

and

$$\tau_d \approx R_f C_f \quad (2-39)$$

Equation (2-30) may now be rewritten as

$$R_{B(\text{opt})} = \frac{\frac{\tau_d}{C_N}}{2 + \ln \frac{\tau_r}{\tau_d}} \quad (2-40)$$

C. Oscillation stability

The characteristic equation for the closed-loop preamplifier, the denominator of Equation (2-8), page 15, set equal to zero, is

$$0 = s^2 + s \frac{\tau_2 + RC_f g_m A_{2m} + R(C_f + C_T)}{\tau_2 R(C_f + C_T)} + \frac{1 + \frac{R}{R_f} g_m A_{2m}}{\tau_2 R(C_f + C_T)} . \quad (2-41)$$

Roots of the characteristic equation are poles of the closed-loop transfer function and therefore determine the stability of the preamplifier.

The behavior of the roots of the characteristic equation as R_B varies may be studied by applying root-locus techniques.²² Rewriting Equation (2-41) in the proper form for root-locus analysis and using Equations (2-3) and (2-4), page 13, yields

$$0 = 1 + R_B C_N \frac{s^2 + s \left[\frac{1}{R(C_f + C_T)} + \frac{g_m A_{1v} C_f}{C_N (C_f + C_T)} \right] + \frac{g_m A_{1v}}{C_N (C_f + C_T) R_f}}{s + \frac{1}{R(C_f + C_T)}} . \quad (2-42)$$

If the approximations of Equations (2-23) - (2-27), pages 18 and 19, are applied, and if the additional results of those approximations as noted in Equations (2-37) and (2-38), pages 22 and 24, are used, the characteristic equation may be written approximately as

$$0 = 1 + R_B C_N \frac{(s + \frac{1}{\tau_r})(s - \frac{1}{\tau_d})}{(s + \frac{1}{\tau_{in}})} \cong 1 + R_B \frac{P(s)}{Q(s)} . \quad (2-43)$$

The time constant τ_{in} appearing in Equation (2-43) is the product of the total resistance and the total capacitance at the input of the open loop preamplifier, i. e.

$$\tau_{in} \equiv R(C_f + C_T) = \frac{R_f R_l}{R_f + R_l} (C_f + C_T) . \quad (2-44)$$

The root-locus resulting from Equation (2-43) is sketched in Figure 2-5. The root locus for positive values of R_B is shown in Figure 2-5(a) while Figure 2-5(b) exhibits that for negative values of R_B .

Note from Figure 2-5(b) that two poles will cross into the right half-plane (i.e., oscillations will occur) when R_B is negative and is increasing toward zero. The question to be answered is whether or not the previously calculated optimum value of R_B , $R_{B(opt.)}$ as shown in Equation (2-40), is close enough to zero to cause oscillations.

The value of R_B at which the poles cross the j -axis may be found by replacing s by $j\omega$ in Equation (2-43) and solving for R_B . Carrying this out and designating that value of R_B as $R_{B(osc)}$ yields

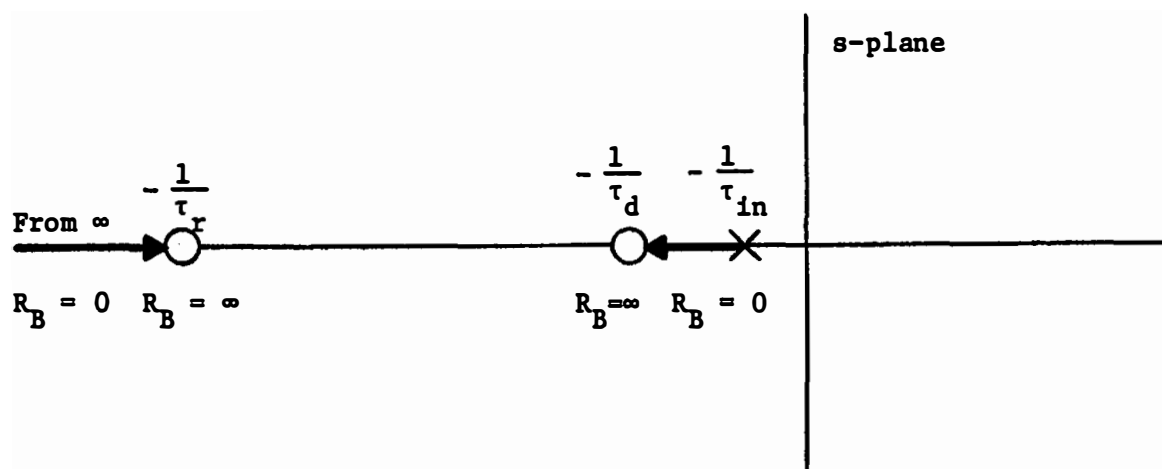
$$R_{B(osc)} = \frac{-1}{C_N \left(\frac{1}{\tau_r} + \frac{1}{\tau_d} \right)} . \quad (2-45)$$

Oscillations will not occur for R_B equal to $R_{B(opt.)}$ if

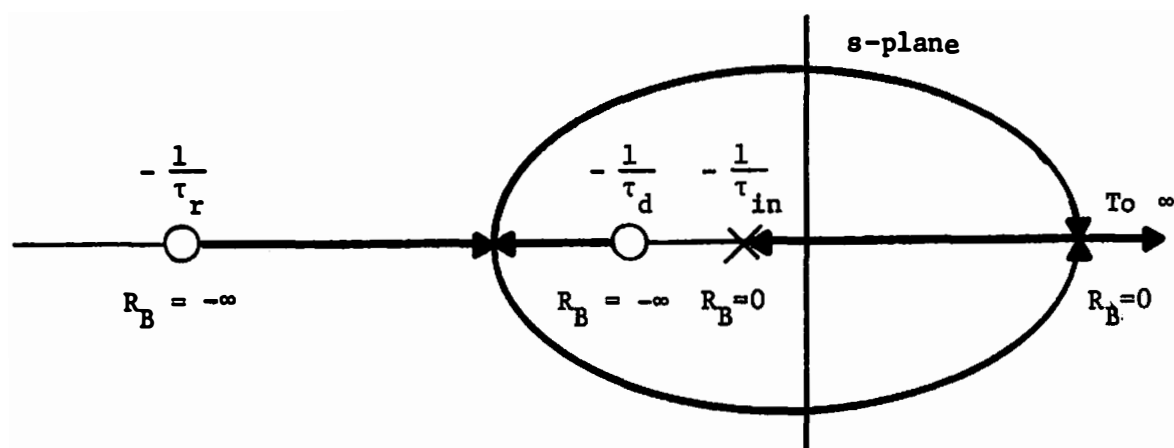
$$R_{B(opt.)} < R_{B(osc)} . \quad (2-46)$$

Equation (2-46) may be rewritten by using the relationships of Equations (2-40) and (2-46) as

$$\ln \frac{\tau_d}{\tau_r} < \frac{\tau_d}{\tau_r} + 2 . \quad (2-47)$$



(a) R_B starting at zero and increasing to infinity.
(Not drawn to scale)



(b) R_B starting at minus infinity and increasing to zero.
(Not drawn to scale)

Figure 2-5. Root locus of closed-loop preamplifier with R_B variable.

The inequality of Equation (2-47) is satisfied since

$$\tau_d \gg \tau_r . \quad (2-48)$$

Oscillations will not occur, therefore, for R_B equal to $R_{B(opt)}$.

In fact, for any case of practical interest, the value of $R_{B(opt)}$ will be of such magnitude that the approximations of Equations (2-23) - (2-27), pages 18 and 19, will be valid and the closed-loop poles with positive feedback will be approximately the same as without positive feedback. Thus, the output pulse rise- and decay-time constants will be approximately the same with and without positive feedback and will be given approximately by Equations (2-38) and (2-39), page 24.

D. Enhancement of the Output Pulse Rise-Time by Application of Positive Capacitance Feedback

It has been pointed out in previous sections that the output pulse rise-time constant is given approximately by Equation (2-38), which is repeated here for convenience.

$$\tau_r \approx \frac{C_N (C_f + C_T)}{g_m A_i A_v C_f} . \quad (2-49)$$

The 10 to 90 percent rise-time of the preamplifier is given by

$$t_r \approx 2.2\tau_r \approx \frac{2.2 C_N (C_f + C_T)}{g_m A_i A_v C_f} . \quad (2-50)$$

The term C_T in Equation (2-50) is composed of the sum of the detector capacitance, C_d , the capacitance to ground at the FET gate, C_1 , and the Miller capacitance associated with the FET, C_m , as shown in Equation (2-1), page 13, and in Figures 2-2 and 2-3, pages 12 and 14. The cascode input configuration, which keeps the Miller capacitance small, therefore aids in keeping the rise-time small. Note also that the rise-time varies with changes in detector capacitance. Both of these effects were previously stated in the introduction.

In any experiment where a time fiducial is to be derived from the preamplifier's output pulse, it is important that the rise-time be kept as short as possible. This is because the timing error is inversely dependent on the rate-of-change of output voltage with respect to time.²³

It is obvious from Equation (2-50) that the output pulse rise-time can be made smaller by reducing the capacitance at the high impedance point, C_N . Low values of C_N are initially obtained by utilizing low-capacitance active devices and careful construction techniques.

Even lower values of effective capacitance may be realized by application of positive capacitance feedback (PCF) in the same manner in which positive resistance feedback was used in previous sections to control the impedance at the high impedance point.

One of the first uses of PCF was made by P. R. Bell,²⁴ for increasing the speed-of-response of amplifiers. The technique, when used for this purpose, was aimed at reducing the capacitance at the

input of an amplifier. For the present purpose, the technique will be used to reduce the capacitance at a point other than the amplifier input.

In order to arrive at the limitations on output pulse rise-time with positive capacitance feedback applied, the equations of the foregoing sections must be modified. This is because those equations were derived assuming that the preamplifier had only a single pole at the high impedance point. Since positive capacitance feedback will be used to reduce the capacitance at the high impedance point, the dominant pole occurring at that point will be moved to a higher frequency and the next most dominant open-loop pole will eventually play a part in determining the closed-loop high-frequency performance.

The required equations may be developed by following the same analysis procedure outlined in Equations (2-1) - (2-8), pages 13 and 15, if the gain A_2 , given in Equation (2-2), is modified to be

$$A_2 \equiv \frac{V_o}{I_2} = \frac{A_1 R_B A_v}{(1 + R_B C_N s)(1 + \tau s)} \equiv \frac{A_{2m}}{(1 + \tau_2 s)(1 + \tau s)} , \quad (2-51)$$

thereby including the effects of the second most dominant pole, $\frac{1}{\tau}$. The $\frac{1}{\tau}$ pole may occur in either the A_1 or the A_v gain blocks shown in Figure 2-3, page 14.

Performing the analysis of Equations (2-1) - (2-8), with Equation (2-2) replaced by Equation (2-51) yields the new charge gain expression

$$\frac{V_o}{Q} = \frac{\frac{-g_m A_{2m}}{\tau(C_f + C_T)}}{s^2 + \left[\frac{1}{\tau} + \frac{1}{\tau_{in}} + \frac{g_m A_{2m} C_f}{\tau(C_f + C_T)} \right] s + \left[\frac{1}{\tau \tau_{in}} + \frac{g_m A_{2m}}{\tau(C_f + C_T) R_f} \right]} , \quad (2-52)$$

$$\frac{1+C_N R_B}{s^2 + \left[\frac{1}{\tau} + \frac{1}{\tau_{in}} + \frac{g_m A_{2m} C_f}{\tau(C_f + C_T)} \right] s + \left[\frac{g_m A_{2m}}{\tau \tau_{in} + \tau(C_f + C_T) R_f} \right]}$$

where the definition of τ_{in} , Equation (2-43), page 25, has been used. The charge gain expression has been written so that its denominator, the characteristic equation, is in the most convenient form for applying root-locus techniques.

If the approximations

$$\left| \frac{g_m A_{2m} C_f}{\tau(C_f + C_T)} \right| \gg \frac{1}{\tau} + \frac{1}{\tau_{in}} \quad (2-53)$$

and

$$\left| \frac{g_m A_{2m}}{\tau(C_f + C_T) R_f} \right| \gg \frac{1}{\tau \tau_{in}} \quad (2-54)$$

are applied, the quadratic term in the characteristic equation may be conveniently factored and the characteristic equation, set equal to zero, will have the approximate form

$$0 = 1 + C_N R_B \frac{s(s + \frac{1}{\tau})(s + \frac{1}{\tau_{in}})}{(s + \frac{A\beta_h}{\tau})(s + \frac{1}{\tau_f})} \quad (2-55)$$

where

$$\tau_f \equiv R_f C_f \quad (2-56)$$

is the feedback time constant,

$$A \equiv g_m A_{2m} \equiv g_m A_1 R_B A_v \quad (2-57)$$

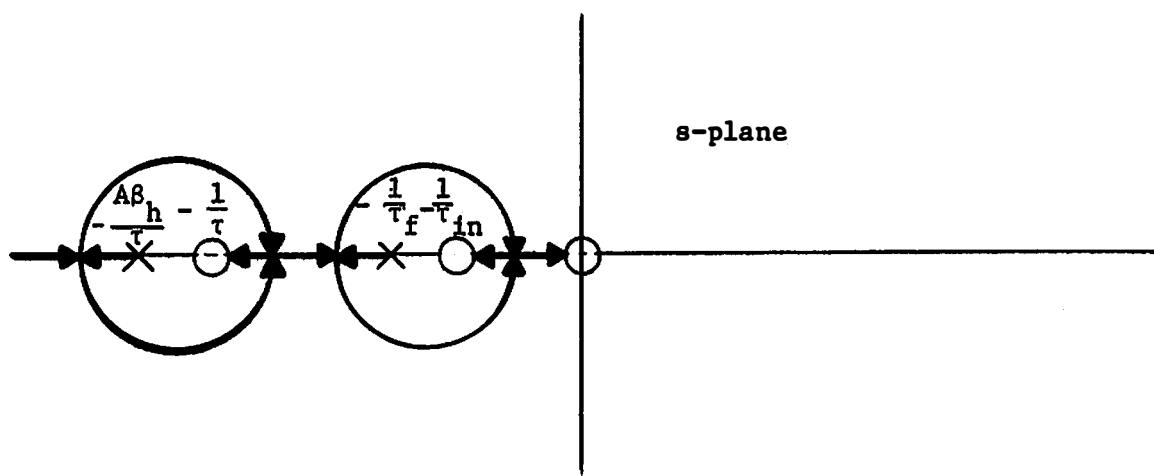
is the open-loop gain, and

$$\beta_h \equiv \frac{C_f}{C_f + C_T} \quad (2-58)$$

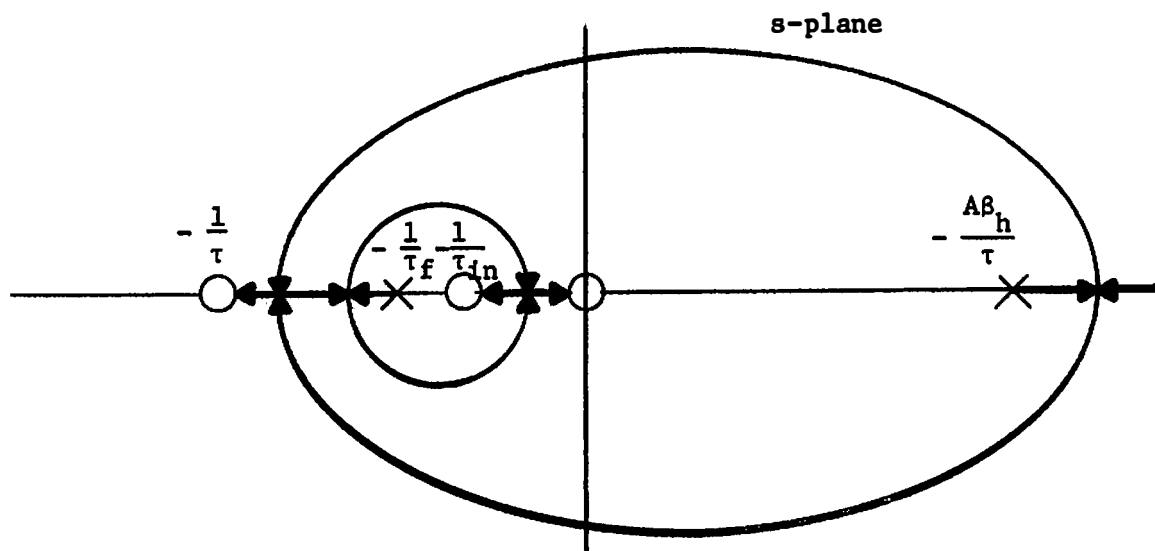
is the high-frequency feedback ratio.

The root loci for the characteristic equation, Equation (2-55) with C_N variable are shown in Figures 2-6 and 2-7. Loci for positive values of C_N are shown in Figure 2-6 while Figure 2-7 shows the loci for negative values of C_N . In either case, R_B may be a positive number, shown in Figures 2-6(a) and 2-7(a), or a negative number, shown in Figures 2-6(b) and 2-7(b). Since, as shown in Figure 2-7, there is always a pole in the right half-plane for negative values of C_N , the only cases of interest are those where C_N is positive.

It can be seen from Figure 2-6(b) that for C_N positive and R_B negative there will be two poles in the right half-plane unless C_N is

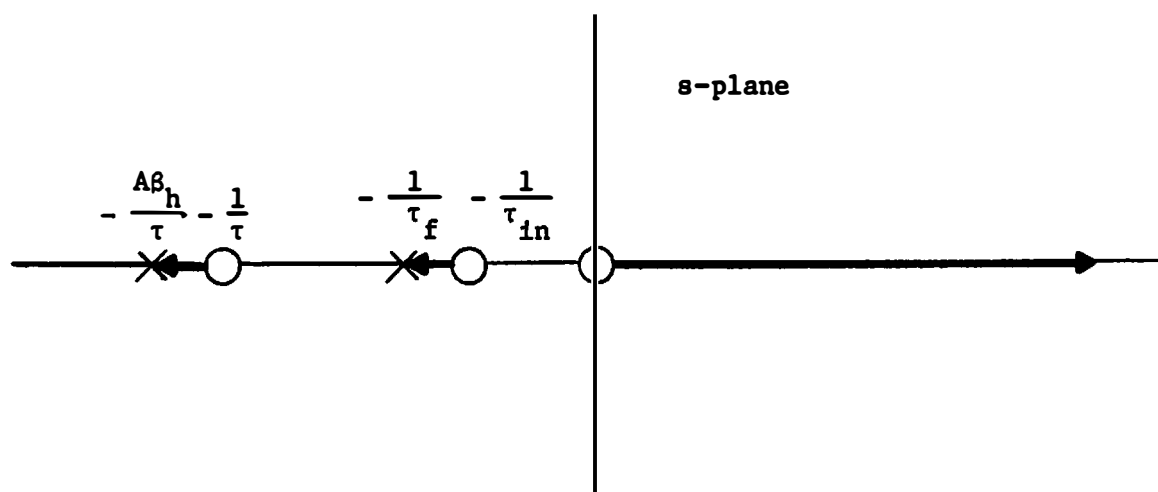


(a) C_N starting at zero and increasing to infinity with R_B a positive number. (Not drawn to scale)

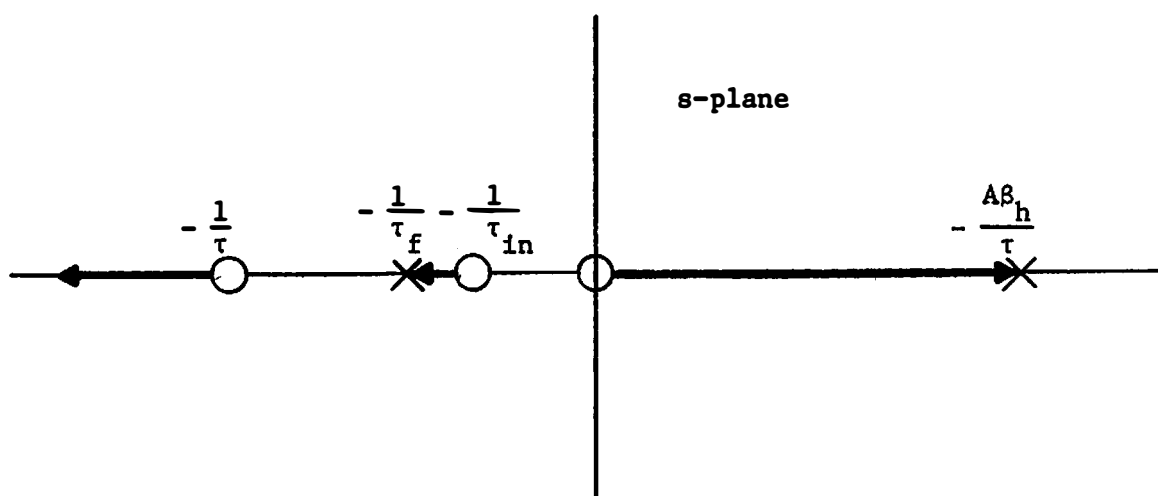


(b) C_N starting at zero and increasing to infinity with R_B a negative number. (Not drawn to scale)

Figure 2-6. Root locus of closed-loop preamplifier with C_N variable and positive.



- (a) C_N starting at minus infinity and increasing to zero with R_B a positive number. (Not drawn to scale)



- (b) C_N starting at minus infinity and increasing to zero with R_B a negative number. (Not drawn to scale)

Figure 2-7. Root locus of closed-loop preamplifier with C_N variable and negative.

larger than some minimum value $C_{N(\min)}$. This minimum value of C_N may be found by replacing s by $j\omega$ in Equation (2-55) and solving for C_N .

If the approximations

$$\frac{1}{\tau} \gg \frac{1}{\tau_{in}} \quad (2-59)$$

and

$$\frac{1}{\tau} \gg \frac{1}{\tau_f} \quad (2-60)$$

are made, the minimum value of C_N is found to be

$$C_{N(\min)} = \frac{\tau}{-R_B} \quad (2-61)$$

The foregoing discussion shows that when positive capacitance feedback is used to reduce the capacitance C_N , with R_B a negative number, the resulting value of C_N must be such that the pole at the high impedance point, $\frac{1}{-R_B C_N}$, is less than the next most dominant open loop pole, $\frac{1}{\tau}$, or oscillations will occur. Since, in a practical case, the preamplifier can have other open-loop poles higher than $\frac{1}{\tau}$, oscillations may occur before C_N is made as small as $C_{N(\min)}$.

It should also be noted that although the root-locus of Figure 2-6(a), page 33, for C_N and R_B both positive, indicates no instability for any value of C_N , the presence of open-loop poles higher than $\frac{1}{\tau}$ will result in oscillations as C_N approaches zero.

The amount of separation between the $\frac{1}{\tau}$ pole and the other high-frequency open-loop poles determines how close C_N may approach $C_{N(\min)}$ for negative R_B , or zero for positive R_B , without the occurrence of oscillations.

A more important question is how small C_N should be in order to produce the fastest output pulse rise-time while maintaining good output pulse shape. In the region of interest, the closed-loop preamplifier will have two high-frequency poles and a third pole at a much, much lower frequency. The closed-loop preamplifier may therefore be treated as a simple second-order system for purposes of determining the rise-time and front edge shape of the output pulse.

A simple second-order system, with poles located equally as far from the real axis as from the j -axis in the s -plane (i.e., for the damping ratio, ζ , equal to .707), produces an output pulse having less than 5 percent overshoot and whose 10-90 percent rise-time is given by

$$t_{rp} \approx \frac{1.55}{\omega_p} \quad (2-62)$$

where ω_p is the pole spacing from either the real or j -axes.

The value of C_N required to place the high-frequency poles so that the damping ratio associated with them will be 0.707 may be found by replacing s by $\omega_p + j\omega_p$ in Equation (2-55), page 32, and solving for $C_N \equiv C_{Np}$. Carrying out that procedure, utilizing the approximation

$$\left| \frac{AB_h}{\tau} \right| > > \frac{1}{\tau_f} \quad (2-63)$$

and assuming

$$\left| \frac{A\beta_h}{C_{Np} R_B} \right| > > \frac{1}{\tau_{in}} \quad , \quad (2-64)$$

$$\frac{1}{\tau} > > \left| \frac{1}{C_{Np} R_B} + \frac{1}{\tau_{in}} \right| \quad , \quad (2-65)$$

and

$$\left| \frac{A\beta_h}{C_{Np} R_B} \right| > > \frac{1}{\tau_f} \quad , \quad (2-66)$$

yields

$$C_{Np} \approx \frac{2 A\beta_h \tau}{R_B} = \frac{2 g_m A_i A_v C_f \tau}{C_f + C_T} \quad . \quad (2-67)$$

The value of ω_p may now be found, by replacing C_N in Equation (2-55) by the value given in Equation (2-67), with s again replaced by $\omega_p + j\omega_p$. If the approximations of Equations (2-63) - (2-66) are observed, the result is

$$\omega_p \approx \frac{1}{2\tau} \quad . \quad (2-68)$$

Using positive capacitance feedback to make C_N equal to the value C_{Np} , given in Equation (2-67), will therefore produce a preamplifier whose output pulse has less than 5 percent overshoot and whose 10-90 percent rise-time is found, by using Equation (2-68) in Equation (2-62), to be

$$t_{rp} \approx 3.1\tau \quad . \quad (2-69)$$

By examination of Equation (2-55) with C_N replaced by the value of C_{Np} in Equation (2-67), and using the fact that for this value of C_N the conjugate poles are located at $\omega_p \pm j\omega_p$, and using Equations (2-63) - (2-66), it can be shown that the real axis pole for $C_N = C_{Np}$ has not moved appreciably from its location without positive feedback as given in Equation (2-39), page 24.

E. Effects of Positive Feedback on Noise Performance

If the noise sources in the preamplifier are represented as noise current sources located at the gate and drain terminals of the input FET, the noise sources may be conveniently added to the equivalent circuit of Figure 2-3, page 14, as shown in Figure 2-8.

The effect of positive feedback on noise performance may be ascertained by determining the transfer functions from the appropriate noise source to the preamplifier output for the two noise sources shown in Figure 2-8.

With $\overline{i_{df}^2}$ removed and $\overline{i_{gf}^2}$ replaced by I_{gf} , the circuit is the same as that from which the charge gain expression was derived where the input current source was Q as shown in Figure 2-3. The transfer function $\frac{V_o}{I_{gf}}$ may therefore be written by inspection of Equation (2-15), page 16, as

$$T_{gf} \equiv \frac{V_o}{I_{gf}} = \frac{-g_m A_{2m}}{(C_f + C_T) \tau_2} \cdot \frac{1}{(s + \frac{1}{\tau_r})(s + \frac{1}{\tau_d})} \quad , \quad (2-70)$$

where the definitions and manipulations of Equations (2-1) - (2-14), pages 13 - 16, have been observed. Using the approximations of Equations (2-37) and (2-38), pages 22 and 24, which result from Equations (2-23) - (2-27), pages 18 and 19, T_{gf} can be written, after simplification, as

$$T_{gf}(j\omega) = \frac{R_f}{\left[1 + j\omega \frac{C_N(C_f + C_T)}{g_m A_i A_v C_f} \right] [1 + j\omega R_f C_f]} \quad (2-71)$$

where s has been replaced by $j\omega$.

The required noise power transfer function is, then,

$$G_{gf}(\omega) \equiv \frac{\overline{e_{ogf}^2}}{\overline{i_{gf}^2}} \equiv T_{gf}(j\omega) T_{gf}^*(j\omega) = \frac{R_f^2}{\left[1 + \omega^2 \frac{C_N^2 (C_f + C_T)^2}{g_m^2 A_i^2 A_v^2 C_f^2} \right] [1 + \omega^2 R_f^2 C_f^2]} \quad (2-72)$$

Making the "mid-band" approximation,

$$\frac{1}{R_f C_f} \ll \omega \ll \frac{g_m A_i A_v C_f}{C_N (C_f + C_T)} \quad , \quad (2-73)$$

yields the mid-band noise transfer function,

$$G_{gfm}(\omega) = \frac{1}{\omega^2 C_f^2} \quad (2-74)$$

The transfer function for the $\overline{i_{df}^2}$ source is found by removing $\overline{i_{gf}^2}$, replacing $\overline{i_{df}^2}$ by I_{df} , and writing the following equations for the resulting circuit:

$$I_2 = I_{df} - g_m V_g \quad (2-75)$$

$$V_g \left(\frac{1}{R_i} + sC_T \right) + (V_g - V_o) \left(\frac{1}{R_f} + sC_f \right) = 0 \quad (2-76)$$

$$\frac{I_2 A_i A_v}{\frac{1}{R_B} + sC_N} = V_o \quad (2-77)$$

Solving Equations (2-75) - (2-77) and again making use of Equations (2-1) - (2-5) and Equations (2-9) - (2-14) yields

$$T_{df} \equiv \frac{V_o}{I_{df}} = \frac{\frac{A_{2m}}{\tau_2} \left(s + \frac{1}{\tau_{in}} \right)}{\left(s + \frac{1}{\tau_r} \right) \left(s + \frac{1}{\tau_d} \right)} \quad (2-78)$$

Following the same procedure outlined in Equations (2-69) - (2-73) gives

$$T_{df}(j\omega) = \frac{\frac{R_f}{g_m R} [1 + j\omega R(C_f + C_T)]}{\left[1 + j\omega \frac{C_N(C_f + C_T)}{g_m A_i A_v C_f} \right] [1 + j\omega R_f C_f]} \quad (2-79)$$

$$G_{df}(\omega) \equiv \frac{\overline{e_{odf}^2}}{\overline{i_{df}^2}} \equiv T_{df}(j\omega) T_{df}^*(j\omega) =$$

$$\frac{\frac{R_f^2}{g_m^2 R^2} [1 + \omega^2 R^2 (C_f + C_T)^2]}{\left[1 + \omega^2 \frac{C_N^2 (C_f + C_T)^2}{g_m^2 A_i^2 A_v^2 C_f^2} \right] [1 + \omega^2 R_f^2 C_f^2]} \quad (2-80)$$

and

$$G_{dfm}(\omega) = \frac{(C_f + C_T)^2}{g_m^2 C_f^2} \quad (2-81)$$

Since R_B does not appear in either noise power transfer function, it is obvious that the value of R_B , even if controlled by positive feedback, will not affect the noise performance so long as R_B is such that the approximations made in obtaining the noise transfer functions are still valid. The approximations which were made, and which were also functions of R_B , are Equations (2-23) and (2-24), page 18. These approximations may be rewritten to more clearly show their dependence on R_B . Equation (2-23) becomes

$$|R_B| > > \frac{1}{g_m A_i A_v \frac{R_i}{R_f + R_i}} \quad (2-82)$$

Equation (2-24) becomes

$$|R_B| > > \frac{1}{g_m A_i A_v \frac{C_f}{C_f + C_T}} \quad (2-83)$$

where

$$C_N < < C_f g_m A_i R A_v \quad (2-84)$$

and the triangle inequality,²⁵

$$|a + b| \leq |a| + |b| \quad (2-85)$$

have been used. Close inspection of Equations (2-82) and (2-83) show that they simply state that the high- and low-frequency loop gains must be large.

It can therefore be stated that, as long as R_B is such that the loop gain remains large, controlling R_B by the application of positive feedback should not affect preamplifier noise performance.

It can be seen from Equations (2-72) and (2-80) that the noise transfer functions will be affected if positive feedback is used to control C_N . The alterations, however, will only occur in the high-frequency range and will not, as shown in Equations (2-74) and (2-81), affect the mid-band noise transfer functions. This argument is valid if C_N is such that the approximations contained in Equations (2-24) and (2-26), page 18, are satisfied and, as for R_B , it can be shown that the approximations are satisfied if the loop gain is large. Thus, under the condition of large loop gain, the use of positive feedback to control C_N should not affect the preamplifier's mid-band noise performance.

CHAPTER 3

IMPROVING THE ABILITY OF CHARGE-SENSITIVE PREAMPLIFIERS TO TERMINATE LONG INPUT CABLES BY APPLICATION OF POSITIVE FEEDBACK

In this chapter the problem of providing an acceptable cable termination at the input of a charge-sensitive preamplifier is considered.

The input impedance expression for a charge-sensitive preamplifier is developed and the preamplifier performance criteria for providing the best input cable termination are derived.

A. Charge-Sensitive Preamplifier Input Impedance

It is often stated that the input impedance to a charge-sensitive preamplifier is a large capacitance. This statement is justified by considering the preamplifier to be represented by the simple current integrator configuration shown in Figure 3-1. The circuit of Figure 3-1 is simply a voltage amplifier of gain $-A$ and a shunt feedback capacitor, C_f .

By inspection of Figure 3-1, it is seen that

$$I_{in} = sC_f(V_{in} - V_o) \quad (3-1)$$

and

$$V_o = -A V_{in} \quad (3-2)$$

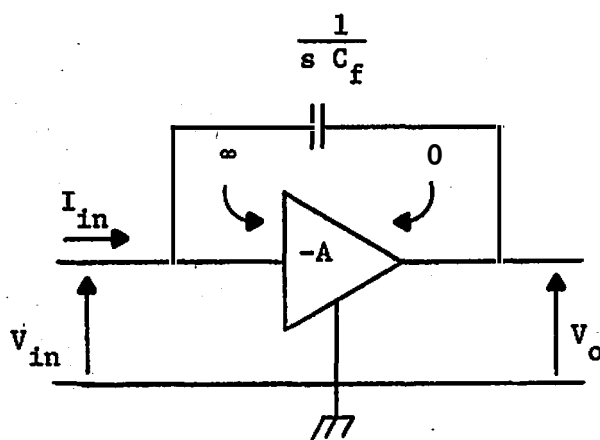


Figure 3-1. Simple current integrator representation of charge-sensitive preamplifier.

Solution of Equations (3-1) and (3-2) yields the input impedance expression,

$$Z_{in} \equiv \frac{V_{in}}{I_{in}} = \frac{1}{s C_f (1 + A)} \quad , \quad (3-3)$$

which shows that the input impedance is a capacitance equal to the feedback capacitance times one plus the gain magnitude. If the voltage gain is large, therefore, the input impedance can be said to be a large capacitance.

Since, in order to terminate an input cable, the input impedance should be a resistance equal to the cable's characteristic impedance, Z_0 , it is apparent that the preamplifier represented above cannot provide an acceptable termination.

However, the preamplifier representation shown in Figure 3-1 has a serious limitation. It is only valid at frequencies below the dominant pole of the open-loop amplifier. Since most charge-sensitive preamplifiers have dominant open-loop poles at fairly low frequencies, which is a direct result of having very large low-frequency open-loop gains, the representation of Figure 3-1 is not adequate for deriving the input impedance.

A more useful input impedance expression can be derived from the basic charge-sensitive preamplifier equivalent circuit of Figure 2-3, page 14, which includes the dominant open-loop pole.

Equations for the circuit of Figure 2-3 have been written previously. They are Equations (2-1) - (2-7), page 13.

Solving Equations (2-6) and (2-7) and using the definitions of Equations (2-1) - (2-5) yields the input admittance,

$$Y_{in} \equiv \frac{I_{in}}{V_g} = s(C_i + C_m + C_f) + \left(\frac{1}{R_i} + \frac{1}{R_f} \right) + \left(s C_f + \frac{1}{R_f} \right) \left(\frac{g_{m1} R_B A_v}{1 + R_B C_N s} \right). \quad (3-4)$$

Input admittance, rather than impedance, is stated only for convenience in writing the equation. The detector capacitance, C_d , has been eliminated from the equation since it is now separated from the preamplifier input by the intervening cable.

The components of the input admittance may be more easily identified if Equation (3-4) is put in the form

$$Y_{in} = s(C_i + C_m + C_f) + \left(\frac{1}{R_i} + \frac{1}{R_f} \right) + \frac{\frac{1}{C_N}}{\frac{g_{m1} A_v}{s} + \frac{1}{s g_{m1} R_B A_v C_f}} + \frac{1}{s \frac{C_N R_f}{g_{m1} A_v} + \frac{R_f}{g_{m1} R_B A_v}}. \quad (3-5)$$

A circuit representation of Equation (3-5) is shown in Figure 3-2.

B. Constraints on the Preamplifier for Providing Input Cable Termination

As discussed previously, the values of R_B and C_N may be controlled by the application of positive feedback. Hence, positive feedback can be used to control the component values in the last two terms of Equation (3-5).

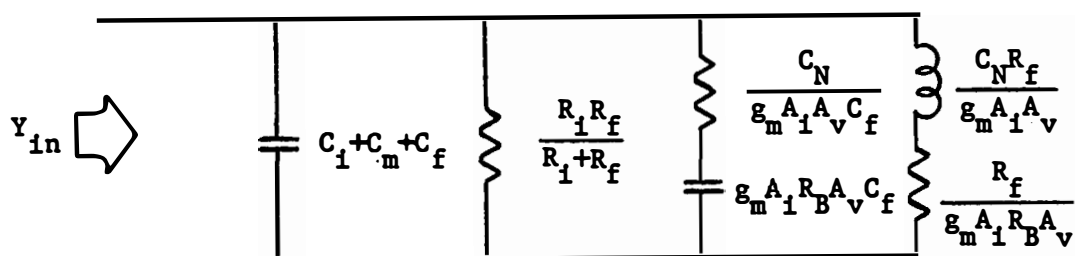


Figure 3-2. Input admittance for the basic charge-sensitive preamplifier.

If R_B and C_N are chosen according to the definitions

$$R_{BT} \equiv \frac{R_f}{g_m A_i A_v Z_o} \quad (3-6)$$

and

$$C_{NT} \equiv g_m A_i A_v Z_o C_f, \quad (3-7)$$

where Z_o is the characteristic impedance of the cable to be terminated, Equation (3-5) becomes

$$Y_{inT} = s(C_i + C_m + C_f) + \left(\frac{1}{R_i} + \frac{1}{R_f} \right) + \frac{1}{Z_o + \frac{1}{s \frac{R_f C_f}{Z_o}}} + \frac{1}{s Z_o R_f C_f + Z_o}, \quad (3-8)$$

where Y_{inT} now denotes the value of Y_{in} under the conditions imposed by Equations (3-6) and (3-7). Rewriting Equation (3-8) reveals that

$$Y_{inT} = s(C_i + C_m + C_f) + \left(\frac{1}{R_i} + \frac{1}{R_f} \right) + \frac{1}{Z_o}. \quad (3-9)$$

Since, for any practical case,

$$\frac{1}{Z_o} > \frac{1}{R_i} + \frac{1}{R_f}, \quad (3-10)$$

Equation (3-9) may be rewritten as

$$Y_{inT} = s C_I + \frac{1}{Z_o} \quad (3-11)$$

where

$$C_I \equiv C_1 + C_m + C_f \quad . \quad (3-12)$$

The circuit representation of Equation (3-11) is shown in Figure 3-3.

Because C_I has not been eliminated from Y_{inT} , the termination is not perfect. Since the values of C_I and Z_o will usually be quite small, however, the $Z_o C_I$ time constant will be rather short. This means that the mistermination due to C_I will be a high-frequency phenomenon. It will be argued later, in fact, that the effect of the mistermination on the preamplifier's output pulse will be to add a sequence of very short perturbations to the desired output pulse, and that those perturbations will be essentially ignored by the pulse shaping amplifier which will follow the preamplifier.

C. Development of the General Output Pulse Expression for a Charge-Sensitive Preamplifier with a Long Input Cable

The circuit to be used in developing the output pulse expression is shown in Figure 3-4. The detector output current pulse, i_d , is shown in Figure 3-4 advanced in time by the cable time delay, T , simply to allow the preamplifier output voltage pulse, e_{op} , to begin at $t = 0$. The preamplifier is represented in Figure 3-4 by its equivalent input admittance, Y_{in} , previously calculated and given in Equation (3-5), page 47, and its open-loop voltage gain, A_{OL} . The open-loop gain for the basic charge-sensitive preamplifier of Figure 2-3, page 14, was

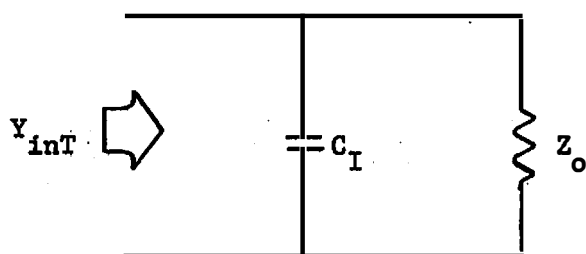
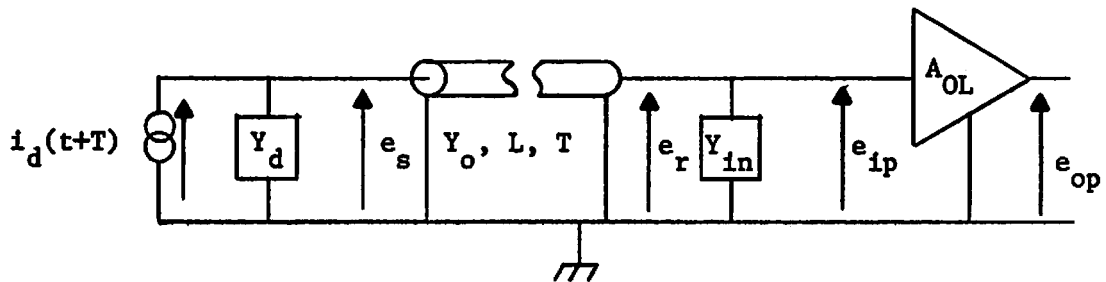


Figure 3-3. Input admittance for basic charge-sensitive preamplifier adjusted to terminate an input cable.



- i_d = Detector output current pulse
- Y_d = Detector output admittance
- e_s = Voltage at the sending end of the input cable
- Y_o = Cable characteristic admittance
- L = Cable length
- T = Cable time delay
- e_r = Voltage at the receiving end of the cable
- Y_{in} = Equivalent input admittance of the preamplifier
- e_{ip} = Preamplifier input voltage
- A_{OL} = Open-loop voltage gain of the preamplifier
- e_{op} = Preamplifier output voltage

Figure 3-4. Equivalent circuit for a charge-sensitive preamplifier with a long input cable.

discussed in Chapter 2 and, by making use of Equations (2-2) and (2-7), pages 13, can be written as

$$A_{OL} \equiv \frac{E_{op}}{E_{ip}} = - \frac{g_m A_R A_v}{1 + R_B C_N s} , \quad (3-13)$$

where E_{op} and E_{ip} are the Laplace transformed preamplifier output and input voltages, respectively. The cable is assumed to be lossless.

A step-by-step development of the preamplifier output pulse will now be undertaken. Laplace transformed quantities will be used throughout.

The detector output current is

$$\mathcal{L}[i_d(t + T)] = \epsilon^{Ts} \mathcal{L}[i_d(t)] = \epsilon^{Ts} I_d . \quad (3-14)$$

The initial voltage pulse at the sending end of the cable is

$$E_{so} = \frac{\epsilon^{Ts} I_d}{Y_d + Y_o} . \quad (3-15)$$

The initial incident voltage pulse at the receiving end of the cable is the initial voltage pulse at the sending end delayed by T , and is

$$E_{ro}^+ = \epsilon^{-Ts} E_{so} = \frac{I_d}{Y_d + Y_o} . \quad (3-16)$$

Since the cable is not perfectly terminated at the receiving end, a portion of E_{ro}^+ will be reflected. The portion reflected will be

determined, as reference to any standard textbook on transmission lines will show,²⁶ by the reflection coefficient at the receiving end which is defined by

$$P_r = \frac{Y_o - Y_{in}}{Y_o + Y_{in}} \quad (3-17)$$

The initial reflected voltage pulse at the receiving end is

$$E_{ro}^- = P_r E_{ro}^+ = \frac{P_r I_d}{Y_d + Y_o} \quad (3-18)$$

The total initial voltage pulse at the receiving end, and therefore at the preamplifier input, is the sum of the initial incident and reflected pulses and is

$$E_{ipo} = E_{ro} = E_{ro}^+ + E_{ro}^- = \frac{(1 + P_r) I_d}{Y_d + Y_o} \quad (3-19)$$

At the preamplifier output, the total initial voltage pulse is

$$E_{opo} = A_{OL} E_{ipo} = \frac{A_{OL} (1 + P_r) I_d}{Y_d + Y_o} \quad (3-20)$$

Now, after a delay period T , the initial reflected voltage pulse at the receiving end returns to the sending end and is reflected at the sending end by the sending-end reflection coefficient,

$$P_s = \frac{Y_o - Y_d}{Y_o + Y_d} \quad (3-21)$$

Finally, after another delay T , it arrives at the receiving end where it becomes the first-reflection incident voltage pulse at the receiving end,

$$E_{r1}^+ = \epsilon^{-Ts} P_s \epsilon^{-Ts} E_{ro}^- = \frac{\epsilon^{-2Ts} P_s P_r I_d}{Y_d + Y_o} \quad (3-22)$$

Following the same development given for the initial incident pulse yields the following equations for the effects of the first-reflection incident pulse.

$$E_{r1}^- = P_r E_{r1}^+ = \frac{\epsilon^{-2Ts} P_s P_r^2 I_d}{Y_d + Y_o} \quad (3-23)$$

$$\begin{aligned} E_{ip1} &= E_{r1}^+ + E_{r1}^- = \frac{(1 + P_r) I_d}{Y_d + Y_o} P_s P_r \epsilon^{-2Ts} \\ &= E_{ipo} P_s P_r \epsilon^{-2Ts} \end{aligned} \quad (3-24)$$

$$\begin{aligned} E_{op1} &= A_{OL} E_{ip1} = \frac{A_{OL} (1 + P_r) I_d}{Y_d + Y_o} P_s P_r \epsilon^{-2Ts} \\ &= E_{opo} P_s P_r \epsilon^{-2Ts} \end{aligned} \quad (3-25)$$

The expressions for the contributions to the preamplifier output pulse due to the additional reflections may be found by following the procedure outlined above. The results of this analysis indicate that

the preamplifier output pulse contribution due to the n^{th} reflection pulse is

$$\begin{aligned} E_{\text{opn}} &= E_{\text{op}(n-1)} P_s P_r \epsilon^{-2Ts} \\ &= E_{\text{opo}} [P_s P_r \epsilon^{-2Ts}]^n. \end{aligned} \quad (3-26)$$

The total preamplifier output voltage may now be written as the sum of the contributions from the initial pulse and all of the reflection pulses.

$$\begin{aligned} E_{\text{op}} &= \sum_{n=0}^{\infty} E_{\text{opn}} = E_{\text{opo}} \sum_{n=0}^{\infty} [P_s P_r \epsilon^{-2Ts}]^n \\ &= \frac{A_{\text{OL}} (1 + P_r) I_d}{Y_d + Y_o} \sum_{n=0}^{\infty} \left[\frac{Y_o - Y_d}{Y_o + Y_d} \cdot \frac{Y_o - Y_{\text{in}}}{Y_o + Y_{\text{in}}} \epsilon^{-2Ts} \right]^n. \end{aligned} \quad (3-27)$$

D. Development of the Output Pulse Expression for the Special Case in Which a Charge-Sensitive Preamplifier has been Designed to Provide the Best Possible Termination for its Long Input Cable

In order to obtain a somewhat more specific result, consider the following special case. Let the detector output current pulse be

$$i_d(t) = -Q \delta(t) \quad (3-28)$$

so that

$$I_d = -Q \quad (3-29)$$

and let the detector output admittance be

$$Y_d = s C_d \quad (3-30)$$

The sending end reflection coefficient, Equation (3-21), becomes

$$P_{sT} = \frac{Y_o - s C_d}{Y_o + s C_d} = \frac{-s + \frac{1}{\tau_s}}{s + \frac{1}{\tau_s}} = -1 + \frac{\frac{2}{\tau_s}}{s + \frac{1}{\tau_s}} \quad (3-31)$$

where

$$\tau_s \equiv \frac{C_d}{Y_o} = Z_o C_d \quad (3-32)$$

The term $Y_d + Y_o$ in Equation (3-27) becomes

$$Y_d + Y_o = s C_d + \frac{1}{Z_o} = C_d \left(s + \frac{1}{\tau_s} \right) \quad (3-33)$$

In addition, consider that the preamplifier is designed to provide as good a termination as possible; i. e., the conditions of Equations (3-6) and (3-7), page 49, are satisfied. Under these conditions, as has been shown previously, Y_{in} becomes Y_{inT} , where Y_{inT} was shown in Equation (3-11), page 49. Using Equation (3-11), the receiving end reflection coefficient, Equation (3-17), page 54, becomes

$$P_{rT} = \frac{Y_o - s C_I - \frac{1}{Z_o}}{Y_o + s C_I + \frac{1}{Z_o}} = \frac{-s}{s + \frac{2}{\tau_I}} = -1 + \frac{\frac{2}{\tau_I}}{s + \frac{2}{\tau_I}} \quad (3-34)$$

where

$$\tau_I \equiv Z_o C_I . \quad (3-35)$$

The preamplifier open-loop gain expression, Equation (3-13), page 53, under the conditions of Equations (3-6) and (3-7), becomes

$$A_{OLT} = - \frac{R_f}{Z_o} \frac{1}{1 + R_f C_f s} = - \frac{1}{Z_o C_f} \frac{1}{s + \frac{1}{\tau_f}} \quad (3-36)$$

where

$$\tau_f \equiv R_f C_f . \quad (3-37)$$

Substituting Equations (3-28) - (3-37) into Equation (3-27) gives the preamplifier output voltage expression for the case outlined above which, after simplification, is

$$E_{opT} = \frac{\frac{Q}{C_f} \cdot \frac{2}{\tau_s \tau_I}}{(s + \frac{1}{\tau_f})(s + \frac{1}{\tau_s})(s + \frac{2}{\tau_I})} \sum_{n=0}^{\infty} \left\{ \left[\frac{(s - \frac{1}{\tau_s}) s}{(s + \frac{1}{\tau_s})(s + \frac{2}{\tau_I})} \right] e^{-2Ts} \right\}^n . \quad (3-38)$$

Separation of the first term in Equation (3-38) from the others allows Equation (3-38) to be written as

$$E_{opT} = E_{opoT} + \sum_{n=1}^{\infty} E_{opnT} , \quad (3-39)$$

where

$$E_{opoT} = \frac{\frac{Q}{C_f} \cdot \frac{2}{\tau_s \tau_I}}{(s + \frac{1}{\tau_f})(s + \frac{1}{\tau_s})(s + \frac{2}{\tau_I})} \quad (3-40)$$

is the output due to the initial input pulse and

$$E_{\text{opnT}} = \frac{\frac{Q}{C_f} \cdot \frac{2}{\tau_s \tau_I} s^n (s - \frac{1}{\tau_s})^n e^{-n2Ts}}{(s + \frac{1}{\tau_f})(s + \frac{1}{\tau_s})^{n+1} (s + \frac{2}{\tau_I})^{n+1}} \quad (3-41)$$

is the output due to the n^{th} reflection pulse.

The time-domain expression for the output due to the initial input pulse is obtained by taking the inverse Laplace transform from Equation (3-40).

$$e_{\text{opoT}} = \mathcal{L}^{-1} [E_{\text{opoT}}] = \frac{Q}{C_f} \frac{2}{\tau_s \tau_I} \left[\frac{\epsilon^{-\frac{t}{\tau_f}}}{(\frac{2}{\tau_I} - \frac{1}{\tau_f})(\frac{1}{\tau_s} - \frac{1}{\tau_f})} + \frac{\epsilon^{-\frac{2t}{\tau_I}}}{(\frac{1}{\tau_f} - \frac{2}{\tau_I})(\frac{1}{\tau_s} - \frac{2}{\tau_I})} + \frac{\epsilon^{-\frac{t}{\tau_s}}}{(\frac{1}{\tau_f} - \frac{1}{\tau_s})(\frac{2}{\tau_I} - \frac{1}{\tau_s})} \right] u(t). \quad (3-42)$$

Since

$$\tau_f \gg \frac{\tau_I}{2} \quad \text{and} \quad \tau_s \quad (3-43)$$

for any practical case, Equation (3-42) may be expressed as

$$e_{\text{opoT}} \approx \frac{Q}{C_f} \left[\epsilon^{-\frac{t}{\tau_f}} - \frac{\tau_s \epsilon^{-\frac{t}{\tau_s} - \frac{\tau_I}{2}} \epsilon^{-\frac{2t}{\tau_I}}}{\tau_s - \frac{\tau_I}{2}} \right] u(t). \quad (3-44)$$

The time domain expression for the output due to the n^{th} reflection pulse is obtained by taking the inverse Laplace transform of Equation (3-41). This may be done by first expanding Equation (3-41) by the partial fraction method.²⁷

$$\begin{aligned}
 E_{\text{opnT}} = & \frac{\frac{Q}{C_f} \frac{2}{\tau_s \tau_I} \left(-\frac{1}{\tau_f}\right)^n \left(-\frac{1}{\tau_f} - \frac{1}{\tau_s}\right)^n}{\left(-\frac{1}{\tau_f} + \frac{1}{\tau_s}\right)^{n+1} \left(-\frac{1}{\tau_f} + \frac{2}{\tau_I}\right)^{n+1}} \cdot \frac{1}{s + \frac{1}{\tau_f}} e^{-n2Ts} \\
 & + \sum_{k=0}^n \frac{1}{k!} \left\{ \left[\frac{d^k}{ds^k} \frac{\frac{Q}{C_f} \frac{2}{\tau_s \tau_I} s^n \left(s - \frac{1}{\tau_s}\right)^n}{\left(s + \frac{1}{\tau_f}\right) \left(s + \frac{2}{\tau_I}\right)^{n+1}} \right]_{s = -\frac{1}{\tau_s}} \frac{1}{\left(s + \frac{1}{\tau_s}\right)^{n+1-k}} + \right. \\
 & \left. + \left[\frac{d^k}{ds^k} \frac{\frac{Q}{C_f} \frac{2}{\tau_s \tau_I} s^n \left(s - \frac{1}{\tau_s}\right)^n}{\left(s + \frac{1}{\tau_f}\right) \left(s + \frac{1}{\tau_s}\right)^{n+1}} \right]_{s = -\frac{2}{\tau_I}} \frac{1}{\left(s + \frac{2}{\tau_I}\right)^{n+1-k}} \right\} e^{-n2Ts}. \quad (3-45)
 \end{aligned}$$

Taking the inverse Laplace transform of Equation (3-45) and using the inequality of Equation (3-43) yields

$$\begin{aligned}
e_{opnT} &\approx \frac{Q}{C_f} \left(\frac{\tau_I}{2\tau_f} \right)^n \epsilon^{-\frac{t-2nT}{\tau_f}} u(t-2nT) \\
&+ \frac{Q}{C_f} \sum_{k=0}^n \left\{ \left[\frac{d^k}{ds^k} \frac{\frac{2}{\tau_s \tau_I} s^n (s - \frac{1}{\tau_s})^n}{(s + \frac{1}{\tau_f})(s + \frac{2}{\tau_I})^{n+1}} \right]_{s = -\frac{1}{\tau_s}} \frac{(t-2nT)^{n-k} \epsilon^{-\frac{t-2nT}{\tau_s}}}{k! (n-k)!} \right. \\
&\quad \left. + \left[\frac{d^k}{ds^k} \frac{\frac{2}{\tau_s \tau_I} s^n (s - \frac{1}{\tau_s})^n}{(s + \frac{1}{\tau_f})(s + \frac{1}{\tau_s})^{n+1}} \right]_{s = -\frac{2}{\tau_I}} \frac{(t-2nT)^{n-k} \epsilon^{-\frac{2(t-2nT)}{\tau_I}}}{k! (n-k)!} \right\} u(t-2nT) . \quad (3-46)
\end{aligned}$$

Inspection of the first term of Equation (3-44) and the first term of Equation (3-46) reveals that, in view of the inequality of Equation (3-43), the first term of Equation (3-46) may be neglected for any $n \geq 1$. The total time-domain output pulse expression, which is

$$e_{opT} = e_{opoT} + \sum_{n=1}^{\infty} e_{opnT} , \quad (3-47)$$

may therefore be written, using Equation (3-44) and Equation (3-46) without its first term, as

$$\begin{aligned}
e_{opT} = & \frac{Q}{C_f} \left[\epsilon^{-\frac{t}{\tau_f}} - \frac{\epsilon^{-\frac{t}{\tau_s}} - \frac{\tau_I}{2} \epsilon^{-\frac{2t}{\tau_I}}}{\tau_s - \frac{\tau_I}{2}} \right] u(t) \\
& + \frac{Q}{C_f} \sum_{n=1}^{\infty} \sum_{k=0}^n \left\{ \left[\frac{d^k}{ds^k} \frac{\frac{2}{\tau_s \tau_I} s^n (s - \frac{1}{\tau_s})^n}{(s + \frac{1}{\tau_f})(s + \frac{2}{\tau_I})^{n+1}} \right] \frac{(t-n2T)^{n-k} \epsilon^{-\frac{t-n2T}{\tau_s}}}{k! (n-k)!} \right. \\
& \quad \left. s = -\frac{1}{\tau_s} \right. \\
& + \left. \left[\frac{d^k}{ds^k} \frac{\frac{2}{\tau_s \tau_I} s^n (s - \frac{1}{\tau_s})^n}{(s + \frac{1}{\tau_f})(s + \frac{1}{\tau_s})^{n+1}} \right] \frac{(t-n2T)^{n-k} \epsilon^{-\frac{2(t-n2T)}{\tau_I}}}{k! (n-k)!} \right\} u(t-n2T). \quad (3-48) \\
& \quad \left. s = -\frac{2}{\tau_I} \right.
\end{aligned}$$

Some general comments concerning the total output pulse expression, Equation (3-48), may now be made. The portion of the output which is due to the initial input pulse (i.e., the first bracketed term of Equation (3-48)) is a pulse whose decay-time is controlled by the long time constant τ_f and whose rise-time is controlled by the short time constants τ_s and $\frac{\tau_I}{2}$. The exact shape and the 10-90 percent rise-time of the front edge will depend on the relative magnitudes of τ_s and $\frac{\tau_I}{2}$ which, as can be seen from Equations (3-32) and (3-35), pages 57 and 58, depend on the relative magnitudes

of capacitances C_d and C_I . Decay time constant τ_f is the preamplifier feedback time-constant, as shown in Equation (3-37), page 58.

The reflection inputs appear in the output as a sequence of fast perturbations superposed on the initial output. The term "fast perturbation" is used because the perturbations are controlled by time-constants τ_s and $\frac{\tau_I}{2}$, both of which are very short compared to the decay time-constant of the initial pulse, τ_f .

The first perturbation occurs at time $2T$ after the beginning of the initial output, where T is the time-delay of the input cable. Succeeding perturbations occur, in turn, at time intervals of $2T$ thereafter.

Generally, the charge-sensitive preamplifier will be followed by a pulse shaping filter. If the shaping filter time-constants are such that

$$\frac{\tau_I}{2} \text{ and } \tau_s \ll \tau_{\text{FILTER}} \ll \tau_f, \quad (3-49)$$

then the filter will "see" neither the fast rise, the fast perturbations, nor the slow decay of the preamplifier output. That is, if Equation (3-49) is satisfied, the preamplifier output will appear to the shaping filter, effectively, as

$$e_{\text{opT}} \approx \frac{Q}{C_f} u(t). \quad (3-50)$$

E. A Simple Example of Mismatch Effects

The most simple type of mismatch would be that where the resistive part of the preamplifier input admittance was not equal to the cable characteristic admittance. This would occur if the preamplifier parameters R_B and C_N were made to be, for example,

$$R_{BM} = \frac{R_f}{g_m A_i A_v 2Z_o} \quad (3-51)$$

and

$$C_{NM} = g_m A_i A_v 2Z_o C_f, \quad (3-52)$$

instead of the correct values for termination which are given in Equations (3-6) and (3-7), page 49. In this case, the preamplifier input admittance is found, from Equation (3-5), page 47, to be

$$Y_{inM} \approx s C_I + \frac{1}{2Z_o} \quad (3-53)$$

where the approximation of Equation (3-10), page 49, has again been used.

Also for this case, the receiving end reflection coefficient is found, from Equation (3-17), page 54, to be

$$P_{rM} = \frac{-s + \frac{1}{2\tau_I}}{s + \frac{3}{2\tau_I}} = -1 + \frac{\frac{2}{\tau_I}}{s + \frac{3}{2\tau_I}}, \quad (3-54)$$

and the preamplifier open-loop gain is found from Equation (3-13), page 53, to be

$$A_{OLM} = - \frac{1}{2 Z_o C_f} \cdot \frac{1}{s + \frac{1}{\tau_f}} , \quad (3-55)$$

where the definitions of Equations (3-35) and (3-37), page 58, apply.

The preamplifier output pulse for this mistermination case may be obtained by replacing P_{rT} and A_{OLT} by P_{rM} and A_{OLM} , respectively, in the development of Equations (3-28) - (3-50), pages 56 - 63. Carrying out that development, including approximations, and assuming in addition that

$$\tau_f \gg n2T \quad (3-56)$$

yields the following expression for the preamplifier output pulse which is "seen" by the shaping filter that follows the preamplifier.

$$e_{opM} \approx \frac{Q}{C_f} \frac{2}{3} \sum_{n=0}^{\infty} \frac{1}{3^n} u(t-n2T) . \quad (3-57)$$

The effective output shown in Equation (3-57) is not a single step-function, which, as shown in Equation (3-50), was true for the terminated case, but rather a sequence of step-functions of decreasing amplitude. Successive steps occur, in turn, at time intervals of $2T$ after the initial step where, again, T is the time delay of the input cable.

The shaping filter, therefore, would not "see" each detector event as a single event, but as many events spaced apart in time by $2T$ seconds. The output data would thus be meaningless.

As previously stated, the mistermination case discussed above is the most simple type. The situation would be worse if R_B and C_N were not chosen so that their product was equal to τ_f .

F. Effects of Long Input Cables on Preamplifier Output Noise

The noise power transfer functions for the charge-sensitive preamplifier shown in Figure 2-8, page 39, were developed in Chapter 2 and are shown in Equations (2-72) and (2-80), pages 40 and 41. Application of the "mid-band" approximation given in Equation (2-73), page 40, allowed the noise power transfer functions to be written in the forms shown in Equations (2-74) and (2-81), pages 40 and 42.

The preamplifier mid-band output noise power spectral density function may now be written, using Equations (2-74) and (2-81) and the noise sources shown in Figure 2-8, as

$$\begin{aligned} \overline{e_{no}^2} &= \overline{i_{gf}^2} G_{gfm} + \overline{i_{df}^2} G_{dfm} \\ &= \overline{i_{gf}^2} \frac{1}{\omega^2 C_f^2} + \overline{i_{df}^2} \frac{(C_f + C_T)^2}{g_m^2 C_f^2}. \end{aligned} \quad (3-58)$$

The portion of the preamplifier output noise spectrum which is of interest is that portion which will be "seen" by the pulse shaping filter following the amplifier. If the electrical length of the input cable is short compared to the wavelength of those frequencies "seen" by the filter, the cable will not exhibit transmission line properties at those frequencies. The cable may then be represented by a capacitance

to ground at the preamplifier input. The capacitance will be the lumped capacitance of the cable, C_{CL} , where

$$C_{CL} \cong \frac{\text{Cable capacitance}}{\text{ft.}} \times \text{Cable length in ft.} \quad (3-59)$$

As an example, consider an input cable consisting of 100 feet of RG62A/U driving a preamplifier which is followed by an RC-RC filter having 1 $\mu\text{sec.}$ shaping time constants. The propagation velocity for RG62A/U is approximately 84 percent of the velocity of light, while the center frequency of the filter is

$$f_c = \frac{1}{2\pi\tau} \approx 159 \text{ KHz} . \quad (3-60)$$

The cable length corresponding to one wavelength at the center frequency of the filter may now be calculated using the relationship²⁶ between wavelength, λ , propagation velocity, v , and frequency, f .

$$\lambda = \frac{v}{f} \quad (3-61)$$

$$\lambda_c = \frac{.84 (3 \times 10^8)}{159 \times 10^3} \approx 1585 \text{ meters} \approx 5200 \text{ ft.} \quad (3-62)$$

The 100 ft. cable length is therefore much shorter than one wavelength for frequencies "seen" by the filter. Thus, the cable may be represented by its total lumped capacitance which, for 100 feet of RG62A/U, is

$$C_{CL} \approx 13.5 \frac{\text{pf.}}{\text{ft.}} \times 100 \text{ ft.} = 1350 \text{ pf.} \quad (3-63)$$

Since this lumped cable capacitance adds directly to the total capacitance at the preamplifier input, C_T , it increases the portion of the output noise spectrum due to the $\overline{i_{df}^2}$ noise source as can be seen from Equation (3-58). For cable capacitances such as that shown in Equation (3-63) the increase in noise would be drastic.

One result of the increase in the portion of output noise due to the $\overline{i_{df}^2}$ source is that the $\overline{i_{gf}^2}$ source may be allowed to become larger than would usually be the case and still have its contribution to the output noise be only a small part of the total.

The noise current generator associated with feedback resistor R_f ,²⁸

$$\overline{i_{R_f}^2} = \frac{2kT}{\pi R_f} \frac{\text{Amps}^2}{\text{rad/sec}}, \quad (3-64)$$

is a component of $\overline{i_{gf}^2}$. Normally, R_f is constrained to be very large so that its contribution to the total output noise will be small. It would be desirable, however, to reduce R_f in order to lessen the problem of pulse pile-up at high count-rates.

Reducing R_f allows higher count-rates without pulse pile-up because the decay time constant of the preamplifier output pulse is a direct function of R_f as shown in Equation (2-39), page 24.

It would be informative to know how small R_f can be and still have its noise contribution negligible. In order to obtain a useful expression for the minimum value of R_f some generalization will be necessary.

First, assume that the $\overline{i_{gf}^2}$ noise source is dominated by $\overline{i_{R_f}^2}$, Equation (3-64); the total input capacitance is dominated by the lumped cable capacitance, C_{CL} ; and $C_{CL} \gg C_f$. The output noise power spectral density function, Equation (3-58), may now be rewritten as

$$\overline{e_{no}^2} \approx \frac{2kT}{\pi R_f \omega^2 C_f^2} + \overline{i_{df}^2} \frac{C_{CL}^2}{g_m^2 C_f^2} \quad (3-65)$$

Now assume that the $\overline{i_{df}^2}$ source is due only to white noise from the FET,²⁹ i.e.

$$\overline{i_{df}^2} \approx \frac{2kT}{\pi} (.7) g_m \frac{\text{Amps}^2}{\text{rad/sec}} \quad (3-66)$$

The $\overline{e_{no}^2}$ expression, Equation (3-65), then becomes

$$\overline{e_{no}^2} \approx \frac{2kT}{\pi R_f \omega^2 C_f^2} + \frac{2kT (.7) C_{CL}^2}{\pi g_m C_f^2} \quad (3-67)$$

Let the minimum value of R_f , R_{fmin} , be defined as the value at which the output noise power due to R_f , evaluated at the filter center frequency, is 10 percent of the output noise power due to FET white noise. That is,

$$\left. \frac{2kT}{\pi R_{fmin} \omega^2 C_f^2} \right|_{\omega = \frac{1}{\tau}} \equiv .1 \frac{2kT (.7) C_{CL}^2}{\pi g_m C_f^2} \quad (3-68)$$

Solving Equation (3-68) for R_{fmin} yields

$$R_{fmin} = \frac{g_m \tau^2}{.07 C_{CL}^2} \quad (3-69)$$

This equation may be used to find R_{fmin} without the long input cable by simply replacing C_{CL} by the total input capacitance.

There is also another requirement for R_f . For correct pulse shaping by the shaping filter, the decay time constant of the preamplifier output pulse should be large compared to the shaping time constant. That is,

$$R_f C_f \gg \tau \quad (3-70)$$

In many practical systems the value of R_{fmin} given in Equation (3-69) will be smaller than the value required by Equation (3-70). This is not a contradiction. It simply means that, in such a case, the value of R_f may be made as near as desired to the value required by Equation (3-70) with no appreciable increase in noise.

For convenience, the information contained in the equation for R_{fmin} , Equation (3-69), is shown in nomographic form in Figure 3-5. The use of Figure 3-5 will be illustrated by the following example.

Consider again the example system used above for calculating C_{CL} and assume the preamplifier's input FET to have a transconductance of 15 mmhos. Enter Figure 3-5 at the previously given value of C_{CL} , 1350 pf., and drop vertically to the line representing the previously given value of τ , 1 μ sec. From that point draw a horizontal line to intersect a line representing the transconductance, 15 mmhos. Now,

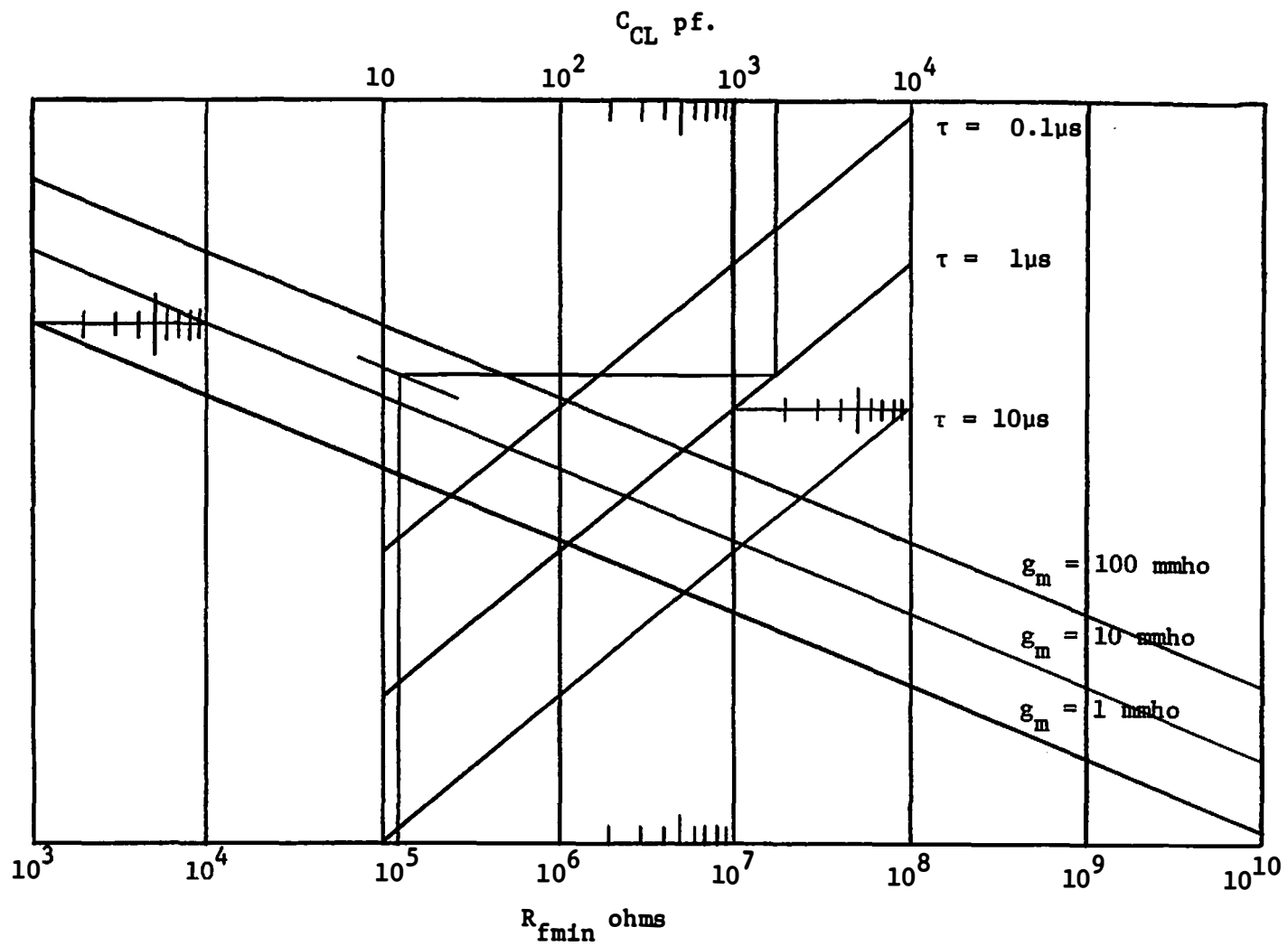


Figure 3-5. Nomographic display of Equation (3-69) for calculating R_{fmin} .

dropping vertically to the R_{fmin} scale, the value of R_{fmin} is found to be approximately 120 K ohms.

If the preamplifier had a 5 pf. feedback capacitor, however, Equation (3-70) would require R_f to be much larger than 200 K ohms. Since this value is larger than the value found above for R_{fmin} , any value of R_f which provides acceptable pulse shaping may be used with no ill effect on noise performance.

The value of R_{fmin} given by Equation (3-69) is pessimistic since the $\overline{i_{df}^2}$ noise source will, in general, have contributions from other sources besides that from the FET white noise.

The results of the above discussion indicate that with a long input cable, the noise performance of a charge-sensitive preamplifier is apt to be considerably worse than it would have been without the long cable.

By making use of multiple input devices, the noise performance with long input cables could be improved. Improvement would be realized because the effective g_m would be increased by the use of multiple devices, while the total input capacitance, dominated by C_{CL} , would not change materially.

CHAPTER 4

EXPERIMENTAL RESULTS

In this chapter the design of a practical charge-sensitive preamplifier with an internal positive feedback loop is discussed.

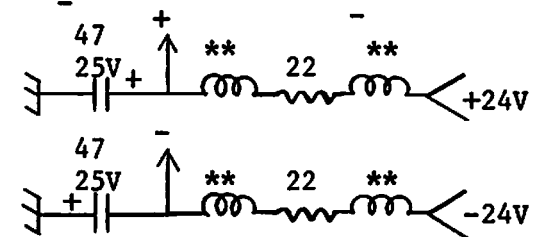
The general design criteria and the operation of the internal positive feedback loop are considered.

Preamplifier performance is determined for the cases of no positive feedback, positive feedback for reducing output pulse height variations with input capacitance changes, and positive feedback for reducing output pulse rise-time. Actual performance is compared to that predicted from the equations of Chapter 2 for each case.

Long input cables are introduced and, with positive feedback used to provide termination for the cables, the preamplifier output pulse shapes are evaluated and compared to the pulse shapes predicted by the equations of Chapter 3.

A. General Discussion of the Experimental Preamplifier Design

The schematic diagram of the experimental preamplifier is shown in Figure 4-1. Comparison of Figure 4-1 and Figures 2-1, 2-2, and 2-3, pages 10, 12, and 14, will permit easy identification of the functional blocks which were used, in the preceding chapters, in discussing theoretical preamplifier performance.



**** Ferrite bead on resistor lead.**

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The input FET is Q_1 . Transistors Q_2 and Q_3 comprise the current amplifier, A_i . Complementary emitter-followers Q_6 , Q_7 , Q_8 , and Q_9 make up the voltage amplifier, A_v .

Transistors Q_4 and Q_5 serve as a constant-current source for dc bias and also function as part of the internal positive feedback loop, whose operation will be described presently.

The TIS75 transistor was chosen for the input FET, Q_1 , because of its large ratio of transconductance-to-input capacitance. The device is not operated at zero volts gate-to-source bias for two reasons. First, power dissipation in the device would be quite large if the drain current was equal to I_{DSS} , since I_{DSS} for the TIS75 is large (80 ma. max., 38.5 ma. for the device used). Second, temperature stability of the transconductance is poor for $V_{GS} = 0$.

The required negative gate-bias voltage was obtained by introducing diodes D_5 and D_6 into the dc emitter-return circuit of the output emitter-follower Q_8 , and taking the dc feedback signal from the cathode of D_5 . This technique was used, instead of simply letting the quiescent preamplifier output voltage be the required negative value, because it was desired that the quiescent output voltage be zero in order to allow dc coupling at the preamplifier output.

Transistors Q_2 - Q_7 , whose type numbers are given in Figure 4-1, were chosen for their high gain-bandwidth products, f_T . Additionally, transistors Q_3 - Q_7 were required to have very low base-collector capacitance, C_{ob} , since these capacitances contribute to the total effective

capacitance to ground at the collector of Q_3 , which is the location of the dominant open-loop pole.

The complementary emitter-follower configuration was chosen for the output group because of its excellent drive capability, good linearity, and good high-frequency response. Diodes D_1 - D_4 provide low-impedance dc voltage separation for the base terminals of complementary emitter-followers. The output complementary emitter-followers, Q_8 and Q_9 , have separate dc bias circuits for their emitters, with ac coupling between their emitters, in order to provide stability against thermal runaway. Diode D_7 prevents large bias shifts at high pulse repetition rates.

One facet of the experimental preamplifier design is worthy of special mention. Two transistors were used in each of the Q_2 - Q_3 and Q_5 - Q_4 common base current gain stages. This was done in order to make the handling of the effects of the internal positive feedback loop as easy as possible, since those effects are of primary interest.

The effective output impedances of Q_3 and Q_4 will not be functions of input FET bias circuitry and value of positive feedback impedance because of the isolation provided by Q_2 and Q_5 . In addition, very small values of positive feedback resistance, R_{PF} , may be used without upsetting the bias conditions since both ends of R_{PF} are at approximately the same dc potential.

If the preamplifier had been designed with low noise as a prime requisite, the Q_2 - Q_3 and Q_5 - Q_4 current gain stages would certainly have consisted of only one transistor each.

One of the prime factors considered in selecting the active devices, in addition to those stated above, was low cost. All of the devices are very inexpensive epoxy or plastic packaged types.

The experimental preamplifier was housed in a steel box (5-1/2 × 2-1/2 × 1-1/2 inches) and BNC type signal connectors were used. The circuit was constructed on a glass epoxy board which was copper plated on both sides. Point-to-point wiring techniques were used. Transistors were mounted in miniature teflon sockets.

B. Description of the Open-Loop Gain of the Experimental Preamplifier

The open-loop gain, utilized in the discussions of Chapters 2 and 3, is given in Equation (2-7), page 13. For convenience, Equation (2-7) is rewritten here, including the definition shown in Equation (2-2), page 13.

$$A_{OL} \equiv \frac{V_o}{V_g} = - \frac{g_m A_i R_B A_v}{1 + R_B C_N s} \quad (4-1)$$

The terms in Equation (4-1) will now be described quantitatively for the experimental preamplifier. Reference should be made to Figures 2-1, 2-2, and 2-3, pages 10, 12, and 14; and Figure 4-1, page 74, throughout the following development.

The measured transconductance of the input FET at the bias point used ($I_D \approx 14.5$ ma., $V_{DS} \approx 4.9$ V) was

$$g_m = 15 \text{ mmhos} \quad (4-2)$$

As previously stated, transistors Q_2 and Q_3 comprise the current amplifier A_1 . The current gain is the product of a current division term between R_D and h_{ib2} , α of Q_2 , a current division term between the collector load resistor of Q_2 and h_{ib3} , and α of Q_3 , where h_{ib2} and h_{ib3} are the common-base equivalent input resistances of Q_2 and Q_3 , respectively. Neglecting base spreading resistance, $h_{ib} = r_e = kT/qI_E$. The current gain is found to be

$$A_1 = 0.95 \quad (4-3)$$

for

$$I_{E2} = 1.5 \text{ ma.} ,$$

$$\beta_2 = 40 ,$$

$$I_{E3} = 8 \text{ ma.} ,$$

and

$$\beta_3 = 70 . \quad (4-4)$$

Because the emitter terminals of the common-base stages, Q_2 and Q_3 , are driven from impedances which are large compared to their own input impedances, the frequency response of each stage, and therefore of A_1 , will be close to the transistor's gain-bandwidth product, f_T .³⁰ A typical value of f_T for Q_2 (2N3563) is 900 MHz (600 MHz minimum). Similarly, the typical f_T for Q_3 (2N4258) is 1.1 GHz (500 MHz minimum).

The resistor R_B and the capacitor C_N represent, as stated in Chapter 2, the total equivalent resistance and capacitance to ground at

the output of the current amplifier. Equations for R_B and C_N , including the effects of positive feedback elements R_{PF} and C_{PF} will now be developed.

The circuitry which loads the current amplifier, A_1 , is shown, with dc bias components removed, in Figure 4-2. For convenience, complementary emitter-followers Q_6 and Q_7 have been replaced by one equivalent transistor Q_{67} , a procedure shown to be valid in Appendix B, and the input resistance to complementary emitter-followers Q_8 and Q_9 has been represented by the resistor R_{189} .

Before proceeding, some simplifications will be made which will greatly simplify the calculation of R_B and C_N .

Since transistors Q_5 and Q_4 form a current amplifier of the same type comprised by Q_2 and Q_3 , they may be represented by a current source $A_{154} I_{54}$ as shown in Figure 4-3. Current gain A_{154} is calculated in the same manner outlined above for calculating A_1 with the exception that the 75 ohm resistor, which is a parasitic suppressor, should be added to the input resistance of Q_4 . With

$$I_{E4} = 8 \text{ ma. ,}$$

$$\beta_4 = 70 \text{ ,}$$

$$I_{E5} = 2 \text{ ma. ,}$$

and

$$\beta_5 = 70 \text{ ,} \tag{4-5}$$

the current gain is found to be

$$A_{154} = 0.92. \tag{4-6}$$

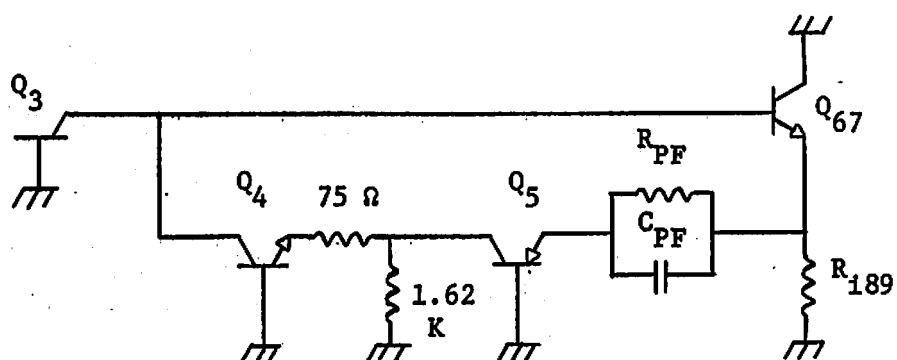


Figure 4-2. Load circuit for current amplifier A_1 .

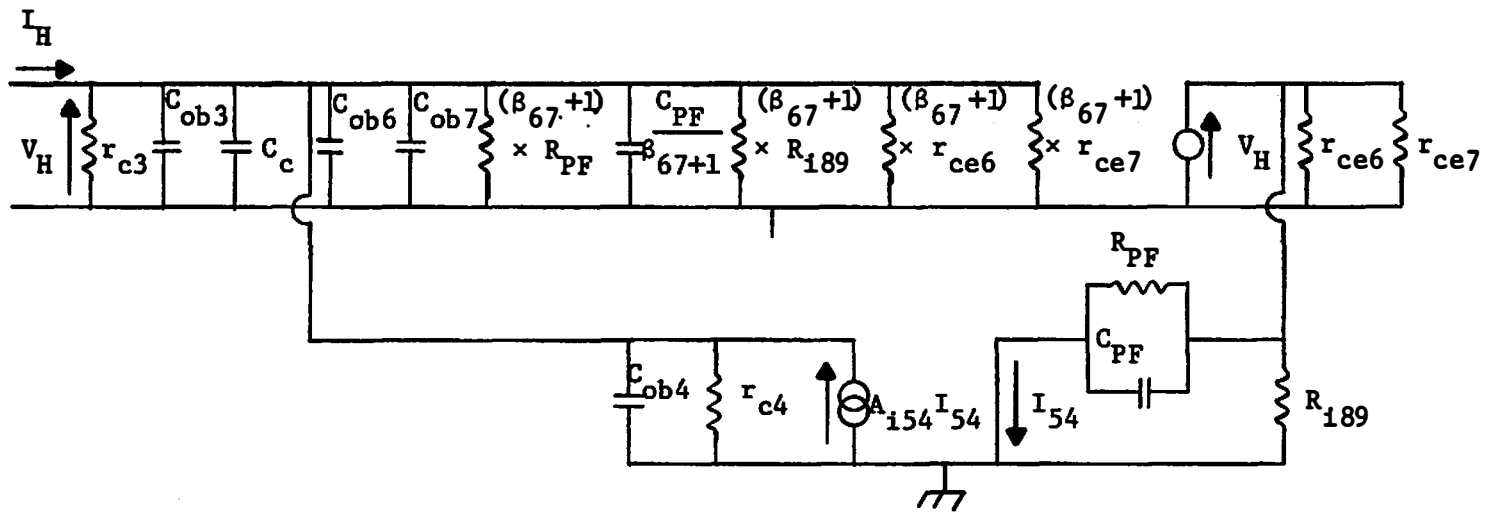


Figure 4-3. Circuit for calculating R_B and C_N .

Frequency response for A_{154} , as for A_1 , is near f_T of the transistor involved. For Q_4 , a 2N4255, the value of f_T is 1.4 GHz maximum (600 MHz minimum) and for Q_5 , a 2N4258, f_T is typically 1.1 GHz (500 MHz minimum).

If the total emitter load impedance for Q_{67} is assumed to be much, much larger than r_{e67} , emitter-follower Q_{67} may be replaced by a voltage amplifier with a gain of one, an input impedance equal to β_{67+1} times the total effective emitter load impedance, and a frequency response near f_{T67} .³⁰ The total effective emitter load impedance includes the equivalent collector-to-emitter resistances, r_{ce} , of Q_6 and Q_7 , as shown in Appendix B. This representation is shown in Figure 4-3. Transistor Q_6 , a 2N4255, and Q_7 , a 2N4258, have values of β of 90 and 125, respectively, and values of r_{ce} of 20 Kohms and 7 Kohms, respectively. From Equation (B-17), Appendix B, the effective β of Q_{67} is found to be $\beta_{67} = 105$.

Also appearing in Figure 4-3 are the collector-to-base capacitances, C_{ob} , of Q_3 , Q_4 , Q_6 , and Q_7 , and the stray capacitance, C_c . For Q_3 and Q_7 , both 2N4258's, C_{ob} is typically 2 pf. (3 pf. maximum) and for Q_4 and Q_6 , both 2N4255's, C_{ob} is 0.1 pf. minimum (0.65 pf. maximum).

Resistors r_{c3} and r_{c4} in Figure 4-3 are the equivalent common base output resistances of Q_3 and Q_4 , respectively, and are related to the equivalent collector-to-emitter resistances, r_{ce} , by the equation⁶

$$r_c = r_{ce} \left[1 + \beta \left(1 - \frac{\beta r_e + r_b}{\beta r_e + r_b + R_{os}} \right) \right], \quad (4-7)$$

where r_b is the base spreading resistance and R_{os} is the total resistance in the emitter circuit. With $r_{ce3} = 3.5$ Kohms, $r_{ce4} = 8$ Kohms, $\beta_3 = \beta_4 = 70$, $I_{E3} = I_{E4} = 8$ ma., $R_{os3} = 1.62$ Kohms, $R_{os4} = 1695$ ohms, and neglecting r_b , the values for r_c are found to be

$$r_{c3} = 218 \text{ Kohms} \quad (4-8)$$

and

$$r_{c4} = 501 \text{ Kohms} . \quad (4-9)$$

The input resistance to complementary emitter-followers Q_8 and Q_9 , R_{189} , may be found by replacing Q_8 and Q_9 by an equivalent transistor, Q_{89} , and following the same procedure outlined previously in the treatment of Q_{67} . The needed parameters of Q_8 , an MPS6531, are $\beta_8 = 135$ and $r_{ce8} = 30$ Kohms and of Q_9 , an MPS6534, are $\beta_9 = 230$ and $r_{ce9} = 4$ Kohms. Including the two 4.7 Kohm emitter bias resistors of Q_8 and Q_9 as part of the total emitter load, and assuming that the preamplifier output drives an unterminated cable, the effective input resistance to Q_8 and Q_9 is found to be

$$R_{189} = 241 \text{ Kohms} . \quad (4-10)$$

As for the Q_6 - Q_7 emitter-followers, the frequency response of the Q_8 - Q_9 emitter-followers will be near the transistor's f_T if the total emitter load is much larger than the r_e of the transistor. Values of f_T for Q_8 and Q_9 are, typically, 390 MHz and 260 MHz, respectively.

The parameters R_B and C_N may now be found from the circuit of Figure 4-3, page 81, by writing the two node equations,

$$I_H = V_H \left\{ \left[\frac{1}{r_{c3}} + \frac{1}{r_{c4}} + \frac{1}{\beta_{67}+1} \left(\frac{1}{R_{PF}} + \frac{1}{R_{189}} + \frac{1}{r_{ce6}} + \frac{1}{r_{ce7}} \right) \right] + s \left[C_{ob3} + C_{ob4} + C_{ob6} + C_{ob7} + C_c + \frac{C_{PF}}{\beta_{67}+1} \right] \right\} - A_{154} I_{54} \quad (4-11)$$

and

$$I_{54} = V_H \left(\frac{1}{R_{PF}} + s C_{PF} \right), \quad (4-12)$$

and solving for

$$\frac{I_H}{V_H} \equiv \frac{1}{R_B} + s C_N. \quad (4-13)$$

The solution is found to be

$$\frac{I_H}{V_H} \equiv \frac{1}{R_B} + s C_N = \left(\frac{1}{R_p} - \frac{K_{PF}}{R_{PF}} \right) + s(C_p - K_{PF} C_{PF}), \quad (4-14)$$

where

$$\frac{1}{R_p} \equiv \frac{1}{r_{c3}} + \frac{1}{r_{c4}} + \frac{1}{\beta_{67}+1} \left(\frac{1}{R_{189}} + \frac{1}{r_{ce6}} + \frac{1}{r_{ce7}} \right), \quad (4-15)$$

$$C_p \equiv C_{ob3} + C_{ob4} + C_{ob6} + C_{ob7} + C_c, \quad (4-16)$$

and

$$K_{PF} \equiv A_{154} - \frac{1}{\beta_{67} + 1} \quad . \quad (4-17)$$

Therefore

$$R_B = \frac{1}{\frac{1}{R_p} - \frac{K_{PF}}{R_{PF}}} = R_p || - \frac{R_{PF}}{K_{PF}} \quad (4-18)$$

and

$$C_N = C_p - K_{PF} C_{PF} = C_p || - K_{PF} C_{PF} \quad . \quad (4-19)$$

Using the parameter values given in the preceding discussion, the values of K_{PF} and R_p are found to be

$$K_{PF} \approx 0.91 \quad (4-20)$$

and

$$R_p \approx 118 \text{ Kohms} \quad . \quad (4-21)$$

Equation (4-18) may therefore be written as

$$R_B \approx \frac{1}{\frac{1}{118K} - \frac{.91}{R_{PF}}} \approx 118K || - \frac{R_{PF}}{.91} \quad . \quad (4-22)$$

Explicit values for the capacitances which make up C_p , shown in Equation (4-16), were not given in the preceding discussion. The value of C_p , determined experimentally, is

$$C_p \approx 4 \text{ pf.} \quad . \quad (4-23)$$

Equation (4-19) may therefore be written as

$$C_N \approx 4 \text{ pf.} - .91 C_{PF} = 4 \text{ pf.} - .91 C_{PF} . \quad (4-24)$$

The open-loop gain expression, Equation (4-1), page 77, may now be rewritten by using Equation (4-2), page 77; Equation (4-3), page 78; and the fact that the voltage gain of the output complementary emitter-followers is approximately one.

$$A_{OL} \equiv \frac{V_o}{V_g} \approx - \frac{(15 \times 10^{-3})(0.95) R_B}{1 + R_B C_N s} . \quad (4-25)$$

The values of R_B and C_N are dependent on R_{PF} and C_{PF} as shown in Equations (4-22) and (4-24).

C. General Performance Characteristics of the Experimental Preamplifier Without Positive Feedback

Values for R_B and C_N without positive feedback, i.e., with R_{PF} and C_{PF} removed from the circuit, may be obtained from Equations (4-22) and (4-24) with $R_{PF} = \infty$ and $C_{PF} = 0$. The values thus obtained,

$$R_B \approx 118 \text{ Kohms} \quad (4-26)$$

and

$$C_N \approx 4 \text{ pf.} , \quad (4-27)$$

may then be used in Equation (4-25) to obtain the open-loop gain without positive feedback,

$$A_{OL} \equiv \frac{V_o}{V_g} \approx - \frac{(15 \times 10^{-3}) (.95) (11.8 \times 10^3)}{1 + (118 \times 10^3) (4 \times 10^{-12}) \text{ s}}$$

$$\approx - \frac{1680}{1 + (472 \times 10^{-9}) \text{ s}} \quad (4-28)$$

Examination of Equations (2-2) - (2-4), page 13 and (4-28) allows identification of the following quantities, which were defined in Equations (2-3) and (2-4).

$$g_m A_{2m} = g_m A_i R_B A_v \approx 1680 \quad (4-29)$$

$$\tau_2 = R_B C_N \approx 472 \times 10^{-9} \text{ sec.} \quad (4-30)$$

These two open-loop parameters will be used in calculating the charge gain from Equation (2-19), page 17. The other parameters needed in Equation (2-19) will now be determined.

The overall feedback resistor and capacitor are found from Figure 4-1, page 74.

$$R_f = 10^9 \text{ ohms} \quad (4-31)$$

$$C_f = 1 \text{ pf.} \quad (4-32)$$

Resistor R_i , shown in Figure 2-2, page 12, is assumed to be approximately equal to the gate leakage resistance of the FET and is

$$R_i \approx 10^{10} \text{ ohms.} \quad (4-33)$$

From Equations (2-5), page 13, (4-31) and (4-33), the parameter R is found to be

$$R \approx 0.91 \times 10^9 \text{ ohms} . \quad (4-34)$$

The effective capacitance from the FET gate to ground, excluding the detector capacitance, C_d , and shown in Figure 2-3, page 14, is found experimentally to be

$$C_1 + C_m \approx 13 \text{ pf.} \quad (4-35)$$

The parameter C_T , defined in Equation (2-1), page 13, may therefore be written as

$$C_T \equiv C_d + C_1 + C_m \approx C_d + 13 \text{ pf.}, \quad (4-36)$$

and for $C_d = 0$ is

$$C_T \approx 13 \text{ pf.} . \quad (4-37)$$

With the parameter values given in Equations (4-29) - (4-37), it may be shown that the inequalities and approximations of Equations (2-23) - (2-27), pages 18 and 19, are satisfied. Under these conditions Equation (2-19), page 17, may be rewritten in the approximate form

$$A_c \equiv \frac{V_{op}}{Q} \approx - \frac{1}{C_f} \quad (4-38)$$

so that, using Equation (4-32), the nominal charge gain is

$$A_c \approx -10^{12} . \quad (4-39)$$

The nominal charge gain was measured by applying the output of an ORNL Model Q-1212 mercury relay pulser (10-90 percent rise-time $\approx$ 3 nsec., decay-time constant $\approx$ 300 μ sec., and repetition rate = 60 Hz) to the preamplifier's test input. The input charge, Q , was therefore

$$Q = v_{in} C_{test} \quad (4-40)$$

where v_{in} is the pulser's peak output voltage and C_{test} is the preamplifier's test input capacitor. From Figure 4-1, page 74, the test input capacitor is

$$C_{test} = 1 \text{ pf.} \quad (4-41)$$

so the charge gain is

$$A_c \equiv \frac{v_{op}}{Q} = \frac{v_{op}}{v_{in}} 10^{12} . \quad (4-42)$$

A Hewlett Packard Model 180A oscilloscope was used to measure the input and output peak voltage levels. This technique, though not extremely accurate, suffices to determine the nominal charge gain. The nominal charge gain thus measured is

$$A_c \approx -0.95 \times 10^{12} , \quad (4-43)$$

which compares excellently with the calculated value of -1×10^{12} shown in Equation (4-39), especially in view of the fact that the 1 pf. test- and feedback-capacitors had a ± 25 percent tolerance.

The calculation and measurement of charge gain carried out above are for the case of zero detector capacitance, C_d . Variations in charge gain which result from changes in C_d will be considered later.

Approximate expressions for the rise- and decay-time constants associated with the preamplifier output pulse are given in Equations (2-38) and (2-39), page 24. These approximate expressions are valid since, as previously stated, the inequalities in Equations (2-23)-(2-27) are satisfied. Using Equations (2-38) and (2-39), the parameter values given in Equations (4-2), (4-3), (4-27), (4-32) and (4-37), and taking the emitter-follower voltage gain, A_v , to be approximately one, the preamplifier output pulse rise- and decay-time constants are calculated to be

$$\tau_r \approx 3.94 \text{ nsec.} \quad (4-44)$$

and

$$\tau_d \approx 1.0 \text{ msec.} , \quad (4-45)$$

respectively. The calculated 10-90 percent rise-time of the output pulse is³⁰

$$t_r \approx 2.2 \tau_r \approx 8.6 \text{ nsec.} \quad (4-46)$$

The 10-90 percent rise-time of the experimental preamplifier was measured by driving the preamplifier's test input with a 100 nsec. wide pulse from a Hewlett-Packard Model 215A pulse generator (10-90 percent

rise-time ≈ 1 nsec.) and viewing the preamplifier output pulse on a Tektronix Model 661 sampling oscilloscope. The HP 215A pulser was used to insure that the signal source rise-time would not affect the measured preamplifier rise-time. The preamplifier 10-90 percent rise-time and the corresponding rise-time constant were found to be

$$t_r \approx 9 \text{ nsec.} \quad (4-47)$$

and

$$\tau_r \approx \frac{9 \text{ nsec.}}{2.2} \approx 4.1 \text{ nsec.}$$

which agree with the calculated values shown in Equations (4-46) and (4-44).

The decay-time constant of the experimental preamplifier's output pulse was measured by driving the preamplifier's test input with a 100 Hz square wave from a Hewlett Packard Model 3301A function generator and viewing the preamplifier output pulse on a Hewlett Packard Model 180A oscilloscope. The square wave input was used to insure that the measured decay-time constant would be due only to the preamplifier. The decay time constant was found to be

$$\tau_d \approx 1 \text{ msec.}, \quad (4-48)$$

which agrees with calculated value shown in Equation (4-45).

The linearity of the experimental preamplifier was measured by using a differential summing technique whose circuitry is shown in

Figure 4-4. The shaping amplifier peak output and the direct output from the mercury relay pulser are compared by viewing their differential sum on the oscilloscope as the pulser output is varied over the desired range. Oscilloscope overload is prevented by the clamp diodes so the differential sum may be viewed with maximum oscilloscope sensitivity (5 mv/division for the Hewlett Packard 180A). Nonlinearity of the system excluding the preamplifier is measured first. That nonlinearity subtracted from the nonlinearity measured with the preamplifier included represents the nonlinearity of the preamplifier alone.

With the preamplifier driving a load of approximately 1 K ohms over a range of ± 5 volts, the nonlinearity was approximately 0.08 percent. For a 93 ohm load the nonlinearity was approximately 0.1 percent over a range of ± 2.5 volts.

The noise performance of the experimental preamplifier was determined by using the RMS meter method³¹ to measure the equivalent noise charge (ENC). The experimental set-up is shown in Figure 4-5.

Equivalent noise charge is defined as

$$\text{ENC} \equiv \frac{E_{\text{no}}}{A_c} \text{ rms coulombs} \quad (4-49)$$

where E_{no} is the rms noise in the shaping amplifier output due to noise from the preamplifier and A_c is the charge gain including the pulse gain of the shaping amplifier.

The value of E_{no} is determined from

$$E_{\text{no}} = (E_{\text{no1}}^2 - E_{\text{no2}}^2)^{1/2} \quad (4-50)$$

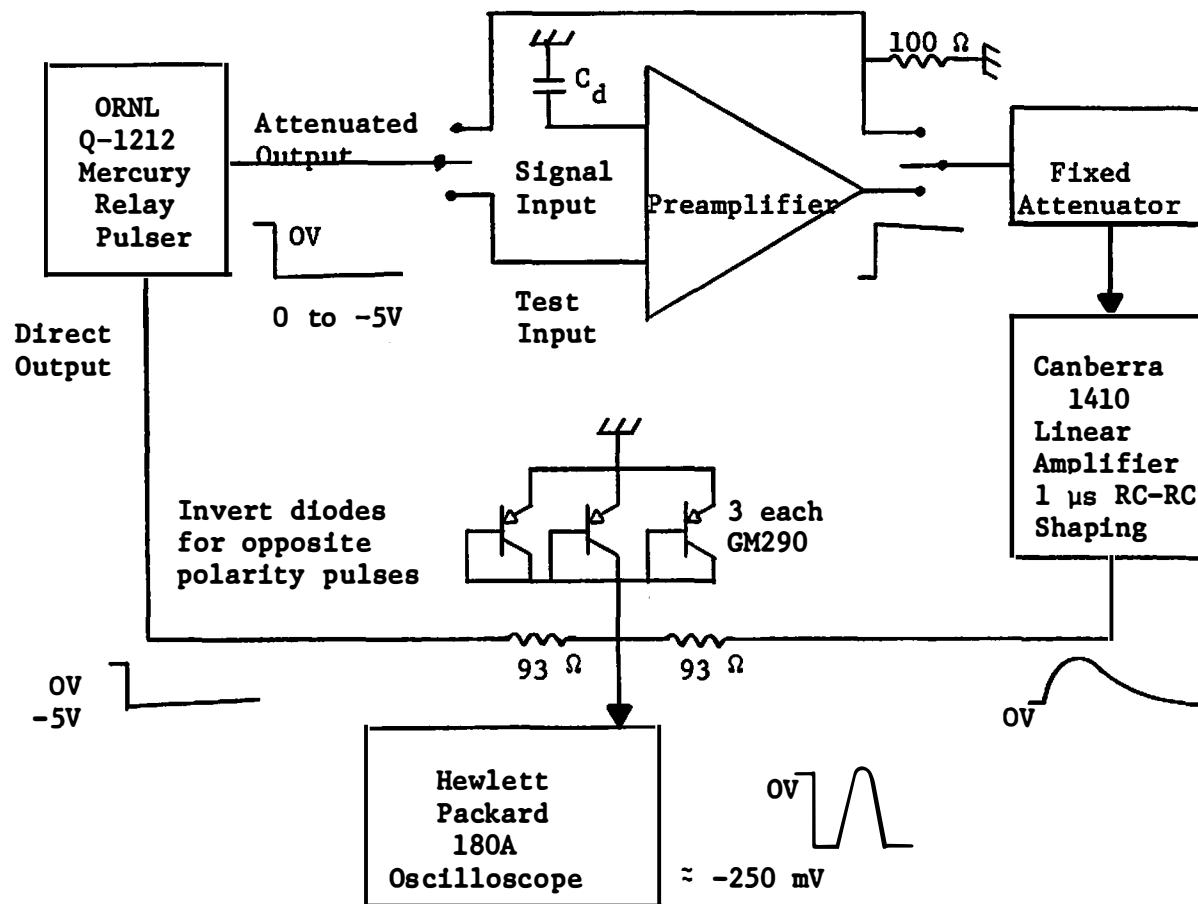


Figure 4-4. Differential summing system for observing small variations in output pulse height.

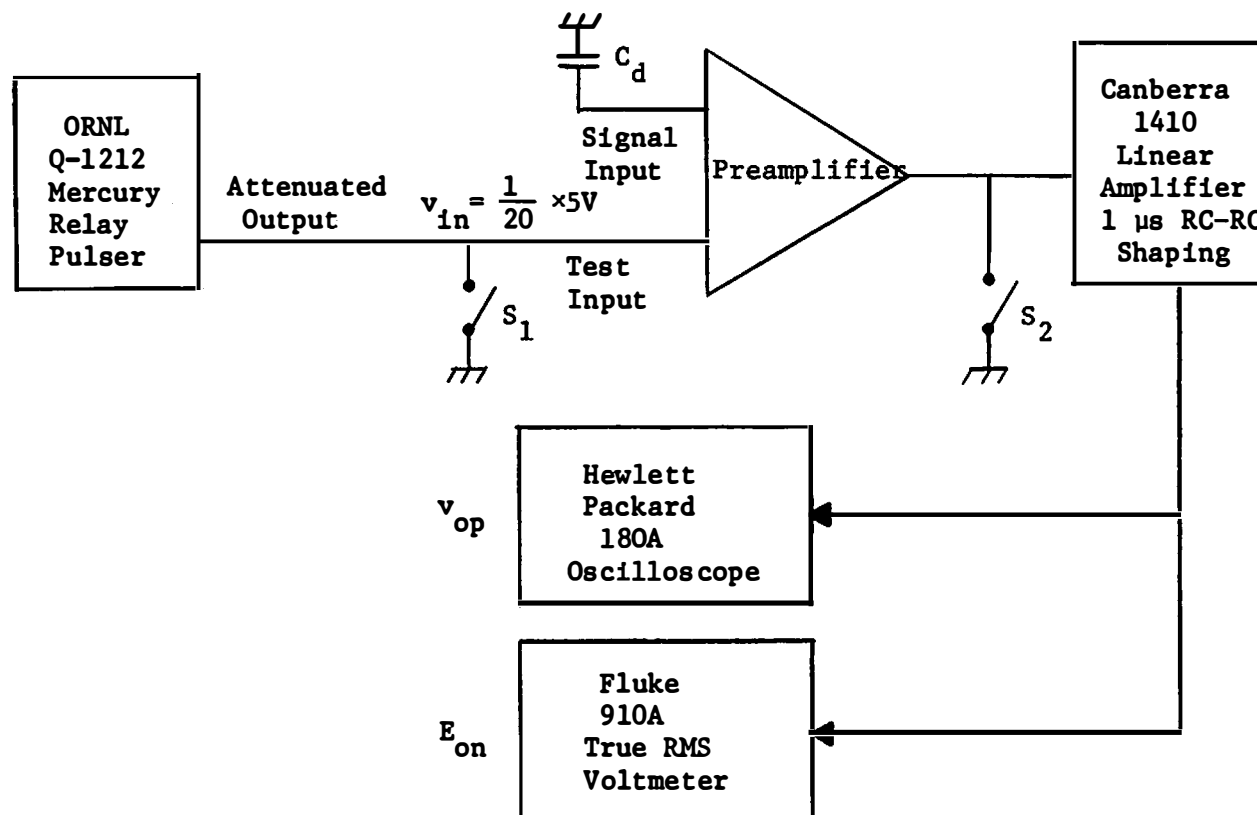


Figure 4-5. System for measuring equivalent noise charge.

where E_{no1} is the rms output noise of the system including the preamplifier and E_{no2} is the rms output noise of the system excluding the preamplifier. The true rms voltmeter measures E_{no1} when switch S_1 is closed and E_{no2} when switch S_2 is closed.

Charge gain has been defined previously and, with a preamplifier test input capacitance, C_{test} , of 1 pf., is

$$A_c = \frac{v_{op}}{v_{in}} \times 10^{12} \quad . \quad (4-51)$$

The values of ENC found for various values of detector capacitance C_d , simulated by a lumped capacitance at the preamplifier's signal input, are shown by the curve labeled "experimental" in Figure 4-6.

The noise performance of the preamplifier may be calculated by referring to Figure 2-8, page 39, and Equations (2-74), page 40, and (2-81), page 42. Equations (2-74), G_{gfm} , and (2-81), G_{dfm} , are the mid-band noise power transfer functions for the noise current sources at the gate and drain terminals, respectively, of the input FET as shown in Figure 2-8. Using those equations, the mid-band noise power spectral density at the preamplifier output may be written as

$$\begin{aligned} \overline{e_{nop}^2} &= \overline{i_{gf}^2} G_{gfm} + \overline{i_{df}^2} G_{dfm} \\ &= \overline{i_{gf}^2} \frac{1}{\omega^2 C_f^2} + \overline{i_{df}^2} \left(\frac{C_f + C_T}{g_m C_f} \right)^2 \quad . \quad (4-52) \end{aligned}$$

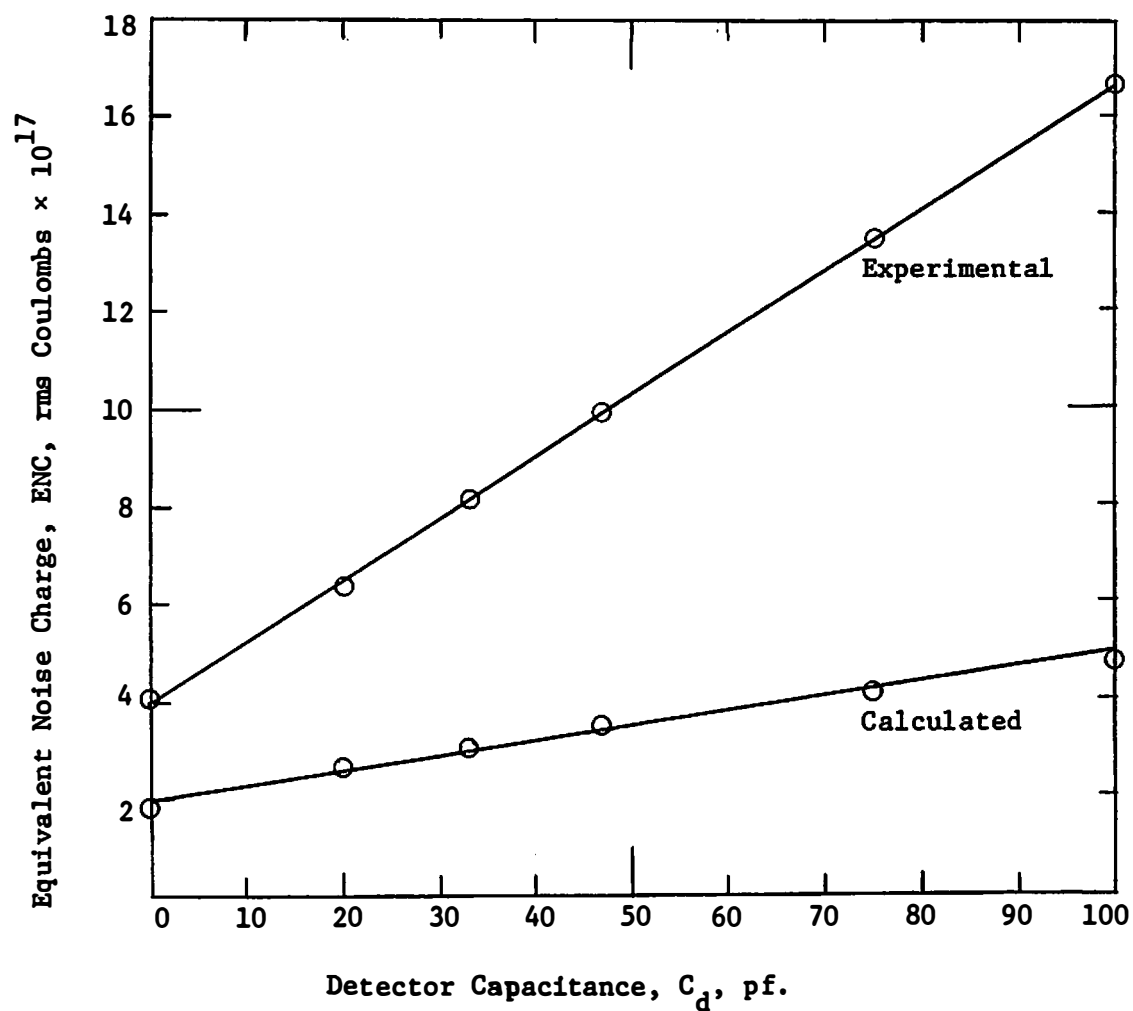


Figure 4-6. Equivalent noise charge, ENC, vs detector capacitance, C_d , for the experimental preamplifier.

The noise current source at the FET gate, $\overline{i_{gf}^2}$, is given approximately by the sum of the mean squared noise currents due to the feedback resistor, R_f , the effective resistance to ground at the FET gate, R_i , and the gate leakage current of the FET, I_G . The expression for $\overline{i_{gf}^2}$ is^{28,29}

$$\overline{i_{gf}^2} \approx \frac{2kT}{\pi R_f} + \frac{2kT}{\pi R_i} + \frac{e I_G}{\pi} \frac{\text{amps}^2}{\text{rad/sec}} \quad (4-53)$$

where k is Boltzman's constant, T is the absolute temperature, and e is the electronic charge. Noise due to lossy dielectrics at the FET gate³² and noise due to channel-to-gate feedback in the FET³ have been neglected.

The noise current source at the FET drain, $\overline{i_{df}^2}$, is given approximately by the sum of the mean squared noise current due to white noise in the FET channel, the mean squared noise current due to flicker noise in the FET channel, the mean squared equivalent noise currents of transistors Q_2 - Q_5 in Figure 4-1, page 74, and the mean squared noise currents of all the bias resistors connected from the signal path to ground in Figure 4-1. The expression for $\overline{i_{df}^2}$ is^{28,29,33}

$$\begin{aligned} \overline{i_{df}^2} \approx & \frac{2kT (.7g_m)}{\pi} + \frac{2kT}{\pi} \left(\frac{g_m^2 F}{|\omega|} \right) \\ & + \frac{e}{\pi} \left[\frac{I_{E2}}{\beta_2} + \frac{I_{E3}}{\beta_3} + \frac{I_{E4}}{\beta_4} + \frac{I_{E5}}{\beta_5} \right] + \frac{2kT}{\pi R_{eq}} \frac{\text{amps}^2}{\text{rad/sec}} \end{aligned} \quad (4-54)$$

where g_m is the FET transconductance, F is the $1/f$ noise constant of

the FET, I_E is dc emitter current, and R_{eq} is the parallel combination of all the bias resistors connected to ground along the signal path extending from the FET drain to the emitter follower bases in Figure 4-1, page 74.

Flicker noise and noise due to I_{cbo} for transistors Q_2 - Q_5 have been neglected.³⁴ The equivalent noise voltage generator and the noise due to base spreading resistance for transistors Q_2 - Q_5 ³⁴ have also been neglected since, in each case, the transistor is driven from an impedance which is much larger than its own input impedance.

Using Equations (4-53) and (4-54) in Equation (4-52), the mid-band noise power spectral density at the preamplifier output may now be written as

$$\begin{aligned} \overline{e_{nop}^2} \approx & \frac{2kT}{\pi C_f^2} \left\{ \left[\frac{1}{R_f} + \frac{1}{R_i} + \frac{e I_G}{2kT} \right] \frac{1}{\omega^2} \right. \\ & \left. + [F(C_f + C_T)^2] \frac{1}{|\omega|} \right. \\ & \left. + \left[.7g_m + \frac{e}{2kT} \left(\frac{I_{E2}}{\beta_2} + \frac{I_{E3}}{\beta_3} + \frac{I_{E4}}{\beta_4} + \frac{I_{E5}}{\beta_5} \right) + \frac{1}{R_{eq}} \right] \frac{(C_f + C_T)^2}{g_m^2} \right\} \frac{\text{volts}^2}{\text{rad/sec}} . \end{aligned} \quad (4-55)$$

Making the following definitions,

$$K_2 \equiv \frac{2kT}{\pi} \left[\frac{1}{R_f} + \frac{1}{R_i} + \frac{e I_G}{2kT} \right] \quad (4-56)$$

$$K_1 \equiv \frac{2kT}{\pi} F(C_f + C_T)^2 \quad (4-57)$$

$$K_o \equiv \frac{2kT}{\pi} \left[.7g_m + \frac{e}{2kT} \left(\frac{I_{E2}}{\beta_2} + \frac{I_{E3}}{\beta_3} + \frac{I_{E4}}{\beta_4} + \frac{I_{E5}}{\beta_5} \right) + \frac{1}{R_{eq}} \right] \frac{(C_f + C_T)^2}{g_m^2}, \quad (4-58)$$

allows Equation (4-55) to be simplified to

$$\overline{e_{nop}^2} \approx \frac{1}{C_f^2} \left(\frac{K_2}{\omega^2} + \frac{K_1}{|\omega|} + K_o \right) \frac{\text{volts}^2}{\text{rad/sec}}. \quad (4-59)$$

The shaping amplifier, with equal differentiation and integration time constants, has a Laplace transformed voltage transfer function³ of

$$T_a(s) = \frac{A_a \tau_a s}{(1 + \tau_a s)^2} \quad (4-60)$$

where A_a is a gain constant and τ_a is the shaping time constant.

The noise power transfer function of the shaping amplifier is therefore given by

$$G_a(\omega) \equiv T_a(j\omega) T_a^*(j\omega) = \frac{A_a^2 \tau_a^2 \omega^2}{(1 + \tau_a^2 \omega^2)^2}. \quad (4-61)$$

Multiplying Equation (4-59) by Equation (4-61) gives the noise power spectral density at the shaping amplifier output due to the output noise of the preamplifier.

$$\overline{e_{noa}^2} = \frac{A_a^2 \tau_a^2 (K_2 + K_1 |\omega| + K_o \omega^2)}{C_f^2 (1 + \tau_a^2 \omega^2)^2} \frac{\text{volts}^2}{\text{rad/sec}}. \quad (4-62)$$

The rms value of $\overline{e_{noa}^2}$ is

$$E_{noa} = \left[\int_0^{\infty} \frac{e_{noa}^2}{\omega} d\omega \right]^{1/2} \text{ rms volts .} \quad (4-63)$$

Evaluation of Equation (4-63)³⁵ yields

$$E_{noa} = \frac{A_a}{C_f} \left[\frac{\pi \tau_a K_2}{4} + \frac{K_1}{2} + \frac{\pi K_o}{4 \tau_a} \right]^{1/2} \text{ rms volts.} \quad (4-64)$$

The charge gain of the system is the product of the preamplifier's charge gain, given approximately in Equation (4-38), page 88, and the shaping amplifier's pulse gain, which is³ $A_a \exp(-1)$, therefore

$$A_{c(\text{system})} \approx \frac{A_a}{\epsilon C_f} . \quad (4-65)$$

Dividing Equation (4-64) by Equation (4-65) yields the equivalent noise charge.

$$ENC = \frac{E_{noa}}{A_{c(\text{system})}} \approx \epsilon \left[\frac{\pi \tau_a K_2}{4} + \frac{K_1}{2} + \frac{\pi K_o}{4 \tau_a} \right]^{1/2} \text{ rms coulombs.} \quad (4-66)$$

For the FET used in the experimental preamplifier the gate leakage current was

$$I_G \approx 100 \text{ pa.} \quad (4-67)$$

and the 1/f noise constant was

$$F \approx 1.3 \times 10^7 , \quad (4-68)$$

both determined experimentally.

The value of R_{eq} , the parallel combination of all the bias resistors connected to ground along the signal path extending from the FET drain to the emitter follower bases in Figure 4-1, page 74, is

$$R_{eq} \approx 1.47K || 1.62K || 1.62K || 12.1K \approx 500 \text{ ohms.} \quad (4-69)$$

Constants K_0 , K_1 , and K_2 , given in Equations (4-56) - (4-58), may be evaluated using the parameter values given in Equations (4-2), (4-4), (4-5), (4-31), (4-33), and (4-67) - (4-69). These constants may then be used in Equation (4-66), along with the time constant of the shaping amplifier

$$\tau_a = 1 \text{ } \mu\text{sec.} \quad (4-70)$$

to calculate the ENC.

The resulting values of ENC, for various values of detector capacitance, C_d , are shown by the curve labeled "calculated" in Figure 4-6, page 96.

The discrepancy between the calculated and experimental curves of Figure 4-6 is probably due to excess noise produced in transistors Q_1 - Q_5 and to the noise sources which were neglected in the calculations. Noise in excess of the theoretically predicted value is not an uncommon phenomenon in electronic devices. The white noise resistance of the FET, for example, was found experimentally to be almost 40 percent greater than the value predicted theoretically.

D. Sensitivity of Charge Gain to Variations in Input Capacitance for the Experimental Preamplifier With and Without Positive Feedback

The necessary conditions for reducing the sensitivity of charge gain to variations in input capacitance were discussed in Chapter 2. It was found there that the current amplifier's total effective load resistance, R_B in Figures 2-2 and 2-3, pages 12 and 14, should be made equal to the optimum value given in Equation (2-30), page 19. It was also found that, if the approximations of Equations (2-23) - (2-27), pages 18 and 19, are satisfied, the expression for $R_{B(opt)}$ could be simplified to the form shown in Equation (2-40), page 24, which is repeated here for convenience.

$$R_{B(opt)} = \frac{\frac{\tau_d}{C_N}}{2 + \ln \frac{\tau_r}{\tau_d}} \quad (4-71)$$

This form will be used since, as has been stated previously, the approximations of Equations (2-23) - (2-27) are satisfied for the experimental preamplifier.

In Equation (4-71), τ_d and τ_r are the decay time-constant and the rise time-constant, respectively, of the preamplifier output pulse and C_N is the current amplifier's total effective load capacitance as shown in Figures 2-2 and 2-3.

The measured output pulse decay-time constant, as stated in Equation (4-48), page 91, was 1 msec. This value will not be used in calculating $R_{B(opt)}$ for test purposes, however, because the output pulse decay time-constant was only

$$\tau_d \approx 250 \text{ } \mu\text{sec.} \quad (4-72)$$

in the differential summing system of Figure 4-4, page 93, which was used to measure the small variations in charge gain due to changes in input capacitance. This occurred because the mercury relay pulser which was used as a test signal source in the differential summing system had its own decay time-constant of approximately 300 $\mu\text{sec.}$, which essentially determined the preamplifier decay time-constant.

Values for τ_r and C_N are given in Equations (4-27), page 86, and (4-47), page 91. Using those values and the value of τ_d given in Equation (4-72), $R_{B(\text{opt.})}$ may be calculated from Equation (4-71) to be

$$R_{B(\text{opt.})} \approx - 6.94 \text{ Meg ohm} . \quad (4-73)$$

As previously stated, the value of R_B in the preamplifier is controlled by the positive-feedback resistor R_{PF} . The relationship between R_B and R_{PF} for the experimental preamplifier is given by Equation (4-22), page 85. The value of R_{PF} required to make R_B equal to $R_{B(\text{opt.})}$ is found from Equation (4-22) to be

$$R_{PF(\text{opt.})} \approx 105.6 \text{ K ohms} . \quad (4-74)$$

The differential summing system shown in Figure 4-4 was used to measure the percent change in charge gain resulting from changes in detector capacitance, simulated by placing lumped capacitors C_d at the preamplifier's signal input, and taking, in each case, the charge gain for $C_d = 0$ to be the reference value.

Measurements were made with positive feedback resistor R_{PF} removed from the circuit, with $R_{PF} = 105.6 \text{ K ohms} \approx R_{PF(opt)}$ as given in Equation (4-74), and with $R_{PF} = 102.6 \text{ K ohms}$. Two resistors in series were used for R_{PF} in order to minimize the effects of end-cap capacitance. The results are shown by the curves of Figure 4-7.

The curves of Figure 4-7 show that the change in charge gain is much less with positive feedback than without.

The fact that the curve for $R_{PF} \approx R_{PF(opt)}$ shows a charge gain increase for small values of C_d indicates that the calculated values of $R_{B(opt)}$ and, therefore, $R_{PF(opt)}$ were too small. This error is not surprising since, as can be seen from Equations (4-7) - (4-22), pages 82 - 85, the calculated relationship between R_B and R_{PF} involved several transistor parameters whose values were not known to extreme accuracy.

The curve for $R_{PF} \approx R_{PF(opt)}$ also shows that the change in charge gain is not zero for all values of C_d . This was to be expected since, as pointed out in Chapter 2, $R_{B(opt)}$ is a function of C_d .

The curve for $R_{PF} = 102.6 \text{ K ohms}$ indicates that the value of C_d for zero change in charge gain can be controlled by R_{PF} . Smaller values of R_{PF} produce zero change in charge gain at larger values of C_d and vice versa.

The equivalent noise charge (ENC) of the preamplifier with $R_{PF} \approx R_{PF(opt)}$ was measured using the RMS meter method discussed previously. Values of ENC obtained for various values of C_d were the same as those obtained without positive feedback. The "experimental"

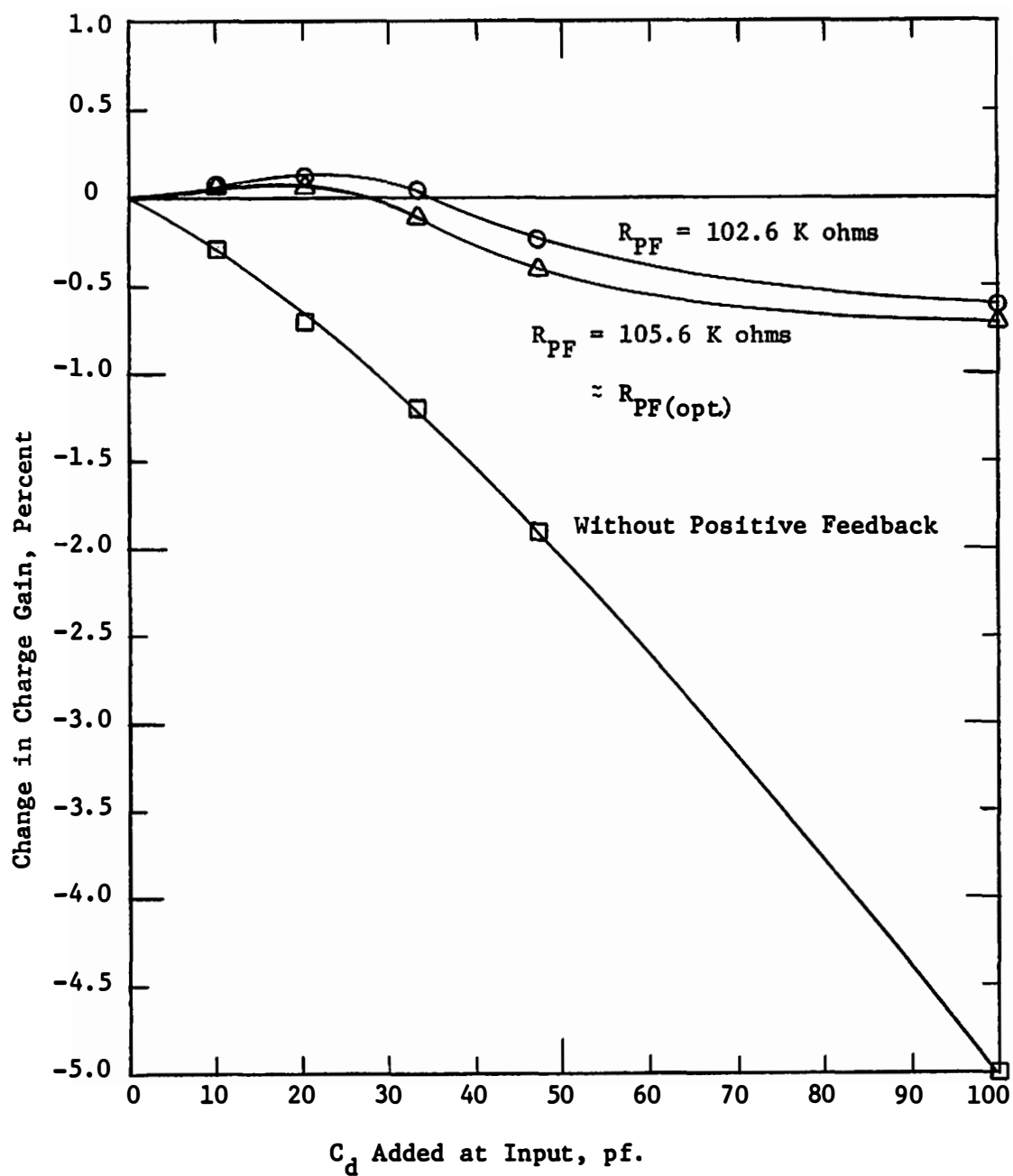


Figure 4-7. Sensitivity of charge gain to changes in detector capacitance for the experimental preamplifier.

ENC vs C_d curve in Figure 4-6, page 96, therefore represents the noise performance of the preamplifier with, as well as without, positive feedback.

The value of R_B for which oscillations should occur, $R_{B(osc)}$, is given in Equation (2-45), page 26. Using the parameter values shown in Equations (4-27), (4-47), and (4-48) to evaluate $R_{B(osc)}$ from Equation (2-45) yields

$$R_{B(osc)} \approx -1025 \text{ ohms.} \quad (4-75)$$

From Equation (4-22), the corresponding value of R_{PF} is

$$R_{PF(osc)} \approx 925 \text{ ohms.} \quad (4-76)$$

The actual experimental value of R_{PF} required for instability was 950 ohms.

E. Effects of Positive Capacitance Feedback on the Rise-Time Performance of the Experimental Preamplifier

Enhancement of preamplifier rise-time by the application of positive capacitance feedback was discussed in Chapter 2. In Chapter 2, it was found that the 10-90 percent rise-time attainable with approximately 5 percent overshoot, t_{rp} , was related to the time-constant associated with the second most dominant open loop pole, τ , by Equation (2-69), page 38, which is repeated here for convenience.

$$t_{rp} \approx 3.1 \tau \quad (4-77)$$

The second most dominant pole for the experimental preamplifier is due to the f_T 's of the output emitter-followers, Q_8 and Q_9 , which are typically 390 MHz and 260 MHz, respectively. Using 300 MHz for the second dominant pole frequency, the rise time for 5 percent overshoot calculated from Equation (4-77) is

$$t_{rp} \approx 3.1 \frac{1}{2\pi \times 300 \times 10^6} \approx 3.1 (0.53 \text{ nsec.}) \approx 1.64 \text{ nsec.} \quad (4-78)$$

The rise-time given by Equation (4-78) is not attainable for the experimental preamplifier because the additional poles due to the f_T 's of transistors Q_2 - Q_7 are, at best, only an octave or two above the second most dominant pole. Values of f_T for Q_2 - Q_7 cover the minimum-to-maximum range of 500 MHz to 1.4 GHz.

The rise-time performance of the experimental preamplifier with and without positive capacitance feedback, and for various values of simulated detector capacitance, is shown in Figure 4-8.

For the case of positive capacitance feedback, the positive feedback capacitor was a 1-8 pf. trimmer capacitor which was adjusted, for each value of C_d , to give an output pulse with 5 percent overshoot.

The curves of Figure 4-8 show that the application of positive capacitance feedback provides a dramatic improvement in preamplifier rise-time performance, especially for large values of detector capacitance.

The equivalent noise charge (ENC) of the preamplifier was measured, using the RMS meter method, for various values of C_d with, in

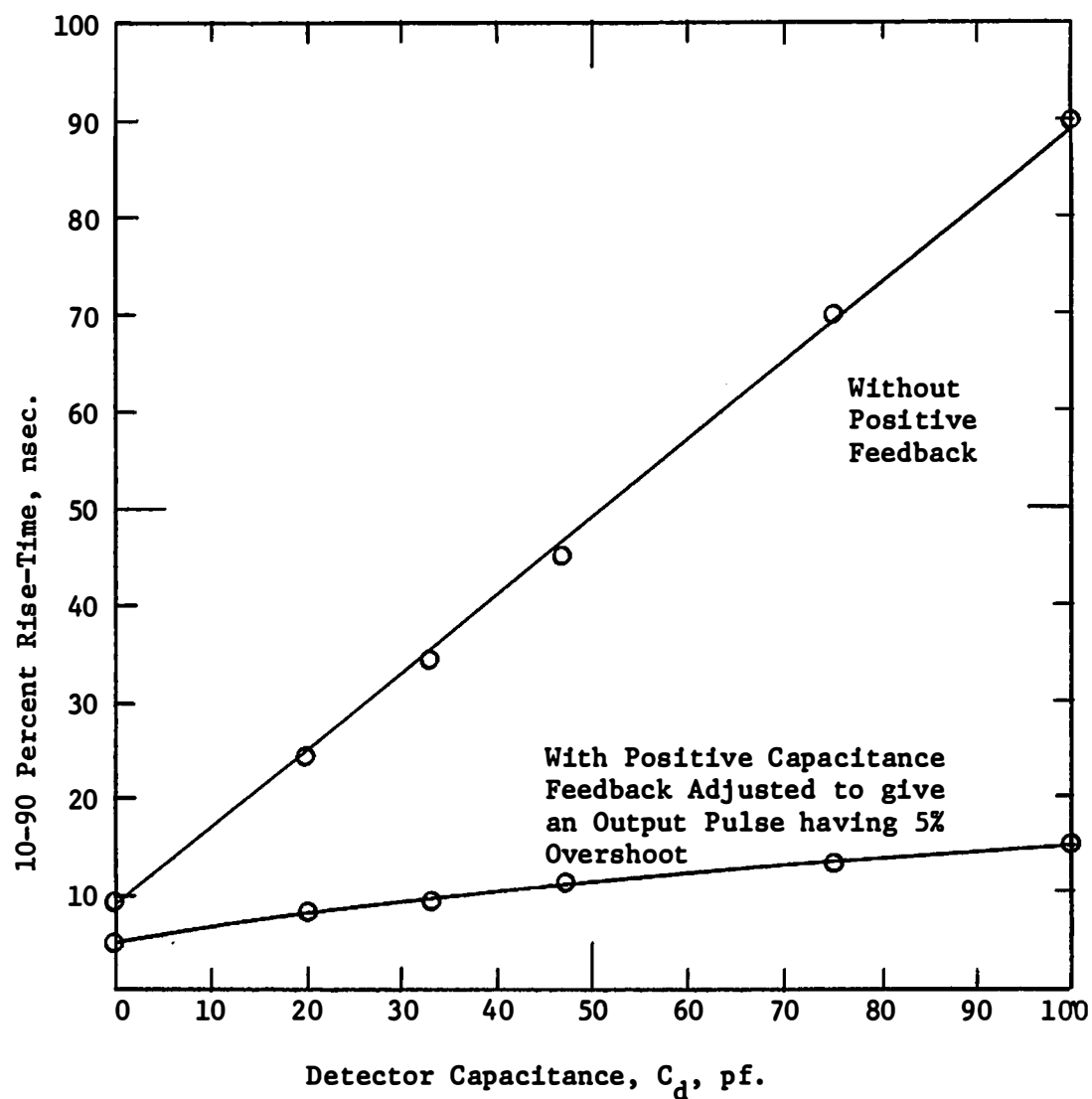


Figure 4-8. Preamplifier output pulse 10-90 percent rise-time vs detector capacitance.

each case, C_{PF} adjusted to give a preamplifier output pulse with 5 percent overshoot. Values of ENC thus obtained were the same as those obtained without positive feedback. The "experimental" curve in Figure 4-6, page 96, therefore represents the noise performance of the preamplifier with positive capacitance feedback, as well as without positive feedback.

It was stated in Chapter 2 that oscillations would occur for any negative value of C_N when R_B was positive. The value of C_{PF} required to reduce C_N to zero, and therefore cause oscillations, may be calculated from Equation (4-24), page 86, and is

$$C_{PF(osc)} \approx \frac{4 \text{ pf.}}{.91} \approx 4.4 \text{ pf.} \quad (4-79)$$

No oscillations occurred for $C_{PF} \approx 4.7 \text{ pf.}$, however oscillations did occur for $C_{PF} \approx 5.1 \text{ pf.}$ Ceramic capacitors having a 5 percent tolerance were used. The actual value of $C_{PF(osc)}$ is therefore close to the calculated value.

F. Effects of Long Input Cables on the Output Pulse Shape of the Experimental Preamplifier With and Without Positive Feedback

The necessary conditions for providing an effective termination for long cables at the input of a charge-sensitive preamplifier were discussed in Chapter 3. It was found there that effective termination can be achieved by adjusting the total equivalent resistance, R_B , and capacitance, C_N , which load the current gain section, A_1 , of the preamplifier. These parameters are shown schematically in Figures 2-2 and

2-3, pages 12 and 14. The proper values of R_B and C_N for termination, R_{BT} and C_{NT} , were given in Equations (3-6) and (3-7), page 49.

As previously stated, the values of R_B and C_N may be controlled by positive feedback. The values of R_B and C_N as functions of the positive feedback elements R_{PF} and C_{PF} are given by Equations (4-22) and (4-24), pages 85 and 86, for the experimental preamplifier.

The values of R_{BT} , C_{NT} , R_{PF} , and C_{PF} required for effective termination of cables having characteristic impedances, Z_0 , of 50, 93, 125, 185, and 950 ohms have been calculated for the experimental preamplifier and are shown in Table I. The calculations were made using Equations (3-6), (3-7), (4-2), (4-3), (4-22), and (4-24). The emitter-follower voltage gain, A_v , was taken to be one.

The test system shown in Figure 4-9 was used to observe the effects of providing input cable termination on both the preamplifier output pulse and the pulse obtained by applying simple RC-RC shaping to the preamplifier output pulse.

Photographs of the preamplifier output and the shaping amplifier outputs for various shaping time constants, and for $C_d = 0$, are shown in Figures 4-10 through 4-14. In each case the output signals obtained without positive feedback and those obtained with positive feedback set to provide input cable termination are shown.

For the positive feedback cases, fixed resistors were used to set R_{PF} at the values specified in Table I. A 1-8 pf. trimmer capacitor was used for C_{PF} and it was adjusted to give the best preamplifier output pulse shape for the 50, 93, 125, and 185 ohm cables. As noted in Table I, C_{PF} was not required for the 950 ohm cable. Instead, C_N was

TABLE I
CIRCUIT PARAMETERS REQUIRED IN THE EXPERIMENTAL PREAMPLIFIER
TO PROVIDE TERMINATION FOR INPUT CABLES

	Cable Characteristic Impedance, Z_o , Ω				
	50	93	125	185	950
R_{BT} , M Ω	1403	755	562	380	74
C_{NT} , pf.	0.7125	1.325	1.781	2.636	13.53
R_{PF} , K Ω	107.4	107.4	107.4	107.4	107.5
C_{PF} , pf.	3.615	2.94	2.439	1.5	*

* For this case C_{PF} is not used. Instead, C_N without positive capacitance feedback is padded by ≈ 9.5 pf. This procedure is necessary whenever $C_{NT} > 4$ pf.

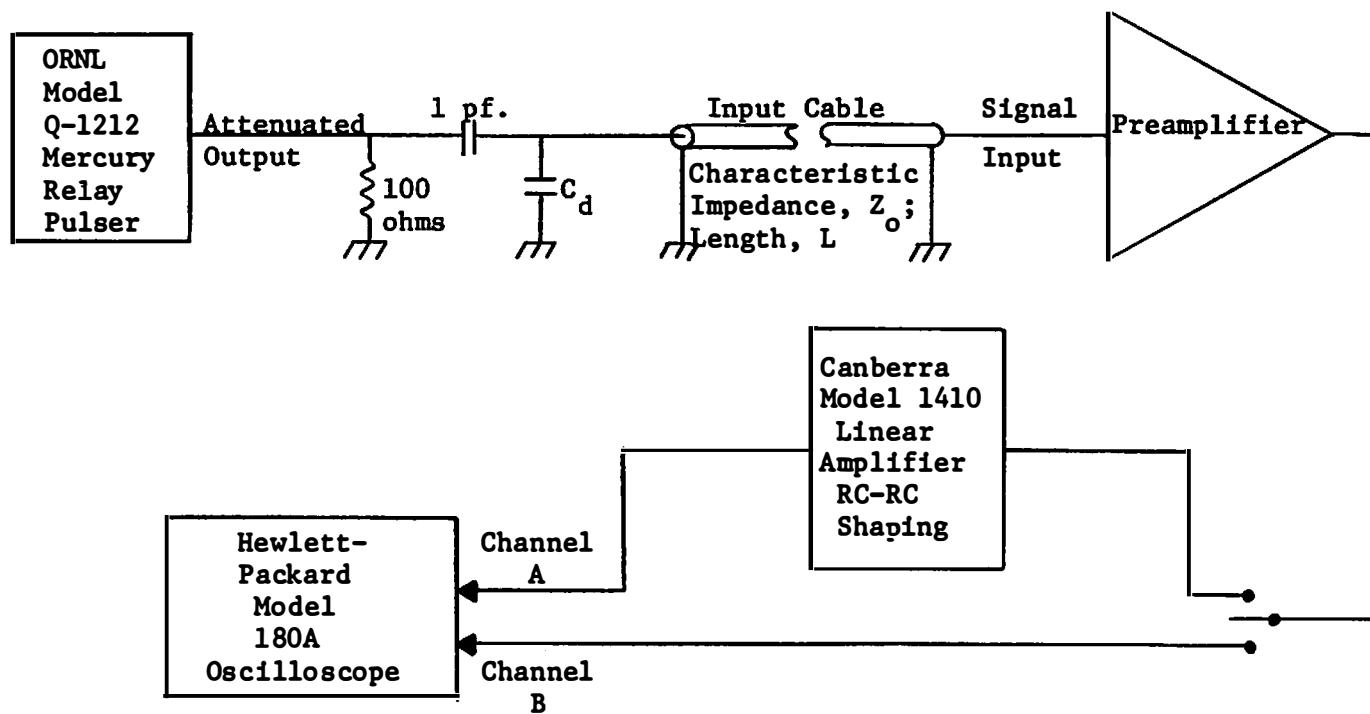
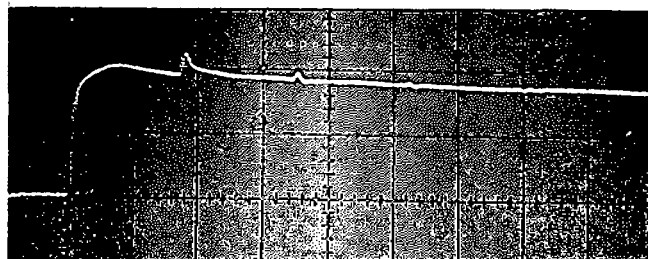


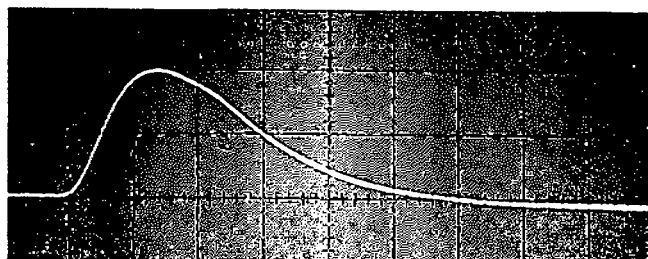
Figure 4-9. Test system for examining the effects of preamplifier input cable termination.



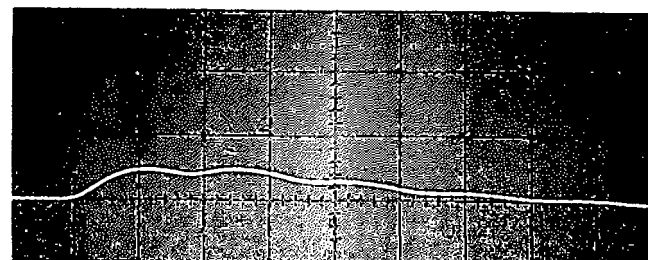
(a) Preamplifier output pulse with positive feedback. (0.5 V/Div., 0.2 μ sec./Div.)



(b) Preamplifier output pulse without positive feedback. (0.5 V/Div., 0.2 μ sec./Div.)

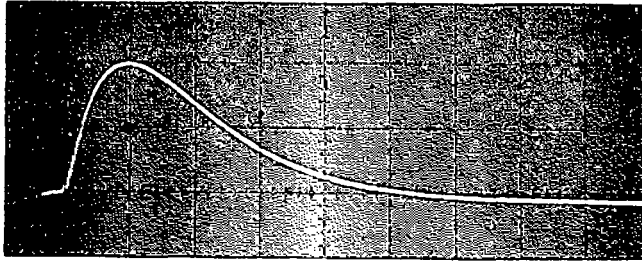


(c) Shaping amplifier output pulse with positive feedback and with shaping time constants of 0.2 μ sec. (5 V/Div., 0.2 μ sec./Div.)

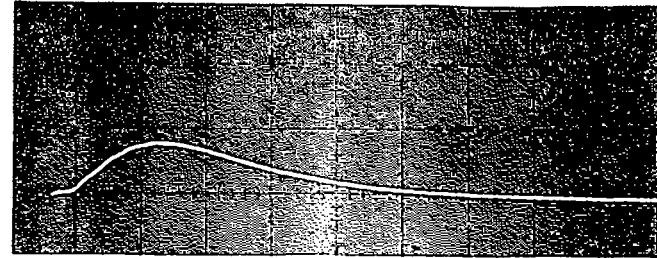


(d) Shaping amplifier output pulse without positive feedback and with shaping time constants of 0.2 μ sec. (5 V/Div., 0.2 μ sec./Div.)

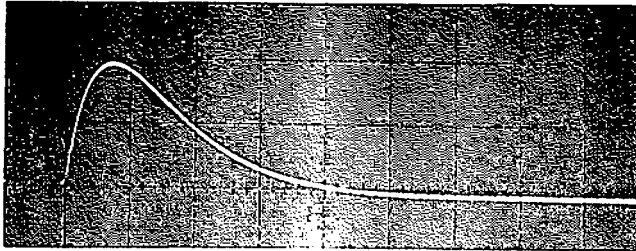
Figure 4-10. Preamplifier and shaping amplifier output pulses with and without positive feedback for a 110 ft. RG58C/U (50 ohm) input cable and $C_d = 0$.



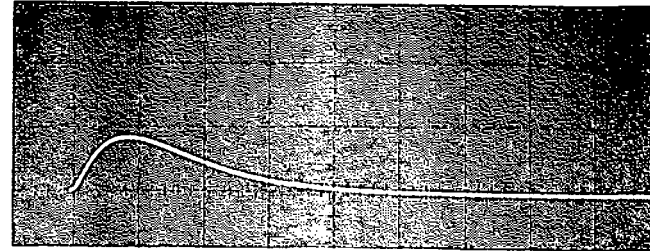
(e) Shaping amplifier output pulse with positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)



(f) Shaping amplifier output pulse without positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)

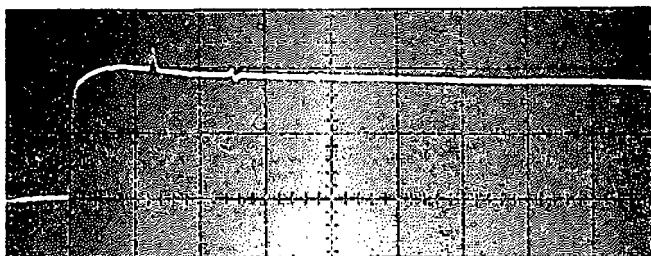


(g) Shaping amplifier output pulse with positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)

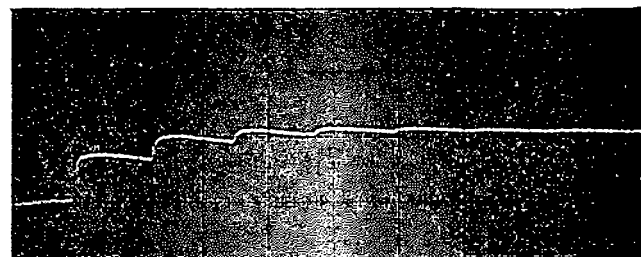


(h) Shaping amplifier output pulse without positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)

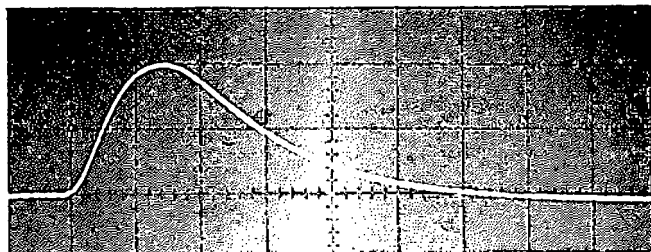
Figure 4-10 (continued)



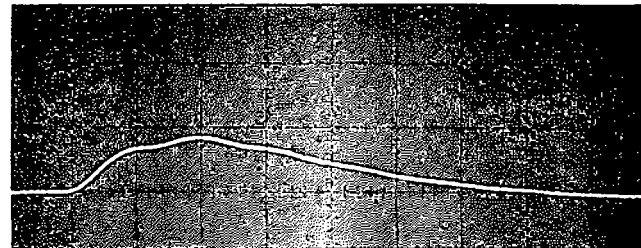
(a) Preamplifier output pulse with positive feedback. (0.5 V/Div., 0.2 μ sec./Div.)



(b) Preamplifier output pulse without positive feedback. (0.5 V/Div., 0.2 μ sec./Div.)

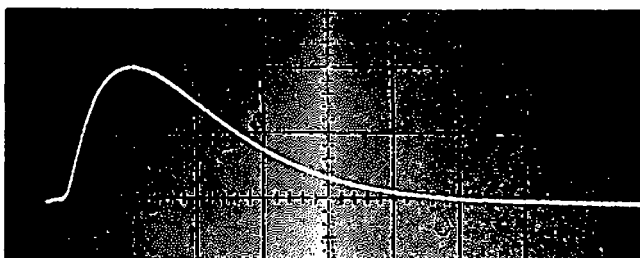


(c) Shaping amplifier output pulse with positive feedback and with shaping time constants of 0.2 μ sec. (5 V/Div., 0.2 μ sec./Div.)



(d) Shaping amplifier output pulse without positive feedback and with shaping time constants of 0.2 μ sec. (5 V/Div., 0.2 μ sec./Div.)

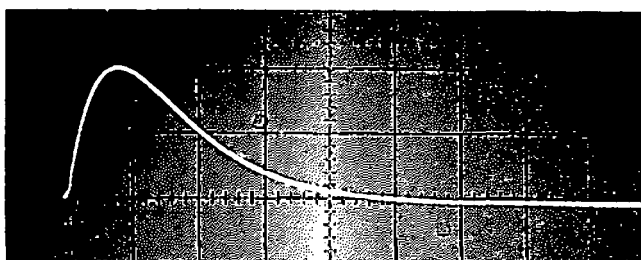
Figure 4-11. Preamplifier and shaping amplifier output pulses with and without positive feedback for a 103 ft. RG62A/U (93 ohm) input cable and $C_d = 0$.



(e) Shaping amplifier output pulse with positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)



(f) Shaping amplifier output pulse without positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)

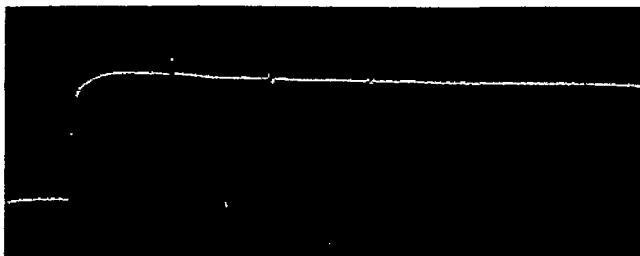


(g) Shaping amplifier output pulse with positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)



(h) Shaping amplifier output pulse without positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)

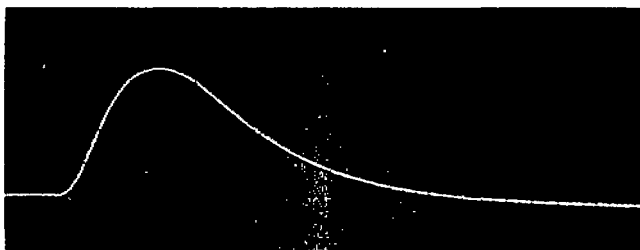
Figure 4-11 (continued)



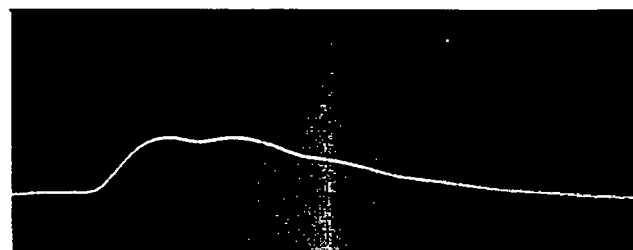
(a) Preamplifier output pulse with positive feedback. (0.5 V/Div., 0.2 μ sec./Div.)



(b) Preamplifier output pulse without positive feedback. (0.5 V/Div., 0.2 μ sec./Div.)

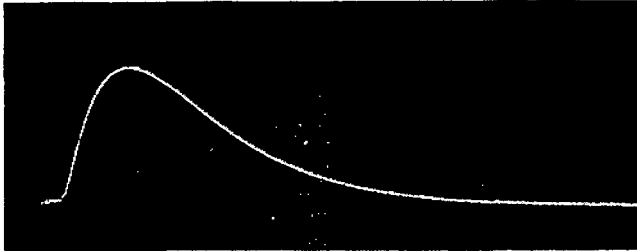


(c) Shaping amplifier output pulse with positive feedback and with shaping time constants of 0.2 μ sec. (5 V/Div., 0.2 μ sec./Div.)

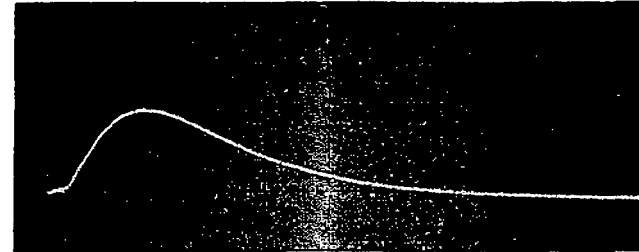


(d) Shaping amplifier output pulse without positive feedback and with shaping time constants of 0.2 μ sec. (5 V/Div., 0.2 μ sec./Div.)

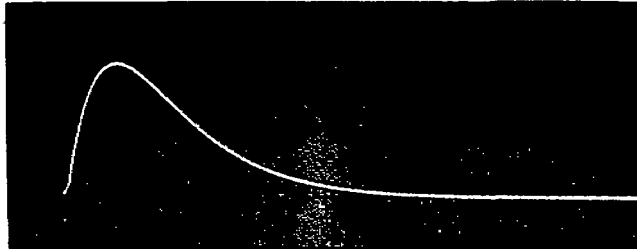
Figure 4-12. Preamplifier and shaping amplifier output pulses with and without positive feedback for a 124 ft. RG63B/U (125 ohm) input cable and $C_d = 0$.



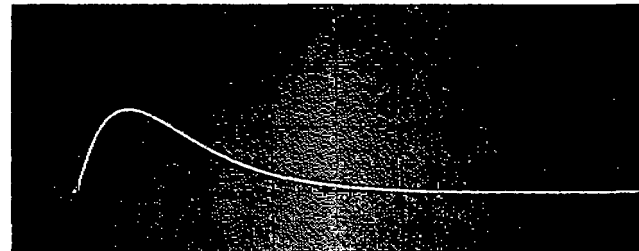
(e) Shaping amplifier output pulse with positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)



(f) Shaping amplifier output pulse without positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)

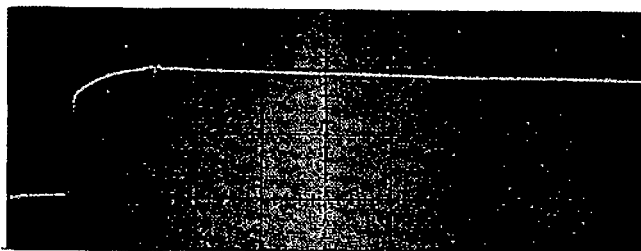


(g) Shaping amplifier output pulse with positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)

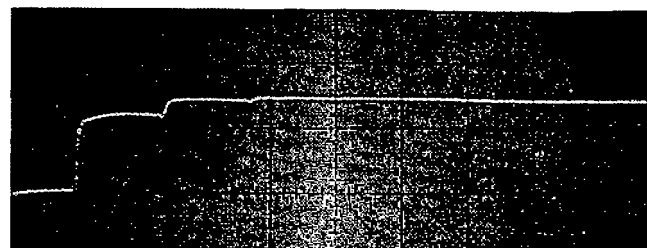


(h) Shaping amplifier output pulse without positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)

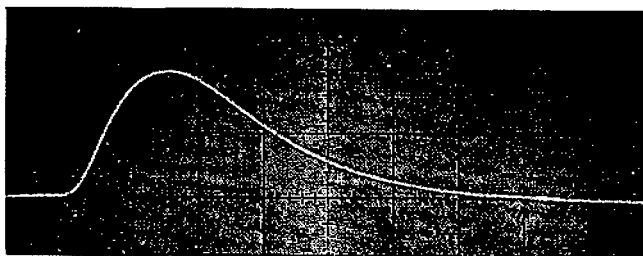
Figure 4-12 (continued)



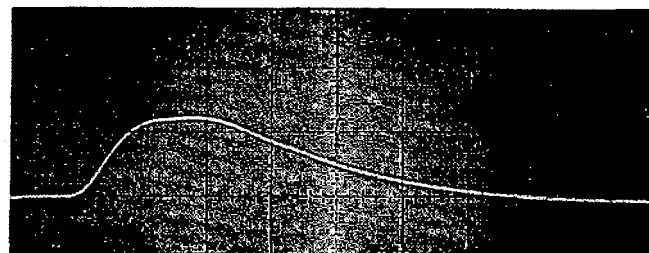
(a) Preamplifier output pulse with positive feedback. (0.5 V/Div., 0.2 μ sec./Div.)



(b) Preamplifier output pulse without positive feedback. (0.5 V/Div., 0.2 μ sec./Div.)

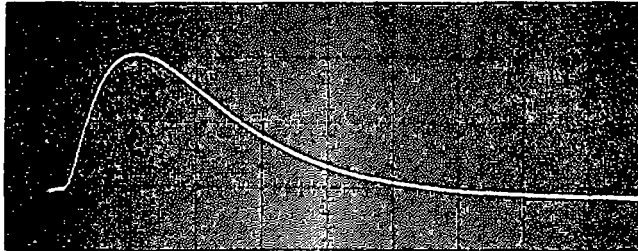


(c) Shaping amplifier output pulse with positive feedback and with shaping time constants of 0.2 μ sec. (5 V/Div., 0.2 μ sec./Div.)

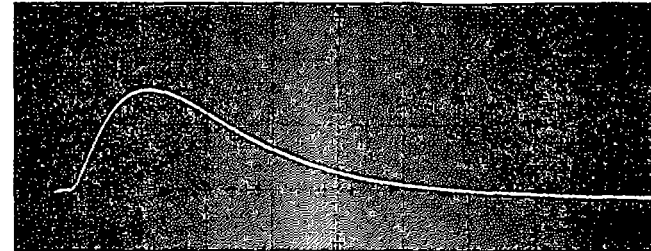


(d) Shaping amplifier output pulse without positive feedback and with shaping time constants of 0.2 μ sec. (5 V/Div., 0.2 μ sec./Div.)

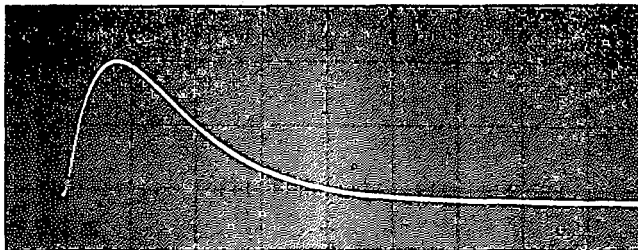
Figure 4-13. Preamplifier and shaping amplifier output pulses with and without positive feedback for a 110-ft. RG114/U (185 ohm) input cable and $C_d = 0$.



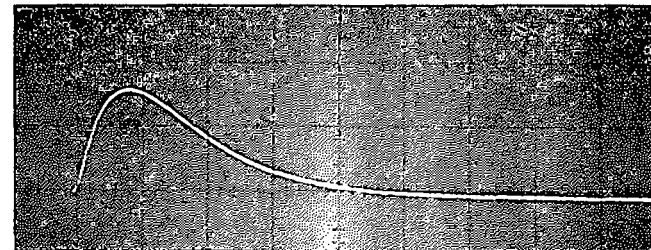
(e) Shaping amplifier output pulse with positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)



(f) Shaping amplifier output pulse without positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)

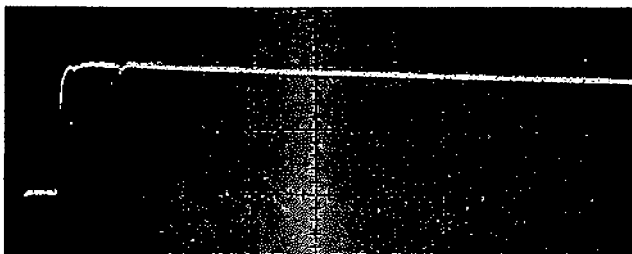


(g) Shaping amplifier output pulse with positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)

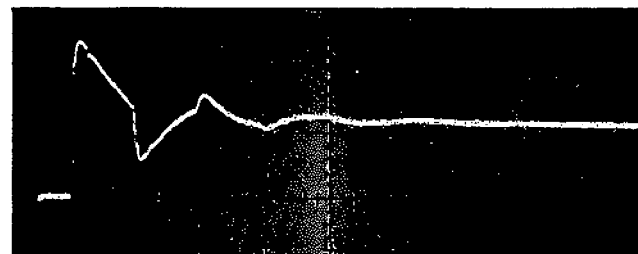


(h) Shaping amplifier output pulse without positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)

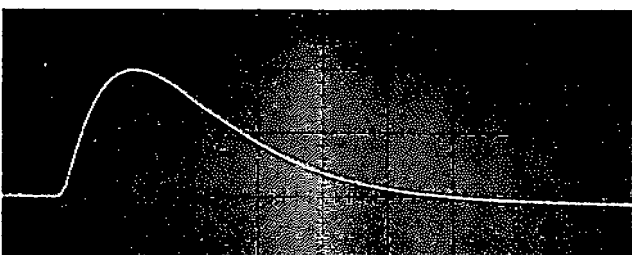
Figure 4-13 (continued)



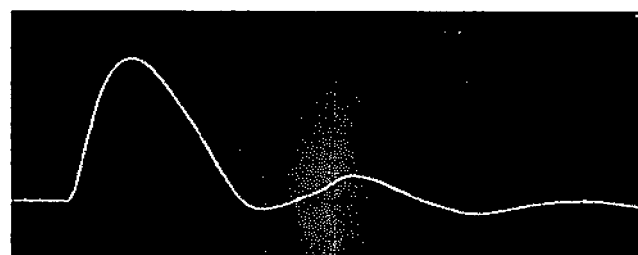
(a) Preamplifier output pulse with positive feedback. (0.5 V/Div., 2 μ sec./Div.)



(b) Preamplifier output pulse without positive feedback. (0.5 V/Div., 2 μ sec./Div.)

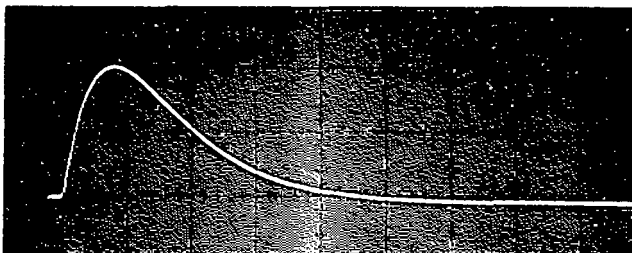


(c) Shaping amplifier output pulse with positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)

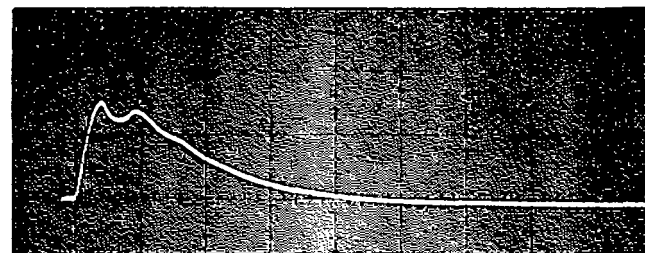


(d) Shaping amplifier output pulse without positive feedback and with shaping time constants of 1 μ sec. (5 V/Div., 1 μ sec./Div.)

Figure 4-14. Preamplifier and shaping amplifier output pulses with and without positive feedback for a 23 ft. RG65A/U (950 ohm) input cable and $C_d = 0$.



(e) Shaping amplifier output pulse with positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)



(f) Shaping amplifier output pulse without positive feedback and with shaping time constants of 4 μ sec. (5 V/Div., 5 μ sec./Div.)

Figure 4-14 (continued)

padding with a 5-25 pf. trimmer capacitor which was adjusted to give the best preamplifier output pulse shape.

The pulser output level and shaping amplifier gain settings were the same for all of the cable impedances except 50 ohms. For the 50 ohm cable, the pulser output level was increased approximately 10 percent. This was necessary because the attenuation of the 50 ohm cable used, RG58C/U, was somewhat higher than that of the other cables.

Comparison of the preamplifier output pulses with and without positive feedback, shown in Figures 4-10 through 4-14, shows the tremendous improvement in cable termination resulting from the use of positive feedback.

It was stated in Chapter 3 that the input cable termination would not be perfect, even with positive feedback, because of the capacitance C_I , at the preamplifier input. It was further stated that the effect of this slight mistermination would be to add to the desired preamplifier output pulse a sequence of short perturbations which occur at time intervals of $2T$, where T is the time delay of the input cable. These short perturbations are evident in the photographs of the preamplifier output pulses with positive feedback shown in Figures 4-10 through 4-14.

The short perturbations superposed on the preamplifier output pulse will not, as stated in Chapter 3, affect the shaping amplifier output if the shaping time constants are long compared to the time constants controlling the perturbations. The shaping amplifier output pulses with positive feedback, shown in Figures 4-10 through 4-14, show this to be true for all of the cable impedances tested except 50 ohms.

For the 50 ohm cable, the shaping amplifier output pulse with positive feedback and with shaping time constants of 0.2 μ sec., Figure 4-10(c), page 113, has a distorted shape.

This problem arises because the value of C_N required for effective termination of the 50 cable is very small, as shown in Table I, page 111. That is, a large amount of positive capacitance feedback is required. When large amounts of positive capacitance feedback are employed, the higher order open-loop preamplifier poles begin to play a larger role in determining the preamplifier response. This is the same problem which was discussed in connection with the application of positive capacitance feedback for the purpose of improving the preamplifier output pulse rise-time. The experimental preamplifier is simply not fast enough to provide the best termination for a 50 ohm cable.

With regard to the problem discussed above, the following general statement may be made. Provision of effective termination for long input cables having progressively lower and lower characteristic impedances by employing positive feedback requires progressively faster and faster open-loop preamplifiers.

Referring again to Figures 4-10 through 4-14, pages 113 through 122, the improvement in shaping amplifier output pulse waveforms due to application of positive feedback is most evident for the 0.2 μ sec. shaping time constants. The differences between the shaping amplifier output pulse waveforms with and without positive feedback are not so dramatic for the longer shaping time constants. In fact, if the shaping time constants are much longer than the time-delay of the input

cable, the shaping amplifier will not "see" the reflection produced steps on the front edge of the output pulse of the preamplifier without positive feedback. This was true of the 4 μ sec. shaping time constants for all of the cable impedances tested except 950 ohms. The time delay of the 950 ohm cable was so long that the shaping amplifier output pulse was distorted even for the 4 μ sec. shaping time constants.

Thus, the following general statement may be made regarding the above discussion. The effects of input cable mistermination on the shaping amplifier output pulse wave-shape may be eliminated without the use of positive feedback by making the shaping time constants much longer than the time-delay of the input cable, if the reduced count rate capability resulting from the longer shaping time constants can be tolerated.

An important side effect of applying positive feedback to provide input cable termination is that the positive resistance feedback improves the stability of charge gain against variations in input capacitance. This is an important effect because, at steady-state, the total cable capacitance, capacitance-per-foot times cable length, appears to be a lumped capacitance at the preamplifier input.

This lumped capacitance can be quite large for long input cables. For the 103 foot length of 93 ohm RG62A/U used in one of the tests, for example, the total capacitance is approximately 1400 pf.

For the cables used in the tests, the signal amplitudes obtained without positive feedback were from approximately 30 percent to approximately 60 percent below the amplitudes obtained with positive feedback,

depending on the particular cable in use. This can be seen by comparing the final amplitudes of the preamplifier output pulses with and without positive feedback shown in Figures 4-10 through 4-14, pages 113 through 122.

As previously stated, all of the waveforms shown in Figures 4-10 through 4-14 were recorded with C_d , in Figure 4-9, page 112, equal to zero. That is, the simulated detector capacitance was zero.

The effects of a nonzero C_d on the preamplifier output pulse waveshape with positive feedback would be to increase the rise-time and to change the shape of the perturbations caused by reflections on the input cable.

The rise-time would increase because of the $C_d Z_o$ time constant which would be present at the sending end of the cable. This time constant, defined to be τ_g in Equation (3-32), page 57, was included in the derivation of the preamplifier output pulse given in Chapter 3. The shape of the reflection caused perturbations would be altered because C_d affects the sending and reflection coefficient as shown in Equation (3-31), page 57.

The preamplifier output pulse with positive feedback and with the same 93 ohm RG62A/U cable used in the previous test but with $C_d = 100$ pf. is shown in Figure 4-15.

The preamplifier output pulse waveshape of Figure 4-15, for $C_d = 100$ pf., should be compared to that of Figure 4-11(a), page 115, for $C_d = 0$.

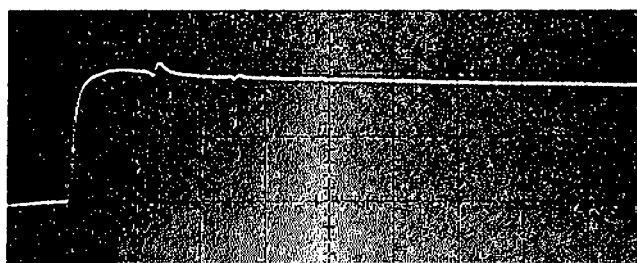


Figure 4-15. Preamplifier output pulse under the same conditions as for Figure 4-11(a), page 115, except $C_d = 100$ pf. (0.5 V/Div., 0.2 μ sec./Div.)

In attempting to provide acceptable input cable termination with a charge-sensitive preamplifier, the size of the input coupling capacitor (C_c in Figure 4-1, page 74) is important. In order to provide correct termination by using the values of R_B and C_N given in Equations (3-6) and (3-7), page 49, it is necessary that the input coupling capacitor be made large compared to the lumped capacitance of the cable. This was accomplished in the experimental preamplifier by using a 0.01 μf . input coupling capacitor.

If the input coupling capacitance is too small, Equations (3-6) and (3-7) will not give the correct values of R_B and C_N for good termination. Good termination may still be attained, however, by using more positive resistance feedback.

The problem may be eliminated altogether by simply returning the overall feedback capacitor, C_f , back to the opposite side of the input coupling capacitor, C_c . That is, connect the feedback capacitor directly to the preamplifier's signal input connector rather than to the gate terminal of the input FET.

It was stated in Chapter 3 that the presence of a long cable at the input to a charge-sensitive preamplifier would result in an increase in output noise from the amplifier. It was further stated that this would occur because the cable would not exhibit transmission line properties in the frequency range covered by the shaping filter for the commonly used shaping time constants. The long input cable, in that frequency range, could be represented simply by its lumped capacitance to ground (capacitance-per-foot times cable length in feet).

Using the measuring system described previously (pp. 92-95), the equivalent noise charge (ENC) of the experimental preamplifier with a 103 foot RG62A/U (93 ohms) input cable was measured. The measured ENC was found to be

$$\text{ENC} \approx 197 \times 10^{-17} \text{ rms coulombs.} \quad (4-80)$$

For this measurement the preamplifier's internal positive feedback was set to provide cable termination.

The lumped capacitance of this cable was

$$C_{CL} \approx (13.5 \text{ pf./ft.})(103 \text{ ft.}) \approx 1390 \text{ pf.} \quad (4-81)$$

With the input cable removed and a capacitance of 1400 pf. placed at the preamplifier input, the ENC was measured again. For this condition the ENC was the same as the value found with the cable in place as given in Equation (4-80).

These measurements show that, within the bandwidth of the filter, the long input cable can be represented simply by its lumped capacitance for purposes of determining preamplifier output noise.

The ENC given in Equation (4-80) is also within 10 percent of the value found by the extrapolating the experimental ENC vs detector capacitance curve shown in Figure 4-6, page 96.

CHAPTER 5

CONCLUSIONS

A. Summary

The objective of this thesis was to examine ways of improving the performance characteristics and versatility of charge-sensitive preamplifiers by the application of positive feedback techniques. Improvements in stability of charge gain against variations in input capacitance, output pulse rise-time, and ability to terminate long input cables were considered.

Chapter 1 presented background material concerning the basic function required of the charge-sensitive preamplifier, i.e., extraction of information from the output signal of a nuclear particle detector. The basic principle of operation of charge-sensitive preamplifiers, as opposed to that of current-sensitive preamplifiers and voltage-sensitive preamplifiers, was briefly discussed.

In Chapter 2, the basic charge-sensitive preamplifier circuit was presented. Equations were derived for the output voltage pulse, charge gain, and rate of change of charge gain with respect to input capacitance of the basic circuit. It was shown that the performance of the basic circuit could be improved by controlling the effective resistance, R_B , and capacitance, C_N , at the point in the preamplifier where the dominant open-loop pole was determined.

Improved stability of charge gain against input capacitance variations was shown to result from controlling R_B , and enhanced output pulse rise-time was shown to result from controlling C_N . Approximate equations for the optimum values of R_B , Equation (2-40), page 24, and C_N , Equation (2-67), page 37, were derived.

The optimum value of R_B was found to be negative and the optimum value of C_N was found, for most practical cases, to be smaller than the value which would be present in the basic preamplifier circuit. This implies that positive feedback must be used in order to realize the optimum values.

The effects of controlling R_B and C_N on the preamplifier's oscillation stability and mid-band output noise were examined. Arguments were presented which indicated that adjustment of R_B and C_N to their optimum values should produce neither oscillation instability nor significant increase in output noise.

Improvement in ability of the charge-sensitive preamplifier to terminate long input cables was considered in Chapter 3.

An equation for the input admittance of the basic charge-sensitive preamplifier circuit, Equation (3-5), page 47, was derived. Modification of the input impedance was shown to be possible by controlling the effective resistance, R_B , and capacitance, C_N , at the point in the basic preamplifier circuit where the dominant pole is determined. Equations were developed for determining the proper values of R_B and C_N , Equations (3-6) and (3-7), page 49, for providing the best impedance match between the preamplifier input and a long input cable.

A general expression, Equation (3-27), page 56, was derived for the output pulse from a preamplifier having a long input cable. The general output pulse expression was then evaluated for the special case where R_B and C_N were equal to the values required for best impedance match at the input. The resulting equation is Equation (3-48), page 62. For this case, the output was found to consist of a sequence of fast perturbations superposed on the desired output. Arguments were then presented to indicate that, for most practical cases, the extraneous fast perturbations would not be passed by the pulse shaping filter which would follow the preamplifier.

A simple case of input mismatch was then considered and the effect of the mismatch on the preamplifier output pulse were discussed.

The effects of using long input cables on the preamplifier's output noise were considered. It was shown that the presence of long input cables would drastically increase the output noise.

Experimental results were presented in Chapter 4. The design of an experimental preamplifier containing an internal positive feedback loop was discussed. It was shown that the internal positive feedback loop could be used to control the effective resistance, R_B , and capacitance, C_N , at the point in the experimental preamplifier where the dominant open-loop pole was determined.

General performance characteristics of the experimental preamplifier were measured with and without internal positive feedback. The effects of positive feedback were evaluated by comparing the performance characteristics found with and without positive feedback.

Positive feedback was found to have no effect on preamplifier noise performance. In a system employing 1 μ sec. RC-RC shaping time constants, the equivalent noise charge of the experimental preamplifier with 0 pf. detector capacitance was approximately 4×10^{-17} rms coulombs. Equivalent noise charge as a function of detector capacitance is shown in Figure 4-6, page 96.

The noise performance of the experimental preamplifier was not particularly outstanding. It was pointed out in Chapter 4, however, that the experimental preamplifier was not designed with low noise as a prime requisite. Some facets of the design were, in fact, incorporated for the sole purpose of making the evaluation of positive feedback effects as simple as possible with full realization that noise performance would suffer.

The percent change in charge gain as a function of input capacitance change is shown in Figure 4-7, page 105. With positive feedback applied to set R_B at its optimum value the percent change in charge gain for an input capacitance change of 100 pf. was almost an order of magnitude smaller than that obtained without positive feedback.

The output pulse rise-time of the experimental preamplifier as a function of detector capacitance is shown in Figure 4-8, page 108. Using positive feedback to control C_N the output pulse rise-time with 100 pf. detector capacitance was approximately 15 nsec. as opposed to almost 90 nsec. without positive feedback.

The equations developed in Chapter 3 were used to determine what positive feedback adjustments were necessary in the experimental pre-amplifier to provide acceptable termination of long input cables.

Using long input cables of several different characteristic impedances the preamplifier output pulse shapes, with and without positive feedback, were recorded and are shown in Figures 4-10 through 4-14, pages 113 - 122. Comparison of the output pulse shapes with positive feedback to those without positive feedback indicate the tremendous improvement afforded by the use of positive feedback. The output of a pulse shaping filter, which followed the charge-sensitive preamplifier, was also recorded in each case.

Noise measurements were made for the experimental preamplifier with a long input cable. The effect of the long input cable was to greatly increase the noise, as had been predicted in Chapter 3.

In general, very good correlation was found between the experimental findings of Chapter 4 and the theoretical relationships of Chapter 2 and 3.

B. Suggestions for Further Study

Upon consideration of the material presented in this thesis, several areas where additional work would be desirable become evident.

Problems of temperature stability were not considered in this thesis. Previous work¹⁴ has indicated that temperature stability will not be a problem when using positive feedback to improve the stability of charge gain against variations in input capacitance. The theoretical

treatment, however, of temperature stability with this use of positive feedback is still lacking.

For the rise-time enhancement and long input cable matching applications of positive feedback, neither theoretical treatment nor practical evaluation of temperature stability problems has been attempted.

The successful use of positive feedback to improve the output pulse rise-time of a charge-sensitive preamplifier might lead one to ask the following questions. Could positive feedback techniques be employed to improve slewing rates of amplifiers? If so, how much improvement could be realized?

Possible applications would be high level cable drivers, fast operational amplifiers, integrated circuit operational amplifiers, etc. In fact, any system which required high slewing rate and, simultaneously, small bias currents would benefit from such a technique.

The poor noise performance of the charge-sensitive preamplifier with a long input cable provides another opportunity for additional study. Noise performance is poor because, usually, the input cable will not be long enough to exhibit transmission line properties at frequencies near the center frequency of the shaping filter. The effect, therefore, is that the lumped capacitance of the cable appears at the preamplifier input thereby causing the preamplifier's output noise to increase.

If the input cable could be made to exhibit transmission line properties at the center frequency of the shaping filter, the impedance

shunting the preamplifier input due to the cable could be controlled by adjusting the cable length. This is the same technique which has been used by DeLorenzo³⁶ to improve the noise performance of voltage-sensitive preamplifiers driven from long cables.

A problem with this technique would be that, for normal shaping time constants, the required cable lengths would be very long. In that case, the loss resistance of the cable itself might be a considerable noise source.

If the shaping time constants are made short in order to keep the required cable length reasonable, the noise bandwidth of the filter becomes large.

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APPENDIXES

APPENDIX A

COMPARISON OF TWO METHODS OF APPLYING POSITIVE FEEDBACK FOR CHARGE GAIN STABILIZATION

In the interest of clarity, the comparison between the two positive feedback techniques will be carried out on the most simple basis possible.

Consider the charge-sensitive preamplifier to be represented by the circuit of Figure A-1. The circuit of Figure A-1 is a mid-band equivalent circuit and is the same as the equivalent circuit shown in Figures 2-2 and 2-3, pages 12 and 14, except that resistors R_f and R_i , which determine the low frequency feedback factor, and capacitor C_N , which determines the dominant open-loop pole, have been eliminated and noise current sources $\overline{i_{gf}^2}$ and $\overline{i_{df}^2}$ have been added. The circuit parameters in Figure A-1 are defined in Figures 2-2 and 2-3.

Application of standard circuit analysis techniques to the circuit of Figure A-1 yields the charge gain,

$$A_c \equiv \frac{V_o}{Q} = - \frac{1}{s C_f \left[\frac{C_d + C_i + C_f + C_m}{g_m A_i R_B A_v C_f} + 1 \right]}, \quad (A-1)$$

the noise power transfer function for the $\overline{i_{gf}^2}$ noise source,

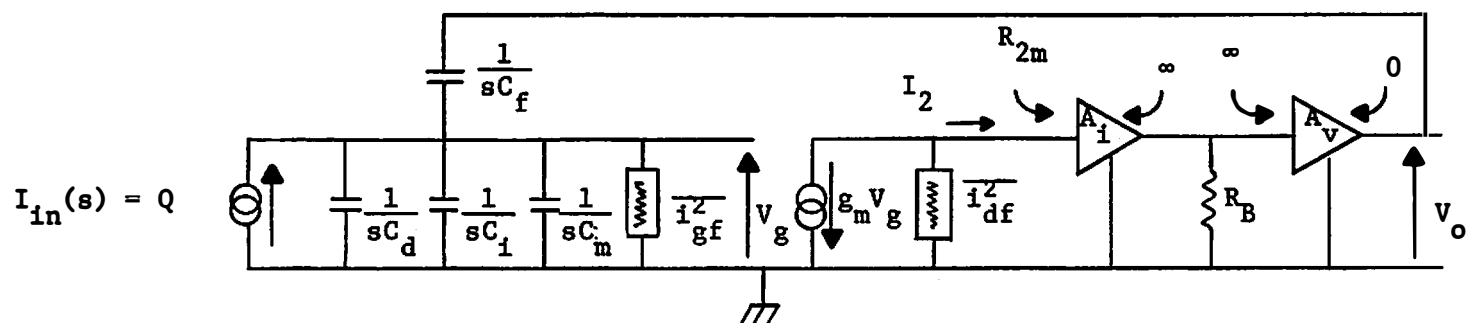


Figure A-1. Mid-band equivalent circuit model for the basic charge-sensitive preamplifier.

$$G_{gf} \equiv \frac{\overline{e_{no}^2}}{i_{gf}^2} = \frac{1}{\left[\frac{C_d + C_i + C_f + C_m}{g_m A_i R_B A_v C_f} + 1 \right]^2 \omega^2 C_f^2}, \quad (A-2)$$

and the noise power transfer function for the $\overline{i_{df}}$ noise source,

$$G_{df} \equiv \frac{\overline{e_{no}^2}}{i_{df}^2} = \frac{\left[\frac{C_d + C_i + C_f + C_m}{g_m C_f} \right]^2}{\left[\frac{C_d + C_i + C_f + C_m}{g_m A_i R_B A_v C_f} + 1 \right]^2}. \quad (A-3)$$

It will be helpful to define the second stage current-to-output voltage gain,

$$A_{2m} \equiv \frac{V_o}{I_2} = A_i R_B A_v, \quad (A-4)$$

as was done in Chapter 2.

The Miller capacitance, C_m , is the gate-to-drain capacitance of the input FET times one minus the voltage gain of the FET as defined in Figure 2-3, page 14.

$$C_m = C_{gd} (1 + g_m R_{2m}). \quad (A-5)$$

Using Equations (A-1) and (A-5) in Equations (A-1) - (A-3), the charge gain may be written as

$$A_c = - \frac{1}{s C_f \left[\frac{C_d + C_i + C_f + C_{gd} (1 + g_m R_{2m})}{g_m A_{2m} C_f} + 1 \right]} \quad (A-6)$$

and the mean squared output noise voltage as

$$\begin{aligned} \overline{e_{on}^2} &= \overline{i_{gf}^2} G_{gf} + \overline{i_{df}^2} G_{df} \\ &= \frac{\overline{i_{gf}^2} \left[\frac{1}{\omega C_f} \right]^2 + \overline{i_{df}^2} \left[\frac{C_d + C_i + C_f + C_{gd} (1 + g_m R_{2m})}{g_m C_f} \right]^2}{\left[\frac{C_d + C_i + C_f + C_{gd} (1 + g_m A_{2m})}{g_m A_{2m} C_f} + 1 \right]^2} . \end{aligned} \quad (A-7)$$

The dependence of charge gain on detector capacitance, C_d , is shown in Equation (A-6).

Both of the positive feedback techniques to be considered affect the second stage current-to-output voltage gain, A_{2m} . The second and succeeding stages of the circuit of Figure A-1, page 143, are shown, with positive feedback added, in Figure A-2. The effect of this type of positive feedback is to control the resistance which loads the current amplifier A_1 . Let this type of positive feedback be Type A.

The second stage current-to-output voltage gain for Type A positive feedback is found to be

$$A_{2mA} \equiv \frac{V_o}{I_2} = \frac{A_1 R_{AV}}{1 - \frac{A_{1PF} R_{AV}}{R_{PF}}} . \quad (A-8)$$

Application of Type A positive feedback does not change the Miller capacitance since it does not alter the second stage input resistance.

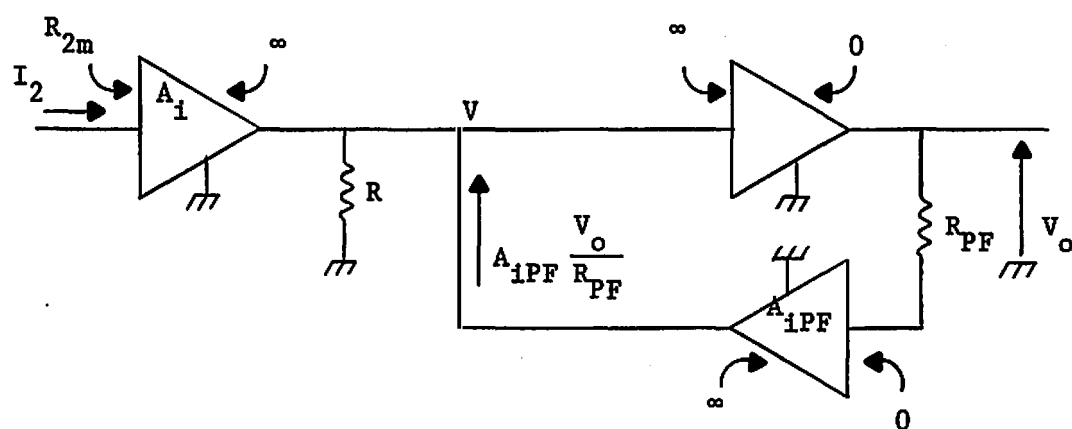


Figure A-2. Type A positive feedback.

The charge gain and mean squared output noise voltage for Type A positive feedback are obtained by replacing A_{2m} in Equations (A-6) and (A-7) by A_{2mA} .

$$A_{cA} = - \frac{1}{s C_f \left[\frac{C_d + C_i + C_{gd} (1 + g_m R_{2m})}{A_1 R_A v} + 1 \right] \left(\frac{A_{1PF} R_A v}{1 - \frac{A_{1PF} R_A v}{R_{PF}}} \right) C_f g_m} \quad (A-9)$$

$$\overline{e_{onA}^2} = \frac{\overline{i_{gf}^2} \left[\frac{1}{\omega C_f} \right]^2 + \overline{i_{df}^2} \left[\frac{C_d + C_i + C_f + C_{gd} (1 + g_m R_{2m})}{g_m C_f} \right]^2}{\left[\frac{C_d + C_i + C_f + C_{gd} (1 + g_m R_{2m})}{A_1 R_A v} + 1 \right]^2 \left(\frac{A_{1PF} R_A v}{1 - \frac{A_{1PF} R_A v}{R_{PF}}} \right) C_f g_m} \quad (A-10)$$

If the positive feedback resistor, R_{PF} , is adjusted so that

$$R_{PF} = A_{1PF} R_A v, \quad (A-11)$$

the charge gain becomes

$$A_{cA} = - \frac{1}{s C_f}, \quad (A-12)$$

independent of C_d , and the mean squared output noise voltage becomes

$$\overline{e_{onA}^2} = \overline{i_{gf}^2} \left[\frac{1}{\omega C_f} \right]^2 + \overline{i_{df}^2} \left[\frac{C_d + C_i + C_f + C_{gd} (1 + g_m R_{2m})}{g_m C_f} \right]^2. \quad (A-13)$$

The second and succeeding stages of the circuit of Figure A-1, page 143, are shown, with Type B positive feedback added, in Figure A-3. The effect of this type of positive feedback is to control the second stage input resistance.

The second stage current-to-output voltage gain for Type B positive feedback is found to be

$$A_{2mB} \equiv \frac{V_o}{I_2} = \frac{A_i R_B A_v}{1 - \frac{A_i R_B A_v - R_{2m}}{R_x}} \quad (A-14)$$

and the second stage input resistance is found to be

$$R_{in2B} \equiv \frac{V_2}{I_2} = \frac{R_{2m}}{1 - \frac{A_i R_B A_v - R_{2m}}{R_x}} \quad (A-15)$$

Application of Type B positive feedback changes the Miller capacitance since it alters the second stage input resistance.

The charge gain and mean squared output noise voltage for Type B positive feedback are obtained by replacing A_{2m} by A_{2mB} and R_{2m} by R_{in2B} in Equations (A-6) and (A-7).

$$A_{cB} = - \frac{1}{s C_f \left[C_d + C_i + C_f + C_{gd} \left[1 + \frac{g_m R_{2m}}{1 - \frac{A_i R_B A_v - R_{2m}}{R_x}} \right] + 1 \right]} \quad (A-16)$$

$$\left[\frac{A_i R_B A_v}{1 - \frac{A_i R_B A_v - R_{2m}}{R_x}} \right] C_f g_m$$

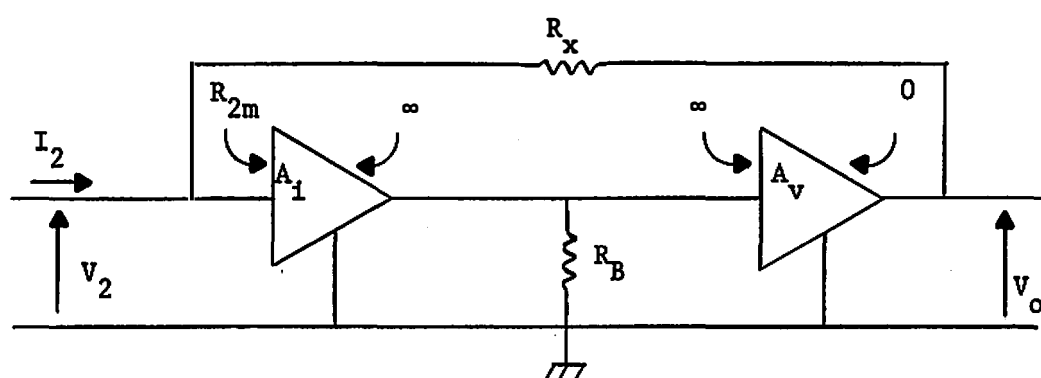


Figure A-3. Type B positive feedback.

$$\overline{e_{onB}^2} = \frac{\overline{i_{gf}^2} \left[\frac{1}{\omega C_f} \right]^2 + \overline{i_{df}^2} \left[\frac{C_d + C_i + C_f + C_{gd} \left(1 + \frac{g_m R_{2m}}{1 - \frac{A_1 R_B A_v - R_{2m}}{R_x}} \right)}{g_m C_f} \right]^2}{\left[\frac{C_d + C_i + C_{gd} \left(1 + \frac{g_m R_{2m}}{1 - \frac{A_1 R_B A_v - R_{2m}}{R_x}} \right)}{\left(\frac{A_1 R_B A_v}{1 - \frac{A_1 R_B A_v - R_{2m}}{R_x}} \right) C_f g_m} + 1 \right]^2} \quad (A-17)$$

If the positive feedback resistor, R_x , is adjusted so that

$$R_x = A_1 R_B A_v - R_{2m} \quad , \quad (A-18)$$

the charge gain becomes

$$A_{cB} = - \frac{1}{s C_f \left(\frac{C_{gd} R_{2m}}{A_1 R_B A_v C_f} + 1 \right)} \quad , \quad (A-19)$$

independent of C_d but still dependent on parameters other than C_f . The mean squared output noise voltage, under the condition of Equation (A-18), is

$$\overline{e_{noB}^2} = \frac{\overline{i_{df}^2} \left[\frac{1}{\omega C_f} \right]^2 + \overline{i_{df}^2} \left[\frac{C_{gd} R_{2m}}{C_f} K \right]^2}{\left[\frac{C_{gd} R_{2m}}{A_1 R_B A_v C_f} + 1 \right]^2} \quad (A-20)$$

where

$$K \rightarrow \infty . \quad (A-21)$$

Equation (A-20) indicates that the noise tends to infinity for Type B positive feedback. Actually, this will not be true since the resistors R_f and R_1 , which have not been included in this analysis, will ultimately control the feedback ratio at very low frequencies. The significance of Equation (A-20) is that the noise, at mid-band, for Type B positive feedback can be quite large.

For Type A positive feedback, the noise is very nearly the same as that without positive feedback. This may be seen by comparing Equations (A-7) and (A-13) realizing that the term $g_m A_{2m}$ in the denominator of Equation (A-7) is the preamplifier's open-loop gain which is generally quite large.

It may be concluded from the analysis above that either Type A or Type B positive feedback can be used to make the charge gain independent of detector capacitance. Type A is to be preferred over Type B, however, because of the increased noise associated with Type B due to the increased Miller capacitance.

Having decided upon the use of Type A positive feedback, it should be pointed out that the preamplifier's output pulse rise-time and decay-time will affect the charge gain and those parameters were not included in the foregoing mid-band analysis. The complete treatment of Type A positive feedback is given in Chapter 2.

APPENDIX B

COMPLEMENTARY EMITTER FOLLOWERS

A complementary emitter follower circuit is shown in Figure B-1 with all dc bias components removed. The ac small signal equivalent circuit is shown in Figure B-2 using a simplified hybrid- π model²⁴ for the transistors.

The following equations may be written by inspection of the equivalent circuit of Figure B-2.

$$V_{b'e} = V_{b'e1} = V_{b'e2} = V_{in} - V_{out} \quad (B-1)$$

$$I_{in} = V_{b'e} \left(\frac{1}{r_{b'e1}} + \frac{1}{r_{b'e2}} \right) \quad (B-2)$$

$$I_{in} + g_{m1}V_{b'e1} + g_{m2}V_{b'e2} = V_{out} \left(\frac{1}{r_{ce1}} + \frac{1}{r_{ce2}} + \frac{1}{R_L} \right) . \quad (B-3)$$

Solution of Equations (B-1) and (B-3) for the input resistance yields

$$R_{in} \equiv \frac{V_{in}}{I_{in}} = \frac{r_{b'e1} r_{b'e2}}{r_{b'e1} + r_{b'e2}} + \frac{1 + (g_{m1} + g_{m2}) \left[\frac{r_{b'e1} r_{b'e2}}{r_{b'e1} + r_{b'e2}} \right]}{\frac{1}{r_{ce1}} + \frac{1}{r_{ce2}} + \frac{1}{R_L}} . \quad (B-4)$$

If the dc emitter currents, I_{E1} and I_{E2} , are equal, then

$$r_{e1} = \frac{kT}{qI_{E1}} = r_{e2} = \frac{kT}{qI_{E2}} \equiv r_e . \quad (B-5)$$

Using Equation (B-5) and the following relationships for the hybrid- π model,

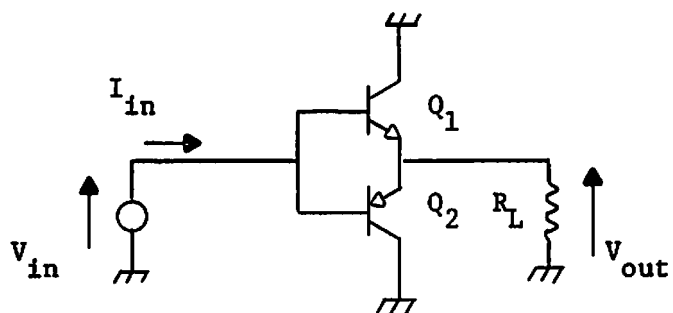


Figure B-1. Complementary emitter follower circuit.

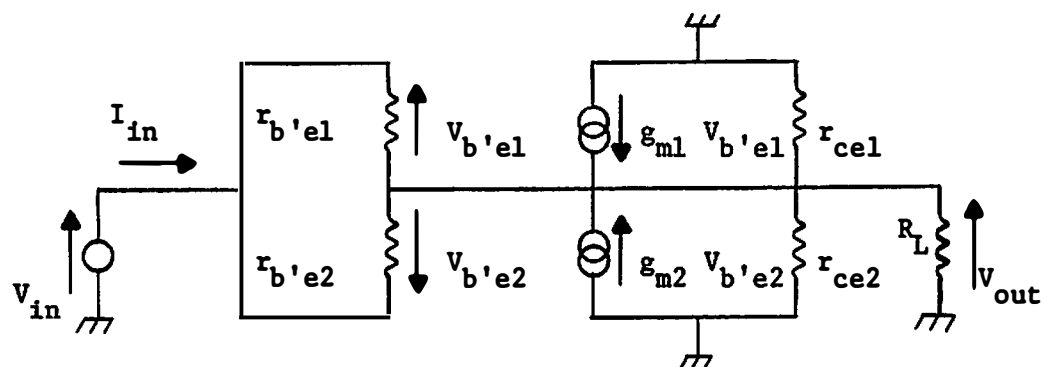


Figure B-2. Complementary emitter follower ac small signal equivalent circuit.

$$r_{b'e} = r_e (1 + \beta) \quad (\text{B-6})$$

$$g_m = \frac{\beta}{r_e (1 + \beta)} \quad , \quad (\text{B-7})$$

the input resistance expression, Equation (B-4), may be rewritten as

$$\begin{aligned} R_{in} &= \left(1 + \frac{2\beta_1 \beta_2 + \beta_1 + \beta_2}{\beta_1 + \beta_2 + 2} \right) \left(\frac{r_e}{2} + \frac{1}{\frac{1}{r_{ce1}} + \frac{1}{r_{ce2}} + \frac{1}{R_L}} \right) \\ &= (1 + \beta_{eff}) \left(r_{e_{eff}} + \frac{r_{ce_{eff}} R_L}{r_{ce_{eff}} + R_L} \right) \end{aligned} \quad (\text{B-8})$$

where

$$\beta_{eff} \equiv \frac{2\beta_1 \beta_2 + \beta_1 + \beta_2}{\beta_1 + \beta_2 + 2} \quad , \quad (\text{B-9})$$

$$r_{e_{eff}} \equiv \frac{r_e}{2} = \frac{r_{e1}}{2} = \frac{r_{e2}}{2} \quad , \quad (\text{B-10})$$

and

$$r_{ce_{eff}} = \frac{r_{ce1} r_{ce2}}{r_{ce1} + r_{ce2}} = r_{ce1} || r_{ce2} \quad . \quad (\text{B-11})$$

The voltage gain is found by solution of Equation (B-1) - (B-3)

to be

$$A_v \equiv \frac{V_{out}}{V_{in}} = \frac{\frac{1}{r_{b'e1}} + g_{m1} + \frac{1}{r_{b'e2}} + g_{m2}}{\frac{1}{r_{b'e1}} + g_{m1} + \frac{1}{r_{b'e2}} + g_{m2} + \frac{1}{r_{ce1}} + \frac{1}{r_{ce2}} + \frac{1}{R_L}} \quad . \quad (\text{B-12})$$

Using the relationships in Equations (B-5) - (B-7) and the definitions of Equations (B-10) and (B-11), Equation (B-12) may be rewritten as

$$A_v \equiv \frac{V_{out}}{V_{in}} = \frac{\frac{r_{ce\,eff} R_L}{r_{ce\,eff} + R_L}}{\frac{r_{ce\,eff} R_L}{r_{ce\,eff} + R_L} + r_{e\,eff}} \quad (B-13)$$

A simple emitter follower circuit and its ac small signal equivalent circuit are shown in Figures B-3 and B-4, respectively. Analysis of the simple emitter follower in the same manner outlined above will show that the input resistance and voltage gain are given by

$$R_{in} \equiv \frac{V_{in}}{I_{in}} = (1 + \beta) r_e + \frac{r_{ce} R_L}{r_{ce} + R_L} \quad (B-14)$$

and

$$A_v \equiv \frac{V_{out}}{V_{in}} = \frac{\frac{r_{ce} R_L}{r_{ce} + R_L}}{\frac{r_{ce} R_L}{r_{ce} + R_L} + r_e} \quad (B-15)$$

By comparing Equations (B-8) and (B-13) to Equations (B-14) and (B-15), it can be seen that, for purposes of ac analysis, a complementary emitter follower circuit may be replaced by a simple emitter follower which has effective β , r_e , and r_{ce} given by Equations (B-9) - (B-11).

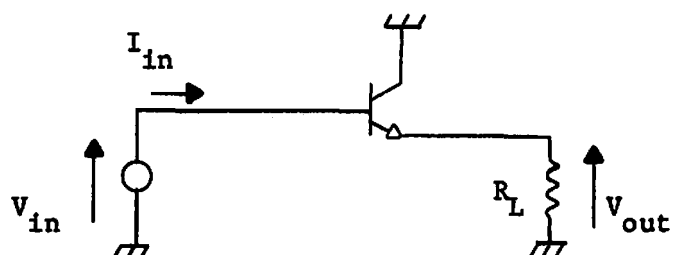


Figure B-3. Simple emitter follower circuit.

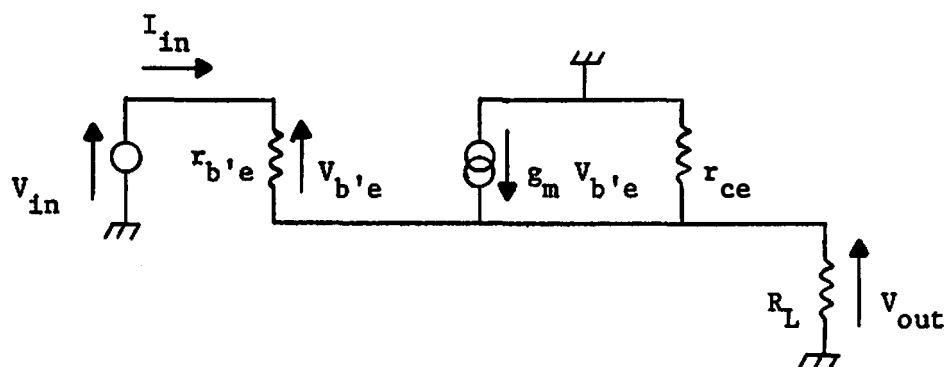


Figure B-4. Simple emitter follower ac small signal equivalent circuit.

Further simplifications may be made if

$$\beta_1 \gg 1 ,$$

$$\beta_2 \gg 1 . \quad (B-16)$$

and

$$\frac{r_{ce\text{eff}} R_L}{r_{ce\text{eff}} + R_L} \gg r_{e\text{eff}} .$$

Under those conditions

$$\beta_{\text{eff}} \approx 2 \frac{\beta_1 \beta_2}{\beta_1 + \beta_2} , \quad (B-17)$$

$$R_{\text{in}} \approx \left(1 + 2 \frac{\beta_1 \beta_2}{\beta_1 + \beta_2} \right) \frac{r_{ce\text{eff}} R_L}{r_{ce\text{eff}} + R_L} . \quad (B-18)$$

and

$$A_v \approx 1 . \quad (B-19)$$

VITA

William Pinkston Albritton, Jr., son of William Pinkston and Currie Cumby Albritton, was born in Frisco City, Alabama, on August 12, 1940. He attended Frisco City, Alabama public schools and graduated from Frisco City High School in May, 1958.

In September, 1958 he entered Auburn University and was awarded the Bachelor of Electrical Engineering degree in August, 1962 and the Master of Science degree in June, 1965. From September, 1962 to August, 1965 he was employed as an Instructor in the Electrical Engineering Department of Auburn University.

In September, 1965 he was admitted to The University of Tennessee to do further graduate work in electrical engineering and was employed as an Instructor in The University of Tennessee Electrical Department. During the summers of 1966, 1967, and 1968 he was employed as an Associate Development Engineer in the Instrumentation and Controls Division of the Oak Ridge National Laboratory.

He married the former Letty Williams in December, 1962. They have two sons, William Scott and Nathaniel Grayson.

