To the Graduate School:

I am submitting herewith a dissertation written by Yun-Ching Tsuei entitled "A Minimum Fuel Minimum Range Error Controller for a Surface-to-Surface Missile." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Engineering Science.

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We have read this dissertation and recommend its acceptance:

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Accepted for the Council:

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The Graduate School

A MINIMUM FUEL MINIMUM RANGE ERROR CONTROLLER FOR A SURFACE-TO-SURFACE MISSILE

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Yun-Ching Tsuei

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ABSTRACT

The purpose of this investigation was to conduct a sensitivity analysis of a surface to surface optimally controlled guided missile and compare it with one controlled by a conventionally guided missile. In particular, the problem was to find how much the performance of this missile was adversely affected if some of the parameters did not adhere to those utilized in the optimal simulation and how well the conventional scheme compared.

Two methods were employed in measuring the performance of "cost" of the missile. First a cost function was selected which was a function of the summation of all the deviations from an optimal trajectory and the amount of side thrust that was used by the steering rocket. Secondly, selected states at rocket burnout of the optimal trajectory were compared with those of the nominal trajectory to determine the magnitude of error as the missile began its ballistic path back to earth.

The method for determining the optimal trajectory and all the experimental data was obtained by mathematically simulating the missile on a digital computer. The aerodynamic equations of the missile and other parameters affecting its flight were obtained from a previous study of a typical surface-to-surface guided missile. Its range was approximately 30 miles and it had a powered flight lasting for 5.12 seconds.

The parameters that were varied in the sensitivity analysis were:

1. drag or axial force coefficient, C_A;
2. lift or normal force coefficient, C_N;
3. length from the tail to the center of mass;
4. length from center of mass to center of pressure, X;
5. thrust misalignment.

The exact procedure was to vary the above parameters one at a time by plus or minus 5% and then simulate the missile flight on the computer under several different winds. At the completion of each flight, the data for the cost function and selected states was analyzed to determine the effect of the perturbation.

In conclusion, it was found that the missile attitude was the parameter having the most significant effect on the missile's performance. The conventional scheme had a much smaller miss distance but in some cases (thrust misalignment) used more fuel.

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LIST OF SYMBOLS

$A(t)$	$(\partial \underline{f}/\partial \underline{x})^*$ (n x n matrix)
A_D	drag or axial force acting on missile (lb_f)
A_e	exit area of nozzle (ft^2)
$B(t)$	$(\partial \underline{f}^*/\partial \underline{x})$ (n x m matrix)
C	measurement matrix (p x n matrix)
C_A	drag or axial force coefficient (dimensionless)
C_N	Lift or normal force coefficient (dimensionless)
F	terminal weighing matrix of cost function J (n x n matrix)
F_c	$F_c^* + \delta F_c$ side thrust (lb_f)
F_{NSL}	nominal thrust at sea level (lb_f)
F_T	thrust of missile (lb_f)
F_{TA}, F_{TN}	axial and normal thrust components, respectively, due to thrust misalignment (lb_f)
$\underline{f}$	nonlinear vector function representing the state model of the system (n-vector)
f_h	vertical velocity of the missile (ft/sec)
f_v	acceleration of missile along flight path (ft/sec^2)
f_γ	rate of change of flight path angle (rad/sec)
f_θ	rate of change of heading angle (rad/sec)

$f_{\dot{\theta}}$	second time derivative of missile heading angle (rad/sec^2)
G	feedback gain vector (n-vector)
G_{FC}	gain for error in side thrust (dimensionless)
$G_{F\dot{C}}$	gain for error in side thrust rate (sec)
G_h	gain for error in altitude (lb_f/ft)
G_V	gain for error in velocity (lb_f-sec/ft)
G_γ	gain for error in flight path angle (lb_f/rad)
G_θ	gain for error in heading angle (lb_f/rad)
$G_{\dot{\theta}}$	gain for error in heading angle rate (lb_f-sec/rad)
g	acceleration of gravity (32.2 ft/sec^2)
H	Hamiltonian function (scalar function)
h	altitude of missile (ft)
I	polar moment of inertia of missile (lb_f-ft-sec^2)
I	identity matrix (square matrix)
J	cost function
$K(t)$	matrix Riccatti function (n x n matrix)
K_p	fixed gain for directional control
k	scalar Riccatti function
L	integrand of integral in cost function
l	length from tail to the center of mass of missile (ft)

l_{CP}	distance to center of pressure of missile (ft)
M	Mach number (dimensionless)
m	mass of missile (slugs)
m	dimension of control vector
$N = N'\alpha$	lift or normal component of force acting on missile (lb_f)
n	dimension of state vector
p	atmospheric pressure (lb_f/ft^2)
$\underline{p} = \underline{p}^* + \delta\underline{p}$	costate vector (n-vector)
p_{SL}	atmospheric pressure at sea level (lb_f/ft^2)
Q	weighing matrix in cost function for state vector deviations (n x n matrix)
R	control vector weighing matrix in cost function (m x m matrix)
r	$r^* + \delta r$ = range of missile (ft)
r^*	nominal range of missile (ft)
r_V, r_γ, $r_{\dot\theta}$, r_θ, r_h	range of missile for only perturbations of $V(t_b)$, $\gamma(t_b)$, $\dot\theta(t_b)$, $\theta(t_b)$, and $h(t_b)$ from the nominal values, respectively (ft)
S	form area of missile (ft^2)
s	complex Laplace variable (1/sec)
t	time (sec)
t_b	propellant burnout time (sec)
t_f	final time or propellant burnout time (sec)
t_i	initial time of missile launch (sec)

t_{impact}	time of missile impact (sec)
t_o	time zero or time of missile launch (sec)
$\underline{u}$	control vector acting to change the trajectory $\underline{x}$ (m-vector)
$\underline{u}^*$	nominal or optimum control law (m-vector)
V	velocity of the missile with no winds (ft/sec)
V_{RW}	velocity of missile relative to the winds (ft/sec)
V_W	velocity of the wind (ft/sec)
$w(t)$	instantaneous weight of missile (lb_f)
W_T	total initial weight of missile
w_N	nominal rate at which propellant is being consumed (lb_f/sec)
$\underline{x}$	$\underline{x}^* + \delta x$ - state vector of the system (n-vector)
$\bar{x}$	length between the center of mass of the missile and the center of pressure (ft)
$\underline{\hat{x}}$	estimated value of the state vector (n-vector)
Y	transformation matrix for the Riccatti equation (n x n matrix)
$\underline{y}$	measurement vector with elements that may be easily measured such as θ and $\dot{\theta}$ (p-vector)
α	$\theta - \gamma$ = angle of attack (rad)
γ	flight path angle (rad)
δ	thrust misalignment (rad)
$\delta(\ )$	first variation of quantity

$\delta\underline{u}$	$\underline{u} - \underline{u}^*$ = first variation in control variable (m-vector)
$\delta\underline{x}$	$\underline{x} - \underline{x}^*$ = first variation in the state vector (n-vector)
θ	$\theta^* + \delta\theta$ = heading angle of missile (rad)
θ_R	reference heading angle of missile (rad)
$\dot{\theta}$	$d\theta/dt = \dot{\theta} + \delta\dot{\theta}$ = missile heading angle rate (rad/sec)
ρ	density of the atmosphere (slugs/ft^3)
ϕ	terminal quantity in cost function to be minimized
ξ	damping factor for the second order controller (dimensionless)
ω_n	natural frequency for the second order controller (rad/sec)
$(\ )^*$	nominal quantity evaluated along the optimal path
q	an aerodynamic variable termed dynamic pressure

CHAPTER I

PROBLEM FORMULATION

Background Information

In general a surface-to-surface guided missile has the mission of impacting a warhead upon a selected target with a certain accuracy. For a given range with the initial conditions of the launch site known and the final conditions of the target specified, one determines a thrust program and control program such that some mission criteria, such as fuel usage, velocity, or time of flight is optimized. This is a somewhat standard form for an optimization problem that may be solved by the calculus of variations. However, because of noise and disturbances one cannot expect the vehicle to terminate on the target if the precalculated control program is applied without any corrections or feedback.

Typical sources of disturbances are:

1. imperfect implementation of the thrust program;
2. imperfect knowledge of the environment;
3. imperfect knowledge of the initial launch conditions;
4. imperfect knowledge of system parameters.

Thus while formulating the optimum trajectory one must at the same time synthesize some form of corrective control action along the way. In this thesis this corrective action will be developed by a neighborhood optimal controller.

One problem that faces an engineer then in designing such systems is to find some means of determining how "good" the systems are. This problem can be aided by modern control theory because:

1. the "best that a designer can achieve is evident";
2. by using the method of the "2nd variation" a canonical form for the control scheme can be developed allowing a systematic examination of the desirability of various schemes.

To apply these techniques to a practical system, a surface-to-surface missile as shown in Figure 1.1 will be utilized. Two problems that are inherent in the use of optimum control theory are:

1. the conversion of design specifications into a meaningful mathematical performance index is not straightforward;
2. existing algorithms require complex computer control.

However, a surface-to-surface missile can be cast in a mathematical tractable form that alleviates these problem areas and still retains most of the realistic aspects of a practical control problem.

The cost function for this missile will be of the form

$$J = \frac{1}{2} (\underline{x} - \underline{x}^*)^T F(\underline{x} - \underline{x}^*)\Big|_{t_f} + \frac{1}{2} \int_0^{t_f} F_c^{\,2}\, dt \qquad (1.1)$$

and is discussed in Chapter IV. As shown in Chapter II the final condition for the matrix Riccatti equation is the matrix $F(t_f)$ and the control law is given by

$$\delta F_c = G(t)\ \delta\underline{x}\ . \qquad (1.2)$$

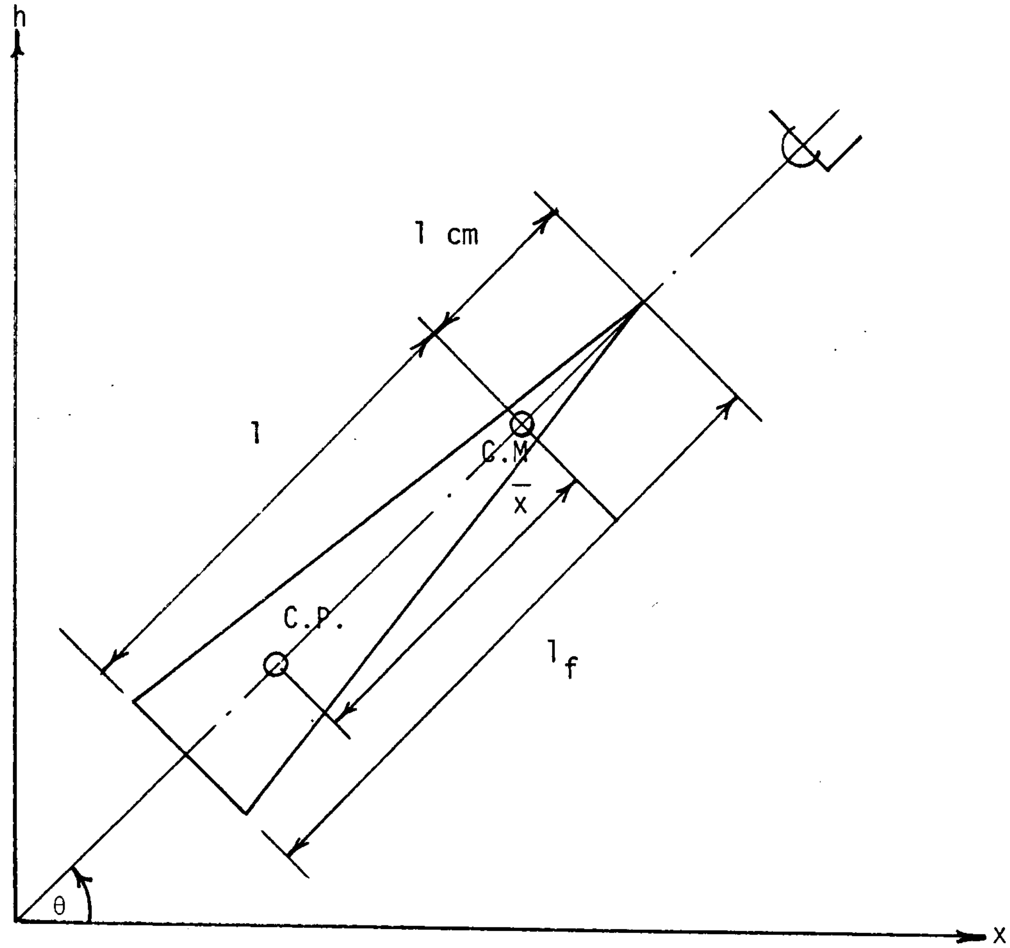

Figure 1.1. System Configuration.

l_f = 7.715 ft. = total length of missile;

l = 3.101 ft. = control force moment arm;

$\bar{x}$ = 1.086 ft. = lift force moment arm;

l cm = 4.6.4 = distance to center of mass from tip;

W_t = 434.1 lb. = total weight;

W_p = 176.7 lb. = propellant weight;

t_b = 5.12 sec. = propellant burn time;

A_e = .2799 ft.^{2} = exit area of nozzle;

s = .54542 ft.^{2} = form area.

The block diagram for this control scheme is shown in Figure 1.2 which is the state regulator for the linearized system. This approach allows a comparison between the optimal control law and other sub-optimal control schemes. The suboptimal scheme examined in this thesis is described by the equation

$$F_c = -K \quad q \; \alpha \tag{1.3}$$

where q is an aerodynamic variable termed dynamic pressure and α is the missile angle of attack.

For the case where there is a second order lag in the controller, it is possible to compensate for this lag in the control law by incorporating two more elements in the state vector.

Review of Literature

The control problem which arises in connection with surface-to-surface guided missiles is one of minimizing the miss distance in range at impact. Since the control is usually effective only during propellant burn time, the problem consequently reduces to that of minimizing the "miss distance" at propellant burnout. The "miss distance" in this case is the deviation of the actual state from the nominal state. This is a terminal control problem and is closely related to the problem of Bolza in the calculus of variations (2). The beginning and end times are fixed as:

$t_o = 0$

t_f = end of propellant burning = t_b

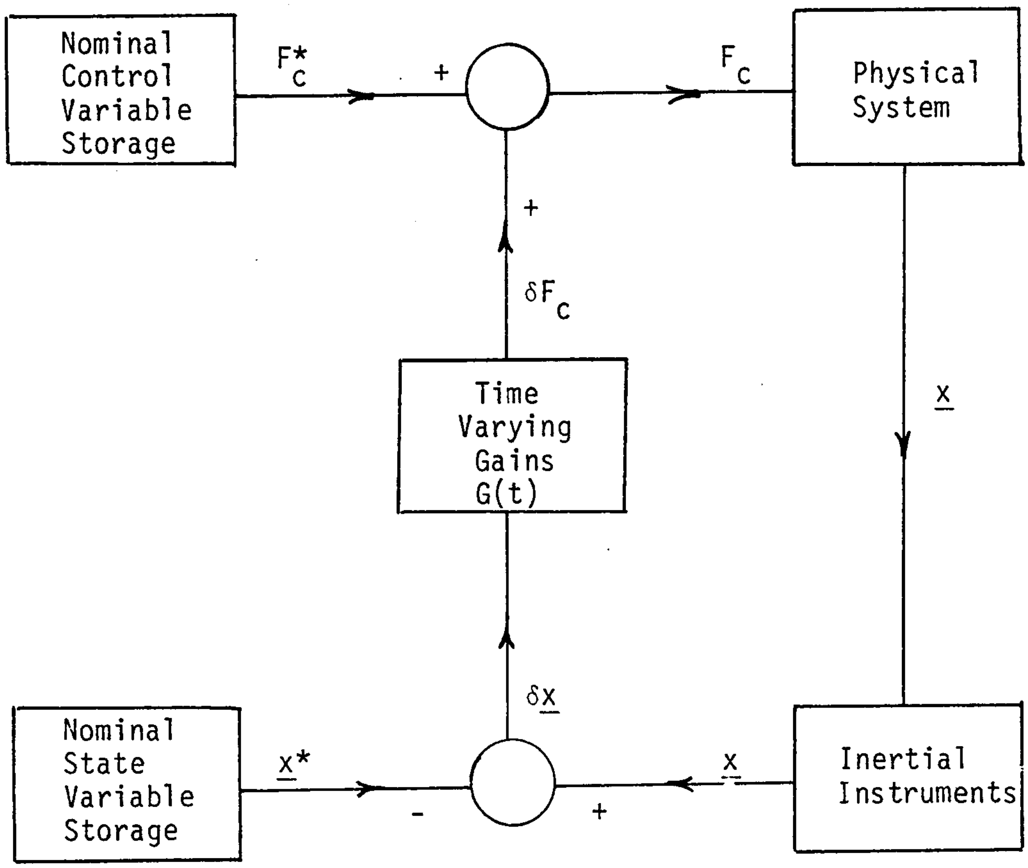

Figure 1.2. Block Diagram for System Simulation.

The initial value of the state vector is given as

$$\underline{x}(t_o) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \theta \\ 0 \end{bmatrix}_{t=t_o} \tag{1.4}$$

and the terminal value of the state vector is

$$x(t_f) = \begin{bmatrix} V \\ 0 \\ \dot{\theta} \\ \theta \\ h \end{bmatrix}_{t=t_b} \tag{1.5}$$

The initial angle θ_i is the initial attitude that will allow the missile to impact a desired range while flying a ballistic path, i.e., $F_c = 0$. The values along this ballistic path are considered to be optimal and at the end of propellant burn the optimal terminal value of the state vector is given as:

$$\underline{x}^*(t_b) = \begin{bmatrix} V \\ \gamma \\ \dot{\theta} \\ \theta \\ h \end{bmatrix}_{F_c=0} \tag{1.6}$$

There are no inequality constraints and no terminal constraints. If the equations of motion are linearized about the nominal path, and the performance index is expanded up to second order in perturbations away from the nominal path, the result is an optimization problem for a linear system with a quadratic performance index (2,3). Such problems have been treated extensively in (4,5) and given a linear feedback law, but also requires that the nonlinear Riccatti equation be integrated backwards.

The two major divisions of this thesis will be:

1. the formulation of conventional control techniques in terms of the second variation;
2. the comparison of various suboptimal performances of surface-to-surface guided missiles with the optimum performance.

Surface-to-Surface Missiles

Surface-to-surface missiles of this general range (30 miles) usually have trajectories consisting of two phases: (1) the powered flight with propellant and (2) a ballistic path with auxiliary controls to maintain the proper trajectory. A typical missile might utilize air varies during this portion of the trajectory. With present day miniaturization techniques, inertial measuring units (IMU) are quite common. In general, such units allow the measurement of $\dot{\theta}$, θ, $\ddot{x}$ and $\ddot{y}$ and hence the calculation of V, γ and α. In this thesis the optimal system controls the missile during the powered flight portion of the trajectory. Thus the "miss distance" relates to the difference between the normal state variables at "burn out" and their actual values. It is this error that must be corrected in the ballistic portion of the flight trajectory.

The errors at "burnout" can be related to actual impact range errors. A range error gradient can be defined as

$$\frac{\partial r}{\partial \underline{x}} = \left[\frac{\partial r}{\partial v}, \frac{\partial r}{\partial \gamma}, \frac{\partial r}{\partial \theta}, \frac{\partial r}{\partial h} \right] \tag{1.7}$$

This vector can be obtained by

1. simulating the flight of the entire nominal trajectory and determining the value of r;
2. implanting a 1% deviation in one of the state variables as missile burnout while keeping the other state variables at their nominal value;
3. simulating the flight to impact again but with the deviation incorporated and noting the change in r at impact;
4. repeating 2 and 3 above for all of the state variables.

The range error RE then becomes

$$RE = \frac{\partial r}{\partial \underline{x}} \; [x - x^*]_t = t_b \tag{1.8}$$

This range error of course assumes a nominal missile with no correction during the ballistic flight.

CHAPTER II

THEORETICAL CONSIDERATIONS

Mathematical Model

As shown in Chapter III, the equations of motion for the missile are

$$m\dot{V} = (F_T - A_D)\cos\alpha + (F_c - N)\sin\alpha - mg\sin\gamma \tag{2.1}$$

$$mV\dot{\gamma} = (F_T - A_D)\sin\alpha - (F_c - N)\cos\alpha - mg\cos\gamma \tag{2.2}$$

$$I\ddot{\theta} = F_c\ell - N\bar{x} \tag{2.3}$$

If we now consider the state vector to be

$$\underline{x} = \begin{bmatrix} V \\ \gamma \\ \dot{\theta} \\ \theta \\ h \end{bmatrix} \tag{2.4}$$

and F_c to be the control variable, then the state model is of the form:

$$\frac{d}{dt}\begin{bmatrix} V \\ \gamma \\ \dot{\theta} \\ \theta \\ h \end{bmatrix} = \begin{bmatrix} f_V \\ f_\gamma \\ f_{\dot{\theta}} \\ f_\theta \\ f_h \end{bmatrix}$$

$$\begin{bmatrix} f_V \\ f_\gamma \\ f_{\dot\theta} \\ f_\theta \\ f_h \end{bmatrix} = \begin{bmatrix} [(F_T - A_D)\cos\alpha + (F_c - N)\sin\alpha - mg\sin\gamma]/m \\ [(F_T - A_D)\sin\alpha - (F_c - N)\cos\alpha - mg\cos\gamma]/mV \\ (F_c \ell - N\bar{x})/I \\ \dot\theta \\ V\sin\gamma \end{bmatrix} \tag{2.5}$$

The System as a Bolza Problem

The problem of guiding the missile only during the time that there is propellant available to burn may be considered as a Bolza problem since:

1. there is fixed beginning and terminal times;
2. there are no inequality constraints; and
3. there are no constraints at the terminal time.

The beginning time is

$$t_o = t_i = 0 \tag{2.6}$$

and the final time is

$$t_f = t_b = \text{end of propellant burning.}$$

At the beginning time the state vector is given by

$$\underline{x}(t_o) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \theta \\ 0 \end{bmatrix}_{t = t_o} \tag{2.7}$$

and at the known final time the nominal state vector is:

$$\underline{x}^*(t_f) = \begin{bmatrix} V \\ \gamma \\ \dot{\theta} \\ \theta \\ h \end{bmatrix}_{t = t_f} \tag{2.8}$$

where θ is the initial attitude that will allow the missile to impact a desired range while flying a ballistic path, i.e., $u^* = 0$. The state $\underline{x}^*$ (t_f) are the values of the state vector along this trajectory at the end of the propellant burn.

The Bolza problem with no inequality constraints (7, p. 64) is that of finding the minimum of

$$J = \phi[\underline{x}(t_f), t_f] + \int_0^{t_f} L[\underline{x}(t), \underline{u}(t), t]dt \tag{2.9}$$

for a system described by

$$\dot{\underline{x}} = f[\underline{x}(t), \underline{u}(t), t], \ \underline{x}(t_o) = \underline{x}_0 \tag{2.10}$$

with t_o and t_f fixed. If a Hamiltonian is defined as:

$$\begin{aligned} H[\underline{x}(t), \underline{u}(t), \underline{p}(t), t] &= L(\underline{x}(t), \underline{u}(t), t] \\ &+ \underline{p}^T(t)\underline{f}[x(t), u(t), t] \ ; \end{aligned} \tag{2.11}$$

the necessary conditions for the state vector is

$$\frac{\partial H}{\partial \underline{p}} = \dot{\underline{x}}, \qquad \underline{x}(t_o) = \underline{x}_o \tag{2.12}$$

and the necessary conditions for the costate vector is:

$$\frac{\partial H}{\partial \underline{x}} = - \dot{\underline{p}} \tag{2.13}$$

$$\underline{p}(t_f) = \frac{\partial \phi}{\partial \underline{x}(t_f)} [\underline{x}(t_f), t_f] \tag{2.14}$$

Control Law

Equation (2.5) is of the form:

$$\dot{\underline{x}} = \underline{f}(\underline{x},\underline{u}) \tag{2.15}$$

which is a set of nonlinear ordinary differential equations. If we now consider the optimal trajectory $\underline{x}^*$ to be that as provided by the optimal control $\underline{u}^*$ such that the cost function

$$J = 1/2(\underline{x} - x^*)^T F(\underline{x} = \underline{x}^*)|_{t_f} + 1/2 \int_{t_o}^{t_f} u^T Ru\ dt \tag{2.16}$$

is minimized, then we form the Hamiltonian function from this cost function as:

$$H = 1/2\ \underline{u}^T R\underline{u} + \underline{p}^T\underline{f} \tag{2.17}$$

Substituting the Hamiltonian function as given by Equation (2.17) into Equation (2.16) results in the following:

$$J = 1/2(\underline{x} - \underline{x}^*)^T F(\underline{x} - \underline{x}^*)|_{t_f} + \int_{o}^{t_b} (H - \underline{p}^T\underline{f})dt \tag{2.18}$$

If the cost function is now expanded about the nominal trajectory x* with a Taylor's series where only the second variation terms are retained then since $\delta \underline{x} = \underline{x} - \underline{x}^*$, we have

$$\delta J = (\underline{x} - \underline{x}^*)^T F(\underline{x} - \underline{x}^*)\Big|_{t_f} + \int_0^{t_f} [(\frac{\partial H}{\partial \underline{x}} - \underline{p}^T \frac{\partial f}{\partial \underline{x}})\partial x$$

$$+ (\frac{\partial H}{\partial \underline{p}} - \underline{f}^T)\ \delta p + (\frac{\partial H}{\partial \underline{u}} - \underline{p}^T \frac{\partial f}{\partial \underline{u}})\ \delta u]dt \tag{2.19}$$

For arbitrary $\delta \underline{x}$, $\delta \underline{p}$ and $\delta \underline{u}$ the integrand will be zero provided that:

$$\frac{\partial H}{\partial \underline{x}} = \underline{p}^T \frac{\partial f}{\partial \underline{x}} = -\ \dot{\underline{p}} \tag{2.20}$$

$$\frac{\partial H}{\partial \underline{p}} = \underline{f}^T = \dot{\underline{x}}^T \tag{2.21}$$

$$\frac{\partial H}{\partial \underline{u}} = \underline{p}^T \frac{\partial f}{\partial \underline{u}} \tag{2.22}$$

From Equation (2.14) the final condition for the costate vector is:

$$\underline{p}\ (t_f) = \frac{\partial \phi}{\partial \underline{x}\ (t_f)} [\underline{x}(t_f)^f, t_f] \tag{2.23}$$

and from Equation (2.9)

$$\phi[\underline{x}(t_f), t_f] = 1/2(\underline{x} - \underline{x}^*)^T F(\underline{x} - \underline{x}^*)\Big|_{t_f} \tag{2.24}$$

Along any trajectory $\underline{x}(t)$ the final value of the costate vector is:

$$\underline{p}(t_f) = F(\underline{x} - \underline{x}^*) \tag{2.25}$$

Along the nominal trajectory $\underline{x}^*(t)$, the final value of the costate is

$$\underline{p}^*(t_f) = F(\underline{x}^* - \underline{x}^*)\Big|_{t_f} = 0 \ . \tag{2.26}$$

If Equation (2.13) is now integrated backwards from $\underline{p}^*(t_f) = 0$ to an arbitrary value of t, the result is:

$$\underline{p}^*(t) = 0 \tag{2.27}$$

Now if $\partial H/\partial \underline{u}$ is evaluated with Equation (2.17) then the result is

$$\frac{\partial H}{\partial \underline{u}} = \underline{u}^T R + \underline{p}^T \frac{\partial f}{\partial \underline{u}} \tag{2.28}$$

and

$$\left(\frac{\partial H}{\partial \underline{u}}\right)^* = \underline{u}^{*T} R + \underline{p}^{*T} \left(\frac{\partial f}{\partial u}\right)^* \tag{2.29}$$

By Equation (2.22) and Equation (2.27), $(\partial H/\partial \underline{u})^* = 0$, therefore:

$$\underline{u}^{*T} R = 0 \tag{2.30}$$

Since $R > 0$ by Equation (2.4), then $\underline{u}^* = 0$.

Another way to arrive at this optimal trajectory is to let $\underline{u}^* = 0$ and let the resulting trajectory $\underline{x}^*(t)$ be the nominal trajectory. The cost is given as:

$$J^* = 1/2(\underline{x}^* - \underline{x}^*)^T F(\underline{x}^* - \underline{x}^*) + 1/2 \int_{t_o}^{t_f} 0.dt = 0 \tag{2.31}$$

which is a minimum since J is a quadratic cost function. Now since the costate variable is given by the equation

$$\underline{p} = \underline{p}^* + \delta\underline{p} \tag{2.32}$$

then

$$\underline{p} = \delta\underline{p} \tag{2.33}$$

and since the control variable $\underline{u}$ is given by the equation

$$\underline{u} = \underline{u}^* + \delta u \tag{2.34}$$

then

$$\underline{u} = \delta\underline{u} \tag{2.35}$$

For the first variation of the cost δJ to be zero, it is required that

$$\underline{x}(t_f) = \underline{x}^*(t_f) \tag{2.36}$$

and

$$\underline{u}^* = 0 \tag{2.37}$$

$$\underline{p}^* = 0 \tag{2.38}$$

The second variation of the cost function with respect to $\underline{x}$, $\underline{u}$, and $\underline{p}$ is given by the equation

$$\delta^2 J = 2(\underline{x} - \underline{x}^*)^T F(\underline{x} - \underline{x}^*)\Big|_{t_f} + 2\int_0^{t_f} \delta\underline{u}^T R\, \delta\underline{u}\, dt \tag{2.39}$$

Since the second variation is in the form of a quadratic then it will be positive definite provided that the matrices

$$F(t_f) > 0 \tag{2.40}$$

and

$$R(t) > 0 \tag{2.41}$$

The second variation of the cost function causes Equations(2.12 (2.12) and (2.13) to be perturbed from the nominal trajectory or

$$\frac{\partial^2 H}{\partial x^2}\,\delta \underline{x} + \frac{\partial^2 H}{\partial \underline{p}\,\partial x}\,\delta \underline{p} + \frac{\partial^2 H}{\partial \underline{u}\,\partial \underline{x}}\,\delta \underline{u} = -\,\delta \dot{\underline{p}} \tag{2.42}$$

and

$$\frac{\partial^2 H}{\partial \underline{x}\,\partial \underline{p}}\,\delta \underline{x} + \frac{\delta^2 H}{\delta \underline{u}\,\delta \underline{p}}\,\delta \underline{u} + \frac{\delta^2 H}{\delta p^2}\,\delta \underline{p} = \delta \dot{\underline{x}} \tag{2.43}$$

Since the Hamiltonian is given by Equation (2.17), the six second partials given in the above equations (2.42) and (2.43) are given as

$$\left(\frac{\partial^2 H}{\partial \underline{x}^2}\right)^* = \underline{p}^{*T}\left(\frac{\partial^2 f}{\partial \underline{x}^2}\right)^* = 0 \tag{2.44}$$

and

$$\left(\frac{\partial^2 H}{\partial \underline{u}\,\partial \underline{x}}\right)^* = \underline{p}^{*T}\left(\frac{\partial^2 f}{\partial \underline{u}\,\partial \underline{x}}\right)^* = 0 \tag{2.45}$$

Since $\underline{p}^*(t) = 0$ (see Equation 2.27), then

$$\left(\frac{\partial^2 H}{\partial \underline{p}\,\partial \underline{x}}\right)^* = \left(\frac{\partial \underline{f}}{\partial \underline{x}}\right)^* \tag{2.46}$$

$$\left(\frac{\partial^2 H}{\partial \underline{x}\,\partial \underline{p}}\right)^* = \left(\frac{\partial \underline{f}}{\partial \underline{x}}\right)^* \tag{2.47}$$

and

$$\left(\frac{\partial^2 H}{\partial \underline{u}\,\partial \underline{p}}\right)^* = \left(\frac{\partial \underline{f}}{\partial \underline{u}}\right)^* \tag{2.48}$$

where the partials are all evaluated along the nominal trajectory as indicated. Substituting the expressions for the partials as given by Equations (2.44) through (2.48) into Equations (2.42) and (2.43) results in:

$$\delta\dot{\underline{x}} = \left(\frac{\partial f}{\partial \underline{x}}\right)^{*} \delta\underline{x} + \left(\frac{\partial f}{\partial \underline{u}}\right)^{*} \delta\underline{u} \tag{2.49}$$

and

$$\delta\dot{\underline{p}} = -\left(\frac{\partial f}{\partial \underline{x}}\right)^{*T} \delta\underline{p} \tag{2.50}$$

as linear equations in the perturbations of the state $\delta\underline{x}$ and in the perturbations of the costate $\delta\underline{p}$, or

$$\delta\dot{\underline{x}} = A(t)\,\delta\underline{x} + B(t)\,\delta\underline{u} \tag{2.51}$$

and

$$\delta\dot{\underline{p}} = -A^{T}(t)\,\delta\underline{p} \tag{2.52}$$

From the optimality condition:

$$\frac{\partial^2 H}{\partial \underline{u}^2}\delta\underline{u} + \frac{\partial^2 H}{\partial \underline{x}\,\partial \underline{u}}\delta\underline{x} + \frac{\partial^2 H}{\partial \underline{p}\,\partial \underline{u}}\delta\underline{p} = 0 \tag{2.53}$$

or

$$\delta\underline{u} = -R^{-1}\left(\frac{\partial f}{\partial \underline{u}}\right)^{T*}\delta\underline{p} \tag{2.54}$$

Substituting the above equation (2.54) into Equation (2.51) results in:

$$\delta\dot{\underline{x}} = A(t)\ \delta\underline{x} - B(t)\ R^{-1}\ B^T(t)\ \delta\underline{p} \tag{2.55}$$

Since it has been shown in (5) that

$$\delta\underline{p} = K(t)\ \delta\underline{x} \tag{2.56}$$

then upon differentiation with respect to time

$$\delta\dot{\underline{p}} = K\delta\dot{\underline{x}} + \dot{K}\delta\underline{x} \tag{2.57}$$

By substituting the expression given for $\delta\dot{\underline{p}}$ as given by Equation (2.52) and the expression for $\delta\underline{p}$ as given by Equation (2.56) into Equation (2.57) the result is:

$$(-A^TK - KA + KBR^{-1}\ B^TK - \dot{K})\ \delta\underline{x} = 0 \tag{2.58}$$

since $\delta\underline{x}$ is arbitrary it is necessary that K satisfy the matrix differential equation

$$\dot{K} = -A^TK - KA + KBR^{-1}\ B^TK \tag{2.59}$$

It is shown in (8, p. 302) that at the final time $t = t_f$ the costate equation must satisfy the transversality conditions:

$$\underline{p}(t_f) = \frac{\partial}{\partial \underline{x}} [1/2(\underline{x} - \underline{x}^*)^T\ F(\underline{x} - \underline{x}^*)]|_{t_f} \tag{2.60}$$

and since $\underline{p}^* = 0$ the first variation in the costate variable must satisfy the equation:

$$\delta\underline{p}(t_f) = F(\underline{x} - \underline{x}^*)|_{t_f} \tag{2.61}$$

At the final time the costate variable is given by:

$$\delta \underline{p}(t_f) = K(t_f)\ \delta \underline{x}(t_f) \tag{2.62}$$

and therefore

$$K(t_f) = F(t_f) \tag{2.63}$$

is the final value of the matrix Riccatti equation. It is therefore necessary to integrate the matrix Riccatti equation backwards in time from the final condition

$$F(t_f) = K(t_f) \tag{2.64}$$

to the time t_o in order to determine the matrix Riccatti function K(t). When this is done the control law is given by:

$$\delta \underline{u} = -R^{-1} \left(\frac{\partial f}{\partial \underline{u}}\right)^{*T} K(t)\ \delta \underline{x}\ . \tag{2.65}$$

<u>A More General Cost Function</u>

A somewhat more general cost function would have the form:

$$J = 1/2\ \underline{x}^T F \underline{x} + 1/2 \int (\underline{x}^T Q \underline{x} + \underline{u}^T R\ \underline{u})dt \tag{2.66}$$

The matrices F, Q, and R may be time varying and symmetric. They must also be semidefinite. The matrix Riccati equation (7) becomes

$$\dot{K} = -K(t)A(t) - A^T(t)K(t) + K(t)B(t)R^{-1}\ B^T(t)K(t) - Q(t) \tag{2.67}$$

This cost function would not only attempt to minimize the deviations from the optimal trajectory at the final time but also the integrated deviations from the optimal trajectory throughout the flight.

CHAPTER III

THE MISSILE EQUATIONS OF MOTION

Equations of Motion

The law of conservation of linear and angular momentum was used to derive the equations of motion. As shown in Figure 3.1, the forces acting on the body are: thrust, F_T; drag, A_D; control, Fc; lift, N; and gravity, mg. By summing the forces in the V direction and perpendicular to the V direction:

$$m\dot{v} = (F_T - A_D) \cos \alpha + (Fc - N) \sin \alpha - mg \sin \gamma \qquad (3.1)$$

$$mV\dot{\gamma} = (F_T - A_D) \sin \alpha - (Fc - N) \cos \alpha - mg \cos \gamma \qquad (3.2)$$

By summing the moments about the center of gravity, the following equation results:

$$I \ddot{\theta} = -N\bar{x} + F_c\ell \qquad (3.3)$$

rom the geometry of the figure the velocity relative to the wind is given as:

$$V_{RW} = [\dot{h}^2 + (V \cos \gamma + V_w)^2]^{1/2} \qquad (3.4)$$

where + V_w is a tailwind and - V_w is a headwind. The angle of attack is given as:

$$\alpha = \theta - \gamma \qquad (3.5)$$

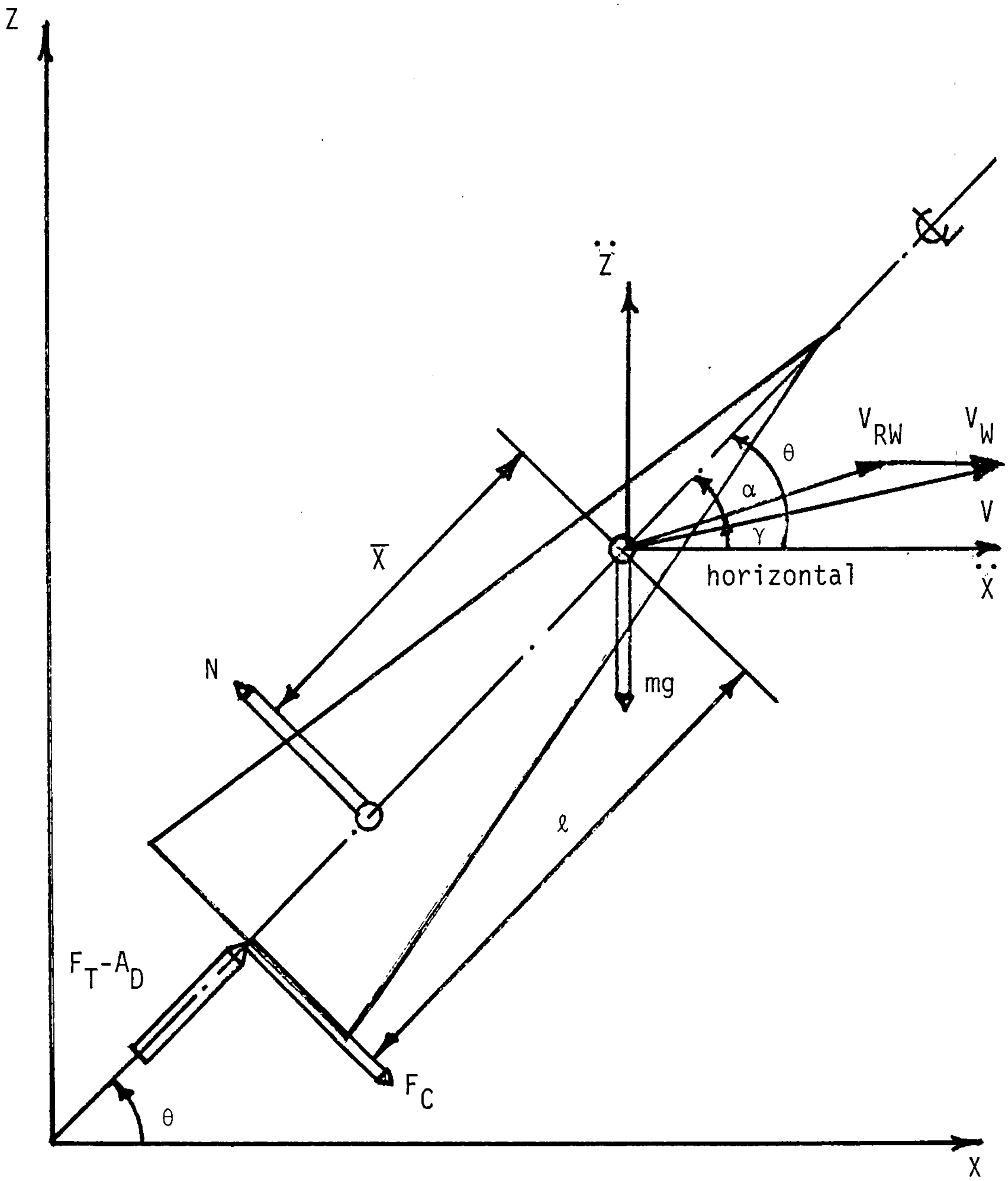

Figure 3.1. Force Diagram.

Thrust Equations

The equation of thrust is given as:

$$F_T = [F_{NSL} + Ae(P_{s\ell} - P)]\dot{W}/\dot{W}_N \tag{3.6}$$

where F_{NSL} is the nominal thrust at sea level (8700 lb_f), $\dot{W}$ is the actual weight change w.r.t. time and $\dot{W}_N$ is the normal weight change w.r.t. time (34.5 lb_f/sec). The propellant weight is given by the equations:

$$W_{pk} = W_p - \dot{W}_N t, \qquad t < W_p/\dot{W}_N, \tag{3.7}$$

$$W_{pk} = 0, \qquad t > W_p/\dot{W}_N \ . \tag{3.8}$$

Weight and Inertia

The weight of the missile is given by the equations (assuming $\dot{W}_N$ is constant):

$$W = W_T - \dot{W}_N t$$

The mass of the missile is given as

$$m = W/g$$

where a constant acceleration of gravity of g = 32.2 ft/sec was used. The polar moment of inertia of the missile about its center of gravity was taken to be

$$I = 0.0554\ W + 32.9\ .$$

Lever Arms

As shown in Figure 3.1, the lever arms ℓ and $\bar{x}$ are used in the equations of motion. The length of the missile was ℓ_F = 7.715 ft. and the equation for ℓ is given as

$$\ell = \ell_F - \ell_{cm} . \tag{3.9}$$

The length to the center of mass is given by

$$\ell_{cm} = 0.00224\ W + 3.642 . \tag{3.10}$$

The length between the center of pressure and the center of mass is given by

$$\bar{X} = \ell_{cp} - \ell_{cm} \tag{3.11}$$

where ℓ_{cp} is a function of Mach number as given by the table of values:

Mach No.	ℓ_{cp}
0	5.7
0.5	5.725
0.8	6.01
1.2	7.02
1.5	6.70
2.0	5.93
3.0	4.58
4.0	4.49

Atmosphere

A standard IGY atmosphere was used for obtaining pressure p and density ρ. This is given in Table 3.1. The equation for the speed of sound is:

$$V_s = (1.4\ p/\rho)^{1/2} \tag{3.12}$$

and the Mach number is given by:

$$M = V_{RW}/V_s \tag{3.13}$$

The dynamic pressure is calculated by

$$q = 1/2\ \rho V_{RW}^2\ . \tag{3.14}$$

Aerodynamic Forces

The lift force is calculated as

$$N = 1/2\ SC_N\ \rho\ V_{RM}^2 \tag{3.15}$$

where S = 0.54542 ft, and C_N is given by the equation:

$$C_N = C_{N2}\ \alpha/57.35 \tag{3.16}$$

The coefficient C_{N2} as a function of Mach number is given by the table of values:

TABLE 3.1

STANDARD IGY ATMOSPHERIC DATA

Altitude, h ft.	Pressure, P Inches of Hg	Density, P lbm/ft^3
0	29.921000	0.076474
20,000	13.760000	0.040773
40,000	5.538000	0.018826
60,000	2.117780	0.007199
80,000	0.815462	0.0027171
100,000	0.321922	0.0010438
120,000	0.131789	0.00040196
140,000	0.0572731	0.00016315
160,000	0.0261996	$7.1292\text{x}10^{-5}$
180,000	0.0120901	$3.3610\text{x}10^{-5}$
200,000	$5.40654\text{x}10^{-3}$	$1.5755\text{x}10^{-5}$
400,000	$6.34070\text{x}10^{-7}$	$1.1640\text{x}10^{-9}$

Mach No.	C_{N2}
0	4.2
0.5	4.233
0.8	4.51
1.2	5.70
1.5	5.14
2.0	4.67
3.0	4.53
4.0	4.504

The drag force is given by the equation

$$A_D = 1/2\ S \rho V_{RW}^2\ C_A \tag{3.17}$$

when Ft ≠ 0. The coefficient C_A is a function of Mach number is given as the following table of values:

Mach No.	C_A
0	0.215
0.25	0.215
0.50	0.197
1.00	0.291
1.50	0.258
2.00	0.234
2.50	0.205
3.00	0.181
4.00	0.160

when $F_T = 0$ the drag force is given by the formula

$$A_D = 1/2\ S\rho\ V_{RW}^2\ C_{A2} \tag{3.18}$$

where C_{A2} as a function of Mach number is given as the following table of values:

Mach No.	C_{A2}
0	0.358
0.25	0.358
0.50	0.334
1.00	0.479
1.50	0.414
2.00	0.363
2.50	0.311
3.00	0.267
4.00	0.221

Typical Trajectories

Complete range-time trajectories are now shown in Figures 3.2-3.5. The computer program written in CSMP (Continuous System Modeling Program) is also included.

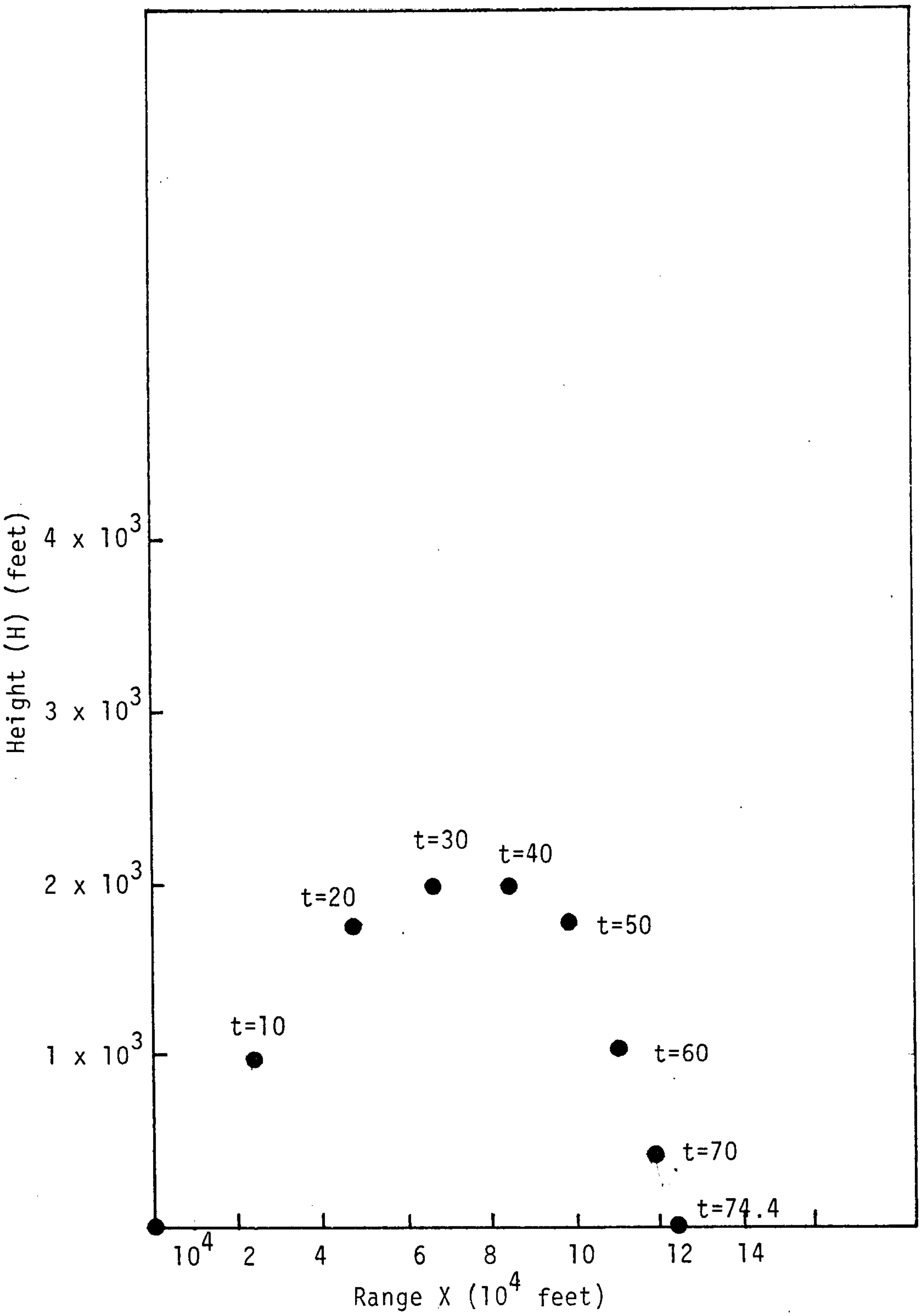

Figure 3.2. Complete-Time Trajectory for Nominal Missile Fc=0 and θo=30°.

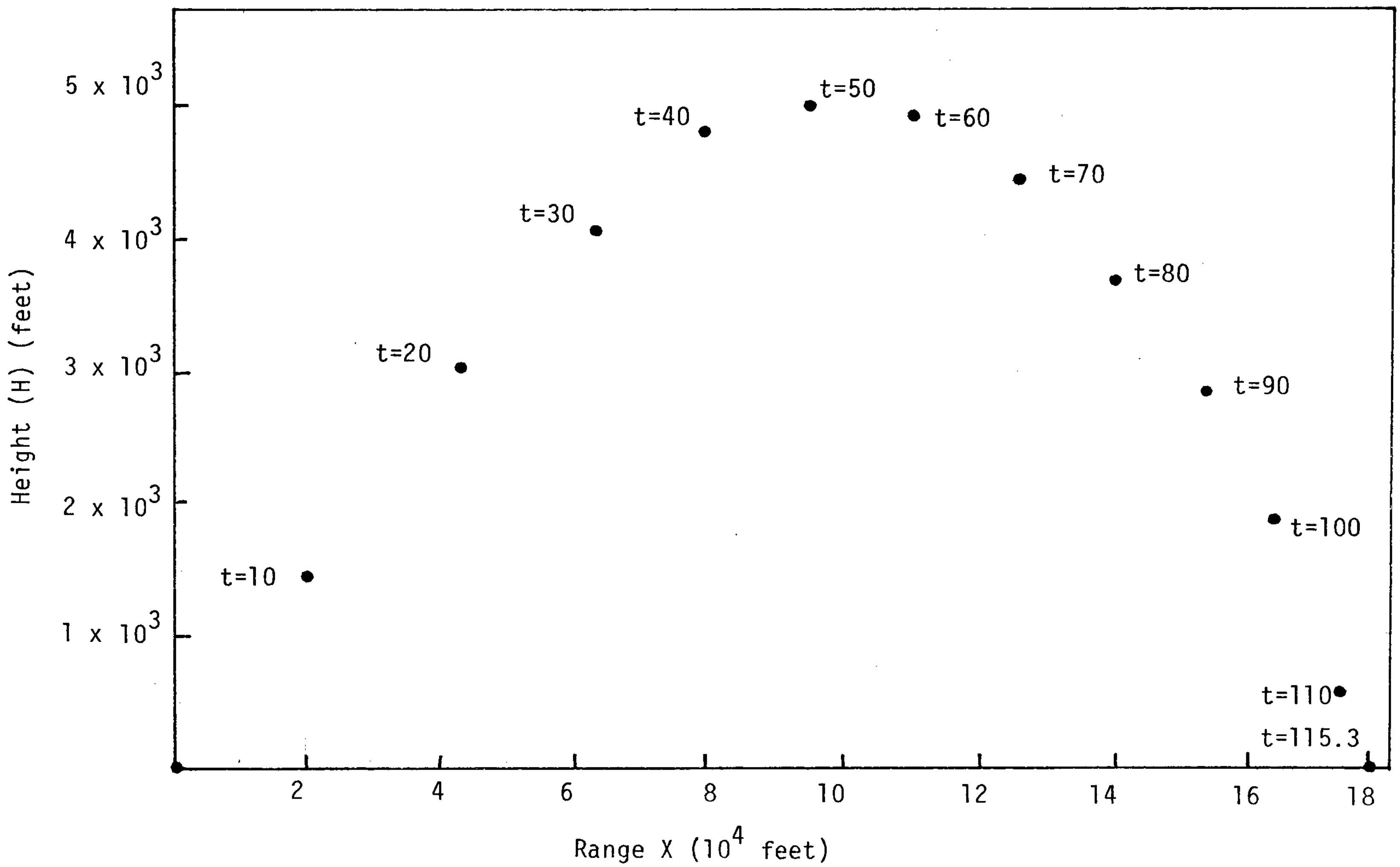

Figure 3.3. Complete-Time Trajectory for Nominal Missile Fc=0 and θo=45°.

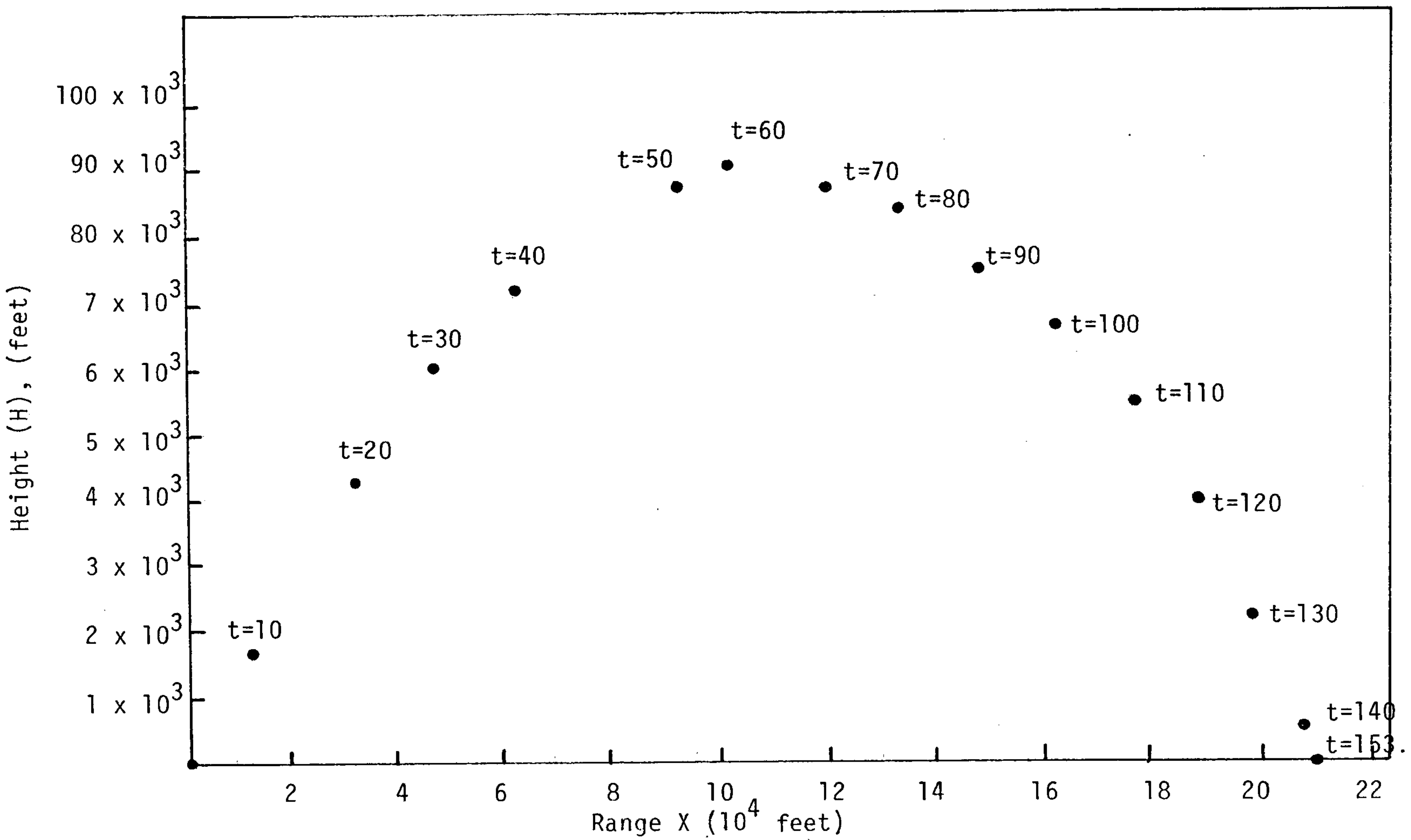

Figure 3.4. Complete-Time Trajectory of Nominal Missile with Fc=0 and θo=60°.

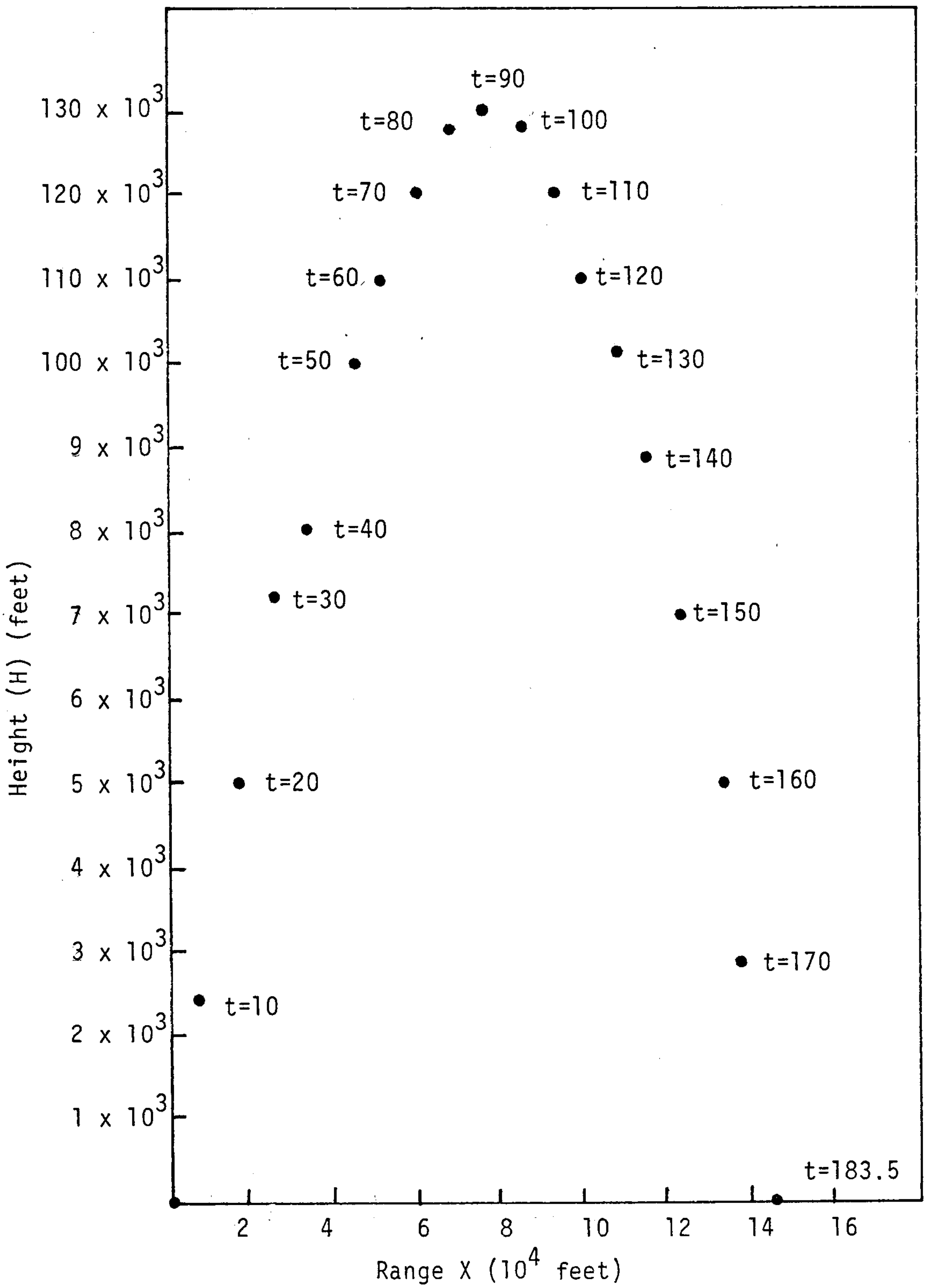

Figure 3.5. Complete-Time Trajectory for Nominal Missile Fc=0 and θo=75°.

```
//MISSILE JOB ()48961,******,R-192,G-X),TSUEI
***CSMP  PROGRAM  FOR  CALCULATING  ANGLE  OG  ATTACK-
ALPHA=THETA-GAMMA
//SYS IN DD *


    INITIAL
    CONSTANT FNSL= 8700.,AE=.2799,PSL=2116.8,S=.54542,WT=434.1,...
         WDNOM=34.5,LF=7.715,P1=.02,P2=.02,G=32.2
    FUNCTION PVSZ=0.,29.921,20000.,13.76,40000.,5.538,60000.,...
         2.1178,80000.,.815462,100000.,.321922,120000.,.131789,...
         140000.,.0572731,160000.,.0261996,180000.,.0120901,...
         200000.,.00540654,400000.,.63407E-6
    FUNCTION RHOZ=0.,.076474,20000.,.040773,40000.,.018826,60000.,...
         .007199,80000.,.0027171,100000.,.0010438,120000.,...
         .00040196,140000.,.00016315,160000.,.00071292,180000.,...
         .0003361,20000.,.000015755,400000.,.1164E-8
    FUNCTION CN2MAC=0.,4.2,.5,4.233,.8,4.51,1.2,5.7,1.5,5.14,2.,...
         4.67,3.,4.53,4.,4.504
    FUNCTION CAMACH=0.,.215,.25,.215,.5,.197,1.,.297,1.5,.258,2.,...
         .234,2.5.,.205,3.,.181,4.,.160
    FUNCTION LCPMCH=0.,5.7,.5,5.725,.8,6.01,1.2,7.02,1.5,6.7,2.,...
         5.93,3.,4.58,4.,4.49
    INCON VO=1.,TDF=0.,THETAD=90.,H0=0.,X0=0
    GAMMAD=THETAD
    GAMMAO=0.5237
    THETAO=0.5237
    PARAMETER DELTA=0.,KP=3500.
    PARAM XBF=1.,LFF=1.,CNF=1.,CAF=1.,FTF=1.,VW=0
    PARAM FTF = 1.0
    DYNAMIC
    W=WT-WDNOM*TIME+(WDNOM*TIME-176.64)*BOT
    LCM=.00224*W+3.64
    I=.0554*W+32.9
    L=LF-LCM
    SUMFX=(FT-A)*COS(THETA)+(FC-N)*SIN(THETA)
SUMFH= (FT-A)*SIN(THETA)-(FC-N)*COS(THETA)-M*G
    XC=INTGRL(0.0,SUMFH/M)
    XDOT=INTGRL(XO,XDOT)
    HDOT=INGRL(0.0,SUMFH/M)
    HC=INTGRL(HO,HDOT)
    FUEL=INTGRL(0.0,FC)
    XBAR=(LCP-LCM)*XBF
    CN=AFGEN(CN2MAC,MA)*CNF
    CA=AFGEN(CAMAOH,MA)*CAF
    FT=FT1*COS(DLETA)*FTF
    FT1=(FNSL+AE*(PSL-P))*(L-BOT)
    BOT=STEP(5.12)
    SUMFV=(FT-A)*COS(ALPHA)+(FC-N)*SIN(ALPHA)-M*G*SIN(GAMMA)
    V=INTGRL(VO,SUMFV/M)
```

```
SUMFG=(FT-A)*SIN(ALPHA)-(FC-N,(*COS(ALPHA)-M*G*COS(GAMMA)
GAMDOT=SUMFG/(M*V)
GAMMA=INTGRL(GAMMO,SUMFG/M/V)
SUMMO=-N*XBAR*FC*L
THEDOT=INTGRL(THDOTO, SUMMO/I)
THETA=INTGRL(THETAO,THEDOT)
H=INTGRL(HO,V*SIN(GAMMA))
X=INTGRL(XO,V*COS(GAMMA))
VS=(1.4*P/RHO)**.5
ALPHA=THETA-GAMMA
M=W/G
LCP=AFGEN(LCPMCH,MA)
RHO=AFGEN(RHOZ,H)/G
P=13.6*62.4/12.*AFGEN(PVSZ,H)
VRW=((V*COS(GAMMA)+VW)**2+(V*SIN(GAMMA))**2)**.5
A=S/2.*RHO*CA*VRW**2
N=S/2.*RHO*CN*ALPHA*VRW**2
MA=VRW/VS
TERMINAL
TIMER FINTIM=200,PRDEL=1.0,OUTDEL=1.0,DELMIN=1.0E-08
PRINT ALPHA,THETA,GAMMA,V,X,H,FT,N,M,XBAR,MA,VS,I,L,CA,CN,LCP,...
LCM,P,RHO,THEDOT,GAMDOT,FUEL,BOT,W
END
END
STOP
```

Program for Evaluating A and B Matrices

Calculating the A matrix. The state model equations are given as:

$$\dot{V} = \frac{\sum F_{\parallel V}}{m} = f_V = [(F_T - A_D)\cos\alpha + (F_C - N)\sin\alpha - mg\sin\gamma]/m$$

$$\dot{\gamma} = \frac{\sum F_{\perp V}}{mV} = f_\gamma = [(F_T - A_D)\sin\alpha - (F_C - N)\cos\alpha - mg\cos\gamma]/mV \qquad (3.19)$$

$$\ddot{\theta} = \frac{\sum M_{C.G.}}{I} = f_{\dot{\theta}} = (F_C \ell\ N\bar{x})/I$$

$$\dot{\theta} = \qquad f_\theta = \dot{\theta}$$

$$\dot{h} \qquad f_h = V\sin\gamma$$

and the A matrix in the linearized equations is obtained by taking the partial derivative of the above vector function with respect to each element of the state vector, and evaluating these partials along the nominal optimum trajectory. As shown in Chapter II, the nominal optimum trajectory is that trajectory which results for $u^* = 0$ and in the case where there are no lags in the controller $F_c = 0$, the above vector function reduces to:

$$\underline{f}^* = \begin{bmatrix} f_V \\ f_\gamma \\ f_{\dot\theta} \\ f_\theta \\ f_h \end{bmatrix}^* = \begin{bmatrix} [(F_T - A_D)\cos\alpha - N\sin\alpha - mg\sin\gamma]/m \\ [(F_T - A_D)\sin\alpha + N\cos\alpha - mg\cos\gamma]/mV \\ -N\bar{x}/I \\ \dot\theta \\ V\sin\gamma \end{bmatrix}^* \tag{3.20}$$

The lift and drag forces and $\bar{x}$ are all functions of velocity and attitude and in addition the lift force is also a function of angle of attack, which itself is a function of heading angle and flight path angle. The elements of the A matrix are:

$$A = \begin{bmatrix} \frac{\partial f_V^*}{\partial V} & \frac{\partial f_V^*}{\partial \gamma} & 0 & \frac{\partial f_V^*}{\partial \theta} & \frac{\partial f_V^*}{\partial h} \\ \frac{\partial f_\gamma^*}{\partial V} & \frac{\partial f_\gamma^*}{\partial \gamma} & 0 & \frac{\partial f_\gamma^*}{\partial \theta} & \frac{\partial f_\gamma^*}{\partial h} \\ \frac{\partial f_{\dot\theta}^*}{\partial V} & \frac{\partial f_{\dot\theta}^*}{\partial \gamma} & 0 & \frac{\partial f_{\dot\theta}^*}{\partial \theta} & \frac{\partial f_{\dot\theta}^*}{\partial h} \\ 0 & 0 & 1 & 0 & 0 \\ \frac{\partial f_h^*}{\partial V} & \frac{\partial f_h^*}{\partial \gamma} & 0 & 0 & 0 \end{bmatrix} \tag{3.21}$$

The nonzero elements of the first row are therefore given as:

$$\frac{\partial f_V^*}{\partial V} = [-\frac{\partial A_D}{\partial V} \cos \alpha - \frac{\partial N}{\partial V} \sin \alpha]/m$$

$$\frac{\partial f_V^*}{\partial \gamma} = [(F_T - A_D) \sin \alpha + N \cos \alpha - mg \cos \gamma]/m = \frac{\partial f_V^*}{\partial \alpha} \frac{\partial \alpha}{\partial \gamma}$$

$$\frac{\partial f_V^*}{\partial \theta} = [-(F_T - A_D) \sin \alpha - N \cos \alpha]/m = \frac{\partial f_V^*}{\partial \theta} \frac{\partial \theta}{\partial \alpha} \tag{3.22}$$

$$\frac{\partial f_V^*}{\partial h} = [(\frac{\partial F_T}{\partial h} - \frac{\partial A_D}{\partial h}) \cos \alpha - \frac{\partial N}{\partial h} \sin \alpha]/m$$

The nonzero elements of the second row are given as

$$\frac{\partial f_\gamma^*}{\partial V} = - [(F_T - A_D) \sin \alpha + N \cos \alpha - mg \cos \gamma]/mV^2 + [\frac{\partial A_D}{\partial V} \cos \alpha + \frac{\partial N}{\partial V} \sin \alpha]/mV$$

$$\frac{\partial f_\gamma^*}{\partial \gamma} = \frac{\partial f_\gamma^*}{\partial \alpha} \frac{\partial \alpha}{\partial \gamma} \tag{3.23}$$

$$\frac{\partial f_\gamma^*}{\partial \theta} = \frac{\partial f_\gamma}{\partial \alpha} \frac{\partial \alpha}{\partial \theta}$$

$$\frac{\partial f_\gamma^*}{\partial h} = [(\frac{\partial F_T}{\partial h} - \frac{\partial A_D}{\partial h}) \sin \alpha + \frac{\partial N}{\partial h} \cos \alpha]/mV$$

The nonzero elements of the third row are given as

$$\frac{\partial f^*_{\dot\theta}}{\partial V} = [- \frac{\partial N}{\partial V}\bar{x} - N\frac{\partial \bar{x}}{\partial V}]/I$$

$$\frac{\partial f^*_{\dot\theta}}{\partial \gamma} = N'\bar{x}/I \tag{3.24}$$

$$\frac{\partial f^*_{\dot\theta}}{\partial \theta} = - N'\bar{x}/I$$

$$\frac{\partial f^*_{\dot\theta}}{\partial h} = - [\frac{\partial N}{\partial h}\bar{x} + N\frac{\partial \bar{x}}{\partial h}]/I$$

The nonzero element of the fourth row is given as

$$\frac{\partial f^*_{\theta}}{\partial \dot\theta} = 1 \tag{3.25}$$

The nonzero elements of the fifth row are given as

$$\frac{\partial f^*_{h}}{\partial V} = \sin\gamma \tag{3.26}$$

$$\frac{\partial f^*_{h}}{\partial \gamma} = V\cos\gamma \tag{3.27}$$

If there is a controller with a second order lag such that its transfer function is given as

$$F_C = \frac{\omega_n^{\;2}u}{S^2 + 2\xi\omega_n S + \omega_n^{\;2}} \tag{3.28}$$

then the state model for this controller is:

$$\frac{d}{dt}\begin{bmatrix} F_C \\ \dot{F}_C \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}\begin{bmatrix} F_C \\ \dot{F}_C \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} u \tag{3.29}$$

and there must be added to the state vector the additional elements of F_c and dF_c/dt. The state model now becomes:

$$\frac{d}{dt}\begin{bmatrix} V \\ \\ \gamma \\ \\ \dot{\theta} \\ \theta \\ h \\ F_C \\ \dot{F}_C \end{bmatrix} = \begin{bmatrix} [(F_T - A_D)\cos\alpha + (F_C - N)\sin\alpha \\ \qquad - mg\sin\gamma]/m \\ [(F_T - A_D)\sin\alpha - (F_C - N)\cos\alpha \\ \qquad - mg\cos\gamma]/mV \\ (F_C\ell - N\bar{x})/I \\ \dot{\theta} \\ V\sin\gamma \\ \dot{F}_C \\ -\omega_n^2 F_C - 2\xi\omega_n\dot{F}_C + \omega_n^2 u \end{bmatrix} \tag{3.30}$$

and the nonzero element of the sixth row is 1, and the nonzero elements of the seventh row are:

$$\frac{\partial f_{F_C}}{\partial F_C} = -\omega_n^2 \tag{3.31}$$

$$\frac{\partial f_{\dot{F}_C}}{\partial \dot{F}_C} = -2\xi\omega_n \tag{3.32}$$

The angle of attack appears in the vector function $\underline{f}$ and is given as the difference between the heading angle and the flight path angle:

$$\alpha^* = \theta^* - \gamma^* \tag{3.33}$$

Additional partial derivatives which are needed to evaluate the A matrix are

$$\frac{\partial \alpha^*}{\partial \gamma} = -1 \tag{3.34}$$

$$\frac{\partial \alpha^*}{\partial \theta} = 1 \tag{3.35}$$

The drag force is given as:

$$A_D^* = \frac{S}{2} \rho C_A V^2 \tag{3.36}$$

and

$$\frac{\partial A_D^*}{\partial V} = \frac{S}{2} \rho (C_A 2V + \frac{\partial C_A}{\partial V} V^2) \tag{3.37}$$

The lift or normal force is given as

$$N^* = N'\alpha = \frac{S}{2} \rho C_N V^2 \alpha \tag{3.38}$$

and

$$\frac{\partial N^*}{\partial V} = \frac{S}{2} \rho\alpha (2VC_N + \frac{\partial C_N}{\partial V} V^2) \tag{3.39}$$

the thrust is given by the equation:

$$F_T^* = F_{NSL} + A_e(P_{SL} - p) \tag{3.40}$$

and

$$\frac{\partial F_T^*}{\partial h} = -A_e \frac{\partial p}{\partial h} \tag{3.41}$$

The lift and drag forces are both functions of altitude since they are functions of the lift and drag coefficients and these coefficients are functions of Mach number. The Mach number is a function of the velocity and the speed of sound. The speed of sound is given by the equation:

$$V_s = (1.4\ p/\rho)^{1/2} \tag{3.42}$$

which shows that it is a function of density and pressure. Both density and pressure and functions of the altitude are therefore:

$$\frac{\partial V_s}{\partial h} = \frac{V_s}{2} \left(- \frac{1}{\rho} \frac{\partial \rho}{\partial h} + \frac{1}{p} \frac{\partial p}{\partial h}\right) \tag{3.43}$$

Since the Mach number is given by the equation

$$M = V/V_s \ , \tag{3.44}$$

it is a function of both velocity and altitude, or:

$$\frac{\partial M}{\partial V} = \frac{1}{V_s} \tag{3.45}$$

and

$$\frac{\partial M}{\partial h} = - \frac{V}{V_s^{\ 2}} . \tag{3.46}$$

The lift and drag coefficients are both functions of Mach number as shown in the preceding sections and therefore the partials of these coefficients must be evaluated numerically. With respect to velocity, the partials of the lift and drag coefficients are:

$$\frac{\partial C_A}{\partial V} = \frac{\partial C_A}{\partial M} \frac{\partial M}{\partial V} \tag{3.47}$$

and

$$\frac{\partial C_N}{\partial V} = \frac{\partial C_N}{\partial M} \frac{\partial M}{\partial V} \tag{3.48}$$

The partials of these coefficients with respect to the Mach number are approximated by

$$\frac{\partial C_A}{\partial M} \simeq \frac{C_A(M + \Delta M_V) - C_A(M)}{\Delta M_V} \tag{3.49}$$

and

$$\frac{\partial C_N}{\partial M} \simeq \frac{C_N(M + \Delta M_V) - C_N(M)}{\Delta M_V} \tag{3.50}$$

Since the approximation to the partial differential is given by:

$$\Delta M_V = \frac{\partial M}{\partial V} \Delta V \tag{3.51}$$

the approximation to the partial derivatives in Equations (3.47) and (3.48) above are

$$\frac{\partial C_A}{\partial V} = \frac{C_A(M + \Delta M_V) - C_A(M)}{\Delta V \frac{\partial M}{\partial V}} \frac{\partial M}{\partial V} \tag{3.52}$$

and

$$\frac{\partial C_A}{\partial V} = \frac{C_N(M + \Delta M_V) - C_N(M)}{\Delta V \frac{\partial M}{\partial V}} \frac{\partial M}{\partial V} \tag{3.53}$$

If the partial of M with respect to V is cancelled in both numerator and denominator, the result is:

$$\frac{\partial C_A}{\partial V} = \frac{C_A(M + \Delta M_V) - C_A(M)}{\Delta V} \tag{3.54}$$

$$\frac{\partial C_N}{\partial V} = \frac{C_N(M + \Delta M_V) - C_N(M)}{\Delta V} \tag{3.55}$$

In a similar manner the partial of the lift and drag coefficients w.r.t., the altitude is given by

$$\frac{\partial C_A}{\partial h} = \frac{C_A(M + \Delta M_h) - C_A(M)}{\Delta h} \tag{3.56}$$

and

$$\frac{\partial C_N}{\partial h} = \frac{C_N(M + \Delta M_h) - C_N(M)}{\Delta h} \tag{3.57}$$

Since the moment arm $\overline{x}$ is also a function of Mach number, the following approximations are valid:

$$\frac{\partial \bar{x}}{\partial V} = \frac{\bar{x}(M + \Delta M_V) - \bar{x}(M)}{\Delta V} \tag{3.58}$$

$$\frac{\partial \bar{x}}{\partial h} = \frac{\bar{x}(M + \Delta M_h) - \bar{x}(M)}{\Delta h} \tag{3.59}$$

The time derivative of velocity is given as:

$$\frac{dV^*}{dt} = [(F_T - A_D)\cos\alpha - N\sin\alpha - mg\sin\gamma]/V \tag{3.60}$$

and the approximation to the differential dV is given as:

$$dV \stackrel{\sim}{=} \Delta V = [(F_T - A_D)\cos\alpha - N\sin\alpha - mg\sin\gamma]\Delta t/m \tag{3.61}$$

In a similar manner the approximation to the differential dh is:

$$dh \simeq \Delta h = [(F_T - A_D)\sin\alpha + N\cos\alpha - mg\cos\gamma]\Delta t/mV \tag{3.62}$$

<u>Calculating the B matrix</u>. The B matrix is given as:

$$B = (\partial \underline{f}/\partial \underline{u})^* \tag{3.63}$$

For the case where there are no lags in the controller, the control variable $\underline{u}$ is F_c and $\partial \underline{f}/\partial F_c$ is a nx1 matrix, i.e.:

$$(\partial \underline{f}/\partial F_C)^* = \begin{bmatrix} \frac{1}{m}\sin\alpha^* \\ -\frac{1}{mV}\cos\alpha^* \\ \ell/I \\ 0 \\ 0 \end{bmatrix} \tag{3.64}$$

For the case where there is a second order lag in the controller [see Equation (3.30)]:

$$F_C = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2} u \tag{3.65}$$

and

$$(\partial \underline{f}/\partial \underline{u})^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \omega_n^2 \end{bmatrix} \tag{3.66}$$

Logic diagram. In Figure 3.6 is shown the logic diagram for calculating the A and B matrices. In the first block the missile data, atmospheric data, and initial conditions are read into core storage. In the second block the system equations are integrated along a nominal trajectory from the initial time to $t = k\ \Delta t$, and $k = 1$. With the values of the state vector at this time, the elements of the A and B matrices are calculated and stored in an array. Since there is a limited amount of core storage available, it is necessary to set up files on a direct access device and store the elements of the A and B matrices—along with the elements of nominal state vector—at each time step. The actual program is included in Appendix A.

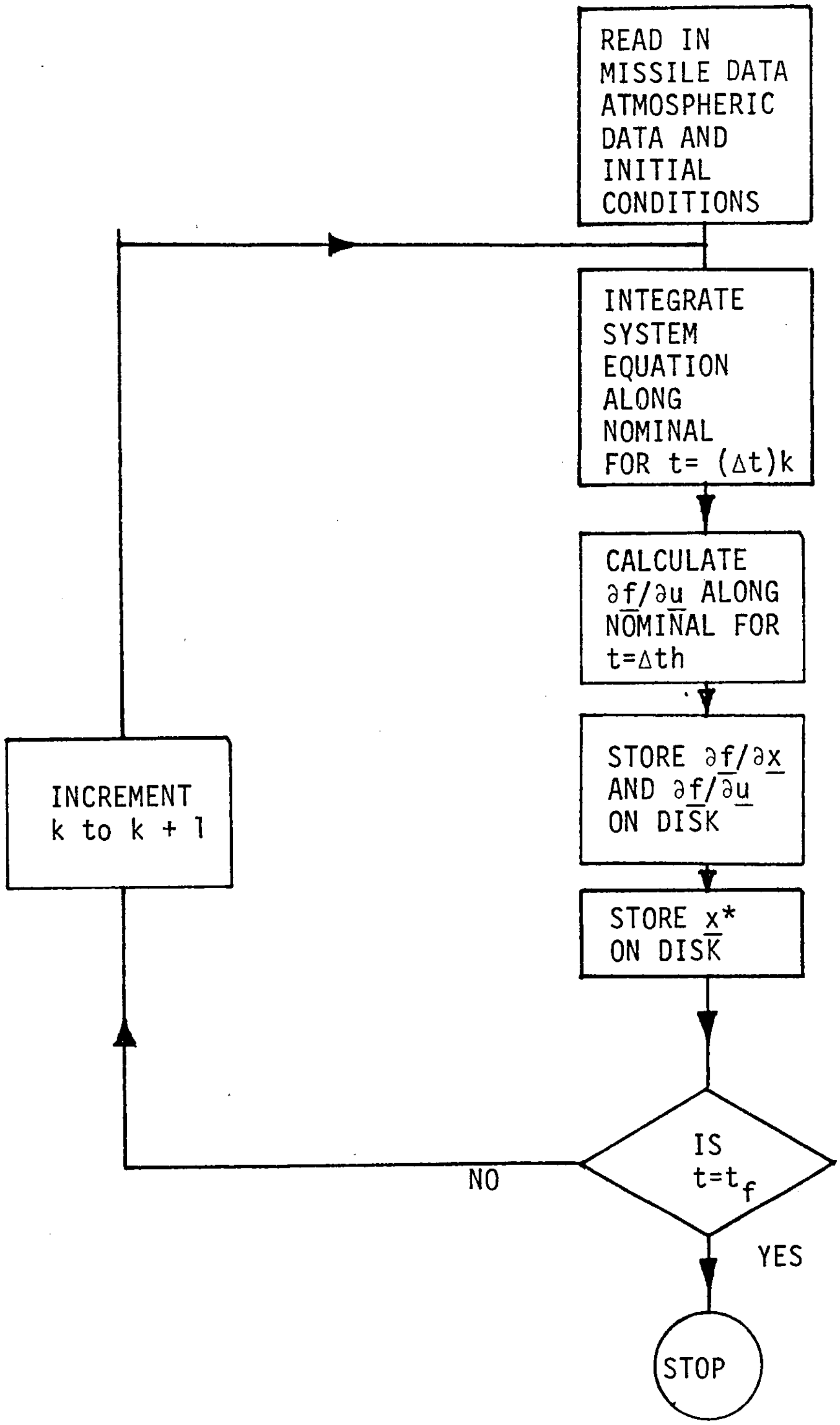

Figure 3.6. Logic Diagram for Calculating the A and B Matrices.

Calculating the Time Varying Feedback Gains

As shown in Chapter II, the variation from the nominal state vector $\delta\underline{x}$ and the variation from the nominal costate vector $\underline{p}$ are related by:

$$\delta\underline{x} = K(t)\delta\underline{p} \tag{3.67}$$

where the matrix K(t) satisfies the nonlinear matrix differential equation

$$\dot{K} = -KA - A^TK + KBR^{-1}B^TK \tag{3.68}$$

with the final condition

$$K(t_f) = F(t_f) \tag{3.69}$$

Taking the transpose of Equation (3.68) results in:

$$\dot{K}^T = -A^TK^T - K^TA + K^TB(R^{-1})^TB^TK^T \tag{3.70}$$

Since K^T satisfies the same matrix differential equation

$$K^T = K \ . \tag{3.71}$$

Since K is symmetric there are only (n)(n + 1)/2 elements of the K matrix that are distinct. This means that a system of n(n + 1)/2 nonlinear differential equations must be solved simultaneously backwards, starting from the final condition, to determine the elements $K_{ij}(t)$ of the matrix K(t).

Once the elements of the K matrix have been calculated, the feedback gains may be calculated as

$$G^T(t) = B^T(t)K(t) \tag{3.72}$$

For the case of optimal control with no lags the feedback gains are:

$$G^T(t) = (G_V, G_\gamma, G_{\dot\theta}, G_\theta, G_h)^T \tag{3.73}$$

and the K matrix is a 5 x 5 symmetric matrix with 15 distinct functions of time and the B vector has 5 functions of time as its elements. For the case of optimal control with a second order lag the gains would be

$$G^T(t) = (G_V, G_\gamma, G_{\dot\theta}, G_\theta, G_h, G_{F_c}, G_{\dot F_c})^T \tag{3.74}$$

and the K matrix would be a 7 x 7 symmetric matrix with 28 distinct elements. The B vector would have 7 elements. The logic diagram is shown in Figure 3.7. The program is shown in Appendix B.

Evaluating Miss Distances and Fuel Factors

In Figure 3.8 is shown the logic diagram for the program which integrates the system differential equations for a particular control law and evaluates the miss distance at the propellant burn time and the fuel factor at the propellant burnout time. A listing of the CSMP program which performs these calculations is included in Appendix C.

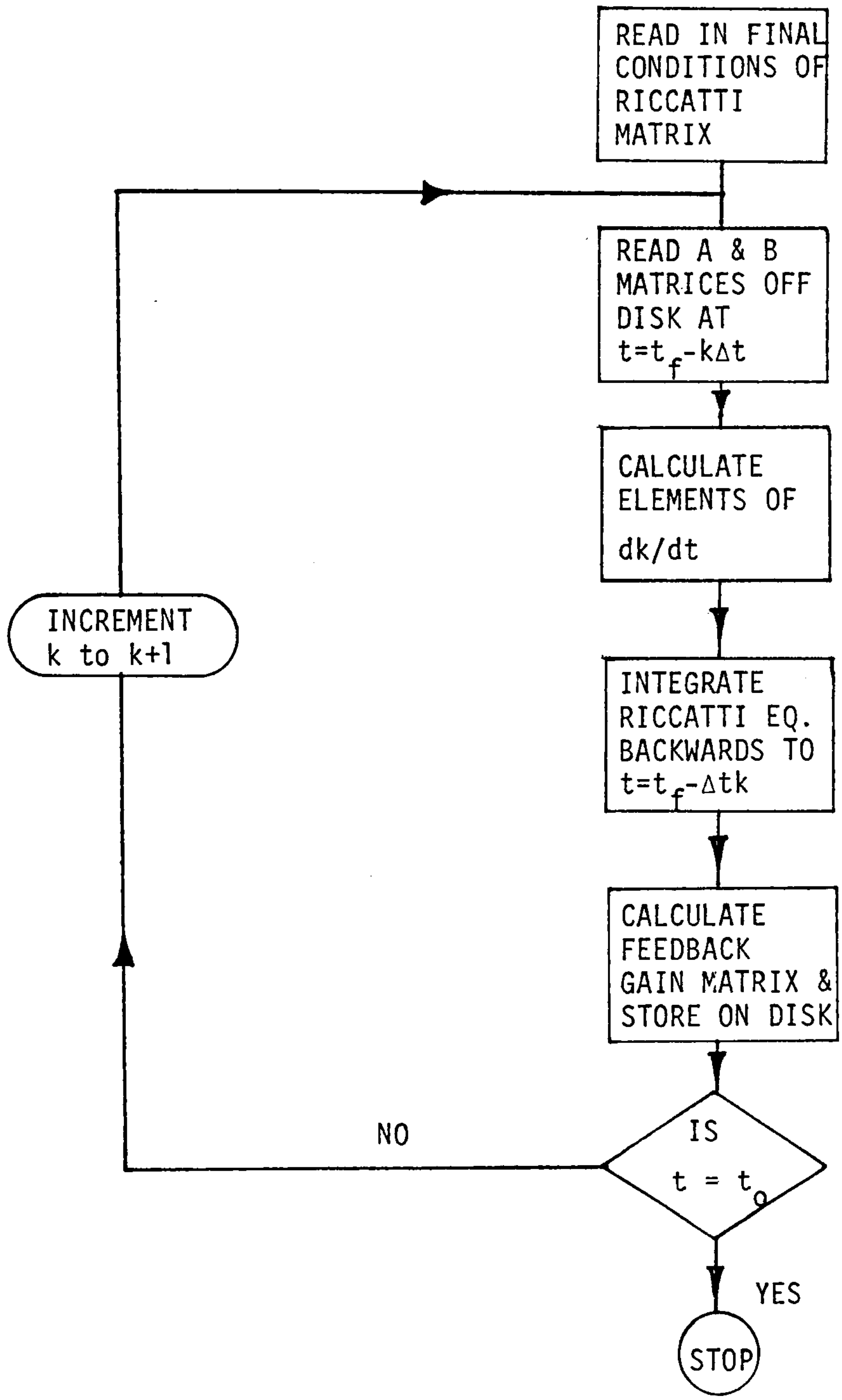

Figure 3.7. Logic Diagram for Integrating the Matrix Riccati Equation Backwards and Calculating the Time Varying Feedback Gains.

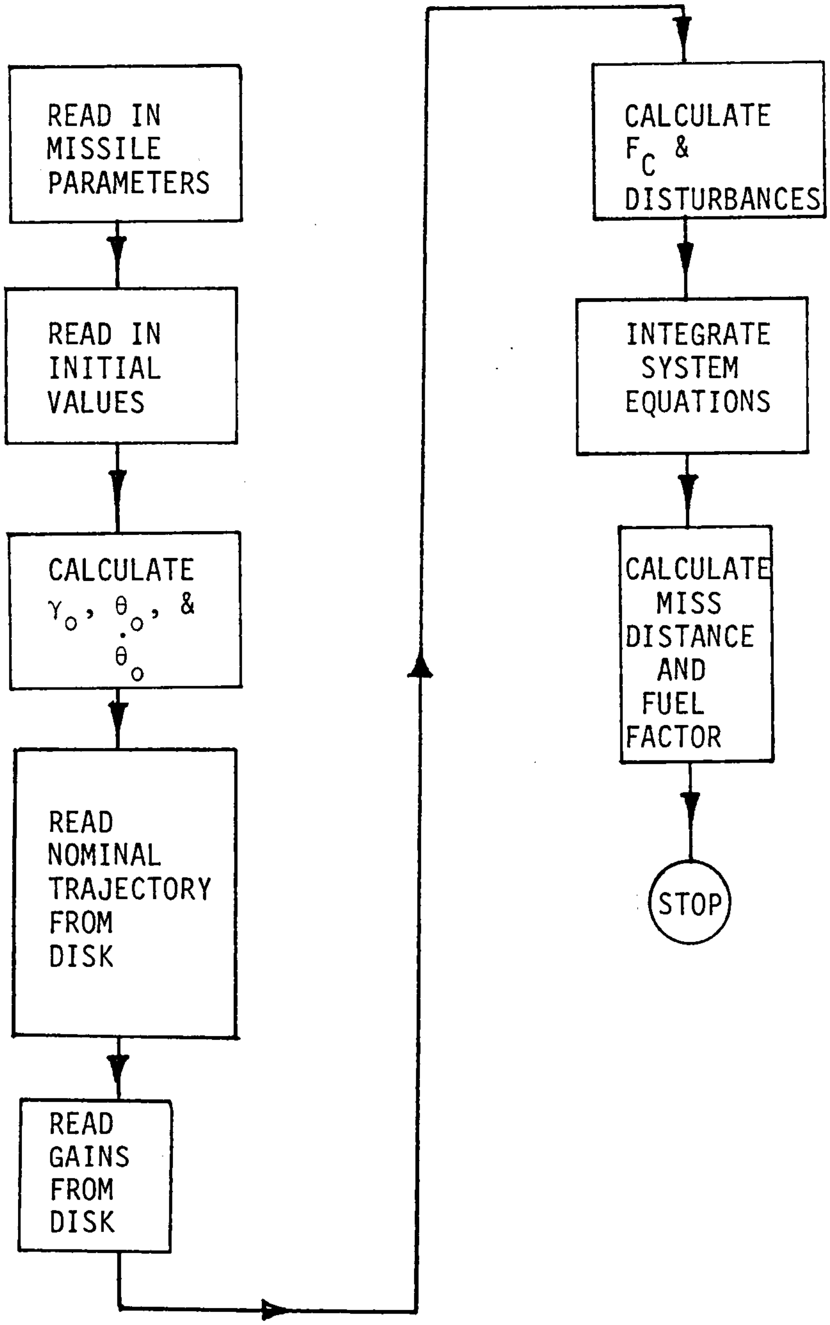

Figure 3.8. Logic Diagram for "Flying" the Missile in the Presence of Disturbances and Calculating the Miss Distance and Fuel Factor.

CHAPTER IV

DESIGNING THE SUBOPTIMAL CONTROLLER

Introduction

While no controller design is truly complete without extensive simulation of all nonlinearities, an excellent starting point is the linearized equations of the missile response. To obtain these three assumptions can be made starting with Equation (2.5).

1. The forward velocity V does not change significantly during the controller response time.
2. The angle attack α is small such that

   $\sin \alpha \simeq \alpha$

   $\cos \alpha \simeq 1$
3. Gravity may be neglected and the pertinent equations become

$$\dot{\gamma} = [(F_T - A) \sin \alpha - (Fc - N) \cos \alpha]/mV \tag{4.1}$$

$$\ddot{\theta} = (F_C \ell - N\bar{x})/I \tag{4.2}$$

$$\alpha = \theta - \gamma \tag{4.3}$$

The two aerodynamic variables N and A are given by

$$N = 1/2\, S\rho\, V^2\, C_N \alpha \tag{4.4}$$

$$A = 1/2\, S\rho\, V^2\, C_A \tag{4.5}$$

Using small angle approximations these equations become

$$\dot{\gamma} = [\frac{F_T - Sq\ C_A}{mV}]\ \alpha + \frac{Sq\ C_N}{mV}\ \alpha - \frac{F_c}{mV} \tag{4.6}$$

$$\ddot{\theta} = - \frac{Sq\ C_N\ \overline{x}}{I}\ \alpha + Fc\ \frac{\ell}{I} \tag{4.7}$$

$$\alpha = \theta - \gamma \tag{4.8}$$

Deriving the Transfer Functions

Since Equations (4.6)-(4.8) are linear these can be transformed to algebraic equations in the Laplace Transform Variable "s." In matrix form these equations become

$$\begin{bmatrix} s & 0 & -[\frac{F_T - Sq\ (C_A - C_N)}{mV}] \\ 0 & s^2 & \frac{SqC_N\overline{x}}{I} \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{\alpha}{Fc}\ (s) \\ \frac{\theta}{Fc}\ (s) \\ \frac{\alpha}{Fc}\ (s) \end{bmatrix} = \begin{bmatrix} -\frac{1}{mV} \\ \frac{\ell}{I} \\ 0 \end{bmatrix} \tag{4.9}$$

The value of the determinant Δ associated with this matrix is

$$\Delta = s\ [s^2 + \frac{F_T - Sq\ (C_A - C_N)}{mV} + \frac{SqC_N\overline{x}}{I}\ ] \tag{4.10}$$

The three transfer functions now become

$$1.\ \ \frac{\theta}{F_c}\ (s) = \frac{\frac{\ell}{I}\ [s + \frac{F_T - Sq\ (C_A - C_N)}{mV} + \frac{SqC_N\overline{x}}{\ell mV}\ ]}{s^2 + s[\frac{F_T - Sq\ (C_A - C_N)}{mV}] + \frac{SqC_N\ \overline{x}}{I}} \tag{4.11}$$

$$2. \quad \frac{\dot{Y}}{Fc}(s) = \frac{-\frac{1}{mV}\left[s^2 + \frac{Sq\,C_N\bar{x}}{I} - \frac{mV\ell}{I} Z_1\right]}{s^2 + D_1\, s + \frac{SqC_N\bar{x}}{I}} \tag{4.12}$$

where

$$Z_1 = \frac{1}{mV}\left[F_T - Sq\left(C_A - C_N + \frac{C_N\bar{x}}{\ell}\right)\right]$$

and

$$D_1 = \frac{F_T - Sq\,(C_A - C_N)}{mV}$$

$$3. \quad \frac{\alpha}{F_c}(s) = \frac{\frac{1}{mV}\left[s + \frac{mV\ell}{I}\right]}{s^2 + D_1\, s + \frac{SqC_N\bar{x}}{I}} \tag{4.13}$$

Choosing the Controller

Since the aerodynamic parameters N and A both are functions of angle of attack it was decided to implement the controller utilizing the third transfer function α/F_c. The block diagram is shown in Figure 4.1. As stated in Chapter I, an IMU is available and angle of attack is calculated from the output of the two orthogonal accelerometers and the gyro. Defining the accelerometer outputs as $\ddot{x}$ and $\ddot{y}$ the calculations would be

$$\dot{x} = \int \ddot{x}\, dt \tag{4.14}$$

$$\dot{y} = \int \ddot{y}\, dt \tag{4.15}$$

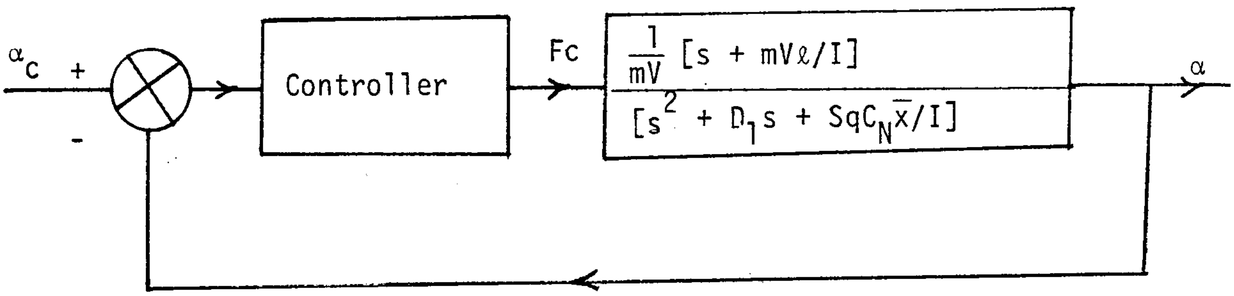

Figure 4.1. The Conventional Controller.

$$V = \sqrt{\dot{x}^2 + \dot{y}^2} \tag{4.16}$$

$$\gamma = \tan^{-1} \left(\frac{\dot{y}}{\dot{x}}\right) \tag{4.17}$$

$$\alpha = \theta - \gamma \tag{4.18}$$

The variation of the pair of open loop poles and the zero with a nominal ($F_c = 0$) trajectory is shown in Figures 4.2 and 4.3 and in Tables 4.1 and 4.2 (P_1 and P_2 are the poles of Eq. (4.13)). As an example, the transfer function at 2 seconds into a 30° trajectory is

$$\frac{\alpha}{F_c}(s) = 0.6507 \times 10^{-4} \left(\frac{s + 938.84}{s^2 + 0.975s + 1.088}\right) \tag{4.19}$$

The open loop poles are located at

$$s_1, s_2 = -0.4875 \pm j\,0.922 \tag{4.20}$$

The transfer function can be expressed in a form suitable for constructing a Bode plot as

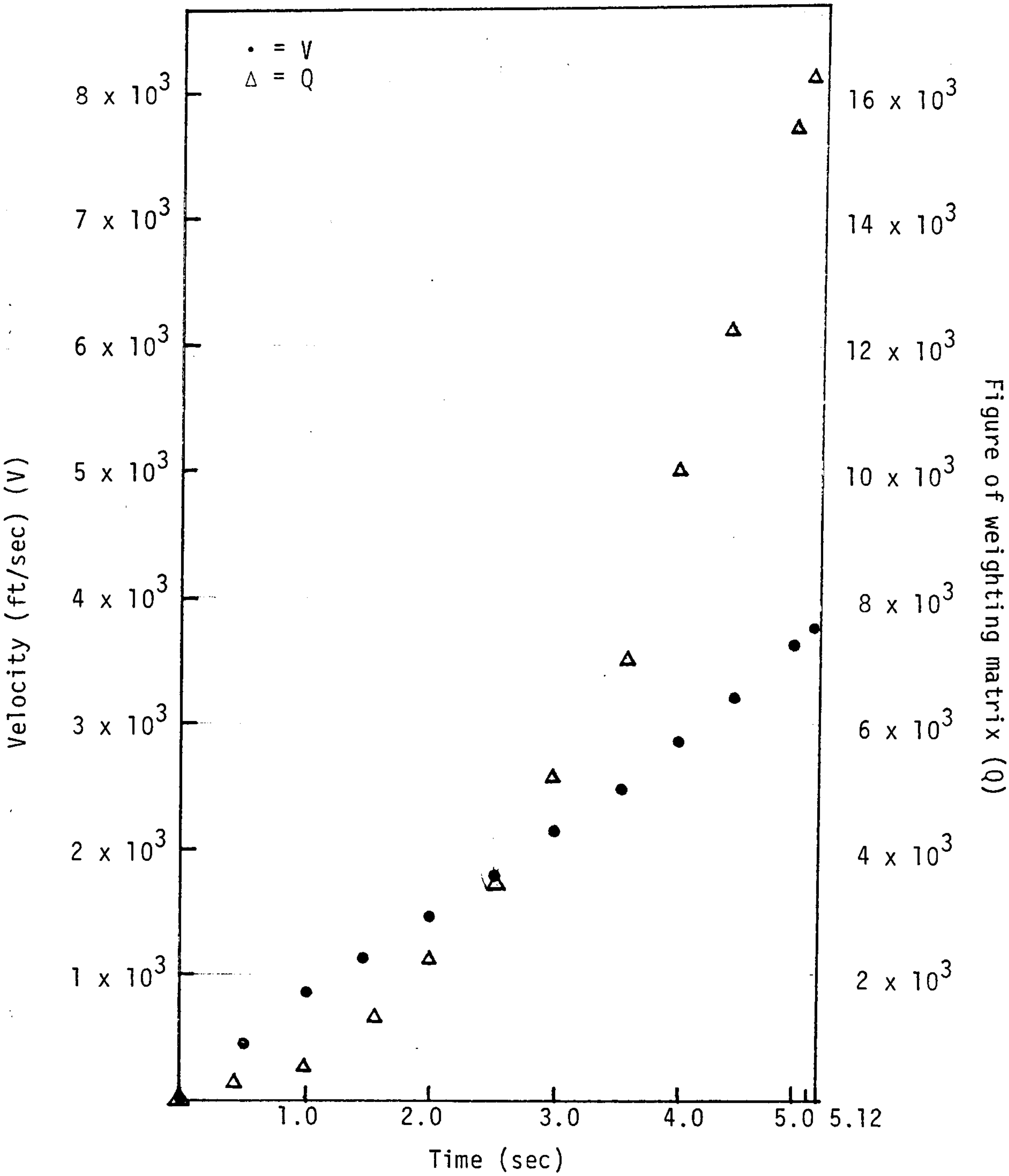

Figure 4.2. Velocity and Weighting Versus Time for the Missile at $\theta o=30°$.

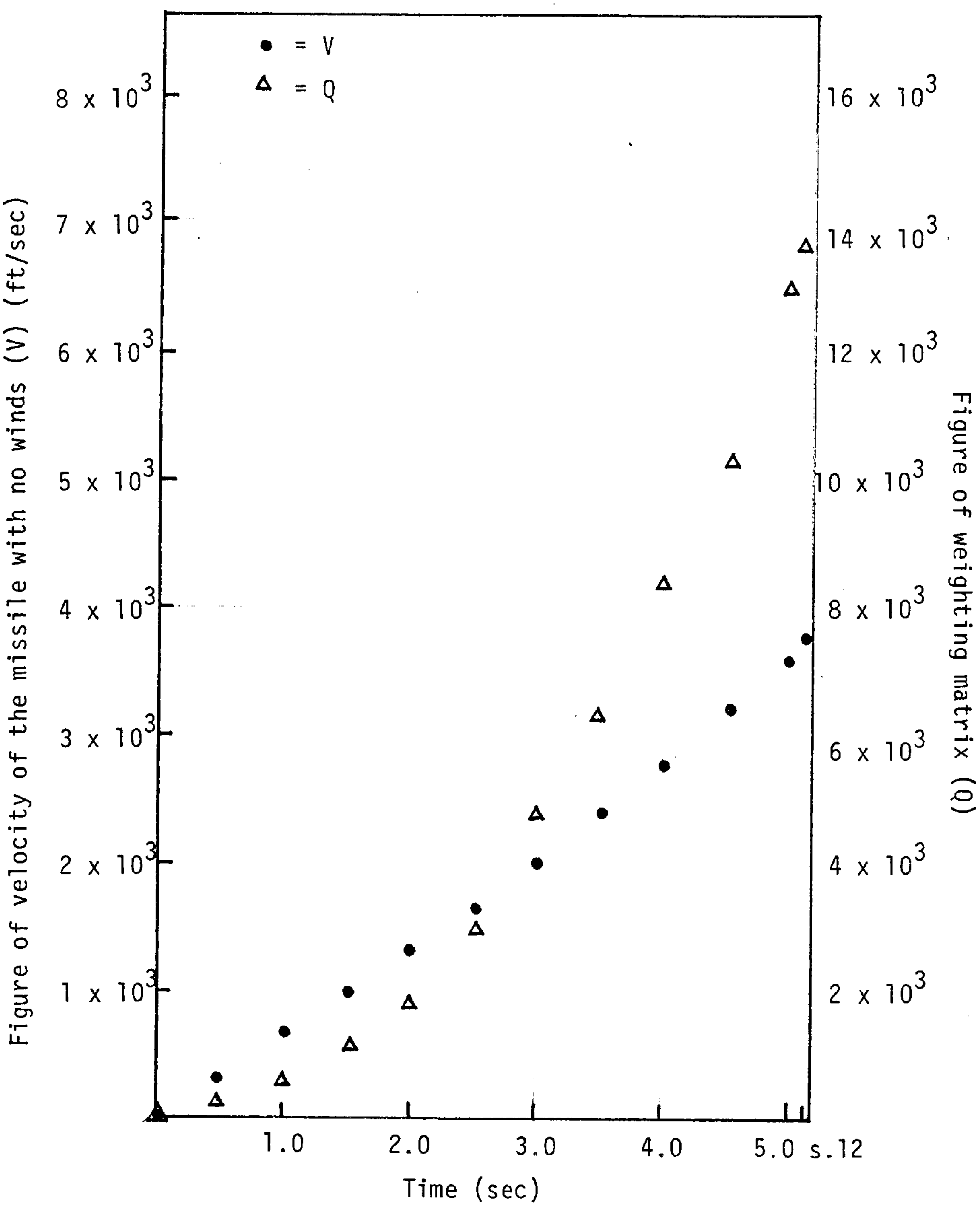

Figure 4.3. Velocity and Weighting Versus Time for the Missile at $\theta o=75°$.

TABLE 4.1

PARAMETERS (ℓ/I, Z1, D1, $SqC_N\bar{x}/mV$, P_1, P_2, 1/mV, AND $mV\ell/I$) VERSUS TIME FOR MISSILE AT $\theta o=30°$

t	(ℓ/I)	Z1	D1	$\frac{SqC_N\bar{x}}{mV}$	P_1	P_2	1/mV	$mV\ell/I$
1	0.0578	1.1567	1.1567	0.1900	-0.2682	-0.9310	1.2380E-4	467.0459
2	0.0630	0.6428	0.9750	1.0880	-0.4875+j0.9221	-0.4875-j0.9221	0.6507E-4	938.8386
3	0.0650	0.6679	0.9891	1.0600	-0.4946+j0.9015	-0.4946+j0.9015	0.4690E-4	1384.867
4	0.0693	0.9760	1.1950	0.7100	-0.5975+j0.5941	-0.5975+j0.5941	0.3720E-4	1860.455
5	0.0735	1.3700	1.4800	0.3900	-1.0215	-1.0215	0.3255E-4	2263.9287

TABLE 4.2

PARAMETERS (K_i, Z1, D1, $SqC_N\bar{x}/mV$, P_1, P_2, 1/mV, AND $mV\ell/I$) VERSUS TIME FOR MISSILE AT $\theta o=75°$

t	$K_i(\ell/I)$	Z1	D1	$\frac{SqC_N\bar{x}}{mV}$	P_1	P_2	1/mV	$mV\ell/I$
1	0.0577	1.1900	1.2100	0.1800	-0.1737	-1.0363	1.2285E-4	470.6576
2	0.0609	0.6552	0.9722	1.0000	-0.4861+j0.2341	-0.4861-j0.2341	0.5578E-4	1094.5822
3	0.0652	0.6585	0.9678	1.0271	-0.4839+j0.8905	-0.4839-j0.8905	0.4778E-4	1362.7500
4	0.0692	0.9100	1.1100	0.6722	-0.555+j0.6035	-0.555-j0.6035	0.3829E-4	1807.4935
5	0.0735	1.2253	1.3300	0.3453	-0.3537	-0.7535	0.3307E-4	2228.3302

$$\frac{\alpha}{Fc}(s) = \frac{\ell}{SqC_N\,x}\left[\frac{\frac{s}{mv\ell/I} + 1}{\left(\frac{s}{\sqrt{sqC_N\bar{x}/I}}\right)^2 + \frac{D_1}{SqC_N\bar{x}/I}\,s + 1}\right] \tag{4.21}$$

The important results obtained from Equations (4.19) and (4.21) can be summarized as:

1. For the two trajectories examined, the open loop poles have negative real parts.
2. The zero is two orders of magnitudes that of the poles.
3. The d-c or low frequency gain is inversely proportional to dynamic pressure q.

From these facts it was decided to implement the controller as a gain directly proportional to dynamic pressure. The control action then becomes

$$Fc = -1000q\,(\alpha_c - \alpha) \tag{4.22}$$

where the commanded angle of attack α_c was set to zero.

Typical Trajectories

Typical trajectories are shown in Figures 4.4, 4.5, 4.6, 4.7 and Tables 4.1 and 4.2. It should be noted that given the same heading angle, i.e., θo the nominal missile i.e. $Fc = 0$ will not have the same range. In the comparison with the optimal controller, however, percent errors will be calculated. Figures 4.8 through 4.14 show the trajectories at $Fc = -10^3 * \theta * \alpha$.

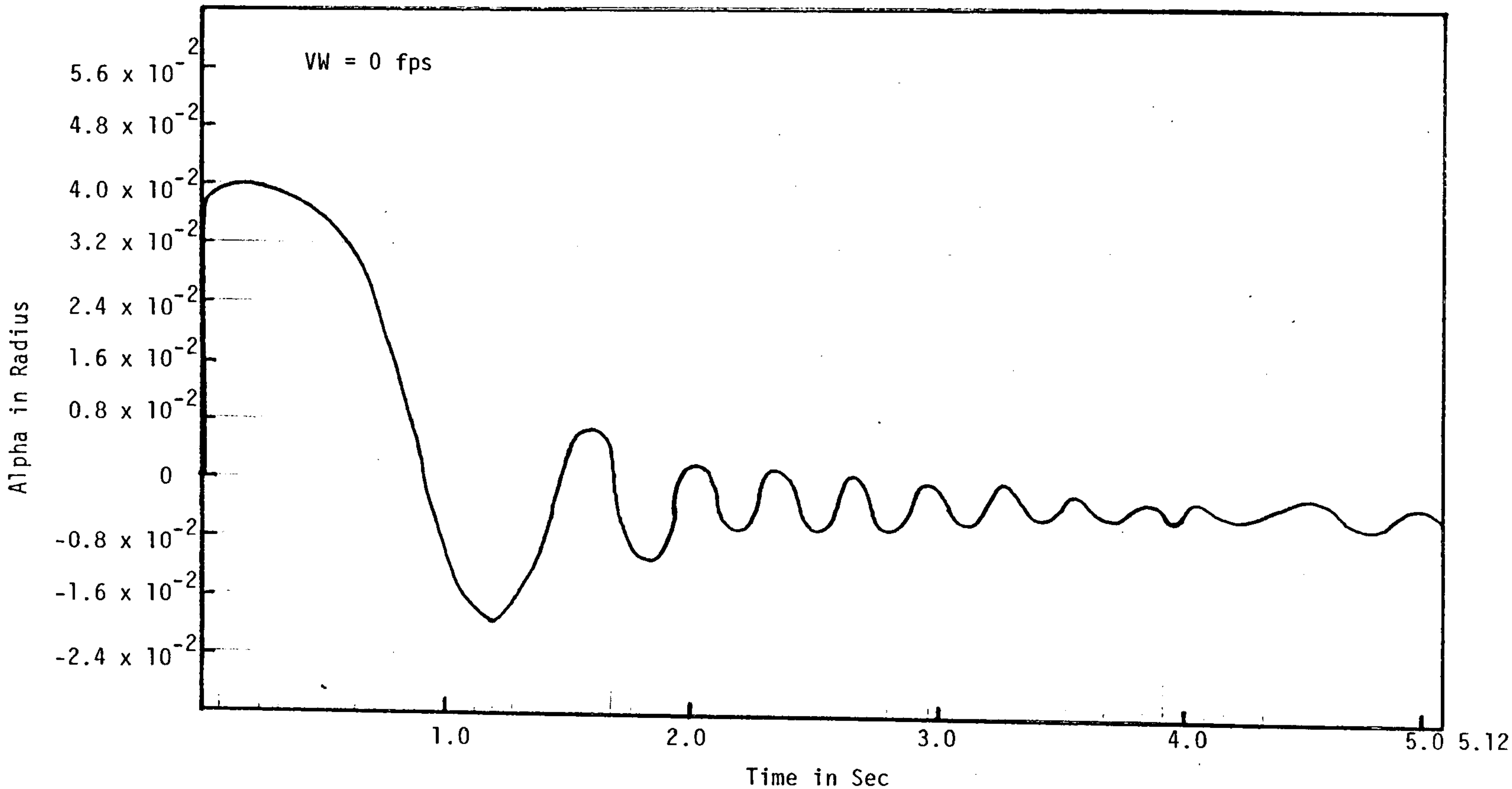

Figure 4.4. Alpha (α) Versus Time for Nominal (Fc=0) Missile θo=30°.

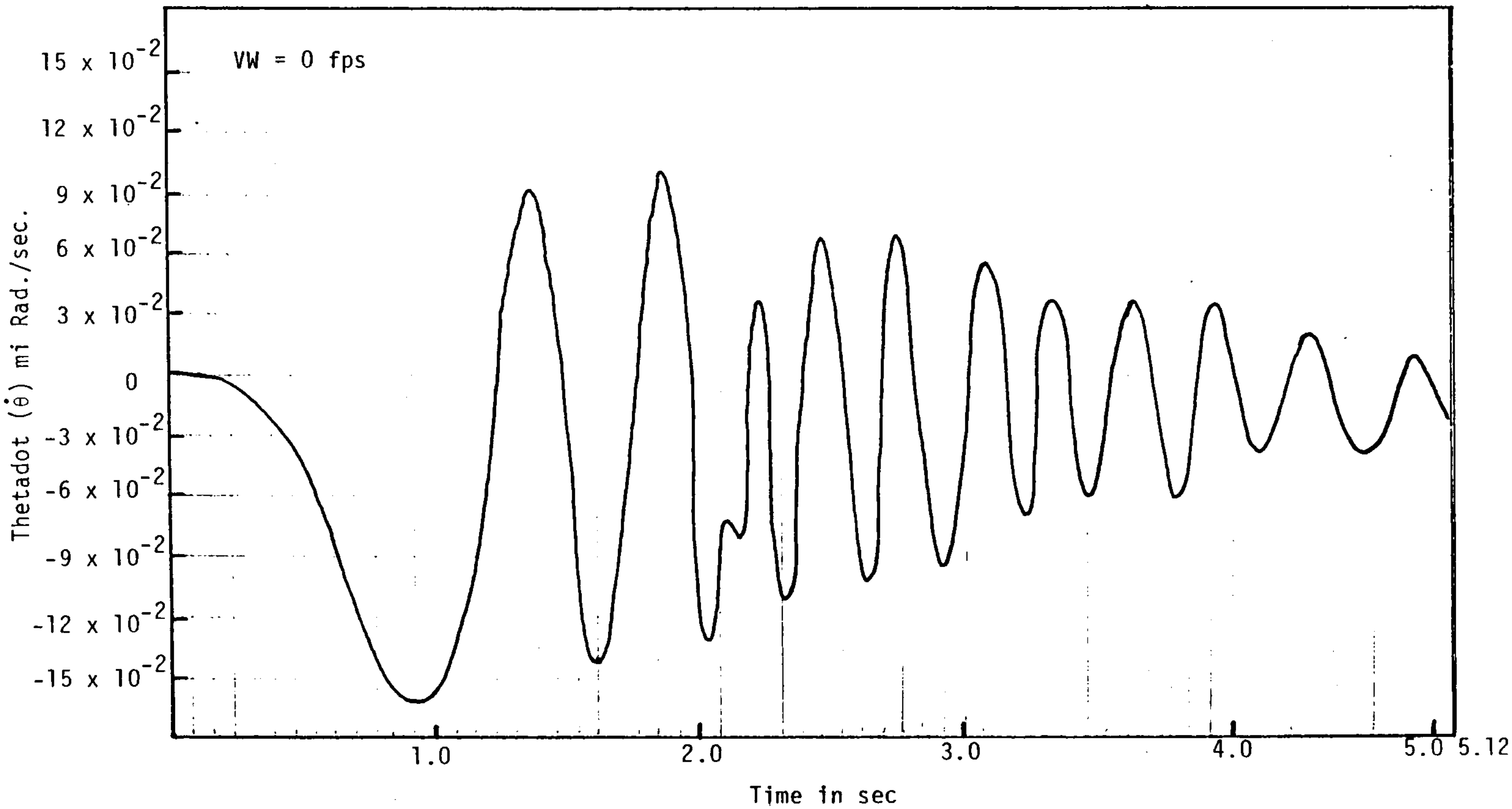

Figure 4.5. Thetadot ($\dot{\theta}$) Versus Time for Nominal (Fc=0) Missile θo=30°.

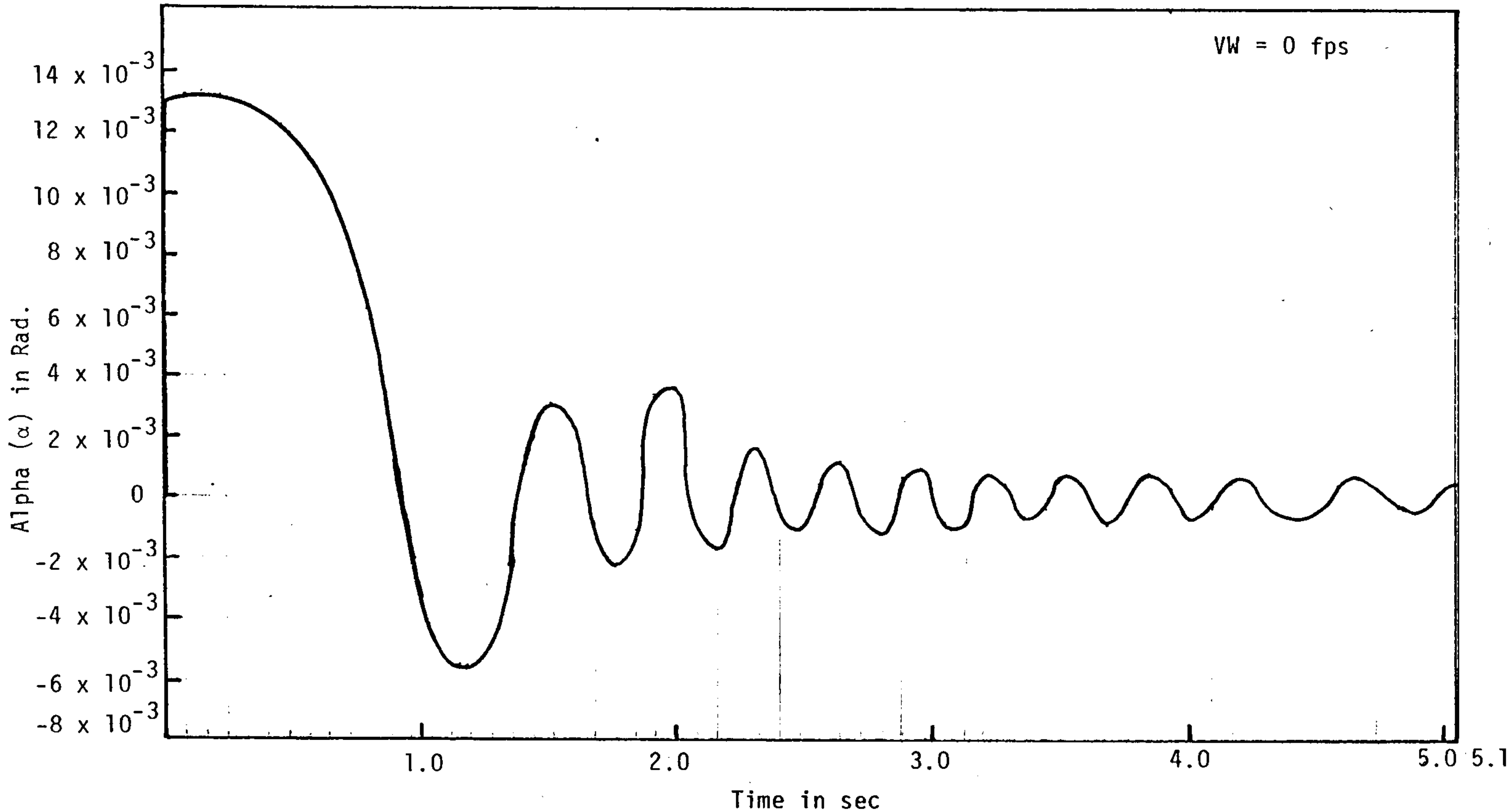

Figure 4.6. Alpha (α) Versus Time for Nominal (Fc=0) Missile θo=75°.

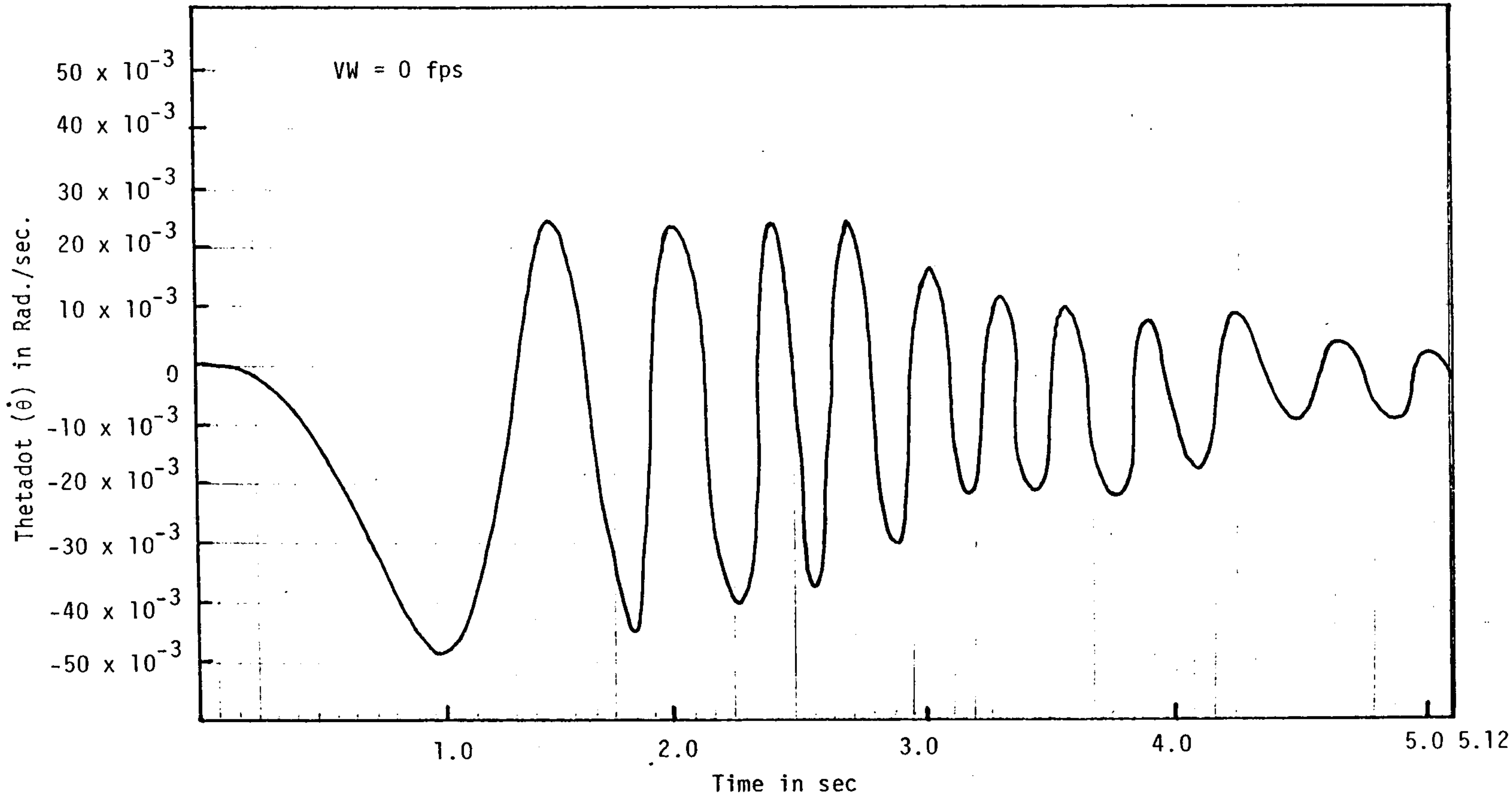

Figure 4.7. Thetadot ($\dot{\theta}$) Versus Time for Nominal Missile $\theta o=75°$.

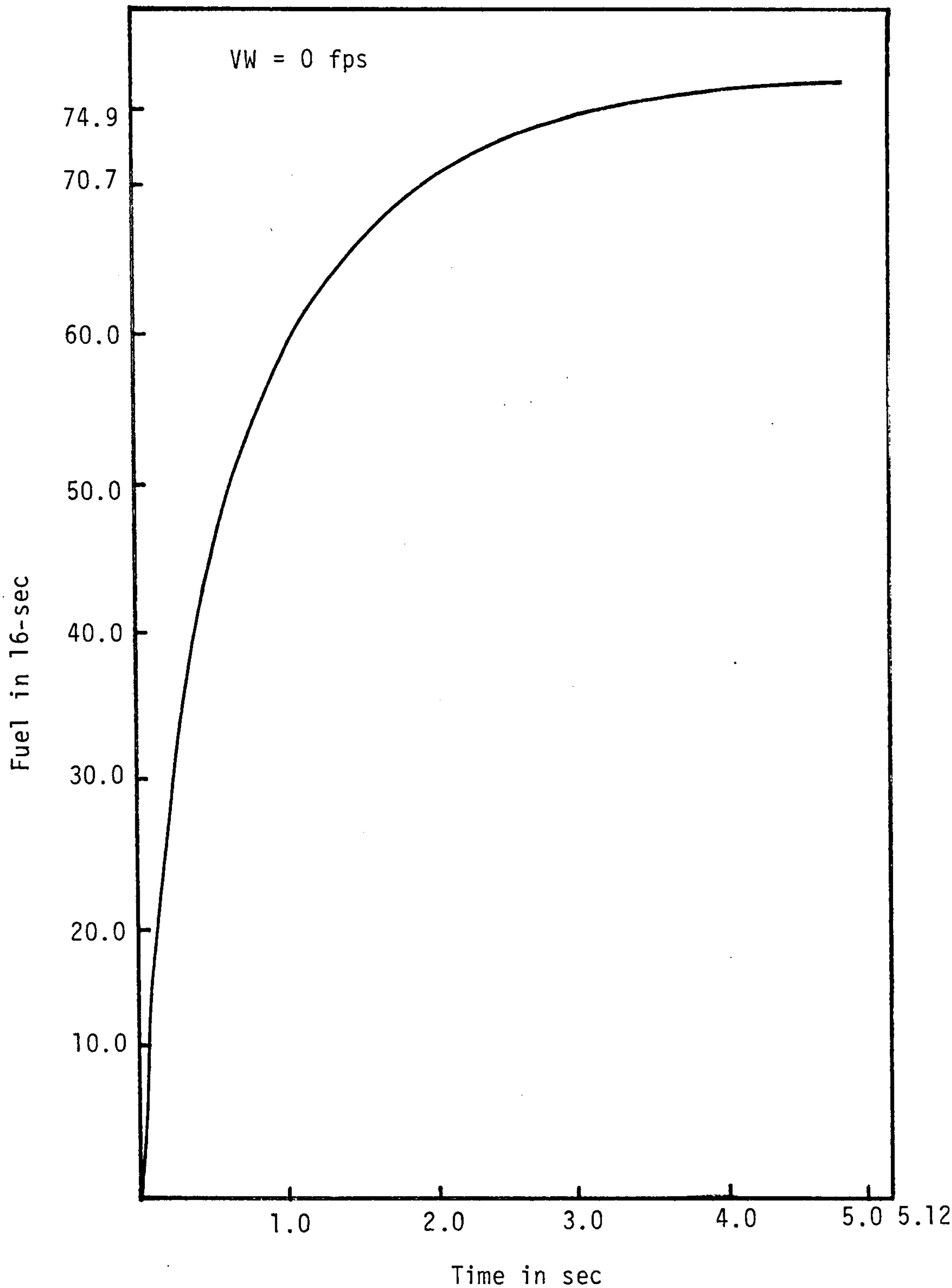

Figure 4.8. Fuel Versus Time for Missile with Conventional Autopilot ($Fc=-10^3*Q*\alpha$) $\theta o=30°$.

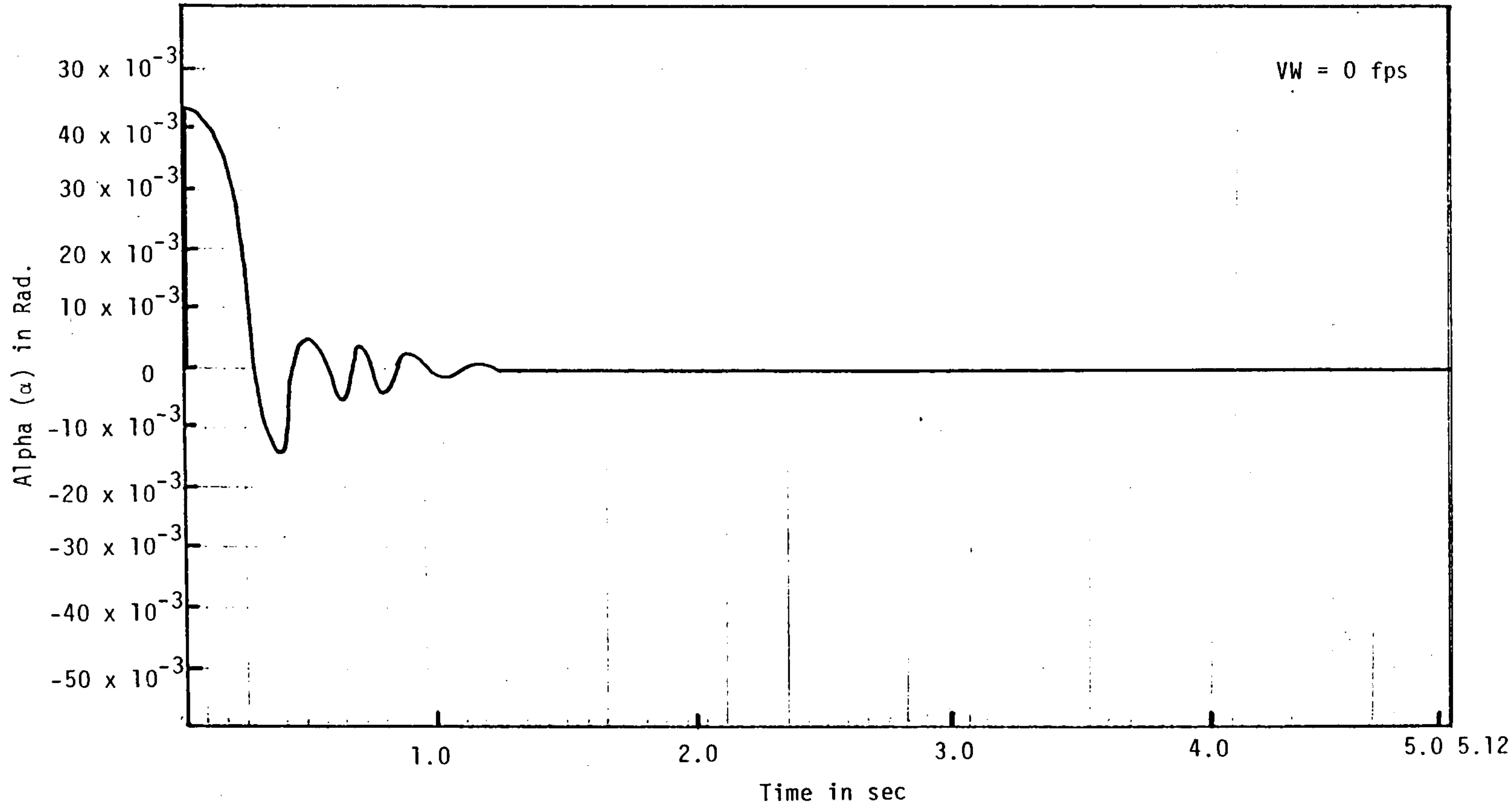

Figure 4.9. Alpha (α) Versus Time for Missile with Conventional Autopilot ($Fc=-10^3*Q*\alpha$) $\theta o=30°$.

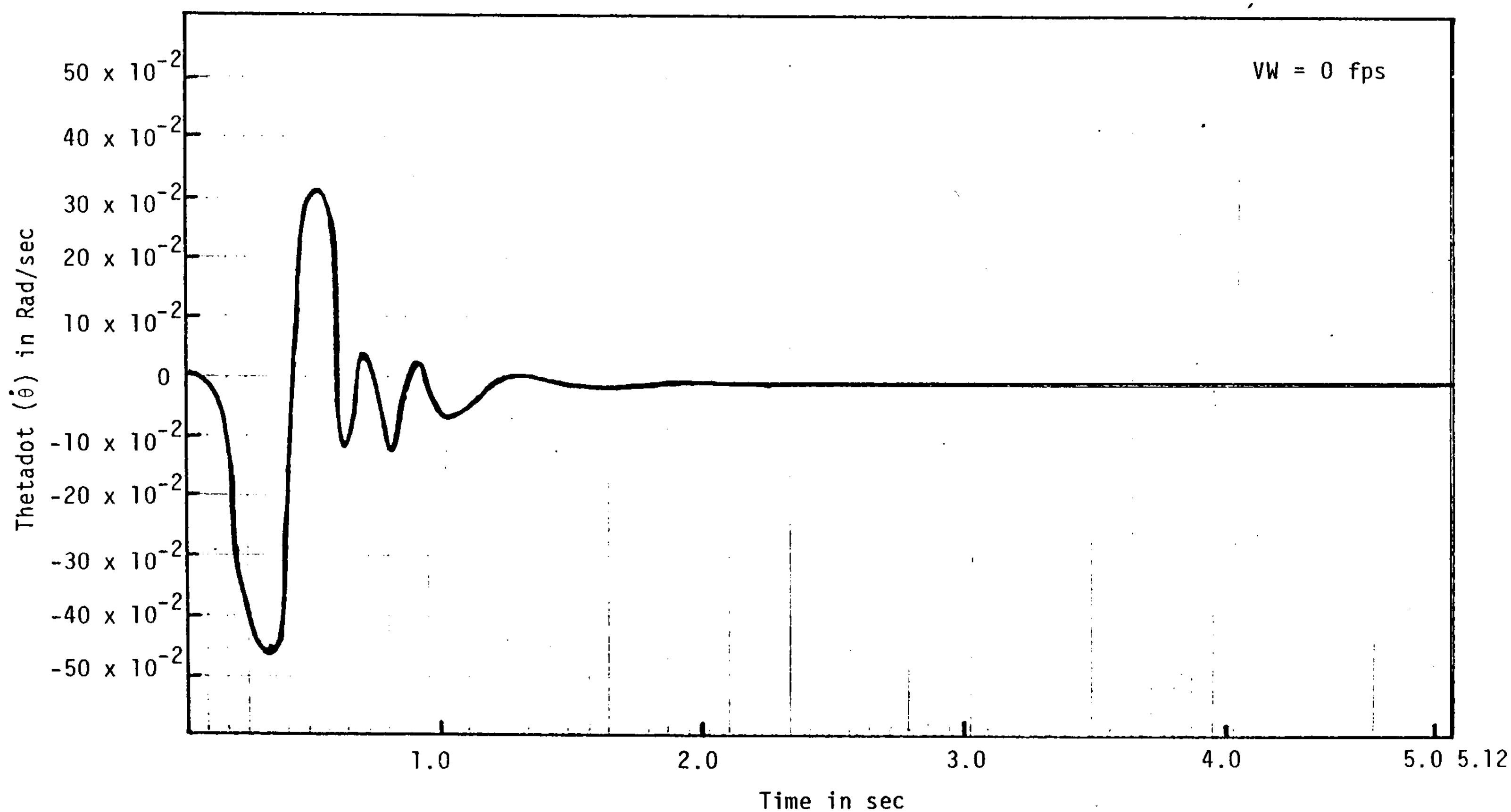

Figure 4.10. Thetadot ($\dot{\theta}$) Versus Time for Missile with Conventional Autopilot ($Fc=-10^3*Q*\alpha$) $\theta o=30°$.

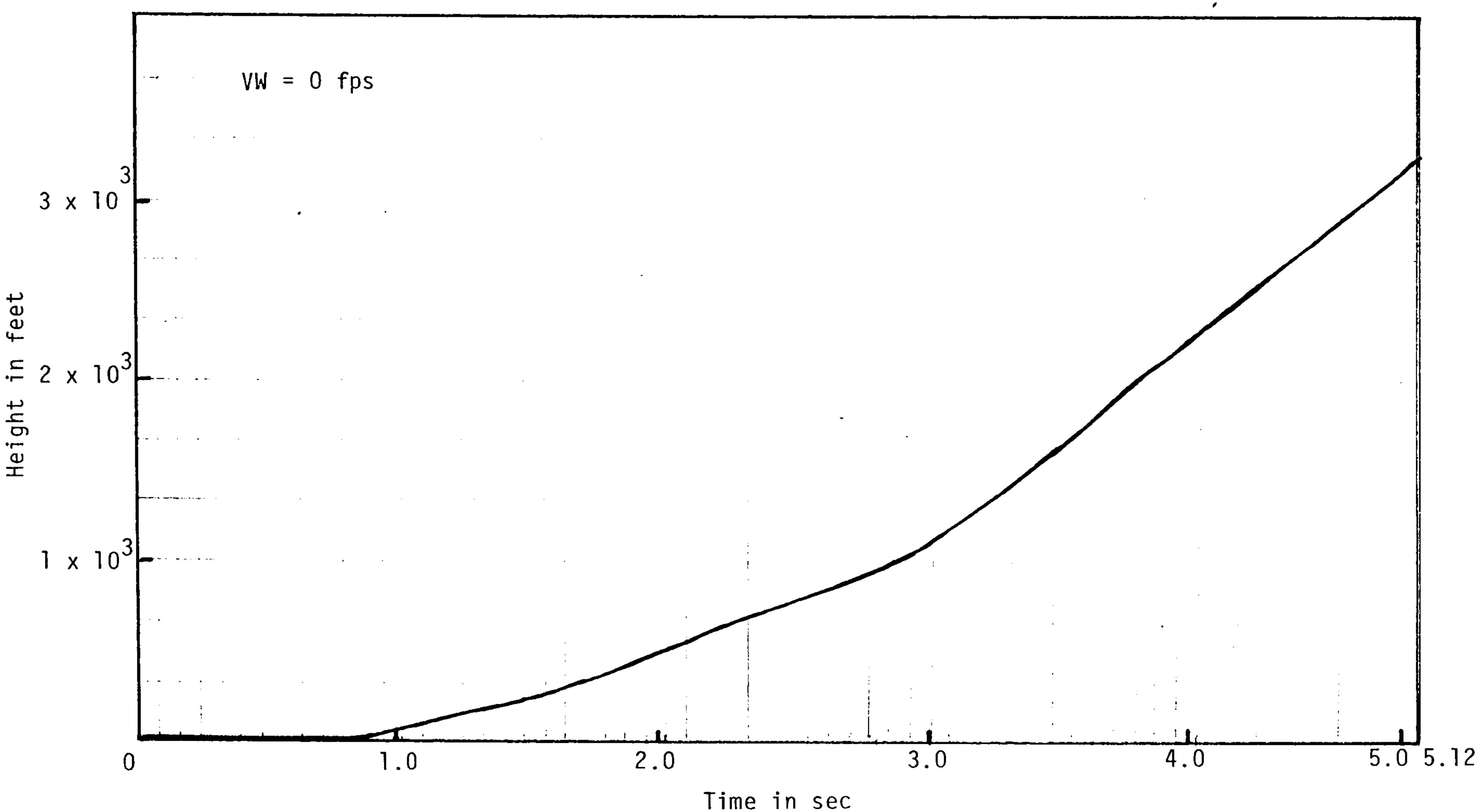

Figure 4.11. Height Versus Time for Missile with Conventional Autopilot ($Fc=-10^3*Q*\alpha$) $\theta o=30°$.

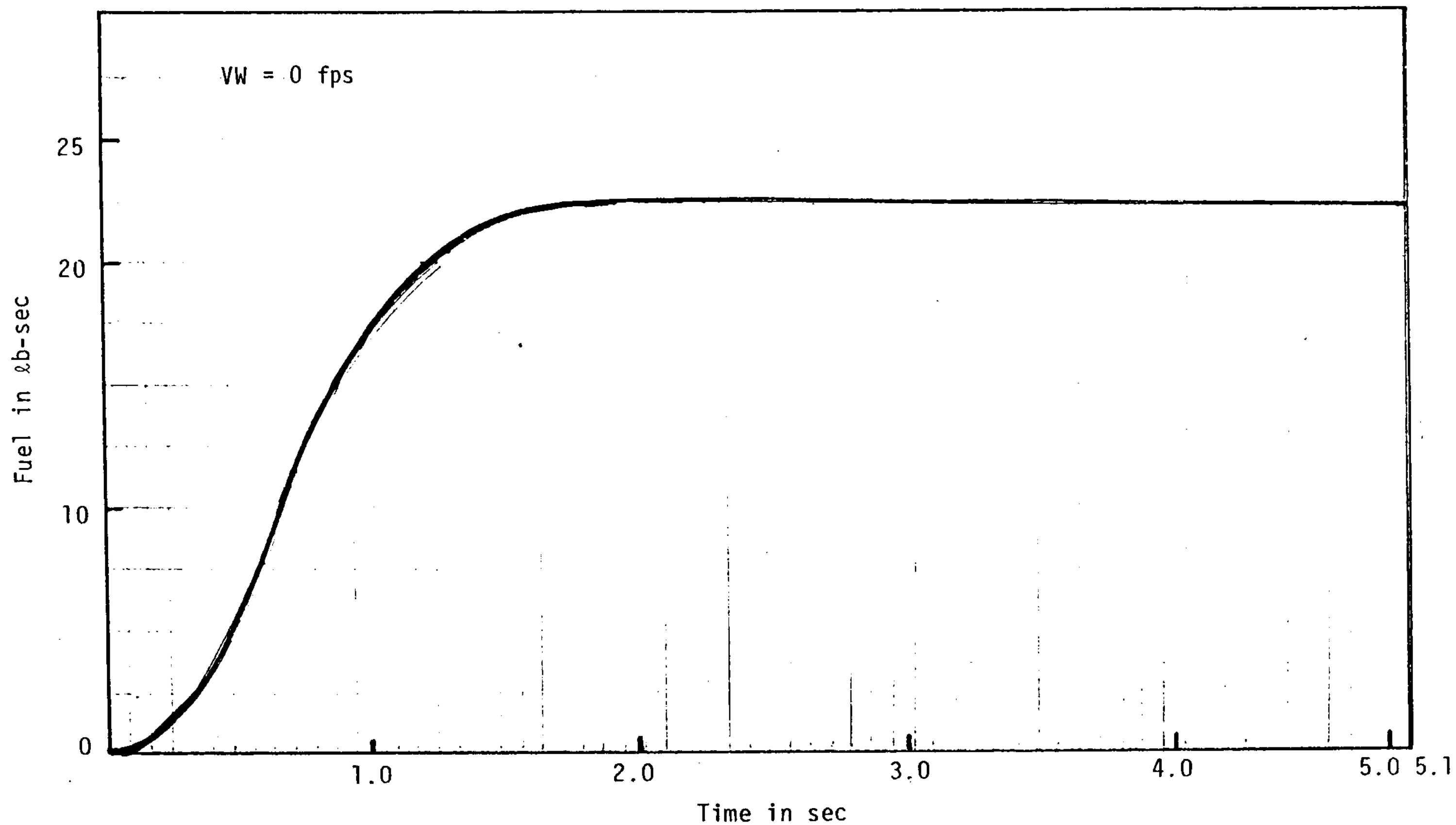

Figure 4.12. Fuel Versus Time for Missile with Conventional Autopilot ($Fc=-10^3*Q*\alpha$) $\theta o=75°$.

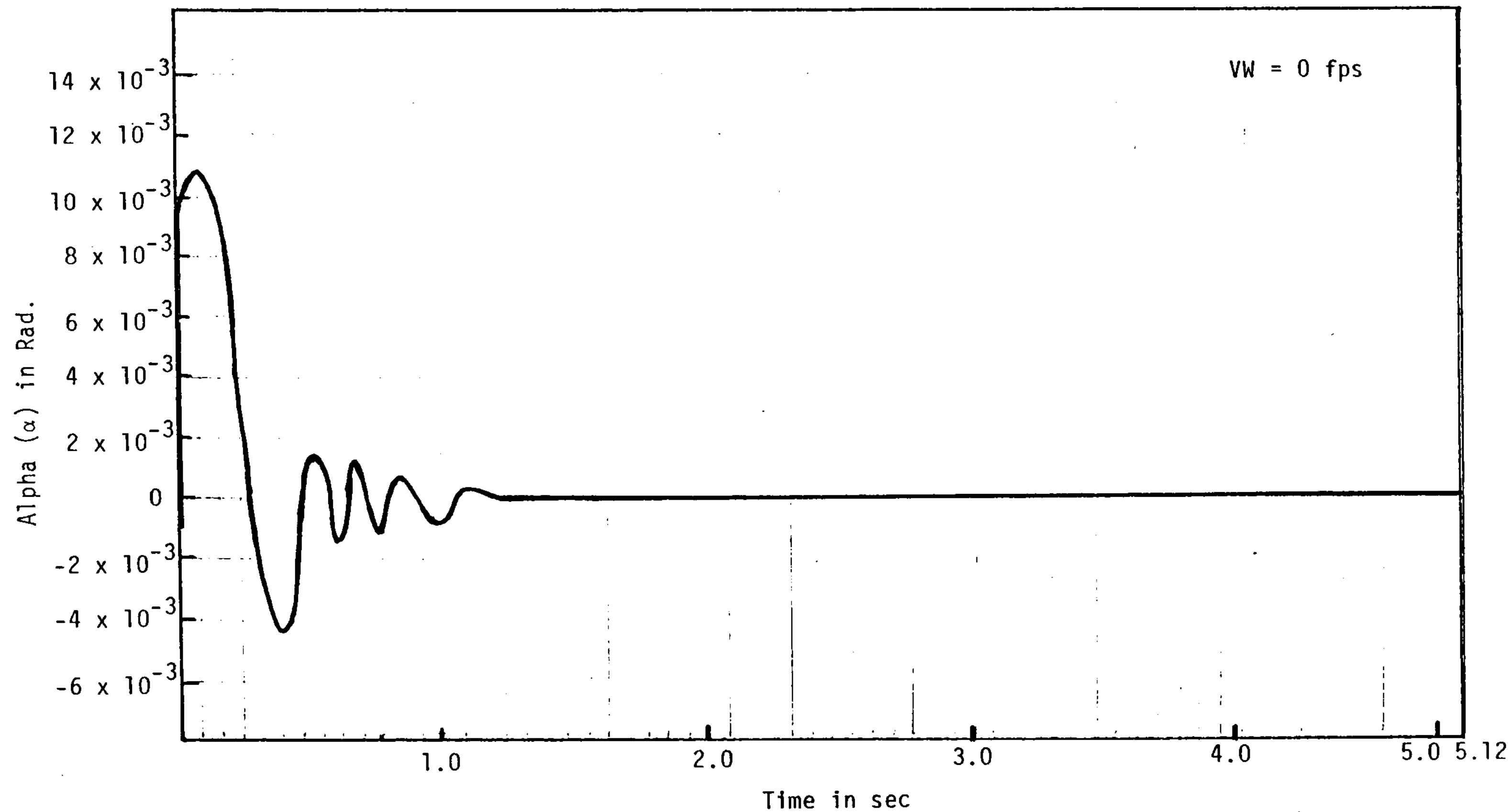

Figure 4.13. Alpha (α) Versus Time for Missile with Conventional Autopilot ($Fc=-10^3*Q*\alpha$) $\theta o=75°$.

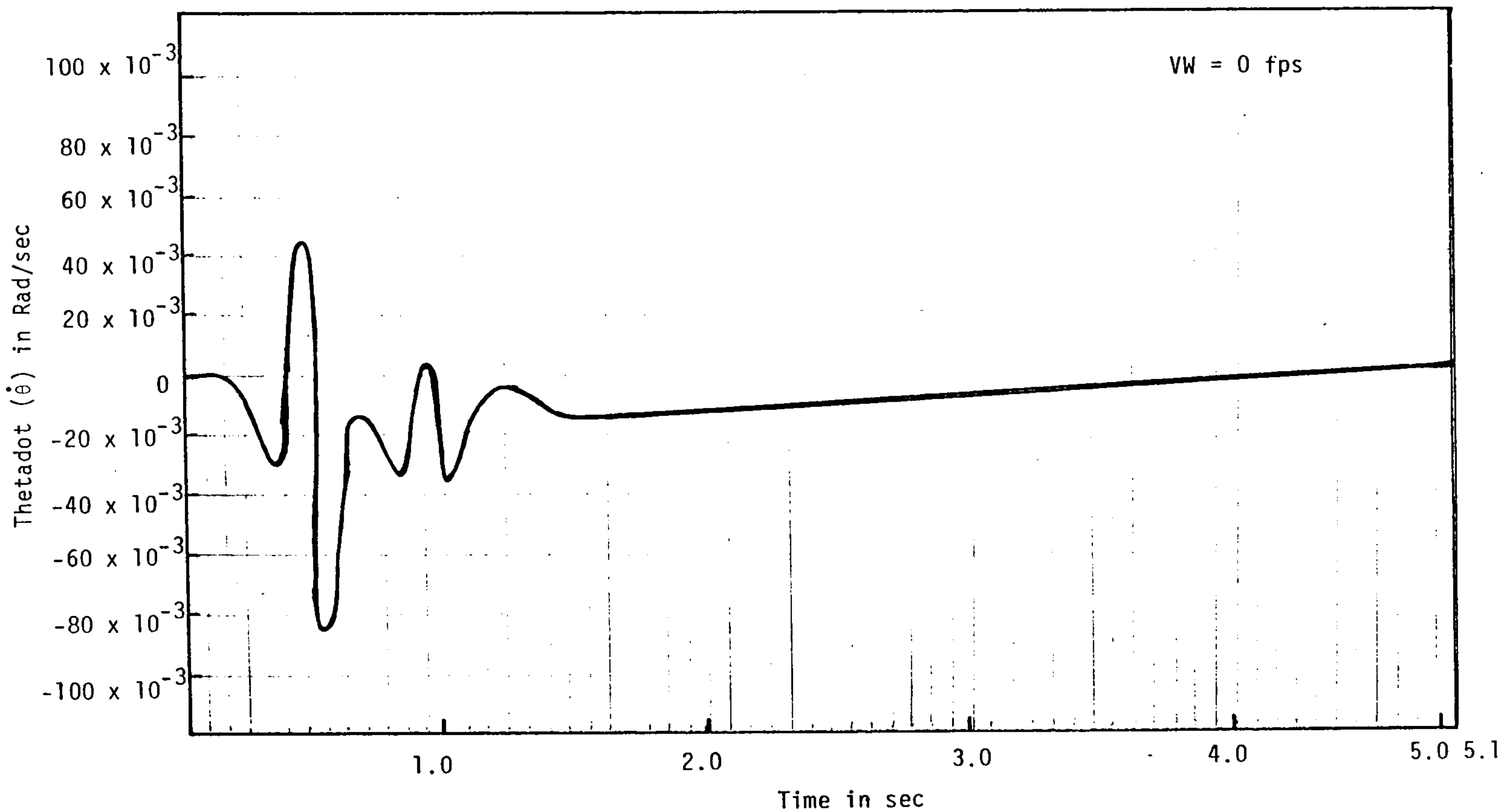

Figure 4.14. Thetadot ($\dot{\theta}$) Versus Time for Missile with Conventional Autopilot ($Fc=-10^3*Q*\alpha$) $\theta o=75°$.

CHAPTER V

SIMULATION RESULTS AND CONCLUSION

The Simulations

While typical trajectories are shown in Figures 3.2 to 3.5 (pp. 28-31) it was decided to concentrate on two: a 30° and a 75° flight. These two were chosen because of the extreme spread in flight time. It was expected that the range sensitivity to changes in the state variables at "burnout" would have wide variations.

Three autopilot or controller configurations were flown:

1. No Controller ($F_c = 0$);
2. Alpha Autopilot ($F_c = -1000q$);
3. "Optimal" Controller ($F_c = -K^T \Delta \underline{x}$).

The Alpha Autopilot was optimized for the 30° trajectory even though it was utilized for the 75° trajectory also. It should be noted, however, that the "gains" associated with the "optimal" controller differ with the trajectory.

The system parameters that were advanced for the simulation and their ranges are summarized in Table 5.1. The aerodynamic parameters L, $\overline{X}$, CN and CA are usually determined by a combination of analytical techniques and wind tunnel testing. A ±5% accuracy may be obtained in most cases with a minimum of such testing. The propulsion parameters F_T and δ are usually obtained by strict quality control. Plus or minus 2%/control F_T is a straightforward problem but 0.001

radius on thrust misalignment δ begins to become a problem but is not unreasonable.

TABLE 5.1

PARAMETER VARIATIONS

Parameter	Variation	Units
L	± 5%	-
$\overline{X}$	± 5%	-
C_N	± 5%	-
C_A	± 5%	-
F_T	± 2%	-
δ	0.001, 0.003	Radius
V_W	16, 48, 100	Feet/sec

For weapon systems of this type one of the primary disturbances is that due to winds. Usually the wind mode, against which they are tested, is a statistical envelope of wind speeds encountered at various altitudes. Such an envelope is often termed a Sissenwene Profile (1). In the system studied in this thesis a constant profile was utilized. It should be noted that the winds were applied up to propellant burnout. The unpowered flight encountered zero winds.

The amount of fuel necessary to produce the required side force F_C was calculated by

$$\text{Fuel} = \int_0^{t_b} |F_C|^2 \, dt \tag{5.1}$$

and hence has the units of lb-sec.

Simulation Results

The simulation results are summarized in the tables in Appendix E. These tables allow a comparison between the three systems enumerated in the previous section. To use these results, range sensitivity parameters were derived by the techniques outlined in Chapter I. The parameters are summarized in Table 5.2 for both the 30° and 75° trajectory. The range error is taken to be RE = r-r* where r* is the nominal (no disturbances) value and r is actual range with the 1% perturbated state variable. It should be noted that wide variations occur between two trajectories.

TABLE 5.2

RANGE SENSITIVITY

Parameter	$\theta = 30°$	$\theta = 75°$
$\frac{\alpha r}{\alpha V}$	1274 ft.	3281 ft.
$\frac{\alpha r}{\alpha \gamma}$	331 ft.	-4971 ft.
$\frac{\alpha r}{\alpha \theta}$	-2413 ft.	10 ft.
$\frac{\alpha r}{\alpha h}$	-105.3 ft.	332 ft.

The relative merits of the three systems can adequately be compared by considering the miss distance effects of three parameters:

1. an aerodynamic coefficient $\bar{x}$;
2. a thrust parameter δ;
3. head winds VW.

The procedure consists of first obtaining the percent change in the state variables due to perturbating the particular parameter. The percent in each state variable is then multiplied by the range sensitivity obtained from Table 5.2. The miss distance is next obtained by algebraically combining the range errors due to each state variable. To assist in the comparison process another table is included for each parameter that shows the fuel usage.

It was obvious that the major contributor to miss distance was the perturbation of $\dot{\theta}$. The miss distance tables thus were calculated with and without the error due to $\dot{\theta}$ (see Tables 5.3-5.12).

Overview

Before discussing the results of the simulation results, a review of the optimizing procedure is in order. A figure of merit J was chosen such that

$$J = 1/2(\underline{x} - \underline{x}^*)^T F(\underline{x} - \underline{x}^*) + 1/2 \int_0^{t_b} F_c^2 \, dt \qquad (5.2)$$

where t_b is the time at which the thrust terminates. The rationale for this selection was that it would balance fuel usage and the state variable errors at "burnout." F was chosen to be the unit matrix which means that each state variable error was given equal weight in the optimization process. It was shown in Chapter II that utilizing the second variation for Eq. (5.2) allowed a control law to be developed that would tend to face the system to remain on the optimal trajectory symbolized by $\underline{x}^*$. The gain of this control law

TABLE 5.3

PERCENT CHANGE IN STATE VARIABLE DUE TO $\bar{x}$
(VW = 0)

Variation State Variables	$\bar{x}$ + 5%	
	θ_0 = 30	θ_0 = 75°
% V_{NOM}	0	0
% V_α	0	0.0235
% V_{OPT}	-0.0026	0
% γ_{NOM}	0.2066	0.0079
% γ_α	-0.0317	0.4050
% γ_{OPT}	0.4209	0.0237
% θ_{NOM}	0.2096	0.0315
% θ_α	-0.0317	0.4050
% θ_{OPT}	0.4243	0.0474
% $\dot{\theta}_{NOM}$	228.9203	-58.7267
% $\dot{\theta}_\alpha$	0.6321	276.0000
% $\dot{\theta}_{OPT}$	-441.1290	-96.0391
% H_{NOM}	0.1336	0.0057
% H_α	0	0.2120
% H_{OPT}	0.3046	0.0091

TABLE 5.4

RANGE ERROR DUE TO STATE VARIABLE PERTURBATION
(VW = 0)

	$\bar{x}$ + 5%	
	θ_0 = 30°	θ_0 = 75°
V_{NOM}	0	0
V_α	0	77.1035
V_{OPT}	-3.3124	0
γ_{NOM}	68.3846	-39.2709
γ_α	-10.4927	-2013.2550
γ_{OPT}	139.3179	-117.8127
θ_{NOM}	-51.3520	-0.8505
θ_α	7.7665	-10.9350
θ_{OPT}	-103.9535	-1.2798
$\dot{\theta}_{NOM}$	-55238.4680	-587.2670
θ_α	-152.5257	2760.0000
θ_{OPT}	106444.4300	-960.391
H_{NOM}	-14.0681	1.8924
H_α	0	70.3840
H_{OPT}	-32.0744	3.0212

TABLE 5.5

MISS DISTANCE DUE TO $\bar{x}$ PERTURBATION

	With $\dot{\theta}$ Error		Without $\dot{\theta}$ Error	
	VW = 0, $\bar{x}$ + 5%		VW = 0, $\bar{x}$ + 5%	
	$\theta_0 = 30°$	$\theta_0 = 75°$	$\theta_0 = 30°$	$\theta_0 = 75°$
$F_c = 0$	-55170.0830	-625.496	68.3850	-38.2290
$F_c = 10^3\theta\alpha$	-155.2519	883.2975	-2.7262	-1876.7025
$F_c = -k^T\Delta\underline{x}$	106444.410	-1076.4623	-0.0200	-116.0713

TABLE 5.6

FUEL USAGE DUE TO $\bar{x}$ PERTURBATION
(VW = 0)

Systems	Fuel (lb sec)	
Fc =	$\theta_o = 30°$	$\theta_0 = 75°$
$Fc = -10^3 Q\alpha$	45.920	15.1670
$F_c = -k^T \Delta \underline{x}$	1.1885	4.6979

TABLE 5.7

PERCENT CHANGE IN STATE VARIABLES DUE TO WIND

	$\theta_0 = 30°$			$\theta_0 = 75°$		
	$V_w = 16$	$V_w = 48$	$V_w = 100$	$V_w = 16$ fps	$V_w = 48$ fps	$V_w = 100$ fps
% V_{NOM}	-0.06980	-0.2093	-0.43680	-0.209	-0.0653	-0.1463
% V_α	-0.07485	-0.2220	-0.46500	-0.0261	-0.0706	-0.1620
% V_{OPT}	-0.06460	-0.19600	-0.0235	-0.0235	-0.0679	-0.1490
% γ_{NOM}	+0.29800	-0.44600	-1.70311	+0.0237	-0.0473	-0.1420
% γ_α	-0.02220	-0.00317	-0.04753	-0.0565	0.0081	-0.0484
% γ_{OPT}	-0.91189	-2.89100	-6.56100	-0.0316	-0.0949	-0.2293
% θ_{NOM}	0.25765	-0.53825	-1.68618	-0.0079	-0.0473	-0.1419
% θ_α	0.02218	-0.00317	-0.04753	-0.0565	0.0081	-0.0484
% θ_{OPT}	-0.96969	-3.0069	-6.12354	-0.6316	-0.0948	-0.2608
% $\dot{\theta}_{NOM}$	-17.03791	-58.30400	-117.45100	+187.2340	+559.5745	1224.4681
% $\dot{\theta}_\alpha$	0.89058	0.89058	0.11450	-39.7163	7.8014	129.7872
% $\dot{\theta}_{OPT}$	19.8582	67.61229	133.13633	115.5844	474.0260	1237.6623
% H_{NOM}	0.22956	-0.51251	-1.76441	-0.0216	-0.0740	-0.1754
% H_α	-0.06471	-0.19737	-0.41739	-0.0483	-0.0460	-0.1563
% H_{OPT}	-0.79919	-2.50946	-5.54638	-0.0274	-0.0890	-0.2064

TABLE 5.8

RANGE ERRORS DUE TO STATE VARIABLE PERTURBATION

	θ_0 = 30°			θ_0 = 75°		
	V_w = 16 fps	V_w = 48 fps	V_w = 100 fps	V_w = 16 fps	V_w = 48 fps	V_w = 100 fps
V_{NOM}	-88.9252	-266.6482	-556.4832	-68.5729	-214.2493	-480.0103
V_α	-95.3589	-282.828	-592.410	-85.6341	-231.6386	-531.5220
V_{OPT}	-82.3004	-249.704	-533.4238	-77.1035	-22.7799	-488.8690
γ_{NOM}	98.638	-147.626	-563.7294	-117.8127	+155.1913	+465.9020
γ_α	-7.3482	-1.04927	-15.7324	+185.3765	26.5761	+158.8004
γ_{OPT}	-301.8356	-956.921	-2270.991	+157.0836	+471.7479	+1139.8503
θ_{NOM}	-63.1243	131.8713	413.1141	0.2133	1.2771	3.8313
θ_α	-5.4341	0.7767	11.6449	1.5255	-0.2187	1.3068
θ_{OPT}	237.5741	735.1691	1500.2637	0.8532	2.5596	7.0416
$\dot{\theta}_{NOM}$	4111.2477	14068.755	28340.926	1872.3400	5595.7450	12244.6810
$\dot{\theta}_\alpha$	-214.8970	-214.8970	-27.6289	-397.1630	78.0140	1297.8720
$\dot{\theta}_{OPT}$	4791.7837	16314.846	32125.796	1155.8440	4740.2600	12376.6230
H_{NOM}	-24.1727	53.9673	185.7924	-7.1712	-24.5680	-58.2328
H_α	6.8140	20.7831	43.9512	-16.0356	-15.2720	-51.8916
H_{OPT}	84.1547	264.2461	584.0338	-9.0968	-29.5480	-68.5248

TABLE 5.9

MISS DISTANCE DUE TO WIND

	$\theta = 30°$			$\theta = 75°$		
	V_w = 16 fps	V_2 = 48 fps	V_2 = 100 fps	V_2 = 16 fps	V_w = 48 fps	V_2 = 100 fps
			With $\dot{\theta}$ Error			
$F_c = 0$	4033.66	13987.95	27819.62	1678.9965	5513.3961	12176.171
$F_c = -10^3\theta\alpha$	-316.22	-477.21	-580.16	-311.9307	-142.5392	874.5656
$F_c = -k^T\Delta\underline{x}$	4729.38	16107.64	31405.68	1227.5805	4962.2396	12966.1210
			Without $\dot{\theta}$ Error			
$F_c = 0$	-77.5	-80.q	521	-193.3435	-82.3489	-68.5100
$F_c = -10^3\theta\alpha$	-101.32	-262.3	-553.16	85.2323	-64.5252	-423.3064
$F_c = -k^T\Delta\underline{x}$	-62.4	-207.2	-720	71.7365	221.9796	589.498

TABLE 5.10

MISS DISTRIBUTION FOR MISALIGNMENT PERTURBATION WITH $\dot{\theta}$ ERROR VW = 0

	θo = 30°		θo = 75°	
	Δ=0.001	Δ=0.003	Δ=0.001	Δ=0.003
		With $\dot{\theta}$ Error		
$F_c = 0$	-17077.4450	706.6265	-183127.3400	-514318.8700
$F_c = -10^3 Q\alpha$	-192.7432	-70.2885	-175.0393	-1776.0291
$F_c = -k^T \Delta \underline{x}$	-65.5056	-180.1402	-3045.7837	-5759.9106
		Without $\dot{\theta}$ Error		
$F_c = 0$	-585.3140	83.5175	-36755.0	-11267.38
$F_c = -10^3 Q\alpha$	-8.5348	-8.8776	80.2797	-1435.6031
$F_c = k^T \Delta \underline{x}$	1.0449	0.4970	-1671.7577	-5017.0536

TABLE 5.11

RANGE ERROR DUE TO STATE VARIABLE PERTURBATIONS

	θo = 30°		θ = 75°	
	Δ=0.001	Δ=0.003	Δ=0.001	Δ=0.003
V_{NOM}	-210.7196	898.8452	-257.2304	-1046.2366
V_{α}	-6.6248	3.3124	-68.5729	-85.6341
V_{OPT}	-6.6248	-23.0594	-8.5306	-25.5918
γ_{NOM}	7432.9691	22338.5610	-37008.1010	-112200.44
γ_{α}	244.4104	710.0281	160.5633	-1363.5453
γ_{OPT}	498.5853	1494.8622	-1689.6429	-5059.4258
θ_{NOM}	-5891.7355	-17750.177	-204.9894	-617.1012
θ_{α}	-180.908	-525.623	0.8721	-7.4061
θ_{OPT}	-372.253	-1116.171	-9.1746	-27.3132
$\dot{\theta}_{NOM}$	-16492.131	623.1090	-14637.3400	-401691.49
θ_{α}	-184.2084	-61.4109	-255.3190	-340.4260
$\dot{\theta}_{OPT}$	-66.5505	-180.6372	-1374.0260	-742.8570
H_{NOM}	-1915.8282	-5403.7117	715.3241	1236.4012
H_{α}	-65.4124	-196.5951	-12.5828	20.9824
H_{OPT}	-118.6626	-355.1348	35.5904	105.2772

TABLE 5.12

PERCENT CHANGE IN STATE VARIABLE FOR MISALIGNMENT PERTURBATIONS VW = 0

		$\theta_o = 30^0$		$\theta = 75^0$	
		Δ=0.001	Δ=0.003	Δ=0.001	Δ=0.003
%	V_{NOM}	-0.1654	0.70553	-0.0784	-0.4286
%	V_{α}	-0.0052	0.0026	-0.0209	-0.0261
%	V_{OPT}	-0.0052	-0.0181	-0.0026	-0.0078
%	γ_{NOM}	22.4561	67.4881	7.4448	22.5710
%	γ_{α}	0.7384	2.1454	-0.0323	0.2743
%	γ_{OPT}	1.5063	4.5162	0.3399	1.0198
%	θ_{NOM}	24.0479	72.4497	7.5922	22.8556
%	θ_{α}	0.7384	2.1454	-0.0323	0.2743
%	θ_{OPT}	1.5194	4.5558	0.3398	1.0116
%	$\dot{\theta}_{NOM}$	68.3470	-2.5823	-14637.234	-40169.149
%	$\dot{\theta}_{\alpha}$	0.7634	0.2545	-25.5319	-34.0426
%	θ_{OPT}	0.2758	0.7486	-137.4026	-74.2857
%	H_{NOM}	18.1940	51.3173	2.1611	3.7241
%	H_{α}	0.6212	1.8670	-0.0379	0.0632
%	H_{OPT}	1.1269	3.3726	0.1072	0.3171

was determined by matrices A and B which were functions of the optimal trajectory.

Since the system discussed in this thesis was a surface-to-surface missile, it was decided that a reasonable optimal trajectory $\underline{x}^*$ would be obtained by first setting the missile to some predetermined heading angle θo, and then allowing it to follow a ballistic trajectory ($F_c = 0$) to impact. Thus, for any range within the capability of the missile there would be a unique heading angle. It should be noted that this is now the only type of trajectory that could be utilized. As an example, the original Pershing Missile System flew the trajectory shown in Figure 5.1. The initial heading angle is always 90°. The missile was forced to follow the trajectory shown. The desired range was obtained by terminating the propellant (case venting) at a preselected time t_i. If the missile is not on a nominal trajectory, a guidance law adjust t_i accordingly. The nominal trajectory is optimized to minimize range effects of the state variable perturbations.

Discussion of Results

For each of the perturbated parameters the angle of attack Autopilot ($F_c = -1000\ q\alpha$) was by far the most successful for minimizing miss distance. The major contributor to the large miss distances developed by the "optimal" controller is that due to thetadot ($\dot{\theta}$) errors. To illustrate this preponderance, the miss distance is shown with and without the thetadot errors.

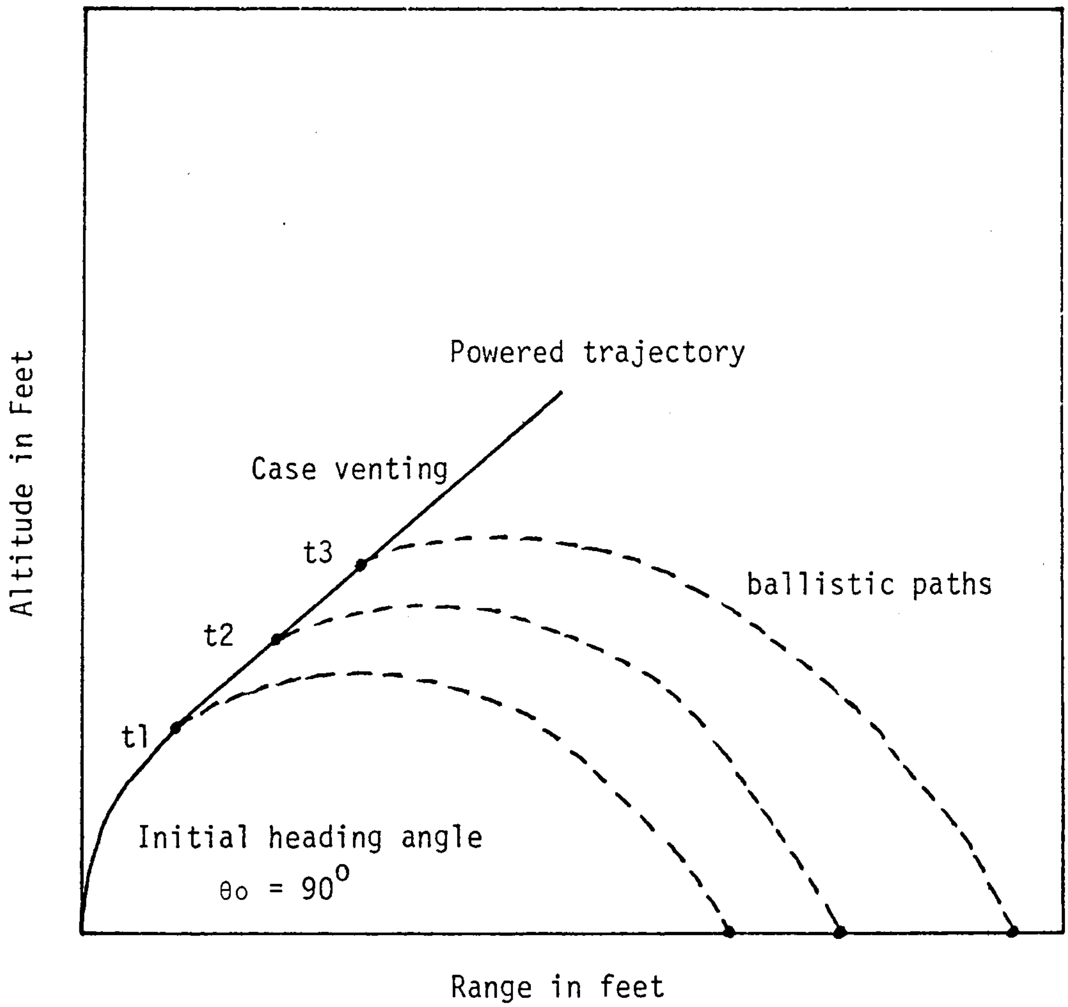

Figure 5.1. An Alternate Trajectory.

The physical mechanism whereby such errors arise is shown in Figures 4.5, 4.7, 4.10, and 4.14 (pp. 59, 61, 64, 68). For the nominal missile, thetadot is similar to the response of a lightly damped system. At the "burnout" time, 5.12 seconds, theatdot is oscillating about a small negative value but its magnitude is comparatively large. Unfortunately for this application the magnitude of the oscillations are larger at a heading angle of $\theta = 75°$ where the range sensitivity is small conversely at $\theta = 30°$ where the magnitude of oscillation is smaller the range sensitivity is large.

For the angle of attack autopilot the thetadot response is similar to the response of a highly overdamped system at the "burn-out" time. Thus, perturbations in the system parameters are not near as significant.

The amount of fuel for perturbations in $\bar{x}$ followed the "optimal" concept; that is, the optimal controller used less fuel than the angle of attack autopilot. The effect of winds as fuel usage is shown in Figure 5.2. For winds less than 24 ft/sec the optimal controller at $\theta = 30°$ uses the least amount of fuel. However, the Angle of Attack Autopilot fuel usage relatively insensitive to winds. For $\theta = 75°$, however, the optimal controller uses the least amount of fuel for winds. The same type of behavior occurs for misalignment errors as shown in Figures 5.3, 5.4, and 5.5.

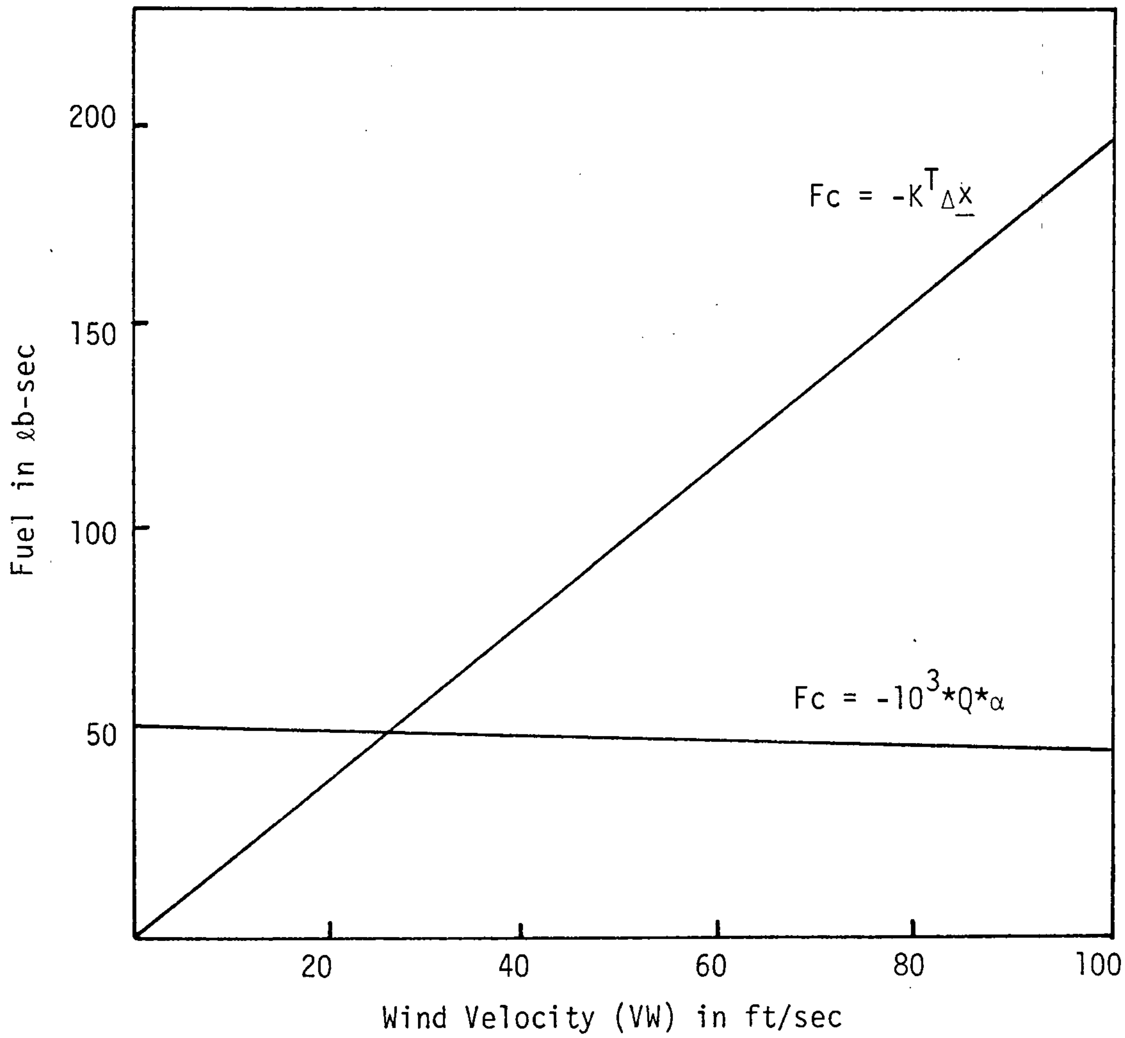

Figure 5.2. Fuel Versus Wind Velocity $\theta o = 30°$.

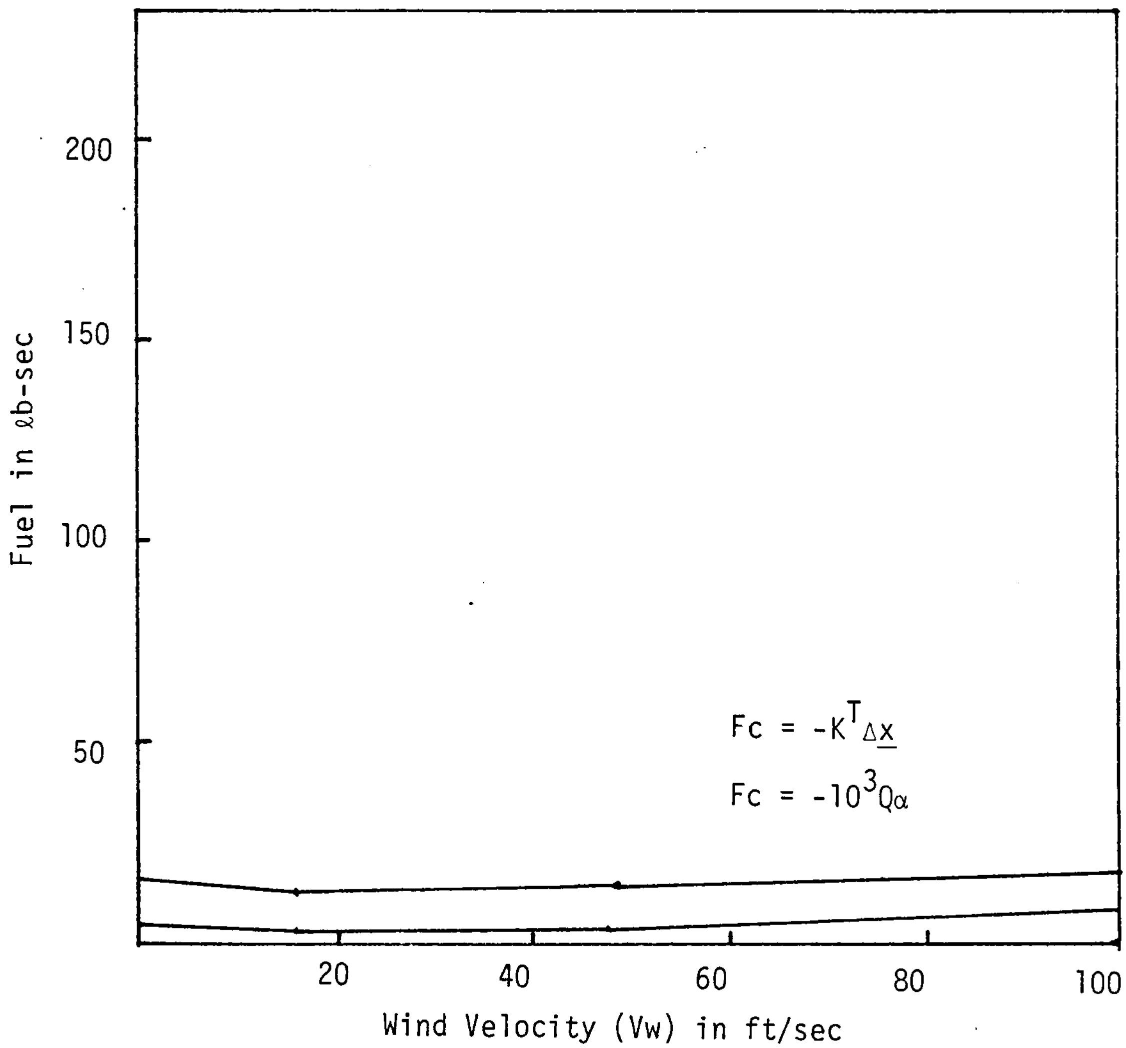

Figure 5.3. Fuel Versus Wind Velocity θo=75°.

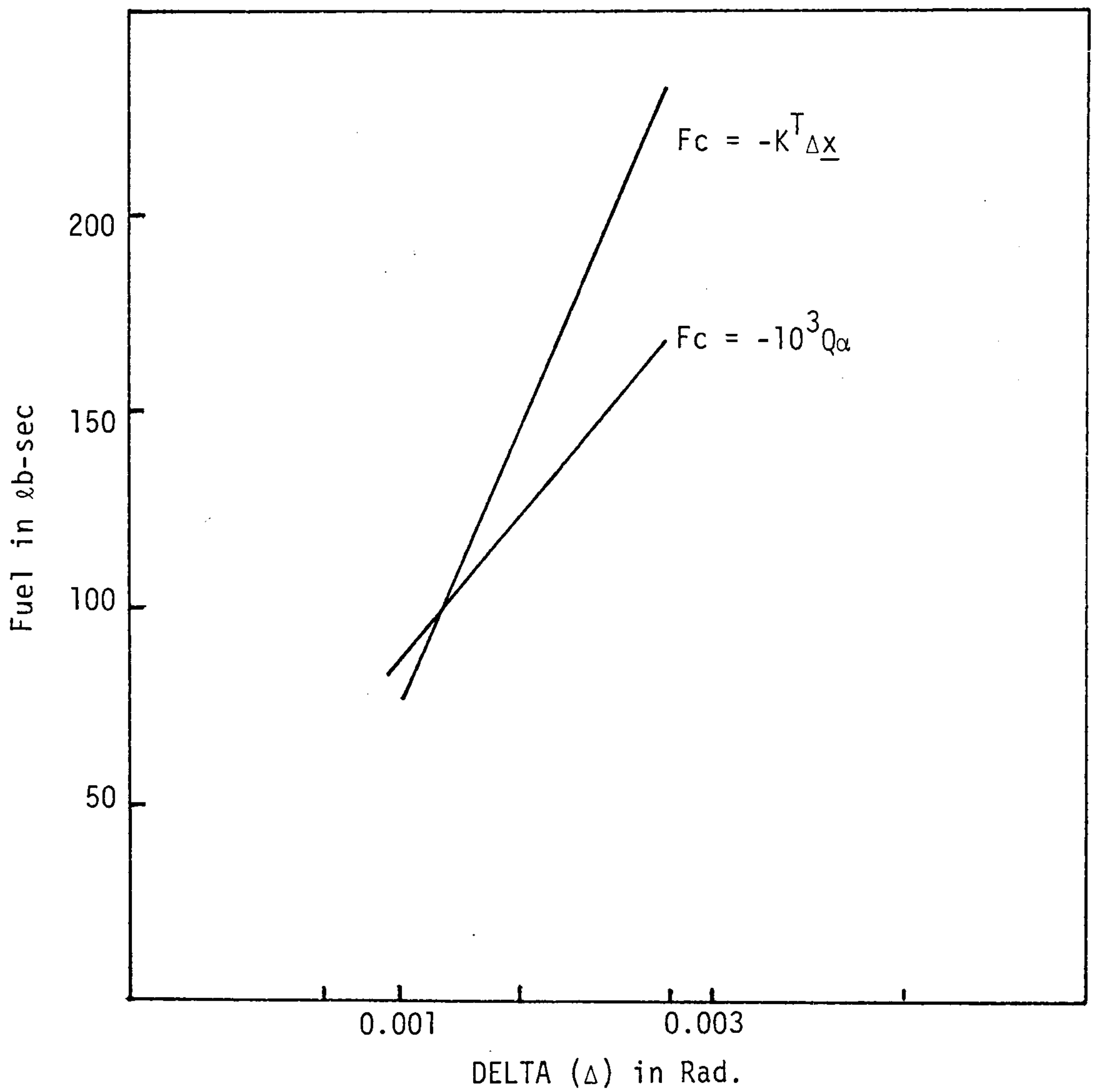

Figure 5.4. Fuel Versus Delta (Δ) θo=30°, VW = 0.

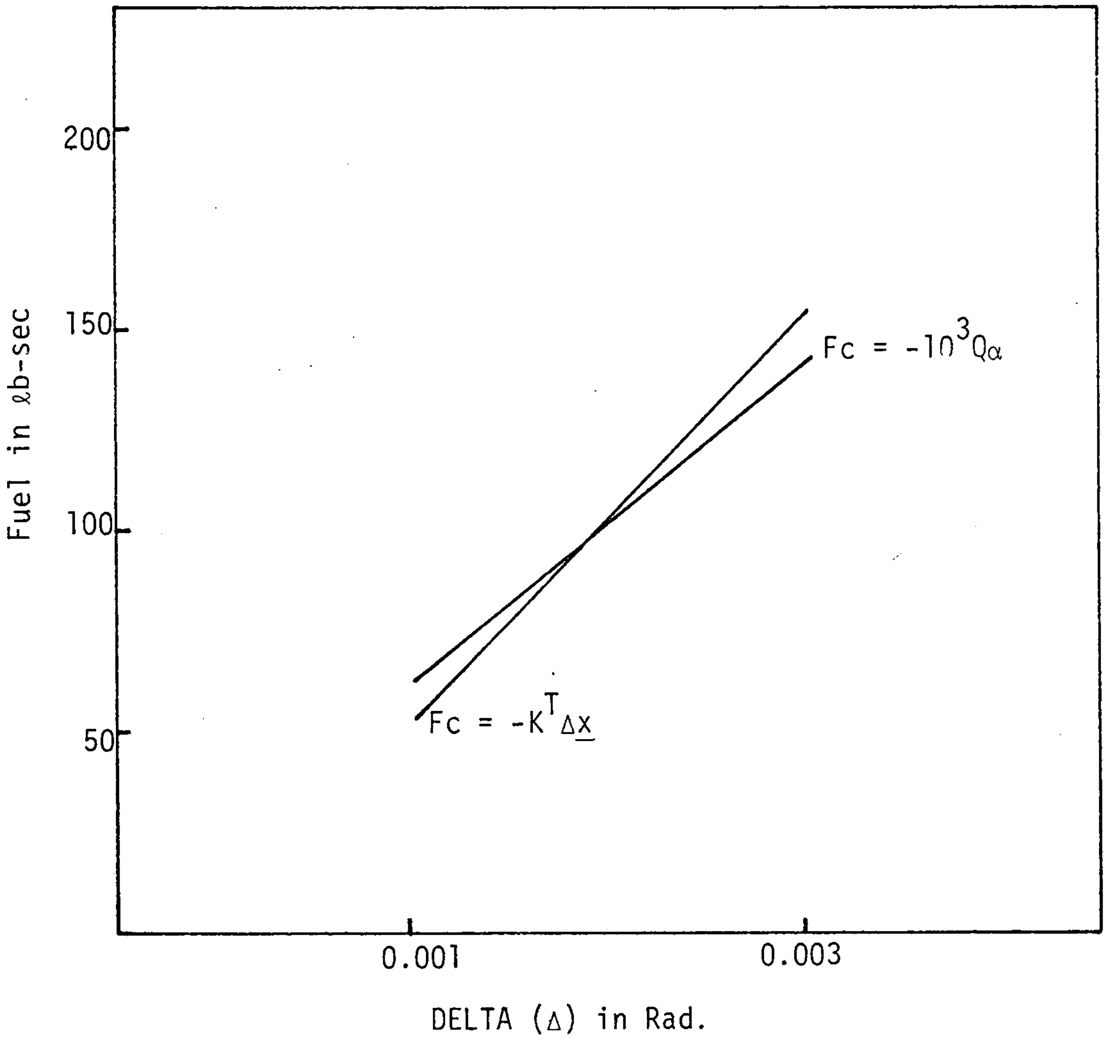

Figure 5.5. Fuel Versus Delta (Δ) θo=75°, VW = 0.

Conclusions

The purpose of this thesis as outlined in Chapter I was:

1. the development of an "optimal control law;

2. a comparison between the optimal controller and one designed by conventional techniques has been accomplished. The basis of the comparisons was chosen to be impact miss distance and fuel usage.

In the two trajectories examined the conventional autopilot had significantly smaller impact miss distances than the so-called optimal controller. For fuel usage there were regions where the optimal controller did use less fuel. However, as constituted in this thesis, the optimal controller is not a viable alternative to the Angle of Attack Autopilot.

The reason for the poor showing of the optimal controller lies in the oscillating nature of the thetadot (θ) response. This is compounded by the range sensitivity magnitude on some of the optimal trajectories.

In the performance index J chosen for this thesis expressed in Eq. (5.2) F is a 5 x 5 unity matrix. This means that that the state variable errors $\underline{x} - \underline{x}^*$ are of equal importance. From this thesis, it would seem fruitful to examine the effect of weighting the state variable errors according to their range error sensitivity. In particular, it is suggested that the new matrix F be expressed as

$$F = \left(\frac{\alpha r}{\alpha \pm}\right)^T \frac{\alpha r}{\alpha \underline{x}} \qquad (5.3)$$

where

$$\frac{\alpha r}{\alpha \underline{x}} = \left[\frac{\alpha r}{\alpha v}, \frac{\alpha r}{\alpha \gamma}, \frac{\alpha r}{\alpha \theta}, \frac{\alpha r}{\alpha \theta}, \frac{\alpha r}{\alpha r}\right] . \qquad (5.4)$$

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BIBLIOGRAPHY

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APPENDICES

APPENDIX A

PROGRAM FOR EVALUATING THE A AND B MATRICES

Program for Evaluating the A and B Matrices

In order to set up the files on the direct access device, the following Job Control Language (JCL) statements were used:

```
//CSMP1.FT20F001 DD DSN=J48961.ABMA,DISP=(NEW,CATLG),SPACE=(88,5122),
//DCB=(DSORG=DA),UNIT=ONLINE
//CSMP1.SYSIN DD *
//DSN=J9
//SPACE
//CSMP1.SYS
```

In the first line . . . FT2F001 . . . , 2 is the number given to the file . . . is the name given to this file 2 . . . SPACE=(88,5122) . . . is the amount of space allocated to this file. The number 88 is the length of the record in bytes. This number of bytes was necessary because there were a total of 22 words in the array to store the 14 nonzero elements of the A matrix, the 3 elements of the B matrix and the 5 elements of the nominal state vector. The number of records was 5122 because the array was stored at t = 0, and after each time interval of 0.0001 seconds for a total of 5.12 seconds.

In the second line . . . ON LINE . . . is the disk where the array was stored.

After the values of A and B matrices were stored on the disk at the completion of the first time step, the process was repeated for succeeding time steps until the final time was reached.

On the following pages of this appendix is shown the CSMP statements for evaluating the A and B matrices and the nominal state vector and then storing them on the disk.

```
//MISSILE JOB,TSUEI,GROUP=J48961,USER=P22718,
// MSGLEVEL=(1,1),
// PASWORD=MISSILE,TIME=2
/*ROUTE PRINT RMT1
/*JOBPARM LINES=5
// EXECUTE CSMP
```

```
//MISSILE JOB ,TSUEI,GROUP=J48961,USER=P22718,
// MSGLEVEL=(1,1),
// PASSWORD=MISSILE,TIME=2
/*ROUTE PRINT RMT1
/*JOBPARM LINES=5
// EXEC CSMP
//CSMP1.FT20F001 DD DSN=J48961.AB40,DISP=(NEW,CATLG),SPACE=(88,5122),
// DCB=(DSORG=DA),UNIT=ONLINE
//CSMP1.SYSIN DD *
      INITIAL
      CONSTANT FNSL=8700.,AE=.2799,PSL=2116.8,S=.54542,WT=434.1,...
      WDNOM=34.5
      CONSTANT LF=7.715,P1=.02,P2=.02,G=32.2,K28=1
      INCON VO=1.,GAMMAD=30.,THDOTO=0.,THETAD=30.,HO=0.,XO=0.
      FUNCTION PVSZ=0.,29.921,20000.,13.76,40000.,5.538,60000.,...
               2.1178,80000.,.815462,100000.,.321922,120000.,.131789,...
               140000.,.0572731,160000.,.0261996,180000.,.0120901,...
               200000.,.00540654,400000.,.63407E-6
      FUNCTION RHOZ=0.,.076474,20000.,.040773,40000.,.018826,60000.,...
               .007199,80000.,.0027171,100000.,.0010438,120000.,...
              .00040196,140000.,.00016315,160000.,.000071292,180000.,...
              .00003361,200000.,.000015755,400000.,.1164E-8
      FUNCTION CN2MAC=0.,4.2,.5,4.233,.8,4.51,1.2,5.7,1.5,5.14,2.,...
               4.67,3.,4.53,4.,4.504
      FUNCTION CAMACH=0.,.215,.25,.215,.5,.197,1.,.291,1.5,.258,2.,...
               .234,2.5,.205,3.,.181,4.,.160
      FUNCTION LCPMCH=0.,5.7,.5,5.725,.8,6.01,1.2,7.02,1.5,6.7,2.,...
               5.93,3.,4.58,4.,4.49
      FIXED K28
      GAMMO=GAMMAD/57.35
      THETAO=THETAD/57.35
      CP=13.6*62.4/12.
      DYNAMIC
```

```
      SUMFV=(FT-A)*COS(ALPHA)-N*SIN(ALPHA)-M*G*SIN(GAMMA)
      V=INTGRL(VO,SUMFV/M)
      SUMFG=(FT-A)*SIN(ALPHA)+N*COS(ALPHA)-M*G*COS(GAMMA)
      GAMMA=INTGRL(GAMMO,SUMFG/M/V)
SUMMO=-N*XBAR
      THEDOT=INTGRL(THDOTO,SUMMO/I)
      THETA=INTGRL(THETAO,THEDOT)
      H=INTGRL(HO,V*SIN(GAMMA))
      VV=(-AV*COS(ALPHA)-NV*SIN(ALPHA))/M
      VG=VA*AG-G*COS(GAMMA)
     VT=VA*AT
      VH=((FTH-AH)*COS(ALPHA)-NH*SIN(ALPHA))/M
      GV=-((FT-A)*SIN(ALPHA)+N*COS(ALPHA)-M*G*COS(GAMMA))/M/V**2+...
      (-AV*SIN(ALPHA)+NV*COS(ALPHA))/M/V
      GG=GA*AG+G*SIN(GAMMA)/V
      GT=GA*AT
      GH=((FTH-AH)*SIN(ALPHA)+NH*COS(ALPHA))/M/V
      TDV=(-NV*XBAR-N*XV        )/I
      NP=S/2.*RHO*CN*V**2
      TDT=-NP*XBAR/I
      TDG=NP*XBAR/I
      TDH=-(NH*XBAR+N*XH        )/I
      HV=SIN(GAMMA)
      HG=V*COS(GAMMA)
      VA=(-(FT-A)*SIN(ALPHA)-N*COS(ALPHA))/M
      GA=((FT-A)*COS(ALPHA)-N*SIN(ALPHA))/M/V
      AG=-1.
```

```
 AT=1.
 A=S/2.*RHO*CA*V**2
 AV=S/2.*RHO*(CA*2.*V+CAV*V**2)
 N=S/2.*RHO*CN*ALPHA*V**2
 NV=S*RHO*ALPHA*(2.*V*CN+CNV*V**2)/2.
 FT=FNSL+AE*(PSL-P)
 FTH=-AE*PH
 AH=S/2.*(RHO*CAH+RH*CA)*V**2
NH=ALPHA *S/2.*(RHO*CNH+RH*CN)*V**2
 CAV=(AFGEN(CAMACH,MA+MAV)-AFGEN(CAMACH,MA))/DV
 CNV=(AFGEN(CN2MAC,MA+MAV)-AFGEN(CN2MAC,MA))/DV
 CAH=(AFGEN(CAMACH,MA+MAH)-AFGEN(CAMACH,MA))/DH
 CNH=(AFGEN(CN2MAC,MA+MAH)-AFGEN(CN2MAC,MA))/DH
 MA=V/VS
MV=1./VS
 PH=(PD-P)/DH
 RH=(RD-RHO)/DH
 PD=CP*AFGEN(PVSZ,H+DH)
 RD=AFGEN(RHOZ,H+DH)/G
 VS=(1.4*P/RHO)**.5
 MH=MVS*VSH
 VSH=VS*(-1./RHO*RH+1./P*PH)/2.
 MVS=-V/VS**2
 XV=(AFGEN(LCPMCH,MA+MAV)-LCP        )/DV
 XH=(AFGEN(LCPMCH,MA+MAH)-LCP        )/DH
 MAV=MV*DV
 MAH=MH*DH
 DV=SUMFV/M*DELT
 DH=V*SIN(GAMMA)*DELT
 CA=AFGEN(CAMACH,MA)
 CN=AFGEN(CN2MAC,MA)
ALPHA=THETA-GAMMA
 W=WT-WDNOM*TIME
```

```
      M=W/G
      XBAR=LCP-LCM
      LCP=AFGEN(LCPMCH,MA)
      LCM=.00224*W+3.64
      L=LF-LCM
      I=.0554*W+32.9
      RHO=AFGEN(RHOZ,H)/G
     P=13.6*62.4/12.*AFGEN(PVSZ,H)
B1=SIN(ALPHA)/M
B2=-COS(ALPHA)/(M*V)
B3=L/I
      XK=F(VV,VG,VT,VH,GV,GG,GT,GH,TDV,TDG,TDT,TDH,HV,HG,ALPHA,M,L,I,...
      V,GAMMA,THEDOT,THETA,H,K28)
      TIMER FINTIM=5.12,DELT=.0010,PRDEL=0.01
      PRINT VV,VG,VT,VH,GV,GG,GT,GH,TDV,TDG,TDT,TDH,HV,HG,B1,B2,B3
     METHOD RKSFX
      END
      STOP
      FUNCTION F(VV,VG,VT,VH,GV,GG,GT,GH,TDV,TDG,TDT,TDH,HV,HG,ALPHA,AM,
     1AL,AI,V,GAMMA,THEDOT,THETA,H,K28)
COMMON MEM
      DIMENSION A(22 )
      DEFINE FILE 20(5122,22,U,K28)
      F=H
      IF(KEEP.NE.1)GO TO 60
      I=1
      A(I)=VV
      A(I+1)=VG
```

```
      A(I+2)=VT
      A(I+3)=VH
      A(I+4)=GV
      A(I+5)=GG
      A(I+6)=GT
      A(I+7)=GH
      A(I+8)=TDV
      A(I+9)=TDG
      A(I+10)=TDT
      A(I+11)=TDH
      A(I+12)=HV
      A(I+13)=HG
      A(I+14)=SIN(ALPHA)/AM
      B1=A(I+14)
      A(I+15)=-COS(ALPHA)/AM/V
      B2=A(I+15)
      A(I+16)=AL/AI
      B3=A(I+16)
      A(18)=V
      A(19)=GAMMA
      A(20)=THEDOT
      A(21)=THETA
      A(22)=H
      WRITE(20'K28)A
   60 CONTINUE
      RETURN
      END
ENDJOB
```

APPENDIX B

PROGRAM FOR CALCULATING THE TIME VARYING GAINS

```
//MISSILE JOB ,TSUEI,GROUP=J48961,USER=P22718,
// MSGLEVEL=(1,1),
// PASSWORD=MISSILE,TIME=6
/*ROUTE PRINT RMT1
/*JOBPARM LINES=5
// EXEC CSMP
//CSMP1.FT23F001 DD DSN=J48961.AB75,DISP=OLD
//CSMP1.FT24F001 DD UNIT=ONLINE,
// DSN=J48961.MDA75,SPACE=(20,5122),DISP=(NEW,CATLG),DCB=DSORG=DA
//CSMP1.SYSIN DD *
INITIAL
INCON K110=1.,K120=0.,K130=0.,K140=0.,K150=0.,K220=1.,K230=0.,...
K240=0.,K250=0.,K330=1.,K340=0.,K350=0.,K440=1.,K450=0.,K550=1.
CONSTANT WN=100.,R=.01,ZETA=.5
CONSTANT K28=5123,K32N=5123
FIXED K28,K32N
K11=K110
K12=K120
K13=K130
K14=K140
K15=K150
K22=K220
K23=K230
K24=K240
K25=K250
K33=K330
K34=K340
K35=K350
K44=K440
```

```
K45=K450
K55=K550
DYNAMIC
B11,B12,B13,B14,B15,B22,B23,B24,B25,B33,B34,B35,B44,B45,B55,B1,B3,B5...
     =F(K28,    K11 ,K12 ,K13 ,K14 ,K15 ,K22 ,K23 ,K24 ,K25 ,K33 , ...
K34 ,K35 ,K44 ,K45 ,K55     )
K11=INTGRL(K110,-B11)
K12=INTGRL(K120,-B12)
K13=INTGRL(K130,-B13)
K14=INTGRL(K140,-B14)
K15=INTGRL(K150,-B15)
K22=INTGRL(K220,-B22)
K23=INTGRL(K230,-B23)
K24=INTGRL(K240,-B24)
K25=INTGRL(K250,-B25)
K33=INTGRL(K330,-B33)
K34=INTGRL(K340,-B34)
K35=INTGRL(K350,-B35)
K44=INTGRL(K440,-B44)
K45=INTGRL(K450,-B45)
K55=INTGRL(K550,-B55)
KX=G(K11,K12,K13,K14,K15,K22,K23,K24,K25,K33,K34,K35,K44,K45,K55,...
K32N,    B1,B3,B5       )
TIMER DELT=1.0E-3,PRDEL=.0100,FINTIM=5.12
PRINT K11,K12,K13,K14,K15,K22,K23,K24,K25,K33,K34,K35,K44,K45,K55,B5,...
B3,B1,B55,B45,B44,B35,B34,B33,B25,B24,B23,B22,B15,B14,B13,B12,B11
METHOD RKSFX
END
STOP
      SUBROUTINE F(K28N,     AK11,AK12,AK13,AK14,AK15,AK22,AK23,AK24,
     3AK25,
```

```
     1AK33,AK34,AK35,AK44,AK45,AK55,B11,B12,B13,B14,B15,B22,B23,B24,
     2B25,B33,B34,B35,B44,B45,B55,B1,B3,B5)
COMMON MEMORY
      DIMENSION D(22)     ,AK(5,5),A(5,5),B(5,5),BK(5,5),AD(5,5),W(5)
      DEFINE FILE 23(5122,22,U,K28N)
      IF(KEEP.NE.1) GO TO 60
      IF(K28N.EQ.2) GO TO 60
      READ(23'K28N-2)D
      A(1,1)=D(1)
      A(1,2)=D(2)
      A(1,3)=0.
      A(1,4)=D(3)
      A(1,5)=D(4)
      A(2,1)=D(5)
      A(2,2)=D(6)
      A(2,3)=0.
      A(2,4)=D(7)
      A(2,5)=D(8)
      A(3,1)=D(9)
      A(3,2)=D(10)
      A(3,3)=0.
      A(3,4)=D(11)
      A(3,5)=D(12)
      A(4,1)=0.
      A(4,2)=0.
      A(4,3)=1.
      A(4,4)=0.
      A(4,5)=0.
      A(5,1)=D(13)
      A(5,2)=D(14)
```

```
A(5,3)=0.
A(5,4)=0.
A(5,5)=0.
W(1)=D(15)
B1   =D(15)
W(2)=D(16)
B3   =D(16)
W(3)=D(17)
B5   =D(17)
W(4)=0.
W(5)=0.
AK(1,1)=AK11
AK(1,2)=AK12
AK(1,3)=AK13
AK(1,4)=AK14
AK(1,5)=AK15
AK(2,2)=AK22
AK(2,3)=AK23
AK(2,4)=AK24
AK(2,5)=AK25
AK(3,3)=AK33
AK(3,4)=AK34
AK(3,5)=AK35
AK(4,4)=AK44
AK(4,5)=AK45
AK(5,5)=AK55
AK(2,1)=AK(1,2)
AK(3,1)=AK(1,3)
AK(4,1)=AK(1,4)
AK(5,1)=AK(1,5)
AK(3,2)=AK(2,3)
```

```
      AK(4,2)=AK(2,4)
      AK(5,2)=AK(2,5)
      AK(4,3)=AK(3,4)
      AK(5,3)=AK(3,5)
      AK(5,4)=AK(4,5)
      DO 9 I=1,5
      DO 9 J=1,5
      BK(I,J)=0.
      DO 9 L=1,5
    9 BK(I,J)=BK(I,J)+W(I)*W(L)*AK(L,J)
      DO 10 I=1,5
      DO 10 J=1,5
      AD(I,J)=0.
      DO 10 L=1,5
   10 AD(I,J)=AD(I,J)-AK(I,L)*A(L,J)-A(L,I)*AK(L,J)+AK(I,L)*BK(L,J)
      B11=AD(1,1)
      B12=AD(1,2)
      B13=AD(1,3)
      B14=AD(1,4)
      B15=AD(1,5)
      B22=AD(2,2)
      B23=AD(2,3)
      B24=AD(2,4)
      B25=AD(2,5)
      B33    =AD(3,3)
      B34    =AD(3,4)
      B35    =AD(3,5)
      B44    =AD(4,4)
      B45    =AD(4,5)
      B55    =AD(5,5)
```

```
      GO TO 60
   60 CONTINUE
      RETURN
      END
      FUNCTION   G(A11,A12,A13,A14,A15,A22,A23,A24,A25,A33,A34,A35,A44,
     1A45,A55,K32,    B1,B3,B5     )
COMMON MEMORY
      DIMENSION GAIN(250),GA(5),AK(5,5),B(5)
      DEFINE FILE 24(5122, 5 ,U,K32)
      G=A11
      IF(KEEP.NE.1) GO TO 60
      AK(1,1)=A11
      AK(1,2)=A12
      AK(1,3)=A13
      AK(1,4)=A14
      AK(1,5)=A15
      AK(2,2)=A22
      AK(2,3)=A23
      AK(2,4)=A24
      AK(2,5)=A25
      AK(3,3)=A33
      AK(3,4)=A34
      AK(3,5)=A35
      AK(4,4)=A44
      AK(4,5)=A45
      AK(5,5)=A55
      AK(2,1)=AK(1,2)
      AK(3,1)=AK(1,3)
      AK(4,1)=AK(1,4)
      AK(5,1)=AK(1,5)
      AK(3,2)=AK(2,3)
```

```
      AK(4,2)=AK(2,4)
      AK(5,2)=AK(2,5)
      AK(4,3)=AK(3,4)
      AK(5,3)=AK(3,5)
      AK(5,4)=AK(4,5)
      B(1)=B1
      B(2)=B3
      B(3)=B5
      B(4)=0.
      B(5)=0.
      DO 20 I=1,5
      GA(I)=0.
      DO 20 J=1,5
   20 GA(I)=GA(I)+B(J)*AK(J,I)
      IF(K32.EQ.2)GO TO 60
      WRITE(24'K32-2) GA
   60 CONTINUE
      RETURN
      END
ENDJOB
```

APPENDIX C

PROGRAM FOR EVALUATING MISS DISTANCE AND FUEL FACTOR

```
//MISSILE JOB , TSUEI,GROUP=J48961,USER=P22718,
// MSGLEVEL=(1,1),
// PASSWORD=MISSILE,TIME=6
/*ROUTE PRINT RMT1
/*JOBPARM LINES=8
// EXEC CSMP
//CSMP1.FT23F001 DD DSN=J48961.AB75,DISP=OLD
//CSMP1.FT24F001 DD DSN=J48961.MDA75,DISP=OLD
//CSMP1.SYSIN DD *
CONSTANT FNSL=8700.,AE=.2799,PSL=2116.8,S=.54542,WT=434.1,WDNOM=34.5,...
LF=7.715,P1=.02,P2=.02,G=32.2
FUNCTION PVSZ=0.,29.921,20000.,13.76,40000.,5.538,60000.,2.1178,...
80000.,.815462,100000.,.321922,120000.,.131789,140000.,.0572731,...
160000.,.0261996,180000.,.0120901,200000.,.00540654,400000.,.63407E-6
FUNCTION RHOZ=0.,.076474,20000.,.040773,40000.,.018826,60000.,...
.007199,80000.,.0027171,100000.,.0010438,120000.,.00040196,140000.,...
.00016315,160000.,.000071292,180000.,.00003361,200000.,.000015755,...
400000.,.1164E-8
FUNCTION CN2MAC=0.,4.2,.5,4.233,.8,4.51,1.2,5.7,1.5,5.14,2.,4.67,3.,...
4.53,4.,4.504
FUNCTION CAMACH=0.,.215,.25,.215,.5,.197,1.,.291,1.5,.258,2.,.234,...
2.5,.205,3.,.181,4.,.160
FUNCTION LCPMCH=0.,5.7,.5,5.725,.8,6.01,1.2,7.02,1.5,6.7,2.,5.93,3.,...
4.58,4.,4.49
INCON V0=1.,GAMMAD=30.,TDF=0.    ,THETAD=30.,H0=0.,X0=0.
PARAM VW=(0.,16.,48.,100.)
PARAMETER K40=1,K41=1,KP=3500.
INCON LD=1.,XBF=1.,CNF=1.,CAF=1.,FTF=1.,DELTA=0.001
FIXED K40,K41
INITIAL
W=WT-WDNOM*TIME
LCM=.00224*W+3.64
```

```
I=.0554*W+32.9
L=(LF-LCM)*LD
GAMMO=GAMMAD/57.35
THETAO=THETAD/57.35
THDOTO=-WT*L/I*DELT*TDF
      BOT=STEP(5.12)
DYNAMIC
VN,GN,TDN,TN,HN,KV,KG,KTD,KT,KH=F(K40,K41)
XBAR=(LCP-LCM)*XBF
L=(LF-LCM)*LFF
CN=AFGEN(CN2MAC,MA)*CNF
CA=AFGEN(CAMACH,MA)*CAF
FT=FT1*COS(DELTA)*FTF
FC=-((KV*(V-VN)+KG*(GAMMA-GN)+KTD*(THEDOT-TDN)+KT*(THETA-TN)+...
KH*(H-HN))+FT1*SIN(DELTA))*(1-BOT)
FT1=FNSL+AE*(PSL-P)
MISSD=(V-VN)**2+(GAMMA-GN)**2+(THEDOT-TDN)**2+(THETA-TN)**2+(H-HN)**2
FUEL=INTGRL(0.,FC**2)
COST=MISSD+FUEL
PROP=INTGRL(0.,ABS(FC))
SUMFV=(FT-A)*COS(ALPHA)+(FC-N)*SIN(ALPHA)-M*G*SIN(GAMMA)
V=INTGRL(VO,SUMFV/M)
SUMFG=(FT-A)*SIN(ALPHA)-(FC-N)*COS(ALPHA)-M*G*COS(GAMMA)
GAMMA=INTGRL(GAMMO,SUMFG/M/V)
SUMMO=FC*L-N*XBAR
THEDOT=INTGRL(THDOTO,SUMMO/I)
THETA=INTGRL(THETAO,THEDOT)
H=INTGRL(HO,V*SIN(GAMMA))
```

```
X=INTGRL(XO,V*COS(GAMMA))
XN=INTGRL(O.,VN*COS(GN))
VS=(1.4*P/RHO)**.5
ALPHA= THETA-GAMMA
W=WT-WDNOM*TIME
LCM=.00224*W+3.64
I=.0554*W+32.9
M=W/G
LCP=AFGEN(LCPMCH,MA)
RHO=AFGEN(RHOZ,H)/G
P=13.6*62.4/12.*AFGEN(PVSZ,H)
VRW=((V*COS(GAMMA)+VW)**2+(V*SIN(GAMMA))**2)**.5
A=S/2.*RHO*CA*VRW**2
N=S/2.*RHO*CN*ALPHA *VRW**2
MA=VRW/VS
VE=V-VN
GE=GAMMA-GN
TDE=THEDOT-TDN
TE=THETA-TN
HE=H-HN
XE=X-XN
ZD=V*SIN(GAMMA)
XD=V*COS(GAMMA)
XDN=VN*COS(GN)
ZDN=VN*SIN(GN)
MISSDI=(X-XN)**2+(H-HN)**2+(THETA-TN)**2+(XD-XDN)**2+(ZD-ZDN)**2+...
(THEDOT-TDN)**2
COSTI=MISSDI+FUEL
DV=V-VN
DG=GAMMA-GN
DTD=THEDOT-TDN
DT=THETA-TN
DH=H-HN
TERMINAL
```

```
      K40=1
      K41=1
TIMER  FINTIM=5.12,DELT=.001,PRDEL=0.08,OUTDEL=0.08
PRTPLT  KG,KV,KT,KH,KTD,VN,GN,TDN,TN,HN
PRINT  MISSD,COST,PROP,FUEL,KV,KG,KTD,KT,KH,FC,V,GAMMA,THEDOT,THETA,H,...
VN,GN,TDN,TN,HN,VE,GE,TDE,TE,HE,X,XN,XE,COSTI,MISSDI,CA,CN,ALPHA,W,...
M,A,N,MA,VRW,VW,P,RHO,LCP,LCM,L,I,VS,FT,XBAR
METHOD  RKSFX
END
PARAM  DELTA=(-3.,3.),FTF=1.
STOP
      SUBROUTINE   F(K40,K41,VN,GN,TDN,TN,HN,AV,AG,ATD,AT,AH)
COMMON MEM
      DIMENSION A(22),G(5)
      DEFINE FILE 23(5122,22,U,K40),24(5122,5,U,K41)
      IF(KEEP.NE.1) GO TO 40
      READ(23'K40) A
      READ(24'K41) G
      VN=A(18)
      GN=A(19)
      TDN=A(20)
      TN=A(21)
      HN=A(22)
      AV=G(1)
      AG=G(2)
      ATD=G(3)
      AT=G(4)
      AH=G(5)
   40 CONTINUE
      RETURN
      END
ENDJOB
```

APPENDIX D

EXPERIMENTAL RESULTS

TABLE D.1

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF ℓ (LD) ($Fc = 0$; $\theta o = 30°$)

State	$\ell + 5\%$	ℓ	$\ell - 5\%$	$\ell + 5\%$	ℓ	$\ell + 5\%$
		Wind = 0 fps			Wind = 16 fps	
V	3869.4	3869.4	3869.4	3866.7	3866.7	3866.7
γ	.39286	.39281	.39281	.39398	.39398	.39398
θ	.39201	.39201	.39201	.39302	.39302	.39302
$\dot{\theta}$	-.92559	-.02559	-.02559	-.02123	-.02123	-.02123
H	3746.3	3746.3	3746.3	3754.9	3754.9	3754.9
		Wind = 48 fps			Wind = 100 fps	
V	3861.3	3861.3	3861.3	3852.5	3852.5	3852.5
γ	.39106	.39106	.39106	.38612	.38612	.38612
θ	.38990	.38990	.38990	.38540	.38540	.38540
$\dot{\theta}$	-.01067	-.01067	-.01067	.004468	.004468	.004468
H	3727.1	3727.1	3727.1	3680.2	3680.2	3680.2

TABLE D.2

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF $\overline{X}$ (XBF) ($Fc = 0$; $\theta o = 30°$)

State	$\overline{X}$ - 5%	$\overline{X}$	$\overline{X}$ - 5%	$\overline{X}$ + 5%	$\overline{X}$	$\overline{X}$ - 5%
		Wind = 0 fps			Wind = 16 fps	
V	3869.4	3869.4	3869.4	3866.7	3866.7	3866.6
γ	.39200	.39281	.39333	.39321	.39398	.39454
θ	.39119	.39201	.39451	.39269	.39302	.39551
$\dot{\theta}$	-.00778	-.02559	-.01429	.009595	-.02123	-.01792
H	3741.3	3746.3	3751.5	3750.0	3754.9	3759.9
		Wind = 48 fps			Wind = 100 fps	
V	3861.4	3861.3	3861.3	3852.5	3852.5	3852.5
γ	.39029	.39106	.39175	.38543	.38612	.38696
θ	.39044	.38990	.39217	.38632	.38540	.38644
$\dot{\theta}$	-.010698	-.01067	-.02405	.00021	.00447	-.02226
H	3721.9	3727.1	3732.5	3674.4	3680.2	3686.2

TABLE D.3

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_N (CNF) (Fc = 0; θo = 30°)

State	C_N + 5%	C_N	C_N - 5%	C_N + 5%	C_N	C_N - 5%
		Wind = 0 fps			Wind = 16 fps	
V	3869.4	3869.4	3869.4	3866.7	3866.7	3866.6
γ	.39203	.39281	.39330	.39324	.39398	.39451
θ	.39127	.39201	.39456	.39275	.39302	.39555
$\dot{\theta}$	.00664	-.02559	-.01488	.00833	-.02123	-.01873
H	3741.5	3746.3	3751.3	3750.2	3754.9	3759.7
		Wind = 48 fps			Wind = 100 fps	
V	3861.4	3861.3	3861.3	3852.5	3852.5	3852.5
γ	.39033	.39106	.39172	.38547	.38612	.38693
θ	.39046	.38990	.39217	.38629	.38540	.38636
$\dot{\theta}$	.00940	-.01067	-.2532	-.000314	.004468	-.02338
H	3722.1	3727.1	3732.2	3674.8	3680.2	3685.8

TABLE D.4

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_A (CAF) (Fc = 0; θo = 30°)

State	C_A + 5%	C_A	C_A + 5%	C_A + 5%	C_A	C_A - 5%
		Wind = 0 fps			Wind = 16 fps	
V	3854.8	3869.4	3884.1	3851.9	3866.7	3881.5
γ	.39272	.39281	.39288	.39390	.39398	.39407
θ	.391.88	.39201	.39214	.39290	.39302	,39314
$\dot{\theta}$	-.02510	-.02559	-.02593	-.02053	-.02123	-.02174
H	3738.5	3746.3	3754.2	3746.9	3754.9	3762.9
		Wind = 48 fps			Wind = 100 fps	
V	3846.3	3861.3	3876.4	3837.1	3852.5	3868.0
γ	.39096	.39106	.39114	.38603	.38612	.38622
θ	.38980	.38990	.39000	.38535	.38540	.38547
$\dot{\theta}$	-.00988	-.01067	-.01143	.00494	.00447	.00401
H	3719.0	3727.1	3735.3	3671.8	3680.2	3688.7

TABLE D.5

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF FTF ($F_c = 0$; $\theta_o = 30°$)

State	99FTF%	FTF	98FTF%	99FTF%	FTF	98FTF%
		Wind = 0 fps			Wind = 16 fps	
V	3831.4	3869.4	3793.4	3828.7	3866.7	3790.7
γ	.39168	.39281	.39052	.39285	.39398	.39168
θ	.39089	.39201	.38972	.39189	.39302	.39071
$\dot{\theta}$	-.02596	-.02559	-.02576	-.02159	-.021231	-.021314
H	3701.2	3746.3	3756.0	3709.6	3754.9	3664.2
		Wind = 48 fps			Wind = 100 fps	
V	3823.4	3861.3	3785.4	3814.6	3852.5	3776.7
γ	.38987	.39106	.38864	.38485	.38612	.38354
θ	.38871	.38990	.38748	.38415	.38540	.38288
$\dot{\theta}$	-.01079	-.01067	-.01025	.004520	.00447	.00514
H	3681.7	3727.1	3636.2	3634.5	3680.2	3588.8

TABLE D.6

STATE VARIATION RESULTING FROM 3 PERCENT VARIATION OF Δ ($F_c = 0$; $\theta_o = 30°$)

State	Δ	0.003Δ	Δ	0.003Δ
	Wind = 0 fps		Wind = 16 fps	
V	3869.4	3842.1	3866.7	3841.3
γ	.39281	.65791	.39398	.65007
θ	.39201	.67602	.39302	.66773
$\dot{\theta}$	-.02559	.04049	-.02123	-.00463
H	3746.3	5668.8	3754.9	5611.9
	Wind = 0 fps		Wind = 100 fps	
V	3861.3	3839.3	3852.5	3834.9
γ	.39106	.63431	.38612	.60875
θ	.38990	.64931	.38540	.61649
$\dot{\theta}$	-.01067	-.07804	.00447	-.11725
H	3727.1	5496.4	8762.5	5307.1

Maximum Percent Variation for Each State:

States	Percent Variation	Occurrence
V	.4568	
γ	-56.7660	
θ	-59.9611	Wind = fps
$\dot{\theta}$	-2523.0425	
H	39.4340	

TABLE D.7

STATE VARIATION RESULTING FROM 1 PERCENT VARIATION OF Δ ($F_c = 0$; $\theta_o = 30°$)

State	Δ	0.001Δ	Δ	0.001Δ
	Wind = 0 fps		Wind = 16 fps	
V	3869.4	3863.0	3866.7	3860.7
γ	.39281	.48102	.39398	.47739
θ	.39201	.48628	.39302	.48175
$\dot{\theta}$	-.01067	-.05545	.00447	-.03242
H	3746.3	4427.9	3754.9	4396.8
	Wind = 48 fps		Wind = 100 fps	
V	3861.3	3856.1	3852.5	3848.3
γ	.39106	.46998	.38612	.45777
θ	.38990	.47249	.38540	.45825
$\dot{\theta}$	-.01067	-.05545	.00447	-.03242
H	3727.1	4333.5	8762.5	4229.8

Maximum Percent Variation for Each State:

States	Percent Variation	Occurrence
V	.1091	
γ	-18.5564	
θ	-18.9024	Wind = fps
$\dot{\theta}$	-625.2796	
H	51.7284	

TABLE D.8

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF ℓ (LD) ($Fc = 0$; $\theta o = 75°$)

State	ℓ + 5%	ℓ	ℓ - 5%	ℓ + 5%	ℓ	ℓ - 5%
		Wind = 0 fps			Wind = 16 fps	
V	3826.5	3826.5	3826.5	3825.7	3825.7	3825.7
γ	1.2680	1.2680	1.2680	1.2679	1.2683	1.2679
θ	1.2684	1.2684	1.2684	1.2683	1.2683	1.2683
$\dot{\theta}$	-.00376	-.00376	-.00376	-.00108	-.00108	-.00108
H	8777.9	8777.9	8777.9	8776.0	8776.0	8776.0
		Wind = 48 fps			Wind = 100 fps	
V	3824.0	3824.0	3824.0	3820.9	3820.9	3820.9
γ	1.2674	1.2674	1.2674	1.2662	1.2662	1.2662
θ	1.2678	1.2678	1.2666	1.2666	1.2666	1.2666
$\dot{\theta}$	-.00248	-.00248	-.00248	-.00498	-.00498	-.00498
H	8771.4	8771.4	8771.4	8762.5	8762.5	8762.5

TABLE D.9

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF $\overline{X}$ (XBF) ($F_c = 0$; $\theta_o = 75°$)

State	$\overline{X}$ - 5%	$\overline{X}$	$\overline{X}$ - 5%	$\overline{X}$ - 5%	$\overline{X}$	$\overline{X}$ - 5%
		Wind = 0 fps			Wind = 16 fps	
V	3826.5	3826.5	3826.5	3825.7	3825.7	3825.7
γ	1.2679	1.2680	1.2682	1.2677	1.2679	1.2680
θ	1.2680	1.2684	1.2680	1.2677	1.2683	1.2679
$\dot{\theta}$	-.00911	-.00376	-.00310	-.00916	-.00108	-.00334
H	8777.4	8777.9	8778.4	8775.5	8776.0	8776.5
		Wind = 48 fps			Wind = 100 fps	
V	3824.0	3824.0	3824.0	3820.9	3820.9	3820.9
γ	1.2672	1.2674	1.2676	1.2660	1.2662	1.2664
θ	1.2672	1.2678	1.2675	1.2658	1.2666	1.2665
$\dot{\theta}$	-.00894	-.00248	-.00366	-0.00767	-.00498	-.00337
H	8770.9	8771.4	8772.0	8761.9	8762.5	8763.1

TABLE D.10

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_N (CNF) ($F_c = 0$; $\theta_o = 75°$)

State	$C_N + 5\%$	C_N	$C_N - 5\%$	$C_N + 5\%$	C_N	$C_N - 5\%$
		Wind = 0 fps			Wind = 16 fps	
V	3826.5	3826.5	3826.5	3825.7	3825.7	3825.7
γ	1.2679	1.2680	1.2682	1.2677	1.2679	1.2680
θ	1.2680	1.2684	1.2680	1.2678	1.2683	1.2678
$\dot{\theta}$	-.00875	-.00376	-.00364	-.00872	-.00108	-.00384
H	8777.5	8777.9	8778.4	8775.5	8776.0	8776.5
		Wind = 48 fps			Wind = 100 fps	
V	3824.0	3824.0	3824.0	3820.9	3820.9	3820.9
γ	1.2672	1.2674	1.2675	1.2660	1.2662	1.2664
θ	1.2672	1.2678	1.2674	1.2658	1.2662	1.2665
$\dot{\theta}$	-.00852	-.00248	-.00403	-.00726	-.00498	-.00374
H	8771.0	8771.4	8772.0	8761.9	8762.5	8763.1

TABLE D.11

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_A (CAF) (Fc = 0; θo = 75°)

State	$C_A + 5\%$	C_A	C_A	$C_A + 5\%$	C_A	$C_A - 5\%$
		Wind = 0 fps			Wind = 16 fps	
V	3813.2	3826.5	3839.9	3812.4	3825.7	3839.1
γ	1.2680	1.2680	1.2681	1.2678	1.2679	1.2679
θ	1.2684	1.2684	1.2684	1.2683	1.2683	1.2683
$\dot{\theta}$	-.0007695	-.00376	-.0000132	-.00141	-.00108	-.0006565
H	8761.1	8777.9	8794.9	8759.0	8776.0	8793.0
		Wind = 48 fps			Wind = 100 fps	
V	3810.6	3824.0	3837.5	3807.4	3820.9	3834.6
γ	1.2674	1.2674	1.2674	1.2662	1.2662	1.2663
θ	1.2678	1.2678	1.2678	1.2665	1.2666	1.2666
$\dot{\theta}$	-.00276	-.00248	-.00208	-.00538	-.00498	-.00466
H	8754.3	8771.4	8788.6	8745.1	8762.5	8780.0

TABLE D.12

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF FTF ($F_c = 0$; $\theta_o = 75°$)

State	99FTF%	FTF	98FTF%	99FTF%	FTF%	98FTF%
		Wind = 0 fps			Wind = 16 fps	
V	3787.6	3826.5	3748.7	3786.8	3825.7	3747.9
γ	1.2677	1.2680	1.2673	1.2675	1.2679	1.2671
θ	1.2681	1.2684	1.2677	1.2679	1.2683	1.2676
$\dot{\theta}$	-.00057	-.00376	-.000524	-.00123	-.00108	-.00117
H	8687.2	8777.9	8696.5	8685.3	8776.0	8594.5
		Wind = 48 fps			Wind = 100 fps	
V	3785.1	3824.0	3746.2	3782.0	3820.9	3743.1
γ	1.2670	1.2674	1.2666	1.2658	1.2662	1.2654
θ	1.2674	1.2678	1.2670	1.2662	1.2666	1.2658
$\dot{\theta}$	-.00264	-.00248	-.00268	-.00531	-.00498	-.00542
H	8680.7	8771.4	8590.0	8671.8	8762.5	8581.1

TABLE D.13

STATE VARIATIONS RESULTING FROM 3 PERCENT VARIATION OF Δ ($Fc = 0$; $\theta_o = 75°$)

State	Δ	0.003Δ	Δ	0.003Δ
	Wind = 0 fps		Wind = 16 fps	
V	3826.5	3810.1	3825.7	3810.2
γ	1.2680	1.5542	1.2679	1.5529
θ	1.2684	1.5583	1.2683	1.5570
$\dot{\theta}$	-.000376	.15066	-.00108	.15038
H	8777.9	9104.8	8776.0	9105.1
	Wind = 48 fps		Wind = 100 fps	
V	3824.0	3810.6	3820.9	3811.0
γ	1.2674	1.5484	1.2662	1.5384
θ	1.2674	1.5535	1.2666	1.5455
$\dot{\theta}$	-.00248	.15012	-.00498	.14808
H	8771.4	9105.7	8762.5	9105.5

TABLE D.14

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF Δ ($Fc = 0$; $\theta o = 75°$)

State	Δ	0.001Δ	Δ	0.001Δ
	Wind = 0 fps		Wind = 16 fps	
V	3826.5	3823.5	3825.7	3822.9
γ	1.2680	1.3624	1.2679	1.3616
θ	1.2684	1.3647	1.2683	1.3641
$\dot{\theta}$	-.000376	.05466	-.00108	.05442
H	8777.9	8967.6	8776.0	8965.1
	Wind = 48 fps		Wind = 100 fps	
V	3824.0	3821.8	3820.9	3819.7
γ	1.2674	1.3594	1.2662	1.3548
θ	1.2674	1.3625	1.2666	1.3588
$\dot{\theta}$	-.00248	.05250	-.00498	.04520
H	8771.4	8959.	8762.5	8946.4

TABLE D.15

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF ℓ ($Fc = -1000*\theta*\alpha$; $\theta o = 30°$)

State	ℓ+5%	ℓ	ℓ-5%	ℓ+5%	ℓ	ℓ-5%
		Wind = 0 fps			Wind = 16 fps	
V	3874.6	3874.6	3874.5	3871.8	3871.7	3871.7
γ	.31496	.31556	.31641	.31492	.31563	.31637
θ	.31496	.31556	.31641	.31492	.31563	.31637
$\dot{\theta}$	-.00789	-.00786	-.00788	-.00792	-.00793	-.00792
H	3084.4	3090.6	3097.0	3082.5	3088.6	3095.1
		Wind = 48 fps			Wind = 100 fps	
V	3866.1	3866.0	3866.0	3856.7	3856.6	3856.6
γ	.31484	.31555	.31629	.31471	.31541	.31615
θ	.31484	.31555	.31629	.31471	.31541	.31615
$\dot{\theta}$	-.00790	-.00793	-.00794	-.00794	-.00795	-.00796
H	3078.4	3084.5	3091.0	3071.7	3077.7	3084.2

TABLE D.16

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF $\bar{x}$ ($Fc = -1000*\theta*\alpha$; $\theta o = 30°$)

State	$\bar{x}$-5%	$\bar{x}$	$\bar{x}$+5%	$\bar{x}$-5%	$\bar{x}$	$\bar{x}$+5%
		Wind = 0 fps			Wind = 16 fps	
V	3874.6	3874.6	3874.6	3871.7	3871.7	3871.8
γ	.31566	.31556	.31566	.31563	.31563	.31563
θ	.31566	.31556	.31566	.31563	.31563	.31563
$\dot{\theta}$	-.00791	-.00786	-.00791	-.00792	-.00793	-.00788
H	3090.6	3090.6	3090.5	3088.6	3088.6	3088.6
		Wind = 48 fps			Wind = 100 fps	
V	3866.0	3866.0	3866.1	3856.6	3856.6	3856.6
γ	.31555	.31555	3.1555	.31142	.31541	.31540
θ	.31555	.31555	.31555	.31542	.31541	.31540
$\dot{\theta}$	-.00793	-.00793	-.00793	-.00796	-.00795	-.00795
H	3084.6	3084.5	3084.5	3077.8	3077.7	3077.7

TABLE D.17

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_N (Fc = -1000*θ*α; θo = 30°)

State	C_N+5%	C_N	C_N-5%	C_N+5%	C_N	C_N-5%
		Wind = 0 fps			Wind = 16 fps	
V	3874.6	3874.6	3874.6	3871.8	3871.7	3871.8
γ	.31566	.31556	.31566	.31553	.31563	.31562
θ	.31566	.31566	.31566	.31563	.31563	.31562
$\dot{\theta}$	-.00794	-.00786	-.00793	-.00791	-.00793	-.00793
H	3090.6	3090.6	3090.5	3088.6	3088.6	3088.6
		Wind = 48 fps			Wind = 100 fps	
V	3866.0	3866.0	3866.0	3856.6	3856.6	3856.6
γ	.31555	.31555	.31555	.31541	.31541	.31541
θ	.31555	.31555	.31555	.31541	.31541	.31541
$\dot{\theta}$	-.00793	-.00793	-.00791	-.00792	-.00795	-.00793
H	3084.6	3084.5	3084.5	3077.7	3077.7	3077.7

TABLE D.18

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_A ($Fc = -1000*\theta*\alpha$; $\theta o = 30°$)

State	C_A+5%	C_A	C_A-5%	C_A+5%	C_A	C_A-5%
	Wind = 0 fps			Wind = 16 fps		
V	3859.8	3874.6	3889.5	3856.8	3871.7	3886.8
γ	.31558	.31556	.31575	.31554	.31563	.31571
θ	.31558	.31556	.3157	.31554	.31563	.31571
$\dot{\theta}$	-.00792	-.00786	-.00789	-.00793	-.00793	-.00784
H	3084.0	3090.6	3097.1	3081.9	3088.6	3095.3
	Wind = 48 fps			Wind = 100 fps		
V	3850.9	3866.0	3881.3	3841.0	3856.6	3872.3
γ	.31546	.31555	.31564	.31531	.31541	.31551
θ	.31546	.31555	.31564	.31531	.31541	.31551
$\dot{\theta}$	-.00797	-.00793	-.00794	-.00798	-.00795	-.00786
H	3077.7	3084.5	3091.4	3070.6	3077.7	3084.9

TABLE D.19

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF FTF ($F_c = -1000*\theta*\alpha$; $\dot{\theta}_o = 30°$)

State	98FTF%	FTF	98FTF%	99FTF%	FTF	98FTF%
		Wind = 0 fps			Wind = 16 fps	
V	3836.7	3874.6	3798.9	3833.9	3871.7	3796.1
γ	.31370	.31556	.31170	.31367	.31563	.31166
θ	.31370	.31556	.31170	.31367	.31563	.31166
$\dot{\theta}$	-.00799	-.00786	-.00809	-.00802	-.00793	-.00808
H	3044.7	3090.6	2998.9	3042.7	3088.6	2996.8
		Wind = 48 fps			Wind = 100 fps	
V	3828.2	3866.0	3790.4	3818.9	3856.6	3781.1
γ	.31358	.31555	.31158	.31344	.31541	.31144
θ	.31358	.31555	.31158	.31344	.31541	.31144
$\dot{\theta}$	-.00801	-.00793	-.00807	-.00804	-.00795	-.00810
H	3038.7	3084.5	2992.9	3032.0	3077.7	2986.2

TABLE D.20

STATE VARIATIONS RESULTING FROM 3 PERCENT VARIATION OF Δ ($Fc = -1000*\theta*\alpha$; $\theta o = 30°$)

State	Δ	$.003\Delta$	Δ	$.003\Delta$
	Wind = 0 fps		Wind = 16 fps	
V	3874.6	3874.7	3871.7	3871.3
γ	.31556	.32233	.31563	.32230
θ	.31556	.32233	.31563	.32230
$\dot{\theta}$	-.00786	-.00788	-.00793	-.00791
	3090.6	3148.3	3088.6	3146.3
	Wind = 48 fps		Wind = 100 fps	
V	3866.0	3865.6	3856.6	3856.2
γ	.31555	.32223	.31541	.32210
θ	.31555	.32223	.31541	.32210
$\dot{\theta}$	-.00793	-.00789	-.00795	-.00795
H	3084.5	3142.3	3077.7	3135.5

TABLE D.21

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF Δ ($Fc = -1000*\theta*\alpha$; $\theta o = 30°$)

State	Δ	.001	Δ	.001Δ
	Wind = 0 fps		Wind = 16 fps	
V	3874.6	3874.4	3871.7	3871.6
γ	.31556	.31789	.31563	.31785
θ	.31556	.31789	.31563	.31785
$\dot{\theta}$	-.00786	-.00792	-.00793	-.00789
H	3090.6	3109.8	3088.6	3107.8
	Wind 48 fps		Wind = 100 fps	
V	3866.0	3865.9	3856.6	3856.5
γ	.31555	.31777	.31541	.31764
θ	.31555	.31777	.31541	.31764
$\dot{\theta}$	-.00793	-.00791	-.00795	-.00791
H	3084.5	3103.8	3077.7	3097.0

TABLE D.22

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF ℓ ($F_c = -1000*\theta*\alpha$; $\theta_o = 75°$)

State	ℓ+5%	ℓ	ℓ-5%	ℓ+5%	ℓ	ℓ-5%
		Wind = 0 fps			Wind = 16 fps	
V	3826.6	3826.7	3826.5	3825.6	3825.7	3825.6
γ	1.2391	1.2395	1.2393	1.2380	1.2388	1.2387
θ	1.2391	1.2395	1.2393	1.2380	1.2388	1.2387
$\dot{\theta}$	-.00302	-.00282	-.00300	-.00562	-.00170	-.00350
H	8697.3	8698.9	8697.3	8693.3	8699.7	8695.1
		Wind = 48 fps			Wind = 100 fps	
V	3823.8	3824.0	3823.7	3820.3	3820.5	3820.7
γ	1.2391	1.2396	1.2394	1.2377	1.2389	1.2398
θ	1.2391	1.2396	1.2394	1.2377	1.2389	1.2398
$\dot{\theta}$	-.00292	-.00304	-.00154	-.00152	-.00648	-.00300
H	8691.9	8694.9	8691.5	8680.1	8685.3	8689.1

TABLE D.23

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF $\bar{x}$ ($Fc = -1000*\theta*\alpha$; $\theta o = 75°$)

State	$\bar{x}$+5%	$\bar{x}$	$\bar{x}$-5%	$\bar{x}$+5%	$\bar{x}$	$\bar{x}$-5%
	Wind = 0 fps			Wind = 16 fps		
V	3825.8	3826.7	3826.7	3825.8	3825.7	3825.7
γ	1.2345	1.2395	1.2397	1.2396	1.2388	1.2391
θ	1.2345	1.2395	1.2397	1.2396	1.2388	1.2391
$\dot{\theta}$	-.00075	-.00282	-.00299	-.00313	-.00170	-.00293
H	8680.5	8698.9	8700.0	8697.6	8694.7	8695.4
	Wind = 48 fps			Wind = 100 fps		
V	3823.7	3824.0	3823.5	3820.0	3820.5	3820.4
γ	1.2391	1.2396	1.2372	1.2361	1.2389	1.2389
θ	1.2391	1.2396	1.2372	1.2361	1.2389	1.2389
$\dot{\theta}$	-.00243	-.00304	-.00609	-.00612	-.00648	-.00193
H	8690.4	8694.9	8687.6	8676.3	8685.3	8683.9

TABLE D.24

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_N (Fc = $-1000*\theta*\alpha$; $\theta o = 75°$)

State	C_N+5%	C_N	C_N-5%	C_N+5%	C_N	C_N-5%
		Wind = 0 fps			Wind = 16 fps	
V	3826.7	3826.7	3826.3	3825.7	3825.7	3825.3
γ	1.2395	1.2395	1.2376	1.2388	1.2388	1.2369
θ	1.2395	1.2395	1.2376	1.2388	1.2388	1.2369
$\dot{\theta}$	-.00282	-.00282	-.00601	-.00170	-.00170	-.00235
H	8698.9	86989	8692.4	8694.7	8694.7	8687.5
		Wind = 48 fps			Wind = 100 fps	
V	3824.0	3824.0	3823.8	3820.5	382p.5	3820.4
γ	1.2396	1.2396	1.2389	1.2389	1.2389	1.2393
θ	1.2396	1.2396	1.2389	1.2389	1.2389	1.2393
$\dot{\theta}$	-.00304	-.00304	-.00296	-.00648	-.00698	-.00305
H	8694.9	8694.9	8693.2	8685.3	8685.3	8684.8

TABLE D.25

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_A (Fc = -1000*θ*α; θo = 75°)

State	C_A+5%	C_A	C_A-5%	C_A+5%	C_A	C_A-5%
		Wind = 0 fps			Wind = 16 fps	
V	3813.2	3826.7	3839.5	3812.4	3825.7	3839.0
γ	1.2387	1.2395	1.2631	1.2392	1.2388	1.2387
θ	1.2387	1.2395	1.2631	1.2392	1.2388	1.2387
$\dot{\theta}$	-.00329	-.00282	-.00205	-.00305	-.00170	-.00352
H	8679.6	8698.9	8702.6	8680.3	8694.7	8710.6
		Wind = 48 fps			Wind = 100 fps	
V	3810.3	3824.0	3837.2	3806.9	3820.5	3834.2
γ	1.2394	1.2396	1.2378	1.2388	1.2389	1.2395
θ	1.2394	1.2396	1.2378	1.2388	1.2389	1.2395
$\dot{\theta}$	-.00303	-.00304	-.00534	-.00316	-.00648	-.00294
H	8675.0	8694.9	8707.0	8667.1	8685.3	8703.9

TABLE D.26

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF FTF ($Fc = -1000*\theta*\alpha$; $\theta o = 75°$)

State	99FTF%	FTF	98FTF%	99FTF%	FTF	98FTF%
		Wind = 0 fps			Wind = 16 fps	
V	3787.7	3826.7	3748.9	3787.0	3825.7	3747.7
γ	1.2388	1.2395	1.2380	1.2391	1.2388	1.2373
θ	1.2388	1.2395	1.2380	1.2391	1.2388	1.2373
$\dot{\theta}$	-.00288	-.00282	-.00309	-.00291	-.00170	-.00335
H	8606.5	8698.9	8517.3	8608.3	8694.7	8511.0
		Wind = 48 fps			Wind = 100 fps	
V	3785.0	3824.0	3745.6	3781.6	3820.5	3742.8
γ	1.2387	1.2396	1.2363	1.2382	1.2389	1.2376
θ	1.2387	1.2396	1.2363	1.2382	1.2389	1.2376
$\dot{\theta}$	-.00288	-.00304	-.00463	-.00299	-.00648	-.00293
H	8602.3	8694.9	8503.0	8594.8	8685.3	8504.5

TABLE D.27

STATE VARIATIONS RESULTING FROM 3 PERCENT VARIATION OF Δ (Fc = -1000*θ*α; θo = 75°)

State	Δ	0.003Δ	Δ	0.003Δ
	Wind = 0 fps		Wind = 16 fps	
V	3826.7	3825.7	3825.7	3824.6
γ	1.2395	1.2429	1.2388	1.2419
θ	1.2395	1.2429	1.2388	1.2419
$\dot{\theta}$	-.00282	-.00186	-.00170	-.00418
H	8698.9	8704.4	8694.7	8697.9
	Wind = 48 fps		Wind = 100 fps	
V	3824.0	3823.2	3820.5	
γ	1.2396	1.2947	1.2389	
θ	1.2396	1.2447	1.2389	
$\dot{\theta}$	-.00304	-.01627	-.00648	
H	8694.9	8704.2	8685.3	

TABLE D.28

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF Δ ($F_c = -1000*\theta*\alpha$; $\theta_o = 75°$)

State	Δ	0.001Δ	Δ	0.001Δ
	Wind = 0 fps		Wind = 16 fps	
V	3826.7	3825.9	3825.7	3825.0
γ	1.2395	1.2391	1.2388	1.2389
θ	1.2395	1.2391	1.2388	1.2389
$\dot{\theta}$	-.00282	-.00210	-.00170	-.00642
H	8698.9	8695.6	8694.7	8691.5
	Wind = 48 fps		Wind = 100 fps	
V	3824.0	3823.1	1.2389	
γ	1.2396	1.2388	1.2389	
θ	1.2396	1.2388	1.2389	
$\dot{\theta}$	-.00304	-.00185	-.00648	
H	8694.9	.8688.0	8685.3	

TABLE D.29

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF ℓ ($Fc = -K^T \Delta \underline{X}$; $\theta o = 30°$)

State	ℓ + 5%	ℓ	ℓ - 5%	ℓ + 5%	ℓ	ℓ + 5%
		Wind = 0 fps			Wind = 16 fps	
V	3869.2	3869.2	3869.2	3866.7	3866.7	3866.7
γ	.39369	.39369	.39369	.39010	.39010	.39010
θ	.39291	.39291	.39291	.38910	.38910	.38910
$\dot{\theta}$	-.02538	-.02538	-.02538	-.02034	-.02034	-.02034
H	3753.8	3753.8	3753.8	3723.8	3723.8	3723.8
		Wind = 48 fps			Wind = 100 fps	
V	3861.6	3861.6	3861.6	3853.0	3853.0	3853.0
γ	.38231	.38231	.38231	.36856	.36786	.36856
θ	.38112	.38112	.38112	.36786	.36786	.36786
$\dot{\theta}$	-.00822	-.00822	-.00822	-.00841	.00841	.00841
H	3659.6	3659.6	3659.6	3545.6	3545.6	3545.6

TABLE D.30

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF $\bar{x}$ ($Fc = -K^T \Delta \underline{X}$; $\theta o = 30°$)

State	$\bar{X} + 5\%$	$\bar{X}$	$\bar{X} - 5\%$	$\bar{X} - 5$	$\bar{X}$	$\bar{X} - 5\%$
		Wind = 0 fps			Wind = 16 fps	
V	3869.3	3869.2	3869.2	3866.8	3866.7	3866.6
γ	.39204	.39369	.39510	.38838	.39010	.39160
θ	.39125	.39291	.39627	.38792	.38910	.39255
$\dot{\theta}$	.00744	-.02538	-.01410	.01071	-.02034	-.01868
H	3742.4	3753.8	3765.7	3711.9	3723.8	3736.2
		Wind = 48 fps			Wind = 100 fps	
V	3861.6	3861.6	3861.5	3853.1	3853.0	3852.9
γ	.38053	.38231	.38408	.36668	.36856	.37066
θ	.38080	.38112	.38439	.36768	.36786	.36991
$\dot{\theta}$	.01187	-.00822	-.02410	.00109	.00841	-.02031
H	3646.5	3659.6	3673.0	3530.6	3545.6	3561.1

TABLE D.31

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_N ($Fc = -K^T \Delta \underline{X}$; $\theta o = 30°$)

State	$C_N + 5\%$	C_N	$C_N - 5\%$	$C_N + 5\%$	C_N	$C_N - 5\%$
		Wind = 0 fps			Wind = 16 fps	
V	3869.3	3869.2	3869.2	3866.8	3866.7	3866.6
γ	.39212	.39369	.39502	.38846	.39010	.39152
θ	.39138	.39291	.39628	.38804	.38910	.39254
$\dot{\theta}$	.00637	-.02538	-.01461	.00945	-.02034	-.01954
H	3743.0	3753.8	3765.0	3712.5	3723.8	3735.5
		Wind = 48 fps			Wind = 100 fps	
V	3861.6	3861.6	3861.5	3853.1	3853.0	3852.9
γ	.38062	.38231	.38399	.36680	.36856	.37055
θ	.38087	.38112	.38432	.36772	.36786	.36974
$\dot{\theta}$	-.01052	-.00822	-.02540	.00058	.00841	-.02131
H	3647.2	3659.6	3672.2	3531.5	3545.6	3560.2

TABLE D.32

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_A ($Fc = -K^T \Delta \underline{X}$; $\theta o = 30°$)

State	$C_A + 5\%$	C_A	$C_A - 5\%$	$C_A + 5\%$	C_A	$C_A - 5\%$
	Wind = 0 fps			Wind = 16 fps		
V	3854.6	3869.2	3884.0	3852.0	3866.7	3881.5
γ	.39351	.39369	.39387	.38989	.39010	.39026
θ	.39269	.39291	.39314	.38884	.38910	.38929
$\dot{\theta}$	-.02483	-.02538	-.02576	-.01960	-.02034	-.02099
H	3745.4	3753.8	3762.3	3715.3	3723.8	3732.4
	Wind = 48 fps			Wind = 100 fps		
V	3846.6	3861.6	3876.7	3837.6	3853.0	3868.6
γ	.38210	.38231	.38251	.36832	.36856	.36879
θ	.38091	.38112	.38133	.36768	.36786	.36806
$\dot{\theta}$	-.00731	-.00822	-.00912	.00890	.00841	.00786
H	3650.8	3659.6	3668.3	3536.6	3545.6	3554.6

TABLE D.33

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF FTF ($Fc = -K^T \Delta \underline{X}$; $\theta o = 30°$)

State	99FTF%	FTF	98FTF%	99FTF%	FTF	98FTF%
		Wind = 0 fps			Sind = 16 fps	
V	3831.4	3869.2	3793.5	3828.9	3866.7	3791.0
γ	.38891	.39369	.38399	.38520	.39010	.38018
θ	.38815	.39291	.38323	.38420	.38910	.37918
$\dot{\theta}$	-.02480	-.02538	-.02383	-.01957	-.02034	-.01837
H	3682.0	3753.8	3609.9	3651.5	3723.8	3579.0
		Wind = 48 fps			Wind = 100 fps	
V	3823.8	3861.6	3786.0	3815.3	3853.0	3777.5
γ	.37723	.38231	.37199	.36312	.36856	.35751
θ	.37606	.38112	.37086	.36249	.36786	.35699
$\dot{\theta}$	-.00711	-.00822	-.00542	.00957	.00841	.01133
H	3586.3	3659.6	3512.6	3470.5	3545.6	3395.1

TABLE D.34

STATE VARIATIONS RESULTING FROM 3 PERCENT VARIATION OF Δ ($F_c = -K^T \Delta \underline{X}$; $\theta_o = 30°$)

State	Δ	0.003Δ	Δ	0.003Δ
	Wind = 0 fps		Wind = 16 fps	
V	3869.2	3868.5	3866.7	3865.9
γ	.39369	.41147	.39010	.40814
θ	.39291	.41081	.38910	.40724
$\dot{\theta}$	-.02538	-.02557	-.02043	-.02107
H	3753.8	3880.4	3723.8	3852.7
	Wind = 48 fps		Wind = 100 fps	
V	3861.6	3860.8	3853.0	3852.3
γ	.38231	.40095	.36856	.38823
θ	.38112	.39987	.36786	.38758
$\dot{\theta}$	-.00822	-.01003	.00841	.00589
H	3659.6	3793.2	3545.6	3687.9

TABLE D.35

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF Δ ($Fc = -K^T \Delta \underline{X}$; $\theta o = 30°$)

State	Δ	0.001Δ	Δ	0.001Δ
	Wind = 0 fps		Wind = 16 fps	
V	3869.2	3869.0	3866.7	3866.4
γ	.39369	.39962	.39010	.39609
θ	.39291	.39888	.38910	.39512
$\dot{\theta}$	-.02538	-.02545	-.02034	3766.9
H	3753.8	3796.1	3723.8	3766.9
	Wind = 48 fps		Wind = 100 fps	
V	3861.6	3861.3	3853.0	3852.8
γ	.38231	.38852	.36856	.37512
θ	.38112	.38737	.36786	.37443
$\dot{\theta}$	-.00822	-.00883	.00841	.00756
H	3659.6	3704.2	3545.6	3593.1

TABLE D.36

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF ℓ ($F_c = -K^T \Delta \underline{X}$; $\theta_o = 75°$)

State	ℓ + 5%	ℓ	ℓ - 5%	ℓ + 5%	ℓ	ℓ - 5%
		Wind = 0 fps			Wind = 16 fps	
V	3826.4	3826.4	3826.4	3825.5	3825.5	3825.5
γ	1.2649	1.2649	1.2649	1.2645	1.2645	1.2645
θ	1.2653	1.2653	1.2653	1.2649	1.2649	1.2649
$\dot{\theta}$	-.000385	-.000385	-.000385	-.00083	-.00083	-.00083
H	8768.1	8768.1	8768.1	8765.7	8765.7	8765.7
		Wind = 48 fps			Wind = 100 fps	
V	3823.8	3823.8	3823.8	3820.7	3820.7	3820.7
γ	1.2637	1.2637	1.2637	1.2620	1.2620	1.2620
θ	1.2641	1.2641	1.2641	1.2624	1.2620	1.2624
$\dot{\theta}$	-.00221	-.00221	-.00221	-.00515	-.00515	-.00515
H	8760.3	8760.3	8760.3	8750.0	8750.0	8750.0

TABLE D.37

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF $\bar{X}$ ($F_c = -K^T \Delta \underline{X}$; $\theta_o = 75°$)

State	X - 5%	$\bar{X}$	$\bar{X}$ - 5%	$\bar{X}$ + 5%	$\bar{X}$	$\bar{X}$ - 5%
		Wind = 0 fps			Wind = 16 fps	
V	3826.4	3826.4	3826.4	3825.5	3825.5	3825.5
γ	1.2646	1.2649	1.2651	1.2643	1.2645	1.2648
θ	1.2647	1.2653	1.2648	1.2643	1.2649	1.2646
$\dot{\theta}$	-.00972	-.000385	-.00327	-.00979	-.00083	.00350
H	8767.3	8768.1	8768.7	8765.0	8765.7	8766.5
		Wind = 48 fps			Wind = 100 fps	
V	3823.8	3823.8	3823.8	3820.7	3820.7	3820.7
γ	1.2635	1.2637	1.2640	1.2617	1.2620	1.2623
θ	1.2634	1.2641	1.2639	1.2615	1.2624	1.2623
$\dot{\theta}$	-.00979	-.00221	.00371	-.00823	-.00515	.00330
H	8759.6	8760.3	8761.1	8749.1	8750.0	8750.7

TABLE D.38

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_N ($Fc = -K^T \Delta \underline{X}$; $\theta o = 75°$)

State	C_N + 5%	C_N	C_N - 5%	C_N + 5%	C_N	C_N - 5%
V	2826.4	3826.4	3826.4	3825.5	3825.5	3825.5
γ	1.2646	1.2649	1.2651	1.2643	1.2645	1.2648
θ	1.2647	1.2653	1.2648	1.2644	1.2649	1.2646
$\dot{\theta}$	-.00930	-.000385	.00360	-.00939	-.00083	.00393
H	8767.4	8768.1	8768.7	8765.1	8765.7	8766.5
V	3823.8	3823.8	3823.8	3820.7	3820.7	3820.7
γ	1.2635	1.2637	1.2639	1.2617	1.2620	1.2622
θ	1.2635	1.2641	1.2638	1.2615	1.2624	1.2623
$\dot{\theta}$	-.00952	-.00221	.00415	-.00785	-.00515	.00376
H	8759.6	8760.3	8761.0	8749.2	8750.0	8750.6

TABLE D.39

STATE VARIATIONS RESULTING FROM 5 PERCENT VARIATION OF C_A ($Fc = -K^T\Delta \underset{\sim}{X}$; $\theta o = 75°$)

State	$C_A + 5\%$	C_A	$C_A - 5\%$	$C_A + 5\%$	C_A	$C_A - 5\%$
		Wind = 0 fps			Wind = 16 fps	
V	3813.1	3826.4	3839.7	3812.2	3825.5	3838.9
γ	1.2648	1.2649	1.2650	1.2644	1.2645	1.2647
θ	1.2652	1.2653	1.2654	1.2648	1.2649	1.2651
$\dot{\theta}$	.00058	-.000385	-.000046	-.00106	-.00083	-.00051
H	8751.1	8768.1	8785.2	8748.6	8765.7	8783.0
		Wind = 48 fps			Wind = 100 fps	
V	3810.4	3823.8	3837.3	3807.2	3820.7	8384.3
γ	1.2636	1.2637	1.2638	1.2619	1.2620	1.2621
θ	1.2640	1.2641	1.2643	1.2623	1.2624	1.2625
$\dot{\theta}$	-.00254	-.00221	-.00186	0.00540	-.00515	-.00441
H	8743.1	8760.3	8777.7	8732.5	8750.0	8767.6

TABLE D.40

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF FTF ($Fc = -K^T \Delta \underline{X}$; $\theta o = 75°$)

State	99FTF%	FTF	98FTF%	99FTF%	FTF	98FTF%
		Wind = 0 fps			Wind = 16 fps	
V	3837.5	3826.4	3748.7	3786.7	3825.5	3747.8
γ	1.2569	1.2649	1.2542	1.2593	1.2645	1.2539
θ	1.2599	1.2653	1.2545	1.2596	1.2649	1.2541
$\dot{\theta}$	.00061	-.000385	.00150	.000033	-.00083	.00114
H	8665.5	8768.1	8563.2	8663.3	8765.7	8560.7
		Wind = 48 fps			Wind = 100 fps	
V	3784.9	3823.8	3746.1	3781.8	3820.7	3742.9
γ	1.2585	1.2637	1.2531	1.2566	1.2620	1.2511
θ	1.2588	1.2641	1.2533	1.2569	1.2624	1.2513
$\dot{\theta}$	-.00137	-.00221	.000099	-.00327	-.00515	-.00204
H	8658.0	8760.3	8555.4	8647.2	8750.0	8544.2

TABLE D.41

STATE VARIATIONS RESULTING FROM 3 PERCENT VARIATION OF Δ ($F_c = -K^T \Delta \underline{X}$; $\theta_o = 75°$)

State	Δ	0.003Δ	Δ	0.003Δ
		Wind = 0 fps		Wind = 16 fps
V	3826.4	3826.1	3825.5	3825.3
γ	1.2649	1.2778	1.2645	1.2775
θ	1.2653	1.2781	1.2649	1.2779
$\dot{\theta}$	-.000385	-.000099	-.00083	-.00065
H	8768.1	8795.9	8765.7	8793.9
		Wind = 48 fps		Wind = fps
V	3823.8	3823.7	3820.7	3820.7
γ	1.2637	1.2769	1.2620	1.2775
θ	1.2641	1.2773	1.2624	1.2785
$\dot{\theta}$	-.00221	-.00174	-.00515	-.00351
H	8760.3	8789.2	8750.0	8779.9

TABLE D.42

STATE VARIATIONS RESULTING FROM 1 PERCENT VARIATION OF Δ ($Fc = -K^T \Delta \underline{X}$; $\theta o = 75°$)

State	Δ	0.001Δ	Δ	0.001Δ
		Wind = 0 fps		Wind = 16 fps
V	3826.4	3826.3	3825.5	3825.5
γ	1.2649	1.2692	1.2645	1.2690
θ	1.2653	1.2696	1.2649	1.2694
$\dot{\theta}$	-.000385	.000144	-.00083	-.000619
H	8768.1	8777.5	8756.7	8775.3
		Wind = 48 fps		Wind = 100 fps
V	3823.8	3823.7	3820.7	3820.7
γ	1.2637	1.2682	1.2620	1.2666
θ	1.2641	1.2686	1.2624	1.2669
$\dot{\theta}$	-.00221	-.00204	-.00515	-.00422
H	8760.3	8770.1	8750.0	8760.1

VITA

Yun-Ching Tsuei was born October 4, 1937, in Ho-Nan, Republic of China. He graduated in 1956 from The Provincial Yuan-Lin High School in Taiwan and entered the Chung-Cheng Institute of Technology the same year. Mr. Tsuei received a Bachelor of Science degree in Electrical Engineering in February, 1961. In March, 1961, he became an assistant electrical engineer at the National Kuang-Hsing plant in Taipei. He became a supervisor at Utility Division of this plant in 1969. In the fall of 1971, he entered the Graduate School of The University of Tennessee, Knoxville, and received the Master of Science degree with a major in Electrical Engineering in 1973. After six years, he returned to complete his education, receiving the Ph.D. degree with a major in Engineering Science and Mechanics in December, 1983. His area of specialization is that of control theory.

He is married to the former Lin Chen and they have two sons, Kuo-Lun and Kuo-Jen.