

Mechanics and Design of Stiffness-Modified Continuum Robots and Endoscopic Instruments

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To my past and present labmates and the friends I've made in engineering.

Without your encouragement, cheer, and support— technical and not—

I could not have done this.

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Abstract

This dissertation contributes to the growing literature on continuum robot modeling, design, and application.

Chapter 2 addresses tendon robots, beginning by deriving analytical models of robot deflection due to external loads in terms of load location, tendon stiffness, backbone stiffness, and tendon routing. These formulas clarify relationships between robot design and loaded deflection; they show that moving the tendon termination point closer to the robot centerline vastly improves stiffness. For robotic control and large deflections, it develops a Cosserat-rod-based model that uses tendon displacement inputs and accounts for factors such as tendon slack and stretch. Simulations demonstrate significant advantages in robot stiffness with tendon displacement control vs. tension control. These models are validated experimentally.

Chapter 3 introduces Notched-Tube Continuum Robots (NTCRs), a novel continuum actuation paradigm. The chapter derives a kinetostatics model and explores the design space of NTCRs with a design process for achieving desired planar shapes. Prototypes of varying cross sections, materials, and geometries were created with this process and used to experimentally validate a varying-curvature kinematics model, with generally good agreement. The design scheme is then extended to intake a desired tip pose and validated in simulation. The chapter ends by discussing implementation and offers practical advice for those seeking to utilize NTCRs.

Chapter 4 details the design and evaluation of an at-scale, NTCR-based tool for use in minimally invasive surgery via flexible endoscopes. Endoscopic instruments

must be compliant in bending but axially and torsionally stiff, motivating the development of a model that analytically relates the anisotropic stiffness of a symmetrically notched tube to its notch parameters. This enables notch designs that attain specified axial, torsional, and bending rigidities. For the tool, colorectal lesion removal determines the rigidity targets. Two versions of the instrument are prototyped, both with a passive, flexible transmission and an NTCR tip. The study evaluates the tip output response for base inputs of rotation, translation, and actuation when deployed through a mock colonoscope channel, demonstrating that the concept is effective. The design and evaluation methodologies utilized here are adaptable for tool development for other endotherapies.

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Chapter 1

Introduction

Continuum robots arose from a desire to attain organic, bending actuated shapes that evoke the capabilities of elephant trunks, snakes, vines, and other natural phenomenon. Given the need for compliant underlying structures, motors could not be hosted on the body of the robot, and offset actuation was developed as an essential aspect of continuum robotics. Some systems pull on tendons embedded along the length of an elastic element to generate bending [1, 2, 3]; others have pneumatic chambers that are actuated by pressurization [4, 5, 6]; and others still combine elastic elements that interact with each other, arriving at various equilibrium positions due to asymmetry in the components or their configuration [7, 8]. A variety of these robots are shown in Figures 1.1 through 1.4. Benefits of continuum robots due to their flexibility, soft structures, and offset actuation include intrinsic safety for human interaction [9, 10], better gripping of delicate or irregularly shaped objects [11, 12, 13], and the ability to be made at slender, millimeter-scale sizes for access in hard-to-reach areas [14, 15]. Their development has allowed modern robotics to expand beyond the strictly-regimented factory floor and into uncertain, unstructured environments, ranging from space [16] to medicine [17].

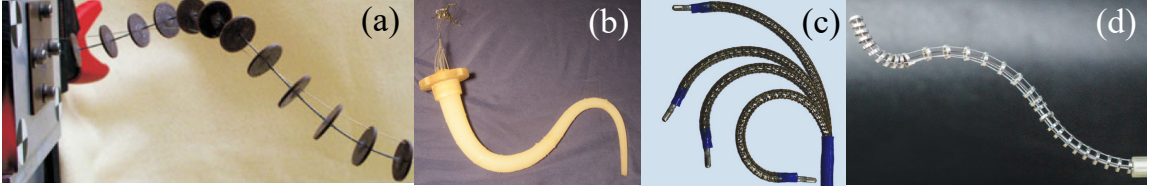


Figure 1.1: Examples of tendon-driven continuum robots. (a) is from [18], (b) is from [2], (c) is from [19], and (d) is from [20].

This dissertation focuses on three primary subgroups of continuum robots: tendon-driven robots, tube-based robots, and continuum robots used in minimally invasive surgery.

Tendon-Driven Continuum Robots

Tendons are a primary means of offset actuation transmission for many systems, from rigid-link mechanisms [21], to hybrid compliant-rigid robots [22, 23, 24], to fully-flexible continuum robots [25, 26, 27, 2, 28, 29, 30, 31]. Tendon-driven continuum systems have been proposed for use in surgery [32, 33, 3], aerospace applications [34, 16], and humanoid robots [35], among many others.

While traditional, rigid-link robots with discrete joints benefit from being able to use classic robotic kinematic approaches such as Denavit-Hartenberg, continuum robots form continuously bending actuated shapes that require more complex modeling to fully capture. For a continuum robot with a uniform backbone cross-section and straight tendon paths parallel to the backbone from the base to the tip, the constant-curvature actuated shape is well-known [19, 36, 17]. However, the presence of even small external loads dramatically changes the robot shape, and a tendon robot no longer exhibits constant-curvature behavior. Static and dynamic models of the externally loaded behavior have been derived and validated by several groups [37, 38, 18, 27, 2, 28, 29] with regards to tendon-tension-controlled systems. However, the robot stiffness strongly depends on how the proximal end of the tendons are controlled. If tendon tensions are held constant or dictated by a control scheme to

remain constant (e.g. [39, 40]), external loading will generate a change in position of the proximal end of the tendons in order to maintain a constant tension, effectively lengthening the tendon. On the other hand, if the tendon’s proximal end is dictated by a constant position or displacement, the tendon tension will change as the load is applied but not the position. This difference in actuation causes a displacement-dictated tendon robot to deform much less than a tension-dictated robot.

Displacement-input systems where tendon length is prescribed form the basis for many constant-curvature kinematic models without external loads [41, 42, 19, 36]. This approach makes sense, as a very common means of building tendon robots is with non-backdrivable, servomotor systems that use position as the input. However, all existing models for externally loaded continuum tendon robots have assumed tension inputs, leading to systems that must either use expensive force-controllable motors or have a high-level control loop to translate force information from sensors into a displacement command for the motor (e.g [43, 16, 19, 44, 32, 2, 45]). Additionally, prior work has yet to explore the ramifications of using a displacement-input kinematics model, aside from a brief exploration in [46].

The control of tendon displacement leads to not only simpler physical systems but also marked improvement in stiffness under external loading, and Chapter 2 of this dissertation presents statics models that consider this type of actuation for tendon robots under external loads. It covers analytical, small-deflection formulas that are useful for design as well as a numerical, large-deflection model for control of displacement-input tendon robots.

Tube-Based Continuum Robots

While a central backbone is a common feature for tendon-driven robots, many continuum robots push their support structure to the device periphery by using tubes as their underlying form factor, a few of which are pictured in Figure 1.2 and 1.3. This hollow central lumen is useful for a variety of tasks, from passing tools through the manipulator [49] to operating as a vacuum port [50]. Tube-based continuum robots

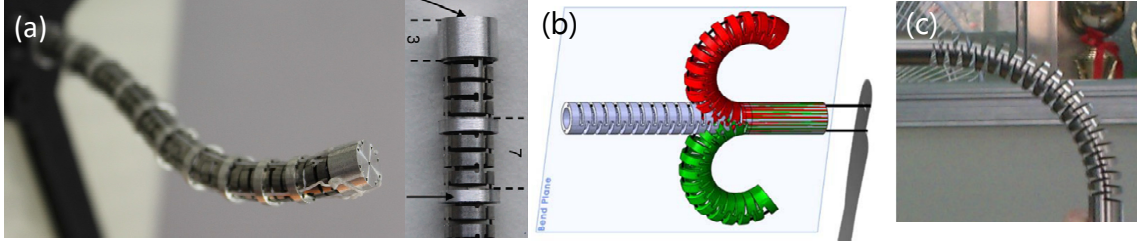


Figure 1.2: Examples of notched tube-based continuum robots that utilize notching to increase their articulation range. (a) is from [47], (b) is from [3], (c) is from [48].



Figure 1.3: Examples of notched tube-based continuum robots with offset backbones. (a) is from [26], (b) is from [51], and (c) is from [25].

can also be tendon driven [51, 25, 26, 28, 30, 31], many of which have notches in their sidewall in order to increase angulation and generally lower the stiffness of the tube to reduce actuation forces [28, 30, 31, 52]. Some designs, such as a wrist by York et. al. [51] and Swaney et. al. [25], use the notches to offset the neutral axis of the tube. The wrist design then uses a pull-wire on the opposite side of the backbone to apply a force at the tip, which generates bending in the structure. Eastwood et. al. present a similar design in [53] and then optimize the notch shape to achieve high angulation at the edge of the manipulator workspace in [26]. Kim et al use asymmetric notching in nested tubes to create a variable stiffness mechanism that can be controlled by changing the alignment of the notched portions [54].

Another major class of tube-based continuum robots are Concentric-Tube Robots (CTRs). CTRs translate and rotate nested, precurved tubes with respect to one another, generating a change in tip position at different equilibrium states without tendons or any other force transmission element. CTRs are strong and can be built at mm-scale sizes, often down to the size of a needle. They have been explored for use in surgical procedures throughout the body, including transnasal skull base surgery

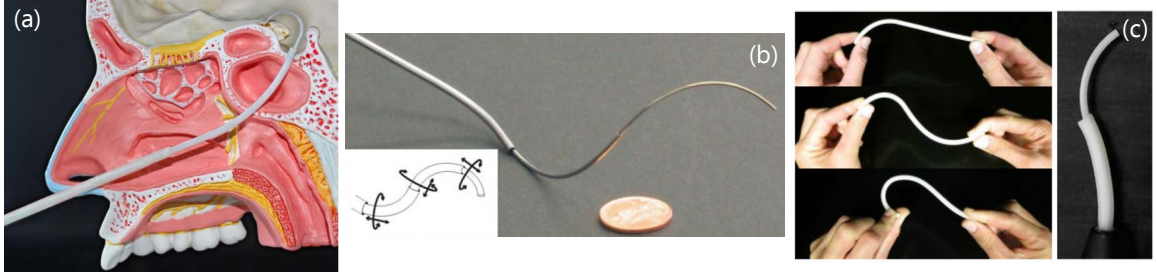


Figure 1.4: Examples of concentric tube robots. (a) is a custom-designed CTR proposed for transnasal brain operations [47], (b) is a nitinol CTR, which is their most common from [66], and (c) shows additively manufactured CTRs from [57].

[14], cardiac procedures [55], thermal ablation in the liver [56], and transurethral resection of the prostate [15], among others. While CTRs are primarily made of superelastic nitinol, recent work has been done exploring the use of polymers and additive manufacturing to construct CTRs. Additively-manufactured CTRs were evaluated by Motorimoto et al in [57], and Amanov et. al. use additive manufacturing to generate patient-specific manipulators in [47]. Noh et. al. present the workspace optimization of a CTR made from heat-shrink tubing for use in transnasal brain surgery [58]. While versatile and easily miniaturized, CTRs have a major drawback: elastic instability, where the robot transitions from one state to another, minimal-energy state so rapidly that it is referred to as “snapping” [59, 60]. A novel take on this phenomenon by Riojas et al in [61] argues it could be useful for suture throwing, but otherwise, it is widely regarded as an undesired behavior. Various approaches have been taken to address this stability issue in CTR design, including optimal control [60]. Dupont et al. and Rucker et. al. [55, 62] found that reducing the bending-to-torsional stiffness ratio of the tubes can increase the stability of the robot, which some groups have accomplished by notching the tube [63, 64, 65].

Chapter 3 of this dissertation introduces a novel type of tube-based continuum robot, Notched-Tube Continuum Robots (NTCR). The actuation principle of NTCRs is similar to the push-pull motion of multi-backbone robots [7, 67, 68], and it retains the compact form factor of tube-based robots. NTCRs remove material from tube

walls to enable actuation, similar to other tube-based robots, but does not have additional components such as tendons, resulting in a manipulator similar to a CTR without stability issues. Additionally, Chapter 4 includes modeling for modification of the global stiffness behavior of a tube, which is relevant to controlling the stability of CTRs.

Continuum Robots in Flexible-Access Surgery

The presence of robots in the medical field grows every year, with engineers and researchers continually working to improve medical technology. Within surgical technology, this growth is driven by the advent of minimally-invasive surgery (MIS), where long, slender tools and a camera are used to perform an operation through a few small incisions rather than cutting one large one. While MIS was pioneered with manual instruments and still frequently deploys them, robotic tools have been successfully implemented in MIS procedures, as well. A general overview of actuator-driven robots in surgery was done by Le et. al. [69], and they review both rigid-link and flexible robots used in a variety of procedures at both the clinical and research stages. Continuum robots in particular have been proposed for use in medical procedures in nearly every area of the body, as surveyed by Burgner-Kahrs et al in [17]. Their flexibility shows promise in facilitating NOTES, natural orifice transluminal endoscopic surgeries. The central concept of NOTES relies on using flexible endoscopes— long, slender devices with a light, camera, and tool ports at the end— to follow natural pathways within the body to the surgical site, further reducing the number of incisions needed to operate. It enhances the patient benefits of MIS, which includes lowered costs, shorter recovery time, less collateral damage to the surrounding tissue, and reduction or elimination of incisions and scars [70, 71, 72, 73].

NOTES is particularly promising for the gastrointestinal tract, where practitioners already utilize flexible endoscopes for screenings and simple surgical procedures, where tools such as biopsy graspers and electrocautery knives are passed through tool channels that run within the length of the scope. However, only the distal

section of a flexible endoscope can be maneuvered, which, for example, means that the operator can move only the last four inches of a six-foot colonoscope. This tip control allows the operator to adjust the direction of motion as the scope travels through the body and to realign the camera’s field of view to examine the surgical site or any abnormalities discovered during the procedure. Unfortunately, it is also the only means of performing the work of surgery and articulating the tools passed through the ports. Standard endoscopic tools are passive and unable to move independently from the endoscope aside from rotation within the tool channel and insertion/retraction. Thus, in order to steer the tool and aim it toward the surgical target, the entire tip of the scope must be articulated, and the camera’s field of view moves with it, too, as illustrated in Figure 1.5(b). This coupling presents challenges for the operator, and it fundamentally limits the capability of NOTES procedures and increases the burden of training it takes to perform one successfully [74].

Prior efforts to provide enhanced dexterity to endoscopes largely sought to integrate specialized, non-changeable instruments or articulating mechanisms into the endoscope structure, such as the NOTES-scope, R-Scope, and EndoSamurai (Olympus) [75, 76] and the MASTER robotic system [77, 78], or to robotize the scope control knobs themselves [79, 80]. Few attempts have been made to use standard colonoscopes, though a notable design by Gafford et al. consists of a cap that clips on to the tip of commercially-available colonoscopes and contains actuators that laterally deflect tools after they emerge from the endoscope ports [81, 82].

Chapter 4 of this dissertation designs and evaluates an NTCR on the end of an elongated, flexible transmission, intending to address the challenges currently facing NOTES procedures. Using the NTCR for endoscopic deployment offers several key advantages: it can be made at sizes compatible with the tool channels of commercial endoscopes, currently available passive tools can pass through its hollow lumen, which then enables the NTCR to act as an articulating sheath, and the straightforward NTCR actuation principle allows the tip to be articulated intuitively without a control scheme, thus it can be made into manually-driven and robotic tools. These

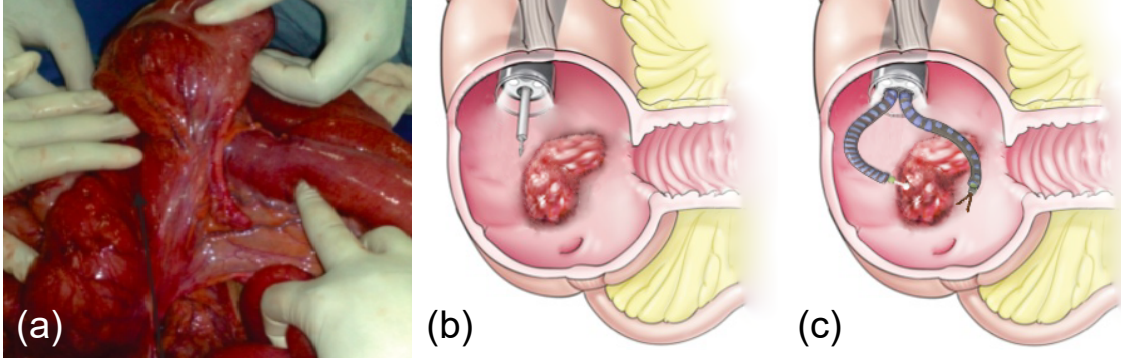


Figure 1.5: Treatment options for colorectal lesions and proposed approach. (a) A partial colectomy involves removing a portion of the colon and stitching the remaining ends together. This is the route taken when a large polyp is found that cannot be removed endoscopically. (b) Currently available tools used in polyp removal have no dexterity of their own; they rely on the articulation of the scope tip for maneuverability. (c) This work evaluates the use of an NTCR to act as a sheath around the tools, providing them with motion independent from the scope and enabling triangulation of tools for two-channel scopes.

factors reduce costs as compared to systems that reinvent the endoscope, and the cost reduction also increases accessibility. The design in this work focuses on colorectal lesion removal.

Colorectal Lesion Removal

Colorectal cancer (CRC) kills 50,000 Americans each year, and 1 in 20 people will be diagnosed with it in their lifetime, making it the most common life-threatening disease of the digestive tract [83]. In response to this risk, doctors perform 6.3 million colonoscopic CRC screenings each year [84] to search for the cancer in its earliest stages, which includes benign polyps that are regarded as pre-cancer that can immediately be removed with a snare [85]. Through these screenings, 6.6% of patients find a large polyp or tumor (generally regarded as a lesion of length greater than 9 mm) that requires a biopsy and referral to a specialist [86], either an endoscopist or surgeon. The next step depends on the results of the biopsy; while early-stage cancer and benign lesions can often be treated by either specialist, late-stage cancer requires surgery and more aggressive forms of intervention than what

endoscopy can offer. Of the 135,000 cancer cases annually discovered in screenings, 39% are diagnosed at the local stage [83], where it is still anatomically possible to remove the cancer endoscopically, meaning nearly 53,000 people every year have the option of endotherapy to treat their CRC [83]. Endotherapies are also available to the vast majority of the 280,000 patients who discover non-cancerous polyps that cannot be removed during the screening [83, 84, 86]; in total, well over 300,000 patients per year have the option of endoscopic intervention for lesion removal.

The alternative to endotherapies is surgical treatment by means of a partial colectomy, which involves the removal of a section of colon and anastomosing the remaining ends together. It is a highly invasive surgical procedure even when performed laparoscopically. Partial colectomies entail high complication rates such as infection (27%), the need for revision surgery (10%), and non-infectious complications (14%) that contribute to comorbidity [87]. Comparatively, endotherapies offer significantly lower costs [70, 71] and a reduced risk of adverse events [73, 72]. Strong scientific evidence supports the patient benefits of endotherapies, especially in the case of non-cancerous polyps, with Peery et al stating that “The literature to date is clear that endoscopic resection is the preferred management of nonmalignant colorectal polyps based on evidence that almost all (>90%) complex nonmalignant colorectal polyps, regardless of size, can be safely resected endoscopically with an outpatient procedure” [88]. This contrasts with the reality that around 28,000 people per year undergo a partial colectomy for nonmalignant polyps, and that number is on an upward trend [88]. This does not include the roughly 29,000 cases of early-stage cancer patients who undergo a colectomy despite being suitable for endoscopic approaches [88, 83]. Thus, around 57,000 patients annually— nearly one in five patients eligible for treatment via endotherapy— undergo a risky partial colectomy instead.

Endotherapies, despite their demonstrated benefits to the patient, are hindered by problems stemming from poor tool dexterity. Flat learning curves due to technical difficulties of operating through an endoscope demonstrate problems as early as a surgeon’s training stage [74]; additionally, in-practice difficulties like poor scope

maneuverability, poor counter-traction efficacy, and the inability to reach around large polyps/lesions make endotherapies difficult for surgeons to perform [89]. These issues lead to both increased required training hours and longer procedure times for endotherapies than laparoscopic approaches [74], ultimately acting as a strong deterrent for endoscopic procedures in what would be a relatively simple laparoscopic procedure from the viewpoint of surgeon [88].

Chapter 4 of this dissertation focuses on Endoscopic Submucosal Dissection (ESD), an advanced colonoscopic endotherapy currently inhibited by poor tool dexterity, to design an endoscopic tool based on NTCRs. ESD was selected as the targeted procedure because of its relative complexity; any tool that can perform ESD can also perform other colonoscopic endotherapies. Performance requirements from ESD were used to design a manual instrument to act as an articulate-able sheath for colonoscopic tools. Chapter 4 of this dissertation details those requirements and how the design addresses them, including the development of a model for an elongated, notched-tube transmission that holds implications for other types of continuum robots, primarily CTRs.

Outline and Contributions

Overall, this dissertation contributes to the field of continuum robotics from modeling, design, and application perspectives. Chapter 2 derives new modeling for tendon robots and examines displacement-input tendon robots under external loading, addressing their deflection response as well as effects of tendon routing, tendon stretch, and tendon slack. Both analytical modeling for design assessment and a numerical, large-deflection model for robotic control are presented. Simulations and experiments demonstrate the significant stiffness advantages of using prescribed tendon displacements over prescribed tensions, as well as demonstrating that converging the tendon termination point (i.e., moving it closer to the robot centerline) increases robot stiffness. Chapter 3 explores a novel continuum, tube-based robot and lays the foundations of understanding its mechanics. Both statics and kinematics

models are derived for constant- and varying-curvature NTCRs, and a design process for attaining a desired actuated planar shape is presented and experimentally validated, with results showing generally good agreement. That process is then used as the basis of a scheme to attain a desired planar tip pose, which is validated in simulation. Concluding the chapter is a discussion on practicalities of and guidance for NTCR implementation. Chapter 4 focuses on the development of an at-scale endoscopic instrument for use in NOTES procedures, using the NTCR as the basis for distal articulation. The target application—endoscopic colorectal lesion removal—requires low bending rigidity but very high torsional and axial rigidity, motivating the derivation of a model for determining anisotropic stiffness behavior of a symmetrically-notched tube based on the notch parameters. This analytical modeling, derived in Section 4.3, can be used to design more stable CTRs as well as other types of continuum structures. The resulting endoscopic tool, with a passive, flexible transmission and distal NTCR tip, is assessed for input base actuation vs. output tip motion alongside another candidate design and an existing colonoscopic biopsy grasper, with the results showing that the novel tool offers significant dexterity advantages. Chapter 5 concludes the dissertation and points to future work.

Chapter 2

Stiffness Analysis of Tendon Robots

Disclosure of Previously Published Material

The work presented in this chapter has been published in IEEE Transactions on Robotics [1]. The dissertation author served as first author on this publication; John Till was second author, providing the tendon slack model and assisting in the large-deflection simulations; D. C. Rucker, the chair of this dissertation’s committee, was third author and provided guidance and support throughout the paper writing process.

2.1 Overview

The constant-curvature shape of tendon-driven robots with displacement-controlled, parallel tendons has been extensively studied [19, 36, 17], as have tendon robots with tension-dictated systems under external loads [37, 38, 18, 27, 2, 28, 29]. However, a displacement-input model that includes external loading is absent from the literature. The distinction in how the tendon is controlled has important implications for the deflection under external loads, as illustrated by the simulation results in Figure 2.1.

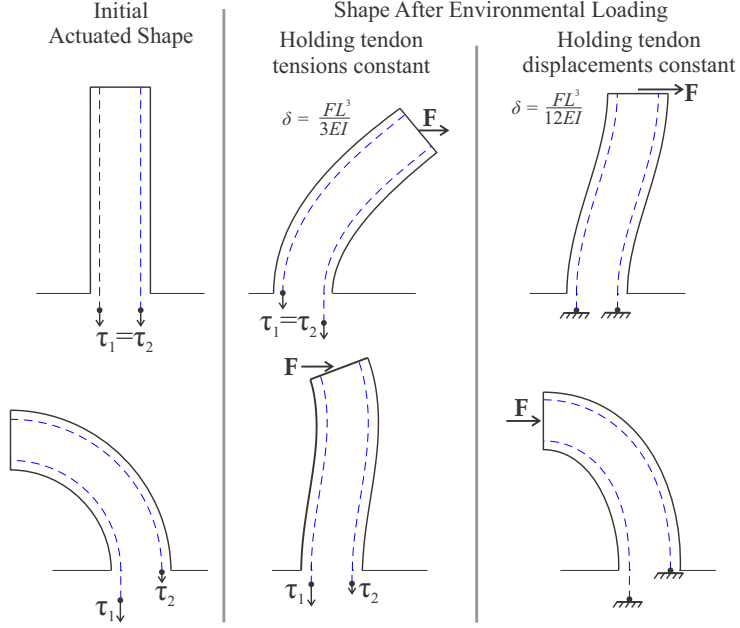


Figure 2.1: Simulation results demonstrate the differences between tension-controlled and displacement-controlled tendon bases. In both the straight and actuated cases, the robot is significantly stiffer when the proximal tendon ends are held at a constant position.

When the tendon tensions are held constant, the proximal tendon displacements will change under external loads, and thus the robot will deform accordingly. On the other hand, when the proximal displacements are held constant, an external load will change the tendon tension, and the robot will generally deflect less. Only a model that can handle both displacement inputs and external loads can predict the response of a robot controlled in such a manner. All existing models for externally-loaded robots have assumed tendon tensions are the prescribed inputs to the system, though this historic modeling emphasis on prescribed tension inputs is somewhat incongruous with the majority of robotic systems themselves. Systems commonly use non-backdrivable, servo-actuation systems that receive high-level position commands and prescribe the associated tendon displacements, even if the actuation tension is ultimately measured (e.g [43, 16, 19, 44, 32, 2, 45]).

In this chapter, we seek to remedy this imbalance by providing models that can predict the deflection of tendon-displacement-controlled continuum robots under

environmental loads. In Section 2.2, we begin by deriving simple models for the loaded deflection of initially straight, Bernoulli-Euler beams that are constrained by displacement-controlled tendons. This analysis provides analytical formulas and dimensionless plots that enable an intuitive understanding of the role of tendon stiffness, routing path radius, backbone compressibility, and load location on the beam’s deflection curve. These results should be generally useful for predicting and adjusting the properties of a continuum manipulator during the design process. We further analytically describe the potential stiffness benefits of non-parallel tendon routing. Additional deflection formulas for this more general case are provided in Section 2.3, and we show that routing paths that converge to the neutral axis at the tip of a robot segment can markedly improve stiffness. Section 2.4 summarizes the coupled Cosserat rod/tendon model of [18], and Section 2.5 extends it to include prescribed displacement inputs, tendon stretch, and nonlinear slack, which is accomplished by modifying the algorithmic structure and boundary conditions of the original model. This extended model is then implemented to produce simulation results for external loading of an actuated robot segment in 3D to extend and further illuminate the principles of Sections 2.2 and 2.3. Section 2.6, presents experimental validation the derived deflection formulas and the extended Cosserat model by measuring the response of prototype structures to a set of known loading conditions.

2.2 Analytical Results for Small Deflections with Parallel Tendon Routing

In this section, expressions for the deflection of an elastic member under a transverse external load are derived with classical beam theory. The member contains ideal flexible tendons passed through frictionless guide channels along its length, which are fixed at the tip and the base of the member (i.e. a dictated displacement of zero), as depicted in Figure 2.2 for a single tendon. The tendon material is allowed

to slide axially within its channel. The following analysis covers only the straight “unactuated” state and assumes small deflections from it. These assumptions allow the derivation of simple analytical deflection formulas that can inform intuition about more complex, actuated scenarios and guide general design choices. Cases considered include: (1) a single extensible tendon running parallel to the member’s central line, (2) the limiting case of an inextensible tendon, (3) backbone compression, and (4) multiple pretensioned tendons. These are experimentally validated in Section 2.6.

2.2.1 Analysis with a Single Stretchable Tendon

The schematic in Figure 2.2 depicts the variables in the problem statement. An initially straight, cantilevered elastic member of length L , Young’s modulus E , and second area moment I is subjected to a force at a distance z from the base. Arc length along the member is denoted by s . A flexible tendon runs through a straight channel at a radial distance r from the center. The proximal tendon end is fixed in space, and the tendon has a total spring constant k , which is given in the figure by the classical formula for axial stiffness in terms of tendon cross sectional area A_t , Young’s modulus E_t , and length L_t . The tendon channel is assumed to be frictionless, which implies that the tendon tension is constant along the length. For now, the tendon is assumed to be relaxed when the member is straight and in tension when the external load is applied, but pretension is examined in Subsection 2.2.4 in the case of multiple tendons.

Examining the free-body section diagrams in Figure 2.2, we can write a static moment balance and solve for the internal moment (about the x axis) as a function of arc length. Assuming small deflection, we obtain

$$m(s) = \begin{cases} F(s - z) + r\tau & \text{if } s < z \\ r\tau & \text{if } s \geq z . \end{cases} \quad (2.1)$$

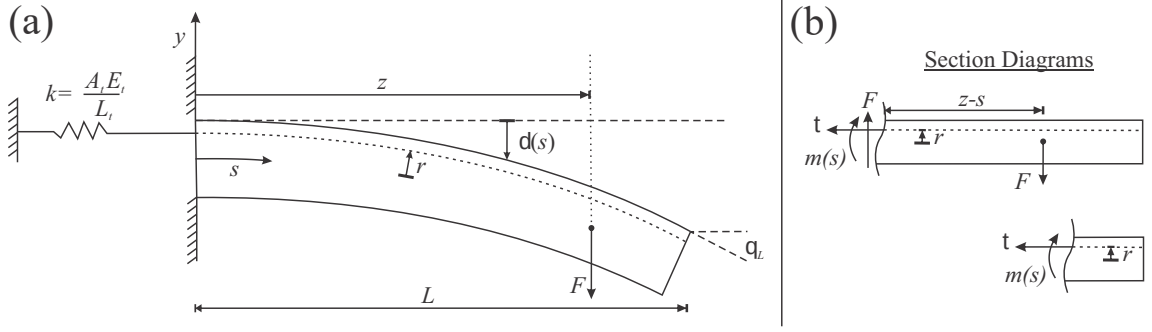


Figure 2.2: Diagrams for the analytical tendon robot model. Problem schematic (a) and free-body diagrams (b) were used to derive the analytical results, shown here with exaggerated deflections for clarity.

where τ is the force in the tendon. We assume the classical linear constitutive law that the internal moment is proportional (via the flexural rigidity EI) to the rate of change of the tangent angle with respect to arc length,

$$m(s) = -EI \frac{d\theta}{ds}, \quad (2.2)$$

where the negative comes from the sign conventions established in Figure 2.2. Then integrating the moment equation, enforcing the boundary condition $\theta(0) = 0$, and enforcing continuity in $\theta(s)$ at $s = z$, we get

$$-EI\theta(s) = \begin{cases} \frac{1}{2}Fs^2 + (r\tau - Fz)s & \text{if } s < z \\ r\tau s - \frac{1}{2}Fz^2 & \text{if } s \geq z. \end{cases} \quad (2.3)$$

At this point, the tendon reaction tension is unknown, but it can be obtained from the tendon stretch caused by the deflection of the beam, $\Delta = r\theta_L$, and the tendon spring constant k :

$$\tau = kr\theta_L. \quad (2.4)$$

Substituting the above into (2.3) and evaluating at $s = L$, solving for θ_L obtains

$$\theta_L = \frac{Fz^2}{2(EI + r^2kL)}, \quad (2.5)$$

and consequently,

$$\tau = \frac{Fkrz^2}{2(EI + r^2kL)}. \quad (2.6)$$

Note that the F and r vectors are designated positive in opposite directions in Figure 2.2, and the tension will be positive if this is the case. If not, then the tension computed by the above expression will be negative, indicating compression, which is impossible for an ideal flexible tendon. Thus, application of this analysis must be restricted to cases with tendons that can support some compressive force without buckling (e.g. metal rods) or where tension is always positive. With multiple tendons, pretension can be applied to ensure all the tendon tensions remain positive during subsequent external loading, and Subsection 2.2.4 analyzes this case.

Finally, making the small angle approximation $\theta \approx \frac{d\delta}{ds}$, we can integrate once more, enforce the boundary condition $\delta(0) = 0$, and enforce continuity of $\delta(s)$ at $s = z$ to obtain

$$-EI\delta(s) = \begin{cases} \frac{1}{6}Fs^3 + \frac{1}{2}(r\tau - Fz)s^2 & \text{if } s < z \\ \frac{1}{2}r\tau s^2 - \frac{1}{2}Fz^2s + \frac{1}{6}Fz^3 & \text{if } s \geq z. \end{cases} \quad (2.7)$$

Equation (2.7), when combined with (2.6), is the most general form of the results in this section, providing deflection as a function of arc length, load magnitude, load location, total member length, flexural rigidity, tendon location, and tendon stiffness. However, examining some special cases of this result can also provide useful insights that can be encapsulated in simpler formulas. For instance, we can consider the deflection at the same location where the lateral loading is applied ($s = z$). In this case, we get

$$\delta(z) = \frac{Fz^3}{3EI} - \left(\frac{r^2kz}{EI + r^2kL} \right) \frac{Fz^3}{4EI}. \quad (2.8)$$

For the special case of tip loading ($z = L$), the tip deflection is

$$\delta_{tip} = \frac{FL^3}{3EI} - \left(\frac{r^2kL}{EI + r^2kL} \right) \frac{FL^3}{4EI}. \quad (2.9)$$

We see that the classical cantilevered deflection formula $\frac{FL^3}{3EI}$ appears as the first term, and that δ_{tip} is equal to this in the case of $k = 0$ (no tendon) or $r = 0$ (tendon located at the centerline). Increasing either k or r will tend to reduce the deflection (increasing the effective stiffness at the tip), and the dependence on r^2 is illuminating for the purposes of design; increasing r by a factor of 2 has the same effect as increasing k by factor of 4.

To illustrate our results in a generally useful way throughout the paper without specifying manipulator or material properties, we define the following dimensionless quantities, representing tendon stiffness, compliance at the loaded point, and load location, respectively:

$$\alpha := \frac{r^2 k L}{EI} \quad \sigma := \left(\frac{12EI}{L^3} \right) \frac{\delta}{F} \quad \zeta := \frac{z}{L}.$$

The nondimensional form of Equation (2.8) is then

$$\sigma = 4\zeta^3 - 3 \left(\frac{\alpha}{1 + \alpha} \right) \zeta^4. \quad (2.10)$$

When $\zeta = 1$, this equation becomes the dimensionless form of Equation 2.9. These dimensionless quantities will be used as the axes of the plots in this section, but they are labeled with the full dimensionless expressions for clarity. We plot the above relationship of dimensionless tip compliance versus dimensionless tendon stiffness for a parallel tendon in Figure 2.7, where it is compared to a converging tendon design developed in Section 2.3. The plot shows that the robot's dimensionless tip compliance approaches an asymptote of 1 as the tendon stiffness increases to infinity.

2.2.2 Inextensibility

Many models assume the tendons are inextensible (such as the model in [90]), as there are various cable materials with a sufficiently high stiffness, and we can consider the

effect of having an inextensible tendon by letting $k \rightarrow \infty$. In this case, we see that

$$\lim_{k \rightarrow \infty} \theta_L = 0,$$

and the deflection at the loaded point is

$$\delta(z) = \frac{Fz^3}{3EI} - \left(\frac{z}{L}\right) \frac{Fz^3}{4EI}. \quad (2.11)$$

Figure 2.3 plots this relationship nondimensionally. From this, we see that the tip deflection in response to loads at the tip is

$$\delta_{tip} = \frac{FL^3}{12EI}, \quad (2.12)$$

which is depicted in Figure 2.1. This is exactly one fourth of the deflection experienced by the same member with no tendons. Interestingly, and somewhat counter-intuitively, the reaction tension of an *inextensible* tendon in response to transverse external loading is only a function of the path radius, total length, and the loading; it is not influenced by the member’s flexural rigidity:

$$\lim_{k \rightarrow \infty} \tau = \frac{Fz^2}{2rL}. \quad (2.13)$$

In contrast, the tension required to actively bend the member to a certain angle in free space is proportional to the flexural rigidity.

2.2.3 Backbone Compression

The prior formulas consider only members with an incompressible backbone, but for “soft” manipulators and some catheter robots, the effects of backbone compression are non-negligible [16, 45]. To model these cases, we define the total compression force on the backbone c_B as

$$c_B = -k_B \Delta_B = \tau, \quad (2.14)$$

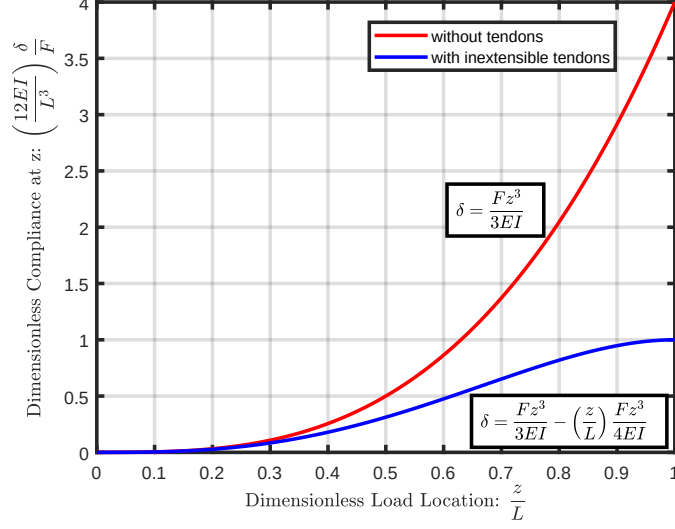


Figure 2.3: Dimensionless Compliance of the Loaded Point of a Parallel Tendon Robot. With reference to Figure 2.2, this plot shows the derived nondimensional relationship between the load’s distance z from the base and the deflection δ at the point of loading. The red line depicts the classical formula for the deflection of a solid elastic member while the blue line shows the new result for the deflection of a member with inextensible actuation tendons.

where k_B is the axial backbone stiffness and Δ_B is the axial stretch of the backbone. This affects the stretch of the tendon Δ . Without backbone compression, $\Delta = r\theta_L$, but including it makes

$$\Delta = \Delta_B + r\theta_L. \quad (2.15)$$

The reaction tension can be found by substituting (2.14) and (2.15) into $\tau = k\Delta$ and isolating τ . This leads to the tip angle as a function of backbone and tendon stiffness:

$$\theta_L = \frac{Fz^2}{2(EI + r^2Lk_{eq})}, \quad (2.16)$$

where

$$k_{eq} = \frac{k_B k}{k_B + k}. \quad (2.17)$$

Equation (2.16) is the same form as Equation (2.5) with a different tendon stiffness value. Similarly, expressions for tendon tension τ and the deflection of the loaded

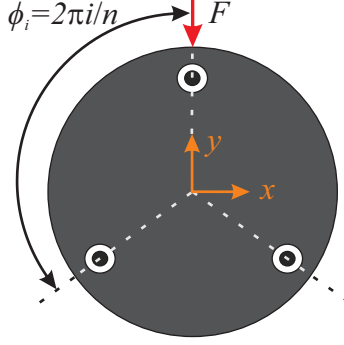


Figure 2.4: Cross section of a continuum robot with equally spaced tendons.

point $\delta(z)$, respectively Equations (2.6) and (2.8), are the same form as in the incompressible case except with $k = k_{eq}$, implying that a compressible backbone acts like a spring in series with an extensible tendon in this single-tendon case. Thus, an equivalent stiffness can be used in the equations presented for the inextensible case in order to handle an axially compliant backbone.

2.2.4 Multiple Pretensioned Tendons

The above analysis assumed only a single, initially relaxed tendon. However, continuum robots and tendon-actuated catheters usually employ at least two tendons for bending in a plane and often three or four symmetrically spaced tendon channels for bending in two planes, as shown in Figure 2.4. For three or more equally spaced tendons, the deflection of an initially straight, symmetric member will be in the direction of the lateral load vector, and the deflection will be equal for unit loads in all lateral directions. If there are $n > 2$ equally spaced, equally pretensioned tendons at equal radii r from the centroidal axis with equal k and equal pretension τ_p , then the internal moment in the direction perpendicular to the lateral load in the cross sectional plane is

$$m(s) = \begin{cases} F(s - z) + rT & \text{if } s < z \\ rT & \text{if } s \geq z, \end{cases} \quad (2.18)$$

where

$$T = \sum_{i=1}^n \cos\left(\frac{2\pi i}{n}\right) \tau_i,$$

and τ_i is the tension of the i^{th} tendon in response to the load, including any pretension. The trigonometric term accounts for the perpendicular distance from the bending plane to the tendon location in the cross section. In terms of the deflected tip angle θ_L and the pretension τ_p (equal for all tendons so that the initial shape is straight), the individual reaction tension τ_i is given by

$$\tau_i = kr \cos\left(\frac{2\pi i}{n}\right) \theta_L + \tau_p. \quad (2.19)$$

This is also the correct expression in the case of multiple tendons with a compressible backbone as long the pretension is high enough for all tendons to remain in positive tension during loading and the modified tendon stiffness expression is used.

By inspection, we see that all terms involving τ_p cancel out in T due to the trigonometric coefficients and the equal spacing of the tendons. Thus, pretension has no affect on the internal moment and no effect on the displacement under load. Pretension could increase static frictional forces and thus indirectly affect stiffness, and it can also eliminate slack tendons, as we show in Section 2.5. However, under the ideal assumptions of no friction and no slack, one cannot make a tendon-driven manipulator with an elastic, passive backbone stiffer by increasing tendon pretension, which has also been observed by Kim et al [91]. Exceptions to this include some multi-link structures in [91] and pneumatic backbone structures that stiffen with a combination of increased pressure and tension [6].

The rest of the derivation follows the single tendon case, and our result for the displacement at the loaded point is

$$\delta(z) = \frac{Fz^3}{3EI} - \left(\frac{r^2 kz \sum_{i=1}^n \cos^2 \phi_i}{EI + r^2 kL \sum_{i=1}^n \cos^2 \phi_i} \right) \frac{Fz^3}{4EI}, \quad (2.20)$$

where

$$\phi_i = \frac{2\pi i}{n}$$

as defined in Figure 2.4. This confirms the intuitive result that adding more tendons can increase stiffness, but the upper limit of tip stiffness given by Equation (2.12) still holds, even if infinitely many tendons are used.

Again, this result assumes all tendons remain in tension during deformation, which is achievable through an appropriate constant pretension. In the worst-case scenario for potential slack, where the member deflects purely in the direction of a single tendon, the required pretension to prevent slack in that tendon is the same as in Equation (2.6). All other tendons will undergo smaller deflections and will thus not experience slack, given equal pretension of all tendons.

2.3 Nonparallel Linear Tendon Paths

Motivated by the intriguing but limited result that parallel inextensible tendons increase the stiffness of an elastic member by a factor of four, this section explores what further stiffness gains may be obtained by considering tendon paths that do not run parallel to the member's central axis or to each other. We derive similar formulas to those obtained in the previous section for parallel tendons, considering inextensibility, multiple pretensioned tendons, and the effect that nonparallel tendon routing has on the manipulator shape and required actuation force.

Here, the tendon channel radius is defined as a linear function of s (as shown in Figure 2.5) and Equation (2.1) is restated as

$$m(s) = \begin{cases} F(s - z) + r(s)\tau & \text{if } s < z \\ r(s)\tau & \text{if } s \geq z, \end{cases} \quad (2.21)$$

where $r(s) = as + b$, a is the slope of the tendon path, and b is the initial radius at the base. After applying the constitutive law in Equation (2.2), integration of (2.21)

and enforcement of the boundary condition $\theta(0) = 0$ and continuity in $\theta(s)$ at $s = z$ yields

$$-EI\theta(s) = \begin{cases} \frac{1}{2}(F + a\tau)s^2 + (b\tau - Fz)s & \text{if } s < z \\ \frac{1}{2}a\tau s^2 + b\tau s - \frac{1}{2}Fz^2 & \text{if } s \geq z. \end{cases} \quad (2.22)$$

Given the initial assumption of small deflections, we additionally assume that $a \equiv \frac{dr}{ds} \ll 1$, and thus the total tendon stretch Δ (assuming backbone incompressibility) is

$$\Delta = \int_0^L r(s) \left(\frac{d\theta}{ds} \right) ds, \quad (2.23)$$

where $\frac{d\theta}{ds} = -\frac{m(s)}{EI}$. Integrating this and once more enforcing the boundary conditions and continuity gives

$$\Delta = \frac{1}{EI} \left[\frac{Fa}{6}z^3 + \frac{Fb}{2}z^2 - \left(\frac{a^2L^2}{3} + abL + b^2 \right) \tau L \right]. \quad (2.24)$$

The tendon reaction tension can then be calculated as $\tau = k\Delta$, resulting in

$$\tau = \frac{kF(az^3 + 3bz^2)}{6EI + kL(2a^2L^2 + 6abL + 6b^2)}. \quad (2.25)$$

Substituting the tension into (2.22) and once more integrating, we obtain the final deflection formula:

$$-EI\delta(s) = \begin{cases} \frac{1}{6}(F + a\tau)s^3 + \frac{1}{2}(b\tau - Fz)s^2 & \text{if } s < z \\ \frac{1}{6}a\tau s^3 + \frac{1}{2}b\tau s^2 - \frac{1}{2}Fz^2s + \frac{1}{6}Fz^3 & \text{if } s \geq z. \end{cases} \quad (2.26)$$

Thus, the deflection of the loaded point is

$$\delta(z) = \frac{Fz^3}{3EI} - \frac{\tau}{6EI} (az^3 + 3bz^2), \quad (2.27)$$

and for tip loading,

$$\delta_{tip} = \frac{FL^3}{3EI} - \frac{\tau}{6EI} (aL^3 + 3bL^2), \quad (2.28)$$

where τ is given by (2.25). When the tendons are assumed to be inextensible, these equations become:

$$\delta(z) = \frac{Fz^3}{3EI} - \left(\frac{(az + 3b)^2 z}{3L(a^2L^2 + 3abL + 3b^2)} \right) \frac{Fz^3}{4EI} \quad (2.29)$$

and

$$\delta_{tip} = \frac{FL^3}{3EI} - \left(\frac{(aL + 3b)^2}{3(a^2L^2 + 3abL + 3b^2)} \right) \frac{FL^3}{4EI}. \quad (2.30)$$

The plot in Figure 2.6 shows how increasing the magnitude of the slope a can increase the overall stiffness of the manipulator, where all other variables remain unchanged.

2.3.1 Converging Tendon Path

Figure 2.6 reveals a special case of nonparallel tendon routing that has zero tip compliance, which we have named converging routing; in this case, the tendon path begins at a radial offset of b at the base and ends at the neutral axis centerline at the tip, meaning that $a = -b/L$. As shown by Figure 2.5, the backbone and the tendon form a truss-like structure, illustrating why this design has significantly greater resistance to tip loads compared to parallel tendon designs. For a single converging tendon, the deflection at the loaded point simplifies to

$$\delta(z) = \frac{Fz^3}{3EI} - \left(\frac{(3bLz^2 - bz^3)^2}{3EI + b^2kL} \right) \frac{kF}{12L^2EI}, \quad (2.31)$$

and the deflection of the end for a tip load is

$$\delta_{tip} = \frac{FL^3}{3EI} - \left(\frac{b^2kL}{3EI + b^2kL} \right) \frac{FL^3}{3EI}. \quad (2.32)$$

Figure 2.7 depicts the relationship between tip compliance and tendon stiffness in dimensionless form for this style of routing, compared to the same formula for parallel tendons from Section 2.2. Whereas the parallel tendon case has an asymptotic tip

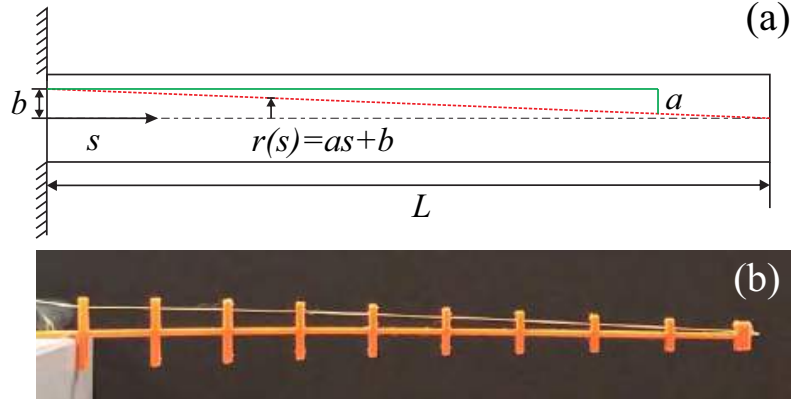


Figure 2.5: A nonparallel tendon routing path is defined by a linear function of s . The diagram (a) and a prototype (b) depict the particular choice of $a = -b/L$, which results in the tendon path converging to the backbone center at the tip of the segment.

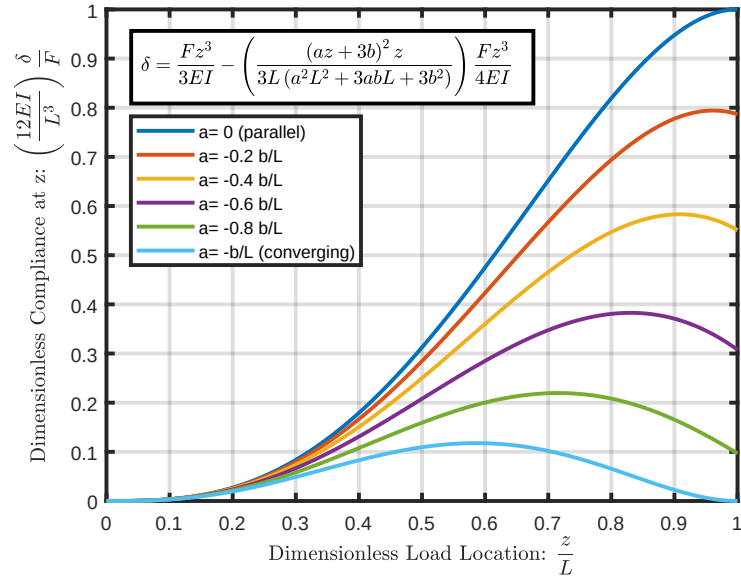


Figure 2.6: The nondimensional relationship between a load's distance z from the base and the deflection δ at the point of loading for multiple slopes of nonparallel tendon routing cases. The tendons are assumed to be inextensible. The line with $a = 0$ depicts our result for the deflection of a member with inextensible parallel tendons, as in Figure 2.3 (but note the y-axis scaling difference). Increasing the slope means terminating the tendon closer to the member's centerline, which this plot shows will increase overall stiffness. The bottommost line, where $a = -b/L$, represents a tendon that is terminated at the centerline, and the compliance is exactly zero for loads at the tip under the assumption of inextensible tendons.

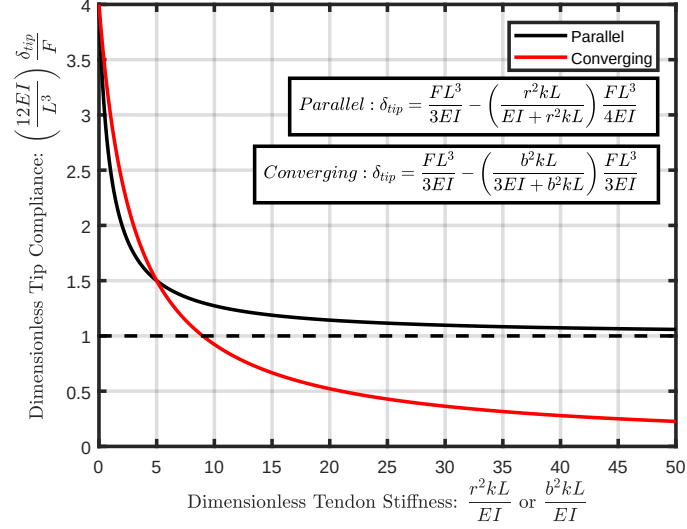


Figure 2.7: The relationship between tendon stiffness and deflection response for a tip load. Both parallel routing and converging routing are shown. Tendon stretch plays a larger role when a tendon is routed in a converging manner, but as the tendon stiffness increases, the robot tip compliance goes to zero (infinite stiffness).

compliance of 1, the tip compliance for converging tendons actually approaches zero as tendon stiffness is increased. This confirms that a converging tendon design could be significantly stiffer than a parallel tendon design with all other factors equal, as long as the effective tendon stiffness is high enough. For a dimensionless tendon stiffness greater than 5, a converging-tendon robot will be stiffer than a parallel tendon robot.

2.3.2 Converging Tendon Inextensibility

Assuming the tendons can be considered inextensible, the deflection equation reduces further:

$$\delta(z) = \frac{Fz^3}{3EI} - \left(\frac{z}{L}\right) \left(\frac{3L-z}{\sqrt{3}L}\right)^2 \frac{Fz^3}{4EI}. \quad (2.33)$$

Figure 2.6 depicts Equation (2.33) (converging) and Equation (2.11) (parallel, previously graphed as the blue line in Figure 2.3) alongside other tendon slope values. The deflection with converging routing is smaller than the parallel case regardless of the load location, and of particular interest is its ability to handle tip loads, as it

produces a tip deflection of zero regardless of the load magnitude:

$$\delta_{tip} = \frac{FL^3}{3EI} - \left(\frac{2L}{\sqrt{3}L} \right)^2 \frac{FL^3}{4EI} = 0. \quad (2.34)$$

This is obviously an impractical result, as some tendon stretch is inevitable, and the effective tendon stiffness must be very high in order to approach this behavior according to Figure 2.7. Regardless, the stiffness can be markedly improved; this is shown experimentally in Section 2.6, where converging tendons increased stiffness by almost a factor of 2 versus parallel tendons (i.e. a factor of 8 improvement over the member's stiffness with no tendons).

2.3.3 Multiple Pretensioned Converging Tendons

For multiple pretensioned tendons that take a converging path, the derivation for the deflection begins similarly to the parallel case with Equation (2.18), where the radius r is defined as $r(s) = as + b$ and $a = -b/L$. The definition of T is then

$$T = \sum_{i=1}^n (k\Delta_i + \tau_p) \cos \phi_i, \quad (2.35)$$

where ϕ_i is defined in Figure 2.4,

$$\Delta_i = \Delta \cos \phi_i,$$

and Δ is as defined in Equation (2.23). Again, the pretension terms cancel out in T due to the trigonometric factor and the equal spacing, and the derivation follows what has been previously done. The resulting formula for the deflection of the loaded point is

$$\delta(z) = \frac{Fz^3}{3EI} - \left(\frac{(3bLz^2 - bz^3)^2 \sum_{i=1}^n \cos^2 \phi_i}{3EI + b^2kL \sum_{i=1}^n \cos^2 \phi_i} \right) \frac{kF}{12L^2EI}. \quad (2.36)$$

2.3.4 Routing Effects on Actuation, Shape, and Design

The significant stiffness advantage of converging routing, especially the theoretically infinite tip stiffness for inextensible tendons, raises the question of whether there is any significant tradeoff associated with a converging tendon design. In particular, one might suspect that the required actuation force for articulation of the tip in free space might be increased by a converging tendon path. In order to characterize this, we examine the deflection equations without an external load for both tendon path cases. Taking Equations (2.3) and (2.22), we set the load force to zero, set $s = L$, and reverse the sign convention on θ for convenience of expression to find actuation forces. The parallel tendon actuation force required to achieve a particular tip angle θ_L in free space is

$$\tau_{parallel} = \frac{EI}{rL} \theta_L. \quad (2.37)$$

In contrast, the required actuation force for the converging path is

$$\tau_{converging} = 2 \frac{EI}{bL} \theta_L. \quad (2.38)$$

For two members that have the same initial base offset, (i.e., $b = r$), the converging tendon member requires twice the actuation force of the member with parallel routing to achieve the same tip angle.

If we compare the tension required to achieve the same tip displacement δ_L in free space, the tradeoff is less significant. Based on Equations (2.7) and (2.26), the required tensions are

$$\tau_{parallel} = \frac{2EI}{rL^2} \delta_L \quad (2.39)$$

and

$$\tau_{converging} = \frac{3EI}{bL^2} \delta_L. \quad (2.40)$$

Thus, the converging tendon case only requires 50% greater actuation force beyond the parallel case. The relative stiffness gain, however, could potentially be much

greater according to Figure 2.7, and actuation systems with higher force capacity and positional precision can easily be designed to handle the increased requirements.

The routing also affects the actuated and deflected shapes of the manipulator. The actuated shape of a parallel-routed manipulator is constant curvature, which has been explored extensively and summarized in [36]. With a converging tendon path, actuation creates a shape with linearly varying curvature (higher curvature at the base) due to the linearly varying moment arm of the tendon. This shape is identical to the shape of a simple cantilevered beam under tip load (for small deflections) because a tip load also creates a linearly varying internal moment. Figures 2.16 and 2.19 in Section 2.6 show the shape of an externally-loaded manipulator for both path routing cases from the experimental validation. The deflected shape in the parallel case is s-shaped (maintaining approximately constant tip angle) while the s-shape is less pronounced in the converging design.

The images in Figure 2.8 demonstrate shape variations caused by routing choices. In the top image, both a parallel- and converging-tendon manipulator are actuated to $\pm 90^\circ$ tip angles. The parallel-path manipulator produces the expected constant-curvature shape, but it requires more tendon displacement and less tension to reach the tip angle than does the converging-routed one. The bottom image shows both initially straight manipulators placed under a 0.9 N tip load. From their deflected shapes, they were actuated until their tip returned to the zero position of the initial state. The parallel-tendon manipulator deformed further under the load, which agrees with our analytical results, and thus it took much more tendon displacement to return the tip to its initial position. This picture shows that it also cannot return to its straight, unloaded shape through actuation; the load causes the s-shape to remain even while actuated. Conversely, the converging-tendon manipulator deflected very little, required less tendon displacement to return, and restored its original shape almost completely.

The choice of routing scheme offers clear differences in stiffness and shape, and it impacts other aspects of robot design, too. For example, parallel routing offers

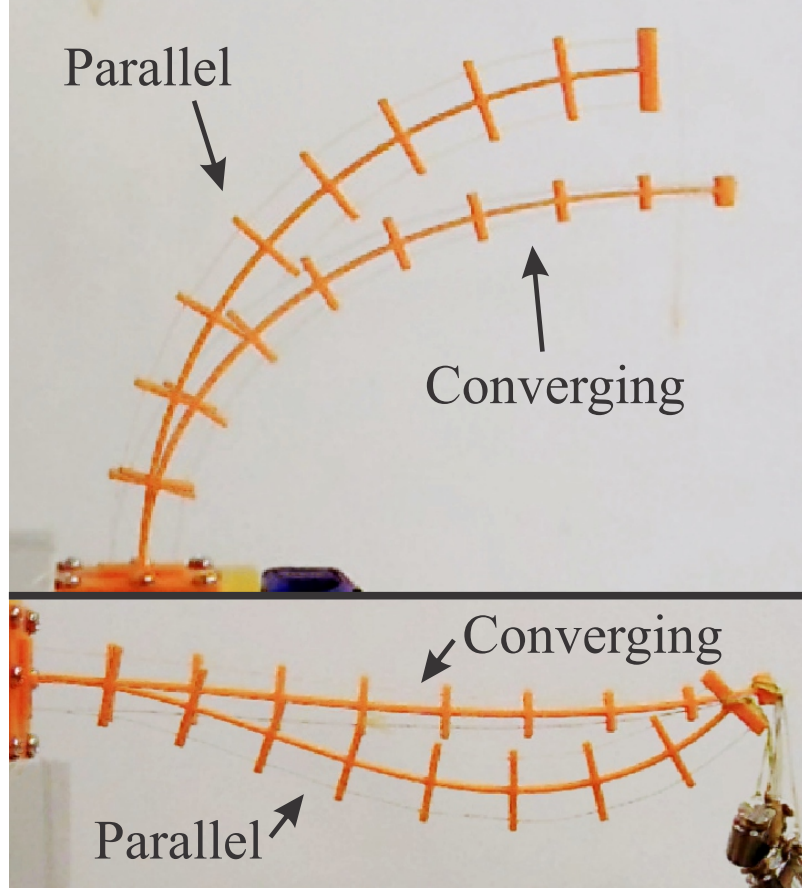


Figure 2.8: Shape differences between converging and parallel routing. (Top) Both manipulators are actuated to a tip angle of 90° with the constant-curvature shape of the parallel-routed manipulator shown against the linearly-varying curvature shape of the converging-tendon manipulator. The parallel manipulator required a tendon displacement 7.1% of its total length to reach this position while the converging manipulator required 5.3%. (Bottom) This overlay shows the results when the manipulators were placed under a 0.9 N tip load and actuated back to the zero position as closely as possible. Clearly, the manipulator with converging tendons recovered its shape much better than the one that has parallel tendons.

simpler construction for robots that embed tendons in a thin tube wall, which is commonly done; a true converging path as defined here cannot be attained in a robot with a central lumen since the tendon could not end at the centerline. However, the non-parallel linear path equations at the start of Section 2.3 describe the deflection of any compliant member with a linearly sloping tendon, which we call semi-converging, and such a manipulator is still stiffer overall as shown in Figure 2.6. Non-constant cross-sections could also be considered, and the approach taken here can be adapted to the case where I is a function of arc length s .

2.3.5 Improving the Deflection Response: Backbone Stiffness vs. Tendon Slope

Locating the tendon termination point closer to the backbone centerline provides clear stiffness advantages. However, it brings about the question of whether the robot benefits more from a change in backbone rigidity— through either stronger materials or a thicker cross-section— or from changing this tendon path. A sensitivity analysis using the Morris method [92] was carried out to investigate this question. We examine the sensitivity of the tip deflection for tendons with inextensible tendons, which is described by Equation (2.30).

The local sensitivity is defined as the partial derivative of the model $Y = f(\mathbf{X})$ with respect to the i – th parameter in a set of n parameters, $\mathbf{X} = X_1, \dots, X_n$, and evaluated at a specific set of parameter values, $\bar{\mathbf{X}}$:

$$S_{l,i}(\bar{\mathbf{X}}) = \left. \frac{\partial Y}{\partial X_i} \right|_{\bar{\mathbf{X}}} \quad (2.41)$$

where $S_{l,i}$ is the sensitivity of Y to the parameter X_i . Then, to form a global estimate of the sensitivity, the local sensitivity is evaluated across a range of parameter values,

and the mean is taken as the sensitivity index:

$$S_i = \frac{1}{m} \sum_{j=1}^m \left. \frac{\partial Y}{\partial X_i} \right|_{\bar{\mathbf{X}}_j} \quad (2.42)$$

where m is the number of evaluations of $\bar{\mathbf{X}}_j$, where all parameters but X_i are held constant. The partial derivative of Equation (2.30) with respect to tendon slope, a , is

$$\frac{\partial \delta}{\partial a} = \frac{(La + 3b)FL^4b(La + b)}{4(L^2a^2 + 3Lab + 3b^2)^2EI} \quad (2.43)$$

and with respect to EI is

$$\frac{\partial \delta}{\partial EI} = \frac{-FL^3(La + b)^2}{4(L^2a^2 + 3Lab + 3b^2)EI^2}. \quad (2.44)$$

These expressions were evaluated at the parameters for the robot geometry, material, and loading of the parallel-routed robot used in the experiment in Section 2.6 to generate a sensitivity measure of deflection to backbone rigidity EI and tendon slope a . The parameters for that robot are the following: $F = 0.5$ N, $L = 201$ mm, $b = 8$ mm, $a = 0$, and $EI = 11.2 \times 10^3$ N-mm². The parameter EI was allowed to range from 10% to 200% of the prototype value, and a ranged from 0 to $-b/L$, which is the total possible range of tendon slope values. For this robot, the resulting sensitivity to EI was -0.022 (N-mm)⁻¹, while the sensitivity to a was 721 mm. Note that the sign difference exists because making the tendon slope *more negative* moves it closer to the robot centerline and decreases the robot compliance.

An example helps to illustrate these results. The prototype in Section 2.6 is made from Amphora ColorFabb HT, which has a modulus of elasticity $E = 1.575$ GPa. That parallel-tendon robot under a 0.5-N tip load has a deflection of 30 mm. Moving the tendon termination three-quarters of the way to the robot centerline where $a = -2.99 \times 10^{-2}$ would reduce the deflection to 4.3 mm, an 85% reduction in deflection. Moving the tendon 80% of the way to the centerline where $a = -3.18 \times 10^{-2}$ reduces the deflection to 2.9 mm (a 90% reduction), and the results in Figure 2.6 show

that a tendon terminated at the centerline with $a = -3.98 \times 10^{-2}$ theoretically has a tip deflection of zero. On the other hand, changing the backbone material to spring steel, which has $E = 207$ GPa and provides around a hundred-fold increase in flexural rigidity, would reduce the tip deflection to 0.24 mm, a 99% reduction in deflection. While that is a much lower deflection than the baseline, it is also a much larger jump in parameter value than what it takes to get a similar result by increasing the slope.

On a practical note, Equations (2.38) and (2.40) show that increasing the bending rigidity of the robot backbone linearly increases the required actuation forces, while the converging tendon case requires only twice the magnitude of actuation force than that of the parallel case for tip angulation, and 1.5 times greater actuation force for equivalent tip deflection. Thus, increasing the EI a hundred-fold for a less compliant load response would also increase the required actuation forces by a hundred fold, while the same load response can be achieved by converging the tendon and only increasing the actuation force marginally, comparatively.

2.4 Review of Spatial Cosserat Model Framework

The analytical analysis is useful for small deflections from an initially straight robot shape, and the dimensionless deflection charts will help designers quickly evaluate the stiffness of candidate robot designs without significant modeling effort. However, high-magnitude external loads and initially curved actuated shapes invalidate the assumptions used. The true robot behavior will diverge from these analytical predictions as the robot gets further away from a nominal straight shape (whether due to actuation or external loads). As an example, Figure 2.9 compares the results of the small-deflection analytical model to the large-deflection 3D model developed in Section 2.5 for an initially straight robot shape. The large-deflection scenarios are not captured well by the analytical formulas. To address this, we extend the Cosserat rod model of tendon-driven robots derived in [18] to include prescribed tendon displacement inputs, tendon stretch, pretension, and slack in the following section.

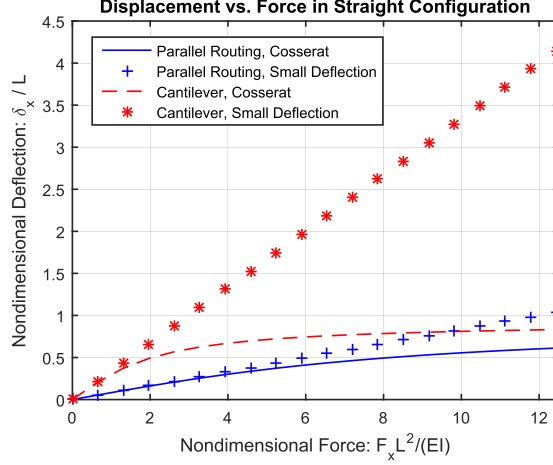


Figure 2.9: Performance of analytical, small-deflection model versus numerical, large-deflection model. The analytical formulas of Section 2.2 are a good approximation for initially straight robot shapes under small loads. At larger loads and significantly curved actuated shapes, the nonlinear Cosserat model should be used.

This nonlinear model framework is geometrically exact for all robot configurations and large loads. We use the notation and conventions established in [18] for the sake of consistency.

This section, for the sake of completeness and clarity, re-presents the model equations that were derived and experimentally validated in [18]. Readers are encouraged to see [18] for an in-depth explanation of the derivations. The definitions and equations given here are nonetheless self-contained and sufficient to reproduce an implementation of the model. The next section builds on this basis to include tendon displacements as inputs and account for tendon stretch and slack.

We characterize a robot by its centerline position $\mathbf{p}(s) \in \mathbb{R}^3$ and its material orientation $\mathbf{R}(s) \in \text{SO}(3)$, which are functions of a reference arc length parameter $s \in [0, L]$. Kinematic variables $\mathbf{v}(s)$ and $\mathbf{u}(s)$ represent the linear and angular rates of change of $\mathbf{p}(s)$ and $\mathbf{R}(s)$ with respect to s expressed in coordinates of the “body frame” $\mathbf{R}(s)$. Thus,

$$\dot{\mathbf{R}}(s) = \mathbf{R}(s)\hat{\mathbf{u}}(s), \quad \dot{\mathbf{p}}(s) = \mathbf{R}(s)\mathbf{v}(s), \quad (2.45)$$

where the dot denotes a derivative with respect to s , and the $\hat{\cdot}$ and operator maps $\mathbb{R}^3$ to $\mathfrak{so}(3)$ as

$$\hat{\mathbf{u}} = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}, \quad (2.46)$$

defined in [93]. In [18], a full Cosserat model and a simplified Kirchhoff model that neglects shear strain and axial stretch or compression of the robot backbone were formulated. For simplicity, we here present the Kirchhoff version, and in this case $\mathbf{v} = [0 \ 0 \ 1]^T = \mathbf{e}_3$. Reference variable $\mathbf{u}^*$ represents the robot shape in its initial, stress-free state. Thus $\mathbf{u}^* = [0 \ 0 \ 0]^T$ for an initially straight robot. Standard linear constitutive laws map $\mathbf{u}$ to the internal moment vector $\mathbf{m}$:

$$\mathbf{m} = \mathbf{K}(\mathbf{u} - \mathbf{u}^*),$$

where $\mathbf{K} = \text{diag}(EI_{xx}(s), EI_{yy}(s), GI_{zz}(s))$ is the stiffness matrix for bending and torsion deformations, $A(s)$ is the cross sectional area, $E(s)$ is Young's modulus, $G(s)$ is the shear modulus, and $I_{xx}(s)$, $I_{yy}(s)$, and $I_{zz}(s)$ are the second moments of area of the principal axes.

We define the routing path of the i^{th} tendon by two functions $x_i(s)$ and $y_i(s)$ that are the coordinates of the tendon in the cross section (constants for parallel tendon routing but can be any general function of s). The body-frame vector from the robot centerline to the tendon is given by

$$\mathbf{r}_i(s) = \begin{bmatrix} x_i(s) & y_i(s) & 0 \end{bmatrix}^T. \quad (2.47)$$

The space curve defining the tendon path in the global frame is then

$$\mathbf{p}_i = \mathbf{R}\mathbf{r}_i + \mathbf{p}. \quad (2.48)$$

In the coupled tendon-rod model derivation in [18], the forces and moments that a tendon applies to the robot as functions of the robot's actual deformed shape were

formulated. This leads to a coupled tendon-rod model that is defined in terms of the following quantities:

$$\begin{aligned}
\dot{\mathbf{p}}_i^b &= \hat{\mathbf{u}}\mathbf{r}_i + \dot{\mathbf{r}}_i + \mathbf{e}_3, \\
\mathbf{A}_i &= -\tau_i \frac{\left(\hat{\mathbf{p}}_i^b\right)^2}{\|\dot{\mathbf{p}}_i^b\|^3}, & \mathbf{B}_i &= \hat{\mathbf{r}}_i \mathbf{A}_i, \\
\mathbf{A} &= \sum_{i=1}^n \mathbf{A}_i, & \mathbf{B} &= \sum_{i=1}^n \mathbf{B}_i, \\
\mathbf{G} &= -\sum_{i=1}^n \mathbf{A}_i \hat{\mathbf{r}}_i, & \mathbf{H} &= -\sum_{i=1}^n \mathbf{B}_i \hat{\mathbf{r}}_i, \\
\mathbf{a}_i &= \mathbf{A}_i \left(\hat{\mathbf{u}}\dot{\mathbf{p}}_i^b + \hat{\mathbf{u}}\dot{\mathbf{r}}_i + \ddot{\mathbf{r}}_i \right), & \mathbf{b}_i &= \hat{\mathbf{r}}_i \mathbf{a}_i, \\
\mathbf{a} &= \sum_{i=1}^n \mathbf{a}_i, & \mathbf{b} &= \sum_{i=1}^n \mathbf{b}_i, \\
\mathbf{c} &= \mathbf{K}\dot{\mathbf{u}}^* - \hat{\mathbf{u}}\mathbf{K}(\mathbf{u} - \mathbf{u}^*) - \hat{\mathbf{e}}_3 \mathbf{R}^T \mathbf{n} - \mathbf{R}^T \mathbf{l}_e - \mathbf{b}.
\end{aligned}$$

where τ_i is the tension of the i^{th} tendon, and $\mathbf{f}_e$ and $\mathbf{l}_e$ are external force and moment distributions along the length. These definitions allowed us to easily write the governing differential equations for the coupled tendon-rod system as

$$\begin{aligned}
\dot{\mathbf{p}} &= \mathbf{R}\mathbf{e}_3, \\
\dot{\mathbf{R}} &= \mathbf{R}\hat{\mathbf{u}}, \\
\dot{\mathbf{u}} &= (\mathbf{H} + \mathbf{K})^{-1} \mathbf{c}, \\
\dot{\mathbf{n}} &= -\mathbf{R}(\mathbf{a} + \mathbf{G}\dot{\mathbf{u}}) - \mathbf{f}_e.
\end{aligned} \tag{2.49}$$

When tendon i terminates at $s = t_i$ along the length of the robot, it applies a point force $\mathbf{F}_i$ and moment $\mathbf{M}_i$ to the robot given by

$$\begin{aligned}
\mathbf{F}_i &= -\tau_i \mathbf{R}(t_i) \frac{\dot{\mathbf{p}}_i^b(t_i)}{\|\dot{\mathbf{p}}_i^b(t_i)\|}, \\
\mathbf{M}_i &= -\tau_i \mathbf{R}(t_i) \left(\mathbf{r}_i(t_i) \times \frac{\dot{\mathbf{p}}_i^b(t_i)}{\|\dot{\mathbf{p}}_i^b(t_i)\|} \right).
\end{aligned} \tag{2.50}$$

If at some location $s = \sigma$, point loads $\mathbf{F}(\sigma)$ and $\mathbf{L}(\sigma)$ (resulting from any tendon terminations and/or external loads) are applied to the backbone, the internal force and moment change across the boundary $s = \sigma$ by

$$\begin{aligned}\mathbf{n}(\sigma^-) &= \mathbf{n}(\sigma^+) + \mathbf{F}(\sigma), \\ \mathbf{m}(\sigma^-) &= \mathbf{m}(\sigma^+) + \mathbf{L}(\sigma),\end{aligned}\tag{2.51}$$

where σ^- and σ^+ denote locations just before and just after $s = \sigma$. If such point loads are applied to the distal end of the robot ($s = L$), then this condition becomes

$$\begin{aligned}\mathbf{n}(L) &= \mathbf{F}(L), \\ \mathbf{m}(L) &= \mathbf{L}(L).\end{aligned}\tag{2.52}$$

Any combination of external point loads and tendon termination loads can be accommodated in this way. The differential equations (2.49) with known proximal boundary conditions $\mathbf{p}(0) = \mathbf{0}$ and $\mathbf{R}(0) = \mathbf{I}$ and general boundary conditions (2.52) represent a forward kinetostatic model where the tendon tensions τ_i and external loads are known constant inputs.

2.5 Modeling Tendon Displacement Inputs in Spatial Cosserat Model Framework

In this section, we now extend the model in [18] that the prior section summarize. The inputs of the original model are prescribed tendon tensions and external loads, and the output is a framed 3D curve representing position and orientation of the backbone material. The top flowchart in Figure 2.10 depicts a typical shooting algorithm for solving the model equations. A guess for the values of the reaction forces at the base is iteratively updated by a general purpose nonlinear solver (e.g. a Levenberg-Marquardt routine) in order to eventually satisfy the distal boundary conditions that

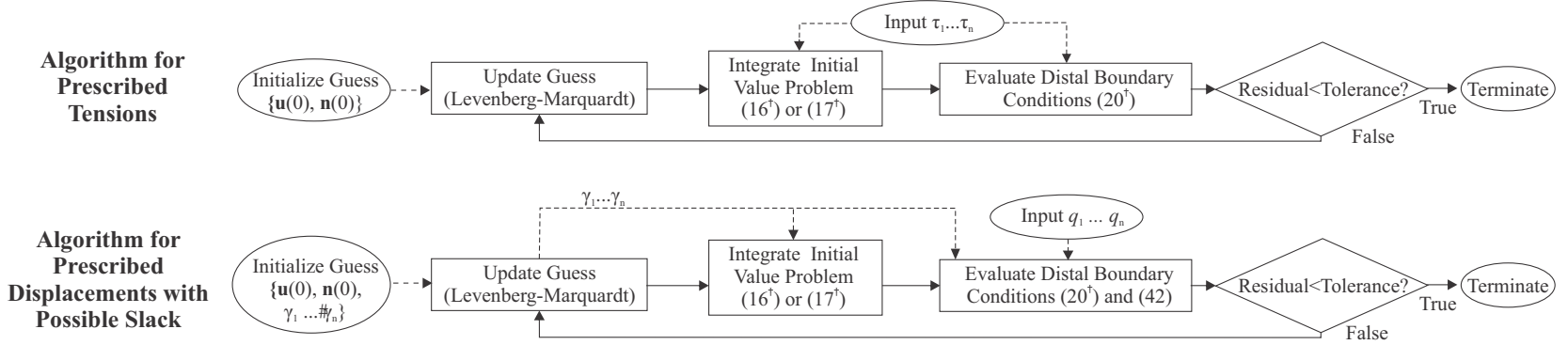


Figure 2.10: Algorithm flowchart comparison of original tendon model and new model extension. These algorithmic flowcharts depict methods for solving the coupled Cosserat rod/tendon model equations subject to prescribed tendon tensions (top chart) or prescribed tendon displacements (bottom chart). The top chart is the approach described in [18], and equation numbers with the “†” symbol refer to that paper. The bottom chart method extends the model of [18] to prescribed tendon displacements with possible tendon slack by including new variables γ_i for the unknown slack/tension and additional constraint equations (2.54) to be satisfied.

the sum of forces and moments at the robot tip is zero for static equilibrium. In order to evaluate the boundary conditions, the differential equations describing the tendon robot are integrated from base to tip using a standard routine for initial value problems (e.g. 4th order Runge-Kutta) [18]. The tendon tensions appear in both the differential equations and the boundary conditions.

In the case of prescribed tendon displacements, the tendon tensions are initially unknown, and there are additional global length constraints that are dependent on the tendon displacements $q_1, \dots, q_n$, measured forward from the initial positions of the tendon bases. In the deformed manipulator state, the arc length s_i along a tendon guide path is implicitly a function of the unknowns $\mathbf{n}(0)$, $\mathbf{u}(0)$, and $\tau_1 \dots \tau_n$ through the following differential equation:

$$\dot{s}_i = \frac{ds_i}{ds} = \|\dot{\mathbf{p}}_i\| = \|\mathbf{u} \times \mathbf{r}_i + \dot{\mathbf{r}}_i + \mathbf{e}_3\|,$$

where $\mathbf{p}_i(s) \in \mathbb{R}^3$ is the global position of the i th tendon, $\mathbf{r}_i(s) \in \mathbb{R}^3$ is the tendon path location with respect to the robot centerline in the local frame (a predefined function), and $\mathbf{u} \in \mathbb{R}^3$ is the robot curvature vector in the local frame. Note again that a dotted variable indicates its derivative with respect to arc length, s . Integrating s_i along with the other state variables gives the total path arc length $s_i(t_i)$ when the tendon terminates at $s = t_i$. The reference path length $s_{r,i}$ can be computed similarly using the reference curvature, $\mathbf{u}^*$, and if the reference shape and guide paths are initially straight and parallel, then $s_{r,i}(t_i) = t_i$. By Hooke's law, and ignoring the possibility of slack, the tendon arc length $s_i(t_i)$ must be

$$s_i(t_i) = \begin{cases} \left(1 + \frac{\tau_i}{(EA)_i}\right) s_{r,i}(t_i) + q_i & \text{(extensible)} \\ s_{r,i}(t_i) + q_i & \text{(inextensible)} \end{cases} \quad (2.53)$$

where $(EA)_i$ is the tendon stiffness, τ_i is the unknown tendon tension, and q_i is a linear actuator displacement to retract the tendon. These equations must be enforced as

an additional constraint to resolve the unknown tensions. Note, that in the special case of inextensible tendons, $(EA)_i \rightarrow \infty$, and the term in parentheses goes to 1. While the assumption of inextensibility appears to eliminate tendon tension from the constraint equation, the tension is still implicitly involved as an unknown since the deformed path length s_i is a function of it, as shown in the equations of [18]. The algorithm framework depicted in Figure 2.10 is the same regardless of whether the tendons are stretchable, and we always solve for the unknown tendon tension that satisfies the appropriate constraint equation. The simulations presented in Figures 2.1, 2.11(b) and (c), 2.15a, and 2.15b all assumed inextensible tendons, while Figures 2.12, 2.13a, 2.13c, 2.13b, and 2.14 considered compliant tendons, thus demonstrating the model's ability to handle both finite and infinite tendon stiffnesses.

In addition to tendon stretch, it is possible to develop slack in one or more tendons. If slack occurs, the integrated path arc length $s_i(t_i)$ will be less than the actual tendon length. We can account for this by introducing an unknown variable β_i for the amount of slack. The length constraint is then

$$s_i(t_i) = \begin{cases} \left(1 + \frac{\tau_i}{(EA)_i}\right) s_{r,i}(t_i) + q_i - \beta_i & (\text{extensible}) \\ s_{r,i}(t_i) + q_i - \beta_i & (\text{inextensible}) \end{cases} \quad (2.54)$$

The inclusion of slack introduces an additional unknown variable for each tendon, but we can reduce the number of unknowns by recognizing that tendon tension and slack are mutually exclusive and restricted to be positive; that is, $\beta_i > 0 \implies \tau_i = 0$ and $\tau_i > 0 \implies \beta_i = 0$. Thus, both effects can be represented with a single unknown variable γ_i :

$$\begin{aligned} \tau_i &= \begin{cases} \gamma_i^2, & \gamma_i \geq 0 \\ 0, & \gamma_i < 0 \end{cases}, \\ \beta_i &= \begin{cases} 0, & \gamma_i \geq 0 \\ \gamma_i^2, & \gamma_i < 0 \end{cases}. \end{aligned} \quad (2.55)$$

Parameterizing tension and slack with a single, continuous variable eliminates the need to explicitly identify the appropriate constraints during model solves. The squaring of γ_i is a choice made so that τ_i and β_i have continuous first derivatives with respect to γ_i . The slack constraint residual error is most simply formulated as

$$E_i^s = s_i(t_i) + \beta_i - \left(1 + \frac{\tau_i}{(EA)_i}\right) s_{r,i}(t_i) - q_i,$$

but for better scaling of the shooting problem we implement the error as

$$E_i^{s^*} = (1 + \tau_i) E_i^s.$$

This seems to result in better solver convergence, and we note that the $(1 + \tau_i)$ term will never nullify E_i because τ_i is positive semidefinite.

For a robot with n tendons, our approach provides the right number of equations to solve for the $6 + n$ unknowns: $\mathbf{u}(0)$, $\mathbf{n}(0)$, and $\gamma_1 \dots \gamma_n$, where $\mathbf{n}(s) \in \mathbb{R}^3$ is the internal force of the backbone. To solve this, we modify the shooting algorithm as shown in the bottom flow chart in Figure 2.10. The tension/slack variables $\gamma_1 \dots \gamma_n$ are simply appended to the set of unknowns that is iteratively updated and obtained by the Levenberg-Marquardt routine. The iteratively updated variables are all used in the differential equations and boundary condition evaluations, which now include the new constraints in (2.53) based on the prescribed displacements $q_1 \dots q_n$.

2.5.1 Spatial Simulation and Discussion

Before providing non-dimensional displacement charts based on the Cosserat model, we illustrate the use of our new approach in three simulations computed using the methods described above and rendered in Figure 2.11. The simulated robot was modeled with an incompressible backbone (Kirchhoff rod) of length $L = 0.1\text{m}$, diameter 1.04mm , Young's modulus $E = 210\text{GPa}$ and shear modulus $G = 80\text{GPa}$, and three equally spaced parallel tendons with guide channels offset $r = 0.01\text{m}$ from

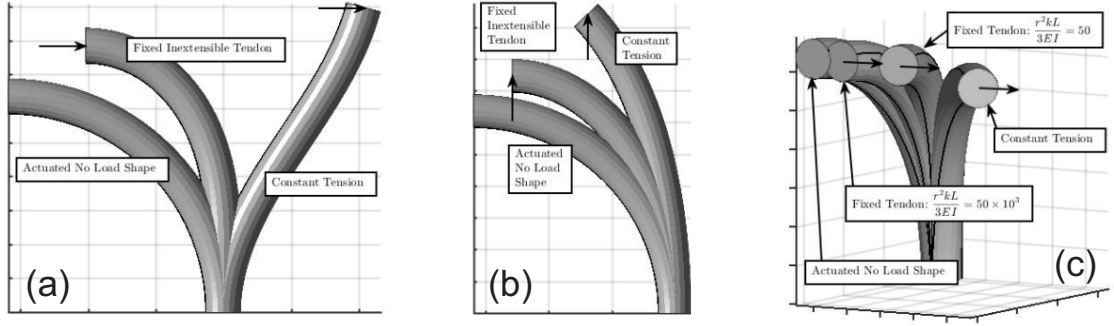


Figure 2.11: Simulations with Cosserat-rod-based, numerical tendon robot model. In each plot, simulations of tendon-driven manipulators are shown where the initial actuated tip angle is 90° . A load is applied, and the resulting displacements are calculated and compared using both prescribed tensions and prescribed displacements. (a) and (b) show single-inextensible tendons manipulators with in-plane loads while (c) has three evenly-spaced, stretchable tendons with an out of plane load.

the centerline. Computation time using an unoptimized MATLAB implementation on a standard laptop computer with i7 processor is typically the order of 1 second or less, depending on how close the initial guess is to the solution, but note that optimized implementations of Cosserat rod models can run in real-time as demonstrated in [94]. Figure 2.11 (a) and (b) show a manipulator actuated to a 90° tip angle by inextensible tendons. Transverse and vertical loads are then applied, and the plot compares the resulting deformed shape in the case of prescribed displacements and prescribed constant tension. This affirms the main conclusion of our analytical results of previous sections and extends it to the actuated case: prescribed tendon displacements significantly increase the output stiffness.

2.5.2 Sensitivity to Tendon Stiffness in Special Loading Cases

We now use our model to illustrate the fact that in certain special cases, even a very small amount of tendon stretch can significantly affect a robot's loaded shape. In Figure 2.11 (c), we simulate an out-of-plane load applied to the same initial

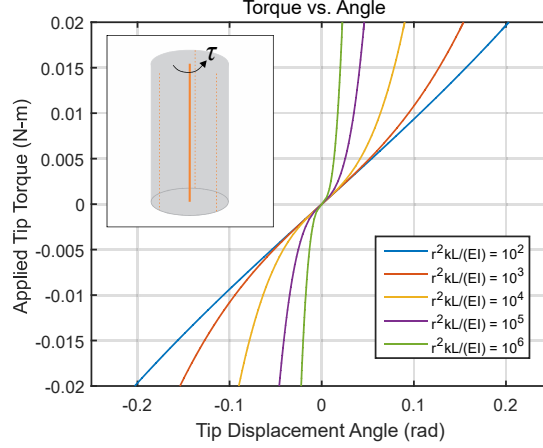


Figure 2.12: Example of an artificial singularity in the torsion case to motivate use of tendon stretch in the model. A torque is applied at the end effector of a three-tendon robot with an incompressible backbone in the straight configuration. There is a singularity $\lim_{c \rightarrow 0} \partial\theta/\partial\tau = 0$. Interestingly, the slope at $\theta = 0$ is constant regardless of the tendon compliance.

configuration with three symmetrically spaced, stretchable tendons. The out-of-plane loading causes torsion as well as bending, and the torsion is a significant contributor to the overall shape and amount of tip deflection. However, torsion has a minimal effect on the stretch of individual tendons. These two observations imply that only highly stiff tendons can mitigate the significant torsional effects caused by out-of-plane loads, and this is illustrated in the plot. For the in-plane loads, increasing the dimensionless tendon stiffness beyond $\frac{r^2 kL}{3EI} = 50$ has a negligible effect, but in the out-of-plane load case, the plot shows a significant deflection difference between $\frac{r^2 kL}{3EI} = 50$ and $\frac{r^2 kL}{3EI} = 50 \times 10^3$. Since robots encounter such loads in many practical scenarios, this example underscores the importance of actually modeling the tendon stretch (and/or similar effects such as backbone compression) in order to accurately predict robot performance.

Further, we can conceive another a special case in which tendon inextensibility actually creates a paradox. Figure 2.12 considers three straight tendons on a straight, incompressible backbone subjected to a pure axial moment at the tip. If we keep increasing the tendon stiffness, the torque/displacement curve becomes more

nonlinear, and we see larger resistance for finite displacements, but the slope at zero is apparently unaffected. In the limit as tendon stiffness approaches infinity, the member cannot exhibit any finite angular displacement (since any angular displacement of the tip must either elongate the tendons or compress the backbone), yet the slope of the torque/displacement curve at zero remains constant, and if there is zero angular displacement, geometry dictates that neither the backbone material nor the tendons can possibly provide a resistance torque to balance the applied moment, creating a singular paradox. This again shows the importance of modeling tendon and backbone compliance, even if they are small, in order to avoid singularities introduced by assuming zero axial strain in both the backbone and tendons.

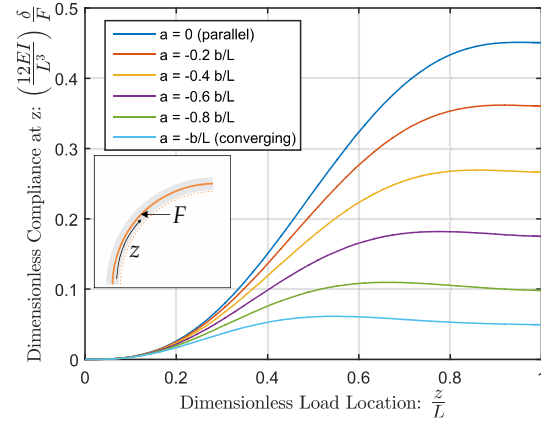
2.5.3 Dimensionless Deflection Plots for the 3D Actuated Case

We now provide dimensionless stiffness plots for a typical 3D actuated case, analogous to the analytical results using the planar small-deflection assumptions in the previous section. These plots are generated by the Cosserat model with parameters given in the previous section and for a robot actuated to a 90° tip angle and subjected to loads in the transverse (horizontal) direction in the bending plane, the axial (vertical) direction in the bending plane, and loads orthogonal to the bending plane (out-of-plane), as depicted by the force arrows in the figure insets. Following the discussion in the previous section, we note that poor numerical conditioning in the case of inextensible tendons caused convergence issues in some cases, so a high dimensionless stiffness of 8.29×10^6 was used instead. Figure 2.13 shows the dimensionless compliance (for small loads) at a loaded point z in each load direction (transverse, axial, and out-of-plane). As in the previous analytical results, the robot's stiffness increases with converging tendon routing, except in the out-of-plane load, where the routing path seems to have a negligible effect.

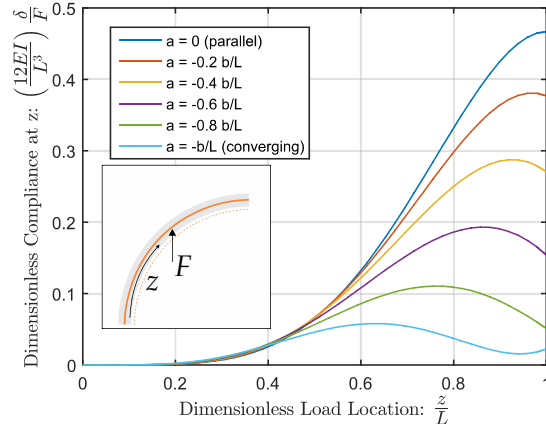
The effect of tendon stiffness in the 3D model is further examined in Figure 2.14, which shows the relationship of tendon stiffness to tip compliance in the three directions for a robot with a single tendon (in both parallel and converging cases) actuated to a 90° tip angle. For the axial loading direction, the converging tendon design results in greater tip stiffness if the dimensionless tendon stiffness is larger than 15. For the transverse loading direction, the converging design is stiffer for all tendon stiffnesses. The effect of tendon stiffness and routing path for loads in the out-of-plane direction is nearly imperceptible, but the results from the simulations in Figure 2.11 suggest that these lines do have a small slope.

2.5.4 Simulation with Slack

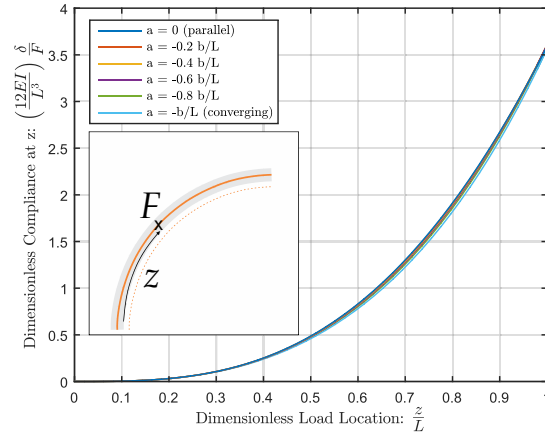
In certain circumstances, it is possible for external loads to induce tendon slack, and this can significantly affect the displacement. While robots with opposing tendons can generally avoid slack by employing a fixed amount of pretension, robot segments with a single tendon or with tendons only on one side have no redundancy available for pretension to exploit, and thus slack is more likely to occur. Such a scenario is illustrated in Figure 2.15 (a). An incompressible robot backbone has a single inextensible tendon with a prescribed displacement such that the tip is brought to a 90° angle. When a vertical external load is applied, the tendon initially remains in tension, and the tip angle remains at 90° while the tip displaces downward. Above the critical load of 3.6N, the tendon tension goes to zero, slack is induced, and the tip is free to rotate. In the induced-slack regime, the compliance at the tip is a factor of 3 higher than it was when the tendon remained in tension. To illustrate the benefit of employing opposing tendons, the same simulation experiment is shown in Figure 2.15 (b) with two opposing tendons and enough pretension to avoid the load-induced slack in the shorter tendon. The avoidance of slack results in a significantly smaller deflection at 0.5N and constant tip orientation throughout the loading.



(a) Compliance in X-Direction



(b) Compliance in Z-Direction



(c) Compliance in Z-Direction

Figure 2.13: Dimensionless compliance for forces applied to a 90-degree actuated robot in the x-, z-, and y-directions. The dimensionless tendon stiffness is 8.29×10^6 for all plots.

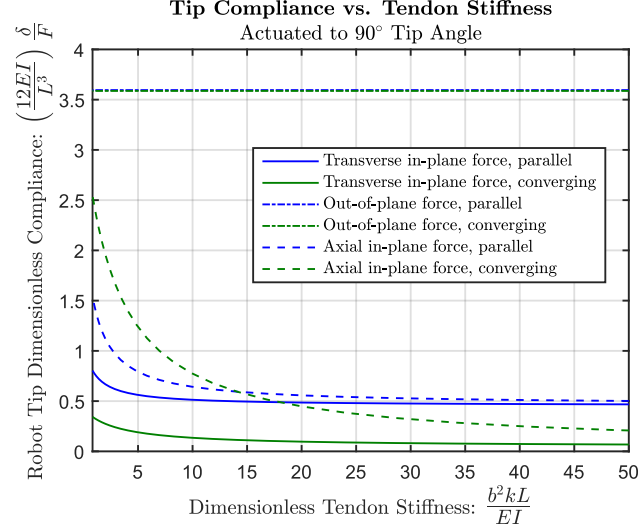


Figure 2.14: Simulated results for the robot tip compliance versus dimensionless tendon stiffness using numerical model. This plot includes both parallel and converging designs in the 3D case where the robots are actuated to a 90° tip angle and subjected to loads in the axial, transverse, and out-of-plane directions. The results with the spatial rod model match the analytical results in Figure 2.7.

For the simulations used to generate Figure 2.15, the robot backbone length is $L = 0.2\text{m}$ and the tendon offset is 0.01m . The Young’s modulus and shear modulus are 207GPa and 79.6GPa respectively, and the diameter of the backbone is 0.8mm .

2.6 Validation of Models

2.6.1 Validation of 2D Analytical Models

We validated the formulas from Sections 2.2 and 2.3 in a set of experiments where known weights were hung from a cantilevered elastic member with tendon guide channels, shown in Figure 2.16, for both a member with parallel tendon routing and one with converging tendon routing. Aside from the tendon paths, the members and experimental procedures were identical between the two routing cases, and the dimensions are provided in Figure 2.16(a). They were 3D printed using Amphora ColorFabb HT on a MakeIt-Pro M printer. The members had a square cross section

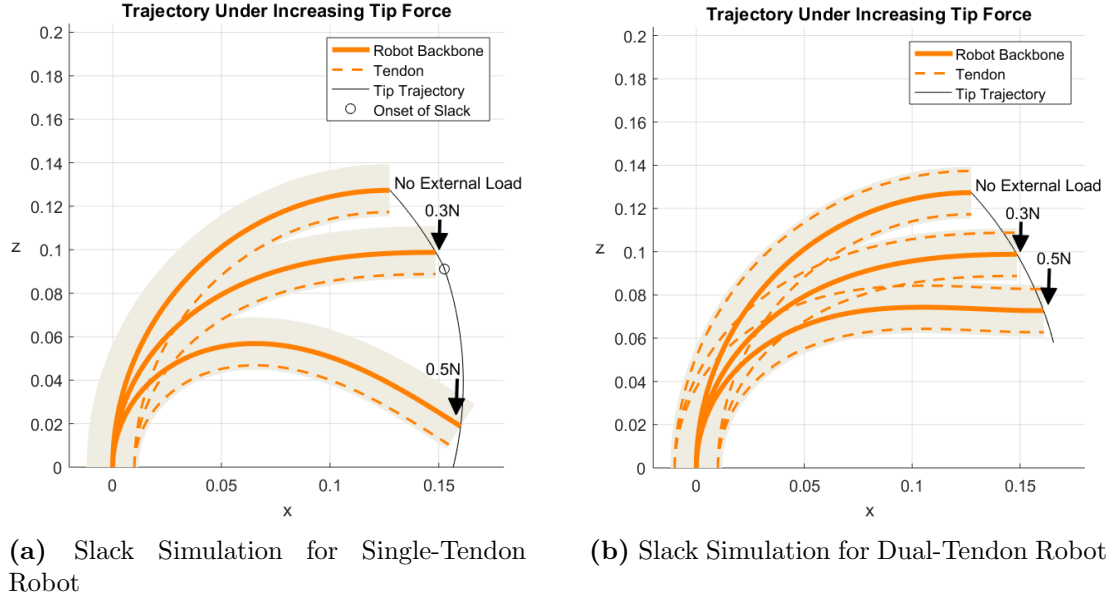


Figure 2.15: Simulations demonstrating tendon slack. In both plots, the x- and z-axis have units of meters. **(a)** A simulated tendon robot is actuated to 90° , and the actuator position is locked. Initially, the robot maintains its orientation under an increasing tip force, but a sufficiently large tip force results in tendon slack, and the orientation is no longer constrained. After the load induces slack, the tip compliance is about 3 times greater than when the tendon was taught. **(b)** Repeating the simulation with an additional opposing tendon yields a drastically different result. Due to a constant pretension, both tendons remain in tension and continue to constrain the tip orientation to 90° , resulting in much less displacement at 0.5 N.

of 3 mm side length. Ten evenly spaced tendon guides were added with tendon routing holes at a radius of 8 mm from the centerline for the parallel tendon case and holes beginning at the base with a radius of 8 mm and ending at the tip with a radius of 1.875 mm for the converging case. The tendon guide on the tip was 3 mm thick to mitigate deformation of the support at the point at where the tendons were attached, and all others were 2 mm thick, making the total length 201 mm. Ending the converging tendon exactly at the centerline (as specified in Section 2.3) would diminish the structural integrity of the tip, as the tendon hole would have cut through the member's cross-section, so instead, the distal end of the converging tendon was terminated with the smallest offset from the centerline that could be

created without cutting through the member. We installed a single tendon (size 69 Kevlar thread) through the top set of guides on each member.

For each routing case, we performed eight baseline measurements with no tendon attached, measuring the deflection due to a 20-gram calibration weight hung at various locations along the robot length. The load locations were at evenly spaced points along the member (i.e., at each tendon guide except the two closest to the base). Unique markers were attached to each of the tendon guides that were used as a location for a load, as shown in Figure 2.16(b), and a MicronTracker H3-60 (Claron Technology, USA) was used to capture displacement data at all 8 points on the member for each load. It has a reported calibrated accuracy of 0.20 mm, with jitter contributing an additional possible 0.07 mm at most for static targets. We calibrated the effective flexural rigidity EI of the member by performing a least squares fit of the data from these 8 displacement experiments, where the standard error of the fit was calculated by taking the square root of the sum of the squared errors divided by the number of data points. The results are shown by the red dashed line in Figures 2.17 (a) and (b). In the parallel tendon case, the resulting calibrated rigidity of $EI = 11.2 \times 10^3 \text{ N mm}^2$ had a standard error of fit of 1.33 mm and was just over 5% higher than the nominal value computed from the cross section dimensions and an E of 1.575 GPa for ColorFabb HT. In the converging tendon case, the calibrated rigidity was found to be $EI = 11.3 \times 10^3 \text{ N mm}^2$, which is just 6% higher than the nominal value, with a standard error of fit of 1.46 mm. Slight differences in the cross section dimensions from print discretization, variability in the material properties and print conditions, and small misalignments of the load application points can explain the minor deviations from the nominal flexural rigidity value in the two prototypes.

Next, we rigidly attached the tendons to an Omega LC70 3-10 load cell with a 3D printed bracket, pulling the tendon taught enough to avoid slack while not inducing bending in the structure before fixing it to the bracket. The force data was streamed through the Arduino IDE serial monitor using a SparkFun Load Cell Amplifier HX711 and an Arduino Mega 2560. The load cell was then anchored to a rigid base, and we

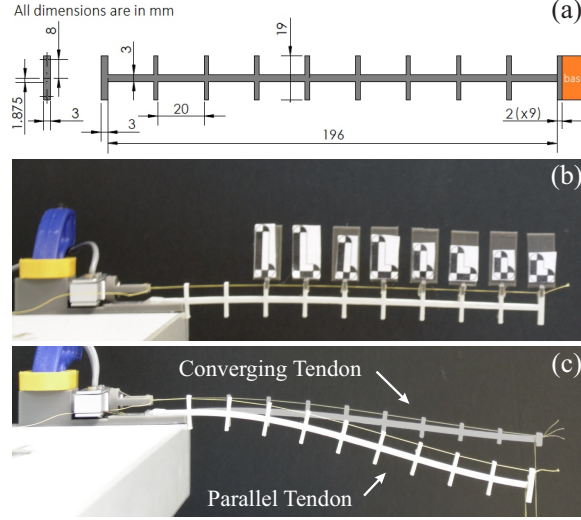
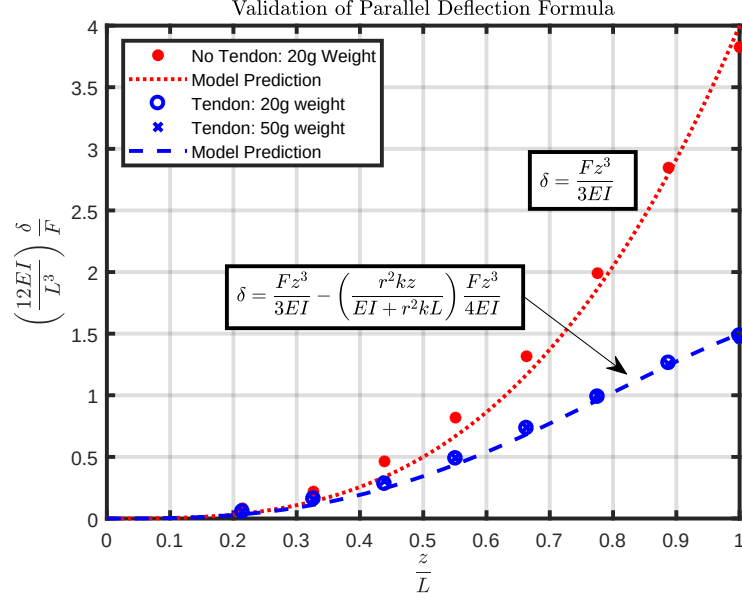


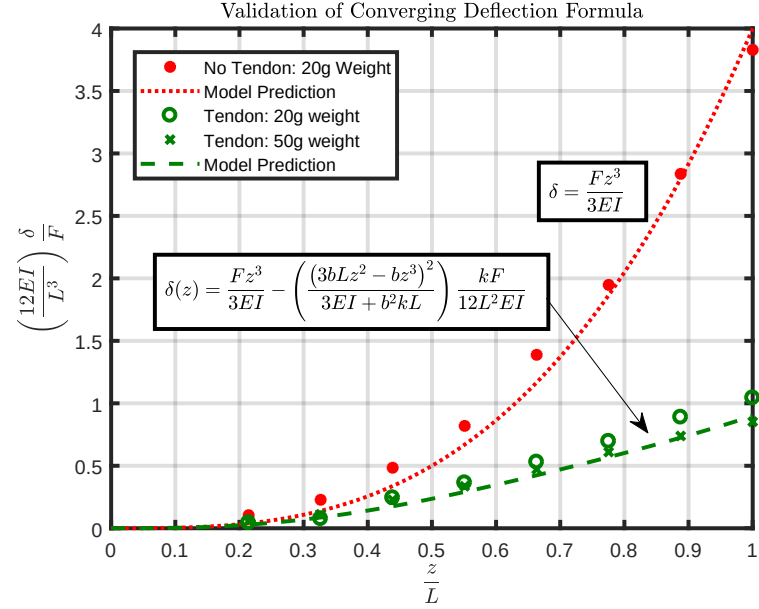
Figure 2.16: Setup for analytical tendon robot experiments. Shown are the tendon-embedded structures, dimensioned in (a), where the loads were applied at eight evenly spaced points along the member length (the inner corners of the 8 most distal tendon supports). The radial offset in (a) is the offset for the parallel tendon structure and the initial offset in the converging structure, and the terminal offset for the converging tendon is also shown. The markers used to track deflection are shown in (b), and the overlaid image in (c) shows both structures under a 0.5-N tip load; the parallel-routed member deflects around twice as much as the converging-routed one.

performed the same eight displacement tests on both members, except a 50g weight was used to apply the forces. To calibrate each effective tendon spring constant k , we performed a least squares fit of the data from these 8 experiments for each member, using its previously calibrated value of EI . The resulting prediction is compared to the data in Figures 2.17 and 2.19.

For the member with a parallel tendon, the calibrated value of k was 4.5 N/mm with a standard error of the fit of 1.20 mm, resulting in the nondimensional parameter $\frac{r^2 k L}{EI} = 5.03$. We note that the actual Kevlar thread has a much higher stiffness than this calibrated value, so the calibration is likely accounting for other factors that create an effect similar to a reduced tendon stiffness, such as finite clearance between the tendon and the guide holes, axial compression of the central backbone (which we showed in Section II can be modeled by an effective tendon stiffness), and compliance of other structures such as the tendon guide where the tendon was fixed at the tip. For



(a) Validation of Parallel Routing Case



(b) Validation of Converging Routing Case

Figure 2.17: Results for analytical tendon robot experiments. (a) Data from the loading experiments for the parallel routing case are shown with the curves of the analytical formulas. (b) Experimental data for the converging routing case is shown alongside the curve for the analytical formula. The tendon-less member data that was used for the calibration of EI is shown in red.

the member with a converging tendon, the calibrated tendon stiffness was $k = 8.55$ N/mm with a standard error of fit of 1.07 mm (making for a dimensionless tendon stiffness of 9.52). The effective stiffness is lower in the parallel routing case due to a small amount of bending in the final tendon guide where the tendon is fixed at the tip. In the converging case, there is practically no bending in the final guide because the moment arm is nearly zero, another advantage of the converging tendon design that affects stiffness.

Finally, we cross-validated the performance of our calibrated model on a third, independent set of 8 data points for each robot, which were generated with a different weight (20g) hung at each point on both members with fixed tendon displacements. These data agreed with the prediction of the model using the two previously calibrated parameters EI and k , as shown in Figure 2.17 (a) for parallel tendons and Figure 2.17 (b) for converging tendons. The tendon force data, averaged from 100 points taken during each loading scenario, can be seen in Figure 2.18. The predicted model again uses the calibrated EI and k , and the measured tensions follow closely to the prediction. If friction were a significant factor, the measured tension would be much different from our model prediction, as we do not account for it; thus, the tension measurements illustrate that our assumption of negligible friction is valid for our test system.

Figure 2.19 shows the experimental data from the 20-gram and 50-gram weights superimposed on the theoretical manipulator shapes for the last four loading cases in Cartesian space. Integration of the sine and cosine of theta (Equations (2.3) and (2.22) with their respective equations for τ substituted in) with respect to arc length produces the theoretical curve in each case. These equations also used the respective calibrated EI and k values from the experiment for the parallel and converging shapes. The plotted experimental data points show the measured deflection in the y-axis of the eight tracked points at the appropriate x location calculated by integration to the load arc length. The plots demonstrate the loaded shape differences between the two routing paths; the parallel path produces an s-shape when deflected, and the

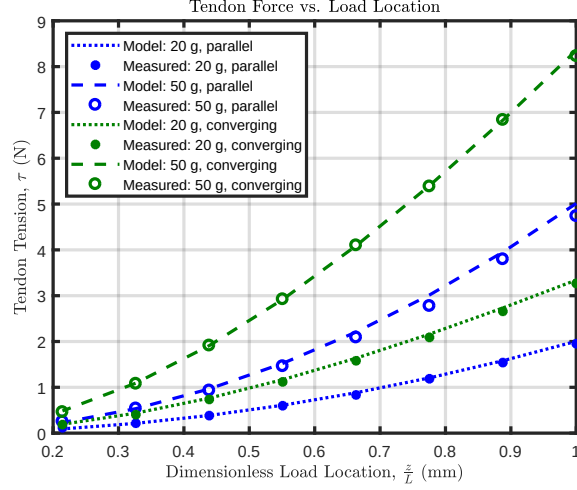


Figure 2.18: Tendon force data versus model prediction. For each applied load and load location, the tendon tension was measured. The model-predicted curves use Equations (2.6) and (2.25), which do not account for friction, and the calibrated EI and k values from the experiment. These results show that friction can be safely neglected for our setup, at least in the initially straight case. As the robot becomes more curved, friction plays an increasing role as indicated by Figure 2.21

converging tendon path produces smaller deflection along the entire length of the manipulator than the parallel path for the same load. In particular, the converging tendon design is shown to be stiffer than the parallel design by a factor of two, which is consistent with the deflection formulas and the gains predicted by Figure 2.7 at the calibrated effective tendon stiffnesses.

2.6.2 Validation of 3D Model for In-Plane Loads

We also validated the 3D numerical Cosserat model using members of the same geometry and fabrication method as the prior experiments. A motor mount structure was made with 3D printed parts, and Dynamixel MX-28 servo motors were used to actuate each member using a single tendon, as shown in Figure 2.20. Tests were carried out with the robots at a 30° and 60° tip angle. The motor was gripped in a vice and clamped to the table to ensure stability. A tracking marker was placed on the tip and base of the robot in order to measure tip deflection in the robot base frame.

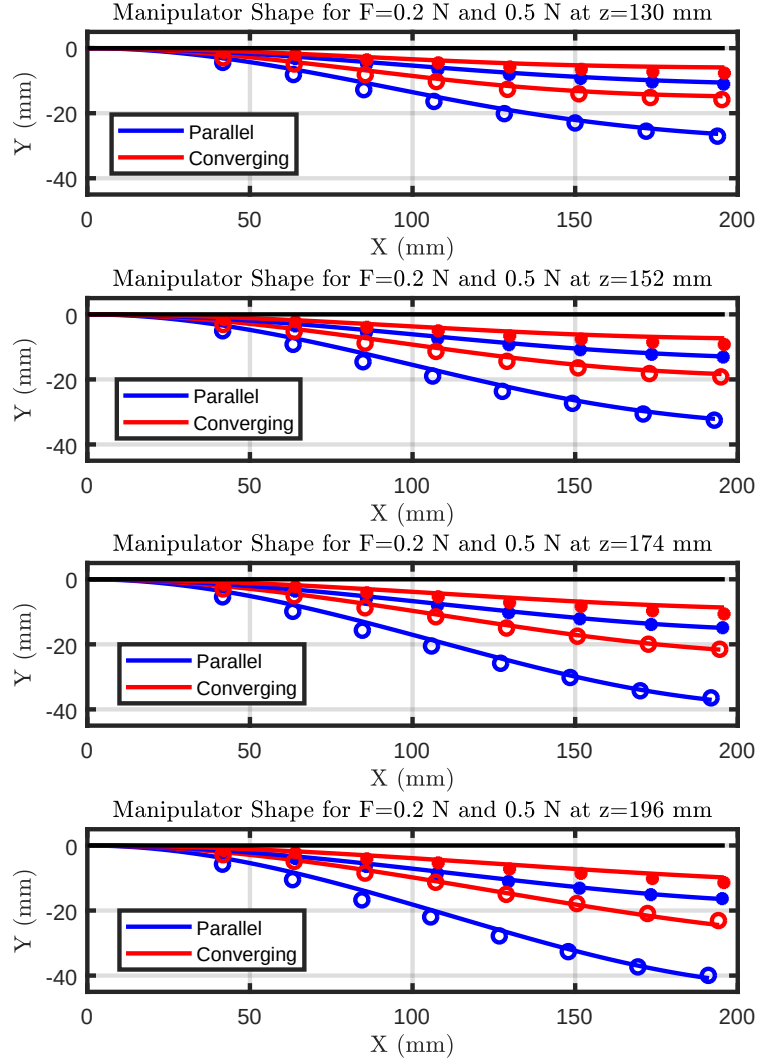


Figure 2.19: Loaded tendon robot shape from initially straight position. Shown here are the shapes of parallel- and converging-routed prototypes when loaded at different points along the manipulator. For a load of 0.2 N and 0.5 N on each manipulator, the shape was found by integrating the equations for $\theta(s)$ with respect to arc length. The circles on each plot represent the experimental data of the y-displacement of the measured points plotted at the correct arc length; the solid circles are the 0.2 N data, and the rings are the 0.5 N data.

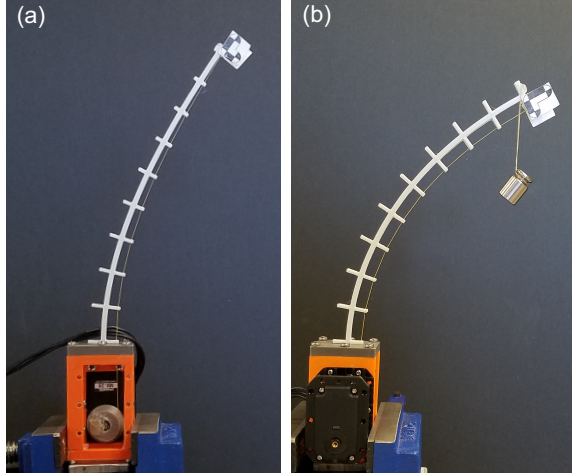


Figure 2.20: Setup for numerical model tendon robot experiments. The robots were actuated to 30° and 60° and loaded. **(a)** shows the converging robot unloaded, and **(b)** shows the parallel-routed robot under a load, both actuated to a 60° tip angle.

The MicronTracker was used to capture displacement data as the tip was loaded in the axial (vertical) direction using weights of 10 to 50 grams in 10-gram increments. The direction of loading meant tendon slack could develop, but it was not observed at these magnitudes of tip forces. The deflection data is shown in Figure 2.21.

The frictionless model predicts the deflection well for the tests carried out at a 30° tip angle and slightly less accurately in the 60° actuated case. While the predicted stiffness difference between the parallel and converging cases is well matched, the data shows slightly stiffer behavior than the model prediction. This discrepancy is likely due to the stiffening effects of static friction between the guide holes and the tendon, which is not considered in our models. We note that these robots were not specifically designed to minimize tendon channel friction. The further the robot is actuated, the more pronounced frictional effects will be, so designs that are intended for larger ranges of motion would benefit from lubricious materials for the tendon channels or friction-reducing coatings. The effects of static friction can also potentially be reduced by “dithering” during robot control as described in [19].

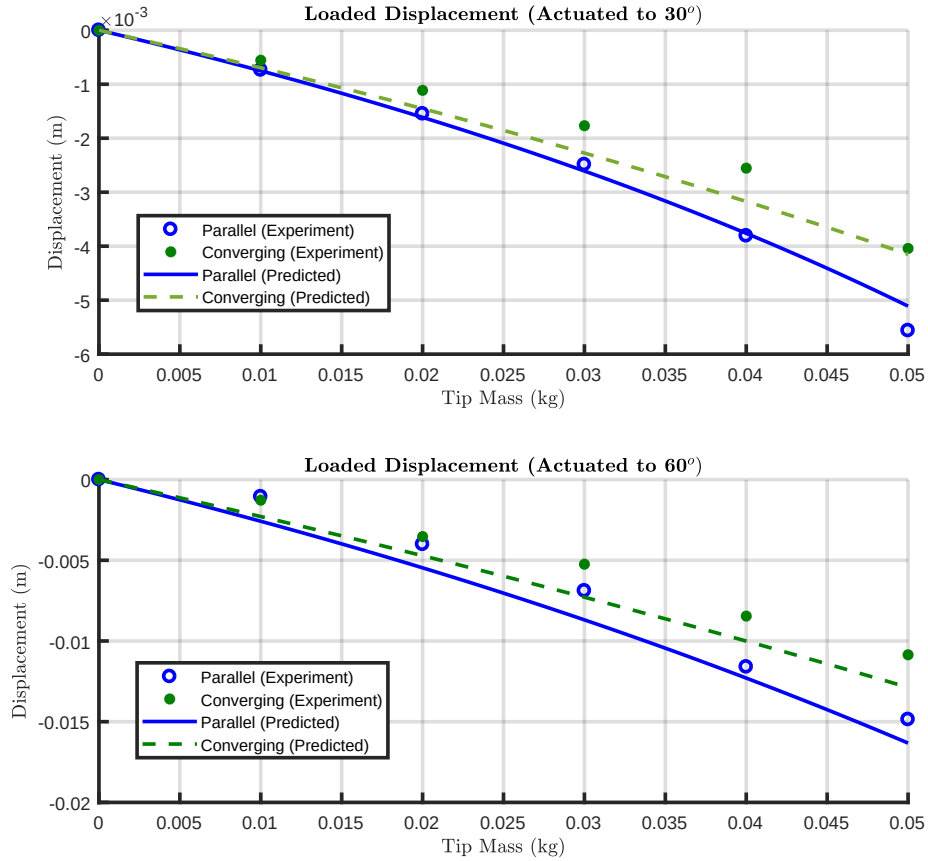


Figure 2.21: Results for numerical model tendon robot experiments. The robots were actuated to 30° and 60° tip angles and loaded in the axial direction. The results agree well with our model prediction for the 30-degree case, but friction in the 60-degree case causes slightly less deflection than predicted. This points to the need for including friction when lubricious materials or friction-reducing coatings are unavailable for robots that are actuated to high angular displacements.

2.7 Conclusions

This chapter explored the behavior of continuum robots driven by prescribed tendon displacements under external loads. Using Bernoulli-Euler beam equations, we derived analytical equations for the loaded deflections of a compliant member embedded with one or more tendons. This provided useful insight into the role of tendon stretch, backbone compression, and tendon location on the overall stiffness. We additionally investigated non-parallel routing paths and showed that using a converging tendon path can significantly increase the stiffness of the member along its entire length, though it also increases the required actuation force for tip articulation and induces a shape with linearly varying curvature. We believe that the effects of non-parallel tendon routing can be exploited to improve continuum robot design by creating stronger, more controllable robots, and our aim is that the results presented here will serve designers well as a tool for selecting design parameters and control schemes.

To account for large-deflection scenarios, we also extended the general Cosserat rod/tendon model in [18] to accept prescribed tendon displacement inputs, making it useful for simulating the majority of tendon-driven manipulators, which are tendon-displacement controlled. The model extension also newly accounts for tendon slack, stretch, and pretension. Simulations show that accurately predicting out-of-plane loading requires modeling the tendon stretch, and that load-induced slack can increase compliance. We used the extended model to generate numerically-simulated results for nondimensional compliance in x, y, and z directions along the length of an actuated robot with various tendon routing. Similarly, we explored how the nondimensional tendon stiffness affects the compliance of an actuated robot in all three directions for parallel and converging routing schemes.

Finally, we validated the analytical models with a set of experiments, showing that friction is negligible when considering small deflections from an initially straight shape. The Cosserat model was validated using position-dictated actuation for two

actuated shapes (30° and 60° tip angle) and five external load magnitudes. The results showed that the model, which does not include friction, predicts the externally-loaded shape of the robot well when actuated to 30° , but friction plays a larger role as curvature increases, and the results for 60° agree well but begin to display the limitations of our model, one of which was that we neglected static tendon friction in all the models and analysis for the sake of tractability. Static friction can effectively stiffen a tendon-driven robot, and this effect is greater at larger tensions and configurations with higher curvatures, as our experimental results showed. Modeling and estimation of tendon friction in robots without external loads has been recently addressed in [95], and those results may eventually be adaptable to models with external loading in the future.

Chapter 3

Modeling and Design of Notched-Tube Continuum Robots

Disclosure of Previously Published Material

The work in this chapter has been partially published with the dissertation author as first author on all publications; a preliminary form of the work in Section 3.3 was presented at the 2017 SPIE Medical Imaging conference [96], and the work in Sections 3.2, 3.4, 3.5, 3.6, and 3.8 compose a manuscript submitted to IEEE Transactions on Robotics, which is currently in review at the time of writing of this dissertation [97]. The work in Section 3.7 was original to this dissertation at the time of writing.

D. C. Rucker, the chair of this dissertation’s committee, was final author on all publications, again providing guidance and support throughout the projects. The material for the 2017 SPIE Medical Imaging conference [96] was created with the help of second author Zane Epps, who supported initial model formulation efforts. Adam Daniel served as second author on the IEEE T-RO submission [97], assisting with prototyping, model troubleshooting, and figure creation.

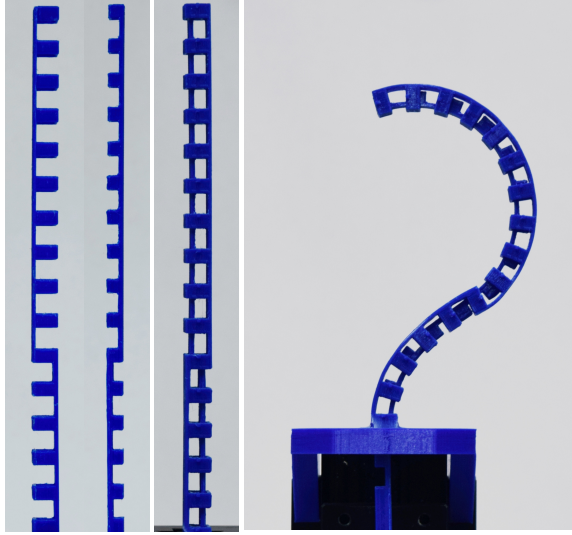


Figure 3.1: The basic construction of the NTCR. Two tubes with asymmetrically notched sidewalls are nested inside one another with their backbones parallel, and their tips are fixed to each other. Then, translating one tube with respect to the other generates bending that varies with the notch profile.

3.1 Overview

There are various means of actuating continuum robots, many of which allow the actuators to remain at the base of the robot— some systems pull on tendons down the length of an elastic element to generate bending [1, 2, 3], others have pneumatic chambers that are actuated by pressurization [4, 5, 6], and others still combine elastic elements that interact with each other, arriving at various equilibrium positions due to asymmetry in the components or their configuration [7, 8]. Concentric-tube robots (CTRs) [55, 98, 56, 14] and multi-backbone robots [67, 68] fall into this latter category. The concept explored in this chapter, the notched-tube continuum robot (NTCR), combines beneficial characteristics from both concentric-tube and multi-backbone approaches.

The basic form of the NTCR consists of two nested tubes, each with rectangular notches removed from its wall as shown in Figure 3.1. This removal creates sections of tube with a much smaller bending moment of inertia than the unmodified stock, and the asymmetric geometry offsets the neutral bending plane to the remaining backbone

of tube, which is a circular segment. The uncut segments then effectively act as rigid blocks, given their much larger bending moment of inertia, and the neutral plane of the backbone is considered the neutral plane for the whole tube. Its design is inspired by a tendon-driven wrist design by York et al. in [51] and Swaney et al. in [25], which is also studied by Eastwood et al. in [53]. These devices and the NTCR both use asymmetric material removal to shift the neutral axis of a primary tube; however, an NTCR uses a second tube fixed to the tip to induce bending in the structure instead of the pull-wire used in the wrist designs. This strengthens the overall structure and adds bi-directional actuation capability, which can increase both workspace and force capacity.

Selective removal of tube wall material has been used to modify the properties of metal and plastic tubes for a number of purposes in robotics, such as shifting the neutral axis [51, 53], generating high angulation for compact devices [26, 52], reducing bending stiffness for tendon-actuated designs [28, 30, 31], and increasing the torsional stability of concentric tube robots [63, 64]. The NTCR concept leverages asymmetric material removal to enable the tubes themselves to function as actuation transmission elements, which simplifies the design and enables bi-directional bending through push-pull actuation without any additional components. This concept is analogous to push-pull multi-backbone actuation, but in a different form factor which preserves a central working channel through which flexible instruments can be deployed. It also inherits the simplicity and ease of miniaturization of precurved concentric-tube robots [55, 98, 56, 14] while eliminating the issue of elastic instability [59] and enabling more straightforward control of curvature.

In addition to its compact form factor and straightforward actuation principle, the NTCR represents a versatile and robust design concept that can be created with a variety of materials and through both additive and subtractive fabrication methods (milling, wire EDM, laser-cutting, etc.). Additive manufacturing of concentric tube robots has been explored by Motorimoto et al in [57], and Amanov et. al. use it to generate patient-specific manipulators in [47]. Similar customization frameworks can

be used for the design of the NTCR in principle, and this work develops a model-based approach to designing an NTCR that lays the groundwork for that possibility.

This chapter of the dissertation introduces the NTCR concept, defines relevant geometric relationships of the parameters of the notch profile, derives an approximate constant-curvature kinematic model, and presents a more general, variable-curvature kinematics model for planar NTCRs. It also describes a design method for generating notch parameters along the length given a desired actuated shape in Section 3.5, accounting for material strain limits to achieve a robust and feasible design. Section 3.6 details experiments performed on several variable-curvature NTCR prototypes with multiple notch patterns, cross-sections, and materials that were developed using the proposed design method, showing that the results have good agreement between both the desired curved and the experimental prototype’s actuated shape as well as the kinematics and experiments. Section 3.7 presents a design method for achieving specified tip poses that leverages the established shape design process. Section 3.8 discusses some of the considerations and practicalities in building an NTCR and offers some guidance on design.

3.2 Geometric Relationships of Backbones

The bending plane location and stiffness profile along the backbone are determined by the notch parameters, as shown in Figure 3.2: the number (n), length (h), spacing (c), depth (g), and orientation (ϕ) of each individual notch. The parameters of h and c determine when the notches will self-interfere at the edges of the workspace, and their relationship dictates how closely an arc can approximate the overall shape of the manipulator (i.e., if h is small and c is large, the manipulator will have long, straight sections, whereas if the opposite were true, the straight sections would be small and negligible). The notch depth, g , determines the offset of the neutral axis (and thus the location of the bending plane) as well as the strain in the notch, which is typically the limiting factor for the workspace unless one uses high-strain materials such as

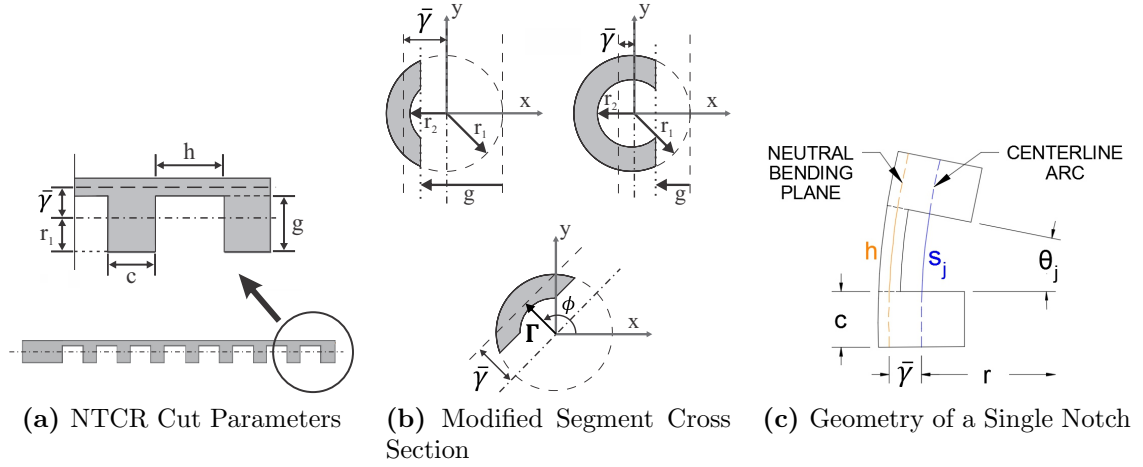


Figure 3.2: The NTCR notch parameters and geometry of a single notch. The curvature of a single notch is assumed to be constant with respect to arc length along that notch, though each notch may have its own individual curvature.

nitinol. The orientation, ϕ , determines the rotation of the bending plane about the axis of the tube. While both the inner and outer tubes share the same values of n , h , and c , they have different values for g and ϕ . The notched tubes are concentrically aligned, with the two backbones being parallel to each other on opposite sides of the manipulator (see Figure 3.1) and with the tips rigidly attached to each other. When the base of one tube translates with respect to the other, the tubes are placed in agonist-antagonist compression and tension, which, in turn, causes bending in the device. When the notches are uniform along the length, the manipulator shape can be approximated by a constant-curvature arc. However, when the notches are varied along the length, the actuated shape is non-constant curvature.

3.2.1 Neutral Axis Offset Geometry

The notch depth, g , is the most important factor in determining the stiffness of a given notched section, and varying it along the tubes generates a variable curvature shape. This parameter determines the second moment of area, I , as well as the neutral axis

offset, Γ , which we define as a vector within the plane of the cross section:

$$\mathbf{\Gamma} = \bar{\gamma} \begin{bmatrix} \cos(\phi) \\ \sin(\phi) \end{bmatrix}, \quad (3.1)$$

where ϕ is the notch angle measured from the x-axis and $\bar{\gamma}$ is the magnitude of the offset, as shown in Figure 3.2. Here, we consider only planar segment curves in the $x - z$ plane, so ϕ is restricted to be either 0 or π depending on the direction of the cut. Thus, only the x-component of $\mathbf{\Gamma}$ is nonzero, and we write

$$\gamma = \bar{\gamma} \cos(\phi),$$

where here and throughout the rest of the paper, the symbol γ denotes the signed neutral axis location in the x direction, whereas $\bar{\gamma}$ denotes the absolute value of γ .

The equations to calculate $\bar{\gamma}$ and I in terms of the notch depth are provided in the Appendix for circular, square, and hexagonal tube cross sections, though provided that the cross-section has an axis of symmetry in the direction of the notch, the tubes from which a planar NTCR is created can have any shape.

3.2.2 Backbone Curvature Relationships

The relationship between the curvature of the inner and outer tube backbones are dictated by their opposing, offset backbones. Since the outer and inner tubes have notches in opposite directions,

$$\phi_i = \phi_o - \pi \quad (3.2)$$

where ϕ_o and ϕ_i are the angles of the outer and inner tube neutral axis offsets, respectively, measured from the positive x-axis. This implies that $\text{sgn}(\gamma_o) = -\text{sgn}(\gamma_i)$. From that relationship, we also define the signed distance between the two, d , as

$$d = \gamma_i - \gamma_o, \quad (3.3)$$

which is also

$$d = r_o - r_i = \frac{1}{\kappa_o} - \frac{1}{\kappa_i},$$

where r_o and r_i are the radii of curvature of the outer and inner tubes, respectively, and κ_o and κ_i are the corresponding curvatures, as illustrated in Figure 3.3.

We assume that the curvatures are constant over the length of a single notch and that the curves of the tubes' neutral axes share a common center of curvature outside the body of the manipulator (which merely excludes curvatures so large that self-contact would occur). These assumptions imply the following relationships and constraints:

$$r_o = r - \gamma_o \quad r_i = r - \gamma_i$$

and

$$\frac{1}{\min(\gamma_i, \gamma_o)} < \kappa < \frac{1}{\max(\gamma_i, \gamma_o)}$$

This results in a relationship between the curvatures

$$\kappa_i = \frac{\kappa}{1 - \gamma_i \kappa} \quad \text{and} \quad \kappa_o = \frac{\kappa}{1 - \gamma_o \kappa}, \quad (3.4)$$

and the denominators can never be zero according to the above restrictions. Note that the curvatures and radii of curvature are allowed to be negative, which would indicate that the segment is curving in the opposite direction than shown in Figure 3.3 (a).

3.3 Approximate Constant-Curvature Model

If all notches are uniform along each tube, i.e., all notches in a tube are cut to the same depth and height, the Notched-Tube Continuum Robot forms a circular arc. This is useful for quick analysis of simple designs and for studying the effect of clearances on the manipulator.

Uniform cuts in both tubes generates an approximately constant-curvature arc when actuated, and the following equation for its kinematics holds true:

$$\theta = \theta_0 + \Delta \mathbf{q}(\bar{\gamma}_i + \bar{\gamma}_o)^{-1} \quad (3.5)$$

where θ is the angle of the end effector from the horizontal, θ_0 is the angle of precurvature, $\Delta \mathbf{q}$ is the translation of one tube with respect to the other, and $\bar{\gamma}_i$ and $\bar{\gamma}_o$ are the distances from the center of the assembly to the centroids of the outer and inner tube backbones, respectively. This equation is useful for the examination of the effect of clearances, as they always cause the inner backbone to deform to the inner wall of the outer tube. As this happens, this increases the distance between the backbones since the tubes are no longer perfectly concentric, effectively increasing $\bar{\gamma}_i + \bar{\gamma}_o$. This equation shows that in this case, it causes the actuation q to generate a smaller angular displacement than expected.

Since constant curvature is assumed, the centerline arc length of a single deflected notch can be multiplied by the number of total notches in order to find the total deflection of the manipulator. Using the geometry shown in shown in Figure 3.2 (c), the centerline arc length of a single notch of the outer tube, which is assumed to be fixed, is

$$s_j = h - \bar{\gamma}_o \theta_j \quad (3.6)$$

where s_j is the deflected centerline arc length and θ_j is the angle of a single notch. The uncut sections of the tube are considered rigid and non-deformed, and $\theta = n\theta_j$, giving the total manipulator length equation as

$$s = (nh - \bar{\gamma}_o \theta) + c(n - 1) + b \quad (3.7)$$

where c is the height of the uncut section and b is the length of the manipulator's end effector past the last notch. Using this equation for the centerline arc and Equation (3.5), we can find the position of the end effector at any given Δq value using the

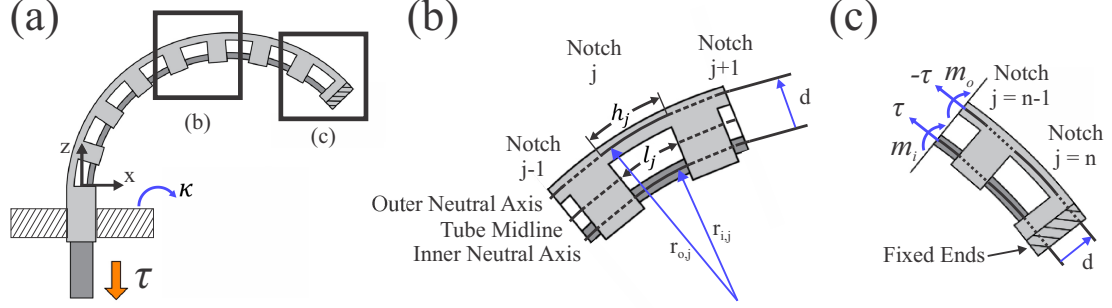


Figure 3.3: Diagrams for the NTCR kinetostatics derivation. (a) The coordinate definitions, where pulling on the inner tube creates a positive curvature, κ . (b) A close-up of the j^{th} notch, showing the relationship between inner and outer tube curvatures and the manipulator centerline. (c) A free-body diagram of the tip.

following homogeneous transformation for motion in the x-z plane, based on the constant curvature model reviewed by Webster and Jones [36]:

$$T_{base}^{tip} = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & \frac{s(1-\cos(\theta))}{\theta} \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & \frac{s\sin(\theta)}{\theta} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3.8)$$

3.4 NTCR Kinetostatics Derivation

For non-uniform notch profiles, a more complex model is required, and here we develop a kinetostatics model to capture non-constant curvature behavior. This static model does not include friction forces, which increase as curvature increases and can play a significant role at the edges of the manipulator's workspace but are generally minimal through most of the robot's range of motion. Material nonlinearities, which may be significant factor when using materials such as nitinol, are also neglected in this model. York et al and Swaney et al use an approximate, piecewise stress-strain relationship and Castigliano's theorem to address this issue with favorable results [51, 25].

3.4.1 Mechanics

As shown in Figure 3.3, pulling on the inner tube is defined as positive actuation. With that defition, summing the moments about the outer tube's neutral axis as shown in the free-body diagram in Figure 3.3(c) gives

$$-d\tau + m_o + m_i = 0 \quad (3.9)$$

where τ is the tension carried by the inner tube, d is the distance between the neutral axes of the tubes, and m_i and m_o are the internal moments of the inner and outer backbones, respectively. According to the linear constitutive law of classical beam mechanics,

$$m_i = E_i I_i \kappa_i \quad \text{and} \quad m_o = E_o I_o \kappa_o,$$

where E is Young's Modulus and I is the second moment of area. Using this and the relationship between the tube curvatures in Equation (3.4), we can rewrite (3.9) in terms of centerline curvature as

$$-d\tau + E_o I_o \frac{\kappa}{1 - \gamma_o \kappa} + E_i I_i \frac{\kappa}{1 - \gamma_i \kappa} = 0. \quad (3.10)$$

This moment equation is valid for all notches along the manipulator. In terms of only the outer tube curvature and for the j^{th} notch, τ is:

$$\tau = \left(E_o I_{o,j} \kappa_{o,j} + E_i I_{i,j} \frac{\kappa_{o,j}}{1 - d\kappa_{o,j}} \right) d_j^{-1}. \quad (3.11)$$

If tension inputs are known, this equation can be used with Equations (3.13) through (3.17) to attain the robot position.

3.4.2 Kinematics

Often, displacement is used to control robot actuation, so we continue the model to find the kinematic relationship between the relative displacement of the tubes and

robot position. The actuation variable, q , is the relative displacements of the tube bases, which is found by subtracting the length of the two backbones when actuated:

$$q = s_o - s_i.$$

This can be reduced using the geometric relationships defined in Subsection 3.2.2 and written in terms of the outer tube notch curvatures, κ_o :

$$q = \sum_{j=1}^n d_j h_j \kappa_{o,j}. \quad (3.12)$$

where h_j is the notch length of the j^{th} notch and d_j is the distance between the axis offsets of the corresponding j^{th} notches on the inner and outer backbones. To solve the forward kinematics problem, this equation must be solved numerically to find the array of κ_o values for a given q . In addition to satisfying this kinematic equation, the set of curvatures must also satisfy the requirement that τ is constant throughout the backbones (since friction is assumed to be negligible). This provides us with $n + 1$ variables ($\kappa_{o,1}, \kappa_{o,2}, \dots, \kappa_{o,n}, \tau$) and $n + 1$ equations (Equation (3.12) and Equation (3.11) for $j = 1 \dots n$). Actuation in the negative direction is defined as the inner tube being pushed through the outer tube, and once this translation is large enough, the solid segment of tube below the base of the first notch of the inner tube prevents the first notch of the outer tube from bending. In this case, the number of variables reduces by one, removing κ_o and τ of the first notch from the set of equations and unknowns.

Once the outer tube curvatures are known, we can rearrange Equation (3.4) for the j^{th} notch to find the centerline curvature, κ_j :

$$\kappa_j = \frac{\kappa_{o,j}}{1 + \gamma_{o,j} \kappa_{o,j}}. \quad (3.13)$$

Since the outer tube notch length remains constant as the notch bends, the centerline length l_j over the notch changes with the curvature according to

$$l_j = \frac{h_j}{(1 - \gamma_{o,j}\kappa_j)} \quad (3.14)$$

for each notch. The transformation from the base of notch j to the base of notch $j + 1$ consists of a constant curvature bending transformation

$$\mathbf{T}_{b,j} = \begin{bmatrix} \cos(\kappa_j l_j) & 0 & \sin(\kappa_j l_j) & (1 - \cos(\kappa_j l_j)) \kappa_j^{-1} \\ 0 & 1 & 0 & 0 \\ -\sin(\kappa_j l_j) & 0 & \cos(\kappa_j l_j) & \sin(\kappa_j l_j) \kappa_j^{-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.15)$$

followed by a rigid translation

$$\mathbf{T}_{r,j} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c_j \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3.16)$$

Then, the transformation from the base of the first notch to the tip is

$$\mathbf{T} = \prod_{j=1}^n \mathbf{T}_j = \prod_{j=1}^n \mathbf{T}_{b,j} \mathbf{T}_{r,j}, \quad (3.17)$$

fully defining the position of the robot in space using the relative tube translation, q .

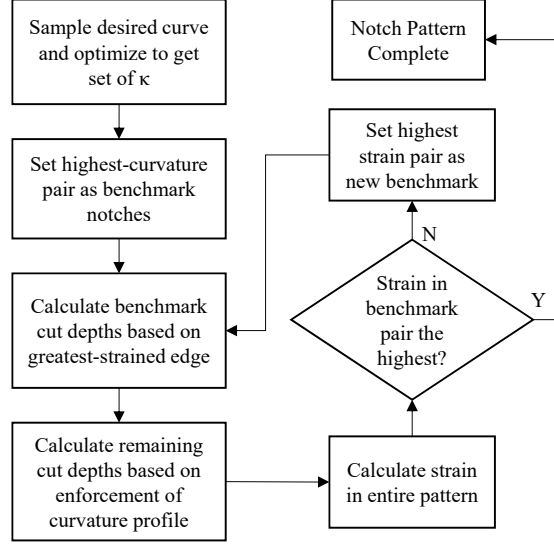


Figure 3.4: The NTCR shape design process. A flow diagram illustrating the design process for creating an NTCR for a desired actuated shape.

3.5 Notch Design for Variable Curvature Actuated Shapes

The NTCR can be designed to achieve many actuated shapes by dictating the notch spacing (c), length (h), depths (g_o & g_i), and orientation (ϕ_o & ϕ_i). The following procedure generates a full NTCR notch pattern from a given desired planar centerline shape. Aside from defining the desired curve, the designer need only choose the tube geometry, number of notches, n , and percentage of the centerline arc length, β , that is composed of the non-notched, non-bending NTCR segments. The design process is depicted in flowchart form in Figure 3.4. This process optimizes the design to maximize stiffness while remaining within the material strain limit at the desired maximum actuation.

To summarize the process, first, a set of curvatures consistent with the NTCR kinematics is fit to the desired shape. Next, we initially assume that the notched segment with the highest curvature will also be the notch with the highest bending strain. We design the notch depths of the inner and outer tubes at this benchmark

segment by maximizing stiffness while enforcing a maximum strain constraint (with a factor of safety) and an inner/outer stiffness ratio of 1. Once this benchmark pair of cut depths is determined, we use the mechanics model to enforce the correct ratio of curvatures between the benchmark notch and all remaining notches. Finally, we check the magnitude of bending strain in all notches to verify the assumption that the benchmark notch experiences the highest strain. If it does not, the procedure is repeated with the true highest-strain notch as the benchmark notch. The final result is a notch pattern that (1) generates the desired curve when actuated and (2) ensures that the device is as stiff as possible without exceeding the factor of safety on the material strain limit.

3.5.1 Curve Sampling & Fitting

The desired actuated shape will generally be defined by some continuous centerline curve $\mathbf{p}_d(s)$ in the $x-z$ plane, parameterized by arc length s from 0 to L . The desired curve is sampled at $n + 1$ evenly spaced points:

$$s_j = (j - 1) \frac{L}{n} \quad \text{for} \quad j = 1 \dots n + 1.$$

Each discrete segment of length $\frac{L}{n}$ represents a robot segment containing a notched portion followed by a rigid (straight) portion. The lengths of the rigid segments, c and curved segments l are defined by the design input parameter β , which is the percentage of the centerline curve that is considered to be straight:

$$c = \frac{L}{n} \beta, \quad l_j = \frac{L}{n} (1 - \beta). \quad (3.18)$$

β is generally kept around 0.5, as discussed in Section 3.8. Starting at the base of the robot, we sequentially choose the desired curvature κ_j of each curved segment that minimizes error at each sampled point. Specifically, $\mathbf{p}_j$, the position of the end of the j^{th} notched-rigid segment pair, can be calculated as a function of the curvature κ_j

using the transformations in Equations (3.15), (3.16), and (3.17). We then select k_j to minimize e_{j+1} , the component of the error which is normal to the desired curve at s_{j+1} , i.e.

$$e_j = N_j^\top (\mathbf{p}_j - \mathbf{p}_d(s_j)) \quad (3.19)$$

where

$$N_j = R_y\left(\frac{\pi}{2}\right) \mathbf{p}'(s_j)$$

represents the vector normal to the curve at s_j , $\mathbf{p}'(s_j)$ is the arc-length derivative of position at s_j , and $R_y\left(\frac{\pi}{2}\right)$ is the rotation matrix associated with a 90-degree rotation about the y-axis. Since we proceed sequentially from $j = 1$ to n , we only solve a single nonlinear equation in the unknown κ_j at each step, and we use MATLAB's *fsolve* function to accomplish this. These calculated values of κ_j are then checked to verify that they produce a curve that fits the desired curve and are subsequently used throughout the rest of the design process.

3.5.2 Determination of Benchmark Notch

For each notch pair, the two notch depths g_o and g_i together determine the relationship between the actuator displacement and the centerline curvature of that notch pair, which is shown to be dependent on both backbone stiffnesses in Equation (3.10). Thus, there is some redundancy in the choice of two parameters that will achieve a single desired segment curvature. To resolve this redundancy, we could impose any additional constraint on the depths. Through preliminary implementations, we have found that constraining the flexural rigidities of the inner and outer notched segments to be equal to each other results in designs that are more mechanically robust, so we set

$$E_o I_o = E_i I_i. \quad (3.20)$$

For a physically possible design, the g values are bounded to be positive and no greater than the tube's outer diameters, as well.

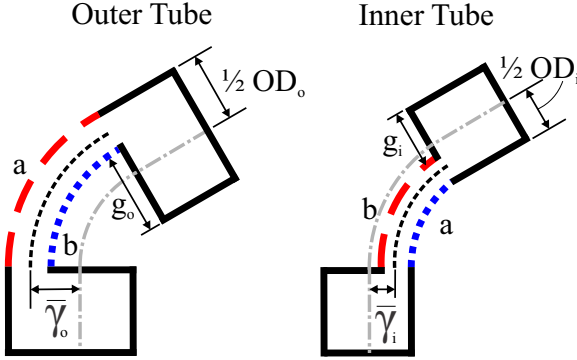


Figure 3.5: Strain in the NTCR. An illustration of a single notch shows the strain pattern in the inner and outer tubes at the same curvature. The outer tube edge, a , and the inner edge formed by the notch, b , experience strain depending on the direction of actuation. Under positive actuation, the inner tube has tension on b while the outer tube experiences tension in a . The opposite is true for negative actuation.

Once the relationship between notch pairs are chosen, a rule that governs the benchmark pair must be selected. Generally, we want the manipulator to be as stiff as possible while still being able to achieve the desired range of motion within the material strain limits. We achieve this design criteria by enforcing that the maximum tensile strain experienced in any notch is less than the material strain limit ϵ_f by a factor of safety FS :

$$\epsilon_{FS} = \epsilon_{max}/FS.$$

We assume initially that the outer tube of the notch pair experiencing the highest magnitude curvature, κ_m , will also experience the greatest strain. The magnitude of the strain experienced under actuation is expressed as

$$|\epsilon_{x,j}^a| = |\kappa_{x,j}| \left(\frac{OD_x}{2} - \bar{\gamma}_{x,j} \right) \quad (3.21)$$

and

$$|\epsilon_{x,j}^b| = |\kappa_{x,j}| \left(\frac{OD_x}{2} + \bar{\gamma}_{x,j} - g_{x,j} \right), \quad (3.22)$$

where (3.22) represents the strain in the inner wall formed by the notch, a , (3.21) represents the strain of the outside edge of the tube, b , and x is a placeholder for either the subscript for the inner tube, i , or for the outer tube, o . As illustrated in Figure 3.5, the outer edge, a of the outer tube is in tension when actuation is positive while the inner edge, b , is in compression. The reverse is true for the inner tube. The initial calculation assumes the tensile strain will be higher and calculates the notch depth based on (3.22):

$$\epsilon_{FS} = |\epsilon_{o,m}^b| = |\kappa_{o,m}| \left(\frac{OD_o}{2} - \bar{\gamma}_{o,m} \right).$$

Then, we use our relationship in (3.20) to calculate $g_{i,m}$. With the pair now fully defined, we test the strain of edges a and b for both the inner and outer tube. If any edge experiences higher-magnitude strain than the one assumed, we re-calculate the cut depths according to the strain limit and either (3.21) or (3.22) depending on the edge. The complementary tube's notch depth is then derived from the constraint in (3.20). Once those are determined, we calculate d_m , $I_{o,m}$, and $I_{i,m}$ for the benchmark pair for use in the next design step.

3.5.3 Calculation of Full Notch Pattern

The Curvature-Ratio Method

In one method of determining the remaining notch parameters, we first take Equation (3.10) and assume $\gamma_o\kappa$ and $\gamma_i\kappa$ are much smaller than unity (i.e. a slender robot or relatively small curvature). In the limit as these values approach zero, $\kappa_o = \kappa_i = \kappa$, (3.10) becomes linear in κ , and we have the solution

$$\kappa = \frac{d\tau}{E_o I_o + E_i I_i}. \quad (3.23)$$

Assuming the inner tube tension τ is constant down the length of the manipulator (i.e. assuming no friction), Equation (3.23) is rewritten for the j^{th} notch as:

$$\kappa_j = \alpha_j \tau \quad (3.24)$$

where

$$\alpha_j = \frac{d_j}{E_o I_{o,j} + E_i I_{i,j}}.$$

Then, to ensure that the notches achieve the desired curvatures, we enforce the following constraint based on Equation (3.24) to calculate the remaining notch depths:

$$\left| \frac{\kappa_j}{\kappa_m} \right| = \left| \frac{\alpha_j}{\alpha_m} \right|, \quad (3.25)$$

where κ_m and α_m are the benchmark notch pair properties. For tubes made of the same material, where $E_o = E_i$, Young's modulus within α cancels out. This ratio is dependent on both inner and outer notch depths. However, the relationship defined by Equation (3.20) allows it to be defined solely in terms of outer tube notch depth, which can be found with a nonlinear solver such as MATLAB's *fsolve* function. For the initial guess for the solver, we use

$$g_{o,j} = g_{o,m} \left| \frac{\kappa_j}{\kappa_m} \right|.$$

The Actuation-Force Method

Another means of determining notch depths is to use Equation (3.11), which calculates the actuation force. Rewriting this in terms of the benchmark notch and with Equation (3.20) substituted in yields

$$\tau_m = \left(E_o I_{o,m} \kappa_{o,m} + E_i I_{o,m} \frac{\kappa_{o,m}}{1 - d \kappa_{o,m}} \right) d_m^{-1}.$$

where

$$\kappa_{o,m} = \frac{\kappa_m}{1 - \gamma_{o,m}\kappa_m}.$$

Since the system is assumed to be frictionless, all notches must experience the same actuation force, and thus, this can be applied to all other notches such that

$$\tau_m = \tau_j = \left(E_o I_{o,j} \kappa_{o,j} + E_i I_{o,j} \frac{\kappa_{o,j}}{1 - d\kappa_{o,j}} \right) d_j^{-1} \quad (3.26)$$

for all notches. The only unknown is cut depth of the j^{th} notch since the unknowns in this equation ($\kappa_{o,j}$, $I_{o,j}$, and $\gamma_{o,j}$) are functions of cut depth and other known variables such as the set of optimized curvatures, κ , and tube geometry. Thus, Equation (3.26) can be used to determine each remaining cut depth, which can once again be found with a nonlinear solver such as MATLAB's *fsolve* function. The initial guess for the solver can be the same as the one for the Curvature-Ratio method.

Notch Length

Once the depth pattern is determined, the notch lengths, h , can be found using the relationship between centerline curvature and the outer tube neutral axis offset in Equation (3.14), resulting in a complete notch profile. From that equation we can see that

$$l_j \leq h_j < 2l_j$$

since $0 \leq \bar{\gamma}_o < \frac{OD_o}{2}$ and $0 \leq \kappa \leq \left(\frac{OD_o}{2}\right)^{-1}$, as self-interference occurs at $\kappa = \left(\frac{OD_o}{2}\right)^{-1}$.

3.5.4 Final Strain Evaluation

The design process began with the assumption that the highest-curvature notch pair would also experience the greatest strain. This assumption must be verified. Thus, after an initial pattern is determined, the entire pattern is evaluated for strain using Equations (3.21) and (3.22) to check the initial assumption and, if needed, update the design. If a notch pair is found that experiences higher-magnitude strain than

the initial benchmark pair, the design is re-evaluated with the new highest-strain pair as the benchmark. One or two iterations of this method typically generates a pattern that remains under the strain limit when actuated to the desired shape. Once a design is determined, the required actuation, q , that attains the desired shape can be calculated with Equation (3.12), where the set of $\kappa_{o,j}$ values are calculated with Equation (3.4) and the optimized set of desired κ_j values.

3.6 Experimental Validation

The design process detailed in Section 3.5 was used to create prototypes of varying desired shapes, cross sections, and materials. The chosen NTCR parameters are listed in Table 3.1. Six prototypes, shown in Figure 3.7, were created using the Curve-Ratio Method design process and 3D printed on a MAKEiT Pro-M printer using manufacturer-recommended print settings for the given material. Small ridges along the manipulator centerline at the base of each notch were added to the CAD model of the 3D-printed designs; these ridges were then painted with a contrasting hue to ensure consistent identification. An additional, mm-scale design was milled from nitinol to demonstrate small-scale implementation.

For all prototypes, the distal notched tubes ends were affixed together with cyanoacrylate and the proximal ends were mounted to a linear stage, as shown in Figure 3.6. A Nikon D3300 DSLR camera was positioned parallel to the bending plane of the robot at a distance of approximately 14 feet from the actuated setup, and a Tamron A17NII lens was used to zoom into the setup, reducing image distortion due to perspective. Using a remote trigger, a picture was taken at even steps of actuation for each prototype. An image taken in the unactuated state was used to register the base frame of the robot to image space, and measured lengths were used to convert pixel coordinates to millimeters. The images were processed using MATLAB’s image processing toolbox. The *ginput* function, which records pixel coordinates from user-input locations, was used to select the painted ridges at the base of each notch,

Table 3.1: Chosen parameters of the NTCR prototypes. These parameters were used in the Curvature-Ratio method of the design process in Section 3.5 to create the experimental prototypes. Designs 1-5 had a wall thickness of 1 mm ($t_o = t_i = 1$ mm). Design 7 had a wall thicknesses of $t_o = 0.255$ mm and $t_i = 0.145$ mm.

Design #	Cross Section	Material	E (GPa)	ϵ_{FS}	OD_o (mm)	OD_i	L_{curve} (mm)	β (mm)	n (mm)
1	Square	ColorFabb HT	1.58	5.80%	8.0	5.2	120	0.5	10
2	Circle	ColorFabb HT	1.58	6.36%	6.0	2.8	100	0.55	12
3	Hexagon	ColorFabb nGen	1.80	3.00%	10.0	6.54	150	0.5	10
4	Square	Nylon Alloy 910	0.502	6.67%	8.0	4.8	120	0.5	12
5	Hexagon	Nylon Alloy 910	0.502	6.67%	8.0	4.3	150	0.65	15
6	Square	ColorFabb nGen	1.80	3.91%	8.0	4.8	150	0.45	15
7	Circle	Nitinol	58	3.00%	3.43	2.54	50	0.5	8

which were typically 2x2 pixels on the 1920x1080 pixel monitor; the image was a more pixel-dense 24.2 MP, thus the ridges were 23x23 pixels in terms of image pixels, which is what *ginput* records. The mm-to-image-pixel conversion factor for each dataset is given in Table 3.2. The process is summarized in Figure 3.6. The resulting experimentally measured points are plotted and compared with the kinematic model prediction in Figures 3.8 through 3.11. The root-mean-square position error of each shape, which used the error of the notch positions compared to the model in each configuration, is provided in Table 3.2 as a percentage of robot length.

The prototype NTCRs designed to achieve higher-magnitude curvatures at the base (Design 4, Design 6) had the best shape-fitting behavior overall. Since Designs 4 and 6 bend the most at notches close to the actuator, friction from surface finishing, which develops along the length of the manipulator and causes diminished bending, had less of an impact. A small amount of friction was present in all designs, particularly in higher-curvature actuated shapes, which saw diminishing performance as actuation (and therefore curvature and friction) increased. Surface finish artifacts also play a role in error due to frictional effects— in Design 1, the negative half of the workspace is affected by the interaction of a surface artifact near the base, which increases friction and causes the robot to bend more at the base and less at the tip.

Tube clearances also have some small contribution to the error, as the inner tube will press against the inner wall of the outer tube when actuated, rather than remaining perfectly concentric with it, as discussed in Section 3.3. This increases the distance between the backbones, thus increasing the effective moment arm between the neutral axes of the tubes and leading to smaller angular displacements for the same actuator displacements. This effect causes all notches to bend less than the model predicts. Some error can also be attributed to the camera being slightly out of alignment with the robot bending plane, which means that motion would appear smaller than predicted, as it would be a projection onto the camera plane. Overall, the NTCR fit the shape of the desired cure well. Position errors at the desired shape,

Table 3.2: RMS position error as a percentage of robot length for NTCR shape experiments. Error was calculated at each notch for each prototype at various magnitudes of actuation, $|q|$, in both the positive and negative directions. The positive direction of Configuration 5 is the robot actuated to its desired curve, which was calculated for each NTCR using Equation (3.12) and the desired centerline curvatures. Each design was stepped by even increments for Configurations 1-4. For Designs 1-4 and 6, q was stepped by 3 mm to 12 mm. For Design 5, q was stepped by 4 mm to 16 mm. For Design 7, q was stepped by 1 mm to 3 mm, with Configuration 4 defined by the strain limit at $q = 3.8$ mm. The final column gives the mm per pixel ratio that was used to convert each image from pixels to mm, which varied based on the zoom of the camera per experiment.

RMS Position Error as % of robot length											
Design	Config. 1		Config. 2		Config. 3		Config. 4		Config. 5		mm/pixel
Number	q_1	$-q_1$	q_2	$-q_2$	q_3	$-q_3$	q_4	$-q_4$	q_5^*	$-q_5$	Conv.
1	0.46	0.28	0.43	0.71	1.06	3.21	2.53	6.57	3.18	8.09	0.06
2	1.00	0.78	1.78	1.17	2.16	0.92	4.35	2.11	5.06	2.05	0.07
3	0.77	0.20	0.94	0.61	1.81	3.24	2.33	3.71	3.33	4.80	0.06
4	0.75	0.50	1.19	1.69	2.24	2.98	3.31	1.76	4.07	1.89	0.07
5	0.74	0.73	1.67	1.53	3.37	2.14	4.60	2.15	5.33	2.30	0.07
6	0.72	0.47	1.07	4.17	1.02	5.96	1.13	6.77	1.12	8.70	0.07
7	0.73	1.05	0.90	0.86	1.89	1.05	2.18	1.74	—	—	0.02
* denotes that the robot is actuated to achieve desired curve											

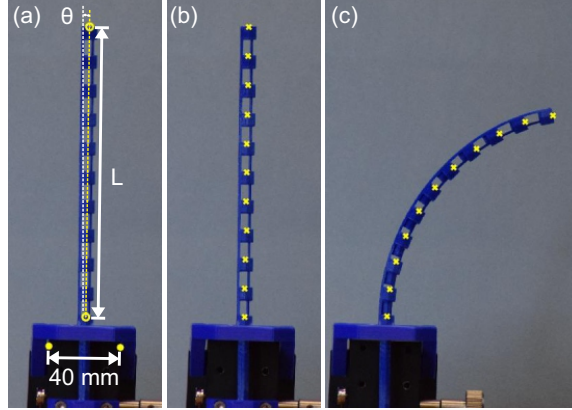


Figure 3.6: Image registration for NTCR shape experiment. MATLAB's *ginput* function was used to process images of the prototypes. (a) The yellow circles show the points selection on a straight, unactuated robot that were used to create a pixel-to-mm conversion and find the angle of camera tilt in order to register the experimental data to the model. The yellow “x” marks the user-selected points that were saved in (b) and (c).

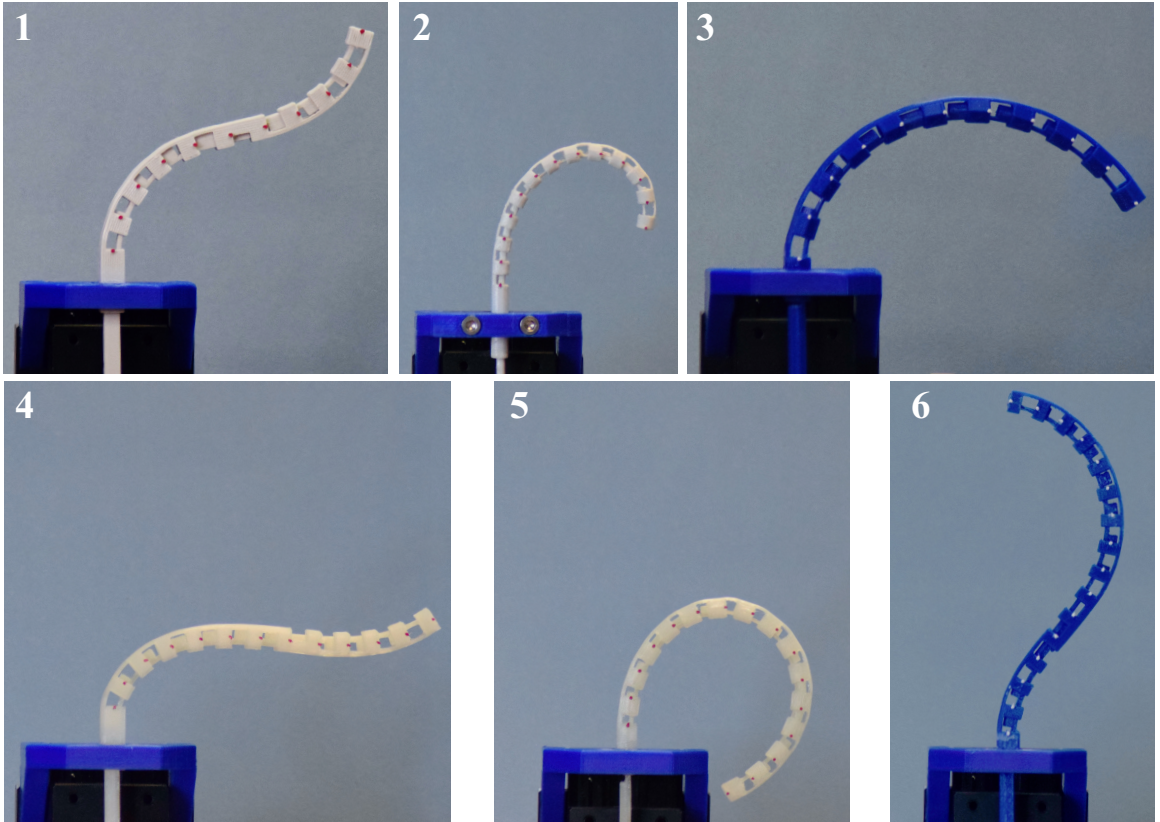
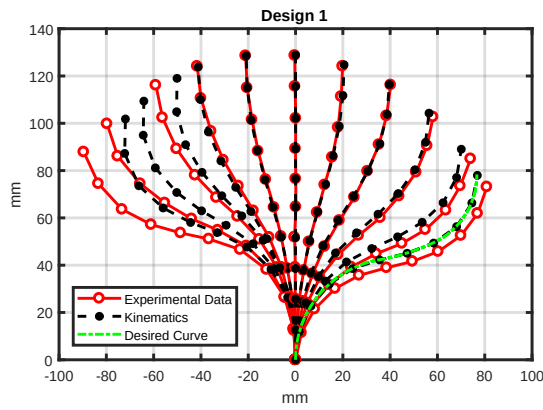
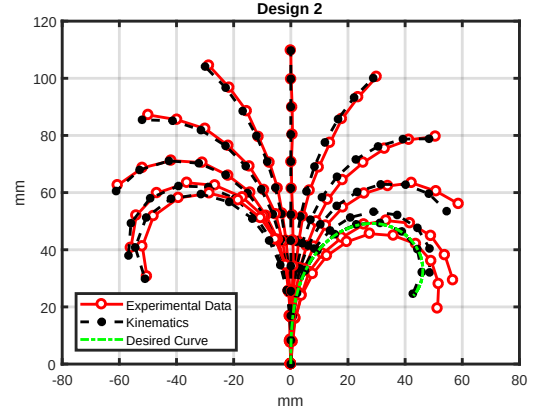


Figure 3.7: Designs 1-6 actuated to their designed curves. The designs were determined using the Curvature-Ratio method of the process detailed in Section 3.5.

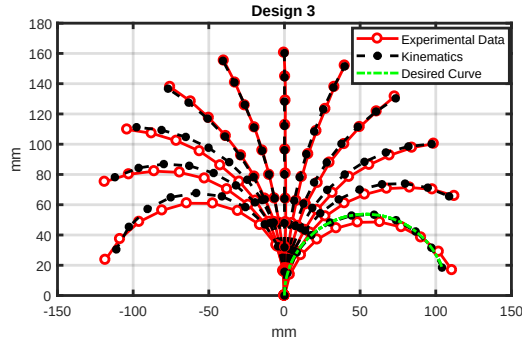


(a) Design 1 Results

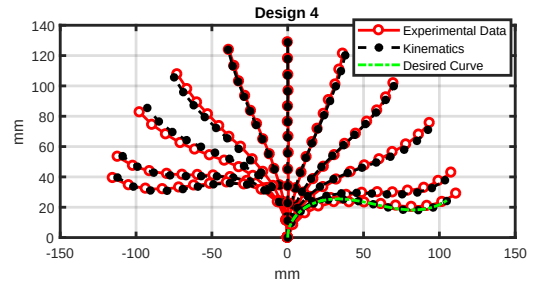


(b) Design 2 Results

Figure 3.8: The experimental results for Designs 1 and 2 for the entire workspace compared to the numerical kinematic model. Interference from a printing artifact caused inaccuracy in the workspace of both prototypes. For Design 1, The desired shape provides near-constant tip orientation. Design 2 exhibits tip-first bending.



(a) Design 3 Results



(b) Design 4 Results

Figure 3.9: The experimental results for Designs 3 and 4 for the entire workspace compared to the numerical kinematic model. Design 3 is a constant-curvature design with a desired tip angle of 160 degrees. Design 4, with base-first bending, generates a wide-reaching NTCR that has minimal change in tip orientation.

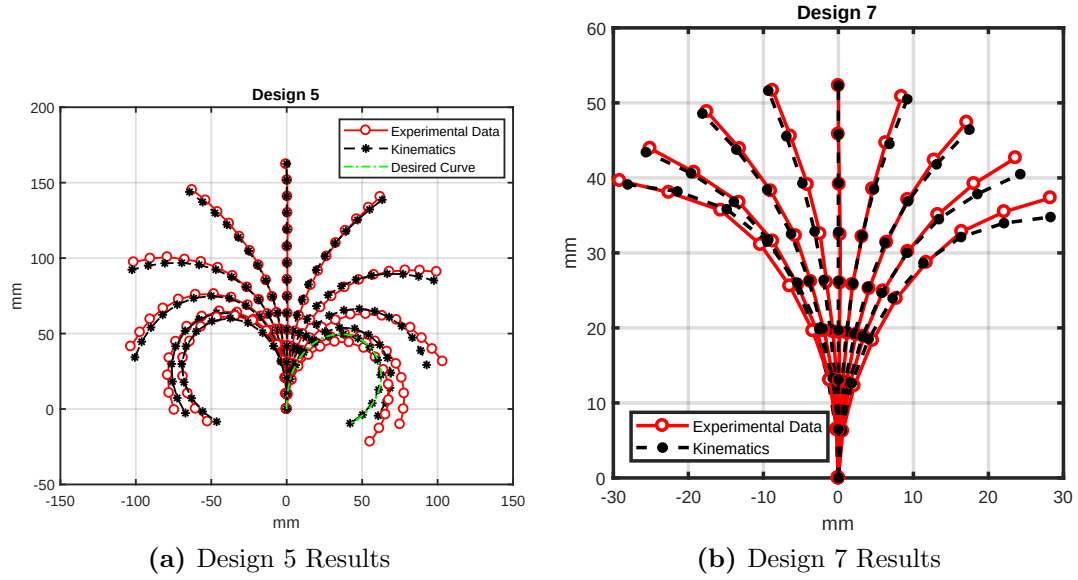


Figure 3.10: The experimental results for Designs 5 and 7 for the entire workspace compared to the numerical kinematic model. The desired curve for Design 5 bends most in the middle, allowing retroflexion. Design 7 is a mm-scale prototype made from milling notches into nitinol tube stock.

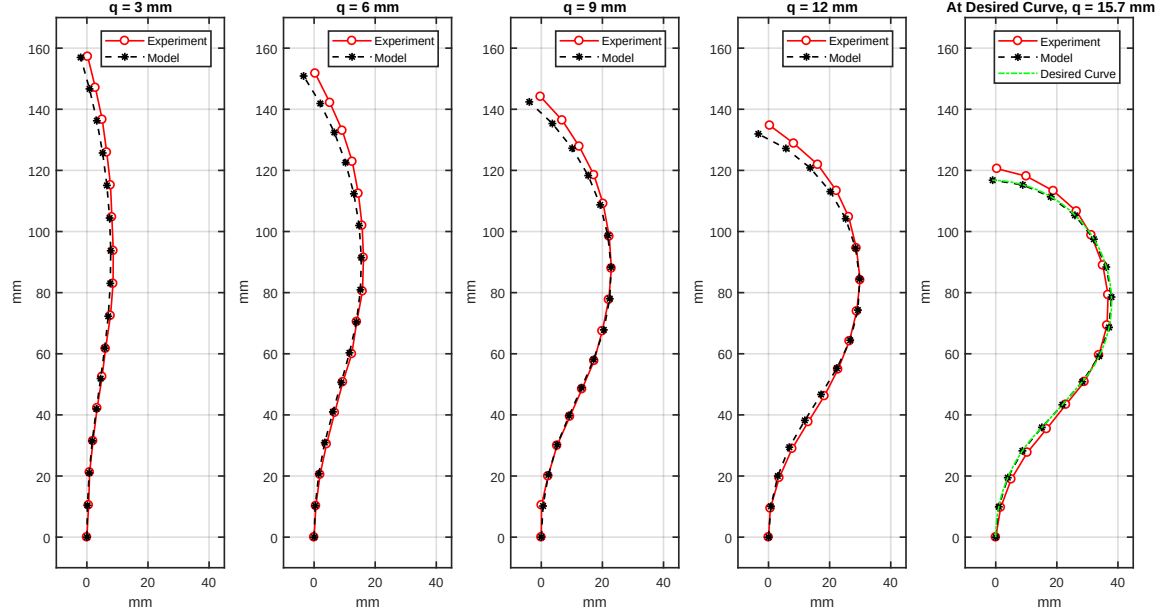


Figure 3.11: The experimental results for Design 6 compared to the numerical kinematic model as it steps through actuation. This design changes orientation but has a near-constant position in x.

which the NTCR was designed to attain, range from 1.12% (Design 6) to 5.33% (Design 5) when normalized by robot length.

3.7 Notch Design for Desired Tip Pose

Many applications are more concerned with the tip pose of a robot than its actuated shape. Thus, this section develops a method of designing an NTCR that attains a desired tip pose by leveraging the shape design process. The process in this section ultimately minimizes the maximum value of curvature, κ , of a Bézier curve and can be summarized in the following way:

1. Input a desired tip pose (position and tip angle).
2. Use the vector between specified base and tip positions form initial Bézier control points.
3. Optimize select control points to minimize the maximum curvature while maintaining tangency requirements.
4. Incorporate the generated curve into the shape design process with a modified objective function.

Note that while the following procedure to generate a curve from the desired tip pose can handle the full spatial description of the tip pose (i.e., specification of the full 6D pose, beyond the planar 3D pose), the shape design algorithm is planar. Extension of the shape design method is necessary before design of full 3D curves can be done, which is left to future work. As such, all examples in this section are planar.

3.7.1 Definition of Bézier Curves

Many means of describing a curve based on two points and their corresponding tangent vectors exists, and excellent resources can be found in [99, 100, 101]. With

the many options available, a Bézier curve was selected for its balance of descriptive power and straightforward implementation. A Bézier curve is defined as

$$\mathbf{B}(t) = \sum_{i=0}^n \binom{n}{i} (1-t)^{n-i} t^i \mathbf{P}_i \quad 0 \leq t \leq 1, \quad (3.27)$$

where n is the order of the curve, $\mathbf{P}_i$ are the control points, and $\binom{n}{i}$ is the binomial coefficient, defined as

$$\binom{n}{i} = \frac{n!}{i!(n-i)!}.$$

The set of $n + 1$ points, $[\mathbf{P}_0, \dots, \mathbf{P}_n]$, define the vertices of the control polygon of the curve, which is plotted with its associated Bézier curve in Figures 3.12 and 3.13. Changing the location of any one of these vertices reshapes the entire curve.

Highly favorable characteristics of Bézier curves are that the curve ends at defined points, $\mathbf{P}_0$ and $\mathbf{P}_n$, and those ends are tangent to their respective control polygon edges. Therefore, constraining the second and next-to-last points, $\mathbf{P}_1$ and $\mathbf{P}_{n-1}$, to lie on the desired tangent lines of the base and tip, respectively, enforces the tangent direction requirements. Cubic Bézier curves ($n = 3$) minimally describe a shape that meets these tangent constraints for any end pose, but there is a strong chance of creating sharp angles or short edges in the control polygon, which leads to high curvatures in the resulting curve. Since high curvature directly relates to high strain in the NTCR, this process needs to provide some means of reducing the curvature of the shape, and thus a quartic curve of $n = 4$ is implemented. It provides an additional point in the control polygon not constrained to the end poses, and it can be utilized purely for the reduction of curvature.

For any parametric curve such as a Bézier curve, the curvature function is

$$\kappa(t) = \frac{\|\mathbf{B}'(t) \times \mathbf{B}''(t)\|}{\|\mathbf{B}'(t)\|^3}. \quad (3.28)$$

where $'$ indicates a derivative with respect to t . The total arc length of a Bézier curve is calculated as

$$s = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^1 \left\| \frac{d}{dt} \mathbf{B}(t) \right\| dt, \quad (3.29)$$

for which there is no analytical solution for cubic Bézier curves or higher ($n \geq 3$), though it can be solved efficiently numerically.

3.7.2 Constrained Optimization

A quartic Bézier curve is defined by the set of five points, $[\mathbf{P}_0, \dots, \mathbf{P}_4]$. The NTCR base defines $\mathbf{P}_0$, and the desired end position defines $\mathbf{P}_4$. The midpoint, $\mathbf{P}_2$, is unconstrained by the desired end poses and can have any value. Points $\mathbf{P}_1$ and $\mathbf{P}_3$ must lie on the tangent lines of the base and tip, respectively, though the magnitude of the distance from the ends can vary. Those distances are defined as L_1 and L_3 , and thus

$$\mathbf{P}_1 = \mathbf{P}_0 + L_1 \mathbf{V}_0 \quad \text{and} \quad \mathbf{P}_3 = \mathbf{P}_4 - L_3 \mathbf{V}_4$$

where $\mathbf{V}_0$ and $\mathbf{V}_4$ are the unit tangent vectors of the base and tip, respectively.

Constraints and Initialization

As defined up to this point, the optimization of the Bézier curve with respect to minimizing curvature can and will generate positions of $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$ at coordinates orders of magnitude higher than the positions of $\mathbf{P}_0$ and $\mathbf{P}_4$ for some pathological cases of tip pose inputs (e.g., a tip pose that represents retroflexion, where the tip of the robot points at its own base). To prevent this, constraints are placed on the optimization. Here, the designer-defined points in the problem, $\mathbf{P}_0$ and $\mathbf{P}_4$, are used in a simple and robust method of both scaling these constraints and forming the initial guess for the control polygon. The vector between the base and tip positions is defined as

$$\mathbf{PP} = \mathbf{P}_4 - \mathbf{P}_0 = L_{PP} \mathbf{V}_{PP} \quad (3.30)$$

where L_{PP} is the magnitude of the vector and $\mathbf{V}_{\mathbf{PP}}$ is the unit vector that defines the direction from $\mathbf{P}_0$ to $\mathbf{P}_4$. From this, the constraints on the values of L_1 and L_3 can be formulated, ensuring that the control polygon edges that enforce the tangent requirements are appropriately scaled:

$$\frac{L_{PP}}{10} \leq L_1, L_2 \leq 2L_{PP},$$

where the lower bound ensures that the optimized values are much greater than zero, which is necessary to avoid short edges in the control polygon, and the upper bound prevents runaway magnitudes. These bounds, while somewhat arbitrary, performed well during testing of the algorithm. It is worth noting that most end pose inputs are not defined by these limits, and only certain pathological cases require these constraints.

To initialize the process, $L_1 = L_3 = \frac{L_{PP}}{2}$ is used. The initial guess for $\mathbf{P}_2$ can be formulated from the same line by placing it on the perpendicular bisector of vector $\mathbf{PP}$:

$$\mathbf{P}_{2,initial} = \mathbf{P}_0 + 0.5L_{PP}\mathbf{V}_{\mathbf{PP}} + L_{PP}\mathbf{N}_{\mathbf{PP}} \quad (3.31)$$

where $\mathbf{N}_{\mathbf{PP}}$ is the unit vector that runs normal to $\mathbf{V}_{\mathbf{PP}}$.

Optimization Problem Statement

In order to reduce the maximum curvature, and thus strain, in any potential design for a given desired end pose, the optimization is formulated:

$$\begin{aligned} \min_{L_1, L_2, \mathbf{P}_2} \quad & \max(\kappa(t)^2) \\ \text{subject to} \quad & 0.1L_{PP} \leq L_1, L_2 \leq 2L_{PP} \\ & \mathbf{P}_2 \leq \mathbf{P}_{2,initial} + 2[L_{PP}, L_{PP}]^T \\ & \mathbf{P}_2 \geq \mathbf{P}_{2,initial} - 2[L_{PP}, L_{PP}]^T \end{aligned} \quad (3.32)$$

where $\leq$ and $\geq$ define component-wise inequalities, $\mathbf{P}_{2,initial}$ is the initial guess for $\mathbf{P}_2$ as defined in Equation (3.31), and $\kappa(t)$ is the curvature function defined in Equation (3.28). These constraints ensure the creation of a shape that is scaled to the inputs and free of folds, cusps, and loops, all of which are undesirable for the NTCR centerline. This scheme, when using the initial guesses as outlined, has a convergence time on the order of minutes using non-optimized code in MATLAB and a i7 processor.

If the resulting curve is in the small subset of curves unfeasible for an NTCR design (e.g., it contains regions of sharp curvature), flipping the direction of the bisection line for the calculation of $\mathbf{P}_{2,initial}$ as shown in Equation (3.31) can lead to a different curve. Changing the bounds of the constraints can also lead to an improved shape, though care must be taken that $\mathbf{P}_1$ and $\mathbf{P}_3$ are not allowed to be coincident with the end points, which would eliminate the enforcement of tangent constraints. Increasing the order of the curve to a quintic curve ($n = 5$) can provide an additional point for optimization, but the initial guess would need to be reformulated to account for this additional control point, and the complexity of the problem increases as the number of control points increases. If a quartic curve that meets the end poses and can feasibly be used for an NTCR cannot be found, the designer could create a composite curve by inserting an artificial middle pose (or poses) that acts as the “end” of the first curve and the “base” of the second curve, then perform this optimization on those curves individually.

3.7.3 Incorporation into Shape Design Process

The process in Section 3.5 can be utilized from this point, provided that the error function in Equation (3.19) for the shape sampling method is modified. It must be modified because it does not consider the tangent vector at the tip of the robot. In order to ensure the 3-DOF tip pose is reached, the last two notches must be treated in a different manner, where their curvatures (κ_n and κ_{n-1}) and centerline lengths

Table 3.3: NTCR Tip Pose Simulation Error.

Design	$\mathbf{P}_{err} [x, z]$ (mm)	θ_{err} (degrees)
A	[0.015, 0.040]	0.003
B	[0.091, 0.042]	0.235
C	[0.006,0.006]	0.010
D	[0.104,0.118]	0.021

$(l_{c,n}$ and $l_{c,n-1})$ must be optimized with the following error metric:

$$e = \left\| \begin{bmatrix} \mathbf{P}_{tip} - \mathbf{P}_{des} \\ (\log(\mathbf{R}_{tip}^T \mathbf{R}_{des}))^\vee \end{bmatrix} \right\| \quad (3.33)$$

where $\mathbf{P}_{des}$ and $\mathbf{R}_{des}$ are the desired position and rotation matrix composed by the desired tip angle, respectively, $\log()$ is the matrix logarithm, and the $\vee$ operator is the inverse mapping from $\mathbb{R}^3$ to $\mathfrak{so}(3)$, which maps a skew-symmetric matrix in group $\mathfrak{so}(3)$ onto a 3x1 vector in $\mathbb{R}^3$ as defined in [93].

3.7.4 Simulation Examples

Four candidate designs were chosen to demonstrate the tip pose design scheme. Their chosen parameters and desired tip poses are listed in Table 3.4. The curves created using the optimization scheme are plotted in Figures 3.12 and 3.13, with their tip poses illustrated with arrows indicating the end tangents. A variety of tip positions and angles were chosen to illustrate the versatility of the curve optimization method and the breadth of the NTCR notch design space, even for strictly planar curves. Once the optimized curve was found, each candidate design was processed using the modified shape design process to generate a notch profile. Finally, the forward kinematics were simulated for each notch profile, and Table 3.3 shows the error between the forward kinematics result and the desired tip pose. The errors are all well below 1 mm in both the x - and z -directions and below a half a degree for the

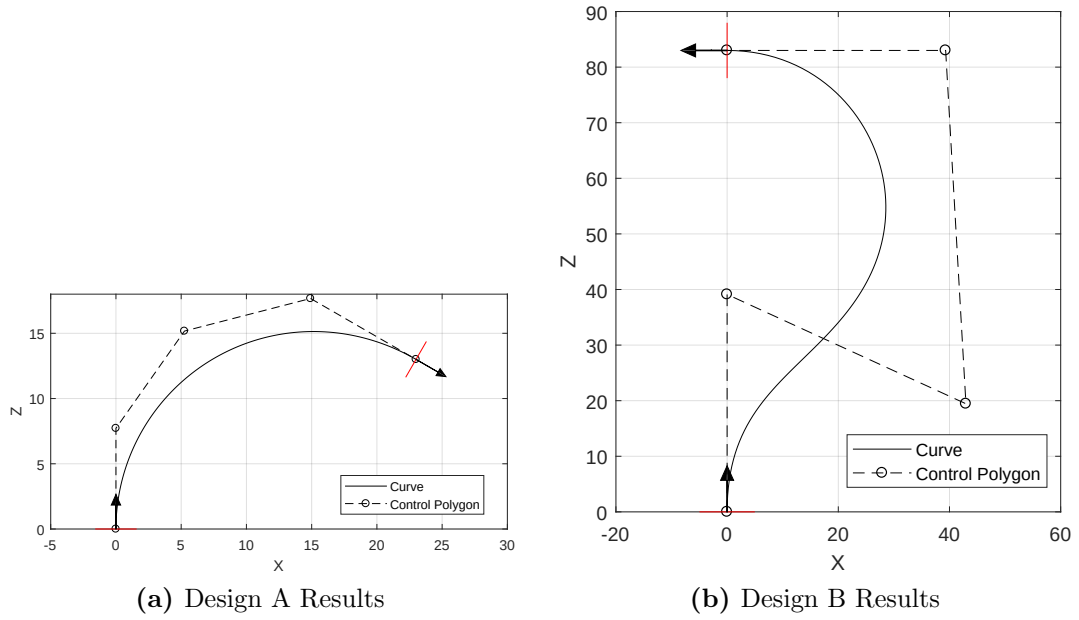


Figure 3.12: The resultant curve for desired tip pose for Designs A and B. Dimensions are in millimeters.

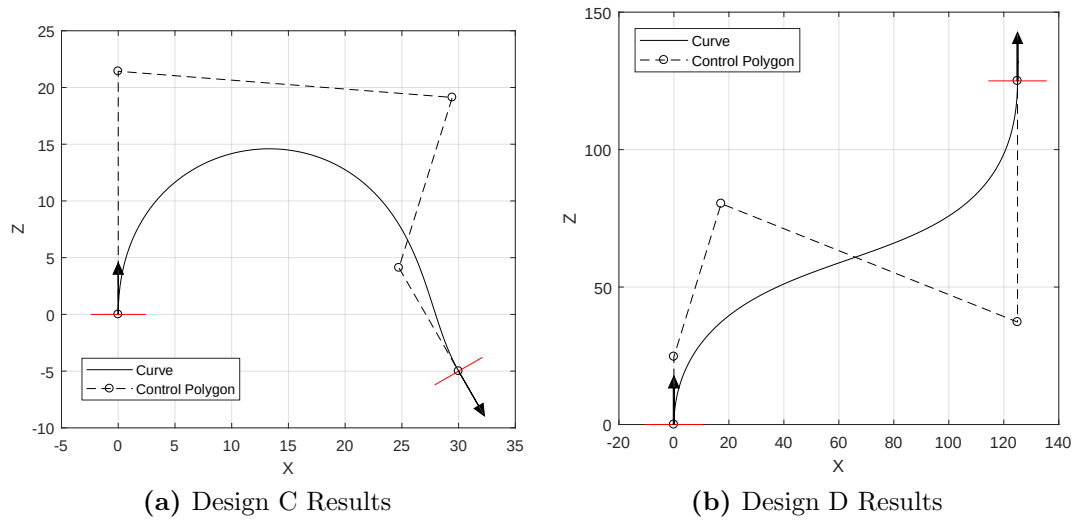


Figure 3.13: The resultant curve for desired tip pose for Designs C and D. Dimensions are in millimeters.

Table 3.4: NTCR Tip Pose Design Parameters. These are the chosen parameters for the designs used in the tip pose design scheme and verified via simulation, where θ is the tip angle measured as clockwise from the z -axis. For all designs, $\beta = 0.5$.

Design	Cross Section	Material	E (GPa)	ϵ_{FS}	OD_o (mm)	OD_i (mm)	n (mm)	$\mathbf{P}$ [x, z] (mm)	θ (rad)
A	Circle	Nitinol	58	3.00%	3.43	2.54	9	[23,13]	$2\pi/3$
B	Hexagon	Nylon Alloy 910	1.58	6.67%	4.0	2.15	15	[0,83]	$3\pi/2$
C	Circle	PEEK	4.60	4.55%	2.31	1.65	18	[30,-5]	$5\pi/6$
D	Square	ColorFabb HT	1.58	4.67%	6.0	2.8	12	[125,125]	0

tip angle, thus providing evidence that NTCRs can be designed for demanding tip poses. Experimental validation of this tip-pose scheme modification is left to future work, though the modifications to the shape design process are few enough that it is unlikely that the results from new experiments would differ much from the generally good agreement seen in the shape experiments in Section 3.6.

3.8 Discussion on Implementation

The NTCR is a versatile design concept, and creating a prototype requires careful consideration of many factors. This section offers further guidance to help designers narrow down the unknowns and create a successful NTCR. First, material selection should focus on lubricious materials or those compatible with lubricators to combat friction, which impacts accuracy at the edges of the workspace. Also, material selection must consider the relationship between strain, curvature, and notch depth, as shown in Equations (3.21) and (3.22).

A maximum notch depth, which corresponds to a minimum backbone width, can be determined from strength requirements or other design factors, and the neutral axis offset created by this notch drives the relationship between curvature and strain. Using the strain limit of a given material, the maximum curvature of a notch of this depth can be calculated with Equations (3.21) and (3.22), allowing designers working with a specific material to select an achievable curve. Conversely, if the desired curve cannot be adjusted or reconfigured, one can select a material based on the strain limit as calculated by the maximum desired curvature and notch depth requirements.

The choice of desired curve can also be used to mitigate the effects of friction and interference between the notches. A designer should avoid large changes in curvature that also coincide with a change in notch direction, as this can cause a point contact between an outer-tube notch edge and the inner-tube backbone and can generate significant friction. A single notch designed for low curvature either before or after the direction change can mitigate this effect with very little change to the overall curve.

An advantage of using Bézier curves to define the centerline is that they smoothly transition between curvatures, as a Bézier function is a form of interpolation.

A curve discretization parameter was introduced in this process, β , which influences the notch length and defines the notch spacing. It can be changed to slightly modify the desired notch depths (increasing β increases the notch length, and a longer notch is less stiff in bending). Generally, this value should be kept near 0.5—decreasing β to below 0.5 means each discrete segment of the NTCR has more of its centerline as straight, rigid sections than of bending, actuated portions. The opposite is true for $\beta > 0.5$, and while this leads to actuation influencing more of the robot shape, having large notch lengths increases the risk of the inner tube buckling, particularly for thin, high-curvature notches. Essentially, a large h is the same as increasing the column length when viewed in light of classic Euler buckling. After a pattern is determined, a designer can verify their design will not buckle using the actuation force and the equation for pinned-pinned column loading:

$$h_{cr,j} = \sqrt{\frac{\pi EI_{i,j}}{\tau_{max}}}$$

where $h_{cr,j}$ is the maximum notch length, τ_{max} is the actuation force at the desired curve as calculated by Equation (3.11), and $EI_{i,j}$ is the stiffness of the notch. The true end conditions for the notch are more constrained than the pinned-pinned assumption since they can support a moment, so this equation provides a conservative measure since pinned-pinned columns fail at lower loads than other end conditions. Generally, with β near 0.5, the notch lengths produced by the design process are much lower than this critical length.

Selecting proper tube clearances is a balance of material, fabrication method, and friction reduction. For the 3D-printed prototypes, additional clearance helped to mitigate the effects of printing imperfections and snags; however, it also leads to lower-than-expected curvatures when actuated, as seen in the Design 5 results. It can also have a detrimental impact on the negative half of the workspace, where the

inner tube is pushed through the outer one, and the model accounts for the absence of bending in the first notch. Large clearances allow this notch to bend, and this additional angular displacement at the base of the robot propagates to large position errors at the tip.

This chapter introduced the notched-tube continuum robot, and the groundwork of modeling and design has been laid. Future work on this project can including modeling nonlinear materials and external loads, as well as 3D shape design and kinematics. Multi-segment designs were not explored, but two nested NTCR segments that can rotate, translate, and actuate independently create a 6-DOF robot, even if the individual NTCR segments are planar. The modeling work completed in this chapter was leveraged to design an endoscopic instrument in the next chapter, and its novel actuation paradigm offers abundant potential for application in other areas.

Chapter 4

Development of an NTCR-based Endoscopic Instrument: Transmission Stiffness Modeling and Experimental Evaluation

4.1 Overview

Natural Orifice Transluminal Endoscopic Surgery (NOTES) holds great promise in improving patient outcomes, including significantly lower costs [70, 71] and a reduced risk of adverse events [73, 72], but working down a flexible endoscope to reach surgical sites poses clear challenges to surgeons. A lack of tool dexterity is among the most difficult of these obstacles [74]. This chapter develops and experimentally evaluates an endoscopic instrument based on NTCRs in order to address this need, focusing on Endoscopic Submucosal Dissection (ESD) of colon lesions, a difficult-to-master technique with exceptional patient benefits as compared to alternatives.

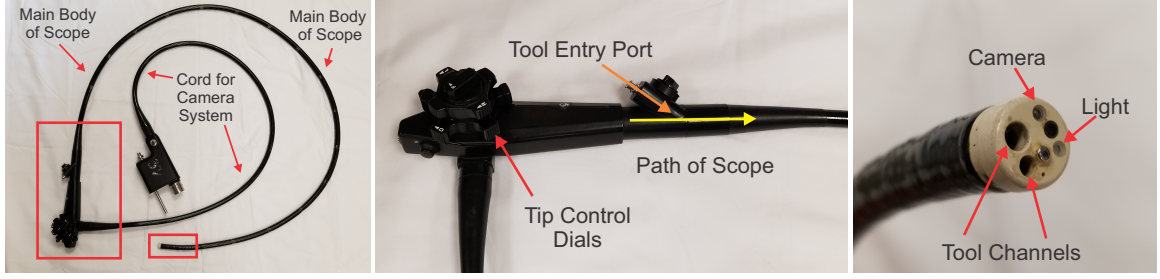


Figure 4.1: Colonoscope geometry. Colonoscope length is approximately 1.85 m (just over 6 ft) from the tool entry port to the distal end. The tool entry at the proximal end of the colonoscope contains a sharp angle to pass through. The distal end of the scope has a small actuatable section that is controlled by dials on the handle.

4.1.1 Requirements of Endoscopic Submucosal Dissection (ESD)

Deploying instruments down the tool channels of commercially-available colonoscopes, such as the one pictured in Figure 4.1, enforces stringent design requirements. The colonoscope itself is approximately 185 cm (just over six feet) long from tool port to tip, and thus the instrument requires an elongated transmission segment. This transmission must be compliant in bending to avoid applying excessive forces to the colon during the procedure; however, it must also have sufficient axial and torsional stiffness to overcome the friction within the tool channel and output the necessary forces for actuating the distal end. Additionally, the actuation transmission scheme must be suited for the winding, tortuous pathway of the colon, and the tools themselves must fit into port sizes of 3.8 mm (0.15 inches) and 2.8 mm (0.11 inches) [102]. The manipulator at the distal end must meet a separate set of requirements, as well. The colon forms a cylindrical workspace approximately 4-6 cm in diameter and 8-10 cm long, though it is worth noting that these are estimates and that a “typical” colon cannot be well defined [103]. Studies that have determined lateral forces (orthogonal to the tool axis) required to lift and cut the tissue from the surrounding submucosa show that electrosurgery, the most common means of cutting tissue, requires contact forces in the range of 300-400mN [81]. Retraction, the means

of pulling tissue away from the colon wall, was measured to require lifting forces of up to 0.89 N in *in-vivo* testing [104].

4.1.2 An NTCR-Based Tool

Its simplicity in both construction and actuation makes the NTCR well-suited for the task. It can be made at the challenging mm-scale sizes required of the application, and given that only two tubes are needed to construct it, there are minimal components to take up valuable space within the tool ports. The push-pull actuation generates intuitively-understood tip motion without the need for complex mathematics to operate the manipulator in task space, creating the opportunity for both manually and robotically actuated versions of this instrument.

In addition to providing articulation at the tip, the tool must also feature an elongated, flexible transmission. It must be rigid in torsion in order to transmit rotation with minimal reduction yet compliant in bending so that no undue force is applied to the patient's colon. No single, unmodified material stock can meet the stiffness requirements of colonoscopic deployment, and thus a custom structure will be required for the transmission component of the tool. One means of attaining specific material properties is braid-reinforced tubing, where thin, plastic tubes are embedded with high-strength wires along their length, the path of which determines the overall stiffness behavior. Often, these wires are run in a helical path of varying pitch, and they can be run axially, as well. Layering multiple levels of wire and plastic can generate highly directional stiffness responses. Similar to those, helical-hollow strand (HHS) tubes are tubes created out of helically-wound wires. They, too, can be used in multiple layers of multiple wire diameters and helix pitches to attain finely-tuned stiffness behavior. These were investigated by Ponten in [105] for use in a colonoscope, but their axial compliance proved to be too high to operate an NTCR.

Given the importance of maintaining the lumen of the working tool channel through which traditional instruments will be passed, a promising option for the

design is to modify high-strength materials via radially symmetric notching, as proposed by Ponten et. al. [106] and Black [107]. These high-strength materials, such as stainless steel or nitinol, are able to use thinner-walled tubes than lower-strength counterparts. Symmetric notching of tubes in order to modify the global stiffness behavior has been used for the purpose of improving concentric-tube robot stability [63, 64, 65], which is determined by the bending-to-torsional stiffness of the tubes [55, 62]. With no model to serve as a basis, most use FEA software and trial-and-error to determine the notch parameters that yield the desired stiffness behavior [63, 64]. Lee et al in [63] derive a spring-based model of material behavior for this purpose as well, and they provide an analytical framework for determining the ratio of stiffnesses based on ratios of the notch dimensions; however, to relate the notch dimensions directly to an effective Young’s modulus or Shear modulus, one must tune a proportional term using FEA software first. Luo et. al. use topology optimization to design patterns to cut into the tubes, which generates a modest improvement of the stiffness ratio [65] but required multiple iterations of evaluation and post-processing into a realizable design. There currently exists no model in the robotics literature that analytically relates global stiffness behavior of a tube to a notch pattern cut into it.

To address this gap in the literature, this chapter of the dissertation derives a model that relates notch dimensions (length, width, depth, and number) to effective bending and torsional stiffness values and compares it to FEA simulations. Given the push-pull actuation of the NTCR, it also address axial stiffness to ensure that actuation is properly transmitted through the structure. The model considers typical loading scenarios, shown in Figure 4.3 (a), and equates expressions for the deflection of the notched structure to classic expressions for the deflection of a solid body with some EI , GJ , and EA under identical loading conditions. Solving that equation for the relevant stiffness parameter (EI , GJ , or EA) then provides analytical formulas for effective stiffness behavior of the notched structure as defined by notch dimensions.

4.1.3 Estimation of Torsion

A brief investigation of endoscopic deployment of an NTCR in collaboration with Vanderbilt used a tool made from PEEK (polyetheretherketone) deployed down a 84-cm bronchoscope to successfully remove a simulated tumor from a sheep trachea [108], which is shown in Figure 4.2. Additional tests were conducted on this prototype, including tests of axial and torsional transmission of motion down the length in both straight and bent configurations, which showed minimal issue with actuation and translation but significant issues in transmission of rotation. While straight, the prototype reached at worst a 70-degree lag at the tip compared to the base input. This was used in the classic equation for torque of a circular shaft

$$\theta = \frac{TL}{GJ}$$

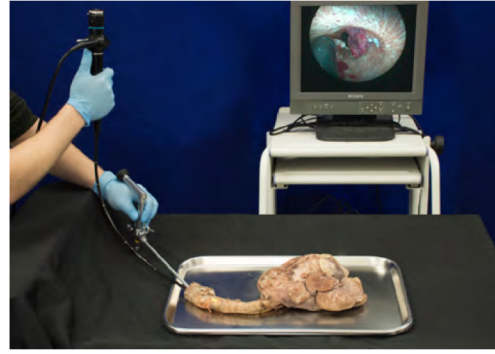
in order to estimate the frictional torque experienced by that prototype, which was calculated as 5.6×10^{-3} N-m. An allowable lag of 10° was then used to calculate an effective GJ for a tool the length of the colonoscope (185 cm), arriving at a torsional rigidity target of 0.0593 N-m². For perspective, a Gauge 12 extra-thin walled 304 Stainless Steel hypodermic tube (OD=2.77 mm, t=0.15 mm) has $GJ = 0.4393$ N-m², and a tube of the same geometry made of PEEK has $GJ = 3.62 \times 10^{-3}$ N-m².

4.2 Decoupled Transmission Shape and Distal Tip Actuation

Central to the working concept of the tool is the passive, elongated transmission that takes relative translation at the base, where a surgeon might operate a tool handle or where motors may be attached, and transmits it with minimal reduction to the distal tip. Ideally, any elongated design decouples the motion of the transmission and that of the tip (i.e., changing the shape of the transmission does not actuate the tip, nor



(a) NTCR Deployed in a Bronchoscope



(b) The Setup for Simulated Lesion Removal



(c) Images from the Tests

Figure 4.2: Prior Work in Deployment down a Bronchoscope. These images demonstrate the experiment performed with an NTCR down a bronchoscope. All images are from [108].

does actuating the tip change the shape of the transmission). Thus, both the form of the transmission and distal actuating segment must be carefully considered.

4.2.1 Passive Concentric Transmission

The first condition of an ideal tool is that changing the shape of the transmission does not actuate the tip. In order for this to be true, three lines must be coincident: the centroidal axis, the neutral axis, and the line-of-action of the actuating forces.

For any element whose stress-free state is straight, the centroidal axis is coincident with the neutral axis, which is true for all candidate designs considered here. (This is not the case for plastically deformed structures or precurved elements of significant thickness.) The neutral axis by definition experiences no strain, and so it is constant in length no matter what shape the transmission takes, meaning that there is no relative

change in length for two concentric tubes even while taking the shape of winding, natural pathways like those in the colon. The proposed transmission modifies solid tube sections in a symmetric manner, meaning that the centroidal axis of the tube is unchanged even while its stiffness characteristics change drastically. Finally, since the tubes themselves also act as the primary force transmission elements, there are no actuation forces acting at an offset from the transmission’s centroid, meaning there are no actuating elements that must change length as the transmission changes shape. Thus, symmetrically modified concentric tubes meet the requirement of decoupling the distal tip actuation from the transmission shape.

A contrasting example are tendon-driven transmissions, where the line of action for the actuating forces (the tendons) lie on the outer edges of the transmission elements, offset from the structure’s centroidal axis. Only one fiber of a bending element can remain constant length— the neutral axis— and so these edges on which the tendons lie change length as the transmission bends. Thus, the tendon lengths are dependent on the shape of the transmission, which affects the shape of the distal tip unless the lengths are compensated in some manner. An estimate of the transmission shape, which can be difficult to attain, enables calculation of the new tendon lengths required to prevent actuation of the distal tip. Actuating via controlled tendon force can also help, though friction can be a significant issue and is also dependent on the transmission shape. Sufficient initial slack, tendon stretch, or a combination of both can overcome this issue without calculations, but those characteristics are generally considered detrimental to the performance of the distal tip. Colonoscopes are an example of a system that has a flexible transmission with a tendon-driven distal tip, and they experience significant play in their controls due to the tendon slack and stretch. Catheter robots are another example, and significant research effort has been spent on modeling and compensating for the effects of the flexible transmission shape [109, 110, 111, 112, 113].

4.2.2 Distal Tip Actuation

While the concentric transmission shape does not affect actuation, it is important that actuation also does not greatly affect the transmission shape, either, if it is to be considered truly passive. Absent of external loads, this means that the distal tip cannot, at equilibrium and through actuation alone, pass a net moment or offset force to the transmission elements.

The NTCR is uniquely suited for this. Its actuating principle is that the offset of the two tubes' neutral axes causes a net moment in the structure at equilibrium, as evidenced by the moment balance in Equation (3.9) derived from the free-body diagram in Figure 3.3, rearranged and repeated here for clarity:

$$m_o + m_i = d\tau$$

where m_o is the moment in the outer tube, m_i is the moment in the inner tube, τ is the actuation force, and d is the offset in the neutral axes of the two backbones. This causes the asymmetrically-notched portions of the structure to curve, generating tip motion. When the actuating force's line of action in both tubes returns to the centroidal axis of the structure, as it does at the base of the robot and throughout the concentric transmission, that offset is eliminated ($d = 0$) and the net moment goes to zero. This means that when the NTCR is actuated, even down a long transmission, there is no net moment wherever there is no offset in the neutral axis from that of the structure's centroidal axis. In a real system, clearances between tubes will cause some small moment arm to appear, as the inner and outer tubes cannot be perfectly concentric, but the magnitude of the moment is negligible compared to that which is generated in the backbones. Effectively, the NTCR actuation will not generate a change in shape of the concentric transmission, thus fulfilling the second requirement of the desired tool behavior.

The following sections are dedicated to the development of a model that relates symmetric notch parameters to global stiffness characteristics, followed by the design and evaluation of an endoscopic tool based on these principles.

4.3 Simplified Analytical Models for Effective Stiffness of Tubes with Alternating, Symmetric Notches

While much prior work on notching tubes to modify stiffness characteristics has centered around improving the stability of CTRs [63, 64, 65] and developing novel manipulators [28, 30, 114], reducing the torsion-to-bending stiffness ratio is also useful for design of elongated, flexible transmissions. Such transmissions can enable NOTES through flexible endoscopes and catheters. Towards that end, this section develops analytical models directly relating notch parameters to effective stiffness behavior for bending, torsion, and axial loading. These models can be used to reveal the relationships of parameters to desired behavior, giving designers better insight into the properties and opening the possibility for automated design optimization. Additionally, we use this model to show that alternating, anisotropic notches produce isotropic bulk behavior, which can be used to create structures that are considerably flexible in bending but relatively stiff in torsion, a key relationship in both CTR design and flexible transmissions.

The following model is formulated for a varying number of radial notches, but the discussion and analysis focuses on the case of 2 radial notches, as shown in Figure 4.3c, as it can lead to dramatic increases of the bending-to-torsional stiffness ratio, and it has been unexplored thus far in the literature.

4.3.1 Model Basis: Castigliano's Theorem and Parameter Definitions

The notch parameters, shown in Figure 4.3, are as follows: h for notch length, d for the spacing, w for the width of remaining material, m for number of radial notches, and n for number of axial notches. The radial notches are rotated by π/m as it goes along the length, and our analysis restricts n to be an even number. The tube is assumed to be circular and of diameter OD and thickness t .

To analyze the behavior of such a complex structure, we utilize Castigliano's Theorem, a strain energy approach. It states that:

$$\delta = \frac{\partial U}{\partial F} \quad \text{and} \quad \theta = \frac{\partial U}{\partial M} \quad (4.1)$$

where

$$U = \int_V u_o dV,$$

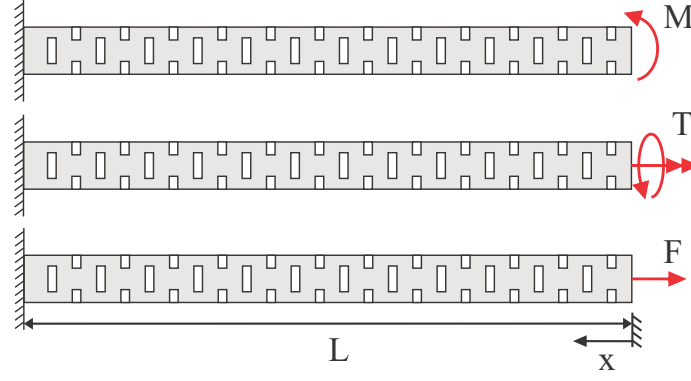
u_o is the strain energy density, V is the volume, F is an applied load, M is an applied moment, and δ and θ are the deflection at the point of the load or moment application defined positive in the direction of the load or moment, respectively. The total strain energy, U , can be found using the principle of superposition, and different loading modes can be summed to find the total energy. For typical loading scenarios and isotropic materials, the strain energy is well known and easily written. The bending strain is

$$U_{bending} = \int \frac{M^2}{2EI} dx \quad (4.2)$$

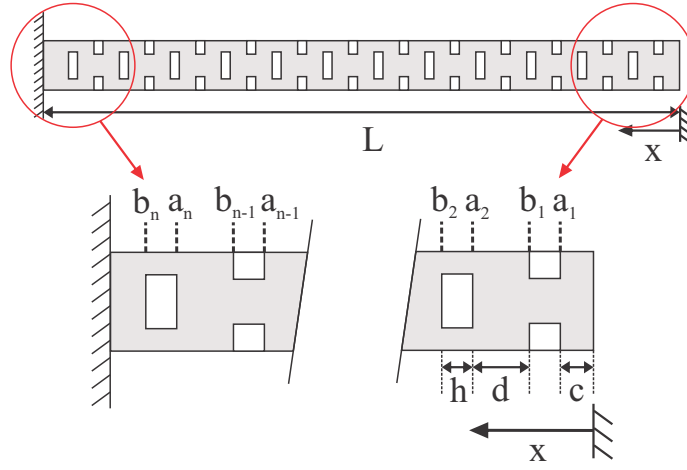
where M is the internal moment, E is the Young's Modulus, I is the second moment of area. For shear,

$$U_{shear} = \int \frac{f_s V^2}{2AG} dx \quad (4.3)$$

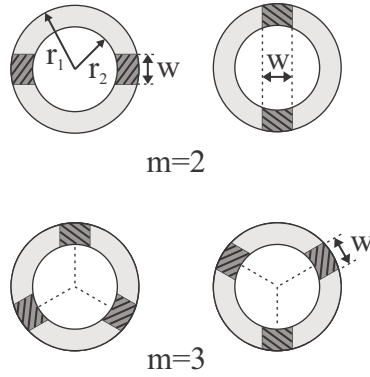
where A is the cross-sectional area, G is the shear modulus, V is the shear force, and f_s is the shear factor, which is based on geometry and assumptions of shear in the



(a) Problem Setup



(b) Parameter Definitions



(c) Radial Notches

Figure 4.3: Setup for Stiffness Modeling. (a) The loading setup for each scenario. (b) The definition of h , d , c , and L . (c) The number of radial notches m and definition of w .

cross section. Exact values for f_s are not generally available, and in this paper it is approximated as $f_s = 1.5$, which is the ratio of the shear stress at the neutral surface of a rectangular beam over the average shear stress [115]. For axial loading, the strain energy is written as

$$U_{axial} = \int \frac{F^2}{2EA} dx \quad (4.4)$$

where F is the axial load. Finally, for loading in torsion, the strain energy is

$$U_{torsion} = \int \frac{T^2}{2GJ} dx \quad (4.5)$$

where T is the applied torque and J is the torsional constant of the cross-section, which is equal to the polar moment of inertia for circular cross-sections.

For the given structure, the total strain energy must be found in a piecewise fashion, since the cross-sectional properties are different for the modified and unmodified sections. Using the notation in Figure 4.3(c), we can state

$$U = \sum_{i=1}^n U_{m,i} + \sum_{i=1}^n U_{d,i} \quad (4.6)$$

where U_m is the strain energy of all modified cross sections and U_d is for the unmodified tube sections. Integration is then carried out from edge-to-edge, e.g. for bending,

$$U_{m,i} = \int_{a_i}^{b_i} \frac{M^2}{2EI_m} dx$$

and

$$U_{d,i} = \int_{b_{i-1}}^{a_i} \frac{M^2}{2EI_d} dx.$$

where I_m and I_d are the second moments of area for the modified and unmodified cross sections, respectively. In order to evaluate the integrals, we define the beginning

and end of each notch as

$$\begin{aligned} a_i &= c + \sum_{j=1}^{i-1} h_j + \sum_{j=1}^{i-1} d_j \\ b_i &= c + \sum_{j=1}^i h_j + \sum_{j=1}^{i-1} d_j, \end{aligned} \tag{4.7}$$

where $b_0 = 0$, c is the length of the end cap of the structure, h_j is the length of the j^{th} notch, and d_j is the distance between notches j and $j - 1$. These equations apply to both varying notch parameters as well as constant parameters, but when h and d are constant down the length, they simplify to

$$\begin{aligned} a_i &= c + h(i - 1) + d(i - 1) \\ b_i &= c + hi + d(i - 1), \end{aligned}$$

where still, $b_0 = 0$. It is worth noting that

$$\begin{aligned} h_i &= b_i - a_i, \\ d_i &= a_i - b_{i-1}, \\ c &= a_1 - b_0, \end{aligned}$$

which simplifies most of the expressions once integration has been carried out.

4.3.2 Bending Stiffness

To assess bending stiffness, we apply a pure moment to the tip of the structure, as shown in Figure 4.3 (a). We assume that the notched portions of the tube are the primary source of deflection and that the unmodified discs between notches do not warp. Equation (4.2) applies to both the notched and unmodified sections. Applying this to the notch geometry as in Equation (4.6), integrating with respect to x , and

taking the partial with respect to M yields the angular deflection formula:

$$\theta_{bending} = \frac{M}{E} \left[\sum_{i=1}^n \frac{b_i - a_i}{I_m} + \sum_{i=1}^{n+1} \frac{a_i - b_{i-1}}{I_d} \right], \quad (4.8)$$

which simplifies to

$$\theta_{bending} = \frac{M}{E} \left[\sum_{i=1}^n \frac{h_i}{I_m} + \sum_{i=1}^n \frac{d_i}{I_d} + \frac{c}{I_d} \right] \quad (4.9)$$

where I_d is the second moment of area for the unmodified section of tube and I_m is the second moment of area of the modified section, which changes as the pattern alternates:

$$I_m = \begin{cases} I_1 & i = 1, 3, 5, \dots \\ I_2 & i = 2, 4, 6, \dots \end{cases} \quad (4.10)$$

For the case of $m = 2$, $I_1 = I_x$ and $I_2 = I_y$ since the notches alternate in 90° rotations.

The equation for deflection of a uniform cantilever beam assuming no shear is

$$\theta = \frac{ML}{EI}$$

from Euler beam theory. Equating the deflections and solving for EI yields the effective bending stiffness:

$$(EI)_{eff} = LE \left[\sum_{i=1}^n \frac{h_i}{I_m} + \sum_{i=1}^n \frac{d_i}{I_d} + \frac{c}{I_d} \right]^{-1} \quad (4.11)$$

where L is the overall length, h is the notch length, d is the spacing between notches, c is the end camp, I_m is defined by Equation (4.10), and I_d is the second moment of area of the tube.

Behavior of Anisotropic Notches in Off-Axis Bending

The above analysis assumes that for $m = 2$, the deflection in bending is equal in all directions, and the derivation applies a load orthogonal to one of the notch directions

(thus parallel to the other) to attain an equivalent stiffness. Individually, these notches act in an anisotropic manner, as at least 3 remaining tube segments ($m \geq 3$) are required to be strictly isotropic. However, the alternating pattern accounts for the directionality of each individual notch, as two notches act in tandem. While one is very compliant in the direction of the load, the other is relatively stiff, and their balance is equivalent to a cross section with isotropic behavior.

We demonstrate this by analyzing a non-orthogonal load. If such a moment is applied, it can be resolved into components of principle axes:

$$M_{xy} = \sqrt{M_x^2 + M_y^2}$$

where M_x is applied along the x-axis and M_y is applied along the y-axis. The resultant deflection in the direction of the moment can be gained from the components:

$$\theta_{xy} = \sqrt{\theta_x^2 + \theta_y^2}$$

The equation for deflection in bending, Equation (4.9), can be further simplified for constant notch parameters:

$$\theta = \frac{M}{E} \left[h \left(\frac{n/2}{I_1} + \frac{n/2}{I_2} \right) + \frac{nd}{I_d} + \frac{c}{I_d} \right],$$

where I_1 is defined by the orientation of the odd-numbered notches and I_2 corresponds to the orientation of the even-numbered notches as defined in Equation (4.10). For forces applied along the x-axis, $I_1 = I_x$ and $I_2 = I_y$, and the reverse is true for loads applied along the y-axis due to the 90-degree rotation of every other notch. This makes the x- and y-components of deflection due to an off-axis moment

$$\begin{aligned} \theta_x &= \frac{M_x}{E} \left[h \left(\frac{n/2}{I_x} + \frac{n/2}{I_y} \right) + \frac{nd}{I_d} + \frac{c}{I_d} \right] \\ \theta_y &= \frac{M_y}{E} \left[h \left(\frac{n/2}{I_y} + \frac{n/2}{I_x} \right) + \frac{nd}{I_d} + \frac{c}{I_d} \right], \end{aligned}$$

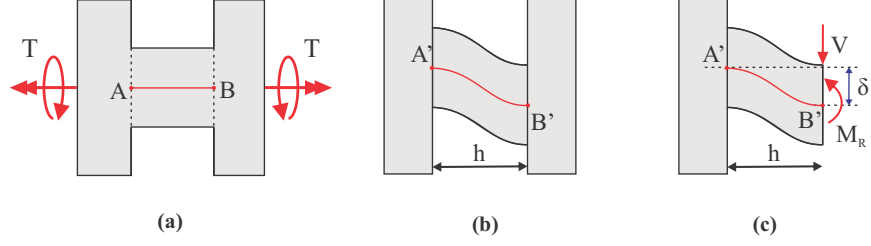


Figure 4.4: The setup for the beam approximation of torsion behavior.

where clearly, $\left(\frac{n/2}{I_x} + \frac{n/2}{I_y}\right) = \left(\frac{n/2}{I_y} + \frac{n/2}{I_x}\right)$. For non-alternating notches of $m = 2$, this would not be true, as M_y would only act on I_y and M_x would only act on I_x , producing significant directionality in the displacement, as $\frac{n}{I_x} \neq \frac{n}{I_y}$. Since the notches alternate and the sums are equivalent, the following can be shown:

$$\begin{aligned}
 \theta_{xy} &= \sqrt{\left(\frac{M_x}{E} \left[h \left(\frac{n/2}{I_x} + \frac{n/2}{I_y} \right) + \frac{nd_i}{I_d} + \frac{c}{I_d} \right]\right)^2 + \left(\frac{M_y}{E} \left[h \left(\frac{n/2}{I_y} + \frac{n/2}{I_x} \right) + \frac{nd_i}{I_d} + \frac{c}{I_d} \right]\right)^2} \\
 &= \sqrt{(M_x^2 + M_y^2) \left(\frac{1}{E} \left[h \left(\frac{n/2}{I_x} + \frac{n/2}{I_y} \right) + \frac{nd_i}{I_d} + \frac{c}{I_d} \right] \right)^2} \\
 &= \sqrt{(M_x^2 + M_y^2)} \left(\frac{1}{E} \left[h \left(\frac{n/2}{I_x} + \frac{n/2}{I_y} \right) + \frac{nd_i}{I_d} + \frac{c}{I_d} \right] \right) \\
 &= M_{xy} \left(\frac{1}{E} \left[h \left(\frac{n/2}{I_x} + \frac{n/2}{I_y} \right) + \frac{nd_i}{I_d} + \frac{c}{I_d} \right] \right),
 \end{aligned}$$

which is the same equation as displacement for an orthogonal load. Therefore the displacement under a bending moment is not direction-dependent, as it would be for non-alternating notches, and therefore the structure behaves isotropically.

4.3.3 Torsional Stiffness

The cross-section of the notched tube segments is not well-described by torsion models, which can be stated for solid general cross sections or closed, thin-walled hollow sections. This geometry is open, thin, and hollow. Thus, rather than approaching this as a torsion problem, we propose another modeling approach: that of a beam in bending. The torque applied to the structure can be calculated as a

shearing force on the tip of a beam, as shown in Figure 4.4(c). That force is

$$V = \frac{T}{\bar{y}m} \quad (4.12)$$

where V is the resulting shear load, T is the applied torque, $\bar{y}$ is the distance from the tube centerline to the centroid of the segment, and m is the number of remaining material segments. In addition to the shearing force, the “beam” segment must remain parallel at both ends, creating a boundary condition of an applied moment at the tip. The resulting total strain energy can be stated as

$$U = \int_0^h \frac{M_B^2}{2EI} dx + \int_0^h \frac{f_s V^2}{2AG} dx.$$

where f_s is approximated as 1.5.

Before solving for deflection, M_B must be found. This setup creates a statically indeterminate system, but the Principle of Least Work can be used to find the moment, since it is a redundant reaction force:

$$\frac{\partial U}{\partial M_B} = \theta_B = 0$$

resulting in

$$M_B = \frac{Vh}{2}.$$

Now, applying Castiglano’s Theorem, the resulting deflection at the point of load application can be found as

$$\delta_{b,i} = \frac{V(b_i - a_i)^3}{12EI_s} + \frac{f_s V(b_i - a_i)}{GA_s} \quad (4.13)$$

where I_s and A_s are the second moment of area and the area of a single remaining segment of material, respectively. The angle of rotation for a single notch can then

be approximated as

$$\theta_{torsion,i} = \frac{\delta_b}{r_o}$$

which implies that the beam deflection approximates a circular arc. Finally, the complete deflection of the structure under torsion can be calculated by the following, which includes the strain energy from the solid segments as derived from Equation (4.5), as well:

$$\theta = \sum_{i=1}^n \frac{Vh_i^3}{12EI_s r_o} + \sum_{i=1}^n \frac{f_s V h_i}{GA_s r_o} + \sum_{i=1}^n \frac{T d_i}{GJ_d} + \frac{Tc}{GJ_d} \quad (4.14)$$

where J_d is the polar moment of inertia of the unmodified tube.

Setting Equation (4.14) equal to the classical expression for a rod in torsion

$$\theta = \frac{TL}{GJ}$$

provides an equivalent stiffness expression,

$$GJ_{eff} = L \left[\frac{1}{\bar{y}mr_o} \left(\sum_{i=1}^n \frac{h_i^3}{12EI_s} + \sum_{i=1}^n \frac{f_s h_i}{GA_s} \right) + \sum_{i=1}^{n+1} \frac{d_i}{GJ_d} + \frac{c}{GJ_d} \right]^{-1}. \quad (4.15)$$

For a pattern with a constant h and d , this simplifies to

$$GJ_{eff} = L \left[\frac{n}{\bar{y}mr_o} \left(\frac{h^3}{12EI_s} + \frac{f_s h}{GA_s} \right) + \frac{nd}{GJ_d} + \frac{c}{GJ_d} \right]^{-1}.$$

A drawback of this method for modeling torsion is the reliance on a shear correction factor, f_s . With this beam model, as the ratio of h to w grows smaller, shear becomes the dominant term, and the approximation of the value of f_s plays a larger role in the accuracy. Generally, it will underestimate the torsional deflection as that ratio gets smaller because the geometry becomes less like a beam in bending and more like a member in torsion. The height-to-width ratio at which shear is the dominant term in the deflection can be approximated by assuming that the cross section of a single remaining section of material is rectangular, making $A_s = tw$ and

$I_s = tw^3/12$. Setting the bending and shear deflection terms from Equation (4.13) equal to one another, the result is

$$\frac{h}{w} = \sqrt{2f_s(1 + \nu)} \quad (4.16)$$

where ν is Poisson's ratio for the material. Above this ratio value, which depends on f_s , the bending term is dominant. Below it, shear dominates. For $f_s = 1.5$ and $\nu = 0.3$, this limit is $\frac{h}{w} = 1.97$, meaning h needs to be at least twice of w , which is not all that restrictive. If model accuracy is necessary at a ratio near to this, there are other means of finding this coefficient: it can be calculated as the inverse of the Timoshenko shear factor, which is reviewed in [116], or it can be calculated as the ratio of the shear stress at the neutral surface over the average shear stress (which motivated the use of $f_s = 1.5$) [115], though special considerations must be taken for the shear in short beams [117, 118, 119]. Finally, it could be tuned in FEA as it is done in [63].

4.3.4 Axial Stiffness

Axial loading of the structure generates significant strain not only in the axial direction but also in shear in the unmodified sections due to the alternating cut pattern. While the stiffness is primarily determined by the axial strain energy and shear could be considered negligible, it is straightforward to model and generally improved results, thus is it included here. Considering both modes, the total deflection of the structure is calculated as

$$\delta_{axial} = \delta_{tensile} + \delta_{shear}. \quad (4.17)$$

Integrating Equation (4.4) and taking the partial with respect to F yields

$$\delta_{tensile} = \frac{F}{E} \left[\sum_{i=1}^n \frac{b_i - a_i}{A_m} + \sum_{i=1}^{n+1} \frac{a_i - b_{i-1}}{A_d} \right] \quad (4.18)$$

where b_i and a_i are defined in Equation (4.7), A_m is the cross-sectional area of the modified segments, and A_d is the cross-sectional area of the unmodified tube sections.

Shear occurs in significant magnitude in the unmodified cross section. Using symmetry, we examine only one radial section of the structure and multiply the resulting deflection by the number of radial sections (which is the number of notches, m). Thus, under these assumptions, $\delta = m \frac{\partial U}{\partial F}$. Applying Castigliano's Theorem using polar coordinates results in

$$U_{shear,i} = \int_0^{2\pi/m} \frac{f_s V^2 r_m}{2GA_{um}} d\theta$$

where $V = F/2m$, $r_m = (OD - t)/2$ is the median line of the tube, $A_{um} = td_i$, and the integral is evaluated over half of the circle. Applying this equation to the entire structure and then using Castigliano's Theorem yields the resulting deflection:

$$\delta_{shear} = \frac{f_s F r_m \pi}{2m^2 G t} \left(\sum_{i=1}^n \frac{1}{d_i} + \frac{1}{c} \right) \quad (4.19)$$

and for patterns with constant d :

$$\delta_{shear} = \frac{f_s F r_m \pi}{2m^2 G t} \left(\frac{n}{d} + \frac{1}{c} \right),$$

where f_s is approximated as 1.5, t is the wall thickness of the tube, and d_i is the length of the uncut section. The equivalent deflection model for a uniform rod is

$$\delta = \frac{FL}{EA}$$

and equating the two expressions for deflection, the effective axial stiffness is then

$$EA_{eff} = LE \left[\sum_{i=1}^n \frac{h_i}{A_m} + \sum_{i=1}^n \frac{d_i}{A_d} + \frac{c}{A_d} + \frac{f_s r_m \pi}{2m^2 G t} \left(\sum_{i=1}^n \frac{1}{d_i} + \frac{1}{c} \right) \right]^{-1}. \quad (4.20)$$

For patterns with constant parameters, it reduces to

$$EA_{eff} = LE \left[\frac{nh}{A_m} + \frac{nd}{A_d} + \frac{c}{A_d} + \frac{f_s r_m \pi}{2m^2 G t} \left(\frac{n}{d_i} + \frac{1}{c} \right) \right]^{-1}.$$

4.3.5 Comparison to Finite Element Analysis

Eight designs were drafted in CAD and simulated in Abaqus FEA software. A quadratic tetrahedral mesh was applied to each model, and an encasture boundary condition was enforced at the base. For the torsion simulations, a pinned end condition was applied to the tip, and an x-asymmetry boundary condition was enforced for the axial cases to ensure the conditions matched the assumptions of the model (i.e., that the tube did not warp in and out of the plane and that deflection was only in the axial direction). The load in bending was $M = 1.36 \times 10^{-2}$ N-mm, for axial it was $F = 4.448$ N, and for torsion it was $T = 1.130$ N-mm. The material for all tubes was 304 Stainless Steel ($E = 196$ GPa, $\nu = 0.29$). All designs had $m = 2$. Two tube geometries were used: for Designs 1-4, $OD = 3.581$ mm and $t = 0.279$ mm, and for Designs 5-8, $OD = 2.769$ mm and $t = 0.152$ mm. All simulations had a length of $L = 30.5$ cm except for Designs 4 and 5, where $L = 307.3$ mm. The pattern geometries as well as the deflection results are listed in Table 4.1.

The FEA results showed generally good agreement with all candidate designs; however, significant deviation in the torsional loading case occurred for the two designs with the smallest notch lengths, Designs 1 and 5. This is due to the modeling of torsion as a beam problem, rather than using traditional torsion approaches. The shorter the “beam” formed by the remaining element, the greater the influence of shear and the approximations made therein, as discussed at the end of Subsection 4.3.3. For consistency with the other tests, f_s was not changed for these two designs.

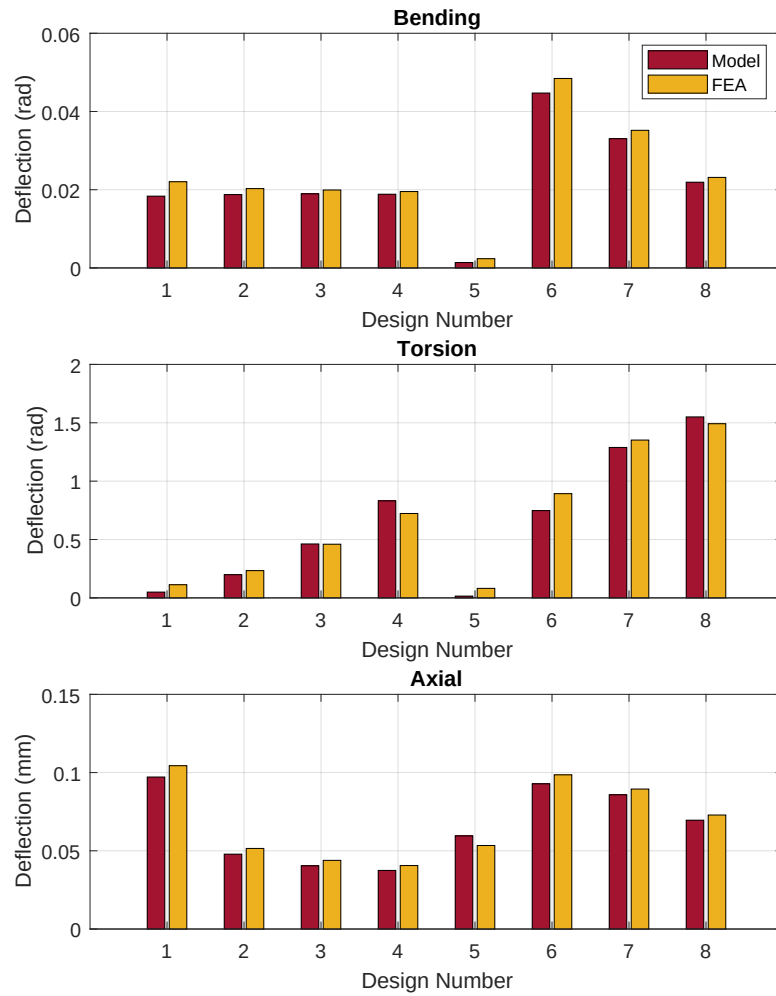


Figure 4.5: The model results for deflection in bending, torsion, and axial loading compared to the FEA output.

Table 4.1: Deflection results from the model output compared to the FEA results. For Designs 1-4, $OD = 3.581$ mm and $t = 0.279$ mm. For Designs 5-8, $OD = 2.769$ mm and $t = 0.152$ mm. All designs had $m = 2$. Length of each sample was 304.8 mm except for Designs 4 and 5, where $L = 307.3$ mm.

Design Number	Pattern Geometry (mm)				Bending Deflection (rad)			Torsion Deflection (rad)			Axial Deflection (mm)		
	c	h	w	d	Model	FEA	% Error	Model	FEA	% Error	Model	FEA	% Error
1	3.251	0.630	0.179	0.711	0.0184	0.0220	16.71%	0.0496	0.1126	55.95%	0.0972	0.1044	6.93%
2	1.016	1.397	0.179	1.524	0.0187	0.0203	7.55%	0.1985	0.2335	15.00%	0.0479	0.0515	7.09%
3	2.540	2.159	0.179	2.286	0.0189	0.0199	4.94%	0.4620	0.4598	0.47%	0.0404	0.0439	7.92%
4	2.540	2.921	0.179	3.175	0.0188	0.0195	3.49%	0.83%	0.7230	14.95%	0.0375	0.0406	7.62%
5	3.251	0.635	0.470	1.219	0.0014	0.0024	41.67%	0.0153	0.0816	81.25%	0.0595	0.0534	11.48%
6	3.048	1.397	0.166	1.397	0.0447	0.0485	7.70%	0.7476	0.8928	16.26%	0.0928	0.0986	5.82%
7	2.794	2.159	0.194	1.524	0.0330	0.0352	6.09%	1.2888	1.3520	4.68%	0.0858	0.0895	4.11%
8	7.620	2.921	0.221	2.032	0.0219	0.0232	5.31%	1.5504	1.4920	3.92%	0.0696	0.0729	4.53%

4.3.6 Future Work

This model is the first to analytically relate symmetric notch parameters to the bending, torsional, and axial rigidity of a notched tube. It also shows that while notches with two radial segments have anisotropic bending behavior individually, alternating them by 90 degrees in succession along a tube creates an overall isotropic bending response. This is important, as no exploration of a cut of this pattern has been conducted yet, and it shows promise for drastically reducing the bending rigidity of a tube while maintaining a relatively high rigidity under torsion and axial loads. The following section uses this model for the design of an elongated transmission, but broader application and full experimental validation of this model is left to future work, which could include the use of the model to create and demonstrate a concentric tube robot with improved stability.

4.4 Design and Feasibility Study of NTCR-based Endoscopic Tool with Elongated Transmission

In this section, a feasibility study of the proposed tool concept is presented. The data collected in this study are the first at-scale evaluations of the tool, using geometries and materials that are practical for a real tool that could be used in a clinical setting. Two candidate transmission designs were fitted with identical nitinol NTCR distal tips. Their input/output responses to translation, rotation, and actuation inputs were recorded using a stereoscopic vision system, and the results can be seen in Figures 4.11 and Figure 4.12. This is compared to the translation and rotational performance of a commercially-available biopsy grasper, a standard tool used in colonoscopic procedures. This section ends with a discussion of the results and directions for future work.

4.4.1 Design

The goal of the tool is to create an colonoscopic tool that can meet the demanding requirements outlined in Subsection 4.1.1. In order to ensure that no undue forces are applied to the patient colon, the maximum allowable stiffness for the tool is that of the colonoscope, which was found in [105] to be $EI = 0.0261 \text{ N-m}^2$. The tool must also be sufficiently rigid enough in torsion to transmit rotation with minimal lag, which preliminary testing outlined in Subsection 4.1.3 determined was a minimum target of $GJ = 0.0593 \text{ N-m}^2$. The axial stiffness was designed to handle 0.2% strain under the actuation load of of the distal tip of 30 N, which was a minimum target at $EA = 15 \times 10^3 \text{ N}$.

Hypodermic 304 Stainless Steel tubes were chosen as the base stock for the designs, as they are a sufficiently strong base material to achieve the desired torsional stiffness; are biocompatible and frequently used in the medical device industry; are available in small diameters and thin walls; and are highly cost effective over other material choices that also meet the other criteria. Hypodermic tubing in particular adheres to strict standards for finishing, tolerances, and dimensions, helping to narrow down the choices for wall thickness and tube diameter. This design focuses on the 3.80 mm channel of the colonoscope, and so the tubes selected are a 10 gauge extra-thin wall outer tube and a 12-gauge extra-thin wall inner tube, the dimensions of which are given with other design parameters in Tables 4.2 and 4.3. These diameters leave 0.12 mm radial clearance between the inner and outer tubes and 0.18 mm radial clearance between the outer tube and the channel. A PET heat-shrink liner with an 0.025-mm wall thickness was applied to both the inner tube and outer tube to prevent the notch edges from interfering with each other. Additionally, this pairing of tubes has an innermost lumen large enough to fit a standard endoscopic grasper like the one evaluated alongside the tool designs in Figure 4.11.

The distal NTCR tip, pictured in Figure 4.9 (a), is made of superelastic nitinol. Its design parameters are the same as Design 7 from the experimental validation in

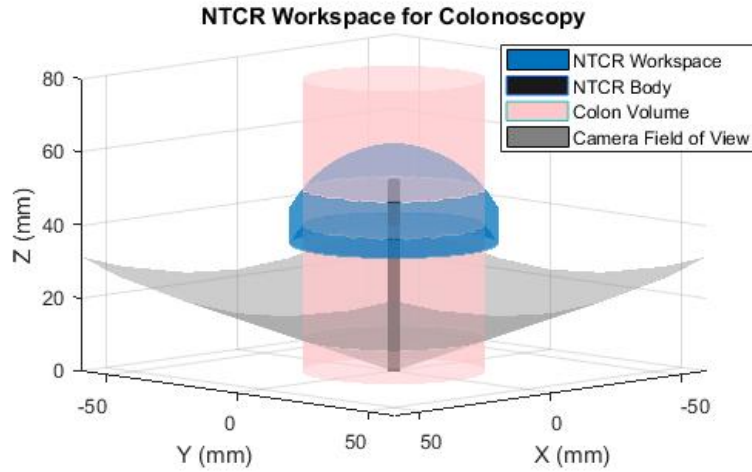


Figure 4.6: Workspace of NTCR Distal Tip Compared to Colon and Colonoscope Field of View. The workspace of the chosen NTCR design, shown in blue, considers actuation via pulling on its inner tube to its maximum value ($q = 4$ mm), rotation about the tube axis from 0° to 360° , and translation from 0 to 10 mm. The colon (in pink) has a diameter of 50 mm, and the colonoscope’s field of view (in gray) is 140° .

Chapter 3, and its notch pattern parameters can be found in Table 3.1. To reach a tip deflection of just under 90° , which is within its linear strain limit (taken to be 3%), it takes 4 mm of relative tube displacement or 30 N of actuation force. Both prototype tools use this manipulator as the distal segment. Figure 4.6 shows a workspace simulation of this design in blue, with the unactuated NTCR itself shown in black. The simulation considers actuation via pulling on the inner tube to its maximum value ($q = 4$ mm), rotation about the tube axis from 0° to 360° , and translation of 0-10 mm. The small amount of translation was shown to demonstrate that axial translation makes the workspace cylindrical, and further axial translation simply makes the cylinder longer. The colon (in pink) has a diameter of 50 mm, and the colonoscope’s field of view (in gray) is 140° .

Two candidate transmission cut designs were chosen for this feasibility study: an alternating notch (AN) Design, which uses the notch pattern examined in Section 4.3, and an Interrupted Spiral (IS) Design, which is widely used in catheter systems but is not common in endoscopic instruments. Both are pictured in Figure 4.7, with a

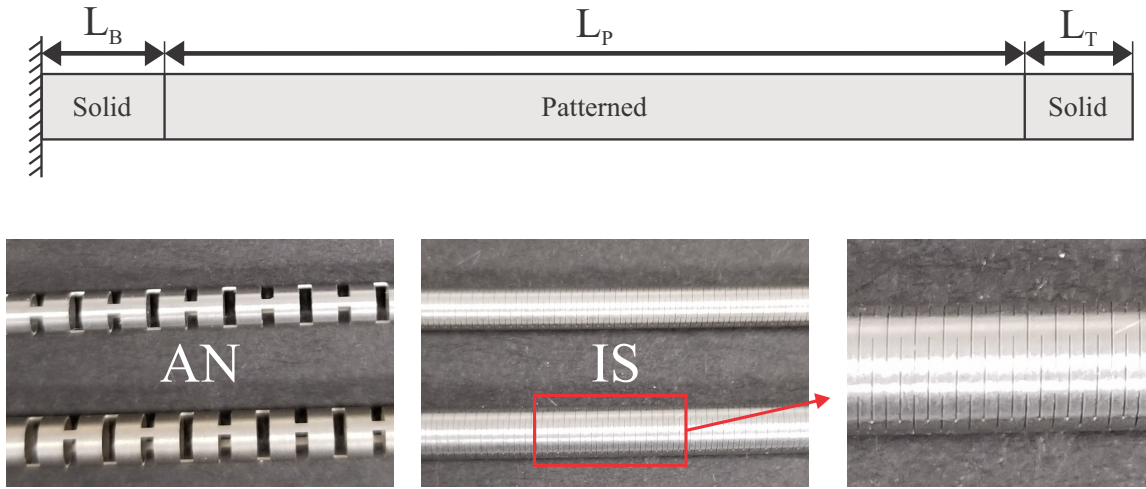


Figure 4.7: Cut Patterns and Geometry of Elongated Transmission. The top shows the definitions of L_P , L_T , and L_B , and the bottom images are pictures taken of the patterned segments.

Table 4.2: Parameters of the Alternating Notch Transmission Design. The inner and outer tubes are gauge 12 extra-thin walled and gauge 10 extra-thin walled hypodermic tubing, respectively. L_P , L_B , and L_T are defined in Figure 4.7. All dimensions in millimeters.

Tube	OD	t	h	w	d	L_P	L_B	L_T
Outer	3.40	0.20	0.89	0.51	1.65	1980	76.20	19.05
Inner	2.77	0.15	0.89	0.58	1.65	1980	101.60	19.05

Table 4.3: Parameters of the Interrupted Spiral Transmission Design. The inner and outer tubes are gauge 12 extra-thin walled and gauge 10 extra-thin walled hypodermic tubing, respectively. IP=Interruption pattern of X°/Y° where X is the angle of revolution of the spiral that is cut and Y is the revolution left uncut. CPR= Cuts per rotation. L_P , L_B , and L_T are defined in Figure 4.7. All dimensions in millimeters.

Tube	OD	t	Pitch	IP	CPR	L_P	L_B	L_T
Outer	3.40	0.20	0.36	$81^\circ/21.85^\circ$	3.5	1980	76.2	19.1
Inner	2.77	0.15	0.36	$81^\circ/21.85^\circ$	3.5	1980	101.6	19.1

close-up provided of the interrupted spiral cut. The interrupted spiral pattern makes an incredibly thin helical cut (typically the width of a laser kerf, near 0.001”) that contains “interruptions” of uncut gaps along the spiral. Changing the pitch of the helix and the width and frequency of the interruptions changes the properties of the resulting design.

The model of Section 4.3 was used to design the notches for the AN design to be below the bending target and above the torsional and axial targets. Additionally, though data on typical curvatures of the colon were not available, it was designed to withstand a 14.6-cm radius of curvature with 0.2% strain, with the bending strain modeled as

$$\epsilon = \frac{\Delta l}{l_o} = \frac{w}{2r} \quad (4.21)$$

where w is the width of the notch, r is the radius of curvature, and the neutral axis is assumed to be centered on the notch. The 14.6-cm radius is just below the estimated radius of curvature required to pass through the tool port entry on the colonoscope pictured in Figure 4.1.

While many cut patterns meet the above criteria, for the sake of reducing fabrication effort, a design which featured fewer, larger notches was selected, which is detailed in Table 4.2, the definitions of which follow what was laid out in Figure 4.3 in Section 4.3. Without a model to use as a basis, finite element analysis was used to choose an IS design that would hit the torsional and bending stiffness requirements, detailed in Table 4.3. Significant differences between the two designs are the ability to handle strain– the IS design can achieve much higher curvatures– and the axial stiffness of the elements, of which the AN has significant advantage due to the spring-like and thus extensible geometry of the interrupted spiral.

4.4.2 Verification of Bending Stiffness

The bending rigidity is a key factor in the performance of the tool, as it is desirable that it is below that of the scope so that we can avoid over-stiffening the total system,

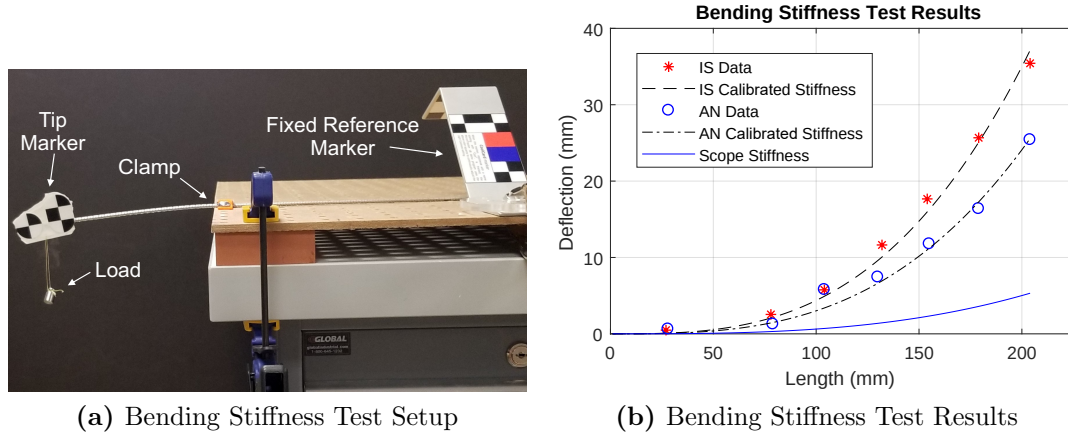


Figure 4.8: The setup and results for testing the bending stiffness of the transmission prototypes. Both designs were tested for bending stiffness, and deflection measurements were taken at 7 different lengths. The data was fit to a classic cantilever beam equation using least-squared error to determine the effective bending stiffness of each prototype. The resulting solution for EI is then plotted next to the experimental data. Also plotted is the deflection of a beam with the measured stiffness of the colonoscope, illustrating that the prototypes are more flexible than the scope.

which could generate excessive forces in the colon. To test this, the tubes were fixed together at their ends and placed in a cantilever beam setup with a 5-g tip load, as shown in Figure 4.8 (a). A marker was centered on the tip of the notched portion of the beam (as there is a small solid section at the end of each design to allow for mating with the NTCR), and the load was applied at that point. Each prototype was tested at seven lengths of the beam. For each length, both an unloaded and loaded tip position was recorded to account for self-weight, and the initial, unloaded position was subtracted from the loaded one to isolate deflection due to the load alone. The resulting data were fit to the classical cantilever equation

$$\delta = \frac{FL^3}{3EI}$$

using least squared error to find the effective EI of the beam, where the standard error of the fit was calculated by taking the square root of the sum of the squared errors divided by the number of data points. For the AN design, $EI = 0.0054 \text{ N-m}^2$

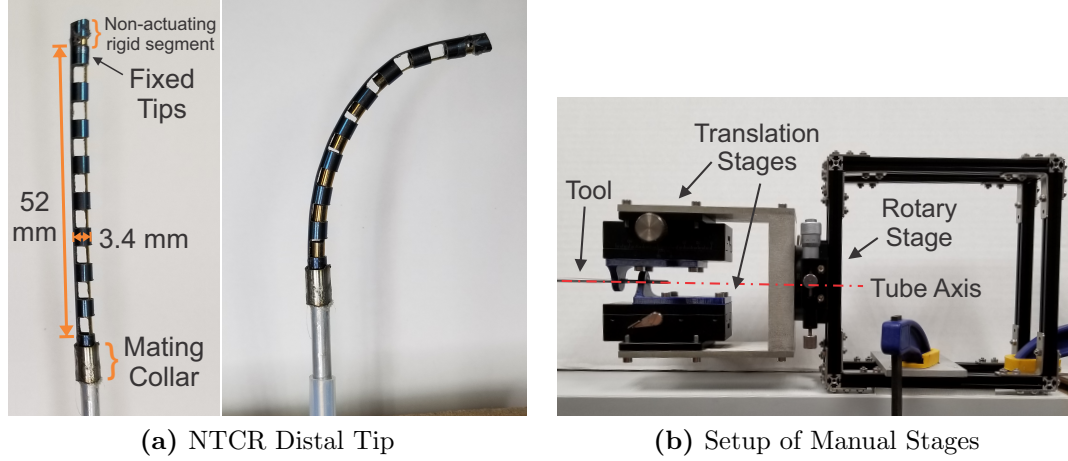


Figure 4.9: NTCR distal tip and manual stage setup for experimental evaluation of tool. **(a)** The NTCR tip design shown in unactuated and actuated shapes. The rigid, unactuated segment at the end has an alignment notch to assist with assembly, helping to ensure the NTCR backbones are parallel. It was attached to the transmission with cyanoacrylate using a mating collar on the outer tubes (pictured) and a solid rod on the inside of the inner tubes. **(b)** The stages were mounted to enable translation and rotation of the entire tool as well as relative translation of the tube for actuation. It was mounted to a rigid frame, which was then clamped firmly to an unmoving base.

with a standard error of the fit of 1.06 mm. For the IS design, $EI = 0.0037 \text{ N-m}^2$ with a standard error of the fit of 1.18 mm. Figure 4.8 (b) shows the experimental data for each prototype plotted with the curve of the cantilever beam expression using the calculated EI values. Also shown is the expected deflection for a beam the stiffness of the colonoscope ($EI = 0.0261 \text{ N-m}^2$) [105] under an identical loading scenario, which is significantly lower than that of the two prototypes.

4.4.3 Experimental Evaluation

Previous work to deploy a continuum manipulator through the working channel of a flexible endoscope demonstrated the importance of the ability to transmit rotational and translational motion of the entire tool in addition to actuation [105, 107, 108]. In particular, rotational motion proved to be a challenge to provide while also attaining the target bending stiffness. As such, the feasibility test of the two candidate designs will focus on the ability of the distal tip to potentially fill the workspace in a reliable

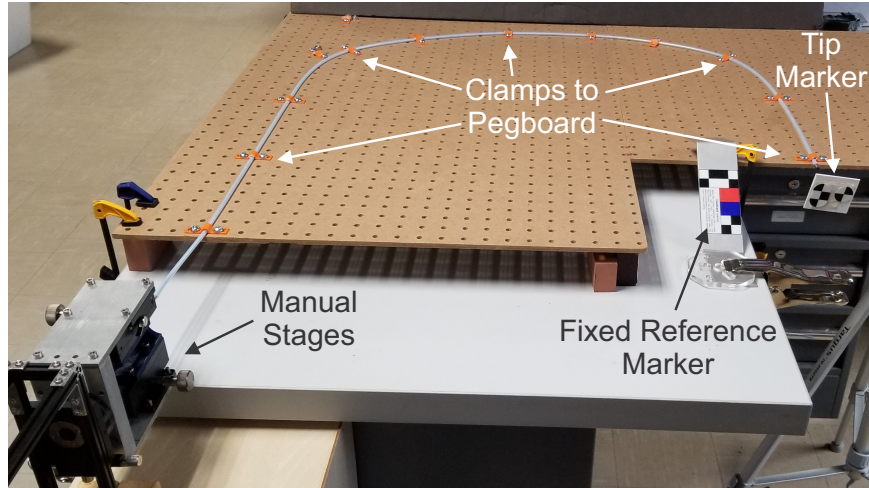


Figure 4.10: The setup for testing the input/output response of the tool prototypes. This image depicts the “Bent” configuration of the mock channel, meant to roughly mimic the shape of a colon. The PTFE mock channel was clamped down to the pegboard at approximately every 20 cm with a 3D-printed brace lined with foam to ensure a firm grip that would not deform the tube. A marker was glued to the tip of the tool, and a fixed reference frame was placed in the same field of view of the camera as the tip marker. The setup was identical for the straight configuration aside from the path of the mock channel.

manner by showing the input/output response to motion that makes up its 3-DOF actuation. Both designs were tested in a mock endoscope channel made of PTFE, which lines the tool channel in the colonoscope. The mock channel diameter is 3.76 mm, thus slightly smaller than the actual channel for which the design was made. It was clamped to a pegboard in both a straight and bent configurations, the latter of which is pictured in Figure 4.10. The clamps were necessary to ensure repeatability of tests and counteract the precurvature of the PTFE tubing, which was more stiff than either of the tool prototypes.

The two designs were affixed to 3D-printed mounts, then bolted to manual stages shown in Figure 4.9 (b), which allowed for translation and rotation of the entire tool and actuation via relative translation of the two tubes. Testing also included the evaluation of a standard endoscopic grasper, providing a comparison to currently available equipment used in colonoscopic procedures.

As a baseline, each tool was first tested with the mock channel laid straight. A MicronTracker H3-60 (Claron Technology, USA), which has a reported accuracy of 0.2 mm, was used to track both a fixed reference frame clamped to the table as well as a marker glued to the tip of the tool, both of which can be seen in Figure 4.10. A round of testing included rotational and translational measurements for all three devices, and actuation measurements for the two articulating tools. These same tests were then carried out with the scope held in a “colon-like” bent shape, pictured in Figure 4.10. This bent configuration provided a strong contrast to the straight configuration and represents a characteristic winding pathway.

For the rotational tests, the tool was rotated about its axis in 45-degree increments to positive 360° , down to -360° , and then returned to zero. Translation tests pushed the device forward 20 mm then back to zero in increments of 4 mm. The response to these inputs is plotted in Figure 4.11 for the AN design, IS design, and standard endoscopic grasper. Clearly, none of the tools had an issue with translational inputs. There is no significant difference between the straight or bent configurations, though there is a small zero offset on the return to the initial position for the bent configurations of the two articulating tools; this is likely due to tool shifting within the clearance between it and the channel.

Each tool offered a markedly different response to rotational input in the bent configuration. The standard grasper effectively offered no control over its rotation while in the bent configuration. The IS design, on the other hand, showed only slight deviation from a 1:1 input-output response. This is likely due to some slight curvature in the tubes giving the device a preferred shape, as it is difficult to create a perfectly straight prototype of this length. However, the deviation from 1:1 input/output was small enough that a teleoperator could easily compensate for the difference.

For the Alternating Notch design, the rotational response mirrors the description of the rotational response of a precurved tube from work done by Ponten [105]. This result is considered a flaw in the implementation, not the design, as this indicates that some segment(s) of the transmission were plastically deformed. The components

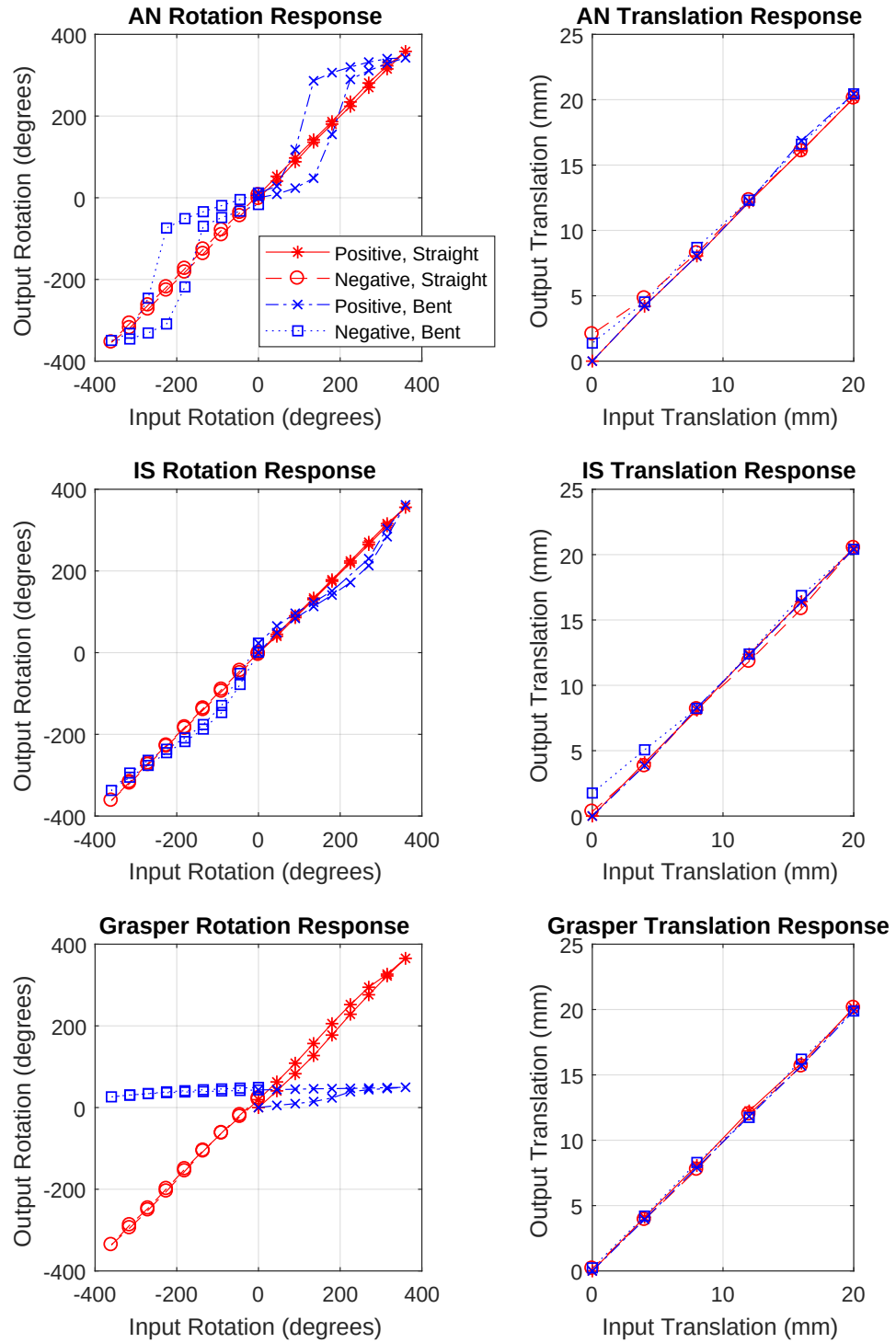
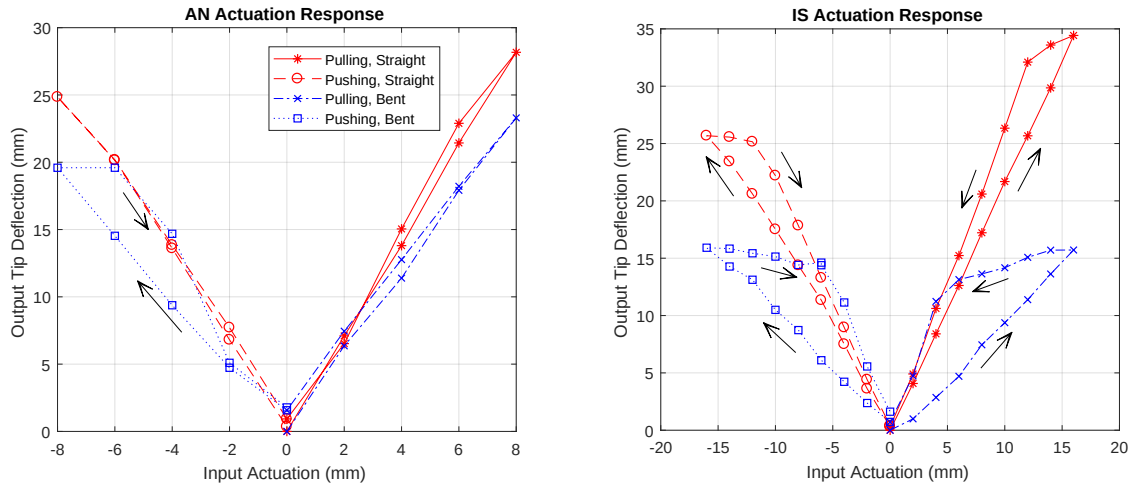


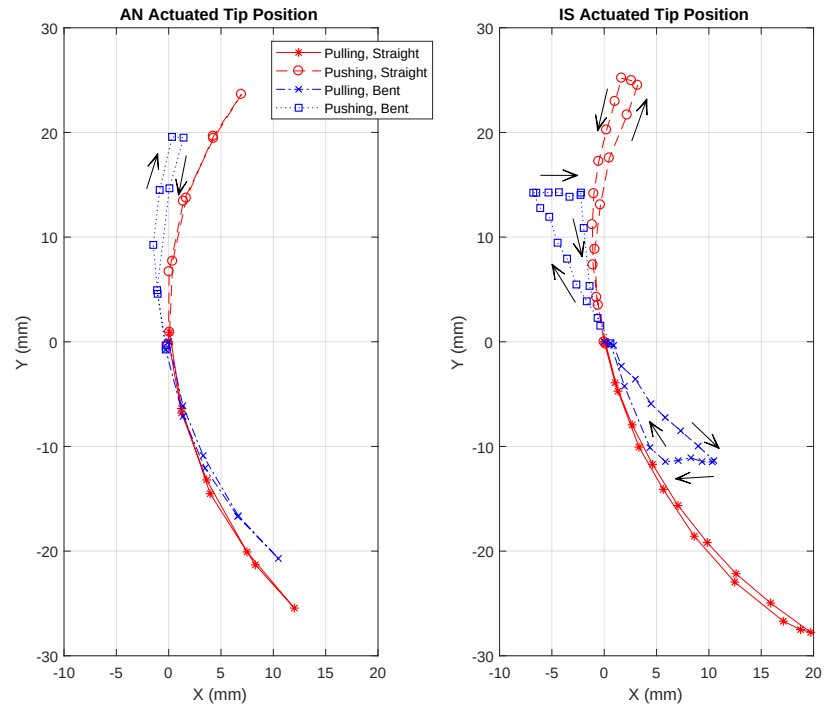
Figure 4.11: The input/output response plots for both tool designs and an endoscopic grasper.

did not arrive from the manufacturer perfectly straight, and though efforts were made to straighten them, the results suggest some precurvature of the deformed notches remained or was induced through handling at some point through the experiment. During testing, the inner tube was observed to be rotating at a different rate than the outer tube, as the inner backbone in the NTCR segment was twisted and no longer parallel with the outer backbone before the “jump” that appears in the plot. The direction of the twist indicated that the inner tube was lagging behind the outer tube. After the “jump” in rotation output angle, the backbones were aligned properly once more. Ultimately, the data and observations suggest there was significant precurvature at some point along the inner tube of the transmission, giving it a preferred shape that the outer tube did not share. Thus, the outer tube rotated as necessary, but the inner tube lagged behind. While addressing the plastic deformation of the transmission is a primary concern, another means of mitigating this issue would be to use a non-circular cross section of either the transmission or the NTCR, or to key the mating parts between the NTCR tip and the transmission elements. This would prevent relative rotation between the tubes at the tip of the device and would improve the torsional stiffness overall.

The two articulating tools were also evaluated for actuation performance. Actuation moved the inner tube only—the outer tube was held constant—and the range q was determined by pulling the inner tube by 2 mm increments in the straight channel configuration until the tip reached an approximately 90-degree tip angle, as pictured in Figure 4.9 (a). Once that position was determined, the corresponding relative translation was used for all other tests on that prototype. This upper limit was gauged visually, as stretch along the transmission was sufficient enough that the forward kinematics estimate of $q_{max} = 4$ mm was no longer valid; this is expected, as even 0.2% strain along 185 cm of material is 3.7 mm. For the Alternating Notch design, $q_{max} = 8$ mm. For the Interrupted Spiral design, $q_{max} = 16$ mm, demonstrating the axial compliance of that design.



(a) Actuation Response (Tip Deflection)



(b) Actuation Response (Tip Position)

Figure 4.12: The Actuation Response as Tip Deflection and Tip Position. Note that the NTCR has an inherently asymmetric workspace, and pulling and pushing of the same magnitudes does not generate the same tip deflections. Arrows indicate the progression of the test for tests that showed significant hysteresis. (a) The plot of input translation vs. tip deflection magnitude. (b) This plot shows the position of the tip within its plane of actuation.

The actuation response is represented in two plots: Figure 4.12 (a) shows the tip deflection from the unactuated state as a function of relative tube translation (input actuation), and Figure 4.12 (b) shows the x-y tip position in the plane of actuation. Arrows were included on the plots with significant hysteresis in order to indicate progression. Note that the NTCR has an inherently asymmetric workspace, and pulling and pushing of the same magnitudes does not generate the same tip deflections. Figure 4.12 (a) is a useful metric of how effective input actuation is on total tip motion, as ideally, greater input actuation leads to greater tip deflection. These plots also show that in total, the IS design was actuated slightly farther than the AN design, but the AN design still required less relative translation to reach the same magnitude of tip deflection. Clearly, the shape of the mock channel had an effect on both prototypes, but the AN design showed better resilience to the change due to its higher axial stiffness and ability to overcome the increased friction forces. The most significant hysteresis was seen in the AN Design was while pushing the inner tube in the bent configuration, where the manipulator clearly remains more deflected on the return path than it should; this is consistent with friction between the backbones, which can cause slight offsets such as this and was witnessed in the experiments in Section 3.6.

The bent configuration presented a significant challenge for the Interrupted Spiral design, shown clearly in its overall deflection reduction and wide hysteresis band. When the scope is bent, the normal forces acting on the transmission are higher between the channel and the outer tube as well as between the outer and inner tubes, leading to higher friction forces. This, paired with its axial compliance, meant the actuation force was diminished significantly before it reached the tip, and thus tip deflection was lowered. This also led to a re-shaping of the workspace, as the data show evidence that the transmission stretched and compressed according to the direction of actuation: for the pulling motion, tip position moves closer to the robot base than expected, and the opposite is true for the pushing configuration. Only on the release of the actuation does the tip position return to the expected path, as the

friction generated through actuation is eventually broken after a significant reduction in actuating force. It is possible that other combinations of parameters could lead to a less compliant IS design; future model development is necessary in order to optimize the design for all relevant factors.

4.4.4 Implication for Future Development

This study is the first at-scale proof of concept of an NTCR-based tool designed for colonoscopic deployment. The concept of a passive concentric transmission design was shown to be effective; changing the shape from straight to bent did not actuate the distal tip. While friction was a factor in diminishing performance of actuation in the bent configuration for both prototypes, the NTCR was still effective at covering most of its workspace for the Alternating Notch design. The torsional response of the AN design revealed flaws in implementation, but the concept proved useful, as it was well under the bending stiffness target and transmitted actuation force well. Addressing tube relative rotation as well as initial deformation and/or precurvature of the notches will lead to improved results. Further examination of the Interrupted Spiral design and its cut parameters offers a promising path toward addressing the axial compliance that was a detriment to actuation in these tests, especially given the design's combination of low bending and high torsional stiffnesses. An ideal tool may even potentially use an inner tube of one cut pattern and outer tube of another—or perhaps another cut pattern completely. These data show that axial stiffness is equally important to torsional stiffness, and they can be used to help further define the requirements for the next iteration of the design.

Additionally, the actuation strategy of the NTCR warrants further exploration. Since it is a bi-directional mechanism, it does not need the ability to operate in both a push-pull fashion and rotate a full 360-degrees about its own axis to fill its workspace; ± 90 -degree rotation is sufficient to cover its entire workspace if both directions of actuation are used. This reduces the requirements of rotational transmission.

However, the workspace is asymmetric under that paradigm, so certain applications may be best suited to pulling only. This consideration affects the transmission design, as a pulling-only scheme would require an inner tube that is axially stiff under tension and an outer tube well-suited for compression.

The next step in development of the tool is to evaluate its capacity to output the required forces, which is a function of the NTCR material, the backbone and tube geometry, and the axial compliance of the transmission. This study used a nitinol distal tip because it offers superior force output as compared to other high-strain materials considered for NTCR base stock, and it was certain to meet the goals laid out in Subsection 4.1.1. There is an inherent tradeoff between force capacity of the NTCR and its required actuation force, which the transmission must be able to withstand with minimal deflection. As the data show, the axial stiffness of the transmission is critical to tip actuation, and using a more compliant material could allow for a wider range of transmission designs due to a lower axial stiffness requirement. A statics model for the NTCR capable of addressing external loads would facilitate design optimization of the tip and help inform the choice of materials. Other materials, such as the PEEK that was used in [108], may deliver adequate tip forces for certain geometries, and this should be explored in future work.

The underlying design concept can be adapted to meet the demands of other endoscopic procedures, and the diameters and lengths can be varied to fit into other tool ports. We have demonstrated the passive transmission concept and effectiveness of the NTCR distal tip, which can be optimized for criteria such as the geometry of specific procedures or tip force output. Estimations of the strain the tool will undergo during a real colonoscopy needs further research; geometries of the colon are difficult to define, and testing in a simulated colon or *ex-vivo/in-vivo* environment is needed. Integration with a manual handle or robotic system can lead to user studies to allow further refinement through feedback with end users.

Chapter 5

Conclusions

Continuum robots are a versatile class of robotics that have the potential to expand and enhance the abilities of humankind. This dissertation contributes to the growing body of knowledge on how best to design, build, and model these types of robots.

The advances made in modeling tendon robots in Chapter 2 allows for quick, reliable design of tendon robots and provides an efficient numerical model of complex, displacement-input tendon robots with large deflections and external loads. This aligns the state of modeling with the common practice of using position-dictated motors to actuate the robot, and it allows for less complicated systems without force sensors. This work investigated the stiffness advantages of converging the tendon path, which points to the opportunity for further exploration of routing benefits to robot performance beyond workspace considerations. The model's ability to predict slack and tendon stretch allows accurate calculations of tendon tension in the displacement-input systems without force sensor data, and it can be used to find the appropriate pretensions required to avoid slack at any actuated, loaded configuration. Simulations demonstrated the importance of modeling tendon stretch, which other models often neglect, especially for cases of out-of-plane and torsional loading.

The introduction of NTCRs ushers in a new paradigm of continuum robots that are easy to build and operate, and the shape and tip design procedures have been

written to enable engineers to utilize them for a broad variety of tasks. NTCRs combine beneficial characteristics of other notched tube robots and concentric tube robots (easy to miniaturize and an open, central lumen) with those of multi-backbone robots (simple, push-pull actuation and bi-directional bending). The ability to dictate an NTCR’s shape with the design of its notch pattern offers a powerful means of customization. The experiments performed in Chapter 3 showed that designers are not constrained to a few materials or base stock choices; tests covered a variety of materials and showed additively and traditionally manufactured prototypes, both large and small scale. A detailed discussion on implementation was included to make the concept even more accessible in the hopes of reducing any time spent troubleshooting and fine-tuning.

The medical application of NTCRs to colonoscopic procedures sets the stage for enhanced dexterity in all flexible endoscopes, particularly given its effectiveness when paired with a passive transmission. The modeling used to design the flexible transmission is the first in the robotics literature to analytically relate notch parameters to the effective stiffness behavior of a symmetrically notched tube. This approach uses comparatively little computation effort vs. current means of determining modified tube stiffness, can lead to better designer insight into the effects of notch dimensions, and provides the basis for automated design. The demonstration that anisotropic notches behave in an isotropic manner in bending when alternated sequentially opens the door to new patterns that can attain better ratios of bending-to-torsional rigidity than those that only considered strictly isotropic notches, potentially benefiting concentric tube robots and other continuum structures.

The symmetric notch modeling and NTCR work informed the design and construction of the first at-scale, articulating colonoscopic tool made of a passive transmission and distal NTCR tip, which was tested for its ability to fill its workspace when deployed in a mock colonoscope channel. The decoupled transmission-articulation system creates a simpler, more intuitive design for an elongated tool that is suitable for robotic and manual implementation. Modifying a high-strength

material (hypodermic 304 stainless steel) to achieve transmission flexibility goals was shown to be effective for two cut patterns, and opportunities exist to find potentially better cut patterns provided they do not modify the neutral axis location of the structure. The NTCR tip filled the nominal colon workspace, and future iterations can be optimized for simple geometries or complicated, patient-specific goals; the modeling in this work sets the foundation for such a task. The design approach and evaluation methodology in Chapter 4 can be applied to other flexible endoscopes for procedures in other parts of the body, allowing efficient testing of candidate designs before moving to more lifelike environments. Providing enhanced dexterity to endoscopic instruments for use in NOTES procedures has the potential to increase access to life-saving procedures, reduce undesired side effects, and enable surgeons to better serve their patients, and this particular approach of using existing endoscope tool channels will keep costs low, as well.

5.1 Future Work

The expanded modeling in Chapter 2 can be applied to many tendon-driven robot systems. Further exploration of the effects of converging tendon routing in a multi-DOF system and perhaps on a 3D path may reveal fascinating relationships between deflection response, tendon stiffness, and actuation. Furthermore, the large-deflection model enables future work on designing control schemes to leverage the superior stiffness of a displacement-prescribed system.

Future work on the NTCR will expand beyond the planar shapes explored here; 3D shape design and kinematics are the next step forward, and design optimization of a nested, multi-segment design is fertile ground for exploration, especially considering that it can act in 6-DOF. The models in Chapter 3 do not consider any material nonlinearities or external loads, both of which will require investigation as applications for the NTCR are discovered and the modeling boundaries are pushed. While this

work focused on medical applications, areas such as inspection and maintenance can benefit from slender, dexterous manipulators such as NTCRs, as well.

The symmetric notching model in Section 4.3 requires experimental validation, and implementing it to design a stable concentric tube robot would be a valuable contribution to that work.

Chapter 4 closes with an extensive discussion on the feasibility study’s implications for future design iterations. This study represents the first at-scale implementation of an articulating colonoscopic tool that uses the standard working channels, and it is time to leave the proof-of-concept phase and move towards *ex-vivo* and *in-vivo* studies, with the goal of eventually conducting clinical trials. Determination of a manual handle and/or robotic teleoperation system is essential in order to begin user studies, which will be the true filter for what features of the tool need to be refined. The path towards application to other endoscopic procedures, beyond those that use a colonoscope, should be clearer now with the data generated by this study, as the design presented here can act as a springboard for future development.

Ultimately, the aim for this dissertation was to provide useful insights, analyses, and tools to enable the work done here to be expanded upon and continued. Future work such as that detailed here will pave the way forward so that robots may continue to find new applications, new forms, and new capabilities for solving challenging problems.

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Appendix

Equations for Neutral Axis Offset Based on Notch Depth for Various Cross Sections

The notch depth g directly affects the neutral axis location $\bar{\gamma}$, and moment of inertia I of the notched segment, which are calculated as follows. For any given symmetric shape, the appropriate equations can be derived from geometry, and here we include the equations for circular, square, and hexagonal cross sections. These equations are expressed in the form such that for any symmetric shape, the neutral axis offset and modified second moment of area can be described for each shape as:

$$\bar{\gamma} = \begin{cases} \bar{\gamma}_{seg} & g \geq r_1 \\ \bar{\gamma}_{mod} & g < r_1 \end{cases} \quad (1)$$

and

$$I = \begin{cases} I_{seg} & g \geq r_1 \\ I_{mod} & g < r_1 \end{cases} \quad (2)$$

where r_1 and r_2 are defined in the following subsections for each cross-section. These parameters are calculated as

$$\begin{aligned} A_{seg} &= A_1 - A_2 \\ \bar{\gamma}_{seg} &= \frac{\bar{\gamma}_1 A_1 - \bar{\gamma}_2 A_2}{A_{seg}} \\ I_{seg} &= I_1 - I_2 - \bar{\gamma}_{seg}^2 A_{seg}, \end{aligned} \quad (3)$$

and

$$\begin{aligned} A_{mod} &= A_{tube} - A_1 + A_2 \\ \bar{\gamma}_{mod} &= \frac{\bar{\gamma}_1 A_1 - \bar{\gamma}_2 A_2}{A_{mod}} \\ I_{mod} &= I_{tube} - I_1 + I_2 - \bar{\gamma}_{mod}^2 A_{mod}, \end{aligned} \quad (4)$$

where A_1 , $\bar{\gamma}_1$, and I_1 are the properties of the segment formed by the outer tube wall, A_2 , $\bar{\gamma}_2$, and I_2 are the properties of the modified solid formed by the inner wall of the tube, and A_{tube} and I_{tube} are the properties of the unmodified tube. Note that the second moments of area I_1 and I_2 are calculated about the unmodified cross-section's centerline, and the parallel axis theorem is used so that I_{seg} and I_{mod} are calculated about the modified neutral axis. The following subsections detail how to calculate A_1 , $\bar{\gamma}_1$, I_1 , A_2 , $\bar{\gamma}_2$, and I_2 for various cross-sections to use in these equations.

Circles

Note that this derivation uses circular cord geometry, assuming the cut will form a straight line across the tube, which is the geometry that milling a notch into a tube would generate, rather than a cut that forms radial lines from the center, which would be the result of most laser cutting processes. Particularly at small scales, this difference will affect the results of the model and should be carefully considered.

First, as shown in Figure 3.2 we compute the neutral axes and areas of two circular segments formed by the cut:

$$\begin{aligned}\alpha_1 &= \arccos(a/r_1) & \alpha_2 &= \arccos(a/r_2) \\ A_1 &= r_1^2(\alpha_1 - \sin \alpha_1 \cos \alpha_1) & A_2 &= r_2^2(\alpha_2 - \sin \alpha_2 \cos \alpha_2) \\ \bar{\gamma}_1 &= \frac{2r_1 \sin^3 \alpha_1}{3(\alpha_1 - \sin \alpha_1 \cos \alpha_1)} & \bar{\gamma}_2 &= \frac{2r_2 \sin^3 \alpha_2}{3(\alpha_2 - \sin \alpha_2 \cos \alpha_2)},\end{aligned}\tag{5}$$

where r_1 is the outer radius of the tube, r_2 is the inner radius, $a = |g - r_1|$, and A_1 and A_2 are the outer and inner areas of the circular segments formed by the notch. The resulting second moment of area about the unmodified tube centerline can be calculated using the following:

$$\begin{aligned}I_1 &= \frac{r_1^4}{4}(\alpha_1 - \sin \alpha_1 \cos \alpha_1 + 2 \sin^3 \alpha_1 \cos \alpha_1) \\ I_2 &= \frac{r_2^4}{4}(\alpha_2 - \sin \alpha_2 \cos \alpha_2 + 2 \sin^3 \alpha_2 \cos \alpha_2)\end{aligned}\tag{6}$$

If $a > r_1 - t$ where t is the thickness of the tube, Equations (5) and (6) can be used without subtracting the quantities for the inner tube radius (i.e., $A_{seg} = A_1$, $\bar{\gamma}_{seg} = \bar{\gamma}_1$, and $I_{seg} = I_1$).

Squares

The following equations assume the square and notch are oriented so that the backbone is a flat surface, not on the diagonal. The appropriate equations could be derived for the opposite case, if desired.

For notch depths greater than the wall thickness and up to the opposite inner sidewall ($t < g < w_1 - t$), the following hold true:

$$\begin{aligned} r_1 &= \frac{w_1}{2} & a &= |g - r_1| \\ A_1 &= w_1 (r_1 - a) & A_2 &= (w_1 - 2t) (r_1 - t - a) \\ \bar{\gamma}_1 &= a + \frac{r_1 - a}{2} & \bar{\gamma}_2 &= a + \frac{r_1 - t - a}{2} \end{aligned} \tag{7}$$

where w_1 is the side length of the outer square, t is the wall thickness of the tube, and g is the notch depth. The second moment of area of each section is calculated using the parallel axis theorem:

$$\begin{aligned} I_1 &= \frac{w_1 (r_1 - a)^3}{12} + \bar{\gamma}_1^2 A_1 \\ I_2 &= \frac{(w_1 - 2t) (r_1 - t - a)^3}{12} + \bar{\gamma}_2^2 A_2 \end{aligned} \tag{8}$$

These equations are true for notch depths beyond the opposite inner sidewall ($g \geq w_1 - t$) and for notch depths less than the wall thickness ($g \leq t$) when evaluated with $\bar{\gamma}_2 = 0$ and $A_2 = 0$.

Hexagons

Similar to the treatment of squares, we have defined the notches in a way that makes a flat edge of the hexagon form the backbone, not the vertex. These equations also assume the hexagon is regular (i.e., all sides are the same length). For notch depths greater than the wall thickness and up to the opposite inner sidewall ($t < g < w_1 - t$), the following hold true:

$$\begin{aligned}
 r_1 &= \frac{\sqrt{3}}{2}w_1 & a &= |g - r_1| \\
 h_1 &= r_1 - a & h_2 &= r_1 - t - a \\
 b_1 &= w_1 + \frac{2}{\sqrt{3}}h_1 & b_2 &= w_2 + \frac{2}{\sqrt{3}}h_2 \\
 A_1 &= \frac{h_1}{2}(b_1 + w_1) & A_2 &= \frac{h_2}{2}(b_2 + w_2) \\
 \bar{\gamma}_1 &= a + \frac{h_1(2w_1 + b_1)}{3(w_1 + b_1)} & \bar{\gamma}_2 &= a + \frac{h_2(2w_2 + b_2)}{3(w_2 + b_2)}
 \end{aligned} \tag{9}$$

where w_1 is the side length of the outer hexagon, t is the wall thickness of the tube, $w_2 = w_1 - \frac{2}{\sqrt{3}}t$ is the side length of the inner hexagon, and g is the notch depth. The second moment of area of each section about the unmodified centerline is calculated as follows:

$$\begin{aligned}
 I_1 &= \frac{h_1(w_1^2 + 4w_1b_1 + b_1^2)}{36(w_1 + b_1)} + \bar{\gamma}_1^2 A_1 \\
 I_2 &= \frac{h_2(w_2^2 + 4w_2b_2 + b_2^2)}{36(w_2 + b_2)} + \bar{\gamma}_2^2 A_2.
 \end{aligned} \tag{10}$$

These equations are true for notch depths beyond the opposite inner sidewall ($g \geq 2r_1 - t$) and for notch depths less than the wall thickness ($g \leq t$) when evaluated with $\bar{\gamma}_2 = 0$ and $A_2 = 0$.

Vita

Kaitlin Oliver Butler was born and raised in Muhlenberg County, KY, and attended the Carol Martin Gatton Academy for her final two years of high school. She met her now-husband, Phillip Butler, at the Academy, and they graduated together in 2010. She completed her BS in Mechanical Engineering at Western Kentucky University in 2015, where she won the Evan McCasland award in Mechanical Engineering. She started in the REACH Lab at the University of Tennessee, Knoxville, the semester following graduation. This dissertation outlines the bulk of her technical work while there; in addition to research, she was president of the Graduate Association of MABE for the 2018-2019 year and president of Tech CarniVOL for 2017-2018 and 2018-2019. She won the MABE Outstanding Graduate Student award in 2019. After completion of her doctorate, she plans to work in industry and eventually become faculty at a teaching-oriented university.