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An Empirical Model of Thermal Updrafts Using Data Obtained From a Manned Glider

A Thesis Presented For
Masters of Science Degree
The University of Tennessee

Christopher E. Childress
May 2010

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DEDICATION

I would like to dedicate this thesis to my wonderful wife and children for putting up with my hectic schedule and allowing me to take the necessary time away from family time to finish this work.

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I would like to thank Dr. Frank Collins and Dr. Stephen Corda for their support in my fits and starts on this thesis. Thank you for your patience and understanding that it took to see this through to completion. Thanks to Maj Mark Jones for looking over my math and making sure it was sound, and to Mr. Randolph Thompson for help with the cantankerous MATLAB® program. I would finally like to thank the men and women of the United States Air Force Test Pilot School and the Air Force Flight Test Center for continuing to push the boundaries of what is possible in flight and providing an environment where new ideas and concepts are embraced and accepted.

The views and opinions in this thesis are not necessarily the views of the United States Air Force Test Pilot School, the Air Force Flight Test Center, the United States Air Force or the Department of Defense and are purely the author's own.

ABSTRACT

Various methods have been used, including airborne radars, LIDAR, observation of flying birds, towers, tethered balloons, and aircraft to gain both a qualitative and quantitative representation of how heat and moisture are transported to higher altitudes and grow the boundary or mixing layer by thermal updrafts. This paper builds upon that research using an instrumented glider to determine the structure and build a mathematical model of thermals in a desert environment. During these flights, it was discovered that the traditional view of a thermal as a singular rising plume of air did not sufficiently explain what was being observed, but rather another phenomenon was occurring. This paper puts forth the argument and a mathematical model to show that thermals actually take the form of a hexagonal convection cell at higher levels in the convective boundary layer when the thermal acts as if unrestrained by borders as in non-linear cases of free convection.

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ABBREVIATIONS AND SYMBOLS

AGL = Altitude, Above Ground Level, m

AOA = Angle of Attack

AOS = Angle of Sideslip

b_x = Aircraft body reference frame x-axis inertial velocities

b_y = Aircraft body reference frame y-axis inertial velocities

b_z = Aircraft body reference frame z-axis inertial velocities

CBL = Convective Boundary Layer

d_L = Thermal diameter, as calculated by Lenschow & Stephens, defined in Equation (3)

d_T = New thermal diameter term of entire thermal, Childress diameter, defined in Eq (4)

d_1 = Thermal core diameter, defined in Equation (5)

e_s = water vapor saturation partial pressure, Pa

e_x = Earth reference frame x-axis inertial velocities

e_y = Earth reference frame y-axis inertial velocities

e_z = Earth reference frame z-axis inertial velocities

GPS = Global Positioning System

INS = Inertial Navigation System

K = Degrees Kelvin

KIAS = Knots Indicated Airspeed

LIDAR = Light Detecting and Ranging

m = meters

mb = millibars

MSL = Altitude, Mean Sea Level, m

m/s = Meters per second

P = Roll rate component of the aircraft angular-velocity vector

p_o = Reference atmospheric pressure, 1000 mb

p_s = Surface atmospheric pressure, mb

$\frac{\partial \overline{p_T}}{\partial z}$ = Average rate of change of atmospheric pressure with height

Q = Pitch rate component of the aircraft angular-velocity vector

R = Yaw rate component of the aircraft angular-velocity vector

RTD = Resistive Temperature Detector

r_1 = Radius of thermal core region

r_2 = Radius of entire thermal

s = seconds

T_s = Surface temperature, deg K

U = X-axis Body Fixed Velocity

V = Y-axis Body Fixed Velocity

θ_v = Virtual Potential Temperature, deg K, $\theta_v = T_s \left(\frac{p_o}{p_s} \right)^{.286} \times (1 + 0.61w_s)$

θ_{vi} = Individual virtual potential temperature data point within a thermal

$\overline{\theta_v}$ = Mean virtual potential temperature within a thermal

θ'_v = Virtual potential temperature flux within a thermal, $\theta'_v = \frac{1}{N} \sum_{i=1}^N (\theta_{vi} - \overline{\theta_v})$

W = Z-axis Body Fixed Velocity

$\bar{w}$ = Mean vertical velocity within a thermal

w'_i = Vertical velocity flux within a thermal, $w'_i = \frac{1}{N} \sum_{i=1}^N (w_i - \bar{w})$

w^* = Deardorff Velocity Scale (convective scaling parameter), m/s, defined in Eq (5)

w_d = Thermal deceleration scaling term, defined in Equation (12)

w_D = Thermal core collapse term, defined in Equation (13)

w_i = Individual vertical velocity data point within a thermal

w_{peak} = Calculated peak vertical velocity of a thermal, defined in Equation (9)

ω_s = water vapor mixing ratio, kg/kg, $\omega_s = 0.622 \times \left(\frac{e_s}{p_s - e_s} \right)$

$\bar{w}_T$ = Average vertical velocity of a thermal, defined in Equation (8)

$\overline{\dot{w}\theta_v}$ = kinematic heat flux, deg K*m/s, defined in Equation (6)

$w(r)$ = Vertical velocity profile of plume-type thermal, defined in Equation (14)

$w(r_1)$ = Vertical velocity profile of core portion of thermal, defined in Eqs (15a, 16a)

$w(r_2)$ = Vertical velocity profile of thermal outside of core, defined in Eqs (15b, 16b)

z^* = vertical scaling parameter, defined in Equation (1)

z = AGL altitude at test conditions, m

z_i = Convective Boundary Layer (CBL) thickness, m

Θ = Pitch Euler angle

Ψ = Yaw Euler angle

Φ = Roll Euler angle

1. Introduction

Considerable research has been done regarding convection as defined by environmental updrafts (thermals) and downdrafts in the Convective Boundary Layer (CBL). Various methods have been used, including airborne radars, Light Detecting and Ranging (LIDAR) sets, observation of flying birds, towers, tethered balloons, and aircraft to gain both a qualitative and quantitative representation of how heat and moisture are transported to higher altitudes and grow the boundary or mixing layer. This paper builds upon that research using an instrumented glider to determine the structure and build an empirical model of thermals in a desert environment. Through the course of flight tests that were executed from 5-22 September 2006 to research whether dynamic maneuvering in a vertical wind shear could reap greater energy gains than traditional methods of soaring, a great deal of data was gathered through many thermals. In addition to the dynamic maneuvering runs, constant airspeed runs were made through thermals at various altitudes to help characterize each individual thermal. During data analysis of these runs, it was discovered that the traditional view of a thermal as a singular rising plume of air did not sufficiently explain what was being observed, but rather another phenomenon was occurring. This paper puts forth the argument and a model to show that thermal updrafts actually take the form of a hexagonal convection cell at higher levels in the mixing layer when the thermal acts as if unrestrained by borders as in non-linear cases of free convection (Busse, 1978).

Environmental data being taken from an instrumented aircraft is not a new idea, but doing so in a glider has rarely been done. Experiments by Lenschow and Stephens (Lenschow & Stephens, 1980) in 1980 took measurements from a powered aircraft over the surface of the ocean. Young (Young, 1988) took an aircraft over continental plains of eastern Colorado in 1987 in the Phoenix 78 experiment. The advantages of using a glider are that any bias from the propulsion system of the aircraft is eliminated, and the effects of the air mass movements are more pronounced on the lighter weight aircraft. The atmosphere is the sole source of propulsion, and changes in the air mass surrounding the glider are immediately apparent. This makes a physical measurement of the local mixing layer extremely accurate, as the air noticeably smooths out when above the mixing layer. The disadvantages are that it is extremely difficult to attain the same altitude for subsequent runs through the same thermal, and one must use whatever thermals are present, which can eliminate any choosing of altitudes or thermals of a specific strength.

2. Data Collection

Data were collected on test flights over Rogers Dry Lake and land to the north of the dry lake over Edwards Air Force Base, California from 5-22 September 2006 in an instrumented L-23 Super Blanik glider. The primary component of the instrumentation was an Athena Controls GuideStar™ 111m Flight Control System. The GS-111m was equipped with accelerometers, angular rate sensors, and magnetometers in all three axes, an internal GPS receiver, and air data sensors. A real-time, multi-state Kalman filter was used to integrate the different sensors. Pitot-static pressures from a nose-mounted 5-hole probe were measured by the GS-111m to determine airspeed, altitude, angle-of-attack (AOA), and angle-of-sideslip (AOS), which was calibrated using a trailing cone. Total air temperature was measured by a resistive temperature detector (RTD) mounted under the right wing. The RTD voltage was sampled by a 14 bit analog-to-digital input on the GS-111m. Data were output to two tablet PCs with test team developed software used for flight displays and data recording. The GS-111m updated its navigation solution at 50 hertz, and the data from all outputs were sampled at 50 hertz for testing. The GS-111m was modified to accept a digital signal from an analog-to-digital converter that was wired to the pressure transducers. This hardware modification consisted of a circuit board housed in a generic black box that could be mounted anywhere in the proximity of the GS-111m and connected to the GS-111m using an RS-232 serial cable. (Childress et al, 2006)

The data used were from the flights on 20 and 22 September 2006. These dates in particular were used due to the large number of quality data collection runs accomplished. There was also a very well defined Convective Boundary Layer (CBL) on those dates. While the glider was being towed to altitude, the air was consistently turbulent, sometimes violently so, in the CBL. There was always a very distinct transition when the top of the CBL was reached and smooth air began. The glider would then be towed an extra 300 feet before release, so as to have time to get free from the influences of the tow aircraft and set up for entry into the CBL where the thermals resided. The post flight reports also stated where the smooth air began, so there is a high level of confidence of the actual, observed altitude of the CBL.

Once released from tow, the glider would begin a search pattern to look for a thermal. When one was found, one or two turns were made to find the area of strongest lift, as well as to ascertain the direction and speed of travel of the thermal as the winds aloft would push and elongate the thermal downwind. Once the best axis of entry was determined, the glider would be set up to make a straight line run directly downwind or upwind at a constant airspeed of 65 knots indicated airspeed (KIAS) with a tolerance of ± 2 knots, to eliminate acceleration effects of the glider in the data, and to characterize the structure of the thermal. Airspeed was maintained by using aircraft pitch as necessary to maintain the proper airspeed and tight tolerances through the very turbulent thermals. Aircraft Pitot-static instruments exhibited significant pressure-lag, which, coupled with poor fidelity, made them difficult to use for flight test, so GS-111m outputs were sent to a tablet PC, where the test team created software interface acted as a digital “glass cockpit”

display for high fidelity attitude, airspeed, altitude, and vertical velocity indications. Data recording was initiated just prior to estimated thermal entry time, and stopped after it was clear that the glider was no longer in the thermal. This further serves to remove the biasing effects of mesoscale motion. Subsequent runs on reciprocal headings would be one or two maneuvering runs, followed by another constant airspeed run if the thermal could still be located. The task was very difficult, and usually after two runs through the thermal, it would be lost or die out. Therefore, the data sample was too small to ascertain the true time dependence of a thermal's lifespan. This paper will use an altitude dependant model using a mixed layer scaling parameter z^* which is equal to the ratio of above ground altitude z , to the above ground altitude of the top of the CBL, z_i . The scaling parameter z^* is a common parameter that has been used in every paper referenced in this thesis and serves to homogenize results for a wide variety of conditions where the CBL is either growing or shrinking, and essentially removes the bias of a changing CBL.

$$z^* = \frac{z}{z_i} \quad (1)$$

3. Data Analysis

The scaling parameters z^* and a convective velocity scale w^* (see Equation 7), also known as the Deardorff velocity scale (Wallace & Hobbs, 2006) were used to scale the varying atmospheric conditions that occurred during each data run. The use of the Deardorff velocity scale is useful for attempting to characterize the turbulent mixing due to convection in the boundary layer in a universal case for the atmosphere. It mainly relies upon the use of the kinematic heat flux, $\overline{w\theta_v}$, to capture mean potential temperature and vertical velocity changes with time (Wallace & Hobbs, 2006). The Deardorff velocity scale and its components are detailed more in section 4.2.

Thermal boundaries and thresholds to determine when an updraft is truly a thermal rather than a secondary eddy have been an area where there has been no standardization among different research activities. Lenschow (Lenschow & Stephens, 1988) used a variance in humidity as well as a size threshold to define a thermal. Others have defined them as simply a convective updraft (Young, 1987). The latter method of using the first change of vertical velocity to positive was used to define a thermal for this data set. Further, for this test, there were no size criteria set, and the narrow definition used by Lenschow and Young actually led to the exclusion of the full structure of some thermals and focused them on a narrow band of simple, single buoyant plumes. This led to uncertainties and confounding of their data near the top of the CBL and statements such as Young's assertion in his 1987 Part II paper on the Phoenix 78 aircraft experiment, "This implies that thermals cluster or that small downdrafts often occur in the midst of

thermals.” This phenomenon is exactly what was observed during this test. It became evident in looking at the vertical velocity profiles of a number of thermals that either thermals occur in pairs about 60% of the time, or there are other mechanisms at work. In reviewing a data strip chart from Lenschow (Lenschow & Stephens, 1980), the two dimensional convection cell construct clearly appears in the vertical velocity chart. Other instances of the “double peak” of vertical velocity towards the top half of the CBL appeared in a paper using LIDAR to measure convective plume updrafts and downdrafts. (Gibert et. al., 2007) Some of their results from that paper are shown in Figure 1. The double peaked plumes also appear to originate from a single plume, further bolstering the convection cell argument.

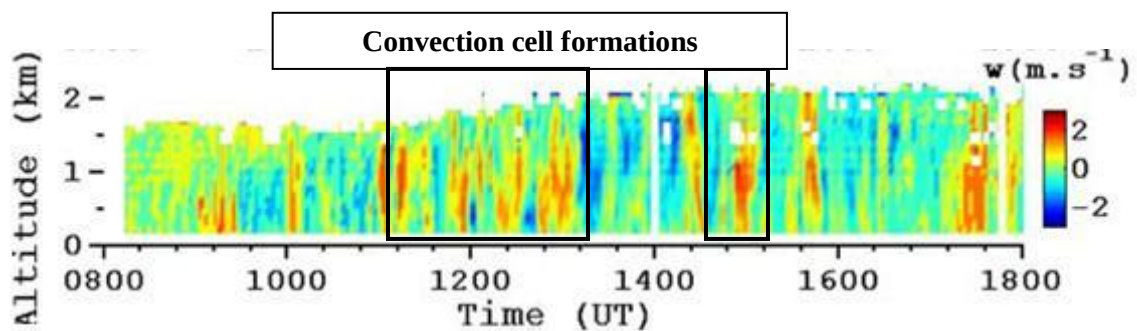


Figure 1: LIDAR measured Vertical Velocity time series, (Gibert et. al., 2007)

The data acquisition system recorded the glider inertial velocities (North / East / Down), inertial positions (North Position / East Position / Down Position), Euler rates (P / Q / R), and Euler angles (θ / Φ / ψ) through a blended GPS/INS navigation solution. A standard coordinate transformation matrix was then used to convert inertial velocities, e_x , e_y , and e_z to body fixed velocities b_x , b_y , and b_z (U / V / W) assuming a flat non-rotating Earth reference frame as shown in Equation 2. This assumption was valid due to the slow speed of the sailplane and the short time frame and size of the maneuver with respect to the earth's surface. All data were automatically logged individually per maneuver by date and time on the tablet PC in the form of a comma separated variable spreadsheet. The comma separated variable spreadsheet was imported into MATLAB[®] (Childress et al, 2006). The resulting data file was then plotted for analysis of thermal shape, entry time, and trend information. The still air sink rate of the glider at the test airspeed of 65 Knots Indicated Airspeed (KIAS) was 1.5 m/s, this bias was added to the plots and the vertical velocity data to ascertain true thermal strength. Once targeted areas were found, the comma separated variable spreadsheet was opened in Microsoft Excel[®] and actual values of time through thermal, entry altitude, corrected vertical velocity, and average ground speed from the raw data were used in the analysis.

$$\begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix} = \begin{bmatrix} \cos \theta \cos \Psi & \cos \theta \sin \Psi & \sin \theta \\ \sin \Phi \sin \theta \cos \Psi + (-\cos \Phi) \sin \Psi & \sin \Phi \sin \theta \sin \Psi + \cos \Phi \cos \Psi & \sin \Phi \cos \theta \\ \cos \Phi \sin \theta \cos \Psi + \sin \Phi \sin \Psi & \cos \Phi \sin \theta \sin \Psi + (-\sin \Phi) \cos \Psi & \cos \Phi \cos \theta \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_z \end{Bmatrix} \quad (2)$$

Environmental data was gathered from the weather station at Edwards Air Force Base, less than 5 miles away from the test site. Temperature, Dew point Temperature, Relative Humidity, surface winds, and pressure were used from this station to derive

virtual potential temperatures, θ_v , and mixing ratios, ω_s . The environmental data was recorded in 30 minute intervals, and temperature data was double checked with the temperature sensor on the aircraft and found to be accurate. A linear interpolation of surface pressure, relative humidity, and dew point temperature were used for runs that fell in between measurement times when conditions were changing rapidly, which only occurred once during a single one hour period during the data collection.

4. Results

In an attempt to explain the observational data, the phenomena of convection cells came to the forefront. Convection cells are hexagonal shaped, and have been observed in cloud formations in numerous areas by satellites and radars. A 1978 paper by FH Busse (Busse, 1978) on Non-linear convection explains how the simplest state of an asymmetric temperature dependent convection system takes the form of a hexagon. When symmetries exist and dominate, and in the limit of a semi-rigid barrier at the top (the two-dimensional case), convection takes the form of rolls, allowing a classical plume shape to form between the rolls. This is seen in the atmosphere when gravity waves or surface roughness drives an oscillation in the horizontal winds at higher altitudes. Weak circulations of symmetrical, rising and sinking air allow “cloud streets” to form above regularly spaced, plume type thermals. The oscillating horizontal flow acts as a barrier to the vertical development of the plume in regions inside of the rolls, and the circulations act as walls. In the absence of these rolls, asymmetries in the cylindrical structure of the plume about its midpoint are allowed to develop, and the hexagonal shape emerges from the plume. In gasses, the hexagonal solution begins to break down in the center due to increased viscosity at the origin (bottom, higher temperature region) of the gas, and updrafts from the edges begin to collapse inward. It is also of significant note that there is always a region where both rolls (plumes) and hexagons can exist in the absence of a barrier. As asymmetries to the cylindrical structure of the plume increase such as increasing and directionally changing winds with altitude distorting the plume, the simplest form is assumed, and the hexagonal case forms. Busse elaborates in his

concluding remarks, “In the Earth’s atmosphere convection cells become visible in the form of patterns of cumulus clouds and the horizontal scale varies from 100 m to nearly 100 km.” (Busse, 1978) This lends credence to the phenomena occurring in the CBL as a naturally occurring, simplest state solution in atmospheric convection. The lower end of the scale to which Busse gave as an example are the sizes that are relevant to this thesis, and those which were observed in the data sets examined.

When viewed in an altitude or time slice in two dimensions, the resulting hexagonal region of updrafts and downdrafts would look like a region of updraft with a lesser updraft or even a downdraft in the center, followed by another region of updraft. This is exactly the observations that were seen in this data set. Examples are in Figures 2, 3, 6, and 7. A traditional, plume type thermal is shown in Figures 4 and 5.

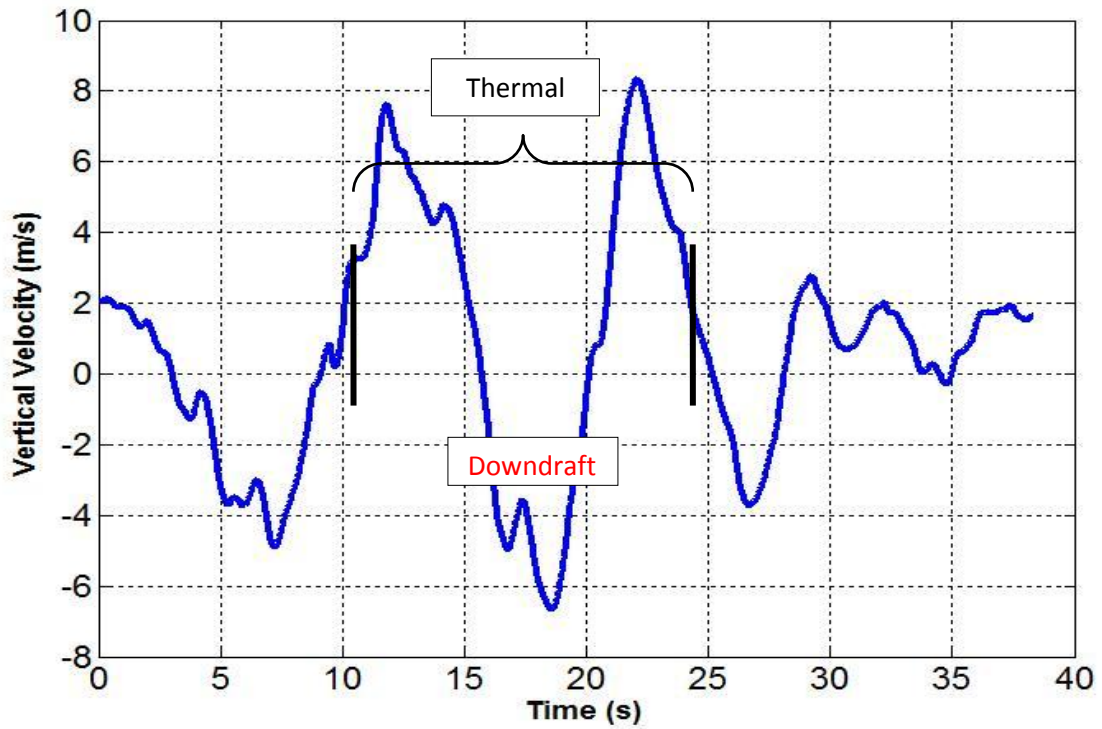


Figure 2: Convection Cell, 22 Sep 1301 $z^* = .87$, Time Domain

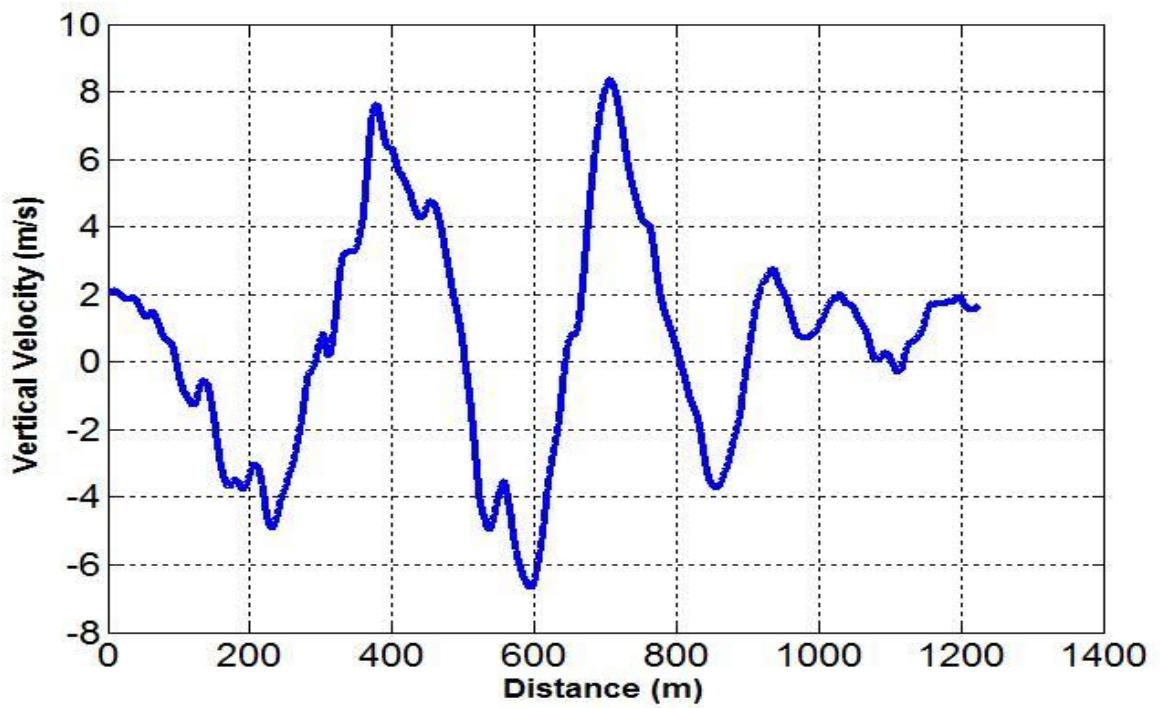


Figure 3: Convection Cell, 22 Sep 1301 $z^* = .87$, Space Domain

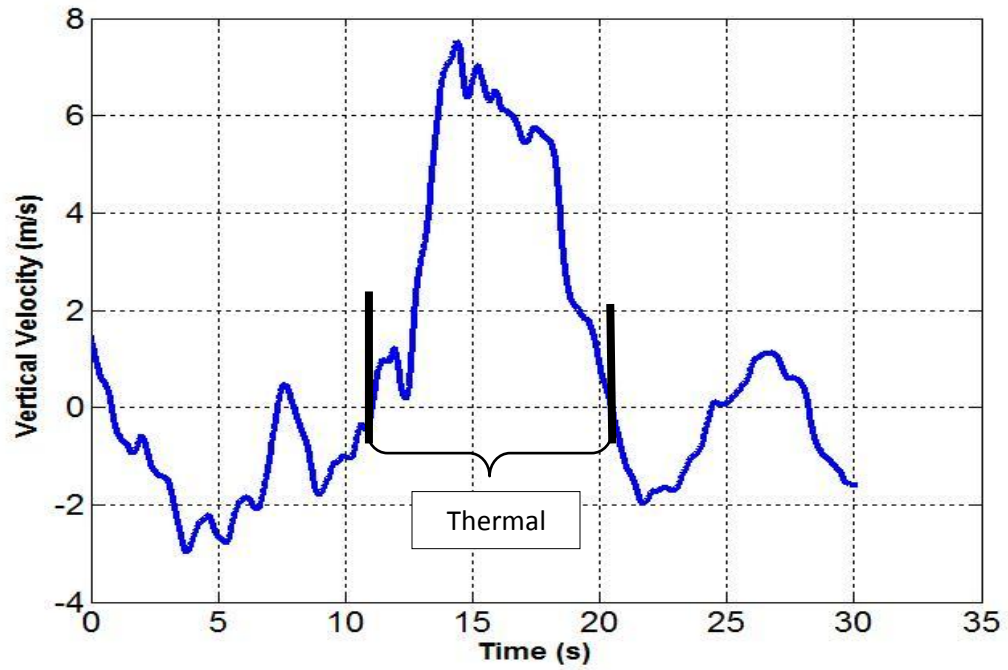


Figure 4: Plume, 20 Sep 1304, $z^* = .60$, Time Domain

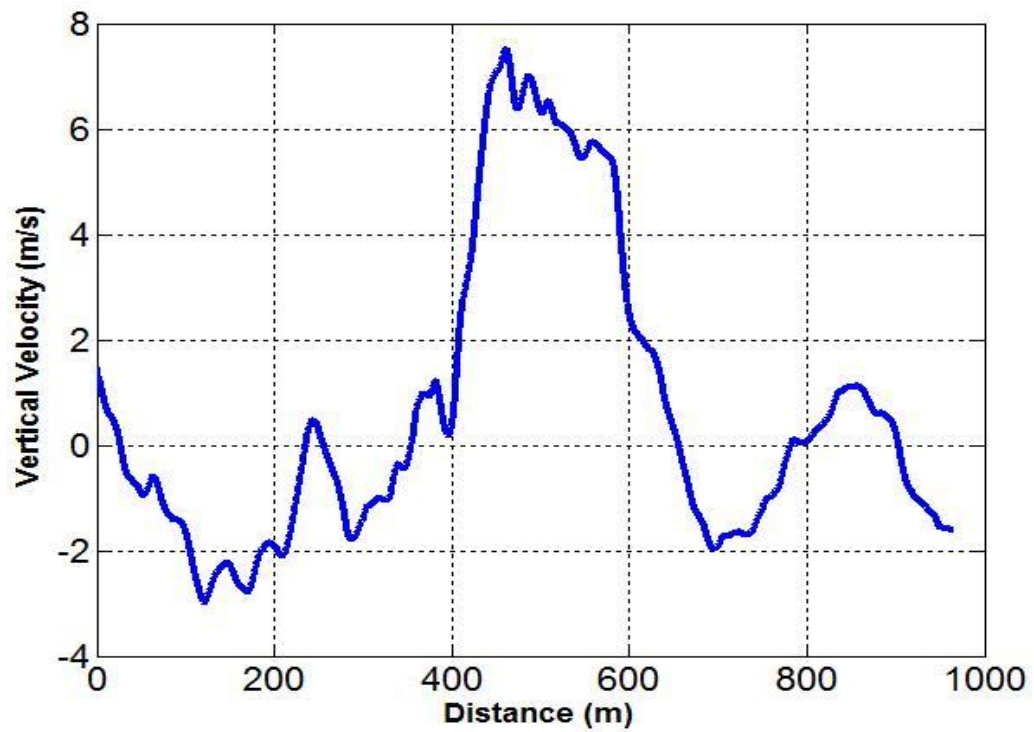


Figure 5: Plume, 20 Sep 1304, $z^* = .60$, Space Domain

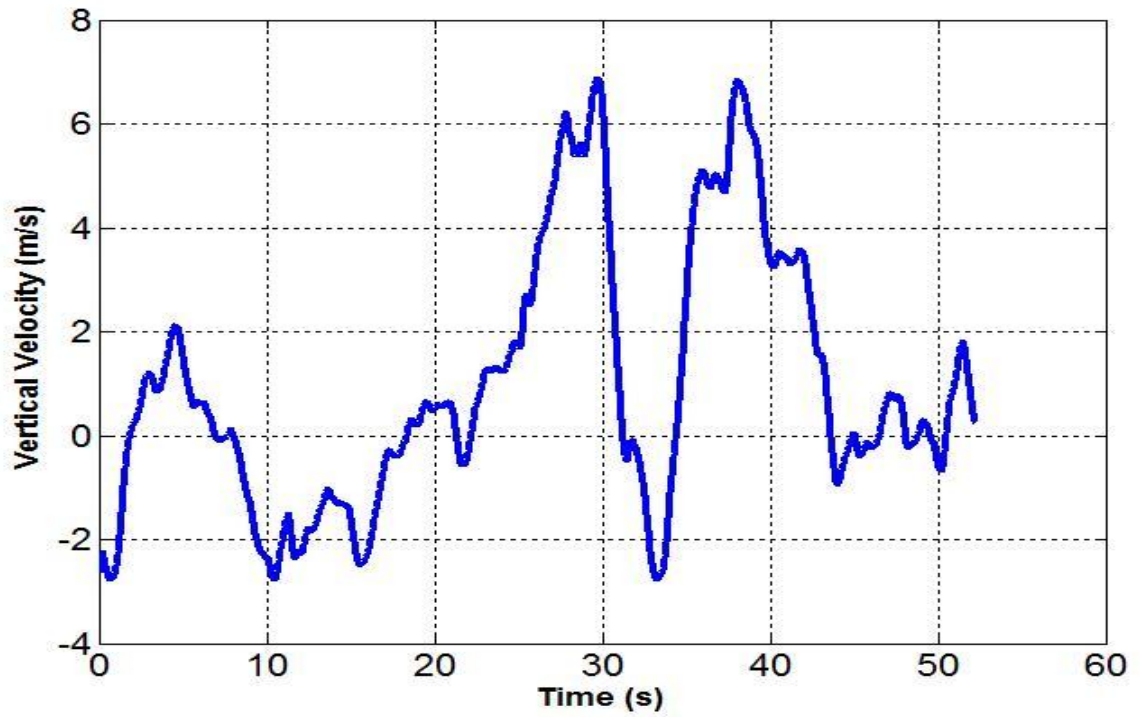


Figure 6: Convection Cell, 22 Sep 1323L, $z^* = .41$, Time Domain

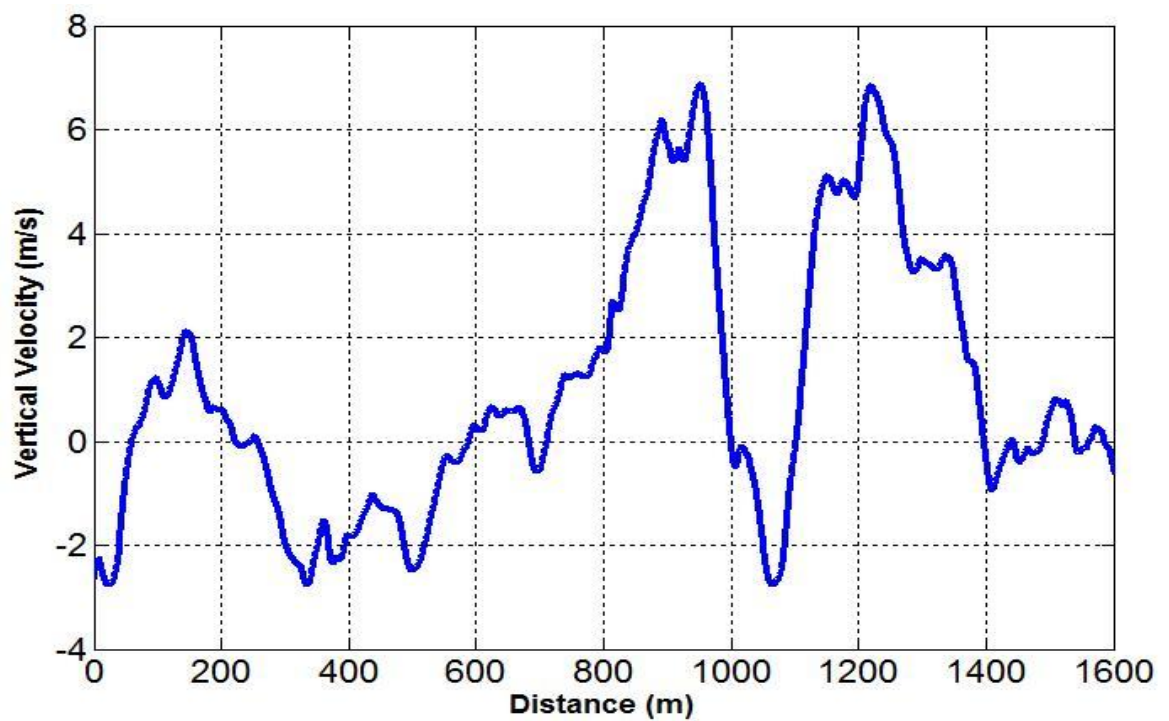


Figure 7: Convection Cell, 22 Sep 1323L, $z^* = .41$, Space Domain

4.1 Updraft Shape and Diameter

Throughout all of the data flights, both plume and hexagonal shaped thermals were encountered at all levels within the CBL. At levels above approximately $z^* = 0.6$, hexagonal convection cells dominated, though they were observed at levels as low as $z^* = 0.41$, as shown in Figures 6 and 7, when the surface winds and lower level winds aloft were strong, due to asymmetries forming at a lower altitude in the CBL. The triggering mechanisms for asymmetries to form were most certainly winds aloft, but due to a lack of fidelity in instrumentation, those winds were unable to be recorded. In their place, surface winds are a good indicator of when the convection cells will begin forming as the mixing layer was deep and well defined with no strong horizontal wind shears developing in the test area. From limited data points, it appears that when the surface winds are two times stronger than the Deardorff velocity scale, w^* (Equation 7), convection cells could begin forming at lower levels as shown in Figures 6 and 7. Plume type thermals were also observed at higher levels in the CBL, but they generally had weaker vertical velocities and appear to be the downwind, dying sides of the convection cells. The downwind side of convection cells in general had stronger and bigger updraft portions, lending credence to the theory that horizontal winds do not weaken the vertical updrafts, but tilt them with the wind. This pattern remained consistent throughout all data runs, and the downwind side remained the more pronounced feature which was the last to weaken as the thermal gained height and began to break down in the upper CBL.

Thermals continued to grow in diameter as they rose within the CBL, as observed in all of the experiments previously mentioned. However, the restrictive definition of

Lenschow and Young led to drastic under predictions of actual thermal size. The Lenschow diameter equation is shown in Equation 3. When compared with actual measurements, it was shown to under estimate the actual physical dimensions of a full thermal by up to a factor of 3. These differences were more pronounced when the CBL was fairly shallow, and at levels higher than $z^* = 0.6$.

$$d_L = z_i \times \{0.16\sqrt[3]{z^*} \times (1 - 0.25z^*)\} \quad (3)$$

Therefore, a new thermal diameter formula was derived from an approach that took into account the changing nature of the thermal at different levels and the continued growth of the thermal until the top of the CBL. The result was an adaptation of the Lenschow formula (Lenschow & Stephens, 1980) with different constants to overcome his restrictive definition of thermals. This formula is shown in Equation 4. The constants were changed from Lenschow's original formula (Lenschow & Stephens, 1980) to account for the more rapid expansion of the thermals in the lower CBL that was observed. The second term accounts for the continuing growth of the convection cell in the upper CBL at levels above $z^* = 0.6$. While the new formula does not match observed results precisely, the results are consistent with observations at both higher and lower levels in the CBL, and between classical plume type thermals and convection cell type thermals. The turbulent flow, entrainment/detrainment, thermal merging, and varying winds with altitude that accompany atmospheric phenomena make a simple solution nearly impossible. It is thought that a highly accurate predictor would require significant Large Eddy Simulations and Computational Fluid Dynamics studies that are beyond the

scope of this paper and this test. Table 1 and Figure 8 compare actual observed results with both sets of predictions.

$$d_T = z_i \times \{0.4 \sqrt[3]{z^*} \times (1 - 0.5z^*)\} + \left\{ \frac{(z^* - 0.6)(z^*)z}{\pi} \right\} \quad (4)$$

Table 1: Thermal Diameter in meters, Actual vs. Predicted, Sorted by z^*

Shape	z^*	Actual Diameter	Childress Diameter	Lenschow Diameter
Plume	0.31	563.3	479.69	219.76
Plume	0.34	482.5	483.99	224.88
Cell	0.41	526.1	476.70	227.69
Plume	0.48	232.6	474.54	232.12
Plume	0.48	501.7	474.54	232.13
Cell	0.53	744	498.97	246.89
Cell	0.54	604.4	499.39	247.30
Plume	0.60	402	252.65	125.76
Cell	0.60	695.2	491.92	244.87
Cell	0.63	363.3	495.51	246.45
Cell	0.64	458	489.44	243.31
Plume	0.74	350.5	267.91	129.46
Cell	0.75	468.8	478.68	230.52
Plume	0.75	215.8	269.22	129.61
Cell	0.75	687.9	524.19	252.16
Cell	0.76	554.4	482.36	230.90
Cell	0.78	561.7	487.59	231.35
Cell	0.79	516.3	550.69	260.39
Plume	0.79	312.4	529.57	249.66
Cell	0.80	444.7	493.78	231.80
Plume	0.83	288.3	547.78	250.70
Cell	0.87	463.2	575.68	255.08
Cell	0.88	1184.1	569.05	251.46

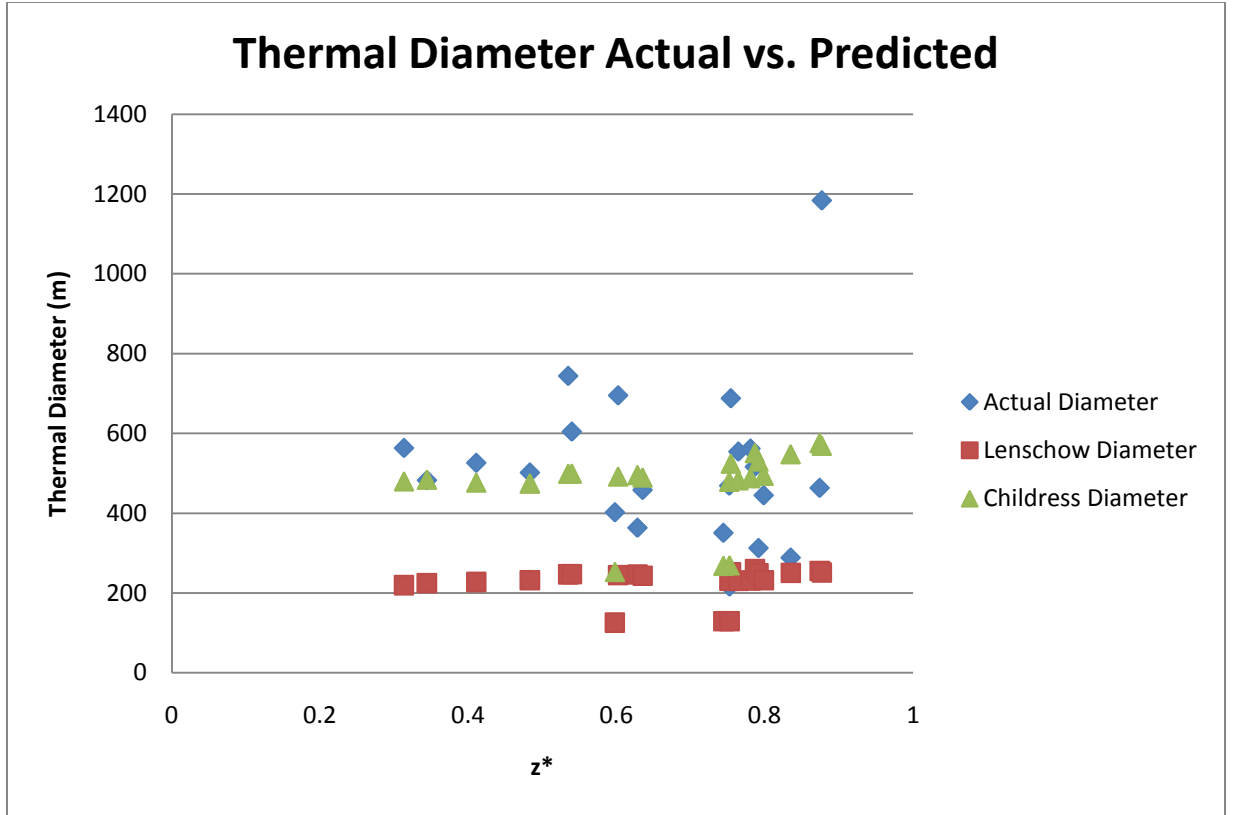


Figure 8: Actual vs. Predicted Thermal Diameter

4.1.1 Updraft Convection Cell Formation

As the formation of convection cells is dependent on asymmetries developing in an unrestrained portion of the mid-CBL, and there is a region where both plume type and convection cell type thermals can be present, it is impossible to determine with certainty when the cell begins formation. Therefore, the following equations and formulas will not be true in all cases, and may not even be true between adjacent thermals, but should nonetheless hold true in the general case.

The core of the updraft as measured from the center point of the updraft, which eventually collapses into itself, will be called d_1 , with the remaining portion called d_2 , the

total region of the updraft will be called d_T , defined in Equation 4. The collapse of the core, d_1 begins at approximately the $z^* = 0.5$ level, with definite negative vertical velocities becoming apparent at around $z^* = 0.6$. The initial collapse inward comes from a partial inflow from all sides simultaneously, at first rapidly decreasing the upward vertical velocity of the thermal, then eventually becoming a downdraft in the center of the convection cell. Across all observed convection cells, the minimum diameter of this region was 17% of the total diameter, consistent with the length of one side of a hexagon (1/6). The collapsing region then grows in a linear progression to a maximum of 33% of the total diameter, when the entire thermal begins to lose vertical velocity, most likely due to mass flux and pressure forces from the top of the CBL. The equation governing the start and the growth of the core downdraft area of the convection cell is outlined in Equation 5.

$$d_1 = .17d_T + 0.5 \times (z^* - 0.6) \times d_T \quad (5)$$

A comparison of the actual central core diameters of the convection cells with the predictions in Equation 5 is provided in Table 2. Again, the predictions are confounded with convection cells that form at levels lower than $z^* = 0.6$ due to the effort to fit the general case, but in the case of strong surface winds, the constant of 0.6 can be adjusted an appropriate amount, to account for the formation of convection cells lower in the CBL. Other outliers are due to the previously discussed, extremely complicated turbulent flow that exists within the thermals themselves.

Table 2. Predicted vs. Actual Convection Cell Core Diameters, sorted by z^*

Shape	z^*	Predicted Core-d_1	Actual Core-d_1
Cell	0.41	40.17	89.44
Cell	0.53	75.35	171.12
Cell	0.54	76.70	193.41
Cell	0.60	91.24	180.75
Cell	0.63	98.20	72.66
Cell	0.64	98.60	73.28
Cell	0.75	120.01	93.76
Cell	0.75	131.83	206.37
Cell	0.76	123.03	99.79
Cell	0.78	127.01	174.13
Cell	0.79	144.48	129.08
Cell	0.80	131.35	133.41
Cell	0.87	163.79	152.86
Cell	0.88	162.22	367.07

4.2 Updraft Vertical Velocities

To evaluate the vertical velocities of the thermal updrafts one must begin with the buoyant forces that drive convective forcing and try to apply them to a more general case. This has been accomplished in the past by Lenschow (Lenschow & Stephens, 1980), Young (Young, 1988), Allen (Allen, 2006), and others by use of the convective, or Deardorff scaling parameter w^* (Wallace & Hobbs, 2006). While this is not the only effect that drives vertical atmospheric motion, it is recognized as the dominant force in the lower CBL, and the momentum which buoyant forcing imparts upon the airmass is the critical starting place of overall thermal motion. Allen (Allen, 2006) used measured radiative sensible heat flux from a specific site to come up with a matrix of possibilities of the convective scaling parameter. Young (Young, 1988) used the kinematic heat flux, $\overline{w\theta_v}$, which comprised the fluxes of the virtual potential temperature, θ_v and vertical

velocities, w , at the measurement altitudes over a large area that the data were gathered over. This data set used the variances from the mean vertical velocities and temperatures as measured within each individual thermal to derive the kinematic heat flux. Equation 6 shows the kinematic heat flux equation. The use of this term in the Deardorff velocity scale, acts as a zeroth-order turbulence closure that remains valid for the unstable CBL (Wallace & Hobbs, 2006).

$$\overline{\dot{w}\theta_v} = \frac{1}{N} \sum_{i=1}^N (w_i - \bar{w}) (\theta_{vi} - \bar{\theta}_v) \quad (6)$$

Although the Deardorff velocity scale uses virtual temperature for the temperature term, this model uses actual surface temperature, T_s in calculating the scale. This was deliberate to simplify the inputs necessary to run the model. As virtual temperature and actual surface temperatures generally only differ by 1-2 degrees K, this substitution is a valid approximation, and was shown through back testing of the data to only make a difference of approximately .003-.005 m/s in the calculated Deardorff velocity scales. Equation 7 shows the form of the Deardorff scaling velocity as used here.

$$\omega^* = \sqrt[3]{z_i \overline{\dot{w}\theta_v} \frac{g}{T_s}} \quad (7)$$

The updrafts themselves were very resilient, and penetrated with significant vertical velocities quite high into the CBL as evidenced by Figure 1 at $z^* = .87$. Lenschow's (Lenschow & Stephens, 2006) derivation of average thermal strength in Equation 8 shows that the average vertical velocity becomes negative at $z^* = 0.9$, and despite the fact that he focused exclusively on plume type updrafts and overly limited the

scope of what was defined as a thermal, this appears to be accurate, and further bolsters the convection cell argument, as positive vertical velocities can still exist above $z^* = 0.9$, while the net average goes negative due to the large downdraft within the center of the convection cell. Maximum vertical velocities were expected at $\sim 0.4 - 0.5 z^*$, but because a thermal was not followed throughout its entire life cycle in this data set, it is unknown if this is the case. Numerous studies have confirmed this relationship, and higher vertical velocities were seen at other parts of the CBL in this test, but these could be the product of other forces at work and not the buoyant forcing. The data gathering by use of a glider precluded continuous runs at a certain z^* level to ascertain average thermal strength encountered. Further, it prevented a continuous time series of an individual thermal due to the need to turn the aircraft around after a pass through a thermal to find the thermal again. By the time another thermal penetration was initiated, several minutes had elapsed along with the case that the penetration would occur at a lower altitude due to needing to maintain a safe flying airspeed with pitch. Predictions for peak vertical velocity were consistent with observed peak vertical velocities at various levels throughout the CBL indicating that buoyant forces continue to dominate within the larger updraft area throughout the CBL.

$$\overline{w_T} = w^* * \sqrt[3]{z^*} (1 - 1.1z^*) \quad (8)$$

4.2.1 Peak Vertical Velocities

Allen (Allen, 2006) used flight test results from Konavalov (Konavalov, 1970) to determine a toroid shape for thermal updrafts as they increased in height, which proved to

be very accurate in many respects. The major difference is that his core section remains the strongest as the thermal decays similar to closed cell convection (Wallace & Hobbs, 2006), while the actual mechanism is the formation of convection cells. Nevertheless, Allen's (Allen, 2006) revolved trapezoid argument has the same physical structure of the observed thermals, with the core and side vertical velocities reversed. Allen (Allen, 2006) determined the average and peak vertical velocities by dividing the volume of his revolved trapezoid by its base as follows in Equation 9.

$$\bar{w}_T = \frac{1}{\pi r^2} \int_0^{2\pi} \left[\int_0^{r_1} w_{peak} * r * dr + \int_{r_1}^{r_2} \frac{w_{peak} (r_2 - r_1)}{r_2 - r_1} r * dr \right] d\Psi \quad (9)$$

Evaluating Equation 9 and solving for w_{peak} Allen (Allen, 2006) found the relationship shown in Equation 10.

$$w_{peak} = \frac{3\bar{w}_T (r_2^3 - r_1^3)}{(r_2^3 - r_1^3)} \quad (10)$$

This approximation was much more accurate than Lenschow's (Lenschow & Stephens, 1980) predictions when compared to observed peak vertical velocities when using all of the above relationships in concert, lending even more credence to the convection cell shape argument. When adding in actual observed data for all thermal diameters, the accuracy improves in some cases and accounts for the higher observed vertical velocities as shown in Table 3 and compared in Figure 9.

Table 3. Peak Vertical Velocities, Predictions vs. Actuals, sorted by z^*

Shape	z^*	Peak VV (m/s) (Actual)	Peak VV- Allen (Derived)	Peak VV - Allen (Observed Data)
Plume	0.31	5.86	4.99	4.99
Plume	0.34	7.01	5.49	5.49
Cell	0.41	6.86	5.01	3.99
Plume	0.48	5.48	3.69	3.69
Plume	0.48	6.64	4.05	4.05
Cell	0.53	6.09	4.33	3.38
Cell	0.54	6.49	4.32	2.31
Plume	0.60	7.52	1.98	5.27
Cell	0.60	7.15	4.50	3.57
Cell	0.63	9.74	4.32	4.30
Cell	0.64	5.5	2.59	2.94
Plume	0.74	5.51	2.69	5.55
Cell	0.75	6.51	3.51	4.20
Plume	0.75	3.61	3.78	5.54
Cell	0.75	5.36	3.95	3.22
Cell	0.76	9.1	3.45	4.46
Cell	0.78	10.13	3.55	2.85
Cell	0.79	4.16	1.94	2.03
Plume	0.79	4.17	3.24	4.23
Cell	0.80	6.57	3.46	2.99
Plume	0.83	8.79	3.45	4.37
Cell	0.87	8.34	3.08	2.46
Cell	0.88	6.58	2.73	2.43

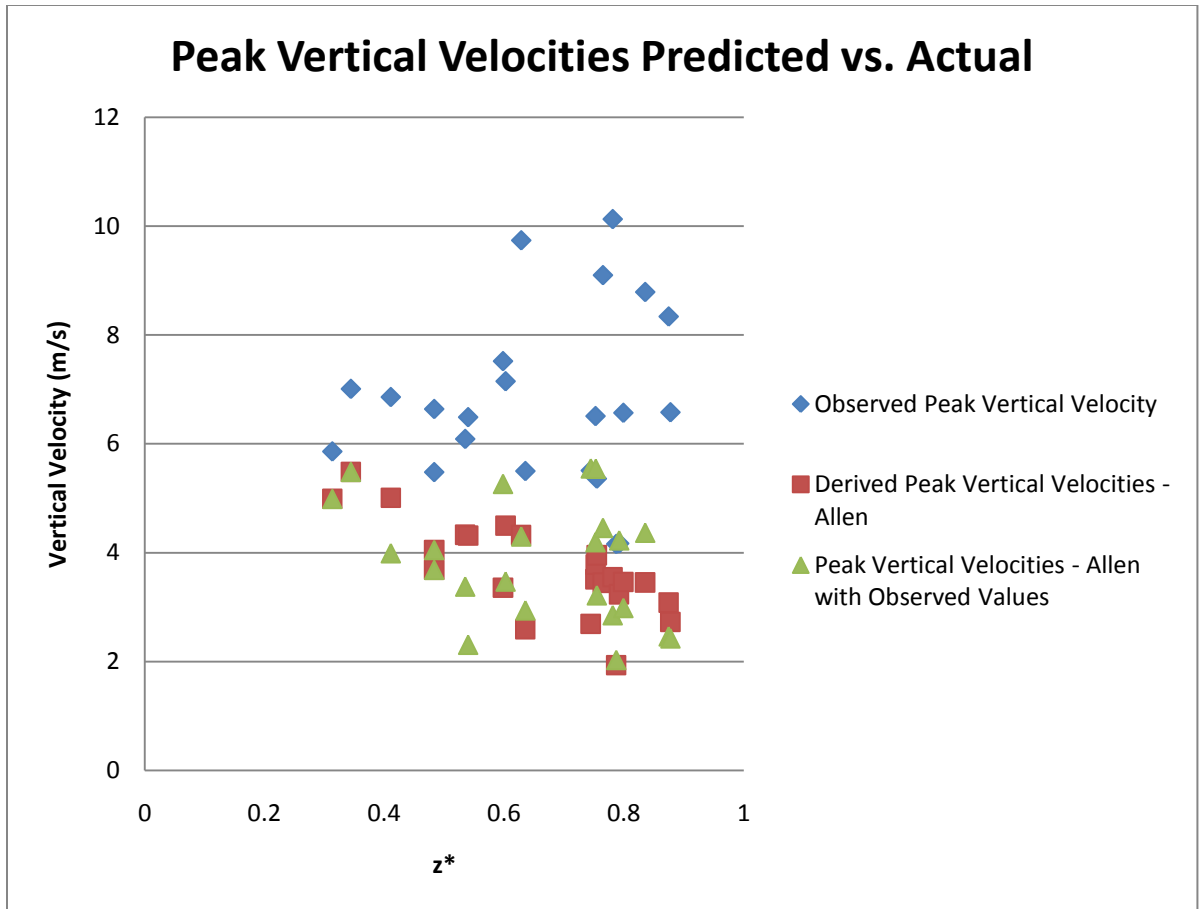


Figure 9: Actual vs. Predicted Peak Vertical Velocities

4.2.2 Vertical Velocities and Core Collapse

The formation of the convection cell begins to show the influence of downdraft from the edges starting at approximately $z^* = 0.5$ as previously mentioned. Typically, by the $z^* = 0.6$ level, a downdraft has begun to appear in the center of the thermal, which deepens as the downdraft area increases with height. The magnitude of this core collapse event is a function of the peak vertical velocities of the outer walls of the hexagon, as well as a deceleration term that serves to slow the thermal due to pressure effects as it

risers. Lenschow found this term to be as shown in Equation 11 (Lenschow & Stephens, 1980), which will be called the dimensionless deceleration scaling term, w_d .

$$w_d = \frac{-z_i}{\rho w^{*2}} \times \frac{\overline{\partial P_T}}{\partial z} \quad (11)$$

Substituting the standard day density of dry air which is 1.275 kg/m^3 for ρ , and using the general rule that atmospheric pressure falls at the rate of about 4% per 304.8 meters to substitute for the $\frac{\overline{\partial P_T}}{\partial z}$ term, Equation 12 follows.

$$w_d = \frac{-z_i}{1.275 w^{*2}} \times \frac{12.192}{z} \quad (12)$$

Using the deceleration scaling term, the core collapse term can now be derived to take into account when a convection cell forms and the magnitude of the downdraft can now be estimated. The core collapse term, w_D , is a function of z^* , with a 0.45 bias added in to account for the thermal behaving like a plume at z^* levels below 0.5. Also, the effects of the sides collapsing into the core are accounted for by taking half of the peak vertical velocity expected in the cell updrafts. This yields the core collapse term shown in Equation 13.

$$w_D = w_d (z^* + 0.45) - 0.5 w_{peak} \quad (13)$$

4.2.3. Vertical Velocity as a Function of Radius

With all of the above derivations complete, a modeling function of the thermal updraft vertical velocity as a function of radius can be completed. Since thermals and atmospheric conditions are extremely variable, it is nearly impossible to model these

complex interactions with 100% accuracy for all cases. This will present a method for the general case, which will give a more accurate representation of the size and vertical velocity structure of a thermal than has been accomplished in the past. Due to the changing structure of the thermal, thresholds of z^* have been established to better show the way in which the vertical velocity will change with altitude as well. In the following equations, the terms r_1 and r_2 will be used, r_1 represents the radius of the core portion of the thermal that is subject to the inward collapse, r_2 is the total radius of the thermal, from the thermal center point to the edge of the thermal. The first phase of the thermal is the classic plume. It will be assumed that the thermal rises in this state until an altitude of approximately $0.5z^*$, where the convection cell begins to form. The observed velocity profiles of this data set ranged from a spike shaped maxima to a “top hat” type of thermal, but the majority of the vertical velocity profiles fell off gently to the edge of the updraft area. Therefore a cosine dependency was assumed, with a fall off to zero as shown in Equation 14, for altitudes less than $z^* = 0.5$.

$$w(r) = w_{peak} \cos\left(\frac{r}{r_2} \times \frac{\pi}{2}\right) \quad (14)$$

For levels in the CBL with z^* between 0.5 and 0.9, a convection cell may form if the horizontal winds create enough of an asymmetry in the absence of roll type circulation borders. In this case, there will be a core that collapses starting at $z^* = 0.5$, and this type of structure is retained until approximately $z^* = 0.9$. Making use of the core collapse term, w_D in Equation 13, you get a two part curve which consists of a cosine dependent term at the core, transitioning to a sine dependent curve beginning at the outer

edge of the collapsing core of the thermal. Equations 15a and 15b, show the form of a convection cell type thermal that exists between $z^* = 0.5$ to $z^* = 0.9$.

$$w(r_1) = w_D \times \cos\left(\frac{r}{r_1} \times \frac{\pi}{2}\right); \text{ for } 0 < r < r_1 \quad (15a)$$

$$w(r_2) = w_{peak} \times \sin\left(\left(\frac{r-r_1}{r_2}\right) \times 1.212 \pi\right); \text{ for } r_1 < r < r_2 + r_1 \quad (15b)$$

Once above $z^* = 0.9$, the positive vertical velocity of the thermal rapidly decays until reaching the top of the CBL when positive vertical velocity should trend to zero. While no data were collected at this level, based on evidence gathered in the rest of the CBL, an educated guess can be made at the form of thermal decay at the top of the CBL. This is shown in Equations 16a and 16b.

$$w(r_1) = \frac{w_D}{2} \times \cos\left(\frac{r}{r_1} \times \frac{\pi}{2}\right); \text{ for } 0 < r < r_1 \quad (16a)$$

$$w(r_2) = (1 - z^*) \times w_{peak} \times \sin\left(\left(\frac{r-r_1}{r_2}\right) \times 1.212 \pi\right); \text{ for } r_1 < r < r_2 + r_1 \quad (16b)$$

Together, the above equations create a piecewise picture of the vertical velocity structure of a thermal from a few simple initial conditions, surface temperature, the kinematic heat flux, the height of the convective boundary layer, and the height at which the thermal is encountered. A MATLAB® script is included in Appendix A which will produce a visual representation of this model with the inputs mentioned. Further, a comparison against actual data gathered in this test is overlaid on a modeled thermal updraft to give a feel for the model's accuracy in Figures 10 through 12.

5. Conclusion

In this thesis the argument was put forth that a new type of convection was naturally occurring that did not meet the “plume” model of thermal updrafts, but rather satisfied all of the criteria of “open cell” convection (Wallace and Hobbs, 2006), forming convection cells. A new, more accurate method of estimating thermal diameter was introduced from observed data, and a means of estimating peak vertical velocity was also re-introduced. All signs point to the fact that convection cell type thermal updrafts do in fact occur, and account for the majority of thermal updrafts that are able to penetrate significantly into the CBL. While there are many natural, turbulent and unknown atmospheric phenomena associated with thermal updraft creation and sustainment, a general construct has been made that fits observed data quite well. A MATLAB® model has been constructed and is located in Appendix A, which can output a vertical velocity profile as a function of radius from the center of the thermal based on the level of entry z^* . The only input required to run the model are CBL height, surface temperature, altitude of entry, and the kinematic heat flux term. It is hoped that this new understanding of the naturally occurring convection can be used to more accurately make CBL predictions of vertical transport of water vapor and aerosols, and improve weather forecasting and modeling. This model could also be used to create better methods of soaring for glider pilots or autonomous unmanned aerial vehicles.

REFERENCES

References

1. Busse, FH, "Non-linear properties of thermal convection", Rep. Prog. Phys., Vol. 41, 1978.
2. Lenschow, D. H., and Stephens, P. L., "The Role of Thermals in the Convective Boundary Layer," *Boundary-Layer Meteorology*, 19, 1980, pp. 509–532.
3. Young, George S., "Turbulence Structure of the Convective Boundary Layer Parts 1-3," American Meteorological Society, 15 Feb 1988
4. Childress, Chris et. al., "A Limited Evaluation Of Dynamic Soaring Through Thermal Updrafts (Project ThUMP)," Air Force Flight Test Center –Technical Report #TIM-06-07, December 2006
5. Wallace, John M. and Hobbs, Peter V., "Atmospheric Science: An Introductory Survey-Second Edition," © 2006, Academic Press, ISBN 0-12-732951-X
6. Gibert, F., Cesta, J., Yano, J.I., Arnault, N., Flamant, P., "On the correlation between convective plume updrafts and downdrafts, lidar reflectivity and depolarization ratio," *Boundary-Layer Meteorology*, 14 July 2007, pp. 553-573.
7. Allen, Michael J., "Updraft Model for Development of Autonomous Soaring Uninhabited Air Vehicles," AIAA 2006-1510, *44th AIAA Aerospace Sciences Meeting and Exhibit 9 - 12 January 2006, Reno, Nevada*,
8. Konovalov, D. A., "On the Structure of Thermals," 12th OSTIV Congress, Alpine, USA, 1970.

APPENDIX

Appendix A. MATLAB® Code for Example Application

```
% An Empirical Model of Thermal Updrafts
% Christopher E. Childress, 2010
% University of Tennessee Space Institute, Tullahoma, TN

% Input Variables
Ts = 300.15;           %Surface Temperature in Deg K
zi = 1500;             %CBL height in m
z = 1000;              %Thermal penetration height in m
zstar = z/zi;
heatflux = .40;        %Kinematic heat flux (Deg K*m/s)

%% Thermal Diameter terms

dt = zi*(0.4*zstar^(1/3))*(1-(0.5*zstar))+(((zstar-
0.6)*zstar*z))/3.14159);

% Convection Cell core growth term

d1 = 0.17*dt + 0.5*(zstar-0.6)*dt;
d2 = dt
r1 = d1/2;
r2 = d2/2;

%% vertical velocity terms

% Deardorff Velocity Scale
x = zi*heatflux*(9.86/Ts);
wstar = x^(1/3);

% Average Vertical Velocity

wtavg = wstar*(zstar^(1/3))*(1-(1.1*zstar));

% Peak Vertical Velocity

wpeak = ((3*wtavg)*((r2^3)-(r2^2*r1)))/((r2^3)-(r1^3));

% Downdraft Core fomation and dimensionless deceleration term

wd = (-zi/(1.275*wstar^2))*(.04/(z/304.8));
wcore = (wd*(zstar+.45))-(wpeak*0.5);
```

```

%% vertical velocity vs. radius plots

rt = dt/2;
rs = 0:0.1:rt;
rc = 0:0.1:r1;
ro = r1:0.1:(r2+r1);

if zstar < 1.0;
    if zstar < .499
        w = wpeak*cos((rs/rt)*(pi/2));
        figure('Color','w');clf
        plot(rs,w,'b','LineWidth',3); ylabel('Vertical Velocity...
(m/s)','FontSize',14,'FontWeight','bold');grid;xlabel('Radius...
(m)','FontSize',14,'FontWeight','bold'), grid on
    elseif zstar > .500 & zstar < .900
        w = wcore*cos((rc/r1)*(pi/2));
        z = wpeak*sin(((ro-r1)/r2)*(1.212*pi));
        figure('Color','w');clf
        plot(rc,w,'b','LineWidth',3); ylabel('Vertical Velocity...
(m/s)','FontSize',14,'FontWeight','bold');grid;xlabel('Radius...
(m)','FontSize',14,'FontWeight','bold'), grid on, hold
        plot(ro,z,'b','LineWidth',3); ylabel('Vertical Velocity...
(m/s)','FontSize',14,'FontWeight','bold');grid;xlabel('Radius...
(m)','FontSize',14,'FontWeight','bold'), grid on
    elseif zstar > .901
        w = (wcore/2)*cos((rc/r1)*(pi/2));
        z = (1-zstar)*wpeak*sin(((ro-r1)/r2)*(1.212*pi));
        figure('Color','w');clf
        plot(rc,w,'b','LineWidth',3); ylabel('Vertical Velocity...
(m/s)','FontSize',14,'FontWeight','bold');grid;xlabel('Radius...
(m)','FontSize',14,'FontWeight','bold'), grid on, hold
        plot(ro,z,'b','LineWidth',3); ylabel('Vertical Velocity...
(m/s)','FontSize',14,'FontWeight','bold');grid;xlabel('Radius...
(m)','FontSize',14,'FontWeight','bold'), grid on
    end
elseif zstar > 1.0;
    w = 0;
end

```

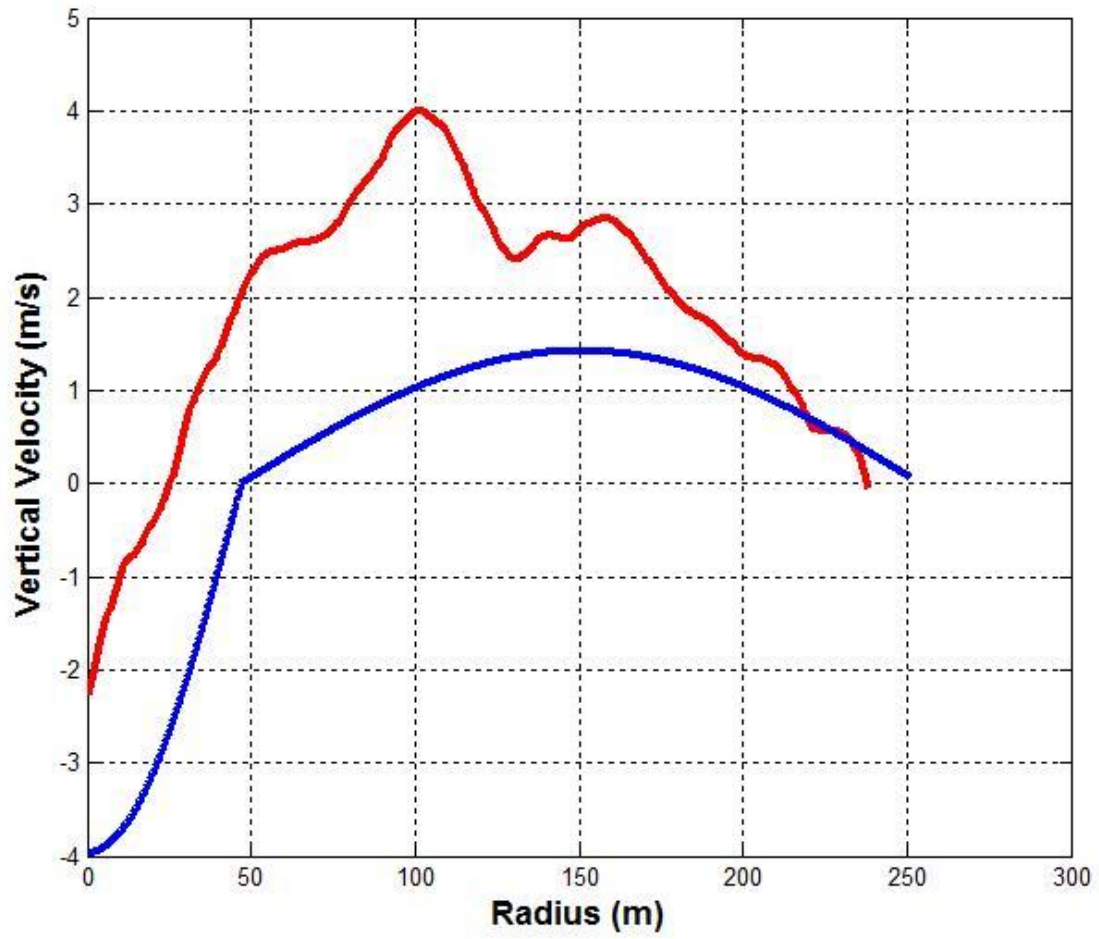


Figure 10: 20 Sep 1716, Model Comparison (Model in Blue), $z^* = .64$, $T_s = 300.75$, $\overline{\dot{w}\theta_v} = 0.163$, Actual Thermal Data from 20 Sep 1716 in Red

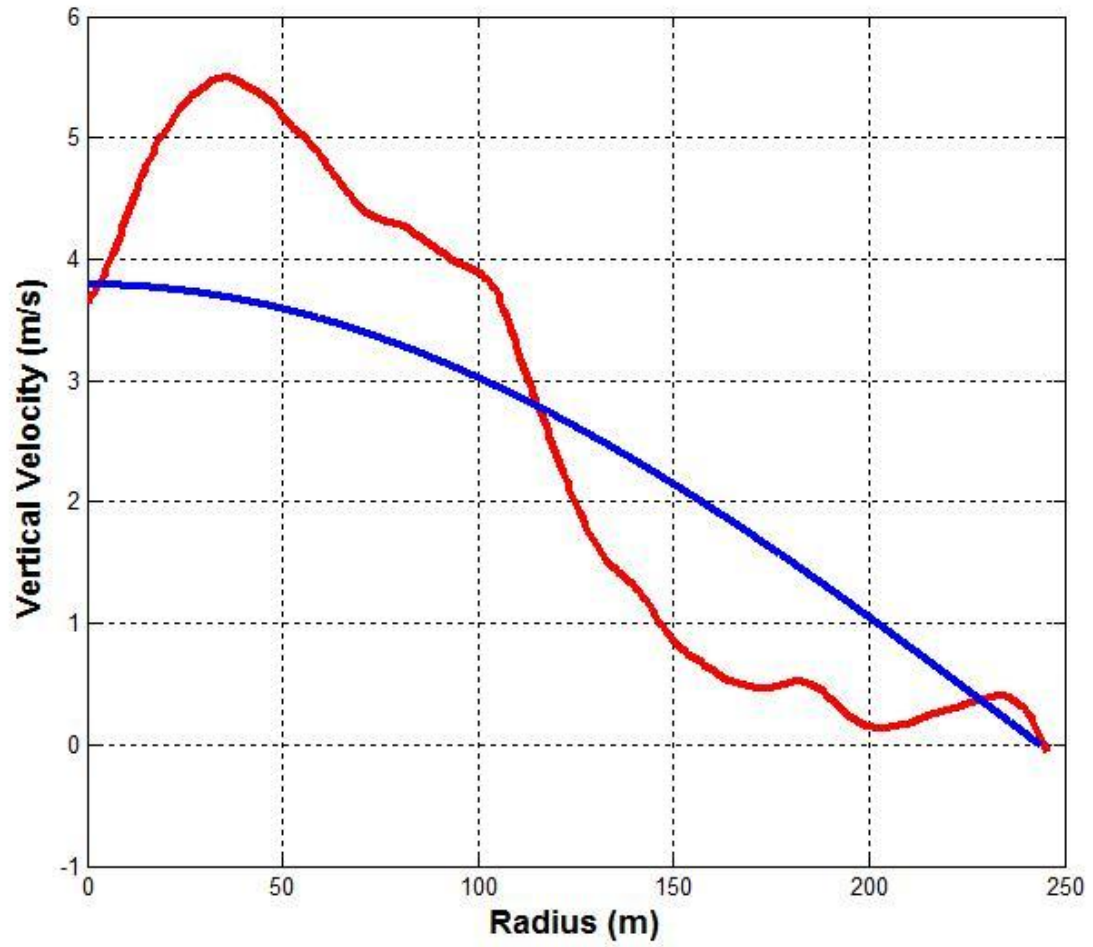


Figure 11: 22 Sep 1509, Model Comparison (Model in Blue), $z^* = .34$, $T_s = 300.65$, $\overline{\dot{w}\theta_v} = 0.385$, Actual Thermal Data from 20 Sep 1716 in Red

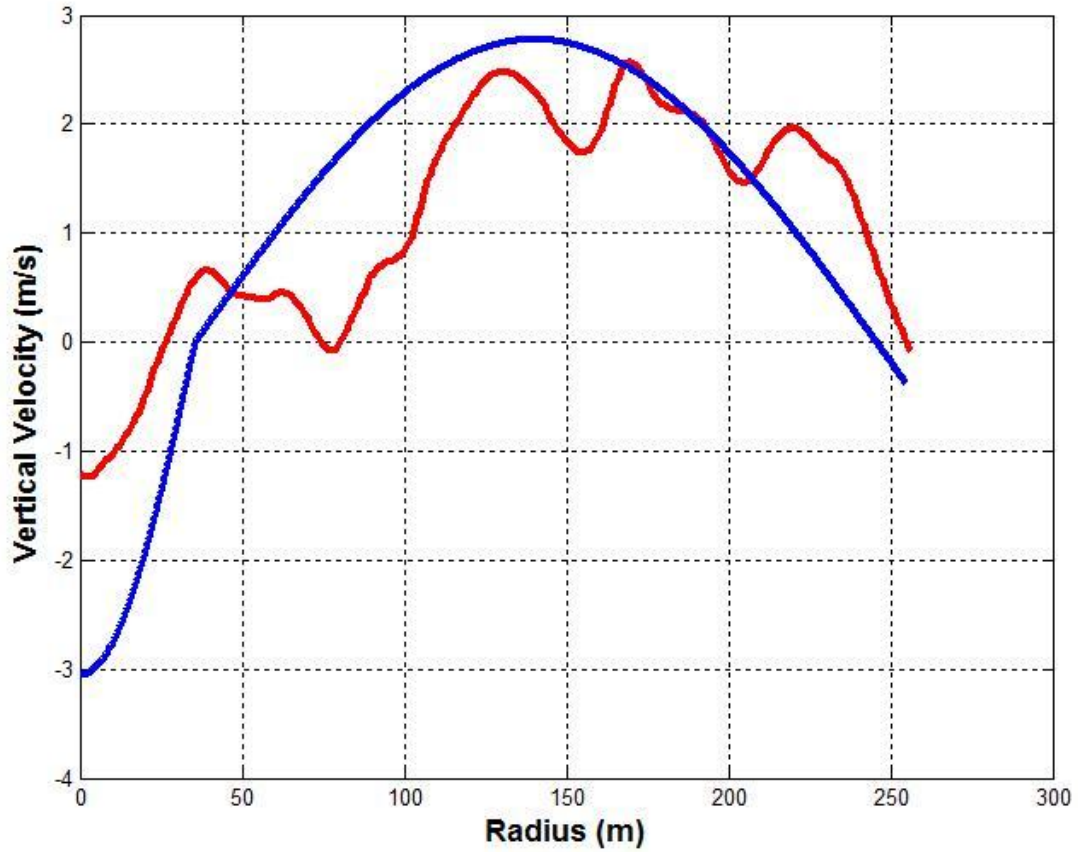


Figure 12: 22 Sep 1452, Model Comparison (Model in Blue), $z^* = .54$, $T_s = 300.65$, $\overline{\dot{w}\theta_v} = 0.473$, Actual Thermal Data from 20 Sep 1716 in Red

VITA

Christopher E. Childress is a graduate of the University of Missouri-Columbia with a B.S. in Physics. Upon graduation, he joined the United States Air Force where he became a pilot. After serving in various positions flying cargo aircraft, he was selected to attend the United States Air Force Test Pilot School. Chris has served in various positions as an Experimental Test Pilot for the USAF and continues in that capacity today. If you wish to obtain the raw data obtained from the glider during this experiment, he can be reached at Christopher.childress@us.af.mil.