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ENTROPY BASED SHAPE DISCRIMNATION

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ABSTRACT

Object recognition based on morphological properties is a routine task that a human being performs with intelligence. In an attempt to integrate such capability in an automatic vision system, this work presents a new approach to shape understanding. This approach is based on using the shape characteristic function, which reduces the task of 2-D shape analysis to 1-D function analysis. Moreover, shape normalization with respect to location, scale, and orientation becomes a reasonable task. Based on the normalized shape, entropy, being the self information in a probability density, is shown to measure the degree of circularity in any shape. Other shape properties are evaluated using the mutual information concept which measures the degree of similarity between different shapes. Shape recognition and classification is achieved once prototypes are selected. In the process of shape discrimination, the adjustment of the size of samples is achieved using a nearest neighbor substitution algorithm in order to maximize the information content in a shape. An introduction for analyzing 3-D objects based on their 2-D cross-sections is given for future work. Finally, different types of applications are considered for testing the descriptor's performance in analyzing global as well as local variations.

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CHAPTER 1

INTRODUCTION

In general, the capability of a vision system to describe a scene through pre-specified measures such as texture, shape descriptors, ect., is essential in scene understanding. When describing a scene, two options are usually encountered. The first is referred to as internal because only the interior of the object, in a given scene, is considered in the analysis. This option is selected when the main interest is the extraction of regional properties from the object considered. Texture analysis is an example of such a situation. The second option, often referred to as an external study, is considered when the object boundary is the focus of study. Another label for the above approach is shape analysis.

The wide range of applications which have used shape analysis techniques include motion detection, target tracking, recognition and classification of objects, etc. For instance, time varying scenes are common examples of boundary change studies. The nature of some types of motions can be determined by the variations measured from one frame to another [15]. Similar applications exist in the area of biomedicine where the morphological changes of organs can be used to determine critical behavior. Understanding heart motions using approaches using shape variations is one example [26]. In the area of microbiology, isolating

specific microstructures in a large variety of objects can be a tedious and difficult task if approached purely through region internal characteristics. However, some shape analysis techniques have shown promising results in the characterization of biological shapes [10]. In the field of microscopy, several shape descriptors have been used to classify phase particles in metallographic images [7]. The topic of shape study is also identified with the area of industrial inspection and quality control. For instance, it is often desirable to build an automatic vision system which is capable of detecting shape defects in mechanical parts.

Even though the topic of shape analysis has long been one of special interest in image processing, the problem of shape description is still unsolved. In general, much of the work done in this area dealt with global characteristics of shape, and most of the approaches emphasized such classical properties as compactness, elongation, symmetry, etc. Because such measures did not require complete shape recovery, the shape measures were not reliable when considering complicated shapes [6,34]; moreover, the shape descriptor was not capable of providing additional information about the shape. Most importantly, some limitations were, in general, imposed on the type of shape that should be considered, in addition to the small range of properties that could be identified. Therefore, a shape descriptor that is capable of discriminating among several basic groups of closed contours, as well as measuring any desirable characteristic, is desired for a countless number of real-world applications.

Developing a shape descriptor is usually a two-stage process, a representation stage and a description stage. Since, in most situations, it is difficult to operate

directly on the actual contour, a data transformation process is required. At this stage, some data reduction is preferred for simpler analysis. In the second step, the actual shape descriptor is established based on the newly represented contour.

This work presents a different approach to shape analysis. A shape descriptor has been developed. It is capable of classifying objects in different groups and tracking minor variations among objects within the same group. Once a prototype is selected, the degree of its similarity to another shape can be evaluated using the same descriptor. A mathematical analysis is established, then followed by implementation. Sample results for different applications are given for two types of shape models: analytically and arbitrarily defined models, (see p.69).

The body of this thesis is composed of three main parts: the first part of Chapter 2 includes a review for previous shape analysis techniques. The second part of this chapter discusses the importance of a representation scheme when a shape descriptor is being developed. The final part of Chapter 2 provides an analysis and an evaluation for a shape signature approach. In the following part of the thesis, Chapter 3, theoretical background for a shape descriptor that is based on information theory, is discussed. A general formulation of shape discrimination is presented. The final part of this work is Chapter 4, where the implementation and results are presented. Some examples of shape properties are illustrated including two categories: geometrical properties in regular and irregular shapes, and non-geometrical properties where possible situations can apply in different real world situations.

CHAPTER 2

REPRESENTATION AND DESCRIPTION OF TWO DIMENSIONAL CONTOURS

One of the early tasks of automatic image analysis is to simplify a picture through the elimination of unnecessary data. In most cases, it is advantageous to use schemes that reduce the data into representations which are useful in the computation of a particular scene descriptor. In shape analysis, morphological features are of interest and object boundaries are of great importance, thus, the objective is to construct a descriptor that best identifies them those features and boundaries. Therefore, the selection of a representation scheme for the boundaries greatly determines the performance of the boundary descriptor. Because the information contained in the object boundary is important in the description process, selecting the most suitable representation usually depends on two factors: the computational expenses and the reliability.

This chapter gives an overview of the algorithms used in shape analysis. It also discusses different shape representation schemes and their significance regarding the capabilities of the shape descriptor. The last part of this chapter focuses on a specific representation technique and discusses its features and its limitations.

2.1 Shape: Definition and restrictions

Shape is a general term that includes all forms of information pertaining to a closed contour. The concept of shape is usually associated with three main attributes: scale, location, and rotation invariance. That is, two objects have the same shape if they coincide exactly after properly adjusting the location, the size, and the orientation of either object. Therefore, if a descriptor is going to be used to represent the shape of an object, it has to be invariant with respect to position, size, and orientation of such an object. The shape descriptor should also be reproducible (i.e., when translated, magnified, and rotated, the descriptor of the resulting object should remain unchanged). Two objects with different shapes should produce different descriptions. Developing a descriptor that differentiates between shapes requires full shape information to be preserved in order to guarantee the uniqueness of the description.

2.2 2-D Shape analysis: A review

Because shape recognition has been the subject of research for many years, the number of related publications is large. The range of applications includes recognition of characters [11,4,27], monitoring boundary changes as a function of time [15,2,1,26], identification and inspection of parts, and others, e.g., [3,12,10,7]. It is not surprising to encounter difficulty in categorizing the countless number of methods proposed in the literature. In his latest review, Pavlidis listed a variety

of algorithms used for shape analysis [9], some of which he classified to whether the algorithm operated on the boundary or the region of the object.

In an area such as shape analysis, it is often difficult to label every approach under a certain category; there are many methods that combine various approaches. The literature surveyed here can be primarily classified into spatial domain techniques and transform domain techniques, depending on whether local measurements or global measurements are extracted in the analysis of the shape. The spatial domain techniques refer to those dealing with the physical domain of the shape, (i.e., the operations performed on the shape that are purely spatial, such as distance between points, their relative order, etc.). The transform domain techniques, however, usually involve structural transformation of the shape, such as an evaluation of the mean or the variance from shape information. Each has its advantages and limitations as discussed in the following.

2.2.1 Transform domain techniques

A transform domain technique is based on transforming a given shape into a set of parameters describing the shape. The Fourier descriptors have long been used for the purpose of either reconstructing or matching shapes [11,9,15,28]. The descriptors are defined as the Fourier series of contour points; they have more than one form, but the most common form is derived from the parametric representation of the curve in question. Because of the properties of Fourier series, normalizations with respect to location, scale, and rotation are easily achieved.

Like most transform domain techniques, the Fourier approach is supported by a well-established theory. However, the most difficult task in working with this approach lies in the lack of physical interpretations of high-order Fourier coefficients. When local shape information is of importance, selecting the appropriate number of terms becomes difficult. Theoretically, one can have an infinite number of terms; but, enormous number of computations are unavoidable in such cases.

The discrete Fourier transform has been used to identify airplanes [28], recognize characters [11], and track boundary variations in time [15]. A similar representation is suggested using elliptical Fourier descriptors [16], where a closed curve is treated as a set of ellipses of fixed axes and locations; this modification adds physical interpretation to the shape decomposition with the use of the Fourier domain method. In addition it speeds the analysis.

The method of moments is another transform domain technique that has been used for shape description [9,28,29]. Statistical moment functions were derived and used to characterize the process of varying shapes through the use of shape signature [5,28]. In principle, it is possible to generate the original shape from an infinite number of moments. In practice, however, regenerating the original shape from its moments requires a full understanding of high-order moments, which is not an easy task. (While the four first moments describe the mean, variance, skewness, and kurtosis of the shape, it is hard to explain the physical meaning of higher moments.) The main advantage of such approaches is the well-established theoretical foundations which made them popular. On the other

hand, the difficulty in describing local characteristics, which is contained in high order moments, is enormous.

Expansion of the Walsh series was proposed for the description and discrimination of polygonal curves, and tested on hand-written characters [27]. The Walsh descriptors are sensitive to rotation, so normalization is required. Even though the Walsh descriptors can be evaluated from any one-dimensional periodic mapping of the shape, they are best fitted for polygonal curves with few components. Compared to the Fourier descriptor, the Walsh method requires fewer components for the description, therefore, it is faster and easier to reconstruct the original curve.

Among other transform domain techniques, the Hough transform was used for recognizing simple shape structures like lines and curves [36,37,38]. The Hough transform has been extended to detecting analytical and arbitrary shapes [39,40].

2.2.2 Spatial domain techniques

The number of spatial domain methods exceed by far that of transform domain techniques. Depending on specific applications, different techniques have been considered in order to identify best the desired shape features.

The polygonal approximation of a 2-D curve is one of the basic methods for representing a shape and identifying its critical points for curvature behavior and other shape features analysis [31,28]. The chain code is an advanced form of representing a closed contour; it originated with Freedman and was later adopted by several authors for shape description and discrimination. Different forms of

chain codes have been used in addition to the four and eight directional codes; subtended angle chain codes consist of vectors of variable length; when generated, the result is plotted as a graph of the code value versus the arc length and used for analyzing dynamic scenes [1]. Based on the chain code, the order of the shape number is derived for basic shape description. It was shown [13] that the latter quantity is invariant with respect to shape location, scale, and rotation and the order of the shape was extended to arithmetic shape operation and mathematical analysis for shapes and surfaces [14].

The chord length distribution is another spatial technique that was suggested for shape matching; it was tested on mechanical tools and geographical maps in the presence of different types of distortion [12]. The distribution is generated by joining all the line segments and evaluating the distance between each pair of boundary points, then, normalized by the longest chord, the result is treated as a distribution. A test statistic is used as the dissimilarity measure between the shapes. This approach has the advantage of being invariant under location, size, and rotation. In addition to this advantage, this method avoids the problem of feature selection. It was observed that the chord length method works well in the presence of noise and distortion, but it is computationally expensive and does not preserve all shape information. However, compared to other methods, like Fourier descriptors and moments, it was found that the chord length distribution performs reasonably well [12].

A natural way to describe a contour is to identify the critical points that control it. Critical points are defined differently, but they generally describe

curvature changes (magnitude or sign) to determine local features such as corners. The use of critical points has many advantages: (i) it involves a tremendous data reduction, (ii) it exercises control over local information in shapes, therefore, it can be useful in analyzing shape variations in a time sequence, (iii) it allows critical points to be inputted into a syntactical recognition system. Critical points can be used for curve fitting [31], spline approximation of contours [4], etc. A comparison between control points and Fourier descriptors, as discussed in Paglieroni's work, suggests the use of control points when the local shape information is of importance [30,29]. The determination of critical points on planar shapes of different degrees of detail has been discussed [32,3]. Because the critical points depend on shape changes (e.g., location, and rotation), normalizing the shape representation is usually necessary. One solution to such a problem was proposed using a set of coordinate axes that are shape dependent [32].

With any reasonable shape descriptor, however, there are unavoidable problems, some of which are, complexity, efficiency, and reliability. The complexity of the method can be a disadvantage even though it may not imply slow processing. Almost every approach in shape analysis has some limitation as to the type of shape that would yield the best results. The range of shape features to be evaluated is usually restricted. The following section addresses the question of preserving shape information.

2.3 Preserving shape information

Preserving shape information is one among several criteria which evaluate the effectiveness of an approach. A reasonably good method usually has the capabilities of recovering or storing all shape information. It is usually preferrable for a given shape analysis method to have one or more parameters that control the amount of information transferred. The amount of information is, in general, directly proportional to the processing time. In practice, most approaches are information non-preservative.

The selection of a representation scheme used in transforming the desired shape features is strongly related to precision and reliability in the description stage. A general approach to 2-D and 3-D shape representations and the considerations to be accounted for have been discussed by several researchers, e.g., [23,22].

The basic problem in shape representation has always been and remains, how 'best' to preserve the shape through the transformation process and still meet the following criteria:

1. The shape descriptor should contain enough data to be able to reconstruct an image which appears similar to the original when viewed by human observers.

2. The shape descriptor should contain only information which is invariant under all perturbations of the image, leaving the shape looking generally

the same.

With the wide range of applications involving the study of shapes, adopted schemes of representation range from simple to complicated [22]. Most of the methods used in representing a two-dimensional contour fall in one of two categories: information preservative or information nonpreservative. Information nonpreservative methods lack the ability to reconstruct the original shape after the process of data transformation. For instance, a shape can be represented by a basic rectangle which is defined by the minor and major axes of the shape. The number of different shapes that can be represented by this rectangle is uncountable because shape details are neglected. Finding the skeleton of an object is also a technique that completely ignores the information pertaining to the shape and emphasizes only its basic structure. Shape discrimination based solely on global properties among objects can be achieved in most cases. The major limitation with information non-preservative methods is the fact that most of the local information is lost and only the basic skeleton is preserved which does not make it attractive, especially when shape details are of significance.

2.3.1 Shape signature

It is easier to analyze a two-dimensional shape when it is viewed in a different domain. For instance, it is easier to study an ellipse using cylindrical coordinates rather than Cartesian coordinates. In general, finding an analytical expression for a given shape is only wishful thinking because most objects in nature are not easily analytically defined. Nevertheless, transforming a two-dimensional contour

into a one-dimensional function can still be of help. A common term used for such a process is called a signature. One way of generating a signature is to sample the contour at a fixed angle in a given direction and plot the distance from the centroid (x_0, y_0) to the boundary point as shown in Figure 2.1,

$$f(\theta) = \sqrt{(x_s - x_0)^2 + (y_s - y_0)^2},$$

where, (x_s, y_s) is a sampled boundary point, and (x_0, y_0) are the centroid coordinates given by:

$$x_0 = \frac{1}{N}\sum_{i=1}^{N} x_i, \quad \text{and} \quad y_0 = \frac{1}{N}\sum_{i=1}^{N} y_i,$$

where N is the total number of points in the boundary.

The sequence of points $f(\theta)$ has a fixed length M determined by the sampling rate (N and M are in general different). The smaller the angle θ is, the more details are revealed through the signature, and, therefore, the larger M is. The shape may not be preserved if θ is not small enough. Therefore, different shapes may have identical representations. The problem of a multi-valued function is inevitable if the contour is not simple (i.e., it is self intersecting).

2.4 Characteristic function

2.4.1 Principle

Using a slightly different scheme for generating the signature, a more accurate representation of a shape can be obtained. When every point on the shape boundary is considered, the resulting signature is, in general, different from the

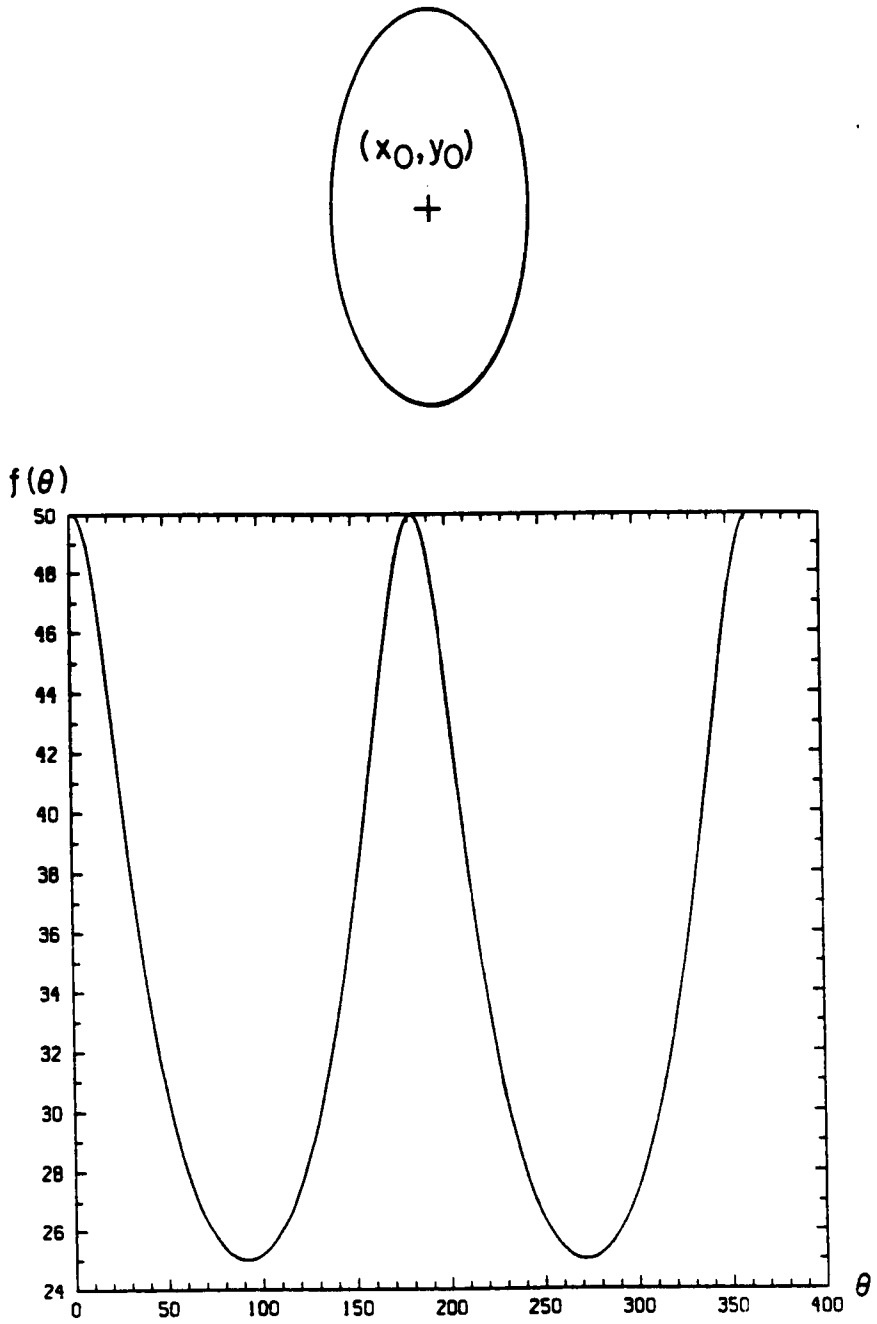


Figure 2.1: Example of contour signature: (a) an ellipse, (b) its signature.

signature obtained for the same shape through the process of equiangular sampling. When the contour is closed, which is the case for shapes, the sequence obtained is periodic. This type of signature is referred to as the characteristic function of the contour or shape. The two-dimensional shape is described by an ordered sequence obtained as follows: starting at a selected point, the coordinates of the N contour pixels of a digitized shape are described by an ordered set $\{(x_i, y_i), i = 1, 2, ..., N\}$. The number of samples, N, depends on the size of the contour and the resolution used in the digitization process. Then, the Euclidean distance $f(i), i = 1, 2, ..., N$, between the centroid (x_0, y_0) and the point (x_i, y_i) form an ordered one-dimensional representation of the contour. Since the choice of the starting pixel can be arbitrary, different choices shift the distance function $f(i)$.

The sequential representation is obtained using an algorithm that tracks the contour in a clockwise direction starting at a selected point on the boundary. Once the object boundary is tracked, the distances between the centroid and the pixels traced are evaluated. Examples of contour sequences of several shapes are shown in Figure 2.2.

2.4.2 Advantages

Because the shape is traced in one direction, not only the relative distances from every point to the centroid are recorded, but the order among the points is preserved. As a result, most of the information contained in a given shape is contained in its characteristic function. Some of the advantages of the characteristic

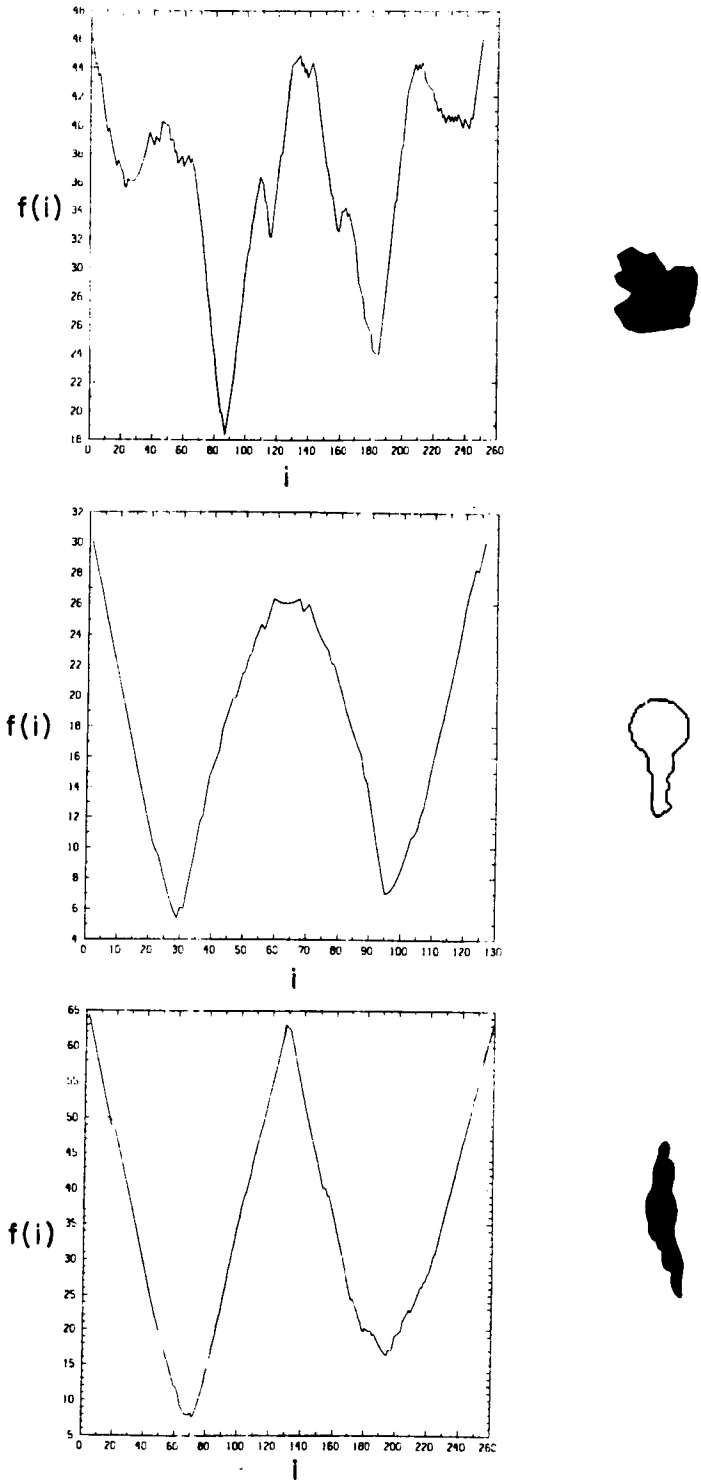


Figure 2.2: Examples of characteristic functions for different shapes.

function are as follows:

1. The characteristic is a natural form of shape representation; it is an unfolding of the boundary around its centroid.

2. The problem of shape analysis reduces to 1-D function analysis; therefore, shape discrimination becomes an easier task. The well established theory of 1-D functions makes any feature extraction reliable, efficient, and simple.

3. The processing time is reduced when the task of analyzing is reduced from two-dimensional array to a one-dimensional array.

4. No extra processing is required for shapes having spiral or concave contours; contours with holes can also be considered.

The uniqueness of this representation scheme implies that shapes of the same group have 'similar' characteristic functions. As a result, the degree of similarity between shapes can be determined from the degree of similarity between measures extracted from these one-dimensional functions.

2.4.3 Location, scale, and rotation normalization

Once the characteristic function is chosen to represent a shape, the next task is to normalize the function with respect to shape changes, such as changes in location, scale, and orientation. Any feature extracted from the normalized function

is normalized against the same shape changes as a result. The choice of a representation is, therefore, often related to the ease of normalizing it. Normalizing the characteristic function with respect to the above three operations is done in the following:

Location: Any feature that can be extracted from the characteristic function is invariant with respect to the location of the object. This is already established since $f(i)$ depends only on the relative position between the centroid of the object and the pixel considered; therefore,

$$f(i) = f_t(i), i = 1, 2, ..., N,$$

where $f_t(i)$ is the translated shape.

Scale: A shape descriptor should have the same value regardless if the shape size has changed or not, provided that the image resolution is kept unchanged when the object is scaled. Moreover, the scaling must be uniform in all directions (x and y in our case). Suppose a given shape has been scaled by a factor α, the resulting characteristic function is a scaled version of the original function. The size of boundary samples as well as the function values are magnified (reduced) by α,

$$f_s(i) = \alpha f(i), \quad N_s = \alpha N.$$

Normalization against the scaling effect can be done, in theory, through a second scaling process where the new scaling factor is $1/\alpha$. In this work, scaling the magnitude is achieved by normalizing the area so that the area becomes unity. This is accomplished in two steps: first, the original area is approximated using

Simpson's rule; then, each $f(i)$ is scaled by the area resulting in a unit area function (approximately). The number of samples, N, is reduced to the data size of the prototype at hand using the technique of nearest neighbor substitution (see Chapter 3 for details). Once the number of points N is set, the original shape and its scaled version have the identical characteristic functions. Figure 2.3 shows an example of an enlarged shape, and its normalized function is compared with the function of the original shape.

Rotation: Because the boundary is always traced in one direction, rotating a shape is analogous to a circular shift in the ordered sequence, and, therefore, a shift in the characteristic function,

$$f_r(i) = f(r + i),$$

where r corresponds to the degree of rotation from the original position.

In order to obtain the same function when the object is rotated, starting point normalization is needed. The selection of a starting point is critical. Different criteria can be imposed in order to guarantee the uniqueness of the starting point, for example, the closest point to the centroid which lies on the minor axis of the shape could be used as the starting point. Depending on specific types of shapes, different criteria can be imposed. In this work, the point with the largest distance from the centroid is chosen as the starting point. Therefore, the characteristic function is always shifted based on the absolute maximum point as the starting point in order to normalize rotational effects.

In principle, the above procedure yields perfect normalization (i.e., exact over-

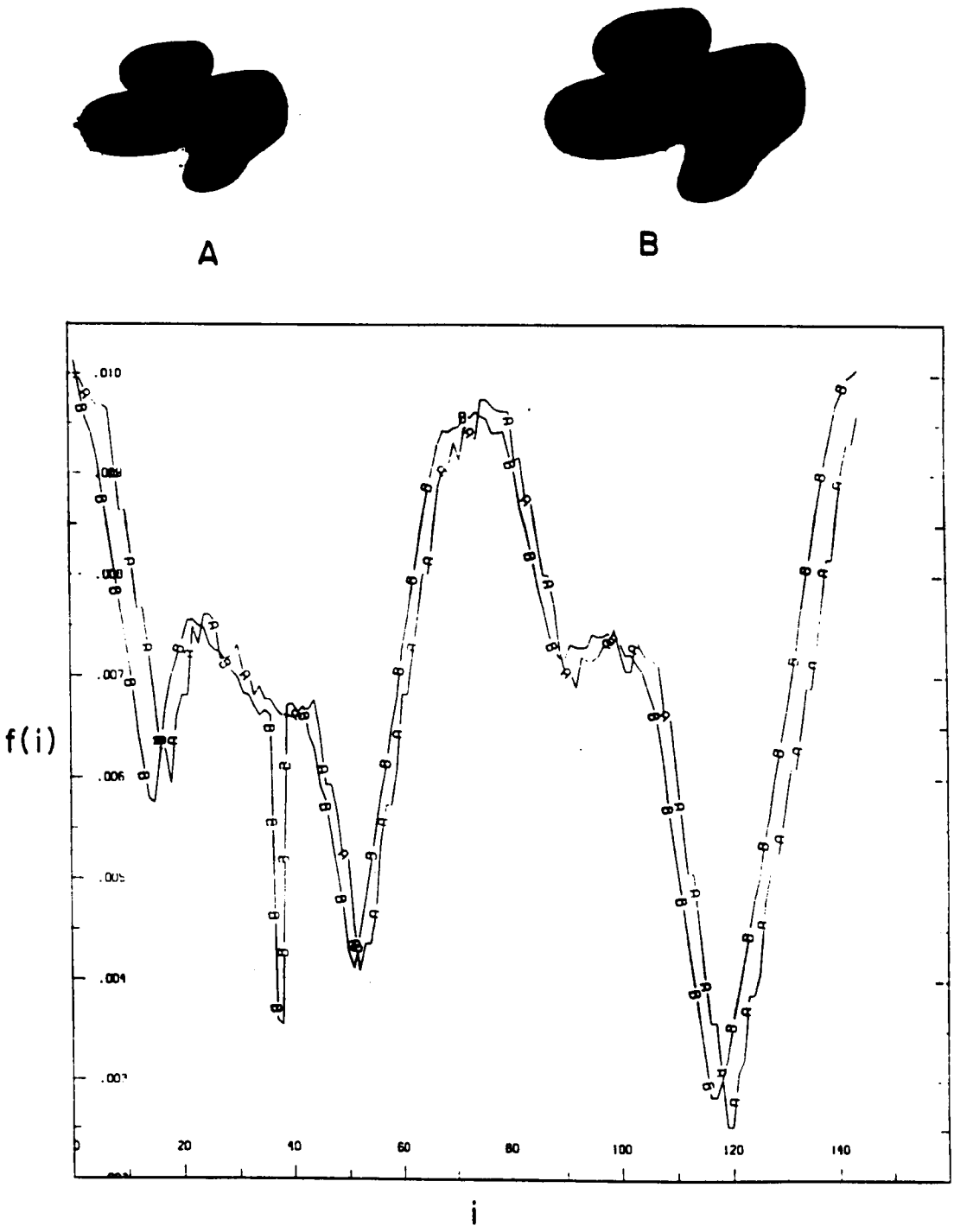


Figure 2.3: Scale normalization: Curve A corresponds to the original shape, curve B corresponds to the enlarged shape.

lap between the functions). However, limitations due to present technology are always sources of error. The digitization process, for example, yields imperfect normalization with respect to rotation. Depending on the orientation of the same object, different boundaries are obtained, and, therefore, different functions are generated for the same object when rotated. As a matter of fact, the number of boundary points may not be the same in such case. Figures 2.4 and 2.5 show typical results of starting point normalization. It is important to note the amount of error introduced by the digitization process. The limitation of any descriptor extracted from such a function to distinguish two similar shapes is unavoidable. These differences can be large enough to treat two orientations of the same object as two different looking objects. Discriminating between shapes based on their local variations and accounting for the possibility of rotation, can be a tedious and unreliable task. Therefore, based on this normalization procedure, there is a limit to the level of differences between shapes in order for any shape descriptor to detect it properly, as shown in Figure 2.6. Even though this type of error is unavoidable, it can be alleviated through the use of higher resolution; the error decreases as the resolution increases, but the processing time also increases.

The selection of the starting point as the absolute maximum in the characteristic function can lead to confusing results when that point is not stable in the shapes considered. This type of situation often appears when treating a time sequence of similar looking shapes where local variations are the main focus. Figure 2.7 shows plots of the characteristic functions after normalizing each function using the maximum distance point as the starting point. Due to the multiplic-

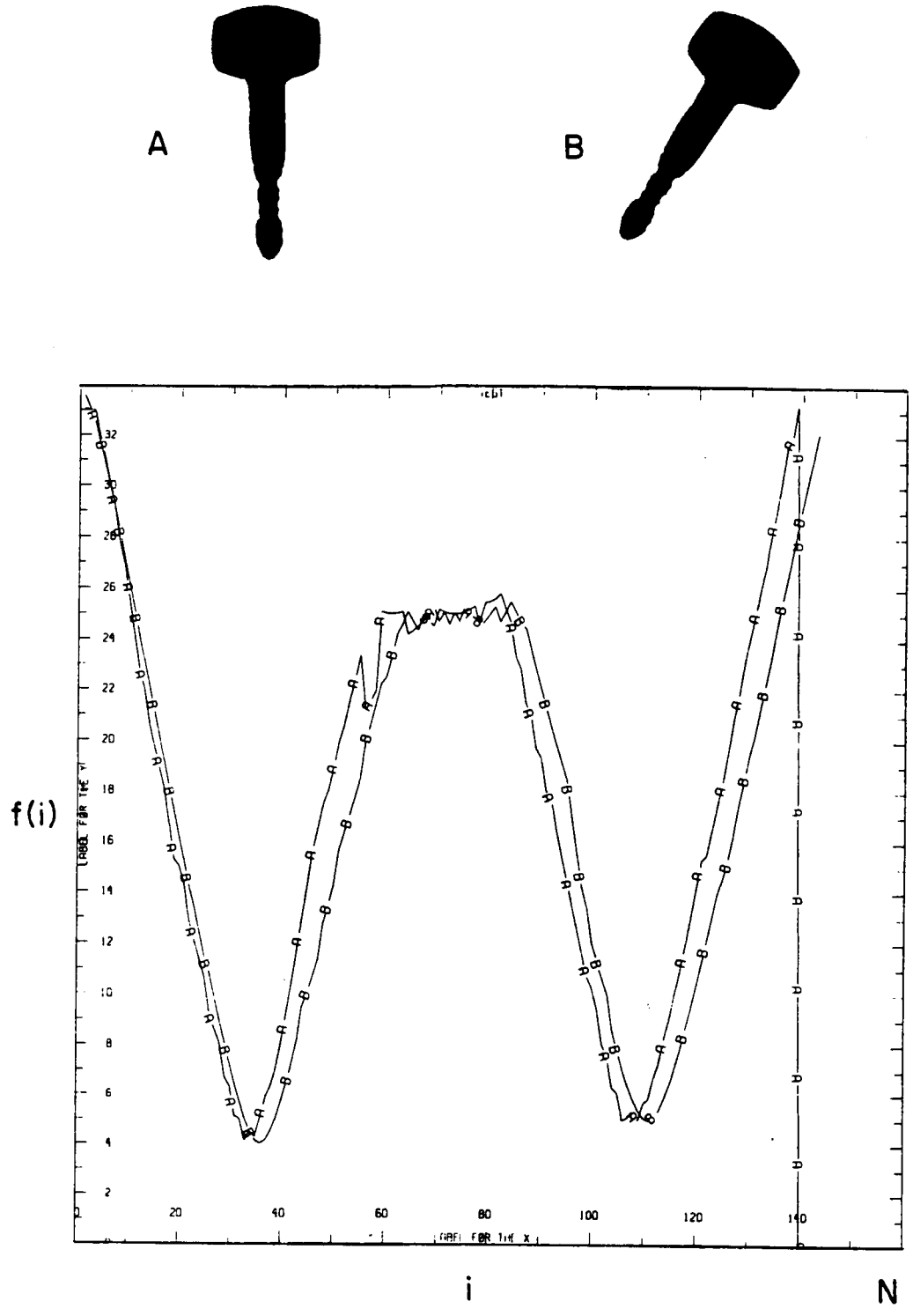


Figure 2.4: Rotation normalization: example of normalizing the characteristic function of a rotated object.

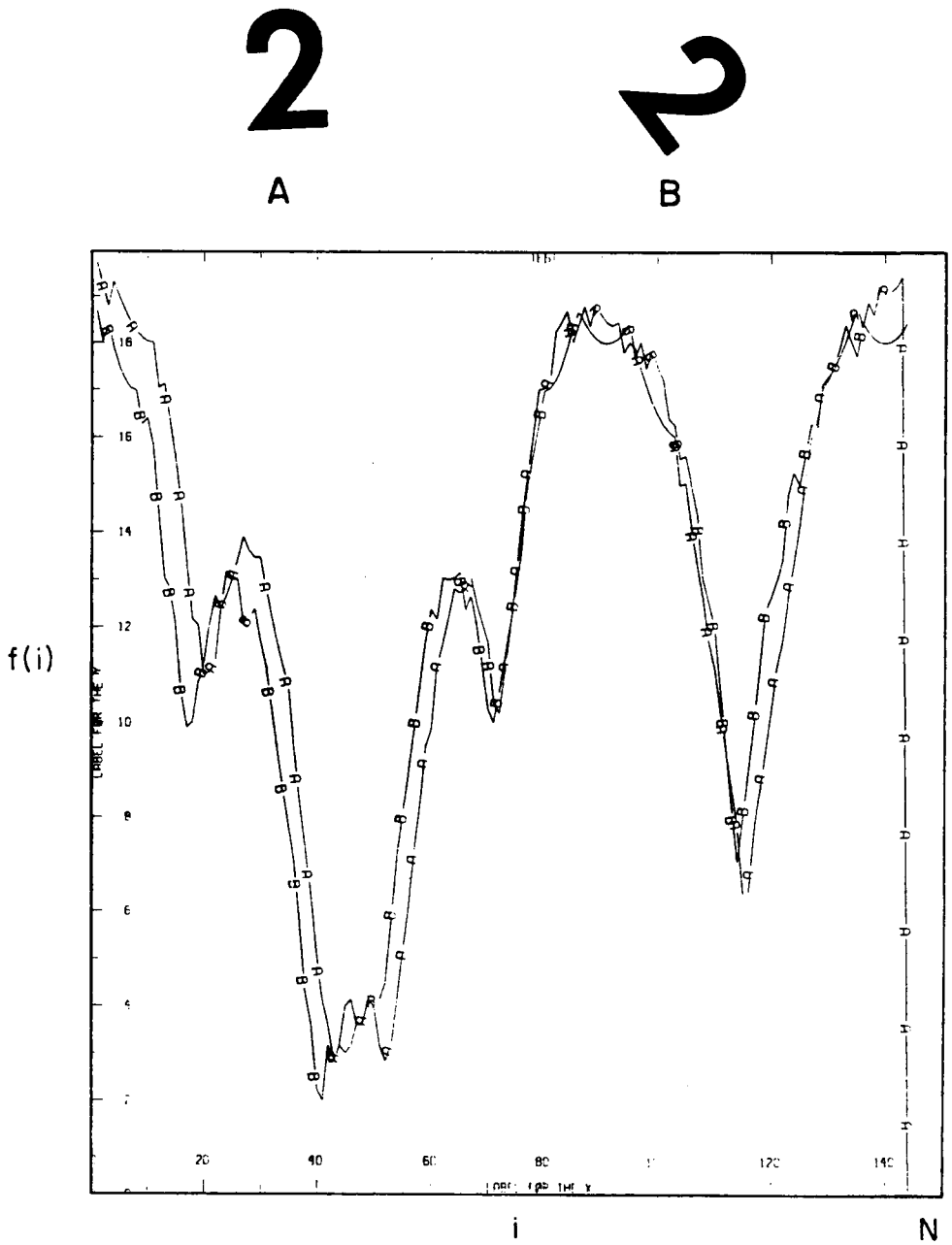


Figure 2.5: Example of normalizing the characteristic function of rotated object.

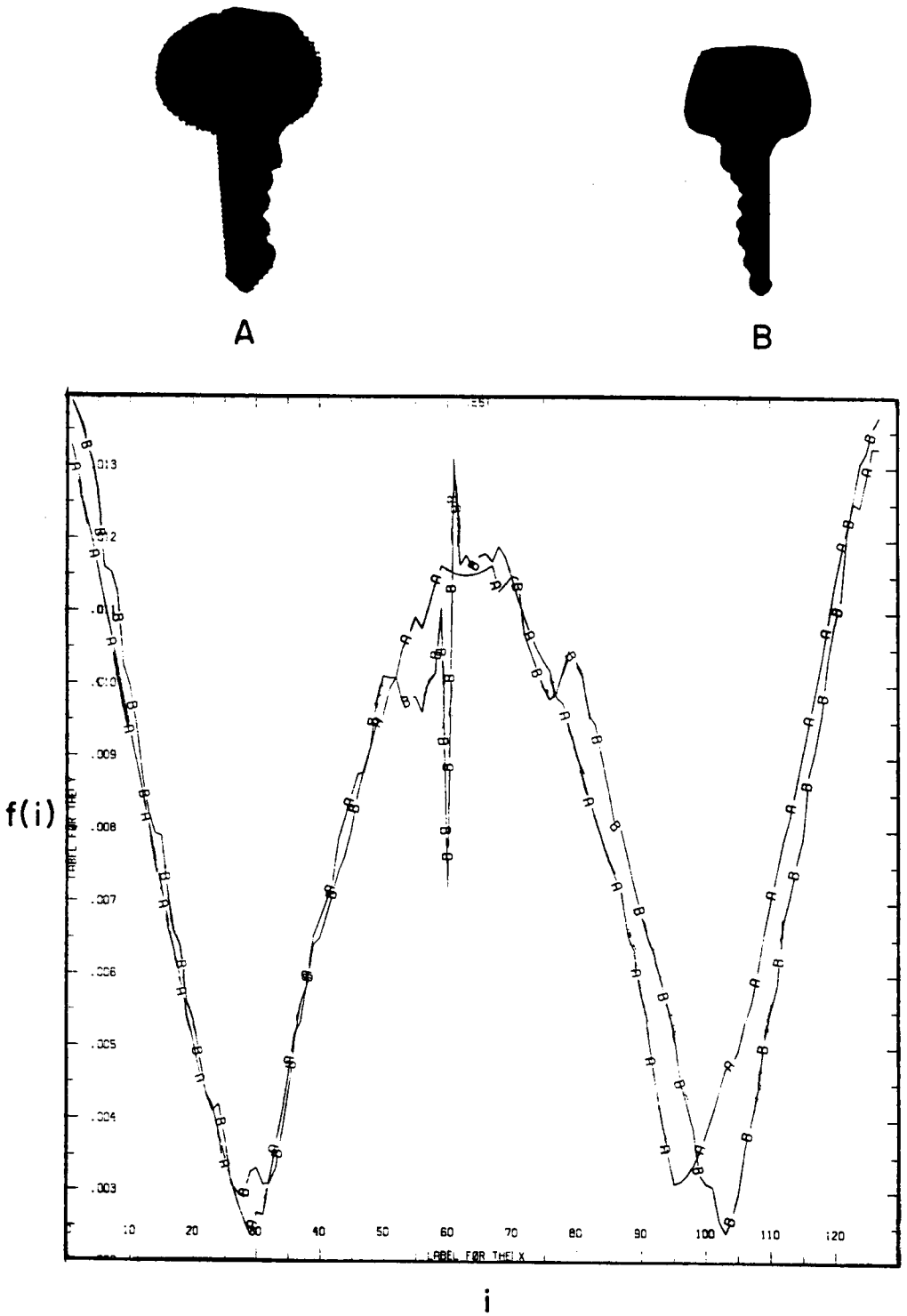


Figure 2.6: The effect of rotational error in the case of slightly different objects (see Figure 2.5 for comparison).

ity and instability of the starting point, those functions do not properly reflect the differences among the shapes. Overcoming this problem can be achieved by avoiding the possibility of rotation, which is usually not needed because all of the frames are recorded from the same point of view. Figure 2.8 illustrates one way of dealing with this situation, the fixed starting point is taken as the pixel with the smallest x coordinate in the image. When compared, the characteristic functions shown in Figure 2.8 properly reflect the similarities between the shapes; the curve structure is the same for all shapes but local variations are apparent.

In conclusion, the descriptor's reliability greatly depends on the stability of the characteristic function and its performance during the normalization process. Based on the representation scheme proposed in Section 2.3 and on the approach dealing with shape perturbations described in this section, a new shape descriptor is proposed in the next chapter.

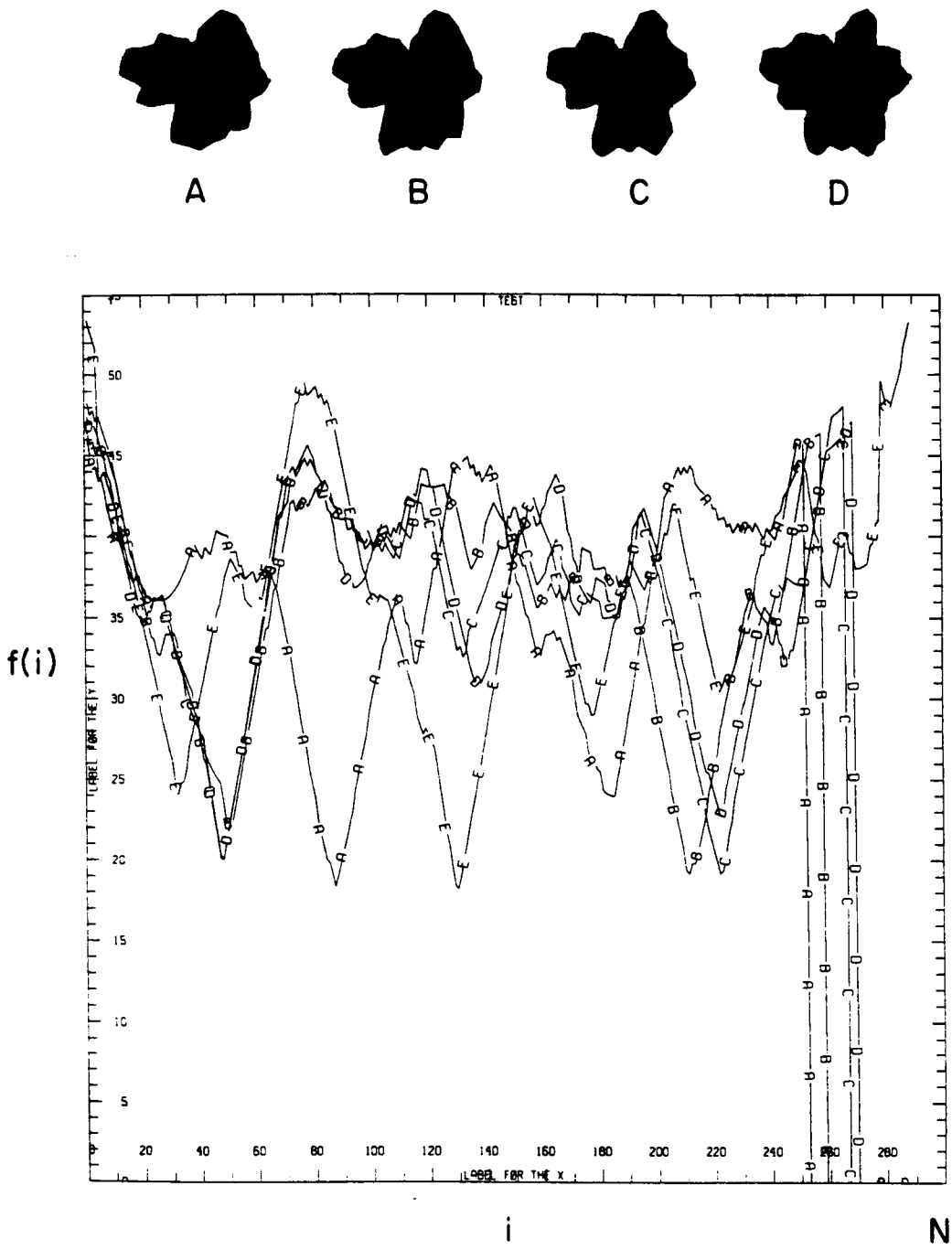


Figure 2.7: Starting point instability resulting in rotationally unnormalized characteristic functions.

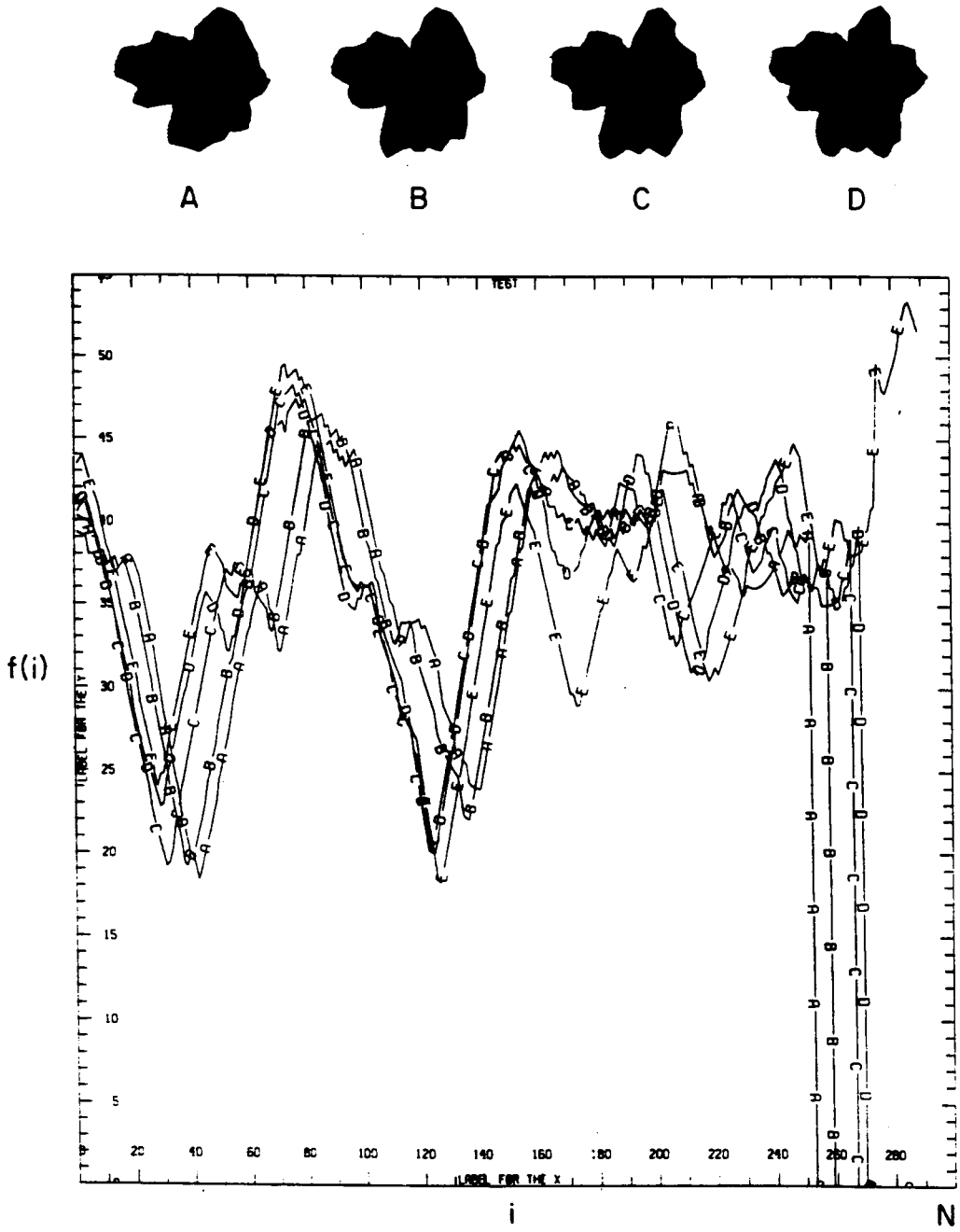


Figure 2.8: Characteristic functions for objects taken from a single orientation.

CHAPTER 3

ENTROPY AS A SHAPE DESCRIPTOR

In more than one situation, considering the probability theory is promising when solving various engineering problems. Even though probabilistic experiments are usually related to simple acts like tossing dice or drawing cards, the variety of situations where probabilistic models can be applied is large. Probabilistic approaches are realistic, conservative, and, therefore, reliable in solving problems.

In this work, the analogy between the shape signature and probability function is underlined. For instance, circularity in shapes is identified with entropy in random experiments and different shape realizations are linked with stochastic processes. Results that have been established in information theory are then adopted for shape discrimination.

Based on the representation scheme developed in the previous chapter, Section 3.1 focuses on the derivation of a measure that is optimum for a circle. Section 3.2 introduces the concept of information measure and defines entropy. Measuring the degree of dependence between two densities or the closeness between two shapes is defined in Section 3.3. A general formulation of shape comparison, using conditional entropy, is given in Section 3.4, along with an introduction to

3-D shape analysis.

3.1 Circularity measure in 1-D functions

In engineering applications, an evaluation of the degree of circularity of an object is often required because the circle represents such a significant shape and has unique properties; for instance, it has the highest compactness and bending energy of any shape [17,34]. The aim of this section is to develop a descriptor from the normalized characteristic function that can compare a given shape to a circle. In order to determine the degree of circularity, one has to identify the unique properties of a circle. Using the representation method developed in the previous chapter, it is interesting to note the special characteristic function of the circle: for a circle of N points, the function $f(i)$ is uniform and has the value of $1/N$ (see Figure 3.1). The objective, then, is to develop a function, H, that is maximized for this special function. First, a functional expression C is developed to 'describe' any point f_i of the characteristic function. In order to achieve this goal, four conditions are imposed on C:

1. For a degenerate characteristic function, (i.e., N=1), C=0.

2. C is a strictly decreasing function of f, that is,

$$\text{if } f_i < f_j, \quad \text{then} \quad C(f_i) > C(f_j),$$

where $i, j = 1, 2, .., N$.

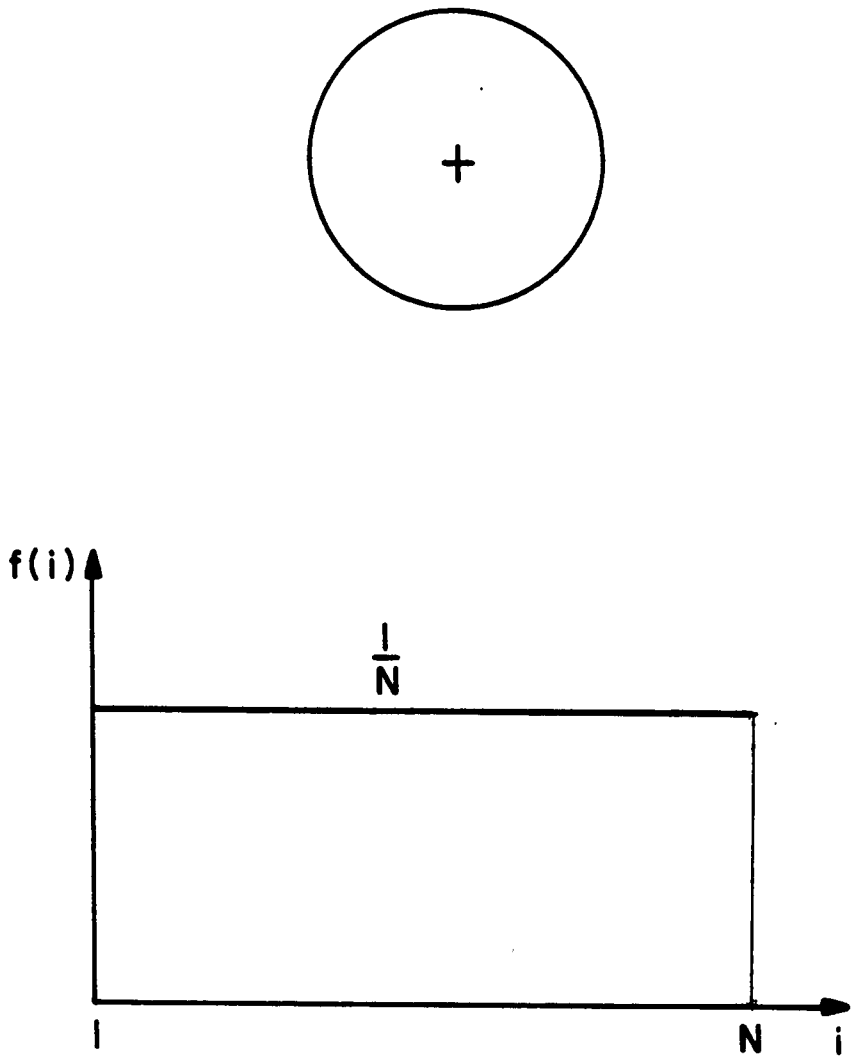


Figure 3.1: Characteristic function for a circle of N points.

3. C is additive, that is, $C(f_i)$ is unaffected by $C(f_j)$ for any distinct $i, j = 1, 2, ..., N$. In other words, the description obtained from both f_i and f_j is the sum of their individual descriptions. This implies:

$$C(f_i, f_j) = C(f_i) + C(f_j).$$

The above statement is also recognized as the 'independence' requirement among all N points of a given shape.

4. C is a continous function of f (i.e., a small variation in f must result in a small variation of C).

It is important to note that in addition to the above four conditions, all characteristic functions have the following property:

$$\sum_{i=1}^{N} f_i = 1, \text{ and } 0 \leq f_i \leq 1, \ 1 \leq i \leq N.$$

In order for C to satisfy all the above stated conditions, C must have the form of [35]

$$C = -K \log f_i,$$

where K is a positive constant, for simplicity, let $K = 1$.

Now that any point of the characteristic function can be described by C, our goal is to use C for a description of the entire function $f_i, i = 1, ...N$. Taking the expected value:

$$E[C] = -\sum_{i=1}^{N} f_i \log f_i.$$

In order to understand the behavior of $E[C]$, a simple example, $N = 2$, is given. In this case,

$$H = E[C] = -\sum_{i=1}^{N} f_i \log f_i = -f_1 \log f_1 - f_2 \log f_2.$$

From the normalization process, $\sum f_i = 1$; therefore,

$$H = -f_1 \log f_1 - (1 - f_1)\log(1 - f_1).$$

Differentiating H with respect to f_1 and solving yields

$$f_1 = f_2 = 1/2.$$

Figure 3.2 illustrates the behaviour of H as a function of f_i for this particular case; H_{max} occurs at 1/2. For a shape with N points, proving that H has its maximum only for a circle described by N points (i.e., when all the f_i are equal) requires the use of the Lagrange multiplier since the function H is subject to the constraint $\sum f_i = 1$. Defining $G(f_i) = \sum f_i - 1 = 0$, then optimizing H using the Lagrange multiplier technique yields the following two differential equations:

$$\nabla H = \lambda \nabla G(f_i)$$

$$G(f_i) = 0.$$

Solving the above two equations is equivalent to solving the following $(N + 1)$ system of differential equations:

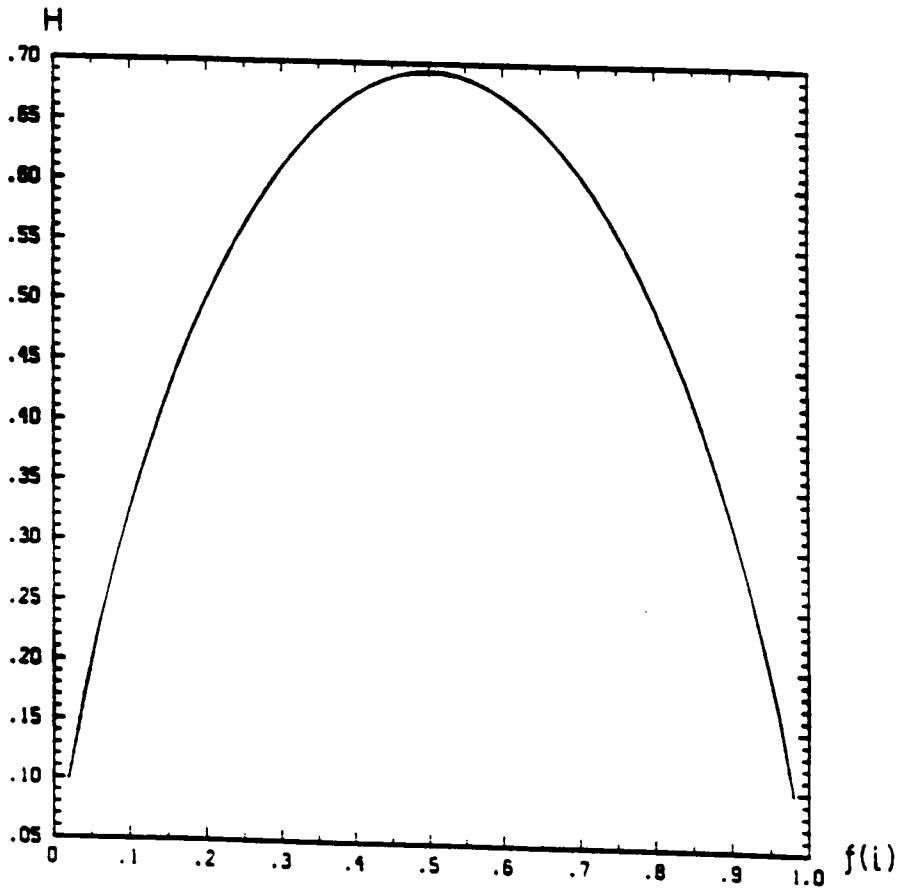


Figure 3.2: Plot of H versus f for N=2.

$$\begin{aligned}
\frac{\partial}{\partial f_1} H &= \lambda \frac{\partial G(f_1)}{\partial f_1} \\
\frac{\partial}{\partial f_2} H &= \lambda \frac{\partial G(f_2)}{\partial f_2} \\
\vdots & \quad \vdots \\
\frac{\partial}{\partial f_N} H &= \lambda \frac{\partial G(f_N)}{\partial f_N}
\end{aligned}$$

$$f_1 + f_2 + ... + f_N = 1.$$

Substituting for H and differentiating with respect to each f_i, the above $(N+1)$ equations reduce to:

$$\begin{aligned}
-\log f_1 - 1 &= \lambda \\
-\log f_2 - 1 &= \lambda \\
\vdots & \quad \vdots \\
-\log f_N - 1 &= \lambda
\end{aligned}$$

$$f_1 + f_2 + ... + f_N = 1.$$

The first N equations are of the same form, therefore, if a solution exists, all the f_i's are identical. Solving for λ,

$$\lambda = -\frac{N+1}{N},$$

and substituting for λ, gives the desired result:

$$f_1 = f_2 = ... = f_N = 1/N.$$

In conclusion, H is maximum for the uniform function, which is the characteristic function for a circle of N points. Evaluating H for this special case gives

$$H_{max} = -\sum_{i=1}^{N} \frac{1}{N} \log \frac{1}{N} = \log N.$$

Figure 3.3(**a**) shows the plot of H_{max} as a function of N. The increase in the number of points, N, results in a logarithmic increase of the circularity measure H_{max}. This is a desired feature in any descriptor and can be stated as the following: the more one knows about the shape, the better the description becomes and therefore, the larger the descriptor gets.

It is interesting to realize that the situation discussed above has been treated in the context of probability and information theory. What was defined as circularity in a shape can be identified as entropy in a probability function. We must note, however, that the physical interpretations of a probability function and a shape signature are different, even though the mathematical analysis is identical.

3.2 Entropy as a measure of circularity

In every field, the term information is widely used but rarely defined. In probability theory, information is regarded as the amount of unexpectancy and uncertainty in a message. In 1959, Shannon introduced the concept of entropy and defined it as a measure of information [35].

The amount of information about an event is closely related to its probability of occurrence. Events with high probability of occurrence, that is, those

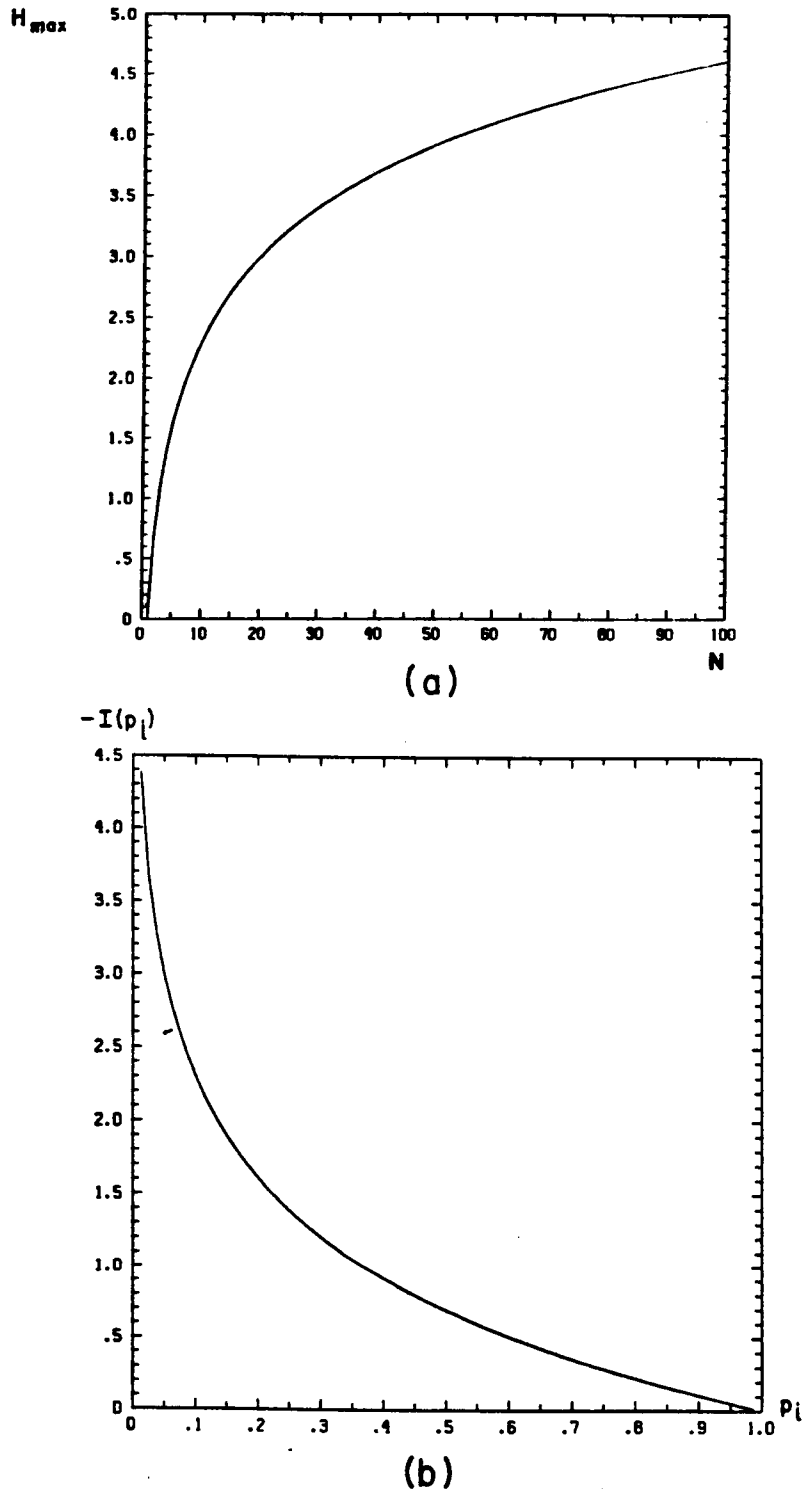


Figure 3.3: **(a)**: H_{max} variation with the number of samples, N, **(b)**: Information function for an event of probability p_i.

with little uncertainty in their outcome, convey relatively little information. In contrast, those events or messages with low probability convey a relatively large amount of information. Therefore, the information received from the knowledge of occurrence of an event is inversely related to its occurrence. The above situation is equivalent to the second requirement in Section 3.1, where C is required to be a decreasing function.

The measure of information is real, monotonic, and additive for events which are statistically independent. A certain event x (i.e., an event with a probability of 1) has zero information, while an impossible event has an infinite amount of information. Anything else has a positive finite content of information. Information I_x is therefore inversely proportional to probability p_x and was proven to have the following form [21],

$$I_x = \log(1/p_x).$$

As shown in Figure 3.3 **(b)**, in order to generalize from one event to N events, entropy is defined as the average information. By taking the expected value, the entropy, H_x, of a set of N events is

$$H_x = I_{avg} = E[I_x]$$

$$H_x = -\sum_{i=1}^{N} p(x_i)\log(p(x_i)).$$

An average information content is assigned even though the information for each event might fluctuate. There is an upper limit to information and therefore to

entropy,

$$H_x < I_x max.$$

The maximum entropy principle states that the inequality in the above equation becomes an equality when events are equiprobable, implying that H is maximum when all events have the same probability of occurrence $1/N$ [24,18]. As a result, H has the property that the flatter the distribution, the more information (on average) is contained in the sequence, and the larger H is; the more peaked the distribution, the less information contained in the sequence, and the smaller H is.

In the previous sections, a 2-D shape was transformed into a 1-D function, then the analysis of the shape became an analysis of a discrete function. The process of normalizing the area under the characteristic function justified its treatment as a density function (in a mathematical sense). It is natural to treat different shapes as different densities, and those densities vary as given shapes undergo changes. The probability associated with a random variable, p_i, is equivalent to the distance from the centroid to a point on the shape, f_i. A 2-D shape with N points can be identified with a set of N independent random events E_i. The event E_i represents the distance f_i from the centroid of the shape to the boundary point i. The probability of the occurrence of E_i equals the normalized distance.

3.3 Entropy based descriptor

Entropy was shown to discriminate between shapes based on their degree of circularity. However, analysis of other properties that can relate shapes to each other are often required. Therefore, a definition of information that takes note of the relation between shapes is desired, be it circularity or other shape properties. In other words, the objective is to define a descriptor that evaluates the degree of similarity of one shape to a reference shape selected arbitrarily.

Measuring the closeness of a shape to another is equivalent to measuring the similarity between two characteristic functions. Assuming the first shape is the reference and defined by $f(x_i)$, and the second shape, defined by $g(x_i)$, is to be compared to the reference, then the closeness is defined as the difference between the information contained in the reference characteristic function and the information contained in the second function about the reference $f(x_i)$. The average information that would be computed using the g density, when the f density is the reference, is defined as H and is obtained in the following manner:

$$H = \sum_{i=1}^{N} f(x_i) \log(g(x_i)).$$

The closeness of the density g to f is, then, defined as [19,20],

$$H_{ref}(x) - H(x) = \sum_{i=1}^{N} f(x_i) \log(f(x_i)) - \sum_{i=1}^{N} f(x_i) \log(g(x_i))$$

$$\mathrm{MS} = -\sum_{i=1}^{N} f(x_i) \log \frac{g(x_i)}{f(x_i)}. \tag{3.1}$$

Measure MS has the following properties: (i) it is zero for identical densities,

and (ii) it increases as the dissimilarity of density g to and a reference density f increases.

These properties are verified by taking into account the following:

A logarithmic function is concave down; this can be seen by examining its second derivative $-1/x^2$. Therefore, any tangent forms an upper bound for the log function; for instance, consider the tangent at $x = 1$:

$$y = \log'(x = 1)x - 1 \quad = \quad x - 1, \quad \text{it follows, then}$$

$$\log(x) < x - 1, \tag{3.2}$$

$$\log(x) = x - 1, \text{if and only if} \quad x = 1, \tag{3.3}$$

Replacing x by $g(x_i)/f(x_i)$, Equations (3.2) and (3.3) can be rewritte n as

$$\log \frac{g(x_i)}{f(x_i)} \quad < \quad \frac{g(x_i)}{f(x_i)} - 1, \tag{3.4}$$

$$\log \frac{g(x_i)}{f(x_i)} \quad = \quad \frac{g(x_i)}{f(x_i)} - 1, \quad \text{if and only if} \quad f(x_i) = g(x_i), \quad i = 1, 2, ..., N. \tag{3.5}$$

Applying summation to both sides

$$-\sum_{i=1}^{N} f(x_i) \log \frac{g(x_i)}{f(x_i)} \quad > \quad -\sum_{i=1}^{N} g(x_i) - f(x_i) = 0 \tag{3.6}$$

because $\sum_{i=1}^{N} f(x_i) = \sum_{i=1}^{N} g(x_i) = 1$.

Equation (3.5) becomes

$$\text{MS} = -\sum_{i=1}^{N} f(x_i) \log \frac{g(x_i)}{f(x_i)} \quad = 0, \tag{3.7}$$

if and only if f and g are identical.

As a result, MS is always positive. The only case where it is zero occurs when the two functions are identical. This result allows one to evaluate the degree of similarity between two shapes represented by the same data size N.

When the function is normalized to generate a probability density, approximations are made when evaluating the area, and, therefore, the result is an approximation of a density rather than a perfect density. Due to this approximation, the measure MS can take slightly negative values when it is supposed to be zero.

In real world situations, the numbers of data points on the boundaries of different objects can vary greatly. As was seen in Chapter 2, simple operations like rotating an object result in different boundaries, which are usually associated with a difference in the number of points. In practice, it is very unlikely that two objects have exactly the same number of boundary points. The questions then become: how to compare two shapes where one has more points than the other, what kind of criteria should be used in adding or eliminating data, and what is the 'best' way to achieve this with a minimum loss of information. The following section addresses this problem.

3.3.1 Generic problems

In order to make a 'fair' comparison between two shapes, the shapes must be of the same level of detail. Given two different shapes, represented by their corresponding characteristic functions $f(i), i = 1, 2, ..., N$ and $g(j), j = 1, 2, ..., M$

where $M \neq N$, there are three possibilities:

1. N≈M,

2. $N < M$ or $M < N$, within a specified range,

3. $N << M$ or $M << N$.

The first case is a desirable one since it deals with an unbiased comparison, due to the fact that both shapes have their information expressed by the same number of boundary points. The few extra pixels can be attributed to noise, rotational effects, etc.

The second and the third cases, however, require extra processing based on some assumptions (to be justified experimentally and/or theoretically). In this work, one is interested in both local and global information in a given shape. Therefore, one has to be aware of the effects of the approach used in adding and eliminating data in the characteristic function. When N and M are not in the same range, two possibilities can exist:

1. a scaled version of the same object is present, or

2. a different object is present.

The nearest point substitution appears to be an appropriate method in this type of problem. Suppose $N < M$, generating $f(j), j = 1, ..., N$, given $f(i), i = 1, ..., M$ is done using the following algorithm:

1. Set $step = M/N$,

2. for each $j = 1, ..., N$, if $i \leq step \leq i + 0.5$, then $f(j) = f(i)$,

3. else, $f(j) = f(i + 1)$,

4. j=j+1, then go to 2.

The end points are unchanged. The new characteristic function is guaranteed to have N points. The new function values have not changed even though some data are eliminated through the process. The experimental results of this algorithm have shown that, in all cases, global as well as local object information is preserved as much as possible. Therefore, depending on specific applications, the nearest neighbor algorithm will not only normalize the size of samples under scaling, but also adjust the level of detail in one function to a second one. The following figures illustrate the performance of the algorithm for different cases. In Figure 3.4, a curve represented by 90 data points is increased by 20 points based on the principle of nearest neighbor substitution, resulting in two similar curves. Not only is the basic shape preserved, the local information is emphasized. The extreme effect of the algorithm is illustrated in Figure 3.5 where a curve of 200 data points is transformed into its closest curve represented by 100 points. The two curves are superimposed for comparison. Most of the local information is lost, while the basic skeleton of the curve remains the same. Figure 3.6 shows another example of data reduction in given shape.

In this work, the proposed nearest neighbor algorithm shows reasonable results for the applications considered; however, depending on specific shape applications, one is free to use the best method fitted for that application. In the

next section, a general mathematical formulation to this problem is presented.

3.4 Mutual information: Statistical independence in shapes

While in Section 3.3 an algorithm is suggested for adding or eliminating data points, this section presents a formulation of the general problem of extracting information from two densities which are generated from a different number of events. The implication of any result of this type allows one to compare two shapes of different number of data. This problem has been addressed in entropy theory, and a few basic results are given here (more material regarding this topic can be found in [41]).

Let S_1 be a shape with N boundary points, and the associated characteristic function values be

$$f_1, .., f_N \quad \text{with} \quad f_j \geq 0,$$
$$\sum_{j=1}^{N} f_j = 1.$$

Let S_2 be a shape with M boundary points, and the associated function values be

$$g_1, ..., g_M \quad \text{with} \quad g_k \geq 0,$$
$$\sum_{k=1}^{M} g_k = 1.$$

The mutual information $I(S_1, S_2)$ between the two shapes S_1 and S_2 is defined by

$$I(S_1, S_2) = \sum_{j=1}^{N} \sum_{k=1}^{M} h_{jk} \log \frac{h_{jk}}{f_j g_k},$$

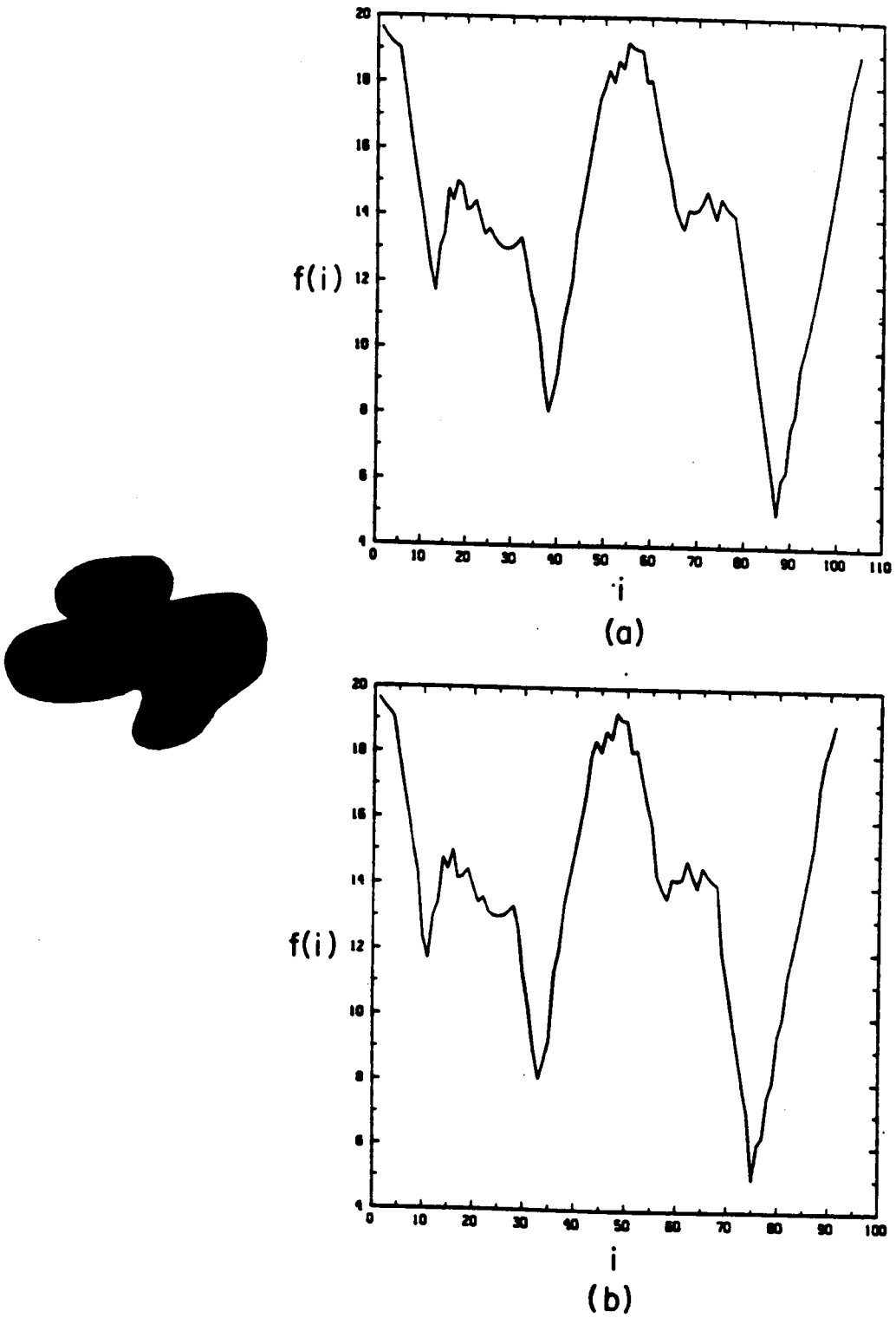


Figure 3.4: Effects of nearest neighbor substitution algorithm on a shape represented through 90 points in (a), then increased to obtain 105 points in (b).

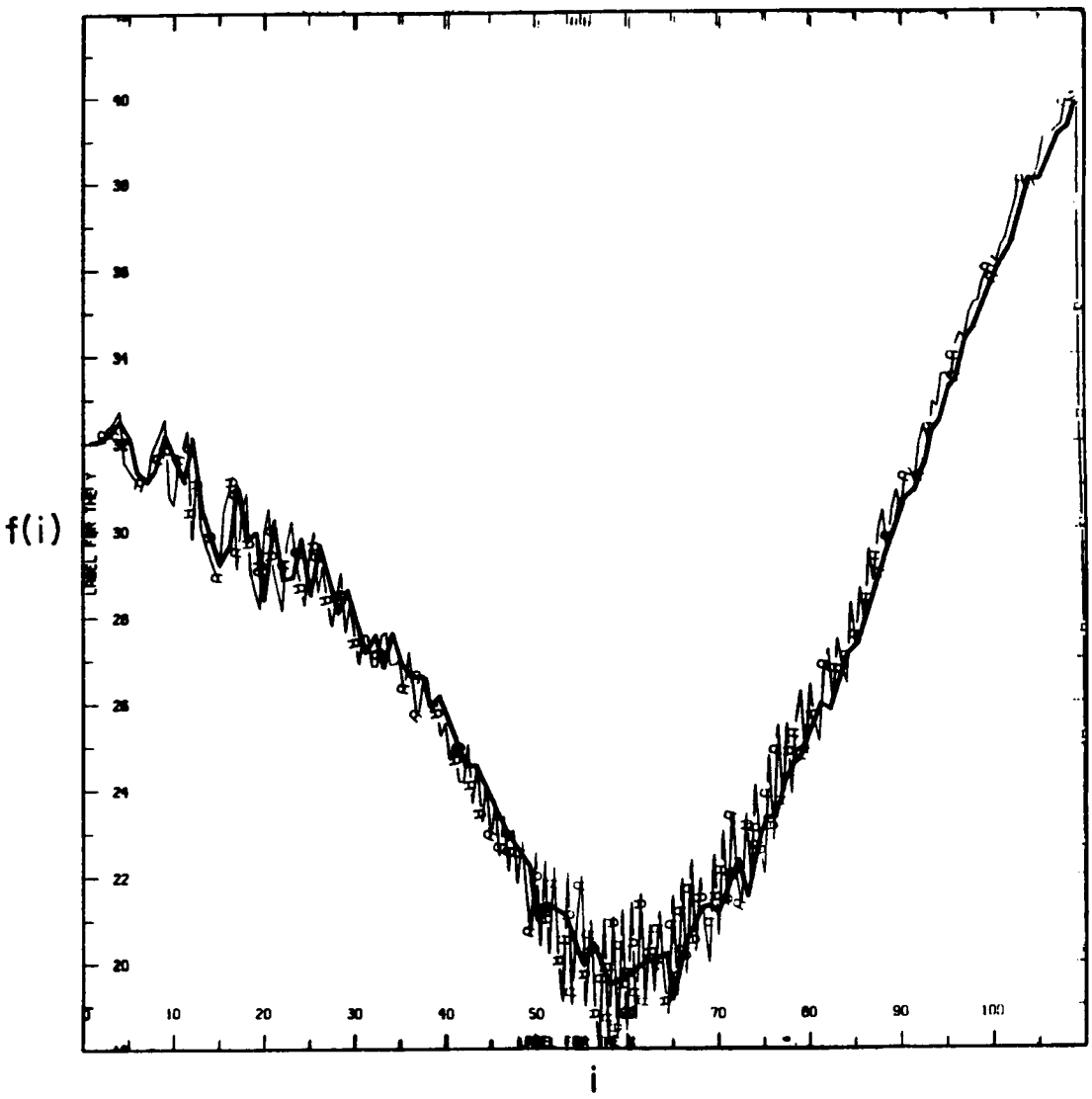
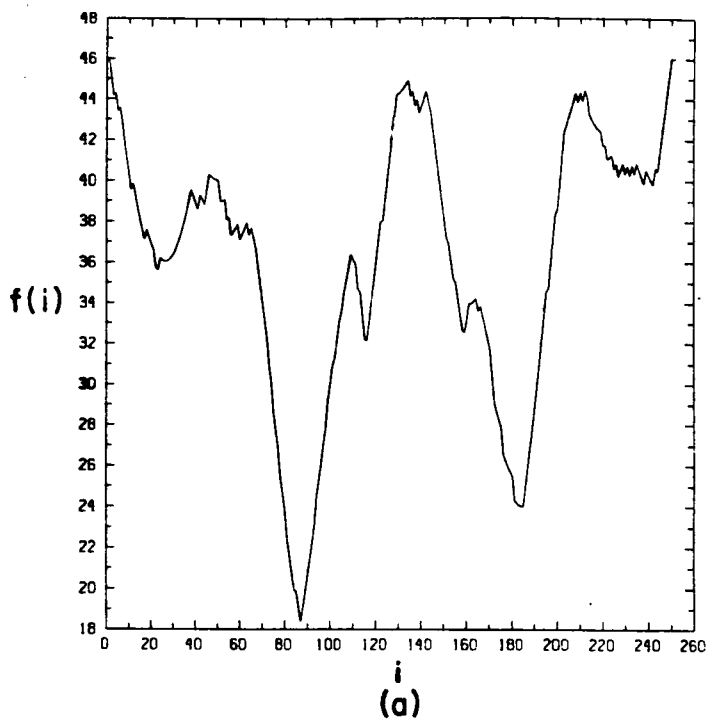


Figure 3.5: Example of the results obtained using the nearest neighbor algorithm to eliminate half of the data from a shape of 200 points.





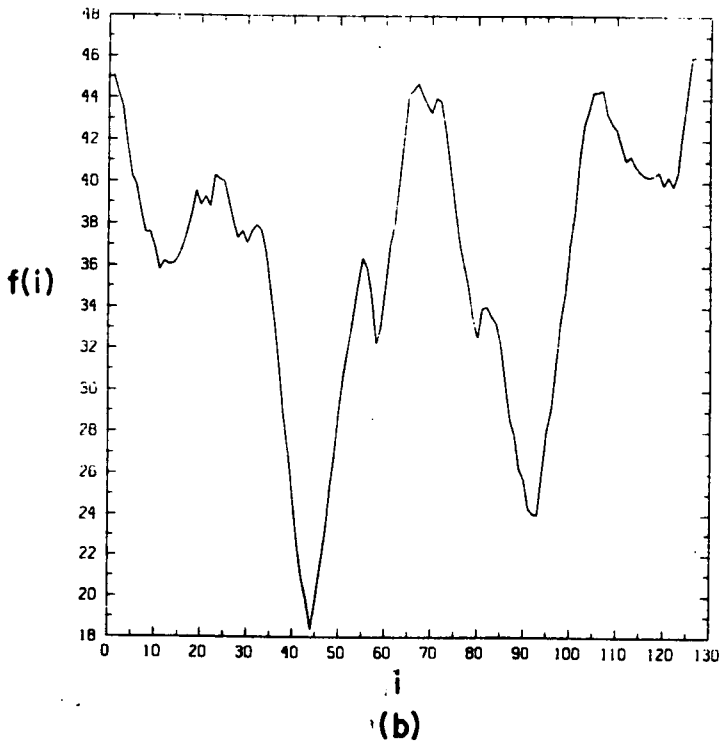


Figure 3.6: Effects of nearest neighbor algorithm in eliminating 130 points from shape in plot (a).

which is the average of the mutual information between shapes in terms of shape entropy. In a similar way, conditional entropy is defined. The connection between the two shapes is obtained by specifying

$$h_{jk} \quad (j = 1, ..., N;\ k = 1, ..., M)$$

subject to $h_{jk} \geq 0$, and $\sum_{j=1}^{N} \sum_{k=1}^{M} h_{jk} = 1$.

The quantities h_{jk}, f_j, and g_k are related in the same manner marginal densities are related to the their joint density:

$$f_j = \sum_{k=1}^{M} h_{jk},$$

and similarly,

$$g_k = \sum_{j=1}^{N} h_{jk}.$$

Just as f_j and g_i have physical meaning as the distance between the centroid and a boundary point of a two dimensional shape, the term h_{ij} also represents the distance from the centroid of a three dimensional shape to boundary point on its surface.

Even though the direct application of such results requires knowledge about the process that generates these shapes, the potential of using the above formulations and others from information theory is clear. In other words, h_{jk} terms are specified or assumed based on the knowledge of the 3-D shape from which the cross sections S_1 and S_2 are generated. The following section illustrates an application of the above result.

3.4.1 From 2-D to 3-D shapes

Operating directly on three dimensional objects is often an impractical task, but it is necessary for solving real world problems. In this section, an attempt to understanding the shape of three dimensional objects, based on the shapes that constitute their cross-sections, is presented.

Considering two shapes S_1 and S_2 is equivalent to having two probability densities of single random vectors, $\mathbf{X}$ and $\mathbf{Y}$ respectively. The joint densities of S_1 and S_2 are related to the marginal densities as follows:

$$S_1(\mathbf{X}) = \int S(\mathbf{X}, \mathbf{Y}) d\mathbf{Y},$$

$$S_2(\mathbf{Y}) = \int S(\mathbf{X}, \mathbf{Y}) d\mathbf{X}.$$

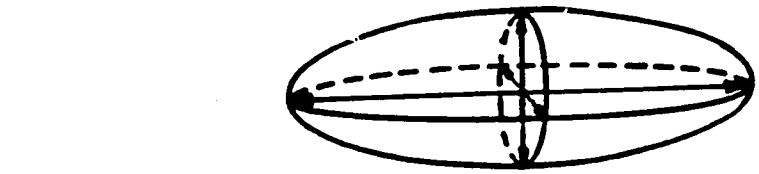
The joint density can be viewed as the characteristic surface of a three-dimensional object related to its cross-sectional contours, S_1 and S_2. If S_1 and S_2 are 'independent', then

$$S(\mathbf{X}, \mathbf{Y}) = S_1(\mathbf{X}) S_2(\mathbf{Y}).$$

Consider a sphere, for instance, all its cross-sections are circular. If only two cross-sections (based on an orthogonal coordinate system) are considered, their characteristic functions are two uniform densities S_1 and S_2. The joint density is simply a plane surface bounded by the range of $\mathbf{X}$ and $\mathbf{Y}$. This surface represents the distance function from the center to each point on the sphere. In another

example, the signature of an ellipsoid can be found by multiplying the signature of a circle by that of an ellipse (see Figure 3.7).

Understanding three dimensional objects, purely through their shapes, can be achieved using two-dimensional views. Depending on whether the object is regular or irregular, the marginal densities are 'dependent' or 'independent'. An identical analysis to 2-D shape discrimination can be applied to 3-D shapes with the difference of two variables $\mathbf{X}$ and $\mathbf{Y}$.



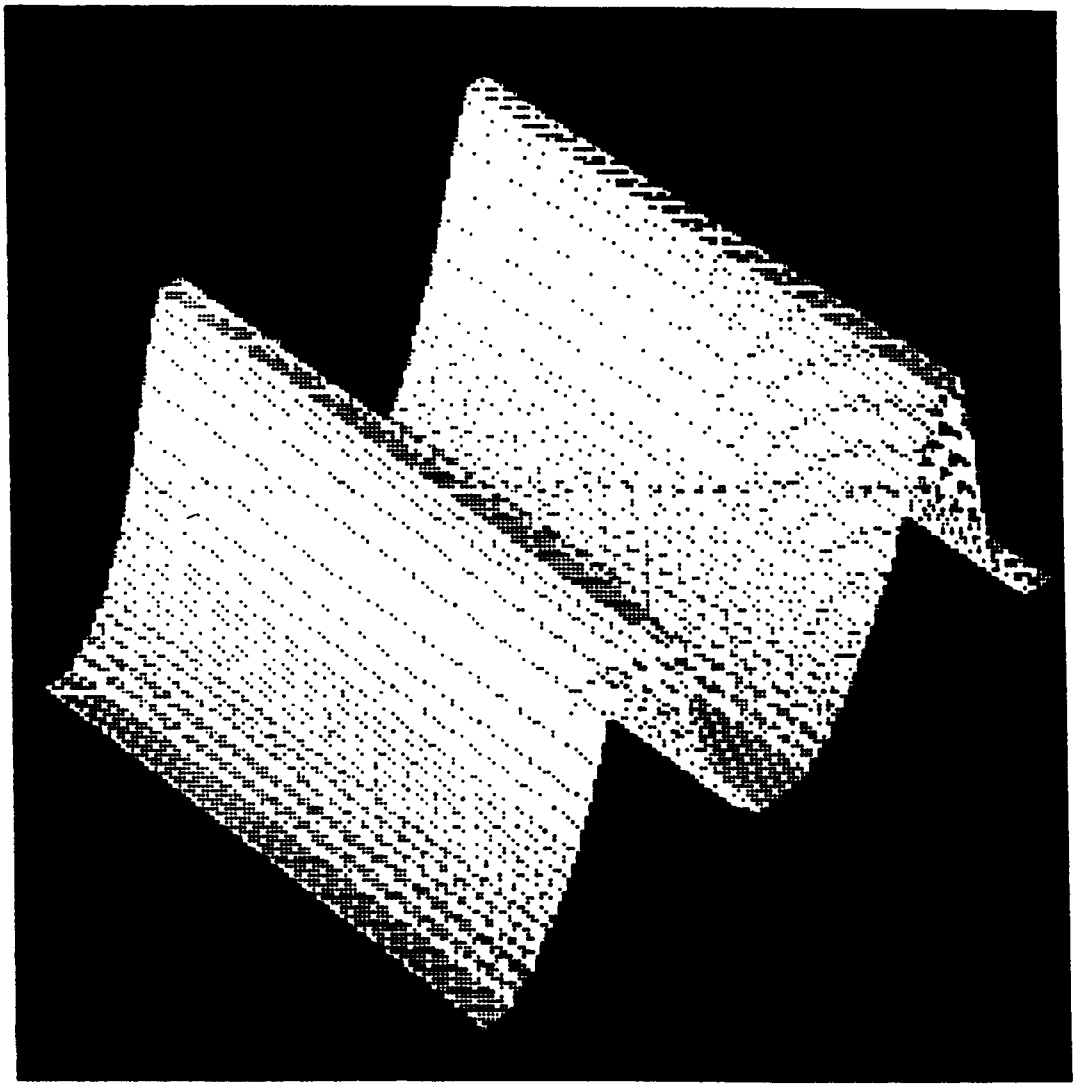


Figure 3.7: Characteristic surface of an ellipsoid based on 2-D cross-sections.

CHAPTER 4

IMPLEMENTATION AND RESULTS

In this chapter, the implementation of the developed descriptor is presented. Various types of applications are divided into two categories: (i) comparison of a given shape to an analytically defined model, (ii) comparison to 'natural' shapes, where natural is taken in the meaning of non-analytical. A variety of examples is considered for different shape applications.

4.1 Analytically defined models

Analytically defined models present the advantage of complete prediction of the contour data once they are specified; for example, in order to generate an ellipse of 100 points, it is sufficient to specify the major and minor axes and the sampling rate that will result in 100 data points. The same type of ellipse can be generated for any number of points N.

The degree of similarity between two shapes where one of them is analytically defined can be measured using Equation (3.1). Once the model is known, comparing it to a given shape can be achieved by providing that model with the same amount of detail that was provided for the shape. Consider, for instance, measuring the circularity in shapes. As demonstrated in the previous chapter,

a circle of N points has a uniform function, $f_i = 1/N$, implying that Equation (3.1) can be rewritten as

$$MS = \sum_{i=1}^{N} f_i \log f_i - \sum_{i=1}^{N} f_i \log g_i$$

$$= \log N - \frac{1}{N} \sum_{i=1}^{N} \log g_i.$$

It is worth noting that in this case rotational adjustments are automatically done because the circle function values are the same (i.e., $1/N$), whether the analyzed shape has been rotated or not. Figure 4.1 shows the evaluation of the above measure for four different shapes. The measure reflects the differences between each one of the shapes and a circle. Because the first shape is a circle, the measure reflects complete matching and takes its lowest value, zero, as predicted in the previous chapter. The degree of closeness of shapes 2, 3, and 4 to a circle decreases gradually from shape 2 to shape 3, increasing the measure appropriately.

Figure 4.2 shows results obtained when measuring the degree of ellipticity in six shapes. It should be noted that in this case all contours have the same number of samples. Finally, the graph in Figure 4.3 shows the descriptor property increasing as the dissimilarity increases between the triangle prototype and the rest of the shapes.

In order to further demonstrate the descriptor performance, three types of models were used, a circle, an ellipse, and a triangle. The classification of a given shape is based on the nearest measure value. Figure 4.4 is an example of classification where ten contours are considered, then, each shape is compared to each of the models. The result is presented in a hierarchical manner. Starting

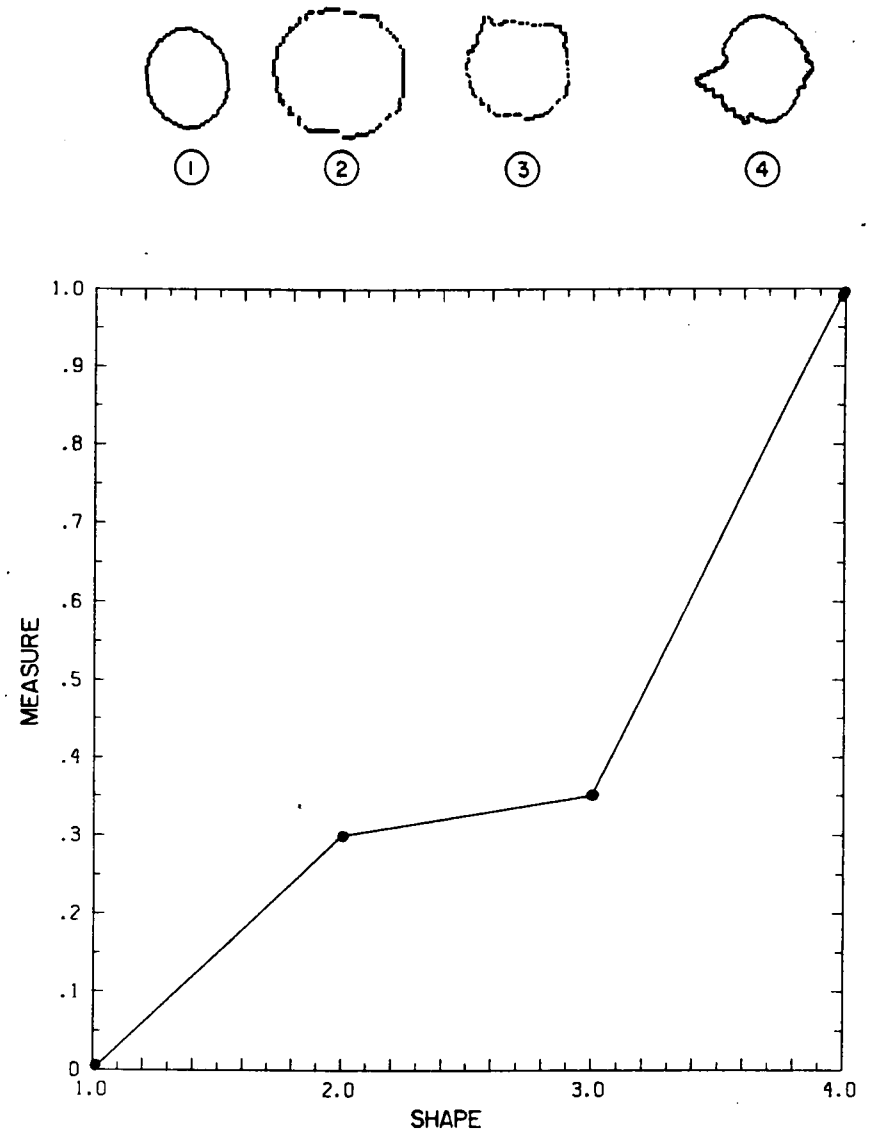


Figure 4.1: Shape discrimination based on entropy where the model is given by shape 1.

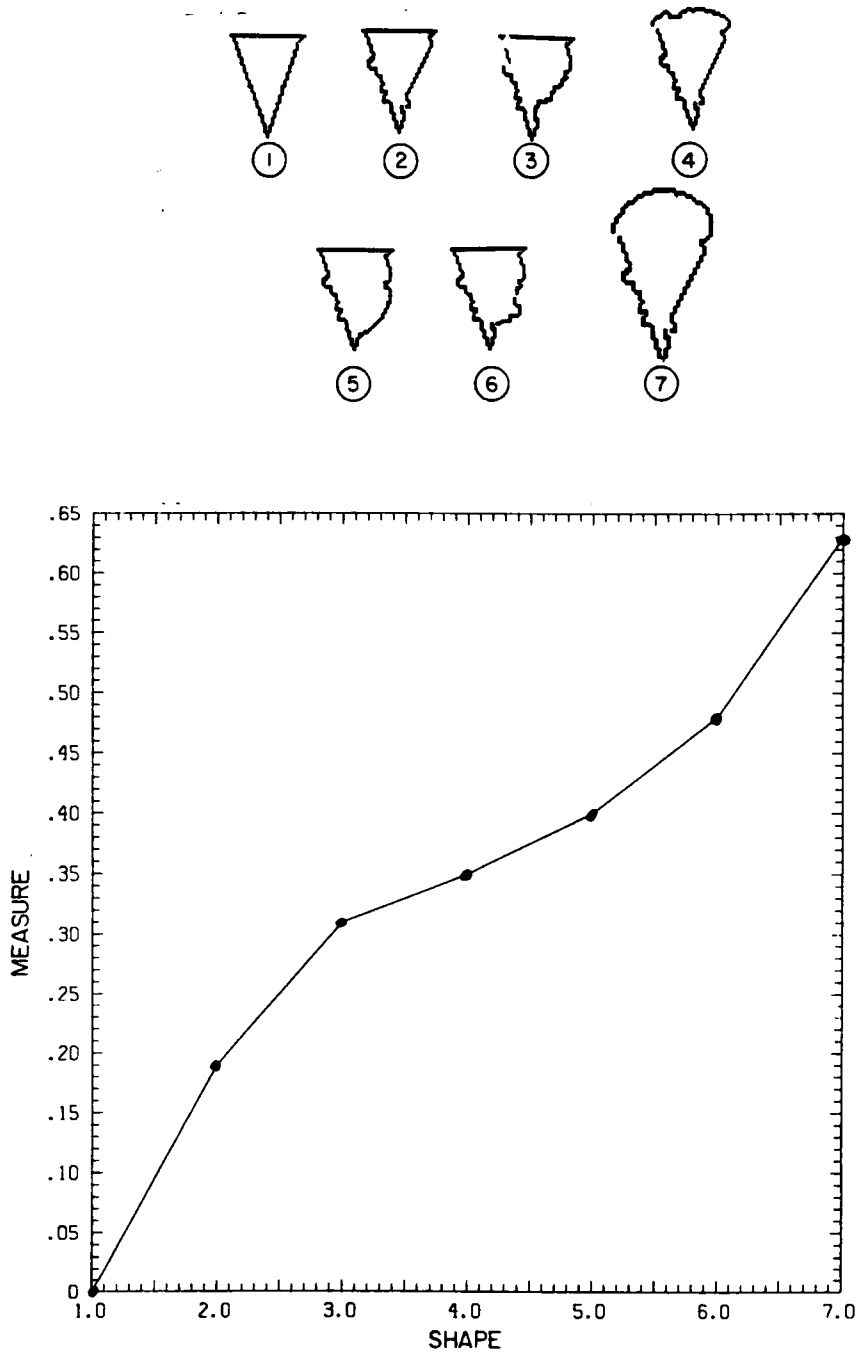
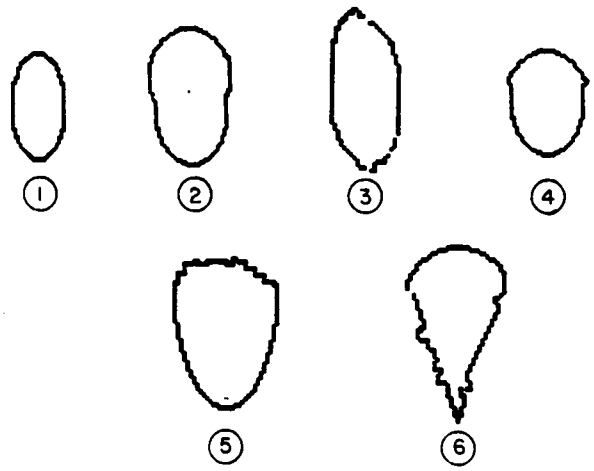


Figure 4.2: Shape discrimination based on entropy where shape 1 is the prototype.



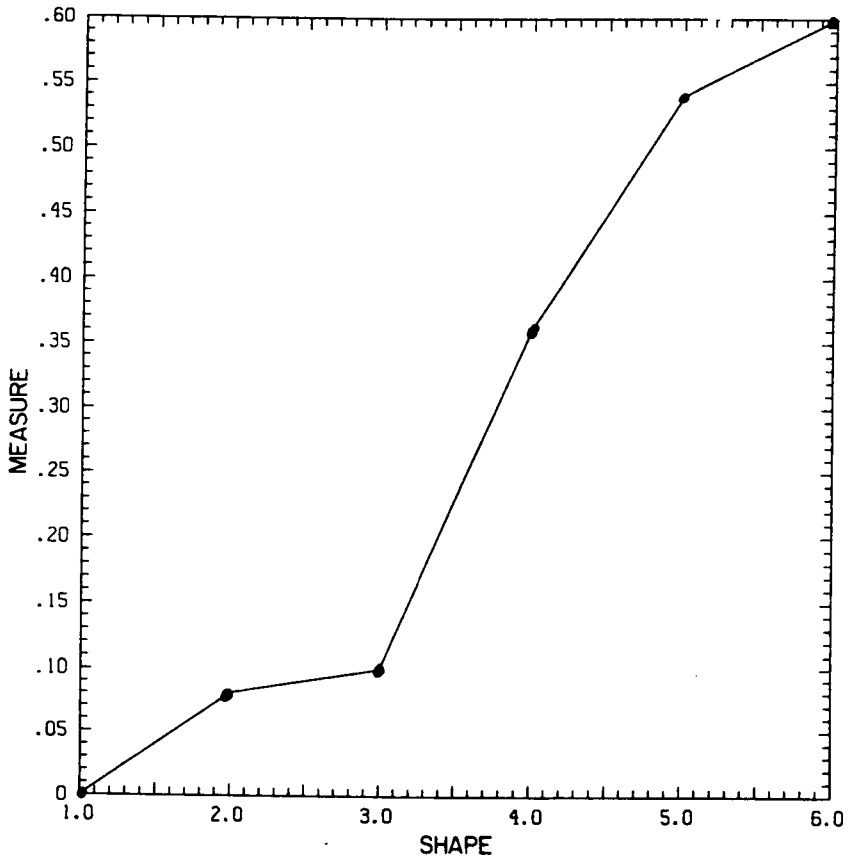


Figure 4.3: Shape discrimination based on entropy where shape 1 is the prototype.

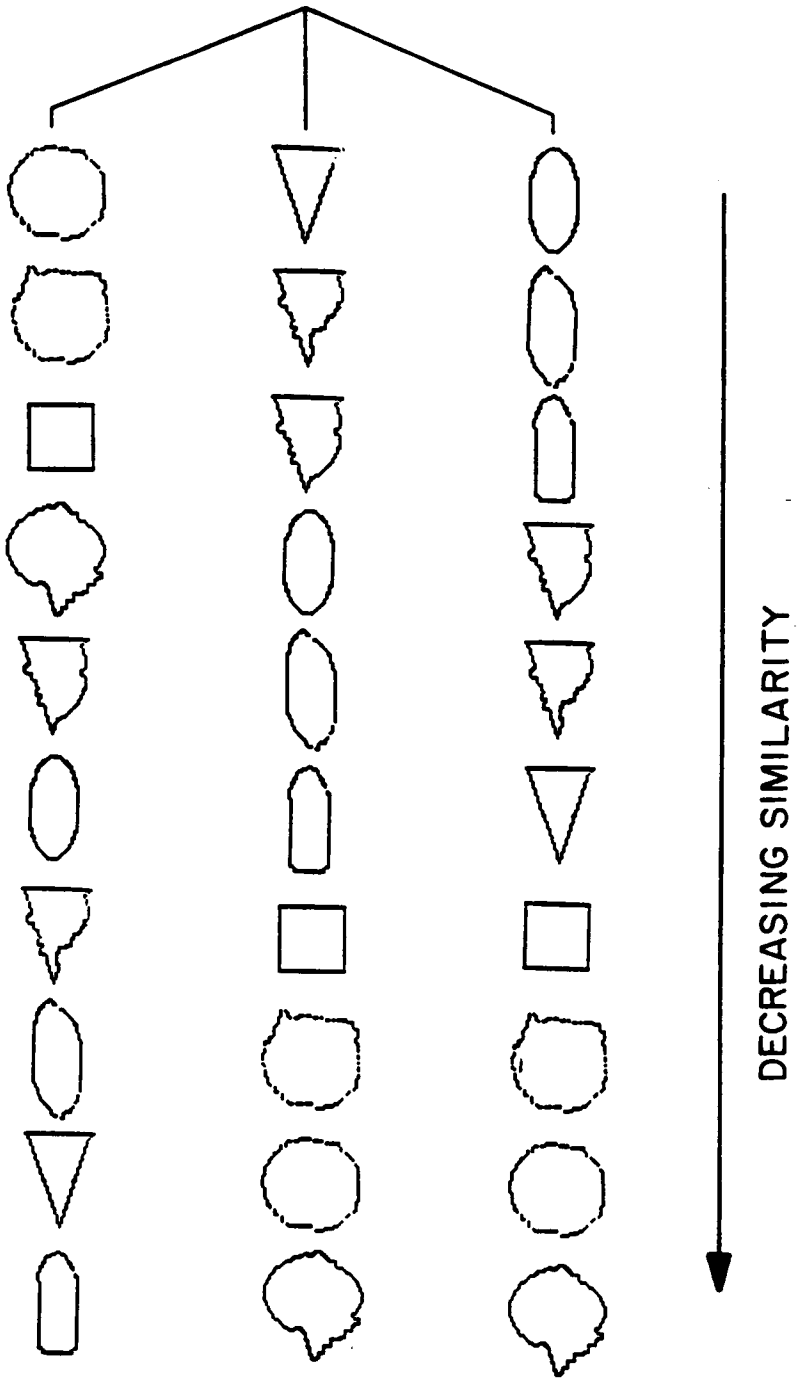


Figure 4.4: Shape classification based on 3 models: circle, triangle, and ellipse.

with the model, the remaining shapes are ordered according to their degree of similarity to the model. The resulting order is very similar to the way humans would classify. In order to properly group this ordered sequence of shapes, several thresholding techniques are available in the literature to accomplish optimal grouping [33].

Starting with Figure 4.5, geometrical properties are studied using different types of shapes that appeared in Veillon's work on a systematic study to shape measures [6]. Figure 4.5 describes the differences between similar looking shapes that were not discriminated using two-measures normally used for evaluating the circularity in two dimensional shapes: the ratio of the perimeter squared to the area and the ratio of the mean to the the standard deviation [17]. In principle, those measures are minimal for a circle and deviate (i.e., increase or decrease) from that optimum value as the closed continuous curve deviates from a circle; however, working with digitized contours and discrete data was shown [34] to result in unreasonable description. For instance, the shapes appearing in Figure 4.5 were reported in [6] to have the same value when the two measures were used. It is important to note the discriminating power entropy has as a circularity measure; its performance is better than previously used circularity measures. This was observed in every set of shapes we have tested (see Figures 4.5 through 4.12 for examples). One should note that other analytical models could be defined and used for evaluating properties other than circularity.

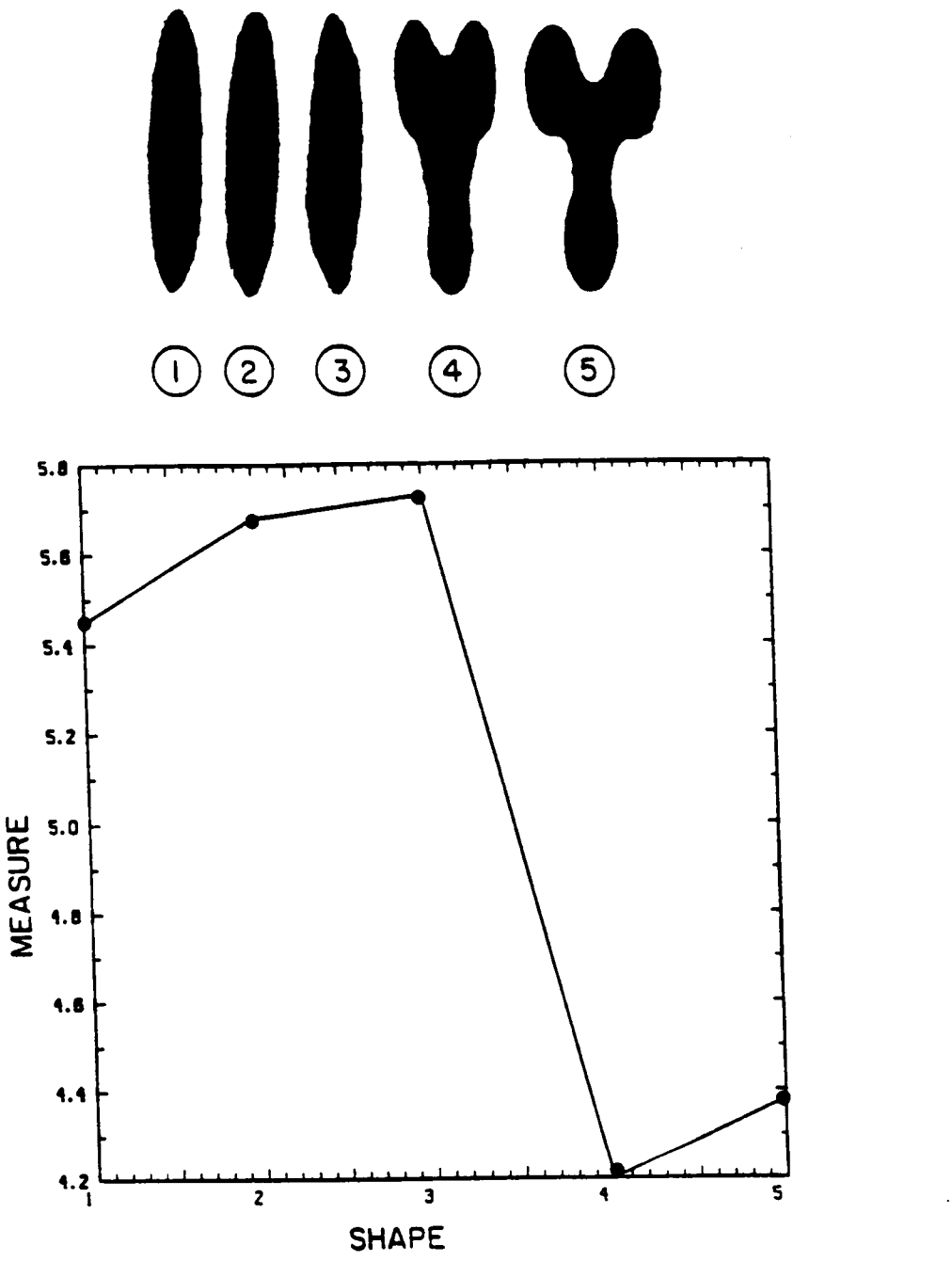


Figure 4.5: Shape discrimination using entropy as a measure of circularity.

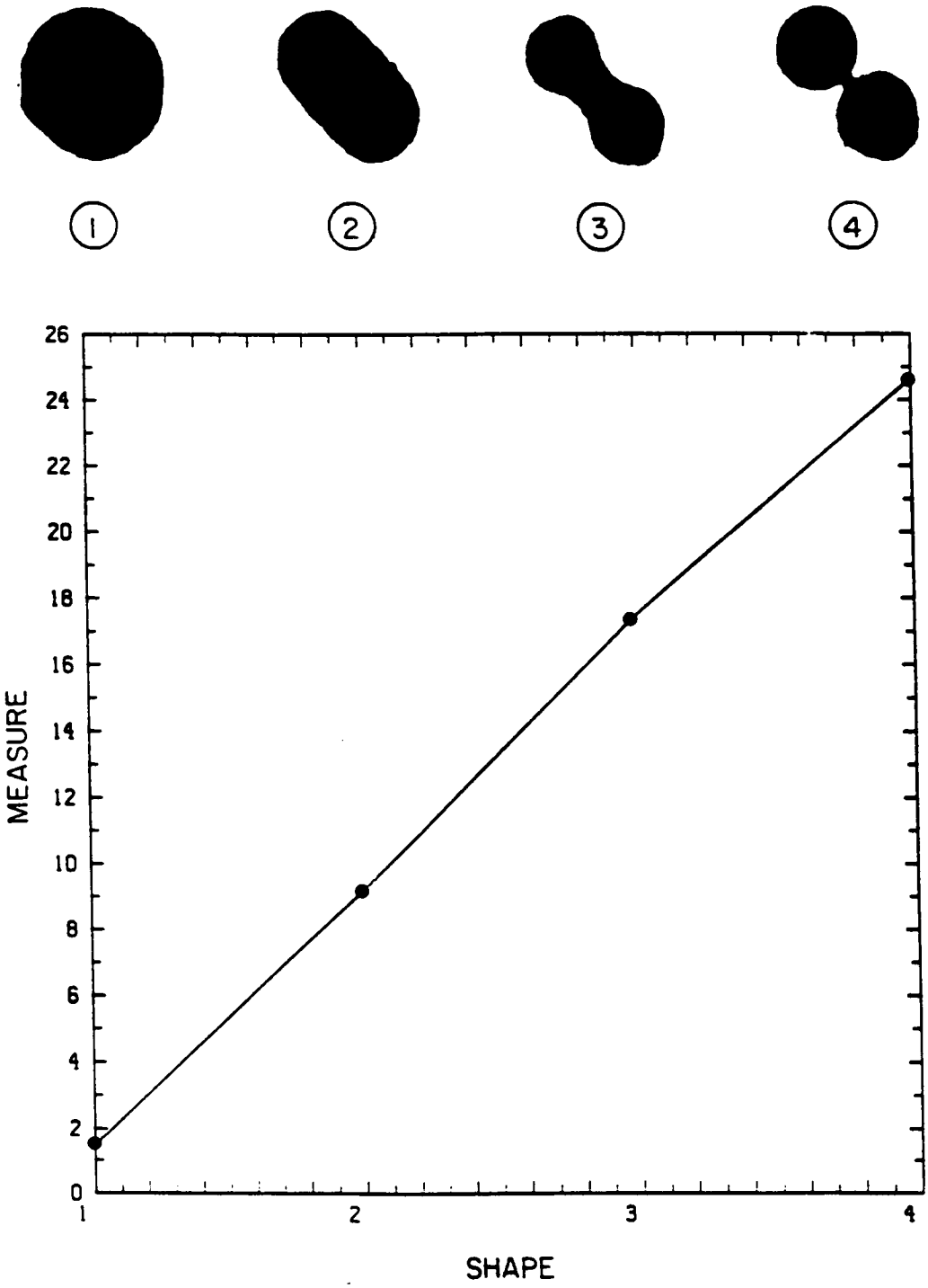


Figure 4.6: Shape discrimination based on entropy: Example 1.

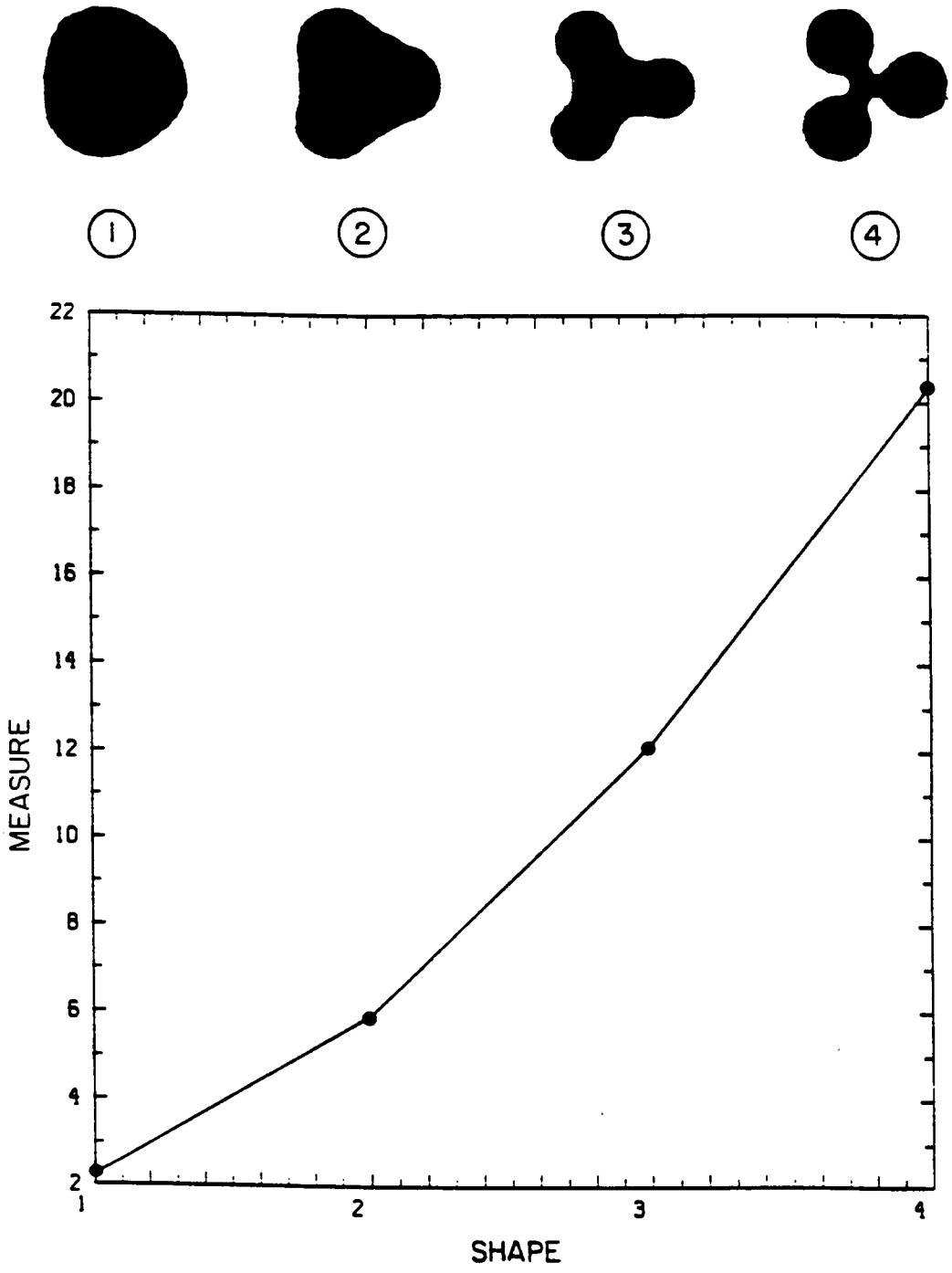


Figure 4.7: Shape discrimination based on entropy.

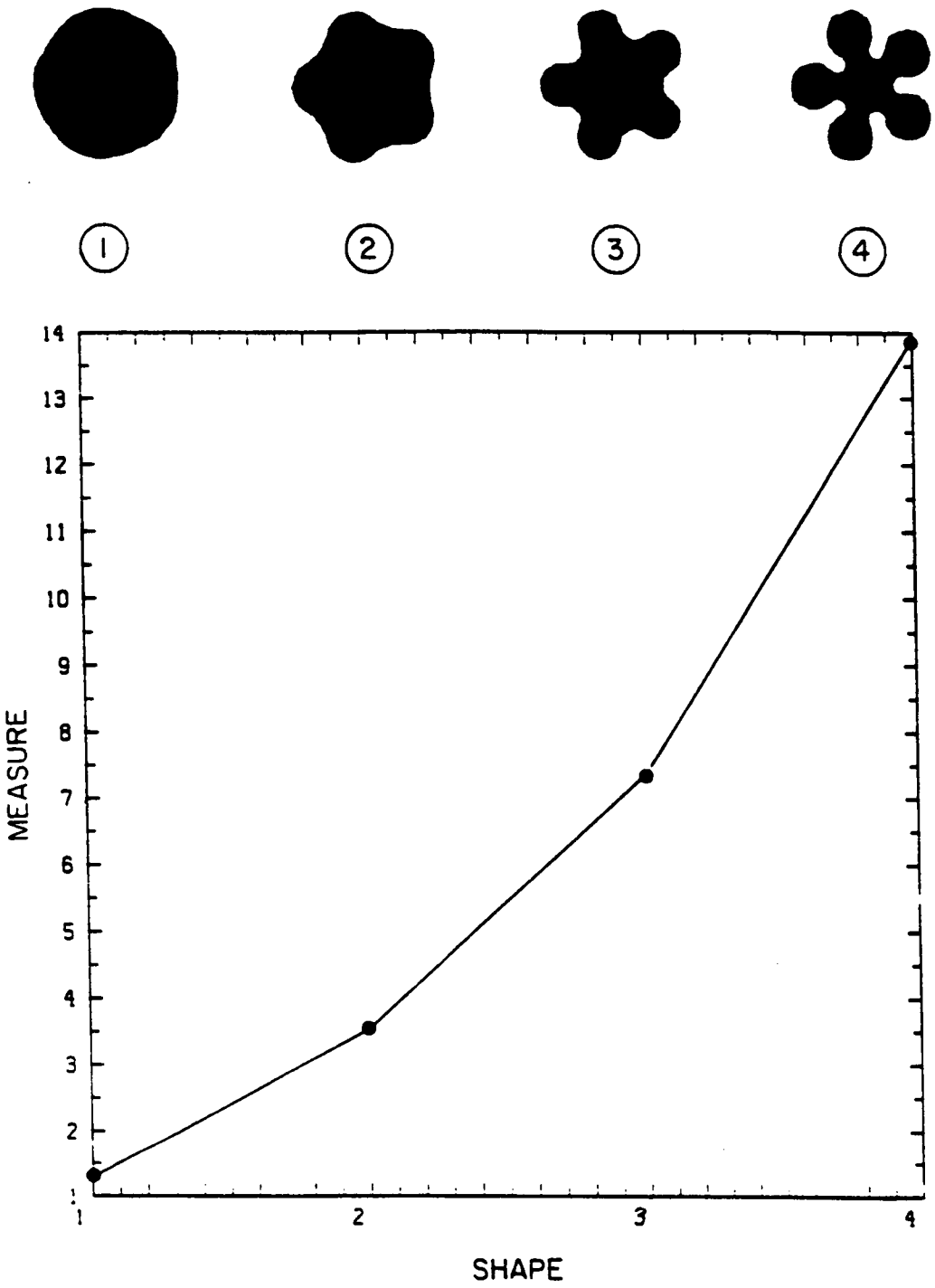


Figure 4.8: Shape discrimination based on entropy.

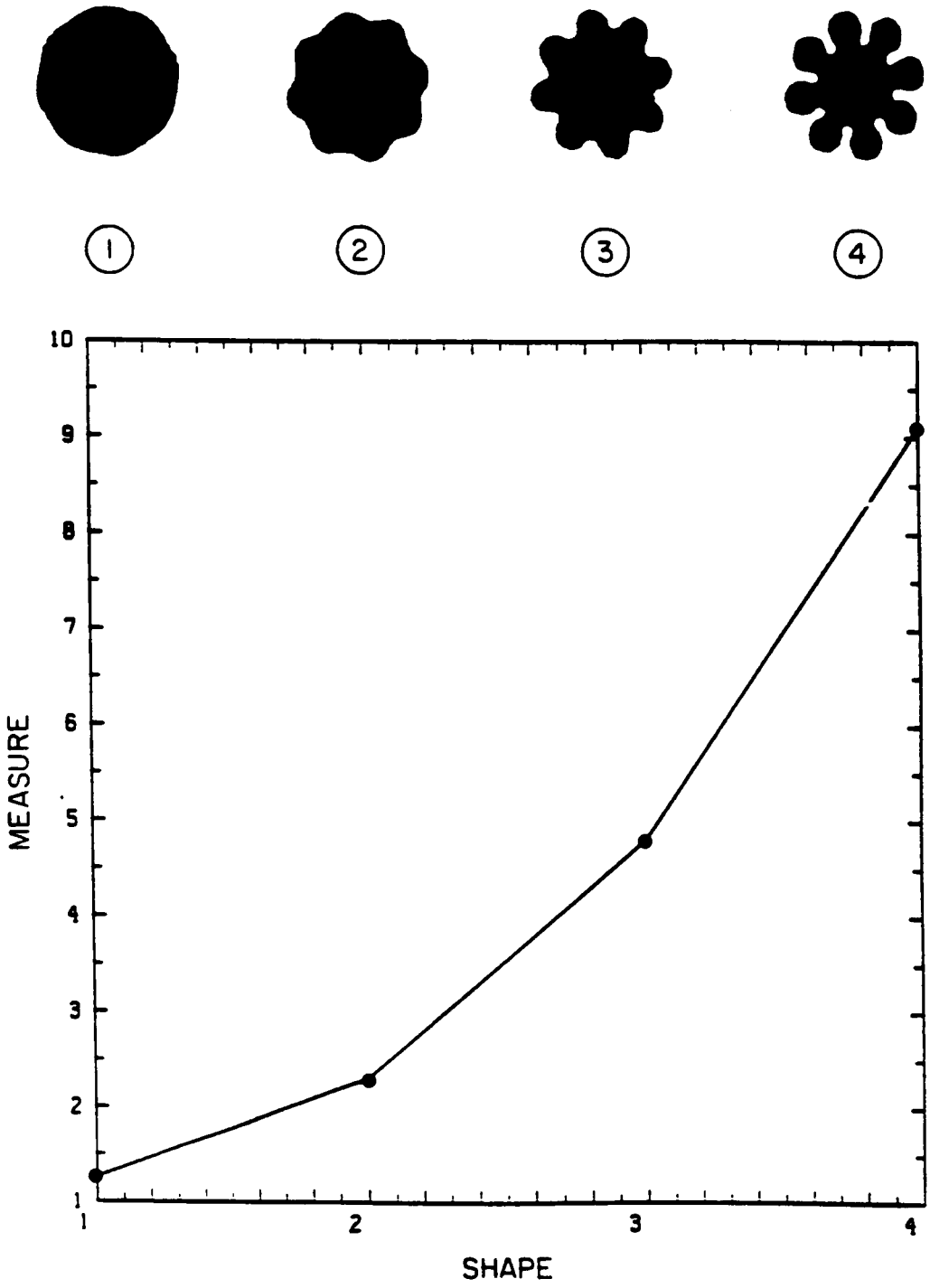


Figure 4.9: Shape discrimination based on entropy.

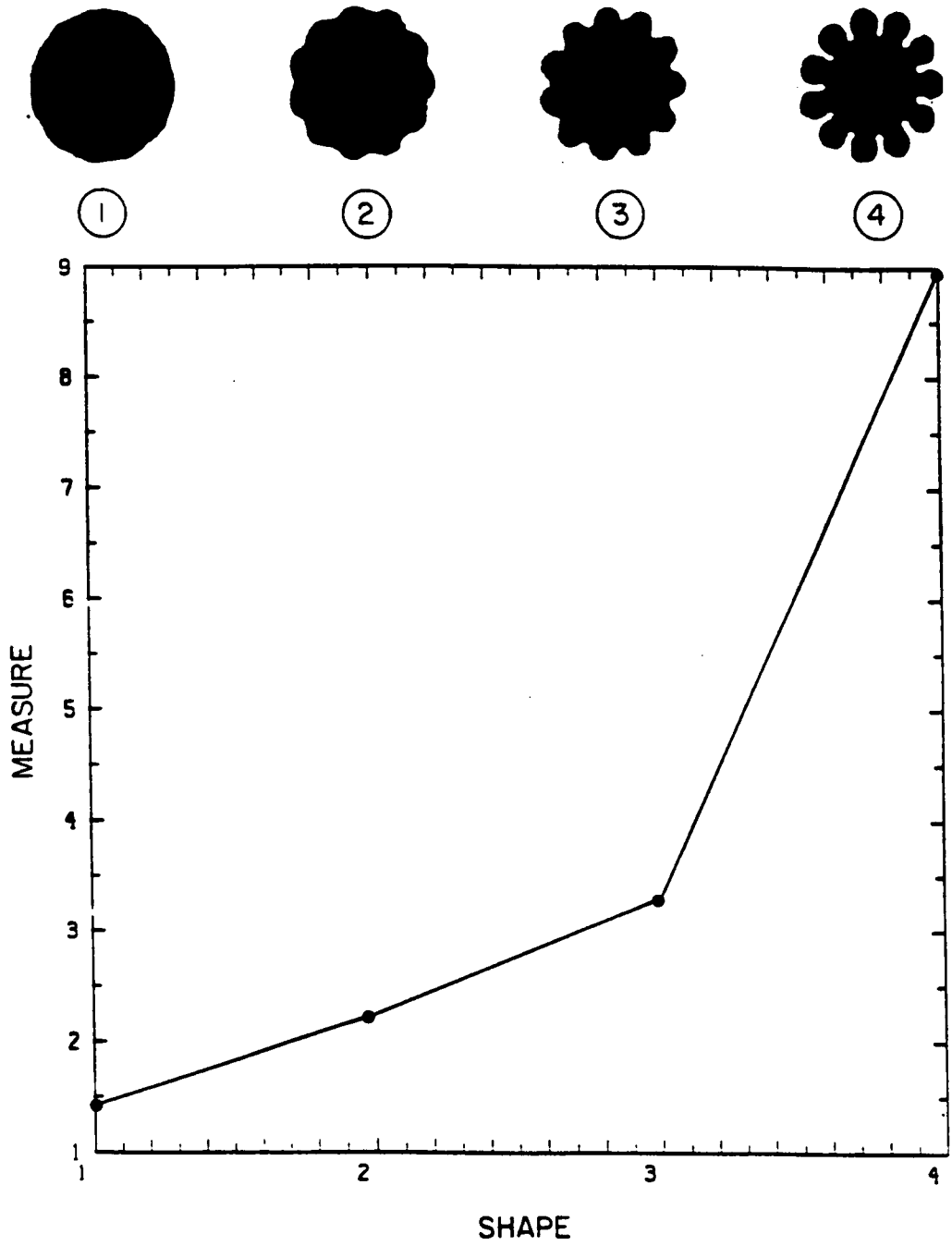


Figure 4.10: Shape discrimination based on entropy.

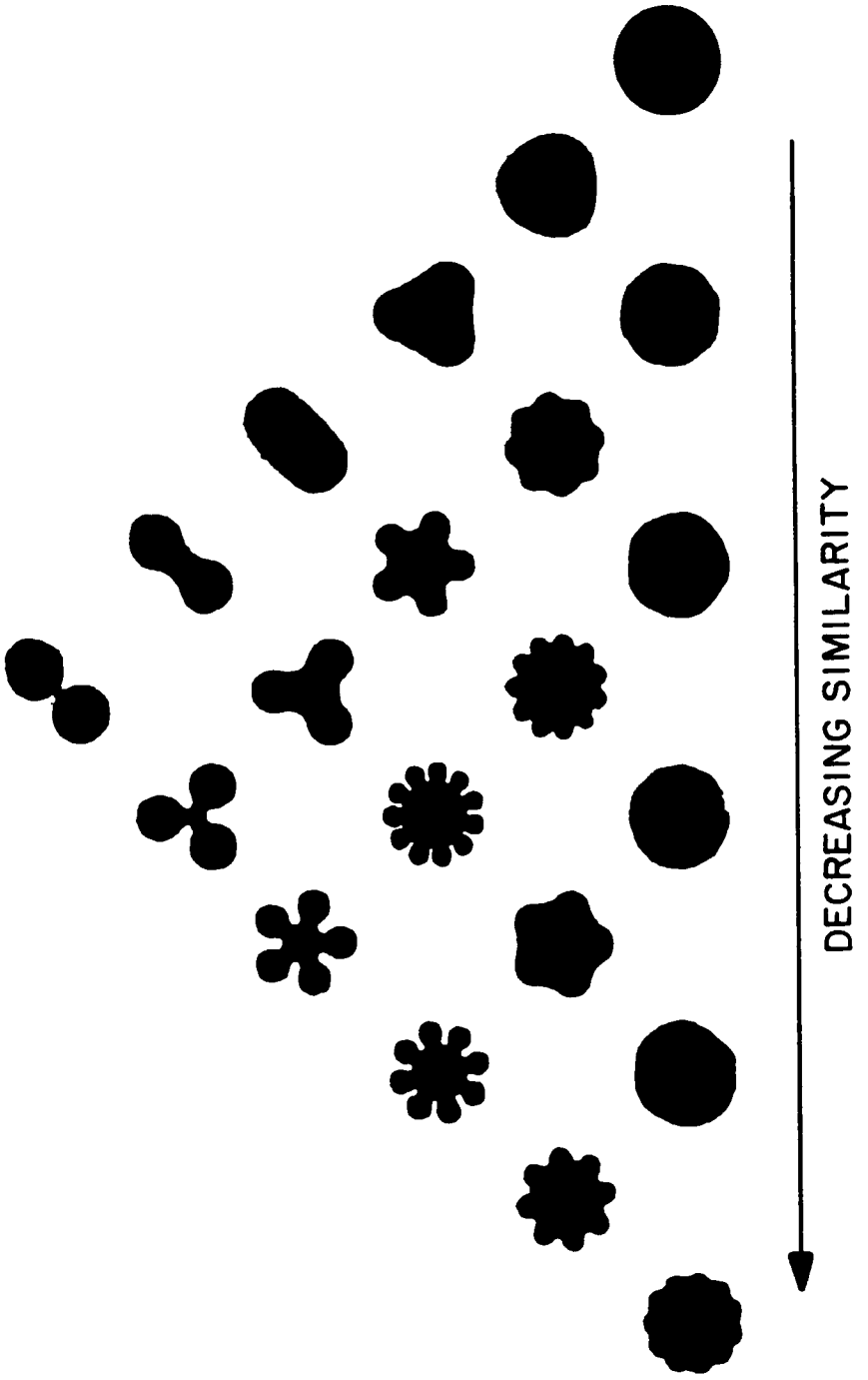


Figure 4.11: Order of similarity between the shapes shown in Figures 4.6-10, starting with a perfect circle and ending with the least circular shape.

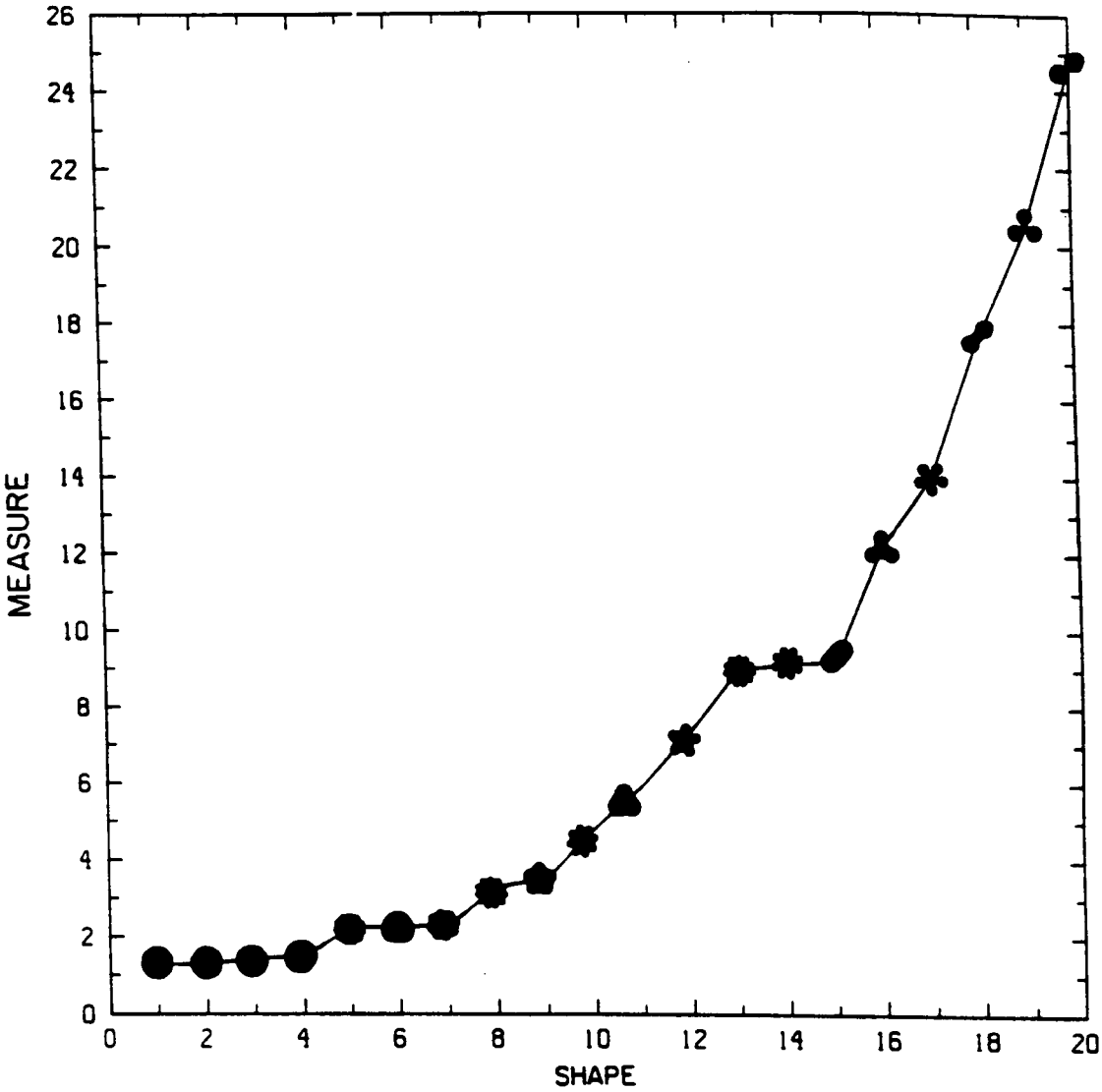


Figure 4.12: Performance of the entropy based descriptor in discriminating the shapes of Figure 4.11.

4.2 Arbitrarily chosen models

Comparing irregular shapes using a model defined analytically can be achieved without difficulty. However, assuming an analytical model may not be a natural choice for solving realistic problems. For instance, when matching keys, one is interested in identifying possible matches of several keys to a specific key. The assumption of an analytical model for the reference key implies an approximation that may not be reasonable. Therefore, when a shape descriptor is developed, it is important to examine its behavior with arbitrary shapes when they are compared to non-analytical models (i.e., arbitrarily chosen models). A variety of test shapes are considered in this section. One example examines the situation of matching keys: both the single orientation and the multi-orientation cases are considered. Another example shows a typical airplane identification problem in noise free and noisy environment. Tracking local boundary changes is illustrated using a time sequence of heart cross-sections; an example of local and global shape classification is shown through a variety of shapes. Recognizing machine printed numerals and hand-written numerals is also considered.

An interesting application for determining the discriminating power of a shape descriptor is tested when one tries to match keys. In the next set of figures, the performance of the entropy based descriptor is investigated using a few keys. As predicted in Chapter 2, the error resulting from rotation can be large enough to cause false classification (see Figure 2.6). Therefore, it is necessary to con-

sider keys with considerable differences whenever rotational effects are allowed, or maintain the same orientation when dealing with small variations between the keys. Using a reference key, keys of the same group are being compared, as Figure 4.13 shows. When there is complete match, the descriptor takes its lowest value of zero; as the difference increases, so does the descriptor. The orientation of the keys in Figure 4.13 is kept constant. The second example considered also deals with shape recognition and matching; four airplanes: A_1, A_2, A_3, and A_4, are chosen as reference shapes, while four other shapes, B_1, B_2, B_3, and B_4, are generated from the references with the addition of noise; (all eight airplanes are shown in Figure 4.14). The objective is to differentiate between the references, of course, and then to match the noisy airplanes to the referenced ones. The first four statistical moments were used [5] to identify the distorted airplanes. In [5], the authors assumed that the number of boundary points remains unchanged when the noise is added (this assumption is highly unrealistic). The same reference airplanes were tested using the descriptor proposed in this work. Table 4.1 shows the discrimination among them, and it is clear from the diagonal array of zeros that all four airplanes are distinguished. It should be noted that the four model planes have a different number of boundary points, which is believed to cause inappropriate differences between the airplanes. Table 4.2 shows the matching results of the noisy airplanes when they are compared to the references. The added noise is measured (as seen from the nonzero diagonal values), but in this example, the number of points is not preserved when the noise is added. The decision for classification is made based on the nearest neighbor

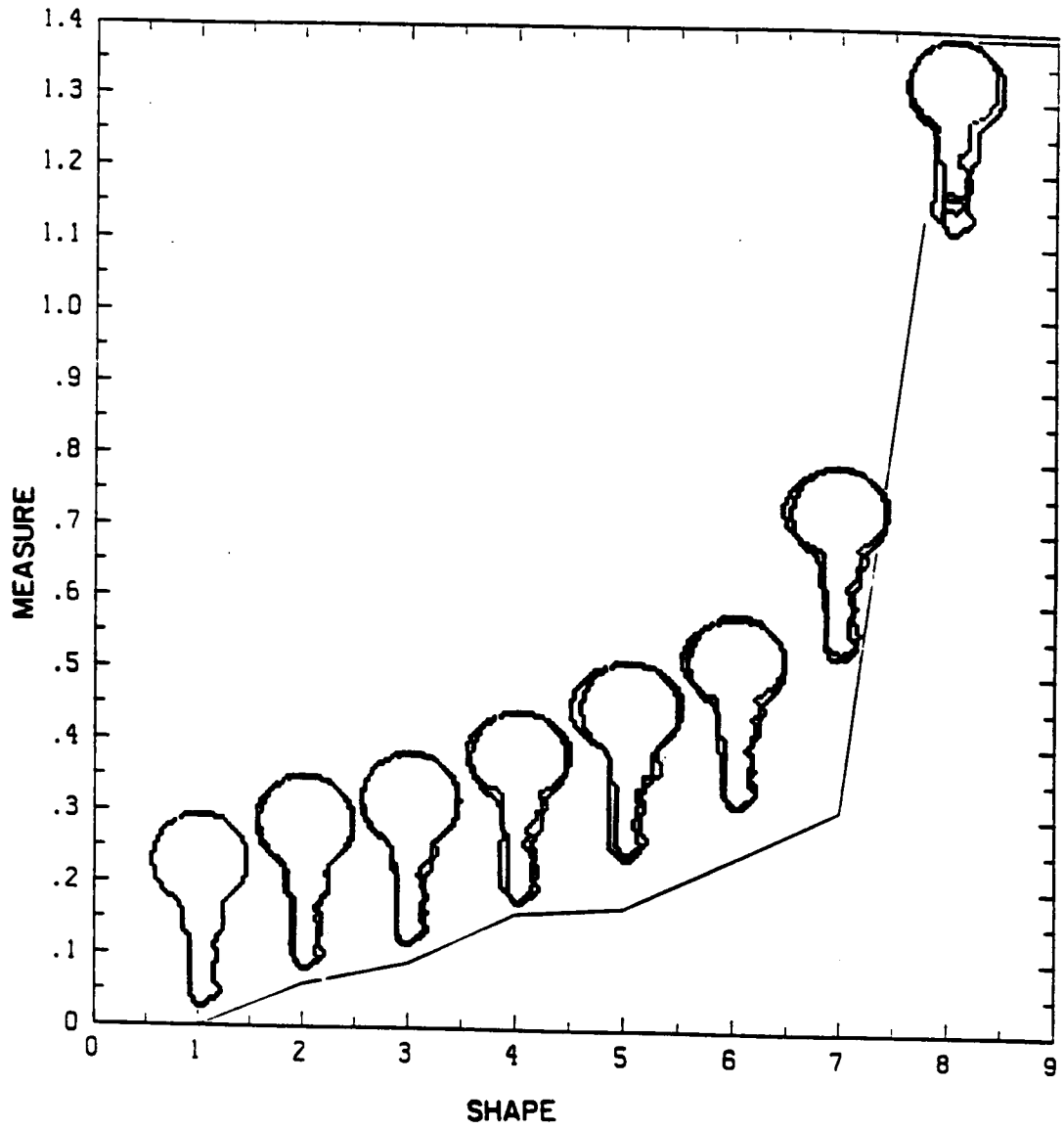


Figure 4.13: Matching keys to the reference key while maintaining the same orientation.

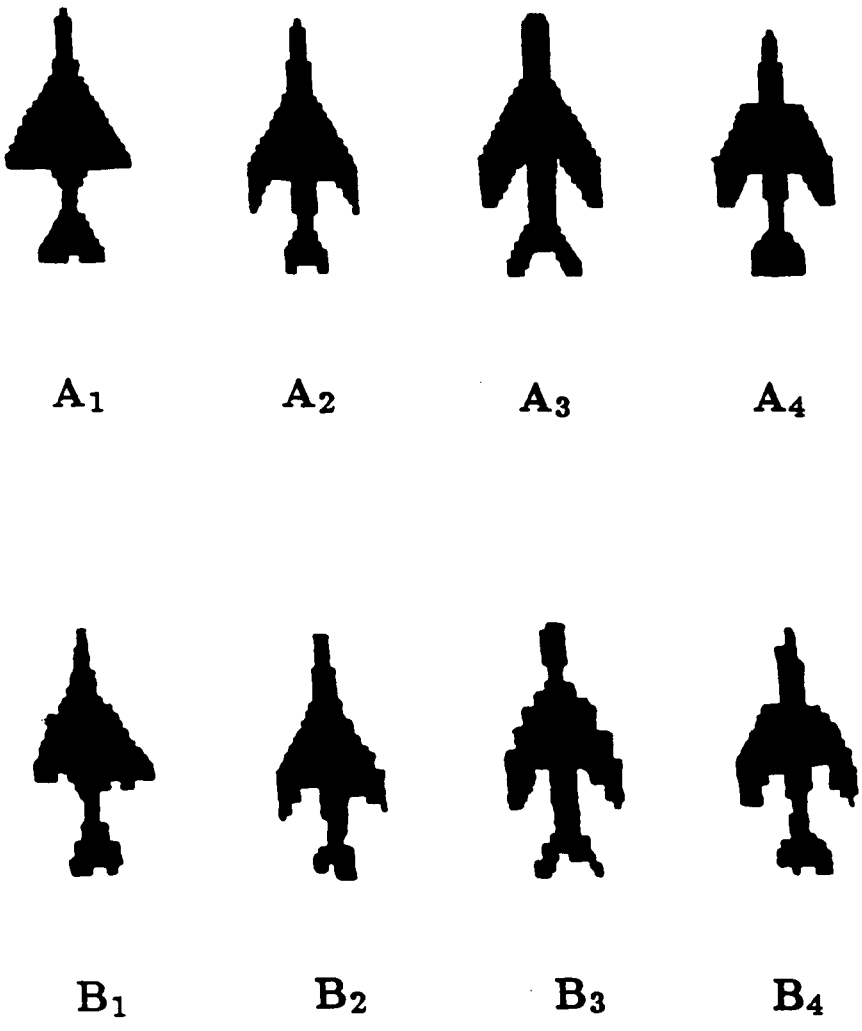


Figure 4.14: (a) Model airplanes, (b) noisy airplanes.

rule. From this example, it is apparent that the entropy based descriptor discriminated among all four model airplanes. The results of Table 4.2 are similar to Table 4.1, indicating the recognition of all four noisy airplanes except for B_3 where it can be recognized as A_3 or A_4. This result is expected because of the high similarities between A_3 and A_4. It is apparent, however, that the entropy based descriptor is more beneficial because it avoids the problem of multi-feature selection such as the use of four moments.

Figure 4.15 shows a sequence of a heart cross-section taken at different times [26,25]. Their corresponding characteristic functions are plotted. The general structure of all the shapes is reflected through their similar looking functions. In addition, local variations are apparent, and, therefore, can be extracted using the descriptor. The plot in Figure 4.16 shows the differences between the shapes that the descriptor detected. These shapes originally appeared with a different shape analysis approach [26]. The approach was based on the decomposition of boundaries into segments, then the correspondences between successive frames were established for detecting minimal change. The complexity of such an approach requires a relatively long processing time. The descriptor proposed in our work did not in all cases exhibit perfect results. Incorrect classification was caused by the variable data points in different shapes, which implied partial loss of shape information.

Tracking local shape variations is desired in many situations, however, multi-group classification is usually a more elementary stage in understanding scenes. Several arbitrary shapes are tested for global properties using only the circle

Table 4.1: Matching airplanes.

Measure	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$
$\mathbf{A_1}$	0.0	1.12	0.71	0.82
$\mathbf{A_2}$	1.02	0.0	1.56	0.61
$\mathbf{A_3}$	0.64	1.811	0.0	0.93
$\mathbf{A_4}$	0.86	0.67	0.86	0.0

Table 4.2: Recognizing noisy airplanes.

Measure	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$
$\mathbf{B_1}$	0.37	1.13	0.80	0.63
$\mathbf{B_2}$	0.93	0.35	2.03	0.98
$\mathbf{B_3}$	0.95	1.26	0.75	0.74
$\mathbf{B_4}$	0.80	0.57	0.59	0.12

provided with the shapes. The results are shown in Figures 4.17 and 4.18. A simple thresholding of the result yields the grouping of the Figure 4.18, where it is obvious that similarly looking shapes are within the same range of the descriptor value. It is important to note that similar groupings should result from any arbitrarily chosen model.

Recognizing characters is one of the attractive subjects of pattern recognition. Different types of methods have been implemented for the purpose of teaching computers to read machine-typed text or hand-written text. The machine-generated characters are standard and regular which makes them easier to recognize. Recognizing hand-written characters is a more complex task because of the variety involved in every individual's way of writing a character. We wanted to test the entropy based descriptor with a few machine-typed and hand-written numerals. Being able to extract a closed contour from the edges of a numeral is the only assumption needed in applying the descriptor. Table 4.3 presents the matching of five numerals, 1, 2, 3, 5, and 7; the diagonal of the table shows zeros indicating that each numeral is being recognized. We also tested the descriptor's ability to identify a numeral when it scaled or rotated. Table 4.4 shows the results; the closest match is considered to be recognizable numeral. Table 4.5 gives the results of comparing hand-written and machine-generated numerals.

It is the purpose of this work to demonstrate the capability of the entropy measure in discriminating shapes. The shapes were selected in order to observe the entropy based descriptor's discriminating power.

It is apparent from these results that the descriptor is at an elementary stage and more elaborate work is needed in order to obtain robust results. This can be achieved once the range of application is narrowed to specific shape application.

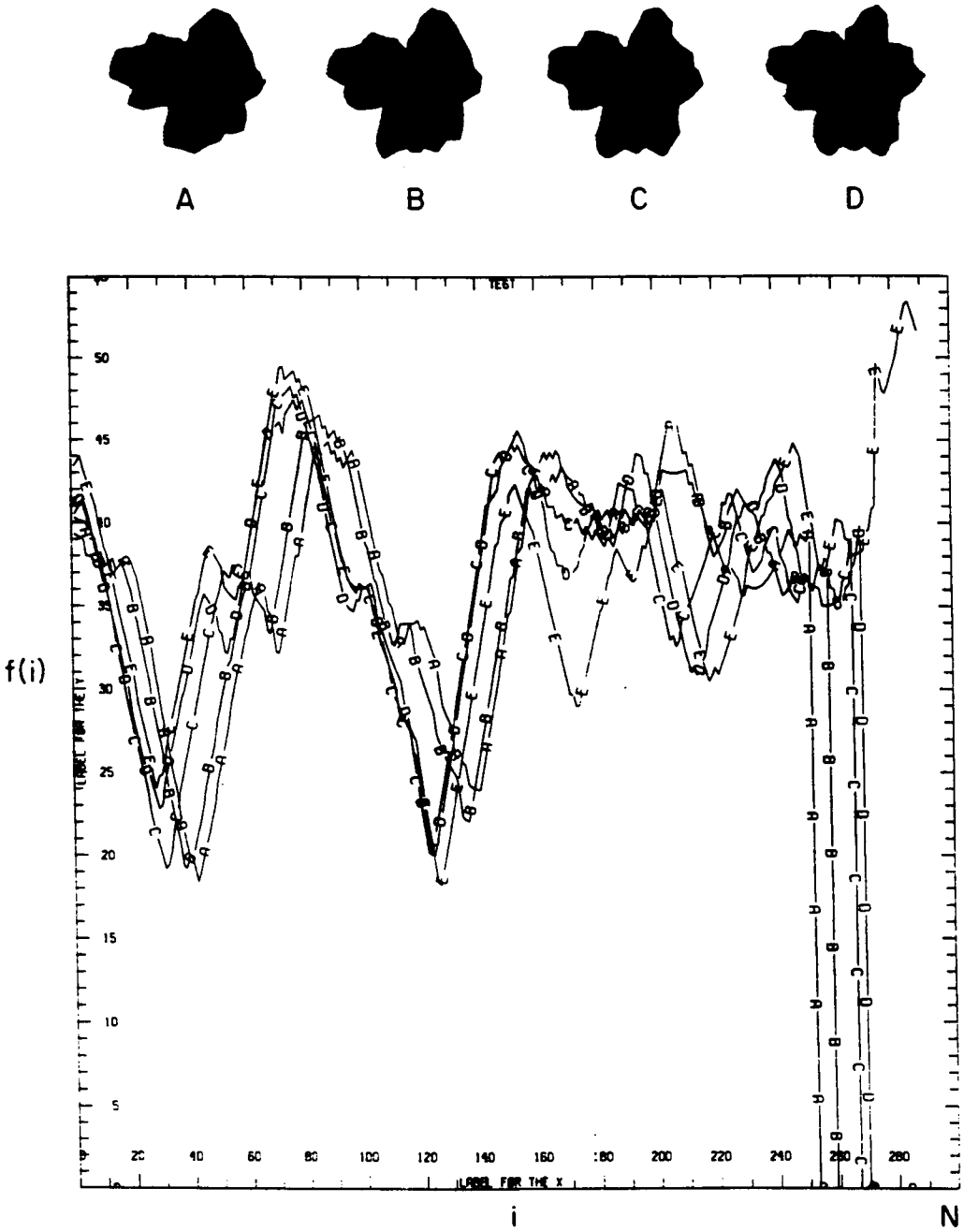


Figure 4.15: Characteristic functions for objects of the same class.

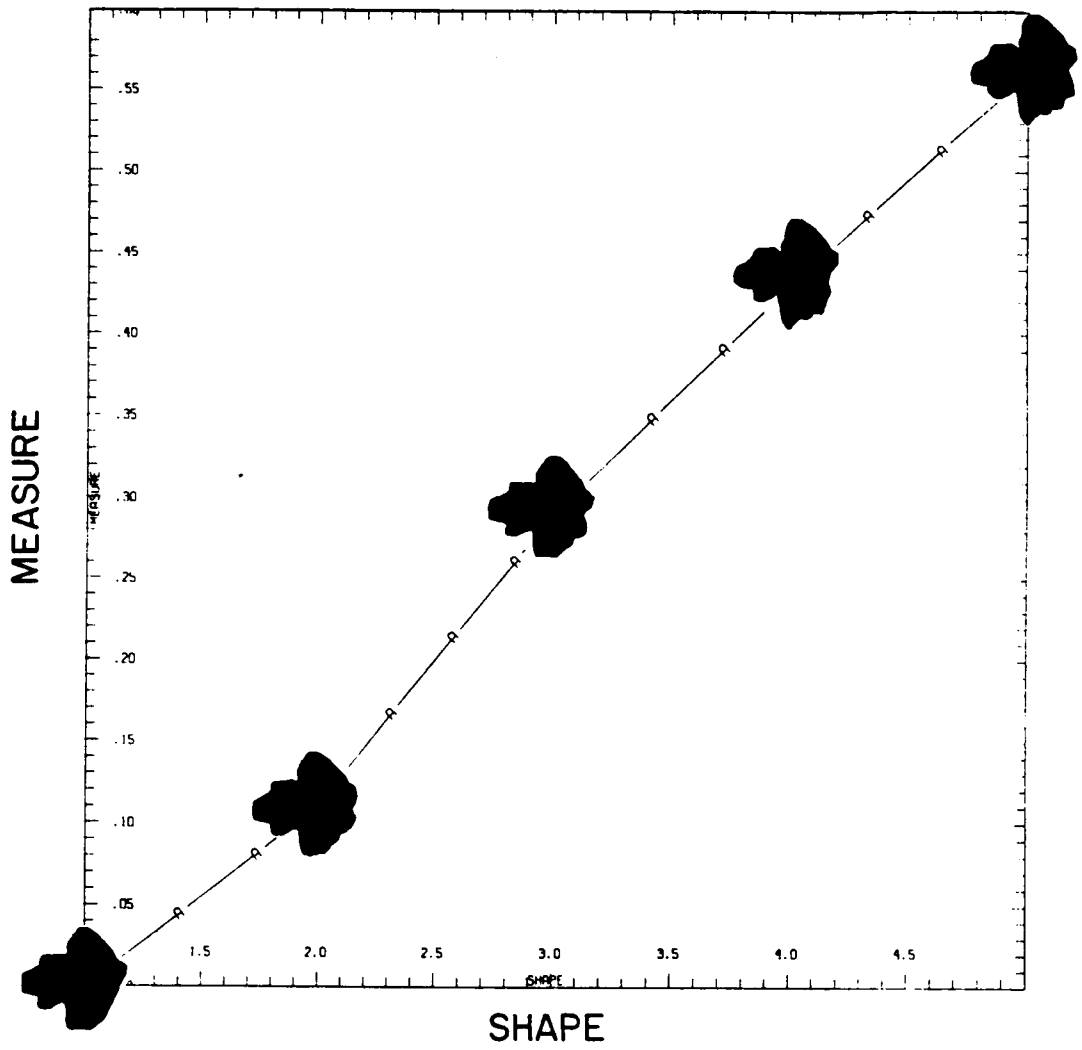


Figure 4.16: Tracking local variations within the same class.

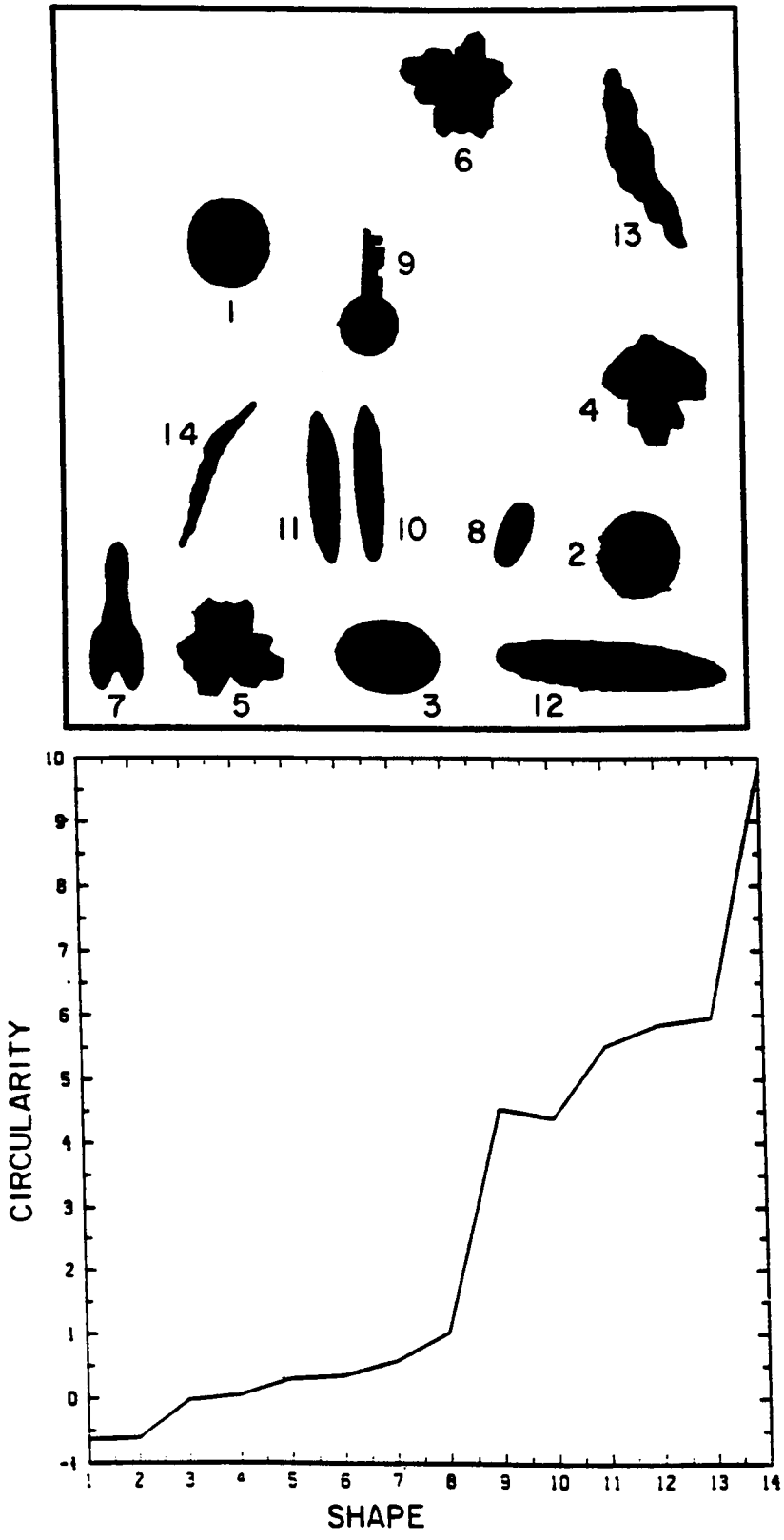


Figure 4.17: Plot of the descriptor values for the shapes shown above.

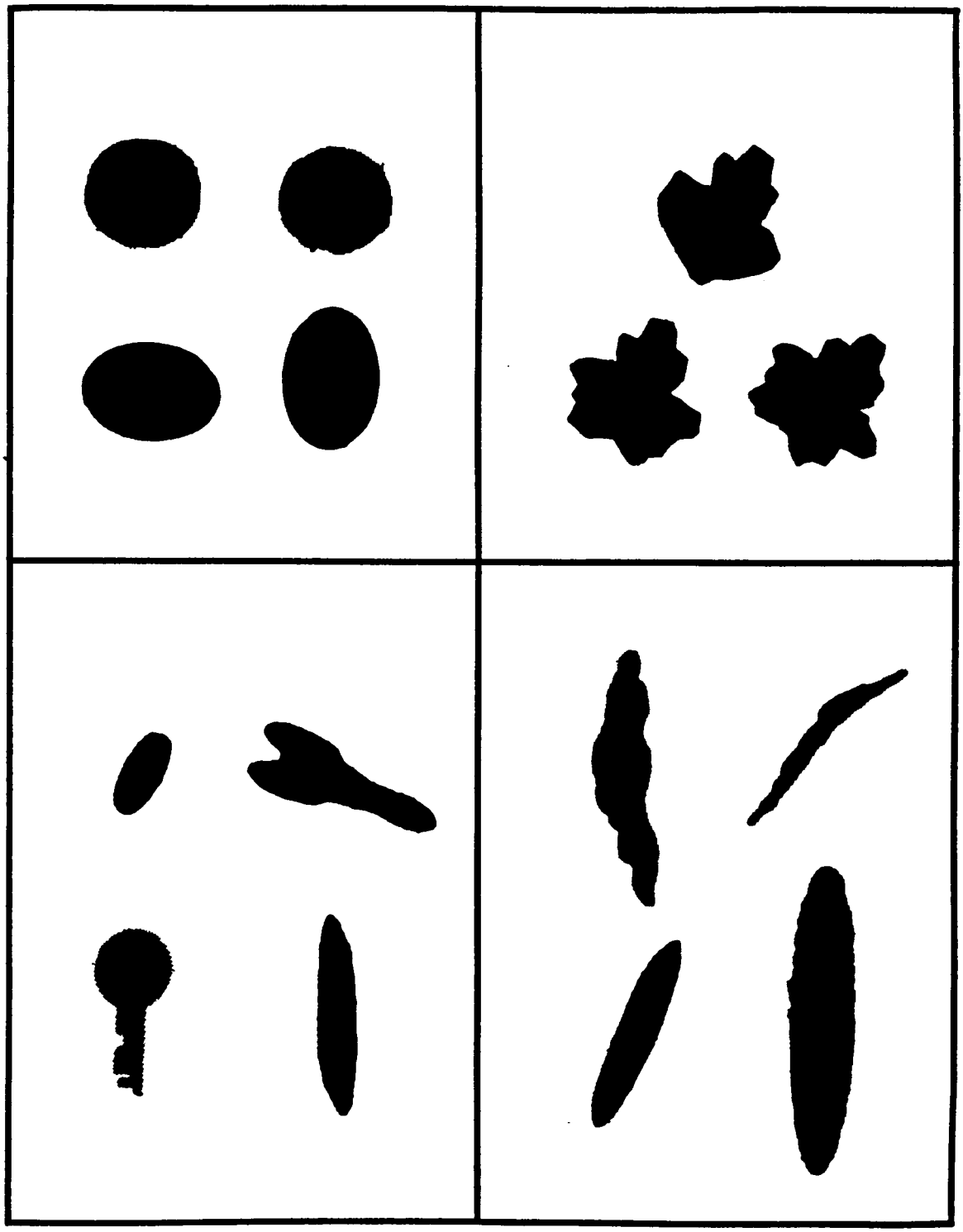


Figure 4.18: A possible classification based on the plot of Figure 4.17.

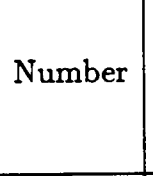
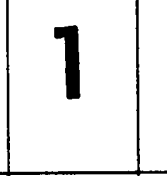
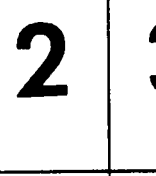
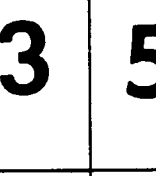
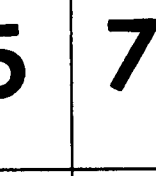
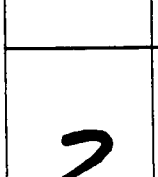
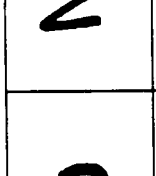
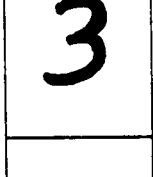
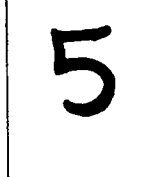
Table 4.3: Matching Numerals.

Number	1	2	3	5	7
1	0.0	9.4	2.7	7.8	3.4
2	11.2	0.0	12.4	10.5	15.7
3	2.8	7.5	0.0	10.3	3.4
5	7.3	9.6	18.3	0.0	19.2
7	3.4	8.2	3.9	7.3	0.0

Table 4.4: Matching scaled and rotated numerals.

Number	1	2	3	5	7
3	2.9	7.52	0.0	10.3	3.4
3	2.4	8.1	0.1	10.2	3.4
3	2.9	9.1	0.5	10.8	3.3

Table 4.5: Examples of matching hand written numerals.

Number					
	1.0	7.9	5.3	4.9	4.5
	12.7	2.1	9.8	5.7	8.9
	1.9	7.5	1.7	7.7	2.1
	3.4	6.9	6.2	2.3	5.0
	4.6	6.7	4.5	6.9	0.7

CHAPTER 5

CONCLUSION

The purpose of this work has been the development of a shape descriptor that has the ability to discriminate between closed contours and able to evaluate the difference between any two shapes, independently of their location, scale, and orientation. A shape signature was selected for representing a given shape because it facilitates shape normalization with respect to location, size, and orientation. The main reason for using the characteristic function (one form of shape signature) lies in the availability of advanced theories for correlating probability functions, which are equivalent to the characteristic functions of given shapes. As a result, entropy was shown to be a circularity measure in any closed contour while mutual information was useful in measuring differences between two shapes. Because actual implementations of the descriptor requires the same number of samples from the shapes under consideration, an algorithm which is based on nearest neighbor substitution was used for adding or eliminating extra samples. Even though reasonable results were obtained using the algorithm, a formulation of a descriptor based on two shapes with different size of samples is given at the end of Chapter 3. Applying the generalized form of the descriptor requires knowledge of the process that relates the shapes in consideration.

The same principle used in comparing two dimensional shapes can be ex-

tended to three-dimensional shape analysis. In practice, great difficulties could be encountered in evaluating the two-dimensional characteristic function in a 3-D object, especially if the object is not geometrically regular. To overcome this difficulty, the joint density concept is used to generate this surface using only two-dimensional cross-sections of the 3-D shape. This idea was introduced for future work. The entropy based descriptor was implemented for a variety of shape analysis applications. Local as well as global differences were registered in all cases.

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BIBLIOGRAPHY

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VITA

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In September 1987, she joined the graduate program and began her study toward a Master of Science in Electrical and Computer Engineering, at the University of Tennessee, Knoxville. During her study, she was a teaching and a research assistant; her research concentrated on the area of Image Processing and Pattern Recognition. In August 1989, Ms Ghozi received her Master degree.