To the Graduate Council:

I am submitting herewith a thesis written by Robert Alan Metcalf entitled "Development of Bayesian Procedures for the Calibration of Film Badge Dosimeters." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Masters of Science, with a major in Nuclear Engineering.

Dr. Peter Groer, Major Professor

We have read this thesis
and recommend its acceptance:

Accepted for the Council:

Associate Vice Chancellor
and Dean of the Graduate School

Development of Bayesian Procedures for the Calibration of Film Badge Dosimeters

A

Thesis

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ABSTRACT

Calibration of film badges uses data ($\underline{z}$) from a "calibration" experiment. These data consist of pairs $z_i=\{x_i,y_i\}$ where x_i are exposures fixed and chosen before the experiment and y_i are the corresponding film densities. To describe the saturation of film density at higher exposure levels the following model was used relating exposure and film density:

$$y=\beta_o+\beta_1 x e^{-\beta_2 x}+\mu$$

where β_0, β_1, and β_2 are parameters and $\mu \sim N(0,\sigma)$ is the normally distributed observational error.

Using Bayes' Theorem and a non-informative prior for β_1, β_2, and σ the expression for the joint posterior density of these parameters is obtained, assuming negligible background density β_0. Integration of the likelihood of a new observation, multiplied by the joint posterior density of β_1, β_2, and σ yields the predictive density which serves as a new "likelihood" in the derivation of the calibrative density.

This density describes the remaining uncertainty about a new unknown exposure X_n corresponding to a new observed film density Y_n. The advantage of the procedure described here stems from the fact that the uncertainty about X_n is described quantitatively by the calibrative density. This procedure is

applied to film badge calibration data sets used at Oak Ridge National Laboratory in the 1960's and 1970's and the results are compared to other calibration analysis.

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CHAPTER 1

Introduction

Film dosimeters are worn by individuals to determine the external radiation dose the given individual might receive. Calibration of film dosimeters entails determining the relationship between the density and radiation doses. This process involves exposing film from a given batch to known radiation doses. The densities of the film are then used to estimate a new, unknown radiation dose to another dosimeter from the same batch.

As calibration data have been collected for many years much information has been gathered on the radiation doses that individuals received. This information is necessary for epidemiological studies that try to determine the likelihood of health risks to the exposed individuals. Such studies attempt to quantitatively characterize the radiation risk for the individuals wearing the badges.

The Center for Epidemiological Research (CER) of the Oak Ridge Institute for Science and Education, in Oak Ridge, Tennessee, performs studies to determine the potential long term risks from occupational exposures to chemicals and radiation. Individuals who have been or are currently working

at Department of Energy (DOE) facilities are being studied to determine the potential health effects of their past radiation exposure. Estimates, based on the dosimetry data which exist, must be made to determine the radiation dose these individuals have received. This allows these studies to determine the potential risks for occupational exposure to radiation.

The purpose of this thesis is to develop better calibration methods, using Bayesian statistics, for film dosimeters. It will apply statistical methods developed by Aitchison [1975] to determine the distribution of a new unknown radiation exposure based on a film density and a set of calibration data. It will fit a model to the calibration data set and determine, posterior, predictive, and calibrative densities for use in determining radiation exposures. This was accomplished by obtaining actual calibration data sets and applying a model to the data by using mathematical derivations and numerical methods to integrate over the various parameters.

1.1 Film Badge Calibration

In the past calibration has been accomplished by drawing or calculating a function to a set of calibration data to produce a model of the dosimeter's response. The plotted line or curve is then used to determine a single value for a new response. A more detailed analysis of calibration data can be

used to find a distribution for the exposure corresponding to a new density reading and thus provide more quantitative information. Statistical methods using both classical [Snedecor, 1980] and Bayesian [Aitchison, 1975] techniques for linear regression have been used. Although these methods provide excellent results for systems which produce a linear response they do not describe the non-linear response of film to radiation.

Over 600 calibration data sets were obtained from CER. These data sets are from the years 1961, '63, and '72 and were used to determine exposures for workers at Oak Ridge National Laboratory (ORNL). The characteristics of these data sets were studied in order to determine a mathematical model which can be used to model the response of film to varying levels of radiation exposure.

Such a mathematical model provides the basis for Bayesian methods, which are used to determine a calibrative density for the calibration data. Once the method for determining the calibrative density is developed for this nonlinear model, it can then be compared to the linear regression model of calibration. The calibrative density characterizes the uncertainty of the radiation exposure to a new exposed film dosimeter.

1.2 Organization of Thesis

This thesis consists of three parts. The first describes film badges, their uses, and how the information gained from them has been used to assign radiation exposures in the past. The second provides a description of the Bayesian methods used for calibration. The last shows applications of the developed methods and comparisons with past methods.

The description of film badge dosimetry given in Chapter 2 is written with an emphasis on calibration. It first describes the interactions of radiation with film which allow it to be used as a radiation detector. It then describes how film is used as a dosimeter. Finally it describes past methods which have been used to calibrate film.

The next chapter describes the Bayesian Methods used for calibration. First it provides a brief introduction into Bayesian Statistics. Next it gives a derivation of the calibrative density using these methods. Then it shows how the chosen mathematical model for film calibration data is incorporated into the derivations of the calibrative density.

The final chapter applies the methods developed in Chapter 3 to actual calibration data sets. Two data sets are chosen for analysis. The first is chosen because it is typical of most of the many data sets obtained from ORNL. The other is chosen because it displays linear characteristics. They are both then compared to a linear regression calibration

method to highlight similarities and differences in the two methods.

Included in the appendices are: the results from other calibration data sets analyzed with the developed method, a listing and description of the computer programs written to calculate the probability densities and calculations of the linear regression method for the various sets of data as described in Aitchison [1975].

CHAPTER 2

Description of Film Badge Dosimetry

Ionizing radiations ability to expose covered film was first discovered by Becquerel who used it to describe the ability of radiation to penetrate and interact with matter [Rhodes, 1986]. The mechanisms which cause film to react to radiation determine its characteristic response. This response in turn allows film to be used as a dosimeter. But it must be calibrated to be useful in such a role.

2.1 Photographic Emulsions

There are two parts to understanding film and its interaction with radiation. First it involves understanding how radiation interacts with the film. Second is understanding how the reaction is brought forth to form an image on the film. To understand how film can be used to measure radiation exposure these two processes must be understood.

Film is usually comprised of a cellulose acetate or polyester strip coated on one or both sides by an emulsion. The emulsion is the sensitive material and consists of

microscopic silver halide crystals dispersed in a gelatin. The reaction of light or other forms of radiation with the crystals cause a latent image to be formed [Cember, 1988; Dudley, 1966].

The process of forming a latent image provides insight into the unique response of film to radiation. As the silver halide crystals are bombarded by radiation, energy is transferred to electrons in the crystals which cause the formation of electron holes. The formation of these holes along with the positive silver ions causes the silver to agglomerate into small clumps in the crystal. These clumps are the essence of the latent image [Cember, 1988; Dudley, 1966].

The development of the latent image is a process which finds crystals which have been exposed to a sufficient amount of radiation and chemically converts the entire grain into elemental silver. The developer consists of electron donating molecules which chemically reduce the positive silver ions. The electron donating capability of the developer determine what kind of image is brought forth in the film [Dudley, 1966].

The image which is formed on the film is a factor of many things. The size of the crystals as well as the chemical make-up of the gelatin in which it is suspended cause differences in clarity and sensitivity of the image. The intensity and type of radiation reaching the film cause it to

react differently. The type of development used is usually a function of the properties of the film and is usually recommended by the manufacturer, as different methods provide varying degrees of development [Cember, 1988; Dudley, 1966].

Because of the complex interactions and chemistry required to form a latent image and bring it out on film, small changes in the process can cause large differences in the final result. For this reason it is considered wise to treat the properties of each batch of film produced as unique, even if the next batch is produced by the same materials and processes [Cember, 1988]. This explains the existence of so many sets of calibration data, one for each batch used as dosimeters.

2.2 The Reactions of Film to Radiation

Film is sensitive to many types of radiation. Its use as a dosimeter is determined by the type of radiation being monitored. The types of film used and the construction of the film holders allow for the discrimination between the different radiations.

Film is primarily used for the detection of gamma and X-rays [Barber, 1966]. They show a high energy dependence below 200 keV caused by the large photoelectric cross section of silver at those energies [Cember, 1988; Dudley, 1966]. This energy dependence can be accounted for by the use of screens

and filters in the film badge holder.

Beta radiation can also be measured by film badges. Its interaction with film is the same as that for gamma and X-rays. The effect of beta radiation can be measured by providing an open window on the film badge holder. The comparison of the film densities corresponding to shielded and unshielded parts of the film allows the determination of the beta exposure [Dudley, 1966; Ehrlich, 1962].

Neutrons can also be detected on film. Fast neutrons are detected by configuring the film and holder to produce proton recoil tracks from elastic collisions of the neutrons with hydrogen nuclei [Cember, 1988]. Thermal neutrons can be made to produce proton tracks from capture by nitrogen which decays to carbon producing a proton. Counting the proton recoil tracks provides an estimate of the neutron dose [Cember, 1988; Dudley, 1966; Ehrlich, 1962].

2.3 Film Calibration Methods

Calibration of film is accomplished by determining the response of film to radiation as a function of exposure. The response varies with the radiation dose to the film and the type of film being used. To determine this response the film is exposed to doses from a known radiation source (typically Co-60). Several pieces of film are exposed to different doses from the source and the exposure versus the density of the

developed pieces of film is plotted to make a calibration curve. A new exposure is found using this calibration curve by first developing a new piece of film which has received the new exposure. Then the curve is used to relate the film density of the developed film to an exposure [Cember, 1988].

Different types of film will produce different calibration curves, but the general response of all types of film to radiation is similar. An example of a typical film response curve is shown in figure 2.1. At low doses the film can under respond due to insufficient formation of a latent image which does not produce a film density when developed. At higher doses film exhibits a linear relationship between film density and exposure. At still higher exposures film becomes less responsive to increases in the exposure. This is caused by the formation of a sufficient latent image in almost all of the film crystals that none are left to react to the increase in exposure. Many types of film show complete saturation at extremely high doses, but some have shown a failure to record extremely high exposures thus producing a reduction in the film density at these levels [Ehrlich and McLaughlin, 1956; Morgan, 1944; Becker, 1961].

Different mathematical models have been used to describe the calibration curve. The most common is:

$$D=kaN(1-e^{-aF})$$

which relates the properties of the film being used to its radiation response. The symbol k is a constant, N and a are

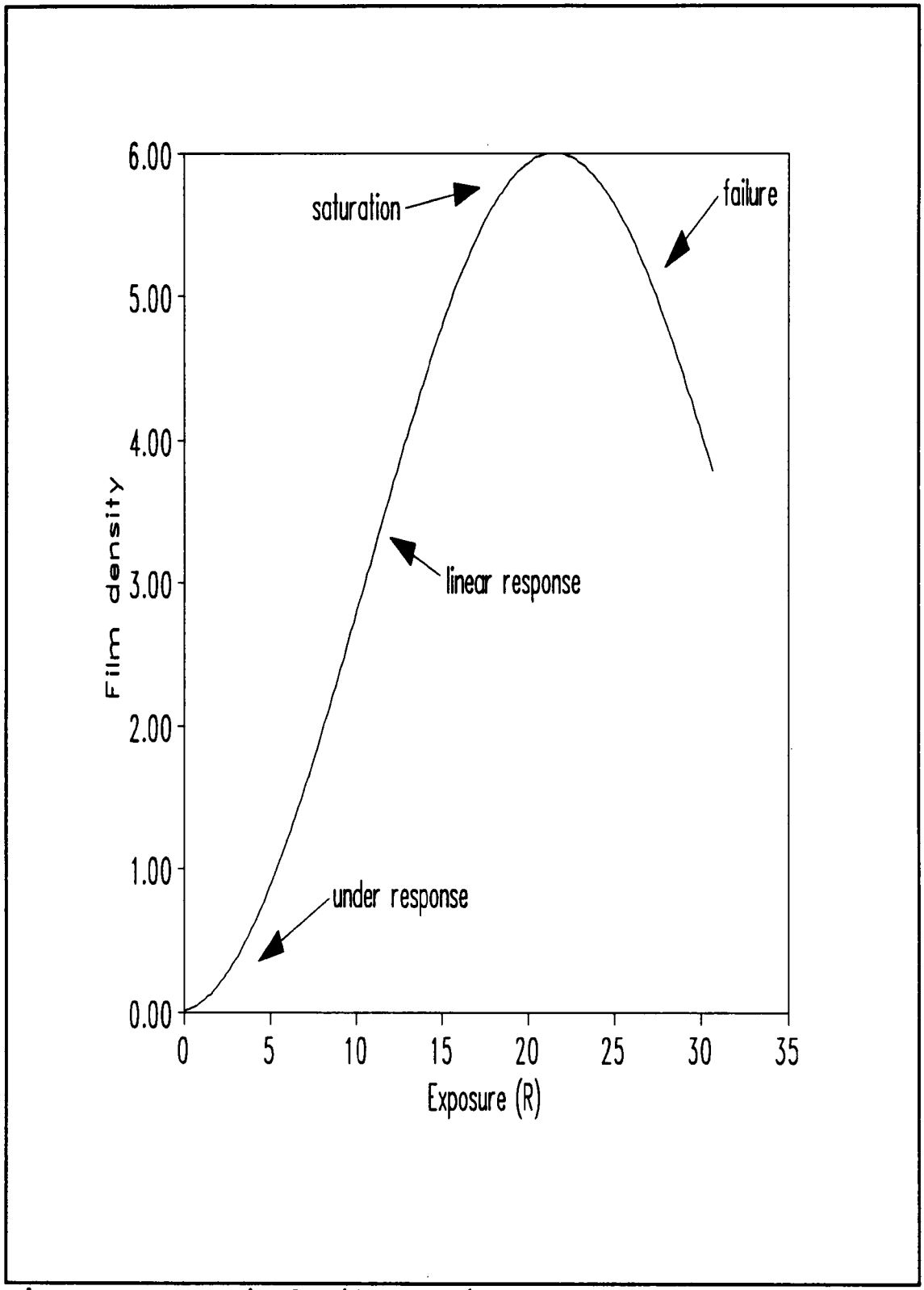


Figure 2.1 Typical film density response curve. The four main regions shown vary for different types of film.

the number and size of the film grains and E is the exposure [Dudley, 1966; Miyanaga, 1962]. Another model uses the concept of an apparent dose which has a linear relation to film density. This model is:

$$AD=\beta\,[1+\alpha_1 R+(\alpha_2 R)^2+(\alpha_3 R)^3]\,[\log R_{max}-\log(R_{max}-R)]$$

Where AD is the apparent dose, R is the film density R_{max} is the saturation film density, and α_1, α_1, α_1, and β are parameters determined by a computer fit of calibration data [Mota, 1990]. These models show that the film density response to radiation is often described by variations on an exponential model.

CHAPTER 3

Bayesian Techniques for Calibration

Chapter 2 described the way in which film and ionizing radiation interact and calibration of the film to a radiation exposure is performed. It also discussed the various methods which have been used to describe film reaction to radiation and yield a calibration curve. What is not described is the uncertainty which is inherent in assigning a radiation exposure to a given film density. This chapter discusses how this uncertainty is quantified for calibration procedures and the methods needed to provide a probability density of the radiation exposure corresponding to a given film density.

As described in section 2.1 the composition of film and processes used to produce and develop a latent image on it are sensitive to slight changes. To compensate for this each batch is calibrated separately. The film can come in contact with other things to which it is sensitive. These include abrasion, chemicals, light, temperature, and even humidity [Ehrlich, 1961; Kathren and Brodsky; 1963]. There are still more elements which could differ within a batch not covered in section 2.1 such as the thickness of the emulsion or the rate or difference in developing chemicals which can cause non-

uniform development of the film [Ehrlich, 1956].

The influence of these unknown factors are what necessitates a statistical analysis of the calibration. Choosing a single exposure based on a given film density does not describe the uncertainty inherent in the calibration. Fitting curves to the response of film to radiation helps in more precise determinations of the exposure based on the given calibration but it still does not describe the remaining uncertainty.

This chapter will introduce the term μ, the error in the film density, y, from the actual exposure, x, incident on the film. This error will be assigned a normal distribution. The relationship between exposure and the error allow the exposure to be related to a probability density. Evaluation of this density using Bayesian methods is what allows the calculation of a calibrative density for the exposure.

3.1 Introduction to Bayesian Calibration

Calibration can be defined as the process of relating the response of an "instrument" to the property of some "agent" which causes the instrument to respond. The term agent refers to any physical entity having properties can be measured and the term instrument stands for any physical device that can be affected by the agent such that the effects can be observed. To accomplish a calibration, a calibration experiment is

performed using agents with known properties. This provides the data necessary to attach meaningful values to the instrument's scale. This is a simple and straight forward process assuming that each response measured corresponds to only one place on the scale. However this is not usually the case. The random error which is inherent to the process of an instrument measuring an agent causes a distribution of values for the instrument for a given agent with constant properties. To account for this uncertainty a calibrative distribution is needed.

The first step in obtaining a calibrative distribution is to determine the type of the calibration experiment being performed. There are two types of calibration experiments, natural and designed [Aitchison, 1975]. In the natural case the values from the independent variable (agent) used in the calibration provide information concerning future values for this variable. An example is using two instruments to measure an independent variable where one of the instruments is known to be more accurate than the other. The independent variable is naturally occurring and thus is uncontrolled by the experimenter using the instruments. Assuming that any future values for the independent variable arise in the same way as the values of the independent variable used to form the calibration data, information as to the arrival pattern of the independent variables is provided. Thus the less accurate measurement can be used to give information as to the expected

measurement of the more accurate instrument. In a designed experiment the values of the independent variable are selected and fixed by the experimenter and thus they provide no direct information concerning future values. An example of this are measurements of the same variable with one instrument for preselected values of the independent variable. A future value of the independent variable is not related to the values which have been selected in the past. Film badge calibration is therefore a designed calibration experiment and the subsequent derivation of the calibrative density will be based on this fact.

3.2 Derivation of the Calibrative Density

A calibrative density can be found by using a predictive density as a new likelihood and multiplying this by the prior density for the independent variable. For film badge calibration a predictive density is the density function for the film density given a known new exposure and a calibrative density is the density function of the exposure given a known new film density [Aitchison, 1975]. To derive the calibrative density the predictive density must first be found.

The following derivation is given for the case of film badge dosimetry with the following assumptions:

1) The probability model describing the experiments

which have been performed (i.e. film exposure to radiation) and the probability model which describes the future experiment have the same index or parameter set θ, and the true models have the same index θ^*.

2) For a given index θ the calibration experiment which has been performed and the future experiment are independent.

3) The calibration experiment is a designed calibration experiment.

These assumptions are true in the case of film badge calibration.

4) All marginal prior distributions are uniform distributions.

Using these assumptions the derivation starts by considering the calibration data set, $\underline{z}$, which consists of n pairs of exposures, x_i, and corresponding film densities, y_i, , i=1,2,...,n. The contribution of one pair (x_i,y_i) to the likelihood is:

$$P((x_i,y_i)|\theta) \qquad 3.1$$

The joint posterior density is the product of the likelihood

and the prior density. Thus with assumption 4 it becomes:

$$P(\underline{z}|\Theta) \propto \prod_{i=1}^{n} P(x_i, y_i|\Theta) \qquad 3.2$$

The likelihood of a new observation, Y, is:

$$P(Y|X,\Theta) \qquad 3.3$$

Equation 3.3 multiplied by the joint posterior density and integrated over the index Θ becomes the predictive density:

$$P(Y|X;\underline{z}) \propto \int_{-\infty}^{\infty} P(Y|X,\Theta)\, P(\underline{z}|\Theta)\, d\Theta \qquad 3.4$$

Again using assumption 4 the predictive and the calibrative densities are proportional and thus the calibrative density is:

$$P(X|Y;\underline{z}) \propto P(Y|X;\underline{z}) \qquad 3.5$$

The general derivation of the calibrative density is provided by Actchison [1975].

3.3 Description of the Exposure Density Relationship

To make use of the derivations described in section 3.2

a model for the likelihood is required. Linear regression models have been used and the results for such models can be found in Aitchison [1975], but, as discussed in chapter 2, a linear description of the film calibration curve is not a good approximation at higher exposures.

The model chosen is:

$$y_i=\beta_0+\beta_1 x_i e^{-\beta_2 x_i} \qquad 3.6$$

β_0 and β_1 are the y intercept at $x_i=0$ and slope. The exponential term containing β_2 decreases the slope until it reaches a maximum and becomes negative. The index or parameter set, θ, is therefore $\{\beta_0, \beta_1, \beta_2, \sigma\}$. Figure 3.1 shows a representation of the model for given values of the model parameters.

This model was chosen to fit the calibration data for several reasons. First, although it was stated in section 2.3 that film may not respond linearly to radiation at low exposures the values of calibration data obtained from ORNL did not show this behavior. Second it allows for saturation as the exposure increases and at higher exposures for a reduction of the response.

Finally a few assumptions simplify the calculations needed to obtain the different densities. Because of the characteristics of the model as they pertain to the film dosimeter response to radiation, β_1 and β_2 are always positive

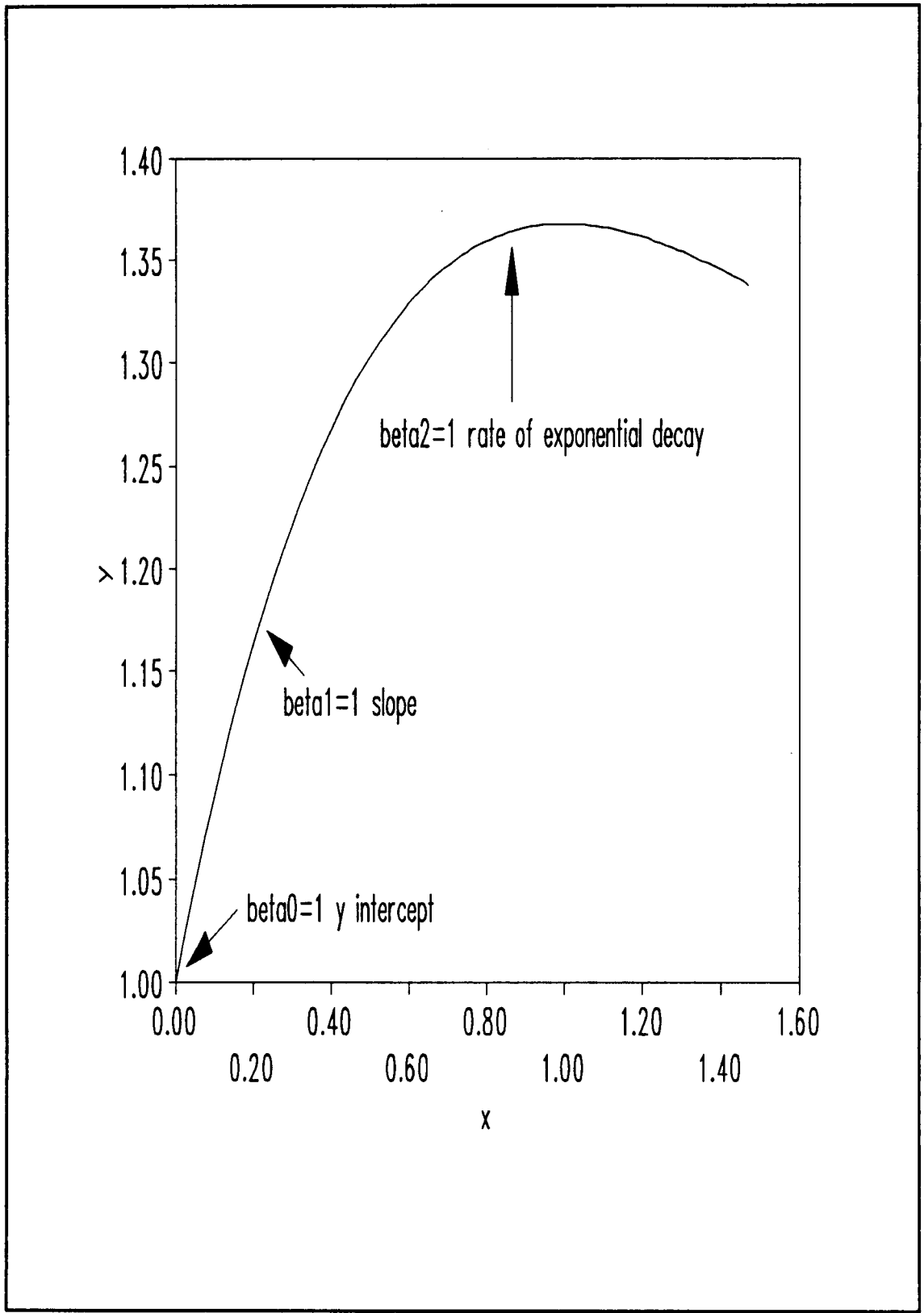


Figure 3.1 The calibration model for β_0=1, β_1=1, and β_2=1.

and a range from 0 to ∞ is assumed. The following assumptions are made concerning the relationship of film dosimeters and the model.

1) The background density of film (i.e. unexposed film) is negligible. Thus β_0, the y intercept is approximately 0.

2) The error μ is the same for all measurements and is normally distributed with mean 0 and standard deviation σ ($\mu \sim N(0,\sigma)$).

From these assumptions one obtains:

$$\mu_i = y_i - \beta_1 x_i e^{-\beta_2 x_i} \qquad 3.7$$

substituting equation 3.7 into the normal distribution gives the contribution of one observation to the likelihood:

$$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(y_i - \beta_1 x_i e^{-\beta_2 x_i})^2} \qquad 3.8$$

and the joint posterior density with a constant prior density becomes:

$$P(z|\beta_1, \beta_2, \sigma) \propto \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(y_i - \beta_1 x_i e^{-\beta_2 x_i})^2}$$

$$\propto \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{n} e^{\sum_{i=1}^{n} -\frac{1}{2\sigma^2}(y_i-\beta_1 x_i e^{-\beta_2 x_i})^2} \qquad 3.9$$

The marginal posterior densities of the model parameters are obtained by integrating over the other two parameters from 0 to ∞ (σ is always positive). The integration over σ can be done analytically as shown in Appendix I and the joint posterior density for β_1 and β_2 becomes:

$$\int_0^{\infty} P(z|\beta_1, \beta_2, \sigma)\, d\sigma \propto$$

$$\left[\sum_{i=1}^{n} (y_i-\beta_1 x_i e^{-\beta_2 x_i})^2\right]^{\frac{1}{2}-\frac{n}{2}} \qquad 3.10$$

Using the same normal distribution for the error the likelihood of a new observation, Y, is:

$$P(Y|X, \beta_1, \beta_2, \sigma) \propto \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(Y-\beta_1 X e^{-\beta_2 X})^2} \qquad 3.11$$

The predictive density is the product of the joint posterior and the likelihood integrated over the parameters β_1, β_2, and σ:

$$P(Y|X;Z) \propto \int_0^\infty \int_0^\infty \int_0^\infty \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{n+1} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i-\beta_1 x_i e^{-\beta_2 x_i})^2}$$

$$e^{-\frac{1}{2\sigma^2}(Y-\beta_1 X e^{-\beta_2 X})^2} d\sigma d\beta_1 d\beta_2 \qquad 3.12$$

This also can be integrated exactly over σ by the same method as shown in Appendix I and it becomes:

$$P(Y|X;Z) \propto$$

$$\int_0^\infty \int_0^\infty \left[\sum_{i=1}^{n}(y_i-\beta_1 x_i e^{-\beta_2 x_i})^2 + (Y-\beta_1 X e^{-\beta_2 X})^2\right]^{-\frac{n}{2}} d\beta_1 d\beta_2 \qquad 3.13$$

As stated before the predictive density is proportional to the calibrative density and thus equations 3.12 and 3.13 hold for it as well. To obtain the predictive density a new value for X (exposure) is chosen and the model is integrated over β_1 and β_2. The calibrative density is obtained by providing a value for Y (film density) and performing the same integration. The integrations for both β_1 and β_2 require numerical methods which will be discussed in depth in the next chapter.

CHAPTER 4

Data Analysis

This chapter will use the techniques developed in chapter 3 on actual film badge calibration data sets. Over 600 sets of calibration data were obtained for ORNL. Enough sets of data were collected to obtain an overall idea of film performance and calibration curves that are common from ORNL. Several of these data sets were chosen for analysis, and their differences and similarities are discussed.

The detailed analysis of two of these data sets is shown in the text of this thesis, while the calculation for an additional data set is in Appendix II. These data sets were chosen because they exhibit the range of attributes found throughout all of the data sets.

4.1 History and Origin of the Calibration Data

The data was used to determine doses for workers at ORNL during the years 1961, 1963, and 1972. There are two reasons for the ages and spread in years of the data. The first reason is to provide a data set sufficiently old such that a statistical distribution of the doses incurred is

epidemiologically meaningful. Since the latent periods for radiogenic concerns are often a decade or longer, past have exposures to be considered. The second reason is to study how consistent the methods used to perform the calibration remained over time. A sufficient number of data sets were collected and evaluated to determine that the data are representative of the calibration experiments which took place at ORNL during the time span indicated (1961-1972).

Figure 4.1 is a replica of a typical calibration data sheet. As each batch of film arrived weekly from the Eastman-Kodak company it was exposed to various known radiation doses and a sheet similar to this was filled out. A calibration was done for each new batch to obtain more precise dose information since the film response to radiation changes from batch to batch. This data was then transferred to graph paper and a calibration curve was plotted by hand along the data points.

4.2 Calculation Methods

Integrations over β_1 and β_2 and σ are needed to obtain the various marginal densities. No exact method of performing these integrations was found, therefore numerical integration methods were used. To perform the calculations several computer programs were written in the C language.

CALIBRATION DATA

Dev. Temp._____°F Film Meters ( ) Week or Process No. _____

Dev. Time _____ Mis. Film ( ) Batch or Series No. _____

Dark Room _____ Date _____

	Sensitive Film Density				Insensitive Density		
Exposure in (mr)	Beta OW	Gamma OW	Gamma S		Gamma OW	Gamma S	Remarks

Figure 4.1 Replica of a calibration data sheet typical of those studied.

4.2.1 Numerical Integration Methods

Numerical methods are not exact therefore several precautions were taken to assure adequate accuracy. Two problems were identified as important, those dealing with the range of integration and those dealing with calculational errors. These problems cannot be eliminated but steps can be taken to minimize their effects.

While the range of integration for both β_1 and β_2 is from 0 to ∞, most of the mass of the densities for β_1 and β_2 is concentrated on small intervals. Most of the mass of β_1 lies between 0.0005 and 0.0015 and for β_2 between 0.00001 and 0.0005 for all the data sets studied. Thus a method for assuring that the integration interval covers the area where the mass is concentrated is needed.

To minimize calculational errors from numerical integration methods precautions need to be taken. Numerical methods such as Gauss quadratures give exact values for polynomials of the n^{th} degree if a certain number of points (n or larger depending on the method) are used [Chaney, 1991], but if the function chosen for the model is exponential exact integration is not possible.

Two fundamentally different methods of integration were chosen to ensure that the calculational error is small. Integration methods based on approximate representations of the function such as the mid-point method or the trapezoidal

rule, and Gauss quadrature methods, provide the different approaches. Simpson's rule was selected over the mid-point method because it offers a better approximation [Chaney, 1991] and because it has been used in other solutions to this type of problem. Although Gauss-Laguerre seems to be the obvious quadrature choice since it applies to the integration interval $(0, \infty)$ a Gauss-Legendre quadrature was chosen because: (1) it has finite limits of integration thus facilitating the calculations, (2) it is easier to use than Gauss-Laguerre which requires the function being integrated to be of the form $f(x)e^{-x}$.

Since the two methods chosen use finite limits of integration adjustments are necessary to cover the range of maximum concentration. This limits the number of calculations needed, but care has to be taken to assure that the interval with the maximum concentration of the density is covered. Trial and error was used to determine the number of integration points which were necessary to provide a smooth integration over the main support. This number ranged from 50 to 100. Therefore to be certain that the numerical method covered the range of integration all integrations were preformed at 200 points (n=200).

To assure that the calculations were performed over the main supports several different calculations were necessary. First a calculation of the posterior densities of β_1 and β_2 was performed to determine the range where the probability mass

concentrates. This was done by trial and error until the range was found. Then in the calculations of the predictive and calibrative densities these ranges were used. Thus a study of the ranges of β_1 and β_2 was necessary for each calibration data set.

4.2.2 Computer Programs

The computer programs which performed the calculations were written in C. Four densities were needed for each data set studied, posterior densities of β_1 and β_2, and predictive and calibrative densities of the film density and exposure. This section is a description of the similarities and differences in calculating each of these densities.

Determination of all the needed densities requires the calculation to be performed with 200 points for reasons stated in section 4.1.2. For Simpson's Rule this is a simple matter, but the Gauss-Legendre Quadrature requires the use of points between -1 and 1 and their corresponding weights. These are calculated by using complex sets of quadrature equations. The 200 points needed were obtained using the IMSEL mathematics library on the U.T. mainframe. The procedure used and the points obtained are shown in Appendix III (part A).

All of the programs were written to provide an output file of the numbers necessary to obtain a graph of the distribution. To run the programs information on the data set

and the ranges for calculation were needed. Input files were created for these purposes and an example and explanation of the input files for the calculation of each density is provided in Appendix III (part B).

The calculation of the posterior distributions of β_1 and β_2 is very similar. They both require integration over the other to calculate the points for the distribution. This single integration is preformed at 200 points with 200 calculations for both numerical methods at each point. The programs are shown in Appendix III (part C).

The calculation of the predictive and calibrative densities is also similar. Both densities require a double integration, over β_1 and β_2, with a known exposure provided to determine the predictive density of the film density and a known film density provided to determine the calibrative density of exposure. The double integration is preformed at 200 points with 200 calculations being preformed at the lowest level of integration and the results of each put into separate arrays and then numerically integrated using the midpoint method. This second level of integration is preformed at 200 points to provide the data for each density. This lengthy calculation was necessary to assure accuracy but methods could be developed to shorten the calculation for practical use. The programs for calculating the predictive and calibrative densities are shown in Appendix III (part D).

4.2.3 Validation of Computer Programs

The plots of the various densities are shown in the next sections. The numerical methods used to produce these plots are satisfactory because they address both of the problems associated with applying numerical methods as stated in section 4.2.1. These problems were range of integration and the error in the methods.

In all the plots of the various probability densities shown in the next sections both Simpson's Rule and the Gauss-Legendre quadrature methods were applied to the same integration. The figures seem to show one graph even though the results for both methods are plotted. The two methods provide close results in all cases so that the error between them not perceptible. For this reason it is concluded that the methods used provide the correct integrations.

The range of integration which was used for β_1 and β_2 changes for each calibration data set as described in section 4.2.1. For each calibration data set discussed in the following sections the integration ranges of β_1 and β_2 are given. A comparison of these ranges to the plots of the posterior distributions of β_1 and β_2 shows that the main support of each distribution is sufficiently covered by the integration methods.

4.3 "Saturation" Data Set

The first data set to be explored will be named the saturation data set because it displays saturation of the film at higher radiation exposures. As the level of radiation increases the number of ionizing events in the film increases. This ionization continues with increasing exposure but less and less of the ionizing events are recorded as a latent image due to the saturation of all the reactions which produce the latent image on the film. This data set was chosen because it is typical of the majority of the data sets studied. Table 4.1 shows the saturation data, obtained from ORNL. The range and selection of radiation exposures for calibration is also typical of the majority of the data sets studied. Figure 4.2 is a plot of the data. From this figure the non-linear response of film to gamma radiation can be seen.

Table 4.1 "Saturation" data set

exposure (mR)	film density
500	0.41
1000	0.7
1500	0.96
2000	1.08
5000	1.73

4.3.1 Posteriors densities of β_1 and β_2

The posterior densities of the model parameters were

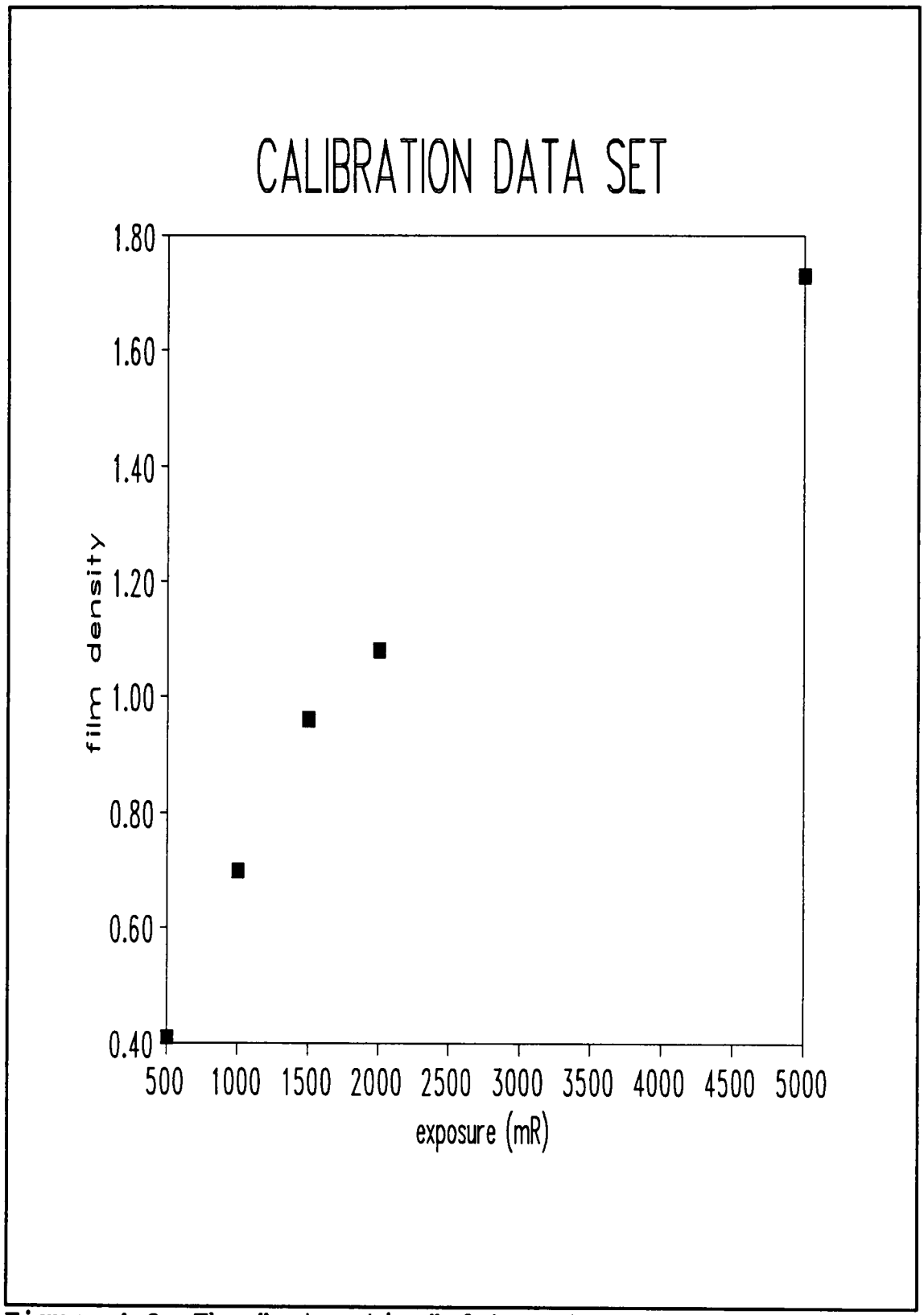


Figure 4.2 The "saturation" data set.

calculated using numerical methods in computer programs as described in section 4.2. Figure 4.3 shows the marginal posterior density of the model parameter β_1. Figure 4.4 shows the marginal posterior density of the model parameter β_2. From these plots the limits of integration for β_1 and β_2 were chosen to be from 0.0006 to 0.001 and 0.00005 to 0.0003 respectively. Because the prior densities of the model parameters are assumed to be constant a comparison of the model to the calibration data points is possible. This is accomplished by inserting the mode of the posterior densities of $\beta 1$ and $\beta 2$ in the model equation and plotting it along with the calibration data points. The posterior modes were found to be 0.0008 and 0.000165 for β_1 and β_2 respectively. Figure 4.5 shows this plot. From this comparison it can be seen that the model using these values for β_1 and β_2 fits the data closely.

4.3.2 Predictive Density

Figure 4.6 shows the predictive density found using the methods described in section 4.2. It shows the uncertainty about the film density when the film has been exposed to a radiation exposure of 3000 mR. The mode of this density was found to be at a film density of 1.45. Comparing this value for film density to the plot of the model shown in figure 4.5 shows it corresponds approximately to a radiation exposure of

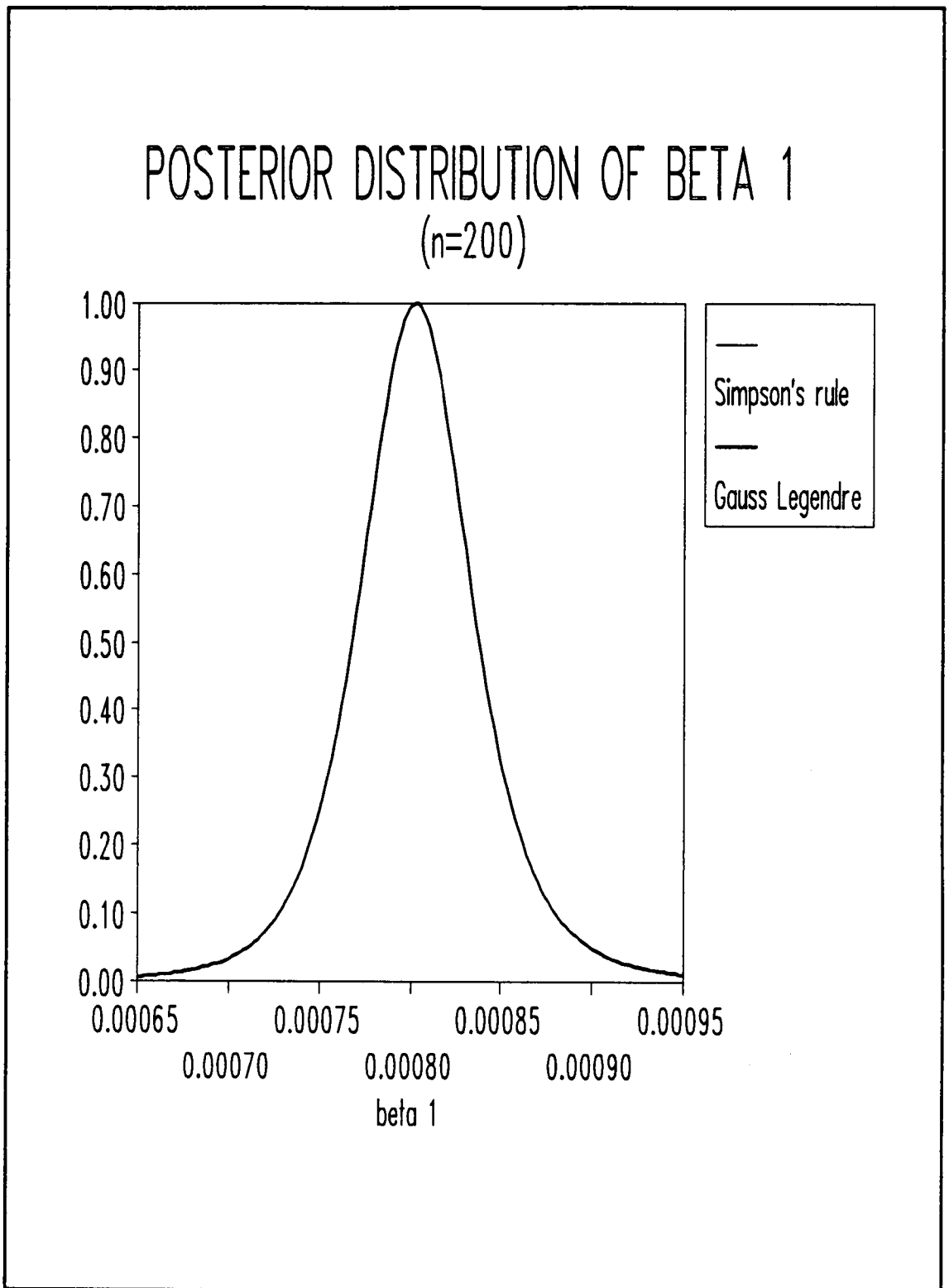


Figure 4.3 Posterior distribution for the parameter β_1 found by integrating the joint posterior density over the parameters σ and β_2.

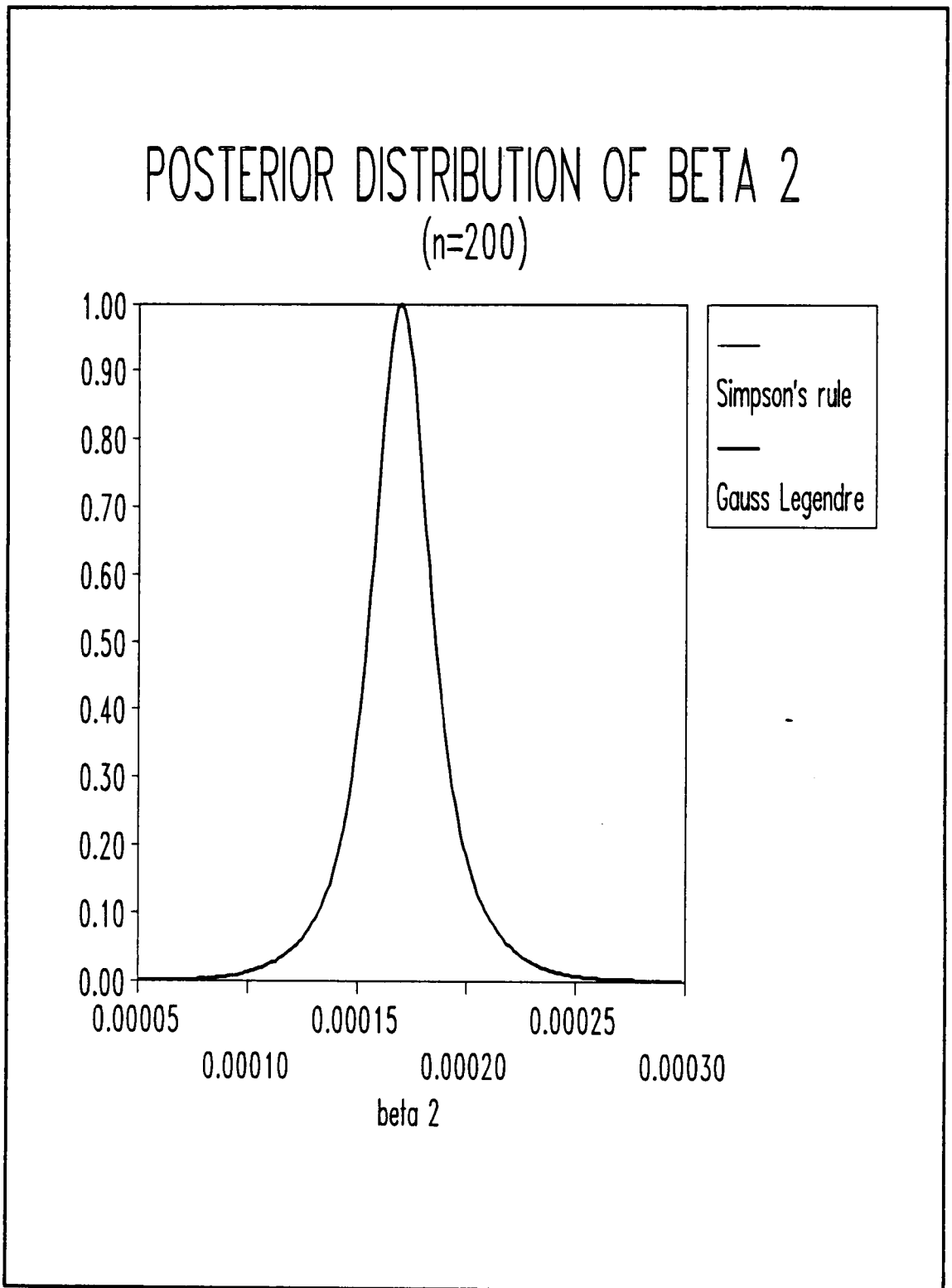


Figure 4.4 Posterior distribution for the parameter β_2 found by integrating the joint posterior density over the parameters σ and β_1.

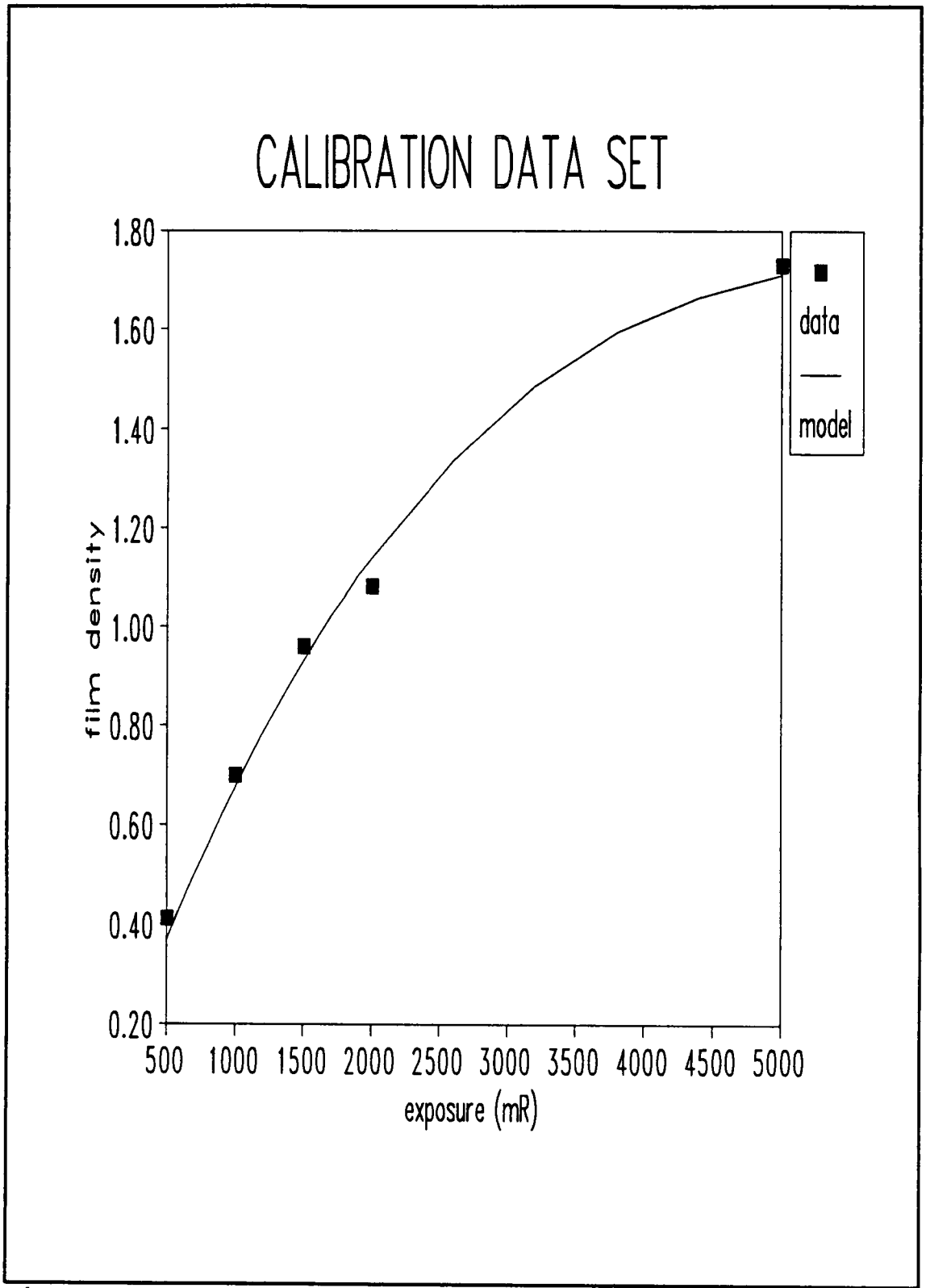


Figure 4.5 Comparison of the "saturation" data set and the model chosen.

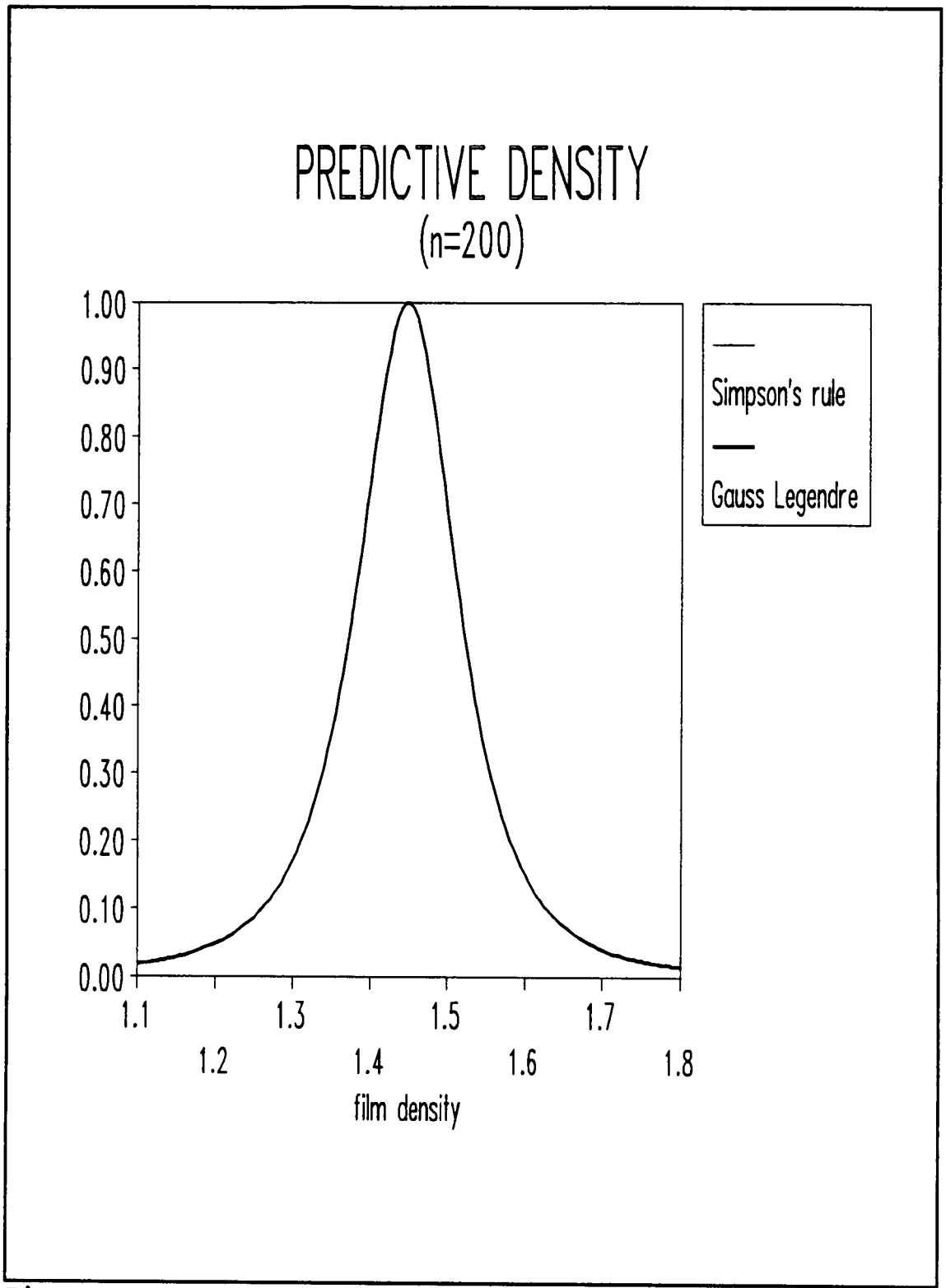


Figure 4.6 Predictive density of film density for a radiation exposure of 3000 mR found by integration over the parameters σ, β_1, and β_2.

3000 mR, which is to be expected if the calculations are correct.

4.3.3 Calibrative Density

Figure 4.7 shows the calibrative density found using the methods described in section 4.2. It shows the uncertainty about the radiation exposure when the film density is 1.45. The mode of this density was found to be at an exposure of 3000 mR which corresponds to the values found for the predictive density. Also a comparison of this value to the model in figure 4.5 shows it corresponds to a film density of 3000 mR, which is to be expected if the calculations are correct.

4.4 "Linear" Data Set

Table 4.2 "Linear" data set

exposure (mR)	film density
500	0.36
1000	0.71
1500	1.0
2000	1.22
5000	2.64

The second data set to be analyzed will be named the linear data set because it displays an almost linear density exposure relationship. This data set was chosen to provide a comparison to calibration methods using linear regression. Table 4.2 shows

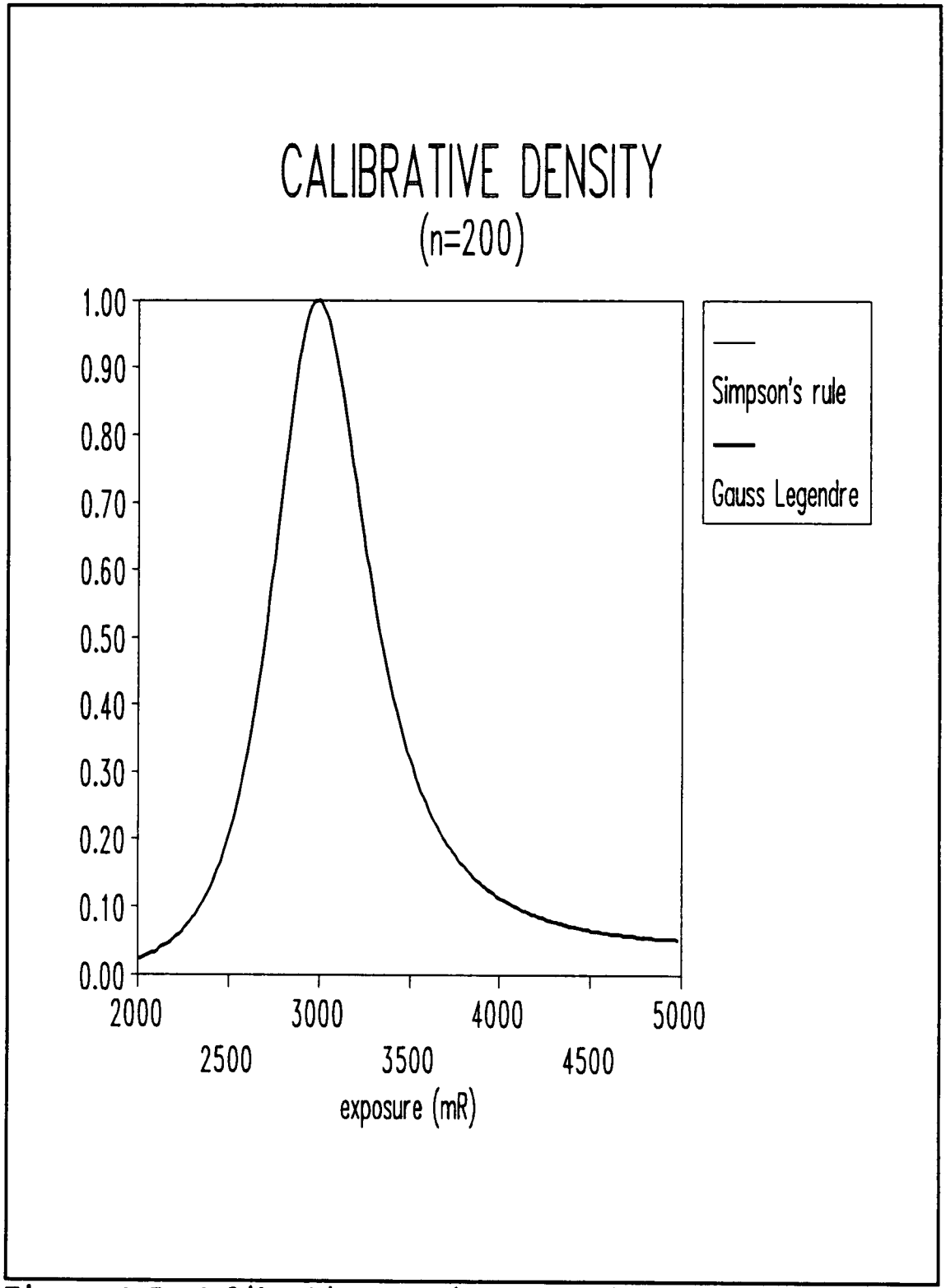


Figure 4.7 Calibrative density of radiation exposure for a film density of 1.45 found by integration over the parameters σ, β_1, and β_2.

the linear data, obtained from ORNL. Figure 4.8 is a plot of the data. From this figure the linear response of film to gamma radiation can be seen.

4.4.1 Posteriors densities of β_1 and β_2

The posterior densities of the model parameters were calculated using numerical methods as described in section 4.2. Figure 4.9 shows the marginal posterior density of the model parameter β_1. Figure 4.10 shows the marginal posterior density of the model parameter β_2. From these plots the limits of integration for β_1 and β_2 were chosen to be from 0.0005 to 0.001 and 0.0 to 0.00012 respectively. A comparison of the model and the calibration data was done by using the mode of the posterior densities of $\beta 1$ and $\beta 2$ in the model and plotting it along with the calibration points. The modes were found to be 0.000715 and 0.0000615 for β_1 and β_2 respectively. Figure 4.11 shows this plot. From this comparison it can be seen that the model using these values for β_1 and β_2 fits the data closely which is to be expected if the posterior densities were calculated correctly.

4.4.2 Predictive Density

Figure 4.12 shows the predictive density found using the

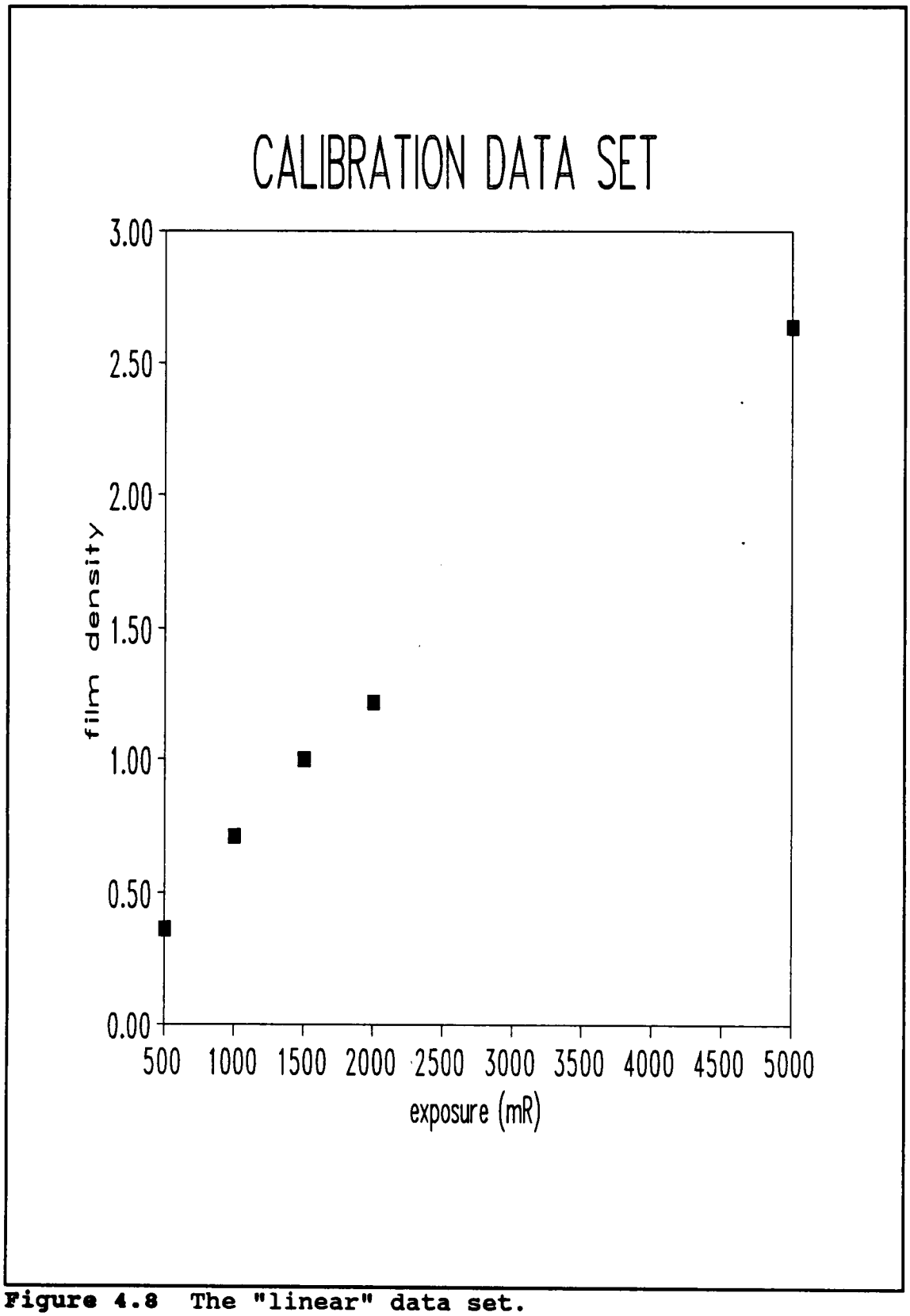


Figure 4.8 The "linear" data set.

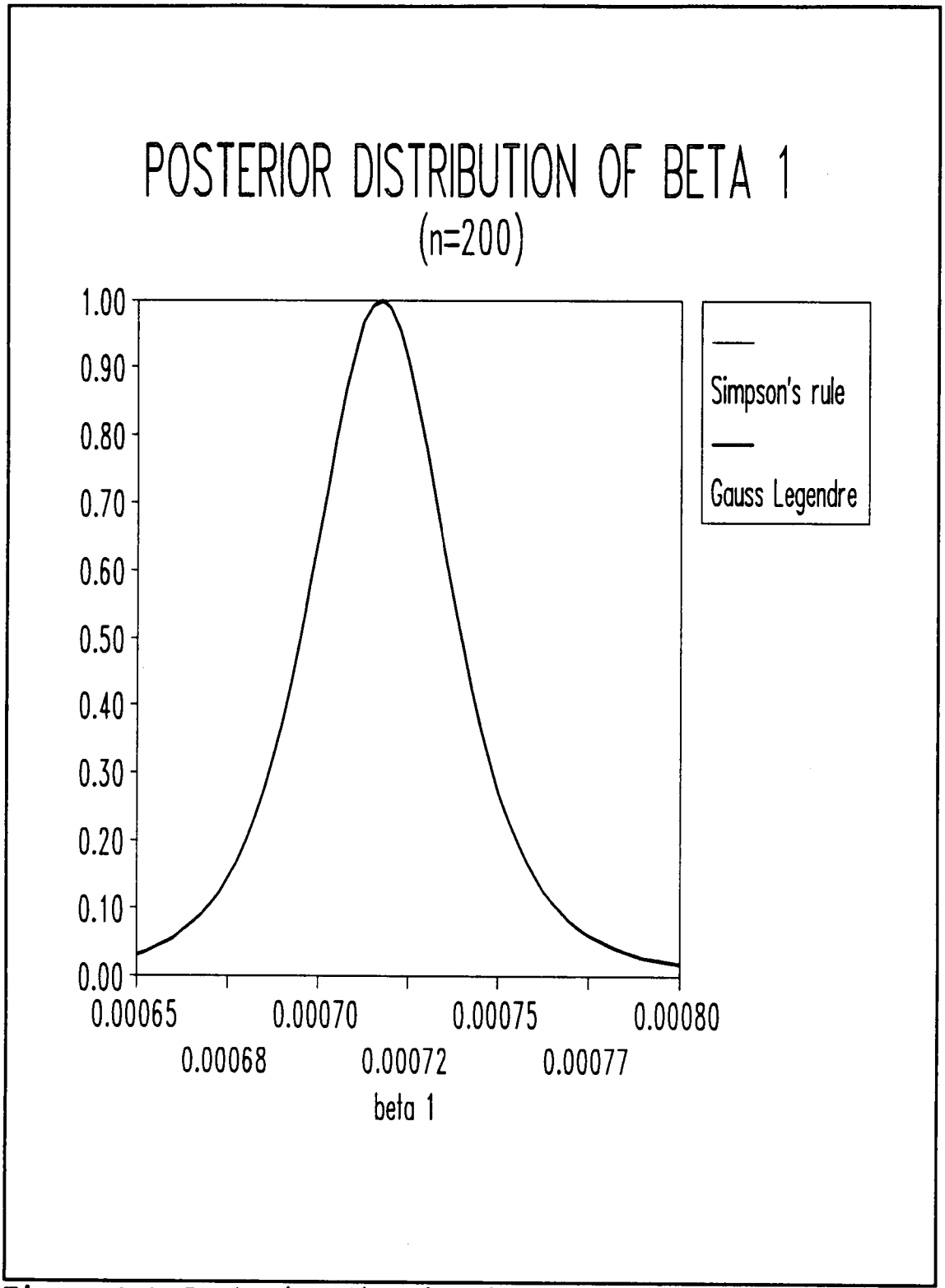


Figure 4.9 Posterior distribution for the parameter β_1 found by integrating the joint posterior density over the parameters σ and β_2.

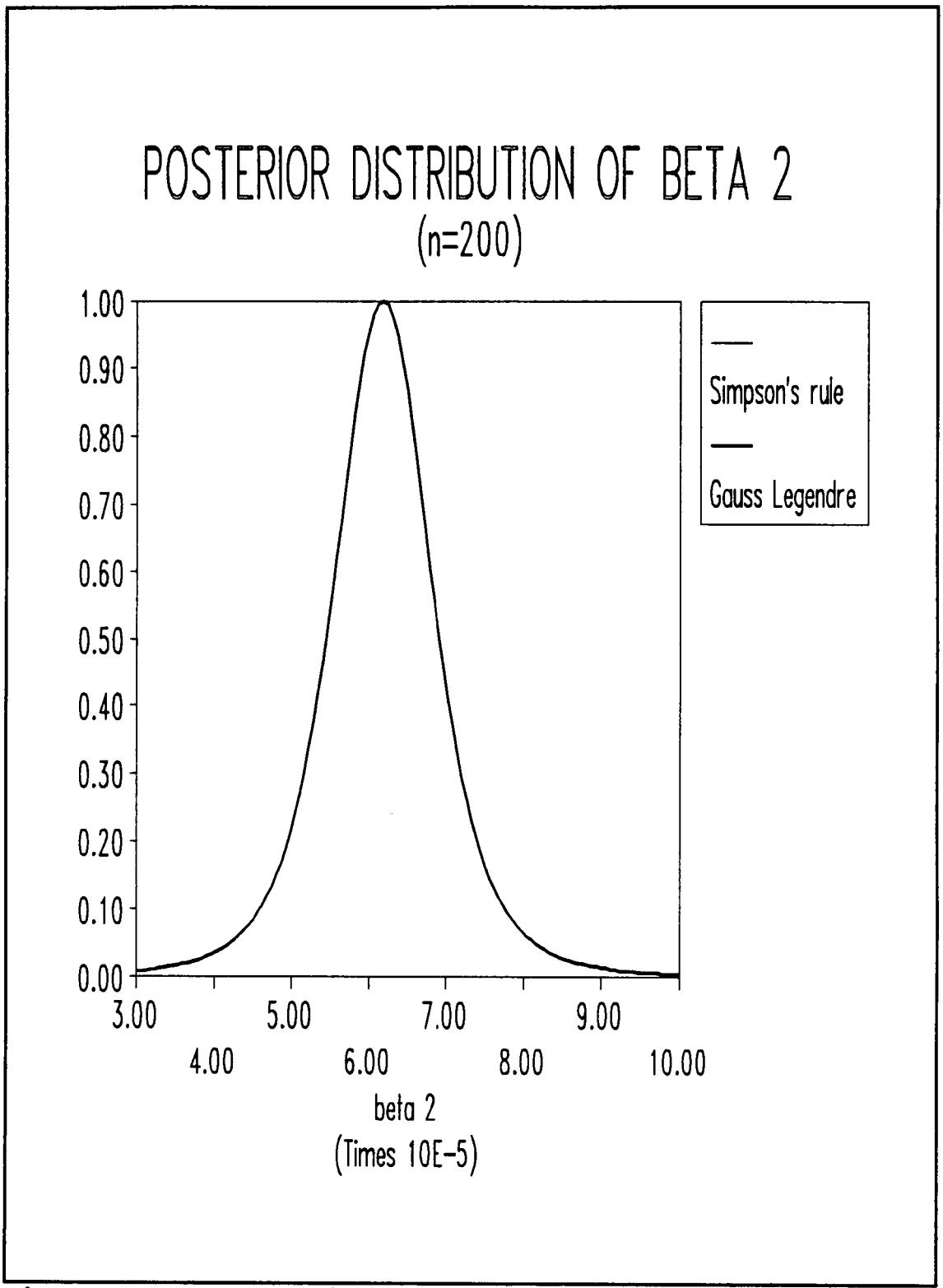


Figure 4.10 Posterior distribution for the parameter β_2 found by integrating the joint posterior density over the parameters σ and β_1.

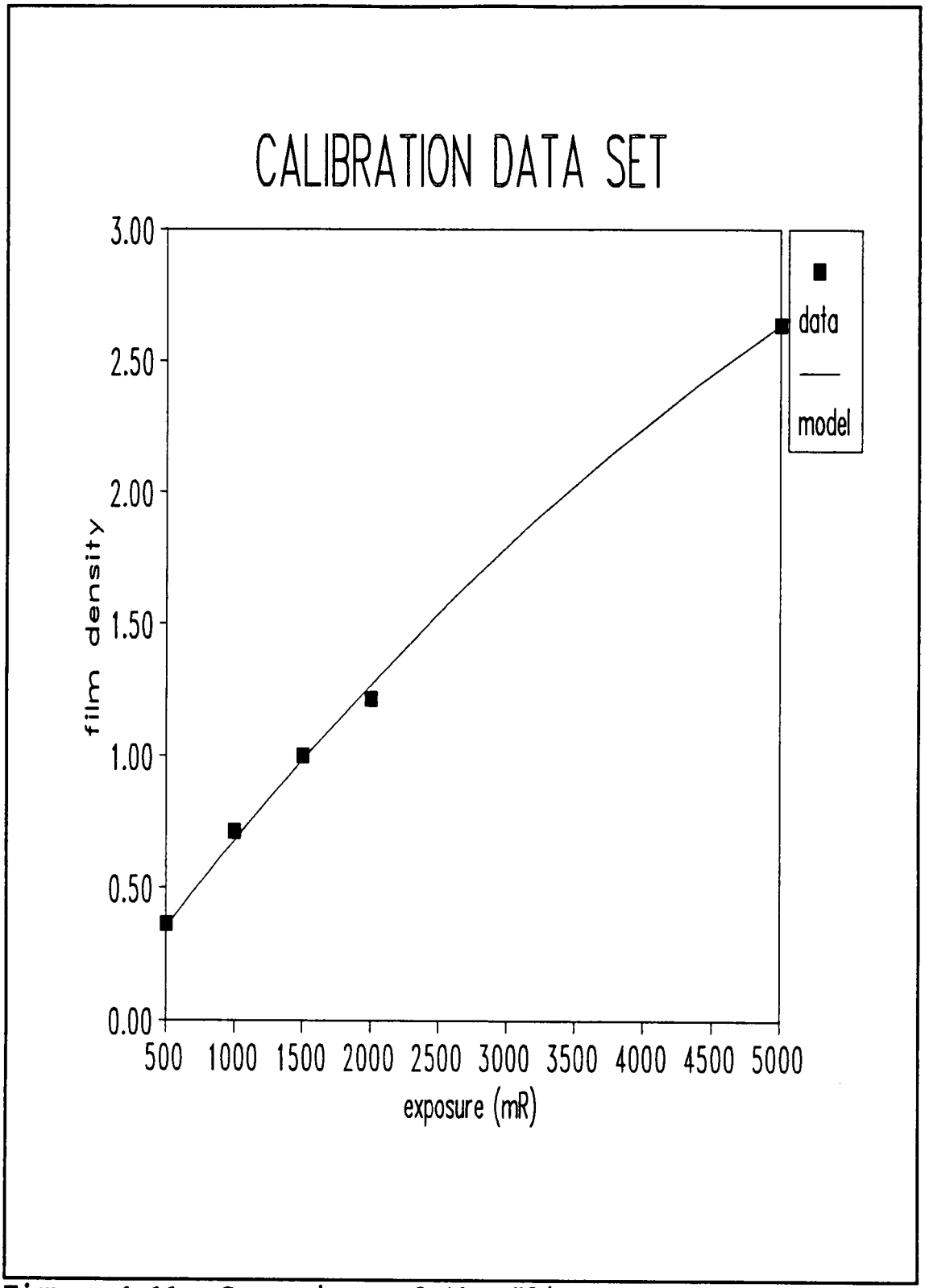


Figure 4.11 Comparison of the "linear" data set and the model chosen.

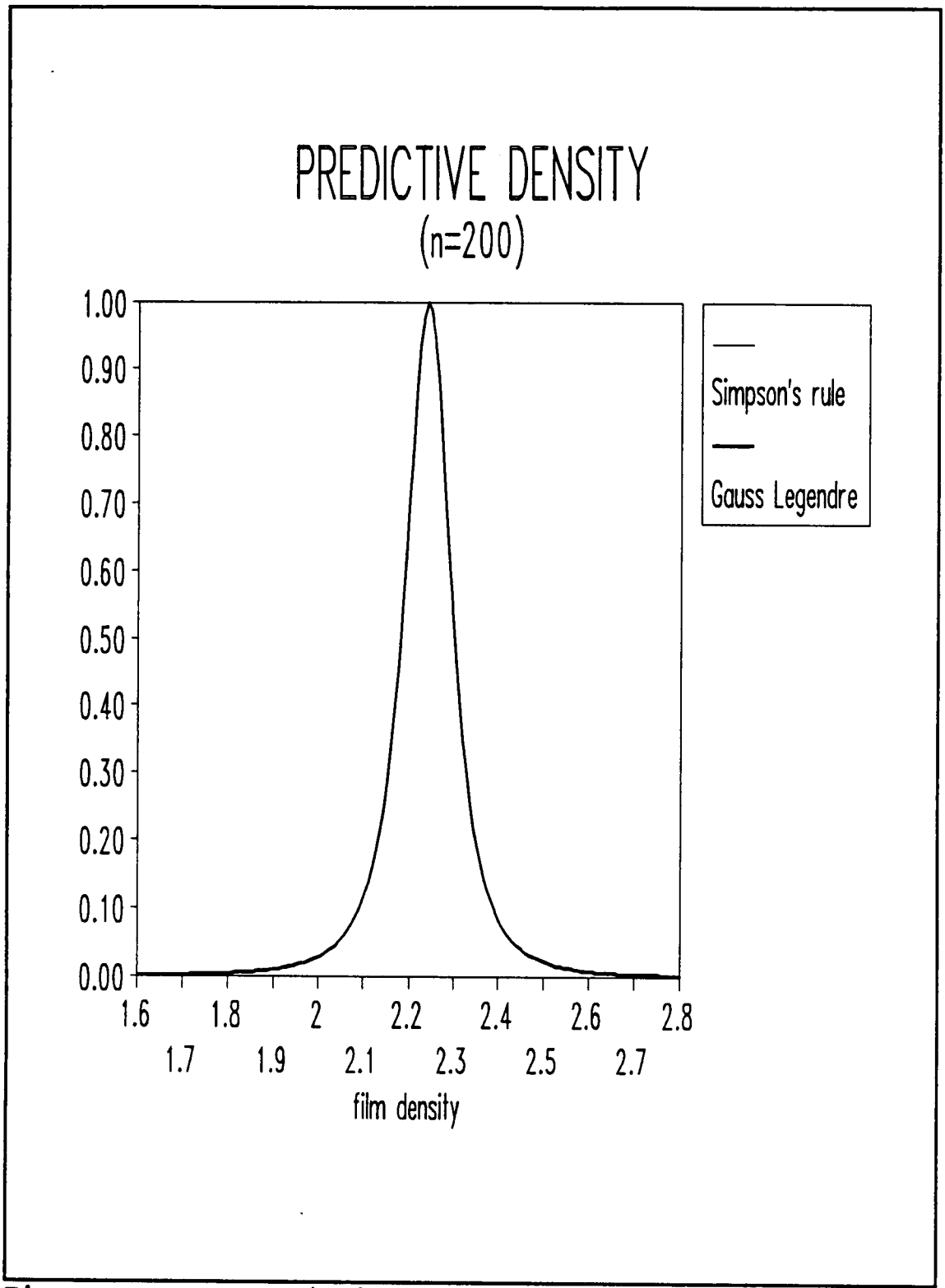


Figure 4.12 Predictive density of film density for a radiation exposure of 4000 mR found by integration over the parameters σ, β_1, and β_2.

methods described in section 4.2. It shows the uncertainty about the film density when the film has been exposed to 4000 mR. The mode of this density was found to be at a film density of 2.25. Comparing this value to the model shown in figure 4.11 shows it corresponds to a radiation exposure of 4000 mR, which is to be expected if the calculations are correct.

4.4.3 Calibrative Density

Figure 4.13 shows the calibrative density found using the methods described in section 4.2. It shows the uncertainty about the radiation exposure when the film density is 2.25. The mode of this density was found to be at an exposure of 4000 mR which corresponds to the values found for the predictive density. Also a comparison of this value to the model shown in figure 4.11 shows it corresponds to a film density of 4000 mR, which is to be expected if the calculations are correct.

4.5 Comparison of Exponential and Linear Regression Calibrative Densities

As a consistency check the model developed in this thesis is compared to the model described in Chapter 10 of Aitchison

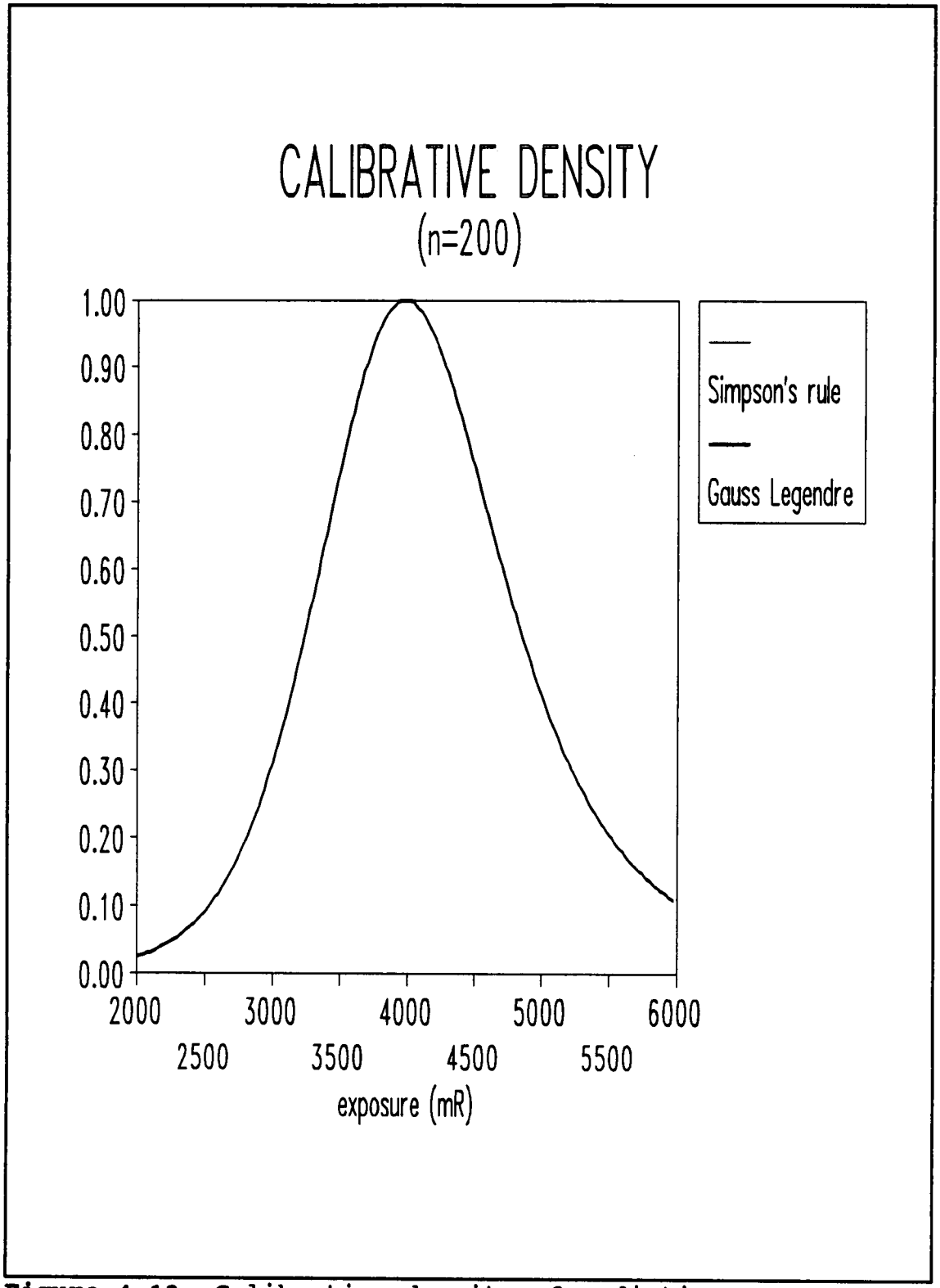


Figure 4.13 Calibrative density of radiation exposure for a film density of 2.25 found by integration over the parameters σ, β_1, and β_2.

[1975]. Aitchison derives a calibrative density for a linear regression model applied to a set of calibration data. To demonstrate the differences between the linear regression model and the exponential model developed in this thesis the results using both models for the "saturation" and "linear" calibration data sets are discussed.

4.5.1 "Saturation" Data Set

A calibrative density for the linear regression model was found using the "saturation" data set described in section 4.2. The fit of both models to the data set is shown in figure 4.14. The calibrative density found in section 4.2.3 and the calibrative density for the linear model are shown in figure 4.15.

From figure 4.15 it can be seen that the mode for the exponential model occurs at a lower exposure than the mode for the linear regression model. This is consistent with figure 4.14 which shows that at a film density of 1.45 corresponds to a lower exposure for the exponential model than for the linear regression model. It should also be noted from figure 4.14 that at low and high film densities the linear regression model underestimates the actual exposure.

The width of the calibrative density for the linear regression model is also greater than the width of the

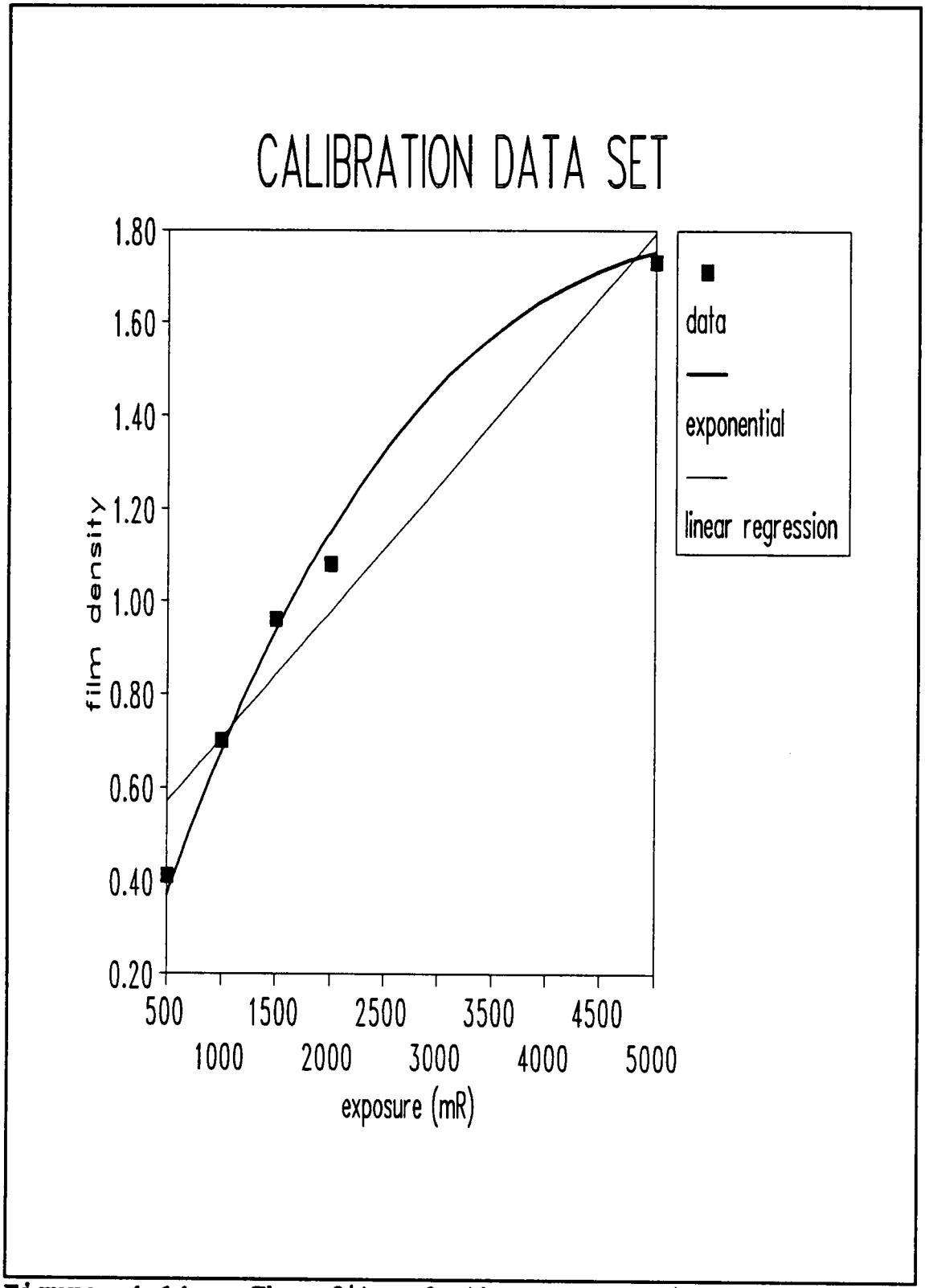


Figure 4.14 The fit of the exponential and linear regression models to the "saturation" data set.

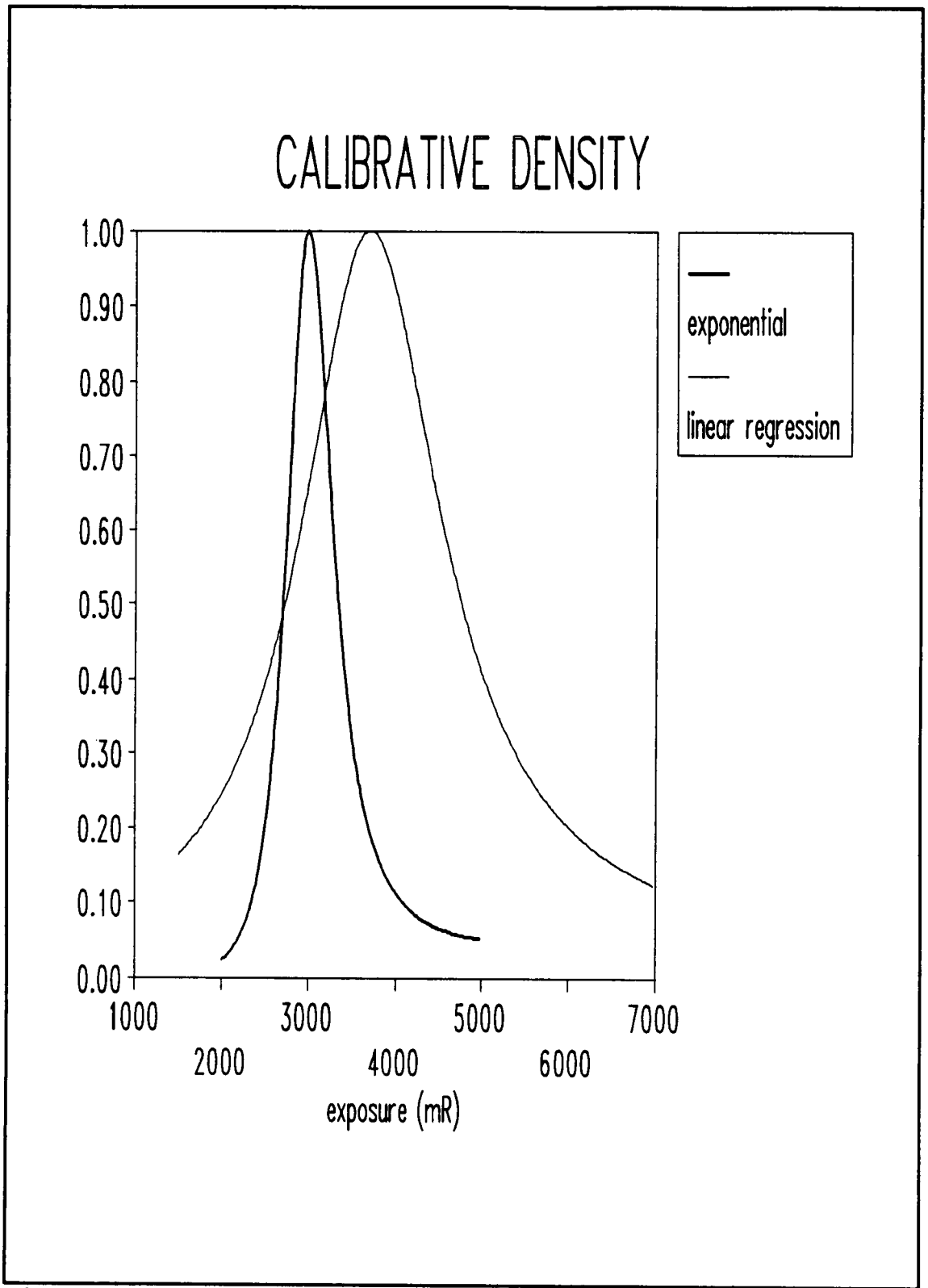


Figure 4.15 Calibrative density for a new film density of 1.45 for the exponential and linear regression models.

corresponding density for the exponential model. This is due to the poorer fit of the linear regression model to the data points.

Figures 4.16, 4.17, and 4.18 are comparisons of the two methods over the film density range of the calibration data set. From them it can be seen that the differences between the two models discussed for figure 4.15 occur throughout the film density range.

The upward turn in the tail of the calibrative density from the exponential model in figure 4.18 is caused by the chosen film density corresponding to two exposures. This upward turn is the center of a bimodal density caused by the exponential model and is shown completely in figure 4.19. Mathematically the exponential model reaches a maximum and descends as shown in figure 4.20. When the exponential model is used with typical film badge calibration data from ORNL the exposure range of the data almost always lies entirely within the increasing portion of the model. Under these circumstances the density is unimodal because of the truncation of the calibration data.

4.5.2 "Linear" Data Set

A calibrative density for the linear regression model was found using the "linear" data set described in section 4.3. The fit of both models to the data set is shown in figure

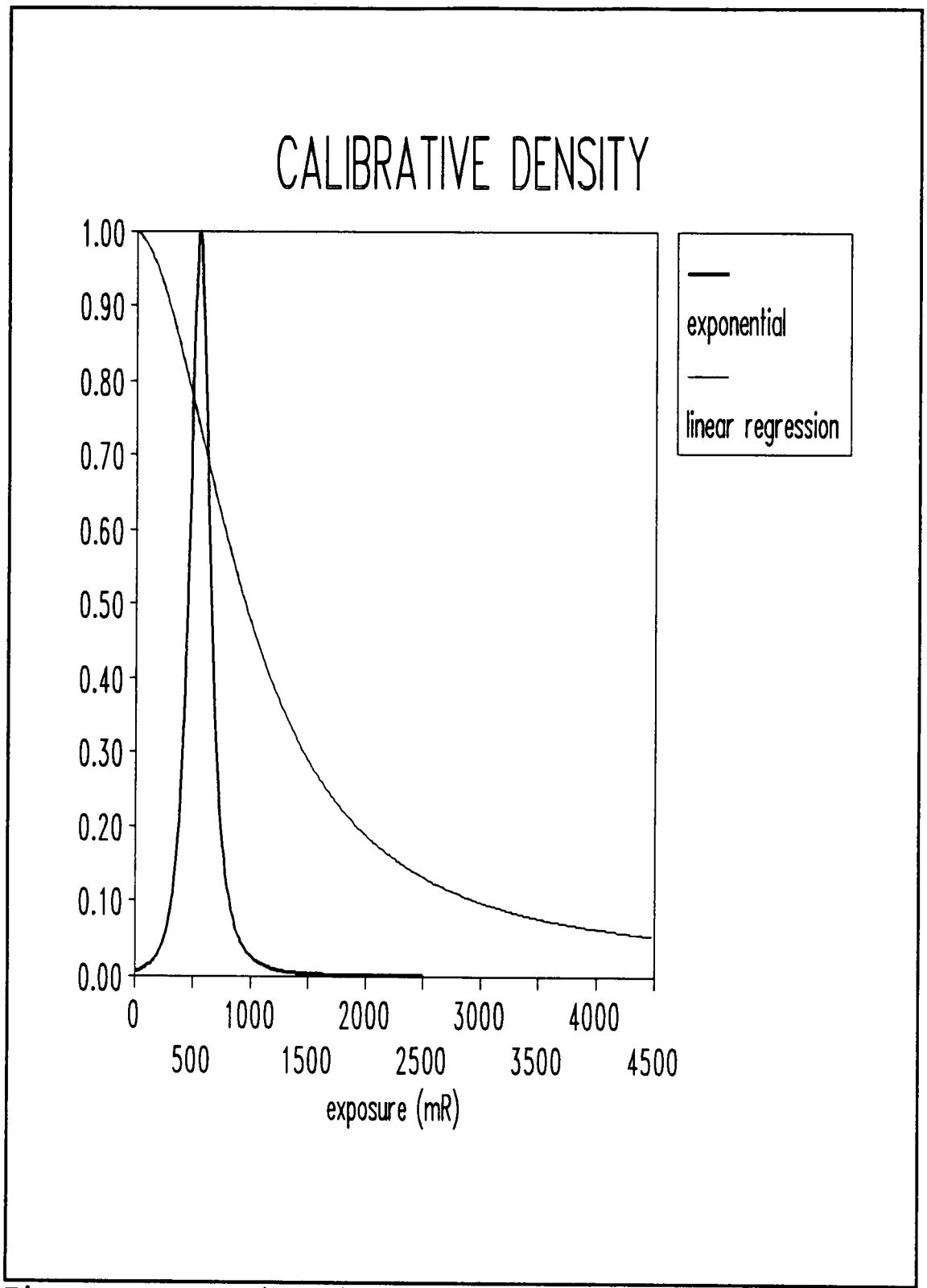


Figure 4.16 Calibrative density for a new film density of 0.4 for the exponential and linear regression models.

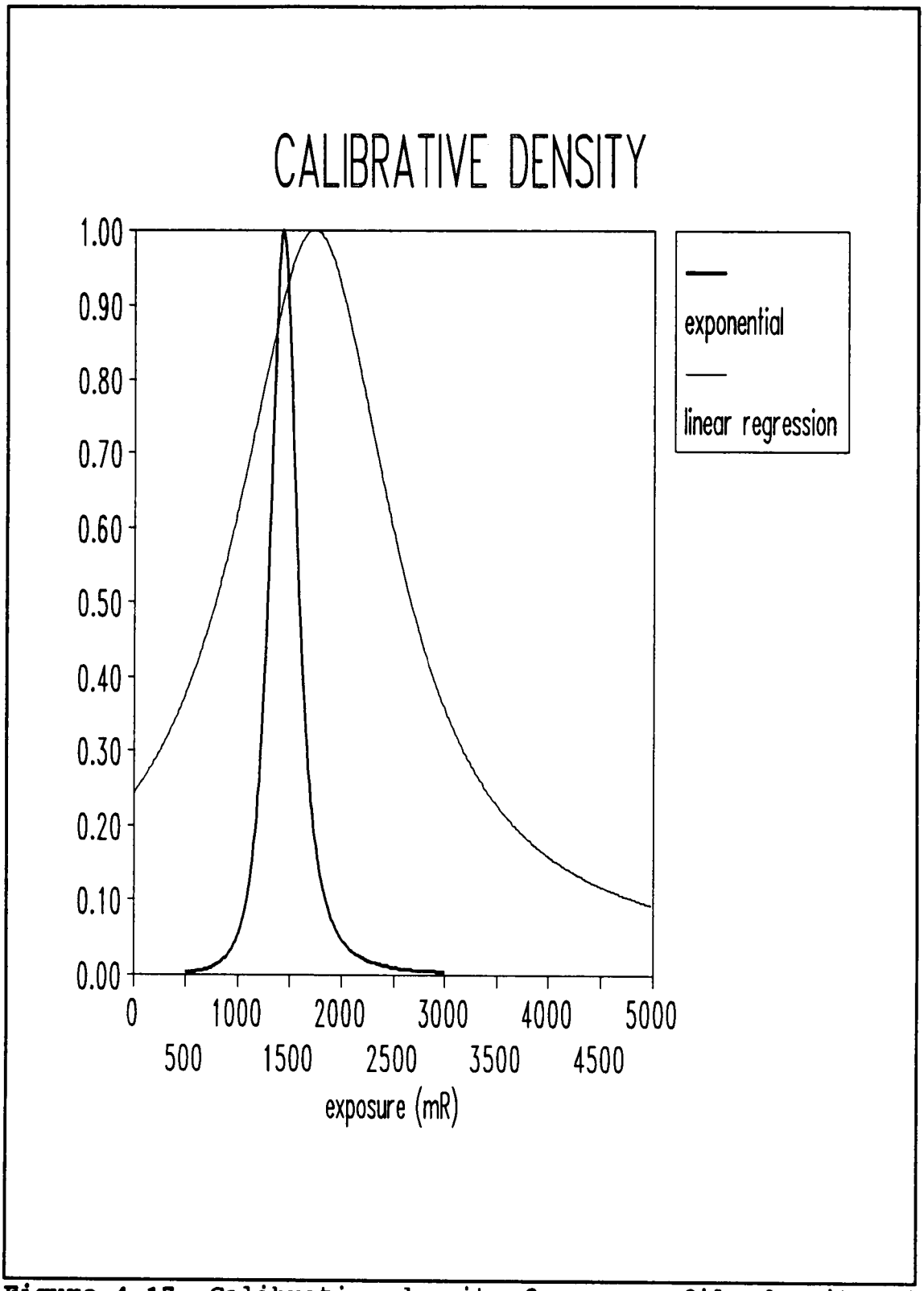


Figure 4.17 Calibrative density for a new film density of 0.9 for the exponential and linear regression models.

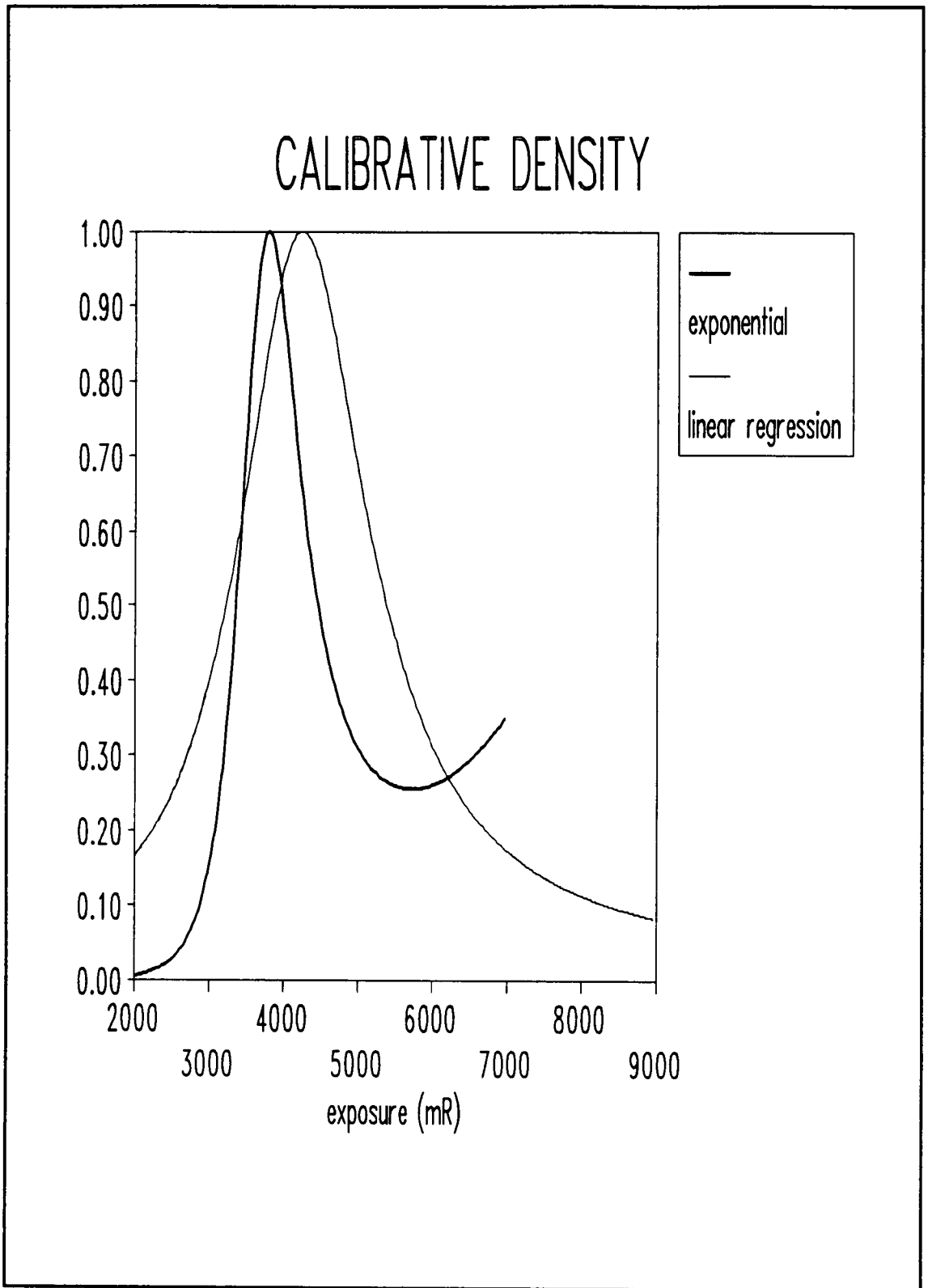


Figure 4.18 Calibrative density for a new film density of 1.6 for the exponential and linear regression models.

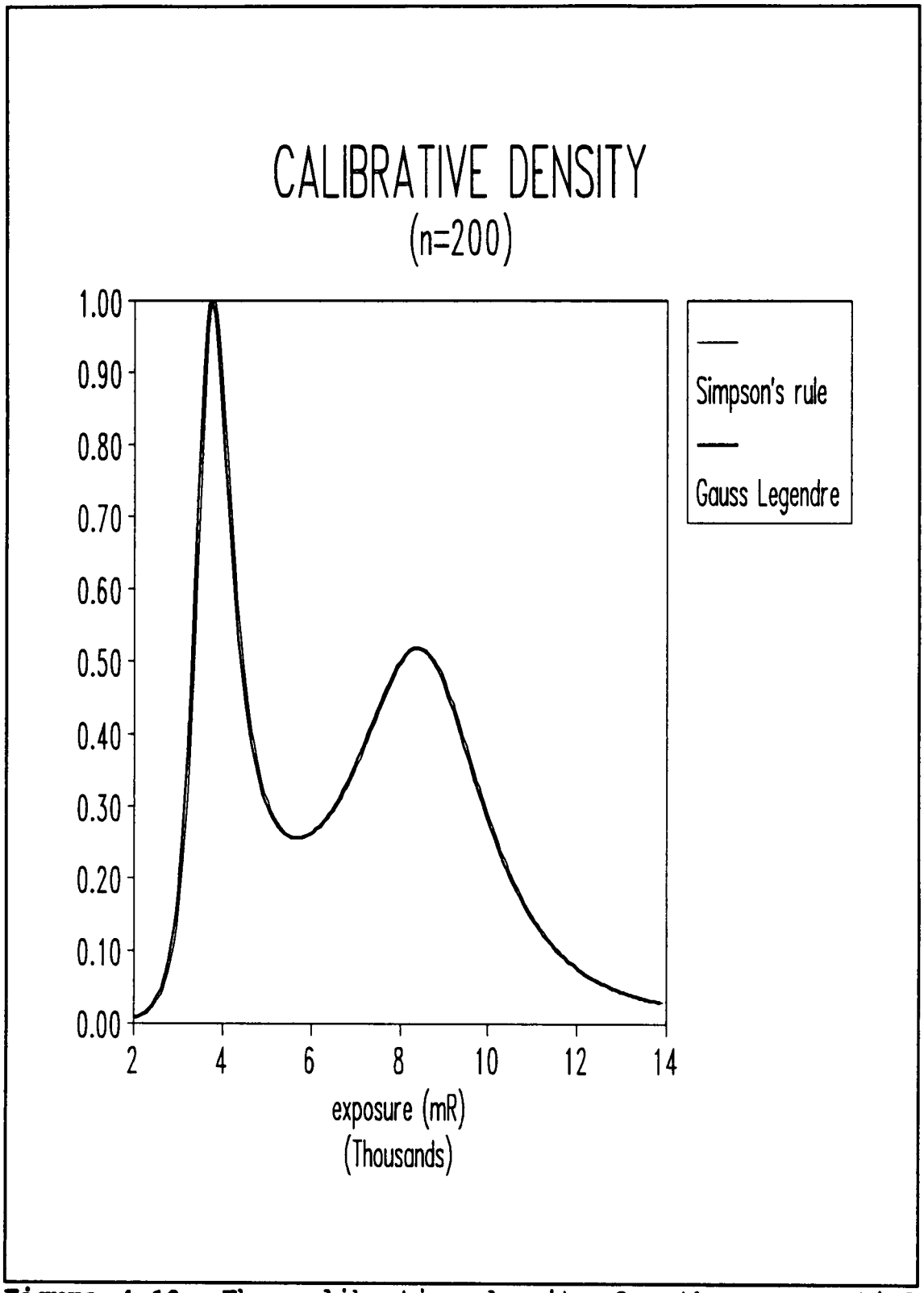


Figure 4.19 The calibrative density for the exponential model at a film density of 1.6.

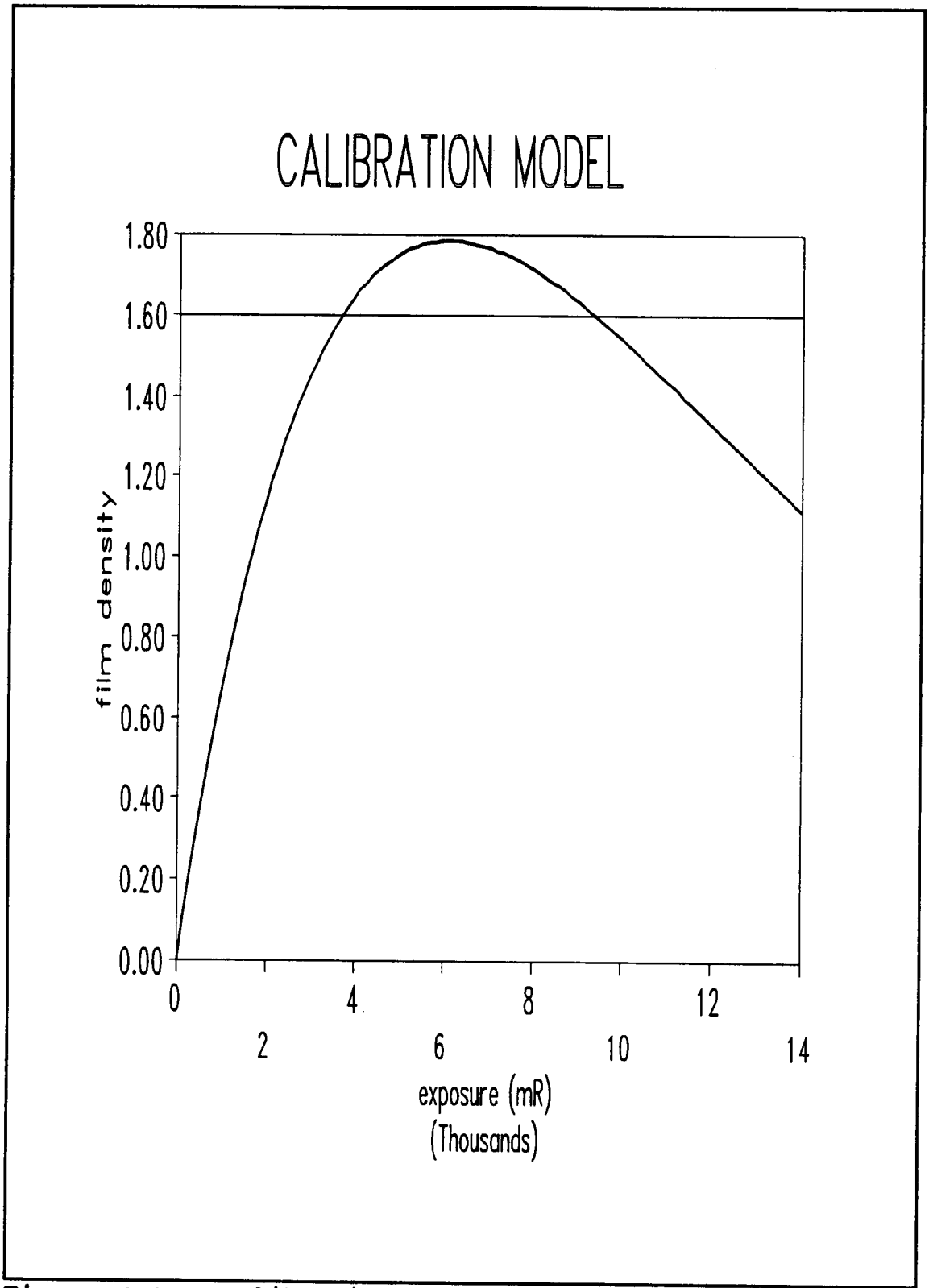


Figure 4.20 Calibration model for the "saturation" data set.

4.21. The calibrative density found in section 4.3.3 and the new density are shown in figure 4.22.

In figure 4.22 the mode of the calibrative density for the exponential model occurs at almost the same exposure as for the linear model. This can also be seen in figure 4.21 which shows that at a film density of 2.2 corresponds to almost exactly the same exposure for the exponential model and the linear regression model. The calibrative density for the linear regression model is more peaked than the corresponding calibrative density for the exponential model.

Figures 4.23, 4.24, and 4.25 are comparisons of the two models covering the range of film densities for the "linear" calibration data set. They show that the modes for both models occur close to each other. Also the densities for the exponential model have a slightly smaller widths than the corresponding densities for the linear regression model at low film densities. In contrast the density for the linear regression model has a smaller width at higher film densities. The differences in the widths are small when compared to the differences seen with the "saturation" data set.

The similarity of the two models for the "linear" data set is apparent and expected since the linear regression model is contained within the exponential model. For extreamly small values of β_2 the exponential term in the exponential model approaches unity and the model becomes linear with a y intercept of 0 and slope β_1. The exponential model is

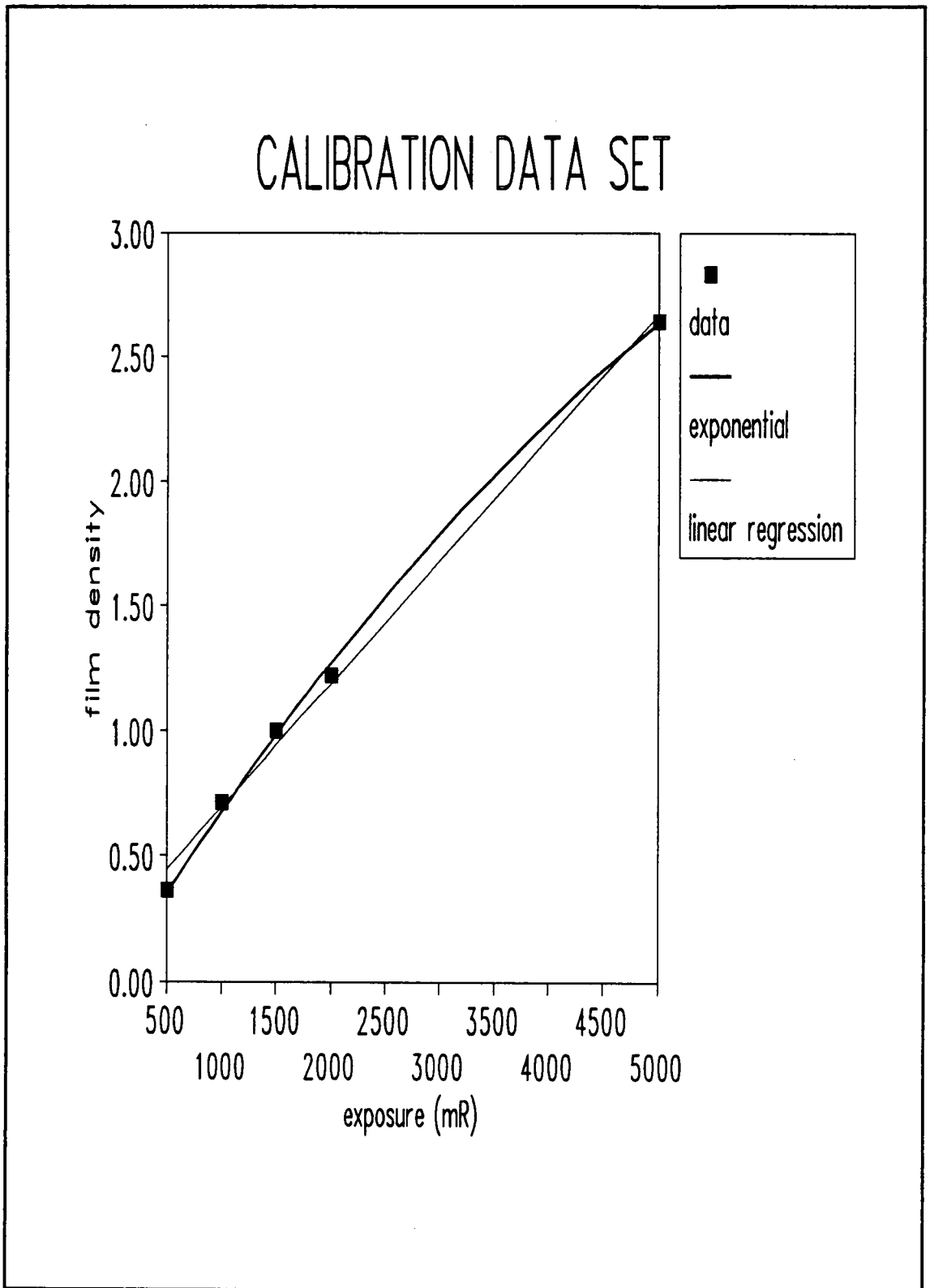


Figure 4.21 The fit of the exponential and linear regression models to the "linear" data set.

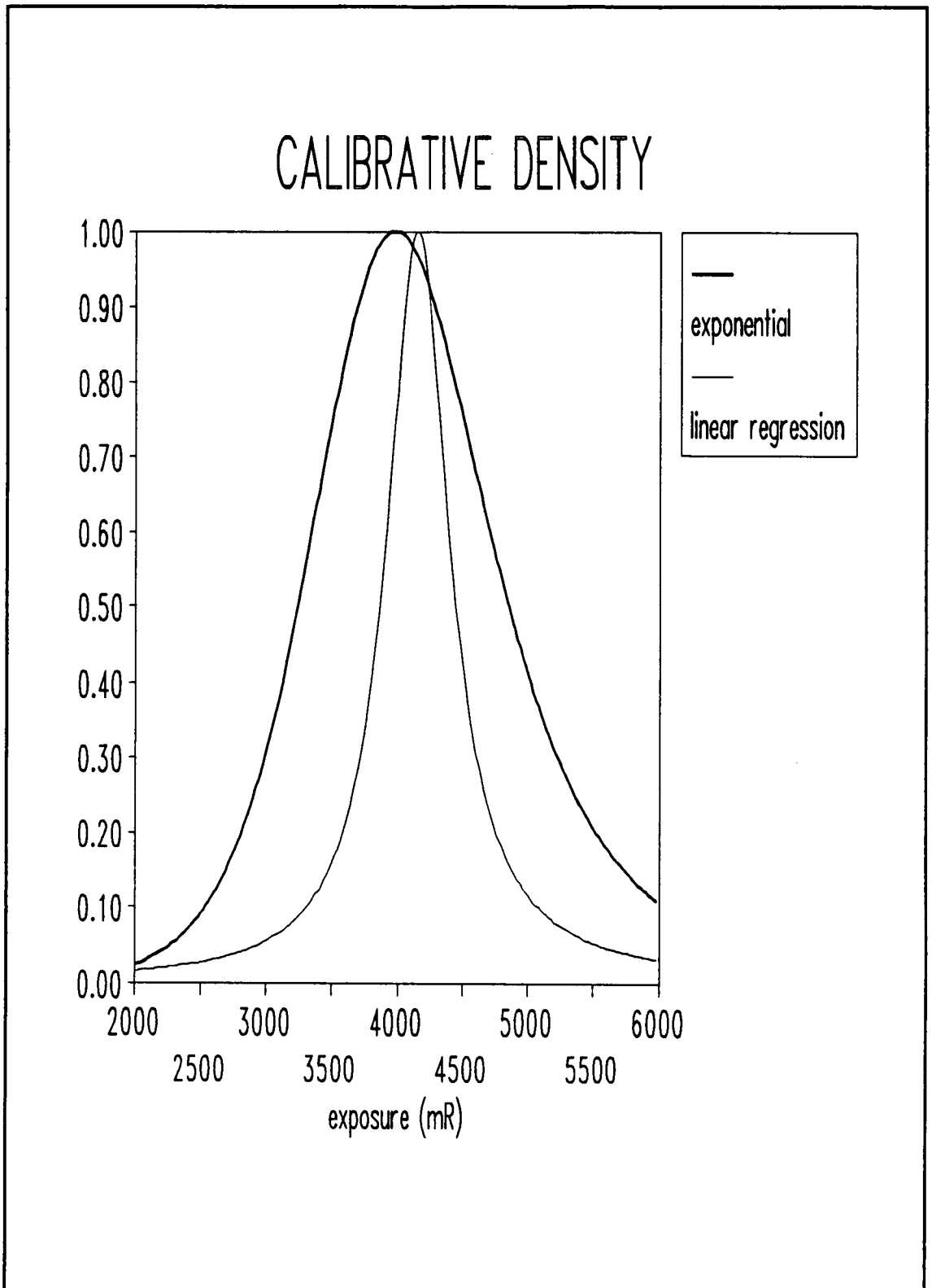


Figure 4.22 Calibrative density for a new film density of 2.25 for the exponential and linear regression models.

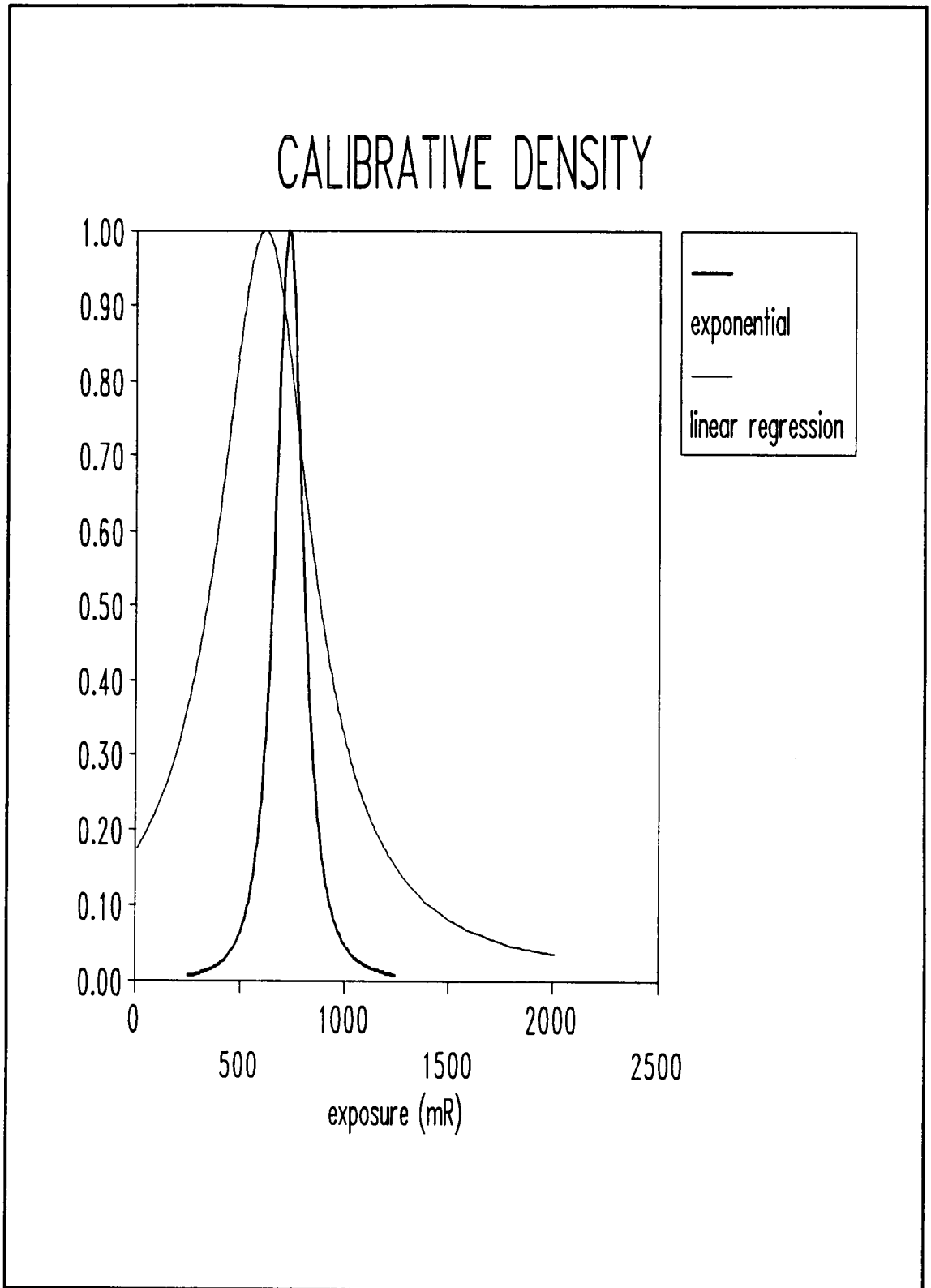


Figure 4.23 Calibrative density for a new film density of 0.5 for the exponential and linear regression models.

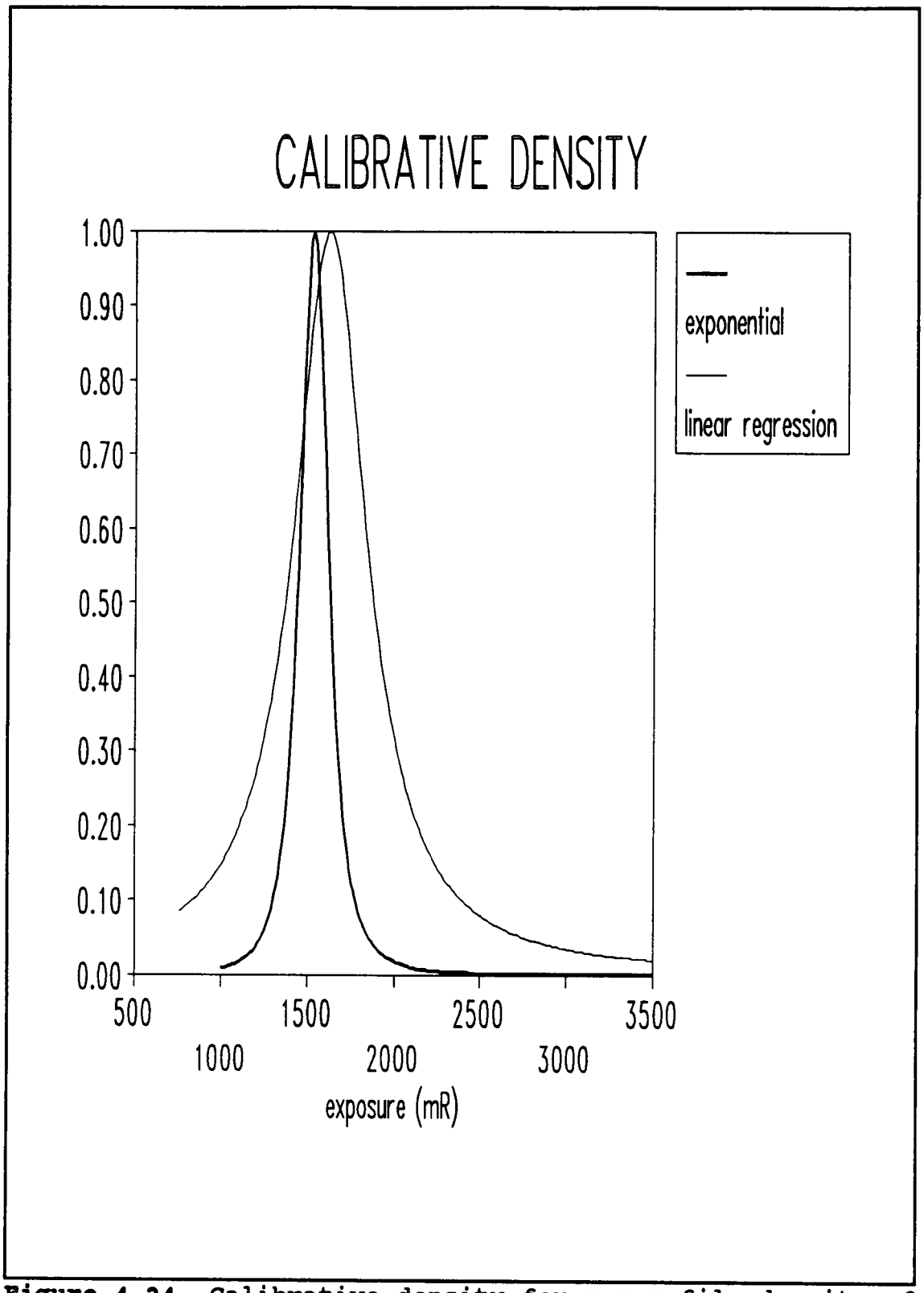


Figure 4.24 Calibrative density for a new film density of 1.0 for the exponential and linear regression models.

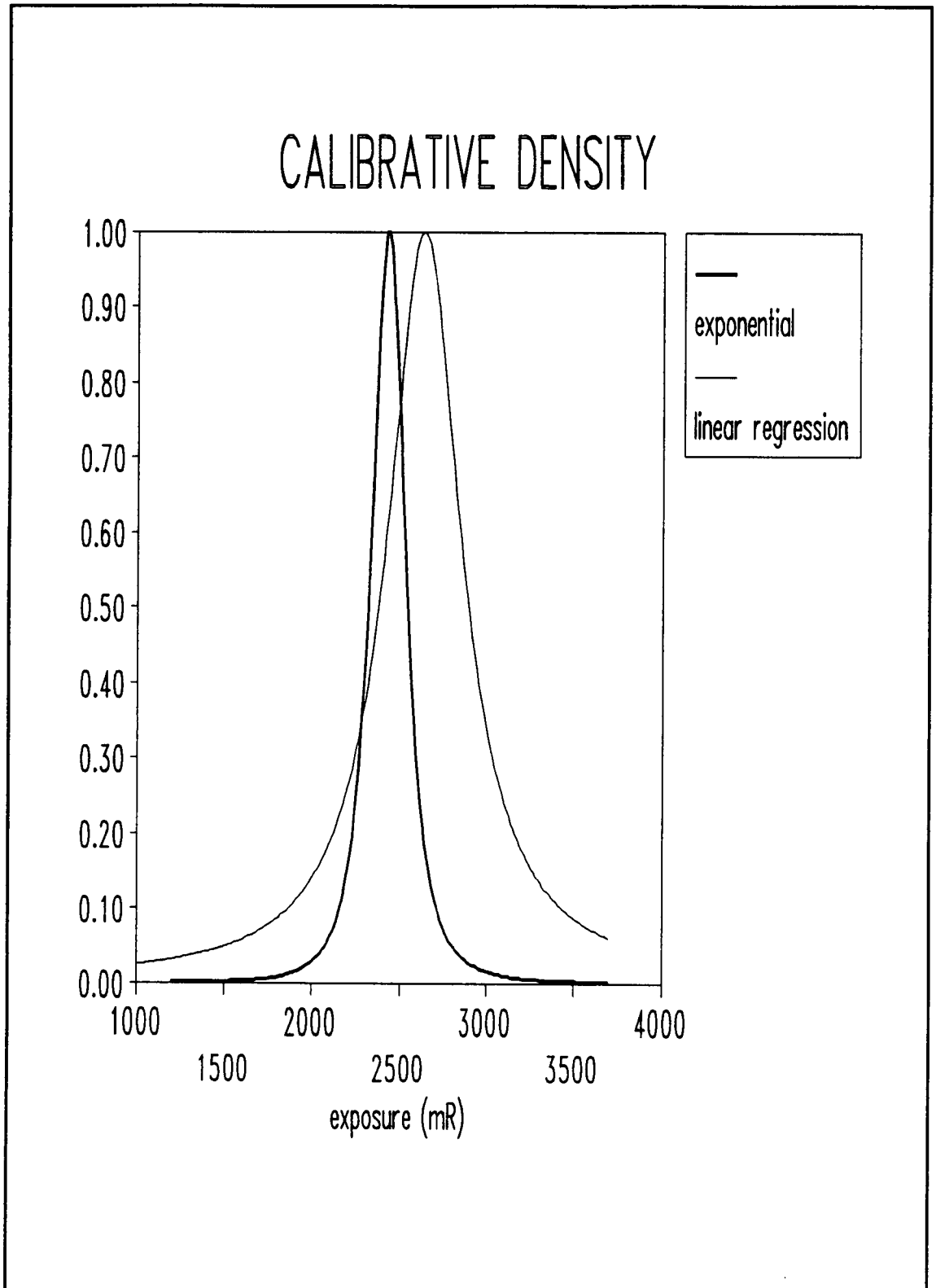


Figure 4.25 Calibrative density for a new film density of 1.5 for the exponential and linear regression models.

therefore an extreamly close approximation to the linear model in the case of data sets displaying linear characteristics.

CHAPTER 5

Conclusions

This thesis discussed the calibration of film badge dosimeters. It provides a quantitative description of the uncertainty of the exposure given a film density. The uncertainty in the exposure could be related to a dose for an individual wearing a film badge and thus quantitatively describe the dose that individual has received.

First it provided background on the interactions between film and ionizing radiation. It discussed the mechanisms which produce an image on film, how different types of radiation produce this effect, and how this allows film to serve as a dosimeter. Also it provided an example of how film has been calibrated for use as a dosimeter.

Second it described how Bayesian techniques can be used in film dosimetry to provide more quantitative results. It provided a brief description and derivation of Bayesian calibration methods. Then it described the model chosen to fit film calibration data and the reasons why the model was chosen.

Next it discussed the application of the model with film calibration data sets to derive a calibrative density. This

necessitated the use of numerical methods. The numerical methods were implemented with computer programs written in C and a detailed discussion of these programs was also given.

Finally it showed the results of an analysis of two film badge calibration data sets from ORNL and made comparisons to other calibration methods which have previously been used. The first data set was chosen as representative of most of the 600 calibration sets reviewed for this thesis. The second was chosen because it showed near linear characteristics thus allowing a demonstration of the models ability to handle linear as well as curved data sets. The analyses of both data sets were compared with a Bayesian calibration technique using linear regression to estimate the parameters of the calibration curve.

The model developed in this thesis provides a method of quantitatively describing the relationship of film density to radiation exposure. It provides an accurate description of the film badge calibration data from ORNL. The accuracy is reflected by a more peaked calibrative density than the corresponding density based on a linear description of the data. The procedure described here permits a characterization of the uncertainty in the dose received by an individual wearing a film badge. This characterization can be used for an estimation of the relationship between the radiation dose received by an individual and the potential health effects related to the exposure.

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APPENDIXES

Appendix I

INTEGRATION OVER σ

The integral is of the form:

$$\int_0^{\infty} \prod_{i=1}^{n} \frac{A}{\sigma} e^{-\frac{B_i}{2\sigma^2}} d\sigma$$

where A, and B are constants. This becomes:

$$\int_0^{\infty} \left(\frac{A}{\sigma}\right)^n e^{\sum_{i=1}^{n} -\frac{B_i}{2\sigma^2}} d\sigma$$

let:

$$t = \frac{\sum_{i=1}^{n} -B_i}{2\sigma^2}$$

then:

$$dt = -\sum_{i=1}^{n} \frac{B_i}{\sigma^3} d\sigma$$

As σ goes to infinity t goes to 0 and vice versa. With these substitutions the integral can be manipulated to look like:

$$A' \sum_{i=1}^{n} B_i^{\frac{1}{2}-\frac{n}{2}} \int_0^{\infty} t^{\left(\frac{n}{2}-\frac{1}{2}\right)-1} e^{-t} dt$$

Where the constant $A'=A^n(2^{-3/2+n/2})$. The definition of the gamma function is:

$$\int_0^\infty t^{m-1}e^{-t}dt=\Gamma(m)$$

Thus one obtains for the integral:

$$A'\sum_{i=1}^{n} B_i^{\frac{1}{2}-\frac{n}{2}}\Gamma(\frac{n}{2}-\frac{1}{2})$$

This result can be used for the derivation of the posterior densities of β_1, β_2, and the predictive and calibrative densities.

Appendix II

CALCULATION OF THE CALIBRATIVE DENSITY FOR OTHER DATA SETS

The data set shown in table I will be called the "many points" data set because it contains eight calibration points as opposed to the more usual five. The posterior distributions are shown in figures 1 and 2. The limits of integration for β_1 and β_2 were chosen to be from 0.0005 to .001 and 0.00004 to 0.00011 respectively. Using these limits the predictive density for an exposure of 4000 mR and a calibrative density for a film density of 2.2 were calculated and are shown in figures 3 and 4.

exposure (mR)	film density
500	.48
1000	.82
1500	1.1
2000	1.32
3000	1.74
5000	2.81
10000	3.18
20000	3.66

Table I "Many points" calibration data set.

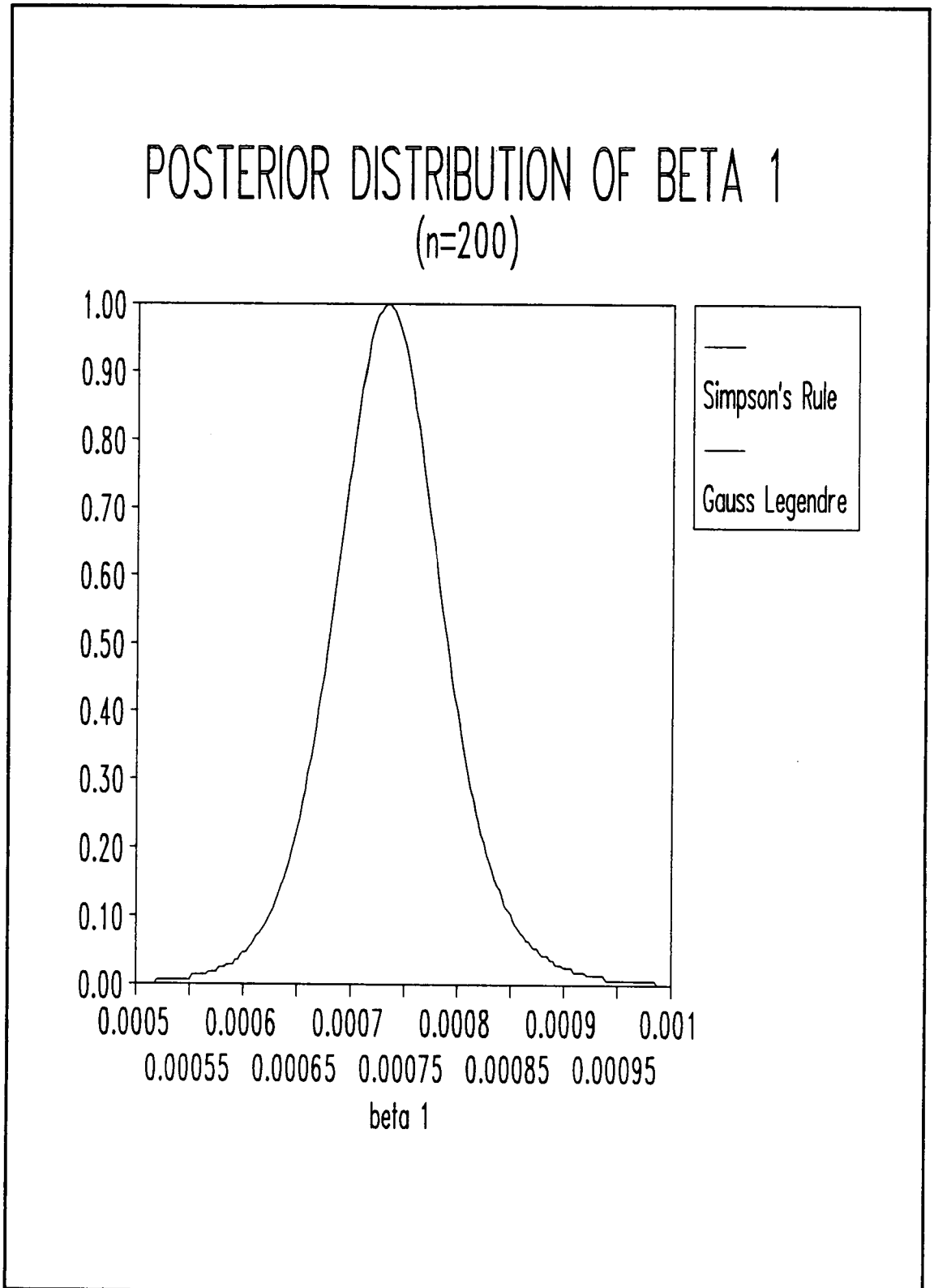


Figure 1 Posterior density of β_1 for the "many points" calibration data set.

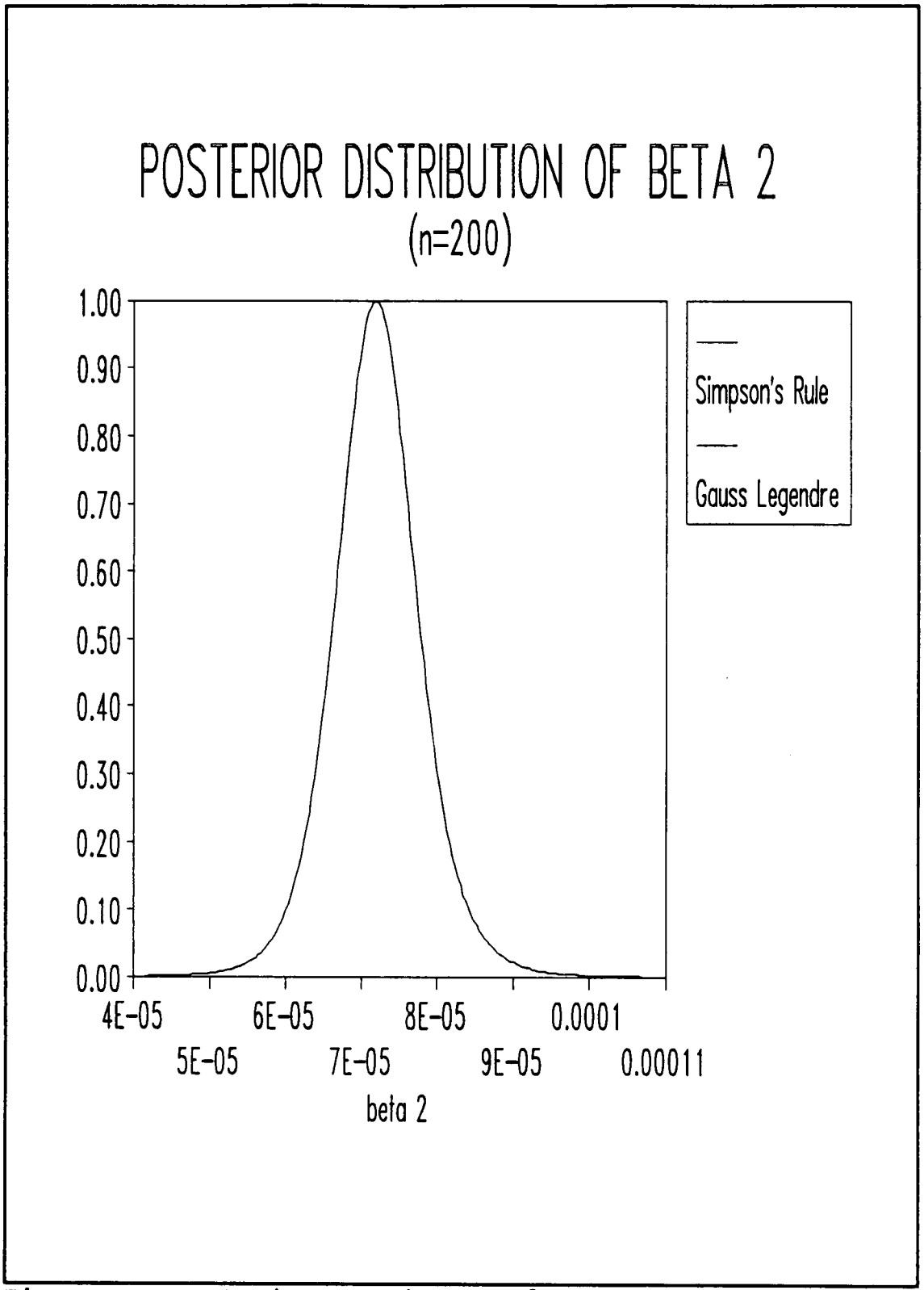


Figure 2 Posterior density of β_2 for the "many points" calibration data set.

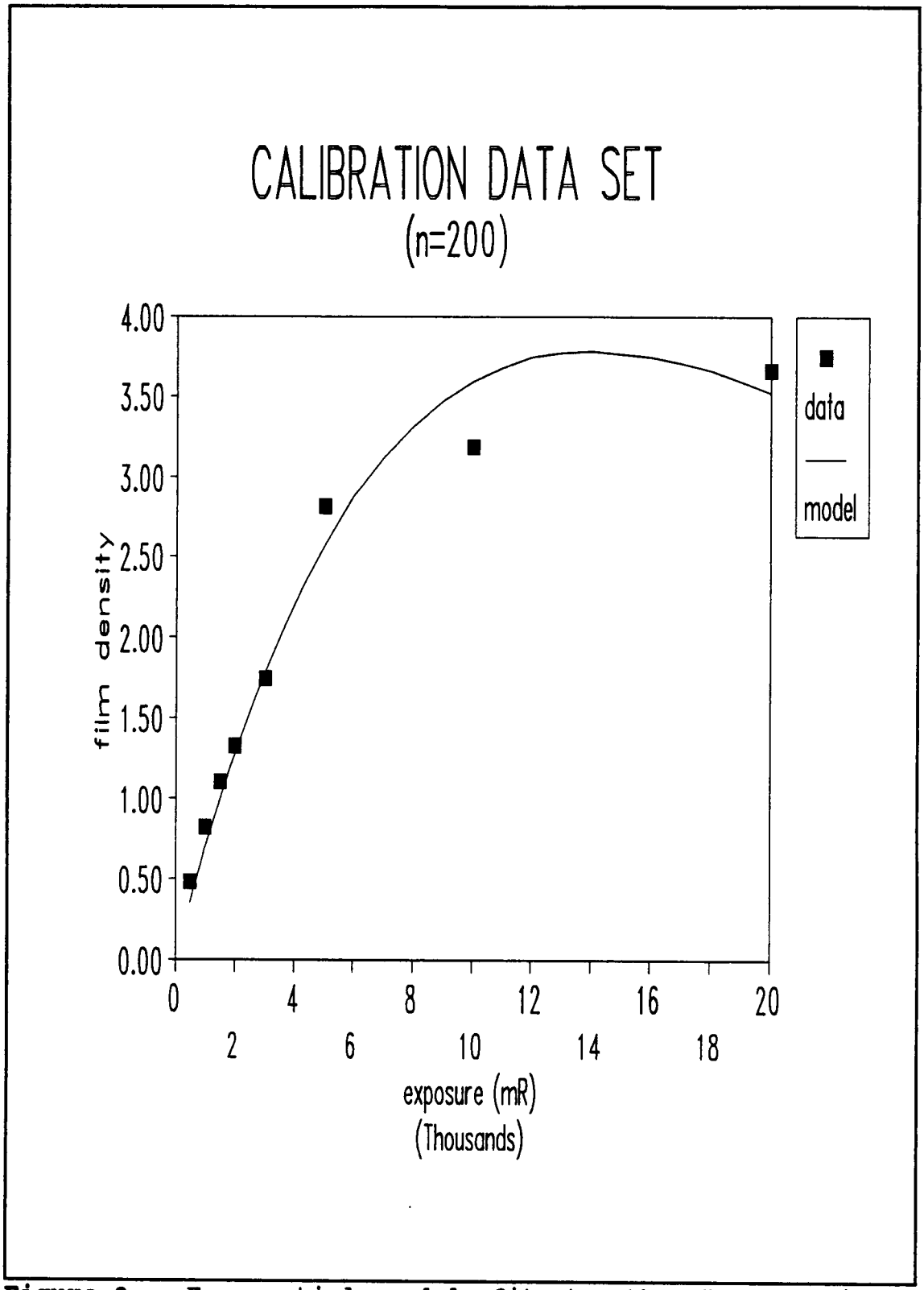


Figure 3 Exponential model fit to the "many points" calibration data set.

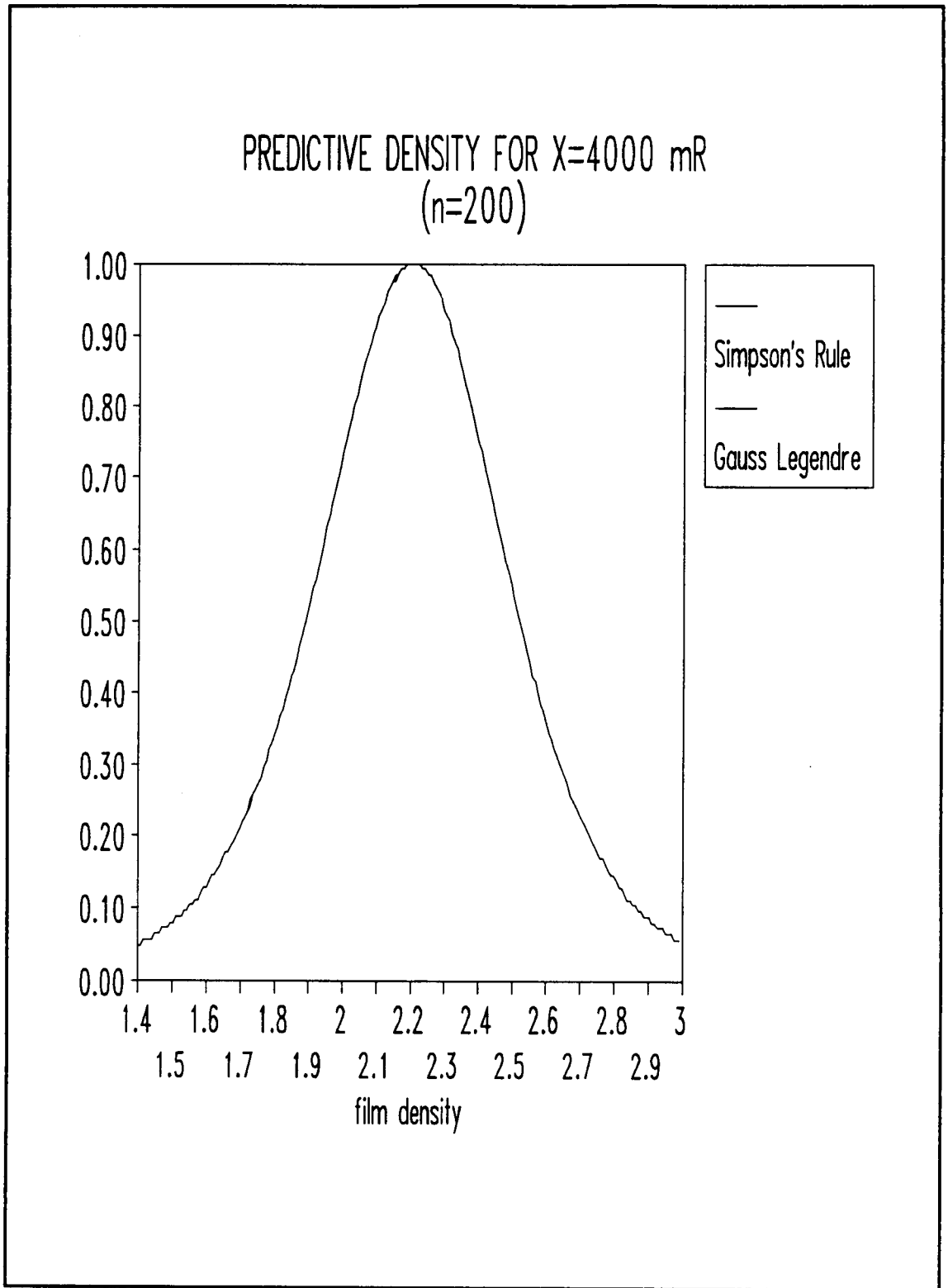


Figure 4 Predictive density of the film density for a new exposure of 4000 mR for the "many points" calibration data set.

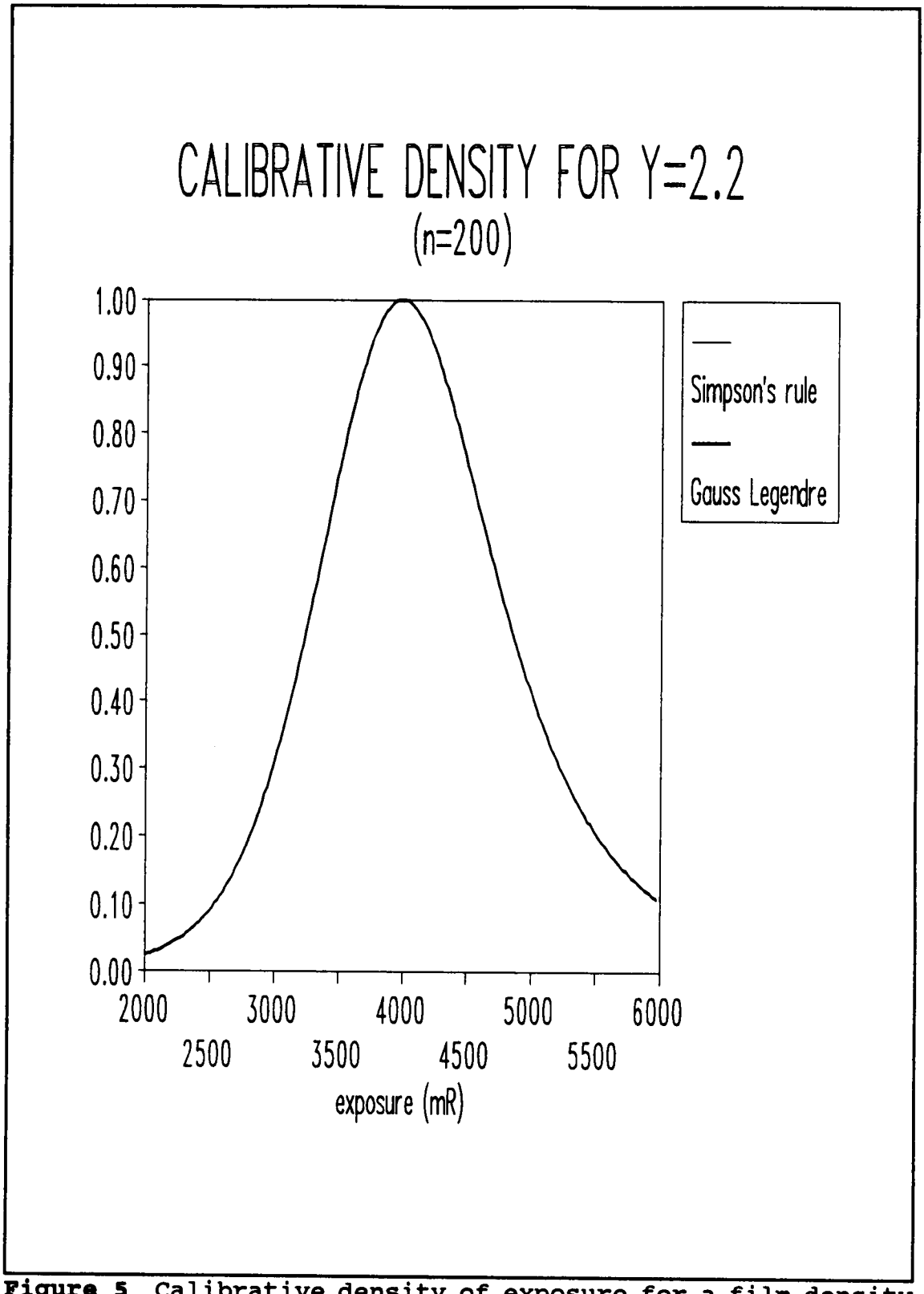


Figure 5 Calibrative density of exposure for a film density of 2.2 for the "many points" calibration data set.

Appendix III

COMPUTER PROGRAMS

A. Gauss Legendre Quadrature Points

To obtain Gauss Legendre quadrature points the IMSL library on the UTK VAX was used. IMSL is a collection of numerical subprograms for general applied mathematics, statistics, and special functions [1]. A Fortran program was written which called the GQRUL subprogram and printed out the abscissa and weights for the specified number of quadrature points. The program follows:

```
      PROGRAM GLQUAD
C     Declaration of variables
      IMPLICIT NONE
      INTEGER I,N,IWEIGH,NFIX
      PARAMETER (N=200)
      REAL  QX(N),QW(N),ALPHA,BETA,QXFIX
      OPEN (1,FILE='GLQUAD.OUT',STATUS='NEW')
C     Assignment of variables
      IWEIGH=1
      ALPHA=0
      BETA=0
      NFIX=0
      QXFIX=0
C     Call to GQRUL subprogram
      CALL GQRUL(N,IWEIGH,ALPHA,BETA,NFIX,QXFIX,QX,QW)
C     Output of abscissas and weights
      DO I=1,N
        WRITE(1,*) QX(I),QW(I)
      END DO
C     Close file and end program
      CLOSE(1)
      STOP
      END
```

Where the values for the GQRUL variables are defined by IMSEL as follows:

IMSL Name: GQRUL/DGQRUL (Single/Double precision version)

Purpose: Compute a Gauss, Gauss-Radau or Gauss-Lobatto quadrature rule with various classical weight functions.

Usage: CALL GQRUL (N, IWEIGH, ALPHA, BETA, NFIX, QFIX, QX, QW)

Arguments:

N - Number of quadrature points. (Input)

IWEIGH - Index of the weight function. (Input)

IWEIGH	WT(X)	Interval	Name
1	1	(-1,+1)	Legendre
2	$1/(1-X^2)^{1/2}$	(-1,+1)	1° Chebyshev
3	$(1-X^2)^{1/2}$	(-1,+1)	2° Chebyshev
4	$e^{(-X^2)}$	(-inf,+inf)	Hermite
5	$(1-X)^{ALPHA}(1+X)^{BETA}$	(-1,+1)	Jacobi
6	$e^{-X}X^{ALPHA}$	(0,+inf)	Laguerre
7	1/COSH(X)	(-inf,+inf)	COSH

ALPHA - Parameter used in the weight function with some values of IWEIGH, otherwise it is ignored. (input)

BETA - Parameter used in the weight function with some values of IWEIGH, otherwise it is ignored. (input)

NFIX - Number of fixed quadrature points. (Input) NFIX = 0, 1 or 2. For the usual Gauss Quadrature rules NFIX = 0.

QXFIX - Array of length NFIX (ignored of NFIX = 0) containing the present quadrature point(s). (Input)

QX - Array of length N containing quadrature points. (Output)

QW - Array of length N containing quadrature weights. (Output)

The Gauss Legendre quadrature points output to the file GLQUAD.OUT from this program are:

QX QW

-0.9999305	1.8331656E-04
-0.9996235	4.2941220E-04
-0.9990714	6.7436678E-04
-0.9982739	9.1999146E-04
-0.9972310	1.1653956E-03
-0.9959434	1.4091905E-03
-0.9944115	1.6549610E-03
-0.9926354	1.8977952E-03
-0.9906158	2.1410973E-03
-0.9883525	2.3843045E-03
-0.9858470	2.6270545E-03
-0.9830993	2.8685837E-03
-0.9801105	3.1093846E-03
-0.9768806	3.3495978E-03
-0.9734116	3.5889198E-03
-0.9697027	3.8281400E-03
-0.9657564	4.0652496E-03
-0.9615725	4.3018754E-03
-0.9571528	4.5373831E-03
-0.9524980	4.7717257E-03
-0.9476095	5.0058253E-03
-0.9424882	5.2370867E-03
-0.9371355	5.4671629E-03
-0.9315531	5.6976983E-03
-0.9257416	5.9256707E-03
-0.9197029	6.1511500E-03
-0.9134387	6.3767717E-03
-0.9069499	6.6001723E-03
-0.9002387	6.8219993E-03
-0.8933066	7.0422641E-03
-0.8861548	7.2610485E-03
-0.8787852	7.4775480E-03
-0.8712004	7.6920618E-03
-0.8634015	7.9050064E-03
-0.8553910	8.1162471E-03
-0.8471701	8.3248364E-03
-0.8387411	8.5319541E-03
-0.8301066	8.7372325E-03
-0.8212678	8.9397253E-03
-0.8122275	9.1406107E-03
-0.8029880	9.3383174E-03
-0.7935513	9.5341522E-03
-0.7839199	9.7282510E-03
-0.7740960	9.9192429E-03
-0.7640818	1.0108457E-02
-0.7538798	1.0294615E-02
-0.7434934	1.0478201E-02
-0.7329239	1.0659721E-02
-0.7221749	1.0838121E-02
-0.7112486	1.1014375E-02
-0.7001474	1.1187641E-02
-0.6888744	1.1357529E-02

-0.6774323	1.1525999E-02
-0.6658239	1.1690530E-02
-0.6540519	1.1852839E-02
-0.6421192	1.2012391E-02
-0.6300288	1.2168165E-02
-0.6177837	1.2321343E-02
-0.6053872	1.2471618E-02
-0.5928422	1.2618082E-02
-0.5801515	1.2762385E-02
-0.5673185	1.2903223E-02
-0.5543460	1.3040826E-02
-0.5412382	1.3175272E-02
-0.5279971	1.3306070E-02
-0.5146264	1.3434676E-02
-0.5011296	1.3558954E-02
-0.4875101	1.3680524E-02
-0.4737700	1.3798614E-02
-0.4599133	1.3913319E-02
-0.4459444	1.4025176E-02
-0.4318654	1.4131621E-02
-0.4176804	1.4236717E-02
-0.4033933	1.4337682E-02
-0.3890069	1.4434557E-02
-0.3745250	1.4528898E-02
-0.3599514	1.4618867E-02
-0.3452894	1.4705033E-02
-0.3305423	1.4787619E-02
-0.3157143	1.4867007E-02
-0.3008086	1.4943022E-02
-0.2858294	1.5015259E-02
-0.2707799	1.5083796E-02
-0.2556638	1.5148092E-02
-0.2404850	1.5209263E-02
-0.2252471	1.5265964E-02
-0.2099538	1.5319666E-02
-0.1946089	1.5368872E-02
-0.1792167	1.5415023E-02
-0.1637804	1.5456885E-02
-0.1483037	1.5495675E-02
-0.1327908	1.5529964E-02
-0.1172448	1.5560579E-02
-0.1016704	1.5587406E-02
-8.6070642E-02	1.5610915E-02
-7.0450403E-02	1.5629832E-02
-5.4812316E-02	1.5645623E-02
-3.9161183E-02	1.5656697E-02
-2.3500090E-02	1.5665047E-02
-7.8333821E-03	1.5668770E-02
7.8354552E-03	1.5668644E-02
2.3501959E-02	1.5664205E-02
3.9162897E-02	1.5656639E-02
5.4814160E-02	1.5645063E-02

7.0452102E-02	1.5629599E-02
8.6072512E-02	1.5611002E-02
0.1016721	1.5587811E-02
0.1172463	1.5560757E-02
0.1327920	1.5530006E-02
0.1483051	1.5495795E-02
0.1637819	1.5457268E-02
0.1792183	1.5416208E-02
0.1946108	1.5368826E-02
0.2099553	1.5319505E-02
0.2252487	1.5266167E-02
0.2404861	1.5208421E-02
0.2556648	1.5147918E-02
0.2707807	1.5083444E-02
0.2858304	1.5014844E-02
0.3008097	1.4942273E-02
0.3157152	1.4867418E-02
0.3305433	1.4788203E-02
0.3452899	1.4705363E-02
0.3599524	1.4618302E-02
0.3745261	1.4528793E-02
0.3890078	1.4434122E-02
0.4033940	1.4337611E-02
0.4176811	1.4236084E-02
0.4318660	1.4132041E-02
0.4459444	1.4024477E-02
0.4599135	1.3913162E-02
0.4737698	1.3798689E-02
0.4875098	1.3680719E-02
0.5011300	1.3559222E-02
0.5146272	1.3434388E-02
0.5279981	1.3306663E-02
0.5412393	1.3175807E-02
0.5543479	1.3040536E-02
0.5673201	1.2903773E-02
0.5801530	1.2762886E-02
0.5928434	1.2618527E-02
0.6053888	1.2470957E-02
0.6177848	1.2321191E-02
0.6300297	1.2167080E-02
0.6421198	1.2011888E-02
0.6540521	1.1852378E-02
0.6658241	1.1690440E-02
0.6774323	1.1525312E-02
0.6888745	1.1358363E-02
0.7001474	1.1187610E-02
0.7112483	1.1014526E-02
0.7221748	1.0838178E-02
0.7329237	1.0659890E-02
0.7434930	1.0477750E-02
0.7538797	1.0294064E-02
0.7640814	1.0107824E-02

0.7740952	9.9193351E-03
0.7839192	9.7281700E-03
0.7935506	9.5344819E-03
0.8029874	9.3375491E-03
0.8122270	9.1404701E-03
0.8212670	8.9395167E-03
0.8301053	8.7371934E-03
0.8387403	8.5317362E-03
0.8471690	8.3257575E-03
0.8553899	8.1154061E-03
0.8634005	7.9057505E-03
0.8711995	7.6922150E-03
0.8787844	7.4772872E-03
0.8861533	7.2610117E-03
0.8933053	7.0414799E-03
0.9002374	6.8231691E-03
0.9069487	6.5999795E-03
0.9134372	6.3778581E-03
0.9197018	6.1520552E-03
0.9257404	5.9251236E-03
0.9315516	5.6971153E-03
0.9371342	5.4671015E-03
0.9424869	5.2372324E-03
0.9476080	5.0050314E-03
0.9524965	4.7715944E-03
0.9571514	4.5373738E-03
0.9615709	4.3017650E-03
0.9657547	4.0647304E-03
0.9697011	3.8276601E-03
0.9734097	3.5895808E-03
0.9768792	3.3490178E-03
0.9801089	3.1101673E-03
0.9830980	2.8680991E-03
0.9858455	2.6266989E-03
0.9883512	2.3849406E-03
0.9906142	2.1421891E-03
0.9926341	1.8979900E-03
0.9944100	1.6538963E-03
0.9959421	1.4096129E-03
0.9972294	1.1655213E-03
0.9982722	9.1977930E-04
0.9990698	6.7502703E-04
0.9996219	4.3006823E-04
0.9999292	1.8542704E-04

B. Input File

All programs request an input file name and the value of a gamma function for some argument. The later is requested because it was felt it would be easier to look up a value of the gamma function for a given argument, which changes depending on the number of calibration data sets and the density being integrated, than include a library of the values. The file is requested to provide all the necessary data for the program to run. The input file is an ASCII file and may have any standard DOS name (<file_name>.<ext>). The file itself is a series of numbers, one in each line. A sample input file is:

```
5
500
1000
1500
2000
5000
0.36
0.71
1
1.22
2.64
0.0005
0.0000025
0.0
0.0000006
1.2
0.005
3000
```

Where each line contains the following values:

Line #	Purpose
1	number of calibration data sets
2-6	exposure data pts (depends on value in line 1)
7-11	film density data pts (depends on value in line 1)
12	lower limit of integration for $\beta 1$
13	Δ for $\beta 1$ ($200*\Delta$=upper limit of integration for $\beta 1$)
14	lower limit of integration for $\beta 2$
15	Δ for $\beta 2$ ($200*\Delta$=upper limit of integration for $\beta 2$)
16	lower limit for film density when determining the predictive density*
16'	lower limit for exposure when determining the calibrative density*
17	Δ for film density when determining the predictive density ($200*\Delta$=upper limit for film density)*
17'	Δ for exposure when determining the predictive density ($200*\Delta$=upper limit for film density)*
18	value of exposure for predictive density*
18'	value of film density for calibrative density*

* These values are not used in the programs calculating the posterior densities of $\beta 1$ and $\beta 2$ but they <u>can</u> be in the input files to these programs, they simply will not be read.

There are two things to note about the input file. (1) Depending upon the number of calibration points, the numbers past the first row will vary, in this example there are 5 sets of calibration data points, if there are more or less rows than this value rows 12-18 will be shifted accordingly. (2) The limits and Δ specified for $\beta 1$ and $\beta 2$ are limits of integration and care must be taken in choosing these limits to cover the main supports of the distributions. The limits and

▲ specified for film density and exposure determine the starting value and range of points plotted for the distribution and are not limits of integration.

C. Posterior Densities of β_1 and β_2

The two programs are very similar and can use the same input file. Using the limits determined from the input file they integrate using Simpson's rule and a Gauss Legendre quadrature over one variable at 200 points and print to an output file the corresponding values of the other variable. The output file for the posterior distribution of $\beta 1$ is b1_out.dat, and for $\beta 2$ is b2_out.dat. All comments are within the /* and */ symbols.

POSTERIOR DISTRIBUTION OF β_1

```
#include <stdio.h>      /*this program integrates over beta 2
*/
#include <math.h>       /*to give a posterior distribution */
main()                  /*of beta 1*/
{
/*declaration of variables*/
FILE *fopen(), *in_file, *out_file;
char f_name[13];
int a, b, num;
float upper, lower, gamma, temp;
float abeta2, beta1, dbeta1, beta2, dbeta2, quadval, outval;
float funct(), quad();
float x[10], y[10];
float xi[202] = {-0.999931, -0.999623,
/*...these are the 200 quadrature abscissa points...*/
                0.999622, 0.999929};
float wi[202] = {0.000183, 0.000429,
/*...these are the 200 quadrature weight points...*/
                0.000430, 0.000185};
/*asking for, receiving, and opening the input file*/
printf("\nwhat is the name of the input file\n");
gets(f_name);
in_file=fopen(f_name,"r");
/*reading the input file*/
```

```
fscanf(in_file,"%d", &num);
temp=0;
for (a=0;a<num;++a)
{
    scanf(in_file,"%f", &x[a]);
    temp=temp+1;
}
for (a=0;a<num;++a)
    scanf(in_file,"%f", &y[a]);
fscanf(in_file,"%f %f %f %f",&beta1,&dbeta1,&abeta2,&dbeta2);
/*printing the input file to the screen for verification*/
printf("%d\n",num);
for (a=0;a<num;++a)
    printf("%f %f\n",x[a],y[a]);
printf("%15.7f\n",beta1);
printf("%15.7f\n",dbeta1);
printf("%15.7f\n",abeta2);
printf("%15.7f\n",dbeta2);
/*requesting and receiving the value for the gamma function*/
/*needed in the calculations*/
printf("\nwhat is the gamma function of %f\n", temp/2-.5);
scanf("%f", &gamma);
/*opening the output file*/
out_file=fopen("b1_out.dat","w");
/*setting upper and lower integration limits*/
lower=abeta2;
```

```
upper=abeta2+200*dbeta2;
/*outer loop calculates one distribution plot point for
beta1*/
/*each iteration*/
for (a=0;a<200;++a)
    {
    outval=0;
    quadval=0;
    beta2=abeta2;
/*inner loop integrates over beta2 for each beta1*/
    for (b=0;b<200;++b)
    {
/*Simpon's rule*/
outval=outval+(funct(temp,gamma,num,x,y,beta1,beta2)+4*funct
(temp,gamma,num,x,y,beta1,beta2+dbeta2/2)+funct(temp,gamma,n
um,x,y,beta1,beta2+dbeta2));
/*Gauss Legendre quadrature*/
quadval=quadval+wi[b]*quad(temp,gamma,num,x,y,upper,lower,be
ta1,xi[b]);
        beta2=beta2+dbeta2;
    }
/*print values out to b1_out.dat*/
    fprintf(out_file,"%15.7f                %15.7f
%15.7f\n", beta1, dbeta2*outval/6, (upper-lower)*quadval/2);
    printf("%15.7f              %15.7f              %15.7f\n",
beta1, dbeta2*outval/6, (upper-lower)*quadval/2);
```

```
    beta1=beta1+dbeta1;
    }
fclose(out_file);
}
/*called function calculates the value of the density*/
/*function for Simpson's rule*/
float funct(temp,gamma, num, x, y, one, two)
int num;
float temp, gamma, x[], y[], one, two;
{
float con, val, temp1;
int a;
temp1=-temp/2;
con=pow(2,-1.5)*pow(3.141592654,temp1)*gamma;
/*printf("\nfunct con=%f",10000*con);*/
val=0;
for (a=0; a<num; ++a)
{
    val=val+pow(y[a]-one*x[a]*exp(-two*x[a]),2);
    /*printf("\nfunct x=%f  y=%f", x[a], y[a]);*/
}
temp1=temp1-.5;
val=con*pow(val,temp1);
/*printf("\nfunct temp= %f  gamma= %f\n", temp, gamma);*/
return(val);
}
```

```
/*called function which calculates the value of the*/
/*density function for the Gauss Legendre quadrature*/
float quad(temp, gamma, num, x, y, up, low, one, two)
int num;
float temp, gamma, x[], y[], up, low, one, two;
{
float con, val, lambda, temp1;
int a;
temp1=-temp/2;
con=pow(2,-1.5)*pow(3.141592654,temp1)*gamma;
/*printf("\nquad 2^-3/2=%f",pow(2,-1.5));*/
val=0;
lambda=(up-low)*two/2+(low+up)/2;
for (a=0; a<num; ++a)
{
    val=val+pow(y[a]-one*x[a]*exp(-lambda*x[a]),2);
    /*printf("\nquad x=%f  y=%f", x[a], y[a]);*/
}
temp1=temp1-.5;
val=con*pow(val,temp1);
/*printf("\nquad temp= %f  gamma= %f\n", temp, gamma);*/
return(val);
}
```

POSTERIOR DISTRIBUTION OF β_2

```
#include <stdio.h>     /*this program integrates over beta1 */
#include <math.h>       /*to give a posterior distribution */
main()                  /*of beta 2*/
{
/*declaration of variables*/
FILE *fopen(), *in_file, *out_file;
char f_name[13];
int a, b;
float upper, lower, gamma, temp;
float abeta1, beta1, dbeta1, beta2, dbeta2, quadval1, outval;
float funct(), quad();
float x[10], y[10];
float xi[202] = {-0.999931, -0.999623,
/*...these are the 200 quadrature abscissa points...*/
                0.999622, 0.999929};
float wi[202] = {0.000183, 0.000429,
/*...these are the 200 quadrature weight points...*/
                0.000430, 0.000185};
/*asking for, receiving, and opening the input file*/
printf("\nwhat is the name of the input file\n");
gets(f_name);
in_file=fopen(f_name,"r");
/*reading the input file*/
fscanf(in_file,"%d", &num);
```

```
temp=0;
for (a=0;a<num;++a)
{
    scanf(in_file,"%f", &x[a]);
    temp=temp+1;
}
for (a=0;a<num;++a)
    scanf(in_file,"%f", &y[a]);
fscanf(in_file,"%f %f %f %f",&abeta1,&dbeta1,&beta2,&dbeta2);
/*printing the input file to the screen for verification*/
printf("%d\n",num);
for (a=0;a<num;++a)
    printf("%f %f\n",x[a],y[a]);
printf("%15.7f\n",abeta1);
printf("%15.7f\n",dbeta1);
printf("%15.7f\n",beta2);
printf("%15.7f\n",dbeta2);
/*requesting and receiving the value for the gamma function*/
/*needed in the calculations*/
printf("\nwhat is the gamma function of %f\n", temp/2-.5);
scanf("%f", &gamma);
/*opening the output file*/
out_file=fopen("b2_lin.dat","w");
/*setting upper and lower integration limits*/
lower=abeta1;
upper=abeta1+200*dbeta1;
```

```
/*outer loop calculates one distribution plot point for
beta2*/
/*each iteration*/
for (a=0;a<200;++a)
    {
    outval=0;
    quadval=0;
    beta1=abeta1;
/*inner loop integrates over beta1 for each beta2*/
    for (b=0;b<200;++b)
    {
/*Simpson's rule*/
outval=outval+(funct(temp,gamma,num,x,y,beta1,beta2)+4*funct
(temp,gamma,num,x,y,beta1+dbeta1/2,beta2)+funct(temp,gamma,n
um,x,y,beta1+dbeta1,beta2));
/*Gauss Legendre quadrature*/
quadval=quadval+wi[b]*quad(temp,gamma,num,x,y,upper,lower,xi
[b],beta2);
        beta1=beta1+dbeta1;
    }
    fprintf(out_file,"%15.7f                %15.7f
%15.7f\n", beta2, dbeta1*outval/6, (upper-lower)*quadval/2);
    printf("%15.7f              %15.7f            %15.7f\n",
beta2, dbeta1*outval/6, (upper-lower)*quadval/2);
    beta2=beta2+dbeta2;
    }
```

```
fclose(out_file);
}
/*called function calculates the value of the density*/
/*function for Simpson's rule*/
float funct(temp,gamma,num,x,y,one, two)
int num;
float temp,gamma,x[],y[],one, two;
{
float con, val,temp1;
int a;
temp1=-temp/2;
con=pow(2,-1.5)*pow(3.141592654,temp1)*gamma;
val=0;
for (a=0; a<num; ++a)
    val=val+pow(y[a]-one*x[a]*exp(-two*x[a]),2);
temp1=temp1-.5;
val=con*pow(val,temp1);
return(val);
}
/*called function which calculates the value of the*/
/*density function for the Gauss Legendre Quadrature*/
float quad(temp,gamma,num,x,y,up, low, one, two)
int num;
float temp,gamma,x[],y[],up,low,one, two;
{
float con, val, lambda,temp1;
```

```
int a;
temp1=-temp/2;
con=pow(2,-1.5)*pow(3.141592654,temp1)*gamma;
val=0;
lambda=(up-low)*one/2+(low+up)/2;
for (a=0; a<num; ++a)
    val=val+pow(y[a]-lambda*x[a]*exp(-two*x[a]),2);
temp1=temp1-.5;
val=con*pow(val,temp1);
return(val);
}
```

D. Predictive and Calibrative Densities

Predictive Density

```
include <stdio.h>     /*this program integrates over beta 1 &
2*/
#include <math.h>      /*to give a predictive distribution */
main()                      /*of film density given an input
exposure*/
{
/*declaration of variables*/
FILE *fopen(), *in_file, *out_file;
char f_name[13];
int a, b, c, d, num;
float upper, lower, gamma, temp, temp4, con;
float abeta2, abeta1, beta1, dbeta1, beta2, dbeta2, quadval,
outval;
float funct(), quad();
float x[10], y[10];
float X, Y, dY, simp, gauss;
float temp1[201], temp2[201];
float xi[202] = {-0.999931, -0.999623,
/*...these are the 200 quadrature abscissa points...*/
                0.999622, 0.999929};
float wi[202] = {0.000183, 0.000429,
/*...these are the 200 quadrature weight points...*/
```

```
                0.000430, 0.000185};
/*asking for, receiving and opening the input file*/
printf("\nwhat is the name of the input file\n");
gets(f_name);
in_file = fopen(f_name,"r");
/*reading the input file*/
fscanf(in_file,"%d",&num);
temp=0;
for (a=0;a<num;++a)
{
    fscanf(in_file,"%f",&x[a]);
    temp=temp+1;
}
for (a=0;a<num;++a)
    fscanf(in_file,"%f",&y[a]);
fscanf(in_file,"%f %f %f %f %f %f %f", &abeta1, &dbeta1,
&abeta2, &dbeta2, &Y, &dY, &X);
fclose(in_file);
*/printing the input file to the screen for verification*/
printf("%d\nx       y\n",num);
for (a=0;a<num;++a)
    printf("%f %f\n",x[a],y[a]);
printf("%15.7f\n",abeta1);
printf("%15.7f\n",dbeta1);
printf("%15.7f\n",abeta2);
printf("%15.7f\n",dbeta2);
```

```
printf("%15.7f\n",Y);
printf("%15.7f\n",dY);
printf("%15.7f\n",X);
/*requesting and receiving the value for the gamma function*/
/*needed for the calculations*/
printf("\nwhat is the gamma function of %f\n", temp/2);
scanf("%f", &gamma);
temp4=-(temp+1)/2;      /*calculation of constant*/
con=pow(2,-1.5)*pow(3.141592654*temp4)*gamma;
/*opening the output file*/
out_file=fopen("tot_int3.dat","w");
lower=abeta2;        /*defines limits of integration*/
upper=abeta2+200*dbeta2;
/*outer loop calculates one distribution plot point for*/
/*film density each iteration*/
for(c=0;c<200;++c)
    {
    beta1=abeta1;
/*second loop integrates over beta1*/
    for (a=0;a<200;++a)
     {
     outval=0;
     quadval=0;
     beta2=abeta2;
/*third loop integrates over beta2*/
     for (b=0;b<200;++b)
```

```
        {
outval=outval+(funct(X,Y,temp,num,x,y,beta1,beta2)+4*funct(X
,Y,temp,num,x,y,beta1,beta2+dbeta2/2)+funct(X,Y,temp,num,x,y
,beta1,beta2+dbeta2));
        /*simpson's rule*/
quadval=quadval+wi[b]*quad(X,Y,temp,num,x,y,upper,lower,beta
1,xi[b]);
        /*gauss quadrature*/
        beta2=beta2+dbeta2;
        }
     temp1[a]=10000*con*dbeta2*outval/6;
     temp2[a]=10000*con*(upper-lower)*quadval/2;
     beta1=beta1+dbeta1;
     }
    simp=0;
    gauss=0;
/*calculation of midpoint method for second loop*/
    for(d=0;d<199;++d)
     {
     simp=simp+dbeta1*(temp1[d]+temp1[d+1])/2;
     gauss=gauss+dbeta1*(temp2[d]+temp2[d+1])/2;
     }
/*print results to outfile*/
    fprintf(out_file,"%f          %15.7f          %15.7f\n",
Y, 100*simp, 100*gauss);
    Y=Y+dY;
```

```
    }
fclose(out_file);
}
/*called function calculates the value of the density*/
/*function for Simpson's rule*/
float funct(X,Y,temp, num, x, y, one, two)
int num;
float X,Y,temp, x[], y[], one, two;
{
float val, temp1;
int a;
val=0;
for (a=0; a<num; ++a)
{
    val=val+pow(y[a]-one*x[a]*exp(-two*x[a]),2);
}
temp1=-temp/2;
val=pow(val+pow(Y-one*X*exp(-two*X),2),temp1);
return(val);
}
/*called function which calculates the value of the*/
/*density function for the Gauss Legendre Quadrature*/
float quad(X,Y,temp, num, x, y, up, low, one, two)
int num;
float X, Y, temp, x[], y[], up, low, one, two;
{
```

```
float val, lambda, temp1;
int a;
val=0;
lambda=((up-low)*two+(low+up))/2;
for (a=0; a<num; ++a)
{
    val=val+pow(y[a]-one*x[a]*exp(-lambda*x[a]),2);
}
temp1=-temp/2;
val=pow(val+pow(Y-X*one*exp(-lambda*X),2),temp1);
return(val);
}
```

Calibrative Density

```
include <stdio.h>     /*this program integrates over beta 1 &
2*/
#include <math.h>      /*to give a predictive distribution */
main()                      /*of film density given an input
exposure*/
{
/*declaration of variables*/
FILE *fopen(), *in_file, *out_file;
char f_name[13];
int a, b, c, d, num;
float upper, lower, gamma, temp, temp4, con;
float abeta2, abeta1, beta1, dbeta1, beta2, dbeta2, quadval,
outval;
float funct(), quad();
float x[10], y[10];
float X, Y, dY, simp, gauss;
float temp1[201], temp2[201];
float xi[202] = {-0.999931, -0.999623,
/*...these are the 200 quadrature abscissa points...*/
                0.999622, 0.999929};
float wi[202] = {0.000183, 0.000429,
/*...these are the 200 quadrature weight points...*/
                0.000430, 0.000185};
*/asking for, receiving, and opening of the input file*/
```

```
printf("\nwhat is the name of the input file\n");
gets(f_name);
in_file = fopen(f_name,"r");
/*open input file*/
fscanf(in_file,"%d",&num);
temp=0;
for (a=0;a<num;++a)
{
    fscanf(in_file,"%f",&x[a]);
    temp=temp+1;
}
for (a=0;a<num;++a)
    fscanf(in_file,"%f",&y[a]);
fscanf(in_file,"%f    %f    %f    %f    %f    %f
%f",&abeta1,&dbeta1,&abeta2,&dbeta2,&Y,&dY,&X);
fclose(in_file);
/*printing the input file to the screen for verification*/
printf("%d\nx       y\n",num);
for (a=0;a<num;++a)
    printf("%f %f\n",x[a],y[a]);
printf("%15.7f\n",abeta1);
printf("%15.7f\n",dbeta1);
printf("%15.7f\n",abeta2);
printf("%15.7f\n",dbeta2);
printf("%15.7f\n",Y);
printf("%15.7f\n",dY);
```

```
printf("%15.7f\n",X);
/*requesting and receiving the value for the gamma function*/
/*needed in the calculations*/
printf("\nwhat is the gamma function of %f\n", temp/2);
scanf("%f", &gamma);
temp4=-(temp+1)/2;      /*calculation of constant*/
con=pow(2,-1.5)*pow(3.141592654,temp4)*gamma;
/*opening the output file*/
out_file=fopen("tot_int3.dat","w");
lower=abeta2;       /*defines limits of integration*/
upper=abeta2+200*dbeta2;
/*outer loop calculated one distribution plot point for*/
/*exposure each iteration*/
for(c=0;c<200;++c)
    {
    beta1=abeta1;
/*second loop integrates for beta2*/
    for (a=0;a<200;++a)
     {
     outval=0;
     quadval=0;
     beta2=abeta2;
/*inner loop integrates over beta2*/
     for (b=0;b<200;++b)
         {
outval=outval+(funct(X,Y,temp,num,x,y,beta1,beta2)+4*funct(X
```

```
,Y,temp,num,x,y,beta1,beta2+dbeta2/2)+funct(X,Y,temp,num,x,y
,beta1,beta2+dbeta2));
          /*simpson's rule*/
quadval=quadval+wi[b]*quad(X,Y,temp,num,x,y,upper,lower,beta
1,xi[b]);
          /*gauss quadrature*/
          beta2=beta2+dbeta2;
          }
      temp1[a]=10000*con*dbeta2*outval/6;
      temp2[a]=10000*con*(upper-lower)*quadval/2;
      beta1=beta1+dbeta1;
      }
     simp=0;
     gauss=0;
/*calculation of midpoint method for second loop*/
     for(d=0;d<199;++d)
      {
      simp=simp+dbeta1*(temp1[d]+temp1[d+1])/2;
      gauss=gauss+dbeta1*(temp2[d]+temp2[d+1])/2;
      }
/*print values to outfile*/
     fprintf(out_file,"%f          %15.7f          %15.7f\n",
Y, 100*simp, 100*gauss);
     Y=Y+dY;
     }
fclose(out_file);
```

```
}
/*called function calculates the value of the density*/
/*function for Simpson's Rule*/
float funct(X,Y,temp, num, x, y, one, two)
int num;
float X,Y,temp, x[], y[], one, two;
{
float val, temp1;
int a;
val=0;
for (a=0; a<num; ++a)
{
    val=val+pow(y[a]-one*x[a]*exp(-two*x[a]),2);
}
temp1=-temp/2;
val=pow(val+pow(Y-one*X*exp(-two*X),2),temp1);
return(val);
}
/*called function calculates the value of the density*/
/*function for the Gauss Legendre Quadrature*/
float quad(X,Y,temp, num, x, y, up, low, one, two)
int num;
float X, Y, temp, x[], y[], up, low, one, two;
{
float val, lambda, temp1;
int a;
```

```
val=0;
lambda=((up-low)*two+(low+up))/2;
for (a=0; a<num; ++a)
{
    val=val+pow(y[a]-one*x[a]*exp(-lambda*x[a]),2);
}
temp1=-temp/2;
val=pow(val+pow(Y-X*one*exp(-lambda*X),2),temp1);
return(val);
}
```

VITA

Robert Alan Metcalf was born in Columbia, Missouri, on July 28, 1966. His parents are Dr. Virgil Alanzo Metcalf and the former Janice Ann Maples. Mr. Metcalf graduated from Corona del Sol High School in Tempe, Arizona, in June 1984. He received a Bachelor of Science in Nuclear Engineering from Arizona State University in December 1991. Before entering college, Mr. Metcalf studied for a year in Rovaniemi, Finland on a Rotary International Scholarship. While in college, Mr. Metcalf Studied Chemical Engineering for one year at H.E.I. in Lille, France. He was also employed as a summer intern for the C.E.A in Cadarache, France.

Mr. Metcalf entered the Graduate School of the University of Tennessee in January 1991. He accepted a graduate research assistantship working for Dr. Peter Groer under contract for the D.O.E. He was employed as a summer intern by the internal dosimetry division of Martin Mareitta Energy Systems in Oak Ridge, Tennessee in 1993. As of the writing of this vita Mr. Metcalf is unemployed.

Mr. Metcalf is single, enjoys traveling, movies, and good books. Most of all, Mr. Metcalf loves to snow ski.