

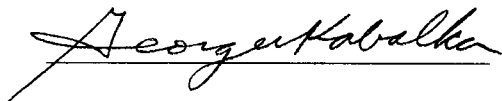
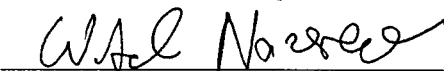
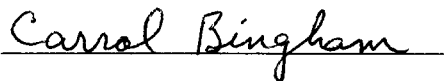
To the Graduate Council:

I am submitting herewith a dissertation written by Wilhelm Friedrich Mueller entitled "High-spin Spectroscopy on $^{179,181}\text{Au}$ and a Systematic Study of Moments of Inertia in Prolate Rotational Bands from Dy to Tl." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Physics.



Leo L. Riedinger, Major Professor

We have read this dissertation
and recommend its acceptance:



Accepted for the Council:



Associate Vice Chancellor and
Dean of The Graduate School

High-Spin Spectroscopy on $^{179,181}\text{Au}$
and
A Systematic Study of Moments of Inertia in
Prolate Rotational Bands from Dy to Tl

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Wilhelm Friedrich Mueller

May 1997

This dissertation
is dedicated to
my mother,
Dr. Marilyn Ruth Mueller

ABSTRACT

High-spin states have been studied in ^{181}Au and ^{179}Au nuclei for the first time. These are the most neutron-deficient Au isotopes which have been studied in detail. Excited states in ^{181}Au were populated by the $^{150}\text{Sm}(^{35}\text{Cl},4n)$ heavy-ion fusion reaction, and studied at the Oak Ridge Holifield Heavy-Ion Research Facility with the Spin Spectrometer and 19 Compton-suppressed Ge detectors. Eight prolate rotational bands have been established in the analysis of this nucleus. Bands based on $\pi h_{9/2}$, $\pi f_{7/2}$, $\pi i_{13/2}$, and $\pi h_{11/2}$ configurations have been clearly identified. In addition, there is a structure of high-lying states feeding at high spins into the $\pi i_{13/2}$ band in ^{181}Au . Of the two observed side bands, one has been identified as a rotational band base on the three quasiparticle excitation $\pi \frac{1}{2}[660] \otimes \nu \frac{1}{2}[521] \otimes \nu \frac{5}{2}[512]$, and the second has been suggested as the unfavored signature of the $\pi i_{13/2}$ band. A final observed band has been suggested as the unfavored signature of the $\pi f_{7/2}$ configuration.

The experiment to study high-spin states in ^{179}Au took place at the ATLAS facility at Argonne National Laboratory. The AYE-ball array of 19 Compton-suppressed Ge detectors was used in conjunction with the Fragment Mass Analyzer and a double-sided silicon strip detector to make the first positive identification of excited states in ^{179}Au by the recently developed method of Recoil Decay Tagging. These states were populated in a $^{124}\text{Te}(^{58}\text{Ni},p2n)$ reaction. From the study, the $\pi i_{13/2}$ band was clearly established, and a lower limit of 592 keV is determined for the bandhead energy. Three other bands have been observed in this nucleus with suggested configurations assignments of $\pi h_{9/2}$ and $\pi d_{3/2}$.

In addition to the study of ^{181}Au and ^{179}Au , a systematic analysis was made of moments of inertia of ground-state rotational bands in even-even nuclei and $\pi h_{9/2}$ and $\pi i_{13/2}$ bands in odd- Z even- N nuclei. From the study of even-even nuclei, an empirical relationship was developed from experimental data where by the deformation of a nucleus can be inferred from the moment of inertia with very good correlation to other observables. This relationship is applied to measured moments of inertia in intruder ($\pi h_{9/2}$ and $\pi i_{13/2}$) bands using theoretically determined deformation param-

eters. From this comparison, it can be concluded that the amount of deformation enhancement caused by the occupation of an intruder orbital is underpredicted in some regions.

ACKNOWLEDGEMENTS

As the writing of this dissertation is coming to an end, I want to consider all the support from people that have helped make this dissertation a reality. I would first like to thank my primary advisor, Dr. Lee Riedinger. He has given me the freedom to develop my own research style, yet at the same time provided considerable guidance to keep me on the right track. Despite having many administrative duties, he always found time to sit down and discuss physics. I would also like to thank Dr. Walter Reviol, who taught me many of the details of detector design and experimental analysis (in spite of the fact that Eintracht Frankfurt fell into the 2nd league). I am very grateful to Dr. Witek Nazarewicz. My understanding of physics has been greatly enhanced by his patient teachings and advice concerning the theoretical models of the nucleus.

I would also like to acknowledge Drs. Carrol Bingham, Jing-ye Zhang, and Haoqiang Jin for their many fruitful discussions. I thank Dr. Jan Wauters for his advice on the subtleties of α decay, as well as, introducing me to Belgian culture. The experiments presented in this dissertation were performed at the Holifield Heavy Ion Facility at Oak Ridge National Laboratory and the ATLAS facility at Argonne National Laboratory. This dissertation would not have been possible without the dedication and effort of the staff at these facilities. I would particularly like to thank Drs. R. V. F. Janssens and M. P. Carpenter at Argonne for their advice and encouragement. Special recognition goes to the physics department faculty and fellow students at my undergraduate institution, Appalachian State University. It was there where I first became truly interested in the field of physics.

The poker gang, Brad, Max, Jeff, Mizutori, Raymond, *et al.*, is acknowledged for being a reminder that all work and no play makes for a boring physicist (though perhaps with more money). In addition, fellow student, Brad Smith deserves recognition for introducing me to the Macintosh religion and for his advice on the finer points of computer programming. While there are many more people I would like to acknowledge, I must stop here in the interest of brevity.

I reserve this last paragraph for the extra special recognition of my family and my wife. My mother, Dr. Marilyn Ruth Mueller, had particularly difficult challenge of raising five teenagers essentially on her own. She accepted this challenge and has been an inspiration and a role model to me. Her perseverance and determination has provided the foundation for me to succeed. I also would like to recognize the support of my fellow "troublesome teenagers," Hans, Venus, Erich, and Karl, who have all grown up now but still have a profound influence on my life. Finally, I want to gratefully acknowledge my wife, Candy Jane Hayden. She has been the keystone that has kept me together not only through this dissertation, but through all my studies. Even though she insists that her support is merely a loan and that I owe her "big time," I cannot thank her enough. This dissertation is a result of her work as much as it is mine.

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CHAPTER 1

Introduction

The field of nuclear structure physics is concerned with the study of the composition of the atomic nucleus and interaction of the constituent particles. This branch of physics has its roots early in the twentieth century when, in 1911, Rutherford discovered that the bulk of matter is contained within small core at the center of the atom [1]. This atomic nucleus was later determined to be composed of two types of “nucleons:” protons and neutrons. The “strong force” provides the glue that binds the nucleons together and is the primary mechanism through which they interact. Nuclei are composed of many nucleons (sometimes hundreds), and the interplay of these particles introduces a wide variety of ground-state and excited configurations. In some regimes, nucleons can be treated to a good approximation as independent particles, orbiting in an average potential well generated by all the other nucleons. In other areas, nuclear properties are dominated by nucleons acting collectively.

One particularly interesting property is that many nuclei exhibit stable deformations in the ground state. The evolution of nuclear shapes throughout the known nuclei has been a subject of study for many years. This subject has presented challenges for both the experimental as well as the theoretical physicists. There have been many different techniques developed in order to determine nuclear deformation. Despite all these methods, the accurate determination of deformation, particularly of excited configurations, remains a challenge to physicists. The most direct evidence that nuclei are deformed comes from the observation of sets (or “bands”) of rotational states, i.e. nuclear levels that correspond to the deformed nucleus rotating with different frequencies. Rotational states in nuclei were first observed by Huus and Zupancic [2] in 1953. Since that time, the study of rotational states has been

the focus of many nuclear structure physicists. This field is often referred to as high-spin nuclear physics because states with very large angular momentum¹ are observed in these studies. The subject of this dissertation is in the field of high-spin nuclear structure physics.

The work presented here is organized into several chapters. A summary of some of the theoretical models that are used to interpret the results of observed high-spin states is given in Chapter 2. Chapter 3 covers some of the unique techniques used to observe high-spin states in nuclei, as well as discusses specific setups for the experiments reported in Chapters 4 and 5. Excited states in ^{181}Au and ^{179}Au were observed for the first time in experiments by the Nuclear Physics Group at the University of Tennessee, as part of the research for this dissertation. The results of these experiments are presented in Chapters 4 and 5. A strong test of a nuclear model is its ability to reproduce observed data over wide ranges of nuclei. A survey of the moment of inertia (one of fundamental properties of rotational states) for bands in nuclei from Dy ($Z = 66$) to Tl ($Z = 81$) was undertaken in order to test these models. The results of this survey and the ramifications for nuclei in the transitional region from well deformed to spherical shapes ($78 < Z < 81$) are discussed in Chapter 6. Chapter 7 discusses some of the interesting properties of rotational bands based on $\pi h_{9/2}$ and $\pi f_{7/2}$ single-particle orbitals observed in Au and Ir nuclei. The last chapter (Chapter 8) presents a summary of this dissertation. The following sections of this introduction are included to present an short overview of the field.

1.1 Theoretical Developments

The description of the nucleus has a number of difficulties that make the interpretation of observed phenomena a challenging problem. Because of the large, but countable, number of particles, the nucleus must be treated as a many-body system. In addition to the difficulty of the many-body problem, the fundamental strong interaction is not entirely understood. As pointed out by Bohr and Mottelson [3], however, the complete formulation of the strong force is not necessary to explain a wide range of

¹Excited states with as much as $60\hbar$ of angular momentum have been observed in some nuclei.

nuclear phenomena. For example, similar to atomic electrons, nuclei with certain “magic numbers” of nucleons have a much lower binding energy than the neighboring nuclei. The introduction of a nuclear shell model (see e.g. [4, 5]) was very successful at interpreting existence of these magic numbers.

One of the earliest theories to interpret nuclear dynamics was the liquid-drop model [6]. This phenomenological model is useful in describing many collective aspects of nuclei such as rotational and vibrational excitations, as well as such dramatic features as nuclear fission. The pure liquid-drop model, however, predicts that all nuclei would have a spherical deformation in their ground state. The interpretation of stable ground-state deformation came initially from Bohr and Mottelson [7], and Hill and Wheeler [8]. This was extended by Nilsson [9], where he developed a nuclear shell model for the deformed regime. He was able to illustrate that nuclei with nucleon numbers away from the spherical “magic gaps” could achieve a lower total energy by conforming to a deformed configuration. Contemporary to the work by Nilsson, others, e.g. Bohr and Mottelson [7], developed models to link the single-particle motion with the collective aspects of the nucleus. Also evident in the early nuclear data was that short-range two-body correlations would be very important. Interpretation of these two-body correlations comes from analogy with the theory of superconductivity by Bardeen, Cooper, and Schieffer [10]. It was shown by several authors (e.g. [11, 12, 13]) that the pairing concepts of superconductivity theory can be applied to many of the features observed in nuclei.

One of the fundamental properties in determining that nuclei are deformed comes from the observation of nuclear rotational bands containing states with very large moments of inertia and connecting transitions with enhanced probabilities. Excited states in rotating nuclei have been observed having angular momenta sometimes as high as $60\hbar$. The understanding of the structure of these nuclei at high spin was enhanced by the introduction of a model where single-particle orbitals are “cranked” by the application of a rotation operator ($-\omega J_x$; see Sec. 2.5 for details). The cranking model was first introduced by Inglis [14, 15] in the 1950’s, but it was not until the early seventies that this model became widely used.

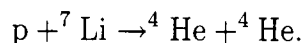
Not only have interesting and unique models been introduced to help un-

derstand nuclear structure, but there have also been significant developments in the techniques for solving the many-body problem. One of the most popular techniques is the so-called Hartree-Fock method. Originally developed in an independent manner by Hartree [16] and Fock [17, 18] as a method to solve the atomic electron many-body problem, these formulations have equal applicability in the nuclear case. The solutions to the Hartree-Fock equations can be very computationally intensive, and thus the true power of this technique could not be utilized until the advent of powerful computers. This technique has also been expanded to self-consistently account for the strong pairing correlations in nuclei. The theories using this method are referred to as Hartree-Fock-Bogoliubov theories, because they include a transformation to a “quasiparticle” space that was introduced by Bogoliubov [19]. A summary of this model as well as other discussed above are presented in Chapter 2

1.2 Experimental Developments

While theorists have to use new and unique methods to successfully describe nuclear phenomena, experimental physicists have an equally difficult challenge to create and detect nuclei. The early experiments of Rutherford [1] involved directing α -particles from a naturally radioactive source onto a Au foil and observing the resulting scattering on photographic plates that surrounded the foil. The use of natural radioactive sources, however, has limited applicability. Such sources are not well collimated, have very low intensity, and are often not monoenergetic. It was soon desired to be able to have greater control over the source. This led to the development of several types of particle accelerators in the early 1930's.

One of the first utilized accelerator designs was one of electrostatic type where a large potential difference was achieved by alternatively charging a number of capacitors in parallel and discharging them in series. This technique, perfected by Cockroft and Walton in 1932 was used in the first nuclear reaction experiment:



Also occurring in the same period was the development of three other successful de-

signs: the Van de Graaff generator, the cyclotron, and the linear accelerator. Early designs only allowed the acceleration of light ions, such as protons and He^+ . As knowledge of nuclear structure grew, it became desirable to accelerate much heavier ions to much higher energies. Today there are facilities that can accelerate nuclei as heavy as uranium to very high energies. Examples of such facilities are the Argonne Tandem-Linac Accelerator System (ATLAS) at Argonne National Laboratory (ANL), the 88" cyclotron at Lawrence-Berkeley National Laboratory (LBNL). Another group of accelerators at Oak Ridge National Laboratory (ORNL), the 25 MV "folded" tandem and Oak Ridge Isochronous Cyclotron (ORIC), have been modified so that, soon, even short lived radioactive isotopes can be accelerated and utilized as beams in nuclear reaction measurements.

The development of heavy-ion accelerators has enabled physicists to create and study well over a thousand different nuclei. The identification and the spectroscopic measurement of quantal states within these nuclei has also involved a large amount of ingenuity. Because of the many different types products emitted in nuclear processes and the wide range of energies involved, a large number of different specialized detectors have been developed. Of particular interest to high-spin physicists are γ -ray detectors, since enhanced γ -ray transitions are observed between members of nuclear rotational bands. Thallium-activated sodium iodide (NaI(Tl)) scintillator detectors, developed in the early 1950's, were the first practical devices for high efficiency spectrometry of nuclear γ -ray sources. Early experiments utilizing these devices are some that identified rotationally excited states in nuclei for the first time. A later advance in γ -ray spectroscopy was the development of lithium-drifted germanium semi-conductors detectors (Ge(Li)) in the early 1960's. These detectors achieved the finest resolution presently available in γ -ray spectroscopy ($\Delta E_\gamma \sim 2 \text{ keV}$). Since the development of the Ge(Li) counter, further refinements in detector design allow the use now of high-purity germanium material. The total detection efficiency of Ge spectrometers have been continually improved with the development of large arrays of detectors. The Ge arrays for the projects presented here contain nearly twenty individual γ -ray detectors (see Chapter 3), however, recently developed detector arrays, such as GAMMASPHERE, have greater than 100 individual devices. Such devices allow

the probing of some of the very finest features of nuclear structure.

CHAPTER 2

Theoretical Models of the Nucleus

2.1 Nuclear Deformation

Considerable evidence exists to suggest that nuclei can form stable deformed shapes. In many cases, the nucleus can be deformed even in the ground state. For a discussion of nuclear deformations, it becomes necessary to parameterize the shape of the nucleus.

2.1.1 Spherical Harmonic Expansion

One common practice used to parameterize the nuclear shape is to define a radius vector from the origin to a point on the nuclear surface as an expansion of spherical harmonics. Thus the radius vector is represented as

$$R(\theta, \phi) = C(\alpha_{\lambda\mu})R_0 \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right]. \quad (2.1)$$

R_0 is the radius of a sphere with the same volume. Due to the incompressibility of nuclear matter, the volume of the nucleus must remain constant with deformation. The parameter $C(\alpha_{\lambda\mu})$ is a normalization factor, chosen to be

$$C(\alpha_{\lambda\mu}) = \left\{ \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} [1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \phi)]^3 \sin \theta d\theta d\phi \right\}^{-1/3}, \quad (2.2)$$

so that this fact is held true.

Because the value of the radius must be a real number, the choice of the parameter $\alpha_{\lambda\mu}$ is restricted such that

$$\alpha_{\lambda\mu}^* = \alpha_{\lambda-\mu}.$$

$R(\theta, \phi)$ should be invariant under rotations of the coordinate system, which implies that $\alpha_{\lambda\mu}$ can be transformed in the same manner as spherical tensors,

$$a_{\lambda\mu} = \sum_{\mu'} D_{\mu'\mu}^{\lambda}(\alpha, \beta, \gamma) \alpha_{\lambda\mu'}, \quad (2.3)$$

where $D_{\mu'\mu}^{\lambda}(\alpha, \beta, \gamma)$ are the matrix elements of the finite rotation operator.¹ It is thus possible to define a coordinate system along the three principal axes of the nucleus. If the three planes defined by the principal axes are symmetric the following simplifications can be applied:

$$a_{\lambda\mu} = a_{\lambda-\mu}$$

and

$$a_{\lambda\mu} = 0 \text{ for odd } \lambda \text{ or } \mu.$$

The radial vector for the nuclear surface to order $\lambda = 2$ can then be represented as

$$R(\theta', \phi') = C(a)R_0[1 + a_{20}Y_{20}(\theta', \phi') + 2a_{22}Y_{22}(\theta', \phi')]. \quad (2.4)$$

The parameters a_{20} and a_{22} are generally expressed in terms of the Hill-Wheeler coordinates β and γ [8]. Using the ‘‘Lund’’ sign convention, a_{20} and a_{22} become

$$a_{20} = \beta_2 \cos \gamma \quad (2.5)$$

and

$$a_{22} = -\frac{1}{\sqrt{2}}\beta_2 \sin \gamma. \quad (2.6)$$

In this formalism, β_2 represents the amount the nucleus is stretched, and γ represents the degree of triaxiality. Fig. 2.1 illustrates the relation of β_2 and γ to the shape of the nucleus. A β_2 of zero is at the origin in the $\beta_2 - \gamma$ plane and corresponds to a spherical nucleus. Positive values of β_2 represent the amount of deformation. When $\gamma = n * 60^\circ$, where n is any integer, two of the three axes are equivalent, implying spherical symmetry in the plane of those axes. For values of γ that deviate from multiples of 60° , all three principal axes are different. When $\gamma = 0^\circ$, β_2 corresponds to prolate deformation along the collective axis. For $\gamma = 60^\circ$, β_2 represents the amount of oblateness that the nucleus assumes.

¹The definition of the rotation operator is derived and discussed in depth by Edmonds in Ref. [20].

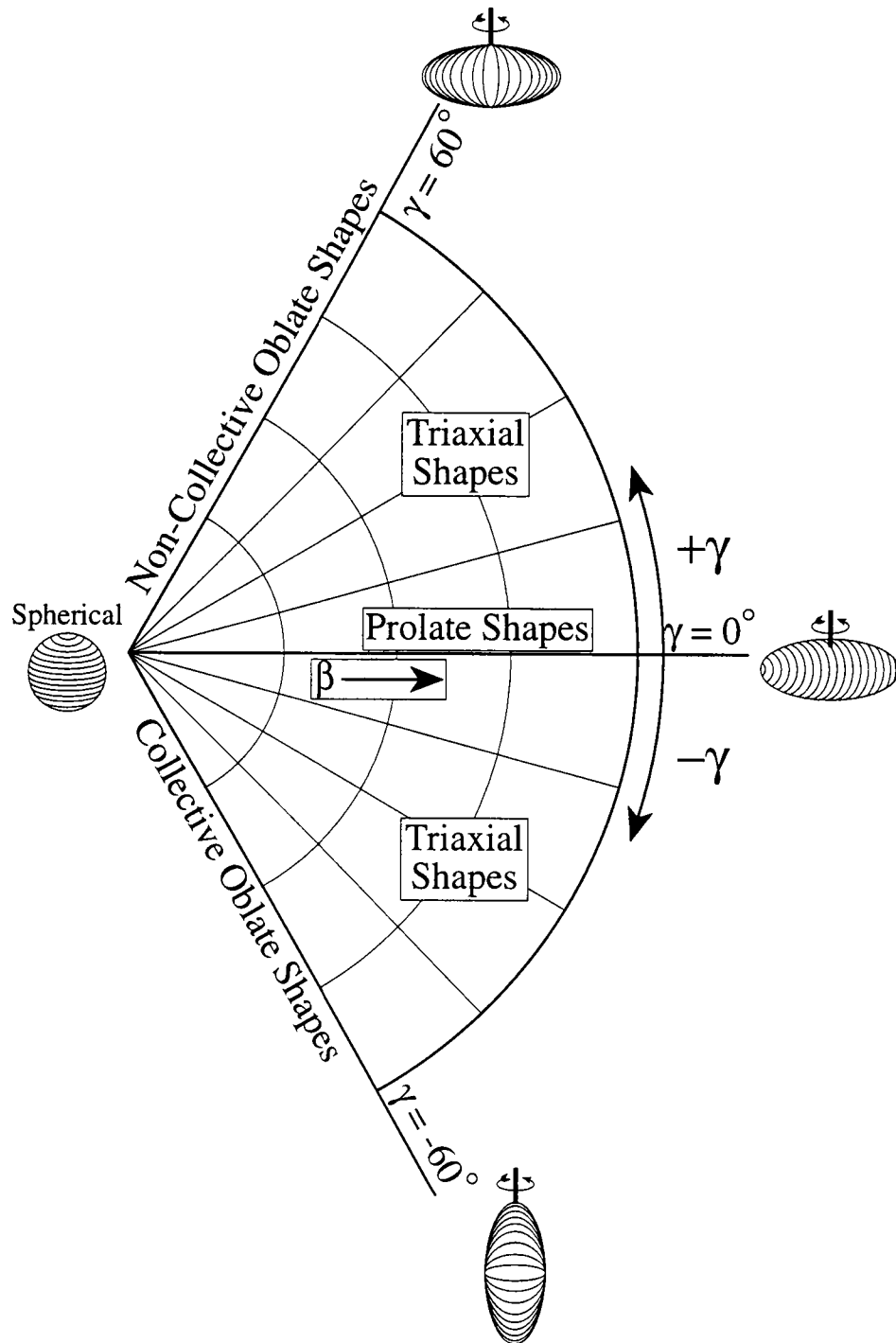


Figure 2.1: Nuclear shapes in relation to the parameters β_2 and γ . The parameters are expressed as polar coordinates.

2.2 Deformed Shell Model

In the study of the atomic nucleus, the effect of the nuclear binding energy being enhanced at certain proton and neutron numbers in a manner analogous to electron binding energies of the noble gases was discovered. These “magic numbers” of increased binding energy are 2, 8, 20, 28, 50, 82, and 126. There is also a trend in the quadrupole moment related to the magic numbers as well. Those nuclei that have nucleon numbers that are different from the magic numbers can exhibit very large quadrupole moments, indicating a large deformation. Those nuclei near the magic numbers, however, have small quadrupole moments. This indicates that these magic numbers also attract the nucleus to sphericity. These properties provide an indication that a shell model is necessary in order to interpret these increased binding energies.

The basic assumption of the shell model is that the nucleons orbit independently within the nucleus as dictated by an average potential created by the other nucleons. Because of this assumption, this model is often referred to as the single-particle model. Proton and neutron scattering experiments have yielded information about the phenomenological properties of the nuclear potential. Nucleons near the center of the nucleus have approximately uniform potential energy with a relatively abrupt change near the surface of the nucleus. Due to the short range of the nuclear force, the nuclear potential outside the nucleus is approximately zero. Two commonly used nuclear potentials in the formulation of a Hamiltonian characterizing the average field of the nucleus will be described in the following sections.

2.2.1 Modified Harmonic Oscillator

The first potential to be discussed is called the modified harmonic oscillator (MHO) potential and was introduced in 1955 by Nilsson [9]. As the name suggests, this potential is based on the spheroidal harmonic oscillator, which for axially symmetric nuclei can be represented as

$$V_{h.o.} = \frac{m}{2} [\omega_{\perp}^2 (x^2 + y^2) + \omega_z^2 z^2]. \quad (2.7)$$

By the introduction of the average oscillator length, $\bar{\omega} \equiv (2\omega_{\perp} + \omega_z)/3$, and deformation parameter, $\delta \equiv (\omega_{\perp} - \omega_z)/\bar{\omega}$, Eq. 2.7 can be rewritten to

$$V_{h.o.} = \frac{m\bar{\omega}^2}{2} \left[\left(1 + \frac{\delta}{3}\right)^2 (x^2 + y^2) + \left(1 - \frac{2\delta}{3}\right)^2 z^2 \right]. \quad (2.8)$$

Nilsson noted that by transforming the cartesian coordinates into the stretched coordinate system, $x_{t_i} = (m\omega_i/\hbar)^{1/2}x_i$, that the potential could be separated into a spherical and deformed component. With this transformation the potential becomes

$$V_{h.o.} = \frac{\hbar\bar{\omega}}{2} \left[r_t^2 - \delta \frac{4}{3} \sqrt{\frac{\pi}{5}} r_t^2 Y_{20}(\theta_t, \phi_t) \right], \quad (2.9)$$

where r , θ , and ϕ are spherical coordinates in the stretched frame. In this notation, only the second term within the square brackets depends on the deformation parameter, δ .

The harmonic oscillator potential is infinite, so this potential is only good for strongly bound nucleons. Also, it does not have a flat bottom, which seems to be dictated by experimental observation. To accommodate this second deficiency, a term proportional to $\vec{\ell}^2$ is added to the nuclear potential. Two different groups, Haxel, *et al.* in Ref. [5] and Mayer in Ref. [4], noted that incorporation of a strong spin orbit term into the nuclear potential reproduces the observed magic gaps. With the inclusion of $\vec{\ell}^2$ to mimic the flat bottom and a term proportional to $\vec{\ell} \cdot \vec{s}$, the modified harmonic oscillator takes the form

$$V_{m.h.o.} = V_{h.o.} - 2\kappa\hbar\bar{\omega}[\vec{\ell}_t \cdot \vec{s}_t - \mu(\vec{\ell}_t^2 - \langle \vec{\ell}_t^2 \rangle)]. \quad (2.10)$$

The factors κ and μ are adjusted for each major oscillator shell such that the experimentally observed single-particle levels are reproduced. Bengtsson and Ragnarsson published a set of “universal” κ and μ parameters that are commonly used in model calculations [21]. Zhang *et al.* [22] reported on an improved set of parameters to be used in the Au-Pt region. The Coulomb force, present between protons, is not explicitly represented in $V_{m.h.o.}$. The presence of this force is accounted for by modifications in κ and μ , thus protons and neutrons will have different parameter sets.

2.2.2 Woods-Saxon Potential

To more realistically describe the nuclear potential, Woods and Saxon introduced the spherical Woods-Saxon potential [23]. This potential phenomenologically reproduces the overall characteristics of the average nuclear field in that it has a flat bottom at small radii, a relatively sharp cutoff at the nuclear surface, and becomes zero at very small distances outside the nuclear radius. The function is represented as

$$V_{w.s.} = -V_0 \left[1 + \exp \left(\frac{r - R_0}{a} \right) \right]^{-1}. \quad (2.11)$$

The parameter V_0 denotes the depth of the potential well, R_0 is the nuclear radius, and a dictates the diffuseness of the potential surface. Typical choices of parameters for this function can be found in the paper by Dudek, *et al.* [24]. With deformation, the $(r - R_0)$ is replaced by a function $\xi(\vec{r}; \beta)$, where ξ is the distance from a point on the nuclear surface, described by Eq. 2.1, to the point of interest in space. In the full description of the nuclear potential, the Coulomb term as well as the spin-orbit orbit term are explicitly expressed,

$$V = V_{w.s.} + V_{s.o.} + V_{Coul}. \quad (2.12)$$

In this formalism the spin-orbit term is defined as

$$V_{s.o.} = -\lambda_{s.o.} \left(\frac{\hbar^2}{2Mc} \right) (\nabla V_{w.s.} \times \vec{p}) \cdot \vec{\sigma}, \quad (2.13)$$

and the Coulomb potential is that of a uniformly charged spheroid

$$V_{Coul} = \frac{Z-1}{\frac{4}{3}\pi R_0^3} \int_V \frac{d\tau'}{|\vec{r} - \vec{r}'|}. \quad (2.14)$$

This formalism of the nuclear potential is a more “realistic” approach than that of Nilsson in that it does not contain explicitly unphysical terms (such as the $\vec{\ell}^2$ of the Nilsson model).

2.2.3 Basis States

The basis state wavefunctions often chosen in shell model calculations are those of the spherical harmonic oscillator. In this basis, the principal quantum number, N , the

orbital angular momentum, $\vec{\ell}$, and the total angular momentum, $\vec{j}$, are good quantum numbers. At spherical shape these quantum numbers are constants of motion, and thus the single particle states are labeled: $N\ell_j$, where the spectroscopic notation, $s, p, d, f, g, \dots$ are used to identify $\ell = 0, 1, 2, 3, 4, \dots$. Each energy level represented by these states is $(2j + 1)$ -fold degenerate.

With deformation, N, ℓ , and even j , are no longer good quantum numbers. The projection of total angular momentum on the axis of deformation, $j_z \equiv \Omega$, is a good quantum number of the total Hamiltonian. The single-particle wavefunctions are still invariant under time reversal, thus states with $\pm\Omega$ are degenerate.

In the spherical basis, the wavefunctions at a given deformation are expressed

$$\Phi_{\Omega}^{\pi} = \sum_{j\ell} C_{j\ell\Omega} |N\ell j\Omega\rangle. \quad (2.15)$$

The coefficients, $C_{j\ell\Omega}$, have been tabulated by Chi [25]. At large deformations, the deformed potential terms, $V_{h.o.}$ or $V_{w.s.}$ become much greater than the $\vec{\ell} \cdot \vec{s}$, and $\vec{\ell}^2$ terms. In this case, one can neglect the $\vec{\ell} \cdot \vec{s}$ and $\vec{\ell}^2$ terms, and the quantum numbers of the anisotropic harmonic oscillator basis become constants of motion. In this basis, the principal quantum number, N , the projection of principal quantum number on the deformation axis, n_z , and projection of the orbital angular momentum, $l_z \equiv \Lambda$, are constants of motion, as well as Ω and π . These *asymptotic quantum numbers* are then used to label a given Nilsson orbital, even though these numbers are not strictly “good” in a region of small deformation. A given Nilsson orbital is then labeled

$$\Omega^{\pi}[Nn_z\Lambda].$$

An example of calculations of the single-particle matrix elements as a function of deformation in the anisotropic harmonic oscillator basis is that by Boisson and Pieperbring in Ref. [26].

2.2.4 Single-particle Diagrams

Figs. 2.2 and 2.3 indicate the neutron and proton single-particle states in the region of deformed rare-earth nuclei. The breaking of the degeneracy in Ω is evident as deformation increases. For example, the $i_{13/2}$ spherical shell state in Fig. 2.2 breaks into 7 doubly-degenerate orbitals that can be labeled by the asymptotic quantum numbers $1/2^+[660]$, $3/2^+[651]$, $5/2^+[642]$, $7/2^+[633]$, $9/2^+[624]$, $11/2^+[615]$, and $13/2^+[606]$. This is a particularly good example to observe the splitting, because it is the unique parity orbital between the major shell gaps of 82 and 126. As such, the $i_{13/2}$ does not mix with the other orbitals in the same shell. Likewise, $h_{11/2}$ is the unique parity orbital between the 50 and 82 particle shell gaps.

Another feature that can be observed in the normal parity orbitals of a given shell is that states that differ in j by one tend to group together energetically in pairs. For example, in Fig. 2.2 the negative parity $h_{9/2}$ and $f_{7/2}$ spherical subshell states differ in energy by only a few hundred keV. Another pair of states are the $f_{5/2}$ and $p_{3/2}$, while a singlet $p_{1/2}$ state is observed highest in the shell. This close proximity in energy continues even with deformation, for example, the $\frac{3}{2}[532]$ and $\frac{1}{2}[530]$ orbitals, originating from the $h_{9/2}$ and $f_{7/2}$ states respectively, are nearly degenerate even up to the highest deformation. These nearly degenerate features of the spherical shell states have led to the idea of recoupling the “normal” parity orbital of a given shell into a new set of “pseudo” basis states. R. D. Ratna Raju *et al.* [27] noted, for example, that the $h_{9/2}$ ($\ell = 5, j = \ell - \frac{1}{2}$) and $f_{7/2}$ ($\ell = 3, j = \ell + \frac{1}{2}$) could be recoupled to have the same pseudo-orbital angular momentum, $\tilde{\ell} = 4$. Thus the natural parity orbitals in this shell, $h_{9/2}$, $f_{7/2}$, $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$, become $\tilde{g}_{9/2}$, $\tilde{g}_{7/2}$, $\tilde{d}_{5/2}$, $\tilde{d}_{3/2}$, and $\tilde{s}_{1/2}$ in the pseudo coupling scheme. One large advantage of this coupling scheme is that the pseudo spin-orbit coupling ($\tilde{\ell} \cdot \tilde{s}$) is significantly smaller than the standard spin-orbit coupling, allowing, in some cases, the spin-orbit coupling to be neglected altogether. More details on the pseudo-spin coupling is given in Chapter 7.

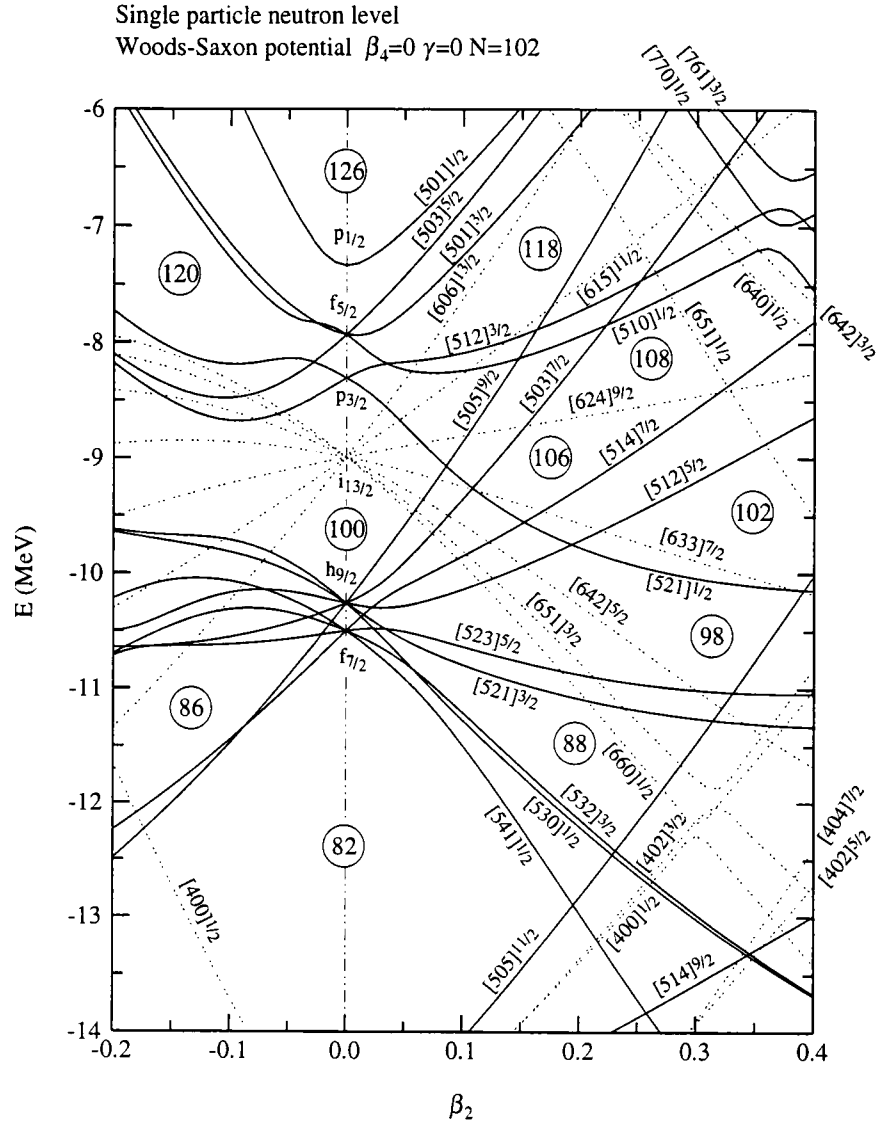


Figure 2.2: Neutron single-particle orbitals as a function of quadrupole deformation, β_2 for states between the $N = 82$ and $N = 126$ spherical shell gaps. The calculations were performed using the Woods-Saxon potential of Eq. 2.12. States are labeled by the asymptotic quantum numbers and, at $\beta_2 = 0$, the spherical quantum numbers. Negative (positive) parity orbitals are indicated with solid (dashed) lines. Spherical and deformed shell gaps are noted with circles. Figure courtesy of H.-Q. Jin.

Single particle proton level
Woods-Saxon potential $\beta_4=0$ $\gamma=0$ $Z=77$

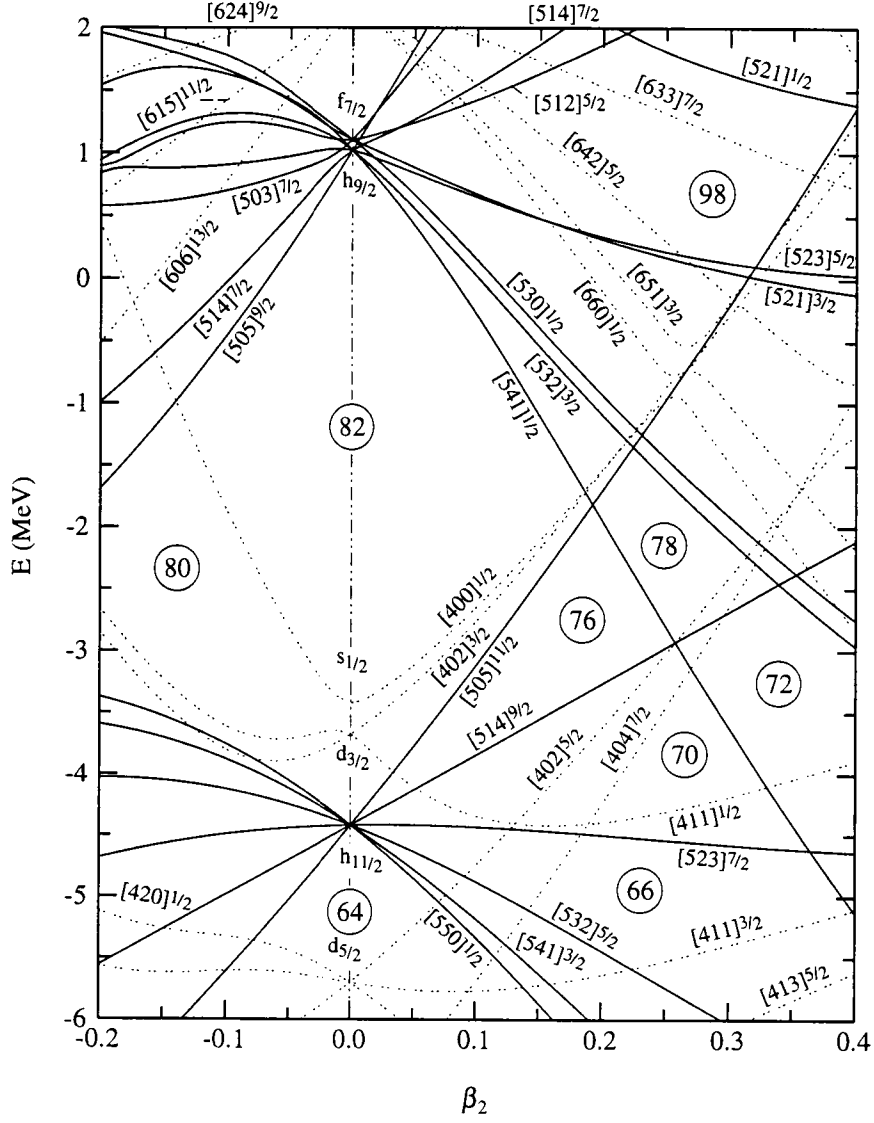


Figure 2.3: Proton single-particle orbitals as a function of quadrupole deformation, β_2 for states around the $Z = 82$ spherical shell gap. The calculations were performed using the Woods-Saxon potential of Eq. 2.12. States are labeled by the asymptotic quantum numbers and, at $\beta_2 = 0$, the spherical quantum numbers. Negative (positive) parity orbitals are indicated with solid (dashed) lines. Spherical and deformed shell gaps are noted with circles. Figure courtesy of H.-Q. Jin.

2.3 Pairing

The shell model from Sec. 2.2 was developed using the assumption that the single nucleons only interact with other nucleons by an average potential field created by all the other nucleons. However, considerable evidence suggests a two-body residual interaction is also necessary to explain many features of nuclear structure. Two such features that cannot be explained by a simple independent-particle model are

- the ground state spin and parity of *every* nucleus with an even number of protons and neutrons are $I^\pi = 0^+$, and
- a systematically smaller binding energy of odd mass nuclei compared to neighboring even-even nuclei.

A two-body residual interaction would be the greatest when the wavefunctions of the two states are maximally overlapped. In the case of nucleons, the maximum overlap can be achieved when they occupy time reversed orbitals. If a particular nucleon can be characterized by the wave function $\phi(N, \ell, j, \Omega)$, the respective time reversed orbital is $\bar{\phi} = T\phi = (-1)^{2\Omega}\phi(N, \ell, j, -\Omega)$. Thus, the respective eigenvalues differ only in the sign of the total angular momentum projection.

The BCS model

In 1957, Bardeen, Cooper, and Schrieffer presented an interpretation of superconductivity in which electron pairs, coupled so that they have a net momentum of zero, are subject to a two-body residual interaction [10]. By analogy, the nucleons can be coupled into “conjugate” pairs with net angular momentum zero and be subject to a similar theoretical treatment as that of the electrons of Bardeen, Cooper, and Schrieffer. In the second-quantization formalism, the Hamiltonian including the two-body interaction, introduced by Bardeen, *et al.* and adapted for the nuclear model, is

$$H = \sum_{k>0} e_k (a_k^\dagger a_{k'} + a_{\bar{k}}^\dagger a_{\bar{k}'}) - G \sum_{k, k'>0} a_k^\dagger a_{\bar{k}}^\dagger a_{k'} a_{\bar{k}'}. \quad (2.16)$$

The second term on the right-hand side of the equation represents the annihilation of a particle and its conjugate pair from states k' and $\bar{k}'$, and their subsequent creation

in states k and $\bar{k}$. In principle the scattering strength, represented by the factor “ G ,” is configuration dependent, but calculations are simplified by assuming an average pairing strength, $G = \langle G_{kk'} \rangle_{ave}$, for the region around the Fermi surface. A variational approach is used in calculating energy eigenvalues, thus an appropriate ansatz wavefunction must be chosen. The ansatz used for the ground-state of an even-even nucleus is chosen to be

$$|\text{BCS}\rangle = \prod_{k>0} (u_k + v_k a_k^\dagger a_{\bar{k}}^\dagger) |0\rangle, \quad (2.17)$$

with the terms u_k and v_k used as variation parameters. These two parameters are not independent, however, and are related by

$$|u_k|^2 + |v_k|^2 = 1. \quad (2.18)$$

In this notation, $|v_k|^2$ is interpreted as the probability that the state $(k, \bar{k})$ is occupied. A particularly undesirable effect of the choice of Eq. 2.17 as the ansatz for this model is that particle number is no longer a conserved quantity. This creates an additional requirement on the variation parameters, that the average particle number must be equal to the actual number of nucleons,

$$\langle \text{BCS} | \hat{N} (= \sum_k a_k^\dagger a_k) | \text{BCS} \rangle = 2 \sum_{k>0} v_k^2 \equiv N. \quad (2.19)$$

This restriction can be observed by introducing the Fermi energy, λ , into the model Hamiltonian as a Lagrange multiplier,

$$H' = H - \lambda \hat{N}. \quad (2.20)$$

With the inclusion of an operator to ensure proper particle number, the energy expectation value is

$$\langle \text{BCS} | H' | \text{BCS} \rangle = 2 \sum_{k>0} (\tilde{\epsilon}_k v_k^2 + \frac{1}{2} G v_k^4) - \frac{\Delta^2}{G}. \quad (2.21)$$

Two new parameters are introduced in this equation, the gap parameter,

$$\Delta = G \sum_{k>0} u_k v_k, \quad (2.22)$$

and the quasiparticle energy,

$$\tilde{\epsilon}_k = \epsilon_k - \lambda - G \cdot v_k^2. \quad (2.23)$$

In calculations the terms Gv_k^4 and $G \cdot v_k^2$ are typically neglected, because these terms only amount to a renormalization of the single-particle energies. In terms of Δ and $\tilde{\epsilon}_k$, the occupation probabilities can be represented

$$|u_k|^2 = \frac{1}{2} \left[1 + \frac{\tilde{\epsilon}_k}{\sqrt{\tilde{\epsilon}_k^2 + \Delta^2}} \right], \quad (2.24)$$

and

$$|v_k|^2 = \frac{1}{2} \left[1 - \frac{\tilde{\epsilon}_k}{\sqrt{\tilde{\epsilon}_k^2 + \Delta^2}} \right]. \quad (2.25)$$

Substitution of Eqs. 2.24 and 2.25 back into Eq. 2.22, yields the gap equation:

$$\Delta = \frac{G}{2} \sum_{k>0} \left[\frac{\Delta}{\sqrt{\tilde{\epsilon}_k^2 + \Delta^2}} \right]. \quad (2.26)$$

This equation can be solved simultaneously with the particle number equation (derived from Eq. 2.19),

$$N = \sum_{k>0} \left[1 - \frac{\tilde{\epsilon}_k}{\sqrt{\tilde{\epsilon}_k^2 + \Delta^2}} \right], \quad (2.27)$$

to determine λ and Δ for a given system.

Quasiparticle transformation

The concept of the quasiparticle was developed by Bogoliubov and Valatin [19, 28]. To simplify calculations, one transforms the particle creation and annihilation operators into the *quasiparticle* creation and annihilation operators:

$$\alpha_k^\dagger = (u_k a_k^\dagger - v_k a_{\bar{k}}), \quad \alpha_k = (u_k a_k - v_k a_{\bar{k}}^\dagger),$$

and

$$\alpha_{\bar{k}}^\dagger = (u_k a_{\bar{k}}^\dagger + v_k a_k), \quad \alpha_{\bar{k}} = (u_k a_{\bar{k}} + v_k a_k^\dagger).$$

The above equations utilize the phase convention

$$u_{\bar{k}} = u_k > 0,$$

and

$$v_{\bar{k}} = -v_k < 0, \text{ for } k > 0.$$

With this transformation, the occupation of a given pair of conjugate states is characterized by the amount of “particle” and “hole” mixture of a given quasiparticle. For example, $|u_k|^2 = 0$ and $|v_k|^2 = 1$ corresponds to a fully occupied particle state. Likewise, $|u_k|^2 = 1$ and $|v_k|^2 = 0$ would correspond to an unoccupied particle state. With the transformation to quasiparticle space, the BCS ground-state wavefunction of Eq. 2.17 becomes the vacuum state, thus quasiparticle excitations are expressed with respect to the nuclear ground-state.

2.4 Particle-Rotor Model

In 1953, Bohr and Mottelson developed what is known as the Collective Model of the nucleus [7], the idea of which led to their sharing the Nobel prize for physics with Rainwater in 1975. The model assumes that for certain excitations the nucleons within a nucleus do not act individually but in phase with one another. Collective excitations include vibrations and giant resonances. Another form of collective excitation is the rotation of the nucleus. With the onset of deformation, the nucleus can exhibit long “bands” of excited energy states that can be characterized as a collective nuclear rotation. In depth coverage of the development of the collective model can be found in the second volume of the book by Bohr and Mottelson [29].

The particle-rotor model was developed as a way to understand the collective excitations observed in odd- A nuclei. The assumption of this model is that the Hamiltonian can be described as

$$H_{tot} = H_{core} + H_{s.p.} + H_{int}, \quad (2.28)$$

where H_{core} is the Hamiltonian for the even-even core nucleus, $H_{s.p.}$ describes the valence particle, and H_{int} represents the interaction between the core and the valence particle. The core and single-particle systems individually are assumed to be well

understood and characterized by their own sets of basis states. e. g.

$$H_{core}|\mu\rangle = e_{core}|\mu\rangle \quad (2.29)$$

and

$$H_{s.p.}|\nu\rangle = e_{s.p.}|\nu\rangle. \quad (2.30)$$

From these basis states, a set of product states are constructed to be used as the basis for the particle-core system,

$$|\mu, \nu\rangle = |\mu\rangle \otimes |\nu\rangle. \quad (2.31)$$

The Core

The Hamiltonian for the core derives from the classical kinetic energy of a rotating body, $E = \frac{I^2}{2\mathcal{J}}$, which in quantized form is

$$H_{core} = \sum_{i=1}^3 \frac{\hat{I}_i^2}{2\mathcal{J}_i}. \quad (2.32)$$

The numbers i correspond to the three principle axes of the nucleus, $\hat{I}$ represents the core angular momentum operator, and the quantities $\mathcal{J}_i$ are the moments of inertia about the respective axis 1, 2, or 3. Typically the tensor of the moment of inertia is defined by the hydrodynamic relation,

$$\mathcal{J}_i = \frac{4}{3}\mathcal{J}_0 \sin^2\left(\gamma + i\frac{2\pi}{3}\right). \quad (2.33)$$

With the choice of a Hamiltonian, an appropriate set of basis states must be chosen. If the wavefunction describing the nucleus in its intrinsic coordinate system is represented as ϕ_K , then in the limit of strong coupling the nuclear wavefunction in a rotated reference frame is

$$|IMK\rangle = \sqrt{\frac{2I+1}{16\pi^2}} [\phi_K D_{M,K}^I(\alpha, \beta, \gamma) + (-1)^{I+K} \phi_{\bar{K}} D_{M,-K}^I(\alpha, \beta, \gamma)]; (K > 0). \quad (2.34)$$

The rotational wavefunction, $|IMK\rangle$, is characterized by three quantum numbers, the total angular momentum (I), the projection of the total angular momentum on the

z axis of the rotated coordinate system (M), and the projection of the total angular momentum on the z axis of the intrinsic coordinate system of the nucleus (K). In this case, the z axis of the intrinsic system is the axis of nuclear deformation. An assumption of this formulation is that the intrinsic wavefunction, ϕ_K , is rotationally invariant. That is

$$\mathcal{R}_x(\pi)\phi_{r,K} = e^{-i\pi\hat{I}_x}\phi_{r,K} = r\phi_{r,K}. \quad (2.35)$$

The eigenvalues, $r = e^{-i\pi\hat{I}_x}$, for specific values of I are

$$r = +1 \text{ for } I = 0, 2, 4, \dots; \quad r = -1 \text{ for } I = 1, 3, 5, \dots; \quad (2.36)$$

and

$$r = -i \text{ for } I = \frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \dots; \quad r = +i \text{ for } I = \frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \dots \quad (2.37)$$

This quantum number is identified as the signature quantum number. A more commonly used value for the signature quantum number is the signature exponent, α , where

$$r = e^{-i\pi\alpha}. \quad (2.38)$$

The α quantum number eigenvalues are then

$$\alpha = 0 \text{ for } I = 0, 2, 4, \dots; \quad \alpha = 1 \text{ for } I = 1, 3, 5, \dots; \quad (2.39)$$

and

$$\alpha = +\frac{1}{2} \text{ for } I = \frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \dots; \quad \alpha = -\frac{1}{2} \text{ for } I = \frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \dots \quad (2.40)$$

An advantage of using α as the signature quantum number rather than r is that α is an additive quantity. For example, in the case of an odd-odd nucleus, if the proton wavefunction is characterized by $I = 1/2$ ($\alpha = \frac{1}{2}$) and the neutron by $I = 7/2$ ($\alpha = -\frac{1}{2}$), then the signature of the resulting rotational band is $\alpha_r = \alpha_p + \alpha_n = 0$.

In addition to the rotational invariance of the intrinsic wavefunctions, another assumption is that the rotation operator, $\mathcal{R}$, is identical in both the intrinsic and the rotated reference frames. This leads to the existence of the second term on the right hand side of the equation, where $\phi_{\bar{K}} \equiv \phi_{-K}$. For the case where $K = 0$, it can be seen that this second term differs from the first term only by a phase,

$(-1)^I$. Thus for spins $I = 0, 2, 4, \dots$ these two terms act constructively, while for odd spins, $I = 1, 3, 5, \dots$, the rotational wavefunction, $|IMK\rangle$, becomes identically zero. This is the reason why the ground-state rotational band of deformed even-even nuclei is always characterized by states of even spin.

2.5 The Cranking Model

Rather than consider the phenomenological core of the particle-rotor model, Inglis reported in Refs. [14] and [15] on a more microscopic treatment of the rotating nucleus. As in the shell model discussed in Sec. 2.2, individual nucleons are treated as independent particles orbiting in a potential created by the other nucleons. In this case, however, the potential is rotating with angular frequency, ω , about a given axis.

If the axial deformation is assumed and the nucleus rotates about an axis of symmetry, then the average potential in spherical coordinates can be represented as

$$V(r, \theta, \phi, t) = V(r, \theta, \phi - \omega t, 0). \quad (2.41)$$

The time-dependent single-particle Hamiltonian is then

$$h(t) = \frac{\vec{p}^2}{2m} + V(r, \theta, \phi - \omega t, 0). \quad (2.42)$$

The explicit time-dependence of the Hamiltonian can be eliminated by applying the unitary transformation

$$Uh(t)U^{-1} = h(0), \quad (2.43)$$

where $U = e^{i\omega j_x t}$ and j_x is the angular momentum about the axis of rotation. Following transformation, the wave equation,

$$h(t)\psi(\vec{r}, t) = i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t), \quad (2.44)$$

becomes

$$h_\omega \tilde{\psi} = (h(0) - \omega j_x) \tilde{\psi} = \epsilon_\omega \tilde{\psi}, \quad (2.45)$$

where $\tilde{\psi} = U\psi$ and ϵ_ω is the time-independent eigenvalue. The many-body Hamiltonian of the nucleus can then be represented as a sum of the single-particle Hamilto-

nians:

$$H_\omega = \sum_{i=1}^A h_\omega^{(i)} = \sum_{i=1}^A \left(h^{(i)}(0) - \omega j_x^{(i)} \right). \quad (2.46)$$

The total wavefunction, Ψ_ω , resulting from this Hamiltonian is represented by a Slater determinant of the single-particle wavefunctions, $\tilde{\psi}^{(i)}$. The reason for the Slater determinant representation arises from the fact that the fermion wavefunctions of the system are anti-symmetric under particle exchange.

2.6 Shell Correction Method

Deformed shell model theories, such as those discussed in Sec. 2.2, are successful at predicting deformed configurations in the nucleus by considering the sum of single particle energies in a phenomenologically deduced average potential. This model alone, however, is insufficient to describe other features of the nucleus, such as nuclear binding energies. It was noted, for example by Myers and Swiatecki [30], that the deformed shell model could be combined with the phenomenological liquid-drop model, where binding energies from experimental data are reproduced. This idea was expanded by Strutinsky [31] by assuming that the total energy of the nucleus can be expressed as

$$E_{\text{total}}(Z, N; I) = E_{\text{ld}} + \delta E_{\text{shell}}(\text{protons}) + \delta E_{\text{shell}}(\text{neutrons}), \quad (2.47)$$

where E_{ld} is the binding energy defined by the liquid drop model, and δE_{shell} are corrections dictated by the shell model. These parameters are individually discussed in the following sections. This approach is variously referred as the macroscopic-microscopic model or the Nilsson-Strutinsky method.

2.6.1 Liquid-Drop Model

It is possible to effectively represent the bulk properties of the nucleus as a classical charged droplet of incompressible liquid. The contributions to the total energy in this model is represented, for example, by

$$E_{\text{ld}} = E_{\text{vol}}(Z, N) + E_{\text{surf}}(\beta, A) + E_{\text{Coul}}(\beta, Z, A), \quad (2.48)$$

where E_{vol} is a volume dependent term representing nucleon masses, E_{surf} accounts for the energy related to the surface tension, and E_{Coul} reflects the Coulomb repulsion of protons.

Nuclear rotations can be considered with the inclusion of a rotational energy term,

$$E(\beta, Z, N; I) = E_{\text{ld}} + \frac{(I\hbar)^2}{2\mathcal{J}(\beta, Z, N)}, \quad (2.49)$$

where $\mathcal{J}$ denotes the moment of inertia of the droplet. It can be noted that this classical picture predicts that rotating nuclei will favor oblate shapes at low angular momentum (I), but have a shape change to prolate shapes at high spins. Many nuclei can be characterized at low spin (or even in the ground state) by prolate shapes. This is a result of quantal effects, and one of the primary reasons for considering shell model corrections.

2.6.2 Strutinsky Shell Correction

A method was developed by Strutinsky [31] where an average value of the sum of single-particle energies is subtracted from the total value, thus the remaining energy accounts only for local fluctuations. Specifically this shell correction term is defined as

$$\delta E_{\text{shell}}(n) = \sum_i e_i - \tilde{E}, \quad (2.50)$$

where n corresponds to protons ($n = Z$) or neutrons ($n = N$), e_i are the single-particle energies defined, for example, by a Woods-Saxon potential, and $\tilde{E}$ is the average shell-model energy defined in the following paragraph.

The determination of $\tilde{E}$ is achieved by “smearing” out the level density function, defined by Dirac δ -functions as

$$g(\epsilon) = \sum_i \delta(\epsilon - e_i), \quad (2.51)$$

with the formulation

$$\tilde{g}(\epsilon) = \frac{1}{\gamma} \int_{-\infty}^{+\infty} g(\epsilon') f\left(\frac{\epsilon' - \epsilon}{\gamma}\right) d\epsilon'. \quad (2.52)$$

In this equation, γ is defined as the smoothing range and $f(x)$ is a function defining a “smearing” width of a level. From this smooth density function, $\tilde{g}(\epsilon)$, $\tilde{E}$ at an energy, $\tilde{\lambda}$, is determined to be

$$\tilde{E} = \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(\epsilon) \epsilon d\epsilon. \quad (2.53)$$

2.6.3 Total Routhian Surface Calculations

A specific application of the macroscopic-microscopic model is one where the total Routhian energy at a rotational frequency, ω , and deformation, $\hat{\beta}$, is obtained from

$$E^\omega(Z, N, \hat{\beta}) = E_{\text{macr}}^\omega(Z, N, \hat{\beta}) + \delta E_{\text{shell}}^\omega(Z, N, \hat{\beta}) + \delta E_{\text{pair}}^\omega(Z, N, \hat{\beta}), \quad (2.54)$$

where E_{macr}^ω is a rotating liquid drop energy, $\delta E_{\text{shell}}^\omega$ is a shell correction to the Woods-Saxon potential, and $\delta E_{\text{pair}}^\omega$ is a pairing correction energy determined in a similar formalism to the shell correction. The development and applications of aspects of this model can be found in Refs. [32, 33, 34].

2.7 Transformation of Experimental Data

In the study of physical observables, it is often necessary to consider separate properties that contribute to the resulting value of the observable. Examples of contributions to the total energy of a nuclear quantum state are energy from single particle excitations, collective effects such as rotation or vibrations, and pairing correlations. Transformation of the total energy and angular momentum of nuclear levels into other forms more readily illustrate the underlying nuclear structure.

From basic assumptions, the total angular momentum and excitation energy of a level can be transformed into other coordinates that can help provide some hints to the underlying nuclear structure. For the following analysis, the assumption is made that the nucleus is cylindrically symmetric prolate or oblate spheroid and rotating on an axis perpendicular to the symmetry axis. With this assumption the projection of the total angular momentum on the rotational axis (I_x) can be shown to be:

$$I_x(I) = \sqrt{(I(I+1) - K^2)}. \quad (2.55)$$

I is the total angular momentum of a given state and K is the projection of the total angular momentum onto the symmetry axis. Note that the determination of K is often based on “best guess” estimates of the Nilsson orbitals that the valence particles occupy.

Another necessary quantity to calculate is the rotational frequency of the nucleus. Rotational frequency is defined as

$$\hbar\omega = dE/dI_x, \quad (2.56)$$

where E is the excitation energy. Experimentally, the rotational frequency at a given energy and angular momentum can be estimated as

$$\hbar\omega(E, I) = \frac{E(i) - E(f)}{I_x(i) - I_x(f)}, \quad (2.57)$$

where $E(i)$ and $I_x(i)$ are the total energy and projected angular momentum of a particular state, and $E(f)$ and $I_x(f)$ are energy and spin of a state connected to state i by a stretched $E2$ transition ($I(f) = I(i) - 2\hbar$).

Often, results from theoretical calculations are presented in the rotating frame of the nucleus. In general, it is more convenient to convert the experimental total energy from the laboratory frame into the rotating frame, rather than converting the calculations into the lab frame, in order to compare the theoretical calculations with experiment. The total energy can be transformed in the following manner:

$$E'(E, I) = E - \hbar\omega I_x. \quad (2.58)$$

E' is called the total Routhian.

While one can transform spins and energies into I_x and E' , both these values contain all contributing factors to the total angular momentum and energy. The contributions of collective effects are removed by subtracting a model dependent reference from I_x and E' :

$$i(\omega) = I_x(\omega) - I_{\text{ref}}(\omega) \quad (2.59)$$

$$e'(\omega) = E'(\omega) - E'_{\text{ref}}(\omega). \quad (2.60)$$

In these equations, I_{ref} and E'_{ref} are reference functions that simulate the contribution of collective rotation to their respective quantities. The resulting physical quantities

$i(\omega)$ and $e'(\omega)$ are called the quasiparticle aligned angular momentum (or simply alignment) and the quasiparticle Routhian energy (usually referred to as just the Routhian energy).

In the calculation of I_{ref} and E'_{ref} , the nucleus is assumed to have a moment of inertia that varies as a function of rotational frequency as defined by Harris [35]:

$$\mathcal{J} = \mathcal{J}_0 + \mathcal{J}_1 \omega^2. \quad (2.61)$$

$\mathcal{J}_0$ and $\mathcal{J}_1$ are parameter constants that are adjusted for a given nucleus. From the above moment of inertia the equations for I_{ref} and E'_{ref} become:

$$I_{\text{ref}}(\omega) = \omega \mathcal{J} = \omega \mathcal{J}_0 + \omega^3 \mathcal{J}_1 \quad (2.62)$$

$$E'_{\text{ref}}(\omega) = - \int I_{\text{ref}} d\omega = -\frac{1}{2} \omega^2 \mathcal{J}_0 - \frac{1}{4} \omega^4 \mathcal{J}_1 + \frac{1}{8 \mathcal{J}_0}. \quad (2.63)$$

The removal of a core contribution is very useful to illustrate the non-collective contributions, however, care must be taken in using these references. If the levels used to fit the $\mathcal{J}_0$ and $\mathcal{J}_1$ parameters do not conform to a rotational pattern, this can lead to an improper reference for the rest of the band. In a comparison of the alignment or Routhian energy of two different bands it is appropriate to use the same reference for both bands. If, however, the bands have significantly different cores the resulting comparison may produce effects that are not non-collective in nature, but simply the result of using an improper reference.

Moment of Inertia

Classically, the angular momentum is directly proportional to the rotational frequency by the factor defined as the moment of inertia ($\vec{I} = \mathcal{J} \vec{\omega}$). By analogy, the moment of inertia associated with a given nuclear rotational band can be expressed as:

$$\mathcal{J}^{(1)} = I_x / \hbar \omega. \quad (2.64)$$

I_x and $\hbar \omega$ for a given level are calculated from Eqs. 2.55 and 2.57 respectively. This resulting moment of inertia is referred to as the kinematic (or first) moment of inertia.

An alternative definition is to express the moment of inertia as the derivative of I_x with respect to ω :

$$\mathcal{J}^{(2)} = \frac{dI_x}{d(\hbar\omega)}. \quad (2.65)$$

This quantity is called the dynamic (or second) moment of inertia. Fig. 2.4 illustrates the differences between the two definitions. Initial inspection seems to indicate that the definitions give completely different results. Investigation of the underlying structure of the rotational bands, however, reveals that the differences arise primarily from the non-collective contributions to the moment of inertia. The total angular momentum of a given level can be represented as

$$I_x = i + I_c, \quad (2.66)$$

where i is the non-collective angular momentum (alignment) and I_c is angular momentum resulting from collective rotation. The collective rotation can be parameterized in the same manner as Eq. 2.62. The kinematic and dynamic moments of inertia can then be represented as

$$\mathcal{J}^{(1)} = \frac{i}{\hbar\omega} + \mathcal{J}_0 + \mathcal{J}_1(\hbar\omega)^2, \quad (2.67)$$

and

$$\mathcal{J}^{(2)} = \frac{di}{d(\hbar\omega)} + \mathcal{J}_0 + 3\mathcal{J}_1(\hbar\omega)^2. \quad (2.68)$$

From this parameterization, it can be seen that the dramatically rising $\mathcal{J}^{(1)}$ for the $\pi h_{9/2}$ band in Fig. 2.4 is the result of the contribution of the alignment (i) of the valence proton in the first term. For $\mathcal{J}^{(2)}$, if the alignment is constant, the first term will be zero. In the case of even-even Hf, all nucleons are paired, so there is no single-particle contribution ($i = 0$) to $\mathcal{J}^{(1)}$ at low values of ω .

2.8 Angular Distribution of Electromagnetic Radiation

In the analysis of electromagnetic radiation, the electromagnetic field can be treated as a sum of individual multipole moments. The electromagnetic field can be considered to be composed of a summation of a dipole field, a quadrupole field, a hexadecapole

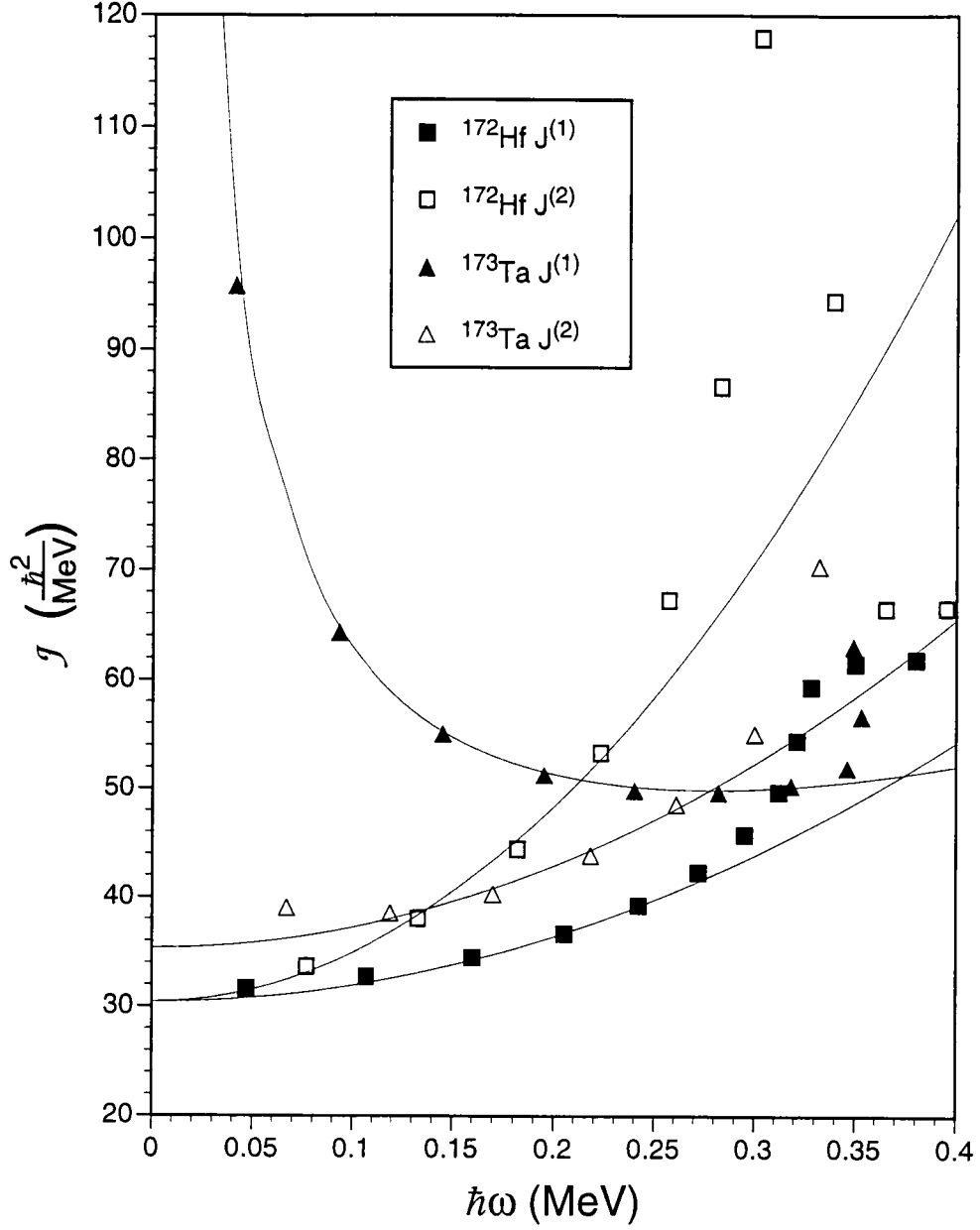


Figure 2.4: Calculated $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ moments of inertia as a function of rotational frequency for the ground-state band in ^{172}Hf and the $\pi h_{9/2}$ band in ^{173}Ta . The solid lines are the results of a least squares fit of the $\mathcal{J}^{(1)}$ values. Eq. 2.67 was used for the fit, with i , $\mathcal{J}_0$, and $\mathcal{J}_1$ used as fitting parameters. For ^{172}Hf , i was set to zero, and $\mathcal{J}_0$ and $\mathcal{J}_1$ were fit to be $35.33 \frac{\hbar^2}{\text{MeV}}$ and $63.00 \frac{\hbar^4}{\text{MeV}^3}$ respectively. The parameters resulting from the fit of the $\pi h_{9/2}$ band in Ta are $i = 2.7\hbar$, $\mathcal{J}_0 = 30.41 \frac{\hbar^2}{\text{MeV}}$, and $\mathcal{J}_1 = 149.6 \frac{\hbar^4}{\text{MeV}^3}$.

field, and so on. With this assumption the effect of different multipole operators (denoted as $\mathcal{M}(\sigma\lambda, \mu)$, where σ represents E or M for electric or magnetic, λ is the multipole order of the representation as determined by 2^λ , and μ is the magnetic substate projection, $\lambda \geq \mu \geq -\lambda$) on nuclear transition rates can be examined.

Photon multipole emission in quantal systems is governed by the following parity and angular momentum selection rules.

1. Since angular momentum is conserved, the photon multipole order must obey angular momentum addition rules. This implies that:

$$\vec{I}_i = \vec{L} + \vec{I}_f \quad (2.69)$$

where $\vec{I}_i$ and $\vec{I}_f$ are the total angular momentum of the initial and final nuclear states, and $\vec{L}$ is the angular momentum of the photon, the magnitude of which is related to the photon multipole order as $|\vec{L}| = \lambda\hbar$. This restricts the possible multipole orders to be $(I_i + I_f) \geq \lambda \geq |I_f - I_i|$. For example, if $I_i = 4\hbar$ and $I_f = 2\hbar$, the possible values of λ are 2, 3, 4, 5, and 6. The radiation field observed would contain a mixture of these multipoles. The transition probability, however, drops dramatically for higher multipoles, so that while all allowed multipoles of λ order are possible only those of lowest order are important. Another consequence of the vector addition is the additivity of the magnetic substates, $\mu = m_i - m_f$, where m_i and m_f are the magnetic substate projections of the states. Photons have spin $1\hbar$, with projections (helicity (τ)) of ± 1 . No $\tau = 0$ projection exists because of the transverse nature of electromagnetic radiation. The γ radiation therefore only connects levels where the magnetic projections differ by 1. Except in the presence of very strong magnetic fields, the magnetic substates are degenerate.

2. Parity of the radiation field is dictated by:

$$\begin{aligned} \pi(E\lambda) &= (-1)^\lambda \\ \pi(M\lambda) &= (-1)^{\lambda+1}. \end{aligned} \quad (2.70)$$

These equations indicate that electric and magnetic fields of the same multipole order always have opposite parity. A transition that is electric dipole in character would change the parity between the initial and final states, whereas a magnetic dipole would not. To use the example from point one, if the spin and parity (denoted as $I^\pi=\text{parity}$) of the initial and final states were 4^+ and 2^+ respectively, the possible $\sigma\lambda$ values are $E2$, $M3$, $E4$, $M5$, and $E6$. The multipoles, $M2$, $E3$, etc. would not be allowed since these radiations would change the parity.

A particular exception to rule 1 is monopole order ($\lambda = 0$). The monopole moment is simply the Coulomb field produced by the electric charge. No radiation is produced in this order, so for cases where $I_f = I_i$ the lowest multipole order γ ray observed would be dipole ($\lambda = 1$). The Coulomb field, however, does have a significant effect on the atomic electrons, so the lowest order multipole ($E0$) can be observed by conversion electrons emitted from such a transition.

Weisskopf estimates of multipole transition rates indicate that electric multipole radiation is greater than magnetic radiation of the same order by 10^2 . It follows that the most commonly observed radiations are magnetic and electric dipole, and electric quadrupole.

2.8.1 Methods of Nuclear Orientation

While γ decay from quantum states produces characteristic multipole angular distributions that can lead to identification of the angular momentum carried away by the radiation, measurement of it is often not possible in experimental situations. In general, no particular orientation of nuclei is found, therefore, the γ -decay angular distribution is isotropic in space. Excited nuclei, however, can be prepared in preferentially oriented states, such that the decay can be anisotropic in space. Two methods for orienting nuclei are low-temperature orientation or orientation via compound nucleus reactions. These two methods are summarized below in the following sections.

Low-temperature nuclear orientation

One way of preparing preferentially oriented states is a technique known as low-temperature nuclear orientation. When nuclei are exposed to a magnetic field, their spins will align in the direction of the magnetic field, and as a result of the Zeeman effect, the normally degenerate magnetic substates (m_i) are split by a magnitude $\Delta E = m_i \mu B$, where μ is the magnetic moment of a particular state and B is the external magnetic field. There is an uneven population of the resulting m -substates based on a statistical Boltzmann distribution. The resulting m -substate population ($p(m_i)$) is proportional to $e^{-m_i \mu B / kT}$. Because of the very small value of the nuclear magnetic moment ($\mu_N \approx 3.15 \times 10^{-8}$ eV/T), even very large magnetic fields produce very small m -substate splitting. Magnetic fields on the order of 10 Tesla produce energy splittings of only $\sim 10^{-6}$ eV. At room temperature ($kT \approx \frac{1}{40}$ eV) with the above magnetic field, essentially no preferential population of m -substates is created. Cooling nuclei to temperatures such that $kT \sim \mu B$ would counter the effects of thermal redistribution sufficiently to create a measurable difference in m -substate population. Such temperatures down to $kT \sim 10^{-6}$ eV can be achieved by helium dilution refrigerators such as one used by the UNISOR (now UNIRIB) consortium at ORNL.

Nuclear orientation via heavy-ion fusion reactions

One fortunate consequence of compound nuclear reactions is that the spins of nuclei prepared in these reactions are oriented in a plane perpendicular to the beam axis radiating from the target location. Such a preparation initially produces nuclei in energy states with angular momentum projections of $m = 0$. It has been shown by Diamond, *et al.* [36] that the subsequent particle and statistical γ decay result in a gaussian distribution of m -substate population symmetric around $m = 0$. Yamazaki [37] presented the m -state population distribution as:

$$P_m(I) = e^{-\frac{m^2}{2\sigma^2}} / \left(\sum_{m'=-I}^I e^{-\frac{m'^2}{2\sigma^2}} \right), \quad (2.71)$$

where σ is the width parameter of the distribution. It was shown by Krämer-Flecken, *et al.* [38] that choice of $\sigma = 0.3I$ is appropriate for the $70 < Z < 82$ mass region.

2.8.2 Angular distribution measurement

If the m -substate populations of energy levels in nuclei are prepared in an oriented state such as those listed above, γ rays emitted from these states will show an angular distribution in space defined by equation 12.197 from [39]:

$$W(\theta_1) = \sum_{\lambda \text{ even}} B_\lambda(I_1) A_\lambda(\gamma_1) Q_\lambda(\gamma_1) P_\lambda(\cos\theta_1). \quad (2.72)$$

$B_\lambda(I_1)$ are the parameters that describe the initial orientation of the state I_i . These parameters are dependent on the the preparation of the m -substate population. The equation describing the initial orientation is:

$$B_\lambda(I_1) = (2I_1 + 1)^{1/2} \sum_{m=-J}^J (-1)^{I_1+m} \langle I_1, -m, I_1, m | \lambda 0 \rangle P_m(J). \quad (2.73)$$

The parameter $P_m(J)$ describes the normalized population distribution which, in the case of a heavy-ion fusion reaction, is represented by Eq. 2.71.

$A_\lambda(\gamma_1)$ are directional distribution coefficients that depend on the multipole content of the γ radiation. In experimental practice, these are the values that are measured, and used to determine angular momentum carried away by the radiation, thus fixing the relative spin of a nuclear level. Theoretically, the directional distribution coefficients are functions of the multipole mixing of a given transition. For example, the second rank distribution coefficient ($A_{\lambda=2}$) for a $\Delta I = 1$ transition from I_i to I_f is given by:

$$A_2(\gamma) = \frac{F_2(1, 1, I_f, I_i) + 2\delta F_2(1, 2, I_f, I_i) + \delta^2 F_2(2, 2, I_f, I_i)}{1 + \delta^2}. \quad (2.74)$$

The multipole mixing ratio is defined as $\langle I_f || E2 || I_i \rangle / \langle I_f || M1 || I_i \rangle$, and denoted in this equation as δ . The F -coefficient angular momentum coupling terms contain Wigner 6- j and 9- j symbols, resulting from the coupling of four angular momenta. The generalized formulae for the F -coefficients are given in Eq. 12.163 of Ref. [39] and Eq. 46 in Ref. [40].

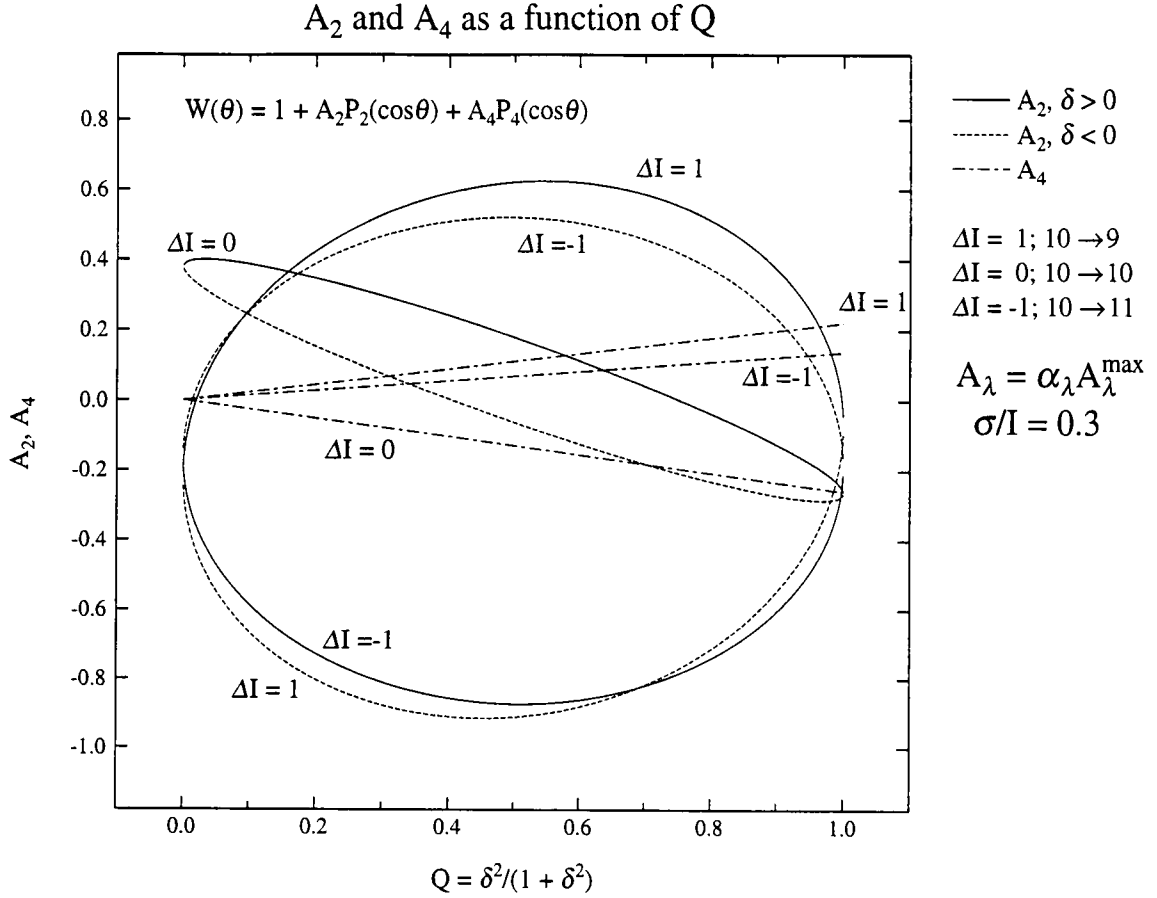


Figure 2.5: Directional distribution coefficients $A_{\lambda=2}$ and $A_{\lambda=4}$ of $\Delta I = |1|$ and 0 as a function of quadrupole content ($Q = \delta^2 / (1 + \delta^2)$). δ is the mixing ratio as defined in the text.

$Q_\lambda(\gamma_1)$ is a solid angle correction factor that accounts for the fact that detectors are not point sources and have a non-zero angle of acceptance for radiation. Derivation of the formula for calculating the Q -values for a coaxial Ge detector is presented by Krane in Refs. [41, 42]. $P_\lambda(\cos\theta_1)$ are Legendre polynomials that describe the different characteristic radiation patterns of different multipole orders.

As was shown in Eq. 2.74, the directional distribution coefficients can be used to extract the ratio of intensity of a given multipole with respect to another multipole. Fig. 2.5 shows the behavior of $A_{\lambda=2}$ and $A_{\lambda=4}$ coefficients as a function of the quadrupole content of $\Delta I = 1, 0$, and $-1\hbar$ transitions.

2.8.3 Directional Correlations from Oriented States (DCO)

Because of the γ -decay selection rules stated at the beginning of this section, the decay of a γ ray into a state followed by the observation of subsequent decay out of the state can yield correlation information. If the state from which the initial γ ray had originated was aligned from low-temperature orientation, from the dynamics of the reaction, or some other method, the unambiguous determination of multipole mixing ratios can often be obtained.

In its general form the DCO function is given by eq. 11 in Ref. [40] as:

$$W(\theta_1, \theta_2, \phi) = \sum_{\lambda_1, \lambda, \lambda_2} B_{\lambda_1}(I_1) A_{\lambda}^{\lambda_2 \lambda_1}(\gamma_1) A_{\lambda_2}(\gamma_2) H_{\lambda_1, \lambda, \lambda_2}(\theta_1, \theta_2, \phi). \quad (2.75)$$

$B_{\lambda_1}(I_1)$ is the initial orientation coefficient of Eq. 2.73. The directional distribution coefficients, $A_{\lambda_2}(\gamma_2)$, are defined in Eq. 2.74. $A_{\lambda}^{\lambda_2 \lambda_1}(\gamma_1)$ are the generalized form of the directional distribution coefficients from Eq. 60 in Ref. [40]. $H_{\lambda_1, \lambda, \lambda_2}(\theta_1, \theta_2, \phi)$ is an angular function describing the angles of the two emitted γ rays. For the case where polarization is not measured, $\lambda_1 + \lambda + \lambda_2 = \text{even}$ and the angular function is represented as

$$H_{\lambda_1, \lambda, \lambda_2}(\theta_1, \theta_2, \phi) = \sum_{q \geq 0}^{\lambda'} (2 - \delta_{0q}) \langle \lambda_1, 0, \lambda, q | \lambda_2, q \rangle \left[\frac{2\lambda+1}{2\lambda_2+1} \right]^{\frac{1}{2}} \left[\frac{(\lambda-q)!(\lambda_2-q)!}{(\lambda+q)!(\lambda_2+q)!} \right]^{\frac{1}{2}} P_{\lambda}^q(\cos \theta_1) P_{\lambda_2}^q(\cos \theta_2) \cos(q\phi). \quad (2.76)$$

θ_1 and θ_2 are the angles at which the incoming and outgoing γ rays of a given state are observed. They are given in spherical coordinates. In heavy-ion fusion reactions $\theta = 0$ would correspond to the axis of the incident beam. ϕ is the azimuthal angle between the two observed radiations (i.e. $\phi = \phi(\gamma_{\text{in}}) - \phi(\gamma_{\text{out}})$).

DCO Ratio

The accurate measurement of the angular distribution function or the directional correlation function requires that the intensities of the γ rays of interest be measured at several angles. Large detector arrays, such as the first generation Spin Spectrometer and the current generation GAMMASPHERE, provide increased high-resolution detection efficiency of γ rays, but the sacrifice is that the angles of the detectors are

fixed. The Ge detectors in the Spin Spectrometer, for example, are arranged in rings of fixed θ 's of 24.7° , 41.4° , 63.4° , 116.6° , 138.6° , and 155.3° . Since standard coaxial Ge detectors are insensitive to polarization of the γ rays, the angular distribution will be symmetric around $\theta = 90^\circ$. While three angles are sufficient to get an angular distribution and directional correlation, the uncertainties would be very large.

Because of few angles involved, an alternative method for determining the multipolarity of a γ -ray transition is to take the ratio of DCO functions. That is, measure the DCO function at a given θ_1 and θ_2 , and then divide by the measured DCO function with the angles reversed. The DCO ratio is thus defined as:

$$R_{DCO} = \frac{W(\theta_1, \theta_2, \phi)}{W(\theta_2, \theta_1, \phi)}. \quad (2.77)$$

Experimentally, R_{DCO} is extracted by measuring the intensity of a γ ray, γ_1 , at an angle θ_1 when gating on a second γ ray, γ_2 , at an angle θ_2 . This value is then divided by the intensity of γ_1 at the angle θ_2 , when γ_2 is detected at θ_1 . The experimental DCO ratio can be represented then as

$$R_{DCO} = \frac{I_{\theta_1}^{\gamma_1}(\text{Gate}_{\theta_2}^{\gamma_2})}{I_{\theta_2}^{\gamma_1}(\text{Gate}_{\theta_1}^{\gamma_2})}. \quad (2.78)$$

Fig. 2.6 illustrates the dependence of R_{DCO} on the multipole mixing ratio for transitions to states that differ in spin by -1 , 0 , and $+1\hbar$. It can be seen that a pure $\Delta I = |1|$ transition will have a DCO ratio of about 0.6 . The DCO ratio deviates relatively quickly from 0.6 with only a small quadrupole mixture in the transition probability. For a transition to a state of the same spin, R_{DCO} will be greater than 1 , if the transition is pure dipole. It is clear that a single DCO ratio is insufficient to uniquely determine the change in spin of a given transition. A pure $\Delta I = 1$ dipole is indistinguishable from a $\Delta I = -1$ transition. A $\Delta I = 0$ transition with only a small quadrupole moment mixture produces the same DCO ratio as that of a $\Delta I = 2$ transition. Regardless of this deficiency, the DCO ratio is a useful quantity, when used with other physical observables, to determine the change in angular momentum between two states. Because of its sensitivity to δ , R_{DCO} can also be used to infer the amount of quadrupole mixture for a given transition.

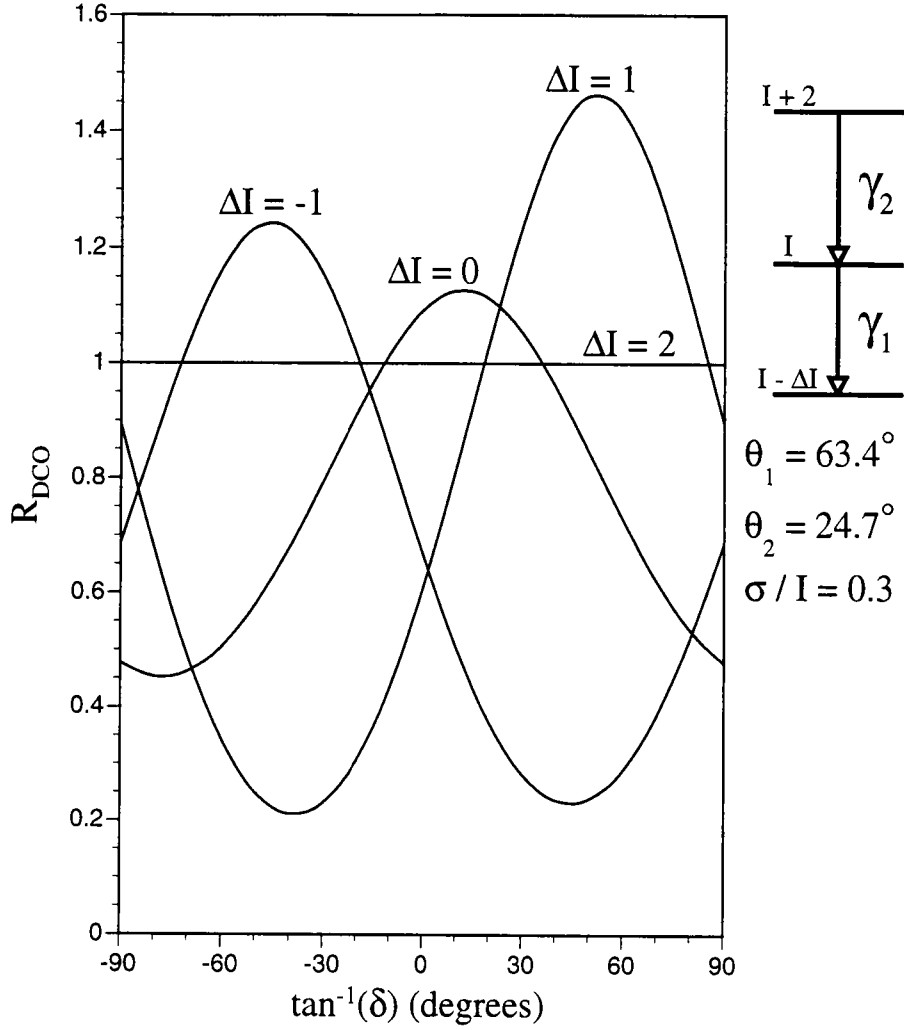


Figure 2.6: R_{DCO} as a function of the inverse tangent of the dipole-quadrupole mixing ratio ($\tan^{-1}(\delta)$). The angles, θ_1 and θ_2 , used in the calculation are appropriate for the Spin Spectrometer described in Sec. 3.3.1. The DCO ratio was calculated assuming that γ_2 is a pure quadrupole and γ_1 is a mixed dipole-quadrupole transition decaying to a state differing in spin by $-1, 0$, or $+1\hbar$. The line denoted by $\Delta I = 2$ is present to emphasize that R_{DCO} for a quadrupole is one. The chosen value for the m -state distribution parameter, σ , is appropriate for the nuclei in the region around $Z = 79$.

CHAPTER 3

Experimental Techniques

3.1 Heavy-Ion Fusion Evaporation Reactions

3.1.1 General Considerations

Nuclear structure studies are performed on excited nuclei by measuring their decay properties. One method used for exciting nuclei is to direct a mass analyzed highly energetic beam of a particular ion sort onto a target of isotopically enriched material. The type of reaction that will occur is highly dependent on the energy of the beam, and the charge and mass of both the incident and the target nucleus. Fig. 3.1 illustrates an accelerated nucleus on a path to collide with a target nucleus. As the incident nucleus approaches the target nucleus, they will interact via the electromagnetic force. If the geometrical impact parameter, b , is greater than the combined radii of the two nuclei, they will only interact electromagnetically. In the case where the impact parameter is smaller than the combined radii, the interaction will depend on the energy of the incoming nucleus. If the energy is less than the Coulomb barrier the two nuclei will still interact only by the electromagnetic force. The Coulomb barrier (in MeV) is defined

$$E_{CB} = \frac{Z_i Z_t e^2}{r_i + r_t}, \quad (3.79)$$

where $Z_i e$ and $Z_t e$ are the charge of the incident and target nuclei respectively, and r_i and r_t are their respective radii. This represents the minimum energy at which the nuclear matter of an incident nucleus, on a path with impact parameter zero, will overlap with the matter of the target nucleus. It is possible to populate excited states within both nuclei when the incident energy is below E_{CB} , and the common term applied to this type of "reaction" is Coulomb excitation. Increasing the energy, of

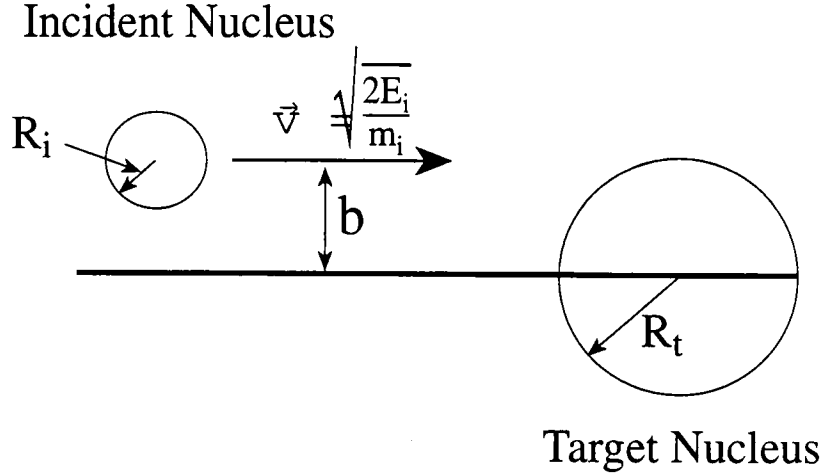


Figure 3.1: Schematic drawing of an incident nucleus impacting a target nucleus.

the incoming nuclei, above the Coulomb barrier allows the nuclear matter to overlap, and the nucleons within the individual nuclei can interact via the strong force. It is then possible that nucleons can be transferred from one nucleus to the other. With a further increase in the energy of the incident particle, it is possible that two nuclei will overlap such that a complete fusion of can occur. In such a reaction, the nucleons inside the resulting compound nucleus come to a thermal equilibrium such that the subsequent deexcitation will be essentially independent of the reaction mechanism.

A compound nucleus is assumed to come to thermal equilibrium with respect to all degrees of freedom if it exists for a time much longer than the orbital period of the individual nucleons ($\sim 10^{-22}$ s). After thermal equilibrium is reached, the compound nucleus is highly excited and usually has a very large angular momentum. The excitation energy of the compound nucleus is

$$E_{ex}^{max} = E_{CM} + (M_i + M_T - M_{c.n.})c^2, \quad (3.80)$$

where $E_{CM} = \frac{A_i}{A_i + A_T} E_i [\text{MeV}]$ is the center of mass energy of the system, and M_i , M_T , and $M_{c.n.}$ are the masses the incident, target, and compound nuclei respectively. The term $(M_i + M_T - M_{c.n.})c^2$ is often referred to as the Q value. Pelte and Schwalm showed in Ref. [43] that the maximum angular momentum contributing to fusion can

be estimated from the sharp cut-off model as

$$\ell_{max}(\hbar) \approx 0.219 R_{fus} [A_\mu (E_{CM} - E_{CB})]^{1/2}, \quad (3.81)$$

where R_{fus} is an empirically derived radius of fusion,

$$R_{fus} = 0.95(A_i^{1/3} + A_T^{1/3})[\text{fm}], \quad (3.82)$$

$A_\mu = \frac{A_i A_T}{A_i + A_T} [\text{amu}]$ is the reduced atomic mass, E_{CM} is the center of mass energy of Eq. 3.80, and E_{CB} is the Coulomb barrier energy defined by Eq. 3.79. Fig. 3.2 schematically illustrates the decay of an excited compound nucleus. Included in this figure are approximate estimates of E_{ex} and I_{max} for heavy-ion fusion reactions creating nuclei with $A \sim 180$.

Typically when $A \sim 180$ compound nuclei are populated with angular momenta near ℓ_{max} there will be a strong competition between fission and particle emission. The ratio of fission to neutron evaporation probability can be estimated [43] as

$$\frac{T_{fission}}{T_n} \approx \exp \left(\frac{B_n - \varepsilon(I)}{t} \right), \quad (3.83)$$

where B_n is the neutron binding energy, $\varepsilon(I)$ is the rotational liquid drop barrier defined, for example, by Eq. (1.86) in Ref. [44], and t is the temperature of the nucleus (typically 1-2 MeV). When $\varepsilon(I) > B_n$, the compound nucleus will decay primarily by fission. The critical spin (defined from $\varepsilon(I_{crit}) = B_n$) below which single-particle evaporation will dominate over fission is about $0.7 I_{max}$ for $A \sim 180$. If the angular momentum of the compound nucleus is less than I_{crit} then nuclei with $A > 100$ near the valley of stability will “cool off” primarily by emitting neutrons. Neutrons, rather than protons, are emitted from these nuclei because the protons have the addition of a Coulomb barrier that binds them more to the nucleus; however, for nuclei that are very deficient in neutrons, the emission of protons and even α particles can compete with neutrons in single-particle evaporation. The probability of decay by a charge particles versus neutrons can be estimated [43] by

$$\frac{T_\xi}{T_n} \approx \exp \left(\frac{B_n - B_\xi - V_{CB\xi}}{t} \right), \quad (3.84)$$

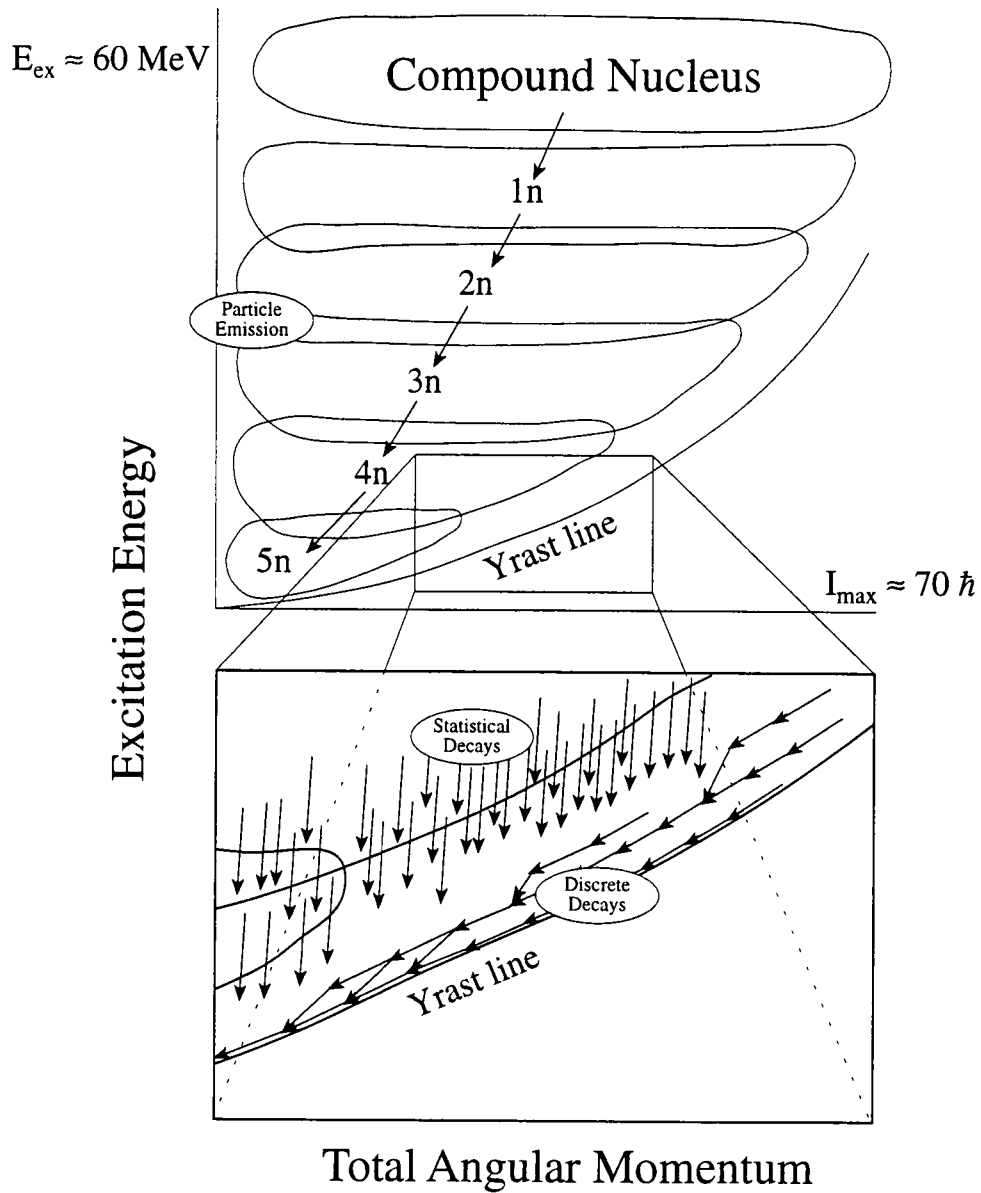


Figure 3.2: Angular momentum versus energy illustration of the decay of a compound nucleus. The nuclear excitation is plotted as total angular momentum versus excitation energy. The labels “ $E \approx 60 \text{ MeV}$ ” and “ $I \approx 70\hbar$ ” indicate the excitation energy and maximum angular momentum for a compound nucleus with $A \sim 180$ and $A_i \sim A_t$.

where ξ denotes a proton or an α particle respectively. B_n and B_ξ are the separation energies, V_{CB_ξ} is the appropriate Coulomb barrier for the charged particle, and t is the average temperature of the compound nucleus. The emission of particles dramatically deexcites the nucleus (~ 10 MeV/nucleon), but only carries away a small amount of angular momentum ($\sim 1\hbar$).

The level which, for a given energy, has the lowest angular momentum is called the yrast level. A parabolic line in the angular momentum-energy plane connecting the levels of lowest angular momentum is thus known as the yrast line.¹ When the excitation energy of a residual nucleus ($\equiv$ compound nucleus minus emitted charged particles) is less than the particle separation energy above the yrast line γ decay will occur. Because of the large level density at 5-10 MeV above the yrast line, the γ decay will be characterized by unresolved statistical decays. For excitation energies about 1 MeV above the yrast line, the level density is low enough that discrete transitions can be resolved above the background of statistical γ rays. This is the region of discrete high-spin spectroscopy. The various stages of decay are schematically illustrated in Fig. 3.2.

3.1.2 Statistical Model Calculations with EVAPOR

While the formulae in the previous section provide qualitative descriptions of formation and decay of a compound nucleus, it is necessary to model the conditions for the complete reaction in detail when one wishes to study a specific nucleus. One such model is the “projection angular-momentum coupled evaporation” model. This statistical model was implemented in the monte-carlo simulation program PACE. This program was developed by Gavron at LASL. The fusion cross-section for a particular reaction is calculated based on a suggestion by Bass in Ref. [45]. The determination of Q -values and other related parameters that dictate the probability for specific decay channels are based on the mass tables compiled by Wapstra and Audi in Ref. [46]. For cases where the mass was not measured, the liquid-drop mass from Myers and Swiatecki in Ref. [30] is used. Several versions of PACE have been developed over

¹The word “yrast” comes from the Swedish language and translates roughly as “dizziest.”

time. The version of the code used for the following analysis has been developed by N. G. Nicolis of Washington University and J. R. Beene of Oak Ridge National Laboratory [47]. The modifications, while maintaining the same basic core as PACE, had evolved sufficiently that it was given a new name, EVAP.

The results of these calculations have been very successful in predicting the results of heavy-ion fusion-evaporation reactions in several regions of stable nuclei. As one goes further from the valley of stability, however, the results of the calculations become less reliable, particularly when one cannot rely on measured nuclear masses. Nevertheless, programs like EVAPOR provide useful information for optimizing reaction conditions for which at high spins the cross section for the nucleus of interest is maximum, while the yields for the other decay channels are minimized. For the two experiments presented in this dissertation, different combinations of beam and target were explored to determine the most favorable reaction for each experiment. Fig. 3.3 shows the results of calculations from EVAPOR for the two reactions chosen in the ^{181}Au and ^{179}Au experiments. A common feature in both graphs is that fission is about zero low in energy but, at a particular point, increases very rapidly. This is the point where the average angular momentum produced in the reaction is equal to the critical angular momentum described in Sec. 3.1.1. For the case of the $^{58}\text{Ni} + ^{124}\text{Te}$ reaction, the channel of interest, ^{179}Au , is maximized at an energy greater than point where fission becomes the dominant decay channel. The γ -ray decay from fission products creates a large background making the identification of γ rays associated with ^{179}Au decay impossible with Ge-detector arrays alone. Another feature is that the cross-section for all products is near zero at $E = 150$ MeV for Fig. 3.3(a) and $E = 240$ MeV for Fig. 3.3(b). These two energies correspond to the Coulomb barrier for each reaction. Below these respective energies, a compound nucleus will not form. For the reaction in Fig. 3.3(a), the pure neutron channels (Au nuclei) are much stronger than the proton out channels (Pt nuclei). In Fig. 3.3(b), however, the proton out channels (also Au nuclei) are much stronger than pure neutron channels (Hg nuclei). In fact, the two proton (or one α) channels (Pt nuclei) are also stronger than the pure neutron channel. This illustrates that the proton binding energy for the compound nucleus in reaction (a) (^{185}Au) is still relatively large. For the compound

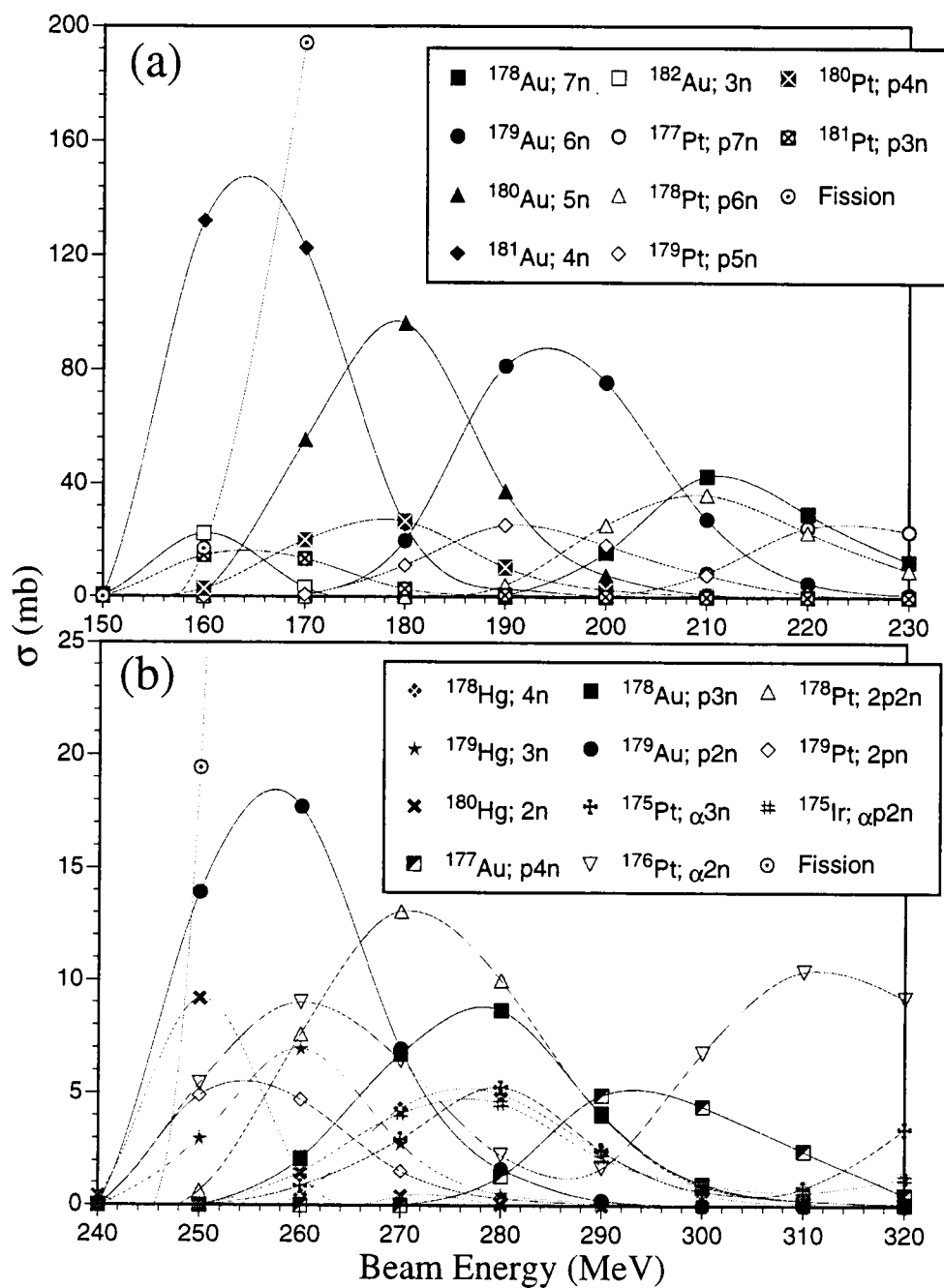


Figure 3.3: Calculated cross section of residue nuclei and fission products as a function of beam energy for the (a) $^{35}\text{Cl} + ^{150}\text{Sm}$ and (b) $^{58}\text{Ni} + ^{124}\text{Te}$ reactions.

nucleus in reaction (b) (^{182}Hg), however, the proton binding energy is much smaller than the neutron binding energy. It can be noted in Fig. 3.3(b) that some reaction channels show two maxima, e. g. ^{176}Pt . For the case of ^{176}Pt , these two maxima correspond to a particle emission of $\alpha 2n$ and $2p4n$. At a beam energy of 260 MeV, the three particle emission is dominant ($3n$, $p2n$, $2pn$, and $\alpha 2n$), but as the energy is increased it becomes more favorable to create ^{176}Pt by the emission of six particles ($2p4n$).

3.2 Radiation Detection

3.2.1 Introduction

The products resulting from the deexcitation of nuclei can be classified in to four basic groups:

- X rays and γ rays
- Heavy ($A \geq 4$) charged particles
- Fast electrons, positron, and protons
- neutrons.

Each of these different products have unique properties that require different strategies in order to detect them with the greatest efficiency. Some special types of counters used to detect these emitted products will be discussed in the following subsections, with emphasis given to Germanium Detectors. Since the detection of γ rays is of particular interest in this dissertation, a short summary of photon interaction with matter is given below. For indepth coverage of detectors and measurement techniques the reader is referred to the text by Knoll [48].

3.2.2 Photon Interaction with Matter

Photons interact with matter in three basic ways: photoelectric absorption, Compton scattering, and electron-positron pair production. The photoelectric effect is the absorption of the γ -ray energy by a bound atomic electron which results in the emission

of that electron from the atom. This effect involves the total absorption of the γ ray and is the most useful interaction for the purpose of detecting the complete energy of the photon. Compton scattering involves the scattering of a photon off free electrons. The scattered electron does not absorb the full energy of the γ ray, and if the photon escapes from the detector after being scattered, the total energy of the γ ray will not be deposited. The chance that a photon will be photoabsorbed is roughly dependent on Z^5 ($Z \equiv$ number of protons) of a given material, while the chance for Compton scattering is approximately linear in Z . Thus high- Z materials are desirable for γ -ray detection to favor the photoabsorption process. Pair production involves creation of an electron and a positron from γ rays with energies greater than 1.022 MeV. For the studies presented in this dissertation, this effect is not a major factor and will not be discussed further.

3.2.3 Germanium Detectors

Semiconductor materials, like germanium, offer very good energy resolution because of the low ionization energy. The number of free charge carriers (electrons and holes) created for a given energy of incident radiation is directly related to the resolution of the detector. i.e. the greater number of free electrons means the detector is less susceptible to statistical fluctuations, assuming all other factors are equivalent. For germanium semiconductor material, the energy gap to create an electron-hole pair is ~ 3 eV. This is a much lower energy compared with gaseous ionization (~ 30 eV)² or photoelectrons from a scintillator material (~ 1 keV). The resulting energy resolution for a Ge detector is about 2 keV for the 1.33 MeV γ ray emitted by a ^{60}Co source, while the 1.17-1.33 MeV doublet can hardly be resolved with a scintillation device. Two semiconductor materials are typically used in detectors, silicon and germanium. For the purposes of γ ray detection, germanium ($Z = 32$) is superior over silicon ($Z = 14$) because of the Z^5 dependence of the electromagnetic photoabsorption. For this reason, experiments where good energy resolution of γ rays is of primary

²Ionization detectors are not used in γ -ray detection, but is included here for comparison of the energy gap.

importance detectors of Ge material are used.

The low ionization energy of semiconductor detectors, however, has a disadvantage in that probability to thermally excite electrons into the conduction band is quite high (leakage current). For Ge detectors to be useful in γ -ray spectroscopy the counters are cooled to liquid nitrogen temperature (77 K). This greatly reduces the leakage current. To enlarge the sensitive volume for radiation detection, an $n - p$ junction is typically created in the semiconductor material, and a reverse bias voltage is applied to the device.³ With the application of the reverse bias, the total collection time for electron-hole pairs created by an incident γ -ray is ~ 100 ns per 1 cm of detector material. In the electronics that process the detector signals, the charge collection is manifested as a voltage pulse with a rise time equivalent to the collection time. Compared to other types of detectors (e.g. NaI scintillators) 100 ns is a slow rise time. An important factor contributing to the rise time of a given pulse in a Ge detector is the location of the event within the detector. Events occurring near the edge of the detector have nearly twice the rise time of events in the center of the spectrometer. Therefore, while they have excellent energy resolution, Ge detectors have poor timing properties.

There are two basic types of geometries used in Ge detector design, planar and coaxial. Each has particular advantages and disadvantages. Recent designs involve using composite detectors of coaxial Ge counters, some of which are electronically segmented. These detector designs are discussed below.

Planar Ge detectors

Planar detectors are often called Low-Energy Photon Spectrometers (LEPS). These detectors have a relatively small volume compared to the surface area facing the reaction. The result of this is that collection time for charge carriers is relatively fast and the signal to rise ratio comparatively large (due to a small detector capacitance). Therefore these detectors have better time and energy resolution than coaxial Ge

³Most present day γ -ray detectors are equipped with high-purity n-type germanium material. i.e. the detector signal is based on the collection of electrons

detectors. The greatest disadvantage of these detectors is that because of the small sensitive volume, high-energy γ rays pass through the detector without interacting. The result is that LEPS detectors are typically used for detecting γ rays with energies up to about 200 keV.

Coaxial Ge detectors

In order to present a larger detector volume for a given surface area, the coaxial detector was designed. This design provides a much wider range for an efficient γ -ray detection (~ 50 keV–10 MeV). The sacrifice is that the energy and time resolution are reduced compared to planar designs. The larger energy range and the overall greater efficiency of the coaxial geometry makes it the most popular and is found in all large γ -ray spectrometers around the world. Currently in use are two generations of high-purity Ge detectors. The older detectors have an efficiency of 20 to 30% relative to a 3 by 3 inch NaI(Tl) detector for 1.33 MeV γ rays from a ^{60}Co placed at 25 cm from the crystal, while recent developments have achieved large volume Ge detectors with $> 70\%$ relative efficiency.

Segmented Ge detectors

One of the problems with large volume Ge detectors is that they have such a large angular acceptance that the Doppler shift of γ rays emitted from a recoiling nucleus decreases the resolution, particularly at 90° angles. One technique that has been used to effectively reduce the size of the angular acceptance without reducing efficiency is to electronically segment a detector [49]. For an n-type Ge detector, such a segmenting is realized by still having a single anode as an inner electrode, but two cathodes. Signals from the outer contact provide low resolution position information for either the right or left side of the detector. The side which received the most charge would be considered the side on which the event occurred. This effectively cuts the solid angle in half.

Clover and Cluster Ge detectors

Another technique to increase the detector area presented to the target while minimizing Doppler broadening is to pack several coaxial detectors in a close configuration, thus getting a much larger detector with a resolution that is no worse than a single coaxial Ge detector. One design involves packing four detectors together in a common liquid nitrogen cryostat. Such a design is known as a clover detector. Another design that extends this idea further is "the cluster." This involves putting seven detectors into one package. Since these detectors are so closely packed there is nothing to prevent a Compton scattered γ ray from entering a neighboring detector. This feature of the design is, however, advantageous, because one can add energies from events detected in adjacent detectors under the assumption that they are from the same γ ray ("add-back" mode). This technique is very effective when data rates are such that the chance of "true" events occurring in adjacent detectors is small. In "add-back" mode, one can gain in efficiency up to 50% of the value obtained in "singles" mode.

3.2.4 Auxiliary Detectors

While for purposes of spectroscopy of high-spin states, the Ge detector plays the most significant role, picking out the reaction channel of interest from the flood of γ -ray data being emitted from the target position is often difficult. As discussed in Sec. 3.1, several nuclei are often produced in a single compound reaction, and as one goes farther from stability the number of evaporation residues is expected to increase. In order to sort out the information of interest from the rest of the reaction, the use of auxiliary detectors becomes important.

There are several strategies that can be useful in enhancing yield, and even uniquely determining reaction products. Some of these techniques and the detector used in these techniques are reviewed in the following paragraphs.

Scintillator Detectors as Multiplicity/Sum Energy Filters

Before the advent of semiconductor γ -ray detectors, the most popular devices for γ -ray spectroscopy were scintillation counters. In a scintillation device, incident γ

rays can excite electrons from their normal position in the valence band into the conduction band. Rather than attempting to collect these free electrons, they are allowed to recombine with the atoms or molecules of the detector. Because of the properties of the scintillator material, the free electrons will recombine with the atom, or crystalline system, in an excited state. The decay of this excited state to the ground-state produces visible light that is then collected and converted into an electron with a photomultiplier tube, which also amplifies these secondary electrons. Because of the generally high densities of scintillation materials compared to Ge, these detectors are more efficient for detecting γ rays, but the energy resolution is far less than what can be achieved with semiconductor devices.

Examples of scintillator crystals that are commonly used in the detection of γ rays are NaI(Tl), Bismuth germanate ($\text{Bi}_4\text{Ge}_3\text{O}_{12}$, often abbreviated as BGO), and BaF_2 . Arrays using large numbers of these detectors (sometimes 50 or more) have been developed in an effort to collect the total γ ray yield from a reaction in order to measure the sum energy and multiplicity of γ -ray cascades. As discussed in Sec. 3.1, the first products typically released in a compound nuclear reaction are charged particles and neutrons. When the final nucleus is populated, deexcitation proceeds through γ -ray emission. The total number and sum energy of γ rays can then be detected by the efficient scintillator detectors and mapped back to the origin of the decay in a plot of E vs. I for a final nucleus (See Fig. 3.4). Scintillators can also serve as Compton suppression shields for Ge detectors. This application is discussed in Sec. 3.4.1.

Mass Analyzers with Position Sensitive Gaseous Ionization Chambers

A mass spectrometer used in conjunction with detection devices can be a very powerful tool for identifying γ rays originating from a specific nucleus. Nuclei formed in heavy-ion fusion reactions recoil with a velocity a few percent of the speed of light. Most of these nuclei recoil in a small cone with a relatively small velocity distribution. These nuclei are also highly ionized, so their trajectories can be manipulated with electric and magnetic fields to separate recoil nuclei on the basis of mass of nucleus

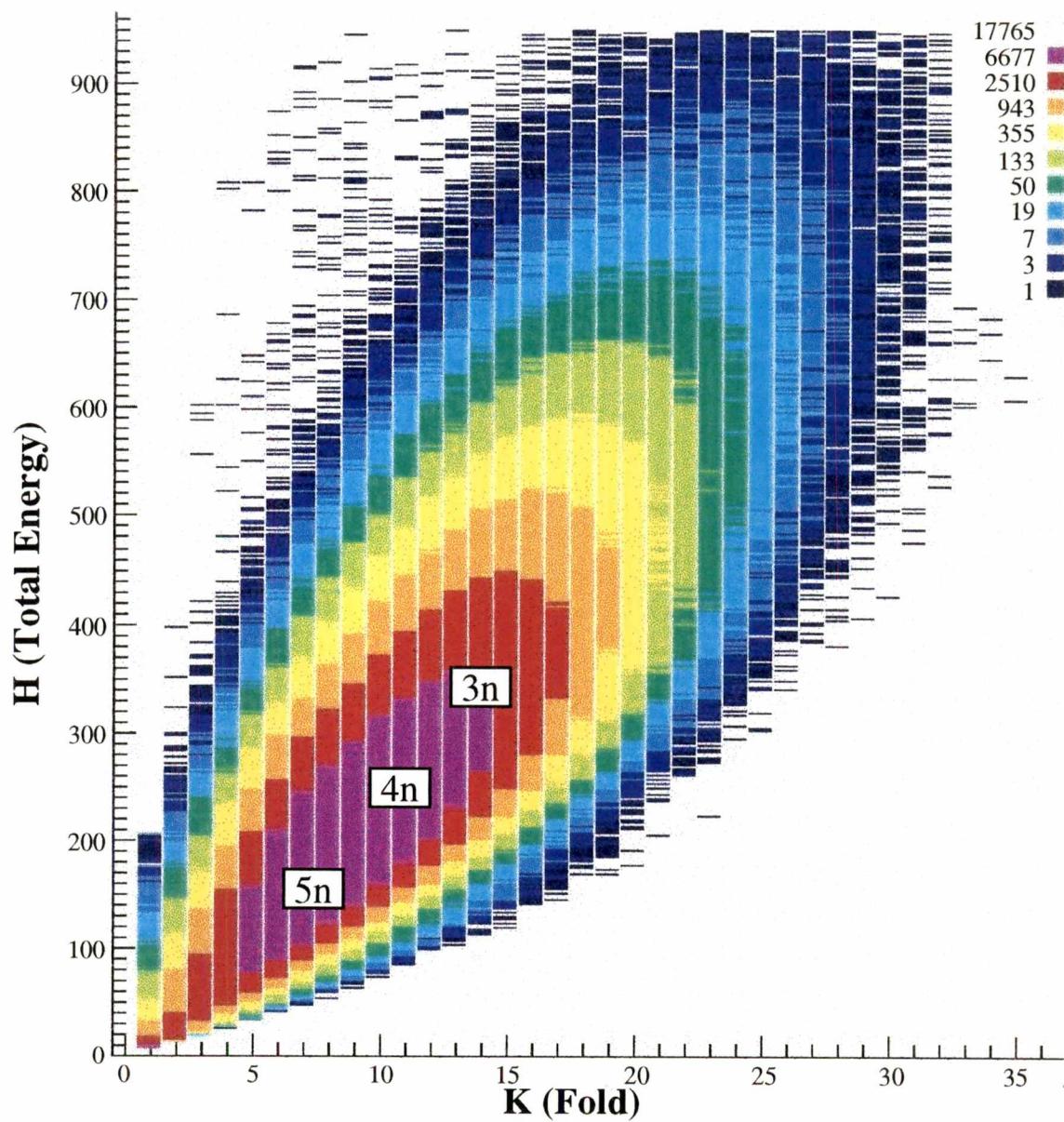


Figure 3.4: K vs. H from the NaI detectors in the Spin Spectrometer

divided by the charge state of the nucleus (M/q). A common and very effective mass spectrometer design is the Fragment Mass Analyzer (FMA) at Argonne National Laboratory. A description of the specifications of the FMA is presented in Sec. 3.3.2.

A counter at the focal plane of the mass separator is necessary to detect the M/q position of recoils. A very effective position-sensitive counter is based on a design variously known as a Multi-Wire Proportional Counter (MWPC), Parallel Plate Avalanche Counter (PPAC), Position Sensitive Avalanche Counter (PSAC). This device is a gaseous ionization chamber in which a large amount of secondary electron pairs (avalanche) is produced, when the recoil passes through the detector. The specifics of the chamber is the design of the electrode planes, which provide sensitivity for x and y position of the incident recoil. A typical design for the electrodes consists of a plane of two planes equally-spaced horizontal and vertical wires sandwiched between anode and cathode planes. In addition to x and y position signals, the PPAC provides information on the energy lost by the recoils as well a timing information. The latter is essential to establish a recoil- γ coincidence condition.

Double-sided Silicon Strip Detectors

The Double-sided Silicon Strip Detector (DSSD) is a specially designed silicon wafer that has been etched to divide the wafer into strips. The strips are created vertically on the front surface and horizontally on the back. These horizontal and vertical strips create a matrix of several hundreds of detectors providing sensitive position information. Like Ge detector pixels. Due to the high pixelation, this detector is very suitable for detection at high count rates. DSSD's are semiconductor detectors and radiation interacting with the detector produces electron-hole pairs that are collected on the electrodes and measured. DSSD counters are most useful in the detection of α particles, because α particles are immediately stopped. Most DSSD designs are far too small to have much sensitivity to β particles or γ rays.

Particle Detectors at the Target Position

The purpose of these detectors is to detect prompt emission of particles from a nucleus just after formation in a compound nucleus reaction. The most commonly used detectors are those for detecting charged particles. One such device is the MICROBALL. The reader is referred to Ref. [50] for a discussion of the MICROBALL. This is a device that uses CsI scintillator material for detecting charged particles, and is used heavily in conjunction with GAMMASPHERE. Other designs use silicon as the detector material. An example of such a detector is the one used at Niels Bohr Institute [51]. Charged particle detectors are most useful in the neutron deficient region, where proton and α emission compete effectively with neutron emission.

Prompt neutron detectors are also used in many experiments, but these counters are less common than charge particle devices. The reason is that neutrons are very difficult to detect. They have a very large interaction length because they do not interact via the electromagnetic force. Prompt neutrons are emitted in the energy regime $100 \text{ keV} < E < 100 \text{ MeV}$. Such neutrons interact with material by inelastic scattering with protons via the strong force. Protons freed by such a collision then interact with the detector material electromagnetically. Thus neutrons are indirectly detected. Materials commonly used for neutron detection are plastics and liquid scintillators. These materials are typically sensitive to γ rays as well, and special considerations are required to properly isolate a γ -ray interaction from a neutron interaction.

3.2.5 Detectors in Coincidence

Individually the detectors described in Sec. 3.2 can yield significant information about the structure of atomic nuclei. The real power of these detectors, however, become apparent when they are linked to work together. Linked detectors are often referred as operating in coincidence. The principle of coincidence is that events occurring in different detectors within a given time window are the result of correlated physical processes of the system being observed. This principle is schematically illustrated by the decay scheme in Fig. 3.5. In this simple example, a decay into the middle level

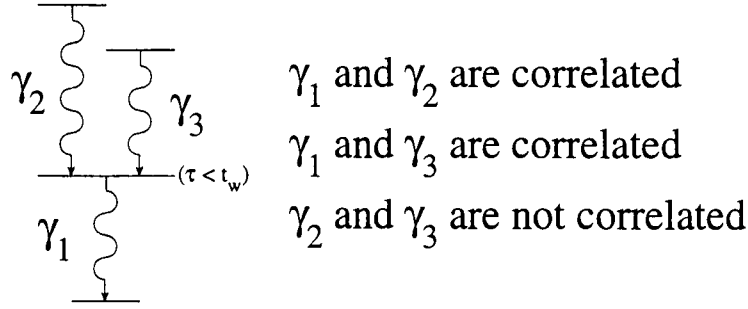


Figure 3.5: Schematic decay scheme illustrating correlated decays. The lifetime (τ) of the middle energy level is much less than the coincidence time window (t_w) of the detectors observing decay.

by *either* γ_2 or γ_3 is followed at some later time, characterized by the half-life, τ , by γ_1 . By choosing a time window (t_w) greater than τ , two detectors will observe γ_1 and γ_2 to occur in coincidence, likewise for γ_1 and γ_3 . Since γ_2 and γ_3 are not correlated, observation of a coincidence between these two would be a random occurrence. By choosing t_w to be only marginally greater than τ , the random coincidence of γ_2 and γ_3 would be far less frequent than the correlated coincidences. Over the course of many such decays, the γ_1 - γ_2 and γ_1 - γ_3 correlations will appear greatly enhanced over γ_2 - γ_3 coincidences.

3.3 Experimental Devices

3.3.1 Spin Spectrometer

The Spin Spectrometer is a multi-detector device, located at Oak Ridge National Laboratory (ORNL), that has in its full configuration 72 NaI(Tl) detectors arranged in a 4π geometry. The total solid angle coverage around the target location is 96.8%. The 3.2% loss is due to the thickness of the casing (1 mm) that holds the individual detectors. An in-depth description of the Spin Spectrometer can be found in Jääskeläinen, *et al.* [52]. Some details that are specific to this dissertation are discussed here.

The large solid angle coverage of the Spin Spectrometer allows measurements of total pulse height (H) and coincidence fold (K) that are very close to the actual total energy and γ multiplicity of the reaction. To take advantage of their good

energy resolution, Compton-suppressed Ge detectors can be installed in the Spin Spectrometer by removing an equal number of NaI detectors. A total of 19 Ge detectors can be substituted into the Spin Spectrometer. The remaining 53 NaI detectors only cover 71.2% of the solid angle when all 19 Ge detectors are in place, but this still provides coverage to measure H and K sufficiently to resolve different particle out reaction channels.

Originally 12 Ge detectors with BGO Compton-suppression shields were designed with geometries that were appropriate for fitting into the Spin Spectrometer [53]. The final configuration uses 19 Compton-suppressed Ge detectors.

3.3.2 AYE-ball and FMA

AYE-ball was a Ge-detector array that was developed to be placed at the target position of the Fragment Mass Analyzer (FMA) located at the ATLAS facility at Argonne National Laboratory (ANL). This array was originally proposed by W. Gelletly from the University of Surrey, U. K. and C. J. Lister from ANL. A description of the detector system is given in Ref. [54]. The configuration of the array used in the ^{179}Au experiment described in Chapter 5 consisted of nine $>70\%$ efficient coaxial Ge detectors, ten $>25\%$ coaxial Ge detectors, and two planar Ge (LEPS) detectors. With the exception of one LEPS counter, all Ge detectors had Compton-suppression shields. This array had a total photopeak efficiency of 1.2% at $E_\gamma = 1.33$ MeV (^{60}Co). This is about 2-3 times greater efficiency compared to the Ge array in the Spin Spectrometer. The whole AYE-ball spectrometer had a P/T (Photopeak to Total number of events) of about 53% for ^{60}Co . The electronics for the Compton suppression of the $>70\%$ Ge detectors is discussed in Sec. 3.4.1. A similar electronic setup was used for the $>25\%$ detectors, with minor changes being in brand of the amplifiers. The electronics for AYE-ball were setup so that it could be triggered by Ge-Ge events alone or by a Ge events plus a PPAC event.

The Argonne Fragment Mass Analyzer was designed by Davids, *et al.* [55] and has been in use since 1992. This so-called three focus mass spectrometer separates reaction recoils from the incident beam and focuses them in a plane by mass

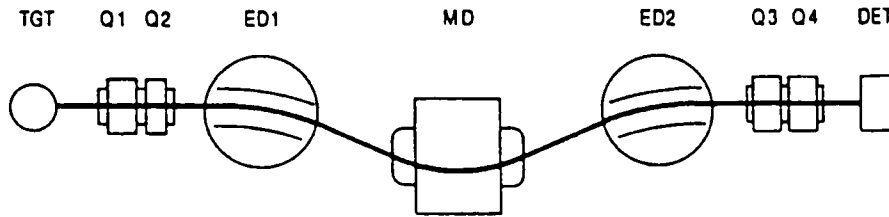


Figure 3.6: Schematic diagram of the Fragment Mass Analyzer at ANL. Q1 and Q2 indicate the field coils for the first electric quadrupole lens, while Q3 and Q4 indicate the coils for the second. ED1 and ED2 are the two electric dipoles, and MD is the magnetic dipole.

over charge (m/q). The design uses focusing double electric quadrupole lenses to enhance the solid angle acceptance and provide additional focusing capabilities, in conjunction with electric and magnetic dipoles for mass and charge state separation. The schematic layout of the FMA is shown in Fig. 3.6. The m/q resolution of the FMA is about 340:1.

The positions of the m/q dispersed recoils are detected with a parallel plate avalanche counter (PPAC) as described in Sec. 3.2.4, which is located at the position labeled “DET” in Fig. 3.6. The PPAC uses isobutane gas at a pressure of 3 Torr to interact with the ionizing recoils. Its electronics are described in Sec. 3.4.3.

Also in place at the focal plane area was a DSSD. This device was placed 40 cm downstream of the PPAC. The DSSD used in the ^{179}Au experiment was divided into 40 front and 40 back strips. Each strip was 1 mm wide, making the total detector 4 cm on a side. The detector was 60 μm thick. The device has a ~ 34 keV resolution for internal decays (α and proton) at 6.5 MeV. The electronics used for the DSSD are described in Sec. 3.4.4.

3.4 Electronics for AYE-ball and the FMA

3.4.1 Compton Suppression for Ge detectors

As mentioned in Sec. 3.2.1, photons, in the energy range of interest, interact with matter primarily by both photoelectric absorption and Compton scattering. Because of the low stopping power of Ge material for photons, it is quite likely that a Compton-scattered γ ray will escape from the detector. If this occurs the Ge detector will not record the full energy of the γ ray. The result is that the spectrum for a given decay will have a sharp photopeak at the channel that represents the energy (E_γ) of the decay, and an approximately continuous background of compton-scattered γ rays at energies $\approx 0.8E_\gamma$. A typical measure that is used to determine the efficiency for a particular detector for photoabsorption versus Compton scattering is the peak-to-total ratio. The peak-to-total is defined as

$$P/T = \frac{\text{Sum of counts in all photopeaks}}{\text{Sum of counts in the total spectrum}}. \quad (3.85)$$

The standard for measuring the peak-to-total ratio is the γ radiation from ^{60}Co . ^{60}Co β^- decays primarily to the 4_1^+ state in ^{60}Ni . The deexcitation from the 4_1^+ state to the ground state of ^{60}Ni produces two γ rays with energies of 1173.237 keV and 1332.516 keV. Thus the decay of ^{60}Co produces only two γ rays of consequence from which P/T can be measured. A typical coaxial Ge detector has a peak-to-total ratio for the ^{60}Co lines of $\sim 20 - 25\%$.

Fig. 3.7 illustrates the effect of Compton suppression. Without suppression there is a large background resulting from the Compton scattering of γ rays. When the suppression shields are activated, a large number of the scattered events are rejected (REJECTED EVENTS in Fig. 3.7), and due to a significantly reduced background (COMPTON SUPPRESSED SPECTRUM in Fig. 3.7) a P/T typically of 50 – 60% is achieved. It can also be seen that the photoabsorption peak is not significantly reduced since the γ ray is completely absorbed in the Ge detector. The only loss to the photopeak comes from rare random coincidences of unrelated γ -ray events in the Ge detector and Compton suppressor.

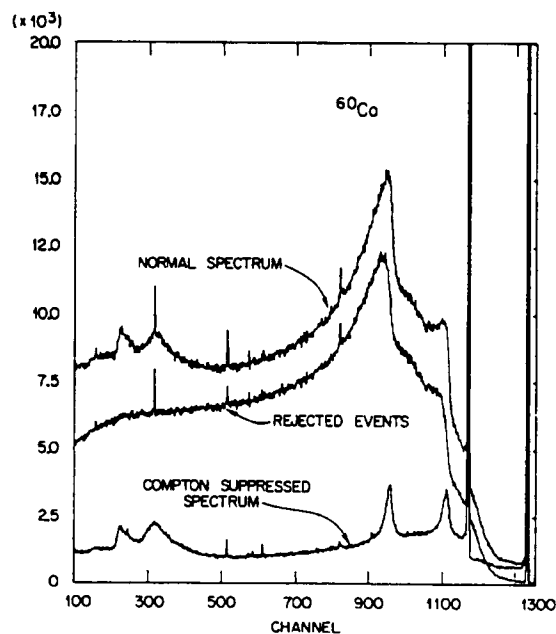


Figure 3.7: Illustration of the effect of Compton suppression. The three spectra in the figure are of a ^{60}Co source using a Spin Spectrometer Ge detector. The Compton-suppressed and normal spectra represent Ge events with and without the suppressors (BGO material) activated. The rejected events are the Ge events with the suppressors activated. (This is Figure 2.100 from Ref. [53].)

It is now common practice to surround Ge detector crystals with high-efficiency scintillator material (such as BGO) to detect scattered γ rays. A typical design is that shown in Fig. 3.8. A particular electronics setup to do the suppression is shown in Fig. 3.9. When an event occurs in the Ge detector an identical signal is passed down the cables labeled "E" and "T". To determine coincidence of events in separate detectors, it is naturally very important obtain the time at which an event occurs in a detector. The time that an event occurs is determined by the rise time of the output pulse from the detector, while the area of the pulse is proportional to the energy deposited by an event. To achieve a good energy resolution, proper pulse shaping leading to a high signal-to-noise ratio is essential rather than a fast amplification of the signal. In addition, pulses are shaped such that the trailing edge is minimized to limit pile up of consecutive pulses. However, this process also distorts the leading edge of the pulse. The slow amplification and distortion of the leading edge requires two routes in the electronics be used to extract and process energy and timing information separately.

The signals from the BGO Shield and one from the Ge detector (labelled "T") in Fig. 3.9 are amplified with "fast amplifiers," generally referred to as Timing Filter Amplifiers (TFA), to form a time signal. TFA's quickly amplify the pulse to preserve the rise time. The amplified signals from the BGO are passed to a discriminator (DISC). The discriminator is a device that produces a logic pulse when the voltage in a circuit exceeds a set level. If the rise time of a given pulse is slow, this can cause the time of the pulse to be significantly dependent on the signal's amplitude (proportional to the energy of event). For scintillator detectors such as BGO devices, the rise time is dependent primarily on the time constant of the associated photo-multiplier tube. The rise time for these detectors is typically very fast, so "simple" discrimination is often sufficient for most timing purposes. Because of the slow charge collection properties of Ge detectors, however, formation of a logic signal is much more dependent on the signal's amplitude. To reduce this dependence (often referred to as "walk"), a so-called Constant Fraction Discriminator (CFD) is used. In the CFD, the input signal is formed into two pulses. The first pulse is attenuated by a certain constant percentage (about 70%) of the full amplitude, and the second pulse

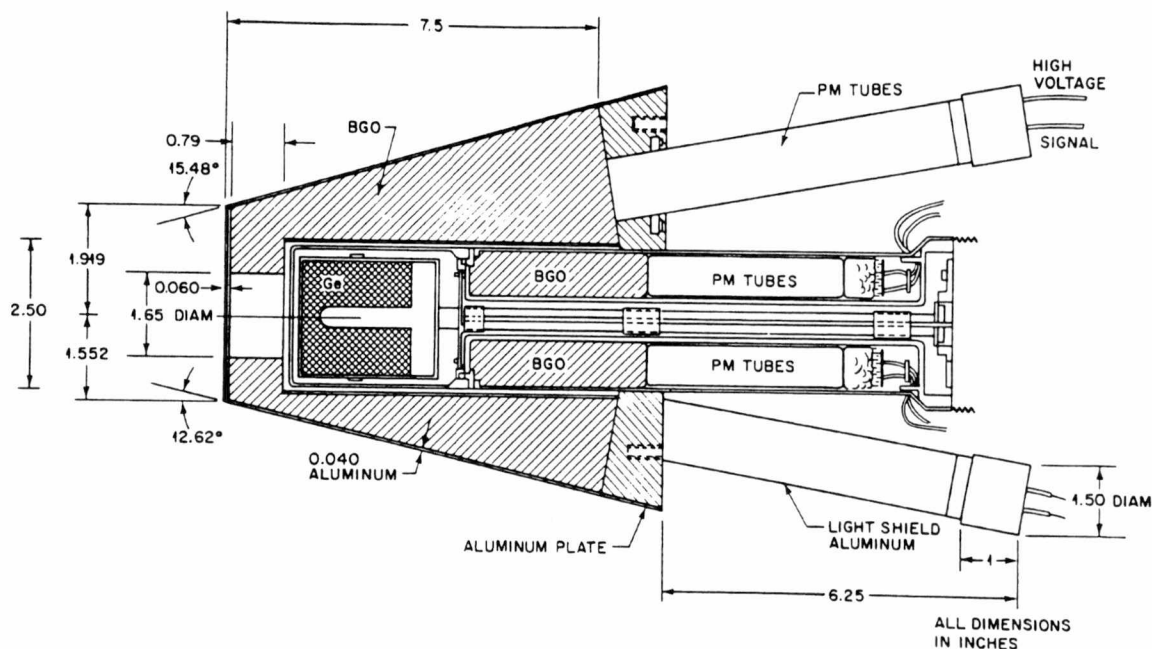


Figure 3.8: A cross-section of a Ge detector with BGO Compton suppressors. This particular design is for the Ge detectors used with the Spin Spectrometer. (This is Figure 2.99 from Ref. [53].)

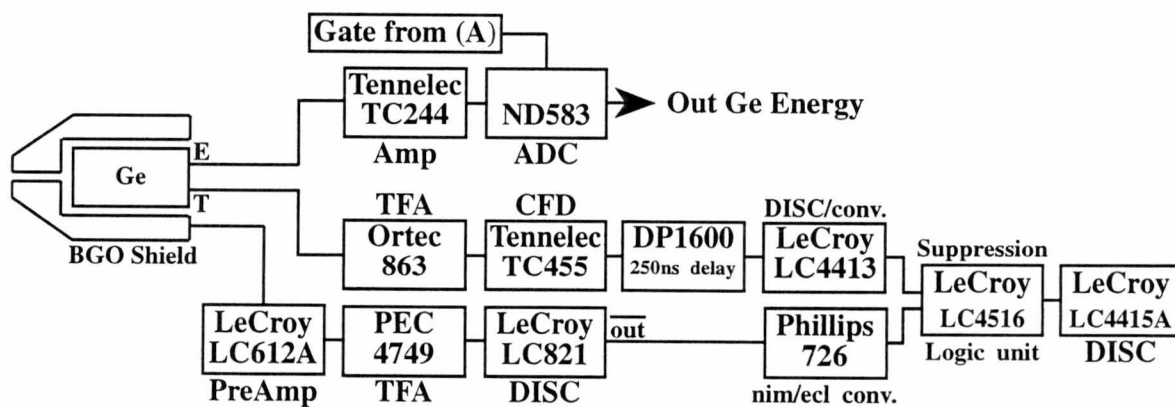


Figure 3.9: Compton-suppression electronics for the large volume detectors used in AYE-ball.

is inverted and delayed a small amount (~ 50 ns for Ge pulses). The two signals are then added together. The resulting bipolar signal is initially negative, and the zero crossing point is essentially independent of the amplitude of the input pulse. This crossing triggers an output logic pulse from the CFD module.

The width of the logic pulse (amount of time that the pulse exists) is a variable that is adjusted in the discriminator. This width defines the “time window” with which to establish a coincidence. For the design in Fig. 3.9, the BGO signal window was set to width of 500 ns and converted to a different electronics standard in the Phillips 726 module. To achieve an anti-coincidence (or veto) in the particular circuit, the BGO signal was further processed as an anti-logic pulse ($\overline{\text{out}}$). The Ge signal is delayed by 250 ns, to overlap with the BGO signal, and passed through special discriminator that also converts the electronics standard (LeCroy 4413 module). The Ge pulse was set more narrow with a width of 40 ns to ensure a high rate of Compton suppressed events. The BGO and Ge signals are then compared in the LC4516 logic unit. Because of the setup of the time windows, if an event in the BGO shield occurs within 250 ns of the Ge, the event is suppressed and no signal leaves the logic unit. If the Ge event occurs alone, the 40 ns pulse is passed to the LC4415A discriminator (i.e. the event is accepted). In this discriminator the pulse width is again modified to 160 ns. The pulse can now compared with timing signals from other detectors. A description of the electronics setup for AYE-ball is discussed in Sec. 3.4.2.

3.4.2 AYE-ball Coincidence Circuitry

The γ rays originating from the same cascade in a single compound nucleus can be virtually considered as simultaneous. If, for example, the γ rays are detected by the Ge detectors at almost identical time, they are considered to be in coincidence. The purpose of a coincidence circuit is to determine the overlap from timing measurements and assure that all the required information (energies, hit patterns, times, etc.) is recorded properly. The coincidence circuit used for the AYE-ball array in the ^{179}Au experiment for the Ge detectors (after passing the Compton-suppression anti-coincidence logic) is illustrated in Fig. 3.10. The LC4415A module with the label

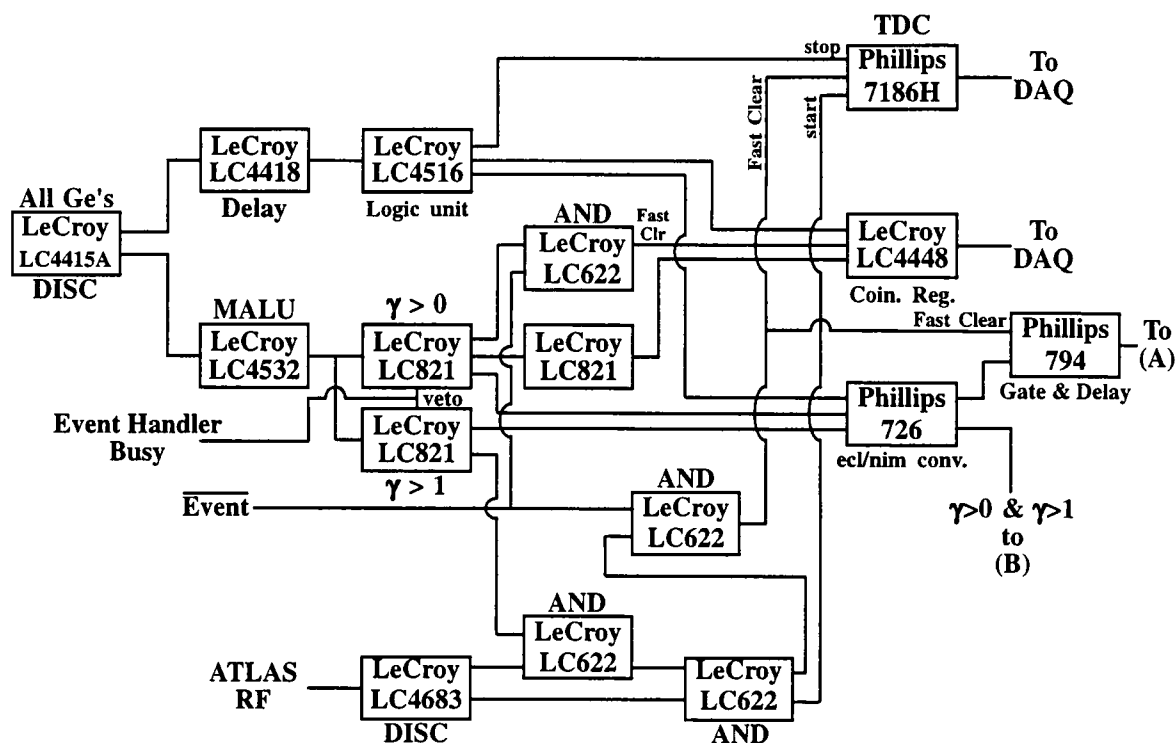


Figure 3.10: Schematic of the AYE-ball electronic setup. Definitions for abbreviations are given in the text. "To DAQ" represents digitized information that is available for the data acquisition computer. "To (A)" goes to the entry "Gate from (A)" in Fig. 3.9. " $\gamma > 0$ & $\gamma > 1$ to (B)" go to the entries " $\gamma > 0$ from (B)" and " $\gamma > 1$ from (B)" in Fig. 3.14. (The electronics were designed and implemented by the physics division staff at ANL.)

"All Ge's" is the same discriminator on the far right of Fig. 3.9. This discriminator represents the individual time signals from all 19 AYE-ball detectors. The signals are fanned out into two paths. The lower path will be discussed first.

The pulses from the LC4415A discriminator are passed to a Multiplicity Analog Logic Unit (MALU). This device produces a sum of the logic pulses from the detectors and the result is passed to the LC821 units. This summing process is illustrated in Fig. 3.11. Like a discriminator the LC821 module emits a logic pulse when the voltage exceeds a set value. The voltage settings for the units labeled " $\gamma > 0$ " and " $\gamma > 1$ " are also illustrated in Fig. 3.11. These two settings correspond to the multiplicity of the coincidence, thus a pulse from " $\gamma > 0$ " means that at least

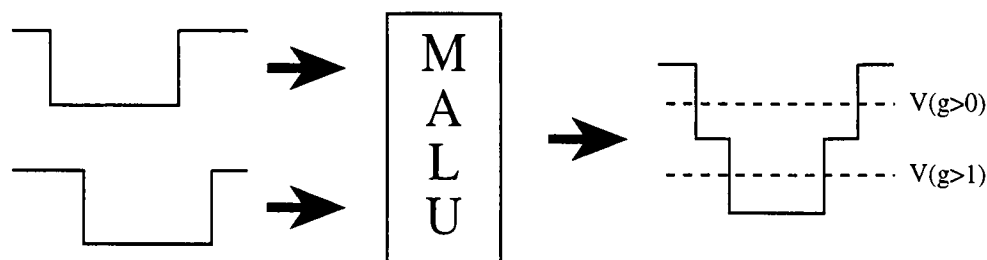


Figure 3.11: Illustration of summing by the MALU module.

one detector has fired. “ $\gamma > 1$ ” means at least *two* detectors fired within 160 ns (the pulse width of one signal) of each other. Because the data acquisition computer can only process one event at a time, the event handler sends a “busy” signal to the LC821 module to veto the output pulses. The output from both “ $\gamma > 0$ ” and “ $\gamma > 1$ ” are sent to different Phillips ecl/nim converter modules then passed to other areas (entry (B) in Fig. 3.14).

The upper path coming out of the initial discriminator pulses are delayed, to allow processing by the MALU and other electronics, and then fanned out to three modules. On one path the signals are converted in the Phillips ecl/nim converter and passed to a Phillips gate and delay generator. If the signal is not cleared (see below for definition of clear), a gate is created allowing the ND583 Analog-to-Digital converter in Fig. 3.9 to digitize the energy signal of the Ge detector. A second path for the discriminator signals leads to an LC4448 Coincidence Register. If this signal not cleared, this module will set a specific bit of a 16-bit data word corresponding to a specific detector. Thus, this module creates a data word that identifies the coincident detectors. This word is then past to the data acquisition computer when requested. On the third path, each discriminator signal is passed to a Phillips time-to-digital converter (TDC). The pulses are used to stop the individual TDC’s to provide a measurement of the time period between two logic signals, one of which comes from the ATLAS resonator frequency (RF). The RF signal is passed through some logical AND gates and used as a common start for all coincident Ge detectors.

Because AYE-ball is used in conjunction with other devices, a Ge event is not necessarily sufficient to trigger an event. To prevent time to be lost by unnecessarily

processing signals, an $\overline{\text{Event}}$ signal is sent from the triggering electronics in Fig. 3.14. $\overline{\text{Event}}$ is false (zero voltage) if trigger conditions are not satisfied. This clears the modules so they are available new signals.

3.4.3 PPAC Electronics

The device used to detect the recoil products separated by the FMA is the PPAC. This device is a position sensitive ionization detector essentially consisting of an anode, cathode, and two planes of parallel wires in the horizontal and vertical directions respectively. These wire planes surround the device's anode and are at a similar potential as the anode. The cathode is upstream (toward the FMA) of the other planes. Fig. 3.12 illustrates the electronics design for the PPAC. A recoil entering the PPAC causes a signal at the anode which is converted to a digital signal in a discriminator and starts a TDC (Time-to-Digital Converter). The anode signal is also used to determine coincidences with other detectors, so the pulse is fanned off at points (C) and (D) to be used in other areas of the electronics. See Figs. 3.14 and 3.13 respectively for the use of these signals

The point within the detector where a recoil deposits energy is determined by the difference in time it takes for a signal to be produced in the "UP" and "DOWN" ends for horizontal position and "RIGHT" and "LEFT" ends for vertical position. The pulses for the four position signals are individually formed into logic pulses and are used as stop signals in the TDC to produce a time for each end with respect to the anode. The PPAC is designed such that the maximum horizontal and vertical delays are 120 ns and 40 ns respectively, and the PPAC TDC is typically calibrated to 0.1 ns per channel. The energy loss as the recoil passes through the PPAC is obtained from the cathode. Recording of this energy loss through the PPAC can provide an additional parameter to separate scattered beam that passes through the FMA from reaction recoils.

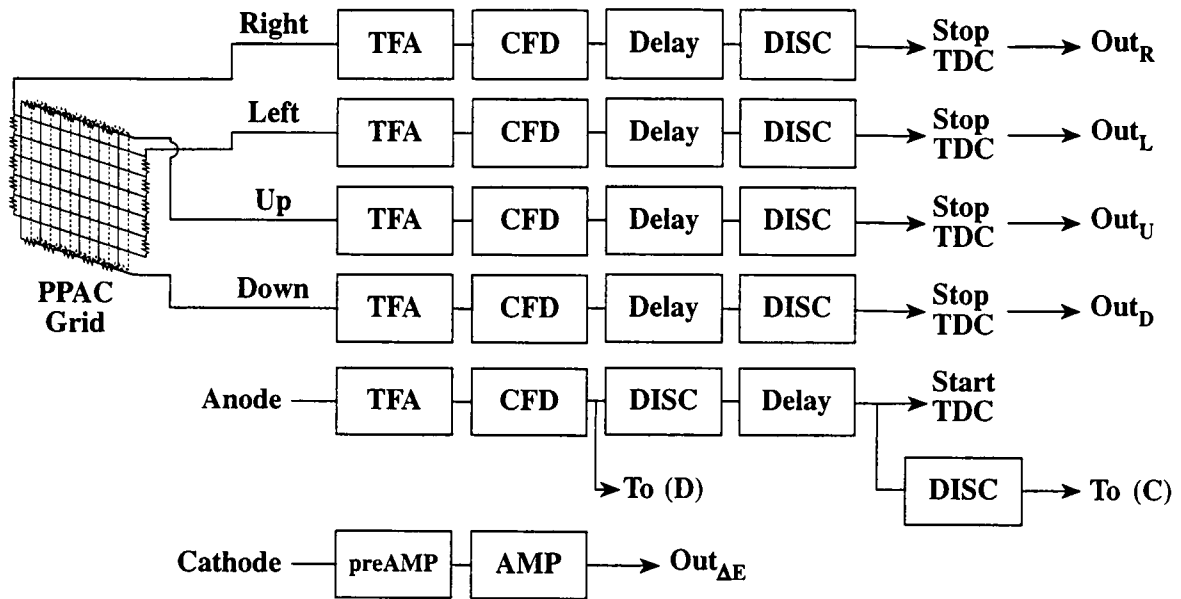


Figure 3.12: Electronics schematic for the PPAC at ANL. The entry “To (C)” goes to the “PPAC from (C)” in Fig. 3.14. The output “To (D)” goes to “From (D)” in Fig. 3.13. The electronics were designed and implemented by physics division staff at ANL.

3.4.4 DSSD Electronics

The DSSD is a “pixelated” silicon detector designed for identifying α and proton decay of recoil nuclei at the focal plane of the FMA. The particular DSSD design used in the ^{179}Au experiment was composed of 40 horizontal and 40 vertical strips. This effectively creates 1600 tiny charged particle detectors. A decay can be associated with a particular recoil mass by observing a coincidence with the PPAC. Because α decay lifetimes are often several seconds, measuring a time coincidence between the PPAC and an α decay is impossible. What is observed instead is a *position* coincidence. Recoils passing through the PPAC are implanted with nearly 100% efficiency into the DSSD. One can readily observe a time coincidence between the PPAC and the recoil impact on the DSSD. Since the silicon detector is so finely divided, the average time before another recoil impacts the same pixel is large when compared to a non-divided detector. If the lifetime of the implant is small compared to the average time between implants, the decay can be correlated with the specific implant. If the average time

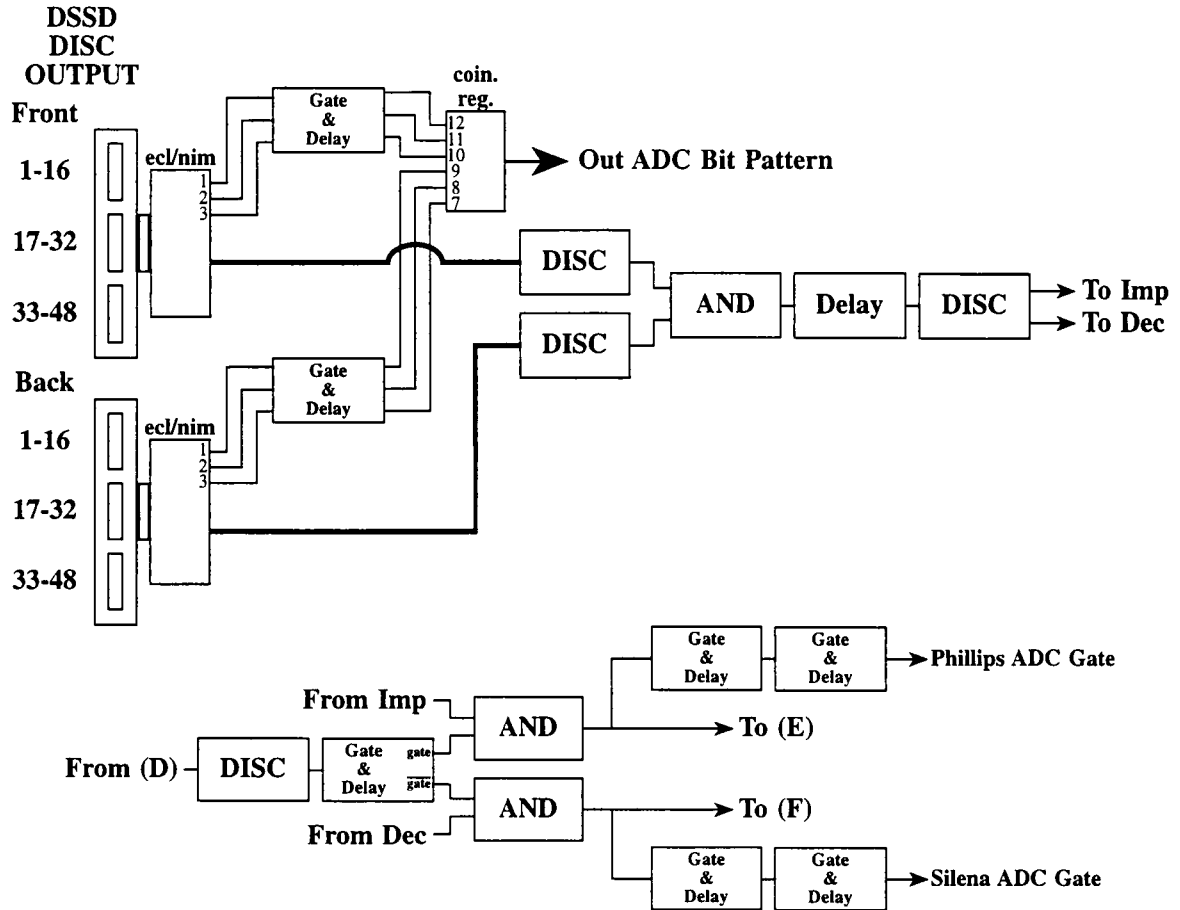


Figure 3.13: Electronics schematic for the DSSD at ANL. The input "From (D)" is the PPAC anode signal coming from "To (D)" in Fig. 3.13. The output signals "To (E)" and "To (F)" go to Fig. 3.14. The electronics were designed and implemented by physics division staff at ANL.

between implants is not large compared to the decay time, accidental correlations will result. By recording the clock time of the implant and the decay, one can measure the half-life of the α -decaying implant and thus both the α -decay spectrum and half-life of the recoils observed inbeam can be determined. In addition, a particular decay can be associated with a particular mass by a PPAC coincidence with the implant signal. The electronics used to make these coincidences are illustrated in Fig. 3.13.

The events in the DSSD are fanned out for energy and time processing. The discriminator output for the time signals is shown in left-hand side of Fig. 3.13.

There are 40 separate signals for the front strips and 40 for the back. Most electronics modules accommodate a maximum of 16 signals, but, since there are only 40 strips for the detector described here, the last signals are grouped in units of 8 channels. The signals are passed from the discriminator to an ecl-to-nim converter. The first three outputs from the converters are sent to a coincidence register where a particular bit identifies the group from which the signal occurs (e. g. 1-16, 17-32, or 33-40). These signals are fed to an additional module to change the pulse width. The pulse from the front and back sets are logically "ANDed" together. Thus if a event occurs in one of the front strips, but there is not an event in a back strip, the event processing is effectively stopped at this point.

After it is determined that at least one front and one back strip signal exists, the single pulse, corresponding to an accepted AND, is passed through additional electronics and fanned out to two paths labeled "Imp" and "Dec." These two paths are fed to the lower part of Fig. 3.13. There is an additional signal, "From (D)," that represents the time of a recoil passing through the PPAC. The PPAC signal passes through a gate-and-delay generator. The output logic signal (gate) is passed to an AND module and compared with the "Imp" signal. The anti-logic pulse ($\overline{\text{gate}}$) is ANDed with the "Dec" signal. If a PPAC signal is present the "Imp" signal is accepted, and the event is considered to be a recoil implanting into the DSSD. If a PPAC signal is not present, however, the event is considered to be a charged particle decay and the "Dec" pulse is accepted.

The results of this AND process are then fed, via paths (E) and (F), to the logic circuit of Fig. 3.14. The resulting signal is also passed as a gate pulse to either a Phillips ADC for an implant event or a Silena ADC for a decay event. While not denoted in the figure, the energy signals from the DSSD strips are fed to two different sets of amplifiers before being passed to Phillips and Silena ADC's. Because of the significantly different energies associated with an implant or a decay, amplifiers with different gain settings are required. For example, in the ^{179}Au experiment a typical recoil implant had an energy of ≈ 56 MeV. Conversely, the typical α -decay energy is only ≈ 5 MeV. As a consequence the amplifiers associated with the Phillips ADC's and an implant event have a full-scale gain of 200 MeV, whereas those associated

with the Silena's and a decay have a 20 MeV full-scale gain.

3.4.5 The electronic layout for the RDT experiment

The previous sections have described the electronics for several detector systems. In Sec. 3.4.2, the electronics used for measuring the prompt decay γ rays was described. In Sec. 3.4.3, the circuit used to identify a mass separated by the FMA was discussed, and in Sec. 3.4.4, it was explained how the PPAC signal could be used to differentiate between an implant and a decay in the DSSD detector.

The combination of the DSSD and PPAC have been used extensively in experiments to identify α and proton decays of previously unobserved nuclei. For example, these devices were used in the recent identification of ^{166}Pt and ^{167}Pt by α decay [56]. If the devices are used to tag on known α or proton decays, these charged particle decays could be used as a gate to identify the γ rays associated with the parent nucleus. This method is known as Recoil Decay Tagging (RDT). The first RDT experiment using a setup very similar to the system at the FMA was reported by Paul *et al.* in Ref. [57]. The key to the success of RDT is to correlate γ rays promptly emitted from excited residue nuclei at the target position with their α decay. The electronics logic used in the ^{179}Au RDT measurement is shown in Fig. 3.14.

The timing signals from the three detection devices (AYE-ball, PPAC, and DSSD) are fed in from the left in Fig. 3.14. From the AYE-ball array, two signals come from point (B) in Fig. 3.10. One signal corresponds to at least one γ event, and other at least a γ - γ coincidence. The PPAC signal comes in at point (C), and there are two signals from the DSSD at points (E) and (F). The two DSSD signals correspond to an implant and decay event respectively. The PPAC signal is fanned out to be used as a start signal for two time-to-amplitude (TAC) converters. The stop signal for one of the TAC's comes from the DSSD Implant signal (E). This establishes a PPAC-DSSD coincidence and can also serve as the time of flight information of the recoil from the PPAC to the DSSD. The stop signal for the second TAC comes from the $\gamma > 0$ signal from the AYE-ball. Since the γ decay occurred before the recoil was detected in the PPAC, the AYE-ball signal must be delayed. Both AYE-ball signals ($\gamma > 0$ and $\gamma > 1$)

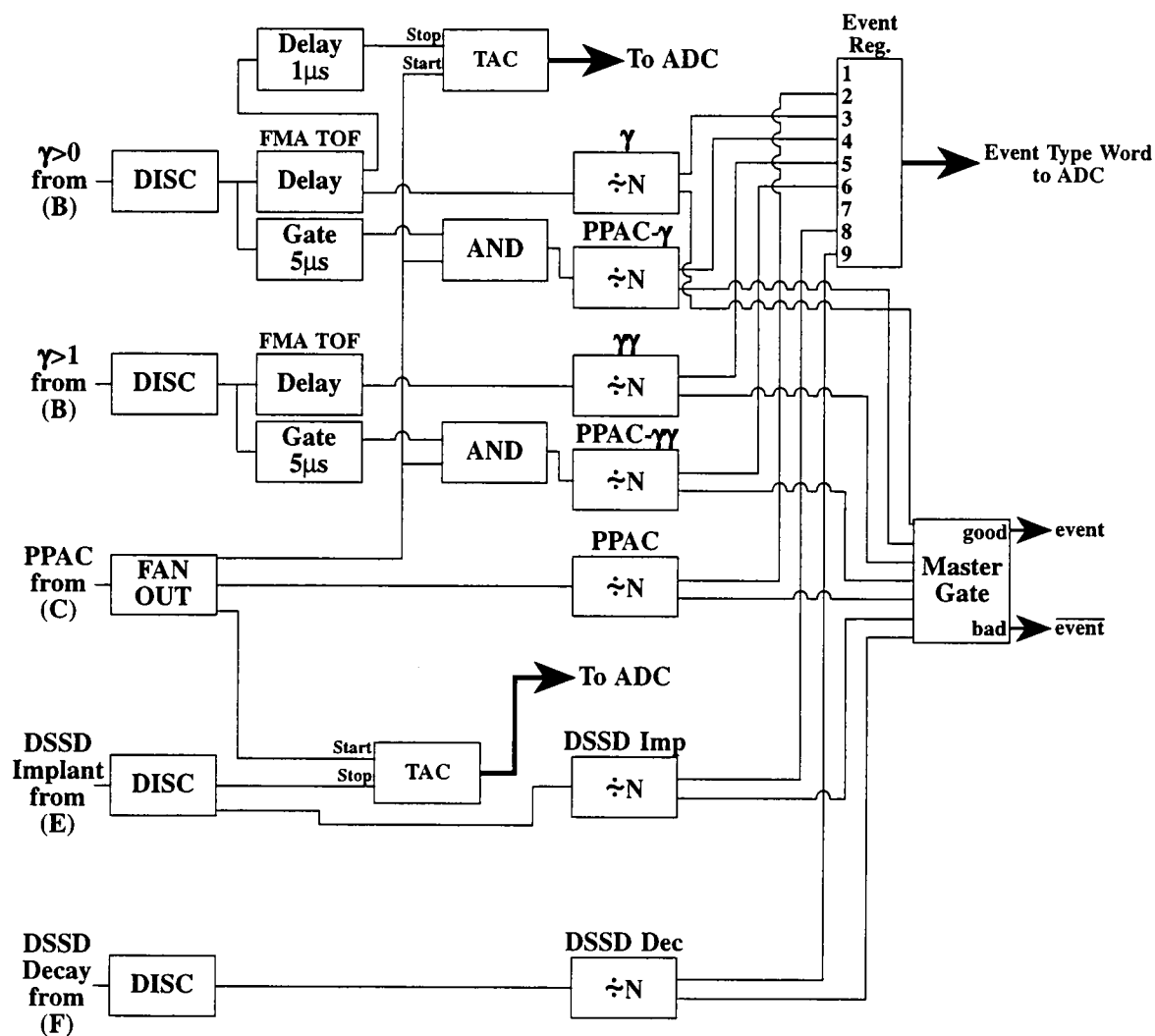


Figure 3.14: Schematic of the logic for the ^{179}Au experiment. (B) comes from Fig. 3.10, (C) from Fig. 3.12, and (E) and (F) from Fig. 3.13. (The electronics setup was carried out by the physics division staff at ANL.)

are passed through a delay generator. The amount of delay is defined by the time of flight through the FMA. For the ^{179}Au experiment, the residual nuclei had an average recoil velocity of $0.03c$, which corresponds to a time of flight through the FMA of $0.92\ \mu\text{s}$. Since this is an average value for the time of flight, there is still a possibility that the AYE-ball event arrives before the PPAC, thus the signal is delayed further by $1\ \mu\text{s}$. The output of the second TAC thus represents $1.92\ \mu\text{s}$ minus the time of flight of the recoil. The output of both TAC's are sent to ADC's to be digitized.

The time-of-flight delayed AYE-ball signals are sent to a multiplicity logic unit (represented by N) and from there go to a coincidence register and the master gate. The PPAC and DSSD signals are also fanned into the coincidence register and master gate. Two additional signals are created by making large time gates from the " $\gamma > 0$ " and " $\gamma > 1$ " signals and combining them with the PPAC signal by a logical AND.

The coincidence register produces a data word where a given bit corresponds to a given signal being present in the event. Whether a given event is accepted is determined in the master gate. The master gate can be programmed to trigger a good event based on a given condition of the input parameters. For example, a good event can be triggered by a DSSD decay event, and a separate good event trigger caused by PPAC- γ plus DSSD implant event. If the trigger conditions are not satisfied, the $\overline{\text{event}}$ signal is emitted to clear all the ADC's and TDC's used for AYE-ball, PPAC, and DSSD signals, so that another event can be accepted. For a good event, the ADC's and TDC's are triggered to digitize the input signal, and the "event" logic pulse is sent to the event handler.

When conditions in the electronics trigger an "event" signal, the event handler will read the appropriate ADC's and TDC's and send the information to the data acquisition computer. While in the computer, the event is typically passed to a program that decodes the event so that spectra may be viewed while the experiment is in progress. More importantly, the event data are written to magnetic storage tapes for a detailed offline analysis at a later date. The computer is typically the slowest aspect of the entire experimental setup, so at times when the data rate is high, much of the online sorting is sacrificed to maximize the amount of data that is written to

tape.

3.5 Offline Analysis

The Purpose of the offline analysis is to establish a level scheme of the nucleus of interest. The essential tool to derive the level scheme is the γ -ray coincidence information. Many powerful software packages have been introduced to sort and evaluate these coincidence data with unprecedented efficiency and precision. In this section, the general principles for constructing the level scheme will be discussed and the software used for these tasks will be described.

3.5.1 Data Structure and Sorting

The typical event will contain information about the detectors that were coincident including the energy and time. One example of how events can be formatted is shown in Tables 3.1 and 3.2. The event, shown in these tables, is an example of a DSSD implant event from phase 1 of the ^{179}Au experiment at ANL using the Michigan State University data acquisition system. To examine correlations between detectors, the events must be properly decoded and incremented into a format useful for analysis. There are many program packages available for handling event data. These programs have functions that handle the low-level tasks such as tape handling and memory management and make the event data available for the user to manipulate as desired. Examples of such programs include DAPHNE from ANL and SCANU from ORNL. The tape scanning package that was used for the analyses presented in this dissertation was TSCAN [58]. The TSCAN program will typically read data from magnetic tape and pass it to a function that is user defined. It is in this user defined function that the event is decoded and sorted.

Quantities from decoded events are incremented in binary histograms where the value of each word (defined as a 16 or 32 bit integer) in that file corresponds to the number of events that occurred with a particular channel number for the quantity of interest. For example, if one was interested in viewing the energy signal from detector five in the AYE-ball array, each event is checked to see if a signal from detector five

Table 3.1: First part of a typical implant event from ^{179}Au experiment at ANL. The experimental setup is described in Sec. 3.3.2. The event format is from the Michigan State data acquisition system, that is currently in use at ANL.

#	Word (Hex)	Mask		Value (Decimal)	Description
1	001C	XXXX		28	Event Length
2	0080	XXXX	bits -	8	Event Type Bit Pattern
3	4B19	XXXX		19225	Low 16 bits of 48 bit Scalar
4	FA78	XXXX		64120×2^{16}	Middle 16 bits of 48 bit Scalar
5	00B0	XXXX		176×2^{32}	Upper 16 bits of 48 bit Scalar
6	0440	XXXX	bits -	7 & 11	DSSD ADC hit pattern
7	0400	XXXX	bits -	11	ADC strips hit pattern (first ADC)
8	A09E	YYYY	X-	10	Strip number (first strip)
			Y-	158	Strip Energy Channel (first strip)
9	4200	XXXX	bits -	10 & 15	ADC strips hit pattern (second ADC)
10	E044	YYYY	X-	14	Strip number and energy (first strip)
			Y-	68	Strip Energy Channel (first strip)
11	90A4	YYYY	X-	9	Strip number and energy (second strip)
			Y-	164	Strip Energy Channel (second strip)
12	1FCE	YYYY	X-	1	ADC ID of Focal plane Ge Energy
			Y-	4046	Energy (no signal)
13	9FCE	YYYY	X-	9	ADC ID of Focal plane Ge Time
			Y-	4046	Time (no signal)

Table 3.2: Second part of a typical implant event from ^{179}Au experiment at ANL. The experimental setup is described in Sec. 3.3.2. The event format is from the Michigan State data acquisition system, that is currently in use at ANL.

#	Word (Hex)	Mask		Value (Decimal)	Description
14	1325	XXXX	X-	1	TDC id of PPAC right time
			Y-	805	PPAC right time
15	33A7	XXXX	X-	3	TDC id of PPAC left time
			Y-	935	PPAC right time
16	521F	XXXX	X-	5	TDC id of PPAC up time
			Y-	543	PPAC up time
17	7242	XXXX	X-	7	TDC id of PPAC down time
			Y-	578	PPAC down time
18	0567	XXXX		1383	PPAC Cathode Energy
19	9450	XXXX	X-	9	TDC id of ToF from PPAC to DSSD
			Y-	1104	Time of flight from PPAC to DSSD
20	22F2	XXXX	X-	2	ADC id from TAC from PPAC to DSSD
			Y-	754	TAC from PPAC to DSSD
21	B0B7	XXXX	X-	11	TDC id for ToF from ATLAS RF to PPAC
			Y-	183	Time from ATLAS RF to PPAC
22	3FCE	XXXX	X-	3	ADC id from TAC from AYE-ball to PPAC
			Y-	4046	TAC from AYE-ball to PPAC
23	0002	XXXX		2	Pileup register (No pileup)
24	8010	XXXX	X-	8	Register id for first Ge hit pattern
			Y-	bit 5	First Ge hit pattern
25	8000	XXXX	X-	8	Register id for second Ge hit pattern
			Y-	0	Second Ge hit pattern
26	0383	XXXX		899	Ge Energy
27	06AA	XXXX		1760	Ge Time
28	DEAD	XXXX			End of event marker

is present. If a signal for detector five is not present the event is discarded. If it is present, such as the event shown in Tables 3.1 and 3.2, the appropriate channel in the histogram is incremented one unit. The detector is identified in word 24. In this data word, the fifth bit is set. This signifies that detector five is part of the event. Word 26 corresponds to the energy value for the detector, and thus channel 899 in the binary histogram is incremented one unit.

For the analysis of prompt γ -decay cascades, the most common histograms are created by sorting Ge-Ge events into a two-dimensional energy spectrum (matrix). That is, when two or higher-fold Ge coincidences occur in a given event, the cell defined by the row/column of Ge 1/Ge 2 energies is incremented. An event containing three γ rays would be resolved into three pairs (Ge 1/Ge 2, Ge 1/Ge 3, and Ge 2/Ge 3). The newest arrays, such as GAMMASPHERE, have so many detectors that three and higher-fold Ge coincidences are common. In these cases, cubes (3-d histograms) of Ge-Ge-Ge events are typically created.

3.5.2 Energy and Efficiency Calibration of Ge spectra

The materials used in detectors are chosen for their predictable relationship for generating signals that are proportional to the energy deposited in the detector. Electronics are designed to preserve this proportionality. Because of the unique characteristics of the detectors and the electronics setup, the devices must be calibrated to determine this proportionality relation. The response of a Ge detector for photons is a function of γ -ray energy. Thus it is not only important to calibrate the energy but also the efficiency of the counters. A metal absorber is often placed over the face of a Ge detector to limit the amount of atomic x rays detected. Naturally, these absorbers also have an effect on the detection efficiency.

Detectors are calibrated by using "standard" radioactive sources that have been carefully studied to very accurately determine energies and intensities of their associated γ -ray decays. For γ radiation, some of the most common calibration standards are ^{60}Co , ^{152}Eu , and ^{182}Ta sources. The energies and relative intensities for these sources can be found in the Table of Isotopes [59]. ^{60}Co has only two peaks

in the γ decay and is useful in P/T analyses such as that described in Sec. 3.4.1. ^{152}Eu has well separated γ -ray peaks from 121.8 keV to 1408.0 keV, thus covering a wide energy range. ^{182}Ta has a large number of γ rays at low energy (< 250 keV) where the efficiency changes rapidly due to electrons thresholds in addition to the absorbers. Fig. 3.15 illustrates the spectra observed from the ADC connected to a Ge detector when sources of ^{152}Eu and ^{182}Ta , respectively are placed at the target position. The peak at 511.0 keV, observed in both spectra, is due to γ rays created from electron-positron annihilation.

Energy Calibration

To make offline analysis easier, the amplifier gains for all detectors are matched so that their peak positions in different spectra overlap. A typical calibration is 0.5 keV/channel gain with zero offset to be set in the ADC's. Examples of such a calibration can be seen in the Eu and Ta spectra of Fig. 3.15. While gain matching of the electronics can provide a good first approximation of the calibration, it is often necessary to fine tune the gain offline. This is done by fitting a gaussian function to the observed peaks to obtain the centroids. These centroids are compared to the true energies by performing a least-squares fit to a polynomial expression, $E_{\text{true}} = f(\text{channel})$. Typically, only a linear function is necessary, but sometimes the inclusion of a quadratic term is needed because of non-linearities in the electronics. The evenly spaced and relatively large range of γ -ray energies of ^{152}Eu make it an ideal source for energy calibrations.

Efficiency Calibration

Figure The relative efficiency of a detector at a given γ -ray energy is determined from

$$\epsilon(E_{\gamma}) = A_{\text{peak}}/I_{\text{true}}, \quad (3.86)$$

where A_{peak} is the area under a gaussian fit of a given peak in a source spectrum, and I_{true} is the true relative intensity of the source peak. A coaxial Ge detector has a maximum efficiency typically about 200 keV. At about this energy, the efficiency

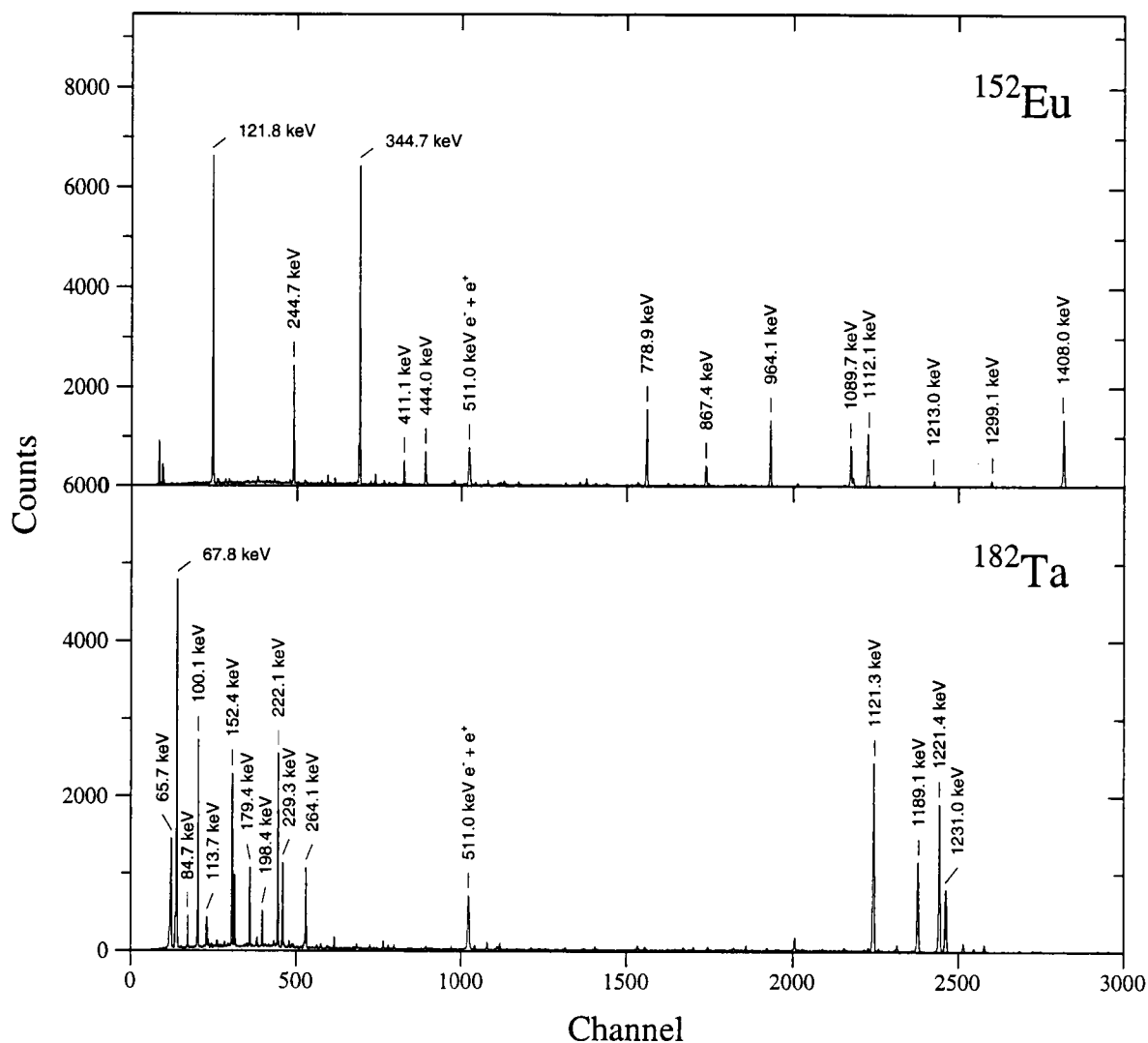


Figure 3.15: Spectra of standard sources ^{152}Eu and ^{182}Ta . Data are taken with an AYE-ball detector. The horizontal axis represents the ADC channel, and peaks are labelled by their γ -ray energy.

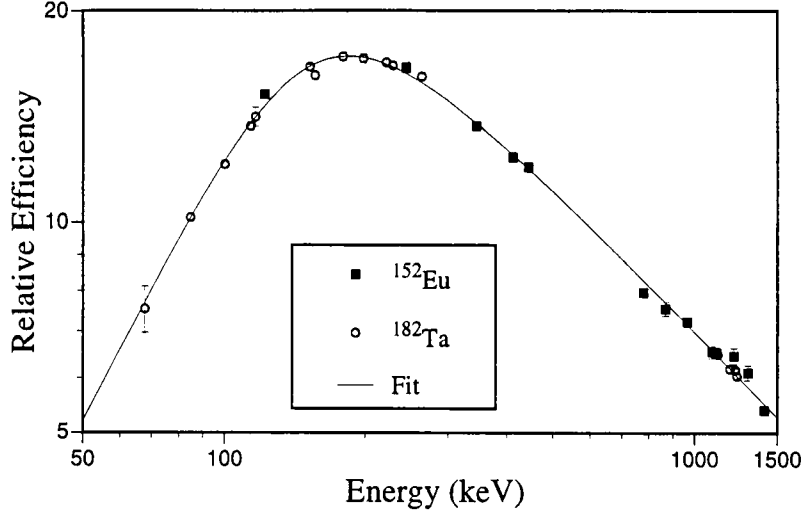


Figure 3.16: Relative efficiency for the sum of the coaxial detectors in AYE-ball. The calculated relative efficiency for the two sources, ^{152}Eu and ^{182}Ta , are normalized, and the solid line represents a least-squares fit to the experimental points using Eq. 3.88.

function turns over and drops rather quickly, while the high energy side has a comparatively shallow drop. Both the low-energy and high-energy efficiency curves can be fit with a function of the type:

$$\epsilon = \exp \left\{ A + B' \ln(E_\gamma) + C' \ln^2(E_\gamma) \right\}, \quad (3.87)$$

where A , B' , and C' are fitting parameters. Because of the different slopes of the low energy and high energy parts of the efficiency curve, however, different fitting parameters are needed for both sections. An additional parameter is then needed to connect the two functions. The final function used to fit the efficiency is

$$\epsilon = \exp \left\{ \left[\left(A + B \ln(E_\gamma/100) + C \ln^2(E_\gamma/100) \right)^{-G} + \left(D + E \ln(E_\gamma/1000) + F \ln^2(E_\gamma/1000) \right)^{-G} \right]^{-1/G} \right\}, \quad (3.88)$$

where A , B , and C are the parameters to fit the low-energy part, D , E , and F fit the high-energy part, G is the interaction parameter between the two functions. To properly normalize the two functions, E_γ is divided by 100 in the low-energy part, and 1000 in the high-energy part [60]. Fig. 3.16 illustrates the fit of Eq. 3.88 to ^{152}Eu and ^{182}Ta sources for all the coaxial detectors in the AYE-ball array. The parameters used

to produce the fit in Fig. 3.16 are $A = 2.5634$, $B = 1.3186$, $C = 0.0000$, $D = 1.9450$, $E = -0.6886$, $F = -0.0264$, and $G = 6.4500$.

3.5.3 Background Subtraction in Ge Spectra

While γ - γ coincidence techniques increases the “resolving power” of a Ge array by reducing the background associated with random decays, there is still a considerable background. There are three occurrences that contribute to this background:

1. random coincidences,
2. true coincidences with unresolved continuum γ rays, and
3. true coincidences with Compton-scattered γ rays.

Two techniques used to reduce the background are described below.

Energy-Time Gates

Random coincidences can be reduced by the choice of the width of the coincidence window in the electronics (See Sec. 3.2.5). There are, however, practical limits to the amount that a coincidence window can be decreased: If the window is chosen too narrow, there can be significant loss of true coincidences. One technique used to reduce random coincidences is to set an energy dependent time gate. For this purpose, an energy-time matrix was scanned offline. Fig. 3.17 displays such a matrix where the energy of detector 6 (a coaxial design) in AYE-ball is incremented on the horizontal axis and the corresponding time signal on the vertical axis. The time signal is centered about y-channel 325 for most energies, but there is a tailing at low energies. The spread around y-channel 325 is related to the time resolution of the detector. At the low end of the energy scale, detectors lose in time resolution because of the slow rise time in the signal pulse compared to the amount of charge collected. Random events will be approximately evenly spread throughout the energy-time plane. A free-form gate can be drawn around the prompt time peak, as illustrated by the solid line in Fig. 3.17, and used as a sorting requirement. When data are sorted on an

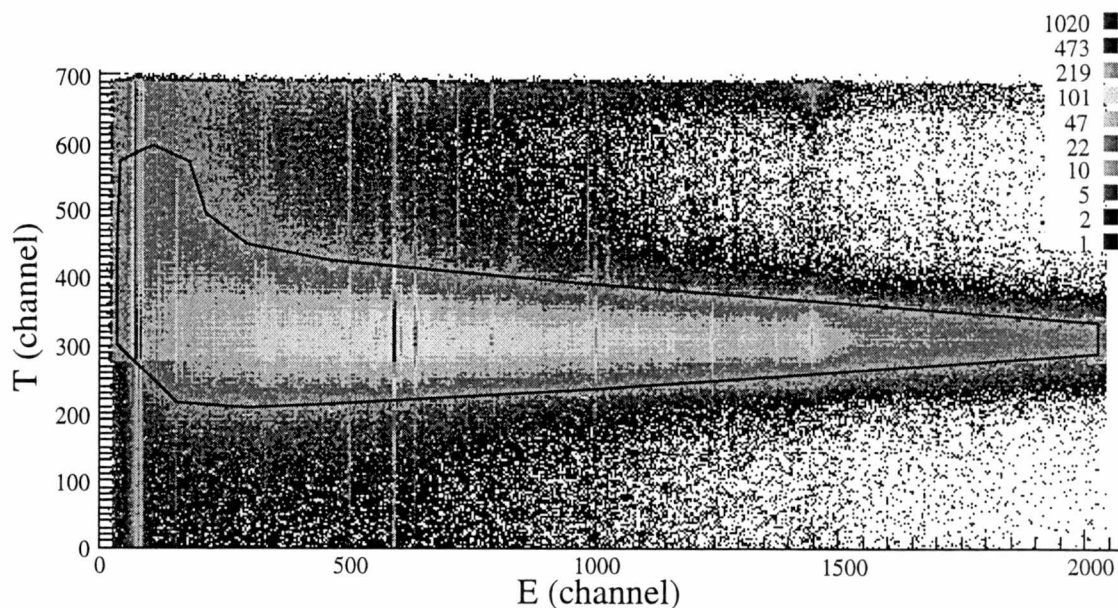


Figure 3.17: Energy vs. Time matrix for detector 6 in AYE-ball.

event-by-event basis, the energy and time for a given detector is checked to be within the free-form gate. If this is true, the energy is accepted, otherwise the event is rejected. This gating technique eliminates much of the random coincidences while still accepting the true prompt coincidences.

Palameta-Waddington background subtraction

The energy-time gate described above will reduce a large amount of the background of type (1). However, it also does nothing to effect the backgrounds of type (2) and (3). Palameta and Waddington [61] have introduced a background-subtraction technique to remove the contributions from all three factors to a Ge-Ge coincidence matrix so that essentially only photopeak-photopeak coincidences remain. This technique is supported by the analysis programs used and involves the following steps. First, a background spectrum is defined from smooth fit, $f(i)$, of the total projection. This is illustrated in Fig. 3.18. The solid line under the spectrum in Fig. 3.18 is an example of a smooth background fit. There are two “kinks” in this otherwise smooth function, occurring at ≈ 600 keV and ≈ 830 keV. These bumps correspond to the

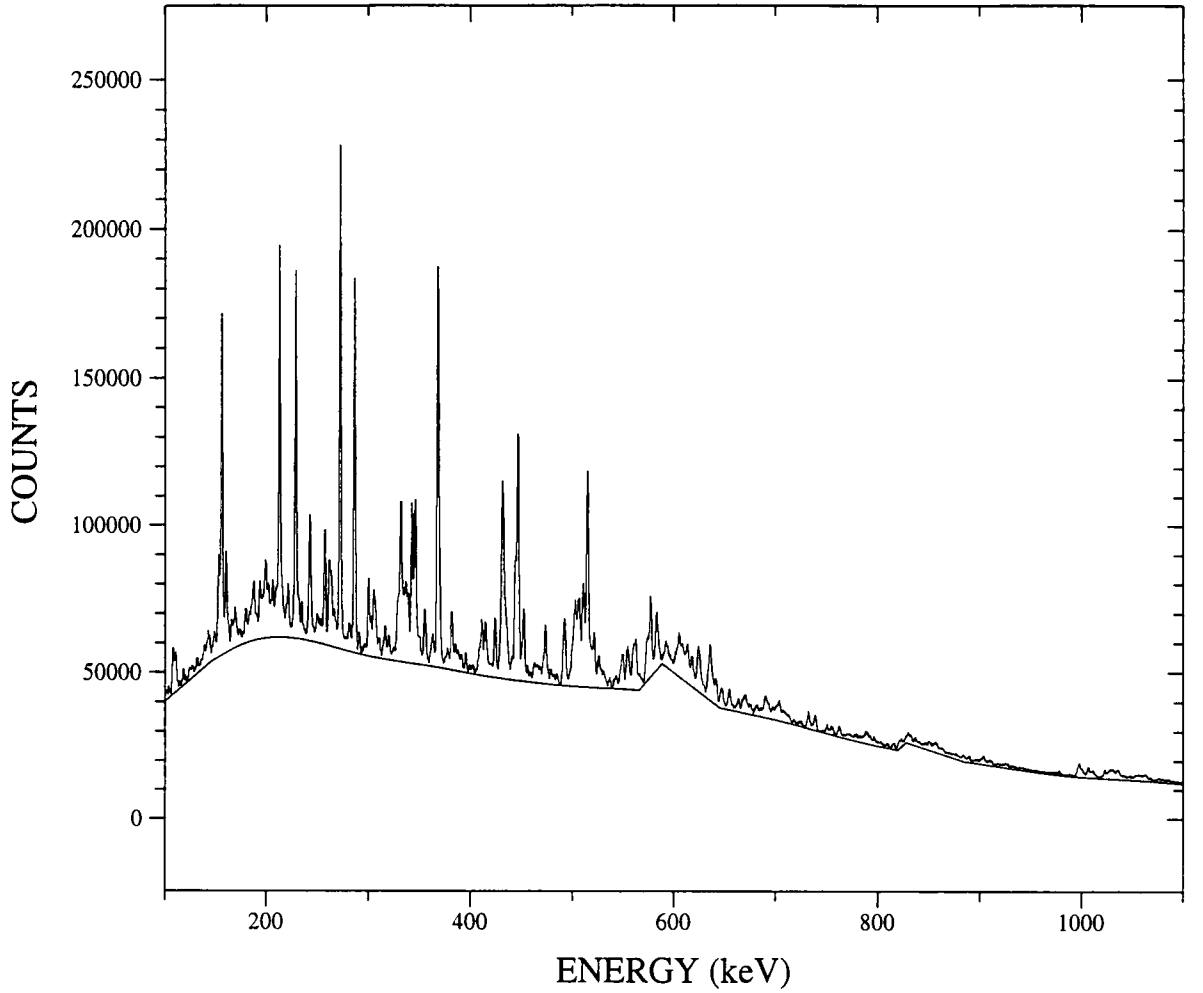


Figure 3.18: Total projection and background fit of γ - γ matrix from the ^{181}Au experiment. The background fit is the smooth solid line beneath the γ -ray peaks.

$2^+ \rightarrow 0^+$ transition in ^{74}Ge and ^{72}Ge respectively. The 2^+ states are excited by inelastic scattering of neutrons in Ge detector. Unlike the γ rays from the residual nuclei, the γ rays from the Ge's are not doppler shifted. Thus the Ge γ rays are spread out in energy when a doppler correction is applied.

The background spectrum is created by

$$b(i) = \sum_{j \in K} C(i, j), \quad (3.89)$$

where $C(i, j)$ represents the coincidence matrix, and $j \in K$ signifies that the summation is taken over the background channels, K . A background channel is defined as a particular channel, i , where $|P(i) - f(i)| < \epsilon \approx 0$. $P(i)$ is the total projection

spectrum, $P(i) \equiv \sum_{j=1}^N C(i, j)$, and $f(i)$ is the background function. A background subtracted matrix can be created in the following manner:

$$C'(i, j) = C(i, j) - f(i)b(j) / \left(\sum_j b(j) \right). \quad (3.90)$$

This subtraction, however, does not remove Compton background from beneath the peaks. Palameta and Waddington noted that the Compton background can also be approximated by the background function, $f(i)$, and pointed out that a complete background subtraction could be obtained by applying the following:

$$C''(i, j) = C'(i, j) - f(j) \left(\sum_{j \in K} C'(i, j) \right) / \left(\sum_j b(j) \right). \quad (3.91)$$

The resulting matrix, $C''(i, j)$, will, on average, be zero for background channels, and contain only photopeak-photopeak coincidences. See Chapters 4 and 5 for examples of projected spectra where the background has been subtracted by this method.

3.5.4 Radware

Once matrices are created, γ - γ coincidence relationships are studied by examining slices of these matrices. Slices, or gates, are created by summing the contents of a set of channels that represents a peak. The resulting one-dimensional histogram is then further analyzed to determine what events occurred in coincidence with the gated γ ray. Radford developed a package of programs, collectively known as Radware, that automates this gating process, and adds several powerful features that enhances γ -ray analysis. Descriptions of Radware programs can be found in Ref. [60]. The two primary programs of the Radware package are “escl8r” and “levit8r.” These programs perform the gating procedure and display the resulting spectrum on a computer terminal or write the spectrum to a data file. “escl8r” handles two-dimensional histograms, and “levit8r” is the version that is used for the analysis of cubes (three-dimensional histograms).

One of the powerful features of “escl8r” and “levit8r” is the integration of peak fitting routines. Based on coincidences, one constructs a level scheme of

correlated γ decays. This level scheme can be passed to "escl8r," where automated routines will use the decay paths and level branchings to fit peak centroids and areas for all the γ rays present in the level scheme. While it is always a concern whether such an automated global fit can be accurate, this is a powerful tool for fitting many γ rays when the results are critically analyzed.

Another powerful feature of these programs comes with a successful fit. Once energies and intensities of peaks in a level scheme have been established, the programs will calculate spectra based on these fits and the proposed γ -ray multipolarities of the transitions. Branching ratios are calculated taking into account the electron conversion coefficients. Fig. 3.19(a) shows a gate on the 273 keV γ ray from ^{181}Au with the calculated spectrum from the "escl8r" program overlaid. The quality of the fit can be easily checked by comparing the difference between the observed and calculated spectra. This difference is shown in Fig. 3.19. By being able to subtract out the known coincidences, one can more effectively concentrate on the features of the data set that are not known.

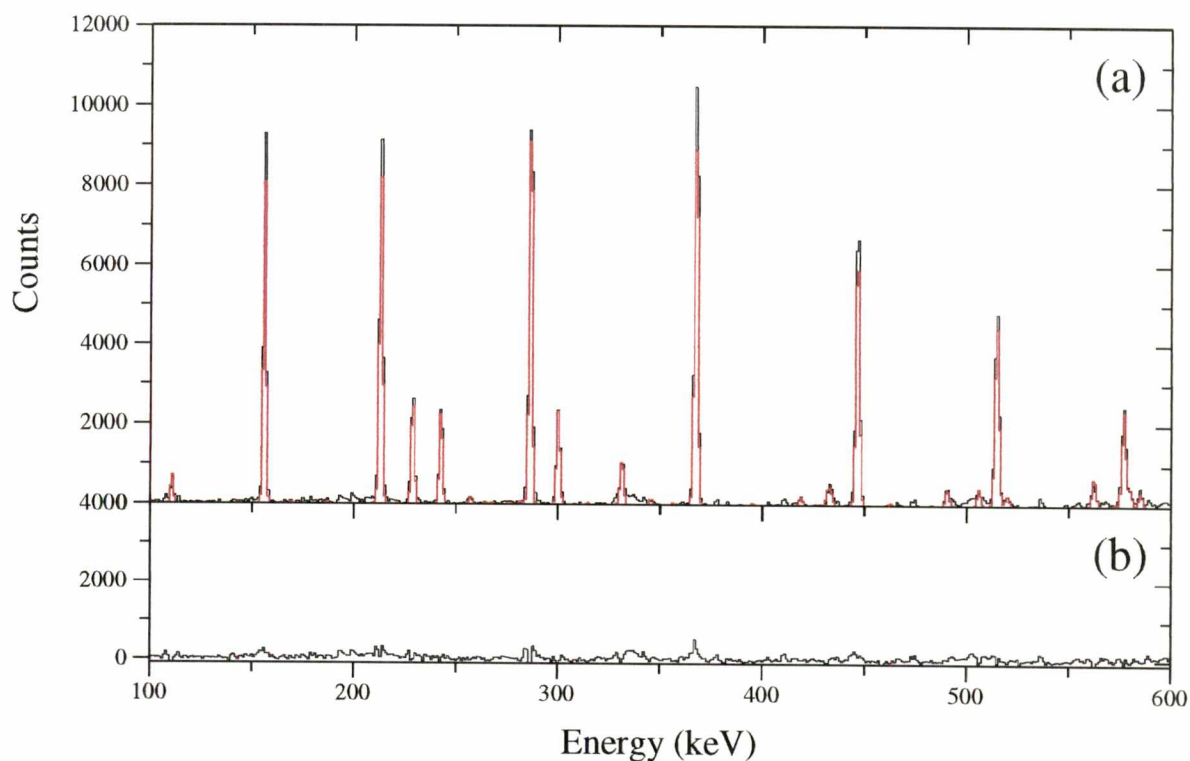


Figure 3.19: Comparison of observed and calculated spectra. This is a gate on the 273 keV γ ray from ^{181}Au . Panel (a) shows the observed spectrum with the calculated spectrum overlaid in red. Panel (b) shows the difference between the observed and calculated spectra.

CHAPTER 4

Experiment to Study ^{181}Au

4.1 Experimental Setup

The object of this experiment was to study the high-spin states from their discrete γ decay. The experiment was performed at the Holifield Heavy Ion Research Facility at Oak Ridge National Laboratory with the Spin Spectrometer array loaded with 19 Compton-suppressed $\sim 25\%$ -relative-efficiency Ge detectors and 52 NaI detectors. A description of this device is given in Sec. 3.3.1, and Table 4.1 lists the specific positions and angles of the Ge detectors within the Spin Spectrometer array. Excited states in ^{181}Au were produced in the heavy-ion fusion reaction $^{150}\text{Sm}(^{35}\text{Cl}, 4n)$ at a beam energy of 168 MeV. The target consisted of two self-supporting 0.5-mg/cm^2 -thick foils to allow the recoiling nuclei to decay in flight. The ^{150}Sm target material was enriched to 95%. In this reaction, ^{181}Au is the primary residue, with the largest competition coming from the p3n evaporation channel (^{181}Pt) and fission. Ge-Ge coincidences were collected under the condition that at least six NaI elements fired in prompt coincidence.

4.2 Analysis Basis

There were ~ 95 million triggered events recorded on tape and utilized in the subsequent analysis. Identification of ^{181}Au from other reaction byproducts was achieved by examination of different cuts on NaI fold (K) and total energy (H) in coincidence with Ge counters. Additional discrimination came from observing the transitions in coincidence with Au K_{α} x rays, which could be distinguished from the $\sim 2\text{-keV}$ [59] lower Pt X rays.

Table 4.1: Location of Ge detectors within the Spin Spectrometer array.

Position	$\theta(^{\circ})$	$\phi(^{\circ})$
A0	0.00	0.00
A2	24.70	85.11
A4	24.70	-130.89
A5	24.70	-58.89
D3	41.36	50.24
E3	41.36	-21.76
C0	63.43	108.00
D0	63.43	36.00
E0	63.43	-36.00
F0	63.43	-108.00
I0	116.57	72.00
J0	116.57	144.00
K0	116.57	-144.00
L0	116.57	-72.00
I3	138.64	57.76
K3	138.64	-158.24
G2	155.30	94.89
G4	155.30	-49.11
G5	155.30	-121.11

The data were presorted to gain match the Ge signals as well as correct for the Doppler shift caused by the decay of the in-flight recoils. Initial analysis of the presorted data was to determine the optimum channels in total energy (H) and fold (K) of the NaI detectors for ^{181}Au . The data were sorted into matrices, K vs. H , H vs. Ge energy, and K vs. Ge energy. Fig. 4.1 shows the spectra of the NaI total energy and fold from this experiment. Although a hardware threshold of $N \geq 6$ NaI elements was required, there was still some "leak through" of 4- and 5-fold events presumably due to random coincidences with the very intense γ -ray lines from Coulomb excitation. Certain reaction channels are enhanced at given gates on H and K , as illustrated in Fig. 4.2. This figure shows the γ -ray spectra resulting from gates on the H -Ge matrix in panels labeled (1) and from gates on the K -Ge matrix in panels labeled (2). As expected, γ -ray lines from ^{181}Au dominate all the gates. Contributions from other reaction products can be reduced (or enhanced) very selectively by H and K software gates. Nice examples are the γ -ray yields of the ^{180}Pt and ^{182}Pt nuclei, populated by the p4n and p2n reaction channels respectively. Gates on the lower portions of the H and K spectra enhance ^{180}Pt (Figs. 4.2(a) and 4.2(b)). By contrast, gates on the higher channels of H and K reduce ^{180}Pt and enhance ^{182}Pt , since the latter is populated at higher excitation energy and spin. It should be mentioned that the multiplet of γ -ray lines around 330 keV result from the 334-keV ground-state transition in Coulomb-excited ^{150}Sm target nuclei. The peak is spread out over several energies when the Ge γ -ray energies are Doppler corrected, because the recoil velocity of ^{150}Sm is significantly different from that of the ^{185}Au compound nuclei.

From the analysis of the γ -ray intensities in the H and K gates, it was decided that a gate of $K \geq 10$ would be used. This corresponds to the sum of panels (2b), (2c), and (2d) in Fig. 4.2. With this gate, most of the intensity of second largest byproduct, ^{180}Pt , is removed, and the ^{150}Sm Coulomb excitation lines are reduced significantly, while still keeping a significant proportion of the ^{181}Au population. With this choice the Ge energy data were sorted into a K -gated ($10 \leq K \leq 30$) total symmetrized γ - γ matrix. An angle-sorted γ - γ matrix was also created with a gating requirement of $8 \leq K \leq 30$. For this matrix the six detectors at angles 24.70°

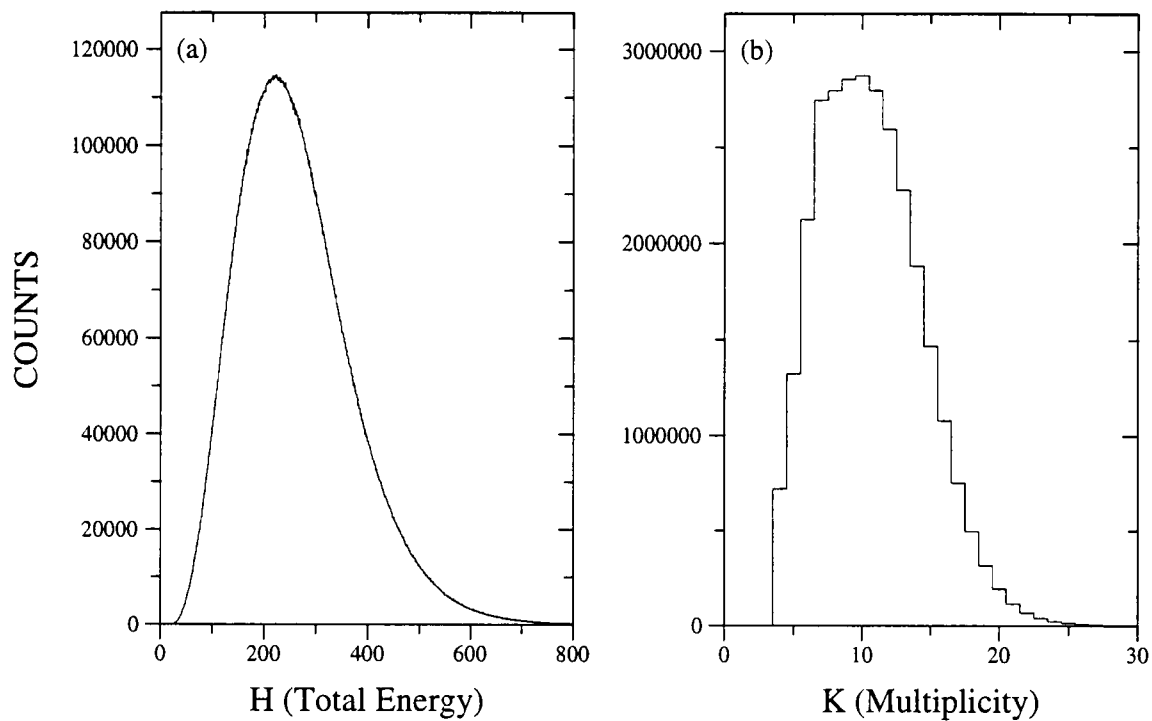


Figure 4.1: Total projections of H - K (total energy-fold) matrix sorted for the NaI detectors of the Spin Spectrometer. Panel (a) shows the projection on the H axis, and panel (b) displays the projection on the K axis.

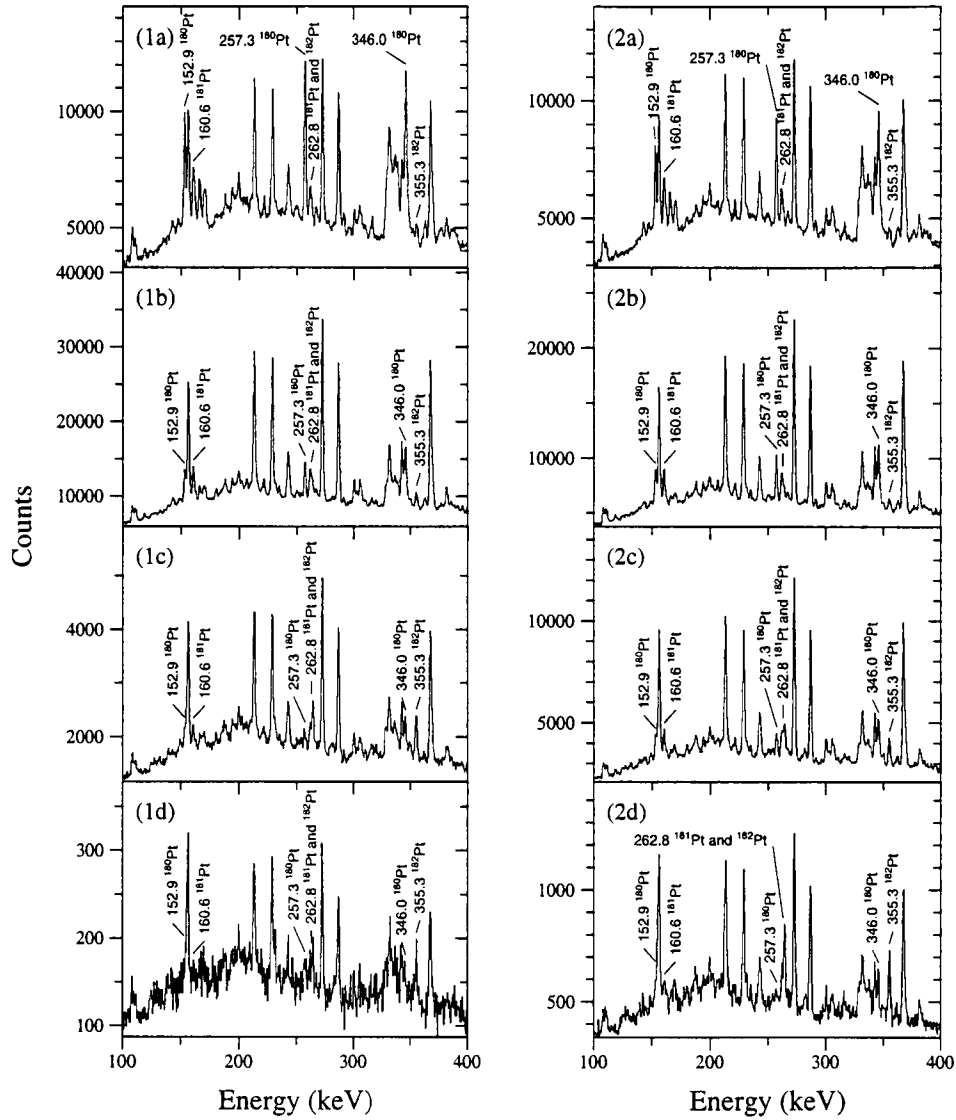


Figure 4.2: The γ -ray spectra resulting from gates on the total energy (H) and fold (K) of the NaI detectors in the Spin Spectrometer. The figures labeled “1” result from gates on the total energy, and those labeled “2” are from gates on fold. The channels used in the H gates for figures (1a) through (1d) are 50-200, 201-400, 401-600, and 601-1000, respectively. Figures (2a) through (2d) are from K gates of 7-9, 10-13, 14-18, and 19-30 detectors. The most intense clean peaks from decay of ^{180}Pt , ^{181}Pt , and ^{182}Pt are noted in each figure. The other strong peaks are associated with the decay of ^{181}Au .

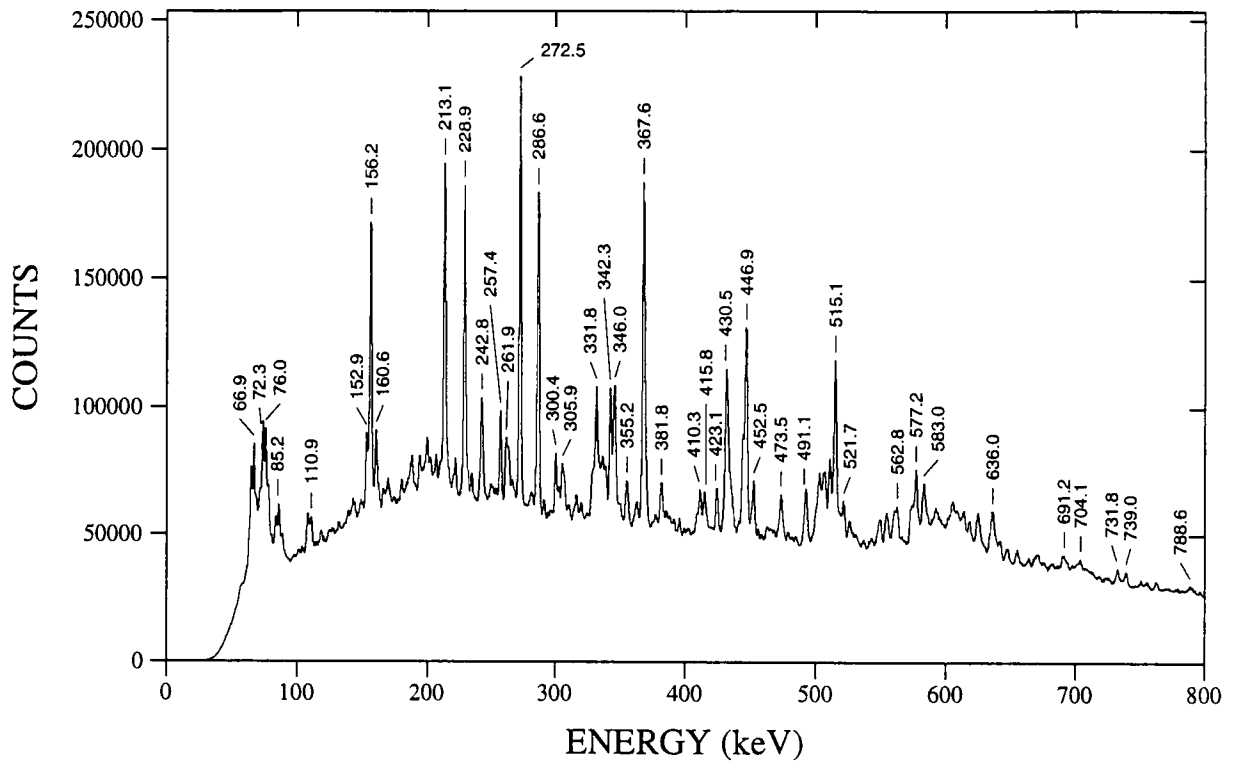


Figure 4.3: Total projection of the symmetrized γ - γ matrix from the ^{181}Au experiment. The matrix was created with the gating condition $10 \leq K \leq 30$.

and 155.30° (referred to as the 24° axis) were sorted against the eight detectors at 63.43° and 116.57° (63° axis). The lower K requirement for the angle-sorted matrix was chosen to increase the intensity of ^{181}Au lines as a compensation for the fewer number of detectors. The total projection of the total γ - γ matrix is shown in Fig. 4.3.

4.3 Results

Analysis of the γ - γ -coincidence matrices (total and angle-sorted) revealed nine bands or band fragments of stretched E2 decays. The resulting level scheme for ^{181}Au is shown in Fig. 4.4. This level scheme represents the first establishment of excited states observed in ^{181}Au . Decays of γ rays possibly associated with ^{181}Au were reported by Sauvage *et al.* [63], however, a level scheme was not established and none of the γ rays from that report correspond with the transitions presented here. Specific γ -ray energies and other spectroscopic information are listed in Tables 4.2,

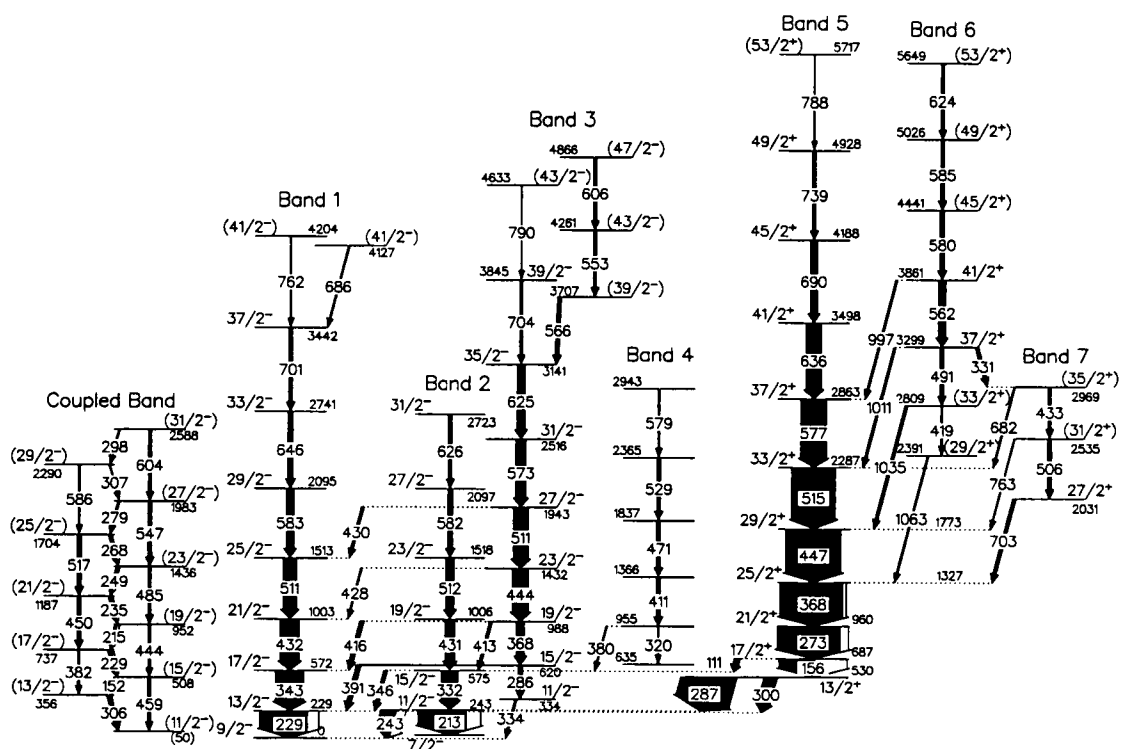


Figure 4.4: Level scheme for ^{181}Au . The level energies in Bands 1 through 7 are relative to the $9/2^-$ level in Band 1. The energy of the $11/2^-$ level in the coupled band is arbitrarily set to 50 keV, and the energies of the other levels in this band are relative to the energy of $11/2^-$. The widths of the arrows represent the relative intensity of the transitions, and the black and white shading correspond to the γ -ray (measured) and internal-conversion (calculated) intensity respectively. Assignments of spin and parity of levels are discussed in the text.

Table 4.2: γ -ray energies, intensities, and DCO ratios for Bands 1 and 2 in ^{181}Au . Uncertainty in the γ -ray energies is 0.2 keV when the decimal point is present, and 0.5 keV when the decimal point is not present. Intensities of γ -ray transitions are relative to the 272.5-keV transition in Band 4. The values in column “Intensity (converted)” are the total decay intensity for the given transition, accounting for internal conversion by using the coefficients from Rösler *et al.* [62].

E_γ (keV)	Band	I_γ relative	Intensity (converted)	R_{DCO}
228.9	1	133.1 $\pm$ 1.1	165.2 $\pm$ 1.4	0.90 $\pm$ 0.04
342.6	1	74.7 $\pm$ 2.5	79.9 $\pm$ 2.7	
431.5	1	51.7 $\pm$ 1.8	53.7 $\pm$ 1.9	1.19 $\pm$ 0.08
510.5	1	36.9 $\pm$ 1.6	37.8 $\pm$ 1.6	
582.6	1	21.1 $\pm$ 0.9	21.5 $\pm$ 0.9	1.21 $\pm$ 0.28
646.4	1	14.0 $\pm$ 0.6	14.2 $\pm$ 0.6	1.14 $\pm$ 0.4
700.6	1	10.0 $\pm$ 0.5	10.1 $\pm$ 0.5	1.75 $\pm$ 1.24
762	1	3.4 $\pm$ 0.3	3.5 $\pm$ 0.3	
686	1	4.6 $\pm$ 0.4	4.6 $\pm$ 0.4	
213.2	2	138.6 $\pm$ 4.5	181.0 $\pm$ 5.9	0.90 $\pm$ 0.1
331.9	2	45.2 $\pm$ 1.8	48.7 $\pm$ 1.9	0.92 $\pm$ 0.05
431.4	2	28.6 $\pm$ 1.4	29.6 $\pm$ 1.4	1.07 $\pm$ 0.16
512.3	2	24.3 $\pm$ 1.1	24.9 $\pm$ 1.1	0.83 $\pm$ 0.32
582.2	2	15.4 $\pm$ 0.9	15.7 $\pm$ 0.9	1.81 $\pm$ 0.4
626.4	2	10.1 $\pm$ 0.6	10.2 $\pm$ 0.6	1.03 $\pm$ 0.54
242.6	2 $\rightarrow$ 1	43.8 $\pm$ 0.0	70.7 $\pm$ 0.0	0.78 $\pm$ 0.04
345.6	2 $\rightarrow$ 1	8.8 $\pm$ 0.6	10.8 $\pm$ 0.7	

Table 4.3: γ -ray energies, intensities, and DCO ratios for Bands 3 and 4 in ^{181}Au .
See Table 4.2 for definitions of the column entries.

E_γ (keV)	Band	I_γ relative	Intensity (converted)	R_{DCO}
286.5	3	11.1 ± 0.7	12.4 ± 0.7	
367.7	3	21.0 ± 1.0	22.2 ± 1.1	
444.0	3	44.4 ± 1.6	46.0 ± 1.7	1.26 ± 0.4
510.7	3	41.8 ± 1.7	42.8 ± 1.7	
553.2	3	8.7 ± 0.5	8.8 ± 0.5	
566.1	3	11.0 ± 0.5	11.2 ± 0.5	
573.1	3	29.7 ± 1.1	30.3 ± 1.1	1.69 ± 0.4
605.7	3	7.9 ± 0.5	8.1 ± 0.5	
624.9	3	21.9 ± 0.9	22.2 ± 0.9	0.64 ± 0.8
703.8	3	7.6 ± 0.5	7.7 ± 0.5	
790	3	1.5 ± 0.3	1.6 ± 0.3	
334.0	3 $\rightarrow$ 1	$5.0 \pm$	6.3 ± 0.0	
391.2	3 $\rightarrow$ 1	12.0 ± 0.7	14.0 ± 0.8	0.37 ± 0.1
416.4	3 $\rightarrow$ 1	11.5 ± 0.5	13.1 ± 0.6	0.34 ± 0.4
428	3 $\rightarrow$ 1	4.5 ± 0.6	5.1 ± 0.7	
430	3 $\rightarrow$ 1	7.4 ± 0.7	8.3 ± 0.8	
413.4	3 $\rightarrow$ 2	8.2 ± 0.5	8.5 ± 0.5	
319.9	4	3.1 ± 0.6	3.3 ± 0.6	
411.4	4	11.5 ± 0.8	12.0 ± 0.8	
471.3	4	10.8 ± 0.6	11.1 ± 0.6	
528.7	4	10.2 ± 0.5	10.4 ± 0.6	
578.9	4	5.4 ± 0.5	5.4 ± 0.5	
380.4	4 $\rightarrow$ 2	3.4 ± 0.6	4.0 ± 0.7	

Table 4.4: γ -ray energies, intensities, and DCO ratios for Bands 5 and 6 in ^{181}Au .
See Table 4.2 for definitions of the column entries.

E_γ (keV)	Band	I_γ relative	Intensity (converted)	R_{DCO}
156.3	5	101.1 ± 3.2	191.9 ± 6.0	0.89 ± 0.02
272.5	5	177.4 ± 5.4	201.9 ± 6.2	1.04 ± 0.02
367.5	5	178.0 ± 5.4	188.3 ± 5.7	1.04 ± 0.02
446.7	5	151.9 ± 4.6	157.1 ± 4.8	1.11 ± 0.03
514.9	5	120.3 ± 3.7	123.2 ± 3.8	0.95 ± 0.03
577.0	5	71.1 ± 2.2	72.4 ± 2.3	0.90 ± 0.06
635.8	5	43.4 ± 1.4	44.1 ± 1.4	0.95 ± 0.09
690.4	5	20.7 ± 0.8	21.0 ± 0.8	1.10 ± 0.15
739.2	5	9.7 ± 0.5	9.8 ± 0.5	1.32 ± 0.5
788	5	1.9 ± 0.3	1.9 ± 0.3	
286.7	5 $\rightarrow$ 1	149.4 ± 4.7	153.9 ± 4.9	0.76 ± 0.12
111.1	5 $\rightarrow$ 2	13.0 ± 0.5	17.1 ± 0.6	0.86 ± 0.02
300.4	5 $\rightarrow$ 2	35.7 ± 1.2	36.6 ± 1.2	1.02 ± 0.08
419	6	4.4 ± 0.3	4.5 ± 0.3	
490.7	6	9.9 ± 0.5	10.2 ± 0.5	
562.5	6	21.5 ± 0.8	21.9 ± 0.8	1.20 ± 0.2
579.9	6	13.0 ± 0.6	13.3 ± 0.6	
585.2	6	10.5 ± 0.5	10.6 ± 0.5	
624	6	8.7 ± 0.5	8.8 ± 0.5	
996.8	6 $\rightarrow$ 5	5.7 ± 0.4	5.7 ± 0.4	2.61 ± 1.5
1010.7	6 $\rightarrow$ 5	6.8 ± 0.4	6.8 ± 0.4	1.14 ± 0.6
1035.2	6 $\rightarrow$ 5	8.6 ± 0.5	8.7 ± 0.5	
1063	6 $\rightarrow$ 5	4.9 ± 0.4	4.9 ± 0.4	
331.1	6 $\rightarrow$ 7	8.2 ± 0.4	10.3 ± 0.6	

Table 4.5: γ -ray energies, intensities, and DCO ratios for Band 7 and the coupled band (C.B.) in ^{181}Au . See Table 4.2 for definitions of the column entries.

E_γ (keV)	Band	I_γ relative	Intensity (converted)	R_{DCO}
432.6	7	8.6 ± 0.6	8.9 ± 0.6	0.47 ± 0.3
506.2	7	11.1 ± 0.6	11.4 ± 0.6	
682	7 $\rightarrow$ 5	4.2 ± 0.4	4.4 ± 0.4	
703.5	7 $\rightarrow$ 5	9.5 ± 0.6	9.8 ± 0.6	
763	7 $\rightarrow$ 5	3.4 ± 0.4	3.5 ± 0.4	
153.1	C.B.	4.2 ± 0.5	9.7 ± 1.1	
215.5	C.B.	8.3 ± 0.5	15.5 ± 0.9	
228.7	C.B.	9.9 ± 0.7	17.1 ± 1.3	
232.1	C.B.	3.8 ± 0.3	$\pm$	
234.8	C.B.	7.6 ± 0.4	12.8 ± 0.7	
250.2	C.B.	6.8 ± 0.4	10.7 ± 0.6	
267.1	C.B.	7.1 ± 0.4	10.4 ± 0.6	
280.5	C.B.	4.5 ± 0.3	6.3 ± 0.5	
297	C.B.	3.4 ± 0.3	4.6 ± 0.4	
305.4	C.B.	$6.0 \pm$	8.0 ± 0.0	
308	C.B.	2.8 ± 0.3	3.6 ± 0.4	
381.6	C.B.	5.9 ± 0.6	6.2 ± 0.6	
444	C.B.	6.5 ± 0.7	6.7 ± 0.7	
450.3	C.B.	10.2 ± 0.7	10.5 ± 0.7	
458.6	C.B.	7.9 ± 1.1	8.1 ± 1.1	
484.7	C.B.	8.6 ± 0.6	8.9 ± 0.6	
516.7	C.B.	11.6 ± 0.7	11.8 ± 0.7	
547.4	C.B.	9.6 ± 0.6	9.8 ± 0.6	
586	C.B.	4.6 ± 0.6	4.6 ± 0.6	
604	C.B.	8.7 ± 0.7	8.8 ± 0.7	

4.3, 4.4, and 4.5. The spins of the levels in ^{181}Au are based on the assumption that the lowest level observed in Band 1 has a spin and parity of $I^\pi = \frac{9}{2}^-$.

This assignment for the spin of the lowest level is based on the systematics between levels observed in ^{181}Au , ^{183}Au , and ^{185}Au . Fig. 4.5 shows partial level schemes for ^{185}Au [64] and ^{183}Au [65], as well as the levels in ^{181}Au and ^{179}Au established in the work for this dissertation. The illustrated energy states are those observed in compound nuclear reactions. Spins and parities for the levels indicated by asterisks in ^{185}Au were established in a β^+/EC experiment reported by Bourgeois *et al.* [66]. A spin and parity of $I^\pi = \frac{9}{2}^-$ was established for the asterisked level in ^{183}Au in a similar experiment by Macias-Marques *et al.* [67]. As is indicated above, the lowest states observed in the heavy-ion fusion experiments of ^{185}Au and ^{183}Au was the $\frac{9}{2}^-$ level in the $h_{9/2}$ band. As seen in Fig. 4.5, the $\frac{9}{2}^-$ levels are only a few keV above the ground state. For the high-spin experiments of ^{185}Au and ^{183}Au as well as this project, the low-energy efficiencies of the detector arrays were not sufficient to observe γ -ray transitions below 20 keV. The comparison of the observed energy levels in ^{181}Au with those in similar experiments for ^{183}Au and ^{185}Au lead to the conclusion that the lowest level in ^{181}Au has $I^\pi = \frac{9}{2}^-$.

Assignment of spins for the excited states are based on DCO ratio measurements of γ decays. Experimental DCO ratios are calculated from Eq. 2.78:

$$R_{DCO} = \frac{I_{24^\circ}^{\gamma_1}(\text{Gate}_{63^\circ}^{\gamma_2})}{I_{63^\circ}^{\gamma_1}(\text{Gate}_{24^\circ}^{\gamma_2})},$$

where $I_{24^\circ}^{\gamma_1}(\text{Gate}_{63^\circ}^{\gamma_2})$ is the intensity of γ_1 in a gated spectrum of γ_2 projected on the axis of 24° detectors, and $I_{63^\circ}^{\gamma_1}(\text{Gate}_{24^\circ}^{\gamma_2})$ is the γ_1 intensity from a γ_2 -gated projection on the 63° axis. For the case where γ_2 is a stretched quadrupole transition, the theoretical DCO ratios are $R_{DCO} = 1.0$ when γ_1 is a stretched E2 and $R_{DCO} = 0.6$ when γ_1 is a pure $\Delta I = 1$ dipole transition. The measured DCO ratios for most γ -ray transitions are displayed in Fig. 4.6. Parity assignments for states are determined based on comparison with analogous states in known nuclei. Justification for specific cases are presented below.

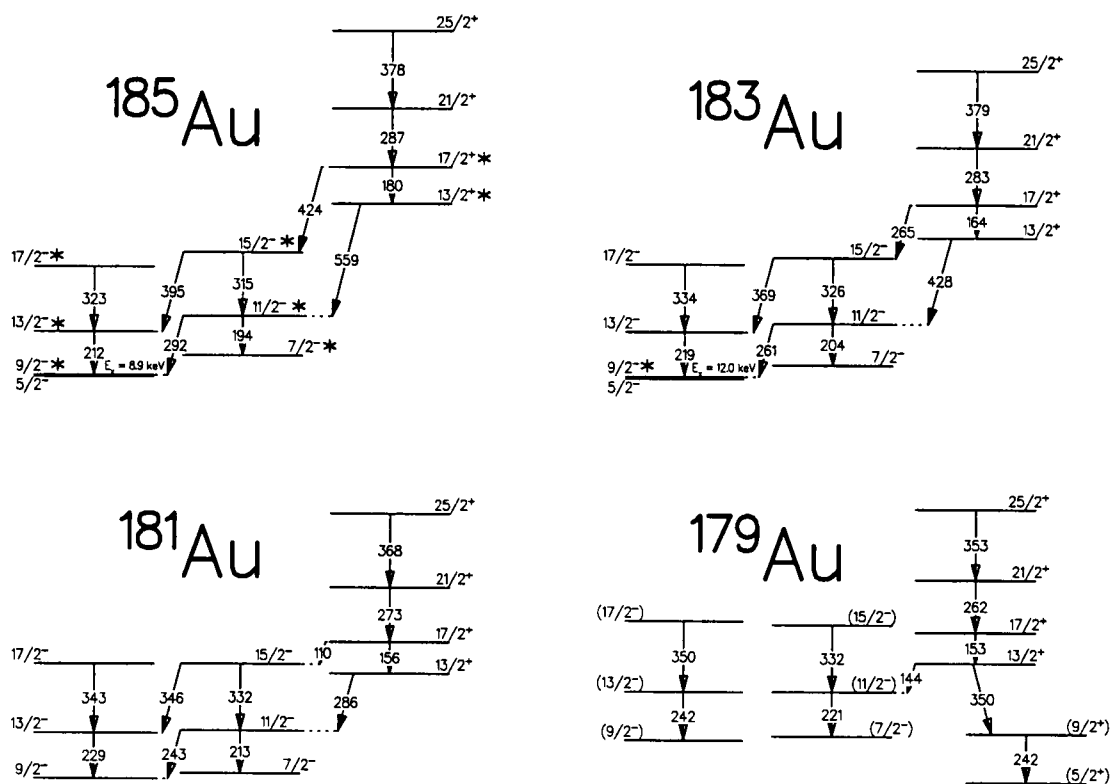


Figure 4.5: Partial level schemes of ^{185}Au , ^{183}Au , ^{181}Au , and ^{179}Au . The excitation energy (E_x) of the $9/2^-$ level above the $5/2^-$ ground-state for ^{185}Au and ^{183}Au are denoted in the figure.

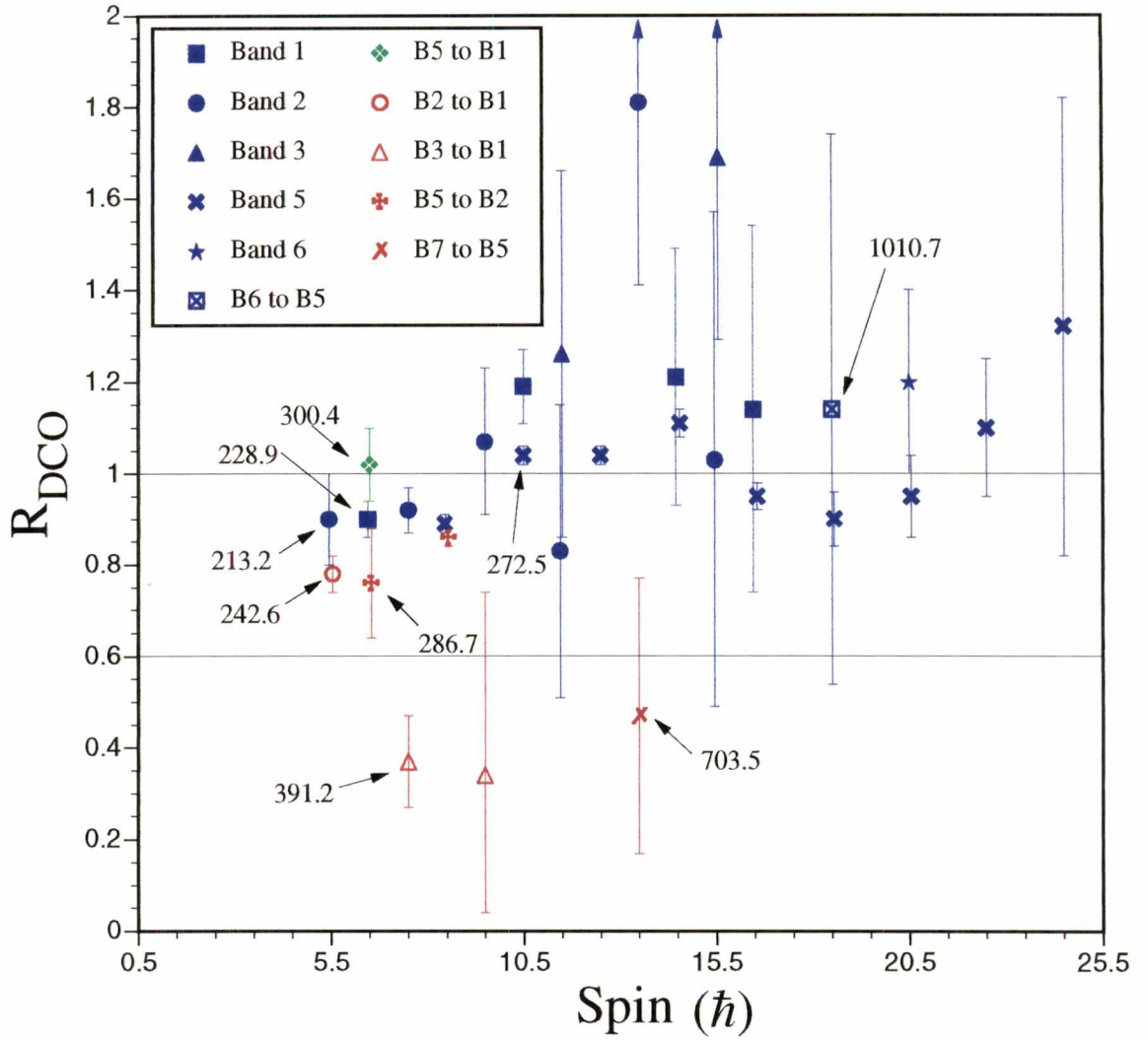


Figure 4.6: Measured DCO ratios for γ rays in ^{181}Au . The initial spin of these transitions (according to labels in Fig. 4.4) are given on the x axis. The lines indicate expected DCO ratios for pure γ multipoles: $R_{DCO} = 1.0$ corresponds to a $\Delta I = 2$ quadrupole transition, $R_{DCO} = 0.6$ to $\Delta I = 1$ dipole. Legend: In-band transitions are specified by the label of the band. Interband transitions are specified in abbreviated form (e.g. Transitions from Band 3 to Band 1 are noted by “B3 to B1”).

4.3.1 Bands 1 through 3 – negative parity structures

A series of γ -ray transitions was observed feeding directly into the $\frac{9}{2}^-$ state. These γ rays are labeled Band 1 in Fig. 4.4. A gate on the 342.6-keV transition in this band is shown in Fig. 4.7(a). Three of the γ rays in this band are energetically identical to those in Band 2, thus making it difficult or impossible to determine accurate DCO ratios for many of the transitions. Those transitions for which ratios were established are shown in Fig. 4.6, and confirm that the γ rays are quadrupole in character. The rotational character of the γ -ray sequence provides confirmation that these transitions are of stretched E2 character. The multipolarity of the 762-keV and 686-keV transitions feeding in at the top of the band could not be established.

Fig. 4.7(b) shows the spectrum from a gate on the 331.9-keV transition in Band 2. This gate indicates not only γ rays in Band 2, but also those in Band 5 that result from the 111.1-keV transition that feeds from Band 5 into Band 2. Decays from Bands 3 and 4 are also observed in this gate because of the 413.4-keV and 380.4-keV transitions that feed the level directly above the gated γ ray. The 228.9-keV γ ray of Band 1 appears in this gate because of the 331.1-keV doublet in Band 7. As mentioned in the previous paragraph three of the transitions in Band 2 are doublets with γ rays in Band 1. Those transitions where DCO measurements are possible are indicated by the filled circles in Fig. 4.6, and illustrate that the γ rays in this band have quadrupole multipolarity. The spins in this band are fixed by the measured multipolarity of the 242.6-keV transition that connects Band 2 to Band 1. The DCO ratio for this transition, indicated by the open circle in Fig. 4.6, is 0.78 ± 0.04 , which is the expected ratio for a $\Delta I = 1$ mixed dipole-quadrupole transition. An assignment of this as a mixed $\frac{11}{2}^- \rightarrow \frac{9}{2}^-$ transition is consistent with this ratio, and agrees well with similar transitions in neighboring nuclei (see Fig. 4.5).

The γ -ray transitions in Band 3 are clearly established from the gate on the 391.2-keV transition, despite the large number of doublets in this band. The 286.5, 367.7, and 444.0-keV are all doublets or near doublets with transitions in Band 5. The 510.7-keV γ ray is nearly identical to ones in both Bands 1 and 2, and the 624.9-keV also has an identical counterpart in Band 2. DCO measurements were only

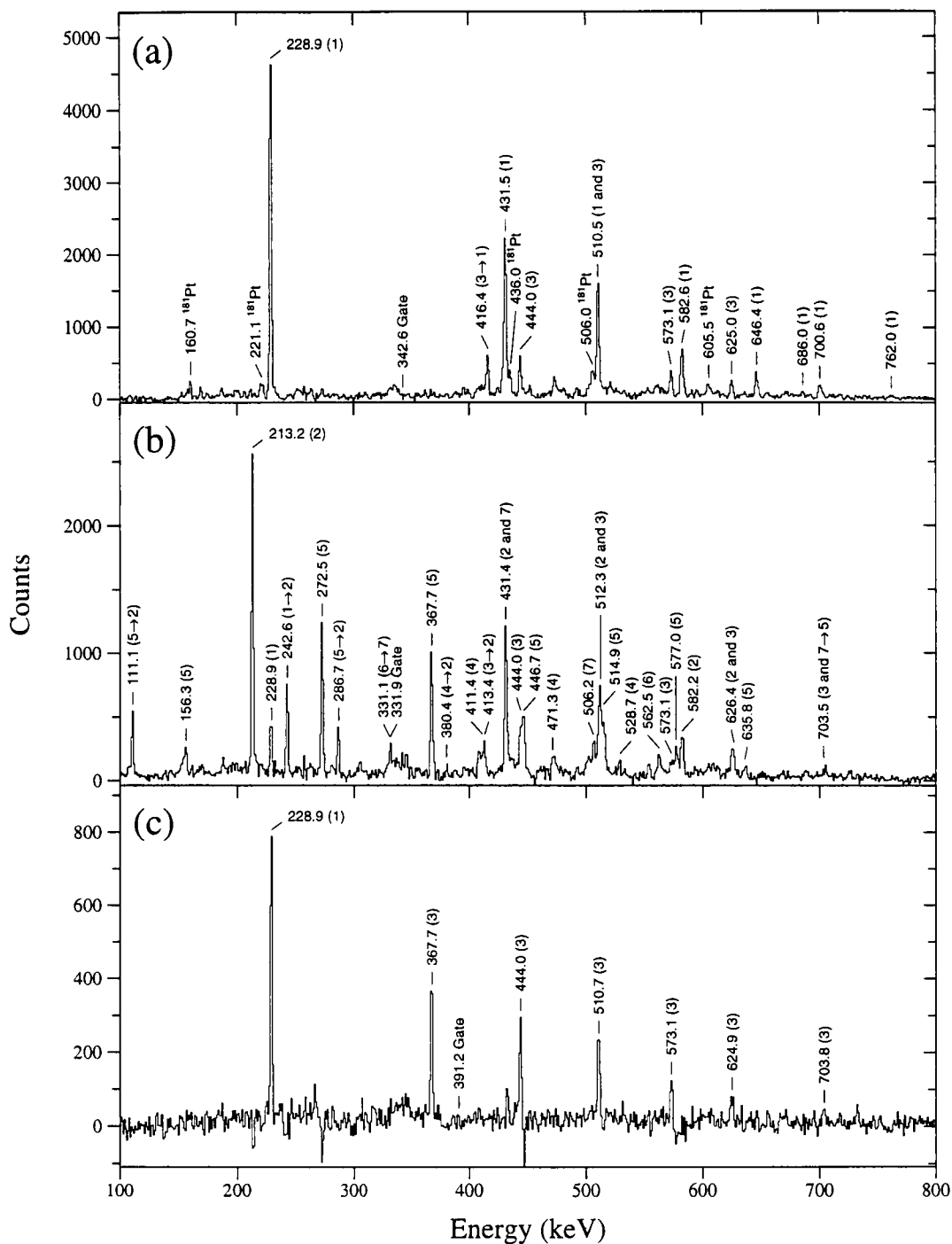


Figure 4.7: Representative coincidence spectra illustrating Bands 1, 2, and 3 with gates on the (a) 342.9, (b) 331.9, and (c) 391.2-keV transitions respectively. The values in parentheses following the peak marker indicate the band from Fig. 4.4 in which the γ -ray transition is found.

possible for three transitions in this band, two of which are shown in Fig. 4.6 as filled triangles. The rotational character of these γ rays clearly establishes these transitions as stretched E2's at least up to the level labeled $35/2^-$. For γ rays above $35/2^-$, the multipolarities are not known, but are postulated as being E2 transitions. See There are five transitions identified as decaying from Band 3 to Band 1. The open triangles in Fig. 4.6 indicate the resulting DCO ratios for the 391.2 and 416.4-keV transitions. These ratios are below the $\Delta I = 1$ pure dipole value, and show that these γ rays have a dipole/quadrupole multipolarity (for $I \rightarrow I - 1$) characterized by a large negative mixing ratio.¹ These ratios clearly fix the spins for the levels in Band 3, and provide strong indication that the transitions are of M1/E2 type thus fixing the parity as well. A transition is also observed from the $19/2^-$ level in Band 3 to the $15/2^-$ level in Band 2, and results from the fact that the $19/2^-$ levels of Bands 2 and 3 are only 18 keV apart. This interaction between the two bands lends further support for the spin and parity assignments.

4.3.2 Band 4

Band 4 is a set of weak transitions representing about 4% of the total decay intensity of the nucleus. Fig. 4.8 shows a gate on the 411.4-keV transition in the band. The band is connected with the rest of the level scheme via a 380-keV transition from the level fed by the 411-keV transition. This 380-keV transition, as well as several transitions from Bands 1 and 2 can be seen in Fig. 4.8. The γ rays in this band, including the decay-out transition, are too weak to extract DCO ratios, so the spins cannot be deduced. Based on the energy spacing the in-band transitions are likely stretched E2's of a collective rotational band.

4.3.3 Bands 5 through 7 – positive parity structures

For excitation energies $500 < E_x < 4000$, the γ rays in Band 5 are the most intense transitions in the nucleus. DCO ratios measured for the transitions in Band 5 (represented by crosses in Fig. 4.6) clearly establish the γ rays as having E2 multipolarity.

¹See Sec. 2.8 for the definition of the mixing ratio.

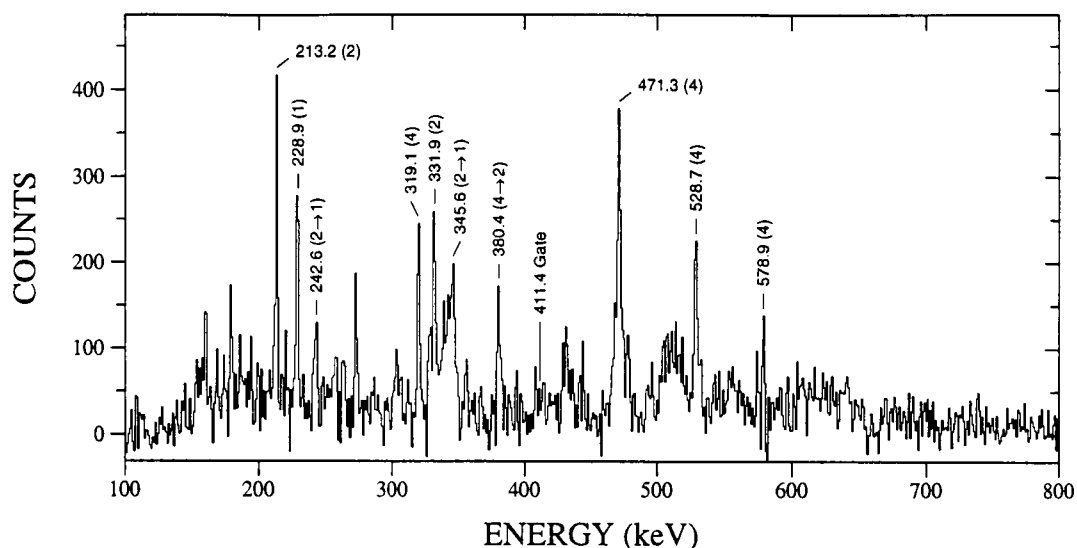


Figure 4.8: Representative coincidence spectrum illustrating Band 4 with a gate on the 411.4-keV transition. The values in parentheses following the peak marker indicate the band from Fig. 4.4 in which the γ -ray transition is found.

Fig. 4.9(a) shows the spectrum from a gate on the 272.5-keV γ ray. Visible in this gate are the 111.1, 286.5, and 300.4-keV transitions that decay out of Band 5 into Bands 2 and 1. The 111.1 and 286.5-keV γ rays have DCO ratios of 0.86 and 0.76 respectively. These values indicate that the two transitions have primarily $\Delta I = 1$ dipole multipolarity. This would set the lowest spin of Band 5 as $\frac{13}{2}$. Further support for this assignment comes from the DCO measurement of the 300.4-keV transition from Band 5 to Band 1. This transition has a ratio of 1.02 ± 0.08 . Such a ratio indicates either a $\Delta I = 2$ quadrupole transition or a dipole distribution of a γ ray connecting two levels of the same spin. Since the 300.4-keV transition feeds the $\frac{13}{2}^-$ level of Band 1, an assignment of $\frac{13}{2}$ for the spin of the lowest level in Band 5 is consistent with the ratios for all three decay-out transitions. Structures similar to Band 5 in ^{181}Au are also observed in ^{183}Au and ^{185}Au (see Fig. 4.5). In ^{183}Au and ^{185}Au , the respective bands were assigned positive parity. Based on the systematic comparison of these bands with bands in ^{181}Au , Band 5 is assigned positive parity. From the spin assignments, it is clear that Band 5 is the yrast sequence for spins $\frac{21}{2}$ and greater.

For the first time in Au nuclei, side feeding, in the form of decay-out transi-

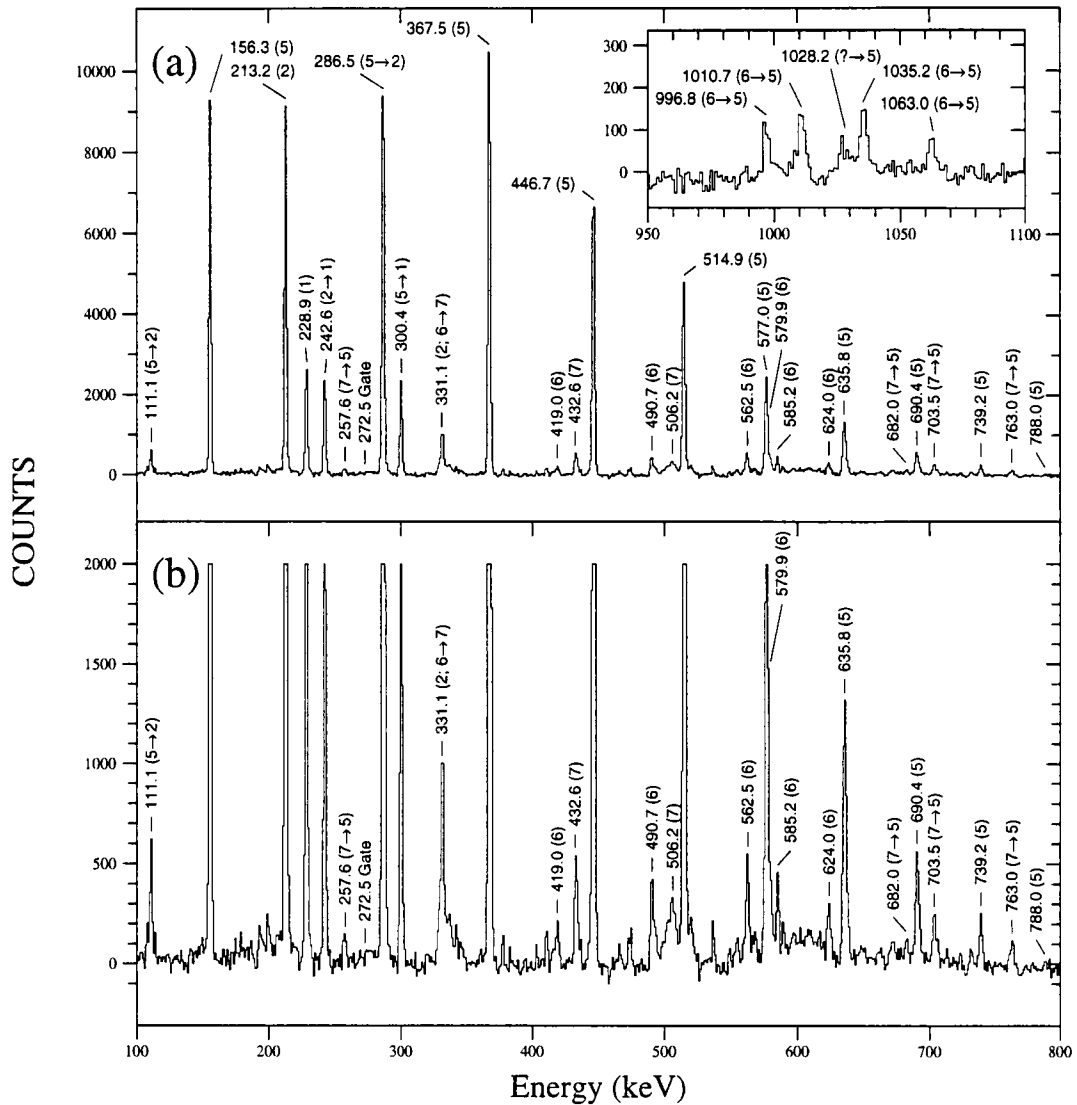


Figure 4.9: Representative coincidence spectrum illustrating Bands 5, 6, and 7 with a gate on the 272.5-keV transition. Panel (a) has a full scale y-axis to emphasize transitions in Band 5, while (b) has a restricted scale y-axis to observe Band 6 and 7 γ -ray lines. Inset: Spectral range of this projection is around 1 MeV to highlight linking transitions from Band 6 to 5. The values in parentheses following the peak marker indicate the band from Fig. 4.4 in which the γ -ray transition is found.

tions from Bands 6 and 7, into the positive-parity yrast configuration is observed. The transitions in Band 6 and 7 are clearly seen in Fig. 4.9(b). The 562.5-keV transition in Band 6 is the only transition in this band where a DCO measurement is possible, and has a value consistent with the assignment as a $\Delta I = 2$ quadrupole. Band 6 is connected with Band 5 by four γ ray transitions with energies ~ 1 MeV. These transitions are clearly observed in the gate of the 272.5-keV γ ray in Band 5. The peaks are illustrated in the inset of Fig. 4.9(a). Since the peaks around 1 MeV are relatively isolated, a DCO measurement was possible for 1010.7-keV and 996.8-keV γ rays, although with large uncertainty. These transitions have ratios of 1.14 ± 0.6 and 2.61 ± 1.5 , respectively. These values are consistent with the transitions being $\Delta I = 2$ quadrupoles or $\Delta I = 0$ dipoles. The quadrupole multipolarity was chosen based on comparison of the intensities of the γ decays at the highest levels in Band 5 with those of Band 6. For spins greater than $45/2\hbar$, Band 6 is more intense than Band 5. In heavy-ion fusion reactions the yrast sequence is typically the most strongly populated (particularly at high spin). With the spins indicated for Band 6 in Fig. 4.4, the 5649-keV level is the yrast level for $I = 53/2$.

While only two transitions within Band 7 were observed, decay-out transitions from all three levels have been established. Multipolarity measurements were not possible for the transitions within the band, but the relative spin for Band 7 is established by measured DCO ratio for the 703.5-keV decay-out transition (illustrated by the stylized cross in Fig. 4.6). This ratio, 0.47 ± 0.30 , suggests that the 703.5-keV γ ray corresponds to a $\Delta I = 1$ transition. The series of decays out of Band 7 (i.e. the 705.3, 763, and 682-keV transitions) is consistent with the depopulation of one rotational band to another (see e.g. decay from Band 3 to Band 1 or Band 6 to Band 5), thus it is likely that the 763 and 682-keV transitions have the same multipolarity as the 705.3-keV transition. This leads to the proposed spins assignments for the $E_x = 2535$ and 2969-keV levels in Band 7 to be $I = 31/2$ and $35/2$, respectively.

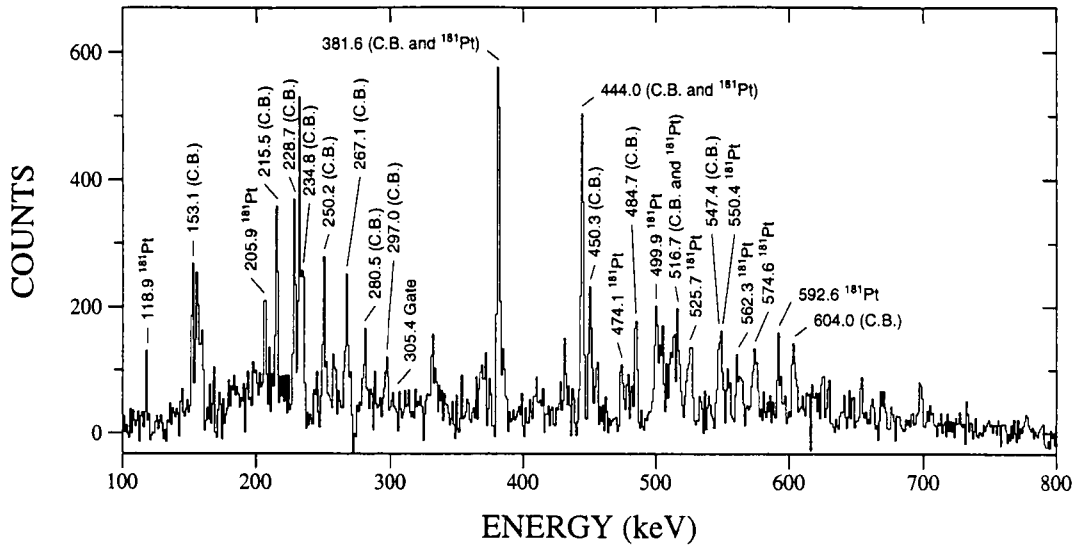


Figure 4.10: Representative coincidence spectrum illustrating the Coupled Band (C.B.) with a gate on the 305.4-keV transition. The values in parentheses following the peak marker indicate the band from Fig. 4.4 in which the γ -ray transition is found.

4.3.4 Coupled Band

A strongly coupled band with no connections to main level scheme has been observed associated with ^{181}Au decay. Fig. 4.10 shows a gate on the 305.4-keV transition at the bottom of the band. The 305-keV transition is a doublet with a transition in ^{181}Pt . Those peaks resulting from the decay of ^{181}Pt are indicated. The peaks labeled (C.B.) can be ruled out as being part of the decay ^{181}Pt since this nucleus is well studied, and the (C.B.) transitions do not correspond to any known Pt γ rays. There are no transitions observed that establish the relative excitation energy or spin of the “Coupled Band” with the other levels in ^{181}Au . Thus the excitation energy of the levels in this band are based on the lowest level being arbitrarily set to 50 keV. The spin assignments of the levels are postulated from arguments made in Sec. 4.4.4

4.4 Configuration Assignments

Examination of the developed level schemes of ^{180}Pt [68], ^{183}Au [65], and ^{185}Au [64] provide clear indication that the level structures observed in ^{181}Au originate from

prolate deformed configurations. In ^{180}Pt , rotational states were established based on a prolate deformed core. Likewise, decoupled bands resulting from $\pi h_{9/2}$, $\pi f_{7/2}$, and $\pi i_{13/2}$ valence protons coupled to a prolate core are established in ^{183}Au and ^{185}Au . Potential-energy-surface calculations [69] indicate that corresponding prolate states in ^{181}Au have deformations of $\beta_2 \sim 0.2 - 0.3$. The expected single-particle states for ^{181}Au can be seen in Fig. 2.3 as those orbitals around the gap labeled “78.” Specifically, single-particle orbitals that are expected to play a role are negative-parity $\frac{1}{2}[541]$, $\frac{3}{2}[532]$, $\frac{1}{2}[530]$, $\frac{11}{2}[505]$, and the positive-parity $\frac{1}{2}[660]$ configurations. Experimental quasiparticle alignment and Routhian energy² of the individual rotational bands provide insight as to the underlying configuration of each structure, and resulting calculations of all bands observed in ^{181}Au are displayed in Fig. 4.11. Specific spectroscopic assignments for individual bands are discussed in the following sections.

4.4.1 Bands 1 through 3 – $\pi h_{9/2}$ - $\pi f_{7/2}$ configurations

Experiments on ^{185}Au [70, 66, 64] and ^{183}Au [67, 65] have clearly identified that the lowest negative-parity positive-signature rotational band³ observed in these nuclei are based on a $\pi h_{9/2}$ quasiparticle configuration. Other low-lying rotational bands identified in these nuclei are based on the negative-signature $\pi h_{9/2}$ configuration and the positive-parity $\pi i_{13/2}$ orbital. The lowest levels of the rotational bands based on these configurations are illustrated in Fig. 4.5. From a comparison of these bands in ^{185}Au with low-lying states in ^{181}Au , it is clear that Band 1 is based on a $\pi h_{9/2}$ quasiparticle configuration. The alignment and Routhian energy of Band 1, denoted by the filled squares in Fig. 4.11, also support this assignment. From the extrapolation to $\hbar\omega = 0$ in Fig. 4.11(a), one can see that Band 1 exhibits an alignment of $\approx 2.5\hbar$. The expected alignment of a fully oriented $\frac{1}{2}[541]$ single particle is $4.5\hbar$, however, examination of the single-particle diagram of Fig. 2.3 indicates that the Fermi level for ^{181}Au ($Z = 79$) is within the $\pi h_{9/2}$ shell, thus Band 1 has significant contribution for the $\frac{3}{2}[532]$ orbital, as well as $\frac{1}{2}[541]$. Thus the reduced alignment observed in

²See Sec. 2.7 for definitions of alignment and Routhian energy.

³These bands are characterized by the spins and parities $I^\pi = \frac{9}{2}^-, \frac{13}{2}^-, \frac{17}{2}^-, \dots$

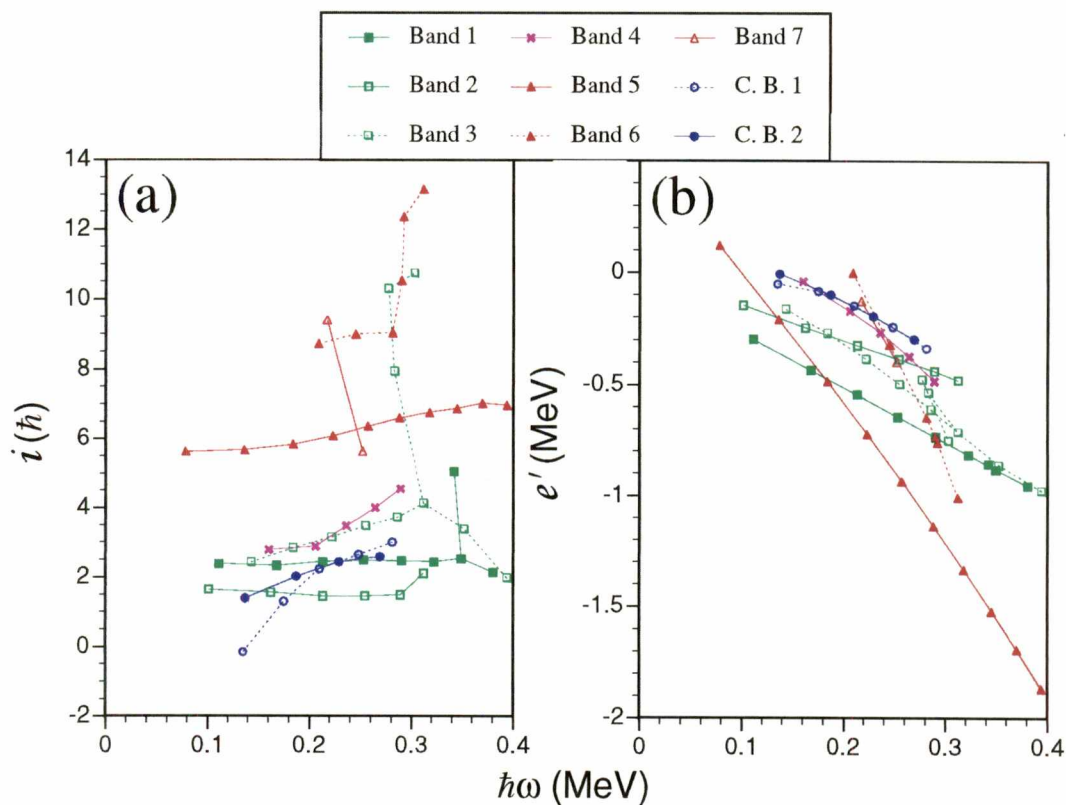


Figure 4.11: Calculated alignment (Panel “a”) and Routhian energy (Panel “b”) for experimentally measured rotational bands in ^{181}Au . The labels in the legends indicate the bands as they are labeled in Fig. 4.4. The Harris reference parameters are chosen to be $\mathcal{J}_0 = 31.6\hbar^2/\text{MeV}$ and $\mathcal{J}_1 = 136.2\hbar^4/\text{MeV}^3$ such that the Band 5 is approximately flat.

the $\pi h_{9/2}$ band here is a result of mixing with less aligned orbitals. From angular momentum coupling rules [3], the lowest signature of a decoupled $\pi h_{9/2}$ rotational band would be $\alpha = +\frac{1}{2}$. By examining the Routhian energies in Fig. 4.11, one can see that Band 1 is the lowest energy negative parity band observed. Since Band 1 has a signature of $\alpha = +\frac{1}{2}$,⁴ it clearly conforms to the energetically favored signature of the $\pi h_{9/2}$ configuration.

There are two negative-parity $\alpha = -\frac{1}{2}$ rotational bands (Bands 2 and 3) that have been clearly identified in ^{181}Au . The alignment and Routhian energy for these two bands are shown in Fig. 4.11. It is clear that neither can be associated with high- K configurations such as prolate $\pi h_{11/2}$ or oblate $\pi h_{9/2}$ (see Fig. 2.3), because of the decoupled nature of these two bands. Possible configurations must be related to the $\pi h_{9/2}$ or $\pi f_{7/2}$ orbitals. The energetically favored signature of the $\pi f_{7/2}$ band has $\alpha = -\frac{1}{2}$; this and the unfavored $\pi h_{9/2}$ configuration are possible configurations for Bands 2 and 3. Rotational structures similar to Bands 2 and 3 have also been observed in ^{183}Au and ^{185}Au , and interpretation [65, 64] of the bands in these two nuclei was indeed that one is the unfavored signature $\pi h_{9/2}$ while the second structure is related to $\pi f_{7/2}$. The bands identified as the unfavored $\pi h_{9/2}$ bands in ^{185}Au and ^{183}Au are illustrated in Fig. 4.5. Based on the comparison of these unfavored $\pi h_{9/2}$ bands to Band 2 in ^{181}Au , it is clear that such an assignment for Band 2 can also be made. By analogy, Band 3 can be established as a rotational band based on the $\pi f_{7/2}$ configuration. Because of the close proximity in energy of the $h_{9/2}$ and $f_{7/2}$ spherical shell states,⁵ the corresponding deformed single-particle configurations of rotational bands from these orbitals are heavily mixed. As a result, special consideration is taken for the examination of other properties, such as branching ratios and band interaction, in these two bands. Detailed interpretation of these two bands as well as analogous bands in other nuclei is given in Chapter 7.

As can be seen in Fig. 4.11(a), a large increase in alignment at $\hbar\omega \approx 0.31 - 0.32$ MeV is observed Bands 1 and 3. The last transition in Band 2 also indicates

⁴The signature of a rotational band is determined by the spin of the band. See Eq. 2.39 for the relation that identifies the signature quantum number.

⁵This proximity can be seen in Fig. 2.3 at $\beta_2 = 0.0$.

that the alignment in this band may be starting a significant increase. This dramatic change in alignment is related to the well known breaking of a pair of $i_{13/2}$ neutrons and subsequent arrangement of their spins along the axis of rotation.

4.4.2 Band 4

The weak intensity and lack of spin assignments for Band 4 make the spectroscopic assignment of this band difficult. Since this band feeds the negative parity Band 2 rather than the positive-parity yrast-structure Band 5, it is likely that Band 4 has negative parity. The alignment and Routhian energy are extracted for Band 4 assuming $K = \frac{1}{2}$ and that the spin of the lowest level is $\frac{13}{2}$. The results are shown as crosses in Fig. 4.11. As can be seen in the figure, the alignment of the Band 4 is gradually increasing with respect to the chosen reference and has a value at high rotational frequency that is larger than all other negative-parity states. From the Routhian energy, it may be possible that this band is the signature partner of Band 3. The larger alignment and the fact that no decay is observed from Band 4 to 3 would have to be explained.

4.4.3 Bands 5 through 7 – $\pi i_{13/2}$ band and side bands

The extracted alignment and Routhian energies for Band 5, 6, and 7 are displayed as triangles in Fig. 4.11. The alignment at $\hbar\omega = 0$ of Band 5 is $\approx 5.5\hbar$. If only one-quasiparticle configurations are considered, the only orbital close enough to the ^{181}Au Fermi level with sufficient alignment is $\pi i_{13/2} \frac{1}{2}[660]$. The low-lying positive-parity rotational bands observed in ^{185}Au and ^{183}Au have been clearly demonstrated as arising from this orbital. The lowest three levels of $\frac{1}{2}[660]$ bands in ^{185}Au and ^{183}Au are compared with the corresponding three levels of Band 5 in Fig. 4.11. Based on the alignment and comparison with similar bands in heavier Au nuclei, it is clear that Band 5 in ^{181}Au is based on the one-quasiparticle $\frac{1}{2}[660]$ orbital, as opposed to a multi-quasiparticle excitation.

Excluding one point in Band 7, Band 6 has the largest aligned angular momentum below $\hbar\omega = 0.25$ MeV with a value of $\approx 8.0\hbar$. As observed in Bands 1

and 3, Band 6 also appears to be influenced by the $\nu i_{13/2}$ alignment. The fact that a signature partner for this band is not observed indicates that the configuration for the band is likely low K . To form such a large alignment, Band 6 must be based on a configuration of at least three quasiparticles. The coupling of these quasiparticles should form a low- K positive-parity state. Since a backbend occurs at a similar rotational frequency as the $\nu i_{13/2}$ alignment in Bands 1 and 3, the $\nu i_{13/2}$ orbital is not part of the low- ω configuration of Band 6. This is due to the fact that a one quasiparticle occupation of the lowest $\nu i_{13/2}$ state would inhibit the normal $\nu i_{13/2}$ alignment from occurring at the expected rotational frequency. This well known blocking effect can be seen in odd- N nuclei throughout the rare-earth region. Bark *et al.* [71] reported on several positive-parity three-quasiparticle bands observed in ^{177}Re . In that report, the suggested configurations all involved an $i_{13/2}$ neutron. For reasons stated above, such a choice for Band 6 is unlikely. A possible configuration for Band 6 is a rotational band built on the $\pi \frac{1}{2}[660] \otimes \nu \frac{1}{2}[521] \otimes \nu \frac{5}{2}[512]$ set of single-particle excitations. As can be seen in Fig. 2.2, the $\frac{1}{2}[521]$ and $\frac{5}{2}[512]$ are both near the Fermi level at $N = 102$. The low energy of these two configurations with respect to $N \sim 102$ has also been observed experimentally in ^{181}Pt [68]. The $\frac{1}{2}[521]$ orbital is observed to be the ground-state configuration in ^{181}Pt , and $\frac{5}{2}[512]$ orbital is observed at an excitation energy of 166.8 keV. The energetically favored signatures for the three configurations are: $\alpha(\pi i_{13/2}) = +\frac{1}{2}$, $\alpha(\nu f_{5/2}) = +\frac{1}{2}$, and $\alpha(\nu f_{7/2}) = -\frac{1}{2}$. From the addition of signature, the favored signature for this configuration would be $\alpha = +\frac{1}{2}$, which corresponds to the signature of Band 6. Most of the alignment would be contributed by the $\pi i_{13/2}$ state ($\sim 5.5\hbar$), while a small amount would be added by the $\nu p_{3/2}$ and $\nu f_{7/2}$ orbitals. Both the $\frac{1}{2}[660]$ and $\frac{1}{2}[521]$ configurations have large signature splitting, but there is no splitting observed for the $\frac{5}{2}[512]$ configuration in ^{181}Pt . Thus the signature splitting of Band 6, should be determined by the $\frac{5}{2}[512]$ quasi-neutron. The likely reason why the signature partner is not observed is that the intensity of this band is too weak to be resolved.

Only three levels are observed in Band 7. It can be seen in Fig. 4.11(a) that this band is observed in the midst of an apparent backbend. The frequency where this backbend occurs ($\hbar\omega \approx 0.24$ MeV) is too low to be easily considered a $\nu i_{13/2}$ alignment.

Rather than a quasiparticle alignment, another consideration is that some of these levels are perturbed by unobserved states, thus causing an appearance of a backbend. If this is the case, Band 7 could be interpreted as the unfavored signature of the $\pi i_{13/2}$ band (Band 5). Band 7 has an excitation of about 500 keV above Band 5 in the Routhian energy plot, Fig. 4.11(b). This is a comparable energy splitting to that seen in low- K $\nu i_{13/2}$ bands reported in Ref. [72]. A possible source of the perturbation would be from the unobserved signature partner of Band 6. Based on the excitation energies of the $33/2^+$ and $37/2^+$ states (2809 and 3299 keV respectively), the expected energy of the $35/2^+$ level in the (strongly coupled) signature partner would be ~ 3050 keV. This is less than 100 keV from the observed $35/2^+$ level in Band 7, thus the perturbation in this band is likely the result of an accidental near degeneracy with a level the unseen partner of Band 6. This would also explain why a decay is observed from the 3299-keV level in Band 6 to the 2969-keV level in Band 7.

Without additional information on Band 7 further conclusions cannot be drawn.

4.4.4 Coupled Band – $\pi h_{11/2}$ band

Possible spectroscopic assignments for the strongly-coupled band in ^{181}Au arise from consultation of the proton single-particle diagram of Fig. 2.3. Such configurations include the oblate $\frac{9}{2}[505]$ state, as well as the prolate $\frac{11}{2}[505]$ and $\frac{5}{2}[402]$ orbitals. Rotational bands resulting from oblate $\pi h_{9/2}$ orbitals have been observed as the yrast structures of most odd- A Tl nuclei [73, 74, 75, 76]. From Total Routhian Surface calculations by Wyss *et al.* [34], these configurations in Tl are predicted have a β_2 parameter of ~ 0.15 (oblate). The energy of γ -ray transitions observed in these bands are on the order of 700 keV for the stretched E2's. The oblate $\pi h_{9/2}$ configuration in Au nuclei is predicted to have a deformation similar to that of Tl, thus one would expect γ -ray transition energies to also be the same order. The transitions in the Coupled Band of ^{181}Au , however, have energies $\sim 0.6E_\gamma(\text{Tl})$, thus the oblate $\frac{9}{2}[505]$ can be ruled out as a possible configuration.

To aid in the determination of whether the Coupled Band is prolate $\pi h_{11/2}$

or $\pi d_{5/2}$, one can compare the extracted $B(M1)/B(E2)$ ratios for this band with strongly-coupled bands in odd- A Ir nuclei. Fig. 4.12 illustrates the resulting $B(M1)/B(E2)$ ratios for Coupled Band in ^{181}Au , as well as the $\pi h_{11/2}$ and $\pi d_{5/2}$ bands observed in ^{177}Ir [77], ^{179}Ir [78], and ^{181}Ir [79]. The $B(M1)/B(E2)$ ratio was extracted from reported γ -ray energies and intensities using the expression:

$$\frac{B(M1; I \rightarrow I-1)}{B(E2; I \rightarrow I-2)} = 0.693 \frac{E_\gamma^5(I \rightarrow I-2) I_\gamma(I \rightarrow I-1)}{E_\gamma^3(I \rightarrow I-1) I_\gamma(I \rightarrow I-2)} (\mu/eb)^2. \quad (4.92)$$

The values for the Coupled Band in ^{181}Au are plotted in Fig. 4.12, assuming the spin of the lowest level is $\frac{11}{2}\hbar$ as denoted in Fig. 4.4. Motivation for this spin assignment is discussed below. It should be noted that the $B(M1)/B(E2)$ ratio is independent of the relative angular momentum of the band, thus the choice of spin will only shift the “ ^{181}Au C.B.” trend in Fig. 4.12 left or right. It can be seen in this figure that the $\pi h_{11/2}$ configurations in Ir produce ratios of $\approx 1 \left(\frac{\mu}{eb}\right)^2$. The $\pi d_{5/2}$ orbitals show a bit more scatter but have consistently lower $B(M1)/B(E2)$ ratios than the $\pi h_{11/2}$ states. The extracted ratios for the Coupled Band in ^{181}Au , indicated by the filled squares, have values that are comparable to the $\pi h_{11/2}$ bands in Ir. This provides support for the assignment of the Coupled Band as the $\pi h_{11/2}$ configuration. Additional confirmation for this assignment comes from the observation of the prolate $\pi h_{11/2}$ band in ^{185}Au [64]. In this case, the strongly-coupled prolate band was observed interacting with a lower-lying oblate band based on the $\pi h_{11/2} \frac{1}{2}[550]$ orbital.

Fig. 4.11 shows the calculated alignment and Routhian energy for the Coupled Band where the two sequences of stretched E2 transitions are denoted by open and filled circles. These calculations were made assuming the initial spin is $\frac{11}{2}$ and the excitation energy is arbitrarily set to 50 keV. “C.B. 1” denotes the $\alpha = -\frac{1}{2}$ band ($I = \frac{11}{2}, \frac{15}{2}, \dots$), while “C.B. 2” the $\alpha = +\frac{1}{2}$ band ($I = \frac{13}{2}, \frac{17}{2}, \dots$). It is evident from this figure the Coupled Band shows deviations from strong coupling at both high and low rotational frequencies. The deviations at the high frequencies are likely a result of the uncertainty of the placement of the γ rays for these weakest transitions. The deviations at low rotational frequencies are likely the result of the perturbation of these levels by the unobserved oblate $\pi h_{11/2}$ orbital.

It was demonstrated by Wood *et al.* [80] that the lowest members of the

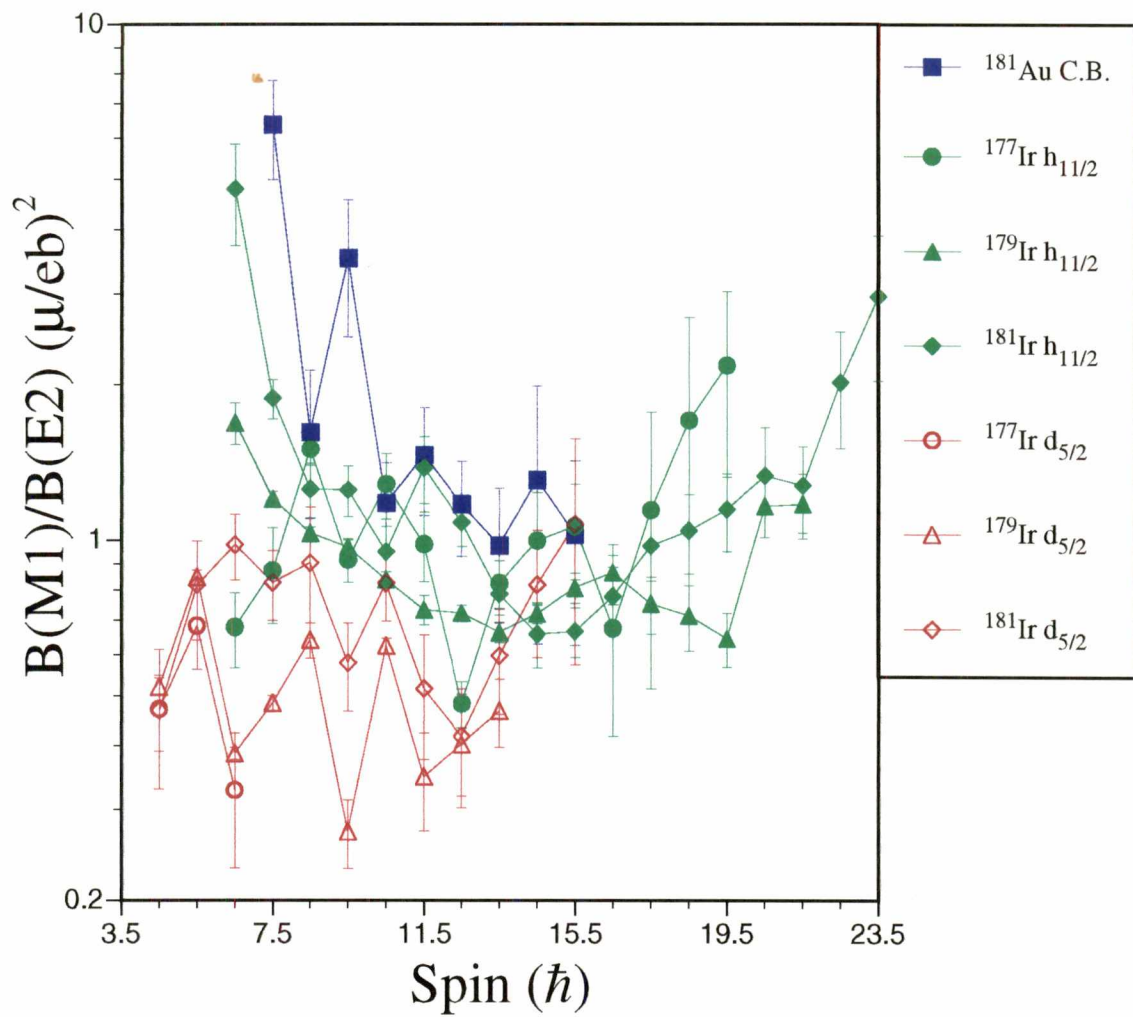


Figure 4.12: Measured $B(M1)/B(E2)$ values for $\pi h_{11/2}$ and $\pi d_{5/2}$ bands in $^{177,179,181}\text{Ir}$ and the Coupled Band in ^{181}Au . References are provided in the text.

ground-state rotational band in ^{180}Pt are perturbed by mixing of prolate and oblate configurations. This perturbation is clearly evident in the quasiparticle Routhian energy of this band. Fig. 4.13 shows the calculated Routhian energy of the ground-state band of ^{180}Pt as triangles. A reference was chosen such that the energies of the high-spin levels before the backbend are approximately flat as a function of rotational frequency. A horizontal line is drawn to illustrate the deviation of the lowest points for ^{180}Pt . As denoted in the figure, the deviation of the lowest point from the reference line is ~ 0.040 MeV. Also shown in Fig. 4.13 are Routhian energies of the two stretched E2 sequences of the Coupled Band in ^{181}Au . This band is sloped because of the alignment of the unpaired proton in Au, nevertheless, a straight line can be traced through the points as a common reference. The deviation of the lowest point in "C.B. 1" from the Coupled Band reference line is ~ 0.045 MeV. It can be seen that the positive signature of the Coupled Band, "C.B. 2," is not perturbed. The oblate $\pi h_{11/2}$ state is low- K and thus would produce a decoupled rotational band. For $\pi h_{11/2}$, the energetically favored signature would be $\alpha = -\frac{1}{2}$, thus only the levels with the same signature would be perturbed. This is the motivation for the choice of spins in the Coupled Band.

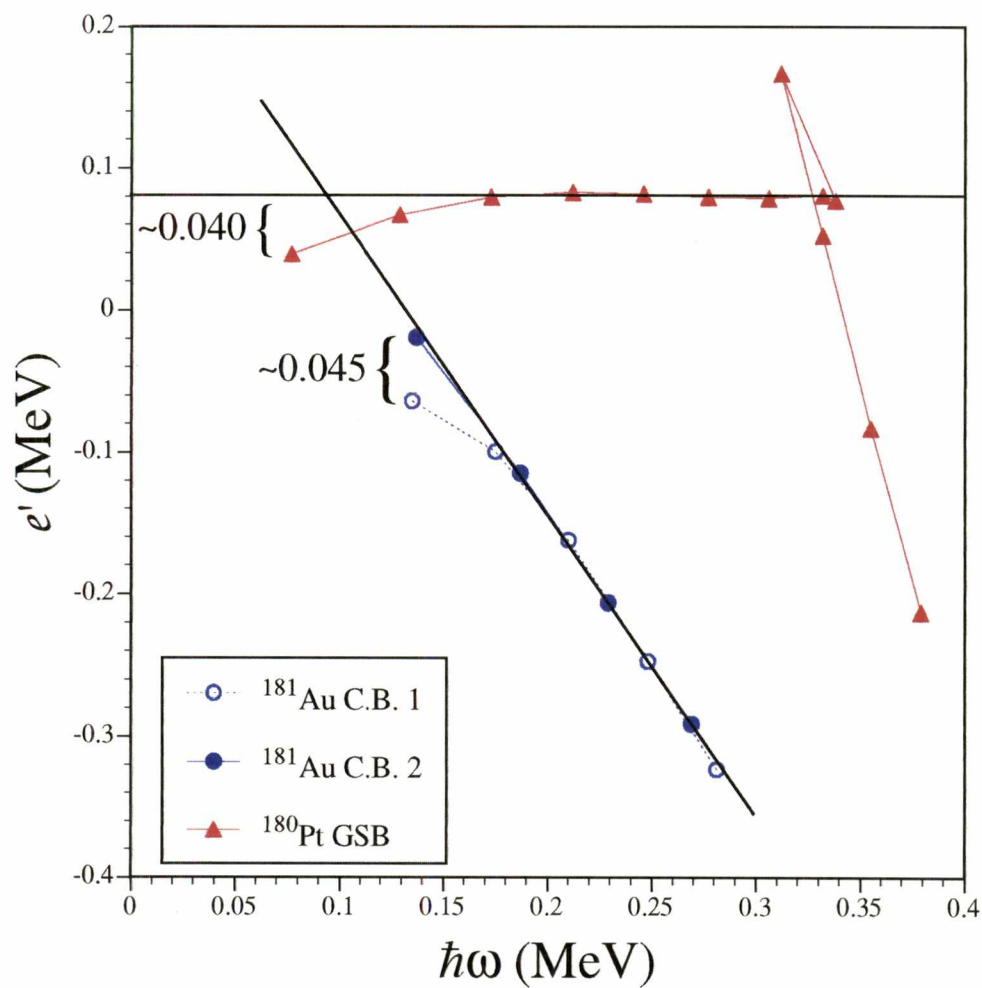


Figure 4.13: Calculated quasiparticle Routhian energy of ^{180}Pt compared with the quasiparticle Routhian of the Coupled Band in ^{181}Au . The Harris reference parameters are $\mathcal{J}_0 = 27.47 \frac{\hbar^2}{\text{MeV}}$ and $\mathcal{J}_1 = 179.2 \frac{\hbar^4}{\text{MeV}^3}$.

CHAPTER 5

Experiment to Study ^{179}Au

5.1 Experimental Setup

The experiment to observe excited states in ^{179}Au involved two phases; identification and spectroscopy. Both phases of the experiment were performed at ANL's ATLAS facility. Excited states in ^{179}Au were produced with a ^{58}Ni beam, accelerated to 260 MeV, directed onto a target of ^{124}Te . In the identification phase, a ^{124}Te target with a thickness of $385\text{ }\mu\text{g}/\text{cm}^2$ was used. Tellurium is a rather delicate material and is not self-supporting at this thickness, so the target was coated with $40\text{ }\mu\text{g}/\text{cm}^2$ of carbon on the front (towards the beam) surface, and $10\text{ }\mu\text{g}/\text{cm}^2$ of carbon on the back. The thicker carbon layer provides support for the tellurium, while the back layer helps stop the material from sputtering away. The setup for this phase utilized the AYE-ball array in conjunction with the Fragment Mass Analyzer (FMA), a Parallel Plate Avalanche Counter (PPAC), and a Double-sided Silicon Strip Detector (DSSD). The setup of the AYE-ball array with the FMA and DSSD is described in Sec. 3.3.2. In this configuration, 500,000 Ge+FMA and FMA+DSSD events were recorded. This was sufficient to unambiguously identify γ rays associated with the decay of ^{179}Au . After identification of some γ ray lines was established, the second phase of the experiment was undertaken. The second phase utilized a $500\text{ }\mu\text{g}/\text{cm}^2$ ^{124}Te target backed with $60\text{ mg}/\text{cm}^2$ Pb. With this target, the AYE-ball was used with 12 neutron counters to provide additional channel selection. Here 75 million Ge-Ge and higher fold events were recorded. An unfortunate difficulty with this phase of the experiment was that the Pb backing used in the target was contaminated with oxygen. The predicted cross section for ^{179}Au is about 20 mb. The corresponding cross sections for products produced by ^{58}Ni on ^{16}O are on the order of 300 mb. Due to the different reaction

cross sections, only a small amount of oxygen is sufficient to severely contaminant the ^{58}Ni on ^{124}Te reaction. Nevertheless, sufficient statistics were obtained to allow the further identification and spectroscopy of γ -ray lines in ^{179}Au .

5.2 Phase One Results

Before this experiment, excited states in ^{179}Au had not been previously identified. In order to identify γ rays associated with the decay of ^{179}Au , the Recoil Decay Tagging method, described in Sec. 3.4.5 was used. ^{179}Au is at the limit of stability where the chance of fission is much greater than the chance of a nucleus reaching thermal equilibrium after emission of only a few particles. In the case of the chosen reaction for this experiment (see Sec. 5.1), the fission cross section is calculated to be about 75% of the total reaction cross section. Thus most of the γ -ray background results from γ emission from the fission fragments.

Fig. 5.1 shows α -decay events in the DSSD associated with a particular PPAC event. Two charge states (20^+ and 19^+ left to right) are focused by the FMA onto the PPAC, thus there are two peaks related to each atomic mass. The ground state to ground state α decay in this mass region is well known, thus one can use those known peaks to identify a specific residual nucleus. While the α decay energy is enough to uniquely define a residual nucleus, the additional PPAC position requirement reduces random correlations of α decay with PPAC events related to scattered beam or fission byproducts, and thus reduces the overall background.

Table 5.1 summarizes previously known α decay energies, branching ratios, and halfives for the residual nuclei. The right side of Table 5.1 compares the relative cross sections observed in the PPAC and DSSD with calculations from EVAPOR. The EVAPOR calculations are summarized in Sec. 3.1.2. The experimental values assume that all nuclei are transmitted through the FMA with equal efficiency. The ^{179}Hg , ^{180}Hg , and ^{178}Pt experimental values are all about twice as large as predicted in calculation. This may be an indication that the ^{179}Au value is not a good normalization to the calculation. Renormalization of the calculated cross sections to ^{179}Hg , ^{180}Hg , and ^{178}Pt experimental cross sections indicates that the ^{179}Au cross section is only

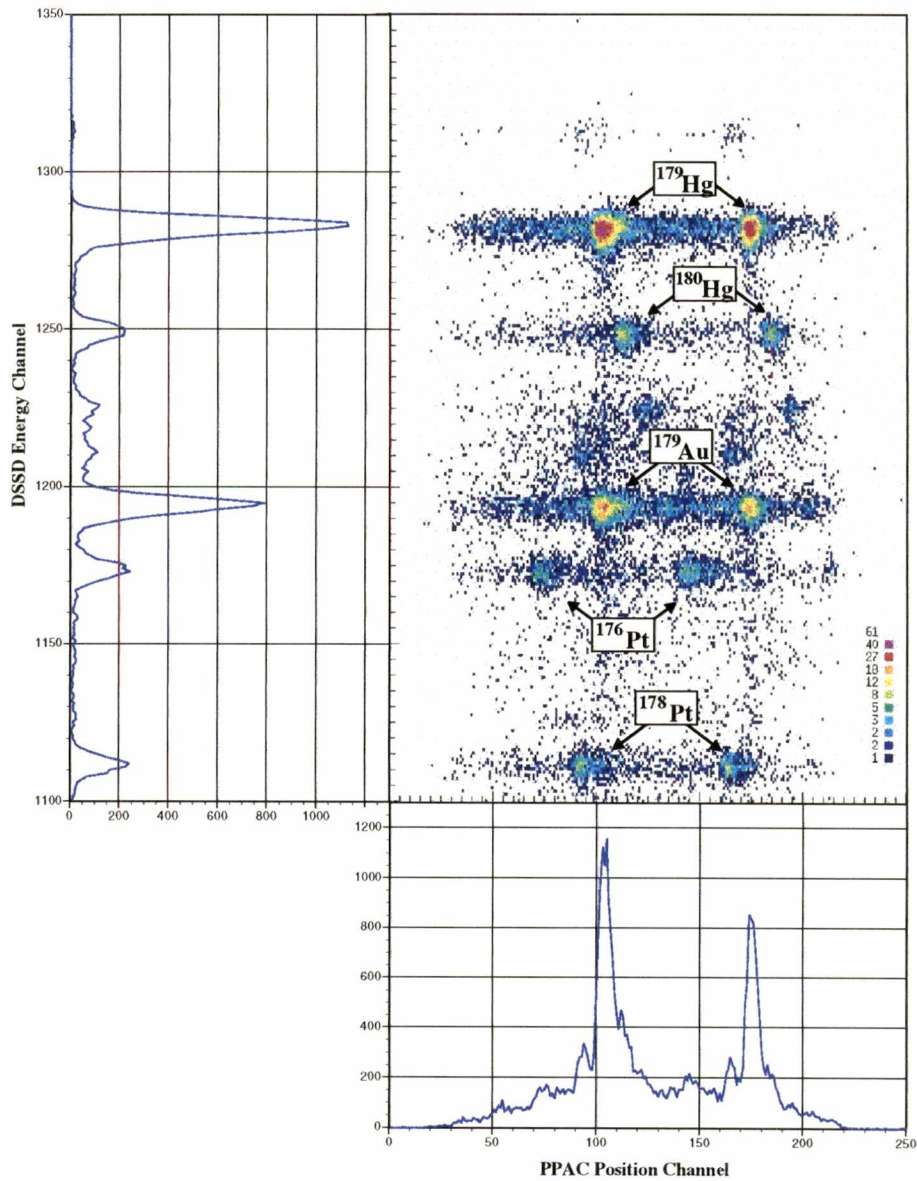


Figure 5.1: Compressed PPAC position versus the channel from the energy ADC for decays measured in the DSSD. The energy calibration with respect to the DSSD decay ADCs' channels conforms to the following formula: $E_{\alpha}[\text{MeV}] = 0.005(\text{channel}) - 0.12$. The figures below and to the left of the central figure are the total projections on the respective axes. The identity of the nuclei related to the ten largest peaks are denoted in the central figure. The left peak in the PPAC position corresponds to an atomic charge state of 20^{+} and the right 19^{+} .

Table 5.1: Known α -decay energies, lifetimes, and branching ratios for residual nuclei observed in the RDT experiment are displayed in the first three columns. The data are extracted from Ref. [59]. The column, "Int.," indicates the measured intensity of the α decays relative to ^{179}Au α decay. "Rel. σ " indicates the relative cross section calculated from the intensity (Int.) and α -branching ratio (α Branch). "Calc. σ " are the tabulated relative cross section calculated from the multi-particle evaporation code, EVAPOR, discussed in Sec. 3.1.2.

Nucleus	E_α (MeV)	$\tau_{1/2}(\text{sec})$	α Branch(%)	Int.	Rel. σ	Calc. σ
^{179}Hg	6.288(5)	1.09	≈ 0.53	6095(0)	0.82(7)	0.41
^{180}Hg	6.120(5)	2.8	0.48(4)	1004(25)	0.15(2)	0.08
^{179}Au	5.848(5)	7.1	0.220(9)	3103(58)	$\equiv 1(4)$	$\equiv 1$
^{176}Pt	5.753(3)	6.33	0.38(3)	985(29)	0.18(9)	0.52
^{178}Pt	5.404(3)	21.1	0.077(3)	1018(20)	0.94(7)	0.39
^{179}Pt	5.194(10)	21.2	0.0024(3)	16667(3800)	1.18(31)	0.29

50% that of the prediction. Regardless of either normalization, the ^{176}Pt cross section is significantly overpredicted by the calculations. A possible error in the calculation of the experimental cross sections is the assumption that all residual nuclei are transmitted through the FMA with equivalent efficiency. This may not necessarily be the case. For example, β -decay experiments of $^{181,183,185,187}\text{Hg}$ [63, 67, 66, 81] indicate that odd- A Au nuclei in this mass region have many low-lying energy levels. Such low-lying levels can have lifetimes on the order of tens of nanoseconds. For example, Berg *et al.* reported a $\frac{11}{2}^- \rightarrow \frac{9}{2}^-$ γ -ray transition with a lifetime of 26 nsec [82]. Low energy transitions often proceed via internal conversion, particularly if the transition is $\Delta I = 2\hbar$. Such transitions naturally change the charge state of the recoiling atom. There is a thin carbon foil 3 cm downstream of the target location to stabilize the charge state of the recoiling atoms. The recoiling nuclei pass through the reset foil after only a few nanoseconds. Only a few decays by electron conversion after the carbon reset foil are sufficient to result in the loss of the recoil through the FMA.

Nevertheless, ^{179}Au is one of the most abundant nuclei produced in the reaction, and sufficient data exist to identify ^{179}Au . Fig. 5.2 displays γ -ray spectra

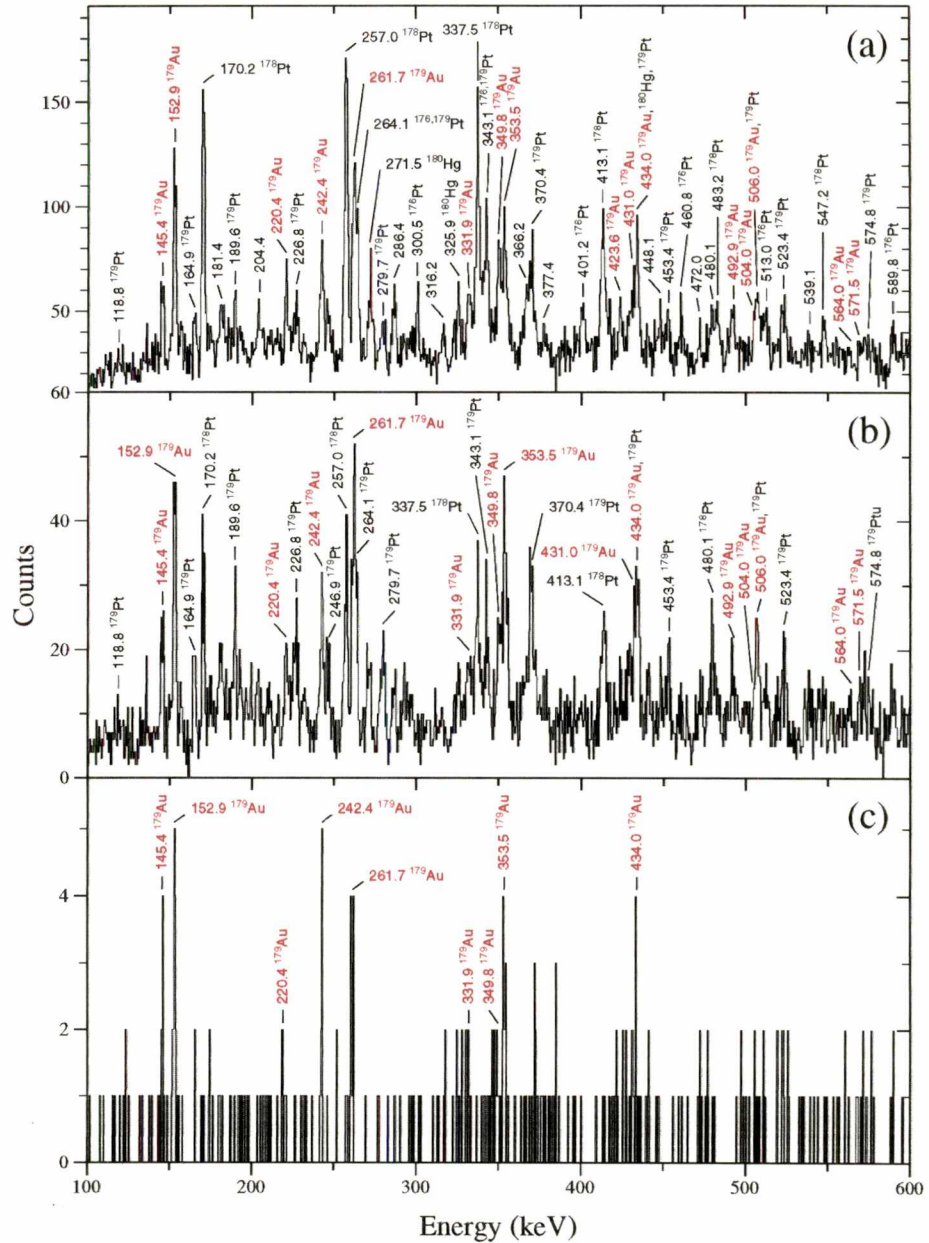


Figure 5.2: (a) γ rays from AYE-ball Ge detectors in coincidence with a PPAC event (no gating condition on the PPAC). (b) Same as panel (a) except that the PPAC has a gating condition on two peaks associated with $M/q = 179/20^+$ and $179/19^+ \frac{\text{a.m.u.}}{e}$ respectively. (c) γ rays correlated with α decay of ^{179}Au . Random coincidences were minimized by correlating ^{179}Au α -decay energies with the M/q peaks stated above. The Pt and Hg labeled peaks are those known from previous experiments [83, 84, 85].

with various gating conditions on the PPAC and DSSD as indicated in the figure caption. Part (c) of this figure identifies ^{179}Au γ rays from correlations with the known ^{179}Au α decay energy. There are six γ rays that can be identified as resulting from ^{179}Au deexcitation: 145, 153, 242, 262, 353, and 434 keV. The additional γ rays labeled as ^{179}Au in Fig. 5.2 have been identified in phase 2 (see Sec. 5.3) of the experiment.

5.3 Phase Two Results

From the RDT analysis (see Sec. 5.2), six γ rays were identified as being associated with the deexcitation of ^{179}Au . Those data are in Ge singles, therefore no coincident γ spectroscopy is possible in that phase of the experiment. The second phase, which utilized a backed Te target, provided the γ - γ data necessary to do coincidence spectroscopy. A total symmetrized γ - γ matrix was created for this analysis. Fig. 5.3 shows the total projection of this matrix. Contamination from decays of oxygen reaction byproducts is extreme, as is evident in the figure. The identification of γ -ray lines from phase 1 of this experiment, however, allows one to locate the peaks of interest. In this phase of the experiment, four rotational bands are identified as being associated with ^{179}Au decay. The level scheme of ^{179}Au is displayed in Fig. 5.4, and the γ -ray energies and intensities are listed in Table 5.2. Because discrete peaks for lowest transitions in Bands 2, 3, and 4 could not be resolved in the total projection, only lower limits could be deduced for the intensity of the lowest transition in each of these bands. Due the large background, accurate DCO measurements could not be achieved for transitions in this nucleus. Therefore, spin and parity assignments for the levels in ^{179}Au are made on the basis of comparison with systematics. Support for specific assignments are discussed in the following sections.

5.3.1 Band 1

The strongest decay sequence identified in ^{179}Au are the γ rays in Band 1. These transitions form a band that is clearly rotational in nature. A gate on the 261.6-keV transition within this band is illustrated in Fig. 5.5(a). As can be seen in the level

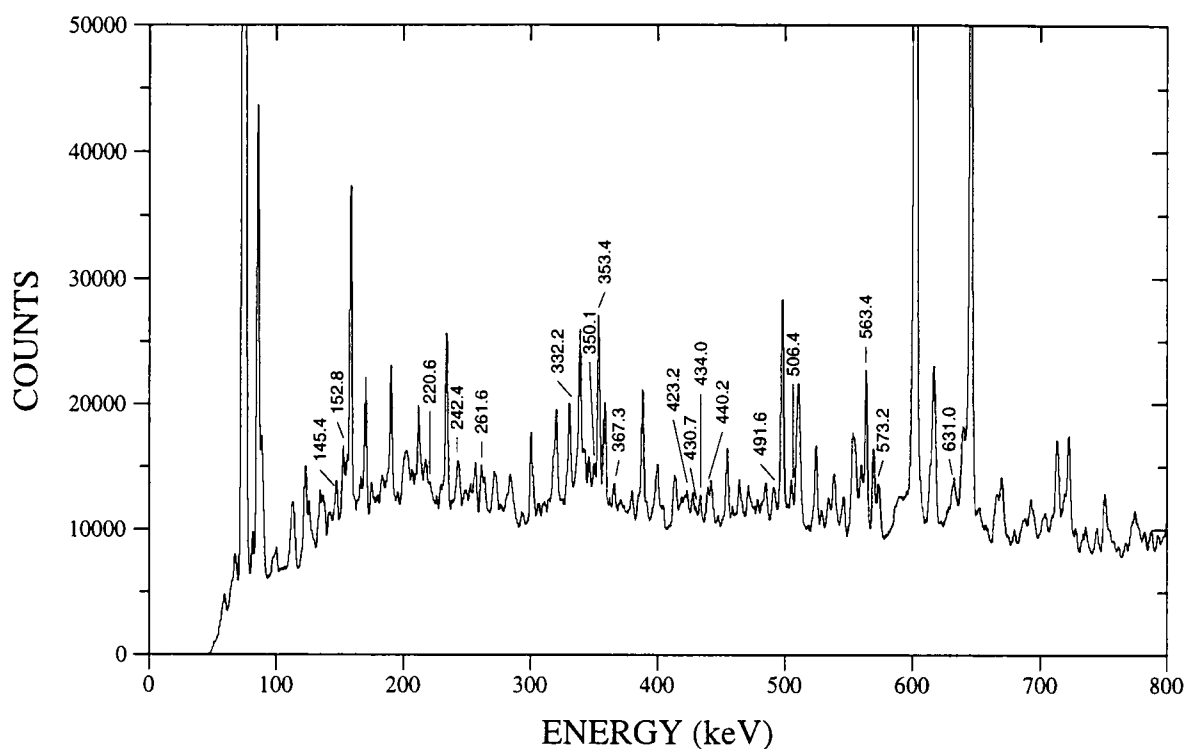


Figure 5.3: Total projection of the γ - γ matrix from phase 2 of the ^{179}Au experiment. γ rays associated with the decay of ^{179}Au are marked. Other peaks are the result of decay of products created by reactions of ^{58}Ni on ^{16}O . The large peaks at 603 and 646 keV transitions are from coulomb excitation of ^{124}Te .

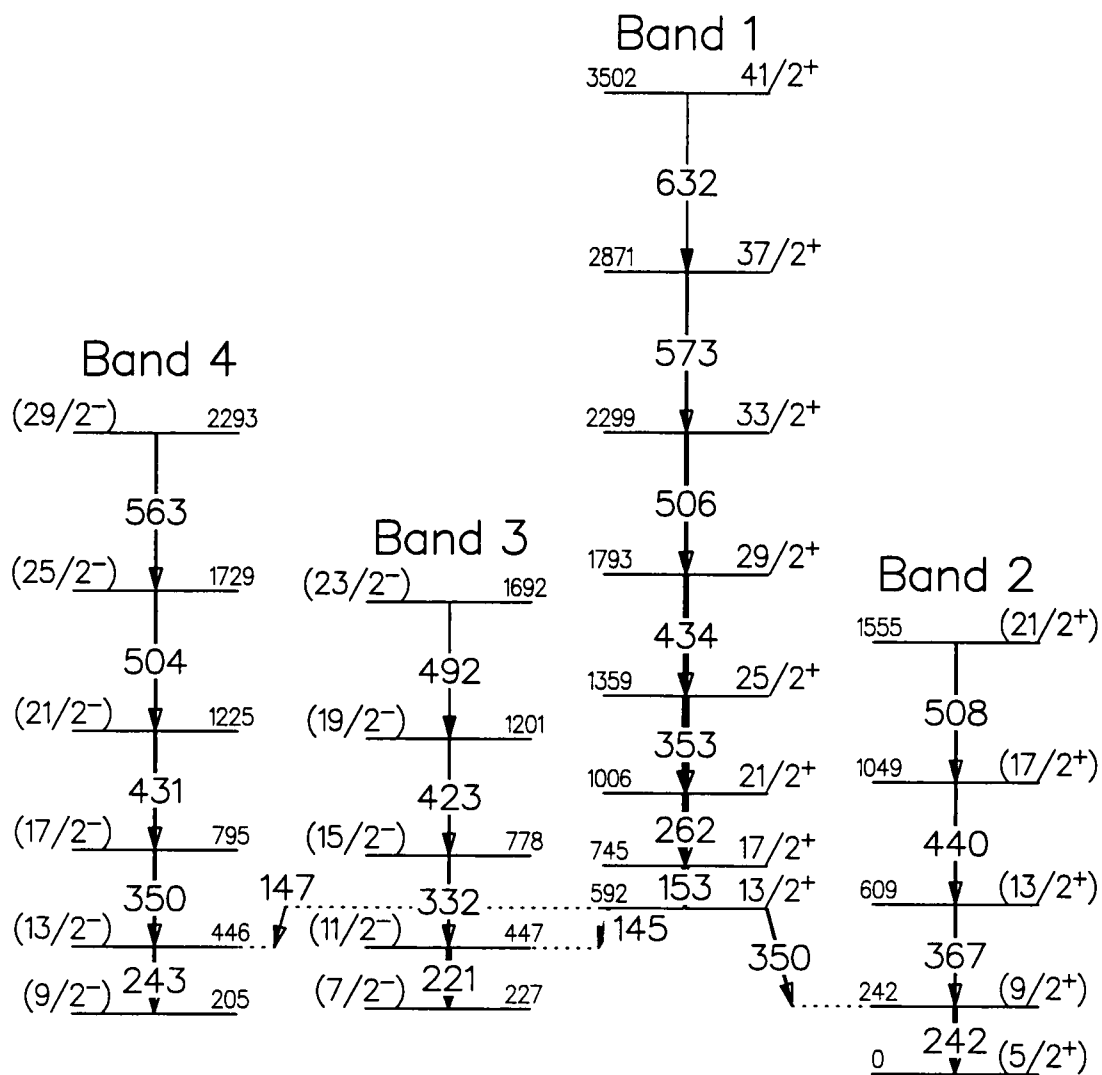


Figure 5.4: ^{179}Au level scheme. Excitation energies are relative to the lowest energy level observed in Band 2. The widths of the arrows represent the relative intensity of the transitions, while the black and white shading correspond to the γ -ray (measured) and internal-conversion (calculated) intensity respectively.

Table 5.2: γ ray energies and intensities for ^{179}Au . Uncertainty in the γ -ray energies is 0.3 keV when the decimal point is present, and 0.6 keV when the decimal point is not present. The values in column “Intensity (converted)” are the total decay intensities for the given transitions, accounting for internal conversion by using the coefficients from Rösler *et al.* [62].

E_γ (keV)	Band	I_γ relative	Intensity (converted)
152.8	1	109±8	217±16
261.6	1	156±9	180±11
353.4	1	197±10	205±11
434.0	1	134±8	138±8
506.4	1	94±7	96±7
573.2	1	46±6	47±6
631	1	8±5	8±5
350.1	1→2	53±8	56±9
146.7	1→3	50±7	58±8
145.3	1→4	13±7	15±8
242.1	2	>122	>147
367.3	2	39±9	43±9
440.2	2	41±7	43±8
507	2	40±7	41±8
220.6	3	>80	>102
332.3	3	37±9	40±9
423.2	3	36±7	37±7
491.6	3	18±6	18±6
242.6	4	>53	>64
349.7	4	65±14	69±14
430.7	4	67±8	69±8
504.4	4	58±8	60±8
563.4	4	41±8	42±8

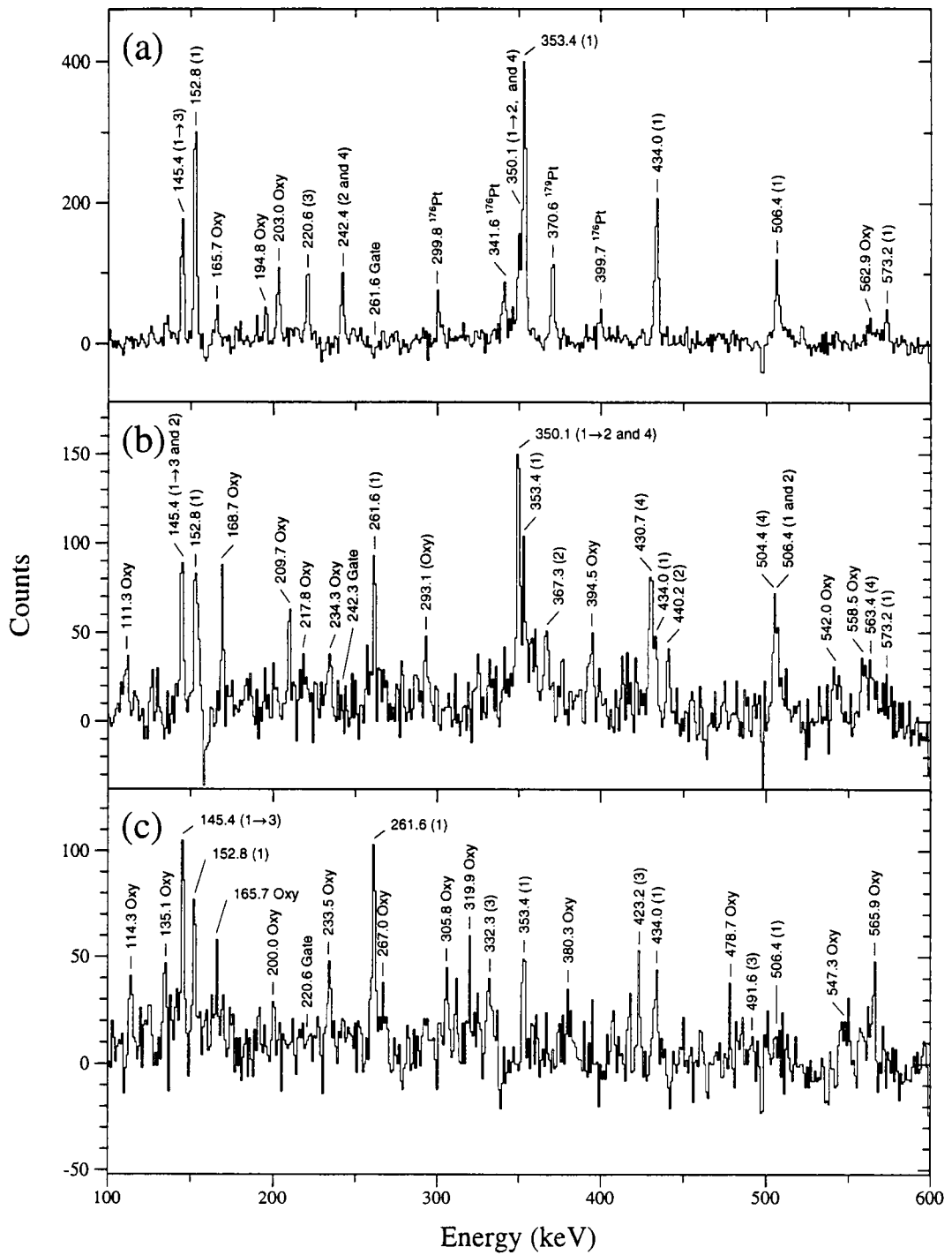


Figure 5.5: Representative coincidence spectra gated on the (a) 261.6, (b) 242.5, and (c) 220.6 keV transitions in ^{179}Au . The values in parentheses following the peak marker indicate the band in which the γ -ray transition is found (see Fig. 5.4). The γ rays denoted with “Oxy”, indicate that they are contaminants related to the decay of byproducts of the $^{58}\text{Ni}+^{16}\text{O}$ reaction.

scheme of ^{181}Au (Fig. 4.4), the strongest sequence in that nucleus is a rotational band based on the $\pi i_{13/2}$ configuration and is the yrast sequence for spins $I = \frac{21}{2}$ through $\frac{49}{2}$. Similar positive-parity bands are also observed in ^{183}Au and ^{185}Au (Fig. 4.5). From the comparison of lowest transitions in ^{185}Au , ^{183}Au , and ^{181}Au , the spin and parity of the lowest level of Band 1 in ^{179}Au has been assigned $I^\pi = \frac{13}{2}^+$. From the rotational character of the band, the γ ray transitions within the sequence are assumed to be of stretched E2 multipolarity. Decay out of Band 1 is observed to occur at the $\frac{13}{2}^+$ level into Bands 2, 3, and 4.

5.3.2 Band 2

Fig. 5.5(b) shows a spectrum gated on the transition at the bottom of Band 2. There are four transitions identified in this band. The 350-keV γ ray from the $\pi i_{13/2}$ band is observed to feed the second level of the band, which then subsequently decays within the band by a 242-keV transition. The level fed by the 242-keV γ ray is the lowest energy level observed in the decay. This transition is a doublet with the lowest transition in Band 4, accounting for the increased intensity observed in the 350 and 506-keV peaks. A 145-keV transition is also observed feeding the first excited state of Band 2 from an unknown level. The basis for the spin assignments of this band are discussed in Sec. 5.4.

5.3.3 Band 3

Four γ rays are observed in Band 3 which is connected to Band 1 via a 146.7-keV transition, as seen in Fig. 5.5(c). While the peaks in this gate are rather weak, the 332.3, 423.2, and 491.6-keV transitions are well separated from other peaks in the illustrated gate. No transitions are observed below the 220.6-keV γ ray. Possible reasons for the decay out of Band 3 not being observed are discussed in Sec. 5.4.

5.3.4 Band 4

As noted in Sec. 5.3.2, the lowest transition in Band 4 is a doublet with a transition in Band 2. Likewise, the second transition is also a doublet, however, the three

transitions observed above these γ rays identify the band. These transitions can be seen in Fig. 5.5. This band is connected to Band 1 by a 146.7-keV transition from the 592-keV level in Band 1 to the 446-keV level. Additional support for associating this band with the deexcitation of ^{179}Au , come from the presence of the 430.7, 504.4, and 563.4-keV transitions in the mass spectrum from phase 1 (Fig. 5.2(b)) of this experiment. These transitions are noted in Fig. 5.2(a) and (b). The level schemes of the potential contaminant nuclei ($^{178,179}\text{Pt}$ and ^{179}Hg) are well known. Since Band 4 cannot be associated with the decay of these products, thus this band is associated with the decay of ^{179}Au .

5.4 Configuration Assignments

The spin and parity assignments of Band 1 were made on the basis of comparison with $\pi i_{13/2}$ bands in heavier Au nuclei. The calculated alignment and Routhian energy for Band 1 are illustrated as triangles Fig. 5.6. It can be seen in Fig. 5.6(a), that the alignment of $\approx 5.5\hbar$ is consistent with the large alignment of $\pi i_{13/2}$ bands observed in other nuclei (e.g. ^{181}Au in Fig. 4.11).

The spectroscopic assignment for the other three bands observed ^{179}Au are somewhat more difficult to ascertain. By examining the low-spin structure of ^{185}Au , ^{183}Au , and ^{181}Au (Fig. 4.5), one can see that decays from the yrast $\pi i_{13/2}$ bands feed the two signatures of the $\pi h_{9/2}$ band in all three cases. A similarity to this type of feeding is also observed in ^{179}Au . It can be readily seen, however, that Bands 2 and 3 do not fit the systematic observed in the bands of the other nuclei. For example, the $\pi h_{9/2}$ signature partners in ^{181}Au have levels that are nearly degenerate. Based on the excitation energies, Bands 2 and 3 do not fit this pattern. From this comparison, it is apparent that the Band 2 is a different structure from those previously observed in Au nuclei.

From potential energy surface calculations by Cwiok [86], the ground-state configuration in ^{179}Au is predicted to be very different from ^{181}Au and heavier Au nuclei. Fig. 5.7 displays the results of these calculations as a function of neutron number where the predicted bandhead energies are labeled by their respective asymptotic

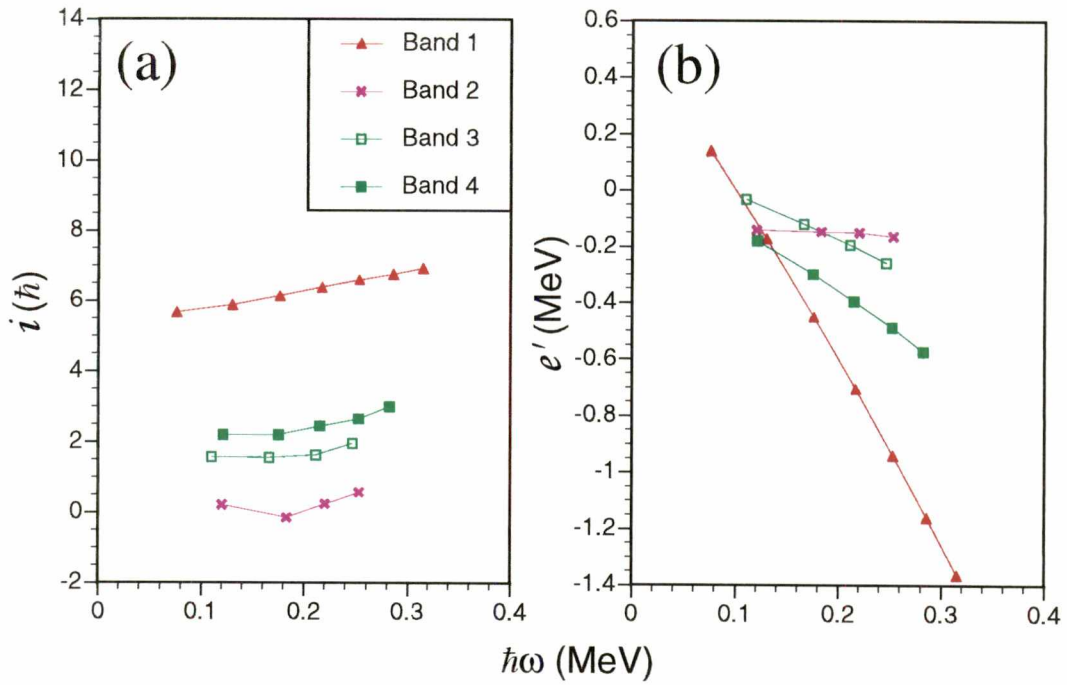


Figure 5.6: Calculated alignment (Panel “a”) and Routhian energy (Panel “b”) for experimentally measured rotational bands in ^{179}Au . The labels in the legends indicate the bands as they are labeled in Fig. 5.4. The Harris reference parameters are chosen to be $\mathcal{J}_0 = 33.5\hbar^2/\text{MeV}$ and $\mathcal{J}_1 = 130.6\hbar^4/\text{MeV}^3$ such that the Band 1 is approximately flat.

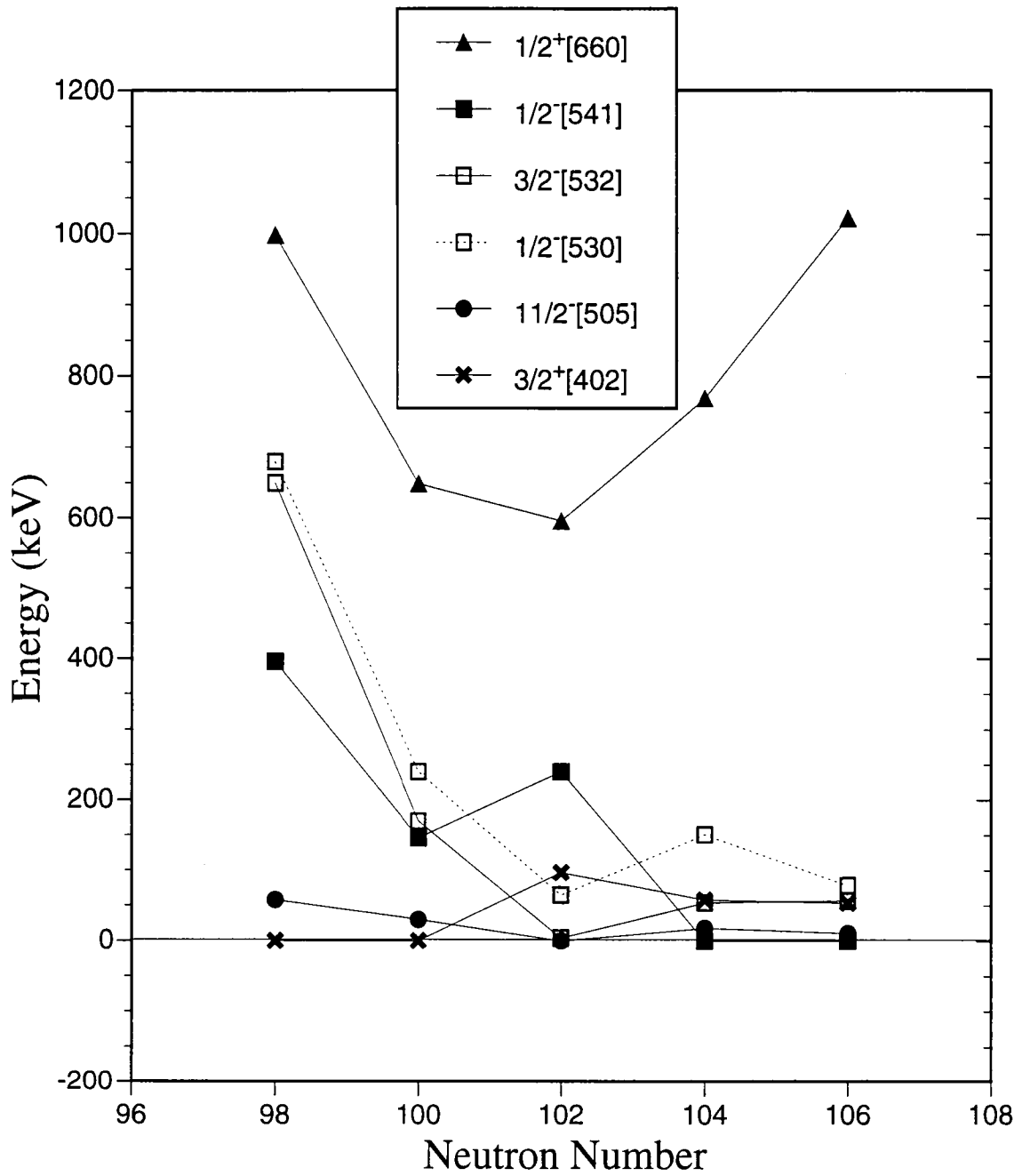


Figure 5.7: Theoretical bandhead energies [86] in even- N Au nuclei from $N = 98$ to 106 for selected Nilsson configurations.

quantum numbers. The model used for the calculations is the same as that used in Ref. [69]. For ^{185}Au , the ground-state spin and parity was measured by Ekström *et al.* [70] to be $\frac{5}{2}^-$. Later experiments and discussion [66, 87] demonstrated that the ground-state can be interpreted as predominantly $\pi h_{9/2}$. Such a configuration for ^{185}Au is also shown in the calculations presented here where the ground-state is associated with the $1/2^- [541]$ state. Similar observations and conclusions have been made in ^{183}Au [67, 65]. The new data for ^{181}Au (Chapter 4 in this dissertation) also provide a clear indication that the structure of this nucleus is similar to ^{183}Au and ^{185}Au . Indeed this is also predicted by the model calculations, where $\pi h_{9/2}$ configurations are still very low in energy. This trend changes, however, in ^{179}Au . The $\pi h_{9/2}$ configurations in ^{179}Au are predicted to increase in energy with respect to the ground-state, where the ground-state configuration is calculated to be based on a moderately deformed ($\beta_2 \sim 0.13$) $3/2^+ [402]$ orbital.

Based on the calculations presented above, Band 2 is interpreted as a rotational band based on the $3/2^+ [402]$ configuration. The expected signature partner for this band is not observed, because of the weak intensity associated with Band 2. Postulated spins for this band are presented in parentheses in Fig. 5.4. Band 1 is observed feeding Band 2, because of the configuration mixing of the close lying $\frac{13}{2}^+$ levels in both bands.

In ^{181}Au , decay out of the $\pi i_{13/2}$ bandhead (Band 5) most strongly feeds the $\frac{11}{2}^-$ state of the unfavored signature of the $\pi h_{9/2}$ configuration (Band 2). By analogy, Band 3 in ^{179}Au is interpreted as the unfavored signature $\pi h_{9/2}$ rotational band. The 146-keV γ ray from Band 1 to 3 would be equivalent to the 287-keV $\frac{13}{2}^+ \rightarrow \frac{11}{2}^-$ transition in ^{181}Au . The spins for Band 3 are denoted in Fig. 5.4 to reflect this interpretation.

If Band 4 is interpreted as the favored signature $\pi h_{9/2}$ rotational band, then the spin and parity assignments for the observed levels would be those denoted in Fig. 5.4 (i.e. $\frac{9}{2}^-, \frac{13}{2}^-, \dots$). The calculated alignment for Bands 3 and 4, with the assumed spins, are illustrated as open and filled squares, respectively, in Fig. 5.6(a). The slightly downsloping trend of the alignment of these two bands with respect to the $\pi i_{13/2}$ structure (Band 1) would be consistent with other observed $\pi h_{9/2}$ configurations

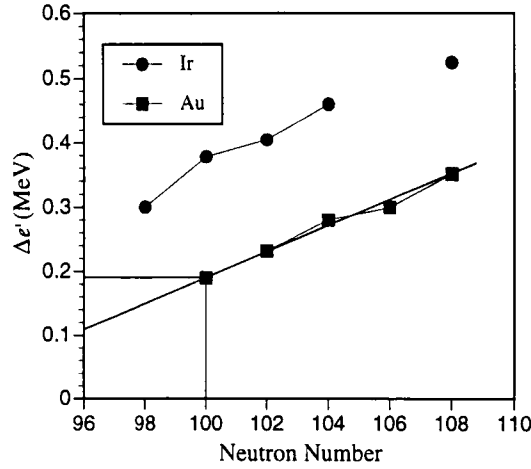


Figure 5.8: Energy signature splitting for known $\pi h_{9/2}$ bands in odd- A Ir and Au at $\hbar\omega = 0.22$ MeV.

(see e.g. Bands 1 and 2 in Fig. 4.11). The parallel paths traced by the alignments of Bands 3 and 4 are also consistent with the assignment of these structures as signature partners.

An additional support for the assignment comes from the signature splitting systematic between $\pi h_{9/2}$ partners. The signature splitting can be extracted in a quantitative way by taking the difference of the Routhian energies of signature partners at a given rotational frequency. For example, the signature splitting (denoted as $\Delta e'$) at $\hbar\omega = 0.22$ MeV between Bands 1 and 2 in ^{181}Au is extracted from Fig. 4.11(b) as 0.232 MeV. Fig. 5.8 shows the trend in signature splitting of $\pi h_{9/2}$ bands in Ir and Au as a function of neutron number. In both isotopic chains, there is a clear smooth decrease in the splitting toward smaller neutron numbers. From the plot of Routhian energies (Fig. 5.6(b)) signature splitting between the $\pi h_{9/2}$ partners in ^{179}Au is measured to be 0.19 MeV. This value is consistent with the systematic trend observed in Fig. 5.8.

CHAPTER 6

The Moment of Inertia as a Fingerprint of Deformation

6.1 Introduction

With the onset of axial quadrupole deformation, the $(2j + 1)$ -fold degeneracy of the spherical shell states is broken into doubly degenerate orbitals with good Ω quantum number. For axially symmetric prolate shapes the low- Ω orbitals drop sharply in energy as deformation increases. In high- j orbitals these configurations are strongly down-sloping with deformation, and collective rotational states based on these single particle states can be observed in nuclei with particle numbers below the shell gap of the originating orbital. Examples of these “intruder” orbitals are the $\frac{1}{2}[660]$ and $\frac{1}{2}[541]$ orbitals originating from $i_{13/2}$ and $h_{9/2}$ spherical shell states respectively.¹ Both the $i_{13/2}$ and $h_{9/2}$ are in the $82 < N < 126$ major shell, but with deformation the $\frac{1}{2}[660]$ and $\frac{1}{2}[541]$ orbitals drop dramatically in energy, and rotational bands originating from these states are observed in odd- Z nuclei from Re ($Z = 75$) to Tl ($Z = 81$) for $\frac{1}{2}[660]$ and from Ho ($Z = 67$) to Tl for $\frac{1}{2}[541]$.

One of the well known properties of intruder bands is that, due to their down-sloping character, the occupation of these orbitals enhances the deformation of the nucleus to larger prolate shapes. This increased deformation in intruder bands was demonstrated theoretically in potential energy surface calculations by Nazarewicz, *et*

¹The definition of an intruder orbital in deformed nuclei should not be confused with the intruder state of the spherical shell model. In the spherical shell model, an intruder state (also referred to as an unusual parity state) is defined as the configuration that is lowered in energy by the spin-orbit interaction into a major shell of different parity.

al. in Ref. [69]. Experimentally, the larger deformation of intruder bands can be deduced from lifetime measurements of rotational levels of a given band. Such deformation enhancement has been seen from lifetime measurements in the $\pi h_{9/2}$ band, e.g. Gascon *et al.* in Ref. [88]. Likewise, small lifetimes that are indicative of increased deformation have also been recently reported for $\pi i_{13/2}$ bands by Walpe *et al.* [89], Müller *et al.* [90], and Jin *et al.* [78]. Such a direct measurement of deformation is often very difficult or impossible, however, and alternative methods of inferring the nuclear deformation are often necessary. For example, one such property that shows a clear deformation dependence is the energy splitting between signature partners of a rotational band. Energy signature splitting in $\nu i_{13/2}$ bands in odd- N even- Z nuclei for $62 < Z < 78$ are presented by Mueller, *et al.* [72], where a deformation dependence is clearly illustrated. Another property that shows a dependence on deformation is the rotational frequency at which a time-reversed pair of $i_{13/2}$ neutrons is broken and aligned along the axis of rotation.² The dependence of the $\nu i_{13/2}$ crossing frequency ($\hbar\omega_c$) has been discussed, for example, by Jin [91]. While these two properties have an illustrated dependence on deformation, these two measurements are limited to cases where two signatures (for signature splitting) or the $\nu i_{13/2}$ bandcrossing are observed.

As a further test, it is desirable to investigate other properties that demonstrate a deformation dependence. One such quantity is the moment of inertia of a rotational band. The goal, in this chapter, is to demonstrate the validity of the moment of inertia as a fingerprint of deformation, and use this quantity to interpret the behavior of intruder rotational bands. The following section (Sec. 6.2) will discuss the extraction of the moment of inertia. The resulting extracted values for ground-state rotational bands in even-even rare-earth nuclei are illustrated in Sec. 6.3. These values are compared with extracted moments of inertia for observed $\pi h_{9/2}$ and $\pi i_{13/2}$ rotational bands in odd- Z even- N nuclei in the same region. Sec. 6.4 presents a parameterization of existing data such that from the measured moment of inertia of new bands one can infer the deformation of the nucleus to a high degree of accuracy. Deformation values from theoretical calculations are used with this deduced

²This property is often referred to as the $\nu i_{13/2}$ backbending, $\nu i_{13/2}$ alignment, or alternatively the $\nu i_{13/2}$ bandcrossing.

parameterization to infer properties of intruder bands.

6.2 Extraction of the Moment of Inertia

In Sec. 2.7, several quantities that can be calculated from the experimental data are discussed. Two examples are the quasiparticle alignment and Routhian. These two quantities aid in the identification of the quasiparticle character of specific bands, because the collective properties of the nucleus are minimized by the use of an appropriate reference (the Harris formula, Eq. 2.61). A more suitable quantity to study in order to explore the collective properties of the nucleus (such as deformation) is the moment of inertia. As was summarized in Sec. 2.7, there are two definitions for calculating the moment of inertia of a rotational band, kinematic (Eq. 2.64) and dynamic (Eq. 2.65). The method for obtaining these values from experimental data, and an example of such a calculation are also presented in Sec. 2.7.

The experimental moment of inertia contains contributions from single-particle effects as well as collective dynamics. These individual contributions can be represented by the parameters in Eqs. 2.67 and 2.68, repeated here for convenience:

$$\mathcal{J}^{(1)} = \frac{i}{\hbar\omega} + \mathcal{J}_0 + \mathcal{J}_1(\hbar\omega)^2,$$

$$\mathcal{J}^{(2)} = \frac{di}{d(\hbar\omega)} + \mathcal{J}_0 + 3\mathcal{J}_1(\hbar\omega)^2.$$

In these equations, the parameter i is the quasiparticle alignment and its term in the moment of inertia equations accounts for the one-body contributions to the moment of inertia. The parameter $\mathcal{J}_0$ represents the static collective moment of inertia, and $\mathcal{J}_1$ accounts for properties such as stretching of the nucleus and the Coriolis anti-pairing effect.³ While each of these three parameters are used to represent certain aspects of the nuclear moment of inertia, these quantities are not completely independent. This is a necessary consideration when comparing these values to theoretical calculations.

³As the nucleus rotates, Coriolis forces tend to de-align nucleons in time-reversed orbitals. The result of this effect is that pairing correlations gradually decrease with increasing rotational frequency. This effect, known as Coriolis anti-pairing (CAP), causes a gradual increase in the moment of inertia.

To extract a given i , $\mathcal{J}_0$, and $\mathcal{J}_1$ for a rotational band, one can use Eqs. 2.67 and 2.68 to fit the experimentally calculated moments of inertia, $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$, as a function of rotational frequency. The three quantities are parameters in a χ^2 fit. In this case, the quasiparticle alignment is assumed to be constant for the range of the fit, therefore, the first term of the $\mathcal{J}^{(2)}$ parameterization $\left(\frac{di}{d(\hbar\omega)}\right)$ is zero. In principle, $\mathcal{J}_0$, and $\mathcal{J}_1$ can be extracted using either definition of the moment of inertia, $\mathcal{J}^{(1)}$ or $\mathcal{J}^{(2)}$. An example of this for the cases of the ground-state band in an even-even nucleus and an intruder band in an odd- Z nucleus is shown in Fig. 2.4 from Sec. 2.7. In practice, one can judge the quality of the parameters by alternatively fitting both the $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ trends and comparing the results. The quasiparticle alignment parameter, i , can only be obtained from a fit of $\mathcal{J}^{(1)}$. In the case of even-even nuclei, i is assumed to be zero.

The model presented here is only one way of many to parameterize the experimentally observed moment of inertia. One example of an alternative formulation comes from the variable-moment-of-inertia model (VMI), initially proposed by Mariscotti *et al.* [92]. In addition, there are also other experimental quantities related to the moment of inertia that are also useful in determining nuclear deformation. Two examples of such quantities, in even-even nuclei, are the $B(E2 : 2_1^+ \rightarrow 0_1^+)$ and $E(4_1^+)/E(2_1^+)$ values. Comparison of the properties of these quantities has been studied extensively, for example, by Casten and Zamfir *et al.* [93, 94, 95, 96]. These values are useful as a probe of collectivity at low angular momentum and excitation energy. The advantage of the $(i, \mathcal{J}_0, \mathcal{J}_1)$ expansion discussed here is that it allows one to probe nuclear deformation at higher rotational frequencies, thus the procedure is very complementary to the work of Casten *et al.*.

6.3 Experimental Moments of Inertia

6.3.1 Even-Even Nuclei

Before investigating the moment of inertia of one-quasiparticle bands it is necessary to consider the moment of inertia of bands in fully paired even-even nuclei. The collective

effects in these nuclei are well studied and typically understood.⁴ The parameters $\mathcal{J}_0$ and $\mathcal{J}_1$ have been extracted from measured $\mathcal{J}^{(1)}$ values for prolate even-even ground-state bands for nuclei from Dy to Hg. These parameters are compared with the values obtained from a $\mathcal{J}^{(2)}$ analysis of the same measured bands, and variations between the two fits are reflected as an uncertainty in the extraction. Other sources of uncertainty are the number of experimental points used and the “goodness” of the fit (χ^2). The points available to be fit are those of smoothly varying behavior as a function of rotational frequency, thus transitions that are clearly influenced by non-collective effects (e.g. backbends) are omitted. In some cases this results in only a very few experimental points (< 4) being available to fit. Thus, for these cases while a very good fit might be possible, because of the small number of points, uncertainty is increased since the few points might not accurately reflect the true collective moment of inertia of the band. In some cases, the number of unperturbed points were so few that a reliable extraction of $\mathcal{J}_0$ and $\mathcal{J}_1$ was impossible (e.g. ^{180}Os).

Because of the possibility of interactions at low angular momentum, the first one or two points of a rotational band were typically omitted from the fit of the $\mathcal{J}_0$ and $\mathcal{J}_1$ parameters. Particular examples where the omission of these points is necessary is in Os, Pt, and Hg nuclei. These nuclei near the $Z = 82$ spherical shell gap are in the region where energetically low-lying oblate states are known to coexist with prolate states. Wood *et al.* [80] illustrated that the prolate and oblate states can mix and result in perturbations in the observed rotational bands. It was also demonstrated that only the lowest rotational states are significantly perturbed by the shape mixing.

The results of these fits are tabulated in Table 6.1. There are several features to note for $\mathcal{J}_0$ and $\mathcal{J}_1$ in the trends versus neutron number. These trends are summarized in Figs. 6.1(a) and 6.1(b) for $\mathcal{J}_0$ and $\mathcal{J}_1$ respectively. Around the neutron midshell ($N = 104$) where quadrupole deformation is the greatest, $\mathcal{J}_0$ parameters are maximum, with smoothly decreasing $\mathcal{J}_0$ as N becomes farther from midshell.

⁴There are still some aspects of collectivity in even-even nuclei that are not well understood. A particular example is the “hexadecapole anomaly” in certain Hf, W, and Os nuclei, where very large experimentally measured $\lambda = 4$ moments are not reproduced by theoretical calculations. See pages 109-112 in Ref. [97] for a short discussion of this problem.

Table 6.1: Experimental Harris parameters ($\mathcal{J}_0$, $\mathcal{J}_1$) extracted from ground-state rotational bands in even-even Dy through Hg nuclei. The data for Dy isotopes (in order of increasing N) come from Refs. [98, 99, 59, 59, 59], Er from Refs. [59, 100, 101, 102, 103, 104, 105, 59], Yb from Refs. [106, 107, 108, 109, 110, 111, 112, 113, 59, 59, 59], Hf from Refs. [114, 115, 115, 116, 117, 118, 119, 59, 59, 59, 59], W from Refs. [120, 121, 122, 123, 123, 124, 125, 126, 127, 128], Os from Refs. [129, 130, 130, 131, 132, 59], Pt from Refs. [84, 83, 68, 59], and Hg from Refs. [85, 133, 134, 134, 135].

N	Dy ($Z = 66$)		Er ($Z = 68$)		Yb ($Z = 70$)		Hf ($Z = 72$)	
	$\mathcal{J}_0$	$\mathcal{J}_1$	$\mathcal{J}_0$	$\mathcal{J}_1$	$\mathcal{J}_0$	$\mathcal{J}_1$	$\mathcal{J}_0$	$\mathcal{J}_1$
88			3.0 ± 3.0	225 ± 28	2.3 ± 2.3	207 ± 21		
90	21.2 ± 2.4	251 ± 16	14.0 ± 2.3	221 ± 15	10.1 ± 2.4	179 ± 16	7.9 ± 2.1	147 ± 14
92	29.9 ± 1.3	174 ± 8	23.3 ± 1.4	173 ± 10	17.0 ± 1.8	173 ± 12	12.8 ± 1.9	152 ± 13
94	34.2 ± 1.2	128 ± 8	29.0 ± 1.3	131 ± 9	24.0 ± 2.0	156 ± 13	18.0 ± 1.6	176 ± 11
96	37.0 ± 1.2	94 ± 8	32.6 ± 1.2	97 ± 8	29.0 ± 1.5	127 ± 10	23.6 ± 1.6	178 ± 10
98	40.6 ± 1.2	111 ± 8	36.9 ± 1.5	130 ± 10	34.0 ± 1.7	133 ± 12	29.1 ± 1.5	207 ± 10
100			37.5 ± 1.2	58 ± 8	35.4 ± 1.5	80 ± 10	31.3 ± 1.5	128 ± 10
102			38.0 ± 1.3	62 ± 8	38.0 ± 1.5	69 ± 10	32.7 ± 1.5	116 ± 10
104					39.1 ± 1.5	67 ± 10	33.7 ± 1.5	91 ± 10
106					36.4 ± 1.5	56 ± 10	32.0 ± 1.5	70 ± 10
108					35.5 ± 1.5	62 ± 10	32.0 ± 1.5	38 ± 10
110							30.6 ± 1.5	46 ± 10
N	W ($Z = 74$)		Os ($Z = 76$)		Pt ($Z = 78$)		Hg ($Z = 80$)	
	$\mathcal{J}_0$	$\mathcal{J}_1$	$\mathcal{J}_0$	$\mathcal{J}_1$	$\mathcal{J}_0$	$\mathcal{J}_1$	$\mathcal{J}_0$	$\mathcal{J}_1$
94	13.4 ± 1.8	171 ± 12						
96	17.8 ± 1.8	214 ± 12						
98	23.2 ± 2.0	278 ± 14	25.4 ± 3.6	336 ± 24	25.7 ± 3.2	294 ± 21		
100	25.6 ± 1.8	205 ± 12	26.7 ± 2.9	228 ± 19	29.1 ± 2.4	172 ± 16	30.3 ± 1.9	166 ± 13
102	26.8 ± 1.8	157 ± 12	21.3 ± 2.9	293 ± 19	27.5 ± 2.6	180 ± 17	29.6 ± 2.3	156 ± 16
104	27.8 ± 1.5	120 ± 10			22.4 ± 2.3	268 ± 15	27.7 ± 2.3	185 ± 15
106	28.7 ± 1.5	90 ± 10	23.1 ± 2.5	133 ± 8			24.3 ± 3.0	246 ± 20
108	29.9 ± 1.2	45 ± 8	25.2 ± 2.3	77 ± 15			19.4 ± 3.7	249 ± 25
110	26.9 ± 1.2	47 ± 8	21.9 ± 2.3	70 ± 15				
112	24.5 ± 1.7	47 ± 11						

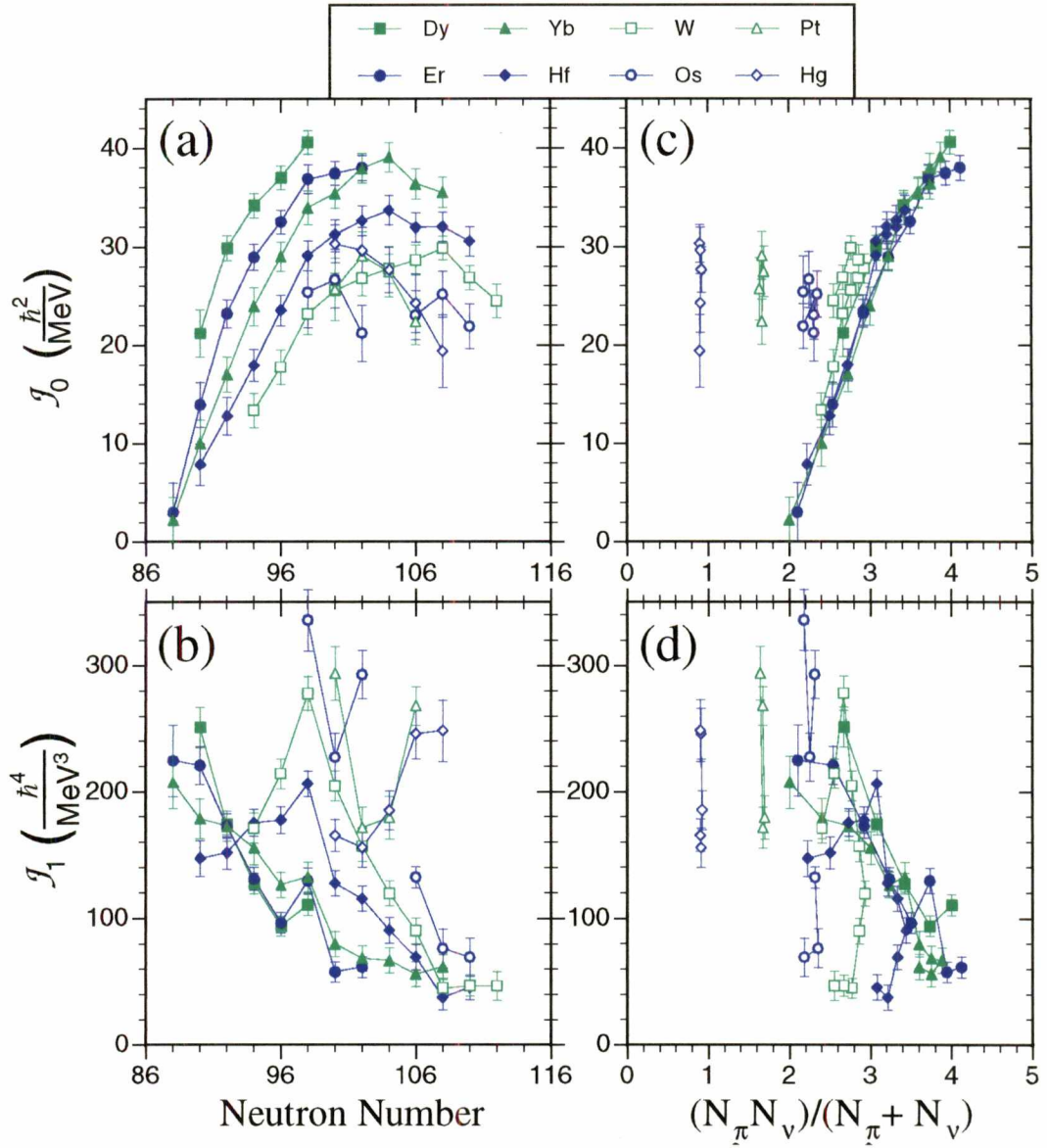


Figure 6.1: Experimental Harris parameters ($\mathcal{J}_0, \mathcal{J}_1$) for prolate even-even ground-state bands for Dy through Hg. Panels (a) and (b) are $\mathcal{J}_0$ and $\mathcal{J}_1$ values plotted vs. neutron number, respectively. Panels (c) and (d) show $\mathcal{J}_0$ and $\mathcal{J}_1$ vs. the promiscuity parameter $(N_\pi N_\nu)/(N_\pi + N_\nu)$ as defined in the text. References for experimental data are given in Table 6.1.

This clearly indicates the deformation dependence of the moment of inertia. $\mathcal{J}_0$ is also seen to decrease as a function of increasing proton number. For example, for $N = 98$ the $\mathcal{J}_0$ values from Table 6.1 are 40.6 ± 1.2 , 36.9 ± 1.5 , 34.0 ± 1.7 , 29.1 ± 1.5 , and $23.2 \pm 2.0 \hbar^2/\text{MeV}$ for Dy through W. For Os, Pt, and Hg, however, this trend of decreasing $\mathcal{J}_0$ is not strictly true. This is due to the involvement of intruder bands in the formation of the prolate states, and will be discussed later in the text.

From the comparison in Figs. 6.1(a) and 6.1(b), one can see that $\mathcal{J}_0$ and $\mathcal{J}_1$ are roughly inversely proportional. For example, the $\mathcal{J}_0$ values for Yb ($Z = 70$) isotopes show increasingly larger values from $2.3 \pm 2.3 \hbar^2/\text{MeV}$ for $N = 88$ to a maximum value of $39.1 \pm 1.5 \hbar^2/\text{MeV}$ at $N = 104$. For these same isotopes $N = 88$ has the largest $\mathcal{J}_1$ value, and this parameter is the smallest for $N = 104$. This inverse trend is indicative that nuclei with larger deformation tend to be more rigid, and the moment of inertia thus has less dependence as a function of rotational frequency. It can be noted that there is an increase in the $\mathcal{J}_1$ at $N = 98$ compared to adjacent values. This increase is particularly interesting because this is a common feature in $\mathcal{J}_1$ for *all* measured isotope chains. The explanation for this “bump” can be seen by examining Fig. 6.2. In this figure the extracted $\mathcal{J}_1$ values (Fig. 6.2(b)) are compared with a measure of the interaction slope of the $\nu i_{13/2}$ bandcrossing (V_{slope}). In this measure, V_{slope} is calculated as $\Delta\omega/\Delta I_x$ when the rotational band is plotted in the total alignment (I_x) versus rotational frequency ($\hbar\omega$) reference frame. In this parameterization, $V_{\text{slope}} < 0$ corresponds to the case of a sharp backbending, while $V_{\text{slope}} > 0$ implies a smooth upbend. One can see from Fig. 6.2(a) that all the isotope chains have a maximum V_{slope} at $N = 98$, corresponding to the same point where $\mathcal{J}_1$ increases. Thus the likely cause for the bump in $\mathcal{J}_1$ arises from the gradual $\nu i_{13/2}$ alignment taking place over a wide frequency and spin range. In general the first five numbers ($I = 2^+, \dots, 10^+$) of the rotational bands are used to extract the moments of inertia, but the resulting $\mathcal{J}_1$ value is affected when the bandcrossing affects lower spin states. One might expect that with a sharply increasing $\mathcal{J}_1$ value there would be a corresponding decrease in $\mathcal{J}_0$. This, however, is not the case, as can be seen in Fig. 6.1(a) that there is very little (if any) deviation from the trend of smoothly increasing $\mathcal{J}_0$ values at $N = 98$.

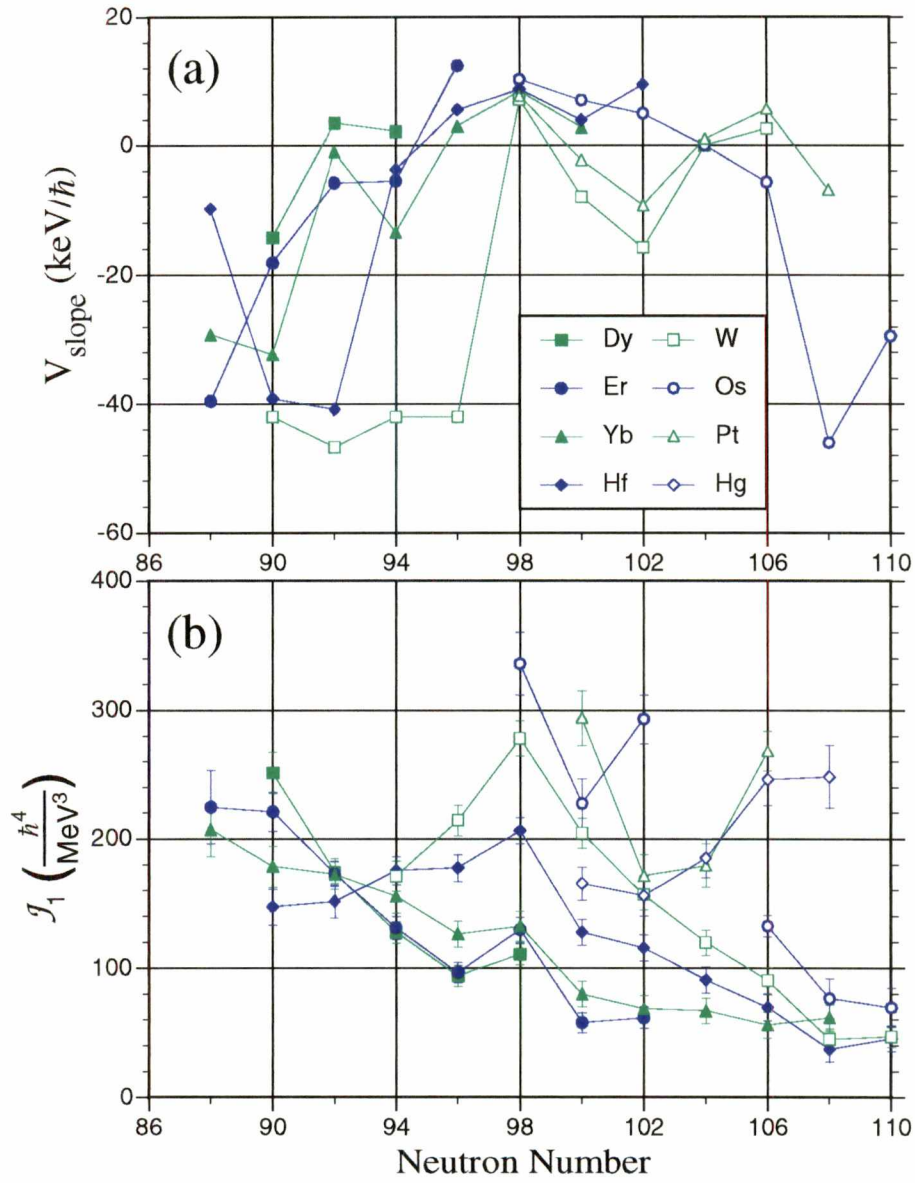


Figure 6.2: (a) Experimental interaction slope (V_{slope}) of the $\nu i_{13/2}$ band crossing in even-even nuclei for Dy through Pt. (b) J_1 vs. neutron number. (Panel (b) is identical to Fig. 6.1(b), but included here for comparative purposes.)

An alternative way to investigate the $\mathcal{J}_0$ and $\mathcal{J}_1$ parameters is to plot them as a function of the product of valence proton and neutron bosons divided by their sum ($P \equiv N_\pi N_\nu / (N_\pi + N_\nu)$). This parameter is referred to as the promiscuity factor and was first introduced by Casten *et al.* [94]. The procedure for determining the number of valence bosons is to count the number of pairs of nucleons from the nearest shell closure. For example, ^{160}Yb ($Z = 70$ and $N = 90$) has an N_π value of 6 $((82 - 70)/2)$ and an N_ν value of 4 $((90 - 82)/2)$ producing $P = 2.4$. The magnitude of the numerator of P ($N_\pi N_\nu$) describes the distance that a nucleus is from a closed shell, whether proton or neutron. Since the amount of collectivity that nuclei exhibit is to first order related to the distance from a closed shell, the quantity $N_\pi N_\nu$ is also a measure of the amount of collectivity expected for a given system. The number of valence bosons in a system also effects the two-body pairing interactions. The average effect of these interactions can be accounted by dividing by the total number of valence pairs.

The extracted $\mathcal{J}_0$ and $\mathcal{J}_1$ values are plotted against P in Figs. 6.1(c) and 6.1(d) respectively. The trend of $\mathcal{J}_0$ in Fig. 6.1(c) is a striking example illustrating the relation of the moment of inertia to the collectivity of nuclei. For Dy through Hf the $\mathcal{J}_0$ values follow nearly identical traces. For W deviations from the path set by the lighter nuclei begin to appear, and $\mathcal{J}_0$ begins to vary around a value of about $25 \hbar^2/\text{MeV}$ for Os through Hg. This deviation can be understood from the scattering of proton pairs into intruder orbitals. For $Z \approx 74$, the $h_{9/2}$ intruder orbital is close enough to the Fermi level that pairs can be scattered into it. Because of the deformation driving characteristic of the intruder orbital, scattering into the $h_{9/2}$ state will increase the deformation and, as a consequence, increase $\mathcal{J}_0$ (see Ref. [136] for an example of a discussion of the enhancement of deformation by intruder configurations around the Pb region).

The trend of $\mathcal{J}_1$ with respect to P is not as striking as the $\mathcal{J}_0$ trend, but nevertheless there are clear correlations. As can be seen in Fig. 6.1(d), there is a tendency for $\mathcal{J}_1$ to be minimum for large P and gradually become larger as one comes closer to a closed shell. There is significant deviation from the regular trend of $\mathcal{J}_1$ for Hf, W, and Os nuclei with neutron numbers greater than 108. For these nuclei,

$\mathcal{J}_1$ is approximately equal to $50 \hbar^4/\text{MeV}^2$. The reason for these deviations in nuclei with $N \geq 108$ may be related to the deformed shell gap that occurs at $N = 108$ (see Fig. 2.2). It could be postulated that this gap decreases the pairing and thus CAP effects are not as prevalent, thus reducing $\mathcal{J}_1$. Marshalek detailed, in Refs. [137, 138], second order effects of nuclear rotation to the nuclear Hartree-Fock-Bogoliubov mean field. Marshalek considered the expansion of rotational energy in the following way:

$$E_I = \frac{\hbar^2}{2\mathcal{J}} I(I+1) + \mathcal{B} I^2(I+1)^2 + \dots \quad (6.93)$$

This expansion is different from the one used by Harris in Ref. [35] which is the basis for the $\mathcal{J}_0$ and $\mathcal{J}_1$ parameters shown here. While the Harris parameter $\mathcal{J}_1$ is not directly related to $\mathcal{B}$, the parameter considered by Marshalek, the qualitative aspects of Marshalek's calculations can be considered. Marshalek noted that the centrifugal stretching of the nucleus and Coriolis anti-pairing effects contribute to the parameter, $\mathcal{B}$, with nearly equal weight. Thus it would be difficult to make a judgment as to the cause of the suppression of $\mathcal{J}_1$ for $N > 108$ without further detailed mean field calculations.

6.3.2 Odd-*A* Nuclei

The $\mathcal{J}_0$ and $\mathcal{J}_1$ values for $\pi h_{9/2}$ and $\pi i_{13/2}$ bands in odd- Z , even- N nuclei from Ho to Tl have been extracted from experimental data in a manner similar to the even-even nuclei in the previous section. One particular difference for these odd- A nuclei, however, is the addition of alignment caused by the unpaired valence proton. Thus a fit of the kinematic moment of inertia ($\mathcal{J}^{(1)}$) involves a third parameter, the quasiparticle alignment (i). The results of the fit to $\mathcal{J}^{(1)}$ are compared with the values extracted from $\mathcal{J}^{(2)}$ as a self-consistency check. These results for Ho, Tm, and Lu nuclei are summarized Fig. 6.3 and Table 6.2; Ta, Re, and Ir in Fig. 6.4 and Table 6.3; and Au and Tl in Fig. 6.5 and Table 6.4. Included in each of the figures is the average $\mathcal{J}_0$ or $\mathcal{J}_1$ value of the $Z - 1$ and $Z + 1$ even-even cores for the respective nucleus. For example, the points denoted by "Ave." in Fig. 6.3(b) represent the average of Er and Yb $\mathcal{J}_0$ values. Thus "Ave." trends represent the expected $\mathcal{J}_0$ and $\mathcal{J}_1$ values if single-particle effects are neglected.

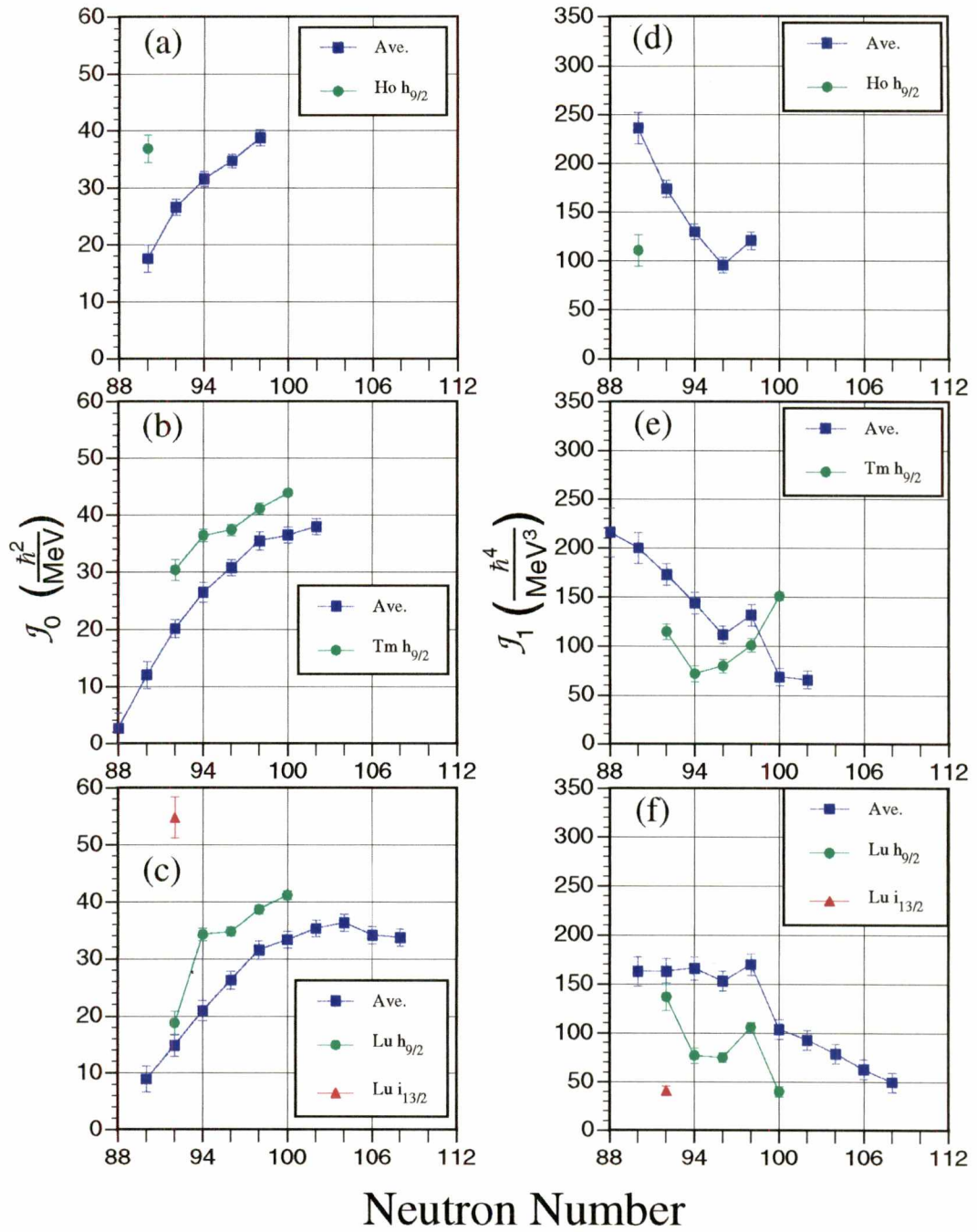


Figure 6.3: Extracted $\mathcal{J}_0$ and $\mathcal{J}_1$ for intruder bands in odd- A Ho, Tm, and Lu. The label “Ave.” denotes the average $\mathcal{J}_0$ or $\mathcal{J}_1$ value from the ground-state bands of respective particle and hole cores of each nucleus plotted. References for experimental data are given in Table 6.2.

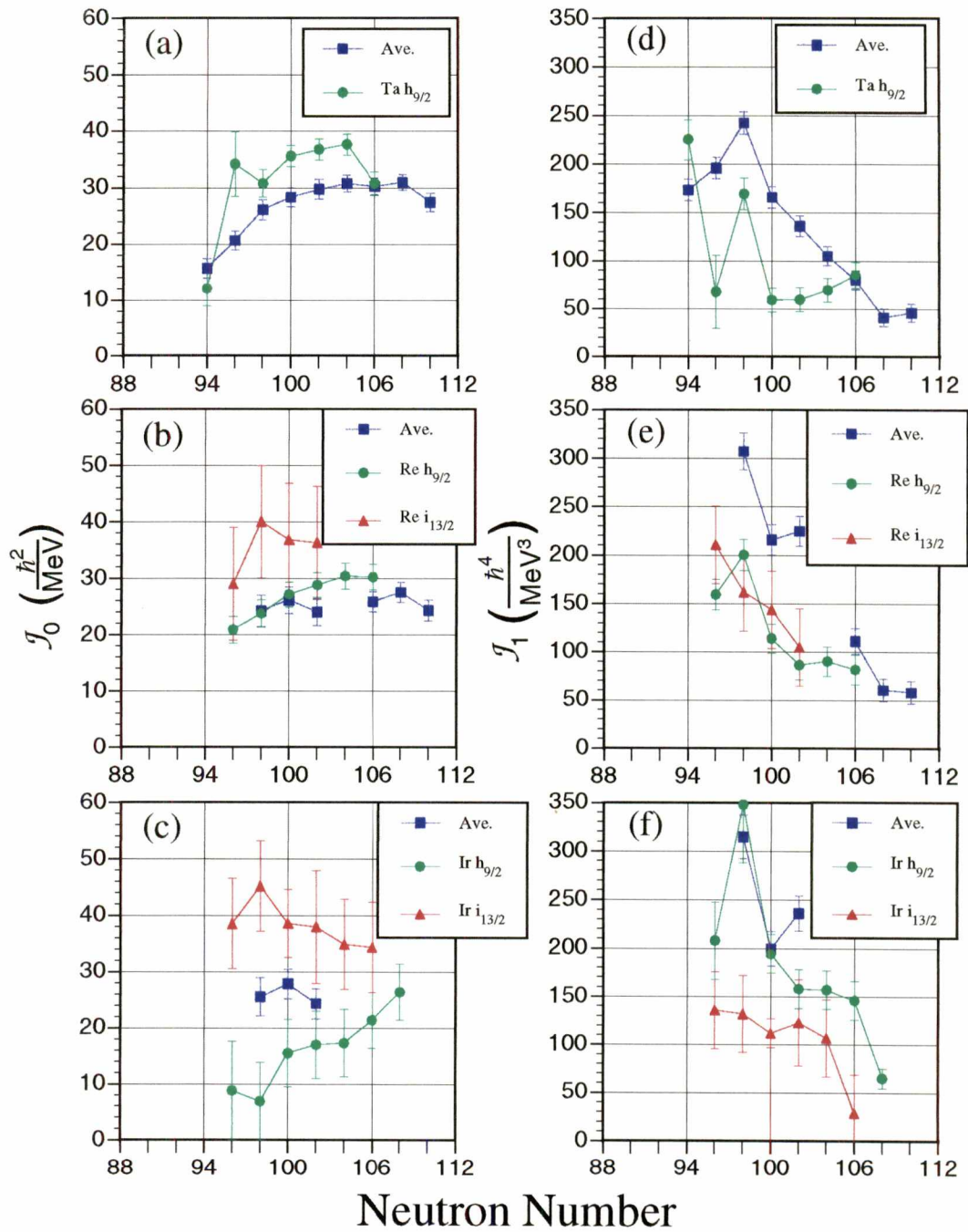


Figure 6.4: Extracted J_0 and J_1 for intruder bands in odd- A Ta, Re, and Ir. The label “Ave.” denotes the average J_0 or J_1 value from the ground-state bands of respective particle and hole cores of each nucleus plotted. References for experimental data are given in Tables 6.3 and 6.4.

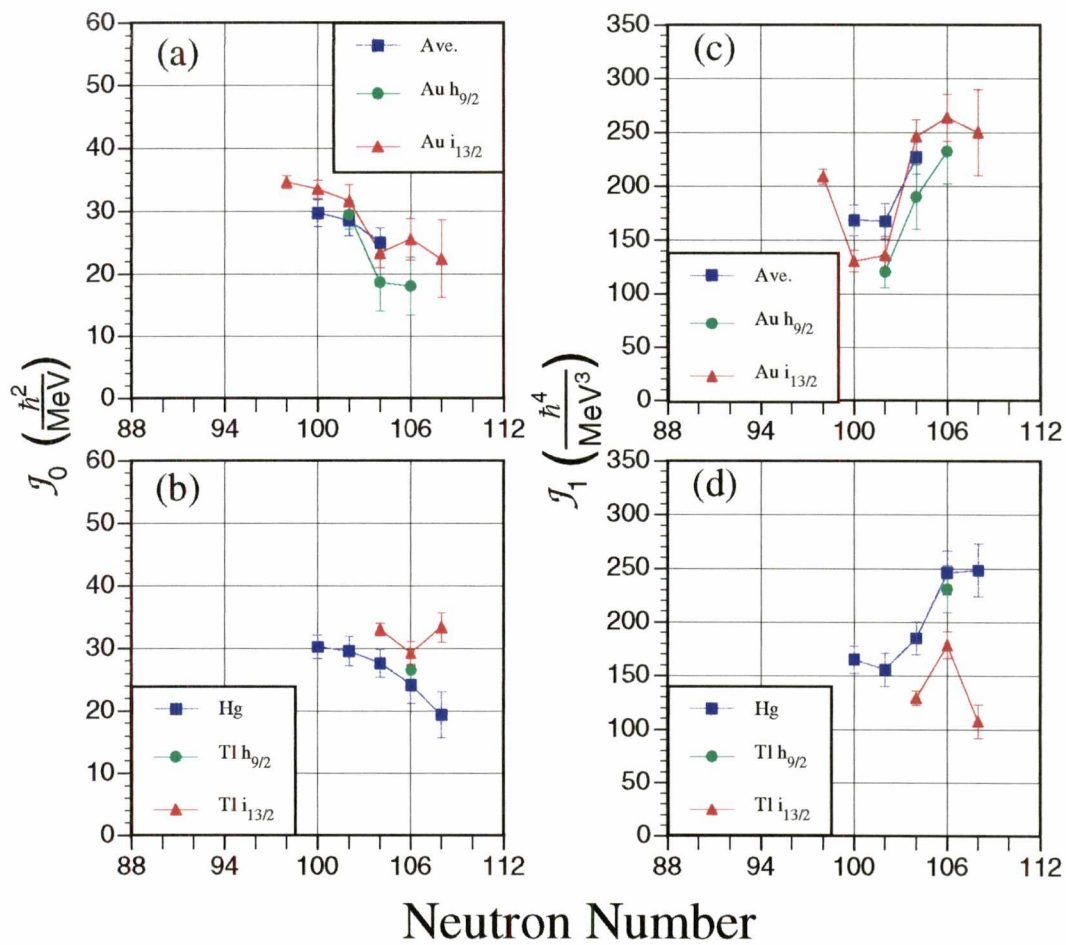


Figure 6.5: Extracted $\mathcal{J}_0$ and $\mathcal{J}_1$ for intruder bands in odd- A Au and Tl. The label “Ave.” in panels (a) and (c) denotes the average $\mathcal{J}_0$ or $\mathcal{J}_1$ value from the ground-state bands of Pt and Hg. “Hg” in panels (b) and (d) indicates values from Hg. References for experimental data are given in Table 6.4.

Table 6.2: Experimental i , $\mathcal{J}_0$, and $\mathcal{J}_1$ values for $\pi h_{9/2}$ and $\pi i_{13/2}$ intruder bands in odd- A Ho, Tm, and Lu. The data for Ho come from Ref. [88], Tm isotopes (in order of increasing N) from Refs. [139, 140, 141, 59, 59], and Lu from Refs. [142, 143, 144, 145, 59].

N	Ho ($Z = 67$) $\pi h_{9/2}$			Tm ($Z = 69$) $\pi h_{9/2}$		
	i	$\mathcal{J}_0$	$\mathcal{J}_1$	i	$\mathcal{J}_0$	$\mathcal{J}_1$
90	1.96 ± 0.24	36.9 ± 2.4	111 ± 16			
92				2.60 ± 0.18	30.4 ± 1.8	115 ± 8
94				1.95 ± 0.09	36.4 ± 1.1	72 ± 8
96				2.09 ± 0.09	37.4 ± 1.0	80 ± 7
98				2.13 ± 0.09	41.1 ± 1.0	101 ± 7
100				2.18 ± 0.05	43.9 ± 0.5	151 ± 4
N	Lu ($Z = 71$) $\pi h_{9/2}$			Lu ($Z = 71$) $\pi i_{13/2}$		
	i	$\mathcal{J}_0$	$\mathcal{J}_1$	i	$\mathcal{J}_0$	$\mathcal{J}_1$
92	1.59 ± 0.21	18.8 ± 2.1	137 ± 14	1.18 ± 0.36	54.8 ± 3.6	41 ± 5
94	1.90 ± 0.10	34.3 ± 1.1	77 ± 8			
96	2.20 ± 0.08	34.8 ± 0.8	75 ± 5			
98	2.25 ± 0.08	38.7 ± 0.8	106 ± 5			
100	2.31 ± 0.08	41.2 ± 0.8	40 ± 5			

Table 6.3: Experimental i , $\mathcal{J}_0$, and $\mathcal{J}_1$ values for $\pi h_{9/2}$ and $\pi i_{13/2}$ intruder bands in odd- A Ta and Re. The data for Ta isotopes (in order of increasing N) come from Refs. [146, 147, 148, 148, 149, 150, 151], and Re from Refs. [152, 153, 154, 154, 155, 132].

N	Ta ($Z = 73$) $\pi h_{9/2}$		
	i	$\mathcal{J}_0$	$\mathcal{J}_1$
94	9.43 ± 0.31	12.1 ± 3.1	225 ± 21
96	1.90 ± 0.57	34.2 ± 5.7	68 ± 38
98	2.79 ± 0.24	30.8 ± 2.4	169 ± 16
100	2.62 ± 0.18	35.6 ± 1.8	59 ± 12
102	2.80 ± 0.18	36.7 ± 1.8	60 ± 12
104	3.00 ± 0.18	37.6 ± 1.8	70 ± 12
106	1.74 ± 0.20	30.8 ± 2.0	85 ± 13

N	Re ($Z = 75$) $\pi h_{9/2}$			Re ($Z = 75$) $\pi i_{13/2}$		
	i	$\mathcal{J}_0$	$\mathcal{J}_1$	i	$\mathcal{J}_0$	$\mathcal{J}_1$
96	3.25 ± 0.24	20.9 ± 2.4	159 ± 16	4.34 ± 1.00	29.1 ± 10.0	211 ± 40
98	3.21 ± 0.24	23.9 ± 2.4	200 ± 16	4.2*	40.1 ± 10.0	162 ± 40
100	3.04 ± 0.24	27.2 ± 2.3	114 ± 15	4.14 ± 1.00	36.9 ± 10.0	144 ± 40
102	3.14 ± 0.24	28.9 ± 2.3	87 ± 15	4.1*	36.4 ± 10.0	105 ± 40
104	3.33 ± 0.24	30.5 ± 2.3	90 ± 15			
106	3.59 ± 0.24	30.3 ± 2.3	82 ± 15			

* These values were fixed in the fit of $\mathcal{J}^{(1)}$.

Table 6.4: Experimental i , $\mathcal{J}_0$, and $\mathcal{J}_1$ values for $\pi h_{9/2}$ and $\pi i_{13/2}$ intruder bands in odd- A Ir, Au, and Tl. The data for Ir isotopes (in order of increasing N) come from Refs. [156, 157, 154, 78, 79, 158, 159], ^{177}Au from Ref. [160], $^{179,181}\text{Au}$ from this dissertation, $^{183,185,187}\text{Au}$ from Refs. [65, 64, 161], and $^{185,187,189}\text{Tl}$ from Refs. [76, 76, 75].

N	Ir ($Z = 77$) $\pi h_{9/2}$			Ir ($Z = 77$) $\pi i_{13/2}$		
	i	$\mathcal{J}_0$	$\mathcal{J}_1$	i	$\mathcal{J}_0$	$\mathcal{J}_1$
96	4.1*	8.8 ± 8.8	208 ± 40	4.0*	38.6 ± 8.0	136 ± 40
98	4.1*	6.9 ± 6.9	348 ± 60	4.06 ± 0.80	45.2 ± 8.0	132 ± 40
100	4.11 ± 0.60	15.5 ± 6.0	194 ± 20	4.72 ± 0.60	38.6 ± 6.0	112 ± 15
102	4.10 ± 0.60	17.0 ± 6.0	158 ± 20	4.6*	38.0 ± 10.0	123 ± 45
104	4.25 ± 0.60	17.3 ± 6.0	157 ± 20	4.50 ± 0.80	34.9 ± 8.0	107 ± 40
106	4.12 ± 0.50	21.4 ± 5.0	146 ± 20	4.47 ± 0.80	34.4 ± 8.0	29 ± 40
108	3.94 ± 0.50	26.4 ± 5.0	65 ± 10			
N	Au ($Z = 79$) $\pi h_{9/2}$			Au ($Z = 79$) $\pi i_{13/2}$		
	i	$\mathcal{J}_0$	$\mathcal{J}_1$	i	$\mathcal{J}_0$	$\mathcal{J}_1$
98				5.08 ± 0.10	34.7 ± 1.0	209 ± 7
100				5.36 ± 0.15	33.5 ± 1.4	131 ± 10
102	2.45 ± 0.24	29.4 ± 2.3	121 ± 15	5.43 ± 0.27	31.6 ± 2.7	136 ± 18
104	4.25 ± 0.50	18.7 ± 4.6	190 ± 30	5.94 ± 0.24	23.4 ± 2.4	246 ± 16
106	4.33 ± 0.50	18.0 ± 4.6	232 ± 30	5.49 ± 0.33	25.6 ± 3.3	264 ± 22
108				5.48 ± 0.62	22.4 ± 6.2	250 ± 42
N	Tl ($Z = 81$) $\pi h_{9/2}$			Tl ($Z = 81$) $\pi i_{13/2}$		
	i	$\mathcal{J}_0$	$\mathcal{J}_1$	i	$\mathcal{J}_0$	$\mathcal{J}_1$
104				5.76 ± 0.10	33.1 ± 1.0	130 ± 7
106	2.14 ± 0.80	26.6 ± 8.0	231 ± 40	5.54 ± 0.19	29.3 ± 1.9	179 ± 12
108				4.98 ± 0.24	33.4 ± 2.4	108 ± 16

* These values were fixed in the fit of $\mathcal{J}^{(1)}$.

From the comparison of the $\pi h_{9/2}$ intruder bands (circles) from Ho to Ta with their respective cores (squares), one can see that, for all these nuclei (with only two exceptions in Ta), the $\pi h_{9/2}$ values show an enhanced $\mathcal{J}_0$ over the expected value from the even-even cores. For example, the $\mathcal{J}_0$ values for $\pi h_{9/2}$ bands in Lu are as much as 20% larger than the $(\text{Yb} + \text{Hf})/2$ core values. But, then a change occurs, as the difference between $\mathcal{J}_0$ for the $\pi h_{9/2}$ bands in Re and the respective cores are very small or negligible (see Fig. 6.4(b)). This indicates that the Fermi level in Re has come very close to the $\pi h_{9/2}$ shell, and the deformation-driving characteristic of the $\pi h_{9/2}$ band becomes less important. The $\pi h_{9/2}$ bands in Ir, Au, and Tl also exhibit $\mathcal{J}_0$ values equivalent or even less than their respective cores. In analogy to the $\pi h_{9/2}$ bands, the $\mathcal{J}_0$ values for $\pi i_{13/2}$ also show qualitatively similar trends. For the observed $\pi i_{13/2}$ band in ^{163}Lu as well as bands in Re and Ir, $\mathcal{J}_0$ is greatly enhanced over the core values. For corresponding bands in Au and Tl nuclei the enhancement is very small.

As was noted in Sec. 6.3.1, the trends in $\mathcal{J}_1$ are approximately the inverse of those in $\mathcal{J}_0$. This general trend is also observed in the odd- A intruder bands. For example, while the $\mathcal{J}_0$ values in Lu show an enhancement over the core values, the $\mathcal{J}_1$ values are suppressed with respect to the corresponding core values. One set that violates this trend, however, is the chain of isotopes of Tm (see Fig. 6.3(b) and (e)). For $N = 94 - 100$, the $\mathcal{J}_1$ values are steadily increasing. This is contrary to the expected trend, since for this same range the $\mathcal{J}_0$ values are also increasing.

The systematic trend for the extracted quasiparticle alignment (i) parameters is shown in Fig. 6.6. Those bands where i was set to a fixed value (see Tables 6.3 and 6.4) are not plotted in the figure. Considering $\pi h_{9/2}$ bands in Fig. 6.6(a), one can see that, for nuclei where the $\frac{1}{2}[541]$ configuration is far above the Fermi level (i.e. Ho, Tm, and Lu), average alignment is $\approx 2\hbar$. As proton number increases the average alignment also increases to a maximum value $\approx 4\hbar$ for Ir.

This trend in alignment can be understood by considering a simple example of a single- j shell. If a particular state is characterized by $|j, \Omega\rangle$, where Ω is the projection on the axis of deformation, the wavefunction can be transformed to a basis

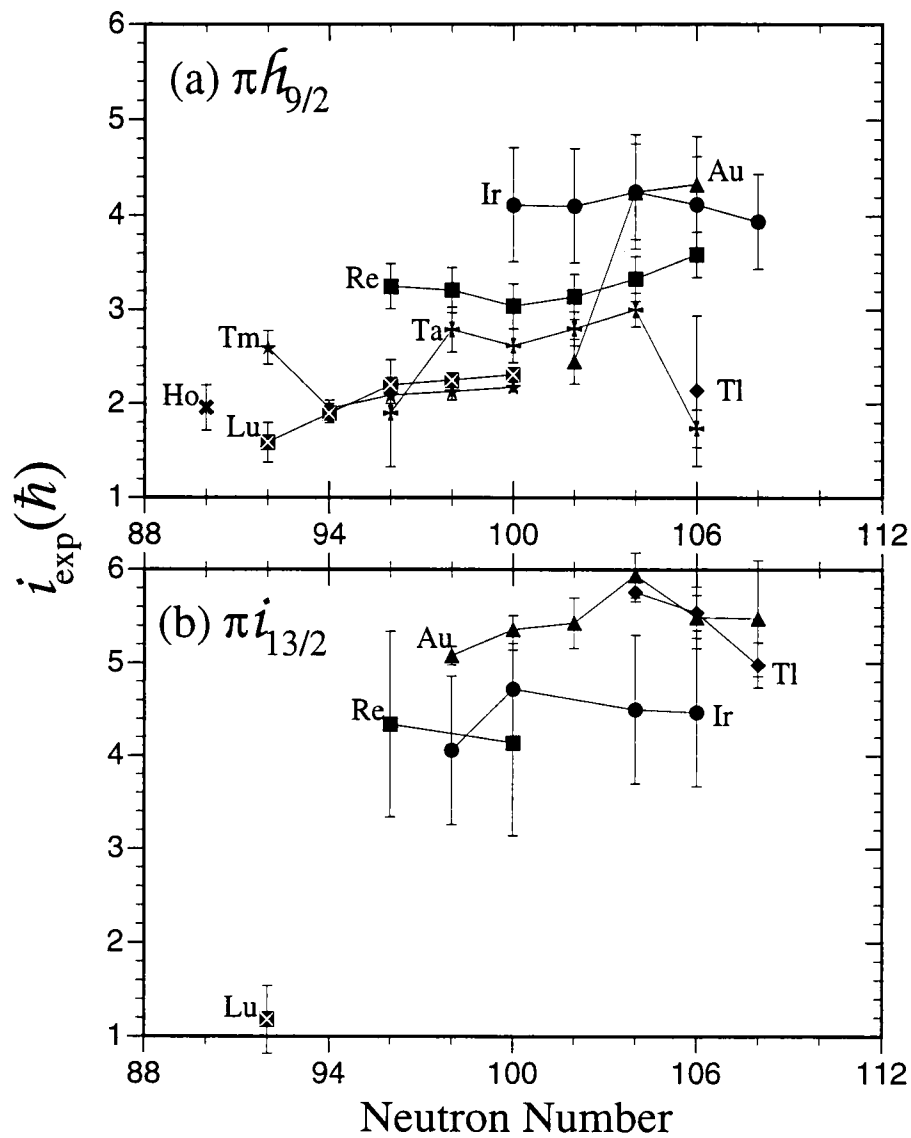


Figure 6.6: Experimental quasiparticle alignment (i) for (a) $\pi h_{9/2}$ and (b) $\pi i_{13/2}$ bands in odd- A Ho through Tl.

of good angular momentum projection on the rotation axis (I_x) by

$$|j, I_x\rangle = \sum_{\Omega} d_{I_x\Omega}^j \left(\frac{\pi}{2}\right) |j, \Omega\rangle,$$

where $d_{I_x\Omega}^j \left(\frac{\pi}{2}\right)$ are the matrix elements of finite rotations. Thus, a state, $|j = 9, I_x = 9\rangle$ becomes

$$|9, 9\rangle = 0.431|j = 9, \Omega = 0\rangle + 0.409|9, 1\rangle - 0.348|9, 2\rangle + \dots$$

in the good- I_x basis. Even for the lowest Ω state, the maximally aligned state ($I_x = 9$) is only $\approx 20\%$ of the total wavefunction. Maximum alignment is achieved when contributions from all $(2j + 1)$ states of good- Ω are included.

In the case of the $\pi h_{9/2}$ intruder orbital, the large- Ω states are very far from the Fermi surface (see Fig. 2.3) and do not contribute significantly to the alignment. Nuclei with Fermi levels within the $\pi h_{9/2}$ shell (e.g. Ir) should have the greatest amount of contribution from higher-lying states and thus the greatest alignment. Contributions become progressively less farther from the shell. This trend can be seen in the continually reducing alignment for Re through Lu. A similar trend is observed in $\pi i_{13/2}$ bands (Fig. 6.6(b)). To within the uncertainty of the results, the alignment is roughly constant for a given isotope chain.

One aspect that is not understood is the very low alignment of the $\pi h_{9/2}$ bands in ^{181}Au and ^{187}Tl (see Fig. 6.6(a)). It is expected that $\pi h_{9/2}$ bands in these nuclei would have alignments comparable to or greater than Ir, as is the case for ^{183}Au and ^{185}Au . One possibility for this lower alignment may arise from the contribution from the $\pi f_{7/2}$ configuration.

6.4 Parameterization of Even-Even Nuclei

It is clear from the discussion in Sec. 6.3 that there is a deformation dependence in the first order moment of inertia ($\mathcal{J}_0$). In order to more readily quantify this dependence, one must consider other correlations as well. It is well known that the moment of inertia depends not only on deformation but also residual interactions. The results of these effects were discussed by Migdal [13] and Belyaev [12, 162] within the frame

work of the Inglis [15] cranking model. In his work, Migdal presented a qualitative formulation of the moment of inertia that accounts for pairing correlations:

$$\mathcal{J} = \mathcal{J}_{\text{rig}} \left[1 - f \left(\frac{\delta \hbar \omega_0}{2\Delta} \right) \right], \quad (6.94)$$

where

$$f(x) \equiv \frac{\ln \left(x + (1 + x^2)^{1/2} \right)}{x(1 + x^2)^{1/2}}. \quad (6.95)$$

$\mathcal{J}_{\text{rig}}$ is defined as the rigid moment of inertia, δ is the deformation parameter from the deformed modified harmonic oscillator potential expressed in spherical coordinates (see Sec. 2.2.1 for the definition), $\hbar \omega_0$ is the characteristic harmonic oscillator frequency which to first order can be represented as

$$\hbar \omega_0 = \frac{41}{A^{1/3}} [\text{MeV}], \quad (6.96)$$

and Δ is the pairing energy-gap parameter discussed in Sec. 2.3.

To lowest order the rigid moment of inertia is proportional to $A^{5/3}$, and for small deformation $\delta \propto \beta$, where β is the Hill-Wheeler deformation parameter discussed in Sec. 2.1.1. The function, $f(x)$, of Eq. 6.95 can be expanded around zero to first order as

$$f(x) \approx 1 - \frac{2}{3}x^2. \quad (6.97)$$

Using this expansion in Eq. 6.94 and applying the above approximations for $\mathcal{J}_{\text{rig}}$ and δ , one obtains

$$\mathcal{J} = K A \left(\frac{\beta}{\Delta} \right)^2, \quad (6.98)$$

where K is a constant of proportionality. It should be noted that, for well deformed nuclei, $f(x)$ is better represented by a linear expansion (i.e. $f(x) \approx a - bx$). To better reflect this dependence one can take the square root of Eq. 6.98, and thus the final relation becomes

$$\mathcal{F}_{\mathcal{J}} = 6.5 A^{1/2} \left(\frac{\beta}{\Delta} \right), \quad (6.99)$$

where the constant of proportionality was chosen such that this factor ($\mathcal{F}_{\mathcal{J}}$) will be approximately the same magnitude as $\mathcal{J}_0$.

To test the quality of the relationship in Eq. 6.99, one can compare $\mathcal{F}_{\mathcal{J}}$, calculated with experimentally deduced deformation and pairing values, with the extracted $\mathcal{J}_0$ values from Sec. 6.3.1. To this end, experimental β and Δ values are obtained from compiled sources (Ref. [163] for β and Ref. [46] for Δ). These values are illustrated in Fig. 6.7. It should be noted that the β values of Fig. 6.7(a) are calculated from experimental $B(E2; 0^+ \rightarrow 2^+)$ values in the manner by Raman *et al.*

$$\beta = (4\pi/3ZR_0^2) \{B(E2; 0^+ \rightarrow 2^+)/e^2\}^{1/2}. \quad (6.100)$$

Thus this parameterization represents contributions from not only quadrupole deformation, but also higher order terms such as hexadecapole correlations. The effective Δ from proton and neutron contributions was determined in the same manner as Ref. [164]. Specifically: $\Delta_{eff} \equiv \sqrt{(\Delta_n^2 + \Delta_p^2)/2}$. The procedure is used rather than an arithmetic average, because the pairing energy depends on Δ^2 .

Fig. 6.8 illustrates $\mathcal{J}_0$ plotted as a function of the $\mathcal{F}_{\mathcal{J}}$ parameter, where $\mathcal{F}_{\mathcal{J}}$ is calculated using the experimental values from Fig. 6.7. It can be seen from this figure that there is a remarkably high degree of correlation between the $\mathcal{J}_0$ and $\mathcal{F}_{\mathcal{J}}$ parameters. The only significant deviations from the correlated trace are the two Hg nuclei. This deviation arises from the prolate-oblate shape coexistence in Hg as was summarized in Sec. 6.3.1. The β used in $\mathcal{F}_{\mathcal{J}}$ is obtained from $B(E2; 0^+ \rightarrow 2^+)$ which is a measure of the nuclear deformation at low spin, while $\mathcal{J}_0$ was extracted for $I > 2^+$ and a measure at higher angular momentum. Hg nuclei undergo substantial shape changes at low spin, thus resulting in a significant deviation from the trace. The solid line through the trace of well correlated nuclei is the result of a χ^2 fit of these nuclei, and is defined as

$$\mathcal{J}_0 = -0.432\mathcal{F}_{\mathcal{J}}^2 + 3.61\mathcal{F}_{\mathcal{J}} - 33.7. \quad (6.101)$$

With only a few exceptions, this fit is within the uncertainty of nearly all points.

With the clearly demonstrated correlation of $\mathcal{J}_0$ to $\mathcal{F}_{\mathcal{J}}$, it is now possible compare this trace to calculated deformation values. For this analysis, deformation parameters are taken from the results of a cranked-triaxial-Nilsson-Strutinsky approach with a Woods-Saxon potential. An overview of this model, typically referred to as the total Routhian surface (TRS) calculation, is presented in Sec. 2.6.3. In the

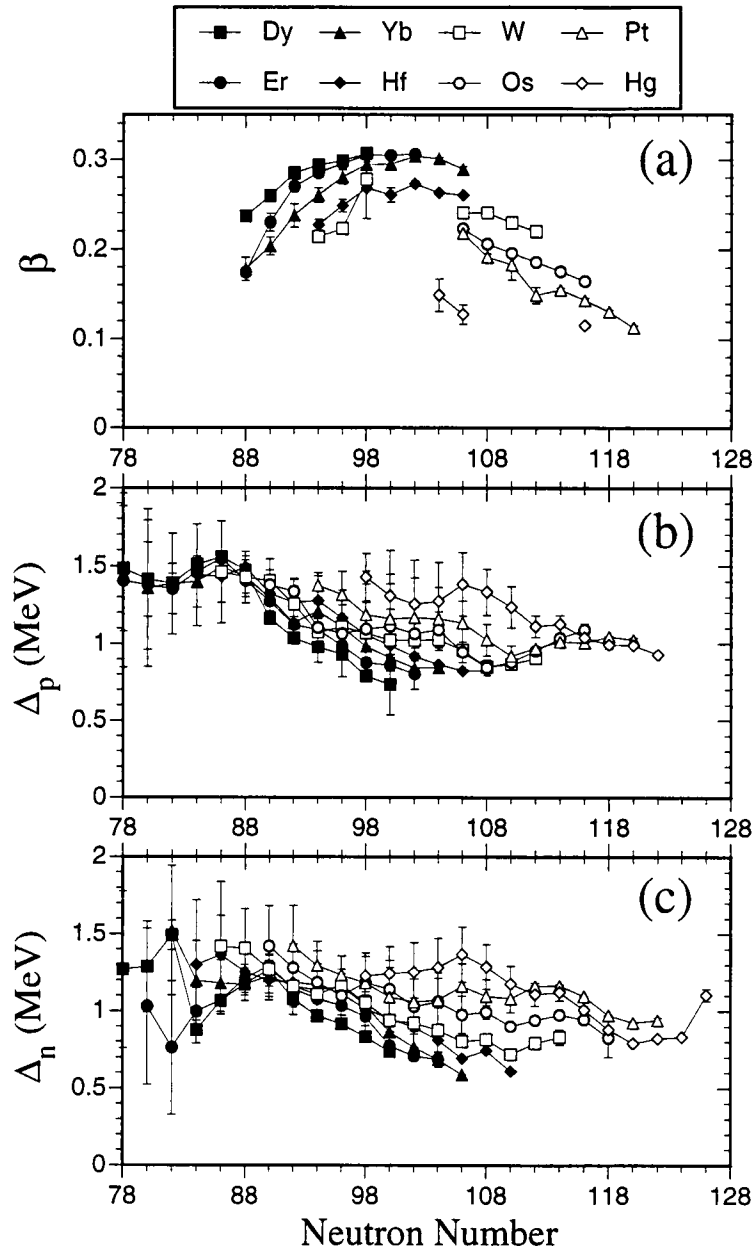


Figure 6.7: (a) Experimental deformation, (b) proton pair gap, (c) neutron pair gap values for even-even nuclei from Dy to Hg. The deformation parameters are obtained from the compilation of measured $B(E2)$ values by Raman *et al.* [163]. Pair gap parameters are taken from even-odd mass differences from the collection of Wapstra and Audi [46].

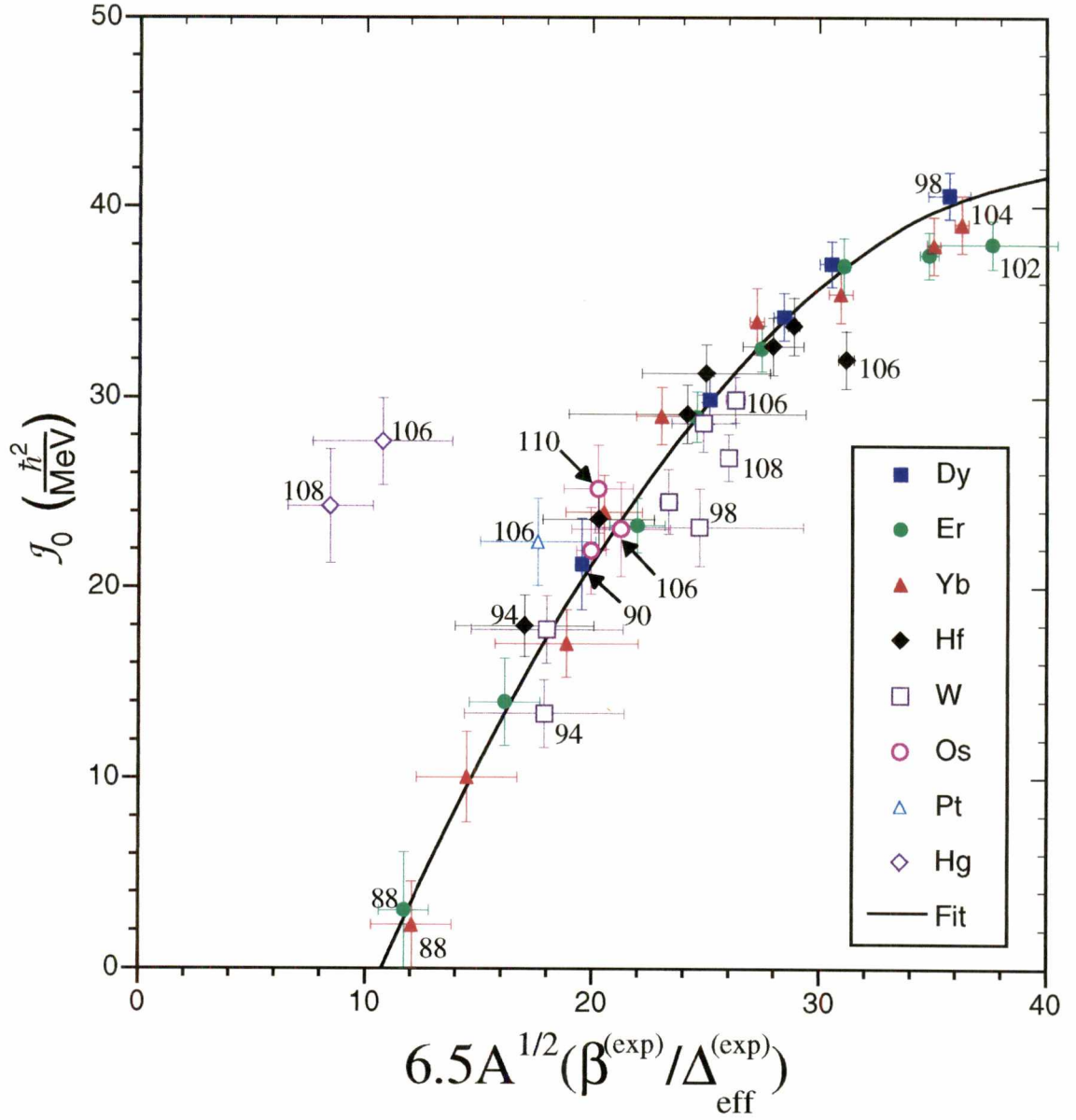


Figure 6.8: Experimental $\mathcal{J}_0$ plotted against $\mathcal{F}_{\mathcal{J}}$ (see Eq. 6.99) for even-even nuclei from Dy to Hg. The label “Fit” corresponds to a χ^2 fit to the trace of nuclei with $N < 104$. The values in the figure indicate the neutron number of selected nuclei.

execution of the TRS calculations, neighboring nuclei are organized into groups, referred to as meshes. These meshes tend to be limited in the number of nuclei that they contain, and for each mesh the Woods-Saxon parameters are optimal for the central nucleus. This produces slightly improper single-particle levels and wave functions for other nuclei particularly at the edges of the mesh. In addition to this approximation, pairing was treated self-consistently only at $\hbar\omega = 0$, and is varied according to a simple phenomenological formula. Furthermore, total Routhian energies are calculated only at prescribed points of a deformation grid, and deformation values at a energy minimum are obtained through interpolation.

Of the two meshes used in this analysis, one is centered around ^{168}Yb [165], and spans nuclei from $Z = 60 - 74$ and $N = 84 - 104$. For deformation parameters for heavier nuclei, a mesh centered around ^{182}Pt [34] is utilized. This mesh contains nuclei from $Z = 72 - 82$ and $N = 94 - 118$. The values from these calculations are extracted at $\hbar\omega = 0.17$ MeV. For two nuclei beyond the limits of the meshes presented here (^{176}Yb and ^{178}Yb), deformation parameters are taken from Ref. [69]. These results are summarized in Fig. 6.9.

To compare the theoretical calculations with the experimental trace defined by Eq. 6.101, one must consider the total deformation. The parameter β from Eq. 6.100 can be represented theoretically as

$$\beta^{(th)} = \beta_2 + \frac{2}{7}\sqrt{\frac{5}{\pi}}\beta_2^2 + \frac{12}{7\sqrt{\pi}}\beta_2\beta_4 + \frac{20}{77}\sqrt{\frac{5}{\pi}}\beta_4^2 + O(\beta^3). \quad (6.102)$$

This as well as other shape parameterizations can be found in Ref. [97]. The deformation, $\beta^{(th)}$, depends on triaxiality as $\cos(\gamma)$. As can be seen in Fig. 6.9(c), for most of the nuclei considered γ is less than 5° , which corresponds to a less than 1% effect for this analysis. Therefore, triaxiality will be neglected.

The results using $\beta^{(th)}$ are displayed in Fig. 6.10. So that the number of theoretical variables introduced can be minimized, the $\mathcal{F}_{\mathcal{J}}^{(th)}$ was calculated using the same experimental effective pair gap, $\Delta_{eff}^{(exp)}$, as that of Fig. 6.8. This is a slightly improper treatment of the pair gap since the experimental $\Delta_{eff}^{(exp)}$ contains contributions from deformation effects, which have already been included, in principle, through $\beta^{(th)}$. The uncertainty on the ordinate axis was calculated assuming

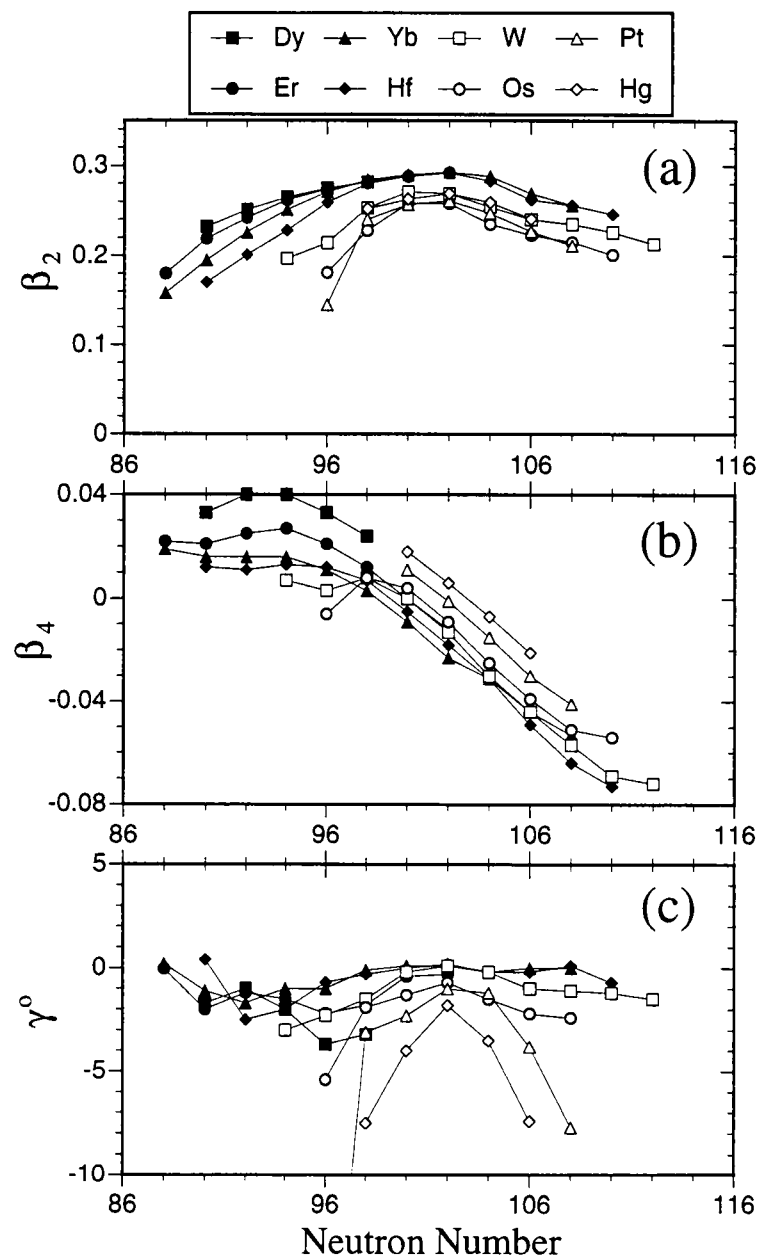


Figure 6.9: Theoretical deformation parameters from TRS calculations for even-even Dy through Hg. References for the calculations are given in the text.

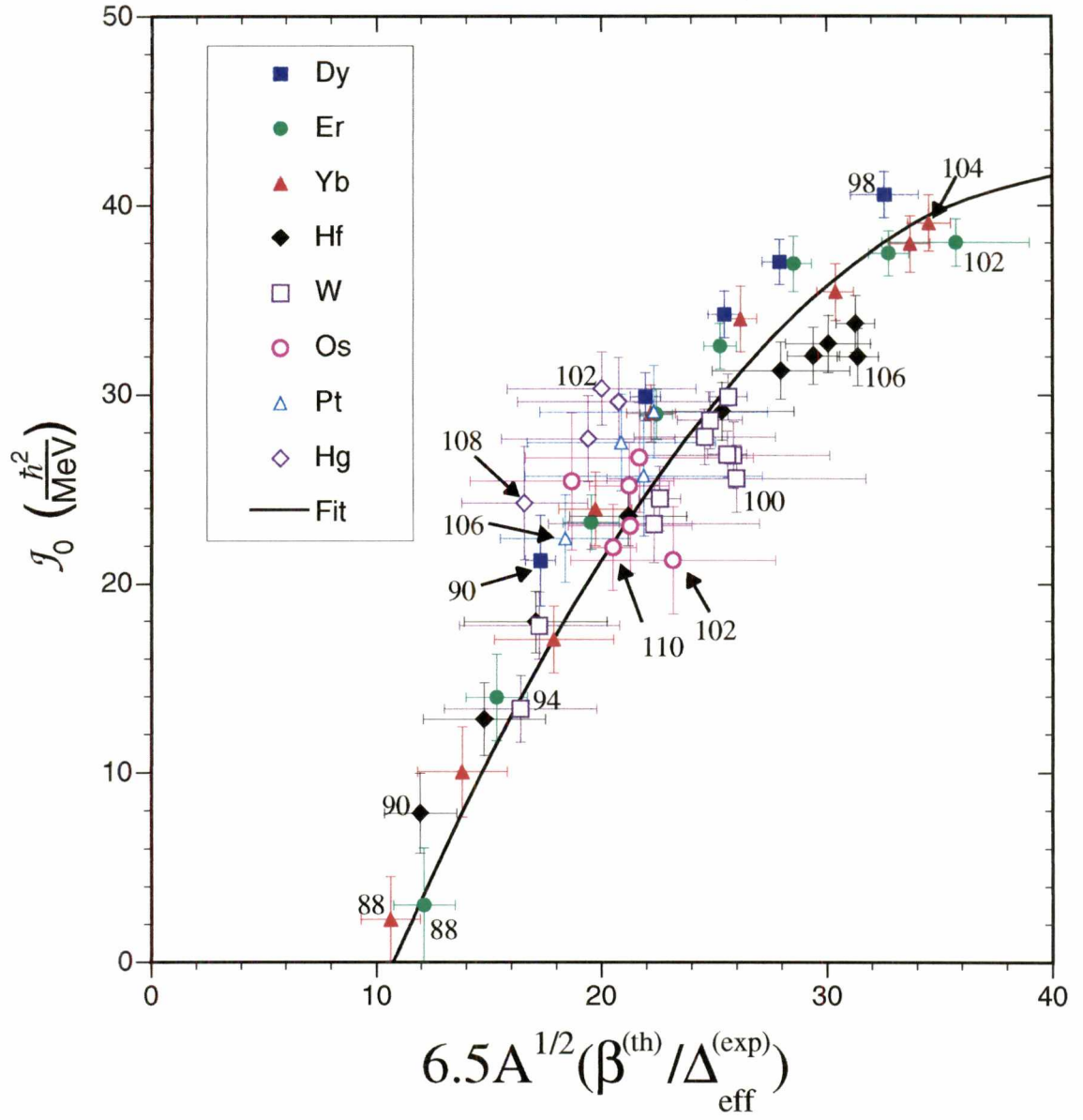


Figure 6.10: Experimental $\mathcal{J}_0$ plotted against $\mathcal{F}_{\mathcal{J}}^{(th)} = 6.5A^{1/2}(\beta^{(th)}/\Delta_{eff}^{(exp)})$ for even-even nuclei from Dy to Hg. The $\Delta_{eff}^{(exp)}$ parameter is the experimental effective pair gap, calculated in the manner presented in the text. The label “Fit” corresponds to the function from Eq. 6.101 and Fig. 6.8. The values in the figure indicate the neutron number of selected nuclei.

$\delta\beta^{(th)} = \pm 0.007$ [69]. As can be seen in Fig. 6.10 the agreement with the experimentally defined function is very good. A noteworthy improvement is the much better agreement for Hg nuclei. Since the theoretical deformation parameters were extracted at a rotational frequency where the prolate shape is well established, an appropriate comparison is made. The greatest amount of disagreement from the experimental trace occurs in Dy and some Er nuclei, where $\beta^{(th)}$ are slightly underpredicted. A possible explanation for these differences is related to the previously mentioned fact that the experimental pair gap contains additional energy related to deformation differences.

A more proper comparison would be to use theoretically calculated pair gaps as well as deformation parameters. For this mass region, however, the standard BCS approach for calculating Δ underpredicts experimental values by $\approx 20\%$. For example, theoretically calculated values for the proton gap parameter of even-even Dy isotopes from $N = 90$ to 98 are 0.911, 0.843, 0.787, 0.725, and 0.678 MeV. These values can be compared to the experimental Δ_p values: 1.163 ± 0.014 , 1.036 ± 0.002 , 0.979 ± 0.002 , 0.930 ± 0.002 , and 0.790 ± 0.032 MeV. The observed deviation of Dy and Er nuclei, however, is only $\approx 4\%$, thus use of BCS calculated pair gaps will overcorrect by 15-20%, which is clearly not appropriate. Regardless of the deficiencies in the theoretical pairing values, the good correlation of the nuclei in Fig. 6.10 when the experimental pair gap is used provides sufficient confidence to compare $\beta^{(th)}$ with the experimental $\mathcal{J}_0$ for odd- A intruder bands. This will be discussed in the following section.

6.5 Moments of Inertia in Odd- A Intruder Bands

In the previous sections the clear correlation of $\mathcal{J}_0$ with respect to $\mathcal{F}_{\mathcal{J}}$ was demonstrated for even-even nuclei. In this section this parameterization is extended to $\pi h_{9/2}$ and $\pi i_{13/2}$ intruder bands in odd- Z nuclei from Ho to Tl. For these cases, only a few quadrupole moment measurements have been made, thus a quantitative comparison of intruder band deformations with theoretical calculations has been limited. One can now use Eq. 6.101 as a measure of the nuclear deformation to test theoretical models.

Theoretical deformation parameters have been extracted for odd- A bands from two sources. The parameters β_2 and β_4 for Ho, Tm, Lu, and Ta are taken from Ref. [69]. For Re, Ir, Au, and Tl nuclei, β_2 , β_4 , and γ are obtained from the Pt centralized TRS mesh [34]. The consistency of these two models was checked in overlapping regions, and, for cases where the triaxiality is small, they produce essentially identical results. The extracted deformation parameters for the $\pi h_{9/2}$ band are shown in Fig. 6.11 and those for $\pi i_{13/2}$ in Fig. 6.12. A comparison of these deformation parameters with those from even-even nuclei clearly indicate the predicted enhancement of deformation by intruder orbitals. For example, the predicted β_2 deformation for the $\pi h_{9/2}$ band in ^{165}Tm is 0.291. This is a 7% increase over deformation of the even-even core nuclei (0.274 and 0.271 for ^{164}Er and ^{166}Yb respectively). Similar predictions of deformation enhancement are made for $\pi i_{13/2}$ bands as well (e.g. the deformation of the $\pi i_{13/2}$ band in ^{179}Ir ($\beta_2 = 0.295$) is 13% larger than the prediction for the even-even neighbors). It can be noted for some configurations, particularly in Au and Tl, that the triaxial degree of freedom becomes relatively significant. Despite this, the contribution of γ is assumed to be zero. The gap parameter, $\Delta_{eff}^{(exp)}$, for odd- A nuclei is calculated from even-even pair gaps with

$$\Delta_{eff}^{(exp)} = \sqrt{\frac{[0.8\Delta_p(Z)]^2 + [\Delta_n(Z)]^2}{2}}, \quad (6.103)$$

where

$$\Delta_{p,n}(Z) = \sqrt{([\Delta_{p,n}(Z-1)]^2 + [\Delta_{p,n}(Z+1)]^2)/2}. \quad (6.104)$$

In this representation, pairing reduction caused from the blocking by the unpaired proton is simulated by using only 80% of the average proton value. Motivation for this choice of pairing reduction can be found, for example, in Ref. [166]. An exception to this pairing treatment is the Tl nuclei, where $\Delta_{eff}^{(exp)}$ is taken only from Hg, rather than the interpolation of Hg and Pb. Comparisons of experimental $\mathcal{J}_0$ values with respect to $\mathcal{F}_{\mathcal{J}}$, where $\beta^{(th)}$ is calculated from theoretical parameters using Eq. 6.102, are presented in Figs. 6.13 and 6.14.

Before considering the results, it is necessary to consider two different types of deviations from the trace of even-even nuclei that are evident in the data. The

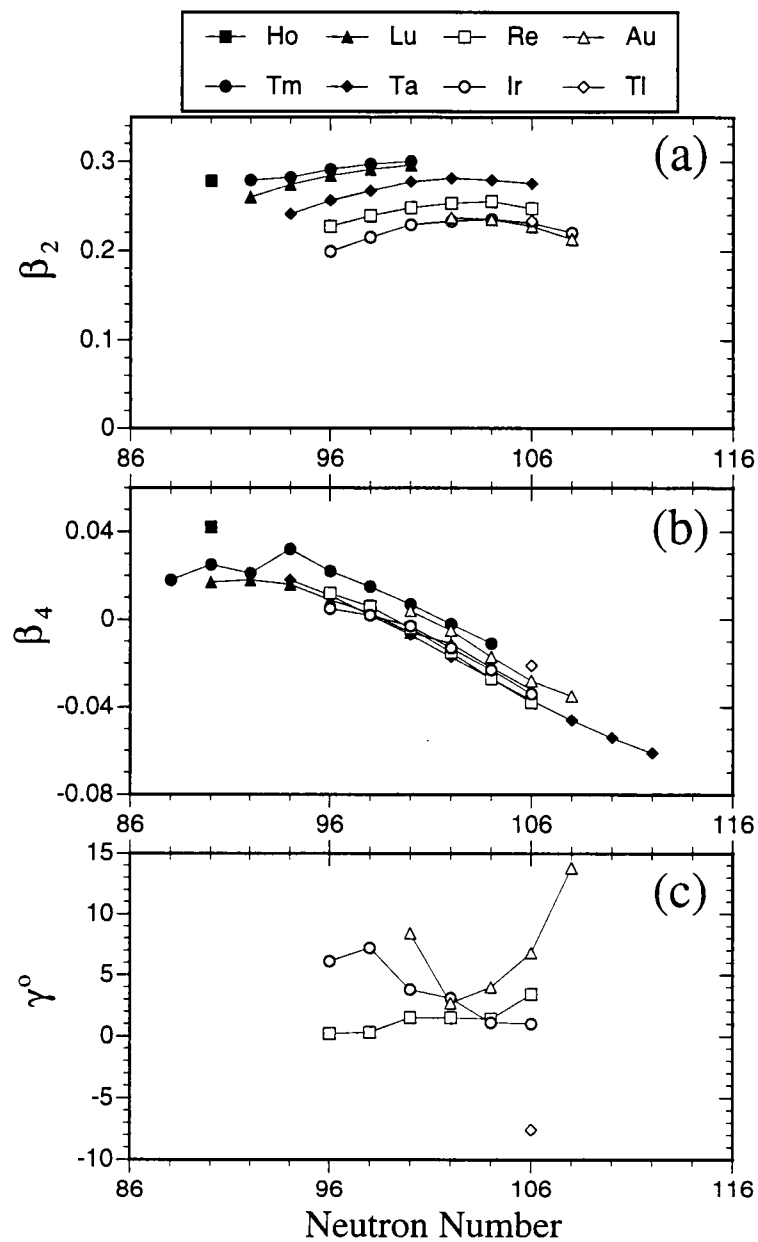


Figure 6.11: Theoretical deformation parameters, (a) β_2 , (b) β_4 , and (c) γ , from model calculations for $\pi h_{9/2}$ bands in Ho through Tl. References for the calculations are given in the text.

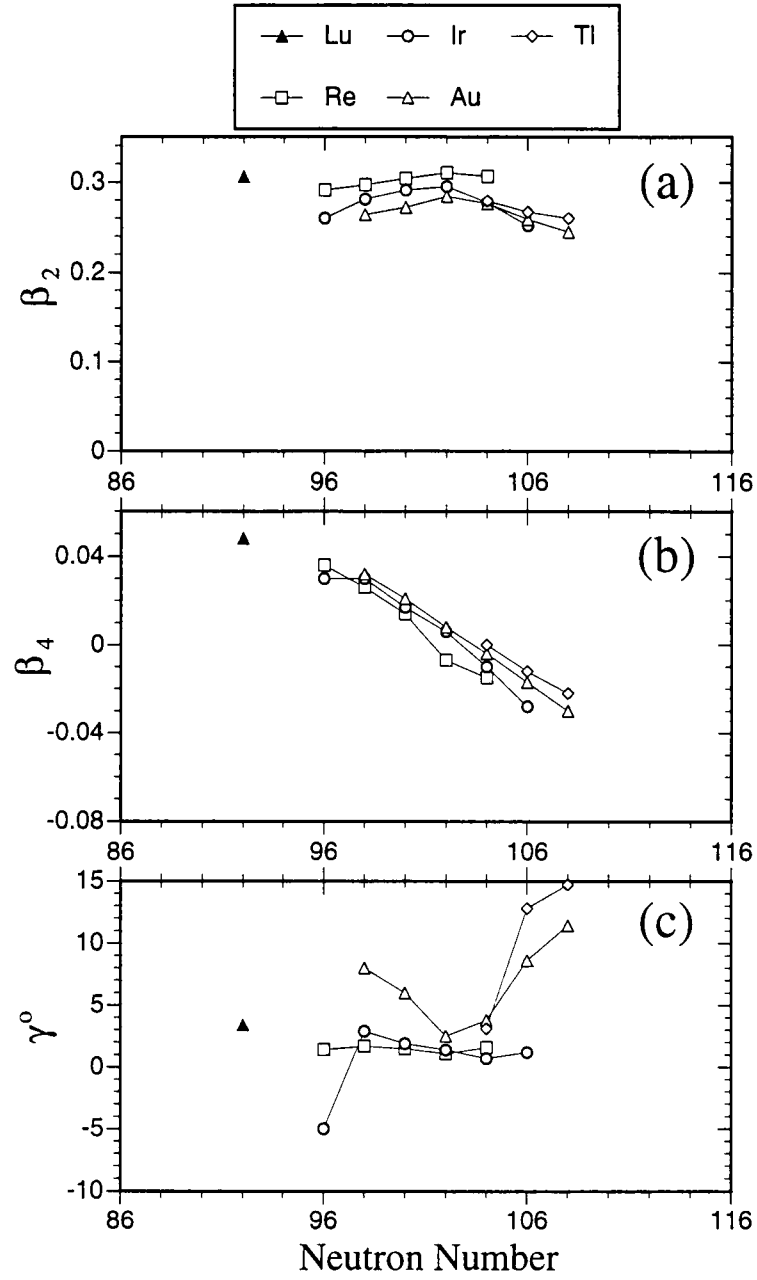


Figure 6.12: Theoretical deformation parameters, (a) β_2 , (b) β_4 , and (c) γ , from model calculations for $\pi i_{13/2}$ bands in Ho through Tl. References for the calculations are given in the text.

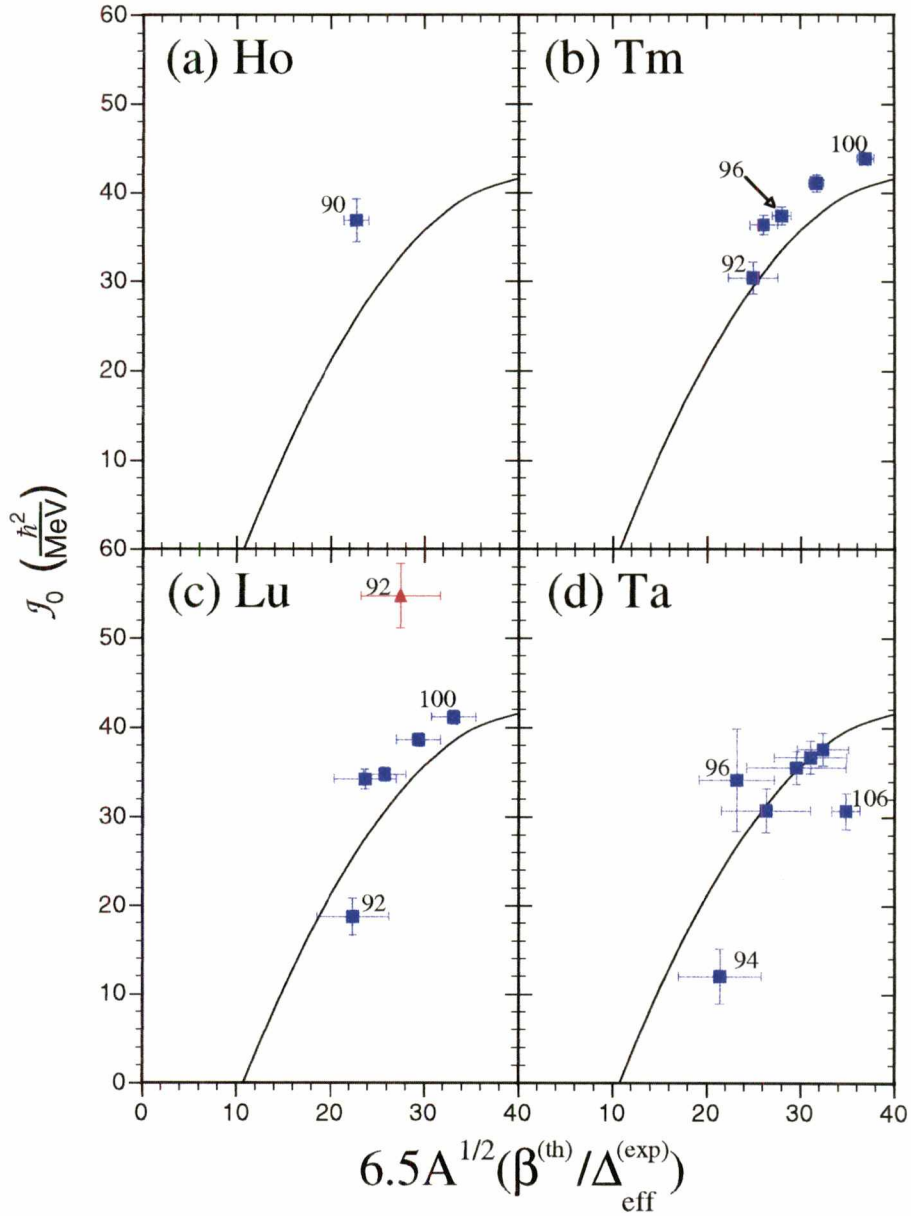


Figure 6.13: Experimental $\mathcal{J}_0$ vs. $\mathcal{F}_{\mathcal{J}}$ for $\pi h_{9/2}$ and $\pi i_{13/2}$ intruder bands from odd- Z even- N nuclei from Ho to Ta. The blue squares represent values from $\pi h_{9/2}$ bands, and the red triangles $\pi i_{13/2}$ bands. The neutron numbers for selected data are shown in the figure. The solid line in the four panels represents the trace of even-even nuclei represented by Eq. 6.101 and drawn in Fig. 6.8.

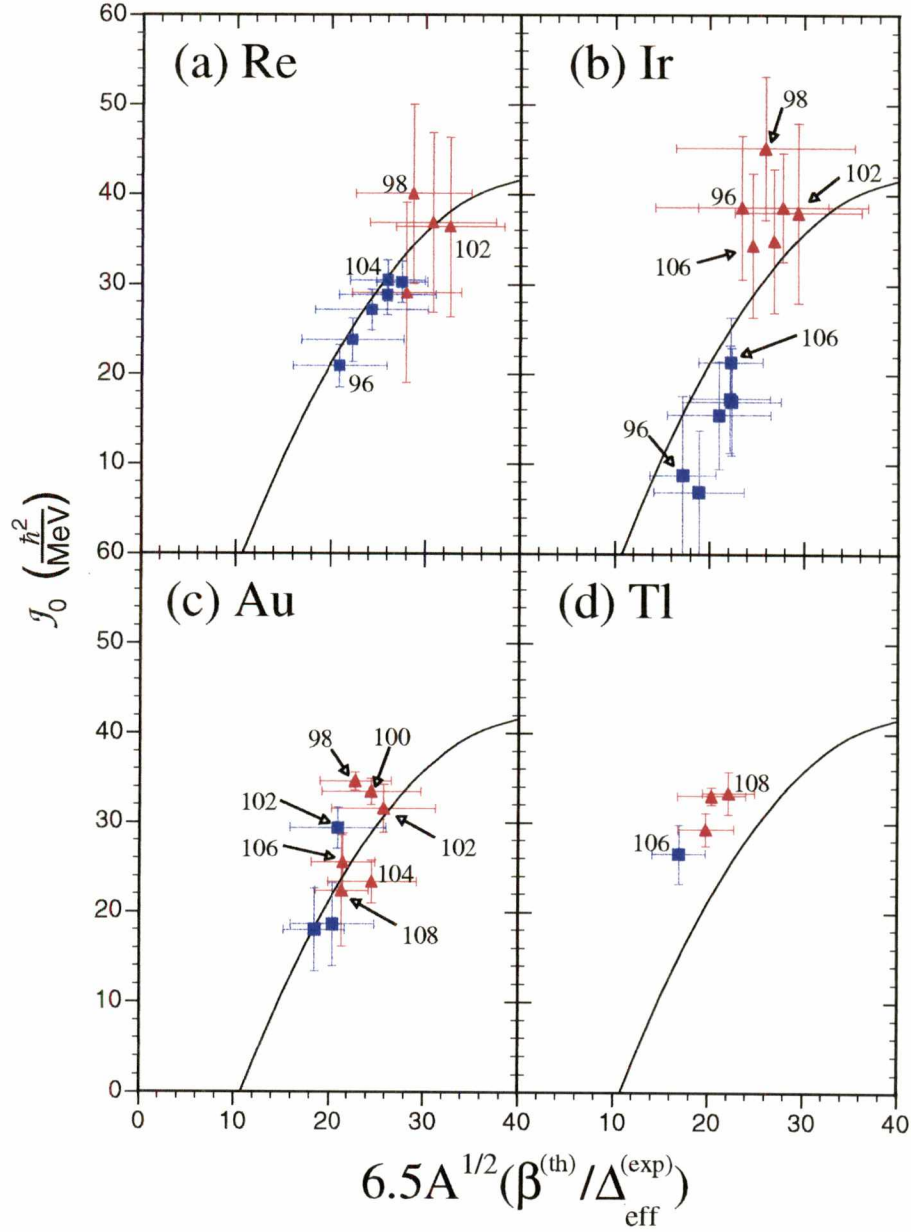


Figure 6.14: Experimental J_0 vs. $\mathcal{F}_J$ for $\pi h_{9/2}$ and $\pi i_{13/2}$ intruder bands from odd- Z even- N nuclei from Re to Tl. The blue squares represent values from $\pi h_{9/2}$ bands, and the red triangles $\pi i_{13/2}$ bands. The neutron numbers for selected data are shown in the figure. The solid line in the four panels represents the trace of even-even nuclei represented by Eq. 6.101 and drawn in Fig. 6.8.

first are systematic differences. These are differences that occur consistently over a range of nuclei. An example of such a deviation is that which occurs in Tm isotopes. As can be seen in Fig. 6.13(b), the $\mathcal{F}_{\mathcal{J}}$ for these data are consistently $\approx 15\%$ less than the trace defined by Eq. 6.101. It will be demonstrated that many of these differences can be interpreted as resulting from a prediction of $\beta^{(th)}$ that is too small. A second type of deviation are isolated differences. These are cases where, not only do the data deviate from the trace, but also from the systematic trend of other odd- A nuclei. Examples of these are the points labeled by $N = 94, 96$, and 106 in Fig. 6.13(d). These data represent cases where the Harris parameterization of the moment of inertia is insufficient to appropriately model the experimentally observed bands.

The best examples where the $\beta^{(th)}$ values systematically reproduce the experimentally observed data are for the $\pi h_{9/2}$ bands in Ta and Re isotopes, seen in Figs. 6.13(d) and 6.14(a) respectively. While there are three cases where large isolated deviations are observed in Ta nuclei, the $N = 98$ to 104 isotopes are very well represented by the trace. Very good agreement is also obtained for all Re $\pi h_{9/2}$ bands presented here. For these nuclei, however, the $\pi h_{9/2}$ bands are very close to the Fermi level and an enhanced deformation is not predicted. The quality of the comparison of theoretical values to the experimental $\mathcal{J}_0$ confirms this prediction.

For the lighter nuclei, Tm and Lu, the calculated $\mathcal{F}_{\mathcal{J}}$ for the $\pi h_{9/2}$ bands is systematically $\approx 15\%$ less than the trace (see Figs. 6.13(b) and (c)). As mentioned in the example above, $\pi h_{9/2}$ bands in these are predicted to have enhanced deformation. The larger $\mathcal{J}_0$ for these bands confirms that these bands are indeed more deformed than their even-even neighbors, however, the theoretical values underpredict the amount of this enhancement. By rearrangement of Eq. 6.101, a relation can be established from which an experimental deformation, $\beta^{(exp)}$, can be inferred from $\mathcal{J}_0$. This relation is

$$\beta^{(exp)} = \frac{\Delta_{eff}^{(exp)} [\text{MeV}]}{6.5A^{1/2}} \left(41.8 - 11.6\sqrt{13.0 - 0.173(33.7 + \mathcal{J}_0 [\hbar^2/\text{MeV}])} \right), \quad (6.105)$$

where the values in the square brackets represent the units for the appropriate quantity. As an illustrative example, this equation is applied to the case of ^{165}Tm [141],

which is denoted in Fig. 6.13(b) by $N = 96$. For this nucleus, $\mathcal{J}_0 = 37.4 \pm 1.0 \hbar^2/\text{MeV}$ is taken from Table 6.2. The experimental pair gap is calculated from the data in Figs. 6.7(b) and (c) with Eq. 6.103 to be $\Delta_{\text{eff}}^{(\text{exp})} = 0.978 \pm 0.015 \text{ MeV}$. Through the use of these quantities, the experimental deformation is determined to be $\beta^{(\text{exp})} = 0.372 \pm 0.019$. This compares to $\beta^{(\text{th})} = 0.328$, calculated from theoretical β_2 (0.291) and β_4 (0.022) values. Thus the systematic trend in $\pi h_{9/2}$ bands in Tm and Lu will match the trace with deformations $\approx 15\%$ greater than theoretical predictions. For the case of ^{157}Ho ($N = 90$ in Fig. 6.13), the value of $\mathcal{F}_{\mathcal{J}}$ is nearly 30% less than the trace for the measured $\mathcal{J}_0$. To explain this as a deformation effect would require a $\approx 40\%$ increase over the predicted $\beta^{(\text{th})}$. The deformation of ^{157}Ho can be inferred, by the same procedure as for ^{165}Tm , from the measured $\mathcal{J}_0$ to be $\beta^{(\text{exp})} = 0.437 \pm 0.054$, compared with the theoretical value of $\beta^{(\text{th})} = 0.318$.

For $\pi h_{9/2}$ bands of Ir nuclei in Fig. 6.14(b), the $\mathcal{F}_{\mathcal{J}}$ values appear overpredicted. The extracted $\mathcal{J}_0$ values, however, have considerable uncertainty. This, in addition to the large uncertainty associated with $\Delta_{\text{eff}}^{(\text{exp})}$, provides an indication that this deviation may be anomalous. The relatively parallel trend of the $\pi h_{9/2}$ values in Ir isotopes with respect to the trace, however, provides a case for these differences to be true. The $\mathcal{F}_{\mathcal{J}}$ values for $\pi h_{9/2}$ bands in Au, however, are generally well reproduced. Of these examples, the $\mathcal{J}_0$ that deviates most from the trace is that for $N = 102$ Au. This nucleus also corresponds to the isotope that has an anomalously low quasiparticle alignment (see Fig. 6.6 and ^{181}Au in Table 6.4). As mentioned in Sec. 6.3.2, the $\pi h_{9/2}$ band in ^{181}Au may be influenced by other interactions not modeled by the Harris formula (e.g. bands from $\pi f_{7/2}$ configurations (see Chapter 7)). There is also a discrepancy in the $\pi h_{9/2}$ band in ^{187}Tl . This however, may be the result of $\Delta_{\text{eff}}^{(\text{exp})}$ not being properly averaged.

Unlike the $\pi h_{9/2}$ bands, the observed trends in $\pi i_{13/2}$ bands are not as clear due to the large uncertainty associated with a number of the $\mathcal{J}_0$ values. In general, the comparison of $\mathcal{F}_{\mathcal{J}}$, calculated with the theoretical deformation parameters, to experimental $\mathcal{J}_0$ values matches the trace rather well. The most systematic trend of values for $\pi i_{13/2}$ bands are those found in Au isotopes. The $\mathcal{J}_0$ values for $N = 108, 106$,

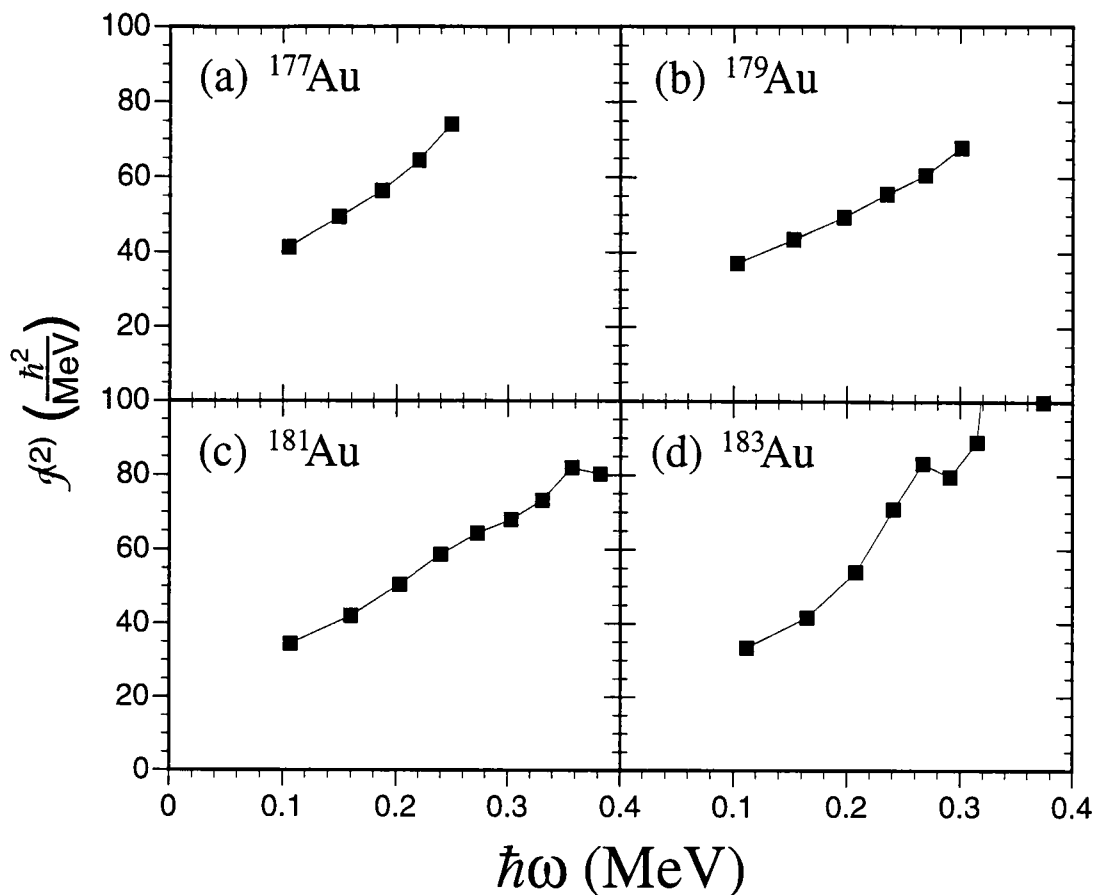


Figure 6.15: Dynamic moments of inertia ($\mathcal{J}^{(2)}$) for $\pi i_{13/2}$ bands in ^{177}Au [160], ^{179}Au , ^{181}Au , and ^{183}Au [65].

and 102 are well reproduced by the theoretical deformation parameters. The value for $N = 104$ Au deviates from the systematic trend. This is due to an interaction in the $\pi i_{13/2}$ band in ^{183}Au that results in the extracted $\mathcal{J}_0$ appearing reduced from the true value. Evidence for this interaction can be seen in Fig. 6.15(d), where the $\mathcal{J}^{(2)}$ moment of inertia is plotted. At a rotational frequency of about 0.26 MeV this band is clearly perturbed, thus affecting the extraction of $\mathcal{J}_0$ for this band. It can be seen in Fig. 6.15(c) that there is similar evidence of a perturbation in the $\pi i_{13/2}$ band of ^{181}Au . This perturbation is very small, however, and is accounted for in the uncertainty of the measurement.

For Au nuclei with $N < 102$ the measured $\mathcal{J}_0$ values continue to rise, but predicted deformation values suggest that this quantity should decrease. The $\pi i_{13/2}$

bands in ^{177}Au and ^{179}Au do not show any unusual perturbations that would indicate that this increase is due to something other than deformation (see Figs. 6.15(a) and (b)). With the assumption that the deviation from the trace is a deformation effect, the deformation of ^{179}Au and ^{177}Au can be inferred from $\mathcal{J}_0$ to be $\beta^{(exp)} = 0.348 \pm 0.089$ and 0.380 ± 0.074 respectively. The large uncertainty of these values arises primarily from the pair gap contribution. By accounting for the predicted β_4 for the isotopes (0.021 and 0.032 for ^{179}Au and ^{177}Au respectively), the quadrupole deformation is determined to be 0.308 ± 0.079 and 0.330 ± 0.064 . These values are 14% and 21% greater than the predicted deformations (0.264 and 0.272 respectively). From qualitative examination of $\mathcal{J}_0$ values in $\pi i_{13/2}$ bands in Ir and Re nuclei, one can see that the greatest deviations from the trace also occur for $N \sim 98$ isotopes. It should be noted that the $\pi i_{13/2}$ bands in these two isotopes are predicted to have $\gamma > 5^\circ$ (Fig. 6.12), which could have a significant contribution to the $\mathcal{J}_0$. This is unlikely since $N = 106$ and 108 Au are predicted to have a greater amount of triaxiality and these data are well correlated with the trace.

The $\pi i_{13/2}$ bands in Tl deviate from the trace by a similar amount as the $\pi h_{9/2}$ band in $N = 106$ Tl. This provides further evidence that the source of deviations in Tl nuclei could be an improper pairing treatment. A final datum point for $\pi i_{13/2}$ bands in this survey is a result in ^{163}Lu [142]. The measured $\mathcal{J}_0$ for this band is greater than $50 \hbar^2/\text{MeV}$ (see Fig. 6.13(c)). This result is greater than the empirical fit presented here, and a determination of deformation from $\mathcal{J}_0$ would require a more careful consideration of the moment of inertia model.

As mentioned at the beginning of this section, there are a few cases where transition quadrupole moment measurements have been performed for intruder bands and thus provide a more direct measure of the nuclear deformation. Those cases where measurements have been made are summarized in Table 6.5. The deformations listed in column " β_Q " of this table are obtained from Q_t values using the relation from Ref. [167],

$$\beta_Q = -7\sqrt{\frac{\pi}{80}} + \sqrt{\frac{49\pi}{80} + \frac{7\pi Q_t}{6Zr_0^2 A^{2/3}}}. \quad (6.106)$$

The values listed in column " $\beta^{(exp)}$ " are those obtained from extracted $\mathcal{J}_0$ quantities

Table 6.5: Experimental deformation parameters (β_Q) for $\pi h_{9/2}$ and $\pi i_{13/2}$ bands deduced from lifetime measures compared with values inferred from $\mathcal{J}_0$ ($\beta^{(exp)}$) and theoretical predictions ($\beta^{(th)}$).

Nucleus	band	Q_t (eb)	β_Q	Ref.	$\beta^{(exp)}$	$\beta^{(th)}$
^{187}Au	$\pi h_{9/2}$	6.21 ± 0.44	0.220 ± 0.016	[89]		0.223
	$\pi i_{13/2}$	8.35 ± 0.59	0.296 ± 0.021	[89]	0.25 ± 0.10	0.260
^{179}Ir	$\pi h_{9/2}$	6.0 ± 0.5	0.224 ± 0.019	[90]	0.200 ± 0.100	0.250
	$\pi i_{13/2}$	7.7 ± 0.6	0.289 ± 0.023	[90]	0.36 ± 0.15	0.328
^{165}Tm	$\pi h_{9/2}$	8.3 ± 0.5	0.37 ± 0.02	[141]	0.372 ± 0.019	0.328
^{163}Lu	$\pi h_{9/2}$	10.7 ± 0.7	0.432 ± 0.028	[142]		0.355
^{157}Ho	$\pi h_{9/2}$	5.88 ± 1.51	0.27 ± 0.07	[88]	0.437 ± 0.054	0.318

using Eq. 6.101. Most lifetime measurements listed in Table 6.5 were performed on nuclei where the uncertainty in the $\mathcal{J}_0$ measurement is rather large. Thus only a qualitative comparison can be made. There is one case, however, where uncertainty in the measurement of $\mathcal{J}_0$ is small, as well as in the measurement of the quadrupole moment. This case is the $\pi h_{9/2}$ band in ^{165}Tm . The deformations determined by the two different methods are identical. Thus from two separate methods of determining deformation, one can conclude that the deformation in ^{165}Tm is underpredicted by the model calculations by 15%. For the case of ^{157}Ho , there is a relatively substantial difference between the quadrupole moment measurement and the $\mathcal{J}_0$ analysis (0.27 ± 0.7 and 0.432 ± 0.028 for the two methods respectively). The predicted deformation for this band (0.318), however, is between the two experimental values.

To summarize, the $\mathcal{J}_0$ for $\pi h_{9/2}$ and $\pi i_{13/2}$ bands have been compared with the parameterization $\mathcal{F}_{\mathcal{J}}$. For cases where the intruder configuration is not predicted to enhance the deformation of the nucleus (e.g. $\pi h_{9/2}$ bands in Ta and Re, and $\pi i_{13/2}$ bands for $N \geq 102$ Au) the theoretical predictions correlate very accurately to the observed $\mathcal{J}_0$ values. Systematic deviations from the trace are observed for $\pi h_{9/2}$ bands in Ho, Tm, and Lu. In the latter cases of Tm and Lu, the deviations indicate that

the deformation is underpredicted by $\approx 15\%$ for these nuclei. This result is also confirmed for the case of ^{165}Tm by lifetime measurements. While the uncertainties for values obtained from $\pi i_{13/2}$ bands are, in some cases, rather large, there are also deviations from the trace observed in these bands for nuclei with $N \leq 100$. In particular, the moment of inertia analysis indicates that the deformation in ^{179}Au and ^{177}Au continues to increase compared to heavier Au isotopes (^{181}Au , ^{183}Au , etc.). This is contrary to theoretical predictions, where the deformation is predicted to be the greatest in ^{181}Au .

CHAPTER 7

Alignment and Deformation Properties in $\pi h_{9/2}$ - $\pi f_{7/2}$ Bands

As the Fermi level increases into the $\pi h_{9/2}$ shell, the bandhead energy becomes low enough that the unfavored signature of this intruder band is observed. As the Fermi level increases further, evidence of the $\pi f_{7/2}$ band becomes apparent. In 1986, the first identification of a low-lying $\pi f_{7/2}$ rotational band in a nucleus below lead was reported in ^{185}Au [64]. Since that time, $\pi f_{7/2}$ bands have been identified in ^{183}Au [65], and ^{179}Ir [78]. A fourth case has now been presented for ^{181}Au in Sec. 4.3 of this dissertation.

The $\pi h_{9/2}$ and $\pi f_{7/2}$ bands have a unique relationship in neutron-deficient odd- A Au and Ir compared to other observed bands in these nuclei. For example, Jin, *et al.* [78] identified several rotational bands in ^{179}Ir with spectroscopic assignments $\frac{1}{2}^-$ [541], $\frac{1}{2}^-$ [530], $\frac{9}{2}^-$ [514], $\frac{5}{2}^+$ [402], and $\frac{1}{2}^+$ [660]. In the latter three cases, each configuration is rather pure in that each orbital has a unique combination of K and parity. The $\pi h_{9/2}$ ($\frac{1}{2}^-$ [541]) and $\pi f_{7/2}$ ($\frac{1}{2}^-$ [530]) orbitals, however, are both low- K and negative parity, and the classification of these bands in the basis of asymptotic quantum numbers becomes less clear since the configurations are heavily mixed. Because of this configuration mixing, enhanced correlations between these bands are observed, thus further increasing the difficulty in classifying these bands. The reason for the existence of this pair of negative-parity low- K structures is a direct result of the strong nucleonic spin-orbit coupling present in the nuclear system. Because of the spin-orbit coupling, the $h_{9/2}$ and $f_{7/2}$ single-particle spherical shell states are nearly degenerate in energy. This can be clearly seen at $\beta_2 = 0$ in the calculated single-particle proton level diagram (Fig. 2.3). As discussed in Sec. 2.2.4, the onset

of quadrupole deformation does not break the near degeneracy of the related $\pi h_{9/2}$ and $\pi f_{7/2}$ orbitals.

One would expect that rotational bands based on these orbitals would be quite similar since the configurations are so mixed. On the contrary, rotational bands from these configurations exhibit different properties, and interact in a manner that is not readily understood. Discussion of the details of these differences follow in the sections below. Specific properties to be addressed are the alignment trends and branching ratios of the $\pi h_{9/2}$ - $\pi f_{7/2}$ bands, as well as the observed interactions of these rotational structures. The properties are analyzed in the framework of cranking and particle rotor models. Another example where mixed configurations are observed are the $\frac{5}{2}^+[402]$ and $\frac{7}{2}^+[404]$ rotational bands in ^{175}Re [168].

7.1 Summary of Observed Bands

In the four nuclei where $\pi h_{9/2}$ and $\pi f_{7/2}$ bands have been reported, three decoupled negative-parity rotational bands have been observed. The low-spin levels of these sequences are illustrated in Fig. 7.1. The labels for the bands in Fig. 7.1 are from the configuration assignments made in the appropriate reference for each nucleus. These assignments were initially made by Larabee, *et al.* [64] for ^{185}Au , and the classification of the bands in ^{183}Au , ^{181}Au , and ^{179}Ir are made by analogy. In all four cases a single negative signature ($\alpha = -\frac{1}{2}$) band is observed. In addition, there is a possible $\alpha = +\frac{1}{2}$ signature partner for $\pi f_{7/2}$ in ^{181}Au , and this is included here for comparison. Spin assignments for Band 4 are not known, but the labels in parentheses represent the expected spin and parity if this is the unfavored $\pi f_{7/2}$ band.

Due to a number of near level degeneracies in ^{181}Au , many of the connecting transitions in Fig. 7.1 are ambiguous. The transitions for ^{181}Au are summarized here to resolve these ambiguities. There are two transitions (243 and 346 keV) observed decaying from the $\alpha = -\frac{1}{2}$ to the $\alpha = +\frac{1}{2}$ $\pi h_{9/2}$ band. The 413-keV transition connects the $19/2^-$ level in the $\pi f_{7/2}$ band to the $15/2^-$ level in the $\alpha = -\frac{1}{2}$ $\pi h_{9/2}$ band. The five transitions, 334, 391, 416, 428, and 430 keV, all decay from the favored $\pi f_{7/2}$ band to the $\alpha = +\frac{1}{2}$ $\pi h_{9/2}$ band. Finally, the 380-keV decay out of the $(17/2^-)$

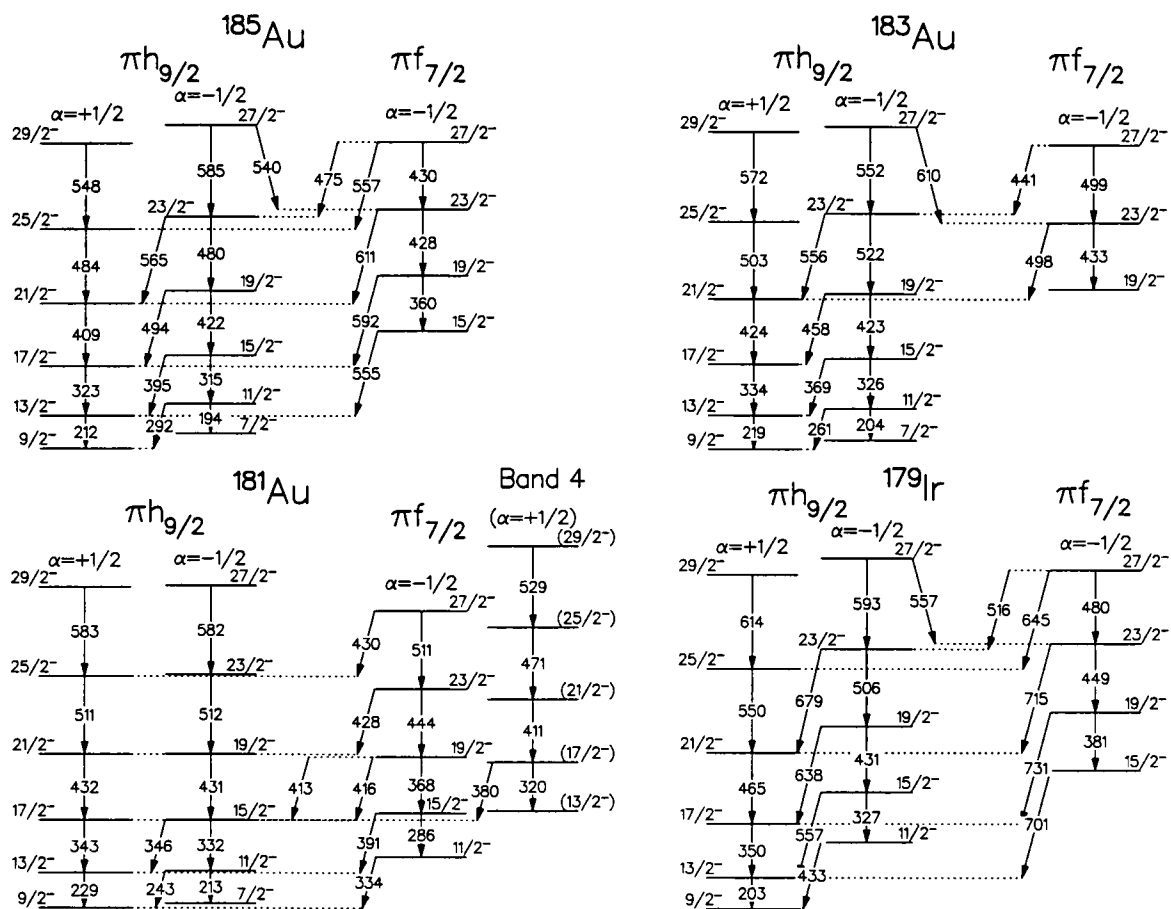


Figure 7.1: Partial level schemes for ^{185}Au [64], ^{183}Au [65], ^{181}Au , and ^{179}Ir [78]. The labels $\pi h_{9/2}$ and $\pi f_{7/2}$ reflect the quasiparticle assignments from each reference.

level of Band 4 goes to the $15/2^-$ level in the $\alpha = -\frac{1}{2} \pi h_{9/2}$ band.

From angular momentum coupling rules, the energetically favored signatures for the $\pi h_{9/2}$ and $\pi f_{7/2}$ configurations are $\alpha = +\frac{1}{2}$ and $-\frac{1}{2}$, respectively. It is clear that the positive signature band is the lowest energy structure of the three, and can clearly be associated with the favored $\pi h_{9/2}$ band. The positive-signature $\pi f_{7/2}$ rotational band would be shifted much higher in energy with respect to the other bands making it more weakly populated (as is the case for Band 4 in ^{181}Au). From the comparison of the negative-signature bands, it can be seen that the $\pi h_{9/2}$ structure is initially lower in energy than the $\pi f_{7/2}$ levels, however, at some higher spin a crossing occurs and the situation becomes reversed. It can also be observed that all negative-signature bands feed the respective positive-signature bands by several dipole transitions. The interpretation of these features are presented in the following sections.

7.2 Quasiparticle Routhian Energy and Alignment

Fig. 7.2 shows the experimental Routhian energies and alignments for nuclei where $\alpha = -\frac{1}{2} \pi h_{9/2}$ and $\pi f_{7/2}$ bands have been reported. Panels (1a) through (4a) indicate cranked shell model Routhian energies for negative-parity bands in these nuclei. The calculation is a pairing self-consistent cranking model [169], where the particle-hole mean field is approximated by an ω -independent Woods-Saxon potential. The deformations used in each nucleus were extracted from the lowest $(\pi, \alpha) = (-, +\frac{1}{2})$ configuration from Total Routhian Surface calculations (see Sec. 2.6.3). For ^{179}Ir , ^{181}Au , ^{183}Au , and ^{185}Au , respectively, these values are 0.233, 0.237, 0.235, and 0.227 for β_2 , -0.013, -0.005, -0.017, and -0.028 for β_4 , and 3.1° , 2.7° , 4.0° , and 6.9° for γ . The labeling for the theoretical bands is consistent with Ref. [64]. i.e. “a” labels the lowest $\alpha = -\frac{1}{2}$ band, “b” the lowest $\alpha = +\frac{1}{2}$ band, and “c” the second lowest $\alpha = -\frac{1}{2}$ band. The doubly-degenerate band related to the strongly coupled $\pi h_{11/2}$ orbital is labeled in each panel.

From the comparison of the theoretical Routhian energies with the experimental values, it is clear that the two $\pi h_{9/2}$ signature partners (the blue squares and green circles) can be associated with the “b” and “a” configurations, respectively,

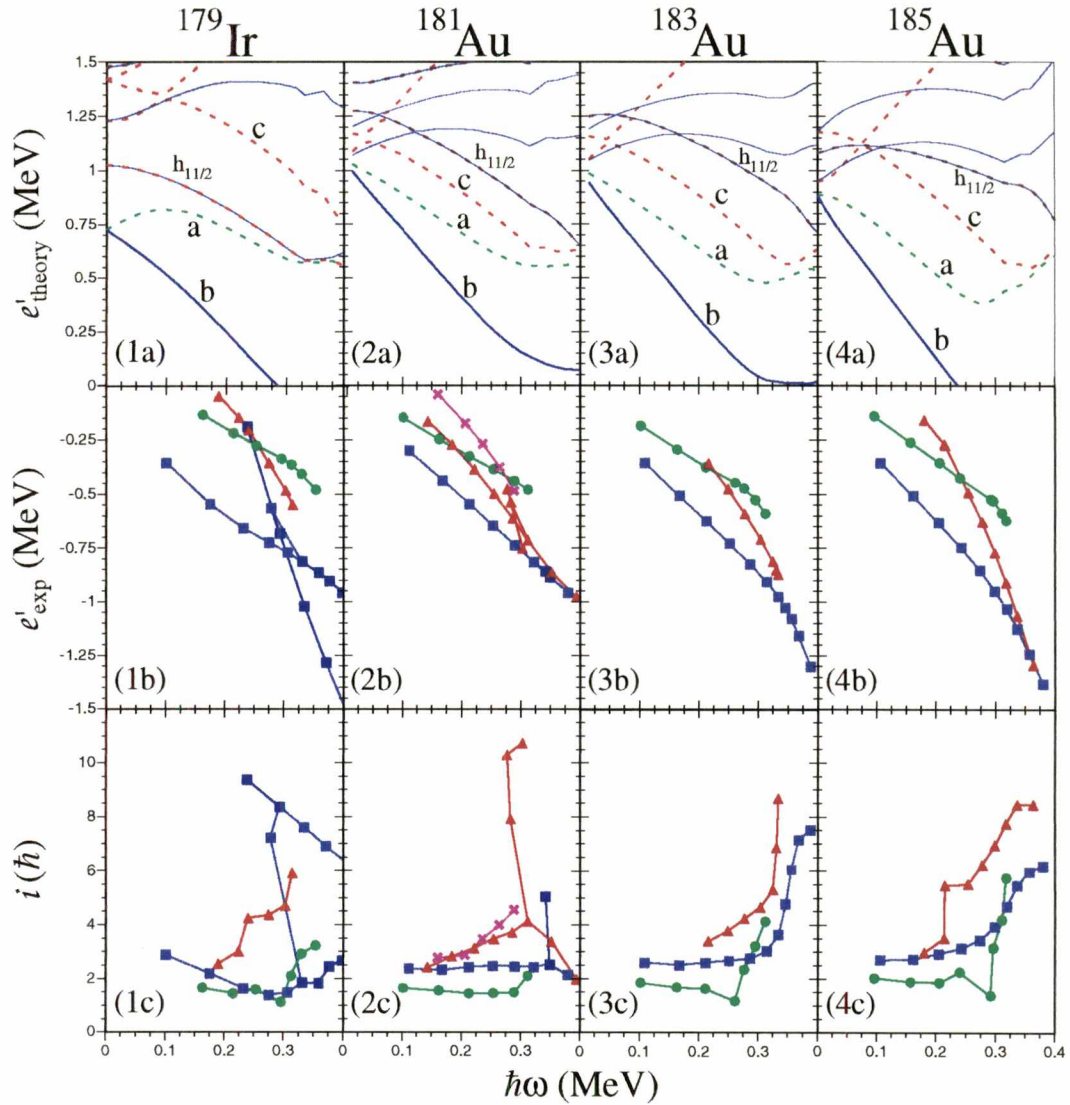


Figure 7.2: Ruthian and alignment plots of $\pi h_{9/2}$ and $\pi f_{7/2}$ bands for ^{179}Ir , ^{181}Au , ^{183}Au , and ^{185}Au . The figures labeled (1a) through (4a) are Ruthians for negative parity states resulting from a cranked shell model calculation. The solid and dashed lines in (a) panels indicate states with $\alpha = +\frac{1}{2}$ and $-\frac{1}{2}$ respectively. Details of the calculation are provided in the text. Panels (b) and (c) show experimental Ruthians and alignments extracted from observed bands using the method described in Sec. 2.7 with Harris parameters $\mathcal{J}_0 = 29.8 \hbar^2/\text{MeV}$ and $\mathcal{J}_1 = 132.1 \hbar^4/\text{MeV}^2$. The blue square symbols represent the $\alpha = +\frac{1}{2} \pi h_{9/2}$ bands, the green circles $\alpha = -\frac{1}{2} \pi h_{9/2}$, and the red triangles $\alpha = -\frac{1}{2} \pi f_{7/2}$. The crosses in (2b) and (2c) denote Band 4 from ^{181}Au .

in the calculations. Not only is the converging trend of the two signatures toward lower frequency reproduced, but there is good qualitative agreement of the energy splitting between the bands. Of course, the calculations are made assuming that the quasi-particle mean field is the same for all configurations, thus such direct comparison of states is only appropriate when deformation parameters and pairing fields are similar. The good agreement of configurations “b” and “a” with the experimental values clearly demonstrates the similar deformation and pairing field between the two signatures. The comparison of “c” with the experimental $\pi f_{7/2}$ band is an example where the approximation of a similar mean-field does not appear to be valid. In the calculation, The “a” and “c” configurations are nearly parallel to one another as a function of $\hbar\omega$, which is clearly not the case in experimental observation. Larabee *et al.* in Ref. [64] attributed the gradually increasing alignment in the $\pi f_{7/2}$ band of ^{185}Au to breaking of a pair of $h_{9/2}$ protons.

Evidence of a $\pi h_{9/2}$ alignment (a $\pi h_{9/2}$ crossing) is clearly observed in ^{187}Au [161], where a sharp backbend occurs at $\hbar\omega = 0.23$ MeV in the $\pi i_{13/2}$ band. In lighter Au nuclei, this crossing appears to occur at progressively higher rotational frequencies. In ^{181}Au , there is no evidence for a low-frequency $\pi h_{9/2}$ alignment in the $\pi i_{13/2}$ band. It is clear from Fig. 7.2 that a gradually increasing alignment of the $\pi f_{7/2}$ band with respect to the $\pi h_{9/2}$ structures is a common feature for all observed cases.

An alternative interpretation is to consider that the difference in the alignments is related to shape changes. The relationship of the first order moment of inertia, $\mathcal{J}_0$, to quadrupole deformation was demonstrated in Chapter 6. Using the procedures described in that chapter, one can extract the i , $\mathcal{J}_0$, and $\mathcal{J}_1$ parameters from a fit of the $\mathcal{J}^{(1)}$ moments of inertia for the three bands. The results of this fit are $i = 2.45 \pm 0.24\hbar$, $\mathcal{J}_0 = 29.4 \pm 2.3 \hbar^2/\text{MeV}$, and $\mathcal{J}_1 = 121 \pm 15 \hbar^4/\text{MeV}^2$ for the $\alpha = +\frac{1}{2} \pi h_{9/2}$ band, $1.88 \pm 0.50\hbar$, $26.2 \pm 3.0 \hbar^2/\text{MeV}$, and $140 \pm 12 \hbar^4/\text{MeV}^2$ for the $\alpha = -\frac{1}{2} \pi h_{9/2}$ band, and $1.25 \pm 0.50\hbar$, $37.0 \pm 3.0 \hbar^2/\text{MeV}$, and $132 \pm 15 \hbar^4/\text{MeV}^2$ for the $\pi f_{7/2}$ band. To within the uncertainties of the extraction from $\mathcal{J}^{(1)}$, the $\mathcal{J}_0$ values for the $\pi h_{9/2}$ signature partners are approximately the same, but the $\pi f_{7/2}$ band has $\mathcal{J}_0$ that is clearly much larger than both $\pi h_{9/2}$ values. From Eq. 6.101 the $\mathcal{J}_0$ values

29.4 ± 2.3 , 26.2 ± 3.0 , and 37.0 ± 3.0 would correspond to deformations, $\beta^{(exp)}$, of 0.36 ± 0.08 , 0.32 ± 0.08 , and 0.45 ± 0.11 respectively. These numbers can be compared to the theoretically predicted value of $\beta^{(th)} = 0.256$. With the possible exception of the unfavored $\pi h_{9/2}$ band, the $\beta^{(exp)}$ values are much too large to be able to interpret the $\mathcal{J}_0$ values as resulting from a difference in deformation between $\pi h_{9/2}$ and $\pi f_{7/2}$.

An alternative method for inferring deformation of a band is from the frequency at which $\nu i_{13/2}$ alignment occurs ($\hbar\omega_c$). The $\nu i_{13/2}$ alignment is observed as a backbend in the $\pi f_{7/2}$ band (triangles) in Fig. 7.2(2c). The beginning of the $\nu i_{13/2}$ alignment is also observed in the $\pi h_{9/2}$ band (squares) in the same figure. It has been demonstrated (e.g. Jin *et al.* [78]) that a larger relative deformation delays the frequency at which the $\nu i_{13/2}$ alignment occurs. In the case of ^{181}Au , the backbend in the $\pi f_{7/2}$ band occurs earlier ($\hbar\omega_c = 0.30$ MeV) than for the favored $\pi h_{9/2}$ band ($\hbar\omega_c = 0.35$ MeV), implying that the $\pi f_{7/2}$ band is less deformed than the $\pi h_{9/2}$ band.

As can be seen in the calculations of Fig. 7.2(2a), the second lowest decoupled $\alpha = +\frac{1}{2}$ band is predicted at greater than 1 MeV above the quasiparticle vacuum. This band shows very little alignment in the calculation. By comparison, Band 4, the proposed unfavored $\pi f_{7/2}$ signature, in ^{181}Au (denoted by crosses in Figs. 7.2(2c)), has a relative alignment very similar to the $\pi f_{7/2}$ band. Despite this difference conclusive support for or against the assignment of Band 4 cannot be made until the alignment differences of the favored $\pi f_{7/2}$ band with respect to the $\pi h_{9/2}$ configurations are resolved.

In summary, the pattern that the alignment of the $\pi f_{7/2}$ band smoothly increases with respect to the $\pi h_{9/2}$ alignment is consistent for all observed cases. This effect, however, cannot be easily interpreted as a strongly interacting $\pi h_{9/2}$ crossing, and it is clearly not an artifact of a deformation difference between the $\pi h_{9/2}$ and $\pi f_{7/2}$ bands. The reason for this increase in alignment remains an unresolved issue.

7.3 B(M1)/B(E2) branching ratio

In the previous section, the identification of the various decoupled negative-parity bands was discussed. As can be seen in Fig. 7.1, there are a number of $\Delta I = 1$ transitions observed decaying from both the unfavored $\pi h_{9/2}$ and the $\pi f_{7/2}$ band into the favored $\pi h_{9/2}$ band. To facilitate the discussion, transitions from the $\pi f_{7/2}$ to the favored $\pi h_{9/2}$ band will be referred to as interband transitions, and unfavored $\pi h_{9/2}$ to favored $\pi h_{9/2}$ as intraband transitions. In this section, the $B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)$ reduced transition ratios (referred to henceforth as $B(M1)/B(E2)$) for these transitions in ^{179}Ir , ^{181}Au , and ^{183}Au are extracted from the data and compared to theoretical models. The experimental and theoretical $B(M1)/B(E2)$ ratios are illustrated in Fig. 7.3. Also included in this figure are the branching ratios for $\pi h_{11/2}$ bands observed in ^{179}Ir and ^{181}Au . The $B(M1)/B(E2)$ values are extracted from the experimental data in accord with the formula:

$$\frac{B(M1; I \rightarrow I - 1)}{B(E2; I \rightarrow I - 2)} = 0.693 \frac{E_\gamma^5(I \rightarrow I - 2)}{E_\gamma^3(I \rightarrow I - 1)} \frac{1}{\lambda(1 + \delta^2)} (\mu/eb)^2, \quad (7.107)$$

where E_γ represents the γ -ray energies in MeV, λ is the intensity branching ratio [$= T(I \rightarrow I - 2)/T(I \rightarrow I - 1)$], and δ is the $E2/M1$ mixing ratio for the $\Delta I = 1$ transition. The mixing ratio is not known for most transitions presented in Fig. 7.3 and, for those bands where the mixing ratio is known, the effect on the $B(M1)/B(E2)$ ratio is less than 10%. Since this effect is less than the uncertainty in most values, and to maintain consistency where the ratios are not known, δ was assumed to be zero in Eq. 7.107.

A comparison of the experimental transition ratios of the interband and intraband γ rays indicates that the intraband rates are larger than the interband values in ^{181}Au and ^{183}Au . These transition rates are reversed in ^{179}Ir . The expectation is that transition matrix elements between signature partners would be the largest since the wavefunctions of these bands essentially differ by only a rotation of 180° . Interband transitions would occur as a result of configuration mixing, but these rates are in general smaller than for intraband transitions. It is clear that the wavefunctions of the $\pi f_{7/2}$ and $\pi h_{9/2}$ bands are very mixed and thus strong intraband transitions

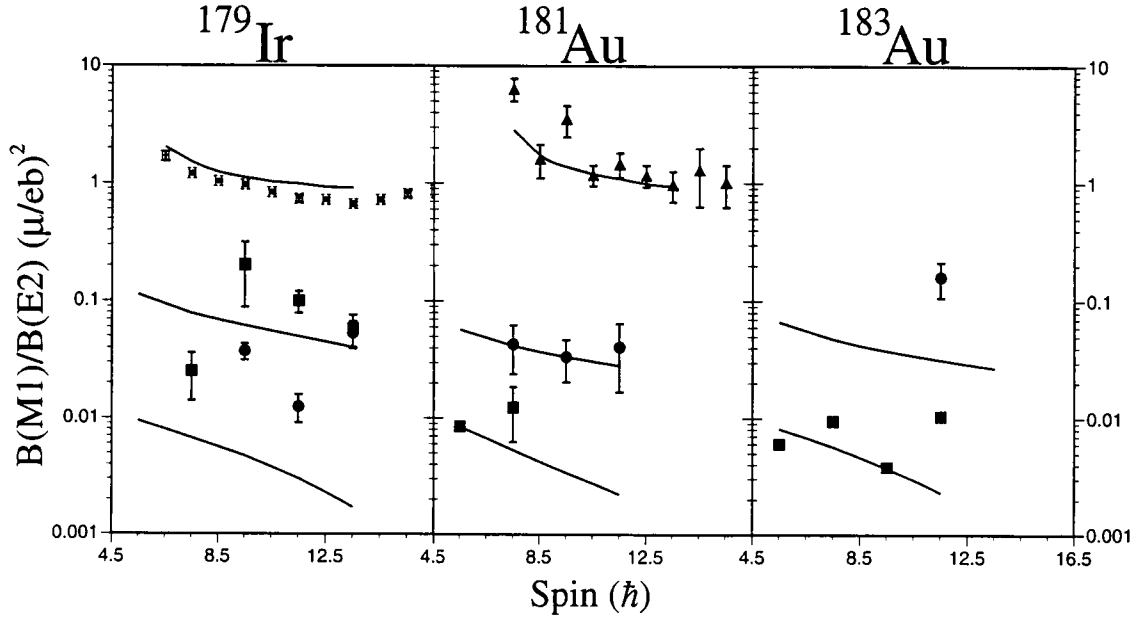


Figure 7.3: Experimental $B(M1)/B(E2)$ compared with calculations for ^{179}Ir , ^{181}Au , and ^{183}Au . The squares correspond to $B(M1)/B(E2)$ values for transitions within the $\pi h_{9/2}$ bands. Circles correspond to $\pi f_{7/2} \rightarrow \pi h_{9/2}$ transitions, and triangles $\pi h_{11/2}$ intraband transitions. The solid lines indicate the results from particle rotor calculations. Details of the calculation are presented in the text.

are likely, but it is unexpected that these intraband transitions are stronger than the interband transitions. Theoretical calculations were performed for these bands such that this problem could be better understood.

A particle rotor model with a Woods-Saxon potential is used in the calculation of the theoretical $B(M1)/B(E2)$ ratios. The deformation of the core was chosen from the TRS predictions for the lowest $(\pi, \alpha) = (-, +\frac{1}{2})$ configuration for the individual nuclei. These are the same parameters chosen for the cranked shell model calculations in Sec. 7.2 with the exception that γ was set to zero for all three nuclei. Since γ was predicted to be less than 5° for all three, this is a reasonable assumption. The value $E(2_1^+)$, the excitation energy of the first 2^+ level in the effective core, is required for parameterizing the effective core. To estimate this parameter, Grodzin's formula,

$$E(2_1^+) = \frac{1225}{\beta_2 A^{7/3}}, \quad (7.108)$$

was employed. For ^{179}Ir and ^{181}Au , the same core deformations as the $\pi h_{9/2}$ bands were used to calculate the matrix elements of $\pi h_{11/2}$ bands. From the output of the model calculations, a clear identification of an $\pi f_{7/2}$ and $\alpha = -\frac{1}{2}$ $\pi h_{9/2}$ band could not be established, because the two configurations are so mixed. For purposes of comparison, the lower energy levels of a given spin are defined as the unfavored $\pi h_{9/2}$ band and conversely the higher levels the $\pi f_{7/2}$ band. For example, in the calculated level scheme for ^{181}Au in Fig. 7.4, the bands labeled $(-, -1/2)_1$ and $(-, -1/2)_2$ are considered to be the unfavored $\pi h_{9/2}$ and favored $\pi f_{7/2}$ bands, respectively. The results of these calculations are denoted by solid lines in Fig. 7.3.

In the case of ^{181}Au , the $B(M1)/B(E2)$ ratios are reproduced very well. The ratios are also reproduced at least qualitatively for ^{183}Au . For ^{179}Ir , these values are less well predicted. While it is possible to make some comparisons of the calculations to the experimental data, a detailed analysis is difficult. The very different alignment between the $\pi f_{7/2}$ and $\pi h_{9/2}$ bands, for example, is not reproduced. Another difficulty can be seen by comparing the calculated level scheme (Fig. 7.4) with the experimental levels in Fig. 4.4; one can readily see that the levels are not accurately reproduced. In this calculation, the moment of inertia of the core was dictated by the hydrody-

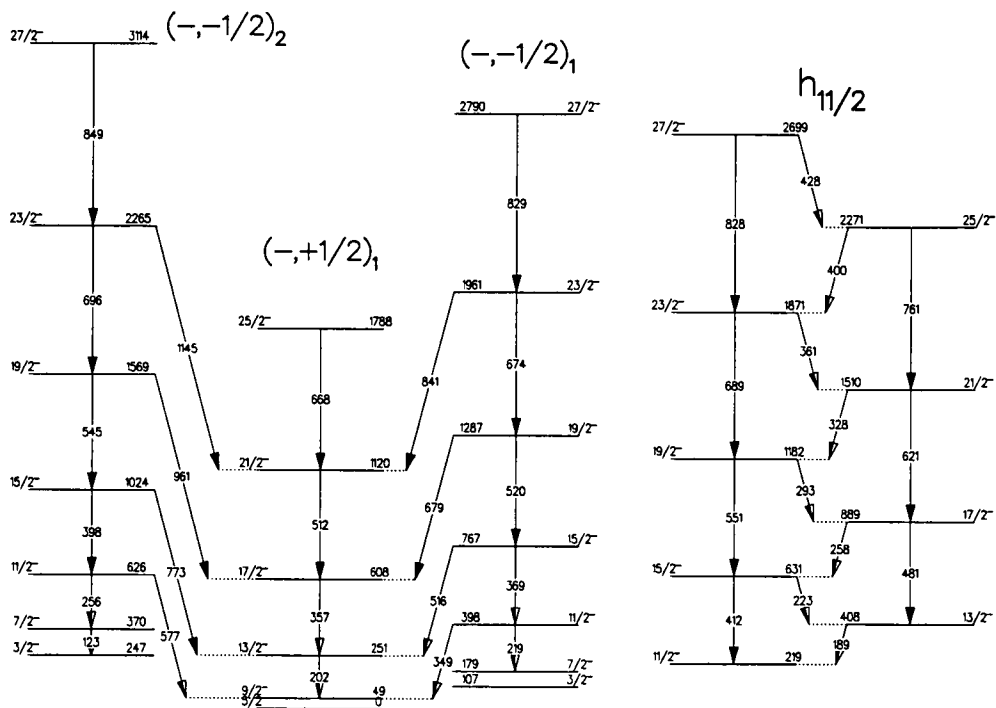


Figure 7.4: Predicted level scheme from particle-rotor calculations for ^{181}Au . Arrows connect the states that have relatively large $B(E2)$ and/or $B(M1)$ transition probabilities. The parameters used to create this scheme are discussed in the text.

dynamic relation, $\mathcal{J} \sim \beta_2^2$, and was assumed to be constant. It is thus not expected that the levels within a rotational band are well reproduced. There is an additional problem in that the excitation energies for the two bands labeled $(-, -1/2)$ are about 150 keV larger than the experimental values. The predicted level schemes for ^{179}Ir and ^{183}Au , likewise, poorly reproduce the observed energy levels. Nevertheless, the $B(M1)/B(E2)$ ratios resulting from these calculations indicate that, due to the large amount of configuration mixing, transition rates for both interband and intraband γ rays are of the magnitude observed in experiment.

7.4 Interaction between $\alpha = -\frac{1}{2}$ bands

As indicated previously, the two negative-signature bands of the $\pi h_{9/2}-\pi f_{7/2}$ system are seen to interact in all four of the nuclei presented here. In some cases, the energy levels for the particular bands are highly perturbed by the interaction, e.g. ^{185}Au , while for other nuclei the levels are negligibly perturbed, e.g. ^{181}Au . It was recently reported by Reviol *et al.* [170], that the magnitude of the level perturbation is proportional to the interaction strength and the levels of a rotational band can be transformed by the formula:

$$\Delta^4 E = \frac{1}{16} [E_\gamma(I+4) - 4E_\gamma(I+2) + 6E_\gamma(I) - 4E_\gamma(I-2) + E_\gamma(I-4)]. \quad (7.109)$$

Fig. 7.5 shows the $\mathcal{J}^{(2)}$ and $\Delta^4 E$ values for the negative signature bands in ^{179}Ir , ^{181}Au , ^{183}Au , and ^{185}Au . The dynamic moment of inertia, $\mathcal{J}^{(2)}$, is very sensitive to perturbations in the energy levels of rotational bands, and this can be readily observed in Fig. 7.5, by the large fluctuations in the moment of inertia of the bands. Because of the gradually increasing nature of $\mathcal{J}^{(2)}$, however, it is difficult to quantify a relative stagger between the different nuclei. The quantity $\Delta^4 E$ is essentially a fourth-order derivative of the given rotational band, thus the gradually rising feature of $\mathcal{J}^{(2)}$ is averaged out, and a staggering related to the level perturbations is all that remains. As is illustrated in Fig. 7.5, the interaction between the $\pi h_{9/2}$ and $\pi f_{7/2}$ bands is greatest in ^{185}Au where the staggering is as great as 16 keV. For ^{179}Ir and ^{183}Au , the interaction is about 60% that of ^{185}Au . In ^{181}Au , the staggering is essentially zero

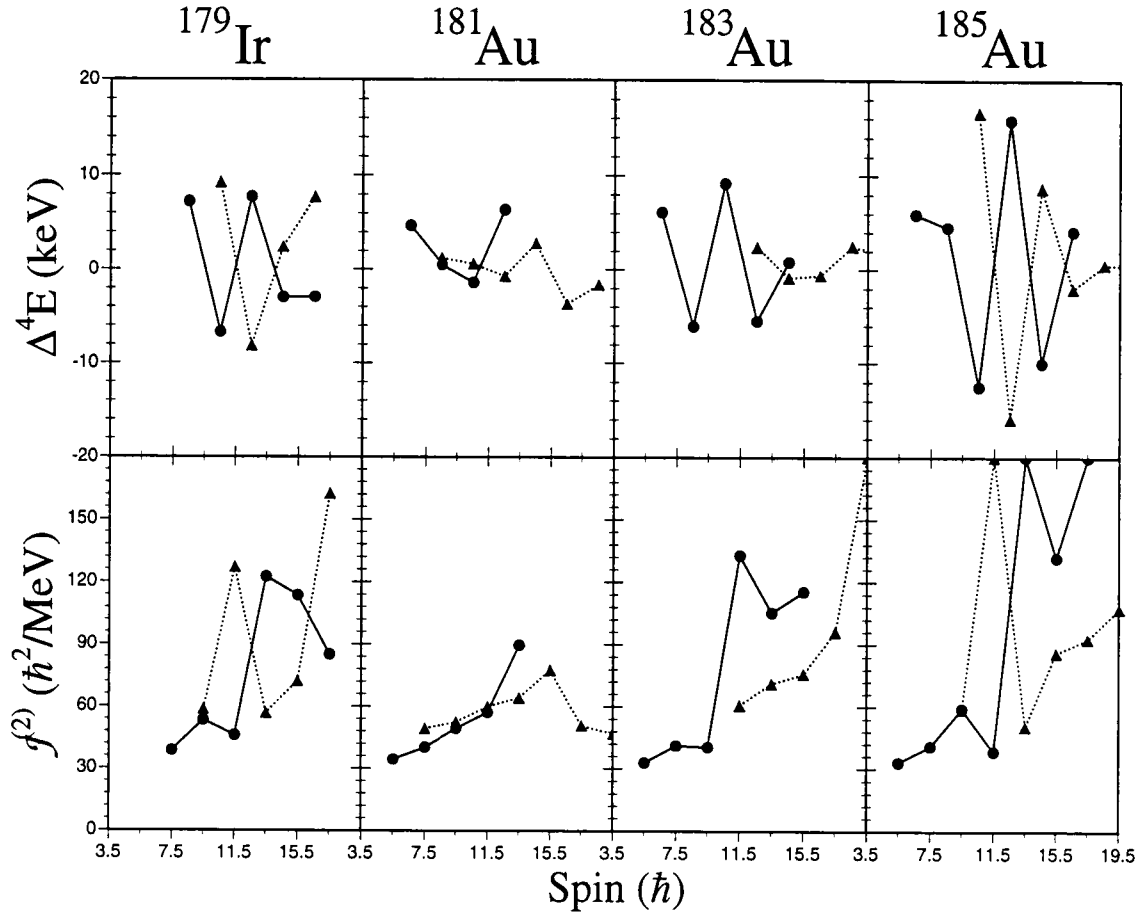


Figure 7.5: $\mathcal{J}^{(2)}$ and $\Delta^4 E$ vs. $\hbar\omega$ for $\alpha = -\frac{1}{2} \pi h_{9/2} - \pi f_{7/2}$ bands in ^{179}Ir , ^{181}Au , ^{183}Au , and ^{185}Au . The circles represent the $\pi h_{9/2}$ band and the triangles correspond to $\pi f_{7/2}$. The lower panels indicate the $\mathcal{J}^{(2)}$ calculated from observed bands using Eq. 2.65, and the upper panels represent $\Delta^4 E$ staggering extracted from experimental levels using Eq. 7.109.

at the point where the two bands cross (levels $15/2$ and $19/2$). This indicates that there is only a very small interaction between the close lying $\pi h_{9/2}$ and $\pi f_{7/2}$ bands in ^{181}Au .

The reason for this weak interaction in ^{181}Au may be related to the point where the $\pi h_{9/2}$ and $\pi f_{7/2}$ bands cross. As can be seen in Fig. 7.1, the unfavored $\pi h_{9/2}$ and $\pi f_{7/2}$ bands cross at about spin $19/2^-$. This compares with a crossing at about $23/2^-$ to $27/2^-$. This can be seen in Routhian space as well. The rotational frequency at which the two bands cross can be determined from Figs. 7.2(2b), (3b), and (4b) for ^{181}Au , ^{183}Au , and ^{185}Au to be 0.20, 0.21, and 0.22 MeV. Thus for progressively higher rotational frequencies, the interaction strength increases. As with the other aspects of the $\pi h_{9/2}$ - $\pi f_{7/2}$ system, this is a feature that is not understood, and cannot be reproduced by the present models.

CHAPTER 8

Conclusions

The purpose of the study presented in the dissertation was to observe deformation effects in the lightest Au nuclei yet studied by understanding the spectroscopy of high-spin states based on different single-proton orbitals. ^{79}Au is in the large Z portion of the well established region of strong prolate deformation and close to nuclei (^{80}Hg , ^{81}Tl) where ground states are slightly oblate and Pb where there is a closed shell (and thus spherical). Of particular interest for this study is how much the high- j proton orbits “restore” the strong prolate shapes. These “intruder” orbits were observed to be going to lower energies for Au isotopes down to ^{183}Au , so the desire was to study even lighter Au isotopes to sample the fuller impact of intruder states.

To this end, excited states have been studied in ^{181}Au and ^{179}Au nuclei for the first time. These are the most neutron-deficient Au isotopes which have been studied in detail. They represent the latest in a series of experiments performed by the University of Tennessee Nuclear Physics Group or in collaboration with other laboratories on the high-spin states of neutron deficient Au nuclei (^{187}Au [161], $^{186,188}\text{Au}$ [171], ^{185}Au [64], ^{183}Au [65]). One purpose of these experiments is to study the evolution of nuclear shape from the region of predominately oblate deformations ($N > 108$) through the region of prolate configurations ($100 < N < 108$).

High-spin states in ^{181}Au were populated by the $^{150}\text{Sm}(^{35}\text{Cl}, 4n)$ heavy-ion fusion reaction, and studied at the Oak Ridge Holifield Heavy-Ion Research Facility with the Spin Spectrometer and 19 Compton-suppressed Ge detectors. Eight prolate rotational bands have been established in the analysis of this nucleus. Bands based on $\pi h_{9/2}$, $\pi f_{7/2}$, $\pi i_{13/2}$, and $\pi h_{11/2}$ configurations have been clearly identified. In addition, there is a structure of high-lying states feeding at high spins into the $\pi i_{13/2}$ band in ^{181}Au . Of these two observed side bands, one has been identified as a rotational band

based on the three quasiparticle excitation $\pi \frac{1}{2}[660] \otimes \nu \frac{1}{2}[521] \otimes \nu \frac{5}{2}[512]$, and the second has been suggested as the unfavored signature of the $\pi i_{13/2}$ band.

The experiment to study high-spin states in ^{179}Au took place at the ATLAS facility at Argonne National Laboratory. The AYE-ball array of 19 Compton-suppressed Ge detectors was used in conjunction with the Fragment Mass Analyzer and a double-sided silicon strip detector to make the first positive identification of excited states in ^{179}Au by the recently developed method of Recoil Decay Tagging [57]. These states were populated in a $^{124}\text{Te}(^{58}\text{Ni}, p2n)$ reaction. From this study, the $\pi i_{13/2}$ band was clearly established, and a lower limit of 592 keV is determined for the band-head energy. Three other bands have been observed in this nucleus with suggested assignments as two signatures of $\pi h_{9/2}$ and $\pi d_{3/2}$ configurations.

A preliminary analysis of the moment of inertia of the $\pi i_{13/2}$ band in ^{179}Au provided an indication that the deformation (β) associated with this band increases with respect to the similar bands in heavier Au nuclei. This fact, among others, prompted a more thorough study of moments of inertia of prolate rotational bands throughout the rare-earth region. The “standard” method for determining nuclear deformation involves measuring level lifetimes from which the quadrupole moment (and deformation) can be calculated. These measurements are difficult and usually have large uncertainties. Thus, alternative methods for inferring deformation are important to be able to more readily examine a wide range of nuclei. From the study in this dissertation, a method was developed to correlate the moment of inertia of a rotational band to the nuclear deformation.

The first aspect of this survey was the analysis of prolate ground-state rotational bands even-even nuclei from Dy to Hg. The various contributions to the experimental moment of inertia ($\mathcal{J}$) were separated by the Harris parameterization,

$$\mathcal{J} = \mathcal{J}_0 + \mathcal{J}_1 \omega^2,$$

where the static-deformation-dependent portion ($\mathcal{J}_0$) could be established. A fit to the energies of the three to five levels below the first backbend of these ground bands yielded $\mathcal{J}_0$ values which were correlated with experimentally determined deformation (from $B(E2; 0_1^+ \rightarrow 2_1^+)$) and pair gap parameters (from even-odd mass differences) in

even-even nuclei leads to the following amazingly correlated empirical relation:

$$\beta = \frac{\Delta [\text{MeV}]}{6.5A^{1/2}} \left(41.8 - 11.6\sqrt{13.0 - 0.173(33.7 + \mathcal{J}_0 [\hbar^2/\text{MeV}])} \right).$$

This is the first such correlation developed, and allows an estimate of deformation to be extracted from $\mathcal{J}_0$ values in these even-even nuclei.

This strong correlation of $\mathcal{J}_0$ to β has been applied to infer the deformation of intruder states in odd- A nuclei. Nuclear models predict an enhancement of β due to the excitation of a proton into an intruder state. This prediction has been confirmed by a limited number of lifetime measurements. In this dissertation, a more extensive analysis of the moments of inertia of intruder bands, however, indicates that the models underpredict the amount of enhancement in some nuclei.

For cases where the intruder configuration is not predicted to enhance the deformation of the nucleus (e.g. $\pi h_{9/2}$ bands in Ta and Re, and $\pi i_{13/2}$ bands for $N \geq 102$ Au) the theoretical predictions correlate very accurately to the observed $\mathcal{J}_0$ values. Systematic differences between deformation deduced from the experimental moment of inertia and from theoretical calculations are observed for $\pi h_{9/2}$ bands in Ho, Tm, and Lu. In the latter cases of Tm and Lu, the deviations indicate that the deformation is underpredicted in the current models by $\approx 15\%$ for these nuclei. This result is also confirmed for the case of ^{165}Tm by lifetime measurements. For ^{157}Ho , the inferred deformation ($\beta = 0.432 \pm 0.028$) is larger than the predicted value (0.318) by a substantial 36%. This conclusion, however, is contrary to lifetime measurements from which the extracted deformation is 0.27 ± 0.07 .

While good agreement between experimental and theoretical deformations was observed for $\pi i_{13/2}$ bands in $N \geq 102$ Au isotopes, differences occur in Au nuclei with $N \leq 100$. From the analysis of the moments of inertia the deformation of $\pi i_{13/2}$ bands in ^{179}Au and ^{177}Au are inferred to be $\beta = 0.348 \pm 0.089$ and 0.380 ± 0.074 respectively. These values are correspondingly 17% and 25% greater than the predicted values of 0.298 and 0.304. A similar conclusion that the deformation for $N \leq 100$ nuclei in Re and Ir can also be made, however, only qualitatively due to large uncertainty in $\mathcal{J}_0$. The $\pi i_{13/2}$ band in ^{163}Lu (the smallest Z where a $\pi i_{13/2}$ has been observed) has a measured $\mathcal{J}_0 = 54.8 \pm 3.6$. This is a value that is greater than

the maximum $\mathcal{J}_0$ for which the correlation developed here is valid and a deformation can be deduced. This implies that $\beta > 0.540$ for ^{163}Lu . This is significantly greater than the value determined from lifetime measurements (0.437 ± 0.054) or predicted theoretically (0.318).

While the correlation $\mathcal{J}_0$ to $\beta^{(th)}$ and Δ indicates that the deformation of intruder bands in some nuclei differ by $\approx 15\%$, there are at least two cases where the deviations indicate as much as 37% or more (e.g. ^{157}Ho and ^{163}Lu). Lifetime measurements for ^{157}Ho conflict with the β extracted from $\mathcal{J}_0$, while similar measurements in ^{163}Lu correlate qualitatively with $\mathcal{J}_0$. These deviations appear much larger than what can easily be explained as a common effect. In order to confirm that the deviations are systematic throughout a given isotope chain (thus indicating a deformation effect), it is important to investigate additional Ho and Lu nuclei to search for additional $\pi h_{9/2}$ and $\pi i_{13/2}$ intruder bands. Also important in this search is the need for lifetime measurements in order to compare extracted deformation with those deduced from $\mathcal{J}_0$.

An increasing deformation for $N < 102$ suggested by a larger $\mathcal{J}_0$ in $\pi i_{13/2}$ bands in Au, Ir, and Re is not predicted by theoretical calculations. To determine whether the increasing $\mathcal{J}_0$ is truly a deformation effect it is necessary to perform lifetime measurements on some the lightest Au nuclei yet observed (e.g. ^{179}Au or ^{177}Au). Due to the large background from fission and multiple particle evaporation channels such measurements will be very difficult, and likely require new techniques for measuring nuclear level lifetimes. In addition to lifetime measurements, it would be interesting to push to even lighter nuclei (^{175}Au) to determine where the observed $\mathcal{J}_0$ in $\pi i_{13/2}$ bands will begin to decrease. Evidence in Re and Ir suggest that such a decrease will occur at $N = 96-98$, however, the large uncertainty in the measurements for these bands makes the quantitative assessment difficult.

This systematic study should be extended to other mass regions in order to determine the extent this correlation of $\mathcal{J}_0$ to β and Δ . For example, the $A \sim 130$ region is another area where strong prolate deformation is observed. In this case, the $\frac{1}{2}[550]$ orbital originating from the $\pi h_{11/2}$ spherical shell state has intruder characteristics. A study similar to the project presented here can provide further

evidence to test the consistency of theoretical calculations to observed phenomena. In addition, this study can also be expanded to include superdeformed rotational bands and be used as a consistency check for lifetime measurements in the second well.

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Vita

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