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ANALYSIS OF FLUCTUATIONS IN A COMBUSTION-DRIVEN
OPEN-CYCLE MHD GENERATOR

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ABSTRACT

Fluctuations present in MHD generators may cause significant degradation in the generated power. The fluctuations may result from three sources. First, the mass flow rates of the components' input to the combustor vary. Second, the combustor initiates its own variations which are functions of the combustor geometry and injection techniques. Third, the generator action, i.e., flow of plasma in a magnetic field, introduces variations in the plasma variables.

The purpose of the study is to investigate the fluctuations of MHD output signals, which may either arise from the combustor fluctuations propagating into a conducting channel, or are inherent in the generator dynamics. The analysis of fluctuations is based on the analytical and empirical models. Both models assume that stochastic processes take place within the MHD plasma, and both models yield results in the form of autocorrelation, crosscorrelation, and power spectral density functions of the system variables.

The study showed that fluctuations, whose frequencies exceed 200 Hz, in the plasma density, velocity, pressure, current and voltage variables are acoustic in nature, and are caused by longitudinal standing waves present in the generator. The analysis proved that Hall generators develop fluctuations mainly in the range 700 Hz to 2000 Hz, whereas Faraday and DCW generators are favorable for the low frequency fluctuations. Parametric study of the plasma disclosed that stronger magnetic fields and larger Hall parameters increase the frequency range of fluctuations. Changes in plasma specific heat ratio or in inlet steady-state parameters may increase or decrease the

intensities of some odd harmonics of the standing waves. The fluctuations that originate in the combustion chamber also affect the plasma variables. A white noise character of these fluctuations guarantees a fairly uniform distribution of energy in the fluctuations of the plasma variables in the frequency range up to 200 Hz.

Future research in the field of fluctuations should concentrate on the influence of the viscous effect, heat conduction in plasma, and wall friction on the generator performance. These effects were not included in the modeling. Extension to two-dimensional dynamics will also be useful in studying flow characteristics in diverging channels.

TABLE OF CONTENTS

CHAPTER	PAGE
1. INTRODUCTION	1
2. STATE OF THE ART IN THE ANALYSIS OF FLUCTUATIONS AND WAVE PROPAGATION PHENOMENA IN THE MHD GENERATOR.	8
3. ANALYTICAL MODELING OF THE MHD PLASMA.	14
1. Formulation of the Plasma Model.	14
2. Linearization of the Model Using the Langevin Technique.	18
3. Nodal Form of the Plasma Model	22
4. Solution of the Steady State Plasma Equations.	29
4. SOLUTION OF THE MODEL IN THE FORM OF AUTOCORRELATION AND CROSSCORRELATION FUNCTIONS	32
1. Response of a Linear System to Stochastic Inputs	32
2. Similarity Transformation of a State Parameter Matrix.	37
3. Analytical Correlation and Power Spectral Density Functions: The Computer Program MHD CR	42
4. Summary.	51
5. RESULTS OF ANALYTICAL MODELING OF THE MHD PLASMA DYNAMICS.	54
1. Selection of the MHD Data.	54
2. Analysis of the State Vector Variables	58
3. Analysis of the Current and Voltage Variables.	85
4. Summary.	94
6. PARAMETRIC STUDY OF THE MHD PLASMA DYNAMICS.	96
7. EMPIRICAL STOCHASTIC TIME-SERIES MODELING OF THE MHD PLASMA DYNAMICS.	117
1. Estimation of Time and Frequency Domain Signatures	117
2. Results of Empirical Modeling of the MHD Plasma.	122

CHAPTER	PAGE
8. SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE WORK. . . .	134
1. Summary.	134
2. Conclusions.	135
3. Suggestions for Future Work.	137
BIBLIOGRAPHY	138
APPENDIXES	141
APPENDIX A. MODEL COEFFICIENTS.	142
APPENDIX B. FORTRAN PROGRAM	150
VITA	163

LIST OF TABLES

TABLE	PAGE
1. Time dependent components of the integrand	46
2. Time dependent components of the output crosscorrelation matrix	49
3. Frequency dependent components of the output spectral density matrix	52
4. Analysed MHD generators and the corresponding current data	57
5. Odd harmonics and the corresponding wavelengths of the MHD channel.	68

LIST OF FIGURES

FIGURE	PAGE
1. Types of the MHD generators.	4
2. The APSD functions of the pressure variable for different axial currents at $x = 36$ cm.	59
3. The APSD functions of the pressure variable for different axial currents at $x = 106$ cm	60
4. The APSD functions of the pressure variable at different axial locations for $J_x = .60$ A/cm ²	61
5. The APSD functions of the pressure variable at different axial locations for $J_x = 1.10$ A/cm ²	62
6. The crosscorrelation functions between the pressure variables for $J_x = .60$ A/cm ²	64
7. The crosscorrelation functions between the pressure variables for $J_x = 1.10$ A/cm ²	65
8. The CPSD functions between the pressure variables for $J_x = 1.10$ A	66
9. The APSD functions of the density variable for different axial currents at $x = 36$ cm.	70
10. The APSD functions of the density variable for different axial currents at $x = 106$ cm	71
11. The APSD functions of the density variable at different axial locations for $J_x = .10$ A/cm ²	72
12. The APSD functions of the density variable at different axial locations for $J_x = 1.10$ A/cm ²	73
13. The crosscorrelation functions between the density variables for $J_x = .60$ A/cm ²	74
14. The crosscorrelation functions between the density variables for $J_x = 1.10$ A/cm ²	75
15. The CPSD function between the density variables for $J_x = .60$ A/cm ²	76
16. The APSD functions of the velocity variable for different axial currents at $x = 36$ cm.	78

FIGURE	PAGE
17. The APSD functions of the velocity variable for different axial current at $x = 106$ cm.	79
18. The APSD functions of the velocity variable at different axial locations for $J_x = .40$ A/cm ²	80
19. The APSD functions of the velocity variable at different axial locations for $J_x = 1.10$ A/cm ²	81
20. The crosscorrelation functions between the velocity variables for $J_x = .40$ A/cm ²	82
21. The crosscorrelation functions between the velocity variables for $J_x = 1.10$ A/cm ²	83
22. The CPSD function between the velocity variables for $J_x = 1.10$ A/cm ²	84
23. The APSD functions of the various axial currents at $x = 36$ cm.	87
24. The CPSD functions between the axial currents at $x = 36$ cm and $x = 72$ cm.	88
25. The CPSD functions between the axial voltages at $x = 36$ cm and $x = 72$ cm.	89
26. The crosscorrelation functions between the axial currents at $x = 36$ cm and $x = 72$ cm	90
27. The crosscorrelation functions between the y-currents at $x = 36$ cm and $x = 72$ cm.	91
28. The crosscorrelation functions between the axial voltages at $x = 36$ cm and $x = 72$ cm	92
29. The APSD functions of the pressure variable resulting from various noise sources $\exp(- p \tau)$; $x = 106$ cm, $J_x = .40$ A/cm ²	97
30. The APSD functions of the density variable of various Hall parameters; $x = 106$ cm, $J_x = .40$ A/cm ²	99
31. The autocorrelation functions of the density variable of various Hall parameters; $x = 106$ cm, $J_x = .40$ A/cm ²	100
32. The APSD functions of the velocity variable of various Hall parameters; $x = 106$ cm, $J_x = .40$ A/cm ²	101

33.	The autocorrelation functions of the velocity variable of various Hall parameters; $x = 106$ cm, $J_x = .40$ A/cm ²	102
34.	The APSD functions of the pressure variable of various Hall parameters; $x = 106$ cm, $J_x = .40$ A/cm ²	103
35.	The autocorrelation functions of the pressure variable of various Hall parameters; $x = 106$ cm, $J_x = .40$ A/cm ²	104
36.	The APSD functions of the pressure variable of various specific heat ratios; $x = 106$ cm, $J_x = .40$ A/cm ²	106
37.	The APSD functions of the velocity variable of various specific heat ratios; $x = 106$ cm, $J_x = .40$ A/cm ²	107
38.	The APSD functions of the density variable of various specific heat ratios; $x = 106$ cm, $J_x = .40$ A/cm ²	108
39.	The CPSD functions between the velocity variables at $x = 36$ cm and $x = 106$ cm, for various input velocities . . .	110
40.	The crosscorrelation functions between the velocity variables at $x = 36$ cm and $x = 106$ cm, for various input velocities	111
41.	The APSD functions of the pressure variable at $x = 106$ cm, for various input pressures.	112
42.	The autocorrelation functions of the pressure variable at $x = 106$ cm, for various input pressures.	113
43.	The APSD functions of the density variable at $x = 106$ cm, for various mass flow rates.	114
44.	The autocorrelation functions of the density variable at $x = 106$ cm, for various mass flow rates.	115
45.	The APSD functions of the MHD signals, resulting from the empirical modeling	124
46.	The CPSD functions between the MHD signals, resulting from the empirical modeling	125
47.	The coherence functions between the MHD signals, resulting from the empirical modeling.	126
48.	The coherence functions between the MHD signals, resulting from the empirical modeling.	127
49.	The SCR functions; contributions to current spectral density.	128

50.	The SCR functions; contributions to combustor pressure spectral density	129
51.	The SCR functions; contributions to plasma luminosity spectral density	130

LIST OF SYMBOLS

A	State parameter matrix
A_i	Time-series model parameter matrix
$\overline{B}$	Magnetic field vector
C_{ij}	Crosscorrelation function between the variables at the nodes i, j
$C(k)$	Correlation matrix at time lag k
$\overline{E}$	Electrical field vector
E_x	x-direction component of an electrical field vector
E_y	y-direction component of an electrical field vector
$\underline{F}$	Forcing function vector
G	Mass flow rate
$G(f)$	Transfer function
$H(f)$	Transfer function
J	Matrix obtained from a similarity transformation based on the Jordan canonical form
J_k	Jordan block
$\overline{J}$	Current density vector
J_x	x-direction component of a current density vector
J_y	y-direction component of a current density vector
L	Channel length
	Langevin operator
P	Similarity transformation matrix
Q	Noise covariance matrix
	Similarity transformation matrix

R	Gas constant
R_{FF}	Forcing function correlation matrix
R_{XX}	Output correlation matrix
S_{ij}	Cross-spectrum density function
S_{FF}	Input spectral density function matrix
S_{XX}	Output spectral density function matrix
S_{YY}	Spectral density function matrix of MAR model
T	Plasma temperature
$T_{o(i)}$	Steady state plasma temperature at i-th node
$\underline{V}$	White noise sequence vector
V_i	Ionization potential
$\underline{X}$	State vector
$\underline{Y}(k)$	Vector of observations at time lag k
C_p	Specific heat at constant temperature
e	Elementary charge
e_x	Fluctuating component of an x-direction electrical field vector
e_y	Fluctuating component of an y-direction electrical field vector
f	Frequency
$\underline{f}(t)$	Vector of driving forces
i	Node number
j_x	Fluctuating component of x-direction current density vector
j_y	Fluctuating component of y-direction current density vector
k	Boltzmann's constant
n	Plasma electron number density
n'	Fluctuating plasma electron number density
n_o	Steady-state plasma electron number density

p	Plasma pressure
p'	Fluctuating plasma pressure
p_0	Steady-state plasma pressure
q	Collision cross section
t	Time
$\bar{u}$	Plasma velocity vector
u'	Fluctuating plasma velocity
u_0	Steady-state plasma velocity
$\underline{w}$	Electron drift velocity vector
x	x-axis, $x = 0$ at channel inlet
y	y-axis
Λ	Matrix obtained from a similarity transformation
Λ_i	Eigenvalue
Ω	Hall parameter
α	Time
β	Imaginary part of an eigenvalue
γ	Specific heat ratio
γ_ρ	Coefficient of equations (3.19)
γ_u	Coefficient of equations (3.19)
γ_p	Coefficient of equations (3.19)
δ_ρ	Coefficient of equations (3.19)
δ_u	Coefficient of equations (3.19)
δ_p	Coefficient of equations (3.19)
ϵ	Error
ϵ_0	Permittivity of vacuum
λ	Eigenvalue

λ	Wavelength
μ_0	Permeability of vacuum
ρ	Plasma density
ρ'	Fluctuating plasma density
ρ_0	Steady-state plasma density
ρ_e	Electron number density
σ	Plasma conductivity
	Real part of an eigenvalue
σ'	Fluctuating plasma conductivity
σ_0	Steady-state plasma conductivity
τ	Time
ω	Electron cyclotron frequency
	Angular velocity
AIC	Akaike information criterion
APSD	Auto-power spectral density
CPSD	Cross-power spectral density
DCW	Diagonally connected wall
MAR	Multivariate autoregressive
MHD	Magnetohydrodynamic
MISO	Multiple-input single-output
SCR	Signal contribution ratio
WKB	Wentzel-Kramers-Brillouin

CHAPTER 1

INTRODUCTION

A magnetohydrodynamic (MHD) generator's importance to the United States fuel economy lies in its possible potential for producing more energy from the direct conversion of coal than any currently available power conversion system. Performance studies of advanced coal-fired power generating systems indicate that a combined MHD-steam plant cycle is the only coal-based cycle capable of achieving a 45-50% efficiency, which is much better than the 35% efficiency of a modern, coal-fired steam plant. The efficiency of a steam plant's thermal cycle is limited by the ability of turbines' rotating components to withstand high stresses at high temperatures. An MHD generator has no rotating parts, therefore, it can operate at higher than turbine temperatures. This results in its higher efficiency. The higher efficiency implies significant coal savings, which translate directly into reduced total costs and reduced environmental pollution. Further environmental benefits from MHD energy production stem from the close management of SO_x and NO_x emissions, which are not easily controlled in a conventional plant. Two other features of an MHD generator are its extremely rapid start-up and its even more rapid response to load variations.

For a large-scale, coal-fired MHD power plant, the basic system design choice is between an open-cycle and a closed-cycle system. Currently, an open-cycle system concept using linear MHD channels is favored for electrical power plants. A closed-cycle system, together with other channel design concepts, is viewed as an alternative for second generation MHD power plants.

MHD electrical power generation was first discovered by Michael Faraday during his investigation of electromagnetic induction in 1831. The first recorded attempt to develop an MHD generator was conducted by Westinghouse Corporation before and during World War II. During the last fifteen years, several countries have supported MHD research on a relatively large scale. The most impressive MHD effort today exists in the Soviet Union. It is the only country operating an MHD facility connected to the national energy grid. The United States has recently started testing 20 MWth prototypes. One of the designs is under development at the University of Tennessee Space Institute (UTSI), and the other in Butte, Montana.

A coal-fired, open-cycle MHD generator consists of two main parts: a combustion chamber and a conducting channel. The fuel, pulverized coal enriched with kerosene and preheated air, is mixed with small amounts of alkali metal. The alkali metal, usually potassium or cesium, has an ionization potential 3 to 4 times smaller than any of the other combustion products. As a result of combustion, the gases reach temperatures in the 2500 to 3000 K range, which is adequate for the seed material to convert the gases into slightly ionized plasma. The plasma is in thermal equilibrium and has a conductivity of 10^{-6} that of copper. The electrically conducting fluid is then directed into the conducting channel at a high velocity, $\bar{u}$. A strong magnetic field in the 2 to 3 Tesla (T) range is applied in a direction perpendicular to the flow. In accordance with Faraday's Law, an electromotive force, $\bar{u} \times \bar{B}$, where $\bar{B}$ is a magnetic field vector, is induced in the moving fluid. If suitable electrodes are used to make contact with the fluid, and if they

are connected to a load, then current flows through the fluid and energy is delivered to the load. The current, $\bar{J}$, causes a body force of magnitude $\bar{J} \times \bar{B}$ to be exerted on the fluid. The body force retards the net motion of the fluid, thereby causing a difference in pressure. Thus, as in a conventional turbine, an MHD generator extracts energy from the moving fluid at the expense of a pressure drop.

An MHD system generates DC electrical power rather than AC power needed for transmission on national grids. This deficiency can be remedied by specially designed inverters.

As a result of the conditions present inside an MHD generator, the drift velocity of the current carrying electrons is not negligible; in fact, it contributes significantly to the total potential drop. In an MHD generator, therefore, there is an electric field, $\bar{u} \times \bar{B}$, due to the fluid velocity, $\bar{u}$, and an induced electric field, $\bar{w} \times \bar{B}$, due to the electron drift velocity, $\bar{w}$, relative to the neutrals. The presence of this additional electric field, called the Hall effect, gives rise to a number of different channel configurations, four of which are shown in Figure 1. Two are of the Faraday type, in which the load(s) is(are) connected so that power is extracted from the plasma via the y direction only. In the first type (Figure 1a), a single, continuous electrode is used; no potential appears axially along the channel. The Hall (axial) current flows downstream in the gas and returns through the electrode walls, causing energy dissipation and a reduction in the power output. The geometry is attractive in spite of these losses because of its simplicity and single current output. Another configuration, the segmented-electrode Faraday generator (Figure 1b), is

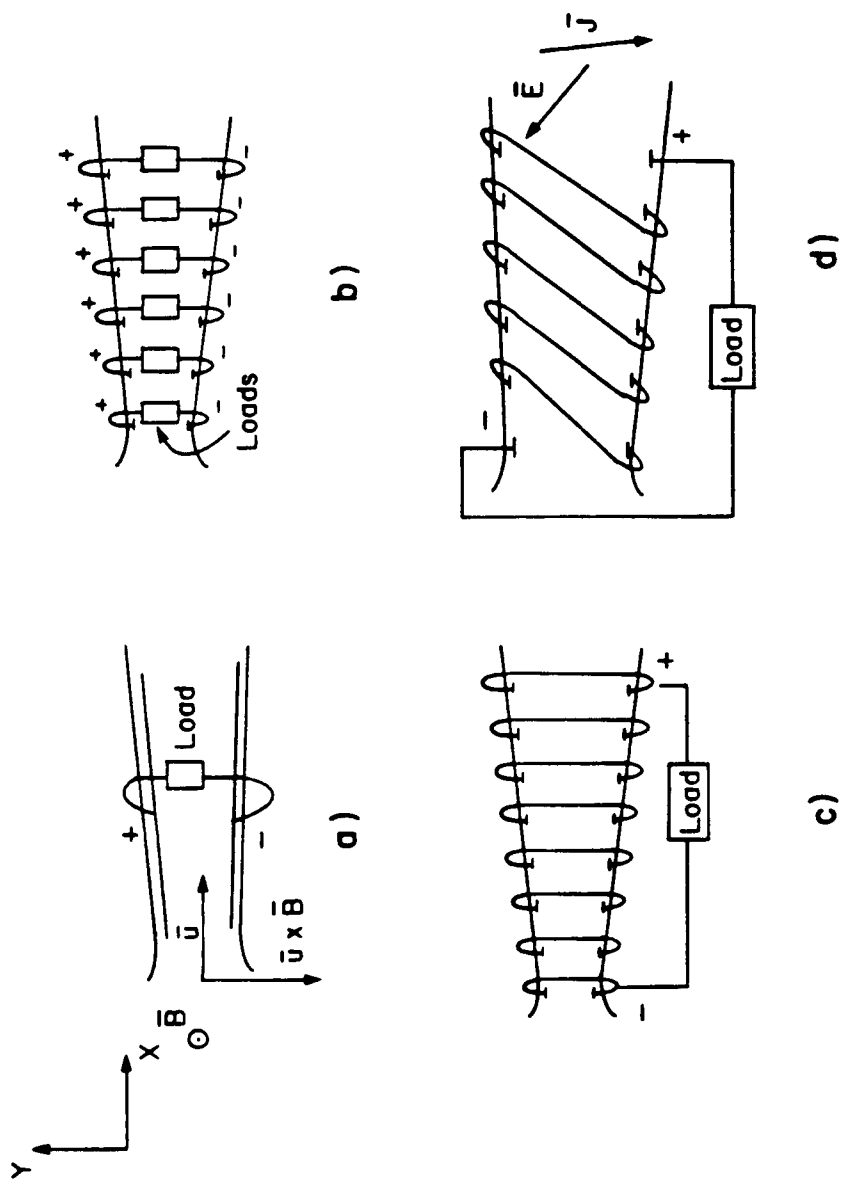


Figure 1. Types of the MHD generators

- a) A Faraday generator with continuous electrodes.
- b) A Faraday generator with segmented electrodes.
- c) A linear Hall generator.
- d) A diagonally connected walls generator.

more efficient. The channel is segmented, preventing the flow of the Hall current, and power is delivered to several external loads. The design is not practical, however, because of its load multiplicity. In the third configuration (Figure 1c), the load is connected so that power is extracted via the Hall (axial) current only, but this Hall generator is less efficient than the segmented-electrode Faraday generator. The last configuration (Figure 1d) is the diagonally connected wall (DCW) generator. In the DCW generator, those electrodes which would lie on the surfaces of equal potential in the segmented-electrode Faraday generator are connected together. As a result, one obtains both the efficiency advantage of the segmented-electrode Faraday generator and the reduced number of outputs of the Hall connection. High efficiency and simplicity of operation, therefore, make the DCW generator the most desirable design for commercial application.

Even though an MHD generator would have numerous advantages as an alternative energy source, it is not yet available for industrial use. The magnet and inverter designs, as well as the choice of the best materials for the electrodes and insulators, present problems still to be solved. Other problems awaiting solution involve the fluctuations and wave phenomena within the generator system.

The experimental results indicate that fluctuations are present in an MHD generator which cause significant degradation in the produced power. These fluctuations of the MHD output signals, both current and voltage, probably result from three sources. First, the mass flow rates of the components' input to the combustor vary. Second, the combustor introduces its own variations, which are functions of

the combustor geometry and injection techniques. Third, the generator action introduces variations, due mainly to the plasma nonuniformities.

The purpose of this study is to assess the cause of the generator current and voltage fluctuations which either arise from the combustor fluctuations propagating into the conducting channel, or are inherent in the generator dynamics. The study answers the questions whether, and under what conditions, waves are present inside the channel, and describes how they affect the current and voltage.

Most of the results presented in this study were obtained under conditions characteristic of a DCW generator, since this is the generator most favored by industry. The model developed for the system plasma is very general, however, and can be applied without modification to all of the types of MHD generators discussed above. The originality of the research lies in this general plasma model coupling the combustor with the conducting channel, and also in the methodology developed for an analytical study of stochastic systems. This methodology, which yields results in the form of crosscorrelation functions and power spectral density functions of the system variables, does not involve any approximation. Each result presented in the study is described by a true analytical solution which is a function of the system eigenvalues and of assumed different conditions at the entrance to the conducting channel. The same methodology, with slight modifications, could be helpful in the dynamic analysis of other stochastic systems.

Chapter 2 discusses the state of the art in the analysis of fluctuations and wave propagation phenomena in MHD generators. The

equations governing MHD plasma appear in Chapter 3. This chapter also describes the Langevin technique applied to the equations, in order to linearize them. These linearized space and time dependent equations are presented next in a nodal form which constitutes the MHD plasma model. The solution to the model, based on a similarity transformation of a state parameter matrix, appears in Chapter 4. This chapter also introduces the computer program MHDCR which solves the model in the form of analytical autocorrelation and crosscorrelation functions. The results of the analytical modeling are presented in Chapters 5 and 6. Chapter 5 contains analyses of the fluctuating density, velocity, pressure, current, and voltage variables of the MHD plasma, performed under sets of data characteristic of various types of MHD generators. Chapter 6 contains a parametric study of the plasma, performed for a DCW generator. Chapter 7 describes a totally different approach to plasma dynamics: empirical modeling, using stochastic time-series models. The results of this empirical modeling are based on the data obtained from the UTSI MHD test facility. Finally, Chapter 8 gives the summary of the dissertation, conclusions from analyses, and suggestions for future research.

CHAPTER 2

STATE OF THE ART IN THE ANALYSIS OF FLUCTUATIONS AND WAVE PROPAGATION PHENOMENA IN MHD GENERATORS

Several papers have been published discussing the influence of fluctuations on the overall MHD performance. Their theoretical methods of analysis are based on the traditional first order perturbation theory applied to the continuity, momentum, and energy equations coupled with the generalized Ohm's Law.

Velikov [1] appears to have been the first to perform a parametric study of MHD plasma. Considering variations in the Hall parameter (see chapter 3), he concluded that acoustic waves propagating transverse to the direction of flow are present inside Faraday generators. An acoustic wave is initiated by a small shift of fluid elements in a compressible medium. If this initial disturbance induces similar small movements in adjacent elements, the acoustic wave propagates then through the medium at a relatively high speed.

McCune [2], investigating the influence of plasma conductivity fluctuations, noted that these fluctuations can provide the dominant mechanism for transverse wave growth.

Locke and McCune [3] were the first to recognize that the Hall type generator might present very favorable conditions for axial wave propagation. They derived analytically two such waves, which they called the magneto-acoustic downstream mode and the magneto-acoustic upstream mode. They concluded that the amplitude of the downstream

mode is always damped, whereas that of the upstream mode can sometimes be amplified if large plasma conductivity fluctuations are present.

Powers and Dicks [4], performed analyses of conductivity and Hall parameter fluctuations in the DCW generator (including the Hall generator as a special case). Their solutions demonstrated the occurrence of three acoustic modes: magneto-acoustic upstream, magneto-acoustic downstream, and thermal.

The four works cited above treated wave growth in an infinite medium. Fishman [5] was the first to consider entrance and exit conditions in his investigation of wave phenomena within the Hall channel. His analysis confirmed the presence of magneto-acoustic waves inside the system. His boundary conditions, however, were not suitable for a coal-fired, combustion-driven generator.

Houghen and McCune [6] were the first to apply a two-fluid model to plasma in thermal nonequilibrium. A two-fluid model assumes separation of the main body of the plasma into two components, free electrons and heavy particles (composed of gas ions and molecules). The two components have different temperatures. The Houghen and McCune analysis was performed for a closed-cycle MHD generator utilizing a noble gas as the working fluid. Very complex calculations revealed five modes of waves propagating within the system: magneto-acoustic upstream, magneto-acoustic downstream, thermal, and, in addition, electrothermal and ionization rate. The waves, under certain conditions, could become unstable.

Most of the papers summarized above dealt with Faraday or Hall type generators, whose dynamics can be described by a relatively simple

set of equations. The low efficiencies of Faraday and Hall type generators, however, make them less than optimal choices for energy production. Furthermore, all but one of these papers treated the MHD channel as a separate unit, whereas, in practice, the channel is connected to the combustor; this coupling affects the output current and voltage.

In all of the papers summarized above, the WKB (Wentzel-Kramers-Brillouin) approximation was used to solve the dynamic equations governing the MHD plasma. This approximate method, using a Fourier transformation, reduces the space and time dependent system variables into the frequency domain. The set of dynamic equations is then solved for those frequencies whose determinant is zero. In the case of a three-equation set, there are three such frequencies depending on a magnetic field strength, and, hence, three modes of solution. These modes are called the magneto-acoustic upstream, the magneto-acoustic downstream, and the thermal.

The approximate WKB method would be useful for assessing the amplitude amplification rate if it were known that traveling waves are present within the system. The experimental results from the various MHD test facilities [10], however, do not confirm that such waves are present. They indicate, rather, that if waves are present, they are standing waves, whose lengths are integer fractions of the channel length.

Random noise analysis techniques of signal processing are a more recent method of investigating fluctuations and wave propagation phenomena. Studies of experimental measurements recorded on magnetic tape

have indicated that a random process does take place within MHD plasma, and that generator signals, such as current, voltage, plasma pressure, and plasma temperature, show good agreement with a Gaussian distribution [7,8]. Random noise analysis techniques of signal processing, therefore, show promise in the evaluation of relationships among plasma variables.

Experimental data acquired at the UTSI MHD facility have been evaluated in terms of auto-correlation, cross-correlation, auto-power, and cross-power spectral density (APSD, CPSD) functions [9]. The analysis was applied to the voltage signal between adjacent electrodes and to the plasma luminosity signal. The voltage signal was analysed in the frequency range up to 5 kHz; the APSD showed that the spectral composition at very low frequencies is about 20-30 dB above that at high frequencies. The APSD of the luminosity signal was uniform in the frequency range up to 1 kHz.

The analysis also confirmed the importance of coupling the MHD channel with the combustor. In experiments with poor combustion conditions due to large variations in the fuel and oxidizer flow rate, a noticeable difference existed among the spectral densities of the variables.

Experimental test data collected from the Stanford University M-2 MHD generator have also been evaluated in terms of functions derived from stochastic theory [10]. The analysis of the pressure signal exhibited resonances characteristic of the acoustic standing waves of an MHD channel. The pressure signals correlated strongly from one axial position to another, with maximum correlation near zero delay

time; this confirmed the presence of standing waves. A traveling wave would be expected to have its maximum axial correlation at time lag $\tau = x/c$, with x equal to a distance between two measurements, and c being the wave velocity. The experimental results also indicated an increase in pressure and Hall voltage fluctuations with higher $\bar{J} \times \bar{B}$ and mass flow rate.

The existence of MHD experimental data has added a new dimension to plasma analysis. The processing of this data has been limited, however, to correlation and spectral density analysis, which do not provide sufficient information about overall system dynamics. The recent paper by Upadhyaya and Skorska [11] discussed a new approach to generator investigation: stochastic time series modeling. This method assumes division of the generator into subsystems based on the expected flow of the signals. Each subsystem is then analysed using an empirical model constructed from the data. Proper combination of the models and analysis in the spectral domain should provide information about cause-effect relationships among the dynamic variables.

Stochastic modeling has been successfully applied in communication systems, in the aerospace industry, and, more recently, in the testing and diagnostics of nuclear reactors. The success of the method depends on the proper configuration of the signals. Upadhyaya and Skorska assumed the multiple-input, single-output (MISO) model for the generator current analysis, taking the current as the output. The MISO model can be applied only when no feedback exists between the system output and inputs; such feedback does exist when the generator current is the output.

This analysis of the stochastic model was performed only up to 200 Hz. The frequency range was too low to exhibit any resonance related to the presence of standing waves in the MHD channel.

CHAPTER 3

ANALYTICAL MODELING OF THE MHD PLASMA

1. Formulation of the Plasma Model

The gas which is ionized by heating is in thermal equilibrium, i.e., temperature of its electrons is the same as that of its ions and neutrals. A one-fluid model, therefore, is adequate to represent the dynamics of the combustion-driven MHD generator.

The plasma model formulated in this study is based on assumptions that electron pressure gradients, radiation, and thermal conductivity of the gas are negligible. Viscous effects on the electrodes and the side walls are also neglected, and the model is limited to free stream flow.

Different types of particles are present in an MHD generator. Each type has a different cross section for momentum-transfer collisions with electrons. The ions, in particular, due to the coulomb forces, have very large collision cross sections with other charged particles. The electron-ion collisions, however, are meaningful when the degree of ionization of a working gas exceeds roughly 0.1 percent [12]. The plasma under consideration is seeded by potassium having a number density ratio much smaller than 0.1 percent. A small fraction of potassium is ionized only. This implies that the electron-ion collisions are negligible, and the electron-neutral collisions dominate in a momentum transfer mechanism.

The MHD plasma is characterized by a magnetic Reynolds number much smaller than unity, a Hall parameter in the range between one and

three, conductivity about 20 mhos per meter, and electron number density about 10^{20} per cubic meter.

The magnetic Reynolds number can be interpreted as a ratio between the magnetic field induced by plasma motion and the applied magnetic field. The small magnetic Reynolds number, therefore, indicates that gas motion does not modify the original field, i.e., the magnetic field is constant.

Another important quantity in gaseous conductors is the frequency of the circular orbit which an electron describes in the magnetic field, called the cyclotron frequency ω . The dimensionless product of the cyclotron frequency and mean free time τ between the electron-neutral collisions is called the Hall parameter. The Hall parameter Ω , written in the a form of thermodynamic properties of the plasma, is given by [13]

$$\Omega = \omega\tau = .388 \frac{T^{1/2} B}{p (10^{15} q)} , \quad (3.1)$$

where T is the plasma temperature (K), p is the plasma pressure (atm), B is the magnetic field strength (Tesla), and q is the electron-neutral collision cross section (m^2).

The conductivity of the plasma, given by the Saha equation, is also a function of thermodynamic properties [13]

$$\sigma = 2.53 \frac{T^{3/4}}{p^{1/2} (10^{15} q)} \exp(-V_i/2kT) , \quad (3.2)$$

where V_i is the ionization potential, i.e., amount of heat expressed in eV necessary to ionize the seed, and k is the Boltzmann's constant (J/K).

The electron number density also depends on thermodynamic properties of plasma and is given by

$$n = \sqrt{x} \cdot 1.3 \cdot 10^{22} T^{1/4} p^{1/2} \exp(-V_i/2kT), \quad (3.3)$$

where x is the ratio between seed and parent gas densities.

The plasma dynamic state is represented by four variables: the plasma pressure p , the density ρ , the temperature T , and the velocity $\bar{u}$. These variables are related through the following four equations:

1. Conservation of mass equation

$$\frac{D\rho}{Dt} + \rho \bar{\nabla} \cdot \bar{u} = 0, \quad (3.4)$$

2. Conservation of momentum equation

$$\rho \frac{D\bar{u}}{Dt} + \bar{\nabla} \cdot p = \bar{J} \times \bar{B}, \quad (3.5)$$

3. Conservation of energy equation

$$\rho \frac{D}{Dt} (C_p T + 1/2 u^2) - \frac{\partial p}{\partial t} = \bar{J} \cdot \bar{E}, \quad (3.6)$$

4. Equation of state

$$p = \rho RT, \quad (3.7)$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \bar{u} \cdot \bar{\nabla}$ is a substantial derivative, C_p is the specific heat at constant pressure, and R is the gas constant. The conservation of momentum equation involves the electric body force retarding the flow of plasma, $\bar{J} \times \bar{B}$, where $\bar{J} = \bar{i} J_x + \bar{j} J_y$ is a vector of current

density, $(\frac{A}{m})$, $\vec{B} = \vec{k} B_z$ is a vector magnetic field, in (T). The power per unit volume extracted from the plasma is $\vec{J} \cdot \vec{E}$, where $\vec{E} = \vec{i}E_x + \vec{j}E_y$ is a vector of electrical field (Volts/m).

The conservation equations are coupled with the generalized Ohm's equation relating the current, voltage, velocity, Hall parameter and plasma conductivity. The generalized Ohm's equation has the form

$$\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B}) - \Omega(\vec{J} \times \vec{B})/B. \quad (3.8)$$

The plasma dynamic equations have to be implemented by Maxwell's equations of electromagnetics

$$a. \quad \text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$b. \quad \text{curl } \left(\frac{\vec{B}}{\mu_0} \right) = \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$c. \quad \epsilon_0 \text{div } \vec{E} = \rho_e, \quad (3.9a-c)$$

where ρ_e is the electron density, ϵ_0 and μ_0 are the permittivity and permeability of vacuum, respectively. The small Reynolds magnetic number reduces equation (3.9a) to the form $\text{curl } \vec{E} = 0$.

Equations (3.4) - (3.9) applied to free stream flow form the nonlinear set of equations (3.10), which is called in this study the MHD plasma model. The temperature variable is eliminated from the model through the equation of state.

$$a. \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) = 0$$

$$b. \quad \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = J_y B$$

$$c. \quad \frac{\rho C}{R} \left[\frac{\partial}{\partial t} \left(\frac{P}{\rho} \right) + u \frac{\partial}{\partial x} \left(\frac{P}{\rho} \right) \right] + \rho u \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right] - \frac{\partial p}{\partial t} = E_x J_x + E_y J_y$$

$$d. \quad J_x = \frac{\sigma}{1 + \Omega^2} [E_x - \Omega (E_y - uB)]$$

$$e. \quad J_y = \frac{\sigma}{1 + \Omega^2} [E_y - uB + \Omega E_x]$$

$$f. \quad \frac{dE_y}{dx} = 0$$

$$g. \quad \epsilon_0 \frac{dE_x}{dx} = -en$$

$$h. \quad \frac{\partial J_x}{\partial x} = e \frac{\partial n}{\partial t} \quad (3.10a-h)$$

2. Linearization of the Model Using the Langevin Technique

The equations in the MHD plasma model are nonlinear. To linearize them, one may apply an interesting technique relevant to noise problems, developed by Langevin during his study of Brownian motion. The Langevin technique has been used successfully in modeling the nuclear reactors where various disturbance sources arise from fluctuations in inlet coolant temperature and pressure, flow rate, mechanical vibrations of control rods and fuel elements, etc.

The Langevin technique can be represented by the equation

$$L(\underline{\theta})\underline{x}(t) = \underline{f}(t) , \quad (3.11)$$

where L is an operator containing informations about a system, $\underline{\theta}$ is a vector of system parameters, and $\underline{f}(t)$ is a vector of driving forces. If either of the two vectors $\underline{\theta}$ or $\underline{f}(t)$ is random, then the state variable $\underline{x}(t)$ is also random, and the variables form a set of stochastic random processes. The Langevin technique requires that each of the sets $\underline{\theta}$, $\underline{f}$ and $\underline{x}$ be separated into its mean value and a random component as follows:

$$\begin{aligned}\underline{\theta}(t) &= \langle \underline{\theta}(t) \rangle + \underline{\theta}'(t) \\ \underline{f}(t) &= \langle \underline{f}(t) \rangle + \underline{f}'(t) \\ \underline{x}(t) &= \langle \underline{x}(t) \rangle + \underline{x}'(t) ,\end{aligned}\tag{3.12a-c}$$

where $\langle \rangle$ indicates time averaging. Equations (3.12) inserted into a stochastic system model, yield a set of coupled Langevin equations subject to a random disturbance vector. This stochastic model can be solved if information concerning statistics of the random disturbance vector is available.

Following the requirements of the Langevin technique the plasma parameters, the thermodynamic variables, and the electric variables are written as summations of two components

$$\begin{aligned}\Omega &= \Omega_0 + \Omega' \\ \sigma &= \sigma_0 + \sigma' \\ n &= n_0 + n' \\ \rho(x,t) &= \rho_0(x) + \rho'(x,t) \\ u(x,t) &= u_0(x) + u'(x,t) \\ p(x,t) &= p_0(x) + p'(x,t) \\ \overline{E}(x) &= \overline{i}(E_x + e_x) + \overline{j}E_y * \\ J(x) &= \overline{i}(J_x + j_x) + \overline{j}(J_y + j_y) ,\end{aligned}\tag{3.13a-h}$$

* Maxwell equation $\frac{dE_y}{dx} = 0$ implies that $e_y = 0$

where $\rho_0(x)$, $u_0(x)$, $p_0(x)$, E_x , E_y , J_x , J_y are steady state solutions of the plasma model. After inserting equations (3.13) into the model 3.10, neglecting products of second-order terms, and eliminating steady-state gradient terms, one obtains the linearized plasma equations for the fluctuating thermodynamic variables and electric variables. The fluctuating thermodynamic variables are expressed by

$$\frac{\partial \rho'}{\partial t} + u_0 \frac{\partial \rho'}{\partial x} + \rho_0 \frac{\partial u'}{\partial x} + u_0 \frac{\partial \rho_0}{\partial x} \left(\frac{u'}{u_0} - \frac{\rho'}{\rho_0} \right) = 0$$

$$\rho_0 \frac{\partial u'}{\partial t} + \rho_0 u_0 \frac{\partial u'}{\partial x} + \frac{\partial p'}{\partial x} - u_0^2 \frac{\partial \rho_0}{\partial x} \left(\frac{u'}{u_0} + \frac{\rho'}{\rho_0} \right) = j_y B$$

$$\rho_0 \frac{\partial H'}{\partial t} + \rho_0 u_0 \frac{\partial H'}{\partial x} + \rho_0 u_0 \frac{\partial H_0}{\partial x} \left(\frac{u'}{u_0} + \frac{\rho'}{\rho_0} \right) - \frac{\partial p'}{\partial t} = J_x e_x + j_y E_y + j_x E_x, \quad (3.14a-c)$$

where

$$H' = \frac{\gamma}{\gamma - 1} \frac{p_0}{\rho_0} \left(\frac{p'}{p_0} - \frac{\rho'}{\rho_0} \right) + u' u_0$$

$$H_0 = \frac{\gamma}{\gamma - 1} \frac{p_0}{\rho_0} + \frac{u_0^2}{2},$$

where γ is the ratio of specific heat capacities.

The fluctuating electric variables are expressed by

$$j_x = J_x \frac{\sigma'}{\sigma_0} - \frac{J_y + \Omega_0 J_x}{1 + \Omega_0^2} \Omega' + \frac{\Omega_0 \sigma_0 B}{1 + \Omega_0^2} u' + \frac{\sigma_0}{1 + \Omega_0^2} e_x$$

$$j_y = J_y \frac{\sigma'}{\sigma_0} + \frac{J_x - \Omega_0 J_y}{1 + \Omega_0^2} \Omega' - \frac{\sigma_0 B}{1 + \Omega_0^2} u' + \frac{\Omega_0 \sigma_0}{1 + \Omega_0^2} e_x$$

$$\epsilon_0 \frac{de_x}{dx} = -en'. \quad (3.15a-c)$$

In order to solve the sets of equations (3.14) and (3.15), the fluctuating parameters σ' , Ω' , and n' have to be written as functions of plasma thermodynamic properties. To accomplish this, one may write

$$\sigma' = \left(\frac{\partial \sigma}{\partial T}\right)_0 T' + \left(\frac{\partial \sigma}{\partial p}\right)_0 p'$$

$$\Omega' = \left(\frac{\partial \Omega}{\partial T}\right)_0 T' + \left(\frac{\partial \Omega}{\partial p}\right)_0 p'$$

$$n' = \left(\frac{\partial n}{\partial T}\right)_0 T' + \left(\frac{\partial n}{\partial p}\right)_0 p' , \quad (3.16a-c)$$

where $\left(\frac{\partial}{\partial T}\right)_0$, $\left(\frac{\partial}{\partial p}\right)_0$ are partial derivatives evaluated for steady-state conditions. After evaluating partial derivatives from the equations (3.1) through (3.3), and eliminating the temperature variable through the state equation, one obtains pressure- and density-dependent plasma conductivity, Hall parameter, and electron number density as given by

$$\sigma' = \sigma_0 \left[\left(.25 + \frac{V_i}{2kT_0} \right) \frac{p'}{p_0} - \left(.75 + \frac{V_i}{2kT_0} \right) \frac{\rho'}{\rho_0} \right]$$

$$\Omega' = -.5\Omega_0 \left(\frac{p'}{p_0} + \frac{\rho'}{\rho_0} \right)$$

$$n' = n_0 \left[\left(.75 + \frac{V_i}{2kT_0} \right) \frac{p'}{p_0} - \left(.25 + \frac{V_i}{2kT_0} \right) \frac{\rho'}{\rho_0} \right] . \quad (3.17a-c)$$

Substituting equations (3.17) in equations (3.15), yields the following current and voltage-dependent relationships:

$$\begin{aligned}
j_x &= [J_x (.25 + \frac{v_i}{2kT_o}) + \frac{J_x \Omega_o^2 + J_y \Omega_o}{2(1 + \Omega_o^2)}] (\frac{p'}{p_o}) \\
&- [J_x (.75 + \frac{v_i}{2kT_o}) - \frac{J_x \Omega_o^2 + J_y \Omega_o}{2(1 + \Omega_o^2)}] (\frac{\rho'}{\rho_o}) \\
&+ \frac{\sigma_o \Omega_o BU_o}{1 + \Omega_o^2} (\frac{u'}{u_o}) + \frac{\sigma_o}{1 + \Omega_o^2} e_x , \\
j_y &= [J_y (.25 + \frac{v_i}{2kT_o}) + \frac{J_y \Omega_o^2 - J_x \Omega_o}{2(1 + \Omega_o^2)}] (\frac{p'}{p_o}) \\
&- [J_y (.75 + \frac{v_i}{2kT_o}) + \frac{\Omega_o J_x - \Omega_o^2 J_y}{2(1 + \Omega_o^2)}] (\frac{\rho'}{\rho_o}) \\
&- \frac{\sigma_o BU_o}{1 + \Omega_o^2} (\frac{u'}{u_o}) + \frac{\sigma_o \Omega_o}{1 + \Omega_o^2} e_x ,
\end{aligned}$$

$$\frac{de_x}{dx} = - \frac{en_o}{\epsilon_o} [(.75 + \frac{v_i}{2kT_o}) \frac{p'}{p_o} - (.25 + \frac{v_i}{2kT_o}) \frac{\rho'}{\rho_o}] . \quad (3.18a-c)$$

Thus, the linearized MHD plasma model consists of the set of equations (3.14) and (3.18); the state variable vector includes the fluctuating density, velocity and pressure.

3. Nodal Form of the Plasma Model

The solution of the linearized plasma model, given by equations (3.14) and (3.18) can be accomplished via a nodal method.

The nodal method is a convenient way of solving space dependent system equations. The method divides a physical system into several

segments (nodes), and replaces spatial derivatives of the system equations with finite difference approximations. Each of the nodes has its own boundary conditions which can be determined using various methods. The conditions at the ends (the first and the last node) must agree with the true boundary conditions coupling the system under consideration with other systems. The agreement between the space dependent analysis and nodal analysis improves as the number of nodes increases.

Following the nodal method, the MHD channel is divided into n nodes of length L each. The boundary conditions of the nodes are derived using "well-stirred" plasma approach. This approach assumes that the variables within a node have the same values as the node's output variables. After substituting space partials by their finite difference approximations, equations (3.14) and (3.18) yield the set of $3n$ coupled time dependent equations:

1. n density differential equations

$$\begin{aligned} \frac{d}{dt} \left(\frac{\rho'(i)}{\rho_o(i)} \right) &= \frac{1}{L} u_o(i) \frac{\rho_o(i-1)}{\rho_o(i)} \left(\frac{\rho'(i-1)}{\rho_o(i-1)} \right) + \frac{1}{L} u_o(i-1) \left(\frac{u'(i-1)}{u_o(i-1)} \right) \\ &\quad - \frac{1}{L} u_o(i) \frac{\rho_o(i-1)}{\rho_o(i)} \left(\frac{\rho'(i)}{\rho_o(i)} \right) \\ &\quad - \frac{1}{L} u_o(i) \left(2 - \frac{\rho_o(i-1)}{\rho_o(i)} \right) \left(\frac{u'(i)}{u_o(i)} \right) \end{aligned}$$

where $i = 1, 2, \dots, n$, indicates a node number. (3.19a)

2. n velocity differential equations

$$\begin{aligned}
 \frac{d}{dt} \left(\frac{u'(i)}{u_{o(i)}} \right) &= \frac{\sigma_o \Omega_o e n_o L B}{(1 + \Omega_o^2) \epsilon_o \rho_{o(i)} u_{o(i)}} \left(.25 + \frac{v}{2kT_{o(i-1)}} \right) \left(\frac{\rho'(i-1)}{\rho_{o(i-1)}} \right) \\
 &+ \frac{1}{L} u_{o(i-1)} \left(\frac{u'(i-1)}{u_{o(i-1)}} \right) \\
 &+ \frac{1}{L} \left[\frac{\rho_{o(i-1)}}{\rho_{o(i)} u_{o(i)}} - \frac{\Omega_o \sigma_o e n_o L^2 B}{(1 + \Omega_o^2) \epsilon_o \rho_{o(i)} u_{o(i)}} \left(.75 + \frac{v}{2kT_{o(i-1)}} \right) \right] \left(\frac{\rho'(i-1)}{\rho_{o(i-1)}} \right) \\
 &+ \frac{1}{L} \left[u_{o(i)} \left(1 - \frac{\rho_{o(i-1)}}{\rho_{o(i)}} \right) + \frac{LB \gamma_{\rho(i)}}{u_{o(i)}} \right] \left(\frac{\rho'(i)}{\rho_{o(i)}} \right) \\
 &+ \frac{1}{L} \left[-u_{o(i)} \frac{\rho_{o(i-1)}}{\rho_{o(i)}} + \frac{LB \gamma_u(i)}{\rho_{o(i)}} \right] \left(\frac{u'(i)}{u_{o(i)}} \right) \\
 &+ \frac{1}{L} \left[-\frac{\rho_{o(i)}}{\rho_{o(i)} u_{o(i)}} + \frac{LB \gamma_p(i) \rho_{o(i)}}{\rho_{o(i)} u_{o(i)}} \right] \left(\frac{\rho'(i)}{\rho_{o(i)}} \right)
 \end{aligned}$$

$$i = 1, 2, \dots, n. \quad (3.19b)$$

3. n pressure differential equations

$$\begin{aligned}
 \frac{d}{dt} \left(\frac{\rho'(i)}{\rho_{o(i)}} \right) &= \left[(A_{(i-1)} - A1_{(i-1)}) \frac{\gamma-1}{\rho_{o(i)}} - A2_{(i-1)} + \gamma u_{o(i)} \frac{\rho_{o(i-1)}}{\rho_{o(i)}} \right] \left(\frac{\rho'(i-1)}{\rho_{o(i-1)}} \right) \\
 &+ \left[\gamma u_{o(i-1)} - \frac{\gamma-1}{\rho_{o(i)}} B_{(i-1)} - B1_{(i-1)} \right] \left(\frac{u'(i-1)}{u_{o(i-1)}} \right) \\
 &+ \left[(C_{(i-1)} - C1_{(i-1)}) \frac{\gamma-1}{\rho_{o(i)}} - C2_{(i-1)} \right] \left(\frac{\rho'(i-1)}{\rho_{o(i-1)}} \right)
 \end{aligned}$$

$$\begin{aligned}
& + [(D_{(i)} - D1_{(i)}) \frac{\gamma-1}{p_{o(i)}} - D2_{(i)} - \gamma u_{o(i)} \frac{\rho_{o(i-1)}}{\rho_{o(i)}}] \left(\frac{\rho'_{(i)}}{\rho_{o(i)}} \right) \\
& + [(E_{(i)} - E1_{(i)}) \frac{\gamma-1}{p_{o(i)}} - E2_{(i)} - \gamma u_{o(i)} \left(2 - \frac{\rho_{o(i-1)}}{\rho_{o(i)}} \right)] \left(\frac{u'_{(i)}}{u_{o(i)}} \right) \\
& + [(F_{(i)} - F1_{(i)}) \frac{\gamma-1}{p_{o(i)}} - F2_{(i)}] \left(\frac{p'_{(i)}}{p_{o(i)}} \right)
\end{aligned}$$

$$i = 1, 2, \dots, n. \quad (3.19c)$$

where

$$A_{(i-1)} = \left[\frac{en_o L^2}{\epsilon_o} (J_{x(i)} + \frac{\sigma_o}{1+\Omega_o} (\Omega_o E_{y(i)} + E_{x(i)})) \left(.25 + \frac{v}{2kT_{o(i-1)}} \right) \right]$$

$$A1_{(i-1)} = \frac{\gamma}{\gamma-1} u_{o(i)} p_{o(i-1)} \frac{\rho_{o(i)}}{\rho_{o(i-1)}}$$

$$A2_{(i-1)} = \frac{(\gamma-1) \sigma_o \Omega_o en_o L^2 B u_{o(i)}}{(1+\Omega_o^2) \epsilon_o p_{o(i)}} \left(.25 + \frac{v}{2kT_{o(i-1)}} \right)$$

$$B_{(i-1)} = \rho_{o(i)} u_{o(i)} u_{o(i-1)}^2$$

$$B1_{(i-1)} = (\gamma-1) u_{o(i)}^2 \rho_{o(i)} \frac{u_{o(i-1)}}{p_{o(i)}}$$

$$C_{(i-1)} = \left[- \frac{en_o L^2}{\epsilon_o} (J_{x(i)} + \frac{\sigma_o}{1+\Omega_o} (\Omega_o E_{y(i)} + E_{x(i)})) \left(.75 + \frac{v}{2kT_{o(i-1)}} \right) \right]$$

$$C1_{(i-1)} = - \frac{\gamma}{\gamma-1} u_{o(i)} p_{o(i-1)} \frac{\rho_{o(i)}}{\rho_{o(i-1)}}$$

$$C2_{(i-1)} = \frac{(\gamma-1)u_o(i)}{P_o(i)} \left[p_{o(i-1)} - \frac{\sigma_o \Omega_o \epsilon_o L^2 B}{(1+\Omega_o^2)\epsilon_o} \left(.75 + \frac{V}{2kT_{o(i-1)}} \right) \right]$$

$$D_{(i)} = \left[\frac{\epsilon_o L^2 J_x(i)}{\epsilon_o} \left(.25 + \frac{V_i}{2kT_{o(i)}} \right) + LE_{y(i)} \gamma_{\rho(i)} \rho_{o(i)} + LE_{x(i)} \rho_{\rho(i)} \rho_{o(i)} \right]$$

$$D1_{(i)} = \left[\rho_{o(i)} u_o(i) \frac{u_o^2(i) - u_o^2(i-1)}{2} - \frac{\gamma \rho_{o(i)} u_o(i) P_{o(i-1)}}{(\gamma-1) \rho_{o(i-1)}} \right]$$

$$D2_{(i)} = \frac{\rho_{o(i)} u_o^2(i) (\gamma-1)}{P_o(i)} \left[u_o(i) \left(1 - \frac{\rho_{o(i-1)}}{\rho_{o(i)}} \right) + \frac{LB \gamma_{\rho(i)}}{u_o(i)} \right]$$

$$E_{(i)} = (E_{x(i)} L \delta_{u(i)} + E_{y(i)} + E_{y(i)} L \gamma_{u(i)}) u_o(i)$$

$$E1_{(i)} = (\rho_{o(i)} u_o^3(i) + \frac{\gamma}{\gamma-1} P_{o(i)} u_o(i) + \rho_{o(i)} u_o(i) \frac{u_o^2(i) - u_o^2(i-1)}{2} - \frac{\gamma \rho_{o(i)} u_o(i) P_{o(i-1)}}{(\gamma-1) \rho_{o(i-1)}})$$

$$E2_{(i)} = \frac{(\gamma-1) \rho_{o(i)} u_o^2(i)}{P_o(i)} \left(\frac{LB \gamma_{u(i)}}{\rho_{o(i)}} - u_o(i) \frac{\rho_{o(i-1)}}{\rho_{o(i)}} \right)$$

$$F_{(i)} = - \left[\frac{J_x(i) \epsilon_o L^2}{\epsilon_o} \left(.75 + \frac{V}{2kT_{o(i)}} \right) - E_{y(i)} L \gamma_{\rho(i)} P_{o(i)} - E_{x(i)} L \delta_{\rho(i)} P_{o(i)} \right]$$

$$F1_{(i)} = \frac{\gamma}{\gamma-1} P_{o(i)} u_o(i)$$

$$F2_{(i)} = u_o(i) (\gamma-1) (LB \gamma_{\rho(i)} - 1)$$

$$\gamma_{\rho(i)} = -[J_{y(i)} \left(.75 + \frac{V}{2kT_{o(i)}} \right) + \frac{\Omega_o (J_{x(i)} - \Omega_o J_{y(i)})}{2(1+\Omega_o^2)} - \frac{\sigma_o \Omega_o e n_o L}{\epsilon_o (1+\Omega_o^2)} \\ \left(.25 + \frac{V}{2kT_{o(i)}} \right)] \frac{1}{\rho_{o(i)}}$$

$$\gamma_{u(i)} = - \frac{\sigma_o B}{1+\Omega_o^2}$$

$$\gamma_{p(i)} = [J_{y(i)} \left(.25 + \frac{V}{2kT_{o(i)}} \right) - \frac{\Omega_o (J_{x(i)} - \Omega_o J_{y(i)})}{2(1+\Omega_o^2)} - \frac{\sigma_o \Omega_o e n_o L}{\epsilon_o (1+\Omega_o^2)} \\ \left(.75 + \frac{V}{2kT_{o(i)}} \right)] \frac{1}{p_{o(i)}}$$

$$\delta_{\rho(i)} = -[J_{x(i)} \left(.75 + \frac{V}{2kT_{o(i)}} \right) - \frac{\Omega_o (J_{x(i)} \Omega_o + J_{y(i)})}{2(1+\Omega_o^2)} - \frac{\sigma_o e n_o L}{\epsilon_o (1+\Omega_o^2)} \\ \left(.25 + \frac{V}{2kT_{o(i)}} \right)] \frac{1}{\rho_{o(i)}}$$

$$\delta_{p(i)} = [J_{x(i)} \left(.25 + \frac{V}{2kT_{o(i)}} \right) + \frac{\Omega_o (J_{x(i)} \Omega_o + J_{y(i)})}{2(1+\Omega_o^2)} - \frac{\sigma_o e n_o L}{\epsilon_o (1+\Omega_o^2)} \\ \left(.75 + \frac{V}{2kT_{o(i)}} \right)] \frac{1}{p_{o(i)}}$$

$$\delta_{u(i)} = \frac{\sigma_o \Omega_o B}{1+\Omega_o^2} .$$

The time dependent differential equations can be put into a compact matrix form

$$\frac{d\underline{X}(t)}{dt} = \underline{A}\underline{X}(t) + \underline{F}(t) , \quad (3.20)$$

where $\underline{X}(t)$ is a vector of state variables, $F(t)$ is a vector of forcing functions, and A is a nonsingular matrix of coefficients. The matrix A of the MHD plasma model is determined according to the equations (3.19) and has six groups of elements. One group of elements forms a main diagonal, five other groups form five co-diagonals. All other elements of A are zero. The plasma nodal equations in matrix notation are written as

$$\frac{d}{dt} \begin{bmatrix} \frac{\rho'(1)}{\rho_o(1)} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\rho'(n)}{\rho_o(n)} \\ \frac{u'(1)}{u_o(1)} \\ \cdot \\ \cdot \\ \cdot \\ \frac{u'(n)}{u_o(n)} \\ \frac{p'(1)}{p_o(1)} \\ \cdot \\ \cdot \\ \cdot \\ \frac{p'(n)}{p_o(n)} \end{bmatrix} = A \begin{bmatrix} \frac{\rho'(1)}{\rho_o(1)} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\rho'(n)}{\rho_o(n)} \\ \frac{u'(1)}{u_o(1)} \\ \cdot \\ \cdot \\ \cdot \\ \frac{u'(n)}{u_o(n)} \\ \frac{p'(1)}{p_o(1)} \\ \cdot \\ \cdot \\ \cdot \\ \frac{p'(n)}{p_o(n)} \end{bmatrix} + \begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ f_2 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ f_3 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \quad (3.21)$$

The forcing function vector $F(t)$ has three non-zero component f_1 , f_2 , f_3 which depend on the fluctuating density, velocity and pressure variables input to the channel (i.e., output from the combustor) as given by

$$f_1 = a\left(\frac{\rho'(o)}{\rho_o(o)}\right) + b\left(\frac{u'(o)}{u_o(o)}\right)$$

$$f_2 = c\left(\frac{\rho'(o)}{\rho_o(o)}\right) + d\left(\frac{u'(o)}{u_o(o)}\right) + e\left(\frac{p'(o)}{p_o(o)}\right)$$

$$f_3 = f\left(\frac{\rho'(o)}{\rho_o(o)}\right) + g\left(\frac{u'(o)}{u_o(o)}\right) + h\left(\frac{p'(o)}{p_o(o)}\right) . \quad (3.22a-c)$$

The non-zero elements of the matrix A , and the coefficients a , ..., h of equations (3.22) are listed in Appendix A. The assumptions made about the fluctuating variables in (3.22) provide a tool for coupling the channel with the combustor.

4. Solution of the Steady State Plasma Equations

The coefficients of the matrix A of the plasma model depend on the steady-state profiles of the density, velocity, and pressure variables. These profiles have to be obtained from the steady-state plasma equations which, according to the set (3.10), may be written as

$$a. \quad \rho u = G = \text{const}$$

$$b. \quad \rho u \frac{du}{dx} + \frac{dp}{dx} = J_y B$$

$$c. \quad \rho C_p u \frac{dT}{dx} + \rho u^2 \frac{du}{dx} = J_x E_x + J_y E_y$$

$$d. J_x = \frac{\sigma}{1+\Omega^2} [E_x - \Omega(E_y - uB)]$$

$$e. J_y = \frac{\sigma}{1+\Omega^2} [E_y - uB + \Omega E_x]$$

$$f. E_y(x) = \text{const}$$

$$g. J_x(x) = \text{const} \quad (3.23a-g)$$

To solve the above equations, one must either take advantage of the iterative techniques, or make assumptions about the model in order to simplify the equations and solve them analytically. The author decided to make the assumption that the steady-state plasma temperature remains constant along the MHD channel. This decision has been dictated by necessity to keep the solution in a simple form suitable for implementation into the nodal equations. After inserting data, the assumption about the constant temperature profile was checked against the temperature profile obtained from the equation of state. The maximum error did not exceed 2 percent.

Assuming constant plasma temperature, the steady-state solutions of the velocity, pressure, and density variables are equal to

$$a. u(x) = u_0 + \frac{x}{Gu_0} [J_x^2 \left(\frac{1 + \Omega_0^2}{\sigma_0} \right) + 2 \Omega_0 J_x E_y + \sigma_0 E_y^2 - B u_0 (\Omega_0 J_x + \sigma_0 E_y)]$$

$$b. p(x) = p_0 - \frac{1}{u_0} (a - bu_0)x + (\sigma_0 B E_y + \Omega_0 J_x B)x - \sigma_0 B^2 u_0 x - \frac{\sigma_0 B^2}{Gu_0} (a - bu_0) \frac{x^2}{2}$$

$$c. \quad \rho(x) = \rho_o u_o / u(x) \quad (3.24a-c)$$

where:

$$a = J_x^2 \left(\frac{1 + \Omega_o^2}{\sigma_o} \right) + 2\Omega_o J_x E_y + \sigma_o E_y^2$$

$$b = B(\Omega_o J_x + \sigma_o E_y)$$

CHAPTER 4

SOLUTION OF THE MODEL IN THE FORM OF AUTOCORRELATION AND CROSSCORRELATION FUNCTIONS

1. Response of a Linear System to Stochastic Inputs

A common method of analysis of stationary stochastic processes is based upon the autocorrelation and crosscorrelation functions. The correlation analysis gives information about periodic components present in a system, transmission paths of the signals, and time required for the signals to pass through a system.

The autocorrelation function of a random signal, R_{xx} , describes the dependence of the values of this signal at one time on the values at another time. The crosscorrelation function of two random signals, R_{xy} , describes the dependence of the values of one signal on the other. In mathematical form

$$\begin{aligned} R_{xx}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)x(t+\tau)dt \\ R_{xy}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)y(t+\tau)dt . \end{aligned} \quad (4.1a-b)$$

The autocorrelation and crosscorrelation functions transferred into a frequency domain give the auto-power spectral density and cross-power spectral density functions, S_{xx} and S_{xy} , respectively. In mathematical form

$$\begin{aligned} S_{xx}(\omega) &= \int_{-\infty}^{\infty} e^{-j\omega\tau} R_{xx}(\tau)d\tau \\ S_{xy}(\omega) &= \int_{-\infty}^{\infty} e^{-j\omega\tau} R_{xy}(\tau)d\tau , \end{aligned} \quad (4.2a-b)$$

where for real random process, S_{xx} is real and non-negative, whereas S_{xy} is complex, and gives information about phase relationship between the signals.

The equations (4.1) are based on assumptions that the random variables are wide-sense stationary and the process is ergodic. For a wide-sense stationary process the mean value is constant, and the autocorrelation is a function of the time difference τ , and not of absolute time. The process is ergodic if all time averages of the variables are equal to corresponding ensemble averages.

The MHD data indicate that the plasma variables are wide-sense stationary, and the stochastic processes in the conducting channel are ergodic.

The correlation analysis applied to a linear differential system subjected to random disturbances, yields the output crosscorrelation functions in terms of the input crosscorrelation functions. Suppose that the time-invariant system

$$\frac{d\underline{X}(t)}{dt} = \underline{A}\underline{X}(t) + \underline{F}(t)$$

$$\underline{X}(t_0) = \underline{X}_0, \quad (4.3)$$

is driven by a piecewise continuous noise source $\underline{F}(t)$. The solution of this system, in the form of the state transition matrix e^{At} , is

$$\underline{X}(t) = e^{A(t-t_0)} \underline{X}(t_0) + \int_{t_0}^t e^{A(t-t')} \underline{F}(t') dt'. \quad (4.4)$$

If the system's eigenvalues have negative real parts, i.e., the system is stable, and if it was "switched-on" a long time in the past, one may

neglect the initial condition $\underline{X}_0$. Then multiplying $\underline{X}(t)$ by its transpose gives

$$\underline{X}(t_1)\underline{X}(t_2)^T = \int_0^\infty e^{A\alpha_1} \underline{F}(t_1-\alpha_1)d\alpha_1 \int_0^\infty (e^{A\alpha_2} \underline{F}(t_2-\alpha_2))^T d\alpha_2, \quad (4.5)$$

where $\alpha_1 = t_1 - t'$, $\alpha_2 = t_2 - t'$. Taking the ensemble average (or time average) of equation (4.5), one obtains the following relationship

$$R_{xx}(\tau) = \int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 e^{A\alpha_1} R_{FF}(\tau+\alpha_1-\alpha_2) (e^{A\alpha_2})^T, \quad (4.6)$$

where $\tau = t_2 - t_1$, R_{FF} and R_{xx} are the matrices of the input and output crosscorrelation functions, respectively. Thus, equation (4.6) establishes relations between the system parameters, and the input and the output noise phenomena. The corresponding cross-power spectral density functions are obtained from

$$S_{xx}(\omega) = (j\omega I - A)^{-1} S_{FF}(\omega) (j\omega I - A)^{-T*}, \quad (4.7)$$

where $*$ indicates complex-conjugate, I is identity matrix, $S_{FF}(\omega)$ and $S_{xx}(\omega)$ are the matrices of input and output power spectral density functions.

Input noise sources, contained within the matrix R_{FF} , are modeled often by a white noise process. A white noise process is a wide-sense stationary process; its autocorrelation function is a Dirac function, and its spectral density function is constant at all frequencies. The assumption of the white noise character of input sources is modified to include band-limited noise inputs.

The noise source matrix, R_{FF} , of the MHD plasma model (3.21), is a $3n \times 3n$ matrix having three non-zero rows and three non-zero columns as given by

$$R_{FF}^{(\tau+\alpha_1-\alpha_2)} = \begin{bmatrix} c_{11} & 0... & c_{12} & 0... & c_{13} & 0... \\ 0 & & & & & \\ \vdots & & & & & \\ c_{21} & 0... & c_{22} & 0... & c_{23} & 0... \\ 0 & & & & & \\ \vdots & & & & & \\ c_{31} & 0... & c_{32} & 0... & c_{33} & 0... \\ 0 & & & & & \\ \vdots & & & & & \\ 0 & & 0 & & 0 & \end{bmatrix} \quad (4.8)$$

where c_{ij} , $i = 1,2,3$, $j = 1,2,3$, are crosscorrelation functions involving input to the MHD channel fluctuating density, velocity, and pressure variables. The autocorrelation functions of these fluctuating variables are modeled in this study by an exponential function $\exp(-|p|\tau)$. The parameter "p", by taking different values, affects the character of the driving noise, i.e., the character of coupling the combustor with the channel. As "p" approaches infinity, the driving noise approaches a white noise process. Assuming the uncorrelated noise sources, and assuming exponential, zero-mean autocorrelation

functions, the functions C_{ij} of the noise source matrix R_{FF} are equal to

$$C_{11} = \left[\left(a \frac{\rho'(o)}{\rho_o(o)} \right)^2 \sigma_\rho^2 + \left(b \frac{u'(o)}{u_o(o)} \right)^2 \sigma_u^2 \right] \exp \left[-p(\tau + \alpha_1 - \alpha_2) \right]$$

$$C_{22} = \left[\left(c \frac{\rho'(o)}{\rho_o(o)} \right)^2 \sigma_\rho^2 + \left(d \frac{u'(o)}{u_o(o)} \right)^2 \sigma_u^2 + \left(e \frac{p'(o)}{p_o(o)} \right)^2 \sigma_p^2 \right] \exp \left[-p(\tau + \alpha_1 - \alpha_2) \right]$$

$$C_{33} = \left[\left(f \frac{\rho'(o)}{\rho_o(o)} \right)^2 \sigma_\rho^2 + \left(g \frac{u'(o)}{u_o(o)} \right)^2 \sigma_u^2 + \left(h \frac{p'(o)}{p_o(o)} \right)^2 \sigma_p^2 \right] \exp \left[-p(\tau + \alpha_1 - \alpha_2) \right]$$

$$C_{12} = \left[ac \left(\frac{\rho'(o)}{\rho_o(o)} \right)^2 \sigma_\rho^2 + bd \left(\frac{u'(o)}{u_o(o)} \right)^2 \sigma_u^2 \right] \exp \left[-p(\tau + \alpha_1 - \alpha_2) \right]$$

$$C_{21} = | C_{12} | \exp \left[-p(\alpha_2 - \tau - \alpha_1) \right]$$

$$C_{13} = \left[af \left(\frac{\rho'(o)}{\rho_o(o)} \right)^2 \sigma_\rho^2 + bg \left(\frac{u'(o)}{u_o(o)} \right)^2 \sigma_u^2 \right] \exp \left[-p(\tau + \alpha_1 - \alpha_2) \right]$$

$$C_{31} = | C_{13} | \exp \left[-p(\alpha_2 - \tau - \alpha_1) \right]$$

$$C_{23} = \left[cf \left(\frac{\rho'(o)}{\rho_o(o)} \right)^2 \sigma_\rho^2 + dg \left(\frac{u'(o)}{u_o(o)} \right)^2 \sigma_u^2 + eh \left(\frac{p'(o)}{p_o(o)} \right)^2 \sigma_p^2 \right] \exp \left[-p(\tau + \alpha_1 - \alpha_2) \right]$$

$$C_{32} = | C_{23} | \exp \left[-p(\alpha_2 - \tau - \alpha_1) \right], \quad (4.9a-i)$$

where σ_ρ^2 , σ_u^2 , σ_p^2 are variances of the density, velocity, and pressure noise sources.

Given the noise source correlation matrix R_{FF} , one has to evaluate the integral (4.6) in order to find the response of the plasma system to stochastic inputs.

2. Similarity Transformation of a State Parameter Matrix

To obtain a response of a linear system to stochastic inputs, one has to perform integration of the product of three time-dependent matrices, as indicated in equation (4.6)

$$R_{xx}(\tau) = \int_0^{\infty} d\alpha_1 \int_0^{\infty} d\alpha_2 e^{A\alpha_1} R_{FF}(\tau + \alpha_1 - \alpha_2) (e^{A\alpha_2})^T.$$

The state parameter matrix A , and its transpose A^T , are general square matrices, whose eigenvalues can be real or complex, distinct or of some multiplicity. Solution of equation (4.6) poses difficult problems; the multiplication and integration of the large dimensional matrices are very cumbersome, and, in addition, equation (4.6) cannot be easily evaluated using a computer code. These problems, however, can be remedied by applying a proper similarity transformation to the state parameter matrix.

A similarity transformation of a matrix A is not laborious if A is nondefective, i.e., has a linearly independent set of eigenvectors, and all its eigenvalues are real. For such a case, a similarity transformation is given by Theorem 1 [17].

Theorem 1:

An $n \times n$ matrix A can be reduced to a diagonal matrix Λ by a similarity transformation $\Lambda = P^{-1}AP$ if and only if A has a linearly independent set of eigenvectors; these eigenvectors are the columns of P and their corresponding real eigenvalues are the diagonal entries in Λ .

According to Theorem 1, the similarity transformation reduces matrix A to a diagonal matrix Λ , and since

$$e^{\Lambda \tau} = \begin{bmatrix} e^{\Lambda_1 \tau} & & & \\ & e^{\Lambda_2 \tau} & & \\ & & \ddots & \\ & & & e^{\Lambda_n \tau} \end{bmatrix}, \quad (4.10)$$

the solution to equation (4.6) can be written in a compact form

$$r_{il}(\tau) = \sum_{jkm} p_{ik} p_{lm} s_{kj} s_{mt} \int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 e^{\Lambda_t \alpha_1 + \Lambda_k \alpha_2} f_{jt}(\tau + \alpha_1 - \alpha_2), \quad (4.11)$$

where r_{il} belongs to R_{XX} , $i=1, \dots, n$, $\ell=1, \dots, n$;

f_{jt} belongs to R_{FF} , $j=1, \dots, n$, $t=1, \dots, n$;

p_{ik} belongs to P , $i=1, \dots, n$, $k=1, \dots, n$;

s_{mt} belongs to P^{-1} , $m=1, \dots, n$, $t=1, \dots, n$;

and $\Lambda_1, \dots, \Lambda_n$ are the eigenvalues of A .

Analytical integration in equation (4.11) is simple, especially for the exponentially correlated input noise, i.e., for the case when $f_{jt} \sim \exp[-p(\tau + \alpha_1 - \alpha_2)]$.

The eigenvalues of the MHD dynamics matrix of equations (3.21) are not real. Various sets of steady-state operating data that are characteristics of all four types of the MHD generators, yield complex eigenvalues whose real parts are always negative. In general, when a matrix has complex eigenvalues it can still be transformed into another, more convenient form for integration. This form, called the modal form of matrix A [18], is non-diagonal for the complex eigenvalues, however, it contains off-diagonal terms equal to the imaginary parts of the eigenvalues. According to Ref. [18], if A has m real distinct eigenvalues $\Lambda_1, \dots, \Lambda_m$ and k sets of complex conjugate eigenvalues

$\Lambda_i = \sigma_i \pm j\beta_i$, for $i = 1, \dots, k$, the modal matrix Λ , obtained as a result of the transformation $P^{-1}AP$, is given by

$$\Lambda = \left[\begin{array}{ccc|ccc} \Lambda_1 & & & & & \\ & \Lambda_2 & & & & \\ & & \dots & & & \\ & & & \Lambda_m & & \\ \hline & & & & \underline{\Lambda_1} & \\ & & & & & \underline{\Lambda_2} \\ & & & & & \dots \\ & & & & & \underline{\Lambda_k} \end{array} \right]_{n \times n}, \quad (4.12)$$

where

$$\underline{\Lambda_i} = \begin{bmatrix} \sigma_i & \beta_i \\ -\beta_i & \sigma_i \end{bmatrix}, \quad i = 1, 2, \dots, k,$$

and the transformation matrix P has columns formed of eigenvectors of A ; the i -th column of P corresponds to the real part of the i -th eigenvector, whereas the $(i+1)$ -th column of P corresponds to the imaginary part of the i -th eigenvector. Taking advantage of the modal matrix Λ , the MHD correlation equation (4.6) transfers into

$$R_{XX}(\tau) = \int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 Pe^{\Lambda\alpha_1} P^{-1} C_{FF}(\tau + \alpha_1 - \alpha_2) (Pe^{\Lambda\alpha_2} P^{-1})^T, \quad (4.13a)$$

where

$$e^{\Lambda\alpha} = \left[\begin{array}{cc|cccc} e^{\Lambda_1\alpha} & 0 & & & & 0 \\ \cdot & \cdot & & & & \\ 0 & e^{\Lambda_m\alpha} & & & & \\ \hline & & e^{\sigma_1\alpha}\cos\beta_1\alpha & e^{\sigma_1\alpha}\sin\beta_1\alpha & & \\ & & -e^{\sigma_1\alpha}\sin\beta_1\alpha & e^{\sigma_1\alpha}\cos\beta_1\alpha & & \\ 0 & & 0 & \cdot & \cdot & 0 \\ & & & & e^{\sigma_k\alpha}\cos\beta_k\alpha & e^{\sigma_k\alpha}\sin\beta_k\alpha \\ & & & & -e^{\sigma_k\alpha}\sin\beta_k\alpha & e^{\sigma_k\alpha}\cos\beta_k\alpha \end{array} \right], \quad (4.13b)$$

and $\Lambda_1, \dots, \Lambda_m$ are real eigenvalues of A , $\sigma_1 \pm j\beta_1, \dots, \sigma_k \pm j\beta_k$ are complex conjugate eigenvalues of A . Once the eigenvalues are known, equation (4.13a) can be solved analytically for any size of the parameter matrix. Section 3 of this chapter introduces a computer code handling the analytical solution of equation (4.13a). This is applicable for the general case of complex eigenvalues of the parameter matrix, and when the input noise is exponentially correlated.

If the system eigenvalues are real and have multiplicity, the parameter matrix can still be simplified, using a similarity transformation based on the Jordan canonical form. The multiple eigenvalue case does not occur in the MHD modeling. This case, however, is outlined in order to give a complete scheme for solving analytically any system with stochastic input. The similarity transformation based on the Jordan canonical form is given by Theorem 2 [17].

Theorem 2:

If A is a general $n \times n$ matrix then a nonsingular matrix Q exists such that

$$Q^{-1}AQ = \begin{bmatrix} J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_k \end{bmatrix} = J, \quad (4.14)$$

where J_i are $n_i \times n_i$ Jordan blocks. The same eigenvalues may occur in different blocks, but the number of distinct blocks corresponding to a given eigenvalue is equal to the number of eigenvectors corresponding to that eigenvalue and forming an independent set. The number k and the set of numbers $n_1, \dots, n_k$ are uniquely determined by A . The transformation matrix Q has columns formed of independent eigenvectors of A , and generalized eigenvectors.

A Jordan block in Theorem 2 is a square matrix whose elements are zero except for those on the principal diagonal, which are all equal to an eigenvalue, and those on the first off-diagonal, which are all equal to unity. Taking advantage of the Jordan canonical form, the correlation equation (4.6) transfers into

$$R_{XX}(\tau) = \int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 Q e^{J\alpha_1} Q^{-1} C_{FF}(\tau + \alpha_1 - \alpha_2) (Q e^{J\alpha_2} Q^{-1})^T, \quad (4.15a)$$

where

$$e^{J\alpha} = \left[\begin{array}{c|c} \begin{array}{cc} e^{\Lambda_1 \alpha} & 0 \\ \dots & \\ 0 & e^{\Lambda_m \alpha} \end{array} & \begin{array}{c} 0 \\ \\ \end{array} \\ \hline \begin{array}{c} e^{\Lambda_{m+1} \alpha} \left[\begin{array}{ccccc} 1 & \alpha & \frac{\alpha^2}{2} & \dots & \frac{\alpha^{r-1}}{(r-1)!} \\ 0 & 1 & \alpha & \dots & \frac{\alpha^{r-2}}{(r-2)!} \\ & \dots & & & \\ 0 & 0 & 0 & \dots & i \end{array} \right] \\ \\ 0 & 0 & \dots & 0 \end{array} \\ \hline \begin{array}{c} e^{\Lambda_k \alpha} \left[\begin{array}{ccccc} 1 & \alpha & \frac{\alpha^2}{2} & \dots & \frac{\alpha^{r-1}}{(r-1)!} \\ 0 & 1 & \alpha & \dots & \frac{\alpha^{r-2}}{(r-2)!} \\ & \dots & & & \\ 0 & 0 & 0 & \dots & i \end{array} \right] \end{array} \end{array} \right], \quad (4.15b)$$

and $\Lambda_1, \dots, \Lambda_m$ are real distinct eigenvalues of A , $\Lambda_{m+1}, \dots, \Lambda_k$ are real multiple eigenvalues of multiplicity r of A .

Once the eigenvalues are known, equation (4.15) can be solved analytically for any size of system parameter matrix.

3. Analytical Correlation and Power Spectral Density Functions: The Computer Program MHD CR

The eigenvalue analysis of the MHD plasma model indicates that the plasma system is stable. The eigenvalues, obtained under various

steady state conditions, and for data characteristic of all four types of the MHD generators, are real negative eigenvalues, or are complex eigenvalues with negative real parts. Hence, the solution of the MHD plasma model is based on equation (4.13)

$$R_{XX}(\tau) = \int_0^{\infty} d\alpha_1 \int_0^{\infty} d\alpha_2 P e^{\Lambda \alpha_1} P^{-1} C_{FF}(\tau + \alpha_1 - \alpha_2) (P e^{\Lambda \alpha_2} P^{-1})^T,$$

where the matrix Λ is the modal form of the plasma dynamics parameter matrix. The solution of equation (4.13) in the form of system output crosscorrelation functions, can be evaluated with the aid of a computer program.

The computer program handling the correlation analysis of a system with real or complex conjugate eigenvalues was developed during the course of this study. The program, called MHDCCR, gives the autocorrelation and crosscorrelation functions of the plasma density, velocity, and pressure variables at any axial location of the MHD channel, for desired positive or negative time values. These correlation results are presented in the form of eigenvalue-dependent functions, and the solution depends on the coupling between the MHD channel and the combustor, i.e., on the assumed autocorrelation functions of the input variables.

The program MHDCCR, listed in Appendix B, is based on the following steps:

Step 1.

The state parameter matrix is set up according to equations (3.19) and (3.21). The elements of this matrix consist of the generator's steady state profiles given by equations (3.24). Next the IMS library

routine EIGEN evaluates eigenvalues of this matrix. Given the eigenvalues, the similarity transformation matrix P is formed from the corresponding eigenvectors.

Step 2.

Multiplication of the two matrices, $e^{\Lambda\alpha_1} P^{-1}$, is performed, where P^{-1} is the inverse of the similarity transformation matrix P , and the structure of the matrix $e^{\Lambda\alpha_1}$ can be predicted according to the eigenvalue analysis (equation (4.13b)). If the $n \times n$ state matrix has m real distinct eigenvalues $\{-\Lambda_1, \dots, -\Lambda_m\}$, and k sets of complex-conjugate eigenvalues $-\Lambda_i \pm j\beta_i$, for $i = m+1, \dots, m+k$, then the product of the two matrices equals

$$e^{\Lambda\alpha_1} P^{-1} = \begin{bmatrix} a_{11}e^{-\Lambda_1\alpha_1} & a_{12}e^{-\Lambda_1\alpha_1} & \dots & a_{1n}e^{-\Lambda_1\alpha_1} \\ a_{21}e^{-\Lambda_2\alpha_1} & a_{22}e^{-\Lambda_2\alpha_1} & \dots & a_{2n}e^{-\Lambda_2\alpha_1} \\ \vdots & \vdots & & \vdots \\ a_{m1}e^{-\Lambda_m\alpha_1} & a_{m2}e^{-\Lambda_m\alpha_1} & \dots & a_{mn}e^{-\Lambda_m\alpha_1} \\ A_{m+1,1}e^{-\Lambda_{m+1}\alpha_1} & A_{m+1,2}e^{-\Lambda_{m+1}\alpha_1} & \dots & A_{m+1,n}e^{-\Lambda_{m+1}\alpha_1} \\ B_{m+1,1}e^{-\Lambda_{m+1}\alpha_1} & B_{m+1,2}e^{-\Lambda_{m+1}\alpha_1} & \dots & B_{m+1,n}e^{-\Lambda_{m+1}\alpha_1} \\ \vdots & \vdots & & \vdots \\ A_{m+k,1}e^{-\Lambda_{m+k}\alpha_1} & A_{m+k,2}e^{-\Lambda_{m+k}\alpha_1} & \dots & A_{m+k,n}e^{-\Lambda_{m+k}\alpha_1} \\ B_{m+k,1}e^{-\Lambda_{m+k}\alpha_1} & B_{m+k,2}e^{-\Lambda_{m+k}\alpha_1} & \dots & B_{m+k,n}e^{-\Lambda_{m+k}\alpha_1} \end{bmatrix} \quad (4.16a)$$

where a_{ij} , $i=1,\dots,n, j=1,\dots,n$ are the elements of the matrix P^{-1} , and the time dependent coefficients A_{lj} and B_{lj} , $l = m+1, \dots, m+k, j=1, \dots, n$ are given by

$$\begin{aligned} A_{lj} &= a_{lj} \cos \beta_l \alpha_l + a_{l+1,j} \sin \beta_l \alpha_l, \\ B_{lj} &= a_{l+1,j} \cos \beta_l \alpha_l - a_{lj} \sin \beta_l \alpha_l. \end{aligned} \quad (4.16b)$$

Step 3.

Multiplication of the three matrices,

$$(e^{\Lambda \alpha_1 P^{-1}}) C_{FF}(\tau + \alpha_1 - \alpha_2) (e^{\Lambda \alpha_2 P^{-1}})^T,$$

is performed. The matrix $(e^{\Lambda \alpha_1 P^{-1}})$ is given by equation (4.16), the matrix $(e^{\Lambda \alpha_2 P^{-1}})^T$ is the transpose of the result (4.16) (with different time dependence), and the matrix of the input correlation functions, $C_{FF}(\tau + \alpha_1 - \alpha_2)$, is given by equation (4.8). The result of this multiplication is the $n \times n$ matrix B whose elements can be classified into four groups with several cases. All elements of B are composed of various functions of time; the elements classified into one group have similar time dependence. Table 1 describes the elements of the matrix B and shows the classification. The position-dependent coefficients $C_{1ij}, \dots, C_{4ij}$ appearing in Table 1 are defined in Appendix A.

Step 4.

Integration in time domain of the product of three matrices is performed, i.e.,

$$\int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 (e^{\Lambda \alpha_1 P^{-1}}) C_{FF}(\tau + \alpha_1 - \alpha_2) (e^{\Lambda \alpha_2 P^{-1}})^T = \int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 B$$

TABLE 1
TIME DEPENDENT COMPONENTS OF THE INTEGRAND

Group, Case	Time dependence of an element	Position of an element
1	$b_{ij} = e^{-\Lambda_i \alpha_1 - \Lambda_j \alpha_2 - p} \tau + \alpha_1 - \alpha_2 C1_{ij}$	b_{ij} belongs to the row and the column which does not contain any complex eigenvalues
2	a $b_{ij} = e^{-\Lambda_i \alpha_1 - \Lambda_j \alpha_2 - p} \tau + \alpha_1 - \alpha_2 \cdot [C1_{ij} \cos \beta_{i1} + C2_{ij} \sin \beta_{i1}]$	b_{ij} belongs to the row of a complex eigenvalue and the column of a real eigenvalue*
	b $b_{i+1,j} = e^{-\Lambda_i \alpha_1 - \Lambda_j \alpha_2 - p} \tau + \alpha_1 - \alpha_2 \cdot [C2_{ij} \cos \beta_{i1} - C1_{ij} \sin \beta_{i1}]$	$b_{i+1,j}$ belongs to the row of a complex conjugate eigenvalue and the column of a real eigenvalue
3	a $b_{ij} = e^{-\Lambda_i \alpha_1 - \Lambda_j \alpha_2 - p} \alpha_2 - \tau - \alpha_1 \cdot [C1_{ij} \cos \beta_{j2} + C3_{ij} \sin \beta_{j2}]$	$b_{i,j}$ belongs to the row of a real eigenvalue and the column of a complex eigenvalue
	b $b_{i,j+1} = e^{-\Lambda_i \alpha_1 - \Lambda_j \alpha_2 - p} \alpha_2 - \tau - \alpha_1 \cdot [C3_{ij} \cos \beta_{j2} - C1_{ij} \sin \beta_{j2}]$	$b_{i,j+1}$ belongs to the row of a real eigenvalue and the column of a complex-conjugate eigenvalue
4	a $b_{ij} = e^{-\Lambda_i \alpha_1 - \Lambda_j \alpha_2 - p} \tau + \alpha_1 - \alpha_2 \cdot \{ [C1_{ij} \cos \beta_{i1} + C2_{ij} \sin \beta_{i1}] \cos \beta_{j2} + [C3_{ij} \cos \beta_{i1} + C4_{ij} \sin \beta_{i1}] \sin \beta_{j2} \}$	b_{ij} belongs to the row of a complex eigenvalue and the column of a complex eigenvalue
	b $b_{i,j+1} = e^{-\Lambda_i \alpha_1 - \Lambda_j \alpha_2 - p} \tau + \alpha_1 - \alpha_2 \cdot \{ [C3_{ij} \cos \beta_{i1} + C4_{ij} \sin \beta_{i1}] \cos \beta_{j2} + [-C1_{ij} \cos \beta_{i1} - C2_{ij} \sin \beta_{i1}] \sin \beta_{j2} \}$	$b_{i,j+1}$ belongs to the row of a complex eigenvalue and the column of a complex conjugate eigenvalue

TABLE 1 (Continued)

Group, Case	Time dependence of an element	Position of an element
4	c $b_{i+1,j} = e^{-\Lambda_i \alpha_1 - \Lambda_j \alpha_2 - p} \tau + \alpha_1 - \alpha_2 $ $\cdot \{ [C2_{ij} \cos \beta_{i1} - C1_{ij} \sin \beta_{i1}] \cos \beta_{j2}$ $+ [C4_{ij} \cos \beta_{i1} - C3_{ij} \sin \beta_{i1}] \sin \beta_{j2} \}$	$b_{i+1,j}$ belongs to the row of a complex conjugate eigenvalue and the column of a complex eigenvalue
	d $b_{i+1,j+1} = e^{-\Lambda_i \alpha_1 - \Lambda_j \alpha_2 - p} \tau + \alpha_1 - \alpha_2 $ $\cdot \{ [C4_{ij} \cos \beta_{i1} - C3_{ij} \sin \beta_{i1}] \cos \beta_{j2}$ $+ [-C2_{ij} \cos \beta_{i1} + C1_{ij} \sin \beta_{i1}] \sin \beta_{j2} \}$	$b_{i+1,j+1}$ belongs to the row of a complex conjugate eigenvalue and the column of a complex conjugate eigenvalue

* See the structure in the equation (4.16a)

The elements b_{ij} of the product matrix B are listed in Table 1. Each group of elements is integrated separately, since each group constitutes a different function of time. Integration process is broken into two parts to accommodate the absolute value function, that is, $\exp | -p(\tau + \alpha_1 - \alpha_2) |$. The elements f_{ij} of the resulting matrix F are listed in Table 2. The coefficients A, B, E, F, G appearing in Table 2 are listed in Appendix A.

The integration results presented in Table 2 are obtained after extensive reduction. The accuracy of presented results is confirmed by two facts. First, if one assumes that the imaginary parts β_i of the eigenvalues are equal to zero, then the results in the second, third, and fourth groups reduce to the result in the first group for any time τ . Secondly, for τ equal to zero, the resulting matrix F is symmetric, i.e., $f_{ij} = f_{ji}$.

Step 5.

Multiplication of the three matrices, PFP^T , is performed. The matrix P is the similarity transformation matrix formed in Step 1, P^T is its transpose, and F results from the integration in Step 4. The matrix R_{XX} , which is the product of the multiplication, contains the crosscorrelation functions of the plasma density, velocity, and pressure variables at various axial locations. Each element r_{ij} of R_{XX} is a time dependent function of the system eigenvalues, the assumed input noise autocorrelation functions, and the position along the MHD channel. The output of the program MHDGR gives the cross-correlation functions between any desired primary variables at any positions in the channel.

TABLE 2

TIME DEPENDENT COMPONENTS OF THE OUTPUT CROSSCORRELATION MATRIX

Group, Case	Results of integration	
1	$f_{ij} = C1_{ij} \left[\frac{2pe^{-\Lambda_j \tau}}{(p^2 - \Lambda_j^2)(\Lambda_i + \Lambda_j)} - \frac{e^{-p\tau}}{(p - \Lambda_j)(p + \Lambda_i)} \right]$	
2	a	$f_{ij} = \frac{2pe^{-\Lambda_j \tau} [C1_{ij}(\Lambda_i + \Lambda_j) + C2_{ij}\beta_i]}{(p^2 - \Lambda_j^2)[(\Lambda_i + \Lambda_j)^2 + \beta_i^2]} - \frac{e^{-p\tau} [C1_{ij}(p + \Lambda_i) + C2_{ij}\beta_i]}{(p - \Lambda_j)[(p + \Lambda_i)^2 + \beta_i^2]}$
	b	$f_{i+1,j} \text{ same as 2a case with } C1_{ij} \text{ replaced by } C2_{ij}$ $C2_{ij} \quad - \quad - \quad - C1_{ij}$
3	a	$f_{ij} = \frac{e^{-p\tau} [C3_{ij}\beta_j - C1(p - \Lambda_j)]}{(p + \Lambda_i)[(p - \Lambda_j)^2 + \beta_j^2]} + (A \cos \beta_j \tau + B \sin \beta_j \tau) e^{-\Lambda_j \tau}$
	b	$f_{i,j+1} \text{ same as 3a case with } C1_{ij} \text{ replaced by } C3_{ij}$ $C3_{ij} \quad - \quad - \quad - \text{ by } C1_{ij}$
4	a	$f_{ij} = Ee^{-p\tau} + (F \cos \beta_j \tau + G \sin \beta_j \tau) e^{-\Lambda_j \tau}$

TABLE 2 (Continued)

Group, Case		Results of integration		
4	b	$f_{i,j+1}$ same as 4a case with $C1_{ij}$ replaced by $C3_{ij}$ $C2_{ij} \quad - \quad - \quad C4_{ij}$ $C3_{ij} \quad - \quad - \quad -C1_{ij}$ $C4_{ij} \quad - \quad - \quad C2_{ij}$		
	c	$f_{i+1,j}$ same as 4a case with $C1_{ij}$ replaced by $C2_{ij}$ $C2_{ij} \quad - \quad - \quad -C1_{ij}$ $C3_{ij} \quad - \quad - \quad C4_{ij}$ $C4_{ij} \quad - \quad - \quad -C3_{ij}$		
	d	$f_{i+1,j+1}$ same as 4a case with $C1_{ij}$ replaced by $C4_{ij}$ $C2_{ij} \quad - \quad - \quad -C3_{ij}$ $C3_{ij} \quad - \quad - \quad -C2_{ij}$ $C4_{ij} \quad - \quad - \quad C1_{ij}$		

Step 6.

The output crosscorrelation functions are converted into frequency domain, yielding the corresponding power spectral density functions. Table 3 lists the expected components of any PSD function of the density, velocity, and pressure variables. The phase relationship between the signals is evaluated from

$$\phi = \text{arctg} \frac{\text{Im}(S_{XY})}{\text{Re}(S_{XY})}, \quad (4.17)$$

where $\text{Im}(S_{XY})$ and $\text{Re}(S_{XY})$ are imaginary and real parts of the PSD function.

4. Summary

Chapter 4 discusses the solution of the MHD plasma model. The solution is based on the assumptions that processes inside the conducting channel are stochastic, and input noise sources have their autocorrelation functions in the form $\exp(-|p|\tau)$, where the parameter 'p' varies. The solution, i.e., the response of the plasma variables to the stochastic inputs, involves integration in time domain of the product of three matrices, which is very cumbersome and cannot be easily accomplished using a computer code. The problem is remedied by applying a similarity transformation to the state dynamics matrix. The similarity transformation depends on the character of the system eigenvalues, and is composed of the system eigenvectors. The computer program MHDGR performs the eigenvalue analysis of the plasma model, forms the similarity transformation, calculates the product of the matrices, and performs the analytical integration in time domain.

TABLE 3
FREQUENCY DEPENDENT COMPONENTS OF THE OUTPUT
SPECTRAL DENSITY MATRIX

Time domain $R_{xy}(\tau)$	Frequency domain $S_{xy}(\omega)$
$\alpha a e^{- \Lambda_i \tau}$	$\alpha \frac{\Lambda_i(a_1+a_2) + j\omega(a_1-a_2)}{\Lambda_i^2 + \omega^2}$
$\alpha b e^{- \Lambda_i \tau} \sin \beta_i \tau$	$\alpha \frac{\beta_i(\Lambda_i^2 + \beta_i^2 - \omega^2)(b_1+b_2)}{(\Lambda_i^2 + \omega^2)^2 + 2\beta_i^2(\Lambda_i^2 - \omega^2) + \beta_i^4}$ $+ \frac{j2\Lambda_i\omega\beta_i(b_1-b_2)}{(\Lambda_i^2 + \omega^2)^2 + 2\beta_i^2(\Lambda_i^2 - \omega^2) + \beta_i^4}$
$\alpha c e^{- \Lambda_i \tau} \cos \beta_i \tau$	$\alpha \frac{\Lambda_i(\Lambda_i^2 + \beta_i^2 + \omega^2)(c_1+c_2)}{(\Lambda_i^2 + \omega^2)^2 + \beta_i^2(\Lambda_i^2 - \omega^2) + \beta_i^4}$ $+ \frac{j\omega(\Lambda_i^2 + \omega^2 - \beta_i^2)}{(\Lambda_i^2 + \omega^2)^2 + 2\beta_i^2(\Lambda_i^2 - \omega^2) + \beta_i^4}$

The matrix R_{XX} , which is the result of the MHDCR, contains the correlation functions of the plasma density, velocity, and pressure variables at various axial locations. The correlation functions transferred into the frequency domain give the power spectral density functions. Thus, one obtains the output autocorrelations, crosscorrelation, and power spectral density functions of the variables, which depend on the system eigenvalues, assumed input autocorrelation functions, and position along the MHD channel.

CHAPTER 5

RESULTS OF ANALYTICAL MODELING OF THE MHD PLASMA DYNAMICS

1. Selection of the MHD Data

The results of time and frequency domain analyses of the MHD plasma model dynamics developed in Chapters 3 and 4 are presented here. The model, being an integral part of the program MHD CR, is solved on a PDP-11 minicomputer for a typical set of steady state values:

- a. entrance velocity, $u_o = 1500 \frac{\text{m}}{\text{s}}$
- b. entrance density, $\rho_o = .09 \frac{\text{kg}}{\text{m}^3}$
- c. entrance pressure, $p_o = .75 \text{ atm}$
- d. entrance temperature, $T_o = 2700^\circ\text{K}$
- e. applied magnetic field strength, $B = 2\text{T}$
- f. Hall parameter, $\Omega = 2$
- g. ionization potential, $V = 4.32 \text{ eV}$
- h. specific heat ratio, $\gamma = 1.2.$ (5.1)

These values are close to the steady state values of the UTSI MHD test facility [5]. The solution to the model is given in the form of autocorrelation, crosscorrelation, APSD, and CPSD functions of the plasma state vector. These time and frequency domain functions, evaluated for different positions along the conducting channel, depend on the system eigenvalues and input noise sources. The eigenvalues, computed under the steady state conditions (5.1) and values of current characteristic of different types of the MHD generators, are always negative

real numbers or complex conjugate numbers with negative real parts. The negative real parts guarantee stability of the system; they indicate also that the correlation functions of the fluctuating variables approach zero ultimately. The real eigenvalues and real parts of the complex conjugate eigenvalues depend on the number of nodes used in the modeling; the average eigenvalue is equal to -15,000 for twelve nodes. Inverse of this eigenvalue compares very well to the time that takes plasma to pass through a node, i.e., .06 msec. The short passage time postulates the high frequency analysis of the spectral density functions; it implies also a millisecond scale for the correlation analysis. These postulated time and frequency ranges of analyses are in agreement with the ranges of the experimental data analyses [7,10]. The spectral results in this study, therefore, are analyzed up to 5 kHz, and the correlation results are plotted in the range (-2, 2) msec.

The time and frequency domain functions of the state variable vector depend also on the autocorrelation functions of the fluctuating density, velocity, and pressure variables at the entrance to the MHD channel. These autocorrelation functions are modeled by an exponential function having the form $e^{-|p|\tau}$, where the parameter 'p' takes different values. If 'p' is equal to or greater than 30,000, the input noise sources have character of the white noise (see Chapter 6). Most of the results presented here have been obtained under the white noise character of the input sources. The cases with 'p' much smaller than 30,000 are considered separately in Chapter 6.

The plasma model is very general and can be applied to various types of MHD generators. The type of the generator can be determined

by a proper selection of the current or voltage data used in the modeling. The steady state plasma model includes the conditions that the current J_x and voltage E_y are constant along the channel; these conditions arise from the Maxwell's equations of electromagnetics. By changing the steady state current J_x (or voltage E_y), one may alter the generator's character. Since, according to equation (3.10),

$$J_y(x) = \sigma (E_y - u(x)B) + \Omega J_x,$$

and the generator's action requires that $(E_y - uB) < 0$ and $J_y < 0$, then large values of J_x make $|J_y|$ small, and small values of J_x make $|J_y|$ large, if the input E_y , u , and B are held constant. The first case, i.e., large J_x and very small $|J_y|$, corresponds to the linear Hall generator, where power is extracted via the axial direction. The second case, i.e., very small J_x and large $|J_y|$, corresponds to the Faraday type generator, where power is extracted via the y direction. The case where J_x is smaller than $|J_y|$ by a factor of up to four corresponds to the diagonally connected wall generator, where power is extracted via both x and y directions. Table 4 specifies types of MHD generators selected for the analysis and lists the corresponding current values. All analyses have been performed for the voltage E_y equal to 225 V. These current and voltage values, together with the steady state conditions (5.1) and the channels' lengths equal to 106.68 cm, constitute various sets of data of the program MHD CR.

The analysis of the MHD plasma, based on the correct interpretation of the frequency and time domain plots, is divided into several steps. The first step includes the separate analyses of the fluctuating density, velocity and pressure variables, performed under the same

TABLE 4

ANALYZED MHD GENERATORS AND THE CORRESPONDING CURRENT DATA

Type of the MHD generator	J_x current density, $\frac{A}{cm^2}$	Total current density, $\frac{A}{cm^2}$	$ J_y $ and J_x currents ratio
Faraday divided electrodes	.10	1.81	$ J_y \cong 18 J_x$
Diagonnally connected wall $\beta \cong 70^\circ$ *	.40	1.34	$ J_y \cong 3 J_x$
Diagonnally connected wall $\beta \cong 60^\circ$	.60	1.13	$ J_y \cong 1.5 J_x$
Linear Hall	.86	1.03	$J_x \cong 1.5 J_y $
Linear Hall	1.10	1.13	$J_x \cong 5 J_y $

* β is the angle between the x axis and connected walls.

steady state conditions, for the MHD generators described in Table 4. The next step includes the analysis of the fluctuating current and voltage variables, performed also for the various MHD generators. The final step contains the parametric study of the MHD plasma where the steady state conditions (5.1) have been slightly altered. This parametric study is performed for one DCW generator; the DCW generators are the best choice for the industrial energy generation.

2. Analysis of the State Vector Variables

The APSD functions of the fluctuating pressure variable are shown in Figures 2 through 5. Figures 2 and 3 demonstrate the APSD plots of the variable as a function of the generator's current, i.e., as a function of the different channel configurations. The plots in Figure 2 apply to the axial point located at one-third of the channel's length, whereas those in Figure 3 apply to the end of the channel. The generators whose axial currents are small have clearly a concentration of fluctuations in the 10 to 200 Hz range, particularly in the first section of the channel. The frequency range of fluctuations increases with axial currents; the high frequency fluctuations may even exceed those in the range up to 200 Hz, for very large axial currents. The Hall generators, therefore, are more susceptible to the high frequency pressure fluctuations than the Faraday and DCW generators.

Figures 4 and 5 illustrate the APSD plots of the pressure variable as a function of the axial positions; the plots were obtained for the consecutive points along the channel separated by 18 cm. The plots in Figure 4 are characteristic of the DCW generator, whereas those in Figure 5 are characteristic of the linear Hall generator. The figures

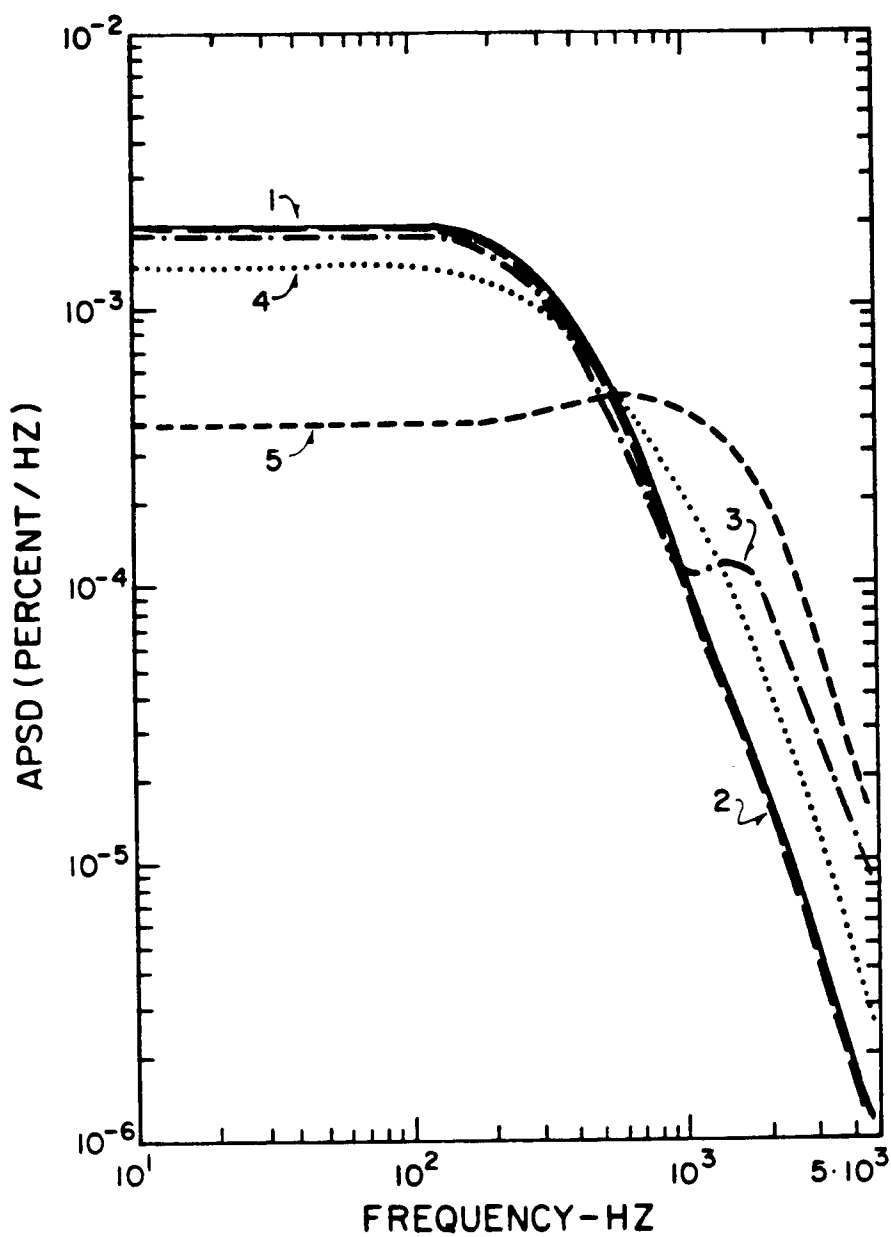


Figure 2. The APSD functions of the pressure variable for different axial currents at $x = 36$ cm.

1. $J_x = .10 \frac{A}{cm^2}$

2. $J_x = .40 \frac{A}{cm^2}$

3. $J_x = .60 \frac{A}{cm^2}$

4. $J_x = .86 \frac{A}{cm^2}$

5. $J_x = 1.10 \frac{A}{cm^2}$

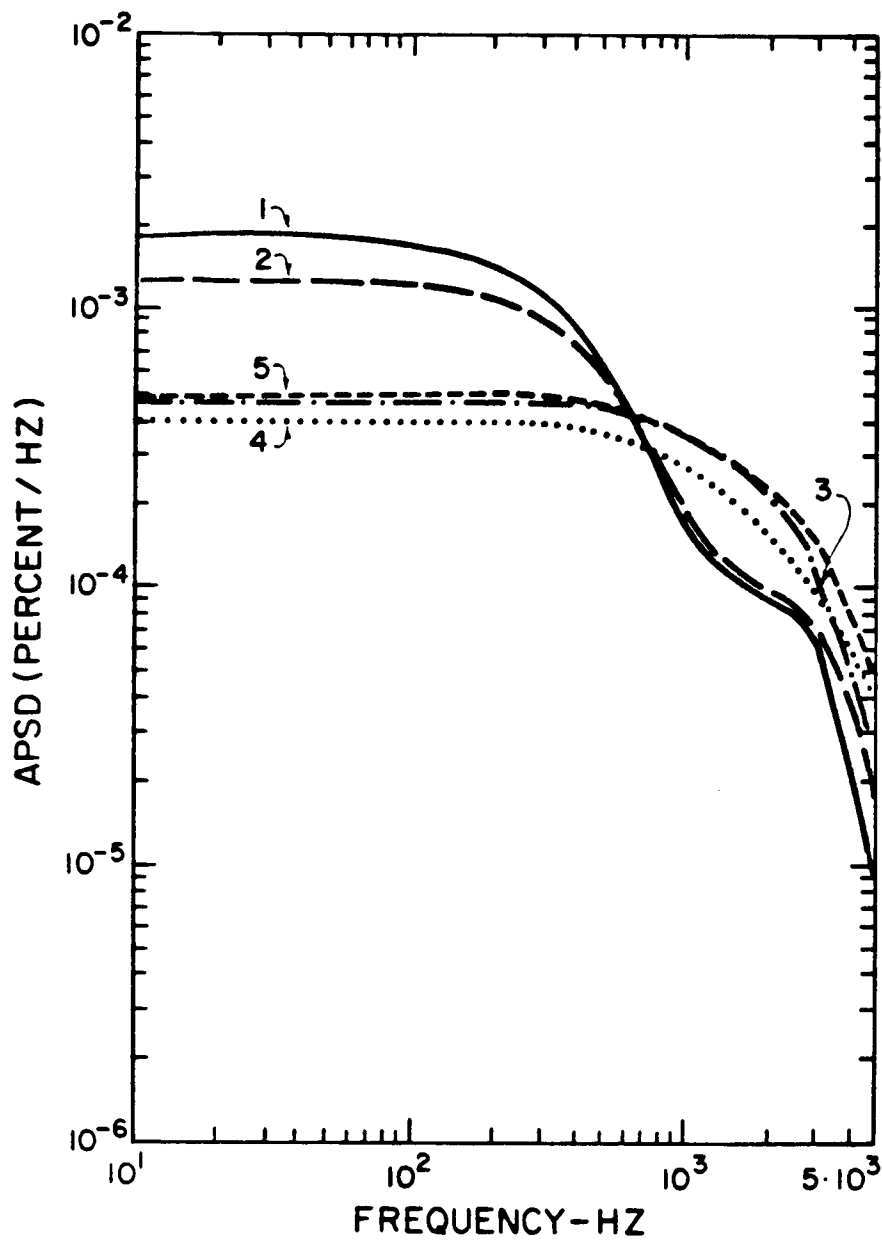


Figure 3. The APSD functions of the pressure variable for different axial currents at $x = 106$ cm.

- | | |
|--------------------------------|-------------------------------|
| 1. $J_x = .10 \frac{A}{cm^2}$ | 2. $J_x = .40 \frac{A}{cm^2}$ |
| 3. $J_x = .60 \frac{A}{cm^2}$ | 4. $J_x = .86 \frac{A}{cm^2}$ |
| 5. $J_x = 1.10 \frac{A}{cm^2}$ | |

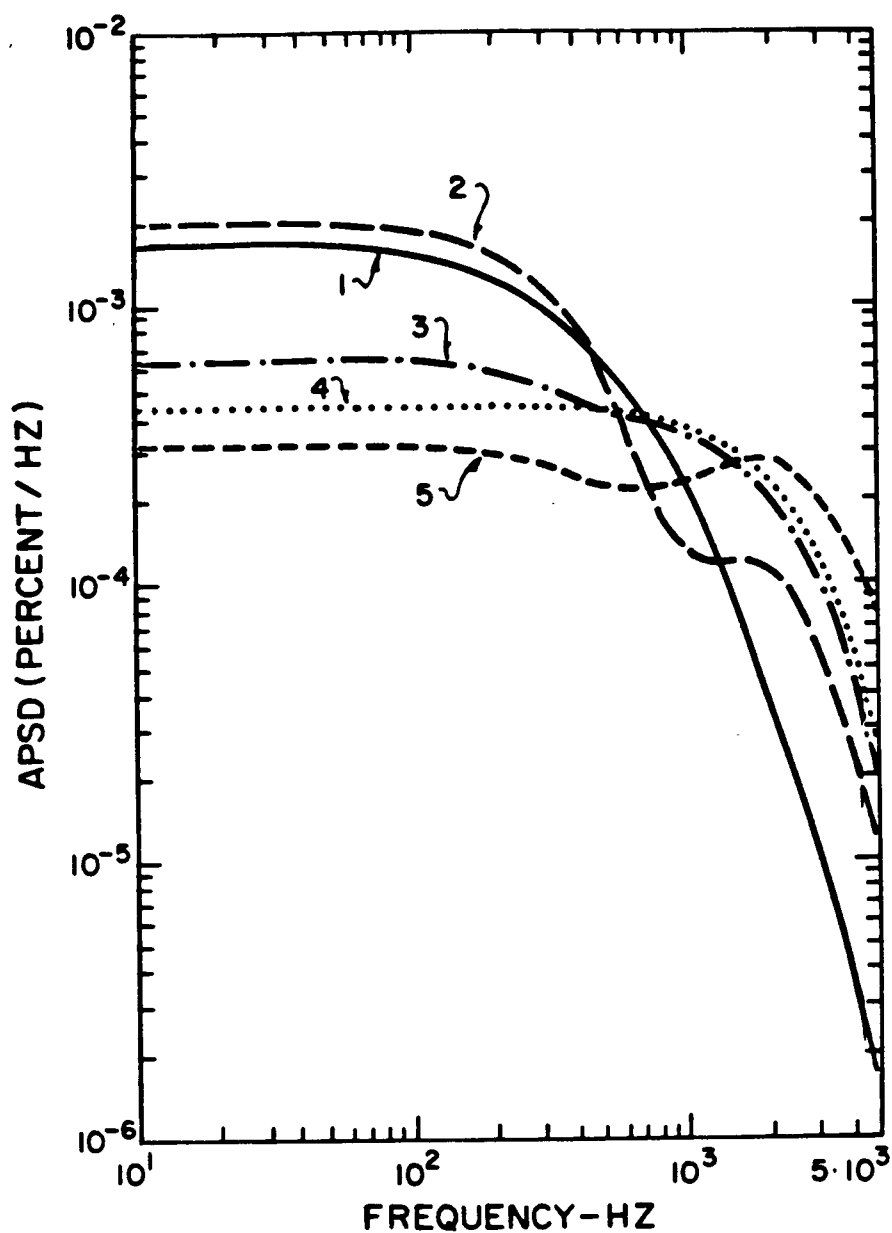


Figure 4. The APSD functions of the pressure variable at different axial locations for $J_x = .60 \frac{A}{cm^2}$.

- | | |
|-----------------|----------------|
| 1. $x = 18$ cm | 2. $x = 36$ cm |
| 3. $x = 54$ cm | 4. $x = 90$ cm |
| 5. $x = 106$ cm | |

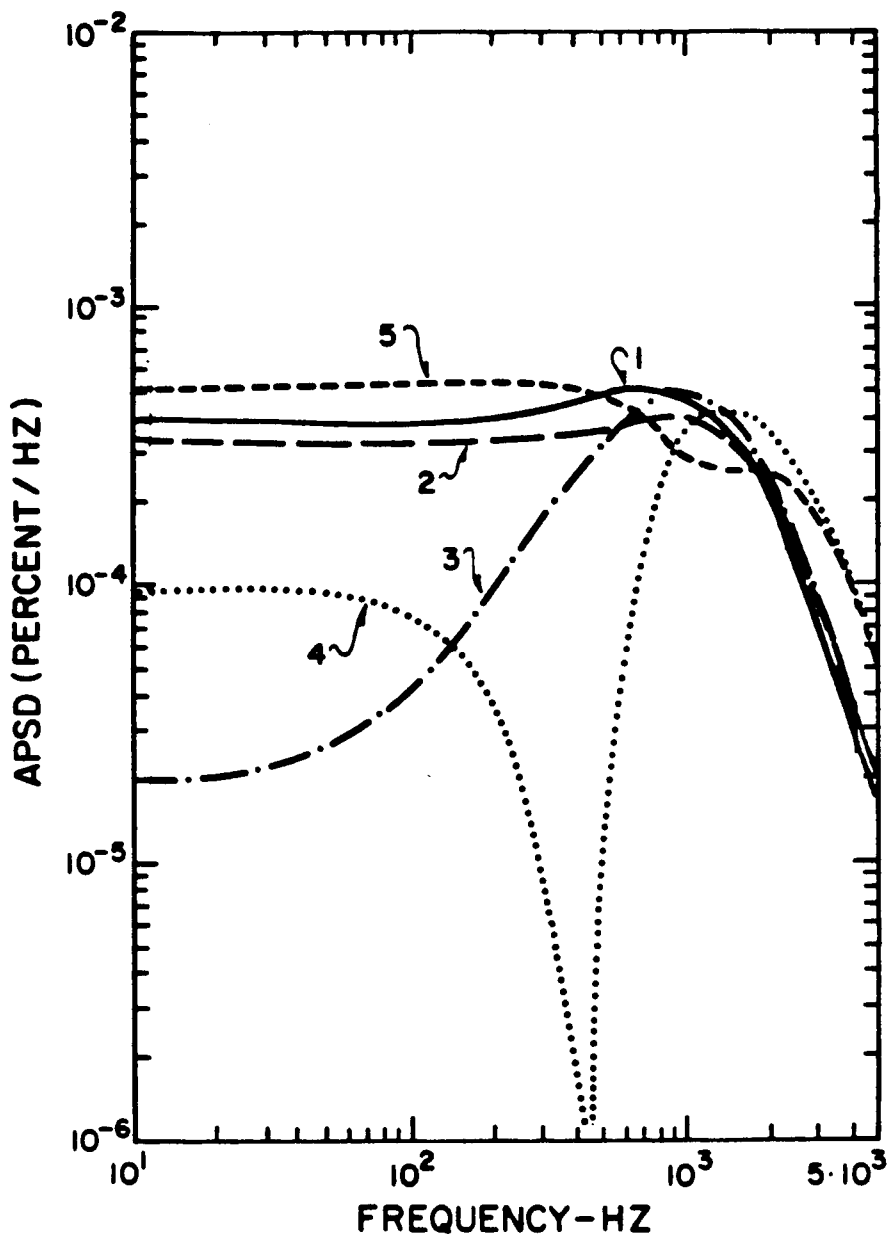


Figure 5. The APSD functions of the pressure variable at different axial locations for $J_x = 1.10 \text{ A/cm}^2$.

- | | |
|-------------------------|------------------------|
| 1. $x = 18 \text{ cm}$ | 2. $x = 36 \text{ cm}$ |
| 3. $x = 54 \text{ cm}$ | 4. $x = 90 \text{ cm}$ |
| 5. $x = 106 \text{ cm}$ | |

indicate that the range of fluctuations increases downstream from the channel's inlet for both cases. They show also that the Hall generator has a concentration of fluctuations in the 700 to 2000 Hz range, especially for the end section of the channel. The DCW generator may exhibit the broad resonances in the same range of frequency.

The crosscorrelation and CPSD functions of the fluctuating pressure between various axial locations are presented in Figures 6-8. Figure 6 demonstrates the normalized crosscorrelation plots of the DCW generator's channel between the fluctuating pressure at point A, located 18 cm from the entrance, and points set away 18, 36, 54, 72 and 88 cm from point A. The pressure variables correlate strongly from one axial position to another with maximum correlation near zero-delay time, indicating that the variables have acoustic character. Other nonacoustic noise would be expected to correlate poorly in the axial direction. Some of the functions in Figure 6 had originally a minimum at zero-delay time, which indicates that the variables at the two channel's locations are out-of-phase. These out-of-phase functions are inverted in this and a few other figures, in order to make a comparison of the assembled plots easier. An in-phase then out-of-phase behavior of the functions together with near zero-delay time suggest that the fluctuating pressure forms acoustic standing waves inside the MHD channel. The same conclusions can be drawn from Figure 7. It demonstrates the normalized crosscorrelation plots between the fluctuating pressure variables at various locations in the Hall generator channel.

Figure 8 presents the typical CPSD plots of the pressure variables. One of the plots has the first break frequency around 200 Hz. Both of

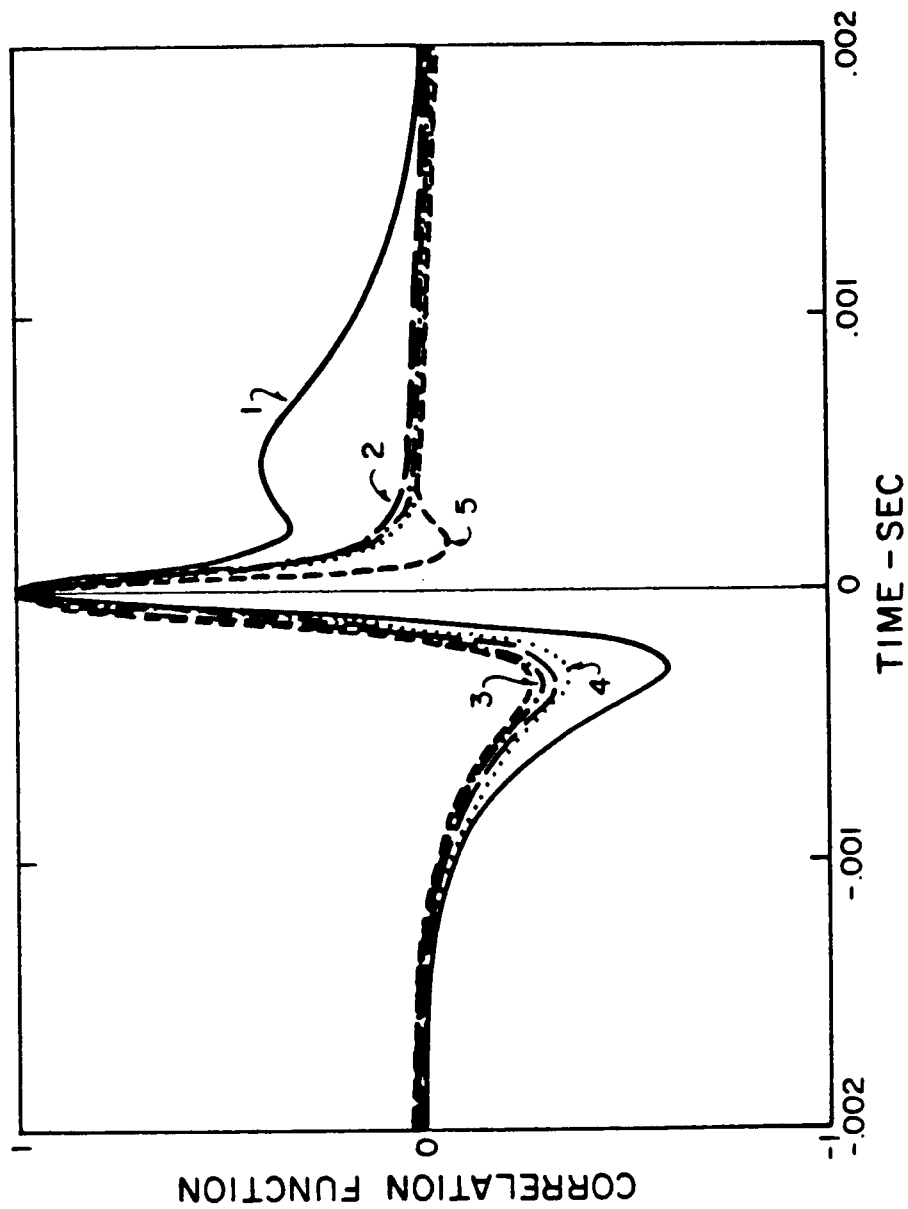


Figure 6. The crosscorrelation functions between the pressure variables for $J_x = .60 \text{ A/cm}^2$.

- | | |
|------------------------------------|-------------------------|
| 1. C_{AB} (A = 18 cm, B = 36 cm) | 2. C_{AC} (C = 54 cm) |
| 3. C_{AC} (D = 72 cm) | 4. C_{AE} (E = 90 cm) |
| 5. C_{AF} (F = 106 cm) | |

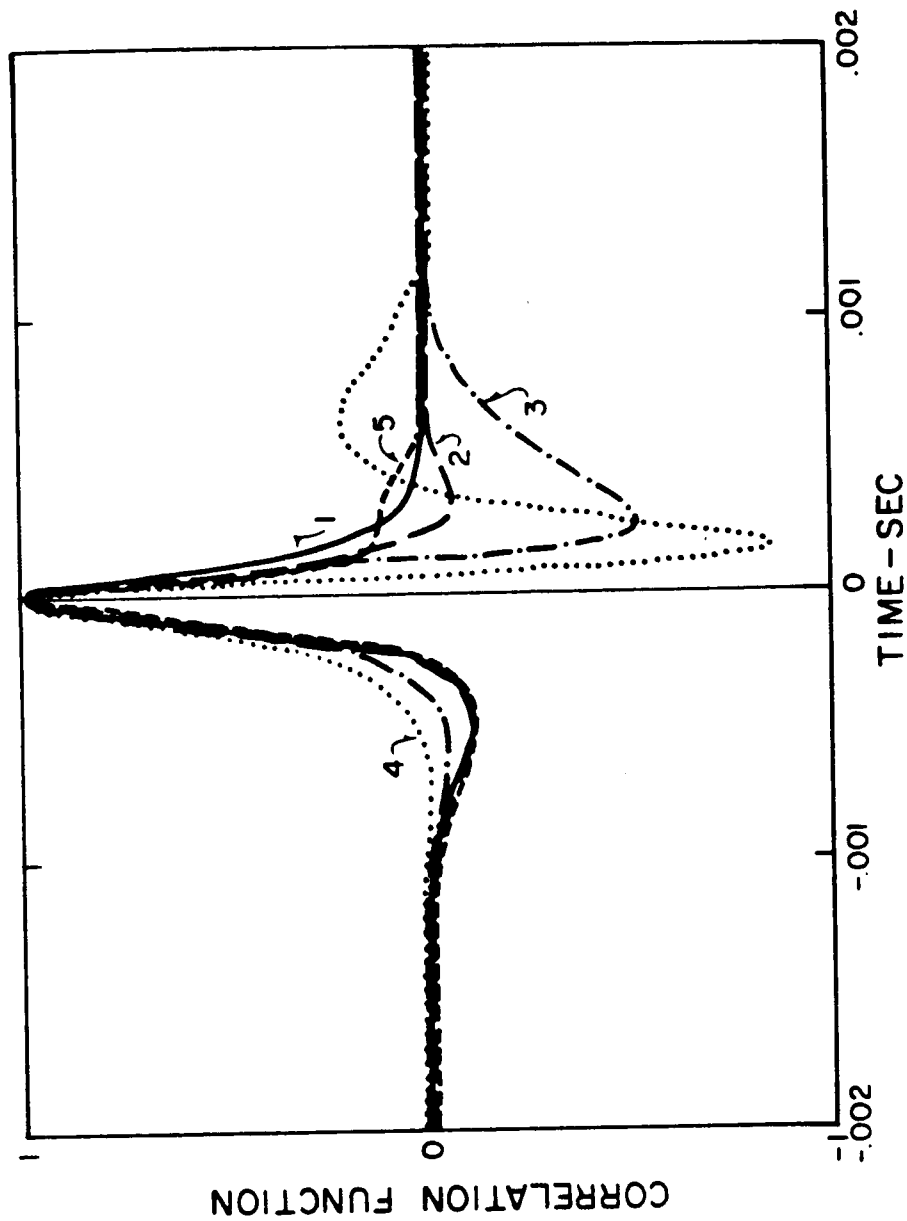


Figure 7. The crosscorrelation functions between the pressure variables for $J_x = 1.10 \text{ A/cm}^2$.

1. C_{AB} ($A = 18 \text{ cm}$, $B = 36 \text{ cm}$)
2. C_{AC} ($C = 54 \text{ cm}$)
3. C_{AD} ($D = 72 \text{ cm}$)
4. C_{AE} ($E = 90 \text{ cm}$)
5. C_{AF} ($F = 106 \text{ cm}$)

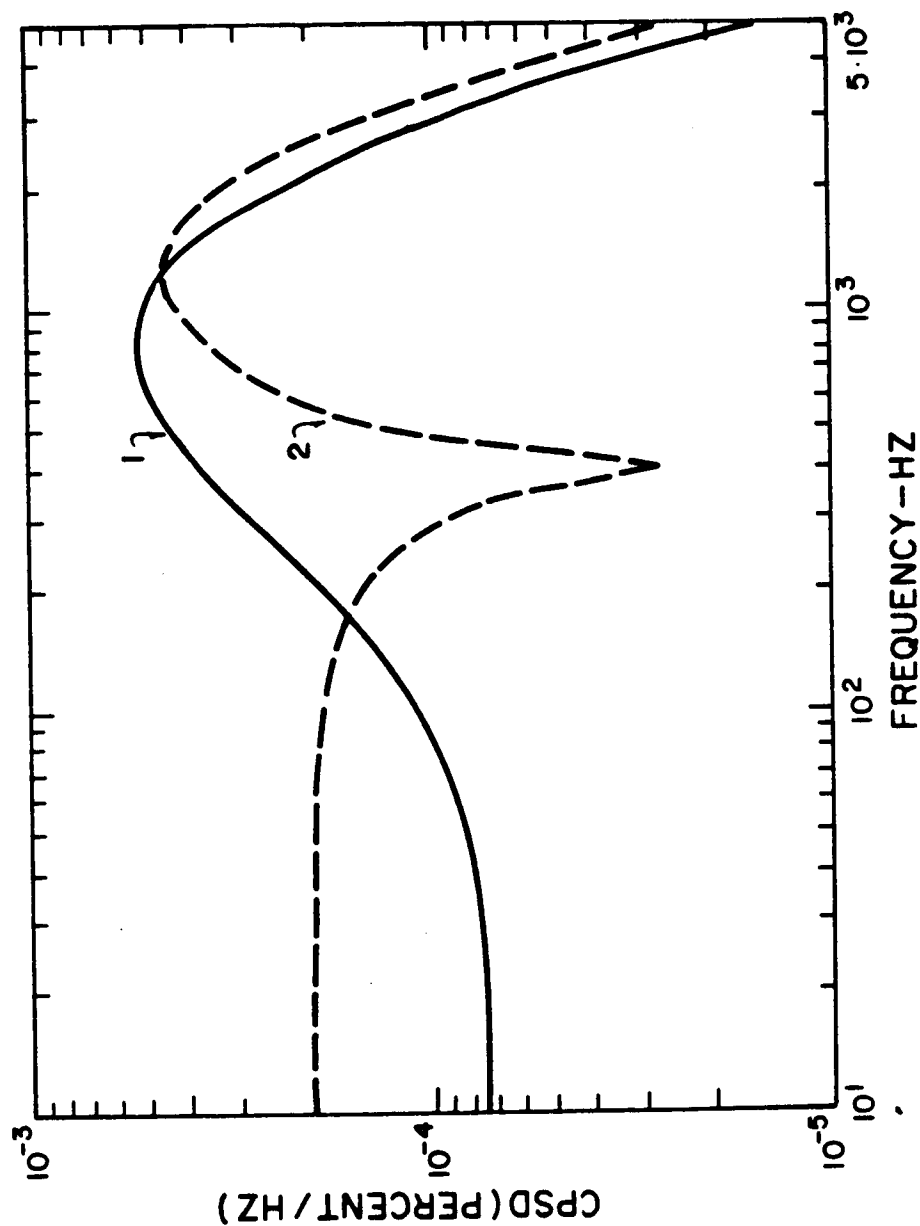


Figure 8. The CPSD functions between the pressure variables for $J_x = 1.10A$.

1. C_{AB} ($A = 40$ cm, $B = 72$ cm)
2. C_{AC} ($C = 90$ cm)

the plots exhibit clearly a very broad peak in the range 700 to 2000 Hz. These frequencies compare very well with the frequencies of the longitudinal standing waves present in the channel. The standing waves result from an interference between the waves traveling in opposite directions. When a wave travels along a channel of finite length, it is reflected at the ends, thereby giving rise to a wave traveling in opposite direction; this causes the interference. The reflection taking place at a closed end produces a displacement node, whereas that taking place at an open end produces a displacement antinode if a channel is narrow compared to the wavelength [20]. The displacement of the particles at nodes is always zero, whereas the displacement at antinodes is maximum. In a channel having one end closed and the other open, the fundamental frequency is equal to $\frac{c}{4L}$, where c is the speed of sound and L is the channel's length; only odd harmonics are present in such a channel. The closed-end and open-end conditions are characteristic of the MHD model. The channel's entrance, being under the highest pressure and being subject to the forcing functions, forms the closed end, whereas the channel's exit forms the open end. Table 5 lists first odd harmonics and the corresponding lengths of the standing waves that may be present in the MHD channel. (The table is valid for the steady state conditions (5.1); changes in pressure and density affect the calculated numbers.) The broad peaks in the CPSD plots of Figure 8 can be formed due to the odd higher harmonics of the standing waves. The figure confirms that the Hall linear generator presents especially favorable conditions for the high frequency standing waves.

TABLE 5

ODD HARMONICS AND THE CORRESPONDING
WAVELENGTHS OF THE MHD CHANNEL

Harmonic (Hz)	Wavelength (m)
Fundamental frequency $f_1 = \frac{c}{4L} = 230$ $c = \sqrt{\frac{\gamma P}{\rho}}$	Fundamental length $\lambda_1 = 4L = 4.35$
$f_3 = \frac{3c}{4L} = 690$	$\lambda_3 = \frac{4L}{3} = 1.45$
$f_5 = \frac{5c}{4L} = 1150$	$\lambda_5 = \frac{4L}{5} = .87$
$f_7 = \frac{7c}{4L} = 1610$	$\lambda_7 = \frac{4L}{7} = .62$
$f_9 = \frac{9c}{4L} = 2070$	$\lambda_9 = \frac{4L}{9} = .48$
$f_{11} = \frac{11c}{4L} = 2530$	$\lambda_{11} = \frac{4L}{11} = .40$

The APSD functions of the fluctuating density variable are shown in Figures 9 through 12. Figures 9 and 10 demonstrate the APSD plots of the variable as a function of the generator's current. The plots in Figure 9 apply to the axial point located at one-third of the channel's length, whereas those in Figure 10 apply to the end of the channel. Figures 11 and 12 show the APSD plots of the variable as a function of the axial position. The plots in Figure 11 are characteristic of the Faraday generator, and those in Figure 12 are characteristic of the Hall generator. The conclusions drawn from the analysis of these APSD figures are similar to those obtained from the spectral analysis of the fluctuating pressure variable. The generators whose axial currents are small have a concentration of fluctuations in the 10 to 200 Hz range, particularly in the first section of the channel. The frequency range of fluctuations increases together with axial currents; the 700 to 2000 Hz fluctuations may even exceed those in the range up to 200 Hz, for very large currents.

The crosscorrelation and CPSD functions of the fluctuating density variables between various axial locations are presented in Figures 13 to 15. Figure 13 demonstrates the normalized crosscorrelation plots of the DCW generator, Figure 14 demonstrates the similar plots of the Hall generator, and Figure 15 shows the typical CPSD plot. The density variables correlate strongly from one axial position to another with maximum correlation near zero-delay time. This zero-delay time together with an in-phase then out-of-phase behavior of the plots (a few of the plots in Figures 13 and 14 are inverted) suggest that the

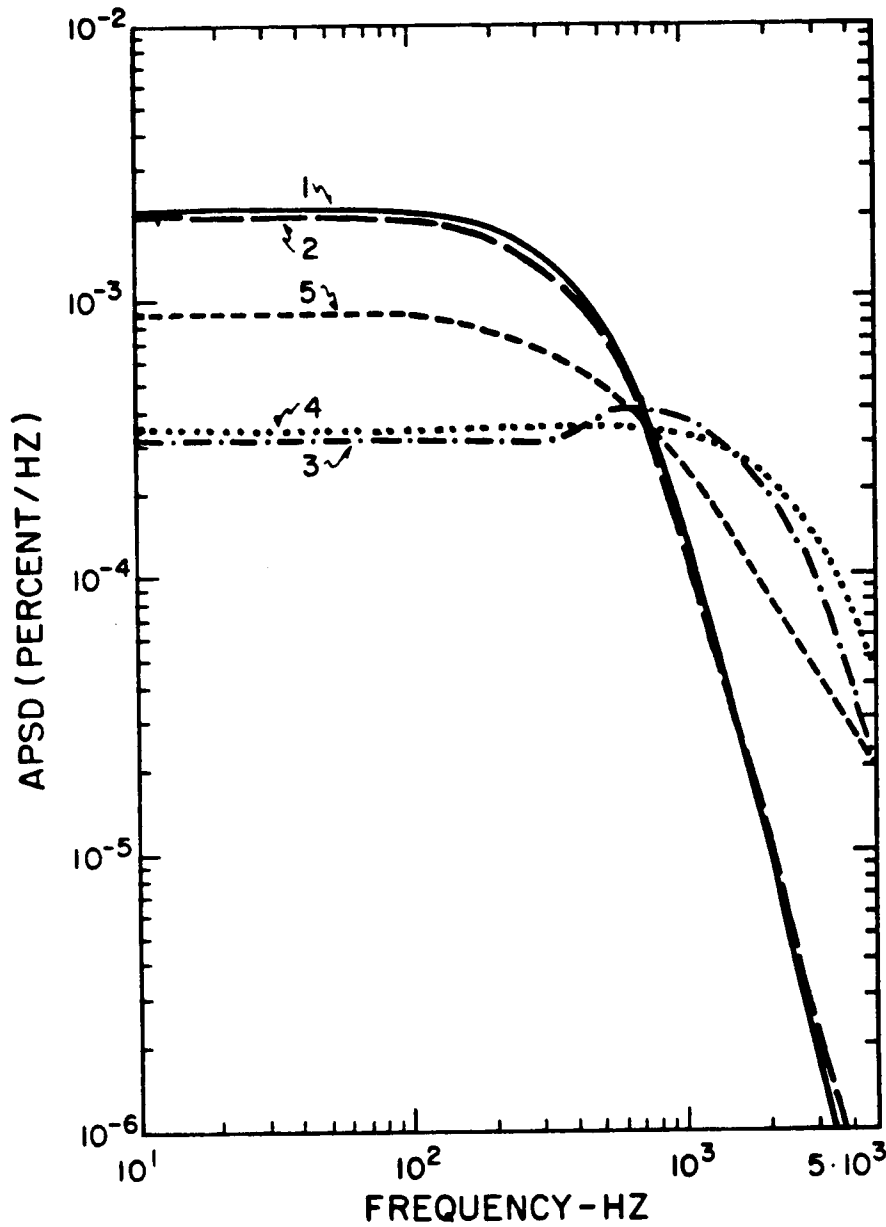


Figure 9. The APSD functions of the density variable for different axial currents at $x = 36$ cm.

1. $J_x = .10 \frac{A}{cm^2}$

2. $J_x = .40 \frac{A}{cm^2}$

3. $J_x = .60 \frac{A}{cm^2}$

4. $J_x = .86 \frac{A}{cm^2}$

5. $J_x = 1.10 \frac{A}{cm^2}$

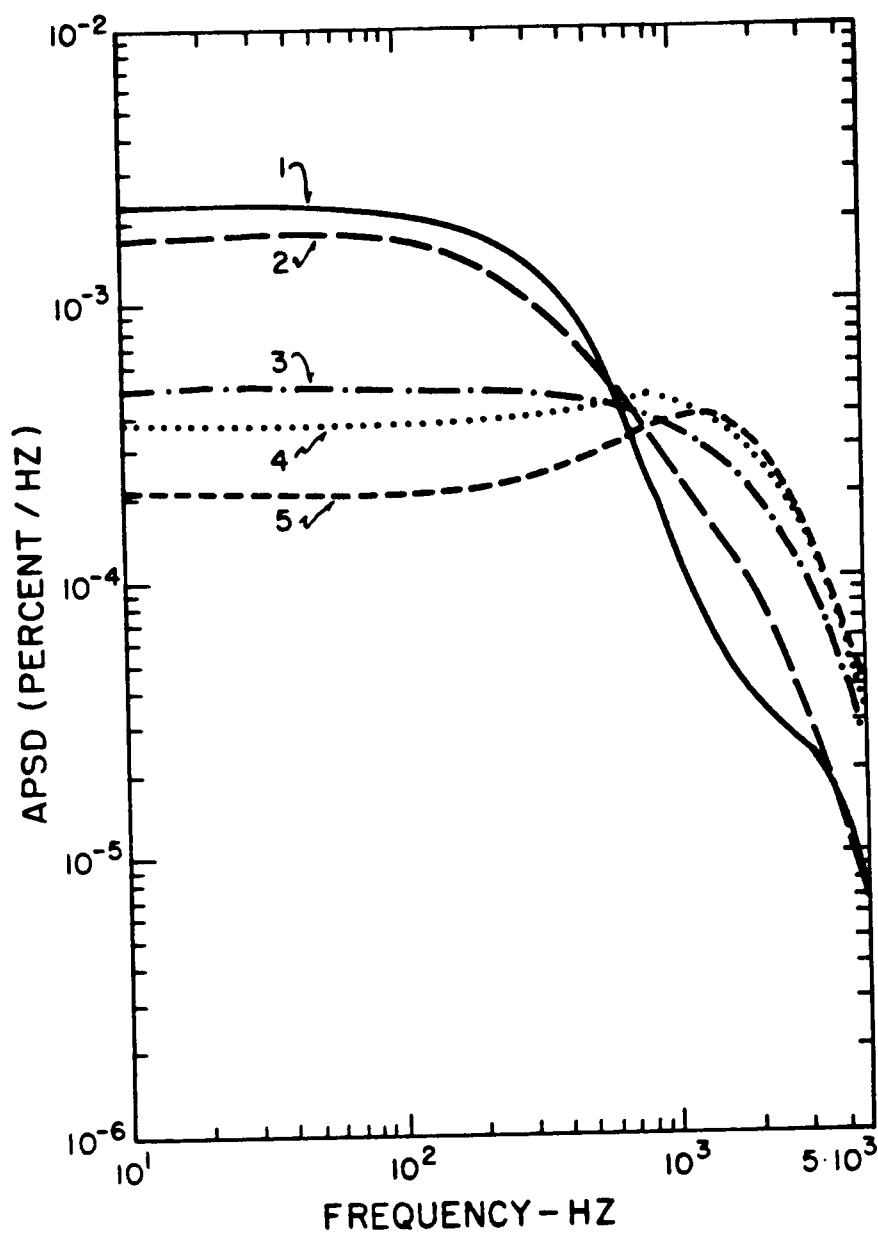


Figure 10. The APSD functions of the density variable for different axial currents at $x = 106$ cm.

$$1. \quad J_x = .10 \frac{A}{cm^2}$$

$$2. \quad J_x = .40 \frac{A}{cm^2}$$

$$3. \quad J_x = .60 \frac{A}{cm^2}$$

$$4. \quad J_x = .86 \frac{A}{cm^2}$$

$$5. \quad J_x = 1.10 \frac{A}{cm^2}$$

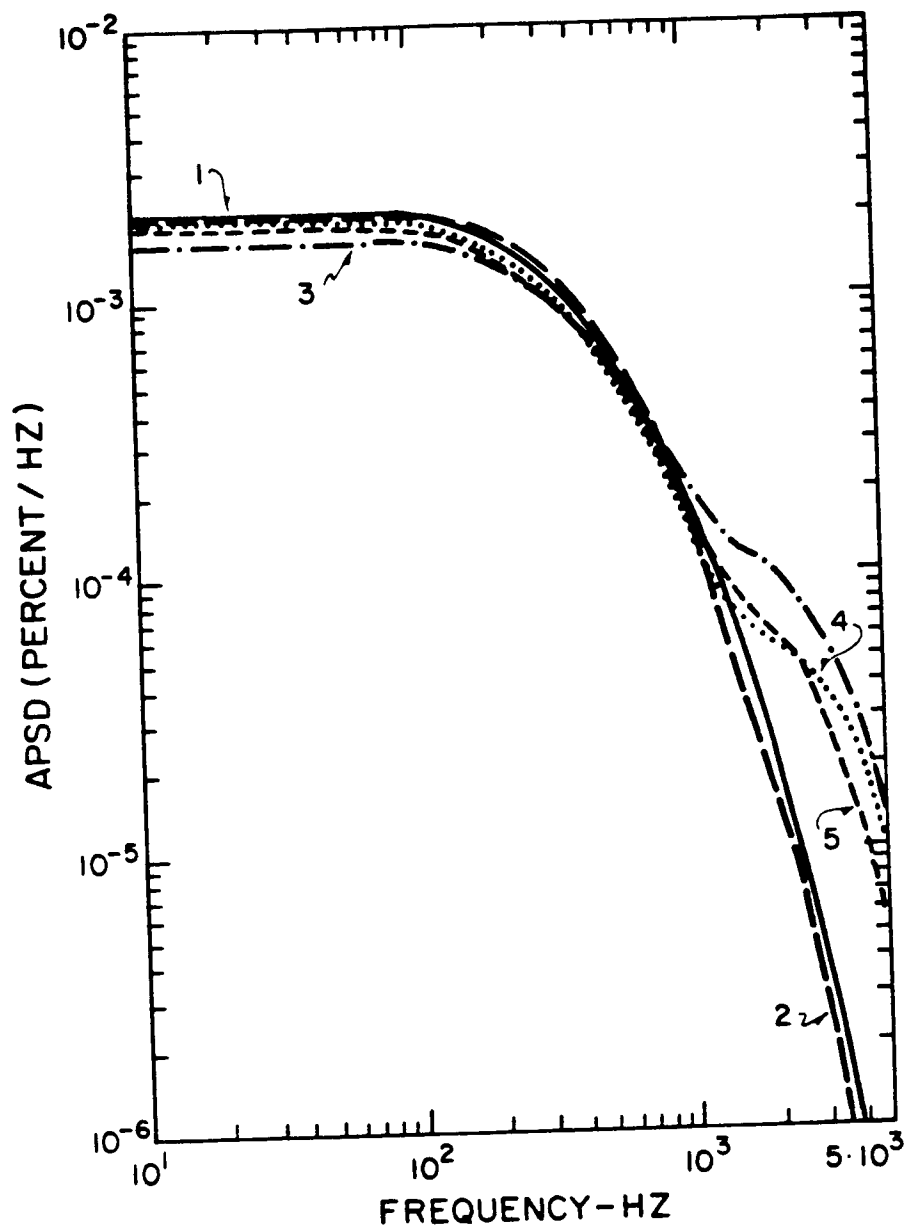


Figure 11. The APSD functions of the density variable at different axial locations for $J_x = .10 \text{ A/cm}^2$.

- | | |
|-------------------------|------------------------|
| 1. $x = 18 \text{ cm}$ | 2. $x = 36 \text{ cm}$ |
| 3. $x = 54 \text{ cm}$ | 4. $x = 90 \text{ cm}$ |
| 5. $x = 106 \text{ cm}$ | |

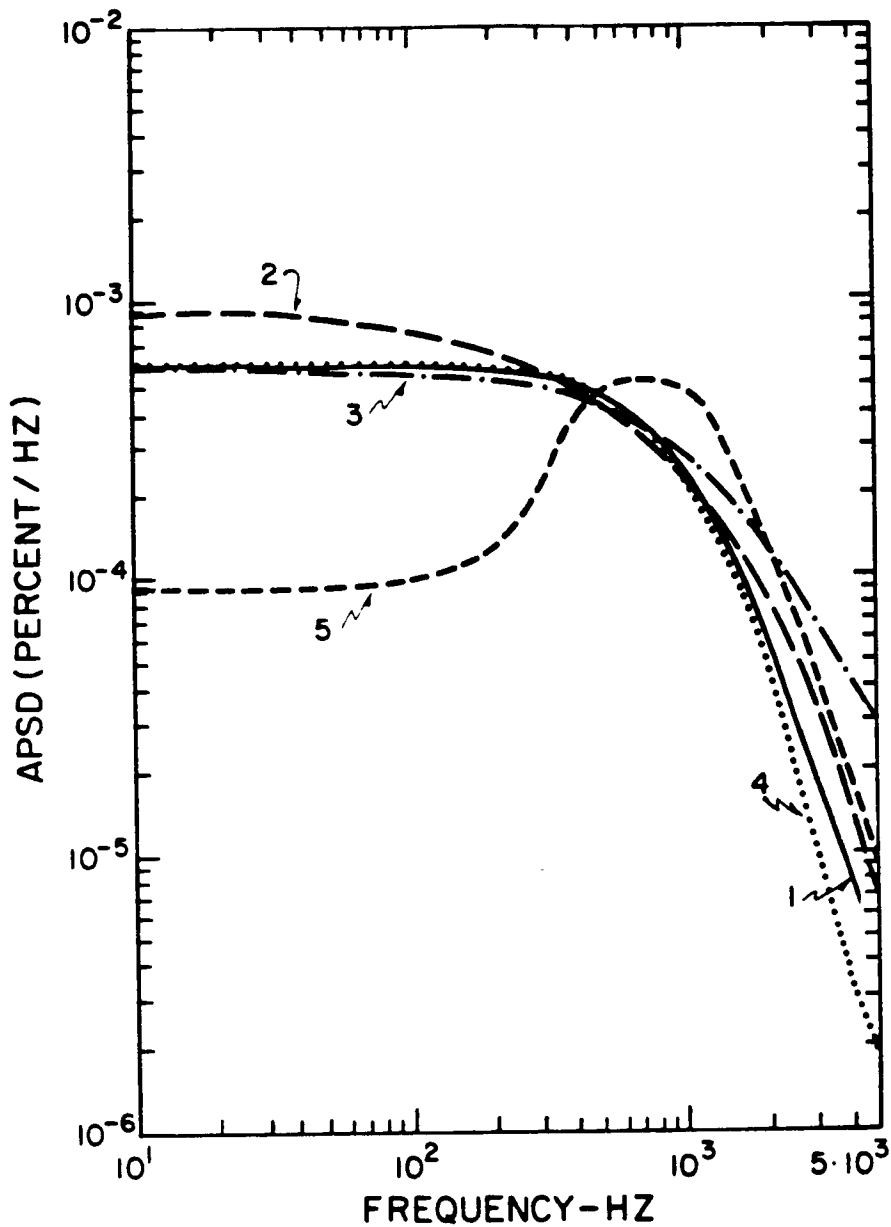


Figure 12. The APSD functions of the density variable at different axial locations for $J_x = 1.10 \text{ A/cm}^2$.

1. $x = 18 \text{ cm}$
3. $x = 54 \text{ cm}$
5. $x = 106 \text{ cm}$

2. $x = 36 \text{ cm}$
4. $x = 90 \text{ cm}$

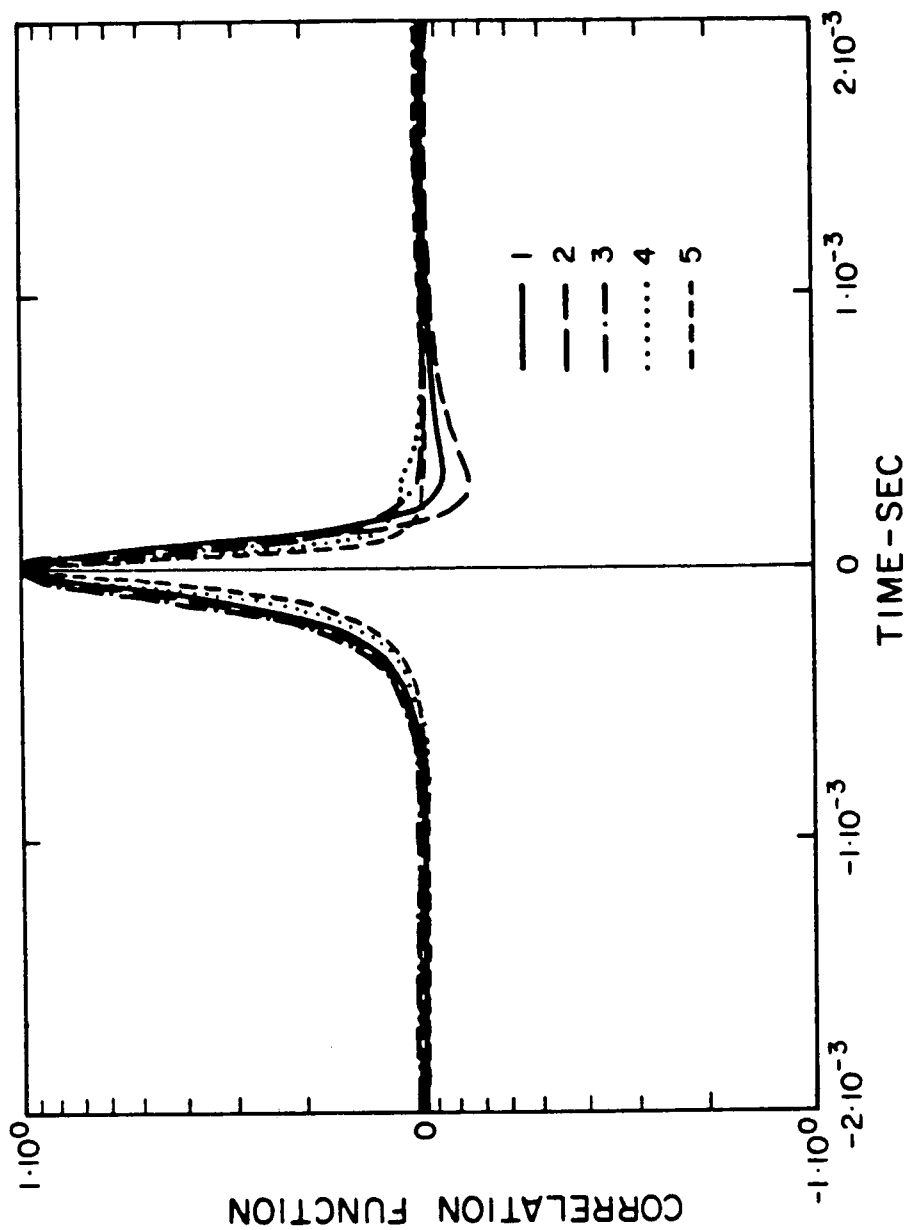


Figure 13. The crosscorrelation functions between the density variables for $J_x = .60 \text{ A/cm}^2$.

- | | |
|------------------------------------|-------------------------|
| 1. C_{AB} (A = 18 cm, B = 36 cm) | 2. C_{AC} (C = 54 cm) |
| 3. C_{AD} (D = 72 cm) | 4. C_{AE} (E = 90 cm) |
| 5. C_{AF} (F = 106 cm) | |

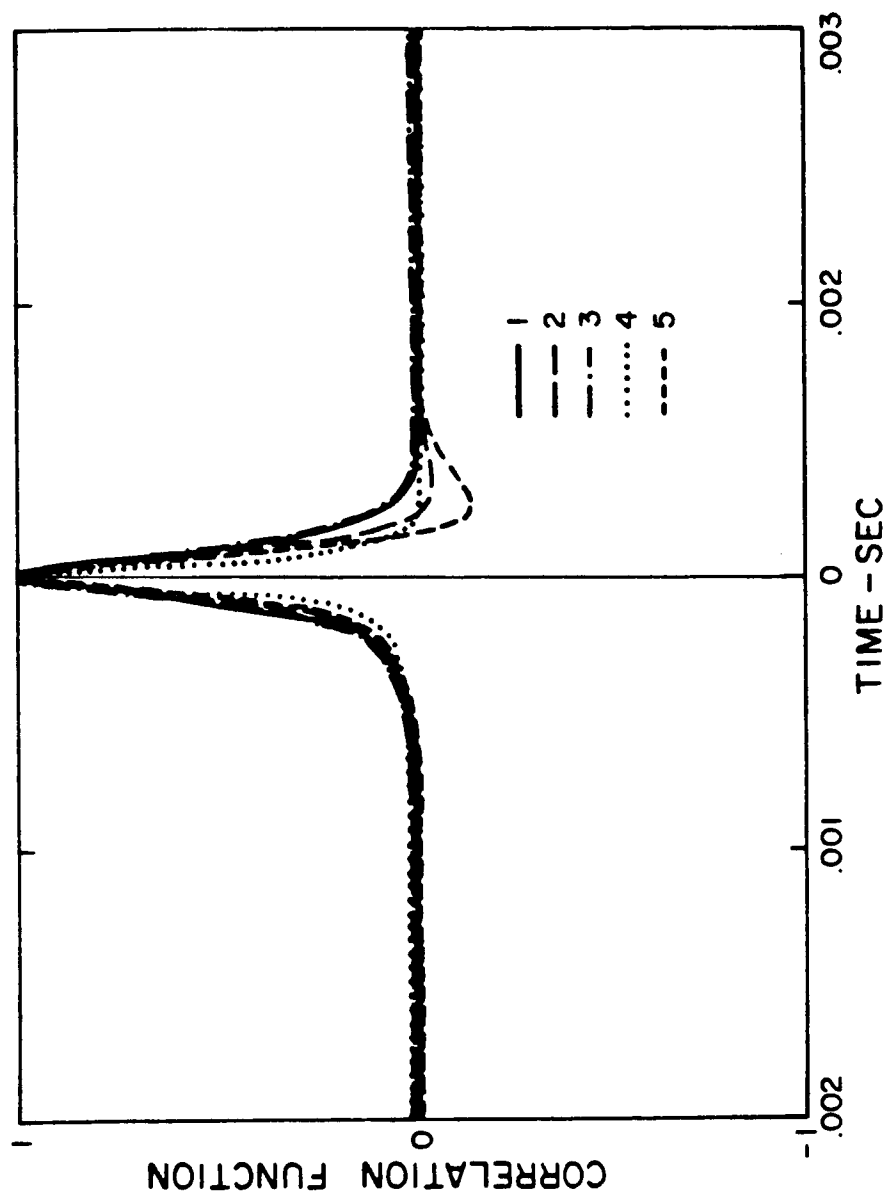


Figure 14. The crosscorrelation functions between the density variables for $J_x = 1.10 \text{ A/cm}^2$.

1. C_{AB} ($A = 18 \text{ cm}$, $B = 36 \text{ cm}$)
2. C_{AC} ($C = 54 \text{ cm}$)
3. C_{AD} ($D = 72 \text{ cm}$)
4. C_{AE} ($E = 90 \text{ cm}$)
5. C_{AF} ($F = 106 \text{ cm}$)

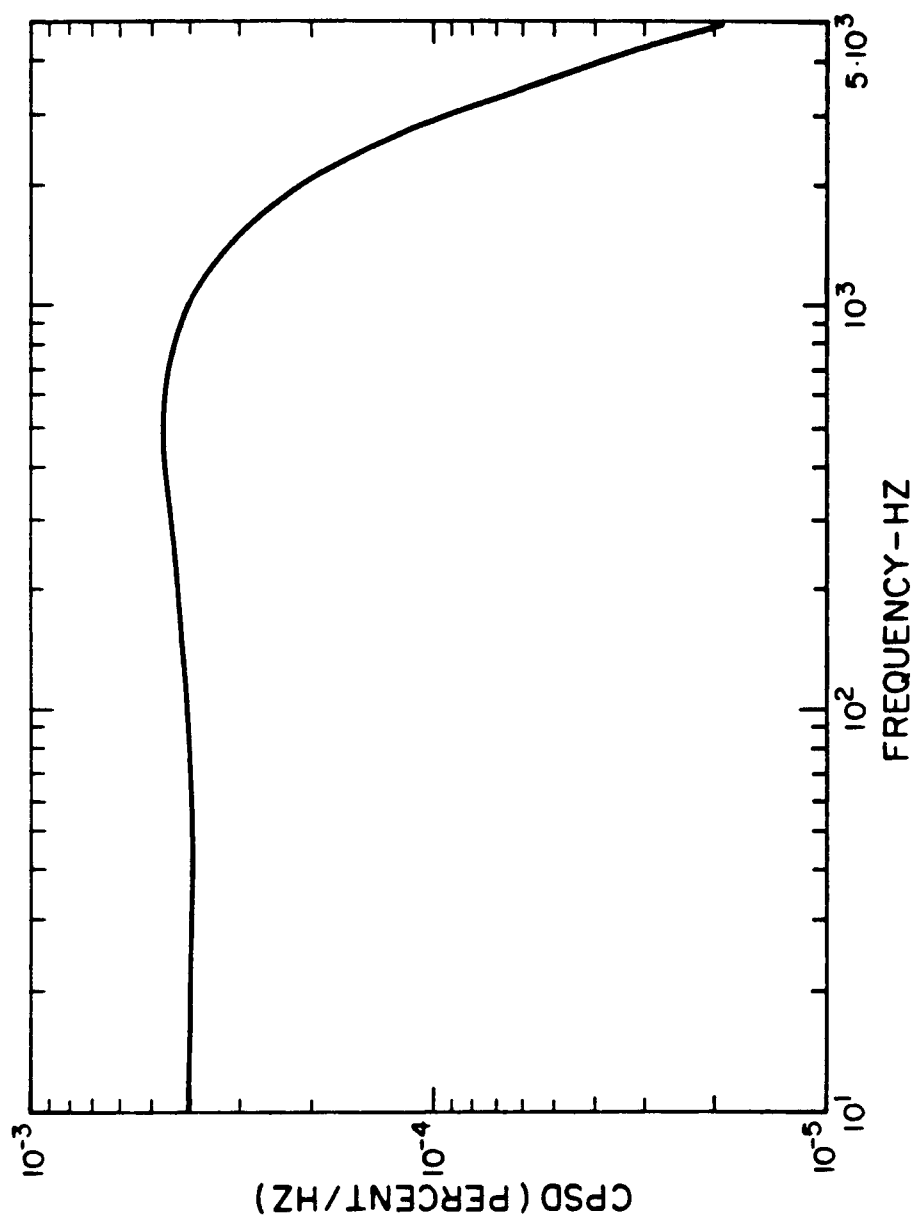


Figure 15. The CPD function between the density variables for
 $J_x = .60 \text{ A/cm}^2$.

$$C_{AB} \text{ (A = 18 cm, B = 106 cm)}$$

fluctuating density forms acoustic standing waves inside the MHD channel.

The APSD functions of the fluctuating velocity variable are shown in Figures 16-19. Figures 16 and 17 demonstrate the APSD plots of the variable as a function of the generator's current. The plots in Figure 16 apply to the axial point located at one-third of the channel's length, whereas those in Figure 17 apply to the end of the channel. The figures prove that the Faraday and DCW generators are favorable for the low (up to 200 Hz) frequency fluctuations, especially in the first section of the channel. The Hall generator develops fluctuations mainly in the range 700 to 2000 Hz; the amplitude of these fluctuations may exceed that of the low frequency fluctuations. The changes in the APSD functions of the velocity variable along the MHD channel are shown in Figures 18 and 19. Figure 18 applies to the DCW generator, and Figure 20 applies to the Hall generator. The figures indicate that the higher frequency fluctuations are more intensive toward the end of the channel.

The crosscorrelation and CPSD functions of the fluctuating velocity variables between the various axial locations are presented in Figures 20-22. Figure 20 demonstrates the normalized crosscorrelation plots for the DCW generator, Figure 21 demonstrates similar plots for the Hall generator, whereas Figure 22 shows the CPSD plot of the Hall generator. The figures indicate that the standing waves are present inside the MHD generator. They indicate also that the crosscorrelation functions of the DCW generator decay slower than those of the Hall generator, confirming that DCW generators present favorable

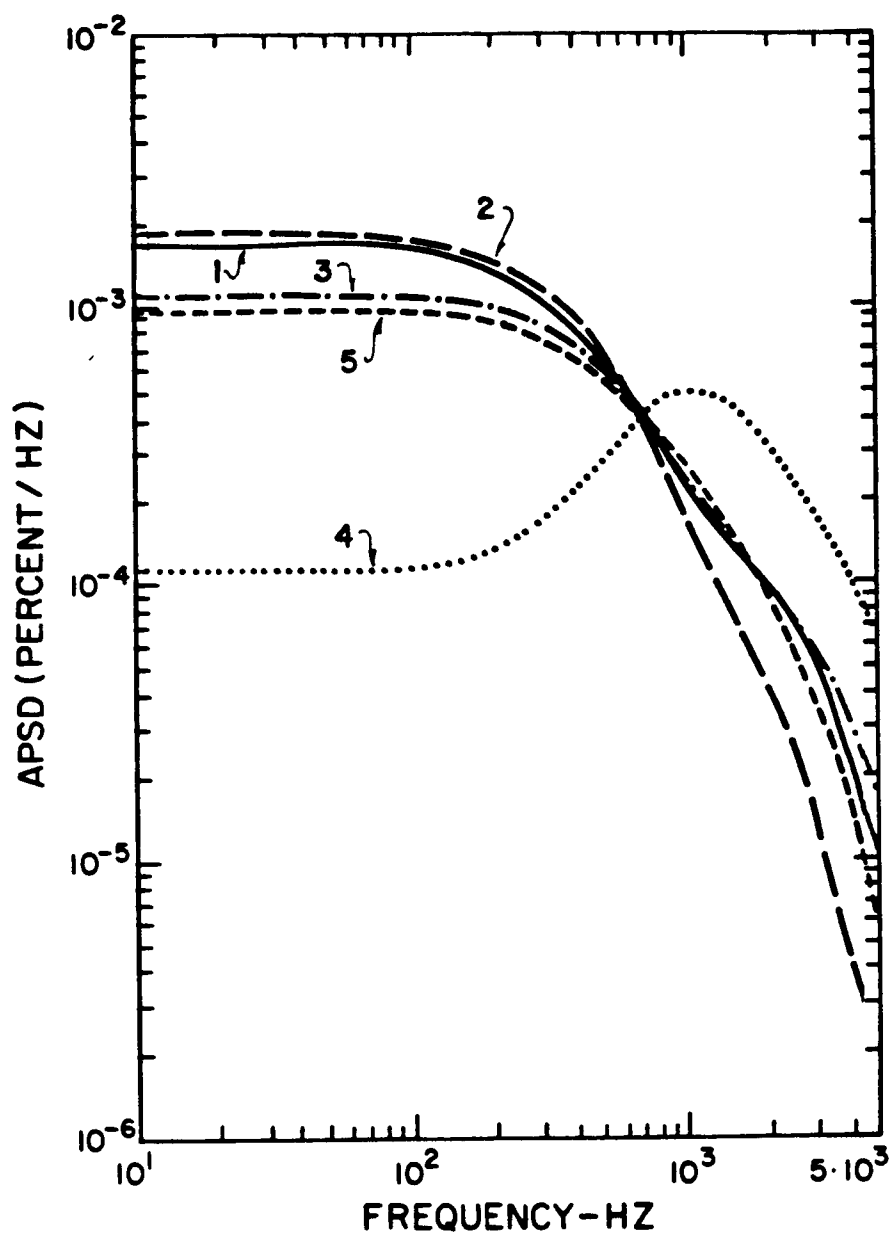


Figure 16. The APSD functions of the velocity variable for different axial currents at $x = 36$ cm.

$$1. \quad J_x = .10 \frac{A}{cm^2}$$

$$2. \quad J_x = .40 \frac{A}{cm^2}$$

$$3. \quad J_x = .60 \frac{A}{cm^2}$$

$$4. \quad J_x = .86 \frac{A}{cm^2}$$

$$5. \quad J_x = 1.10 \frac{A}{cm^2}$$

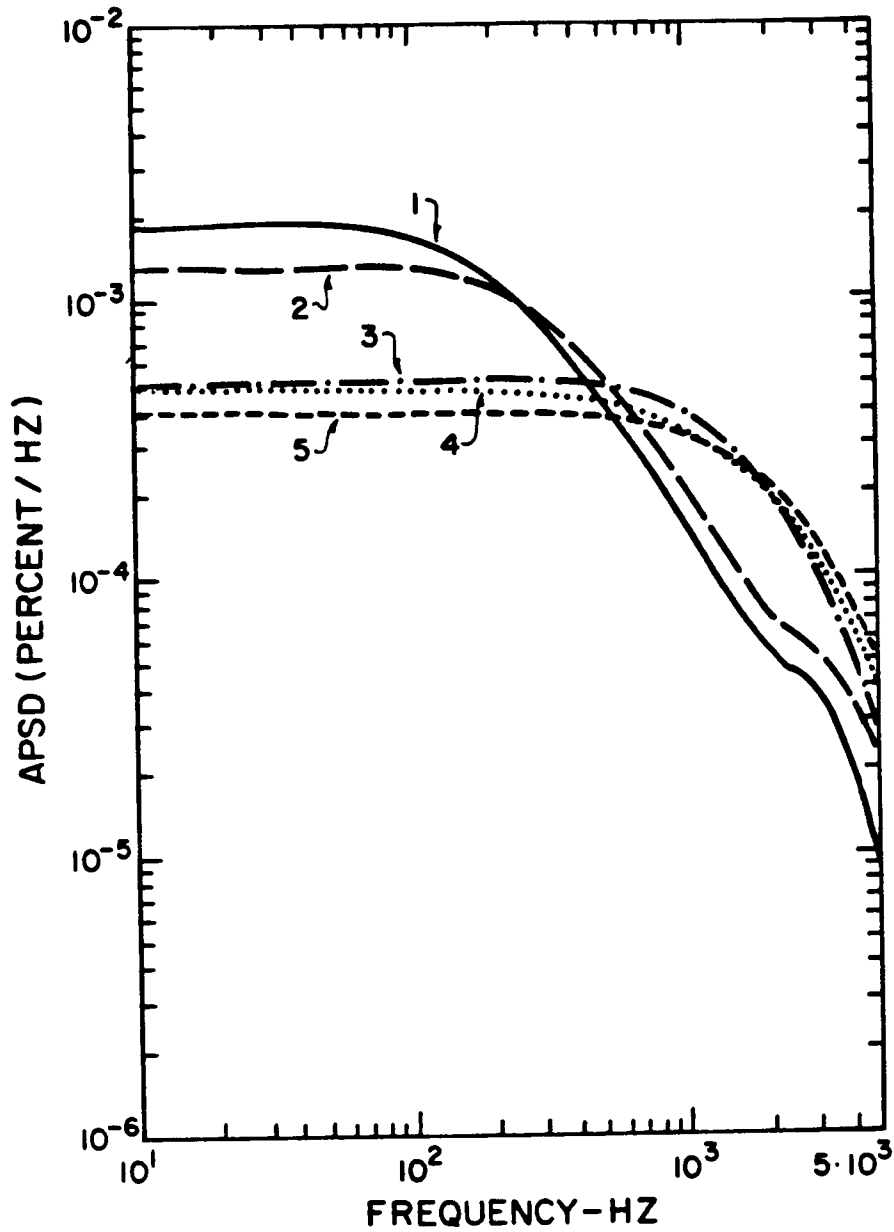


Figure 17. The APSD functions of the velocity variable for different axial current at $x = 106$ cm.

$$1. \quad J_x = .10 \frac{A}{cm^2}$$

$$2. \quad J_x = .40 \frac{A}{cm^2}$$

$$3. \quad J_x = .60 \frac{A}{cm^2}$$

$$4. \quad J_x = .86 \frac{A}{cm^2}$$

$$5. \quad J_x = 1.10 \frac{A}{cm^2}$$

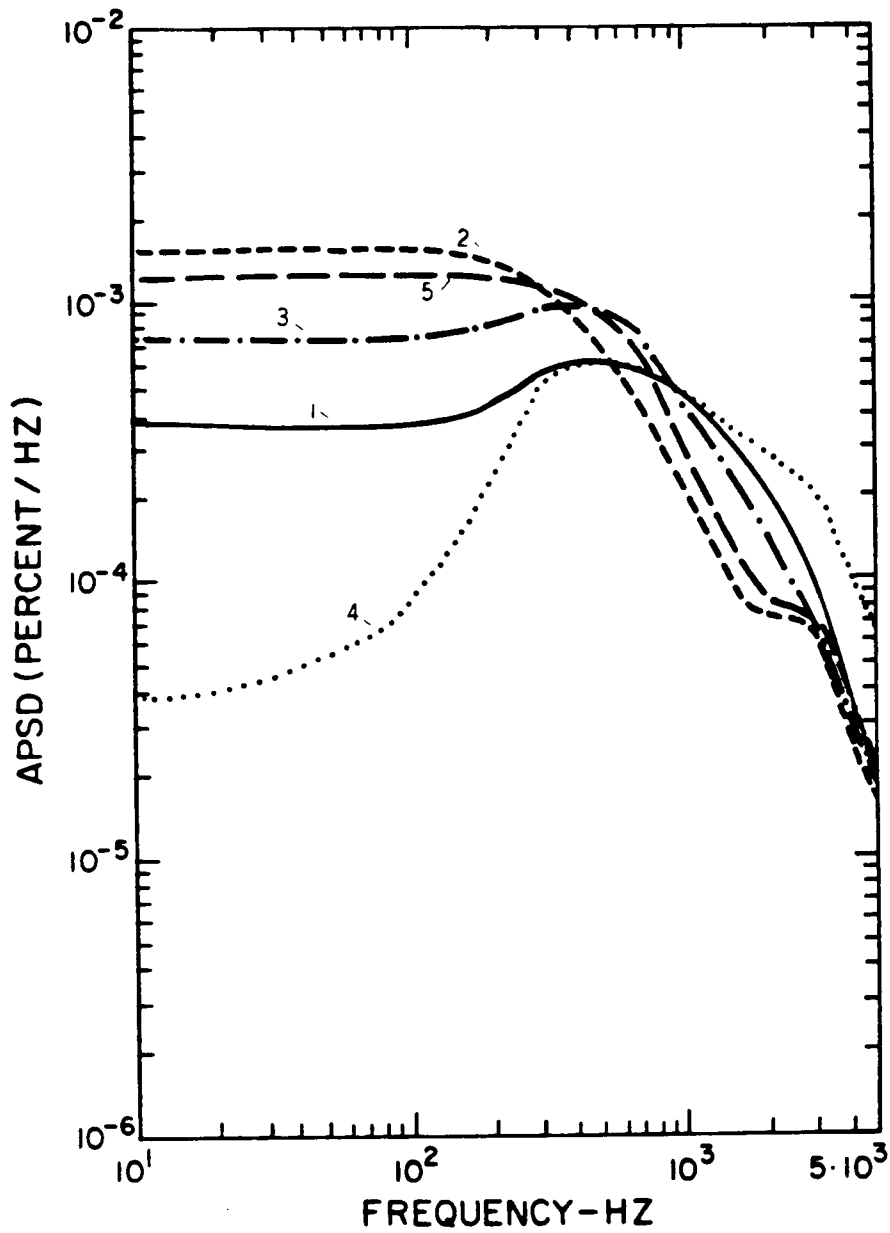


Figure 18. The APSD functions of the velocity variable at different axial locations for $J_x = .40 \text{ A/cm}^2$.

- | | |
|-------------------------|------------------------|
| 1. $x = 18 \text{ cm}$ | 2. $x = 36 \text{ cm}$ |
| 3. $x = 54 \text{ cm}$ | 4. $x = 90 \text{ cm}$ |
| 5. $x = 106 \text{ cm}$ | |

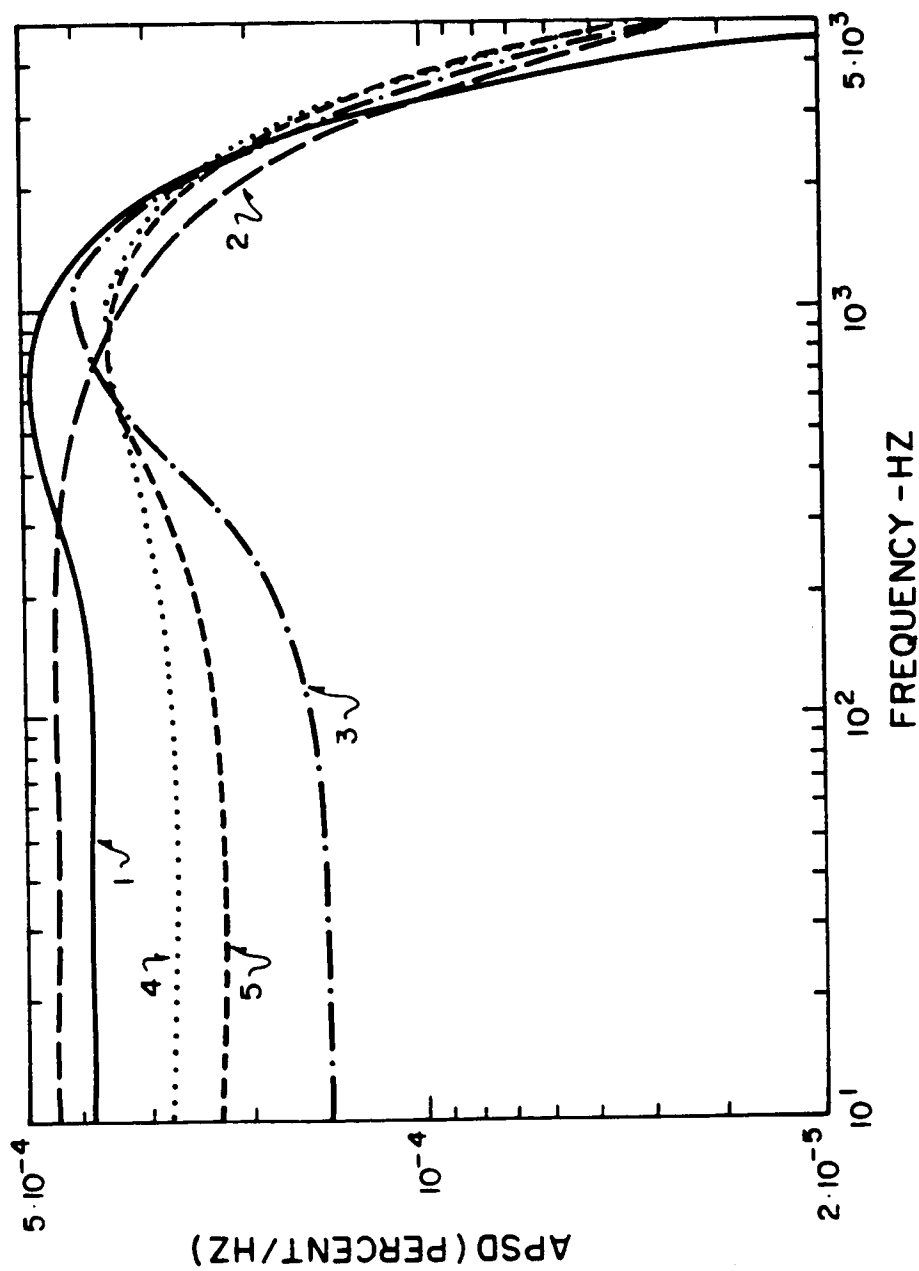


Figure 19. The APSD functions of the velocity variable at different axial locations for $J_x = 1.10 \text{ A/cm}^2$.

- | | |
|-------------------------|------------------------|
| 1. $x = 18 \text{ cm}$ | 2. $x = 36 \text{ cm}$ |
| 3. $x = 54 \text{ cm}$ | 4. $x = 90 \text{ cm}$ |
| 5. $x = 106 \text{ cm}$ | |

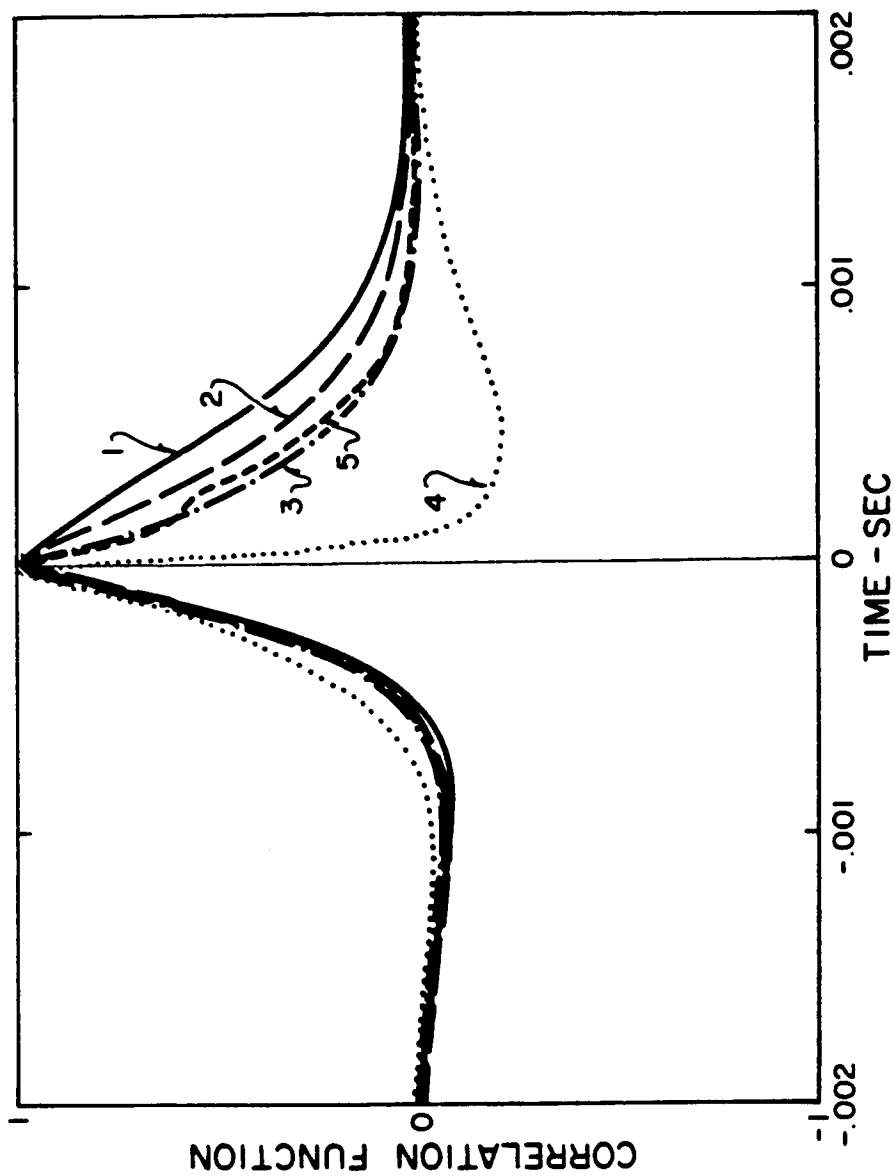


Figure 20. The crosscorrelation functions between the velocity variables for $J_x = .40 \text{ A/cm}^2$.

- | | |
|---|-------------------------------------|
| 1. C_{AB} ($A = 18 \text{ cm}$, $B = 36 \text{ cm}$) | 2. C_{AC} ($C = 54 \text{ cm}$) |
| 3. C_{AD} ($D = 72 \text{ cm}$) | 4. C_{AE} ($E = 90 \text{ cm}$) |
| 5. C_{AF} ($F = 106 \text{ cm}$) | |

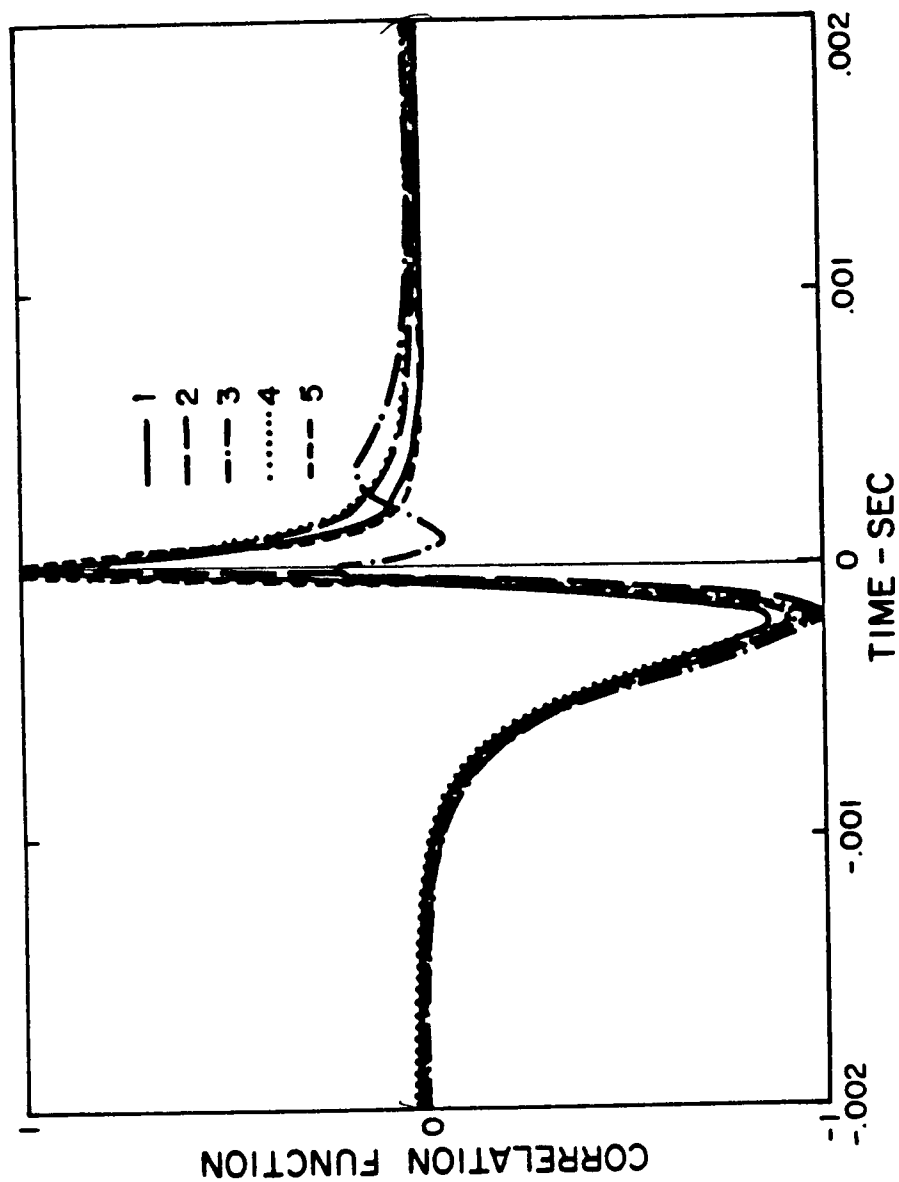


Figure 21. The crosscorrelation functions between the velocity variables for $J_x = 1.10 \text{ A/cm}^2$.

1. C_{AB} ($A = 18 \text{ cm}$, $B = 36 \text{ cm}$)
2. C_{AC} ($C = 54 \text{ cm}$)
3. C_{AD} ($D = 72 \text{ cm}$)
4. C_{AE} ($E = 90 \text{ cm}$)
5. C_{AF} ($F = 106 \text{ cm}$)

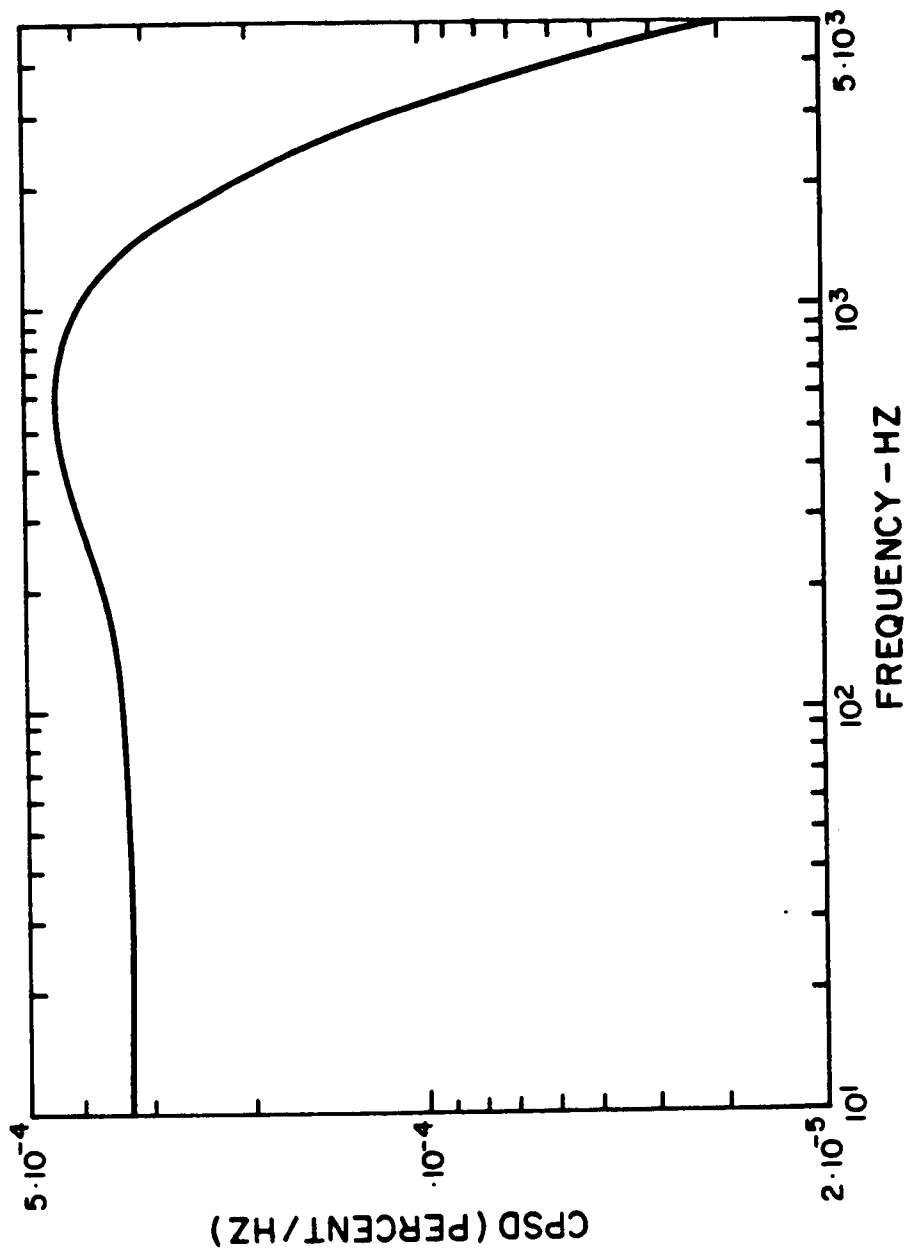


Figure 22. The CPSD function between the velocity variables for $J_x = 1.10 \text{ A/cm}^2$.

$$C_{AB} \text{ (A = 36 cm, B = 106 cm)}$$

conditions for the low-harmonic waves, whereas Hall generators present favorable conditions for the higher-harmonic standing waves.

3. Analysis of the Current and Voltage Variables

The MHD generator's current and voltage depend on the pressure, density, and velocity of the plasma in the conducting channel. The nodal fluctuating currents in the x and y directions are equal to

$$\begin{aligned} \frac{j_{x(i)}}{J_{x(i)}} &= a_{x(i)} \frac{\rho'_{(i-1)}}{\rho_{o(i-1)}} + b_{x(i)} \frac{\rho'_{(i)}}{\rho_{o(i)}} + c_{x(i)} \frac{p'_{(i-1)}}{p_{o(i-1)}} + \\ &\quad d_{x(i)} \frac{p'_{(i)}}{p_{o(i)}} + e_{x(i)} \frac{u'_{(i)}}{u_{o(i)}} \\ \frac{j_{y(i)}}{J_{y(i)}} &= a_{y(i)} \frac{\rho'_{(i-1)}}{\rho_{o(i-1)}} + b_{y(i)} \frac{\rho'_{(i)}}{\rho_{(i)}} + c_{y(i)} \frac{p'_{(i-1)}}{p_{o(i-1)}} + \\ &\quad d_{y(i)} \frac{p'_{(i)}}{p_{o(i)}} + e_{y(i)} \frac{u'_{(i)}}{u_{o(i)}} , \end{aligned} \quad (5.2)$$

where i is the node number, and the coefficients $a_{x(i)}$, ..., $e_{x(i)}$, $a_{y(i)}$, ..., $e_{y(i)}$ are listed in Appendix A. The autocorrelation and crosscorrelation functions of these variables can be obtained directly from the definition

$$\begin{aligned} C_{ij}(\tau) &= \int_0^{\infty} \frac{j_{(i)}(t)}{J_{(i)}} \frac{j_{(j)}(t+\tau)}{J_{(j)}} dt = \\ &a_{(i)} a_{(j)} C_{\rho(i-1)\rho(j-1)} + b_{(i)} b_{(j)} C_{\rho(i)\rho(j)} + \\ &c_{(i)} c_{(j)} C_{p(i-1)p(j-1)} + d_{(i)} d_{(j)} C_{p(i)p(j)} + \end{aligned}$$

$$\begin{aligned}
& e_{(i)} e_{(j)} C_{u(i)u(j)} + a_{(i)} b_{(j)} C_{\rho(i-1)\rho(j)} + \\
& a_{(j)} b_{(i)} C_{\rho(j)\rho(i-1)} + a_{(i)} c_{(j)} C_{\rho(i-1)p(j-1)} + \\
& + a_{(j)} c_{(i)} C_{p(j-1)\rho(i-1)} + b_{(i)} d_{(j)} C_{\rho(i)p(j)} + \\
& b_{(j)} d_{(i)} C_{p(j)\rho(i)} + b_{(i)} e_{(j)} C_{\rho(i)u(j)} + \\
& b_{(j)} e_{(i)} C_{u(j)\rho(i)} + c_{(i)} d_{(j)} C_{p(i-1)p(j)} + \\
& c_{(j)} d_{(i)} C_{p(j)p(i-1)} + d_{(i)} e_{(j)} C_{p(i)u(j)} + \\
& d_{(j)} e_{(i)} C_{u(j)p(i)} , \tag{5.3}
\end{aligned}$$

where i and j are the node numbers, $C_{ij}(\tau)$ is the axial position dependent x-current or y-current crosscorrelation function, and $C_{\rho(i)p(j)}$, $C_{p(i)u(j)}$, etc., are the crosscorrelation functions between the density, pressure, and velocity, (Equation (5.3) does not include terms made of the products of very small coefficients. The analyses proved that the contribution from these terms to the $C_{ij}(\tau)$ was minimal). The crosscorrelation function between the fluctuating x-voltage variables has the structure similar to that in (5.3).

The results of the current and voltage fluctuation analysis are shown in Figures 23 through 28. Figure 23 demonstrates the APSD plots of the various axial currents, calculated for one-third the distance from the channel's inlet. These plots are very consistent with similar plots of the pressure, density, and velocity variables. The first break frequency occurs around 200 Hz for the plots characteristic of the Faraday and DCW generators. These plots exhibit small resonances

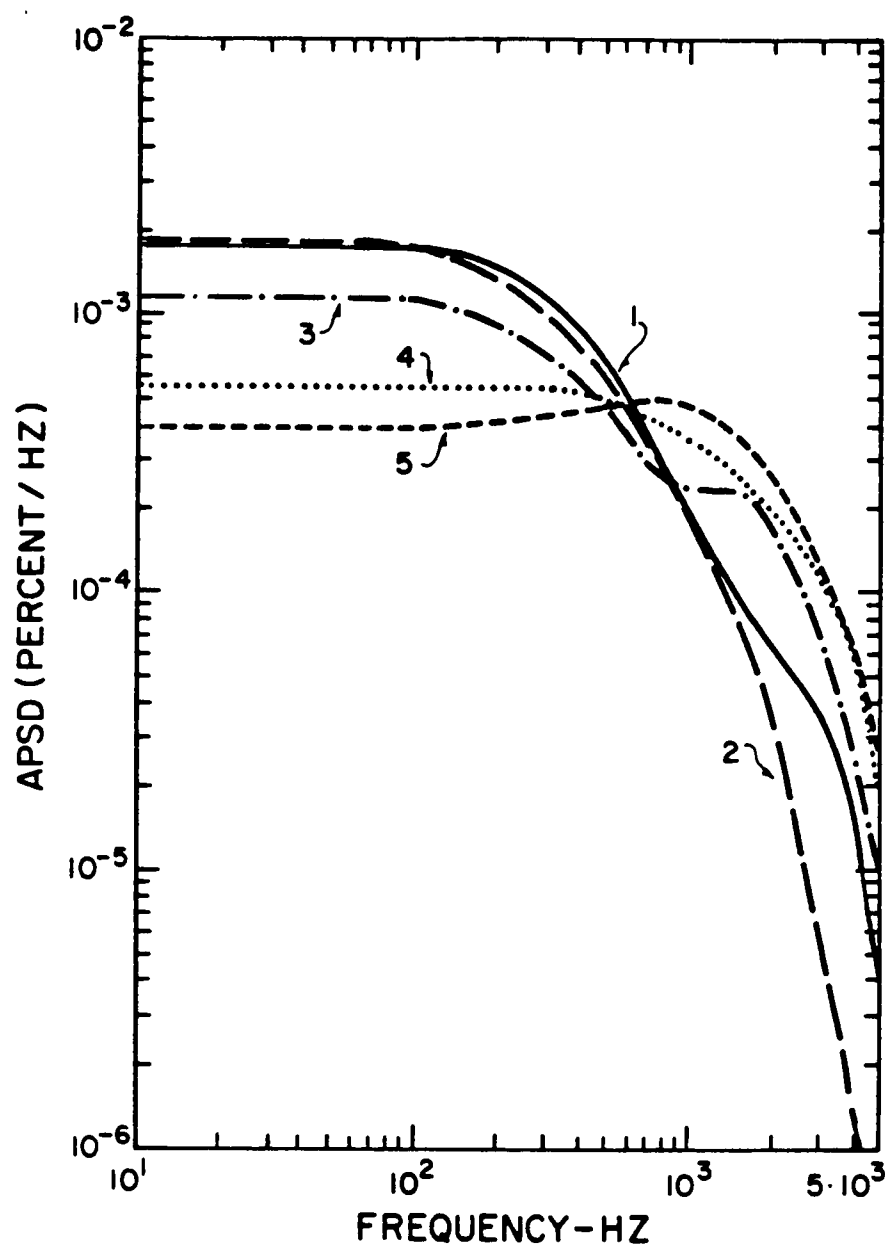


Figure 23. The APSD functions of the various axial currents at $x = 36$ cm.

$$1. \quad J_x = .10 \frac{\text{A}}{\text{cm}^2}$$

$$2. \quad J_x = .40 \frac{\text{A}}{\text{cm}^2}$$

$$3. \quad J_x = .60 \frac{\text{A}}{\text{cm}^2}$$

$$4. \quad J_x = .86 \frac{\text{A}}{\text{cm}^2}$$

$$5. \quad J_x = 1.10 \frac{\text{A}}{\text{cm}^2}$$

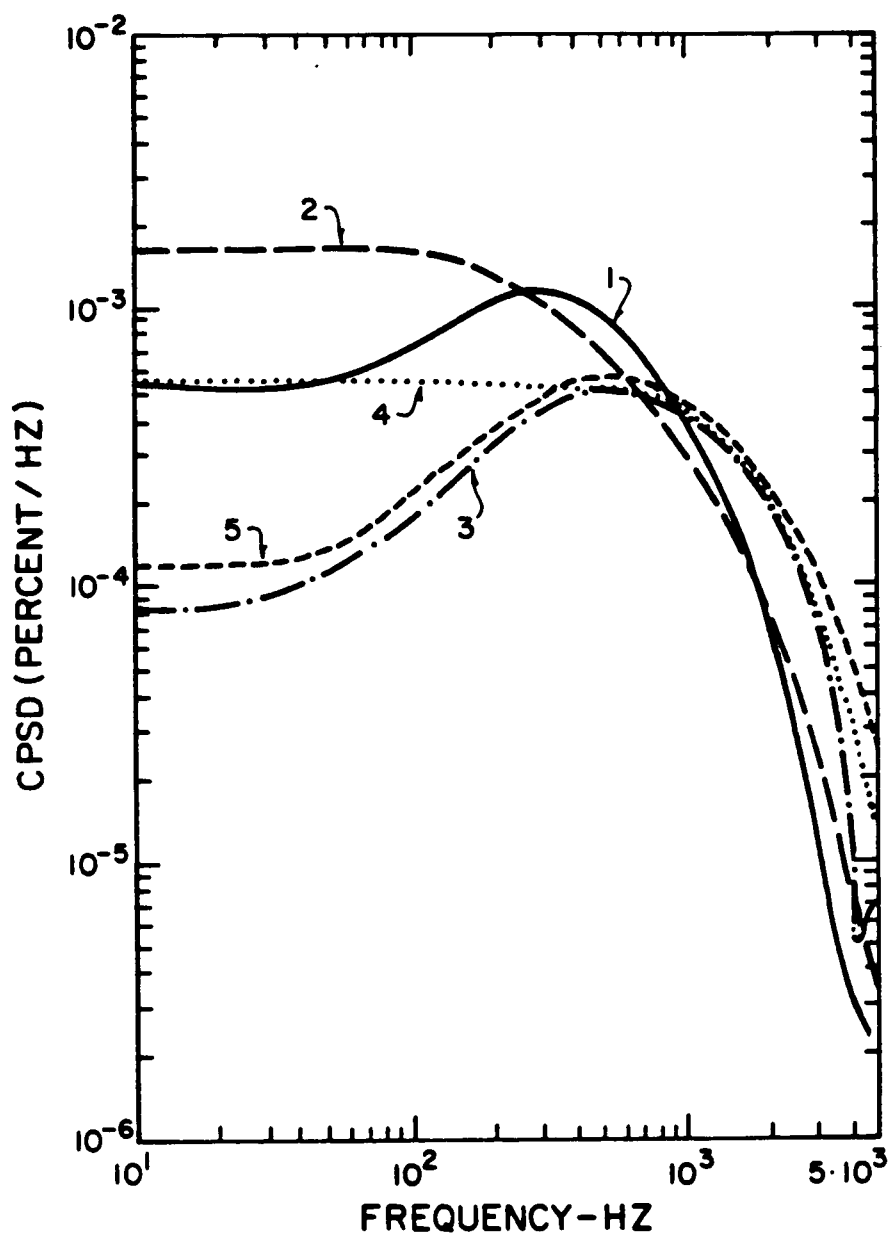


Figure 24. The CPD functions between the axial currents at $x = 36$ cm and $x = 72$ cm.

$$1. \quad J_x = .10 \frac{A}{cm^2}$$

$$2. \quad J_x = .40 \frac{A}{cm^2}$$

$$3. \quad J_x = .60 \frac{A}{cm^2}$$

$$4. \quad J_x = .86 \frac{A}{cm^2}$$

$$5. \quad J_x = 1.10 \frac{A}{cm^2}$$

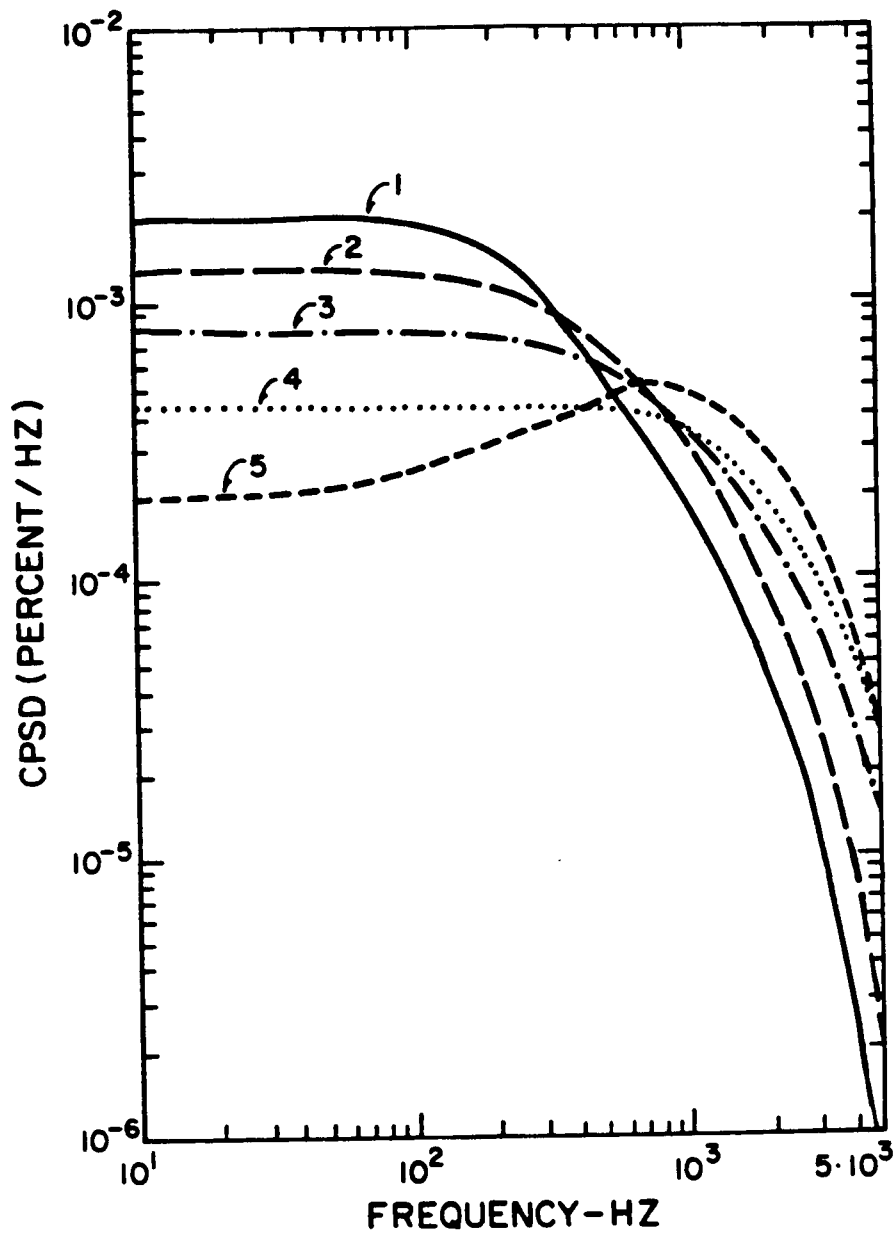


Figure 25. The CPD functions between the axial voltages at $x = 36$ cm and $x = 72$ cm.

$$1. \quad J_x = .10 \frac{A}{cm^2}$$

$$2. \quad J_x = .40 \frac{A}{cm^2}$$

$$3. \quad J_x = .60 \frac{A}{cm^2}$$

$$4. \quad J_x = .86 \frac{A}{cm^2}$$

$$5. \quad J_x = 1.10 \frac{A}{cm^2}$$

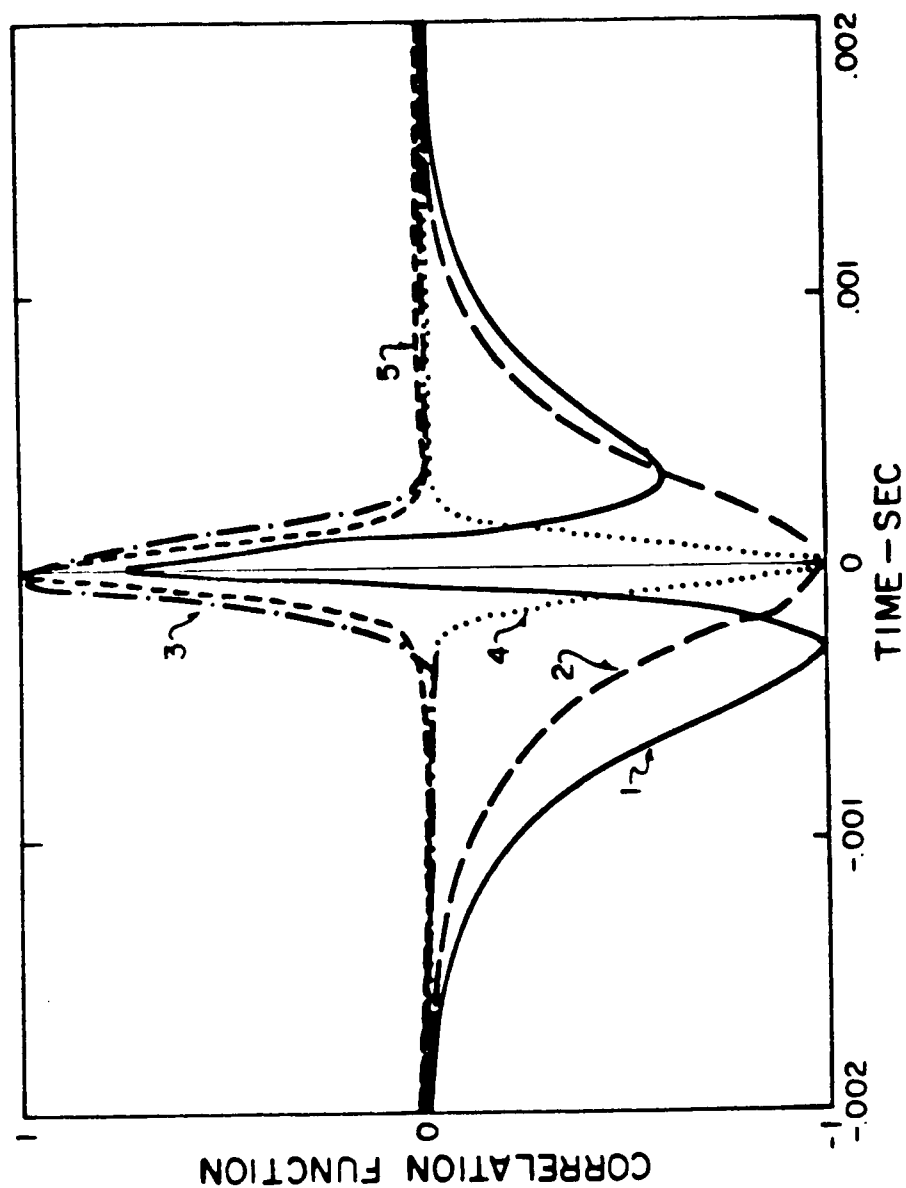


Figure 26. The crosscorrelation functions between the axial currents at $x = 36$ cm and $x = 72$ cm.

- | | | |
|---------------------------------------|--|---------------------------------------|
| 1. $J_x = .10 \frac{A}{2 \text{ cm}}$ | 2. $J_x = .40 \frac{A}{2 \text{ cm}}$ | 3. $J_x = .60 \frac{A}{2 \text{ cm}}$ |
| 4. $J_x = .86 \frac{A}{2 \text{ cm}}$ | 5. $J_x = 1.10 \frac{A}{2 \text{ cm}}$ | |

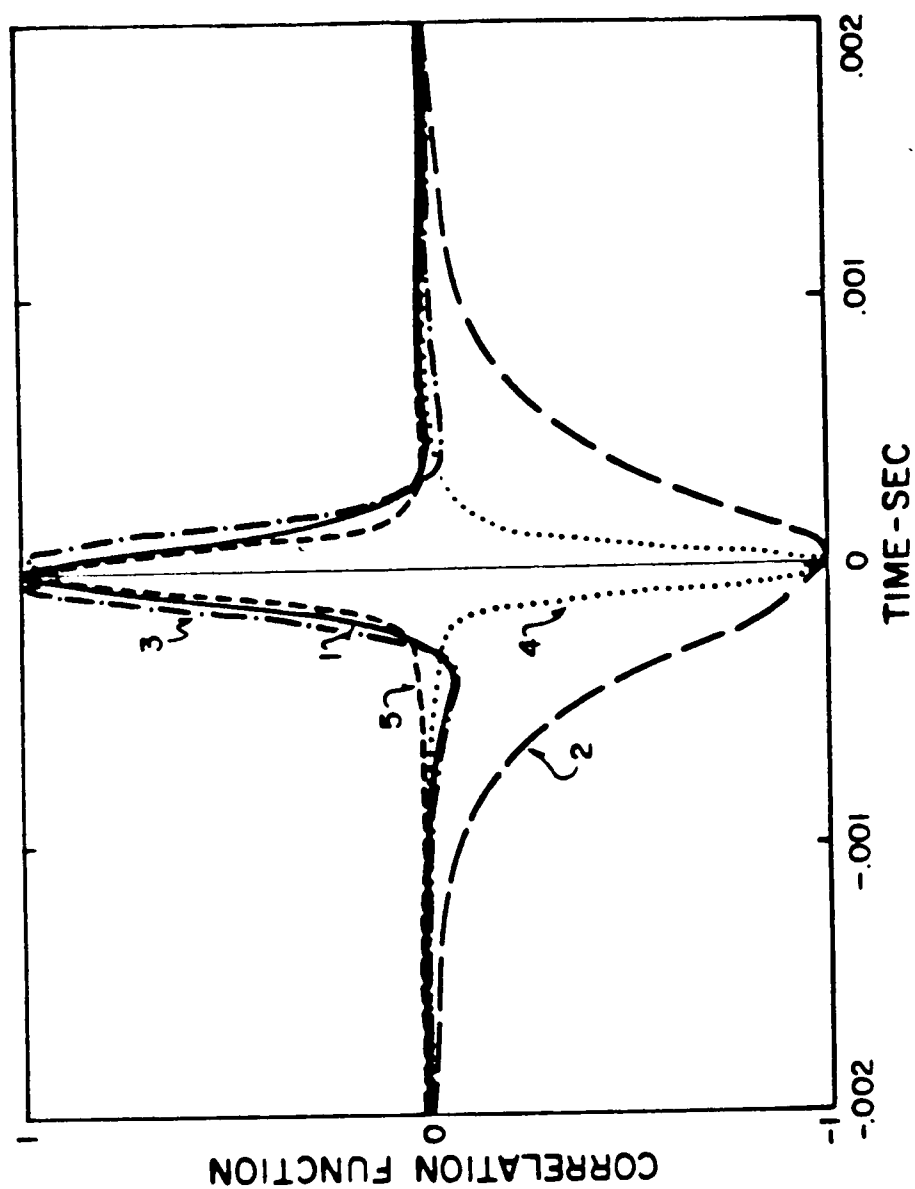


Figure 27. The crosscorrelation functions between the y-currents at $x = 36$ cm and $x = 72$ cm.

- | | | |
|-------------------------------|--------------------------------|-------------------------------|
| 1. $J_x = .10 \frac{A}{cm^2}$ | 2. $J_x = .40 \frac{A}{cm^2}$ | 3. $J_x = .60 \frac{A}{cm^2}$ |
| 4. $J_x = .86 \frac{A}{cm^2}$ | 5. $J_x = 1.10 \frac{A}{cm^2}$ | |

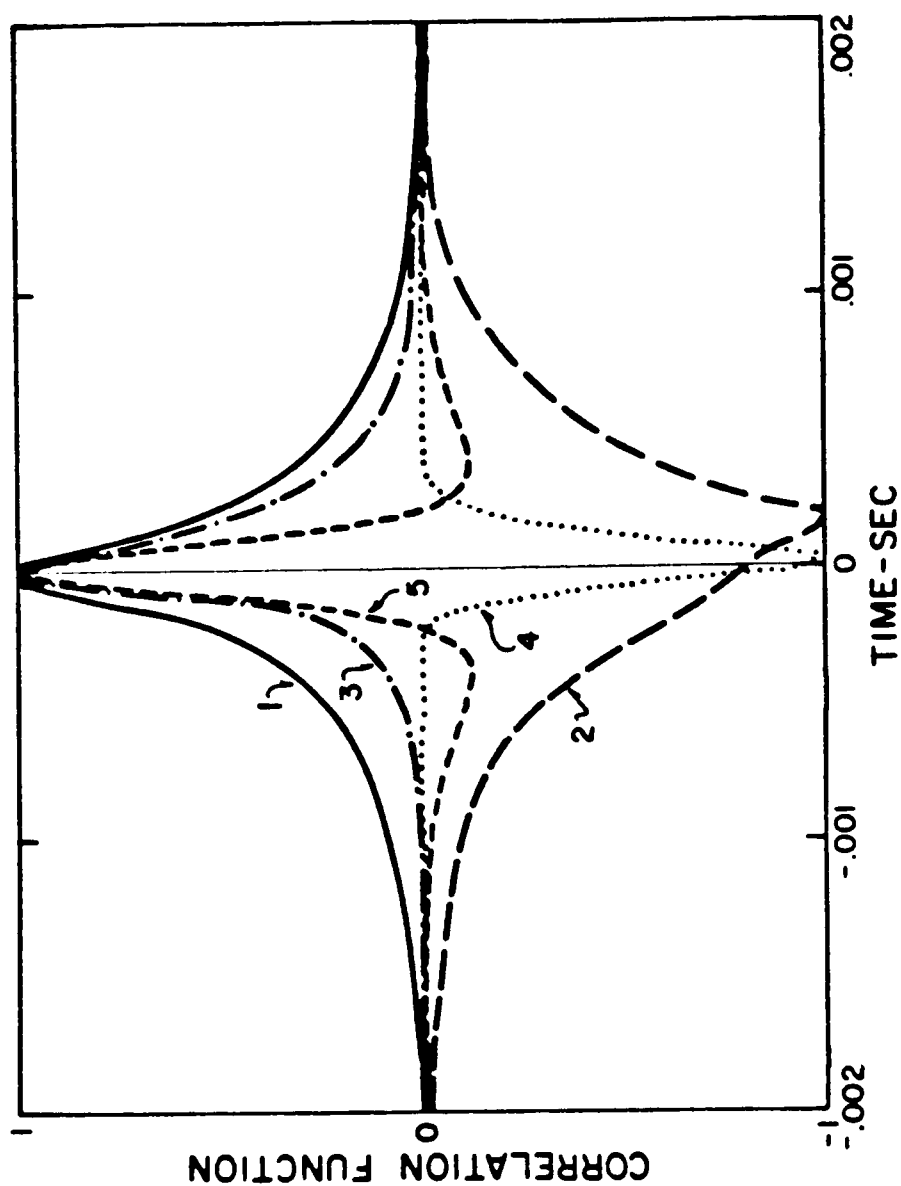


Figure 28. The crosscorrelation functions between the axial voltages at $x = 36$ cm and $x = 72$ cm.

1. $J_x = .10 \frac{A}{2 \text{ cm}}$ 2. $J_x = .40 \frac{A}{2 \text{ cm}}$ 3. $J_x = .60 \frac{A}{2 \text{ cm}}$

4. $J_x = .86 \frac{A}{2 \text{ cm}}$ 5. $J_x = 1.10 \frac{A}{2 \text{ cm}}$

above 1000 Hz, which can be related to the presence of the 5th, 7th, and the higher harmonics of the standing waves. The APSD plots characteristic of the Hall generators have their first break frequencies above 700 Hz, which proves that these generators are susceptible to the high frequency fluctuations.

The CPSD plots between the axial currents at two channel positions ($x = 36$ cm and $x = 72$ cm) are shown in Figure 24. The broad peaks in the range 200 to 2000 Hz may consist of the 3rd, 5th, 7th, and other odd harmonics of the standing waves present in the channel. These peaks, with one exception, disappear in Figure 25 which demonstrates the CPSD plots between the fluctuating axial voltages. The only peak present in this figure corresponds to the Hall generator. Figures 24 and 25 indicate that the fluctuating current concentrates above 200 Hz and does not necessarily follow the fluctuating voltage.

The crosscorrelation functions between the axial currents are shown in Figure 26. The axial currents correlate strongly near zero-delay time proving that the waves present inside the MHD channel are the standing waves. The functions may have at zero time a minimum; this indicates that the waves at the two channel's locations are out-of-phase. Figure 27 demonstrates the crosscorrelation functions between the y-direction currents. The plots in this figure apply to the same axial positions as the plots in Figure 26. They correlate at zero-delay time and show the same phase agreement as the plots of the axial currents. Both figures show agreement between the fluctuating x and y-direction currents of the DCW and Hall generators. The axial currents of the Faraday generator, however, correlate with a strong

oscillation, whereas the y-direction currents of the same generator decay faster without significant oscillations.

Figure 28 demonstrates the crosscorrelation functions between the voltages at different axial positions. The variables have high correlation at near zero-delay time and show the same phase agreement as the plots of the axial currents. The fluctuating voltages, however, have narrow band character, whereas the currents have rather wide band character.

4. Summary

The chapter presents the results of time domain and frequency domain analyses of the MHD plasma model dynamics. The analyses, performed under the steady-state conditions (5.1) and for the MHD generators described in Table 4, include the fluctuating pressure, density, velocity, current and voltage variables.

The analyses revealed that the fluctuations whose frequencies exceed 200 Hz are acoustic in nature, and are caused by the standing waves present in the generator. The 200 Hz frequency limit is imposed by the fundamental frequency of the standing waves. The occurrence of the standing waves is proved by the character of the crosscorrelation and power spectral density functions. The plasma variables correlate strongly from one axial position to another with minimum or maximum near zero-delay time; this indicates that the waves are standing waves. The power spectral density functions exhibit distinct peaks centered around the fundamental frequency and its odd harmonics.

The analyses indicate that Faraday and DCW generators are favorable for the lower frequency fluctuations, whereas Hall generators

develop fluctuations mainly in the 700 Hz to 2000 Hz range. The fluctuating x- and y-direction currents of DCW and Hall generators have similar correlation and power spectral density functions; these functions are different for the fluctuating x- and y-direction currents of Faraday generators. The analysis indicates that the fluctuating currents, in general, have wide band character, whereas the fluctuating voltages have rather narrow band character.

CHAPTER 6

PARAMETRIC STUDY OF THE MHD PLASMA DYNAMICS

The results to be discussed in this chapter apply to DCW generators, since such generators will be preferred for industrial energy production. The parametric analysis is presented for the data set (5.1) with some of the data slightly changed, for the axial current equal to $.40 \text{ A/cm}^2$. These small changes in the data allow us to investigate the sensitivity of the plasma with respect to its chosen parameters. Similar analysis can be performed for other types of MHD generators.

Figure 29 demonstrates the APSD plots of the pressure variable, resulting from the input noise sources having autocorrelation functions in the form $e^{-|p|\tau}$, where the parameter 'p' takes on different values. The figure shows that the pressure fluctuations are concentrated in the 10 to 200 Hz region for small 'p', i.e., equal to 1000; 1000 is a small number compared to system's eigenvalues. When the parameter increases, the range of fluctuations increases also. The values of 'p' larger than 30,000, however, do not cause any visible changes in the plots. One may conclude, therefore, that the parameter 'p' equal to 30,000 gives the white noise character to the input noise sources, for the MHD plasma. The character of the input noise sources depends strongly on variations in the flow rates of the components input to the combustor, and on the combustion process itself. If the flow is nonuniform or the combustion is incomplete due to poor mixing of the components, then the input noise sources do not have the white noise

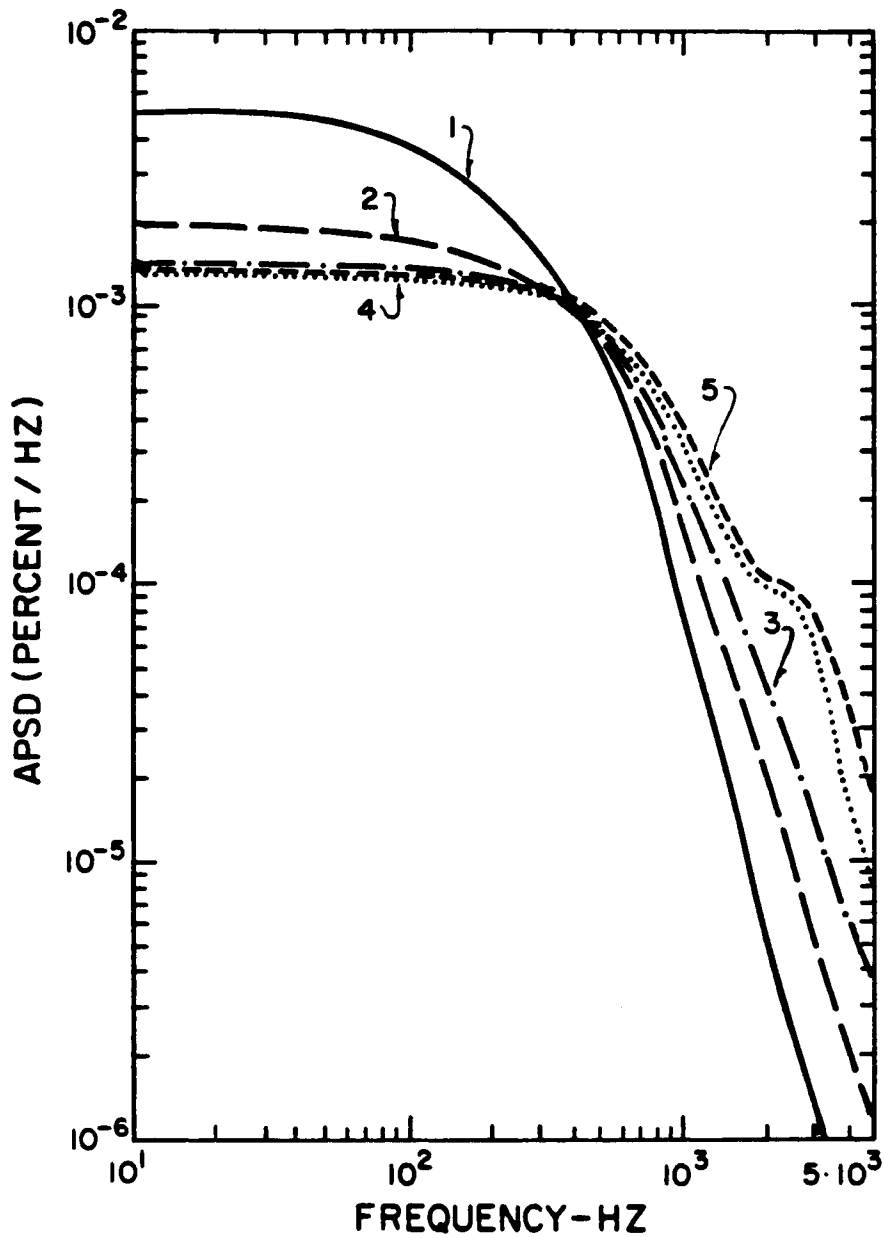


Figure 29. The APSD functions of the pressure variable resulting from various noise sources $\exp(-|p|\tau)$; $x = 106$ cm, $J_x = .40$ A/cm².

1. $p = 1000$

2. $p = 5000$

3. $p = 10000$

4. $p = 20000$

5. $p = 30000$

character. In such a case fluctuations concentrate in the low frequency region, i.e., up to about 200 Hz. The spectral density functions of the generator's variables may exhibit some peaks in this frequency range, these peaks, however, will not be acoustic in origin, since 230 Hz constitutes the fundamental frequency of the standing waves present in the generators under consideration.

Figures 30-35 demonstrate the spectral and time domain functions of the generator's variables subjected to magnetic field and Hall parameter perturbations. (Since Hall parameter characterizes plasma in the magnetic field, and it is linearly proportional to this field, the analysis involves simultaneous perturbations in these two parameters). Figure 30 demonstrates the APSD functions of the plasma density characteristic of the MHD plasma subjected to different magnetic field strengths and Hall parameters. The plots indicate that the density fluctuations concentrate in the range up to 700 Hz for the relatively small field strengths and Hall parameters. Stronger fields and larger Hall parameters extend the range of fluctuations above 2000 Hz. One of the plots ($\Omega = 1.60$) exhibits a clear peak around 2000 Hz. This wide peak can result from the presence of the 7th, 9th, and 11th harmonics of the standing waves in the channel. Figure 31 shows the autocorrelation functions of the density variable, characteristic of the plasma subjected to different magnetic field strengths and Hall parameters. The figure confirms that the density fluctuations have fast changing character for the stronger fields and larger Hall parameters.

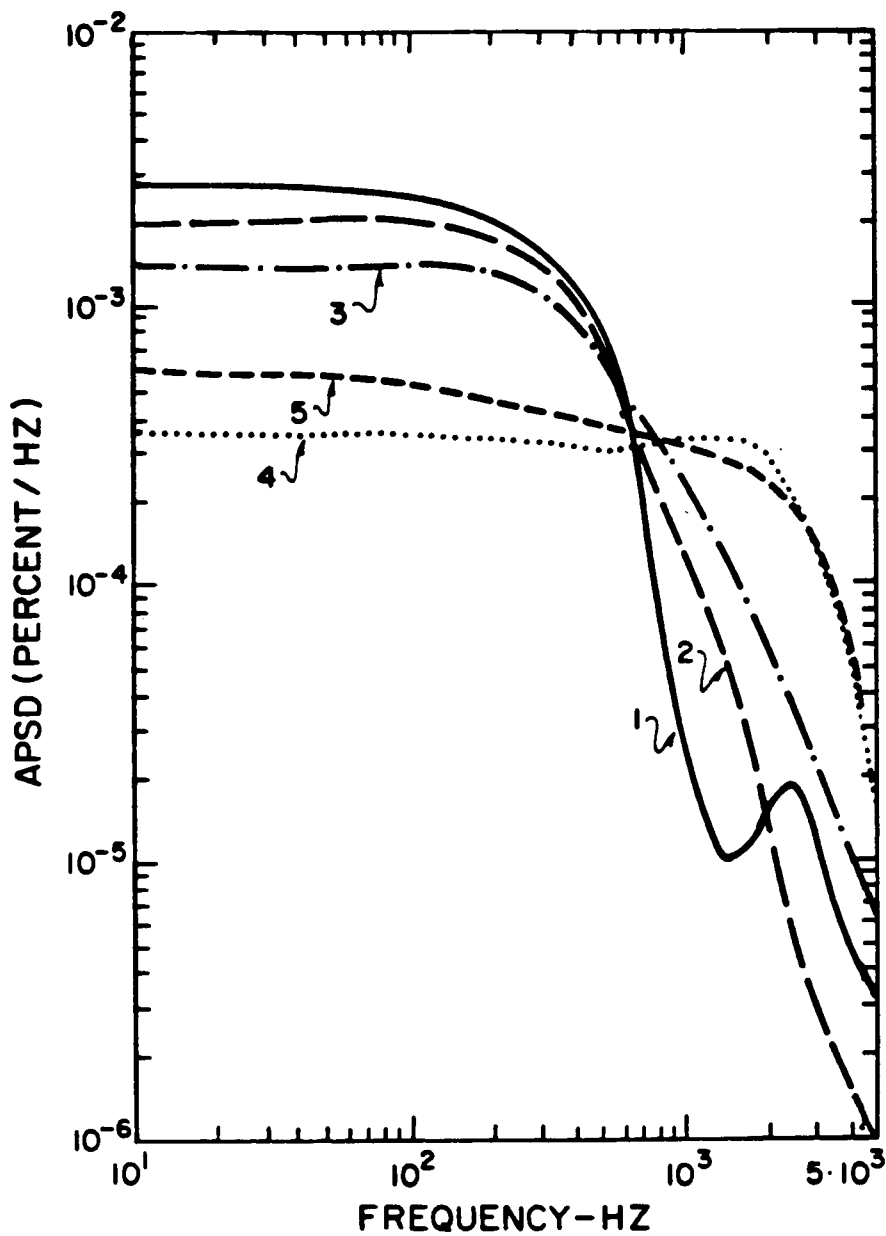


Figure 30. The APSD functions of the density variable of various Hall parameters; $x = 106 \text{ cm}$, $J_x = .40 \text{ A/cm}^2$.

1. $\Omega = 1.60$
3. $\Omega = 2.00$
5. $\Omega = 2.40$

2. $\Omega = 1.80$
4. $\Omega = 2.20$

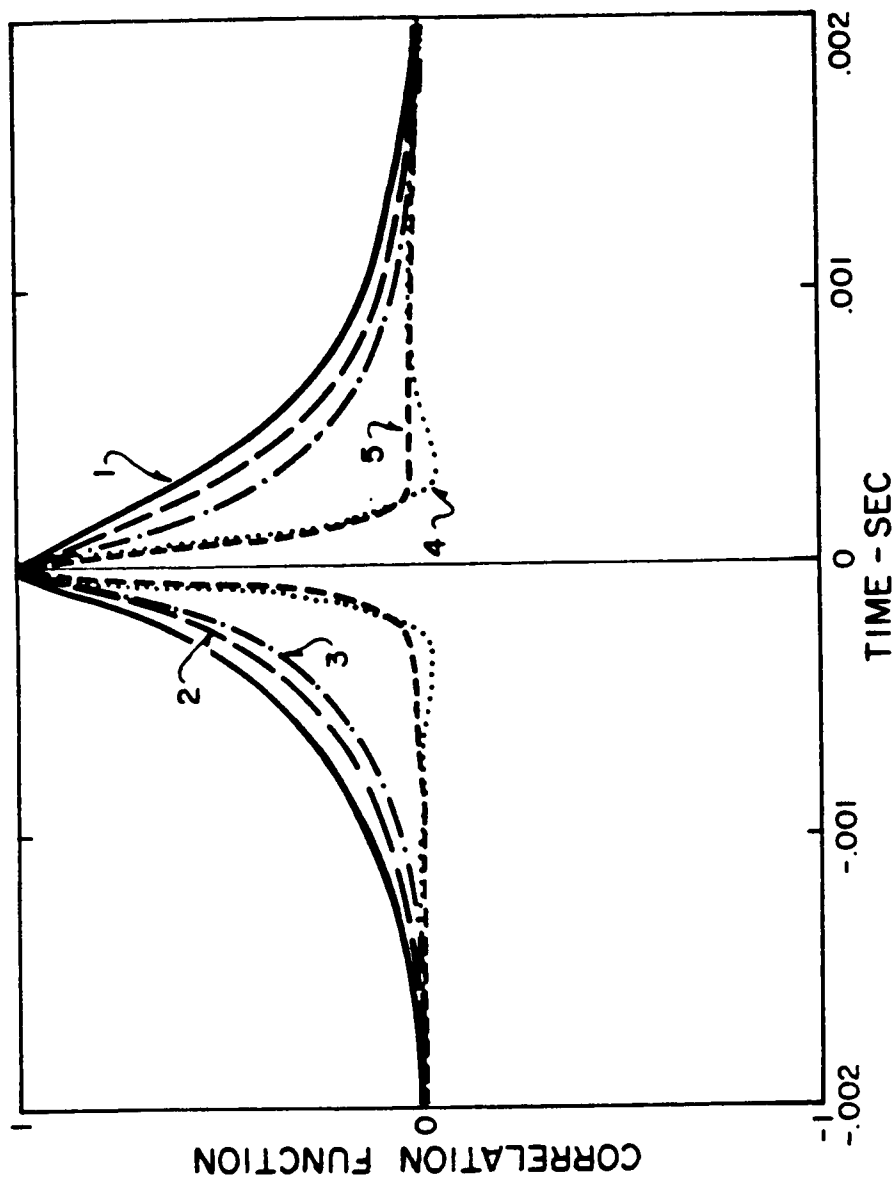


Figure 31. The autocorrelation functions of the density variable of various Hall parameters; $x = 106 \text{ cm}$, $J_x = 0.40 \text{ A/cm}^2$.

- | | |
|--------------------|--------------------|
| 1. $\Omega = 1.60$ | 2. $\Omega = 1.80$ |
| 3. $\Omega = 2.00$ | 4. $\Omega = 2.20$ |
| 5. $\Omega = 2.40$ | |

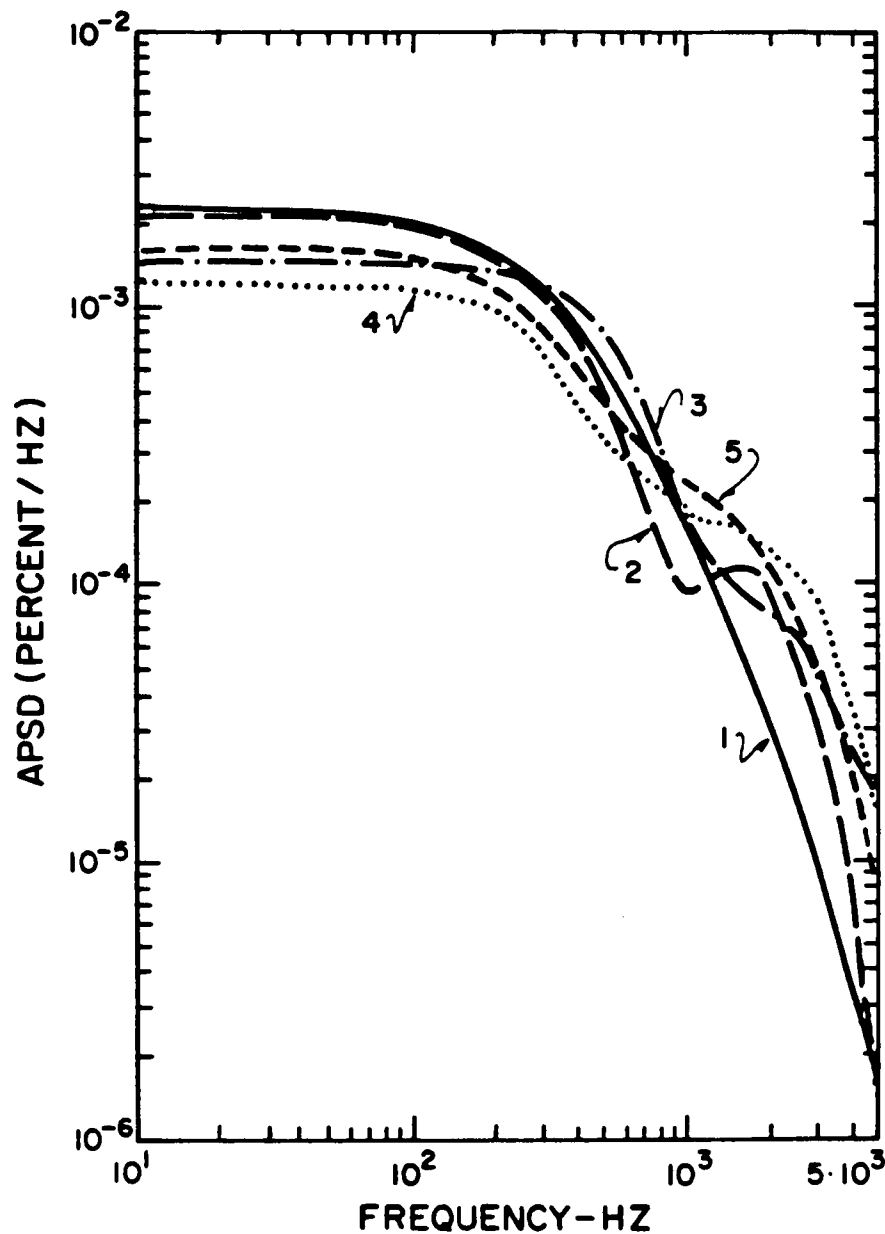


Figure 32. The APSD functions of the velocity variable of various Hall parameters; $x = 106$ cm, $J_x = .40$ A/cm².

1. $\Omega = 1.60$

2. $\Omega = 1.80$

3. $\Omega = 2.00$

4. $\Omega = 2.20$

5. $\Omega = 2.40$

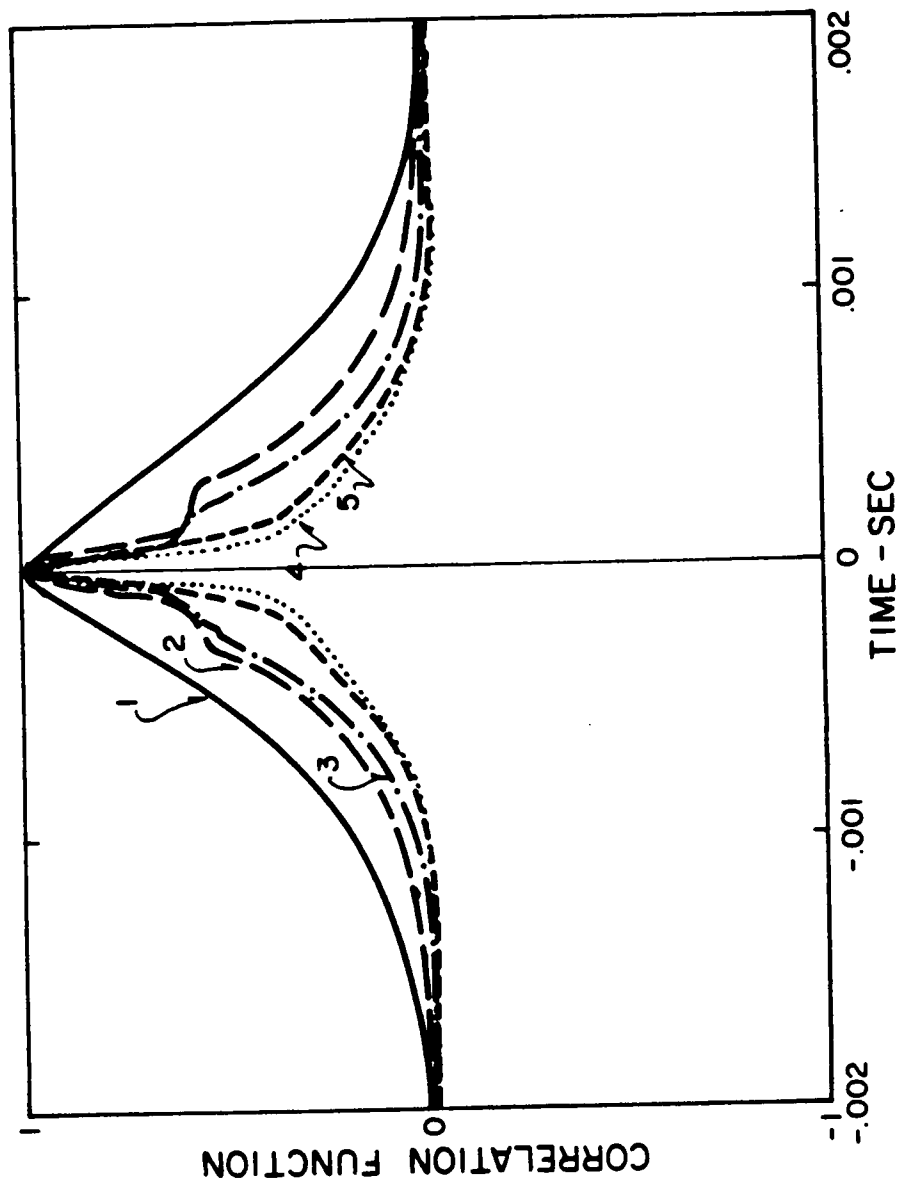


Figure 33. The autocorrelation functions of the velocity variable of various Hall parameters; $x = 106 \text{ cm}$, $J_x = .40 \text{ A/cm}^2$.

- | | |
|--------------------|--------------------|
| 1. $\Omega = 1.60$ | 2. $\Omega = 1.80$ |
| 3. $\Omega = 2.00$ | 4. $\Omega = 2.20$ |
| 5. $\Omega = 2.40$ | |

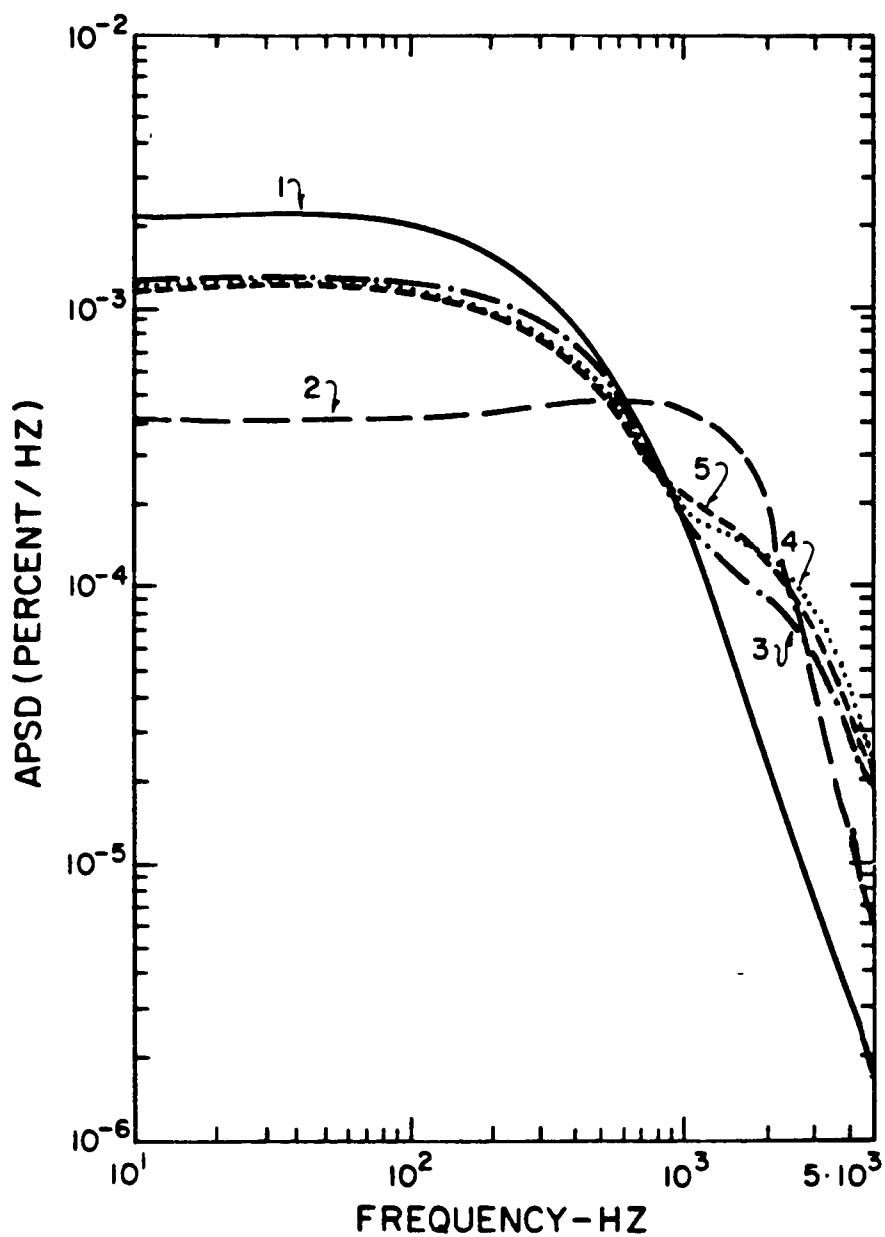


Figure 34. The APSD functions of the pressure variable of various Hall parameters; $x = 106$ cm, $J_x = .40$ A/cm².

1. $\Omega = 1.60$

2. $\Omega = 1.80$

3. $\Omega = 2.00$

4. $\Omega = 2.20$

5. $\Omega = 2.40$

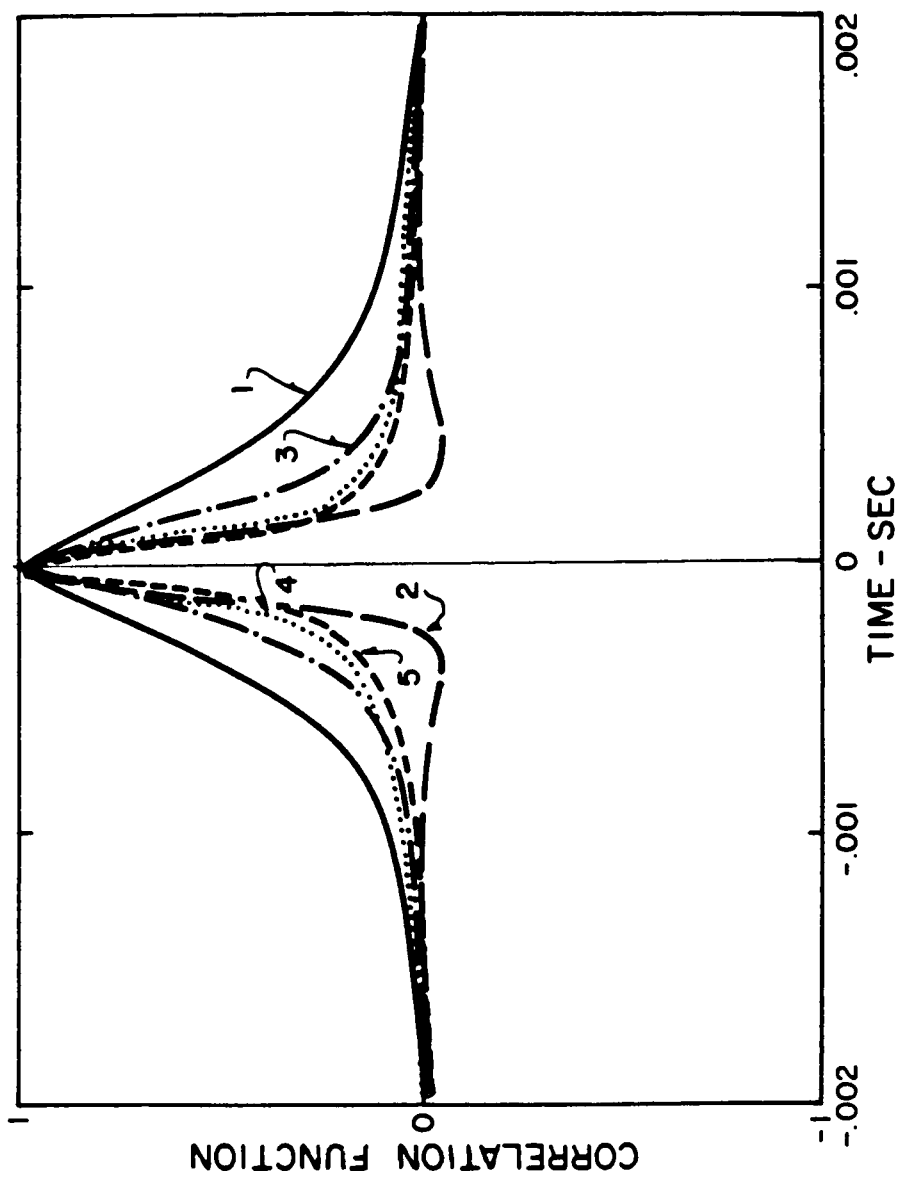


Figure 35. The autocorrelation functions of the pressure variable of various Hall parameters; $x = 106 \text{ cm}$, $J_x = .40 \text{ A/cm}^2$.

- | | |
|--------------------|--------------------|
| 1. $\Omega = 1.60$ | 2. $\Omega = 1.80$ |
| 3. $\Omega = 2.00$ | 4. $\Omega = 2.20$ |
| 5. $\Omega = 2.40$ | |

Figures 32 and 33 demonstrate how the frequency and time domain plots of the fluctuating velocity vary, when the magnetic field and Hall parameter increase. The APSD functions of the variable do not exhibit any significant changes due to the changes in the magnetic properties; the range of the fluctuations increases only slightly. Some of the plots exhibit small peaks around 700 Hz and 1100 Hz. These peaks may result from the presence of the 5th and 7th harmonics of the standing waves in the channel.

Figures 34 and 35 illustrate how the frequency and time domain plots of the fluctuating pressure vary, when the magnetic field and Hall parameter increase. The figures show an increase in the range of fluctuations; this increase, however, is not very large.

Figures 36-38 demonstrate the APSD functions of the generator's variables subjected to changes in plasma specific heat ratio. Specific heat ratio is a strong function of temperature; if the plasma temperature increases, this ratio will decrease. The specific heat ratio may vary also with the plasma composition. An increase in the mass flow rate of air or nitrogen will increase its value, whereas an increase in the mass flow rate of combustion products, like carbon dioxide or carbon monoxide, will decrease its value [21]. The figures indicate that the range of fluctuations extends up to 2000 Hz for small values of the plasma specific heat ratios. As the ratio increases, due to changes in the plasma temperature or composition, the fluctuations concentrate in the region up to 200 Hz. The APSD functions of the pressure variable exhibit clear peaks around 700 and 2000 Hz. Similar peaks are observed in the APSD plots of the velocity variable. The

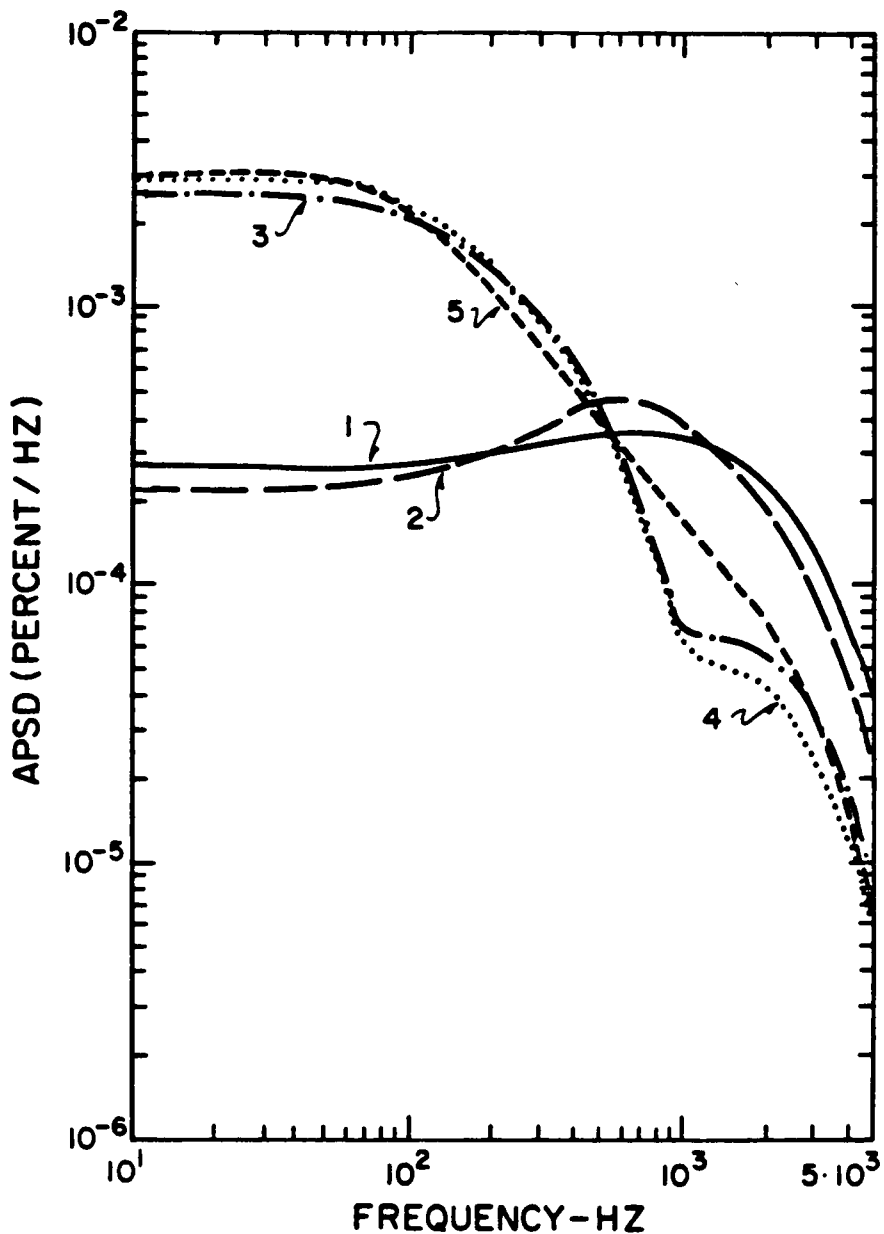


Figure 36. The APSD functions of the pressure variable of various specific heat ratios; $x = 106$ cm, $J_x = .40$ A/cm².

1. $\gamma = 1.10$
3. $\gamma = 1.40$
5. $\gamma = 1.60$

2. $\gamma = 1.30$
4. $\gamma = 1.50$

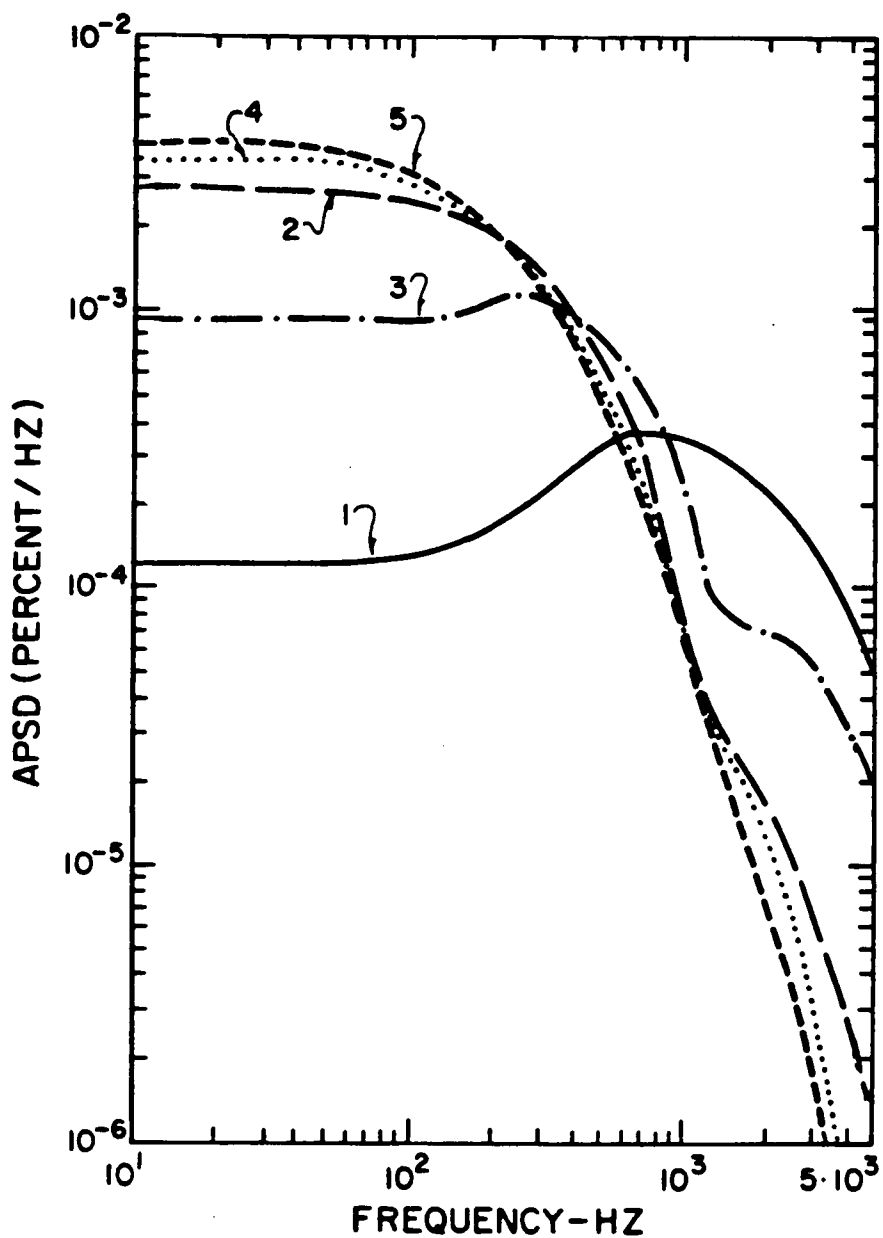


Figure 37. The APSD functions of the velocity variable of various specific heat ratios; $x = 106$ cm, $J_x = .40$ A/cm².

1. $\gamma = 1.10$

2. $\gamma = 1.30$

3. $\gamma = 1.40$

4. $\gamma = 1.50$

5. $\gamma = 1.60$

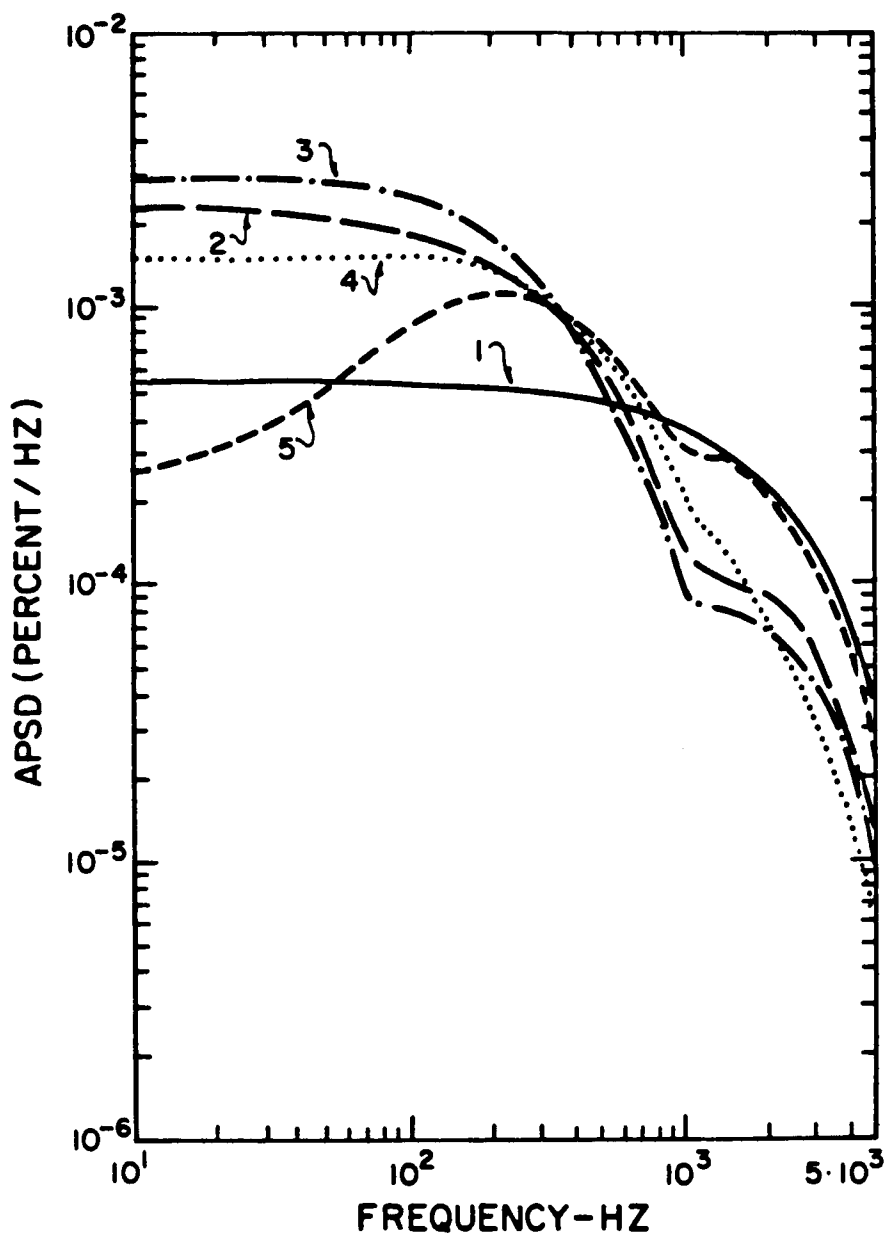


Figure 38. The APSD functions of the density variable of various specific heat ratios; $x = 106$ cm, $J_x = .40$ A/cm².

1. $\gamma = 1.10$
3. $\gamma = 1.40$
5. $\gamma = 1.60$

2. $\gamma = 1.30$
4. $\gamma = 1.50$

spectral plots of the density variable may also exhibit the peak around 200 Hz. The occurrence of these peaks can be caused by the harmonics of the standing waves, i.e., the 1st harmonic corresponding to 230 Hz, 3rd harmonic corresponding to 690 Hz, 5th harmonic corresponding to 1150 Hz, etc.

Figures 39-44 demonstrate how the changes in the inlet plasma velocity, pressure, and mass flow rate affect the fluctuating variables. Figure 39 shows the CPSD functions between the velocity variables at different channel locations, characteristic of various input plasma velocities. When the input velocity decreases from 1500 m/sec to 1200 m/sec, the low frequency fluctuations are gradually eliminated from the variable, and the range of fluctuations concentrates around 700 Hz forming a wide peak. This peak, resulting from the presence of standing waves, includes the 3rd and 5th harmonics corresponding to 690 Hz and 1150 Hz. The crosscorrelation functions between the velocity variables at different channel locations for different inlet velocity conditions are shown in Figure 40. The plots indicate an in-phase or an out-of-phase behavior, and have their maximum or minimum near zero-delay time.

Figures 41 and 42 demonstrate the APSD and autocorrelation functions of the pressure variable, characteristic of various inlet pressures. The figures indicate that fluctuations may concentrate around 1000 Hz for some values of the input pressures; the other values may eliminate the high frequency components from the variable.

Figures 43 and 44 demonstrate the APSD and autocorrelation functions of the density variable, characteristic of various inlet mass

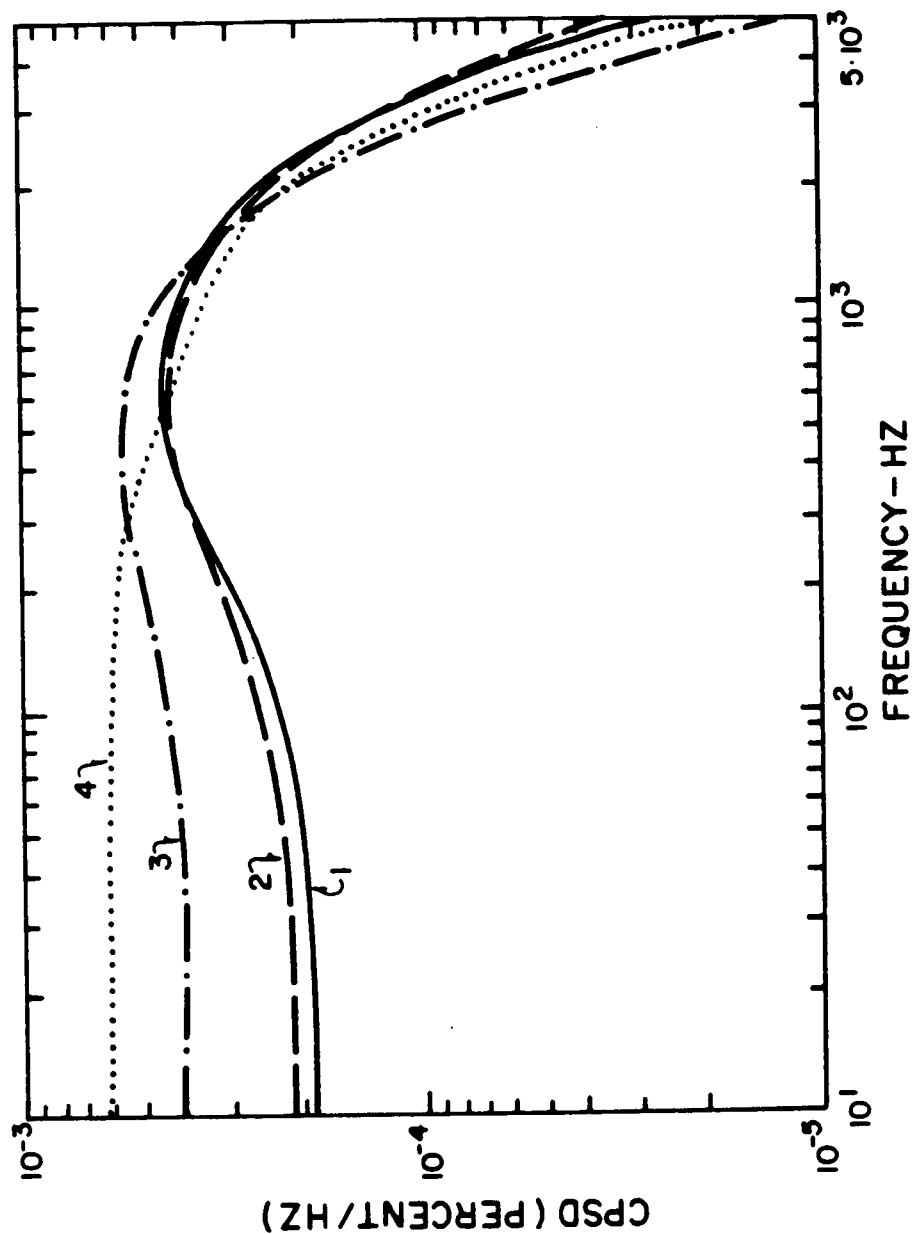


Figure 39. The CPSD functions between the velocity variables at $x = 36$ cm and $x = 106$ cm, for various input velocities.

1. $U_0 = 1200 \frac{\text{m}}{\text{s}}$ 2. $U_0 = 1300 \frac{\text{m}}{\text{s}}$

3. $U_0 = 1400 \frac{\text{m}}{\text{s}}$ 4. $U_0 = 1500 \frac{\text{m}}{\text{s}}$

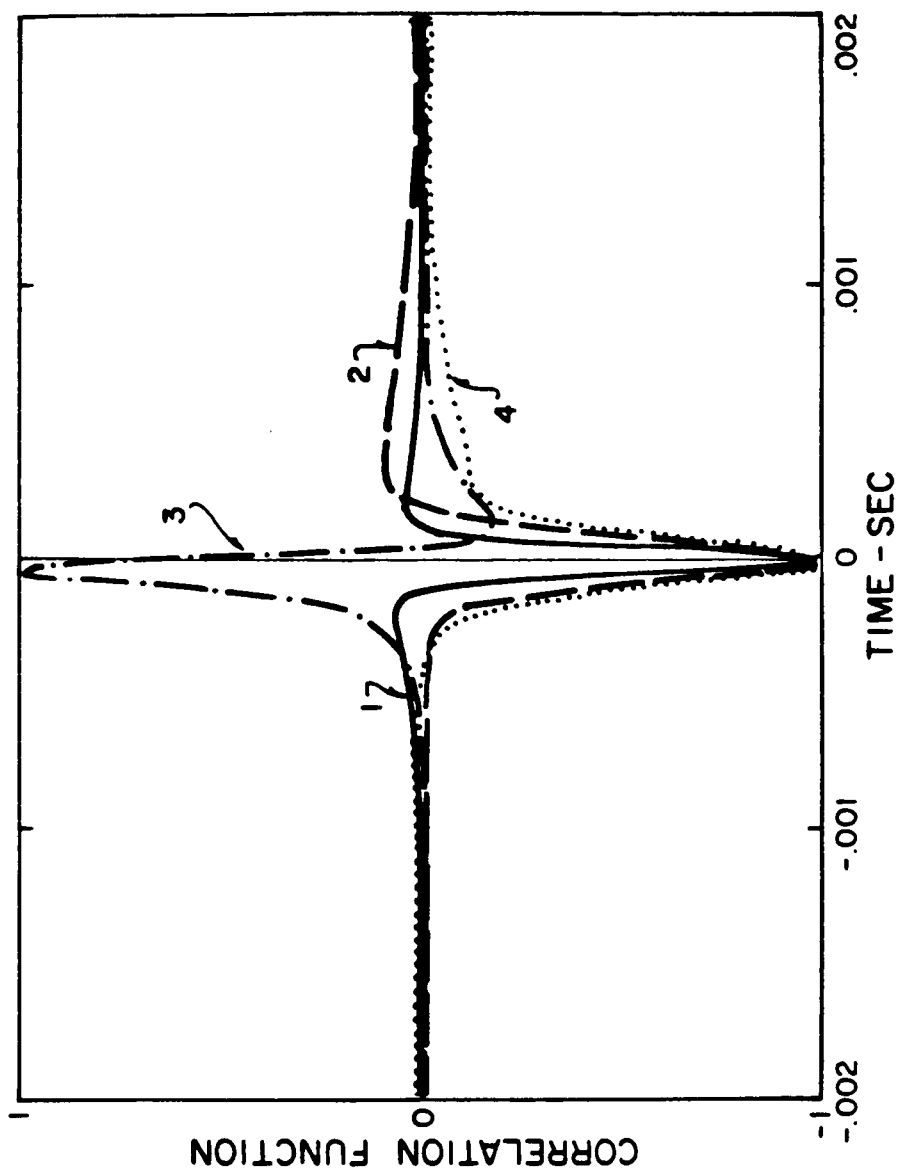


Figure 40. The crosscorrelation functions between the velocity variables at $x = 36$ cm and $x = 106$ cm, for various input velocities.

- | | |
|-----------------------------|-----------------------------|
| 1. $U_0 = 1200 \frac{m}{s}$ | 2. $U_0 = 1300 \frac{m}{s}$ |
| 3. $U_0 = 1400 \frac{m}{s}$ | 4. $U_0 = 1500 \frac{m}{s}$ |

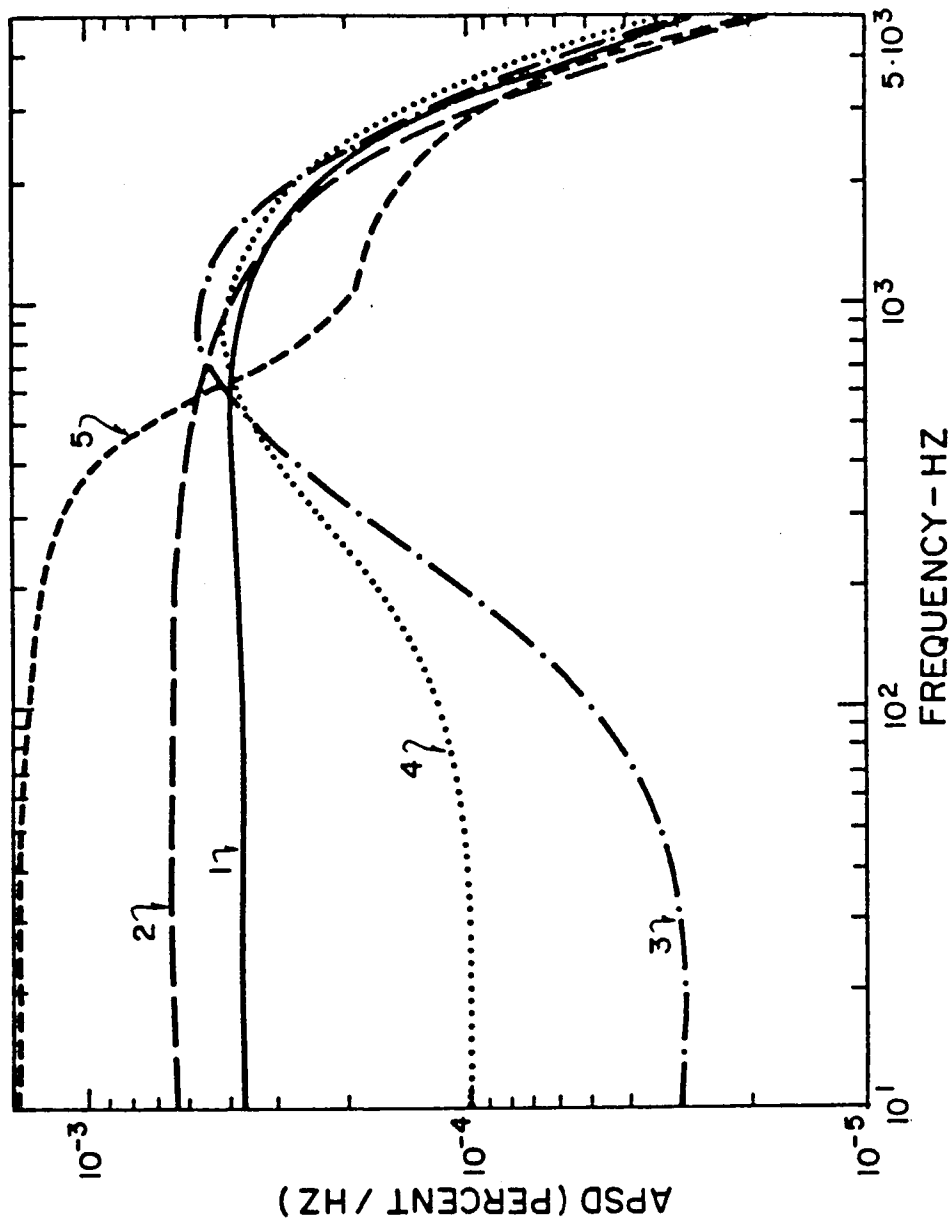


Figure 41. The APSD functions of the pressure variable at $x = 106$ cm, for various input pressures.

- | | | |
|--------------------|--------------------|--------------------|
| 1. $p_o = .40$ atm | 2. $p_o = .50$ atm | 3. $p_o = .60$ atm |
| 4. $p_o = .80$ atm | 5. $p_o = .90$ atm | |

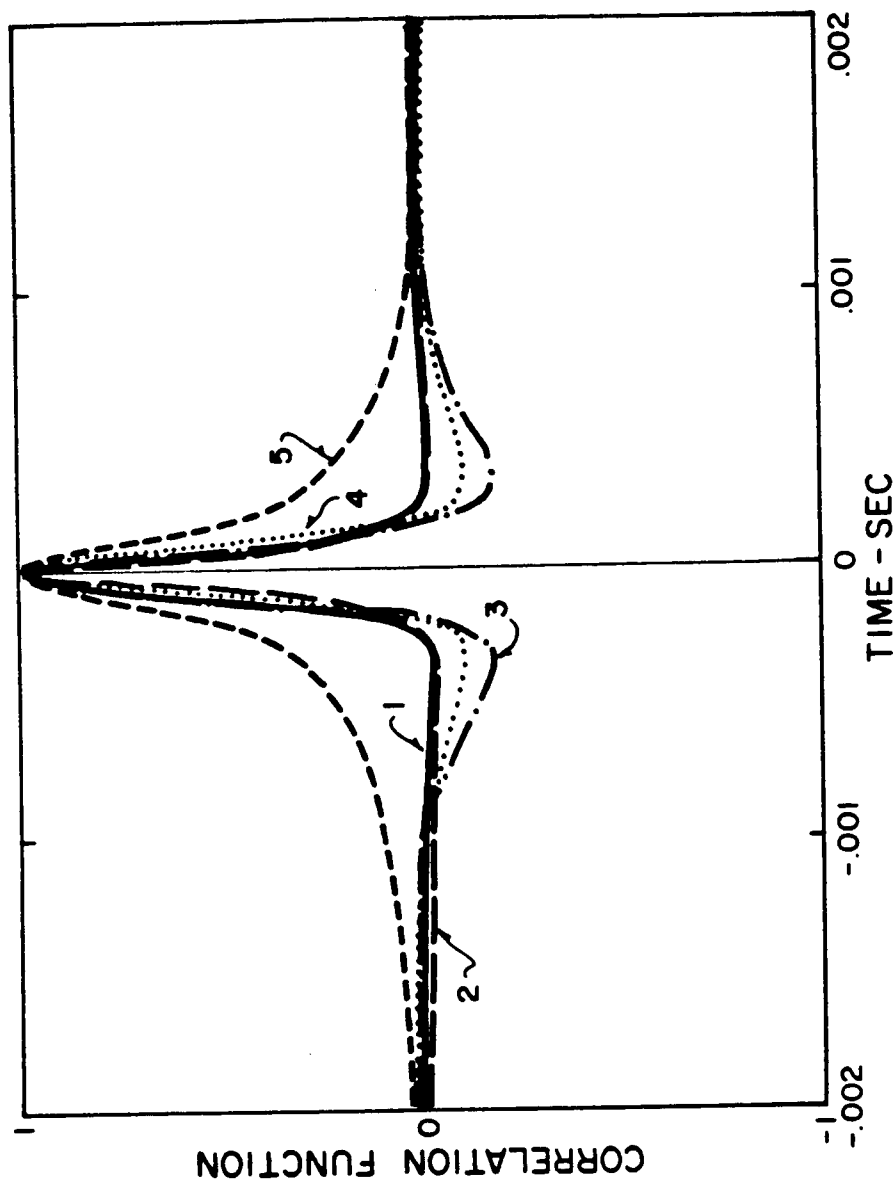


Figure 42. The autocorrelation functions of the pressure variable at $x = 106$ cm, for various input pressures.

- | | |
|--------------------|--------------------|
| 1. $p_o = .40$ atm | 2. $p_o = .50$ atm |
| 3. $p_o = .60$ atm | 4. $p_o = .80$ atm |
| 5. $p_o = .90$ atm | |

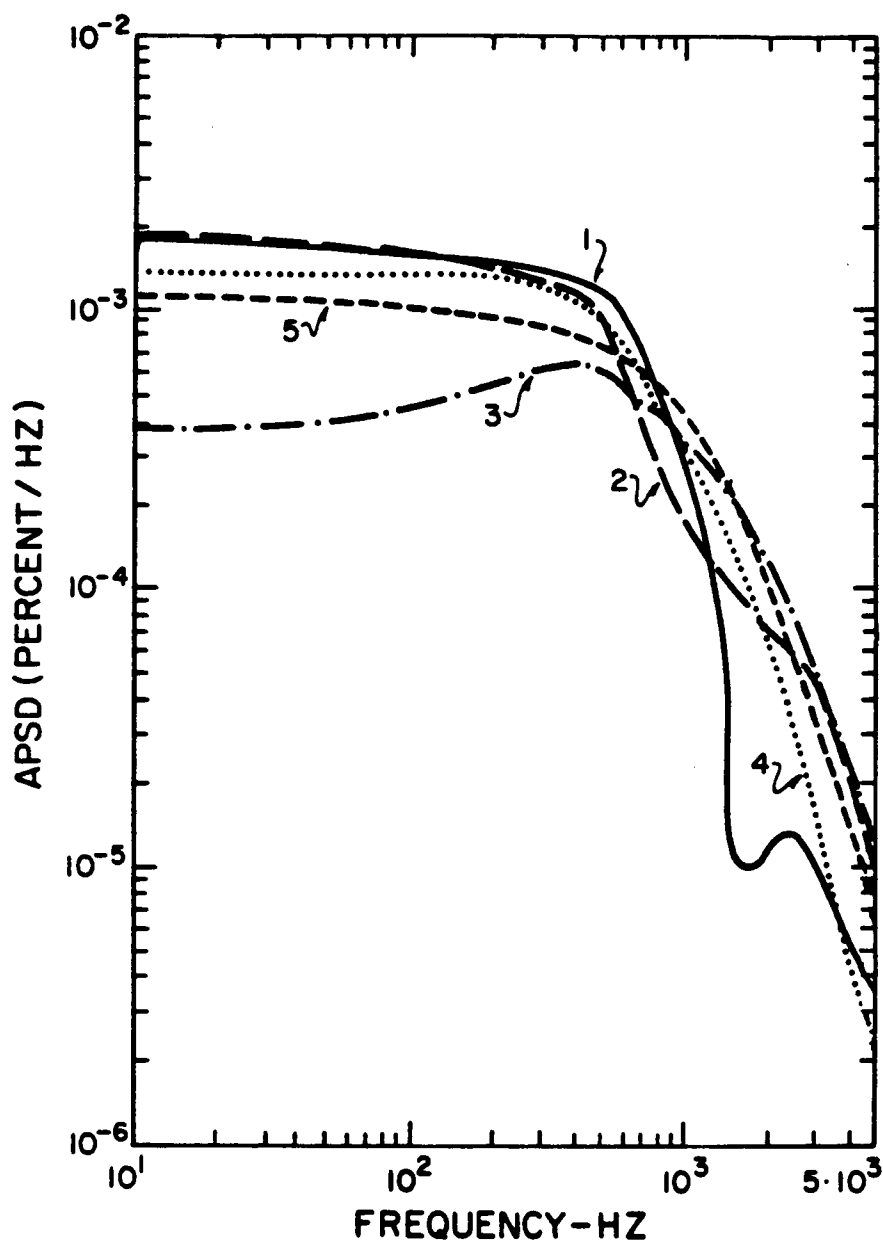


Figure 43. The APSD functions of the density variable at $x = 106$ cm, for various mass flow rates.

1. $w = .60 \frac{\text{Kg}}{\text{s}}$

2. $w = .80 \frac{\text{Kg}}{\text{s}}$

3. $w = .90 \frac{\text{Kg}}{\text{s}}$

4. $w = 1.00 \frac{\text{Kg}}{\text{s}}$

5. $w = 1.20 \frac{\text{Kg}}{\text{s}}$

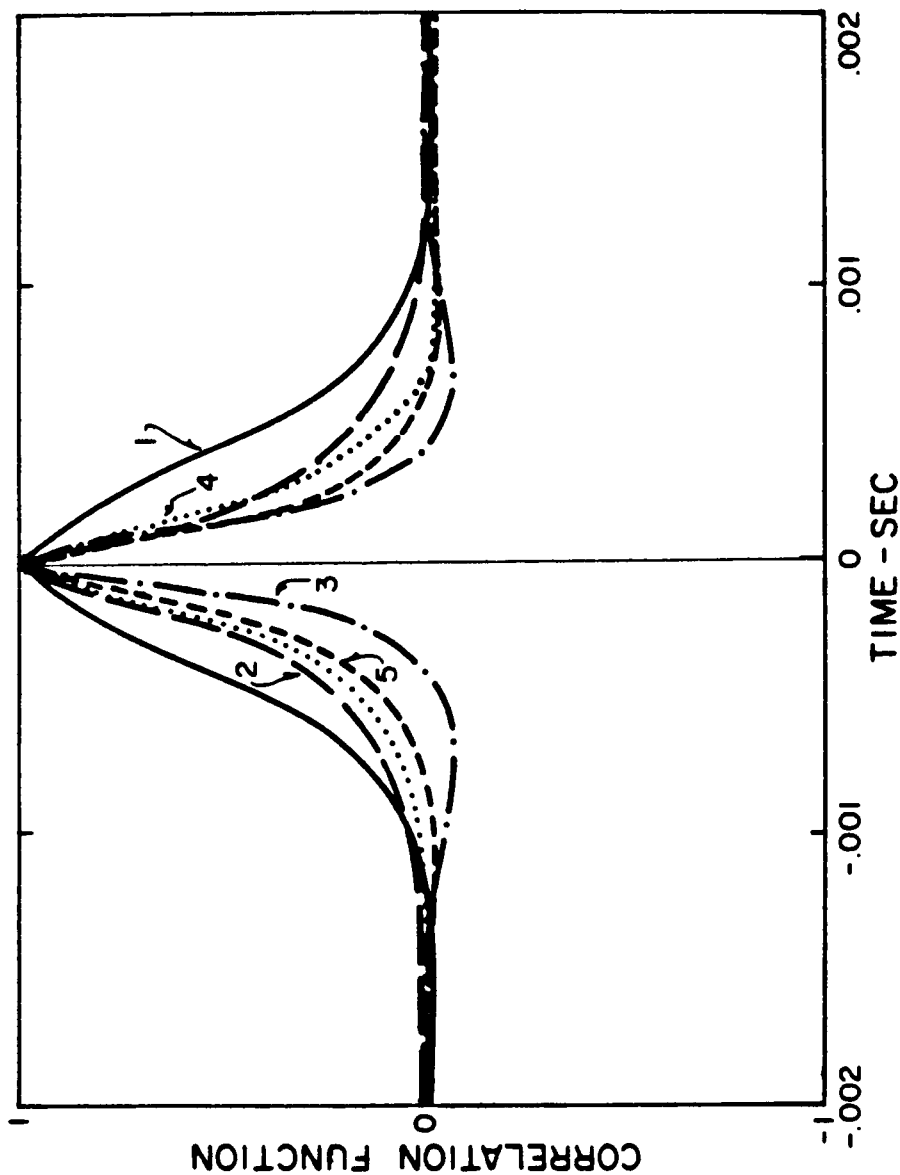


Figure 44. The autocorrelation functions of the density variable at $x = 106$ cm, for various mass flow rates.

- | | |
|--|--|
| 1. $w = .60 \frac{\text{Kg}}{\text{sec}}$ | 2. $w = .80 \frac{\text{Kg}}{\text{sec}}$ |
| 3. $w = .90 \frac{\text{Kg}}{\text{sec}}$ | 4. $w = 1.00 \frac{\text{Kg}}{\text{sec}}$ |
| 5. $w = 1.20 \frac{\text{Kg}}{\text{sec}}$ | |

flow rates. Most of the fluctuations are in the range up to 700 Hz, i.e., up to the 3rd harmonic, according to the figures. Changes in the inlet mass flow rates do not significantly affect the density variable.

The results discussed in this chapter apply to the plasma variables subjected to perturbations in the chosen parameters. The study indicates that the APSD or CPSD functions of the variables often exhibit large peaks in the regions of the odd harmonics of the fundamental frequency 230 Hz. When the plasma parameters change, some of the peaks are shifted into the regions of higher or lower harmonics, the other peaks are eliminated.

The results indicate that the changes in the input noise sources, plasma magnetic parameters, plasma specific heat ratio, and inlet plasma velocity, pressure and mass flow rate affect the fluctuating variables. The results do not cover perturbations in all plasma parameters, nor discuss how these perturbations affect all variables along the MHD channel. The analysis presented here constitutes a sample of parametric study, and can be expanded according to the specific interests.

CHAPTER 7

EMPIRICAL STOCHASTIC TIME-SERIES MODELING OF THE MHD PLASMA DYNAMICS

1. Estimation of Time and Frequency Domain Signatures

Random fluctuations of the process variables carry dynamic information about the process parameters and changes in these parameters, distribution of noise sources, sensor response characteristics, and stability information. Modeling based on operational data can be used to extract desired dynamic information; this information may not be always available using deterministic modeling methods. Empirical models, as opposed to physical models, are those that are formulated by appropriate data fitting to observed performance without any attempt to evoke the laws of physics. Empirical models are computationally efficient, however, they have to agree with a structure of the modeled system.

The MHD plasma variables are represented best by a multivariate autoregressive (MAR) time series model. This is a general feedback model of several jointly stationary random processes. The choice of the MAR model in this study is justified by presence of a feedback in the physical model of the MHD plasma. The MAR model of m random variables is given by

$$\underline{Y}(t) = \sum_{i=1}^n A_i \underline{Y}(t-i) + \underline{V}(t), \quad (7.1)$$

where t denotes the t -th sample,

$\underline{Y}(t) = \{y_1(t), \dots, y_m(t)\}$ is a signal vector,

$\underline{V}(t) = \{v_1(t), \dots, v_m(t)\}$ is a white noise sequence with
finite variance,

$A_i, i=1, \dots, n$, are $m \times m$ parameter matrices.

Usual assumptions about the MAR model are:

$$\begin{aligned} E[\underline{V}(t)] &= 0 \\ E[\underline{V}(j)\underline{V}(t)^T] &= Q\delta_{jk}, \end{aligned} \quad (7.2)$$

where $E[\cdot]$ indicates expected value, and Q is the $m \times m$ noise covariance matrix. These assumptions about first and second moments of the noise vector are correct for the white noise sequence.

After assuming a model configuration, the empirical modeling problem is stated as follows:

Given the set of observations $\underline{Y}(1), \dots, \underline{Y}(N)$,

1. Estimate the parameter set $A_i, i=1, \dots, n$.
2. Select the optimal model order.
3. Derive the spectral domain quantities for interpretation of the relationship between the signals.

Multiplying (7.1) by $\underline{Y}^T(t-k)$ and taking mean of the product gives

$$\begin{aligned} E[\underline{Y}(t) \underline{Y}^T(t-k)] &= \sum_{i=1}^n A_i E[\underline{Y}(t-i) \underline{Y}^T(t-k)] \\ k &> 0. \end{aligned} \quad (7.3)$$

If the correlation matrix is defined by

$$\begin{aligned} C(k) &= E[\underline{Y}(t) \underline{Y}^T(t-k)] \\ k &> 0, \end{aligned} \quad (7.4)$$

then equation (7.3) simplifies to

$$C(k) = \sum_{i=1}^n A_i C(k-i)$$

$$k = 1, \dots, n. \quad (7.5)$$

Equations (7.5), called the Yule-Walker equations, can be solved for the model parameter matrices $A_1, \dots, A_n$.

The noise covariance matrix is obtained from

$$Q = C(0) - \sum_{i=1}^n A_i C(i)^T. \quad (7.6)$$

The selection of the optimal model order is based on the function called Akaike information criterion (AIC). For autoregressive modeling problems AIC is given by [15]

$$AIC(n) = N \ln |Q_n| + 2m^2n, \quad (7.7)$$

where N is the sample size, and $|Q_n|$ is the determinant of the empirical noise covariance for the model of order n . The signals are assumed to have Gaussian distribution in deriving the AIC. The AIC is estimated for each order, and the n -th value corresponding to the minimum AIC indicates the optimal model order.

Frequency domain signatures such as APSD and CPSD functions, coherence, and signal contribution ratio (SCR) functions are derived from the parameters of the optimal order. The $m \times m$ spectral density matrix is defined by

$$S_{yy}(f) = \begin{bmatrix} S_{11}(f) & S_{12}(f) & \dots & S_{1m}(f) \\ S_{21}(f) & S_{22}(f) & \dots & S_{2m}(f) \\ \vdots & \vdots & \ddots & \vdots \\ S_{m1}(f) & S_{m2}(f) & \dots & S_{mm}(f) \end{bmatrix}, \quad (7.8)$$

where S_{ii} are auto-spectra and S_{ij} are cross-spectra.

The matrix $S_{yy}(f)$ is obtained from

$$S_{yy}(f) = H(f)^{-1} Q H(f)^{-1*}, \quad (7.9)$$

where * indicates complex conjugate transpose and

$$H(f) = I - \sum_{k=1}^n A_k \exp(-j2\pi f k \Delta t)$$

$$f < \frac{1}{2\Delta t}, \quad (7.10)$$

where Δt is the sampling interval (sec).

The relationship between signals i and j is given by the coherence function

$$\gamma_{ij}^2(f) = \frac{|S_{ij}(f)|^2}{S_{ii}(f) S_{jj}(f)}. \quad (7.11)$$

The SCR characterizes the relative contribution of the source v_k to the power spectrum of signal y_i , and is helpful in revealing the cause-and-effect relationships between the signals. This estimation requires mutual independence of the noise sources, i.e., diagonality

of the covariance matrix Q . If Q is diagonal, the power spectrum of signal i is equal to

$$S_{ii} = \sum_{k=1}^m |H(f)_{ik}^{-1}|^2 q_{kk}$$

$$i = 1, \dots, m, \quad (7.12)$$

where q_{kk} are the diagonal elements of Q . Then we define the fractional contribution ratio as

$$(SCR)_{ik} = \frac{|H(f)_{ik}^{-1}|^2 q_{kk}}{S_{ii}(f)}$$

$$k = 1, \dots, m. \quad (7.13)$$

The assumption of diagonality of the covariance matrix can be validated by applying an orthogonal transformation [15]

$$R = B Q B^T. \quad (7.14)$$

If the orthogonal transformation matrix B is formed from the eigenvectors of matrix Q , then

$$S_{yy}(f) = [B H(f)]^{-1} R [B H(f)]^{-*}$$

$$= G(f) R G(f)^*, \quad (7.15)$$

and

$$S_{ii}(f) = \sum_{k=1}^n |G_{ik}|^2 \lambda_k$$

$$i = 1, \dots, m, \quad (7.16)$$

where λ_k are the eigenvalues of the covariance matrix. The exact SCR' is defined then as

$$(\text{SCR}_{ik})' = \frac{|G_{ik}|^2 \lambda_k}{S_{ii}(f)}$$

$$k = 1, \dots, m. \quad (7.17)$$

The error between exact SCR' and SCR based on assumption of diagonality given by

$$\epsilon_{ik} = \frac{|(\text{SCR})_{ik} - (\text{SCR}_{ik})'|}{(\text{SCR}_{ik})'}, \quad (7.18)$$

and can be used as a means of model validation.

2. Results of Empirical Modeling of the MHD Plasma

The results of empirical modeling presented in this chapter apply to data taken at the UTSI MHD test facility. This facility includes a DCW type generator operating under the typical thermodynamic conditions close to those in the set (5.1) and a wide range of electrical parameters; the applied electric field can vary between 100 to 1000 V, which gives the total current in the range 20 to 80 A and power 30 to 90 kW. The harsh high-temperature environment in MHD generators restricts available MHD data to the signals that can be measured by devices external to the conducting channel. The data may include the coal flow rate, oxygen flow rate, nitrogen flow rate, heater oil pressure drop, etc. These signals are useful in estimating the influence of fluctuations in the external variables on the generator's performance. They do not, however, help in understanding the fluctuations that are inherent of the generator's action. The signals chosen in the study for the empirical modeling include the total generator current resulting from

an applied electric field, center-line plasma luminosity, and dynamic combustor pressure. The plasma luminosity signal is proportional to a narrow band of spectral radiation from potassium present in the plasma, and is sensed by an optical probe mounted in the channel's sidewall. The combustor pressure signal is measured by a piezoelectric transducer located in the combustor close to the entrance to the channel.

The chosen signals were digitized with the sampling time equal to 0.0004 sec, i.e., 2500 Hz, which permitted aliasing-free spectral analysis up to 1000 Hz. The digitized data were analysed on a PDP-11 minicomputer, using the Fortran programs ARMULT and ARMLT1. (Both programs belong to the Nuclear Engineering Department's computer library.) The program ARMULT chose the model order equal to 16 as an optimal multivariate model for the set of MHD signals. The spectral domain results of the empirical modeling of the variables are shown in Figures 45-51.

Figure 45 demonstrates the APSD functions of the chosen MHD signals. The figure indicates that the generator current and plasma luminosity plots decrease approximately at one decade per decade between 100 Hz and 1000 Hz, whereas the combustor pressure plot decreases at two decades per decade in the same region of frequency. The plasma pressure exhibits a clear peak centered at 700 Hz; the second peak around 200 Hz is less visible. The 200 Hz peak can be also observed in the APSD plot of the generator current.

Figure 46 demonstrates the CPSD functions for generator current-plasma luminosity and generator current-combustor pressure. The CPSD plot between the current and luminosity decreases slightly between

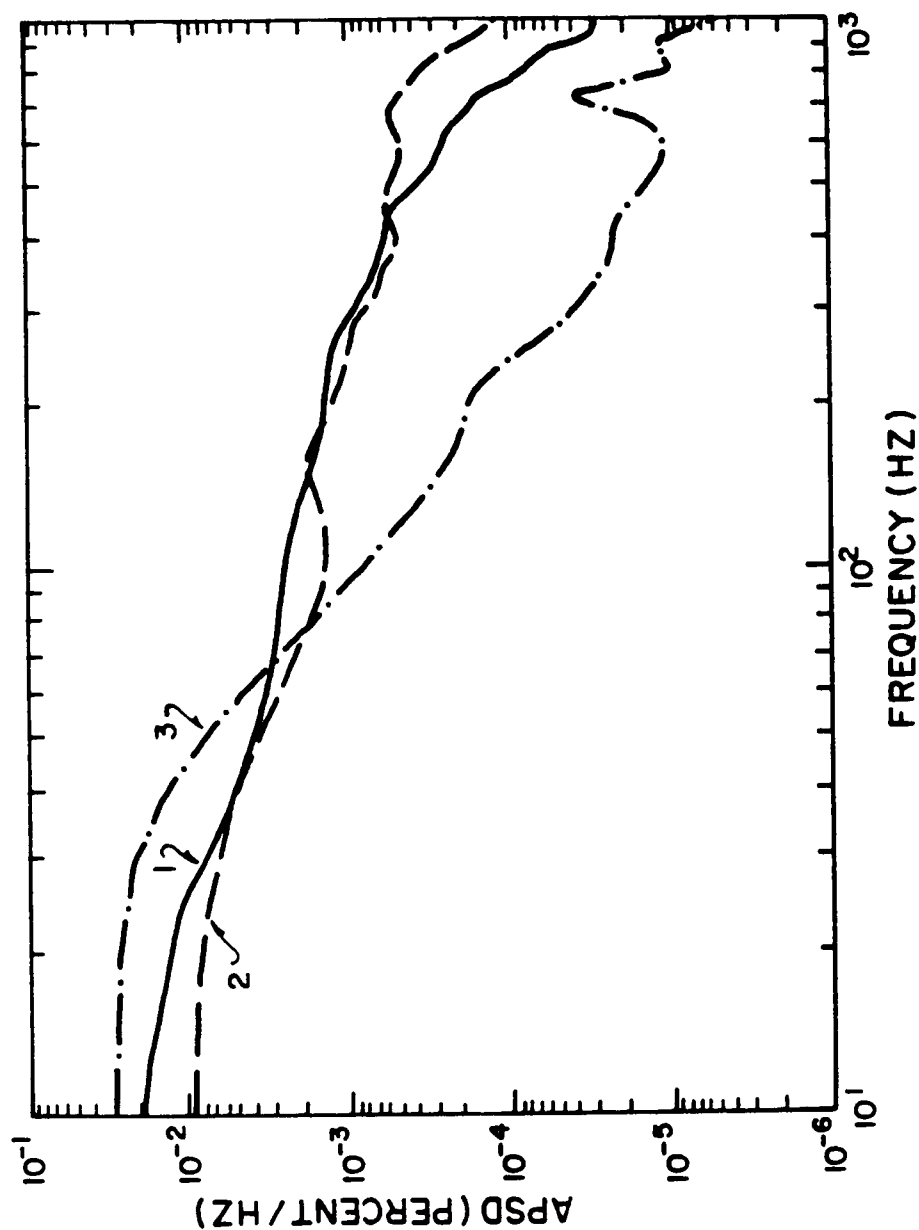


Figure 45. The APSD functions of the MHD signals, resulting from the empirical modeling.

1. Generator current
2. Center-line luminosity
3. Combustor pressure

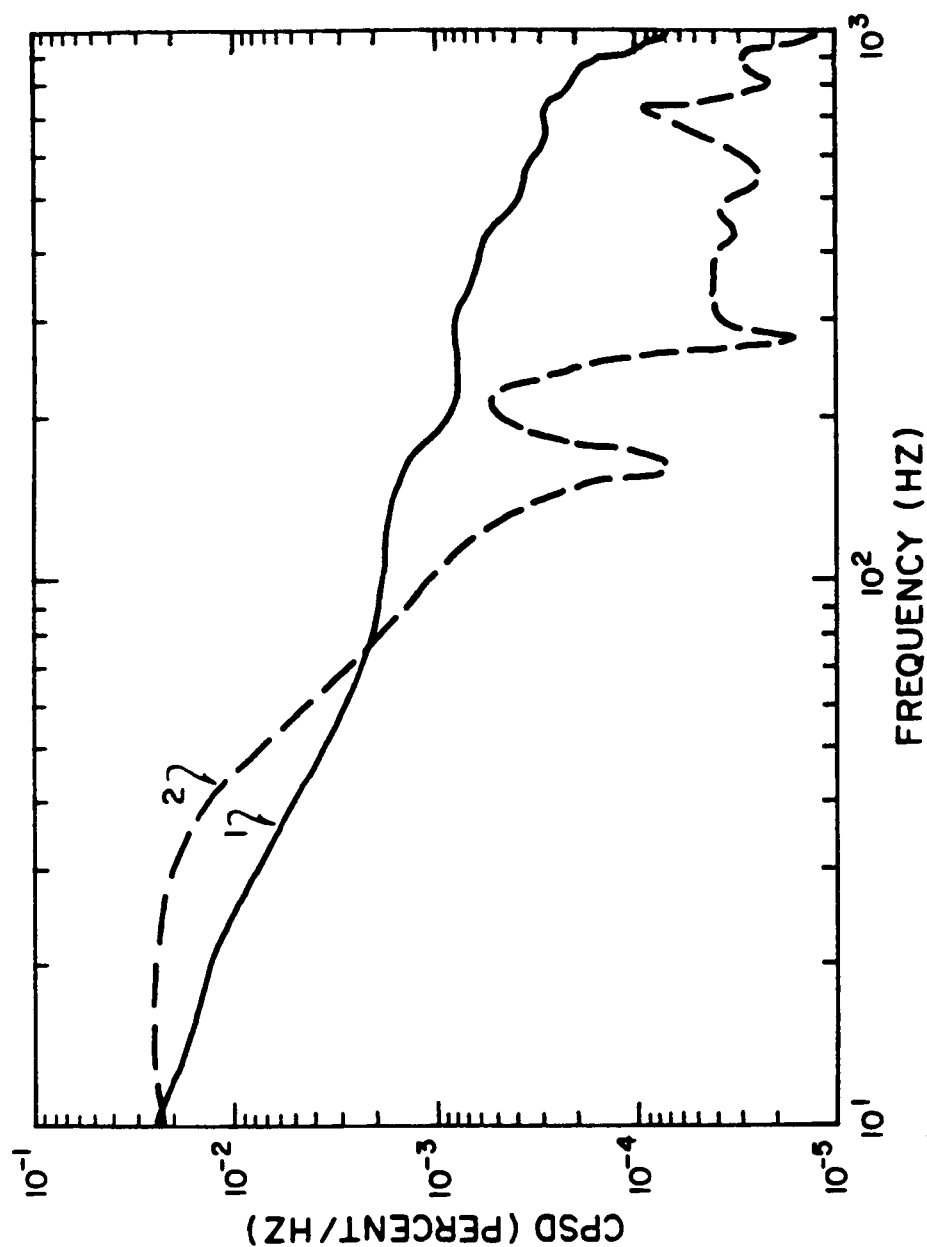


Figure 46. The CPSD functions between the MHD signals, resulting from the empirical modeling.

1. Generator current - center-line luminosity.
2. Generator current - combustor pressure.

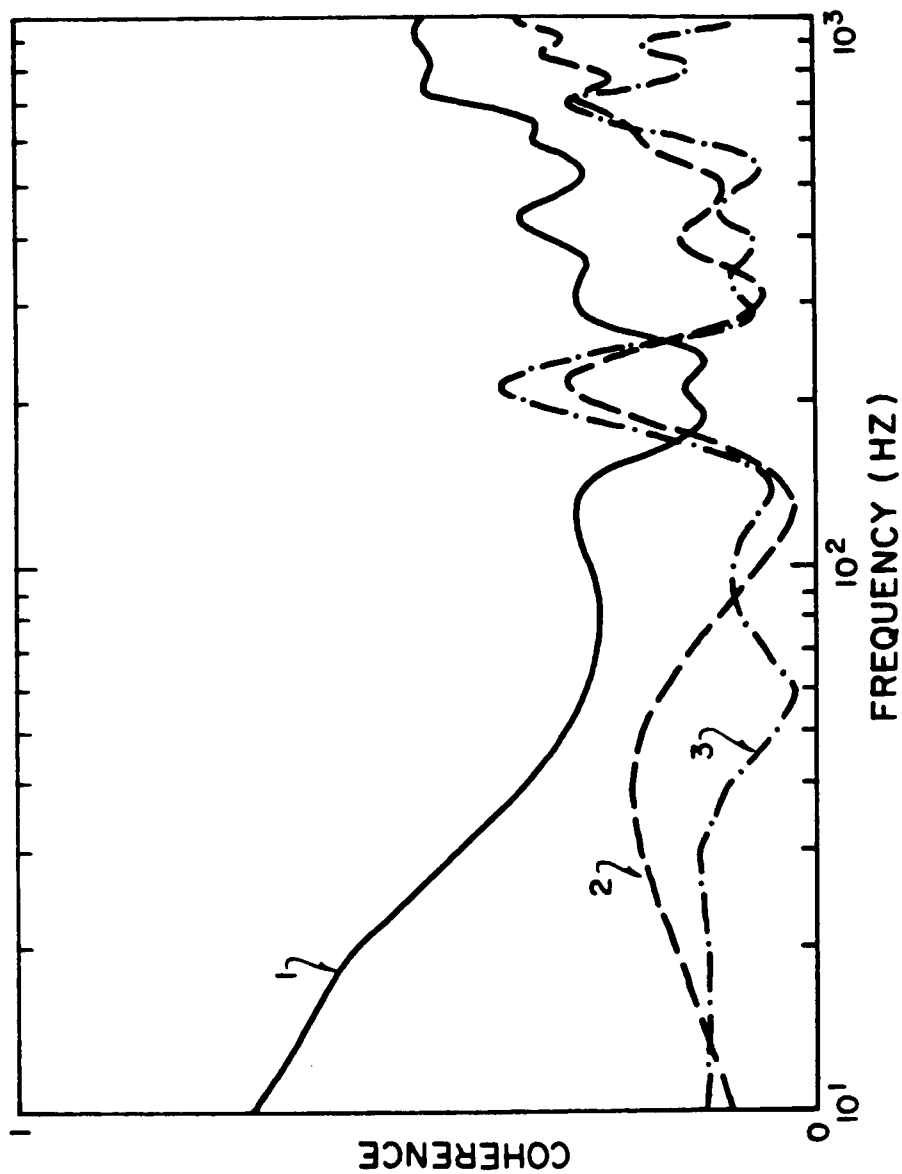


Figure 47. The coherence functions between the MHD signals, resulting from the empirical modeling.

1. Generator voltage - center-line luminosity.
2. Center-line luminosity - combustor pressure.
3. Generator voltage - combustor pressure.

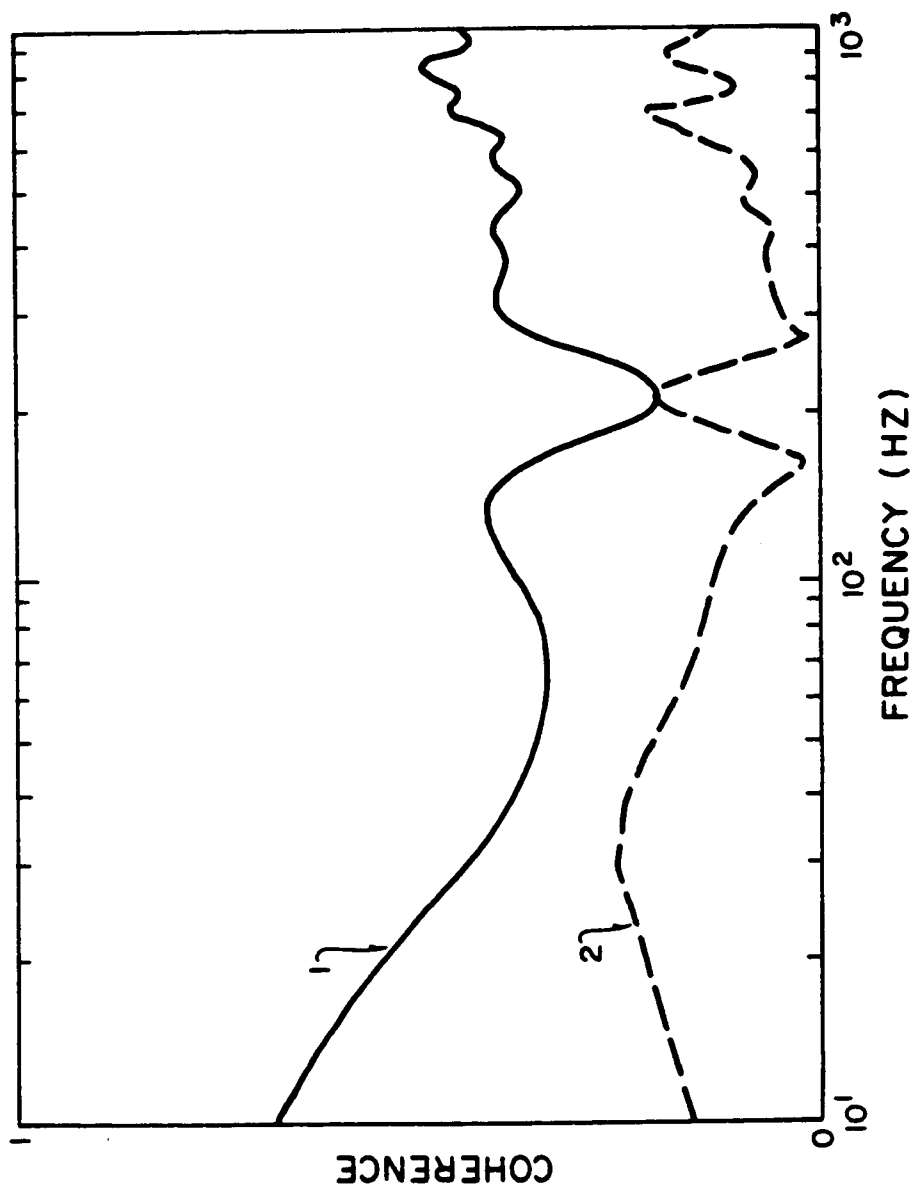


Figure 48. The coherence functions between the MHD signals, resulting from the empirical modeling.

1. Generator current - center-line luminosity.
2. Generator current - combustor pressure.

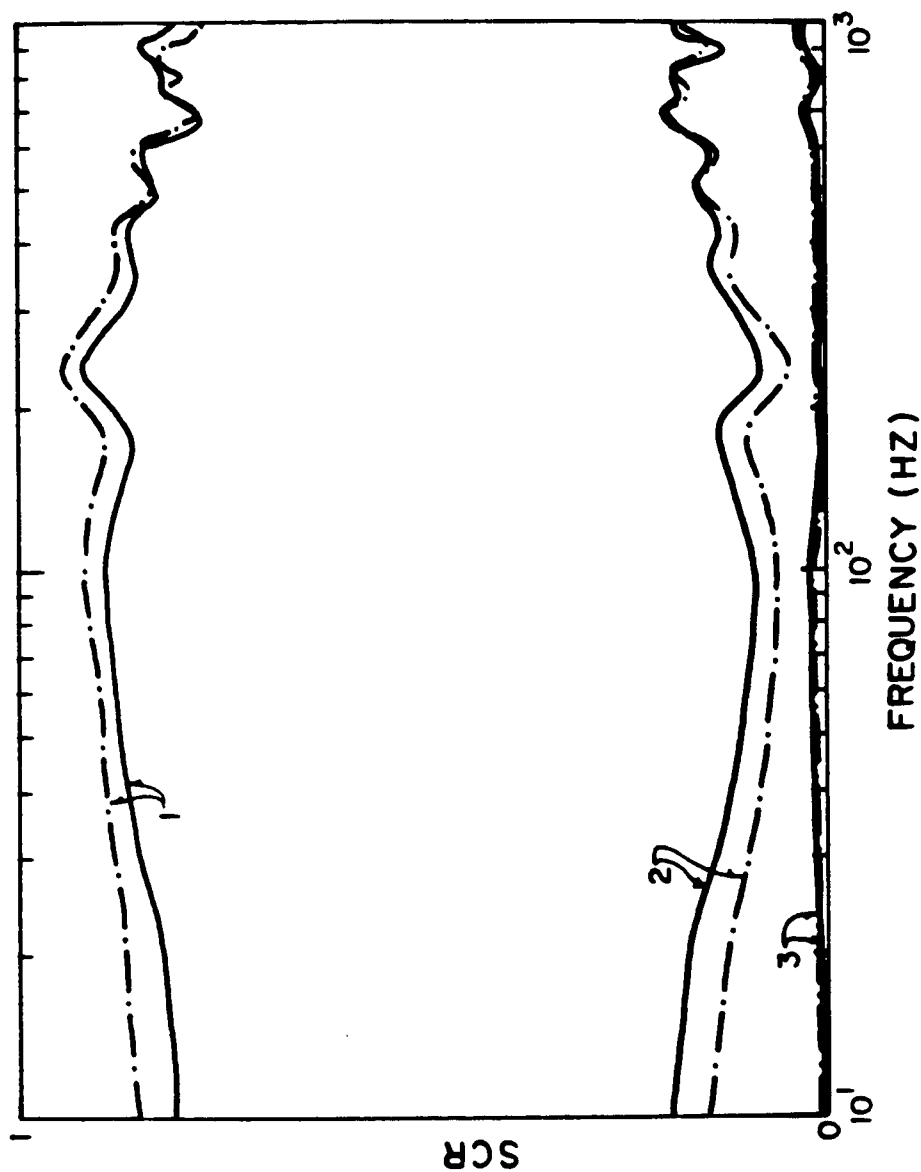


Figure 49. The SCR functions; contributions to current spectral density:

1. from current noise source,
2. from luminosity noise source,
3. from pressure noise source,

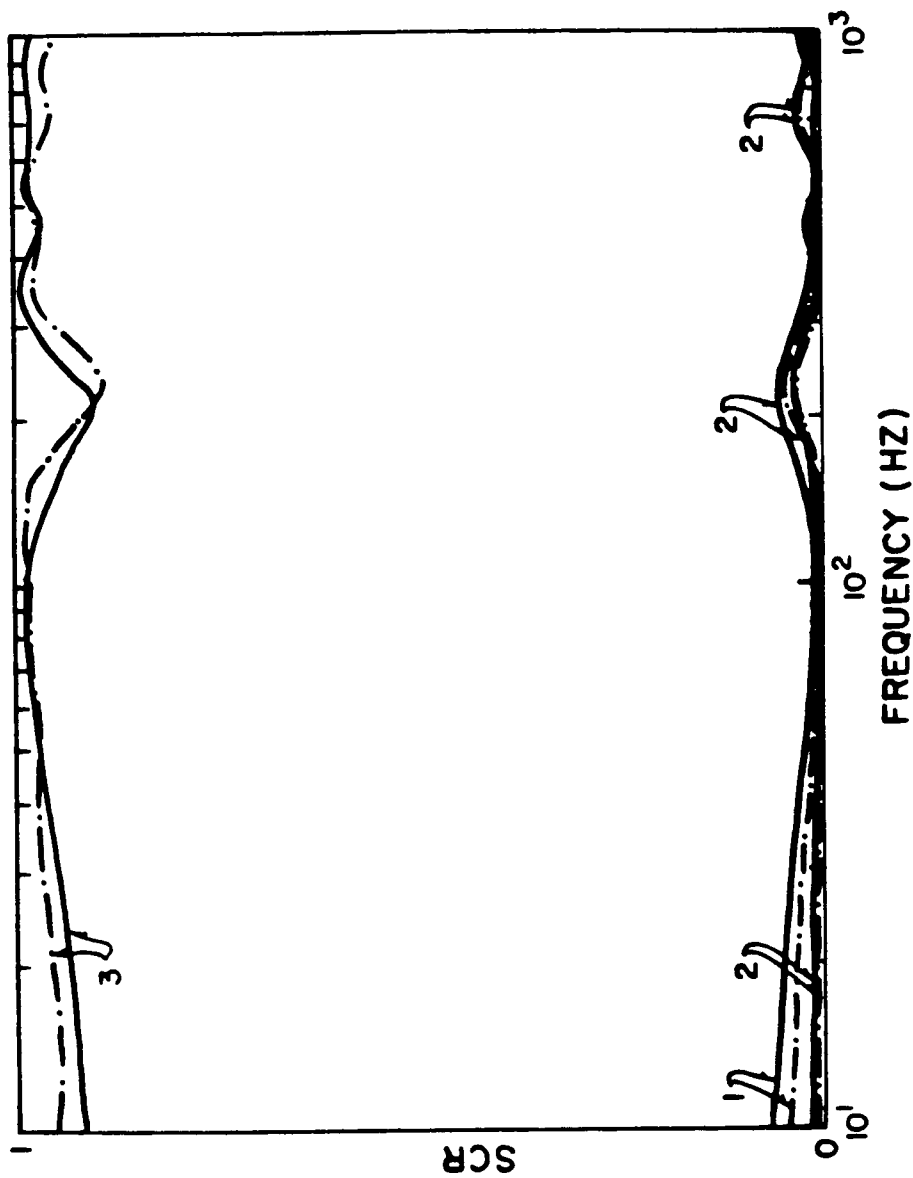


Figure 50. The SCR functions; contributions to combustor pressure spectral density:

1. from current noise source,
2. from luminosity noise source,
3. from pressure noise source.

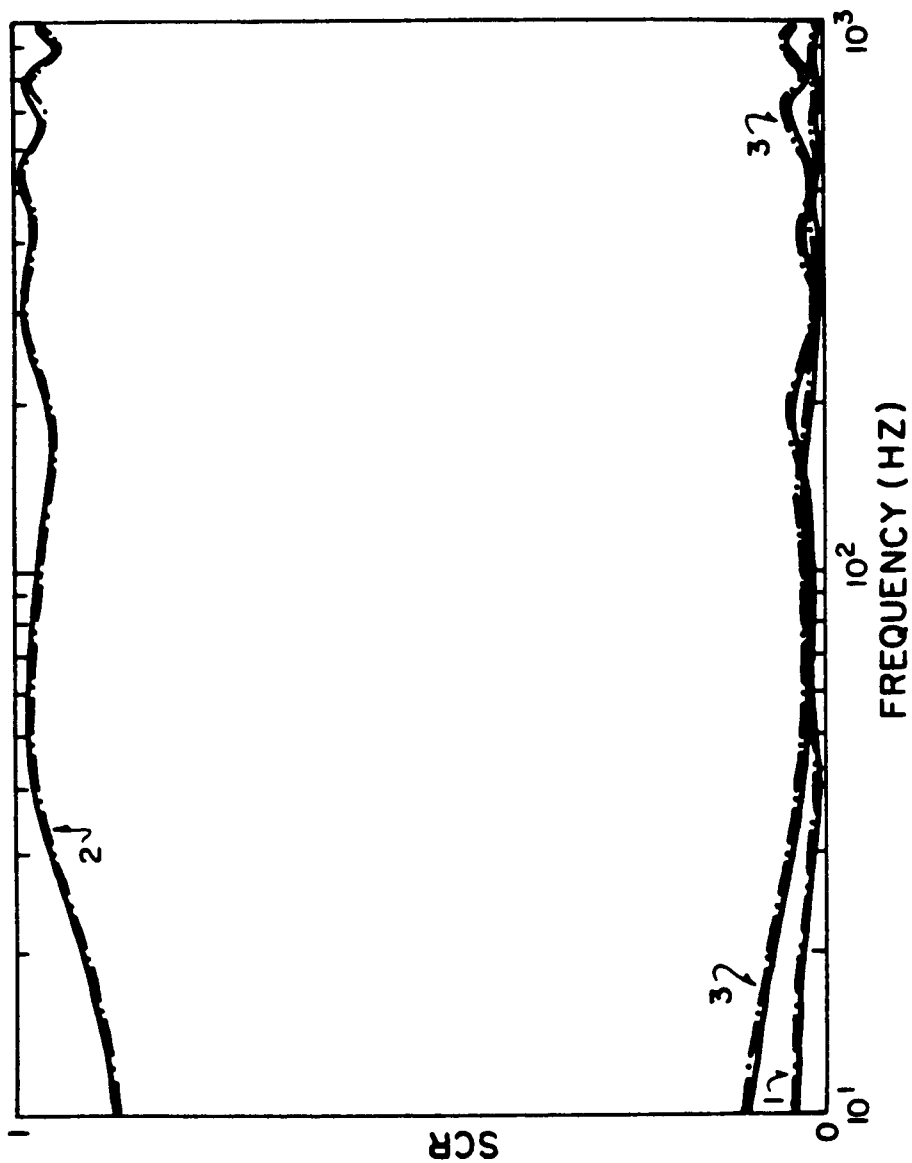


Figure 51. The SCR functions; contributions to plasma luminosity spectral density:

1. from current noise source,
2. from luminosity noise source,
3. from pressure noise source.

100 Hz and 1000 Hz; no peaks occur in this region of frequency. The CPSD plot between the current and pressure, however, exhibits two clear peaks centered at 200 Hz and 700 Hz. The occurrence of these peaks in the CPSD and APSD plots can probably be related to the presence of the standing waves in the MHD channel.

Figures 47 and 48 demonstrate the coherence functions between the MHD signals. The figures indicate that the generator voltage and combustor pressure, generator current and combustor pressure, and plasma luminosity and pressure signals are coherent in the low frequency region. The coherence above 200 Hz decreases; however, it peaks around 200 Hz and 700 Hz, for the signal pairs involving combustor pressure. This can be explained by the presence of the standing waves.

The signal contribution ratios for the various plasma signals are presented in Figures 49-51. Figure 49 demonstrates contributions to the APSD of current signal from the noise source of current, that of plasma luminosity, and that of combustor pressure. Although the self-contribution to the APSD of current is large, the luminosity noise source contributes significantly in the low frequency range and around 700 Hz; the contribution from the combustor pressure is very small. The figure shows the comparison between the signal contribution ratios obtained using the diagonal noise covariance matrix, and those using the similarity transformation. The plots indicate that the assumption of diagonality of Q is clearly satisfied for this case.

Figure 50 demonstrates contributions to the APSD of combustor pressure signal from various noise sources. The self-contribution to

the combustor pressure is large; however, the other two signals contribute also in the low frequency region (current), and around 200 Hz and 700 Hz (plasma luminosity). The assumption of diagonality of Q is also satisfied for this case.

Figure 51 demonstrates contributions to the APSD of plasma luminosity signal from various noise sources. The self-contribution to the plasma luminosity is large; the current and combustor pressure contribute slightly in the low frequency region and around 700 Hz. There is no significant difference between the plots based on the assumption of diagonality of Q , and those obtained using the similarity transformation.

The analysis based on the multivariate autoregressive modeling indicates a strong relationship between the generator current, voltage, and center-line luminosity representing the temperature of the plasma's main body. These signals have large coherence functions, and their noise sources clearly contribute to the signals. The lower coherence between the generator current and plasma pressure is probably influenced by the fact that the pressure transducer was located not in the conducting channel but in the combustion chamber, when the experimental data were recorded. The pressure signal, however, is coherent with the other signals around 200 Hz and 700 Hz, and its contribution to the plasma luminosity signal is not negligible. The signal contribution ratio to the APSD of pressure from the current noise source, visible in the low frequency range and around 200 Hz, may indicate on the feedback effect from the current to the pressure. The occurrence

of the 200 Hz and 700 Hz peaks in most of the plots can be related to the presence of standing waves within the MHD generator channel.

CHAPTER 8

SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE WORK

1. Summary

The results presented in the study were obtained under the assumption that stochastic processes take place within the MHD plasma. The developed plasma model applies to one-dimensional flow, and does not include phenomena occurring on the electrodes and channel walls. It does not analyze the influence of the nonlinear effects, and assumes that the initial amplitudes of all fluctuations are small. The model is very general, however, and can be applied without any modification to all of the types of MHD generators. The originality of the research lies in this general plasma coupling the combustor with the conducting channel, and also in the methodology developed for analytical study of stochastic systems.

The study investigates the plasma fluctuations which either are inherent in the generator dynamics, or arise from the combustor fluctuations propagating into the conducting channel. Third source of plasma fluctuations, variations in the mass flow rates of components input to the combustor, is not investigated.

The analysis of fluctuations of the MHD variables included a development of the plasma model based on the conservation of mass, momentum, and energy equations, coupled with the generalized Ohms' law and Maxwell's equations of electromagnetics. The current and voltage variables, as well as the Hall parameter, conductivity, and electron density were presented as functions of the pressure, density, and velocity variables. The Langevin technique was introduced for the

model linearization. The spatial dependency of the variables was approximated via the nodal method. The linearized and time dependent equations were put into a compact matrix form, with the state vector consisting of the fluctuating pressure, density, and velocity variables. The solution of the equations was based on a similarity transformation of the state parameter matrix. This similarity transformation was implemented into the computer program MHD CR; the program gives the results of the modeling in the form of analytical autocorrelation, crosscorrelation, and power spectral density functions. These functions depend on the system eigenvalues and assumed conditions at the entrance to the channel.

Investigation of fluctuations of the MHD variables also included modeling based on the empirical data acquired at the UTSI MHD test facility. The generator current, plasma luminosity, and combustor pressure signals were modeled by the stochastic multivariate time-series model. This modeling resulted in the power spectral density, coherence, and signal contribution ratio functions of the signals.

2. Conclusions

Analyses of the plasma primary variables, i.e., the pressure, density and velocity, revealed that the fluctuations whose frequencies exceed 200 Hz are acoustic in nature, and are caused by the standing waves present in the generator. The 200 Hz frequency limit is imposed by the fundamental frequency of the standing waves in the analyzed generators. The occurrence of the standing waves is proved by the character of the crosscorrelation and power spectral density functions. The plasma variables correlate strongly from one axial position to

another with minimum or maximum near zero-delay time; this indicates that the waves are standing waves. The power spectral density functions exhibit distinct peaks centered around the fundamental frequency and its odd harmonics. The plasma primary variables influence behavior of the secondary variables, i.e., current and voltage. Fluctuations in the latter two variables are also acoustic in nature, and are centered around the odd harmonics.

The analysis proved that Hall generators develop fluctuations mainly in the range 700 Hz to 2000 Hz, whereas Faraday and DCW generators are favorable for the low frequency fluctuations. Parametric study of the variables indicated that stronger magnetic fields and larger Hall parameters increase the range of fluctuations. Changes in plasma specific heat ratio or in input steady-state parameters may increase or decrease the intensities of some harmonics. The fluctuations that originate in the combustion chamber also affect the plasma variables. A white noise character of these fluctuations guarantees a fairly uniform distribution of energy in the plasma variables up to 200 Hz. If the driving noise sources do not possess broad-band characteristics (this can be due to the nonuniform mass flow rate or incomplete combustion), the spectral density functions may exhibit some peaks below 200 Hz. These peaks will not be acoustic in origin; they will be related to the combustion process or the external inputs to the combustor.

The results of the empirical modeling of combustor pressure, plasma luminosity and generator current signals indicate the existence of the standing waves. The signals used in this modeling did not

include the plasma density or in-channel measured plasma pressure, i.e., the primary variables of the analytical modeling. The results of the empirical and analytical models, therefore, are not directly compared. Both of them, however, suggest the presence of the same phenomenon, the acoustic standing waves.

3. Suggestions for future work

A subsequent study in the field of MHD fluctuations should concentrate on the influence of the viscous effects, heat conduction in plasma, and wall friction on the plasma variables. The analytical study of these effects will require a complete two-dimensional model. The solution of the model, however, would be extremely complex. The alternative approach to plasma fluctuations, the multivariate time series modeling, looks like a more feasible method. This method, however, will require more in-channel signals measured for various operating conditions.

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APPENDIXES

APPENDIX A

MODEL COEFFICIENTS

The state parameter matrix A of equation (3.21) has the form

$\begin{matrix} a_{11} & & & & \\ b_{21} & a_{22} & & & \\ & \cdot & & \cdot & \\ & & \cdot & & \cdot \\ & & & \cdot & \\ & & & & b_{n,n-1} & a_{nn} \end{matrix}$	$\begin{matrix} a_{1,n+1} & & & & \\ b_{2,n+1} & a_{2,n+2} & & & \\ & \cdot & & \cdot & \\ & & \cdot & & \cdot \\ & & & \cdot & \\ & & & & b_{n,2n-1} & a_{n,2n} \end{matrix}$	
$\begin{matrix} c_{n+1,1} & & & & \\ d_{n+2,1} & c_{n+2,2} & & & \\ & \cdot & & \cdot & \\ & & \cdot & & \cdot \\ & & & \cdot & \\ & & & & d_{2n,n-1} & c_{2n,n} \end{matrix}$	$\begin{matrix} c_{n+1,n+1} & & & & \\ d_{n+2,n+1} & c_{n+2,n+2} & & & \\ & \cdot & & \cdot & \\ & & \cdot & & \cdot \\ & & & \cdot & \\ & & & & d_{2n,2n-1} & c_{2n,2n} \end{matrix}$	$\begin{matrix} c_{n+1,2n+1} & & & & \\ d_{n+2,2n+1} & c_{n+2,2n+2} & & & \\ & \cdot & & \cdot & \\ & & \cdot & & \cdot \\ & & & \cdot & \\ & & & & d_{2n,3n-1} & c_{2n,3n} \end{matrix}$
$\begin{matrix} e_{2n+1,1} & & & & \\ g_{2n+2,1} & e_{2n+2,2} & & & \\ & \cdot & & \cdot & \\ & & \cdot & & \cdot \\ & & & \cdot & \\ & & & & g_{3n,n-1} & e_{3n,n} \end{matrix}$	$\begin{matrix} e_{2n+1,n+1} & & & & \\ g_{2n+2,n+1} & e_{2n+2,n+2} & & & \\ & \cdot & & \cdot & \\ & & \cdot & & \cdot \\ & & & \cdot & \\ & & & & g_{3n,2n-1} & e_{3n,2n} \end{matrix}$	$\begin{matrix} e_{2n+1,2n+1} & & & & \\ g_{2n+2,2n+1} & e_{2n+2,2n+2} & & & \\ & \cdot & & \cdot & \\ & & \cdot & & \cdot \\ & & & \cdot & \\ & & & & g_{3n,3n-1} & e_{3n,3n} \end{matrix}$

where

$$a_{ii} = -u_{0(i)} \frac{\rho_{0(i-1)}}{\rho_{0(i)} L}, \quad i = 1, \dots, n$$

$$a_{i,n+1} = -u_{0(i)} \left(2 - \frac{\rho_{0(i-1)}}{\rho_{0(i)}} \right) \frac{1}{L}$$

$$b_{ij} = u_{0(i)} \frac{\rho_{0(i-1)}}{\rho_{0(i)} L}, \quad i = 2, \dots, n, \quad j=1, \dots, n-1$$

$$b_{i,n+j} = u_{0(i)} \frac{1}{L}$$

$$c_{n+i,i} = u_{0(i)} \left(1 - \frac{\rho_{0(i-1)}}{\rho_{0(i)}} \right) \frac{1}{L} + \frac{B_{\gamma\rho(i)}}{u_{0(i)}}, \quad i = 1, \dots, n$$

$$c_{n+1,n+i} = -u_{0(i)} \frac{\rho_{0(i-1)}}{\rho_{0(i)} L} + \frac{B_{\gamma u(i)}}{\rho_{0(i)}}$$

$$c_{n+i,2n+i} = \frac{B_{\gamma p(i)} \rho_{0(i)}}{\rho_{0(i)} u_{0(i)}} - \frac{\rho_{0(i)}}{\rho_{0(i)} u_{0(i)} L}$$

$$w_{(i)} = \frac{\sigma_0 \Omega_0 e n_0 L^2 B}{(1 + \Omega_0^2) \epsilon_0 u_{0(i)} \rho_{0(i)}}$$

$$d_{n+i,j} = w_{0(i)} \left(.25 + \frac{v}{2kT_{0(i-1)}} \right) \frac{1}{L}, \quad i=2, \dots, n, \quad j=1, \dots, n-1$$

$$d_{n+i,n+j} = u_{0(i-1)} \frac{1}{L}$$

$$d_{n+j,2n+j} = \frac{P_0(i-1)}{L \rho_0(i) u_0(i)} - w_{0(i)} \left(.75 + \frac{V}{2kT_{0(i-1)}} \right) \frac{1}{L}$$

$$e_{2n+i,i} = (D_{(i)} - D1_{(i)}) \frac{\gamma-1}{P_0(i)} - D2_{(i)} - \gamma u_{0(i)} \left(2 - \frac{\rho_0(i-1)}{\rho_0(i)} \right),$$

$$i = 1, \dots, n$$

$$e_{2n+i,n+i} = (E_{(i)} - E1_{(i)}) \frac{\gamma-1}{P_0(i)} - E2_{(i)} - \gamma u_{0(i)} \left(2 - \frac{\rho_0(i-1)}{\rho_0(i)} \right)$$

$$e_{2n+i,2n+i} = (F_{(i)} - F1_{(i)}) \frac{\gamma-1}{P_0(i)} - F2_{(i)}$$

$$g_{2n+i,j} = (A_{(i-1)} - A1_{(i-1)}) \frac{\gamma-1}{P_0(i)} - A2_{(i-1)} + \gamma u_{0(i)} \frac{\gamma_0(i-1)}{\gamma_0(i)},$$

$$i = 2, \dots, n$$

$$j = 1, \dots, n-1$$

$$g_{2n+i,n+j} = \gamma u_{0(i-1)} - \frac{\gamma-1}{P_0(i)} B_{(i-1)} - B1_{(i-1)}$$

$$g_{2n+i,2n+j} = C_{(i-1)} - C1_{(i-1)} \frac{\gamma-1}{P_0(i)} - C2_{(i-1)}$$

The coefficients appearing in the forcing-function vector (3.22) are equal to

$$a = \frac{1}{L} u_{0(1)} \frac{\rho_0(0)}{\rho_0(1)}$$

$$b = \frac{1}{L} u_{0(0)}$$

$$c = w_{(1)} \left(.25 + \frac{v}{2kT_{0(0)}} \right) \frac{1}{L}$$

$$d = \frac{1}{L} u_{0(0)}$$

$$e = \frac{p_{0(0)}}{p_{0(1)} u_{0(1)} L} - w_{(1)} \left(.75 + \frac{v}{2kT_{0(1)}} \right) \frac{1}{L}$$

$$f = (A_{(0)} - A_{1(0)}) \frac{\gamma-1}{p_{0(1)}} - A_{2(0)} + \gamma u_{0(1)} \frac{p_{0(0)}}{p_{0(1)}}$$

$$g = \gamma u_{0(0)} - \frac{\gamma-1}{p_{0(1)}} B_{(0)} - B_{1(0)}$$

$$h = (C_{(0)} - C_{1(0)}) \frac{\gamma-1}{p_{0(1)}} - C_{2(0)}$$

The coefficients appearing in Table 1 are equal to

$$\begin{aligned} c_{1ij} &= a_{j1} (a_{i1} | c_{11} | + a_{i,n+1} | c_{21} | + a_{i,2n+1} | c_{31} |) + \\ &\quad a_{j,n+1} (a_{i1} | c_{12} | + a_{i,n+1} | c_{22} | + a_{i,2n+1} | c_{32} |) + \\ &\quad a_{j,2n+1} (a_{i1} | c_{13} | + a_{i,n+1} | c_{23} | + a_{i,2n+1} | c_{33} |) \\ c_{2ij} &= a_{j1} (a_{i+1,1} | c_{11} | + a_{i+1,n+1} | c_{21} | + a_{i+1,2n+1} | c_{31} |) + \\ &\quad a_{j,n+1} (a_{i+1,1} | c_{12} | + a_{i+1,n+1} | c_{22} | + a_{i+1,2n+1} | c_{32} |) + \end{aligned}$$

$$\begin{aligned}
& a_{j,2n+1} (a_{i+1,1} |C_{13}| + a_{i+1,n+1} |C_{23}| + a_{i+1,2n+1} |C_{33}|) \\
C^3_{ij} = & a_{j+1,1} (a_{i1} |C_{11}| + a_{i,n+1} |C_{21}| + a_{i,2n+1} |C_{31}|) + \\
& a_{j+1,n+1} (a_{i1} |C_{12}| + a_{i,n+1} |C_{22}| + a_{i,2n+1} |C_{32}|) + \\
& a_{j+1,2n+1} (a_{i1} |C_{13}| + a_{i,n+1} |C_{23}| + a_{i,2n+1} |C_{33}|) \\
C^4_{ij} = & a_{j+1,1} (a_{i+1,1} |C_{11}| + a_{j+1,n+1} |C_{21}| + a_{i+1,2n+1} |C_{31}|) + \\
& a_{j+1,n+1} (a_{i+1,1} |C_{12}| + a_{i+1,n+1} |C_{22}| + a_{i+1,2n+1} |C_{32}|) + \\
& a_{j+1,2n+1} (a_{i+1,1} |C_{13}| + a_{i+1,n+1} |C_{23}| + a_{i+1,2n+1} |C_{33}|),
\end{aligned}$$

where n is the number of nodes, a_{ij} , $i = 1, \dots, 3n$, $j = 1, \dots, 3n$ are the elements of the p^{-1} matrix, and $|C_{k\ell}|$, $k = 1, 2, 3$, $\ell = 1, 2, 3$ are the amplitudes of the input crosscorrelation functions appearing in (4.18).

The coefficients appearing in Table 2 are equal to

$$\begin{aligned}
A = & \frac{C^1_{ij}(p-\lambda_j)(\lambda_i+\lambda_j) - C^3_{ij}\beta_j(\lambda_i+\lambda_j) + C^1_{ij}\beta_j^2 + C^3_{ij}(p-\lambda_j)\beta_j}{[(p-\lambda_j)^2 + \beta_j^2][(\lambda_i-\lambda_j)^2 + \beta_j^2]} + \\
& \frac{C^1_{ij}(p+\lambda_j)(\lambda_i+\lambda_j) + C^3_{ij}\beta_j(\lambda_i+\lambda_j) - C^1_{ij}\beta_j^2 + C^3_{ij}(p+\lambda_j)\beta_j}{[(p+\lambda_j)^2 + \beta_j^2][(\lambda_i+\lambda_j)^2 + \beta_j^2]} \\
B = & \frac{C^1_{ij}\beta_j(\lambda_i+\lambda_j) + C^3_{ij}(p-\lambda_j)(\lambda_i+\lambda_j) - C^1_{ij}(p-\lambda_j)\beta_j + C^3_{ij}\beta_j^2}{[(p-\lambda_j)^2 + \beta_j^2][(\lambda_i+\lambda_j)^2 + \beta_j^2]} + \\
& \frac{-C^1_{ij}\beta_j(\lambda_i+\lambda_j) + C^3_{ij}(p+\lambda_j)(\lambda_i+\lambda_j) - C^1_{ij}(p+\lambda_j)\beta_j - C^3_{ij}\beta_j^2}{[(p+\lambda_j)^2 + \beta_j^2][(\lambda_i+\lambda_j)^2 + \beta_j^2]}
\end{aligned}$$

$$E = \frac{C1_{ij}(\lambda_j - p)(\lambda_i + p) + C3_{ij}\beta_j(p + \lambda_i)}{[(p - \lambda_j)^2 + \beta_j^2][(\lambda_i + p)^2 + \beta_i^2]} +$$

$$\frac{C2_{ij}(\lambda_j - p)\beta_i + C4_{ij}\beta_j\beta_i}{[(p - \lambda_j)^2 + \beta_j^2][(\lambda_i + p)^2 + \beta_i^2]}$$

$$F = \frac{1}{2[(p + \lambda_j)^2 + \beta_j^2]} \left\{ \frac{(A1 + D1)(\lambda_i + \lambda_j) + (C1 - B1)(\beta_j - \beta_i)}{[(\lambda_i + \lambda_j)^2 + (\beta_j - \beta_i)^2]} + \right.$$

$$\left. \frac{(A1 - D1)(\lambda_i + \lambda_j) + (C1 + B1)(\beta_j + \beta_i)}{[(\lambda_i + \lambda_j)^2 + (\beta_j + \beta_i)^2]} \right\} +$$

$$\frac{1}{2[(p - \lambda_j)^2 + \beta_j^2]} \left\{ \frac{(A2 + D2)(\lambda_i + \lambda_j) + (C2 - B2)(\beta_j - \beta_i)}{[(\lambda_i + \lambda_j)^2 + (\beta_j - \beta_i)^2]} + \right.$$

$$\left. \frac{(A2 - D2)(\lambda_i + \lambda_j) + (C2 + B2)(\beta_j + \beta_i)}{[(\lambda_i + \lambda_j)^2 + (\beta_j + \beta_i)^2]} \right\}$$

$$G = \frac{1}{2[(p + \lambda_j)^2 + \beta_j^2]} \left\{ \frac{(C1 - B1)(\lambda_i + \lambda_j) - (A1 + D1)(\beta_j - \beta_i)}{[(\lambda_i + \lambda_j)^2 + (\beta_j - \beta_i)^2]} + \right.$$

$$\left. \frac{(C1 + B1)(\lambda_i + \lambda_j) + (D1 - A1)(\beta_j + \beta_i)}{[(\lambda_i + \lambda_j)^2 + (\beta_j + \beta_i)^2]} \right\} +$$

$$\frac{1}{2[(p - \lambda_j)^2 + \beta_j^2]} \left\{ \frac{(C2 - B2)(\lambda_i + \lambda_j) - (A2 + D2)(\beta_j - \beta_i)}{[(\lambda_i + \lambda_j)^2 + (\beta_j - \beta_i)^2]} + \right.$$

$$\left. \frac{(C2 + B2)(\lambda_i + \lambda_j) + (D2 - A2)(\beta_j + \beta_i)}{[(\lambda_i + \lambda_j)^2 + (\beta_j + \beta_i)^2]} \right\}$$

where

$$A1 = C1_{ij}(p+\lambda_j) + C3_{ij}\beta_j$$

$$B1 = C2_{ij}(p+\lambda_j) + C4_{ij}\beta_j$$

$$C1 = C3_{ij}(p+\lambda_j) - C1_{ij}\beta_j$$

$$D1 = C4_{ij}(p+\lambda_j) - C2_{ij}\beta_j$$

$$A2 = C1_{ij}(p-\lambda_j) - C3_{ij}\beta_j$$

$$B2 = C2_{ij}(p-\lambda_j) - C4_{ij}\beta_j$$

$$C2 = C3_{ij}(p-\lambda_j) + C1_{ij}\beta_j$$

$$D2 = C4_{ij}(p-\lambda_j) + C2_{ij}\beta_j$$

The coefficients appearing in Equations (5.2) are equal to

$$a_{x(i)} = \frac{en_o(i)L\sigma_o}{J_{x(i)}\epsilon_o(1+\Omega_o^2)} \left(.25 + \frac{v}{2kT_o(i-1)} \right)$$

$$b_{x(i)} = \frac{\delta\rho(i)\rho_o(i)}{J_{x(i)}}$$

$$c_{x(i)} = - \frac{en_o(i)L\sigma_o}{J_{x(i)}\epsilon_o(1+\Omega_o^2)} \left(.75 + \frac{v}{2kT_o(i-1)} \right)$$

$$d_{x(i)} = \frac{\delta p(i)p_o(i)}{J_{x(i)}}$$

$$e_{x(i)} = \frac{\delta u(i)U_o(i)}{J_{x(i)}}$$

$$a_{y(i)} = \frac{en_o(i)L\sigma_o\Omega_o}{J_{y(i)}\epsilon_o(1+\Omega_o^2)} \left(.25 + \frac{v}{2kT_o(i-1)} \right)$$

$$b_{y(i)} = \frac{\gamma_{\rho(i)} \rho_{o(i)}}{J_{y(i)}}$$

$$c_{y(i)} = - \frac{e n_{o(i)} L \sigma_o \Omega_o}{J_{y(i)} \epsilon_o (1 + \Omega_o^2)} \left(.75 + \frac{v}{2 k T_{o(i-1)}} \right)$$

$$d_{y(i)} = \frac{\gamma_{p(i)} p_{o(i)}}{J_{y(i)}}$$

$$e_{y(i)} = \frac{\gamma_{u(i)} u_{o(i)}}{J_{y(i)}}$$

APPENDIX B

FORTRAN PROGRAM

The program MHD CR evaluates the correlation functions between the fluctuating density, velocity, and pressure variables of the MHD generator.

```

00001      PROGRAM
00002      WRITTEN BY
00003      - MMDCR
00004      - MALGORZATA SKORSKA
00005      - UNIVERSITY OF TENNESSEE
00006      - NUCLEAR ENGINEERING DEPARTMENT
00007      - JANUARY 1, 1987
00008      LATEST REVISION
00009      PURPOSE
00010      - THE PROGRAM EVALUATES THE CORRELATION
00011      FUNCTIONS BETWEEN THE FLUCTUATING
00012      DENSITY, VELOCITY, AND PRESSURE VARIABLES
00013      OF THE MMD GENERATOR.
00014      - REQUIRED SURROUTINES: EIGEN, INVERT.
00015      - THE PROGRAM READS THE STEADY STATE MMD
00016      PROFILES FROM THE DATA FILE. THE PROGRAM
00017      STORES THE EVALUATED PARAMETERS IN THE
00018      CORRELATION DATA FILE.
00019
00020      DIMENSION VEL(15),DENS(15),PRES(15),TEMP(15),YCURD(15),V(15),
00021      1 XVELTD(15),XAKIS(15),GPRES(15),DPRES(15),
00022      2 GDENS(15),PC(36,36),A(45),B(45),C(45),D(15),E(15),EPS1(16),
00023      3 R(15),DDENS(15),TINT(15),PEIG(36,36),COR(3,3),CONST(36,3,2),
00024      4 CONST(2),MODE(2),MSIG(2)
00025      LOGICAL HEAD(60)
00026      LOGICAL SING,SKIPJ,SKIPI
00027
00028      DATA HALL/2.00/,COND/20.0/,BFIELD/2.00/,XLENGT/1.0668/,
00029      1 ATM/1.0132E+05/,AD/51.6128E-04/,TD/2700./,G/1.2/,ELV/6.32/
00030
00031      FORMAT(4I4)
00032      FORMAT(2F12.5)
00033      FORMAT(//)
00034      FORMAT(15X,
00035      1 *****
00036
00037      WRITE(7,100)
00038      FORMAT(1) ENTER ISOTHERMAL PROFILES FILE NAME',/)
00039      CALL ASSIGN(1,0,-1,'RD0','NC',1)
00040      DEFINE FILE 1(1000,2,U,ILOCK)
00041      WRITE(7,50)
00042      FORMAT(1) ENTER CORRELATION PARAMETERS FILE NAME',/)
00043      CALL ASSIGN(2,0,-1,'NEW','NC',1)
00044      DEFINE FILE 2(3000,4,U,ILOCK)
00045      WRITE(7,104)
00046      FORMAT(1) ENTER NO. OF NODES ALONG THE CHANNEL, N = ',S)
00047      READ(5,10) N
00048
00049      XLENGT=XLENGT/(FLOAT(N))
00050      K=1
00051      READ(1*K,ERR=320)FLOW
00052      K=2
00053      READ(1*K,EQR=320)VELO
00054      K=3
00055

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FORTRAN IV      V01C-03E+  MON 09-MAY-83
0026      READ(1,K,ERR=320)PRESO
0027      K=4
0028      READ(1,K,ERR=320)XCURD
0029      K=5
0030      READ(1,K,ERR=320)YVOLT
0031      WRITE(7,105)
0032      FORMAT(60A1)
0033      105  ENTER 60-CHARACTER TITLE',/)
0034      106  FORMAT(60A1)
0035      READ(5,106)(HEAD(I),I=1,60)
0036      C
0037      N1=N+1
0038      ICOUNT=0
0039      CONTINUE
0040      DO 115 I=1,M1
0041      K=K+1
0042      READ(1,K,ERR=320,END=320)Y(I)
0043      CONTINUE
0044      ICOUNT=ICOUNT+1
0045      IF(ICOUNT-NE. 1) GO TO 120
0046      DO 116 I=1,M1
0047      VEL(I)=Y(I)
0048      GO TO 110
0049      CONTINUE
0050      IF(ICOUNT-NE. 2) GO TO 125
0051      DO 117 I=1,M1
0052      DEMS(I)=Y(I)
0053      GO TO 110
0054      CONTINUE
0055      IF(ICOUNT-NE. 3) GO TO 130
0056      DO 118 I=1,M1
0057      PRES(I)=Y(I)
0058      GO TO 110
0059      CONTINUE
0060      DO 119 I=1,M1
0061      TEMP(I)=Y(I)
0062      C
0063      WRITE(6,12)
0064      WRITE(6,13)
0065      FORMAT(6,135)(HEAD(I),I=1,60)
0066      WRITE(6,13)
0067      WRITE(6,136)N,FLOW,VELD,PRESO,XCURD,YVOLT
0068      136  FORMAT(15X,NO. OF NODES ALONG THE CHANNEL, N = ,I4,/,
0069      1 15X,MASS FLOW RATE IN KG/S, FLOW = ,E12.3,/,
0070      2 15X,CHANNEL ENTRANCE VELOCITY IN M/S, VELO = ,E12.3,/,
0071      3 15X,CHANNEL ENTRANCE PRESSURE IN ATM, PRESO = ,E12.3,/,
0072      4 15X,X-CURRENT DENSITY IN A/M*2, XCURD = ,E12.3,/,
0073      5 15X,Y-VOLTAGE DROP IN V/M, YVOLT = ,E12.3,/)
0074      WRITE(6,15)
0075      15  FORMAT(15X,NODE NUMBER',4X,VELOCITY',9X,DENSITY',9X,
0076      1 15X,PRESSURE',9X,V-CURRENT',9X,X-VOLTAGE',/)
0077      C
0078      DO 102 I=1,N1
0079      YCURD(I)=(COND*(YVOLT-VEL(I)*8FIELD))+HALL*XCURD
0080      102  CONTINUE

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```

FORTRAN IV      VOIC-OJE+   MON 09-MAY-83
0074      XVOLD(I)=(XCURD+HALL*YCURD(I))/COND
0075      CONTINUE
C
0076      DO 140 I=1,N1
0077      I1=I-1
0078      WRITE(6,16)I1,VEL(I),DENS(I),PRES(I),YCURD(I),XVOLD(I)
0079      CONTINUE
0080      FORMAT(15X,18,5X,5(E12.5,5X))
C
0081      WRITE(7,150)
0082      FORMAT(1,1X,150)
0083      1 FROM COMBUSTOR : '//
0084      WRITE(7,151)
0085      FORMAT(1,1X,151)
0086      1. COMB. VEL/VEL(1) = ',S)
0087      WRITE(7,152)
0088      FORMAT(1,1X,152)
0089      2. COMB. DENS/DENS(1) = ',S)
0090      READ(5,11)CVEL
0091      WRITE(7,153)
0092      FORMAT(1,1X,153)
0093      3. COMB. PRES/PRES(1) = ',S)
0094      READ(5,11)CPRES
0095      WRITE(7,157)
0096      FORMAT(1,1X,157)
0097      1. ENTER VARIANCE OF THE NOISE, VAR = ',S)
0098      READ(5,11)VAR
0099      WRITE(6,155)CVEL,CDENS,CPRES,VAR
0100      FORMAT(30X,155)PERTURBATION PARAMETERS (AMPLITUDE RATIOS) : ',//
0101      15X,1. COMB VEL/VEL(1) = ',E12.5,/,
0102      15X,2. COMB DENS/DENS(1) = ',E12.5,/,
0103      15X,3. COMB PRES/PRES(1) = ',E12.5,/,
0104      15X,4. VARIANCE OF THE NOISE = ',E12.5,//
0105      30X,4. FORCING VECTOR CORRELATION FUNCTIONS : ',//
C
0106      GVEL=-COND*BFIELD/(1.+HALL**2)
0107      DVEL=-GVEL*HALL
0108      EJ=ELC*ELV/(2.*BOLTZ)
0109      G1=G-1
0110      DO 160 I=2,N1
0111      I1=I-1
0112      EPS1(I)=COND*HALL*(XVOLD(I)-XVOLD(I1))/(1.+HALL**2)
0113      EPS1(I1)=EPS1(2)
0114      EVALUATE CURRENT FEEDBACK COEFFICIENTS.
0115      C
0116      DO 200 I=2,N1
0117      I1=I-1
0118      EPS21=EPS1(I1)*XLENG1
0119      EPS2=EPS1(I)*XLENG1
0120      A1=(C-25+EJ/TEMP(I1))
0121      A10=(C-25+EJ/TEMP(I1))
0122      A2=(C-75+EJ/TEMP(I1))
0123      A20=(C-75+EJ/TEMP(I1))
0124      A3=HALL*(XCURD-HALL*YCURD(I))/(2.+2.*HALL**2)
0125      A4=HALL*(XCURD+HALL*YCURD(I))/(2.+2.*HALL**2)

```

VO1C-03E+ MON 09-MAY-83

FORTRAN IV

```

0116 GPRES(I)=(XCURD(I)*A1-A3-EP51(I)*A2)/(PRES(I)*ATM)
0117 GDENS(I)=-XCURD(I)*A2+A3-EP51(I)*A1/DENS(I)
0118 DPRES(I)=(XCURD(A1+A2-EP51(I)/HALL*A1)/PRES(I)*ATM)
0119 DDENS(I)=-XCURD(A2-A4-EP51(I)/HALL*A1)/DENS(I)
C
0120 A(I1)=EPS21*(XCURD+COND/(1.+HALL**2))*(HALL*YVOLT D+
1 XVOLT D(I)))*A10
0121 B(I)=EPS2*(XCURD*A1+YVOLT D*XLENGT*GDENS(I)*DENS(I)+
1 XVOLT D(I)*XLENGT*DDENS(I)*DENS(I)
0122 C(I1)=-EPS21*(XCURD+COND/(1.+HALL**2))*(HALL*YVOLT D+
1 XVOLT D(I)))*A20
0123 D(I)=-EPS2*(XCURD*A2-YVOLT D*XLENGT*GPRES(I)*PRES(I)
1 +ATM-XVOLT D(I)*XLENGT*DPRES(I)*PRES(I)*ATM)
0124 E(I)=(XVOLT D(I)*XLENGT*DVEL*YVOLT D*XLENGT*GVEL)*VEL(I)
0125 CONTINUE
C
0126 DO 210 I=2,M1
0127 I1=I-1
0128 EPS11=EPS1(I1)*XLENGT*9FIELD
0129 EPS31=EPS1(I1)*XLENGT*3FIELD
0130 AA1=G/G1*DENS(I)*VEL(I)*PRES(I1)*ATM/DENS(I1)
0131 AA2=VEL(I)*G1*EPS31/(PRES(I)*ATM)*(.25*EJ/TEMP(I1))
0132 A(I1)=(A(I1)-AA1)*G1/(PRES(I)*ATM)-AA2*G*VEL(I)*DENS(I1)
1 /DENS(I)
C
0133 F1=DENS(I)*VEL(I)*VEL(I)*2-VEL(I1)*2/-G/G1*DENS(I)*
1 VEL(I)*PRES(I1)*ATM/DENS(I1)
0134 F11=DENS(I)*VEL(I)*2*G1/(PRES(I1)*ATM)*(VEL(I)*2-
1 DENS(I1)/DENS(I))*XLENGT*GDENS(I)*8FIELD/VEL(I)
0135 B(I1)=(B(I)-F1)*G1/(PRES(I)*ATM)-F11*G*VEL(I)*DENS(I1)
1 /DENS(I)
C
0136 CJ1=-G/G1*DENS(I)*VEL(I)*PRES(I1)*ATM/DENS(I1)
0137 CJ2=VEL(I)*G1/(PRES(I)*ATM)*(PRES(I1)*ATM-EP51*
1 (.75*EJ/TEMP(I1)))
0138 C(I1)=(C(I1)-CJ1)*G1/(PRES(I)*ATM)-CJ2
C
0139 D11=G/G1*PRES(I)*ATM*VEL(I)
0140 D12=VEL(I)*G1*(XLENGT*4FIELD*GPRES(I)-1.)
0141 D(I1)=(D(I)-D1)*G1/(PRES(I)*ATM)-D12
C
0142 R1=DENS(I)*VEL(I)*2*G1*VEL(I)/(PRES(I)*ATM)
0143 R2=-DENS(I)*VEL(I)*VEL(I)*2
0144 R(I1)=G*VEL(I1)-G1/(PRES(I)*ATM)*R2-R1
C
0145 H1=DENS(I)*VEL(I)*2*G/G1*PRES(I)*ATM*VEL(I)*
1 DENS(I)*VEL(I)*2-VEL(I1)*2/2
0146 -G/G1*DENS(I)*VEL(I)*PRES(I1)*ATM/DFNS(I1)
0147 S1=DENS(I)*VEL(I)*2*G1/(PRES(I)*ATM)*XLENGT*4FIELD
1 *GVEL/DENS(I1)-VEL(I)*DFNS(I1)/DENS(I1)
0147 E(I1)=(E(I)-H1)*G1/(PRES(I)*ATM)-S1*G*VEL(I)*2-
1 DENS(I1)/DENS(I1)
0148 CONTINUE
C
210

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FCRTAN IV      VOIC-03E+  MON 09-MAY-83
C
C      SET UP MATRIX OF COEFFICIENTS.
0149      N3=1+N
0150      DO 215 I=1,N3
0151      DO 216 J=1,N3
0152      P(I,J)=0.
0153      CONTINUE
C
0154      DO 220 I=1,N
0155      IT=I+1
0156      IN=I+N
0157      P(I,IT)=-VEL(IT)*DENS(IT)/DENS(I1)
0158      P(IT,IN)=-VEL(I1)*(2.-DENS(I)/DENS(I1))
0159      CONTINUE
0160      DO 225 I=2,N
0161      IT=I+1
0162      IO=I-1
0163      ION=I-1+N
0164      P(I,IO)=VEL(I1)*DENS(I)/DENS(I1)
0165      P(I,ION)=VEL(I)
0166      CONTINUE
C
0167      N2=2*N
0168      K=0
0169      DO 230 I=N1,N2
0170      K=K+1
0171      K1=K+1
0172      KN=K+N
0173      KN2=K+2*N
0174      P(I,K)=VEL(K1)*(1.-DENS(K1)/DENS(K)))+XLNGT*BFIELD
0175      *GDENS(K1)/VEL(K1)
0176      P(I,KN)=XLNGT*BFIELD*GVEL/DENS(K1)-VEL(K1)*DENS(K)
0177      /DENS(K1)
0178      P(I,KN2)=(XLNGT*BFIELD*GPRES(K1)-1.)*PRES(K1)*ATM/
0179      (DENS(K1)*VEL(K1))
0180      CONTINUE
0181      N1=N1+1
0182      K=0
0183      DO 235 I=N11,N2
0184      K=K+1
0185      K1=K+1
0186      K2=K+1
0187      KN=K+N
0188      KN2=K+2*N
0189      EPS3=EPS1(K1)*XLNGT*BFIELD
0190      P(I,K)=EPS3/(DENS(K2)*VEL(K2)))+(.25*EJ/TEMP(K1))
0191      P(I,KN)=VEL(K1)
0192      P(I,KN2)=PRES(K1)*ATM/(VEL(K2)*DENS(K2))-EPS3/(DENS(K2)
0193      *VEL(K2)))+(.75*EJ/TEMP(K1))
0194      CONTINUE
0195      N21=2*N+1
0196      K=0
0197      DO 240 I=N21,N3

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V01C-03E+ MON 09-MAY-83

FORTRAN IV

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0287 DO 515 L=1,2
0288 DO 516 I=1,N3
0289 DO 518 J=1,3
0290   CONST(I,J,L)=0
0291   CONTINUE
0292   CONST(L)=0
0293   SKIPJ=.FALSE.
0294   SKIPJ=.FALSE.
0295 DO 550 I=1,N3
0296   IF(SKIPJ) GO TO 547
0298 DO 551 J=1,N3
0299   IF(SKIPJ) GO TO 548
0301   I1=I+1
0302   J1=J+1
0303   CONST1=0.
0304   CONST2=0.
0305   CONST3=0.
0306   CONST4=0.
0307 DO 520 L=1,3
0308   L1=1
0309   IF(L1 .EQ. 2) L1=N1
0310   IF(L1 .EQ. 3) L1=N21
0311   CONST1=CONST1+PEIG(J,L1)*PEIG(I,1)*COR(1,L)*PEIG(I,1,N1)*
0312   COR(2,L)*PEIG(I,N21)*COR(3,L)
0313   1 IF(I1 .GT. N3) GO TO 530
0314   CONST2=CONST2+PEIG(J,L1)*PEIG(I1,1)*COR(1,L)*PEIG(I1,N1)*
0315   COR(2,L)*PEIG(I1,N21)*COR(3,L)
0316   1 CONTINUE
0317   IF(J1 .GT. N3) GO TO 520
0318   CONST3=CONST3+PEIG(J1,L1)*PEIG(I,1)*COR(1,L)*PEIG(I,1,N1)*
0319   COR(2,L)*PEIG(I,N21)*COR(3,L)
0320   1 IF(I1 .GT. N3) GO TO 520
0321   CONST4=CONST4+PEIG(J1,L1)*PEIG(I1,1)*COR(1,L)*PEIG(I1,N1)*
0322   COR(2,L)*PEIG(I1,N21)*COR(3,L)
0323   1 CONTINUE
0324   IF(NSIG(1) .EQ. 1) IF=NODE(1)
0325   IF(NSIG(1) .EQ. 2) IF=NODE(1)+N
0326   IF(NSIG(1) .EQ. 3) IF=NODE(1)+N2
0327   IF(NSIG(2) .EQ. 1) JF=NODE(2)
0328   IF(NSIG(2) .EQ. 2) JF=NODE(2)+N
0329   IF(NSIG(2) .EQ. 3) JF=NODE(2)+N2
0330   DEN1=(CE-A(J1))*2+R(J1)*2
0331   DEN2=(CE-A(J1))*2+R(J1)*2
0332   DEN3=(CE-A(J1))*2+R(J1)*2
0333   DEN4=(CE-A(J1))*2+R(J1)*2
0334   SUMN=CE+A(J)
0335   SUMN=CE+A(J)
0336   SUM1=CE-A(J)
0337   SUM1=CE-A(J)
0338   SUM2=A(I)+A(J)
0339   SUM2=A(I)+A(J)
0340   SUM3=B(J)-R(I)
0341   SUM3=B(J)-R(I)
0342   SUM4=B(J)-R(I)
0343   SUM4=B(J)-R(I)
0344   SUM5=B(J)-R(I)
0345   SUM5=B(J)-R(I)
0346   SUM6=B(J)-R(I)
0347   SUM6=B(J)-R(I)
0348   SUM7=B(J)-R(I)

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PAGE 039

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FORTRAN IV      V01C-03E+  MON 09-MAY-83
0349      SUM4=R(J)+B(1)
0350      IF(R(1).NE.0.)GO TO 560
0352      IF(B(J).NE.0.)GO TO 559
C
0354      DO 533 K1=1,2
0355      IF1=IF
0356      JF1=JF
0357      IF(K1.EQ.2) IF1=JF
0359      IF(K1.EQ.2) JF1=JF
0361      CONSTC(K1)=CONSTC(K1)-P(IF1,I)*P(JF1,J)*CONST1/((CE-A(J))*
1      (CE+A(I)))
0362      1 CONST(J,I,K1)=CONST(J,I,K1)+P(IF1,I)*P(JF1,J)*CONST1+
2      *CE/(CE+2-A(J)*2)*SUM2)
0363      CONTINUE
0364      GO TO 551
0365      559 CONTINUE
C*****
0366      DO 534 K1=1,2
0367      IF1=IF
0368      JF1=JF
0369      IF(K1.EQ.2) IF1=JF
0371      IF(K1.EQ.2) JF1=JF
0373      DO 535 L=1,2
0374      IF(L.EQ.2)GO TO 536
0376      J1=J
0377      CONSTA=CONST1
0378      CONSTB=CONST3
0379      GO TO 537
0380      CONTINUE
0381      J1=J+1
0382      CONSTA=CONST3
0383      CONSTB=-CONST1
0384      CONTINUE
0385      537 CONSTC(K1)=CONSTC(K1)+P(IF1,I)*P(JF1,J1)*((CONSTB*B(J))-CONSTA*
1      SUM1)/(DEN1*SUMN)
0386      1 CONST(J,I,K1)=CONST(J,I,K1)+P(IF1,I)*P(JF1,J1)*((CONSTA*SUM1*SUM2
2      -(CONSTB*B(J))*SUM2+CONSTA*SUM1*B(J))/(DEN1
3      *DEN3)+(CCONSTA*SUM+SUM2+CONSTB*B(J)*SUM2-CONSTA*B(J))*2+
1      CONST(J,I,K1)=CONST(J,I,K1)+P(IF1,I)*P(JF1,J1)*((CONSTA*B(J)*SUM2
2      +CONSTB*SUM1*SUM2-CONSTA*SUM1*B(J))*2)/(DEN1+DEN3)+
3      (CONSTA*B(J))*SUM2+CONST4*SUM*SUM2-(CONSTA*SUM+9(J))-
1      (CONSTB*B(J))*2)/(DEN2+DEN3))
0388      CONTINUE
0389      CONTINUE
0390      SKIPJ=.TRUE.
0391      GO TO 551
C*****
0392      CONTINUE
0393      IF(B(J).NE.0.)GO TO 570
0395      DO 561 K1=1,2
0396      IF1=IF
0397      JF1=JF
0398      IF(K1.EQ.2)IF1=JF

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VOIC-D3E+ MON 09-MAY-83

FORTRAN IV

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0400      IF(K1.EQ. 2)JF1=IF
0401      DO 562 L=1,2
0402      IF(L .EQ. 2)GO TO 563
0403      I1=I
0404      CONSTA=CONST1
0405      CONSTB=CONST2
0406      GO TO 564
0407      CONTINUE
0408      I1=I+1
0409      CONSTA=CONST2
0410      CONSTB=-CONST1
0411      CONTINUE
0412      I1=I+1
0413      CONSTA=CONST1
0414      CONSTB=-CONST2
0415      1 CONSTC(K1)=CONSTC(K1)-P(IF1,I1)*P(JF1,J)*((CONSTA*SUMN
0416      +CONSTB*B(I))/((SUM1=DEN22)*P(IF1,I1)*P(JF1,J)*((CONSTA*SUM2
0417      +CONSTB*B(I))*2.*CE/(DEN33*(CE**2-A(J)**2)))
0418      CONTINUE
0419      SKIP1=TRUE.
0420      GO TO 551
0421      CONTINUE
0422      C*****
0423      DO 580 K1=1,2
0424      IF1=IF
0425      JF1=JF
0426      IF(K1 .EQ. 2)IF1=JF
0427      IF(K1 .EQ. 2)JF1=IF
0428      DO 581 L=1,2
0429      IF(L .EQ. 2)GO TO 583
0430      IF(L .EQ. 3)GO TO 584
0431      IF(L .EQ. 4)GO TO 585
0432      I1=I
0433      J1=J
0434      CONSTA=CONST1
0435      CONSTB=CONST2
0436      CONSTE=CONST3
0437      CONSTD=CONST4
0438      GO TO 586
0439      CONTINUE
0440      I1=I+1
0441      J1=J+1
0442      CONSTA=CONST3
0443      CONSTB=CONST4
0444      CONSTE=-CONST1
0445      CONSTD=-CONST2
0446      GO TO 588
0447      CONTINUE
0448      I1=I+1
0449      J1=J+1
0450      CONSTA=CONST2
0451      CONSTB=-CONST1
0452      CONSTE=CONST4
0453      CONSTD=-CONST3
0454      GO TO 588
0455

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161

VOIC-03E+ MON 09-MAY-83

FORTRAN IV

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0471 KA=KA+1
0472 WRITE(2,'KA,ERR=320)CONST(J,I,1),CONST(J,I,2)
0473 A(J)=-A(J)
0474 CONTINUE
0475 DO 660 K1=1,2
0476 WRITE(6,661)CONSTC(K1),CE
0477 JSKIP=.FALSE.
0478 DO 667 J=1,M3
0479 IF(JSKIP)GO TO 671
0499 IF(B(J) .NE. 0.)GO TO 660
0501 WRITE(6,661)CONST(J,1,K1),A(J)
0503 JSKIP=.FALSE.
0504 GO TO 667
0505 CONTINUE
0506 WRITE(6,662)CONST(J,2,K1),B(J),CONST(J,3,K1),B(J),A(J)
0507 JSKIP=.TRUE.
0508 GO TO 667
0509 CONTINUE
0510 JSKIP=.FALSE.
0511 CONTINUE
0512 WRITE(6,12)
0513 CONTINUE
0514 FORMAT(15X,E12.5,' * EXP (',E12.5,' T)')
0515 FORMAT(15X,' (',E12.5,' COS (',E12.5,' T) ) + ',E12.5,' SIN (',
0516 E12.5,' T) ) * EXP (',E12.5,' T)')
0517 DO 669 J=1,M3
0518 A(J)=-A(J)
0519 RES1=CONSTC(1)
0520 RES2=CONSTC(2)
0521 DO 603 J=1,M3
0522 DO 610 I=1,2
0523 RES1=RES1+CONST(J,I,1)
0524 RES2=RES2+CONST(J,I,2)
0525 CONTINUE
0526 CONTINUE
0527 WRITE(6,530)RES1,RES2
0528 FORMAT(15X,'*****',E12.5,'*****',E12.5)
0529 CONTINUE
0530 CALL CLOSE(1)
0531 CALL CLOSE(2)
0532 GO TO 299
0533 CONTINUE
0534 WRITE(7,321)
0535 FORMAT(' ERROR IN PARAMETER FILE!')
0536 STOP
0537 CONTINUE
0538 END

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VITA

Malgorzata Bozena Skorska was born in Warsaw, Poland. She attended Lyceum No. 34, a foremost secondary school in Poland, concentrating on courses in mathematics and physics. She graduated from the Lyceum in June, 1971, with a Golden Medal, the highest award presented to approximately 20 graduates per year throughout Poland. In the fall of 1971 she was accepted to the Warsaw Polytechnic Institute in Warsaw, Poland. She obtained a M.Sc. degree in 1976, from the Electrical Engineering Department with a major in Energy Conversion and Power. After graduation she worked as a construction engineer in the Polish Institute of Electrotechnics in Warsaw. She did research in the field of automatic control of drive systems. In 1978 she entered the Nuclear Engineering Department at the University of Tennessee in Knoxville. She began study toward a Doctor of Philosophy Degree and worked as a teaching and research assistant. The Ph.D. degree was awarded in Summer 1983.

The author is interested in the fields of control and modeling, random noise analysis, magnetohydrodynamics, and, in general, the energy related problems.